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TYPE AND VARIABILITY IN CORN

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Three conceptions of type should rest in the breeder's mind:

1. The ideal, or standard for selection; attained by few individuals, perhaps by none. Page 2

2. The mode, or prevailing type as represented by the highest proportion of what the breeder actually produces. Page 6

3. The mean, or average of all the breeder produces. Page 7

Variability is deviation from type. It is best indicated by the *standard deviation,* a mathematical expression involving the deviation of every individual. Page 11

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The coefficient of variability is a purely abstract expression for variability, so that by its means the variability of one character may be compared with that of another either in the same or different races. Page 12

The effect of selection is to shift the type without greatly reducing variability. Page 17

Each character of every race has a variability that is natural, and this variability cannot be greatly reduced by selection. Page 19

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TYPE AND VARIABILITY IN CORN

By EUGENE DAVENPORT, Professor of Thremmatology,
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The purpose of the present bulletin is to outline and define a clearer conception of type and variability than commonly rests in the breeder's mind, and to present certain data showing conditions that influence type and variability in corn.

TYPE AND VARIABILITY IN GENERAL

The subject is treated by the statistical method, now everywhere employed for the study of the more complicated questions in variation and heredity.[1] This method was first used by Galton in his study of stature of English people (See Natural Inheritance) and afterward elaborated by Pearson and others and applied to the study of heredity problems generally. No excuse is offered for employing the method of treatment here, because it is the only proper one for these purposes and because the time has come when breeders generally are expected to be somewhat familiar with this method of study. The reader is therefore urged not to pass by this form of study because it may happen to be new and unfamiliar.

The technical terms and conceptions, such as standard deviation and coefficient of variability, are no more difficult than are interest and percentage, and a little careful attention will enable the reader to become fully acquainted, not only with their meaning and the method of determination, but with the larger conceptions of heredity that come with their habitual use.

What is Meant by Type

A farmer plants corn from an ear, say ten inches in length. What he gets is not a crop of ears all ten inches long, nor of any other even length, but rather a mass of ears ranging in length all the way from three or four inches up to perhaps eleven or twelve, and very unevenly distributed between the extremes. The same

[1]For a more complete statement of this method of study of breeding problems the reader is referred to chapters X and XI of Pearson's "Grammar of Science," published by A. and C. Black, London, or Part III of Davenport's "Principles of Breeding," Ginn and Company, Boston.

principle would have held if the ear planted had been nine inches long instead of ten except that the distribution would have been different, lengths running in general slightly lower; that is to say the length of ear in the offspring is not the same as that of the parent but it constitutes a "distribution" extending both above and below that length.

So far as known this principle of transmission holds true in all races and for all characters. Stated in more general terms, applying to all breeding, we may say that the *offspring as a whole is not the same as the immediate parents but it constitutes a distribution extending from near the lower to approximately the upper limits of the race. This suggests at once the idea of type and that deviation from type which we call variability.*

What now is our conception of type? If ten inch ears will not bring ten inch ears but something else, and not only something else but a considerable variety of lengths; and if what we get extends both above and below the parent, then we arrive at once at a double conception as to type; that is to say the type of the offspring is not the same as that of the parent. The type of the parent is very definite, representing an ideal; but if the offspring is distributed both above and below that ideal, some better and some not so good, then a close analysis of the real character of that offspring becomes necessary in order to make any just comparison between the two or to arrive at any adequate conception of type in a mixed population, even in one arising from a selected ancestry.

A concrete case will serve best to illustrate the principle involved. In the year 1906, some Leaming corn was raised on good ground from seed ears of ten inches in length. A "random sample"[1] of this crop consisting of 327 ears gave the following distribution as to length:

One ear was 3.0 inches long; one was 4.0 inches; two were 5.0 inches; three were 5.5 inches; nine were 6.0 inches; eight were 6.5 inches; twelve were 7.0 inches; nineteen were 7.5 inches; thirty-two were 8.0 inches; forty were 8.5 inches; sixty-seven were 9.0 inches; sixty-three were 9.5 inches; thirty-eight were 10 inches;

[1]By a random sample is meant a sufficient portion of the whole and taken so much at random as to fairly represent the entire crop, or total "population" as the technical phrase goes. Statistical problems were first studied with reference to people and the term population was thus a natural one. As the studies have been extended to other fields, even of inanimate nature, we still retain the old terms and "population" in this sense is applicable as well to animals as to men; to bricks or stems as to either.

twenty-one were 10.5 inches; eight were 11.0 inches; two were 11.5 inches, and one was 12.0 inches long.[1]

Put in tabular form as it appears in actual work we have the following:[2]

LENGTH OF EARS OR VALUE – V		NO. OF EARS OR FREQUENCY – f
3.0	/	1
3.5		0
4.0	/	1
4.5		0
5.0	//	2
5.5	///	3
6.0	𝍸 ////	9
6.5	𝍸 ///	8
7.0	𝍸 𝍸 //	12
7.5	𝍸 𝍸 𝍸 ////	19
8.0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 //	32
8.5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	40
9.0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 //	67
9.5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ///	63
10.0	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ///	38
10.5	𝍸 𝍸 𝍸 𝍸 /	21
11.0	𝍸 ///	8
11.5	//	2
12.0	/	1

Here we have a "frequency distribution" representing the entire "population" or crop, and as it lies spread out before the eye a glance is sufficient to afford considerable information as to the prevailing type.

[1]Measurements might be taken at quarter inches with a seeming higher degree of accuracy, but repeated trials show that the same final results follow whether measurements are taken at the quarter inch or at the half inch. The main point is that the numbers be sufficient and that the sample be representative. Judgment must dictate as to the accuracy of the sample, but the number depends upon the degree of reliability desired. This matter will be fully discussed in the appendix under probable error, but experience shows that in studies with corn excellent results can be gotten with 300 to 400 ears, and very fair results may generally be had from half that number.

[2]This is the most convenient form in which to make the original record. A mark is made for every individual examined, after which the additions are readily made. These totals constitute the "frequency distribution" and each group (as 12, 19 etc.,) is spoken of as a "class," and its measurement (as 7, 7.5, etc.,) is known as the class mark or value.

It will be noted at once that there are more ears 9.0 inches long than of any other length and that the distribution decreases in both directions from this "highest frequency."

The Mode.—This highest frequency or most common length is called the mode. It shows clearly what is the prevailing type as to length in the crop, as distinct from the selection type in the seed ear. This mode represents the value or measurement that is of the most common occurrence, and it is held by statisticans and by students generally to be the best obtainable single expression for *type.* When it is ascertained, therefore, we know at once what one might conceive as *the natural type of the race or variety* so far as the character in question is concerned. *When this is determined for a number of important characters we shall have a good knowledge of racial type as a whole.* Thus we might after the same manner obtain the mode for circumference, number of rows, weight of ear, color of grain, percent of corn to cob, or any other desired character. Having done so a *typical ear of this variety could be definitely described.* We thus arrive at an accurate idea of type and of its definite measurement as well.[1]

For the purpose of comparing the variability of races we use the "coefficient of variability" to be described later. The modal coefficient is chiefly valuable for comparing one type with another *within the race,* which is all that is required in practical breeding.

Practical Value of the Frequency Distribution, the Mode, and Modal Coefficient.—The practical importance of the information afforded by these values must be apparent. By means of the frequency distribution the breeder is enabled at any time when he can secure sufficient numbers, to spread out before his eyes a good and fair representation of the whole population of the variety or race he is breeding, and in respect to any character which he can measure or accurately estimate.

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[1]*The Empirical and the Theoretical Mode.*—It is evident by inspection of the frequency table that if measurements had been taken at the quarter inch, or some less fraction, the highest frequency would have fallen not at the nine inch point, but slightly above it, say at 9.25 for example, for the next frequency above (63) is greater than the next one below (40); that is to say the mode is to some extent dependent upon the scheme of measurements adopted. A mode so determined is therefore only a close approximation to the actually most common length, and it is known as the empirical mode. If, however, the theoretical curve should be platted then all values would be accurately represented (see appendix) and the highest point in this curve would be the actual, or as it is called, the theoretical mode. In practical breeding operations the empirical mode arising from convenient measurements is sufficiently accurate. It leads to no error because a convenient scheme of measurements once found is generally employed by all observers, so that empirical modes are comparable. Thus the scheme of half inch measurements is the one likely to be universally employed for corn.

When he has ascertained its mode he knows what is the natural type, for mode indicates type; and he then knows by how much, if any, it differs from the type which he has chosen as the standard for selection. By this he may judge whether and to what extent he is operating at variance with nature.

The Mean.—There is still another conception of type as to this distribution, and that is the average or mean as it is technically called. It will be noted that the distribution does not decline uniformly both above and below the mode; that is to say there are twelve values below and only six above, from which we conclude that the average length of ear is somewhat different from the most usual length. By multiplying each separate length by the number of ears of that length and adding the products, (or, what is the same thing, adding together the lengths of all the ears) then dividing by the total number of ears we find the average or mean length to be 8.83 inches.

Accordingly we have the following for the determination of the mean.[1] Multiply each value by its frequency, add the results and divide the sum by the number of individuals or variates.

Applying this principle to the case in hand we have:[2]

V		f		f V
3.0	×	1	=	3.0
3.5	×	0	=	0.0
4.0	×	1	=	4.0
4.5	×	0	=	0.0
5.0	×	2	=	10.0
5.5	×	3	=	16 5
6.0	×	9	=	54.0
6.5	×	8	=	52.0
7.0	×	12	=	84.0
7.5	×	19	=	142.5
8.0	×	32	=	256.0
8.5	×	40	=	340.0
9.0	×	67	=	603.0
9.5	×	63	=	598.5
10.0	×	38	=	380.0
10.5	×	21	=	220.5
11.0	×	8	=	88.0
11.5	×	2	=	23.0
12.0	×	1	=	12.0
		327		2887.0

2887.0 ÷ 327 = 8.83—the mean length of ear in inches.

[1]By "mean" is here meant the "arithmetical average" which is the average most commonly accepted.

[2]In this table "V" stands for "values" or "magnitudes"—in this case length—and "f" stands for frequency, or the number of varieties (ears) of each separate class. The heading "f V" means the products of the values (lengths) multiplied by the corresponding frequencies.

Here we have a third valuation for type (8.83) representing the average as distinct from 9.0 of the highest frequency representing the most usual length, and both distinct from the 10 inches of the ear planted.

Practical Use of the Mean.—The mean gives a good average value of the character, and establishes the practical or commercial value of a race or variety, for it shows what it will do *on the average*. It is not always, however, a good index of the prevailing type, for as often happens, the variety with the higher mean may have the lower mode. Neither is the mean always a good index of conditions; for example, in a population of one thousand paupers and one millionaire, the mean wealth is fair, but the type is clearly that of the pauper.

Here then are three separate and very definite conceptions of type, all of which have distinct applications to the practical affairs of breeding: 1. The ideal, which is used in selecting the parentage. 2. The prevailing type of the offspring as represented by the highest frequency (the mode). 3. The average of the offspring as represented by the mean. These distinctions apply not only to length of ears in corn, but to all characters and all races; that is, to breeding in general.

The breeder of pedigreed stock is interested primarily in the ideal and in the mode or highest frequency, while the general farmer who multiplies or raises it for the open market is most interested in the mean or average production.[1]

VARIABILITY, OR DEVIATION FROM TYPE

Having established definite distinctions as to type the student of breeding problems should form equally clear conceptions as to deviation from type, commonly known as variability.[2].

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[1]It is to be noted that the generation to which the selected parent belonged had also its own mode and mean which may have been quite different from those of the offspring.

[2]The term variability should not be understood as expressing departure in the sense of wandering from a fixed standard. Students sometimes gain the impression that if the law of heredity were infallible all offspring would be of a common type, and that any departure from the type of the race, variety, or breed is to be regarded as by so much a failure of heredity and a concession to variation.

The truth is that all transmission is heterogeneous in the sense that the individuals of any race, whether parents or offspring belong not to a fixed type but to a frequency distribution similar to the one now under discussion, and the *idea of type arises out of the distribution.*

The chief conception to rest in the mind of the breeder is that whatever the parentage, the offspring will constitute a distribution extending through a considerable range, and that the parent itself also belonged to and was drawn from some portion of a frequency distribution is not very different from that of the race in general.

Variability is therefore not the opponent of heredity but its inevitable accompaniment in transmission and our problem is to devise methods of accurately measuring and expressing its range and extent in any particular instance.

In the study of variability it is worse than useless to study a few scattered individuals here and there. What we seek is a measure of what may be called the average tendency to deviate from type. Some individuals deviate but little, others more, and still others very much; and we seek a measure of this non-conformity to type. To find this we must study groups of individuals sufficiently large to be representative of their race. This brings us back to the frequency distribution and what it can teach as to variability.

Again the concrete serves well as a medium of teaching a principle. In this connection we refer once more to our distribution of 327 ears and note that *every ear in the lot deviates somewhat from the mean of 8.83 inches.* The range and extent of this deviation are shown in the following table, column D.

V	f	D[1]
3.0	1	−5.83
3.5	0	−5.33
4.0	1	−4.83
4.5	0	−4.33
5.0	2	−3.83
5.5	3	−3.33
6.0	9	−2.83
6.5	8	−2.33
7.0	12	−1.83
7.5	19	−1.33
8.0	32	−0.83
8.5	40	−0.33
9.0	67	0.17
9.5	63	0.67
10.0	38	1.17
10.5	21	1.67
11.0	8	2.17
11.5	2	2.67
12.0	1	3.17
	327	

The practical question now is to reduce this column of deviations to a single expression denoting the variability of the population of which this distribution is representative. Manifestly when this is done the variability of this distribution can be compared directly with that of any other distribution, and at the present or any future time. Two methods of procedure are possible in thus securing a kind of general expression for the average amount of deviation, giving rise to two similar but slightly different values; viz., the average deviation and the standard deviation.

The Average Deviation.—If each deviation (column D) represented an equal number of ears this "single expression" could be readily derived by adding the deviations and dividing by the total number. But these deviations do not represent equal numbers of ears. The deviation −5.83, for example, represents but one ear

[1] "D" indicates the deviation of the several classes from the common mean of the population 8.83 inches. Thus the first ear deviates the difference between 3 inches and 8.83 inches, or −5.83, and being below it is written with the negative sign. Also, for example, the 21 ears 10.5 inches long deviate 10.5−8.83 or 1.67 inches from the mean and being above the mean we write it with the positive sign, and similarly for other values.

while no less than twelve ears deviated 1.83 inches below the mean and two ears deviated 2.67 inches above, with others unevenly distributed.

Manifestly each deviation should first be multiplied by the number of ears involved, thus:[1]

f		D		Df
1	×	5.83	=	5.83
0	×	5.33	=	0.00
1	×	4.83	=	4.83
0	×	4.33	=	0.00
2	×	3.83	=	7.66
3	×	3.33	=	9.99
9	×	2.83	=	25.47
8	×	2.33	=	18.64
12	×	1.83	=	21.96
19	×	1.33	=	25.27
32	×	0.83	=	26.56
40	×	0.33	=	13.20
67	×	0.17	=	11.39
63	×	0.67	=	42.21
38	×	1.17	=	44.46
21	×	1.67	=	35.07
8	×	2.17	=	17.36
2	×	2.67	=	5.34
1	×	3.17	=	3.17
327				318.41

The result of this calculation is that the total deviation of all the 327 ears from their average length is 318.41 inches, some above and some below the mean.[2] If now we divide 318.41 by 327, the number of ears involved, we have 0.97+ inches, which is a good expression for the *average* deviation of this particular population. If another variety should give a larger quotient we should conclude it to be more variable. In this manner we may reduce the variability of a whole population to a single expression.

Standard Deviation.—Mathematicians have another method of calculating variability. It differs from the one just discussed in only one detail; viz., the deviations are *squared* before multiplying by their respective frequencies, thus:

V	f	D	D^2 [3]	D^2f [4]
3.0	1	−5.83	33.9889	33.9889
3.5	0	−5.33	28.4089	00.0000
4.0	1	−4.83	23.3289	23.3289
4.5	0	−4.33	18.7489	00.0000
5.0	2	−3.83	14.6689	29.3378
5.5	3	−3.33	11.0889	33.2667
6.0	9	−2.83	8.0089	72.0801
6.5	8	−2.33	5.4289	43.4312
7.0	12	−1.83	3.3489	40.1868
7.5	19	−1.33	1.7689	33.6091
8.0	32	−0.83	0.6889	22.0448
8.5	40	−0.33	0.1089	4.3560
9.0	67	0.17	0.0289	1.9363
9.5	63	0.67	0.4489	28.2807
10.0	38	1.17	1.3689	52.0182
10.5	21	1.67	2.7889	58.5669
11.0	8	2.17	4.7089	37.6712
11.5	2	2.67	7.1289	14.2578
12.0	1	3.17	10.0489	10.0489
	327			538.4103

[1]When the variability is to be obtained in this way the minus sign is disregarded.

[2]The reader will note that this total 318.41 is exactly what would have resulted if we had added the deviations of each separate ear of the entire 327 measured from their average length, 8.83.

[3]The column marked D^2 is secured by squaring the various deviations, thus eliminating the minus sign. For example, −5.83 × −5.83 = 33.9889, etc., etc.

[4]The column marked D^2f is obtained by multiplying the squared deviations, each by its respective frequency, on the same principle as before. For example, 8.0089 × 9 = 72.0801,—the seventh number down the last column, corresponding to the frequency 9 and the deviation −2.83.

Dividing 538.4103 by 327 after the manner of finding the average deviation we have the quotient 1.6465, but as the deviations have all been squared during the operation it is necessary to extract the square root of this number in order to arrive at the units in which the measurements were taken. The square root of 1.6465 is 1.28+, and this is the so-called *standard deviation* of the mathematician.

Hence to find the standard deviation we have: Find the deviation of each frequency from the mean; square each deviation, and multiply by its corresponding frequency; add the products, divide by the total number of variates and extract the square root.

Shortening the Method.—The calculations just described necessarily involve large decimals. These large decimals can be avoided and the process of finding *both the mean and the standard deviation* can be very much shortened by *assuming* as a mean the nearest *probable* measurement as determined by inspection of the frequency distribution, and afterward applying the necessary correction. For example, in the present instance, we should judge by inspection that the mean cannot be far from 9.0[1] This we infer from the fact that the distribution reduces both ways from this point and quite evenly. Proceeding with this assumption, denoting our "guess" by G and, reckoning deviation provisionally from this point, we have the following, using exactly the same methods as before:[2]

[1]The advantage of assuming this value from which to reckon deviation lies in the fact that it is exact and contains but one decimal, while the true mean has at least two decimal places, making relatively large numbers to deal with.

[2]The following table will be found useful for obtaining the squares of numbers containing only two significant figures—99 or 9.9, correct to three significant figures.

SQUARES OF NUMBERS.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	
1.	1.00	1.21	1.44	1.69	1.96	2.25	2.56	2.89	3.24	3.61	
2.	4.00	4.41	4.84	5.29	5.76	6.25	6.76	7.29	7.84	8.41	
3.	9.00	9.61	10.2	10.9	11.6	12.2	13.0	13.7	14.4	15.2	
4.	16.0	16.8	17.6	18.5	19.4	20.2	21.2	22.1	23.0	24.0	
5.	25.0	26.0	27.0	28.1	29.2	30.2	31.4	32.5	33.6	34.8	
6.	36.0	37.2	38.4	39.7	41.0	42.2	43.6	44.9	46.2	47.6	
7.	49.0	50.4	51.8	53.3	54.8	56.2	57.8	59.3	60.8	62.4	
8.	64.0	65.6	67.2	68.9	70.6	72.2	74.0	75.7	77.4	79.2	
9.	81.0	82.8	84.6	86.5	88.4	90.2	92.2	94.1	96.0	98.0	

V	f	V–G	f (V–G)		(V–G)²	f(V–G)²
3.0	1	–6	–6.0		36.00	36.00
3.5	0	–5.5	0.0		30.25	00.00
4.0	1	–5.0	–5.0		25.00	25.00
4.5	0	–4.5	0.0		20.25	00.00
5.0	2	–4.0	–8.0		16.00	32.00
5.5	3	–3.5	–10.5		12.25	36.75
6.0	9	–3.0	–27.0		9.00	81.00
6.5	8	–2.5	–20.0		6.25	50.00
7.0	12	–2.0	–24.0		4.00	48.00
7.5	19	–1.5	–28.5		2.25	42.75
8.0	32	–1.0	–32.0		1.00	32.00
8.5	40	–0.5	–20.0		0.25	10.00
9.0	67	0.0		–181.0	0.00	00.00
9.5	63	0.5	31.5		0.25	15.75
10.0	38	1.0	38.0		1.00	38.00
10.5	21	1.5	31.5		2.25	47.25
11.0	8	2.0	16.0		4.00	32.00
11.5	2	2.5	5.0		6.25	12.50
12.0	1	3.0	3	125.0	9.00	9.00
	327		Difference—56.0			548.00

This method gives us both the mean and standard deviation. Considering first the mean: In column f(V–G) we find that after multiplying the deviations from our assumed mean (9.0) by their respective frequencies, the sum of the negative products (–181.0) exceeds the sum of the positive products (125.0) by 56.0; that is the algebraic sum of the products is –56.0. Our assumed mean is therefore *too high* by the amount of –56.0 ÷ 327[1] = –0.171. We then reduce our assumed mean by this amount (9.0 – 0.171 = 8.829) and arrive at the true mean 8.83–.[2]

Considering next the standard deviation: In column f(V–G)² we have 548.00 as the sum of the products of the several frequencies into their respective deviations from the *assumed* mean, derived on the same plan as when working from the true mean D.

Dividing by the total number (327) we have 548.00 ÷ 327 = 1.6758, corresponding to the quotient, 538.4103 ÷ 327 = 1.6465 of the previous calculation when working from the true mean.

The correction made in the mean was –0.171, but as we are now dealing with the second powers it seems but natural that this amount be squared before it be taken from the quotient 1.6758[3]. The square of –0.171 is 0.029241 or 0.0292 +. We have therefore after applying this correction 1.6758 – 0.0292 + =1.6466.

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[1]We divide by the total number (327) because we are dealing with a column of products arising from the introduction of the frequencies.

[2]On the other hand should the sum of the positive deviations exceed the sum of the negative deviations it would indicate that our assumed value is too small and we should *add* the correction in order to arrive at the true mean.

[3]This can be justified by a strictly mathematical proof. It is to be noted that in the case of standard deviations the square of the correction is always to be *subtracted*.

This agrees very nearly with the value 1.6465 previously found, but the shorter method is the more accurate because no decimals have been lost during the process. The square root of 1.6466 is 1.28+, the standard deviation sought, agreeing with the former value and derived by a very much shorter method. The first method is useful for expounding the principles involved but the latter is far preferable for actual use, not only on account of its brevity but its increased accuracy as well.

The farmer is at liberty of course to choose whether he will use the average deviation or the standard deviation as an index of variability. The average deviation is the simpler, but it is seldom used by mathematicians. As the results are different, generally smaller, they cannot be compared with those found in standard literature of this kind.

The standard deviation, obtained by one of the two latter methods, is strongly recommended. It is the one that will be used in all publications of this station. The breeder may employ either the shorter, or the longer and slightly less accurate method. The shorter method is far more convenient and is no more complicated except in making the correction and this, after a little practice, offers no difficulty.

Practical Value of Standard Deviation.—The standard deviation is a good measure of deviation from the mean. It is therefore a good measure of variability reckoned from that point. It is manifest that by the same methods we could calculate the deviation and express the variability from the mode, the selection standard, or any other type on which the mind might rest.

The practical value of standard deviation is that it stands as a definite measure of variability of the population in question, and if records be kept the variability of any race may be compared from year to year. The advantage of being able to make comparisons of this sort is too obvious to require elaboration.

Coefficient of Variability.—It is often desirable to compare the variabilities of different characters measured in different units either within the same race or between separate races; thus, which is more variable, the length, the circumference or the weight of ear? In such cases one standard deviation cannot be compared *directly* with another for two reasons: First one mean is very much larger than another, and second, they are of entirely different units, as inches and pounds, in which cases direct comparison is impossible. We seek, therefore, an *abstract* expression combining the idea both of standard deviation and type. Such an expression is known as the coefficient of variability and is found as follows: Divide the standard deviation by the mean as a base and the result will be an excellent index of variability in the form of a rate percent.

Thus for the case in question we have: $1.28 \div 883 = 0.145-$, indicating the variability of this population to be over 14.5 percent of its own mean. Here we have a mathematical expression for comparing variability on an abstract basis, and by this means we can compare the variability of this population with that of any other from any race, plant or animal; and for any character of which accurate measurements can be made.

For example, the coefficient of variability has been worked out for a large number of characters in man as is shown in the following table:[1]

Nose length	9.49	Head length	2.44
" breadth	7.57	" breadth	2.78
" height	15.2	Upper arm length	6.50
Forehead height	10.4	Forearm "	3.85
Underjaw length	4.81	Upper leg "	5.00
Mouth breadth	5.18	Lower leg "	5.04
		Foot "	5.92

From this we note that the most variable character in these physical measurements of man is the height of the nose from the plane of the face (15.2) and this is the only character that is as variable as is length of ear in the distribution now under discussion (14.5 percent). It is manifest that the variability of the nose in man or of the weight of animals could not be directly compared one with another, because the units are different and because they are reckoned on different means, but when variability is reduced to a coefficient then direct comparison becomes entirely possible and intelligible.

Practical Use of Statistical Constants.—The practical advantage to the breeder in being able to calculate the mode, mean, and variability of the animals and plants he is breeding, and thus to know definitely their behavior from generation to generation under his methods of selection and treatment—all this is too obvious to need discussion. Breeding operations in the past have lacked much in definiteness because of the inability of breeders to possess themselves of this class of knowledge or even to appreciate its bearing upon breeding operations. The successful breeder of the future will be a statistician and a bookkeeper. He will keep himself as accurately and as fully informed as may be as to the type and variability in succeeding generations of the breeds and strains he attempts to improve, and he will know this of all important characters that can be subjected to any form of measurement.

[1]See Var. in An. and Pl. Vernon, p. 24.

Manifestly the methods here given do not avail with characters that cannot be subjected to measurement, nor can they be employed when it is impossible to find sufficient numbers to make the calculations reliable.

The characters that can be classified and measured, at least approximately, are however, more numerous than might at first thought seem possible. Dimensions and weights are in most cases easily taken. Gains in weight, yield of milk, rate of speed, etc., are readily handled by the statistical methods, and even such characters as color, degree of intelligence, and the like, are not impossible of classification and approximate measurement.

While most characters can thus be brought into the form for statistical treatment it is useful to know that present day knowledge of breeding operations seem to indicate that all characters, whether measureable or not, tend to behave after the same general principles as to type and variability, so that we may confidently believe that every character of every individual belongs in some portion of a distribution whether the distribution could or could not be definitely written.

Probable Error

Clearly no calculations based on a portion of the population can represent the entire race with absolute accuracy. If one more ear had been measured it would have fallen somewhere in the scheme of distribution, and wherever it may have fallen it would have slightly changed our calculations.

When we are able to examine all the individuals involved in a problem we can of course determine absolute values to within the limits of measurement, as in the average weight of a bunch of steers or the yield per acre of a field of grain. Our present discussion, however, is of a class of problems in which we can never hope to see and examine more than a fraction of the total population, as when we ask what is the average weight of steers the country over, or the average length or weight of ears of corn.

In practice when dealing with this class of problems we can do no better than to take a random sample and assume it to be representative of the entire population, accepting whatever error may be involved, and there is always an error of some magnitude, for no random sample can be assumed as being completely representative of the entire race to which it belongs.

Now no method can inform us as to the exact magnitude of this error. If it could we should at once correct for it and thus come into possession of the true value; but methods are known by which we may judge fairly well of the degree of confidence which may be reposed in results of this kind. These methods result in deducing what is called the "probable error," written plus or minus E ($\pm$ E). The formulas for calculating $\pm$ E are derived by mathematical methods which are too complicated for discussion here, but which are briefly stated in the appendix, with the following results:

1. *Probable Error of Mean.*—The formula for probable error in determinations for mean is $\pm E = \pm \frac{\text{Standard deviation}}{\sqrt{n}} \times 0.6745$, in which n is the number of variates examined and 0.6745 is a mathematical constant. In words, it is, —Multiply the standard deviation by 0.6745 and divide by the square root of the number of variates examined. Thus in the present instance by substituting values for standard deviation and number we have $\pm E = \pm \frac{1.28}{\sqrt{327}} \times 0.6745 = \pm 0.047$. This probable error ~~0.047~~ is but a small fraction of the value determined ~~(1.28)~~ and indicates that a high degree of confidence can be placed in the result. If, however, another calculation should show a smaller value for probable error we should conclude that a still higher degree of confidence could be placed in its accuracy. The reader will note that the number (n) is the only element of the formula that is under our control, others arising necessarily out of the problem. He will note, too, the overwhelming influence of numbers; that as numbers increase the denominator increases and probable error decreases, and that when the number should reach infinity, E would become zero. 8.8

2. *Probable Error of Standard Deviation.*—The formula for probable error in determinations for standard deviation is, $\pm E = \pm \frac{\text{Standard deviation}}{\sqrt{2n}} \times 0.6745$.

In words, it is,—Multiply the standard deviation by 0.6745 and divide by the square root of *twice* the number examined. Substituting values for the case in hand we have as the probable error of the standard deviation 1.28, $\pm E = \pm \frac{1.28}{\sqrt{2 \times 327}} \times 0.6745 = 0.034$.

3. *Probable Error of Coefficient of Variability.*—The formula for probable error in determinations for coefficient of variability below 10 percent is, $\pm E = \frac{\text{Coefficient of Variability}}{\sqrt{2n}} \times 0.6745$.

In words, it is,—Multiply the coefficient of variability by 0.6745 and divide by the square root of twice the number. In the case in hand, however, the coefficient of variability (14.5) is greater than ten percent and in such cases the following formula is used: $\pm E = 0.6745 \frac{14.5}{\sqrt{2 \times 327}} \left[1 + 2\left(\frac{14.5}{100}\right)^2\right]^{½}$ which equals 0.39 as the corrected probable error.

Meaning of Probable Error.—It is important that the meaning of probable error be not misunderstood. It has no reference to errors in our computations, which are assumed to be correct. It is not the actual magnitude of errors made nor is it the most probable size of any mistake, neither does it set the limits within which errors must lie. Such limits cannot be set, but it does mean that the chances are even that the true value lies *within* the range set by ± E; that is, if the determination (~~as in mean~~) be ~~8.83~~ and the E be ± 0.02 then the chances are even that the true length ~~is~~ not less than 7.23 (7.25–0.02) nor greater than 7.27 (7.25+0.02). 7.25

Of course the chances are also even that the true value may lie outside this range but these chances rapidly decrease as we increase the range. Thus the chances against the true value lying outside of *twice* the probable error are as 4.5 to 1. The following table shows the rapid increase in the chances that the true value lies *within* the range set by ± E, ± 2E, etc. They are as follows:[1]

± E,	the	chances	are	even.
± 2 E,	"	"	"	4.5 to 1.
± 3 E,	"	"	"	21. to 1.
± 4 E,	"	"	"	142 to 1.
± 5 E,	"	"	"	1310 to 1.
± 6 E,	"	"	"	19200 to 1.
± 7 E,	"	"	"	420,000 to 1.
± 8 E,	"	"	"	17,000,000 to 1.
± 9 E,	"	"	"	about a billion to one.

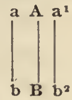
It will be noticed that by the time we have made an allowance of three or four times the probable error we have reached a chance which amounts to practical certainty, and even 21 to 1 involves far less chance than is *involved in most business transactions.*

Degree of Confidence Shown by Probable Error.—There is a popular notion that most affairs of life rest on a positive basis of fact and that unless error or chance can be entirely eliminated from our calculations no confidence is to be placed in the results. This is erroneous. A large element of uncertainty is nearly always involved in all affairs of life whether we recognize the fact or not.

Problems of the class now under discussion differ from ordinary affairs of life, therefore, only in this, that we can calculate the probable error involved and by that determine the degree of confidence that may be reposed in the conclusions. If the probable error is large as compared with the determination then we have nothing but a shrewd guess unless we increase the numbers, but if the probable error is small as compared with the determination then a high degree of confidence may be placed in the results and the facts taught may be relied upon as practically certain.

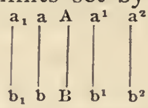
[1] C. B. Davenport, Statistical Methods, p. 14.

For a graphic illustration of the meaning of $\pm$ E: Suppose in the following figure the line AB represents our determination and the lines ab and a^1b^1 represent +E and −E as follows:



Now this means that the chances are even that the line which would represent the true value is not outside the limits set by the lines ab and a^1b^1; representing $\pm$ E.

If now we set other lines at $\pm$ 2 E as follows,–



then we know from the table that the chances are 4.5 to one that the line which represents the true value is not outside the lines a_1b_1 and a^2b^2, each removed twice the probable error from the determination.

The first of the two purposes of this bulletin is accomplished in the text up to this point and in the appendix. It remains to present certain data showing how type and variability of corn behave under various influences.

INFLUENCE OF SELECTION UPON TYPE AND VARIABILITY.

In 1896 Dr. C. G. Hopkins, chemist of the Experiment Station, began a series of breeding experiments to determine whether the chemical composition of corn could be influenced by selection. The mass of data which has accumulated during the ten years of the investigation affords some of the most reliable information that has ever been secured concerning the influence of selection upon both type and variability.

The selections were made in four directions; namely, for high oil, for low oil, for high protein, and for low protein,—giving rise to four strains, known as Illinois' high-oil, Illinois' low-oil, Illinois' high-protein, and Illinois' low-protein. All four strains sprung from the same original stock of 163 good ears of a local strain, known as Burr's White.

Table 1, exhibits the distribution of the original 163 ears and the effects of ten years of selection for oil content. The column headed "Seed" gives the average of the seed ears planted both for high oil and for low oil, and the distributions opposite show the actual oil content of the entire crop of good ears raised therefrom.[1] The column headed "Average" gives the average content of oil as

[1]These different strains were raised in small isolated plots, each ear in a separate row.

determined by chemical analysis of the actual ears, differing slightly from what would be the computed mean of these distributions because in the distribution the ears are made to fall in definite classes.

Nothing can exhibit more clearly than this table the readiness with which the type responds to selection. In the fourth crop the high oil and low oil strains parted company,—that is to say, their distributions no longer overlapped; in the seventh year the entire low oil crop dropped below the lowest ear of the original stock, and in the ninth the entire high oil distribution was above the highest ear of the original stock. That is to say, the two strains of high and low oil, though developed from the same stock, had separated by a space wider than that covered by the original distribution or that of either strain. Both strains had entirely departed from the space occupied by the original stock and the mean oil content of the high oil corn was nearly twice that of the foundation, or almost three times that of the low oil strain.

This illustrates the principle of progression as it is illustrated by no other definite data known to the writers. The distributions began from the very first to separate, and within four years the separation was complete. Not only are these facts clearly established but the separation continued and all the distributions are normal; that is they slope both ways from a maximum that is not far from the middle point. The plain conclusion is that response to selection is rapid and pronounced and by persistent selection the type may be carried entirely beyond the former limits of the race.

Table 2 gives in condensed form the effect of selection upon the mean, standard deviation, and coefficient of variability for all four strains of corn and through the ten years of the experiment. A glance at this table will show that the high and low protein strains followed the same general principle as indicated by the high and low oil strains. It will be noted, however, that the response to selection was upon the whole least prompt in the low protein corn.

TABLE 1.—PROGRESSION IN HIGH AND LOW OIL CONTENT OF "ILLINOIS" CORN

Year.	Seed.	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.00	4.25	4.50	4.75	5.00	5.25	5.50	5.75	6.00	6.25	6.50	6.75	7.00	7.25	7.50	7.75	8.00	8.25	8.50	Ave. %
1896											1	15	20	41	36	25	15	5	4	1											4.70
1897	H. 5.39									1			10	15	19	24	3	7	1												4.73
	L. 4.03									5	7	9	7	7	5																4.06
1898	H. 5.20											1	7	29	37	50	35	24	22	6	2	2	1								5.15
	L. 3.65								8	9	31	31	19	9	1																3.99
1899	H. 6.15												1		2	10	16	22	24	18	12	3									5.64
	L. 3.47						1	2	5	32	36	36	23	8	1																3.82
1900	H. 6.30														1	1	3	9	18	16	30	18	9	2	1						6.12
	L. 3.33					1	3	8	32	39	32	20	6	3																	3.57
1901	H. 6.77															2	3	10	13	26	21	28	15	7	1						6.09
	L. 2.93						2	12	34	43	29	5	1																		3.43
1902	H. 6.95															2	5	3	10	13	18	16	13	9	1						6.41
	L. 3.00				1	6	34	19	16	9	5																				3.02
1903	H. 6.73																	1	10	10	19	18	21	13	5	2	1				6.50
	L. 2.62				1	3	26	33	22	5																					2.97
1904	H. 7.16														1					3	9	8	12	15	18	18	8	6	2	1	6.97
	L. 2.80				3	7	42	31	15	4																					2.89
1905	H. 7.89																				4	9	19	15	21	17	19	7	5	4	7.29
	L. 2.67		1	4	21	42	31	19	1																						2.58
1906	H. 7.86																			1		5	9	24	16	32	20	8	2	3	7.37
	L. 2.20	1		4	15	34	36	19	10	1																					2.66

TABLE 2. VARIABILITY OF CORN BRED FOR OIL AND PROTEIN

Year grown.	Percent Protein in Original Stock, 163 Ears.					
	Mean, 10.93±0.05		Standard Deviation, 1.04±0 04		Coefficient of Variability, 9.50±0.35	
	Percent protein in High protein selection.			Percent protein in Low protein selection.		
	Mean.	Standard Deviation.	Coefficient of Variability.	Mean.	Standard Deviation.	Coefficient of Variability.
1897	10.99±0.07	1.16±0.05	10.90±0.50			
1898	10.98±0.05	1.22±0.04	11.15±0.33	10.49±0.08	1.32±0.06	12.61±0.53
1899	11 62±0.06	1.28±0.04	11.00±0.36	9.59±0.06	1.01±0.04	10.50±0.42
1900	12.62±0.05	1.02±0.03	8.09±0.26	9.13±0.06	1.04±0.04	11.34±0.45
1901	13.78±0.07	1.17±0.05	8.48±0.38	9.63±0.07	1.10±0.05	11.47±0.49
1902	12.90±0.08	1.10±0.05	8 50±0.43	7.86±0.05	0.75±0.04	9.60±0.48
1903	13.51±0.09	1.36±0.06	10.04±0.48	8.00±0.06	0.83±0.04	10.41±0.50
1904	15.04±0.10	1.34±0.07	8.94±0.43	8.17±0.06	0.86±0.04	10.55±0.51
1905	14.71±0.08	1.24±0.05	8.84±0.42	8.55±0.06	1.05±0.05	12.24±0.54
1906	14.25±0.08	1.33±0.06	9.33±0.41	8.66±0.06	0.93±0.04	10.77±0.47

Year grown.	Percent Oil in Original Stock, 163 Ears.					
	Mean, 4.68±0.02		Standard Deviation, 0.41±0.02		Coefficient of Variability, 8.83±0.33	
	Percent oil in high oil selection.			Percent oil in low oil selection.		
	Mean.	Standard Deviation.	Coefficient of Variability.	Mean.	Standard Deviation.	Coefficient of Variability.
1897	4.79±0.03	0.38±0.02	7.87±0 42			
1898	5.10±0.02	0.48±0.02	9.33±0.30	3.95±0.02	0.32±0.01	8.13±0.37
1899	5.65±0.03	0.42±0.02	7.47±0.34	3 85±0.02	0.32±0.01	8.42±0.33
1900	6.10±0.03	0.44±0.02	7.26±0.33	3.57±0.02	0.36±0.01	10.13±0.40
1901	6.24±0.03	0.45±0.02	7.26±0.31	3.45±0.02	0.26±0.01	7.59±0.32
1902	6.25±0.04	0.50±0.03	8.06±0.41	3.00±0.02	0.32±0.02	10.84±0.55
1903	6.51±0.03	0.46±0.02	7.07±0.34	2.99±0.02	0.23±0.01	7.83±0.39
1904	7.12±0 04	0.58±0.03	8.19±0.39	2.91±0 02	0.25±0.01	8.45±0.40
1905	7.30±0.03	0.55±0.02	7.47±0.33	2.56±0.02	0.28±0.01	10.86±0.55
1906	7.37±0.03	0.45±0.02	6.15±0.27	2.66±0.02	0.31±0.01	11.69±0.51

This table shows the effects of selection. It will be noted that the means steadily increase or decrease with selection, but that the variability coefficient of variability does not greatly change. This all shows that the effect of selection is to shift the type without sensibly reducing variability.

The chief interest in Table 2 is with respect to *variability* as shown by the standard deviations or better yet by the coefficients of variability of the different strains. On this point one fact is clear cut and significant; namely, while the different strains differ as to variability,—the high oil upon the whole being least variable and the low protein most variable,—yet in every instance *the variability was not sensibly reduced during the ten years of rigid selection.*

True, it fluctuates from year to year but rarely more than is accounted for by the probable error, and it cannot be said from these figures that the effect of selection is greatly to reduce variability. This agrees with other modern studies in the field of breeding and in connection with the data given in Table 1, tends strongly to confirm the statement that in general the *effect of selection is to shift the type without greatly altering variability.* All of this means that after great improvement has been secured there is still left abundant variability on which to base future selection, and that if the limits of improvement are ever reached it will be for some reason other than the failure of variability.

INDIRECT EFFECTS OF SELECTION

Table 3, gives the physical characters of these four strains of corn for two separate crops, 1905 and 1906. By this we see that the effect of selecting for chemical content has been also to alter the physical characters of the different strains; that is to say, while the ears differed in the two years, 1905 and 1906, yet in both cases the low protein strain had the longest and the high oil strain the shortest ears; the low oil corn had the largest and the high protein the smallest circumference; the low oil corn was the heaviest in both years and the high protein the lightest. Aside from this there seems to be a tendency to affect the number of rows. In any event in both these years the high oil corn had the largest number of rows and the low oil corn the smallest number of rows. While these two years are not enough to determine whether these differences will remain permanent with these strains, the data presented are certainly sufficient to show that these four strains of corn are coming to differ decidedly in respect to physical characters though not as widely as they differ in their chemical composition, for which they were selected.

TABLE 3. EFFECT OF SELECTION FOR CHEMICAL CONTENT UPON PHYSICAL CHARACTERS OF CORN

LENGTH OF EAR.

Chemical strain.	Crop of 1905.			Crop of 1906.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
High Protein	7.21±0.04	1.27±0 03	17.6±0.4	7.880±0.043	1.201±0 030	15.24±0.39
Low Protein.	7.80±0.04	1.54±0.03	19.7±0.4	8.841±0.050	1.364±0.035	15.43±0.43
High Oil.....	6.87±0.04	1.39±0.03	20.2±0.4	7.606±0.042	1.014±0.030	13.33±0 41
Low Oil.....	7.48±0.04	1.30±0.03	17.4±0.4	8.378±0.056	1 510±0.040	18 02±0.49

TABLE 3 CONTINUED.

CIRCUMFERENCE OF EAR.

Chemical strain.	Crop of 1905.			Crop of 1906.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
High Protein	5 76±0.01	0.44±0.01	7.6±0.2	5.863±0.014	0.393±0.010	6.70±0.17
Low Protein.	6.51±0.02	0.61±0.01	9.4±0.2	6.495±0.018	0.478±0.013	7.36±0.19
High Oil	6.05±0.01	0.53±0.01	8.8±0.2	6 134±0.017	0 417±0.012	6.80±0.20
Low Oil.....	6.65±0.02	0.59±0.01	8.9±0 2	6.717±0.018	0.515±0.013	7.52±0.19

WEIGHT OF EAR.

Chemical strain.	Crop of 1905.			Crop of 1906.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
High Protein	7.53±0.04	2.50±0.03	33.2±0.4	8.289±0.081	2.061±0.057	24.86±0 72
Low Protein.	9.66±0.10	3.30±0.07	34.2±0.7	10.77±0.11	2.680±0.071	24.88±0.74
High Oil.....	7.79±0 07	2.43±0.05	31.2±0.6	8.850±0.075	1.763±0.053	19.92±0.63
Low Oil	9 84±0.08	2.87±0.06	29.2±0.7	11.50±0.13	3.349±0.091	29.13±0.84

NUMBER OF ROWS IN EAR.

Chemical strain.	Crop of 1905.			Crop of 1906.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
High Protein	13.72±0 03	1.85±0.02	13.5±0 2	13.774±0.060	1.707±0.042	12.39±0.31
Low Protein.	14.17±0.06	1 94±0.04	13 7±0 3	14 597±0.072	1 922±0.051	13.17±0 35
High Oil.....	15.65±0.06	2 08±0.04	13.3±0 3	14.712±0.073	1.802±0.052	12.25±0.36
Low Oil	12.80±0.05	1.77±0.04	13.8±0.3	13.310±0 067	1 897±0.047	14.25±0.38

EFFECT OF FERTILITY UPON TYPE AND VARIABILITY OF CORN

Upon this point the experiments of Dr. Hopkins upon soil fertility afford considerable information. Tables 4 and 5.

TABLE 4. EFFECT OF FERTILITY UPON YIELD AND PHYSICAL CHARACTERS OF CORN, 3 YEARS ROTATION, CORN, OATS AND CLOVER (LEAMING CORN, CROP OF 1906)

Plot number.	Treatment.	Yield.	Weight of individual ears in ounces. Mean.	Standard deviation.	Coefficient of variability.
201	None	57.0	7.90±0.11	2.657±0.077	33.63±1.09
202	Legume	56.5	7.86±0.11	2.592±0.074	32 98±1.06
203	Manure, 2 tons per acre	70.4	8.807±0.12	2 854±0.080	32.41±1.03
204	Legume and lime	57.6	7.828±0.12	2.798±0.085	35.75±1.25
205	Manure and lime	73.6	8.984±0.11	2.783±0.075	30.98±0.89
206	Legume, lime, phosphorus	84.2	10.167±0.10	2.777±0.071	27.31±0.76
207	Manure, lime, phosphorus	87.4	10.433±0.10	2.812±0.071	26.95±0.76
208	Legume, lime, phosphorus, postassium	85.8	10.267±0.10	2.729±0.071	26.58±0.73
209	Manure, lime, phosphorus, potassium	86.6	10.469±0.11	2.835±0.08	27.08±0.77
210	Extra heavy manure and phosphorus† (1906 only)	71.0*	9.828±0.15	3.362±0.11	34.21±1.19

† With lime and potassium.
* Crop injured by cattle.

TABLE 4. CONTINUED.

Plot.	Length of ears.			Circumference of ears.			Rows on ears.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
201	7.047±0.057	1.620±0.040	22.99±0.60	6.041±0.022	0.557±0.015	9.22±0.25	17.627±0.101	2.490±0.070	14.13±0.412
202	7.231±0.053	1.456±0.037	20.14±0.57	6.121±0.020	0.530±0.014	8.66±0.23	17.484±0.094	2.332±0.066	13.34±0.39
203	7.436±0.054	1.527±0.038	20.54±0.53	6.253±0.021	0.558±0.015	8.92±0.24	17.911±0.095	2.501±0.067	13.96±0.38
204	7.077±0.058	1.616±0.041	22.83±0.60	6.030±0.023	0.585±0.016	9.70±0 27	17.509±0.101	2.470±0.071	14.11±0.42
205	7.533±0.052	1.469±0.037	19.50±0.50	6.304±0.021	0.579±0.015	9.18±0.24	17.895±0.099	2.640±0 070	14.75±0.30
206	8.015±0.046	1.348±0.033	16.82±0.41	6.439±0.018	0.522±0.013	8.11±0.20	18.069±0 083	2.397±0.059	13.27±0.33
207	8.152±0.045	1.265±0.032	15.52±0.40	6.553±0.019	0.524±0.013	8.00±0.20	18.318±0.091	2.525±0.064	13.78±0.36
208	7.981±0.051	1.374±0.036	17.22±0.46	6.505±0.031	0.499±0.022	6.90±0.23	18.164±0.087	2.385±0.060	13.13±0.35
209	8.083±0.053	1.456±0.037	18.01±0.46	6.555±0.019	0.517±0.014	7.89±0.20	18.107±0.089	2.417±0.062	13.35±0.35
210	7.684±0.071	1.756±0.050	22.85±0.68	6.461±0.025	0.589±0.018	9.12±0.28	17.991±0.10	2.326±0.071	12.93±0.41

TABLE 5. EFFECT OF FERTILITY UPON YIELD AND PHYSICAL CHARACTERS OF CORN—2 YEAR ROTATION, CORN AND OATS (LEAMING CORN CROP OF 1906)

Plot number.	Treatment.	Yield.	Weight of individual ears in ounces.		
			Mean.	Standard deviation.	Coefficient of variability.
501	None	58.3	7.360±0.087	2.266±0.062	30.79±0.91
503	Legume	51.9	7.124±0.086	2.075±0.061	29.13±0.94
503	Manure	54.3	6.766±0 084	2.022±0.059	29.88±0.96
504	Legume and lime	48.9	6.591±0.095	2.121±0.067	32.18±1.13
505	Manure and lime	57.8	7.493±0.091	2.332±0.064	31.12±0.93
506	Legume, lime, and phosphorus	63.3	8.028±0.090	2.243±0.063	27.94±0.85
507	Manure, lime, and phosphorus	61 8	7.518±0.091	2.274±0.064	30.25±0.97
508	Legume, lime, phosphorus, and potassium	58.3	7.354±0.098	2.552±0.069	34.70±1.06
509	Manure, lime, phosphorus, and potassium	49.6	6.496±0.092	2.250±0.065	34.64±1.13
510	Extra heavy manure and phosphorus, with lime and potassium	64.9	6.232±0 085	2 408±0.060	38.64±1.10

TABLE 5. CONTINUED.

Plot.	Length of ear.			Circumference of ear.			Rows on ear.		
	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.	Mean.	Standard deviation.	Coefficient of variability.
501	7.171±0.043	1.231±0.030	17.17±0.44	5.959±0.018	0.489±0.013	8.21±0.21	17.055±0.082	2.191±0.058	12.85±0.35
502	7.141±0.047	1.267±0.033	17.74±0.48	5.902±0.017	0.460±0.013	7.79±0.22	16.631±0.084	2.100±0.059	12.63±0.37
503	6.879±0.044	1.203±0.031	17.49±0.46	5.896±0.019	0.480±0.013	8.14±0.22	16.869±0.095	2.377±0.067	14.09±0.41
504	6.952±0.048	1.264±0.034	18.18±0.50	5.840±0.019	0.459±0.013	7.86±0.23	16.686±0.096	2.353±0.068	14.10±0.42
505	7.031±0.049	1.394±0.035	19.83±0.54	6.020±0.018	0.501±0.013	8.32±0.22	17.037±0.093	2.480±0.066	14.56±0.40
506	7.357±0.040	1.148±0.028	15.60±0.40	6.132±0.018	0.497±0.013	8.11±0.21	17.521±0.085	2.279±0.060	13.01±0.36
507	7.020±0.049	1.400±0.035	19.94±0.52	6.142±0.018	0.496±0.013	8.08±0.21	17.737±0.089	2.376±0.063	13.40±0.36
508	6.980±0.053	1 519±0.037	21.76±0.56	6 057±0.020	0.549±0.014	9.06±0.23	17.590±0.087	2.284±0.062	12.98±0.36
509	6.386±0.054	1.462±0.038	22.89±0.63	5.917±0.021	0.533±0.015	9 01±0.26	17.67 ±0.10	2.411±0.071	13.65±0.41
510	6.165±0.048	1.507±0.034	24.44±0.58	5.882±0.018	0.530±0.013	9.01±0.21	17.525±0.086	2.444±0.061	13.95±0.36

Table 4 shows the fertility treatment, the yield and the physical characters of ear on a series of plots (201-210) under a three-year rotation of corn, oats and clover. All the plots were given a uniform stand of two stalks to the hill except plot 210 which had three stalks per hill. Under the system of planting therefore the yield and weight of ear of necessity moved together, but the length and circumference are free to vary somewhat independently. By consulting the columns of means we note that in general the ears were *both longer and larger* in the higher yields, except in the thicker planting where the advantage of increased numbers was in a measure offset by a decrease in size of ear. In general these figures show that increased fertility results in increase in *both* length and circumference and at a rate fairly uniform with each other and with the increased yield. As would be expected fertility has no effect upon the number of rows.

The greater variability of plot 210 is noticeable at once with respect to both length and circumference and therefore to weight. Whether this is due to thickness of planting, to injury, or to the increased fertility is at first a matter of doubt. Referring to Table 5, two years' rotation with the same system of fertilizing as before, we find the same increased variability in the thickly planted plot (510), which was not injured, though it is less evident with respect to circumference than to length. This apparently confines the cause either to increased fertility or to thickness of planting. That it is not wholly or principally due to excessive fertility is likely from the fact that the only plot whose variability approaches that of 210 (Table 4) is plot 200 which is unfertilized. This shows that the increased variability of plots 210 and 510 is due in part at least to thickness of planting, which seems to affect the length of ear rather more than the circumference.

APPENDIX

BY H. L. RIETZ.

GRAPHIC REPRESENTATION OF STATISTICS

A mere tabulation of any considerable number of figures does not make it possible, in general, for the mind to grasp the main facts which the figures represent; in fact, the tabulation of one thousand figures may leave no impression on the mind. By the graphic methods the chief characteristics of a mass of figures are presented to the eye by means of a picture or curve. The graph gives, at a glance, important features which may be overlooked or which can only be obtained from the figures by considerable labor.

The use of the graphic method in statistical work is very extensive, and every student who has to deal with complex groups of figures appreciates more and more this method as it enables him to perceive relations through the eye. It is the object of this section to show how frequency graphs are obtained from given data.

Frequency Curves. Let us consider the following frequency distribution:

Classes	4	4.5	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5	10
Frequencies	1	1	8	33	70	110	176	172	124	61	32	10	2

in which the first line of this table gives the class marks, and the second line the corresponding frequencies; what we propose to do here is to present a significant picture of this frequency distribution. For the benefit of those who find it easier to follow a concrete example, it may be said that this frequency distribution was not taken arbitrarily but actually represents the result of measuring the lengths of 800 ears of corn as a random sample taken from a large group, the class marks representing inches.

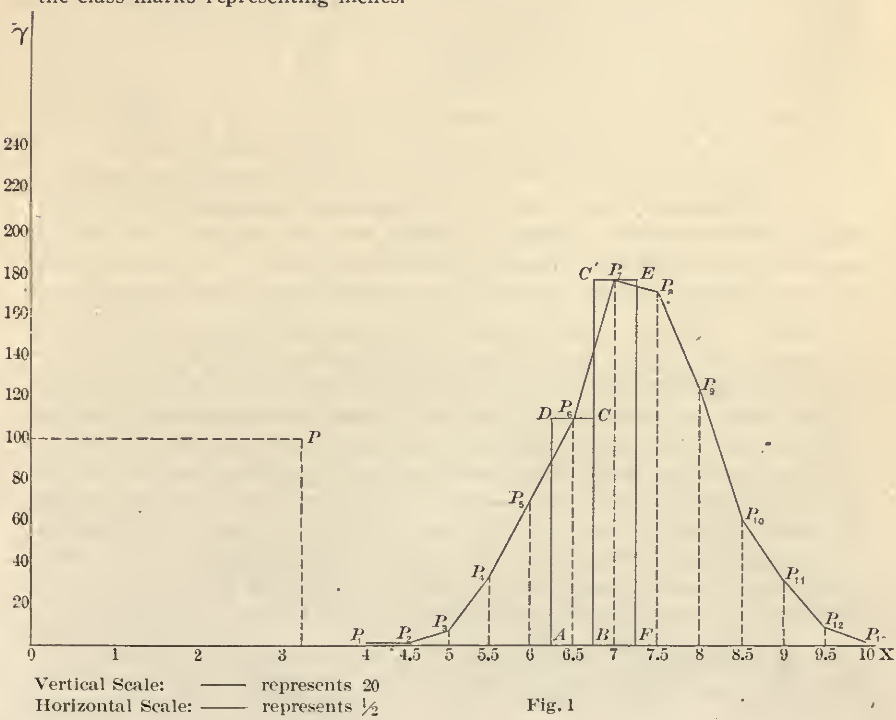


Fig. 1

Draw two lines OX and OY at right angles to each other. These are called coördinate axes. The line OX is called the x-axis and OY the y-axis. Beginning at O imagine equal intervals marked off along the x-axis. In the particular case in hand the numbers under OX indicate the number of inches represented by the distance from O, and the number on the left hand side of OY indicate the frequency represented by the distance from O. The question as to what each division represents is a matter of scale, and the scale must be chosen to suit the particular problem in hand. These numbers may also be so chosen as to be class marks. At each class mark construct a perpendicular of such length, measured upward from the x-axis, as to represent the frequency corresponding to the given class. (See dotted lines in Fig. 1.) The straight lines P_1, P_2, P_2, P_3, P_3P_4 ...., P_{12} and P_{13} joining the upper extremities of the perpendiculars so constructed form what is called "the frequency polygon" of the given distribution. If a smooth curve were drawn as nearly as possible to the points P_1, P_2, P_3,.. and P_{13}, (through them if possible), the curve would resemble the curve in Fig. 2 and is called the *frequency curve* of the given distribution.

Any point such as P (Fig. 1) represents two numbers. The one number is represented by its distance from the y-axis and the other by its distance from the x-axis. The numbers which are represented by the distances of P from the y-axis and x-axis are called the abscissa and ordinate of P respectively. We may well think of each of the points which determine the frequency polygon as having an abscissa and ordinate which give the position of the point.

Significance of Area Under Curve. Construct rectangles such as ABCD, and BC^1EF on the ordinates at class marks as midlines so that the sides AD, BC^1, FE, etc., set the boundaries to measurements belonging to different classes. Suppose now that we define unit area as equal to a rectangle bounded by AB, AD, BC, and a line parallel to AB, and at the distance from AB which represents one unit in frequency. Then the area of ABCD is 110 and the area of all such rectangles together represents the total population.

In drawing the frequency curve, one guide is to make the area bounded by the curve, the x-axis, and two ordinates equal to the sum of the areas of rectangles between the same ordinates. Then the total area under the curve represents the total population. We shall see the importance of this representation of the population by an area in connection with the discussion of probable error.

Choice of Scale. The question always arises in drawing a graph as to what scale is to be used in plotting, and unfortunately no definite rule can be laid down. Attention may, however, be called to a few points. First, we should choose a scale such that all the points can be plotted on the paper used; for, the purpose of the graph may be defeated if it is not visible in its entirety. Secnodly, if the point involved in the investigation is a queston of rate of increase or decrease, we should select such a scale as to make the curve reasonably steep. To illustrate, the sociologist presents the population of a city or country for successive years by a frequency curve in which years are used as class marks, and the corresponding populations are used as ordinates. In this case, the steepness should give at a glance, a good idea of the rate of change of the population.

Application of the Theory of Probability. What is commonly known as a law of nature is a generalization based upon experience. Such a law can be established only in the sense that a high probability may be shown to exist in its favor. To illustrate, we may take one of the best established laws of science; namely, that all bodies are attracted by the earth. The evidence for this statement consists in the fact that of the thousands and even millions of bodies which have been observed, they have followed this rule without an exception. This has established a very high degree of probability in favor of the generalization. It is altogether conceivable, however, that some body is repelled by the earth, and that such a body will at some time be observed. Although experience has established an overwhelming probability against such an occurrence, we must not overlook the fact that experience establishes a law of nature only in the sense of establishing a high degree of probability in its favor. If 1000 pennies be tossed at random, there is nothing more uncertain than that a given penny will turn up heads, but it is a matter of common experience that the ratio of the number of heads to the total number of pennies tossed is, in general, nearly ½, and that, in general, this ratio is more likely to approximate ½ as the number of tossings is increased.

A proper view of the theory of probability is especially needed in statistical work because we deal with occurrences, and characters of such a nature that we wish to make statements in regard to large numbers of them taken together. It is a matter of common experience that results, such as averages, and ratios, obtained from large numbers are nearly stationary. We find the average length of 1000 ears of corn as a random sample taken from a larger population, and are surprised if, upon taking another random sample of 1,000 ears from the same larger population, their average differs materially from that already found. We are not at all surprised if they come out substantially alike. There are probably many causes which influence the growth of each single ear, but when they are all taken together, these small disturbances tend to counterbalance each other. In short, it is regularity in large numbers which we expect. While it is common sense to expect this, we shall later give a mathematical measure known as the probable error to indicate what deviations we should expect in results such as averages.

This discussion leads us to the following definition of probability:

If, in the long run, out of n possible cases in each of which some event (or character) occurs, or fails to occur, it occurs n_1 times, and fails $n-n_1$ times, the probability that the event occurs on a particular occasion in question is $\frac{n_1}{n}$, *and the probability that it fails to occur is* $\frac{n-n_1}{n}$

Hence the expression "relative frequency" is a significant equivalent for "probability." In framing this definition we idealize an actual experience. When we say the probability of a penny falling heads is ½, this may be looked upon as an answer to the following question: What part of the pennies tossed should we expect to find heads up if we should toss an indefinitely large number? This idealization for purposes of definition is analogous to the idealization of the crude chalk mark into the straight line of geometry. Since the sum of the probabilities of occurrence and failure on a particular occasion is $\frac{n_1}{n}+\frac{n-n_1}{n}=1$, the number 1 is the symbol for certainty.

We shall state the following corollary to the definition as it is sometimes easier to apply:

If all the cases in which an event is in question can be analyzed into n^1 cases, each of which is equally likely, and m^1 is the number of these cases in which the *event occurs then* $\frac{m^1}{n^1}$ *is the probability that the event will occur on the occasion in question.*

For instance, in tossing two pennies, what is the probability of one head and one tail?

The four different ways in which the pennies can fall are: Heads-tails, tails-heads, heads-heads, tails-tails. Two of these lead to the occurrence of the event, and the four are equally likely. Hence $\frac{2}{4} = \frac{1}{2}$ is the required probability.

Combination of Probabilities. *The probability that all of a set of independent events will occur on an occasion in which all of them are in question is the product of the probabilities of the separate events.*

Proof: Let $p_1, p_2, \ldots . p_r$ be the separate probabilities of r events. Out of a great number n of cases, the first will happen on p_1n occasion. Out of these the second will happen on $p_2(p_1n)$ occasions. Continuing this process and applying the definition above, the theorem follows. To illustrate the theorem, suppose that among a population of 100,000 people, 30,000 are vaccinated, and that 500 persons have smallpox. If vaccination has no influence on smallpox, what is the probability that a person is both vaccinated and has smallpox?

Since 100,000 is a large number, we may give $\frac{30000}{100000} = \frac{3}{10}$ as the probability that a person is vaccinated, and $\frac{500}{100000} = \frac{1}{200}$ as the probability that a person has smallpox. Then $\frac{3}{10} \times \frac{1}{200} = \frac{3}{2000}$ is the probability that a person is both vaccinated and has smallpox if vaccination has no influence on smallpox. Furthermore, $\frac{3}{2000} \times 100000 = 150$ is the most probable number of vaccinated persons we should expect to have smallpox if vaccination has influence on the number of cases of smallpox.

The Normal Distribution. If 4 pennies are thrown at random, what is the probability that exactly r of them are heads and the rest tails when r takes values 0, 1, 2, 3, 4?

1) $(\frac{1}{2})^4$ = probability of 0 heads and 4 tails.
2) $4(\frac{1}{2})^4$ = probability of 1 head and 3 tails.
3) $6(\frac{1}{2})^4$ = probability of 2 heads and 2 tails.
4) $4(\frac{1}{2})^4$ = probability of 3 heads and 1 tail.
5) $(\frac{1}{2})^4$ = probability of 4 heads and 0 tails.

In 2) the coefficient 4 appears before $(\frac{1}{2})^4$ because with 4 coins there are four different combinations possible each consisting of 1 head and 3 tails. For similar reasons the coefficients 6 and 4 appear in 3) and 4) respectively.

The above illustration may be generalized and the result may be put into the following form: If n coins are thrown upon a table at random, the probability that exactly r of them are heads, and the rest tails, is given by the r+1 st term of the binomial expansion $(\frac{1}{2} + \frac{1}{2})^n$

That is, in other symbols ${}^nC_r \ (\frac{1}{2})^n$

where the symbol ${}^nC_r$. indicates the number of combinations of n things taken r at a time. In order to emphasize the fact that there is a much greater probability of getting an almost equal number of heads and tails than of getting widely different numbers, and in order to lead up to the normal probability curve, we present the following table for n = 999 obtained from Quetelet's Lettres sur la Theorie des Probabilités. As indicated in the table, columns 1 and 2 give the number of heads and tails whose probabilities are in question, while column 3 gives the corresponding probabilities:

1	2	3	1	2	3
499	500	0.025225	450	549	0.0001863
490	509	0.021069	440	559	0.0000209
480	519	0.011794	430	569	0.0000016
470	529	0.004423	420	579	0.00000004
460	539	0.001110			

It may be observed from this table that in the long run one should expect 499 heads and 500 tails more than 600,000 times as often as 420 heads and 579 tails.

In Figure 2 the results in this table are presented graphical by taking as class marks the various combinations $\frac{499}{500}$, $\frac{490}{509}$, etc., and taking as ordinates the probabilities recorded in columns 3 of table.

If we had taken all the intermediate integers from $\frac{499}{500}$ to $\frac{440}{559}$ we should have had ten times as many points which would arrange themselves on the curve in Fig. 2. By increasing the number of coins and decreasing the horizontal scale we can get the plotted points as close together as we please. The curve so obtained is known as the normal probability curve. The curve in Fig. 2 is a close approximation. Clearly, the probabilities can be converted into frequencies by multiplying each of them by the same large number, and then we obtain the normal frequency curve identical with the probability curve by merely adjusting the scale.

The causes of deviations in the case of biological measurements are analogous to the causes which produce deviations in the tossing of pennies, and it has, furthermore, been found by experience that the frequency curves of many populations obtained in biology follow the normal probability curve. While more will be said later about distributions which are not normal, for the present, let us assume that we are dealing with normal distributions, and proceed to justify the standard deviation as a measure of variability.

Geometrical Meaning of Standard Deviations. It should be noted that there are two points, A and B, Fig. 2, on the normal frequency curve such that, as we follow the course of the curve from left to right, the curve changes at A from concave upward to concave downward, and it changes at B from concave downward to concave upward. Such points on a curve are called points of inflexion. The important fact is that ½ the distance between these two points is the standard deviation* of the population represented by the frequency curve, and that this distance determines the curve in a manner analogous to the way in which the radius determines a circle. For this reason, we can say that the standard deviation is a perfect measure of variability for a normal distribution. For, when it is given along with the type, we can draw the curve which is completely descriptive of variability. In other words, the form of the population can be reproduced. This completely justifies the use of standard deviation as a measure of variability for a normal distribution.

When the distribution is not normal, the standard deviation can at most be considered as only approximately descriptive, but it is always a significant measure of variability.

If along the base line of the probability curve (Fig. 3) we measure distances in terms of standard deviation so that when x is any horizontal distance OP in ordinary units, and σ the standard deviation in the same units, we can present

*This is proved by methods involving the calculus.

the following useful table of areas which correspond to $\frac{x}{\sigma}$ For a value of $x = OP$ the area concerned is bounded by the base line, the probability curve, OY, and the line through P parallel to OY. These areas are given in Table 1 for various values of $\frac{x}{\sigma}$ and with such a unit of area that the total area under the curve is unity.

TABLE 1. AREAS CORRESPONDING TO $\frac{x}{\sigma}$

$\frac{x}{\sigma}$	Area	$\frac{x}{\sigma}$	Area	$\frac{x}{\sigma}$	Area	$\frac{x}{\sigma}$	Area
0.00	0.0000	0.30	0.1179	0.60	0.2257	0.90	0.3159
0.01	0.0040	0.31	0.1217	0.61	0.2291	0.91	0.3186
0.02	0.0080	0.32	0.1255	0.62	0.2324	0.92	0.3212
0.03	0.0120	0.33	0.1293	0.63	0.2357	0.93	0.3238
0.04	0.0160	0.34	0.1331	0.64	0.2389	0.94	0.3264
0.05	0.0199	0.35	0.1368	0.65	0.2422	0.95	0.3289
0.06	0.0239	0.36	0.1406	0.66	0.2424	0.96	0.3315
0.07	0.0279	0.37	0.1443	0.67	0.24857*	0.97	0.3340
0.08	0.0319	0.38	0.1480	0.68	0.25175	0.98	0.3365
0.09	0.0359	0.39	0.1517	0.69	0.2549	0.99	0.3389
0.10	0.0398	0.40	0.1554	0.70	0.2580	1.00	0.3413
0.11	0.0438	0.41	0.1591	0.71	0.2612	1.10	0.3643
0.12	0.0478	0.42	0.1628	0.72	0.2642	1.20	0.3849
0.13	0.0517	0.43	0.1664	0.73	0.2673	1.30	0.4032
0.14	0.0557	0.44	0.1700	0.74	0.2704	1.40	0.4192
0.15	0.0596	0.45	0.1736	0.75	0.2734	1.50	0.4332
0.16	0.0636	0.46	0.1772	0.76	0.2764	1.60	0.4452
0.17	0.0675	0.47	0.1808	0.77	0.2794	1.70	0.4554
0.18	0.0714	0.48	0.1844	0.78	0.2823	1.80	0.4641
0.19	0.0753	0.49	0.1879	0.79	0.2852	1.90	0.4713
0.20	0.0793	0.50	0.1915	0.80	0.2881	2.00	0.4772
0.21	0.0832	0.51	0.1950	0.81	0.2910	2.10	0.4821
0.22	0.0871	0.52	0.1985	0.82	0.2939	2.20	0.4861
0.23	0.0909	0.53	0.2019	0.83	0.2967	2.30	0.4883
0.24	0.0948	0.54	0.2054	0.84	0.2995	2.40	0.4918
0.25	0.0987	0.55	0.2088	0.85	0.3023	2.50	0.4938
0.26	0.1026	0.56	0.2123	0.86	0.3051	2.60	0.4953
0.27	0.1064	0.57	0.2157	0.87	0.3078	2.70	0.4965
0.28	0.1103	0.58	0.2190	0.88	0.3106	2.80	0.4974
0.29	0.1141	0.59	0.2224	0.89	0.3133	2.90	0.4981
						3.00	0.4987

*Extra figure in 5th decimal place because we wish to use the result to this number of places later.

PROBABLE ERROR

Scientific results which depend upon measurement, or which are derived from a random sample of a larger group must not be looked upon as exactly representative of the larger group even if the greatest care has been taken to eliminate sources of error. The accuracy of statistical results depends not only upon the nature of the data, and the accuracy of the measurements, but also upon the number of variates. Granted that statistical results are only approximations, it is of first rate importance to have a criterion to indicate what degree of confidence is to be placed in such results. The so-called probable error is designed to serve this purpose.

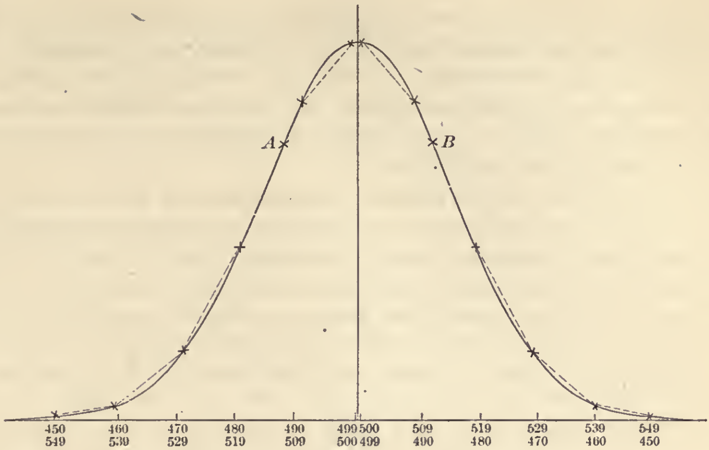


Fig. 2

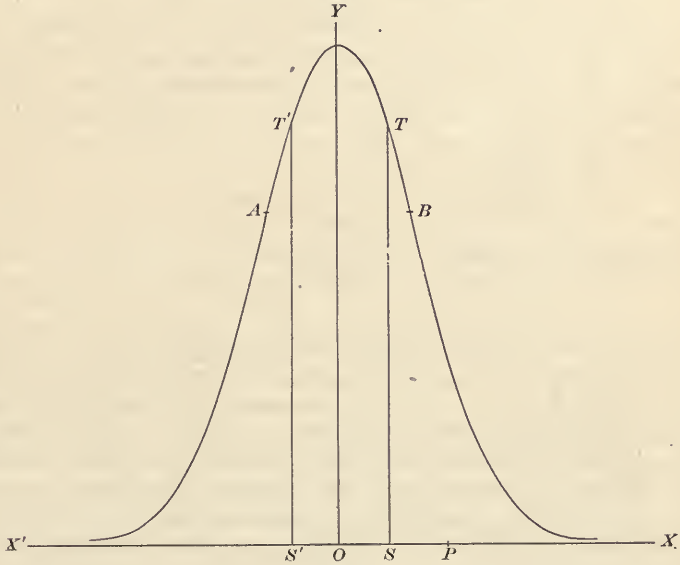


Fig. 3

Probable Error of Single Variate. *The probable error* of a single variate may be defined as that deviation* (± E) *from the mean on either side within which just one-half of the population is contained.* In other words, it is an even wager that a variate taken at random would have a deviation greater or less than the probable error. The conception of the probable error of a single variate is useful only for deriving the probable errors of other results; for, it is impossible to ascertain the probable error of a single variate until a population has

*We are using the term "error" here in the sense of a deviation from the most probable value derived from a set of measurements, and we are using the mean as this most probable value.

been treated. For instance, some one might go to an unknown island for the purpose of finding the height and variability as to height of the population. Suppose that he has no knowledge to start with in regard to the inhabitants, and simply secures his measurements from the first inhabitant he chances to see, and reports the result. There is no way known of finding what the probable error is, until a population has been measured.

When the unit of area is selected as explained above the area under the normal frequency curve becomes very significant in discussing probable error. Remembering that the total area represents the population, if ST and S^1T^1 (Fig. 3) are so drawn as to include between them just one-half the area and are equally distant from OY (the line through the mean), the distance OS or OS^1 represents the probable error. From Table 1, $OS = \frac{E}{\sigma}$ when the area in the table is 0.25. But, referring to the table, we have by interpolation that $\frac{E}{\sigma} = 0.6745$ corresponds to an area 0.25.

That is, the probable error of a single variate is found by multiplying the standard deviation of the population by 0.6745.

The approximate value of the probable error of a single variate may be found without calculation by simply marking the magnitude S^1 below which ¼ the population lies, and S above which ¼ of the population lies. Then $\frac{S-S^1}{2}$ is the probable error of a single variate. Applied to the population given on p 27 we have that $S = 7.88$

$$S^1 = 6.65$$
$$S - S^1 = 7.88 - 6.65 = 1.23$$
$$\frac{S-S^1}{2} = 0.61$$

Probable Error of the Mean. Given the mean of n variates taken at random from a larger group, we set the problem of finding the probable error in the mean.

Imagine that we continue selecting random samples of n variates from this group until we find a considerable number m of means from these different samples. Let M_1, M_2, ... M_m be these means. They will not all be equal to each other but will themselves constitute a population which can be represented by a frequency curve. Such a frequency curve of means will, of course, be much steeper than the frequency curve of the original observations. The standard deviation of this population of means can be shown, by calculus methods, to be equal, to the standard deviation of the single population divided by the square root of n.

Now, we can apply to this population of means the same definition of probable error that we have applied to a single observation, and if E_M represents the probable error of the mean, $E_M = \frac{0.6745\sigma}{\sqrt{n}}$

Probable Error of Standard Deviation. If we have found the standard deviations of m different population each of n variates and obtain $\sigma_1, \sigma_2, \ldots, \sigma_n$. These constitute a population whose standard deviation can be shown to be $\frac{\sigma}{\sqrt{2n}}$, and just as in the case of the probable error of a single variate, the probable error E_σ of the standard deviation is $0.6745 \frac{\sigma}{\sqrt{2n}}$

Probable Error of Coefficient of Variability. If we have determined C_1, C_2, ..., C_m as the coefficients of variability of m different random samples, these constitute a population in which C_1, C_2, ..., C_m take the place of the measurments in an ordinary population, and it can be shown by calculus methods that the probable error in this statistical constant is $E_c = \frac{0.6745C}{\sqrt{2n}}\left[1+2\left(\frac{C}{100}\right)^2\right]^{\frac{1}{2}}$

This can be taken in the simpler form $E_C = \frac{0.6745}{\sqrt{2n}}$ if C does not exceed 8 or 10 per cent.

The probable error E in any result is sometimes defined as such a deviation from the true value (either above or below) that it is an even wager that the result obtained deviates more or less than E from the true value. The expression "true value" is difficult of accurate definition, but may be thought of as meaning what would be obtained if an infinite number could be included in the sample. As the "true value" is an unattainable ideal so far as any important statistical results are concerned, it seems better to use the expression most probable value for our purposes.

To be sure, there is a class of problems concerning which one may well hold a different view as to the true value of a determination. In dealing with certain populations, we may take all the individuals of the group in question, so that in one sense we have not limited ourselves to a random sample. For instance, we could easily count the number of rows on each ear of a certain plot of corn. Then one may say the true value of the mean can be found so far as this plot is concerned. But this result cannot be applied with absolute accuracy to any larger group of similar individuals, and such a result is of little importance for our problems. It is, on the contrary, of first rate importance in statistical work to be able to consider the given sample as representative of a larger group to which the results may be applied with a certain degree of accuracy.

The significance of the probable error as a measure of accuracy can be further shown if we ask what odds should be given in a wager as to a deviation 2 E, 3 E, ... These results can at once be obtained from Table I and are given on p. 15 of this bulletin. We may say, in short, that an error in a result is as likely as not to be as great as E but it is very unlikely to be much greater.

Number Required to give Good Representative Sample—Paucity of Data— In making a determination of type, mean, and variability of any measurable quantities by selecting a representative random sample, an important question arises as to the number which should be included in the random sample. Nearly all our quantitative knowledge of nature depends upon measuring only a portion of a population. For instance, we say that the average height of male caucasions is 68.5 inches. We know that not all the statures have been taken to get this result, but rather a comparatively small number. The question is: how many should be taken from the whole group of male caucasions to provide a result which can be applied with reasonable accuracy to the whole group. Similarly, in taking measurements on a population made up of ears of corn the same questions arise as to the number to be taken. Of course, we should say "the more the better" if theory and accuracy were the only considerations. But in order to make the method practical, it is desirable to save labor by taking only enough measurement to insure a certain degree of accuracy.

The number to be taken should depend, to a certain extent, upon the variability of the material. If the material shows but little variability a smaller number needs be taken than if the population were much more variable.

In the work contained in this bulletin, the probable errors are such as to indicate that the number of variates is large enough to insure satisfactory results. After such a determination has been made, it is not difficult in general, by considerations of the probable error, and the nature of the frequency distribution, to decide whether enough individuals have been taken for the purpose in view. We should, however, if possible, know something of the numbers to be taken before carrying through the work. In this connection, it is important to remember that the probable error of an average varies inversely as the square root of the number of observations. We shall, furthermore, give a table (Table 2) of probable errors of the coefficient of variability. By experience, it is, in general, possible to estimate this coefficient in a case to within a few per cent. Then the table gives the probable errors for different numbers of variates.

As to the special work on the characters in question in this bulletin, we can say that we have found by experience with many distributions, and by the use of the probable error that three or four hundred variates give results of value.

TAKING THE MEASUREMENTS.

a) **Devices.** For taking the length and circumference of ears, we have designed a simple caliper. This caliper is provided with two scales, the one of which reads the lengths and the other the circumference of ears. However, we suggest that any corn grower desiring to take such measurements get a shoemaker's device for getting the length of shoes. This will serve to give lengths. For getting circumference, a tape rule will serve the purpose if not a very extensive investigation is to be made. Or, diameters may be measured and converted into circumferences by means of the following table:

CORRESPONDING DIAMETERS AND CIRCUMFERENCES.

Diam.	Circum.	Diam.	Circum.	Diam.	Circum.	Diam.	Circum.
1.	3.14	1.7	5.34	2.4	7.54	3.1	9.74
1.1	3.46	1.8	5.65	2.5	7.85	3.2	10.05
1.2	3.77	1.9	5.97	2.6	8.17	3.3	10.37
1.3	4.08	2.0	6.28	2.7	8.48	3.4	10.68
1.4	4.40	2.1	6.60	2.8	8.80	3.5	11.00
1.5	4.71	2.2	6.91	2.9	9.11		
1.6	5.03	2.3	7.23	3.0	9.42		

For taking weight of ears, we have used postal scales although any kind of scales which weigh accurately to the nearest ounce serves the purpose.

b) **Accuracy.** In the first place, it seems desirable to define what we mean by length, circumference, and weight. Shall we mean by length the length of that portion of the ear which has grown to maturity or the length of the cob? Similarly, shall we mean by the circumference the greatest distance around the ear? Likewise, do we mean by weight that taken at husking time, or that taken at a later date?

As a matter of fact, we mean by length the length of the cob, and by circumference the circumference taken one-third the distance from the large end of the ear towards the small end, and by weight the weight taken shortly after husking time. These measurements may, of course, be taken at any time, but it was convenient for our investigations to take the measurements at this time.

TABLE 2. PROBABLE ERRORS OF THE COEFFICIENT OF VARIABILITY C. FOR VARIOUS NUMBERS OF VARIATES.

Number of variates	Coefficient of Variability in Per cents.																								
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
25	0.10	0.19	0.29	0.38	0.48	0.57	0.67	0.77	0.87	0.96	1.06	1.16	1.26	1.36	1.46	1.57	1.67	1.77	1.88	1.98	2.09	2.20	2.31	2.42	2.53
50	0.07	0.13	0.20	0.27	0.34	0.40	0.47	0.54	0.61	0.68	0.75	0.82	0.89	0.96	1.03	1.11	1.18	1.25	1.33	1.40	1.48	1.55	1.63	1.71	1.79
75	0.06	0.11	0.17	0.22	0.28	0.33	0.39	0.44	0.50	0.56	0.61	0.67	0.73	0.79	0.84	0.90	0.96	1.02	1.08	1.14	1.21	1.27	1.33	1.40	1.46
100	0.05	0.10	0.14	0.19	0.24	0.29	0.33	0.38	0.43	0.48	0.53	0.58	0.63	0.68	0.73	0.78	0.83	0.89	0.94	0.99	1.05	1.10	1.15	1.21	1.27
200	0.03	0.07	0.10	0.13	0.17	0.20	0.24	0.27	0.31	0.34	0.38	0.41	0.45	0.48	0.52	0.55	0.59	0.63	0.66	0.70	0.74	0.78	0.82	0.85	0.89
300	0.03	0.06	0.08	0.11	0.14	0.17	0.19	0.22	0.25	0.28	0.31	0.34	0.36	0.39	0.42	0.45	0.48	0.51	0.54	0.57	0.60	0.63	0.67	0.70	0.73
400	0.02	0.05	0.07	0.10	0.12	0.14	0.17	0.19	0.22	0.24	0.27	0.29	0.32	0.34	0.37	0.39	0.42	0.44	0.47	0.50	0.52	0.55	0.58	0.60	0.63
500	0.02	0.04	0.06	0.09	0.11	0.13	0.15	0.17	0.19	0.22	0.24	0.26	0.28	0.30	0.33	0.35	0.37	0.40	0.42	0.44	0.47	0.49	0.52	0.54	0.57
600	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.43	0.45	0.47	0.49	0.52
700	0.02	0.04	0.05	0.07	0.09	0.11	0.13	0.15	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.29	0.32	0.33	0.35	0.37	0.39	0.42	0.44	0.46	0.48
800	0.02	0.03	0.05	0.07	0.08	0.10	0.12	0.14	0.15	0.17	0.19	0.21	0.22	0.24	0.26	0.28	0.29	0.31	0.33	0.35	0.37	0.39	0.41	0.43	0.45
900	0.02	0.03	0.05	0.06	0.08	0.10	0.11	0.13	0.14	0.16	0.18	0.19	0.21	0.23	0.24	0.26	0.28	0.30	0.31	0.33	0.35	0.37	0.38	0.40	0.42
1000	0.02	0.03	0.05	0.06	0.08	0.09	0.11	0.12	0.14	0.15	0.17	0.18	0.20	0.22	0.23	0.25	0.26	0.28	0.30	0.31	0.33	0.35	0.36	0.38	0.40

In beginning this investigation, the measurement of length and circumference were taken to the nearest tenth inch. Great care must, however, be taken to get such a close measurement, and, in general, results derived from such masurements are no better than results derived from measurements taken with less apparent accuracy. In fact, our experiments have shown that length may well be taken to half inches, circumference to three-tenths inches and weights to ounces. The closeness of measurements is closely connected with the question of grouping measurements into classes.

c) **Grouping into Classes.** In forming the frequency distribution the measurements are grouped into classes as has been shown in this bulletin. There is no object in taking measurements with extreme accuracy and then grouping them into broad classss. In fact, the nature of the frequency distribution with a given grouping must help to settle the question of grouping, and this in turn the closeness of the measurements. In short, measurements should be so grouped as to show the variability and at the same time to leave the frequency distribution fairly smooth. In the matter of grouping, there are two opposing tendencies—grouping into too few classes to show variability, and grouping into too many classes to give a smooth distribution. In short, the law of distribution is hidden because of too much detail.

We may lay it down as a general rule that the classes should be only just broad enough to make the distribution fairly smooth, that is, there should be no vacant classes except very near the extremes of the range, and a gradual increase from one extreme up to a maximum and then a gradual decrease to the other extreme, if there is only one maximum in the distribution as is, in general, the case with these populations.

In respect to grouping into classes the characters treated in this bulletin, we have settled upon one-half inch classes for length of ears, three-tenths inch for circumference, one ounce for weight and even numbers for rows. This classification or grouping was decided upon after experimenting with classes taken at more frequent intervals.

There is a further danger of error in grouping besides the narrowness and broadness of classes. For example, at first we measured ears to the nearest tenth inch in length, then suppose we had made quarter inch groupings as follows:

4, 4.25, 4.50, 4.75, 5.00, 5.25, 5.50, 5.75, 6.00, etc.

At 5.75 would be grouped all ears which measured 5.7 and 5.8 while at 5.00 would be grouped those which measured 4.9, 5.0, and 5.1. In the long run, this would clearly result in placing more ears at 5.0 than at 5.25 other things being equal. If we should group measurements taken to the nearest tenth inch in 0.5 inch or 0.3 inch classes, no such difficulty arises. Such a grouping as that into quarter-inch group would not greatly disturb the mean and variability, but would destroy the smoothness of the distribution. Again, if we measure to quarter inches, but group to half inches, some measurements fall on the division lines between classes. Then one-half a variate may be recorded in each of the classes between which the variate falls, or if we are dealing with large numbers one can alternately put such a variate into a class above, and below such a measurement.

While many other questions may arise in taking the measurements of a certain character, this brief discussion covers the main difficulties in obtaining measurements for this bulletin.