THE CONTROL OF WATER

AS APPLIED TO IRRIGATION, POWER AND TOWN WATER SUPPLY PURPOSES

BY

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WITH FULL DIAGRAMMATIC ILLUSTRATIONS

LONDON
GEORGE ROUTLEDGE & SONS LIMITED
BROADWAY HOUSE, 68–74 CARTER LANE, E.C.
1913

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PREFACE

THIS book is essentially the product of actual engineering experience, and is mainly based on a collection of notes and formulæ accumulated in some eighteen years of professional work, during the major portion of which I was engaged in independent practice. It must therefore be regarded not as a text-book, but rather as a manual for engineers in active work.

Although the initial knowledge assumed in the reader may be considered to be somewhat unusual, many portions of the book have stood the test of everyday office requirements in the hands of draughtsmen and assistants; and I consequently trust that on the whole it will prove useful to all technically trained engineers.

The treatment of the theoretical parts is purposely cursory, and may even appear somewhat incomplete; but, after considerable experience of precise hydraulic measurements both under field and laboratory conditions, I have come to the conclusion that at the present date the results obtained from well-conducted observations are more accurate than the assumptions made in the most modern mathematical treatment of hydraulics. To illustrate my meaning, I need only refer to the errors introduced by the usual theoretical assumption of uniform velocity, and to the undoubted fact that no weir capable of practical construction discharges water according to the law of $(\text{head})^{1.5}$.

With a view to providing a bibliography an attempt has been made to give systematic references to original authorities; but it is as well to mention that in several cases the reference is not to the original authority, but to some later paper which contains a more useful presentation of the subject. In some cases, however, search for the original authority has proved fruitless, and I must consequently apologise to all engineers who may find their work utilised without any definite

acknowledgment on my part, and can assure them that such omission has been quite unintentional.

I must further acknowledge my general indebtedness to Prof. W. C. Unwin, and the late Prof. Kernot of Melbourne University; also to Messrs. Middleton, Hunter, and Duff.

For the facilities afforded by nearly every officer of the Punjab Irrigation Branch, and especially by my former superior officers, Messrs. Floyd and Tickell, I am greatly indebted.

For assistance in the preparation of the book, and for much helpful criticism, I have also to thank Messrs. R. E. Middleton, P. H. Fish, R. E. Reeves, M. Mawson, H. C. Booth, and W. R. Pettit, whose special knowledge has been extremely useful.

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Preliminary Data of Hydraulics.—Density and weight of a cube foot of pure water—Conversion tables—Rain-fall and run-off figures—Conversion from metric into English measure.

PRELIMINARY DATA OF HYDRAULICS

In most cases the figures required for conversion purposes are considered and tabulated on the pages where they are likely to be wanted. The following figures are used so frequently that they are tabulated together in order to save reference. The column headed "Approximate Values" will be found to indicate very rapid methods of conversion, which are liable to but small errors either absolutely or in arithmetical working.

DENSITY AND WEIGHT OF A CUBE FOOT OF PURE WATER AT DIFFERENT TEMPERATURES

Temperature in Degrees Fahr.	Density.	Weight of a Cube Foot in Lbs.
32·0	0·99987	62·416
39·3	1·00000	62·424
45·0	0·99992	62·419
50·0	0·99975	62·408
55·0	0·99946	62·390
60·0	0·99907	62·366
65·0	0·99859	62·336
70·0	0·99802	62·300
75·0	0·99739	62·261
80·0	0·99669	62·217
85·0	0·99592	62·169
90·0	0·99510	62·118
100·0	0·99418	62·061
105·0	0·99318	61·998
110·0	0·99214	61·933
115·0	0·99105	61·865
120·0	0·98870	61·719

CONVERSION TABLE FOR QUANTITIES FREQUENTLY USED IN HYDRAULICS

	Accurate Values.		Approximate Values.
	Number.	Log.	
Cube feet into imperial gallons	6·24	0·7952	$\frac{100}{16} = \frac{25}{4} = 6·25$
Cube feet into U.S. gallons .	7·49	0·8744	$\frac{15}{2} = 7·5$
Imperial gallons into cube feet	0·1603	$\bar{1}$·2048	$\frac{16}{100}$
U.S. gallons into cube feet .	0·1336	$\bar{1}$·1256	$\frac{1}{10} + \frac{1}{30} = 0·1333$
U.S. gallons into imperial gallons	0·8333	$\bar{1}$·9208	$\frac{5}{6}$
Imperial gallons into U.S. gallons	1·200	0·0792	$\frac{6}{5}$
Feet head of water into lbs. per square inch	0·434	$\bar{1}$·6375	
Lbs. per square inch into feet head of water	2·304	0·3625	
Kilograms into pounds . .	2·2046	0·3433	$2 + \frac{2}{10} = 2·2$
Kilos per square centimetre into lbs. per square inch	14·225	1·1530	
Kilos per square metre into lbs. per square foot	0·2048	$\bar{1}$·3114	
Lbs. per square inch into kilos per square centimetre	0·0703	$\bar{2}$·8470	
Lbs. per square foot into kilos per square metre	4·8829	0·6886	
Lbs. per cube foot into kilos per cube metre	16·019	1·2046	
Metres into feet. . . .	3·2808	0·5160	$3 + \frac{1}{4} + \frac{3}{100} = 3·28$
Square metres into square feet	10·764	1·0320	$10 + \frac{1}{2} + \frac{1}{4} = 10·75$
Cube metres into cube feet. .	35·315	1·5480	$\frac{100}{3} + 2 = 35·333$
Inches into centimètres	2·54	0·4048	$2 + \frac{1}{2} = 2·5$
Square inches into square centimetres	6·45	0·8096	6·5
Imperial gallons into litres .	4·544	0·6575	4·5
U.S. gallons into litres . .	3·785	0·5780	$3\frac{3}{4}$
Feet into metres . . .	0·3048	$\bar{1}$·4840	
Square feet into square metres	0·0929	$\bar{2}$·9680	
Cube feet into cube metres .	0·02832	$\bar{2}$·4521	

In actual practice the use of systematic tables saves both time and errors. The following works are useful :

Bellasis' *Hydraulics with Tables* (Rivington's), which contains a very complete set of tables usually employed in Hydraulics. In my opinion the book also gives the soundest general treatment of theoretical questions that exists in English.

Horton's *Weir Experiments, Coefficients, and Formulæ* (United States Geological Survey: apply to the Superintendent of Public Documents, Washington, D.C., U.S.A.) is indispensable for weir work, and contains many other very useful tables.

Kennedy's *Graphic Hydraulic Diagrams* (apply to Thomason College, Roorkee, U.P., India). This volume is intended purely for irrigation purposes. If silty waters are considered it is indispensable.

EQUIVALENTS FOR USE IN RAINFALL AND RUN-OFF

Calculations.—These are calculated on the assumption that $6\frac{1}{4}$ gallons = 1 cube foot, and 365 days = 1 year.

1 inch of run-off in a year	= 0·0736 cusecs per square mile.
1 inch of run-off in an hour	= 640 cusecs per square mile.
	= 1 cusec per acre.
1 cusec per square mile	= 13·56 inches yearly run-off.
	= 31·54 million cube feet yearly.
1 inch depth over 1 square mile	= 2·32 million cube feet yearly.
100,000 cube feet storage per acre	= 27·5 inches depth on 1 acre.
100,000 imperial gallons per acre	= 4·40 inches depth on 1 acre.
1000 imperial gallons per acre per day	= 16·08 inches yearly run-off.

CONVERSION OF AN EQUATION FROM ENGLISH INTO METRIC UNITS, AND *VICE VERSA*

The most usual form for a hydraulic equation is one or other of the two following:

$$Q = C_1 l h^{1.5} \qquad \text{or, } Q = A l^n h^m$$
$$v = C\sqrt{rs} \qquad \text{or, } v = B r^p s^q$$

As a rule it is necessary to transform from metric into English units. Now:

Q is in cubic metres	$=(3·2809)^3$ cubic feet.
l is in metres	= 3·2809 feet.
h is in metres	= 3·2809 feet.

Thus the new value of C_1 (say C_e), or of A (say A_e), is given by:

$$(3·2809)^3 Q = C_e l h^{1.5}(3·2809)^{2.5} = A_e l^n h^m (3·2809)^{n+m}$$

Therefore $C_e = \sqrt{3·2809}\,\frac{Q}{lh^{1.5}} = 1·811C.$

$$A_e = A(3·2809)^{3-m-n}.$$

So also, proceeding to the second equation,

$$v = C\sqrt{rs}$$

v is in metres per second	= 3·2809v feet per second.
r is in metres	= 3·2809r feet.
s is a pure number	$=\frac{\text{metres}}{\text{metres}}=\frac{\text{feet}}{\text{feet}}$.

So that $C_e = C1·811$. $\qquad B_e = B(3·2809)^{1-p}$.

The principles are now clear.

More complicated cases are better treated by successive substitution.

Let us consider Maxime Levy's equation for the velocity of water in metres per second in cast-iron pipes with a radius of R metres. We have

$$v = 20·5\sqrt{Rs(1+3\sqrt{R})}$$

Let us transform this into English measure, and substitute r (the hydraulic mean radius) for R, *i.e.* we obtain:

$$v = C\sqrt{rs(1+x\sqrt{r})} \text{ in feet, and feet per second.}$$

In metres $r = \frac{R}{2} = \frac{3 \cdot 28}{2} R$ feet.

$$1+3\sqrt{R} = 1+x\sqrt{r}, \text{ if } x\sqrt{1 \cdot 64R} = 3\sqrt{R}, \text{ or } x = \frac{3}{\sqrt{1 \cdot 64}} = 2 \cdot 34.$$

We have consequently disposed of the factor $1+3\sqrt{R}$, and we get:

$$C = \frac{v}{\sqrt{rs(1+x\sqrt{r})}}$$

$$= \frac{3 \cdot 28v}{\sqrt{1 \cdot 64Rs(1+3\sqrt{R})}} = \frac{3 \cdot 28}{\sqrt{1 \cdot 64}} 20 \cdot 5 = 20 \cdot 5 \times 2 \cdot 56 = 52 \cdot 5.$$

Or, $V = 52 \cdot 5\sqrt{rs(1+2 \cdot 34\sqrt{r})}$ is the equation in English measure.

CHAPTER II

GENERAL THEORY OF HYDRAULICS

Definitions.—PRESSURE—Gauge and Absolute—HEAD—Velocity head—**Viscosity**—Theoretical equations of hydraulics.
VELOCITY.—Definition of mean velocity—Velocity—Practical measurement—Resultant velocity—Steady and uniform motion.
Periodic Unsteady Motion.
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UNITS

THROUGHOUT this book, unless otherwise definitely stated, the units employed are feet, pounds, and seconds. Thus areas are measured in square feet, volumes in cube feet, velocities in feet per second, etc.

Pressures, however, are usually measured in feet of water, so that a pressure of 1 foot of water corresponds to a pressure of 62·5 lbs. per square foot, or 0·433 lbs. per square inch.

Since practical utility rather than uniformity is considered of primary importance, the inch unit is occasionally employed when discussing structures such as pipes, or metal work, which are bought and sold by this unit; also where the adoption of a foot would lead to very small figures or to long strings of zeros. These cases are invariably indicated by the word **Inch** being printed opposite to the formula in which the symbol occurs, besides being defined in the letterpress. In the actual working of examples feet and decimals of a foot are almost exclusively employed. This practice is already adopted by many hydraulic engineers, and where quantities of water are measured accuracy can hardly be attained if feet and inches are employed.

The term CUSEC is used as an abbreviation for cubic feet per second. The American equivalent is SECOND-FOOT.

The term acre-foot is occasionally used to denote a volume of 43,560 cube feet, which is obviously the content of a reservoir 1 acre in area and 1 foot deep.

It may be noted that for all practical purposes: 1 cusec flowing for 24 hours delivers 2 acre-feet.

The actual figures are—86,400 and 87,120 cubic feet respectively.

Also 1 cube foot per minute = 9000 imperial gallons per day of 24 hours, and therefore 1 cusec = 540,000 gallons per day.

Definitions.—The ordinary definition of a fluid is a "substance which yields continually to the slightest tangential stress." Consequently, "when the fluid has come to rest the stress across any surface in the fluid must be normal to the surface."

Pressure.—In all cases considered in this book (which is exclusively devoted to the discussion of engineering practice) the above stress must be a pressure. The intensity of the pressure is measured by the number of units of force per unit of area. Thus if P be the force in pounds acting on an area of a, square feet, the mean pressure over this area is $p_m = \frac{P}{a}$ lbs. per square foot, and, as usual in mechanics, the pressure at a given point is defined as $p = \text{Lt}\frac{P}{a}$, when a decreases to an indefinitely small area surrounding the point considered. The pressure across any surface being normal to the surface, it follows that the pressure at any point is the same in all directions about that point.

As already stated, pressures are usually measured in feet of water. Thus the pressure at a point in a body of water at rest, y feet below the free surface of the body of water, is y feet of water,

or $62{\cdot}5y$ lbs. per square foot,
or $0{\cdot}433y$ lbs. per square inch.

This is the "pressure" usually considered by engineers, since it is that measured by a pressure tube open to the atmosphere, or by a pressure gauge, and is that which engineering structures are usually required to sustain. The pressure of the atmosphere, however, can be measured by a barometer, and is found to vary from day to day, and also to depend upon the height above the sea level. On the average, however, the absolute pressure of the atmosphere is about 2116·8 lbs. per square foot, or 32·9 feet of water. Thus, the height of the water barometer is about 33 feet, and the absolute pressure at a point where the gauge pressure is y feet is about $y+33$ feet of water. In certain calculations it is found that reckoning the pressure in this manner obviates all necessity for a consideration of "negative pressures." The term "absolute pressure" is then employed, and "gauge pressure" signifies the pressure as first defined. The term pressure when used without qualification denotes gauge pressure.

The term head is frequently employed by engineers, and the following discussion will show that head is in reality a generalised expression for the energy due to pressure and position, and its mechanical transformations into velocity.

Head.—The term head was primarily used by millwrights in order to describe the differences in elevation between the water surfaces in the head and tail races of a mill.

The term as at present used may be best defined as follows:

Let a vertical tube of a size which will prevent any measurable capillary elevation be placed in communication with a mass of water in such a manner that the velocity of the water does not affect the height to which the water rises in this tube. Let the free water surface in this tube stand at a height H feet above a fixed datum plane. Then the water at the point where the tube opens into the mass of water considered is said to be under a head H relative to the fixed datum plane. It will be evident that if p_g represent the gauge pressure in feet of water and z the height of the mass of water considered above the datum plane:

$$H=p_g+z$$

and, as remarked later on, we are not concerned with the absolute magnitude of H, but rather with the changes in its absolute magnitude. Hence we may also say that:

$$H=p_a+z$$

where p_a is the absolute pressure in feet of water, provided that we adhere to this notation throughout the whole investigation.

In actual practice it is found that the orifice of the tube must be somewhat carefully shaped and adjusted relatively to the direction of the velocity of the water, in order to prevent this velocity from having any influence on the height of the water in the tube. If the orifice is so adjusted as to permit the velocity to have its greatest possible effect, it will be found that the water surface rises a certain additional amount, which (errors and imperfections in the tube orifice being neglected) is equal to $\frac{v^2}{2g}$ feet, where v is the velocity of the water in feet per second.

It will therefore be evident that H represents the portion of the energy of a unit mass of the water considered, which depends on its position, and that the portion which depends on its velocity is represented by $H_v=\frac{v^2}{2g}$.

As a matter of fact, it is well known that the total energy of a body cannot be measured, and all that engineers are really concerned with is the measurement of the change of energy. Thus, if strict accuracy be desired the variation of H represents the variation of that portion of the energy which is due to the position and pressure of the unit mass considered, and similarly the variation of H_v represents the variation of the energy due to the velocity of the unit mass.

Water being a highly incompressible fluid, and the alterations in temperature which occur in practical hydraulics being small, we can usually regard the changes in its energy due to compression and changes of temperature as negligible. Hence it can be stated that the whole energy of a unit mass of water relative to the datum plane is represented by $H+H_v$. If the matter is regarded in this manner, and the above definitions are accepted, we see that $H+H_v$, is constant for the mass of water considered, and that Bernouilli's equation:

$$H+\frac{v^2}{2g}=z+p_g+\frac{v^2}{2g}=\text{a constant}$$

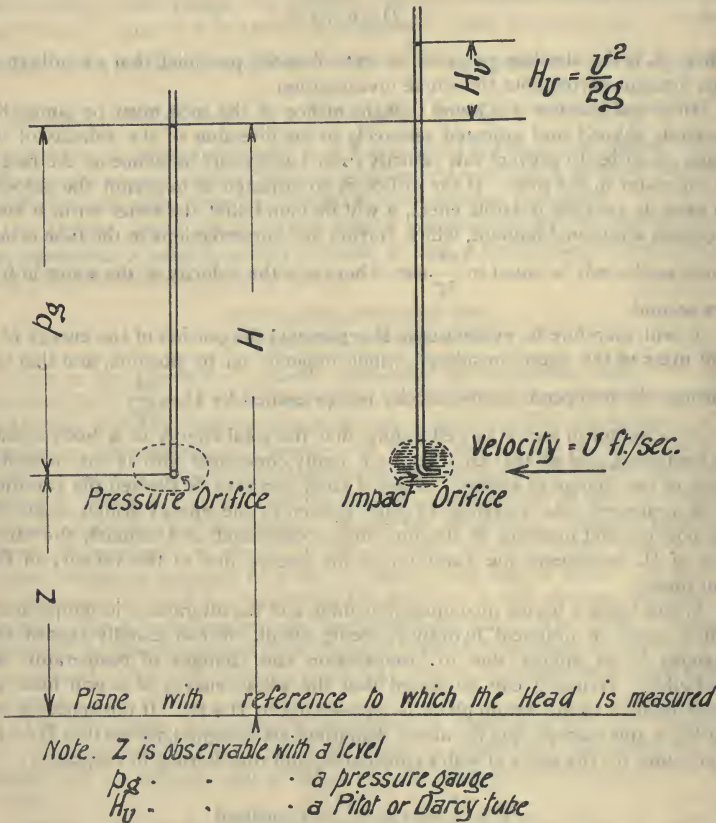
follows at once. It must, however, be understood that this is no proof, as the definition of the velocity cannot be regarded as complete. Still less is it a proof of Bernouilli's equation as applied to two unit masses of water, which at the same instant of time are at different points in a stream of water in motion.

When thus applied we find in practice that

For the first point . . . . . $H_1+\frac{{v_1}^2}{2g}=A_1$ say

For the second point . . . . $H_2+\frac{{v_2}^2}{2g}=A_2$.

The difference $A_1 - A_2$ (which is positive if the first point be upstream of the second) is defined as the head lost by friction or shock. The definition must be regarded merely as a statement of observational fact. A profound ignorance exists as to the precise manner in which this energy is dissipated. All that can really be said is that in certain cases the lost energy has been accounted for by an observed rise in the temperature of the water. This loss



SKETCH No. 1.—Relations between Head, Pressure, and Position, and between Velocity Head and Velocity.

of energy is believed to be produced by the internal motion of the water, which gradually dies out, and is transformed into heat energy. In the majority of cases the observed rise of temperature does not fully account for the loss in energy. This may be explained either by imperfections in the experimental arrangements, or by the internal motions which still exist but are not observed.

Viscosity.—The term Viscosity is employed for that property of fluids in virtue of which the change in form of the fluid under the action of a continued stress proceeds gradually, and is opposed by a resistance which increases as the relative velocity of adjacent particles of the fluid increase. While it is probable that the whole series of experimental coefficients discussed in this book are in reality caused by the fact that water is a viscous fluid, viscosity *per se* does not generally obtrude itself upon the notice of engineers, and is therefore discussed only in connection with critical velocities.

In treatises on Hydraulics certain proofs concerning such relations as the connection between the pressure at an orifice and the velocity with which water flows through the orifice are usually given. The mathematical demonstration of these proofs invariably rests on Bernouilli's equation (see p. 13). As this book is largely concerned with a discussion of the corrections that must in practice be applied to these theoretical equations, I have thought it best to assume the theoretical equations and to at once consider the experimental corrections. The process has practical advantages, as the theoretical relations are almost invariably so masked by experimental coefficients that their application in an uncorrected form would usually lead to highly erroneous results.

VELOCITY.—The velocity of water at any time t, and at any point P, and in any direction, is defined as follows :

Consider a small plane area, denoted by a. Let the quantity of water that passes through this area in the interval between the times t and $t+T$ seconds after a fixed epoch be given by $Q=avT$. Then, if Q is expressed in cube, and a in square feet, v is defined as the mean velocity in feet per second over the area a, during the time interval T, in a direction normal to the plane of a.

Following the usual rules of mechanics, the velocity at the time t at a point P in this direction is

$$\text{the limit of } \frac{Q}{aT}$$

when a and T become indefinitely small, and P is the centre of the indefinitely small area a.

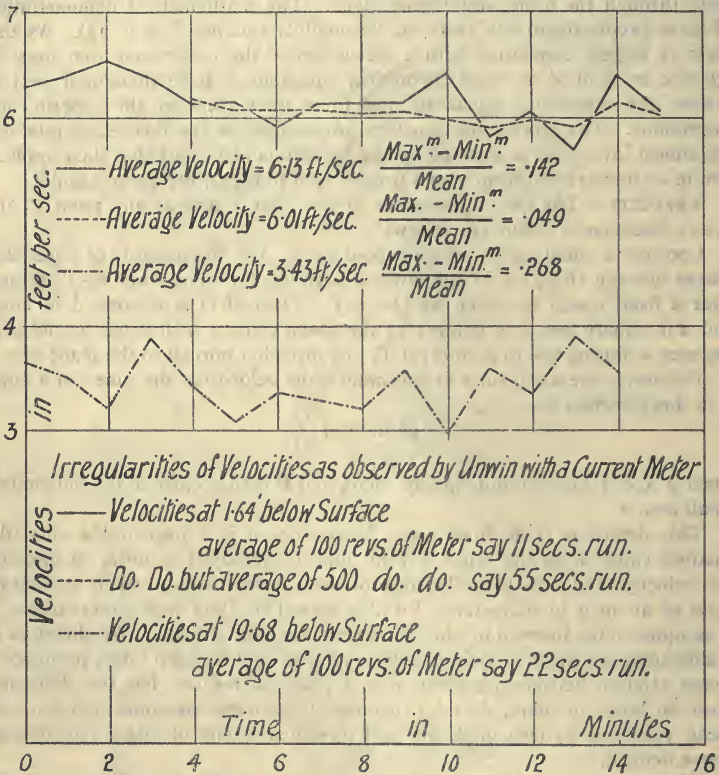
This definition is an ideal one. In practice a is a measurable area (the smallest value occurring when a Pitot tube is employed in order to measure the velocity, and a is then the area of the pressure orifice, which is at least $\frac{1}{32}$nd of an inch in diameter). So also, except in Pitot tube observations, T is an appreciable interval of time, say twenty seconds at least. We therefore in reality measure mean velocities only. This, as will be seen later, produces a closer relation between observation and practical results ; but the difference must be borne in mind, since all theoretical equations are concerned not with mean velocities as measured, but with the ideal limits of mean velocities as above defined.

We can similarly define the resultant mean velocity (usually termed the velocity) at P, as the value of the mean velocity obtained when the area a is so oriented that the velocity through it is greater than that through any equal area which is not similarly oriented ; or, more precisely, when the velocity through all areas in planes which are perpendicular to the plane of a is zero. The direction of this resultant mean velocity is obviously normal to the plane of a as thus defined. Similarly, the resultant velocity at the time t is the maximum limit of the mean velocity when the interval T and the area a become indefinitely small.

We may also define the motion of water as steady when the magnitude and direction of this resultant velocity is independent of t.

In uniform motion the magnitude of the resultant velocity is the same for the same particle of water at every point of its path during the motion.

In practical hydraulics (with the possible exception of motion in capillary tubes) the motion is always unsteady and non-uniform. Engineers are, however, accustomed to consider the motion of water as steady or uniform when the above definitions are fulfilled, if we consider the motion of the entire stream of water as a whole. The unsteadiness or non-uniformity of the motion of individual particles is thus ignored, provided that the motion of the whole body



SKETCH No. 2.—Diagram of Irregularities of Velocity.

of water is steady or uniform. The phraseology cannot be regarded as incorrect, as it is precisely similar to the use of the term "homogeneous," as applied to concrete or brickwork, or even to steel, if the results of microscopic investigations are considered.

The assumption that these substances are homogeneous permits certain mathematical deductions to be obtained, which are found to agree very well with the results of observations conducted by the ordinary commercial methods, If, however, the observations are conducted with the utmost possible refinement, differences between the results of the approximate theory and the observations are disclosed, which are found to be explicable by mathematical investigations based on the assumption that the substances are not perfectly homogeneous. The difference between the tensile strength of steel plates when measured along and perpendicular to the direction of rolling forms a very pronounced example.

Similarly, the assumption that the motion of water in a pipe or channel of constant cross-section is uniform and steady may be made, and will lead to results which agree fairly closely with observation so long as the motion of large masses of water only is considered. If, however, the motion of individual particles of water is investigated by accurate methods, the assumption is not sufficiently in accordance with the real physical facts, and the divergence between the approximate theory and observation will be found to be more and more marked the greater the accuracy of the observations.

When the distinction between the instantaneous values of the velocity at a point in a given direction, and the mean value of the same velocity over a period of time which is sufficiently long to eliminate the effect of the accidental irregularities is of importance, the term mean local velocity (see p. 12) will be used for the latter quantity, and the differences positive or negative between the mean local velocity and the instantaneous values of the velocity will be termed the irregularities.

Thus in symbols: Mean local velocity$=\frac{Q}{aT}$ where a is very small, when T is sufficiently large to eliminate irregularities, and the irregularity at the time t is given by $\frac{Q}{aT}-\text{Limit}\left(\frac{Q}{aT}\right)$.

As a rule, we wish to observe the mean local velocity, and employ the term velocity to denote its value.

Periodic Unsteady Motion.—In ordinary channels with rough boundaries the velocity at any given point is found to vary from moment to moment both in magnitude and direction. On the other hand, the resultant mean velocity during a sufficiently long period (say 1 to 5 minutes) varies but little either in magnitude or direction. Hence the time variations of the direction and magnitude of the velocity are, in a certain sense, periodic. Consequently, if at each point of the stream the resultant mean velocities, as defined above, are regarded as being the velocities which actually exist at every moment, this average of the actual motions may be considered as steady, and calculations based on this assumption will prove to be fairly reliable.

THEORETICAL INVESTIGATION.—The definitions given above suffice for practical purposes, and the following distinctions need only be considered if extreme precision is desired. They are, however, fundamental, and although complicated must be taken into account if any advance in the accuracy of hydraulic measurements is desired. The use of mathematical symbols is therefore legitimate.

Let any point in the body of water considered be denoted by its co-ordinates (x, y, z).

In considering a motion which is steady in the broad sense already defined, the velocity in any direction, say Oz, for clearness, is a function of t, due to the irregularities of the motion only, and v, the resultant velocity, is given by the equation:

$$v=\phi(x, y, z, t)$$

so that v depends on t as well as on x, y, and z. In practical cases the motion of the water is such that the mean value of v over a sufficiently long period is independent of t, or :

$$V=\frac{\int_t^{t+T}\phi(x,y,z,t)dt}{T}=\chi(x,y,z)$$

and V, the resultant mean velocity normal to the plane x, y, is independent of t or T.

Boussinesq terms V the mean local velocity at the point (x, y, z) in the direction Oz. Thus v may be regarded as a quantity which fluctuates irregularly about its mean value V. The meaning of the term periodic steady motion is now obvious.

Certain opinions (they possess no firmer foundation) concerning the length of the period T are given later.

In practice, engineers are almost entirely concerned with V, and do not wish to measure v. The difference $V-v$ is termed the irregularity of the velocity, and in all practical measurements the smaller the influence which $V-v$ (which may include not only variations in absolute magnitude, but also in direction, *i.e.* $V-v$, is a vector quantity) has on the indications of the instrument used to observe V, the better. The importance of this condition is best realised by the statement that $V-v$ may amount to $\pm$ 50 per cent. of V. (See Sketch 2.)

Now, considering Q, the discharge through any area in the plane $z=0$, which is supposed to be perpendicular to the direction of V,

$$Q=\iint V\,dy\,dx=\frac{1}{T}\iiint_t^{t+T} v\,dx\,dy\,dt$$

where the integration is performed over the whole area, gives the mean discharge during the period t to $t+T$; but any element of the discharge is represented by $dQ=v\,dx\,dy\,dt$. Thus, any instrument which is really accurate for the purpose of discharge measurement must measure V, and not v.

Assuming that the instrument measures V, we can in practice only measure V at definite points. Thus, in place of $Q=\iint V\,dx\,dy$, we really have :

$Q=\Sigma V_m a$, where V_m is the mean value of V over the area represented by a.

Now, the practical assumption is that, if V be measured at a point P, the value thus obtained represents V_m for practical purposes over a certain area. Thus we have to consider not only the possible variations of v, for a fixed point (x, y) as t varies, but the possible variations of V, as x and y, vary over the area represented by a. These we may shortly term the permanent differences of the local velocities at adjacent points.

The various formulæ used in practice when calculating Q, from a series of observations of V, are discussed later. Mathematically speaking, the most complicated of these formulæ leads to a correct value of the discharge if the local velocities over the whole of each partial area a can be represented by :

$$V=a_0+a_1x+b_1y+a_2x^2+b_2xy+c_2y^2$$

Thus, any very marked difference between the values of V, at three adjacent points of observation, may be regarded as indicating a possibility of an error in Q due to insufficiently close spacing of the points at which V is observed. The question therefore at once arises,—how closely should the points of observation be spaced, and is it better to observe some quantity $\frac{\int_t^{t+T_1} v\,dt}{T_1}$ at many points, taking the risk that T_1 is not sufficiently long to eliminate the effect of irregularities typified by $V-v$, or to observe the velocity at fewer points, and take every possible means by repetition of observations to secure that the mean local velocity is determined? The alternative is fundamental in all hydraulic work, unless the discharge is obtained by weir or volumetric methods.

A general answer cannot be given. My own experience is mainly confined to the gauging of large quantities of water (50 cusecs and over), flowing in earthen channels. In such cases irregularities in motion and differences between the observed velocities at consecutive points are both very pronounced. A mathematical study of the observations, conducted by Pearson's curve-fitting methods, has led me to believe that a close spacing of the points at which the velocities are observed is preferable to a repetition of observations at a few selected points.

The question, however, deserves careful study, especially when a station for systematic gauging observations is being selected. It is probable that in some cases irregularities are more marked than differences, and then fewer points and repeated observations at these points should produce better results.

HYDRAULIC CALCULATIONS.—The equation usually employed by hydraulic engineers is that known as Bernouilli's. Using feet, the weight of a cube foot of water, and seconds, as our foundamental units, this equation in the case of water can be written as follows:

$$h+z+\frac{v^2}{2g}=\text{a constant}$$

where v is the velocity in feet per second, $2g=64{\cdot}4$ approximately, but varies with the latitude and height above sea level, and h is the pressure at any point measured in feet of water, i.e.

$$h=\text{lbs. per square inch} \times 2{\cdot}34=\frac{\text{lbs. per square foot}}{62{\cdot}5}$$

when the weight of water is taken as 62·5 lbs. per cube foot, and z is the height in feet of the point considered above a fixed plane.

This equation can be proved mathematically for a perfect fluid moving under gravity. The proof extends to fluid motion, whether vortices exist or not, provided that the motion is steady, *i.e.* is the same at a fixed point at all instants of time, and that points in the same stream line alone are considered. It is somewhat doubtful whether a rigid proof can be given for such motion as usually takes place in cases considered in practical hydraulics. This doubt, however, is not a matter of great practical importance, as the form actually employed in hydraulics is:

$$h+z+\frac{v^2}{2g}+\text{losses by friction, etc.}=\text{a constant.}$$

These "losses by friction," etc. are calculated from the results of actual experiments under the assumption that Bernouilli's equation in the above corrected form holds during the motion.

The general form of the equation can consequently be regarded as valid, and we may proceed to investigate its practical applications.

Losses by Friction, etc.—The friction losses (*i.e.* those which occur when the velocity remains unchanged both in magnitude and direction) are usually assumed to be proportional to v^2, although there is ample experimental evidence to show that this assumption is only true under exceptional circumstances. The correct law is that the losses by friction (excluding those in tubes where v is less than the critical value) vary as v_n, where n ranges from about 1·7 to 2·1 according to the smoothness of the sides of the channel and its size, the lower values corresponding to small smooth channels, and the higher to rough channels, and probably also to large channels, whether smooth or rough.

For losses by shock (*i.e.* more or less sudden changes in velocity, or in the direction of velocity) the evidence that the losses vary as v^2 is far more conclusive; although, in certain cases (usually concerned more with a change in the direction of the velocity than with a change in its magnitude), the divergence from the v^2 law is greater than the possible errors in the measurements.

Practical Equation.—For practical convenience, however, it is usual to express the motion of water from one cross-section of the channel (defined by the suffix 1) to another cross-section lower down the channel (defined by the suffix 2) by an equation of the following form:

$$h_1+z_1+\frac{v_1^2}{2g}-\left(h_2+z_2+\frac{v_2^2}{2g}\right)=(\zeta_f+\zeta_o+\zeta_d)\frac{v^2}{2g}$$

where v is a velocity which is some fraction of v_1 or v_2, and is determined by the geometrical form of the channel between the cross-sections denoted by suffixes 1 and 2.

ζ_f is a coefficient determining the loss by friction during the intermediate motion.

ζ_o is a coefficient determining the loss by obstructions, or alterations of cross-sections during the intermediate motion.

ζ_d is a coefficient determining the loss caused by changes in the direction of the velocity during the intermediate motion.

Now, these three coefficients depend upon the hydraulic character of the boundaries of the channel, and also upon such geometrical quantities as the length and hydraulic mean radius of the channel between 1 and 2, the angle of deflection and radius of the bends, and the magnitude and position of the various alterations in its cross-section. The most important question is: How closely do these losses follow the law of proportionality to v^2? Assuming that the channel is completely defined in its geometrical and physical condition, we have to investigate the variations in the values of ζ, which depend solely upon variations in the quantity of water passing along the channel. That is to say, the question becomes: How does each ζ vary as v or v_1 varies?

We find experimentally that ζ_f is usually a variable coefficient, depending on the magnitude of v_1 or v_2, and this portion of the question is fully discussed later under the headings Pipes and Open Channels (see pp. 422 and 469).

ζ_o is usually fairly constant compared with ζ_f and the available information is given under the heading Contractions and Enlargements (see p. 796).

As regards ζ_d, we have little definite information. The older experimenters considered it to be constant, but the bulk of later evidence tends to show that it varies quite as much as ζ_f and probably follows the same laws.

Engineers are also accustomed to write:

$$v_1A_1=vA=v_2A_2$$

where A_1 and A_2 are the areas of the cross-sections of the channel at the points 1 and 2, and A is any intermediate area, and v is the corresponding velocity, and to express v and v_2 in terms of v_1 by these relations.

When such a substitution is made in the above equation we really assume that the sum of the kinetic energies (*vis vivæ*) of all the particles of water (which at a given instant are found in any cross-section) is equal to the energy of a quantity of water equal to that which passes through this cross-section in unit time, moving with a velocity equal to the mean velocity of the water across this cross-section. For example, if $v+\eta$, be the actual velocity occurring over a small element da of the area A of the cross-section, we assume that:

$$v\mathrm{A}v^2 = \int(v+\eta)^3 da$$

provided that,

$$\int\eta\ da = 0,$$

so that v is the mean velocity over the whole cross-section A.

The discussion already given concerning the irregularities of velocity will suffice to show that if we consider the momentary values of the quantity $v+\eta$ this assumption is untrue. If, however, $v+\eta$ represents a mean local velocity, so that η represents the difference between the mean local velocity at the point considered and the mean velocity over the whole cross-section, the conditions are not only more favourable, but also more closely represent the quantities we actually observe in practice. We have:

$$\int(v+\eta)^3 da = v\mathrm{A}v^2 + 3v\int\eta^2 da + \int\eta^3 da.$$

Now, $v\int\eta^2 da$ is always positive, and $\int\eta^3 da$ may be either positive or negative.

The matter has been experimentally investigated by Darcy and Bazin and others. The results are conflicting; but, as a general rule, we may assume that $\int(v+\eta)^3 da = \mathrm{A}v \cdot v^2(1+a)$, where a is positive, and is usually not far removed from 0·06. For definiteness we may call a the coefficient of local distribution.

We thus see that if v_1 and v_2 differ, the usual assumption may introduce an error in the values deduced for the losses equal to about 6 per cent. of the head corresponding to the change in the mean velocity.

In the experiments above referred to the flow was not markedly obstructed, and the losses corresponding to the terms $\zeta_0 v^2$, and $\zeta_d v^2$ were small. When these terms are large, the values of a are quite unknown, but it is possible that not only does a markedly exceed 0·06, but that it varies from cross-section to cross-section.

The errors thus shown to be possible are not usually allowed for, and since measurements are usually taken at sections where v_1 and v_2 are fairly equal, they may not be very great unless the coefficient a varies considerably. Bazin's values of a are tabulated on page 481.

Summing up the available evidence, it would appear that the present methods of calculating frictional and other losses are liable to certain errors due to the assumption that the energy corresponding to the mean velocity properly represents the whole velocity energy of the fluid. These errors may amount to 6 per cent. (possibly more) of the velocity head, and wherever the nature of the motion is considerably changed errors of this character may be suspected.

For instance, let us consider the heads lost at curves, bends, or elbows in pipes. The present experimental values are widely divergent, although each experimenter's results agree very fairly well *inter se*, and the laws may be considered as entirely unknown. Actual errors (in the ordinary sense) in experimenting may be considered as unlikely, but no experimenter has as yet (with a few exceptions due to Saph and Schroder, *Trans. Am. Soc. of C.E.*, vol. xlvii. p. 301) investigated the distribution of the velocities over the cross-section of the pipe before and after passing the bend. The losses of head observed are (in pipes of large diameter, at any rate) only small fractions of the velocity head. We may conclude that what has been observed is not so much the curve resistance typified by $\zeta_d v^2$ as a combination of this and the terms :

$$a_1\frac{v^2_1}{2g} - a_2\frac{v^2_2}{2g},$$

where owing to the disturbance produced by the curve, a_1 and a_2 are probably not the same, so that even if $v_1 = v_2$ we have a possible error of :—

$$\frac{v^2_1}{2g}(a_1 - a_2).$$

This quantity may be either positive or negative, and there is evidence to show that it may amount to as much as $0{\cdot}15\,\frac{v^2_1}{2g}$, although it is believed that so large a value can only be obtained when special care is taken to produce conditions favouring irregular motion.

In some cases experimenters have endeavoured to correct for this source of error by observing the loss of head at four points, two above the curve and two below, and considered ζ_d as correctly determined when, the points being equidistant, it was found that :

Loss from 1 to 2 = Loss from 3 to 4.

And then assumed—

$\zeta_d v^2$ = Loss from 2 to 3 − Loss from 1 to 2 (or 3 to 4).

It will, however, be obvious that all that is really proved is that the value of the quantity $a_1 - a_2$ above the curve is equal to that of the similar quantity $a_3 - a_4$ below the curve, and that a_3 does not differ so markedly from a_1 as to materially influence the friction losses in the straight lengths of pipe. These facts do not at all prove that a_2 and a_3 are equal, or that the disturbances produced by the curve have died out. The assumption that $a_2 = a_3$ is probably correct in small pipes, provided that the length 12, or 34, exceeds 100 diameters. In large pipes (say over 12 inches in diameter) we have no knowledge which enables us to say how great a length of straight pipe is required in order to wipe out the curve disturbances. The impression left on my mind is that, unless the character of the motion above and below the curve is fixed in some manner, errors of the character now discussed do occur, and, being constant, are not disclosed by mean square calculations. As a practical matter, where the observations are applied in order to calculate losses in pipes of the same size as those experimented on, the results are probably worthy of confidence. If we endeavour to extend the formulæ thus obtained outside the limits of existing observations we are liable to very great errors, and I regard all calculations concerning curve losses in pipes 12 inches or more in diameter as waste of time and paper.

The question is referred to under the heading of Backwater Calculations, and it may be stated that a comparison of observed and calculated backwater curves leads to similar results. The practical importance of the matter is best illustrated by a case which actually occurs on the Upper Swat Canal (Punjab). Here $v_1 = 3{\cdot}1$ feet per second, $v_2 = 14$ feet per second, and the change occurs in a length of 70 feet. $\frac{v_2^2 - v_1^2}{2g} = 2{\cdot}89$ feet, and all calculations regarding the drop in the water surface required to produce this change in velocity are plainly uncertain to about $0{\cdot}06 \times 2{\cdot}89$ feet $= 0{\cdot}18$ foot. The question is complicated by the fact that v_1 occurs in an earth channel (Bazin's $\gamma = 1{\cdot}6$ about), and v_2 in a masonry channel (Bazin's $\gamma = 0{\cdot}7$ about).

My final views on the question were arrived at after studying individual rod float velocity observations in similar channels, and I believe that the actual drop in the water surface, after all allowances for friction in the intermediate length have been made, will be found to be $3{\cdot}15 \pm 0{\cdot}10$ feet. The value calculated by the designer, after very careful investigation, is 3·40 feet. Since the various coefficients other than a_1 and a_2 are very accurately known, and were used in both calculations, the difference in opinion regarding what is theoretically a fairly simple case admirably illustrates the real uncertainties affecting the work. The rod float observations were only employed since current meter (point) velocity observations were not available.

General Laws of Resistance to the Motion of Fluids.—The object of the present section is to discuss the changes in the mathematical laws expressing the physical characteristics of the resistance to fluid motion that are produced by variations in the velocity of the fluid.

The matter has only been fully investigated in connection with the motion of fluids in pipes, and the distinction between the laws of motion in a capillary tube and those holding for a pipe of ordinary dimensions are discussed on page 20.

It will, however, be shown that there are well-marked indications that similar changes in the mathematical expression of the laws of resistance occur in other cases of fluid motion.

It may at once be stated that the whole question is at present of but little practical importance, and were it not for the fact that engineers are accustomed to apply the results of small scale laboratory experiments to the calculation of the dimensions of large works it could be entirely ignored.

It is, nevertheless, necessary to discuss the general laws, and to make allowance for the results thus obtained when small scale laboratory experiments (especially those relating to bends and constrictions in pipes) are used, in order to deduce rules for practical calculations.

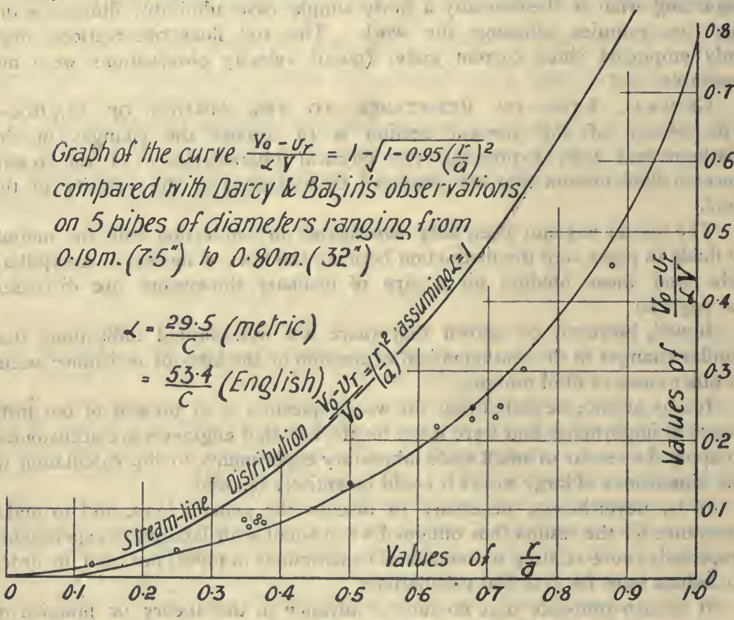
It is also probable that no radical advance in the theory or practice of experimental hydraulics will be made until similar questions concerning the motion of water in open channels, and over sharp edges (as in weirs or orifices), are thoroughly investigated.

Broadly speaking, the whole matter is included in the question of stream line and turbulent motion. The ordinary methods of mathematical hydrodynamics lead to solutions of problems regarding the movement of fluids which are characterised by the existence of stream lines. Physically regarded, in a steady stream line motion, if at a time t, we find a particle of water at a point P, and later on at a time $t + T$, we find this same particle at Q, we can rest assured that the particle now at P, will arrive at Q, at a time $t + 2T$. The water particles, in fact, move through the channel in regular order, just as soldiers in files, where no man quits the ranks. In turbulent motion, on the other hand, this regularity does not exist, and the motion may be compared to that of a disorderly crowd, where there are no regular files and where each man leaves his position as he chooses.

In a straight circular pipe of uniform section the stream line theory shows that the particles of water move with velocities which entirely depend upon their distance from the sides of the pipe, and are parallel to the axis of the pipe. Pursuing the simile of a regiment, the files close to the sides of the pipe move slowly and the central files very much more rapidly, the graduation being quite regular and represented by the equation:

$$v_r = V_0\left\{1-\left(\frac{r}{a}\right)^2\right\}$$

Consequently, V_0, the maximum (central) velocity, is twice the mean velocity, it being assumed that there is no "slip" at the walls of the pipe, *i.e.*



SKETCH No. 3.—Relations between the Mean Local Velocities over the Cross-sections of Pipe.

$v_a = 0$, where a, is the radius of the pipe and v_r, is the velocity of a particle of water at a distance r, from the axis of the pipe (Sketch No. 3).

Now, in turbulent motion the facts are quite different. The velocities are no doubt greater near the centre of a pipe than close to its sides, but a particle which at one moment is near the walls, an instant later is close to the axis, and *vice versa.*

Each individual particle therefore moves not only parallel to the axis of the pipe, but can also move in a plane perpendicular to the axis. These minor velocities perpendicular to the axis are entirely accidental in their nature, cannot be calculated, and have not as yet been studied observationally with any degree of accuracy. In consequence, the following discussion is solely devoted to a consideration of the velocities parallel to the axis of the pipe.

The law of the velocities parallel to the axis of the pipe is approximately (see Bazin's discussion, *Trans. Am. Soc. of C.E.*, vol. 47, p. 258, or *Mem. Sav. Étrangers*, tome 32. p. 258):

$$V_o - v_r = aV\left\{1 - \sqrt{1 - \beta\left(\frac{r}{a}\right)^2}\right\}$$

where V_o, is the central and maximum velocity, V, the mean velocity, and v_r, the velocity at a distance r, from the centre, in a pipe of radius a, whilst α and β are constants, where α is about $\frac{53{\cdot}4}{C}$ for English measure, and where C is the constant in the equation $V = C\sqrt{\frac{as}{2}}$ and β is 0·95.

When the velocities are mean local velocities, defined as on page 12, the agreement with observation is very close, as is shown by Bazin's diagram (Sketch No. 3). But it must always be realised that each particle in addition possesses irregular and constantly changing velocities, in all directions in space, which can be considered as super-imposed on v_r, as given by the above equation.

Still following out the military analogy, it is plain that if we clothe one file of soldiers in a different uniform, this file will preserve its individuality, and will persist as a coloured streak; while if we colour each individual of the crowd that passes a fixed point, the coloured units will rapidly become mixed up with the remainder of the crowd, and will finally be equally distributed throughout the mass.

The actual conditions determining whether the motion of water in a pipe is stream line or turbulent were first systematically investigated by Osborne Reynolds (*Phil. Trans.*, 1883), who introduced colouring matter into water by means of a fine tube.

I give Reynolds' results with the remark that they cannot be considered as exact, since Coker (*Proc. Roy. Soc.*, vol. 174) and other experimenters have found that his figures are not absolutely accurate. The causes of these divergencies are obscure, but can probably be explained by the previous history of the motion of the water.

Reynolds found as follows:

(i) If the mean velocity of water does not exceed

$$v_c = \frac{{\cdot}0388P}{D}$$

where D, is the diameter of the pipe in feet, and

$$P = \frac{1}{1 + {\cdot}0337T + {\cdot}000221T^2} \quad . \quad . \quad . \text{ (Poisseuille's ratio)}$$

where T, is the temperature of the water in degrees Centigrade, the movement is always stream line in character; and even if eddies (*i.e.* turbulent motion) are artificially produced, they rapidly disappear, and the motion again becomes stream line. The motion being regular and ordered, the resistance is but small, and varies as v, *i.e.* $v = k_1 \frac{h}{l}$ where h is the head in feet lost in a pipe l feet long.

The value of k_1 may be expressed by

$$k_1 = \frac{cd^2}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Inches]}$$

where Reynolds gives $c=361$, when d is in inches and P is the temperature correction given above.

Hazen (*Trans. Am. Soc. of C.E.*, vol. 51, p. 317) uses the formula :

$$k_1=c_1 d^2 \frac{(t+10)}{60} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Inches]}$$

where t is in degrees Fahr. ; so that, roughly, $c_1=c\times 1{\cdot}339$, or $\frac{4c}{3}$. Hazen also obtains for brass pipes :

with $d=0{\cdot}107$ to $0{\cdot}631$ inch, $c_1=462$ to 584, or $c=347$ to 438
Average, 495 Average, 372

and for another series of Hazen's, with $d=0{\cdot}11$ inch, average $c_1=450$, or $c=338$.

It may incidentally be remarked that if we apply the mean value in Hazen's formula for percolation through sand (p. 25) we find that the effective size of the grains of a layer of sand is about 3·0 times the mean diameter of the capillary passages through the layer, assuming that their length is equal to the thickness of the sand.

(ii) Above this first critical velocity we have a transition stage where stream line motion can exist, but if turbulent motion is artificially produced it may continue.

Thus, in any given case, the occurrence of stream line, or turbulent motion, is a more or less accidental matter. The resistance is therefore also fortuitous, and no rule can be given.

The transition stage ends when :

$$v_d=\frac{0{\cdot}2458\text{P}}{\text{D}} \text{ feet per second. D being expressed in feet.}$$

(iii) Above this velocity the motion is always turbulent. (Although Coker, *ut supra*, has procured stream line motion at velocities 50 per cent. above those given by Reynolds' formula.)

All velocities usually occurring in practical engineering fall into this class, and we have :

$$\frac{h}{l}=kv^n$$

where n lies between 1·70 and 2·10.

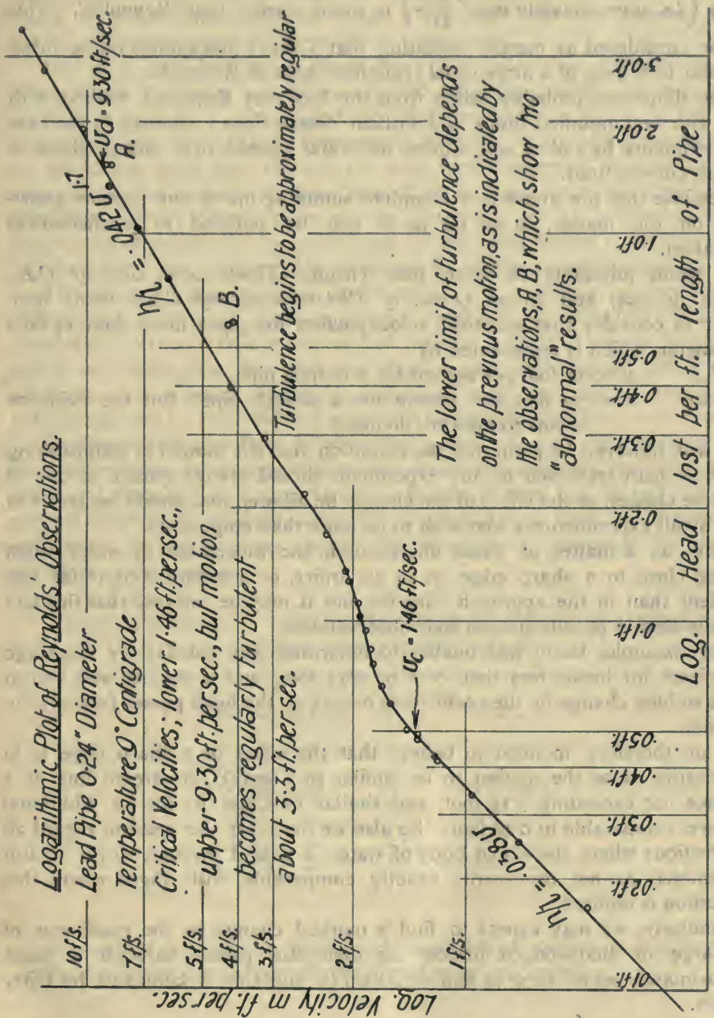
The motion being irregular and disorderly, the resistance is usually increased, and, as Unwin (*Hydraulics*, p. 39) states :

"Take a pipe of 12 inches diameter, with a virtual slope of 1 in 1000. If in such a pipe non-sinuous (*i.e.* stream line) motion were possible, the velocity would be 72 feet per second. But the actual velocity, the motion being turbulent, is only $1\frac{1}{4}$ foot per second."

Actual values of the critical velocities at 0 degrees Cent. are as follows :

Diameter of Pipe {	½ 0·0417	1 0·0833	1½ 0·125	2 inches. 0·1667 feet.
First critical velocity, v_c . .	0·928	0·465	0·310	0·232 feet per second.
Second critical velocity, v_d .	5·90	2·95	1·97	1·47 ,,

Sketch No. 4, a logarithmic plot of h/l, and v, taken from Reynolds' experiments on a lead pipe 0·242 inch in diameter, shows the conditions. I have added v_c, and v_d, as given by equations No. (i), in order to show the theoretical turning-points more closely.



SKETCH No. 4.—Losses of Head in a Pipe at Velocities below and near to the Critical Velocities.

As already stated, for practical calculations of pipe sizes, motion of class (iii) alone is of importance. If, however, we consider such small scale experiments as Fliegner's (*Civil Ingénieur*, 1875, p. 98) on constrictions in pipes, we find in many cases that the velocities at some point of the pipe fall below v_c or v_d.

Such experiments should be rejected, as they do not fulfil the conditions existing in the larger scale examples to which engineers usually wish to apply the results obtained. It is perhaps permissible to state that, so far as can be gathered from a study of such cases, we may assume that Coker's value for v_a $\left(i.e. \text{ approximately } v_a = \frac{0{\cdot}36P}{D}\right)$ is more correct than Reynolds'. This may be considered as merely indicating that Coker's precautions for securing a regular flow were of a more usual type than those of Reynolds.

The difference probably arises from the fact that Reynolds worked with pipes with bell-mouthed inlets and without obstructions; whereas, in the case of experiments by Coker and others, the water passed over sharp edges or through constrictions.

I believe that the above is a complete summing up of our present knowledge on the matter, in so far as it can be reduced to mathematical calculation.

It seems advisable to state that Thrupp (*Trans. Am. Soc. of C.E.*, vol. 47. p. 234) and Bilton (*Proc. of Victorian Soc. of C.E.*, 1908) both appear to consider that a critical velocity exists for pipes more than 1½ inch in diameter, which is represented by:

$v = 0{\cdot}14$ foot per second for a 12-inch pipe

rising to $v = 0{\cdot}70$ foot per second for a 30-inch pipe. But the evidence seems to me very doubtful.

It will, however, be plain that the condition that the motion of water during the whole path traversed in any experiment should always remain in one of the three classes, or the effect of the change be allowed for, should be borne in mind by all experimenters who wish to be more than empiricists.

Now, as a matter of visual observation, the movement of water when passing close to a sharp edge, as in an orifice or standard weir, is far less turbulent than in the approach channel, and it may be inferred that this fact explains certain peculiarities in weir observations.

For example, Bazin was unable to determine any satisfactory discharge coefficients for heads less than 0·16 or 0·15 foot, and it is fairly well known that a sudden change in the coefficients occurs as the head passes from 0·35 to 0·45 foot.

I am therefore inclined to believe that the effect of a sharp edge is to temporarily cause the motion to be similar to Class (i), or stream line at a distance not exceeding 0·15 foot, and similar to Class (ii) for an additional distance comparable to 0·40 foot. So also we must, for the present, regard all observations where the whole body of water is passed through sieves (to still oscillations) as not necessarily exactly comparable with those where this precaution is omitted.

Similarly, we may expect to find a marked change in the coefficient of discharge of sharp-edged orifices as their size passes through a value approximately equal to 0·30 foot ($=2 \times 0{\cdot}15$), and this is known to be fairly correct.

It may therefore be hoped that, as knowledge accumulates, we shall be able to distinguish between other hydraulic observations in a manner similar to that in which Reynolds has (approximately at any rate) classified pipe observations.

Fortunately, the question is more of theoretical than practical interest, since the differences in cases other than pipes appear to be of the order of 2 to 3 per cent. only. In this connection it seems advisable to mention Thrupp's pioneer work on open channels. Here, according to Thrupp, the surface slope, and not the dimensions of the channel, is of importance (*P.I.C.E.*, vol. 171, p. 346). Speaking from the point of view of pure theory, this is somewhat improbable, but there is no doubt that some change in the law of resistance occurs at low velocities (see p. 477).

Similarly, consideration of these principles makes the otherwise peculiar observations of Bazin on depressed and adhering nappes somewhat less puzzling. Here it would appear that the form of the nappe is far more dependent upon the pressure observed as existing on the sill of the weir than on the actual head over the sill, *i.e.* on a factor observed at a point where presumably the change in the class of motion has occurred (see p. 124). Hamilton Smith's observations on the form of a jet as influenced by the irregularity of supply, *i.e.* the turbulence of the water before it reaches the jet (*Hydraulics*, p. 51), are probably explained by similar considerations.

The foregoing may possibly raise visions of a hydraulic engineer loaded with colour tubes and pressure gauges, which is as alien to the present generation as the current meter was to the last. But this cannot be avoided, and if we realise that the ultimate effect is the abolition of tables of coefficients, and personal judgment, combined with economy in construction, the change may be regarded not only with equanimity, but even with favour.

Capillary Motion or Percolation.—The general laws of the motion of water in capillary tubes have already been discussed, since capillary motion is merely motion at velocities less than Osborne Reynolds' first critical velocity. The general subject of capillary tubes has a certain importance in observational hydraulics, since a pressure gauge may be rendered erroneous by capillary elevation or depression.

CORRECTION FOR CAPILLARY ELEVATION IN A MERCURY GLASS PRESSURE GAUGE.

Diameter of Tube.	Add.
0·08 inch.	0·18 inch.
0·16 „	0·08 „
0·32 „	0·027 „
0·40 „	0·016 „

In practical engineering, however, the question of motion in capillary tubes arises only in the case of percolation through sand or gravel.

Formulæ.

Poisseuille's Ratio.

$$P=\frac{1}{1+0·0337T+0·000221T^2},$$ where T, is in **degrees Cent.**

Or, if t is expressed in **degrees Fahr.,**

$$f=\frac{1}{P}=1+0·0187(t-32)+0·0000682(t-32)^2=0·4736+0·0143t+0·0000682t^2$$

Hazen's approximate form $f=\frac{t+10}{60}$

Critical Velocities.

First, $v_c=\frac{0·0388P}{D}$ Second, $v_a=\frac{0·2458P}{D}$

Resistance Formulæ for Velocities less than v_c.

$v=52100fD^2\frac{h}{l}$; D, in feet, and $f=1$ at 32 degrees Fahr.

$v=360$ to $370\frac{d^2}{P}\frac{h}{l}$; **d, in inches.** P = 1 at 32 degrees Fahr. or 0 degree Cent.

$=480$ to $490\frac{(t+10)d^2}{60}\frac{h}{l}$; **d, in inches,** at t, degrees Fahr.

Hazen's Values for Sand.

$v=210d^2\frac{h}{l}\frac{(t+10)}{60}$ for sand of an effective size equal to **d, hundredths of an inch,** where v, is the effective velocity (see p. 25) in **feet per day.**

$v=3280d^2\frac{h}{l}\frac{(t+10)}{60}$ if **d, is in millimetres.**

Percolation of Water through Sand or Gravel.—The passage of water through a layer of sand or fine gravel is effected by capillary flow through the small irregular tubes that are formed by the void spaces in the sand. It is therefore necessary to consider the laws of capillary flow.

The capillary motion of water is essentially the motion of water through pipes at velocities, which are less than the critical velocity.

Let h, be the pressure in feet of water producing such a flow through a pipe l_1, feet long, and d_1, feet in diameter. The velocity of the water in feet per second is given by:

$$v_1=52,100fd_1^2\frac{h}{l_1}\text{ feet per second.}$$

$$\text{where }f=1+0·02(t-32)+0·00007(t-32)^2$$

where t, is the temperature in degrees Fahr.; v_1, must not exceed

$$\frac{0·039}{fd_1}\text{ feet}$$

per second, or the flow may cease to be capillary (see p. 20).

Now, in sand or gravel the length l_1, cannot be measured, but, on the average, it is a certain multiple of l, the length of the path of percolation through the sand. Similarly, d_1, cannot be measured, but bears, on the average, a certain ratio to the mean diameter of the grains of sand. Consequently, the expression of the quantities contained in the above formula in terms of quantities which are easily measurable, can only be effected by certain assumptions concerning the average values of the ratios $\frac{l}{l_1}$, and $\frac{d}{d_1}$, where l, and d, represent the length of the path through which percolation occurs, and d, represents some measurable quantity (say, the mean diameter of the sand grains), which is proportional to d_1, the average diameter of the interstitial passages through the sand.

The percolation properties of sand or gravel are best defined by the quantities known as the effective size and the uniformity coefficient. The sand can be separated into grades by sifting through sieves, and if the sizes of the holes in the successive sieves are sufficiently close together the diameters of all the particles in a grade will be approximately equal.

The effective size is defined as the mean diameter of a grain such that 10 per cent. (by weight) of the sand is composed of smaller particles, and 90 per cent. of larger particles.

The uniformity coefficient is the ratio which the mean diameter of a grain such that 60 per cent. (by weight) of the sand is composed of smaller particles bears to the effective size of the sand (see Sketch No. 264, p. 962).

It is plain that the effective size is an approximate measure of the mean size of the smaller grains of the sand, and that the uniformity coefficient is an indication of the ratio between the sizes of the larger and smaller particles of the sand.

It is also plain that these two quantities cannot be regarded as rigidly specifying the properties of the sand, and that their practical importance is solely due to the fact that sands and gravels as they occur in Nature are approximately similar substances, so that a coarse sand may be regarded either as a magnified small sand, or as a fine gravel on a diminished scale.

Subject to these remarks, Hazen (*Filtration of Water*) has found experimentally that the effective velocity of percolation is given by:

$$v = Cd^2 \frac{(t+10)}{60} \frac{h}{l}$$

where v, is the equivalent velocity at which the water passes through the sand, i.e. v, is not the velocity of the water in the pores of the sand (which is denoted by v_1), but is the velocity of a solid column of water of the same area as that through which the percolation occurs which would deliver the quantity of water which actually percolates through the sand.

l, is the length of the path along which the percolation occurs, and h, is the head producing percolation, measured in feet.

d, is the effective size of the sand.

The expression $\frac{t+10}{60}$, is an approximate representation of the factor denoted by f, a theoretically more accurate expression for which has already been given (see p. 24).

C, is a constant depending on the units employed.

If v, is expressed in feet per 24 hours, and d in millimetres, C = 3280.

If d, be expressed in hundredths of an inch, C = 210.

v, could also be expressed in feet per second, but C would then become an inconveniently small fraction.

The equation is subject to exceptions. For example, if p be the percentage of voids in the sand, it is plain that $v = 100\frac{v_1}{p}$, where v_1 depends on d and $\frac{h}{l}$. Now, on page 970 figures are given which show that the form of the sand grains may cause p to vary from 25·6 to 34·6. Thus we may infer that the general form of the grains alone may cause v to vary as much as 20 per cent. either way. In practice, however, the equation is found to apply with fair accuracy to sands occurring in Nature, in which d lies between 0·10 mm. (0·004 inch) and 3·00 mm. (0·12 inch), and with a uniformity coefficient less than 5. The equation also applies with equal accuracy in gravels up to 5 or 7 mm. (0·20 to 0·28 inch) effective size, so long as $v_1 = 100\frac{v}{p}$ is not too large. In this last case the limit at which the equation ceases to hold is fairly accurately ascertained by estimating the critical velocity, v_c, in a tube of a diameter equal to $\frac{4d}{7}$. (See p. 19.)

In most cases we can take $t = 50$ degrees, and then the bracketed expression becomes unity.

We can also express this formula in terms of d_m, the mean diameter of the sand grains, with a very small degree of error, by changing the value of C in the ratio $\left(\frac{d}{d_m}\right)^2$ in ordinary sand. Since $d_m = d\sqrt{3}$, we have $C_m = \frac{C}{3}$, and actual values as given by Seelheim, Hazen, and Kröber, are as follows:

d_m, in millimetres:	0·16	0·23	0·28	0·48	0·54	0·68	0·70	0·90	1·35	2·1
C_m, in feet per 24 hours:	1060	1047	1016	1076	1205	1063	1158	1395	1030	1165

so that the above relation is quite close to the truth.

All these experiments were conducted on clean sand, such as is used in filters. The following results show the influence of a small quantity of clayey or dirty matter in diminishing percolation (*Trans. Am. Soc. of C.E.*, vol. 48, p. 302).

The experiments being recorded in terms of the effective size, we get:

d in Millimetres.	Value of v in Feet per Day as ascertained experimentally when $\frac{h}{l} = 1$.	Value for v for Sand of same effective Size according to Hazen's Rule.	Percentage of Flow with dirty instead of clean Sand.
0·55	758	991	76
0·46	154	695	22
0·45	30	663	5
0·45	92	663	14
0·40	131	525	25
0·38	49	472	10
0·37	36	449	8

Although the information is incomplete, it seems as well to record the values of Cd^2, or $C_m d_m^2$, obtained when $\frac{h}{l} = 0·036$, in the alluvial deposits at:

Lyons.	Strassburg.	Gladbach.	Augsburg.	Vienna.	Bucharest.
545	1511	701	1180	273	403

The value of d, not being given, C cannot be calculated.

These values are obtained on strata of a markedly water-bearing character, and should consequently be considered as maxima, and as unlikely to occur in strata suitable for dam foundations.

The Hazen formula has been very severely criticised of late years by many experimenters, especially by Baldwin Wiseman (*P.I.C.E.*, vol. 181, p. 29). As I have based many designs on the results of the formula, I consider that the following remarks are not out of place.

The correction for temperature is probably somewhat faulty, and the Poisseuille ratio used in investigations of critical velocities in pipes is a more accurate method of allowing for the influence of temperature. I consider that Hazen's results are approximately correct for a range of 50 to 70 degrees Fahr.

Formula of the type suggested by Slichter or Baldwin Wiseman are certainly more accurate in the case of small-scale experiments. Their weak point, however, is that the work necessary in order to ascertain the coefficients is more laborious than a series of determinations of the actual flow under varying heads.

In large scale experiments the porosity, surface area per unit volume, and other qualities determined by Baldwin Wiseman vary from point to point. In view of these variations any formula more accurate than that given by Hazen is unnecessary. The following is a table of the velocities in feet per 24 hours when $t=50$ degrees Fahr.:

EFFECTIVE SIZE OF THE SAND GRAINS IN MILLIMETRES.

$\frac{h}{l}$	0·10	0·20	0·30	0·35	0·40	0·50	1·00	3·00
0·001	0·033	0·13	0·30	0·41	0·524	0·82	3·28	29·5
0·005	0·164	0·66	1·48	2·06	2·62	4·10	16·40	147·6
0·010	0·328	1·31	2·96	4·12	5·24	8·2	32·8	295·0
0·050	1·64	6·56	14·8	20·6	26·2	41·0	164·0	...
0·100	3·28	13·1	29·5	41·2	52·5	82·0	328·0	...
1·000	32·80	131·2	295·2	411·8	524·8	820·0	...	...

Hazen states that a similar law holds for the motion of water in beds of gravel (see p. 525). There is a very fair amount of experimental evidence (supported by experiments on flow in pipes) which shows that the law is really more complex, and that

$$\frac{h}{l}=av+bv^2$$

better represents the results. Fortunately, such large grained beds are not likely to form a dam foundation, so that the theory (which is obviously very complex) need not be investigated.

Additional information concerning this subject will be found under the heads of Wells (see p. 258) and Percolation under a Dam (see p. 293).

In these cases the formula employed is $v=K\frac{h}{l}$

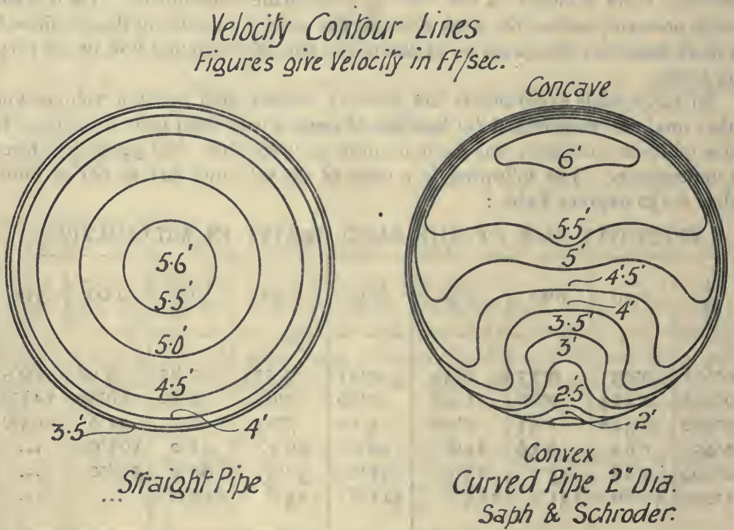
$$\text{so that } K=Cd^2\,\frac{(t+10)}{60}$$

A table of values of K, when $t=50°$ Fahr., in terms of the number of meshes per lineal inch of a sieve that retains 10 per cent. by weight of the sand (i.e. Cd^2 expressed in terms of the mesh of the sieve), is given on page 269.

CURVE RESISTANCE.—The experiments of Saph and Schroder (*Trans. Am. Soc. of C.E.*, vol. 47, p. 301), and Brightmore (*P.I.C.E.*, vol. 169, p. 315), show very clearly that curve resistance is caused by a redistribution of the velocities over the cross-section of the pipe.

The maximum velocity in the case of motion in a straight pipe is found at, or close to, the centre of the pipe. Whereas, when the water has passed through a sufficient length of curved pipe, the maximum velocity is found close to (about $\frac{2}{7}$ths of the radius away from) the concave side of the pipe.

It may be stated in general terms that the velocities in a straight pipe tend to be uniform over the cross-section, but owing to the resistance of the sides of



SKETCH No. 5.—Distribution of Velocities in Straight and Curved Pipes.

the pipe the actual distribution is as already discussed. Likewise, in a curved pipe, the velocities tend to arrange themselves as in a free vortex, where v, being the velocity at a distance x, from the centre of the circle formed by the curve of the pipe, vx=constant. Owing to the influence of the sides of the pipe the final distribution is of the form shown in Sketch No. 5 (which is an example given by Saph and Schroder).

Experimentally it would appear that the change from the form of motion appropriate to straight pipes to that occurring in curved pipes does not entail any marked loss of head. The head is lost in a gradual wiping out of the curve distribution of velocities by the friction of the sides of the straight portion of the pipe which succeeds the curve.

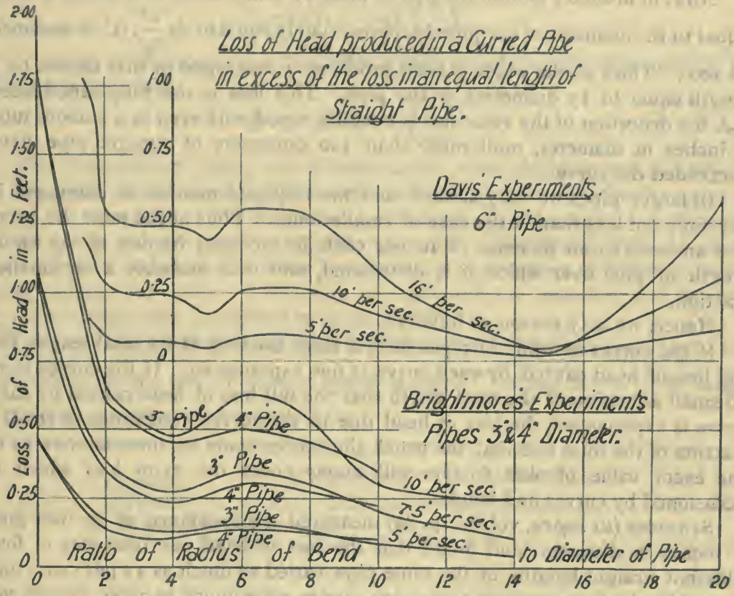
The head lost at a curve is therefore in a certain sense a species of head lost by skin friction, and is subject to the same laws. Consequently, it cannot be considered as being accurately proportional to v^2. This has been amply proved by Alexander (*P.I.C.E.*, vol. 159, p. 341), who found that for smooth wooden tubes, 1¼ inch in diameter, in which the friction loss was $\frac{h}{l}=kv^{1.777}$ the resistance of all curves was given by

$$h_d=k_1v^{1.777}$$

For a knee (*i.e.* a curve of radius = radius of the pipe), and an elbow where the loss is mostly due to shock, Alexander found that :

$$h_d=k_2v^2$$

Now, it will be evident that the problem is complicated. In the first place, the length of the curve must be considered, since it determines the amount of redistribution of velocities that takes place.



SKETCH No. 6.—Losses of Head in Curved Pipes.

Secondly, the length of straight pipe below the curve must be sufficient to eliminate this redistribution.

Thus if l be the length of the curve which is sufficient to entirely effect the redistribution of velocities, and l_1, be the length of straight pipe in which this distribution is wiped out, we see that :

(i) All curves of a length greater than l, produce the same curve resistance.

(ii) If any other curves or obstructions occur before the water has traversed a length l_1, downstream of the end of the curve, another redistribution of velocities takes place, and the observed resistance is then affected by this factor, and is therefore not completely determined by the circumstances of the original curve.

Sketch No. 6 shows Brightmore's and Davis' observations on the loss of head in curves of 90 degrees in pipes, 3, 4, and 6 inches in diameter. They are interesting as showing that the bend with least resistance has a radius of about four times the diameter of the pipe. This fact is confirmed by the experiments of Schoder and Davis (*Trans. Am. Soc. of C.E.*, vol. 62, p. 67) on pipes 6 and $2\frac{1}{16}$ inches in diameter.

A study of Schroder's paper will show the extreme complexity of the question.

I consider that the whole subject is useless from a practical point of view.

For bends of 90 degrees with radii such as are found in practical work, the loss of head caused by the bend rarely, if ever, exceeds $\frac{1}{3}\frac{v^2}{2g}$. In pipes of diameters greater than 6 inches this figure is probably never reached.

Now, in ordinary motion the loss of head by skin friction in a length of pipe equal to the diameter of the pipe is approximately equal to $\frac{1}{39}\frac{v^2}{2g}$, (C is assumed as 100). Thus, at the worst, a bend produces a loss equal to that caused by a length equal to 13 diameters of the pipe. This loss is not fully experienced (*i.e.* the distortion of the velocities is not fully wiped out), even in a smooth tube 2 inches in diameter, until more than 120 diameters of straight pipe have succeeded the curve.

In larger pipes we may assume that the requisite number of diameters is certainly not less than in the case of smaller ones. Thus at the most the curve loss amounts to an increase of 10 per cent. in the total friction of the whole length of pipe over which it is distributed, and it is probably a far smaller fraction.

Hence, we may reason as follows :

If the curves in a line of pipes form a large fraction of its total length, the full loss of head caused by each curve is not experienced. If the curves form so small a fraction of the total length that the full loss of head caused by each curve is experienced, the loss of head due to the curves constitutes so small a fraction of the total loss that the usual allowances made for uncertainties as to the exact value of skin friction will amply cover the extra loss which is occasioned by curves and bends.

Schroder (*ut supra*, vol. 62, p. 89) measured the resistance of an iron pipe 8 inches in diameter, and found that the loss of head per 1000 feet in four different straight lengths of the same pipe varied as much as 15 per cent., and that while the four portions containing curves gave more variable results, yet only one of these four lengths showed a loss of head per 1000 feet which exceeded the mean loss per 1000 feet for the four straight portions.

The only exception to the above rule is where a series of curves follow one another in reverse directions. A considerable loss of head may then occur in a short length of pipe, since the reverse curve very rapidly wipes out the distortion of the velocity which is produced by the preceding curve. Such arrangements of piping are not likely to occur in practice, except in the case of turbines, when the available installation space is small. Here, owing to the disturbances produced by the turbine, it is doubtful what influence the curves really have. A study of tests of turbines under such circumstances suggests that any curve action will be amply allowed for by assuming a loss of head equal to $0{\cdot}10\frac{v^2}{2g}$.

For elbows in small pipes the formula $h_e = 1{\cdot}17\frac{v^2}{2g}$ is given by Brightmore (*ut supra*), and probably holds good for all sizes of pipes in which elbows occur in practice.

The formulæ of Weisbach (*Die Experimental Hydraulik*) for the head lost by curves, and angular deflections in pipes, have been frequently quoted. They refer to small pipes 1¼ inch in diameter, and are as follows:

(i) For an angular deflection with a deflection angle represented by ϕ;

$$h_d = \zeta_d \frac{v^2}{2g}, \text{ where } \zeta_d = 0{\cdot}9457 \sin^2\frac{\phi}{2} + 2{\cdot}047 \sin^4\frac{\phi}{2}$$

(ii) For a curve with radius equal to ρ, in a pipe with a diameter equal to d, we have:

$$\zeta_d = 0{\cdot}131 + 1{\cdot}847\left(\frac{d}{2\rho}\right)^{3.5}$$

In a pipe of rectangular section, with a side parallel to ρ, represented by s, Weisbach finds that:

$$\zeta_d = 0{\cdot}124 + 3{\cdot}104\left(\frac{s}{2\rho}\right)^{3.5}$$

The circumstances under which these results were obtained are not known. If they were similar to those discussed when referring to Weisbach's experiments on valves (p. 781) it is probable that these formulæ are not applicable to the cases which are usually met with in practice. The results obtained when these formulæ are applied in order to calculate the resistance of bends in large pipes, confirm this view. I therefore merely quote the formulæ, and believe that they should only be applied to small pipes.

Bellasis (*Engineering*, May 26, 1911) has recently discussed all recorded experiments on the loss of head at bends in pipes. Putting $h_d = \zeta_d\frac{v^2}{2g}$, he finds the following approximate values of ζ_d:

Authority.	Diameter of Pipe in Inches = D.	Value of ζ_d when the Radius of Bend is								
		Elbow Radius =0	2·5 D	3·5 D	5 D	7 D	10 D	14 D	15 D	20 D
Weisbach	...	0·89	0·14	0·135	0·13	...	...	...	...	...
Brightmore	3	1·17	...	0·29	...	0·39	...	0·15	...	...
Schroder	6	...	0·12	0·11	...	0·14	0·08	0·25	0·015	0·14
Williams, Hubbell and Fenkell	12	...	...	0·35	...	...	...	...	...	...
	30	...	...	0·40	...	...	...	...	...	...

Having attempted to prepare a similar table, I am of the opinion that these values are most unreliable. They form, however, useful indications of the order of magnitude of h_d.

CHAPTER III

GAUGING OF STREAMS AND RIVERS

The following chapter is devoted to a consideration of the methods usually employed in the measurement of large quantities of water. Measurement by weirs or orifices will be considered separately, as these methods are generally employed for smaller volumes, and are less subject to errors caused by irregularities in the motion of the water.

The standard method of determining a quantity of water is by actual measurement of its volume or weight. Very accurate results can be obtained by this method, and all other methods are directly or indirectly tested by a comparison with a volumetric or gravimetric measurement.

The precautions required are enumerated in most text-books of physics.

From the point of view of an engineer the volumetric method alone is of importance. The most accurate results are obtained by observing the time which the stream of water which it is desired to measure takes to fill the portion of the volume of a measured vessel contained between two horizontal planes. The precautions required to accurately determine the level of the water surface are discussed under the heading Weirs.

My own experience leads me to believe that, under favourable circumstances, field observations can be secured which are subject to an error of 0·4 or 0·5 per cent. only. The main source of error is the difficulty in accurately observing the rise of the water. The level of a moving surface of water can hardly be observed with an accuracy exceeding 0·005 foot. Thus, if the water rises 1 foot during the observations, errors of 1 per cent. from this source alone are possible. Whereas, if the water rises 10 feet during the observation, this error is reduced to 0·1 per cent. The volumetric method, however, is usually only applicable to small quantities of water, and can rarely be employed in the field.

Methods of Stream Gauging.—These are as follow :

(i) *By Current Meter.*—This method is applicable to all streams, and is the only possible method in large rivers, with the doubtful exception of the chemical system.

(ii) *By Rod Floats.*—This method is especially adapted to regular canals, and is only approximate if the bed of the stream is at all irregular.

(iii) *By Surface Floats.*—This method is only approximate, but is very rapid ; and when it is occasionally, but systematically, checked by more accurate methods, is very useful.

(iv) *By Pitot Tubes.*—This method is most applicable to pipes, or to very regular channels ; but it is useful in the case of streams which are either too rapid for current meters, or where the beds of the streams are too irregular in cross-section for rod floats.

Special methods requiring more or less permanent installations of measuring apparatus are :

(v) *Gauging by Chemical Means.*
(vi) *Gauging by Weirs* (see Chap. iv.).
(vii) *By Venturi, or other Meter.*
(viii) *By a Travelling Screen.*

The best system in any case depends upon the character of the stream, and to a smaller extent upon the skill of the observer, and upon the intelligence of the available labour.

(i) Gauging by current meters requires a considerable amount of skill on the part of the observer, but little extra apparatus beyond a boat is necessary.

(ii) Gauging by rod floats demands less skill on the observer's part, but at least two men are necessary for wading, or two boats when wading is impossible. A complete set of rod floats is less portable than a current meter.

(iii) Gauging by surface floats is the simplest and quickest method, and can be carried out by a single observer ; but in order to obtain any real degree of accuracy the ratio between the surface and mean velocities must be previously determined by more accurate methods.

(iv) Gauging by Pitot tubes is extremely accurate, and the method can be employed under conditions where a current meter would be destroyed, and rod floats would prove inaccurate.

As a rule, the discharge can be obtained far more rapidly by means of rod floats, or by a current meter, than by a Pitot tube. This latter instrument is only necessary when more detailed information is wanted than is required for the determination of the discharge, and is not well adapted for gauging streams the flow of which is irregular.

In very regular channels, lined with smooth masonry, or in pipes, where the geometrical distribution of the velocities over a cross-section can be accurately estimated, a fixed Pitot tube will record the momentary discharge with an accuracy which is only surpassed by a weir or Venturi meter; but the necessary preliminary studies are tedious.

The following methods require certain fixtures to be installed at each gauging site.

(v) The necessary apparatus for carrying out the chemical method is easily installed, and is portable; but the observations require special knowledge which is not usually possessed by engineers, although it can be fairly rapidly acquired.

(vi) Gauging by a weir necessitates a somewhat expensive permanent construction, and the sacrifice of a certain head, which may be a disadvantage.

Engineers are, I think, inclined to somewhat overestimate the accuracy of the weir method; and if it is applied to measure the flow of a natural stream throughout the year, difficulties occur, since, a weir which correctly measures the low-water discharge, is quite unfit for gauging the flood discharge, and *vice versa*.

It is, however, one of the few systems with pretensions to extreme accuracy, which permits the observations to be taken by an untrained man, and afterwards calculated at leisure.

(vii) The Venturi meter is only adapted for measuring volumes of water which are capable of being passed through a pipe. The apparatus is relatively more costly than either the chemical, or weir, or screen systems. The method is very accurate, and no great sacrifice of head is entailed. Further, the method adapts itself more readily than any other to continuous recording, although weirs are not far inferior in this respect.

(viii) Measurement by a screen is new, and experimental. Consequently, a definite statement should be made with caution. Nevertheless, it is promising, and has advantages which will be later discussed.

Soundings.—With the exception of chemical and weir gaugings, all methods of measuring the discharges of streams require a previous knowledge of the cross-section of the stream channel.

A cross-section of the river is obtained by soundings, and the mean local velocity perpendicular to the cross-section is observed at as many points as possible. The discharge is then calculated by multiplying each of these velocities by the area over which it is supposed to occur, so that the discharge is:

$$Q = \Sigma va$$

where v, is the observed velocity at any point, and a, is the corresponding partial area of the cross-section of the stream over which v, is assumed to represent the mean velocity.

The process for obtaining the areas is comparatively simple. The only difficulty lies in observing the depths.

So long as the depth does not exceed about 10 feet (more or less according to the velocity of the current), a sounding pole is used, which is best divided into feet and tenths (or even hundredths), and is provided with a flat base so as to prevent it from sinking into the softer parts of the bottom.

In the case of depths exceeding 10 feet, a sounding-line with a weight has generally to be employed.

The sounding-line is usually a hemp cord, graduated into feet, or fathoms, by tags of leather, or cloth, inserted between the strands of the cord. A hemp cord is very easily handled, but errors can arise owing to alterations in the length of the cord caused by soaking in the water. In really accurate work it will be found that this alteration is not uniform over the whole length of the cord, and that frequent checking is essential. In some cases this has led to the use of piano wire, in place of hemp cord. The greater accuracy thus obtained is undeniable. Wire should always be employed in swift rivers where the velocity is sufficiently high to require a heavy weight. If a hemp cord is then used, the thick rope is caught by the water, and the depth observed exceeds the truth.

In ordinary cases, however, the fact that a comparatively unskilled man can handle the cord is of great advantage; and although the varying corrections entail somewhat greater labour on the part of the observer, I believe that the cord is really the more practical method, except possibly in a very rapid stream.

The weight used for carrying the sounding-line to the bottom is a very important portion of the apparatus, especially in the case of swift streams. The most common form adopted is a frustrum of a cone, weighing about three pounds. This is easily handled, and may be used in streams with a current not exceeding 3 to 4 feet per second.

For more rapid streams, soundings made with such a weight usually over-estimate the depth, and the torpedo-shaped sinker shown in Sketch No. 7 should consequently be employed. This is by no means easy to use, but the greater accuracy obtained justifies the time expended in training the leadsman.

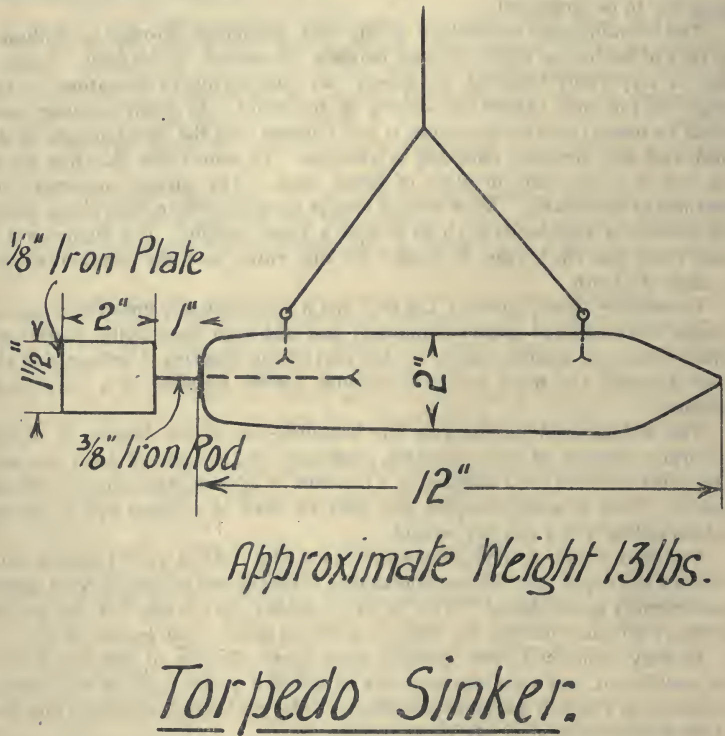
In very rapid and deep streams, even 40-lb. weights of the above form are insufficient, and some device in the nature of a pulley fixed in the bows of the observer's launch becomes necessary. The problem is a difficult one, and as yet it has not been satisfactorily solved.

In sounding a rapid river some difficulty may be experienced through the water banking up against the pole. The best method is to put the pole on the bottom, and then withdraw it until its lower end just dips in and out of the waves. The distance which the pole is raised in order to effect this, as measured against a fixed point on the bridge, or against the support from which the sounding is taken, gives the depth unaffected by any banking up of the water, due to currents.

The determination of the position of each sounding, and its correction for the rise or fall of the river during survey, is the business of a surveyor, and will not be discussed.

In small streams, it is usual to stretch a cord across from bank to bank and to determine the depth at equal intervals along the cord. The mean of three such cross-sections at intervals of say 50, or 100, feet along the course of the stream is taken as the cross-section. In large rivers, the points at which the depths are observed are usually fixed by observations taken with a theodolite, and two observers are then required.

I may, however, remark that very accurate determinations may be made even in wide rivers, by the use of a sextant, and the two angle method. Only one skilled observer is then required. In my own practice I have found that this nautical system is most convenient.



SKETCH No. 7.—Torpedo-shaped Sinker.

IRREGULARITY IN THE FLOW OF WATER.—The flow of water in Nature is always irregular, or turbulent, as defined on page 11. Such irregularities affect the indications of all instruments used to observe the velocity of the water to a greater or less degree. Thus, except in laboratory observations, where special precautions are taken to regularise the flow, it is necessary to consider the errors which may thus be introduced. The following discussion is only approximate, and the figures given are in reality only relative, since it is believed that the matter is fundamental in all hydraulic measurements, other than those of a gravimetric or volumetric nature.

If we merely consider the forward velocity (*i.e.* the component parallel to the main direction of the stream), momentary variations amounting to 20 per cent. of the mean velocity will be found at those points where the motion is steadiest; and near the bottom the variations exceed 50 per cent. (see Sketch No. 2). These figures are less than the true values, since, the inertia and friction of whatever instrument is employed to observe the velocities must tend to diminish the irregularities. It is therefore plain that the forward velocity at any given point (which in future will be termed the velocity at that point), can never be ascertained by one observation only, and its mean value must be obtained by averaging a large number of the momentary values.

This averaging can be effected mechanically, as in the case of the current meter, which indicates the mean velocity over the whole filament that passes through the vanes during its run (*i.e.* a cylinder say 3 inches in diameter, and over 100 feet long); or by means of the rod float which moves with a velocity nearly equal to the mean of the velocities existing in the column of water surrounding it (which is probably not a circular cylinder, but a more or less elongated elliptical cylinder). The mean velocity can also be obtained arithmetically, as when the average of the velocities of a certain number of surface floats is taken. The problem is, in fact, of exactly the same character as that discussed in connection with rainfall (see p. 176).

If the forward velocity at one point in the cross-section of a large stream be required, the observations of Cunningham (*Roorkee Hydraulic Experiments*) indicate that the difference between the means of the velocities of 25 and 50 consecutive floats is not likely to exceed 0·05 foot per second. Thus, at least 25 float observations, or Pitot tube readings, would be required to ascertain the velocity with an accuracy of 1 per cent. In cases where the instrument itself effects a certain amount of averaging, a smaller number of separate observations will suffice.

In discharge observations, however, the velocity at any one point is not a very important factor, and such tedious repetitions are not required. The following rules are adopted in practice:

For Current Meters.—For all practical purposes a run of three minutes gives the velocity at the point.

For Rod Floats.—The mean of the velocities of five floats, which, during their path through the length over which the velocity is observed, move in a direction which is approximately parallel to the general direction of the stream, is assumed to be the mean velocity over the depth occupied by the float.

A good deal of doubt exists in the case of surface floats. The results of some systematic observations are given on page 42.

In view of the fact that the method of surface floats requires the selection of a multiplier in order to reduce the observed velocity to the mean, the mean velocity obtained by five float observations per point is probably sufficiently accurate for practical purposes.

These rules have no very great observational basis, and in reality they represent the amount of time which experience shows can be devoted to ascertaining the velocity at any one point. They should not therefore be blindly applied. Unless some special reason exists which renders the accurate determination of the velocities desirable, it is probably better to obtain a value of the average velocity during a short period at each individual point as rapidly as possible, and to devote the time thus saved to observing the short period average velocities at more numerous points.

The matter can be best illustrated by an example. Consider a channel 60 feet wide. The usual process of measuring the discharge would be to take 10 soundings, 6 feet apart, across the canal, and to observe the depths at intermediate points wherever the 10 original soundings indicate irregularities in the bed. The mean velocities over the 10 original verticals would then be determined by one or other of the methods given later. In ordinary work, whether by current meter or by float, the velocities thus obtained would usually be averages over periods of 3 to 5 minutes; or averages over a larger area (due to the floats not running in exactly the same paths) for a somewhat shorter period, as in rod float work. If the work is accurately carried out, it is probable that the individual mean velocities obtained by repeating the work over the same verticals will differ by 5 or 6 per cent. from those previously obtained. On the whole, however, some differences being positive and others negative, the discharges calculated from the two sets of observations should not differ by more than 1 or 2 per cent. Now, the mean velocities on two consecutive verticals may differ by as much as 10 per cent. even when these means are long-period averages. It is therefore probable that better results could be attained in either set of observations by observing the mean velocities during half the period of 3 to 5 minutes over 12 verticals; since the two discharges would probably agree quite as accurately as when the velocities on 6 verticals only were observed, and the chances of missing a marked irregularity in the flow of the stream would be greatly minimised. Such practical tests as I have been able to effect confirm this view. The experiments were carried out with rod floats in fairly regular channels (Bazin's $\gamma=1{\cdot}5$, or Kütter's $n=0{\cdot}020$).

The real question is whether it is more important, to eliminate the effects of the momentary irregularities in the velocity at individual points, or to discover permanent differences between the mean velocities which prevail over adjacent portions of the cross-section of the stream (see p. 12). The matter deserves careful consideration whenever a permanent gauging station is installed; and in such cases the velocities and depths obtained in the early observations should be plotted, and the graphs should be studied for indications of sudden and permanent differences in the velocities at points close to one another.

My own studies indicate that while it is desirable to observe the point velocities with great accuracy in this preliminary work, routine gaugings are best effected by quick determinations of the velocities at as many points as possible, the points being selected so as to represent the areas in which the preliminary studies show that permanent differences are most likely to exist.

I believe that this principle is applicable to all discharge observations without exception, and there is little doubt that no engineer accustomed to gauge earthen channels or natural streams will dispute it. Exceptions may occur in laboratory work, or when observers of great skill are gauging extremely smooth and regular channels. For example, Francis (see p. 58) appears to have been satisfied with far less closely spaced rod float observations in smooth channels than I should be disposed to make.

Calibration of Instruments (General Principles).—The term calibration is used to denote the process of comparing the indications of an instrument with the true values of the quantity which it is proposed to observe.

In discussing the calibration of the instruments for observing the velocity of water, the precautions which are necessary in order to eliminate the effect of irregularities in the flow of the water as it occurs in Nature require consideration.

The indications given by any instrument are (to a greater or less degree) dependent on its construction, and on the irregularity of the flow of the water.

Calibration is usually effected by moving the instrument through still water, at a measured velocity. Thus, the calibration takes place under circumstances which are probably equivalent to a water motion which is entirely devoid of noticeable irregularities of velocity; hence, a consideration of the probable effects of the irregularity of the motion of running water in which the instruments are used is necessary.

The irregularities of velocity distribution, both as regards the momentary variations of velocity at a fixed point, and as regards the permanent differences between the mean velocities at points close to one another, increase in proportion to the size of the cross-section of the channel and the roughness of its sides and bed. It is probable that each stream has its own peculiar constitution which governs the degree and character of the irregularity of the flow.

We do not, except in very rare circumstances, wish to measure the momentary variations of the velocity. We require to ascertain the mean result (*i.e.* the mean local velocity, as defined on p. 11, or its direction) as quickly as possible, apart from variations; and all methods of calibration essentially consist in comparing the indications of the instrument with this mean value as obtained by some other process. In a well designed instrument momentary oscillations are prevented, or the indications are so damped that, as a rule, the irregularities in the quantity observed do not produce visible fluctuations in the reading, and an average result is alone indicated.

We cannot, however, ignore the fact that it is quite possible that the total effect of the irregularities may influence this average reading of the instrument.

So far, it must be confessed, our knowledge of hydraulics is not sufficiently precise to allow other than general statements to be made on the question. For instance, it is permissible to state that the present method of calibrating current meters by moving them through still water, is, theoretically speaking, defective; but the method is justified by the fact that the results obtained are sufficiently accurate for practical purposes.

As our knowledge of hydraulics advances, many of the uncertainties which at present exist will undoubtedly be recognised as caused by a lack of sufficient care in calibrating instruments under conditions similar to those existing during their practical use.

The matter is probably of most importance in the case of Pitot tubes, where we can at present state that the coefficients obtained by motion through still water may differ by as much as 5 per cent. from those obtained by observing the mean velocities at different points in the cross-section of a stream, and comparing the discharge thus obtained with the discharge determined by a weir, or volumetric observation.

This class of error is difficult to discover, for the irregularities in flow are most marked in streams of large volume, and the greatest discharge that has ever been measured volumetrically, or over an accurate weir, does not at present exceed 300 cusecs. Hence, a careful observer is quite justified in going to some extra trouble and expense in attempting to carry out the calibration observations under circumstances as closely resembling practical conditions as possible.

Thus, instruments for use in pipes should be calibrated in water moving through a pipe, or in a smooth channel (even although of a different size from those in which they are to be used), in preference to still water.

If, for lack of the necessary opportunities, calibration must be carried out in still water, it is plainly advisable to calibrate instruments intended for shallow streams in shallow water,—and *vice versa*; while any opportunity of checking an actual gauging in moving water volumetrically, or by standard weir measurements, should be taken.

A careful comparison between a current meter and a weir, or other accurate gauging of a large river, is one of the most urgent requirements of the present day.

The following figures show the errors which are likely to occur in good work when large streams, or small rivers, are gauged.

So far as is possible only the effects of irregularities in motion are now referred to, and the question of the errors caused by imperfections in the instruments themselves is discussed separately.

Current Meters.—Murphy (*Trans. Am. Soc. of C.E.*, vol. 47, p. 370) finds that 50 current meter gaugings (where the velocities were taken at one point per 2·3 square feet of cross-section) when compared with simultaneous weir gaugings, give:

Maximum difference	4·73 per cent.
Minimum difference	0·13 ,,
Mean difference	+0·93 ,,

The observed discharges ranged from 197 to 225 cusecs; and one of the meters was obviously less accurate than the others, since, if only the two most reliable instruments are considered, the mean difference is 0·74 per cent.

Allen (*ibid.*, vol. 66, p. 131), gives six observations of a similar character, as follows:

Maximum difference	3 per cent.
Minimum difference	0·5 ,,
Mean difference	Under 2 ,,

A theoretical estimate of the effects of irregularities in the absolute magnitude of the velocity can be made as follows:

Let $v=an+b$, be the rating formula of the current meter (see p. 49).

Let the water flow with a velocity equal to v_1, for t_1 seconds, and then at a velocity v_2, for t_2 seconds. The total number of turns recorded on the dial of the meter is

$$n_1t_1+n_2t_2=\frac{v_1t_1+v_2t_2}{a}-\frac{b(t_1+t_2)}{a}.$$

The true mean velocity is $\frac{v_1t_1+v_2t_2}{t_1+t_2}=v_m$ say, and the velocity deduced from the readings is :

$$v_d = \frac{n_1 t_1 + n_2 t_2}{t_1 + t_2} + b = a\left\{\frac{v_1 t_1 + v_2 t_2}{a(t_1 + t_2)} - \frac{b}{a}\right\} + b$$

$$= a\left\{\frac{v_m}{a} - \frac{b}{a}\right\} + b = v_m$$

So that, assuming the rating curve is of the above form, variations in the absolute magnitude of the velocities do not affect the results.

For rating curves of any other form, the effect of similar irregularities may be calculated for any given case ; but, since a rating curve of the form $v = an + b$, represents the results of calibration with sufficient accuracy, it appears unnecessary to go through the work.

The question is somewhat more complicated when irregularities or variations in the direction of the velocity are considered. If the axis of meter makes an angle θ with the true direction of the velocity, it records a velocity which is approximately equal to $v \cos \theta$, and for $\theta = 20$ degrees, this would be 94 per cent. of the true velocity. Actually, the meter swings to and fro with the current, and may be assumed to follow the momentary direction of the current fairly closely, since its inertia is but small. Thus, we may assume that the meter records slightly less than the average of the velocities at the point of observation (without respect to direction). As we usually wish to observe only the forward velocity (since that is the important factor in discharge observations), it is possible that the meter will record slightly more than the forward velocity. The meters used by Murphy (*ut supra*) recorded more than the weir discharge in 34 cases, with an average error of +1·94 per cent. ; and less than the weir discharge in 16 cases, with an average error of −0·91 per cent., so that, so far as these observations go, the above theory is confirmed.

As these observations were taken in a very regular channel, they are likely to be less affected by variations in the direction of velocity than those obtained by a current meter in natural streams.

The angle through which the meter swings may be considered to roughly indicate the possibilities of error. So far as I have been able to observe, swings in excess of 30 degrees, *i.e.* 15 degrees on either side, corresponding to a possible maximum error of 3·5 per cent., are unusual. It must be noted that a study of the variation of absolute velocities shows (*e.g.* the curves given in Sketch No. 2) that the greater irregularities occur near the bottom of the stream, where it is hard to see what the meter is doing.

While the above investigations appear to confirm the general accuracy of current meters, they are certainly defective in one respect, and possibly in several. They take no account of the probability of velocities varying both in magnitude and direction at different points on the vanes of the meter. If the motion of a stream which bears so little silt that individual particles are visible, is observed, the possibility of such differences is evident ; and, as already stated, we can usually only investigate the surface portion of the stream, which is known to be least subject to irregularities.

Rod Floats.—The effect of irregularities in motion is probably quite as marked on rod floats as in the case of a current meter. In actual practice, however, those observations which are most influenced by irregularities in direction are rejected, since floats the paths of which markedly diverge from the general direction of the current, are condemned, and only the results obtained by "fair runs" are recorded.

It should be noted that the percentage of floats which make fair runs is far less in large rivers than in small streams.

We may therefore consider that the automatic averaging obtained in deeper channels, where the longer rod floats are affected by a greater volume of water, does not entirely compensate for the increased irregularity of the motion of the water. The probable errors of gaugings by rod floats are tabulated on page 59.

Since, in really first class work, the probable error caused by incorrect timing of the runs, and sounding of the stream, does not greatly exceed 1 per cent., we may consider that a large fraction of the excess above 1 per cent. which occurs in the larger streams is due to the increased irregularity in the motion. The figures given are applicable to very regular canals, rather than to natural streams; so that the effect of irregularities is generally greater than is indicated in the table.

Surface Floats.—Surface floats are considerably affected by irregularities, and in the case of a large river it is often almost impossible to obtain fair runs. Such figures as 10 or 15 per cent. of fair runs only, are by no means unusually poor results.

I have usually been content with the mean of 5 fair runs; but, where time permits, I have endeavoured to secure 10 fair runs. In 92 examples (where I was able to secure 10 runs satisfactorily) I have worked out the mean surface velocity for the first 5, and also for the first 10 runs, and have expressed the difference as a percentage. As might be expected, the larger percentages are found where the total number of floats observed was least. When 5 points were selected for observation, the mean of the surface velocities as obtained from 25 floats shows a maximum difference of 6 per cent. from that obtained from 50 floats, and the mean difference is 2·1 per cent.

For 50 floats and 100 floats (*i.e.* 10 points observed), the maximum difference between the mean velocities is 4 per cent. and the mean difference is 1·6 per cent.

For the few 75 and 150 float observations which I have been able to secure, the maximum difference between the mean velocities is 7 per cent. (probably due to a slight change in the discharge of the canal), and the mean difference is 2·4 per cent.; or, if the doubtful observation is rejected, 1·8 per cent.

Pitot Tubes.—In Pitot tube work, the effect of the irregularity of motion is of primary importance; and it may be stated that calibration in still water is, owing to this cause, subject to an inaccuracy which may exceed 10 per cent., and which is rarely less than 2 to 3 per cent.

Even this statement does not fully disclose the possibilities of error when the discharge of earthen channels is observed. Working with a Pitot tube, I obtained the following results:

(*a*) Calibration in moving water by traversing a semicircular galvanised iron channel, and checking against a triangular weir, gave:

$$v = 1{\cdot}02\sqrt{2gh}$$

This calibration was the result of 7 discharge observations, and the tube was read at 12 points per square foot of area. The mean of 3 readings per point was taken as the velocity at that point.

(*b*) Calibration in a small earth channel with a bed of fine sand, checked against a Cippoletti weir, gave:

$$v = 0{\cdot}97\sqrt{2gh}$$

This was the result of 7 discharge observations, and the tube was read at 6 points (3 readings per point) per square foot of area.

(*c*) Calibration in a channel with a bed 20 feet wide, consisting of fine sand, carrying up to 60 cusecs, checked against chemical methods, and rod floats, gave:

$$v=0{\cdot}97\sqrt{2gh}$$

as the result of 18 discharge observations; the tube being read at 1 point per square foot, with 3 readings at each point.

The system of checking was very complete, the triangular weir being volumetrically verified, and the Cippoletti weir being checked against a series of triangular weirs. In the case of the chemical and rod float determinations, a portion of the discharge was diverted over the Cippoletti weir, and the difference as indicated by chemical or float methods was compared with the weir readings.

The Pitot tube was badly designed,—but by no means more so than those used by several other experimenters,—and it would appear that whenever a Pitot tube is employed to measure velocities in natural channels, very careful and systematic calibration in moving water under similar conditions must be made.

Summing up, and bearing the possible errors in timing and sounding carefully in mind, we may state as follows:

The current meter and rod float are probably not affected by the irregularities of the flow to such an extent as to materially influence the results. The present evidence suggests that the discharge obtained by a current meter is likely to be slightly greater than the truth. The rod float also, as will be seen later, usually overestimates the discharge.

The surface float is only useful when the observations at each point are systematically averaged; and, where possible, 10, rather than 5, fair runs should be obtained at each point, or, still better, the points at which the velocities are observed should be very closely spaced.

The Pitot tube is liable to appreciable errors, which can probably be diminished by careful design, but which at present cause it to be unreliable when used in any channels except those which are very regular and smooth.

The possible errors indicated above will evidently affect all measurements to much the same degree, and will only be discovered by weir or volumetric checking. When considering their importance, it should be remembered that an engineer is probably more concerned with the relative than with the absolute accuracy of his various results. Hence, a constant error will seldom lead to confusion, even if its existence is undetected.

CALCULATION OF THE DISCHARGE.—The velocity at any point has been defined on page 9.

Assuming that the observations have been taken, and that the calibration of the instrument is carried out in such a manner as to eliminate the effects of irregularities both in the absolute magnitude, and in the direction of the velocity, the quantity of water passing through a small element of the area of the cross-section of the stream is given by the equation:

$$q=va \text{ cusecs}$$

where a, is the elementary area in square feet, and v, is the velocity normal to the area in feet per second.

The total discharge of the stream is given by the equation:

$$Q=\Sigma va \text{ cusecs}$$

A study of the rate at which the observed values of v, vary from point to point will, in any given case, permit a selection to be made of the size of the partial areas typified by a, which will render this equation sufficiently correct for practical purposes. The usual practical rules are given on page 52; and, as already indicated, it is probable that in most cases too much attention is devoted to eliminating the momentary variations in v, and consequently the partial areas are larger than they should be.

If short intervals of time, such as one-tenth of a second, are considered, so that v, is a momentary velocity, and irregularities are not eliminated, the elementary discharge given by the first expression may vary between $0{\cdot}50q$ and $1{\cdot}50q$. Even in the case of such periods as 20 seconds, it is probable that q varies as much as 10 per cent., if a, be a small fraction of the total cross-section of the stream.

The equation:

$$Q = AV_m$$

where A, is the total area of the cross-section, and Q, is the total discharge of the stream, averaged over a period of sufficient length to eliminate the effect of irregularities, may be considered as defining V_m, the mean velocity of the stream. A similar equation defines v_m, the mean velocity over any portion of the whole area A.

This expression "sufficiently long," as applied to intervals of time in the above definitions, cannot be exactly defined. I believe that in small streams it may be as small as 5 seconds; while in large rivers, there is a certain amount of evidence to show that it may be as many minutes. The question, however, is not of practical importance, since no method of gauging in which the velocities are measured is so rapid as to permit of Q, being ascertained in an interval less than at least ten times the "sufficiently long period." It is also quite possible that my estimates of 5 seconds and 5 minutes are excessive.

Actual observations on the distribution of velocity, as defined on p. 11, over a cross-section, show that it varies from point to point, in accordance with more or less regular laws. The discharge of a stream may be geometrically represented by a solid with no marked irregularities, constructed on the cross-section of the stream as base, the height at each point being proportional to the velocity.

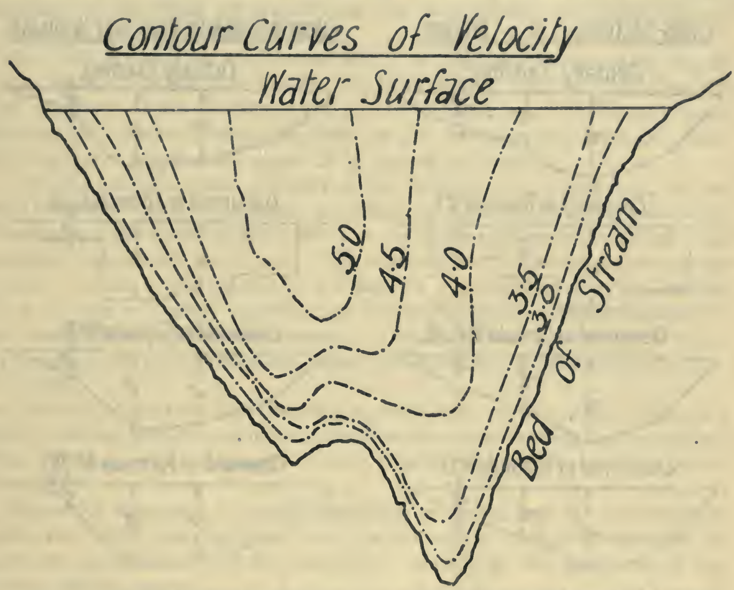
The obvious method of determining the discharge is therefore to plot curves of equal velocity at frequent intervals over the cross-section, to ascertain the areas contained by these curves, and to multiply each area by the velocity that exists across it. We thus get a series of terms, the sum of which represents the total discharge of a stream.

Such methods were actually employed by Harlacher and others, and Sketch No. 8 shows an example.

The labour entailed is obvious, and it is plain that unless the points at which the velocity observations are taken are very closely spaced, the curves of equal velocity may differ greatly from the truth. In fact, the process is almost as subject to error as contour lines, drawn by a draughtsman who had not seen the natural ground, would be.

Thus, for ordinary discharge observations, some method which is less dependent upon personal opinion must be employed. The method usually adopted in practice is that of "mean velocities over verticals." This is a convenient process, and is liable to but small errors. Theoretically, it is as accurate as the accuracy of the individual observations justify. It may, however, be stated that the same observations can be made to produce results which differ to the extent of 5 per cent., or more, by selecting one or other of the formulæ given. Thus, when working up observations the underlying assumptions must be carefully borne in mind, and the information obtained by determining the discharge from the same set of observations by all four formulæ, and comparing the results, forms a very valuable check on the methods adopted in selecting the points at which the velocities were observed. If differences much in excess of 2 per cent. exist, the points have not been sufficiently closely spaced.

The velocity observations are taken in sets arranged in vertical lines; and the mean velocity in each of these verticals is obtained either graphically or arithmetically, and is termed the mean velocity over that vertical (see p. 52).



SKETCH No. 8.—Contour Curves of Velocity in a Channel.

Assuming then a series of soundings, so that d_1, d_2, d_3, etc., are the depths of the water at a series of points spaced at a distance l feet apart along a line perpendicular to the general flow of the river, and that v_1, v_2, v_3 are the mean velocities over the verticals d_1, d_2, d_3, any one of the following formulæ may be employed:

(i) $Q = \Sigma\, d_1 v_1 l$

(ii) $Q = \Sigma\, \frac{d_1 + 6d_2 + d_3}{8} v_2 l$

(iii) Calculate $v^1 = \frac{v_1 + v_2}{2}$, and $d^1 = \frac{d_1 + d_2}{2}$:

$Q = \Sigma\, d^1 v^1 l = \frac{1}{4} \Sigma\, (v_1 + v_2)(d_1 + d_2)\, l$

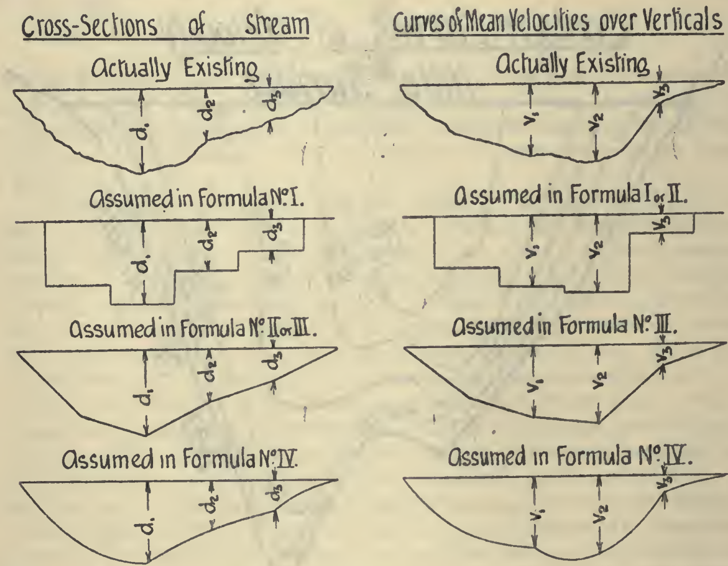
(iv) Calculate $v_n = \frac{v_1 + 4v_2 + v_3}{6}$, and $d_n = \frac{d_1 + 4d_2 + d_3}{6}$:

$Q = \Sigma\, v_n d_n 2l$

It will be plain that l, need not necessarily be constant, the only modification required being that $\frac{l_{12}+l_{23}}{2}$ is substituted for l, where l_{12} and l_{23} are the distances from d_2, to d_1, and d_3 respectively. So also, if the direction of the line of soundings is not perpendicular to the general direction of the river, we must put $l_{12}=L_{12} \sin \phi$, where L_{12}, is the distance between two consecutive points, and L_{12}, makes an angle ϕ with the direction of the flow of the water.

The formulæ are deduced under various assumptions as to the manner in which the mean velocities and depths vary across the stream (see Sketch No. 9).

In (i) we assume that the velocities and depths vary by sudden steps, the change occurring half-way between the verticals where the observations are



SKETCH No. 9.—Diagram illustrating the Assumptions made in deducing Discharge Formulæ.

taken. This assumption is no doubt open to criticism, but I believe that in practice the error introduced is inappreciable.

In (ii) we assume that the velocities vary by steps, the change occurring half-way between the verticals as before, but that the stream bed is composed of straight lines, joining the bottom of d_2, to the bottom of d_1, and d_3. I cannot see that this assumption possesses sufficient theoretical advantages to justify the additional labour entailed.

In (iii) the adjacent velocities and depths are averaged, the assumption being that both the depths and velocities vary as ordinates to straight lines. So far as theory can assist, the results are neither better nor worse than those of (i). It is, however, a very convenient method of obtaining the discharge through areas of the channel near the banks where the soundings are not equally spaced. Personally, I have been accustomed to use the equation in cases where the depths or velocities at consecutive points vary rapidly, but I do not pretend that this practice has the slightest theoretical foundation.

In (iv) the bed of the stream and the mean velocity curves are assumed to be composed of parabolic arcs; and in contrast with the other formulæ, each term of the summation expresses the discharge over a length $2l$, in place of l. This assumption agrees fairly closely with actual observations. This formula, therefore, may be regarded as standard, and in considering the labour involved in its application, notice should be taken of the fact that only half the number of multiplications occurring in the other formulæ are now required.

In any given case, a study of the observations of velocity and depth, enables us to determine the assumption that most closely coincides with the facts, and so to select the best formula.

As a rule, (i), with an application of (iii), or (iv), near any marked irregularities, leads to satisfactory results. It is as well to bear in mind that the usual observations for discharge are not so accurate as to justify any great refinement in mathematical treatment by an over-laborious formula, which may give rise to arithmetical errors of far greater magnitude.

A skilled computer will probably select formula (iv) as standard, but he must remember that systematic gaugings necessitate computations being performed on the spot, by men who are but little accustomed to other than the usual arithmetical processes. Such men can rarely do more than handle formula (i), and if other formulæ are used in important gaugings, I believe that it is best to permit the observer to select the type which appears to most closely fit the actual observations. The methods of obtaining the values of v_1, v_2, v_3, etc., are discussed under the headings Current Meters, Rod Floats, etc.

CURRENT METERS.—The essential portions of a current meter consist of a wheel, or screw, which is set in rotation when the meter is immersed in moving water, and a counting apparatus which records the number of revolutions produced.

The whole apparatus is usually mounted on pivots, and is provided with a tail vane. The rotating portion of the apparatus is thus directed so as always to be influenced by the maximum velocity at the point where the observations are made.

In modern forms of current meters many complications are introduced in order to secure sensitiveness and accuracy. The rotating parts are made hollow, and their weight is adjusted so that it is water-borne, and the recording apparatus is electrical. Thus, the bearing friction is greatly reduced. So also, shields and guides are placed so as to protect the wheel from the impact of drift; and, in some cases, to prevent oblique currents in the water from affecting the records.

It is not proposed to describe these details. As a general rule, most current meters are somewhat too sensitive for engineers' field work, and as a personal expression of opinion I am accustomed to select the simplest possible form, to calibrate it carefully, and to use it roughly, but frequently, in the field; on the principle that twenty fairly concordant gaugings are more useful than five taken with extreme accuracy. In all cases, however, it is desirable to systematically test every current meter for errors produced by oblique currents. Thus, a meter should not only be calibrated when moved straight ahead through the water, but comparative tests should be made at the same speed while moving the meter up and down, or to the right or left of its general direction. The best meter for field work is that which is least affected by such handling; for in large rivers, at any rate, these vibratory motions represent the circumstances under which the meter works far more closely than the ordinary straight ahead motion does.

The meters which are most usually employed in practice are those of Price, Haskell, and Fteley. Of these, the Fteley is probably the most accurate, and is best adapted for the registration of low velocities. Price's type permits of the registration of all velocities usually met with in river gauging, and is the pattern which is almost universally employed by the American, Egyptian, and Indian Governments. The general adoption of the Price current meter is due to the fact that it combines a sufficient degree of accuracy for practical gaugings, with adequate constructional strength to guard against damage by the unavoidable incidents of field work, provided that reasonable care is taken of it.

A current meter must be considered as an instrument liable to a fair amount of rough usage, and in the Price meter a certain degree of accuracy and sensitiveness has been sacrificed in order to obtain the necessary strength and reliability for practical operations. For laboratory work, with skilled and careful observers, the Fteley, or the Warren, are a little more accurate; but, personally, I should be reluctant to expose either to ordinary engineering handling.

As regards their adaptation to local conditions, both the Price and Haskell meter can be suspended on a weighted sounding-line, and are consequently suitable for rivers of all sizes, although the Haskell is probably somewhat better fitted for use in very large streams.

The Fteley and the Harlacher meters must be fixed to rigid rods, and are therefore only applicable to rivers that can either be waded, or are spanned by bridges, and do not greatly exceed 10 feet in depth at the bridges.

Rating.—All meters require a preliminary rating, or calibration. This is accomplished by moving them through still, or nearly still, water at a definite velocity; and is usually effected by fixing the meter in a frame, either hung from a vehicle which travels along the bank, or installed well forward of a boat or steam launch, travelling at a known speed.

In selecting the method of rating, it is as well, where possible, to be guided by the use to which the meter is to be put. For example, were I rating meters which would afterwards be employed in gauging small streams, I should prefer the system of a vehicle on the bank, since the possible retardation due to the proximity of the bank would also be present in actual work. Whereas, for meters intended for determining the discharge of large rivers, the method of a boat on a wide and deep canal is preferable.

Similarly, the general effects of irregularities in the motion of the water can be investigated as indicated on page 41; and the method of allowing for possible currents in the "still" water is dealt with on page 49.

In discussing the rating of a meter, let:

v, represent the observed velocity through still water, in feet per second.

n, represent the number of revolutions per second indicated on the meter dial or counter.

A preliminary graphical plot shows that the relation between v and n may take the forms:

$$\text{(i)}\ v=an+b$$
$$\text{(ii)}\ v^2=cn^2+d$$
$$\text{(iii)}\ v=e+fn+gn^2$$

Each of these relations may be discussed by the methods of least squares; and on the usual assumptions of that method, we can (for any individual case) select that which leads to the most accurate results by calculating the probable errors of the constants a, b, etc.

If this work is carried out, it will usually be found that formula (i) fits slightly less well than either (ii) or (iii), but in no case that I am aware of, is this difference of such a magnitude as to introduce appreciable errors.

Hence, although formula (ii), probably best represents the actual relation, I see no reason for abandoning the comparative simplicity of formula (i).

Dowson (*Measurement of Volumes discharged by the Nile in* 1905 *and* 1906) also discusses the formula:

$$v=an+b\pm u$$

where u, represents a possible current in the apparently still water used for rating. The effect is that for runs in one direction:

$$v=an+b-u$$

and for runs in the reverse direction:

$$v=an+b+u$$

and by the method of mean squares he gets for his observations:

$$u=-0{\cdot}0092 \text{ foot per second}$$
$$\text{with } a=4{\cdot}4521 \qquad b=0{\cdot}1086 \text{ foot per second}$$

so that the rating formula when applied to river gaugings is really represented by:

$$v=4{\cdot}4521n+0{\cdot}1085$$

whereas, if u were ignored:

$$v=4{\cdot}4405n+0{\cdot}1105$$

which shows that such a velocity has practically no effect on the rating curve.

So also, Dowson investigated the effect on a "small Price" meter, of motion not in a horizontal line, but in a curve with vertical sinuosities. He demonstrates mathematically that in rough water (waves about 6·5 feet long, and 16 inches high) the velocities deduced may be too high by 10 per cent. when compared with calm water calibrations; and cites experimental results that confirm his calculations.

The error introduced varies as the square of the height of the waves, and inversely as the square of their length, and may be by no means inappreciable in turbulent streams. This error may occur in all meters which rotate when moved up and down in a vertical direction in still water. So far as I am aware, the Fteley meter is the only type in which this does not take place; although, in the Haskell meter (*Trans. Am. Soc. of C.E.*, vol. 47, p. 380), wave motion will lead to an underestimation of the velocity.

An investigation of the probable errors of the constants a, and b, as ascertained by skilled observers, for 14 different Price current meters, leads me to believe that the velocities deduced from readings taken with a carefully rated current meter are rarely subject to a greater error than 1 per cent. so long as the velocity exceeds 0·8 foot per second, unless the motion of the water in which the observations are made is far more irregular than that in which the calibration is effected. When the velocity is less than 0·5 foot per second, current meters are not usually employed.

As will be shown later, irregularities in the water motion certainly induce errors of 5 per cent. in the velocities, and errors of 10 per cent. probably occur. These errors are, however, almost as likely to be positive as negative, so that the nett effect on the discharge is usually by no means as great as 5 per cent. The available evidence is given on page 56.

Meters, when carefully handled, can be used to determine velocities as high as 10, or 12, feet per second, provided that the stream carries no drift or submerged bodies. The rating curves indicate (when the water is not turbulent) that the accuracy is as great, or greater than, that for low velocities.

In a Fteley meter the general accuracy is greater, but the limits of use range from 0·3 foot per second, to 5·5 or 6 feet per second.

In discussing the calibration observations of Price meters, it is as well to note that while the form

$$v=an+b$$

represents the observed values with sufficient accuracy for all velocities from 0·8 to 6 feet per second, a better result can be obtained when the velocities range from 0·3 to 10, or 12, feet per second, by taking

$v=a_1n+b_1$ up to $v=3{\cdot}5$ to 4 feet per second

and, $v=a_2n+b_2$ for greater velocities.

The tables given by Hoyt (*Trans. Am. Soc. of C.E.*, vol. 66, p. 90) show that the manufacturers' rating, even when not specially determined for individual instruments, might be accepted if constant errors of 1 per cent. are permissible.

Hoyt also states that the rating of meters is but slightly affected by use. My own observations, however, lead me to believe that rough handling should be avoided at all costs, and that where a river carries silt, the pivots and bearings of the instrument should be scrutinised before each observation, as a particle of silt in the mechanism may entirely change the rating.

In the case of the Fteley meter, it appears that the relation

$$v=an+b$$

holds very fairly well for velocities greater than 1·2 foot per second, but for lower speeds a curve such as

$$v=\frac{a_1}{n}+b_1$$

is more accurate (see Diamant, *Trans. Am. Soc. of C.E.*, vol. 66, p. 107).

I shall not discuss the electrical recording apparatus, but would refer to Hoyt's paper as giving a very practical and reliable resumé.

Many other forms of meter are used in Europe, such as the Harlacher, Amsler, etc., but, having no practical experience of these instruments, I do not propose to discuss them.

Accuracy of Results.—When, besides the imperfections in a current meter, we also take into account those due to errors in soundings, and the other measurements necessary to ascertain the cross-section of a river, such as those caused by the measured velocities not being perpendicular to the cross-section, and finally the fact that a gauge reading alone does not necessarily completely determine the condition of a river (so that the discharge on a falling stage may differ from that at the same gauge in a rising flood), we obtain the following results.

Dowson (*ut supra*), for the Nile at Sarras, indicates a probable error of 4·3 per cent., as given by the discharge curve and the 126 discharge observations taken to secure it. In this case the mean velocities were actually observed, so that errors only arise from uncertainties as to the depth and actual defects in the meter.

The experiments of Prasil (*Schweizerische Bauzeitung*, 1906) and Barrows (*Proc. Am. Soc. of C.E.*, vol. 59, p. 501), indicate that systematic current meter measurements, where the area of the stream section can be exactly determined, and the mean velocity is obtained by actual observation (no assumptions as to its position, or ratio, to selected velocities, being made), are probably quite as accurate as the weir measurements usually made by engineers, the probable error being 0·9 per cent. in careful laboratory experiments, and 3 per cent. in field work.

Where, however, the quicker methods of gauging, later discussed, are employed, errors of 5 per cent. are possible, and if, in addition, the velocity of the stream, or its depth, is so great that the area is uncertain, mistakes of 10 per cent. seem probable.

Murphy's experiments (see p. 40) seem to indicate that somewhat better results (0·93 per cent. average error) than the 3 per cent. mentioned, can be obtained. The experimenters were apparently very skilful, and the conditions more favourable than those usually met with in field work.

It will be evident that while the results of any one observation (especially flood discharges) may be open to criticism, it is unlikely that the total of, say, a month's flow of a river (when obtained by careful current meter gaugings) is seriously in error, unless the month is one of continued high floods, or the bed of the stream is shifting, and the movement is not allowed for.

Summing up these results, it seems fair to infer that errors in current meter gaugings are principally due to:

(i) Irregularities in the motion of the water.

(ii) Errors in the partial areas caused either by actual errors in the observed depths (such as occur in rapid streams); or by the soundings being too widely spaced, so that irregularities in the bed are overlooked.

(iii) Errors in the mean velocities caused by the points where the velocities are observed being too widely distributed.

Errors due to the first of these causes can be minimised by careful calibration under circumstances which imitate irregular motion.

The second and third class of errors are less easily avoided.

In my own practice I am accustomed to personally observe the soundings, and during the velocity observations employ an assistant to systematically search for irregularities in the bed. If any marked irregularities are discovered, special velocity observations are taken round them, in order to discover if they produce notable differences in the velocities. Dowson's results, however, show how hard it is to obtain accurate flood discharges; and my own observations in the Thames when in flood, indicate errors of 7 and 8 per cent. under more favourable circumstances, but with less skilful observers.

Observation of the Mean Velocity over a Vertical.—The accurate determination of the discharge of a river requires velocity observations to be taken at many points distributed over its cross-section.

For example, in Dowson's Nile gaugings, there was a velocity observation for every 200 to 550 square feet of cross-section, according to the height of the flood.

In the Sudbury culvert gaugings, one observation for every 0·5 square foot.

In Harlacher's work, one observation for every 2 square feet in small, up to 20 square feet in larger streams.

The method of obtaining the discharge from velocity observations has already been indicated (see p. 44).

(*a*) Harlacher's graphical method may be considered to be the most accurate. It is so laborious, that it is only adopted in cases where the velocity observations are very closely distributed over the cross-section of the stream, and where the bed is hard and free from deposits, so that the area of the cross-section can be accurately measured.

(*b*) Considering the method by mean velocities over verticals, and for the future denoting the mean velocity over the vertical by v_m, we find in actual practice that each v_m, is usually the mean of, say, 10 observations at 0·05 depth, 0·15 depth, etc. up to 0·95 depth; and if 8 verticals are dealt with, there are 80 observations in all, and 8 partial areas *a*.

Sometimes this work is reduced by first observing 10 velocities on 3 representative verticals, and assuming that the mean velocity v_m, for the other 5, occurs at the same fraction of the depth as it does on the 3 representative verticals. We then only take $3 \times 10 + 5 = 35$ observations.

Either method necessitates the observation of velocities at from 4 to 10 points on each vertical, or say 80 to 100 points in the cross-section of the stream; and in large rivers even this number does not give a very close distribution of the verticals.

Also, the time required to make such a series of observations is not only a disadvantage (as being costly and tending to discourage frequent observations), but may lead to errors due to the discharge of the river altering materially during the time taken to complete the gauging.

Thus, methods requiring a smaller number of observations per vertical are needed, and should be employed even if only approximately correct.

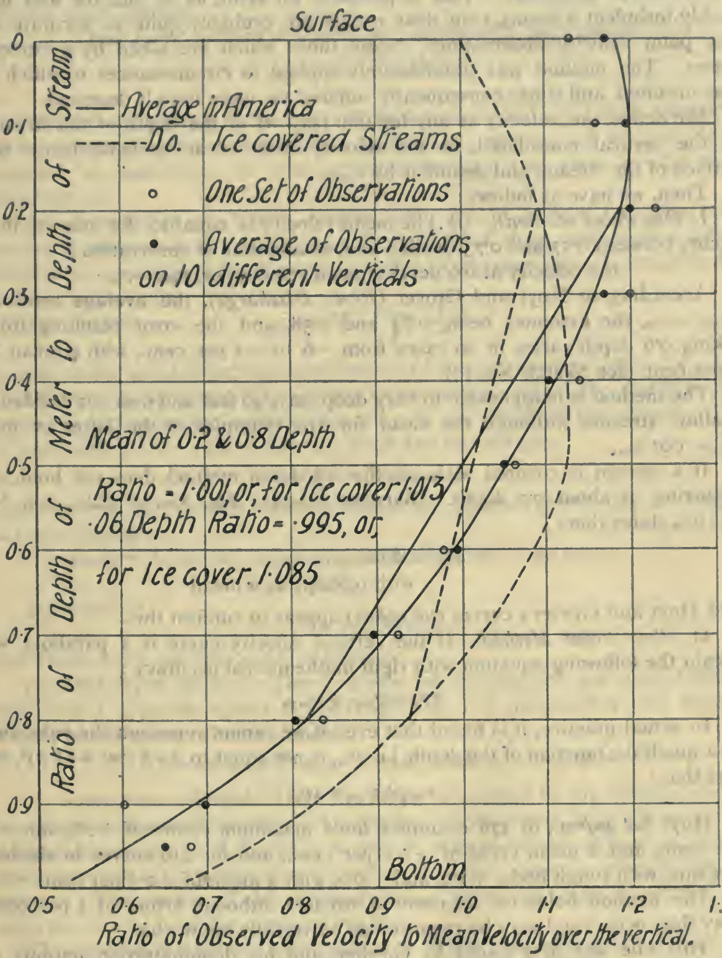
The general discussions on the irregularity of the motion of water will have made it plain that, for the same number of observations, the best distribution is probably secured by approximately determining the mean velocity over as many verticals as possible, rather than by accurately determining the mean velocity over a few widely spaced verticals. So far as either theory or experiment can be applied to what is essentially a series of accidents, it would appear that no advantage is gained by multiplying the point velocity observations on a vertical until the horizontal spacing of the verticals is less than the mean depth of the river.

The most effective method of selecting the appropriate points is to examine the vertical velocity curves, as determined by velocity observations taken at regular intervals, usually 1 foot (or one-tenth the depth) apart, in a vertical from the surface to the bottom of the stream.

When these velocities are plotted as abscissæ, and the depths as ordinates, a vertical velocity curve is obtained; and it is found that under the most varied conditions of depth, velocity, surface slope, and roughness of bed, these curves approximately assume the form of a parabola the axis of which is parallel to the water surface (see Sketch No. 10).

If the curve was an accurate parabola, it would be possible to ascertain the mean velocity over the vertical by observations at comparatively few points. As a matter of observation, founded on long and careful studies, the velocities taken at certain points geometrically selected on a vertical, are so intimately connected with the mean velocity over that vertical that the mean velocity can be obtained from these velocities with very fair accuracy.

A very excellent resumé of the general principles will be found in Cunningham's



SKETCH No. 10.—Typical Curves showing the Ratios of the Velocities at various Depths in a Vertical.

Roorkee Observations. The methods employed by Cunningham deserve careful study, and will be found very useful when abnormal circumstances are met with in gauging observations.

The following rules, however, are entirely based on the results of modern current meter work; for the figures given by Cunningham are now known to be subject to errors which are greatly minimised in modern practice. For example, Cunningham's twin float observations, although very accordant relatively to each other, are inaccurate; and in consequence the twin float is now entirely abandoned. This is probably an error, as in shallow and not highly turbulent streams, twin float results are probably quite as accurate as any point velocity observations, except those which are taken by a current meter. The method was unfortunately applied to circumstances to which it was unsuited, and it has consequently suffered far more than it deserved.

We define the velocity at any fraction (say $\cdot n$) of the depth of the stream, in the vertical considered, as the velocity observed at $\cdot n$ depth below the surface of the stream, and denote it by $v_{\cdot n}$.

Then, we have as follows:

I. *One Point Methods.*—(i) The mean velocity is equal to the velocity that occurs between 0·57 and 0·73 depth; and, as a matter of observation;

the velocity at 0·6 depth = mean velocity, or, $v_{0\cdot6} = v_m$.

According to Hoyt and Grover (*River Discharge*), the average result is $v_{0\cdot61} = v_m$, the extremes being 0·73 and 0·58, and the error resulting from taking 0·6 depth varies in 90 cases from -6 to $+4$ per cent., with a mean of 0 per cent. (see Sketch No. 10).

The method is inapplicable to very deep (say, 40 feet and over) or decidedly shallow streams, although the mean for 210 examples of the latter is only $v_{0\cdot6} = 1{\cdot}01\ v_m$.

If a stream is covered with ice, the 0·6 depth method does not hold, v_m occurring at about 0·7 depth. Barrows (*Trans. Am. Soc. of C.E.*, vol. 66, p. 110), states that:

$$v_m = 0{\cdot}82 \text{ to } 0{\cdot}92\ v_{0\cdot5},$$
$$\text{with } 0{\cdot}88 v_{0\cdot5} \text{ as a mean}$$

and Hoyt and Grover's curves (*ut supra*) appear to confirm this.

II. *Two Point Method.*—If the vertical velocity curve is a parabola, we obtain the following equation with rigid mathematical accuracy:

$$2v_m = v_{0\cdot21} + v_{0\cdot79}$$

In actual practice, it is found that even if we cannot represent the velocities as a quadratic function of the depth, i.e. $v_{\cdot n}$, is not equal to $a + b\,(\cdot n) + c\,(\cdot n)^2$, we find that:

$$v_{0\cdot2} + v_{0\cdot8} = 2v_m$$

Hoyt (*ut supra*), in 478 examples, finds maximum errors of $+2{\cdot}6$, and -3 per cent., and a mean error of $+0{\cdot}1$ per cent., and for 219 curves in shallow streams, with rough beds, $+1{\cdot}6$, and $-0{\cdot}0$, with a mean of $+0{\cdot}5$ per cent.

The method holds for ice-covered streams, although errors of 4 per cent. may then occur, and may be regarded as universally applicable.

This rule was first stated by Gordon, and his demonstration permits of extension. If a, b, c, are quadratic functions of the horizontal distance from the centre of the stream, i.e. $a = a_1 + b_1x + c_1x^2$, etc., where x, is the distance of the vertical from the centre of the stream, the mean of the mean velocities on the verticals at points 0·21, and 0·79 across the stream, is the mean velocity over the whole cross-section of the stream if the depth be uniform; so that on this assumption observations at 4 points would give the mean velocity for the whole stream.

The old method was :

$$\frac{v_{0\cdot0}+v_{1\cdot0}}{2}=v_m$$

I do not presume to indicate how $v_{1\cdot0}$, the velocity at the bottom, can be observed ; but since errors of 20 to 30 per cent. may occur, the question is not of very great interest.

III. *Three Point Method.*—Theory and experiment both indicate that :

$$v_m=\frac{v_{0\cdot2}+2v_{0\cdot6}+v_{0\cdot8}}{4}$$

The error is a mean of the error of the one point and two point methods ; and in cases where the two point method is correct, and the 0·6 depth method is inaccurate, we may actually increase the error by the third observation.

An examination of Hoyt's results leads me to believe that the extra accuracy secured is not worth the expenditure of time, and that a better distribution is obtained for the same number of observations by increasing the number of verticals.

The old rule :

$$v_m=\frac{v_{0\cdot0}+2v_{0\cdot5}+v_{1\cdot0}}{4}$$

is open to the usual objections, and errors of 6 to 9 per cent. occur.

IV. *Special Methods.*—(*a*) Surface Observations.—Here the meter is held 0·5 to 1·0 foot below the surface, according to the depth of the stream. This is most useful in the case of floods. Hoyt and Grover give :

$$v_m=0{\cdot}78 \text{ to } 0{\cdot}98v_{\text{surface}}$$

and state that the deeper the stream, and the greater the velocity, the larger is the coefficient.

For average American streams, in moderate freshet, we have :

$$v_m=0{\cdot}90v_{\text{surface}}$$

For floods :

$$v_m=0{\cdot}90 \text{ to } 0{\cdot}95v_{\text{surface}}$$

But Harlacher's rule (see p. 61) is better.

(*b*) *Mid Depth Method.*—This is a relic of old practice, and should not be used, but I give the figures in the hope that the information will prevent further observations of this character.

$v_m=0{\cdot}96$ to $0{\cdot}90\ v_{0\cdot5}$ As a mean $v_m=0{\cdot}91v_{0\cdot5}$

V. *Summation Method.*—Here the meter is lowered to the bottom, and is raised again at a uniform rate. The reading of the meter (which is evidently a mechanical average of the rates at which it turns during its journey up and down the vertical) is assumed to correspond to the average velocity over the vertical. The results of the ordinary rating curve are used to obtain this average velocity from the observed reading.

This method is therefore only applicable in the case of meters (such as the Fteley), which are unaffected by lateral currents.

The above figures give the errors produced by adopting any of the shorter methods of determining v_m, when compared with the results obtained when v_m, is ascertained by 6 or 10 observations per vertical. Now, it is known that the discharge thus obtained may differ by 3 per cent. from the results of a weir gauging. We may, therefore, infer that either the 0·6 depth method, or the 0·2+0·8 depth method, will produce a result which probably does not differ more from a weir gauging than would the theoretically more accurate 6 or 10 points per vertical methods.

Nevertheless, comparisons of the results obtained by the shorter methods with weir observations are greatly to be desired, since it is plain that the agreement between all good current meter methods is too close to permit comparisons between them to disclose any systematic errors.

The only systematic investigation of the above question appears to be that undertaken at Cornell Hydraulic Laboratory (see *Report on Barge Canal of the State of New York*, p. 932).

An abstract of the results is as follows:

Mean Depth in Feet.	Ratio: $\frac{\text{Weir discharge}}{\text{Discharge by 0·6 depth method}}$, on 6 verticals		Remarks.
	Observations worked up graphically.	Observations worked up analytically.	
9·5 to 8·9 .	0·962	0·956	Mean of 4 tests.
8·5 „ 8·4 .	0·961	0·970	
7·7 „ 7·5 .	0·973	0·972	
6·3 „ 6·0 .	0·964	0·962	

Depth in Feet.	Ratio: $\frac{\text{Weir discharge}}{\text{Discharge by summation method on 6 verticals}}$			
9·5 to 8·9 .	0·994	0·949	0·986	0·861
8·5 „ 8·4 .	0·937	0·913	0·973	0·719
7·7 „ 7·5 .	0·951	0·951	0·961	0·819
6·4 „ 6·0 .	0·956	0·959	0·971	0·929

In the first three cases of the last column the velocities were less than 0·5 foot per second, so that the large errors are readily explained.

The "ordinary" method consisted of velocity observations at 6 points on each of 6 verticals, and the results obtained were as shown in table on top of page 57.

There is a certain amount of evidence to show that the weir discharges were subject to constant errors (see Horton, *Weir Experiments Coefficients and Formulæ*, p. 96).

Making all possible allowance for such errors, the results are far worse than might be expected, and contrast markedly with those obtained with rod floats as given on page 59. I am inclined to consider that the whole set is affected by the smoothness of the channel and the slow velocities, and consequently the figures are of but slight importance in discussions concerning the discharge of rough channels such as occur in Nature. Errors in the actual observations are extremely improbable, so that the figures are of value if a smooth channel is gauged by such methods.

Depth in Feet.	Ratio : Weir discharge / Current meter discharge by "ordinary" method				Remarks.
9·5 to 8·9	0·974	0·982	1·002	1·149	These are affected by velocities being less than 0·5 feet per second.
	0·980	0·981	0·994	1·080	
8·5 ,, 8·4	1·056	1·094	1·080	1·236	,,
	1·012	1·072	0·999	1·079	
7·7 ,, 7·5	1·022	1·037	0·999	1·072	,,
6·4 ,, 6·0	1·043	1·069	1·086	1·083	,,
	1·023	1·040	1·043	1·048	

Summing up,—it may be inferred that the 0·6 depth, or the 0·2+0·8 depth methods will probably, under favourable circumstances, agree with weir observations within 2 per cent., although individual observations may differ by 5 per cent. Some portion of these differences can probably be attributed to errors in the weir observations.

I consider that it is extremely doubtful whether any additional accuracy can be attained by such preliminary work as observing the velocities at 10 or more points per vertical, and then selecting the depth at which the mean velocity is found to occur, for all future observations. This method, however, was employed by Dowson, and may be useful in large rivers where, as already indicated, the 0·6 depth method may possibly lead to errors.

ROD FLOATS.—It is obvious that the velocity of a rod float, extending in a vertical line from the surface to the bottom of a stream, must be a fairly close approximation to the mean velocity over that vertical.

The matter has been investigated by Cunningham (*Roorkee Hydraulic Experiments*) under the following assumptions :

(i) The force acting on any small element of the rod is proportional to the square of the difference between the velocity of the rod and the velocity of the water in contact with the element.

(ii) The vertical velocity curve is a parabola.

Cunningham finds mathematically that v_r, the velocity of the rod, is equal to v_m, when l_i, the immersed length of the rod is from 0·950 to 0·927 the depth of the water; the exact value depending upon the position of the maximum velocity in the vertical velocity curve.

He takes as a mean : $v_m = v_r$, when the immersed length of the rod is equal to 0·94 depth, i.e. $l_i = 0{\cdot}94d$.

An arithmetical study of 38 vertical velocity curves obtained by current meters leads me to believe that if Cunningham's first principle is accepted as correct, the fact that a vertical velocity curve is not an exact parabola is of small importance. The mean result obtained was :

$$v_r = v_m, \text{ when } l_i = 0{\cdot}952d,$$

and the maximum value was $l_i = 0{\cdot}97d$, and the minimum $l_i = 0{\cdot}91d$.

The adoption of $l_i = 0{\cdot}94d$ led to a mean error of $+1$ per cent., and maximum errors of $+4$ per cent. and -3 per cent.

When practically tested, the rod float does not compare quite so well with the current meter, as the above figures indicate.

As already stated, this must mainly be ascribed to the irregularity in flow which affects all floats more markedly than current meters, since a float practically at rest in relation to the water, and is therefore mostly influenced by the irregularities of a small volume of water; while a current meter is acted on by a fresh volume of water at every instant.

Thus, taking Cunningham's observations:

In a current meter the mean of 6 observations gave $v = 4{\cdot}13$ feet per second.
The mean of the next 6, $v = 4{\cdot}11$ „ „
the maximum and minimum individual values being 4·36 and 3·84.
While, for 50 floats the mean was $v = 3{\cdot}95$ feet per second.
The mean of the next 50, $v = 3{\cdot}86$ „ „
and the maximum and minimum individual values were 4·44, and 3·33.

The figures do not allow of an exact comparison being made, since the current meter observed the velocity at 5 feet depth, while the rod floats determined the mean over a depth of 10 feet. But it will be plain that 6 observations of a current meter are more effective in the elimination of irregularities, than 50 floats.

So also, Cunningham's investigation of the relation between v_r, and v_m, does not appear to hold good in practice; and, as a rule, rod float gaugings are found to give too high a result when checked by weir or current meter methods.

The only systematic comparison of rod float gaugings and weir measurements was undertaken by Francis (*Lowell Hydraulic Experiments*). The discharge was measured by rod floats in a smooth, timber-lined channel, and also over a weir. Putting Q_r for the rod float discharge, and Q_w for the weir discharge, Francis finds that:

$$Q_w = Q_r \{1 - 0{\cdot}116 (\sqrt{D} - 0{\cdot}1)\}$$

where $D = \frac{d - l_i}{d}$. That is to say, D, is the ratio of the portion of the depth which is not covered by the rod, to the total depth.

Francis assumes that $v_m = v_r \{1 - 0{\cdot}116 (\sqrt{D} - 0{\cdot}1)\}$, and his experiments cover the range $D = 0{\cdot}004$ to $D = 0{\cdot}129$. The circumstances are in no way comparable to those under which rod floats are usually employed. Francis is known to have been a most careful experimenter, and there is not the least doubt that he observed the velocity on a sufficient number of verticals to secure substantially accurate values of Q_r. Nevertheless, if his observations represented a rod float gauging of an earthen channel, they would be rejected in accurate work on the ground that the verticals were too widely spaced. Thus, the relatively smoother gauging channel in which Francis' observations were made has evidently caused the horizontal distribution of the velocities to differ widely from that which obtains in earthen channels; and it is therefore probable that the vertical distribution of velocities (and consequently the relation between v_r and v_m) is also altered.

The following results were obtained at the Cornell Hydraulic Laboratory (see *State of New York Barge Canal Report*, p. 923):

Average Depth of Channel.	Ratio : $\frac{\text{Weir measurement}}{\text{Mean of 5 rod floats per vertical}}$	
	h=0·75 depth.	h=0·90 depth.
Feet.		
9·3	0·989	1·003
8·3	0·955	0·973
7·5	0·962	0·980
6·3	0·960	0·971

It will be seen that Francis' statements are generally confirmed, and that the accuracy is somewhat better than I later indicate.

The experiments appear to have been made in a smooth, concrete lined channel, and are therefore not rigidly comparable with the results obtained under the more irregular conditions occurring in earth channels (even if smoother than usual). Certain special experiments of my own indicate that the results obtained with floats immersed to only 75 per cent. of the depth would, in the case of regular earth channels (where Bazin's γ=1·5, and Kütter's n=0·020), usually be some 5 per cent. greater than the values obtained with a 90 per cent. immersion of the floats. In rougher channels (where Bazin's γ=2·3, and Kütter's n=0·027) the difference is even greater, but these last observations were taken under circumstances which were not at all favourable to accurate work.

In actual work, the errors which specially affect rod float gaugings are mainly caused by the fact that the irregularities which exist in the beds of natural streams are usually sufficiently marked to prevent a rod of a length equal to 0·90 or 0·94 of the mean depth being used to observe the velocities.

In the Punjab Irrigation Branch the rod float is the standard gauging instrument, and the conditions existing are those to which the method is best adapted.

The gaugings are taken in channels of regular section, usually specially trimmed, or lined with brick-work; so that it is possible to run a float only 3 inches, or at the most 6 inches, shorter than the depth of the water. It is always possible to select a site with a straight stretch of canal up-stream, so that cross currents are infrequent. The labour available is cheap, but is unskilled, and gaugings are frequently taken by comparatively untrained observers.

When a good site is selected, the method is a very excellent one; and so far as can be judged from results (checked where possible by other methods) I believe that a good gauging rarely errs by more than:

1 per cent. for discharges up to	60 cusecs			The more or less constant error hereafter discussed being neglected, so that the comparison is really with other rod float gaugings.
2 ,, ,,	300 ,,			
3 ,, ,,	500 ,,			
4 ,, ,,	1000 ,,			
5 ,, ,,	2000 ,,			

Above 2000 cusecs it appears that the method is not very accurate ; but this, I believe, is owing to the fact that it is almost impossible to find a gauging length where the bed is sufficiently uniform to permit a rod being run which is only 6 inches less than the depth. If this were possible, I consider that the method would prove equally satisfactory.

It is not customary in the Punjab to correct the velocities by Francis' formula (p. 58).

I think that this is a mistake. My own experiments, and those of at least two other officers, lead me to consider that this correction is a very valuable one, and it should certainly be employed wherever justified by the accuracy of the rest of the work. If it is applied to a series of gaugings, I believe that the relative errors, above tabulated, may be reduced by one half.

Rod floats are usually of wood, ¾ inch, or in the larger lengths 1 inch square, weighted so as to show only 1 to 1½ inch above water. A useful set starts at 1 foot immersion, and proceeds by steps of 3 inches up to 4 feet; and then by steps of 6 inches up to 8 feet, with three rods of each length. Greater lengths are best made of watertight tin tubes, weighted with shot ; indeed, this material should be adopted wherever gaugings are taken in close succession ; for wooden floats, if used so frequently as to get water-logged, are liable to sink.

The Punjab instructions are that 5 rods should be run in each vertical, and the mean be taken as the velocity in that vertical. The verticals are equally spaced across the canal, 10 feet apart in large, and 5, 4, or 2 feet apart in smaller channels ; and there are usually 10 verticals in the total width.

The relative errors of good gaugings in the Punjab have already been given. There is, however, very little doubt that all rod float gaugings overestimate the discharge, and that the whole evidence shows very clearly that all Punjab gaugings are, on the average, some 3 per cent. in excess of the truth. In good observations, such as were alone considered when obtaining the above figure, $\frac{d-l_i}{d}$, is rarely greater than 0·05 ; so that the application of Francis' correction would leave about one-half the difference unexplained. While the individual observations are probably subject to at least this amount of error, I consider that the mean result is quite sufficiently accurate to permit the statement to be made that a correction formula :

$$Q = Q_r \{1 - 0{\cdot}2(\sqrt{D} - 0{\cdot}1)\}$$

where D, represents the mean value of $\frac{d-l_i}{d}$, for all the floats observed, is probably more accurate than Francis' when applied to gaugings in earthen channels, provided that D, does not greatly exceed 0·10. The formula has been systematically checked for discharges up to about $Q = 150$ cusecs. Above this value the checkings are less reliable, and the principal evidence in its favour is the fact that if the discharge of a canal is observed with say $D = 0{\cdot}20$, and simultaneously with say $D = 0{\cdot}05$, the two values of the discharge are found to agree very closely when corrected by this method.

It is believed that the value of the coefficient which Francis gives as 0·116, and I give as 0·2, increases with the roughness of the bed. The figure 0·2, corresponds approximately to Bazin's $\gamma = 1{\cdot}54$, or Kütter's $n = 0{\cdot}020$.

Summing up, the rod float system of gauging is a very practical method for systematic work, and untrained men can rapidly be taught to do good work.

In my opinion, the smaller size and weight of a current meter constitutes its only decided advantage in gauging regular canals. For natural streams and rivers, however, the current meter should be adopted.

Surface Floats.—The use of surface velocities in estimating the discharge of a river can only be considered as a makeshift. The method is justifiable under the following conditions:

(*a*) In floods, when a boat cannot be accurately manœuvered on the river, and where the soundings are consequently known to be subject to errors which render any more accurate method of obtaining the velocities unnecessary.

(*b*) In rough reconnaissance work, when time, material, and labour for the more accurate systems are not available.

The surface velocity is probably less affected by irregularities of the water motion than any other velocity of the stream. On the other hand, it is greatly influenced by wind and bends in the course of the stream.

In order to avoid wind effects I have found it best to use globular floats, rather than flat pieces of circular board, such as are usually recommended. Where a supply of oranges is available, they form ideal surface floats, and have the great advantage that they can be thrown to the desired position with fair accuracy.

The best method of treating the observations is due to Harlacher, and appears to accord very closely with the real facts.

Harlacher (*P.I.C.E.*, vol. 91, p. 399) states as follows:

No very constant ratio exists either between V_m, the mean velocity over the whole cross-section, and the maximum surface velocity, or v_{ms}, the mean of the surface velocities; but if v_s, be the surface velocity at any point, the total discharge of the stream is represented by:

$$Q = p\Sigma v_s \times \text{corresponding partial area}$$

Thus, pv_s, may be considered as a *quasi* mean velocity over the vertical below it, although it is not equal to v_m, the mean velocity in that vertical as obtained by direct observation.

In 28 gaugings in the Danube and Bohemian rivers, with widths ranging from 160 to 1400 feet, maximum velocities varying from 2 to 10 feet per second, and depths between 2·5 and 25 feet, the value of p, was always between 0·79 and 0·91, and lay between 0·83 and 0·88 in 23 cases.

Harlacher also states that p, is greatest for sandy beds, and that the minimum value occurred with beds of gravel of fist size.

He suggests that p, may generally be taken as 0·85.

In Switzerland, for 200 cases, the mean value is 0·835, and it is evident that this smaller figure is due to mountain streams, possessing gravelly or stony beds.

For the Rhine, in Holland, the value rises to 0·87, owing to the finer quality of the sand.

For the Punjab rivers, where the sand is extremely fine, the ratio is usually taken as 0·93; but I consider that this is somewhat high, since these gaugings are taken with a view to estimating flood discharges, and a slight overestimation is recognised as by no means undesirable.

Thus, if the surface velocities are observed at points distant l feet from each other, all across the river, and if d_m, be the mean depth corresponding to the width $\frac{l}{2}$, on each side of the point where v_s, is the surface velocity, then:

$$\text{Discharge} = p\Sigma l d_m v_s$$

The rule given above may be supplemented by Grunsky's results for 25 Californian streams (*Trans. Am. Soc. of C.E.*, vol. 66, p. 123). Grunsky assumes that $\frac{v_m}{v_s}$, is approximately the same for all verticals in a river, and consequently we can put:

$$\frac{v_m}{v_s} = \frac{\Sigma v_m}{\Sigma v_s} = p, \text{ as in Harlacher's rule.}$$

In streams with sandy bottoms, Grunsky finds that p, depends only upon the ratio:

$$\frac{\text{Width of stream}}{\text{Mean depth}}$$

and gives the following table:

$\frac{W}{d}$	p	$\frac{W}{d}$	p
5	1·01	30	0·89
10	0·97	40	0·87
15	0·94	50	0·85
20	0·92	100	0·82

An examination of the individual results shows that 14 cluster very closely round p=0·90, and these 14 include values of $\frac{W}{d}$, from 18 to 32. The evidence afforded is therefore not in conflict with Harlacher's rules, and classification by the character of the bed appears to be more likely to produce accurate results.

Hoyt and Grover (*River Discharge*) give a large number of values of $\frac{v_m}{v_s}$. The maximum value is 0·98, and the minimum 0·78, but the average of 138 curves is 0·85, and the figures cluster closely round this value. For small streams with rough beds the maximum value is 0·89, and the minimum 0·78, and the average of 219 curves is 0·84.

In practice these authors only employ the method in gauging floods, and state:

The deeper the stream, the larger is the coefficient.

For average (American) streams, in moderate freshet, 0·90 will generally give fairly accurate results. In floods, 0·90 to 0·95 should be adopted (see p. 55).

Special Methods of Gauging.—The following methods are approximate. The only justification for giving any details of such methods lies in the fact that a bad gauging is better than none at all. In actual work, good observers should obtain their own values of the ratios now enumerated from the results of two or three careful preliminary gaugings conducted by accurate methods. Discharges which are obtained in this manner may be expected to agree *inter se* within about 5 to 7 per cent. of error. Thus, the following tables are in reality suggestions for preliminary observations.

If the ratios are taken from the table, and are blindly applied, the comparative errors will not of course be increased, but the absolute errors may be doubled.

It is usual to give certain tables showing the probable ratio of the mean velocity in a vertical v_m, to the surface velocity v_s, or the maximum velocity v_{max}, or the velocity near the bottom $v_{1\cdot0}$.

So far as can be ascertained, these ratios are extremely variable, and are considerably influenced by irregularities in the motion of the water. I have been unable to trace any case in which ratios obtained by one observer were found to agree accurately with those obtained by another observer, even under circumstances which were apparently quite similar.

Central Vertical Method.—Bazin's experiments, as calculated by Bellasis, (*Hydraulics*, p. 165) give :

RATIO.						
$\frac{\text{Mean width}}{\text{Mean depth}}$	1	1·5	2	3	4	5
$\frac{\text{Mean velocity}}{\text{Mean velocity in central vertical}}$	0·86	0·87	0·88	0·89	0·90	0·91
$\frac{\text{Mean width}}{\text{Mean depth}}$	6	7	10	20	30	50
$\frac{\text{Mean velocity}}{\text{Mean velocity in central vertical}}$	0·92	0·93	0·94	0·95	0·96	0·97
$\frac{\text{Mean width}}{\text{Mean depth}}$	90	...	...	...	...	...
$\frac{\text{Mean velocity}}{\text{Mean velocity in central vertical}}$	0·98	...	...	...	...	...

These are applicable to rectangular and trapezoidal sections, and are probably correct to 1 or 2 per cent. when the $\frac{\text{mean width}}{\text{mean depth}}$ is less than 10, and to 0·5 per cent. when this value is exceeded, except when the side slopes are very flat, provided always that the channel upstream of the point of observation has no marked irregularities for a length equal to at least 20 times the width. If there are great irregularities, say 5 times the width above the point of observation, errors of 5, or even 10, per cent. may occur in either direction.

Maximum Velocity Method.—For a single vertical, it is usual to state that $\frac{v_m}{v_{max}}$ has values varying from 0·98 for the deepest portion of large rivers, down to 0·85 for shallow and gravelly streams. On investigating the original authorities for these statements I am inclined to believe that no reliance can be placed on these figures. In any case, I am totally unable to conceive what practical object can be attained by a knowledge of their values. I have already stated what I believe to be the most useful function connecting v_s and v_m, the mean velocity over the vertical.

Surface Velocity Method.—Bellasis (*ut supra*, p. 169) gives a table of values of $\frac{v_m}{v_s}$, for verticals not too close to the banks, classified according to the depth, and Kütter's *n*. The general law is fairly well known, $\frac{v_m}{v_s}$, increases as the depth increases, and as *n* decreases.

The following portion of his table is probably as accurate as any estimation of *n*, or γ, without systematic measurements, will be :

d in feet	Kütter's.				
	n=0·020	*n*=0·0175	*n*=0·015	*n*=0·013	*n*=0·010
0·9	0·83	0·86	0·88	0·89	0·91
1·1	0·84	0·87	0·89	0·90	0·91
1·25	0·85	0·87	0·89	0·91	0·91
1·50	0·87	0·88	0·90	0·91	0·92
Bazin's γ.	1·54	0·833	...	0·290	0·109

For values of *n*, greater than *n*=0·020, better information is now available from the results of the current meter gaugings undertaken of late years in the United States.

d	Kütter's.	
	n=0·030	*n*=0·025
1	0·78	0·82
2	0·80	0·86
3	0·83	0·88
5	0·85	0·89
10	0·86	0·90
15	0·87	0·91
20	0·88	0·92
Bazin's γ.	3·17	2·35

Bellasis' figures for *n*=0·025, show a decrease for depths greater than 10 feet. The more modern figures do not confirm this, and Bellasis probably relied too much on old twin float results. A test of this table on 100 curves taken at random allows me to state that these figures are probably accurate to 3 per cent. when there are no marked disturbances upstream. The ratios are, however, almost useless, as Harlacher's *p*, is better adapted for practical purposes.

Central Surface Velocity.—There is a certain amount of evidence to show that the ratio between v_{cs}, the central surface velocity, and V_m, the mean velocity, is approximately constant.

In a regular channel with no marked irregularities,

$$V_m = 0{\cdot}81 \text{ to } 0{\cdot}89\ v_{cs},$$

or, as a mean, $V_m = 0{\cdot}84\ v_{cs}$.

The ratio is a useful one, and if determined for a well selected site by special experiments, it will be found to be but slightly affected by small alterations in the water level.

Bottom Velocities.—The ratio $\frac{v_{1\cdot0}}{v_m}$ has been stated to range from 0·68 to 0·70. As a matter of fact, what has probably been observed is not $v_{1\cdot0}$, the velocity at the bottom, but $v_{0\cdot9}$, or $v_{0\cdot95}$, according to the depth of the river, since it is hardly safe to allow a current meter to be less than 6 inches from the bottom of the river.

The following values are obtained from Bellasis' suggestions :

Kütter's n.	0·030 to 0·0275	0·020	0·015	0·010
Depth ..	5 to 18 feet	1 to 1·5 foot	1 to 1·25 foot	1 foot
$\frac{v_{1\cdot0}}{v_m}$ . . .	0·50 to 0·55	0·50 to 0·55	0·60	0·65

My own experiments on silt-carrying canals with $n = 0{\cdot}019$, or $\gamma = 1{\cdot}5$ approximately, give $\frac{v_{1\cdot0}}{v_m} = 0{\cdot}50$ to $0{\cdot}62$; and as a mean, 0·58 for depths ranging from 0·8 to 2·4 feet. I believe that the method used to observe the velocities is likely to give results which are less than the truth.

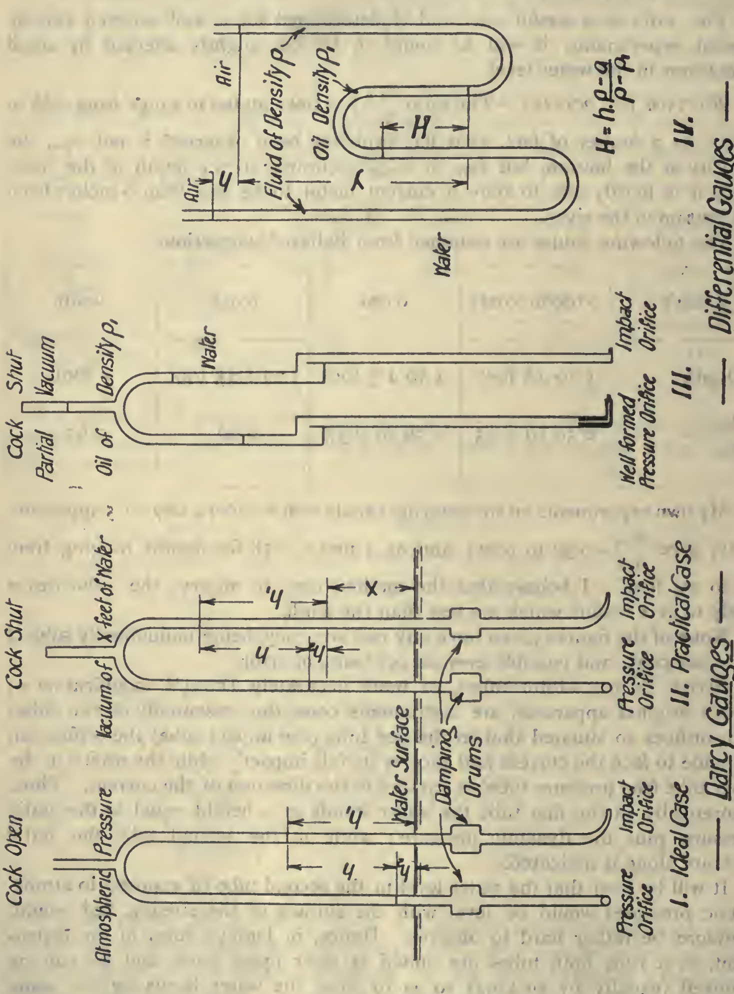
None of the figures given have any real accuracy, being undoubtedly subject to 10 per cent., and possibly even 20 per cent., of error.

Pitot Tubes.—Pitot tubes, or more accurately Darcy's modification of Pitot's original apparatus, are instruments consisting essentially of two tubes with orifices so situated that in the one tube (the impact tube) the orifice can be made to face the current and receive its full impact ; while the orifice in the other tube (the pressure tube) is parallel to the direction of the current. Thus, theoretically, in the first tube the water stands at a height equal to the static pressure, plus the dynamic pressure ; while in the second tube the static pressure alone is indicated.

It will be seen that the water level in the second tube (if exposed to atmospheric pressure) would be level with the surface of the stream, and would, therefore, be rather hard to observe. Hence, in Darcy's form of the instrument, as a rule, both tubes are united at their upper ends, and air can be removed (usually by sucking) so as to raise the water levels by the same amount.

Theoretically, if h, be the difference in level of the water columns, then $v^2 = 2gh$, gives the velocity of the current. In actual practice, it is extremely difficult to prevent the pressure orifice from being exposed to some action by the current, usually in the nature of suction, producing a depression of the corresponding water column. Hence, as a rule, $v = C\sqrt{2gh}$, where C, is a coefficient.

It may at once be stated that all, or nearly all, our difficulties in using Pitot tubes arise from the pressure tube. White (*Journ. of Assoc. of Eng. Societies*, August 1901) has proved that the water level in the gauge con-



SKETCH NO. II.– Diagrams of Pilot Tubes and Differential Gauges.

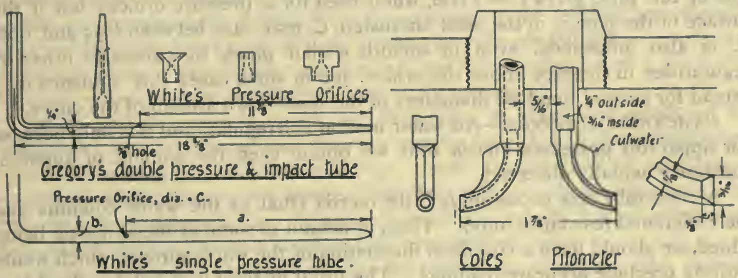
nected with the impact tube (when exposed to atmospheric pressure) always stands at a height h_1 above the surface of the water in the channel, given by $v^2 = 2gh_1$, whatever be the form of the orifice. White's orifices included such diverse forms as knife-edged orifices in finely tapered pipes, conical trumpet mouths, and small holes in wide flat surfaces. Consequently, it may be said that it would be very difficult to design an impact tube orifice which did not indicate an excess of pressure equal to $\frac{v^2}{2g}$ relative to the free water surface.

On the other hand, it appears to be a difficult matter to construct a pressure tube which does not show either a slight rise, or a small depression, caused by the velocity of the water passing its orifice. If this rise be represented by $h_2 = k\frac{v^2}{2g}$, it is plain that the difference of level read in the two columns, when lifted above the water level for convenience in observation by exhausting air, is :

$$h = h_1 - h_2 = (1 - k)\frac{v^2}{2g}$$

So that $v = C\sqrt{2gh}$, where $C = \frac{1}{\sqrt{1-k}}$; and k is usually negative, so that C is less than 1. (See Sketch No. 11, Figs. 1 and 2.)

The information at present available on the laws affecting the value of C,



SKETCH No. 12.—Forms of Orifices in Pitot Tubes.

is not of much practical value. It was believed that the form of the impact orifice had the greatest effect on C, and it was not until White (*ut supra*) carefully investigated the matter, that the paramount importance of the circumstances of the pressure orifice became known.

The most favourable position for the pressure orifice appears to be in the side of a tapering pointed rod, as indicated in Sketch No. 12, which shows a combination of pressure and impact orifices in one piece, which has certain advantages as regards compactness.

Here White, in flowing water, with $a = 3\frac{1}{2}$ inches, $b = \frac{1}{2}$ inch, and $c = \frac{1}{8}$th inch, obtained C = 1·0071 in one instrument, and C = 0·993 in a "duplicate copy"; which would suggest that the true value for both was C = 1·000.

The first tube was afterwards provided with a linseed oil differential gauge (theoretical magnification 12·8), and was tested at 26 known velocities, in flowing water, against a Price current meter which had been previously rated. The results were :

For the Pitot tube,

$$v = 0{\cdot}993\sqrt{2gh}, \text{ in place of } 1{\cdot}007.$$

The maximum value was C = 1·028, and the minimum C = 0·910, which is almost certainly an error in reading.

In the case of the current meter,

the true velocity = 0·983 indicated velocity,

the maximum being 1·018, and the minimum (probably erroneous) 0·893.

We may, therefore, consider that a properly designed Pitot tube is capable of giving results which are as accordant *inter se*, as those of a current meter.

So also, Gregory (*Trans. Am. Soc. of Mech. Eng.*, vol. 25, p. 184), in flowing water, obtained with $a = 11\frac{7}{8}$ inches, $b = \frac{3}{4}$ inch, $c = \frac{1}{8}$th inch, C = 1·003, and C = 0·995 in a "duplicate."

The experiments of Lawrence and Braunworth (*Trans. Am. Soc. of C.E.'s*, vol. 57, p. 273), who obtained C = 1·005 with a blunt ended tube, where $a = 0{\cdot}03$ inch, $b = 0{\cdot}17$ inch, and $c = 0{\cdot}1$ inch, indicate that the taper form of the tube is of little importance, provided that the orifice is in the side of the tube, and is not too close to other tubes.

In the case of a smooth pipe under pressure, it would appear that a smooth hole $\frac{1}{8}$th of an inch in diameter (with all burrs carefully removed) bored in the side of the pipe, gives C = 1·000, when used for a pressure orifice; but if the surface of the pipe is in the least encrusted, C, may vary between 0·95 and 1·06. C, is also influenced, even in smooth walled pipes, by curves or other irregularities in the pipe above the orifice, and in some cases this influence may extend for several hundred diameters of the pipe, down stream of the curve.

Practical Details.—All water motion is irregular, and it is only because our apparatus possesses inertia that we obtain even the amount of apparent constancy actually observed.

A Pitot tube has exceedingly little inertia (that of the water columns and their frictional resistance only). Thus, in default of some artificial inertia being added, we should have a continual fluctuation of the water surface, which would entirely preclude accurate readings. The usual method is to enlarge the tubes, just above the orifice, into a drum-shaped vessel, as per Sketch No. 11.

For values of v, exceeding 4 feet per second, h, is fairly large, 3 inches or over, and can therefore be observed with some exactitude; but where it is desired to accurately observe small velocities, a differential gauge must be used. Here, if water and a liquid of a density ρ_1 be employed, h, is increased in the ratio $\frac{1}{1-\rho_1}$; but in actual work, tests must be undertaken in order to see whether capillary attraction, or viscosity, alter this ratio.

The general formula for a differential gauge containing liquids with densities equal to ρ_1 and ρ is:

Observed $h = h$ for a water gauge $\times \frac{\rho}{\rho-\rho_1}$, and if air is included, as in Sketch No. 11 (Fig. IV.), the factor is $\frac{\rho-a}{\rho-\rho_1}$ where a, is the density of air.

As an example, take kerosine, $\rho_1 = 0{\cdot}80$ approximately. In the first case, the factor is $\frac{1}{1-0{\cdot}80} = 5$. In the second case, $\frac{1-0{\cdot}00125}{1-0{\cdot}80} = 4{\cdot}994$ say.

Actually, the best method is to observe the ratio when the height on the water gauge is sufficiently large to be read with accuracy, as shown in Fig. IV., Sketch No. 11.

In Williams' experiments (*Trans. Am. Soc. of C.E.*, vol. 47, p. 1), the observed ratio was about 4 per cent. greater than the calculated ; and the small alterations in ρ_1 due to the change in temperature did not appear to affect this excess. In White's (*ut supra*) linseed oil gauge, $\rho_1 = 0·922$, and the increase was probably 2·8 per cent.

A Pitot tube, as described above, probably forms the most accurate method of observing velocities.

Reference may be made to papers by Stanton (*P.I.C.E.*, vol. 156, p. 86), and by Smith (*Proc. of Victorian Inst. of Engineers*, November 1909) for details of such instruments.

I am, however, inclined to believe that this very possibility of intense refinement renders a Pitot tube unsuitable for purely engineering purposes, where, as is almost invariably the case, the water is in turbulent motion. An engineer generally wishes to obliterate the effects of this turbulence, and desires a mean result corresponding to the undisturbed portion of the motion. He also usually wants to observe the average velocity of the water, and not an average of the squares of the momentary velocities as given by a Pitot tube. As indicated by the second formula on page 49, the current meter also appears to average the squares of its own momentary velocities ; but owing to its greater inertia, the results do not materially diverge from the average of the momentary velocities of the water, since the vanes of the current meter do not follow the momentary variations of the water velocity so closely as the water columns of a Pitot tube do, even when the tube is enlarged, as already suggested.

If the various cases in which the Pitot tube is practically employed are considered, it will be found that they are usually restricted to the measurement of velocities in pipes, and that the best results are obtained at, or close to, nozzles or other orifices ; and further, that the motion in these cases is almost invariably less turbulent than in open channels, especially if these have rough boundaries, such as occur in earthen channels, or river beds.

Thus, it will readily be inferred that the calibration of a Pitot tube presents difficulties analogous to those found in the calibration of current meters, but in a more marked degree.

The experiments of Darcy and Bazin (*Recherches Hydrauliques*), or of Murphy (*Trans. Am. Soc. of C.E.*, vol. 47, p. 197), are the most complete.

Putting $v = C\sqrt{2gh}$ in the following cases we have :

(1) 92 ratings in moving water, v, being obtained by floats.

Mean value of C = 1·006. Maximum value, 1·039. Minimum value, 0·981.

(ii) 87 ratings in moving water, tube used to determine the discharge, and checked by a weir.

Mean value of C = 0·993. Maximum value, 1·029. Minimum value, 0·965.

(iii) 32 ratings, in still water.

Mean value of C = 1·034. Maximum value, 1·053. Minimum value, 1·015.

With another tube they obtained the following results :

I. Impact orifice directed against the current, pressure orifice parallel to current.

$$C = 0·848 \text{ by floats.}$$
$$C = 0·797 \text{ in still water.}$$

II. Both orifices directed against the current, but the pressure orifice plugged, and a small hole 0·04 inch in diameter pierced laterally.

$$C = 0·875 \text{ by floats.}$$
$$C = 0·864 \text{ in still water.}$$

III. Impact orifice directed against the current, pressure orifice facing downstream.

C = 0·998 by floats.
C = 0·991 in still water.

Williams, Hubbell, and Fenkell (*Trans. Am. Soc. of C.E.*, vol. 47, p. 199) find as follows:

(i) Pitot tube No. 3.

Rated in still water. 85 observations.

Mean value of C = 0·914. Maximum value of C = 0·975. Minimum value of C = 0·821.

(ii) Rated in a 2-inch brass tube (the impact tube held in centre of pipe), and the volume obtained by weighing the discharge, so that the absolute indications are subject to some uncertainty. The pressure orifice was a hole in the side of the tube.

13 Ratings. Mean value of C = 0·729. Maximum value of C = 0·740. Minimum value of C = 0·718.

(iii) Do., but pressure orifice a circumferential slit in the side of the tube.

13 Ratings. Mean value of C = 0·779. Maximum value = 0·789. Minimum, value = 0·764.

II. Pitot tube No. 5. Gives under circumstances similar to the above:

(i) 133 observations. Mean of C = 0·835. Maximum, 0·916. Minimum, 0·677.

Or, with another pressure tube:

112 observations. Mean of C = 0·851. Maximum, 0·942. Minimum, 0·705.

(ii) 29 observations. Mean of C = 0·694. Maximum, 0·727. Minimum, 0·647.

III. Pitot tube No. 6. As above.

(i) 110 observations. Mean of C = 0·966. Maximum, 1·091. Minimum, 0·895.

(ii) 13 observations. Mean of C = 0·683. Maximum, 0·695. Minimum, 0·659.

(iii) 13 observations. Mean of C = 0·784. Maximum, 0·799. Minimum, 0·749.

The final coefficients employed were:

Tube No.	In Still Water.	In Pipes.	Diameter of Pipe in Inches.
3	0·926	0·89	2, 5, and 12
5	0·859	0·75	30
6	0·950	0·846	16 and 30

The skill and care displayed by the experimenters was most remarkable, but a study of the above figures seems to afford ample proof that their tubes were unreliable when applied to absolute measurements of the velocity at a point.

The experiments are otherwise of great interest. I have studied them very carefully by mean square methods, and am inclined to believe that the effect of the irregularities is very nearly eliminated in their discharge measurements, and that their discharge values are rarely subject to 1·5 per cent. of error. Thus, the application of even such tubes as the above, to obtain discharges, appears permissible.

The whole available experiments (without exception) indicate that a still water rating of the normal Darcy tube will give a higher value of C than that obtained in flowing water. There are also indications which show that the difference is greater, the more C (as found in flowing water) diverges from unity; but there are exceptions to this rule. Also in each case, the values of C, vary somewhat irregularly, through rather wide limits.

It will be observed that rating in flowing water is productive of far more concordant results, and many of the divergencies there observed are to be ascribed to imperfections in the method of rating. This is especially the case when the method used by Williams and others is adopted, which consists in observing the central velocity by a Pitot tube, and calculating its ratio to the volumetrically observed mean velocity by equations which are themselves subject to a certain amount of probable error (roughly 0·85 that of C). We may thus assume that the C, given by ratings in flowing water, is far more constant than that given by still water ratings.

Comparing these results with those of White (see p. 67), we are led to infer that the variations in C, from run to run, are not entirely due to errors of observation, but have a real physical existence dependent on the irregularity of the motion of the water, and are the more marked the greater the difference between the mean coefficient and unity.

When accuracy is desired, it is therefore essential to calibrate Pitot tubes in flowing water, and to design them so as to obtain $C \approx 1{\cdot}000$, which is best attained by following the lines of Sketch No. 12.

The general impression left on my mind is that each Pitot tube (or rather the pressure orifice of each tube), even when supposed to be an accurate duplicate of one already calibrated, must be individually calibrated. There is no doubt that this is the case when the accuracy of duplication is equal to that of ordinary handwork.

In order to avoid these uncertainties, Gregory (*Trans. Am. Soc. of Mech. Eng.*, September 1908) and Cole, have designed Pitot tubes where the pressure tube is an exact reduplication of the impact tube, except that its orifice is turned downstream. The experimenters assume that the indications of duplicate instruments of this construction, will agree; and if this is the case, the device has made the Pitot tube a standard instrument which can be manufactured in quantities, and distributed for use. Nevertheless, definite proof is desirable; since, at present, general practical employment forms the only basis for the above statement.

Gregory obtains for his type . . . $v = 1{\cdot}133\sqrt{2gh}$

Cole obtains for his type . . . $v = 1{\cdot}19\sqrt{2gh}$

apparently by calibration in flowing water in pipes (see Sketch No. 12).

Thus, I consider that the usual field of utility of a Pitot tube will be found to lie in gaugings of pipes and other extremely smooth channels of small dimensions.

The preliminary studies require time. We must (so I believe) calibrate the tube in the actual pipe which it is intended to use, and fix the instrument at a point in the cross-section of the channel where its indications are found to bear a definite ratio to the mean velocity of the water flowing in the channel. Then, as the tube offers an extremely small resistance to the motion of the water, it can be permanently left in the pipe, and can be used as a recording instrument. Cole, by employing photography to record the oscillations of the water columns, has constructed a water meter which records the total quantity passing through the pipe each hour, or each day.

The method is open to many theoretical objections. In the first place, Williams (*ut supra*) and Bilton (*Proc. of Victorian Inst. of Engineers*, 1909) have shown that the distribution of velocities over the cross-section of a pipe greatly depends on the mean velocity, and Bazin (*Trans. Am. Soc. of C.E.*, vol. 47, p. 258) has shown that it depends on the roughness of the pipe. Thus, the Cole pitometer will probably record a velocity which is only equal to the mean velocity for one particular value, and this value will vary from year to year as the pipe becomes more encrusted. On the other hand, with the exception of the more costly, and less easily fixed Venturi meter, no other simple instrument exists which will permit the momentary flow to be as easily ascertained as time is read from a watch; and personally, I should prefer to know the quantity of water used within 10 per cent. without trouble, at any moment, rather than to ascertain it to within 1 per cent. once a week, with difficulty.

The original apparatus used by Pitot consisted of one tube only, *i.e.* that which has here been termed the impact tube. As already stated, this apparatus may be regarded as needing no calibration, provided that the zero of h_1, can be accurately ascertained.

In the case of open channels, the free water level can be very closely determined by means of Bazin pits (see p. 101), or ordinary gauge wells (see p. 103). When studying silt problems I have found the simple impact tube most useful in obtaining velocities near to the bottom of a canal. These velocities being low, the two fluid gauge must be employed. So far as my experience goes, the method is the best which we possess for ascertaining bottom velocities in silt-laden water, as it has none of the defects of current meters or of sub-surface floats; although, where the bottom can be seen, these latter (preferably in the form of red currants) are very simple, but less reliable.

So also, an impact tube attached to a mercury column, or the ordinary Bourdon pressure gauge, is very useful for the study of jets, such as are employed in impulse wheels. Here, a design is required possessing sufficient strength and stiffness to resist the pressure of the flowing water (which may amount to values equal to 100 lbs. per square inch), and of such a form that spattering and splashing from the jet does not occur. Eckhart's design (*Proc. of Inst. of Mech. Engineers*, 1910) seems very excellent. A piece of sheet steel $\frac{3}{4}$ inch thick, and $2\frac{1}{2}$ inches wide, is drilled through its $2\frac{1}{2}$ inch width with a $\frac{3}{32}$ inch hole, and the back of this hole is connected by a fair curve with an $\frac{1}{8}$ inch steel tube, soldered to the back of the plate. The front of the plate, and the tip of the front end of the hole, are sharpened off to a knife edge; and the whole plate can be slid in and out of the jet by a screw and lock nuts working in a fixed grooved clamp.

Gauging by Chemical Methods.—The principle is very simple. Suppose that a weight of w lbs. of a chemical be added each second to a stream discharging Q cusecs, and that after thorough mixture a sample taken from the stream is found to contain 1 lb. of the chemical per n lbs. of water, then evidently:

$$\frac{w}{62 \cdot 5Q} = \frac{1}{n} \qquad \text{or, } Q = \frac{nw}{62 \cdot 5}.$$

The practical details require somewhat careful consideration, and investigations of the necessary conditions lead me to believe that, when these are properly fulfilled, the method is a very excellent one.

The two crucial points are: Firstly, that the chemical is added at a definite and constant rate. Secondly, that thorough mixture takes place before the sample is drawn off for analysis.

In order to ascertain the necessary conditions for ensuring these results, I carried out 84 tests on streams varying from 15 to 96 cusecs, flowing in earthen channels 5 to 20 feet in width, and with mean velocities ranging from 1·5 to 5 feet per second.

The obvious method of adding the chemical (which in 78 of my tests was common salt, and in the other 6, calcium chloride), is to dissolve it in water, and deliver the solution through a small orifice under a constant head. While a saturated solution is liable to deposit crystals in the measuring orifice, and so interfere with the regularity of the flow, it is plain that it is advisable to use a concentrated solution in order to handle as small a volume as possible.

Such a solution has a specific gravity greater than unity, and consequently the volume discharged cannot be calculated by ordinary rules, but must be observed by weighing the quantity discharged in a given period.

Owing to evaporation and impurities existing in all commercial chemicals, it is impossible to make up, day after day, a solution of a sufficiently constant specific gravity to produce a constant discharge, so that it is necessary to observe the discharge of the orifice with each fresh batch of solution.

In order to ascertain the conditions for a satisfactory mixture of the solution, and the water of the stream, samples were taken at points at various distances below the point where the solution flowed into the stream, and at different intervals during the period when the solution was in flow.

The solution was assumed to be thoroughly mixed with the stream when the proportions of the added chemical in the various samples thus obtained did not differ by more than 1 per cent. It is believed that under the circumstances of the analyses a variation of $\frac{1}{2}$ per cent. could be detected. Thus, it is possible that the following rules would require modification if they were employed by a more skilful analyst.

It would be impossible to quote details of over 1000 analyses, but the general results are as follows:

Let V_m, represent the mean velocity of the stream, and b, its breadth. Then, for streams with depths between $\frac{b}{10}$, and $\frac{b}{3}$, complete mixture (as above defined) does not occur until a distance of at least $6b$, has been traversed, and the discharge of the solution has continued for a period equal to at least $24 \frac{b}{V_m}$ seconds. Also, if, under these conditions, samples are taken more than

$\frac{6b}{V_m}$ seconds after the discharge of chemical has ceased, a diminution in the content of chemical occurs, owing to the stoppage of the addition of the chemical.

These results suggest that the ratio between the maximum and minimum velocities existing in the cross-sections of the streams experimented on, is always less than 2 to 1; which agrees very fairly with our general knowledge of the subject.

Let us now consider the practical effect of these conditions.

Assume a stream 20 feet wide, and 3 feet deep, carrying 90 cusecs, *i.e.* $V_m = 1{\cdot}5$ foot per second, or $\frac{24b}{V_m} = 320$.

Thus, we must add solution for 320 seconds, and sample about 120 feet below the point where the solution enters the stream.

Using the ordinary volumetric methods for the determination of chlorine, I found that it was possible to determine the fraction $\frac{1}{n}$ with an accuracy of 1 per cent. so long as n did not exceed 30,000; and that the task was less difficult when $n = 20{,}000$, or less. Thus, for the determination of a 90 cusec flow, with an accuracy of 1 per cent., it was necessary to add salt (NaCl) at the rate of at least 0·19 lb. per second, and preferably at 0·28 lb. per second. Hence, one gauging required an expenditure of at least 61 lb., and preferably 90 lbs. of salt; and the solution containing approximately ¼ lb. of salt per pound of solution (saturated solutions possessing 0·312 lb. per pound of solution), the least possible volume of solution was approximately 4, or 6 cube feet, and was added at the rate of 0·013 to 0·02 cusec.

In practice, since the available orifices discharged approximately 0·005, 0·01, 0·02, and 0·04 cusecs, the minimum quantity of salt consumed in this gauging was about 90 lbs., contained in 6 cube feet of solution; and the actual results were 110 lbs. and 7·2 cube feet, since the experiment continued for nearly 400 seconds.

This particular stream was the most difficult example dealt with, owing to its low velocity; but as a favourable case I found with:

A stream 3 feet deep, and 10 feet broad, with a mean velocity of 3 feet per second, *i.e.*

$$Q = 90 \text{ cusecs as before, but } 24\frac{b}{V_m} = 80,$$

only one quarter of the above quantities were required.

The method is capable of great accuracy. Thus, in determining a stream of approximately 7 cusecs, which was passed over a weir (not for measurement, but as being the best method of keeping the discharge constant), the results of eight observations gave:

$$\text{Discharge} = 7{\cdot}473 \pm 0{\cdot}15 \text{ cusec},$$

i.e. a probable error of 0·2 per cent.

If this be the true accuracy of a chemical gauging, it is plain that competition by any other method is hopeless.

The real criterion for the adoption of this system is: Does thorough mixture occur? Thus, it is admirably adapted for such purposes as the determination of the coefficients of discharge of weirs or orifices, and of the quantity of water utilised by all types of hydraulic machinery. It is less suitable for the gauging of rivers ; large and placid streams requiring considerable quantities of chemical, and the preparation of much above 8, or 9 cube feet of solution, of uniform composition, is a difficult matter.

If a bulk of solution much exceeding 4 or 5 cube feet is necessary, very careful and systematic mixture of the solution is needed.

As an example, a volume of 11 cube feet of salt solution, was discharged at a rate of about 0·01 cusec, and samples were taken every 100 seconds. The analyses were as follows :

Time.	Per cent. Content in Salt.	Time.	Per cent. Content in Salt.
0	22·8	600	22·4
100	22·7	700	22·6
200	22·6	800	22·8
300	22·7	900	22·8
400	22·9	1000	22·9
500	22·6		

The maximum content is 22·9 per cent., and the minimum 22·4 per cent., or a variation of 2 per cent. in the strength of the solution occurs. The stirring and mixing was as systematic as was possible with unlimited labour, but no mechanical appliances were employed. The volumetric method used was certainly capable of detecting variations in the strength of the solution of 0·5 per cent., and is believed to be accurate to 0·2 per cent. Thus, it may be inferred that at least 1·5 per cent. variation in strength occurred in certain portions of the solution, and these differences cannot be avoided unless special mixing tanks are provided.

The above information is entirely founded on experiments conducted by the addition of chlorides to the stream. The chlorine was estimated volumetrically by silver nitrate, with potassium chromate as indicator. Reference is made to Sutton's *Volumetric Analysis* for details.

The method has many practical advantages, and was well adapted to the water experimented on, which was naturally almost free from chlorides. If chemical difficulties alone are considered, greater accuracy can be obtained by adding acid (preferably sulphuric acid), or an alkali (preferably caustic soda). The best substance depends on the salts originally present in the water.

The following table is derived from one given by Stromeyer (*P.I.C.E.*, vol. 160, p. 351).

Column 3, 4, 5, and 6, indicate the figures which I believe should be obtainable by volumetric methods, by an engineer who is not a skilled chemist. Column 5 is calculated on the supposition that dilutions only two-thirds as great as those employed by Stromeyer are advisable, and the volume of solution in Column 6, is calculated on the basis that a solution is employed which is only five-sixths saturated.

Such concentrated solutions of common salt, and calcium chloride, carbonate and bicarbonate of soda, etc., are easily handled ; but caustic soda and sulphuric acid should be used in a more diluted form, as the concentrated solution is very viscous.

Chemical.	Weight of Anhydrous Chemical in 1 cubic Foot of saturated Solution.	Dilution of this saturated Solution that will just give an accuracy of 1 p.c. by Volumetric Methods.	Ratio: Weight of Chemical to Weight of Water under these Circumstances.	Lbs. of Chemical per Second per Cusec of Stream Discharge that should be used in Practice.	Approximate Volume of partially saturated Solution used per Second per Cusec of Stream Discharge.	Maximum Value of *n* that permits 1 p.c. accuracy to be attained by gravimetric Methods.	Remarks.
	Lbs.						
Common salt (NaCl) . .	19·5	1 in 12,160	1 in 39,000	0·0024	$\frac{1}{6700}$	77,000	Reliable and cheap.
Chloride of calcium ($CaCl_2$) . .	29·4	1 in 21,490	1 in 45,700	0·0020	$\frac{1}{12000}$	125,000	Reliable; less bulky than brine.
Chloride of magnesium ($MgCl_2$).	29·3	1 in 24,360	1 in 52,000	0·0018	$\frac{1}{13500}$	140,000	Reliable.
Sulphuric acid (H_2SO_4) . .	115·0	1 in 132,300	1 in 132,300	0·0007	(?)	500,000	Reliable, but dangerous to handle.
Caustic soda (NaHO) . .	76·4	1 in 213,500	1 in 175,000	0·00054	(?)	...	,, ,,
Carbonate of soda (Na_2CO_3) . .	27·8	1 in 29,500	1 in 66,000	0·0014	$\frac{1}{16000}$	...	...
Bicarbonate of soda ($NaHCO_3$).	5·1	1 in 6,250	1 in 77,000	0·0012	$\frac{1}{3300}$	...	Reliable, but bulky.

Column 7, is calculated for gravimetric methods, on the assumption that a 1 litre sample of the mixture is taken, and that the precipitate obtained should then weigh 0·01 gramme, a saturated solution being used.

My own experience with gravimetric work has been unfortunate, but I attribute this entirely to lack of skill, and consequently am unable to state whether these figures are as reliable as the others.

It will be noted that Mr. Stromeyer, being a skilled analyst, is able to get good results with considerably more dilute solutions than my limited experience permitted. It will be seen, however, that even with his values we must be prepared to deliver fairly large volumes of solution, unless the costly and tedious gravimetric methods are employed.

The above considerations permit us to state that the chemical method is:

(*a*) Best adapted for systematic gaugings of a stream in such work as turbine testing, where a suitable installation for chemical gaugings would allow us to dispense with the cost of installing a weir, and the loss of head entailed.

(*b*) The necessary apparatus entails greater first cost, but the expense of each gauging is far less than that of a current meter gauging, and requires a shorter time.

(*c*) The probable error, in the case of quantities under 100 cusecs, is about one half that of a current meter gauging, and is approximately equal to that of a good weir gauging.

Having expressed the opinion that the method is quite inapplicable to pioneer, or reconnaissance work, I feel it due to Mr. Stromeyer to state that he gives instructions for what he terms a "gulp method"; which consists in pouring solution containing a known weight of chemical as quickly as possible into a stream, and taking a continuous sample of the resulting mixture.

My tests of this system (when its results were checked by those of weir and rod float determinations) were decidedly adverse, as I found that errors of 10, to 15 per cent. occurred. I must, nevertheless, acknowledge that the process obviously needs very careful and well thought out preparations for securing continuous samples of the mixture, and that my arrangements were by no means satisfactory.

It appears that such a sampling apparatus, when practical, lacks portability, and is therefore unlikely to be employed in reconnaissance work.

The method has one very practical application. It can be used to rapidly and cheaply determine the coefficients of discharge of weirs, or other falls of water. Here the conditions for producing a thorough mixture are very efficient, and an addition of the chemical for periods much exceeding one minute is unlikely to be required.

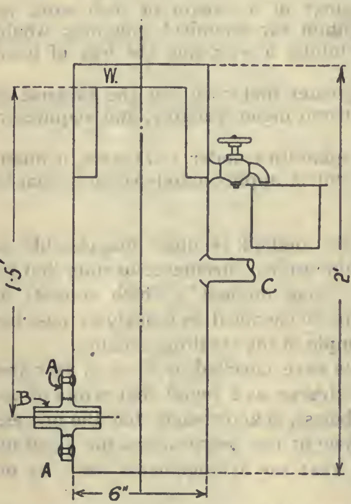
Let it be assumed that we wish to determine a 1000 cusec flow, and are using common salt.

A solution containing 16 lbs. per cube foot is easily made up, and such a solution can be detected with an accuracy of 1 per cent. when diluted to 1 in $12160 \times \frac{16}{19 \cdot 5} = 1$ in 10,000.

Thus a flow of 0·1 cusec will suffice, or we can use about 6 cube feet of solution and 96 lbs. of salt per minute.

For gravimetric methods, let us suppose that an accuracy of 0·5 per cent. is required, and that the balance available will indicate 0·1 mgrm. We therefore wish to have 20 mgrms. of precipitate, and find that with a 16 lbs. per cube foot solution, 1·64 cube mm. will give 1 mgrm. of precipitate; or, that our sample must contain the equivalent of 32·8 c.mm. of concentrated solution. Now, assume that we take a litre sample, equal to 1000 c.c., or 1,000,000 c.mm. Thus a dilution of $\frac{1,000,000}{32\cdot 8} = 30,000$, is permissible, or say approximately three parts per 100,000. Or, we could use 2 cube feet per minute, and 32 lbs. of salt; and if we chose (as a skilled chemist probably would) to work with 5 litres, and a balance indicating $\frac{1}{10}$th of a mgrm., we could work with 0·04 cube foot per minute, and about 0·64 lb. of salt.

Any of these dilutions, however, are approximately identical with the quantities of salt normally occurring in British waters, and in such cases it is plain that sulphuric acid will probably give more accurate results. In any actual case, a preliminary study of the natural chemical contents of the water is required, and the advice of a skilled chemist is useful.



SKETCH NO. 13.—Apparatus for Chemical Gauging.

PRACTICAL DETAILS.—The only real difficulty lies in the steady discharge of the solution. Sketch No. 13 shows the arrangement which I finally adopted The vessel was 6 in. × 6 in. × 2 feet high, and discharged through a small Borda mouthpiece B, screwed into the plate AA, riveted to its side.

The solution was fed to the vessel by a pipe C, which was provided with a tap (not shown) for approximately regulating the flow. Any excess in the discharge at B, was allowed to escape by the overflow weir W, and was drawn off by the second tap.

The calculations are fairly plain. The head over the orifice measured from B, to W, is 1·5 foot. The maximum discharge which it is intended to pass is 0·04 cusec. Suppose that C, supplied 0·05 cusec, then 0·01 cusec must escape over the lip W, which is 18 inches long. The rise in the water surface above W, is then given by:

$$1\cdot 5 \times 3\cdot 33\, h^{1.5} = 0\cdot 01, \quad h^{1.5} = 0\cdot 002. \quad \text{Therefore, } h = 0\cdot 015 \text{ foot,}$$

and the discharge through B, is consequently:

$$0\cdot 04 \times \sqrt{\frac{1\cdot 515}{1\cdot 5}} = 0\cdot 04 \times 1\cdot 005 \quad \text{cusec.}$$

So that, even with a very rough regulation of the tap on C, the discharge through B, will not vary ½ per cent.

Care being taken to accurately regulate the tap, the actual discharge is easily kept constant to 0·1 per cent.

I found that a set of orifices discharging approximately 0·005, 0·01, 0·02, and 0·04 cusec were quite sufficient for practical use.

So far as my experience goes, discharges up to 100 cusecs can always be gauged with an accuracy of 1 per. cent., provided that 10 cube feet of solution can be made up, and can be thoroughly mixed, and that orifices delivering 0·005, 0·01, 0·02, and 0·04 cusec (approximately) are available. For larger discharges the table will permit the necessary quantities to be estimated.

The chief difficulties (other than chemical ones) will be found to arise as follows :

(i) In preparing a large volume of dosing solution of uniform strength.

(ii) In determining when complete mixture has occurred.

The chemical difficulties are probably easily surmounted by a practical chemist. To an engineer unprovided with an accurate balance for making up the test solutions the following points appear to be the most important :

(i) The determination of the end point of the reaction.

This is very marked if an acid or alkali be employed, but is somewhat blunt when chlorides are estimated.

(ii) In hot climates the strengths of the solutions employed are affected by evaporation. This can be allowed for (since the quantities required are relative and not absolute) by blank experiments, but these are tedious. There is, however, little doubt that any large scale gauging operations of sufficient importance to occupy two engineers, could be better effected by an engineer assisted by a chemist. Judging by my own experiments, the cost in labour and material will be less than half that required for current meter gaugings of the same magnitude. The only possible exception is that of a large and very slow flowing river ; and such cases are easily detected by a preliminary experiment with colouring matter, as described under the heading Pipes.

VENTURI METERS.—The Venturi meter was first applied to water measurement by Clemens Herschell. It consists of a double cone forming a constriction in a pipe, as shown in Sketch No. 14.

Let the area of the unconstricted pipe at S, be represented by A_s, and let A_t, represent the area of the throat, or smallest portion of the constriction. Put $\rho = \frac{A_s}{A_t}$. Let v_s, and v_t, be the corresponding velocities, and p_s, and p_t, be the pressures in feet of water at these points.

Then, assuming that the flow is regular :

If Q be the quantity of water passing per second, we have :

$$Q = v_s A_s = v_t A_t, \text{ or } v_t = \rho v_s$$

Neglecting friction, and other losses in the cone ST, we have :

$$p_s + \frac{v_s^2}{2g} = p_t + \frac{v_t^2}{2g} \qquad \text{or, } \frac{v_s^2}{2g}(\rho^2 - 1) = p_s - p_t$$

$$\text{Thus,} \quad Q = A_s v_s = A_s \sqrt{\frac{2g(p_s - p_t)}{\rho^2 - 1}}$$

Similarly, if we assume that the frictional losses between S, and T, are equal to h_1 feet of water :

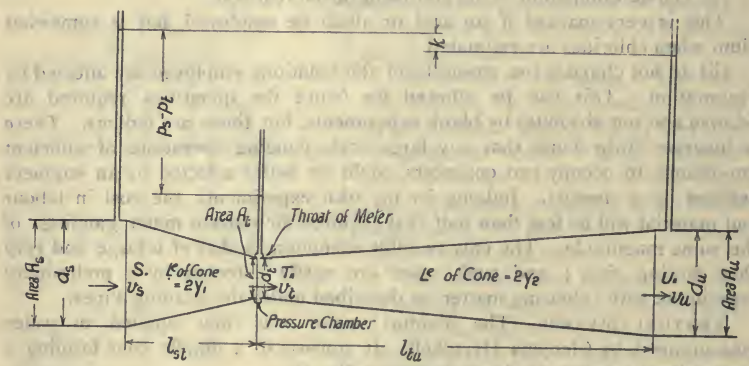
$$\text{We have,} \quad Q = A_s \sqrt{\frac{2g(p_s - p_t - h_1)}{\rho^2 - 1}}$$

Now, h_1, is small. Thus, in practice a corrective factor is allowed for the neglected shock and skin friction. So we get:

$$Q = CA_s\sqrt{\frac{2g(p_s - p_t)}{\rho^2 - 1}}$$

In actual working it is found that C, is approximately equal to unity. It will also be plain that since the skin friction of water moving in smooth pipes does not vary exactly as the square of the velocity, C varies with v_s. Actually, however, these variations are very small; and for all practical purposes C, is constant.

Herschell (*Trans. Am. Soc. of C.E.*, vol. 17, p. 228) gives a very careful series of experiments on two Venturi meters, each with $\rho = 9$. But in one case A_s, was approximately a circle of 9 feet in diameter, while in the other A_s, was approximately a circle of 1 foot in diameter.



SKETCH No. 14.—Theory of the Venturi Meter.

The curves for C, show marked variations when v_t, is less than 10 feet per second. The values for the 9-foot pipe rise in a quasi-parabolic curve to C = 1·08, when v_t = 2 feet per second; and, in the case of the 1-foot pipe, fall in a similar curve to C = 0·75, when v_t = 1-foot per second.

For values of v_t, greater than 10 feet per second, however, C, is as follows:

v_t	Herschell's Values, $\rho = 9$.		Coker's Values for a 1·6-inch Pipe, $\rho = 18·4$.
	9-foot Pipe.	1-foot Pipe.	
10	0·980	0·980	1·368
20	0·970	0·990	1·003
30	0·960	0·992	0·978
40	...	0·995	0·967
50	...	0·999	0·956

Coker's values are taken from a smoothed curve of his results, as given in *Proc. of Can. Soc. of C.E.*, October, 1902.

The Venturi meter is probably a far more accurate water measurer than a weir, so that only the last two columns (which are obtained from direct volumetric measurements) need be considered as of use in determining the law of the variations of C.

Since the meter is so accurate, the following investigation will be found of practical use.

Assuming that $v=C_F\sqrt{rs}$, represents the law for all the losses in the Venturi meter, then the head lost in a frustrum of a cone of which the initial and terminal diameters are d_s, and d_t, and the vertical angle is 2γ, is given by the equation:

$$h_f = \frac{v_t^2}{C_F^2 \tan \gamma}\left\{1-\left(\frac{d_t}{d_s}\right)^4\right\}$$

where C_F may be taken as having a value very close to that which occurs in a pipe of an area A_t, when the velocity is v_t.

Thus, the frictional head lost in the upstream cone ST, is very approximately equal to:

$$h_1=\frac{v_t^2}{C_F^2 \tan \gamma_1}\left\{1-\frac{1}{\rho^2}\right\}$$

and the similar loss in the down-stream cone TU, is:

$$h_2=\frac{v_t^2}{C_F^2 \tan \gamma_2}\left\{1-\frac{1}{\rho^2}\right\}$$

and provided that the area at S, is equal to the area at U.

Thus, if a third gauge be established at the point U, and the difference between p_s, the pressure in the gauge at S, and the pressure indicated by the gauge at U, be k, we have:

$$k = h_1 + h_2 \qquad \text{and } h_1 = \frac{k \cot \gamma_1}{\cot \gamma_1 + \cot \gamma_2}$$

where γ_1, and γ_2, are the semi-vertical angles of the two cones. This last equation holds if the law of frictional resistance is of the form, $h_f = \frac{Kv^n}{d^m}$; provided that m, and n, are constant.

$$\text{Now, } \cot \gamma_1 = \frac{2l_{st}}{d_s - d_t} \qquad \cot \gamma_2 = \frac{2l_{tu}}{d_s - d_t}$$

$$\text{Thus, } h_1 = k\,\frac{l_{st}}{l_{st} + l_{tu}}$$

Hence, h_1, can be calculated.

In practice, the areas at S, and U, are not always exactly equal; and this should be allowed for by decreasing the observed difference in the pressures by the quantity $\frac{v_u^2 - v_s^2}{2g}$, regard being paid to the sign of this quantity.

It is, however, doubtful whether the investigation is entirely reliable in this case.

An experimental proof of the relation, $C^2 = \frac{p_s - p_t - h_1}{p_s - p_t}$, on a large scale is greatly to be desired.

Working with a very badly proportioned and roughly made constriction (it would be unfair to call it a Venturi meter), I found as follows :

v_t, feet per Second.	Calculated Value of C.	C, by volumetric checking.
11·7	0·983	0·987
12·8	0·991	0·988
14·2	0·984	0·986
21·4	0·973	0·977

The agreement is satisfactory ; but the methods of observation were not sufficiently accurate to enable any real reliance to be placed on the results, as the volumetric measurements are known to be subject to an error of 0·7 per cent. It is, however, plain that the method of correction adopted enabled an otherwise unreliable instrument to produce results which were quite as accurate as a volumetric measurement in the field.

It is believed that calculations founded on this corrected form of the Venturi equation permit greater accuracy in water measurement than any other method, and any inaccuracies are probably due to the difficulties of observing k, quite as much as to defects in the theory. It is, however, certain that the calibrated commercial Venturi meter which can now be obtained from many firms is so accurate that only volumetric methods, or the very best weir observations, can be employed to check it.

The following investigation of the possibilities of changes in C, is therefore only useful when extreme accuracy is required.

The losses represented by h_1, are due to skin friction, and changes in the direction of the velocities as the water passes through the cone. These last Alexander (*P.I.C.E.*, vol. 159, p. 341) has shown to be probably expressed by terms of the same form in v, as the frictional losses. Thus,

$$h_1 = \mathrm{K}v_s^n \text{ (where } v_s\text{, is the velocity in the pipe).}$$

Now, $p_s - p_t$, is approximately proportional to v_s^2.

Hence, approximately $C^2 = \dfrac{v_s^2 - \kappa v_s^n}{v_s^2}$

Now, so long as v_s, is greater than Osborne Reynold's higher critical value, n, varies between 1·75 and 2·1. (See p. 20.)

Thus, we have :

(i) v_s, is greater than v_d (or, as practically discovered by Herschell when v_t, is greater than a certain value),

$$C^2 = 1 - \frac{\kappa}{v_s^{2-n}}$$

So, for smooth, new pipes where n, is less than 2, we may expect that C, is less than 1 ; and that C, increases as v_s, or Q, increases.

For encrusted pipes, $n = 2$, and C, should be constant. For old, encrusted pipes, where n exceeds 2, C may be expected to decrease as v_s, increases.

(ii) If, however, v_s, is so small that the velocity falls below the critical value at some point in the meter (which Coker's experiments have shown may probably occur in cones, for values which considerably exceed those at which it occurs in cylindrical pipes) we have:

$$C^2 = \frac{(1 + \alpha)v^2 - \kappa v^n}{(1 + \beta)v^2}$$

where α and β are coefficients of the character discussed on page 15, and express the fact that the square of the mean velocity no longer accurately represents the mean energy of the motion of the water. Any estimation of the values of α and β is impossible; but it will be plain that the peculiar values of C, found by Herschell, when v_t, is small, do not conflict with theoretical results.

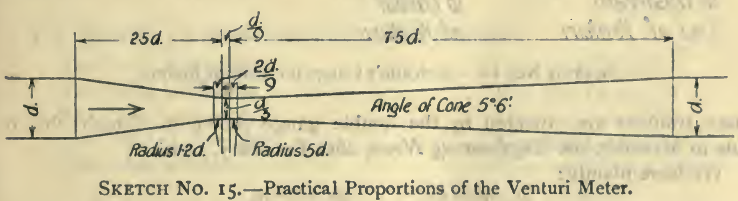
The practical effects of the above investigation may be summed up as follows:

(i) For each Venturi meter a minimum value of v_t, or Q, exists, and the meter coefficient C, varies very rapidly when Q, is less than this value. Under these circumstances, C, is dependent upon the temperature of the water, and upon the character of its motion before reaching the meter; so that the meter may be regarded as useless for measuring quantities less than this minimum value. The minimum value of Q, may be assumed to be determined by the fact that the friction law for a velocity $\frac{Q}{A_s}$, differs from that for a velocity $\frac{Q}{A_t}$; and this probably occurs when $\frac{Q}{A_s} = v_s$, is approaching Osborne Reynold's critical velocity, so that the minimum value of v_s, possibly varies as $\frac{1}{d_s}$, where d_s, is the diameter of the pipe.

This matter has been recognised by Herschell, and his rules are given later. They may be regarded as specifying v_s, as greater than 1 foot per second, and probably apply to pipes 12 inches in diameter, or larger. Coker's results show that v_s, should be still greater in the case of smaller pipes.

(ii) For values of Q, exceeding this minimum, C, is very nearly constant, and may be expected to increase slightly as Q, increases, for new pipes; but this increase will diminish, and may even possibly become a decrease as the meter grows encrusted, and ages like a pipe.

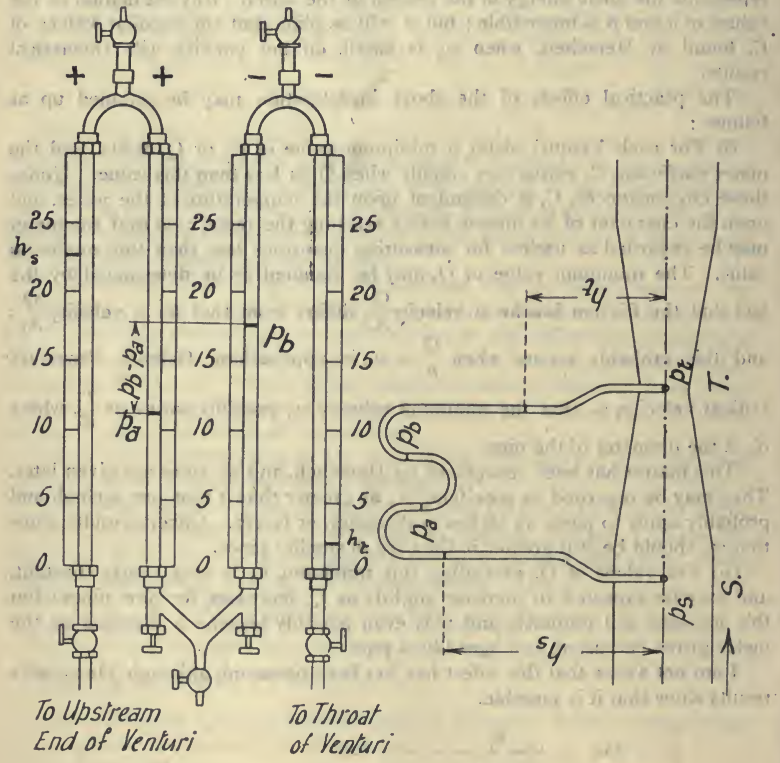
I am not aware that this effect has yet been observed, although Herschell's results show that it is possible.



SKETCH NO. 15.—Practical Proportions of the Venturi Meter.

Herschell has laid down standard proportions for the Venturi meter (see Sketch No. 15). When the approximate theory is followed, these should be adhered to, in order to obtain the advantage of his careful calibrations. The necessity is not so acute where the corrected theory is employed, and I have been unable to ascertain whether Herschell's proportions are founded on experiments. Herschell states that for values of v_t, exceeding 10 feet per second, C_r may be taken as 0·99 without very much error; and for values down to $v_t = 2$ feet per second, it is probable that C_r does not greatly depart from values between 0·99 and 0·96.

The gauge requires some consideration. The pipe may be under so great a pressure that the water columns would be inconveniently long; or, on the other hand, the pressure may be such that air would be sucked into it at T.



SKETCH No. 16.—Metcalfe's Gauge for Venturi Meters.

These troubles are obviated by the double gauge shown in Sketch No. 16 (due to Metcalfe, see *Engineering News*, 28th February, 1901).

We have plainly:

$$p_a = p_s - h_s \qquad p_b = p_t - h_t$$

Therefore, $p_s - p_t = (p_a - p_b) + (h_s - h_t)$,
which are all easily read on a scale of convenient length.

MEASUREMENT OF WATER BY A TRAVELLING SCREEN.—The principle of this method is simple. A very light screen of varnished canvas with a stiff framing of angle iron is hung from a wheeled carriage, and is allowed to move with the water along a short length of regular channel of rectangular section. The velocity of the screen is observed electrically; and, after certain corrections for leakages past its edges, this velocity is taken as the mean velocity of the water.

The method is described in the *Zeitschrift des Deutscher Ingenieure Verein*, of the 20th April, 1907. In the actual installation used for turbine tests, by Voith at Heidenheim, the channel is 2·992 metres wide, and the screen 2·972 metres wide; so that there is a clearance of 0·4 inch at each side. The depth of the stream is observed on either side of the screen, and the mean is taken for calculating the area of the stream.

The pressure producing leakage through the clearances is estimated from the rolling resistance of the screen, and is stated to be less than 0·0001 metre (0·004 inch) of water; and the coefficient of discharge through the clearance is taken as 0·65, both at the sides and at the bottom of the channel.

It is maintained that the method is more accurate than a weir gauging.

It is quite evident that the apparatus is complicated, and that the preliminary calibration is arduous. The actual gauging, however, is very simple; and, owing to the velocity being electrically recorded, the results are obtained directly in a sense that mere written data of a gauge reading cannot be. The only uncertainty is in respect to leakage, which, in Voith's application of the method, is so small a fraction of the total discharge, as to be of little moment.

It would therefore appear that for permanent work on water which is absolutely free from drift and silt, the method is probably one of the best yet employed; since, once installed, it does not require the presence of a trained observer, nor is any head lost, as in the case of a weir. Its adoption is well worth consideration in such cases as large town water supplies, or power schemes where the water is clear, and passes through an open conduit, as the records can be worked up at a distance, and at leisure.

DISCHARGE CURVES.—The obvious method of determining the daily discharge of a river is to use the individual observations of discharge obtained by the various processes discussed above, in order to determine a relation between the quantity discharged, and H, the height of the river surface, as read on a fixed gauge at the point of observation.

We can thus determine a relation:

$$Q = f(H)$$

The following are actual examples of such formulæ:

The Loire at Roanne	$Q = 100(H + 0{\cdot}25)^{1.5}$
The Seine at Mantes	$Q = 95(H + 0{\cdot}70)^{1.5}$
The Isere at Grenoble	$Q = 121(H + c{\cdot}86)^{1.5}$
The Drac at Grenoble	$Q = 280(H - 0{\cdot}70)^{1.5}$
The Adda at Como	$Q = 100H^{1.5} - 3{\cdot}20H^{2.5}$

where Q, is expressed in cubic metres per second, and H, in metres.

The forms arrived at above appear to be somewhat influenced by theoretical considerations. The French rivers are expressed according to Boussinesq's theoretical formula:

$$Q = M(H + c)^{1.5}$$

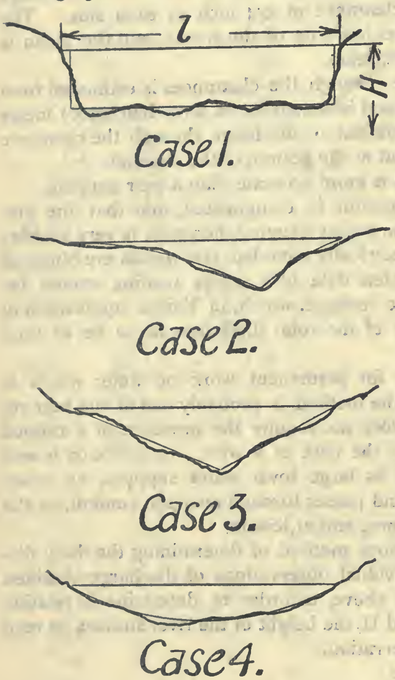
The Adda is expressed in Lombardini's formula:

$$Q = NH^{1.5} \pm PH^{2.5}$$

Evidently, the most general form is:

$$Q = a + bH + cH^2 + dH^3 + \text{ etc.}$$

The simplest method of determining such a formula is to plot the observations graphically, and to determine the form of the function $f(H)$, by trial and error, and then the constants can be ascertained by the method of mean squares.



SKETCH No. 17.—Types of Cross-sections of Streams.

The following mathematical investigation is founded on the assumption that, on the average, s, the slope of the water surface, is very nearly constant. The process permits a prediction of the general trend of the discharge curve to be made, together with possible irregularities in its shape, with a fair degree of accuracy, by a study of the form of the cross-section of the river-bed.

The advantages are plain. Although we must attend the pleasure of the river in order to obtain a discharge observation at any particular gauge, we nevertheless can study the cross-section of its channel at leisure.

The ordinary theory gives:

$$v = C\sqrt{rs}, \text{ and, } Q = vA = vpr$$

Where A, is the area of the cross-section;
r, is its hydraulic mean radius;
p, the wetted perimeter;
and v, the mean velocity.

Now, plot A, and r, graphically, as functions of H, the gauge reading. Let l, be the width of the river at the gauge H, we consequently have:

$$dA = l\,dH$$

Four cases usually occur (Sketch No. 17):

(i) l, is constant, *i.e.* the banks of the river are approximately vertical.

$l = a$, say. Then $A = ag$, where $g = H + c$, and a, is roughly the mean value of l, and c, is easily determined arithmetically.

(ii) The banks of the river become flatter as the gauge height increases. (see Sketch, Case 2).

$l = ag^n$, where n, is less than 1 say, and $A = \frac{a}{n+1} g^{n+1}$

(iii) The river-bed is approximately triangular in section.

$$l=ag, \text{ and } A=\frac{ag^2}{2}$$

(iv) The river-bed has a section resembling that of a saucer, the banks becoming steeper the higher the river rises.

$l=ag^m$, where m, is greater than 1.

$$A=\frac{a}{m+1}g^{m+1}$$

In the last three cases, $g=H+c$, and at first sight the determination of c, may appear to be difficult; but, in actual practice, once the curve of A, and H, is plotted, the matter becomes far clearer than any mathematical investigation can make it.

Now, in all these cases p, is very approximately proportional to l, so that $p=el$.

And $r=\frac{A}{p}$, may be written as $r=\frac{A}{el}=\frac{g}{(m+1)e}$

We thus get as a first approximation to the discharge curve:

$$Q = CA\sqrt{rs} = C\sqrt{s}\frac{a}{m+1}g^{m+1}\sqrt{\frac{g}{(m+1)e}}=K\sqrt{s}\,g^{m+1.5}$$

where $C = K\frac{(m+1)^{1.5}\sqrt{e}}{a}$

and all the quantities comprised in K, can be obtained by surveys of the cross-section of the river-bed.

The four cases above considered lead to the following:

$$\text{(i)} \quad Q = K_1\sqrt{s}\,g^{1.5}$$
$$\text{(ii)} \quad Q = K_2\sqrt{s}\,g^{1.5+n}$$
$$\text{(iii)} \quad Q = K_3\sqrt{s}\,g^{2.5}$$
$$\text{(iv)} \quad Q = K_4\sqrt{s}\,g^{1.5+m}$$

In practice the above theory is best applied by plotting log A, and log r, in terms of log g, and thus obtaining

$$A = a_1g^{m+1} \qquad \text{and } r = r_1g^{2x}$$

directly.

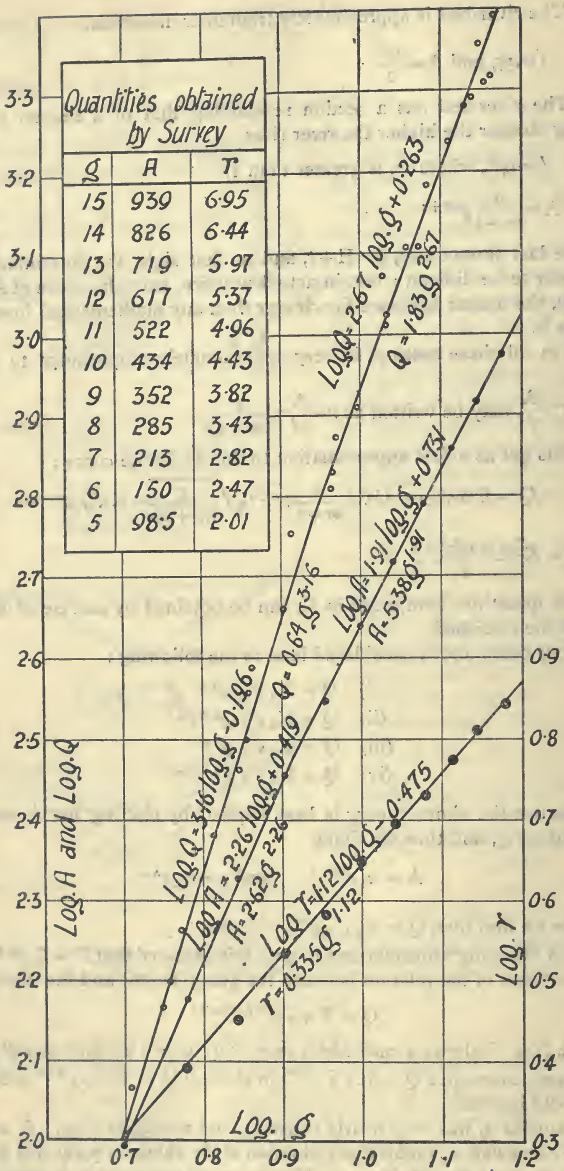
Thence we find that, $Q = K_1\sqrt{s}g^{m+1+x}$, etc.,

and, if as in Manning's formula (see p. 401), it is assumed that $C = C_1r^{0.17}$, we get as the final form of the relation between the gauge height and the discharge:

$$Q = K\sqrt{s}g^{m+x+1.17}$$

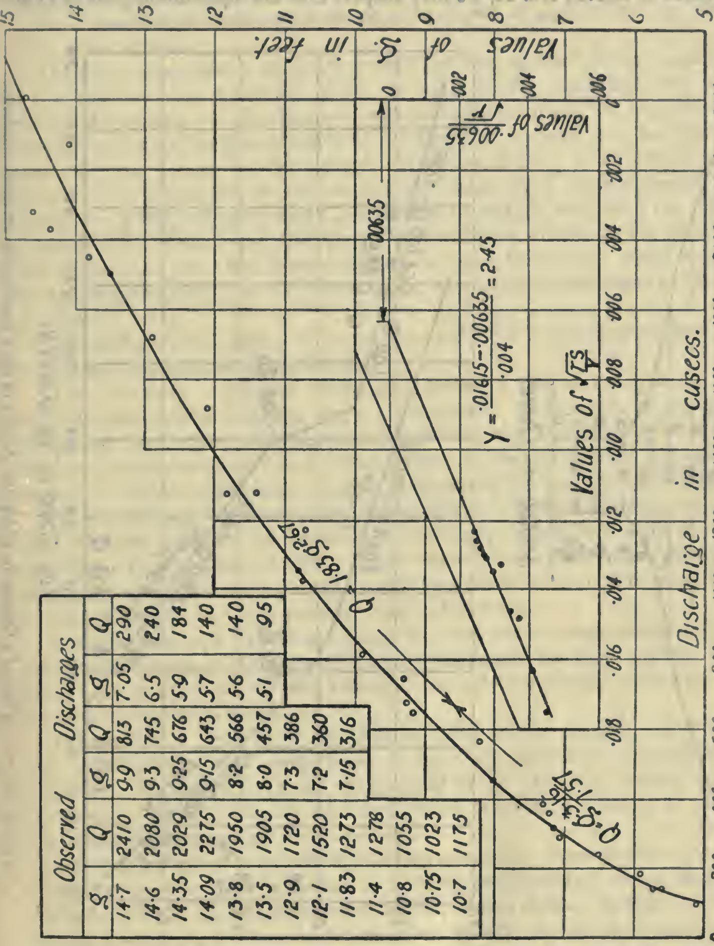
Sketch No. 18 shows a case where $m = 1.91$, $2x = 1.12$, and, as will be seen, the observed values give $Q = K\sqrt{s}\,g^{2.67}$ in place of $Q = K\sqrt{s}\,g^{2.64}$ as the above theory would indicate.

The value of s, was very nearly constant, and averaged $\frac{1}{6300}$. A smoothed discharge curve and a graphic determination of the values of γ, for this smoothed curve are given in Sketch No. 19. The value obtained for γ, is 2·45, and the results agree almost suspiciously well. Some compensating error in the determination of s, may be suspected.



Sketch No. 18.—Logarithmic Plot of Areas and Discharges of a Stream at various Gauge Heights.

It should be remembered that where some attempt at a discharge curve must be given, and only one gauging is available, the value of m, can be obtained by survey, and $K\sqrt{s}$, can be deduced from one observation only, which may be checked to some slight degree by observing s, and seeing whether the value of C, thence calculated, is a likely one.



Observed Discharges

g	Q	g	Q	g	Q
14·7	2410	9·9	813	7·05	290
14·6	2080	9·3	745	6·5	240
14·35	2029	9·25	676	5·9	184
14·09	2275	9·15	643	5·7	140
13·8	1950	8·2	566	5·6	140
13·5	1905	8·0	457	5·1	95
12·9	1720	7·3	386		
12·1	1520	7·2	360		
11·83	1273	7·15	316		
11·4	1278				
10·8	1055				
10·75	1023				
10·7	1175				

SKETCH No. 19.—Gauge Discharge Curve and Determination of Bazin's γ for Case of Sketch No. 18.

The practical value of the work, however, is far greater than such rough approximations indicate.

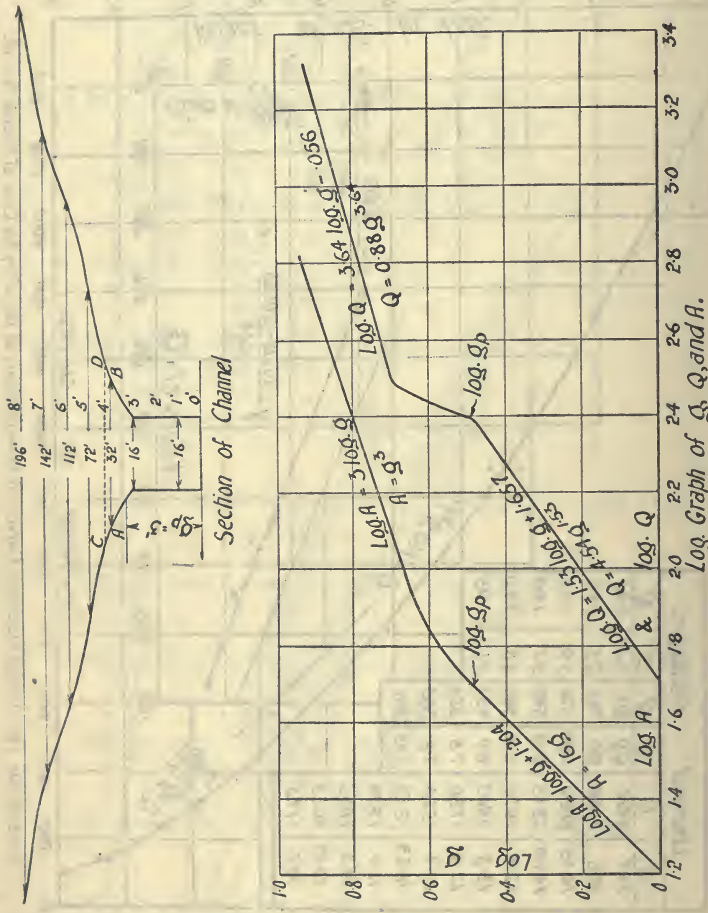
Firstly, by survey we obtain c, in the formula:

$$g = H + c$$

Now, plot the discharge observations against g, in logarithmic form.

The above work shows that the points (log Q_1, log g_1), (log Q_2, log g_2), etc. may be expected to lie very fairly close to a straight line, so long as the general aspect of the cross-section of the river-bed does not materially change inside the limits of g, under consideration.

Secondly, should the aspect of the cross-section change suddenly, as is indicated in Sketch No. 20, we may suspect that the logarithmic plots of Q and



Sketch No. 20.—Case of a Stream in which the Aspect of the Cross-section changes suddenly.

A will consist of two straight lines, probably connected by a curve. It will also be plain that discharge observations near $g = g_p$ are urgently needed.

Above and below $g = g_p$, a few observations may suffice (if in good agreement) to determine the curve over a wide range of H or g.

In my own practice I have found that the results of actual observation agree surprisingly well with the above theories. I have such confidence in the work as to use it in order to obtain by extrapolation the discharge at gauges which are higher than those for which discharges have been observed; and in twelve out of fourteen later checkings I found that my confidence had proved to be justified.

The process, however, must not be considered as universally applicable. The root assumptions are that:

(i) C, is either constant, or varies as some power of g, *i.e.* $C = C_1 g^y$ say.

(ii) s, is constant.

The first assumption is probably correct, so long as the hydraulic character of the bed does not materially alter. A study of the vegetation, and of the quality of the soil or silt deposits, will permit exceptions to be predicted. Alterations (usually in the direction of increased roughness) may be expected, when the river rises in high flood. Nevertheless, it is noteworthy that Bazin's γ is generally far more steady under such circumstances than Kütter's n; and a calculation of C, according to Bazin's rule, with the value of γ obtained in ordinary stages of the river, is sufficient allowance for this factor. Cases where the flood bed is encumbered by trees, fences, etc., must of course be excluded, but under such circumstances no really accurate observations can be taken.

The constancy of s, during large variations of H, or g, is a bigger assumption, and cannot be regarded as justifiable.

If we consider individual discharges, during a rise of the river, s, is increased; and during a falling stage, s, decreases. If, however, we regard the average of the discharges at the same gauge when the river is rising and falling, the assumption leads to but a slight degree of error; and for the practical purpose of obtaining the total quantity discharged, the assumption is justified.

Where damage by momentary flooding is ascribed to backing up of the water surface produced by works in a river bed, calculations founded on the assumption that s, does not widely depart from its usual value must be regarded with suspicion. An engineer will hardly be well advised to fight such a case on the assumption that s, is constant during the flood.

For this reason, a subsidiary gauge situated two or three miles above the observation gauge is valuable, as also are readings from such a gauge during floods; for in many rivers the variation in s, will explain all divergencies from the normal discharge curve.

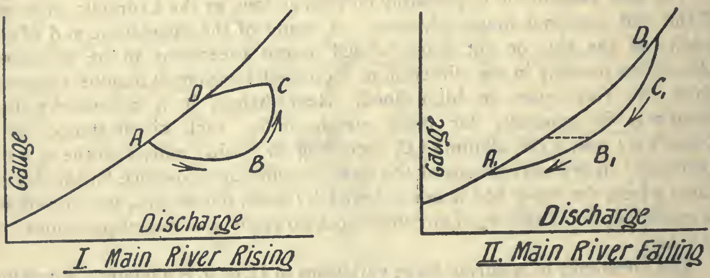
Influence of a Tributary.—In certain cases, as for instance on the Blue Nile at Khartoum, this explanation is quite insufficient to account for the difference. Craig (see Lyons' *Physiography of the Nile Basin*, p. 265), discusses the problem for the Blue and White Niles at Khartoum, in the light of Tommasini's theoretical investigations.

The general effect of a flooded tributary entering a river just below a gauge station (when the river is rising) is to shift the discharge curve as shown; where AD, is the normal curve, and ABCD, is the shifting due to the flooded tributary, where A, is the point where effect is first felt, and B, the point where the tributary begins to ebb (see Sketch No. 21).

When the river is falling, the upward rise of the curve in the portion DC, is not possible, and the loop is now as indicated in Sketch No. 21, Fig. 2. Where the tributary does not rise more rapidly than the main river, the portion DC, may be perpendicular to the gauge axis, thus producing a stage where gauge and discharge are unconnected.

This species of irregularity may be expected in all cases where a gauge is established, close to, and above the junction of two rivers.

The circumstances of a properly selected gauging station are usually such that all variations in discharge caused by rising and falling stages, can be amply allowed for by drawing two discharge curves. One curve represents the falling, and the other the rising stage. The difference does not generally exceed 5, or 7 per cent.; while in a really satisfactory site, 2, or 3 per cent. is more usual.



SKETCH No. 21.—Effect of a Tributary on the Gauge-discharge Curve of a Station upstream of the Junction.

Rivers with Shifting Beds.—The foregoing work refers entirely to rivers with a fairly stable bed. Where the river carries much detritus, the aspect of the cross-section may largely depend on the stage of the river, or on the motion of sand or gravel bars travelling down the stream.

The question of these "deeps" and "shallows" has been the subject of much discussion, and two theories are usually put forward.

According to German ideas, such waves in the bed travel down the river at a fairly constant rate, preserving their form unaltered.

For example :

> In the Rhine between Basle and Coblentz, the velocity is 735 feet (225 metres) per year.
> In the Elbe, 820 feet (250 metres) per year.
> In the Vistula, 1310 feet (400 meters) per year.
> In the Waal, 820 feet to 1640 feet (250 metres to 500 metres) per year.
> In the Loire, 1200 feet (365 metres) per year.

Most Indian engineers would be disposed to agree with this statement.

Lokhtine, in Russia (*Mechanisme du Lit Fluviale*), as also the majority of French engineers, and (so far as I can gather) the American engineers working on the Mississippi, consider that deeps and shallows are fixed with respect to the horizontal meandering of the river, and only alter their position to any marked degree when the river shifts its bed. The divergences may be reconciled by the statement that in a natural river, which is not restricted by training works, the deeps and shallows are mainly fixed by its horizontal plan. If the river is forcibly straightened and is restricted to a determined course, deeps and shallows move according to the German theory; and apparently the usual German method of regulation by spur dykes accentuates this movement somewhat more than regulation by dykes parallel to the general flow of the river.

The explanation is not complete, as the Indian rivers are entirely unregulated ; but it is quite possible that further study will show that fixed deeps and shallows also exist in these rivers, and several cases of a fixed deep are already known.

The question is of little importance as regards its effect on discharge curves. It will be found in certain rivers that the bed alters in form at the gauging site ; and consequently, in place of obtaining one approximately definite relation between Q, and H, we find (after long studies) that a sheaf of two or more discharge curves exists, as in Sketch No. 22.

The question has been investigated by Tavernier (*Études des Grandes Forces Hydrauliques des Alpes*, vol. 1, p. 160). It appears that after three or four years' study it is usually possible to reduce the sheaf to three or four curves, and that the change from one curve to another occurs not gradually, but *per saltum*, one curve being accurate for three or four months at a time, and then another curve becoming applicable the next day.

It would also appear that this sheaf of three or four curves recurs year after year.

Such studies require a longer time than can be afforded by anybody except a Government Department. The practical method is to institute systematic surveys of the cross-section, and to divide the sections thus obtained into three or four classes, and then to treat the discharge observations of each class as though they referred to separate rivers, using the methods already developed.

In such work it would be as well to take into account Tavernier's statement that as the waves of sand or gravel above referred to pass the gauging station, the effective slope may differ considerably from the mean slope of the river bed, or even from the mean slope of the water surface over say a length of a mile. In Tavernier's actual example, s, varies from 0·0086 to 0·0014, the mean value being 0·0050.

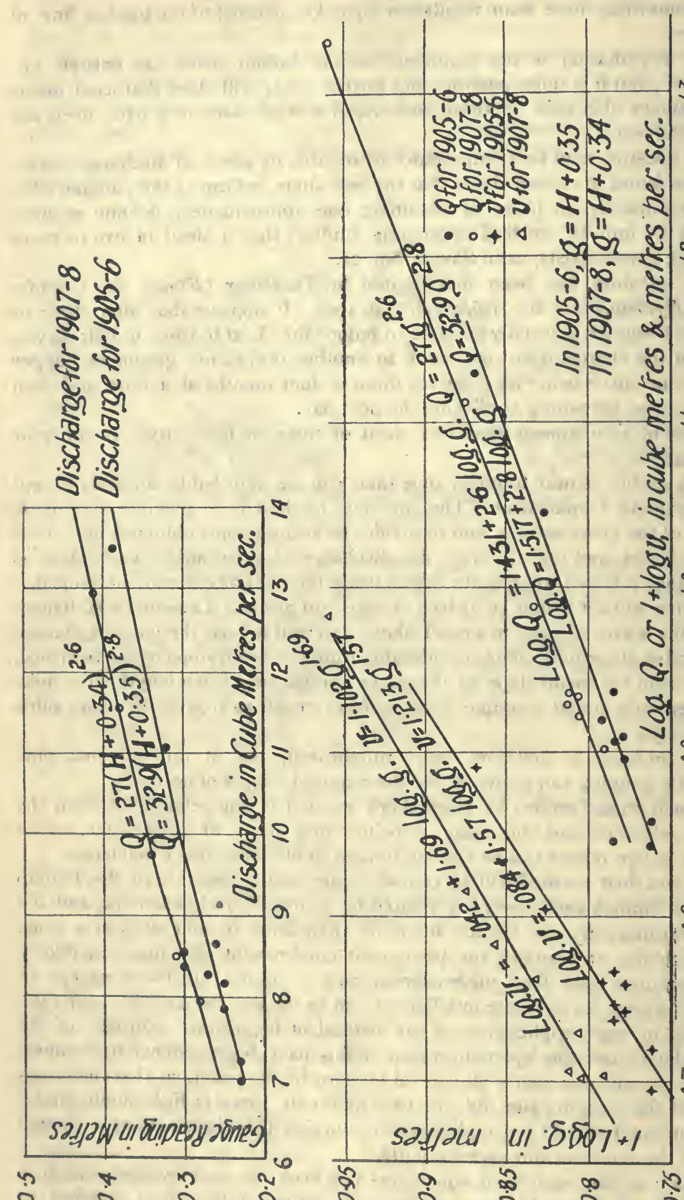
The problem is therefore very complicated, and it appears that only systematic gauging can really secure the required degree of accuracy.

In such cases, studies by Harlacher's method of the relation between the surface velocities and the mean velocities may prove of great value, as an ordinary gauge reader can be rapidly trained to observe surface velocities.

The rod float method will of course secure better results. In the Punjab Irrigation Branch such cases are treated by systematic rod float work, and the gauge readings are not trusted for more than three or four days at a time. This evidently necessitates the permanent employment of a man capable of taking accurate rod float observations, and is hardly justifiable except in countries where the requisite intelligence can be obtained at a fairly cheap rate.

While my own applications of the method of logarithmic plottings of the mean velocity and the hydraulic mean radius have been confined to channels in which the silt was nearly all carried forward by the water, so that the cross-section at the gauging site did not vary markedly, there is little doubt that a similar method should be applied when studying the discharges of a channel of which the cross-section varies rapidly.

So far as any statement which has not been checked by observation is worth making, it would appear that each curve of the sheaf referred to above should be represented by a straight line on the logarithmic diagram



SKETCH No. 22.—Logarithmic Plot of the Discharge of the Isere, showing Influence of Changes in the Bed of a River.

(see Sketch No. 22). Confirmation of Tavernier's statement that the change from one curve to the other occurs *per saltum* would then be easily obtained, and is greatly to be desired.

In this connection it must be noted that the method adopted by the United States Geological Survey for obtaining the gauge-discharge relation at a station where the bed is shifting, assumes that the changes occur gradually. It is hard to believe that the simplification in calculation which is possible if Tavernier's statement is generally true would have been overlooked by the United States experts.

CHAPTER IV

GAUGING BY WEIRS

Weir Formulæ.—Definitions.
Measurement of the Head.—Effect of errors—Gauge pits, or stilling wells—Effect on coefficient.
Weir Discharge Formulæ.—Correction for velocity of approach—Francis formula—Practical formulæ—Definitions—Contraction.
Practical Rules.—Francis' weir formula—Weir formulæ as applied to measurement of discharge—Accuracy.
Formulæ of First Class.—FRANCIS' FORMULA—Description—Conditions.
BAZIN FORMULA.—Description—Conditions—Approximate formula—Shorter formula—Corrections when nappe expands.
General Formulæ.—With end contractions—Without end contractions.
GENERAL AGREEMENT OF WEIR DISCHARGE FORMULA.—Corrections for velocity of approach—Distance at which head is measured—Experimental results—Correction for end contractions.
Sharp-edged Notches of other than Rectangular Form.—Triangular—Trapezoidal—Cippoletti.
Weir Gauging of Water containing Silt.—Triangular and Cippoletti.
Suppression of Contraction.—Experimental weir formulæ—Freeze's formula—Logarithmic plots.
Inclined Weirs.
Oblique Weirs.
WEIR WITH ROUNDED EDGES.
DROWNED WEIRS.—*FRANCIS' FORMULÆ.*
Fteley and Stearns' Experiments.
HERSCHELL'S FORMULA.—Triangular notch.
BAZIN'S FORMULA FOR NOTCHES WITHOUT SIDE CONTRACTIONS.—Nappe forms—Discrimination of Cases.
WEIRS WITH OTHER THAN SHARP-EDGED NOTCHES.
FLAT-TOPPED WEIRS.
TRIANGULAR WEIRS.—Do. with curved downstream prolongation.
WATER-FALLS.
Coefficients of Discharge for Large Weirs.
Sharp-edged Weirs.
Flat-topped Weirs.—With down-stream slopes—Drowned ditto.—Drowned weirs.

SYMBOLS.

A, is the area in square feet of the cross-section of the channel of appoach at the point where D, is measured.
B, is the breadth of the channel of approach in feet.
a and *b* (see p. 102).
C, is the coefficient of discharge of the weir when the formula $Q = CLH^{1.5}$ is used.
C^1, is used in the formula $Q = C^1LD^{1.5}$, when distinction is required, and similarly $Q = KD^{1.5}$ or KD^n, and $Q = \frac{2}{3} MLH\sqrt{2gH}$ are used for distinction if necessary.
D, is the observed head in feet.

d, d_1 (see p. 121).
D_2 (see p. 130).
$h=\frac{v^2}{2g}$, where v, is the velocity of approach in feet per second.
H, is the corrected head = D + ah, etc. (see p. 104).
H_1, and H_2 (see p. 122).
k, is a coefficient used for various ratios (see p. 130).
K (see pp. 105 and 119).
K_1 (see p. 105).
L, is the length of the sill of the weir notch in feet.
L_1, is the width in feet of a triangular notch at a height D, above the vertex.
L_B, L_F (see p. 113).
l (see p. 102).
m (see p. 102).
N, is used in the formula $Q = KH^N$. n (see p. 105).
p, is the height of the notch sill above the bottom of the approach channel.
p_1, and P_0 (see pp. 121 and 124).
Q, is the discharge over the weir in cusecs.
Q_B, Q_F (see p. 113). Q_s (see p. 126).
s, is the slope of the face of a triangular weir (see p. 130).
$t = D + p$ (see p. 118).
w, is the width in feet of the top of a broad-crested weir (see p. 128).
v, is the velocity of approach in feet per second.
z, is the slope $\frac{z}{2}$ horizontal to 1 vertical of the sides of triangular notches (p. 114).
α, is a coefficient in the equation $H = D + \alpha h$.
ϵ (see p. 119). η (see p. 118).
χ (see p. 102).

SUMMARY OF FORMULÆ.

Velocity of approach. $v=\frac{Q}{A}$ $h=\frac{v^2}{2g}$ (see p. 99).

Errors in measurement of the head. $\frac{\delta Q}{Q}=\frac{3}{2}\frac{\delta D}{D}$.

Note.—All formulæ are subject to error if D is less than 0·30 to 0·40 foot.

Francis' Formula (see p. 105).

$$Q = 3{\cdot}33\,(L - nH \times 0{\cdot}1)H^{1{\cdot}5}, \text{ end contractions,}$$
$$Q = 3{\cdot}33LH^{1{\cdot}5}, \text{ no end contractions.}$$

with $H^{1\cdot5} = (D + h)^{1\cdot5} - h^{1\cdot5}$, or $H = D + h\left(1 - \frac{2}{3}\sqrt{\frac{h}{D}}\right)$

or, $Q = 3{\cdot}33\left\{1 + 0{\cdot}25\left(\frac{D}{p + D}\right)^2\right\}LD^{1\cdot5}$, no end contractions.

Bazin's Formula for weirs with no end contractions (see p. 109).

$$Q = \left(3{\cdot}248 + \frac{0{\cdot}0789}{D}\right)\left\{1 + 0{\cdot}55\left(\frac{D}{p + D}\right)^2\right\}LD^{1\cdot5},$$

$$\text{or, } Q = \left\{3{\cdot}41 + 1{\cdot}69\left(\frac{D}{p + D}\right)^2\right\}LD^{1\cdot5}.$$

General Formulæ for weirs with end contractions, D, greater than 0·40 foot (see p. 112).

$Q = 3{\cdot}110\,L^{1\cdot02}\,H^{1\cdot465}$. L, less than 4 feet
$Q = 3{\cdot}122\,L^{1\cdot016}\,H^{1\cdot475}$. L, between 4 and 10 feet
$Q = 3{\cdot}148\,L^{1\cdot013}\,H^{1\cdot485}$. L, greater than 10 feet
(all three) $H = D + 1{\cdot}4h$.

Triangular Notches (see p. 114).—

$z = \frac{1}{2}$. $Q = 0{\cdot}707D^{2\cdot5}$.
$z = 1$. $Q = 1{\cdot}31D^{2\cdot5}$.
$z = 2$. $Q = 2{\cdot}55D^{2\cdot5}$.
$z = 4$. $Q = 5{\cdot}30D^{2\cdot5}$.

Cippoletti.—

$Q = 3{\cdot}367LH^{1\cdot5}$. $H^{1\cdot5}=(D + h)^{1\cdot5} - h^{1\cdot5}$.
$Q = 3{\cdot}301LH^{1\cdot5}$. $H = D + 1{\cdot}4h$.

Constricted Cippoletti (see p. 117).—

$Q = 3{\cdot}82D^{1\cdot57}$. $L = 1$ foot.
$Q = 5{\cdot}42D^{1\cdot57}$. $L = 1{\cdot}5$ foot.
$Q = 7{\cdot}38D^{1\cdot57}$. $L = 2$ feet.

Submerged Weirs.—

$Q = 3{\cdot}33\ L(H_1 - H_2)^{1\cdot5} + 4{\cdot}60LH_2\sqrt{H_1 - H_2}$ (see p. 122).
$Q = 3{\cdot}33\ KLH^{1\cdot5}$. See page 123 for table of K.

Broad-crested weir. $Q = Q_s\left(0{\cdot}7 + 0{\cdot}185\frac{D}{w}\right)$ (see p. 128).

Weirs.—A weir is essentially an irregularity in the stream bed, over which the water falls in a sheet of a certain depth. It is found experimentally that the observation of the absolute value of this depth permits the quantity of water passing over the weir to be calculated by formulæ of a more or less simple character, provided that the weir is properly constructed.

The usual terminology is somewhat redundant, and I therefore suggest the following:

The term "Weir" refers to the whole constructional apparatus used to produce the definite sheet. In a weir there is a more or less well defined and measurable orifice through which the water flows, which determines the form of the issuing stream. This orifice I propose to define by the restricted term "Notch," which was formerly used as an equivalent for a weir, especially in America.

The circumstances and form of the issuing stream have an appreciable effect on the discharge, and the term "Nappe" (used by Bazin—in English, "Sheet") will be employed to define the issuing stream.

The term "Channel of Approach" defines the body of water immediately upstream of the weir in which is situated the gauge on which the thickness of the nappe produced by the weir is measured. The velocity of approach is the mean velocity of the water in the channel of approach at the point where the measurement of the head is made.

The "Head" is the measured height of the water surface in the channel of approach above a fixed point in the weir, usually the lowest portion of the notch. When this is corrected for the velocity of approach by the formulae which will be given later, the term "Corrected Head" is used.

As a matter of experiment, neither the head nor the corrected head accurately represent the nett thickness of the nappe. The measurement of this last quantity is attended by great experimental difficulties, and even Bazin's most exhaustive researches have not satisfactorily determined the relation between the nappe thickness and the head, or the corrected head. It is therefore better to regard the head as an observed quantity, and its corrected value as a quantity which the results of observation show to be more closely connected with the discharge over the weir than the observed head.

The height of the sill or lower boundary of the notch above the bottom of the approach channel is important in some of the discharge formulæ.

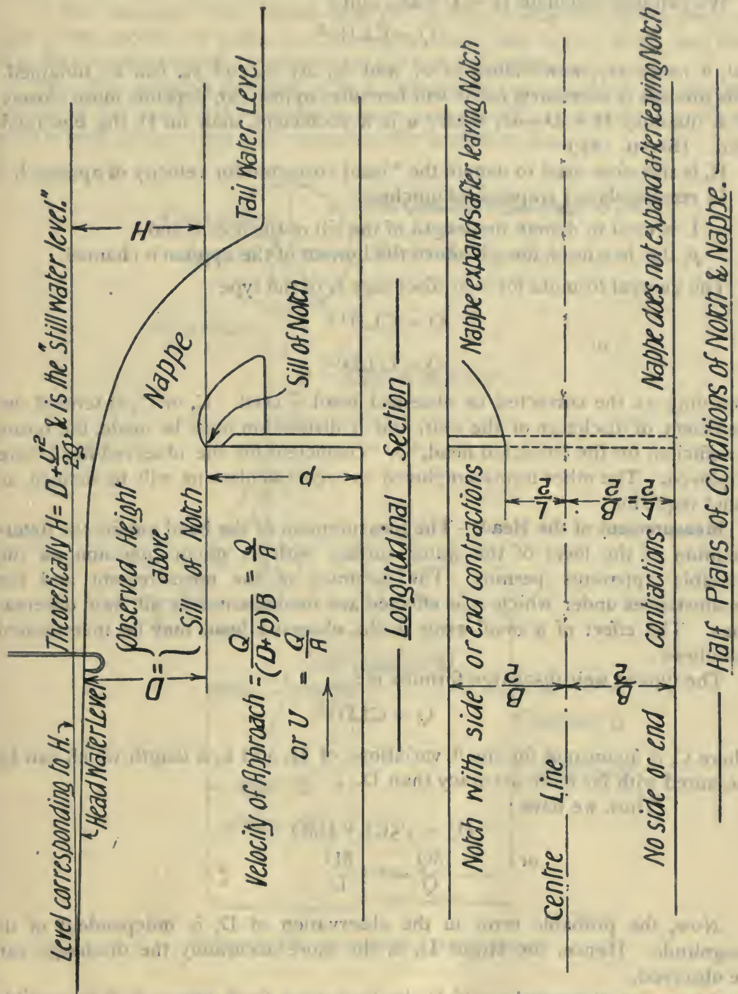
Using feet, feet per second, and cubic feet per second as units, the following symbols are employed (see Sketch No. 23):

The head is denoted by D,

The velocity of approach is denoted by v,

and h_v is used to denote the quantity $\frac{v^2}{2g}$.

The discharge over the weir is denoted by Q.



SKETCH No. 23.—Generalised Sketch of a Weir.

In practice, it will be seen that v, must usually be determined by successive approximation. We have :

$$v = \frac{Q}{A}$$

where A is the area of the cross-section of the approach channel at the point where D is measured. We therefore calculate an approximate value of Q, from the observed value of D, say, $Q_1 = CLD^{1.5}$, and thus obtain

$$v_1 = \frac{Q_1}{A} \text{ and } h_1 = \frac{v_1^2}{2g}$$

We can now calculate $H = D + ah_1$, and:

$$Q_2 = CLH^{1.5}$$

and, if necessary, new values of v_1, and h_1, say v_2, and h_2, can be obtained. This process is necessary, for as will hereafter appear, Q, depends more closely on a quantity $H = D + ah$, where a is a coefficient, than on D, the observed head. (See p. 104.)

H, is therefore used to denote the "head corrected for velocity of approach."

In rectangular or trapezoidal notches,

L is used to denote the length of the sill of the notch, and

p, the height of the sill above the bottom of the approach channel.

The general formula for weir discharge is of the type:

$$Q = CLH^{1.5}$$

or:

$$Q = C'LD^{1.5}$$

according as the corrected or observed head is used. C, or C′, is termed the coefficient of discharge of the weir, and if distinction must be made, the terms "coefficient for the corrected head," or "coefficient for the observed head" are employed. The other terms employed in weir calculations will be defined as found requisite.

Measurement of the Head.—The measurement of the head entails the determination of the level of the water surface with as much precision as the available apparatus permits. The accuracy of the measurement and the circumstances under which it is effected are fundamental in all weir observations. The effect of a small error in the observed head may be investigated as follows:

The typical weir discharge formula is:

$$Q = CLD^{1.5}$$

where C, is a constant for small variations of D, and L, a length which can be measured with far more accuracy than D.

Thus, we have:

$$\delta Q = 1{\cdot}5CL\sqrt{D}\,\delta D$$

or:

$$\frac{\delta Q}{Q} = 1{\cdot}5\,\frac{\delta D}{D}$$

Now, the probable error in the observation of D, is independent of its magnitude. Hence, the larger D, is, the more accurately the discharge can be observed.

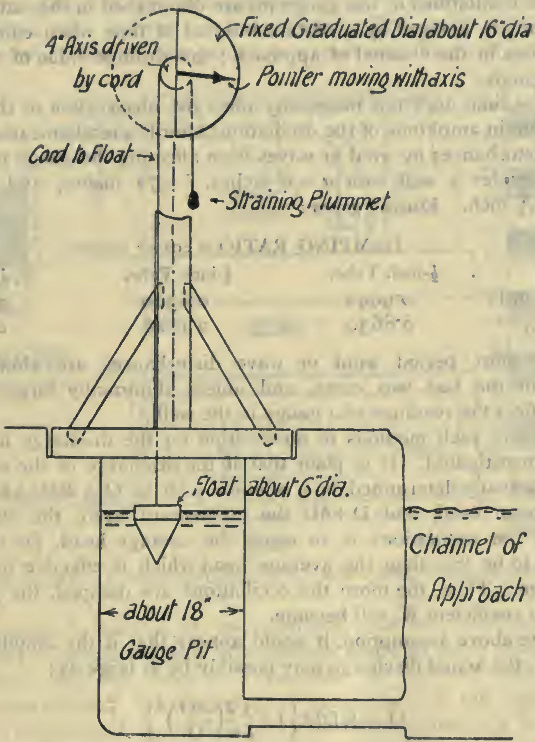
As an actual example:—If D, be read on a fixed gauge, it is impossible to determine the water level with certainty to more than 0·005 foot, so that if an accuracy of 1 per cent. in Q, is desired, D, must be at least 0·75 foot, and this accuracy is only attainable with practice.

If a hook gauge is used, a skilled observer will read to 0·001 foot, (I have never been able to attain 0·0005 foot as Francis states is possible). Consequently, D, must be at least 0·15 foot in order to secure an accuracy of 1 per cent.

The wheel gauge used by Bazin, and now employed for investigating the sieches in lakes, is probably somewhat more accurate. (See Sketch No. 24.)

It has, however, been shown (see p. 23), that owing to certain peculiarities in the motion of water, it is useless to work with values of D, less than 0·15 foot; and values of D, exceeding 0·40 foot are preferable.

Thus, in a properly designed weir, it appears that an accuracy of 1 per cent. (so far as this particular source of error is concerned), can usually be secured with a hook gauge, even with comparatively unskilled observers.



SKETCH NO. 24.—Bazin's Wheel Gauge.

In this connection, the theory of gauge pits as developed by Murray (*The Fresh Water Lochs of Scotland*, vol. 1, p. 51) deserves consideration. The oscillations in the water level caused by wind and waves, or by irregularities in stream motion, may be represented by:

$$\delta D = A \sin nt$$

where $nT = 2\pi$, gives T, the period of oscillation.

Now, let :

a, be the diameter of the gauge well,
b, be the diameter of the pipe connecting the gauge well with the approach channel,
l, be the length of the pipe ; all expressed in inches.

Put $\chi = \frac{7140b^4}{la^2}$ (accurately $\chi = \frac{2813b^4}{la^2}$ where all dimensions are expressed in centimetres). Then the oscillations in the gauge pit are expressed by

$$\delta G = A \cos n\tau \sin n(t - \tau), \text{ where } \tan n\tau = \frac{n}{\chi}.$$

Thus, the oscillations in the gauge pit are diminished in the ratio 1 : cos $n\tau$, and "lag," or are delayed by a certain interval of time when compared with the oscillations in the channel of approach ; the absolute value of this interval being $n\tau$ seconds.

The lag in time does not materially affect the observation of the head, but the diminution in amplitude of the oscillations affords a ready means of preventing slight disturbances by wind or waves from affecting the gauge readings.

Thus, consider a well with $a = 6$ inches, $l = 72$ inches, and $b = \frac{1}{2}$ inch, $\frac{1}{4}$ inch, and $\frac{3}{16}$ inch. Murray gives :

DAMPING RATIO = cos $n\tau$

T.	$\frac{1}{2}$-inch Tube.	$\frac{1}{4}$-inch Tube.	$\frac{3}{16}$-inch Tube.
870 seconds	0·9992	0·8320	0·4213
60 ,,	0·8630	0·1028	0·0320

So that short period wind or wave disturbances are almost entirely eliminated in the last two cases, and unless abnormally large, would not materially affect the readings of a gauge in the well.

The effect of such methods of observation on the discharge formulæ has never been investigated. It is plain that if the discharge of the weir at each moment is actually determined by a relation such as $Q = K(D+\delta D)^{1.5}$ where D, is the mean head, and $D+\delta D$ the momentary head, the effect of any damping of the oscillations is to cause the average head, (as read in the gauge well), to be less than the average head which is effective in producing the discharge. Thus, the more the oscillations are damped, the greater the experimental coefficient K, will become.

Under the above assumption, it would appear that if the amplitude of the waves is ΔD, the actual discharge may possibly be as large as :

$$Q = KD^{1.5}\left\{1 + \frac{3}{8}\frac{2}{\pi}\left(\frac{\Delta D}{D}\right)^2\right\}$$

and if these waves are entirely damped out, the gauge well reading will be D, so that the experimental coefficient is increased in the ratio :

$$\frac{1 + \frac{1}{4}\left(\frac{\Delta D}{D}\right)^2}{1}$$

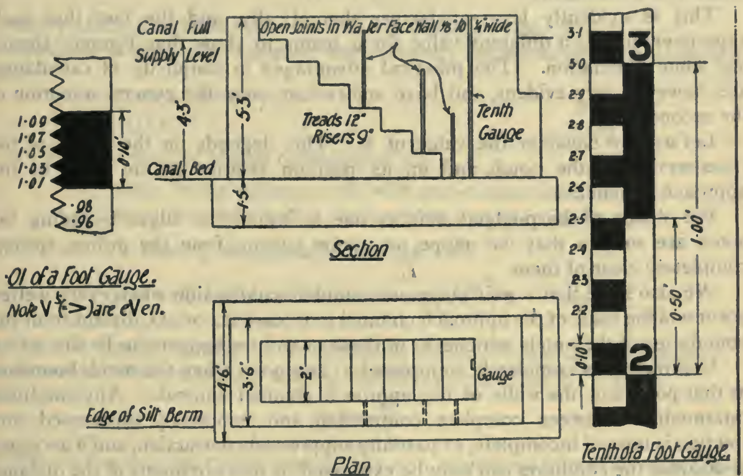
or, taking $\Delta D = 0·04$ foot, and $D = 0·5$ foot, the ratio becomes, 1·0016. But for $\Delta D = 0·08$, $D = 0·25$ (which I have seen in bad cases), the value is 1·0256.

The figures available on this subject are somewhat discordant.

In Bazin's observations . .	$\chi = 294$.
In Francis' ,, . .	$\chi = 21$ for weirs with side contractions.
	$\chi = 0{\cdot}4$ for weirs without side contractions.
In Fteley and Stearns' observations	$\chi = 0{\cdot}1$.

The stardard well gauge of the Punjab Irrigation Branch has $\chi = 0{\cdot}3$ to $0{\cdot}6$ approx. (See Sketch No. 25.)

My own trials lead me to consider that $\chi = 1$, is well adapted to secure accurate work with a fixed or hook gauge. The value used by Bazin is abnormal, and must not be regarded as expressing the whole damping, since



SKETCH No. 25.—Punjab Gauge Well, with typical Gauge Graduations.

his wheel gauge introduces an undetermined, (but very considerable) amount of damping. Comparison with observations made by other methods is therefore difficult, and this must be considered as the one obvious defect in Bazin's observations. In practice, the oscillations of the water level in a gauge pit constructed in accordance with Bazin's rules will be found to be too great to permit really accurate work when the level is observed by a hook or fixed gauge. When Bazin's wheel gauge is used, the observations are very easily made; and I believe that the results are not only more accurate, but that they more closely represent the head which is effective in producing the discharge, than do those which are obtained by any other method.

Weir Discharge Formulæ.—The usual formula for the discharge of water over a weir is that obtained from the ordinary approximate theory:

$$Q = \tfrac{2}{3}MLH\sqrt{2gH}$$

where H, is the corrected head over the weir, and L, is the length of the notch, supposed to be rectangular in section.

Now, the quantity actually measured, is not H, but D, the difference in level between the sill of the weir, and the surface of water when moving with a velocity v, towards the weir, where $v^2 = 2gh$, say, and the first correction that must be made is for the "velocity of approach." (See Sketch No. 23.)

Francis, following an approximate theory, gets :

$$H^{1.5} = (D+h)^{1.5} - h^{1.5}$$

which is obviously a very cumbrous form.

Other experimenters (especially Fteley and Stearns, and Bazin), have abandoned the theory, and used the empirical relation :

$$H = D + ah$$

where a is a coefficient.

This is evidently less satisfactory theoretically, and the fact that each experimenter uses a different value for a seems to show that Francis' theory has some foundation. The practical advantages in simplicity of calculation are, however, very evident, and have sufficed to cause the general adoption of the second form.

Let us now consider the value of M. This depends on the shape of the cross-section of the notch, and on its position relative to the sides of the approach channel.

We define a sharp-edged weir as one in which the edges bounding the notch are so thin that the nappe, or stream issuing from the orifice, springs completely clear of them.

We also state that a weir possesses complete contraction when every wetted portion of the walls of the approach channel is at least 2L, or 2D, distant from the boundaries of the notch, whichever of these quantities happens to be the lesser.

Contraction is completely suppressed at any point when the notch boundary at that point and the walls of the approach channel coincide. Any condition intermediate between complete contraction and completely suppressed contraction is termed incomplete, or partially suppressed contraction, and if accuracy is desired, the condition can only be expressed by measurements of the distance between the boundaries of the notch and of the approach channel.

The following varieties of sharp-edged weirs are used for accurate measurements :

(i) Complete contraction all round the notch. (Sketch No. 23, upper portion.)

(ii) Contraction complete at the sill of the notch, and completely suppressed at the sides; such a weir is usually referred to as being without end contractions. (Sketch No. 23, lower portion.)

Certain formulæ are given on page 118 which permit approximate corrections to be made for the effect of incomplete contraction. The formulæ are, however, unreliable, and accurate results are obtained only by using one or other of the above types of weir.

The general effect of partial suppression of contraction is well known. The discharge is always increased, and the percentage of increase becomes greater the more the contraction is suppressed. Thus, the larger the head the greater is the increase in discharge. The final effect therefore is that while the discharge of a rectangular weir with complete contraction is expressed by:

$$Q = KH^N$$

where N, varies from 1·45 to 1·50, or perhaps even 1·52, the discharge of a similar weir with partially suppressed contraction is expressed by:

$$Q = K_1 H^N$$

where N, varies from 1·5, to 1·6, and K_1, is somewhat larger than K, the increase in K_1, and N, being roughly speaking proportional to the amount the contraction is suppressed. This method of regarding the matter proves very useful when it is desired to obtain a weir formula from actual observations, as although no definite rules for K_1, and N, can be given, the calculations are far easier than those required when a formula:

$$Q = CH^{1.5}$$

is used, and C, is variable. (See p. 118 and Sketches Nos. 18–20.)

Practical Rules.—Francis gives the following equations for these standard cases:

$$Q = 3{\cdot}33(L - nH \times 0{\cdot}1)H^{1.5} \quad . \quad . \quad . \quad . \quad . \quad \text{case (i)}$$
$$Q = 3{\cdot}33LH^{1.5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{case (ii)}$$

where n, is the number of side contractions, *i.e.*

$n = 2$, in the ordinary weir with end contractions,
$n = 4$, in a weir with a sharp-edged pier in its midst,

and if there is a velocity of approach, $v^2 = 2gh$.

$$H^{1.5} = (D + h)^{1.5} - h^{1.5} = D + \left(1 - \tfrac{2}{3}\sqrt{\frac{h}{D}}\right) h \qquad \text{approx.}$$

Now, as already stated, the correction for velocity of approach is cumbrous; but otherwise the formulæ are simple, and easily remembered. Further,

$$3{\cdot}33 = \frac{10}{3}$$

so that in the case of rough calculations and approximate results, where the correction for the velocity of approach need not be considered, these formulæ may be adopted.

Weir Formulæ as applied to Measurement of Discharge.—It is believed that a weir gauging (failing an actual volumetric measurement) is the most accurate method of measuring a discharge. When applied for such purposes, it is necessary to consider the effect of local peculiarities on the discharge of a weir. It can at once be said that a weir is a very accurate measuring instrument, but like all other measuring instruments it must be carefully standardised in order to get the best results from its use; and the standard should be copied in details which at first sight appear to have no real effect upon the discharge.

The two reliable series of standard experiments (when considered from thi point of view) are those of Francis and Bazin, and of these Bazin's are by far the most comprehensive in range.

If we erect a weir, and carefully and systematically copy the details either of Bazin's or Francis' original apparatus, it will be found that the discharges agree with those found by the appropriate formulæ, within ½ per cent. It will also be found that the errors are nearly all attributable to one cause, namely,—the difficulty of accurately observing the level of a water surface to more than 0·005 foot, especially when large volumes of water are dealt with.

If, however, we erect what may be called a generalised weir, *i.e.* a weir constructed in accordance with the specification of Francis or Bazin, but the details of which (say, for instance, the method of observing the water level, or the material lining the approach channel between the weir and the point where the water level is observed) are varied according to local convenience, it will be found that the formulæ given by both Francis and Bazin may lead to results differing from the true discharges by 2, or possibly even 3 per cent.

We cannot therefore state that the formulæ are wrong, but merely that we have used an unstandardised apparatus for measurement.

Besides the experiments of Francis and Bazin, others exist, notably those of Fteley and Stearns, also of Lesbros, and Boileau, etc. A study of their results, assisted by Hamilton Smith's computations, permits me to state that for these generalised weirs a formula can be found which is applicable to weirs with complete side contraction, and which is more easy to calculate than the one adopted by Francis, and which can be employed over a wider range of conditions.

We thus have two series of formulæ :

1. Formulæ applicable to standard weirs :
   - (*a*) Francis' formula for completely contracted weirs.
   - (*b*) Bazin's formula for weirs with suppressed end contractions.
2. Formulæ applicable to generalised weirs :
   - (*a*) For weirs with complete contraction.
   - (*b*) For weirs with side contractions suppressed.

The first class produce results which are correct to about ½ per cent. when the observers are sufficiently skilful to work the weir as it deserves.

The second class give a mean result, and may be as much as 3 per cent. in error. It will, however, be found that when applied to weirs which are not carefully constructed, so as to agree with the standard type, they usually yield results which are less subject to error than those given by the formulæ of the first class.

I. **Formulæ of the First Class** (Francis' Formulæ).—The weir must be constructed as shown in Sketch No. 26 which is copied from Plates 11 and 12 of the Lowell Hydraulic Experiments. The head D, is measured by hook gauges in boxes, 6 feet upstream of the weir, the water being admitted to these boxes by holes 1 inch in diameter. On scaling the original drawing we find that : $p = 4{\cdot}60$ feet.

The formula is :

$$Q = 3{\cdot}33\,(L - n \times H \times 0{\cdot}1)H^{1.5}$$

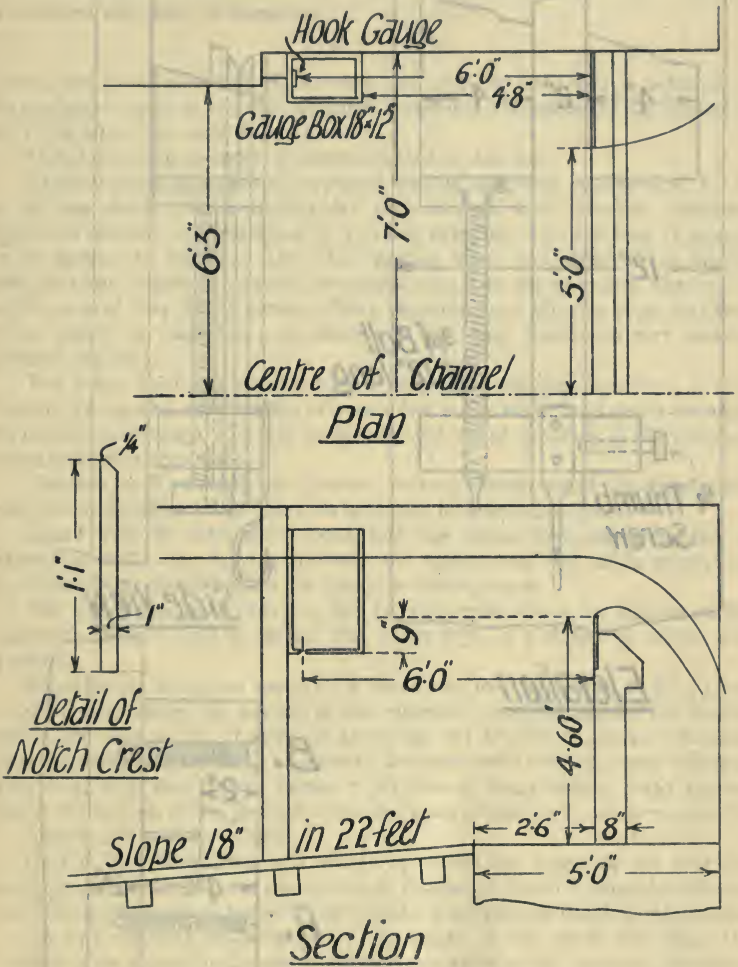
with :

$$H^{1.5} = (D + h)^{1.5} - h^{1.5}$$

and H, was varied in the experiments between 0·60 foot and 1·60 foot ; but the formula is probably applicable between H = 0·50 foot, and 4·00 feet (see Horton, p. 39), provided that p, is greater than 3H.

The side walls of the approach channel were of granite masonry, and the bed of timber, although possibly timber all over will suffice for copies.

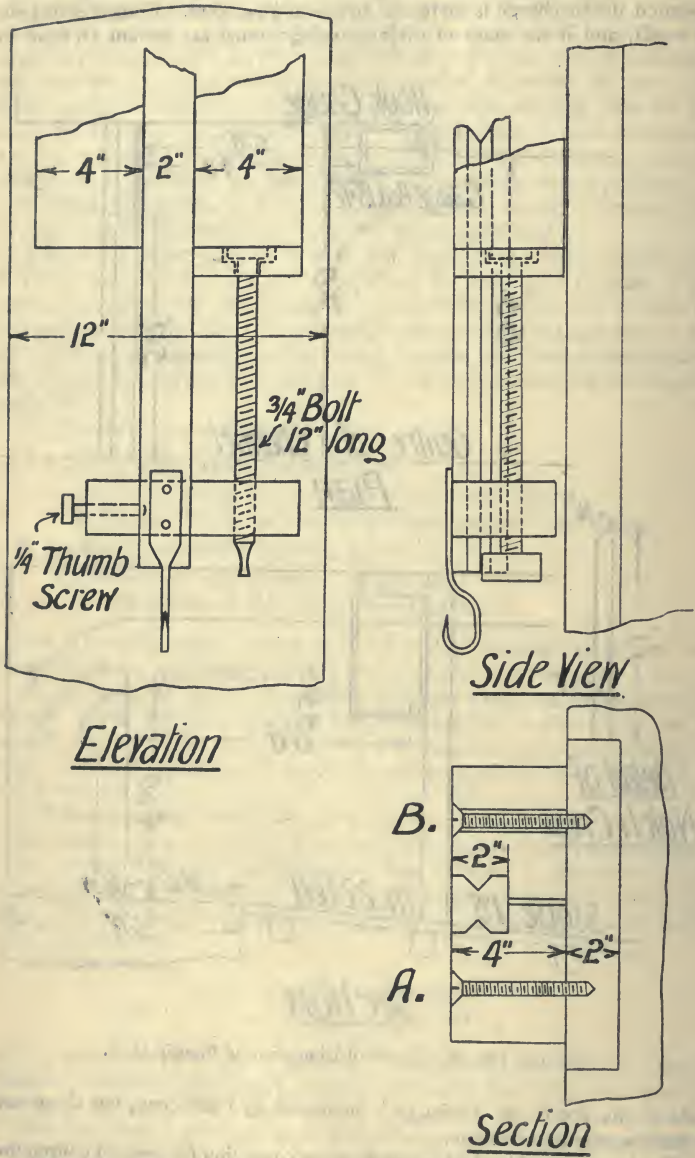
The side walls of the approach channel should be at least 2D, distant from the ends of the notch. The nappe should be allowed to expand freely at its sides after leaving the notch. It is believed that if this side expansion is prevented the discharge is increased by about ½ per cent. Francis states that if $p = 2D$, and if the sides of the approach channel are distant D, from the



SKETCH NO. 26.—Standard Dimensions of Francis' Weir.

ends of the notch, the discharge is increased by 1 per cent., but these statements are only approximate.

The term $-0{\cdot}1nH$, which represents the correction for the end contractions, is not very accurately determined; except when L, is greater than 3D, and D, lies between 0·50, and 1·60 foot. It is probable that the coefficient 0·1, should



SKETCH No. 27.—Hook Gauge.

be increased if H_1 is less than 0·50 feet, and should be decreased if H_1 is greater than 1·60 feet; but the facts stated on page 114 show that the question is not of great practical importance.

A weir of similar construction, without end contractions, can also be used as a standard weir, and the formula:

$$Q = 3{\cdot}33LH_1^{1.5}$$

is accurate for all values of H_1 between 0·50, and 1·60 feet; and is believed to be applicable up to H = 5 feet, although it possibly overestimates the discharge by 1·5 to 2 per cent. near H = 2·5 feet.

The Bazin weir, however, is better adapted to this case.

BAZIN FORMULÆ.—Bazin's standard weir (*Écoulement en deversoir*, Pt. I. p. 9) was erected in a rectangular cement-lined, and smoothly rendered approach channel 6·56 feet wide, by 5·25 feet deep, and 49·2 feet long (2 metres × 1·6 metre × 15 metres). The water surface level was observed in lateral pits, 1·64 foot square (0·5 metre) communicating with the approach channel at a distance of 16·4 feet (5 metres) above the weir, by a circular pipe 0·33 foot (0·10 metre) in diameter, and about 1·00 foot long (scales as 0·31 metre). (Sketch No. 28.)

The water level was observed by a float and indicating quadrant, as per Sketch No. 24, and an alteration of 0·0002 foot in the water level could certainly be observed, although it is not so certain that equal accuracy in the absolute value of D_1 was obtained.

The weir itself was built up of beams, and the sharp-crested sill of the notch was made of 0·28-inch iron plate, as indicated in Sketch No. 28.

There were no side contractions, and the nappe was not permitted to expand laterally after leaving the weir, but special care was taken to prevent a vacuum being formed below the nappe in consequence.

The sill of the notch was 3·72 feet (1·135 metre) above the bottom of the approach channel, and its length was either 6·56 or 3·28 feet (2 metres and 1 metre).

Experiments were also made on a notch 1·64 foot long, with a sill 3·3 feet (1·005 metre) above the bottom of the approach channel, and for the shorter notch one side of the channel of approach was of planed planks. Notches approximately 6·56 feet long (2 metres), and with their sills 2·46 feet, 1·64 foot, 1·15 foot, 0·79 foot, (0·753 metre, 0·502 metre, 0·349 metre, 0·240 metre), above the bottom of the channel were also less systematically experimented on.

The results were as follows:

(i) For D_1 greater than 0·62 foot (0·19 metre) the length of the weir has no appreciable effect on the coefficients of discharge, and it is doubtful whether the differences observed below $D = 0{\cdot}62$ foot are not entirely those of observation.

(ii) The effect of variations in p, the height of the notch sill above the bottom of the approach channel is reduced to a table, which gives the discharge with errors not exceeding 1 per cent., and usually less than 0·3 per cent. except in the case of the notch with a very low sill, $p = 0{\cdot}79$ foot where errors of 2·5 per cent. occur; but the formula given below occasionally introduces errors of 2, to 3 per cent. when p is small, say less than 1·5 D.

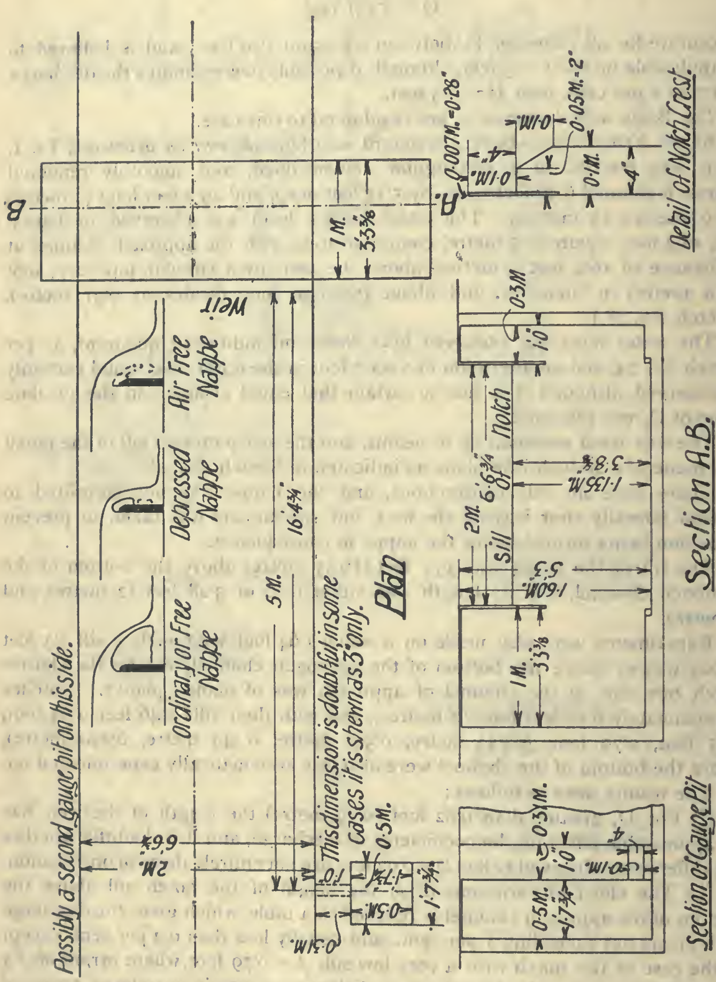
This approximate formula is:

$$Q = \left(3{\cdot}248 + \frac{0{\cdot}0789}{D}\right)\left\{1 + 0{\cdot}55\left(\frac{D}{p+D}\right)^2\right\} LD^{1.5}$$

The errors never exceed 1 per cent. over the range $D = 0{\cdot}30$ foot to $D = 1{\cdot}45$ foot, provided that p, exceeds 1 foot.

The shorter formula:

$$Q = \left\{3{\cdot}41 + 1{\cdot}69\left(\frac{D}{p+D}\right)^2\right\} LD^{1.5}$$

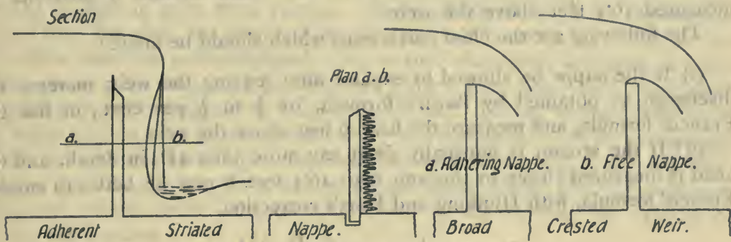


SKETCH No. 28.—Standard Dimensions of Bazin's Weir with Types of Nappe.

may be used for values of D, between 0·33 foot and 1·00 foot if errors of 2, or 3 per cent. are permissible. (See Horton, *Weir Experiments*, p. 32.)

The Bazin weir must be regarded as an instrument of great delicacy, and is therefore easily put out of order if carelessly used. As an example: If the nappe is allowed to expand laterally, after leaving the notch, the discharge increases about 0·5 to 1 per cent. So also, owing to this prevention of lateral expansion, the nappe (under favourable circumstances) may assume various forms. The typical form, and the only one that can be employed in accurate gauging, is the Free Nappe (Nappe Libre), Sketch No. 28. If, however, the under side of the nappe is not properly ærated, or rather, if the circumstances are such as not to produce perfect æration, the nappe becomes Depressed (Deprimée), see Sketch No. 28, and the discharge may be 8, to 10 per cent. greater than that given by the formula. If the air is completely exhausted from under the nappe, the air-free (Noyée à dessous) form may appear, and another 10 per cent. increase in discharge may occur. (See p. 126.)

Under favourable circumstances, and especially if the wall of the weir is inclined upstream, we may obtain the striated (adherente) nappe, see Sketch No. 29. This form permits of a discharge which is 20 to 30 per cent. greater than the normal. The matter is carefully discussed by Bazin (*ut supra*), but the practical result of his work is that such forms should not be permitted in accurate work. (See p. 126.)



SKETCH No. 29.—Striated Nappe and Types of Nappe occurring with broad-crested Weirs.

With great diffidence I venture to suggest that were the nappe allowed to expand after leaving the notch, it is probable that the length of the weir would in no way affect the coefficients of discharge (although these would no longer agree with Bazin's determinations), and it is quite certain that the peculiar forms above enumerated would not occur.

If for any reason, the nappe is allowed to expand laterally after leaving the notch, the Francis form of weir and gauge pits should be adopted, and the formula :

$$Q = 3·33LH^{1.5}$$

with Francis' correction $H^{1.5} = (D+h)^{1.5} - h^{1.5}$, used.

II. **General Formulæ.**—(*a*) Weirs with complete contraction. The general results of all existing experiments have been very carefully discussed by Hamilton Smith. I am inclined to believe that the agreement between the various series is not quite as close as would at first sight appear, since Hamilton Smith seems to select the correction for the velocity of approach with but little regard for the method of observation employed by the original experimenter.

I should not, however, feel justified in proposing a formula differing from that deduced by Hamilton Smith, were it not that I find that the mean result of the observations can be very well represented by formulæ which permit us to obtain the discharge by a multiplication of two figures taken from tables, in place of three required with the coefficients given by Hamilton Smith.

The proposed formulæ are, with $H = D + 1 \cdot 4h$.

(i) For heads less than 0·40 foot.

$Q = 3 \cdot 101\ L^{1.02}\ H^{1.46}$, so long as L, is less than 2·6 feet.

$Q = 3 \cdot 146\ L^{1.005}\ H^{1.46}$, where L, is greater than 2·6 foot.

(ii) For heads greater than 0·40 foot.

$Q = 3 \cdot 110\ L^{1.02}\ H^{1.465}$, so long as L, is less than 4 feet.

$Q = 3 \cdot 122\ L^{1.016}\ H^{1.475}$ for L, between 4 and 10 feet.

$Q = 3 \cdot 148\ L^{1.013}\ H^{1.485}$ for L, greater than 10 feet.

These formulæ hold up to L = 30 feet, and possibly for greater values of L; and for H, as high as 1·7 foot, and possibly further. (See p. 132.)

(*b*) Weirs with side contractions suppressed.

Bazin's formula should be used, and the head should, if possible, be measured 16·4 feet above the weir.

The following are the chief corrections which should be made:

(i) If the nappe be allowed to expand after leaving the weir, increase the discharge as obtained by Bazin's formula, by ¼ to ½ per cent., or use the Francis' formula, and measure the head 6 feet above the weir.

(ii) If the stream is unusually deep, say more than 4D, in depth, and the head is measured closer to the weir than 16·4 feet, it may be better to employ Francis' formula, with Hunking and Hart's correction.

$$Q = 3 \cdot 33 \left\{1 + 0 \cdot 25 \left(\frac{D}{p+D}\right)^2\right\} LD^{1.5}$$

where $\frac{D}{p+D}$ is less than 0·36; while if $\frac{D}{p+D}$ be greater than 0·36, the original Francis formula for the correction for velocity of approach must be used, though in such cases the Bazin formula is certainly more accurate.

General Agreement of Weir Discharge Formulæ.—At first sight it would appear that the various weir formulæ are so discordant as to produce a certain distrust in weirs as a method of measuring water.

For example, taking a weir without end contraction, we have:

Francis . . $Q = 3 \cdot 33\ LH^{1.5}$
Hamilton Smith . $Q = 3 \cdot 31$ to $3 \cdot 51\ LH^{1.5}$
Fteley and Stearns $Q = 3 \cdot 31\ LH^{1.5} + 0 \cdot 007L$.
Bazin . . . . $Q = \left(3 \cdot 25 + \frac{0 \cdot 08}{D}\right)\left\{1 + 0 \cdot 55\left(\frac{D}{D+p}\right)^2\right\} LD^{1.5}$

and it would therefore appear that differences as great as 6 per cent. might occur.

As a matter of practice, if all considerations except the numerical result are disregarded, differences of 2, or 3 per cent. can be obtained; and I am by no means satisfied that such juggling is entirely unpractised in commercial testing. When, however, the matter is treated in a scientific manner, it must first be remarked that the H, in the above formulæ is by no means the same quantity.

Putting $\frac{v^2}{2g} = h$.

Francis defines	$H^{1.5} = (D+h)^{1.5} - h^{1.5}$
and very nearly	$H = D + h\left(1 - \frac{2}{3}\sqrt{\frac{h}{D}}\right)$
Hamilton Smith puts . . .	$H = D + h\ 1{\cdot}33$
Fteley and Stearns	$H = D + h\ 1{\cdot}5$
Bazin	$H = D + h\ 1{\cdot}68$

Thus, we see that with the exception of Hamilton Smith's general formula, the smaller the coefficient C, in the equation $Q = CLH^{1.5}$, the larger will be the value of H, as calculated for the same values of D, and v, the quantities actually observed. The water surface of the stream above the weir is not absolutely horizontal, but drops down in a flat curve towards the weir. Thus, the observed value of D, depends to some degree upon the exact point at which the observations are taken, and the greater the distance of the gauge from the weir, the larger will be the values which are obtained.

The balancing goes even further. Bazin observed D, at a distance of 16 feet from the weir, and obtained the smallest coefficient in the whole series. The other observers generally used 6 feet as their standard; although, in some cases, where other distances (usually less than 6 feet) were used, corrections for surface curvature appear to have been applied. Owing to the curvature of the water surface as it approaches the weir, it is certain that the D, as observed by Bazin, is appreciably larger (my calculations indicate 0·5, to 0·7 per cent. larger; but I would lay no stress on an obviously flimsy piece of evidence) than that which would have been observed by the methods of Francis, or Fteley and Stearns.

I had an opportunity of testing the question, and conducted 18 experiments as follows:

An unknown volume of water was passed over a weir constructed according to Bazin's specification, except that the sides of the channel were of wood, in place of cement plaster. The value of D, was observed according to Bazin's methods, say D_B, at 16·4 feet above the Bazin weir. The water then passed over a standard Francis weir, and D, was observed in a box according to Francis' methods, at 6 feet above the standard Francis weir, say D_F. The lengths of the sills of the two weirs were as follows:

$$L_B = 4{\cdot}08 \text{ feet}, \qquad L_F = 4{\cdot}04 \text{ feet}.$$

D_B, and D_F, were then corrected by the proper formulæ, and H_B, and H_F, were used to calculate the quantity discharged according to Bazin's and Francis' formulæ.

The ratio $\frac{Q_B - Q_F}{Q_B}$ varied between $+1{\cdot}4$ per cent. and $-1{\cdot}6$ per cent., with a mean error of $-0{\cdot}2$ per cent. I was not at the time so skilled an observer as I later became (this being almost my first attempt at large scale weir observations). I am now inclined to believe that a skilled worker would have obtained less concordant results by selecting quantities of water which were better suited to disclose the lack of agreement in the formulæ, (*e.g.* in all my observations the values of D, lay between 0·7 and 1·1 foot, and this range is probably the one over which the formulæ agree best).

8

Nevertheless, the agreement is well within the probable accuracy of my observations, and I believe that the various weir formulæ, when properly applied, lead to results which agree with each other quite as closely as would those obtained by two independent observers working up their separate observations, and using the same formula.

If the reverse method is adopted, and Bazin's formula and methods of observation are applied to calculate the discharge over a Francis weir, where $p = 4{\cdot}6$ feet, the reasoning already employed indicates that the agreement will probably be less accurate, but experiments do not exist.

For weirs with side contractions, the apparent agreement in the formulæ is somewhat better. The formulæ must be considered as less accurate, and the correction :

$$\text{Effective length} = \text{Measured length} - nH \times 0{\cdot}1$$

(see p. 105) must be regarded as subject to some uncertainty.

The only definite experiments, other than the original ones by Francis, were undertaken by Bazin (as yet unpublished).

It would appear that under certain circumstances (usually at low heads) the factor 0·1, may attain the value 0·14. The fact that the French official instructions do not give any other rule than that of Francis, is fair proof that the present formulæ do not depart very widely from the truth, under any circumstances.

Sharp-edged Notches of other than Rectangular Form.—The only notches which have been sufficiently experimented on to be used in accurate measurement are the Triangular Notch, and a special form of Trapezoidal Notch known as the Cippoletti Notch.

(*a*) *Triangular Notches.*—The usual theory leads to the following formulæ :

$$Q = \tfrac{4}{15} M L_1 \sqrt{2g} D^{1.5}$$

and L_1, the width of the notch at the water level varies as D.

$$\text{Let } L_1 = zD \text{ say, so that } Q = \tfrac{4}{15} z M \sqrt{2g} D^{2.5}$$

We find experimentally that M, varies with z, but is quite unaffected by D, probably owing to the fact that the cross-section of the issuing stream remains similar to itself for all values of D.

The following special cases may be given :

$z=\frac{1}{2}$. Or $L_1 = \frac{D}{2}$. $Q = 0{\cdot}707 D^{2.5}$ up to $D = 1{\cdot}6$ foot.

$z=1$. Or $L_1 = D$. $Q = 1{\cdot}31 D^{2.5}$ up to $D = 1{\cdot}5$ foot.

$z=2$. Or $L_1 = 2D$. $Q = 2{\cdot}545 D^{2.5}$ up to $D = 1{\cdot}10$ foot, with the floor of the approach channel at least D, below the vertex of the notch.

$Q = 2{\cdot}56 D^{2.5}$ up to $D = 1{\cdot}10$ foot, with the floor at the level of the notch.

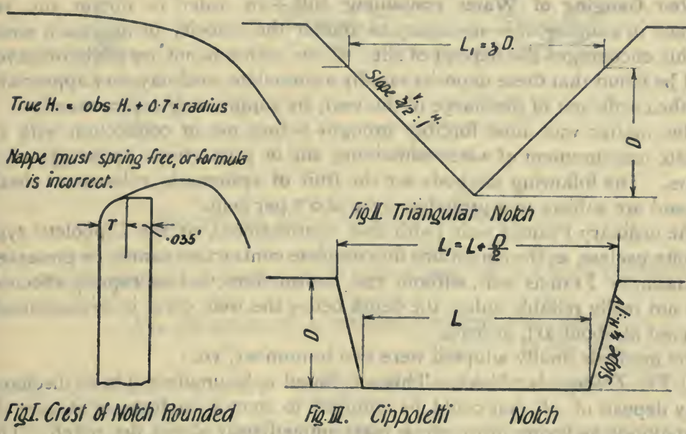
$z=4$. Or $L_1 = 4D$. $Q = 5{\cdot}30 D^{2.5}$ up to $D = 1{\cdot}06$ foot, with the floor well below the vertex.

$Q = 5{\cdot}22 D^{2.5}$ with the floor at the level of the vertex.

The above formulæ are due to Thomson (*Brit. Asscn. Report*, 1861, p. 155, not the usual reference of 1858) and Leslie. The upper limits of D, are the greatest values observed in my own experiments which ranged from D = 0·60 foot upwards, by intervals of 0·01 foot. A special examination was made of each 0·10 foot interval, in order to detect variations in M, with negative results.

I obtained my results by a series of systematic volumetric checkings into a tank of 2900 cube feet capacity, the head being read directly on a carefully adjusted brass scale. The probable error was 0·4 per cent. Consequently, it may be assumed that these notches afford a ready means of gauging water up to a discharge of about 5 cusecs, with an accuracy of 0·4 per cent., and the coefficients given above probably hold good for heads greatly exceeding those stated.

I do not give any formula for the correction for velocity of approach; since, with a properly proportioned approach channel, v, cannot exceed 0·5 foot per second, unless contraction is incomplete; and for $v = 0{\cdot}5$ foot per second, $h = 0{\cdot}002$ foot, which is inappreciable in direct readings on scales.



SKETCH No. 30.—Notch with Rounded Edge, Triangular and Cippoletti Notches.

I have, however, purposely tried the effect of high velocities of approach, and incomplete contraction, and can state that the full results of both effects combined can be allowed for by putting $H = D + \frac{2h}{3}$ in the formulæ.

(*b*) *Trapezoidal Notches.*—The theoretical formula is as follows:

$$Q = \tfrac{2}{3} c_1 \sqrt{2g} L H^{1.5} + \tfrac{8}{15} c_2 z \sqrt{2g} H^{2.5}$$

where c_1, and c_2, are coefficients of discharge, and z, is the slope of one side of the weir to the vertical, L, being the length of the sill of the weir.

Now, if we consider Francis' formula, we see that the effect of side contractions is to decrease the discharge by an amount equal to

$$\tfrac{4}{3} \frac{c_1}{10} \sqrt{2g} H^{2.5}$$

Equating this to the amount passed by the two triangular end pieces, *i.e.*

$\frac{8}{15} z c_2 \sqrt{2g} H^{2.5}$, we get, $z = \frac{1}{4}$. (Sketch No. 30.)

Cippoletti experimented on a weir of these proportions, which otherwise satisfied Francis' specification, and came to the conclusion that its discharge could be represented by :

$$Q = 3 \cdot 367 L H^{1.5}$$

with the Francis velocity of approach correction (*Canale Villoresi, Modulo per le Dispensa della Acque*).

Flinn and Dyers' experiments gave the formula :

$$Q = 3 \cdot 301 L H^{1.5}$$

with $H = D + 1 \cdot 4h$, which is more easily applied (*Trans. Am. Soc. of C.E.*, vol. 32, p. 9).

Weir Gauging of Water containing Silt.—In order to obtain any real accuracy in gauging, it is necessary to render the velocity of approach small, and this encourages the deposit of silt. If the action is not carefully observed, it will be found that these deposits rapidly accumulate, and may very appreciably alter the coefficient of discharge of the weir, by suppressing contraction.

The matter was most forcibly brought before me in connection with the accurate measurement of water containing silt in proportions up to $\frac{1}{300}$ of its volume. The following methods are the fruit of systematic volumetric checkings, and are subject to a probable error of 0·7 per cent.

The ordinary Francis weir (with end contractions), or the Cippoletti type, are quite useless, as the conditions for complete contraction cannot be preserved. The Bazin, or Francis weir, without end contractions, is less rapidly affected ; but is not really reliable unless the depth below the weir crest is systematically preserved at about 2H, at least.

The methods finally adopted were two in number, viz. :

(*a*) *The Triangular Notch.*—This was found to be unaffected in its discharge by any deposit of silt that could be induced to remain in front of it, even by such methods as laying down straw mats immediately above the notch. This statement holds for the $L_1 = D$, $L_1 = 2D$, and $L_1 = 4D$ notches, under heads ranging from 0·60 foot upwards.

The results given by Thomson (see p. 114) indicate that a straw mat or "floor" will have some effect in the case of heads lower than 0·60 foot, but the effect was not observed in my experiments.

This method is the best if the quantity of water, and the available head permits its adoption.

(*b*) For quantities of water greater than those which could be conveniently measured over triangular notches I employed a weir of Cippoletti form, but with partially suppressed contractions. Sketch No. 31 shows the elevation and cross-section, and it may be noted that the somewhat peculiar stopping of the brickwork on either side of the notch was necessary in order to prevent the wooden board in which the weir notch was cut, from warping.

The position of the brick sill in relation to the sill of the notch was fixed so as to cause the natural flow of the water to sweep away any silt deposited on it.

Subject to the above condition, it was found that deposits of silt outside the brickwork had no effect on the discharge, and the following formulæ were adopted :

For a Cippoletti weir with a sill 12 inches long, we find from 23 observations that :

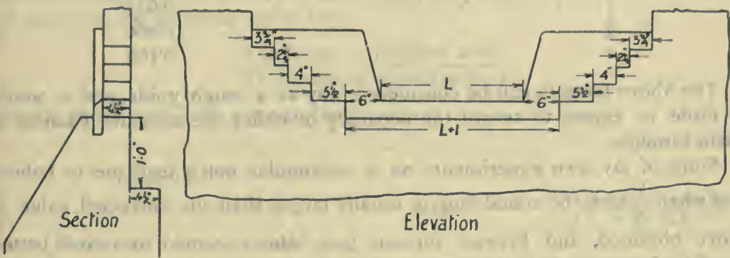
(i) When D, is less than 0·50 foot,

$$\log Q = 1{\cdot}61 \log 10D - 1{\cdot}030.$$

(ii) When D, is greater than 0·50 foot,

$$\log Q = 1{\cdot}57 \log 10D - 0{\cdot}988.$$

The formulæ hold over the range D = 0·15 foot to D = 0·90 foot and the observed discharges agree with the calculated figures in every case if an error of 0·005 foot or less in D, is assumed to occur.



SKETCH No. 31.—Contracted Cippoletti Notch.

For a Cippoletti weir with a sill 18 inches long, we find from 17 observations that :

For all values of D, between 0·25 and 0·75 foot,

$$\log Q = 1{\cdot}57 \log 10D - 0{\cdot}836$$

and all differences can be explained by assuming an error of 0·005 foot in the observed value of D.

For a Cippoletti weir with a 2 foot sill, we find from 14 observations that :

For all values of D, between 0·30 and 0·70 foot,

$$\log Q = 1{\cdot}57 \log 10D - 0{\cdot}702$$

These formulæ may be expressed in the forms :

(i)	1 foot weir	$Q = 3{\cdot}82D^{1\cdot57}$
(ii)	1·5 „	$Q = 5{\cdot}42D^{1\cdot57}$
(iii)	2 „	$Q = 7{\cdot}38D^{1\cdot57}$

If errors of 2 per cent. are permissible, the general formula $Q = 3{\cdot}78L^{0\cdot95}D^{1\cdot57}$ may be used for Cippoletti weirs with contractions as shown for values of L, between 1 and 2 feet

Suppression of Contraction.—The question of the effect of partial suppression of contraction over portions of the notch boundary is obscure. The discharge will be increased in all cases. Hamilton Smith (*Hydraulics*, p. 120) states as follows :

Let X, be the least dimension of the notch, whether L, or H.

Let R = L + 2H, be the wetted perimeter of the notch.

Let Y, be the distance of any boundary of the notch from the corresponding side of the approach channel.

Let S, be the length of the portion of the notch boundary over which this distance Y, occurs.

Then, the discharge of the notch with partially suppressed contraction is $1+\frac{ZS}{R}$ that of a similar notch with complete contraction all round its boundaries (the general formula being used) and

$\frac{Y}{X}$	Z
3	0·000
2	0·005
1	0·025
$\frac{1}{2}$	0·060
0	0·160

The above formula can be considered only as a rough guide, and it would be futile to expect to secure the accuracy of either the accurate Francis or Bazin formulæ.

Some of my own experiments on a rectangular notch lead me to believe that when $\frac{Y}{X}$ is 1, the actual flow is usually larger than the corrected value as above obtained, and Freeze's formula (see below) seemed to accord better. Prasil (*Schweizerische Bauzeitung*, 1905) finds the reverse to be the case, and at present no general formula for a suppressed notch can be regarded as accurate to even 5 per cent. over large ranges of H, or L.

Experimental Weir Formulæ.—The usual method of allowing for the effect of partial suppression of contraction, and other deviations from the standard weir form, is to state the equation in the following manner :

$$Q = CLD^{1.5}$$

and to give an expression for C, in terms of D.

As an example, Freeze's formula may be taken ; which, if B, be the breadth of the assumedly rectangular approach channel, and T, is its depth, so that :

$$T = D + p$$

where p, is the height of the weir crest above the bottom of the channel, is represented by :

$$Q = 5{\cdot}35\mu LD^{1.5} \text{ including velocity of approach.}$$

Where :

$$\mu = \left\{ 0{\cdot}5755 + \frac{0{\cdot}0558}{D+0{\cdot}59} - \frac{0{\cdot}246}{L+3{\cdot}94} \right\} \eta$$

$$\text{and } \eta = 1 + \left\{ 0{\cdot}25\left(\frac{L}{B}\right)^2 + 0{\cdot}025 + \frac{0{\cdot}0375}{\left(\frac{D}{T}\right)^2 + 0{\cdot}020} \right\} \left(\frac{D}{T}\right)^2$$

The agreement with the observations discussed is good, but the practical application of the formula is somewhat wearisome.

On the other hand, a wooden flume with a terminal weir is practically a standard hydraulic apparatus, and considerations of available space usually prevent the weir from being made of the standard Francis or Bazin type.

It therefore becomes necessary to inquire whether some more convenient discharge equation cannot be obtained.

The method of logarithmic plotting, enables me to say that all accessible experiments agree very accurately with formulæ of the type:

$$Q = KD^N$$

provided that D, is over 0·40 foot. The same form holds when D, is less than this value, but the values of K, and N, are changed.

At present I cannot give rules for the values of K, and N; but would observe that the more the contraction is suppressed, the greater is the value of N.

Inclined Weirs.—When the whole barrier forming the weir is inclined in a vertical plane, and the notch is sharp-edged, Bazin (*Ecoulement en deversoir*, ii. p. 43) gives the following ratios for:

$$\frac{\text{Discharge of inclined weir}}{\text{Discharge of standard weir}}$$

the D, and *p*, being the same for both weirs:

Inclination.		Ratio.	Approximate Value of C in $Q = CLD^{1.5}$.
Horizontal.	Vertical.		
1 Upstream	1	0·93	3·097
3 "	2	0·94	3·13
3 "	1	0·96 (Boileau finds 0·973)	3·197
Vertical.		1·00	3·33
3 Downstream	1	1·04	3·463
3 "	2	1·07	3·563
1 "	1	1·10	3·663
1 "	2	1·12	3·730
1 "	4	1·09	3·630

The maximum value of the discharge occurs when the inclination is about 7 horizontal to 4 vertical.

Oblique Weirs.—The case of a weir oblique to the approach channel has been studied by Aichel (*Ztschr. Deutsche Ingenieure Verein*, October 31, 1908).

The notch had no side contractions, and the sill was 10 inches (0·24 metre) above the bottom of the approach channel, the heads ranging from 6 inches upwards. Except for its obliquity the weir was of the standard sharp-edged Bazin type.

Taking L, as the length of the sill of the notch,
p, as its height above the bottom of the channel,
ϵ, as the angle the weir makes with the sides of the channel,
D, as the head,

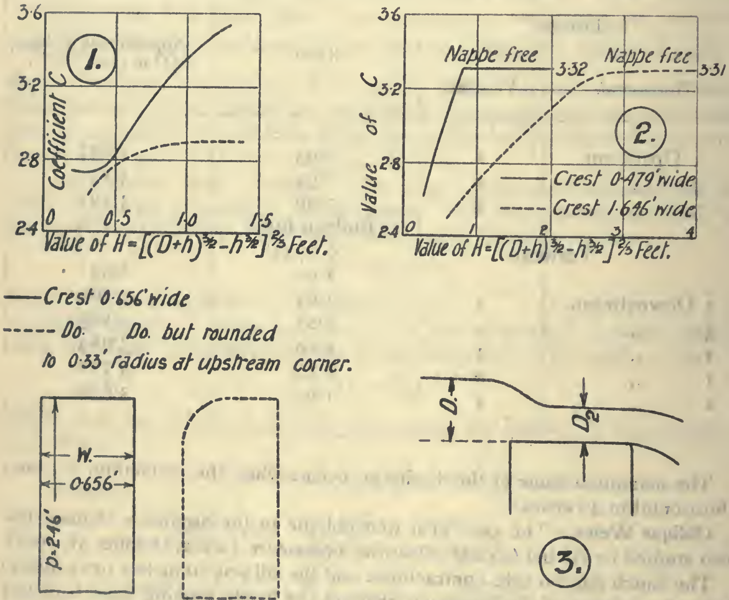
Let Q_B be the discharge over a weir of notch length L, and height p, under a head D, as computed by Bazin's formula (p. 109). Then Aichel finds:

$$Q = \left(1 - \frac{250}{\rho}\frac{D}{p}\right) Q_B,$$

where ρ is given in the following table.

η	For Channel 0·25 Metre (say 10 Inches wide).	For Channel 0·50 Metre (say 20 Inches wide).
15 degrees	305	362
30 ,,	532	700
45 ,,	893	1250
60 ,,	1923	2275
75 ,,	6579	6579

The observations are accurate, but it can hardly be supposed that they disclose the whole law, especially the effect of variations of p.



SKETCH No. 32.—Coefficients of Discharge for Broad-crested Weirs.

We can, however, be fairly certain that where p, is not too small in comparison with D, and especially where the weir has side contractions, the effect of a slight obliquity is not very marked.

Certain escape weirs in America have been constructed with the sill crenellated in plan, so as to give a sill length which is three or four times the width of the escape channel. These are stated to discharge the same quantity as a straight weir of equal length.

Aichel's values show that if p, be large in comparison with D, this is probably approximately correct, and in cases where the nature of the ground is such as to permit a very deep escape channel to be excavated more readily than one which is somewhat wider, but shallower, such construction appears to be desirable.

I should, however, prefer to deduct some 10 per cent. from the sill length in calculating the discharge, and should like to have p, at least equal to 3D. This is not impossible, as assuming that D is equal to 5 feet and that the sill length is three times the width of the channel, it is evident that each foot width of the escape or approach channel must carry at least 100 cusecs (allowing 10 per cent. deduction), and this would require about 10 feet depth, so that the bottom of the channel must be some 15 feet below the sill of the weir in order to avoid drowning the weir.

Weir with Rounded Edges.—In some cases the edges of the notch are not perfectly sharp, although sufficiently sharp to cause the nappe to spring clear of the sill of the notch (Sketch No. 30, Fig. 1). (Compare p. 141.)

Fteley and Stearns (*Trans. Am. Soc. of C.E.*, vol. 12, p. 97) working with a weir the notch sill of which was 0·035 foot wide, and with an upstream edge rounded to radii of ¼ inch, ½ inch, and 1 inch, found that the usual formulæ applied provided that 0·7 radius was added to H. D, must exceed 0·17 foot, 0·26 foot, or 0·45 foot, in order that the nappe may spring free from the sill. When the sill was 4 inches wide the correction is :

$$H = \text{Observed } H + 0{\cdot}41 \text{ radius}$$

When the sill is sufficiently wide, or the radius is sufficiently large to cause the nappe to adhere to the sill of the notch, the formulæ for sharp-edged weirs are inapplicable (see p. 128).

These corrections will be found very useful when weirs in which the notches are constructed from wooden planks are used for measuring small quantities of water.

I have applied the correction 0·70 radius to cases where the radii were 1 inch, and 1½ inch, and find that it leads to results which agree very closely with volumetric checkings, provided that the nappe springs free. Since I could not observe D, more closely than 0·005 foot, I am unable to state that the ratio 0·70 is correct, as 0·65 or 0·75 would have answered equally well.

Drowned Weirs (Sharp-edged).—The following notation will be employed (Sketch No. 33).

d, is the difference in level between the upstream water surface and the sill of the notch.

d_1, is the same quantity for the downstream water surface.

p, is the difference in level between the sill of the notch and the bottom of the upstream channel.

p_1, is the difference in level between the sill of the notch, and the bottom of the downstream channel.

Thus, the depth of water in the upstream channel is represented by $d+p$, and in the downstream channel by d_1+p_1.

D, the head over the weir is :

$$d-d_1$$

The problem is complicated, and has by no means been completely investigated. In the first place, the corrections for velocity of approach are obscure, and in theory at any rate, it is plain that :

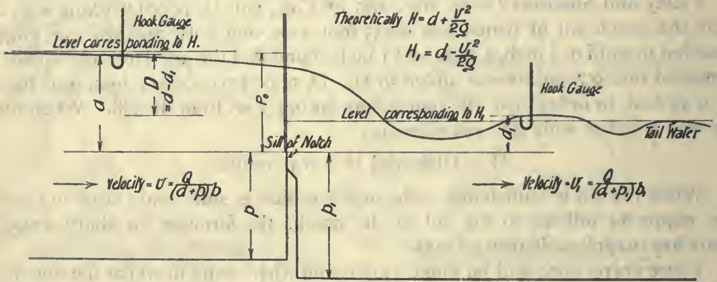
$$H = D + a\frac{v^2}{2g} - a_1\frac{v_1^2}{2g}$$

where v_1, is the velocity of "escape," *i.e.* the mean velocity in the lower channel.

The existing experiments refer solely to weirs without end contractions. Francis (*Trans. of Am. Soc. of C.E.*, vol. 13, p. 303), following the approximate theory that the discharge is the sum of that of an ordinary weir under a head equal to D, and of an orifice with a length L, and a height d_1, under a head D, finds that :

$$Q = 3{\cdot}33L(H_1 - H_2)^{1.5} + 4{\cdot}60LH_2\sqrt{H_1 - H_2}$$

where H_1, is the value of d, measured to a still water surface ; and H_2, is the value of d_1, measured at a point where the oscillations did not affect the result.



SKETCH NO. 33.—Generalised Sketch of a Drowned Weir.

So far as can be gathered from the figures given in Francis' paper, the weir was a standard Francis weir in all respects, but the velocities of approach and escape were so small that $H_1 = d$, and $H_2 = d_1$.

The formula is the result of 24 experiments on a 22·2 feet notch with :

- Q, ranging from 72 to 224 cusecs.
- d, ranging from 0·99 to 2·31 feet.
- d_1, ranging from 0·02 to 1·11 foot.

The least value of d_1, is determined by the condition that air had disappeared from under the nappe, which usually occurred at $d_1 = 0{\cdot}08$ foot, to 0·10 foot, but was delayed until $d_1 = 0{\cdot}17$ foot, when $d = 1{\cdot}96$ foot.

Francis assumes that the coefficient of the first term in the expression for Q, is always 3·33, and under this assumption the minimum value of the coefficient which is represented by 4·60, was 4·55, and the maximum 4·64.

The formula agrees within 1 per cent. with the experiments of Fteley and Stearns on a notch 6 feet wide, with $p = p_1 = 3{\cdot}17$ feet, and d, varying from 0·4 foot to 0·8 foot, and d_1, from 0·01 foot to 0·79 foot in 14 of the 22 cases. In two of the exceptions, d_1 is less than 0·10, and the nappe was probably aerated, and in two other cases $d-d_1$, was less than 0·05 foot, so that accurate results could not be expected. (*Ibid.* vol. 12, p. 103.)

In no case, except the last two, does the difference exceed 2 per cent.

Until further experiments are made, we may therefore adopt Francis' formula as a general basis, subject to the conditions that the nappe is not aerated on the under side, and that there is a wide pool below the weir, so that the nappe not only expands laterally, but falls into a body of water almost at rest.

The most important case in practice is that in which $\frac{d_1}{d}$ is small. Clemens Herschel (*Trans. Am. Soc. of C.E.*, vol. 14, p. 180), has collected the experiments of Francis, and Fteley and Stearns, and gives a formula that can be reduced to:

$$Q = 3{\cdot}33\ KLH^{1.5}$$

where $H = d$.

The values of K, are as follows:

$\frac{d_1}{d}$	K	$\frac{d_1}{d}$	K
0·00	1·000	0·12	1·003
0·01	1·006	0·13	1·000
0·02	1·009	0·14	0·997
0·03	1·009	0·15	0·994
0·04	1·010	0·16	0·991
0·05	1·011	0·17	0·988
0·06	1·010	0·18	0·984
0·07	1·009	0·19	0·981
0·08	1·009	0·20	0·978
0·09	1·008	0·21	0·973
0·10	1·007	0·22	0·970
0·11	1·005		

These values are stated to be accurate to 1 per cent., while the remainder of Herschel's table is subject to errors exceeding 1 per cent. I have been accustomed to apply these ratios to Francis' weirs (with and without end contractions), and to Cippoletti weirs under circumstances where the checking and comparison of the observations with other methods was so systematic that non-systematic errors of 1 per. cent. were certainly detected, and systematic errors of 0·5 per cent. would probably have been detected. No such errors were detected. I therefore believe that accurate comparative gaugings may be made with partially drowned weirs by applying this table. When Herschel's table for values of $\frac{d_1}{d}$, greater than 0·22 was similarly applied, non-systematic errors of 2 and 3 per cent. were discovered, and I therefore abandoned its use. The errors are known to have been caused by waves in the escape channel.

I also systematically checked a drowned ($L_1=2D$), triangular notch against a similar undrowned notch.

I was unable to discover any difference between the gauge readings, so long as $\frac{d_1}{d}$, was less than 0·25; consequently, I believe that this amount of drowning does not alter the discharge by 0·5 per cent. For values of $\frac{d_1}{d}$, greater than 0·30 certain differences could be detected, but these were very irregular. The difficulties were apparently caused by waves in the escape channel, but certain capillary phenomena also occurred. It is believed that accurate field gaugings can be undertaken only when $\frac{d_1}{d}$, is less than 0·20, or 0·25.

Better results can be obtained when the escape channel is of the same breadth as the notch, and the approach channel, provided that the form of the nappe is carefully observed.

The following abstract of Bazin's work is given, as the question of accurately measuring large quantities of water with the smallest possible loss of head is of great importance in modern turbine tests. The difficulties are obvious. The gauging weir must be specially constructed so as to conform to Bazin's standard; and the observers must be skilful, for not only must the form of the nappe be carefully noted, but the difficulties attending the determination of the quantities d, and d_1, are great, and a small error in d_1, will materially influence the results obtained. The method has, however, been systematically employed by French engineers with satisfactory results.

Sketch No. 33 represents a drowned weir and nappe of the Francis, or Fteley and Stearns type, although in the last the sides of the nappe were apparently confined for about 6 inches beyond the notch sill.

Bazin (*ut supra*) employed an escape channel of the same dimensions as the approach channel, so that the level of the water in this channel had a marked effect on the form of the nappe.

The typical form is the waved nappe (*nappe ondulée*), as shown in Sketch No. 34, Fig. I.

My own experiments lead me to believe that this form of nappe is identical with that obtained by the earlier experimenters.

There is also the "drowned" nappe or air-free form (*nappe noyée par dessous*), see Sketch No. 34, Figs. II. and III., and the adherent form (*nappe adherente*) see page 111 and Sketch No. 34, Fig. IV., and the nappe with a standing wave (*nappe à ressaut eloignée*), Sketch No. 34, Fig. V.

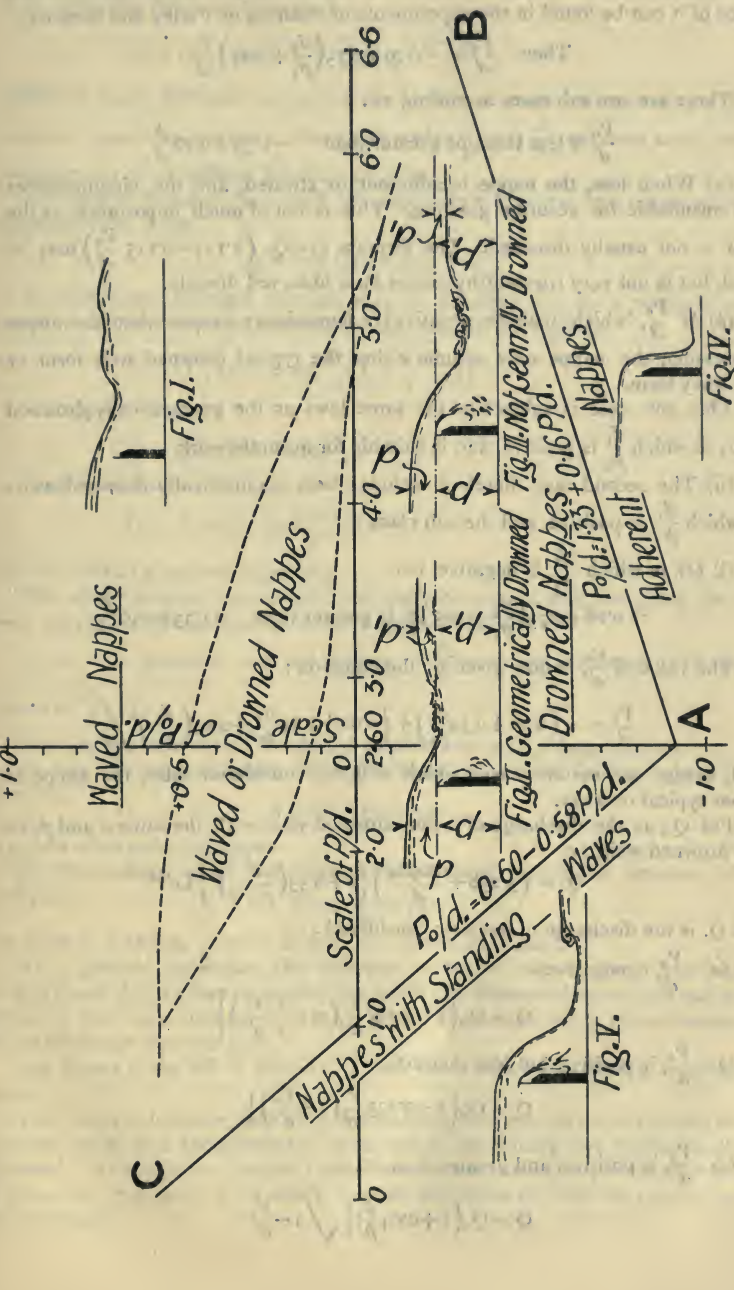
The experiments cover a very large range of values of d, d_1, and p, although p_1, is always equal to p.

The only rational method of discriminating between the various cases is to calculate the quantity P_o, which is actually the pressure existing under the nappe close to the notch sill as observed by a special piezometer (see Sketch No. 33).

The cases run as follows:

$$\text{(i)}\ \frac{d_1}{p_1}\ \text{between} -0{\cdot}00\ \text{and} -0{\cdot}36,$$

i.e. the weir is, geometrically speaking, not drowned, though the under side of the nappe is not aerated; hence owing to the relatively narrow escape channel adopted by Bazin, the tail water influences the discharge (this effect appears to



SKETCH No. 34.—Diagram showing Types of Nappe occurring in a Drowned Bazin Weir.

be entirely due to the relatively narrow escape channel used by Bazin, and no trace of it can be found in the experiments of Francis, or Fteley and Stearns).

$$\text{Then,}\quad \frac{P_o}{d} = -0{\cdot}26 + 0{\cdot}75\left(\frac{d_1}{p_1} + 0{\cdot}05\right)\frac{p}{d}$$

There are two sub cases according as :

$$\frac{P_o}{d} \text{ is less than, or greater than} \quad -1{\cdot}33 + 0{\cdot}16\frac{p}{d}$$

(*a*) When less, the nappe is adherent, or striated, and the circumstances are unsuitable for accurate gauging. This is not of much importance, as the weir is not usually drowned. The formula $Q = Q_s\left(1{\cdot}124 - 0{\cdot}115\,\frac{P_o}{d}\right)$ may be used, but is not very trustworthy unless P_o is observed directly.

(*b*) If $\frac{P_o}{d}$, (which may be negative) is numerically greater than the above expression, the nappe may assume either the typical drowned weir form, or the wavy form.

This sub case is subject to the same laws as the geometrically drowned weir, in which $\frac{d_1}{p_1}$ is positive, and is suitable for accurate work.

(ii) The second case therefore includes both geometrically drowned weirs in which $\frac{d_1}{p}$, is positive, and the sub class

(i) (*b*), in which $\frac{d_1}{p_1}$, is negative, but

$$-0{\cdot}26 + 0{\cdot}75\left(\frac{d_1}{p_1} + 0{\cdot}05\right)\frac{p}{d} \text{ is greater than} \quad -1{\cdot}33 + 0{\cdot}16\frac{p}{d}$$

The value of $\frac{P_o}{d}$, is now given by the equation :

$$\frac{P_o}{d} = -\left(0{\cdot}26 + 0{\cdot}54\frac{d_1}{p_1}\right) + \left\{0{\cdot}02 + 1{\cdot}24\frac{d_1}{p_1} + 0{\cdot}54\left(\frac{d_1}{p_1}\right)^2\right\}\frac{p}{d}$$

and, except for certain cases which will be considered later, the nappe is either typical or wavy.

Put Q_s, as the discharge of a non-drowned weir, with the same d and p, as the drowned weir, *i.e.*

$$Q_s = \left(3{\cdot}258 + \frac{0{\cdot}0984}{d}\right)\left\{1 + 0{\cdot}55\left(\frac{d}{p+d}\right)^2\right\}L d^{1.5}$$

and Q, is the discharge of the weir considered :

(*a*) $\frac{P_o}{d}$, is negative.

$$Q = Q_s\left\{1 - 0{\cdot}235\,\frac{P_o}{d}\left(1 + \frac{1}{7}\,\frac{P_o}{d}\right)\right\}$$

(*b*) $\frac{P_o}{d}$, is positive, but less than 0·6.

$$Q = Q_s\left\{1 - 0{\cdot}235\,\frac{P}{d}\left(1 + \frac{P_o}{d}\right)\right\}$$

(*c*) $\frac{P_o}{d}$, is positive, and greater than 0·6.

$$Q = Q_s\left\{1 + 0{\cdot}04\,\frac{d_1}{p^1}\right\}\sqrt[3]{1 - \frac{P_o}{d}}$$

As an approximation which may be used when the weir is not of the typical Bazin form, the following formula:

$$Q = Q_s\left\{1{\cdot}05\left(1+\frac{1}{5}\frac{d_1}{p_1}\right)\right\}\sqrt[3]{\frac{d-d_1}{d}}$$

covers all cases included under (*b*) and (*c*), with an error not exceeding 1 per cent. to 2 per cent., except in the cases where $\frac{d}{p}$, and $\frac{d_1}{p_1}$, are very small, and these are obviously unfitted for accurate work.

The above formulæ hold until the nappe form changes to a nappe with a standing wave, when

$$\frac{P_o}{d}, \text{ is less than } 0{\cdot}60-0{\cdot}59\frac{p}{d}$$

In such cases, although the weir may be drowned, (*i.e.* d_1, is positive) the nappe itself is not drowned (see Sketch No. 34) and the formula:

$$Q = Q_s\left\{1{\cdot}01-0{\cdot}245\frac{P_o}{d}\left(1+\frac{1}{5}\frac{P_o}{d}\right)\right\}$$

or approximately:

$$Q = Q_s\left(0{\cdot}878+0{\cdot}128\frac{p}{d}\right)$$

and the formula

$$Q = \left\{3{\cdot}77+0{\cdot}06\left(\frac{p}{d}\right)^2\right\}Ld^{1.5}$$

may be used as a general expression.

The above formulæ are complicated, and the graphic diagram given above (No. 34) renders the matter more easily comprehensible.

Here, the abscissæ are the values of $\frac{p}{d}$, and the ordinates are the values of $\frac{P_o}{d}$.

The observations should be plotted as points on such a diagram, and

(i) Any observation which is represented by a point falling below the line AB,

$$y = -1{\cdot}33+0{\cdot}16x$$

is a case of an adhering nappe.

(ii) Proceeding round the diagram, observations which fall between AB and AC:

$$y = 0{\cdot}60-0{\cdot}58x$$

are cases of drowned, or wavy nappes, and the formulæ hold.

As a general principle, the drowned forms of nappe occur near the lines AB and AC, and wavy nappes are found at a distance, but no rule can be given, as the forms depend to a large extent on the preceding circumstances of the discharge over the weir.

(iii) Points to the left of the line AC, are cases of a nappe with a standing wave.

The diagram, however, suffices to discriminate between the cases suitable for accurate work, and the unsuitable cases which are usually not geometrically drowned. Although the precise form of the nappe cannot be predicted, there is never any real doubt as to which formula should be selected for calculating the discharge.

WEIRS WITH OTHER THAN SHARP-EDGED NOTCHES.—It is quite impossible to give any rules for the discharge of such weirs. (See Diagrams Nos. 2 and 3.)

Our accurate knowledge of the subject is mainly due to Bazin (*ut supra*), but a large amount of work has been done of late years in the United States. The whole information has been collated and re-calculated in a most able manner by Horton (*Weir Experiments, Coefficients and Formulæ*), and this book must be regarded as absolutely indispensable for all engineers who have any occasion to consider weirs other than those of standard form.

The coefficients of discharge as determined by various experimenters show large differences. These are probably due to the causes discussed under Sharp-edged Weirs; but such matters as the place where the head was observed, and the method of observation being rarely recorded with sufficient completeness, it is impossible to elucidate fully the obscurities. The following examples must therefore be considered merely as a selection from several hundred recorded observations; and my choice is unfortunately somewhat influenced by accidents such as having had occasion to use or check the results personally, or finding that either Mr. Horton or myself were able to express the facts in a short formula.

It appears that the form of the weir has a very great effect upon the coefficient of discharge when the head is small, and below a certain limit the coefficient of discharge varies very rapidly with the head. Sketch No. 35, Fig. I., shows an actual example, but it must not be taken as typical; the limit where variation ceases being usually well marked as in Fig. II. Sketch No. 32. This limit may be said (very roughly) to occur when the head exceeds twice the longest horizontal dimension of the weir crest. Above this limit the coefficient C, is usually constant, or it may be found that:

$$C = a + bH$$

The variations below this limit are not as a rule reducible to any mathematical formula, although this is probably merely due to lack of adequate and detailed information, and there is a certain amount of evidence to show that a formula of the type:

$$C = kH^{R}$$

is very generally true.

FLAT-TOPPED WEIRS.—Put w, equal to the width of the flat top (sketch No. 32, Fig. I.).

(i) If D, is less than 1·5 w, the nappe adheres to the notch crest, and

$$Q = Q_s \left(0{\cdot}7 + 0{\cdot}185\frac{D}{w}\right)$$

holds for a weir of the Bazin type, and with a very fair approximation for all weirs; Q_s, being the discharge of a sharp edged weir of the same length and height under the same head.

(ii) If D, is greater than 1·5w

the nappe springs free from the upstream edge of the flat top (see Sketch No. 29), and the weir is sharp edged for all effective purposes (see Diagram 2, Sketch No. 32).

The various formulæ often given for flat-topped weirs such as:

$$\left.\begin{aligned} Q &= 2{\cdot}64\ LH^{1.5} \\ \text{and,}\quad Q &= 3{\cdot}09\ LH^{1.5} \end{aligned}\right\}\ \text{where } H = D + h$$

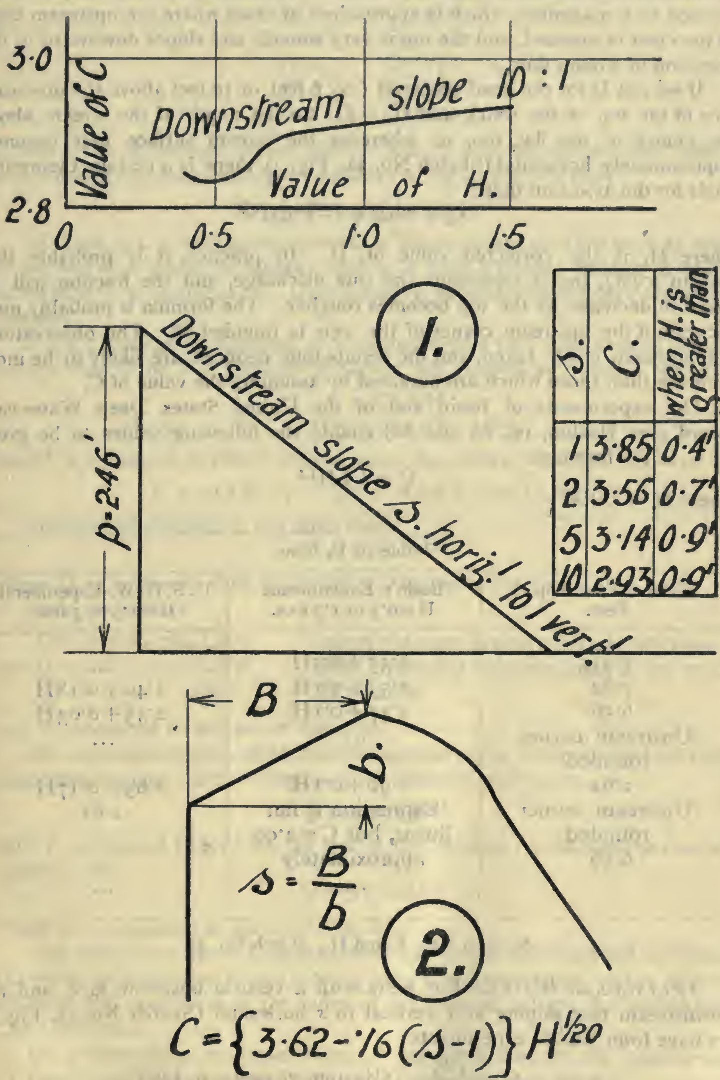
can be applied within certain limits, but these are badly defined.

Horton (*Weir Experiments*, p. 121) considers that:

$$Q = 2{\cdot}64 L H^{1.5}$$

is applicable when w, exceeds 3 feet, and $\frac{H}{w}$, lies between and 0·25, and 1·5.

The weir top must be horizontal, and my own experiments lead me to believe



SKETCH NO. 35.—Diagram of Triangular and Overflow Type of Weir.

that if the water surface on top of the crest is at all wavy, the value 2·64 is exceeded, and 2·73 may be taken as more probable.

The value

$$Q = 3{\cdot}09LH^{1.5}$$

as theoretically obtained (see *Ency. Brit.* "Hydraulics," p. 472) may be considered as a maximum, which is approached in cases where the upstream edge of the crest is rounded, and the top is very smooth and slopes downward in the direction of stream flow.

If we put D for the head observed (say 6 feet or 10 feet above the upstream face of the top of the weir), and $D_2 = kD$ for the depth of the stream above the centre of the flat top, or wherever the stream surface first becomes approximately horizontal (Sketch No. 32, Fig. 3), there is a certain theoretical basis for the assertion that:

$$Q = 8{\cdot}02k\sqrt{1-k}\ LH^{1.5}$$

where H, is the corrected value of D. In practice, it is probable that 0·95 to 0·98Q, better represents the true discharge, and the fraction will be found to decrease as the top becomes rougher. The formula is probably most accurate if the upstream corner of the weir is rounded off. The observations are, however, easily taken, and the results thus deduced are likely to be more accurate than those which are obtained by assuming the value of C.

The experiments of Bazin and of the United States Deep Waterways Board (see Horton, pp. 66 and 88) enable the following values to be given for C, in the formula:

$$Q = CLH^{1.5}$$

where $H = D + h$.

	Value of C, from	
Width of Weir Top in Feet.	Bazin's Experiments. H=0·5 to 1·3 foot.	U. S. D. W. Experiments. H=1·5 to 5 feet.
1·315	2·35+0·5H	...
2·62	2·53+0·1H	2·40+0·18H
6·56	2·47+0·1H	2·35+0·03H
Upstream corner rounded	...	...
2·62	2·90+0·1H	2·65+0·17H
Upstream corner rounded	Expression is not linear, but C=2·90 approximately	2·81
6·56	...	...

See also Figs. I. and II., Sketch No. 32.

Triangular Weirs.—For weirs with a vertical upstream face, and the downstream face sloping at 1 vertical to *s* horizontal (Sketch No. 35, Fig. 1) we have from Bazin's experiments:

$$C = \frac{3{\cdot}85}{s^{0.12}} \qquad \text{(Horton, \textit{ut supra}, p. 125.)}$$

The head should exceed 0·3 foot when s, is less than 2, and 0·75 foot for larger values of s, and Bazin's experiments cease with H = 1·5 foot.

So also, the U.S. Deep Waterways Experiments give for a flat-topped crest 0·67 foot wide, with a vertical downstream face, and the upstream face sloping in 1 in s, the following values for heads exceeding 1·75 foot.

$s=1$	$C=3{\cdot}70$
$s=2$	$C=3{\cdot}74$
$s=3$	$C=3{\cdot}58$
$s=4$	$C=3{\cdot}49$
$s=5$	$C=3{\cdot}39$

For weirs of the type shown (Sketch No. 35, Fig. 2), if the crest radius is large enough to cause the nappe to adhere to the face of the dam :

$$C = \{3{\cdot}62-0{\cdot}16\,(s-1)\}H^{0.05}$$

WATER-FALLS.—The measurement of the discharge of a fall such as occurs at the end of a gutter or launder, and in a more irregular fashion in natural water-falls, is of importance. Unfortunately, experiments are very few and far between.

If the discharge is regarded as occurring over a very wide crested weir the experiments of the U.S. Geological Survey (see Horton, *ut supra*, p. 104) give :

$$Q = CLH^{1.5}$$

where, $$H^{1.5} = (D+h)^{1.5} - h^{1.5}$$

and D, is measured 10 feet to 16 feet above the crest of the weir.

$$C = 2{\cdot}58 \text{ to } 2{\cdot}71 \quad \text{for } H = 0{\cdot}8 \text{ foot to } 4{\cdot}5 \text{ feet}$$

Bellasis (*Hydraulics*, p. 99) finds that :

$$Q = 4{\cdot}74LD^{1.5}$$

where D, is measured close to the fall, *i.e.* on top of the crest.

Bazin finds that :

$Q = 2{\cdot}50$ to $2{\cdot}64\ LH^{1.5}$ where the crest is sharp-edged upstream.
and $Q = 2{\cdot}65$ to $2{\cdot}91\ LH^{1.5}$ with a rounded upstream edge to the crest,
with $H = D+h$

and D, measured 16 feet above the crest.

My own experiments, which are comparable to the theory of Bellasis (being taken in a launder 2 feet × 1 foot deep, of planed boards) give :

$$Q = 4{\cdot}43\ LD^{1.5}$$

with D, measured 1 foot from the end of the launder, *i.e.* as was the case with Bellasis, and

$$Q = 3{\cdot}12\ LD^{1.5}$$

where D is measured 20 feet above the fall.

The difficulty is that if D, is measured far enough above the fall to eliminate the effect of variations in the velocity over the cross-section of the approach channel, the roughness of the sides of the channel affects the observations, and *vice versa*.

On re-computing my observations, it appeared that for the launder used, D, should preferably be measured 10 feet (approx.) above the fall.

Coefficients of Discharge for Large Weirs. — It frequently happens in practical calculations that either L, or D, are far greater than in any accurate experiments. Bazin's experiments were intended to apply to cases where L was very great; and his results agree generally with the formula given by Francis when L is large, and D exceeds 0·60 or 0·70 foot. This is sufficient to render it probable that no very great error will be introduced by the application of either formula to the calculation of such discharges.

Sharp-edged Weirs.—The maximum length of weir over which the discharge has been accurately measured is 29·87 feet. The side contractions were partially suppressed, and Carpenter (*Trans. of Am. Soc. of Mech. Eng.*, vol. 19, p. 255) finds the following values of C_1, in the equation:

$$Q = C_1(L - 0{\cdot}2D)\{(D+h)^{1.5} - h^{1.5}\}$$

H, in Feet.	C_1	H, in Feet.	C_1
1·9 to 0·9	3·53	0·6 to 0·5	3·58
0·9 „ 0·8	3·54	0·5 „ 0·4	3·61
0·8 „ 0·7	3·55	0·4 „ 0·3	3·66
0·7 „ 0·6	3·56		

These values agree very fairly well with those which are obtained by applying Freeze's formula to a short weir similarly suppressed. The length of the weir sill would therefore appear to have but little effect upon the general laws of weir discharge.

For high heads, we find eight observations on a Bazin type of weir conducted at Cornell University, and checked over a standard Bazin weir, which give:

$$Q = 3{\cdot}278\,L\{(H+h)^{1.5} - h^{1.5}\}$$

with H, from 2·00 to 4·88 feet, and a probable error of 0·050, say 1·5 per cent. in the coefficient (see Horton, *Weir Discharges*).

The three cases for heads over 3 feet give C = 3·321.

Thus, for sharp-edged weirs with complete contraction, Francis' formula holds up to H = 4·7 feet at least.

Flat-topped Weirs.—Certain experiments on the Bari Doab, on a weir 80 feet long, as per Sketch No. 208, when compared with rod float observations, gave the following result:

$$Q = 3{\cdot}49LD^{1.5}$$

D, varying between 2·5 and 4·2 feet.

Similar experiments on other weirs of the same section gave the following results:

L = 29 feet	C = 3·52
L = 37 „	C = 3·54
L = 46 „	C = 3·58
L = 50 „	C = 3·59

with $v = 2$ to 3 feet per second. D, ranging in each case from 3 to 3·5 feet.

Bazin's experiments on similar weirs, but with a slope of 1 : 5, in place of 1 in 10, checked by a standard weir, give:

$$C = 3{\cdot}48 \text{ to } 3{\cdot}45 \text{ in the equation}$$

$$Q = CL\left(D + \frac{v^2}{2g}\right)^{1.5}, \text{ with}$$

$$D = 0{\cdot}6 \text{ to } 1{\cdot}30 \text{ feet.}$$

At Cornell, with standard weir checking

$$C = 3{\cdot}43 \text{ to } 3{\cdot}58$$

for $D = 2{\cdot}62$ feet to $4{\cdot}20$ feet.

Thus, for such weirs, the value $C = 3{\cdot}50$, may be considered as well established.

Chatterton (*Hydraulic Experiments in the Kistna Delta*) worked on flat-topped weirs 2 feet wide, with $p = 2$ to 3 feet, and $L = 18$ to 30 feet, and found by current meter observations that:

$$C = 2{\cdot}99 \text{ to } 3{\cdot}21, \text{ in the equation } Q = CLD^{1.5}, \text{ with } D = 3 \text{ to } 4{\cdot}5 \text{ feet.}$$

The higher values are probably affected by suppressed contraction, and for a weir with complete contraction the value $C = 3{\cdot}09$ may be adopted.

Chatterton also experimented on the Kistna Anicut (see Sketch No. 177), in which $L = 3{,}500$ feet approximately. He found as follows:

(i) With the tail water below the crest:

$$Q = 3{\cdot}13 \, LD\sqrt{D + 0{\cdot}035v^2}$$

with $D = 4{\cdot}64$ feet.

(ii) With the tail water above the crest:

$$Q = 3{\cdot}09 \, L\{(D+h)^{1.5} - h^{1.5}\} + C_2 L d_1 \sqrt{D+h}$$

where D, is the difference of level between head and tail waters, and d_1, is the depth of the tail water over the crest.

We thus get the following table:

D, Feet.	d_1, Feet.	v, Feet-seconds.	C_2.	C_2 from the Formula $4{\cdot}90 + 0{\cdot}32d_1$.
2·72	8·70	5·21	7·65	7·68
3·43	6·33	4·63	7·19	6·93
3·15	6·18	4·28	6·86	6·88
3·67	4·48	3·73	6·05	6·09

Lewis, at Rasul, on a flat-topped drowned weir (see Sketch No. 164), 3 feet wide, with $p = 3$ feet, $D = 0{\cdot}7$ foot, $d_1 = 5{\cdot}5$ feet, found a discharge equivalent to that obtained by putting $C_2 = 6{\cdot}57$, in the above formula. The expression $4{\cdot}90 + 0{\cdot}32d_1$, gives 6·66.

In the case of flat-topped drowned weirs, of all kinds, it is therefore probable that the above formula with $C_2 = 4{\cdot}90 + 0{\cdot}32d_1$, is not very far removed from the correct value; and that $C_2 = 8{\cdot}02$ if d_1, exceeds 10 feet (see p. 289).

The available evidence seems to indicate that in the case of sharp-edged weirs, the general formulæ of Bazin or Francis may be applied for large values of L, or D, without any serious error.

For flat-topped weirs, the theoretical formula:

$$Q = 3{\cdot}09 LD^{1.5}$$

is probably sufficiently accurate ; and, if drowned, this formula holds for the unsubmerged portion of the discharge, and the formula for C_2, already given is not likely to introduce serious error.

For flat-topped weirs, with an apron sloping downstream, the formula :

$$Q=3{\cdot}50\ LD^{1.5}$$

appears to be well established.

Horton (*Weir Experiments, Coefficients, and Formulæ*) suggests that the Francis formula is probably applicable to nearly all weirs when D, is large. The experiments tabulated in Horton's book may be advantageously consulted when determining the discharge of ogee, or broad weirs, under high heads.

Further information will be found in a graphic form under Diagram No. 2 (p. 1018). The statements concerning the accuracy of the diagram and the accompanying sketches must be carefully borne in mind. While I have found the accuracy of 3 per cent., which is there stated to be possible, amply sufficient in my own practice, I believe that in many cases the tables given by Horton, if used with judgment, permit better results to be obtained.

CHAPTER V

DISCHARGE OF ORIFICES

SYMBOLS.

a, is the area of the orifice, in square feet.
C, is the coefficient of discharge of the orifice.
c_c, is the coefficient of contraction of the orifice.
c_v, is the coefficient of velocity of the orifice.
C_1, or C_i, is the coefficient of discharge of a partially suppressed orifice, or of an orifice subject to velocity of approach.
C_p (see p. 149).
d, is the vertical height of the orifice, or the diameter of a circular orifice.
d, is also the diameter of a pipe connected with the orifice when this is equal to the diameter of the orifice.
D, is the diameter of the fountain pipes, in feet (see p. 152).
e, is the thickness of the walls of the metal tube forming a Borda mouthpiece (see p. 151).
h, is the head, in feet, measured to the centre of the orifice. H, is used for h, in those equations in which d, is measured in inches.
h_1, and h_2, are the depths of the top of a submerged rectangular orifice, below head and tail water levels.
h_d, is the effective head $= h_1 - h_2$, for a submerged orifice.
H_1, and H_2, are the depths below head water level, of the top and bottom of a large unsubmerged rectangular orifice.

k, is the vertical height of a rectangular orifice.
l, is the length of the tube forming the mouthpiece of an orifice.
m, is the length of the portion of the perimeter of the orifice over which contraction is wholly or partially suppressed.
n, is the ratio $\frac{\text{Diameter of channel of approach}}{\text{Diameter of orifice}}$.
p, is the perimeter of the orifice.
Q, is the discharge of the orifice in cusecs.
u, is the velocity of approach, in feet per second.
v, is the velocity, in feet per second, at the smallest area of the jet issuing from the orifice.
w, is the horizontal width of a rectangular orifice, in feet.

SUMMARY OF EQUATIONS AND FORMULÆ

Velocity of efflux: $\begin{cases} v = c^v \sqrt{2gh} \text{ feet per second.} \\ c_v = 0{\cdot}97 \text{ to } 0{\cdot}99. \end{cases}$

Discharge: $Q = Ca\sqrt{2gh}$ cusecs.

Correction for velocity of approach, or suppression of contraction. $Q = C_1 a\sqrt{2gh}$ cusecs.

CIRCULAR ORIFICES:

Sharp-edged: $Q = 4{\cdot}9a\sqrt{h}$ (see Table, p. 142).

Bell-mouth: $Q = 7{\cdot}8a\sqrt{h}$ (see Table, p. 147).

Tubular, projecting outwards: $Q = 6{\cdot}5a\sqrt{h}$.

Tubular, projecting inwards: (a) Borda. $Q = 4{\cdot}2a\sqrt{h}$. (b) Bidone. $Q = 6a\sqrt{h}$.

Circular orifices, sharp-edged:

$$C = 0{\cdot}5952 + \frac{0{\cdot}018}{\sqrt[3]{d^2}\sqrt{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad \left[\frac{d}{\text{Inches}}\right]$$
(see p. 144)

Correction for velocity of approach:

$$C_1 = \frac{0{\cdot}485}{1 - \frac{0{\cdot}64}{n^2}}\left\{1 + \left(\frac{n^2 - 1}{2n^2}\right)^2\right\}$$

$$\text{or } C_1 = \frac{0{\cdot}97n^2}{\sqrt{2{\cdot}62n^4 - 1{\cdot}62}}.$$

Correction for partial suppression of contraction:

$$C_1 = C\left(1 + 0{\cdot}128\frac{m}{p}\right).$$

Submerged Discharge: $Q = Ca\sqrt{2gh_d}$.

Values of $C\sqrt{2g}$, are on the average 1 per cent. less than those given above (see p. 146).

Fountain Discharge: $Q = 5{\cdot}6D^2\sqrt{h}$ (see p. 152).

RECTANGULAR ORIFICES:

Sharp-edged: $Q = 4{\cdot}8$ to $5a\sqrt{h}$ (see p. 154).

Contraction suppressed on three sides: $Q = 5{\cdot}3$ to $5{\cdot}7a\sqrt{h}$ (see p. 158).

Thick-walled orifices: $Q = 6$ to $6{\cdot}4\,a\sqrt{h}$.

Square orifices: $C = 0{\cdot}598 + \frac{0{\cdot}018}{\sqrt[3]{d^2}\sqrt{H}}$ . . . . . [Inches]
(see p. 156)

Sluices—(i) Submerged:

(a) Bornemann: $Q = Cwk\sqrt{2g\left(h_1 - h_2 + \frac{u^2}{2g}\right)}$ (see p. 165).

With $w = 3$ feet.

$$C = 0{\cdot}664 + 0{\cdot}053\frac{\sqrt{k}}{h_2 + \frac{k}{2}}.$$

or, with $w = 1$ foot to 2 feet.

$$C = 0{\cdot}541 + 0{\cdot}15 \frac{\sqrt{k}}{h_2 + \frac{k}{2}}.$$

(*b*) Benton: $Q = Cwk\sqrt{2gh_d}$.
$C = 0{\cdot}7201 + 0.0074w$ (see Table, p. 167).

(*c*) Chatterton: $Q = Cwk\sqrt{2gh_d}$.
$C = 0{\cdot}83 - 0{\cdot}11h_d$ (see p. 168).

(ii) Unsubmerged: $Q = 5{\cdot}05w\,(H_2^{1.5} - H_1^{1.5})$.

DISCHARGE OF ORIFICES.—For simplicity's sake, let us consider an orifice under a constant head, which is the same at all points in the area of the orifice.

The pressure at the orifice, if it were closed, being that corresponding to h, feet of water, the theoretical velocity of efflux of water through the orifice is given by:

$$v^2 = 2gh$$

Thus, the quantity discharged should be:

$$Q = \text{area of orifice} \times v = a\sqrt{2gh}$$

Actually, we find experimentally that the maximum velocity attained near the orifice (so that the question of free fall may be neglected) is never quite $\sqrt{2gh}$, but is represented by:

$$v = c_v\sqrt{2gh}$$

where c_v, is "near to 0·97" (although modern results for sharp-edged orifices indicate 0·98 to 0·99), and is termed the velocity coefficient. Also, at the point where this velocity is attained the area of the jet is not equal to that of the orifice, but is equal to the area of the orifice multiplied by c_c, where c_c, is termed the coefficient of contraction, and for a sharp-edged circular orifice is not far off 0·62. We thus get for the quantity discharged:

$$Q = c_v \times \text{theoretical velocity} \times c_c \times \text{area of orifice}$$

$$Q = c_v c_c a\sqrt{2gh} = Ca\sqrt{2gh}$$

where $C = c_v c_c$, and is termed the coefficient of discharge of the orifice.

The above is a summary of the explanation of the question usually given. I assume that it tends to elucidate the subject, since it bears little relation to the physical facts.

It is possible to measure c_v, and c_c, for a circular orifice. It is also absolutely certain that c_c, cannot be measured with any degree of accuracy for any other orifice, and that c_v, can only be defined by referring it to the mean velocity over the area of the jet.

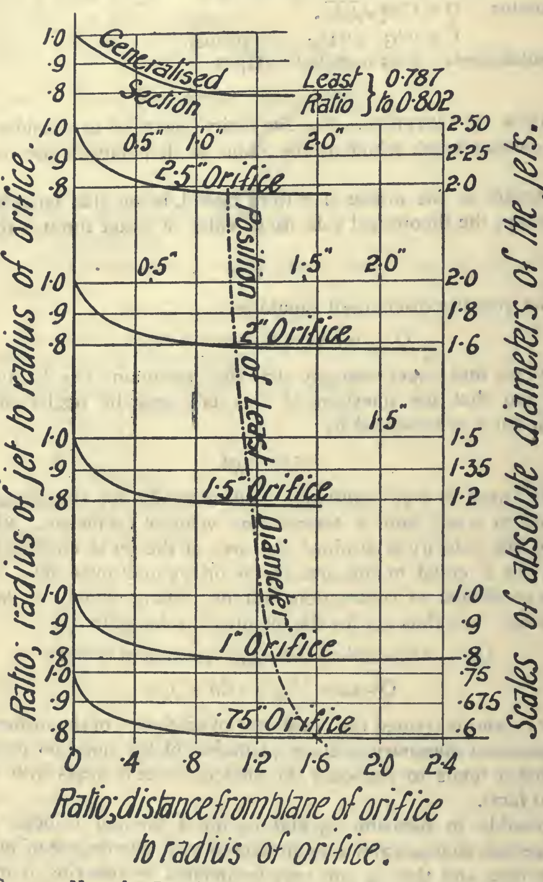
Sketch No. 36 shows the cross-section of jets from circular orifices of various diameters, and is taken from a paper by Messrs. Judd and King (*American Assoc. for Advancement of Science*, 1909) with the addition of Bazin's general contour (*Recherches Hydrauliques*).

For orifices of any cross-section, other than circular in form, the shape of the jet is very complicated, and can only be described as longitudinally ribbed, and swollen at regular intervals.

These forms have been studied by many experimenters (particularly by

Rayleigh, *Proc. Roy. Soc.*, vol. 29), but they concern engineers only in so for as they afford proof of the impossibility of measuring c_c, with any accuracy.

Practically, the coefficient of discharge C, is the important figure (except occasionally c_v, for such matters as Pelton wheels). I shall therefore in future



SKETCH NO. 36.—Longitudinal Sections of Jets from Sharp-edged Circular Orifices.

call C, the coefficient. As will later appear, c_c, possesses a certain somewhat limited theoretical importance.

It is stated in many text-books that when h, is less than $5d$ (where d, represents the vertical height of the orifice), more accurate formulæ than

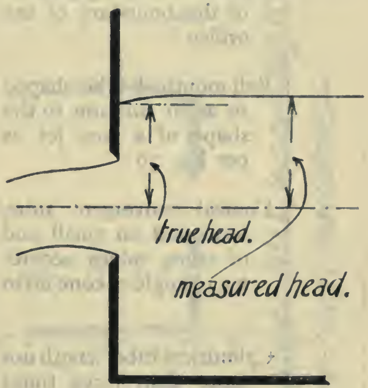
$$Q = C \text{ area } \sqrt{2gh}$$

can be obtained by considering the variation in the pressure from point to point of the orifice. These formulæ are complicated, and the theory upon which they rest is of doubtful accuracy. I have altered the figures (wherever they have been employed by experimenters) so as to cause the simpler formula to be applicable.

CAUSES OF VARIATION OF C.—Boussinesq has proved mathematically that C_1 is constant for all cases hitherto investigated. (For a very excellent précis see Boulanger, "Hydraulique Génerale.") We find by experiment that C_1 varies somewhat both with h_1 and a. These variations are most marked when h_1 and a_1 are small, and this is especially the case when both are small. As they increase C_1 tends to become constant, and (in cases hitherto investigated) this constant value is slightly less than that obtained theoretically.

I therefore believe that true constant values exist for C_1 and that the divergences occurring at low heads and for small sizes of orifices, are due to extraneous influences, such as viscosity, capillary adhesion at the edges of the jet, and also (very probably) to errors in workmanship due to the difficulty of making a small orifice which is truly "sharp-edged" under small heads.

Sketch No. 37 also shows how (when the head over the orifice is small) the measured head may differ from the head producing the velocity at the *vena contracta*, or area where the velocity is a maximum.



SKETCH NO. 37.—Effect of Surface Contraction on the Head observed over an Orifice.

For this reason, I have in some cases added the supposed constant value towards which C_1 tends, as h_1 and a_1 increase.

VELOCITY OF APPROACH.—Theoretically speaking, if the water flows towards the orifice with a velocity u, we have :

$$v^2 = 2gh + u^2$$

so that, $$Q = Ca\sqrt{2g\left(h + \frac{u^2}{2g}\right)}$$

As a matter of fact, the most useful expression is :

$$Q = C_1 a \sqrt{2gh}$$

I shall later discuss this equation as applied to several particular cases.

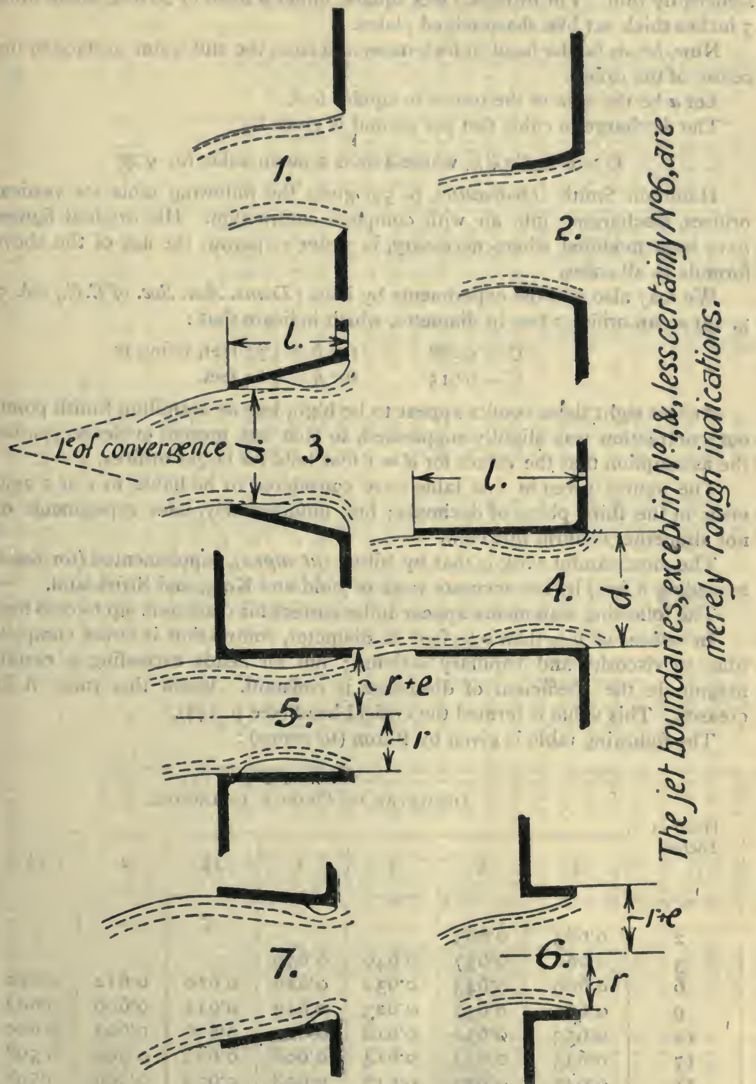
CIRCULAR ORIFICE.—*General Rules.*—Consider a circular orifice, and in future define h_1 as measured from the geometrical centre of the orifice to the surface of the water above it. The following table of Bellasis gives a general idea of the approximate values of C_1 for various forms of the orifice. The values are selected as leading to a first approximation to the size of orifice required, and in accurate work they should be corrected by the rules given hereafter.

Sketch No. 38.	Description.	Remarks.	C.	c_c	c_v
Fig. No. 1	Orifice in a wall whose edges are so thin in comparison with the head and the diameter of the orifice that the jet springs clear and does not wet any portion of the boundary of the orifice	Theoretical value	0·607	...	...
		Fair average for h, about 1 to 6 feet; d, about 0·07 to 0·20 foot	0·61	0·63	0·97
		Constant value to which C, tends	0·598	...	...
2	Bell-mouthed tube shaped so as to conform to the shape of a free jet as per Fig. 36	...	0·97	1·0	0·97
3	Conical convergent tube, measured on small end of cone, values according to angle of cone up to	...	0·94	0·98	0·96
		Fair average, say,	0·90	...	...
4	Cylindrical tube, length not more than three times the diameter	...	0·82	1·0	0·82
5	Ditto., projecting inwardly, with jet adhering	Theoretical	$\frac{1}{\sqrt{2}}$ =	0·707	...
		Experimental	0·75	0·75	1·0
6	Ditto., ditto., with the jet springing clear	Theoretical	$\frac{1}{2}$		
		Experimental	0·51	0·52	0·98
7	Divergent conical tube; calculated for area of the smallest section	Varies, Bellasis' suggestion	1·46	...	...
Sketch No. 40.	Divergent bell-mouth; ditto.	...	2·0	1·0	2·0

The values of c_v are probably somewhat less and those of c_c proportionately greater than the truth. c_v was obtained by observing the path of the jet and is therefore affected by air resistance, and given for use in similar calculations only.

All these figures refer to orifices in a vertical plane. Where the orifices are in a horizontal or inclined plane, the figures are not appreciably altered, but the head should usually be measured from the centre of the jet at the point where it first springs free from the walls of the tube or orifice.

Sharp-edged Orifices.—An orifice is defined as sharp-edged when its edges are so thin that the jet springs free without wetting them. The absolute



SKETCH No. 38.—Typical Forms of Orifices.

sharpness required entirely depends upon circumstances. For example, Bilton (*Proc. of Victorian Inst. of Engineers*) found that for holes $\frac{1}{40}$th of an inch in diameter, a thickness of 0·005 inch was necessary. Whereas for orifices say 6 inches across, under a head exceeding 4 feet, $\frac{1}{16}$th of an inch plates are sufficiently thin. For orifices 2 feet square, under a head of 20 feet, angle irons 3 inches thick act like sharp-edged plates.

Now, let h, be the head in feet, measured from the still water surface to the centre of the orifice.

Let a be the area of the orifice in square feet.

The discharge in cubic feet per second is given by:

$$Q = 8{\cdot}02\ Ca\sqrt{h}, \text{ where } 8{\cdot}02 \text{ is a mean value for } \sqrt{2g}$$

Hamilton Smith (*Hydraulics*, p. 59) gives the following table for vertical orifices, discharging into air with complete contraction. His original figures have been modified where necessary, in order to permit the use of the above formula in all cases.

We may also add the experiments by Ellis (*Trans. Am. Soc. of C.E.*, vol. 5, p. 19) on an orifice 2 feet in diameter, which indicate that:

$C = 0{\cdot}588$ for $h = 1{\cdot}77$ feet, rising to
$C = 0{\cdot}615$ for $h = 9{\cdot}64$ feet.

At first sight these results appear to be high, but as Hamilton Smith points out, contraction was slightly suppressed, so that our present evidence justifies the assumption that the values for $d = 1$ foot hold for larger orifices.

The figures given in this table were considered to be liable to 1 or 2 units error in the third place of decimals; but, unfortunately, later experiments do not altogether confirm this view.

The most careful work is that by Bilton (*ut supra*), supplemented (for heads exceeding 8 feet) by the accurate work of Judd and King, and Strickland.

The following statements appear to be correct for diameters up to 0·20 foot.

In orifices of less than 0·20 foot in diameter, contraction is never complete (due to viscosity and capillary actions); but for heads exceeding a certain magnitude the coefficient of discharge is constant. Below this value it increases. This value is termed the critical head (see p. 144).

The following table is given by Bilton (*ut supra*):

Head in Inches.	Diameter of Orifice in Inches.						
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
2	0·683	0·663					
3	0·680	0·657	0·646	0·640			
6	0·669	0·643	0·632	0·626	0·618	0·612	0·610
9	0·660	0·637	0·623	0·619	0·612	0·606	0·604
12	0·653	0·630	0·618	0·612	0·606	0·601	0·600
17	0·645	0·624	0·614	0·608	0·603	0·599	0·598
18	0·643	0·623	0·613	0·608	0·603	0·599	0·598
22	0·638	0·621	0·613	0·608	0·603	0·599	0·598
45 and over	0·6285	0·621	0·613	0·608	0·603	0·599	0·598

VALUES OF THE COEFFICIENT C IN THE EQUATION

$$Q = Ca\sqrt{2gh}$$

FOR CIRCULAR VERTICAL ORIFICES WITH SHARP EDGES, FULL CONTRACTION AND FREE DISCHARGE INTO AIR.

h Feet.	Diameter of Orifices in Feet. 0·02	0·03	0·04	0·05	0·07	0·10	0·12	0·15	0·20	0·40	0·60	0·80	1·0
0·3	...	...	...	0·637	0·628	0·620	0·612	0·607	...	...	...	...	...
0·4	...	...	0·637	0·631	0·624	0·618	0·611	0·605	...	...	...	...	...
0·5	...	0·643	0·633	0·627	0·621	0·615	0·610	0·605	0·599	0·593	0·585	...	...
0·6	0·655	0·640	0·630	0·624	0·618	0·613	0·609	0·605	0·600	0·594	0·588	0·581	...
0·7	0·651	0·637	0·628	0·622	0·616	0·611	0·607	0·604	0·601	0·595	0·590	0·585	0·579
0·8	0·648	0·634	0·626	0·620	0·615	0·610	0·606	0·603	0·601	0·596	0·591	0·587	0·583
0·9	0·646	0·632	0·624	0·618	0·613	0·609	0·605	0·603	0·601	0·597	0·593	0·589	0·585
1·0	0·644	0·631	0·623	0·617	0·612	0·608	0·605	0·603	0·600	0·598	0·594	0·590	0·586
1·2	0·641	0·628	0·620	0·615	0·610	0·606	0·604	0·602	0·600	0·598	0·595	0·592	0·589
1·4	0·638	0·625	0·618	0·613	0·609	0·605	0·603	0·601	0·600	0·599	0·596	0·593	0·591
1·6	0·636	0·624	0·617	0·612	0·608	0·605	0·602	0·601	0·600	0·599	0·597	0·594	0·592
1·8	0·634	0·622	0·615	0·611	0·607	0·604	0·602	0·601	0·599	0·599	0·597	0·595	0·594
2·0	0·632	0·621	0·614	0·610	0·607	0·604	0·601	0·600	0·599	0·599	0·597	0·596	0·594
2·5	0·629	0·619	0·612	0·608	0·605	0·603	0·601	0·600	0·599	0·599	0·598	0·597	0·595
3·0	0·627	0·617	0·611	0·606	0·604	0·603	0·601	0·600	0·599	0·599	0·598	0·597	0·596
3·5	0·625	0·616	0·610	0·606	0·604	0·602	0·601	0·600	0·599	0·599	0·598	0·597	0·596
4·0	0·623	0·614	0·609	0·605	0·603	0·602	0·600	0·599	0·599	0·598	0·597	0·597	0·596
5·0	0·621	0·613	0·608	0·605	0·603	0·601	0·599	0·599	0·598	0·598	0·597	0·596	0·596
6·0	0·618	0·611	0·607	0·604	0·602	0·600	0·599	0·599	0·598	0·598	0·597	0·596	0·596
7·0	0·616	0·609	0·606	0·603	0·601	0·600	0·599	0·599	0·598	0·598	0·597	0·596	0·596
8·0	0·614	0·608	0·605	0·603	0·601	0·600	0·599	0·598	0·598	0·597	0·596	0·596	0·596
9·0	0·613	0·607	0·604	0·602	0·600	0·599	0·599	0·598	0·597	0·597	0·596	0·596	0·595
10·0	0·611	0·606	0·603	0·601	0·599	0·598	0·598	0·597	0·597	0·597	0·596	0·596	0·595
20·0	0·601	0·600	0·599	0·598	0·597	0·596	0·596	0·596	0·596	0·596	0·596	0·595	0·594
50·0	0·596	0·596	0·595	0·595	0·594	0·594	0·594	0·594	0·594	0·594	0·594	0·593	0·593
100·0	0·593	0·593	0·592	0·592	0·592	0·592	0·592	0·592	0·592	0·592	0·592	0·592	0·592

For orifices smaller than ¼ inch, the critical head increases more rapidly, and so does the constant or normal coefficient for heads greater than the critical.

Bilton tabulates as follows:

Diameter in inches	0·15	0·2	0·25	0·3	0·4	0·5	0·6	0·75	1·00
Critical head in inches	65	55	45	32	25	22·5	20	18	17
Normal coefficient	0·631	0·630	0·628	0·627	0·624	0·621	0·618	0·613	0·608

Bilton's work was extraordinarily accurate, but did not extend to heads exceeding 100 inches.

Judd and King (*ut supra*) experimented with heads varying from 4 feet to 80 or 100 feet. They find as follows:

	Inch.	Inch.	Inch.	Inches.	Inches.
$d=$	¾	1	1½	2	2½
Mean value of C=	0·6111	0·6097	0·6085	0·6083	0·5956

The individual values show slight variations, indicating a rise as the head increases; but the variation is probably well within the limits of observational error.

Strickland (*Proc. Canadian Soc. of Civil Engineers*, 1909, p. 183), finds that for circular orifices less than 3 inches in diameter, under heads of from 1 to 20 feet:

$$C=0{\cdot}5925+\frac{0{\cdot}018}{\sqrt[3]{d^2}\sqrt{H}} \quad \text{[Inches]}$$

where H, is the head in feet, and d, the diameter in inches.

Mair (*P.I.C.E.*, vol. 84, p. 424), for orifices of 1 to 3 inches in diameter, under heads from 0·75 foot to 2 feet finds that:

$$C=0{\cdot}6075+\frac{0{\cdot}0098}{\sqrt{H}}-0{\cdot}0037d \quad \text{[Inches]}$$

I do not consider that a more accurate presentment of the subject is likely to be attained. All experimenters remark that while it is quite easy to get regular results for the same orifice discharging under various heads, it is very difficult to construct a duplicate orifice which will yield the same value of C.

It appears that small differences in the condition of the edge of an orifice, such as are hardly appreciable under a microscope, are (in the case of orifices of less than 3 inches in diameter, at any rate) quite sufficient to produce such variations in the value of C, as 0·002, or 0·003 (see especially Mair's remarks).

The effect of similar differences may be traced in the value given by Judd and King for a 2½-inch orifice. Bilton's values appear to be free from such irregularities, so that his workmanship must have been unusually good.

We may therefore state that in orifices of this size, the errors in workmanship (not in measurement) during construction are more important than the errors of good observations. Thus, for orifices less than 0·20 foot in diameter, we may consider that the values stated by Bilton, or Mair, for small heads, and those given by Judd and King, or Strickland, for larger heads, will probably permit the discharge to be calculated with an accuracy of 0·3, to 0·4 per cent.

In the case of larger orifices, the table drawn up by Hamilton Smith gives the best available information, but is probably subject to errors of 1 per cent.

It may also be remarked that all modern experimenters find that $c_v = 0·99$, or even 0·999. The difference from the older work is entirely due to the fact that in modern practice c_v, or c_c, are measured directly. In older work, c_v, was obtained by observing the path of the jet, and was in consequence diminished by air resistance.

TEMPERATURE.—The effect of the temperature of the water on C_1 is principally due to the diminished action of capillarity. We may consequently consider that it will be most marked at low heads, and with small values of *d*.

Hamilton Smith, with d=0·02 foot, found that C_1 was diminished to the extent of 1·5 per cent. by a rise from 48 degrees to 130 degrees Fahr.

Unwin, with d=0·033 foot, found that C_1 diminished to the extent of 1 per cent. by a rise from 61 degrees to 205 degrees Fahr.

It can hardly be said that any diminution has yet been measured in the case of larger diameters, although some diminution probably occurs.

Horizontal Orifices.—There is no certain evidence to show that a horizontal orifice has a C_1 differing from that of a vertical orifice of the same size under the same head. Bilton's results for heads greater than the critical are generally obtained on horizontal orifices. He also observes orifices inclined at 45 degrees discharging upwards. Where all three cases do not give identical values, the differences are so small, and are so irregular, as to suggest that they are entirely due to errors of observation, and they certainly fall within errors of workmanship.

Effect of Velocity of Approach.—Consider a circular orifice in a diaphragm, at the end of a circular approach channel. Let the ratio of the diameter of the approach channel to the diameter of the orifice be *n*. Rankine gives a formula for c_c, as follows :

$$\frac{1}{c_c}=\sqrt{2·618-\frac{1·618}{n^4}}, \qquad \text{whence } C_1=\frac{0·97n^2}{\sqrt{2·618n^4-1·618}}$$

since he appears to have deduced these values from observed coefficients of discharge, under the assumption that c_v=0·97.

Goodman (*Engineering*, March 11, 1904), by a method which is partly experimental and partly mathematical, obtains :

$$C_1=\frac{c_v}{1-\frac{0·64}{n^2}}\ \frac{1}{2}\left\{1+\left(\frac{n^2-1}{2n^2}\right)^2\right\}$$

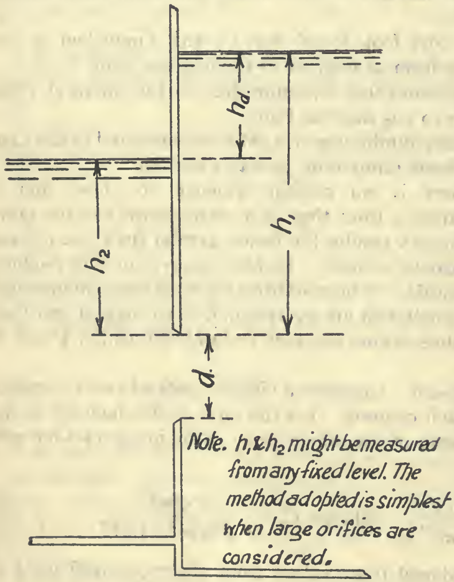
With the assumption that $c_v = 0·97$, we get the following table :

n.	C_1 from Rankine.	C_1 from Goodman.
2	0·652	0·645
3	0·621	0·622
4	0·612	0·615
5	0·607	0·612
6	0·605	0·610
8	0·603	0·609
10	0·601	0·608
100	0·600	0·606

For orifices where Hamilton Smith's coefficient corresponding to complete contraction (i.e. $n=100$) differs from the value 0·606, we may proportionally increase or decrease the coefficient for other values of n.

The difference between the values given by Rankine and Goodman for $n=2$, probably arises from the fact that Rankine takes account of the partial suppression of contraction which occurs, which Goodman's theory neglects.

Where the approach channel is not circular, we take n^2, as equal to the ratio : $\frac{\text{Area of channel}}{\text{Area of orifice}}$, and if any portion of the borders of the channel and orifice are close together (say nearer than $3d$), a certain increase must be allowed for the partial suppression of contraction.



SKETCH No. 39.—Submerged Orifice.

Submerged Discharge (Sketch No. 39).—The appropriate formula is

$$Q = Ca\sqrt{2gh_d},$$

where h_d, represents the difference in the level of the water surfaces above and below the orifice, corrected if necessary for differences in the pressure of the air above the water if the reservoirs are closed.

For sharp-edged orifices, when the jet discharges into water, the coefficient of discharge appears to be diminished by about 0·5 per cent. at high heads, and up to 2 per cent. at low heads, from the coefficient appropriate to the effective head h_d; an equal diminution is believed to occur in other types of orifices. It also appears that this difference diminishes as the area of the orifice increases, the figures given corresponding to $d = 0·20$ to $0·30$ foot, but there is a great deal of uncertainty on this matter. I believe that the uncertainty is explained by the difficulties of correctly measuring the effective head owing to waves set up in the lower reservoir.

Partially Suppressed Contraction.—Since the difference between the area of the vena contracta of the jet and the area of the orifice is caused by the convergence of the streams of water approaching the edge of the orifice from the interior of the vessel in which the orifice is made, any border or thickening of the edge of the orifice will partially prevent this convergence, and will consequently increase the value of c_c. Orifices thus situated are said to have "partially suppressed contraction."

Bidone (see Unwin, *Ency. Brit.*, article on "Hydraulics") states that:

$$c_c = 0·62\left(1 + 0·128\,\frac{m}{p}\right)$$

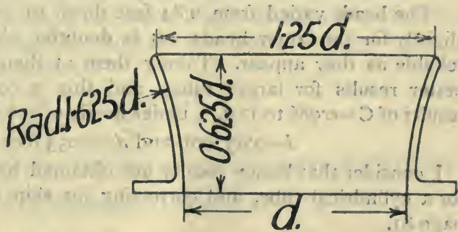
where $\frac{m}{p}$, is the ratio the length of the perimeter over which contraction is suppressed bears to the total perimeter of the orifice. I have been unable to trace the reference, but Bidone experimented on small orifices only. The formula is known not to be very reliable, but the value

$$C_1 = C\left(1 + 0·128\,\frac{m}{p}\right)$$

may be used in default of anything better, where C, represents the coefficient of discharge for an orifice with complete contraction, of the same size and under the same head.

In cases where the border, or thickening, which suppresses the contraction does not coincide with the edge of the orifice, no formula can be given. The ratios given under "Weirs" form the only available information, and the results thus obtained are probably not all accurate.

BELL-MOUTHED ORIFICES.—Sketch No. 40 shows the dimensions usually given as appropriate. Reference to the actual forms of the jets as given in Sketch No. 36 shows that the form should vary with the diameter of the jet, but that the above form is correct, or nearly so, for a diameter of 2½ inches. For greater diameters it is probable that a somewhat larger ratio of expansion and a shorter length may be correct. However, bell mouths of the above proportions up to 4 feet in diameter, were employed at Staines with very satisfactory results. Weisbach gives the following table:



SKETCH NO. 40.—Bell-mouth Orifice.

h	0·66	1·64	11·48	55·77	337·93 feet
C	0·959	0·967	0·975	0·994	0·994

Experiments carried out at Amritsar with a probable error of 0·4 per cent. confirmed this table up to 5·0 feet head on a bell mouth 6 inches in diameter.

Conical Converging Tubes.—The following table is given by Castel (*Annales des Mines*, 1838), where C, is referred to the area of the smallest section (see Sketch No. 38, Fig. 3):

For d=0·05085 Foot, and a Length of Tube equal to 2·6d.

Angle of convergence in degrees and minutes	0°	1°36′	3°10′	4°10′	5°26′	7°52′
	C=0·829	0·866	0·895	0·912	0·924	0·930
,,	8°58′	10°20′	12°4′	13°24′	14°28′	16°36′
	C=0·934	0·938	0·942	0·946	0·941	0·938
,,	19°28′	21°0′	23°0′	29°58′	40°20′	48°50′
	C=0·924	0·919	0·914	0·895	0·870	0·847

For d=0·05085 Foot, l=2·3d.

Angle of convergence in degrees and minutes	9°14′	10°28′	12°42′	16°02′	19°06′
	C=0·929	0·945	0·951	0·940	0·926

For d=0·0656 Foot, l=2·5d.

Angle of convergence in degrees and minutes	2°50′	5°26′	6°54′	10°30′	12°10′
	C=0·914	0·930	0·938	0·945	0·950
,,	13°40′	15°02′	18°10′	23°04′	33°52′
	C=0·956	0·949	0·939	0·930	0·920

For d=0·0656 Foot, l=5d.

Angle of convergence in degrees and minutes	11°52′	14°12′	16·34′
	C=0·965	0·958	0·951

The heads varied from 9·84 feet down to 0·66 foot and C, increased very slightly for the larger heads. It is doubtful whether these experiments are as reliable as they appear. Taking them at their face value, we might predict better results for larger tubes, and this is confirmed by Hamilton Smith's results of C=0·986 to 1·04, under heads of 300 feet approximately, with

l=0·83 foot, and d=0·053 foot to 0·102 foot.

I consider that better results are obtained by calculating the coefficient as for a cylindrical tube, and correcting for skin friction, by the rule given on page 81.

In Castel's experiments, the cone has a sharp inner angle. When this is rounded off, and l=3d, Unwin (*Hydraulics*) states as follows:

Angle of convergence of sides	0°	5°45′	11°15′
	C=0·97	0·95	0·92
,, ,,	22°30′	45°0′	90°0′
	C=0·88	0·75	0·63

Cylindrical Mouthpieces projecting Outwards.—The discharge in such cases is greatly influenced by the form of the corners of the junction of the mouthpiece and the reservoir. If these are sharp and rectangular, the discharge of the orifice is the same as that of a sharp-edged orifice, provided that the length of the mouthpiece does not exceed 1·5 time its diameter. If this length is exceeded, the issuing jet first contracts in a manner very similar to a jet from a sharp-edged orifice, and then expands, and adheres to the sides of the tube (Sketch No. 38, Fig. 4).

The case should be theoretically investigated, in order to produce the best results.

Unwin gives the following equation :

$$c_v = \frac{1}{\sqrt{1+\left(\frac{1}{c_c}-1\right)^2}}$$

where c_c, is the coefficient of contraction for a sharp-edged orifice. From this, taking the approximate value of c_c, as equal to 0·608 (not 0·64 as Unwin does), we get $c_v=0·840$; and in this case c_v, is probably very nearly equal to C.

Castel (as already stated) finds that C=0·829, which would correspond to $c_c=0·599$, which is a lower value than is probable. The difference between theory and experiment can possibly be explained either by friction, or by the small size of Castel's orifice, which may cause c_c, to have a value differing from 0·608.

The full theory would indeed show that in a length equal to the diameter of the pipe, a head equal to $\frac{1}{30}\frac{v^2}{2g}$, is lost in skin friction, if $v=90\sqrt{rs}$ be assumed as the friction equation of the pipe.

The velocity would thus be diminished by at least 1·5 per cent. assuming that the water is in contact with the pipe only over the length

$$\text{KH}=1·1d=(2·6-1·5)d.$$

This correction will bring theory and experiment into almost exact agreement, the value 0·840 being reduced to 0·827 (see Sketch No. 41).

Thus, I believe that for pipes of lengths greater than 1·5d, Unwin's formula with $c_c=0·608$ (as capillarity does not now influence the value), corrected for the friction of a length of pipe equal to the actual length minus 0·8 to 1·5d will give better results than any experiments which are not specially carried out.

The formula proposed is therefore :

$$h=v^2\left\{\frac{1}{2gc_v^2}+\frac{4\,(l-1·5d)}{C_p^2d}\right\}$$

where C_p, is the coefficient in the pipe equation $v=C_p\sqrt{rs}$, appropriate to the size of the pipe (see p. 427). The discharge is $Q=\frac{\pi d^2}{4}v$.

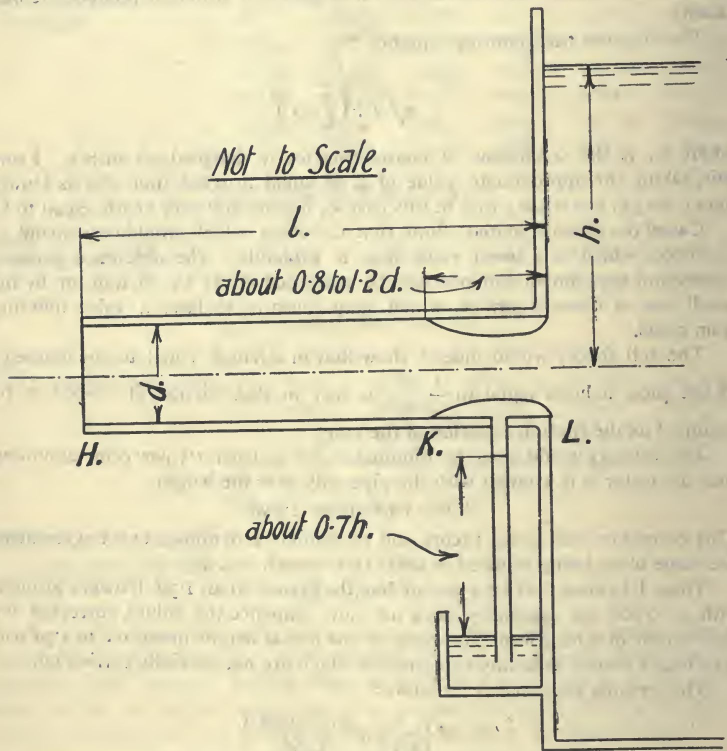
The resemblance to the usual pipe discharge formula is obvious. With the figures already given we find that $\frac{1}{c_v^2}=1·45$. Experimental figures are rare, and in large pipes the difficulties discussed on page 424, probably manifest themselves. To Castel's result may be added those of Weisbach, with $l=3d$, as follows:

d, equal to	0·032	0·066	0·098	0·131 foot.
C, ,,	0·843	0·832	0·821	0·810

Unwin states that the coefficient is also affected by the length of the mouthpiece, and gives as average values :

$\frac{l}{d}$,	equal to	1	2 to 3	12
C,	,,	0·88	0·82	0·77

It will be noticed that the pressure at any point between L and K, Sketch No. 41, is theoretically less than atmospheric, by approximately $0{\cdot}70h$. It



SKETCH No. 41.—Discharge through Cylindrical Pipe.

would thus appear that water can be raised, or air sucked in through an orifice constructed in the tube along this length. This principle is employed in jet pumps, and hydraulic compressors (see p. 813).

It would also appear that if h exceeds 1·4 times the height of the water barometer, say 45 feet, the coefficients of discharge above obtained will no longer hold. Russell (*Text Book of Hydraulics*) states that when h, exceeds 42 feet the flow is "troubled and pulsatory." It is therefore probable that the water jet ruptures, or springs free from the sides of the tube, and that the coefficient of discharge momentarily changes to that of a sharp-edged orifice.

My own experiments lead me to believe that this effect becomes manifest at lower heads, especially if the tube is anything but perfectly smooth. If the tube is shaped internally so as to conform to the form of a jet, the discharge is increased by about 10 per cent.

Cylindrical Mouthpieces projecting Inwards.—Here we have two cases as follows:

(1) That generally known as Borda's mouthpiece (see Sketch No. 38, Fig. 6), where the jet springs free, and does not wet the tube.

Theoretically, if e, be the thickness of the inner end of the tube, and r, its radius, then:

$$C = \frac{(r+e)^2}{2r^2}$$

or C, is 0·50, or slightly greater, Borda finds that $C = 0{\cdot}515$, with $r = 0{\cdot}04$ foot approx. Bidone (*Recherches experimentales*), with $r = 0{\cdot}06$ foot approx., or $d = 0{\cdot}12$ foot finds that $C = 0{\cdot}555$; while the theory would give $C = 0{\cdot}557$. Weisbach finds that $C = 0{\cdot}532$.

(2) In the second case (Bidone's mouthpiece, Sketch No. 38, Fig. 5) the jet expands, and wets the sides of the tube, so that at the outer end the tube flows full bore. If we put μ for the value of C, for Borda's mouthpiece, as calculated above, *i.e.*

$$\mu = \frac{(r+e)^2}{2r^2}$$

we see that theoretically (Bidone, *Recherches*, p. 63):

$$C = \frac{1}{\sqrt{1+\left(\frac{1}{\mu}-1\right)^2}} \qquad \text{or if } \mu=\tfrac{1}{2},\ C = 0{\cdot}707$$

As a rule C, is greater than 0·707.

With $r = 0{\cdot}06$ foot, or $d = 0{\cdot}12$ foot, Bidone finds that $C = 0{\cdot}767$, the theory leading to 0·781.

Bilton (*Proc. Victorian Inst. of Engineers*, 1909) for sharp-edged square cut orifices, with the pipe $2\frac{1}{2}d$ in length, finds that:

d,	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$ inches.
C,	0·91	0·87	0·85	0·83	0·81	0·79	0·77	0·76	0·75

In very large orifices, where $\frac{r}{e}$, is small, 0·71 may be assumed as correct. In a 12-inch pipe, with $e = \frac{1}{2}$ inch, we get $\mu = 0{\cdot}587$, and $C = 0{\cdot}818$. Experimentally, under a head of 14·3 feet, I found that $C = 0{\cdot}79$. The accuracy of the experiment does not justify corrections for friction being applied.

Fountain Flow from Vertical Pipes.—The calculation of the discharge through a vertical pipe is of importance, as the volume discharged by artesian wells and in other cases of fountain flow can thus be easily determined.

The question has been investigated by Lawrence and Braunworth (*Trans. Am. Soc. of C.E.*, vol. 67, p. 265). The conditions of flow may be divided into the two following distinct types:

(*a*) Weir flow, occurring under small heads when the discharge resembles that over a circular weir.

(*b*) Jet flow, where the discharge occurs in a jet or fountain.

If *d*, represent the diameter of the pipe in inches, and H, is the head in fee over the horizontal orifice of the pipe, for pipes of 2, 4, 6, 9, and 12 inches in diameter, we find that:

Type (*a*) occurs so long as H_1 is less than $0{\cdot}028d^{1.04}$ . . . [Inches]
Type (*b*) occurs if H_1 is greater than $0{\cdot}107d^{1.03}$ . . [Inches]

During the intermediate range the discharge bears a fixed relation to the head, but the investigators were unable to deduce a formula, and found it necessary to prepare a table which gave Q_1 in terms of H.

Putting Q = the discharge in cubic feet per second.
D = the diameter of the pipe in feet.
h = the head over the orifice in feet.

The experimenters found that:

I. When h_1 was observed by a pressure gauge in communication with an opening in the side of the pipe 1·90 inch below the top of the pipe:
The weir discharge was represented by $Q = 8{\cdot}8D^{1.29}h^{1.29}$.
The jet discharge was represented by $Q = 5{\cdot}84D^{2.005}h^{0.53}$.

II. When h_1 was observed by sighting across the top of the issuing water:
The weir discharge was represented by $Q = 8{\cdot}8D^{1.25}h^{1.35}$.
The jet discharge was represented by $Q = 5{\cdot}57D^{1.99}h^{0.53}$.
The resemblance between the jet formula and the theoretical formula

$$Q = C\sqrt{2gh}\,\frac{\pi D^2}{4},$$ is obvious.

It is plain that the second jet discharge formula is that which corresponds best with the conditions usually occurring in practice, and for this case C_1 is not far removed from 0·90.

In some field experiments, where the discharge was measured over a weir and compared with the discharge calculated by the weir discharge formula, indicated that the weir condition is not adapted for securing accurate measurements under field conditions (probably owing to the large errors in Q, caused by small errors in the determination of h). Thus, the best results will be obtained by decreasing the diameter of the tube until a jet discharge is produced. The original experiments indicate that if the jet discharge condition is produced by fixing a smaller pipe inside the casing of the well, and blocking up the annular space between the two pipes, an 8 to 10 foot length of the smaller pipe will suffice to wipe out any irregularities of flow which might cause an application of the formulæ to produce erroneous results.

Discharge of Water through Orifices of other than Circular Form.—The figures given in the following discussions are by no means as accurate as are those which relate to circular orifices.

In practice, circular orifices of a size larger than those considered in the tables are not of frequent occurrence, since engineers rarely employ such orifices except for measuring volumes of water.

Square, or rectangular orifices of very large size, frequently occur in practice; and our knowledge of the coefficients of discharge of such orifices is very vague. The available information has been collected.

For preliminary designs, the following simple rules may be used.

The coefficient of discharge of a sharp-edged orifice with complete contraction, is 0·6, and increases as the contraction becomes more and more suppressed. The value $\frac{2}{3}$, may be taken for a case where borders exist around three-quarters of the perimeter, and the value $\frac{3}{4}$, for completely suppressed contraction, or a thick-edged orifice.

All these values are low, and $\frac{5}{8}$, might be used in place of 0·6 ; 0·7 in place of 0·67, and 0·80 in place of 0·75.

In practice, however, the total cost of a large sluice gate and the surrounding masonry is probably not increased by making it a little larger, and there is no doubt that too small a sluice capacity is always troublesome, if not dangerous.

Square, or rectangular orifices are not well adapted for measuring purposes. The experiments of Benton (see p. 167) were intended to form a basis for systematic measurements, and very excellent tables were drawn up and officially promulgated. In practice, however, it was found that the engineers and supervisors preferred to carry out rod float gaugings. The standard of theoretical knowledge in the Punjab Irrigation Branch is high, especially when the overseer or sub-overseer is compared with men performing similar duties elsewhere. Thus, it may be inferred that the complication introduced by the corrections and double entries which are required with even the best system of tables, renders the method unfit for practical purposes.

SQUARE ORIFICES.—For square orifices the formula is :

$$Q = C \times \text{area} \sqrt{2gh}$$

where h, is the head measured to the centre of the orifice.

Hamilton Smith (*ut supra*) gives the following table (p. 154) for sharp-edged square orifices, in a vertical plane with full contraction, discharging into the air, his values being corrected so as to permit the simple formula given above to apply in all cases.

These figures are known to be less accurate than those for circular orifices (being subject to at least 1 per cent. of error), principally due to the difficulty of making a really sharp-edged square orifice of small size.

In view of our present knowledge of the accuracy of the table for round orifices, it is probable that this table is accurate to two figures. The third figure is retained, as Smith's coefficients are so often referred to in experiments.

Rectangular Orifices.—The typical case is an orifice with vertical sides.

Let w = the horizontal width of the orifice, in feet.

Let H_1 = the depth of the top of the orifice, in feet, below the water level.

Let H_2 = the depth of the bottom of the orifice, in feet, below the water level.

The area of the orifice is :

$$w(H_2 - H_1) = wk \text{ square feet, where } k = H_2 - H_1$$

and the theoretical formula is :

$$Q = Cw\sqrt{2gh}(H_2^{1.5} - H_1^{1.5})$$

Now, if $H_2 - H_1$ be small compared with H_1, this is approximately equal to :

$$Q = Cw(H_2 - H_1)\sqrt{2g\frac{(H_2 + H_1)}{2}}$$

or,

$$Q = C \times \text{area} \sqrt{2gh},$$

where $h = \frac{H_2 + H_1}{2}$, and h, is the head measured to the centre of the orifice.

VALUE OF THE COEFFICIENT C IN THE EQUATION

$$Q = Ca\sqrt{2gh}$$

FOR SQUARE VERTICAL ORIFICES WITH SHARP EDGE, FULL CONTRACTION AND FREE DISCHARGE INTO AIR.

Head to Centre of Orifice, in Feet.	Length of the Side of the Square in Feet.												
	0·02	0·03	0·04	0·05	0·07	0·10	0·12	0·15	0·20	0·40	0·60	0·80	1·00
0·3	...	...	...	0·642	0·632	0·624	0·616	0·610	...	...	...	...	...
0·4	...	...	0·643	0·637	0·628	0·621	0·615	0·610	...	...	...	...	...
0·5	...	0·648	0·639	0·633	0·625	0·619	0·614	0·610	0·604	0·597	0·587	...	...
0·6	0·660	0·645	0·636	0·630	0·623	0·617	0·613	0·610	0·605	0·598	0·591	0·584	...
0·7	0·656	0·642	0·633	0·628	0·621	0·616	0·612	0·609	0·605	0·600	0·594	0·589	0·582
0·8	0·652	0·639	0·631	0·625	0·620	0·615	0·611	0·608	0·605	0·601	0·596	0·591	0·586
0·9	0·650	0·637	0·629	0·623	0·619	0·614	0·610	0·608	0·605	0·602	0·598	0·594	0·590
1·0	0·648	0·636	0·628	0·622	0·618	0·613	0·610	0·608	0·605	0·603	0·599	0·596	0·593
1·2	0·644	0·633	0·625	0·620	0·616	0·611	0·609	0·607	0·605	0·604	0·600	0·598	0·595
1·4	0·642	0·630	0·623	0·618	0·614	0·610	0·608	0·606	0·605	0·604	0·601	0·599	0·598
1·6	0·640	0·628	0·621	0·617	0·613	0·609	0·607	0·606	0·605	0·605	0·602	0·600	0·599
1·8	0·638	0·627	0·620	0·616	0·612	0·609	0·607	0·606	0·605	0·605	0·603	0·601	0·600
2·0	0·637	0·626	0·619	0·615	0·612	0·608	0·606	0·606	0·605	0·605	0·604	0·602	0·600
2·5	0·634	0·624	0·617	0·613	0·610	0·607	0·606	0·606	0·605	0·605	0·604	0·603	0·601
3·0	0·632	0·622	0·616	0·612	0·609	0·607	0·606	0·606	0·605	0·605	0·604	0·603	0·602
3·5	0·630	0·621	0·615	0·611	0·609	0·607	0·606	0·605	0·605	0·605	0·604	0·603	0·602
4·0	0·628	0·619	0·614	0·610	0·608	0·606	0·606	0·605	0·605	0·605	0·603	0·603	0·602
5·0	0·626	0·617	0·613	0·610	0·607	0·606	0·605	0·605	0·604	0·604	0·603	0·602	0·602
6·0	0·623	0·616	0·612	0·609	0·607	0·605	0·605	0·605	0·604	0·604	0·603	0·602	0·602
7·0	0·621	0·615	0·611	0·608	0·607	0·605	0·605	0·604	0·604	0·604	0·603	0·602	0·602
8·0	0·619	0·613	0·610	0·608	0·606	0·605	0·604	0·604	0·604	0·603	0·603	0·602	0·602
9·0	0·618	0·612	0·609	0·607	0·606	0·604	0·604	0·604	0·603	0·603	0·602	0·602	0·601
10·0	0·616	0·611	0·608	0·606	0·605	0·604	0·604	0·603	0·603	0·603	0·602	0·602	0·601
20·0	0·606	0·605	0·604	0·603	0·602	0·602	0·602	0·602	0·602	0·601	0·601	0·601	0·600
50·0	0·602	0·601	0·601	0·601	0·601	0·600	0·600	0·600	0·600	0·600	0·599	0·599	0·599
100·0	0·599	0·598	0·598	0·598	0·598	0·598	0·598	0·598	0·598	0·598	0·598	0·598	0·598

In practice the more complicated formula is only applied in cases where the ratio $\frac{H_2 - H_1}{H_2}$ is greater than $\frac{1}{5}$. The coefficient of discharge is not well known, but (see p. 168) C = 0·63 is probably a fair average for sharp-edged orifices.

The typical formula is:

$$Q = C \times \text{area} \sqrt{2g\frac{H_2 + H_1}{2}} = Cw(H_2 - H_1)\sqrt{2gh}$$

and the following table is a condensation of Fanning's values (*A Treatise on Hydraulic and Water Supply Engineering*) for rectangular orifices with vertical sides. The third figures may be regarded as of little, if any, significance.

VALUES OF C FOR RECTANGULAR ORIFICES 1 FOOT WIDE.

Head to Centre of Orifice.	Vertical Height of Orifice.							
	4 feet.	2 feet.	1·5 foot.	1 foot.	0·75 foot.	0·50 foot.	0·25 foot.	0·125 foot.
0·4	...	...	...	...	...	0·614	0·631	0·633
0·6	...	...	...	0·598	0·606	0·616	0·632	0·633
0·8	...	...	0·613	0·600	0·608	0·617	0·632	0·633
1·0	...	...	0·614	0·601	0·609	0·617	0·632	0·632
1·5	...	0·619	0·614	0·603	0·610	0·617	0·631	0·630
2·0	...	0·618	0·614	0·604	0·610	0·617	0·630	0·629
2·5	0·629	0·618	0·614	0·604	0·610	0·616	0·628	0·628
3·0	0·627	0·617	0·613	0·605	0·610	0·615	0·627	0·627
4·0	0·625	0·615	0·611	0·605	0·609	0·614	0·624	0·624
5·0	0·621	0·612	0·609	0·604	0·606	0·611	0·620	0·620
6·0	0·616	0·609	0·606	0·602	0·604	0·609	0·615	0·615
8·0	0·609	0·604	0·602	0·601	0·602	0·603	0·607	0·609
10·0	0·604	0·602	0·601	0·601	0·601	0·601	0·603	0·606
20·0	0·605	0·602	0·601	0·601	0·601	0·602	0·604	0·607
50·0	0·609	0·606	0·603	0·602	0·604	0·605	0·607	0·614

If we compare the fifth column with Hamilton Smith's coefficients for an orifice 1 foot square, the agreement is fairly good; but the figures for heads exceeding 10 feet do not follow the law of steady decrease which Smith considered to be correct for square and circular orifices. The figures for orifices, the vertical side of which is less than the horizontal were subject to special and constant errors, which usually tended to slightly increase the value of the coefficient.

It is therefore probable that the following reasoning leads to values of C, which are quite as accurate as those given by Fanning.

Bazin (*Mem. de l'Academie des Sciences*, tome 32, 1896) finds that for a square of 0·656 foot, under a head of 2·96 feet to 3·27 feet, C = 0·607. For a rectangular orifice 2·62 feet long, and 0·656 feet high, without lateral contractions (*i.e.* practically of infinite length), under the same heads, C = 0·627.

We may therefore deduce that the coefficient for a rectangular orifice, 0·656 foot in breadth, with a length equal to $n \times 0{\cdot}656$, with lateral contractions, is not far off $0{\cdot}627 - \frac{0{\cdot}020}{n}$. This value is correct to two places of decimals, and probably holds for all heads over 3 feet. For widths other than 0·66 foot, we may add or subtract, as is indicated by Hamilton Smith's tables; *e.g.* consider an orifice 1 foot × 0·125 foot, we get if $h = 4$ feet:

$$C = \left(0{\cdot}627 - \frac{0{\cdot}020}{8}\right) + (0{\cdot}606 - 0{\cdot}607) = 0{\cdot}624$$

which agrees with Fanning's table. Similarly, C = 0·622, when $h = 10$ feet, as compared with Fanning's value of 0·606, which is probably erroneous.

Strickland (*Proc. of Canadian Soc. of Civil Engineers*, 1909, p. 185) suggests the following formula for the coefficients of discharge of squares with a side of less than 3 inches, under heads of 1 foot to 20 feet:

$$C = 0{\cdot}598 + \frac{0{\cdot}018}{\sqrt{H}\sqrt[3]{d^2}} \quad . \quad . \quad . \quad . \quad . \quad \text{[Inches]}$$

where H, is in feet, and d, in inches.

If we endeavour to express this in terms of $r = \frac{\text{area}}{\text{wetted perimeter}}$ we get:

$$C = 0{\cdot}598 + \frac{0{\cdot}045}{\sqrt{H}\sqrt[3]{r^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Inches]}$$

which may be applied to rectangles, and does not markedly conflict with existing experimental results.

Royd (*Journ. of the Assoc. of Eng. Societies*, 1895) gives the following results for an orifice 6 inches wide, under a head of 6 inches:

Length of orifice .	6	12	18	24	30 inches.
C, .	0·593	0·607	0·615	0·621	0·626

The orifice was peculiarly constructed, and it may be doubted whether it was really sharp edged. The values of the coefficients, however, indicate that it behaved after the manner of a sharp-edged orifice. The top and bottom edges were formed of 2-inch, and the sides of 1-inch planking, the latter being laid against the water face of the 2-inch planks, so that all four edges were not in one plane.

Ellis (*Trans. Am. Soc. of C.E.*, vol. 5, p. 19) finds as follows:

For a 2 feet by 0·5 foot orifice, C = 0·611, for H = 1·42 feet,
falling to C = 0·600, for H = 16·96 feet.

For an orifice 2 feet wide, by 1 foot high, C = 0·597, for H = 1·82 foot,
rising to C = 0·606, for H = 11·31 feet.

For a 1 foot square orifice, C = 0·582, for H = 1·48 foot,
rising to C = 0·601, for H = 15·13 feet.

When submerged, C, varies between 0·599 for h_d = 2·32 feet, and C = 0·611 for h_d = 14·30 feet, but the mean between h_d = 11·6 and 18·5 feet is C = 0·606.

The differences suggest partially suppressed contractions.

For a 2 feet square orifice, C = 0·611 to 0·597, H, varying between 2·06 and 3·54 feet. The effect of suppressed contraction is very evident.

Lespinasse (quoted by Fanning, *ut supra*) obtained the following values for an orifice 4·265 feet wide :

Height in Feet.	Head in Feet to Centre of Orifice.	C.
1·805	14·55	0·613
1·64	6·63	0·641
1·64	6·25	0·629
1·51	12·88	0·641
1·575	13·59	0·647
1·575	6·39	0·616
1·575	6·22	0·594
1·575	6·48	0·621

The values are probably affected by suppressed contraction, but are consequently more likely to be practically useful than experiments where contraction did not occur. In any case, they are the best available results for really large orifices.

Better results will not be easily obtained. The difficulties arising from errors in workmanship are exceptionally great ; and the duplication of a square orifice with a side of less than 4 inches, so as to obtain accordant values of C (especially for low heads) is almost impossible. Thus, for practical calculations, the above values are sufficiently accurate, and it will also be plain that square orifices are useless for measuring water accurately.

Suppressed Contractions.—Hamilton Smith (*ut supra*) calculates the following percentages of increase in C, from Lesbros' results for an orifice 0·656 × 0·656 foot.

Description of Contraction.	Head. 1 foot.	Head. 2 feet.	Head. 3 feet.	Head. 5 feet.
Nearly suppressed on one side .	1·2	1·0	1·2	1·3
Quite „ „ .	3·8	3·3	3·1	3·5
Nearly „ two sides .	5·7	4·5	4·0	4·0
Quite on one, and nearly on another	5·8	5·3	5·3	5·6
Quite suppressed on two opposite sides	7·3	6·0	5·6	5·6
Quite on one, nearly on two sides	13·3	10·9	9·9	9·8
Quite on three sides . .	15·6	13·4	11·9	11·6

For rectangular orifices, we have the following percentages of increase in C :

Horizontal Side.	Vertical Side.	When the Contraction is					
		Suppressed at the Bottom.			Suppressed at both Vertical Sides.		
l.	w.	Feet. H=1.	Feet. H=3.	Feet. H=5.	Feet. H=1.	Feet. H=3.	Feet. H=5.
0·656 foot	0·656 foot	3·8	3·1	3·5	5·7	4·0	4·0
	0·328 "	5·4	5·2	5·4	3·2	2·4	3·1
	0·164 "	6·3	6·5	7·4	1·6	1·4	2·4
	0·098 "	7·8	7·6	8·5	3·4	2·1	2·4
	0·033 "	8·9	11·1	12·4	4·5	5·1	5·6

Horizontal Side.	Vertical Side.	When the Contraction is					
		Suppressed at the Bottom and partly on one Side.			Suppressed at the Bottom and partly on both Verticals.		
l.	w.	Foot. H=1.	Feet. H=3.	Feet. H=5.	Foot. H=1.	Feet. H=3.	Feet. H=5.
0·656 foot	0·656 foot	5·8	5·3	5·6	13·3	9·9	9·8
	0·328 "	7·0	6·7	7·0	10·7	9·8	10·0
	0·164 "	7·3	7·5	8·4	8·9	8·6	8·5
	0·098 "	8·1	8·8	9·2	9·6	9·2	9·5
	0·033 "	8·5	11·1	12·2	8·5	11·1	12·7

These values are probably less accurate than the results for square orifices.

The term "complete suppression," is employed when a side of the canal conicides with a side of the orifice. For sides where the term "partly suppressed" is used, the distance between the side of the canal and that of the orifice is 0·066 foot. The figures for the second case appear to be erroneous, and somewhat peculiar irregularities occur in all cases.

Bidone (*Recherches Hydrauliques*) suggests the following (compare with circular orifices) :

$$C_1 = C\left(1 + 0{\cdot}152\frac{m}{p}\right)$$

The results are probably more accurate than those given by his formula for circular orifices, but they are obtained from experiments on small orifices only.

Bellasis(*Hydraulics*) gives the following table for rectangular orifices with partial suppression round a portion of their perimeter, where G, is the distance of the side of the channel from the edge of the orifice, and :

$$C_t = C \times \text{coefficient in table}$$

where C_t, is the coefficient of discharge for the orifice with partially suppressed contraction, and C, is the coefficient under the same head, but with complete contraction.

$\frac{m}{p}$	$\frac{G}{d}=3$	2·67	2	1	0·5	0
0·25	1	1·000	1·002	1·006	1·015	1·04
0·50	1	1·001	1·003	1·013	1·030	1·08
0·75	1	1·001	1·004	1·019	1·045	1·12
0·875	1	1·001	1·005	1·04 (?)	1·10 (?)	1·40 (?)
1·0	1	1·002	1·006	1·05 (?)	1·20 (?)	

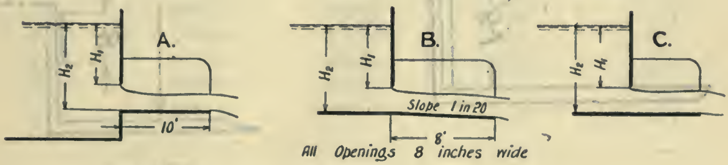
I am not aware what experimental basis these possess.

If a jet be bisected by a metal sheet, which is more than 0·04 inch in thickness, and is afterwards allowed to unite, Smith finds an increase of about 1 per cent. in an orifice with a length of 0·30 foot, and a width of 0·03 foot.

Orifices with Prolonged Boundaries.—The following results are given by Unwin (*Ency. Brit.*, article on "Hydraulics"). I am unable to discover the original source, which is probably French.

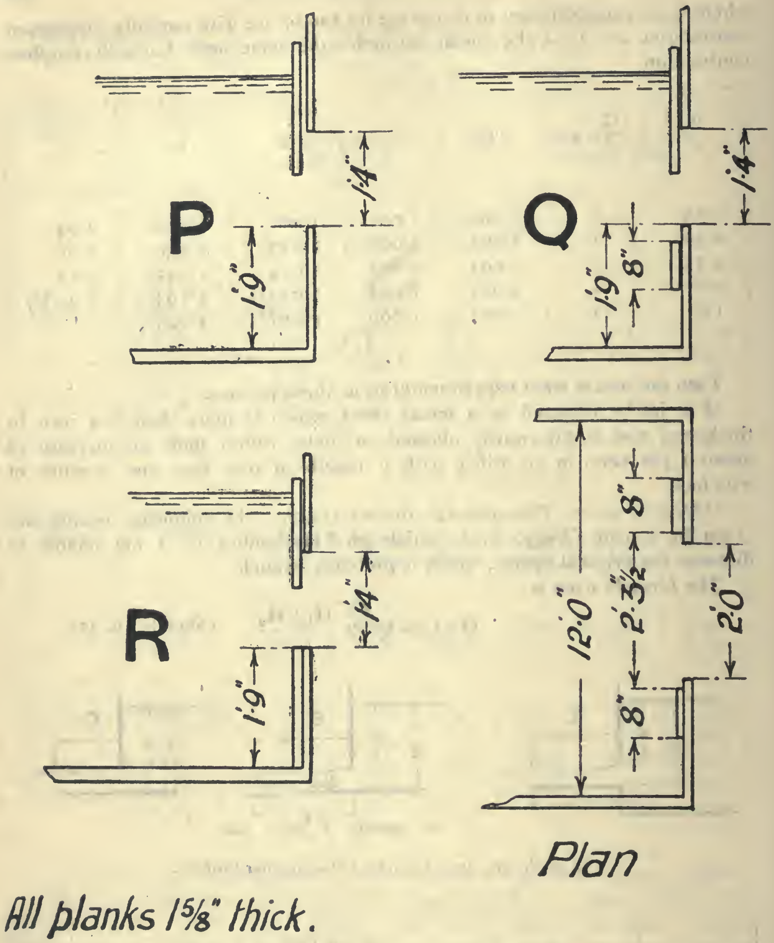
The formula used is :

$$Q = Cwk\sqrt{2g\frac{H_1+H_2}{2}} \qquad \text{(Sketch No. 42).}$$



Sketch No. 42.—Sketch of Rectangular Orifices.

$H_2 - H_1$ in Feet.		H_1 in Feet.								
		0·0656	0·164	0·328	0·656	1·640	3·28	4·92	6·56	9·84
A	1·656	0·480	0·511	0·542	0·574	0·599	0·601	0·601	0·601	0·601
B	1·656	0·480	0·510	0·538	0·566	0·592	0·600	0·602	0·602	0·601
C	1·656	0·527	0·553	0·574	0·592	0·607	0·610	0·610	0·609	0·608
A	0·164	0·488	0·577	0·624	0·631	0·625	0·624	0·619	0·613	0·606
B	0·164	0·487	0·571	0·606	0·617	0·626	0·628	0·627	0·623	0·618
C	0·164	0·585	0·614	0·633	0·645	0·652	0·651	0·650	0·650	0·649



SKETCH No. 43.—Sketch of Rectangular Orifices.

C for figure P.				
Head H_1, above Upper Edge of Orifice in Feet.	Height of Orifice, $H_2 - H_1$, in Feet.			
	1·31	0·66	0·16	0·10
0·328	0·598	0·634	0·691	0·710
0·656	0·609	0·640	0·685	0·696
0·787	0·612	0·641	0·684	0·694
0·984	0·616	0·641	0·683	0·692

[Table continued

Table continued]

C for figure P.

Head H_1, above Upper Edge of Orifice in Feet.	Height of Orifice, $H_2 - H_1$, in Feet.			
	1·31.	0·66.	0·16.	0·10.
1·968	0·618	0·640	0·678	0·688
3·28	0·608	0·638	0·673	0·680
4·27	0·602	0·637	0·672	0·678
4·92	0·598	0·637	0·672	0·676
5·58	0·596	0·637	0·672	0·676
6·56	0·595	0·636	0·671	0·675
9·84	0·592	0·634	0·668	0·672

C for figure R.

Head H_1, above Upper Edge of Orifice in Feet.	Height of Orifice, $H_2 - H_1$, in Feet.			
	1·31.	0·66.	0·16.	0·10,
0·328	0·648	0·668	0·666	0·696
0·656	0·657	0·675	0·688	0·706
0·787	0·659	0·677	0·692	0·708
0·984	0·660	0·678	0·695	0·711
1·968	0·653	0·679	0·697	0·712
3·28	0·634	0·676	0·695	0·705
4·27	0·626	0·675	0·694	0·702
4·92	0·622	0·674	0·693	0·699
5·58	0·620	0·673	0·693	0·698
6·56	0·617	0·672	0·692	0·696
9·84	0·612	0·670	0·690	0·693

C for figure Q.

Head H_1, above Upper Edge of Orifice in Feet.	Height of Orifice, $H_2 - H_1$, in Feet.			
	1·31.	0·66.	0·16.	0·10.
0·328	0·644	0·665	0·664	0·694
0·656	0·653	0·672	0·687	0·704
0·787	0·655	0·674	0·690	0·706
0·984	0·656	0·675	0·693	0·709
1·968	0·649	0·676	0·695	0·710
3·28	0·632	0·674	0·694	0·704
4·27	0·624	0·673	0·693	0·701
4·92	0·620	0·673	0·692	0·699
5·58	0·618	0·672	0·692	0·698
6·56	0·615	0·671	0·691	0·696
9·84	0·611	0·669	0·689	0·693

Submerged Orifices.—The following table gives values calculated from Smith's experiments:

Effective Head in Feet.	Circle $d=0{\cdot}05$ Ft.	Square Ft. Ft. 0·05 × 0·05	Circle 0·1 Ft.	Square Ft. Ft. 0·1 × 0·1	Rectangle Ft. Ft. 0·05 × 0·3
0·5	0·616	0·620	0·602	0·609	0·622
1·0	0·610	0·615	0·602	0·606	0·622
1·5	0·607	0·612	0·601	0·605	0·621
2·0	0·604	0·609	0·600	0·604	0·620
2·5	0·603	0·608	0·599	0·604	0·619
3·0	0·602	0·607	0·599	0·604	0·618
4·0	0·601	0·607	0·599	0·605	...

Smith suggests that the difference between these coefficients, and those for similar orifices discharging into air under the same effective heads, is proportional to:

$$\frac{\text{Wetted perimeter}}{\text{Area} \times \sqrt{\text{head}}}$$

I have been unable to find any difference for orifices 1 foot square, under heads up to 4 feet. I was not able to gauge the quantity of water passing, but the effective heads over the orifices were identical.

The rules given under "Circular Submerged Orifices" may be applied in default of better information.

Coefficients for Orifices which are neither Circular nor Rectangular.—For orifices other than circular, square, and rectangular in form, no very definite information exists. Bovey (*Hydraulics*, p. 40), gives a series of determinations of orifices 0·196 square inch in area, under heads up to 20 feet. The area is far too small to permit any practical application being made, and it is therefore sufficient to state that the mean ratios of the C's, were as follows:

Circular.	Square.		Rectangle Sides 4 : 1.	
	Sides Vertical.	Diagonal Vertical.	Long Side Vertical.	Short Side Vertical.
1	1·011	1·013	1·030	1·033

Rectangle Sides 16 : 1.		Equilateral Triangle.
Long Side Vertical.	Short Side Vertical.	One Side Horizontal.
1·050	1·050	1·023

The arrangements for measuring were extremely accurate, and the figures may be relied on to about three units in the third place of decimals. It is fairly plain that large orifices will probably show no measurable difference.

Large Orifices.—The most complete experiments are those of Stewart (*Bulletin of Univ. of Wisconsin*, March 1908, and *Eng. News*, January 9, 1908) on 4 feet square submerged orifices, under small effective heads.

Here we have as follows :

Where I. refers to square-cornered orifices, prolonged by a tube of a length l.

II. refers to a similar orifice with contraction suppressed at the bottom by a bellmouth of elliptical form.

III. refers to ditto, at one side, and the bottom by a similar bellmouth.

IV. refers to ditto, at two sides, and the bottom by a similar bellmouth.

V. is as IV., but unlike all the other cases, the tube does not simply end in a sharp edge, but in a bulkhead, as though it passed through a thick wall.

VI. is an orifice as No. IV., with contraction completely suppressed on all four sides, and no bulkhead at end of the tube.

We have, $Q = Ca\sqrt{2gh_a}$, and the following are the values for a tube of a length equal to l feet.

h_a Feet.	Case.	l = Length of Tube.						
		0·31 ft.	0·62 ft.	1·25 ft.	2·5 ft.	5·0 ft.	10·0 ft.	14·0 ft.
0·05	I.	0·631	0·650	0·672	0·769	0·807	0·824	0·838
...	II.	0·672	...	...	0·742	0·810	...	0·848
...	III.	0·740	...	...	0·769	0·832	...	0·862
...	IV.	0·834	...	...	0·769	0·875	...	0·890
...	V.	...	...	...	...	...	...	0·875
...	VI.	0·948	...	...	0·943	0·940	0·927	0·931
0·10	I.	0·611	0·631	0·647	0·718	0·763	0·780	0·795
...	II.	0·636	...	...	0·698	0·771	...	0·801
...	III.	0·685	...	...	0·718	0·791	...	0·813
...	IV.	0·772	...	...	0·718	0·828	...	0·841
...	V.	...	...	...	...	...	...	0·830
...	VI.	0·932	...	...	0·911	0·899	0·892	0·893
0·15	I.	0·609	0·628	0·644	0·708	0·758	0·779	0·794
...	II.	0·630	...	...	0·689	0·767	...	0·803
...	III.	0·677	...	...	0·708	0·787	...	0·814
...	IV.	0·765	...	...	0·708	0·828	...	0·839
...	V.	...	...	...	...	...	...	0·829
...	VI.	0·936	...	...	0·910	0·899	0·893	0·894

[*Table continued.*

Table continued]

h_a Feet.	Case.	l = Length of Tube.						
		0·31 ft	0·62 ft.	1·25 ft.	2·5 ft.	5·0 ft.	10·0 ft.	14·0 ft.
0·20	I.	0·609	0·630	0·647	0·711	0·768	0·794	0·809
...	II.	0·632	...	...	0·694	0·777	...	0·819
...	III.	0·678	...	...	0·711	0·796	...	0·833
...	IV.	0·771	...	...	0·711	0·838	...	0·856
...	V.	...	...	...	...	...	...	0·846
...	VI.	0·948	...	...	0·923	0·911	0·906	0·905
0·25	I.	0·610	0·634	0·652	0·720	0·782	0·812	0·828
...	II.	0·634	...	...	0·705	0·790	...	...
...	III.	0·683	...	...	0·720	0·809	...	...
...	IV.	0·779	...	...	0·720	0·854	...	...
...	V.	...	...	...	...	...	...	...
...	VI.	0·965	...	...	0·938	0·928	...	...
0·30	I.	0·614	0·639	0·660	0·731	0·796	0·832	0·850
...	II.	0·639	...	...	...	...	...	...
...	III.	0·689	...	...	...	...	...	...
...	IV.	0·788	...	...	...	...	...	...
...	V.	...	...	...	...	...	...	...
...	VI.	0·980	...	...	...	...	...	...

These experiments were conducted with the help of a carefully calibrated weir, and may be regarded as possessing a very high degree of accuracy.

The values given above are not corrected for velocity of approach, and the form is consequently that which is most useful in practical work. If a correction for the velocity of approach is introduced, the values of C, are reduced by ⅓ per cent. to 1½ per cent., the lesser reduction occurring for square corners, and h=0·05 feet, and the greater for Case VI, and h=0·30 feet.

Sluices and Gates.—These may be considered as orifices, usually rectangular in shape, with completely suppressed contraction along the lower portion of the perimeter, more or less suppressed contraction at the sides, and complete contraction at the upper portion. Also, the issuing jet, or sheet of water, is generally guided, and prevented from expanding on the sides corresponding to those on which contraction is suppressed at entry.

The complexity of the problem is indicated by a mere statement of the above facts. Existing experiments show that the coefficient of discharge varies markedly with the amount of opening of the sluice. The issuing jet often forms a standing wave, and, in such cases, we have the additional problem of specifying where the head is to be measured. (Compare Sketch No. 33.)

The problem is obviously that of a submerged rectangular orifice, and the following general statements can be made.

When the sluice is first opened, the thickness of the bottom of the gate is comparable with the width of the opening, and phenomena occur which are analogous to those of an orifice with a mouthpiece. Coefficients of discharge as high as 1·20, reckoned on the gate opening, have been observed, and there is little doubt that higher values are met with, but are masked by the leakage that takes place at other portions of the gate, and by experimental difficulties connected with the measurement of the head. When this stage ends, (which both theory and experiment indicate occurs when the width of the opening is equal to about three-quarters of the thickness of the bottom of the gate), the flow resembles that through an orifice with partially suppressed contraction, and a coefficient of 0·65 would appear to be approximately correct, although reliable observations occasionally indicate lower values, such as 0·60. These values may be explained by the fact that when the seat of the gate is somewhat raised above the rest of the floor, and the width of the opening is not too great, marked contraction at the bottom of the orifice is produced. Theory then indicates a value of about 0·63, so that the observations are probably accurate. As the gate rises, this bottom contraction has less effect, the coefficient increases, and values ranging up to 0·85 occur.

The only systematic observations are those of Bornemann. Those by Benton, and Chatterton, are more practical, but are less suitable for ascertaining the general laws.

Bornemann (*Civilingenieur*, vol. 26, p. 297) experiments with sluices in open channels. His table shows the following particulars :

No. of Experiments.	Width of Sluice.	Width of Channel.	Height of Opening in Feet.		Effective Heads in Feet.	
	Feet.	Feet.	Max.	Min.	Max.	Min.
16	3·30	3·73	0·58	0·11	0·73	0·06
28	1·70	1·78	0·84	0·30	0·61	0·06
19	2·54	2·83	0·43	0·22	0·57	0·06

With the formula $Q = C \times$ area of opening $\sqrt{2g\left(h_1 - h_2 + \frac{u^2}{2g}\right)}$

where h_1, is the depth of the top of the opening below the upper water surface, h_2, is the depth of the top of the opening below the lower water surface.

Thus, $h_1 - h_2 = h_a =$ effective head.

u, is the velocity of approach.

k, is the height of the opening. (Sketch No. 44.)

Bornemann gets for the 3·30 feet sluice :

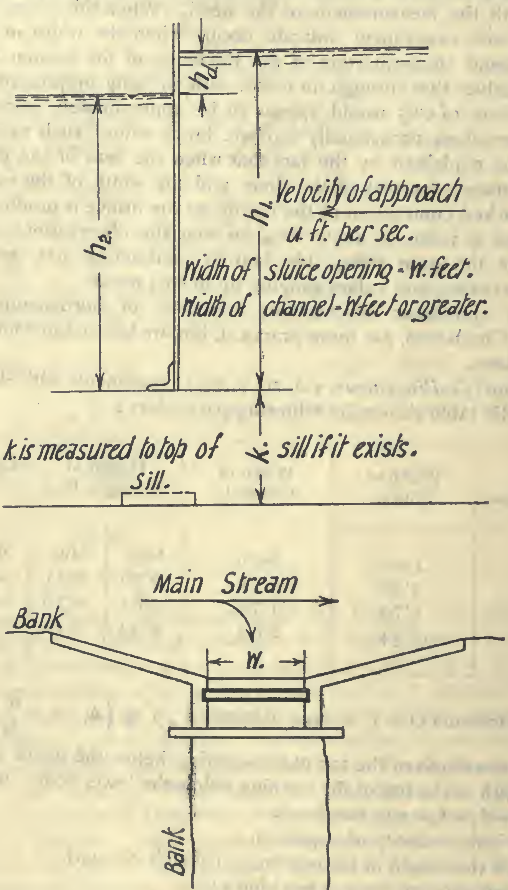
$$C = 0{\cdot}664 + 0{\cdot}053 \frac{\sqrt{k}}{h_2 + \frac{k}{2}}$$

and for all the experiments :

$$C = 0{\cdot}541 + 0{\cdot}150 \frac{\sqrt{k}}{h_2 + \frac{k}{2}}$$

The range of these experiments is by no means as wide as is desirable. Large values of k, are almost universally accompanied by small values of the effective head, but this connection is far less marked than in the case of the experiments which are later discussed. The form arrived at by Bornemann must therefore be considered as the best which is at present available.

Benton (*Punjab Irrigation Branch Papers*, No. 8) experimented on the ordinary sluice gate used for regulating the discharge of irrigation canals.



SKETCH No. 44.—Diagram of Discharge through a Sluice, and Type of Sluice experimented on by Benton.

These gates were 10 feet wide in eleven cases, 8 feet wide in sixteen cases, 6 feet wide in twelve cases, and 4 feet wide in seven cases. The discharges were observed by rod floats, and are probably accurate to 1 per cent., the observer being unusually skilful. The discharge being submerged, and velocity of approach being neglected, the formula is as follows :

$$Q = Cwk \times \sqrt{2gh_d}$$

The effective head h_d is defined as the difference between the water levels upstream and downstream of the gate, as observed in stilling wells.

We have the following table :

w, in Feet.	Max. Q, in Cusecs.	Min. Q, in Cusecs.	Max. h_d, in Feet.	Min. h_d, in Feet.	Max. k, in Feet.	Min. k, in Feet.
10	113·0	30·7	3·88	0·07	2·55	0·46
8	56·7	26·3	1·47	0·16	3·20	0·42
6	49·3	7·2	4·85	0·22	0·92	0·28
4	24·9	10·1	3·51	2·44	0·71	0·19

Benton considers that C is solely affected by w, the width of the gate, and gives for all observations :

$$C = 0·7201 + 0·0074w$$

For all cases where h_d, exceeds 0·50 foot. $C = 0·7162 + 0·0079w$, so that :

$w =$	10 Feet.	8 Feet.	6 Feet.	4 Feet.
C (first case) . .	0·794	0·780	0·765	0·750
C (second case). .	0·795	0·779	0·763	0·748

The results agree very well with the experiments, but it must be remembered that the maximum h_d, almost invariably occurs when k, has its minimum value, and *vice versa*, consequently, Q, as tabulated, varies far less widely than might be supposed.

The experiments of Chatterton (*Hydraulic Experiments in the Kistna Delta*) are subject to the same objection. The gaugings were effected with a current meter, but are probably less accurate than are those of Benton. With one exception, the gates were from 6 feet to 5·25 feet wide ; but, unlike Benton's experiments, the discharge observed was that of a number of gates (in some instances as many as 17) separated by piers 3 feet in thickness. The experiments are somewhat mixed in character, containing submerged orifices similar to those experimented on by Benton, and orifices with free overfall, where the theoretical formula is represented by :

$$Q = Cw\sqrt{2g}\,(H_2^{1·5} - H_1^{1·5})$$

H_1, and H_2, being the depths of the top and bottom of the orifice below upstream water level. Velocity of approach, though neglected, had probably more effect than in Benton's experiments, the circumstances being similar to Sketch No. 42, A.

Chatterton gives the following equation :

$$C = 0·615 + 0·007 \times 2^{5-h_d}$$

for all cases where h_d varies from 0, to 5 feet.

I find approximately that:

(i) For the cases where the discharge is submerged, and the formula:

$$Q = Cwk\sqrt{2gh_d} \qquad \text{is applicable,}$$
$$C = 0{\cdot}83 - 0{\cdot}11 h_d$$

where h_d varies from 0, to 1·2 foot. This compares very well with Benton's experiments under similar heads, and the larger values of C, are plainly due to neglect of the velocity of approach.

(ii) For cases where the discharge is not submerged, and the formula:

$$Q = Cw\sqrt{2g}\,(H_2^{1\cdot5} - H_1^{1\cdot5}) \qquad \text{applies,}$$
$$C = 0{\cdot}63,$$

with $\frac{H_1+H_2}{2}$ varying between 3, and 5 feet.

The results are not very concordant, although they agree well with the theoretical values. The experimental difficulties are great, so that these figures are the best that are likely to be obtained.

The results obtained by Benton and Chatterton show that the shape of the approach channel, or the circumstances affecting side contractions, have extremely little influence on the coefficient of discharge in orifices of the size now under consideration. All these matters may be neglected in practice, without introducing serious errors.

We may therefore sum up this obscure, but important subject, as follows:

The general value of the coefficient of discharge of a submerged orifice with bottom contraction completely suppressed, may be taken from Benton's experiments for heads exceeding 0·6, or 0·7 foot. For lower heads, the values given by Chatterton, corrected by Benton's rule for the width of the opening, may be used. Nevertheless, it is inadvisable to take a greater value of C, than 0·80, although there is little doubt that in wide openings values such as 0·85, or even 0·90 occur. If the bottom contraction is complete (*e.g.* the gate rests on a raised sill), and the orifice is not submerged, a coefficient of 0·63 is safe, but is probably exceeded when the discharge is submerged.

When actual observations have been made, a formula of Bornemann's type, with properly determined coefficients, will probably be found useful; and the velocity of approach should be allowed for.

Comparing these results with the general trend of the evidence afforded by small scale observations, we can rest assured that the application of coefficients derived from work on orifices say 1 foot wide, and 8 inches high, is not likely to lead to serious error, and that the large orifice will almost invariably discharge more than is indicated by the calculations.

Non-circular Orifices, under Large Heads.—The only available experiments are due to Graeff (see p. 788). It appears justifiable to assume that the geometrical form of the orifice has no appreciable influence on the coefficient of discharge, which is determined entirely by the thickness of the walls and the amount of suppression of contraction. In calculations it is desirable to bear in mind that the higher the head the blunter (geometrically considered) the edges of the orifices may become before the orifice ceases to be "sharp-edged" (hydraulically considered).

CHAPTER VI.—(SECTION A)

COLLECTION OF WATER AND FLOOD DISCHARGE

Collection of Water.—Connection between rain-fall and stream discharge.

AVERAGE, OR MEAN VALUE.—Sampling—Application to rain-fall or stream discharge statistics.

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Climate as affecting the Variability of Rain-fall.—Insular and Continental climates—Temperate and Tropical climates—Wet and dry seasons—Variability of the annual rain-fall—Binnie's rules for the relation between the values of the mean annual rain-fall for a short and long period—Ratio of mean annual to maximum and minimum annual rain-fall—**Binnie's rules**—Criticism—Exceptions—Indian and Californian examples—Probable explanation—Space variability of rain-fall—Large scale selection of the sites of rain-gauges—Liability to underestimate the average rain-fall of any area—Local conditions affecting rain-gauges—Effect of eddies—Gauges on hillsides—Nipher shield—Standard rain-gauge—Correction for elevation above the natural surface.

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VARIATION OF THE RAIN-FALL OVER THE YEAR.—Summer and winter rains—Wet and dry season rains.

CONNECTION BETWEEN RAIN-FALL AND RUN-OFF.—Disposal of rain-fall in the forms of: (i) Utilisation by vegetation; (ii) Evaporation; (iii) Topographical flow; (iv) Stored rain—Ground water flow—Period over which the relations hold—Period of a year—Values of ΔR—Possible errors—Approximate estimation of the monthly run-off from evaporation statistics—Values for English catchment areas—For Elbe and Moldau.

Observations supplementing the usual Stream Gaugings.—Subsoil water levels—Survey of permeable beds—Chemical investigations—Seepage water investigations.

Case when rain-fall does not suffice to provide for vegetation and evaporation—Droughts.

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RUN-OFF OF CATCHMENT AREAS.—Circumstances affecting the relation.

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SEASONAL DISTRIBUTION OF RAIN-FALL.

Effect of Geological Structure.—Effect on topography—Permeable beds—Geological and topographic watersheds—Detection of springs, or leaks in stream beds—Detection of underground flows—Dew ponds.

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GLACIER-FED STREAMS.—Monthly discharge curves.

DAILY VARIATIONS OF MOUNTAIN STREAMS.

Relation between mean Yearly Rain-fall and Run-off.—Keller's results for German areas—Flat areas—Partly flat and partly hilly areas—Hilly areas—Alpine areas—Probable errors—British areas.

Relation between the Rain-fall and Rain-fall Loss for Individual Years—Determination of the constant a—Examples—Wet districts.

Distribution of Run-off during the Year.—Visible and invisible reservoirs—Structure of the catchment area—Typical tables of rain-fall and run-off by months.

Subtractive Method.—Monthly rain-fall losses—Summer rain-fall losses.

Proportional Method—Monthly values of the ratio $\frac{\text{Run-off}}{\text{Rain-fall}}$—Practical application.

Third Method taking into Account the Effect of Ground Water Storage on Run-off—Vermeule's investigation—Vermeule's v—Tabulation—Connection with evaporation and rain-fall loss—Classification of catchment areas—Depletion—Determination of monthly run-off in terms of the initial depletion—Practical determination of the run-off and mean depletion curves—Criticism—Application to British catchment areas—Tables of y, and d_i—Example—Special monthly formulæ—The run-off at end of a dry period is a geological phenomenon.

Determination of Reservoir Capacity.—Accuracy of the predicted monthly and yearly run-offs—Three driest consecutive years—Hawksley's rule—Rofe's rule—New rule—Tabulation—German experience—United States experience.

Mass Curve.—Description of monthly mass curve—Period of greatest depletion—Correction for evaporation—Equalising storage for driest year—*Yearly Mass Curve*—Storage capacity derived from the yearly mass curve—Determination of run-off during critical periods—Growing season—Replenishment season—Storage season—Consumption by vegetation—Equalisation of yield over five dry years; probably represents maximum storage capacity—Gore and Brown's investigations—Tables of yearly rain-falls and run-offs.

Catchment Areas situated in Climates of the Second Type—Authorities—Strange's table for India—Criticism—Accuracy of rain-fall records—**Binnie's Method**—Criticism—Observations for a partially permeable catchment area—Table for daily run-offs—Table for flat and permeable areas—Correction for stored water.

Variability of the Wet Season Rain-fall.

Capacity of Reservoirs.

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Records of the Sweetwater catchment area.

Records of Australian and Californian catchment areas.

Victorian records of rain-fall and run-off.

Secondary Catchment Areas.—Capacity of the diversion channel—Vyrnwy secondary catchment areas—Effect of silt—Coghlan's rule—Artificial drainage—Impervious catchment areas.

Collection of Water from Sources other than Stream Flow.—Underflow—Springs—Catchment galleries—Dune sand developments.

Wells in Uniformly Permeable Strata.—Conditions in sand, chalk, and granite—Permeability—Slope of ground-water surface—Velocity of flow—Formulæ for circular well—Catchment gallery—Effect of the size of the well—Variations in permeability—Effect of the well lining, or of a stream or lake near the well—Spacing of wells—Blowing of a well—Natural replenishment of the well—Preliminary studies—Correction in deep beds of sand—Probable yield of large schemes—Reversed filters—Well plugs—Mota wells.

Artesian Wells.—Practical conditions—Thickness of permeable strata—Estimation of probable yield—Permanent diminution of yield—Quality of water—Typical example—Geological conditions as applied to individual wells.

NOTATION FOR RAIN-FALL AND RUN-OFF

The period to which the quantities refer is denoted by a suffix. A symbol without a suffix refers to the year. All quantities are expressed in inches depth over the catchment area.

Suffix 1, 2, etc., refers to calendar months.

m, refers to the 30 or 40 years' mean.

p, refers to any period in general.

c, and h, refer to the cold and hot seasons.

s, and w, refer to summer and winter.
Capitals are occasionally used when two periods of different years are contrasted.
$a_p + b_p x_p = V_p$ is the vegetation and evaporation loss (see p. 185) during the period p, and a similar formula gives Vermeule's v (see p. 219).
$a_p + b_q x_q + b_r x_r$ is used to denote V_p, when the change in temperature during the period p, is too great to permit the simpler form to be used.
A, and A_u (see p. 222).
c (see p. 200).
d_n, or d_i, is the initial depletion (see p. 221) of the month considered. While d_{n+1} is the final depletion of this month, or the initial depletion of the next, and the mean depletion during the month, $\frac{d_n + d_{n+1}}{2}$ is denoted by D_n.
D, 2D, 3D, as suffixes refer to the driest, two consecutive driest, and three consecutive driest years of a long period.
e, is the evaporation from a free water surface, as observed in meteorological observatories.
E (see p. 190).
f, is the ground water flow as calculated on the assumption that $ke_p = V_p$. See p. 190 and g, k, t, u, and v.
g, is the water flowing out from the ground storage to the stream, as actually observed (see p. 187, and also f, t, and u).
$k = \frac{z}{e}$ (see p. 190).
R, is the total quantity of water stored up in the ground. The depletion d, or D, is the difference between the maximum value of R, for the year, and the value of R, at the time considered, when ΔR, is calculated by Vermeule's method.
Also, $\Delta R = s - g$, and $f = -\Delta R$, on the assumptions explained under these symbols.
S, is the maximum depletion ever found to occur, and theoretically, the difference $R_{max} - R_{min}$ calculated according to Vermeule's rules is equal to S.
S_u (see p. 222).
s, is the flow to ground storage from the surface (see p. 187).
t, is the topographic flow (see p. 186).
Neither the s's, g's, nor t's, are ever obtained by calculation, and different symbols are used for the calculated values in order to indicate that these values are only approximations.
u, is the value of g, calculated according to Vermeule's rules. See g, and f.
v, is Vermeule's value of $a + bx$ (see p. 219).
V, is the vegetation and evaporation loss.
x, is the rain-fall.
y, is the run-off.
y_o, and y_n (see p. 222).
z, is the rain-fall loss. $z = x - y$.

Collection of Water.—Apart from a few somewhat doubtful exceptions, all fresh water occurring in nature has at some period of its existence fallen as rain. The present chapter is essentially an endeavour to deal with the following problem. The area of the surface drained by a natural stream, or artificial channel, and the average depth of rain falling on this surface during a given period being known from actual measurement and observation, the total volume of rain water falling on this area (which we shall hereafter term the catchment area of the stream or channel) during this period can be calculated. We wish to determine the relation between this volume and the total volume delivered by the stream in the same period.

No precise solution can be given; and it will be evident that the fact that the problem assumes this particular form is in reality an indication that observations of the discharge of the stream have been neglected. Thus, in a strictly scientific sense, the fact that 85 pages are devoted to a consideration of the question is almost discreditable. In practice, however, observations of rain-fall are made in all countries for many years before the discharge of the streams is systematically measured. Hence, engineers are obliged to consider the question.

Average, or Mean Value.—The conception of the average or arithmetical mean of a number of quantities of the same kind is familiar to all practical men, and the belief that the average value of any quantity common to a class of individuals of not too variable a character (*e.g.* individuals of the same biological species) can be ascertained with sufficient accuracy for practical purposes by selecting a certain number of the individuals at random, and taking the average of the quantity as observed in these individuals, is relied on in practical life to an enormous extent, since it forms the ultimate justification of sales by sample, and analyses by "quartering." The mathematical difficulties attending any proof of this belief are very great, and there is little doubt that in some cases the geometrical mean of the quantity observed in the samples is a fairer representation of the value around which these quantities tend to range themselves when the whole class is considered. In practice, however, a sample of a material taken at random is usually a satisfactory representation of the sampled material, provided that the bulk of the sample is not too small relatively to the total bulk of the material, and that the selection is one made truly at random.

Average Values.—In considering statistics of rain-fall, stream discharge, or other quantities observed by engineers, the problem usually presents itself as follows:

Let P_1, P_2, ... P_N, say, represent the observed quantities, where N, indicates a very large number, say 10,000 if the symbols represent daily values, or 100 if the symbols represent yearly values. The difference is obviously due to the fact that a yearly value is in itself a mean, or total (as the case may be) of 365 daily values.

The mean value is: $$\frac{P_1+P_2+\ .\ .\ .\ +P_N}{N} = {}_mP_N \text{ say.}$$

Now, consider: $$\frac{P_1+\text{etc.}+P_n}{n} = {}_mP_n$$

$$\frac{P_1+\text{etc.}+P_{n+1}}{n+1} = {}_mP_{n+1}$$

$$\frac{P_1+\text{etc.}+P_{n+2}}{n+2} = {}_mP_{n+2}$$

Where n is any number less than N.

What is the relation between ${}_mP_N$, and the various quantities ${}_mP_n$, ${}_mP_{n+2}$, etc.?

As a matter of observation, we usually find that if n, be not too small compared with N, the quantities are very approximately equal to each other. This property is specified by the statement that the "P's" vary more or less regularly about a mean value, and the more regular the variation, the smaller the value of n, required to secure this approximate equality. The differences that still exist between ${}_mP_n$, ${}_mP_{n+1}$, etc., are called the residual irregularities.

Meteorologists have a belief (it is not more) that if the P's, be meteorological quantities observed yearly, n, must be approximately equal to 35. With considerably less justification engineers are accustomed to assume that if P, represents the twenty-four hour discharge of a stream, n, is about 7×365, *i.e.* a seven years' record will suffice.

As a matter of observation, we also find that if:

$$_mP_n \approx {}_mP_{n+1} \approx {}_mP_{n+2} \approx {}_mP_N$$

Then also:

$$_mP_n \approx \frac{P_2 + \text{etc.} + P_{n+1}}{n} \approx \frac{P_3 + \text{etc.} + P_{n+2}}{n}$$

$$\approx \frac{P_4 + \text{etc.} + P_{n+3}}{n} \approx \frac{P_r + \text{etc.} + P_{n+r-1}}{n}$$

$$\approx {}_mP_N$$

From a purely philosophical point of view, the whole series of assumptions rests on very insecure foundations, and it is quite possible that when accurate statistics of 300 or 400 years are available, "century long" climatic changes will be found to have a real existence. At present, 100 years of accurate statistics of any climatic quantity do not exist, and all that can really be said is that some assumption must be made, and that the possibilities of error should be indicated. This has been attempted in the following treatment of the subject.

DEFINITIONS.—We define as follows, all quantities being expressed in inches depth:

The Rain-fall, or mean annual rain-fall, is the mean of the annual rain-fall observed over a period which is sufficiently long to produce a fairly constant mean value. In the British Isles we can state that this period is about thirty to forty years, and that the probable variation of this mean value (*i.e.* the residual irregularity) is $\pm 2{\cdot}5$ per cent. when compared with the mean of another record of equal duration for the same locality.

The Evaporation, or mean annual evaporation, is the mean value, in inches, of the depth of water annually evaporated from a free water surface, the period of observation being of adequate duration to secure approximate constancy, as in the case of rain-fall.

The Run-off, or mean annual run-off, of a catchment area, is the value of the annual volume of water discharged by the stream draining the area, expressed in inches depth of water over the catchment area, the period of observation being sufficiently long to secure a fairly constant mean.

We also define the Rain-fall loss for a catchment area, as the difference between the rain-fall and run-off for any period, both being measured in inches over the catchment area.

The Percolation, or mean annual percolation, is the depth of rain water measured in inches that annually soaks away through the earth; it being presumed that the period of observation is of sufficient length to secure approximate constancy.

As yet we are unaware how long a period of observation is required in order to produce fairly constant mean values of these last four quantities; although it is highly probable that the periods are less than those for rain-fall in the case of percolation, but greater for evaporation and possibly also for rain-fall loss and run-off.

Sources of Information.—Since the above defined quantities vary from year to year, and differ for every locality on earth, it is impossible to give any useful table of their values. Records of rain-fall exist in every civilised country, and usually a very fair value of the rain-fall, and a less accurate one (since observations of these quantities are not so commonly made, and have generally been recently initiated) for evaporation and percolation, is obtainable by consulting such records as :

Symons' *British Rainfall*, for the British Isles. The publications of the United States Weather Bureau, and reports of the various Meteorological Offices for their respective countries.

I have tabulated, and, where possible, given the original references to all published values of the rain-fall loss that I have been able to ascertain, that are based on anything more than a vague assertion.

For run-off statistics, the principal authorities are the publications of the United States Geological Survey, which refer only to the United States. For the British Isles, no complete record of the run-off of any river, except the Thames, has been published. Many must exist stored away in the various waterworks' and engineers' offices. A certain amount of information on the subject (especially values for yearly run-offs) lies scattered throughout the *Proc. Inst. of C.E.*, but it is small compared with that afforded by such papers as those by Fitzgerald, on the Boston Waterworks, in the *Trans. Am. Soc. of C.E.*; or by Freeman, in his paper on the "Report on the Water Supply of New York." This latter work is a model example of the proper use of run-off statistics; and, owing to Mr. Freeman's careful discussion of this record, the possibilities of error remaining even in the case of such complete and carefully handled data, are clearly shown.

In India, most of the big rivers are gauged, more or less systematically, by the Irrigation Department, and by the various railways; and for such matters as the available low water supplies, the information is usually most complete.

In the State of Victoria (Australia), Stuart Murray has instituted a very complete system of run-off records. The available rain-fall records are not very good, so that from the present point of view the figures are not of general interest. They form, however, a very excellent basis for all projects for water supply in Victoria, and, when compared with the available British information, are highly creditable to Mr. Murray.

My own experience is that in most capitals, except London, a fair amount of accessible information exists, either worked up ready for use, or in the form of gauge readings; and this, in conjunction with rain-fall records indicating the probable years of high and low values of run-off, enables a very fair idea of the capabilities of a catchment area to be obtained.

I must here express my thanks to the many practising engineers in countries as far apart as Australia, Japan, the United States, and Great Britain, who have favoured me with copies of private statements of rain-fall and run-off. The rules put forward for British run-offs are avowedly capable of improvement. If I can obtain a careful criticism (even though hostile), supported by only one hitherto unpublished record, I shall feel amply rewarded.

Rain-fall.—The amount of discussion devoted to this subject in a treatise on Hydraulics is, in reality, a measure of the absence, or non-accessibility, of long period records of the discharge of streams and springs. It is to be hoped that, as the art progresses, discussions on the variability of discharge and water yield will gradually supersede the present methods, and that rain-fall will finally be relegated to its rightful, and subordinate positon, which, in my opinion, is but little superior to that of the local temperature, as influencing the daily consumption in a town water supply, or the duty of water in the irrigation of crops.

In the present state of the art we are forced to rely upon rain-fall observations, not so much because they are the most desirable records bearing on hydraulic questions, but because they are one of the few requisite observations that can be taken by an intelligent, but untrained man. Owing to the large influence that rain exerts on personal comfort, this work is undertaken by people who otherwise have not the slightest interest in, or knowledge of, hydraulics. Rain-fall figures indeed, form the one piece of definite information that the non-engineering world is generally able to give the hydraulic engineer when initiating a project. He therefore accepts it gratefully, and uses it to the best of his ability. It is, nevertheless, necessary to bear in mind continually that we are primarily, and almost exclusively, concerned with run-off or discharge statistics; and that to neglect these—even when approximately accurate—for rain-fall records of far greater accuracy, is, as it were "putting the cart before the horse."

Climate as affecting the Variability of Rain-fall.—It will be shown later that the annual rain-fall in any locality varies from year to year within certain limits. These variations are largely determined by the general character of the local climate, and, consequently, it becomes necessary to broadly define the types of climate that influence the probable variations.

I therefore propose to classify climates as Insular or Continental. The distinction is primarily a geographical one. Localities close to the oceans have an Insular climate, while the Continental type of climate occurs either in the interior of Continents, or in places separated from the oceans by high mountain ranges.

The characteristics of the two types are well known. Continental climates have a very hot summer, followed by a relatively cold winter, while the difference between the mean winter and summer temperatures in an Insular climate is by no means so marked, and in some cases is almost imperceptible.

In the Temperate Zone, the climate of the British Isles is typically Insular, while the Middle United States, or Southern Russia, possess a climate of Continental character. The dividing line may be very practically illustrated by the fact that an Englishman's wardrobe does not usually include either furs or white suits, while an American of the same class invariably possesses both.

So also in Tropical Regions, such as the Punjab, fur coats are common in the cold weather, while in the hot weather punkahs, or electric fans, are necessities for Europeans, and are appreciated by all races. The contrast with say, Ceylon, where punkahs or fans are less essential, but are used all the year round by those who can employ them, is very marked.

The distinction between a Tropical and a Temperate climate is somewhat difficult to define. Geographically, for instance, the Punjab is not in the Tropical Zone, yet the temperatures there obtaining are surpassed in very few localities, and, unless I am mistaken, these are also entirely extra Tropical, geographically speaking (*i.e.* the Persian Gulf and the Salton Desert).

From the point of view of an engineer, Tropical climates may be defined as those in which the native workman is unable (during some seasons of the year at any rate) to perform hard manual labour continuously during the hottest portions of the day.

In all Tropical climates (except a few extremely Insular examples), and in most Temperate Continental climates, there are well defined rainy seasons, usually one each year, but in some cases two. In such instances, the major portion of the rain-fall, and all that has any practical influence on the run-off, occurs during well defined periods of the year, usually not exceeding four months in length; and during the remainder of the twelve months the rain that does fall is insignificant in quantity and accidental in occurrence.

Generally, it may be stated that an Insular climate is, (comparatively speaking) a wet one.

A prevalent idea exists that Tropical climates are markedly wetter than Temperate ones. This, I believe, is principally owing to the fact that European habitation in the Tropics is somewhat closely confined to islands and sea coasts, possessing Insular climates. So far as our knowledge permits of any wide statement being made, I believe that when the areas of Tropical Continental climates are as extensively inhabited by civilised races as Tropical Insular climates now are, it will be found that the difference, if it exists, is but small. At present it can be definitely stated that the mean rain-fall of all India is very close to the mean rain-fall of the British Isles, and, so far as my own studies go, I believe that the actual distribution of rain gauges is such as to under-value the rain-fall of the British Isles, and to over-value that of India; although I am well aware that the gentlemen who prepared the estimates of mean rain-falls did in each case allow for this possibility of error to the best of their ability.

Variability of the Annual Rain-fall.—As a matter of observation, the fall of rain at any locality, measured in inches per annum, varies from year to year. A study of rain-fall records extending over periods of many years, such as exist in England, Europe, and the United States, has led to the conclusion that the average of the yearly rain-fall tends towards a constant quantity, as the number of years over which the average is taken increases, and it appears that the average of 30 to 40 years' rain-fall varies but little, whatever period of 30 to 40 years in a long rain-fall record is selected.

The exact figures as given by Hann and Mill (*P.I.C.E.*, vol. 155, p. 368), are:

AVERAGE VARIATIONS FROM THE MEAN FOR 70 YEARS AS GIVEN BY RECORDS OF:—

	10 Years.	20 Years.	30 Years.	40 Years.
	Per cent.	Per cent.	Per cent.	Per cent.
Three European Stations . . .	7·5	5·2	2·6	2·3
Five British Stations	4·7	3·4	2·2	1·7

While Binnie, (*P.I.C.E.*, vol. 109, p. 131) gives for 26 stations, with records of an average length of 53 years, the following:

DEVIATIONS FROM THE MEAN VALUE OF THE ANNUAL RAIN-FALL DURING THE WHOLE PERIOD OF THE RECORD, EXPRESSED AS PERCENTAGES OF THIS MEAN FALL FOR EACH LOCALITY; WHEN THE PERIOD CONSIDERED IS:—

	5 Years.	10 Years.	15 Years.	20 Years.	25 Years.	30 Years.	35 Years.
Maximum positive deviation . .	23·2	14·9	9·2	5·6	7·3	5·2	4·5
Maximum negative deviation . .	29·6	16·1	12·5	9·2	9·0	6·9	4·7
Average positive deviation . .	15·35	8·08	3·87	2·47	2·56	2·17	1·73
Average negative deviation . .	14·52	8·37	5·64	4·08	2·94	2·36	1·86
Minimum positive deviation . .	6·8	1·0	0·0	0·0	0·0	0·0	0·0
Minimum negative deviation . .	7·8	4·7	0·8	0·0	0·0	0·0	0·0
Average deviation .	14·93	8·22	4·77	3·27	2·75	2·26	1·79

These stations are distributed over a large portion of the globe, and may be regarded as including Insular climates, both Tropical and Temperate, together with 4 examples of what may be termed Semi-continental climates. We may therefore consider that these figures are applicable to all Insular climates, and probably, with a fair degree of accuracy, to all except the extreme Continental type.

The results show that, even in so short a period as 15 years, the average annual rain-fall is unlikely to differ materially from the average values for a long period, such as 40 to 50 years. It must also be noted that such short period averages are more likely to be less than the long period average of rain-fall, than in excess of it.

Binnie's figures also indicate that the average deviation may slightly increase if the period exceeds 35 years; the figures being:

For 40 years 2·16 per cent.
" 45 years 2·03 "
" 50 years 1·98 "

Rain-fall records extending for periods much above 50 years are somewhat rare, and statements regarding the rain-fall values over such periods are consequently liable to error. It is nevertheless true that there is apparently a cycle in rain-fall, and that this cycle appears to have a period approximately equal to 36 years.

In my opinion, the actual facts do not permit more than this to be stated, and the cycle is one which refers not to individual rain-falls, but to the average of 3 or 4 consecutive years.

We consequently define the mean annual rain-fall of a locality as: The average taken over a sufficiently lengthy term of years to ensure a fairly constant value, and may assume that 30 to 40 years are generally an adequate period.

It is also evident that if we have a rain-fall record of say 10 to 20 years, which displays a fairly close relationship for this period with the rain-fall at a neighbouring place, where a long period record exists, it is only a matter of simple proportion to arrive at a fairly accurate value of the long period rain-fall for the first locality. In practice, in the case of places fairly close together, and separated by no very marked natural features, such as a range of hills, or a river, this proportional relationship holds with sufficient accuracy to justify its use in supplementing actual records.

If the yearly rain-falls of localities distributed all over the globe are studied with respect to their absolute magnitudes, no rule is disclosed. If, however, we reduce this maze of figures to percentages of the mean annual rain-fall, for each locality, a very striking regularity will be found in nearly every case.

These rules were first systematised by Binnie, in a paper on "The Variation of Rain-fall" (*P.I.C.E.*, vol. 109).

I have taken the figures for some 80 long period records of places in Great Britain, selected by Mill, (*P.I.C.E.*, vol. 155) which may therefore be regarded as extremely accurate, and find that the rain-fall of the wettest of a long series of years is about 146 per cent. of the mean:

The maximum value of this figure being about 170.
The minimum value of this figure being about 125.

The rain-fall of the driest year is about 66 per cent. of the mean:

The maximum value being about 80 per cent.
The minimum value about 55 per cent.

Similar figures for the average rain-fall of the 2 consecutive driest years are:

75 per cent. 86 per cent. and 60 per cent.

and for that of the 3 consecutive driest years:

80 per cent. 87 per cent. and 64 per cent.

I have been unable to trace any relation between the variation of these ratios for British stations, and the absolute value of the mean annual rain-fall.

46 per cent. of the years have a rain-fall above the average, and the average fall of these years is 119 per cent. of the mean fall; while the remaining 54 per cent. have a fall below the average, and their average fall is 83 per cent. of the mean fall.

Also periods of 4, 5, 6, and even 9 years in succession, may have falls less than the average, and the average annual fall of such a period of dry years is about 82 per cent. of the mean.

Binnie's selection of records, as given in Table VII. of the above paper, was obtained at localities distributed all over the globe, and the ratios are as follows. Taking the mean annual rain-fall as above defined, as 100, the value of the rain-fall in other years, on the average, is given by:

Locality.	Number of Stations.	Wettest Year.	Average of Two Consecutive Wettest Years.	Average for Three Consecutive Years.	Average of Three Consecutive Driest Years.	Average of Two Consecutive Driest Years.	Driest Year.	Maximum Number of Consecutive Years with a Fall above the Mean.	Average Fall of these Years.	Maximum Number of Consecutive Years with a Fall less than Mean.	Average Fall of these Years.
British Isles .	44	145	130	123	78	73	66	5·52	117	5·57	84
N. W. Europe .	5	148	133	126	75	66	61	3·80	123	5·40	83
France . .	23	161	142	131	74	68	59	5·22	122	5·43	81
Italy . .	15	159	139	129	76	70	55	4·26	121	5·60	83
N. Germany .	17	139	127	121	77	70	61	5·53	114	5·59	82
S. Germany and Austria .	9	144	133	127	76	68	56	6·11	120	5·55	81
Russia . .	12	166	146	135	68	63	53	5·42	122	7·66	78
India . .	9	162	142	130	72	66	52	4·77	123	5·33	78
Canada and Eastern United States .	10	141	131	125	79	75	68	5·70	119	6·90	85

Binnie also classes his stations by the absolute value of the mean rain-fall. The figures fall into two very sharply defined groups, over 20 inches and under 20 inches mean rain-fall. The values are as follows:

Over 20 inches .	140	149	132	126	76	70	61	5·11	119	5·73	82
Under 20 inches	13	175	149	137	67	62	51	5·62	129	7·38	76

The figures for the individual localities contained in the above table are remarkably concordant, and Binnie considered that the only exceptions likely to occur were those disclosed by the figures concerning rain-falls of less than 20 inches. A study of the information at present available leads me to extend and slightly alter Binnie's deductions. The variations of the individual annual rain-falls from the mean rain-fall are of the order of magnitude indicated by Binnie's figures in the case of Insular climates only. For typically Insular climates, the figures given for the British Isles and N.W. Europe may be taken as very close to the truth for all portions of the globe. For Continental climates, however, the variations are larger, and the ascending scale shown, in the above table, by the graduation through Italy, France, India, and Russia, admirably illustrates the general law. Also, the greater the absolute magnitude of the rain-fall, the smaller is the variation (when expressed as a percentage of the mean annual rain-fall); and *vice versa*, the smaller the absolute magnitude of the rain-fall, the greater the percentage variations. It will be seen that I have been led to attribute more influence to the character of the climate than to the absolute value of the rain-fall. As an example, the following figures hold for the rain-falls of the years 1879–1908, at Amritsar (Punjab), which possesess a typically Continental climate, and a mean rain-fall of 25·26 inches.

Wettest year—309	per cent. of the mean.	
Average of two consecutive wettest years—237	„	„
Ditto. of three consecutive years—214	„	„
Average of three consecutive driest years—52	„	„
Average of two consecutive driest years—48	„	„
Driest year—34	„	„

Maximum number of consecutive years with a fall above the mean—5
Average fall of these years—170 per cent. of the mean.
Maximum number of consecutive years with a fall less than mean—5.
Average fall of these years—54 per cent. of the mean.

Similar cases exist in India, where the average fall is even greater than at Amritsar, and several stations in Siberia and China show even larger variations.

Even if we confine ourselves to Insular climates, the figures given by Grunsky (*Trans. Am. Soc. of C.E.*, vol. 61, p. 498) for the rain-fall at and near San Francisco are:

Maximum annual rain-fall = twice the mean.
Minimum annual rain-fall = one-third to two-fifths of the mean annual rain-fall.

The probable explanation of these abnormalities is to be found in a consideration of the geographical distribution of rain-fall. Amritsar and San Francisco both lie between zones of comparatively heavier rain-fall (Orissa with 59 inches, and Oregon with 80 inches), and zones of far lighter rain-fall (the N.W. Frontier with 8 to 10 inches, and part of Southern California with 3 to 5 inches). While these zones are explained by the topographical features of the country, a study of large scale rain-fall maps shows in each case that a comparatively slight deflection of the rain-bearing currents would produce a considerable alteration in the absolute fall.

We may therefore consider Binnie's rules as generally applicable, but it will be wise to await further studies before accepting them as universally true. Exceptions are most likely to occur where the locality under consideration possesses a dry Continental climate, or lies geographically between regions of markedly lower and higher absolute rain-fall (see also p. 248).

When mathematically regarded, the above facts are conclusive evidence that a yearly rain-fall sequence is not a matter of chance, in the sense that roulette or dice sequences are. In a series of rain-fall observations, negative variations are more frequent than positive, and negative variations (*i.e.* dry years) succeed each other more frequently than they should, if chance ruled the succession of wet and dry years.

Space Variability of Rain-fall.—Having discussed the variation of mean annual rain-fall in time, it is also necessary to consider its variation in space. We have already stated that the rain-falls of two localities, close to each other, and not separated by any marked natural feature, will probably, in any year or series of years, depart from their mean values in much the same proportion. But, it by no means follows that their mean values are likely to be the same.

As a matter of experience, in the British Isles, water-works' engineers prefer to have one rain-gauge to about every 1000 acres of gathering ground; but it must be remembered that the rain-fall of the British Isles (and more especially that of England), varies from place to place far more rapidly, and more patchily, than is the case in countries possessing topographical features on a larger scale. The usual British gathering ground being a deep and frequently a winding valley, surrounded by high hills, possesses precisely the character of surroundings and relative elevation required to accentuate the space variation in rain-fall.

Thus, in countries having large natural features, a wider spacing of rain-gauges is doubtless permissible, but, so far as I am aware, no watersheds except those of Great Britain have, in my opinion, been adequately provided with rain-gauges, the possible exceptions being certain cases in Prussia and Saxony.

The problem of the selection of rain-gauge stations over an area, so as to arrive at a distribution which will approximately represent the average rain-fall over this area, cannot be reduced to general rules. If any records already exist, either in or just outside the area considered, valuable indications can be obtained by plotting these stations on a map, and drawing "contour" lines of equal rain-fall (iso-hyetal lines) across the map. As a rule, these lines will be found to be connected with the natural features of the country, and on a large scale map they are frequently almost undistinguishable from contour lines of equal elevation above sea level. On a small scale map this relationship is not so marked, but it is frequently found to be of assistance in plotting the approximate iso-hyetals.

The iso-hyetals being plotted, it will often be found that irregularities, or indications of irregularities, are disclosed, and these will naturally suggest sites for observation stations.

In a range of hills, the rain-fall generally increases as we proceed towards the crest, but a small area of maximum rain-fall almost invariably exists, not at the exact crest, but a little below it, and to the leeward of the crest in relation to the prevailing rain-bringing wind.

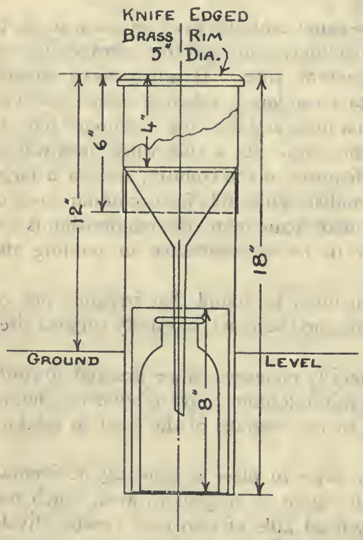
In plains, variation in rain-fall from place to place is generally accidental, due to summer thunder-storms of small extent in respect to area, which not only produce short but heavy and very local falls of rain over comparatively small areas, but tend to follow the track of the first storm of the hot season throughout each summer. This seasonal tendency should not be confused with the general habit of thunder-storms (whether covering a small area and following a sharply defined track, or covering a large and not well defined area) in an undulating country of following, year after year, some natural feature, since this will be more or less clearly disclosed in the mean summer rain-fall records.

In a hill and valley country, it will usually be found that the floors of narrow valleys have approximately the same rain-fall as the adjacent hills, the difference, if appreciable, inclining towards a decrease in fall.

Apart from this exception, and the one mentioned in the first rule, there is a general and undoubted tendency for the rain-fall to increase with the altitude, but such rules as have been proposed seem only to be applicable to limited areas.

The above statements plainly indicate that, unless special precautions are taken, the public will (as a rule) establish rain-gauges mostly in the relatively dry portion of any area, however small. Thus, the engineer should always supplement existing gauges by stations planned on a systematic basis. This usually entails the establishment of new stations in comparatively inaccessible positions, but gauges adapted to hold a month's heavy rain-fall, such as are now procurable, do not throw much labour on the observing staff, and if read regularly every month, give sufficiently reliable information for such supplementary purposes.

Having thus disposed of the result of the natural topography of the country, it becomes necessary to inquire what effect small irregularities, or artificial features, may have on the records of a rain-gauge. Here, a very general and comprehensive rule can be given. Anything likely to produce eddies in the wind near a gauge is liable to cause erroneous and deficient records, and the fewer the eddies, the more accurate the records will be.



SKETCH No. 45.—Standard English Rain-gauge.

Thus, the neighbourhood of a steeple, a tall tree, or a high embankment, is to be avoided; and shrubs, trees, etc. should be removed at least their own height from the gauge. Also, a gauge should not be set on a roof, unless the roof is so large that the eddies produced by its edges have died out before they reach the gauge.

It is as well to place gauges situated on steep hillsides in the centre of a small, level platform, of say 6 feet radius, and to surround the gauge by a wall about 2 feet high, and 3 feet distant from the gauge.

For very accurate work, a Nipher shield (which consists of a large wire gauze funnel, 3 feet in diameter, with its rim at the level of the rim of the gauge) gives very good results.

I append Dr. Mill's drawing of a standard Snowdon gauge (Sketch No. 45). It will be noticed that he specifies the rim of the collecting funnel as 1 foot above the ground; but where, owing to local conditions, this elevation is impracticable, it should be remembered that every extra foot of elevation up to 9 feet, produces, roughly, 1 per cent. decrease in the rain-fall recorded.

Accuracy of Rain-fall Records.—Excluding carelessness in booking, or measuring, which I consider to be more frequent than is believed, the principal sources of error in a Snowdon gauge are those due to snow being blown out of, or into the gauge. A valuable check may be made by measuring the depth of snow in sheltered places, and reckoning 12 inches of snow as equal to 1 inch of rain.

A special warning is necessary against Glaisher gauges. These, when out of order, are liable to collect more rain than has fallen, which is the very last thing an engineer wishes. It may also be noted that the "tube gauge," so common in India, will usually collect more than the actual fall, even when in order. There are forms of rain-gauges which, when out of order, collect less than the true fall, and are therefore not so dangerous.

Everything considered, it is believed that a good rain-fall record is liable to at least 2 per cent. of error; and it is probable that the average record errs from 4, to 6 per cent., some of which could be adjusted if the whole local conditions of the gauge were known.

WATER YEAR.—As a matter of custom, rain-fall records are usually prepared, tabulated, and published, according to calendar years. From an engineer's point of view this is somewhat awkward, since the water year, or period of time during which the total run-off is most closely related to the total rain-fall rarely, if ever, coincides with the calendar year.

It is usual to assume that the water year should begin when stream flow is at its minimum, and should end at a similar period next year.

The minima of stream flows do not succeed each other at rigid intervals of 12 months, far less 365 days; but over a long period of years, it will be apparent that a month can be selected during which the minimum, or nearly the minimum, generally occurs. If the rain-fall and run-off are tabulated by years, starting with this month, or the next succeeding one, it will be found that the connection between these quantities is more regular than when they are tabulated by calendar years.

Such a month is easily selected in any given place, as for example:

In the British Isles, and the Northern United States, it is September to October. In Northern India, it is May.

The above statement as to the beginning of the water year may at first sight appear erroneous. I am of the opinion that if the period at which the water stored up in the ground (see p. 188) is at a maximum could be definitely observed, the relations of rain-fall and run-off for the intervals between these maxima would be more constant than for any other period. The difficulty lies in the fact that these maxima are, so far as present information exists, somewhat more irregular in time than the minima of stream flows, and are less easily observed. The error introduced by the selection of a date, say a month distant from the actual moment of the maximum ground water storage, is considerably greater than that caused by a month's error in the date of minimum stream flow. Consequently, while agreeing that the date of maximum storage of ground water has theoretical advantages, I believe that the time of minima stream flow is practically more useful as a dividing line for the rain-fall and run-off years.

VARIATION OF THE RAIN-FALL OVER THE YEAR.— So far we have regarded the year (whether calendar, or water year), as our unit of time. From a practical engineering point of view, this is too long a period, as when water supplies fail it is but little consolation to discover that the total volume flowing during the year would have sufficed if it had been equally distributed. We are thus face to face with the fact that however variable the annual fall may be, that of periods of less than a year is even more so.

Many studies have been made, and in a given locality it is possible to state that a certain period of the year is usually the wettest, or the driest, as the case may be; but in each individual twelve months the variations are such that, with very few exceptions, the statements are useless for practical purposes. For an engineer's requirements therefore, the best that can be done is to split up the year into periods during which the relations between rain-fall and run-off are markedly different.

The calendar month is usually employed for purposes of convenience, but it is far too short for practical use. In the British Isles a satisfactory division is as follows :

In the winter months (roughly December to April) a very large proportion of the rain-fall appears as run-off, while in the summer months (roughly May to November) the proportion appearing as run-off is but small. In the Northern United States, where the climatic differences are more marked, the following threefold division has been used with advantage :

Storage period, (roughly December to May).
Period of vegetation growth, (roughly June to August).
Replenishing period, (roughly September to November).

In climates such as that of the Punjab, we have : The winter dry season (approximately October to May), when very little rain falls, and when that which does occur is practically without influence on the run-off. In the monsoon or rainy season (approximately June to September), almost the whole of the year's fall occurs, and the year's run-off depends entirely upon this monsoon fall.

A similar division, varying only in regard to the months, holds in nearly all tropical climates, although in some cases, (for example, Ceylon, and S.W. India) there are two wet, and two relatively dry seasons in each year.

These divisions cannot, however, be considered as more than approximate. The exact line of demarcation will be found (if the records are of sufficient length) to oscillate from year to year over a period of as much as three months, and it is a matter of common knowledge that "The weather of each year is abnormal in some respect." Thus, it may be inferred that at least every fourth or fifth year must be abnormal as regards its rain-fall.

CONNECTION BETWEEN RAIN-FALL AND RUN-OFF.—The nature of the relation between the rain-fall on a catchment area, and the discharge of the stream draining that area, is best realised by a general consideration of the manner in which the water which is produced by the rain-fall is disposed of.

When rain falls on a land surface it first wets the upper soil, and it is only after this has become to some extent saturated that water appears in a visible form on the land surface. This visible water collects in small trickles, and runnels, and flows towards the stream channels ; but as it flows, a certain portion of it soaks into the ground. Thus, from the very start, we have a two-fold division of the rain,—that absorbed by the earth, and that which proceeds direct to the stream, without being absorbed.

The fate of the absorbed water now requires consideration. A certain portion is consumed by vegetation, and a further amount is evaporated from the damp surface of the soil by the air ; the remainder slowly soaks into permeable beds underlying the surface, and is finally removed from the influence of vegetation or surface evaporation.

The ultimate fate of this last portion depends on the geological structure of the catchment area. As a rule, we may assume that it finally leaks away to the stream draining the area, although the existence of such phenomena as artesian wells is sufficient to show that exceptions occur, and that in some cases this ground, or subsoil water, never again comes to the surface, but escapes by underground passages (not necessarily larger than pores in the rock, gravel, or sand beds) to the sea, or possibly into the interior of the earth.

We therefore see that the rain is finally disposed of in one or other of four forms :

(i) Utilised by vegetation.
(ii) Evaporated from the surface of the catchment area, either from the earth, or from bodies of water included in the area.
(iii) Appears in a time, measured by days at the most, as stream flow.
(iv) Soaks into the ground, and either appears :
 (*a*) In a time that may be measured by months, or even years, as stream flow, or :
 (*b*) Seeps away through underground channels.

The run-off is plainly the sum of (iii), and (iv), and in the normal catchment area (iv) (*b*), does not exist, so that considered over a sufficiently long period the rain-fall loss is the sum of (i), and (ii), only. But, over a short space of time the rain-fall loss is influenced either positively, or negatively, by (iv) (*a*). The effect of (iv) (*a*), may be compared to that of an invisible reservoir which at certain seasons temporarily increases, and at others diminishes the stream flow.

Let us now consider each of these factors, and express the volumes of water which are thus disposed of, in inches depth over the catchment area :

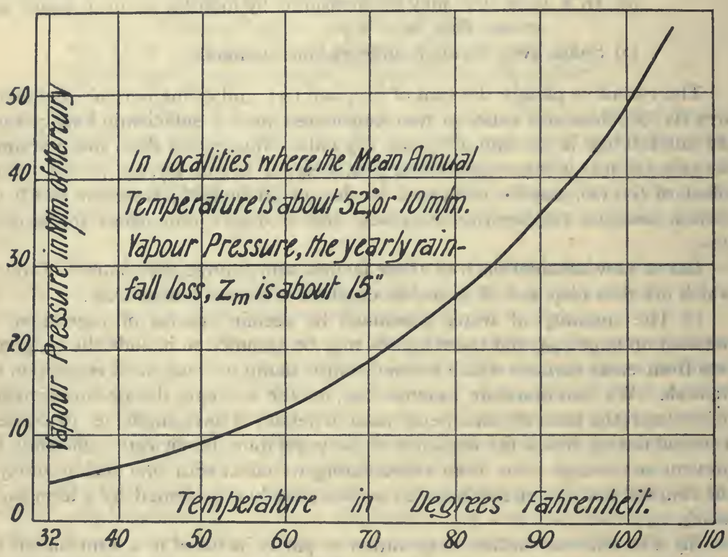
(i) The quantity of water consumed by certain species of vegetation is detailed on page 234, and these figures may be assumed to include the evaporation from earth surfaces which are sufficiently damp to cause such vegetation to flourish. We can therefore assume that, on the average, during any division of the year (the term division being used in default of anything better to express a period during which the demands of the vegetation do not vary sufficiently to prevent an average value from representing its effect with practical accuracy), the rain-fall loss under this head as approximately represented by a term such as a'_p.

(ii) The effect of surface evaporation is partly included in a term similar to the above quantity a'_p. It is evident that the greater the rain-fall, the damper the earth becomes, and the more numerous will be the puddles and other shallow bodies of water exposed to evaporation. The loss by evaporation during a given division of the year is consequently represented by a term of the form $a''_p+b''_p x_p$; where x_p represents the rain-fall during the period considered.

Thus, the first two causes combined may be regarded as producing a loss represented by $a_p+b_p x_p$. Now, both a, and b, depend on the temperature, the amount of sunshine, the manner in which the rain-fall occurs, (*i.e.* in short storms or long drizzles, etc.), the movement of the air, etc., in fact, upon all the meteorological circumstances united. On the average, it may be said that a, and b, are mainly dependent on the mean temperature during the division of the year under consideration. Bearing in mind that while growing vegetation to a certain extent stores up water in its tissues, it disposes of at least 90 per cent. of the liquid by evaporation from its leaves, it may be safely assumed that a, and b, are dependent on the temperature in a manner approximately similar to that in which the vapour pressure of water is affected by the temperature.

Now, the relation between the temperature and the vapour pressure of water, is not even approximately a linear one. An examination of the curve expressing the connection as plotted in Sketch No. 46 shows that a_p, and b_p, can only be considered as even approximately proportional to the duration of the period if the division of the year denoted by the suffix p, is so selected that the temperature of the water which wets the surface soil and is utilised by plants never differs materially from the mean temperature of the whole period during any portion of the interval denoted by p.

For the sake of brevity I propose to term the total loss produced by



SKETCH NO. 46.—Relation between the Temperature and the Vapour Pressure of Water.

vegetation and evaporation the Vegetation Loss, and to denote it by V_p. We are justified in assuming that:

$$V_p = a_p + b_p x_p$$

if the weather during this division of the year is such that the mean temperature can be regarded as in close connection with the probable total evaporation during the period. If this is not the case:

$$V_p = a_p + b_q x_q + b_r x_r + \text{etc.}$$

where the suffixes q, r, etc., refer to less lengthy divisions of the year during which the momentary temperature varies but little from the mean temperature of each sub-period.

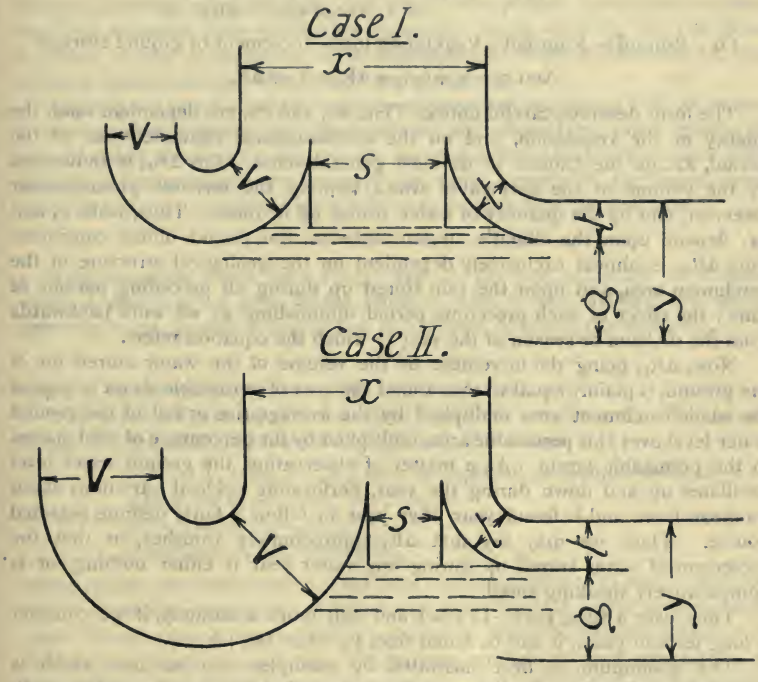
(iii) The quantity of water that flows directly to the river or stream draining the catchment area can, for conciseness, be called the topographical flow, and is denoted by t_p.

(iv) The quantity that soaks into the strata underlying the catchment area, and is there temporarily stored up in the form of ground water, can be called the Stored Rain, and is denoted by s_p.

Thus, we have :

$$x_p = a_p + b_p x_p + t_p + s_p$$

or : Rainfall = Vegetation loss + Topographic flow + Rain stored in the ground. (See Sketch No. 47.)



SKETCH No. 47.—Diagram showing Relations between Rain-fall, Ground Storage, and Run-off.

The two diagrams are drawn so as to represent the disposal of 2·5 inches of rain and the simultaneous production of a run-off of 1·4 inch.

In Case I.—V = 0·7 inch ; s = 1·4 inch ; and t = 0·4 inch.

Thus, g = 1 inch, and the water stored up in the ground is increased by 0·4 inch.

In Case II.—V = 1·25 inch ; s = 0·75 inch ; t = 0·5 inch.

Thus, g = 0·9 inch, and the water stored up in the ground is depleted by 0·15 inch.

The run-off is plainly composed of the two following portions :

(*a*) The topographic flow t_p, and,

(*b*) A contribution from the water stored up in the ground, which we shall term the ground water flow, and denote by g_p.

Thus, $y_p = t_p + g_p$, and the connection between g_p and s_p, is not very evident, for s_p, is the quantity of water supplied to the invisible reservoir formed by the permeable strata; and g_p, is the leakage from this reservoir into the stream channels.

Now, g_p is usually not equal to s_p, and all we can definitely state is that if R, represent the total quantity of water stored in the permeable strata, then:

$$\Delta R_p = s_p - g_p$$

where ΔR_p, represents the positive increment of R, during the period p, and ΔR_p may be a negative quantity.

$$\text{Thus, } y_p = t_p + g_p = x_p - (a_p + b_p x_p) - s_p + g_p$$
$$= x_p - (a_p + b_p x_p) - \Delta R_p$$

Or: Run-off = Rain-fall − Vegetation loss − Increment of ground storage.

$$\text{And } z_p = a_p + b_p x_p + \Delta R_p = V_p + \Delta R_p$$

The form deserves careful notice. The a's, and b's, are dependent upon the quality of the vegetation, and on the meteorological characteristics of the period, *i.e.* on the climate in its most general sense. But ΔR_p, is influenced by the volume of the permeable strata forming the invisible ground water reservoir, and by the quantity of water stored up in them. Thus, while a, and bx, depend upon the climatic circumstances of the period under consideration, ΔR_p, is almost exclusively dependent on the geological structure of the catchment area, and upon the rain stored up during all preceding periods of time; the effect of each preceding period diminishing as we work backwards from the division or season of the year to which the equation refers.

Now, ΔR_p, being the increment of the volume of the water stored up in the ground, is plainly equal to the ratio of the area of permeable strata to that of the whole catchment area multiplied by the average rise or fall of the ground water level over this permeable area, multiplied by the percentage of void spaces in the permeable strata. As a matter of observation, the ground water level oscillates up and down during the year, performing cyclical variations about its mean level, and is found year after year to follow a fairly definite seasonal course. Thus, we may say that ΔR, approximately vanishes, or that the increment of water stored up during any water year is either nothing, or is comparatively speaking small.

Thus, over a year, $y = x - (a + bx)$, and still more accurately, if we consider a long term of years, it will be found that, $y_m = x_m - (a_m + b_m x_m)$.

The assumption is best illustrated by examples. In an area which is underlain by granite, or other compact rocks, the maximum variation of R, when taken between the highest and lowest values of the year, rarely attains 1 inch, as is illustrated by the Nagpur catchment area (see p. 246).

In a chalk, or sandy area, the maximum variation of R, during any year may amount to as much as 6, or 7 inches; and in some cases R, has been observed to increase as much as 20 inches in a period of five years. The value of ΔR, *i.e.* the total change in R, during a year, is of course less than these values, and abnormal years apart, is not more than one-third of the maximum variation.

The matter is of importance. In England we are fairly well aware that over a period of three dry years z_m, is approximately equal to 14, or 15 inches in impermeable areas. But values as high as 20 or 21 inches have been observed during a period of three years in permeable areas, for which z_m, taken over a long period, does not greatly exceed 17 or 18 inches. We may therefore consider that in these cases a quantity of water equivalent to 9 or 12 inches is temporarily stored up; so that the value of ΔR, for each year is about 3 or 4 inches.

In certain chalk districts, a series of 6 or 7 dry years appears to produce a depletion of R, equivalent to 18 or 20 inches at least, and this is probably replenished during the following period of wet years.

It is therefore plain that, so far as eliminating the influence of ΔR_p, is concerned, accuracy is best attained by considering as lengthy periods as possible. But since a_p, and b_p, are both influenced by the temperature (and other meteorological quantities), the longer the periods considered as units, the less accurate their estimation becomes. As a_p, and b_p, are not linear functions of the temperature, it is impossible to express them over a long period even as functions of the mean temperature.

The correct selection of the unit period therefore becomes a difficult matter.

For general studies a year possesses certain advantages, since, (except the year be either abnormally dry, or wet) we may assume that the ground water storage is approximately the same at the same periods of successive years Thus, over a year, and more especially over a water year, we find that:

$$y=x-a-bx, \quad \text{or, } z=a+bx$$

This equation holds very fairly well in practice, although in climates where a well marked cold season (winter), and a well marked hot season (summer) exist, greater accuracy can be assumed by using the equation:

$$y=x-a-b_s x_s-b_w x_w$$

where x_s, is the summer, and x_w, the winter rain-fall.

The possibilities of error in the above equations are obvious. The assumption is that the sum of at least a dozen terms, of the form

$b_1x_1+b_2x_2+\text{etc.}+b_{12}x_{12}$, may be replaced by a single term bx,
where $x=x_1+x_2+\text{etc.} \ldots +x_{12}$,
or by two, $b_s x_s+b_w x_w$,
where $x_s+x_w=x_1+\text{etc.}+x_{12}$,

where the b's, are independent of the relative magnitudes of x_1 and x_2, etc., and this is obviously inaccurate. All that can really be said is that at least twelve years' careful observations would be required to obtain the constants used in the theoretically more exact form. Preliminary studies of such magnitude are impossible. As will be shown later (see p. 232), when the catchment area contains a reservoir of a size adequate to equalise the run-off during the drier years, the assumptions are sufficiently accurate for practical purposes.

The assumption regarding ΔR, cannot be regarded as more than a first approximation. The ground water reservoir is apparently (in an average case) capable of temporarily storing up a quantity of water equivalent to as much as 5, or 6 inches (reckoned over the whole catchment area), and afterwards delivering it to the stream. Even if the water year is taken as the unit period, it appears that after a wet year at least one-third of this volume may be retained in the ground, and passed on to the succeeding year. In a dry year, an extra depletion of about one-quarter may occur, which has to be made up in the following year. Thus, from this cause alone, differences of 1, or even 2 inches may arise in the yearly run-off. These, in the case of a catchment area of small mean run-off, but of large invisible storage, may amount to as much as 6, or 8 per cent. of the annual run-off. On the average, however, the effect is not so great, but an error of 5, or 10 per cent. is quite possible, and may be expected after abnormally dry, or wet years. This error may be increased to 10, or 15 per cent. if there are two abnormally dry or wet years in succession.

For studies of individual catchment areas, which do not contain reservoirs for water storage, we would naturally consider the day as a unit. This is impossible, and the calendar month is the smallest period for which any practical rules can be given. It will be plain that a calendar month is an artificial division, and that it does not accurately define the climate of the period. Thus, better results may be expected if a reservoir exists in the catchment area which is large enough to permit a consideration of seasonal periods only (such as are mentioned on p. 184).

On page 218 I give Vermeule's investigation. This method requires a very large amount of preliminary study before it can be practically applied. The following method obviously departs considerably from the truth, but it utilises observations which can generally be completed before the final designs are made. I therefore put it forward, not as a complete solution, but as the best that can be obtained under ordinary practical circumstances.

We assume the following quantities, relating to the catchment area considered, as known:

(i) The rain-fall loss for one year at least, and that this has been corrected for the nett ground water storage, or depletion during the year, by observations on the ground water level, taken at as many points as are possible. This we call z. Put $z_p = x_p - y_p$, for each division (month, or season) of the year.

(ii) A series of observations on evaporation from a free water surface such as are made at most large meteorological stations, for as many years as practicable, at a locality under as nearly as possible the same climatic conditions as those of the catchment area. Let the sum of these for the year under consideration be denoted by e.

Take the ratio: $k = \frac{z}{e}$. Then, I assume that the term $V_p = a_p + b_p x_p$, for any period p, can be represented by ke_p, where e_p, is the total free water surface evaporation for that period. Thus, for every division of the year for which observations are taken, we can find:

$$f_p = ke_p - z_p$$

where plainly f_p, represents a nett flow of water from the ground, if ke_p, be greater than z_p, and a nett storage of water in the ground if ke_p, be less than z_p, and if my assumption be true:

$$f_p = -\Delta R_p = g_p - s_p$$

Thus, f_p, depends on the geological structure of the catchment area, rather than on the climate.

For the same period of any other year, we have X_p, the observed rain-fall, and E_p, the observed free water surface evaporation, and we assume that Y_p, the run-off, is given by:

$$Y_p = X_p - kE_p + f_p$$

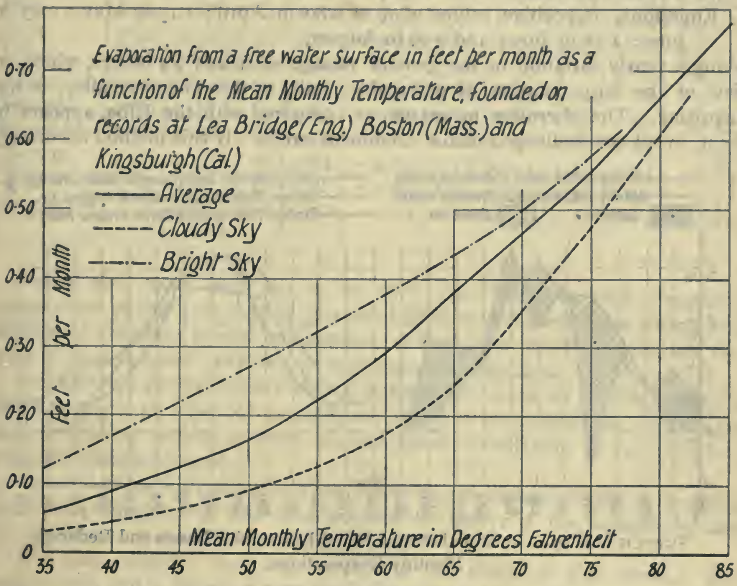
where f_p, is the value obtained for the same period of the year during which the run-off was observed.

The method is subject to one obvious error. While A_p, and B_p, the values of a_p, and b_p, for the fractional period of the year now considered are probably proportional to E_p, we may be fairly certain that if X_p, differs to a marked extent from x_p,

$A_p + B_p X_p$, will differ somewhat from
kE_p, which is probably fairly close to $A_p + B_p x_p$

This is the more likely because, while rain-fall does not in itself have much apparent influence upon evaporation, heavy falls are usually attended by cloudy weather, and the diminution of sunshine thus produced will decrease evaporation, while the term $A_p + B_p X_p$ is probably increased. (See Sketch No. 48.)

The method is useful for the preliminary elucidation of the various factors



SKETCH NO. 48.—Relation between the Mean Monthly Temperature, and the Evaporation from a Free Water Surface.

concerned, and it may be said that it allows (with a very fair degree of accuracy) for :

(i) The vegetation loss ;
(ii) The influence of variations in the mean temperature of the divisions of successive years ;
(iii) Ground water storage ;
(iv) The duration of the rain-fall ;

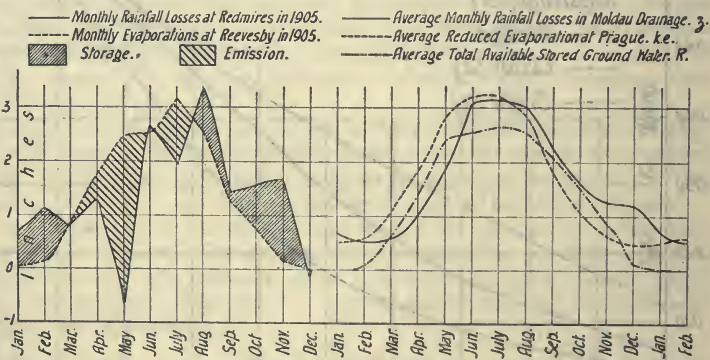
but is liable to error if the absolute magnitude of the rain-fall varies greatly from that of the year of observation. Nevertheless, it tends to over-estimate the loss when the rain-fall is smaller than in the year of the original observations ; and to under-estimate the loss when it is greater. Consequently, the results thus obtained usually possess a certain margin of safety, which is advantageous for practical purposes.

The right half of Sketch No. 49 shows this method in a graphical form, as applied by Penck (*Untersuchungen über Niederschlag und Abfluss*) to the catchment area of the Moldau. The emission of water in the spring of the year to reinforce the stream flow, and its storage in the late summer and autumn, are characteristic of a large, and fairly flat catchment area, with a tolerably severe winter. In the case of a more Insular climate, such as that of the Thames, the figures obtained for the average of 9 years are :

Storages, or negative values of f, of 0·34 in September ; 1·74 in October ; 1·88 in November ; 1·10 in December ; 1·04 in January ; 0·17 in February ; 0·12 in March, and :

Emissions, or positive values of f, of 0·80 in April ; 1·15 in May ; 2·13 in June ; 1·41 in July ; and 0·90 in August,

giving a yearly variation in the ground storage of nearly 6·4 inches, which, in view of the large amount of permeable chalk in the Thames valley, is not surprising. The alteration in season, as compared with the Elbe, appears to occur in all markedly permeable catchment areas. If this method accurately



SKETCH No. 49.—Relation between Monthly Rain-fall Losses and Reduced Monthly Evaporations.

represents the facts, some small and steep catchment areas have two storage and two emission periods during a year. (See left-hand side of Sketch No. 49.)

The following table shows what I believe most closely represents the average values of ke_p, given as percentages of an observed z, for English catchment areas, and probably also for any British catchment area.

January 1 ; February 2 ; March 5 ; April 10 ; May 14 ; June 17 ; July 19 ; August 15 ; September 9 ; October 5 ; November 2 ; December 1.

As an example, let the observed values for the whole year be :

$z = 18$ inches, and for the month of May $z_5 = 1{\cdot}73$ inches, and for October $z_{10} = 2{\cdot}41$ inches.

Thus, during the month of May we have :

$$f_5 = -\Delta R_5 = 18 \times 0{\cdot}14 - 1{\cdot}73 = 0{\cdot}79 \text{ inch}$$

or the ground reservoir contributes 0·79 inch to the observed run-off. For October on the other hand :

$$f_{10} = -\Delta R_{10} = 18 \times 0{\cdot}05 - 2{\cdot}41 = -1{\cdot}51 \text{ inches}$$

or 1·51 inches

are stored up in the ground, and the run-off during the month of October is diminished by that amount.

	Elbe.					Moldau.				
	x	y	z	ke_p	f	x	y	z	ke_p	f
February.	1·24	0·68	0·56	0·60	+0·04	1·16	0·64	0·52	0·60	+0·08
March .	1·76	1·32	0·44	1·12	+0·68	1·68	1·12	0·56	1·12	+0·56
April . .	1·88	1·00	0·88	1·84	+0·96	1·84	0·80	1·04	1·84	+0·80
May . .	2·52	0·68	1·84	2·76	+0·92	2·48	0·64	1·84	2·80	+0·96
June . .	3·48	0·52	2·96	3·16	+0·20	3·60	0·52	3·08	3·20	+0·12
July . .	3·60	0·40	3·20	3·20	0·0	3·48	0·36	3·12	3·24	+0·12
August .	3·36	0·44	2·92	2·84	−0·08	3·44	0·44	3·00	2·88	−0·12
Septem. .	2·80	0·48	2·32	1·80	−0·52	2·84	0·60	2·24	1·80	−0·44
October .	2·16	0·48	1·68	1·04	−0·64	2·08	0·48	1·60	1·04	−0·56
Novem. .	1·76	0·48	1·28	0·64	−0·64	1·72	0·44	1·28	0·64	−0·64
Decem. .	1·80	0·64	1·16	0·48	−0·68	1·72	0·52	1·20	0·48	−0·72
January .	1·32	0·56	0·76	0·52	−0·24	1·20	0·52	0·68	0·52	−0·16

Penck also states that the z, for each year, depends upon the mean temperature of the year, and is increased by 0·70 inch per degree Fahrenheit increase of the mean annual temperature (45·8° F.); so that regarded from this point of view, z, and E, to a certain extent increase together.

Sketch No. 49 also shows that f, varies very much as the flow from a reservoir might be expected to do, being small at first, when the streams are high, increasing rapidly as they fall, and again diminishing as the stored-up water becomes exhausted.

The real importance of the method, however, lies in the fact that, to a certain extent, it permits us to take into account the effect of the topography and geology of the catchment area on the run-off.

We may consider that ke_p, represents the effect of the rain-fall and temperature, and is therefore very much the same for all catchment areas in the same country; f, however, depends on the geology and topography of the catchment area, and is peculiar to each area. We may expect to find large values of f, in flat and permeable districts, while in steep, rocky regions they will be small. Regarded from this point of view, I believe that the method is valuable. If we have obtained the ke_p, terms for a series of years, by actual observation, it is possible to apply them with very fair confidence to an adjacent catchment area. If only one year's records of the second area are known, its monthly f, may be determined, and the run-off by months for other years can then be considered as capable of very fairly accurate estimation.

Observations supplementing the usual Stream Gaugings.—A careful consideration of the principles detailed above will suggest that a great deal of useful information, which is not at present usually obtained, could be secured by special and not abnormally costly observations.

Thus, a very fair idea of the magnitude of the term ΔR_p, could be obtained by systematic observations of the subsoil water level, combined with a survey of the area of the permeable strata existing in the catchment area, and there is

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little doubt that these observations alone would permit a very fair idea of the volume of the equalising reservoir required in the driest year (see p. 236). The matter is of extreme practical importance ; at present, areas largely underlain by permeable beds are generally regarded as unfavourable for development for water supply purposes by storage reservoirs This idea may be correct, as the surface topography of such areas rarely affords advantageous sites for storage reservoirs, but there is but little doubt that if an impermeable reservoir site can be secured, studies of the size of the invisible ground reservoir might enable an engineer to feel satisfied with a smaller visible reservoir than is now considered advisable. Certain American studies (*Trans. Am. Soc. of C.E.*, vol. 27, p. 286) have shown that it is quite possible for the invisible storage to contribute about one-fifth of the supply drawn from the visible reservoir in dry years.

Similarly, the chemical composition of the ground water, as compared with the chemical composition of the river water when the ground flow is known (from subsoil water level observations) to be a minimum, should enable a very fair idea of the value of the ratio $\frac{y_p}{g_p}$ to be obtained. In my own work in the Punjab, I found that alkalinity determinations alone, permitted a very fair idea of the relative proportions of the ground water, and of the water coming down from the hills, to be obtained. Such determinations are useful, if it is desired to predict the flow that is likely to occur in a river-bed at some distance below the headworks of a canal which diverts the whole of the visible flow.

Examples of "seepage water irrigation" with water thus procured are frequent in America, and a very large scale example is likely to occur in a few years in the Punjab. At present, in default of observations of the type suggested, these matters are settled by vague opinions, or by a comparison with similar cases, or, still worse, by legal discussions.

Droughts.—A very important principle must now be considered. We find by observation that z, the rain-fall loss for a year, is very fairly represented by the equation :

$$z = a + bx$$

and better still by :

$$z = a + b_h x_h + b_c x_c$$

where the suffix h, refers to the hot, or summer season, and the suffix c, to the cold, or winter season. We are therefore led to assume (and such observations as do exist justify the assumption) that :

$$V_p = a_p + b_p x_p$$

where the a's and b's, are constants, or at any rate, are approximately so.

Now, as a matter of observation, this vegetation loss has, as a general rule, "priority of right" over the topographic flow, and ground storage disposal of the rain-fall. Of course, exceptions exist, as may be observed wherever a torrential rain-fall occurs, but the statement is as nearly correct as any other that can be made regarding this complex subject.

Now, let us assume that the rain-fall x_p, happens to be unusually small. It is plain that during a dry season we may find that :

$$x_p \text{ is less than } a_p + bx_p$$

Such conditions do occur in practice, and are indicated by the vegetation "wilting," or suffering from drought. The conditions are complex, and obviously depend upon the length of the period which is denoted by the suffix p. They also depend upon the character of the vegetation; but it is plain that periods occur during which the rain-fall is insufficient to provide for the requirements of the vegetation, and for the evaporation which would occur from the surface of the soil if it were wetted sufficiently for these requirements.

We thus arrive at a principle which is extremely important when climates are considered, and which may be stated in general terms as follows.

If the rain-fall during any division of the year is insufficient to wet the soil, the vegetation or evaporation loss may be far less than that which is indicated by the other climatic conditions prevailing during that period.

The case is best illustrated by taking the extreme example of a desert. The rain-fall is normally considerably less than is required to compensate for evaporation, and may be only 2 or 3 inches per annum, while the requirements of the vegetation and the evaporation, as found by observations on irrigated areas existing in the same climate, may be 20 or 30 inches per annum. Nevertheless, when a rainstorm occurs, it produces some run-off, and water can be collected in reservoirs, or can be drawn from the subsoil by wells.

We are consequently led to believe that the relation between rain-fall and run-off in arid climates differs totally from, and is in some respects a simpler matter than, that which exists in Temperate climates such as those of Western Europe and the Eastern United States.

CLIMATE IN RELATION TO RUN-OFF.—The classification of climates, regarded from this point of view, is fairly obvious. If long periods of time alone are considered, the major portion (if not the whole) of the rain-fall loss is caused by vegetation and evaporation. Now, in Insular climates, and more especially in Temperate Insular climates, the amount of rain that falls in the year is sufficiently equably distributed to ensure that during all divisions of the year (abnormally dry years excepted), the rain-fall is always adequate to supply the requirements of the normal vegetation, and to keep the surface soil sufficiently damp to permit of some evaporation. For example, in symbols—x_p is always greater than a_p+bx_p, where the minimum duration of the division of the year expressed by p may be taken as between one and two months, but, in any actual case, depends to a large extent on the thickness of the surface soil. In Continental, and more especially in Tropical Continental climates, this relation does not hold for all seasons, except in abnormal years, and the natural vegetation of these localities has adapted itself to droughts, either, as in the extreme example of the desert cacti, storing up water in its tissues, or, as in the less marked cases of wheat and cotton, being provided with long tap roots permitting it to draw on the subsoil water.

It will therefore be plain that the general investigation given above is only directly applicable (for all seasons of the year) to climates in which the before-mentioned condition (which may be termed a vegetation and evaporation drought), does not normally occur. When such a drought takes place, the terms a and bx may be considerably smaller than the values indicated by the other conditions obtaining during the drought, and a certain depletion of ground water storage which does not appear as run-off, but is consumed by vegetation, may also occur.

It consequently appears logical to divide climates into three classes :

(i) The standard class in which a drought, as above defined, does not occur in normal years at any rate.

(ii) A class in which a drought occurs once a year at least, except in abnormal years.

(iii) A class in which a drought is the normal condition of affairs.

We may therefore have a climate such that the catchment area is normally never thoroughly dry, as (in this sense) is the case with all catchment areas in Temperate climates, except after very intense and long-continued droughts, such as take place at intervals of 30 or 40 years at least.

The expression "thoroughly dry," is perhaps indefinite, but by that term I wish to indicate that the soil of the catchment area is so depleted of water in its upper layers as to reduce evaporation from the ground to a minimum, (I believe, as a matter of fact, that some evaporation, if only of night dew, always happens), which is quite independent of the last rain-fall that occurred.

The second type of climate is the one found in most tropical countries which have a dry season. In such cases, once a year at least, the catchment area becomes thoroughly dry.

The third type of climate, which occurs in its most representative form in desert regions, and in the arid zones of America, South Africa, and India, is one in which the catchment areas are normally thoroughly dry.

Now, when rain falls on a perfectly dry catchment area, it is a matter of observation that the first portion is absorbed by the soil, as by a sponge, and neither runs off, nor soaks deeply into the ground, (in such climates it is very often customary amongst natives to speak of so many "inches" of rain, when the word "inch" does not mean rain-fall, but the depth to which the sides of a newly dug hole are found to be moistened after rain).

Until the soil is soaked to an appreciable depth no run-off can occur, and if the rain is insufficient to effect this, the nett result will be a temporary wetting of the ground, which is sooner or later sucked up by the thirsty air.

Thus, in a dry catchment area, the loss before any run-off can occur may be represented by a constant, which indicates the quantity of water consumed in saturating the surface layers of the soil. The loss that occurs thereafter is proportionally far less, and depends more on the time which the rain takes to fall, and on the distance which the water running off travels before it is collected into a river channel or reservoir, than on the absolute magnitude of the fall.

When rain falls on a damp catchment area the initial loss is, relatively speaking, not large, and the influence both of time and distance factors is less.

Thus, in climates of the first type, it is possible to consider the rain-fall of a month, or even of a season, as a unit ; and, except towards the end of the summer season, it is rarely requisite to consider the rain-falls of individual days.

In climates of the second type, however, we must certainly consider the rain-falls during the wet and dry seasons as independent, and, abnormal torrential downpours apart, can usually neglect the latter entirely. In the wet season we must consider not only the total rain-fall, but also the rain-fall of individual weeks, or, at most, months. As will be seen later, we are led to regard the rain-falls of the successive months, reckoning from the beginning of the wet season, as contributing in increasing proportions to the run-off. Consequently, a fall which had it occurred in the first month would have been regarded as unimportant, might towards the end of the season produce the major portion of the run-off of the season.

In climates of the third type, we regard each rainstorm as a separate entity, and the practical rules will lead us to consider the run-off as produced only by heavy rainstorms, although it must not consequently be assumed that the run-off invariably occurs immediately after a heavy rainstorm.

The general conditions are best illustrated by a detailed study of catchment areas in climates belonging to the first type, and I shall therefore discuss the effect of the geological and topographic structure under that head.

It may be stated that the geological and topographic conditions are equally effective in climates of the second and third types, and that only the lack of detailed information prevents a discussion of their effect under these conditions.

CLIMATES OF THE FIRST TYPE.—The rain-fall loss over a year may be represented with a very fair degree of accuracy by the equation :

$$z = a + bx$$

or, better still, by $z = a + b_s x_s + b_w x_w$, where x_s, is the summer (hot season) rainfall, and x_w, is the winter (cold season) rain-fall.

Now, theoretically, we should of course consider all years, wet or dry, in determining the constants a and bx. But, from an engineer's point of view, it is better to take only the drier years. I have therefore neglected all years in which the annual rain-fall is much above 1·20 times the mean annual rain-fall. Subject to this condition, for catchment areas in Great Britain, Germany, and (less accurately) the Eastern United States, I find that $b = 0{\cdot}16$.

Considering the possibilities of error in the latter country, I am inclined to believe that the above statements are fairly accurate for all Temperate and Non-Continental climates, provided that the mean annual rain-fall does not greatly exceed 60 inches.

As an example, I give the values of z, as observed by Ingham (*Rainfall and Evaporation in Devonshire*) at Torquay, for the years 1878–1900, and those calculated from $z = 9{\cdot}7 + 0{\cdot}16x$.

x observed.	z observed.	z calculated.	Difference.	
			+	−
27·5	13·1	14·0	0·9	...
31·5	14·3	14·5	1·1	...
32·3	8·7	14·8	6·1	...
34·3	18·8	15·1	...	3·7
35·2	18·0	15·3	...	2·7
35·7	14·4	15·3	0·9	...
35·7	18·4	15·3	...	3·1
36·3	10·5	15·4	4·9	...
36·3	16·0	15·4	...	0·6
38·0	12·8	15·7	2·9	...
38·3	14·8	15·8	1·0	...
39·9	5·9	16·1	10·2	...
42·8	17·0	16·5	...	0·5

[*Table continued*

Table continued]

x observed.	z observed.	z calculated.	Difference. +	Difference. −
43·0	13·3	16·6	3·3	...
43·8	17·0	16·7	...	0·3
44·7	16·8	16·8	0·0	0·0
45·3	16·7	16·9	0·2	...
45·5	19·1	17·0	...	2·1
45·8	21·6	17·0	...	4·6
50·4	21·1	17·6	...	3·5
51·1	25·2	17·9	...	7·3
52·0	17·0	18·1	1·1	...
52·2	19·2	18·1	...	1·1

The agreement for individual years is only rough, the average difference being 2·8 inches, or about 17·5 per cent. of the mean value of z, but it will be seen that (except in the case of the wetter years) the average loss in any three years is very close to that calculated. The divergence in the wetter years, and in the two with abnormally small losses, is explained by the fact that the expression $z = a+bx$ is only a contracted form of:

$$z = a_s + b_s x_s + a_w + b_w x_w$$

and since b_s, is far greater than b_w, a year with abnormally heavy summer rain-fall may be expected to have a greater rain-fall loss than one of the same rain-fall with abnormally heavy winter rain-fall.

The following table shows the agreement between:

$$z = 7 + 0{\cdot}4x_s + 0{\cdot}1x_w$$

and the observed values for another British catchment area.

Winter Rain-fall.	Summer Rain-fall.	Observed Loss.	Calculated Loss.	Difference.	
x_w	x_s	z_o	z	+	−
15·8	20·9	15·9	15·4	...	0·5
26·1	18·2	18·3	19·2	0·9	...
20·3	19·7	16·4	17·1	0·7	...
15·8	18·4	16·2	15·1	...	1·1
10·6	18·9	13·8	13·1	...	0·7
16·9	19·5	15·3	15·7	0·4	...
22·5	12·3	15·4	17·2	1·8	...
20·1	21·4	17·7	17·1	...	0·6
21·4	13·6	19·1	17·0	...	2·1
22·3	22·2	15·1	18·1	3·0	...
18·7	21·5	16·6	16·6	0·0	0·0
24·8	21·7	18·4	18·1	...	0·3
19·3	24·2	20·1	17·1	...	3·0
17·6	17·2	15·2	15·7	0·5	...
26·1	19·4	18·3	19·3	1·0	...

The agreement is somewhat closer, the average difference being 1·1 inch, or, say 7 per cent. of the mean rain-fall loss. But it must be remembered that there are now 3 constants at our disposal, as against 2 in the first case; so that a somewhat closer agreement might be expected in any event. It will, however, be observed that the wet years are no longer abnormally divergent, so that in all probability the three term relation is capable of including wet years as well as the drier ones.

A somewhat important distinction must now be drawn. It has been stated that:

$$z = a + 0{\cdot}16x$$

for any individual year, and for any catchment area. It might therefore be supposed that if different catchment areas are considered the mean rain-fall loss over a long term of years z_m, can be expressed in terms of the mean rain-fall over the same period x_m, as:

$$z_m = a + 0{\cdot}16x_m$$

For some reason which is at present unknown, but which is probably connected with the fact that a heavy rain-fall shapes and cuts the topography of a catchment area into steep slopes, and thus favours a speedy arrival at the stream channels, where it is less exposed to evaporation, there appears to be very little doubt that the true relation for the countries enumerated above (so far as rain-fall determines the rain-fall loss), is:

$$z_m = a_1 + 0{\cdot}05x_m \text{ to } 0{\cdot}12x_m$$

It is therefore plain that the value of a, is approximately:

$$a = a_1 + 0{\cdot}11x_m \text{ to } 0{\cdot}04x_m$$

A discussion of the factors affecting a, is consequently necessary.

This is best attained by a consideration of the mean annual run-off of catchment areas generally.

Run-off of Catchment Areas.—The factors having the most influence on the mean yearly run-off of a catchment area, taken in their usual order of importance, are:

(i) The average yearly rain-fall.
(ii) The distribution of rain-fall by seasons.
(iii) The character of the strata beneath the area, whether permeable, or impermeable; and this influences:
The general slope of the catchment area.
(iv) The marshes, lakes, or other bodies of water existing on the catchment area.

The last two have less effect upon the absolute quantity of the run-off than on its variability from month to month during the year.

For catchment areas in which these four factors are approximately similar, the absolute magnitude of the area has a decided influence upon the variability of the run-off, the larger areas having (as a general rule) the less intense floods, and the more abundant dry weather flows. As a matter of experience, while the dry weather flow of a mountain valley, of say 10 square miles, often falls to nothing, the volume of a river draining several hundred square miles, is rarely, if ever, below 0·2 of a cusec per square mile, in Temperate climates. This is, of course, explained by the greater likelihood of an extensive area (especially if flat) including permeable strata to such an extent as to provide a large invisible reservoir.

(i) *Effect of the Absolute Magnitude of the Mean Annual Rain-fall.*—The rain-fall loss can be expressed in the following form :

$$z_m = a_1 + b_1 x_m$$

b_1, is not far from 0·04 to 0·11. Thus, since for the same country z_m, increases less rapidly than x_m, the heavier the rain-fall, the larger the run-off ; and *ceteris paribus*, the fraction of the rain-fall collected each year increases far more rapidly than the absolute value of the rain-fall.

A very important aspect of rain-fall in its relation to run-off is the extreme difficulty of ascertaining its correct value. The question of the location of gauges has already been dealt with, and it is there shown that the most usual mistake is to underestimate the rain-fall. This in itself is not very material, since engineers are principally concerned with the run-off. When, however, the relation between rain-fall and run-off is studied, and the determination of the values of a, and b, in the equation

$$y = x - a - bx$$

is attempted, a correct knowledge of the absolute value of x, is of the greatest importance. It is well known that older engineers invariably (and the practice is as yet by no means extinct), considered the relation as :

$$y = px$$

where p, was a more or less constant quantity.

Now, both rules are avowedly approximate, but at first sight it appears obvious that it should be possible to determine whether a percentage, less a constant quantity, or a fixed percentage of the actual rain-fall, best represents the run-off in any given case, by forming the values of the rain-fall loss and the run-off percentage as given by :

$$\text{Rain-fall loss} = \text{Rain-fall} - \text{Run-off.}$$

and, $$\text{Run-off percentage} = 100 \times \frac{\text{Run-off}}{\text{Rain-fall}}$$

taking the mean values for the available series of years, and investigating their probable errors, by the method of least squares.

A little consideration will show that this is only the case if the "rain-fall" is the actual mean rain-fall on the catchment area. For, assume that, owing either to paucity of rain-gauges, or to a bad selection of their sites, the recorded rain-fall is not the actual rain-fall, but some proportion of this, say

$$\text{Recorded rain-fall} = (1 - c) \text{ true rain-fall.}$$

It should be remembered that this error is likely, since, as already stated, the relation between the annual falls in adjacent localities is one of approximate proportionality, and not a constant, or approximately constant, difference, so that the relation :

Recorded rain-fall = True rain-fall − a constant, is unlikely to occur.

Then, if : . . . . y = the run-off.
x = the true rain-fall.
x_1 = the observed, or apparent rain-fall

we have :

The true rain-fall loss, $z = x - y$

The apparent rain-fall loss $z_1 = x_1 - y = x(1-c) - y = z - cx.$

While the percentages are: $p = \frac{100y}{x}$

$$p_1 = \frac{100y}{x_1} = \frac{100y}{x(1-c)} = \frac{p}{(1-c)}$$

Thus, while z_1, is more variable than z, owing to the inclusion of the variable term cx, p_1 is just as variable as p, and no more so.

Thus, unless it is known that the observed rain-fall represents the actual mean rain-fall over the catchment area as accurately as the observed run-off represents the actual run-off, no valid argument regarding the relation between rain-fall and run-off can be based upon calculations of the mean square errors of p and z.

For this reason, the fact that the percentage method is still largely used by engineers in India and America cannot be considered as a weighty argument in favour of its general adoption. In these countries rain-fall statistics are less reliable than is the case in Europe, the actual observations being frequently less carefully taken; and in nearly every case the duration of the available records is far shorter, and the distribution of the observing stations less satisfactory, both in number and position.

Consequently, the employment of the percentage method merely indicates that the engineers have used unsatisfactory material to the best advantage, and does not in any way show the best method of utilising more accurate records.

As an actual example, take the figures for the Melbourne (Victoria) catchment area, with which Mr. Ritchie, the engineer of the water supply for that city, has favoured me. The portion of his letter relating to the subject under discussion is as follows:

"The catchment of the Wallaby and Silver Creeks, and the Eastern branch of the Plenty River, are composed of mountainous country, ranging from a minimum elevation of about 700 feet above sea level, to a maximum of about 2700 feet. . . .

"There is one rain-gauge at 700 feet elevation, and another at about 1700 feet elevation. Snow falls more or less in winter at the 1700 feet gauge, but does not lie for many days when the falls occur. I append for your information the annual rain-fall, and percentage run-off for stations (*i.e.* the 700 and 1700) averaged, and also on the basis of the 1700 feet station only. I am inclined to think that the record on the 1700 feet station basis would be the most accurate, as the larger portion of the catchment is at that level and over. Of course, to get a really accurate result, you require a number of rain-fall stations in selected sites, within the catchment, but I have no such records. The total catchment areas aggregate 23,000 areas."

If we examine the first series, *i.e.* those in which the average of the two gauges are taken, the method of mean squares gives as follows:

Rain-fall loss = 26·73 inches ± 4·30 inches = 26·73 (1 ± 0·16) inches.
Run-off percentage = 30·77 ± 3·70 = 30·77 (1 ± 0·12).

That is to say, it appears that the assumption of a constant percentage represents the facts better than the assumption of a constant rain-fall loss.

Taking, however, what Mr. Ritchie believes to represent the rain-fall more accurately; that is to say, the 1700 feet gauge record only, we find that:

Rain-fall loss = 34·60 inches ± 5·45 inches = 34·60 (1 ± 0·129) inches.
Run-off percentage = 25·30 ± 3·05 = 25·30 (1 ± 0·119).

The almost complete agreement of the percentages of probable error (11·9 and 12) of the run-off percentage should be noted, since it forms a confirmation of the theory.

The rain-fall loss for the more accurate record is almost as steady as the percentage figure.

I am therefore inclined to go somewhat further than Mr. Ritchie, and to state that I believe that even the 1700 feet record does not fully represent the rain-fall on the catchment area.

My personal knowledge of the catchment area is confined to a 5 days' camp in it, at a time when my experience of these matters was limited. I am not therefore prepared to make a more definite statement for this area, but if these figures related to Great Britain I should feel justified in assuming that the actual mean rain-fall was:

Neither 38·71 (mean of both gauges),
Nor 46·59 (mean of the 1700 feet-gauge),

but more like 52 inches, with a mean annual loss of approximately 40 inches.

I trust, at any rate, that I have made it plain that the fair appearance of the percentage figures is misleading; although, of course, so long as these two gauges alone exist, Mr. Ritchie is perfectly justified in using the percentage method.

It also appears that another gauge is badly required to represent the area between 2000 and 2700 feet elevation.

Treating the matter generally, and assuming that the relation:

$$z = a + bx$$

holds where x, is the true rain-fall, when the recorded rain-fall is $x(1-c)$, in place of the true value z, of the rain-fall loss, an apparent value z_1, is obtained where:

$$z_1 = x - cx - y = x - y - cx = z - cx = a + (b-c)x$$

As definite examples, let us suppose that the correct relation is

$$z = 12 + 0{\cdot}16x,$$

but that the rain-fall is incorrectly observed, and

(i) Is underestimated by 16 per cent. We have,

$$z_1 = 12 + (0{\cdot}16 - 0{\cdot}16)x = 12 \text{ inches.}$$

Thus, owing to underestimation, the value of the rain-fall loss has become apparently constant.

(ii) So large an underestimation may seem improbable. Therefore, let us assume an underestimate of 6 per cent. only. The apparent value of z, is now:

$$z_1 = 12 + 0{\cdot}10x$$

and if the mean rain-fall is 30 inches, in a long series of years the apparent mean loss is 15 inches, while its correct value is 16·8 inches.

(iii) An overestimate of the rain-fall is usually improbable, unless the gauge-stations are very abnormally distributed. But assume an overestimate of 10 per cent., z, now becomes:

$$z_1 = 12 + 0{\cdot}26x$$

and the mean observed rain-fall loss is 19·8 inches, in place of the true value 16·8 inches.

Hence, the following rules can be deduced:

If the rain-fall is underestimated, the calculated value of b, is less than its correct value; and if greatly underestimated, b, may appear to become negative. Consequently, in such cases as the Montaubry Reservoir, or (less markedly so) the Longendale Reservoir, or the Durance River, it would appear probable that the recorded rain-falls are considerably less than the true precipitation, and therefore that the rain-fall losses are underestimated.

The term "precipitation" is employed to indicate the possibility that in some of these cases the water falling on the catchment area may not entirely arrive in the form of rain as collected in a rain-gauge (see p. 207).

Similarly, (although with less certainty), it appears permissible to conclude that in cases where b_r greatly exceeds the usual value 0·16, the recorded rain-fall and the rain-fall losses are overestimated.

These rules are not of much value when applied to the study of existing observations of rain-fall and run-off, although they may lead to good hints for the location of new rain-gauges, and possibly, when some years' of records have been accumulated, to certain corrections in the early figures.

They are, however, very useful when it is desired to apply the records of existing reservoirs to neighbouring catchment areas for which no run-off data exist, but for which good rain-fall records are available.

The rules are obvious. A record of z_r which shows that z_r diminishes as the rain-fall increases, should be regarded as underestimating the rain-fall loss; while one in which z_r increases very rapidly with the rain-fall, may be considered as overestimating the rain-fall loss.

(ii) *Seasonal Distribution of Rain-fall.*—The effect of the seasonal distribution of rain-fall has already been indicated. It is obvious that rain falling during the colder months of the year, when evaporation is at a minimum, and vegetation is inert, will have a far better chance of reaching a stream, or permeable stratum, and appearing sooner or later in the form of run-off, than an equal quantity of rain falling in hotter months, when evaporation and the requirements of vegetation are more intense.

This difference may be regarded as explaining many of the divergences between the observed and calculated rain-fall losses.

Referring to Ingham's table (see p. 197), the year specified by $x=36·3$, $z=10·5$, was one in which the major portion of the rain fell in the winter, while the seasonal distribution in the year given by $x=36·3$, $z=16$, was normal; but the year $x=39·9$, $z=5·9$ cannot be thus explained. So also, taking the four years of heaviest rain-fall:

$x=50·4$, $z=21·1$, was a year of fairly heavy summer rains;
$x=51·1$, $z=25·2$, was a year of very heavy summer rains;
$x=52·0$, $z=17·0$, and $x=52·2$, $z=19·2$, were years in which the distribution between summer and winter rains was more or less normal.

Such abnormal results occur in every table of this character, and it will generally be found that some years of low rain-fall have an unusually low rain-fall loss (compared with the calculated values) owing to the year really having a dry summer, and a winter of normal rain-fall. *Vice versa*, in very wet years, the rain-fall losses are usually higher than the calculated figures, owing to the fact that a year can hardly be pronouncedly wet without having an abnormal amount of summer rain-fall.

The worst possible case, from the point of view of run-off, is that of two successive years of exceptionally small winter rain-fall, when, even though the intervening summer is wet, a very small run-off must be expected.

The Hallington (Northumberland) reservoir, given on page 239, is a very striking example, and cannot be considered as the worst possible case, since the preceding year was exceedingly rainy.

(iii) *Geological Structure.*—The effect of the geological construction of the catchment area is threefold :

(*a*) The geological construction is the cause underlying and creating the topography, and therefore influences the run-off from this point of view.

(*b*) The strata, if permeable, absorb the rain-fall, and surrender it later, acting as concealed reservoirs. From this point of view, the geology influences the absolute value of the run-off to a certain degree, but its most important effect lies in the equalisation of the run-off over the whole year, as discussed on pages 188 and 220.

(*c*) The geological structure may cause the true catchment area to depart widely from the area shown by the surface topography, and thus may influence the run-off to an abnormal extent.

Considering these in order :

(*a*) From the purely topographical point of view, anything that assists rain-water to collect rapidly, increases the run-off. Steep slopes, bare rocks, and numerous well defined small runnels, all indicate a good run-off ; whilst flat slopes, swamps, and bogs, or ill-defined topographical characteristics, are accompanied by a bad run-off.

The question can be briefly summed up : An area underlain by impervious strata allows a large fraction of its rain-fall to run away on the surface, and is cut and shaped by the collected water into a topography favourable to a good run-off. An area underlain by pervious strata disposes of most of its run-off by ground seepage, and therefore is not cut or shaped into a favourable topography, and, in consequence, a certain additional loss is entailed in such topographic run-off as does occur, owing to evaporation during its slower journey to the streams.

It is also plain that an impervious area will have more intense floods, and longer periods of low water than a pervious one, even though the mean yearly run-off is the same. This effect will be even more marked if the mean yearly rain-falls are equal.

(*b*) Returning to the general discussion (see p. 190). The larger the proportion of permeable strata in a catchment area, the greater the individual *f*'s (whether positive or negative), but apparently also the greater the individual *a*'s. This last statement may appear peculiar, but the explanation probably lies in the fact discussed when considering Droughts, namely : A large number of plants are capable of securing and utilising water lying in permeable strata near the ground level, and, where these strata exist, such plants being adapted to the local conditions, either grow naturally, or are cultivated.

A further explanation may be that all, or nearly all, permeable strata communicate with the sea, or with other catchment areas, by underground channels (in which may be included the flow of ground water which usually occurs in alluvial river beds, and which is rarely utilised, although it should obviously be included in the run-off in scientific discussions of the question). Be the explanation what it may, the extra rain-fall loss caused by permeable strata has a real existence in normal cases, and may be roughly estimated at about one-sixth of the average yearly storage in the invisible reservoir, if that is known; although (as is shown in the next section) permeable strata, under certain geological conditions, may increase the apparent run-off.

The above statement should perhaps be somewhat qualified. The records used to investigate the question nearly all refer to catchment areas the average elevation of which is greater than that of the country surrounding them, and the extra loss may be entirely due to leakage through permeable beds into neighbouring catchment areas. The only evidence which conflicts with this view is the fact that the permeable catchment areas with large average rain-fall-losses nearly always yield markedly larger run-offs at the end of a long period (two, or three years) of deficient rain-fall than other permeable areas with smaller average rain-fall losses. This fact suggests that the areas with the greater losses also possess the larger invisible reservoirs. It should, moreover, be noted that submarine discharges of large volumes of fresh water (which probably represent the leakage of permeable beds) are far more common than is generally supposed. Such phenomena are constantly being discovered both by submarine cable engineers, and by physicists studying questions relating to the distribution of deep sea fish.

(*c*) From the purely geological point of view, the effect of permeable and impermeable strata is best illustrated by examples.

If a permeable stratum occurs, which dips into a valley on one side at A, and outcrops again at B, across the topographical watershed W, as shown in section in Sketch No. 50, it is plain that some portion of the rain falling on the area WB, will appear at A. If conditions are favourable, as for instance if the stratum AB, is underlain by a bed of clay, this portion may be quite appreciable.

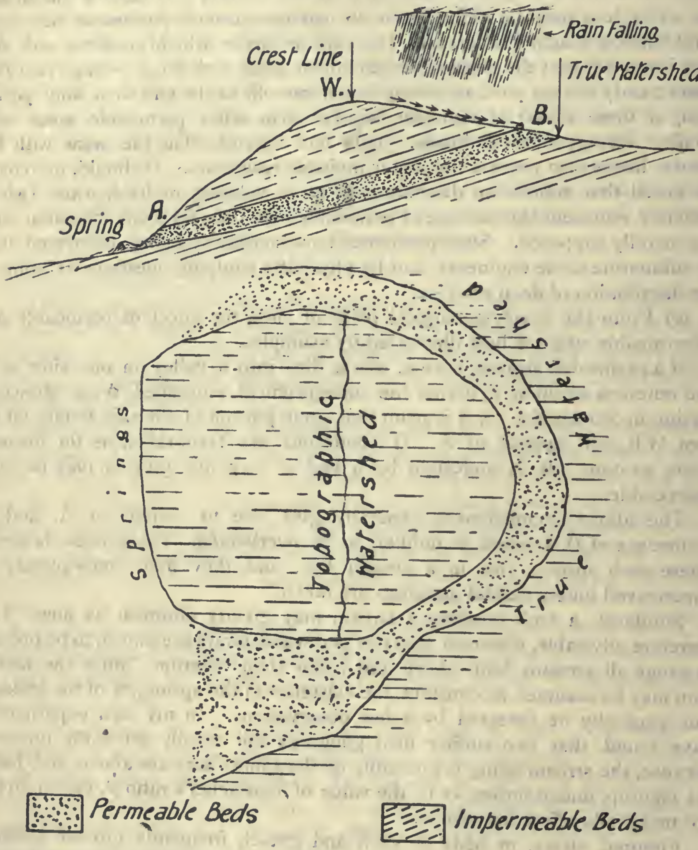
The above circumstances generally give rise to springs at A, and an occurrence of this nature is unlikely to be overlooked. Cases exist, however, where such springs rise in a stream bed, and they may consequently be unperceived unless careful gaugings are taken.

Similarly, a fault crossing a stream may greatly diminish its flow. It is therefore advisable, wherever faults or permeable strata are known to be present, to gauge all streams both above and below their outcrop. Since the loss or gain may be assumed as constant, the existence of the spring, or of the leakage, can generally be detected by a few observations. In my own experience I have found that two surface float gaugings will usually settle the question, because, the stream being presumably of the same character above and below the outcrop, uncertainties as to the value of Harlacher's ratio p, (see p. 61) do not materially affect the results.

Fissured strata, or beds of sand and gravel, frequently provide invisible underground channels for the escape of water. The detection of such channels and the measurement of the flow in them is difficult. This may be effected in a gravel bed by sinking a series of pipe wells across the supposed line of flow, and then one or two more some 100 feet above. A solution of some easily recognisable salt is poured into the upper pipes, and water drawn from the lower pipes is tested for this salt until it is detected.

A better, but more complicated method is electrical. In this case, the upper pipe wells are dosed with ammonium chloride, or common salt, and its advent to the lower pipes is announced by a fall in the electrical resistance measured between plates immersed in two of the lower wells.

It is likely that any important flow of water of the above nature will be previously known, and such local knowledge should be utilised in arranging the tests. Where the quantity of water thus discovered is of sufficient value to justify the expense it may be collected by underground dams, or catchment galleries, or pumped from a series of wells.



SKETCH No. 50.—Effect of Geological Structure on the Boundary of a Catchment Area.

It should be remembered that such pervious beds may be utilised as reservoirs, if the intake of the scheme can be located below the point where they discharge above ground. They are, therefore, not necessarily detrimental.

I shall later refer to the condensing effect which glaciers are believed to exercise, and there is little doubt that such condensations occur less extensively in many other cases. I am not, however, aware of any instance other than glaciers, where the effect is so marked as to have yet been detected in influencing the run-off.

The Dew Ponds of Sussex are worth describing, both as small, but quite practical examples of the possible utilisation of this effect, and as showing under what conditions it may be expected to occur.

Dew Ponds are made as follows:

A basin-shaped hollow is excavated in an open space, well exposed to damp sea winds. The hollow is covered by a layer of straw and twigs, or other non-conducting material, about 18 inches thick. On this is laid a continuous bed of puddled clay, about 2 feet thick, which in its turn is covered by a layer of broken stone.

The object is to provide a surface of stone and clay which rapidly grows cold at night, and the dew thus collected is caught by the layer of puddle clay, and conducted to the central pond.

Such a prepared area, about 200 feet in diameter, under favourable conditions, will keep the pond in the centre about 20 feet in diameter, and say 3 feet maximum depth. So far as can be gathered, in default of systematic observations, the yield is about 0·01 inch per night during the summer, over the prepared area.

(iv) *Effect of Lakes and Swamps.*—The presence of bodies of water in the catchment area has an equalising effect on the run-off. It is generally supposed that the evaporation from a free water surface, during the summer at any rate, is greater that the rain-fall on it. Thus, bodies of water, especially if shallow, and even more so if covered with vegetation, are regarded as tending to diminish the run-off.

The most important example is the loss sustained by the White Nile in the swamps of the Sudd region, between 7 degrees and 9 degrees 30 minutes N. latitude. In this case, the reasoning is founded on approximate gaugings; moreover, the rain-fall being small, and the evaporation intense, everything favours a loss of water.

In more Temperate climates, such as that of Great Britain, the same effect has been assumed to occur.

The later studies referred to on page 740, prove that the absolute magnitude of the evaporation from a free water surface has, until lately, been largely over-estimated, and since no actual gaugings showing the loss assumed to exist have ever been recorded, I am inclined to believe that in Temperate Insular climates at any rate, the loss is inappreciable, or even non-existent. Further evidence is required before this can be accepted as universally accurate, although it may be noted that the value assumed for the mean rain-fall loss ($z_m = 10{\cdot}6$ inches) in the Aker (Norway) project (*Tech. Ugeblad*, 1907, p. 21) which refers to a catchment area of 86 square miles, containing about 18 square miles of lakes, is a very low one for the mean temperature of the locality, and is apparently founded on long-continued gaugings.

Glacier-fed Streams.—The present state of development of water power schemes necessitates a consideration of the conditions of streams of which the catchment areas contain glaciers.

Our knowledge on the subject is unsatisfactory. Such rivers as the Upper Rhone (which includes about 20 per cent. of glacier area in its catchment), are quite abnormal, and the run-off may rise to 90 per cent. of the recorded rain. As Forel has suggested, it is possible that the moisture of the air is condensed on a glacier surface in a form that cannot be recorded by a rain-gauge.

As general rules it may be stated that:

(*a*) The yearly run-off from a glacier is very heavy. Those feeding Lakes Como and Maggiore in Northern Italy give about twenty times the yield of equal adjacent areas where no glaciers occur.

(*b*) Just as the melting snow causes a spring flood, so the more gradual melting of glaciers produces (in a purely glacial stream) a summer period of high water at the season when lowland, or hill streams, are at their smallest. A glacier thus acts as a reservoir, maintaining the summer flow.

I append curves showing the flow of the Upper Rhone, and, as a contrast, that of the Durance, a mountain river which is not glacier fed. (Sketch No. 51.)

DAILY VARIATIONS OF MOUNTAIN STREAMS.—As a glacier-fed stream is highest in summer, so streams issuing from a lofty mountain range are frequently found to possess a well-marked day and night variation in flow, especially in summer weather. Since this often amounts to as much as 20 per cent. of the daily flow, and is sometimes noticeable at points far removed from the mountains, it must be looked for in all such streams, and, if found, will necessitate the installation of a recording gauge, for since the variation occurs at approximately the same period each day, the results of daily, or even twelve hourly gauge readings, will be hopelessly erroneous.

RELATION BETWEEN MEAN YEARLY RAIN-FALL AND RUN-OFF.—The most reliable observations on the relations between the mean rain-fall x_m, and the mean run-off y_m, are to be found in the German and Austrian studies on the subject.

Although very accurate run-off observations exist in America, the corresponding data for rain-fall are by no means so trustworthy.

In view of the great regularity of rain-fall distribution in space existing in the Eastern United States, the fact that gauge stations are sparsely distributed is perhaps not of great importance; but the records are mostly for short periods, and the local circumstances of the rain-gauges (judging by accessible information) are generally less favourable to accurate results than is usual in Europe.

The British rain-fall records, thanks to the labours of Symons and Mill, are probably the best in the world, but the run-off records are usually kept secret.

In Germany, run-off and rain-fall statistics are good, and publication is systematic and customary.

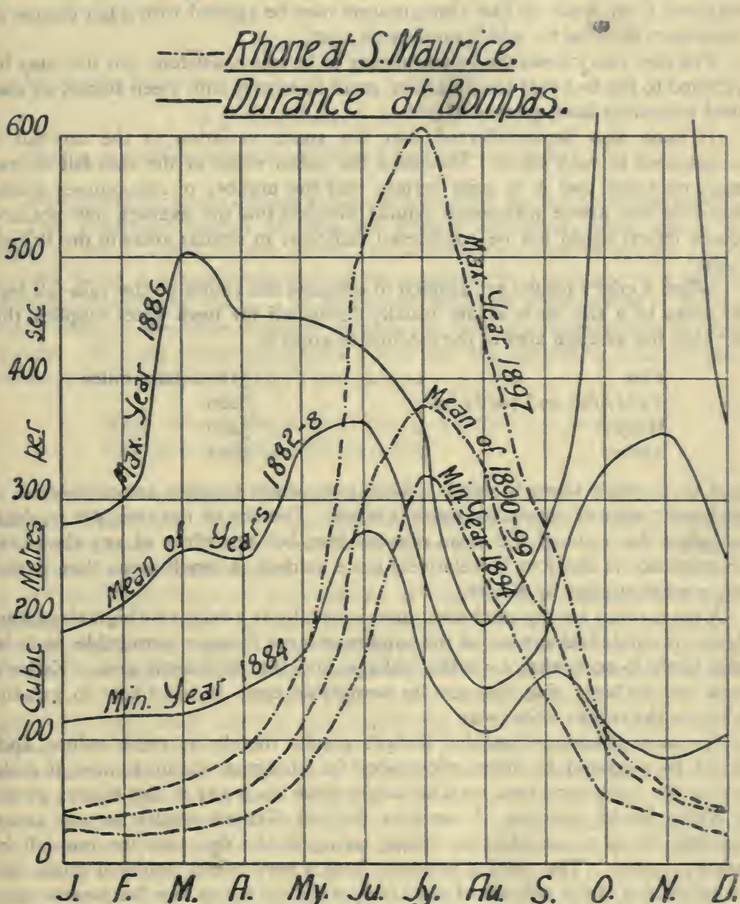
Fortunately, we are able to state that the relations existing in Germany are very similar to those in Great Britain, except in the case of some British areas of unusually heavy rain-fall, so that the labour of comparison will not be wasted.

Taking the results tabulated by Keller, in his paper *Niederschlag, Abfluss und Verdunstung in Mittleuropa*, which are the means of a series of years varying between 6 and 30, and rejecting all observations marked by him as doubtful, we arrive at the following:

Seventeen flat areas, with a mean rain-fall varying from 18·11 to 28·03 inches, give a mean value for the mean rain-fall loss of 15·28 inches, with a probable error of ±7·4 per cent., and a maximum mean loss for individual areas of 17·70 inches, with a minimum of 13·81 inches.

For six areas, partly flat, and partly hilly, with rain-falls varying from 26·20 to 32·30 inches, the mean of all the mean losses is 18·09 inches with a probable error of ±4·9 per cent., a maximum mean loss of 19·30 inches, and a minimum of 17·03 inches.

For twenty-four hilly areas, with rain-falls varying from 24·12 to 49·20 inches,



SKETCH NO. 51.—Monthly Discharge of the Rivers Rhone and Durance.

the mean of all the mean losses is 18·67 inches, with a probable error of ±8·3 per cent., a maximum mean loss of 21·08 inches, and a minimum loss of 15·63 inches. Rejecting the three areas which have rain-falls exceeding 40 inches and small rain-fall losses, the mean loss is 19·06 inches and the probable error only ±6·4 per cent., and the minimum loss is now 16·94 inches.

For eight Alpine areas, with rain-falls varying between 38·80 and 68·10 inches, the mean loss is 17·18 inches, and the probable error ±21 per cent. Here also, rejection of the two wettest areas reduces the loss to 15·62 inches, and the probable error to ±6·6 per cent.

It may therefore be stated that, excluding very rainy areas (over 45 inches), the probable errors of the mean rain-fall loss are but small, and that the values obtained from areas of like configuration may be applied with a fair degree of confidence to those for which no records exist.

For very rainy areas the values appear to be less consistent, but this may be ascribed to the fact that the areas are small in extent, with steep slopes, so that local influences have greater weight.

It must also be recollected that the space variation of the rain-fall is accentuated in such cases. Therefore the mean value of the rain-fall is less easily obtained, and it is quite certain that the number of rain-gauges established in the above mentioned Alpine districts (on the average one per 400 square miles) would not be considered sufficient in similar areas in the British Isles.

When Keller's results are applied to estimate the values of the rain-fall loss for areas of a size such as are usually developed for town water supplies, the fact that the average area of the catchment areas is :

Flat	3180 square miles
Partly flat and partly hilly	6020 „
Hilly	3400 „
Alpine	5610 „

must be carefully borne in mind. When town water supplies are considered, a catchment area of 20 square miles is large. The size of the area, *per se*, does not affect the value of the mean rain-fall loss, but the effect of any abnormal circumstance is likely to be relatively more marked in small areas than in the larger areas studied by Keller.

A town water supply catchment area is usually at a relatively high elevation. Hence, invisible leakage out of the catchment area through permeable beds is more likely to occur than invisible leakage into the catchment area. Keller's areas are so large that leakage by permeable beds is not likely to greatly influence the results either way.

We must therefore consider Keller's results merely as mean values, and should be prepared to make allowances for abnormal circumstances, if such exist in the catchment area, on a far larger scale than any of the figures given by Keller would indicate. I consider that all Keller's results for wet areas probably err to a considerable extent, owing to the fact that the rain-fall is underestimated. The results, however, form a very useful practical guide for the estimation of the run-offs of wet areas, since, so far as can be ascertained, the rain-fall of all abnormally wet European areas is probably underestimated owing to the fact that the very wettest portions of such areas is usually but sparsely inhabited. Mill's paper (*P.I.C.E.*, vol. 155, p. 305), affords two very interesting large scale examples (*i.e.* Central Wales, and Western Northumberland) of this fact. Keller's original paper thoroughly deserves study by all hydraulic engineers.

When the whole of Keller's results are plotted with rain-falls as abscissæ, and rain-fall losses as ordinates, we find that the points form a group with well-marked boundaries. These agree very closely with straight lines, and translated into English measure, Keller's results are as follows, where x_m, represents the mean annual rain-fall in inches, and z_m, the mean annual rain-fall loss in inches. The mean result is :

$$z_m = 0{\cdot}058x_m + 15{\cdot}95. \quad \text{Or, run-off} = \tfrac{16}{17}\ \text{rain-fall} - 16\ \text{inches.}$$

In no case is :

$$z_m \text{ less than } 13{\cdot}80 \text{ inches.}$$

In no case is :

$$z_m \text{ greater than } 0{\cdot}116x_m + 18{\cdot}10 \text{ inches.}$$

Or, the run-off is always larger than $\frac{7}{8}$ rain-fall − 18 inches.

Now, for other than German areas, such detailed figures cannot be given.

The published British records are usually abnormal, being for a dry year, or for a series of dry years. Approximately accurate figures can be arrived at by a comparison with those available in Germany, and also by utilising unpublished data. The difference is almost entirely due to the fact that, owing to the more equable climate, the loss is somewhat less in Great Britain. By using Keller's methods I find that the figures for areas of which the mean rain-fall is less than 60 inches are :

Mean value of the rain-fall loss $z_m = 0{\cdot}06x_m + 14$ inches.
Run-off = 0·94 rain-fall − 14 inches.

As minimum values I have been unable to find a reliable mean loss for any English district less than 12·5 inches, but, following Keller's results, I believe that mean losses as low as 11 inches do occur, and I hope that my statement may result in their publication.

As maximum losses we have figures as high as 22·5 inches, and even 23 inches, but in each case the areas are small (about 500 acres), and the run-off is probably diminished by faults and fissures in the subsoil. These large values are always short period values, being the average of three to five years at the most.

It must not be forgotten that the presence of a limestone stratum, or, more especially, of a chalk stratum in a catchment area, alters the usual relation between rain-fall and run-off. The stratum is probably fissured, and may afford an invisible channel of escape for a large portion of the water that falls on it, or may contribute in the form of springs originating in places which, according to the surface topography, are not included in the catchment area. The stratum (as in the case of bournes such as are found in the limestone and chalk districts of Yorkshire, Derbyshire, and Southern England) may act as a reservoir, and store up large quantities of water, which are delivered in the form of intermittent streams flowing at intervals of years. Since, however, the most natural method of securing water in such districts is by means of wells, catchment areas entirely underlain by permeable strata are unlikely to be utilised for water supply. Where the catchment area is partially occupied by such strata it is wise, unless springs occur, to allow a diminution in the run-off from the chalk area of 3 to 5 inches per annum, in addition to the loss indicated by ordinary rules.

RELATION BETWEEN THE RAIN-FALL AND RAIN-FALL LOSS FOR INDIVIDUAL YEARS.—The above discussion permits us to determine z_m, the mean rain-fall loss over a long period of years for a catchment area the mean rain-fall of which during that period is x_m in the form :

British Isles	$z_m = 14 + 0{\cdot}06x_m$
Germany	$z_m = 16 + 0{\cdot}06x_m$
Eastern United States. . . .	$z_m = 16{\cdot}5 + 0{\cdot}20x_m$

where the first two cases refer to average catchment areas, while the third may be suspected as relating to somewhat more markedly permeable districts, but is founded on the most accurate records that I have been able to discover.

We have now to consider the relation of z, and x, for a given catchment area in individual years. The relation appears to take the form :

$$z = a + 0{\cdot}16x$$

and a, is plainly obtained by putting $z = z_m$, and $x = x_m$, *i.e.* :

$$\begin{aligned} a &= z_m - 0{\cdot}16x_m \\ &= 14 - 0{\cdot}10x_m, \text{ for the British Isles.} \\ &= 16 - 0{\cdot}10x_m, \text{ for Germany.} \\ &= 16{\cdot}5 + 0{\cdot}04x_m, \text{ for the Eastern United States.} \end{aligned}$$

This last may be compared with Vermeule's statement that $z = 15{\cdot}5 + 0{\cdot}16x$. The difference amounts to about 2, or 3 inches for the rain-falls usually occurring in the Eastern United States, and is quite explicable by the fact that the more generally accessible records (such as I have used) are known to refer to somewhat unfavourable cases.

These last relations are very rough, and are put forward in order to invite criticism ; and by way of indicating the slight respect they deserve it may be stated that :

(*a*) All very wet years (usually any years with more than 120 per cent. of the mean fall) have been neglected.

(*b*) When examined by the theory of errors, the figure 0·16 becomes $0{\cdot}16 \pm 0{\cdot}07$; and, the figure 14, $14 \pm 4{\cdot}3$.

The only consoling fact is that these figures indicate that the rule has, in all probability, a physical basis.

(*c*) I would, however, point out that any engineer possessing records will easily obtain figures which are more applicable to his own results by graphically plotting the x and z for each year, as was done in discussing Ingham's results.

A table of the values obtained in this manner is given on page 197.

Penck, (I have been unable to trace the reference) from his studies on the question, has arrived at expressions equivalent to :

$$z = 12 + 0{\cdot}27x \text{ for European catchment areas.}$$
$$z = 10{\cdot}1 + 0{\cdot}20x \text{ for United States catchment areas.}$$

The values given by the first rule do not differ markedly from mine, and any discrepancy is probably due to my neglect of the wetter years, which is justifiable for engineering purposes, although out of place in a purely scientific investigation.

The observations utilised to determine Penck's second equation refer to areas possessing climates of both the first and second types, and the divergence from my rule, is therefore not surprising.

As examples of the methods explained above, let us consider :

(i) A British catchment area of 45 inches mean rain-fall. The mean loss over a series of years is $14+0{\cdot}06\times45 = 16{\cdot}7$ inches. The rule for losses in individual years is expressed by :

$$z = a+0{\cdot}16x$$

$$\text{or, } 16{\cdot}7 = a+0{\cdot}16\times45. \quad \text{Or, } a = 9{\cdot}5, \text{ and } z = 9{\cdot}5+0{\cdot}16x.$$

Estimating the rain-fall according to Binnie's rules, in a long series of years :

The driest year will have a fall of 30 inches, and a loss of 14·3 inches, or a run-off of 15·7 inches.

The average fall of the two driest successive years will be 33 inches. The average loss 14·8 inches, and the average run-off 18·2 inches.

The average fall of the three driest consecutive years will be 35 inches. The average loss 15·1 inches, and the average run-off 19·9 inches.

The mean fall will be 45 inches. The mean loss 16·7 inches, and the mean run-off 28·3 inches.

The average fall of the three wettest consecutive years will be 55 inches. The average loss 18·3 inches, and the average run-off 36·7 inches.

For the two wettest consecutive years the figures are: 58 inches, 18·8 inches, and 39·2 inches respectively.

For the wettest year, the figures are : 65 inches, 19·7 inches, and 45·3 inches.

(ii) Similar figures for a rain-fall of 25 inches, are :

$$\text{mean loss}=14+0{\cdot}06x_m=15{\cdot}5, \text{ loss for an individual year, } z=11{\cdot}5+0{\cdot}16x,$$

and the calculated figures are as follows :

	Rain-fall.	Loss.	Run-off.
Driest year . . .	16·5 (16·8)	14·1 (13·2)	2·4 (3·6)
Driest 2 consecutive years .	18·3 (19·4)	14·4 (14·1)	3·9 (5·3)
Driest 3 consecutive years .	19·5 (20·0)	14·6 (13·9)	4·9 (6·1)
Mean of all years . .	25·0 (25·2)	15·5 (16·0)	9·5 (9·2)
Wettest 3 consecutive years	30·7 (33·0)	16·4 (16·0)	14·3 (17·0)
Wettest 2 consecutive years	32·5 (34·0)	16·7 (15·8)	15·8 (18·2)
Wettest year . . .	36·3 (37·0)	17·3 (18·2)	18·8 (18·8)

(Actually observed figures are given in brackets.)

The records extend over 21 years, and it is believed that these 21 years include rather more than a usual proportion of dry years; thus, while the dry year observations probably show the worst that is likely to occur, wetter years may possibly be observed.

As has already been noted, these rules refer to years in which the seasonal distribution is normal, and we may expect to find that :

(*i*) The run-off for the driest year is underestimated, but the calculated figure does not, necessarily, underestimate the minimum yearly run-off.

(*ii*) The run-off for the wettest year is overestimated.

(*iii*) The sum of the values for two and three consecutive years will agree far more closely with observation than the results of individual years.

Comparing the two areas, we see that in an extremely dry year the run-off of the drier area (when compared with that of the wetter area) is far less than the ratio of either the mean or the individual falls would lead us to believe; and in the wetter years the same divergence occurs, although less markedly, for all the run-offs.

As an example that can also be compared with actual observation, take an American catchment area of 47 inches mean rain-fall. The relation between rain-fall and rain-fall loss being given as $z = 15 + 0{\cdot}25x$. The calculated figures are shown below, while the observed values are given in brackets:

	Rain-fall.	Rain-fall Loss.	Run-off.
Driest year . . .	31·0 (31·2)	22·8 (21·1)	8·2 (10·1)
2 driest consecutive years .	34·3 (35·7)	23·6 (23·2)	10·7 (12·5)
3 driest consecutive years .	36·7 (38·3)	24·2 (23·5)	12·5 (14·8)
Mean of 35 years . .	47·0 (47·0)	26·8 (26·7)	20·2 (20·3)
3 wettest consecutive years	58·8 (52·7)	29·7 (36·6)	29·1 (19·4)
2 wettest consecutive years	61·9 (59·3)	30·5 (40·4)	30·9 (18·9)
Wettest year . . .	64·4 (69·3)	31·6 (42·4)	35·8 (26·9)

The agreement is fair for the dry years, but is abnormally bad for the wet ones. On referring to the monthly records, it will be found that while the proportion of summer and winter rain-fall in the dry years is very close to the normal, the wet years all have an abnormally large proportion of summer rain-fall. It is for this reason that I have selected this adverse example, as it seems necessary to illustrate the errors that may be produced by neglecting the seasonal distribution. Better results might be obtained by considering all years below the average as one group, and those above the average as another, and deducing separate relations of the form:

$$z = a + bx$$

for each group.

Wet Areas.—For very wet districts (roughly those where the mean rain-fall exceeds 70 inches), the above rules are generally considered not to hold good. Actual figures are very rare, since such areas are usually small, and give so good a yield that scarcity of water is but seldom experienced.

A study of 45 yearly records of three such catchment areas has led me to believe that the law is best represented by the following equation:

$$z = a + 0{\cdot}6(x - x_m)$$

Consequently, the yearly run-off is far more constant than that of drier places.

The basis for this rule is obviously not very broad. In such cases it would appear that the total amount of invisible storage (see p. 188) has a considerable influence on the yearly run-off; a very large invisible reservoir tending to increase y, and *vice versa*. This appears to indicate that ground surface evaporation from such thoroughly saturated areas is always active, and the invisible storage being more or less shielded from surface evaporation, the larger the amount of water thus stored, the greater the yearly run-off. I must, however, point out that the measurement of the rain-fall and run-off from such districts is attended by very special difficulties. The rain-fall is heavy and patchy, and the stations at which it is observed are usually sparsely distributed over the area. The monthly rain-falls being heavy, the special gauges erected by the engineers are frequently found to have overflowed. Thus, the rain-fall is likely to be underestimated. A large proportion of the run-off passes off in floods and freshets, so that accurate estimation is difficult. The results obtained at Mercara and at Labugama (see p. 249) are probably the most accurate information available and unfortunately refer to Tropical climates.

Distribution of Run-off during the Year.—The smaller the size of the reservoir provided to equalise the run-off over a year, or series of years, the more important the question of the monthly or other short period distribution of the run-off becomes. In the ordinary British town water supply, the short period distribution may be almost entirely disregarded, since the storage reservoir is sufficiently large to equalise the supply over the whole year. The usual low head power station of the Eastern United States lies at the other end of the scale. Here, at the most, the night flow of the river is stored up for use next day. In such cases the daily values of the run-off are important. In general, however, we can assume that a reservoir of sufficient size to equalise the flow over a month exists.

The ruling factor is of course the structure of the catchment area, as providing either the visible reservoirs formed by topographical features, such as lakes or ponds, or the invisible storage afforded by permeable strata.

An approximate method of obtaining an idea of the influence of this storage (especially the geological storage) has already been given and should be applied in all cases. It is especially valuable when the district under consideration is situated close to a catchment area in respect to which long term records of rain-fall and run-off are available. We can then assume the difference in the f's, for the two catchments as calculated for those years over which simultaneous records exist, as likely to vary but little in other years, and may, (subject to this constant difference), apply the long term records to the catchment area which it is proposed to study. Under such circumstances, we may believe that the deduced run-offs are more than usually accurate.

The process generally adopted by engineers for the determination of the monthly run-offs of a catchment area now requires examination. I give three methods. The first is the most general, and all that can really be said is that it is simple. The second is not so common, and is even less accurate than the first, but is given owing to its frequent employment in the past. The third is a logical and scientific method, but is so complicated as to be almost inapplicable, except to areas that have been very carefully studied for long periods.

It is to be hoped that, before the final designs are prepared, the engineer will possess at least a year's systematic record of the flow of the streams which it is proposed to deal with, and if he is so fortunate as to have statistics for five years at his disposal, he may consider himself lucky.

However, assuming only one year's record, the engineer will then be able to judge by the rain-fall data whether he is dealing with a dry, normal, or wet year, and can draw up a monthly table of the following type :

	Rain-fall.	Run-off.	Loss.	Percentage of Rain-fall appearing as Run-off.
January . . .	2·55	2·46	0·09	96·5
February . . .	0·76	1·04	−0·28	136·8
March . . .	1·76	0·84	0·92	47·7
April . . .	1·38	0·57	0·81	41·3
May	1·81	0·45	1·36	24·9
June . . .	1·30	0·48	0·82	36·9
July	0·76	0·23	0·53	30·3
August . . .	1·77	0·19	1·58	10·7
September . .	2·41	0·23	2·18	9·5
October . . .	1·91	0·24	1·67	12·6
November. . .	3·23	0·44	2·79	13·6
December . .	1·68	0·52	1·16	31·0
Total . .	21·32	7·69	13·63	36·1

Which is the record for the Thames valley for 1887, and is due to Binnie (*Report on the Flow of the Thames*). I regret that I am not permitted to publish in detail a more typical British record, although the rain-fall in the above example is probably far better determined than is usually the case. The above figures are for a very dry year, probably the third driest of the last century. The area to which they refer is some 3,855 square miles, with gentle slopes, and is almost wholly underlain by permeable strata. The typical British reservoir catchment is about 40 square miles, with steep slopes, and is generally underlain by impermeable strata. The flow of the Thames is therefore (and actual comparison of records confirms the statement) more equable throughout the year than is the case in a typical catchment area.

The figures for the mean of nine years, 1883–1891, are :

Rain-fall, 27·01. Run-off, 8·49. Loss, 18·52.

It will be noticed that the dry year loss is less than the average, and this may be taken as holding in nearly all cases. *Vice versa*, the wet year loss is larger than the average ; and, as in the case of this particular record, the driest year is not necessarily the year of minimum run-off, for in 1884 the records were :

Rain-fall, 22·90. Run-off, 6·56. Loss, 16·34.

As another example we may take the table on page 217, which forms a typical American wet year table, being for the Sudbury River drainage of 75 square miles, for the year 1878, the wettest between 1875 and 1897.

The mean records for this area for the twenty-eight years are :

Rain-fall, 45·78. Run-off, 22·22. Loss, 23·56.

	Rain-fall.	Run-off.	Loss.	Percentage of Rain-fall appearing as Run-off.
January . . .	5·63	3·23	2·40	57·0
February . . .	5·97	3·97	2·00	66·0
March . . .	4·69	6·26	− 1·57	133·0
April . . .	5·79	2·81	2·98	48·0
May	0·96	2·49	− 1·53	260·0
June. . . .	3·88	0·87	3·01	22·0
July	2·97	0·23	2·74	8·0
August . . .	6·94	0·84	6·10	12·0
September . .	1·29	0·28	1·01	22·0
October . . .	6·42	0·92	5·50	14·0
November . .	7·02	2·92	4·10	42·0
December. . .	6·37	5·67	0·70	89·0
Total . .	57·93	30·49	27·44	52·6

It will be noticed that the loss in this wet year is above the average, and so also is the run-off. The negative loss in March, due to melting snow, is analogous to the negative loss for the Thames in February, and such large Spring negative losses are characteristic of Northern American drainage areas, and also occur, although less markedly, and with occasional exceptions, in English areas.

Taking the above as a basis, we must now construct similar tables for other years for which we only possess records of the monthly rain-falls.

Two methods are employed by engineers, and I shall give both, for although I am fully convinced that the first is the more accurate, I am well aware that many engineers make use of the second.

I shall term them the Subtractive and the Proportional Methods.

The Subtractive Method.—This consists in subtracting, (or, when the loss is recorded as negative, in adding) the observed monthly loss for the year of observation from the observed monthly rain-falls for the other years, and considering this result as representing the most likely value of the monthly run-off.

The principal pitfalls occur in dealing with summer months. It is well known that in the summer, topographic run-off (as distinct from ground-water flow) is almost entirely produced by rain that occurs on days of great rain-fall only. Hence, it is quite possible that two summer months of identical total rain-fall may give very different run-offs. In the one case, the total rain may fall in a short heavy storm, and (especially on an impervious area), a large fraction may reach the stream; while in the other case, the rain may fall as a succession of slight showers, of short duration, and may all be absorbed by the growing vegetation.

It is therefore necessary to carefully examine the daily rain-falls of the summer months, and to compare their general intensity with those of the year of observation. Such errors are less likely to occur in the winter months, but their possibility should be borne in mind. It must also be remembered that in climates where a marked feature, such as the melting of the snow, or the commencement of the monsoon, occurs, the month does not properly specify the season in so far as it affects the run-off. For example, if in the year of observation the snow melts in April, it is plain that the April loss should be debited to March in years during which the snow melts in that month.

A study of temperature records will often be of great assistance. It is evident that if statistics for three or four years' run-off can be secured the sources of error can be minimised, and, while accurate discharge observations are more useful, a study of records of high and low water, or daily gauge readings, is not to be despised.

Proportional Method.—This is, I believe, a relic of certain early, and now obsolete, rules for estimating run-off as 60, or some other percentage of the rain-fall.

The method, as now applied, consists in calculating the ratio $\frac{\text{Run-off}}{\text{Rain-fall}}$ for each month of the year in which the observations were taken, and obtaining the run-off in other years by multiplying the rain-fall for each month by this ratio.

The system does not appear to rest on any very logical basis, and has one grave defect, namely, a tendency to overestimate the yield of dry months and dry periods. Since the subtractive method tends to underestimate the yields of such periods, it is at any rate safer.

I have taken pains to calculate the monthly ratios for many catchment areas over several years, and find that they vary far more than the corresponding losses. It may be that the method is applicable to certain somewhat peculiar catchment areas, but, so far as I have been able to ascertain, no engineer possessing several years' records of rain-fall and run-off has been led to use it.

The investigation of the question given in connection with annual run-off statistics shows that if the rain-fall records do not, for any reason, correctly give the true fall on the catchment area, the proportional method of estimating the run-off acquires a spurious accuracy, which is apparently, and only apparently, greater than that of the subtractive method.

The real deduction to be drawn is that where the rain-fall is accurately determined, the subtractive method is preferable. Where it is less correctly known, the proportional method has certain advantages, and especially in a case where the recorded value of the rain-fall is approximately proportional to, and somewhat less than the true value, the latter may prove to be the more accurate.

The methods employed in practice by engineers are very accurately indicated by this method of reasoning. In the British Isles, and Germany, where the rain-fall is well determined, the subtractive method is usually employed. In France, the United States (until recently), and India, the proportional method is more common, and in these countries rain-fall observations are relatively less accurate.

Third Method taking into Account the Effect of Ground Water Storage on Run-off.—The importance of the storage of water in pervious strata and swamps has already been considered, and its general effect in partly equalising the monthly, and even (although less markedly) the yearly run-offs, is obvious.

Vermeule (*Report of Geological Survey of New Jersey*, 1894) has endeavoured to treat the question systematically. While the difficulties are apparent, I consider that his methods deserve careful description, and believe that the most promising direction for future studies lies along his line of investigation.

Let us consider the calendar year, and let suffix 1, refer to the month of January, suffix 2, to February, etc.

For each month Vermeule specifies a quantity $v=ax+b$ where x, is the observed rain-fall. Vermeule terms v, the "evaporation," but it does not appear to be proportional to the evaporation from a free water surface, and is essentially analogous to what I have termed the "vegetation and evaporation loss" (see p. 186) for the month. I shall hereafter refer to it as Vermeule's v.

Tabulating we have as follows:

For the month of—

January	$v_1 = 0{\cdot}27 + 0{\cdot}10x_1$	inches.
February	$v_2 = 0{\cdot}30 + 0{\cdot}10x_2$	,,
March	$v_3 = 0{\cdot}48 + 0{\cdot}10x_3$	,,
April	$v_4 = 0{\cdot}87 + 0{\cdot}10x_4$	,,
May	$v_5 = 1{\cdot}87 + 0{\cdot}20x_5$	,,
June	$v_6 = 2{\cdot}50 + 0{\cdot}25x_6$	,,
July	$v_7 = 3{\cdot}00 + 0{\cdot}30x_7$	,,
August	$v_8 = 2{\cdot}62 + 0{\cdot}25x_8$	,,
September . . .	$v_9 = 1{\cdot}63 + 0{\cdot}20x_9$	,,
October	$v_{10} = 0{\cdot}88 + 0{\cdot}12x_{10}$	,,
November . . .	$v_{11} = 0{\cdot}66 + 0{\cdot}10x_{11}$	,,
December . . .	$v_{12} = 0{\cdot}42 + 0{\cdot}10x_{12}$	,,

Similarly, Vermeule gives for the:

Six months, December to May inclusive, $V_i = 4{\cdot}20 + 0{\cdot}12x_i$
Six months, June to November inclusive, $V_f = 11{\cdot}30 + 0{\cdot}20x_f$
and, for the whole year . . $V_y = 15{\cdot}5 + 0{\cdot}16x_y$

Here the V's are also v's, *i.e.* the above three expressions represent rain-fall losses which the individual v_1, v_2, etc. do not.

It is obvious that the three last expressions can only be applied when the rain-fall is distributed month by month according to the proportions generally holding good for the climate of the Eastern United States.

Vermeule also states that these figures can be applied to a climate where the mean annual temperature is approximately 49·7 degrees Fahr., and that, for any other mean annual temperature T, they can be adjusted by multiplying by a factor $0{\cdot}05T - 1{\cdot}48$.

I have tested the method by means of several well determined records, and believe that this statement is probably correct for the Eastern United States, but that the correction is not applicable to British and German examples.

Now, tabulate $x-v$, for each month. This is not the run-off, and Vermeule now proceeds to discuss the ground-water factor.

He divides catchment areas into three classes. (See Sketch No. 52.)

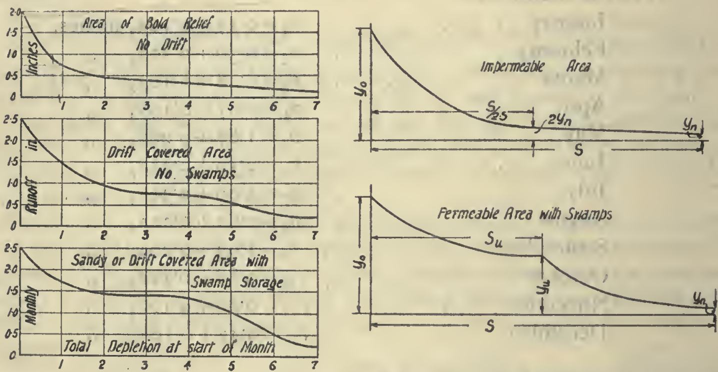
(i) Highland areas, of bold relief with no drift covering. (That is to say, areas which I class as underlain by impermeable strata.)

(ii) Areas with a covering of drift, but no surface storage. (That is to say, areas underlain by permeable strata, and containing no lakes or swamps.)

(iii) Areas covered with drift, and possessing surface storage. (That is to say, areas underlain by permeable strata, and containing lakes and swamps.) This last (except in formerly glaciated areas) is an unusual combination; but it also covers such cases as granite areas with small lakes and large accumulations of peat.

For each of the above Vermeule gives a curve connecting the run-off with $x-v$, and the "storage depletion."

I have found these curves difficult to use, and have consequently tabulated



SKETCH NO. 52.—Curves of Discharge and Depletion, as given by Vermeule (after Vermeule), with Approximate Curves for Typical Permeable and Impermeable Areas.

his data in a more useful form, and must therefore be held responsible for any errors.

Vermeule considers the monthly run-off y, as consisting of two portions, $x-v$ (taken with reference to its sign), and a contribution (positive, or negative) from the ground-water storage. The ground-water storage is supposed to have a maximum possible limit; and the magnitude of this contribution depends on D, the mean amount which the accumulated water storage, in the ground, or as snow, is below this maximum during the month under consideration. The term ground water is perhaps slightly inaccurate, for it is believed that Vermeule's figures are somewhat affected by water stored up in the form of snow.

The reasoning is precisely similar to that on page 190, except that v is written for ke_p. We therefore put

$$y=x-v+u$$

where u is the ground-water flow, and is analogous to the quantity denoted by f_p, or $-\Delta R_p$, on that page.

According to Vermeule, y is a function of $d-\frac{x-v}{2}+\frac{y}{2}$ where d, is the sum of all the u's, since the period when the quantity of water stored up last, had its maximum value. Thus, d, is the depletion of the water storage at the beginning of the month, while $d-(x-v)+y$ is the depletion at the end of the month under consideration, and consequently, $D=d-\frac{x-v}{2}+\frac{y}{2}$, is the mean depletion during the month.

Starting with some winter month (December, January, or February for choice), we assume the ground storage as full, and write $y=x-v$, until about 1st of March (*i.e.*, the melting of the snows).

Thus, for the earlier months of the year, we have :

$$y_1=x_1-v_1 \qquad u_1=0 \qquad d_1=0$$
$$y_2=x_2-v_2 \qquad u_2=0 \qquad d_2=0, \text{ etc.}$$

But when, under American conditions, the snows begin to melt, and $x-v$, is less than 2·0, or 2·5 inches, depletion of the stored water begins, we have :

$$y_n=x_n-v_n+u_n \qquad d_n=0 \qquad d_{n+1}=u_n \qquad D_n=\frac{u_n}{2}$$

where, in the cases considered by Vermeule, n, is usually 3. That is to say, the month of March. So that at the beginning of the second month after depletion commences (April) we have a depletion $d_{n+1}=u_n$; and y_{n+1} is not equal to $x_{n+1}-v_{n+1}$, but has a value corresponding to

$$D_{n+1}=d_{n+1}-\frac{x_{n+1}-v_{n+1}}{2}+\frac{y_{n+1}}{2},$$

which is plainly the mean depletion during the month, and there is a further depletion $u_{n+1}=y_{n+1}-(x_{n+1}-v_{n+1})$. So that at the end of the month the total depletion is $d_{n+2}=u_{n+1}+u_{n+2}$, and y_{n+2} the run-off for the $(n+2)$th month is that corresponding to :

$$D_{n+2}=\frac{d_{n+2}+d_{n+3}}{2}=d_{n+2}-\frac{x_{n+2}-v_{n+2}}{2}+\frac{y_{n+2}}{2}$$

Thus the run-offs can be written down.

As we advance in the year, we finally reach a period (roughly about August or September), when the run-offs corresponding to the obtained mean depletion become smaller than the quantities $x-v$. The ground-water reservoir then begins to fill up, and in a normal year we arrive at no depletion, or the "reservoir" is full up about December. In an abnormally dry year we will obviously finish the year with a certain amount of depletion, and, failing heavy rains in the early part of the next year, the run-offs of the following summer will be materially reduced.

The process is a very excellent one, and the comparisons made by Vermeule with actual observations seem to me most satisfactory, provided that the rain-fall is accurately determined. Difficulties do occur, but they are not of great importance, and seem rather to indicate that calendar months are bad periods to investigate, being too long for the seasons when the depletions are small, and too short for such periods as the end of a dry summer.

Vermeule has shown y, as a graphic function of D (see Sketch No. 52), which necessitates a process of trial and error in every case before y, can be selected.

The general equation is : $D=\frac{y}{2}+d_i-\frac{x-v}{2}$, where d_i, is the initial depletion of the month considered. Thus, if $d_i=0$, $x=1{\cdot}82$, $v=1{\cdot}05$, we get $D=\frac{y}{2}-0{\cdot}38$; and from the curve (Class I.) when $y=1{\cdot}44$, $D=0{\cdot}34$, and the initial depletion of the next month is given by $2D=y-(x-v)=1{\cdot}44-0{\cdot}77=0{\cdot}67$ inches.

The information given permits a table to be drawn up as follows:

D	y	$D-\frac{y}{2}$
0·0	2·0	−1·0
0·05	1·9	−0·90
0·10	1·8	−0·80
0·11	1·7	−0·74
0·18	1·6	−0·62

and $D-\frac{y}{2}=d_i-\frac{x-v}{2}$, so that this transformation permits the tables of y, as a function of $2d_i-x-v$, as given on page 224, to be obtained.

A study of Vermeule's curves of y, and D, however, is very useful, and we can deduce the following general rules for the construction of the curve of y, as a function of D.

(*a*) Determine the minimum month's flow ever recorded in similar catchment areas, and call this y_n.

In America y_n, is about 0·15 inch in small (under 200 square miles), and 0·20 inch in large catchment areas.

In England, in catchment areas of 10 or 12 square miles y_n, occasionally falls as low as 0·10, but 0·15 is closer to the average, although 3 or 4 square miles of impermeable catchment area often yield no run-off for periods of three weeks.

(*b*) Determine the maximum depletion of storage ever recorded, and call this S. S, is about 9 inches in a permeable area, but may be roughly estimated as one-fifth of the maximum oscillation of the ground-water level, and in impermeable areas a proportionate deduction must be made, although it should be remembered that even granite areas can hold about 2 to 3 inches of water in the clay and peat coverings which overlie the granite.

Then (Sketch No. 52):

(i) If there are no visible reservoirs, such as lakes or swamps:

Plot a straight line from $y=y_n$, $D=S$; to $y=2y_n$, $D=\frac{S}{2{\cdot}5}$; and continue this line by a circular arc, or parabola, up to the point $D=0$, $y_o=\frac{y_w}{4}$ where y_w, is the total run-off of the winter months (six in number) during a series of years.

(ii) If lakes or swamps exist, in the area:

Let A, represent the whole area of the catchment area, and A_u, the fraction of the area in which the ground-water level lies lower than the level of the water in the lakes or swamps. Calculate $S_u=S\frac{A_u}{A}$, $y_u=y_o\frac{A_u}{A}$. Then, from

$D=0$ to $D=S_u$, the curve is a parabola through $D=0, y=y_0$,

and $$D=S_u, y=y_u.$$

For greater depletions we construct the curve just as if it referred to an area for which the maximum depletion is $S\left(1-\frac{A_u}{A}\right)$, and the initial point of the curve is $y=y_u$.

Sketch No. 52, shows diagrams thus obtained, and, if necessary, a table of y, and D, can be scaled off, and modified into a table of y, and $2d_t-(x-v)$, as already explained.

These curves agree very closely with those given by Vermeule, although differences of 10, or 15 per cent. in y, are likely to occur, especially when D, is small. The great advantage (which is inherent in Vermeule's method) is that, provided that the curves are constructed with any reasonable adherence to probability, positive errors in one month will be largely compensated by negative errors in the next, and *vice versa*.

We may therefore believe that Vermeule's method produces really practical results provided that S, y_n, and y_0, are determined with some accuracy, and the only factor which requires even ordinary accuracy, is y_0.

The method is therefore a very excellent one, since it permits the run-off to be derived from the rain-fall by a process which takes into account the influence which the rain-falls and run-offs of the previous months undoubtedly possess; while other processes usually regard each month's rain-fall and run-off as isolated facts, or, at the best, as subject to influences which are assumed to be the same every year during corresponding periods of the year.

The great difficulty is the determination of the v's. This is probably best effected by careful studies of the observed run-offs, working with a preliminary curve of y's, and D's, determined as suggested above. In this manner I find that while the mean annual temperature of the Thames valley is about 48·6 degrees Fahr., the v's, for this valley are approximately two-thirds of those for New Jersey. The ground flow begins in March, in place of April, as in America, and if this reduction is applied to other English catchment areas, so far as can be judged these catchment areas fall into two classes, for which figures are given on page 224.

It must be distinctly understood that these figures are merely suggestions. They are deduced from principles which I certainly believe to be correct, and the results obtained by their employment agree better with actual experience than those given by any other methods. It is, however, plain that the data at present available are insufficient for the production of an accurate solution.

The following appear to be the chief differences between American and European catchment areas, when the relation between y, and D, is considered.

Firstly, in the typically Insular climates of the British Isles and Western Germany, the possible amount of storage is not so great as in the more Continental climate of America, because snow neither lies so long, nor accumulates in such masses. This difference may be regarded as climatic in its causes, and could possibly be allowed for.

The second difference is due to artificial reasons. Most British and German watersheds are provided with a system of catchwater drains in the form of field and road ditches, and agricultural tile drains, of a character practically unknown in America.

Vermeule's tables, arranged according to y, the run-off, are :

VALUES OF $2d-(x-v)$.

y	For American Catchment Areas.			For British Catchment Areas.	
	Class I.	Class II.	Class III.	Impermeable Strata.	Permeable Strata.
2·5	...	−2·50	−2·50	...	...
2·4	...	−2·30	−2·30	...	...
2·3	...	−2·05	−2·05	...	...
2·2	...	−1·80	−1·76	...	...
2·1	...	−1·53	−1·46	...	...
2·0	−2·00	−1·27	−1·10	...	...
1·9	−1·79	−0·97	−0·73	...	...
1·8	−1·57	−0·67	−0·27	...	...
1·7	−1·35	−0·33	0·35	...	−1·7
1·6	−1·16	0·0	0·90	...	−1·4
1·5	−0·88	0·30	1·80	...	−1·1
1·4	−0·60	0·70	3·10	...	−0·7
1·3	−0·34	1·10	6·5	−1·3	−0·3
1·2	−0·06	1·55	7·7	−1·15	0·05
1·1	0·24	2·06	8·4	−0·77	0·60
1·0	0·56	2·60	8·9	−0·44	1·00
0·9	0·90	3·34	9·4	−0·10	1·70
0·8	1·25	4·40	9·9	0·24	2·90
0·7	1·66	7·20	10·5	0·67	5·60
0·6	2·14	9·00	11·1	1·10	6·90
0·5	2·66	9·80	12·0	1·65	8·30
0·4	3·50	10·70	12·8	2·55	9·30
0·3	5·30	11·90	...	5·70	10·30

The following example of Vermeule's method refers to a Canadian watershed over which the mean temperature is 46 degrees Fahr. Thus, the v's, for this area are $2·30-1·48=0·82$ of the values of v, calculated from the formula when uncorrected for temperature. The rain-fall is the mean of 5 stations, which represent the area fairly well, and it is stated that the results agree tolerably closely with the actual run-offs (von Schon, *Hydro-Electric Practice*).

Observed rain-falls are stated in Column II., while the v's, are calculated by multiplying the v, for the same month in New Jersey, by 0·82. For example, for January 1903, $v=(0·27+0·10\times 1·36)\times 0·82$.

The filling up of this table (top, p. 225) is fairly obvious. The ground-water storage is assumed to remain full until the beginning of March, so that for the first three months $y=x-v$. In March it is assumed that the ground storage begins to contribute, and $x-v=0·70$. Corresponding to this, for Class I. $y=1·44$, so that u, is 0·74, producing a depletion of 0·74 at the beginning of April. During April, $2d-(x-v)=1·48-0·10=1·38$, and the corresponding run-off is 0·77. Thus each month is successively worked out.

Month.	Rain-fall x.	Vermeule's v.	Difference $x-v$.	Run-off y.	Contribution from the Stored Water u.	Depletion at beginning of Month d_i.	Value of $2d_i-(x-v)$.
December 1902 .	2·18	0·52	1·66	1·66	0·0	0.0	0·0
January 1902 .	1·36	0·33	1·03	1·03	0·0	0·0	0·0
February 1903 .	1·80	0·39	1·41	1·41	0·0	0.0	0·0
March 1903 . .	1·19	0·49	0·70	1·44	0·74	0·0	−0·70
April 1903 . .	0·89	0·79	0·10	0·77	0·67	0·74	+1·38
May 1903 . .	2·74	1·98	0·76	0·62	−0·14	1·41	2·06
June 1903 . .	2·85	2·63	0·15	0·56	0·41	1·27	2·39
July 1903 . .	2·68	3·12	−0·44	0·38	0·82	1·69	3·96
August 1903 .	2·87	2·74	0·13	0·33	1·20	2·51	4·89
September 1903 .	3·54	1·92	1·62	0·39	−1·23	2·71	3·80
October 1903 .	3·92	1·11	2·81	1·13	−1·48	1·48	0·15
November 1903 .	0·96	0·62	0·34	0·89	0·0	Full	0·0
December 1903 .	4·28	0·69	3·59	2·90	0·0	Full	0·0

Month.	Eastern United States.	British Isles.
January . . .	$y_1 = 0{\cdot}40 + \frac{x_1}{2}$	$y_1 = 0{\cdot}50 + \frac{x_1}{3}$
February . . .	$y_2 = 0{\cdot}90 + \frac{x_2}{2}$	$y_2 = 0{\cdot}70 + \frac{x_2}{3}$
March . . .	$y_3 = 1{\cdot}60 + \frac{x_3}{3}$	$y_3 = 0{\cdot}50 + \frac{x_3}{9}$
April	$y_4 = 1{\cdot}05 + \frac{x_4}{2}$	$y_4 = 0{\cdot}15 + \frac{x_4}{3}$
May	$y_5 = 0{\cdot}75 + \frac{x_5}{4}$	$y_5 = 0{\cdot}20 + \frac{x_5}{6}$
June	$y_6 = 0{\cdot}30 + \frac{x_6}{5}$	$y_6 = 0{\cdot}12 + \frac{x_6}{7}$
July	$y_7 = 0{\cdot}15 + \frac{x_7}{10}$	$y_7 = 0{\cdot}20 + \frac{x_7}{20}$
August . . .	$y_8 = 0{\cdot}10 + \frac{x_8}{8}$	$y_8 = 0{\cdot}15 + \frac{x_8}{20}$
September . . .	$y_9 = 0{\cdot}10 + \frac{x_9}{6}$	$y_9 = 0{\cdot}20 + \frac{x_9}{40}$
October . . .	$y_{10} = 0{\cdot}15 + \frac{x_{10}}{7}$	$y_{10} = 0{\cdot}15 + \frac{x_{10}}{10}$
November . . .	$y_{11} = 0{\cdot}20 + \frac{x_{11}}{3{\cdot}5}$	$y_{11} = 0{\cdot}30 + \frac{x_{11}}{5}$
December . . .	$y_{12} = 0{\cdot}40 + \frac{x_{12}}{4}$	$y_{12} = 0{\cdot}30 + \frac{x_{12}}{3}$

Special Monthly Formulæ.—These formulæ are open to the objection which has been already stated, namely, that the month does not really specify the same season in each year. Further, such formulæ cannot take into account the special circumstances obtaining in previous months. Subject to these remarks, we obtain the lower table on page 225.

These figures are only approximations, and it must be remarked that the American results for the months November to March, inclusive, and the British results for November to February, show distinct traces of a term in x^2.

This whole discussion will have been written to very little purpose, unless the reader is by now well aware that at the end of each hot season, and even more markedly at the end of a long period of small rain-fall (*e.g.* the summer of the last year of a period of three dry years), the run-off is almost exclusively dependent on the stored water, and is consequently not so much a meteorological phenomenon as a geological one.

DETERMINATION OF RESERVOIR CAPACITY.—The results obtained by the preceding methods are extremely inaccurate. When applied to existing records, by utilising observations over 5, or even 10 years, to predict the remainder of the series, errors of even 200 per cent. in the values of the flows for individual months are by no means unknown. The results for individual years, however, are better, and I believe that under favourable circumstances an error as large as 15 per cent. should but rarely occur, provided that the rain-falls of individual months (and in the case of heavy storms of rain, of individual days) are carefully considered.

It is plain that the process is at best approximate, and is only sufficiently accurate for practical purposes, where a reservoir is constructed of a size sufficient to equalise the flow over a long period. When such a reservoir exists, errors in the determination of the flows of individual months are of minor importance, because overestimates at one period will probably be balanced by underestimates at another, that is supposing that care is taken to avoid unduly favourable assumptions.

In climates of the type now under discussion, the general practice is to provide a reservoir of a capacity sufficient to equalise the flow over "the 3 driest consecutive years," by which we understand those 3 years in succession of which the total run-off is less than that of any period of 3 years that is likely to occur during say 50 or 60 years.

A solution of the problem can only be obtained by long experience, and the history of the question in Great Britain suggests that it is not yet completely solved.

Hawksley (representing let us say the best available knowledge for the years 1830–1860), gives the following :

The mean annual rain-fall over a long term of years being x_m, the mean annual rain-fall of the 3 driest successive years of this period is $x_{3D} = \frac{5x_m}{6}$, and the mean annual run-off of these three years is $y_{3D} = \frac{5x_m}{6} - 15$ inches, expressed as inches depth over the catchment area. Then, the capacity of the equalising reservoir for 3 dry years, *i.e.* the reservoir permitting constant delivery at the daily rate corresponding to a delivery of the volume represented by y_{3D}, per annum, is represented by $\frac{1000}{\sqrt{x_{3D}}}$ days' supply. That is to say, the reservoir capacity converted into inches depth over the catchment area is :

$$\frac{1000}{\sqrt{x_{3D}}} \times \frac{y_{3D}}{365} \text{ inches} = 2{\cdot}74 \frac{y_{3D}}{\sqrt{x_{3D}}} \text{ inches.}$$

(*Note.*—One inch depth over one square mile = 2,323,000 cube feet, say $2\frac{1}{3}$ million cube feet, and if this volume be delivered at a constant rate in a year the supply is 39,750 imperial gallons, or 47,700 U.S. gallons per day.)

The general history of British waterworks during this period renders it probable that this capacity was insufficient. It is but rarely that we do not find on record that shortage of water occurred once in a generation, as indicated either by the curtailment of the hours of delivery to the houses, or by the installation of temporary additional supplies.

The rule, however, allowed for a supply which satisfied the popular ideas of that generation. About the year 1870, the introduction of the practice of constant supply, combined with the fear of possible pollution, became general. Consequently, it was found necessary to enlarge, or supplement existing reservoirs. Rofe has proposed the following formula :

$$\text{Capacity} = \frac{500}{\sqrt[3]{y_{3D}}} \text{ days' supply} = \frac{500 y_{3D}}{365 \sqrt[3]{y_{3D}}} = 1{\cdot}37 y_{3D}^{0.67} \text{ inches}$$

as representing these results, and also as sufficient to ensure delivery at the rate of y_{3D}, inches per annum during the driest probable three years.

The rule (except in unusual circumstances) affords an adequate capacity for such a supply.

In England, catchment areas of the necessary size, and reasonably free from pollution, affording the required volume, are becoming somewhat difficult to procure. Further, engineers no longer remain satisfied with securing the yield of the three driest years. Present circumstances, therefore, economically justify a larger expenditure of money, in order to obtain a greater yield.

Consequently, we have a third rule, which appears to be very well represented by :

$$\text{Capacity} = 1{\cdot}7 \text{ to } 1{\cdot}8 y_{3D}^{0.67},$$

and which may be considered as the result of experience during the last droughts (1893–5, and 1905–7).

We may classify and tabulate as follows :

(i) Hawksley's rule, applicable to cases where catchment areas are easily secured, and a temporary shortage of supply once in 30 years may be regarded as permissible.
(ii) Rofe's rule, where shortages cannot be permitted.
(iii) An extension of Rofe's rule, which allows of a somewhat greater (10 to 12 per cent.) supply being delivered, but which is only economical when any enlargement of the catchment area is difficult to obtain.

I tabulate the figures, under the assumption that $y_{3D} = x_{3D} - 14$ inches.

			Hawksley's Rule.			Rofe's Rule.			New Rule.	
x_m	x_{3D}	y_{3D}	Number of Days' Supply.	Capacity in Inches.	$\frac{\text{Capacity}}{\text{Average yield } y_{3D}}$	Number of Days' Supply.	Capacity in Inches.	$\frac{\text{Capacity}}{\text{Average yield } y_{3D}}$	Capacity in Inches.	$\frac{\text{Capacity}}{\text{Mean yield}}$
70	56	42	134	15·4	0·37	144	16·6	0·39	21 to 22	0·40
65	52	38	139	14·4	0·38	149	15·5	0·41	19 to 20	0·39
60	48	34	144	13·3	0·39	154	14·3	0·42	18 to 19	0·40
55	44	30	151	12·4	0·41	161	13·2	0·44	17 to 18	0·43
50	40	26	158	11·2	0·43	169	12·0	0·46	15 to 16	0·45
45	36	22	167	10·1	0·46	179	10·8	0·49	14 to 15	0·47
40	32	18	177	8·7	0·48	191	9·4	0·52	12 to 13	0·50
35	28	14	189	7·2	0·52	207	8·0	0·56	10 to 11	0·52
30	24	10	204	5·6	0·56	232	6·4	0·64	8 to 9	0·57
25	20	6	224	3·7	0·61	275	4·5	0·75	6 to 7	0·65
20	16	2	250	1·4	0·69	397	2·2	1·09	3·5	0·70

The average year's run-off in the last column is calculated from

$$y_m = x_m - 15 \text{ inches.}$$

It is believed that the above ratios may be accurately applied even in cases where the mean rain-fall loss differs from 15 inches; since, as a rule, where the rain-fall loss is abnormally low, the run-off is more than usually variable from month to month, and *vice versa*.

A similar process of successive increase in reservoir capacity has taken place in Germany, and the Eastern United States. Intze's earlier designs were for the utilisation of good catchment areas, of unusually large yield, with mean rain-falls from 35 to 45 inches, and losses of 12 to 14 inches. The available records were for 10 or 12 years, at the most, and the capacities, on the average, were approximately those given by Rofe's rule. The later designs had frequently to be adapted to less favourable circumstances, and capacities similar to those given by the third rule occur. I am not, however, disposed to consider this as entirely due to Intze's greater foresight, or more complete information, but believe that the present state of design in Germany is similar to that in England about 1885, and that future experience (combined with a greater development of water storage), will lead to an increase in the present capacities.

A German stream is notably more variable than a British one, under similar circumstances; and I consider that the difference of 10 per cent. between Hawksley's rule, and Intze's earlier designs, and between Rofe's rule, and later German designs, is due to this fact, and that further experience will lead to the adoption of reservoirs of about 10 per cent. greater capacity than those shown by the new English rule.

Catchment areas of the Eastern United States apparently bear the same relation to those of Germany, that those of Germany do to those of Great Britain, and storages 20 per cent. greater than those indicated by the British rules appear to be advisable, allowance being made for the fact that natural storage in lakes is more common in the North-Eastern States, than in either Germany or Great Britain.

MASS CURVE.—The only accurate method, where the necessary records exist, is to construct a mass curve according to the plan laid down by Rippl (*P.I.C.E.*, vol. 71, p. 279). This is a diagram showing the total run-off from a fixed date to any other date as ordinate, with the period elapsed as abscissa. The necessary data are therefore the measured run-offs, usually month by month, and these are best expressed as inches over the catchment area.

A convenient scale for plotting is 1 inch = 10 inches of run-off, and 6 months; but much depends upon the absolute magnitude of the average annual run-off.

Having plotted the mass curve, we can find the storage required to permit any rate of draught up to the maximum possible, as follows:

Lay off on the diagram straight lines at slopes corresponding on the scale of the diagram to the various rates of draught which it is proposed to study. Rule parallel lines from the mass curve at the various humps A, B, C, and note where these, drawn in a positive direction in time, again cut the mass curve at AE, BD, and CF (see Sketch No. 53).

The vertical intercepts between these lines, and the mass curve, represent the total volume of the reservoir which has been emptied of water under such circumstances. The horizontal distances between A, and E, B, and D, C, and F, indicate the periods during which no water escapes from the reservoir, which is evidently full at A, and again full at E, etc.

This having been done, a study of the diagram will show what year (or period of years) covers the time when the reservoir is most depleted, which is consequently the most critical period.

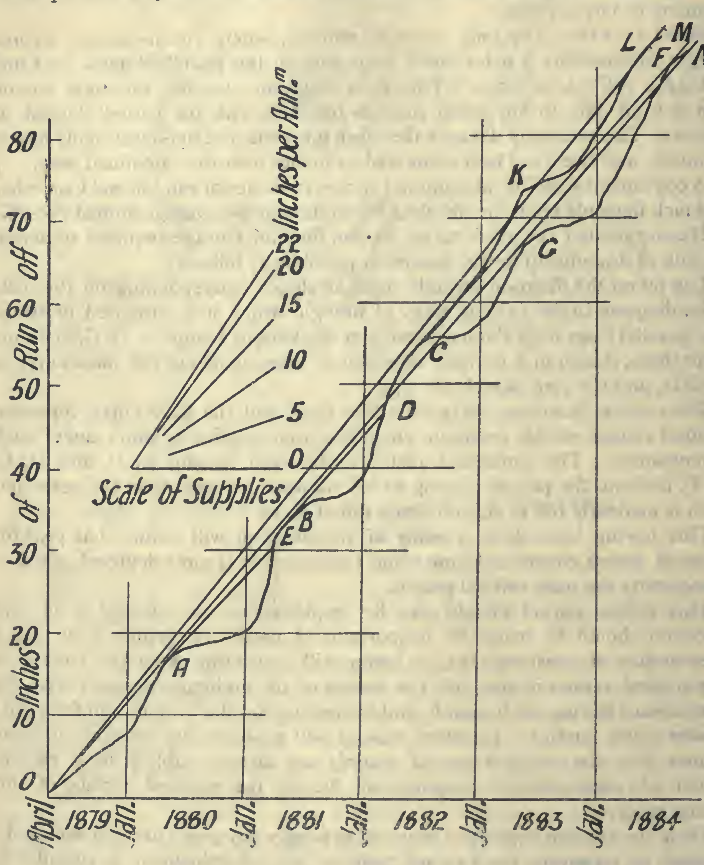
This critical period should now be re-plotted on an enlarged scale, and correction should be made for evaporation if necessary; either assuming a water surface of constant area, or, better still, calculating from the volume of the proposed reservoir site, and the results of the preliminary mass curve, the water surface during each month, and correcting for the evaporation from this variable water surface. In either case, it will probably be necessary to bear in mind that the original run-off records are already subject to a certain amount of water surface evaporation, due to the exposed surface of the existing reservoir.

Then, the storage capacities required to supply any given draught are easily obtained by repeating the original process. Or, if refinement is considered advisable, the result of a variable draught, such as is called for in the supply of water to a town, may be studied, by substituting a curved draught line, plotted like a mass curve, for the straight lines AE, etc.

The results of the process are most interesting. Freeman's Report on the Water Supply of New York shows clearly how important a chance downpour (or rather, its accompanying run-off) may become, when an attempt is made to develop the utmost possibilities of a catchment area by large storage.

Let us now consider the general characteristics of a mass curve. In every year there is a season when the run-off reaches its maximum, and these seasons succeed each other at intervals of approximately 12 months. Thus, we find a series of more or less prominent humps on the mass curve, following each other at intervals of nearly 12 months.

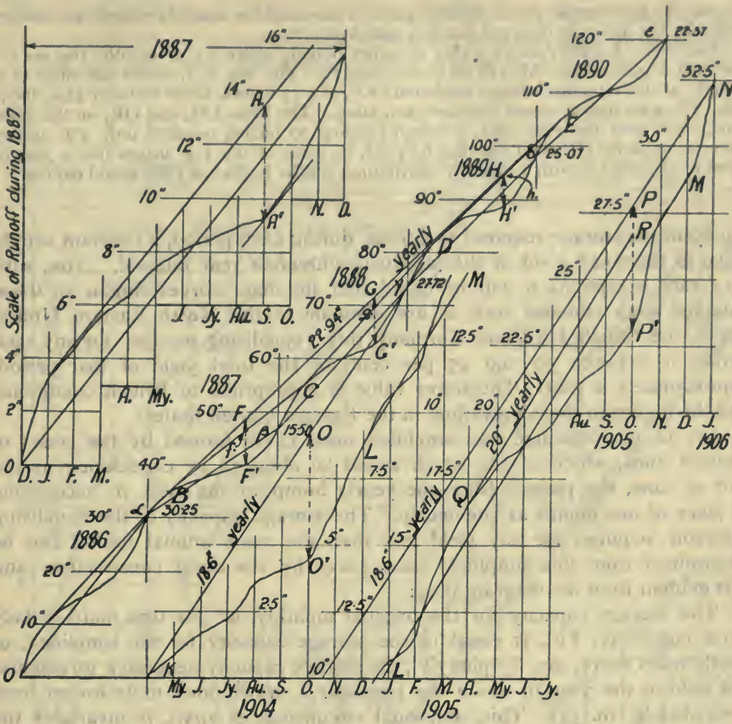
Let us connect up B, C, D, E, &c., the tops of these humps, by a series of straight lines, BC, CD, and DE (See Sketch No. 54). Then, the various intercepts such as *f*F′, *g*G′, and *h*H′, between these lines, and the mass curve,



SKETCH No. 53.—Shows a mass curve plotted for the Croton (New York) records (see Freeman *ut supra*) during the droughty period 1879–84.

The upper curve C K L is that actually observed, the lower curve C G N shows what might have occurred had the heavy rains of September 1882 not replenished the reservoirs. It will be seen that assuming a supply of 18 inches per annum as shown by the line O M; there will be a depletion of 9·7 inches (approximately 217 days supply) about January 1881, and that failing the rains of September 1882 a somewhat larger depletion (approximately 240 days supply) would occur about January 1883. A supply of 17 inches per annum, as shown by the line O N, can however be obtained with safety if a reservoir of 10 inches capacity (allowing 10 per cent. for water below draw-off level) be provided.

The lines A E, B D represent supplies of 16 inches per annum, and this supply can plainly be obtained with certainty since the reservoir is refilled each year under the actual circumstances, though once in a century it is probable that the reservoir will not overflow for two years in succession.



SKETCH NO. 54 is founded on the records of the Redmires (Sheffield) catchment area, as given by Marsh (*P.I.C.E.*, vol. 181, p. 1). The curves plotted are:

(i) Monthly mass curve for the year of minimum run-off (1887).
(ii) Yearly mass curve for the five years' period 1886–90.
(iii) Monthly mass curve (shown in two pieces) for the droughty period May 1st, 1904, to January 31st, 1906.

The first curve shows that the maximum depletion, if the year's run-off of 15·50 inches be drawn off equably, occurs about October 1st, 1887, and that the equalising reservoir should have a capacity of 4·25 inches, as shown by the intercept AA′ (27·4 per cent. of the year's yield).

Similar monthly mass curves are then set off about the yearly mass curve, or polygon O $\alpha\,\beta\,\gamma\,\delta\,\epsilon$ obtained from Marsh's records of the run-offs of the calendar years 1886–90 inclusive. On this assumption, which is plainly a very unfavourable one, we obtain a mass curve with humps B, C, D, E, at the dates January 31st, 1886, January 31st, 1887. Joining up the humps January 31st, 1887, and January 31st, 1890, we obtain the line B, E, representing an equalised supply at a rate of 22·94 inches per year, which, owing to the assumption already indicated, is slightly in excess of 22·73 inches, the mean yield of the three calendar years 1887–1889. If the monthly variation of the run-off be neglected, the depletions are plainly represented by intercepts such as F*f*, G*g*, H*h*. Allowing also for the monthly variations we obtain the depletions FF′, GG′, HH′. The capacity of the equalising reservoir therefore, for these three years, is FF′, the largest of these; and is approximately 8·6 inches.

The period plotted does not include the three driest years in the record, which were 1904–06, and which produced a mean yearly run-off of 21·20 inches. The example, however, forcibly illustrates the theoretical difficulties introduced by tabulating the run-offs by calendar years, although, since records of the monthly run-offs are always available, the difficulty does not occur in actual practice.

The curve KLMN, shows a very droughty period, where for 21 months the run-off averaged 1·55 inch monthly (18·60 inches yearly). The line KN, shows the effect of a drought at this rate, producing a depletion OO′, of 4·77 inches, about October 31st, 1904, and PP′, 4·60 inches, about October 31st, 1905. The lines LM, and QR, on the other hand, show how the year 1905, although yielding 20 inches, required only 3·31 inches reservoir capacity (RP′) to equalise its yield, in place of the 5·48 inches that a year of equal total yield but with a monthly distribution similar to that of 1887 would require.

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represent the storage required to deliver, during each period, a constant supply equal to the mean yield of the (approximately) one year interval. Now, with very rare exceptions it will be found that for mass curves similar to those obtained from climates such as are common to the North Eastern United States, the British Isles, and Germany, this "equalising storage" for any such period is between 30 and 45 per cent. of the total yield of that period, approximately a year. The lower value is appropriate to British conditions, and the higher to those prevailing in the Eastern United States.

Let us now examine the simplified mass curve formed by the series of straight lines, which is that which would be obtained by considering, as the unit of time, the periods from one yearly hump to the next in succession, in place of one month as previously. The storage capacity of the equalising reservoir required for any yield less than the mean annual run-off can be determined from this simplified mass curve by the usual construction, and it is evident from the diagram that:

The storage capacity for the original monthly, or (for that matter) daily mass curve say, FF′, is equal to the storage capacity for the simplified, or yearly mass curve, say, F*f* plus *f*F′, the storage capacity necessary to equalise the yield of the year in which the reservoir is drawn down to its lowest level (see Sketch No. 54). This, abnormal circumstances apart, is invariably the year during which the run-off is a minimum.

Now, we can, with very fair approximation, plot the simplified, or yearly mass curve, from the rain-fall records, by the rules already given.

Thus, we arrive at a graphical construction for the required approximate storage capacity as follows (Sketch No. 55):

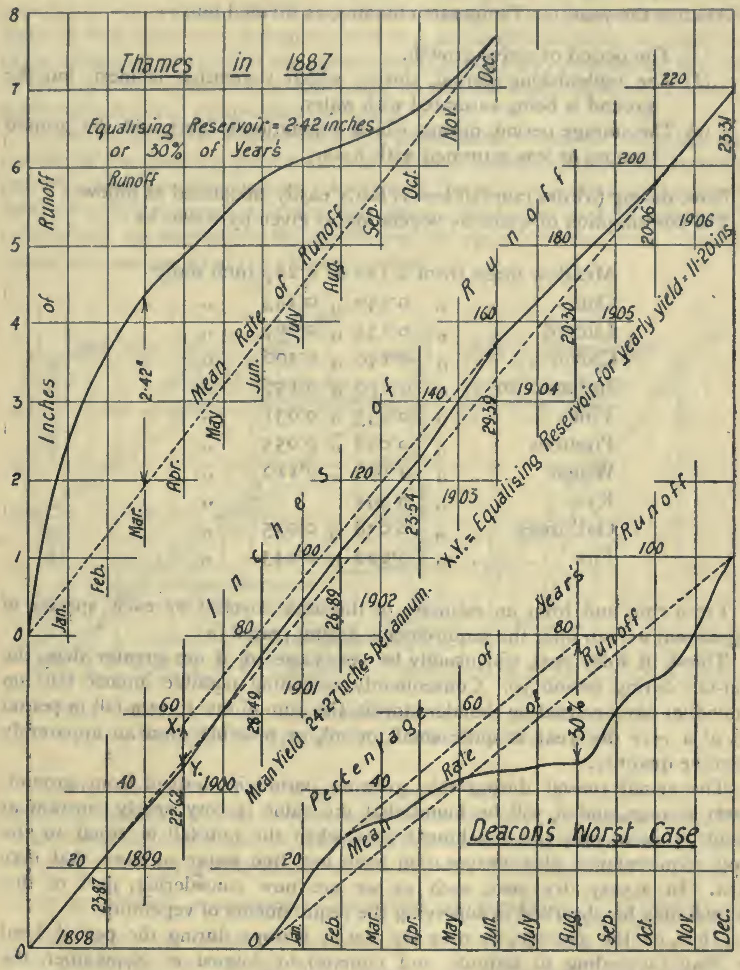
Plot a yearly mass curve, where for definiteness the total run-off of each calendar year is set off on the line representing the 31st December, or, better still, of each water year on the line representing the end of that water year. From this find the storage capacity for the given yield (including evaporation and percolation losses), and note in what year the maximum depletion occurs.

Then, the total storage capacity required is:

> Storage capacity obtained as above, plus 30 to 45 per cent. of the gross yield of the year during which the maximum depletion occurs.

This rule appears somewhat rough at first sight when compared with the more accurate results obtained from the monthly mass curve, but it is in reality more correct than it seems, for two reasons. The only uncertain factor compared with the result of the method of monthly mass curves is the second term, the 30 to 45 per cent. Now, the maximum depletion of the reservoir almost invariably occurs during a very dry year; and the monthly distribution of the run-off during very dry years differs *inter se*, far less than the monthly distribution during years selected at random differ *inter se*, so that the figure 30 to 45 per cent. is far more constant than might at first sight be expected.

We can also make our approximation a little more definite by carefully



SKETCH No. 55.—Yearly Mass Curve, Yearly Equalising Reservoir, for Redmires, 1898–1906, with Mass Curve and Equalising Reservoir for the worst Case recorded by Deacon (after Deacon), and for Thames in 1887.

considering the rain-fall for the two or three years during which the above construction shows maximum depletion as most probable. I am of the opinion that a consideration of the monthly rain-falls is too uncertain to be of practical value, unless it so happens that we possess a good rain-fall and run-off record for a catchment area of very similar characteristics.

The following work is of utility, and is advantageous as fixing the position of the humps more accurately than a mere examination of the average of years.

Assume the year, for Temperate climates, as divided into :

(*a*) The period of active growth.
(*b*) The replenishing period, during which vegetation is inert, but the ground is being saturated with water.
(*c*) The storage period, during which vegetation is inert, and the ground is more or less saturated with water.

Now, during (*a*) the rain-fall loss is fairly easily calculated as follows : The consumption of water by vegetation, is given by Risler as :

Meadow grass	from	0·122	to 0·287	inch daily
Oats	,,	0·140	,, 0·193	,,
Lucern	,,	0·134	,, 0·267	,,
Clover	,,	0·140	,, 0·200	,,
Indian corn	,,	0·110	,, 0·157	,,
Vines	,,	0·035	,, 0·031	,,
Potatoes	,,	0·038	,, 0·055	,,
Wheat	,,	0·106	,, 0·110	,,
Rye	,,	0·091		,,
Oak trees	,,	0·038	,, 0·035	,,
Firs	,,	0·020	,, 0·043	,,

From this, and from an estimate of the area covered by each species of vegetation, we can infer the requirements during period (*a*).

These, in a dry year, will usually be very close to, if not greater than, the rain-fall during period (*a*). Consequently, excluding possible intense falls on individual days, caused by thunder-storms, the run-off due to rain-fall in period (*a*), of a very dry year, is quite small, or nil, or possibly even an apparently negative quantity.

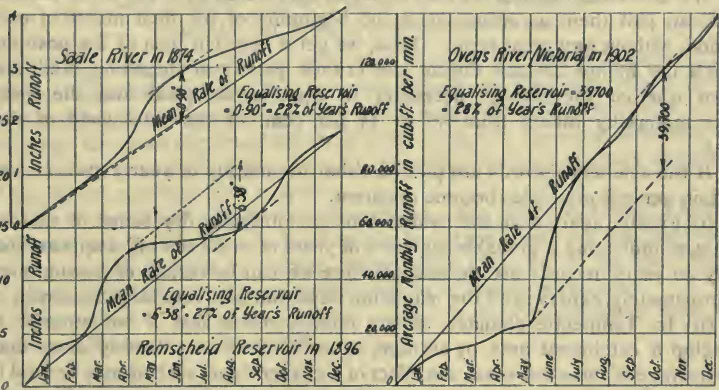
The actual run-off during this growing period is derived from ground-water storage, and it will be found that the value is very nearly constant at about 1½ inch, over the catchment area, when the rain-fall is equal to the crop requirements, plus evaporation from any free water surfaces that may exist. In a very dry year, such as we are now considering, part of this 1½ inch may be absorbed in supplying the requirements of vegetation.

Thus, on the average, we may say that in Europe during the period April or May (according to latitude and climate) to August or September, the following figures represent the bare minima of rain-fall that will suffice to supply the requirements of vegetation (Rafter, *Report on Genesee Storage*).

Tilled land	Average,	10·3 inches
Meadows	,,	16·0 ,,
Woodland and forest	,,	3·7 ,,
Miscellaneous	,,	5·8 ,,

If the rain-fall is equal to these values a total run-off of about 1½ inch will probably occur, but the ground will become so dry that at least 1½ inch of rain will later be absorbed by the dry soil after the drought ends, before any increase in the stream flow can be observed. If the rain-fall is less than these values, the vegetation will wilt and suffer from drought, and while the streams may still yield 1½ inch total run-off (especially if the permeable beds are covered by a thick coating of soil so that the ground water is not easily reached by the roots of the vegetation) yet all the deficiency in rain-fall and the 1½ inch must be made up before the stream-flow increases.

It will therefore be evident that it is inadvisable to consider that the total run-off of the growing period exceeds 1½ inch unless the rain-fall markedly exceeds the values given above. In view of the facts regarding the replenishment of soil moisture it is plain that safety can be secured (if evaporation calculations are neglected) by considering the 1½ inch of run-off as occurring during the last month of the growing period.



SKETCH No. 56.—Mass Curve and Equalising Reservoirs for Saale, in 1874, Remschied in 1890 (German), Ovens (Victoria), in 1906.

During the period (*b*) the ground water has to be replenished. The actual loss mainly depends on the character of the strata, whether permeable, or impermeable, and it may be remarked that when in period (*a*) the run-off is indicated as an apparently negative quantity, this deficiency must be made up before any run-off is assumed. The actual percentage of $\frac{\text{run-off}}{\text{rain-fall}}$, after the 1½ inch has been made up, varies according as the rain falls in bursts, or in steady drizzles. In the first case, a high figure may be assumed. In the second, a very low one, on the average say 10, to 20 per cent. In actual practice, the daily rain-fall records of exceedingly dry years during the first month or so of this period should be studied.

In period (*c*) we can assume that 60, to 70 per cent. of the rain-fall appears as run-off, plus a certain soak out or contribution from the ground storage, if the rain-fall of period (*b*) has been heavy.

Towards the end of period (*b*) this soak out may also occur, but its quantity is uncertain. If the calculated rain-fall loss in the two periods (*a*) and (*b*) greatly exceeds the assumedly known mean annual rain-fall loss, it may be necessary to consider this excessive amount as replaced by a gradual soak out during period (*c*). I believe that this is the only help that ground storage can usually be relied upon to afford towards the end of a very dry period.

The assumptions are obviously unfavourable, especially in that their nett effect during periods (*b*), and (*c*), is to cause the run-off to lag behind the rain-fall, and during this delay the consumption draught continues to deplete the reservoir. In cases where the margin of safety is small, it will be necessary to consider the daily rain-falls carefully, since, in such instances, an intense downpour is frequently the salvation of a water supply. Nevertheless, it should be remembered that after a dry summer the depleted ground storage has, sooner or later, to be made up, and the best we can hope is that the reservoirs being low, they may obtain replenishment from the ground water somewhat more readily than is usually the case.

We thus obtain figures for the run-off as distributed over the three periods, and can plot them as occurring at the beginning of the final month of each period, without any great error. Thus, we get a very fair idea of the positions which the humps assume during the critical period of depletion, and also learn whether the storage necessary to equalise the yield over the driest year materially differs from 30 to 45 per cent. of the total yield of that year.

If the available records are plotted either in monthly or yearly mass curves, certain general principles become apparent.

(i) Firstly, apart from the evidence of an unusually dry series of rain-fall years, as indicating a probable sequence of years of small run-off, it appears that fully 40 years' records are necessary before we can be certain of having, even approximately, experienced the maximum depletion of a very large reservoir.

(ii) In Temperate climates, at any rate, it seems that if we attempt to develop a catchment area by storage, so as to yield considerably more than the average of three dry years, the effect of a chance flood, such as may be (and in the case of the Croton Reservoirs, actually was) produced by a series of summer thunder-storms, is so great that it is inadvisable to equalise the yield over perhaps more than five years. This, it should be remembered, will mean that no water will escape from the reservoir for periods of nearly five years on end.

(iii) It also appears that these developments lead to reservoirs of so large a capacity that, unless the topography is unusually favourable, the exposed water surface forms so high a percentage of the catchment area as to cause the effects of evaporation to be unduly marked.

An attempt has lately been made by Messrs. Gore and Brown (*The Central*, October, 1910), to arrive at a scientifically correct method of calculating the capacities of reservoirs. The method adopted is as follows:

The rain-fall record for a long period is treated by the methods of frequency curves developed by Pearson, and the probable value of the rain-fall for the driest year of a century is calculated. Similarly, the probable values of the average rain-fall of the two consecutive, three consecutive, etc., driest years of a century are calculated.

The method is obviously a refinement of Binnie's studies upon the variability of rain-fall (see p. 176).

The figures arrived at for a century, from a 72 years' British record, are:

Driest years' fall = 0·69 of mean annual rain-fall.
Average of 2 consecutive driest years' fall = 0·74 of mean annual rain-fall.

"	3	"	"	= 0·77	"	"
"	4	"	"	= 0·81	"	"
"	5	"	"	= 0·85	"	"
"	10	"	"	= 0·90	"	"

The annual rain-fall loss is assumed as constant, and the equalising reservoir for the driest year (see p. 232) is taken as 30 per cent. of that year's run-off, as calculated on the assumption of a constant rain-fall loss of 15 inches per year. This figure of 30 per cent. is obtained from the worst recorded English years' flow, as given by Deacon (*Ency. Brit.*, article on "Water Supply"). (See Sketch No. 55).

The method is applied in the following manner:

$x_m = 66{\cdot}5$ inches $z_m = 15$ inches, so that $y_m = 51{\cdot}5$ inches
and the minimum y, is about 30·9 inches.

The equalising reservoir for that year is about 9·3 inches capacity. The reservoir capacities required for any other supply are then determined by the mass curve method already explained. The results agree very well with Rofe's rule when the three dry years' supply is considered, and the equalising reservoir necessary to give a yield at the rate of y_m, over the whole century has a capacity of 103 inches, say 730 days' supply.

The circumstances assumed are favourable. For instance, Pole ("Lectures on Water Supply") found 930 days' supply for a case where x_m, was apparently 45 inches, and y_m, was 14 inches. Freeman's mass curve for the Croton Reservoir indicates that approximately 1040 days' supply is required.

The method is logical, and can be applied to any circumstances when the requisite information is obtainable.

Locality.	Area.	Number of Years.	x_m	z_m	Remarks.	Reference to Original Authority.
Torquay, Devon	961 acres	23	40·8	16·1	Very accurate.	Ingham, *Rain-fall and Evaporation in Devonshire.*
		Min.	27·5	13·1	Driest year also.	
		3D	34·3	15·2		
Thames Valley	3855 square miles	18	26·4	18·3	Very accurate.	*P. I. C. E.*, vol. 167, p. 190.
		9	27·0	18·5	Do.	Binnie, *Report on Flow of Thames.*
		Min.	22·8	17·5		
		1D	21·3	13·6		

[*Table continued*

Table continued]

Locality.	Area.	Number of Years.	x_m	z_m	Remarks.	Reference to Original Authority.
Woodburn, Ireland	3405 acres	14	38·4	13·7		Leslie, *Trans. of Roy. Soc. of Scotland.* 1870–71.
		Min.	28·8	14·2	Driest year also.	
		3D	32·8	13·7		
Wandle	...	14	...	21·5	Tributaries of Thames.	B. Latham, *Q.J. Meteorological Soc.*, 1892.
Graveney	...	14	...	19·7	Accurate.	
Entwhistle	3·2 square miles	24	51·5	17·0	It is suggested that the gauges show less than the true rain-fall.	Swindlehurst, *Trans. Assoc. of Water-Works' Engs.*, vol. 8, p. 12.
Heaton	...	24	39·8	12·7		
Belmont	·2·8 square miles	24	57·0	16·9		
Rivington	...	5	43·9	10·4	These are abnormal, but Mr. Binnie's authority is high.	Binnie, *Lectures on Water Supply.*
Longendale	...	12	52·4	9·5		
Exmouth, Devon	351 acres	3	25·8	20·1	Three dry years.	Hutton, *Engineer.* July 16, 1909
	290 acres	3	26·1	17·7	Do.	
Warrington	...	1	27·5	19·0	Very dry year.	*P. I. C. E.*, vol. 52, p. 34
		1	25·5	18·0	Do.	
Whittledean	...	1	17·7	11·4	Do.	Do.
Sheffield	5000 acres	3	35·0	15·01	Dry years, very accurate.	*P. I. C. E.*, vol. 167, p. 212.
Do.	...	1	...	15·05	Do.	The rain-fall is not given in original reference, but is abstracted from Symon's *Brit. Rain-fall*, 1905.
	...	1	36·5	15·5	Accurate, possible error 0·3 inch.	
	...	1	34·9	16·0		
	Larger area	1	31·7(?)	15·6		
Rotherham	...	2	21·3	18·6	Dry years.	Do.
Doncaster	...	3	25·8	22·3	Do.	Do.

[*Table continued*

Table continued]

Locality.	Area.	Number of Years.	x_m	z_m	Remarks.	Reference to Original Authority.
Northampton	...	1	...	17·0	First year after construction.	...
Boston, Lincs.	1920 acres	2	16·6	13·6	Two dry years.	Rodda, *Notes on Water Supply*.
Leicester .	10,760 acres	3	21·7	15·5	Three dry years.	*Trans. of Water-Works' Engs.*, vol. 7, p. 191.
Unspecified	6000 acres	17	61·0	14·0	...	...
		Min.	41·9	10·6	Driest also.	
		3D	56·8	20·4	...	
Do. .	572000 acres	5	20·4	12·8	...	Unwin, *Hydraulics*, p. 249
Hallington, Northumberland	...	...	38·9	27·6	Loss from $19\frac{4}{72}$ to $1\frac{11}{74}$.] Max. recorded in England	*P. I. C. E.*, vol. 52, p. 34

Catchment Areas situated in Climates of the Second Type.—The second type of climate (*i.e.* a well defined wet season succeeded by a well defined dry season, during which the catchment area becomes thoroughly dry) occurs over nearly the whole of India.

The best records exist in the Reports of the Bombay Public Works (Irrigation) Department, and refer to the large storage reservoirs which are used for irrigation in the Deccan. Some records also exist of reservoirs for town water supply and irrigation in Rajputana, and the Central Provinces. The information has been collected by Strange (*Indian Storage Reservoirs*), and the table on page 241 gives his ideas on the subject.

It will be noticed that if the wet season rain-fall exceeds 48 inches, the increase in run-off produced by 2 inches extra rain-fall is greater than 2 inches. This seems somewhat peculiar, but is not necessarily impossible, especially if the catchment area is permeable. The difficulty does not occur in average, or bad catchment areas. It will be plain that it is hardly safe to reckon on any run-off if the wet season rain-fall is less than 10 or 12 inches, and that the dry season rain-fall has no appreciable effect on the run-off.

The method adopted is plainly an assumption that $y = px$, where p, increases

UNITED STATES CATCHMENT AREAS IN CLIMATES OF FIRST TYPE.

Locality.	Area in Square Miles.	Number of Years.	x_m	z_m	Remarks.
Sudbury	75·20	23	45·8	23·6	All these (except the Croton) and many others are tabulated in the Report on the Erie Canal of 1908. The differences in areas make the facts very useful as showing how small influence this factor has on the yearly means. The generally higher values for *z*, than English or German areas of approximately the same mean temperature, are explicable by the fact that the summer temperatures are much higher and the winter much lower.
$z = 20\cdot0 + 0\cdot083x$		Min. 1D	32·8	21·6	
		2D	34·1	21·4	
		3D	38·8	22·2	
Cochituate	190·0	35	47·0	26·7	
		Min. 1D	31·2	21·1	
$z = 15\cdot3 + 0\cdot24x$		2D	35·8	23·2	
		3D	38·3	23·5	
Mystic Lake	27·70	20	43·8	24·0	
		Min. 1D	31·2	21·9	
$z = 14\cdot3 + 0\cdot23x$		2D	35·2	23·0	
		3D	37·2	21·3	
Passaic	822·0	17	47·1	21·6	
$z = 15\cdot8 + 0\cdot12x$		Min. 1D	35·6	20·4	
		3D	38·9	20·8	
Tohickon	102·20	14	50·1	21·4	
$z = 10\cdot2 + 0\cdot22x$		3D	38·4	24·2	
Croton	338·0	32	48·1	25·1	Freeman's *Report on Water Supply of New York.*
		Min. 1D	36·9	24·3	
$z = 20\cdot5 + 0\cdot1x$		2D	42·3	21·3	
		3D	42·3	20·0	

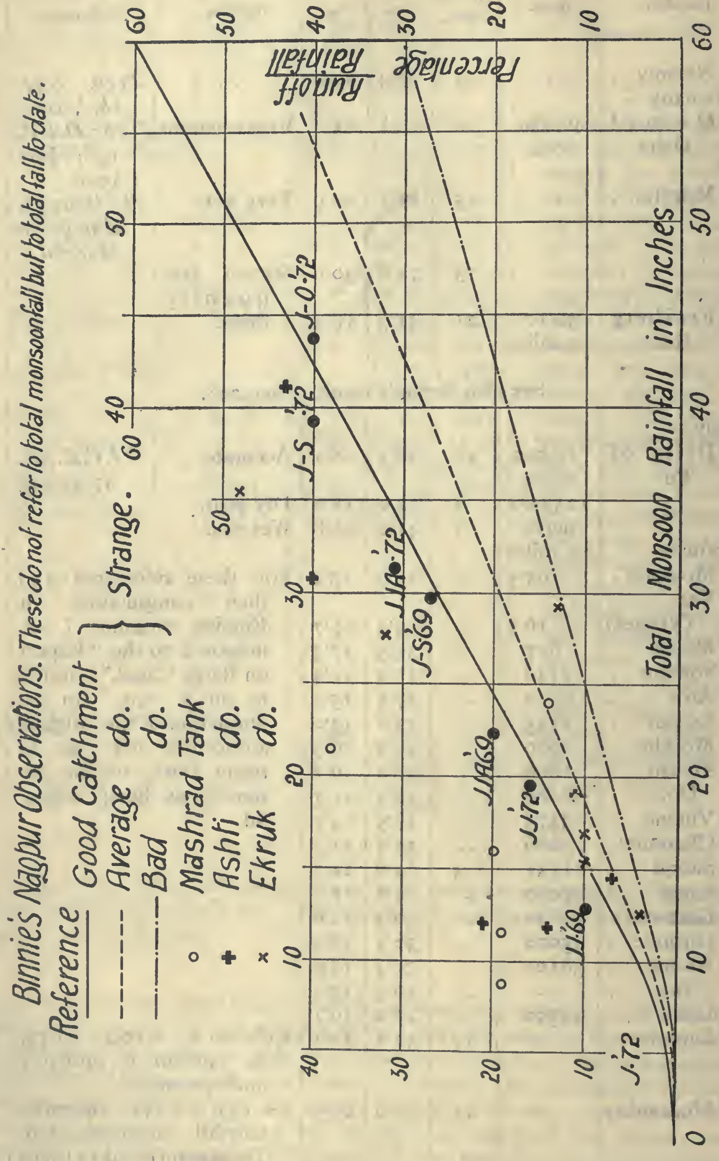
as x, increases. The rule does not agree with the results obtained in climates which belong to the first type. I have therefore endeavoured to find whether any formula of the type :

$$y = x - a - bx$$

can be discovered. The results are not encouraging, and there is no doubt that for the present the proportional method must be adopted for preliminary work. This fact is not surprising, for it must be remembered that the records of rain-fall are not as accurate as could be desired, as the rain-gauge stations are sparsely distributed, and are usually lacking at the very places where the rain-fall is greatest. It may indeed be said that most of the records are those of valley stations, and that most of the run-off is provided by the rain-fall on the neighbouring hills. Thus, *a priori*, the proportional method is

EUROPEAN CATCHMENT AREAS IN CLIMATES OF FIRST TYPE.

Locality.	Area.	No. of Years.	x_m	z_m	Remarks.	Reference.
S. Norway .	...	25	38·4	10·6	...	*Tech. Ugeblad*, 1907
Germany—						
Memel Delta	46,050 acres	8	26·1	15·8	Very accurate.	*Ztschr. D.I.V.*, 13th July 1909.
Murgtal .	...	15	61·1	24·9	Very wet.	*Die Wasserk. Anlage in Murgtal.*
		15	72·8	30·9	Stream frequently dries.	
Freiberg Res.	30·10 sq. miles	21	31·3	19·5		
See also Keller's results, page 208.						
Italy—						
Delta of Po	10,500 acres	10	28·5	18·1	Accurate.	*P.I.C.E.*, vol. 47, p. 147
	125,000 acres	1	24·0	18·0	Dry year.	
		1	41·0	25·8	Wet year.	
France—	Sq. miles					
Mousson .	163·5	...	29·0	17·4	For these references and their computation in English measure I am indebted to the "Report on Barge Canal," referred to on p. 240. An examination of the original authorities led me to reject three of the instances as being doubtful.	
Var (Vosges)	16·7	...	29·5	17·7		
Meuse .	607	...	31·5	17·3		
Somme .	2140	...	25·2	12·4		
Arde .	2510	...	27·5	15·4		
Escaut .	2545	...	23·6	13·0		
Moselle .	2600	...	29·5	20·3		
Meuse .	2896	...	28·3	16·2		
Do. .	8480	...	42·5	21·7		
Vilaine .	3475	...	27·5	13·2		
Charente .	3866	...	33·4	21·7		
Saone .	11551	4	32·6	12·7		
Seine .	27460	...	24·1	17·1		
Garonne .	32820	...	30·4	14·6		
Gironde .	35000	...	32·3	16·2		
Rhone .	38100	...	37·4	15·8		
Do. .	...	...	36·3	13·5		
Loire .	44500	...	27·2	16·1		
Durance .	...	17	32·1	10·1	Relation is $z = 10·2 - 0·07x$, so rain-fall is probably underestimated.	
Montaubry	...	15	33·0	23·5	$z = 13·8 - 0·24x$, therefore rain-fall underestimated. The stream dried 11 times in the 15 years.	



SKETCH No. 57—Typical Relations between Total Monsoon Rain-fall, and Total Run-off, in India.

STRANGE'S TABLE SHOWING THE PERCENTAGE OF RUN-OFF TO MONSOON RAIN-FALL, AND DEPTH OF RUN-OFF DUE TO RAIN-FALL IN INCHES, FOR A GOOD CATCHMENT AREA. (See Sketch 57.)

For an average Catchment Area, take three-quarters of these figures.
For a bad Catchment Area, take half these figures.

Total Monsoon Rain-fall in Inches.	Percentage of Run-off to Rain-fall.	Depth of Run-off due to Rain-fall in Inches.	Total Monsoon Rain-fall in Inches.	Percentage of Run-off to Rain-fall.	Depth of Run-off due to Rain-fall in Inches.
1	0·1	0·001	26	21·8	5·67
2	0·2	0·004	27	22·9	6·18
3	0·4	0·012	28	24·0	6·72
4	0·7	0·028	29	25·1	7·28
5	1·0	0·050	30	26·3	7·89
6	1·5	0·090	31	27·4	8·49
7	2·1	0·147	32	28·5	9·12
8	2·8	0·224	33	29·6	9·77
9	3·5	0·315	34	30·8	10·47
10	4·3	0·430	35	31·9	11·17
11	5·2	0·572	36	33·0	11·88
12	6·2	0·744	37	34·1	12·62
13	7·2	0·936	38	35·3	13·41
14	8·3	1·162	39	36·4	14·20
15	9·4	1·410	40	37·5	15·00
16	10·5	1·680	42	39·8	16·72
17	11·6	1·972	44	42·0	18·48
18	12·8	2·304	46	44·3	20·38
19	13·9	2·641	48	46·5	22·32
20	15·0	3·000	50	48·8	24·40
21	16·1	3·381	52	51·0	26·52
22	17·3	3·806	54	53·3	28·78
23	18·4	4·232	56	55·5	31·08
24	19·5	4·68	58	57·8	33·52
25	20·6	5·15	60	60·0	36·00

likely to be the best suited for practical purposes, so long as the present distribution of the rain-gauge stations continues.

The best method of treating such observations as are usually available is that adopted by Binnie (*P.I.C.E.*, vol. 39, p. 1). The records considered are those of Nagpur (Central Provinces, India). The year has two well marked divisions, namely, the wet, or monsoon season (June to October), and the dry season (November to May). The rain-fall records are as shown in table on next page (see *ut supra*, and *P.I.C.E.*, vol. 110, p. 259).

The following facts are fairly plain, and are characteristic of all small catchment areas in climates such are now considered.

(*a*) Except in extremely abnormal years, which need not be taken into

Year.	Rain-fall during the	
	Monsoon.	Dry Season.
1854–1855 . .	48·40	2·06
1855–1856 . .	24·04	2·03
1856–1857 . .	44·33	2·77
1857–1858 . .	33·46	3·32
1858–1859 . .	31·87	4·21
1859–1860 . .	29·48	1·32
1860–1861 . .	44·50	5·00
1861–1862 . .	40·89	1·15
1862–1863 . .	43·29	3·61
1863–1864 . .	37·46	4·73
1864–1865 . .	28·96	6·87
1865–1866 . .	38·16	2·40
1866–1867 . .	41·01	4·18
1867–1868 . .	53·72	6·26
1868–1869 . .	19·28	0·88
1869–1870 . .	32·11	4·09
1870–1871 . .	37·34	2·29
1871–1872 . .	44·85	1·27
1872–1873 . .	39·82	2·70
Average of 19 Years . .	37·52	3·21

account in practice, the wet season rain-fall alone is effective in producing a run-off.

(*b*) The condition of the catchment area in respect to dryness of surface is certainly the same at the beginning of each successive wet season, and this statement can probably be made with equal accuracy concerning the ground water, except in very permeable catchment areas after abnormally wet years.

Thus, a given fall of rain measured from the beginning of each wet season will produce an approximately constant run-off. Any variations which occur are caused only by the manner in which the rain falls (as is partially indicated in columns 2 to 4, top of p. 247), and are quite independent of the fall during the preceding wet season.

Binnie, when preparing his final designs, possessed rain-fall records to 1873, and the observations on rain-fall and run-off shown in table on page 245.

At first sight the observations only afford two figures, that is to say :

In 1869 a rain-fall of 29·79 inches produced a run-off of 7·87 inches
" 1872 " 43·65 " " 17·46 "

A little consideration will show that each of the entries in Columns No. 4 and 5 can be considered as giving the run-off that would be produced by a wet

Year and Month.	Rain-fall.	Run-off.	Total Rain-fall since Commencement of Wet Season.	Total Run-off since same Date.	Ratio : Total Run-off / Total Rain
Year 1869—					
June 17th to July 31st .	12·76	1·25	12·76	1·25	0·098
August . .	9·61	3·36	22·37	4·61	0·20
September . .	7·41	3·26	29·79	7·87	0·268
Year 1872—					
June . .	6·77	0·32	6·77	0·32	0·047
July . .	12·70	2·88	19·47	3·20	0·16
August . .	11·82	6·59	31·29	9·79	0·31
September .	7·99	5·95	39·28	15·74	0·40
A break in the rains now occurred					
October . .	4·37	1·72	43·65	17·46	0·40

season rain-fall of the same total magnitude as the rain-fall that had occurred up to the end of the month considered.

Thus, up to the end of July 1872, 19·47 inches of rain produced 3·20 inches of run-off, and the observation may be considered as indicating that, since the whole wet season fall of 1868 was 19·28 inches, the run-off of that year was probably about 3·17 inches. Thus, theoretically at any rate, from observations taken over these two years we can assign the probable values of the run-off produced by any rain-fall which is less than 43·65 inches. In actual practice this statement is subject to qualification. The question is best investigated by considering what happens at the end of the wet season. As a matter of observation, the stream flow rapidly decreases in all such catchment areas, and the river draining this Nagpur catchment area (which is 6·6 square miles in area, and consists of steep trap rocks, which are but slightly covered with soil) is generally dry in 4 or 5 hours at the most after the last rainstorm. A study of Binnie's records of the reservoir levels shows that the reservoir surface rarely rises appreciably after 24 hours has elapsed since the last rain-fall. Thus, in this case the assumption is justified.

The question is best settled in any particular case by observations of the duration of stream flow. Thus, if the stream is usually found not to run dry for 5 or 6 days, it would probably be advisable either to correct the partial records obtained as above by allowing for the probable volume of the stored-up water which will later appear as stream flow (as discussed on p. 188), or to treat, by this method, only those total rain-falls at the end of which the stream ran dry before the next fall commenced.

As an example, I give the following record of rain-fall and run-off from a steep catchment area of 18 square miles, approximately 4 square miles of which were underlain by permeable strata :

Date.	Total Rain-fall to Date.	Total Run-off to Date.	Remarks.
June 16 . .	0	0	Rains began
" 30 . .	3·13	0	
July 7 . .	4·18	0·20	
" 15 . .	5·92	0·78	
" 16 . .	6·82	1·20	
" 25 . .	9·84	1·36	
" 28 . .	9·84	1·42	Stream runs dry
" 31 . .	10·15	1·46	Stream begins to run
August 9 . .	11·44	1·58	
" 11 . .	15·32	1·78	
" 19 . .	15·32	2·18	Stream runs dry

Here it is plain that only the following deductions can correctly be made:

A rain-fall of 9·84 inches produced a run-off of 1·42 inches
" 15·32 " " 2·18 "

In fact, Binnie's method is best suited for steep, and impermeable catchment areas; and may lead to entirely erroneous results if blindly applied to large, flat, and permeable areas, where the ground-flow contribution forms a material portion of the run-off.

Where properly applied to suitable areas, the method is a powerful one. Thus, Binnie predicted that the average monsoon fall of 37·52 inches might be expected to yield 14·23 inches of run-off, and that the average monsoon fall of the three driest consecutive years would be about 30 inches, and that one such year would have a run-off of 8·4 inches. The fall and run-off of the driest year were similarly predicted at 19·28 inches and 3 inches. Now, the actual results of observations extending over 18 years (as given by Penny) indicate that 23·3 inches of rain-fall produce an average run-off of 5·30 inches, and that the driest years have a run-off of 3·6 to 4 inches. Thus, the errors in prediction are almost entirely due to incorrect assumptions concerning the rain-fall variability (see p. 248).

Sketch No. 57 shows Binnie's results as plotted with the percentages of $\frac{\text{run-off}}{\text{rain-fall}}$ as ordinates, and the rain-falls as abscissæ, and it will be plain that the observations coincide very closely with Strange's curve for a good catchment area.

The principles are now obvious. We can regard each storm during the rains as a unit, if the stream draining the area runs dry after it ceases, and before the next storm occurs. Thus, if we analyse the results, we can usually arrive at a table of the type shown at top of page 247 (given by Strange).

This table, if properly applied to records of daily rain-falls, will produce results which are less subject to error than those given by Strange's formulæ for annual run-off, and such results are often found to agree extremely well with observation in small, steep, and impermeable areas.

Rain-fall in 24 Hours in Inches.	Ratio Run-off to Rain-fall. State of Catchment Area previous to the Rain-fall.		
	Dry.	Damp.	Wet.
$\frac{1}{4}$	Nil	Nil	0·12
$\frac{1}{2}$	Nil	0·10	0·14
1	0·05	0·14	0·20
2	0·10	0·25	0·34
3	0·20	0·40	0·55
4	0·30 – 0·40	0·50 – 0·60	0·70 – 0·80

In permeable and semi-permeable areas (especially if large) the problem is somewhat more difficult.

Owing to the fact that the river does not rapidly run dry, a year's observations will rarely provide more than two points on a curve of the type used by Binnie (see Sketch No. 57). The following table, which represents the results of observations on a flat catchment area of about 100 square miles in Bengal, may be considered as giving the best available information on this somewhat obscure subject :

Month.	Ratio Monthly Run-off to Monthly Rain-fall.	
	Ordinary Year.	Wet Year.
June . . .	0·05	0·10
July . . .	0·10	0·20
August . . .	0·25	0·50
September . .	0·40	0·50
October . . .	0·40	0·50

The figures are vague (although not more so than the difficulties of the subject warrant), but the difference between the columns fairly accurately represents the influence of the damper surfaces, combined with an increased storage of ground water.

In practice, however, such areas are not usually developed for purposes of water storage, so that the question is not very acute. Valuable information can be obtained by observing the ground-water level, and my own practice has been to estimate the water stored up in the invisible reservoir at various dates, and to tabulate as follows :

Date.	Total Rain-fall.	Total Visible Run-off.	Total Increment in Water Storage since Beginning of Rains.	Probable Total Run-off if Rains were to cease on this Date.

The figure in Column 4 is calculated from the observed rise in ground-water level by the formula given on page 188, and that in Column 5, is the sum of the terms in Columns 3 and 4. The assumption is obvious. We neglect any possible future depletion of the ground water by underground leakage, or by evaporation produced by vegetation.

The likelihood of loss by leakage can be estimated by a survey of the permeable strata. The second form of loss probably occurs, but may be considered to be balanced by the fact that we have neglected the water which is absorbed in producing a partial saturation of the upper layers of permeable soil above the ground-water level. This evaporation is probably only large in those localities where the subsoil-water level comes within 3 or 4 feet of the natural surface; but the nature of the vegetation has a great influence upon its amount.

Variability of the Wet Season Rain-falls.—Nearly all climates of this type fall into the exceptional class discussed on page 180, where the possible annual variability in rain-fall is larger than is generally the case. Thus, in a thirty years' record the minimum wet season rain-fall is frequently as low as 0·33 of the mean, and ratios of 0·20 or 0·25 are not unknown. Similar abnormalities occur when the average of the two or three driest successive years is discussed. The matter is now well understood both in India and in California, but must be borne in mind whenever climates of this type are dealt with in newly settled countries. In some of the earlier Indian projects a short period rain-fall record for the locality considered was compared with a long period record for Calcutta, and the variability ratios for Calcutta were believed to apply. This is now known to be erroneous. A similar error is likely to be made in other countries, unless pointed out; as long period records in a newly developed country are generally confined to the coastal districts, where the variability ratios are usually normal in type.

Capacity of Reservoirs.—The determination of the capacity of the equalising reservoir under such circumstances is a simple matter. In view of the periodic oscillations of the time at which the wet season begins and ends, we must generally assume that no reservoir will suffice, even for one year, unless it holds a supply sufficient for 365 days at the end of the wet season. It is always possible that in one year the stream flow of the wet season may end say in August, and that in the next year the stream flow may not begin until August. A study of the yearly run-offs will enable us to determine whether we can rely upon the reservoir being refilled in the driest year, and if not, a capacity approximately equal to a supply extending over two years, or 730 days, is obviously required. As a matter of practice, a study of the capacities of Indian and Californian reservoirs for town water supplies, or for permanent irrigation (*i.e.* such crops as fruit trees, or lucern, where the crop cultivated is perennial), shows that the rules adopted are very much as follows.

Where calculation indicates that the run-off of the driest years will probably fill the reservoir, a capacity of 650 to 730 days' supply is provided.

If the driest year will not fill the reservoir, a capacity equivalent to about 1000 or 1100 days is provided. Cases exist where a capacity of 1400 and 1500 days' supply is found to be necessary.

For irrigation of such crops as wheat, and cotton, a smaller margin of safety is usually provided; and some Indian and Algerian reservoirs are designed on the assumption that once in 10 or 15 years no irrigation can be effected. The matter is evidently a question of finance, and in view of the fact that in moist climates crops are frequently damaged by excessive rains, the irrigators can hardly be considered to be very adversely situated.

Values of the Rain-fall Loss.—In the typical climate of this class the relation between x and z is not of much practical importance. The following records are of first class accuracy, but refer to Southern Indian climates where two wet seasons occur each year. It is probable that during most years the catchment areas become dry between the monsoons, but during the wetter years this certainly does not occur.

Locality.	Area.	x	y	z	Authority.
Mercara, Southern India, one year	Acres 48	119·18	44·31	74·87	*P.I.C.E.*, vol. 113, p. 312. No run-off in December to February inclusive
Labugama, Ceylon	2385	162·81	84·98	77·83	
	...	127·55	76·66	50·89	Driest year since 1897
Years 1905–7 inclusive.	...	155·79	114·60	41·19	Reports of P.W.D., Ceylon, for years concerned

Climates of the Third Type.—Here the run-off of each separate fall of rain is an independent unit; since, except in abnormal cases, the area is always dry, so that the first column of Strange's second table (see p. 247) may be considered as applicable. For example, Collins (*P.I.C.E.*, vol. 165, p. 271) assumes that near Johannesburg in the Transvaal:

- Falls of less than 1 inch in a day produce no run-off.
- Falls of between 1 and 2 inches per day produce a run-off equal to 0·20 of the rain-fall.
- Falls exceeding 2 inches per day produce a run-off equal to 0·40 of the rain-fall.

The assumption is stated to be favourable, and a yearly run-off equal to 0·07 of the yearly rain-fall cannot be relied upon. The determination of the reservoir capacity is effected by applying these, or similar figures, to the records of daily rain-falls over long periods.

In certain instances, where the catchment area is large, the river will be found to have a perennial, or approximately perennial flow. In such cases gauge records usually exist, and may be employed to estimate the yearly or monthly run-offs. The studies of daily rain-falls may then be applied to investigate whether the river is ever likely to run dry. It must, however, be remembered that the "rain-fall" over a large catchment area of this type is usually only an ideal figure, as those falls of rain which produce any run-off are probably only local, torrential downpours.

The uncertainties are admirably illustrated by the following record of the yearly run-offs of the Sweetwater (California) catchment area.

Since even the deepest natural bodies of water existing in such climates are known to dry up by ordinary evaporation during intense droughts (say three or four times in a century), a permanent water supply, such as can be obtained in moister climates, must probably be set aside as an unattainable ideal.

In cases where it is necessary to provide a permanent water supply in such climates the problem is usually solved in one of the three following ways:

(*a*) Frequently by deep wells, which, in many cases, develop the underground flow of a well-marked subterranean water channel. This is the normal method of supplying cities and oases in desert climates. Any discussion of the methods of discovering such supplies is futile. Where they exist, a desert city or oasis will be found. The publications of the Egyptian Survey Department, and of the United States Geological Survey on Desert Water Supplies may be consulted. Beadnell (*An Egyptian Oasis: Kharga*) describes a case where the supply is semi-artesian, and sufficiently copious to form a basis for large scale agricultural operations.

(*b*) The water is stored in high lands adjacent to the desert, and is delivered by long conduits. The city of Los Angelos, Cal., although hardly a desert city, is supplied in this manner, since the population is now too large to be fed by storage or underflow developments of the adjacent country which has a climate of the second type.

(*c*) Selected areas of rocky ground are rendered impermeable, and the whole rain-fall is collected in deep tanks. The typical example is Aden, and for military reasons Gibraltar is also thus supplied, (see p. 256).

Records of the Sweetwater (*California*) *Catchment Area.*—The following particulars are taken from Schuyler (*Reservoirs*, p. 233). The catchment area is 186 square miles, and the elevation above sea level varies from 220 feet (at the dam) to 5500 feet in the mountains bounding the catchment area. The mean elevation is about 2200 feet. The rain-fall is that which is recorded at the dam, and certainly does not represent the mean rain-fall over the whole catchment area. While the recorded rain-fall may bear some relation to the mean rain-fall over the whole area, it is believed that the run-off of similar Californian catchment areas is mainly produced by the far heavier local rain-fall which occurs in the higher portions of the catchments. The circumstances may therefore be regarded as analogous to those of the Melbourne catchment area (see p. 201), but the difference in rain-fall produced by changes in elevation is probably far greater.

The reservoir originally had a capacity of 18,053 acre-feet (1 acre-foot equals 43,560 cubic feet), say 1½ times the mean annual run-off. In 1896 it was enlarged to 22,566 acre-feet. The reservoir was completely dry by 1899. Tube wells were therefore sunk in the reservoir bed, and infiltration galleries excavated in the river bed below the dam. By pumping from these sources sufficient water was obtained for domestic purposes, and in addition a "depth of water equal to 0·28 feet" was applied to the citrus trees which were usually irrigated from the reservoir. This amount enabled the trees to be kept alive from May to the 23rd November 1899. Similar pumping became necessary in 1900. The history is typical of that of many reservoirs in arid countries, and it is hard to see what more could be done, as the reservoir loses 15 per cent. of its capacity each year by evaporation. The real lesson is that investigations in search of ground-water supplies should be made in all similar cases, and that they should not be deferred until the reservoir shows signs of failure.

Year.	Total Run-off.		Yearly Rain-fall at the Dam in Inches.
	In Acre-Feet.	In Cusecs per Square Mile.	
1887–1888	7,048	0·0524	Not given
1888–1889	25,253	0·1875	13·53
1889–1890	20,532	0·1525	16·52
1890–1891	21,565	0·1602	12·65
1891–1892	6,198	0·0460	9·88
1892–1893	16,261	0·1210	11·62
1893–1894	1,338	0·0099	6·20
1894–1895	73,412	0·5452	16·19
1895–1896	1,321	0·0098	7·29
1896–1897	6,892	0·0512	10·97
1897–1898	4·3	0·00003	7·05
1898–1899	246	0·0018	5·05
1899–1900	0	0·0	5·54
1900–1901	828	0·0061	7·05
1901–1902	0	0·0	4·86
1902–1903	0	0·0	5·72
1903–1904	0	0·0	6·39
1904–1905	13,760	0·1022	15·55
1905–1906	35,000	0·2600	15·52
1906–1907	30,000	0·2228	12·88
Average	12,983	0·0964	9·52

Victorian Records of Rain-fall and Run-off.—The following values of the rain-fall and run-off of various catchment areas in the State of Victoria (Australia) are abstracted from Stuart Murray's publication (*River Gaugings of the State of Victoria*, Melbourne, 1905). The results are extremely interesting for the following reasons:

(*a*) The State of Victoria, north of the Dividing Range represents precisely that class of condition where knowledge of the character now discussed proves most valuable. The rain-fall is sufficient to support a population which desires a good water supply, but the run-off is not so abundant as to permit the designing engineer to guess at random and be none the worse. The supply must be obtained from catchment areas of relatively small size, and snow-fed rivers are absent.

(*b*) As usual, in such cases, the rain-fall records are probably less accurate than the run-off records, which are quite as accurate as any records referring to rivers of the same size.

On the other hand, the climate of the State does not fall into any one of the

River.	Area in Square Miles.	Rain-fall.	Run-off.	Remarks.
YARRA	972	$x_m = 44{\cdot}9$	$y_m = 14{\cdot}7$	14 years.
The name means "ever flowing," and the river probably never dries.	A large proportion of the area is mountainous, and extremely heavily timbered.	Minimum—	Minimum—	Same year.
		$x = 35$	$y = 7.4$	
		Maximum—	Maximum—	Same year, two records.
		$x = 55$	$y = 20{\cdot}4$	
		$x_{2D} = 38{\cdot}5$	$y_{2D} = 9{\cdot}2$	This area is on the coastal side of the dividing range, and the climate is of Class I.
		$x_{3D} = 40$	$y_{3D} = 10{\cdot}1$	
GOULBOURN	3966	$x_m = 34{\cdot}5$	$y_m = 10{\cdot}3$	24 years.
An almost perennial stream. Was dry once during the record.	Mostly undulating, but rises in thickly timbered mountains.	Minimum—	Minimum—	Same year. Dry this year.
		$x = 23$	$y = 3{\cdot}0$	
		Maximum—	Maximum—	Same year.
		$x = 52$	$y = 18{\cdot}2$	
		$x_{2D} = 27$	$y_{2D} = 6{\cdot}5$	
		$x_{3D} = 28$	$y_{3D} = 6{\cdot}5$	
MACKENZIE	29	$x_m = 34{\cdot}1$	$y_m = 12{\cdot}7$	19 years.
This river dries up frequently. During the record it was dry one year, or other, in every month except November, December, and April.		Minimum—	Minimum—	Same year.
		$x = 25$	$y = 4{\cdot}0$	
			$y = 5{\cdot}0$	Another year of equal rain-fall.
		Maximum—	Maximum—	Same year.
		$x = 50$	$y = 17{\cdot}6$	
		$x_{2D} = 27{\cdot}5$	$y_{2D} = 7{\cdot}7$	
		$x_{3D} = 27{\cdot}7$	$y_{3D} = 9{\cdot}1$	
CAMPASPIE	1362	$x_m = 25$	$y_m = 1{\cdot}8$	20 years.
This river dried up in 19 years out of the 20, and sometimes remains dry from November to May.		Minimum—	Minimum—	Same year.
		$x = 16$	$y = 0{\cdot}03$	
		$x = 19$	$y = 0{\cdot}76$	
		$x = 19$	$y = 0{\cdot}95$	

CAMPASPIE (*contd.*)		Maximum—	Maximum—	
		$x = 36$	$y = 5.5$	Same year.
		$x = 36$	$y = 4.2$	Another year.
		$x_{2D} = 18$	$y_{2D} = 0.3$	
		$x_{3D} = 20$	$y_{3D} = 0.8$	
WIMMERA	1530	$x_m = 21.9$	$y_m = 1.5$	17 years.
		Minimum—	Minimum—	No run-off.
		$x = 13$	$y = 0.0$	
		$x = 17$	$y = 0.03$	Next driest year.
		$x = 17$	$y = 0.27$	Same rain-fall.
		Maximum—	Maximum—	Same year.
		$x = 34$	$y = 3.5$	
		$x = 26$	$y = 5.5$	Same year.
		$x = 30$	$y = 4.7$	A year of greater run-off than the wettest year.
		$x = 28$	$y = 3.1$	No other years occur with x, greater than 26 inches.
		$x = 27$	$y = 1.7$	
WIMMERA	768	$x_m = 22.6$	$y_m = 1.7$	21 years.
River ran dry 19 times in the record. Once dry from December to June.		Minimum—	Minimum—	
		$x = 13.5$	$y = 0.07$	Same year.
		Maximum—	Maximum—	
		$x = 31$	$y = 3.47$	Same year.
		$x = 26$	$y = 3.77$	Same year.
		$x = 31$	$y = 3.13$	Same year.
		$x = 29$	$y = 2.4$	
		$x = 26$	$y = 1.25$	
		$x_{2D} = 16.7$	$y_{2D} = 0.3$	
		$x_{3D} = 17.8$	$y_{3D} = 0.5$	

three typical classes. There is no sharp division into wet and dry seasons, as in Northern India; nor is any portion of the State a desert, since wheat is grown without irrigation all over the land, but it is believed that every river in the State, even the largest, ceases to flow once in a century, if not more frequently. The climate is probably best described as belonging to the second class, but the catchment areas are frequently not thoroughly dry for three or four years in succession. Further details are given in connection with the individual areas (see pp. 252–3).

The original paper gives the rain-fall and run-off by months.

An abstract of yearly rain-falls and run-offs, including several rivers of somewhat similar characteristics in the Western United States, is given by Fuertes (*P.I.C.E.*, vol. 162, p. 148). It is believed that the rain-fall records are less accurate than those tabulated by Stuart Murray. The run-off records are also probably slightly less accurate in some cases, but I do not possess the detailed information that is at my disposal for the Victorian records.

Secondary Catchment Areas.—A site is frequently found where a reservoir can be economically constructed of a capacity greater than that required for the catchment area which would naturally feed the reservoir. In such cases, works are sometimes formed in order to divert the flow from other catchment areas into the enlarged reservoir. So also, when the yield of a catchment area has been overestimated, it is frequently necessary to supplement it by diversion from other catchment areas.

Such works usually consist of a diverting dam, or weir, across the natural drainage channel, with a connecting channel to carry the water collected across the natural line of water parting into the main catchment area.

The design of a diversion dam, or weir, is not a difficult matter. The capacity of the diversion channel needs careful consideration, since the circumstances calling for the inclusion of a secondary catchment area require the collection of as much water as is consistent with economy. The construction of the diversion channel will probably prove costly, owing to the deep excavation entailed by crossing the line of natural water parting.

A sufficient number of examples does not exist to enable any useful rules to be drawn from general experience. The Vyrnwy watershed includes two secondary catchment areas, as follows:

> The Cownwy, of 3092 acres, with a diversion channel of 120 million gallons daily capacity.
>
> The Marchnant, of 1650 acres, with a tunnel of 88 million gallons daily.

These two combined areas are said to yield about 10 million gallons daily. Since the yield of the remaining 18,000 acres is about 55 million gallons daily, it would appear that the diverted areas only contribute about 70 per cent. of the normal yield per acre. The rain-fall over the catchment area is irregular, and no very satisfactory deductions can be drawn. The run-off observations are known to have been systematically carried out for more than 20 years. We may consequently say that in this particular case of a catchment area of high, but somewhat irregular yield, a diversion channel with a capacity of approximately twenty times the average daily yield of three dry years proved economical.

Usually, it may be assumed that before the diversion channel need be finally designed, the engineer will have run-off records of approximately four or five years to study. These, combined with a consideration of the capacity of the reservoir (usually only a small one), formed by the diversion dam, will enable him to draw up tables showing the total volume of water diverted when the channels are of various assumed capacities. These, together with the estimates of constructional expense, enable a selection to be made of that channel which yields the greatest volume of water in proportion to its cost. Such a channel may be assumed to be too large, since what we require is a somewhat smaller channel yielding the greatest volume of water during the critical period of depletion (*i.e.* usually the period of three successive dry years). It would generally appear that a channel of about 80 per cent. of the above capacity best suits the circumstances.

There is another aspect of the matter, which is not only a useful guide for preliminary estimates, but must be considered in the final designs. The channel should not be liable to silt up, and a study of the bed of the natural stream will usually enable us to make a very fair estimate of the flow which is not frequently exceeded. This may be roughly estimated as the discharge corresponding to the "bank stage" of the natural channel. If the diversion channel is proportioned so as to carry this discharge with approximately the same velocity as that occurring in the natural channel, we may feel secure against any considerable deposit of silt or stones. If, however, this capacity of channel is much exceeded, a somewhat higher mean velocity is necessary. Failing this, the larger channel may rapidly adjust itself to a discharge capacity which is exactly that which the stream can keep free from deposits. In that case, somewhat costly yearly cleanings will become unavoidable. The area of the natural channel, and the quantity of silt carried by the stream should therefore be carefully studied. If silt is an important factor, it will be found advisable to design the diversion channel of a capacity not greater than that indicated by the discharge of the natural stream when running bank full. Under such conditions, in a British catchment area, we may generally reckon on securing about three-quarters to five-sixths of the total run-off in a dry year. It would therefore appear that the Vyrnwy diversion channels have been proportioned with a somewhat smaller capacity than even this rule indicates, unless the dry year yields of the Vyrnwy catchment area are proportionately higher than is usually the case.

Coghlan (*P.I.C.E.*, vol. 75, p. 187), discusses the question of channels for draining secondary catchment areas in New South Wales, and gives a formula which indicates that a channel with a capacity equal to the mean discharge of a stream, will actually carry off a fraction equal to :

$$\left(0{\cdot}25+\frac{x}{12y}\right) \text{ of the total discharge ;}$$

where x, is the rain-fall, and y, the run-off in inches per annum. He also states that if the capacity is equal to :

75	per cent. the mean discharge,		0.835 of the above quantity is secured.		
50	"	"	0·634	"	"
25	"	"	0·380	"	"
12½	"	"	0·253	"	"
7½	"	"	0·160	"	"
5	"	"	0·118	"	"

Carrying the results to three figures is obviously excessively exact, but the formula is founded on six years' experience of two hilly catchment areas.

Coghlan's actual figures show that in the worst year the fraction delivered is 18 to 20 per cent., when the capacity is equal to the mean yield. This fraction decreases far less rapidly than his table indicates. The mean figures are :

Mean of both Results for	Capacity of Channel.				
	Mean Run-off.	½ ditto.	¼ ditto.	⅛ ditto.	$\frac{1}{20}$ ditto.
Worst year . . .	18·8	14·0	10·3	7·7	4·5
Best year . . .	62·7	37·3	21·3	11·6	5·0

The figures being the percentages of the total run-off actually delivered during that year, and the best year's yield being in the one case four, and in the other 6·5 times that of the worst year.

The above figures refer to a catchment area in New South Wales, but a study of the run-off records does not indicate that the flood and low water periods are more marked than is usually the case in mountainous catchment areas.

If applied to a level area containing a large amount of permeable strata, the channels might possibly be diminished in size. They should certainly be increased in climates of the second class, where the whole run-off occurs during a short period of the year only. It may be suggested that if the main daily yield is then calculated not over 365 days, but over the average duration of the wet season, the results will not err greatly.

In dry climates (especially those belonging to the third class), systematic drainage of the catchment area will greatly increase the run-off. The usual methods consists of contour drains, about 6×4 inches traced by a plough which throws the sod on to the lower side of the drain.

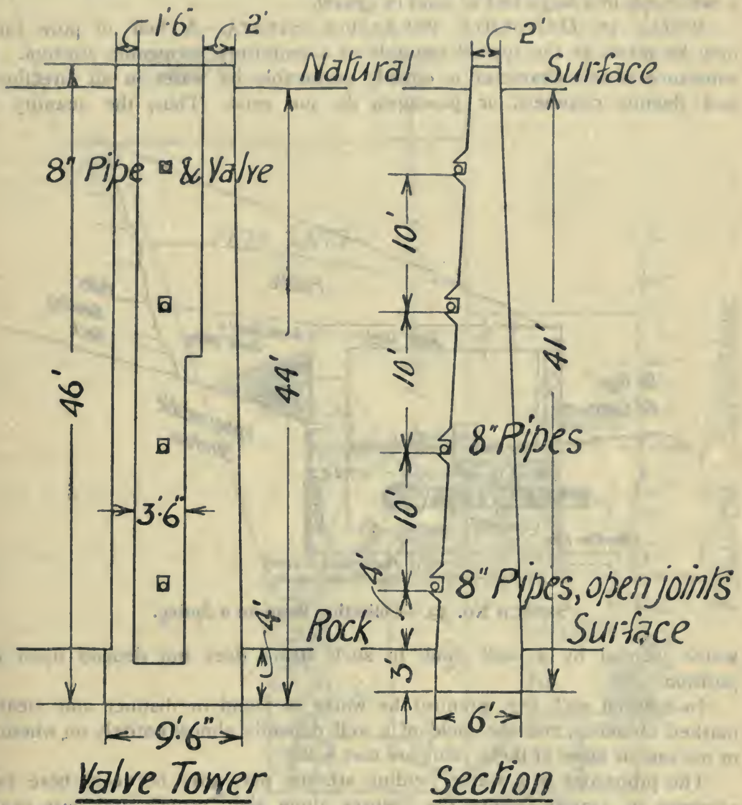
In actual work in the northern parts of Victoria (with $x=35$, to 40, and $z=25$, to 30 inches) I have noticed that such a section, although producing a noticeable increase in run-off, is too small. Better results are obtained if the drains can carry about two cusecs per 100 acres, even in well grassed areas of 300 to 400 acres.

The principle may be extended. The rock areas used for collecting water at Aden and Gibraltar illustrate its extreme development, where, after carefully cementing all cracks, and draining all hollows, a yield equal to 90 or 95 per cent. of the rain-fall is secured.

COLLECTION OF WATER FROM OTHER SOURCES THAN STREAM FLOW.—It is not proposed to discuss the methods of discovering springs and underground flows of water. The only scientific methods are geological, and, unfortunately, the experience acquired in, say, the British Isles can only be utilised in other countries by a skilled geologist. For, while geologists consider strata as permeable, or impermeable, the geological classification of strata pays no regard to this characteristic. Hence, an enumeration of British water-yielding strata would be worse than useless if applied to Indian conditions.

The question of the determination of the volume of water flowing in an alluvial bed has already been considered (see p. 205). Sketch No. 58 shows an underground dam for the collection of the "underflow" in a gravel bed. It may be remarked that the results of such methods are generally disappointing, but it is believed that the preliminary estimates were usually vague and unscientific. So far as I can judge from three personal experiences, an engineer's connection with such schemes will usually be confined to preliminary investigations, and the compilation of a report showing that the scheme is not economically profitable.

Sketch No. 59 shows a design for the collection of water from a spring.



SKETCH No. 58.—Dam for Collecting "Underflow" Water.

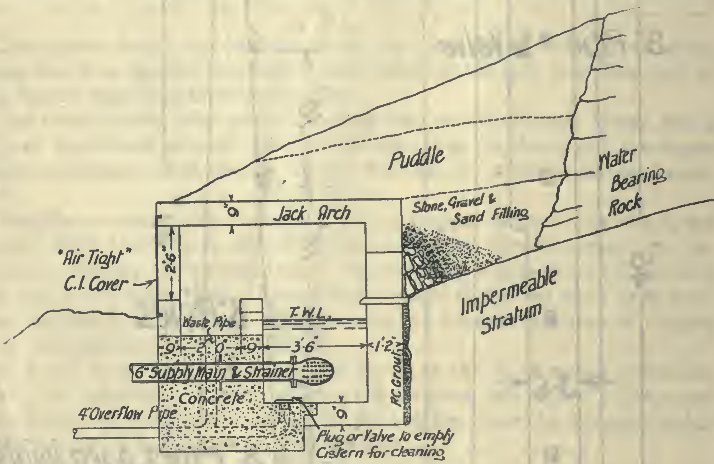
Sketch No. 60 shows two typical German designs for the collection of water by catchment galleries: (*a*) from a permeable stratum underlain by an impermeable stratum, and (*b*) from a deep bed of sand or gravel.

In the above cases geological surveys should be undertaken to ascertain the area that supplies the spring or underground flow. Such developments, if carefully studied, are usually successful, and are very widely adopted for town water supplies in Germany and Austria. Further details are given under Wells.

The dune sand water supplies of the Hague, Leyden, and other Dutch cities, may also be referred to. In these cases (see *Engineering*, 1889, p. 249), the rain water absorbed by the dune sands is collected either by open trenches, or by agricultural drain pipes. The local conditions need careful study, as the fresh water floats on a body of salt water derived from the adjacent sea. In some cases the collection is effected by tube wells.

The principles concerning the collection of all underground water supplies, except springs, are best illustrated by a careful consideration of the problem of a well sunk in a large bed of sand or gravel.

Wells in Uniformly Permeable Strata.—A bed of pure sand may be taken as the typical example of a uniformly permeable stratum. A substance of this character is equally permeable by water in all directions, and definite channels, or passages, do not exist. Thus, the quantity of



Sketch No. 59.—Collecting Basin for a Spring.

water yielded by a well sunk in such strata does not depend upon its position.

In fissured rock (*e.g.* granite) the water is found in distinct and clearly marked channels, and the yield of a well depends almost entirely on whether or not one or more of these veins are met with.

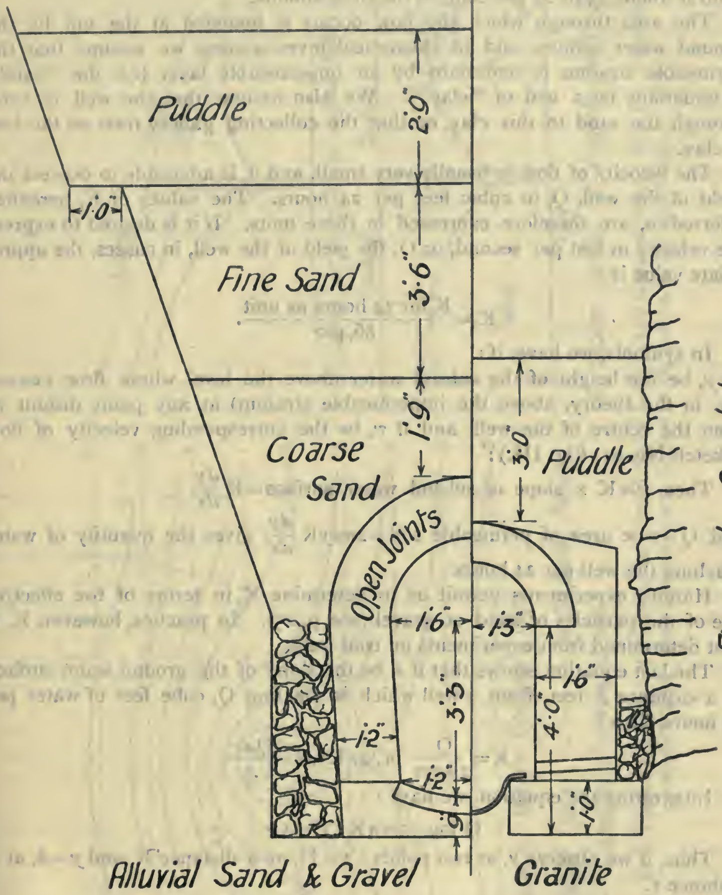
The properties of a water yielding stratum may vary between these two extremes of granite (where the fissures alone yield water), and pure sand, where a defined fissure cannot exist. For example, the chalk which yields so much water in the country round London has properties almost midway between granite and sand. A well sunk in chalk will always give some water, owing to regular percolation similar to that occurring in sand; but, in many cases, the cutting of a fissure, either by the well, or by an adit, greatly increases the yield.

The yield of a well in uniformly permeable strata is determined by two factors:

(*a*) The permeability of the stratum, which depends on the size of the interstices between the individual grains.

(*b*) The natural slope of the ground water surface. This, of course, depends partly on the permeability of the stratum; but, putting this aside, the greater the natural slope, the greater is the available supply of water.

Temporary Yield of a Well.—Consider the capacity of the well from the



SKETCH No. 60.—Cross-sections of Collecting Gallery in Sand or Gravel, and in Fissured Rock.

point of view of a machine for sucking water from the subsoil; and, for the moment, neglect the question whether the subsoil can permanently supply the water which the well can draw.

The flow of water through the pores of the subsoil is evidently capillary. We therefore have:

Velocity of flow = K × slope of subsoil water surface, and the quantity of water delivered is equal to the velocity of the flow multiplied by the permeable area through which flow occurs.

The velocity of flow above defined is consequently that of a solid column of water, and the actual velocity of the water in the pores of the soil is from three to four times as great, since the void space in such substances as wet sand is about 33 to 25 per cent. of the total volume.

The area through which the flow occurs is bounded at the top by the ground water surface, and in theoretical investigations we assume that the permeable stratum is underlain by an impermeable layer (*i.e.* the "sand" is underlain by a bed of "clay"). We also assume that the well is sunk through the sand to this clay, or that the collecting gallery rests on the bed of clay.

The velocity of flow is usually very small, and it is advisable to express the yield of the well, Q, in cubic feet per 24 hours. The values of K, hereafter referred to, are therefore expressed in these units. If it is desired to express the velocity in feet per second, or Q, the yield of the well, in cusecs, the appropriate value is:

$$K_s = \frac{\text{K for 24 hours as unit}}{86{,}400}.$$

In symbols, we have, if:

y, be the height of the subsoil water above the level where flow ceases, (*i.e.* in the theory, above the impermeable stratum) at any point distant x, from the centre of the well, and if v, be the corresponding velocity of flow (Sketch No. 61, Fig. III.):

Then, $v = \text{K} \times$ slope of subsoil water surface $= \text{K}\,\dfrac{dy}{dx}$

and $\text{Q} = v \times$ area of permeable soil $= 2\pi xy\text{K}\,\dfrac{dy}{dx}$, gives the quantity of water reaching the well per 24 hours.

Hazen's experiments permit us to determine K, in terms of the effective size of the particles of sand or gravel (see p. 25). In practice, however, K, is best determined from experiments on trial wells.

The last equation shows that if s, be the slope of the ground water surface at a distance x feet, from a well which is yielding Q, cube feet of water per 24 hours, then:

$$\text{K} = \frac{\text{Q}}{2\pi xys} \text{ or, } 2\pi\text{K}y\,dy = \frac{\text{Q}\,dx}{x}.$$

Integrating the equation, we have:

$$\text{Q} \log_e x = \pi\text{K}\,(y^2 + \text{C})$$

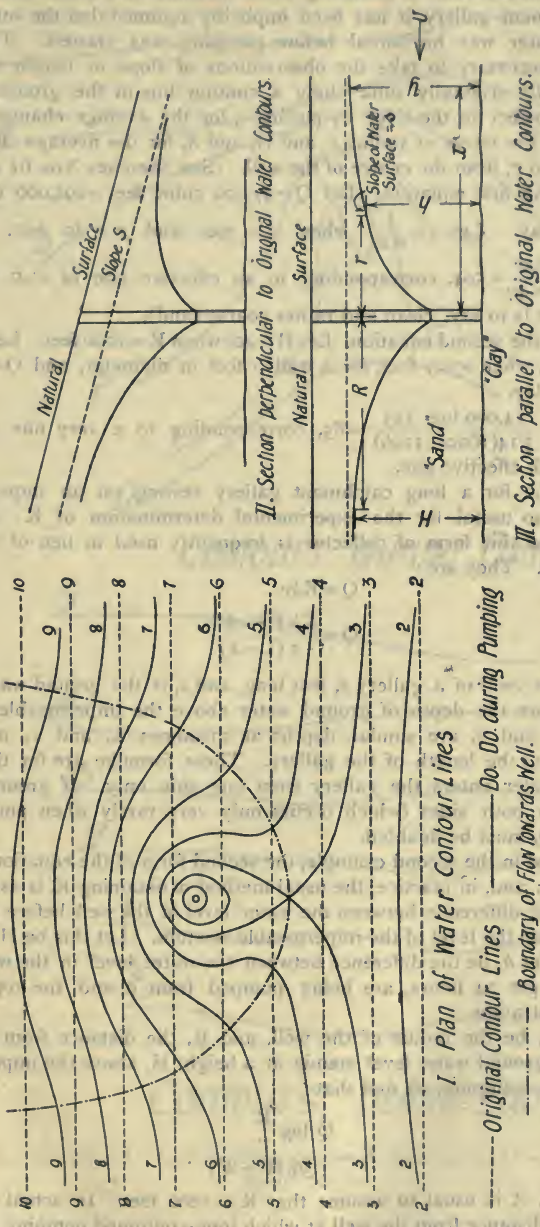
Thus, if we observe y, at two points: $y = \text{H}$, at a distance R, and $y = h$, at a distance r.

$$\text{Q} \log_e \frac{x}{r} = \pi\text{K}\,(y^2 - h^2), \quad \text{whence } \text{K} = \frac{\text{Q} \log_e \frac{\text{R}}{r}}{\pi(\text{H}^2 - h^2)}$$

or, substituting for π, and using logarithms to the base 10, we have:

$$\text{K} = \frac{\text{Q} \log \frac{\text{R}}{r}}{1{\cdot}36\,(\text{H}^2 - h^2)}$$

Either of these equations serves to determine K.



Sketch No. 61.—Theoretical Effect of Pumping on Ground Water Levels, with Cross-sections. Fig. III. also shows Flow towards a Well when Ground Water Surface is originally a Horizontal Plane.

It must be noticed that in deducing these formulæ and those given later on for a catchment gallery, it has been implicitly assumed that the surface of the ground water was horizontal before pumping was started. Thus, in practice, it is necessary to take the observations of slope or height along a line of test wells originally sunk along a contour line in the ground water surface, or to correct for the slope by putting s, for the average change in the slope all round the circle of radius x, and H, and h, for the average depths at distances R, and r, from the centre of the well. (See Sketches Nos. 61 and 62.)

(i) Using the first equation. Let Q = 47,000 cubic feet = 294,000 imperial gallons per day. Let $s=\frac{1}{800}$, when $x = 300$, and $y = 40$ feet. Then, $K=\frac{47{,}000 \times 800}{6{\cdot}28 \times 300 \times 40}=504$, corresponding to an effective size of 0·40 mm. or 0·016 inch (that is to say, clean and rather coarse sand).

(ii) Taking the second equation. Let H = 40, when R = 1000 feet. Let $h=36$ at the well, *i.e.* when $r=3$ feet, for a well 6 feet in diameter, and Q = 14,000 cubic feet per day.

Then, $K=\frac{14{,}000 \log_e 333}{3{\cdot}14(1600-1296)}=85$, corresponding to a very fine sand of about 0·006 inch effective size.

The formulæ for a long catchment gallery resting on an impermeable layer are not so useful for the experimental determination of K; but are necessary, since this form of collector is frequently used in lieu of wells in actual practice. They are:

$$Q = Kbys$$

and,

$$Q=\frac{Kb(H^2-h^2)}{2\,(L-x)}$$

where Q, is the yield of a gallery b, feet long, and s, is the ground water slope at a point where the depth of ground water above the impermeable stratum is y; and H, and h, are similar depths at distances L, and x, measured perpendicular to the length of the gallery. These formulæ are for the usual case where water enters the gallery from one side only. If ground water seeps in from both sides (which occurs only very rarely when pumping is continuous), Q, must be doubled.

As indicated in the second example, the second form of the equation is most easily applied; and, in practice, the usual method of obtaining K, is as follows:

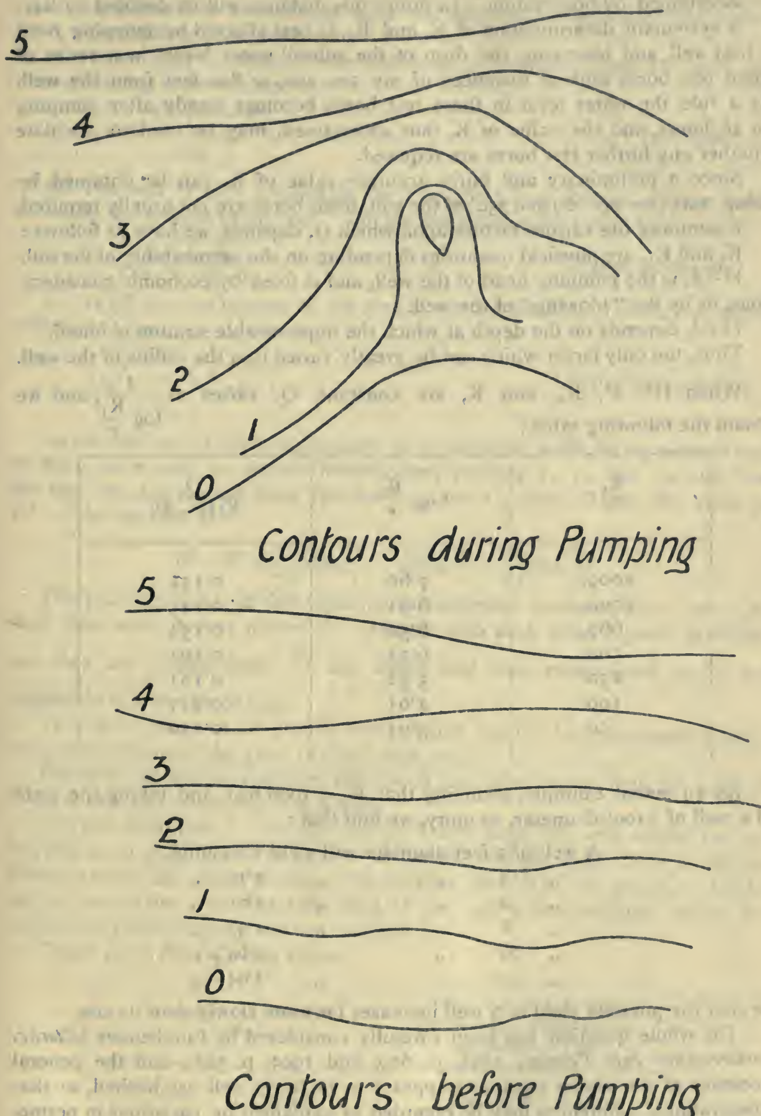
Observe the difference between the water level in the well before pumping commences, and the level of the impermeable stratum. Let this be H.

Similarly, let h, be the difference between the water level in the well when Q, cube feet per 24 hours, are being pumped from it and the top of the impermeable stratum.

Then, if r, be the radius of the well, and R, the distance from the well at which the ground water level stands at a height H, above the impermeable stratum during pumping, we find that:

$$K=\frac{Q \log \frac{R}{r}}{1{\cdot}36(H^2-h^2)}\,.$$

As a rule, it is usual to assume that R = 1000 feet. In actual practice, however, the distance from the well at which long-continued pumping does not



SKETCH No. 62.—Contour Lines, as actually observed in a Case similar to Sketch No. 61.

alter the slope of the subsoil water surface is a very important factor, and should be ascertained by observation. In future this distance will be denoted by R_1.

A systematic determination of K, and R_1, is best effected by pumping from a trial well, and observing the drop of the subsoil water levels in a series of small test bores sunk at distances of say 200, 400, or 800 feet from the well. As a rule, the water level in these test bores becomes steady after pumping for 48 hours, and the value of K, thus ascertained, may be used to calculate whether any further test bores are required.

Since a preliminary and fairly accurate value of K, can be obtained by sizing tests (see pp. 269 and 350) of the soil, fresh bores are not usually required.

Examining the various factors upon which Q, depends, we have as follows:

K, and R_1, are physical quantities depending on the permeability of the soil.

$H-h$, is the pumping head of the well, and is fixed by economic considerations, or by the "blowing" of the well.

$H+h$, depends on the depth at which the impermeable stratum is found.

Thus, the only factor which can be greatly varied is r, the radius of the well.

When H^2-h^2, R_1, and K, are constant, Q, varies as $\frac{1}{\log \frac{R_1}{r}}$, and we obtain the following table:

$\frac{R_1}{r}$	$\text{Log}_e \frac{R_1}{r}$	$\frac{Q}{K(H^2-h^2)}$
2000	7·60	0·132
1000	6·91	0·145
667	6·50	0·154
500	6·21	0·161
250	5·52	0·181
100	4·61	0·217
50	3·91	0·256

As an actual example, assuming that $R_1 = 1000$ feet, and taking the yield of a well of 1 foot diameter, as unity, we find that:

A well of 2 feet diameter will yield 1·10 units.
" 3 " " 1·17 "
" 4 " " 1·21 "
" 8 " " 1·37 "
" 20 " " 1·64 "
" 40 " " 1·94 "

so that the possible yield of a well increases far more slowly than its size.

The whole question has been carefully considered by Forcheimer (*Ztschr. oesterreicher Ing. Vereins*, 1898, p. 629, and 1905, p. 587), and the general accuracy of the above equations appears to be fairly well established, so that observational differences may be regarded as explained by variations in permeability, which my own experiments show may often amount to 10, or even 20 per cent. of K, even over short distances.

The above equations refer to a well through the walling of which water percolates as easily as through the subsoil. This, in fairly coarse sand, can be secured without difficulty; but in fine sand, or where surface pollution is feared, impermeable well linings must be adopted. Forcheimer (*ut supra*) gives the following equations:

(*a*) The supply is derived from a river at a distance a, from the well, and the well lining is permeable:

$$Q = \frac{\pi K (H^2 - h^2)}{\log_e \frac{2a}{r}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \text{ (Accurate)}$$

(*b*) The well is only permeable for a height t, less than h, and no water enters by the bottom of the well. Divide the above values of Q by:

$$\sqrt{\frac{h}{t}} \sqrt[4]{\frac{h}{2h-t}} \quad . \quad . \quad . \quad . \quad . \quad . \text{ (Approximate only)}$$

(*c*) As (*b*), but the bottom of the well is also permeable. Divide the above values of Q, by:

$$\sqrt{\frac{h}{t+\frac{r}{2}}} \sqrt[4]{\frac{h}{2h-t}} \quad . \quad . \quad . \quad . \quad . \quad . \text{ (Approximate only)}$$

As an example of their application to a series of wells, let us assume that we have three wells as already investigated, yielding q_1, q_2, and q_3, and that the last two are distant from the first x_{12}, and x_{13} feet. Then, the value of $H^2 - h^2$ for the first is:

$$H^2 - h^2 = \frac{q_1}{\pi K}\log_e\frac{R_1}{r_1} + \frac{q_2}{\pi K}\log_e\frac{R_1}{x_{12}} + \frac{q_3}{\pi K}\log_e\frac{R_1}{x_{13}}$$

The practical value of this equation is somewhat doubtful, but it serves to show that wells do not materially interfere with each other's yield, provided that they are spaced about $\frac{R_1}{3}$ feet apart, and that many small wells are preferable to a few large ones.

They also permit us to predict the possible yield of large permanent wells from observations of the yield of small tube wells.

Practical experience, however, teaches us that other factors limit the amount of water that can be drawn from a well.

The first condition is that the amount drawn from the well should not be so great as to cause grains of sand to be carried into the well. This condition becomes more and more pressing, the smaller the size of the grains of sand; and in beds of fine sand the safe yield of the well is reached long before the theoretical yields above given are attained.

Thiem gives the following table:

Diameter of Grains in Inches.	Water Velocity, in Feet per Second, which causes the Grains to rise.
0·0 to 0·01	0·0 to 0·10
0·01 ,, 0·02	0·12 ,, 0·22
0·02 ,, 0·04	0·25 ,, 0·33
0·04 ,, 0·08	0·37 ,, 0·56
0·08 ,, 0·12	0·60 ,, 2·60

My own experience on wells up to 20 feet diameter in fine sands of 0·01 inch in diameter, and under, say K ≐ 100, causes me to regard any yield over 0·05r to 0·06r cubic feet per second, as liable to produce failure, the sand being removed from under the well curb, and the well rapidly sinking, and becoming filled with sand. These observations were, however, taken on roughly constructed Indian wells, where the well lining itself did not permit water to pass, and the whole yield entered under the well curb, thus producing a very intense flow around the circumference of the bottom of the well. It is therefore probable that in a well with sides sufficiently permeable to admit water, but capable of retaining the smallest grains of sand, yields equivalent to those given by Thiem's values, calculated over the whole permeable area of the well, might be attained.

So also, in one case, the well bottom being covered with a properly graded reversed filter, I was able to obtain yields equivalent to (0·15 × area of well bottom in square feet) cusecs, without any sign of failure.

It will be evident that such large yields from wells in fine sand, say K = 80 to 100, are only obtainable with high values of $H-h$, or exceedingly deep wells, in which $H+h$, is large. For example, take R_1 = 1000 feet, and

r = 3 feet, and assume that the yield is 2·82 cusecs, or that

Q = 2·82 × 3600 × 24 cubic feet per day. When K = 100, we get:

$$H^2-h^2 = 5184$$

or, if $H-h$ = 20 feet, $H+h$ = 259 feet, or a well 6 feet in diameter must be sunk some 130 feet below the subsoil water level.

Taking what is the more usual Indian practice, for town water supply wells, *i.e.*:

$$H-h = 6 \text{ feet}, \; H+h = 90 \text{ feet}$$

we get:

$$Q = \frac{314 \times 540}{6{\cdot}5} = 26{,}162 \text{ cube feet per day, or about } 0{\cdot}3 \text{ cusec.}$$

Tested by the rule given above, the yield is about twice the safe yield when r = 3 feet; and, as a matter of fact, such wells rarely, if ever, give a regular yield of much above 0·1 cusec.

Permanent Yield.—This introduces us to the second limitation of wells. The above well, regarded as a machine, is capable of taking 0·3 cusec out of the ground, if provided with a proper reversed filter; or, at a smaller depression, (say $H-h$ = 3·5 feet) will yield about 0·16 cusec with safety, it is actually considered to be successful if it yields 0·1 cusec. The explanation is obvious, —the ground water is not replenished sufficiently rapidly.

The maximum possible replenishment of ground water can be roughly calculated as follows. The natural slope of the ground water surface in such districts is about $\frac{1}{1000}$, and this alone replenishes the ground water when the pumping is long continued. The area supplying the well, measured normal to the natural flow, is roughly $2R_1 \times 50$, say 100,000 square feet, and the velocity of flow is $\frac{1}{10}$th of a foot per day. Thus, a continuous yield of 10,000 cube feet per day (= 0·11 cusec approx.) is all that can be expected from day and night pumping, although the same total volume could be obtained with safety in about 16 hours.

The preliminary studies for a well are therefore as follows:

(i) Discover the natural slope of the ground water.

(ii) By pumping from a trial well estimate R_1, and K, and also the safe yield from the point of view of sand "blowing" into the well.

Hence, we can calculate the rate at which ground water will be supplied to the well, and thence its permanent yield. Then we can determine the value of *r*, such that the permanent yield is nearly equal to the safe yield, and so can ascertain the cheapest well (which will generally be the smallest possible).

In cases where no impermeable stratum exists, the problem (if regarded exclusively as a mathematical one) is somewhat more difficult. In Indian practice, as deep a well is sunk as is commercially practicable, which (under Indian conditions) corresponds to a depth of about 50 feet below subsoil water level.

In the studies conducted at Amritsar when preparing a project for irrigation from wells, the sand bed was at least 300 feet deep; except in one case, where a small patch of clay existed about 30 feet below the well curb. The values of K, obtained from observations of the yields of the wells when H, and *h*, were measured from the well curb level, were about 30 per cent. greater than those obtained from small scale experiments on percolation through the sand, or by sizing the sand with sieves (both these methods gave very concordant results).

It may therefore be inferred that the subsoil water was in motion down to a depth below the well curb approximately equal to $\frac{H}{3}$. The sand was very fine, K, as ascertained from small scale tests, varying from 80 to 125, and, as ascertained from the well yields, from 100 to 170.

In practical work, the obvious procedure is to sink the test well to the depth to which the permanent wells are proposed to be sunk, and to calculate the yield of the permanent wells from the experimental value of K, as ascertained by using the equation on p. 262 when H, and *h*, are measured from the well curb.

As a rule, Indian wells are too large for the quantity of water which they can permanently yield, if 24 hours' pumping is contemplated. The wells are proportioned so as to be safe against blowing when yielding the quantity which is initially observed; and, as already stated, this quantity is frequently twice or three times as great as the quantity which the natural slope of the ground water surface supplies to replenish the ground water near the well.

In consequence, "large and permanent falls in the subsoil water level" are produced, and the partial failure in the supply is frequently attributed to a succession of dry years. So far as my experience goes, dry years have but little influence, and the actual facts are that the line of wells is yielding more than the natural replenishment of the ground water can supply.

According to the equations already given, if:

L, be the total length in feet, of a line of wells measured normal to the direction of the natural ground water flow, and if S, be the natural slope of the ground water surface before the wells are installed, the total permanent yield cannot possibly exceed:

$$Q = KS(L+2R_1)H \text{ cube feet per 24 hours}$$

where H, is the depth of the well curb below the natural ground water level, if K, be ascertained from a trial well; or H = 1·25 to 1·30 depth of well curb below the natural ground water level, if K, be ascertained from small scale experiments. This value is the maximum possible permanent yield, and the spacing of the wells and their size should be so selected as to permit this daily yield to be obtained in 8, 12, or 16 hours, or whatever other period is selected for pumping.

The one exception to this rule is where a permanent stream, or lake, exists close to the wells, and the equation:

$$Q = \frac{\pi K(H^2 - h^2)}{\log_e \frac{2a}{r}}$$

is applicable both for temporary and permanent yields.

The possibilities of pollution are obvious, but otherwise the case is very favourable, and should be selected wherever practicable. The actual estimation of the probable yield of an extensive scheme for ground water pumping, is difficult. Rain is the source of the ground water, but the catchment area is ill defined.

The field wells of a large and highly cultivated area in the Sialkote district of the Punjab are capable of supplying at least 6 to 7 inches depth of water yearly, over the whole area, and the only apparent source is a rain-fall of about 20 inches per annum. But, since the water is used for local irrigation, some portion may have been used twice over, and percolation from neighbouring hill torrents may occur. So also, chalk wells in England appear to give yields corresponding to 6, or 8 inches depth; but, after a series of dry years signs of exhaustion, vanishing in wetter years, are noticeable.

The only safe method, therefore, is to actually ascertain the ground water contours over a large area, say 3, or 4 square miles, and to estimate the velocity of flow, either as discussed on page 205, or by the surface slope. The effect of dry years can then be disregarded, unless the draught from the wells is very nearly equal to the calculated supply; since, a general fall of say 4, to 6 feet over the whole area, will amply tide over even a long term of dry years.

The usual installation, in such cases, consists of a line of wells perpendicular to the ascertained direction of ground water flow. The individual yields (after allowing for interference), can be calculated by equations of the form given for a system of three wells; but, in actual practice, the gross yield is very close to that of a catchment gallery of a length equal to $L+2R_1$, where L, is the length of the line of wells.

For calculations of the permanent yield, it would appear safe to take the velocity as given by the natural slope of the ground water, and an area equal to $(L+2R_1) \times H_1$, where H_1, is either the depth of the impermeable stratum below subsoil water level, or $1{\cdot}25 \times$ depth of well curb below the same level, whichever is least. It will also be wise to allow for possible extensions of the line.

If the general equations are considered, it will be evident that the major portion of the pumping head $H-h$, is lost in the soil close to the well. Thus, if the finer grains could be removed from this portion of the soil so as to increase K, near the well, the pumping head for the same yield would be diminished, and failure by blowing would be less likely to occur. The principle has been applied in several ways. The well is surrounded by a reversed filter, of graded material, or the bottom of the well is covered with a similar reversed filter. The most practical method appears to be that produced by "well plugs." These are orifices covered with fine meshed gauze, formed in the sides of the well. When the well is first set to work, steam under pressure is turned through these orifices, and is allowed to escape outside the well. The finer particles of the soil close to the orifices are thus blown away, and a reversed filter is obtained. In practice, the finer particles are sooner or later carried towards the gauze by the flow of water, and clogging occurs. The steam blowing process is then repeated. The method is practical, and should be adopted in all cases where failure by sand blowing is apprehended, or where the calculated permanent yield can only be obtained under a large pumping head.

The system described above is that most usually adopted, but it is useless in fine sands. The only system that is really useful under such conditions consists of a circular removable gauze cylinder, fixed on a frame inside the well, which is in communication with a plug screwed into the well lining. The apparatus chokes more rapidly in fine sand than the usual well plug does in coarse sand, but it is arranged so as to be systematically and frequently blown free by pressure water taken from the main pumps.

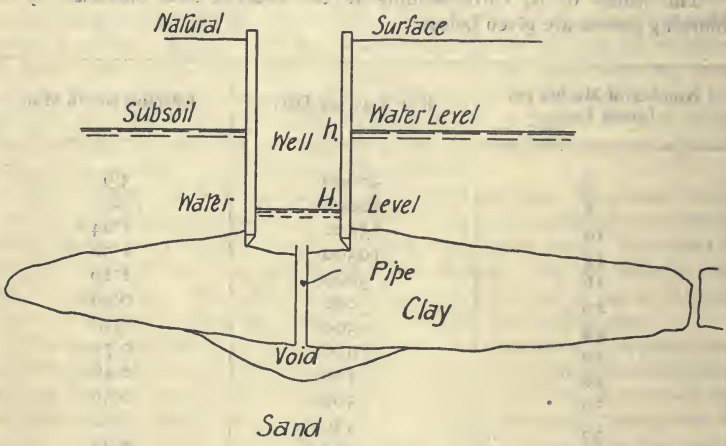
The values of K, corresponding to the effective sizes indicated by the following sieves, are given below:

Number of Meshes per Linear Inch.	K in Feet per Day.	Effective size in Mm.
6	50000	3·9
8	32000	...
10	13500	2·04
12	10500	1·52
16	5800	1·10
20	3000	0·96
24	2500	...
30	1650	0·70
40	700	0·46
50	500	0·39
55	430	...
60	333	0·32
70	190	0·24
80	160	0·22
90	130	0·20
100	105	0·18
120	80	0·155
140	60	0·135
150	55	...
200	40	...

The velocity through the pores of the sand may be taken as about 3K slope, but the factor may vary from 3·7 to 2·2, or even a little less.

Mota Wells—The principles involved in this type of well are illustrated in Sketch No. 63. The well is sunk in sand down to the surface of a lenticular mass of clay, or other hard material lying in the sand. The upper surface of this mass of clay must be below the subsoil water level, and the clay bed must be limited in extent, so that the two beds of sand above and below it are, hydraulically considered, identical. The clay bed is then pierced by a pipe which is not extended below the lower surface of the bed. On pumping from the well all leakages into the well from the upper bed are carefully stopped. If the process is successful a small void space is formed below the clay by removing the sand, and thereafter clear water enters through the pipe under a head equal to H*h*, the difference in the levels between the water inside and outside the well, less any frictional resistance to flow in the sand round the edge of the clay bed. In practice the last term is negligible. The process evidently amounts to the prevention of blowing of the sand, and thus the well can be worked under a far greater head than would otherwise be permissible.

The method is well known to Indian villagers, and roughly speaking a mota well (Urdoo, *mota*=clay) is considered to yield three times the quantity usually obtained from a non-mota well of the same dimensions. Also such wells are found to fail less rapidly when (due say to deficiency in rain) the subsoil water



SKETCH No. 63.—Mota Well.

level falls below its usual height. The circumstances are obviously somewhat peculiar, and the discovery of the clay beds is a difficult matter.

So far as I am aware, the greatest recorded yield of such a well is approximately 1·5 cusec. This I obtained at Amritsar. As a rule, however, yields exceeding 0·5 cusec are rare, and the average of some forty carefully constructed wells was 0·3 cusec.

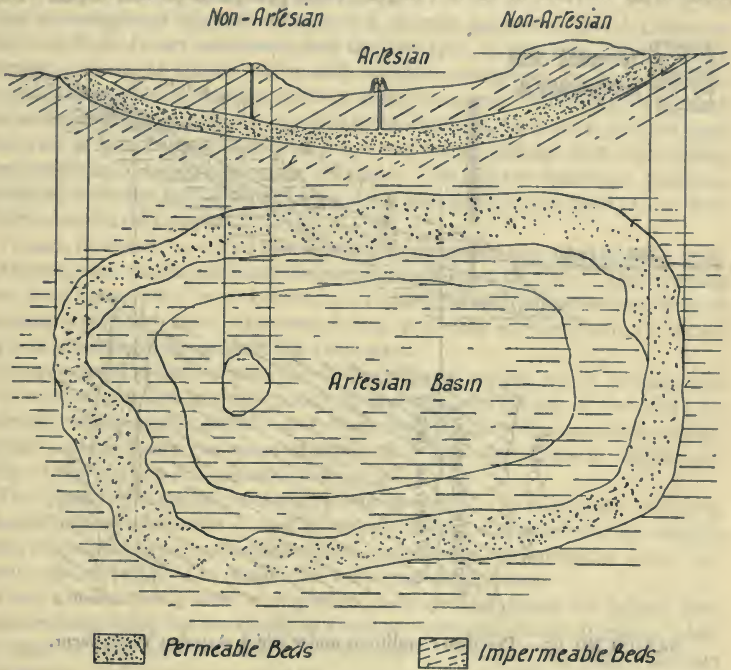
The liability to failure by fracture of the clay bed under excessive pumping is obvious, but distribution of the pressure by steel beams inserted in the clay and built into the well will frequently prevent this.

My own recommendation was to sink all wells down to the clay bed, if such exist at a reasonable depth, and then put in the pipe with a valve; but I do not advocate working the well exclusively as a mota well unless forced to do so owing to deficiency in supply.

The method forms a very useful standby, and wherever clay beds are met with the possibility of its adoption should be investigated.

ARTESIAN WELLS.—Sketch No. 64 shows the ordinary basin theory of an artesian well. The conditions are founded on theory only, and probably but rarely occur. It may also be said that if the permeable stratum was actually full of water which was quite at rest until a well was bored, this water would (through age-long contact with the minerals of the stratum), be so charged with salts as to prove useless for human or agricultural consumption. This was actually the case with the non-artesian waters pumped from the Severn tunnel during the first three or four years after its construction.

Sketch No. 65 shows what is probably the true explanation of all, or nearly all, artesian wells. The water in the permeable stratum is in slow motion towards



SKETCH NO. 64.—Theoretical Artesian Basin.

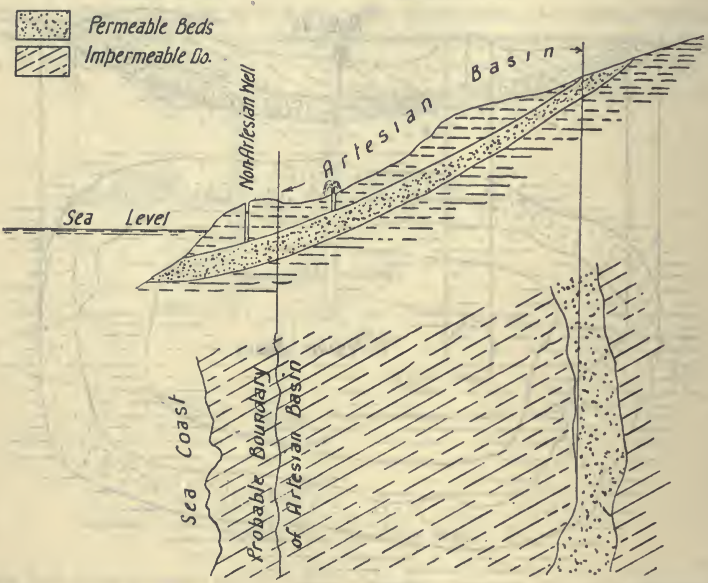
some distant (frequently submarine) outlet. The friction head resisting this motion being greater than that opposing the movement through the bore pipe of the well, the water rises, and either issues under its own pressure, or can be pumped from the well.

The artesian waters of Western Australia and the Atlantic coastal plain of the Eastern United States are certainly of this character, as are probably those of Queensland and New South Wales also.

If observations are taken of the yield of an artesian well, and the pressure of the issuing water; or, in the cases where the water does not rise to ground level, the depression of the water surface, it will be found that in either case the supply is almost directly proportional to the decrease in pressure, or depression below the level of the water when at rest.

This fact permits us to infer that the water issuing from an artesian well is supplied by percolation through the permeable stratum. Consequently, any idea that large cavities filled with water under pressure exist in the subsoil, and are tapped by the bore, must be abandoned; since, if such existed, the supply would be proportional to the square root of the decrease in pressure.

The vertical thickness of the permeable stratum can be estimated by observing the temperature of the water issuing from the bore. The rate at which the temperature in deep bores increases is known to be fairly uniform, and corresponds to an increase of about 1 degree Fahr. per 50 to 60 feet depth. But,



SKETCH No. 65.—Practical Conditions under which Artesian Wells occur.

since hot water is lighter than cold water, the temperature of the issuing water will be very nearly equal to that of the warmest water in the permeable stratum. Thus, while the depth of the well is approximately equal to the depth of the top of the permeable stratum, the temperature of the issuing water corresponds more closely to the temperature prevailing at the bottom of the stratum.

For example, the Queensland artesian waters are usually (see Williams, *P.I.C.E.*, vol. 159, p. 319) some 32 degrees Fahr. (at shallow depths), to 47 degrees Fahr. (at greater depths), hotter than is explained by the depth of the bore. The vertical thickness of the permeable strata may therefore be taken as averaging from 1760 ($=55\times32$) to 2585 feet, ($=55\times47$); the usual temperature gradient in borings being about 1 degree Fahr. per 55 feet increase in depth.

The yield of an artesian well cannot in any way be predicted. The average of 805 Queensland bores in 1901 is given as 444,000 imperial gallons per day, (say 532,000 U.S. gal., or 0·81 cusec), but some go as high as 11 cusecs, while dry bores are not infrequent.

So also, it is at present impossible to estimate the ultimate yield of an artesian basin. No doubt, if a survey of the outcrop of the permeable stratum were made, we could estimate the supply by percolation as a fraction of the rain-fall on the outcrop area; but this does not take into account such matters as extra supplies due to leakage from rivers, or water-bearing surface deposits which cross the outcrop, or losses due to portions of the outcrop being covered with clay, or other impermeable cappings.

As an example of the difficulties found in such problems, David (*Artesian Water in N. S. Wales*), estimates that the nett area of the New South Wales outcrops is about 44 square miles, and on this assumption, he finds that the minimum total yield of artesian water is about 76 cusecs. Allowing for leakage from water-bearing gravels which cross these outcrops, he finds a possible yield at the rate of 3800 cusecs; and finally, by considering leakage from the Darling River, yields up to 44,000 cusecs, are obtained. The last two figures are probably excessive, while the first is some 10 per cent. below the actual yield at the date of Professor David's paper.

It may, however, be stated that there are indications that the artesian wells in Algeria, as a whole, are now discharging less than in 1900. The artesian water level under the City of London has (subject to slight fluctuations due to unusually wet or dry years) also been falling at a rate varying from 18 inches to 3 feet per annum, for at least ten years past.

I am inclined to believe that the deficiencies in supply which occasionally occur in artesian wells are due either to faults in the casing of the well; or, where the supply is permanently and markedly less than that given by neighbouring wells sunk to the same stratum, that local variations in the permeability of the stratum are probably the cause.

The quality of the water yielded by artesian wells is usually good. Organic pollution (except where the casing is leaky) may be disregarded. As already stated, the water may be mineralised, yet cases where the amount of salts contained is so excessive as to render the water useless are rare.

Deep artesian wells must be regarded as sources of water for human consumption only, being far too costly to yield an adequate financial return when the water is employed for irrigation purposes. Shallow artesian wells are occasionally used to irrigate valuable crops, but the conditions giving rise to such wells (say less than 100 feet deep) are rare, and the area enjoying such favourable circumstances is generally small (see p. 250).

As typical figures likely to occur in calculations regarding artesian wells, let us consider the map given by Williams (*ut supra*).

The lines of equal pressure in the artesian bores of Western Queensland are, on the average, spaced about 30 miles apart, per 100 feet fall in pressure.

Taking the unfavourable assumption that the effective size of the grains of the permeable bed is equivalent to a 70 mesh sieve (*i.e.* fine sand), we find a velocity coefficient of 189 at ordinary temperature, or, approximately 400 at a temperature of 122 degrees Fahr., which is roughly that of the artesian water.

The velocity of flow (in a solid column with an area equal to that of the cross-section of the bed), is therefore :

$$\frac{400 \times 100}{160{,}000} \text{ feet per day} = 3 \text{ inches per day},$$

and taking the thickness of the bed as 2000 feet, we find that each 1000 feet horizontal width of bed carries $\frac{2000 \times 1000}{4}$, or 50,000 cube feet per day.

The length over which flow occurs is about 700 miles, or the total available quantity is probably between 150 and 200 million cube feet daily. The yield of the existing wells appears to be about 70 million cube feet per day.

If we endeavour to apply similar calculations to the yield of individual wells, it will be plain that either very great differences in pressure must exist close to the bore tube, in order to force the water through the stratum ; or, that all the finer grains of sand are swept out by the first rush of water, and that the flow in the last 100 to 200 feet near the bottom of the bore tube is through well defined channels, rather than of a capillary nature. The remarkable variability of the yield of individual artesian wells is consequently not surprising, and wells yielding quantities such as 3, or 4 cusecs, are only likely to occur when the strata are coarse-grained, taking the form of beds of large gravel, or greatly fissured rock.

The principles of geology can, however, be applied to indicate certain general laws concerning artesian wells.

The permeable stratum represents the remains of ancient marine or alluvial deposits, and, even if the ancient coast line has been entirely removed, the materials found near the edges of the existing stratum are probably the remains of beds which were laid down near the shores, and the centre of the existing stratum represents beds laid down in deeper water. Thus, the beds at the edges may be composed of all materials ranging from large boulders to fine clay (laid down by an ancient river) ; but, as a general rule, they will be far coarser and more variable than the central beds, which will almost certainly prove to be entirely composed of fine sand (clay beds in the centre may be considered as unlikely to occur, since the stratum as a whole is assumed to be permeable).

Applying these principles to predict the yield of wells, we see that :

Dry wells (representing local clay beds), and wells yielding very large quantities of water (representing local beds of coarse gravel) will generally be found to occur near the edges of the artesian basin.

Near the centre of the basin, dry wells, or very large yields, are unlikely to occur ; but, on the average, the yield of wells will be less than the average yield of wells sunk near the edges.

In view of the great cost of an artesian bore, the fact that artesian wells are usually deep (*i.e.* sunk near the centre of the basin) is not surprising. An investment of £3000 or £4000 (which is fairly certain to yield some water) is more readily undertaken than one of £300 or £400, which may either yield no water, or a far greater quantity than is required.

Similarly, the services of an expert geologist are in reality most necessary when shallow wells are proposed.

CHAPTER VI.—(SECTION B)

FLOODS

Floods.—Relation between the intensity of rain-fall and the time during which the rain-fall continues—Observations required—Bruyn-Kops' values—Connection between intensity of a flood and the absolute size of the area which produces it—Critical period—Relation between intensity and time—General rules.

Flood Discharge of a Stream or Catchment Area.—General principles—Special rules—Estimation of critical period—Ratio of run-off to rain-fall—Shorter formulæ—Examples.

Flood Discharge in a Reservoir.—Gould's table—Examples.

DESIGN OF WASTE WEIRS.—Old and new values of coefficients of discharge.

The Relation between the Intensity of Rain-fall and the Time during which the Rain-fall continues.—Before any logical treatment of the questions concerning flood discharge, and the drainage of small areas can be undertaken, it is necessary to consider the relation between the possible intensity of a rain-storm and the time between the beginning and end of the storm.

Consider any interval of t, minutes, during a fall of rain (where no assumption that the rain begins or ends simultaneously with the period t, is made). During these t, minutes let $R = \frac{tI}{60}$ inches of rain be collected in a rain-gauge at the point considered. Then I, is called the hourly intensity of the rain-fall during the period of t, minutes, and R, is the total rain-fall during the same period.

The present section is devoted to a consideration of R, and I, as functions of t. The values of R, and I, can only be obtained by a systematic study of a long period record of an automatic recording rain-gauge. It will be obvious that R, and I, are extremely variable, but when a long period record is studied, it is possible to lay down certain values of R, or I, which will not be exceeded more than say once in 10 years, twice in 10 years, etc., down to once a year, twice a year, etc.

In this sense, and in this sense only, can we consider R, or I, as functions of t.

Thus, de Bruyn-Kops (*Trans. Am. Soc. of C.E.*, vol. 60, p. 248), from studies of a 17-year record at Savannah (Georgia) (see Sketch No. 66, Fig. II.) where the mean annual rain-fall is about 50 inches, found that:

Maxima records of the 17 years.	$I = \frac{191}{t+30}$	inches per hour
Occur once every two years	$I = \frac{163}{t+27}$	"
Occur once a year	$I = \frac{141}{t+27}$	"

Occur twice a year . . . . $I = \dfrac{104}{t+22}$ inches per hour

Occur 3 times a year . . . . $I = \dfrac{86}{t+19}$ „

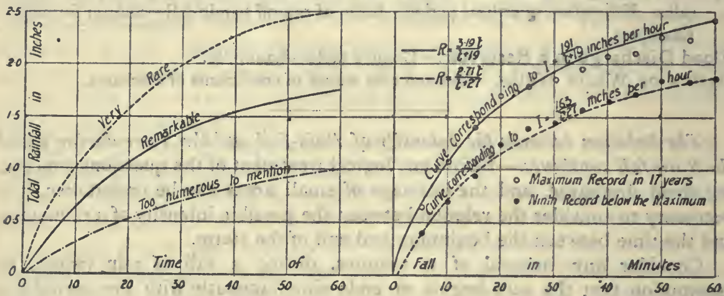
Occur 4 times a year . . . . $I = \dfrac{73}{t+17}$ „

Occur 5 times a year . . . . $I = \dfrac{63}{t+16}$ „

The examples are typical, as may be verified by a study of meteorological journals, and the fact that I, increases as *t*, decreases is the really important part of the investigation.

The importance of the investigation must now be elucidated, before proceeding further.

Consider an asphalted area (*i.e.* a practically water-tight area), A, acres in extent, and of such a configuration that the rain-water falling at the boundaries



SKETCH No. 66.—Relations between Time and Total Rain-fall, in Inches, in England, according to Mill; and at Savannah (Ga.), according to de Bruyn-Kops.

of the area arrives at the drain grating 5 minutes after it reaches the ground. Then, according to de Bruyn-Kops' curves, the drain must be capable of disposing of the water at a rate corresponding to a discharge of

$$A\frac{191}{35} = 5{\cdot}47A$$

cusecs. Otherwise, once at least in 17 years, rain water would (temporarily at any rate) accumulate on the area, and flooding might occur. If, however, an equal area, but of different configuration, such that the rain water falling on the boundaries took 30 minutes to arrive at the drain, be considered, the capacity of the drain need only be $A\frac{191}{60} = 3{\cdot}19A$ cusecs. (*Note.*—1 inch of rain per hour running off in 1 hour = 1·01 cusec per acre = 1 cusec per acre, approximately).

The practical objections to the theory are:

The time *t*, of arrival at the drain, depends on the quantity of water already on the area, the size of the channels, etc.

In the case actually considered, if the drain connected with the first area could only discharge 3·19A cusecs, the flooding at the end of the first 5 minutes would amount to an average depth of $\frac{5{\cdot}47-3{\cdot}19}{12} = 0{\cdot}19$ inch over the whole area, and would be reduced to nothing 25 minutes later, which is not a very important matter.

The practical principle, however, that the maximum discharge per acre, or per square mile, of any area, whether permeable or impermeable, increases as the time decreases is undoubtedly true, and the method of considering the intensity of rain-fall as a function of the time is the most logical process of arriving at the rate of increase.

Let us therefore assume that t, is not necessarily the time water falling as rain takes to arrive at the entry to the drain, or the locality where the flood is measured, but that t, in some way depends on this time, and let us call t, as thus approximately defined, the critical period.

If f, be the fraction of the rain-fall that is actually discharged (i.e. $f = 1{\cdot}00$, for a water-tight area, such as an asphalt, slate, or cement surface, and f, decreases down to say 0·10, for a sandy surface) then the average discharge from an area of A, acres during the time is:

$$Q = I_t f A \text{ cusecs}$$

or, $Q = I_t f M \times 640$ cusecs, if M, be the area in square miles.

If we select I_t, from the maximum recorded curve, it is plain that Q, will represent the maximum flood during the period of the record, while if I_t, is selected from the curve of rain-fall intensity which occurs twice a year, a discharge equal to, or exceeding, Q, cusecs may be expected twice every year.

In practice, t, is usually estimated by calculating the velocity of the water in the stream, or discharge channels, when these are carrying half- or three-quarters of the quantity represented by Q, and, as a rule, the information being deficient, t, is merely estimated by some such rule as:

$$t = \frac{\text{Maximum dimension of the area in feet}}{200 \text{ to } 300}$$

corresponding to a mean water velocity of 3, to 5 feet per second.

Returning to the relation between I, and t, the following information is available:

Symons (*British Rain-fall*, 1892) gives, for Great Britain in general:

t, in Minutes.		10	20	30	40	50	60	90	120
Frequent	R, inches	0·30	0·42	0·52	0·59	0·65	0·72	0·82	0·85
	I, inches per hour	1·80	1·26	1·04	0·89	0·78	0·72	0·55	0·43
Unusual	R, inches	0·55	0·90	1·22	1·44	1·64	1·82	2·10	2·20
	I, inches per hour	3·30	2·70	2·44	2·16	1·97	1·82	1·40	1·10

Mill (*British Rain-fall*, 1908), using further information, states (Sketch No. 66, Fig. I.):

t, in Minutes.		10	20	30	40	50	60
Too numerous to discuss	R, inches	0·30	0·53	0·70	0·82	0·92	1·00
	I, inches per hour	1·80	1·59	1·40	1·23	1·10	1·00
Remarkable	R, inches	0·65	1·06	1·35	1·54	1·67	1·75
	I, inches per hour	3·90	3·18	2·70	2·31	2·00	1·75
Very rare	R, inches	1·00	1·58	2·00	2·26	2·42	2·50
	I, inches per hour	6·00	4·74	4·00	3·39	2·90	2·50

Thirty-three cases of falls exceeding the "Very rare" curve are on record in the 49 years of observations discussed by Mill. The absolute maxima are:

t = . . .	5	15	30	45	60 minutes
R = . . .	1·25	1·46	2·90	3·42 (?)	3·63 inches

For longer periods, we find:

	$1\frac{1}{4}$ hours	2 hours	3 hours	5 hours	9 hours
R =	3·75	4·80	6·70	6·50	4·90 inches

A comparison of the two sets of figures indicates that the short period (5 to 30 minutes) records are likely to be increased, as autographic rain-fall recorders become more common.

The very rare curve is fairly represented by . . . $I = \frac{240}{t+30}$ inches per hour.

and the remarkable curve by . . . . $I = \frac{168}{t+30}$ "

and the too numerous curve by . . . . . $I = \frac{84}{t+30}$ "

Lloyd Davies' observations at Birmingham, which extend over four years (*P.I.C.E.*, vol. 174, p. 48), accord very fairly with . . . . $I = \frac{63}{t+30}$ "

In Berlin the following is stated to occur once a year $I = \frac{36}{t+10}$ "

Talbot gives for the Eastern United States:

Maximum authentic . . . . $I = \frac{420}{t+30}$ "

Probable maximum . . . . $I = \frac{360}{t+30}$ "

At Baltimore the maxima are . . . . . $I = \frac{270}{t+30}$ "

For ordinary falls in the Eastern United States:

Talbot gives . . . . $I = \frac{105}{t+15}$ "

Dorr gives . . . . $I = \frac{150}{t+30}$ "

Kuichling gives . . . . $I = \frac{120}{t+20}$ "

And other records agree fairly well with . . . . $I = \frac{180}{t+30}$ "

The above figures are typical of the general conditions prevailing in Temperate climates, and while individual observations may be better represented by such curves as:

$I = \frac{38{\cdot}64}{t^{0{\cdot}687}}$ . . . . Sherman's maxima,

and $I = \frac{6}{\sqrt{t}}$ . . . . . Gregory's "winter storms,"

and so forth, the curve $I = \frac{A}{t+30}$ is never very far wrong.

For the Tropics, the only systematic information is obtained from records extending over 4 years, at Manilla (Philippines), as follows :

$$\text{Ordinary, } I = \frac{220}{t+30} \quad \text{Maxima, } I = \frac{290}{t+30} \quad . \quad . \quad \text{inches per hour.}$$

The following deductions rest mainly on records obtained in Temperate zones, but are believed not to be contradicted by the unsystematic, and as yet uncollated, information existing in the records of the Indian Meteorologist's Office.

In the first place, if the ordinary curve obtained from records of 3, or 4 years be represented by :

$$I = \frac{A}{t+30} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{inches per hour}$$

then the maximum curve obtained by long observation, and the isolated records made by engineers and other observers, will probably not be very far removed from :

$$I = \frac{3A}{t+30} \text{ to } \frac{2{\cdot}5A}{t+30} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{inches per hour}$$

where both curves refer to an area over which the climate does not vary markedly.

The value of the constant A, is not very far removed from 25 times the maximum fall of 1 day that occurs in a 20 years' record for one locality, not (carefully note) of the maximum day's rain-fall on record in such offices as those of the United States Weather Service, or of the Indian meteorologist. This indicates that, in a long period of time, the quantity of rain that occurs in 1 day once in 15 or 20 years may be expected to occur in about 1½ hours.

It is believed that these rules will permit an engineer to predict the values of I_t, required for estimating maxima flood discharges (*i.e.* I_t = the maximum intensity recorded over a long period), and the drainage capacity necessary to prevent detrimental flooding (*i.e.* I_t = the intensity which occurs once in 4 years, say).

These rules probably hold up to :

$$t = 200 \text{ to } 250 \text{ minutes, or } 3 \text{ to } 4 \text{ hours.}$$

For areas in which the critical period is greater than 3, or 4 hours, no general law can be given. I have usually been accustomed to collect the records of a large number of stations (say 50) adjacent to the locality considered, and assume that :

$$R_t = \frac{tI_t}{60} = \text{the maximum day's rain-fall occurring in the records}$$

for all values of t, up to 1440 minutes or 24 hours. This is probably an under-estimation of the true state of affairs.

While the maxima intensities for short periods are probably produced by thunder-storms of small area, the maxima intensities for such periods as 6, 12, or 24 hours, usually occur during long-continued winter (speaking of Temperate climates) downpours covering a large area. The fact that floodings of streets and railway cuttings usually occur in the summer, while floods covering 10 square miles or more usually occur in the winter or spring, is well known to engineers. Thus, it is probable that the maxima flood discharges are best estimated by putting:

$R_t = \frac{t}{60} I_t =$ maximum day's record, obtained from say 50 stations, and observations extending over 20 years, for $t = 600$ to 1200 minutes.

$R_t = \frac{t}{60} I_t =$ maximum 2 consecutive days' record, as above defined, for $t = 1200$ to 2400 minutes.

The value of the run-off factor f, also needs consideration. The values given on page 282, are fair means. They are probably somewhat high for areas of which the critical period is 1 hour, or less; and may be low for areas with critical periods of 24, to 48 hours. The rules given for I_t, probably provide for this.

Flood Discharge of a Stream, or Catchment Area.—The maximum discharge of a stream is one of its most important hydraulic properties, since the requisite provision for weirs, spillways, bridges, etc., entirely depends upon this discharge.

Also, while an erroneous estimate of the low-water discharge may possibly lead to inconvenience, an underestimate of flood discharge may lead to disaster and loss of life.

The maximum discharge of a stream depends on:

(i) The duration and intensity of rain-fall, and area over which it occurs. Also whether the storm producing the rain-fall moves with, or against the general direction of the stream.

(ii) The storage, both natural (including absorbent strata) and artificial, in the catchment area.

(iii) The size of the catchment area, relative to the area covered by storms producing intense precipitation.

(iv) The general topography of the catchment area, such as its slope, and shape, character of surface, whether forested, cultivated, or impervious.

It should be particularly noted that the combination of steep slopes of tributaries, with a flat slope of the main stream, is very favourable to intense floods.

Since large floods rarely occur more frequently than once in 40 years, actual observation is plainly impossible. The water levels of former high floods are often remembered, and, if reliable, the discharge which should be provided for may be estimated from them.

It must be borne in mind that local report is often absolutely untrustworthy, and invariably needs checking by connecting up the various spots pointed out by actual levelling. Also, assuming that the cross-section and surface slope for the record flood have been accurately determined, even by reliable observers, the selection of the correct discharge formula is frequently a difficult matter, and in some cases (possibly owing to the flow being turbulent, or the bed in motion) an application of any usual formula to well-ascertained flood levels and cross-sections leads to absolutely absurd results.

For example, take the case shown in Sketch No. 20, which is by no means an unusual one. It will be found that a rise of the river from level AB, to level CD, gives, upon calculation, a decreased discharge, owing to the reduction in hydraulic mean radius, due to the extra wetted perimeter BC, DA.

The more correct method of separately estimating the discharge in the flood channel and flats, is fairly obvious, but such a fact does not tend to increase our confidence in these calculations, and it is by no means an insignificant fact that the two records of the most intense floods known to me, which are anything more than rough estimates, were both arrived at by this method. For example, in the flood at Devil's Creek, Iowa, 1300 cusecs per square mile occurred, (see *Floods in United States in 1905*), and in the Oberlausitz (Saxony) records of 1015, and 1160 cusecs per square mile are reported in the *Deutsche Bauzeitung* for 1888, p. 264.

It is not intended to impugn the general accuracy of these records, since well observed floods occurring in certain Japanese rivers are known to have attained a magnitude of similar order (760, 885, 980, etc., cusecs per square mile), and it is believed that these are not absolute maxima, since higher flood levels are on record, but the figures referred to above are probably subject to at least 20 per cent. of error.

As a check, therefore, on such methods, and for application in cases where no reliable flood observations can be secured, it is useful to possess simple formulæ for flood discharges.

For preliminary discussions, the most practical type is :

$$Q = C \times (\text{area})^n$$

where Q, is the total maximum discharge in cusecs.

Here, as a rule, we may say that C, represents the combined effect of the intensity of rain-fall, and the character of the surface of the catchment area ; while n, which is less than unity, seems to depend more on the slope both of the stream and its catchment area, than on the other factors.

As example we may contrast Fanning's formula for New England :

$$Q = 200M^{0.83}$$

where M is the catchment area in square miles ; with Cooley's formula for the Upper Mississippi valley, where the slopes are flatter, and the rain-fall less :

$$Q = 180M^{0.67}$$

while, for Australia, Kernot, by plotting cases of disasters to bridges and culverts, found that :

$$Q = 400M^{0.75} \text{ approximately}$$

in a country of very intense rain-falls, but by no means steep slopes.

Also Dickens' formula for Bengal ;

$$Q = 825\ M^{0.75}$$

where the slopes are not steep.

For the British Isles, owing to the large number of old bridges, such formulæ are not so necessary ; but it may be stated that the floods on impervious catchment areas in the Pennines very closely follow the formula :

$$Q = 500M^{0.83}$$

while for absorbent flat areas the records fall as low as :

$$Q = 100M^{0.67}$$

All these empirical formulæ are merely approximations, and, unless of very limited applicability, are usually approximations tending to overestimate the flood discharge. They form, however, a first step towards the rational method of determining a flood discharge, which I now proceed to give.

Using the rough estimate obtained as above, it is fairly easy to calculate, with approximate accuracy, the time which rain-water falling at the extreme limits of the catchment area would take to reach the point where the flood discharge is required, when the stream and other channels are carrying about half the maximum discharge as above ascertained.

We define this time as the "critical period" for the catchment area under consideration.

Now, from the local rain-fall intensity curves we can estimate the maximum intensity of rain-fall possible in this critical period, and, hence, the whole volume of water falling on the catchment area during the period. From a knowledge of the character of the catchment area we can estimate what fraction of this volume will flow off during the period. We thus get:

If I, be the intensity in inches per hour, or cusecs per acre, of the rain during the critical period of t, minutes and M, the area in square miles of the catchment area:

Then $F = 640 \times 60\ IMt$, is the total number of cubic feet that fall on the catchment area, during the period of t, minutes; and if f, be the fraction that flows away during a period of t minutes:

The flood discharge is:

$$Q = 640 f IM \text{ cusecs}$$

where plainly f depends on the character of the surface of the catchment area being:

- 0·25 to 0·35 for flat country, sandy soil, or cultivated land.
- 0·35 to 0·45 for meadows, and gentle slopes.
- 0·45 to 0·55 for wooded hills, and compact, or stony ground.
- 0·55 to 0·65 for mountainous, or rocky ground, or non-absorbent (*e.g.* frozen soil), surfaces.

The values of I, have already been discussed, and it is plain that in such cases we must use the "Very rare" curve, and must allow some margin even on this.

In this method the estimation of t, the critical period, requires a certain amount of judgment, and it is quite possible in some cases, by omitting the contribution of an isolated and far distant portion of the catchment area, to obtain a far smaller value for t, and therefore a greater value for I, which may produce an absolutely greater flood, in spite of the fact that the flow from a portion of the catchment area has been neglected.

Such conditions are usually hinted at when the intensity of flood obtained by this method, as applied to the whole catchment area, is markedly less than that given by a locally approximate formula, or than that deduced by a comparison with previous observations.

So also, especially in small areas, where the critical period is a few hours only, and the maximum rain-fall intensity is usually produced by thunderstorms; the course of the river in relation to the prevailing direction of rainstorm motion must be considered. Thus, if (as is generally the case in England) the rainstorms move from south-west to north-east, a river which flows from north-east to south-west (*i.e.* against the direction of rainstorm motion) may be expected to have less intense floods than an otherwise identical river which flows in the reverse direction, and is consequently liable to floods produced by the simultaneous arrival of the flood wave from the upper tributaries, and the run-off from the lower portion of the valley produced by a rainstorm which has travelled down the river at approximately the same rate as the flood wave. The motion of the rainstorm, in fact, has decreased the critical period.

Thus, while the general principles are universally applicable, each catchment area must be treated independently.

In town, or agricultural drainage systems, however, general formulæ are frequently required, and since all the minor catchment areas are of nearly similar geometrical form, it is evident that t (and therefore also I), are to a certain degree functions of the mean slope, and the linear dimensions of the area. Hence, the following collection of formulæ is put forward for use in connection with artificial drainage only.

Putting A, for the drainage area in acres, and S, for its mean slope in feet per 1000 feet, we have as follows:

Hawksley's rule, for London .	$Q = 0{\cdot}7A\sqrt[4]{\frac{S}{A}}$
Modified Hawksley's rule, used in New York . .	$Q = 1{\cdot}4A\sqrt[4]{\frac{S}{A}}$
Burkli-Ziegler's rule	$Q = fIA\sqrt[4]{\frac{S}{A}}$ with $f = 0{\cdot}7$ to $0{\cdot}9$ $I = 1$ to 3 inches per hour
M'Math's rule	$Q = fIA\sqrt[5]{\frac{S}{A}}$ with $f = 0{\cdot}1$ to $0{\cdot}8$ $I = 1$ to $2{\cdot}75$
New York tables	$Q = fIA\sqrt[6]{\frac{S^{1\cdot62}}{A}}$ $fI = 1{\cdot}05$ to $1{\cdot}62$
Parmley's rule	$Q = fIA\sqrt[6]{\frac{S^{1.5}}{A}}$ with $f = 0$ to 1 $I = 4$
Gregory's rule	$Q = 2{\cdot}8A\frac{S^{0\cdot186}}{A^{0\cdot14}}$ for impervious areas
Adams' rule	$Q = fIA\sqrt[6]{\frac{S^{\frac{1}{2}}}{AI}}$ when $I = 1$ $f = 1{\cdot}837$

See Gregory's paper, *Trans. Am. Soc. of C.E.*, vol. 58, p. 458.

It is doubtful whether all the authors employ the symbols f and I with precisely the same meaning with which I have defined them, and in many cases the results are applied to town areas where f is probably equal to 0·7 to 0·95. It will, however, be plain that some formula of the type:

$$Q = CA^{0\cdot75 \text{ to } 0\cdot86}\, S^{0\cdot16 \text{ to } 0\cdot25}$$

can usually be arrived at where C depends on the value of f, and on the absolute magnitude of the rain-fall, while the indices of A and S, mostly depend on the form of the rain-fall intensity curve when expressed as a function of *t*.

It is, however, believed that the extended and rational method should be applied to all important cases, and that the deduction of a compressed formula of the type now suggested is only legitimate after a certain amount of experience has been accumulated. The one important fact is that the flood discharge per unit area, *i.e.* $\frac{Q}{A}$, or $\frac{Q}{M}$, for a small area, is more intense than that of a larger area of the same mean slope.

As an example of the erroneous deductions regarding flood discharges, that may be obtained through relying too exclusively on the results deduced from a survey of the old structures existing on a stream, the preliminary studies of the Kali Nadi Aqueduct (*P.I.C.E.*, vol. 95, p. 287) are interesting.

In this case, a road bridge, over one hundred years old, crossed the stream a little below the proposed site. Flood marks above and below the bridge were quite plain, and taking the head thus indicated (1·5 foot), allowing for a velocity of approach of 1·48 feet per second, and a coefficient of discharge of 0·60 through the arches, a flood discharge of 8436 cusecs was obtained.

The engineers concerned evidently did not place complete reliance on this result, and provided for more than twice the quantity. It will be recognised that the calculation is adequate, and even if a coefficient of 0·90 is taken (my own experience leads me to believe that 0·60 may possibly be a trifle low, but no drawings of the bridge are given) the value is only 12,600 cusecs.

As a matter of observation, however, the year after the aqueduct was constructed, a flood of at least 37,000 cusecs occurred, and it was found that the flood breached the approaches to the bridge, but did not damage the structure. The approaches of the aqueduct not affording a similar safety valve, it was badly damaged, a heading up of 3·5 feet occurring.

Next year an even larger flood occurred, and after the water had headed up 9·8 feet the aqueduct arches blew up, but the old bridge was merely submerged. The flood was estimated at 132,475 cusecs, *i.e.* about 11 times greater than the original bridge observations would indicate, even when treated in the most inflated manner.

As a test of the rational method of estimation we may take the following figures: 3,025 square miles of flat and sandy catchment area, and a rain-fall of 17·6 inches on the 16th July 1885, followed by 3 inches on the 17th. The critical period may be estimated as 48 hours, and the run-off factor as 0·20. We thus get a run-off rate of say $\frac{21}{48} \times 0{\cdot}20 = 0{\cdot}09$ inch per hour, or, at the rate of 58 cusecs per square mile, or say 176,000 cusecs; and the aqueduct as now designed appears capable of passing about 200,000 cusecs, and has stood since 1887.

Equalising Effect produced by a Reservoir.—The above discussion permits the volume of water that reaches a reservoir to be estimated with more or less accuracy. It is not, however, necessary that the waste weir should be able to pass off the flood as rapidly as it arrives at the reservoir, since the rise of the top water level of the reservoir which must occur before the waste weir begins to discharge at its maximum capacity, temporarily stores up a certain volume of water. Thus, the discharge over the waste weir continues for a longer period than the flood, and is consequently not so intense.

The question can be mathematically investigated as follows:

Let B, denote the area of the water surface of the reservoir in square feet, at a height of H feet, above the spillway crest.

Then, in any time dt, a volume Qdt, flows into the reservoir of which BdH, is stored up in the reservoir, and $CLH^{1.5}dt$, flows away over a spillway L, feet in length under a head H.

$$\text{Then}: \quad \frac{dt}{B} = \frac{dH}{Q - CLH^{1.5}}$$

Now, this is integrable if B, and Q, are taken as constants, and putting:

$$Q = CLH_a^{1.5} \text{ and } r = \frac{H}{H_a}$$

where H_a, is evidently the head over the weir when the discharge is Q, cusecs we get:

$$T = \frac{2B}{3(C^2L^2Q)^{\frac{1}{3}}}\left[\log_e \frac{\sqrt{1+\sqrt{r}+r}}{1-\sqrt{r}} - \sqrt{3}\left\{\tan^{-1}\frac{2}{\sqrt{3}}(\tfrac{1}{2}+\sqrt{r}) - \frac{\pi}{2}\right\}\right].$$
$$= K\phi(r)$$

Where K is written for $\frac{2B}{3(C^2L^2Q)^{\frac{1}{3}}}$. Gould (*Eng. News*, December 5th, 1901) has tabulated $\phi(r)$ as a function of r (see p. 289).

This equation gives the time during which the reservoir rises from $H = 0$, to $H = rH_a$, and in any practical case there is a certain factor of safety, since B, increases as H, increases.

It is also possible, in cases where our knowledge of Q, the flood discharge into the reservoir, and C, the coefficient of discharge of the spillway, are sufficiently accurate, to obtain a very close approximation to the manner in which the water surface of the reservoir actually rises, by considering B, Q, (and C, if necessary) as constants only during a small variation in H, say 0·5 foot. We can thus for the first half foot calculate K, H_a, and r, and find T_1, the time during which H, rises from 0 to 0·5 foot.

$$\text{Thus, } T_1 = K\left\{\phi\left(\frac{0{\cdot}5}{H_a}\right) - \phi(0)\right\}.$$

Now, with the new B, (and the new value of C, if advisable) calculate the new K, say K_b, and if a new Q, or C, are used, the new H_a, say H_b, and $r_1 = \frac{0{\cdot}5}{H_b}$ corresponding to $H = 0{\cdot}5$ foot and $r_2 = \frac{1}{H_b}$ corresponding to $H = 1{\cdot}0$ foot.

We thus get

$$T_2 - T_1 = K_b\{\phi(r_2) - \phi(r_1)\}$$

as the time during which H, rises from 0·5 to 1·0 foot, and the process may be continued as necessary

As a simple example, consider a Pennine water-shed of say 6 square miles, with a critical period of 200 minutes. I, is, for "very rare" intensity, 1·05 inch per hour. Take 1·15 inch per hour, and take f, as 0·60. We get a very intense flood at the rate of 442 cusecs per square mile.

The general formula gives:

$$\frac{500}{6^{0.166}} = 372 \text{ cusecs per square mile;}$$

so that the discharge is certainly not underestimated.

Now, assume that B=3 per cent. of the catchment area (say 5·2 million superficial feet), and that L=120 feet as Hawksley's rule would give. While C=3, for safety. Q=2652 cusecs.

Thus $360H_a^{1.5} = 2652$: or $H_a^{1.5} = 7·37$. $H_a = 3·79$ feet.

$$K = \frac{10,400,000}{3(9 \times 14,400 \times 2652)^{\frac{1}{3}}} = 4815 \text{ seconds, approximately.}$$

Thus, assuming that there is no change in B, or Q, we find that the surface of the reservoir rises from H=0, to H=3 feet, or r, changes from 0 to 0·79 in 4815×1·862 seconds. Thus T_1=8960 seconds, or just under 3 hours 30 minutes.

Let us assume that before the storm burst a flow of 600 cusecs was already running out over the spillway. We have:

$$360\ H_1^{1.5} = 600 \qquad H_1^{1.5} = 1·67 \qquad H_1 = 1·41 \text{ foot}$$

and $r_1 = 0·37$. Therefore, $\phi(r_1) = 0·614$, and, for a rise to 3 feet, i.e. $r_2 = 0·79$, $\phi(r_2) = 1·862$, and:

$T_2 - T_1 = 4815\,(1·862 - 0·614) = 6010$ seconds, or a little over 1 hour 40 minutes.

Again, take an area of 1 square mile, with a critical period of 140 minutes. We find that:

$$Q = 60 \times 1·4 \times 640 = 538 \text{ cusecs}$$

and, with L=20 feet we find that $H_a^{1.5} = 9·00$ $H_a = 4·33$ feet

Taking B, again as 3 per cent., *i.e.* 20 acres, or 820,000 square feet

$$K = \frac{1,640,000}{3(9 \times 400 \times 538)^{\frac{1}{3}}} = 4370 \text{ seconds}$$

Now, if H=3 feet; $r=0·69$, $\phi(r)=1·44$, T=6300 secs.=1 hour 45 min.

and if H=4 feet; $r=0·92$, $\phi(r)=2·923$, T=13,830 secs.=3 hours 50 min.

and the general deduction can be made, from these examples, that for such run-offs as can occur under the assumptions above stated, Hawksley's rule for the length of spillways, that is to say:

20 feet per square mile

is amply safe with reservoirs of 3 per cent. water surface.

Let us now consider a reservoir of which the area:

at H=0	is 17,500,000 square feet
at H=5 feet	is 20,000,000 ,,
at H=6·5 feet	is 20,500,000 ,,

and L=70 feet. We assume that Q=8,700 cusecs. C=3.

Taking B, for the first portion of the rise as 17,500,000 we get:

$$K = \frac{35,000,000}{3(9 \times 4900 \times 8700)^{\frac{1}{3}}} = 16,000 \text{ seconds}$$

$$H_a^{1.5} = \frac{8700}{210} = 41·5 \qquad H_a = 10·98 \text{ feet.}$$

Thus, the rise to H = 2·20 feet, or $r = 0·2$; $\phi(0·2) = 0·314$ is accomplished in $16,000 \times 0·314$ seconds = 5024 seconds, or 1 hour 24 minutes.

For the rise from H = 2·20 feet, up to H = 5 feet. Take B = 18,700,000. $K_2 = 17,090$. $r_1 = 0·20$, $r_2 = 0·46$; $\phi(r_1) = 0·314$, $\phi(r_2) = 0·802$

Therefore, $T_2 - T_1 = 17,090\ (0·802 - 0·314) = 17,090 \times 0.488$ = 8350 seconds = 2 hours 20 minutes.

Next, for the rise from 5 feet to 6·5 feet. Take B = 20,250,000.

$$K_3 = 18,500. \quad r_2 = 0·45,\ r_3 = 0·59 ;\ \phi(r_3) = 1·113.$$

$$T_3 - T_2 = 18,500(1·113 - 0·802) = 18,500 \times 0·311$$
$$= 5760 \text{ seconds} = 1 \text{ hour } 36 \text{ minutes}.$$

And for the rise from 6·5 feet to 7·25 feet we can take B = 20,500,000 square feet.

$K_4 = 18,750$. $r_3 = 0·59$, $r_4 = 0·66$; $\phi(r_4) = 1·338$.

$T_4 - T_3 = 18,750(1·338 - 1·113) = 10,750 \times 0·225$
= 4217 seconds = 1 hour 10 minutes.

Now, just previous to the Johnstown (Pa.) dam catastrophe, a rise of 9 inches in one hour was observed, when the height above the spillway was about 7 feet The areas and discharge used above are the nearest round figures to those given in the Report of the Commission of the American Society of Civil Engineers on the subject (*Trans. Am. Soc. of C.E.*, vol. 24, p. 447), and on referring to this report it will be found that the process leads to results corresponding with observation. Thus, the rise to 7 feet above the spillway took place in about 8 hours, while the calculation gives the period as about 6½ hours, and it is doubtful whether the flood discharge attained a value of 8700 cusecs throughout the whole period.

The catchment area was 48·6 square miles, and the assumed figure of 8700 cusecs, or 179 cusecs per square mile, is probably far better ascertained than most of the recorded values of intense floods. The formula $Q = 200M^{0.83}$ gives 154 cusecs per square mile, so that the recorded discharge is about 16 per cent. greater than that given by the formula which applies to catchment areas which are on the average somewhat flatter than that now considered. The critical period is probably about 10 hours, and I, from Talbot's curve of "maximum authentic rain-falls" is about 0·67 inch per hour. If we assume that $f = 0·40$, we get a discharge of 171 cusecs per square mile. The assumptions are legitimate, and it may be observed that while the general formulæ might lead to an unduly small length of waste weir, the more logical method yields a result that would certainly have secured a waste weir of adequate dimensions.

DESIGN OF WASTE WEIRS.—The theoretical design of a waste weir is simple, but accurate knowledge of the coefficients of discharge over the weir (especially if partially drowned), and of the relation between the velocity and surface slope in the escape channel, is very deficient.

The difficulties are principally due to the fact that the head over the weir is often considerably greater than is usual in accurate experiments. The flow in the escape channel is always turbulent (due to the agitation produced by the weir), and is frequently varied (as in the case of a weir which is oblique to the escape channel). The deficiency is more apparent than real; since, although the coefficients are mainly selected by the general consensus of engineering opinion, these coefficients have also been employed in the calculation of the volumes discharged by the floods observed; and, consequently, any error equally affects our fundamental assumptions as to the volume that the weir has to discharge.

For this reason I generally employ the coefficients usual among engineers, and regard the newer values (which have lately been obtained by direct experiment) as refinements that do not necessarily require to be introduced into practical calculations. These newer values should, nevertheless, be employed in working up fresh observations, and are therefore put on record in the appropriate places.

Let Q, be the number of cusecs which it is proposed to discharge over a weir L, feet in length.

If the weir be of the free overfall type (Sketch No. 67, Fig. 1), we have:

$$Q = C_1 L D^{1.5}$$

for the discharge produced by a depth D, over the weir sill. If the flow in the escape channel be steady, we also have:

$$Q = vA = CA\sqrt{rs}$$

where A, is the area of the wetted section of the channel,
- r, is the hydraulic mean radius
- s, its slope, which may be taken as equal to the designed bed slope of the channel.

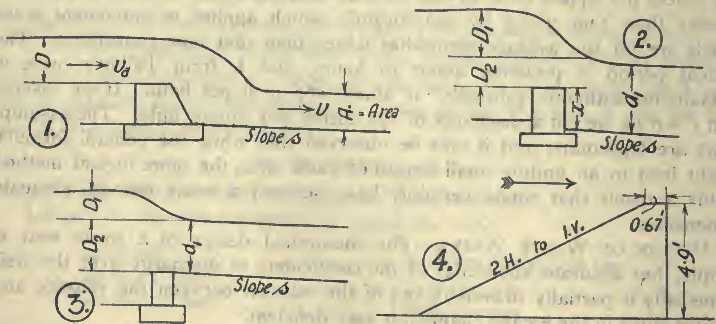
The usual design is a flat-topped weir, and if the crest be less than 3 feet broad, the value:

$$Q = 3{\cdot}09 L D^{1.5}$$

has been very frequently adopted. While for broader crested weirs:

$$Q = 2{\cdot}64 L D^{1.5}$$

The experiments of the United States Deep Waterways Board at Cornell



SKETCH No. 67.—Cross-sections showing conditions occurring in Escape Weirs and Channels.

(*Weir Experiments*, p. 89) indicate that C_1, is in reality variable, and putting $H = D + \frac{v_a^2}{2g}$, we get:

$$Q = C_1 L H^{1.5}$$

and the values of C_1 are tabulated on page 130.

The usual assumptions therefore appear to be safe, and the value $C_1 = 3·09$, is probably a very good mean for the narrower topped weirs when D_1 does not greatly exceed 3 feet, as is usual in British practice.

When the weir is drowned, and is of the form shown in Sketch No. 67, Figs. 2 and 3, the formula are :

$$Q = C_1 L \sqrt{D_1}(D_2 + \tfrac{2}{3}D_1)$$

$$\text{and, } Q = CA\sqrt{rs} = CA\sqrt{d_1 s} \text{ approximately}$$

where, in the second figure, $d_1 = D_2 + x$, and in the third figure $d_1 = D_2$.

The C_1, coefficients are less well determined. For flat-topped weirs, or no weir as in Sketch No. 67, Fig. 3, Strange takes the C_1, coefficient as follows :

D_1+D_2 in Feet	3	4	5	6	7	8	9	10	11	12	13 and over
C_1	3·20	3·31	3·41	3·52	3·63	3·74	3·84	3·95	4·05	4·16	4·27

and neither Chatterton's experiments (*Hydraulic Experiments in the Kistna Delta*) nor Rhind's (*P.I.C.E.*, vol. 154, p. 292), markedly contradict these values.

A more definite statement cannot be made. The values are probably safe, and are likely to be below the truth.

No experiments are available on drowned weirs other than of the flat-topped, or sharp-edged type, except one on a rounded weir section (see Sketch No. 67, Fig. 4) with $D_1 + D_2 = 6·6$ feet (*Report of United States Deep Waterways Board*, p. 291). When undrowned, $C_1 = 3·70$, and diminishes as drowning proceeds, to :

$C_1 = 3·47$, when the tail water is 3·3 feet above the crest

The formulæ are best treated by first calculating D_2, from the equation for the channel discharge and then using the weir discharge equation to calculate D_1, and thus obtaining the total depth over the weir.

VALUES OF GOULD'S FUNCTION $\phi(r)$ (SEE PAGE 285), ARRANGED IN HORIZONTAL LINES FOR EACH TENTH, AND VERTICAL COLUMNS FOR EACH HUNDREDTH OF $r = \frac{H}{H_a}$.

$\frac{H}{H_a}$	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	0·0000	0·0153	0·0306	0·0459	0·0613	0·0766	0·0919	0·1072	0·1226	0·1378
0·1	0·1532	0·1685	0·1838	0·1992	0·2155	0·2319	0·2483	0·2646	0·2810	0·2973
0·2	0·3137	0·3301	0·3464	0·3628	0·3791	0·3955	0·4137	0·4319	0 4501	0·4683
0·3	0·4865	0·5047	0·5229	0·5411	0·5593	0·5775	0·5957	0·6139	0·6321	0·6534
0·4	0·6747	0·6960	0·7173	0·7386	0·7598	0·7811	0·8024	0·8237	0·8450	0·8663
0·5	0·8876	0·9137	0·9399	0·9660	0·9921	1·0183	1·0444	1·0705	1·0966	1·1128
0·6	1·1489	1·1750	1·2012	1·2322	1·2674	1·3027	1·3380	1·3733	1·4086	1·4439
0·7	1·4792	1·5145	1·5498	1·5851	1·6203	1·6556	1·7073	1·7590	1·8107	1·8624
0·8	1·9141	1·9658	2·0176	2·0715	2·1488	2·2262	2·3035	2·3808	2·4582	2·5355
0·9	2·6129	2·7681	2·9233	3·0785	3·2347	3·3889	3·5441	4·0096	4·4750	4·9405

In British practice D, and d_1, rarely exceed 3 feet. The limitation appears to be founded more on a desire to avoid waste of reservoir capacity than on considerations of safety. The design of the weir and the pitching of the escape channel follow the rules given when discussing Falls and Rapids (see pp. 655 and 721). In several of the newer German reservoirs the escape channel is broken up into a series of shallow water cushions, or the slope is even formed into waves. Since escape channels in general are usually dry, and rarely, if ever, run more than 6 inches deep for longer than a week, these designs do no harm, although the experience of irrigation canals shows that if severely tested they would probably be destroyed. The primary function of an escape channel, however, is to remove water, and these refinements obstruct the flow. Since ample opportunity exists for repairing any small erosion that may take place during floods, it appears best, even in very steep channels, to make the channel smooth, and evenly graded, and to prevent erosion by means of a deep curtain wall at the actual weir, with such extra pitching as experience shows to be necessary. A somewhat steeper slope just after the weir does no harm.

I purposely refrain from any discussion of the approximate rules for the determination of the length (channel breadth) of waste weirs. The Gould function method alone has any pretensions to accuracy. One of my earliest professional reminiscences relates to the rejection of a very excellent scheme, on the ground of insufficient waste weir capacity. The designer had, instinctively, applied the principles of Gould's method, but, not being an expert mathematician, was unable to explain them clearly. Thus, what I now consider to have been a perfectly sufficient waste weir appeared far too small. The party I then adhered to was not only devoid of insight, but ignored a very cogent piece of natural evidence. The designed waste weir was an enlarged natural river channel, and no abnormal flood marks could be discovered above or near to the natural constriction. The designer, for want of funds, could merely propose to enlarge and regularise this constricted channel. We totally ignored the existing water marks, which showed clearly that a large temporary storage occurred above the constriction.

CHAPTER VII.—(SECTION A)

DAMS AND RESERVOIRS

Practical proportions—Culvert stops, or plugs—Specification—Details of chases in the plugs—Creeping flanges.

Valve Towers and Culverts.—Head wall and culvert—Central tower—Volume of water rejected when silting must be prevented—Undersluices—Discharge capacity of undersluices.

Silting of Reservoirs. — Desarenador, or scouring gallery — Bombay practice—Assouan reservoir—Removal by dredging—Relation between reservoir capacity and mean annual run-off—Austin reservoir—British reservoir—Preliminary investigations—Willcocks' principles.

Permeability of Earthen Dams.—Mean slope of the saturation plane—Bombay and Croton observations—Drop produced by a masonry core wall—British observations—Drop produced by a puddle wall—Experimental dams.

Hydraulic Fill Dams.—Construction—Grading of material—Proportion of clay—Central drainage tower—Effect of moist climates—Necessity for special plant—Consolidation produced—Ratio of solids deposited to water used.

Rock-fill Dams.—Construction—Initial settlement of the rock fill—Failures probably caused by bad construction—Fissured-rock foundations—Escondido dam—Wood and concrete diaphragm—Otay dam—Steel plate diaphragm.

Stability of an Impermeable Dam and Cut-off Wall on a Permeable Foundation of Indefinite Depth.—An accurate mathematical solution of the above problem can be obtained, on the assumption that the upper boundary of the permeable stratum is a horizontal plane extending indefinitely in both directions from the ends of the dam, as shown in Sketch No. 68. In practice, this condition is sufficiently satisfied provided that marked erosion at the downstream end of the dam is prevented. Thus, the theory can be applied in order to investigate such cases as the following:

(*a*) Earth dams which are never over-topped by the water that they retain.

(*b*) Overflow masonry dams which are rarely exposed to the action of flowing water, so that the tail erosion is but small.

(*c*) Regulators of canals where the action of the flowing water is under control, so that erosion is entirely stopped by the pitching, which is placed downstream of the work.

In works such as weirs and undersluices, tail erosion cannot be kept within definite limits, and although the bed of the channel downstream of the work is pitched, deep scour holes may, and do, occur close to the downstream tail of the work. If certain assumptions are made which agree fairly well with the practical conditions, the theory can be modified so as to be applicable to these cases. Experimental investigations, however, show that the critical points where failure is most likely to begin are not the same as in works which are not exposed to erosion. If erosion does not occur the downstream tail of the dam is the point where failure commences. Whereas, if erosion occurs, failure begins in the neighbourhood of the bottom of the cut-off wall. Thus, while the same theory suffices for the investigation of the two classes of circumstances, the conditions for stability differ somewhat markedly, for in the first, or non-eroded class, a, the depth to which the core wall is sunk will be found to be the factor which mainly determines the stability. Whereas, in the scoured, or eroded class, the depth of the core wall has only an indirect influence on the stability which is almost entirely determined by $2b$, the width of the base of the dam.

The circumstances assumed in the ideal case are shown in cross-section in Sketch No. 68, where the boundary of the impermeable area is hatched.

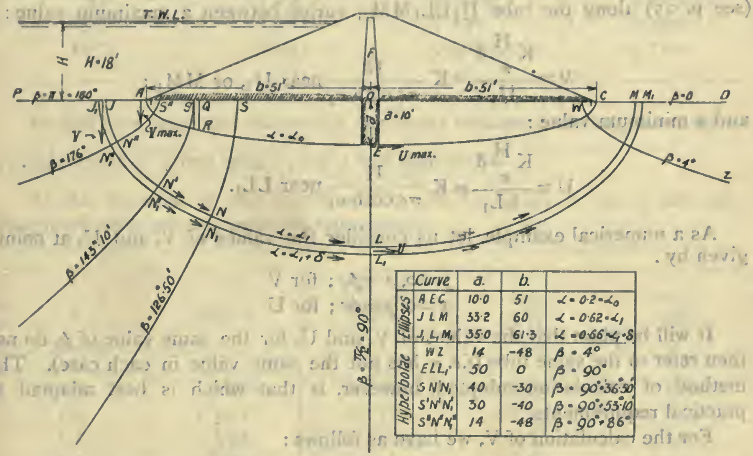
Let $2b$, be the breadth of the impermeable base of the dam, and let a, be the depth to which the impermeable core wall extends below the base of the dam. Put $c^2 = b^2 - a^2$, and assume that the core wall lies in the centre of the base of the dam. For the sake of simplicity, the action of gravity is neglected for the present.

Then, assuming that the stratum below the dam is equally permeable at every point, and that no percolation occurs through the dam or core wall, the lines of flow are the series of confocal ellipses :

$$\frac{x^2}{\cosh^2 a} + \frac{y^2}{\sinh^2 a} = c^2$$

and the lines of equal pressure are the confocal hyperbolæ :

$$\frac{x^2}{\cos^2 \beta} - \frac{y^2}{\sin^2 \beta} = c^2$$



	Curve	a.	b.	
Ellipses	A E C	10·0	51	$\alpha = 0{\cdot}2 = \alpha_0$
	J L M	33·2	60	$\alpha = 0{\cdot}62 = \alpha_1$
	$J_1 L_1 M_1$	35·0	61·3	$\alpha = 0{\cdot}66 = \alpha_1 + \delta$
Hyperbolae	W Z	14	-48	$\beta = 4°$
	$E L L_1$	50	0	$\beta = 90°$
	$S N N_1$	40	-30	$\beta = 90° + 36°50'$
	$S' N' N_1'$	30	-40	$\beta = 90° + 53°10'$
	$S'' N'' N_1''$	14	-48	$\beta = 90° + 86°$

SKETCH No. 68.—Percolation under a Structure not subjected to Tail Erosion.

Let us now consider one foot length of the dam. Let H, be the head of water which the dam holds up. Then the pressure at the hyperbola given by $\beta = \pi$ (*i.e.* along the line AP), is H, and along the line CD (*i.e.* the hyperbola given by $\beta = 0$) the pressure is o. Thus, the pressure along any intermediate hyperbola $\beta = \beta_1$ is H $\frac{\beta_1}{\pi}$, feet of water. Also the quantity of water percolating along the imaginary tube bounded by the ellipses $a = a_1$ and $a = a_2$ is proportional to $a_2 - a_1$, and since the flow is capillary, the quantity delivered per second is :

$$K \frac{H}{\pi} (a_2 - a_1)$$

where K, is the constant dependent on the effective size of the sand grains which occurs in Hazen's formula (see p. 25).

Thus, the rate of flow per square foot is not uniform, but becomes more intense as the normal distance between the ellipses a_1 and $a_1 + \delta$ (where δ is a small constant quantity) decreases.

Now, let us consider two such ellipses, JLM, and $J_1L_1M_1$, as given in Sketches Nos. 68 and 70.

We have $OJ = c \cosh a_1$

$OJ_1 = c \cosh (a_1 + \delta)$

Therefore, $JJ_1 = c\delta \sinh a_1$ if δ be small.

$OL = c \sinh a_1$

$OL_1 = c \sinh (a_1 + \delta)$

Therefore, $LL_1 = c\delta \cosh a_1$, if δ be small.

Any other normal distance NN_1, plainly lies between the values JJ_1, and LL_1, for $NN_1 = c\delta\sqrt{\cosh^2 a_1 - \cos^2\beta_1}$, where $\beta = \beta_1$ represents the hyperbola on which N, and N_1, lie.

Thus, the flow per square foot, or the effective velocity of percolation (see p. 25) along the tube $JJ_1LL_1MM_1$, varies between a maximum value:

$$V = \frac{K\frac{H}{\pi}\delta}{JJ_1} = K\frac{H}{\pi c \sinh a_1} \text{ near } JJ_1, \text{ or } MM_1;$$

and a minimum value:

$$U = \frac{K\frac{H}{\pi}\delta}{LL_1} = K\frac{H}{\pi c \cosh a_1} \text{ near } LL_1.$$

As a numerical example, let us consider the values of V, and U, at points given by:

$$y = 0, x = pc; \text{ for V}$$
$$x = 0, y = pc; \text{ for U}$$

It will be plain that the values of V, and U, for the same value of p, do not then refer to the same tube (*i.e.* a has not the same value in each case). The method of calculation adopted, however, is that which is best adapted to practical requirements.

For the calculation of V, we have as follows:

$$\cosh a = \frac{x}{c} = p \qquad V = \frac{KH}{\pi\, c \sinh a}.$$

$p = \cosh a$	a	$\sinh a$	$\frac{V\pi c}{KH}$
1·0	0·00	0·0000	∞
1·5	0·96	1·1144	0·896
2·0	1·32	1·7182	0·581
3·0	1·77	2·8503	0·351
5·0	2·29	4·8868	0·205
10·0	2·99	9·9177	0·101

The value $\frac{V\pi c}{KH} = \infty$ cannot occur in practice, for since $c^2 = b^2 - a^2$, flow is not possible until p, is at least equal to $\frac{b}{\sqrt{b^2 - a^2}}$.

For the calculation of U, we have as follows:

$$\sinh a = \frac{y}{c} = p \qquad U = \frac{KH}{\pi c \cosh a}.$$

$p = \sinh a$	a	$\cosh a$	$\frac{U\pi c}{KH}$
0·0	0·00	1·0000	1·000
0·5	0·49	1·1225	0·893
1·0	0·88	1·4128	0·709
1·5	1·19	1·7956	0·557
2·0	1·44	2·2288	0·448
3·0	1·82	3·1669	0·315
5·0	2·32	5·137	0·195
10·0	2·99	9·9680	0·100

In this case fractional values of p, can occur because c, can be greater than a, provided that a^2 is less than $\frac{b^2}{2}$, say a less than 0·71b.

As an example of the values of V, and U, which occur in the same imaginary tube, we may take the following:

a	$\cosh a$	$\sinh a$	$\frac{U\pi c}{KH}$	$\frac{V\pi c}{KH}$
0·5	1·1276	0·6211	0·885	1·610
1·0	1·5431	1·1752	0·649	0·850
1·5	2·3524	2·1293	0·426	0·469
2·0	3·7622	3·6269	0·266	0·276
3·0	10·0678	10·0179	0·100	0·100
5·0	74·210	74·203	0·013	0·013

It will be seen that both V, and U, increase as a decreases. There is, however, a minimum value of a, say a_0, which is that corresponding to the ellipse AEC, which passes through the two toes of the dam, and the bottom of the core wall, and is given by the equations:

$$c \cosh a_0 = b, \quad \text{and } c \sinh a_0 = a.$$

Thus, the maximum values of U, and V, are obtained by putting $a = a_0$ and are:

$V_{max} = \frac{KH}{\pi a}$, in a vertical direction at A or C, either toe of the dam, and,

$U_{max} = \frac{KH}{\pi b}$, in a horizontal direction immediately below E, the bottom of the core wall.

The upward pressure of the percolating water on the base of the dam is given by the formula :

$$h = \frac{H\beta}{\pi} = \frac{H}{\pi}\left(\cos^{-1}\frac{x}{c}\right)$$

where x, is positive when measured in the downstream direction.

These formulæ afford a measure of the likelihood of sand being removed from under the dam, and it will be evident that the dam is stable if the pressures and velocities thus determined are insufficient to remove the sand grains, or to blow up the dam.

It is also evident that there is a limit to the total percolation per foot length of the dam, even if the permeable stratum is of very great depth. For, let a_g be the value of a for the ellipse that touches the bottom of the permeable stratum which is g feet deep. Then the total flow per foot length of the dam is $\frac{KH}{\pi}(a_g - a_0)$, and since g, is assumed to be great, we can put

$$\cosh a_g = e^{a_g} = \sinh a_g \text{ or } ce^{a_g} = g. \quad \text{Therefore, } a_g = \log_e \frac{g}{c},$$

and the total flow is $\frac{KH}{\pi}\left(\log_e \frac{g}{c} - a_0\right)$ which increases but slowly as g, increases.

This investigation (at first sight) leads to the rather surprising result that the width of the base of a dam has no influence in stopping upward percolation at the downstream toe of the dam.

This is possibly true in an ideal case, and suggests that the notorious weakness of dams with no core walls is not entirely due to "Leakage along the seat of the dam," or along the "line of junction." In any practical case, there are core walls,—of shallow depth,—but still they exist, and it will be plain on looking at Sketch No. 68 that a shallow core wall, as shown at QR, would theoretically, be equally effective in stopping percolation as the deeper core wall OE, provided that R lies on the ellipse AEC (the dam itself is of course assumed to be impermeable).

Now, take an example, and let us assume that $b = 100$ feet, and $a = 10$ feet. Therefore $c^2 = (99{\cdot}5)^2$ approximately. The equation of the ellipse AEC, is

$$\frac{x^2}{100^2} + \frac{y^2}{10^2} = 1.$$

Now, put $x = 90$. Therefore $y^2 = 100 \times 0{\cdot}19 = 19$. $y = 4{\cdot}3$ feet.

Thus, a core wall with a depth of 4·3 feet at 90 feet from the centre of the base, is, theoretically, as effective as one which is 10 feet deep at the centre of the base.

Again, if $b = 50$ and $a = 10$ feet, we get :

$$\frac{x^2}{50^2} + \frac{y^2}{10^2} = 1$$

$$\text{and when } x = 40, \quad \frac{y^2}{10^2} = \frac{9}{25}; \quad \text{and } y = 6.$$

So that a core wall 10 feet from the toe of the dam now requires to be 6 feet deep, in order to have the same effect as a central core wall which is 10 feet deep.

Let us now treat the case of a blanket of puddle clay 3 feet thick, that starts at 3 feet from the toe of the dam, as shown in Sketch No. 69.

Put $b = 100$.

We have to determine a, where:

$$\frac{x^2}{100^2}+\frac{y^2}{a^2}=1$$

and we know that $x = 97$, and $y = 3$ satisfy this equation; for the ellipse must pass through the corners of the apron.

Thus, $$0{\cdot}97^2+\frac{9^2}{a^2}=1. \qquad a^2=\frac{9}{0{\cdot}06}$$

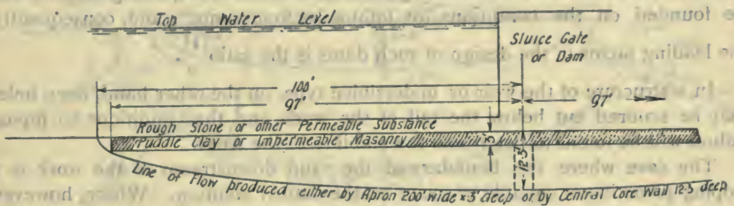
or, $a = 12{\cdot}3$ feet.

So that in such a case a central core wall gives no extra advantage, unless it is more than 12·3 feet in depth.

These figures are of course purely theoretical, but they serve to show the following principles:

(i) A long, shallow blanket or apron is equivalent to a far deeper central core wall, and the theoretical depth of this core wall can be calculated as above.

(ii) The most advantageous position for a vertical core wall is as close to either toe of the dam as possible. This is of course subject to practical



SKETCH No. 69.—Effect of a Shallow Apron in Stopping Percolation.

considerations, and the two following conditions militate somewhat against a core wall being situated near to the toe of the dam.

(*a*) If the core wall be too close to the water surface of the dam, it may be breached by burrowing animals (such as water-rats and crayfish). If it is too far removed, it may become dry during periods when the water is low in the reservoir, and crack.

(*b*) The dam, in practice, is not wholly impermeable, and a wall of puddle, or other impermeable material, must be carried up above the top water level. Thus, if this wall is not placed vertically below the crest of the dam it must be laid on a slope. Puddle walls laid at a slope, or rather, beds of puddle facing a slope, are very difficult to lay clean and unmixed with earth or grit, and are extremely likely to burst and cause the water face of the dam to slip. While masonry walls laid on a newly made slope of earth are certain to crack.

Consequently, I do not consider the fact that the theory does not show the position usually adopted for core walls to be the best, in any way decreases its value.

In other respects, the theory agrees very well with advanced practice, for it clearly shows the *raison d'etre* of the reversed filters at the downstream toe, and the advantages of the concrete base of the core wall together with the accompanying drain and reversed filter.

The practical application of these formulæ greatly depends upon the conditions existing at the downstream toe of the dam. It will be plain that sand may be removed in the two following ways:

(*a*) By piping, that is to say removing the sand in a horizontal direction, such as is indicated as likely to occur near the points L and L_1 below the core wall (see Sketch No. 68).

This (as will later be shown) is largely influenced by the amount of erosion that takes place and otherwise mainly depends upon the ratio: $\frac{H}{b}$.

(*b*) By fountaining, or the upward removal of sand, which is indicated as probable near the points J, and J_1 (see Sketch No. 68). This depends only upon the ratio $\frac{H}{a}$.

If these two actions are independently studied, we find experimentally that removal by piping is far more easily effected. In an actual dam, however, if a horizontal surface of sand exists below the downstream end of the dam, it is plain that piping cannot occur unless fountain failure has previously taken place. Thus, in earthen dams or head regulators, calculations regarding stability can be founded on the conditions for failure by fountaining; and, consequently, the leading factor in the design of such dams is the ratio $\frac{H}{a}$.

In a structure of the weir or undersluice type, on the other hand, deep holes may be scoured out below the tail of the work, and the conditions for piping failure will be found to determine the stability of the work.

The case where the boundary of the sand downstream of the work is a sloping plane is not capable of exact mathematical solution. Where, however, the boundary is one of the confocal hyperbolæ $\beta = \beta_2$ say, as shown in Sketch No. 70, an accurate solution can be obtained. The work need not be given, as it will be obvious to any one who has followed the original investigation that:

$$U = \frac{KH\delta}{(\pi - \beta_2)\, c \cosh a}$$

and the velocity of percolation at any other point N is given by the equation:

$$v = \frac{KH\delta}{(\pi - \beta_2)\, NN_1}$$

where the symbols are those used on page 294.

The exact point where failure is most likely to occur is doubtful.

Immediately under the core wall at E (see Sketch No. 70) we have:

$$U_{max} = \frac{KH}{(\pi - \beta_2)\, b}$$

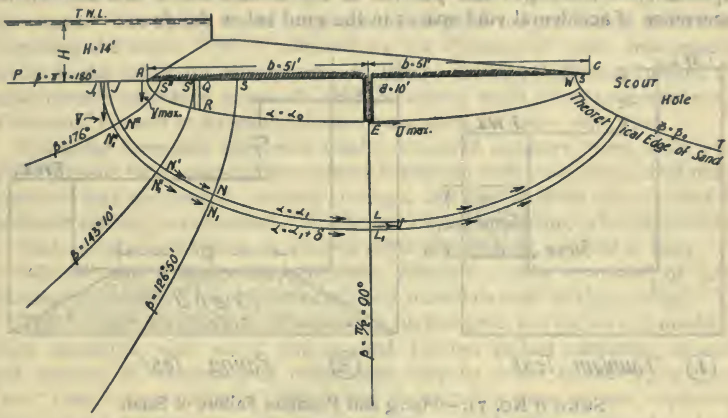
and here, the motion being horizontal, the stability entirely depends upon the friction of the grains of sand on one another. But as we proceed from E, to C, along the ellipse AEC, the velocity of percolation increases, and thus, were it not for the fact that the direction of flow becomes more and more inclined upwards, failure would occur sooner than at E. Consequently, the precise point where failure begins can only be determined experimentally, and it is probable that the weak spot is somewhere close to the line ST, the upstream face of the scour hole. In any case, it is found that the value of a, has but little influence (and that only an indirect one due to the fact that β_2 is dependent upon a), on the stability, and that the important quantities are b, and β_2. This last quantity is principally determined by the ease with which the sand is eroded, and by the amount of erosion which it has sustained.

Actual experimental data are rare :

If we put h, for the head in feet of water that produces removal of sand by fountaining (Sketch No. 71, Fig. 1) through a vertical height of l feet of sand, it will approximately be found that :

(*a*) For sand of a mean diameter of 0·03 inch, $l = 1{\cdot}2h$.
(*b*) For sand of a mean diameter of 0·010 inch, $l = 3{\cdot}6h$.

These figures are confirmed by Beresford (*P.I.C.E.*, vol. 158, p. 77), who states that $l = 3h$ for sand ranging from 0·01 inch to 0·005 inch in diameter, and Russell (*Journ. Ass. of Eng. Socs.*, 1909) (see p. 582) who states that $l = h$, for sand such as is used in mechanical filters. The figures are only approximat ,



SKETCH NO. 70.—Percolation under a Structure subject to Tail Erosion.

and, so far as I can judge, the proportion of the smaller grains has a considerable influence upon the question (*i.e.* the value of n, in the equation $l = nh$, probably depends not only on the effective size of the sand, but also on the uniformity coefficient).

The results are, however, confirmed by practical rules, for we know that dams on coarse sand rarely fail unless $\frac{H}{a}$, is less than $\frac{1}{2}$, and in fine sand unless $\frac{H}{a}$, is less than 1. Now, the quantity l, considered above is equal to πa, so that these rules give $l = 1{\cdot}6h$, for coarse sand ; and $l = 3{\cdot}1h$, for fine sand, which is somewhat coarser than the finer of the two sands above experimented upon. Thus, the rules probably afford a margin of safety equivalent to 50 per cent.

For piping failure, where l, now denotes a horizontal length of sand (Sketch No. 71, Fig. 2), we find that :

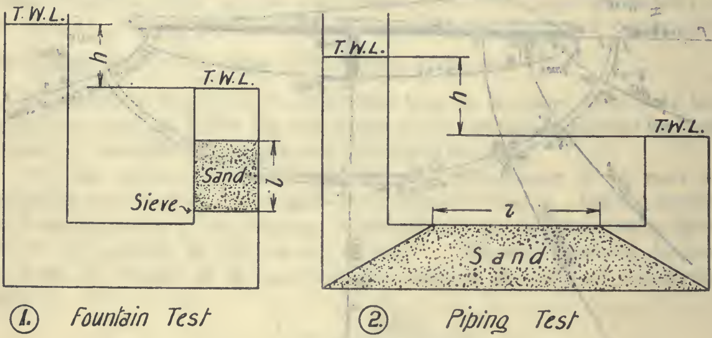
(*a*) For sand of 0·03 inch mean diameter, $l = 2{\cdot}5h$.
(*b*) For sand of 0·01 inch mean diameter, $l = 8h$.

The experimental figures for piping failure are very irregular, and I do not consider that any great reliance can be placed on those given above. If the slightest void is left unfilled with sand in the top of the experimental tube a local flow of water is set up which rapidly removes the sand, and failures have been observed with 0·01 inch sand in cases where $l = 20h$.

The time factor also appears to have a considerable influence, as I have found that if the pressure is left on for an hour or more piping occurs at heads which are easily resisted for 5 or 10 minutes. My apparatus, however, was not sufficiently perfect to enable me to assert that this was not caused by the sand gradually settling, and creating a small void which permitted a flow of water to commence, so starting the failure.

The practical deductions are very clear. If erosion is permitted below the tail of the work failure can occur by piping, and the stability mainly depends upon $2b$, the breadth of the dam.

The failure is more or less fortuitous, as the liability to piping largely depends upon the depth and position of the eroded holes, and also on the occurrence of accidental void spaces in the sand below the dam.



SKETCH NO. 71.—Piping and Fountain Failure of Sand.

If tail erosion does not occur, the stability mainly depends upon the depth of the cut-off wall. Failure occurs by fountaining, and is less dependent upon accidental circumstances.

Regarded in this manner, the figures given by Bligh and my distrust of shallow core walls (see p. 679) are both easily explicable. So also, the utility of shallow core walls for forming cut-offs to accidental vacuities under a dam or impermeable apron, and their probable ineffectiveness if the impermeable apron is not of sufficient length, becomes plain.

Effect of Gravity.—So far the effect of gravity has been neglected. Consider any point (x, y), where y, is positive, when measured downwards from the base of the dam.

Then from the equations:

$$\frac{x^2}{\cosh^2 a} + \frac{y^2}{\sinh^2 a} = 1$$

$$\frac{x^2}{\cos^2 \beta} - \frac{y^2}{\sin^2 \beta} = 1$$

we calculate a and β.

The preceding investigation has shown us that the pressure at (x, y), is $\frac{H\beta}{\pi}$.

Super-adding the effect of gravity, we have a total pressure of:

$$\frac{H\beta}{\pi}+y, \text{ feet of water.}$$

The velocity of flow is represented by:

$\frac{KH}{c\sqrt{\cosh^2\alpha-\cos^2\beta}}$ and is unaltered by the action of gravity.

It will also be plain that the whole theory depends upon the permeability being constant over the whole layer, *i.e.* K = a constant.

So far as I am aware, no variation in K, due to the pressure of superincumbent layers, has been observed; but variations in the size of the grains or in their freedom from dirt may have a considerable influence upon K (see p. 26).

EARTHEN DAMS.—The design of earthen dams is purely a matter of experience. The ruling factor is the means adopted for preventing water from traversing the dam. This is effected by the erection of some impermeable barrier in the substance of the dam. In British practice (including Indian), the impermeable substance is usually clay, or clayey earth; but concrete (either of Portland cement or hydraulic lime), or masonry, has also been used.

The older American engineers usually employed masonry "core walls," but at the present time concrete (sometimes reinforced with steel rods), steel plates protected with concrete or asphalt coatings, and the artificial clay formed by hydraulic sluicing, are also used. Similar devices are appearing in British practice.

Such perishable barriers as wooden sheet piles will be referred to later.

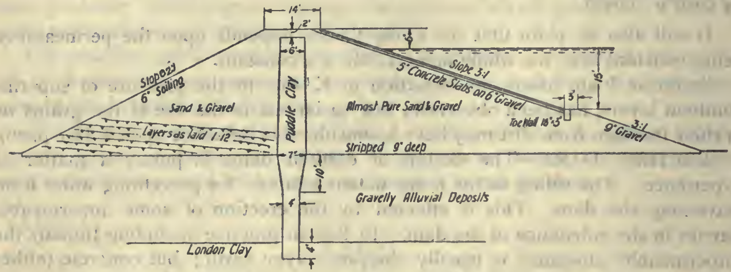
We will now consider the conditions affecting a dam composed of gritty material and clay. The properties of these materials may be thus defined:

The "grits" are pervious to water in a high degree, but do not slip markedly when saturated with water, and a good Thames ballast consisting of sand and pebbles in almost equal proportions may be taken as the typical case. The "clay," when properly treated and deposited, is more or less impervious to water, but is almost incapable of supporting itself when saturated, and will slip until it attains slopes such as 1 in 5, or 1 in 7. London clay may be taken as the typical case. Thus, a "good gritty" material is capable of forming a stable embankment even when saturated with water; and a thin layer of "good clay," if properly deposited, will (for all practical purposes) stop the percolation of water, even if the head producing the percolation is large in comparison with the thickness of the layer. It is evident that the better the quality of the clay the thinner will be the clay wall or layer necessary to stop percolation, and therefore the smaller the danger of slips caused by the pressure of the clay setting the gritty material in motion.

When good clay and good grits are obtainable the construction of a dam is fairly simple. The main body of the dam is composed of the gritty material, and the percolation of the water is stopped by a wall of puddled clay.

When two such materials occur on the dam site, and a junction with a continuous layer of clay in the strata underlying the dam can be secured, the circumstances may be considered as highly favourable. A dam of this kind is shown in Sketch No. 72 (which represents the generalised section of the Staines reservoirs dam). Such dams are absolutely water-tight if the clay layer is properly deposited, and, since the clay portion is small in comparison with the gritty portion, no danger from slips exists.

Let us assume that such excellent clay as London clay cannot be procured, and (since core walls of other substances are too costly) that it is desired to make a dam of black cotton soil and shale. We now have a case resembling the Ashti dam (*P.I.C.E.*, vol. 76, p. 288). The thin puddle wall that suffices in the case of the first class puddle made at Staines is no longer sufficiently thick to prevent leakage, and thus from one-third to one-half (or even more) of the bulk of the dam is made of "clay." The danger of slips is evidently far

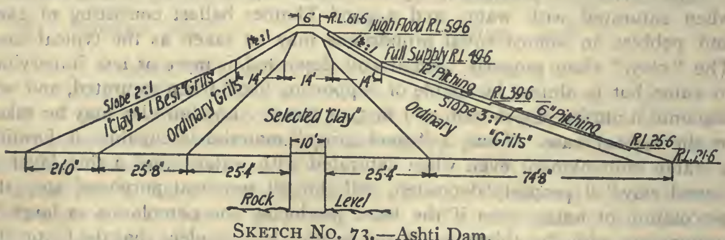


SKETCH NO. 72.—Staines Reservoirs Dam.

greater than in the first type, and increases as the bulk of the clay increases (see Sketch No. 73).

As a matter of history such dams do slip, especially on the downstream face. (The Ashti dam has not slipped so far as I am aware.)

Proceeding further in this direction, we come to the type of dam recommended by Strange (*Indian Storage Reservoirs*, or *P.I.C.E.*, vol. 132). Here, the whole dam is composed of a material which is somewhat more



SKETCH NO. 73.—Ashti Dam.

pervious to water than black cotton soil, but which is also less liable to slip. Strange's specification is a mixture of one part of black cotton soil to one part of shale (Sketch No. 74). In America, Fanning recommends the following:

Coarse gravel	59 per cent.
Fine gravel	20 "
Sand	9 "
Clay	12 "

and tests the mixture by ramming it moist into a bucket, and then ascertaining if it will remain in the bucket when turned upside down.

The above discussion illustrates the variations in the relative proportions of gritty and clayey material that may occur in dams.

We now come to the question of drainage. When saturated with water a pure clay is for all practical purposes absolutely unstable, and even the thinnest clay cores or puddle walls must be drained.

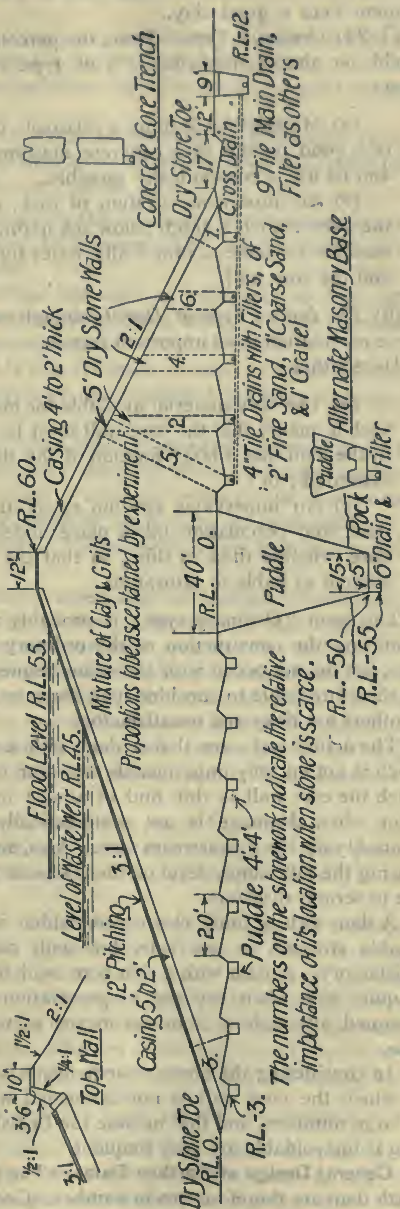
Returning to our three typical sketches, the puddle wall of the Staines reservoirs was not artificially drained, but was erected in material which provided excellent drainage.

The Ashti dam was drained in an unsystematic manner. Strange's type is drained systematically in a way which will be later described.

The principles affecting the design of a dam with a clay core may consequently be stated as follows :

The thickness of the clay core depends on the quality of the available material, and must be sufficient to check detrimental leakage. On the other hand, the thicker the clay core, the greater the care that must be taken to prevent slips. As slips are caused by leakage of water, and especially by leakage through clayey material, the worse the quality of the clay core the greater is the necessity for systematic drainage.

It will also be plain that the more the gritty material diverges from the ideal of a substance which is stable, when saturated with water, the greater is the liability to slips. The matter is not practically so important as the quality of



Sketch No. 74.—Strange's Design for an Earth Dam.

the clay, for the simple reason that a good gritty substance is far more common than a good clay.

(i) *The Ordinary Type.*—Here, the percolation or leakage through the dam should be almost negligible. This type can consequently be constructed when :

(*a*) Material for forming a relatively thin impermeable core wall exists (*e.g.* good puddle clay, concrete, masonry, or well protected steel plates), and its use is economically possible.

(*b*) An impervious stratum of rock, or clay, must exist at the site of the dam at such a depth below the natural surface that it is economically feasible to make a practically water-tight junction between this stratum and the core wall.

(ii) *The Indian Type of Dam* (although not exclusively confined to India), where percolation is an important factor.

Here either :

(*a*) The only material available for making the impermeable core is of such a quality that the core wall must be relatively so thick in proportion to the dam that a large fraction of the dam section is liable to slip when saturated ; or :

(*b*) No impervious stratum exists at a reasonable depth beneath the dam, and percolation takes place under the bottom of the impermeable core (whether thick or thin), so that the whole of the dam must be considered as liable to saturation.

The term "Ordinary type" is probably a misnomer. The circumstances permitting the construction of the ordinary type are common in the British Isles, but do not occur with the same frequency elsewhere. British engineers are therefore liable to consider such dams as the only type, and to believe that all others are risky and unsatisfactory.

The actual facts are that a dam with a relatively thick core wall of clay which is not entirely impermeable to water is more liable to slip than one in which the core wall is thin and of almost impervious clay, but the slips only occur when drainage is not systematically attended to, and then almost invariably on the downstream face. Also, such slips (although in some cases causing the full supply level of the reservoir to be reduced) have never given rise to serious disasters.

A dam with a thick clay centre, which is not carried down to an impermeable stratum, is less safe, but with careful drainage can be rendered satisfactory. A dam with a thin core wall of good clay, masonry or concrete, is quite safe when exposed to percolation under the core wall, if properly designed, although it is by no means as water-tight as one of the ordinary type.

In considering the above statements, it is as well to bear in mind that dams in which the core wall is not connected with an impermeable stratum exist in large numbers, and that outside the British Isles, cases where such construction is unavoidable are very frequent.

General Design of Earthen Dams.—The ruling factors in the design of an earth dam are therefore two in number. Good practice has led to the evolution of three types, which are entirely conditioned by these two factors.

The factors are as follows:

(i) The material available for the construction of an impermeable core wall. This is either clay suitable for making good puddle, or a cheap supply of Portland cement, or hydraulic lime, for forming a good concrete, or masonry core wall.

(ii) The presence of an impermeable stratum of unfissured rock or clay, at such a depth below the dam as to permit the core wall being sunk to form a secure junction with this stratum.

Where both conditions are favourable, the "ordinary type" of dam has been evolved. Such dams are common in all countries, and are frequently held to be the only satisfactory construction.

I shall later endeavour to show that too rigid an adherence to this type is not only very costly, but is unnecessary, whether safety, or the economical utilisation of the stored water, is considered.

Where good core wall material is not available, or where impervious strata are not easily reached, another type of dam (which I propose to term the Indian type) has been evolved.

In this type, percolation under the dam is guarded against by careful drainage, and in some cases the dam itself is porous. While engineers accustomed to the careful and expensive methods employed in the ordinary dam to prevent all percolation, may consider such dams as makeshifts at the best, many satisfactory examples exist. I consider that this type will probably be largely adopted in the future.

The hydraulic-fill dam in theory is essentially an attempt to manufacture an artificial clay by a water-sorting process, analogous to that which on a large scale, and over long periods of time, actually produces clay in Nature.

Ordinary Type of Dam.—The proportions of earthen dams are solely derived from practical experience. Under the conditions usually obtaining, *i.e.* practically no percolation through, or under the core wall, and very little artificial drainage of any portion of the dam except perhaps the puddle trench (see p. 317) slopes of 3 to 1 on the water face, and 2 to 1 on the downstream face, are found to be stable.

The dam is generally carried up at least 5 feet above full supply level, and its width at the top varies from 6 feet in low, to 10, or 12 feet, and even more in high dams, especially if the top of the dam is intended to carry a road, or even a cart track.

The most economical height can be determined by balancing the expense of an extra foot in height of the dam against the cost of the extra excavation required to increase the width of the waste weir in order to pass off the same quantity of water at 1 foot less head. As an example, let us suppose that the length of the waste weir is 200 feet and that the maximum flood likely to occur can be passed off under a 3 feet head. The top of the dam would require to be at least 3 feet above flood level, or 6 feet above the normal full supply level. If the waste weir length is increased to $200 \times \frac{3^{1.5}}{2^{1.5}} = 368$ feet, overtopping will be equally well guarded against with a dam 1 foot lower than that designed. Comparative estimates can be made, and the cheaper solution may be selected.

The matter can be expressed in general terms by stating that when a site suitable for a long waste weir exists, and the depression crossed by the dam is such as to require a long bank to close it, the elevation over the full supply level is least, and as small a value as 4 feet may prove economical.

On the other hand, in localities where the configuration of the ground is precipitous, a long waste weir may prove very costly; and, similarly, it will usually be found that the dam is a short one.

Thus, an elevation of 10, or even 12 feet, above full supply level, may be advisable.

The matter is influenced by the temporary storage of the flood volume in the reservoir. The principles discussed under Flood Discharge (see p. 284) must be considered in drawing up the final design.

A comparison with the designs for an Indian type dam will at once suggest the possibility of a slope steeper than 2 : 1 being employed on the downstream side, provided that drainage is carefully attended to. The question can only be settled by experiments with the actual materials it is proposed to use.

A study of British dams suggests that 2 : 1 is a minimum value in cases where drainage is left to Nature, and 2½ : 1 is frequently adopted; but it must be remembered that British (and to a less degree French and German) practice is still greatly influenced by vague fears resulting from the Dale Dyke failure.

In low dams, the rules given by Strange (see p. 332) may be considered as perfectly safe, and the slopes could be made steeper in good, well drained material, were it not that the economy thus secured is usually quite inappreciable.

CONSTRUCTION.—The practical details of the construction of a dam, including minutiæ, which at first sight appear to be unimportant, have, in reality, a decisive effect on its stability.

Taking the details in order, we have as follows:

(i) Preparation of the dam site.
(ii) Construction of the earthwork. (Drainage will be considered in the section on Indian dams.)
(iii) Construction of the puddle trench and wall, including its drainage.
(iv) Pitching and casing of the dam.
(v) Outlet passages or tunnels, and valve tower.

CONSTRUCTION.—*Preparation of the Dam Site.*—The following specification gives first class work.

(*a*) The whole base of the dam shall be excavated at least 6 inches deep, and as much deeper as may be directed, in order to remove all turf, mould, roots, and vegetable matter, (and pervious soil if directed). The turf and mould to be preserved, and to be used for soiling the outer face of the dam.

(*b*) All tree stumps and roots to be removed, and the voids thus produced to be filled in with well-rammed material, as specified under the heading Embankment.

(*c*) All field drains crossing the site of the dam to be traced out, completely excavated, and to be refilled with puddled clay, as specified under the heading Puddle.

(*d*) The base of the completed excavation to be harrowed in a direction parallel to the length of the dam, so as to promote a satisfactory union between the natural earth and the embankment.

The following comments may be made:

Clause (*a*).—The phrase in brackets is occasionally required in order to enforce the removal of localised pockets or beds of sand or gravel.

Clause (*c*).—This clause is required in cases where the land is artificially drained. It is not usually needed in hill reservoirs.

Clause (*d*).—This clause is of somewhat doubtful utility. It is usually put in in order to prevent the bottom of the stripping being left smooth and polished by the ploughshare, in cases when the turf, etc. is removed by means of ploughs.

The least preparation which can possibly be regarded as good practice consists in the removal of the top 6 inches of the surface soil, and harrowing the surface thus exposed at least twice in directions at right angles to each other.

For small banks holding water up to a height of 5 to 6 feet above ground level, at the most, removal of all growing plants by ploughing and harrowing has been found sufficient. In such instances, the base of the dam is considerably wider in proportion to the depth of water retained than is usually the case. In spite of the fact that the earthwork is by no means so well constructed as is customary in the case of larger dams, breaches almost invariably start with a leak running along the old ground surface.

The junction between the dam and the natural surface is always a weak point, and if there is any doubt as to the quality of the soil exposed by the ordinary 6 or 9 inches of stripping, it is always better to go deeper still. In one example, where work was executed under a specification corresponding to that given, it was considered necessary to remove a quantity of earth equal to 14½ inches over the whole base of the dam, in place of the 9 inches specified.

(ii) *Construction of the Earthwork.*—"Earthen dams rarely fail from any fault in the artificial earthwork, and seldom from any defect in the natural soil —which may leak, but not sufficiently to endanger the dam. In nine-tenths of the cases the dam is breached along the line of the water outlet passages" (MacAlpine).

This dictum forms a very valuable guide in the design of earthen dams.

Nevertheless, in order to secure water-tightness, and to minimise the settlement of the dam after construction, it is necessary to deposit the earth in regular layers, and to roll each layer well, with sufficient watering to ensure adequate consolidation under the rolling.

The following specification is very complete, and certain clauses can usually be omitted:

(*a*) The embankment to be formed in regular layers not exceeding 9 inches in depth at the heart of the embankment, and 18 inches at the outer parts and slopes when spread out. (These thicknesses to be measured before rolling.) The layers are to slope from the outer sides down to the centre at an inclination of 1 in 12. The slope of the earthwork in a longitudinal direction during progress must not exceed 1 in 100.

(*b*) The earth for embanking may be obtained from the excavation for the puddle trench, waste weir channel, etc. so far as it is available, provided that the material is in accordance with the specification. The remainder of the necessary earth must be taken from excavations (from inside the reservoir) as directed, (below top water level), and the excavations are to be left with slopes dressed not steeper than 4 horizontal to 1 vertical, and so as to drain into the main water-course.

(*c*) There is to be no excavation within 100 feet of the toe of the embankment.

(*d*) The most clayey portion of the material is to be used for the heart of the embankment, and more especially on the water side of the puddle wall. The more stoney, gravelly, and sandy portion is to be used in forming the outer parts and slopes, but more especially for the outer side of the embankment.

(*e*) No turf, peat, moss, mud or vegetable matter is to be put into the (heart of the) embankment.

(*f*) The material for embanking may be brought up to the bank site in waggons on rails, but no rails are to be laid above any portion of the embankment site.

(*g*) Every layer must be carefully spread out to the proper thickness, and is to be rolled with a heavy grooved roller, at least 2 tons in weight, till quite consolidated, being kept thoroughly moist by watering as required during the process.

(*h*) In any case when stones are laid on the embankment they must be deposited in a regular layer, each stone being laid on its flattest bed, the rounded side or pointed end being uppermost, no stone being closer to another than 4 inches. The next layer must consist of such material as will bind and entirely fill up the crevices, and the whole must form a compact mass, so as not to be liable to subsidence.

(*i*) The embanking will be measured and paid for to the slopes and sections shown, but the contractor without extra charge is to leave it 12 inches higher than the levels shown at the highest portion, and at corresponding extra heights, according to the depth, in order to allow for subsidence.

The following comments may be made :

Clause (*a*).—As it stands this is first class work, and is probably too stringent. In good practice the sentence in brackets is frequently omitted, and layers 12 inches at the heart, and 2 feet at the slopes, are not really detrimental.

If a deep gorge is left in the bank (*e.g.* in order to pass one year's floods), it is usual to specify that the layers shall slope downwards away from the temporary scar end at 1 in 20, or 1 in 30. Slips are consequently less likely, but the gorge method of disposing of floods is not advisable if it can be avoided.

Clause (*b*).—The bracketed clauses are not always required. In tropical countries, "drainage to the main water-course" may produce erosion, although stagnant water is liable to induce fever, and should therefore be avoided.

Clause (*c*).—The distance depends on the height of the bank, and "two" or "three times this height" appears to be a more logical method of specification.

Clause (*d*).—This is very good practice, but unless marked variations in the quality of the earth procured are anticipated, it must be realised that the contractor will charge from one-eighth to one-sixth extra rates for this clause.

Clause (*e*).—It would be better to entirely reject these materials, using the turf for sodding the outer slopes of the bank. In some cases, "all vegetable earth or mould" is rejected, and there is little doubt that better work is thus produced.

Clause (*f*).—The use of rails on an embankment gives rise to difficulty in spreading and rolling the layers. If good inspection can be relied upon it appears preferable to permit the use of rails, and to specify that they must be shifted at least 6 feet sideways at every rise of two layers.

Rails should not be permitted within 10 or 15 (preferably 20 or 30) feet of the edge of the puddle trench or wall. The erection of trestles on the bank to carry rails should be entirely prohibited.

Clause (*g*).—In some cases, the number of rollings is specified, and rolling across the breadth of the bank by lighter rollers is required.

Clause (*h*).—Stones should be used for the stone drains, and toe wall referred to on page 325. If an excess of stones remains, this clause is useful.

Clause (*i*).—The shrinkage allowed is usually at the rate of 1 foot in 40 or 50 feet height.

The correct fulfilment of the above specification is costly, and, in order to minimise the uncertainties which the contractor has to face, it would seem advisable to systematically investigate the amount of rolling and watering necessary to produce satisfactory consolidation in the manner indicated below, and to append a statement of the results to the specification as an indication of the probable cost. The legal technicalities which must be observed, in order to prevent the experiments being construed as part of the specification are not discussed.

As an indication of the increased expense over ordinary earthwork, I may state that this bank cost about four times as much as the same contractors, in the same locality, charged for ordinary earthwork constructed under somewhat less favourable conditions as regards the quantity of earth per yard forward of the bank; and, judging by other tenders, the reservoir bank prices were closely cut with a view to obtaining the contract.

Since the object of this somewhat costly treatment is to obtain a thorough consolidation of the earthwork, systematic tests should be made by excavating test pits of measured size, in the natural earth and bank, and weighing the excavated material.

The results should be approximately as follows:

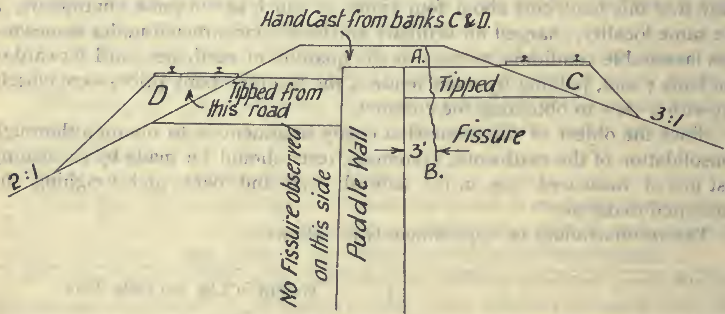
	Weight in Lbs. per Cube Foot.	
	First Example. Mean of 23 Experiments	As given by Bassell (*Earth Dams*).
Natural earth as found in borrow pits	118	116·5
Ditto., as thrown into waggons	78	80·0
Earth in dam as found in trial pits	136	133·0

I am inclined to believe that anything less than 12 per cent. extra weight per cube foot of the dam, when compared with the natural earth, is an indication of bad work, but the fairest method is to compare the results in the dam with those of carefully superintended ramming, carried out in a bricked pit about 6 feet × 6 feet × 18 inches deep (a smaller size is undesirable, as the earth packs against the sides), with smoothly cemented sides and bottom (see p. 322).

It is very important to prevent any lumps being left on the slopes of an embankment, outside the finished section, even as a temporary expedient. Such humps set up stresses, and cracks are likely to occur in the bank just above the hump, as per Sketch No. 75. In one particular case, where a hump some 100 yards long had been made, on which to lay rails for the transport of earth, a crack, approximately one-eighth of an inch wide, and some 50 yards long, formed in the spot marked A, and was in certain places more than 6 feet deep, as tested by probing with a cane. No doubt the shocks of the locomotives and exposure to a temperature of 120° F. had some effect, but the crack occurred in the exact position that theory would indicate. Such a crack may form a starting-point for a slip, even after the hump is removed and forgotten, and two cases where a bad slip in the side of a cutting has been traced to a hump left temporarily in the excavation slope have occurred in my own experience.

(iii) *Puddle Trench and Wall.*—A bank constructed according to the above specifications is by no means water-tight in itself, except under very favourable circumstances; and in order to prevent leakage it is necessary to provide a core wall, or impermeable partition. This is carried up say, 3 feet above the maximum high-water level, and down to solid rock, or other impervious stratum.

While a wall of either puddle clay or concrete, or a masonry core wall (when



SKETCH NO. 75.—Effect of Humps in Cracking a Bank.

of proper construction and proportions) will produce a perfectly satisfactory result, the general practice of engineers appears to favour a puddle wall, wherever clay capable of forming a good puddle is obtainable.

The practice seems logical if the portion of the impervious wall above ground level is alone considered. This is embedded in the dam, and is exposed to stresses and deformations caused by the settlement of the dam. Such actions are best resisted by an easily yielding substance, such as good puddle clay, rather than by rigid, and therefore more easily fractured materials, such as concrete or masonry. If, however, we also consider the filling of the trench, the advantages of puddle are not so manifest, and concrete or masonry may be regarded as equally sound from a constructional point of view.

The following specification shows the method by which first class puddle is made, and laid in position in a temperate climate:

(*a*) The puddle to be made from clay approved by the engineers.

(*b*) All stones (exceeding three inches in maximum dimensions) to be removed, and the clay to be left exposed in layers not more than 12 inches thick for at least 24 hours, and to be watered once or more as directed.

(*c*) The soured clay to be passed through an approved pug mill, or to be otherwise reduced to a homogeneous mass.

(*d*) The broken clay to be deposited in layers, and watered as directed, and to be allowed to weather for at least a week.

(*e*) The puddled clay to be deposited in the trench or wall in layers not exceeding 3 inches in thickness, and to be well cut up by an appropriate tool at least 6 inches long, so as to be incorporated with the lower layer.

(*f*) All clay surfaces in the puddle wall or trench to be covered with bags, or to be otherwise protected against drying, and falling materials. The old surfaces to be well heeled over, or to be otherwise cut up before new puddle is deposited. All puddle which has become dry, cracked, or muddy, or mixed with impurities, to be replaced by approved puddle.

(*g*) The filling of the puddle trench to commence at the deepest point, and to be carried on right and left continuously, but no portion of the trench bottom to be covered up until it has been inspected and passed by the engineers.

(*h*) The filling of the puddle wall to be carried up simultaneously with the construction of the bank, and between properly supported timbers. The top of the puddle clay to be at least 3 inches, and not more than 12 inches, above the earthwork.

(*i*) All timber stringers and walings to be removed from the puddle trench, or wall, as the clay is deposited.

The following comments may be made :

Clause (*a*).—The tests for suitable clay are described on page 312.

Clause (*b*).—The period of "souring" and the amount of watering depend on the properties of the clay. The period specified is a usual one.

(*d*)—This process is very variable. As a rule, a week's weathering in thin layers, with watering three or four times, suffices in a moist climate, like England ; but in some cases better material is secured if the puddle is taken direct from the pug mill and deposited in the trench.

Clause (*e*).—Three-inch layers are, if anything, somewhat thin. A 6-inch layer, if well cut up and spaded during deposition, gives good results. The best results are attained when the puddle is cut into blocks about the size of a brick, which are forcibly thrown into place, while a gang of men cut up the surface of the lower layer with spades, or by means of their heels.

Clause (*f*).—Old surfaces should, as far as possible, be avoided. If puddle has to be deposited on an old surface, a paring, say half an inch thick, should be taken off so as to expose a clean surface. The bags will usually need watering daily, and any mud thus produced should be removed before fresh puddle is deposited.

Clause (*h*).—Unless the puddle is carefully supported both during and after deposition, it is liable to split, and fall asunder. A puddle wall, built between walls of sods, has been known to burst, almost as badly as a wall of water. The clay was probably too wet, but puddle which has once cracked will not re-unite, unless taken out and again kneaded up.

Clause (*i*).—The runners, or poling boards, should be lifted so that their bottoms are slightly above the top of the layer which is being deposited, and lumps of puddle should be thrown into the space thus left vacant. If the runners are drawn after puddle is deposited against them, cracks may be set up. In unstable soil this work may be dangerous, and in such cases concrete is usually employed for filling the "puddle" trench.

The most important tests for clay, suitable for puddle, are those for tenacity, and impermeability. For tenacity, after weathering, watering, and carefully working up, make a roll 1 to 1½ inch in diameter, and 10 to 12 inches long. This should not fall apart when held up by one end. For impermeability, 1 to 2 cube yards, properly prepared, as above, should be formed into a basin to hold 4 to 5 gallons of water, and the loss after 24 hours observed; evaporation being determined in an equal impermeable basin.

Earth suitable for puddle generally gives a clayey odour when breathed upon; is opaque, and not crystalline in fracture; is unctuous to the touch, and when kneaded for three minutes and then formed into balls about 3 inch in diameter does not fall apart under less than 48 hours' soaking in water.

I would, however, point out that the above are the crystallised experience of generations of British Clerks of Works regarding British clays, and that (especially in foreign countries) puddle can be obtained from clay which does not entirely pass these tests.

As an example;—so far as I am aware, no clay exists in the Punjab which passes these tests in a perfectly satisfactory manner; yet I have found that by careful treatment (and especially by a longer weathering than is usually required with British clays) a very fine puddle can be obtained from material which in the raw state looks like a clayey loam; but which, when weathered for three months, with watering every third day, produces a puddle which, in a thickness of one foot only, allowed water to percolate, under 6 feet head, at a rate of 0·3 to 0·6 cusec per million square feet. The earth was not unctuous to the touch, and contained some grit. When rolled for the tenacity tests, the greatest length that could support itself was 6 inches, and 3-inch balls fell in pieces after 30 hours at the most.

It may be noted that, while pure clay forms the best puddle, provided that it is never exposed to evaporation, or to drying by capillary action, yet, where drying by evaporation can occur, a certain admixture of sand, varying from 10 to even 25 per cent. is advisable, in order to prevent cracking. The exact ratio is easily found by experiments on a small scale.

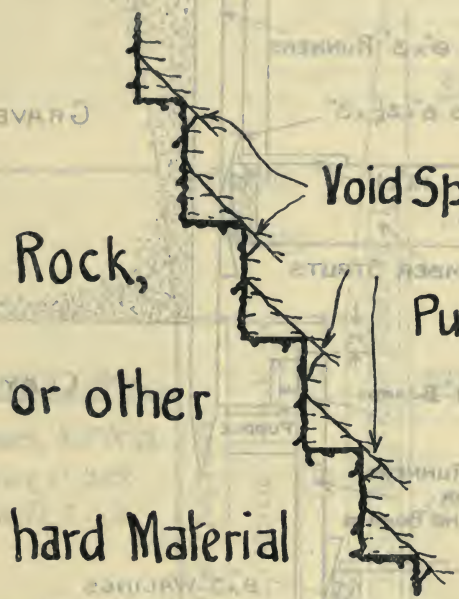
The section of the puddle wall is generally either uniform throughout, or is sloped as in Sketch No. 72. The slope is advantageous, as any slight settlements only tend to consolidate the wall. The minimum thickness in usual practice, (the clay at Staines was exceptionally good) is 5 to 6 feet at the top of the wall, and one-fifth the depth of the water retained at ground level.

Where the clay is not considered to be of first class quality, these thicknesses may be increased to 8 feet, and one-third the depth of the water retained.

Where, as is occasionally the case, screened gravel is mixed with clay, the thickness must be still further increased, but such examples are approaching the Indian type of dam and should consequently be considered under that section.

The puddle trench is carried down at least 2 feet, (and preferably 4 feet) into the impermeable stratum. The width of the excavation will depend on the depth of the trench, and is estimated from the ordinary rules for timbering trenches. Roughly speaking, a timbered trench has a minimum width of $\left(5 + \frac{\text{depth}}{11 \text{ to } 12}\right)$ feet, and such a width is usually amply sufficient for the puddle thickness required to prevent percolation. If a smaller width is advisable, the rules for puddle walls may be followed,—*e.g.* one-third the depth of water at the top, tapering to 5 feet at the bottom.

It is extremely important that right angles should not appear either in the cross, or longitudinal section of the puddle trench. Sketch No. 76 shows what happens during settlement, and further comment is unnecessary. Thus, the section of a trench timbered after the usual walings and runner method (Sketch No. 77) is badly suited for filling with puddle, and certain risky and even dangerous trimming has to be effected. The poling board method (Sketch No. 129) produces a better section, but the actual excavation is less easily effected. When the trench has been taken out to such a width that the puddle would be

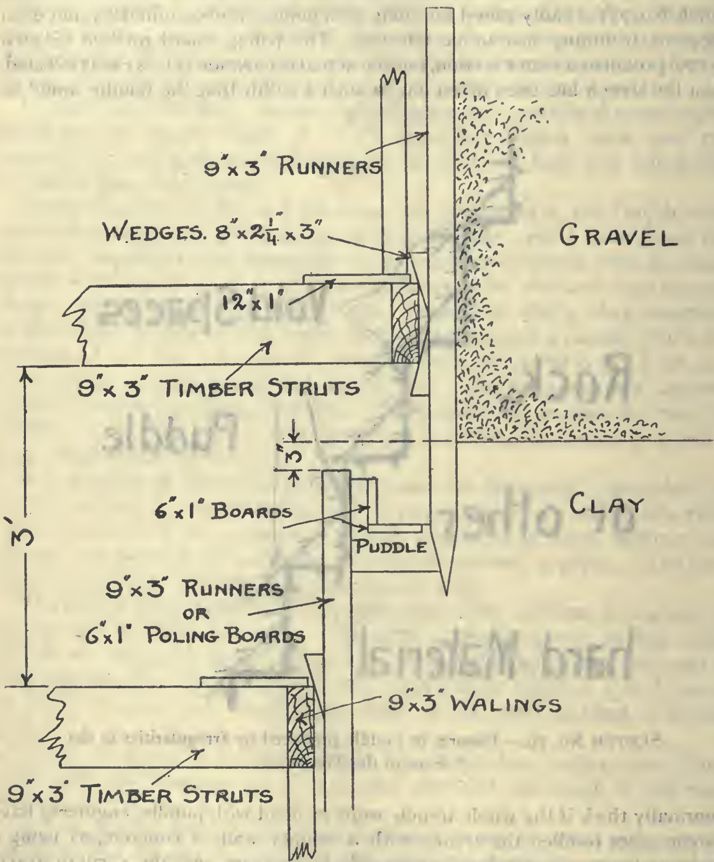


SKETCH No. 76.—Fissures in Puddle produced by Irregularities in the Face of the Trench.

abnormally thick if the whole trench were re-filled with puddle, engineers have in some cases re-filled the trench with a thinner wall of concrete, as being a substance less easily rendered permeable by mixture with the earth or gravel used for re-filling the surplus width. The substitution is quite justifiable, as settlement stresses of a character sufficiently intense to rupture a 5 or 6-foot wall of good concrete are unlikely to occur in a narrow trench below ground level. The only weak points are the junction with the puddle wall above, or near to ground level, and the general impermeability of the concrete. These will be discussed later.

The minutiæ connected with laying the first layer of puddle deserve consideration. In all cases a 9-inch layer should be carefully laid over the whole bottom of the trench, in order to ensure a junction, and should then be removed. This produces a very effective removal of all loose matter and dirt from the bottom of the trench, which might otherwise form a line of weakness.

In cases where the quantity of water flowing into a puddle trench is large, the puddle should not be washed over by the flowing water.

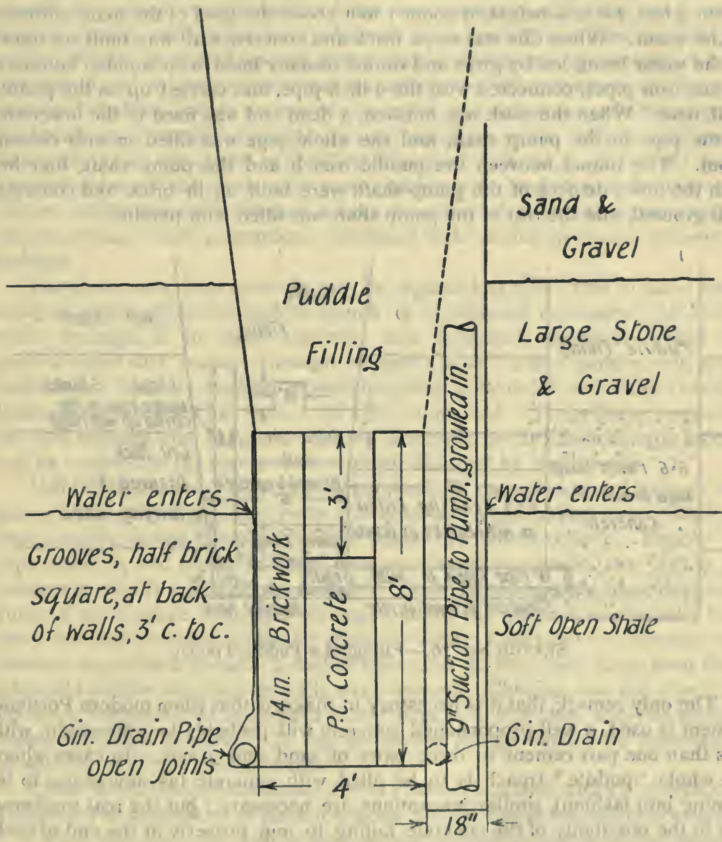


SKETCH No. 77.—Garland for a Puddle Trench.

It is sometimes enough to make a small grip or trench in the puddle, on one side of the trench (or on both if the water enters on two sides), and to allow the water to collect and run off to the nearest pump. If the quantity of water is sufficient to damage or wash away the clay by thus running over it, special devices must be employed.

The ideal method is of course a garland of timber and clay, formed all along the trench at the level of the bottom of the lowest water-yielding stratum. Sketch No. 77 shows the details. This is costly, and either the width of the trench may prove insufficient, or the water may issue in large quantities, almost down to the bottom of the trench.

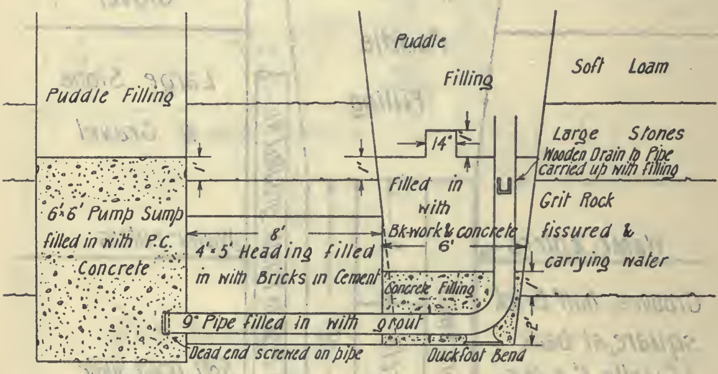
In one case, the problem was solved by laying 6-inch agricultural drain pipes with open joints, on each side of the bottom of the puddle trench, which



SKETCH NO. 78.—Filling of a Puddle Trench.

conducted the water to a pump sump. In front (see Sketch No. 78) two walls of 14-inch brickwork in cement were built, which were carried up above the top of the water-bearing stratum. The space between the walls, to about 3 feet below their top, was filled in with concrete consisting of 1 Portland cement, 2 ballast, and 1 sand. The puddle wall was laid on top of this. When the puddle wall had reached ground level, pumping was stopped; and after the ground water had ceased to rise, the whole of the pump pipes and drains were filled with cement grout. It is open to doubt whether the drain pipes were either properly filled, or really required to be filled.

In another case (Sketch No. 79), where due to some miscalculation, the pump shaft had been sunk on that side of the puddle trench from which the water did not issue, a 9-inch cast-iron pipe with a creeping collar was laid across the trench, and the whole bottom of the trench was then covered with concrete, consisting of 1 Portland cement to 3 gravel and sand, to a depth of about 3 feet, *i.e.* to a height of about 1 foot above the level of the main entrance of the water. When this was set, a brick and concrete wall was built on top of it, the water being led by grips and drains (usually lined with wooden launders) to cast iron pipes, connected with the 9-inch pipe, and carried up as the puddle wall rose. When the work was finished, a dead end was fixed to the lower end of the pipe in the pump shaft, and the whole pipe was filled in with cement grout. The tunnel between the puddle trench and the pump shaft, together with the lower 10 feet of the pump shaft, were built up in brick and concrete, well grouted, and the rest of the pump shaft was filled with puddle.



SKETCH No. 79.—Filling of a Puddle Trench.

The only remark, that it is necessary to make, is that when modern Portland cement is used, a well-proportioned concrete will probably be water-tight, with less than one part cement to three parts of sand and gravel. In cases where the whole "puddle" trench is to be filled with concrete (as now seems to be coming into fashion), similar precautions are necessary; but the real weakness lies in the possibility of the concrete failing to join properly at the end of each day's work. In spite of the obvious disadvantages of night work, wherever possible it is best to lay the concrete day and night, without intermission, until finished. This is, however, frequently quite impracticable, and in such cases the following specification may be adopted:

> "All concrete surfaces over 24 hours old to be picked over, washed with water under 150 lbs. per square inch pressure, and all loose chips removed. Over this surface spread and carefully work into all corners a 1-inch layer of cement mortar (1 cement to 2 of sand), and upon this lay the new concrete, working it well with forks."

Even this amount of precaution, combined with very close inspection, may fail to procure a proper union. On examination of the defective spot a hollow (say ½ to 1 inch deep), will be found in the old work, in which water tends to collect. This water may prove sufficient to drown and wash away the cement from the mortar placed upon it; and small pockets of sand (it may be only $\frac{1}{16}$, or $\frac{1}{32}$ of an inch in thickness) will remain. The action only occurs in shallow, basin-shaped depressions, with sides the slope of which is not steep enough to prevent the pressure of the new mortar forcing the water away against gravity, with a velocity adequate to carry off the finer particles of cement (which, as is well known, alone possess cementing properties). I have therefore found it best to direct that all surfaces on which concrete will later be deposited shall be laid to a slope of 1 in 12, or 15; either towards the edge of the trench, or better still, towards the scar end of the work. The mortar should be as dry as is consistent with filling the small interstices of the old work. Vertical sides of old work also need picking over, and should be laid with chases, which are easily formed by the insertion of a log of timber behind the shuttering. Such precautions, combined with careful inspection, prevent all leakage.

The above described methods may be regarded as applicable to cases where artificial drainage of the puddle trench is considered necessary. In British work, drainage is far more common than is usually reported. There is a general belief that leakage through, or under a well made puddle wall, is discreditable. Such a standard of workmanship is laudable, but water (whether leakage or otherwise) has only a certain value; and it is far better to provide for such leakage than to ignore it, more especially as (if collected) such leakage can be credited to the compensation water allocated by Parliament.

Hill (*P.I.C.E.*, vol. 132, p. 208) acknowledged the existence of leakage, and put in a pipe which for some years delivered, as compensation water, about 0·8 cusec, which leaked through fissures below the puddle trench. Finally, the pipe silted up, and ceased to flow. We may therefore consider that Hill in this manner gradually stanched the leakage. Such stanchings are most desirable; since, if any leakage continues through puddle, the effect is to gradually wash away the clay particles, replacing them by sand. Cases have occurred where vertical, sand filled fissures 2 inches wide, and 40 feet high, have been found when old puddle trenches or walls were opened. The fact that the pipe silted up rather suggests that some such action began in this case, and was arrested before any harm occurred. If this be so, the design should be regarded as productive of a very desirable result.

Core Walls and Trenches of Materials other than Puddle.—In India, and in some parts of America, good puddle clay is not easily obtainable. The water-tightness of a dam is then usually secured by a core wall of masonry or concrete. In late years, armed concrete, asphalte and concrete, or steel plates buried in concrete, have also been used. All these seem to give satisfactory results when proper workmanship is obtained, and no method gives satisfactory results if carelessly applied.

Considerations of cost, together with the available labour and materials, must determine the choice. These walls, unlike puddle, being rigid, do not adapt themselves to the settlement of the earth bank, and must consequently be made of sufficient strength to resist the stresses induced by settlement.

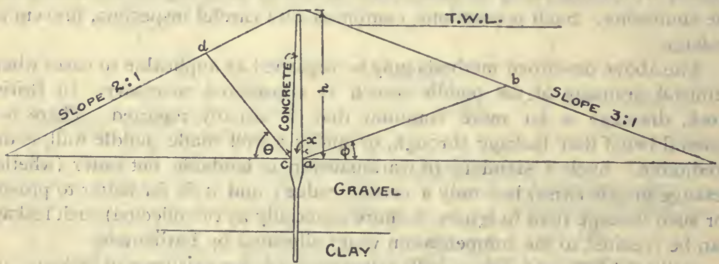
The following investigation, although by no means complete, leads to a satisfactory section, provided that the earth bank is carefully constructed,—*i.e.* is deposited in thin layers (say 18 inches to 2 feet thick at the most); the layers being laid horizontally, or slightly inclined towards the core wall, and being well rolled and watered according to some such specification as that already quoted.

Draw from the base of the core wall (Sketch No. 80):

(i) On the waterside a line *ab*, inclined to the horizontal, at an angle ϕ equal to the angle of repose of the saturated earth, *i.e.* $\phi=20$ to 23 degrees.

(ii) On the downstream side a line *cd*, inclined at θ to the horizontal, where θ is the angle of repose of the rammed earth, *i.e.* $\theta=45$ to 55 degrees.

Calculate the areas of the portions of the cross-section of the dam cut off by these lines. Roughly they are,—with $\phi=20$ degrees, and a 3 : 1 slope, $\frac{3}{4}h^2$, on the water side; and with $\theta=45$ degrees, and a 2 : 1 slope, $\frac{h^2}{3}$, on the downstream side, where h, is the height of the core wall. The weights of the



SKETCH No. 80.—Stability of a Masonry Core Wall.

earth may be assumed as 160, and 132 lbs. per cubic foot, so that the core wall has to sustain a thrust of

$$h^2(160\times\tfrac{3}{4}-132\times\tfrac{1}{3}) \text{ lbs. per foot run,}$$

or $76h^2$, lbs. say; and if the thickness of the wall is x, feet, its ultimate resistance to shear when composed of concrete is about 30,000 x lbs. per foot run.

Thus, for strength only, $x=\frac{fh^2}{400}$, where f, is the factor of safety, equal say to 2, in view of the extremely adverse assumptions made. Unless h, be great, this will usually lead to smaller values of x, than those indicated by practical experience, as requisite to stop percolation through the wall.

Such rules are given by Herschell (*P.I.C.E.*, vol. 132, p. 255) as follows:

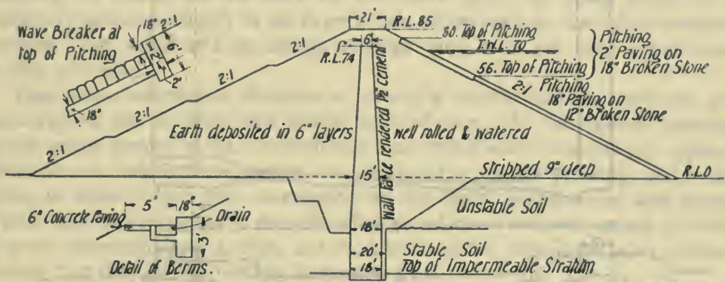
In first class work, 4 to 5 feet thick at the bottom of the trench, enlarging to 8 feet at the natural surface, and tapering off to 4 feet at the top of the wall.

Wegmann (*ibid.*, p. 267) designs the core wall of a proposed dam retaining 96 feet of water, as 6 feet wide at the top, *i.e.* 100 feet above ground level; enlarging to 15 feet at the ground, and 18 feet at 33·33 feet below this level, and then continuing at the same width to the bottom of the trench.

As a contrast, Herschell states that a wall 2 feet thick over its whole height has sufficed to form a satisfactory stop for percolation.

Sketch No. 81 shows a design founded on Wegmann's design (*ibid.*), but modified in accordance with the results of the experiments on permeability referred to on page 348. There is no doubt that a dam of this type can retain 70 feet of water, and *a priori* there is no reason why it should not be as safe as puddle-cored dams which already retain 80 to 90 feet of water satisfactorily. The slopes will be seen to differ considerably from those adopted in puddle cored dams. So far as can be judged these differences are allowable.

Since the masonry core wall is presumably more permeable than a puddle wall, the water face is better drained and therefore less likely to slip, and is partially supported by a rigid wall. Hence, a slope steeper than 3 to 1 is permissible, although whether so steep a slope as 2 to 1 is advisable in a high dam is doubtful. Similarly, the downstream side is likely to be more saturated than in a puddle cored dam; thus, the paved and drained berms form a rational precaution. The core walls should increase in thickness below the natural surface level, until a stable (although not necessarily impermeable) stratum is reached. The earthwork should be constructed with



SKETCH NO. 81.—Dam with Masonry Core.

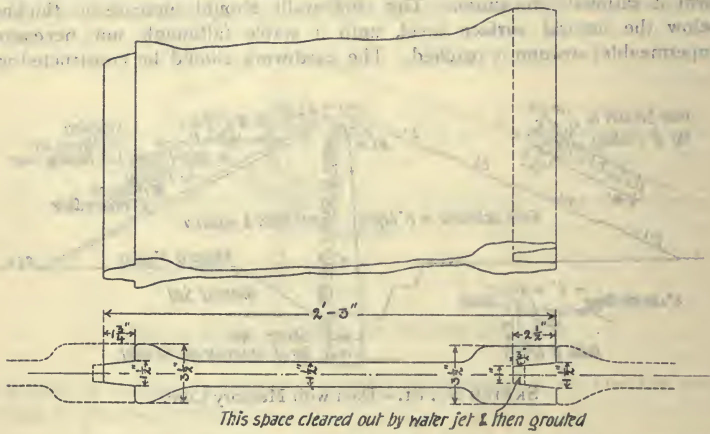
precautions similar to those adopted in puddle core dams, and some such mixture as Fanning's (see p. 302) appears advisable. The wall being rigid is probably less fitted than a good puddle wall to sustain large differences in water pressure. Thus, some arrangement of pipes through the wall connected with drains might be advisable to prevent large differences of water pressure existing on either side of the wall.

The rules given above will probably not suffice if direct water pressure is permitted to act on the core wall, which may happen if the bank is not properly consolidated. The stresses thus developed in a core wall may be approximately estimated by a consideration of the results of the Croton observations (see p. 348).

It appears that if the dam is well made, a pressure equivalent to 10 or 15 feet head of water must be sustained. Thus, calculations on a basis of resisting the bending moment produced by 1500 to 1600 lbs. per square foot over the whole area of the wall will probably lead to a sufficient margin, even if the earth is only moderately well consolidated.

It may be suspected that some such action occurred in the old Bhim Thal Dam core wall, which Ashhurst (*P.I.C.E.*, vol. 75, p. 202) describes as 40 feet high, 10 feet wide at the bottom, and 4 feet wide at the top, buried in a dam 25 feet wide on the top, which failed near the outlet tunnel. Consequently, core walls cannot be considered as substitutes for good workmanship in the earth work; although, if the dam is overtopped, they may save a disaster by temporarily preventing the entire destruction of the dam by erosion.

It is also necessary to refer to the old American method of constructing a core wall and cut off of wooden sheet piling. See Sketch No. 101 for best details. The tendency to eventual decay is obvious; but, in view of the present price of timber, such construction is unlikely to be adopted in the future. Nevertheless, when adopted as a temporary measure (and where the silt content of the stored water is sufficient to ensure eventual stanching by interstitial silt deposit, due to percolation), the construction appears to be justifiable, provided that its limitations are thoroughly realised by the designer.



SKETCH No. 82.—Cast-Iron Piles used in Egypt.

The flat, cast iron piles (Sketch No. 82) used in Egyptian weirs (*e.g.* the Esneh barrage) may be considered as a modern and permanent equivalent of the above construction. The interstices being filled with cement grout, the life of the work is that of thick cast iron under somewhat favourable circumstances, as far as corrosion is concerned. For shallow depths (say not over 25 feet) the method may prove less costly than trenching and filling the trench with puddle or concrete. Visual proof of complete junction with the clay stratum is of course impossible.

Piling built up of rolled steel sections has lately been introduced. The driving gave some trouble at Hodbarrow (*P.I.C.E.*, vol. 165, p. 167), but newer patented and specially rolled sections permit of very good work being done.

The durability of steel is less than that of cast iron, but I doubt if this can be regarded as a serious defect.

In a few instances fissured rock has been rendered "watertight" at small cost by boring a line of vertical holes, say 2 inches in diameter, and 8 inches apart, along the centre line of the dam, and injecting cement grout under pressure. When ocular proof is obtained (such as is afforded by the appearance of new springs above the grouted line), the process may be considered as satisfactory, otherwise a certain amount of distrust is advisable.

Position of the Impermeable Layer.—Theoretically speaking, the best position for any impermeable septum is as close to the water face of the dam as is possible. In nearly all modern dams the core wall or septum is placed vertically below the top of the dam. Thus, the whole water side of the dam is useless qua providing stability. This must be regarded as a defect, but practical considerations compel the designer to adopt the central position.

If a layer of puddle clay is laid on the water face, it will be found to be perforated by burrowing animals such as crayfish, or rats; and consequently the early designs of Telford have rarely been copied in this respect.

If masonry or concrete is placed, instead of puddle, in a similar position, the settlement of the dam invariably causes fractures which permit leakage to occur.

The question deserves investigation, and if an elastic impermeable coating can be found which is not liable to damage from burrowing animals, it should certainly be placed on, or near to, the water face. In small works of a temporary nature economy in earthwork can be secured by the use of bitumen sheeting; but, so far as I am aware, no large dams have yet been constructed on this principle.

The above discussion includes all work in which different methods are employed in the ordinary and Indian type of dam. As the Indian methods permit good results to be obtained with somewhat worse material, it appears advisable to discuss them before considering the casing, pitching, and outlet works of dams, which are constructed on the same principles in both types. It must also be remembered that the adoption of the precautions used in India is never detrimental to a dam, and it is only under very favourable circumstances that they can be entirely dispensed with.

Indian Type of Dam.—A dam of the Indian type is constructed so as to sustain appreciable percolation. This may arise from two causes, as follows:

(i) Either the available material or the climatic conditions do not permit of a good puddle clay, or other substance providing a relatively thin impermeable core, being procured.

(ii) Or, whether the impermeable core be thick or thin, the geological conditions are such that the cut-off trench beneath the dam cannot be carried down to a sufficient depth to unite the impermeable septum (formed by the core wall and trench filling) with an impermeable stratum of clay or unfissured rock.

We thus have two routes by which the percolating water may travel, *i.e.* corresponding to case (i) through the dam itself, or as in case (ii) under the bottom of the cut-off wall or trench sunk below the dam.

As will be seen later, dams exist which are subjected to percolation in both ways, but such dams must be considered as more severely tested than the more normal examples in which material percolation occurs by only one of the above paths.

The typical Indian dam falls under case (i), and for some reason which I am unable to understand, it is held to be safer than a dam which is watertight in itself, but which is subject to percolation below the cut-off trench.

The circumstances under which the Indian dam is generally constructed are as follows :

"Clay" of a second or third rate quality exists, and can be made into fair puddle in the trench, but the climate is such that the manufacture of puddle in the open air (as is required in building the puddle wall) is a difficult matter.

The typical earlier design is well illustrated by Burke's Ashti dam (*P.I.C.E.*, vol. 76, p. 288), see Sketch No. 73. Here there is a puddle trench approximately 10 feet in thickness, carried down to a bed of trap rock. Above the ground level, however, the narrow core wall usually found in British designs is replaced by a triangular mass of puddled "black cotton soil," (*i.e.* 2nd or 3rd class puddle) some 60 feet wide at the base. Outside this is a mixture of weathered trap rock ("Muram") and earth, which may be considered as pervious, but stable, when exposed to water (*i.e.* the Indian equivalent of "gritty material").

As a matter of experience, such dams are pervious to water to a more or less marked degree. Unless the very best workmanship is used during construction, they are apt to slip. Burke followed a specification of practically the same type as that given on page 307, and his finished dam appears to have weighed about 7 per cent. more than the natural earth, and consequently the Ashti dam has not slipped, but slips in the older Indian dams are nevertheless common.

The younger school of Indian engineers have therefore developed the system described by Strange (*Indian Storage Reservoirs*, and *P.I.C.E.*, vol. 132). Percolation through the dam is accepted as a fact, and it is realised that slips occur, not because percolation takes place, but because the percolated water stagnates and saturates the bank, and finally finds its easiest path of escape to be through the dam towards the outer slope, which then slips or cracks.

The following description of a modern Indian dam should be read with reference to Sketches Nos. 74 and 84.

(i) *Preparation of the Site.*—In dams not exceeding 40 feet in height the usual British practice is followed, but drains are constructed. Sketch No. 74 shows a complicated method, where the foundation is cut into angular waves 4 feet in height, and 20 feet from crest to crest, with 4 feet × 5 feet blocks of puddle in the trough of each wave upstream of the puddle trench. This may be considered far too "niggling" where machinery, or even ploughs, are employed for excavation. The downstream portion of the base should be cut into ridges and hollows, approximately as shown, and should be provided with dry stone drains which are connected by a cross drain to the main downstream drain, at all points where the slope of the ground permits.

The principle of these drains resembles that of a filter. It is desired to carry off the percolation water in an absolutely clear state, and to prevent all the silt and clay particles from passing away through the drains. A 4-inch agricultural drain pipe is consequently laid, or a 6 inch by 4 inch dry stone drain is formed, in each of the drain excavations, and this drain is covered by graded layers of gravel or broken stone. On top of these layers from 7 inches to 1 foot of fine sand is placed. The main downstream drain is similarly constructed, but must be proportioned so as to carry off more water. A 12-inch pipe, or a dry stone drain 9 inch by 9 inch usually suffices.

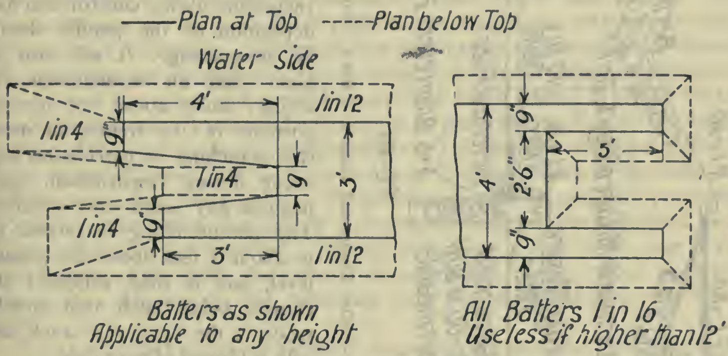
Sketch No. 74, which shows the most systematic drainage system I know of, may be regarded as too costly unless very heavy percolation is anticipated. As a rule, one or at the most two longitudinal drains in the cross-section of the dam will suffice, and dry stone walls are frequently built on top of these in order to collect all water passing over them (see Sketch No. 74).

The correct location of the cross drains, at each valley in the longitudinal section of the dam site, is probably far more important than the number of longitudinal drains, provided that the latter are laid to a uniform grade, and are well constructed.

(ii) *The Proportions of the Puddle Trench.*—Strange's recommendations are shown in Sketch No. 74. In applying these in other localities, it should be remembered that:

(1) The trench is not timbered during excavation, hence the side slopes of $\frac{1}{4}$, or $\frac{1}{8}$ to 1.

(2) The raw material available for puddle is not of first class quality, and owing to the climate the puddle is made by chopping, watering, and rolling the



SKETCH No. 83.—Grooving of Concrete Walls at Junction with Puddle.

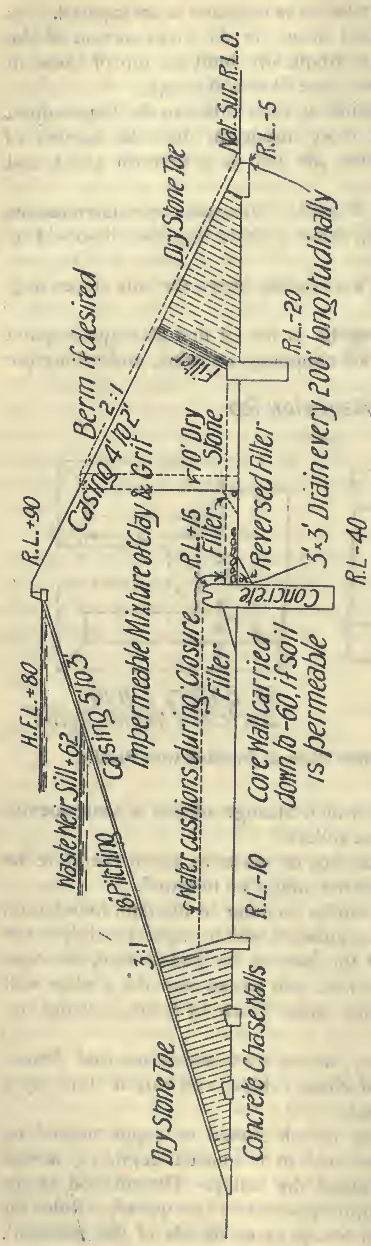
clay in the trench. The width of 14 feet, which Strange adopts, is consequently needed in order to permit the use of horse rollers.

A trench filled with good puddle, concrete, or masonry, could therefore be made narrower, and the usual British practice might be followed.

The puddle trench is drained in a similar manner to the dam foundation. In order to prevent the localisation of percolation which might possibly occur near the drain from setting up erosion of the base of the puddle wall, this base is made of concrete about 4 feet by 10 feet, well keyed into the puddle wall (these details in Sketch No. 74 differ from those given by Strange, being my own).

The drain is constructed of dry stone, of say 4×6 inches internal dimensions, surrounded by about 4×3 feet of clean rubble, and this in turn by a 1 foot layer of clean gravel, or coarse sand.

Strange recommends that the puddle trench should be supplemented by masonry walls at points where it is excavated to an unusual depth, *e.g.* across the bed of the river that previously drained the valley. The method seems rational, but local conditions must determine whether it is required. Rules for such walls are given on page 318 (Sketch No. 94 gives details of the junction).



SKETCH No. 84.—Indian Compound Earth and Rock Dam.

The junction of the puddle with this masonry needs careful consideration. The masonry should batter outwards in all directions, so as to prevent shrinkage cracks in the puddle at the line of junction. Consequently, the masonry should be a frustrum of a cone. So also, chases should be formed at every possible point, as shown in Sketch No. 83. The sketch is a complete solution of the problem, and the chases are easily made in concrete. In brickwork, much cutting of bricks is required, but is unavoidable. Inspection during construction and deposition of the puddle should be unremitting. It will also be plain that an arrangement of silting tanks above the deeper portions of the trench, in order that stanching by percolation may occur during construction, quite justifies any small cost entailed. The puddle filling is carried up to about 3 feet above the ground level, and is then stopped; the climate being such that puddle made in the open will crack and split if formed of pure clay.

(iii) *Construction of the Earthwork.*—As a general rule Strange recommends that the whole body of the dam should be made of equal parts of clay and weathered shale (*i.e.* clay and grits), but in any given case experiment must decide the exact proportions. The specification of workmanship closely resembles that already given, the small variations being due to local conditions. The consolidation obtained with such mixtures is considerably less than is usual with the more gritty materials employed in British practice. Strange states that 106 cube feet of excavation from borrow pits will be required for 100 cube feet of bank. My own experiments give figures ranging from 109 to 105 cube feet, as against 116 to 111 cube feet for purely gritty material.

Dams constructed in this manner are found to be permeable (see p. 348); and, in consequence, slips are of frequent occurrence. These slips always take place on the downstream face (except when local slips are caused at the water face by sudden lowering of the water surface). The slopes adopted (3 : 1 water face, 2 : 1 downstream face) might therefore be modified with advantage; and such values as $2\frac{1}{2}$ to 1, on both faces, or $2\frac{3}{4}$: 1, for the water face, and $2\frac{1}{4}$: 1, for the downstream face have been suggested.

In general practice, it is usual to provide against slips by means of berms on the downstream slope, and to form a heavy toe wall at the lower toe of the dam. Sketch No. 86 shows Jacob's design at Jaipur, which attains stability by a reversed filter at the toe.

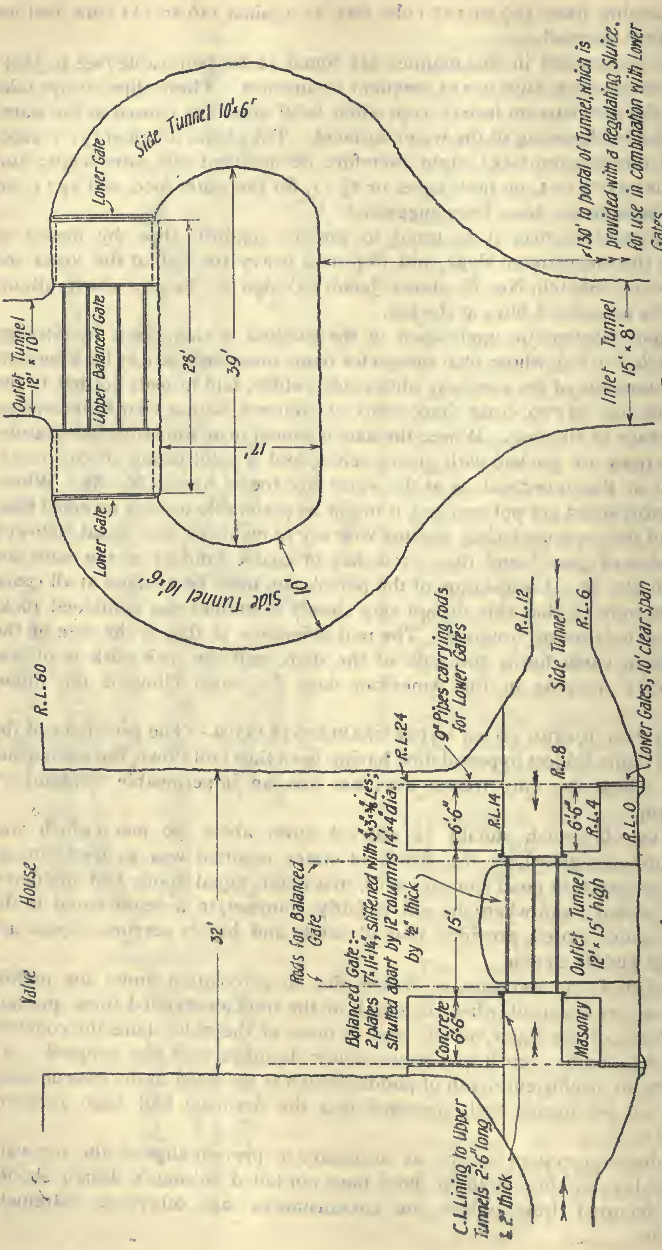
The most systematic application of the method is that given by Strange (see Sketch No. 84), whose final design for dams more than 40 feet high consists of a dry stone toe of the roughest obtainable rubble, laid in beds normal to the slope, with one or two cross chace walls of concrete, with a view to increasing the resistance to slipping. Where the dam is joined to an impermeable stratum, the interstices are packed with clayey schist, and a solid facing of concrete is also built at the upper end, as at the water face toe of Sketch No. 84. Where impermeable strata are not reached, it might be preferable to lay a reversed filter in place of the concrete facing, starting with say 12 inches of road metal, followed by 12 inches of gravel, and then 12 inches of coarse sand as at the outer toe of Sketch No. 84. Localisation of the percolation must be avoided at all costs. It will be noticed that this design very closely resembles the combined rock-fill and earth dams of America. The real difference is that in the case of the Indian dam, earth forms the bulk of the dam, and the rock work is of less importance; whereas in the American dam the earth filling is the minor factor.

Dams not joined to an Impermeable Stratum.—The principles of the design of dams subject to percolation having been thus laid down, their extension to cases where the core trench does not join an impermeable stratum are fairly plain.

The cut-off trench should be carried down about 30 feet (which was Rawlinson's design where the depth of water retained was 30 feet); or, as Strange suggests, in good compact soils, to a depth equal to one half the depth of water stored; and when the soil is fairly compact, to a depth equal to the depth of water stored, provided that all sandy and highly pervious layers are cut by the puddle trench.

So far as I can ascertain, no failure due to percolation under the puddle wall has as yet occurred, when the depth of the trench exceeded three quarters of the depth of the water stored; and in most of the older dams the concrete base of the puddle trench and a systematic drainage were not adopted. No failure due to insufficient depth of puddle trench is recorded in the case of dams founded on permeable soil, provided that the drainage had been properly attended to.

The drainage system is quite as necessary to prevent slips as the toe wall, and while less carefully drained dams have not failed, Strange's design should not be departed from unless the circumstances are otherwise extremely favourable.



SKETCH No. 85.—Balanced Gate Outlet.

This Sketch shows the outlines of the arrangements adopted by Pennycuick at Periyar, to pass 1600 cusecs under a pressure which may attain 49 feet when the gates are closed. In view of the great liability to wear of the upper gate the lower gates must be considered as an integral portion of the design. Since they are hung from rods passing through pipes, these lower gates can never be raised for repair. As a rule this must be considered as a defect. At Periyar these lower gates are only worked when the reservoir is at a low level, never pass more than 500 cusecs, and when open are never exposed to more than 20 feet head; the regulation then being also mainly effected by means of a Stoney sluice at the tunnel portal. Thus, in general the lower gates would need to be replaced by a second balanced gate. The design is an excellent solution of the local problem, and the principles might be generally adopted, since it may be presumed that when discharges computed in hundreds of cusecs are dealt with, slight leakage is permissible. If no leakage can be allowed the gates of the Roosevelt dam (Wilson, *Irrigation Engineering*) may be taken as a basis for design, but the increased cost indicates that as a rule leakage should be permitted. If either type is adopted in a culvert outlet not surrounded by hard rock the stresses in the masonry will plainly require very careful calculation.

The filling of the core trench needs careful thought. Strange is apparently of the opinion that his 14 feet wide trench with $\frac{1}{4}$, or $\frac{1}{8}$ to 1, side slopes will suffice, when filled with puddle, and with the concrete base usually adopted. Personally, I favour concrete, or a double wall of concrete and puddle in two layers.

The dam itself may either be of Strange's type, Sketch No. 84, or if good puddle is procurable water-tightness may be secured by a puddle wall, say 33 p.c. thicker than the usual rules indicate. The fact that settlement will certainly occur seems to preclude the use of masonry or concrete for the core wall.

On referring to the preliminary section, it will be seen that the provision of an impermeable carpet over the whole base of the dam stops percolation almost as effectually as a vertical cut-off trench, and while a good wall of puddle clay or other non-rigid substance gives satisfactory results, there is little doubt that if the whole dam be made of fairly impermeable earthwork, the effect in preventing percolation through the dam is almost equally good; and the percolation under the cut-off trench is probably less than if the thinner wall of more impermeable material forms the only impermeable portion of the dam.

The real dividing line is fixed by the value of labour. In countries such as India, where labour is cheap, the best solution (when good puddle clay is not available) is to roll and puddle the whole dam very carefully. Compare the Ashti dam, and Strange's general section, with the Staines dam.

If labour is dear, a thin wall of strong material is indicated, as in the case of some American dams, where impermeability is secured by thin steel plating, averaging $\frac{3}{8}$ths of an inch in thickness, protected from rust by a 4-inch coating of asphalte on each side.

Such a dam cannot be considered as an innovation in constructional design. Hundreds of examples exist in Ceylon and Southern India, without any puddle trench at all, and are often of great age. As an example of a modern dam subject to heavy percolation, I would instance the Amani Shah dam at Jeypore (Rajputana) constructed by Col. (now Gen.) Jacob. This has no core wall, and is formed of sand, resting on sand and mud. Its dimensions are (*P.I.C.E.*, vol. 115, p. 56).

Height	61 feet.
Breadth at top	30 „
Breadth at base	396 „
Inner slope, *i.e.* water face	4 to 1
Outer slope	2 to 1

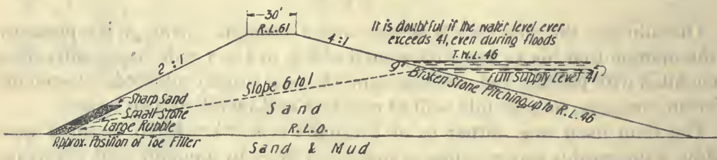
The toe wall was constructed,—to quote Gen. Jacob:

> "Next to the earth a layer of sharp sand about 10 feet wide and 5 feet deep was placed, outside of this a similar layer of small broken stone; and finally a similar mass of large rubble."

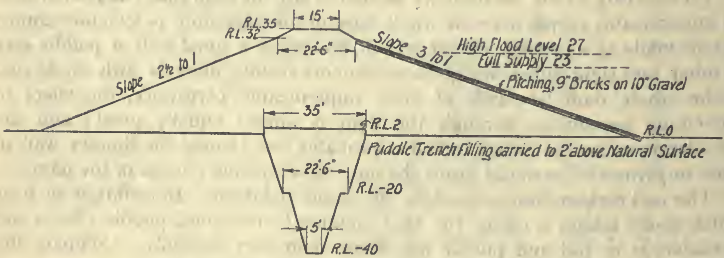
It was anticipated that:

> "It would act as a filtering medium, keeping back the earth, but allowing the water to percolate out free from silt."
>
> "The highest water level yet reached is 31½ feet."

If I may criticise a singularly successful design, I should be inclined to recommend that the water slope should be 3 to 1, or as steep as stability allows; and that the outer slope should be made 3 to 1. My reasons are, that it is



SKETCH No. 86.—Jaipur Dam; for Culvert see Sketch No. 90, Fig. 1.



SKETCH No. 87.—Baroda Dam; for Culvert see Sketch No. 90, Fig. 2.

evident that the dam is saturated up to a line sloping away from top water level at 1 in 7, or perhaps even 1 in 6; and, as at present designed, a line at 1 in 6 drawn from the designed full supply level of 41 feet cuts the outer slope at about 3 feet above the toe; whereas if the slopes were reversed, or were even made 1 in 3, on both sides, the 1 in 6 line would pass well below (about 8 feet for 1 in 3 slopes) the outer toe. (Sketch No. 86.)

The dam at Baroda (see Sketch No. 87), described by Sadasewjee (*P.I.C.E.*, vol. 115, p. 43), shows a more typical section. The dam is of clay and grit mixed, and the puddle trench is filled with good (Indian) puddle, carried down to a clay stratum except for a length of 200 feet, where it stops at depths varying from 25, to 40 feet, being 5 feet wide at the base.

No systematic provision being made for drainage of the dam itself, slips occurred, and an open drain 10, to 12 feet deep was cut just outside the outer toe of the dam for a length of 2900 feet. This discharged a quantity of water varying from 0·6, to 0·9 cusec.

The question was considered by Latham (*P.I.C.E.*, vol. 115, pp. 44 and 122), who gives a rather interesting piece of reasoning. He says that when the leakage takes place below the puddle wall, the apertures by which the water escapes will always be of the same size; if, however, leakage occurs over the puddle wall, the total area of the apertures will vary with the height of the water in the reservoir.

Thus, since:

> "The law which governed the flow of underground water was similar to that which governed the flow of water in pipes and channels,"

a leakage, proportional to the

> "Square root of the height of water above the point at which it was escaping,"

would indicate leakage under, and through, the core wall. But since:

> "By taking into consideration another factor, *i.e.* the actual height of water in the reservoir above the top of the puddle wall,"

it was possible to calculate the escaping water with exactitude, Latham therefore deduces that the leakage was through the body of the dam, and over the top of the core wall.

The reasoning is well worth bearing in mind when leakage occurs; and since Latham is a very accurate experimenter, it would appear that definite channels of escape existed in this particular dam, or that so large a proportion of big stone was present that the usual relation for capillary channels (*i.e.* volume escaping varies as the pressure), did not hold good. *A priori*, we may usually expect to employ the reasoning: Volume escaping varies as the height of the water above the point of escape,—as indicating leakage under, and through, the core wall. While, if the second factor—height of the water above the top of the core wall—also manifests itself, the leakage is over the core wall.

I reproduce the original drawing given by Sadasewjee (*ut supra*), as Latham's observations show that a dam of this type will stand leakage to the extent of 0·85 cusec over a length not greatly in excess of 200 feet without slipping.

The remedies consisted of a clay wall at the outer toe, 12 feet wide, flattening the outer slope near the toe to 3 to 1, and a system of surface drains to carry off rain water falling on the outer slope.

The above may be considered as representing the worst that can happen in dams of this type, when properly constructed, whether the core trench is joined to an impermeable stratum or not.

In order to obtain any instance of a partial failure (such as the above) I have been obliged to include a case where drainage was not well attended to. So far as I am aware, no properly drained dam has failed by reason of percolation. The failures of the older Indian and Cingalese native made dams can almost invariably be traced to over-topping by floods, and no undoubted case of failure due to percolation has yet been recorded.

The following appears to be a fair view of the question. The ordinary

English practice relies too exclusively on puddle and impermeable strata, which can usually be obtained in England at a certain price, owing to the small scale of English geological features. Indian practice is more logical, in that when the above advantages cannot be obtained except at a prohibitive cost, the difficulty is surmounted without any pretence of ignoring it.

Personal experience leads me to believe that puddle clay passing English specifications, or cheap concrete and masonry, combined with easily reached impermeable strata, occur only exceptionally. I confidently look forward to an extensive use of the Indian type of dam in other countries; all the more so since any engineer who has to cope with less favourable climatic conditions and more intense floods than those of India, may consider himself most unfortunate.

The whole matter is one of balance between the dam and the puddle trench; where, (as in the case of the ordinary English dam, great care is taken to produce an absolute junction between the puddle trench and an impermeable stratum), the dam may be considered as not very greatly affected by percolation, and a thin wall of puddle clay is sufficient to stop percolation. Where the puddle trench is known to be carried to an insufficient (according to English ideas) depth, more care and pains must be devoted in order to make the dam as a whole partially impervious by mixing some clay with the gritty materials. A proper system of drainage must be constructed in order to overcome the tendency to slips which exists in this mixture when saturated.

The differences between the above two types having been considered, we can proceed to examine the various portions of the dam that depend not so much on the amount of percolation, as on climatic conditions. These are:

The Casing.
The Pitching.

Casing of the Dam.—The casing of modern dams is thin, and can best be regarded as a layer of stone or gravel which affords a means of draining the pitching.

Strange says that we should construct the casing of one part clay and two parts of shale, and make it 2 feet wide from the top down to full supply level; and below, the width should be increased 1 foot for every 10 feet of vertical height.

This specification is for a climate where the rain-fall, although intense, is limited to about four months in the year. In a country where rain is less heavy, so great a thickness is perhaps useless; but on the other hand, if the rain-fall is distributed over the whole year, the proportion of clay seems excessive, and might produce turbidity in the water.

At Staines (where the rain-fall is about 25 inches annually, and occurs over the whole year), 6 to 9 inches of pure gravel sufficed.

Pitching.—The necessity for pitching is obvious. The most usual type is a layer of roughly hammer-dressed stones, laid on their large ends, and dressed so as to meet all round their bases for a depth of 3 to 4 inches.

The thickness may vary from 6 inches at the bottom, to 18 inches at the top, in reservoirs some 6 square miles in area; and say 9 inches at the bottom, to 2 feet at the top in the case of reservoirs 20 square miles in area.

The following specification of Messrs Hunter & Middleton has produced very satisfactory results at Staines:

> The inner face of the embankment to a depth of 15 feet below the top water level is to be protected by concrete slabs made *in situ* of 4 to 1 concrete, 4 feet square, and 5 inches thick, worked to a good face and resting on 6 inches of gravel; every alternate slab being left out in each line until the subsidence has ceased, when those in position will be packed up to the line of slope, and the work completed. At the foot of the concrete facing there is to be a step or berm 3 feet wide, and below that line the embankments are to be covered with a 9-inch layer of gravel.

The fetch is but small ($1\frac{1}{2}$ mile at the most), but the reservoirs are greatly exposed to wind, and it is doubtful whether any thickness less than 9 inches would have sufficed had the pitching been composed of smaller and rougher material.

In the actual construction, a 3 feet × 18 inch concrete toe wall was built at the bottom of the concrete slabs (Sketch No. 72). The appearance is good, and similar (although probably somewhat thicker) slabs deserve consideration wherever stone is not easily available. Being smooth, and of large size, the thickness need hardly exceed one half of that given by Stevenson's rule, which is obtained from experience in very windy localities.

The following is a good rule:—Thickness $= \frac{1}{3}$ height of waves likely to occur. For this height Stevenson gives:

$$\text{Height in feet} = 1{\cdot}5\sqrt{F} + (2{\cdot}5 - {}^{4}\sqrt{F})$$

where F, is the "fetch" of the wind in miles.

In cases where stone is not procurable, hard burnt blocks of clinkered bricks (say 6 bricks in a block), can occasionally be procured. Pavings of brick, on edge, or on end, are usual in the Punjab, but they are not strong enough if the waves are high, as is likely to be the case if F, is much over a mile.

The wall or wave breaker at the top of the dam (as shown in Sketches Nos. 74 and 81), has been recommended as an effective means of preventing waves from washing over the dam, but is rarely employed. Careful drainage of the top of the dam with drains extending down the side slopes at frequent intervals seems to be a better method of dealing with waves and spray.

The outer slope of the dam requires to be protected from rain wash. This is usually effected in England by covering the slope with vegetable earth, and sowing with grass; or sodding with the sods removed from the base of the dam, special provision for drainage usually being unnecessary. In less equable climates, turf of this character is difficult to maintain, and either careful planting with fleshy plants, or coating with gravel, or small stone, is consequently required. The practice of planting with shrubs, or worse still, with trees, should not be followed, as these conceal leakage if it occurs. Cases have been met with where cracks and fissures in dams have been traced to the action of wind on trees growing on the dam.

The Staines specification gives the usual British practice:

> The whole of the outer slope of the embankment is to be covered with a layer of soil 6 inches thick, resting on 3 inches of gravel, and sown with clean rye grass and white clover seeds.

In hotter climates, a deeper layer of soil, and a more fleshy grass, or even such plants as Mesambryanthemum, are advisable.

A little consideration will make it plain that both the casing and pitching of a dam should in reality be determined by the quality of the gritty material. At Staines, if appearance could be entirely neglected, there appears to be no real necessity for either. Taking the other end of the scale, a dam constructed according to Fanning's or Strange's specification requires thick casing and good pitching to prevent the slopes from being guttered and damaged by rain or waves.

Top Width of a Dam.—The top width is usually taken as 10 to 14 feet. There is very little doubt that in all dams, except the very lowest, it should be of sufficient width to carry a cart road, as the extra cost is rapidly saved by the ease with which repairs and maintenance are carried out. If there is the slightest doubt about the sufficiency of the waste weir (*e.g.* owing to the available records of flood discharges being for a short period only) 14 feet should be considered as the minimum width. To my own personal knowledge, one dam at least has been saved, and a bad disaster averted by temporary heaps of earth erected along the crest, which just sufficed to prevent the destruction of the dam by overtopping during an abnormal and sudden flood.

While the chief credit is due to the local labour which took the risk of accompanying the dam down the valley, the earth was finally secured by excavation in the top of the dam, and, had the top width been insufficient to permit this, procuring the necessary earth from the downstream slope, would have been more tedious and more likely to create a weak spot.

Low Dams.—The previous discussion mainly refers to dams of considerable height. The following proportions are suggested by Strange (*Indian Storage Reservoirs*) for dams under favourable circumstances:

Height of Dam.	Height above High Flood Level.	Top Width.	Slope of Water Face.	Slope of Downstream Face.
Less than 15 feet	4 to 5 feet	6 feet	2 : 1	1½ : 1
15 feet to 25 feet	5 to 6 „	6 „	2½ : 1	2 : 1
25 „ 50 „	6 feet	8 „	3 : 1	2 : 1

Failures of Earth Dams.—The usual cause of a bad failure is the insufficient capacity of the waste weir, as was the case in the Johnstown dam, which is discussed under Floods.

The Holmfirth failure (*P.I.C.E.*, vol. 59, pp. 51 and 57), where the dam had been allowed to settle until its crest was but little above the waste weir sill, can only be regarded as the result of careless maintenance.

Among other disastrous failures the Dale Dyke may be mentioned. This was apparently caused by the bursting of an unprotected line of pipes laid through the dam. Since, however, the earthwork is described as very loose, and more like a quarry tip than a dam, it is difficult to allocate the responsibility accurately.

Small failures usually consist of slips or sloughing of the downstream slope; and if the water in the reservoir is rapidly lowered, similar, but usually more localised slips, may occur on the water face.

OUTLET WORKS.—The crucial point in the design of the earth dam of a reservoir is the method employed for drawing off the water.

The more intensely a catchment area is developed, by increasing the storage on it, the more important it becomes to arrange the outlet so that water can be drawn off at any level. If we in any way limit the depth from which water can be drawn, it is evident that, *pro tanto*, we diminish the effective storage capacity. On the other hand, a large reservoir capacity, (if properly used) entails the reception of large volumes of flood water into the reservoir, and this water, except under very favourable circumstances, is bound to be turbid. It is therefore necessary to be able to draw off the upper layers of the water, which will more quickly become clear, and to reject the lower layers, if abnormally silted. Thus, it is absolutely essential to provide some method of running off large quantities of water when the reservoir is nearly empty, unless it is possible to systematically reject turbid water by means of bye-pass drains. Even then, it is quite possible that the cost of such bye-pass drains may exceed that of a draw-off system, which will permit an equally satisfactory rejection of silt-bearing water, combined with the selection of clear water for use when required.

The systems adopted are as follows :

(i) A valve tower which permits water to be drawn off at any level.

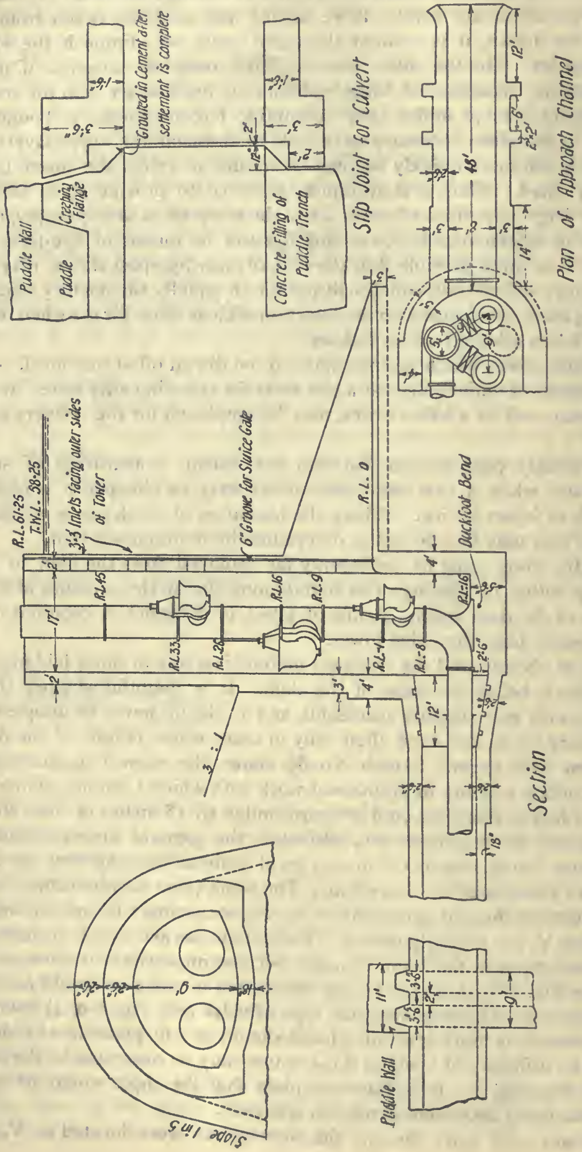
(ii) A series of undersluices at a low level for rejecting silty water, and these (or a separate set), or a valve tower, may be employed for the delivery of water for use.

Valve towers (see Sketch No. 88) are usually constructed of stone or masonry, and while a cast iron valve tower may be cheaper, it is exposed to greater risk of injury by ice. Where the formation of thick ice is unlikely, the question of cost may be allowed to determine the material selected.

The valve tower must be sufficiently far removed from the dam to be safe from injury, either by settling of its foundations, due to the pressure of the dam, or by slips of the dam itself. A line of pipes, or a tunnel, is required to carry away the water from the valve tower.

The most obvious and the cheapest method is a line of pipes laid in the dam or in a trench below the base of the dam. It is doubtful whether this construction is ever permanently successful, and it should never be adopted except for temporary work, and even then only in cases where failure of the dam will result in but little harm. Sketch No. 89, shows the nearest approximation to this construction existing in permanent work with which I am acquainted. The pipes are 3 feet in diameter, and are surrounded by 18 inches of concrete. The design cannot be recommended, although the general arrangements which permit water being drawn off at any level without opening any valve under more than 15 feet head are excellent. The weak point is admirably illustrated by the statement that the dome valves A_1, A_2, etc., cannot be opened unless the lower valves V, are partially closed. Thus, if any serious break occurred in the concrete and pipes in the dam, it might become impossible to close the dome valves (see Fig. No. 3), and then the destruction of the dam would merely be a matter of hours. The pole and plug type of valve (see Fig. No. 4) would really be safer, since it is more certainly closed should any displacement of the pipes occur. The difficulty of opening these valves may be overcome by the bye-pass shown in Fig. No. 2. It is, however, plain that the pipes would be far safer against fracture if they were carried in a culvert.

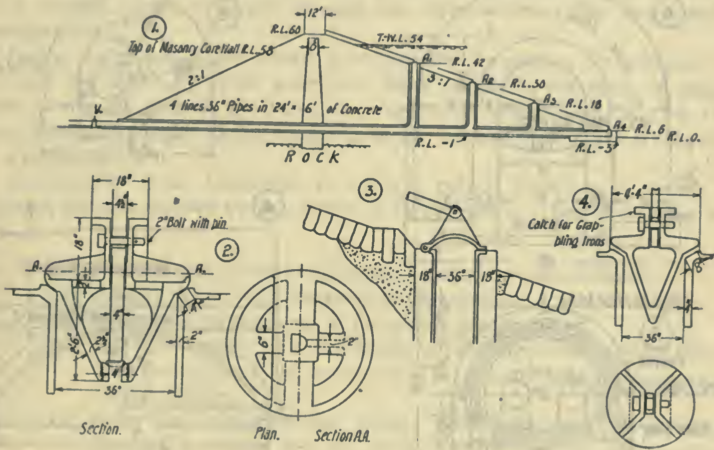
In certain very early designs the only valves were located at V, and the inner orifices A, could not be closed. Such designs are extremely dangerous, and I believe that no examples exist at present. The life of a dam with this type of outlet may usually be reckoned by months.



SKETCH No. 88.—Valve Tower and Culverts.

Even in temporary construction an unprotected line of pipes with the valves correctly placed is extremely objectionable. The line is very liable to fracture; and although the leak can be stopped by shutting the upper valve, repairs are difficult, and may entail letting off all the water in the reservoir. It is also quite evident that the outer skin of the pipe line forms a possible line of leakage for the water, and that the puddle wall crossing may also be breached by the puddle below the pipes settling from beneath the pipes. While wide creeping flanges surrounded by puddle form a fairly efficient stop against leakage along the pipe line, nothing except a concrete wall, built up from the foundation of the puddle trench to the pipe, (as in the method used in carrying the concrete lined channel into the Hampton service reservoir, shown in Sketch No. 83, or that shown in No. 88) is effective against settlement of the puddle.

In pioneer work, where every penny must be saved, it has occasionally been found advantageous to carry a line of pipes through the dam, at or a little (say



SKETCH NO. 89.—Line of Pipes laid through a Dam.

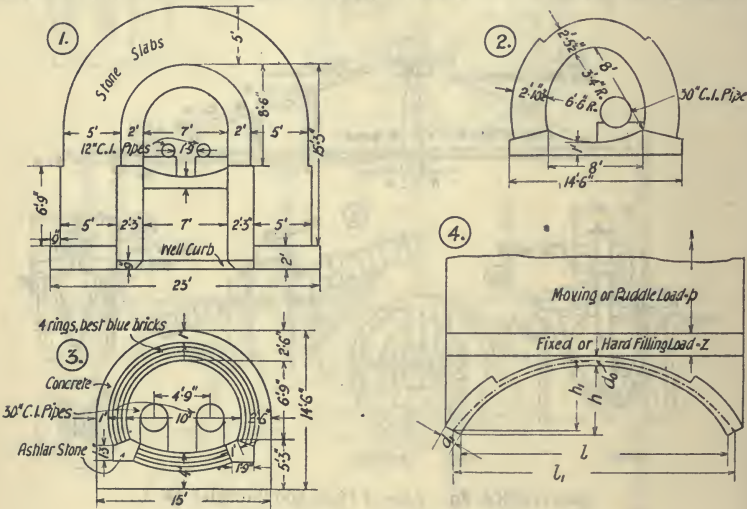
15 feet as a maximum) below the top water level. Under these circumstances the pipe line usually works as a syphon, and is only rarely under pressure. Fractures of the pipes may therefore prove troublesome, but need not be disastrous, and the method has proved very efficient. It is, of course, obvious that any later attempt to utilise more than, say, the top 20 to 30 feet of the reservoir content will entail very expensive works, and the whole of the water in the reservoir may have to be run to waste before a culvert situated at a low level can be constructed.

The next method is to build a culvert of ashlar masonry, or brickwork, in which the pipes are laid (Sketches Nos. 88, 90). A plug of ashlar masonry, or best bricks, is placed across the culvert; usually near the foot of the inner slope of the dam. Both the culvert, and the pipes where they cross the plug, require creeping flanges to prevent the water from leaking along their outer skin. The method is unobjectionable, but the culvert is liable to be fractured by settlement of the earth of the dam, and puddle wall. In high dams, even the best ashlar has proved too weak to resist the forces thus brought into play.

Where (see Sketch No. 88) the culvert lies above the level of the bottom of the puddle trench (as is frequently the case) this action is intensified by the necessity of supporting the culvert where it crosses the puddle trench, on a rigid concrete or masonry pillar, in order to prevent a hollow occurring in the puddle clay beneath the culvert, just as was pointed out when dealing with the pipe crossings. For typical culvert sections see Sketch No. 90.

In some cases the culvert has been built with a slip joint at this point; but even this precaution has not met with invariable success, possibly owing to the pressures produced by settlement being by no means entirely vertical (Sketch No. 88).

The effect of the various failures,—or it would be more correct to say, of the



Sketch No. 90.—Sections of Culverts.

possibilities of failure,—thus disclosed, has led many engineers to consider it preferable to drive a tunnel through the undisturbed hills on one side of the dam, and to lay the outlet pipe in this. The advantages are obvious:—the valve tower and culvert are entirely separate from the dam, and cannot be injured by its settlement, nor can the culvert form a line of weakness in the dam.

On the other hand, the cost is greatly increased, and tunnel work requires a special class of labour, not always easily procurable. It is also quite evident that work done in tunnels, in the dark, is far more liable to be scamped than in a culvert, which is built in the open, and can be inspected inside and out at every stage of construction.

Summarising these points:—It would appear that the tunnel plan should be adopted in large works where skilled labour is abundant. Where unskilled labour only is procurable it is quite possible that bad workmanship may result in unseen defects, which prevent the attainment of the almost absolute safety which the tunnel plan (if well executed) undoubtedly secures.

It must also be remembered that even though a culvert should be slightly fractured by settlement, the defect is by no means irremediable, and probably merely entails the injection of three or four barrels of cement grout under pressure.

I may also mention the method adopted at Staines reservoirs, which seems almost ideal where an impermeable stratum of sufficient thickness is known to exist at a small depth. Here the culvert was carried not only under the dam, but also under the puddle wall, with its top at a depth of some 7 feet below the top of an impermeable stratum of London clay. It was thus possible to combine all the advantages of undisturbed ground secured by the tunnel system, with a length of culvert but slightly in excess of that required to traverse the dam at its base.

The design of the draw-off passage (whether constructed under the dam as a culvert, or as a tunnel through the hillside from the dam) needs consideration.

In the first place, if the culvert is surrounded by puddle, the stresses on the arch are unusually great, since well made puddle is practically a heavy fluid as regards vertical pressures, but cannot be relied upon to give the same horizontal support at the springing of the arch as that which is afforded by a perfect fluid.

The severity of the conditions to which a culvert loaded with puddle is exposed, are best realised by the following investigation. (See Sketch No. 90, Fig. 4.)

Let l, be the span of the arch, in feet, measured to the intrados.
Let h, be its rise in feet, measured to the intrados.
Let d_o, be the arch thickness at the crown in feet.
Let $D_o = d_o + z$, where z, is the height of any rigid non-moving filling that exists above the arch crown, *i.e.* in good work $z = 0$, so as to avoid leakage.
Let p, be the height of the puddle above the crown of the arch in feet, or above the top of the filling if z, is not equal to nothing.

If the weight of the puddle per cube foot differs materially from that of the arch masonry, put:

$$p = \text{Height of puddle} \times \frac{\text{Weight of puddle per cube foot}}{\text{Weight of masonry per cube foot}}.$$

Then, owing to the fact that the puddle may sink unequally, we must consider the arch as exposed to a moving load equivalent to p, feet of masonry per foot run.

For such a case, Tolkmitt (*Entwerfen der Gewoblten Brucken*) finds that:

$$\frac{d_o}{h} = \frac{0{\cdot}5p}{0{\cdot}5p + 0{\cdot}15h + D_o}.$$

Hence, $d_o = h$ approximately, if $z = 0$, whatever value of p be assumed; for we cannot assume that p, is very much less than 20 feet, since, although it may be doubtful whether the top portions of the puddle move sufficiently to be considered as a live load, there is no doubt that the first 20 feet above the culvert must be treated as a live load. This condition secures that there is no tension in the arch, and then the equation :

$$k_o = 1{\cdot}5\ \frac{wl^2}{h}\left(\frac{D_o + 0{\cdot}5p + 0{\cdot}2h}{d_o}\right) \text{ lbs. per square foot}$$

gives the pressure at the crown of the arch ; where w, is the weight of a cubic foot of masonry. The maximum pressure in any portion of the arch does not exceed $2k_o$.

Now, let l_1, be the span, and h_1, the rise of the arch, as measured to the centre line of the arch ; and d_1, the arch thickness at the springing measured perpendicular to the centre line. Muller-Breslau (*Elastizitatstheorie der Tonnengewolbe*) finds that k_1, the pressure at the springing, is given by :

$$k_1 = 5{\cdot}2\ \frac{wl^2_1}{d_1 h_1}\left\{(D_o + 0{\cdot}5p + 0{\cdot}14h)\left(\sec\phi \mp \frac{4\frac{h_1}{d_1}}{\left(\frac{h_1}{d_1}\right)^2 + 1}\right) \mp 0{\cdot}75p\ \frac{h_1}{d_1}\right\} \text{ lbs. per sq. ft.}$$

where $\tan^2\phi = \frac{4h^2_1}{l^2_1}\,\frac{D_o + 0{\cdot}5p + 0{\cdot}5h}{D_o + 0{\cdot}5p + 0{\cdot}14h}$.

The result obtained with the upper sign must not be negative ; and with the lower sign must not be greater than the permissible working pressure.

As a rule, we can put $\frac{d_o}{d_1} = \frac{1}{2}$; but if $\cos\phi$ be greater than $\frac{1}{2}$, take $\frac{d_o}{d_1} = \cos\phi$ for the first trial.

These rules are founded on formulæ deduced by drawing the lines of resistance of various arches ; and the arches investigated were mainly parabolic, or three centered.

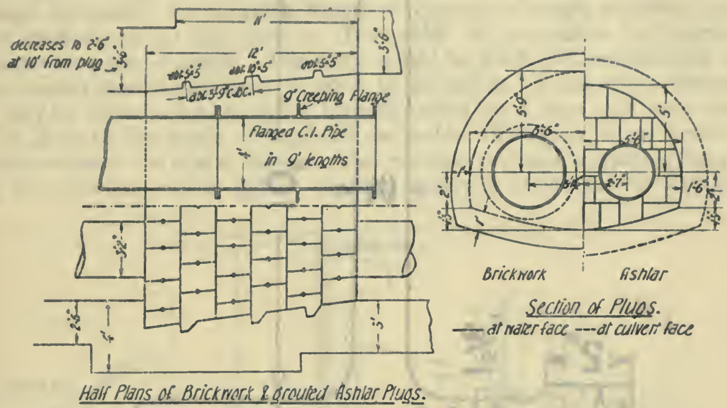
Existing culverts are usually constructed with semicircular arches, and $d_o = \frac{h}{6}$, and $d_1 = \frac{h}{3}$ is a fair indication of their proportions.

It may be inferred, that these dimensions are probably of excessive strength for the portions of the culvert which are loaded with well rammed firm earth, and that they are too weak for the portions under the puddle wall.

I have been unable to find one case where the culvert arches did not crack or settle if the side filling was of puddle clay ; while few appear to have cracked when covered with puddle, and supported at the sides by solid material. The cracking cannot be considered as amounting to a failure, and in most cases was scarcely of such a magnitude as to cause uneasiness to the engineers. It is, however, evident that a side coating of puddle forms an unfavourable condition. I therefore recommend that, wherever possible, the culvert should be bedded in undisturbed material up to the haunch level, and that a water-tight junction between the culvert walls and this material should be secured by heavy and systematic grouting with cement as the side walls are built up. In cases where this method is not practicable, it appears wisest to design the top of the culvert as an arch to carry a pressure equal to that produced by the puddle load ; and to spread out the sides as shown in Sketch No. 88.

While no existing culvert has been constructed of reinforced concrete, this material seems admirably adapted for sustaining the tensile stresses produced by the unequal pressures on the arch. It may be objected that the circumstances are such as to favour corrosion of the reinforcement, but it must be remembered that the tensile stresses probably only act during the settlement of the puddle, so that corrosion of the reinforcement at a later date is of little importance.

The stops, or plugs, which close the water end of the culvert or tunnel, require careful construction, as they are exposed to a head of water equal to the total available depth of the reservoir. Sketch No. 91 shows details.



SKETCH No. 91.—Details of Plug between Valve Tower and Culvert.

It is doubtful whether the general form of the ashlar plug is as satisfactory as that of the brickwork plug. The structure from which the sketch is made gives satisfaction.

The following specification is not only suitable for such work, but may (with the obvious exception of radiated joints) be employed for all brickwork or masonry which is intended to retain a high head of water.

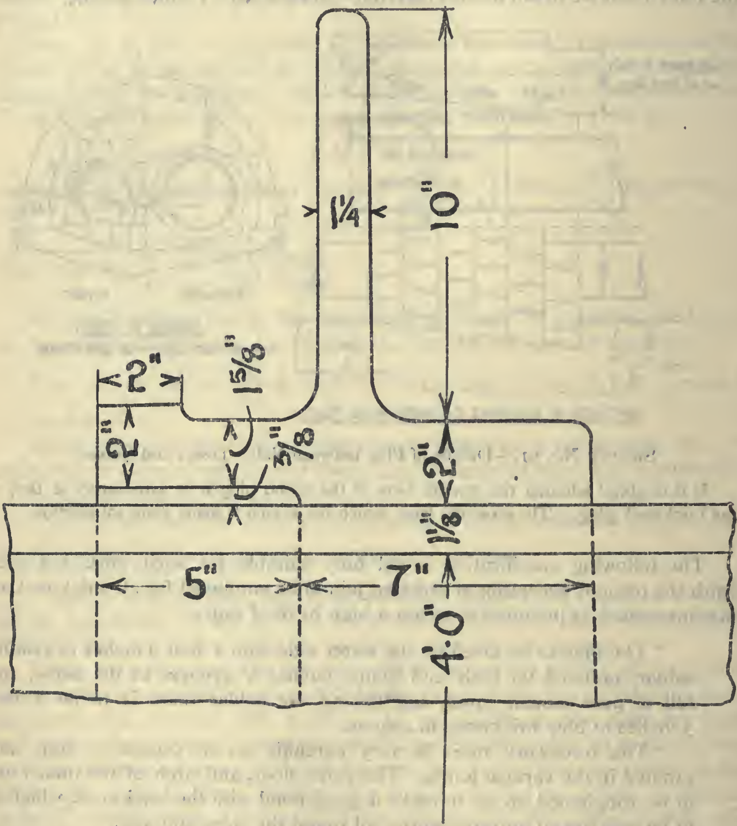
"The plug to be faced on the water side with 1 foot 9 inches of granite ashlar, radiated on beds and joints, having V grooves in the same, run full of pure cement grout, and behind the ashlar there is to be 8 feet 3 inches of blue brickwork in cement.

"The brickwork must be very carefully set in cement mortar, and grouted in the vertical joints. The sides, floor, and arch of the tunnel are to be roughened so as to make a good bond with the brickwork, which is to be well forced into soft mortar all round the sides and arch.

"Three circumferential chases, are to be cut or formed in the arch masonry, and concrete, one to fit the radiated ashlar, and the other two to make a good stopping chase with the brickwork.

"The whole to be made perfectly watertight, and the contractor to cut all bricks to fit the pipes and creeping flanges, grouting in the same in pure cement. All launders, troughs, and cofferdams required to pass the water over the plug during erection, and until the mortar is sufficiently set to allow the water being passed through the pipes, to be provided by the contractor."

Any rule for the thickness of the plugs, or stops, must obviously take into account the area of the exposed surface, as influencing the stresses developed by the water pressure. But it would appear that a thickness of $\frac{1}{6}$th to $\frac{1}{7}$th of the head of water is sufficient to prevent any undue percolation, provided that the stresses are not so heavy as to produce tension in the brickwork. In cases where the stress formulæ indicate a smaller thickness, water-tightness is economically secured by means of a layer of asphalte, or bitumen sheeting

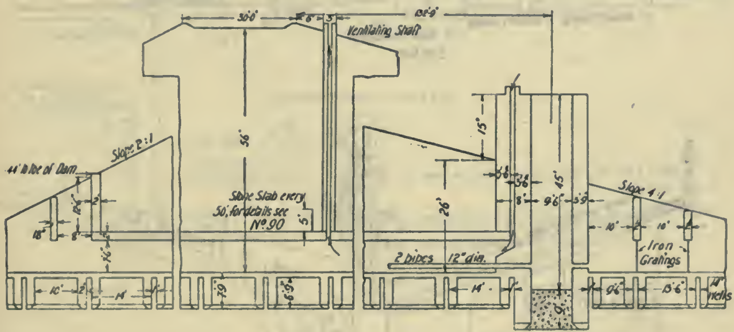


SKETCH NO. 92.—Creeping Flange on Pipe.

sandwiched into the plug. Specifications for such work are given on page 978, and a very excellent detail appears under the head of Service Reservoirs. (See Sketch No. 156.)

The details of the chases deserve consideration. The work is not easily inspected, and necessitates a large quantity of bricks being cut. Thus, there is a great tendency for the masons to fill the chases with a mixture of bats and mortar. The chases should therefore be carefully spaced, so that they will marry easily with courses of the actual bricks used (not with the theory of four courses per foot, etc.). The shape should also be that of a half-brick, so as to encourage the masons to fill in with whole bricks; and experience leads me to believe that bricklayers (as distinct from really skilled masons) make a better job when allowed to lay bricks in mortar, and to fill in the interstices with pure grout at the end of, say, each hour's work.

Creeping Flanges.—The general proportions of creeping flanges for pipes in concrete are shown in Sketch No. 92. The usual error (especially in the case of the creeping flanges to pipes) is to make the radial breadth of the flange too small. For creeping flanges in concrete a width, say, five times the thickness of the pipe suffices. This width will not suffice for creeping flanges for pipes in puddle. The width should be fixed by a consideration of the amount of settlement likely to occur in the puddle. From certain failures it may be inferred that the radial breadth should be at least $\frac{1}{30}$th (or better still, $\frac{1}{20}$th) of the height or depth of the puddle (whichever is the greater) above or below the pipe or culvert. The real object of such a creeping flange is, of course, to provide against slip and movement of the puddle; and the



SKETCH NO. 93.—Valve Tower and Culvert at Jaipur. (See also Sketch No. 90, Fig. 1.)

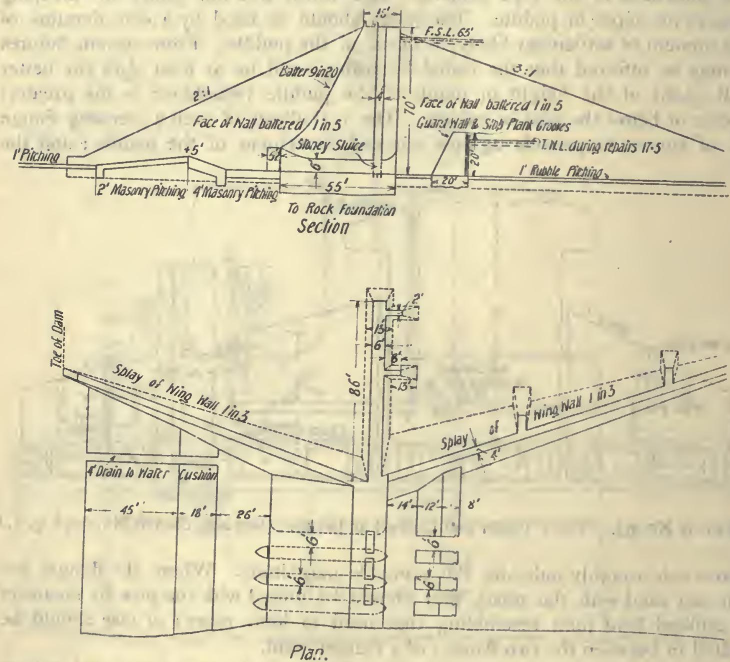
above rule roughly indicates the probable magnitude. Where the flanges are not cast solid with the pipes, they should be united with the pipe by means of a caulked lead joint resembling that used to joint pipes; or else should be bolted in between the two flanges of a flanged joint.

VALVE TOWERS AND CULVERTS.—The typical valve tower and culvert arrangement, with the tower at some distance from the centre of the dam, is shown in Sketch No. 88. The head wall and sluices (see Sketch No. 94) is a somewhat cheaper method of passing the same discharge, but only permits water to be drawn from one, or at the most two levels, and can only be used when firm foundations exist at a shallow depth.

The culvert with a central valve tower and open approach channel is exposed to fracture by settlement of the dam, and can only be considered as good design when the circumstances are such as to favour the construction of a masonry core wall. In such cases it is probably the cheapest solution of the problem; since the valve tower forms part of, and is built with, the masonry core wall.

The various positions of the valve tower produce a design which is well suited for drawing off the quantity of water usually delivered from the reservoir for use either in town supply or in irrigation, *i.e.* a daily volume equal to $\frac{1}{80}$th or $\frac{1}{100}$th of the content of the reservoir.

When it is desirable to prevent silting by the rejection of all turbid water through an outlet at a low level (diversion channels being too expensive) the volumes of water to be dealt with are far larger. Broadly speaking, the problem is to reject, if necessary in one day, a quantity of water equal to a large fraction (one-third, one-half, or possibly even the whole) of the maximum daily flow that usually occurs in each year. This last may be considered as about one-sixth of the maximum flood (which occurs at intervals of 20, or 30 years). In



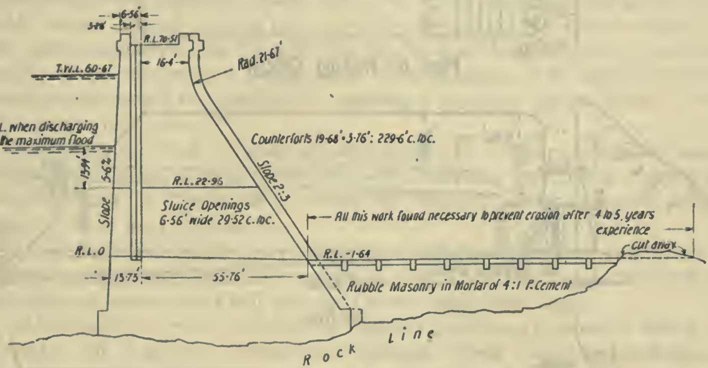
SKETCH No. 94.—Head-wall Outlet.

cases where the reservoir capacity is small compared with the average yearly run-off of the catchment area, it may amount to one-tenth, or to one-twentieth of the capacity of the reservoir. The outlet should, therefore, be large enough to discharge about $\frac{1}{20}$th to $\frac{1}{60}$th of the total reservoir capacity even when the reservoir is nearly empty, so that no marked ponding up of silty water need be permitted. For preliminary designs, the quantity rejected may be considered as that corresponding to the bank stage of the stream draining the catchment area, and the effective head under which the orifices work may be taken as 5, or 10 feet. The final studies will of course take into account the silt content, and the régime of the natural stream.

The problem is not as yet fully understood, and the difficulties attending the preliminary investigations and the preparation of the final designs are very great.

Considerations of cost prevent a valve tower being used for dealing with such large quantities of water, and the present solution usually consist of under-sluices of the type shown in Sketch No. 94. The great difficulty is that the system can hardly be adopted unless a firm rock foundation exists at a moderate depth. Typical examples are the Assouan masonry dam, and the Maladevi earth dam.

Sketch No. 95 shows the present (1910) section of the Assouan dam, and No. 94 the methods used by Strange to connect a head-wall of similar section with an earthen bank. The design is costly, but enables the waste weir either to be shortened, or to be entirely dispensed with. All silt deposits are prevented, so that (correctly regarded) the comparative cost in relation to that of a valve tower is materially reduced.



Sketch No. 95.—Assouan Dam and Repairs below Sluices.

The sketch shows a typical section. On the average the repairs shown are somewhat more bulky, and the stone facing is somewhat thicker than is usually the case. The rock downstream of the dam was cleared out until perfectly sound, and the excavation up to a level of 9·84 feet (3 m.) below the sluice bottom was filled in with rubble masonry in $1^c : 6^s$ mortar. Above this level the mortar was $1^c : 4^s$. The cut stone facing is usually 1·32 feet (0·4 m.) thick, laid in $1^c : 2^s$ mortar, and the bond stones are 2·65 feet (0·8 metre) deep, and at the most exposed points are spaced 5·25 feet (1·6 metre) centre to centre. The quantities of work are stated as about :

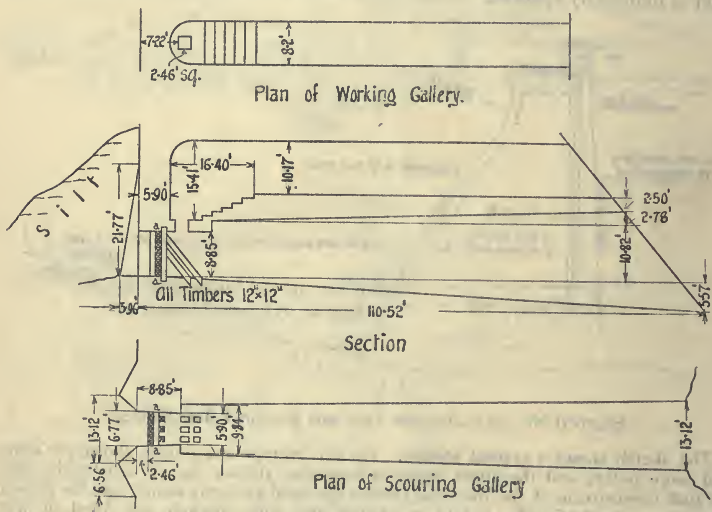
47 cube yards masonry, and 19 square yards facing, per foot run of dam containing sluices.

Very intense erosion is likely to occur in the escape channel, and for this reason alone a rock foundation must be considered almost indispensable. The various sketches of under-sluices (see p. 695) may be consulted for the details of protection against erosion, and the aprons put in at Assouan in 1901–3, are shown in Sketch No. 95.

The design of the wall (see Section in Sketch No. 94) needs careful consideration, since, owing to the existence of the large sluice openings the unit pressures are likely to exceed the permissible values if the height much exceeds 60 feet. For this reason, if for no other, rock foundations under the wall form a necessary condition.

According to the present Indian practice, such under, or scouring sluices, can usually pass off about one-seventh of the maximum flood; and the remainder is dealt with by an escape weir, either drowned, or with a clear overfall. This indicates a method which is economically feasible; and since the problem of silt deposit is forcibly brought before the notice of all Indian designers by the large number of old and abandoned silt-filled reservoirs now in existence, this capacity is probably the largest that is financially profitable. Any scouring sluice of a larger capacity is likely to prove too costly for the advantages reaped, except in unusually favourable circumstances.

Silting of Reservoirs.—It is very hard to state general rules for this subject. Silt deposits can be readily removed in certain cases by means of a scouring gallery, such as is known in Spain as a "desarenador."



Sketch No. 96.—Desarenador, or Scouring Gallery.

The typical, and most successful example, is the Alicante reservoir. Sketch No. 96 shows the dimensions of this gallery, but the upper working gallery is copied from the arrangements existing at Elche. The method of working is very clearly described by Aymard (*Irrigations du Midi de l'Espagne*, pp. 145 and 196). About every fourth year, when the deposits are well consolidated, as is ascertained by boring a hole through the wooden gate, the cross-beams *a*, *a*, are sawn through on both sides, leaving the gate supported only by the inclined struts. Workmen then enter the upper gallery, and cut away these struts by means of long chisels, and remove the beams and gate by hooks. The silt deposits in front of the gallery are then pierced and stirred up by long, iron-shod poles, worked by pulleys from the top of the dam. Once the flow is started, the deposits (which are usually about forty feet thick in front of the dam), are rapidly scoured out, the water initially standing about forty feet above the top of the silt deposits.

So far as can be gathered from the information given by Aymard and Willcocks (which is avowedly approximate only, since no surveys exist), the four years silt deposits represent about 3 to 4 per cent., and the water used to clear it away about 30 per cent. of the reservoir capacity. Such a result indicates very good working, and the proportions of the gallery are important; for in the case of the Elche reservoir, where the gallery is of the same section throughout its length, the process is relatively so unsuccessful that the reservoir filled up in about fifty years; while the Alicante reservoir is some 300 years old. Aymard gives the general slope of the Alicante bed as $\frac{1}{43}$, and the reservoir is narrow, being about 1000 feet wide as a maximum.

It would consequently appear that this method is only applicable to reservoirs situated on steep sites; and certain of the earlier Algerian reservoirs, the bed slopes of which appear to have been about $\frac{1}{300}$, are now silted up, although provided with similar scouring galleries. I cannot, however, state that these galleries were well adapted to local circumstances (as the Alicante proportions were slavishly copied, and this, unless special investigations were made, would appear illogical), or carefully worked, as the records show that the dams themselves gave trouble by cracks and fissures during the whole history of the reservoirs. Under such circumstances, it is doubtful whether any engineer would feel justified in opening a scouring gallery which "works with a noise like cannon."

It is plain that these galleries are only applicable where the reservoir can be completely emptied at more or less frequent intervals; and that the bed of the reservoir must be steeper than would be deemed advantageous if storage capacity alone were considered.

Thus, reservoirs which can be cleared of silt by a scouring gallery are by no means satisfactory from other points of view, since the cost of the dam per cube yard of water stored will obviously be comparatively large.

Many large reservoirs exist in the Bombay Presidency, some of which are subject to silt deposits. In the earlier designs it was considered sufficient to allow about 10 per cent. extra capacity for silting. Later designs, however (the oldest British built reservoir dates from 1868), treat the matter more systematically. The waste weir is supplemented by a set of powerful under-sluices, which pass away the silted water brought down by the early monsoon floods, and it is hoped that this method (if carefully applied) will suffice. Inspection of the large number of ancient silted reservoirs existing in India is not very encouraging. It may be anticipated that existing reservoirs (unless provided with very powerful under-sluices) will also slowly silt up; although it is quite possible that the period necessary for silting to cause appreciable damage may be measured in centuries, rather than years. The final results will probably depend more on the quantity of water which can be passed to waste, than on the actual discharge capacity of the under-sluices. No amount of sluice capacity will prevent marked silting in those reservoirs which store water for two or more years, and are supplied by a catchment area the minimum annual flow of which is insufficient to fill them.

The Nile reservoir at Assouan is worked on the principle of rejecting the silted water of the rising flood, and retaining only clearer after waters. The circumstances, however, are far more favourable than is usual in India. The Nile floods are so regular, and the system of up-river gauge reports so excellent, that it is very rarely necessary for any silted water to be even temporarily retained in the reservoir. While in Indian reservoirs, owing to uncertainty as to the future supply of water, it is frequently necessary to store heavily silted water. Also the Assouan under-sluices can pass off the flood discharge of the river at a mean velocity of about 20 feet per second, so that hardly any ponding up of silted water (and consequent deposition of silt) takes place.

It would therefore appear that designs on similar lines will permit reservoirs to be kept clear of silt; but the practical difficulties of sacrificing all the high flood water in reservoirs the capacity of which is any large fraction of the mean yield of the catchment area, are obvious.

It must also be noted that the Assouan dam is founded on hard granite. Even under these extremely favourable circumstances, extensive and costly repairs have been found necessary below the under-sluices, and it is doubtful whether they will not have to be repeated at frequent intervals. (See Sketch No. 95.)

Suggestions have been made for the mechanical removal of silt by dredging, the requisite power being obtained from turbines worked by the stored-up water and transmitted electrically. The principle is a good one, and it must be remembered that the power obtainable from a storage reservoir the main function of which is irrigation, will rarely be found of much value for manufacturing purposes, owing to the variability both of the discharge, and of the available head of water.

In the case where this electrical dredging was proposed, the local conditions were somewhat peculiar. As a general rule, where such an installation proves necessary, a more economical form of power would undoubtedly be steam, or oil engines, carried on the dredger itself.

It will be seen that there is a certain relation between the ratio which the capacity of the reservoir bears to the mean annual flow of the catchment area, and the probability of damage by silt.

Let us assume that a river (over the whole of the year) deposits in the form of silt 0·5 per cent. of the volume of water entering the reservoir.

Let us first consider a very unfavourable case, such as the Austin reservoir in Texas. Here the volume of the reservoir was $\frac{1}{40}$th that of the mean annual flow, and the dam being of the overflow type, circumstances favoured the disposition of silt. Under the above assumption the yearly silt deposit would be $\frac{40}{200}=\frac{1}{5}$th of the volume of the reservoir. As a matter of fact, in seven years about 49 per cent. of the volume of the reservoir appears to have been silted up.

A reservoir of 40 times the capacity would probably have caused the deposit of a greater proportion of the silt entering it. The deposit in seven years would theoretically have amounted to $7\times0{\cdot}5=3{\cdot}5$ per cent. of the volume of the reservoir, and in practice it might be expected to be at least $\frac{49}{140}\times3{\cdot}5=1{\cdot}3$ per cent. say, and either figure is (comparatively speaking) small, although not very satisfactory.

Consequently, an overflow dam, or, in fact, any dam without powerful under-sluices, is quite unsuited for a reservoir of which the capacity is so small a fraction of the mean annual flow as was the case in the Austin dam. The Hamiz and Habra dams are similarly defective.

On the other hand, the average British town water supply reservoir holds about 33 to 50 per cent. of the mean flow; and although the floods are turbid, it is doubtful whether the mean turbidity over the year amounts to even 0·01 per cent. However, let us assume that 0·1 per cent. is possible. The yearly deposit is at most 0·2 to 0·3 per cent. of the volume of the reservoir, and is probably less than one-tenth of these figures.

Thus, an overflow dam, or a high level escape weir, is quite allowable in such cases.

The general principles are evident.

When the reservoir volume is small in comparison with the mean annual flow (or rather, with the yearly volume of silt carried by the river), the dam must be of the Assouan type, *i.e.* provided with powerful under-sluices. These must be systematically employed during the high water season to pass off heavily silted water, and to scour out deposits. Later on, the clearer waters can be retained. In fact, we should endeavour to store what is mostly ground water flow, coming down after the flood season is finished.

The process is not difficult where the volume of the reservoir is $\frac{1}{40}$th or $\frac{1}{50}$th of the mean annual run-off of a fairly permanent river, such as occurs in tolerably damp climates. Difficulties begin when the volume is one-tenth, or one-fifth of the mean annual flow of a variable river. Even in a fairly moist continental climate, the minimum annual flow may be only about two-fifths of the mean. In such a year, any error in judgment might entail starting the dry season with the reservoir only partially filled; and in a dry continental climate matters are even more difficult. The correct procedure, nevertheless, is plain. Systematic rain-fall observations and gaugings must be carried out during the construction of the reservoir. It should then be possible to estimate the quantity of rain-fall falling towards the end of the rainy season that will certainly produce enough run-off to fill the reservoir, and to determine the relation that this bears to the minimum yearly rain-fall.

Thus, assume a case where the flow of the stream usually ceases in October, and let it be found (by actual observation) that 4 inches of rain falling in August or in September, is sufficient to fill the reservoir, and that the minimum fall between May and October (which is assumed as the flood season) is 12 inches. Then it will be plain that if the under-sluices are closed each year when 8 inches of rain have fallen, it is extremely improbable that the reservoir will not fill, and the longer the records are maintained, the more closely the limit can be fixed. A study of the ground water storage on the methods laid down by Vermeule should be extremely useful in such cases. The important matter is that the designer should state his principles, and should definitely order the required observations to be made, so that the records may be available when the question becomes acute.

Thus, in all cases where the flow of the driest year can be relied upon to fill the reservoir, silt deposits can be materially diminished, or can possibly be entirely prevented if the under-sluices are sufficiently powerful. At present, the experience which would permit any definite rule being given, does not exist. However, this is not very material, as a preliminary stanching of the reservoir bed by silt deposits is actually advantageous, since it will minimise leakage.

The general principles both of design and of observations are very concisely stated by Wilcocks (*Nile Reservoir Dam at Assouan*), as follows:

> "The obstruction to the flood was not to be greater than that of a cataract like Semne."

The natural stream bed must be studied, and if any unusually contracted portions exist, above which silt deposits are not produced, the area of the stream channel in these places can be taken as a basis for the design of the under-sluices. In other cases it appears best to assume an area for the under-sluices, to calculate the heading up necessary in order to pass the greatest flood through this area, correcting for the velocity of approach indicated by the cross-sections of the reservoir, and then to examine the possibilities of silt deposit in the pond formed above the dam. It must always be remembered that a certain amount of silt deposit may be expected in the early years of the life of the reservoir, which is harmless unless it bears too large a ratio to the reservoir capacity.

Permeability of Earthen Dams.—All earth dams are saturated by water up to a plane sloping more or less irregularly away from the water surface. The most exhaustive series of observations on the subject are those made by the Bombay Irrigation Department (*Experiments on the Saturation of High Embankments*). The earlier and less complete observations on the Croton (N.Y.) watershed dams are most useful, as showing that differences in the methods of construction and climatic conditions have little effect on the general results.

The slope of the saturation plane, measured over a short distance, is very irregular, is obviously greatly influenced by accidental circumstances, and fluctuates as the water in the reservoir rises and falls. The average slope measured from the water surface to the saturation level at a point below the downstream toe of the dam, is fairly constant, and its mean value for seven dams (which are practically of uniform material all through) is 0·32, the maximum being 0·46, and the minimum (about which there is some doubt) 0·12.

In dams composed of thick clay cores with more permeable outer casings, the slope in the clay is about 0·32 to 0·35, and in the casings 0·14 to 0·16. In three dams which are very well drained, and are composed of clayey material, the value is 0·20, 0·22, and 0·28.

The figures for the Croton dams are almost identical, ranging from 0·14 to 0·40; the higher figure indicating the very best construction; while the mean is about 0·23, which agrees very fairly with the mean for the whole section of the Bombay dams in which the hearting only is of clay.

The Croton dams have masonry core walls, and although the observation pipes were not spaced sufficiently closely to allow of a definite statement being made, the engineers who made the observations (see *Engineering News*, Nov. 28, 1901) considered that the core wall produced a drop of 10 to 12 feet in the plane of saturation.

My own observations on a well made British dam indicate slopes of 0·30 to 0·37, and that a good puddle wall produces a drop of about 20 to 30 feet. The figures are hardly comparable with the above-mentioned Indian and American results, as the dam was newly made, and the Indian observations show that steeper slopes may be expected in older dams.

The banks of the Punjab irrigation canals are very badly made when compared with the work required in the case of high dams. Slopes as flat as 0·07 and 0·09 occur. Under these circumstances a bank in which the slope of the line joining the full supply level at the water face, and the outer toe, is steeper than 0·15, or 0·16, usually gives trouble by seepage on the outer face, which (if the soil is of a bad quality originally) may cause a slip. (See Sketch No. 217.)

The engineers reporting on the Croton dams make certain deductions from their observations regarding the stability of earth dams. I have submitted their deductions to systematic small scale tests, and believe that they are unwarranted. So far as my observations go, an earth bank is perfectly stable under percolation (however great), provided that the water issuing from the bank does not carry away more particles of the earth than it deposits in the dam. The question of the stability of a dam is therefore intimately connected with the silt content of the water in the reservoir. Putting aside the extreme, and unpractical cases where the leakage is so great as to appreciably reduce the volume stored in the reservoir, the problem is merely to dispose of the leakage in such a manner that the finer particles are not reduced in number. If therefore, the water entering the dam contains much silt, the reversed filter may permit the more minute particles to pass away, provided that a larger volume of similar particles is deposited inside the dam. If the water is very clear, even the slightest removal of fine particles may finally cause a breach.

The stability of barrages or weirs such as are found on the Nile and Punjab rivers, is only explicable by the deposition of silt above the barrage compensating for the removal of finer particles below. The application of such principles to dams may be considered to be risky. It will, however, be plain that if the finest particles can be retained, the dam will be stable whether the water is clear or silted. Thus, a reversed filter properly graded so as to retain the finest particles, will render very intense percolation harmless.

In this connection the very interesting experiments of Saville (*Engineering News*, Dec. 24, 1908) may be referred to. Here an experimental dam about 11 feet high and 6 feet in length was constructed, and its permeability and saturation plane were determined. The figures are not quoted, as they refer to a dam deposited by the hydraulic-fill method, but the whole investigation and the mechanical analysis of the material form a model piece of work. Similar trials should certainly be made by any engineer contemplating the construction of a hydraulic-fill dam, and would also afford valuable information even when the ordinary methods of dam construction are adopted, since the results thus obtained, combined with a survey of the natural ground water levels, permit the leakage through and the stability of the proposed dam to be scientifically determined.

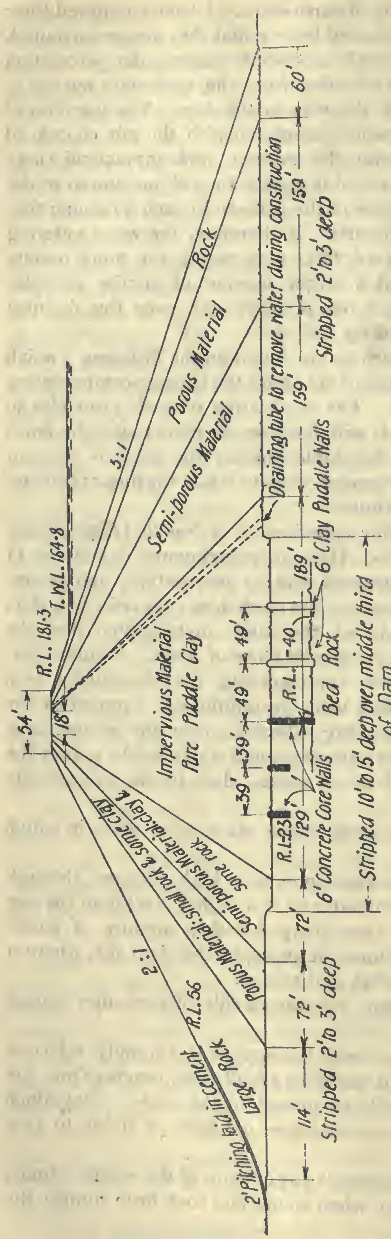
HYDRAULIC-FILL DAMS.—Hydraulic-fill dams are earthen dams in which the earth is deposited hydraulically.

Water is pumped or delivered from reservoirs, at a high pressure, through a nozzle against a bed of earthy material situated at a higher level than the site of the proposed dam. The water is thus charged with a mixture of earth, stones and clay, and is conducted by flumes, or pipes, to the dam site, where it is allowed to deposit its charge of material, and escape.

The principle of the method is plain, but the details of execution require consideration.

In the first place, the earth used should be somewhat carefully selected. What is really required is a mixture of particles of all sizes, ranging from the finest clay up to rocks which can barely be moved by the water. Clay alone cannot be utilised, and earth without any admixture of stones is liable to give trouble by slipping.

It is also stated that when the clay exceeds 50 per cent. of the whole volume, trouble is likely to occur by slips, even when stones and rock form almost the whole of the remaining volume.



SKETCH No. 97.—Hydraulic-fill Dam.

Angular rock is detrimental, since it is not easily moved by the water, and when moved is liable to injure the flumes.

The process evidently very closely imitates the manner in which beds of clay and sand are deposited by rivers ; and, if well carried out, may be relied upon to produce a good bank. Indeed, the chemical composition and physical properties of the clay portion of the deposit markedly resemble those of good puddle clay.

As we wish to obtain a section of material in grades resembling that of an Indian earth dam with a puddle hearting, it is plain that the loaded water should be delivered at the edges of the bank, so that it may there deposit the heavier material, forming a pitching, or rip-rap (see Sketch No. 97). The water should be drawn off near the centre of the dam by a substantial tower, so that clayey particles may be deposited at the place where a puddle hearting would usually be situated.

The method has been principally employed in dry climates. Certain phenomena in hydraulic-fill dams now under construction in climates where rain is somewhat more frequent, lead me to believe that the method will either prove impracticable in a moist climate, or will only be possible with material containing a somewhat smaller proportion of clay than that stated above. But it is evident that each material should be separately considered, and all that can be definitely said is that the more moist the climate, the smaller the proportion of clayey or saturable material that can easily be dealt with.

The danger of slipping being greatest just after deposit (while excess water is oozing from the bank) it is plain that there is a certain element of luck, in that the accidental coincidence of a spell of wet weather with the delivery of material containing a larger proportion of clay, may lead to a partial slip.

It is but fair to the process to state that bad failures have only occurred when the method has been recklessly applied (the fill being deposited on improperly prepared foundations, with inadequate provision for drainage). It would appear that if systematic drainage were universally adopted, after the manner provided for in Indian practice, the proportion of clay at present found advantageous might be greatly exceeded.

I may here refer to the porous conduit adopted in the Crane Valley dam, and described by Schuyler (*Trans. Am. Soc of C.E.*, vol. 58, p. 218), and would remark that the minor accident that occurred was plainly not due to the conduit, which appears to have done its work admirably.

The real advantage of a hydraulic-fill dam is its low cost per cubic yard, when the total quantity of earthwork is large.

The construction of such a dam requires certain somewhat unusual local conditions, and a comparatively large investment in plant of a rather special type. Thus, its adoption is unlikely, except in countries where hydraulic mining is practised, or in the case of large dams where a certain amount of expenditure in preliminary studies is permissible. Under such circumstances, I believe that the method deserves consideration, and if the problem of obtaining good drainage of the base of the dam is properly dealt with, and the available material contains a sufficient proportion of sand, gravel and stones (in order to secure stable slopes), with enough clay to form an impermeable core, the dam should not only be cheaply constructed, but it would appear that a more satisfactory result can be secured than by any other method.

According to Schuyler, the earth deposited in the dam is packed into about 90 per cent. of the volume that it occupied before being washed down, and only the very best rolling and after-consolidation has secured equally good results.

If the method is considered sufficiently infallible to permit of the total neglect of all ordinary drainage and preparation of foundations, the inevitable penalties of bad work can be anticipated.

Very few reliable figures as to the power required can be given at present.

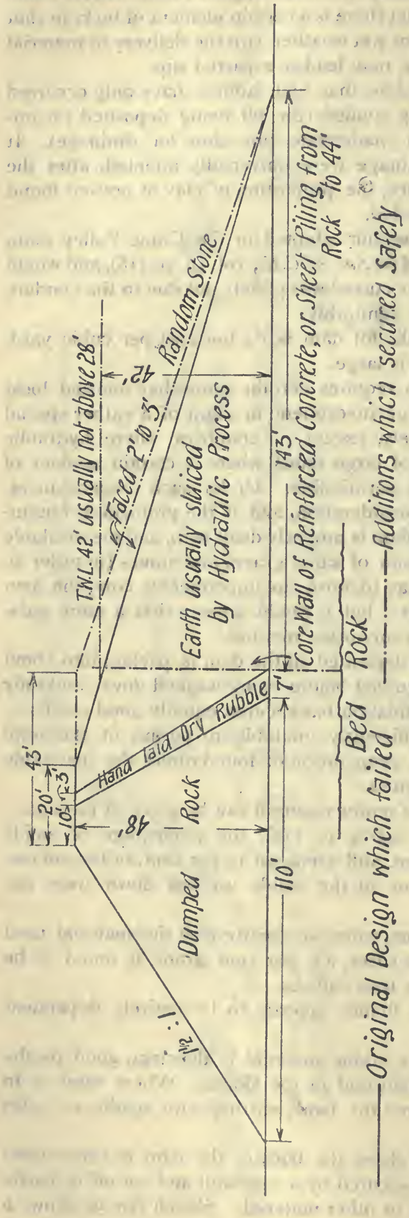
In the Lake Francis dam (*ut supra*, p. 212), the percentage of solids deposited varied from 6 to 48 per cent. and averaged 17 per cent. of the volume of water pumped. About 5 per cent. of the solids washed down were not deposited in the dam.

The friction in the pipes, or flumes, varies so greatly with the material used that no rules can be given. In some cases, a 6 per cent. grade is found to be small, while in others 2, or 3 feet per 1000 suffices.

Similarly, the best shape of the flumes appears to be entirely dependent upon the properties of the material.

It may also be noted, that where stable material is deficient, good results have been obtained by placing brushwood in the slopes. Where sand is in excess, and tends to form layers across the bank, stirring with spades or poles is advisable.

Rock-fill Dams.—In rock-fill dams the body of the dam is constructed of loose rock, and impermeability is secured by a core wall and cut off of earth, clay, steel plates, concrete, masonry, or other material. Sketch No. 98 shows a



SKETCH No. 98.—Rock-fill Dam.

typical section, and also illustrates the fact that an impermeable core wall cannot be dispensed with.

Schuyler (*Reservoirs for Irrigation*, etc.) enumerates seven types of core wall. Study of the examples leads me to believe that, as a rule, the method of constructing the rock work is such that settlements as large as (and in the case of bad work many times larger than) those which occur in earthen dams take place in the rock fill when water is first admitted into the reservoir. Thus, rupture of the impermeable coating has to be guarded against. In the successful examples the impermeable coating is either of earth or clay, and is consequently elastic, and is of considerable thickness; or else it consists of a vertical wall of steel or reinforced concrete of sufficient strength to resist the stresses produced by settlement. Success has also attended the use of thin masonry, concrete, or wooden diaphragms, but only when these are laid against, or are buried in, hand laid rubble walls of considerable thickness.

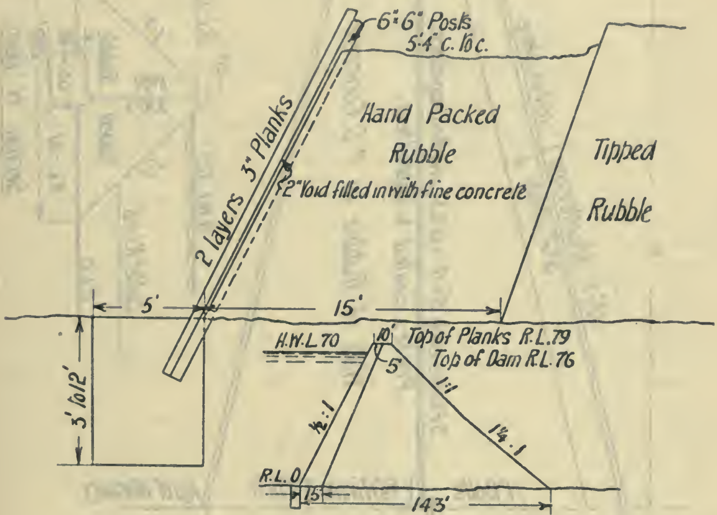
The rock-fill type of dam is at present somewhat discredited, owing to numerous failures having occurred. Such failures, however, are usually clearly traceable to bad construction, and are most frequently caused by the cut-off trench not being taken down to a sufficient depth. Indeed, in most cases, no other type of dam would have been expected to stand such treatment, and the rock-fill dam "made a better show" than could have appeared possible.

It may consequently be stated that the type is serviceable, and appears admirably adapted to cases where the foundation is hard, but permeable, *e.g.* of thick beds of deeply fissured rock.

Percolation must then occur, and so long as it is not sufficient to remove either the foundation rock, or the rockwork of the dam, the dam will stand where an earth dam would be eroded, or where a masonry structure would be destroyed by upward pressure.

The resemblance between such dams as the typical rock-fill dam (see Sketch No. 98), and the Indian composite dam with a dry stone toe, is obvious. Owing to the more careful methods of construction employed in India, the Indian type (height for height) is less bulky.

The sketches of the Escondido and Otay dams are typical of good American practice, although the Otay dam has not yet sustained the full depth of water.

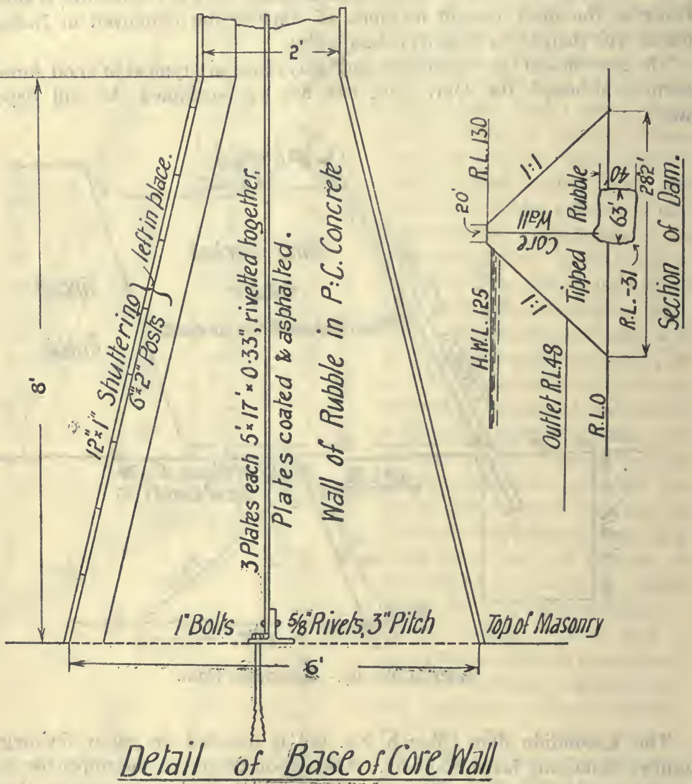


SKETCH No. 99.—Escondido Dam.

The Escondido dam (Sketch No. 99) is founded on partly disintegrated granite, containing large boulders. The cut-off trench at the upper toe is from 3 to 12 feet deep, and is filled with 5 feet thick rubble masonry in Portland cement. The facing is composed of two layers of redwood planks, each three inches thick, for depths of 50 to 76 feet below top water level; 2 inches thick between 25 feet and 50 feet below; and 1½ inch thick when less than 25 feet below. The planks are spiked to 5×6 inch vertical timbers, 5 feet 4 inches apart, embedded in the hand-laid dry rubble facing wall so as to project 2 inches beyond it. This 2 inches of space was filled as the planks were laid with rammed Portland cement concrete. The dry rubble wall was 15 feet thick at the bottom, and 5 feet thick at the top.

If we assume that the cut-off trench is carried down to a sufficient depth, and regard the plank facing as a temporary expedient which is to be replaced by permanent work later on (as local conditions permit), the design may be considered to be first class, and to be well adapted to the locality.

It is somewhat difficult to criticise the Otay dam (Sketch No. 100), as, judging from the foundation, either a masonry, or an earth dam, could have been erected. I shall therefore merely describe the steel core wall. This is of three steel plates, each 0·33 inch thick for the first 15 feet; then ¼ inch thick to 50 feet high, getting thinner towards the top.



SKETCH No. 100.—Otay Dam.

The plates were riveted together, and were caulked on the water face side. They were then brushed over on each side with hot asphalt of a somewhat liquid consistency. On this was placed burlap, which held the soft asphalte in place. The whole was then coated with a harder grade of asphalte, and was then enclosed in a wall of "rubble masonry in Portland cement concrete" (*i.e.* probably concrete with "plums"), 2 feet in thickness, the plate lying in the centre.

The shuttering of the wall (made of 1 inch boards on 2 inch by 6 inch posts) was left in place, and the rock fill was built against it on each side.

The success or failure obviously depends upon the manner in which the rock-work was laid, and since the dam has not yet filled, the case remains unproved.

The details of the steel plating and its treatment are plainly good, although they are capable of improvement if expense has not to be considered.

CHAPTER VIII.—(Section B)

MASONRY DAMS

Masonry Dams.—Gravity and arch dams.

Distribution of Stresses in a Masonry Dam.—Deduction of the "Middle Third" rule—General theory—Application to the calculation of the stresses on a horizontal section of a dam.

Working Units.—Specific gravity of the masonry—Theoretical condition for the minimum section—Preliminary design of the minimum section—Rectangular and parabolic topped dams—Batters and overhangs—Alteration of the thickness when these are neglected—Preliminary estimation of the total volume of the dam—Calculation of the pressures from the preliminary design—Example.

High Dams.—Limiting values of the pressures—Examples—*Shearing Stresses.*

Very High Dams.—Approximate equations—Accurate equation.

Further Theory of Masonry Dams.

Atcherley's Theory—General conditions—Diagram of stresses on a horizontal section—Parabolic distribution of shears on a horizontal section—Diagram of stresses on a vertical section according to Atcherley's theory—Uniform distribution of horizontal shears—Criticism.

Experimental Results.—Tension near water face toe of dam—Construction at and near the water face toe—Values of the tension—Rules for dam design—Algebraic investigation of Atcherley's theory—Tail thickening—General conclusions—Effect of earth pressures.

Fissures in Dams.—General theory—Case of a weak dam—Case where no tension exists in the cracked masonry—Case where the stress vanishes at a point nearer to the water face than the inner end of the crack—Case where the inner end of the crack is exposed to tension—Practical considerations—Permissible length of a crack—Example.

Failures of Masonry Dams.

Puentes Dam.—Failure by Percolation.

Bouzey Dam.—Failure by uplifting pressures due to cracks.

Austin Dam.—Failure by horizontal shear.

Abnormal Loads on Dams.

Practical Construction of Dams—Weak points—Rubble masonry *versus* concrete with plums—Portland cement *versus* hydraulic lime—Water-tightness—Intze's designs—Hollow concrete facings—Jack arch facings—Rich concrete and pure cement facings—Pointing—Dry *versus* wet concrete—Lias lime concrete—"Shearing" and "compressive" strength—Disposition of material with regard to shearing stresses.

Temperature Stresses in Dams.—Theory—Coefficient of expansion of concrete—Practical observations—American observations—Coefficient of expansion of large masses of masonry—Internal temperature variations.

Form of the Downstream Face of Overflow Dams.—Nappe boundaries—Tail portion of the curve—Practical considerations—Flashboards.

Theory of Curved Dams.—Approximate theory—Wade's practice—Values of the compressive stress—Bellet's corrections for the slope of the dam faces—Combined theory of gravity and arch stresses—Values of the ratio $\frac{X}{p}$—Dome-shaped dams.

Arch and Buttress Dams.

Reinforced Concrete Dams.—General principles—Bending moments and shearing forces—Preliminary design of reinforced concrete beam—Graphic diagram of forces.

Foundations.—Core walls—Flooring of the dam.

Earth Pressures on Retaining Walls.—Approximate theory—Graphical determination of the maximum pressure—Practical details.

SYMBOLS

For suffix notation, see page 365.

a, is the distance in feet of the mass centre of the area A, from the water face of the dam.

A, is the area in square feet of the cross-section of the dam above the level x.

b_n, is the area in square feet of the cross-section of the dam between the levels x_{n-1} and x_n.

c, is the vertical "tail" thickness of the dam in feet (see p. 380).

C (see p. 365).

d, is used for the depth in feet below top water level when investigating Atcherley's theory (see p. 381).

E (see p. 365).

H, is the horizontal force acting on the area A. $H=\frac{x^2}{2}$ in the working units.

K (see p. 381).

k (see p. 365).

l, is a suffix (see p. 373).

m (see p. 364).

M_n, is the moment of A_n about the water face end of t_{n-1}; while,

N_n, is the moment of A_n about the water face end of t_n; thus,

N_{n-1}, is the moment of A_{n-1}, about the water face end of t_{n-1}. (See pp. 358 and 368.)

n, when a suffix denotes the section actually considered.

$n=\frac{2}{3}-\frac{y}{t}$ (see p. 385).

p, is the vertical pressure at the water face of the section t. For unit used, see p. 362.

P (see p. 385).

p_e (see p. 363).

p_0 (see p. 373).

p' (see p. 383).

q, is the vertical pressure at the downstream face of t. See under p, for units, and for q_e, q_0, q', and Q.

r_n, is the batter, *i.e.* the distance in feet which the water face end of t_n is shifted towards the water relative to the water face of t_{n-1}.

r, is used for the vertical pressure at any point distant x, from the water face of t. (See p. 360.)

S, and S_1, are used for horizontal shears in Atcherley's investigation. (See p. 380.)

s, is used for the water pressure existing in a crack when investigating fissures, *i.e.* from page 383 onwards.

t, is the horizontal thickness of a dam at a level x, or d (p. 381), below the highest water level.

u (see p. 367).

V, is the vertical force acting on the section t. $V=A\rho$.

W (see p. 380).

x, is used for the depth of the section t, below top water level. x, is also used as a running co-ordinate on page 360 and 380.

$\bar{x}$, and $\bar{y}$ (see p. 381).

y, is the distance of the point where the resultant of H, and V cuts the section t, from the water face end of t. y, is also used as a running co-ordinate on page 380.

z, is the length of the crack in feet.

$\lambda=\frac{z}{t}$

θ, is the angle between the downstream face of the dam and the vertical.

ρ, is the specific gravity of the masonry.

SUMMARY OF EQUATIONS

Vertical pressures.—$r = p + (q - p)\dfrac{x}{t}$ $\qquad V = A\rho$

$$p = \frac{2V}{t}\left(2 - \frac{3y}{t}\right) \qquad q = \frac{2V}{t}\left(\frac{3y}{t} - 1\right)$$

$$a = \frac{M}{A} \qquad y = a + \frac{x^3}{6\rho A}$$

$$p_e = \frac{2V}{t}\left(2 - \frac{3a}{t}\right) \qquad q_e = \frac{2V}{t}\left(\frac{3a}{t} - 1\right)$$

Approximate Section of the Dam.—

$$t = \frac{x^3}{\sqrt{\rho\,(x^4 - 16h^4 + 4\rho A_1^2)}} = \frac{x^3}{\sqrt{\rho\,(x^4 + Eh^4)}} = \frac{x}{\sqrt{\rho}} - \frac{Eh}{2\sqrt{\rho}}\left(\frac{h}{x}\right)^3$$

$$A^2 = \frac{x^4 - 16h^4 + A_1^2}{4\rho}$$

$$A_n = A_{n-1} + \frac{t_n + t_{n-1}}{2}h$$

$$N_{n-1} = A_{n-1}a_{n-1}. \quad M_n = N_{n-1} + \frac{h}{6}\{t_{n-1}^2 + t_{n-1}t_n + t_n^2 - r_n(t_{n-1} + 2t_n)\} = A_n a'_n = A_n(a_n - r_n).$$

$$N_n = M_n + A_n r_n = N_{n-1} + A_{n-1}r_n + \frac{h}{6}\{t_{n-1}^2 + t_{n-1}t_n + t_n^2 + r_n(t_n + 2t_{n-1})\} = A_n a_n.$$

Batter Equations.—

$$u_n = -\frac{3}{2}r_n$$

$$r_n = \frac{2A_n t_n - 6M_n}{6A_{n-1} + (2t_{n-1} + t_n)h}$$

High Dams.—

Maximum pressure $= q\sec^2\theta$, or $p_e\sec^2\theta$

$= 3{\cdot}25x$ approximately

$= 203x$ lbs. per square inch.

Maximum shear $= 101{\cdot}5x$ lbs. per square inch.

Very High Dams.—

$$t = A_l\frac{2\rho}{q_0}e^{\frac{2\rho}{q_0}(x - x_l)}$$

Fissures.—

$$\frac{(1-\lambda)^2}{2} = x\left\{1{\cdot}12(3n + \lambda) - \frac{s}{2x}\lambda\,(4 - \lambda)\right\}$$

The other cases are not summarised, since they but rarely occur.

Masonry Dams.—Masonry dams for retaining water are divided into two classes:

(*a*) Gravity dams, in which the water pressures are resisted by forces brought into action by the weight of the dam only.

(*b*) Arched dams, in which the dam forms an arch, and resists the water pressures in the same manner as an arch sustains the load which is placed upon it.

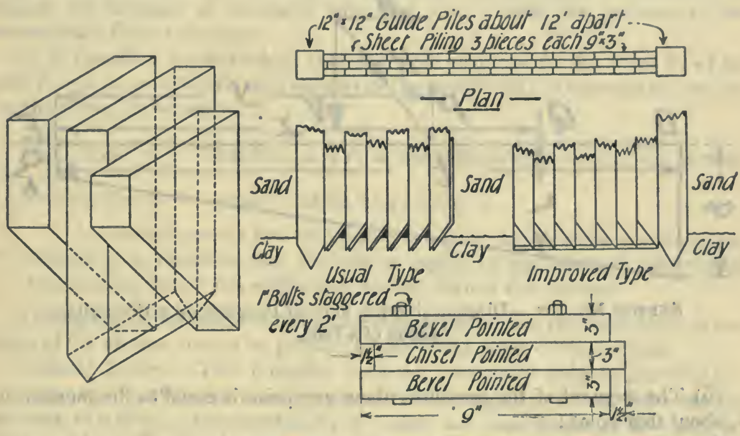
As a matter of fact, all arched dams act partly as gravity dams, and most gravity dams are curved in plan, and are therefore subject, in a greater or smaller degree, to arch stresses.

As a preliminary to the theoretical investigation of the stresses in either type of dam, it is necessary to investigate the internal stresses produced in any section of a gravity dam.

DISTRIBUTION OF PRESSURES IN A MASONRY DAM.—Suppose a force R, to act across a section of unit breadth, perpendicular to the plane of the paper, represented by the line ED, and for the sake of simplicity drawn as horizontal. Let V, and H, be the vertical and horizontal components of R (*i.e.* perpendicular and parallel to ED), and let the line of action of R, cut ED, in P.

Let ED $= t$, and EP $= y$.

The ordinary rules of statics tell us that reactions exist at every point in ED, which may be resolved into vertical pressures and horizontal shears.



SKETCH NO. 101.—Wooden Sheet Piling.

We also know that the resultant of the shears is equal and opposite to H, and that the resultant of the pressures is equal and opposite to V.

Further than this our knowledge does not extend, and any approximate solutions, as yet theoretically obtained, are of somewhat doubtful validity, except under such restrictions that the results are useless for practical purposes.

It is therefore necessary to make some assumption. The assumption selected by engineers finally leads to the result that the pressures vary uniformly across the section ED, as shown by the trapezoidal stress diagram. (See Sketch No. 102.)

This is a simple assumption, and is one which is familiar to engineers, but it cannot be said to have any very strong theoretical foundation. Its real claim to respect lies in the number of satisfactory dams calculated according to its results, and the less numerous unsatisfactory ones which must be considered unsafe when the results of the assumption are applied.

Of late years a certain amount of experimental evidence has been accumulated, which indicates that the assumption leads to smaller stresses than are found experimentally.

Let therefore :

q, be the pressure in lbs. per square foot at D,
p, be the pressure in lbs. per square foot at E.

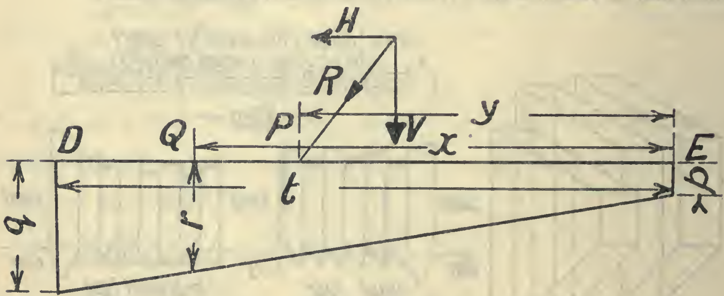
Then, at any point Q, where EQ = x, the pressure is :

$$r=p+(q-p)\frac{x}{t}$$

and we can at once determine p, and q, as follows :

(i) The algebraic sum of the pressures is equal to V, or :

$$V=(p+q)\frac{t}{2}$$



SKETCH NO. 102.—Diagram showing Vertical Pressures on a Horizontal Section of a Dam.

(ii) The moment of the pressures about any point is equal to the moment of V, about that point :

Or, taking moments about E ;—

$$Vy=p\frac{t^2}{2}+(q-p)\frac{t^2}{3}=\frac{t^2}{6}(p+2q)$$

Hence, we have ;

$$p+q=\frac{2V}{t}$$

$$p+2q=\frac{6Vy}{t^2}$$

$$\text{or, } p=\frac{2V}{t}\left(2-\frac{3y}{t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$q=\frac{2V}{t}\left(\frac{3y}{t}-1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and $\frac{V}{t}$, is evidently the mean value of the pressure if distributed uniformly over the area ED.

Now, consider P, to move along the line ED, from E, to D, we have :

$\frac{y}{t}$	p	q	*Remarks*—
0	$\frac{4V}{t}$	$-\frac{2V}{t}$	P, coincides with E. p, is a pressure, but q, is a tension.
$\frac{1}{3}$	$\frac{2V}{t}$	0	P, enters the middle third. q, is zero, but is changing from a tension to a pressure.
$\frac{1}{2}$	$\frac{V}{t}$	$\frac{V}{t}$	P, is at the mid point of ED. Both p, and q, are pressures, and $p=q$.
$\frac{2}{3}$	0	$\frac{2V}{t}$	P, leaves the middle third. p, is zero, and is changing from a pressure to a tension.
1	$-\frac{2V}{t}$	$\frac{4V}{t}$	P, coincides with D. p, is a tension, q, is a pressure.

In all investigations connected with dams the symbol p, is employed to denote the pressure at the water face, and q, to denote the pressure at the downstream face of the dam.

It is therefore quite evident that the only positions of the point P, where both p, and q, are positive and no part of the section ED, is exposed to tension, are those that lie between :

$\frac{y}{t}=\frac{1}{3}$, and $\frac{y}{t}=\frac{2}{3}$. That is to say, P, is inside the middle third of the section.

We thus obtain the usual "Middle Third Rule" ;

> "In order to secure that no tension occurs at any point of a rectangular section, it is requisite that the line of action of the resultant pressure on that section should fall within the middle third of the section."

Although it is not usually stated, the component of the resultant force in the plane of the section should be parallel to one of the sides of the section.

General Theory.—This question does not frequently occur in practice, but let us assume that ED, no longer represents a rectangular section, but a section the area of which is represented by A. Let, the mass centre of this area be distant d, from E, and the moment of inertia about a line through the mass centre and perpendicular to the projection of H on the plane of ED, be Ar^2.

We at once have the following equations :

$$V=pA-(p-q)\frac{Ad}{t} \qquad \text{and } Vy=pAd-(p-q)\frac{A(r^2+d^2)}{t}.$$

Put $\frac{d}{t}=n$, $\frac{r}{t}=m$; and we have ;

$$p(1-n)+qn=\frac{V}{A}$$

$$p\left(1-\frac{m^2+n^2}{n}\right)+q\frac{m^2+n^2}{n}=\frac{Vy}{Ad}$$

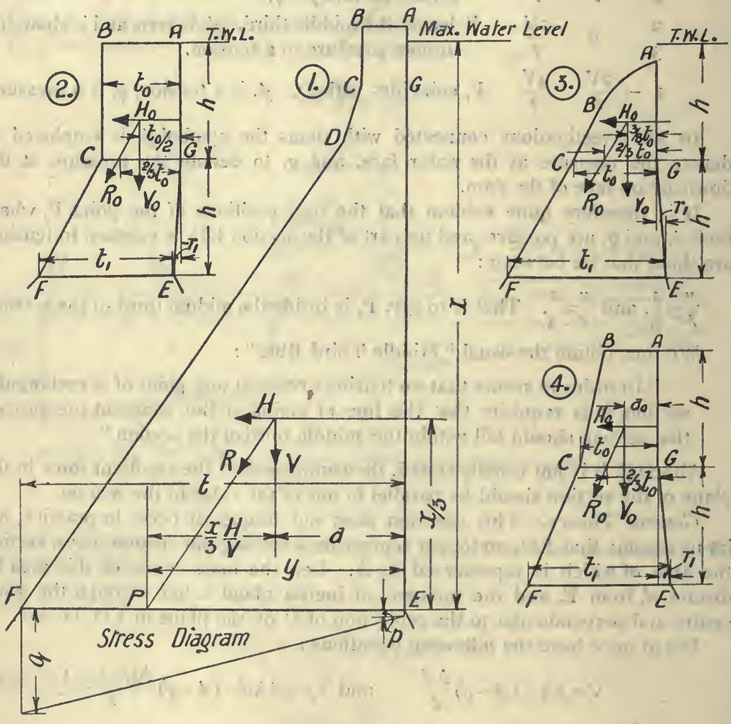
and these equations can be treated just as the simpler equations for a rectangular section were ; and are open to similar theoretical objections.

Application to the Calculation of the Stresses on a Horizontal Section of a Dam.—In this case, if we consider ED, to be a horizontal section of a dam, say 1 foot broad for simplicity, we see that :

V, is the weight of the masonry above ED, and is equal to the area of cross-section of the dam above ED × the weight of 1 cube foot of masonry.

H, is the horizontal thrust of the water on the water face of the dam above ED, and, for the present, we neglect all other forces, such as the vertical component of the water pressure that may exist owing either to the water face of the dam not being vertical, or to cracks in the masonry. So also, such matters as ice pressure, and wave action, etc. are excluded.

Let us take as the unit of weight the weight of one cube foot of water (*i.e.* 62·5 lbs. approx.).



SKETCH No. 103.—Kreuter's Method of Calculating the Section of a Dam.

Then, if ρ be the specific gravity of the masonry of the dam, one cube foot of masonry weighs ρ of these units.

In actual practice, ρ varies from 1·8 (112 lbs. per cube foot), or perhaps a little less, up to 2·5 (156 lbs. per cube foot), or perhaps a little more. The lower figure indicates materials unsuitable for dam construction, and calling for special precautions. The higher figure indicates very good materials, and very careful workmanship.

In preliminary studies, $\rho=2{\cdot}25$ (140·6 lbs. per cube foot) is a fair value to assume.

Now, (Sketch No. 103) let A, be the area of the dam section above EF. Then $V=A\rho$ is the weight of the masonry above EF. Also let the mass centre of this masonry lie at a horizontal distance *a*, from E, which is assumed to be the water face of the dam.

Now, if *x*, be the depth of E, below the maximum water level in the reservoir:

$$H=\frac{x^2}{2}, \text{ and H, acts } \frac{x}{3} \text{ vertically above E.}$$

Hence, referring to the figure, we easily see that R, the resultant of H and V, cuts EF at P, where $EP=y$, and:

$$y=a+\frac{x}{3}\frac{H}{V}=a+\frac{x^3}{6\rho A}$$

Hence, if *t*, be given, *p*, and *q*, can be calculated.

Now, it is evident that economy in material is best secured if the dam be so designed that *p*, and *q*, are always, and only just, positive. Hence, we see that $y=\frac{2t}{3}$. So that

$$p=0,\ q=\frac{2A\rho}{t}$$

Next, consider the reservoir as empty down to E. Then $H=0$. Then, if $a=\frac{t}{3}$,

$$p_e=\frac{2A\rho}{t} \text{ and } q_e=0$$

where p_e, and q_e, represent the values of *p*, and *q*, when the reservoir is empty.

Now, $y=a+\frac{x^3}{6\rho A}$. Hence, we get:

$$t=\frac{x^3}{2\rho A}$$

as the value of the minimum thickness of the dam that is consistent with the condition that no vertical tensions shall exist across the horizontal section EF.

It is plain that cases can be conceived in which this condition would not suffice to produce a satisfactory dam section, and (see p. 375) it is probable that a high dam designed solely in accordance with this condition would fail either by shearing near its base, or by horizontal tension across vertical sections. Nevertheless, under practical conditions a dam of which the horizontal thickness at each vertical depth is calculated by this rule will be found to form a very close approach to the final design, and it is therefore advisable to discuss the methods of laying out such a section before entering into the modifications of the theory. The design of a dam from these conditions is evidently a matter of trial and error. Such methods are laborious, and if once introduced an error will be carried forward, and will affect all the later work. It is consequently advisable to sketch out a preliminary design by a less rigid plan, and to use the exact equations for checking and modifying this design only.

Prof. Kreuter (*P.I.C.E.*, vol. 115, p. 63) has shown how a section, theoretically exact in form, can be laid down. In practical work, it will save time to sketch out such a section, and afterwards to introduce any modifications that practical conditions require.

Let ABCG, (Sketch No. 103, Figs. 2–4) be the upper portion of the dam, which may either be rectangular (Fig. 2), or shaped to an overfall form (Fig. 3). In cases where the dam carries arches for a roadway the pressure produced by the weight of the arches and piers must be combined with the load due to the visible section of the dam.

Let t_o, be the width of the section CG.

Then, if h, be the height of this first section, and ρ the specific gravity of the masonry of the dam:

We have, taking moments about the end of a_o (Fig. 4), when the line of pressure falls at the end of the middle third, for a rectangular top:

$$\rho h t_o \frac{t_o}{6} = \frac{h^3}{6}$$

or: $$h = t_o \sqrt{\rho}$$

and, for a parabolic top: $$h = t_o \sqrt{\frac{7\rho}{6}}$$

where in each case h, and t_o, represent the height and bottom width of this first section. And, as a general rule, if the resultant of all vertical loads above the line CG, be $V_o = n\rho t_o h$ say, and act at a distance a_o, from the point G, where $a_o = m t_o$ (Fig. 4): then, taking moments about the end of a_o;

$$n\rho t_o^2 h\left(\frac{2}{3} - m\right) = \frac{h^3}{6} \quad \text{or,} \quad h = t_o \sqrt{n\rho\,(4 - 6m)}.$$

Next, consider a section below this, cut off at a depth $2h$, and assume the intermediate portion of the dam to be trapezoidal.

The width t_1, necessary to ensure that the centre of pressure lies within the middle third, both when the reservoir is full, and empty, is obtained as follows:

In the case of a rectangular top:

The total weight is, $\rho h t_o + \rho h \frac{t_o + t_1}{2} = \rho \frac{h}{2}(3t_o + t_1)$, and by hypothesis its mass centre lies at $\frac{t_1}{3}$ from the water face. Thus, its moment about the other tri-section of t_1, is:

$$\rho \frac{h}{2}(3t_o + t_1)\frac{t_1}{3}$$

and this is equal to the overturning moment of the water pressure.

Therefore, $\rho t_1(3t_o + t_1) = 8h^2$

Hence, since $h^2 = \rho t_o^2$, we have:

$$t_1^2 + 3t_1 t_o - 8t_o^2 = 0 \quad \text{or, } t_1 = 1{\cdot}7 t_o,$$

and the total area to the depth $2h$, is $A_1 = 2{\cdot}35 \frac{h^2}{\sqrt{\rho}}$. And r_1, the batter of the water face required to cause the mass centre to be at a distance $\frac{t_1}{3}$, from the end of t_1, is given by:

$$h\frac{t_o^2}{2} + h\left(\frac{t_o}{2} \cdot \frac{t_o - r_1}{3} + \frac{t_1}{2} \cdot \frac{t_1 + t_o - 2r_1}{3}\right) = \frac{h}{2}(3t_o + t_1)\left(\frac{t_1 - 3r_1}{3}\right)$$

which gives: $$r_1 = \frac{2t_o\,(t_1 - 2t_o)}{8t_o + t_1} = -0{\cdot}05 t_o$$

Thus, the top portion of the dam should overhang slightly towards the water. This is known to be undesirable, and in practice the thickness t_1 must be slightly in excess of $1{\cdot}7t_o$.

If the top of the dam is parabolic, the overhang is not required, for:

∴ Proceeding on similar lines:

$$t_1 = 2{\cdot}1t_o \quad A_1 = 2{\cdot}08\frac{h^2}{\sqrt{\rho}}$$

and,
$$r_1 = \frac{8t_1t_o - 15t_o^2}{6t_1 + 36t_o} = 0{\cdot}04t_o$$

The upper portion of the dam being thus determined, let t, be the thickness of the dam at a depth x, below the highest water level, and A the total area of the cross-section to that depth. Sketch No. 104.

Then, reasoning just as before, we have:

$$\rho\frac{At}{3} = \frac{x^3}{6}$$

Hence,
$$2At = \frac{x^3}{\rho}$$

Now, plainly $\frac{dA}{dx} = t$. Therefore, $2A\frac{dA}{dx} = \frac{x^3}{\rho}$, or integrating, $A^2 = \frac{x^4 + C^2}{4\rho}$.

We can determine C, by considering that when $x = 2h$, then A, is the sum of the two sections already calculated, $A = A_1$ say, so that we finally get

$$t = \frac{x^3}{\sqrt{\rho(x^4 - 16h^4 + 4\rho A_1^2)}}$$

or:
$$t = \frac{x^3}{\sqrt{\rho(x^4 + 6{\cdot}09h^4)}} \text{ for a rectangular top,}$$

and:
$$t = \frac{x^3}{\sqrt{\rho(x^4 + 1{\cdot}28h^4)}} \text{ for a parabolic top.}$$

In practice, the suffixes must be carefully attended to. A_1, is the area down to the level $x = 2h = x_1$, and, except in very wide-topped dams, we usually find that $x_n - x_{n-1}$, can conveniently be taken as equal to h. Thus, b_2, is the area between $x_1 = 2h$, and $x_2 = 3h$, and A_2, is the total area $= A_1 + b_2$, down to the level $x_2 = 3h$.

If for any reason it is desirable to take $x_n - x_{n-1}$ as not equal to h, but say, $x_n - x_{n-1} = 5$ feet, then b_2, is the area between $x_1 = 2h$, and $x_2 = 2h + 5$, and A_2, is the total area down to the level $x_2 = 2h + 5$.

Similarly t_1, is the horizontal thickness at the level $x_1 = 2h$, and t_2, is the horizontal thickness at the level $x_2 = 3h$, or $x_2 = 2h + 5$ feet.

The zero of x, depends on circumstances. In an overflow dam $x = 0$, at say 5 feet above the crest of the dam. In a non-submerged dam it is usual to take $x = 0$, at the crest of the dam. In final calculations, the allowances made for ice pressure and shocks by floating bodies (see p. 394) must be considered in determining the level $x = 0$.

This equation permits a preliminary section of the dam to be set out at any depth; and if $4\rho A_1^2 - 16h^4 = Eh^4$, we have approximately:

$$t = \frac{x}{\sqrt{\rho}} - \frac{Eh}{2\sqrt{\rho}}\left(\frac{h}{x}\right)^3 = \frac{x}{\sqrt{\rho}} - \frac{kh}{\left(\frac{x}{h}\right)^3}$$

which is quite sufficiently exact when $\left(\frac{x}{h}\right)$, exceeds 5, or 6.

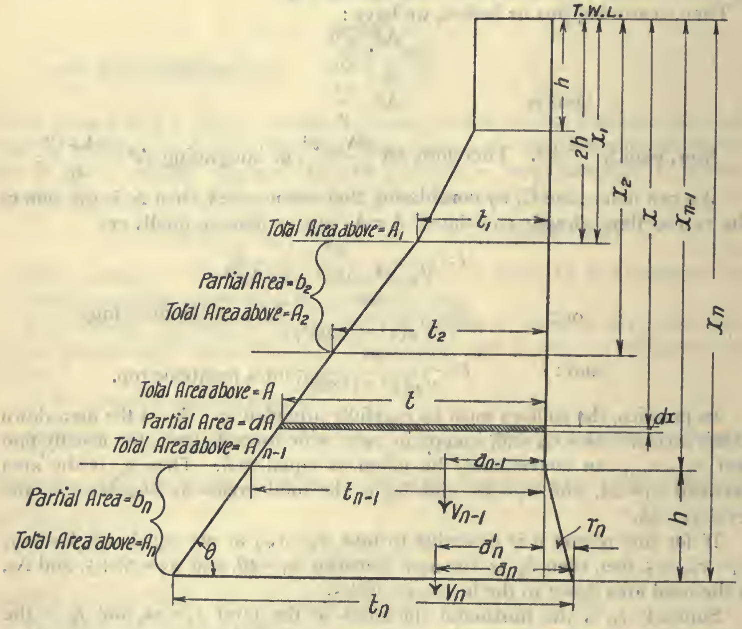
This method is very convenient for studying the effects of modifications in the upper portion of the dam, and saves a good deal of tedious trial and error in any case.

The section obtained, however, is an ideal minimum (subject to the condition that the two upper portions are as assumed), and only satisfies both the middle third conditions if the batter or overhang of the water face is properly adjusted at each of the levels x_1, x_2, . . . and x_n.

The necessary batter, or overhang, at any depth x_n, can be approximately calculated as follows :

Let b_n, be the area of the nth section, *i.e.* between the levels x_{n-1}, and x_n.

Let A_{n-1}, be the total area down to the level x_{n-1}.



SKETCH No. 104.—Kreuter's General Investigation.

Now, assume that $x_n - x_{n-1}$, is so small that the area b_n, may be regarded as a parallelogram. Then, taking moments about the water face end of t_n, if the section above x_{n-1} is correctly adjusted, we have :

$$A_{n-1}\left(\frac{t_{n-1}}{3}+r_n\right)+b_n\left(\frac{t_n}{2}\right)=(A_{n-1}+b_n)\frac{t_n}{3}=A_n\frac{t_n}{3}$$

$$\text{or :}\qquad r_n=\frac{A_n\frac{t_n}{3}-A_{n-1}\frac{t_{n-1}}{3}-b_n\frac{t_n}{2}}{A_{n-1}}$$

where r_n, is positive for a batter, and negative for an overhang.

The above equation is merely a first approximation, and the corrections are obvious when the upper section is not accurately adjusted, or when the mass centre of b_n, is not at the mid point of t_n (see p. 370).

The utility of these batter calculations may be regarded as doubtful, in view of the relatively small importance of the middle third law when the reservoir is empty. When they are not observed, the thickness of the dam is somewhat altered, as is indicated by the following investigation.

Measure all abscissæ from the water face end of t_{n-1}. In the usual notation let unaccented letters refer to the properly adjusted section, and accented letters to a section where t_n, has neither batter nor overhang, and is in consequence increased to t_n+u_n.

Put M_n, as the moment of A_n, about the end of t_{n-1}, and $h = x_n - x_{n-1}$.

We have in the properly adjusted section :

$$y_n=\frac{M_n}{A_n}+\frac{x_n^3}{6\rho A_n}=\frac{2}{3}t_n-r_n.$$

And, in the modified section, without batter or overhang :

$$y'_n=\frac{M'_n}{A'_n}+\frac{x_n^3}{6\rho A'_n}=\frac{2}{3}(t_n+u_n)$$

and, neglecting such terms as $r_n\frac{^2h}{6}$, we have :

$$M'_n = M_n+t_nu_n\frac{h}{2} \qquad A'_n = A_n+\frac{u_nh}{2}$$

Therefore :

$$\frac{2}{3}(t_n+u_n) = \frac{M_n}{A_n}\left(1+u_n\frac{t_nh}{2M_n}-u_n\frac{h}{2A_n}\right)+\frac{x_n^3}{6\rho A_n}\left(1-\frac{u_nh}{2A_n}\right)$$

$$=\frac{2}{3}t_n-r_n+\frac{t_n}{3}\left(\frac{u_nt_nh}{2M_n}-\frac{2u_nh}{2A_n}\right)$$

$$\text{for :}\qquad \frac{M_n}{A_n}=\frac{x_n^3}{6\rho A_n}=\frac{t_n}{3}+\text{small terms.}$$

Now, the section of the dam is nearly triangular in form. Approximately, therefore :

$$M_n = \frac{x_nt_n^2}{6}=\frac{nht_n^2}{6}$$

$$A_n = \frac{x_nt_n}{2}=\frac{nht_n}{2}$$

and, in any given case, if the small corrections are required, they can be calculated.

Therefore :

$$\frac{2}{3}u_n = -r_n+u_n\left(\frac{t_n^2h}{nht_n^2}-\frac{2t_nh}{3nt_nh}\right)$$

$$= -r_n+u_n\left(\frac{1}{n}-\frac{2}{3n}\right)= -r_n+\frac{u_n}{3n}$$

$$\text{or :}\qquad u_n = -\frac{3nr_n}{2n-1} = -\frac{3}{2}r_n, \text{ approximately.}$$

Thus, when a positive batter is required, we could theoretically diminish the thickness, in place of putting in the batter, and still satisfy the middle third condition when the reservoir is full; but in consequence the condition would be violated when the reservoir is empty.

When an overhang, or negative batter, is required, as for example in the second section of a rectangular topped dam, we must increase the thickness, since overhangs are not allowable (see p. 378).

In trial designs, it is best to ignore batters, and any consequent decrease in thickness; but overhangs, or rather the consequent increase in the thickness should be calculated. The nett result is an increase of 2 or 3 per cent. at the most, in the volume of the dam. Whatever method of adjustment, other than a rigid adherence to the calculated batters and overhangs, be adopted, the total volume must be increased if the middle third condition is to hold with the reservoir full and empty.

The whole volume of a dam of varying height is very rapidly approximated to as follows:

We have: $$A^2 = A_1^2 + \frac{x^4 - 16h^4}{4\rho} = \frac{x^4 + Eh^4}{4\rho}$$

and we have calculated $Eh^4 = 6{\cdot}09h^4$ for a rectangular top.

$$Eh^4 = 1{\cdot}28h^4 \text{ for a parabolic top.}$$

Hence, $A^2 = \dfrac{x^4 + \text{a known quantity}}{4\rho}$.

We therefore take the longitudinal section of the dam site, and calculate the fourth powers of the height at each point shown on the section, add the constant quantity, and after division by four times the specific gravity of the masonry, take the square root of the result and sum up by any of the ordinary rules.

In two practical cases, I found that the result agreed within 2 per cent. of that obtained from an accurately calculated table of areas and heights. The approximate result is always somewhat less than the final value.

The preliminary sketch being obtained as above, equations below and p. 369 should be used to calculate the pressures when the reservoir is full and empty: and any slight modifications required to keep p positive; and where advisable q_e, also positive, can easily be made.

When the water face of t_{n-1} is taken as the origin, the required formulæ are:

$$A_n = A_{n-1} + \frac{t_n + t_{n-1}}{2}h, \qquad N_{n-1} = a_{n-1}A_{n-1}$$

$$M_n = N_{n-1} + h\frac{t_{n-1} + t_n - r_n}{6}t_{n-1} + h\frac{t_n - 2r_n}{6}t_n \qquad a'_n = \frac{M_n}{A_n}$$

Thus, measuring from the water face of t_n:

$$N_n = A_{n-1}(a_{n-1} + r_n) + \frac{h}{6}t_{n-1}(t_{n-1} + t_n + 2r_n) + \frac{h}{6}t_n(t_n + r_n) = M_n + A_n r_n = A_n a_n$$

$$a_n = a'_n + r_n \qquad \text{and } y_n = a_n + \frac{x_n^3}{6\rho A_n}$$

and the conditions are:

$$p = \frac{2\rho A_n}{t_n}\left(2 - \frac{3y_n}{t_n}\right) \qquad q_e = \frac{2\rho A_n}{t_n}\left(\frac{3a_n}{t_n} - 1\right)$$

and if p is positive, q_e, may be slightly negative without any great detriment, Also, $q = \frac{2\rho A_n}{t_n}\left\{\frac{3y_n}{t_n} - 1\right\} = \frac{2\rho A_n}{t_n} - p$

$$p_e = \frac{2\rho A_n}{t_n}\left\{2 - \frac{3a_n}{t_n}\right\} = \frac{2\rho A_n}{t_n} - q_e$$

and the work can be carried on with certainty, as any great divergence from the preliminary values of t_n, shows that an error is likely to have occurred.

The final determination of the dam section requires certain conditions to be borne in mind. These are as follows :

(i) We usually wish to make $\frac{y}{t}$, not exactly $\frac{2}{3}$, but somewhat smaller, say, $\frac{y}{t} = 0{\cdot}65$, or $0{\cdot}64$ (see under Fissures, p. 384).

For a similar reason $\frac{a}{t}$, should be about 0·34, or 0·35 ; though this is less essential, and $\frac{a}{t} = 0{\cdot}30$ to $0{\cdot}32$, is not necessarily a bad design.

(ii) The resultant R, should make an angle with the vertical not much in excess of 35 degrees. This is usually secured by the above rules.

(iii) The discussion of Atcherley's theory and Wilson and Gore's experiments suggests (see Sketch No. 106) certain additions to the section as theoretically determined.

In the final calculations, therefore, we usually find that a certain economy can be secured by beginning to batter the water face outwards at about one-third of the total height of the dam above its base, even if the calculations do not indicate any theoretical necessity for this.

As an example, let us suppose that at :

$x_{n-1} = 45$ feet, with $\rho = 2{\cdot}28$. From the preliminary design we find that, $t_{n-1} = 30{\cdot}00$ feet. $A_{n-1} = 668$. $N_{n-1} = 6695$. Therefore $a_{n-1} = 10{\cdot}02$ feet ;

$$\text{and } y_{n-1} = 10{\cdot}02 + \frac{15190}{1523} = 10{\cdot}02 + 9{\cdot}98 = 20{\cdot}00 \text{ feet.}$$

$$\text{Thus, } p = \frac{3046}{30}\left(2 - \frac{3 \times 20}{30}\right) = 0$$

$$q_e = \frac{3046}{30}\left(\frac{30{\cdot}06}{30} - 1\right) = 0{\cdot}203 \text{ units} = 12{\cdot}7 \text{ lbs. per square foot.}$$

Thus, the conditions are satisfied, and :

$q = 101{\cdot}5$ units $= 6340$ lbs. per square foot.

$p_e = 101{\cdot}3$ units $= 6330$ lbs. per square foot.

Thus, the stresses are well within the permissible limits. When $x_n = 50$ feet, t_n is about 33·20 feet, according to the preliminary equation.

Assuming that $r_n = 0$, we at once find that :

$$A_n = A_{n-1} + \frac{5}{2}\, 63{\cdot}20 = 826$$

$$N_n = M_n = M_{n-1} + \frac{5}{6}\,(63{\cdot}20 \times 30 + 33{\cdot}20^2) = 9193$$

Therefore, $a'_n = 11{\cdot}13$ feet $= a_n$, since $r_n = 0$.

$$y_n = 11{\cdot}13 + \frac{20833}{1883} = 11{\cdot}13 + 11{\cdot}07 = 22{\cdot}20 \text{ feet.}$$

Thus, p, is negative, and although the value is small, t_n, must be slightly increased unless a batter be given.

Putting $t_n = 33 \cdot 30$ feet will probably make p, positive.

The values of M_n, and A_n, corresponding to $t_n = 33 \cdot 30$ are :

$$A_n = 826 \cdot 25 \qquad M_n = 9201. \qquad \text{Thus, } a_n = a'_n = 11 \cdot 14, y_n = 22 \cdot 19 \text{ feet.}$$

and p, is now positive.

Let us now investigate the batter required to make $a_n = \frac{t_n}{3}$, in which case t_n could obviously be diminished to $3 \times 11 \cdot 06 = 33 \cdot 18$ without causing p, to become negative.

Equation page 366 (given for obtaining the batter) is obviously too coarse for work where the quantities involved are approximately 0·05 foot, and we must use a more exact equation.

Taking moments about the water face end of t_n, we get in the general case :

$$N_n = A_{n-1}\ (a_{n-1} + r_n) + \frac{h}{6}\{t_{n-1}(t_{n-1} + 2r_n) + t_n(t_{n-1} + t_n + r_n)\}$$

is equal to $A_n \frac{t_n}{3}$, if the middle third is selected, or to $A_n k_n$, if the centre of pressure is to lie at a distance k_n, from the water face when the reservoir is empty.

$$\text{Or,} \qquad r_n = \frac{2A_n t_n - 6M_n}{6A_{n-1} + (2t_{n-1} + t_n)h}$$

where M_n, is the value previously calculated, when $r_n = 0$, for the moment round the water face end of t_{n-1}, of A_n.

In this particular case, we have, when $t_n = 33 \cdot 18$ feet,

$$r_n = \frac{2 \times 825 \cdot 9 \times 33 \cdot 18 - 6 \times 9190}{6 \times 668 + 2 \times 30 \times 5 + 33 \cdot 15 \times 5} = -\frac{343}{4474} = -0 \cdot 08 \text{ foot,}$$

which might also have been obtained from the equation :

$$u_n = -\frac{3}{2}\, r_n, \text{ since } u_n, \text{ is about } 0 \cdot 12 \text{ foot.}$$

Thus, the correct adjustment requires an overhang, and, as a general principle, it is impossible to make $a_n = \frac{t_n}{3}$, without an overhang, if once a_n, becomes greater than $\frac{t_n}{3}$, and the divergence once started increases as the depth below the top of the dam increases.

Thus, in large dams, either the slight overhang indicated as occurring at the second section must be put in, or a certain small excess in the dam section must be allowed to occur throughout.

The example (which is taken from a carefully worked design) shows the trend of affairs. At 45 feet the divergence, $a_n - \frac{t_n}{3}$, is 0·02 feet, increasing to 0·04 feet at 50 feet, and to 0·06 feet at 55 feet. So that t_n, is increased by 0·06, 0·12, and 0·18 feet at these depths.

The increase in bulk of the masonry thus produced is not very great, and it is doubtful whether it is really required. In fact, the real justification for making $t_{50}=33 \cdot 45$, which was the value actually adopted, was that p, is thus increased to 0·55 unit, or 35 lbs. compression per square foot which affords a safeguard against cracks (see p. 387).

For this reason, parabolic topped dams must be considered as leading to a better design when practical conditions permit their construction.

High Dams.—When checked by the exact method this preliminary process suffices for a design of a dam, so long as the height does not exceed a certain quantity. We have very approximately:

$$q=p_e=\frac{2\rho A}{t}=\rho x,\text{—since A, is not far off } \frac{xt}{2}.$$

Hence, the maximum vertical pressure is about 2·25 × the hydrostatic pressure; and, as later indicated, the maximum pressure according to the theory of elasticity is $\rho x \sec^2 \theta$, and $\tan \theta=\frac{t}{x}=\frac{1}{\sqrt{\rho}}$ approximately. So that the maximum pressure is close to:

$$\rho x\left(1+\frac{1}{\rho}\right)=2 \cdot 25 \times 1 \cdot 44 x=3 \cdot 25 x \text{ units;}$$

or, $203x$ lbs. per square foot, where x, is in feet, and q, the vertical pressure is $140x$ lbs. per square foot, approximately.

The limits are usually specified in terms of the vertical pressure.

For example, Rankine gives, $q=15{,}625$ lbs. per square foot, or $x=111$ feet. on the downstream face, and $p_e=20{,}000$ lbs. per square foot, or $x=143$ feet, on the upstream face.

Since the upstream face is nearly vertical, we see that Rankine's rules secure that the maximum pressure does not exceed 22,500 lbs. per square foot approximately.

Other rules for vertical pressure are those of:

Delocre		12,300	lbs. per square feet,	or	$x=88$ feet approx.
Furens Dam .	(1860)	13,280	,,	,,	$x=95$ feet approx.
Fernay Dam .	(1873)	14,320	,,	,,	$x=102$ feet approx.
Ban Dam .	(1870)	16,360	,,	,,	$x=115$ feet approx.

Even if we allow for an increase of 44 per cent., these are all well below the permissible working stresses of first class masonry in compression.

The real determining factor is not resistance to pressure, but resistance to shear. The ordinary theory gives the maximum intensity of shear as inclined at 45 degrees to the maximum intensity of pressure, and one half that maximum. That is to say, the maximum shear is equal to $101 \cdot 5x$ lbs. per square foot approximately, and accurately to: $\frac{q}{2}(1+\tan^2 \theta)$.

This last formula agrees very fairly well with the experiments of Wilson and Gore (*P.I.C.E.*, vol. 172, p. 128), except that they state that θ=angle between the resultant of the forces V and H, and the vertical. The difference may be of importance in the upper portions of the dam, but where the condition of limiting pressure is the determining factor of the design, the resultant and the downstream face of the dam face are practically parallel, so that theory and experiment agree.

Now, according to Bauschinger, the shearing strength of stone is about $\frac{1}{13}$th of its compressive strength, and for cement mortar the ratio is about $\frac{1}{6}$th. The actual figures are :

Concrete (1 Portland cement, 2 sand, 2 gravel) well rammed, shearing strength 70,700 lbs. per square foot.

Granite, shearing strength 92,250 lbs. per square foot.

Old Masonry, shearing strength 94,000 lbs. per square foot.

Allowing for the fact that the shear is about 0·72 of the vertical pressure, the above figures do not provide much over 7, to 9, as factors of safety.

The highest stresses actually existing in a dam appear to be those in the old Almanza Dam, with a vertical, and likewise maximum pressure of 28,660 lbs. per square foot.

The following are examples of maximum pressures actually existing (not vertical) :

	Lbs. per Square Inch.	
Alicante . .	23,080	All calculated by scaling the sections in order to obtain θ, and therefore liable to some degree of error.
Fernay . .	21,500	
Ban . .	28,600	
Furens . .	24,600	
Echapre . .	23,000	
Komotau . .	24,500	

We may therefore consider that a maximum pressure of 25,000 lbs. per square foot can hardly be exceeded, and that the factor of safety against shear is then about 6 to 8.

As a confirmation, it may be noted that the well designed Habra Dam (see Wegmann, *Design and Construction of Dams*) failed under a maximum pressure only slightly in excess of 26,600 lbs. per square foot, although Bouvier states that 29,460 lbs. per square foot maximum pressure is safe for hydraulic lime concrete, when not also exposed to shearing stresses.

In actual work the rules given by Rankine or Wegmann for vertical pressures may be followed, the latter gives :

16,380 lbs. per square foot on downsteam face. $x = 115$ feet.

20,408 lbs. per square foot on upstream face. $x = 146$ feet.

The value, maximum pressure = 29,700 lbs. per square foot, occurs in the proposed Quaker Bridge Dam (see Wegmann, *ut supra*, Plate 77), and if all earth pressures are neglected, 35,000 lbs. per square foot in the dam as constructed.

The assumptions made are unfavourable, so that such a stress is probably not actually sustained.

Very High Dams.—In dams where the above pressures are reached, the intensity of pressures becomes the limiting factor in the lower portions.

The design of the lower portions of such dams is in an unsatisfactory state, and most engineers would wish to have some further experimental evidence, such as the processes of Messrs. Wilson and Gore have given for the ordinary low dam, before erection.

The best method appears to be as follows :

Put q_0, as the limiting vertical pressure, either assumed to be constant as when Rankine's or Wegmann's rules are followed, or calculated from :

$$q_0 = (\text{Maximum permissible pressure}) \cos^2 \theta$$

where θ is the angle which the downstream face makes with the vertical as measured from the design of the upper portion.

Then: $q_o = \frac{2A\rho}{t}$, approximately. And $t = \frac{dA}{dx}$.

$$\text{Therefore,} \qquad \frac{dA}{A} = \frac{2\rho}{q_o}\,dx.$$

Integrating, and remembering that when $x = x_l$, the area is A_l, where x_l is the depth where q first exceeds q_o.

$$\text{Therefore,} \qquad A = A_l e^{\frac{2\rho(x - x_l)}{q_o}}$$

$$\text{and,} \qquad t = \frac{dA}{dx} = A_l \frac{2\rho}{q_o} e^{\frac{2\rho(x - x_l)}{q_o}}$$

and the profile of the dam can be set out from the logarithmic curve.

The profile should be checked at each step (say $h = 5$ feet) and a_n made equal to $\frac{t_n}{3}$ by first adjusting the water face batter r_n, by the equation:

$$r_n = \frac{2A_n t_n - 6M_n}{6A_{n-1} + h(2t_{n-1} + t_n)} = \frac{2A_{n-1}(t_n - 3a_{n-1}) - ht_{n-1}^2}{6A_{n-1} + h(2t_{n-1} + t_n)}.$$

and then determining, $y_n = a_n + \frac{x_n^3}{6\rho A_n}$, and calculating both q and p_e.

It is also evident that if the value of θ has markedly changed at any step, it will be advisable to calculate a new value of q_o, and to use this to determine the succeeding thickness.

The work is laborious, and needs constant checking. The process can be continued until p_e, exceeds its limiting value.

No approximate equation can now be given, and we must proceed in the manner indicated by Wegmann (*ut supra*).

Measure the angles of inclination of both faces, and thence determine the permissible vertical pressures p_o and q_o.

The approximate length of the next section is given by:

$$t_n^2\left(\frac{p_o + q_o}{\rho} - h\right) - 2t_n\left(A_{n-1} + \frac{t_{n-1}h}{2}\right) = x_n^3$$

and the batter by:

$$r_n = \frac{A_{n-1}(4t_n - 6a_{n-1}) + t_{n-1}h(t_n - t_{n-1}) + t_n^2\left(h - \frac{p_o}{\rho}\right)}{6A_{n-1} + h(2t_{n-1} + t_n)}$$

These equations are cumbrous, and the face slopes vary rapidly, so that q_o, and p_o, must be calculated afresh at each step.

I am not aware that these equations have ever been applied in practice, and as the result of actual experiment I am inclined to believe that a return to first principles and trial and error is probably more rapid. The equations, nevertheless, form a very valuable check.

FURTHER THEORY OF MASONRY DAMS.—The preceding methods permit a dam to be designed, which is safe against failure by vertical stresses acting across horizontal sections, (either tensile, or of undue magnitude if compressive). A little consideration, however, will show that failure can conceivably occur:

(*a*) By the dam shearing off horizontally.

(*b*) By horizontal stresses acting across vertical sections.

(*c*) By shear along vertical sections.

The following discussion is mainly concerned with case (*b*), but formulæ which permit cases (*a*) and (*c*) to be investigated are developed. These two cases are not considered in detail, as experimental evidence exists which shows that the really weak points of a dam are not accurately indicated by any theory at present existing, although, fortunately, a dam designed according to the present theories requires very little modification in order to make it safe against failure in the manner indicated by the experiments.

ATCHERLEY'S THEORY.—Until 1904 the theory of masonry dams was considered to be in a very satisfactory state, and the relation between practice and theory was far closer than is usually the case in engineering.

The conditions laid down concerned horizontal sections of the dam only, and were as follows:

(i) The resultant of the water pressure on the dam above each horizontal section, and of the downward forces due to the weight of the dam above this section, should cut this section inside its middle third, thus securing that no vertical tension existed in the masonry.

(ii) This resultant should, at each section, make an angle not exceeding 35 degrees with the vertical, thus securing that the dam has no tendency to slide as a whole against the friction between horizontal layers.

(iii) The maximum pressure per square foot should not exceed about one-tenth of the crushing stress of the masonry employed.

Where the water face of the dam was inclined, it was usual to neglect the vertical component of the water pressure thus brought into play. In condition (iii) engineers generally considered only the maximum vertical pressure, ignoring the fact that the laws of elasticity rendered it perfectly clear that close to either face the maximum pressure is parallel to this face, and is equal to the vertical pressure multiplied by $\sec^2\theta$, where θ is the inclination of the face to the vertical.

The matter does not seem of much importance. Those engineers who considered the point usually employed a higher working stress, and it was only in the case of very high dams that the condition became worth consideration.

In 1904 Messrs. Pearson and Atcherley re-opened the whole question, and showed that, under the assumption that the theory leading to condition (i) was rigidly true, the vertical sections on the downstream side of dams designed according to these rules were under horizontal tension (*Some Disregarded Points in the Stability of Masonry Dams*).

Their paper is somewhat mathematical in form, and the following resumé is given on my own responsibility, but I believe that it is correct as to facts, although the objection may be raised that it is an incomplete presentation of the investigations of these gentlemen.

Let us first consider a dam designed to satisfy condition (i). Let us assume, for the sake of simplicity, that the water face is vertical. Then, any piece of the dam above a horizontal section is subject to the following applied forces:

Water pressure on the face AB, represented by the triangular stress diagram A*b*B.

The weight of the piece of the dam is represented in the same manner by the section ABCD. It will be noticed that if the actual dam section is taken as the stress diagram, the line *b*B is $\frac{\text{BA}}{\rho}$, where ρ is the specific gravity of the masonry (Sketch No. 105).

These are equilibrated by:

(*a*) The vertical upward pressures on the base BC, which, according to the usual theory, are always positive, when condition (i) is satisfied, and when the resultant of the first two forces just lies within the middle third, are represented by the triangle B*c*C, the area of which is equal to that of the dam section ADCB, and in the general case by a trapezoidal diagram of the same area.

(*b*) There is also a set of horizontal forces termed shears, with a resultant equivalent in magnitude to the water pressure. As indicated by the usual theories of elasticity, if the vertical pressures are distributed according to a triangular (or trapezoidal) stress diagram B*c*C, these shears must be distributed according to the parabolic diagram B*f*′C, where the area B*f*′C is equal to the area of the triangle A*b*B, representing the water pressure.

It must be noted that this theory is known to be only approximate, and, since shears and pressures bear a differential relation to each other, it is quite possible to draw a curve of pressures which will generally have the same look and aspect as a triangular one, but which (having a different slope at each point) will produce a very different set of shears. Thus, an experimental proof that the measured pressures do not materially depart from those given by theory, is no assurance that the shears may not greatly depart from the theoretical value. It must also be noted that in every proof of the pressure law that has been attempted, points in the section close to either face are carefully excluded from the theoretical proof, so that we have no mathematical evidence that the shears follow a parabolic law near the faces of the dam. In fact, I believe that I am justified in saying that the theoretical proofs existing are, if anything, adverse to such an assumption, even Pearson's attempt (*Experimental Study of the Stresses in Masonry Dams*) being inconclusive when carefully investigated.

Let us now consider the equilibrium of a portion of the dam cut off by a vertical section, say EFC, where, for simplicity, (and as is almost invariably the case in critical sections) EC, is a straight line.

The stress diagrams of the applied forces are as sketched (see Sketch No. 105), and consist of:

The triangular weight diagram EFC.
The trapezoidal pressure diagram F*fc*C.
The parabolic shear diagram FC*f*′.

Let us now combine the first two, and for the sake of clearness, twist the whole round 90 degress. We then get a quasi dam, subjected to a quasi pressure on the face CF, represented by C*cg*F, where any line K*h* = K*k* − K*h*′, and and to a quasi weight force represented by a parabolic shear diagram C*f*′F, and these are equilibrated by pressures and shears on EF, of the same nature as those on BC, in the original section. Messrs. Pearson and Atcherley show that the resultant of the EF face forces falls outside the middle third, and that therefore, if we calculate the pressures on the vertical face EF, according to the theory employed when considering the horizontal sections, the face EF, is frequently exposed to horizontal tension near F.

In Atcherley's example, this tension is large, and very few existing dams (if they have been designed according to the old horizontal section theory) are found to be entirely free from tension when the stresses on vertical sections are investigated in this manner. The few exceptions occur when the dam itself forms a spillway, and is consequently far thicker than conditions of equilibrium alone require.

So far, I believe that I am in exact agreement with Messrs. Pearson and Atcherley, although my methods differ materially. While they express their doubts as to the absolute accuracy of the parabolic law for shears, it would appear that in many dams (although not all that I have investigated), the assumption of a uniform shear over the horizontal section (which is a far more favourable case than any that theory suggests to be likely to occur) still leads to horizontal tensions over vertical sections. In fact, were not their results contradicted by careful experiment, I should be very diffident in making any criticisms.

I venture to put forward the following :—Firstly, the law of the distribution of the weight forces as represented by ABCD, not only as a sum, but as individual forces, is very doubtful. This may materially alter the quasi pressure diagram C*cg*F ; and, similarly, while the shears represented by C*f*F, have been regarded as playing the role of the weight of the masonry, their points of application are not in the centre of each small section K*h*′ (as would be the case with weights), but at the face K.

This, of course, has been allowed for in Messrs. Pearson and Atcherley's investigations, but it is quite evident that the conditions necessary to cause the ordinary theory to be approximately correct for centrally applied weight forces, may be quite inadequate to make the theory hold good for these face forces. In fact, I believe that actual dams are sufficiently rigid to cause the usual theory to hold good, but are saved from the stresses indicated by this very same theory (as applied by Messrs. Pearson and Atcherley) through insufficient rigidity.

I would point out that Mr. Deacon (with the instinct of an engineer) had, long before the publication of this theory, advocated hydraulic mortar for dams, in place of cement, on the ground of less rigidity.

The theory of Messrs. Pearson and Atcherley did not pass unchallenged. Many engineers relied upon the fact that of all the dams (now amounting to several hundred in number) designed according to the old theory, not one had failed or even shown signs of failure, in the manner indicated.

I consider that the question has been settled, from a practical point of view, by the labours of Messrs. Wilson and Gore, and that their results are in a large measure confirmed by the less accurate, but more easily copied work of Messrs. Otley and Brightmore. My one stricture on the work of these gentlemen is perhaps hypercritical. They were forced to work with markedly non-rigid materials in order to get measurable deformations from which the stresses could be deduced.

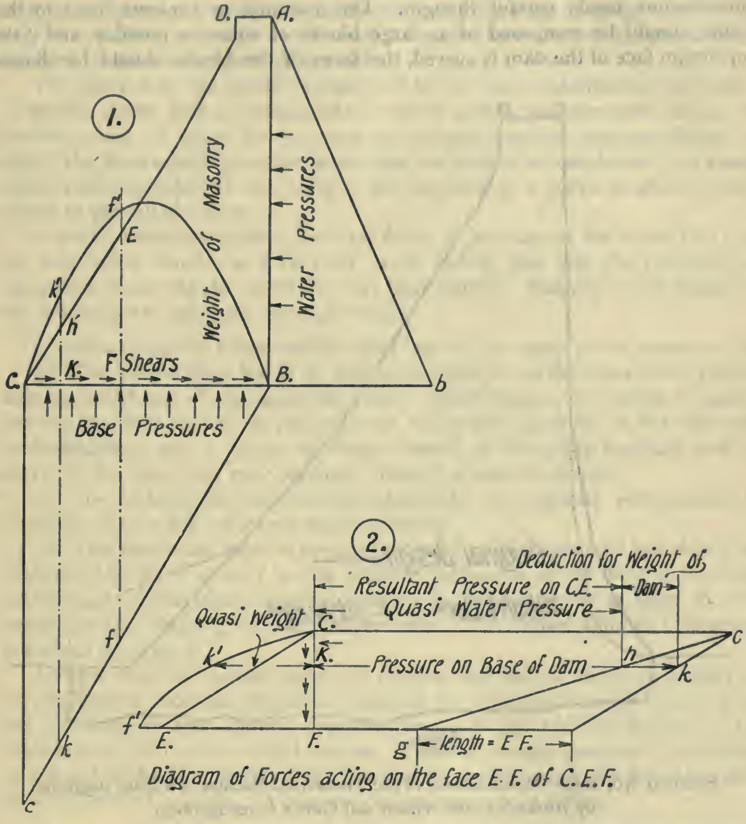
A very full description and discussion of these experiments is given in vol. 172 of *P.I.C.E.*, and should be read by everyone interested in the matter.

The experimental results show that the ordinary theory gives a very fair general idea of the stresses in a dam, at points far removed from the foundations, or faces, and that the stresses calculated by this theory are not likely to be exceeded in any portion of the dam.

In fact, the old theory is shown to be a good working guide, but it is useless to refine it more than engineers have been accustomed to.

Several interesting points appear in detail, and I believe that the following deductions are worthy of careful consideration in future design.

There appears to be a possibility of horizontal tensions existing near the



SKETCH NO. 105.—Diagram showing Forces acting on a Vertical Section of a Dam.

Fig. 1 shows the ordinary forces acting on a dam, and except in the fact that the shears shown by the parabola C$k'f'$B, act along the line CB, needs no explanation.

Fig. 2 shows the portion of these shears Cf'F, which act on the base of the portion CEF, acting as quasi weight forces along the line CF, while the pressure forces obtained by the ordinary theory act as quasi " water " pressures, and are diminished by the forces produced by the weight of the portion CEF, of the dam.

Thus, the elementary forces acting at, say K, are quasi weight forces, represented by

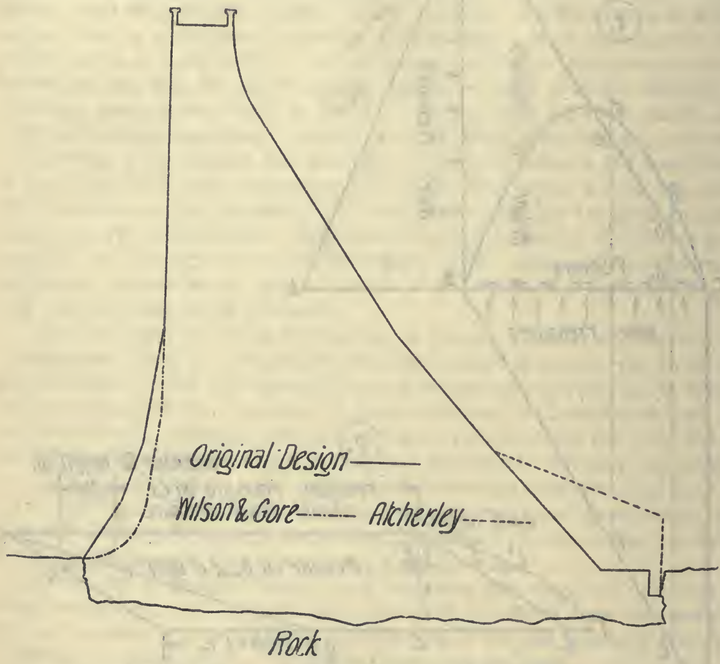
$$Kk' = \text{shear acting at K},$$

and quasi pressure forces, represented by

$$Kh = Kk - hk = Kk - Kh', \text{ i.e. base pressure at K, less weight of dam above K.}$$

water face toe of the dam ; not so much in the dam itself, as in the foundation vertically below the toe. This suggests the advisability of making the water face vertical, or slightly sloping outwards, and any overhang is quite unpermissible, even though this rule may cause the line of pressure (when the reservoir is empty) to pass slightly outside the middle third.

The construction of the corner formed by the water face of the dam and the foundations needs careful thought. The masonry, or concrete, near to this point should be composed of as large blocks of stone as possible, and if the upstream face of the dam is curved, the faces of the blocks should be shaped



SKETCH No. 106.—Modifications of the Theoretical Section of a Dam suggested by Atcherley and Wilson and Gore's Investigations.

to the correct curve, so as to avoid joints. A layer of asphalt, puddle, or other elastic impermeable material should be placed over the corner, so as to prevent percolation of water into the crack.

The experiments indicate that the crack will probably extend in a vertical direction into the foundation, rather than into the dam. The results obtained when investigating the stresses produced by fissures in the dam show that a vertical crack is less dangerous than a horizontal one.

Both Atcherley's theory, and the results of the experiments, indicate that the rock just beneath the base of the dam is more severely stressed than the dam itself. Thus, all available information is decidedly adverse to dams founded on any but the best and strongest rock. Such designs as masonry dams founded on piles driven into a clay stratum have failed too frequently to be good practice, quite apart from any experimental results.

The horizontal shears found by the experimenters appear to be distributed according to an irregular law, and the maximum shear occurs near the down stream face; so that the distribution is not even approximately parabolic or uniform. But, as the tension found by Atcherley does not manifest itself until long after failure by tension at the water face toe has occurred, the matter is of academic rather than practical interest.

The water face toe tension appears to be the cause determining the failure of model dams, and its magnitude is by no means inappreciable, being, in models, about $1\frac{1}{8}$ times (*i.e.* 3·9 tons per square foot for 125 feet depth of water) the hydrostatic pressure when the toe joins the foundation in a sharp angle, and about $\frac{1}{8}$th of this value if the junction is a curve of about 15 feet radius in the full size dam.

It would therefore appear that the form of junction of the water face and the foundation should be a curve of large radius, and that the construction about this point should be of the very best quality. Subject to this condition the following are safe rules for dam design:

(*a*) Adhere rigidly to the middle third law for the condition of reservoir full.

(*b*) The middle third law is of less importance when the reservoir is empty, and any overhang of the upstream face is unadvisable. A coating of impermeable material should be put over the water face junction of the dam and its foundation; and a system of drains should be formed in the dam itself to carry off the water that may percolate through a possible crack.

(*c*) The stability of each section should be investigated with respect to condition (ii), *i.e.* for resistance against sliding.

(*d*) The maximum vertical pressures given by the trapezoidal law should be multiplied by $\sec^2\theta$ where θ is the angle of inclination of the face of the dam, and the values thus obtained should show a factor of safety of at least 10, with regard to the crushing strength of the rock. The values adopted in practice are given on page 371.

(*e*) The vertical sections near the downstream tail of the dam should be investigated to ascertain the mean values of the shearing stresses produced by the resultant of the upward pressures found in (*d*), and the weight of the masonry cut off by the vertical section. This will usually produce a thickening of the tail of the dam of the character investigated on page 382, but if the dam is partly buried in earth, the original design may amply suffice.

I consider that such a dam (when founded on a good rock foundation) will be as safe a structure as any that are made.

As regards practical details, I believe that the real result of the controversy started by Messrs. Pearson and Atcherley is that a dam in hydraulic lime mortar is likely to possess certain advantages over one constructed with cement.

It is also clear that the Cyclopean method of construction in blocks of stone as large as can be handled, and set in good mortar, has many advantages over smaller stones or cut masonry. It appears quite unnecessary to spend any extra money on specially cut stones or thinner joints for the face work. Such outward show is liable to induce unknown stresses, is costly, and (I consider) even æsthetically, is an error. A good dam is independent of any outside prettiness.

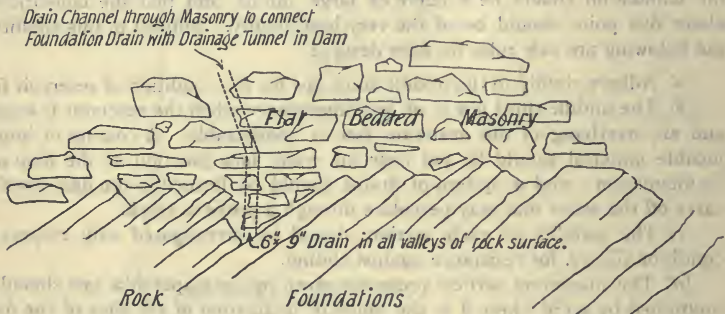
I would also note (although personally adverse to the principle) that bonding between the dam and its foundation, by iron rails, has been employed. It has also been proposed to keep the dam face unwetted by water, by means of a series of slabs of reinforced concrete supported from the dam, leaving a vacant space between the dam and the slabs, so that they and the dam face may be inspected.

The question of the general determination of the stresses across the vertical sections of a dam is best investigated algebraically.

Consider OA, a horizontal section, of length t, at a depth d, below the top of the dam.

Take the horizontal and vertical lines through the downstream end of this section as axes of co-ordinates.

Let $y=mx+c$ be the equation of the line BC, the downstream flank of the dam. As a rule $c=0$, but it will be found that the results obtained indicate that c should be about $\frac{d}{3}$, *i.e.* that the "tail of the dam" should be thickened.



SKETCH No. 107.—Cyclopean Masonry and Drainage of a Dam (after Deacon).

Using the notation for forces and pressures employed when investigating the stresses on horizontal sections of the dam, we find that:

The horizontal shear at any point P, in OA, where OP $=x$, is given by the equation:

$$S=\frac{6H}{t}\,\frac{tx-x^2}{t^2} \text{ if the distribution of the shear is parabolic}$$

(Note, $S_1=\frac{H}{t}$ if the distribution is uniform.)

Thus, the resultant of the shears or quasi weight forces acting on the length OE, where OE $=a$, is:

$$W=\int_o^a S dx = \frac{Ha^2}{t^3}(3t-2a)$$

So also, the vertical pressure at P, is given by the equation, $r=q-(q-p)\frac{x}{t}$.

The weight of the dam above P, partially counteracts this force, and the nett vertical force acting at P, is:

$$r-\rho y=q-\rho c-\left(\frac{q-p}{t}+m\rho\right)x$$

The resultant of these quasi pressure forces acting on the length OE is:

$$(q-\rho c)a-\left(\frac{q-p}{t}+m\rho\right)\frac{a^2}{2}=\text{K say.}$$

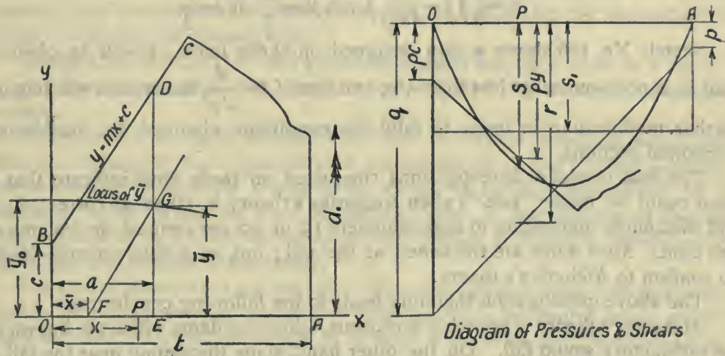
This acts at F, where $OF=\bar{x}$ and:

$$\bar{x}=\frac{(q-\rho c)\frac{a^2}{2}-\left(\frac{q-p}{t}+\rho m\right)\frac{a^3}{3}}{\text{K}}$$

$$\text{Thus,}\quad \text{EF}=a-\bar{x}=\frac{a^2\left\{3(q-\rho c)-a\left(\frac{q-p}{t}+\rho m\right)\right\}}{6\text{K}}$$

The resultant of K, and W, is therefore a force acting along the line FG, and cuts the vertical section ED, at G, and

$$\text{EG}=\bar{y}=(a-\bar{x})\frac{\text{K}}{\text{W}}=\frac{\left\{3(q-\rho c)-a\left(\frac{q-p}{t}+m\rho\right)\right\}t^3}{6(3t-2a)\text{H}}$$



SKETCH No. 108.—Investigation of Atcherley's Theory.

The modifications necessary when the flank of the dam is curved, so that y is not equal to $mx+c$, are obvious.

The horizontal pressures and vertical shears acting at any point in the vertical section ED, can now be written down. It is unnecessary to go through the work. In a dam designed according to the usual rules, *i.e.* where horizontal sections are alone considered, $c=0$, and with very close approximation:

$$q=\rho d \qquad p=0$$

$$t=\frac{d}{\sqrt{\rho}} \qquad m=\sqrt{\rho}$$

$$\text{H}=\frac{d^2}{2}$$

Substituting these values we get:

$$\bar{y}=\frac{\rho\sqrt{\rho}\,(3t-2a)}{3d^2(3t-2a)}\,\frac{d^3}{\rho\sqrt{\rho}}=\frac{d}{3}.$$

Thus, all vertical sections between the tail and the point $a=\frac{d}{2\sqrt{\rho}}$, are exposed to horizontal tensions at and close to their lower edge.

If the circumstances of an actual dam, where q, is slightly less than ρd, where p, is a little greater than 0, and t, is a little less than $\frac{d}{\sqrt{\rho}}$, be investigated, it will be found that the locus of G, is no longer a straight line, but a very flat hyperbola scarcely distinguishable from a straight line.

Since the theory employed is known to be somewhat inaccurate, the actual calculations need not be performed. The general results are unaltered.

The case when c, is not equal to nothing can be similarly treated, and we then obtain the following:

$$\bar{y}=\frac{3\left(t-\frac{c}{\sqrt{\rho}}\right)-2a}{3t-2a}\,\frac{d}{3}$$

Putting $a=0$, we find that:

$$\bar{y}_0=\frac{d}{3}\left(1-\frac{c}{t\sqrt{\rho}}\right) \quad \text{or,} \; \bar{y}_0=\frac{2c}{3} \quad \text{if } c=\frac{d}{3}.$$

Sketch No. 106 shows a dam designed on these lines. It will be obvious tnat m, is now somewhat less than $\sqrt{\rho}$, and that if $t=\frac{d}{\sqrt{\rho}}$, the section will require further modification in order to fulfil the conditions obtained by considering horizontal sections.

The final results of investigations conducted on these lines indicate that a dam could be made "safe" (when Atcherley's theory is taken as correct) by a tail thickening amounting to approximately 15 or 20 per cent. of the volume of the dam. Most dams are thickened at the tail; but, as a rule, not sufficiently to confirm to Atcherley's theory.

The above investigation therefore leads to the following conclusions:

Atcherley's theory is probably erroneous, otherwise dams which are known to be satisfactory would fail. On the other hand, some thickening near the tail is desirable, and a shearing area sufficient to support the stress K, should be provided (see p. 372 for Shearing Stresses).

It will also be plain that the distribution of shears near the downstream face of the dam is of importance, and that the place where the tensions are indicated is doubtless in the dam, but close to its foundation. The circumstances are consequently hardly analogous to those in which tensions exist in a dam at an unsupported face.

The whole evidence indicates that tensions may exist, but their existence depends on a theory which is approximate, and large tensile stresses only occur just where this theory is well known to be doubtful; and, in fact, may almost be said not to hold.

The tensions revealed by this somewhat doubtful theory exist at points where the dam is well supported by the solid rock of its foundations.

I therefore consider that the question is one in which theory alone has no real weight, and believe that the evidence afforded both by the experiments of Messrs. Wilson and Gore, combined with the satisfactory results of general engineering practice, are quite conclusive proof against the theory.

I think that engineers must be grateful to Mr. Atcherley for raising the question, as the result has been to disclose the unsuspected weakness existing at the water face toe of a dam. It will be quite evident that the foundations of a masonry dam must be solid rock, and that any fissures running parallel to the length of the dam (especially near either toe) must be carefully examined, and, unless capable of being properly filled, may necessitate the site of the dam being shifted.

So far we have implicitly assumed that the only horizontal pressures acting on the dam are those produced by the water. Rock of a quality suitable for the foundation of a masonry dam very rarely occurs at the natural surface of the ground. Thus, the lower portions of nearly all existing dams are buried in earth. The earth, in view of its situation under water, or at the best in a very moist state, may be regarded as equivalent to a fluid weighing about 120 lbs. to the cube foot. When the pressures of the earth both up and downstream of the dam are taken into account it is usually found that the final section is approximately triangular down to the level of the top of the earth, and approximately rectangular below this level down to the rock foundation. Under these circumstances, the necessary tail thickening is not very great, but the shearing stresses near the downstream face of the dam, at and about the level of the top of the earth, must be investigated.

Fissures in Dams.—It is well known that good masonry, such as is found in a well constructed dam, can resist a certain, and by no means negligible, tension.

Nevertheless, we have already seen that the whole design of dams (other than such unusually high examples as are subjected in their lower portions to the maximum permissible compression stress) is mainly determined by the condition,—that no tension shall (in theory) exist in any portion of the dam. This appears to be somewhat uneconomical, but the following investigation will show that it is absolutely necessary.

Let us now consider a section of a dam, and assume that, owing either to tension in the masonry, or to faulty workmanship, a horizontal crack exists in the water face, and that this crack is of such a width as to allow water under a pressure s, to penetrate to a depth z, as shown.

Consequently, we have, in addition to the forces formerly considered, an upward pressure s, acting uniformly over the length z, *i.e.* the resultant upward force is sz, acting at a point distant $\frac{z}{2}$ from the water face of the section.

Applying equations Nos. 1 and 2 we get for the tensions produced by this force :

$$\left.\begin{aligned} p' &= \frac{2sz}{t}\left(2 - \frac{3z}{2t}\right) \\ q' &= \frac{2sz}{t}\left(\frac{3z}{2t} - 1\right) \end{aligned}\right\} \quad \text{. . . . . Fig. I.}$$

where t, is the horizontal thickness of the dam.

Hence, the total pressures existing in the dam, due to the combination of the ordinary forces and this uplifting pressure, are (Fig. II.) :

$$p - p' = \frac{2}{t}\left\{V\left(2 - \frac{3y}{t}\right) - sz\left(2 - \frac{3z}{2t}\right)\right\} \text{ at the water face.}$$

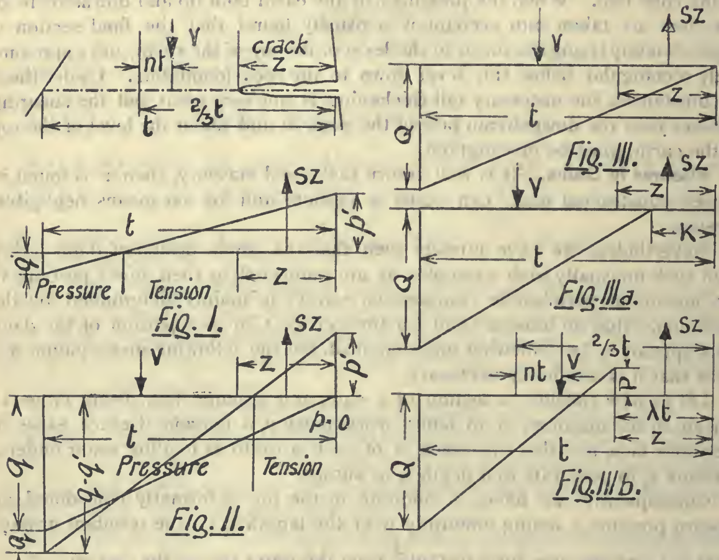
$$\text{and, } q - q' = \frac{2}{t}\left\{V\left(\frac{3y}{t} - 1\right) - sz\left(\frac{3z}{2t} - 1\right)\right\} \text{ at the downstream face.}$$

Therefore, no tensile stress exists in the dam if:

$V\left(2-\frac{3y}{t}\right)-sz\left(2-\frac{3z}{2t}\right)$ is positive; and, in the upper portion of the dam, where the resultant of the ordinary pressures lies well inside the middle third, we can (in any given case) calculate the pressures produced by the ordinary forces, and the extra uplifting pressure due to water in the crack.

Thus, if we put $\frac{z}{t}=\lambda$ we find that $p-p'=0$, when:

$$s=\frac{2V}{z}\cdot\frac{\left(2-\frac{3y}{t}\right)}{4-3\lambda}=\frac{p}{\lambda(4-3\lambda)}$$



SKETCH NO. 109.—Fissures in Dams.

which, when s is given, enables us to find the length of the crack that will just produce tension in the dam, and if, as is the case in most dams, $p=0$, $s=0$, or $\lambda=0$, so that any crack, however small, causes some tension to arise.

It is also evident that absolute safety against any possible crack can be produced by designing the dam so that $p-p'=0$, when $z=t$, for then the dam would be stable, with a horizontal crack extending right across it.

Putting $z=t$, we then get;

$$V\left(2-\frac{3y}{t}\right)-\frac{st}{2}=0$$

and if, for the sake of simplicity, we assume a triangular dam, and that s, is due to a water pressure equal to x, the height of the dam above the section considered, we have :

$$V=\rho\frac{tx}{2}=\rho\frac{ts}{2}. \qquad \text{so that, } \rho\left(2-\frac{3y}{t}\right)-1=0;$$

or, $y=t\left(\frac{2\rho-1}{3\rho}\right)$; $=0{\cdot}52t$ if $\rho=2{\cdot}25$.

Now, for a triangular section, with vertical water face, we have ;

$$y=\frac{t}{3}+\frac{x}{3}\frac{H}{V}=\frac{t}{3}+\frac{x^2}{3\rho t}.$$

Hence, for a triangular dam, safe against all cracks, we have :

$$\frac{t}{3}+\frac{x^2}{3\rho t}=\frac{2\rho-1}{3\rho}t \qquad \text{or, } t^2=\frac{x^2}{\rho-1}.$$

Whereas, for an ordinary triangular dam, complyimg with the middle third law :

$$t^2=\frac{x^2}{\rho}$$

So that, in order to secure absolute safety against horizontal cracks, we must increase the thickness of the dam in the ratio $\sqrt{\frac{\rho}{\rho-1}}$ or, for $\rho=2{\cdot}25$, about 34 per cent.

But, since q, is now $0{\cdot}56\ x\rho$, in place of $x\rho$, it is plain that the limiting depth before the maximum permissible pressures are exceeded is increased in the ratio $\frac{1}{0{\cdot}56}$ or, say $1{\cdot}80$, so that in high dams of weak material, a design of this character deserves consideration.

Such a section would be expensive in the usual case of low dams, and consequently a design such that $p=0$, or a small quantity, is adopted. Hence, $p-p'$ is negative, or tension may occur across the beginning of the crack.

Now, unless we assume the crack to be of limited extension in the direction of the length of the dam, we cannot expect the cracked portion of the masonry to support any tension.

We thus have three further cases to consider:

(i) $$P=p-p'=0=\left\{V\left(2-\frac{3y}{t}\right)-sz\left(2-\frac{3z}{2t}\right)\right\}\frac{2}{t} \quad . \quad . \quad . \quad \text{Fig. III.}$$

That is to say, the stress diagram is as sketched, and no portion of the masonry is in tension ; and if we put :

$$V=\frac{\rho xt}{2}=1{\cdot}12xt, \quad \text{and } \frac{2}{3}-\frac{y}{t}=n, \text{ we get :}$$

$$\frac{z}{t}=0{\cdot}666\pm\sqrt{0{\cdot}444-2{\cdot}24\frac{nx}{s}}$$

The negative sign alone gives a physically existing value, and we have, when $\frac{x}{s}=1$:

$n=0$	$\frac{y}{t}=0{\cdot}666$	$\frac{z}{t}=0$
$n=0{\cdot}033$	$\frac{y}{t}=0{\cdot}633$	$\frac{z}{t}=0{\cdot}06$
$n=0{\cdot}066$	$\frac{y}{t}=0{\cdot}6$	$\frac{z}{t}=0{\cdot}12$
$n=0{\cdot}166$	$\frac{y}{t}=0{\cdot}5$	$\frac{z}{t}=0{\cdot}40$
$n=0{\cdot}198$	$\frac{y}{t}=0{\cdot}47$	$\frac{z}{t}=0{\cdot}67$

(ii) The stress diagram is as sketched in Fig. 111*a*, and k, the distance to the point where the stress is 0, is less than z. Therefore, putting Q, for $q-q'$, we have:

$$\tfrac{1}{2}Q(t-k)\left(\frac{2t+k}{3}\right)=Vy-\frac{sz^2}{2}$$

$$\tfrac{1}{2}Q(t-k)=V-sz$$

Hence, $$k=\frac{V(6y-4t)+sz(4t-3z)}{2(V-sz)}$$

and, $$Q=\frac{4(V-sz)^2}{6V(t-y)-3sz(2t-z)}$$

The crack does not tend to increase in depth, unless tension exists at its end. The condition for this is obtained by putting $k=z$.

Or, since $V=1{\cdot}12xt$, simplifying the equation, we find as the equation for $\frac{z}{t}$, or λ:

$$\lambda^2-\lambda\left(4-2{\cdot}24\frac{x}{s}\right)+6{\cdot}72\frac{nx}{s}=0$$

which permits us to determine λ, when n, and $\frac{x}{s}$, are given;

e.g. $\frac{x}{s}=1$, $n=0{\cdot}033$; gives $\lambda=0{\cdot}14$.

When $z=k$, we have, $Q=\frac{2(V-sz)}{t-z}$, and when z, is given, $k=z$, when:

$$s=\frac{V(4t-6y+2z)}{(4t-z)z}=\frac{p\left(1+\frac{\lambda}{3n}\right)}{\lambda(4-\lambda)}$$

(iii) The dam is so badly cracked that the inner end of the crack is exposed to tension. (Fig. 111*b*.)

Here we merely have to substitute as follows in Equation page 383:

$t-z$, in place of t, $y-z$, in place of y, and $-\frac{z}{2}$, in place of $\frac{z}{2}$, when z, is a co-ordinate, but not in the expression sz.

We thus get:

$$P = \frac{2}{t-z}\left\{V\left(2-3\frac{y-z}{t-z}\right)-sz\left(2+\frac{3z}{2(t-z)}\right)\right\}$$

$$Q = \frac{2}{t-z}\left\{V\left(3\frac{y-z}{t-z}-1\right)+sz\left(\frac{3z}{2(t-z)}+1\right)\right\}$$

or:

$$P = \frac{2}{(t-z)^2}\left\{V(2t-3y+z)-\frac{sz}{2}(4t-z)\right\}$$; where P, is a pressure when positive.

$$Q = \frac{2}{(t-z)^2}\left\{V(3y-t-2z)+\frac{sz}{2}(z+2t)\right\}$$

It should be noticed that the upward force sz, now increases the compression at, and near, the downstream face.

As before, put $V=1{\cdot}12xt \qquad \frac{2}{3}-\frac{y}{t}=n \qquad \frac{z}{t}=\lambda$

We get:

$$\frac{P(1-\lambda)^2}{2}=x\left\{1{\cdot}12(3n+\lambda)-\frac{s}{2x}\lambda(4-\lambda)\right\}$$

$$\frac{Q(1-\lambda)^2}{2}=x\left\{1{\cdot}12\,(1-3n-2\lambda)+\frac{s}{2x}\,\lambda(2+\lambda)\right\}$$

It will be plain that if λ, be assumed at each joint (not necessarily as constant, but rather as a variable value corresponding to a fixed length of crack), since n, is known from the original design, Section No. (ii) permits us to determine whether the end of the crack is exposed to tensile stresses, and these last equations enable us to determine whether the assumed crack is likely to spread.

We may therefore consider that a complete design for a dam will not only include a table of the values of p and q, but also a table of the "permissible length of crack," calculated from Equation page 384, or a table of the stresses existing at the end of a crack of definite length, calculated from the appropriate equation.

The importance of such work cannot be measured solely by the values of the stresses as calculated, for their numerical values alone do not fully disclose the strains that may be induced in the masonry. The stresses occur at a re-entering angle. We have no exact theory of the effect of stresses near the end of a crack, such as are now considered, but there is little doubt that this is a more unfavourable case than that of a spherical flaw in a cylinder under torsion. In the latter we have sound theoretical justification for the statement that the strains may rise to twice the value calculated from the average stress according to the usual rules.

Thus the stresses obtained above are not only high, but act on a weak spot, and are therefore likely to be very effective in extending the fissure.

As examples of the application of the above equations, let us assume the following:

(i) A dam designed to satisfy the middle third law exactly. Then, $\frac{y}{t}=\frac{2}{3}$, or $n=0$, and

$$\frac{P(1-\lambda)^2}{2}=x\left\{1{\cdot}12\lambda-\frac{s}{2x}\lambda(4-\lambda)\right\}$$

Or, assuming successively that:

$$\lambda = 0{\cdot}01 \text{ we get } 0{\cdot}49P = x\left(0{\cdot}0112 - \frac{s}{x}\,0{\cdot}0199\right)$$

$$\lambda = 0{\cdot}02 \quad ,, \quad 0{\cdot}48P = x\left(0{\cdot}0224 - \frac{s}{x}\,0{\cdot}0398\right)$$

$$\lambda = 0{\cdot}03 \quad ,, \quad 0{\cdot}47P = x\left(0{\cdot}0336 - \frac{s}{x}\,0{\cdot}0595\right)$$

$$\lambda = 0{\cdot}05 \quad ,, \quad 0{\cdot}45P = x\left(0{\cdot}0560 - \frac{s}{x}\,0{\cdot}0987\right)$$

$$\lambda = 0{\cdot}10 \quad ,, \quad 0{\cdot}405P = x\left(0{\cdot}1120 - \frac{s}{x}\,0{\cdot}1950\right)$$

Hence, if $s = x$, as will be the case if the crack is of measurable width, even the smallest crack (if horizontally directed) produces a tension at and around its end.

Now, assume the workmanship to be such that an open crack 1 foot long may possibly occur.

When $x = 15$ feet, $t = 10$ feet approx.

Thus, for a 1 foot crack we have $\lambda = 0{\cdot}10$, and the tension when $s = x$, is $0{\cdot}20x = 190$ lbs. per square foot, and *qua* causing an increase in the crack we may consider the tension as being about 380 to 400 lbs. per square foot, which is not likely to break good masonry.

Next, when $x = 75$ feet, $t = 50$ feet approx., and for a 1 foot crack, $\lambda = 0{\cdot}02$, so that when $s = x$, the tension is $0{\cdot}026x = 120$ lbs. per square foot, which is also safe.

Thus, we may conclude that a 1 foot crack, although undesirable, is not necessarily fatal.

If, however, we assume that a 3 foot crack can exist, we get, when $x = s = 75$ feet, $\lambda = 0{\cdot}06$, the tension is $0{\cdot}116x = 547$ lbs. per square foot, which is probably unsafe, since it is in reality equivalent to 1000 to 1100 lbs. per square foot.

We may therefore conclude that the determining factor is the workmanship, in which must be included the methods employed to counteract the changes of temperature which occur.

(ii) It will also be evident without detailed calculation that if we design the dam so that $\frac{y}{t} = 0{\cdot}63$, or $n = 0{\cdot}036$, $\lambda = 0{\cdot}15$, or the crack must be at least $0{\cdot}15t$ long when $s = x$, before tension occurs at its end. Although only 5 to 6 per cent. thicker than the theoretical minimum, the dam is practically safe against cracks such as are likely to occur even with unusually bad workmanship.

The assumptions are, of course, somewhat unfavourable. At 75 feet depth the dam is about 50 feet thick, and a continuous horizontal crack extending 7·5 feet into, and with some extension along the dam, (I find roughly that there is an appreciable beam action unless the extension in this direction exceeds 30 feet) must be considered as indicating very careless workmanship; but is not impossible in a concrete dam built up in horizontal layers. Also, the assumption of uniform pressure over the whole depth of the crack, is a very big one, for if the crack is narrow, leakage and the surface tension of the water in contact with the masonry will prevent the full pressure being carried over the whole depth.

It is difficult to avoid the conclusion that a dam, as ordinarily designed, is liable to fail, unless of first class construction, through the gradual opening of a horizontal crack starting at the water face.

It therefore seems advisable not only for this reason, but also on the ground of the tensile stresses experimentally shown as likely to exist near the junction of the water face and the foundation of the dam, to consider the water face as the portion of the dam requiring the most careful inspection and workmanship. The question of preventing infiltration will be discussed later.

Failures of Masonry Dams.—Owing probably to the fact that masonry dams are rarely constructed without careful technical consideration, actual failures are but few.

Nearly every long masonry dam, which is straight in plan, has cracked, and it may be laid down as a general rule that for this reason, if for no other, the plan of a dam should be slightly curved.

Of actual failures, the most instructive are the Puentes in Spain, the Bouzey in France, and the Austin dam in Texas.

PUENTES DAM FAILURE.—The failure of the Puentes dam is more interesting from the point of view of the properties of permeable strata, than from that of actual dam design. The description and following quotation are taken from Aynard's *Irrigation du Midi de l'Espagne.*

Sketch No. 110 shows the dam to be of unscientific design, which is not surprising in view of the fact that it was constructed in 1785–1791. The design, nevertheless, is a safe one, since no tension occurs, and the maximum compression is 16,250 lbs. per square foot.

The dam is founded on piles 22 feet long, driven into a bed of sand, and gravel, at least 25 feet deep. The original design contemplated reaching solid rock, but this was found to be impossible.

The piling, and the slab of masonry 7·4 feet in thickness, were continued for 131 feet (40 metres) below the dam, thus securing an impermeable coating 283 feet in breadth.

For 11 years the water never rose more than 82 feet above the top of this apron, and the dam showed no signs of failure, the apron evidently being sufficiently broad to prevent deleterious percolation under this pressure.

In 1802, however, the water rose to 154 feet above the level 164·2 (*i.e.* to the full supply level), and, as stated by an eye-witness:

> "On the downstream side of the dam towards the apron, water of an exceedingly red colour was boiling up in great quantities."
>
> "Half an hour later, this boiling up had increased to an enormous mass of water,"

and a definite passage was formed.

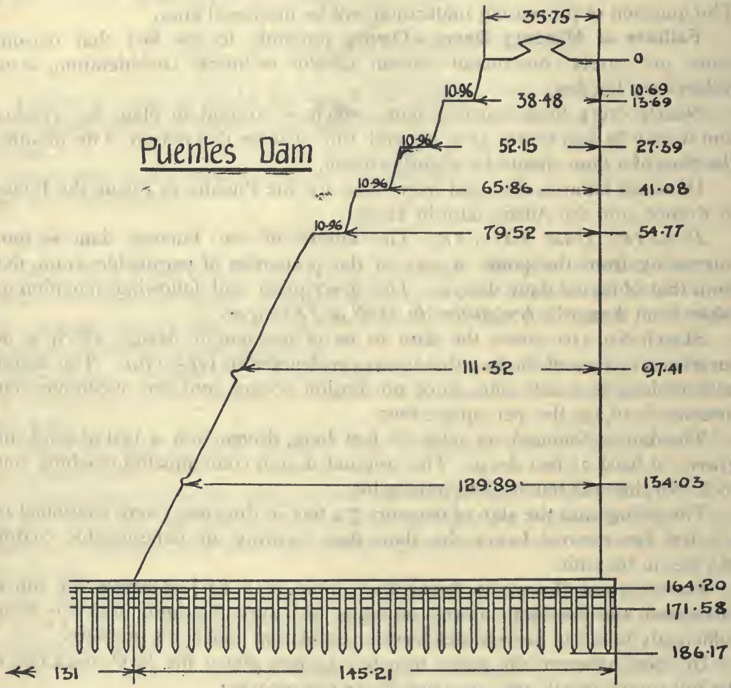
The dam still remains (1864) like a bridge, the opening being about 56 feet broad, by 108 feet high.

I think that it is impossible to describe a failure due to percolation more clearly. The only doubt is as to whether the apron, shown as 131 feet broad, was blown up, or whether there was some portion of the sandy pocket which was not covered by the apron.

In any case, it appears that the dam failed owing to the percolation being .

so excessive as to actually lift up the sand. The example is unusually interesting, because, so far as I am aware, most percolation failures can be traced to the foundations being too narrow. Whereas in this case, the foundations were evidently too shallow. Their effective depth, according to my rules, being about 23 feet, under the assumption that the impermeable core wall is 7 feet deep at 131 feet from the centre of the combined dam and apron (see p. 297).

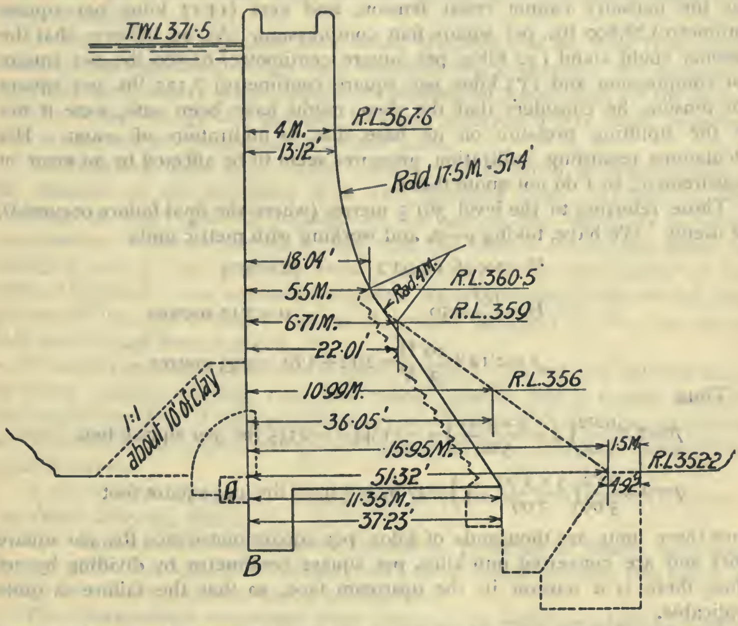
As no engineer is likely to be so reckless as to construct a stone dam on permeable foundations, any further comment is needless.



SKETCH NO. 110.—Puentes Dam.

BOUZEY DAM FAILURE.—The Bouzey failure presents several interesting features. The detailed section taken from Langlois' *Rupture du Barrage de Bouzey*, is as per Sketch No. 111. So far as can be gathered from the figure and reports, the law of the middle third was unknown to the designer, who appears to have been satisfied, provided that the line of the resultant pressure fell within the section of the dam, and that the maximum pressure did not exceed the working compressive stress of the masonry. Consequently, the masonry seems to have been considered as capable of resisting tension, and certain limiting values, both for the tensile and the compressive stresses, appear to have been laid down. From certain statements, these seem to have been 3075 lbs. per square foot (1·5 kilogram per square centimetre) in tension, and 20,500 lbs. per square foot (10 kilos. per square centimetre) in compression. At any rate, some limits were assigned, and were evidently determined by experiment previous to construction. The tensile stress assumed to be permissible appears very great, but there is no doubt that the dam did actually sustain tensions exceeding 2000 lbs. per square foot.

The dam was founded on a layer of micaceous sandstone, traversed by seams and fissures of clay. The guard wall AB was intended to be carried down through this fissured rock into a compact and impermeable stratum. So far as can be ascertained, this was not effected in many places, owing to lack of pumping power, and in certain spots where an impermeable stratum was



SKETCH NO. 111.—Bouzey Dam, original section and repairs.

reached this was only a layer, and permeable rock was known to exist at a still greater depth.

Under such circumstances, it is not surprising that in 1884 (when the water level reached the line 368·80) the dam cracked, moving bodily forward in places. Later, the reservoir was emptied, and it was found that the dam had separated from the guard wall, over a length of 453 feet, and had moved backwards as much as 14 inches at certain points.

The gravity of the situation does not appear to have been fully realised. The cracks were closed with grout, and the repairs, as sketched in dotted lines, were carried out. These can hardly be supposed to have checked percolation along the crack in the guard wall and the consequent uplifting pressure.

The reservoir was again filled, and all seems to have gone well, except for certain leaks, until 1895. Then, the water level rose to the originally designed full supply level 371·5, and the dam cracked horizontally at the levels 361·5 and 358, and failed through the upper portion being swept away over a length of 600 feet. This length was quite distinct from the 345 feet that had previously failed.

The stresses in the original dam appear to have been as high as 11,480 lbs. per square foot (5·6 kilos. per square centimetre) compression, on the downstream side, and to have reached a tension of 3,075 lbs. per square foot (1·5 kilos. per square centimetre) at the level 358 metres on the upstream side. These values are obtained under the assumption that the masonry can resist tension.

Langlois calculates the stresses at the level 349·40 under the assumption that the masonry cannot resist tension, and gets (14·13 kilos. per square centimetre) 28,800 lbs. per square foot compression. As he believes that the masonry could stand (30 kilos. per square centimetre) 61,500 lbs. per square foot compression and (3·5 kilos. per square centimetre) 7,175 lbs. per square foot tension, he considers that the dam might have been safe, were it not for the uplifting pressure on its base, due to infiltration of water. His calculations regarding infiltration pressures seem to be affected by an error in measurement, so I do not quote them.

Those referring to the level 361·5 metres (where the final failure occurred), are useful. We have, taking $\rho=2$, and working with metric units:

$$V=2\times 46{\cdot}1=92{\cdot}2 \qquad M=97{\cdot}4$$

$$H=\frac{10^2}{2}=50 \qquad a=2{\cdot}12 \text{ metres}$$

$$y=2{\cdot}12+\frac{10}{3}\frac{H}{V}=2{\cdot}12+1{\cdot}81=3{\cdot}93 \text{ metres}$$

Thus,

$$p=2\times\frac{92{\cdot}2}{5{\cdot}09}\left(2-\frac{3\times 3{\cdot}93}{5{\cdot}09}\right)=-11{\cdot}44=-2345 \text{ lbs. per square foot}$$

$$q=2\times\frac{92{\cdot}2}{5{\cdot}09}\left(\frac{3\times 3{\cdot}93}{5{\cdot}09}-1\right)=47{\cdot}67=+9772 \text{ lbs. per square foot}$$

since these units are thousands of kilos. per square metre (205 lbs. per square foot), and are converted into kilos. per square centimetre by dividing by 10. Thus, there is a tension in the upstream face, so that the failure is quite explicable.

Consider also the case of a horizontal crack 1 metre deep, with a water pressure of 10 metres acting on it, or a total upward force of 10 units. Then, remembering that in this case no support from the sides of the crack can be relied upon, we must put $t-z=5{\cdot}09-1$, and we get:

$$P=2\times\frac{92{\cdot}2}{4{\cdot}09}\left(2-\frac{3\times 2{\cdot}93}{4{\cdot}09}\right)-\frac{20}{4{\cdot}09}\left(2+\frac{3\times 0{\cdot}5}{4{\cdot}09}\right)=-18{\cdot}35$$

$$=-3762 \text{ lbs. per square foot.}$$

$$Q=2\times\frac{92{\cdot}2}{4{\cdot}09}\left(\frac{3\times 2{\cdot}93}{4{\cdot}09}-1\right)+\frac{20}{4{\cdot}09}\left(\frac{3\times 0{\cdot}5}{4{\cdot}09}+1\right)=+58{\cdot}53$$

$$=+12{,}000 \text{ lbs. per square foot.}$$

M. Langlois also discusses the effect of temperature stresses on the dam, which was not curved in plan, and endeavours to show that the repairs (especially the grouting), really tended to weaken the structure.

I agree with the conclusions of M. Langlois, and would particularly remark that his resume of the advantages of curving a dam in plan is most excellent; but it appears unnecessary to go beyond the above figures for an explanation of the failure.

As is invariably the case when a failure is investigated, the materials appear to have been of second-rate quality. The stone was fissured, and clayey, and the sand very fine, so that the masonry was neither as dense nor as strong as the designer assumed. I do not consider that such statements throw any real light on the laws of dam design. Practical engineers are well aware that not one work in ten is really perfect in construction, and that the other nine, if well designed, are saved from failure by the factors of safety adopted in their calculation.

It may, I think, be taken as a ruling principle that no work fails except when it combines bad design and imperfect workmanship; the amount of imperfection necessary to secure failure depending on the inferiority of the design.

The Bouzey design was radically bad, and therefore a very small imperfection in construction caused failure to take place.

Nevertheless, these facts create a feeling not so much of astonishment at the disaster, as of confidence in a well designed dam. We have a dam violating in every possible manner all the present-day principles of sound construction. The foundation is bad, and the dam is not carried down far enough; yet, it only partially fails. Later, when the dam,—due to this particular defect,—was exposed to upward water pressure, it did not fail until at least 16 lbs., and probably (owing to upward pressure) 30 lbs., per square inch tension had been induced.

It would therefore appear that a dam designed with but one half the factor of safety usually existing, only fails when, besides heavy tensile stresses, percolation occurs. It seems unnecessary to add that both the stone employed, and the mortar used, are stated to have possessed only about two-thirds of the usual strength.

Austin Dam Failure.—This failure is discussed by Gillette (*Engineering News*, May 30, 1901).

The dam was founded on weak limestone, stratified in nearly horizontal layers. This was known to be leaky, and water under pressure was found by boring into the foundations during construction.

The leaks which developed after construction were stanched by careful treatment with clay, and, in view of the large quantity of silt deposited in the reservoir, during its seven years of life, there is little doubt that the leakage and the permeable strata that probably existed beneath the foundation of the dam had little influence on the ultimate failure; although, no doubt, they were hardly conducive to sound sleep on the part of the responsible engineer.

The circumstances existing at the time of the failure are shown in Sketch No. 112, (except for the unknown depth of the silt deposit upstream of the dam).

The horizontal pressure of the water on the upstream face of the dam was equal to 181,500 lbs. per lineal foot of the dam. The back pressure of 37 feet depth of water below the dam is equivalent to 42,800 lbs. per lineal foot; so that the nett force producing sliding is 138,700 lbs. per lineal foot.

Assuming 145 lbs. per cube foot as the weight of the masonry, the weight of the dam was 320,000 lbs. per lineal foot.

Weight of water, 11 feet × 18 feet, on the crest of dam = 1300 lbs. per lineal foot.

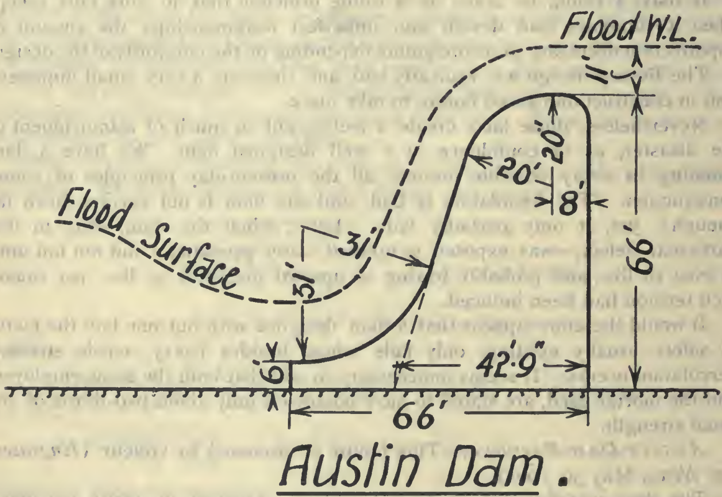
Weight of water, 11 feet × 40 feet, on slope of dam = 2700 lbs. per lineal foot.

Weight of water, 20 feet × 30 feet, on curved toe = 37,000 lbs. per lineal foot.

Total, 360,000 lbs. per lineal foot.

Thus even with this (in my opinion), large over-estimate of the water loads on the dam, we obtain the ratio against sliding as 0·38, and if we allow for a back pressure of only 26 feet depth of tail water, the ratio is 0·44.

Now, we know that there was a fault in the strata under the base of the dam, so that it can hardly be supposed that the limestone rock had any cohesive strength near this fault; and, even if not faulted, such a rock has little cohesive strength along its bedding planes.



SKETCH No. 112.—Austin Dam.

Morin gives the coefficient of friction for limestone on limestone as 0·38, and for stone on wet clay as 0·33, so that it must be acknowledged that there was very little, if any, margin against sliding, unless the limestone had some cohesion.

The reports on the failure of the dam, (which was seen and photographed in a very complete manner), seem to show that a length of about 500 feet did actually break away, and move bodily downstream.

Abnormal Loads on Dams.—Abnormal loads are principally produced by ice, and shocks from floating bodies.

The thrust exerted by the expansion of a sheet of ice 1 foot thick can theoretically amount to as much as 34,000 lbs. per linear foot of the dam. In America (*Trans. Am. Soc. of C.E.*, vol. 53, p. 89), a thrust of 29,000 lbs. appears to have been observed, but the thickness of the ice is not recorded.

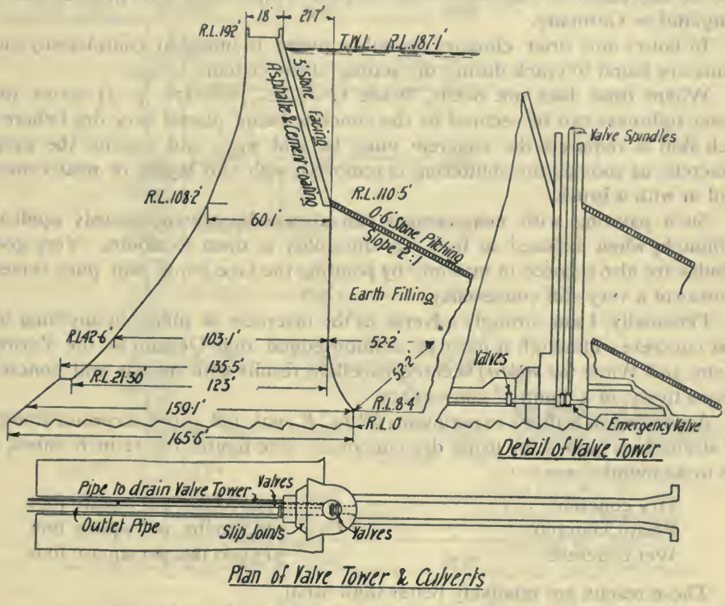
In the Quaker Bridge Dam, a value of 43,000 lbs. was assumed, and at the Columbus Dam, 22,670 lbs. per lineal foot, each including an allowance for shocks from floating blocks of ice.

No rule can be given for shocks from floating bodies, and local circumstances alone can indicate if any allowance should be made; and if so, its exact magnitude.

Practical Construction of Dams.—The discussion of Messrs. Pearson and Atcherley's theory, and the investigation of the effect of horizontal fissures, will have shown the points where weakness is most to be apprehended.

Consequently, the ruling factors are to prevent horizontal cracks at all costs; and to ensure water-tightness, in particular, at and near the junction of the water face of the dam and its foundation.

The first condition excludes any construction such as brickwork, where horizontal joints run through the dam. Modern practice appears to be trending



Sketch No. 113.—Remschied Dam (after Intze).

towards the adoption of mass concrete, with large blocks embedded at random in the concrete, rather than rubble masonry. In view of the fact that well made concrete apparently possesses greater shearing strength than the best masonry, this practice appears to be logical.

The question of hydraulic lime versus Portland cement as the binding material of the concrete or masonry, is a debatable one. Cement is the stronger of the two materials, but the slow setting of the lime allows initial strains to adjust themselves.

Water-tightness is best secured by a correct proportioning of the concrete, and by good workmanship. The designs of Intze (see Sketch No. 113) are costly, but afford a very perfect shield for the junction of the dam and its foundation. In the case of the Remschied Dam, the water face was plastered with cement, and this was covered with asphalt, which was again covered with a wall of brickwork from 1 foot to 2 feet thick. It may be parenthetically remarked that plastering the face with cement appears to be universal in German practice.

The old and leaky Mouche Dam was rendered water-tight by a covering of concrete, about 7 feet thick, in which drains were formed at intervals of about 3·5 feet (vide *A.P.C.*, 1907, vol. 5, p. 136).

Kreuter (*Beitrage zur Berechnung der Staumauren*) recommends that the whole water face of the dam be covered with jack arches; and that the spaces between these arches and the dam face be kept free from water by means of drains.

In British work, water-tightness is usually secured by a skin of slightly richer concrete on the water face. The method is open to objection, but has proved successful in cases where the range in temperature is no greater than in England or Germany.

In hotter and drier climates (possibly owing to unequal contraction) such skins are found to crack during the setting of the cement.

Where frost does not occur, Wade (*P.I.C.E.*, vol. 178, p. 7) states that water-tightness can be secured by the concrete being placed very dry (where a rich skin is required, the concrete must be laid wet), and coating the green concrete, as soon as the shuttering is removed, with two layers of neat cement laid in with a brush.

Such painting with neat cement can always be advantageously applied; although, when exposed to frost, its durability is open to doubt. Very good results are also secured in masonry by pointing the face joints with pure cement mortar of a very stiff consistency.

Personally, I am strongly adverse to the insertion of plums in anything but wet concrete; although it must be acknowledged that Deacon at the Vyrnwy Dam, and Wade (*ut supra*) secured excellent results with mortar and concrete respectively, of a "putty" consistency.

According to Rafter's experiments (*P.I.C.E.*, vol. 178, p. 90), a certain amount of strength is gained by using dry concrete. The figures for 12-inch cubes, at 18 to 24 months, are:

Dry concrete	355,700 lbs. per square foot.
Plastic concrete	330,200 lbs. per square foot.
Wet concrete	313,900 lbs. per square foot.

These results are relatively better than usual.

Wade's figures for dry concrete with a sandstone aggregate are:

1 : 1·66 : 3·33 concrete .	270,000 lbs. per square foot at 90 days.
1 : 2·5 : 4·4 concrete .	248,100 lbs. per square foot at 90 days.
Mortar	230,000 lbs. per square foot at 90 days.

While Bruce obtains with:

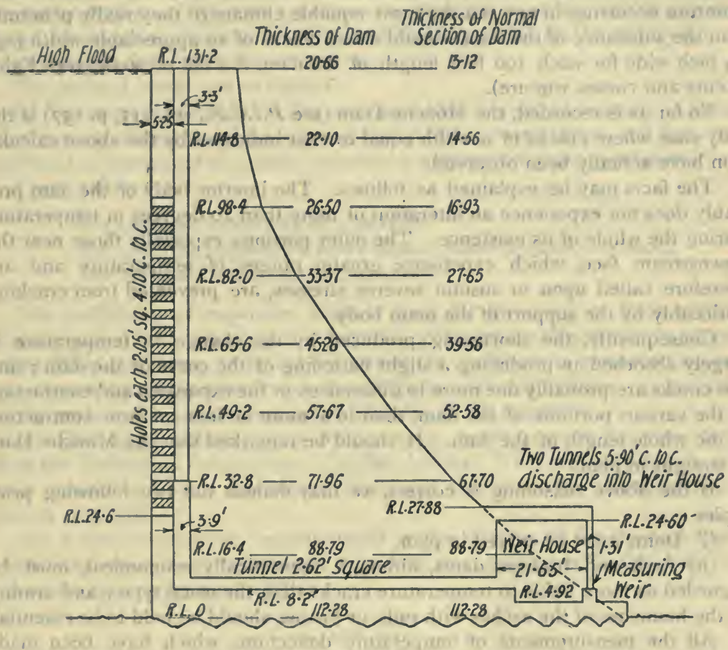
1 : 2½ : 4, and an unsatisfactory aggregate, figures ranging from 282,000 to 335,000 lbs. per square foot at 6 months.

The worst figure reported for concrete of modern Portland cement appears to be 215,000 lbs. per square foot, for a 1 : 1½ : 6½ mixture at 30 days, rising to 327,000 lbs. per square foot at 90 days.

Deacon (*P.I.C.E.*, vol. 126, p. 68) states that hydraulic lime mortar mixed as 3½ : 1 can be made (if precautions are taken) to stand 340,000 to 450,000 per square foot in compression, and 200 to 300 lbs. per square inch in tension.

Hill (*P.I.C.E.*, vol. 126, p. 109) finds that 1 to 3, or 1 to 4, lias lime concrete, 8 feet thick, is quite water-tight in puddle trenches under 150 foot head of water.

The above figures show that failure by crushing is unlikely, even with weak concrete. It has, however, been shown that when strength becomes a determining factor in design, the shearing strength, rather than the crushing strength, should be considered. It is probable that all the test pieces, the strengths of which are recorded above, really failed by shear. Nevertheless, in designing a dam it would appear desirable to test the samples of the proposed



SKETCH No. 114.—Typical Algerian Dam, showing Draw-off Arrangements.

materials in "shear" rather than in "compression" in the sense used by testers. Some results thus obtained are recorded on page 372.

Similarly, the design of the joints in the dam should be conditioned by the shear, rather than by the pressure. Thus, the common practice of laying masonry or plum concrete with the joints of the masonry or the longer dimensions of the plums normal to the flanks and faces of the dam, may be a mistake.

The correct disposition of the material is that in which the direction best fitted to resist shear is at 45 degrees to the faces or flanks. Thus, in a rock with beds along which shear easily occurs, these beds should lie either parallel or perpendicular to the dam faces in all stones close to the faces. The rule usually leads to the disposition above referred to; but, in some varieties of granite, blocks laid normal to the faces would be in the worst possible position to resist shear. Similarly, it will be plain that the relative shearing strengths of the mortar and of the individual stones should be considered before the method of arranging the materials is determined.

Temperature Stresses in Dams.—The question of the influence of temperature on the stability of a dam is puzzling. We have the following figures:

The coefficient of expansion of concrete is about 0·0000076 per 1 degree Fahr., while the value of Young's modulus ranges between 1,400,000 and 2,800,000 lbs. per square inch. Thus, a fall of 20 degrees Fahr. below the temperature at which the concrete was deposited, should theoretically produce tensile stresses ranging from 210 to 420 lbs. per square inch. Consequently, variations of temperature occurring in any but the most equable climate (if they really penetrate into the substance of the dam) should cause cracks of an appreciable width (*e.g.* $\frac{3}{16}$ inch wide for each 100 feet length of the dam, if a fall of 20 degrees Fahr. occurs and causes rupture).

So far as is recorded, the Mouche Dam (see *P.I.C.E.*, vol. 115, p. 157) is the only case where cracks of a width equal to that indicated by the above calculation have actually been observed.

The facts may be explained as follows. The interior body of the dam probably does not experience an alteration of more than 20 degrees in temperature during the whole of its existence. The outer portions, especially those near the downstream face, which experience greater ranges of temperature and are therefore called upon to sustain severer stresses, are prevented from cracking noticeably by the support of the main body.

Consequently, the shortening produced by the change in temperature is largely absorbed in producing a slight flattening of the curve of the dam ; and the cracks are probably due more to differences in the expansion and contraction of the various portions of the dam, than to a more or less uniform contraction of the whole length of the dam. It should be remarked that the Mouche Dam is straight in plan.

If the above reasoning is correct, we may deduce the two following principles :

(i) Dams must be curved in plan.

(ii) Arch and buttress dams, although theoretically economical, must be regarded as more liable to temperature cracks than the usual type ; and arming at the haunches of the arches with rods, or beams, should be held to be essential.

All the measurements of temperature deflections which have been made appear to confirm this statement, although it is incorrect to consider the deflections produced by bending loads (such as the water pressure) as absolutely comparable with those resulting from extensions or compressions arising from changes in the temperature.

As examples, De Burgh (*P.I.C.E.*, vol. 178, p. 64) reports that in the very thin Barren Jack Dam, a change in the temperature from 57 degrees to 100 degrees Fahr. produced an inward deflection of 0·14 inch (the reservoir being empty). The water load is stated to have produced an outward deflection of 0·47 inch.

So also, in the case of the far thicker Vyrnwy Dam (*P.I.C.E.*, vol. 115, p. 117), the deflection due to temperature does not exceed 0·366 mm. ; and that caused by the load produced by the upper 13 feet of water retained, is not more than 0·868 mm.

In the Remschied Dam (Sketch No. 113) Intze reports (*Ztschr. D.I.V.*, June 1, 1895) temperature deflections of $\frac{1}{8}$ inch, and load deflections equal to $1\frac{1}{16}$ inch.

The data collected by Gower (*Trans. Am. Soc. of C.E.*, vol. 61, p. 399) and others, on the effect of temperature changes in American dams, are very complete. Nevertheless, they cannot be considered as universally applicable, since the climate of the United States is such that the difference between the temperature during the period of the deposition of the concrete, or masonry (*i.e.* May to November), and the lowest temperature ever experienced, is far greater than that which occurs in other localities, even when the same annual range of temperature is experienced. In most of these localities the working season is usually the colder portion of the year; whereas, in America it is the hotter portion. Thus, American circumstances are unusually favourable to the production of contraction cracks in masonry or concrete.

The general results are as follows:

(i) The actual expansion on the surface of the concrete or masonry is about 0·6 of that calculated from the range of temperature experienced, and from the coefficient of expansion of similar concrete or masonry in small specimens.

These values are:

For concrete, 0·0000054 to 0·0000081.
For concrete exposed to alterations in humidity also, 0·0000044.
For masonry, 0·0000050 to 0·0000060.

The coefficients of expansion actually observed by Gower in thick granite masonry ranged from 0·00000352 to 0·00000264, with a mean value of 0·00000307 per degree Fahr., and Dana finds that for granite the coefficient of expansion ranges from 0·00000440 to 0·00000480 per degree Fahr. The differences are possibly largely explained by alterations in the humidity of the masonry.

(ii) The actual visible cracks which occur, rarely account for more than 0·6 of the theoretical contraction. (In the Assouan Dam the ratio is only 0·2, and it is highly probable that tensile stresses of 300, to 400 lbs. per square inch exist on cold days.)

(iii) Actual temperature measurements in the interior of the Boonton Dam indicate that the annual variation of temperature at a distance D, feet from the face of the dam, is represented by:

$$x = \frac{R}{3\sqrt[3]{D}}$$

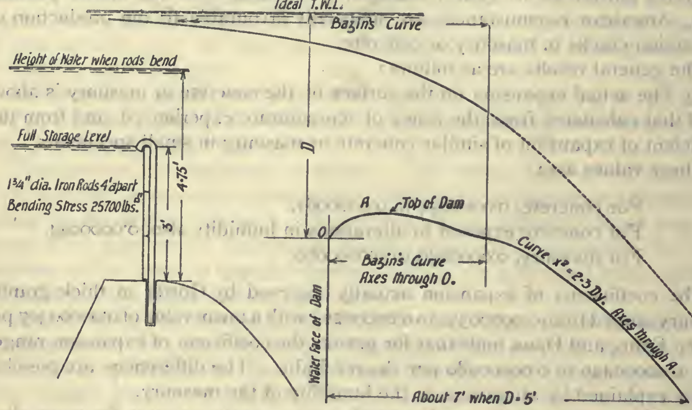
where R, is the annual variation in the surface temperature of the dam.

It would also appear that unless x, exceeds 60 degrees Fahr., cracks either do not occur, or are not sufficiently wide to permit water to pass through them.

Cardew (*P.I.C.E.*, vol. 152, p. 239) attempted to provide for temperature stresses in the Burraga Dam, by burying iron rails in the top. The principle appears to be correct, as the rusting of the iron cannot influence the stability of the dam as a whole. If cracks are prevented in the thin top, where temperature changes are most marked, there is little likelihood of their starting in the interior of the dam.

Form of the Downstream Face of Overflow Dams.—The upper portion (if not the whole), of the overflow face of a dam is usually a parabola. Müller (*Engineering Record*, October 24, 1908), has suggested that since the ordinary parabolic form does not follow the curve which the under surface of the nappe would assume if left to itself, a certain vacuum will exist between the nappe and the dam. If the dam is high, and the head of water passing over it is large, this is undesirable. Certain dams seem to have been exposed to some such action. Müller consequently designed the top of the dam so as to lie inside the nappe boundaries, as obtained by Bazin (*Écoulement en déversoir*).

The errors in detail are plain. Bazin's curves refer to sharp-edged notches, under heads not exceeding 1·7 foot; and Müller applies them to thick notches, under heads of 5, or 10 feet. The principle is a good one, and the process leads to a nice curve.



SKETCH NO. 115.—Müller's Diagram for Face of a Overflow Dam, and Flexible Flashboards.

The nappe boundaries can be plotted from the following tables:

Lower Boundary—

$\frac{x}{D}$ =	0·0	0·05	0·10	0·15	0·20	0·25	0·30	0·35
$\frac{y}{D}$ =	0·0	0·059	0·085	0·101	0·109	0·112	0·111	0·106

$\frac{x}{D}$ =	0·40	0·45	0·50	0·55	0·60	0·65	0·70
$\frac{y}{D}$ =	0·097	0·085	0·071	0·054	0·035	0·013	0·009

Upper Boundary—

$\frac{x}{D}$ =	−3·0	−1·0	0	0·1	0·2	0·3
$\frac{y}{D}$ =	0·997	0·963	0·851	0·826	0·795	0·762

$\frac{x}{D}$ =	0·4	0·5	0·6	0·7
$\frac{y}{D}$ =	0·724	0·680	0·627	0·569

where D_1 is the depth of water over the theoretical weir crest, as observed by Bazin's methods; and x_1 and y_1 represent the co-ordinates of the nappe curves referred to the water face of the dam as vertical axis, and a line through the top corner of the water face as horizontal axis.

Thus, it will be plain that the process amounts to assuming that a Bazin notch exists at the water face of the dam, and making the downstream face conform to the theoretical nappe curve. Thus, the highest point in the dam is situated $0.25D_1$ downstream of the water face, and is $0.112D_1$ above the level at which the sill of the theoretical notch is assumed to lie.

The depth of moving water above this point is only $0.65D$, but it is assumed that the design of the dam secures a discharge equal to that calculated for a head D, from Bazin's weir formulæ (see p. 109).

The tables suffice to determine the dam face for a distance $0.7D_1$ from the water face of the dam.

The mean velocity at this point and its direction can be calculated from the cross-section of the nappe, and the direction of the tangent to the dam face.

It may be assumed that the curve of the dam face should be continued so as to conform to the path of a particle falling under gravity, with an initial velocity equal in magnitude and direction to the calculated mean velocity.

Müller thus finds that the equation:

$$x^2 = 2.3Dy$$

fairly represents the lower portion of the dam face, when referred to horizontal and vertical axes through A, the highest point of the dam.

Müller continues this curve until the tail of the dam is reached (see Sketch No. 115), but an extension much beyond 2D, or 3D, below the crest seems unnecessary.

The real objection to the process seems to lie in the fact that D_1 is assumed to be constant, and is actually variable. For instance, let us assume that $D = 5$ feet. No discharge occurs until the water surface in the reservoir is at least 0·56 foot above the level from which D_1 is measured. A certain small discharge then occurs, but it can hardly be assumed that Bazin's formula is applicable until the water has risen some 1·5 to 2 feet higher. Thus, the dam is really only discharging water properly when the water surface is 2 feet or more above the level from which D_1 is measured, and in the earlier stages it may be doubted (see Horton's *Weir Experiments*, pages 106 *et seq.*) whether it is safe to assume any greater discharge than that given by the equation:

$$Q = 3.30L(D - 0.56)^{1.5}$$

until D_1 is 2·5 or 3 feet.

Thus, the conditions assumed but rarely occur in practice, unless the water level is systematically kept well above the crest of the dam by flash boards, or shutters.

Flashboards.—The use of flashboards, or temporary retaining walls, in order to block the waste weirs of dams is more common in India or America than in Europe. From this it cannot be inferred that the practice is a bad one. Absence of flashboards in European countries is mainly attributable to the fact that the climates are usually too changeable and erratic to enable a flood to be predicted sufficiently in advance to ensure absolute certainty in the removal of the flashboards.

The difficulty of removing the flashboards is to a large extent overcome by adopting the following designs.

(*a*) The flashboards are supported by iron pins, sunk into holes on the weir crest. The pins are calculated so as to bend when the flashboards are materially overtopped.

The following analysis is due to Müller (*Engineering Record*, August 22, 1908):

Let the pins be d, inches in diameter, and spaced s, feet apart.

Let h, be the height of a flashboard, and x, the height of the water over the weir crest when the pins are required to bend, both in feet.

Put $x_1 = x - h$.

The bending moment on a pin is:

$$125sh^2(3x_1+h) \quad . \quad . \quad . \quad . \quad . \quad \text{[inch-lbs.]}$$

The moment of resistance of the pin is $0{\cdot}098fd^3$, where f, is the stress which produces bending. This Müller gives as 70,000 lbs. per square inch.

Thus:

$$125h^2s(3x_1+h) = 6860d^3$$

$$3x_1+h = 3x-2h = 54{\cdot}9\frac{d^3}{h^2s}$$

or,

$$x = \frac{18{\cdot}3d^3}{h^2s} + \frac{2h}{3} \quad . \quad . \quad . \quad . \quad [d, \text{ in inches}]$$

Also the pins should not be unduly strained when the water level reaches the top of the flashboards. Thus, taking a (high) working stress of 25,000 lbs. per square inch, we find that:

$$d = \frac{\sqrt[3]{sh}}{2{\cdot}7} \quad . \quad . \quad . \quad . \quad . \quad [d, \text{ in inches}]$$

(*b*) Sketch No. 229 shows the flashboards used at Tajewala to control a branch of the river Jumna. Here, it will be plain that the boards will fall as soon as the water rises to 3 inches below their top. In practice it is found that small gravel and horse dung (used for dusting the boards in order to render them water-tight), accumulates at the bottom of the boards, and, in consequence, the falling is somewhat irregular. No board ever stood with much more than 6 inches of water above its top. The proportion of boards that fell according to design was quite sufficient to remove all apprehension as to the failure of the system, and in actual tests a man was able rapidly and easily to throw down the boards when nearly overtopped.

The system is reliable, provided that a variation of 3 inches above the designed level of falling can be permitted. It is probable that the iron pin flashboards will fluctuate to an equal extent. This is not a defect, as a flashboard system that was considered automatic in its action would never receive sufficient inspection, and consequently, sooner or later, would fail and cause a bad disaster. A less reliable system which would necessarily be carefully watched would probably be assisted in its fall at the critical moment.

The Tajewala system appears to be better adapted than Müller's type for re-erection before the water has ceased flowing over the weir.

For more elaborate methods the sections on Shutters and Gates should be consulted.

SYMBOLS CONNECTED WITH CURVED DAMS.

a, is the vertical height, in feet, of the horizontal arches into which the dam is divided for purposes of calculation.

A_n, is the area of the cross-section of any one of these arches. $A_n = at_n$ approx.

D_n (see p. 405).

d, is always used for the sign of differentiation.

E, is the modulus of elasticity of the masonry, in tons per square foot, probably E = 100,000 to 120,000, but a knowledge of the precise value is unnecessary.

I, is the moment of inertia of the section of the dam, about a horizontal axis through its mass centre in (feet)^4.

L (see p. 406).

M, is a bending moment (see p. 406) expressed in feet-tons.

m, and n, are used as numbers to indicate the various sections typified by A.

P, is the pressure in tons per square foot; but P_m, or P, with a suffix is the total water pressure, in tons, on the water face of the area A_m.

q, and r (see pp. 406 and 407).

r_d, and r_u, are the radii of the dam, in feet, measured to the downstream and upstream faces.

R, is the radius measured to the mass centre of the section of the dam, but Wade (see p. 404) puts $R = r_u$.

S, is the working compressive stress of the masonry in tons per square foot.

S_1, and ΔS (see pp. 404 and 405).

t, is the horizontal thickness of the dam, in feet.

X_m, is the portion of P_m, which is carried by the dam as a gravity dam.

α (see p. 406).

β, γ, Δ (see p. 407).

μ, is the reciprocal of Poisson's ratio (see p. 405).

ρ, is the density of the masonry $= \dfrac{\text{Weight of a cube foot in lbs.}}{62{\cdot}5}$.

SUMMARY OF FORMULÆ.

Thickness of the dam:

$$t = \frac{PR}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Tons]}$$

Correction for thickness of the dam:

$$S_1 = S\frac{2r_u}{r_u + r_d} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Tons]}$$

Correction for slope of the faces in Wade's type (see p. 405):

$$\Delta S = -\frac{\rho}{3\mu}\left(1 + \frac{r_u}{r_u + r_d}\right)P \quad . \quad . \quad . \quad . \quad \text{[Tons]}$$

The deflection formulæ are not summarised.

THEORY OF CURVED DAMS.—**[In this section the loads are expressed in Tons.]**—I propose to briefly investigate the theory of dams which are so markedly curved in plan, and are so well supported by the sides of the valley which they cross, that they may be considered as acting partly as arches.

Such dams are usually calculated as arches only, and the following very simple formula is used:

$$t = \frac{PR}{S} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{[Tons]}$$

Where t, is the thickness at any level, in feet.

R, is the radius in feet.

P, is the water pressure at that level, in tons per square foot.

S, is the permissible compressive stress in tons per square foot.

$$\text{Plainly } P = \frac{\text{Depth in feet below top water level}}{36}$$

The section of a dam designed by this formula is plainly triangular.

The best practical discussion of this type of dam is given by Wade (*P.I.C.E.*, vol. 178, p. 1), and is founded on his experience of thirteen such dams. Wade adopts a section with a vertical upstream face, and battered downstream face (Sketch No. 116, Fig. 1); as, after trial, this has been found to be more satisfactory than sections with both faces battered, or with a vertical downstream face. The top width is always made 3 feet 6 inches, in place of zero, as indicated by the formula. Water 2 feet deep has passed over dams of this width without causing damage.

The values of S, depend on the rock used in the dam, and on the character of the foundations. For quartzite and sandstone, S, ranges from 10 to 12, or even 15 tons per square foot. For conglomerate, altered slate, or granite, S, is equal to 20 tons per square foot; and for special granite and diorite S, is equal to 24 or 25 tons per square foot, although this last value is not at present employed by Wade.

The dams are made of Portland cement concrete, in the proportion of 1 : 2½ : 5, with large plums of rock inserted; usually from 25 to 30 per. cent. of the volume of the dam being plums.

The concrete being laid very dry, these plums are laid in mortar and "wriggled" into the mass by handspikes, in place of the usual ramming with mauls.

The value of S, is determined by allowing a factor of safety of 5, on the crushing tests of unsupported 6-inch cubes of the concrete; and this probably gives 7½ on the crushing stress of a large mass of similar concrete.

R, is measured to the vertical face of the wall, which increases the factor of safety.

When a dam is designed by these rules, it is found that when S = 20 tons per square foot, and R, is 500 feet, the section obtained is practically as large as that of a similar dam designed so as to resist the water pressure by gravity only.

Thus bearing in mind the extra length entailed by a curved plan, no advantage is gained unless the radius is somewhat less than 500 feet.

The maximum radius of any curved dam yet constructed by Wade is 300 feet.

The most economical curve for a given span of the dam is one with a central angle of about 100 degrees.

The formula is theoretically correct only for a dam battered on both faces, and may be corrected as follows, where:

r_u, is the radius to the upstream face.

r_d, is the radius to the downstream face.

(*a*) Correction for thickness of the dam:

$S_1 = S\frac{2r_u}{r_u+r_d}$ where S, is the stress obtained by the original formula.

(*b*) Correction for vertical pressures due to the weight of the dam. The above value is increased by ΔS, where:

For a vertical downstream face, and a battered upstream face (Sketch No. 116, Fig. 2):

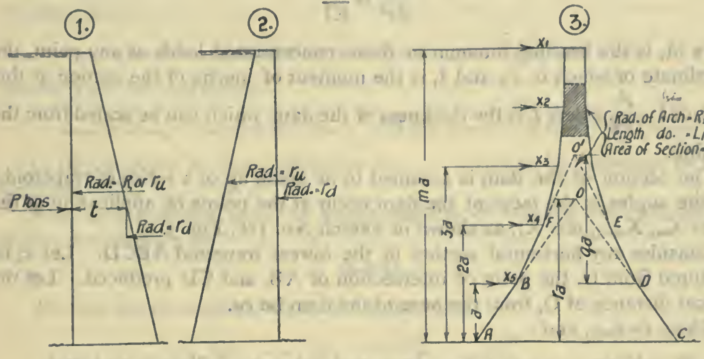
$$\Delta S = \frac{\rho - 1}{3\mu}\left(1 + \frac{r_u}{r_u + r_d}\right)P$$

For Wade's type (vertical upstream, battered downstream), Fig. 1:

$$\Delta S = -\frac{\rho}{3\mu}\left(1 + \frac{r_u}{r_u + r_d}\right)P$$

where $\frac{1}{\mu}$, is Poisson's ratio, and is 0·16 to 0·22 for concrete, *i.e.* in the mean say $\mu = 5$, see Bellet (*Barrages en Maçonnerie*).

These corrections are incomplete, but may be used in order to obtain a preliminary section for treatment by the more complete method now developed. This will be applied in an approximate manner so as to illustrate the method of testing a preliminary design.



SKETCH No. 116.—Curved Dams.

The principles are complete; and, after a dam has been proportioned by the present theory, it is quite possible to select more accurate formulæ for determining the arch deflections, and more closely spaced horizontal sections for determining the cantilever deflections.

The theory is simple. Assume that the dam at its highest point is composed of m (say 5, or 10) horizontal arches, each a, feet high, so that the total height of the dam is ma. Let the total water pressure over one of these, the centre of which is $(m-n+1)a$, above the base of the dam, be P_n, per foot length of the arch. Calculate D_n the deflection in this arch, which is of known radius, and length (measured from the plan of the dam) under a uniform radial load $P_n - X_n$. This gives us an equation as follows:

$$D_n = \frac{(P_n - X_n)R_n L_n \cot a_n}{2EA_n}$$

Where R_n, is the radius of curvature of the dam, at the level considered, measured to the upstream side.

L_n, is the length of the centre line of the arch ring, and varies for each arch according to the cross-section of the gorge across which the dam is built.

a_n, is one-quarter of the angle the arch subtends at the centre from which R_n, is measured :

$$i.e.\ a_n = \frac{L_n}{4R_n}$$

A_n, is the area of the arch ring.

A_n, is best calculated for the preliminary work either from the corrected formulæ already given, or by a consideration of the shear produced at the abutments of the arch by the resultant of the radial forces P_n, acting over its whole length L_n.

Now, consider a vertical section of the dam, at its highest point, under the action of a series of m, concentrated leads, X_1, X_2, ... X_m, acting at distances ma, $(m-1)a$, . . . a, from the base of the dam.

Taking the beam thus obtained as 1 foot wide, we have as an equation for the deflection under these loads :

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

where M, is the bending moment of these concentrated loads at any point, the co-ordinate of which is x; and I, is the moment of inertia of the section at this point, or $I = \frac{t^3}{12}$, where t, is the thickness of the dam, which can be scaled from the drawing.

The section of the dam is assumed to be made up of a series of trapezoids, and the angles in the faces of the dam occur at the points of application of the forces X_m, X_{m-1}, etc. X_1, as shown in Sketch No. 116, Fig. 3.

Consider any horizontal section in the lowest trapezoid ABCD. Let x, be measured from O, the point of intersection of AB, and CD produced. Let the vertical distance of O, from the base of the dam be ra.

Then, $t = b_m x$, and :

$$M = X_m(x - ra + a) + X_{m-1}(x - ra + 2a) + \text{etc.} + X_1(x - ra + ma)$$

Therefore $$\frac{Eb^3_m}{12}\frac{d^2y}{dx^2} = \Sigma X_{m-n+1}\left(\frac{1}{x^2} - \frac{r-n}{x^3}a\right)$$

Integrating, $$\frac{Eb^3_m}{12}\frac{dy}{dx} = \Sigma X_{m-n+1}\left(-\frac{1}{x} + \frac{r-n}{2x^2}a\right) + C$$

Now, $\frac{dy}{dx} = 0$, when $x = ra$.

Therefore, $$C = \Sigma X_{m-n+1}\frac{r+n}{2r^2a}.$$

Integrating again, $$\frac{Eb^3_m}{12}y = \Sigma X_{m-n+1}\left(-\log_e x - \frac{r-n}{2x}a\right) + Cx + C_1$$

and $y = 0$, when $x = ra$.

Therefore, $$C_1 = -raC + \Sigma X_{m-n+1}\left(\log_e ra + \frac{r-n}{2r}\right)$$

Thus, by putting $x = \overline{r-1}a$, we can calculate Δ_m, the beam deflection at the point of application of X_m, and tan β_m, the tangent of the angle of inclination of the central line of the beam at this point.

Plainly Δ_m = value of y, when $x=(r-1)a$,

and tan β_m = value of $\frac{dy}{dx}$, when $x=(r-1)a$.

Next, consider any horizontal section in the second trapezoid BDEF. Take O′, the point of intersection of the lines BF, and DE, as the new origin for x.

We have: $t=b_{m-1}x$, where b_{m-1} is the new value of b_m, and $M=X_{m-1}(x-\overline{q-1}a)+\text{etc.}+X_1(x-\overline{q-m+1}a)$ where qa, is the vertical height of O′, above BD, and X_m, no longer contributes to the moment.

These equations can be treated in precisely the same manner, the conditions now being that $y=0$, and $\frac{dy}{dx}=0$, when $x=qa$, and we obtain, by calculating the values of y, and $\frac{dy}{dx}$ when $x=(q-1)a$, δ_{m-1} and tan γ_{m-1}, the deflection and the angle of inclination of the centre line of the dam at the point of application of X_{m-1} relative to the section BD.

Thus, the absolute deflection and angle of inclination measured from the foundation are:

$$\Delta_{m-1}=\Delta_m+\delta_{m-1}+a\tan\beta_m \quad \text{and} \quad \tan\beta_{m-1}=\tan\beta_m+\tan\gamma_{m-1}.$$

Similarly we can calculate:

$$\Delta_{m-2}=\Delta_{m-1}+\delta_{m-2}+a\tan\beta_{m-1} \quad \text{and} \quad \tan\beta_{m-2}=\tan\beta_{m-1}+\tan\gamma_{m-2}.$$

In this manner all the deflections $\Delta_m \ldots \Delta_1$ can be expressed as linear functions of $X_m \ldots X_1$.

Now, in accordance with the usual methods of treating statically indeterminate structures, put:

$$\Delta_n=D_n$$

We thus obtain m linear equations connecting:

$$X_m \ldots X_1 \quad \text{and} \quad P_m \ldots P_1$$

Solving these equations, we obtain $X_m \ldots X_1$, in terms of the water pressures. And thus the dam sustains the X loads as a gravity dam, and the (P − X) loads as an arch.

The method is laborious, and could be improved at the cost of some extra liability to error by assuming the points of application of X_m, X_{m-1}, etc. as being $\frac{a}{2}$, $\frac{3a}{2}$, etc. above the base of the dam, and the changes in slope of the faces as occurring at a, $2a$, etc

I may refer to a paper by Messrs. Harrison and Woodward on the Lake Cheesman Dam (*Trans. Am. Soc. of C.E.*, vol. 53, p. 89), for two very able discussions of similar methods by the authors, and Mr. Shirreff. I have combined the two methods, as I felt that in order to obtain any accuracy in such calculations it was imperative to use formulæ which are comparatively simple, and are logically deduced. If theoretical advantages alone are considered, a really skilled computer might with advantage follow Mr. Shirreff's method entirely.

Vischer and Wagoner (*Trans. of Technical Society of Pacific Coast*, 1888) have endeavoured to investigate the question in a general fashion. They find that for a triangular dam (top width=o) spanning a gorge with vertical sides (*i.e.* L= constant):

$$\frac{P-X}{P}=\frac{2x^2}{R^2+2x^2}$$

where x, is the height above ground level.

According to Duryea (*ut supra*, p. 180), in a dam 126 feet high, with R, averaging 180 feet, and L, varying from 300 feet at the top, to 25 feet at the bottom, $\frac{X}{P}$ varies from 0·84 at the top, to 0·95 midway down, and is 0·99 at the bottom.

In the Lake Cheesman Dam, which is very close to a gravity section:

$$\frac{X}{P}=0{\cdot}54 \quad 0{\cdot}90 \quad 0{\cdot}94 \quad 0{\cdot}98 \quad 1{\cdot}00$$

at points $\frac{9}{10}$ $\frac{7}{10}$ $\frac{5}{10}$ $\frac{3}{10}$ $\frac{1}{10}$ the height of the dam,

and the radius is 400 feet, with L, varying from 580 feet downwards.

The object of the above theory is to provide a method that will permit some result not too wide of the truth being obtained for preliminary designs, within a reasonable time. This having been done, the design can later be refined on as much as is deemed wise, by assuming more numerous sections. In this connection I would remark that the real assumptions are that the dam is sliced horizontally when the arches are considered, and vertically when the cantilever is considered. This really means that shear is partially neglected in each case. Thus, we may assume that the ratio $\frac{X}{P-X}$ is liable (however fully this theory is developed), to an error of at least 5 per cent. of its own value (more probably 10 per cent., that being equal to about $\left(\frac{1}{2}\frac{\text{modulus shear}}{\text{modulus of elasticity}}\right)$. Consequently, refinements of greater apparent accuracy in calculations based on this theory need experimental justification.

I believe that the theory is fairly accurate, as any treatment following the ordinary principles of elasticity seems to produce approximately the same values of the ratio $\frac{X}{P}$.

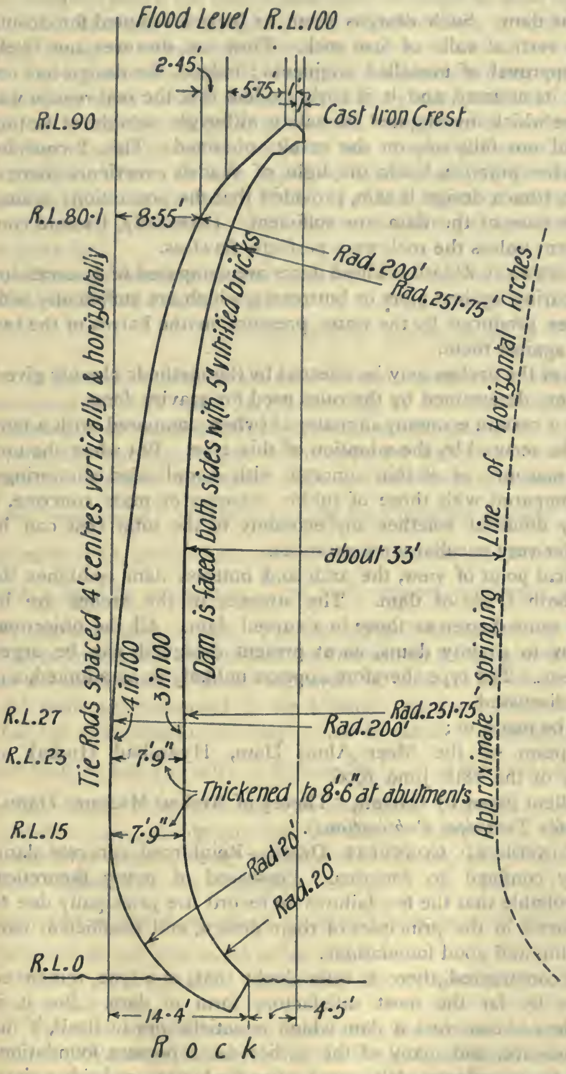
As an actual example, it may be stated that in the Bear Valley Dam the maximum stresses are:

(i) As a pure arch, 40 tons per square foot.
(ii) By the above method, with 5 loads, 33·5 tons per square foot.
(iii) Ditto, 10 loads, 33·1 tons per square foot.
(iv) By correction by the approximate formula already given, 32·7 tons per square foot.

We are consequently justified in assuming that the actual pressures do not materially exceed 33 tons per square foot, and that the shear is therefore about 36,000 lbs. per square foot; which, although high, need not necessarily produce failure.

It is perhaps advisable to state that the value of the ratio $\frac{X}{P}$, is to a certain extent under the control of the designer.

Such dams as Wade's, which are primarily designed as arches, will be found to have small values of $\frac{X}{P}$. Similarly, if a curved dam be designed as a gravity dam, but with a somewhat higher stress than usual; and the section thus obtained is then investigated as above, the values of $\frac{X}{P}$, will be found to be



Radii of Centre Lines of Dam

R.L.	Radius
90	50
87	52
80·1	54·2
71	55·45
63	56·50
55	57·30
47	57·83
39	58·00
31	58·00
23	57·57

SKETCH NO. 117.—Proposed Domed Dam at Ithaca.

large. The relative increase in $\frac{X}{P}$, as we proceed towards the foundation will be found to occur in all cases. The dome shaped arch section adopted by Williams at Ithaca, N.Y. (*Trans. Am. Soc. of C.E.*, vol. 53, p. 182) is singularly bold, and has apparently been designed so as to keep $\frac{X}{P}$, fairly constant all over the dam. Such designs seem to be well adapted for closing gorges with nearly vertical walls of firm rock. They are, however, not likely to meet with the approval of unskilled engineers; indeed, the design has not been carried out in its entirety, and it is fairly evident that the real reason was that the Committee which investigated its safety, although capable of testing the calculations, did not fully rely on the results obtained. This, I consider creditable conservative practice, but in the light of Wade's experience there is little doubt that the Ithaca design is safe, provided that the precautions against under-mining at the base of the dam are sufficient. Personally, I would consider these insufficient unless the rock were perfectly flawless.

Arch and Buttress Dams.—These dams are composed of a succession of arched dams, bearing against piers or buttresses, which are sufficiently wide to carry the stresses produced by the water pressure on the halves of the two arches which abut against them.

The calculation of the arches may be effected by the methods already given, and the buttresses are determined by the rules used for gravity dams.

It is stated that a certain economy in material (when compared with a pure gravity dam) may be secured by the adoption of this type. But when the unit costs of cut stone masonry, or of thin concrete with complicated shutterings, are respectively compared with those of rubble masonry or mass concrete, it becomes extremely doubtful whether any economy in the total cost can be secured except under very peculiar circumstances.

From a theoretical point of view, the arch and buttress dam combines the disadvantages of both types of dam. The stresses in the arches are indeterminate to the same degree as those in a curved dam. All the objections raised by Atcherley to gravity dams, as at present designed, can be urged against the buttresses. The type therefore appears unlikely to be adopted, and will not be further discussed.

Reference may be made to:

(*a*) The description of the Meer Alum Dam, Hydrabad (India), in *Engineering News* of the 18th June 1906.

(*b*) A very excellent paper by Garrett, "Theory of Arched Masonry Dams" (*Government of India Technical Publications*).

Design of Reinforced Concrete Dams.—Reinforced concrete dams are as yet mainly confined to America. Possessed of many theoretical advantages, it is probable that the few failures on record are principally due to over confidence placed in the principles of their design, and insufficient care given to workmanship and good foundations.

When properly constructed, there is little doubt that, as a type, reinforced concrete dams are by far the most satisfactory form of dam. But it is evidently quite useless to construct a dam which is satisfactory in itself, if the foundations are insecure, and many of the earlier dams possess foundations evidently designed in accordance with sound rules for houses, or bridge piers, but which are quite useless when applied to dams.

It therefore appears that success will be attained by utilising American experience for that portion of the dam above ground level, and (like the newer American designs) following the methods adopted in dams of other types as regards design of foundations.

Above ground, the dam consists of a series of buttresses of triangular section, carrying a flat slab of reinforced concrete on their upper face.

The dam may be designed as an overflow dam, or may be provided with a separate waste weir, as circumstances require.

I propose to consider the design of an overflow dam, and the modifications necessary when there is a separate spillway will be evident.

Let the horizontal distance between the centres of the buttresses be l feet.

Let the length of the slope of the upstream face of the dam be s times its height. Then, at any depth h, below the high flood level of the water passing over the spillway (which, as a first approximation, we can assume as 5 feet above the crest of the dam), the pressure per square foot of the slab face is $62{\cdot}5h$ lbs.

Thus, the bending moment per foot width of slab at the centre of the slab, assumed as non-continuous, is :

$$\frac{62{\cdot}5hl^2}{8} \text{ foot-lbs., or say equal to } \frac{K}{8}.$$

If the slab is continuous over several buttresses, theoretically the bending moments in each span vary according to the total number of spans. The variation is of importance only when there are less than seven spans. Owing to the fact that long lengths of concrete are liable to crack by expansion, it is doubtful whether continuous slabs are advisable. If, however, the slabs are built continuous it is safe to provide for a bending moment of $\frac{K}{12}$, at the centre of each span, producing tensile stresses on the downstream side of the slab, and a bending moment of $\frac{K}{20}$, at each support, producing compressive stresses on the downstream side. For the two end slabs, close to the point where the dam is joined with the hillside, the theoretical bending moments largely depend on the exact manner (freely supported or built-in) in which the end buttresses and slabs are connected to the hillside. In good construction it is probable that the connection is so complete as to justify the assumption that the slabs are built in, but it is safer to provide for $\frac{K}{9}$, over the two end buttresses and $\frac{K}{8}$, at the centre of each of the two end slabs. So also, theory shows that the pressures on each buttress are not exactly those given by the rules for non-continuous beams, being roughly $1{\cdot}01 \times 62{\cdot}5hl$, and $0{\cdot}99 \times 62{\cdot}5hl$, alternately. Such differences are negligible, except in the case of the first buttress at each end of the dam, where $\frac{9}{8} \times 62{\cdot}5hl$ (exactly $1{\cdot}134$ for 9 spans) should be provided for per foot width of the slab.

Many theories exist concerning the proportions of reinforced concrete beams and slabs.

The following rules are principally based on the methods adopted by Marsh (*Reinforced Concrete*). Considerations of space prevent a full discussion of the matter, and a design obtained by these rules should always be carefully checked for shearing and adhesion stresses before being finally passed as correct. In my own practice, however, I have invariably found that the rules are sufficient, and believe that in hydraulic work generally the beams and slabs are too thick (owing to the necessity for preventing percolation) to permit the above stresses ever becoming unduly intense. Marsh's treatise, and the *Reinforced Concrete Pocket Book*, by Messrs. Marsh and Dunn, are, however, indispensable, and even in preliminary designs their tables and diagrams save much time and labour.

Let b, represent the breadth of a beam, or slab, in inches.

Let d represent the depth of the beam or slab from the compression face to the mass centre of the reinforcement on the tension side.

Then, if M, be the bending moment in inch-pounds which the beam has to sustain:

$$M = \mu 600bd^2 \text{ (M in inch-pounds)} \quad . \quad . \quad . \quad \text{[Inches]}$$

$$M = \mu 50bd^2 \text{ (M in foot-pounds)} \quad . \quad . \quad . \quad . \quad \text{[Inches]}$$

The analogy with the ordinary formulæ for timber beams is obvious. Just as in the case of timber beams the coefficient of bending strength depends on the species of timber, so in reinforced concrete beams the coefficient μ depends on the ratio of the area of steel to the area bd, the effective area of the concrete.

[*Note.*—The tensile reinforcements being buried from $1\frac{1}{2}$ to 2 inches in the beam, the gross area of the concrete is at least $(d+1\frac{1}{2})b$ square inches.]

Put ω_t = the area of the steel reinforcement which is on the tension side of the beam.

Put ω_c = the area of the steel reinforcement which is on the compression side of the beam.

Then Marsh and Dunn give as follows:

VALUES OF μ

$\frac{\omega_t}{bd}$	$\omega_c=$					
	0	$0{\cdot}2\omega_t$	$0{\cdot}4\omega_t$	$0{\cdot}6\omega_t$	$0{\cdot}8\omega_t$	ω_t
0·005	0·14	0·15	0·15	0·16	0·17	0·17
0·0075	0·16	0·17	0·18	0·19	0·20	0·21
0·010	0·18	0·19	0·20	0·22	0·23	0·24
0·015	0·20	0·22	0·24	0·26	0·28	0·30
0·020	0·22	0·24	0·27	0·30	0·32	0·35
0·025	0·23	0·26	0·30	0·33	0·37	0·40
0·030	0·24	0·28	0·32	0·36	0·40	0·45

The required cross-section can therefore be obtained by trial and error. The original diagrams give three significant figures, and enable the required section to be selected at once.

In the case of continuous slabs ω_t denotes:

(*a*) At the centre of a slab, the steel on the downstream side.

(*b*) Over a support, the steel on the water-face side.

Accurate testing of the design for stresses induced by shear and cohesion requires tables, or lengthy calculations. In preliminary designs let F, denote the greatest shear in pounds. Then, if F_s be less than $50bd$, the design is usually quite safe, and can certainly be rendered absolutely safe by inclining the steel rods, or turning up their ends, without increasing the weight of steel. The patented methods are not considered, as each patentee has his own rules.

In the case of the slabs facing the dam we have to consider a width of one foot. Thus, $b=12$, and $M = \frac{K}{8} = \frac{62 \cdot 5hl^2}{8}$ foot-lbs. per foot width.

$$\text{Thus,} \quad \frac{62 \cdot 5hl^2}{8} = 50\mu \times 12d^2 \quad . \quad . \quad . \quad . \quad . \quad [d, \text{ in inches}]$$

$$\text{or,} \quad d^2 = \frac{hl^2}{77\mu} \quad . \quad . \quad . \quad [d, \text{ inches}] \text{ for a non-continuous slab.}$$

We can thus determine the thickness of the slabs at every point.

Now, it is plain that percolation must be carefully guarded against. This is easily effected in a non-continuous slab, as all the reinforcement lies on the downstream face of the slab. In a continuous slab, however, the water face is in tension over the supports, and some steel must be placed near the water face. Thus, it is probable that the extra thickness of concrete necessary to protect this steel from action by water will counterbalance any decrease in thickness, or percentage of steel, that might theoretically be obtained by continuity.

It also seems probable that future experience will show that a layer of waterproof material at, or near to, the water face of the slabs is advisable, although, so far as I am aware, no such construction has yet been adopted.

We can now proceed to proportion the buttresses. Theoretically, the work is carried out just as for a solid dam, the buttresses having to support water pressures indicated by $62 \cdot 5hls$, lbs. at each foot of height, and their own weight.

In actual practice, the dam is not usually founded on hard rock, and its base is therefore about one and a half times to twice its height. In such cases tension in the buttresses does not occur, as can be seen by merely inspecting the annexed sketch (No. 118).

It will be found by actual trial that the easiest method of design is to proportion the buttresses so as to produce a safe intensity of pressure on the foundation, and to make them of triangular section from this level upwards. It will then be found that such buttresses are of ample strength when tested on any other section; and, as a matter of fact, in actual dams, passage ways and arched openings are frequently made in the buttresses, either to save material or to provide a means of communication along the dam.

Let us therefore assume the following:

The thickness of a buttress at its top is t, feet (usually about 0·8 to 1 foot). At the base, or at the level at which it is proposed to investigate the stresses, let the thickness be represented by d, feet.

Let H_1, be the height, and L_1, the base length of a buttress; each measured at the level where the stresses are to be investigated.

Then its thickness at any height x, above this level, is represented by

$$d-(d-t)\frac{x}{H_1}$$

Thus, the volume of a buttress is :

$$\frac{L_1H_1}{2}\,\frac{2d+t}{3}$$

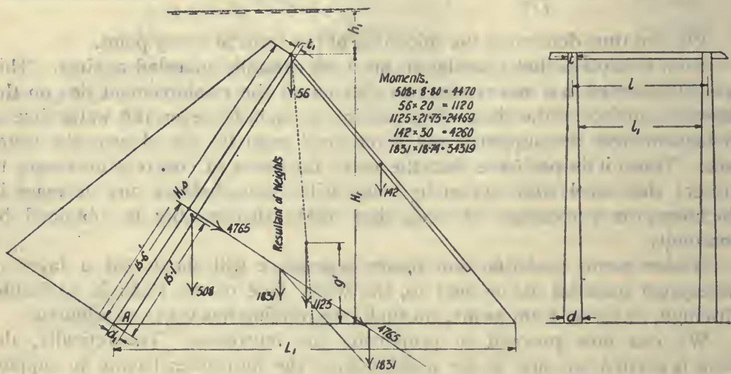
and its centre of gravity lies in the median line of the buttress at a height :

$$\frac{H_1}{2}\,\frac{t+d}{t+2d}$$

So also, the volume of the upstream slab is represented by :

$$\frac{l_1m(t_1+d_1)}{2}$$

where t_1, and d_1, are top and bottom thickness of the slab, and l_1, is the distance between the nearer faces of two adjacent buttresses, and $m=H_1s$ is



SKETCH No. 118.—Diagram of Forces acting on a Reinforced Concrete Dam.

the slant length. The centre of gravity lies very approximately in its face at a distance :

$$\frac{m}{3}\cdot\frac{2t_1+d_1}{t_1+d_1}$$ from the bottom, measured along the slope of the slab.

It is plain that the variation in l_1 ($=l-t$ at top of dam, and $l-d$, at the bottom of the dam) has been neglected. In preliminary calculations the error is inappreciable. If it is desired to use the accurate formulæ they are :

Volume of slab :

$$=m\left\{(l-d)d_1-\frac{(l-d)(d_1-t_1)-d_1(d-t)}{2}-\frac{(d_1-t_1)(d-t)}{3}\right\}$$

Distance of mass centre along median line of slab :

$$=m\left\{\frac{\frac{(l-d)d_1}{2}-\frac{(l-d)(d_1-t_1)-d_1(d-t)}{3}-\frac{(d_1-t_1)(d-t)}{4}}{(l-d)\,d_1-\frac{(l-d)(d_1-t_1)-d_1(d-t)}{2}-\frac{(d_1-t_1)(d-t)}{3}}\right\}$$

We have also the top of the dam to consider ; the thickness of this depends on whether we use it as a bridge, or fix flashboards on it, or merely shape it as a parabola to secure the best discharge.

In this particular case, I take it as $4l_1$ cube feet, *i.e.* 3 feet × 1 foot 4 inches, in section, and assume that the mass centre is immediately over the apex of the buttress.

The downstream face of the dam is (in a spillway type at any rate) covered with slabs. The thickness of these cannot be determined by any ordinary rule. If we regard the depth of water flowing over the dam as the determining factor, we find a pressure of at most 312 lbs. per square foot, assuming a depth over the dam of 5 feet and taking no account of the velocity of the flowing water, which would tend to diminish the pressure.

As a matter of practice, we find in spillway dams, that such damage as occurs is apparently due to a partial vacuum induced by the flowing water; and the form of crest that theoretically, at any rate, prevents the formation of this vacuum is discussed on page 400.

In a hollow dam, such as we are now considering, it is plain that a few holes in the slabs, should prevent any vacuum. I find that the best practice in America usually makes the thickness of the slabs about two-thirds t_1, say 12 inches.

The volume is plainly $\frac{2t_1}{3}ln$, where n, is the length of the downstream face, and the mass centre lies in the middle of the length of the slab.

We have thus estimated all the weights, and can combine them into one resultant, which is best done graphically, as shown in the sketch.

We have now to estimate the water pressure. This is as shown by the trapezoidal stress diagram. Its magnitude in the present units, *i.e.* the weight of 1 cube foot of reinforced concrete is:

$$\frac{lm}{\rho}\,\frac{H_1+2h_1}{2}$$

where h_1, is the overflow depth; and for a first approximation we can take $\rho=2$.

Its line of action is normal to the upstream slab, and cuts it at a distance:

$$\frac{m}{3}\times\frac{H_1+3h_1}{H_1+2h_1}\text{ from the base, measured along the slab.}$$

Sketch No. 118, which is purely diagrammatic, and does not indicate good proportions, shows a case:

$L_1=45$ feet $\quad H_1=30$ feet $\quad h_1=5$ feet $\quad t=1$ foot $\quad d=2$ feet
$l=15$ feet $\quad t_1=0{\cdot}5$ foot $\quad d_1=1{\cdot}5$ foot $\quad m=36{\cdot}0$ feet $\quad n=30{\cdot}5$ feet

and the forces expressed in units of 125 lbs. = weight of 1 cube foot of concrete, as ascertained and laid off from the above formulæ.

The total vertical pressure produced by the weights and water pressures is 4451 units = 556,000 lbs. Allowing for the eccentricity of the resultant the maximum pressure is,

$$2\times 6180\left(\frac{3\times 24{\cdot}25}{45}-1\right)=7622\text{ lbs. per square foot,}$$

and the minimum pressure is:

$$2\times 6180\left(2-3\times\frac{24{\cdot}25}{45}\right)=4738\text{ lbs. per square foot.}$$

These are pressures of a magnitude such that they can be resisted by even the weakest rock.

The total horizontal shear produced by the water pressure is 3945 units, or 493,000 lbs. Thus, the average shear is 5480 lbs. per square foot, which is a greater intensity than most of the weaker rocks can be expected to sustain, especially if fissured. Thus, as already indicated, the foundations of a reinforced concrete dam form its weak point.

The work now proceeds exactly as for a masonry dam, and the pressures and shears can be laid down as in the example worked out.

It should also be noted that the section of the dam should be tested at several points above the base.

In American practice, tensions are permitted in the upper portions of the buttresses. In view of the fact that we are dealing with reinforced concrete this appears allowable, and if the experimental results found by Wilson and Gore, and others, for solid masonry dams, are considered as applying to the buttressed type, it is advisable to reinforce the buttress near the angle A. No rules can be given for this reinforcement.

Similarly, while the horizontal slab reinforcement is easily calculated from the stresses, some vertical reinforcement is needed in the face slabs. So far as can be judged experimentally, temperature cracks are possible, unless the area of steel in the vertical reinforcing bars exceeds 0·3 per cent. of the area of the concrete in the slabs. No dam that I am aware of has so large a percentage of vertical reinforcement, but it must be remembered that reinforced concrete dams are new, and few of the older examples greatly exceed 30 to 40 feet in height. If a smaller percentage of reinforcement be adopted, the designer can at anyrate console himself with the consideration that such cracks as do occur can be repaired without necessarily causing a disaster.

I also notice that some dams are so proportioned as to be in tension, not only in the upper portions of the buttresses, but also at the base. This, I consider, is a departure from good practice, and I believe that such dams are unsafe.

Foundations.—The design of foundations depends on the character of the material on which the dam rests.

In really solid rock, a shallow seepage trench is perhaps all that is necessary; but, in gravel, or fissured rock, it appears to me that the only safe rule is to follow the practice evolved for earth dams. We have one great advantage,—our impermeable wall being of concrete, cannot be injured by burrowing animals, and we can therefore put it right in front of the dam.

I prefer the following design:

A concrete core wall carried down either to an impermeable stratum, or to such depth as investigation of the material, conducted on the lines discussed under earth dams, shows to be necessary. Behind the core wall is a small stone drain, as discussed under earth dams, which should be connected with one or more vent pipes.

The whole floor of the dam is covered with a layer of concrete, the thickness of which need only be 4 inches for good foundations, and, in the case of bad soil, may be reinforced so as to spread the pressure of the buttresses, if any doubt exists as to their foundations being of sufficient width.

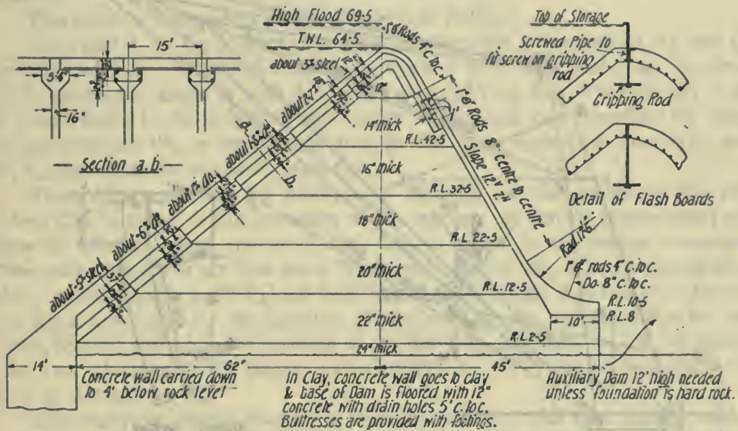
At the tail of the dam is another core wall, the depth of which is fixed by the scour produced below the dam by the overflowing water. The sections on

Falls, and Weirs may be consulted when determining the site and thickness of this tail wall.

It may also be pointed out that lines of steel, or cast-iron sheet piling, may be substituted for the core wall or walls, but such work, unless carefully executed, is liable to prove faulty : and I doubt whether it will be as satisfactory even from the point of view of cost ; since each core wall should probably be replaced by a double line of piles.

Some dams exist which depend on several shallow core walls in place of two deep ones. Personally, I doubt whether such foundations are trustworthy, but they appear to be satisfactory for heads up to 30 or 40 feet.

In such cases, a very wide foundation similar to that of an Indian weir is necessary. It therefore seems doubtful whether expense is saved, more especially as in any soil fit for a dam foundation it is usually possible to sink a deep trench and fill in with concrete.

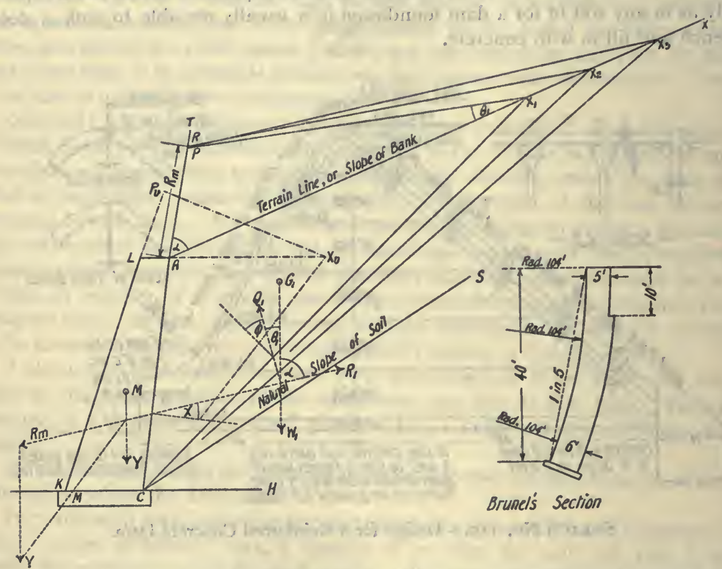


SKETCH No. 119.—Design for a Reinforced Concrete Dam.

EARTH PRESSURES ON RETAINING WALLS.—The pressure of earth or similar materials, differs from that of water in one very marked respect. The pressure is not necessarily normal to the surface across which it acts. If this were the only difference, it would not be difficult to obtain an accurate theory, but all earthy substances also possess a certain amount of cohesion, and therefore neither the magnitude nor the direction of the pressure at any given point can be calculated. Dry sand may be considered as possessing very little, and hard clay, or earth rammed in horizontal layers, a great deal of cohesion. Since the effect of cohesion depends very largely on the state of the earth, and probably varies greatly from time to time, in the same piece of earth, and can certainly be considerably affected by the manner in which the earth is treated, it seems useless to endeavour to define it exactly,—and, consequently, no mathematical method of estimating its effects exists.

The method now put forward neglects the effect of cohesion entirely, and therefore considers earth as differing from water only by the existence of oblique pressures. Thus, the theory is extremely defective, but it errs on the side of safety, and walls which, when treated by this theory are only just stable, may be assumed to actually possess a factor of safety which varies from $\frac{10}{9}$ in the case of artificially dried sand (which possesses no cohesion) to 5, or more, in the case of hard clay not exposed to the weather, where the cohesion is so great that the pressures might be entirely neglected. The theory is believed to be applicable only to earth that has been disturbed; and at great depths in undisturbed earth, such as must be considered when designing timbering of deep trenches, it is absolutely misleading, and as a rule greatly over-estimates the pressure.

In practice, the rules obtained by following this theory lead, in ordinary



SKETCH NO. 120.—Earth Pressures on Retaining Walls.

cases, to retaining walls of safe, but not unduly extravagant sections, and may therefore be considered as guides for design. It must, however, be remembered that the theory assumes a state of things which we know but rarely occurs, and which, when it does occur, usually causes the wall to crack.

As in Sketch No. 120, let CA, be the inner face of the wall, and HCS be the angle of repose of the earth, so that CS, would be the face of the earth if there were no wall.

Let AX, be the terrain line, or top of the earth which rests against the wall.

Now, draw any line CX_1, and let us assume that the earth is about to slip down this plane CX_1, and cause the wall to overturn.

Then, the forces acting on the wedge of earth ACX_1, are as follows :

(i) W_1, its weight, which, considering a length of 1 foot of the wedge or wall, is given by:

$$W_1 = \rho \times \text{area of triangle } ACX_1$$

where ρ is the weight of a cube foot of the earth.

(ii) R_1, the pressure of the wall on the wedge.

(iii) Q_1, the reaction of the undisturbed earth acting across the face CX_1.

Now, since it is assumed that the wedge is just about to slip the ordinary rules of statical friction hold. Thus R_1, makes an angle χ with the normal to the face CA, of the wall, and Q_1, makes an angle ϕ with the normal to CX_1, and since these forces resist the downward motion of the wedge they are both directed upwards. The problem is therefore reduced to the statical problem of determining the magnitude of two forces, acting in given directions, when the magnitude and direction of their resultant (W_1, in this particular case) are known.

The solution is therefore easily obtained as follows:

Let a and θ_1 be the respective angles between the directions of R_1 and Q_1 and the vertical. Draw TA, making an angle a with X_1A, and from X_1, draw P_1X_1 making an angle θ_1 with AX, and cutting AT, in P_1. Then plainly AX_1, is proportional to the weight of the wedge ACX_1, and can therefore be taken as representing W_1, in magnitude, and, on the same scale, the lines AP_1, and P_1X_1, represent the forces R_1, and Q_1.

Thus, AP_1, represents the pressure of the wall on the wedge on the wall when CX_1, is taken as the boundary of the wedge. A similar construction can now be effected when any other line CX_2, is taken as the wedge boundary. The only differences are that since W_1, is not equal to W_2, AX_2, is not equal to AX_1, and the angle $\theta_2 = P_2X_2A$ is not equal to the angle $\theta_1 = P_1X_1A$. Hence, we get a new value for the pressure of the wall on the wedge, $AP_2 = R_2$ say. Now, by trial and error the position of the line CX, say $CX_m = CX_2$ in Sketch No. 120, which gives the largest value of AP_m ($= AP_2$) can be selected. Let this be denoted by R_m.

This theory is almost entirely a pure assumption. All that can be said is that when AX, is horizontal it leads to a position CX_m, which agrees very well with experiment, but when AX, is either sloping in the same direction as CS, or is oppositely directed, Darwin's experiments on sand (*P.I.C.E.*, vol. 71, p. 350) seem to indicate that results agreeing more closely with experiment are obtained by assuming that X_mC bisects the angle ACS.

The difference, however, is not very great, and the construction given above seems safer.

Now, let us consider the forces acting on the wall.

These are R_m, which we know, both as regards magnitude and direction, and which we assume (both on experimental evidence and on the analogy of water pressure) to act one-third of the way up CA, the weight of the wall, and the pressures and shears on its base.

The distribution of the stresses on, say, the cross-section CK of the wall can therefore be calculated by the rules already given under Dams.

We may assume that ϕ is about 30 degrees, and that χ is about 20 degrees. These assumptions are probably somewhat unfavourable. Earth weighs from 112 pounds per cube foot, which is probably a slightly low estimate, up to 125 pounds per cube foot, if heavy clay is considered. Thus $\rho = 1{\cdot}8$ to $2{\cdot}0$.

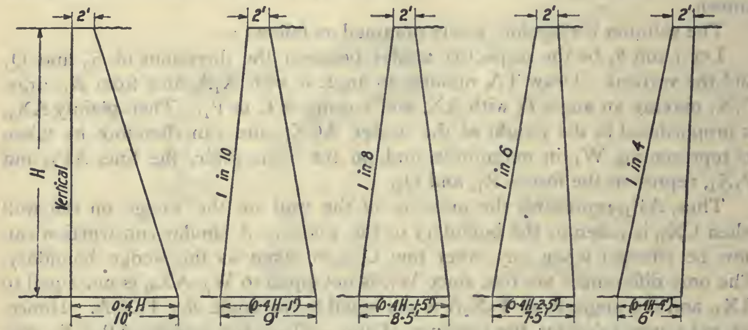
The resultant of R_m, and Y, the weight of the wall, should fall slightly inside the base CK, say at M, where $MK = \frac{CK}{8}$.

Engineers are usually satisfied with testing the stability of the full height of the wall, but the method is plainly applicable to any portion of its height.

Bligh has carefully investigated type sections of walls by this method (*The Practical Design of Irrigation Works*), and deduces that:

> "The back face of a wall should be vertical, or inclined towards the earth, any batter required being given to the front face of the wall."

This is somewhat opposed to ordinary practice, but the reasons are obvious. If the front face of a wall is inclined, either specially cut bricks or stones are required, or the beds of the masonry cannot be set horizontal, and thus provide a possible channel for entry of rain water into the wall.



SKETCH No. 121.—Typical Sections of Retaining Walls.

Nevertheless, the economy obtained by walls with face batters is so considerable that the extra expense of cut stones or bricks is justifiable wherever the walls considerably exceed 20 feet in height.

For example, let us take the sectional area of a wall 25 feet in height, as proportioned by Bligh:—

Face Batter.	Vertical.	1 in 10.	1 in 8.	1 in 6.	1 in 4.
General expression for bottom width	0·4H	0·4H − 1 ft.	0·4H − 1·5 ft.	0·4H − 2·5 ft.	0·4H − 4 ft.
Area of cross-section for 25 ft. height, and 2 ft. top width . .	150	137·5	131	119	100

Sketch No. 121 shows typical designs of walls. Bligh's rules agree very well with the practice adopted in Indian irrigation, and it is believed that the proportions given will suffice to secure safety under unfavourable circumstances. It may indeed be said that were it not for the fact that the top width of a wall is not always 2 feet, and that the terrain line is not always horizontal, the above sketch would provide all that is required in practice. The theory will be found of the greatest utility not in proportioning the cross-sections of walls, but in selecting the design which is best adapted to local conditions, from among a series which has already been determined by eye or by experience. This view is admirably illustrated by Brunel's section, Sketch No. 120. This section is at first sight very thin, and "looks weak." When tested by the above theory it will be found amply stable, and in practice it produces very satisfactory results. In all cases it is as well to remember that a cracked retaining wall is not a disaster, and that a section which will not crack under any imaginable circumstances is an impossibly extravagant ideal.

Practical Details of Construction.—The function of a retaining wall is probably far more that of protecting the cohesion of the earth from being destroyed by the action of the weather, than of actually retaining the earth in the sense that a dam retains water.

The necessary construction therefore is as follows:

(*a*) Keep the water, as far as possible, away from the back of the wall, but give it a free vent through the wall by means of weep holes.

(*b*) If the foundation of the wall becomes saturated, it is only a question of time when the wall will slip.

CHAPTER VIII

PIPES

General Considerations concerning the Motion of Water in a Pipe.
Entrance Head.—Practical Rules.
Formulæ for the Discharge of Pipes.
General Formulæ. — **Tutton's formula** — **Skin-friction equation** — Logarithmic formulæ—Rough rules.
More Exact Formulæ for Existing Mains.—Experimental results—Extension to large pipes.
Measurement of the Discharge of a Pipe.—Use of colouring matter—Practical details.
Effect of age on pipes.—Limpets—Protective coatings—Calcium carbonate incrustations—Slime—Silty, or nodular deposits.
Angus Smith's Process.—Original process—Newer methods.
Cleaning and Scraping Pipes.—Torquay records—Melbourne practice—Table of discharges before and after scraping.
Formulæ expressing the Deterioration of Cast-iron Pipes with Age.—Table of values of C, for old cast-iron pipes—Examples of small uncoated pipes—Exceptions.
Darcy's Formulæ.
Cast-iron pipes.—Thickness of pipe metal—Grashof's value for thickness—Hawksley's and Unwin's rules—American rules.
Pipe Joints.—English and American rules.
Design of Joints in Cast-iron Pipes.—Examples—Summary.
Pipe Laying.—Specification—Remarks.
Large Wrought-iron or Steel Riveted Pipes.
Skin friction coefficients.—Allowances for obstructions by rivet heads and plate edges.
Distortion of Riveted Pipes.
Corrosion of Steel Pipes.
Specification.—Coating—Remarks—Tests for steel.
Locking Bar Pipe.
Construction of Steel Pipes.—Rules for riveted joints—Allowance for corrosion—Minimum thickness of plates.
Anchoring Pipes.
Wood Stave Pipes.—Construction—Calculation—Preliminary experiments.

SYMBOLS EMPLOYED.

A, is a coefficient in the equation $h = AQ^n$.
B, is a coefficient in the equation $h = Bv^n$.

$$\text{Thus,} \quad B = \left(\frac{\pi d^2}{4}\right)^n A$$

C, is the skin friction coefficient in the equation $v = C\sqrt{rs}$.
C_1, is the friction coefficient in Tutton's equation:

$$v = C_1 r^{0 \cdot 67 - \beta} s^{0 \cdot 5 + \beta}$$

d, is the diameter of the pipe in feet, and D, is used when inches are employed as the unit.
d_b, is the diameter of the pipe bands in inches.
e, is the safe pressure on wood staves in pounds per lineal inch of the band (see p. 466).
E″ (see p. 467).
f, is the band spacing in inches. $f=\frac{1200}{N}$.
F, is the safe tensile stress of cast iron = 1850 lbs. per square inch.
f_1, is a factor allowing for the reduction in area caused by rivet holes or corrosion.
h, is the head in feet lost by skin friction in a given length l, of pipe. Thus $\frac{h}{l}=s$.
H, is the maximum internal pressure, in feet of water, to which the pipe is exposed.
k, is a coefficient such that $B=\frac{kl}{d^m}$ (see p. 431).
l, is the length of the pipe in feet.
m, and n, are the indices of d, and v, in the equation $\frac{h}{l}=\frac{kv^n}{d^m}$.
N, is the number of bands per 100 feet length of the pipe. $N=\frac{1200}{f}$.
p, is the pressure corresponding to H, feet of water, measured in pounds per square inch.
q, is the stress in pounds which a band of diameter d_b inches can safely sustain.

$$q=\frac{\pi}{4}d_b^2 s_1.$$

Q, is the stress in pounds actually sustained by a pipe band.
R, is the internal radius of the pipe in inches. $R=\frac{D}{2}$.
$r,=\frac{d}{4}$ is the hydraulic mean radius of the pipe in feet.
r_b, is the radius of the pipe bands in inches $=\frac{d_b}{2}$.
s, is the sine of the slope of the hydraulic gradient of the pipe. $s=\frac{h}{l}$.
s_1, is the safe working stress of the metal of the pipe, or of the pipe bands.
s_2 (see p. 467).
t, is the thickness of the walls of the pipe, in inches.
t_G, t_H, t_m (see p. 445).
T, is the circumferential tension, in pounds per lineal inch, in the walls of the pipe.
v, is the mean velocity of the water in the pipe, in feet per second.
W, is the central load in pounds that a pipe can sustain as a beam under a maximum stress of 1850 pounds per square inch.

SUMMARY OF FORMULÆ

Entrance head $=\frac{v^2}{2g}(1+\alpha)$ $\alpha=0·20$ to $0·50$ (see p. 426).

Tutton's formulæ :

$v=C_1 r^{0.67-\beta} s^{0.50+\beta}$ (see p. 427).

New cast-iron pipes, $v=140 r^{0.66} s^{0.51}$ (see p. 429).

Old cast-iron pipes, $v=105 r^{0.66} s^{0.51}$ (see p. 429).

Skin friction formulæ, $v=C\sqrt{rs}$

$C=C_1 r^{0·17}$. For table (see p. 478).

Logarithmic formulæ, $h=AQ^n=Bv^n$

$$\frac{h}{l}=k\frac{v^n}{d^m}$$

Discharge of a pipe y years old $= \dfrac{\text{Discharge when new}}{\sqrt{1+\dfrac{y}{32}}}$

Cast-iron pipes (see p. 444).

Grashof, $t_G = \frac{D}{2}\left\{\sqrt{\frac{3F+2p}{3F-4p}} - 1\right\}$ F = 1850 lbs. per square inch. . [Inches]

Hawksley, $t_H = 0{\cdot}18\sqrt{D}$ . . . . [Inches]

Unwin, $t_m = 0{\cdot}11\sqrt{D} + 0{\cdot}10$ . . . . . . [Inches]

Steel pipes, $t = \frac{pR}{f_1 s_1}$ (see p. 462) . . . . . . [Inches]

Wood stave pipes, $N = \frac{330HD}{d_b^2 s_1}$ (see p. 465) . . . . . [Inches]

General Considerations concerning the Motion of Water in a Pipe.—Consider the usual case where water is drawn off from a large reservoir through a pipe. Assume that the velocity of water in the pipe, when the motion is steady, is v feet per second.

If the loss of head between two points P and Q, in the pipe be observed, it will be found that (abnormal irregularities, bends, and other obstructions in the pipe being neglected) the loss is proportional to the length of the pipe between P and Q.

The question of the determination of the absolute magnitude of this loss is considered in detail later. The general theory, however, shows that if we consider the motion from a point R, in the reservoir where the velocity is very small, to a point P in the pipe, where the velocity is v; there must, apart from all frictional resistances, be a loss of head equal to $\frac{v^2}{2g}$, or rather, a portion of the initial pressure energy of the water is transformed into velocity, and the pressure consequently diminishes by $\frac{v^2}{2g}$ feet of water. The term Hydraulic Gradient is defined on page 471. Sketch No. 122.

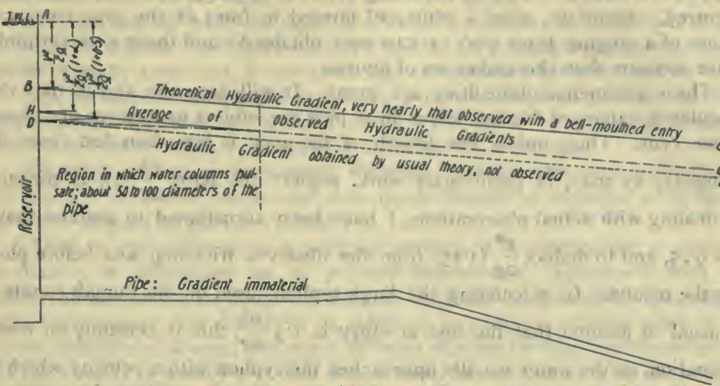
Entrance Head.—We are thus led to consider the "loss of head at entry into the pipe." Reviewing the matter in detail, we find that the total localised loss of head which occurs at and near the entrance to the pipe, and is independent of the length of the pipe, can be expressed by:

$$\frac{v^2}{2g}(1+a)$$

where $a\,\frac{v^2}{2g}$, represents the resistance of the entrance of the pipe considered as an orifice discharging water at a velocity of v feet per second.

In theory, $a = \frac{1}{C_v^2} - 1$, so that a varies from about 0·06 for a pipe with a bell-mouthed entry, to 0·50 for a pipe projecting into the reservoir (see p. 140).

Actual experimental data are rarely given, and modern experimenters have usually found that the flow downstream of the entrance to a smooth pipe does not become turbulent for an appreciable distance, roughly 50 to 100 times the diameter of the pipe. Thus, it is quite possible that a being masked by the decrease in resistance over the length of pipe in which the flow is not turbulent, might be apparently negative. In some experiments of my own on a 12-inch pipe, in which theoretically $a = 0{\cdot}50$, I found an irregular series of values of



SKETCH NO. 122.—Diagram showing "Head lost at Entry" into a Pipe.

This sketch is an attempt to show graphically the difficulties attending a correct estimation of the "head lost at entry" into a pipe, and is founded on observations made on a 12-inch pipe at a mean water velocity of 8 feet per second. The vertical scale of the hydraulic gradients has been made five times the horizontal. Five pressure tubes were established, 1 foot, 10 feet, 45 feet, 230 feet, and 400 feet from entry.

The upper (full) line ABC, shows the gradient obtained by the uncorrected theory. A drop (AB) of $\frac{v^2}{2g} = 1$ foot occurs between the entrance and the first tube, and thereafter the tops of the water columns lie on a straight line BC, sloping downwards at about 2·6 feet per 100 feet. This ideal case was very nearly attained when the entry was bell-mouthed in the manner indicated in Sketch No. 40.

The lowest (dotted) line ADE, shows the circumstances that are usually believed to occur with a cylindrical entry. The initial drop AD, is now about $1{\cdot}5\,\frac{v^2}{2g} = 1{\cdot}5$ foot, and the rest of the hydraulic gradient DE, falls at the same slope as the upper line. This case I was never able to observe. The middle (chain dotted) line ADG, shows the average of the circumstances actually observed with a cylindrical entrance piece projecting 6 inches into the reservoir.

A drop (AD) at entry occurs, which is certainly greater than $\frac{v^2}{2g}$, and is probably less than $1{\cdot}5\,\frac{v^2}{2g}$. Thereafter for at least as far as the third tube (45 feet from the entrance) the water columns pulsated up and down; but on the average the water levels did not lie on a straight line, and the average slope of the curve joining them was less than 2·6 feet per 100 feet.

The columns at 230 feet and 400 feet were, however, steady, and the slope of the hydraulic gradient between these points, was 2·6 feet per 100 feet, and (to the accuracy of the observations) the same as in the first case.

If, however, this slope be prolonged backwards, as shown by the full line GH, we find a calculated drop at entry AH, which is usually less than $1{\cdot}5\,\frac{v^2}{2g}$, and AH, represents the quantity $(1 + a)\,\frac{v^2}{2g}$.

a ranging from 0·15 to 0·60; but the possibilities of error were great, and what was really observed was $a \pm$ all errors in the determination of the friction head over 80 feet of pipe, and it was possible to bring all the values of a inside the range $a = 0{\cdot}20$ to $a = 0{\cdot}30$, by assuming errors of a magnitude that could have occurred. Similarly, when a whirlpool formed in front of the pipe entrance, values of a ranging from 0·90 to 1·10 were obtained; and these were certainly more accurate than the earlier set of figures.

The experimental difficulties are great. It will later be shown that the calculated values of friction losses in a pipe are subject to an error of at least 5 per cent. Thus, unless the length of the pipe is less than 800 times its diameter, we may, in preliminary work, neglect the term $\frac{v^2}{2g}(1+a)$ entirely. In dealing with actual observations, I have been accustomed to assume that: $a = 0{\cdot}25$, and to deduct $\frac{v^2}{2g}(1{\cdot}25)$ from the observed frictional loss before plotting the results. In calculating the large syphons used on the Punjab canals it is usual to assume that the loss at entry is $1{\cdot}5\,\frac{v^2}{2g}$, this is certainly an over-estimation, as the water usually approaches the syphon with a velocity which is a considerable fraction of v. Even if this correction be applied, the results of certain observations show that either a is very small, approximately 0·10 to 0·20, or that the syphons are considerably smoother than the ordinary rules would indicate.

Pasini and Gioppi (*Giornale del Genio Civile*, 1893, p. 49) experimented on three brickwork and concrete syphons, between 4 and 5 feet hydraulic mean radius, and respectively 33 feet (10 metres), 581 feet (177·30 metres), and 861 feet (262·60 metres) in length. The volumes discharged were measured by current meters, and apparently with great accuracy. When $0{\cdot}028v^2 = 0{\cdot}55\,\frac{v^2}{2g}$ (v being the mean velocity in meters per second) is allowed for entrance head, the whole series of experiments fall into line, and agree very well with:

$$v = 112\sqrt{rs}$$

in English measure as the equation for frictional resistance.

This value of the frictional resistance also agrees very well with what would be predicted for brickwork channels of this size by either Bazin's or Kütter's rules. It therefore appears permissible to infer that $\frac{0{\cdot}55v^2}{2g}$, is a fair allowance not only for the entrance head, as above defined, but also for the increase in velocity which occurs as the water quits the earthen canal and enters the syphon.

The matter can be summed up as follows:

In actual observations, the neighbourhood of the entrance to a pipe should be avoided when locating gauges. In preliminary calculations, a can usually be neglected. In working up observations, $a = 0{\cdot}20$ to $0{\cdot}30$, is a fair assumption. In designing structures, $a = 0{\cdot}50$ is amply safe, and probably over-estimates the loss.

FORMULÆ FOR THE DISCHARGE OF PIPES.—It must be confessed that the subject of the discharge of pipes is in a very unsatisfactory state, and that any definite advance seems to be unlikely.

The two chief difficulties are:

(i) The usual commercial description of a pipe is not sufficiently exact to fix its hydraulic condition, so as to enable the discharge to be predicted with an accuracy of even 10 per cent.

(ii) The flow of water in pipes is affected by accidental irregularities to a remarkable degree; and there are very few, if any, existing experiments which, when carefully examined, do not show signs of influence by such conditions.

The commercial method of describing a pipe is never so precise that those experiments only can be selected which were made on pipes of a nature similar to that of which it is desired to calculate the discharge.

Under the above circumstances, therefore, formulæ generally applicable to all pipes of the same commercial description should be regarded as liable to errors of at least 10 per cent. either way.

Thus in the selection of a working formula, it is permissible to consider simplicity in calculation as of primary importance. Agreement with the observations used in its deduction may be regarded as indicating the more or less unconscious skill displayed in selection, in order to include pipes of the same hydraulic character, and reject all cases affected by abnormal disturbances. If the formula is a truly practical one, it must be sufficiently wide in range to include a large proportion of these unusual cases.

On the other hand, it appears advisable to give the most definite indications possible of the manner in which the discharge of a pipe is influenced by alterations of the available head. Consequently, having observed the discharge of an existing pipe under a given head, an engineer can predict its probable discharge under another head, with greater accuracy than a general formula will permit.

GENERAL FORMULÆ.—The most useful formula seems to be the one given by Tutton (*Journ. of Assoc. of Eng. Societies*, vol. 23, 1899), as follows:

$$v = C_1 \sqrt[3]{r^2}\sqrt{s}$$

Where:

- v, is the velocity in feet per second.
- r, is the hydraulic mean radius in feet.
- s, is the sine of the angle of inclination of the hydraulic gradient.
- C_1, is a coefficient which is approximately constant for pipes of the same description.

This equation does not pretend to any very great accuracy, for the reasons given above; and Tutton's actual results, as later indicated, give

$$v = C_1 r^{0.67-\beta} s^{0.50+\beta} \quad \text{where } \beta \text{ varies from 0·00 to 0·08.}$$

We have as follows, for diameters up to 4 feet, and probably for greater diameters (see table on p. 428).

This equation can be easily transformed into the form:

$$v = C_1 r^{\frac{1}{6}} \sqrt{rs} = C\sqrt{rs}$$

where C, is the variable coefficient of skin friction for a pipe $d = 4r$, feet in diameter. The values of $C_1 r^{\frac{1}{6}}$ when $C_1 = 100$ are tabulated on page 478. In future we shall usually specify the hydraulic qualities of a pipe by stating that $C = \ldots$, in the equation $v = C\sqrt{rs}$.

	Description of Pipe.	C_1.	Working Value of C_1.	Remarks.
I.	New cast iron . . . Old cast iron, cleaned. Cement lined pipes, or Angus Smith coated, or tarred	126 to 158	140	The values of C_1 are very evenly distributed irrespective of the radius.
II.	Cast iron, slightly tuberculated, or with mud deposits	87 to 132	105	The majority cluster round 105.
III.	Heavily tuberculated .	30 to 85	...	No value can be given.
IV.	Asphalt coated pipes (new)	140 to 199	170	...
V.	Asphalt coated pipes (old)	140	140	...
VI.	Wood stave pipes .	155 to 129	140	The smaller value applies to square channels.
VII.	Lap riveted pipes, tarred or asphalted, rivets projecting	New 125 to 135 Old 110 to 114	130 112	... ...
VIII.	Large brick conduits .	129 to 110	120	When obstructed by shafts, etc., C_1 may fall to 90.

In applying formulæ No. IV., V., and VII., it should be remembered that old pipes of these kinds were rather rare at the date of Tutton's investigations. The coefficients given must be considered as corresponding to a cast-iron pipe which is but very slightly tuberculated (say $C_1 = 120$), and a further drop of 10 to 15 per cent. (corresponding to $C_1 = 105$ for cast-iron pipes) may be expected when pipes such as are now discussed become badly tuberculated. An asphalt coated pipe, if successfully coated, takes longer to become incrusted than a coated cast-iron pipe, and asphalt coatings being proprietary articles, if the coating is unsuccessful any publication of records relating to the discharge of an old pipe is unlikely. The fact that the discharge capacity of wood stave pipes does not decrease with age appears to be well established.

With all the accuracy required for practical purposes a yard of pipe D, inches in diameter holds $\frac{D^2}{10}$ imperial gallons.

Thus, the discharge of a pipe, when the mean velocity of the water is v, feet per second, is

$$\frac{vD^2}{30} \text{ imperial gallons per second} \quad . \quad . \quad \text{[Inches.]}$$

or, $$2vD^2 \text{ imperial gallons per minute} \quad . \quad . \quad \text{[Inches.]}$$

If U.S. gallons are considered, the figures become :

$$\frac{vD^2}{25} \text{ U.S. gallons per second} \quad . \quad . \quad \text{[Inches.]}$$

or, $$144vD^2 \text{ U.S. gallons per hour} \quad . \quad . \quad . \quad \text{[Inches.]}$$

Tutton's exact formulæ are as follows :

No.		
I.	$v = C_1 r^{0.66} s^{0.51}$	with $C_1 = 140$ say.
II.	$v = C_1 r^{0.66} s^{0.51}$	with $C_1 = 105$.
III.	$v = C_1 r^{0.66} s^{0.51}$	with $C_1 = 30$ to 85.
IV.	$v = C_1 r^{0.62} s^{0.55}$	with $C_1 = 170$.
V.	$v = C_1 r^{0.66} s^{0.51}$	with $C_1 = 140$.
VI.	$v = C_1 r^{0.59} s^{0.58}$	with $C_1 = 140$.
VII.	$v = C_1 r^{0.66} s^{0.51}$	with $C_1 = 130$ or 112.
VIII.	$v = C_1 r^{0.65} s^{0.52}$	with $C_1 = 120$.

These formulæ are best adapted to logarithmic computation. In all practical examples r, and s, are fractions ; hence, the least liability to error is obtained by using the logarithms of $10r$, and $10{,}000s$, as these quantities are usually greater than unity.

The formulæ then become :

I. New cast-iron pipes, etc.

$$\log v = 0{\cdot}66 \log 10r + 0{\cdot}51 \log 10{,}000s - 0{\cdot}5539$$

II. Slightly tuberculated pipes, etc.

$$\log v = 0{\cdot}66 \log 10r + 0{\cdot}51 \log 10{,}000s - 0{\cdot}6788$$

III. Formula is useless.

IV. New asphalt coated pipes,

$$\log v = 0{\cdot}62 \log 10r + 0{\cdot}55 \log 10{,}000s - 0{\cdot}5896$$

V. Old asphalt coated pipes,

$$\log v = 0{\cdot}66 \log 10r + 0{\cdot}51 \log 10{,}000s - 0{\cdot}5539$$

VI. Wood stave pipes,

$$\log v = 0{\cdot}59 \log 10r + 0{\cdot}58 \log 10{,}000s - 0{\cdot}7639$$

VII. Lap riveted pipes,

$$\log v = 0{\cdot}66 \log 10r + 0{\cdot}51 \log 10{,}00s - 0{\cdot}5861, \text{ or } 0{\cdot}6508$$

VIII. Brick conduits.

$$\log v = 0{\cdot}65 \log 10r + 0{\cdot}52 \log 10{,}000s - 0{\cdot}6508$$

It is believed that the application of these coefficients will permit the discharge to be obtained with an accuracy of about 10 per cent., and it is probable that the errors will not exceed 5 per cent. either way. It should also be borne in mind that a pipe, when carefully laid true to a uniform grade, or with few bends either horizontally or vertically, may be expected to have a greater discharging capacity than one which is laid with less care, or has many bends and sinuosities. In good practice, each individual length of pipe is adjusted with extreme care, *e.g.* is laid by level or boning rod to correct grade. In pipes of sufficient diameter to permit a man to get inside them, the spigot end is carefully adjusted to lie centrally in the faucit, so as to present a smooth internal surface to the flow of water. Such precautions entail a good deal of extra labour, but in return a better discharge may be expected. My own experience leads me to believe that this additional discharging capacity remains proportionally constant as the pipe ages, although further studies are greatly to be desired. Consideration of such practical matters leads to the following rules for cast-iron pipes :

(i) A pipe laid for temporary purposes may be calculated with :

$C_1 = 130$ if badly laid . . . . . [Tutton's formulæ]
$C_1 = 145$ if well and carefully laid.

(ii) A pipe for permanent work should be calculated so as to give the required discharge with :

$C_1 = 90$ if badly laid . . . . . [Tutton's formulæ]
$C_1 = 105$ if well laid.

If frequent cleaning is permissible, or if the water is known not to incrust the pipes, the coefficient might be increased ; but it is believed that any marked increase will usually entail a more frequent cleaning or scraping of the pipes than is generally desirable.

(iii) If the water produces severe incrustation, and cleaning is impossible, these values may be reduced to :

$C_1 = 80$, and $C_1 = 90$ . . . . . [Tutton's formulæ]

but, in such cases, it is advisable to design the pipes so as to discharge the required amount for, say the first 10 years, and to provide for laying another line as the discharge falls off.

(iv) Where water is pumped by power, the additional investment entailed by a large pipe should be balanced against the extra cost of pumping through a smaller pipe when incrusted. In such a case, the rate of interest expected, and the cost of a pump horse-power per year, actually determine the size.

The question is best dealt with experimentally, and one measurement of the discharge of a pipe of known age carrying the water which it is proposed to deal with is more valuable than several pages of discussion.

It will also be plain that the allowance for the deterioration in discharging power as the pipe ages should be greater the smaller the pipe ; and, if the very simple formula $v = 100\sqrt{rs}$, be employed, and the diameter thus obtained is increased by 1 inch, cleaning is not likely to be necessary for many years, indeed if ever.

Many engineers are accustomed to consider that v, should increase as the diameter of the pipe increases, and that $v = d + 2$, feet per second (where d, is in feet) gives a very fair practical rule. The rule has no theoretical foundation, but it expresses, in a practical manner, the least costly size of pipe in cases where the available head is not very much more than that which is required to transmit the water.

More exact Formulæ for existing Mains.—Let the discharge of a pipe d, feet in diameter be observed under the various heads, let the quantities discharged be as follows :

Q_1, under a head h_1 } where Q and h are measured in
Q_2, under a head h_2 } any convenient units, preferably
Q_3, under a head h_3 } cubic feet per second, and feet ;

where, in each instance, if necessary, the head has been corrected for the loss at entry (see p. 426). Practically speaking, this correction should be applied if the length of the pipe is less than 2600 times the diameter in rough pipes; or 11,500 times the diameter in very smooth ones. These values give the length for which the head consumed at the inlet is approximately 1 per cent. of the whole head.

Plot these values logarithmically. That is to say, plot the points ($\log Q_1$, $\log h_1$), ($\log Q_2$, $\log h_2$), etc.

In ordinary engineering cases, where the water velocity is not so low (less than say 6 inches per second, in 6 inch pipes) that critical velocities (see p. 19) occur, these points will be found to lie on a straight line. Thus it is evident that the relation is:

$$\log h = n \log Q + \text{a constant.}$$

$$\text{Thus,} \quad h = AQ^n \quad \text{or, since } Q = \frac{\pi}{4}d^2v, \quad h = Bv^n$$

where log A, and log B, are best obtained graphically by finding where the line passing through the plotted points cuts the lines representing

$$Q = 1 \ (i.e. \log Q = 0), \text{ and } v = 1 \ (i.e. \log Q = \log \frac{\pi}{4}d^2) \text{ respectively}$$

Now, this experimental fact permits us to determine rapidly the discharge of a pipe from gauge readings in the reservoirs from which it draws, and into which it discharges, when, say, two or three discharges under different heads have been measured.

My own experiments indicate that so long as the pipe does not alter in character (due to deposits, or tuberculation) the figures thus obtained agree very closely with a series of observations.

We can, however, go further:—The value of n, is connected with the character of the pipe. The following rules may be given:

For a new and smooth pipe, n, lies between 1·73 and 1·87, and is probably, on the average, smallest for wood stave pipes, and largest for cast iron. For all except wooden pipes, n, increases with age, as the interior gets rougher. For slightly tuberculated pipes, n, increases about 0·10 on its original value. For badly tuberculated examples, such values of n, as 2, or even 2·10 are recorded; although it should be mentioned that the cases where n, exceeds 2 are few in number, and that the measurements are usually not very satisfactory.

The experimental relation $h = Bv^n$, may be put into the following form:

$$\frac{h}{l} = k\frac{v^n}{d^m}, \text{ where } B = \frac{kl}{d^m}$$

and l, is the total length of the pipe in feet, so that $\frac{h}{l} = s$.

The term d^m, represents the effect of the diameter of the pipe in determining the velocity, and can evidently only be determined by comparing the results obtained from observations on other pipes (assumedly of the same hydraulic character), and is therefore affected by the uncertainties already indicated.

The following table gives the results obtained by various investigations on pipes. The values of n, will be found useful in checking actual observations. Those of m (although less reliable), should be employed when it is required to predict the discharge of a pipe from experiments made on one of the same construction, but of a different size.

It is fortunate that the irregularities and abnormalities produced by bad adjustment, incrustations, and other less easily recognised factors, do not appear to have any great influence on the value of m. Therefore, if k, is calculated for the observed pipe, we may, with very fair accuracy, assume that the effect of different diameters is sufficiently allowed for by taking m, as equal to 1·25, and assuming that the irregularities influence the value of k, only.

The following formulæ have been proposed :

Unwin (1886), (*Industries*)	$\frac{h}{l}=\frac{0{\cdot}0004\,v^{1{\cdot}85}}{d^{1{\cdot}127}}$	Asphalted cast iron.
	$=\frac{0{\cdot}0007\,v^{2}}{d^{1{\cdot}16}}$	Incrusted cast iron.
Flamant (1892) (*A. P. et C.*, 1892, vol. ii.)	$\frac{h}{l}=k\frac{v^{1{\cdot}75}}{d^{1{\cdot}25}}$	k, for new cast iron, 0·000336. Do. in service, 0·000417. Smooth pipes, lead, glass or wrought iron, 0·000236 to 0·000280.
Lea (1907) (*Hydraulics*)	$\frac{h}{l}=k\frac{v^{n}}{d^{1{\cdot}25}}$	

Description.	Lea's values of		Average Values.	
			k	n
For—				
Clean, cast iron	$k=$ 0·00029 to 0·00042	$n=$ 1·84 to 1·97	0·00036	1·93
Old, cast iron .	$k=$ 0·00047 to 0·00069	$n=$ 1·94 to 2·04	0·00060	2·0
New, riveted pipes . .	$k=$ 0·00040 to 0·00054	$n=$ 1·93 to 2·08	0·00050	2·0
Galvanised pipes . .	$k=$ 0·00035 to 0·00045	$n=$ 1·80 to 1·96	0·00040	1·88
Sheet iron asphalted . .	$k=$ 0·00030 to 0·00038	$n=$ 1·76 to 1·81	0·00034	1·75
Clean, wooden pipes . .	$k=$ 0·00056 to 0·00063	$n=$ 1·72 to 1·75	0·00060	1·75
Brass and lead pipes . .			0·00030	1·75

As actual examples, Lea collects for $\frac{h}{l}=\frac{B}{l}v^n=\frac{k}{d^{1.25}}v^n$:

Description.	Diameter in inches.	$\frac{B}{l}=\frac{k}{d^{1.25}}$	n.
New cast iron	3·22	0·00156	1·97
	5·39	0·00079	1·97
	7·44	0·00062	1·96
	12·0	0·000323	1·78
	16·25	0·000214	1·86
	16·5	0·000267	1·80
	19·68	0·00022	1·84
	30·0	0·00003	2·0
	36·0	0·000062	2·0
	48·0	0·000057	1·92
Cast iron, old	1·41	0·0098	1·99
	3·13	0·0035	1·94
	9·58	0·0009	1·98
	36·0	0·000105	2·0
	48·0	0·000083	2·04
	48·0	0·000085	2·00
Cast iron cleaned	1·43	0·0041	1·85
	3·15	0·00185	1·97
	11·68	0·000375	2·0
	48·0	0·000082	2·02
	48·0	0·000059	1·94
Riveted wrought iron or steel	3·0	0·002450	1·88
	11·0	0·000515	1·81
	11·75	0·000470	1·90
	15·0	0·000270	1·94
	38·0	0·000099	2·00
	42·0	0·00011	1·93
	48·0	0·000090	2·0
	72·0	0·000055	1·99
	72·0	0·000077	1·85
	103·0	0·000036	2·08
Wood	44·0	0·0001254	1·73
	54·0	0·0000830	1·75
	72·5	0·0000610	1·72
	72·5	0·0000480	1·93

Perhaps the most general formula is that given in 1903, by Saph and Schroder (*Trans. Am. Soc. of C.E.*, vol. 51, p. 306), which is as follows:

For ideally smooth pipes, *i.e.* brass, glass, lead, etc.

$$\frac{h}{l}=\frac{0·000296}{d^{1.25}}v^{1.75}\text{; with errors of } \pm 7 \text{ per cent.}$$

For commercial pipes of all sorts, including brick and cement lined, the graphic plots of log d, and log k, cover a zone of some breadth, and they get as follows:

$$\text{smoothest pipes, } \frac{h}{l} = \frac{0.000296}{d^{1.25}} v^{1.82 \text{ to } 1.99}$$

$$\text{central line of zone, } \frac{h}{l} = \frac{0.000469}{d^{1.25}} v^{1.74 \text{ to } 2.00}$$

$$\text{roughest pipes, } \frac{h}{l} = \frac{0.000687}{d^{1.25}} v^{1.82 \text{ to } 1.99}$$

This formula is obviously almost useless for general calculation. It is, nevertheless, worth putting on record, since it shows the general laws with some accuracy, and thus forms a guide for cases outside ordinary experience.

Thus, let us assume that we wish to calculate the discharge of a conduit about 7 feet in diameter. There are only (to date of 1912) six cases of experiments on pipes over 48 inches in diameter, so that the ordinary rules are quite inapplicable. A study of Saph and Schroder's diagram, however, shows fairly clearly that:

For old, heavily tuberculated, cast iron, a value not far off $\frac{h}{l} = \frac{0.0006}{d^{1.25}} v^{2.05}$ is most likely. For heavily tuberculated, say 4 years old, $\frac{h}{l} = \frac{0.0005}{d^{1.25}} v^2$ is most likely.

If we take wood stave pipes, the values are more irregular, but we find that $\frac{h}{l} = \frac{0.0006}{d^{1.25}} v^{1.75}$ will be safe; while for riveted pipes, $\frac{h}{l} = \frac{0.0006}{d^{1.25}} v^{1.90}$ appears to be sound.

These figures are, of course, only approximate, and 20 per cent. of errors are by no means unlikely, but one useful deduction can be made: The law, $\frac{h}{l}$ varies as $\frac{1}{d^{1.25}}$ appears to be well founded. Thus, we are fairly justified in assuming that the discharging power of similarly constructed pipes, under the same hydraulic gradient, varies very approximately as:

$$d^{2.55} \text{ to } d^{2.60}$$

It will be obvious that Tutton's formula could be transformed to a form similar to the above. When this is done, it will be found that $\frac{h}{l}$, varies as $\frac{1}{d^{1.29}}$ for formulæ I, II, III, V, and VII; and as $\frac{1}{d^{1.13}}$ for formula IV.

Measurement of the Discharge of a Pipe.—This is generally one of the easiest operations in hydraulics, provided that the pipe is so long that the water takes more than 200 seconds to pass through it.

The process consists in discharging a small quantity of some easily recognised colouring matter into the upper end of the pipe, and noting when this colouring substance appears at the lower end. The interval of time thus being observed, and the length of the pipe being known, the speed with which the colouring matter traverses the pipe is easily calculated. As a general principle, it can be stated that in smooth channels this speed is the mean velocity of the water, provided that the water is moving at a greater rate than Osborne

Reynolds' higher critical velocity (see p. 20). This statement is not at first sight in accordance with the general ideas as to the distribution of velocities over the cross section of a pipe. If, however, the true circumstances of turbulent motion are considered, it will be seen that a particle which at one instant is moving forward very rapidly at the centre of the pipe, will a second later be, not at the centre, but somewhere else, and travelling less quickly; and that the particle now at the centre will have previously been moving more slowly. This continual alteration in speed causes the mean velocity of any individual particle during, say, one minute to be very much the same wherever it may happen to be situated at the moment of observation.

The best proof of this statement is the practical one of discharging a colouring matter, which is recognisable when greatly diluted, into the pipe, and observing the interval between its first definite appearance and cessation at the other end.

The interval of time that elapses will be found to be roughly proportional to the length of the pipe, and the length of the coloured streak rarely exceeds 1 per cent. of the length of the pipe. Thus, even if the mean velocity be considered as uncertain by an amount corresponding to the total length of the streak, it can be determined to within 1 per cent.

In 68 of my own experiments on a 12-inch pipe, 2074 feet long, the greatest extension corresponded to a coloured streak 14 feet in length. The colour first appeared after 598·2 seconds, and disappeared 4 seconds later. Thus, the mean velocity was less than 3·467, and greater than 3·444 feet per second. The value 3·456 feet per second is probably far more accurate than could be obtained by any other method.

In 25 other observations, the flow was purposely obstructed by partially closing a central valve, and by fixing baulks of timber in the pipe. Obstruction was always found to produce an abnormal lengthening of the coloured portion of the water.

According to my own systematic experiments, this method is applicable to pipes up to 18 inches in diameter, and probably to larger ones also. When tested against weirs, (as in 28 observations) the differences are within the limits of error of the weirs. Benzenberg (*Trans. Am. Soc. of C.E.*, vol. 30, p. 380) has also used it with success to gauge a brick conduit 12 feet in diameter. The method may therefore be considered as universally applicable to smooth pipes. Whether it holds in the case of a badly incrusted main cannot as yet be definitely stated; but there is no reason to believe that it will not, and the results of Benzenberg's gaugings give:

Bazin's $\gamma = 0{\cdot}45$, or Kütter's $n = 0{\cdot}014$ (see p. 474) which are closer to the values for a badly incrusted pipe than for a clean one.

As practical details of the work, we should consider whether the water is intended for human consumption. If so, it is obviously inadvisable to add colour to a noticeable degree. The water presumably being fairly clear, any marked tinting is unnecessary.

In such cases I have been accustomed to use bran, or permanganate of potash, since either is removed by filtration, and both are harmless in any case. Bran is easily strained off by a muslin bag, and permanganate of potash is decolourised by the addition of a little ferrous sulphate.

Where the water is not used for human consumption, such dyes as eosin (Benzenberg), or fluorescin (my own standard) are useful. I have even made very accordant field gaugings with a bottle of red or black ink, the red being slightly more easily recognised.

The conditions for accurate work are obvious :

(i) The colour must be added in one gulp, (say a pint for a 4 feet pipe) to the water, *i.e.* poured into the entrance of the pipe.

(ii) The length of the coloured streak increases very rapidly in the first 100 feet length of the pipe, and is then about four times the diameter of the pipe. Thereafter the length increases but slowly, and is never much greater than 1 per cent. of the length of the pipe. It will, however, be plain that the rapid initial increase renders the method comparatively inaccurate if the pipe is considerably less than 400 or 500 feet in length.

(iii) Usually there is no difficulty in recognising the colour, but a white procelain tile laid below the pipe exit is of assistance.

(iv) The length of the tinted mass should be estimated, and if it considerably exceeds one per cent. of the length of the pipe, obstructions may be considered as likely.

The process is also applicable to smooth, open channels ; but it is very rarely that channels of sufficient length to render the method accurate exist. Difficulties occur in the case of rough earthen channels, owing to streaks of colour being arrested, and delayed by eddies. The quantity of pigment which must consequently be added so as to be easily recognisable some 1000 feet downstream, is considerable. I have not therefore been able to obtain satisfactory results.

The following will be found an effective method of discharging colouring matter into the entrance of a pipe. A glass flask such as the usual thin glass long necked flask used in chemistry, is filled with colouring matter so as to leave no air bubbles, and corked. The full flask is slung by a string, neck downwards, and is lowered until it lies opposite the centre of the pipe, and as close in front as possible. The flask is then smashed, preferably with a rod ; but if the depth is too great, a lead weight with a hole in it is threaded on the suspending string, and dropped. The actual instant of fracture can generally be observed, but occasionally the depth is too great. In practice, the small degree of uncertainty thus introduced is usually negligible ; but if the pipe is a short one, or if extreme accuracy is required, the instant of fracture can generally be noted by fixing a bar below the flask, so as to prevent the weight being lost, and noting the time at which the jerk thus caused occurs. By using piano wire as the suspender, and an 8 lb. weight as traveller, I have been able to get accurate results when the pipe mouth was 40 feet below the point of observation. The most favourable place for discharging the colour is not at the entrance of the pipe, but at a manhole ; the length of wire necessary to permit the flask to lie in the centre of the pipe being measured off before lowering. I have rarely weighted the flask, although, when the water velocity is high, this may be desirable.

Effect of Age on Pipes.—With the doubtful exception of wood stave pipes, all mains as they grow older discharge less water under similar conditions. This decrease is caused by the formation of growths and tuberculations on the inside of, or deposits in, the pipes.

These are due to many causes, and the methods for their removal or prevention are very variable. The classification given by Brown (*P.I.C.E.*, vol. 156, p. 1) seems the most natural, and is as follows :

I. Deposits forming on iron pipes only, wholly or partially consisting of the metal, and therefore localised at or near imperfections in their protective coating.

II. Deposits occurring on the inner surface of pipes, culverts, rock tunnels, etc., formed from substances existing in the water, and therefore not localised by imperfections in the protective coating.

III. Accumulations of loose débris, either natural to the water, or formed from deposits of the first two classes, and therefore accumulated in hollows, irregularities, or dead ends of the mains or channels.

(i) To the first class belong the well-known "limpet" formations, occurring in iron pipes. These appear to originate in all waters, whether acid, or neutral, and are apparently entirely due to pinholes, or flaws in the coating of asphalte, or pitch, which is supposed to protect the metal. Limpets may form a continuous covering over the whole interior surface of pipes which have been badly coated, but as each individual incrustation apparently never attains a size greatly exceeding that of a hemisphere of 1 to 1½ inches diameter, the thickness of the coating does not increase indefinitely. Thus, while small pipes may be entirely blocked by limpet incrustations, a large main is at the worst rarely choked by more than 1½ inches of obstruction all round. As will be noticed in the Table on page 442, pipes much above 12 inches in diameter are seldom scraped.

The only preventive is a good coating of bitumen, or pitch, in smooth and perfect layers. Brown (*ut supra*) specifies that the pitch should be free from hydrocarbons which volatise, or decompose with long exposure to flowing water.

I give Angus Smith's original specification, and would remark that the results (due to the fact that the coal tar of his period is now largely consumed by dyers) are no longer satisfactory. The original process when applied to a good mixture of hydrocarbons, yields satisfactory results.

The first coat should be put on the clean, hot iron, and allowed to cool; then the second should be applied, care being taken that it is not sufficiently hot to melt the first.

Coating should be effected by dipping the pipe into the mixture, and not by painting with a brush immersed in the liquid.

(ii) To the second class belong the more regular, and therefore less detrimental incrustations of carbonate of lime, so frequent in mains conveying water containing bicarbonates of lime.

The best preventive is Clark's water softening process (see p. 591). A sufficient interval of time should be allowed before the water enters the mains, for the completion of the reaction, especially if magnesia salts are present.

The most usual method, however, is to scrape the pipes periodically, in the manner discussed on page 439.

(ii*a*) Slime is a deposit of black substance, occurring not only in pipes, but also in tunnels, brickwork, and masonry channels. Slime contains iron, and is apparently a product of organic life, dependent on the presence of iron in water, It is therefore only found in waters which originally contain iron, although there is a certain amount of evidence to show that an acid water naturally free from iron, can, by contact with uncoated metal pipes, acquire enough iron to support the growth of slime.

The preventives consist in the removal either of the organism, or of its food; protective coatings having no effect on slime deposits.

The most usual method is to neutralise the acidity which exists in such waters by means of lime, or soda. This effectually prevents the growth, probably more by killing the organism than by removing the iron.

This neutralisation is effected by filtration through powdered limestone, either mixed in the sand of ordinary filters, or in special filter beds, working at a high velocity.

Sometimes, (although not invariably) simple sand filtration is effective, and where this is the case, it is unwise to also add limestone, lest the calcium carbonate type of deposit be encouraged.

The older method of straining the water through extremely fine wire gauze, is, according to Brown, only effective for a certain period, and sooner or later slime will form. (See p. 549).

It might be gathered from Brown's paper that slime is a common occurrence in pipes and channels, but my own experience is that it is extremely rare, except in waters drawn from lakes, or storage reservoirs. No rules can be given for predicting its occurrence, but in every instance of which I have heard there is a history of peaty deposits in the catchment area of the watershed or brook from which the supply is procured; and, as already stated, the water in its natural state contains iron, and is acid in reaction. It may also be noticed that slime deposits rarely occur beyond the first five miles' length of a main.

(iii) These deposits are generally of the nature of silt, either drawn from the source of supply, or derived from incrustations of the first, or more usually, the second class.

Certain waters containing manganese, or iron, deposit small nodules composed of oxides and carbonates of these metals. As a rule, such waters are treated by deferrisation (see p. 584), or demanganisation processes, before entering the mains. In certain cases, where the supply pipe is long, the whole of the deposit occurs in the main, and the water as delivered does not contain a sufficient quantity of the above minerals to give rise to complaint. All such loose deposits are very readily brushed out of the pipes by means of the rotary brush used by Deacon on the Vyrnwy main, and described in the *Proceedings of the Institute of Mechanical Engineers*, 1899, p. 502.

The matter is of great importance in a long line of pipes crossing a series of valleys, as the nodules accumulate at the bottom of each depression, and if the water velocity is not great enough to lift them over the next hill, they form a very efficient frictional brake, producing a more and more marked effect as time goes on.

Angus Smith's Process.—Information on the exact nature of the process formerly applied by Angus Smith is somewhat uncertain, but the following details are given by Wood (*Rustless Coatings*).

Coal tar was distilled until the naphtha was removed, and the material was deodorised, and had the consistency of melted wax. Five or six, or even eight per cent. of linseed oil was then added, and was well stirred in. The pipes having been cleaned and freed from all sand, scale, or dirt, and previously heated to 500 degrees Fahr., were dipped vertically into the bath, and remained there until they had attained a temperature approximately equal to that of the bath, *i.e.* 300 degrees Fahr.

In actual practice, it appears to have been more satisfactory (owing to ashes from the furnace becoming attached to the pipes) to dip the cleaned pipes when cold into the bath, and to allow them to attain the temperature of 300 degrees Fahr.

So far as can be judged, the time during which the pipes remained in the bath (roughly 30 minutes for 20-inch pipes, and from 15 to 20 minutes for 4-inch to 12-inch pipes) was the most important factor; although it was found that distillation of the coal tar to a consistency of pitch gave bad results.

The modern equivalents appear to be compositions of pitch and linseed oil, heated to 250 or 300 degrees Fahr.

Wood states that a mixture of nine parts of pitch, with two parts of boiled oil, gives a good coating, and that if the oil is increased to three parts per nine parts of pitch, the coating is thicker and requires baking after immersion, in order to produce satisfactory results.

In this case also, the length of the immersion seems to be of vital importance, and failure must usually be attributed to a too hasty removal from the bath.

In modern practice, pipes are usually coated with an asphaltic coating of the character specified by Goldmark (see p. 460), or Brown (see p. 437).

There are also many proprietary articles (usually paints with an inflammable vehicle) which have apparently given satisfaction under very severe tests. Care is needed during application owing to the inflammable vapour given off by the paint.

The subject is an important one, and but little is really known about it. The actual facts appear to be that any efficient coating is costly, and as the difference between efficient and non-efficient coatings only becomes noticeable after the lapse of time, experience accumulates but slowly. The one definite fact is that "Angus Smith's" coating, is a polite expression for reliance on the maker's experience.

If Angus Smith's coating is specified, the pipes when delivered should be covered with a smooth, adherent coating, showing no flaws or pinholes, and without brush marks. It is doubtful whether any maker pretends to do more, and unless special experiments have been made, it is certainly futile to ask for more.

Cleaning and Scraping Pipes.—The general character of the deposits formed has already been discussed, and approximate indications will be given of the diminution in discharge thus produced. Such incrustations or growths can be removed either manually, or by scrapers; and reference to the formulæ for "cleaned cast-iron" pipes shows that a discharge equal, or very nearly equal, to that obtained in the new pipe, may be expected after thorough cleaning.

I do not propose to describe the methods used either in manual or hydraulic scraping. Hydraulic scraping is far cheaper, and is practicable in all pipes exceeding four inches in diameter; curves and bends of any ordinary radius (3 feet in a 6-inch main) do not affect the process, and it will remove stones, lead, crowbars, and other abnormal obstructions. When the main has been designed for hydraulic scraping, and is properly provided with hatch boxes (see Sketch No. 123), the cost is very small in comparison with the increased discharge obtained.

Amongst British Engineers, therefore, it is customary to scrape pipe mains whenever the discharge falls off sufficiently to seriously interfere with the supply. While it is plain that a Waterworks' Manager is forced to maintain a sufficient supply at all costs, the practice should be entered on with caution.

As already stated, the limpet form of incrustation may occur at every point where there is a flaw in the coating, and once a pipe has been scraped, for all practical purposes it must be regarded as no longer coated. Thus, once mains conveying water producing the limpet form of incrustation have been scraped, it may be anticipated that a renewal of the process will become necessary at very frequent intervals.

The water mains supplying Torquay (Devonshire) are a typical example. Here, according to Ingham (*Engineering*, Nov. 3rd, 1899), a 9-inch and a 10-inch main laid down in 1859 was first scraped in 1866, and thereafter it was found necessary to renew the process yearly.

The quantities discharged by the main were as follows:

	Discharge in gallons per minute.	
	Before Scraping.	After Scraping.
1858 New main	622 approximate	...
1866	317	464
1867	Not observed	564
1868	do.	624
1869	423	659
1870	471	668
1871	496	684
1872	499	581
1896	550	698
1897	600	693
1898	586	708

We thus see that in the first eight years the pipe coated with Angus Smith's composition lost 49 per cent. of its discharging capacity. Thereafter, one year sufficed to reduce the discharging capacity by at least 20 per cent., and often by as much as 30 per cent.

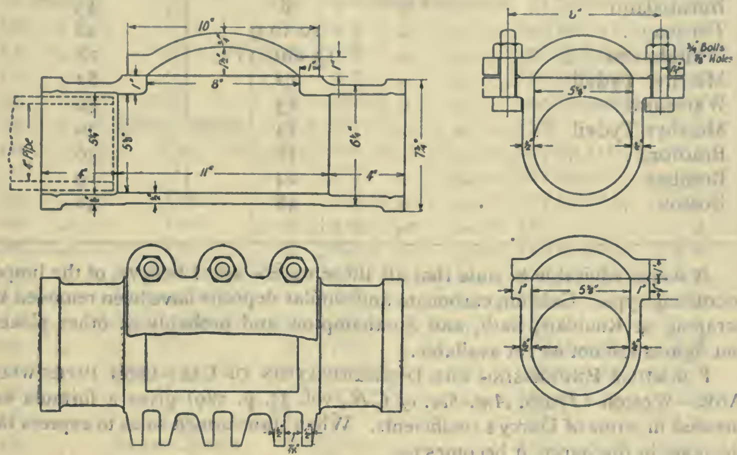
Ingham also gives figures showing that the effect of these thirty-two years' scraping has been to remove a volume of iron equal to ¼-inch thickness all round the circumference of the pipe.

This may be regarded as a case of water possessing abnormal incrusting qualities. At Melbourne, Victoria, the water has similar, but less marked characteristics. Here, it is found necessary to scrape pipes originally properly coated, ten or fifteen years after laying, and thereafter every five or seven years (Ritchie, *P.I.C.E.*, vol. 157, p. 315).

It must also be noted that the Melbourne engineers take extreme care to damage the pipe coating as little as possible, and it is fairly evident that, when compared with the original process at Torquay, they are tolerably successful. There are certain indications that the coating of the Torquay new main was but little damaged by its first scraping; the design of scrapers being now better understood.

We may, nevertheless, consider that in waters of this character scraping is a luxury,—and an expensive one,—unless the pressures in the main are such that a diminution of the pipe thickness by ¼ inch can be regarded as immaterial from the point of view of strength. On the other hand, where the incrustations are derived wholly from the water (*e.g.* in the case of carbonate of lime deposits), scraping is unlikely to lead to an increased rapidity of incrustation, and may therefore be adopted with far less diffidence. It is, however, a rather curious fact that scraping has mostly been adopted in pipes carrying limpet incrusting waters, and it may be inferred that carbonate deposits are less effective in diminishing the discharge.

It therefore appears that in limpet incrusting waters not only should the pipe line be provided with hatch boxes (Sketch No. 123) for inserting the scrapers, but wherever the thickness is determined by strength calculations (and not by practical conditions of casting) an extra ¼ inch to ⅜ inch should be allowed, in order to provide for the gradual consumption of metal by the "limpets." In



SKETCH NO. 123.—Scraper Hatch for 4-inch Pipe.

mains carrying waters which produce incrustations of calcium carbonate, it appears preferable to increase the size of the pipe so as to allow for a deposit of ¼ inch in thickness all round, rather than to remove the growth as formed, by scraping, because a deposit of calcium carbonate increases but slowly in depth after the first ¼ inch has been accumulated, and such deposits are usually not rougher (hydraulically) than a cast-iron pipe.

Few data exist at present for slime deposits. *A priori*, it appears that scraping may be undertaken without increasing the rate of incrustation; but it seems more logical to prevent deposit by previous treatment of the water.

The following table is of value as giving rough indications of the increase in discharge secured by scraping; or, more accurately, of the diminution in discharge which engineers consider sufficiently serious to justify the process. The increase is calculated on the discharge before treatment. The small proportion of pipes exceeding 12 inches in diameter should be noted. The Bombay 24-inch pipe (which, even after scraping, only gave 0·81 of its discharge when new) is interesting, as indicating the possibilities of incrustation in tropical climates.

Place.	Diameter of Main in Inches.	Percentage of Increase.
Aberdeen	4	107
Oswestry	6 and 7	54
Omagh	6	300
Bridge of Allan	6	35
Thurso	6	7
Cowdenbeath	6	23
Cupar, Fife	7	52
Lanark	7	34
Lancaster	8	56
Burntisland	8	43
Torquay	10 to 9	28
Whitehaven	13 and 11	28
Merthyr Tydvil	12	82
Waterford	13	40
Merthyr Tydvil	14	30
Bradford	18	56
Bombay	24	19
Boston	48	30

It seems advisable to state that all these waters are, I believe, of the limpet incrusting type. Calcium carbonate and similar deposits have been removed by scraping at Roublaix, Bath, and Southampton, and probably at other places, but figures are not as yet available.

FORMULÆ EXPRESSING THE DETERIORATION OF CAST-IRON PIPES WITH AGE.—Weston (*Trans. Am. Soc. of C.E.*, vol. 35, p. 289) gives a formula expressed in terms of Darcy's coefficients. When transformed so as to express the decrease in discharge, it becomes :—

$$\text{Discharge at the age of } y \text{ years} = \frac{\text{Discharge when new}}{\sqrt{1+\frac{y}{32}}}.$$

Tutton (*ut supra*) goes into the matter more in detail, and finds for asphalte coated cast-iron pipes :—

New	$v=175\,r^{.62}\,s^{.55}$
Slimy, say one year old	$v=140\,r^{.66}\,s^{.51}$
Very lightly tuberculated, say four years old	$v=132\,r^{.66}\,s^{.51}$
Do., say six years old	$v=124\,r^{.66}\,s^{.51}$
Lightly tuberculated, say eight years old	$v=116\,r^{.66}\,s^{.51}$
The average state of distribution mains, say ten years old	$v=108\,r^{.66}\,s^{.51}$
Fourteen years old, varying with the amount of tuberculation	$v=100\,r^{.66}\,s^{.51}$
Eighteen years old, varying with the amount of tuberculation	$v=90\,r^{.66}\,s^{.51}$
Very heavily tuberculated, twenty-five years old	$v=80\,r^{.66}\,s^{.51}$

This set of formulæ (although doubtless somewhat complicated), in my opinion very accurately represents the actual effects of age. Not only does the constant C_1, decrease, but the loss of head varies as a higher power of the velocity, as the age of the pipe increases.

Hill (*Proc. of American Assoc. of Waterworks' Engineers*, 1907, p. 352) gives a table, which, with the addition of five cases given by Bruce for pipes at Bombay (*P.I.C.E.*, vol. 162, p. 139), and four by Weston (*ut supra*), is as follows :

Diameter of Pipe.	Coating.	Age in Years.	Annual Reduction in C. from Hill's Theoretical Value when clean.	Percentage of Decrease in Discharge per Year.
6 inches	(?)	13	3·6 from values of 75 to 82	4·2
10 ,,	(?)	6	2·3 ,, ,, 95 to 101	2·3
12 ,,	(?)	2	6·8 ,, ,, 96 to 103	6·8
12 ,,	(?)	15	2·7 ,, ,, 96 to 104	2·7
14 ,,	(?)	18	1·2 ,, ,, 104 to 108	1·2
16 ,,	Angus Smith	8·5	2·1 ,, ,, 116	1·8
16 ,,	Tar	18	1·2 ,, ,, 107 to 110	1·1
16 ,,	(?)	18	1·0 ,, ,, 107 to 110	0·9
20 ,,	Tar	11	4·2 ,, ,, 117 to 123	3·7
20 ,,	Tar	5	3·0 ,, ,, 118	2·3
24 ,,	Tar	3	6·7 ,, ,, 118 to 123	5·6
24 ,,	Angus Smith	15	1·1 ,, ,, 123	0·9
24 ,,	Angus Smith	24	1·6 ,, ,, 123	1·3
24 ,,	Angus Smith	16	2·3 ,, ,, 123	1·9
30 ,,	Tar	9	7·3 ,, ,, 118 to 123	6·8
32 ,,	Angus Smith	42	1·0 ,, ,, 118 to 123	0·8
36 ,,	Tar	7	9·1 ,, ,, 124 to 127	7·3
48 ,,	Tar	17	2·1 ,, ,, 143	1·5
48 ,,	Angus Smith	8	3·9 ,, ,, 143	2·8
48 ,,	Angus Smith	10	1·0 ,, ,, 143	0·7

The formula here used is the usual one $v = C\sqrt{rs}$, and the values of C, given are those for a clean pipe.

Where only a single value of C, is given, it is recorded as the result of experiments on a new pipe (not necessarily the same pipe), and some of these values appear to me to be abnormal.

It will be plain that the annual rate of decrease in C, is not constant, but falls off as the age increases.

Hill determines the value of C, for new pipes by a special formula ; which, in some cases, gives values differing as much as 5 per cent. from those obtained from Tutton's formulæ. In calculating the percentage of decrease most weight should therefore be given to figures which agree fairly closely with those stated by Tutton. According to Hill, the percentage of decrease in the discharge of a pipe is affected by the velocity of the water in the pipes ; but the matter is obviously not of great practical importance.

The experiments recorded by Brackett (*Trans. Am. Soc. of C.E.*, vol. 28, p. 269), give the following results :

Diameter of Pipes.	Age.	C.
4 Inches . . .	38 Years	12–19
6 „ . . .	38 „	27–35, rising to 69–82 after scraping

The above figures can hardly be considered as normal, as the pipes were not coated. As a single example of a pipe apparently unaffected by age, Friend's (*P.I.C.E.*, vol. 119, p. 271) value of C = 114, for a 21-inch pipe, nine years old, may be noted. Many others could be collected.

It must also be borne in mind that the character of the incrustation has far more effect on the discharge than the age of the pipe. Limpet incrustations present a very irregular surface to the water, and cause humps and blebs on the interiors of the pipe. They are therefore far more effective in diminishing the discharge than calcium carbonate incrustations, since these are not only smooth in themselves, but form an approximately continuous coating all over the interior of the pipe.

Slime deposits seem to act very much in the same manner as weeds in a river. The strings of slime wave in the water, and produce a braking action.

So far as can be traced from the original records, the figures tabulated above refer to limpet incrustations.

Darcy Formula.—The Darcy formula for the discharge of cast-iron pipes is both frequently referred to and used by engineers.

Putting Unwin's modification (see *Encyc. Brit.*, Article "Hydromechanics," p. 485) into the form $v = C\sqrt{rs}$, we get:

(*a*) Clean and uncoated cast-iron pipes:

$$C = \frac{113\cdot4}{\sqrt{1 + \frac{1}{12d}}}$$

(*b*) Slightly incrusted cast-iron pipes:

$$C = \frac{80\cdot2}{\sqrt{1 + \frac{1}{12d}}}$$

(*c*) New cast-iron pipes coated with pitch, no incrustations:

$$C = \frac{139\cdot2}{\sqrt{1 + \frac{1}{12d}}}$$

where d, is expressed in feet.

The original experiments do not extend beyond about $d = 1\cdot8$ foot. The accuracy of the formula even within these limits is probably not as great as Tutton's. It is, however, very greatly in favour with engineers who maintain their mains in first-class condition. I believe this is due not so much to the fact that the formula applies well to such mains, as that it gives a very excellent "concealed factor of safety," especially in the larger sizes.

Cast-Iron Pipes.—Thickness of Pipe Metal.—The theoretical formula for the thickness of a cylinder D, inches in diameter, exposed to an internal pressure equal to p, lbs. per square inch, is given by Grashof as:

$$t_G = \frac{D}{2}\left\{\sqrt{\frac{3F+2p}{3F-4p}} - 1\right\} \text{ inches} \quad . \quad . \quad \textbf{[Inches]}$$

where F_t is the permissible tensile stress on the material of the cylinder in lbs. per square inch, and where it is assumed that p_i is less than $\frac{3}{4}$ F.

Now, for cast-iron, Unwin (*Machine Design*, 1902, vol. II, p. 10), puts $F=1850$, and thus gets :

p Lbs. per Square Inch.	p Feet Head of Water.	$\frac{t_G}{D}$
75	173	0·021
105	242	0·030
135	311	0·039
165	381	0·048
etc.	etc.	etc.

The actual dimensions of cast-iron pipes can rarely be fixed by this rule. The static head of water on the pipes seldom, if ever, correctly specifies the maximum stress that the mains may be called on to sustain, since the stresses due to unequal earth pressure, or water hammer (to mention only two possible causes), may largely exceed those calculated by the above equation. We are therefore obliged to fall back on practical rules.

Hawksley, some sixty years ago, gave the following rule :

$$t_H = 0{\cdot}18\sqrt{D} \text{ inches.} \quad . \quad . \quad . \quad \text{[Inches]}$$

This may be considered as resulting in but few breaks either from earth or water pressure, or from water hammer. For smaller sizes of pipes, it is also not very far from the minimum thickness that a founder is willing to cast without charging abnormal rates to insure against possible failure.

This minimum thickness is given by Unwin as :

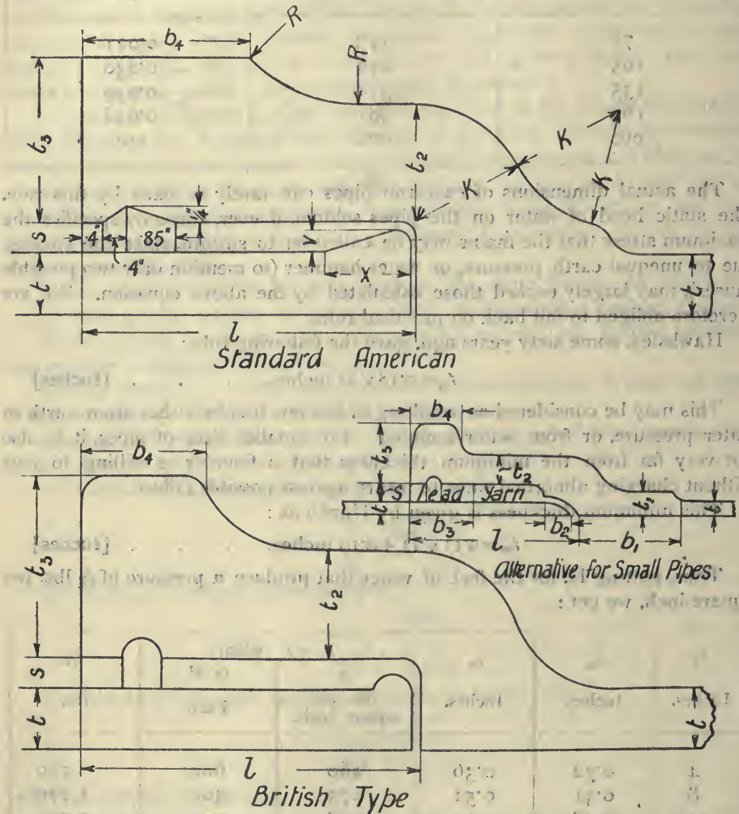
$$t_m = 0{\cdot}11\sqrt{D} + 0{\cdot}10 \text{ inches.} \quad . \quad . \quad . \quad \text{[Inches]}$$

Thus, putting H_i for the feet of water that produce a pressure of p_i lbs. per square inch, we get :

D	t_m	t_H	$t_G = t$ when		W
			p	or H	
Inches.	Inches.	Inches.	lbs. per square inch.	Feet.	lbs.
4	0·32	0·36	260	600	270
8	0·41	0·51	175	400	1,270
12	0·48	0·62	138	315	3,250
16	0·54	0·72	117	270	6,310
20	0·59	0·80	105	240	10,600
24	0·64	0·88	94	215	16,400
30	0·70	0·99	84	190	27,780
36	0·76	1·08	75	170	42,740
42	0·81	1·17	Less than	Less than	61,560
48	0·86	1·25	75	170	84,970
54	0·91	1·32	Less than	Less than	113,300
60	0·95	1·40	75	170	145,000

Now, if we compare these figures with the theoretical value of $t=t_G$ say, found by Grashof's equation, it appears that $t_m=t_G$ for the values of p, or H, tabulated above.

With a view to obtaining an insight into the capacity of pipes of a diameter D, and thickness t_m, to resist unequal earth pressure, I have also tabulated the concentrated central load in pounds, which will produce a stress of 1850 lbs.



SKETCH NO. 124.—Proportions of British and Standard American Pipe Joints.

per square inch, in the metal of the pipe, when regarded simply as a supported beam of 12 feet span, from the formula :

$$W=\frac{\pi}{32}\;\frac{(D+2t_m)^4-D^4}{D+2t_m}\times\frac{1850}{36}=40\,(D+2t_m)^2\,t_m, \text{ approx.} \qquad \text{[Inches]}$$

A consideration of these figures shows that while small pipes of thickness t_m, have ample strength to resist ordinary internal pressures, they are, relatively speaking, more likely to be fractured by straining actions arising from unequal earth pressures than the larger sizes. It is fairly evident that Hawksley's rule gives a pipe which is adequately safe against any ordinary values of water pressure, and also against any probable values of beam loading produced by unequal support.

We may therefore regard Hawksley's rule as a safe standard which may be worked from either up or down, according as circumstances are less or more favourable. Judging by general practice, the excess of strength given by Hawksley's rule is greatest for diameters between 18 inches and 30 inches.

The American Waterworks Engineers' Association recommends the thicknesses given in Table, page 448, for pipes under the tabulated pressures.

The cast iron is specified to bear a central load of 2000 lbs. and to show a deflection of not less than 0·30 inches before breaking, in a bar 26 inches long, 2 inches × 1 inch, loaded on a 24 inch span, lying on its flat side; or to show 20,000 lbs. per square inch in tensile strength.

The pipes are cast in dry sand moulds, in a vertical position, with faucit end downwards.

Pipe Joints.—The opposite sketch (No. 124) represents an English pipe joint, and the American Standard (*Proc. of American Assoc. of Waterworks' Engineers*, 1908, p. 779):

[*All Quantities in Inches*]	[*Notation as in Sketches*]
English (Unwin, *ut supra*)	American (Waterworks Standard)
$t_1 = 1·07\,t + \frac{1}{16}$ inches.	(see Table p. 448).
$t_2 = ·025D + \frac{1}{4}$ inches to $·025D + ·6$ inches.	$·57$ inches $+ ·02D$ approx.
$t_3 = ·045D + ·08$ inches.	$t_3 = 2t_2 = 1·14$ inches $+ ·04D$ approx.
$s = ·01D + ·25$ inches to $·01D + ·375$ inches.	·40 inches up to $D = 14$ inches ·50 inches above.
$b_1 = ·075D + 2\frac{1}{2}$ inches.	
$b_2 = t_2$	
$l = ·09D + 2\frac{3}{4}$ inches to $·10D + 3$ inches.	3·50 inches up to $D = 6$ inches 4·00 inches up to $D = 24$ inches 4·50 inches up to $D = 36$ inches 5·00 inches up to $D = 48$ inches 5·50 inches above.
$b_4 = ·03D + 1$ inch.	1·5 inches up to $D = 14$ inches 1·75 inches up to $D = 20$ inches 2·00 inches up to $D = 48$ inches 2·25 inches up to $D = 72$ inches.
	$R = 1·10$ inches up to $D = 14$ inches 1·15 inches up to $D = 20$ inches.
	$k = t_2 + s$
	$x = \frac{3}{4}$ inches up to $D = 6$ inches 1 inch beyond.
	$V = \frac{3}{16}$ inches up to $D = 6$ inches $\frac{1}{4}$ inch beyond.

The value for t_2, in the American standard, depends somewhat on the pressure; the values being usually increased by 0·05 inch, when the pressures exceed 200 feet head. The tabulated value is a mean.

The dimensions for pipes under pressures greater than 400 feet head require special calculation by Grashof's formula. The general effect is most marked in the dimensions t_2, t_3, and b_4, which should be increased by 0·10 inches for each extra 100 feet head above 400 feet. The American standards run up to 800 feet head, but it is doubtful whether cast iron is an economical material for such pressures, unless the pipes are either of small diameter, or are exposed to intense corrosion.

D Inches.	Pressure.			
	Under 100 Feet Head.	Under 200 Feet Head.	Under 300 Feet Head.	Under 400 Feet Head.
4 . .	0·42	0·45	0·48	0·52
8 . .	0·46	0·51	0·56	0·60
12 . .	0·54	0·62	0·68	0·75
16 . .	0·60	0·70	0·80	0·89
20 . .	0·67	0·80	0·92	1·03
24 . .	0·76	0·89	1·04	1·16
30 . .	0·88	1·03	1·20	1·37
36 . .	0·99	1·15	1·36	1·58
42 . .	1·10	1·28	1·54	1·78
48 . .	1·26	1·42	1·71	1·96
54 . .	1·35	1·55	1·90	2·23
60 . .	1·39	1·67	2·00	2·38
	Test Pressure—			
Up to 20 in. .	300 lb. per sq. in.	300	300	300
Above 20 in. .	150 lb. per sq. in.	200	250	300

Design of Joints in Cast-Iron Pipes.—Sketches No. 125, No. 126, and No. 127 show designs for large pipe joints.

No. 125 is that adopted at Staines, and may be regarded as the practice of the London Water Companies in the year 1898.

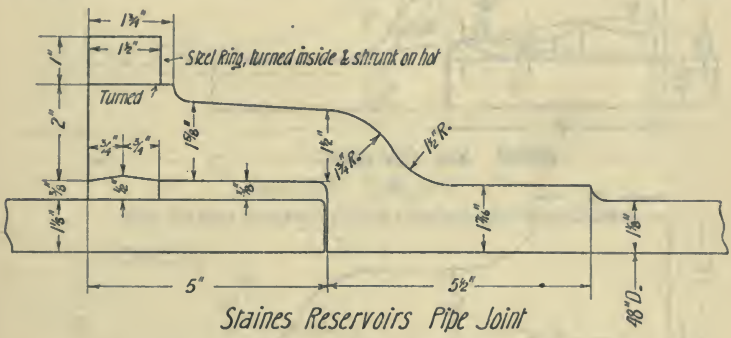
The steel shrunk ring is costly, being practically equivalent to another ton of metal; although, now that the device has ceased to be novel, the expense may be reduced by one-half. It has, however, the great advantage of rendering breakage almost impossible during laying or transit (I have no record of any having occurred). The pipe as a whole is easily cast, and while there is no stop to retain the lead jointing, this is not required with skilled workmen. Similarly, there is no bead on the spigot end, rendering adjustment of each length (so as to secure a continuous inner surface to the pipe), a process necessitating some skill.

The design may be considered as suitable in cases where the labour is efficient, and the trenches are kept quite free from water. I am inclined to believe that the steel ring was unnecessary in this particular case, since all portions of the work were easily accessible. On the other hand, the pipes were laid under railways and roads, carrying heavy traffic, and a break in after years might have been serious.

Sketch No. 126 shows the pipe joint used at Detroit, for 30-inch pipes, with a small stop for lead, and a bead on the spigot end. The latter renders good adjustment of the pipe lengths more certain; but, as designed, renders it difficult to cast the pipe some 4 inches longer than the working length, in order that the accumulated impurities may be removed by cutting off this 4 inches in a lathe. The stop is small, and while it may prevent unskilled work in a joint run with molten lead coming to grief, it is not sufficiently large to be of real assistance in retaining a joint made with lead wool, or leadite, caulked in cold (when the trench is not properly freed from water).

Sketch No. 126 also shows a joint with a deep stop. This is advisable in cases where the jointers are unskilled, or where it is expected that cold caulked joints will frequently occur.

Such a pipe would probably be extremely difficult to cast with the spigot end upwards, and we can only hope to get rid of impurities by large heads over the faucit end, which is obviously a less perfect way than that adopted where the spigot end is cast upwards, and an extra 4 inches of the pipe are cut off.



SKETCH No. 125.—Joint used at Staines.

Modern methods of casting will produce a very satisfactory pipe with head removal only. Broadly speaking, this design asks the foundry to do all the skilled work, and gives the actual pipe layer every possible assistance. The Staines' design, however, gives the foundry man the easier task, and expects the layers and jointers to be first-class workmen, and the trench to be kept very free from water. The Detroit design is probably the most equitable.

Summing up, it would appear that:

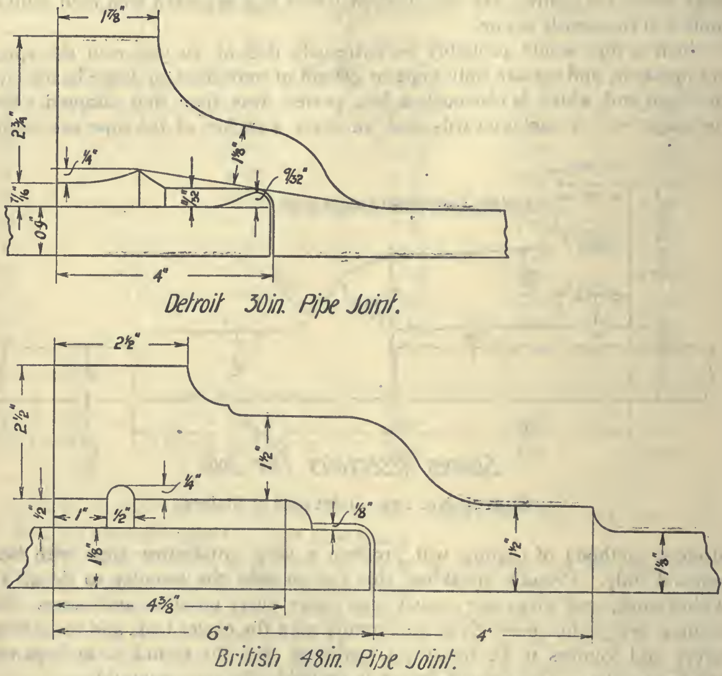
(i) Deep stops and a bead are indicated in cases where the actual laying is difficult, and especially where labour is scarce, and water is likely to occur in the trench. In pipes less than, say, 24 inches in diameter, a bead is almost a necessity, if the pipe line is to be laid with an approximately continuous inner surface.

(ii) A steel, shrunk ring is indicated where the mains are laid in inaccessible places (*e.g.* on a steep hillside, or under rivers), also when laid under roads subject to heavy traffic.

(iii) Deep stops and beads require skilled casting, and therefore good inspection, before the pipes are accepted.

A bead can of course be cast say 5 inches long and the top 4 inches rejected at the cost of some additional lathe work. Similarly, it might be advisable to provide pipes with deep specially turned stops for use in river crossings, and other localities where water cannot be entirely removed, although it must be remembered that cold run joints are undesirable.

Turned and bored joints were frequently employed with satisfactory results in India, when skilled jointers were quite unprocurable. Of late years, however, the lead joint has almost entirely superseded it. A turned and bored joint is quite justifiable where pipe laying is a novelty, but its expense is so great that, as soon as large jobs are undertaken, it becomes cheaper to train jointers specially for the work. (Sketch No. 138, p. 596.)



SKETCH No. 126.—Joints used at Detroit and British Joint with Deep Stop.

The design shown in Sketch No. 128 follows Bateman's practice (*P.I.C.E.*, vol. 126, p. 14). The combination of tubes of uniform thickness and a separate joint ring, also of nearly uniform thickness, is obviously well adapted to prevent internal stresses caused by unequal shrinkage after casting. The design, however, has not been widely copied; and this, I believe, is due both to difficulties in laying, and to greater liability to leakage. The Coolgardie pipe joint is of this type, and the deeper stops for rings exposed to high pressures show a nice graduation in design. (Sketch No. 132.)

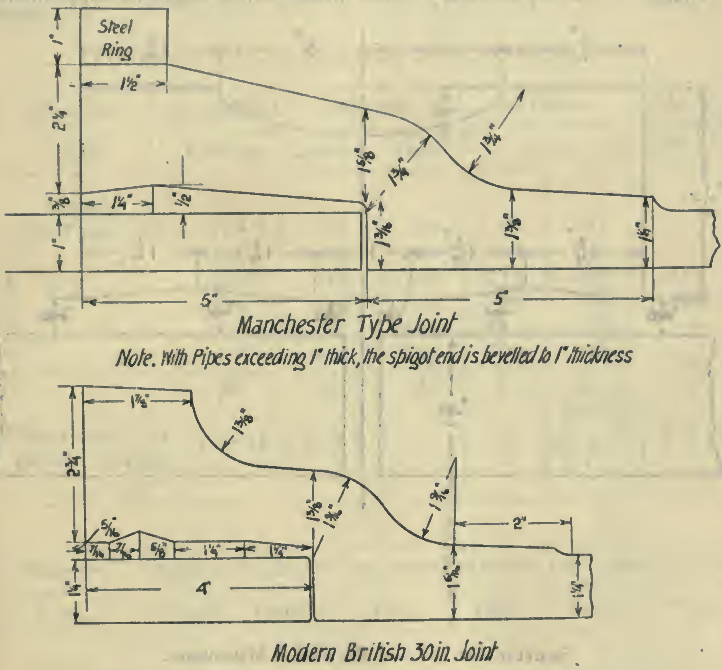
PIPE LAYING.—The following specification was employed at Staines by Messrs. Hunter & Middleton:

(*a*) The pipes are to be cast of best No. 2 pig iron, second melting, cast from a cupola, and are to have the bore perfectly straight and cylindrical, and the thickness exactly uniform.

(*b*) They are to be cast vertically in dry sand, with faucit end downwards, and they must all be clean and solid without sand holes, air holes, or any other flaw.

(*c*) They must be cast with a 6-inch head of metal to secure solidity, and this is to be afterwards cut off in a lathe.

(*d*) The pipe trench must be taken out to a width of 6 inches greater than the outside diameter of the pipes, and a batter of 3 inches per foot of depth will be allowed in the sides.



SKETCH NO. 127.—Joints used at Manchester and London.

(*e*) A faucit hole 3 feet 6 inches long by 9 inches deep, must be excavated round each joint, but the body of the pipe must be bedded solid throughout its entire length.

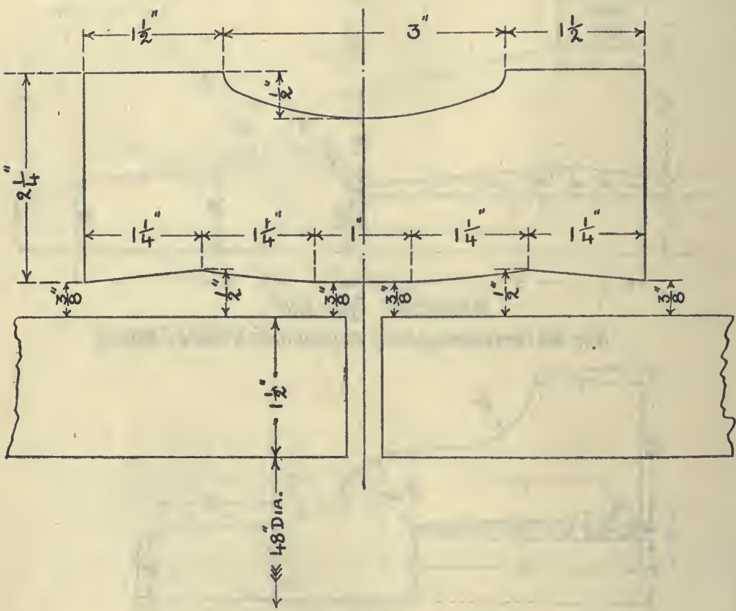
(*f*) The pipes are to be jointed with lead and yarn. The joints are to be made by having the faucit caulked, with good hemp rope yarn, within 2½ inches of the outside, and the space left is to be run full of lead, which must be properly staved up with a caulking iron and a 4-pound hammer, so as to be flush with the faucit when finished, and perfectly water-tight. The lead used must be best soft pig, and a sufficient quantity must be melted at a time to finish each joint at one running.

The following comments may be made:

Clause (*a*). The materials and process specified were those generally used at the time during which the Staines reservoirs were being constructed. It is doubtful whether at present the clause means anything except "best modern practice." The real security against bad metal lies in clauses (*b*) and (*c*), and the test under pressure, and if the founders can satisfy these, any interference with materials or methods seems to be unnecessary.

Clause (*d*). This clause merely specifies the quantities of earthwork to be paid for. The results are usually somewhat in excess of the quantity actually taken out from a timbered trench. Contractors, for some obscure reason, always raise the question of "batters on the faucit holes."

Clause (*e*). A very necessary clause indeed, which might be supplemented



SKETCH. No. 128.—Collar used at Manchester.

by a direction to joint each pipe in place, and adjust true to grade and line by "level and theodolite" in cases where the head available is small.

Clause (*f*). This clause practically means that the whole length of lead joint is to be caulked round twice, and is to be compressed about a quarter of an inch in the process. The use of hemp yarn has been objected to, and it is now believed to be sometimes detrimental to the quality of the water. Jointing by lead alone requires more skill. Jointing with lead wool, or soft wood wedges is a makeshift, which is sometimes resorted to when water cannot be kept out of the trench so as to enable molten lead to be used. Some of the patented lead "wools" will produce very good work in the hands of a skilled man. The joint, however, must be specially designed if these articles are used.

Sketch No. 129 shows the details of timbering and the methods of lowering a 48-inch pipe into a deep trench. The timbering is unusually heavy, but even so it is plain that the excavation should not be kept open longer than is absolutely necessary. It is also plain that a pipe much exceeding 48 inches diameter would be very difficult to handle in a timbered trench unless the circumstances were very favourable.

Large Wrought-Iron or Steel Riveted Pipes.—These pipes are largely employed in America for water works. Hamilton Smith (*Hydraulics*, p. 265) states that their discharge (when coated with asphalt) is the same as that of well made cast-iron pipes, similarly coated. This we now know to be untrue in large sizes. The facts have been collected by Herschel (*115 Experiments on . . . . large riveted Metal Conduits*), and the following table is a resumé :

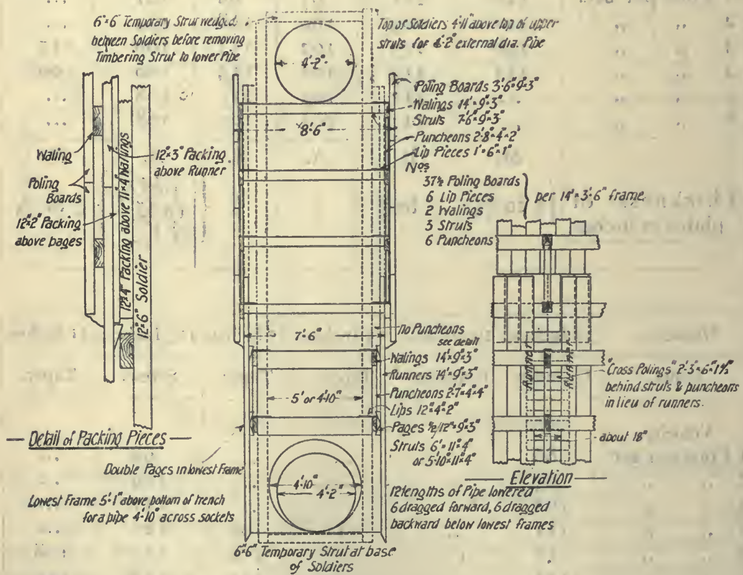
VALUES OF C IN $v=C\sqrt{rs}$ FOR NEW RIVETED PIPES.

Diameter.	72 Inches.	48 Inches.		42 Inches.		38 Inches.
Joints.	Cylinder.	Cylinder.	Taper.	Taper.		Cylinder.
Velocity. 1 Foot per sec.	110	101	97	96	101	...
2 " "	110	109	100	108	104	...
3 " "	108	113	102	113	106	115
4 " "	111	113	104	113	108	109
5 " "	...	112	105	111	108	...
6 " "	...	112	105	100	108	...
	M.	B.	A.	B.	A.	A.
Thickness of plates in inches	$\frac{3}{8}$ to $\frac{11}{16}$	$\frac{1}{4}$ to $\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$ 34% $\frac{5}{16}$ 45% $\frac{3}{8}$ 1%	$\frac{1}{4}$ to $\frac{3}{16}$

Diameter.	36 Inches.	16 Inches.	15 Inches.	13 Inches.	11¼ Inches.	11 Inches.
Joints.	Cylinder.	Cylinder.	Taper.	Taper.	Screw.	Taper.
Velocity. 1 Foot per sec.	86	...	...	...	98	...
2 " "	95	...	...	...	109	...
3 " "	103	...	...	...	117	...
4 " "	111	...	111	108	121	...
5 " "	117	110	113	110	124	108
6 " "	124	...	116	112	126	110
	C.	B.	B.	B.	B.	B.
Thickness of plates in inches	$\frac{1}{4}$	$\frac{1}{4}$	Not recorded	0·07 to 0·08	Not recorded	0·07 to 0·08

For similar old conduits we have:

Diameter.	103 Inches.	72 Inches.	48 Inches.	48 Inches.	36 Inches.	24 Inches.
Joints.	Cylinder.	Cylinder.	Cylinder.	Cylinder.	Cylinder.	Cylinder.
Age.	5 Years.	2 Years.	4 Years.	4 Years.	4 Years.	14 Years.
Velocity in feet per second.						
1 . . .	117	82	97	78	...	...
2 . . .	110	98	103	90	...	...
3 . . .	108	102	105	93	...	80
4 . . .	106	104	104	94	...	...
5 . . .	...	105	104	94	106	...
6 . . .	...	...	104	95	...	...
	B.	M.	A.	A.	C.	B.
Thickness of plates in inches	$\frac{1}{2}$	$\frac{3}{8}$ to $\frac{11}{16}$	$\frac{1}{4}$	$\frac{1}{4}$ to $\frac{3}{8}$	$\frac{1}{4}$	$\frac{3}{16}$



SKETCH No. 129.—Timbering of a Pipe Trench and lowering of a 48-inch Pipe through same.

The letters A,B,C, refer to Herschel's estimates of the accuracy of the experiment, and M, refers to the results obtained by Marx, Wing, and Hoskin (*Trans. Am. Soc. of C.E.*, vol. 44, p. 34).

It is fairly evident that the experiments can hardly be regarded as conducted on pipes of the same hydraulic class, even in the very restricted sense that all "new cast-iron" pipes can be considered as falling under one category.

The question has been dealt with by Kuichling (*Trans. Am. Soc. of C.E.*, vol. 40, p. 535), in discussing the results of the new 72-inch pipe (M, of the above table). This is made with a cylinder joint, so that the walls of the pipe are continuous; but at each joint four rows of rivet heads project into the pipe, and reduce the area. Kuichling considers that the loss of head at such contractions must be allowed for.

No exception can be taken to the principle, and I give the details of the work, although it will be obvious that the coefficients being deduced from Weisbach's experiments on small orifices, cannot pretend to any great degree of accuracy.

We have (see Sketch No. 131, Fig. 1):

A_1, corresponds to a diameter of 72·22 inches.

A_2, is A_1, less the area of the rivet heads, and corresponds to a diameter of 71·78 inches.

$$\frac{A_2}{A_1} = 0{\cdot}98785 = m$$

The loss of head (see p. 786) is given by:

$$h_e = \left(\frac{A_1}{c_c A_2} - 1\right)^2 \frac{v^2}{2g}$$

and Kuichling states that when m, lies between 0·8 and 1, Weisbach's results correspond to $c_c = 1{\cdot}225 + 1{\cdot}45m^2 - 1{\cdot}675m$. Thus, for $m = 0{\cdot}98785$, $c_c = 0{\cdot}98533$, and there being 1984 such constrictions, the loss of head thus produced is 0·34196 feet, when $v = 3{\cdot}846$ feet per second.

When applied to the experimental value, C = 112·6, this correction gives the value of C, that includes skin friction losses only, as C = 118·9.

A more complicated case is that shown in Sketch No. 131, Fig. 2.

Here we have:

$$\frac{A_2}{A_1} = 0{\cdot}97629, \text{ hence } c_c = 0{\cdot}97177,$$

and 14143, such constrictions cause a loss of head of 6·880 feet, and also $\frac{A_1}{A_3} = 0{\cdot}97204$, and the ordinary formula gives a loss of $\left(1 - \frac{A_1}{A_3}\right)^2 \frac{v^2}{2g}$, or, for 6700 constrictions, the head lost is equal to 0·872 feet.

So also, Kuichling uses Weisbach's formula for elbows of a deflection angle ϕ:

$$h_d = \left(0{\cdot}9457 \sin^2\frac{\phi}{2} + 2{\cdot}047 \sin^4\frac{\phi}{2}\right)\frac{v^2}{2g}$$

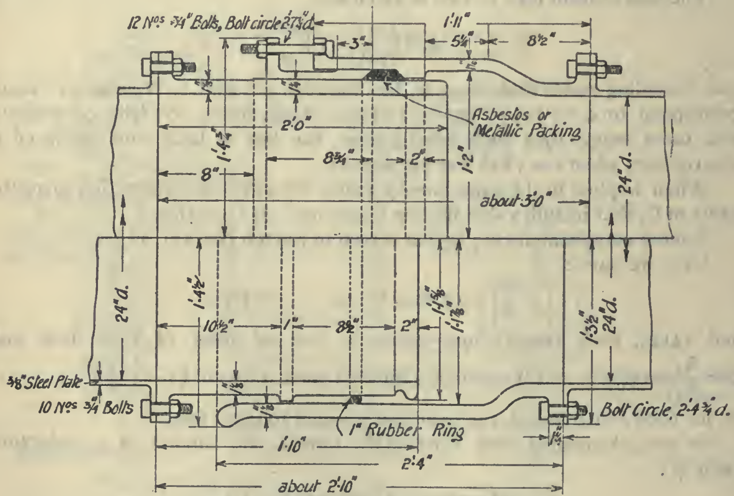
and corrects for stop valves, etc. These last corrections need hardly be applied, since we merely wish to cut out those effects which are directly dependent on the construction of the pipes.

The general result appears to be that in pipes of large diameter, and about ⅜ths of an inch thick, approximately 10 per cent. of the total head lost can usually be explained by resistance due to rivet heads, or discontinuities in the inner surface of the pipe, so that the values of C, given above would be increased in the case of a smooth pipe by some five or six units.

Le Conte (*Trans. Am. Soc. of C.E.*, vol. 40, p. 549), has gone so far as to tabulate the effect on C, of the thickness of the plates (which influences the size of the rivet heads) as follows :

Diameter in Inches.	Plate Thickness in Inches.											
	0·06	0·08	0·10	0·13	0·15	0·16	0·18	0·20	0.24	0·25	0·28	$\frac{3}{8}$.
44 .	...	...	...	120	119	117	116	114	112	111	110	100
42 .	...	...	...	116	115	114	113	111	109	109	108	100
37½ .	...	116	114	112	111	110	109	108	105	104	104	95
30 .	...	114	112	110	109	108	106	105	102	101	100	91
24 .	...	111	109	107	106	104	103	101	98	97	97	87
12 .	100	99	97	95	94	93	91	90	88	87	86	78

Le Conte's table gives the plate thickness in "B.W.G.," and consequently the above thicknesses may err somewhat. It is hardly to be supposed that



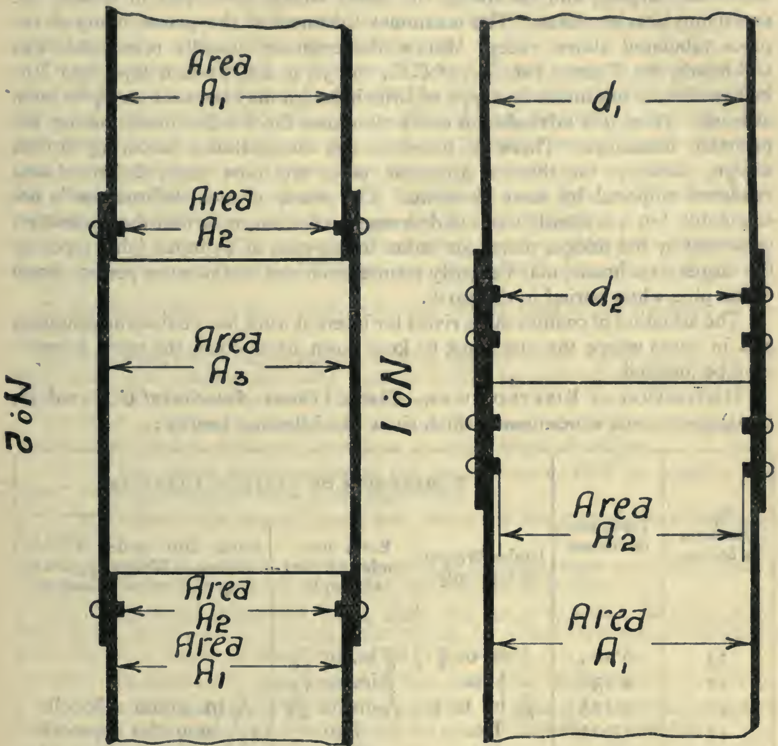
SKETCH NO. 130.—Expansion Joints for Water Pipes.

such a discrepancy will materially affect the coefficients, which apply to pipes about two years' old, subject to little tuberculation, and are mostly obtained from experiments on taper-jointed pipes.

The above facts indicate that the discharge of a riveted iron main may be considered as greatly affected by the type of joint, thickness of plates, and riveting.

When such a pipe has to be calculated, and the plates are abnormally thick, due allowance should be made for the possible diminution in discharge, either empirically (as by Le Conte), or arithmetically (as by Kuichling); and for this reason I have, where possible, tabulated the plate thicknesses.

In new pipes it appears that results not very far from the truth may be obtained by assuming a loss of head between 5, and 10 per cent. greater than that in clean cast-iron mains of the same size, and then allowing for rivet heads, and plate thickness (where the joints are not cylindrical) as above explained.



SKETCH No. 131.—Effect of Rivet Heads and Joints on Discharge of a Steel Pipe.

This, in the case of $\frac{3}{8}$ths-inch plates, and over, finally produces a total increase of lost head equivalent to about 20 per cent., or a decrease in C, of approximately 10 per cent.

Tutton's value is principally founded upon experiments on pipes with plates less than $\frac{5}{16}$ths of an inch thick, so that smaller values of C_1 or C may be expected with heavier plates (page 428).

In considering the possible decrease in discharge, due to age, we can assume that the rivet and joint irregularites are less influential when the remainder of the pipe surface becomes incrusted. A badly incrusted riveted main may be expected to have the same discharging power as a similarly affected pipe of cast iron. Hence, as a value corresponding to Tutton's 105, for a cast-iron pipe with average incrustation, it seems fair to take 100, corresponding to the 10 per cent. extra loss of head unexplained by Kuichling's calculations. Tutton's value 110, for steel pipes refers to pipes from 5 to 8 years' old at the most.

As practical results, we should bear in mind that riveted pipes were, until lately, mostly employed in pioneer work, and that our experience is almost entirely derived from such cases. British engineers are now adopting them somewhat largely, and, following the usual British principles of design, are specifying heavier plates. The maximum thickness of the plates in any of the pipes tabulated above, except Marx's (the engineer actually responsible was Goldmark, see *Trans. Am. Soc. of C.E.*, vol. 38, p. 246) 72-inch pipe, was $\frac{3}{8}$ths inch, while the minimum in a pipe of British design may be taken as $\frac{5}{16}$ths inch, at least. Thus, it is advisable to make allowance for this fact in estimating the probable discharge. There is, however, one circumstance favouring British design, namely,—the thinner American pipes are more easily deformed and rendered elliptical by earth pressure. The effect of such deformation is not calculable, but it evidently tends to decrease the discharge, so that the diminution produced by the thicker plates (or rather in the case of cylinder joint pipes by the larger rivet heads) may be partly counterbalanced by the more perfect shape of the pipe when buried in the earth.

The adoption of counter-sunk rivets for internal work has obvious advantages, and in cases where the size must be kept down, at all costs, the extra expense may be justified.

Distortion of Riveted Pipes.—Clarke (*Trans. Am. Soc. of C. E.*, vol. 38, p. 93) gives some experiments which show the following results :

Pipe Diameter in Inches.	Thickness in Inches.	Compression of Vertical Diameter.		
		Under Weight of Pipe only.	Extra ditto under $5\frac{1}{4}$ Feet of Sand.	Extra ditto under a concentrated Weight applied at Top of Vertical Diameter.
33	0·203	$\frac{1}{4}$ in. or $\frac{1}{8}$	$\frac{3}{8}$ in. or $\frac{7}{16}$	
42	0·238	$\frac{1}{2}$ in.	$\frac{3}{8}$ in. or $\frac{7}{16}$	
42	0·203	$\frac{7}{16}$ in. to $\frac{3}{4}$	$\frac{7}{16}$ in. to $\frac{9}{16}$	$\frac{9}{16}$ in. under 4,800 lb.
42	0·203	Do.	Do.	$1\frac{1}{16}$ in. under 17,000 lb.
42	0·203	Do.	Do.	$4\frac{1}{2}$ in. under 36,000 lb.
42	0·203	Do.	1 in. (sand saturated with water)	

Actual measurements in the trench of 42-inch pipes, 0·203 inches thick, show compressions of $1\frac{7}{8}$ to $2\frac{1}{2}$ inches. It appears that if the earth backfilling is not properly tamped round the main, a large thin pipe may be considerably strained, and possibly stressed beyond the elastic limit, thus producing a permanent set. If, however, the earth is properly rammed up to the top of the pipe, in 6-inch layers, the deformations do not exceed such values as $\frac{1}{2}$, or $\frac{3}{4}$ inch, and cause no undue stress in the metal.

Corrosion of Steel Pipes.—Of late years many cases have occurred (especially in the United States), of marked pitting and corrosion of steel mains. The actual facts are hard to arrive at; but it appears that the steel used in construction was less capable of resisting corrosion than wrought or cast iron, and it has been suggested that this is a general property of steel. The evidence rather appears to suggest that it is a failing common to cheap steel, as delivered by manufacturers who have not been exposed to effective competition. The matter is mentioned as indicating the necessity for obtaining the best possible chemical advice before accepting a tender which may, in other respects, (cost especially) appear to be very satisfactory.

In all cases (above all, where stray currents from tram, or other electric, systems are to be apprehended), the outside of the mains should be carefully protected. The problem is more simple than that of the interior coating. The pipe when hot from the asphalte or tar dip, is carefully wound round with two coatings of burlap, or jute fabric; and this is again tarred over. This coating will generally have to be renewed on the site at each joint, and special precautions should be taken to ensure a good, adherent coating.

Specification.—The following is an abstract of Goldmark's specification for a 72-inch riveted steel pipe (*Trans. Am. Soc. of C.E.*, vol. 38, p. 246):

I. All seams shall be butt seams, with stops exactly fitted to the curvature of the main plates.

II. The round straps uniting adjacent sections shall be placed on the outside of the pipe only. The longitudinal seams shall be united by two butt straps, one on the inside, and the other on the outside of the pipe.

III. The longitudinal joints shall in all cases be placed at the top of the pipe, so that the straps shall be continuous throughout the entire length of the pipe.

IV. All butt straps, both longitudinal and circumferential, shall be rolled to the correct circular curve necessary to fit the pipe closely.

V. The edges of the outside straps, both round and longitudinal, shall be planed for caulking.

VI. The inside longitudinal butt straps shall be the same length as the main plates; they shall be as straight and true as possible, but shall not be caulked.

VII. The outside longitudinal straps, where they are built against the edges of the round straps, shall be planed down to a feather edge for a short distance, and extended under the round straps, the edges of the latter being caulked.

VIII. The splices in the round straps shall be scarphed joints, extending over three rivets.

IX. The under strap at the lap must be scarphed, or thinned by machinery, without being heated; the upper strap is to remain of the original thickness for caulking.

X. The rivet holes shall be punched of $\frac{1}{16}$ inch greater diameter than that of the cold rivet, except in the case of $1\frac{1}{8}$ inch rivets. In this latter case, the rivet holes shall be punched of $1\frac{1}{8}$ inch diameter, on the die side, and reamed to $1\frac{3}{16}$ inch.

XI. All riveting in the shop must be done by machinery, capable of exerting slow pressure, sufficient for the formation of perfect rivet heads.

XII. All burrs caused by punching on the lower side of the plate must be removed by countersinking; all burrs produced by shearing must be removed by filing or chipping.

XIII. The sheets must be pressed closely together, while the rivets are being driven, and until the rivet heads are formed. All rivets which do not properly fill the holes, must be cut out and replaced.

XIV. All riveted seams and joints of every description shall be thoroughly caulked on the outside of the pipe in the best and most workmanlike manner usual in first-class boiler work. The caulking of all seams made in the shop must be done before the coating is applied to the pipe, and every precaution must be taken both in shop and field work, to ensure the utmost strength and tightness.

XV. All plates and rivets must be free from rust, and kept under cover from the time of manufacture of the plates, until the completed pipe is dipped or coated.

XVI. Any plate that shows any defect during the process of punching, bending, riveting, and in manufacturing into pipes, shall be rejected, notwithstanding that the same may previously have been satisfactorily tested.

A tabulation of the dimensions is as follows :

Thickness of— Plates. Inches.	Thickness of— Straps. Inches.	Diameter of Rivets, Cold.	Maximum Stresses in lbs. per Square Inch. Plates. Gross Section.	Plates. Nett Section.	Rivets. Shearing.	Rivets. Bearing.	Maximum Head of Water on Pipe.
$\frac{11}{16}$	$\frac{1}{2}$	$1\frac{1}{8}$	11,000	12,940	5,740	14,600	484 feet
$\frac{5}{8}$	$\frac{1}{2}$	$1\frac{1}{8}$	11,200	13,200	5,200	14,800	448 „
$\frac{9}{16}$	$\frac{3}{8}$	1	11,400	13,200	5,520	15,800	410 „
$\frac{1}{2}$	$\frac{3}{8}$	1	11,600	13,600	5,200	16,400	371 „
$\frac{7}{16}$	$\frac{3}{8}$	$\frac{7}{8}$	11,800	13,500	5,020	15,200	330 „
$\frac{3}{8}$	$\frac{3}{8}$	$\frac{7}{8}$	12,000	14,000	5,300	15,200	288 „

The longitudinal straps were 11 inches wide on the outside, and 16½ inches on the inside ; the circumferential straps being 11 inches in width.

The coating was composed of natural California asphalte, mixed with enough liquid asphalte of over 14 degrees Beaumé gravity to fill the voids in the dry rock.

The mixture was heated by steam coils, and it was found that a good adherent covering was produced on the pipe after one hour's treatment in the bath.

The details of the process evidently require careful previous study, as the time in the bath is longer than is necessary in the case of the typical Angus Smith mixture.

We may further note as follows :

(i) The design secures only one longitudinal joint, at the cost of a circumferential joint at every 9 feet 2 inches. If two longitudinal joints were used, a circumferential joint would occur at every 19 feet : and in view of Kuichling's investigations, the latter seems to be a better system hydraulically.

(ii) The caulking on the outside is not what would be adopted were water-tightness the sole object in view, since inside caulking is far more efficient. The idea is plainly to avoid damage to the pipe coating, and it is satisfactory to know that the main is water-tight.

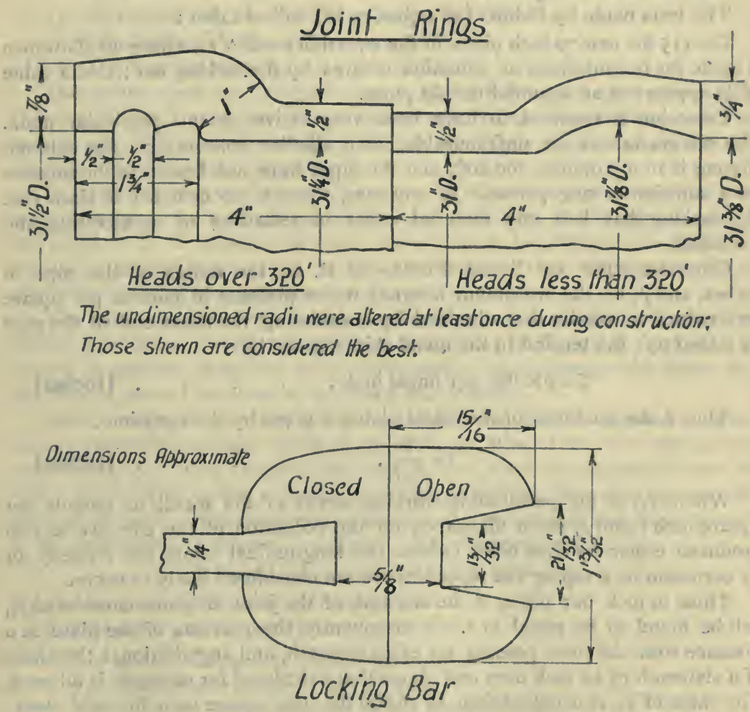
(iii) The first part of paragraph X, is not precisely "first-class work," as specified in the second sentence. The whole paragraph, however, shows a very nice appreciation of the problem of securing satisfactory work at a cheap rate. The "second-class work" is placed where it will do least harm, and where impact stresses will be least. The graduation of the stresses shown in the table is equally commendable. The pipe line is about 4500 feet long, and supplies a power station. Thus, water hammer is possible, and, so far as practicable, the design gives the greater margin of strength at the lower end, where the shock will be worst.

Goldmark's specification of the steel is equally complete. Both for acid and basic steel he specifies the maximum percentages of sulphur, phosphorus, and manganese, and gives the ultimate tensile strength, maximum (65,000 lbs.) and minimum (55,000 lbs.) per square inch, elastic limit (as over one-half of the ultimate strength), elongation (not less than 24 per cent. in 8 inches) and reduction of area (at least 48 per cent.), also character of fracture (as silky) and a bending test (free from crystalline appearance), punching test, and a drifting test.

The two last are:

A row of holes (eight in number), ¾ inch in diameter, and 1¼ inch between centres, shall be punched cold in a plate 1¾ inch wide, and 10 inches long, without any cracks.

Two holes ¾ inch in diameter, 2 inches between centres, shall be punched in a plate 3 × 5 inches and then enlarged cold by drifting to 1¼ inch diameter, without cracks.



SKETCH NO. 132.—Locking Bar and Collars for Locking Bar Pipes.

The rivets are specified as:

Ultimate tensile strength between 56,000 and 64,000 lbs. per square inch, elastic limit not less than 36,000 lbs., shearing stress not less than 72 per cent. of the ultimate tensile stress.

The chemical details of this part of the specification are, of course, somewhat obsolete at the present day, but the information given is very complete, and forms a good model.

Locking Bar Pipe.—This was first introduced by Mephan Ferguson of Melbourne, and has been largely used in Australia.

Several variants have lately been adopted, but none of them appear to possess any real superiority.

Each pipe length is composed of one or two longitudinal plates, bent to the correct radius, and connected by locking bars. Sketch No. 132 shows the burring of the edges of the plates, and the method in which the locking bar is closed, as indicated by Palmer (*P.I.C.E.*, vol. 162, p. 80), and shows the exact dimensions of the bar as used for the Coolgardie 30-inch main of $\frac{1}{4}$ inch and $\frac{5}{16}$ths inch plates. The design is intended to develop the full strength of the plates before failure, and is the fruit of repeated full scale experiments. The working stress is $4\frac{1}{2}$ tons per square inch on the gross area of plates with a minimum thickness of $\frac{1}{4}$ inch.

The tests made by Palmer (*ut supra*, p. 88) indicate that:

C=115 for new 30-inch pipes, in the equation $v=C\sqrt{rs}$, where no allowance is made for irregularities or reduction of area by the locking bar; and a value of 83 appears to be assumed for old pipes.

Corrosion is reported to have been very active on this particular main. The circumstances are unfavourable, since alkaline soils occur; the external coating is in my opinion too soft, and the pipes were not heated in the mixture for a sufficiently long period. In any case, there is no evidence to show that the locking bar had any material effect in retarding or accelerating the corrosion.

CONSTRUCTION OF STEEL PIPES.—If R, be the radius of the pipe in inches, and p, be the maximum internal water pressure in pounds per square inch (which is usually that obtained by considering the lower end of the pipe as closed up), the tension in the metal skin is given by:

$$T=pR \text{ lbs. per lineal inch} \quad . \quad . \quad . \quad . \quad . \quad \textbf{[Inches]}$$

Thus t, the thickness of the metal plates is given by the equation:

$$t=\frac{T}{f_1 s_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \textbf{[Inches]}$$

Where s_1, is the permissible working stress of the metal in pounds per square inch; and f_1 is an allowance for the reduction of the effective section produced either by rivet holes (where the longitudinal joints are riveted), or by corrosion or scraping the pipes if these are considered likely to occur.

Thus, in lock bar pipes, if the strength of the joint is alone considered f_1, will be found to be equal to 1. Consequently, the portions of the plate at a distance from the joint possess no extra strength, and an additional thickness of a sixteenth of an inch over and above that calculated for strength is allowed. The value of s_1, is usually taken as 16,800 lbs. per square inch for mild steel; and, in view of the very equable distribution of the stresses at the joints, this may be considered as a low rather than a high value.

For riveted joints put:

d=The diameter of the rivet in inches . . **[Special Notation]**
p=The pitch of the riveting in inches

$$d=1{\cdot}2 \text{ to } 1{\cdot}3\sqrt{t} \qquad p=2\tfrac{1}{4} \text{ to } 2\tfrac{1}{2}d \quad . \quad . \quad . \quad . \quad . \quad \textbf{[Inches]}$$

Then, for single riveted joints, with the rivets in single shear:

f_1=0·58 to 0·37, say 0·50 as a mean ;

and if two cover plates are used, so that the rivets are in double shear, then :

f_1=0·72 to 0·61, say 0·67 as a mean.

For double riveted joints, with the rivets in single shear, it will be found that :

f_1=0·73 to 0·57, say 0·67 as a mean :

and if the rivets are in double shear :

f_1=0·84 to 0·71, say 0·75 as a mean.

The larger values occur in the thinner plates.

Thus, except at and near the joints, the plates have a large excess of strength, and any allowance for corrosion appears to be unnecessary.

The final design of the joint of course includes the determination of the following stresses (see p. 984) :

(i) The tensile stress in the plate along a line of rivets.

(ii) The shearing stress on the rivets.

(iii) The bearing stress on the rivets.

The values given by Goldmark (see p. 460) are low ; but unless the interior of the joints can be caulked, it is inadvisable to reduce the rivet area, as when outside caulking alone is possible, leakage is mainly prevented by the grip produced by the contraction of the rivets.

In cases where water hammer is likely to occur, the stress thus produced may be estimated ; and, if necessary, the thickness of the plates may be increased so as to provide for this. As a rule, it is allowable to assume a somewhat higher value of s_1, for such stresses, so that an increase in the thickness of the plate is not usually required.

The allowance for corrosion is a matter of experience. Corrosion does not usually produce a general and uniform diminution of the plate thickness, but occurs in patches, small pin holes being eaten right through the plates long before any general decrease in thickness has occurred.

Consequently, under the above circumstances, British engineers usually make a steel plate at least $\frac{5}{16}$ths of an inch, or $\frac{3}{8}$ths of an inch thick, quite apart from any stress calculations ; and do not allow any increase above the value obtained by stress calculations if the thickness thus obtained exceeds $\frac{3}{8}$ths of an inch. American engineers use thinner plates, and their practice is far more suited to pioneer conditions. The German rule, *i.e.* :

Minimum thickness of plates=0·20 inch.

may be adopted in favourable circumstances.

Anchoring Pipes.—Consider a bend or elbow in a pipe.

Let d, be the diameter of the pipe in feet.

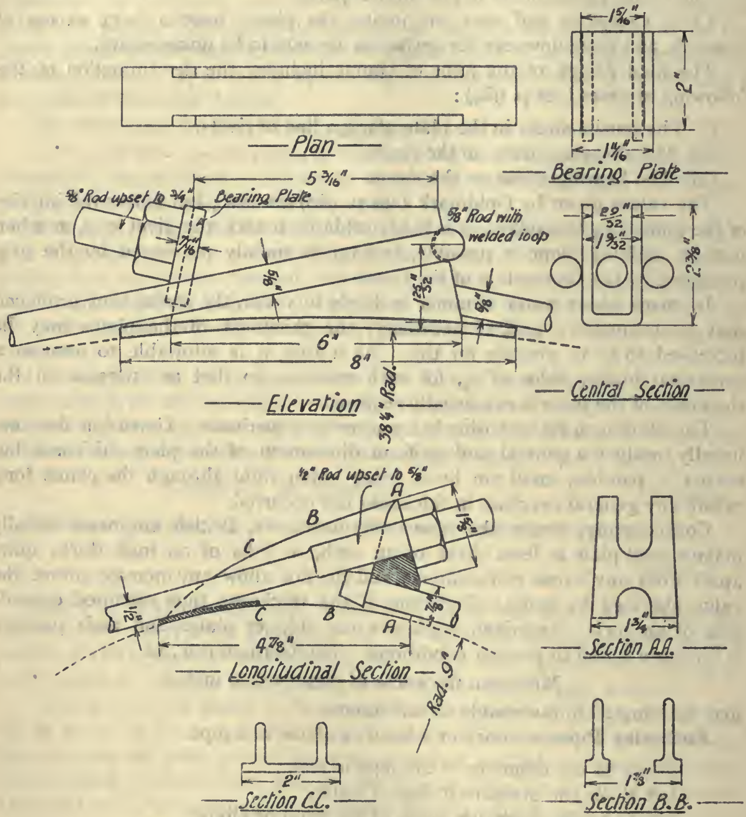
Let H, be the pressure in feet of water.

Let ϕ, be the deflection angle of the bend or elbow.

Then, an outward pressure equal to $2 \frac{\pi d^2}{4} 62{\cdot}5 H \sin\frac{\phi}{2}$ lbs. exists along the line bisecting the angle between the initial and final tangents to the bend. If the velocity be great, H, should be increased by $\frac{v^2}{2g}$.

The stresses thus produced in the pipe metal are not great ; and, as a rule, even the friction of the ordinary lead joints of a cast-iron pipe is sufficient to prevent the bend from being blown out. The stress, however, occurs at a point where the joints are difficult to make, and in good practice it is now usual to bed the outer curve of the bend against a mass of concrete, of sufficient size to prevent motion without producing any stress on the pipe joints. The case where trouble is most likely to occur is at the crest of a hill, in a line of large, riveted pipes under heavy pressure ; and, under such conditions, saddles on the top of the pipes, well bolted down to masses of concrete are sometimes required.

WOOD STAVE PIPES.—Goldmark (*Trans. Am. Soc. of C.E.*, vol. 38, p. 267)



SKETCH NO. 133.—Shoes for Wood Stave Pipes.

describes the construction of a 72-inch main as follows : The timber used was Douglas fir, in place of the usual red wood. This is far harder and stiffer, and trouble was anticipated in using it for staves, on account of the great amount of curvature in the pipe. However, no difficulty was experienced in putting the staves together properly, even in curves of 14 degrees. The specification was severe, requiring the best class of timber, perfectly free from knots, sap holes, season checks, and other flaws. The timber was almost beyond criticism, being practically perfect in appearance. It was, as far as possible, thoroughly seasoned and dried, and was kept under cover until placed in the trench.

The pipe was built of 32 staves, the finished articles being $7\frac{1}{2}$ inches on the outside, $7\frac{1}{8}$ inches inside, and 2 inches thick. The outside was planed to a circle of $38\frac{1}{4}$ inches radius, and the inside to a circle of $36\frac{1}{4}$ inches radius. The radial sides were planes, smoothly finished. No variation of more than $\frac{1}{32}$ inch from the theoretical section was permitted. The staves were specified as 16, 18, and 20 feet long; but actually the lengths used were from 24 to 26 feet, and over. The ends of adjacent staves were at least 12 feet apart; and in all end joints a steel tongue, $1\frac{1}{2}$ inch wide, $\frac{1}{8}$th of an inch thick, and $2\frac{1}{4}$ inches long, was inserted into saw cuts in the staves. The tongue thus lay $\frac{3}{4}$ of an inch in each of the staves jointed, and $\frac{1}{8}$th of an inch in each of the adjacent continuous staves.

The pipe was banded with round steel rods, $\frac{5}{8}$ths of an inch in diameter, for pressures of less than 100 feet; and $\frac{3}{4}$ of an inch for those over 100 feet. The unit stress was 14,500 lbs. per square inch; *i.e.* 4500 lbs. for a $\frac{5}{8}$ths inch, and 6500 lbs. for a $\frac{3}{4}$-inch rod. Thus, for the number of bands per 100 feet length of the pipe under a head of H, feet of water we get:

For $\frac{5}{8}$ths of an inch bands, $N = \dfrac{H \times 62{\cdot}5 \times 6 \times 100}{4500 \times 2} = 4{\cdot}16\ H.$

For $\frac{3}{4}$ of an inch, . . $N = 2{\cdot}9\ H.$

The pipe diameter was assumed to be 6 feet, and the bands are assumed to bear the whole tensile stress.

Sketch No. 133, Fig. I. shows the bands, and steel shoes for their junction.

The design in Sketch No. 133, Fig. II, indicates a malleable iron shoe for a 9-inch pipe, in which the forged loop used by Goldmark is replaced by a bolt-head bearing on the upper horns, A; the upset screwed end being placed between the lower forks, B. The design is decidedly neat and is due to Fuertes (*P.I.C.E.*, vol. 162, p. 154), who states the following formula:

$$N = \frac{330\ HD}{d_b^2 s_1} \quad \text{[Inches]}$$

as giving N, for a pipe D, inches in diameter, where each band has a diameter of d_b, inches, and where s_1, is the permissible working stress in pounds per square inch. The staves in this case had a small bead on one radial side.

Latterly, wood stave pipes have been built up with the bands made of continuous wire wound round in place, under the calculated tension. This appears to be more rational in smaller sizes, although it would be somewhat difficult to devise a means for thus handling mains as much as 6 feet in diameter.

Goldmark's pipe was actually laid with somewhat more than 6 feet vertical dimensions, and slightly less than 6 feet (say 5 feet $11\frac{5}{8}$th inch nett) horizontal diameter; so that deformation under the earth filling may render it approximately circular in form. The pipe was not subjected to more than 120 feet head, and was not exposed to great water hammer, the relief arrangements being extremely well planned.

The detailed design of wood stave pipes has been treated by Adams (*Trans. Am. Soc. of C.E.*, vol. 41, p. 25, and vol. 58, p. 65); the mathematical methods here followed being given by Henny (*Trans. Am. Soc. of C.E.*, vol. 41, p. 71).

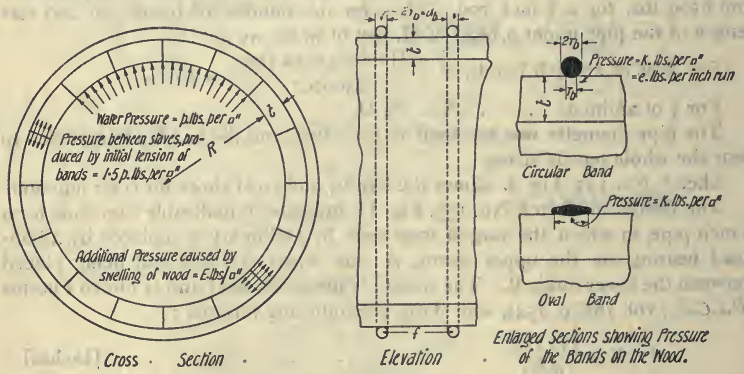
Let:

R = the internal radius of the pipe, in inches.
r_b = the radius of the band section, in inches.
t = thickness of stave, in inches.
f = the spacing between centres of bands, in inches.
Q = the tensile stress in the band, in pounds.
q = the safe ditto.
p = the water pressure, in pounds per square inch.

Now, the bands must be initially stressed when the pipe is empty, so as to prevent leakage between the staves when the water pressure comes on the main. Let this additional stress be represented by X. Then, we assume that $X = \frac{3}{2} ftp$, *i.e.* X, is 150 per cent. of the pressure of the film of water between two staves.

When the pipe is under pressure, we have in addition the tensile stress due to water pressure.

Therefore, $Q = pfR + \frac{3}{2} ftp = q$, say. . . . . . . [Inches]



SKETCH No. 134.—Calculation of Bands for Wood Stave Pipes.

Now, if the tension of the band exceeds $(R+t)e$, where e, represents the safe pressure on the wood per lineal inch of the band, the timber of the staves will be crushed or indented by the band.

Therefore we get:

$$f = \frac{q}{p(R+\frac{3}{2}t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{[Inches]}$$

and f, should not exceed:

$$\frac{(R+t)e}{p(R+\frac{3}{2}t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{[Inches]}$$

or crushing of the staves may occur.

Now, actual experiment appears to indicate that for red wood the permissible value of e, is given by $e = Kr_b$; where K, is not far off 660 lbs. per square inch (although values as high as 800, 900, and 1100 lbs. occur in practice, with satisfactory results), and the width of the portion of the band which presses on the timber is taken as equal to the radius of the band. K, however, varies somewhat with the size of the band, becoming less as the diameter increases. Thus, Henny tabulates as follows :

Diameter of Band in Inches.	e, Pounds per Lineal Inch.
$\frac{3}{8}$	140
$\frac{7}{16}$	153
$\frac{1}{2}$	165
$\frac{5}{8}$	200
$\frac{3}{4}$	232
$\frac{7}{8}$	262

and similar values can be obtained for any other wood by experiment.

The pipe can thus be protected against damage by indentation of the staves by the metal bands. Where the water pressure is small, the spacing is large. Thus, in cases where crushing is feared, oval bands should be used in order to increase the bearing area.

Adams also states that the bands may be fractured by the expansion of the staves on becoming saturated with water.

The stress thus produced is :

$$q_2 = f\{(R+\tfrac{3}{2}t)p + E''t\} \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

and Adams takes E″ = 100 lbs. per square inch.

We thus obtain the following conditions, which should be satisfied in a well designed pipe :

(i) $$Q = fp(R+\tfrac{3}{2}t). \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

and the diameter of the band is given by :

$$Q = \frac{\pi}{4} d_b^2 s_1 = q \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

where s_1, is about 14,000 lbs. per square inch.

(ii) Calculate $q_2 = f\{p(R+\tfrac{3}{2}t)+E''t\}$ . . . . [Inches]

where E″, is an experimental coefficient depending on the expansion of the wood when wetted.

The value of s_2, given by $q_2 = \frac{\pi}{4} d_b^2 s_2$ . . . . [Inches]

should not exceed 20,000 lbs. per square inch.

(iii) Calculate $$e = \frac{Q}{R+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

and ascertain whether the value of the pressure per lineal inch of the bands on the timber exceeds the permissible values.

It will be noticed that very approximately :

$$e = \frac{Kd_b}{2} = Kr_b \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

but the variation is sufficient to justify special experiments with bands of different diameters.

It will be evident that these last two conditions are mainly important when f, and t, are large when compared with R ; *i.e.* in big pipes under small pressures, or in small pipes under heavy pressures.

The thickness of the wood staves is apparently fixed more by stock timber sizes than by any other requirements. Adams gives :

For pipes of 10 to 14 ins. diam. the staves are cut from	$1\frac{1}{2}$ by 4 ins. material.
„ 16 to 48 „ „ „	2 by 6 „
„ 50 to 58 „ „ „	$2\frac{1}{2}$ by 8 „

The maximum head of water rarely exceeds 200 feet, and 180 feet is the more usual value.

The durability of wood stave pipes appears to depend mainly on the way in which they are treated when in use. Like most timber structures, alternate wet and dry conditions are very trying ; and, as a general rule, the pipes should always run full, and should only be emptied when absolutely necessary. Under such circumstances, the wood appears to outlast the bands, and failure finally occurs when the metal is destroyed by rust. It is for this reason that circular, or oval bands, are preferable to the hoop iron type.

There appears to be some difference of opinion as to whether a wooden main should be coated, either on the inside, or on the outside. Inside coating appears to be unnecessary, and outside treatment is not usually adopted in large pipes ; although some of the smaller sizes now manufactured in the Western United States as a stock commercial article are (after banding) systematically coated with asphalte, rolled in sawdust, and again coated. From experience of ordinary wooden structures I am inclined to suggest that if the timber is thoroughly seasoned before the pipe is laid, coating is advisable, and not otherwise. It must be remembered, however, that Goldmark, whose practice must be regarded as most authoritative, and who used wood of a quality not easily procurable under present-day conditions, did not coat his pipes, and is distinctly adverse to the process.

In considering the introduction of wood stave pipes of local material into countries other than the United States, it should be remembered that the timber must be such as is usually considered straight, and long in grain, and fairly flexible ; although this requirement is only vital in curved portions of the pipe. The engineer will have to obtain values for e and E″, by special experiments, and he should be entirely guided by local experience regarding the durability of the particular timber, and in deciding such questions as coating, seasoning, and the amount of tightening to be given to the bands before water pressure is put on the pipes. Broadly speaking, the initial band stress when empty should be about one-fourth that produced by the water pressure. So much depends on the hardness of the timber across the grain (*i.e.* the value of e), and its expansion when wetted (*i.e.* the value of E″), that any general rules are misleading, and more may be learnt from local coopers and an experimental length of pipe under pressure, than from any number of calculations.

I have not discussed the question of beading the radial edges of the staves, in order to prevent leakage. This practice was adopted in some of the earlier wooden mains, but it appears to be now obsolete. Experience in cask making is distinctly adverse to the process, quite apart from the question of economy in labour and material.

CHAPTER IX

OPEN CHANNELS

FORMULÆ FOR THE DISCHARGE OF A CHANNEL IN TERMS OF THE HYDRAULIC MEAN RADIUS AND THE SLOPE.
DEFINITIONS.—Theoretical deduction of formulæ is useless—Kütter's formula—Table of Kütter's *n*—Discussion—**Manning's formula.**
BAZIN'S FORMULA.—Table of Bazin's γ — Classification of channels—Discussion—Classes V and VI have probably no physical existence.
Graphic Solution.—Limits of application of Kütter's or Bazin's formulæ—Thrupp's discussion.
SILT-BEARING WATERS.
Table for Manning's formula.
VARIABLE FLOW IN OPEN CHANNELS.—General formulæ—Possible errors—Values of α—Application to a rectangular channel of uniform breadth—Standing wave—Bore.
PRACTICAL CALCULATION OF BACKWATER CURVES.—Corrections for variations in the cross section or slope of the channel—Examples—Treatment of the case where a standing wave occurs—Drop-down curve.
TRANSPORTING POWER OF CURRENTS OF WATER.—Deacon's experiments—Phases of transport—Lechalas' investigation—Difference between the scouring action of clear and silted water—Relation between depth and velocity in a river carrying silt—Influence of the absolute quantity of silt carried per foot width of the channel—General laws of silt and scour—Comments—Thrupp's values—Comparison—Physical meaning of the equations—Relation between $\frac{\text{Depth}}{\text{Bed width}}$ for a river and a canal —Results obtained by logarithmic plotting of the mean velocity and the mean depth for a river carrying silt—Variation of n in the equation $q = kv^n$ for Phases I, II, and III—Tabulation of the values of V_1.

NOTATION

a, is the area of the cross-section of the channel in square feet.
a_1, and a_2, are the values of a, at specified points (see p. 481).
a_{12}, (see p. 482).
b, is the breadth of the channel in feet.
C, is the coefficient in the formula $v = C\sqrt{rs}$.
C_1, and C_2, (see p. 485).
Throughout this Chapter d, is employed for the sign of differentiation only.
f, is the depth in feet of the water in the channel at any point.
F, is the value of f, appropriate to uniform motion, *i.e.* $v = C\sqrt{Fs}$, and $bvF = Q_t$.
l, is the length of the channel in feet measured along its main stream.
l_{12}, is the value of l, from the point specified by suffix 1, to the point specified by suffix 2.
n, is Kütter's "coefficient of rugosity" (see p. 472), and Manning's formula.
p, is the length in feet of the wetted portion of the boundary of the area a.
Q_t, is the total discharge of the channel in cusecs. In the investigation of Variable Flow Q, is used for the discharge per foot breadth of the channel.
r, is the hydraulic mean radius of the channel in feet. $r = \frac{a}{p}$.
r_{12}, (see p. 482).

R, is used for r, when expressed in metric measure.
s, in open channels is theoretically the sine of the angle of the slope of the water surface. In practice, s, usually refers to the bed slope, and is so used in backwater calculations. In closed channels s, refers to the slope of the hydraulic gradient.
v, is the mean velocity of the water in feet per second.

$$v=\frac{Q_t}{a}, \text{ or } v=\frac{Q}{f}.$$

x, is used for $\frac{f}{F}$.

z_{12}, is the fall of the water surface in feet, measured from the point 2, to the point 1.
α is a coefficient (see p. 481), and
γ is Bazin's coefficient (see p. 474).
ϕ and χ are functional symbols (see p. 483). It must be noted that $\phi(x)$ is tabulated under the argument $\frac{1}{x}$, and $\chi(x)$ under the argument x, the object being to avoid unduly extensive tables.

SUMMARY OF FORMULÆ.

Mean velocity, $v=\frac{Q_t}{a}$.

Hydraulic mean radius, $r=\frac{a}{p}$.

General formula, $v=C\sqrt{rs}$.

Kütter, $C=\dfrac{41{\cdot}6+\dfrac{0{\cdot}00281}{s}+\dfrac{1{\cdot}811}{n}}{1+\dfrac{n}{\sqrt{r}}\left\{41{\cdot}6+\dfrac{0{\cdot}00281}{s}\right\}}$ (see p. 471).

Manning, $v=\frac{1{\cdot}49}{n}\sqrt[3]{r^2}\sqrt{s}$; $C=\frac{1{\cdot}49}{n}r^{0{\cdot}17}$ (see p. 472).

Bazin, $C=\dfrac{157{\cdot}6}{1+\dfrac{\gamma}{\sqrt{r}}}$ (see p. 474).

Backwater curves, $1+\alpha=1+\frac{690}{C^2}$ (see p. 481).

$$\frac{df}{dl}=\frac{s-\dfrac{Q_t^2}{C^2b^2f^3}}{1-\dfrac{1+\alpha}{g}\,\dfrac{Q_t^2}{b^2f^3}}.$$

$l_{12}=\dfrac{f_1-f_2}{s}+F\left\{\dfrac{1}{s}-\dfrac{(1+\alpha)C^2}{g}\right\}\left\{\phi\left(\dfrac{f_2}{F}\right)-\phi\left(\dfrac{f_1}{F}\right)\right\}$ (see Table at end).

Note: $\phi\left(\frac{f_2}{F}\right)$ is found under the argument $\frac{F}{f_2}$.

FORMULÆ FOR THE DISCHARGE OF A CHANNEL IN TERM OF THE HYDRAULIC MEAN RADIUS AND THE SLOPE.—The mean velocity of water in a channel has been defined (p. 44) and is measured in feet per second, and denoted by the symbol v.

Let a, represent the total area of the cross-section of the channel in square feet, and p, be the wetted perimeter in feet, *i.e.* the total length of that portion of the fixed boundary (*i.e.* air boundaries excluded) of a, which is in contact with the fluid. Then

$$r = \frac{a}{p}$$

is defined as the hydraulic mean radius of the channel, and is measured in feet.

In an open channel the slope of the water surface needs no definition, but it is as well to remark that I believe that it cannot be observed. What is usually observed, and is almost invariably used in practical applications of the formulæ, is the bed slope of the channel, which is thus assumed to be of uniform depth.

In a pipe, or closed channel, we can assume that pressure gauges are erected at convenient points, and we may define the slope of the hydraulic gradient as :

$$\frac{\text{The difference of the observed pressures expressed in feet of water}}{\text{The length between the gauges measured along the axis of the pipe in feet}}$$

The symbol s, will be used for the slope.

It is usual in treatises on Hydraulics to give a mathematical investigation showing that $v = C\sqrt{rs}$.

The principles assumed are that water moves as a solid body, and that the laws of friction between this body and the banks and bed of the channel are those usually considered as holding for friction between solid bodies. Since the assumptions depart hopelessly from the truth, I believe that it is more rational to omit the demonstration, and merely to draw attention to its errors.

The formula is probably quite as far removed from a true representation of the facts as its demonstration. Nevertheless, it possesses a certain claim to respect owing to its antiquity, and lends itself to easy calculation, so that practical advantages justify its adoption as a standard.

The equation is usually said to apply to rivers flowing in natural beds ; but this is merely an instance of the conservatism of engineers. In the days when an engineer's field instruments were limited to a level and theodolite, it is possible that the surface slope of a river was really believed to be more easily observed than its discharge.

In the light of the hydraulic knowledge of to-day, it is extremely doubtful whether an uncanalised river possesses a surface slope that can be observed ; and it is quite certain that the discharge can be obtained with less expenditure of time, and greater accuracy.

The $v = C\sqrt{rs}$ equation is suitable for artificial channels of regular section only. When applied to natural channels, it is merely an interpolation formula of very limited range.

I do not propose to discuss any of the earlier equations when C, was taken as constant, or as slightly variable ; such formulæ served their purpose in the past, but are now useless.

The formula most fashionable amongst British engineers is that of Kütter and Ganguillet. Here, $v = C\sqrt{rs}$, and C, is determined by the equation :

$$C = \frac{41{\cdot}6 + \frac{0{\cdot}00281}{s} + \frac{1{\cdot}811}{n}}{1 + \frac{n}{\sqrt{r}}\left(41{\cdot}6 + \frac{0{\cdot}00281}{s}\right)}$$

where it is as well to state that the somewhat peculiar coefficients, such as

1·811, etc., arise from a conversion of the round numbers occurring in the formula when expressed in metric units:

$$C_{\text{metric}}=\frac{23+\frac{0\cdot00155}{s}+\frac{1}{n}}{1+\frac{n}{\sqrt{R}}\left(23+\frac{0\cdot00155}{s}\right)}$$

R, being the hydraulic mean radius expressed in metres.

n, is the "coefficient of rugosity," colloquially termed "Kütter's n."

The table opposite shows the values specified by Kütter and Ganguillet, and also a selection of those determined by other experimenters.

It will be noticed that Kütter himself did not propose to apply his formula to pipes. The fact that other engineers have done so is no doubt complimentary; but I believe that the reason is to be sought not so much in any real physical truth underlying the formula, as in the fact that the statement: Kütter's $n=$ . . . . forms a very convenient description.

The history of this formula is interesting. At the date of its publication very few gaugings of large rivers had been undertaken, and modern methods were either unknown or in an experimental stage. So far as I am aware, none of the large river gaugings accessible to Kütter and Ganguillet were taken by any of the methods which I have discussed; and those on which they placed most reliance (Humphreys & Abbotts' Mississippi Gaugings) were taken by the obsolete double float method, which Bazin (*A.P.C.*, 1884, vol. 7) has shown to largely over-estimate the velocity in deep streams (see p. 54).

The terms in the formula depending on s, were introduced simply to obtain agreement with Humphreys & Abbotts' results, and may therefore be considered as based on very flimsy evidence.

Some of the other observations on smaller rivers were more accurate; and as Kütter and Ganguillet discussed them with great skill, the formula for ordinary slopes (*i.e.* $s=0\cdot01$, to $s=0\cdot00003$) agrees remarkably well with observations, more especially when its date is considered. In fact, I regard it as a very good example of the possibility of obtaining substantial accuracy by a careful discussion of a large number of observations, each individually more or less unsatisfactory.

The continued employment of this formula is therefore not surprising, but it is to be hoped that it may be rapidly abandoned. At present, unfortunately, "Kütter's n," is considered by many engineers as a species of shorthand description of a river, and, consequently, familiarity with the formula is necessary in order to comprehend the work of any engineer employing it.

Diagram No. 5 gives C or n with all the accuracy required in practice. Bellasis' *Hydraulics* contains a very excellent table of C, but since obtaining the diagram I have never required it. Most engineers use a table or some simplified form of Kutter's original equation.

The formula given by Manning (*On the Flow of Water in Channels and Pipes*) is probably nearly as accurate as that given by Kütter, and is easily calculated and remembered. It is:

$$v=\frac{1\cdot49}{n}r^{0.67}s^{0.5}$$

where n, is the value of Kütter's n, for the class of pipe or channel considered.

For example, for earth channels in very good condition, where $n=0\cdot020$,

$$v=74\cdot6r^{0.67}s^{0.5}.$$

Specification of the Channel.	Value of *n*.	Authority.
Timber, well planed and perfectly continuous	0·009	...
Planed timber, not perfectly true	0·010	...
Glazed and enamelled materials with no irregularities, or clean coated pipes	0·010	...
Pure cement plaster	0·010	...
Wood stave pipes	0·010	...
Plaster in cement, one-third sand	0·010	...
Pipes of iron, cement, or terra-cotta, well jointed, and in best order	0·011	Kütter.
Timber unplaned and continuous, new brickwork	0·012	...
Good brickwork, and ashlar, ordinary iron pipes, unglazed stoneware, and earthenware	0·013	Kütter.
Canvas lining on wooden frames	0·015	Kütter.
Foul and slightly tuberculated iron	0·015	...
Rough-faced brickwork	0·015	...
Well dressed stonework	0·015	...
Wooden troughs with battens inside, ½ inch apart	0·015	...
Fine gravel, well rammed	0·017	Kütter.
Rubble masonry in cement, in good order	0·017	...
Tuberculated iron pipes, brickwork or stonework in inferior condition	0·017	...
Earthen channels in faultless condition	0·017	...
Ditto, during heavy silting	0·017	...
Earthen channels in very good order, or heavily silted in the past	0·018	...
Coarse gravel, well rammed	0·020	Kütter.
Wooden troughs with battens inside, 2 inches apart	...	...
Large earthen channels maintained with care	0·0225	Punjab Irrigation Branch.
Small ditto	0·025	Punjab Irrigation Branch.
Channels in average order	0·025	Kütter.
Channels in order, below the average	0·0275	Jackson.
Channels in bad order	0·030	Kütter.
Channels in very bad order	0·035	Kütter.
Channels of worst possible character, with turbulent flow and large obstructions	0·040	Jackson.

I have been accustomed to use this formula for preliminary calculations, and have rarely found that the values obtained by the accurate Kütter form, or by Bazin's equation, differ to such an extent that appreciable errors are introduced.

Bazin's Formula.—Experience has led me to use Bazin's formula of 1897 exclusively, in accurate calculations.

Expressed in English units, we have :

$$v = C\sqrt{rs}.$$

where
$$C = \frac{157.6}{1 + \frac{\gamma}{\sqrt{r}}}$$

Bazin states that ;

Class I. $\gamma = 0.109$ for smoothed cement, or planed wood.
Class II. $\gamma = 0.290$ for planks, bricks, and cut stone.
Class III. $\gamma = 0.833$ for rubble masonry.
Class IV. $\gamma = 1.54$ for earth channels of very regular surface, or reveted with stone.
Class V. $\gamma = 2.35$ for ordinary earth channels.
Class VI. $\gamma = 3.17$ for exceptionally rough earth channels (bed covered with boulders) or weed-grown sides.

The formula is founded on well-selected modern observations only. Two of these, I am aware, were subject to constant errors unknown to M. Bazin, and it so happens that these stand out as markedly less accordant with Bazin's results. Such confirmation of my own private knowledge has greatly increased my confidence in the formula.

$\gamma = 0.109$, Class I, is founded on 42 observations, with r, varying from 0·16 foot to 7 feet, and s, from 0·0001 to 0·0049.

It may be considered as corresponding with

Kütter's formula :—$n = 0.010$; $s = 0.001$: when $r = 0.4$ foot
$n = 0.010$; $s = 0.0001$: when $r = 0.8$ foot
$n = 0.012$; $s =$ anything : when $r = 2.8$ feet.

$\gamma = 0.290$, Class II, is founded on 261 observations, with r, varying from 0·12 foot, to 3·6 feet ; and s, from 0·0001 to 0·0084.

It may be considered as corresponding with

Kütter's formula :—$n = 0.012$; $s = 0.001$: when $r = 0.5$ foot
$n = 0.012$; $s = 0.0001$: when $r = 1.30$ feet

$\gamma = 0.833$, Class III, is founded on 34 observations, with r, varying from 0·3 foot, to 5·0 feet ; and s, from 0·00007 to 0·101.

It practically coincides with

Kütter's $n = 0.017$; for $s = 0.001$; for all values of r.

I should point out that the mere description "Brickwork," or "Rubble," is insufficient to distinguish between this class and Class II. The plotted points representing the actual experiments indicate that two decidedly different classes exist, but the descriptions given by the original experimenters are not sufficient to enable the two classes to be separated before experiments are made.

It would appear that carefully pointed rubble masonry may fall under Class II, but is usually placed in Class III. Similarly, brickwork is generally relegated to Class II ; but, if laid as in tunnel work, or if even small deposits of silt encumber the channel, it crosses over to Class III.

I usually adopt the following classification :

First-class brickwork, or stone laid in the daylight, well inspected, and the whole work laid so as to secure smoothness, can be assumed to rank in Class II ; but may change to Class III, if it carries silted water, and the silt is allowed to deposit in the channel.

Second-class work, laid carelessly ; or first-class work laid as in tunnel work, falls under Class III.

I have found that good work laid by an ordinary builder, who is more accustomed to houses than hydraulic work, seemed to fall into a class approximately represented by $\gamma=0{\cdot}55$. This value has been adopted in Germany for slime-covered sewers, whether made of bricks, masonry, cement, or iron.

$\gamma=1{\cdot}54$, Class IV, is founded on 42 observations ; with r, varying from 0·55 foot to 8 feet, and s, from 0·0001 to 0·014.

It practically coincides with Kütter's $n=0{\cdot}021$; $s=0{\cdot}0001$; for all values of r.

Now, up to this point, the classes may be said, so far as the experiments selected by Bazin show (and I believe that these include practically every recorded experiment that can be considered as of first-class accuracy), to have a real existence. No doubt points representing the experiments do not always fall (even approximately) on the lines representing these classes ; but there is a visible and marked concentration of the points about these lines, along the whole range of the values of r, included in the experiments. In the case of experiments included in the next two classes, which, as will appear, are mostly on natural river channels, the points fall in a very different manner. Concentrations exist near various values of r, and the lines representing these two classes are adjusted so as to pass as close to as many of these concentrations as possible.

A consideration of the graphic plots, or an arithmetical study of the actual results, shows very clearly that these are by no means the only lines that could be chosen ; and, as an illustration, the law :

$$C=\frac{180}{1+\frac{2{\cdot}5}{\sqrt{r}}}$$

seems to lead to a very fair agreement with many series of experiments on channels the descriptions of which are quite sufficiently close to be considered as a class in Bazin's meaning of the word.

It will therefore be apparent that the next two classes are really broad divisions, and that a natural channel may lie anywhere between say $\gamma=2$, and $\gamma=3{\cdot}5$.

$\gamma=2{\cdot}35$, Class V, as Bazin selects it, is founded on 221 experiments, of which only 68 are on artificial channels. r, varies from 0·8 foot to 18·3 feet, and s, from 0·00003 to 0·0146. It may be considered as corresponding with

Kütter's $n=0{\cdot}030$; $s=0{\cdot}001$: when $r=0{\cdot}5$ foot,
$n=0{\cdot}025$; $s=0{\cdot}001$: when $r=6{\cdot}4$ feet,
$n=0{\cdot}025$; $s=0{\cdot}0001$: when $r=18$ feet,

$\gamma=3{\cdot}17$, Class VI, is founded on 74 experiments, of which only 24 are on artificial channels. r, varies from 0·25 foot to 7·4 feet, and s, from 0·00014 to 0·17. It may be taken as corresponding with

Kütter's $n=0{\cdot}030$; $s=0{\cdot}001$: when $r=5$ feet.
$n=0{\cdot}030$; $s=0{\cdot}00001$: when $r=6{\cdot}2$ feet.

The original memoir deserves careful study, and may be found in *A.P.C.* 4me. Trimestre, 1897.

The impression left on my mind is that if a natural river channel permits such a surface slope to exist that it can be observed, it is probable that some formula of Bazin's type will represent the facts fairly accurately. As it is, the s, which we should theoretically use, is the surface slope over a very short length, where the stream is gauged (theoretically speaking, infinitely short; practically, perhaps 100 feet would suffice).

Now, it can be briefly stated that such observations have never been made. What we actually observe is the surface slope over anything between 1000 and 10,000 feet length of the river; and evidently this may, or may not be the s, we assume to be used in the formula. Hence, quite apart from the difficulties of measuring large volumes of water, we really endeavour to compare the discharge with observations of a quantity that may bear little relation to the s, of theory.

Summing up:—In channels of the first four classes, the formula is quite as accurate as the usual hydraulic formulæ; and, with care, it is possible to predict, with a fair degree of exactitude, into what class any given channel falls. In Classes V and VI the formulæ give average values, and all that can really be stated is that in ordinary work, if we observe v, s, and r, we may expect that C, lies between:

The limits $C_1=\dfrac{158}{1+\dfrac{2}{\sqrt{r}}}$ and, $C_2=\dfrac{158}{1+\dfrac{3{\cdot}5}{\sqrt{r}}}$

The real value of Bazin's formula is that for the same channel, and for alterations in r, and v, such as occur in a channel which does not visibly alter its régime, γ is fairly constant. On the other hand, if Kütter's n is calculated under similar circumstances, it will usually be found to vary more than can be explained by possible errors in observation.

The amount of permissible variation in r, and v, depends on circumstances. As general rules, however, v, should not decrease below about 1 foot per second; and if r, increases so much as to alter the general aspect of the channel (*e.g.* if a river overflows its banks, and spreads over wide flats) γ will probably increase materially.

Inside these limits, we have:

$$\frac{\sqrt{rs}}{v}=\frac{1}{C}=\frac{1}{157{\cdot}6}+\frac{\gamma}{157{\cdot}6\sqrt{r}}=0{\cdot}00635\left(1+\frac{\gamma}{\sqrt{r}}\right)$$

Graphical Solution.

Thus, we get a very neat graphical construction (see Sketch No. 135).

Plot $\dfrac{\sqrt{rs}}{v}$, as ordinates, and $\dfrac{0{\cdot}00635}{\sqrt{r}}$ as abscissæ, for each gauging of the river. A large scale must usually be adopted, $0{\cdot}01=2$ inches being none too great in ordinary cases; or, as in sketch the quantities, may be plotted to different scales.

Find the mass centre of these points, either by calculation, or by estimation, according to the accuracy of the observations. Join this point to the point $(0, 0{\cdot}00635)$.

The tangent of the angle between this line and Ox, is γ, and therefore γ is easily measured.

It will be found that the only practical method of calculating Kütter's n, is by trial and error.

Limits of Application of Kütter's or Bazin's Formulæ.—Thrupp (*P.I.C.E.*, vol. 171, p. 346) has collected the evidence on this matter, so far as it exists.

Thrupp states that for slopes flatter than $s=\frac{1}{100{,}000}$

the mean velocity is given by ; $v=\frac{r^{0.7}\ s^{1.25}}{0{\cdot}000002819}$

While I confess myself quite unable to place that confidence in the observations which a formula containing three or four significant figures would indicate, I am at one with Mr. Thrupp in believing that for such slopes the $v=C\sqrt{rs}$, law is not even approximately correct.

Thrupp also states that between

$s=\frac{1}{10{,}000}$ and $s=\frac{1}{100{,}000}$

there is a transition period, where :

$$v=\frac{r^{0.61}\ s^{0.25}}{0{\cdot}1442}$$

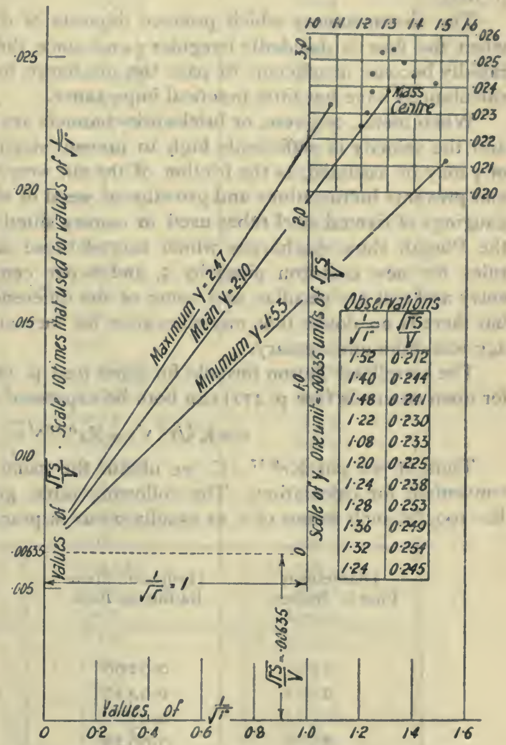
while for s, greater than, $\frac{1}{10{,}000}$, $v=\frac{r^{0.61}\ s^{0.5}}{0{\cdot}01256}$

corresponding to

$$C \doteq 80r^{0.11}$$

in the usual formula.

I agree that, logically speaking, there must be a transition period, but I do not consider that any great error will arise from applying Bazin's formulæ for slopes, as small as $\frac{1}{50{,}000}$, and that is the flattest a practical engineer requires.



SKETCH NO. 135.—Graphical Calculation of Bazin's γ.

There is also fairly good evidence to show that if r, is less than a certain value, the law also alters, very much as is the case in small pipes, due to capillarity. The matter is of small importance, as this critical value of r, is so insignificant that no practical applications are at all likely to occur with such small values of r.

SILT-BEARING WATERS.—The influence of silt on the values of Bazin's γ or Kütter's n, has been incidentally referred to in the preceding work.

As a general rule, it may be stated that water which carries fine silt can be expected to show a somewhat higher value of C, (or lower values of n, and γ) than clear water under similar circumstances. If the silt is allowed to deposit in the channel it may generally be concluded that all channels which, when unsilted, possess values of γ exceeding 1·54, or values of n, exceeding 0·020, will be improved in smoothness; and these values may be considered as the maximum roughness which can occur in a regular channel of which the sides and bed are completely covered with a smooth lining of silt. Similarly, channels which are naturally smoother than the values given will become rougher, and may finally reach a condition specified by $\gamma=1{\cdot}30$, or $n=0{\cdot}018$.

If, however, the silt deposits are so great that waves or ripples of silt form on the bed of the channel, γ may rise to 2, and n, may rise to 0·027 (see p. 703).

The circumstances which produce deposits of this nature generally occur when the flow is decidedly irregular; and since the area of the channel will rapidly become insufficient to pass the discharge for which it is designed, the calculations have but little practical importance.

When metal, concrete, or brickwork channels are used to carry silted water, and the velocity is sufficiently high to prevent deposits, a relatively low value of γ may be assumed, as the friction of the silt wears away small irregularities, and prevents incrustations and growths of weed or slime. Some very accurate gaugings of riveted steel tubes used to convey silted water on certain canals in the Punjab show discharges which exceed those calculated by the ordinary rules for new cast-iron pipes by 5, and 6 per cent. The circumstances at entry and exit are peculiar, and some of the difference may thus be explained; but there is no doubt that any allowance for incrustation and other effects of age was quite unnecessary.

The simplified Tutton formula for pipes (see p. 427) and Manning's formula for open channels (see p. 472) can both be expressed in the form:

$$v=K\sqrt[3]{r^2}\sqrt{s}=Kr^{0\cdot17}\sqrt{rs}$$

Thus, if we put $Kr^{0\cdot17}=C$, we obtain the usual form $v=C\sqrt{rs}$, which is convenient for calculation. The following table gives the values of C, for $K=100$, for such values of r, as usually occur in practice:

Diameter of Pipe in Inches.	Hydraulic Mean Radius in Feet.	$C = 100r^{0\cdot17}$.
1	0·0208	52·4
2	0·0417	58·9
	0·05	60·7
3	0·0625	63·0
4	0·0833	66·1
	0·10	68·1
5	0·1042	68·6
6	0·125	70·7
7	0·1458	72·5
	0·15	72·9
8	0·1666	74·2
9	0·1875	75·6

[*Table continued*

Table continued]

Diameter of Pipe in Inches.	Hydraulic Mean Radius in Feet.	$C = 100r^{0.17}$.
	0·20	76·5
10	0·2083	77·0
11	0·2291	78·2
12	0·25	79·4
14	0·2917	81·4
	0·30	81·8
15	0·3125	82·4
16	0·3333	83·2
	0·35	83·9
18	0·375	84·9
	0·40	85·8
20	0·4166	86·4
21	0·4375	87·1
24	0·50	89·1
	0·55	90·5
27	0·5625	90·9
	0·60	91·8
30	0·625	92·5
	0·65	93·1
33	0·6875	94·0
	0·70	94·2
36	0·75	95·3
	0·80	96·4
39	0·8125	96·6
	0·85	97·3
42	0·875	97·8
	0·90	98·3
45	0·9375	98·9
	0·95	99·2
48	1·00	100·0
	1·10	101·6
	1·20	103·1
	1·30	104·5
	1·40	105·8
	1·50	107·0
	1·6	108·2
	1·7	109·3
	1·8	110·3
	1·9	111·3
	2·0	112·2
	2·1	113·2
	2·2	114·1
	2·3	114·9

[*Table continued*

Table continued]

Diameter of Pipe in Inches.	Hydraulic Mean Radius in Feet.	$C = 100r^{0.17}$.
	2·4	115·7
	2·5	116·5
	2·6	117·3
	2·7	118·0
	2·8	118·7
	2·9	119·4
	3·0	120·1
	3·2	121·4
	3·4	122·6
	3·6	123·8
	3·8	124·9
	4·0	126·0
	4·2	127·0
	4·4	128·0
	4·6	128·9
	4·8	129·9
	5·0	130·8

VARIABLE FLOW IN OPEN CHANNELS.—The question of flow in a channel of non-uniform section is of extreme practical importance in cases where flood damages (caused by backing up of the water levels) need investigation.

The following notation is unusual, but is that which is best adapted for use in practical applications.

Let $v = C\sqrt{rs}$, be the friction equation for the stream under consideration. Let v, be the mean velocity, a, the area of the cross-section of the channel, and r, its hydraulic mean radius at any point distant l, feet downstream of a point designated by the suffix 2.

Then, the ordinary Bernouilli equation, corrected for friction, and for the fact that the square of the mean velocity does not entirely represent the mean energy of the velocity of the water (see p. 15) gives:

$$z_{12} = \frac{(1+a_1)\,v^2{}_1 - (1+a_2)\,v^2{}_2}{2g} + \int_o^{l_{12}} \frac{v^2}{C^2 r}\,dl$$

where z_{12}, is the fall in the surface of the stream, measured from the point 2, to the point 1, and l_{12}, is the distance between these points measured along the course of the stream.

If we assume that $a_1 = a_2$, and differentiate this equation, we get :

$$dz = \frac{(1+a)\,vdv}{g} + \frac{v^2}{C^2 r}dl$$

where dz, represents a decrease in the reduced level of the water surface.

The objections are obvious. We have no assurance that the fact that the motion is varied does not alter the value of C, from that obtained by experiments on steady motion. The value of a is uncertain, even in the case of steady motion, and is consequently still more so under varied motion.

The question was experimentally investigated by Darcy and Bazin (*Recherches Hydrauliques*), and less thoroughly by Ferriday (*Engineering News*, July 11, 1895). The general result is that if a, and C, are assumed to possess the values found by experiments on similar channels in which uniform flow occurs, the observed values of z_{12}, may differ as much as 22 per cent. from the calculated values. The mean error (no regard being paid to sign) is less than 8 per cent., and if the sign is taken into account is less than 3 per cent. Consequently, this last figure best represents the probable agreement of a calculated and an observed backwater curve, when C, and a, have been specially determined by experiments on uniform motion in the channel to which the calculations are applied. In practical applications, errors of 10 per cent. may be regarded as probable, since C, is not likely to be so accurately determined.

The values of $1+a$, according to Darcy and Bazin (*ut supra*) are as follows:

Channel of	$1+a$.	
Rectangular shape; in planed timber	1·052 1·038	In uniform motion. In varied motion.
Do.; with battens, 4 inches apart	1·078	In uniform motion.
Do.; do., 2 inches apart	1·122 1·152	In varied motion. In uniform motion.
Wooden culvert	1·053	do.
Trapezoid; in planed timber	1·048	do.
Masonry walls	1·071	do.
Semicircular channel in cement	1·025	do.
3 cement, 1 sand	1·043	do.
Semicircular; in timber	1·038	do.
Do.; covered with gravel	1·089	do.

a evidently depends on the roughness of the channel, and also to some degree on its form.

Darcy and Bazin suggest the following equation for uniform motion:

$$1+a = 1+\frac{690}{C^2}.$$

Now, let s, be the slope of the bed of the channel, which is assumed to be uniform, and f, be the depth of the water at any point. Then:

$z_{12} = sl_{12}-(f_1-f_2)$. Where f_1, is downstream of f_2.

$dz = sdl-df$. Where df, is positive when the depth increases in the downstream direction.

Now, if Q_t be the quantity of water flowing in the channel, in cusecs, we have in general:

$$v=\frac{Q_t}{a};\quad \text{and in particular, } v_1=\frac{Q_t}{a_1};\quad \text{and } v_2=\frac{Q_t}{a_2}:$$

and we can put:

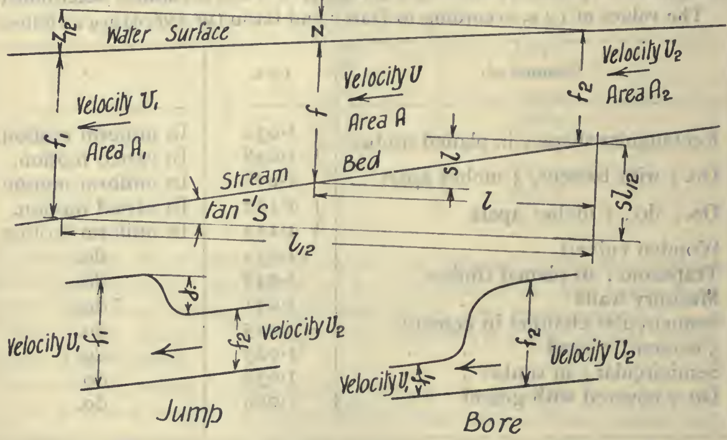
$$\int_0^{l_{12}} \frac{v^2}{C^2 r}dl = \frac{l_{12}Q_t^2}{C^2 a_{12}^2 r_{12}}$$

where a_{12}, and r_{12}, are quantities which are, in a sense, the mean area and the mean hydraulic radius of the channel between the points 1 and 2, and can be calculated when the geometrical relation between a, r, and l, is known for each point of the channel.

We thus obtain the equation :

$$l_{12}=\frac{(f_1-f_2)+\frac{Q_t^2}{2g}\left\{\frac{1+a_1}{a_1^2}-\frac{1+a_2}{a_2^2}\right\}}{s-\frac{Q_t^2}{C^2a_{12}^2r_{12}}}$$

This form of the equation may be applied in cases where the cross-section of the channel alters but little in the length l_{12}. Unless the relation between a and l, be such as to permit a_{12} and r_{12} to be accurately calculated, it is a mere approximation.



SKETCH No. 136.—Diagrams for Variable Flow, Jump, and Bore, in an Open Channel.

In practice, the most accurate solution is obtained from the differential equation which gives :

$$\frac{df}{dl}=s-\frac{(1+a)}{g}\frac{Q^2_t}{a}\frac{d}{dl}\left(\frac{1}{a}\right)-\frac{Q_t^2}{a^2C^2r}$$

This equation can be integrated by assuming that :

$$a=bf:\qquad \text{and } r=f.$$

That is to say, the channel section is assumed to be approximately rectangular, and its breadth b, is assumed to be constant, and to be large in comparison with f.

We then get :

$$\frac{df}{dl}=\frac{s-\frac{Q_t^2}{C^2b^2f^3}}{1-\frac{1+a}{g}\cdot\frac{Q^2_t}{b^2f^3}}$$

or if F, be substituted for the value of f, which occurs in uniform flow when the discharge is Q_t, we have :

$$Q_t = bFC\sqrt{rs} = bF^{1.5}C\sqrt{s}$$

and
$$\frac{df}{dl} = s\frac{1-\left(\frac{F}{f}\right)^3}{1-\frac{(1+a)C^2 s}{g}\left(\frac{F}{f}\right)^3}$$

If we now put $\frac{f}{F} = x$, the integration of this equation leads to :

$$l = F\frac{x}{s} - F\left(\frac{1}{s} - \frac{(1+a)C^2}{g}\right)\left\{\frac{1}{6}\log_e\frac{x^2+x+1}{(x-1)^2} - \frac{1}{\sqrt{3}}\cot^{-1}\frac{2x+1}{\sqrt{3}}\right\} + \text{a constant.}$$

$$= F\frac{x}{s} - F\left(\frac{1}{s} - \frac{(1+a)C^2}{g}\right)\phi(x) + \text{a constant.}$$

Where $\phi(x)$ is termed the backwater function.

In the table on page 1006 (which is due to Bresse) it will be noticed that $\phi(x)$, is tabulated in terms of an argument $\frac{1}{x} = \left(\frac{F}{f}\right)$. This is liable to lead to confusion unless care is taken. With the help of this table we can plot the curve assumed by the water surface. The process is as follows:

The value of f_1, is assumed as known from previous calculation. We assume a value of f_2, and determine the distance l_{12}, between the point where the depth is f_1, and that where it is f_2.

It will be observed that the order now adopted is precisely the reverse of that used in the mathematical discussion. The change of order appears to be necessary, and while it may render the mathematical equations obscure, it greatly assists the practical calculations. Taking the integration from $x = x_2$ (i.e. $f = f_2$) to $x = x_1$, (i.e. $f = f_1$), and reckoning l_{12}, as positive when measured upstream from 1 to 2, the equation becomes :

$$l_{12} = \frac{f_1 - f_2}{s} + F\left(\frac{1}{s} - \frac{(1+a)C^2}{g}\right)\left\{\phi(x_2) - \phi(x_1)\right\}$$

The case most usually considered is where a dam, or other obstruction, exists in the stream.

Thus, f_1 and f_2 are both greater than F, and consequently x_1 and x_2, are greater than 1, although it must be remembered that the function $\phi(x)$, is tabulated by arguments which are less than 1, being $\frac{1}{x_1}$ and $\frac{1}{x_2}$, so that the equation might be better written as :

$$l_{12} = \frac{f_1 - f_2}{s} + F\left(\frac{1}{s} - \frac{(1+a)C^2}{g}\right)\left\{\phi\left(\frac{F}{f_1}\right) - \phi\left(\frac{F}{f_2}\right)\right\}$$

When a sudden fall occurs in the bed of the stream, f_1 and f_2, are less than F, and consequently x, is less than 1.

A similar investigation gives us :

$$l_{12} = -\frac{f_2 - f_1}{s} + F\left(\frac{1}{s} - \frac{(1+a)C^2}{g}\right)\left\{\chi\left(\frac{f_2}{F}\right) - \chi\left(\frac{f_1}{F}\right)\right\}$$

where $\chi(x)$, is another function, which is tabulated as the "drop down function." This function is tabulated under the argument $\frac{f}{F}$.

The above equations are subject to certain exceptions which produce phenomena termed "standing waves," and "bores." These occur when $\frac{df}{dl}$, is infinite.

The standing wave can be produced experimentally, and the following investigation is reliable. The short investigation given on the bore is due to Merriman, and, as he states, must not be regarded as in any way complete or reliable.

(i) The standing wave, or jump, occurs when the value of $\frac{df}{dl}$, is infinite, and positive. Thus, if f_2, represent the depth just before the jump occurs, we have $v_2 = \sqrt{gf_2}$, and s must be greater than $\frac{g}{C^2}$. That is, if C = 100, the slope must be steeper than 0·00322.

The water surface suddenly rises in a wave. The following formulæ for j (the height of this wave) is given by Merriman, and agrees very well with the experiments of Bidone, Darcy and Bazin, and Ferriday.

The velocity-head lost is represented by: $\frac{v_2^2 - v_1^2}{2g}$ and this is expended;

(*i*) In loss in impact, represented by:

$$\frac{(v_2 - v_1)^2}{2g}$$

(*ii*) In raising the whole of the water through a height $\frac{j}{2}$.

Thus, putting $v_2 f_2 = v_1 f_1 = v_1(f_2 + j)$ we get:

$$j = 2\sqrt{f_2 \frac{v_2^2}{2g}} - f_2$$

Since friction is neglected, the computed values are usually a little greater than the observed.

(ii) The Bore.—Here also, $\frac{df}{dl}$, is infinite, but negative. That is to say, $v_2 = \sqrt{gf_2}$, and v_2 is greater than $C\sqrt{f_2 s}$, or s is less than $\frac{g}{C^2}$.

For example, at Johnston, $v_2 = 28$ feet per second. Thus:

$f^2 = \frac{28^2}{32·2} = 24$ feet, and the slope being about $\frac{1}{180}$, we find that C^2, is less than $180 \times 32·2$, or that C, is less than 76.

Practical Calculation of Backwater Curves.—This is most easily effected by dividing the flooded portion of the river into short lengths, over each of which the cross-section can be considered as approximately rectangular, and of uniform breadth, so that the depth of the water alone varies. We then assume that $1 + a = 1$, and use the following formula:

$$l_{12} = \frac{f_1 - f_2}{s} + F\left(\frac{1}{s} - \frac{C^2}{g}\right)\{\phi(x_2) - \phi(x_1)\}$$

in order to calculate l_{12}, for an assumed f_2, the depth at the point $l = 0$, being assumed, or previously calculated as, equal to f_1.

A study of the cross-sections of the river bed will show whether the value of l_{12} thus obtained is sufficiently small, to permit us to consider the assumption regarding the constancy of the breadth of the river over the length l_{12} as correct. If this is not the case, we must assume a new value of f_2, which is somewhat larger than that first obtained, so as to secure a value of l_{12}, which will cause the assumption to be approximately correct. When a satisfactory value of l_{12} is obtained, we must consider the channel above the point 2 (*i.e.* the end of the reach of length l_{12}), and if necessary determine the new values of F, C, and s, say F_1, C_1, and s_1, which are appropriate to the reach above the point 2. We can then determine the length l_{23}, in this reach, at which a depth f_3, occurs, by the equation:

$$l_{23} = \frac{f_2 - f_3}{s_1} + F_1\left(\frac{1}{s_1} - \frac{C^2_1}{g}\right)\left\{\phi\left(\frac{f_3}{F_1}\right) - \phi\left(\frac{f_2}{F_1}\right)\right\}$$

The following calculation will render matters clear:

Take a stream discharging 35·4 cusecs per foot of its width, and let $s = 0·001$, and $C = 100$.

In uniform flow we find that $F = 5$, and that $v = 7·09$ feet per second. Now, let a broad topped weir (*i.e.* weir co-efficient = 2·64 (see p. 128)), 5 feet high, be erected in the channel. The depth over the weir is 5·65 feet, so that the total depth of the stream just above the weir is $f_1 = 10·65$ feet. The flow above the weir will be variable, and we have:

$$l_{12} = (f_1 - f_2)1000 + 5\left(1000 - \frac{100^2}{32·2}\right)\{\phi(x_2) - \phi(x_1)\}$$

$$= 1000(f_1 - f_2) + 3447\{\phi(x_2) - \phi(x_1)\}$$

Assume that $f_2 = 9·65$ feet. We consequently get:

$$\frac{1}{x_1} = \frac{F}{f_1} = \frac{5}{10·65} = 0·469. \qquad \frac{1}{x_2} = \frac{5}{9·65} = 0·518$$

$$\phi(x_2) - \phi(x_1) = 0·1423 - 0·1149 = 0·0274.$$

Therefore, $l_{12} = 1000 + 3447 \times 0·0274 = 1000 + 94·5 = 1094·5$

So that the effect of the back water is to put the depth 9·65 feet, 94·5 feet higher up the stream than would be the case if the water surface was parallel to the bed of the stream.

Proceeding in this manner, we get:

Depth in Feet.	$\frac{1}{x}$	$\phi(x)$.	$\phi(x_2) - \phi(x_1)$.	$1000 + 3447\{\phi(x_2) - \phi(x_1)\}$	Distance above the Weir in Feet.
10·65	0·469	0·1149	...	...	0
9·65	0·518	0·1423	0·0274	1094·5	1094·5
8·65	0·578	0·1818	0·0395	1136·1	2230·6
7·65	0·654	0·2431	0·0613	1211·1	3441·7

This table permits the backwater curve to be set out, and, if necessary, the calculations can now be repeated with shorter intervals between the successive depths, and more correct values for C, and s, if a study of the cross-sections indicates that this is desirable.

Let us, however, suppose that at a distance of 3500 feet above the weir, the channel alters its shape, slope, and roughness, so that it is better represented by $s = 0{\cdot}0005$, and $C = 80$, and that the breadth is now double what it was before. We easily find the new value of F, say $F_1 = 4{\cdot}6$ feet, and the initial depth f_4, can be taken with all necessary accuracy as equal to 7·6 feet.

We now get, if $x_4 = \frac{f_4}{F_1}$, $x_5 = \frac{f_5}{F_1}$;

$$l_{45} = 2000(f_4 - f_5) + 4{\cdot}6\left(2000 - \frac{80^2}{32{\cdot}2}\right)\{\phi(x_5) - \phi(x_4)\}$$

$$= 2000(f_4 - f_5) + 8295\{\phi(x_5) - \phi(x_4)\}.$$

The tabulation is :

Depth in Feet.	$\frac{1}{x}$	$\phi(x)$	$\phi(x_5) - \phi(x_4)$	$8295\{\phi(x_5) - \phi(x_4)\}$	$l.$	Distance above the Change in Section in Feet.
7·6	0·605	0·2019	...	...	...	0
6·6	0·697	0·2852	0·0833	691	2691	2691
5·6	0·822	0·4581	0·1729	1434	3434	6125
5·1	0·903	0·6590	0·2009	1866	2866	8991

The curve can thus be set out, and can be corrected as already suggested, if requisite. The fact that a has been made equal to 0, might also be allowed for, but it is only very rarely that our knowledge of C, is sufficiently accurate to permit this. For instance, were it known that $1+a$ was 1·06, the formulæ merely require us to use $6400(1{\cdot}06) = 6784$ for C^2, or the effective C, would be $\sqrt{6784} = 82{\cdot}4$ say, and accuracy of this character in observations which deal with floods is not likely to be easily attained. Of course, in the case of laboratory experiments, such precision may be arrived at, and may prove a useful exercise.

When s, is equal to, or greater than $\frac{g}{C^2}$ (which will be obvious when the calculation is first attempted), a standing wave occurs. Its position can be fixed by calculating the depth just upstream of the wave, and its height by the formulæ already given. We thus obtain the depth just downstream of the wave, and this can be put for f_2, in the general equation, and l, the distance from the weir to the wave can then be calculated. Standing waves occur in certain cases in irrigation canals. I have applied the formulæ in order to calculate several actual observations. The results are not as satisfactory as might be expected. On the other hand, the drop-down curve before the wave, agrees fairly well with calculation. A change in the value of a may be suspected, or, in such large scale examples shock may play a greater part than in small experimental channels.

The case of a drop-down curve can be similarly treated. Such cases are very infrequent in practice, so that a detailed example is not necessary. As a general principle, the occurrence of drop-down curves should be prevented by a correct design of the channel. Backwater conditions only produce trouble (other than flooding) in water which contains silt. Drop-down curve conditions tend to produce erosion, and if this occurs the water becomes charged with silt, and all the attendant silt troubles may have later to be faced.

Symbols connected with Transport of Material

a, is a coefficient employed by Lechalas in the equation, $P = a\,(v^2 - 0{\cdot}67)$.
b, is the bed width of the river in feet.
d, is the depth of the stream in feet.
k, is a coefficient in the equation $q = kv^n$ (see p. 491).
M, is a coefficient in the equation $d = M\dfrac{v^2 - 1{\cdot}33}{v}$ $\quad M = \dfrac{0{\cdot}0002}{p}$.
As a rule, $d = \dfrac{kv^{n-1}}{p}$, so that $M = \dfrac{k}{p}$, $M\dfrac{v^2 - 1{\cdot}33}{v}$, (see p. 491).
p, is the ratio : $\dfrac{\text{Quantity of silt carried forward}}{\text{Quantity of water carried forward}} = \dfrac{q}{Q}$
P, is the total force transporting the silt.
q, is the number of cubic feet of silt carried forward per foot width of the bed. Thus, the discharge of silt $= bq$ cusecs.
Q, is the number of cubic feet of water carried forward per foot width of the bed. Thus, the discharge of water $= bQ$ cusecs.
v_o, is the velocity of the water at the bottom of the stream, in feet per second.
v_s, is the surface velocity.
v, is the mean velocity of the stream.
V_1 and V_2 (see p. 490).

Summary of Formulæ

Lechalas' investigation of Phases I and II :

$$q = \frac{a}{b}(v_o^2 - 0{\cdot}67) \qquad d = M\frac{v^2 - 1{\cdot}33}{v}.$$

Deacon's investigation of Phase III :

$$q = 0{\cdot}0000028v^5 \qquad d = \frac{0{\cdot}0000028}{p}v^4.$$

General Rules.

I. *Fine Silt*—

Clear water scour when v, is greater than—$0{\cdot}40\sqrt{d}$.
Silted water does not deposit when v, is greater than $1{\cdot}05\sqrt{d}$.
Silted water scours when v, is greater than $1{\cdot}37\sqrt{d}$.

II. *Coarse Sand*—

Clear water scour when v, is greater than $1{\cdot}6d^{0{\cdot}33}$.
Silted water does not deposit when v, is greater than $2{\cdot}2d^{0{\cdot}33}$

III. *Coarse Gravel*—

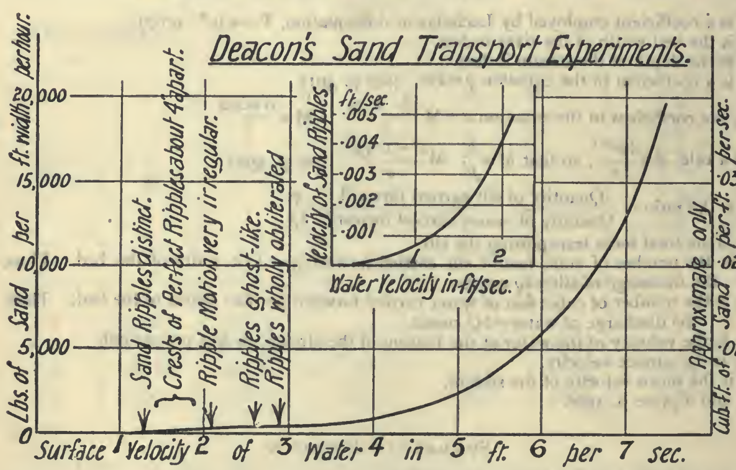
Clear water scours when v, is greater than $2{\cdot}0d^{0{\cdot}25}$
Silted water does not deposit when v, is greater than $2{\cdot}5d^{0{\cdot}25}$

IV. Boulders are moved when v, exceeds $5d^{0{\cdot}25}$

TRANSPORTING POWER OF CURRENTS OF WATER.—This is one of the least understood subjects in hydraulics.

Deacon (*P.I.C.E.*, vol. 118, p. 93) gives a very detailed description of the motion of sand in a trough with glass sides.

The first movement began with a surface velocity of 1·3 foot per second, and was confined to the smaller isolated grains. If this velocity was maintained, the grains arranged themselves in beds perpendicular to the current, in the form of the well-known sand ripples of the seashore. The profile of each ripple had a very slow motion of translation, caused by particles running up the flatter slope, and toppling over the crest. At a surface velocity of 1·5 foot per second the sand ripples were very perfect, and travelled with the stream at a speed of about $\frac{1}{2160}$ of the surface velocity. At a surface velocity of 1·75 feet per second the ratio was reduced to $\frac{1}{1050}$, and at 2 feet per second



SKETCH No. 137.—Deacon's Experiments on the Transportation of Sand by Water.

was reduced to $\frac{1}{480}$. A critical velocity was reached at 2·125 feet per second, the sand ripples becoming very irregular; the particles rolled up the flat slope, and in place of toppling over the steep incline were occasionally carried by the water direct to the crest of the next ripple.

At about 2·5 feet per second another critical velocity was reached, and many of the little projectiles cleared the top of the first, or even of the second crest. At surface velocities of 2·6 to 2·8 feet per second, the sand ripples became more and more ghost like, until at 2·9 feet per second they were wholly merged in particles of sand rushing along in suspension in the water.

The above description clearly shows three phases of sand motion.

Firstly, a discontinuous rolling motion, with surface velocities between 1·3 and 2·1 feet per second.

Secondly, a discontinuous suspension in the lower layers of the current between 2·1 and 2·9 feet per second.

Thirdly, a continuous suspension at surface velocities above 2·9 feet per second.

It will also be plain that the above velocities only apply to the smaller grains of sand. The bottom velocities are not given, nor is the size of the sand grains, so that the observations are qualitative only.

We are thus faced with the following difficulties:—

I. Any substance occurring in Nature is a mixture of grains of different sizes, and visual observations are liable to give figures relating to the smaller grains alone, as their motion renders the movement of the larger grains invisible.

II. The phenomena appear to depend on the magnitude of the velocity at points close to the bottom, possibly (as Flamant suggests) also on the rate of variation of these velocities with the distance from the bottom. All that can be readily measured is the surface, or the mean velocity; and the mere fact that transport of material is taking place will plainly alter the relation which the velocities near the bottom bear to the surface, or mean velocities.

Thus, any general theory of the transport of materials by water is likely to prove very complicated.

The present treatment follows the method of Lechalas' investigation of Sainjon's observations on the motion of sand in the Loire (*A.P.C.*, 1871, vol. 1). The difficulties above referred to are very forcibly brought out, and it can be stated that:—

> If v_o, represent the velocity in feet per second, at or near the bottom of the river
> v_s, the surface velocity of the river
> v, the mean velocity of the river, or probably, more accurately, the mean velocity over the vertical (see p. 52).

Then, for sand with grains of a mean diameter of 0·04 inch we have as follows:

Until v_o, exceeds 0·82 foot per second (or v, exceeds 1·18 foot per second, or v_s, exceeds 1·27 foot per second), there is no motion (corresponding to Deacon's v_s = 1·3 foot per second). Once motion begins, v_o, cannot be directly observed, but the state corresponding to Deacon's $v_s \doteq 2·9$ feet per second, is reached with $v_s = 3·33$ to 3·38 feet per second.

Lechalas assumes that the force available to move the sand, when v_o, exceeds 0·82 feet per second, is:

$$P = a\,(v_o^2 - 0·82^2) = a\,(v_o^2 - 0·67)$$

and this he puts equal to bq, where q, is the amount of sand transported per foot width of the river, which is assumed to have a bed width of b feet.

We thus have $q = \frac{a}{b}(v_o^2 - 0·67)$ so long as v_s, does not exceed 3·33 feet per second, and Lechalas finds that $\frac{a}{b} = ·0004$, where q, is expressed in cubic feet per second per foot width of the bed. This equation refers to a case where the sand "ripples" were some 2·7 feet high; but the scouring action is stated to be normal, the height of the ripples being caused by irregular flow.

Since the quantities thus calculated agree very accurately with the observed velocities of the profiles of the sand ripples, we may infer that the motion of sand or gravel under the influence of a current of water may be very fairly represented as follows:

So long as the velocity measured close to the bottom of the current does not exceed a certain value, which I propose to call that of first scouring, equal to V_1, the particles remain undisturbed.

If this velocity of first scour is exceeded, motion begins according to Phase I, and gradually passes over to Phase II, but there is no abrupt change in character, until Phase III is reached, with a bottom velocity which I propose to call that of the second scouring, equal to V_2. When the bottom velocity exceeds this value, the whole of the bottom layers of water are charged and clouded with sand, and this cloud rises higher and higher as the velocity is increased.

Our knowledge is best for coarse sand, the particles of which have a mean diameter of 0·04 inch approximately.

Here $V_1 = 0·82$ foot per second, and V_2 is about 1·80 foot per second. The values corresponding to V_1, for other substances are fairly well established, and are tabulated in the second column of the annexed table.

Now, in the case of clear water flowing in an earth channel, it is plain that once the bottom velocity exceeds V_1, the channel will be scoured, and scour will continue until by the increase in size of the channel the bottom velocity is reduced to the value of V_1, corresponding to the material of which the channel bed and sides are formed. This action must be provided for in designing the cross-section of the channel.

It is only very rarely that clear water flows in natural, unlined channels. Usually the water carries some silt before it enters the channels; and it is plain that if we design the channel so that the quantity of silt carried forward, is equal to that already in the water we obtain a channel which neither silts nor scours, and which carries both sand and water. The bottom velocity which prevails in this channel may obviously have any value which is greater than V_1, and will depend solely upon the value of q.

Thus, assuming Lechalas' law, we have as follows:

q in Cubic Feet per Second.	Bottom Velocity in Feet per Second.	Remarks.
...	0·82	Clear water, commencement of first phase of transport.
0·00013	1·00	
0·00031	1·20	
0·00052	1·40	
0·00076	1·60	
0·00103	1·80	Probable beginning of third phase of transport.
...	...	
0·00132	2·00	

Lechalas investigated the laws of transport at velocities which exceeded $v_o = 1·80$ foot per second, but his results are not confirmed by Deacon's experiments, and he does not appear to have had personal experience of the case.

The above table requires careful consideration. It must be remembered that q, is an absolute quantity, and is not a ratio. Thus, let us consider that $q=0{\cdot}00031$. We find that a channel which carries 0·00031 cubic feet of sand per second per foot width of its bed must have a bottom velocity of 1·20 foot per second, whatever its depth may be. Let us assume a depth of 1 foot. The mean velocity is about 1·7 foot per second, and the ratio $\frac{\text{silt}}{\text{water}}$ is approximately equal to 0·0002. But if the channel is 10 feet deep, the mean velocity which occurs when the bottom velocity is 1·20 foot per second will, if anything, be slightly greater than 1·7, and the ratio of the silt to the water will be 0·00002, or only one-tenth of that existing in the shallower channel. Now, in water as it exists in Nature, the ratio $\frac{\text{silt}}{\text{water}}$ is usually fairly constant along the whole course of the river, or artificial channel. Hence we arrive at the general principle that water which carries a fixed quantity of sand per unit volume of water must increase its rapidity of flow when the bed width of the channel decreases. Therefore, if the volume of water and silt carried down the channel is constant, the deeper the channel, the greater is the mean velocity required to prevent the deposition of silt. If we assume Lechalas' figures are correct, we have as follows:

Putting Q = the number of cubic feet of water flowing per foot width of the stream bed per second, and d = the depth of the stream in feet:

then, $Q=vd=\frac{v_0 d}{0{\cdot}7}$ say, and $q=pQ$, where p, is the ratio $\frac{\text{silt}}{\text{water}}$.

Thus, $0{\cdot}0004\,(v_0^2-0{\cdot}67)=\frac{pv_0 d}{0.7}$, or substituting in terms of v, we get:

$$d=\frac{v^2-1{\cdot}33}{vp}\times 0{\cdot}0002 = M\frac{v^2-1{\cdot}33}{v} \text{ say,}$$

as the relation between v, and d, when p, is given. This relation may be taken as typical of the conditions during the first and second phases of motion.

During the third phase, it is believed that, $q=kv^n$.

$$\text{Thus we get,} \quad d=\frac{kv^{n-1}}{p}.$$

Through the courtesy of Mr. Martin Deacon I have been enabled to consult the experiments already referred to. In these the water was 8 inches deep, and during the third phase of motion the relation $q=0{\cdot}78\,v_s^5$, was found to hold where q, is expressed in pounds per hour.

Assuming that $v_s=1{\cdot}05\,v$, which is probably approximately true for a trough with glass sides, and that 1 cube foot of sand weighs 100 lbs., we have:

$q=0{\cdot}0000028\,v^5$, or $0{\cdot}0000166\,v_0^5$, in cube feet per second.

The rule bears no resemblance to Lechalas', but this is hardly surprising, as the observations were nearly all made during the third phase of the transport of the sand.

Thus, in the case of the Mersey sand used by Deacon, we find that:

$$d=0{\cdot}0000028\,\frac{v^4}{p}$$

is the relation between the velocity and the depth in a channel which neither scours nor silts.

Similar rules could be deduced for any other substance provided that proper experiments were available. At present it is impossible to give rules which will permit the effect of the size of the individual grains and the ratio $\frac{\text{silt}}{\text{water}}$ to be accurately allowed for. I have, however, for many years noted and plotted the velocities and dimensions of all the silt-carrying channels which I have been able to observe. Taking the term clear water to mean water which is clear to the eye in bulk, as seen in the river, but which probably rolls some silt along the bed of the river, I find that the following rules hold good:

I. For fine silt, with a mean diameter of about 0·01 inch, using feet and feet per second as units, it would appear that:

(i) Clear water scour is of importance if v_r exceeds $0{\cdot}40\sqrt{d}$.
(ii) Heavily silted water gives no deposit if v_r exceeds $1{\cdot}05\sqrt{d}$.
(iii) Heavily silted water begins to scour noticeably if v_r exceeds $1{\cdot}37\sqrt{d}$.

II. For coarse sand, say with a mean diameter of 0·04 inch, we have:
(i) Noticeable scour in clear water if v_r exceeds $1{\cdot}6d^{0.33}$.
(ii) Heavily charged water does not deposit if v_r exceeds $2{\cdot}2d^{0.33}$.

III. For coarse gravel, say pea-size:
(i) Scour begins if v_r exceeds $2{\cdot}0d^{0.25}$.
(ii) Heavily charged water does not deposit if v_r exceeds $2{\cdot}5d^{0.25}$.

IV. Boulders are moved along if v_r exceeds $5d^{0.25}$.

The whole of these results are only approximate, and, except for the first two cases, rest on very slender experimental evidence.

If we may assume that the experiments conducted by Lechalas and Deacon give the general laws of silt transport correctly for the motion during the three phases previously defined, it is fairly plain that the observations on fine silt and coarse sand (Classes I and II) refer to rivers where the ripple method of transport occurs, and that the coarser materials of Classes III and IV (*i.e.* coarse gravel and boulders) were only moved when sand was being shifted continuously as in Phase III (see p. 491). I do not, however, consider that the experimental data are yet sufficiently extensive to permit so definite a statement to be made. The particular case of Punjab silt has been very carefully investigated by Kennedy, and his laws are considered on page 755.

I consider the general rules that the deeper the channel the less likely it is to scour, and that the coarser the material, the less marked the effect of the depth, are quite reliable. In this connection I would refer to a paper by Thrupp (*P.I.C.E.*, vol. 171, p. 346), which is well worth reading:

Thrupp's curves for depths over $d=0{\cdot}4$ foot, agree very well with:

Mud and silt not moved	i v_r is less than	$0{\cdot}40d^{0.5}$
Fine silt moved	if v_r exceeds	$0{\cdot}40d^{0.5}$
Fine sand moved	if v_r exceeds	$1{\cdot}5d^{0.35}$
Coarse sand moved	if v_r exceeds	$1{\cdot}5d^{0.30}$
Pea-sized pebbles moved	if v_r exceeds	$2{\cdot}2d^{0.3}$
Large pebbles (egg-sized) moved	if v_r exceeds	$5{\cdot}0d^{0.25}$
Large stones moved	if v_r exceeds	$17d^{0.15}$

Thrupp's curves show a well marked change in the laws of scouring, and silting, when the hydraulic radius passes through a value approximately equal to 0·40 feet.

This may be regarded as most doubtful, since Deacon's experiments agree very well with the laws deduced from large natural channels. In actual experimental work, however, it would appear advisable to use a fairly big channel, where say $r=0\cdot50$ foot at least. My own experience with small channels leads me to consider that the transport of sand is considerably affected by accidental disturbances such as shaking, or waves in the water.

The general agreement of my rules with those of Thrupp is encouraging, but cannot be taken as indicating extreme accuracy. The terms "pebbles," "large boulders," etc., are mere figures of speech in such cases. Samples of the river bottom can be secured, but it is not fair to infer that these accurately represent what is moved. As an example, I mention that boulders are moved if v, exceeds $5\cdot0d^{0.25}$, and Thrupp states that this is the velocity at which egg-sized pebbles are moved. As a matter of fact, I am aware that trouble from the point of view of maintenance begins at this velocity in rivers which contain boulders, while Thrupp has evidently investigated what produced the trouble. Thrupp's values are therefore the more scientific, while mine are probably the more useful in practical design. As a matter of experience, an engineer may always suspect that any larger stones which are markedly rounder than the smaller-sized material, are not moved. The more angular the material, the more likely it is to be moved.

The physical meaning of these equations deserves some slight consideration. We have the usual "Chezy" equation, as follows:

$v=C\sqrt{ds}$, and $v=ad^n$, where a, and n, depend on the amount of silt carried, and its mean size.

Thus, we get, $s=\frac{a^2d^{2n-1}}{C^2}$, which may be regarded as fixing the slope at which a channel of given depth and roughness will remain in quasi equilibrium.

As an example, take the case, $v=1\cdot05\sqrt{d}$. We have, determining C, as for Bazin's $\gamma=1\cdot54$ class,

When $d=1\cdot0$ C = 61·9 or $\sqrt{s}=\frac{1}{60}$ $s=\frac{1}{3600}$ approx.
When $d=4\cdot0$ C = 89·0 or $\sqrt{s}=\frac{1}{85}$ $s=\frac{1}{7200}$ approx.
When $d=9\cdot0$ C = 103·0 or $\sqrt{s}=\frac{1}{100}$ $s=\frac{1}{10000}$ approx.

and if the slopes exceed these values in any given case, it is a matter of common knowledge that the channel takes up more silt by scouring, and thus, where possible, adjusts itself to the increased slope, and also if able to "meander," increases in length, and so diminishes the slope.

A very wide-spread idea exists among engineers that a canal will be found to be free from troubles arising from silt or scour provided that it is so proportioned that the ratio $\frac{\text{depth}}{\text{bed width}}$ is the same in the canal as in the river from which it takes out. The above investigations may be regarded as an indication that this rule has a certain foundation. The ratio $\frac{\text{silt}}{\text{water}}$, is probably somewhat less in the case of the canal than in the river, and will be considerably smaller if the headworks are properly designed and are intelligently handled. On the other hand, the velocity will probably be somewhat decreased, as the bed slope of the canal will be less than that of the river, and it is plain that if we can arrange to decrease p, (the ratio $\frac{\text{silt}}{\text{water}}$) to the same extent as the depth is decreased, we may succeed in so adjusting the canal that it will carry its own proportion of silt. We may, however, overdo the matter, and (as has actually occurred in some canals) arrive at a canal which will scour its banks owing to the fact that it has not drawn the proportion of silt from the river correspondin to its depth and velocity.

The logical method is to estimate the average silt in the river, and to see what fraction is likely to be drawn into the canal, and then experiment on the transport of the silt in a manner similar to that employed by Deacon or Lechalas, and calculate the required velocity in the canal.

The estimation of the percentage of silt which is carried by the river is best effected by observing its mean velocity, and noting the quantity which is carried in the experimental trough at this velocity, and allowing for the difference in the depth of the trough and the river by the methods which have already been indicated.

The one fact which stands out among all the uncertainties is that if v, is kept constant, d must increase or decrease in the same ratio as $\frac{1}{p}$, increases or decreases.

The preliminary method of dealing with such a question is best illustrated by an actual example. M. Mougnié ("Etudes des variations du lit de l'Isère à Montrigon," published in vol. iii. of the Reports of the French *Service des Études des Grandes Forces Hydrauliques*) gives 22 detailed gaugings for the Isère, for the low-water seasons of 1905–6, 1906–7, and 1907–8, with the cross-sections of the river for each low-water season. Eighteen of these are low-water gaugings of the seasons 1906–7, and 1907–8. From the cross-sections we find by logarithmic plotting (see Sketch No. 22, p. 94) that if H, represents the gauge reading in metres, g, the hydraulic mean radius, or the mean depth in metres (the two quantities only differ by about 0·8 per cent.), and v, the mean velocity in metres per second :

During the low-water seasons of 1907–8 :

$$g = H + 0{\cdot}34 \quad \text{and } v = 1{\cdot}102\, g^{1.69}$$

During the low-water seasons of 1905–6 :

$$g = H + 0{\cdot}35 \quad \text{and } v = 1{\cdot}213 g^{1.57}$$

Thus, taking the mean value for the Isère at this station during low water, we find that the equation of silt transport is given by :

$$q = kv^{1+\frac{1}{1\cdot 63}} = kv^{1.61}$$

During the low water of 1905–6, the river probably carried about 1·06 times as much silt per unit volume of water as in 1907–8. This is confirmed by the fact that the mean surface slope was 0·0008 in 1905–6, and 0·0006 in 1907–8.

The high-water gaugings are not sufficiently numerous to permit a study to be made, but the logarithmic plotting shows with a fair degree of accuracy that approximately 50 per cent. more silt was carried than in 1905–6.

The method is liable to certain difficulties.

Thus, if the silt transport equation is :

$$q=kv^3$$

we find that $v=C\sqrt{d}$.

This is the usual equation for the motion of water which does not carry silt. In practice, however, I believe that this will rarely cause difficulty, since in the 28 cases of silt-bearing rivers which I have studied, in which the relation $v=C\sqrt{d}$, held good, the fact that scour occurred was always quite obvious.

As the method illustrated here is original, it seems advisable to state that I have but rarely found that it leads to results which conflict with actual observations of the quantity of silt carried. The statement is of course a relative one, as it is impossible to measure the absolute quantity of silt carried by a stream with any degree of accuracy. I am of the opinion that the value of n, in the relation $q=kv^n$, largely depends upon the phase of the transporting motion. Where, as in the low-water stages of the Isère, the river carries but little suspended coarse silt, we may expect to find that $n=1{\cdot}5$ to $2{\cdot}5$, and the transport is of the character termed Phase I (see p. 488). Values of n, ranging from 2·5 upwards, indicate Phase II. In Phase III, values of n, as great as 6 have been found to occur. The physical meaning is obscure, and I am inclined to suspect that the necessary inaccuracies of flood gaugings affect the results. The practical applications of the method are generally confined to rivers carrying silt in the modes termed Phases I and II, and in these cases a very useful insight into the laws of silt transport in a river or canal can be obtained.

The following table gives the values of V_1, which is the bottom velocity that just produces motion in the substances under consideration. The values are approximate, but, so far as the information goes, they may be considered as minima values for clear water :

Material.	Velocity.	Remarks.
	Ft. per Second.	
Soft earth . . .	0·25	
Fine clay . . .	0·25	
Soft clay . . .	0·5	Very variable, and depends upon the adhesion of the clay particles.
Finest sand . .	0·50	Such sand as is left when clay is eroded.
Fine sand . . .	0·70	The usual fine sand of rivers.
Coarser sand . .	0·80	
Gravel, or Coarse sand	1·0	Largely depends upon the shape of the grains. Round gravel of say 0·05 inch size is rolled along a smooth surface at 0·30 foot per second. Pea size at 0·60 foot per second, and bean size at 1·1 foot per second.
Pebbles, 1 inch in diameter . .	2·0	

[*Table continued.*

Table continued.]

Material.	Velocity.	Remarks.
	Ft. per Second.	
Pebbles, egg size .	3 to 3·3	At this velocity the shape of the individual masses ceases to have a very marked effect.
Stones, 3 inches in diameter .	5·0	These values are probably not very accurate, and it is extremely doubtful whether the velocities given accurately represent the bottom velocities.
Boulders, 6 inches to 8 inches in diameter	6·6	
Boulders, 1 foot to 18 inches in diameter	10·0	

The following values of V_1, were obtained by current meter measurements in the Rhine (*Deutsche Bauzeitung*, 1883). Being obtained in a deep river, they are probably more reliable under such circumstances than the former values, which were mostly obtained from small scale experiments.

Material.	Velocity in Feet per Second.	
	If Disturbed.	If Undisturbed.
Gravel—		
Pea size	2·46	3·87
Bean size	2·95	4·30
Hazel to walnut size . .	3·48	4·92
Pigeon egg	3·67	...
Weight, ¼ lb.	4·92	...
Weight, 5 lbs.	5·90	...
0·6 foot diameter . . .	6·56	...

The above figures must be considered merely as representing ideal cases. If the water carries silt, scour of the finer materials will not occur until the tabulated velocities have been exceeded in a ratio which depends upon the quantity of silt which is already present in the water. A river which carries a large quantity of sandy silt is able to roll along gravel and boulders at smaller velocities than those indicated in the table. The action appears to be due to the fact that the lower layers of the river being heavily charged with silt, in reality form a fluid which has a density greater than that of pure water. The extra flotation thus obtained renders the stones more easily moved. Exact figures cannot be given.

CHAPTER X

FILTRATION AND PURIFICATION OF WATER

SUMMARY OF PRESSURES REQUIRED IN FILTRATION

BACTERIAL INVESTIGATION OF WATER.—Water-borne diseases—Normal death-rate from water-borne diseases—Bacteriological tests—**Koch's test**—Specification—Criticism—Collection of samples—Importance of after-pollution—Special risks attending concentrated pollution.

APPARENT FAILURES OF FILTRATION SYSTEMS.—Washington filters—Statistics—Criticism—Possible explanations—Unsewered cities.

BACTERIOLOGICAL REPORTS.—Percentage of removal of bacteria—Maximum number of bacteria.

SPECIAL BACTERIAL INVESTIGATIONS.—B. **coli**—Observations at Lawrence—Other bacterial indicators—Thresh's investigations.

CHEMICAL ANALYSIS OF WATER.—Hygenic qualities—Mineral qualities—Interpretation of a chemical analysis—Free ammonia—Albuminoid ammonia—Chlorine—Nitrates and nitrites—Oxygen absorbed—Hardness, temporary and permanent—Organic carbon and nitrogen—Metallic impurities.

SITE OF FILTRATION PLANT.—After pollution—Deposits in pipes—Straining through gauze screens—Treatment of zinc and lead-solvent waters, and acid waters, by neutralisation with chalk or lime.

Methods of Water Improvement.—Must be judged by bacterial tests—Popular standards—German preference for ground water—Not shared by vegetarian races—American standards of bacterial purity—British preference for slow sand filters—Sedimentation—Filtration—Chemical methods—Colloids—Coagulation.

SLOW SAND FILTERS.—Schmutzdecke—Zooglea—Relative value in bacterial removal—Probable rôle of Zooglea—"Raw" and "ripe" sand—Practical conclusions concerning the thickness of the sand layers—Ruptures of the Schmutzdecke.

CONSTRUCTIONAL DETAILS.—Velocity of filtration—German rules—British practice—Removal of population from catchment areas—Area of filters—**Design of filters**—Depth of water—Covering of filters—Thickness of sand layers—"Dalles filtrantes"—Thickness of gravel layers—Specification for a filter—Specification for sand—Head consumed in gravel layers—Spacing and sizes of drain pipes—"Friction discs" or equalisers—Precautions for securing a uniform rate of filtration over the whole area of a filter—Brick and tile floors—Checks in side walls—Precautions round the columns of covered filters.

Working of Filters.—Formation of the Schmutzdecke—Filtration—Cleaning by scraping.

Abstract of Particulars of British Filters.—Depth of water—Sand—Gravel—Cleaning—Description of Derwent filters—Influence of character of water on the rate and interval between cleanings of filters—Fuertes' formula for filter area—Quantity filtered between two cleanings as a function of size of sand—Effect of climate—Depth of sand scraped off—Replacement of sand—Standard of pollution—Effect of turbidity on sand filters—Colours, tastes and odours—Necessity for sedimentation—Rate of deposition of particles—Mechanical entanglement of bacteria—Difference between European and American waters—Houston's views on storage—Criticism—Slow sand filters best adapted for stored, or sedimented water.

SUMMARY OF PRESSURES REQUIRED IN FILTRATION

Any summary of formulæ connected with filtration is impossible. The figures relating to the head or difference of pressure required are the least variable, and since they are important in preliminary investigations concerning the site of the filters or the horse power of the pumps, they are very roughly tabulated below.

Excluding the resistances of the connecting pipes and channels:

(i) A sedimentation basin and its valves consume about 6 inches head.

(ii) Each gravel filter, such as Peuch's dégroisseur (p. 544), requires 6 to 9 inches head. Each fall for aeration between the filters also consumes about 6 inches. A prefilter, such as that used at Steelton (p. 570), or Philadelphia, requires about 2·5 feet when clogged.

(iii) Thorough aeration in most waters can be secured by four falls of 6 inches each. Tray aerators consume about 6 feet. No definite figures can be given for aeration by fountains, but it will usually be found that the larger the individual jets the greater is the head required. The figures in all cases depend considerably on the quality of the water.

(iv) Slow sand filters when clogged consume from 2·5 to 4 feet head. The smaller the available head, the more frequently cleaning is required.

(v) An efficient coagulation basin requires a head of 6 inches to 1 foot, although 3 inches often suffices.

(vi) Mechanical filters when clogged require from 12 to 25 feet head, the usual limits being 15 to 20 feet.

BACTERIAL INVESTIGATION OF WATER.—The main facts as to the bacterial origin of most diseases are generally known. For a waterworks manager, the practical result is that certain diseases are now held to arise from minute organisms conveyed to the human body by means of water. Such diseases are generally termed "water-borne,"—and typhoid, or cholera, may be taken as typical examples. If in any town the death-rate from such diseases rises above the normal, the water supply should be regarded with suspicion. A numerical statement of the normal death-rate is quite impossible. Large groups of American cities are satisfied with a typhoid death-rate which, in Germany, or Great Britain, would probably lead to legal proceedings being taken against the waterworks' authority. My personal experience both of typhoid and cholera, leads me to consider that the water is sometimes unreasonably condemned. But such isolated cases form no ground for the assumption that unfiltered, or badly filtered water, is healthy; and a waterworks' manager must be content to do his best. He is only justified in asking for investigations on other lines when he has proved that the water supply is above suspicion. I may, however, state that there is sufficient evidence, (*e.g.* at Melbourne and Buenos Aires) to show that a perfectly satisfactory water supply may be accompanied by a high death-rate from typhoid, if the drainage is bad. The probable explanation is to be found in the sequence ;—open closets, flies, and food.

The present methods of proving that a water supply is satisfactory (other than the large scale proof of a death-rate from water-borne diseases well below the normal) are bacteriological.

In principle all bacteriological tests consist in placing the bacteria contained in a given volume of water in a situation which is favourable to the development of some or all of the various species. Consequently, each individual of these species generates a group of bacteria, and after an appropriate interval of time has elapsed these groups or "colonies" can be counted, and the various species identified. The details of such bacteriological methods do not concern engineers; but it should be realised that the methods of favouring the growth of the bacteria can be indefinitely varied, and in this way a single species, or a restricted class, of bacteria can be separated out from the others.

Thus, the reports may state the number of bacteria which are found by Koch's method, or by other more specialised tests. Koch's method favours the growth of bacteria which produce the typical water-borne diseases, but some of the resulting colonies are also derived from other allied species which are probably harmless. The special methods can, however, be so adjusted as to select definite species of bacteria for counting purposes. For example, *B. coli* and other sewage bacteria, or special disease-producing bacteria (such as *B. typhosus*), may be separated from other species.

Some European waterworks are managed by skilled bacteriologists, but the usual practice (as also in Great Britain and America) is for a bacteriologist to be retained to examine the water, and report to the responsible engineer or superintendent.

Many filtration works are still managed with success by rule of thumb, and without any bacterial investigations whatsoever, but there is little doubt that daily examinations of the effluent from each individual filter should be carried out wherever the raw water is badly polluted. This is an ideal condition, and a daily examination of the mixed effluent, supplemented by special investigations whenever any irregularity in the working of the filters occurs, is usually considered sufficient.

My own experience leads me to believe that money could be saved by more systematic examinations.

The standard at present generally adopted is that laid down by Koch, who stated that filtered water should not contain more than 100 bacteria per cubic centimetre, as ascertained by counting the colonies after four days incubation on nutrient jelly, at 20 degrees C. Thus, the test really proves that one cubic centimetre of filtered water contains less than 100 individuals capable of propagating themselves under the special conditions laid down by Koch. It may therefore be inferred, and has actually been proved, that many other bacteria which are incapable of reproducing themselves under the above conditions may, in reality, exist in the sampled water. The conditions to which the sample is subjected in Koch's test are those which, in his opinion, are best suited to the propagation of the species of bacteria producing typhoid and cholera, and, as a matter of fact, many other closely related and probably harmless species also flourish under these circumstances.

The standard being a local one (*i.e.* suitable for German conditions), is obviously open to objection, and might be considered as illogical as a description of venomous snakes referring to European species only. As a matter of fact, however, the standard always secures a water which is satisfactory when judged by the death-rate test. Under certain conditions (principally American) it may possibly be too stringent. Likewise, it may eventually be discovered that for waters drawn from rivers flowing through thickly populated Asiatic countries, it is not sufficiently severe. These statements are, however, founded on very scanty and unreliable information, and it will probably be many years before the standard is superseded. The information required for this purpose is neither bacterial, nor engineering, but rather statistics of the public health of well sanitated tropical cities, with a water supply passing Koch's tests.

Certain differences in detail exist in the application of the test. For example, in France a period of 7 to 15 days' incubation is usual. This is apparently a higher standard of purity, but comparative figures cannot be given, although in one case it appeared that a water which produced 120 bacteria per c.c. under the French test, yielded only 55 bacteria per c.c. under the test as carried out in strict accordance with Koch's methods.

The usual way of specifying the test is as follows:

"The number of bacteria per c.c. in the filtered water shall not exceed 100 under Koch's test, the sample being kept in an artificially cooled receptacle (*i.e.* packed in ice) during transit. If the bacteria in the raw water exceed 3300, as tested by Koch's method, the filtered water count may exceed 100, (but the percentage of reduction must not then be less than 97)."

This last clause is illogical, since the fact that the raw water is highly polluted, is, *per se*, an indication that the 3 per cent. of the bacteria that do escape are rather more likely to be pathogenic than if the raw water were less polluted. It has the practical justification that slow sand filters alone (however carefully worked) are unlikely to produce better results unless the supervisor is unusually capable. Nevertheless, I consider that the clause should be omitted, even if in practice it turns out that the latitude must be allowed, since the supervisor should not be encouraged to remit his efforts to obtain a satisfactory filtrate on the very occasion when danger is most probable, and treatment supplementing sand filtration is most required.

The bacteriologist should, of course, give his own instructions for the collection of water for test, or, better still, take his own samples. Where no instructions are given, the glass bottles and stoppers should first be cleaned with strong sulphuric acid, then in freshly boiled distilled water, and should afterwards be steamed for 15 minutes, cooled in a steriliser, and finally stoppered, and wrapped in cotton wool, the water being poured into them without being touched, from a similarly treated tap or vessel.

The bottles should then be packed in ice, which should be renewed until they reach the laboratory.

The chemical methods of testing water are discussed later. At present it is sufficient to state that they can only be regarded as preliminary to bacterial tests. It is occasionally (*e.g.* in isolated localities) necessary to rely on their indications alone. Nevertheless, it cannot be too strongly stated that if the chemical indications are at all doubtful, bacterial tests must be applied.

As already indicated, many species of bacteria exist in water (even the purest natural waters contain some bacteria), and so far as our present knowledge goes, the majority of these species are harmless when consumed by human beings, some indeed actually being beneficial. Even if the above remarks are restricted to those species of bacteria which flourish under Koch's test, it is probable that the majority are quite harmless. Thus, logically speaking, no numerical standard of bacterial purity can be applied to all localities. A proper specification would take into account the probable proportion of disease-producing and harmless bacteria which exist in the raw water. Considering the subject from this point of view, it is evident that a numerical standard which secures safety when applied after filtration to a water drawn from a river which is known to be heavily polluted by sewage, may be far too stringent when applied to a case where the raw water is drawn from a source not exposed to sewage pollution.

Also, the possibilities of an increase of the harmful bacteria after filtration should be considered, since an increase may occur on a large scale in the filtered water, if the conditions in the mains and reservoirs favour the propagation of these bacteria.

Thus, not only should the water be tested immediately after filtration, but tests should also be made of samples drawn from the supply mains, so as to ascertain the bacterial condition of the water at the moment when it is actually used by the consumers.

The results of many tests (which are apparently confirmed by the circumstances attending several epidemics of water-borne disease) indicate that the disease-producing species have a far greater chance of increasing in water to which they gain access, if but a small number of bacteria are originally found in this water. The reason seems fairly obvious: Disease-producing bacteria are not, apparently, those which are best suited to exist in water (their most favourable environment being the human body). When, therefore, they find themselves exposed to the competition of a large number of other species of which the normal home is water, the disease producers are unable to survive the competition of the better adapted species.

This competition is less severe in comparatively pure water, and disease producing species are able to multiply. It is evident, therefore, that an engineer must carefully consider the possibilities of pollution after purification, and should consider after-pollution as a very serious matter, for bacteria are then not only provided with a clear path of access to the human body, but the purified water affords them a favourable incubating ground.

The special risks attending concentrated pollution must be pointed out. The matter is somewhat difficult to define, as the precise conditions are unknown; but it can be best illustrated by the statement that if a "pound" of pollution must enter the water either before or after filtration, it is better, from the point of view of health, to receive the "pound" in 7000 separate "grains" rather than in one mass, even though the water received from the individual sources is afterwards thoroughly mixed, and the "grains" finally reach the consumers simultaneously.

If this principle is once grasped the extreme danger of concentrated pollution, which is also local, needs no discussion. One typhoid patient whose dejecta go straight to the intake of a water supply system is more dangerous than 100 cases whose dejecta reach the river from which the intake derives the water at various isolated points. If a waterworks' manager knowingly permits a person suffering from "summer diarrhœa" to remain on or about the works, even although in the neighbourhood of the raw water channels only, he should not be allowed to retain his position as manager.

Apparent Failures of Filtration Processes.—In certain cities, cases have occurred where the introduction of a purified water supply has not been followed by a decrease in the number of cases of, or deaths from, those diseases which are classed as water-borne. The example which has received most investigation is at Washington, where the circumstances were as follows.

From 1883 to 1903 the typhoid death-rate ranged from 40 to 104, and was usually between 65 and 80 per 100,000.

In October 1905 slow sand filters of a very excellent type were introduced, but the death-rate from typhoid month by month during 1906 was approximately the same as in 1905, and the total number of cases (fatal and non-fatal) was greater.

The public at once inferred that the slow sand filters were useless, and there is no doubt that the figures are extremely disheartening to those who are convinced that a polluted water supply is always a public misfortune.

The exact figures are as follows: The deaths from typhoid for 1906 were 44 per 100,000, and this was the average of the years 1903 to 1905. A figure of 20 to 25 deaths per 100,000 from typhoid in towns situated under circumstances like those found at Washington may be considered to indicate a satisfactory water supply.

Thus, a rate of 19 to 20 deaths per 100,000 needs explanation. A study of the yearly death-rates of the period 1883 to 1906 given in *Trans. Am. Soc. of Civil Eng.*, vol. 57, p. 430, provides a partial explanation.

In the first place, the typhoid death-rate oscillates up and down in very much the same manner as the annual rain-fall of any locality does (it must not be inferred that any connection is intended, the oscillating character of both figures is alone referred to). Judging by the run of the curves, the typhoid death-rate was low from 1903 to 1905, and therefore a rise might have been expected about 1906. Thus, it is possible that the filters did actually produce a real, although not an apparent, decrease in the death-rate. This argument is somewhat flimsy when relied upon to justify an expenditure of many millions of dollars. It is therefore fortunate that the curves show very clearly that after 1895 the sedimentation in the Dalecarlia reservoir produced a marked decrease in typhoid, and that a second, although a less pronounced, decrease is indicated after 1902 or 1903, when another sedimentation basin (the Washington reservoir) was brought into use. It may consequently be inferred that the Washington figures, as they stand, show fairly clearly that an amelioration of the water supply does produce a decrease in the typhoid rates, even although the effect of slow sand filters is not evident. If, however, slow sand filters alone are considered, the results must be regarded as justifying adverse criticism, and it is highly regrettable that such hostile testimony should arise from the results of filtration when introduced into so important a city. It is the more lamentable in that, at the present date, filtration (as compared with European practice) is just being adopted in American cities. I do not, however, believe that the many skilful American experts in filtration will allow this case to remain either unexplained, or without remedy. The circumstances are somewhat obscure, and pollution of the milk or fruit supplies has been suggested as being the true source of the disease.

I suggest three possibilities :

(*a*) As already explained, the character of the raw water may be such as to require a more stringent standard of bacterial purity in this particular case than has been found necessary elsewhere.

(*b*) The daily consumption of water per head in Washington is 200 U.S. gallons, a figure which (for a residential city with comparatively few factories) suggests a large amount of waste, and the possibility of leaky mains, so that after-pollution cannot be considered as impossible.

(*c*) The raw water at Washington is drawn from a river which at certain periods of the year carries large quantities of extremely fine clay. Waters of this type are known to be less easily purified by sand filters than the clearer waters which are found in Europe. As a general rule, such waters have been treated by coagulation processes, or have been passed through dégroisseurs previous to slow sand filtration. The typical method of purification is coagulation followed by mechanical filtration. Now, there is no evidence which permits us to state that the European method of slow sand filtration alone, as practised at Washington, is necessarily effective in removing pathogenic bacteria from waters of this type. Although the Washington methods were in accordance with the very best practice, it is quite possible that the official order prohibiting any addition of coagulant was an error of judgment. This suggestion deserves careful consideration, especially by British engineers, who are notoriously too prone to rely on slow sand filters exclusively.

I put the suggestions forward with some diffidence, but would remark that although they may be conclusively proved to be inapplicable to the case of Washington, they are certainly factors that no investigator could afford to overlook.

It must be remembered that while every case of apparent failure is reported and discussed, the usual result of the introduction of a purified water supply is a marked decrease in water-borne disease, and such successes being normal, they are rarely, if ever, matters of more than local interest.

In this connection it is as well to state that the introduction of a copious supply of water into a city in exchange for a scanty one may be expected to produce an unfavourable effect on public health, unless a sewerage system is simultaneously (or has been previously) introduced. The reason is fairly obvious :—Such cases generally occur in dry climates, and so long as water is economically used, the subsoil is not water-logged, and garbage and refuse are dried up by the air before they become markedly decomposed. On the introduction of a copious water supply, however, the waste liquid saturates the subsoil, and garbage is less rapidly dried up.

The real lesson is therefore to introduce a sewerage system and a water supply simultaneously, and never to consider waste of water as unimportant. Typical examples illustrating the above are Buenos Aires, and Melbourne, and in the latter case proper sewers have produced a satisfactory state of affairs.

To sum up:—The methods of bacteriological testing at present practically employed are (as a rule) excellent guides, but they are by no means infallible. Thus, if the real tests of a process of purification (*i.e.* the health statistics of the community) give an adverse result, an engineer is justified in requiring the bacteriological experts to adopt more searching tests than the routine "bacterial counts"; and until these have been systematically applied in many cases, our present knowledge does not permit us to condemn either the routine methods of filtration, or the customary "bacterial counts," as useless.

I need hardly say that in cases where success is not attained, the filtration process must be carefully overhauled, but under the social conditions usually obtaining, engineers are somewhat too prone to accept routine bacteriological work as sufficient in all cases.

Bacteriological Reports.—The engineer should insist on the actual counts being reported in every case. I have noticed that, especially when reporting the results of tests on proprietary systems of purification, there is a custom (happily a decreasing one) of recording figures other than the actual number of bacteria per cubic centimetre.

A figure commonly reported is the "percentage of removal of bacteria," *i.e.*

$$\left\{1 - \frac{\text{Number of bacteria per c.c. of purified water}}{\text{Number of bacteria per c.c. of the raw water}}\right\} 100.$$

This is quite useless as an index of the safety of the water, since the figure is almost independent of the number of bacteria in the raw water, because any well-arranged system will usually effect a reduction of 99 per cent., and it is very rarely that the figure falls below 95 per cent.

This method may be useful in comparing the relative efficiencies of two different systems of purification, but the fairest way of holding such comparative tests is to supply the same raw water to each system. Consequently, all that can be said is that the figure is not necessarily misleading.

The custom of reporting not the results of each test, but the average result of all the tests during a month, falls under quite a different head. In my own practice I look upon such reports as valuable only as an index of the variability of the process, and believe that in most cases the detailed figures are intentionally concealed. There is very little doubt that an individual who constantly consumes polluted water (should he survive) acquires a certain immunity, and finally can drink water that would in many cases cause a serious illness in any one less accustomed to pollution of this character.

It is therefore evident that a supply which, for say 30 days in the month, attains the requisite standard of purity, but, on the 31st day (owing to carelessness or irregularity in the process of purification), delivers a badly polluted water, is in some respects more dangerous to health than a system of supply that always delivers a slightly polluted water.

Consequently, it is only fair to assume that where the results of bacterial tests are uniformly satisfactory, they are published in detail, and when the literature states average results only, the process is in reality markedly irregular in its working, and is therefore unsuitable for adoption. These remarks cannot of course be applied to reports in which averages are stated, and likewise the maxima counts obtained in each period over which the averages are taken.

Special Bacterial Investigations.—Routine "Standard Bacterial Counts," such as the Koch test, should not be considered as indicating the total number of bacteria present in the water under examination. The method is avowedly such as to favour the propagation of the bacterial species which exist in sewage. If special processes are employed, it is possible to obtain counts of bacteria far exceeding those yielded by the usual methods, but the extra number thus secured consists of species that do not in any way indicate pollution. The results of such counts therefore possess no interest for the waterworks' engineer.

On the other hand, it is also possible to arrange a count of the individuals of a single species, and such investigations are worth consideration. In our present state of knowledge, the most important of these special species counts is the one referring to the **Bacillus coli.** *B. coli* occurs in human intestines and fæcæ, and also, it is believed, in similar situations in several animals. Whilst, therefore, its presence does not necessarily indicate pollution by human beings, its habits and occurrences are such that we may assume that a process of purification which removes *B. coli* will also eliminate the bacilli producing typhoid, and probably those giving rise to other water-borne diseases as well.

The occurrence of *B. coli* is therefore a very valuable index of the efficacy of a process of purification, and of the safety of the filtrate.

An excellent illustration of its efficiency as an indicator is found in a case occurring at Lawrence, Mass. (Clark and Gage, *Significance of Appearance of B. coli communis in Filtered Water*). Here, owing to repairs to the underdrains of a filter, a leak or a weak spot in the sand was produced, in November 1898.

In December 1898 *B. coli* was found in 72 per cent. of the samples examined and 12 cases of typhoid occurred.
In January 1899 the figures were 54 per cent. and 59 cases.
In February „ the figures were 62 per cent. and 12 cases.
In March „ the figures were 8 per cent. and 9 cases, all in the early part of the month.

The example is very instructive, and the figures make the following conclusions fairly clear:

(*a*) That the disease lags behind the appearance of *B. coli.*
(*b*) That *B. coli* is not in itself a disease producer, but some accompanying factor which is only roughly proportionate to the abundance of *B. coli.*

and this is exactly what bacterial investigators would have us believe.

B. coli is an easily recognised indicator of the possible presence of disease-producing organisms which are far less readily discovered, and it is from this point of view that counts of *B. coli* are valuable. Taking 1 c.c. of water for investigation, it would appear that where *B. coli* are found in more than the normal numerical proportion of these counts, danger exists. The danger line, however, cannot be rigidly fixed, since it depends on the results found during the normal working of the filter, and varies with each town. At Lawrence, Mass., 8 per cent. was normal, and was considered satisfactory. In a town supplied with very pure water, a far smaller percentage might indicate a dangerous state of affairs.

It has been proposed to consider such organisms as Klein's *B. enteriditis*, or certain streptococci, as indicators of danger. These, unlike *B. coli*, have never been discovered except in sewage, or sewage-polluted water. The proposal is at present of more or less doubtful utility, except for British waters. So far as can be inferred the difficulty lies in the fact that these organisms are not present in all sewages, although they appear to be characteristic of British sewages. It is plain that a standard which is so local in its application, requires further investigation. It must also be remembered that indicators which only apply locally are likely to prove more useful for those particular localities, than any general indicator such as *B. coli*.

The British semi-official standards are :

For deep wells.—No *B. coli* should be found in 10 c.c. of water.

For moorland and upland waters.—No *B. coli* should be found in 1 c.c. of water.

For shallow wells.—No reliable indication can be drawn from the presence of *B. coli*.

It will consequently be evident that the standard is not an absolutely numerical one, but rather one of variations from the normal number.

The nett result of the discussion is summed up by Thresh (*The Examination of Waters and Water Supplies*), and in explanation it must be stated that he worked with sewage obtained from a town possessing a small supply of water (I believe 15 gallons per head per day), and but little trade waste. His sewage, therefore, is comparatively concentrated, and free from other than fæcal matter.

I. In a water recently polluted by sewage, in a proportion of more than one part per million, both typical *B. coli* and Klein's *B. enteriditis* can be detected.

II. In a water not recently polluted by sewage, or manurial matter, the above bacilli may be absent, but intestinal bacteria occur, *i.e.* other forms, some of which are very closely allied to the typical *B. coli*, and only distinguishable by special tests.

III. Unless some of those forms closely resembling *B. coli* occur, the presence of other "intestinal bacteria" has no significance.

Now, in ordinary tests for *B. coli*, I and II would be classed together ; but I is certainly dangerous, while II is possibly safe.

It may therefore be inferred that towns with a satisfactory death-rate (as regards water-borne diseases), where *B. coli* is found in a fairly large percentage of the samples of filtered water, are really supplied with safe waters, resembling Class II, or are possibly less polluted with sewage than one part in a million. But it will be clear that a sudden increase in the number of *B. coli* contained in samples may indicate a change to the dangerous portion of Class II, or even to Class I, owing to the pollution becoming more intense, or being more rapidly transmitted to the town, *e.g.* by floods, or a breach in some stratum which had previously delayed the progress of the polluted water.

CHEMICAL EXAMINATION OF WATER.—In all calculations connected with the chemical reactions of water it is convenient to note that :

1 part per 100,000 = 4·375 = $4\frac{3}{8}$ grains weight per cubic foot.
= 0·7 grain per Imperial gallon.
= 0·58 grain per U.S. gallon.
Also, 1 pound = 7000 grs.

A chemical analysis of water is undertaken with two objects. The more important is to ascertain its suitability for human consumption from the point of view of health. Such an investigation is really only a substitute for a bacteriological test, and can hardly be regarded as sufficient if the results are at all doubtful. Secondly, as a rule, it is also desired to ascertain whether the water, besides being free from organic pollution, is otherwise satisfactory. In these latter cases it is necessary to determine the content of mineral substances, not only such as arsenic, lead, or zinc, which are obviously undesirable, but also of such salts as calcium and magnesium or iron carbonates, which may cause the water to be unsatisfactory (as being too hard, or likely to produce deposits in the pipes, etc.). The methods of removing, or ameliorating these conditions, where necessary, are dealt with under Softening and Deferrisation.

It is not proposed to describe the methods employed in the chemical analysis of water. An engineer should be able to interpret a chemical analysis, and to draw inferences from the data it affords; and the following notes are directed to that end alone, and are principally concerned with the hygienic qualities of the water.

If a study be made of all the available evidence relating to the connection between the contents of a water as disclosed by chemical analysis, and its fitness for human consumption, it will at first sight appear that no close relationship exists. This, in reality, is merely an instance of the tendency of human nature to give undue weight to exceptional instances.

Deductions drawn from the quantity of any single substance present in a water are liable to prove quite erroneous. In practice, however, the indications are not usually isolated facts, and the inference drawn from, say, the presence of chlorides in a water is usually confirmed either by other chemical indications, or by the results of an examination of the origin of the water and its exposure to possible sources of contamination. Thus, the chemical analysis of a water, combined with information obtained from an examination of its source, almost invariably enables a definite statement to be made as to the fitness of the water for human consumption.

The processes generally employed by chemists when preparing a report on water which is intended for human consumption are as follows:

(i) Determination of the free ammonia.
(ii) „ „ albuminoid ammonia.
(iii) „ „ chlorine.
(iv) „ „ total solids.
(v) „ „ nitrates.
Nitrites are tested for qualitatively, and if present (which is very rarely the case) are estimated quantitatively.
(vi) Determination of the oxygen absorbed.
(vii) „ „ temporary hardness.
(viii) „ „ permanent hardness.

Occasionally (ii) and (vi) are not reported, but the quantities of organic nitrogen and organic carbon are given instead (see p. 512). This is the information usually provided, and if more is required the engineer should ask for it. The additional information needed when coagulation processes are contemplated is considered on pp. 556 and 565. The questions concerning the zinc- and lead-solvent properties of the water are discussed on page 514. In all these cases the engineer should be prepared to afford full information to the chemist, and should state his requirements in a definite form.

Taking the eight quantities detailed above in order:

(i) and (ii) **Free Ammonia and Albuminoid Ammonia.**—These are occasionally reported as ammoniacal nitrogen, and albuminoid nitrogen. The conversion factor is given by:

$$\text{Nitrogen} \times 1{\cdot}214 = \text{Ammonia.}$$

The **albuminoid ammonia** content is by far the most important factor in determining the fitness of a water for human consumption. If less than 0·002 parts per 100,000, the water can be passed as pure, even if large quantities of free ammonia and chlorine are present. If the content of albuminoid ammonia exceeds 0·005 parts per 100,000, the quantities of free ammonia and chlorine should be considered, and if these are high a bacteriological examination is necessary. Anything exceeding 0·008 parts per 100,000 is suspicious, and if the albuminoid ammonia exceeds 0·01 part per 100,000, a bacterial examination is required however favourable the other indications may be.

A proportion of albuminoid ammonia in excess of 0·015 parts per 100,000 requires very strong bacterial evidence before the water can be considered as fit for human consumption. In these cases if the chlorine content is low (say less than 1 part per 100,000) the pollution is probably of vegetable rather than animal origin. While a water so heavily charged with organic matter cannot be considered as first class it may possibly prove bacterially satisfactory, and is certainly a better raw material for filtration processes than a water of equal albuminoid ammonia content, containing chlorine in quantities (say over 3 per 100,000) which indicate pollution of animal origin.

In waters derived from deep wells, **free ammonia** has no definite significance. The average value is 0·01 per 100,000, but 0·03 is not uncommon, and 0·1 is not unknown in bacterially pure deep well waters. The free ammonia in these cases results from the reduction of nitrates, and therefore indicates a pollution of even more remote date.

In safe spring waters, 0·001 is the average, and 0·01 is rarely exceeded.

In upland surface water the average content is 0·002, and 0·008 per 100,000 is rarely exceeded if the water is safe.

In water derived from cultivated land, the average is say 0·005, and 0·025 is rarely exceeded if the water is safe.

In shallow wells, nil to 2 per 100,000 is found.

The figures must be read in conjunction with the albuminoid ammonia content, and the local normal content of free ammonia, as discussed under Chlorides, must be ascertained.

(iii) **Chlorides.**—These usually indicate the presence of common salt, which in itself is unobjectionable. Seventy parts per 100,000 are distinctly perceptible, and anything exceeding this indicates a brackish water, although some people find even 20 parts distasteful. If, as is usually the case, all the chlorine is present as common salt, the weight of the latter is 1·65 times that of the chlorine (if the results are thus reported).

The real importance of chlorides is as an indicator of sewage pollution. Human urine contains about 1 per cent. of salt, and consequently sewage derived from a town supplied with say 30 gallons (36 U.S. gallons) per head per day, will contain about 5 parts per 100,000 more chlorine than the original water. Hence, before we can lay down any standard for a suspicious water, we must know the normal content of unpolluted water in the district under consideration. In England, 4 parts of chlorine per 100,000 may be taken as the probable value for normal ground waters. Therefore, if in a district where well water usually yields 4 parts, we find wells near the centre of population yielding 30 parts (as sometimes occurs), it is hard to avoid the inference that the water (although possibly quite fit for human consumption) is sewage, which has been used over and over again ; and bacteriological examination is obviously necessary in order to make certain that the natural filtration through the ground has destroyed all sewage bacteria.

In the United States it would appear that away (*i.e.* 150 miles or more) from the sea, chlorine contents equal to 0·8 per 100,000 are normal ; so that in such localities a content of even 3 parts (allowing for the fact that the average water supply of an American city greatly exceeds 30 gallons per head, per day) may be regarded as suspicious.

As a contrast to these figures, cases exist of waters of undoubted bacterial purity, containing 50 to 70 parts per 100,000.

It must also be remembered that salt is a very common mineral, and may be present in appreciable quantities in bricks, mortar, or wood (due to soaking in sea water) so that in a new well, the materials composing the lining may affect the analysis.

(iv) **Residue left on Evaporation.**—This may vary from 5 to 6 parts per 100,000 in upland surface waters, to 150, in the case of waters drawn from sandstone strata. Unless the water contains more than 50, or even 60 parts of solids per 100,000, no exception need be taken to the solids as such. The information is mainly useful as indicating the necessity for a complete analysis where the weight of chlorides, nitrates, and hardness found, does not account for all, or nearly all of the weight of the residue.

Charring may be considered as indicating the presence of organic matter. If accompanied by an offensive odour, this is probably of animal origin.

(v) **Nitrates.**—These should be reported as parts of nitric nitrogen per 100,000, not as nitric acid (HNO_3, conversion factor, $\frac{\text{nitric acid}}{4·5}$ = nitrogen), or nitric anhydride (N_2O_5, conversion factor, $\frac{\text{nitric anhydride}}{3·86}$ = nitrogen). Nitrates are derived from the oxidation of nitrogenous matter of animal origin, and may therefore indicate animal pollution at some past period, which, in certain cases at least, has been traced back to as remote a date as 1640.

Owing probably to the fact that nitrates are the final product of the decomposition of organic matter, no definite standard of purity can be given.

A well water containing more than 7 parts per 100,000 of nitric nitrogen must certainly receive a searching bacterial investigation, but many waters containing less than 7 parts per 100,000 are also unsafe. Generally speaking, up to 1·5 parts per 100,000 is considered to be innocuous, but the amount of manuring which the surrounding soil receives must be taken into account ; and, if this does not explain the content of nitrates, bacteriological examination is indicated. In the case of a well, the most favourable time for the recognition of the bacteria, that the presence of nitrates indicates are likely to be found, is soon after heavy rain.

A water is unsafe in which both chlorides and nitrates occur in more than the normal quantity; as also is a well water which becomes opalescent, or turbid, after rain, and in which an excess of nitrates can be traced.

It must also be noted that rain water invariably contains about 0·03 parts per 100,000 of nitric nitrogen.

Nitrites if occurring in deep well waters, have no definite significance; since they are probably produced by the reduction of nitrates, the total content of nitrogen in the nitrates and nitrites should be considered when drawing deductions as to the quality of the water. In a river, however, the presence of nitrites is extremely significant, since it almost invariably indicates so recent a pollution by sewage that the river has not had time to begin to purify itself. Unless the presence of nitrites can be otherwise accounted for, such water is hardly fit for human consumption even after filtration, unless it is also sterilised.

(vi) **Oxygen Absorbed.**—This is usually determined by Forcheimer's test for "Three hours at 80 degrees Fahr." Some chemists make two determinations, at "15 minutes," and at "4 hours." A marked difference between the two results indicates vegetable rather than animal pollution.

Frankland gives the following standards for oxygen absorbed:

	Parts per 100,000	
	In Upland Surface Waters.	Other Waters.
Water of great purity .	Under 0·10	Under 0·05
" medium purity .	" 0·30	" 0·15
" doubtful purity .	" 0·40	" 0·20
Impure water . . .	Over 0·40	Over 0·20

Ceteris paribus, the greater the proportion the oxygen absorbed bears to the organic nitrogen or albuminoid ammonia, the better the water is.

The mere fact that two standards are given is sufficient to show that in considering this quantity we must take into account (even more than is necessary with other chemical data) the nature of the water, and its source.

(vii) and (viii) **Hardness, Temporary and Permanent.**—These are important quantities in coagulation processes. They are usually reported in parts per 100,000 of calcium carbonate, although German chemists report in parts per 100,000 of calcium oxide, and some English chemists in grains per imperial gallon, of calcium carbonate.

1 grain per gallon of calcium carbonate = 1·428 parts per 100,000 of calcium carbonate.

1 part per 100,000 of calcium oxide = 1·786 parts per 100,000 of calcium carbonate.

What chemists really measure is the number of molecules of calcium, or magnesium, present in the water. Consequently, the fact that say 10 parts per 100,000 of calcium carbonate are reported, merely indicates that 2·8 grains per imperial gallon of metallic calcium, or its molecular equivalent in magnesium, exist in the water, combined with one or more of the "acids" enumerated below.

If these metals are combined as carbonates ($CaCO_3$, and $MgCO_3$), or rather, as double carbonates [$Ca(HCO_3)_2$, and $Mg(HCO_3)_2$], the hardness is temporary, and if the metals exist as sulphates ($CaSO_4$, or $MgSO_4$), chlorides ($CaCl_2$, or $MgCl_2$) or other salts, the hardness is said to be permanent.

Using the term degree for 1 part of $CaCO_3$, per 100,000 we find as follows:

Any water under 5 degrees is classed as very soft.
Any water between 5 and 10 degrees, as fairly soft.
„ 10 „ 15 „ neither soft nor hard.
„ 15 „ 20 „ moderately hard.
„ 20 „ 30 „ hard.

Over 30 degrees the water may be considered as objectionably hard, especially for washing purposes, although several large towns use waters which are harder than this.

Moderately hard waters are usually held to be best for public health. Waters with less than 4 degrees of temporary hardness generally act on lead, zinc, and iron, and should consequently be tested for this property.

In those cases where the *Organic Carbon and Nitrogen* are determined in place of oxygen absorbed and albuminoid ammonia, the following standard may be used:

TOTAL ORGANIC CARBON AND NITROGEN IN PARTS PER 100,000

	In Upland Surface Waters.	Other Waters.
Water of great purity .	Under 0·2	Under 0·1
„ medium purity .	„ 0·2 to 0·4	„ 0·1 to 0·2
„ doubtful purity .	„ 0·4 to 0·6	„ 0·2 to 0·4
Impure water . . .	Over 0·6	Over 0·4

Metallic Impurities.—The facts in regard to iron are given on page 584.

Zinc and *Lead* are both dissolved by certain waters, and, in view of their poisonous properties, and use in construction of water pipes, the action of the water on these metals is quite as important as the amount that may already be in solution.

Waters which dissolve zinc are generally very soft (*i.e.* 1 to 4 degrees of temporary hardness), as also are some of the lead solvent waters. In such cases, passing through chalk (see p. 549) is an effective remedy.

The modern bacteriologists' view of chemical analyses is best illustrated by Houston's reports to the London Water Examiner (years 1907–1910). The "chemical" analyses report the following quantities in parts per 100,000, unless otherwise stated:

$$\text{Ammoniacal nitrogen} = \frac{\text{free ammonia}}{1{\cdot}214}.$$

$$\text{Albuminoid nitrogen} = \frac{\text{albuminoid ammonia}}{1{\cdot}214}.$$

Oxidised nitrogen, *i.e.* in nitrates and nitrites, if these latter occur.
Chlorine.
Oxygen absorbed from permanganate, 3 hours at 80 degrees Fahr.
Turbidity in terms of saccharated carbonate of iron.
Colour (by tintometer), Burgess' method, mm. brown, 2 feet tubes.
Total hardness.
Permanent hardness.

Houston states as follows:

"The albuminoid nitrogen, and oxygen absorbed from permanganate, tests are relative measures of the nitrogenous and carbonaceous organic matter in the water."

"The turbidity test measures approximately the suspended matter in the water, and the colour test the degree of brown colouration."

The chemical tests for organic matter must therefore be considered merely as suggestive. The value of such indications is greatest when they are corroborated either by the presence of minerals, such as chlorides, or by the results of an examination of the local conditions affecting the source from which the water is drawn. If the organic chemical tests are isolated facts only, a water which, when thus regarded, is "very pure" may disseminate typhoid, or other diseases, and an "impure water" may be quite harmless.

SITE OF FILTRATION PLANT.—The best location for Purification Works is plainly that which gives least opportunity for after-pollution. Hence, in most cases, the site selected is as close to the town as possible. On the other hand, unpurified water is liable to incrust and foul the supply channels; and where the water is drawn from sources far distant from the town supplied, it is occasionally advisable to submit it to some preliminary process of purification before entering the supply works. Such cases are most frequent in storage reservoirs, and the methods employed are generally designed to prevent action on, or deposits in, the pipes, or conveying channels; and the bacterial purification which may occur is of secondary importance.

The three most usual troubles to be dealt with are (see p. 437):

(i) Slime deposits.
(ii) Deposits of iron or manganese.
(iii) Erosive action, whether on metals or cement.

The first two are somewhat intimately connected. The best method of dealing with either appears to be aeration, followed by a rough filtration, or the installation of Peuch-Chabal or other dégroisseurs.

The Derwent reservoir plant may be considered as typical (see p. 530). It is probably more elaborate than is usually advisable, since it is frequently difficult to find a favourable site for any large installation close to a storage reservoir; and the saving in money expended on pipes, due to the suppression of slime alone, is not very considerable, except in cases where large volumes of water (from the point of view of a town supply) are carried. It must be remembered that slime deposits usually occur only in the first five or ten miles of the channel.

Thus, as a rule, engineers are content with straining the water through fine mesh copper screens. The utility of the process is doubtful (see pp. 438 and 549).

The circumstances which generally prevail close to a storage reservoir (water available at a fairly high pressure and small space for filter) are admirably fitted for rapid or mechanical filtration ; and if this filtration is combined with a properly selected chemical treatment, all types of incrustation and deposits in the supply main can be regarded as impossible. The saving in money thus effected is by no means small, and may (especially in long mains) fully justify an elaborate preliminary treatment, even in cases when the possibilities of after-pollution are such as to render a second filtration nearer to the town absolutely necessary.

Certain waters act on lead, zinc, or other metals. In view of the fact that lead is an accumulative poison, and is not eliminated from the human body, no water that continuously attacks lead can be regarded as safe for human consumption. The tests must, however, be conducted with a view to ascertaining whether the water acts on lead continually, or not ; since many waters are found to attack lead at first contact, but very rapidly cover the metal with a protective coating which prevents any further action. Such waters may be considered as safe, and require no further treatment.

Many waters, principally those drawn from moorlands containing peat bogs, dissolve lead and deposit no protective coating. Such waters, as likewise those which act in a similar manner on zinc or copper, must be treated before they are passed through metal pipes.

The method at present adopted (apparently with universal success) is to neutralise the water. This is generally effected by passing the water through a filter bed containing limestone, or chalk, finely powdered. This process is usually combined with sand filtration, the powdered limestone being mixed with the sand bed.

A similar process has been found useful in the case of waters which attack lime, or cement. These are usually peaty moorland waters of acid reaction, and cases have occurred where their corrosive action has been so intense as to seriously damage the cement linings of the water channels. The danger is especially acute when the concrete aggregate is composed of limestone. (See *P.I.C.E.*, vol. 167, p. 153.)

Methods of Water Improvement.—Any method for the improvement of water intended for human consumption must be judged almost exclusively by the degree in which bacteria are diminished, although popular prejudice as to the suitability of the water is greatly influenced by other, and in reality adventitious qualities, such as clearness, softness, and absence of taste, odour, or colour.

Consequently, it is fortunate that a process which proves successful from a bacterial point of view, generally produces a water gratifying to popular taste. In certain cases, however, special processes have been introduced with a view to softening water, or removing taste, colour, or odour, and these will be described later on.

The conditions usually giving most trouble to officials responsible for public health, are those where the existing water supply is clear, sparkling, and in every way in conformity with popular ideas, but impure from a bacteriological point of view. Such conditions are very frequent, especially in the ground waters of thickly populated countries, and it is often difficult to persuade the community that such a supply is dangerous to health. Popular ideas of a good water vary somewhat in different countries, and this variation has not been without effect on scientific standards. In view of the fact that outside Western Europe and the United States, the official analyst and bacteriologist is frequently a German, either by nationality, or by scientific training, it should be borne in mind that (principally, I believe, due to military considerations) the German standard of good water is a typical, good, ground water. Such experts therefore frequently recommend, in all good faith, a ground water supply, even when better water (from an English or American point of view) is available. While their choice may be assumed to be unexceptionable, from a scientific point of view, it is always well to remember that this preference for a ground water is not shared by other European nationalities, and that (especially among races largely vegetarian in diet), a hard water, even though pure, is liable to be unpopular when introduced to replace a soft one, even though the latter is polluted. Two such cases have come under my personal observation ; in one of them, the introduction of the hard water supply was followed by stomach troubles, to such an extent that the revenue of the Water Supply Company was materially diminished. The inconveniences were rendered more acute because the native population, being accustomed to boil all drinking water, had been but little affected by the pollution existing in the original supply.

Similarly, it must be remembered that American experts are accustomed to deal with waters drawn from sparsely populated (from a European, or Asiatic standard) catchment areas, which contain a large number of bacteria, but apparently not so great a proportion of disease-producing species as is usual in England, or Germany. Hence, it sometimes follows that they are prepared (where economy is the ruling factor) to remain satisfied with a purified water containing a number of bacteria far in excess of any usual European standard. Such opinions, when put forward concerning waters drawn from sources exposed to pollution from a dense population, should be received with suspicion. The danger, however, is not so acute as in the case of Germans and ground water, since the best American practice is as stringent as the best European.

It is somewhat difficult for an English engineer to criticise the methods in which he was trained, but I consider that the English practice is rather too prone to rely exclusively on slow sand filtration, and to regard a moorland water stained with peat somewhat too leniently. From practical experience, I am well aware that peat-stained waters are often quite as objectionable to a population thoroughly unaccustomed to them, as a good German ground water is to a non-German population. I am also inclined to believe that these objections have a very fair foundation in the shape of a somewhat higher death-rate from diseases such as infantile diarrhœa, and other minor stomach ailments. It is therefore always advisable to consider carefully whether the nationality and experience of the bacteriologist recommending the source of supply are such as to enable him to gauge local prejudice accurately.

The methods employed for water purification may be divided into three classes :

(i) Sedimentation.
(ii) Straining, or filtration.
(iii) Chemical methods.

This classification can only be regarded as a practical one ; for, when carefully investigated, even the slow sand filter is found to produce chemical changes, and its working is largely conditioned by the amount of sedimentation that takes place before filtration.

Consequently, it appears more logical to describe and investigate the slow sand filter, and to consider the other methods mainly as supplementary, or alternative.

It should be understood that I do not wish to advocate slow sand filtration in every case. It is a very excellent treatment for most waters, but is not applicable to all. Probably the error most frequently made in hydraulics by English engineers is its application to waters better dealt with by other processes. The idea that a good process of purification necessarily entails the use of a slow sand filter is responsible for much unjustifiable expenditure of money.

The actual facts are that slow sand filters, when properly worked, are capable of satisfactorily removing bacteria. They are, however, readily clogged, and rendered temporarily useless by substances existing in a colloidal state (*i.e.* in a state resembling very diluted glue or jelly). Where the water is turbid (much over 50 parts per million), and the particles producing the turbidity are of, or close to, bacterial size, the sand becomes dirty to such an extent that cleaning is difficult, and a portion of the turbidity is not removed by the filters. In such cases, preliminary treatment is necessary, *e.g.* the colloids are precipitated by aeration, or by treatment with iron ; or the turbidity is rendered filtrable by coagulation with aluminium sulphate or lime, with or without iron sulphate.

Thus, all waters can be rendered fit for purification by the process of slow sand filtration. But it must not be assumed that this method of treatment is necessarily either the most effective, or the cheapest.

SLOW SAND FILTERS.—Practical details of the working of slow sand filters are principally due to English engineers. Our scientific knowledge of the system, and its rationale, is mostly of German origin.

Postponing constructional details for the moment, it may be stated that the effective portion of the filter is not the sand, but the **Schmutzdecke,** and the **Zooglea.** These may be defined as follows :

Schmutzdecke, is the layer of fine sediment containing bacteria and other organised matter (algæ, etc.) that forms on the surface of the sand layer.

Zooglea, is a glutinous coating, also containing bacteria, and probably entirely of bacterial origin, which forms on the individual grains of sand.

Sand, in itself, can hardly be supposed to exercise any straining action on bacteria, for it forms far too coarse a filter. The relative size of a bacterium and the passages between the grains of sand are such, that we might just as well expect a sieve with holes one inch square to retain grains of fine sand.

If a filter which has been at work for some period is examined, a thin crust of "dirt" will be found on the top of the sand. This forms the Schmutzdecke, or rather, the Schmutzdecke is contained in this crust. Also, the individual grains of sand will be found to be no longer sharp and gritty to the touch, but coated with a gelatinous, transparent substance, resembling a whitish coloured glue when viewed under a lens. This is the Zooglea.

The action on bacteria is believed to be somewhat as follows: The passages through the Schmutzdecke are so small that the bacteria are retained in it, and are consumed there by the living organisms composing a portion of the Schmutzdecke. Until lately, this was generally considered to be the only effective portion of the filter. More recent investigations make it probable that a certain fraction of the bacteria pass through the Schmutzdecke, and are arrested by the Zooglea (probably by some action more resembling that of a layer of stones covered with bird-lime, than a sieve), and are there consumed. The relative value of the Schmutzdecke and Zooglea in removing bacteria is somewhat uncertain, and careful investigations are desirable. It is plain that if the Schmutzdecke alone were the active agent, the layer of sand could be made considerably thinner than is usual in good practice; and there is no doubt that of later years there has been a decided tendency to decrease the thickness of the sand layer. On the other hand, should the Zooglea prove to be more important than is at present believed, it is probable that some advantage might be gained by thickening the sand layer.

So far as our present evidence goes, it appears that the Schmutzdecke performs most of the work. I would, however, point out that the evidence is by no means conclusive. The fact that comparatively few living bacteria are found below the Schmutzdecke may only mean that the destructive action of the Zooglea is very rapid. The practical results of filtration through thin layers of sand are not always sufficiently satisfactory to enable us definitely to say that: "The thickness of the sand layer need only be adequate to support the Schmutzdecke, and to prevent accidental fissuring."

The practice which the London Water Companies had arrived at before modern bacteriological investigations had been made, is probably not very far from the truth. If this practice is accepted as correct, we may deduce the following principles.

When the filter is working normally the Schmutzdecke is probably quite capable of destroying all the bacteria existing in the raw water, unless this is abnormally polluted. Every filter, however, is cleaned at fairly frequent intervals, and the efficiency of the filtration system as a whole greatly depends on the rapidity with which a filter attains its normal power of destroying bacteria after cleaning.

Now, cleaning is essentially the removal of the Schmutzdecke, and normal working only commences after the formation of a new Schmutzdecke. The almost universal practice of waterworks' engineers is to replace the sand scraped off in cleaning by old filter sand that has been washed ("ripe sand"), and which is still coated with Zooglea. Since, in many cases, this washed ripe sand is more costly than the freshly dug article, it appears that practical experience favours the idea that some extra expenditure in order to obtain a certain thickness of sand coated with Zooglea is advantageous.

Thus, British engineers had come to the conclusion that while equally good results could be obtained with filters containing only 18 inches or 2 feet of sand as with those containing 3 or 4 feet of sand, yet the results obtained with the thinner layers of sand were less consistent, and were more easily detrimentally affected by carelessness in working, or by sudden changes in the condition of the water, or of the weather.

This practice is still standard, and although a skilled bacteriologist is now able to ascertain the exact causes producing these detrimental effects, the average supervisor of filters is not a skilled bacteriologist, and is therefore likely to be easily led into error by wrongly applying his experience. Thus, the practical view of the matter appears to be that a choice must be made between thick layers of sand and (relatively speaking) a low grade of supervision, or thin layers of sand, and skilled, scientific supervision.

In connection with this question it is also as well to point out that rupture of the Schmutzdecke must occasionally occur, especially in the summer months, and that after such fissuring a satisfactory filtrate can be obtained, provided only that the break is not so marked as to cause the formation of definite channels through the sand. In some instances indeed, it has been found advantageous to purposely rupture the Schmutzdecke, and resume working after say, six or seven hours' rest (see Rutter, *P.I.C.E.*, vol. 146, p. 258).

In both cases it seems hard to avoid the deduction that a fair thickness of sand coated with Zooglea is in itself a very efficient filter.

The question is also of importance in connection with dégroisseurs, and mechanical filters, and will be referred to later on.

Constructional Details.—The area of filters required to purify a given volume of water daily depends entirely upon the quality of the raw water. Systematic preliminary tests should be made, as the necessary expenditure may easily be recouped several times over.

The German Government has definitely laid down that the maximum permissible velocity of filtration must not exceed 4 inches per hour. This means a yield in filtered water of 8 cubic feet, say 50 gallons (60 U.S. gallons) per square foot of filtered area per day; or, taking the usual, but illogical units, 2·18 million gallons (2·61 million U.S. gallons) per acre per day.

It is to be hoped that no such cast-iron rule will be introduced into other countries. Such enactments may be considered as a bureaucratic extension of the principle,—"a ton of bricks we understand, an ounce of brains is beyond our intellect."

Baldwin Wiseman (*P.I.C.E.*, vol. 165, p. 352) has collected the filtration velocities used in forty-one British waterworks.

The mean value is 9·17 feet daily, or say 2·5 million gallons (3 million U.S. gallons) per acre per day.

The maximum value is 20·4 feet per day, at Falkirk. At Ripon, where the water receives some preliminary treatment, it is 17·8 feet. All cases where the velocity is over 12 feet a day occur in small country towns.

The minimum value is 1·4 foot per day, but this, it is believed, is due to the filter being too large for the present supply; and 2 to 2·4 feet daily is probably the true minimum.

I have been unable to trace any connection between these figures, and such of the death-rates from typhoid in the above towns as are accessible to me. I therefore consider that such variations are permissible in good practice, and are entirely caused by variations in the quality of the raw water.

It is, however, possible to classify the figures given by Wiseman according to the source from which the raw water is drawn, as follows:

Waters from storage reservoirs (27 cases).

Mean value	8·93 feet per day.
Maximum value	20·40 " "
Minimum value	2·5 " "

and in this last instance the gathering-ground is thickly populated, and the water is therefore unusually exposed to pollution.

Waters from rivers (7 cases).

Mean value	5·60 feet per day.
Maximum value	9·30 „ „
Minimum value	4·60 „ „

Waters from wells, or springs, i.e. *ground waters* (7 cases).

Mean value	9·60 feet per day.
Maximum value	14·10 „ „
Minimum value	3·5 „ „

The number of cases is hardly sufficient to allow of any very reliable deductions being drawn, more especially in view of the fact that many of the filter beds are known not to be as severely worked as they will be when the population increases.

There is, however, a very clear connection between the density of the population inhabiting the drainage areas, together with the amount of control the Waterworks' Authority possesses, and the velocity of filtration. It appears that it is cheaper to expropriate any small existing population, rather than to build the extra area of filters necessitated by habitation of the catchment area, owing to the reduction in filtration velocity. There is also an apparent connection between a large annual rain-fall on the catchment areas, and high velocities of filtration. Whether this is a real connection, or only indicates that wet catchment areas are thinly populated, is uncertain.

The filtration velocities employed in America vary to a far greater extent, and in most cases these variations are justified when the qualities of the raw waters are considered. American practice, as opposed to English, is not so exclusively founded on experience of slow sand filters alone, and I consequently consider the American figures more applicable to methods which include not only sand filters, but also auxiliary processes.

Having, either by special experiments, or from experience in neighbouring cities using similar waters, selected the appropriate velocity of filtration, we can determine the nett necessary area of filters by the equation :

$$\text{Area in square feet} = \frac{\text{Maximum daily consumption in cubic feet}}{\text{Yield of filters per square foot}}.$$

To this nett figure must be added an allowance for the filter area which is out of use during cleaning. As a matter of experience, filters require cleaning most frequently during the season when the consumption is at a maximum (*i.e.* the hot weather season). Burton (*Water Supply of Towns*) suggests the following rule :

For a population of 2000 :

Allow 2 filter beds, 1 of which can deal with the maximum consumption.

For a population of 10,000 :

Allow 3 filter beds, 2 of which can deal with the maximum consumption.

For	60,000 population,	4 beds.
„	200,000 „	6 „
„	400,000 „	8 „
„	600,000 „	12 „
„	1,000,000 „	16 „

When the number of filter beds exceeds 8, the possibility of 2 of them having to be cleaned simultaneously should be allowed for (see p. 531).

Design of Filters.—It is first necessary to fix the total depth of the whole filter, including the drains, the gravel and sand layers, and the depth of water above the sand, as this generally determines the design and cost of the whole filter.

The depth of water is usually about 3 feet, to 3 feet 6 inches. If a smaller depth than 3 feet is adopted, it will be found that the water tends to become unduly warm in summer, even in so temperate a climate as that of the British Isles. Such a depth, with a freeboard of 6 inches to 1 foot will cause the upper surface of the sand layer to be about 3 feet 6 inches, to 4 feet 6 inches below the top of the filter.

It is probable that a greater depth of water might be advantageous in warmer climates, not so much with the object of keeping the water cool, as to minimise the activity of vegetable and animal life in the Schmutzdecke. This practice has not as yet been largely adopted, and the extra cost entailed is obvious. Nevertheless, tropical installations of slow sand filters are far too frequently designed on lines found suitable in temperate climates, and it is possible that many of the troubles then met with are due to an insufficient depth of water over the sand. Whether the correct solution lies in a greater depth of water, or in the installation of mechanical filters, or in some previous chemical treatment, is a question which the local peculiarities of the water must decide.

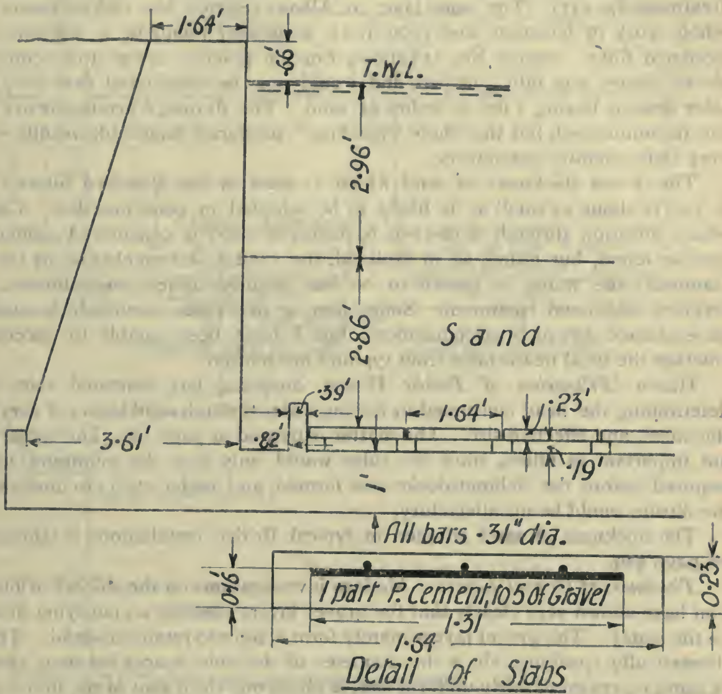
In any case, the adoption of portable ejectors for lifting the sand removed in cleaning, minimises the difficulty previously met with in working filters where the sand layer was much over 5 feet below the top of the filter, this being nearly the maximum height that a wheelbarrow can be rolled up a plank of ordinary size, laid from the sand to the top of the filter. The depths of water in typical British filters are tabulated on page 529.

Covering of Filters.—In cold climates, it is usual to cover the filters with a vaulting of concrete, or brick arches, in order to prevent the water from freezing. Covered filters are more costly, and are bacterially less efficient than the open type (see p. 531). Hazen states that while open filters are worked in climates where the mean temperature of the coldest month is as low as 27 degrees Fahr., yet, where this temperature is less than 31 degrees Fahr., newly constructed filters are almost invariably covered.

It is becoming usual in tropical climates to shade filters by a light roof of galvanised iron, or slate, supported on columns. In many cases, where dust storms are of frequent occurrence, some such shelter is almost indispensable, and there is no doubt that working is rendered more easy in hot weather. Many tropical installations, however, succeed without any shelter being provided. I am not aware that any difference in bacterial efficiency has yet been observed between filters thus protected, and the ordinary unshaded filter.

Thickness of Sand Layers.—The evidence in favour of the belief that ripe sand in itself exercises a destructive action on bacteria, has already been given. A certain minimum thickness of sand is desirable in practical working, if only to prevent the formation of definite channels in the sand, which would permit the passage of unfiltered water if the Schmutzdecke were ruptured.

We have also to consider the working of the filter after it has been cleaned, and while it is well known that a filter does not yield properly purified water for some period (say 24 hours) after each cleaning, yet a satisfactory filtrate is delivered after a far shorter interval than is required to form a good Schmutzdecke. It is therefore considered that the Zooglea is important since it permits earlier delivery of properly purified water. Thus, until more detailed evidence is forthcoming, it is inadvisable to lay too much stress on the theory of the Schmutzdecke alone, and a smaller thickness of sand than say 1 foot 10 inches does not appear to be desirable even in favourable cases. The



SKETCH No. 139.—Filters at Ivry.

matter concerns not only the design of filters, but their working, since it appears advisable to provide a large thickness of sand when the preliminary studies indicate that the filters will have to be cleaned frequently, so as to be able to start the hot weather of each year (or that period during which the filters require most cleaning) with a good depth of sand. This depth may then be diminished to the minimum thickness either by special removal, or by the accumulated effect of scraping, as the season approaches when cleaning is less frequently necessary (see p. 532).

The principles are plain :—In polluted water of a character such that the filters must be cleaned at short intervals, a large thickness of sand is indicated, and this should not be too much diminished by scraping unaccompanied by any replacement of sand. In less polluted water, and where the filters do not need frequent cleansing, the Schmutzdecke may be considered as capable of effecting the whole work of purification without any assistance. Thus, a thin layer of sand, the thickness of which may be largely diminished by scraping, is indicated, and the consequent reduction in the total depth of the filters permits a material saving to be made in first cost.

The Ivry beds (Sketch No. 139) are extremely thin, gravel and drains being replaced by "dalles filtrantes." The water is subjected to a careful preliminary treatment (p. 547). The sand layer at Albany (Sketch No. 140) performs the whole work of filtration, and is as thick as is ever found in a scientifically designed filter. Sketch No. 142 shows London practice before the Schmutzdecke theory was fully grasped, and would now be considered defective, the later designs having 2 feet 9 inches of sand. The drainage arrangements are not recommended, but the whole "machine" produced admirable results with very little scientific assistance.

The 1-foot thickness of sand which is used in the Bamford filters (see p. 530) is about as small as is likely to be adopted in good practice. Cases where filtration through 9, or even 6, inches of sand is considered sufficient can be found, but either, as in Holland, the sand is extremely fine or (as at Bamford) the water is known to be but slightly subject to pollution, or receives additional treatment. Some four or five cases occur which cannot be explained by such circumstances, but I have been unable to ascertain whether the local death-rates from typhoid are normal.

Hazen (*Filtration of Public Water Supplies*) has indicated rules for determining the head consumed in forcing water through sand layers of varying thickness, and effective size. The matter is treated at page 25. The subject is not important in filters, since the rules would only give the minimum head required before the Schmutzdecke was formed, and under such circumstances the filtrate would be unsatisfactory.

The thickness of sand adopted in typical British installations is tabulated on page 529.

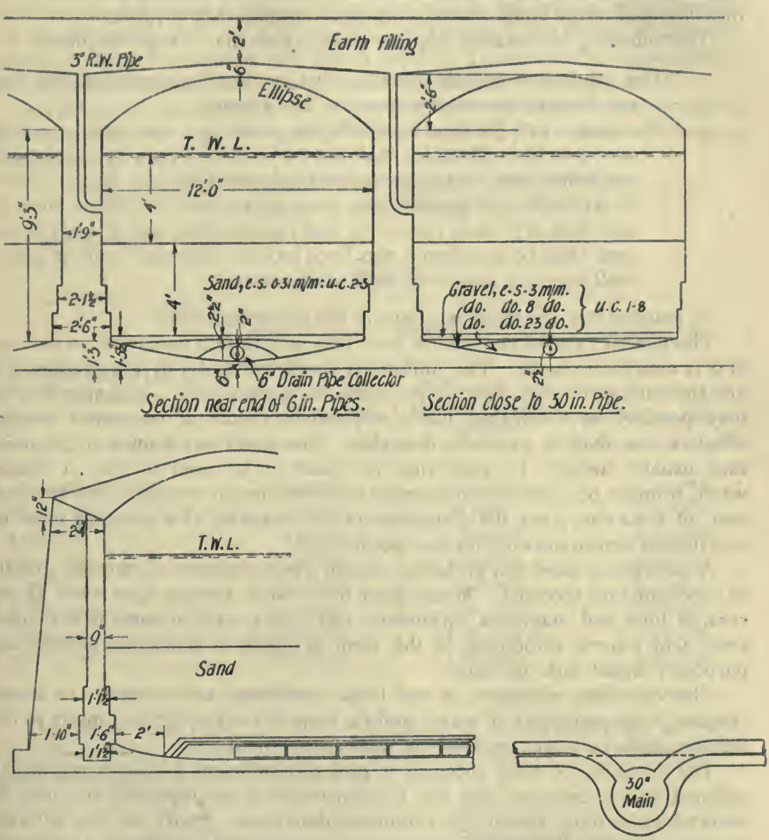
Thickness of Gravel Layers.—Modern investigations on the subject of filtration have shown very clearly that the gravel layers exercise no purifying action on the water. The gravel layers merely form a sieve to retain the sand. Thus, theoretically speaking, since the diameter of the void spaces between grains of sand, or gravel of fairly uniform size, is about one-third that of the individual grains, all that is required is a layer of gravel underneath the sand, the effective size (see p. 25) of which is a little less than three times that of the sand. This, in its turn, might rest on a layer of still larger gravel, the effective size of which is just under three times that of the first layer. This succession of layers might be continued until a size has been reached which cannot enter the open joints of the tile, or brickwork drains.

The annexed specification is a very fair model for a filter suitable for British conditions.

Depth of water over the sand, 2·5 feet. Thickness of sand layer, 2 feet 6 inches as a maximum, reduced to 1 foot 8 inches by scraping.

The sand should have an effective size of 0·36 to 0·42 mm., and a uniformity coefficient between 2·0 and 2·5 (see p. 25), and, except in acid waters, should not contain more than 0·5 per cent. of carbonates of lime, or magnesium, and should also be carefully washed, so that the content of clay is less than 0·4 per cent. by weight.

A layer of 6 inches of gravel should be laid below the sand, approximately $\frac{1}{8}$th of an inch in size (*i.e.* the effective size should be about 0·04 inch), and below this should be a layer 6 inches thick of $\frac{3}{8}$th-inch gravel resting on a continuous layer of brick drains (Sketch No. 141), or a 4-inch layer of $\frac{3}{8}$th-inch



Sketch No. 140.—Covered Filters at Albany.

gravel resting on a 5-inch layer of 1-inch gravel, in which 4-inch agricultural drain-pipes are buried, these being spaced not more than 15 feet apart (Sketch No. 140).

The specification of the uniformity coefficient is probably unnecessary. A high value of the uniformity coefficient indicates that the grains of sand vary considerably in size. Thus, when the uniformity coefficient is large, the void spaces are likely to be smaller than is usually the case, and therefore the water will pass through the sand less readily than is indicated by the effective size of the sand.

Reference is made to this matter on page 530. What is required in filter sand is that the sand shall permit water to pass as easily as is usual in a sand of similar effective size, and this could be secured just as well by specifying the percentage of voids, as the uniformity coefficient, were it not for the questions regarding the wetness of the sand and the amount of shaking previous to the measurement of the voids which a litigious contractor might raise.

The following specification appears to secure all that is really required :

> "The sand shall contain no clay, dust, or organic impurities, and shall not disintegrate when exposed to air or water.
>
> "The quantity of the sand in which the grains are less than 0·13 mm. (0·005 inch) in diameter shall not exceed 1 per cent. by weight, and not more than 10 per cent. by weight shall be less than 0·27 mm. (0·011 inch) in diameter. At least 10 per cent. by weight shall be less than 0·36 mm. (say 0·014 inch) in diameter, and at least 70 per cent. shall be less than 1 mm. (0·04 inch) in diameter, "and no grains shall exceed 5 mm. (0·20 inch) in diameter."

In practice this specification produces the following results :

The effective size of the sand is from 0·29 mm. to 0·32 mm., with an average of 0·31 mm. (0·012 inch). The uniformity coefficient is 2·2 to 2·5, average 2·3, and the void spaces are about 40 per cent. of the total bulk. The specification may possibly be considered likely to produce a sand of somewhat smaller effective size than is generally desirable. The water was known to be more than usually turbid. In specifying for sand to be used to filter a clearer water, it might be advisable to increase the 0·27 mm. to 0·30 mm., and the 0·36 mm. to 0·40 mm. ; but the properties of the available raw material must be ascertained before drawing up the specification.

A satisfactory sand can probably contain a percentage of carbonates greatly in excess of that specified. Many good filter sands contain 2, or even 2½ per cent. of lime and magnesia carbonate ; and (see p. 549) in some filters which treat acid waters, carbonates in the form of chalk or limestone powder are purposely mixed with the sand.

The variations necessary to suit local conditions are obvious. In hotter climates, a greater depth of water, and (in view of the liability to rupture of the Schmutzdecke) a larger thickness of sand are required.

The thickness of sand specified is probably sufficient for all except highly polluted waters, provided that the Schmutzdecke is not ruptured, and may be reduced under more favourable conditions than those usually existing in Great Britain. On the other hand, if the water is occasionally very turbid, a minimum thickness of 3 feet, or even 3 feet 6 inches, of sand may be required to permit of the water being satisfactorily filtered. In the United States 3 feet usually suffices during periods when the turbidity is as high as 125 parts per million, provided that the turbidity does not prevail for longer than two or three days.

It must be pointed out that each layer of gravel means a certain extra depth, and the real object merely being to retain the sand, the thinner the whole series of layers, the better. Looking at the question from this point of view, it will be evident that Sketches No. 139 and No. 140 show somewhat more economical methods of retaining the sand layer than the specification.

Sketch No. 139 indicates the *dalles filtrantes* (filtering paving) adopted at Ivry for the water supply of Paris, and it will be seen that a depth of about 8 inches is economised. Gravel can also be saved by laying the lateral drains in inverts formed in the bottom of the filter. In some cases the laterals are laid in small trenches formed in the concrete lining. So far as is known the system is satisfactory, although it obviously may give rise to difficulties if the rate of filtration is high. The drainage system specified is generally sufficient for velocities of filtration up to 4 million gallons per acre, or 15 feet vertical per twenty-four hours. If higher rates are proposed, the design of the drain system, and the thickness of the lowest layer of gravel, require consideration, and the gravel beds must be made deeper and the drain pipes larger.

According to Hazen (*Filtration of Public Water Supplies*) the head consumed in forcing water through the gravel to the drains may be calculated from the formula (see p. 26):

$$\text{Head lost} = \frac{(\frac{1}{2}\text{ distance between drains})^2 \times \text{rate of filtration}}{2 \times \text{average depth of gravel in feet} \times c} \text{ feet.}$$

The values of c, are as follows:

VALUES OF c.

Effective size of Gravel in Millimetres.	The Rate of Filtration being expressed in—		
	Millions of Gallons per Acre per Day.	Millions of U.S. Gallons per Acre per Day.	Vertical Depth in Feet per Day.
5 = 0·2 inches .	19,000	23,000	70,000
10 = 0·4 ,, .	54,000	65,000	199,000
15 = 0·6 ,, .	92,000	110,000	337,000
20 = 0·8 ,, .	133,000	160,000	490,000
25 = 1·0 ,, .	192,000	230,000	704,000
30 = 1·2 ,, .	250,000	300,000	918,000
35 = 1·4 ,, .	325,000	390,000	1,193,000
40 = 1·6 ,, .	400,000	480,000	1,469,000

The lateral drain pipes are usually spaced about 16 feet apart, and are rarely more than 20 feet distant. Hazen gives the following table of maximum permissible velocities of water in filter drain pipes:

Diameter of Pipes.	Maximum Velocity.
4 inches	0·30 feet per second.
6 ,,	0·35 ,, ,,
8 ,,	0·40 ,, ,,
10 ,,	0·46 ,, ,,
12 ,,	0·51 ,, ,,
Over 12 ,,	0·55 ,, ,,

These formulæ may seem somewhat unnecessary, but a little consideration will show that the low velocities are adopted in order to equalise the total frictional resistance at all points of the filter bed. It will be plain that close to the main drain outlet the resistance to the passage of water is merely that arising from percolation through the vertical thickness of sand and gravel; while at a point midway between the ends of the farthest removed laterals the resistance is that caused by the vertical thickness of sand, about 9 feet of gravel (assuming a 16-feet spacing of laterals) and pipe friction in the full length of a lateral and the main drain. Even with such low velocities as are above specified, this inequality of resistance (especially when the Schmutzdecke is thin) may give rise to great differences in filtration velocity. In normal working the resistance of the Schmutzdecke is usually sufficient to produce practical equality in filtration rates, but the apparently unnecessary size specified for the lateral and main pipes is now known to ensure a rapid attainment of the normal state of affairs.

In large filter beds each lateral is sometimes fitted with a brass "friction disc," in order to equalise frictional resistances. This, however, merely cuts out the disturbing influence of friction in the main drain, and introduces certain constructional complications. Except in very large installations, the expense of a series of discs of various sizes is hardly justified by the economy effected in pipes.

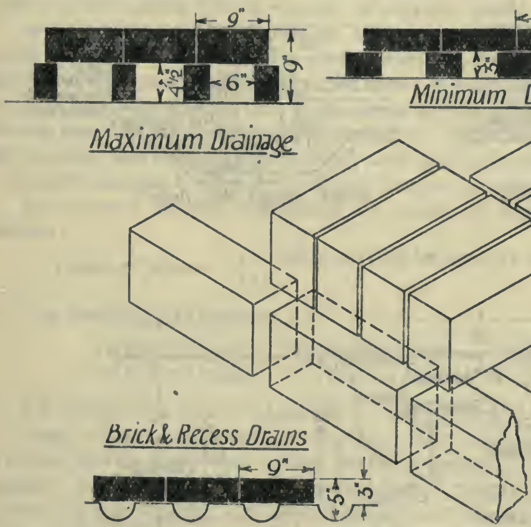
As an example, when a certain filter is working at its maximum rate, the head lost in the main drain is about 1 inch, and the frictional resistance in the sand, before the Schmutzdecke forms, is also about 1 inch. Thus, the rate of flow through the portion of the filter drained by laterals entering the main drain near its exit will be about double that through the area drained by the laterals entering near the far end of the main drain.

A brass disc containing an orifice of a size such that it will pass the quantity of water which the first lateral carries under 1 inch head, is inserted at the junction between the first lateral and the main drain. Similarly, in a lateral half-way along the main drain, the orifice is calculated so as to pass the water under half an inch head, etc.

In the above example, the mean velocity in the main drain is 1·2 foot per second. It is plain that if we had designed the main drain for a velocity 0·6 foot per second, the 1 inch difference would have been reduced to $\frac{1}{4}$ inch, and might have been neglected, but the main drain would have been doubled in area. For formulæ for the loss of head in lateral and main drains, see p. 613.

It will also be plain that when the filter has been working for some weeks, and the head lost in the Schmutzdecke and sand has (owing to the increased thickness of the Schmutzdecke) become say, 1 foot, the friction discs have no appreciable influence on the working. For this reason, both friction discs and the application of Hazen's rules are frequently regarded as unnecessary. There is a good deal to be said for this view, but the period which elapses between scraping a filter and delivery of a bacterially satisfactory filtrate is a critical one. It is then, and then only, that there is any considerable danger of impure water entering the mains. Any small expenditure of money that assists the supervisor to reduce this period is well spent, and devices for equalising the rate of flow through the various portions of the filter area are the one obvious assistance that the designer can give. It must always be remembered that bacterial investigations take time, and, although Houston in London has successfully solved the problem of indicating dangerous (or rather potentially dangerous) filtrates on the same day that the samples are received, it is but rarely that the supervisor receives any bacterial evidence until the third day after filtration. Thus, in practice, filtrates are really classed as safe or unsafe by turbidity tests alone. The method works exceedingly well in practice, but that does not justify a total neglect of the possible danger by the designer.

A study of the proportions of modern filters indicates that a sufficiently uniform rate of filtration over the whole area of the filter bed is secured if the head lost by friction in the longest path through the under-drains (*i.e.* in the full length of a lateral and the full length of the main drain) does not exceed one-quarter, or, at the most, one-third of the head required to force the water from the surface of the sand, through the sand and the gravel, to a lateral drain before the sand has become clogged by the formation of the Schmutzdecke.



SKETCH NO. 141.—Brick Filter Drains.

Hazen's figures (see pp. 25 and 269) may be employed to estimate this head.

The rule is merely empirical, and in actual practice the time that elapses, and the thickness of the Schmutzdecke which forms, before any water is drawn off for use in the town mains should be considered. The rule is valuable, as unequal rates of filtration will probably not cause trouble in a filter thus proportioned unless it is very carelessly handled. Typical actual figures are :

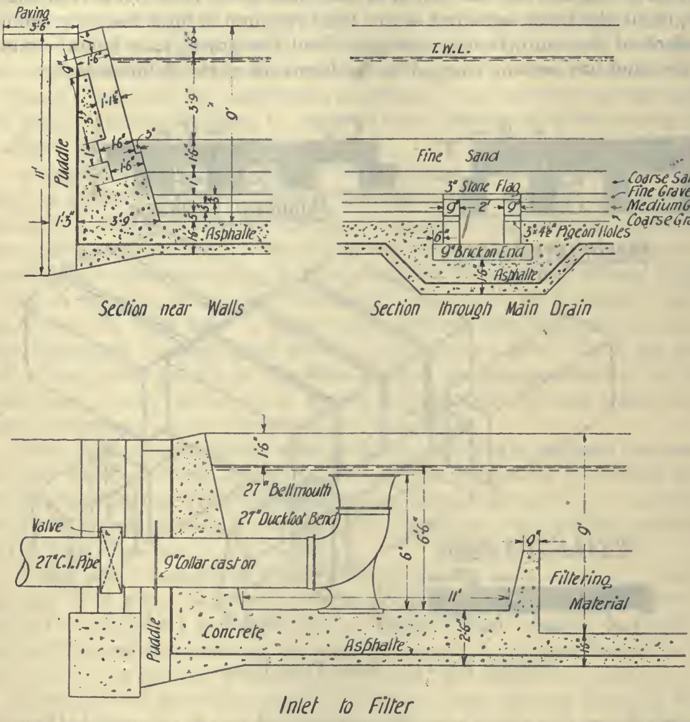
The head required to force water through sand at the commencement of working is about $1\frac{1}{2}$ inch to 2 inches.

The head lost in the under-drain is $\frac{3}{8}$ths of an inch to $\frac{1}{2}$ inch.

It will be evident that frictional inequalities are greatly minimised by such drainage systems as the flooring of brick or tile drains generally adopted in England (Sketch No. 141) or the *dalles filtrantes* used at Ivry. In brick or tile floors, however, this simplicity is gained at a cost of an extra depth of 3 to 7 inches (according as the pipe drains which form the alternative method are, or are not, sunk below the general floor level).

Among other details, it is as well to draw attention to the possibility of unfiltered water creeping down the vertical faces of the side walls. This, I consider, is best prevented by making these faces rough, *i.e.* of rubble masonry, or unrendered concrete.

At Ivry, a circumferential drain (Sketch No. 139) is constructed, but this appears likely to provide a trap for stagnant water, and possibly reduces the



SKETCH No. 142.—London Filters of about 1890.

effective area of the filter bed. The step in the face shown in Sketch No. 142, is usual in English designs, and seems to be equally effective.

The danger is most acute in covered filters, where each pier provides a possible passage. Consequently, in such cases it is usual to stop the gravel layers at some distance from the piers and walls, as shown in Sketch No. 140. This appears safe, and quite unobjectionable, except that sand is employed to replace the less costly gravel.

Working of Filters.—The constitution of the Schmutzdecke and Zooglea has already been discussed. The formation of the Schmutzdecke (as judged by the production of a satisfactory filtrate) occupies from 12 hours to 2 days.

The period of formation depends upon the quality of the raw water, and it may usually be stated that a slightly turbid, and somewhat polluted water, favours the rapid growth of a good coating. As, however, the process is essentially biological in character, the temperature of the water and the weather generally markedly affect the process. There is also a certain amount of evidence to show that ripe sand (*i.e.* well coated with Zooglea) has a favourable influence. Once the coating is formed, filtration (accidents apart) proceeds normally, and regularly.

It will be found that the head necessary to force water through a filter at the required rate, increases daily, but irregularly. Finally, the necessary head becomes too great for economy, and the filter has to be cleaned. This is accomplished by removing the top layer of sand. Usually a depth of three-quarters of an inch is found sufficient, but the practical condition is that all sand which is visibly dirty is scraped off with flat spades (see also p. 542).

British Filters.—The details of filters, as constructed in England, are mainly useful in determining the depth of water over the sand, the thickness of sand, and the size and thickness of the upper layer of gravel. The facts are not of great importance in the case of the lower layers. The designs are by rule of thumb, and take no account of the modern investigations into the rationale of the process, which show that the sizing and spacing of the lower layers may be very badly designed, without much detriment to the results.

An abstract of the particulars as given by Baldwin Wiseman is as follows :

Depth of Water.— { 31 cases varying between 11·5 and 0·75 feet.
21 „ „ 3·0 „ 1·42 „

The details are as follows :

3·0 feet ; 1 case	2·5 feet ; 5 cases	2·0 feet ; 7 cases
1·75 feet ; 3 cases	1·5 feet ; 4 cases	1·42 feet ; 1 case.

All except three cases lie between 3·5 and 1·0 feet.

Sand.—Fine sand is used in 26 cases. The maximum thickness is 4 feet 2 inches, and the minimum 6 inches, but this is underlain by 9 inches of $\frac{1}{16}$th-inch gravel. 1 foot 6 inches is the minimum where $\frac{1}{16}$th-inch gravel is not also used. Between 3 feet 6 inches, and 1 foot 6 inches, we have 21 cases as follows :

3 feet 6 inches ; 4 cases	3 feet 1 inch ; 1 case	3 feet 0 inches ; 5 cases
2 feet 6 inches ; 2 cases	2 feet 0 inches ; 6 cases	1 foot 8 inches ; 1 case
	1 foot 6 inches ; 2 cases.	

Gravel.—In 5 cases the sand is succeeded by :

1 foot 9 inches ; in 3 cases 6 inches ; in 2 cases

of $\frac{1}{16}$th-inch gravel.

In 14 cases sand is succeeded by $\frac{1}{8}$th-inch gravel of the following thickness :

6 inches ; in 6 cases 5 inches ; in 1 case 4 inches ; in 1 case
3 inches ; in 1 case.

In 1 case by 3 inches of $\frac{3}{16}$th-inch gravel ;
In 2 cases by 1 foot and $7\frac{1}{2}$ inches of $\frac{3}{8}$th-inch gravel ;
In 2 cases, 6 inches of $\frac{1}{2}$-inch gravel ;

and in 2 cases the sand rests direct on brick drains.

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Neglecting abnormal cases, the usual layer below the sand is consequently 6 inches of $\frac{1}{8}$th-inch gravel. This is generally followed by 6 inches of $\frac{3}{8}$th-inch gravel, or in some cases by 1 foot of $\frac{3}{8}$th-inch gravel, containing drains.

Cleaning.—The average interval between cleanings is 24 days, the actual figures given being :

7 to 21 days in 1 case	10 to 12 days in 1 case	14 days in 5 cases
18 days in 1 case	21 days in 2 cases	21 to 42 days in 1 case
24 days in 1 case	28 days in 4 cases	30 days in 1 case
31 days in 1 case	42 days in 4 cases	190 days in 1 case
	365 days in 1 case :	

where the 190 and 365 can hardly be considered as good practice.

The following description of the sand filters used by Sandemann at the Derwent Valley reservoirs represents the most advanced British practice, except in the thickness of sand, which is less than usual, owing probably to the character of the water and to the installation of dégroisseurs (see p. 544). Each filter is 125 feet × 200 feet in area, and is worked at the rate of 12 feet per day.

The sand layer is 2 feet thick, and is reduced by scraping to 1 foot before replacement of sand occurs. The sizes of the grains of sand are specified as follows :

All the sand passes a $\frac{3}{16}$th inch circular hole.
70 per cent. passes a $\frac{1}{25}$th inch square hole.
10 per cent. passes a $\frac{1}{70}$th inch square hole.

This is stated to procure a sand of approximately the same percolating properties as that used in the London filter beds. The effective size is 0·35 mm. = 0·014 inch, and the uniformity coefficient about 2·5 or less.

Proceeding downwards, the gravel layers are as follows :

3 inches of fine gravel, passing a $\frac{5}{16}$th-inch sieve, and retained on a $\frac{3}{16}$th-inch sieve.

3 inches of medium gravel, passing a 1-inch sieve, and retained on a $\frac{5}{16}$th-inch sieve.

9 inches of coarse gravel, passing a 2-inch sieve, and retained on a 1-inch sieve.

The whole of the area of the filter is floored with bricks, forming $4\frac{1}{2}$-inch by 3-inch drains, 9 inches apart.

The central drain is 2 feet square, and sunk below the level of the filter bottom.

Thus, the total depth of the filter is 6 feet 6 inches. The velocity of the water in the central drain is about 0·9 feet per second. This is somewhat less than the 2 to 3 feet per second which is usually adopted in British practice, when the filter is floored with brick drains ; but it appears advisable in view of the thin layer of sand and the somewhat high rate of filtration.

In older British filters, the velocity of the water in the tile drains was often as high as 2 feet per second, when the coarse gravel extended to 6 inches over the top of the drains. This is probably somewhat high, unless the water passes through the gravel as well as through the tile drains.

The following figures show the influence which the character of the water has upon the rate of filtration, and upon the frequency of cleaning. The figures are selected from filtration plants which are known to be very scientifically managed.

At Hamburg, the water is highly polluted, black, and muddy. The sedimentation basins have a capacity equal to four days' supply, and the water after three, or occasionally only two days' sedimentation is filtered at the rate of 4·9 feet vertical (1·33 million gallons = 1·6 million U.S. gallons per acre) per 24 hours. The average interval between scrapings of the filters is about 18 days, the maximum being 50, and the minimum 10.

At Berlin, ordinary clear lake water is passed direct on to filters, and is filtered at a rate of 7·9 feet (2·13 million gallons = 2·56 million U.S. gallons per acre) per 24 hours. The filters are scraped, on the average, every 30 days; the maximum period being 80 days, and the minimum 10 days.

At Zurich, a perfectly clear water drawn from a large lake, and entirely free from sediment, was filtered at a rate of 23 feet (6·25 million gallons = 7·5 million U.S. gallons per acre) per 24 hours. The average period between scraping was 21 days, the maximum being 47 days, and the minimum 9 days. Contrary to usual experience, the covered filters are here found to be more efficient than the open filters. At present the Zurich water is treated by a double filtration process.

Fuertes (*Water Filtration Works*) gives for the gross filter area:

$$\text{Area} = \frac{Q}{r}\left\{1 + \frac{1 + \dfrac{cn}{p+c}}{n - 1 - \dfrac{cn}{p+c}}\right\} \text{ square feet}$$

Where n, is the number of beds,

p, the ordinary number of days between cleaning,

c, the number of days taken to clean and get the filter into a fit condition to deliver satisfactory filtrate,

and $\frac{Q}{r}$, represents the area which would be required if the filters worked continuously.

The standard size of individual filter beds where not fixed by such rules as Burton's, may be taken as about 200 feet square to 200 feet × 250 feet, *i.e.* approximately 0·9 to 1·1 acres; $1\frac{1}{3}$ acres being about the maximum.

According to Hazen, the total quantity that can be satisfactorily passed through a filter between two cleanings, is unaffected by the rate of filtration, *i.e.* the greater the rate, the sooner the filter has to be cleaned. In this respect, the working of the filter is influenced by the effective size of the sand, and Hazen's figures may be taken for comparative purposes:

Effective size of Sand in Millimetres.	Total Quantity of Water Filtered between successive Cleanings in Million U.S. Gallons per Acre.
0·39	79
0·29	70
0·26	57
0·20	54
0·14	49
0·09	14

The absolute interval between cleanings is entirely dependent upon the weather and the quality of raw water. Speaking generally, in moderately cold weather (English winter) a filter will run between cleanings about three times as long as in moderately warm weather (English summer). In climates where the seasonal difference is more marked, as in India and China, a ratio of 6 : 1 is not uncommon.

The depth removed by scraping is affected by the weather, but is more influenced by the amount of turbidity existing in the raw water, and even more so by the effective size of the sand. It has been found that if the effective size greatly exceeds 0·40 mm., the sand becomes dirty to such a depth that cleaning is troublesome.

Modern filters are constructed with a view to limiting the head that can possibly be utilised in forcing water through the filter. I consider that this is a mistake, as all available evidence indicates that the rate of filtration alone affects the working of the filter, and any head that can be applied has no appreciable result in consolidating the filter sand. Any constructional limitation of head is therefore only justified when extremely careless management is to be apprehended, and it would appear better to procure a good manager and give him every facility for tiding over seasons of intense demand.

If fresh sand be placed on a filter bed, the effluent is unsatisfactory for a certain period. The duration of this period is increased if the depth of fresh sand placed on the filter increases; and, as a rule, the period is longer when the sand is raw (*i.e.* has never been previously used in a filter) than when the sand is old and ripe (*i.e.* sand which has been removed from a filter and has been washed).

In practice, as already stated, engineers are accustomed to replace sand only at long intervals (say one replacement per 15 to 30 scrapings). This entails replacing approximately 18 inches depth of sand, and some 10 to 15 days may then elapse before the effluent is satisfactory.

Certain experiments at Albany, N.Y., seem to indicate that this period is shortest in the spring months. As a matter of practice, it has long been the custom in England to replace sand as far as possible only in the spring, or in the autumn.

Applicability of Slow-Sand Filters.—The above description cannot be considered as a complete account of the process of water purification by slow-sand filters.

When properly worked, a slow-sand filter will remove a large proportion of the bacteria, and the filtrate will pass Koch's test, and will be satisfactory as regards such matters as freedom from turbidity, tastes, colours, and odours, provided that:

The water which enters the filters:

(*a*) Does not contain much above 2000 bacteria per c.c. (counted by Koch's method);

(*b*) Is fairly free from turbidity, especially from that produced by very minute particles, *i.e.* diameters less than say 0·0005 inches. Definite figures applicable to all cases cannot be given, since the larger particles, although effective in producing turbidity, have but little effect on the working of the filter; but, as a rule, the limits may be stated as follows:

A water containing 125 parts per million of turbidity causes trouble at once, and the filters require to be cleaned if the turbidity exceeds 50 parts per million for more than 36 hours.

(*c*) Is neither very deeply coloured, nor markedly affected by tastes and odours.

It will therefore be plain that very few natural sources exist, other than springs or wells, which, during the whole year, yield a water which can be at once passed on to a slow-sand filter with the assurance that a satisfactory filtrate will be obtained by slow-sand filtration alone. Therefore, in practice, it will be found that a process of sedimentation, or some other form of preliminary treatment, invariably precedes slow-sand filtration. Sedimentation is effected either in special sedimentation basins, or, in cases where the water supply is drawn from a reservoir, storage in the reservoir itself forms a very efficient sedimentation.

The result of sedimentation is twofold :—

(i) The suspended matter sinks to the bottom of the basin, or reservoir, and the turbidity is consequently reduced. The rate at which this action goes on can be calculated when the sizes of the particles producing turbidity are known. The results are not of great importance, as that portion of turbid matter which is prejudicial to filtration is composed of such extremely small particles that their deposition would only be effected in periods of time measured by months or years, and the benefits of sedimentation are really due to the fall of the larger particles which carry with them (mechanically caught) a portion of the extremely fine particles. Roughly speaking, particles exceeding 0·0005 inches in diameter will settle to the bottom of a still water basin 10 feet deep in less than three hours, and would therefore be deposited in any sedimentation basin with the least pretensions to efficiency. Particles of one-tenth this size (*i.e.* under 0·00005 inches in diameter) will remain suspended for at least 300 hours, and will not therefore be deposited in any sedimentation basin of practical size unless they are entangled and carried down by larger particles. Regarded from this point of view, it will be obvious that in many cases the more turbid the water originally, the better the relative improvement that may be expected from a short period of sedimentation. The reasons for the addition of artificial turbidity, as sometimes practised, consequently become obvious.

(ii) The heavier sediment also carries down bacteria, and therefore sedimentation reduces the number of bacteria which remain to be removed by the filters.

(iii) Storage and sedimentation have also certain beneficial effects on the coloration, taste, and odour of water, but cannot be considered as effective remedies when these are marked.

It will consequently be evident that slow-sand filters, combined with more or less efficient sedimentation, produce a satisfactory filtrate in such cases as are usual in England, France, or Germany. The factor mainly determining the quality of the water in the above countries is its bacterial content. The geological structure of these countries is such that their waters very rarely contain an excessive amount of extremely fine turbidity, and the climate being temperate, coloration (excluding that produced by peat), and tastes, or odours, are rarely so marked that anything worse than temporary inconvenience arises.

In countries such as the southern United States, which have not recently (geologically speaking) been exposed to glacial action, the waters are frequently laden with extremely fine particles, which remain undeposited after any reasonable period of natural sedimentation, and therefore processes of coagulation become necessary.

In climates hotter than those of Northern Europe, tastes, odours, and colours often manifest themselves to a marked degree, and special methods are required for their removal.

The various processes can therefore best be illustrated by first considering natural sedimentation (p. 550). It should be remembered that the usual British reservoir with a valve tower, enabling water to be drawn off from various levels at will, forms a very efficient sedimentation basin. Such cases require no further discussion.

The above statements explain the principles usually accepted concerning sedimentation or storage basins. According to Houston (a very excellent précis is found in the *Report of the London Water Examiner for* 1910), systematic storage is the only absolutely certain method of securing a water which is bacterially safe, if the raw water contains large numbers of excremental bacilli. Since sand filters do not apparently exercise a selective action on pathogenic bacteria, but merely reduce the number of all the species of bacteria present in the water, in about equal proportions, it is plain that if one individual pathogenic bacillus can cause infection, drinking the effluent from a filter which secures a reduction of 99 per cent. in bacteria is the same thing as drinking a mixture of one part of the raw water with 100 parts of sterilised water. According to this line of reasoning, therefore, the only way to insure safety is to destroy the pathogenic bacteria before filtration. Houston's tests prove that storage for about 30 days does effect this in the case of Thames water (see p. 552). Houston therefore considers storage as a necessary preliminary to the slow-sand filtration of polluted waters.

The following deductions may be made:

(*a*) The reduction in the number of bacteria previous to filtration secured by storage can generally be attained more cheaply by other treatments.

(*b*) Houston's own experiments prove that it is extremely improbable that the typhoid bacillus exists in the polluted raw water in such quantities that infection (even if producible by one bacillus) would result from consuming the raw water in dilutions of 1 : 100, provided that substances capable of supporting the life of the pathogenic bacilli were removed from the mixture.

(*c*) Slow-sand filtration apparently does remove these substances.

The report deserves careful study, and is encouraging, as showing that storage and sedimentation are more effective than was generally believed to be the case.

On the other hand, the report has greatly strengthened my personal views concerning the inadequacy of slow-sand filters, when unassisted by any other process, in producing an absolutely safe filtrate from water originally highly polluted, except when the water has been drawn from a storage reservoir. The general excellence of the water-borne disease death-rates in British towns has always, in my opinion, been an indication not so much of the efficiency of slow-sand filters *per se*, as of their fitness for treating stored or sedimented water (see also pp. 519 and 552).

Practical Details.—The practical details of filter design are mostly concerned with questions affecting the water-tightness of the walls and bottom of the filter. This is usually secured by a puddle wall and base surrounding the whole filter, as shown in Sketch No. 145 (which is a sedimentation basin). It is extremely doubtful whether such filters are ever entirely water-tight, except for a few years after construction, although the puddle and sand that fill any cracks which may form in the puddle layers probably form a filter and provide a very effective shield against the entrance of any polluting matter. The construction is, however, radically bad, if the subsoil water level is above the bottom of the filter, as the puddle is exposed to variable loads (owing to the alteration in weight produced when water is let into or drawn out of the filter), and the concrete or masonry work will certainly, and the puddle work probably, crack.

Sketch No. 142 shows a design with asphalte or bitumen as a water-proofing material, which produces less noticeable cracking, and which probably remains water-tight, but which does not afford such an effective protection against pollution should cracks occur. The lowest level of the filter should therefore be well above the subsoil water level. It will be noticed that the asphalte layer lies on 6 inches of concrete, and is covered by 12 inches of the same material. This affords greater security against leakage into the filter than the alternative designs where the top layer is thinner than the under layer. Each layer of concrete should be laid in squares, approximately 10 feet by 10 feet, or 12 feet by 12 feet, and the interstices between these squares should be filled in with say ⅜ths of an inch of asphalte. The squares in the two layers should break joint, so that the angle of each square in the upper layer lies vertically above the centre of a square in the lower layer. The side walls should have expansion joints at intervals of 20 feet, and the joints nearest the corners of the filters should be provided with steel plates. It is believed that these precautions will secure a water-tight filter. In London filters are usually surrounded by a puddle wall carried down to unite with an underlying clay stratism, and the included area is pumped dry during construction. When local circumstances permit, this forms an ideal solution of the problem. In some cases the filters are built on the top of clear water reservoirs. Space is thus economised, and, if clear water reservoirs of such a size are really required, the combination is economical. The danger of pollution, however, is not appreciably minimised, since, although the filters to a certain extent shield the bottom of the clear water reservoirs from changes in temperature, cracking is known to occur unless the precautions already indicated are taken. The practice of supporting the filters on sedimentation basins is a very excellent solution as regards any danger of pollution, but it obviously entails an additional complication in the pumping machinery. Further, the power employed in lifting water through so small a height as 12 or 16 feet, is uneconomically expended.

Sand Washing Apparatus.—As already stated, after a filter has been at work for some time, the sand grains become coated with Zooglea, and such sand produces better filtration results. When the sand removed from a filter is washed to cleanse it from dirt, a certain amount of this coating is removed, and this removal is the more marked the longer the sand remains absent from the filter. Nevertheless, the general experience of water works' engineers is that the use of old, washed sand (ripe sand) is advantageous, and it is only in very special circumstances, where fresh sand is cheaply obtainable, that the washing of dirty filter sand can be dispensed with, although (as will later be seen) the cleansing of freshly dug natural sand from dirt is a far easier process.

The simplest method of washing filter sand consists in hosing it while lying on a concrete or brick platform. The lighter dirt is carried away by the escaping water, and with care and a sufficiency of water, good results are obtainable. The method is costly in labour, and entails a large expenditure of water, and is therefore only advisible in very small installations where the total quantity of sand to be washed is so small as to preclude the economical use of machinery.

Machines for washing sand are generally divided into two classes, which may be called the "Trough," and "Ejector" types.

In the first class, the sand is passed along a trough by means of a system of paddles or screw conveyers, and is washed by a current of water passing in the opposite direction. Machines of this type consume but little water compared with the ejector types; and further, the water is not delivered at a high pressure. On the other hand, the rotary paddles or conveyers consume a certain amount of power, and although the actual nett power may be less than that expended in pumping the water under pressure that is used in ejector washers, it is generated by small and inefficient machines, intermittently worked, and the gross power expended (in the form of coal burnt, etc.) may easily exceed that necessary to pump the quantity of water consumed by the ejectors. The use of trough washers is therefore only advisable when economy in filtered water is desired, and especially in cases in which filtered water under pressure is not required for other purposes (*e.g.* in a gravity distribution, where no pumping is done at the filtration works).

In ejector washers, the sand is lifted (usually six to seven times) by water under pressure. The washing is effected by the water carrying the dirt forward faster than the sand, which is caught in troughs or boxes. These are usually provided with internal irregularities to break up and disperse the dirt while collecting the heavier sand. It will be noticed that in these machines relatively clean water acts on comparatively dirty sand, whereas in trough machines the cleaner water acts on the cleaner sand. It is therefore not surprising that all ejector washers consume more water per cube yard of sand than the trough type. On the other hand, ejector washers can easily be combined with a pipe system for conveying the mixture of sand and water to any required point, and especially where portable ejectors are used for lifting the sand out of the filter beds, the large amount of wheel-barrow work entailed in conveying the dirty sand to the washers is dispensed with. Also in cases where the water is distributed by pumping, the power consumed is very cheaply obtained, and in view of the large excess over the normal pump horse-power that must in any case be provided in order to supply the demands caused by hot weather or fires, it is unnecessary to debit the sand washing with any charge for an extra investment in pumping machinery. It is therefore probable that, except in purely gravity distributions, the ejector process always proves the cheaper.

The design of the ejectors and communicating pipes was very carefully considered in the case of the Washington, D.C., filters, and the problem of using clean water to act on clean sand seems to have been partially solved, while the minutiæ of the process have been so carefully considered that an abstract will prove useful.

Ejector Sand Washers.—The problem involved includes three processes:

(i) The sand has to be mixed with water in order to form a quasi-fluid mass.

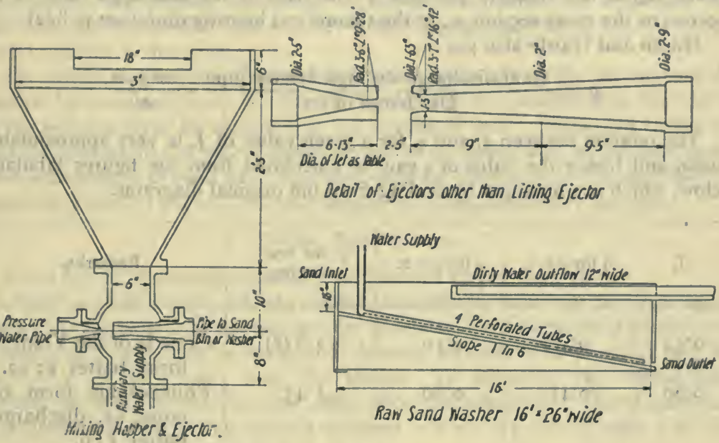
(ii) This fluid mixture has to be lifted up and driven along pipes by the energy of the water issuing from the ejectors.

(iii) Simultaneously with, and after the above process, the sand must be washed, and later when the washing is complete, the sand must be separated from the dirty water.

The Washington experiments of Hazen and Hardy (*Trans. Am. Soc. of C.E.*, vol. 57, p. 307) seem to completely answer all questions.

It is found that process (i) is best effected by shovelling the sand into a hopper the sides of which slope at 1·8 : 1, with the ejector chamber bolted on to the bottom, as shown in Sketch No. 143. At the bottom of the ejector chamber an auxiliary water supply is introduced, which produces a slow, upward current of water through the sand in the hopper, and thus transforms it into a quicksand. In this way it is found that the sand is more readily moved by the ejectors than if (as is usually the case) it is sprayed with water under pressure from above.

(ii) The proportions of the ejector are of great importance.



Sketch No. 143.—Washington Ejectors, and Raw Sand Washer.

As is indicated in the theoretical treatment of the question (see p. 819), the ratio—

$$\frac{\text{Area of jet orifice}}{\text{Area of throat of ejector cone}} = \frac{a}{a_2} = \text{J, say,}$$

is of cardinal importance.

The best value of J, increases as the total lift (including friction) increases. Calculating the ratio for new, unworn cones, we have as follows :

For merely shifting sand along a short length of pipe, with a static lift of 3 to 4 feet . . . . J = 0·34

For the lifting ejector, static lift about 26 feet . . J = 0·48

For the movable ejector, the static lift being about 10 feet but with lengths of pipe up to 100 feet . J = 0·59

The larger the throat (*i.e.* the smaller J) the more sand that can be shifted, but the pressure at D, the entrance to the pipe (see p. 820) measured in lbs. per square inch, decreases steadily, and is approximately proportional to $\frac{1}{a_2}$.

Since the above ratios are found best for practical work, the ratios which would be obtained by theoretical treatment of the question correspond to a somewhat worn throat, and are probably some 5, to 10 per cent. less.

The batter of the diverging discharge cone DE, is found to have a large influence on the working, and the best results are obtained with a batter of 1 : 22, although in the lifting ejector (Case II above) 1 : 14 is used in order to save space, and in earlier designs (such as at Albany, and Philadelphia) fair results were secured with a batter of 1 : 6.

Hazen and Hardy define as follows :

The efficiency of an ejector is the ratio between the pressure of the jet and the pressure at the discharge of the ejector. That is to say :

$$\text{The efficiency} = \frac{H_o - h_b}{H_3 - h_b} = \eta, \text{ say,}$$

where H_3, is the absolute pressure at the base of the discharge cone of the ejector, or the cross section a_3, in the theoretical investigation (see p. 822).

Hazen and Hardy also put :

$$q = \frac{\text{Total discharge through ejector pipe}}{\text{Discharge of jet}} = \frac{av + cu}{av}$$

The relation between q, and η, for a given value of J, is very approximately linear, and hence the value of η can be calculated from the figures tabulated below, which are obtained by scaling from the original diagrams.

J.	η for $q=1$.	η for $q=2$.	q, for best working.	Remarks.
0·54	0·48	0·16	1·8 (?)	Cone is of the Venturi form, batter 1 : 22.
0·60	0·41	0·00	1·45	Philadelphia form of cone, *i.e.* discharge batter 1 : 6.
0·51	0·36	0·07	1·70	Batter 1 : 22.
0·49	0·30	0·06	?	Rough cone, batter 1 : 22.
0·38	0·20	0·085	...	Batter 1 : 22.
0·36	0·21	0·090	...	" "
0·27	0·12	0·06	2·00	" "
0·25	0·10	0·055	...	" "
0·16	0·06	0·04	...	" "

The best point for practical working is obtained when the product $q\eta$, is a maximum, and is given in Column 4.

The percentage of sand is not specified, but seems to have but little influence on the values of η.

The figures are not well adapted to test the theory of the apparatus, but are obviously very well fitted for practical purposes. H_o, is not accurately stated, but $H_o - h_b$, the pressure of the ejector water, was approximately 90 lb. per square inch, or 210 feet head of water.

In driving the mixed sand and water through pipes, certain conditions must be fulfilled. The experimental results are as follows :

Taking V, as the average velocity of the mixture of sand and water in the pipe, *i.e.* :

$$V = \frac{av + cu}{\text{area of pipe section}}$$

If V is less than 2 feet per second : the sand drops, and the pipe "silts solid."

V = 2·5 feet per second : the flow proceeds irregularly, and is sometimes maintained, and is sometimes stopped by "silting."

V = 3 feet per second : stoppages by "silting" almost cease.

V = 4 feet per second, and over : the flow is nearly as steady as for pure water, but the frictional resistance is far greater.

Since these results were obtained on pipes which were 3 inches and 4 inches in diameter, our present knowledge of hydraulics justifies the statement that they will not be found to hold without correction, in pipes which are, let us say, 9 or 12 inches in diameter.

In practical operation, the sand and water taken up by the jet increase the volume by one-third (*i.e.* $q = 1·33$), so that V, may be considered as four-thirds of the velocity given by considering the volume of water discharged by the jet, and the mixture in the pipe is about 75 per cent. water, and 25 per cent. sand. The percentages of sand are calculated without reference to the fact that the sand has about 40 per cent. of void spaces, so that theoretically the ratio :

Water : Sand grains,

is about 85 : 15, assuming that the sand contains 40 per cent. voids.

Such excellent results are only attained by carefully following the Washington rules. At Philadelphia, the best results appear to have been about 82 per cent. of water to 18 per cent. of sand, and this was attained with J = 0·30, and the jet orifice 1¾ inch away from the throat. To judge by the Philadelphia results, this distance has but little effect upon the efficiency. The Washington experimenters seem to have kept it constant at about 2½ inches, when J = 0·59, and at about 3 inches when J = 0·33.

The frictional resistance of the pipes appears to vary very nearly as V, when the percentage of sand is constant. The table at top of p. 540 is scaled from Hazen's diagrams, and it is known that the effective size of the sand influences the results.

Comparison with the Philadelphia results given in the discussion of Hazen's paper seems to indicate that the form of the injectors has some influence on the velocity at which the sand packs ; but the friction is much the same in both cases.

The effective size of the sand used was about 0·40 mm.

Some very accurate experiments were made by Miss Blatch (*ut supra*, p. 406) on the motion of sand in 1-inch pipes, and are compared by her with less accurate figures for 32-inch pipes.

It would appear that while Hazen's table is sufficiently accurate for practical necessities, the correct law of friction is more complicated. Apparently, for velocities which are less than those given in the column headed "Velocity at which Flow becomes steady," the resistance for a given percentage of sand is independent of the velocity, and is approximately constant, and equal to that

	s for V=4 Feet per Second.		*s* for V=6 Feet per Second.		*s* for V=8 Feet per Second.	
Diameter of Pipes .	3 Inch.	4 Inch.	3 Inch.	4 Inch.	3 Inch.	4 Inch.
Percentage of Sand.						
35 . .	0·144	0·137	0·173	0·157	...	...
30 . .	0·130	0·123	0·158	0·141	...	...
25 . .	0·113	0·107	0·141	0·127	0·177	0·154
20 . .	0·098	0·093	0·126	0·112	0·163	0·138
15 . .	0·084	0·078	0·112	0·097	0·146	0·123
10 . .	0·070	0·063	0·097	0·082	0·133	0·108
5 . .	0·054	0·047	0·082	0·067	0·117	0·093

$$\text{Where, } s = \frac{\text{Friction head in feet of water}}{\text{Length of pipe}}$$

which is observed at the velocity of "steady flow." For velocities which exceed that of steady flow, the loss of head is very well represented by the equation:

Loss of head for a mixture of sand and water = Loss of head for pure water at the same velocity + a constant × percentage of sand.

For sands which are carefully sifted so as to consist of grains which are all approximately equal in size, this law ceases to hold at velocities which are but slightly greater than that at which steady flow begins; but for sands with grains of varying sizes, such as occur in Nature, the law holds up to the velocities given in Column 5 of the table below. For greater velocities, the resistance appears to be far in excess of that observed in the flow of pure water. This last result does not agree well with the experiments of Merczyng (*Comptes Rendus*, 1907, p. 70), who finds only small differences between the resistances for pure water and water carrying 11 to 19 per cent. of sand in a clean, steel pipe 15 inches (0·38 metre) in diameter, at velocities ranging from 10·5 to 12·7 feet per second.

The following table is useful:

Diameter of Pipe in Inches.	Value of Constant when Pipe is 1000 Feet long.	Velocities in Feet per Second.		
		When Flow begins but is often Blocked. Resistance = Resistance at Velocity in Column 4.	When Flow is steady, Law begins to hold.	When all Sand is in Suspension and Law ceases to hold.
1	9·6 feet	1·25	3·5	8 to 9
3 to 4	3 ,,	2·5	4	Not observed.
32	1·2 ,,	6 to 7	9	14

The size of the sand is of importance.

The above results refer to sand of about 0·33 to 0·50 mm. in effective size (say 0·022 to 0·034 inch mean diameter). Miss Blatch finds for graded sand moving in a smooth brass pipe one inch in diameter, that:

I. For sand which passes a sieve of 20 meshes per lineal inch, and rests on one of 40 meshes per lineal inch, the constant given in Column 2 above is about 7·8 feet per 1000 feet.

II. For sand that passes a 60-mesh sieve, but is retained by a 100-mesh sieve, the constant is approximately 2·6 feet per 1000 feet.

Washing of the Sand.—There are many types of washer. As a general rule, the sand is lifted 3 or 4 feet, and is swept along a horizontal trough, where it drops and is collected. The water escapes, carrying the dirt with it. Thus, six or seven repetitions of the process are not unusual. The best method is that adopted at Washington, by Hardy (*ut supra*). In this case the actual washing of the sand is effected in hoppers, about 3 feet square at the top, and 6 inches at the bottom, with a vertical height of 2·3 feet. These have an ejector and auxiliary water supply bolted on to their bottom, and the auxiliary water supply is so adjusted that there is no downward flow of water in the hopper. The sand sinks through the water, and is carried away by the ejector, while the dirt is carried away by the overflow water. It is stated that one such washer cleans the sand, but two are provided. The mixture of water and cleaned sand is lifted by the last ejector into a bin, and the sand is deposited there.

It would appear that the quantity of water used is about twelve times the volume of the sand washed, and about 75 per cent. is supplied at a pressure of 80 to 100 lb. per square inch. At Philadelphia the figures are sixteen times the volume of sand, and 70 lb. per square inch, but these figures must, I think, be considered as a minimum, as they are calculated from experimental results.

Actual working results are given as 1 of sand, to 20, or 30 volumes of water, in most cases. In some places, 1 to 14 or 15 is attained with a pressure of 55 to 60 lb. And with a pressure of 80 to 90 lb. a ratio of 1 of sand, to 11 or 12 of water is stated to be obtained, although these latter figures do not appear to include the water used in the lifting ejector.

It will be seen that even the minima results contrast unfavourably with the 1 to 7 or 8 attained in ordinary work with drum or trough washers, although it would appear that such results could be approached at Washington provided that the lifting ejector was not used.

In all calculations respecting the strength of sand bins it appears advisable to assume that pressures equivalent to those produced by a fluid weighing 120 to 125 lb. per cube foot may occur when the sand begins to settle out from the water. The weight of the mixed fluid travelling in the pipes does not materially exceed 80 to 85 lb. per cube foot.

Washing of Raw Sand.—In applying the above results to the washing of sand in mechanical filters, or to the cleansing of natural sand from dirt, it must always be remembered that sand from a slow sand filter is coated with Zooglea, and that even when this has dried up a certain amount of gummy matter still remains on each individual grain. Thus, the washing of sand which has not been used in a slow sand filter is a far more simple process, and where raw sand is cheaply procurable, and the filter beds have been so designed that a thickness of sand can be placed in them sufficient to compensate for the relative inefficiency of raw sand (as compared with ripe sand), such elaborate arrangements are unnecessary.

I give a sketch (No. 143) of the apparatus used at Washington for washing an ordinary raw sand mixed with clay, so as to free it from clay down to 0·2 or 0·1 per cent. by weight. The raw material is washed through a screen of say 0·16 inch to 0·20 inch mesh, and the mixture falls into a box 16 feet long by 24 inches wide, by 16 inches deep at one end, and 4 feet 2 inches at the other. In the bottom of this four perforated pipes were laid, and by means of these about 1 cube foot of water per square foot per minute was passed upwards, and overflowed as shown. The sand was drawn off at the lower end of the box, and was washed at a rate approximately equal to 1 cube yard per square foot per hour.

The expenditure of water was about five to six times the volume of sand washed.

Washing of Filters.—Certain special methods of filter washing (they can hardly be termed sand washing methods) deserve consideration.

At Long Island the filter is washed by turning water into a trough running along one side of the filter bed, with its lip at the level of the sand, so that a thin stream of water flows over the bed, and is drawn off by a similar trough on the other side of the bed. The top layer of the sand is then raked and broken up, so as to loosen the Schmutzdecke and dirt, and this is continued until all the dirt has been carried away by the flowing water, and a clean surface is secured.

The method is essentially that employed for cleaning the early mechanical filters, and the fact that the water flows over the sand in place of rising up through it would appear to render the cleaning still less effective.

No systematic report of the working of filters washed in this manner has yet been published, but the method seems hardly likely to produce consistently satisfactory results, except in cases where the raw water is neither highly polluted nor very turbid. If, however, it is regarded as an expedient for rapidly and temporarily restoring the efficiency of a filter during hot seasons when the growth of organisms in the Schmutzdecke is so rapid as to clog the filters long before the top layer of the sand is really dirty, it appears to be a valuable process, provided that it does not lead to an undue neglect of sand washing later on. A similar process of raking under water is occasionally adopted in the summer in London. Bacterial investigations have not been published, and seem desirable.

At Ivry, where the Anderson process (see p. 547) is applied, the pre-filters are scraped under water every day, and about 3 mm. of sand (say $\frac{1}{8}$th of an inch) is removed by means of a flat nozzle, provided with a guard in order to prevent more than the desired thickness being removed, and connected with a centrifugal pump. This method permits a steady and systematic scraping to take place without stopping filtration, and I am informed that the bacterial results are excellent.

I regret that as the inventor has not yet published any drawings, I cannot give a sketch, as it seems to be a process which is well worth adoption in all cases. It must be remembered that this process is at present only applied to pre-filters, and bacterial tests when used on ordinary single filters (where, if no coagulant is used, the scrapings would hardly require to be so frequent) are greatly to be desired.

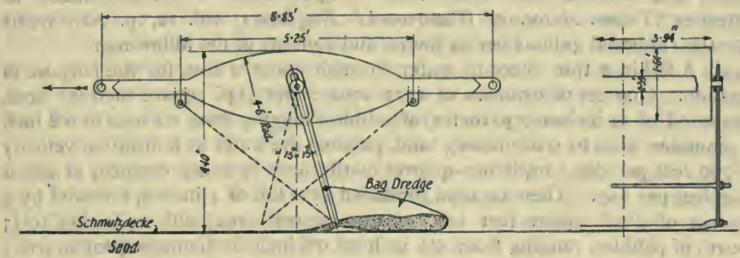
In some cases, the filters are washed by agitating the whole sand bed by means of compressed air and water, introduced into the under-drains, as is usual in mechanical filters. The difficulties of obtaining an equable distribution over a bed of large area are obvious. Judging from experience of mechanical filters, we may expect that after a few washings the sand will become stratified, and that the upper layers will contain a large proportion of finer grains, so that the washing will require to be frequently repeated, or the sand employed must be graded to a uniform size before use. The system therefore promises to be costly not only in maintenance, but in first outlay, and its advantages are open to doubt.

The Torresdale (Philadelphia) pre-filters may be taken as an example.

These are 60 feet by 20 feet 3 inches in area, and consist of 12 inches of sand, varying from 0·032 to 0·04 inch in diameter, resting on the following layers of gravel :

8 inches	of a size	varying	from	$\frac{1}{8}$ to $\frac{1}{4}$ inch.
3	"	"	"	$\frac{1}{4}$ " $\frac{1}{2}$ inch.
4	"	"	"	$\frac{5}{8}$ " $1\frac{1}{2}$ inch.
15	"	"	"	2 to 3 inches.

The grading is very carefully adjusted, and the gravel depth (2 feet 6 inches) is larger than would be required were the washing effected in the ordinary manner.



SKETCH NO. 144.—Bag-Scraper used at Hamburg.

Cleaning in Frosty Weather.—Sketch No. 144 shows a grab scraper used for scraping the sand of the Hamburg filters when the water is covered with ice.

As a rule, the adoption of covered filters renders such methods unnecessary. Where frosts occur which are not sufficiently intense to render covered filters absolutely necessary, but which are hard enough to form thin ice on the water, the following process suffices to keep the filters in regular working order.

Shortly before the advent of the cold weather, the surface of the sand is formed into regular waves about 9 inches high, and 3 feet from crest to crest, so that when the water is drawn down to the sand level the ice is fractured and can be removed. The height and size of the waves must depend on the thickness of ice expected, and any undue delay in scraping may prove disastrous.

PRELIMINARY FILTRATION PROCESSES.—Preliminary processes of filtration vary greatly in character. Filters of coke, brickbats, compressed sponge, etc., have been employed. The general idea is to strain out the coarser particles by means of a filter which can be easily cleaned so as to permit the sand filters to run without cleaning for a longer period. The general ignorance of the principles involved is best illustrated by the fact that it is still open to doubt whether these coarse filters do not remove a greater portion of the finer particles than of the coarse particles. As has already been stated, bacteria, and the particles which cause turbidity and which cannot be rapidly removed by sedimentation, are all far smaller than the passages between the grains of even the finest sand. Thus, at first sight, coarse filters should have no influence upon substances which are difficult to deal with by means of sand filters. As a matter of fact, it may be suspected that coarse filters are usually effective not as filters, but as aerators. The principles are best illustrated by a description of the dégroisseurs of Peuch-Chabal.

DÉGROISSEURS OR ROUGHING FILTERS.—In principle, dégroisseurs are filters of coarse gravel. The theoretical rationale of such filters is at present uncertain. Kemna (*Etudes sur Filtration*, see also *Engineering News*, 26th March 1908) suggests that the action resembles the retention of small particles by the slimy layer of mud that forms on each individual stone of the gravel. Contact action has also been put forward as an explanation, but such terms are merely polite expressions of ignorance.

The most systematic method of pre-filtration is that introduced by Messrs. Peuch and Chabal, under the term "dégroisseurs." The installation at Suresnes (*Trans. Assoc. of Waterworks' Engineers*, vol. 12, p. 200) treats 7,700,000 imperial gallons per 24 hours, and consists of the following:

(i) A fall in a thin sheet of water through about 2 feet, for the purpose of aeration. One set of strainers of 1571 square feet (146 square metres) area, composed of 12 inches (0·30 metre) of pebbles, ranging from 1·2 inch to 0·8 inch in diameter (0·03 to 0·02 metre), and passing the water at a filtration velocity of 790 feet per day; or, if one-quarter of the area is being cleaned, at about 1040 feet per day. Then aeration produced by a fall of 4 inches, followed by a strainer of 2648 square feet (246 square metres) area, with 14 inches (0·35 metre) of pebbles, ranging from 0·4 inch to 0·6 inch in diameter (0·01 to 0·015 metre), with a filtration velocity of approximately 464 feet per 24 hours. This is followed by aeration as before, and treatment in a strainer of 4810 square feet (447 square metres) area, composed of 16 inches (0·40 metre) of pebbles, ranging from 0·28 inch to 0·4 inch in diameter (0·007 to 0·010 metre), at a filtration velocity of 255 feet per 24 hours. Followed by four falls of about 2 inches each, and a fourth strainer of 7972 square feet (741 square metres) composed of 16 inches (0·40 metre) of pebbles, ranging from 0·16 inch to 0·20 inch in diameter (0·004 to 0·007 metre), at a filtration velocity of 154 feet per 24 hours.

It may be suspected that the aeration is not without influence on the working of the system. The water is exceptionally polluted, being drawn from the Seine below Paris.

(ii) The further treatment consists of a double filtration as follows:

(*a*) Through 2 feet of coarse sand at a rate of 65 feet daily.
(*b*) „ 3 „ fine „ 10·7 „.

The bacterial results are excellent, being considerably better than those yielded by the Anderson process at Ivry (see p. 547), and the final filters appear to require cleaning at the most twice a year. The first sand filters are cleaned about once every six weeks. During floods the dégroisseurs are cleansed almost continuously.

From personal inspection, I feel it safe to state that without the aid of dégroisseurs, or an unusually lengthy period of sedimentation, combined with coagulation, it would be impossible to work the filters at all; I also doubt if equally good bacterial results could be obtained from the raw water by any other process of filtration (as distinguished from chemical disinfection).

The system is usually applied to waters similar to those at Suresnes, but has been introduced at the Bamford filters, which treat a moorland reservoir water but little exposed to pollution. In this case the strainers are three in number, as follows:

(i) 12 inches of gravel, from 0·42 inch to 0·62 inch in diameter, working at a rate of 343 feet per 24 hours.
(ii) 14 inches of gravel, from 0·30 inch to 0·42 inch in diameter, working at 264 feet per 24 hours.
(iii) 16 inches of gravel, from 0·18 inch to 0·30 inch in diameter, working at 103 feet per 24 hours.

Aeration appears to be unprovided for, and the further treatment consists of a slow sand filter with 2 feet of sand (reduced by scraping to 1 foot), working at a rate of 12 feet per 24 hours (see p. 530).

Our present information does not permit the variations in the process necessitated by different qualities of raw water being discussed. The Peuch-Chabal dégroisseur is by no means the only type that can be used, and very good results may be obtained by one filtration through ordinary stone broken to about ½-inch size. A typical case exists at Shanghai, where a very turbid and polluted river water, which had to receive at least two days' sedimentation previous to slow sand filtration, is now turned direct into a strainer consisting of 3 feet of ½-inch stone, working with a filtration velocity approximately equal to 100 feet per 24 hours; then slightly aerated, and passed at once to the filters. At present, the process is experimental, but the average life of a filter between cleanings appears to be increased from 14 days to 6 weeks.

Double Filtration.—In some cases the water is filtered twice through slow sand filters. The circumstances which necessitate double filtration are discussed by Goetze (*Trans. Am. Soc. of C.E.*, vol. 53, p. 210). The water of the river Weser at Bremen in times of flood is turbid, and when it arrives at the filters frequently contains over 10,000 bacteria per c.c. (by Koch's test). It is then found that even with such low filtration velocities as 4·9 feet per 24 hours (63 mm. per hour) a bacterially satisfactory effluent cannot be obtained, and the effluent is also liable to be turbid. Under these circumstances, Goetze filters the water twice, at velocities ranging (according to the bacterial content and turbidity) from 8, to 16 feet (even 20 feet has been used) per 24 hours, and obtains a final filtrate which contains only 20 to 40 bacteria per c.c. In addition, the effluent from a freshly cleaned filter can be turned on to an old, ripe filter, and a satisfactory final filtrate is produced.

The advantages are obvious, and it is equally plain that a preliminary sedimentation or coagulation would allow a fairly clear water containing less than 10,000 bacteria per c.c. to be delivered to the filters. Thus, while double filtration can produce a satisfactory effluent from a very bad water, it is extremely doubtful whether it is the best, and it is certainly not the only possible process.

The process is, in my opinion, somewhat in the nature of a makeshift, and has no real advantage except that the second filters require cleaning at long intervals only, so that during a short period of bad water the full capacity of the filter area can be utilised. In considering the rates of filtration to be adopted, the figures for Ivry (see p. 547), and Suresnes (see p. 545) are typical; and, since the water dealt with at these places is unusually polluted, these velocities will, as a general rule, be found to produce satisfactory results with double filtration without any preliminary treatment.

No rules can be given, and local experience alone permits a satisfactory system to be arrived at. It may be suspected that either the typical system of dégroisseurs, or the rough modification used at Shanghai, is usually preferable to double filtration through two sand filters. The Shanghai water is about as bad as is likely to be usually dealt with, and the results are excellent.

Dégroisseurs are believed to have more effect on coloured water, or on waters with odours and tastes, than a sand filter. Double filtration should only be adopted when it is quite clear that it possesses advantages over the simpler method of dégroisseurs and single sand filtration, or coagulation and sand filtration.

Flood Waters.—Most engineers are adverse to passing water drawn from a river in high flood direct to slow sand filters, as the filters are liable to become rapidly clogged, and the necessary cleaning is laborious. Flood water, however, is not in itself objectionable if sufficient sedimentation is provided.

Certainly, the first washings of a catchment area, as carried down by a flood, may be highly polluted, when regarded from a chemist's point of view. It is somewhat doubtful whether such water generally contains far more than the normal quantity of pathogenic bacteria, and being turbid, it is certainly proportionately more improved by sedimentation than are the clearer, normal waters.

If the sedimentation is incomplete, the filters will require frequent cleaning, but this fact may be regarded as an assurance that an engineer will not filter unsedimented flood water if it can be avoided. The water of a flood succeeding a previous flood at a short interval is probably (except for its turbidity) the best of the year.

Flood waters of even such highly cultivated and thickly populated catchment areas as the Thames, and Lea valleys, will produce satisfactory filtrates. Thus, during October 1898 (a month of high and continuous floods) the following figures were obtained:

Raw water,—average 1719 microbes per c.c.: filtered water,—average of 26 samples, 13 microbes per c.c.

These particular results were obtained with very little sedimentation, and it may be inferred that careful slow sand filtration is capable of producing satisfactory filtrates from flood waters. If previous sedimentation can be effected, the circumstances are even more favourable; and, if sedimentation is impossible, coagulation is not only an allowable, but even an advantageous, temporary measure.

Processes Supplementary to Slow-Sand Filtration.—A mere enumeration of the various processes adopted to render water better adapted for purification by slow-sand filters would fill several pages.

Preliminary treatment by roughing filters has already been discussed.

It is now proposed to discuss several processes of a chemical character before discussing such methods as sedimentation or coagulation, which are more or less dependent on the turbidity of the water.

The arrangement is illogical, and has merely been adopted because sedimentation and coagulation processes form an integral portion of the American, or mechanical filtration, processes. As already stated, however, sedimentation, either in special basins or storage reservoirs, is a very important preliminary portion of the slow sand filtration process when applied to turbid waters (see pp. 534 and 552).

Preliminary Chemical Treatments.—(i) *Treatment with Metallic Iron.*—The typical iron process is the Anderson process, in which the raw water is exposed to the action of scrap iron in revolving drums. This treatment is usually applied to turbid waters, and especially to those which are both heavily polluted and turbid. It forms a very powerful means of ameliorating the condition of the water. As a rule, the turbidity is considerably reduced, and settles down more rapidly in the sedimentation basins, and is more easily removed by slow sand and other filters. In fact, the process produces a coagulation.

In addition, many waters of the type referred to contain colloidal (glue-like) substances, which are not easily filtered. Any classification of these substances is at present impossible, but as a matter of experimental knowledge, the Anderson process usually causes colloidal substances to assume a form in which they are easily removed by filtration. The process therefore constitutes a very efficient preliminary to filtration, and while special experiments must be made in all cases, it is usually found that when the water is of a kind which is adapted to the process, the area of the filters can be reduced to about one-half of that which would be required to filter the same quantity of untreated water, and the interval between cleanings is greatly increased.

The most scientifically arranged installation is found at Ivry (near Paris), and supplies a large portion of the water used in Paris. The drums are 26 feet 3 inches by 5 feet 9 inches in diameter (8 metres × 1·75 metres). These each contain 3½ tons of metallic iron in small fragments, and make 15 revolutions per minute. The water passes through the drums at a rate of about 12 cubic feet per minute per drum, and is thoroughly mixed with the iron. The drums are provided with internal shelves which lift the iron and allow it to drop through the water. The water consequently takes up a small quantity of iron, the consumption being about 660 lb. per drum per month.

On leaving the drums, the water passes into a "sedimentation basin," through which it travels in six hours. The after-treatment consists of filtration through 2 feet of coarse sand (all the sand passes through an ⅛-inch (3 mm.) hole, and is retained on a 0·04 inch (1 mm.) sieve). These pre-filters are cleaned every day by pumping the top 0·2, to 0·4 inch of sand away by means of a suction scraper worked by a small centrifugal pump (p. 542). The "sedimentation" basin is only cleaned once a month. Hence, it may be inferred that the "sedimentation" basin is in reality a coagulation tank, and that the pre-filter removes the sediment. The slow sand filters are shown in Sketch No. 139, are worked at a rate of 13·1 feet daily, and are cleaned every six months. Before the installation of the pre-filters, however, these filters were cleaned every 15 days in summer, and every month in winter.

The process described is admirably adapted to the water of the Seine, as found at Ivry. It is not precisely a typical Anderson process, that usually consisting of treatment in revolving drums, followed by 12, to 24 hours' sedimentation (sometimes less than 12 hours), and slow sand filtration at the rates later indicated.

As will be seen from the figures given on pages 547 and 588, each cubic foot of water subjected to the Anderson process dissolves about 0·9 grains of iron. The coagulation effect thus induced is approximately equal to that produced by 4·5 grains of crystallised ferrous sulphate, or 6·6 grains of crystallised aluminium sulphate, though some portion of the iron may become combined with the colloidal substances without producing coagulation, and is obtained without any increase in the hardness of the water. Thus, in waters which are adapted to this process the coagulation is very favourably effected. The power required to rotate the cylinders, and the fact that the supervisor has no control over the process, must be taken into account when balancing the advantages of the Anderson and other methods of coagulation. The process does not appear to be well adapted to turbid waters which are not polluted (either by products of animal or vegetable decomposition), and the typical coagulation process should be employed in such cases.

Occasionally the Anderson process is employed as a preliminary to mechanical filtration, but I understand that this is not recommended by the inventor.

(ii) *Polarite, Oxidium, and other processes.*—These are proprietary articles, consisting principally of iron and lime salts, which are placed in layers buried in the sand of the filters. They appear to permit a higher rate of filtration to be used than would otherwise be found satisfactory.

The published results indicate very satisfactory working, but (like all proprietary articles) full information is difficult to obtain, and it may be suspected that the process has sometimes been applied to waters to which it is not well adapted.

(iii) *Ozone.*—This process has been applied to several waters after filtration. Its efficacy,—which is undeniable,—lies in completely destroying all bacilli when properly carried out.

At present, however, all installations appear to be so costly in power that the process is not economically justifiable. If anything approaching the theoretical yield of ozone per horse-power hour could be obtained, the process would prove economically practicable; and it may be hoped that the advance of electro-chemistry will equip waterworks' engineers with a very efficacious method.

(iv) *Sterilisation by Heat.*—In this process the water is first raised to a temperature sufficient to destroy all organisms, and is then cooled. It is obviously costly, even when the heating is produced by a "multiple effect" heater, and is thus hardly suitable for general use. I would, nevertheless, strongly recommend such an installation in all cities where infection from cholera is likely.

The localities where cholera is endemic are well known, and in view of the enormous loss of life and depreciation to property caused by an outbreak of cholera, the expenditure entailed by a plant which will ensure absolute safety during the periods when infection may be apprehended, is thoroughly justified.

(v) *Chemical Sterilisation.*—The practice of chemically sterilising water is making rapid advance, but the question is one for the chemist and physician, rather than for the engineer.

Sterilisation is usually affected by chlorine, produced by adding bleaching power ("chloride of lime"), or hypochlorite of soda to the water.

At Reading (Mass.), (*Engineering Record*, 9th Oct. 1909), water containing from 120,000 to 1360 bacteria per c.c. (27,127 per c.c. on the average) was treated with quantities of bleaching powder varying from 0·93 to 7·5 grains per cube foot. The "percentages of bacteria removed" varied from 99·5 to 99·8, and the action was apparently complete in five minutes, for after 60 minutes the increase in the percentage of removal never exceeded 0·2 per cent. It will be plain that even the highest percentage of removal will not produce a satisfactory effluent when the bacterial content of the raw water exceeds 50,000. The bleaching powder contained 41·5 per cent. of free chlorine, and if more than 2·5 grains per cube foot was added to the water, a slight odour of chlorine was noticed. *Bacillus coli* was "entirely destroyed," and no free chlorine could be detected in the effluent.

The process is simple. The requisite quantity of free chlorine, or the chemical containing it, is added to the water after as long a sedimentation as can be obtained; or just before filtration, if that process is employed. Engineers must consider it as a very useful means of tiding over temporary emergencies such as abnormal pollution, or threatened outbreaks of cholera and typhoid. It is doubtful whether popular prejudice at present permits an extensive and systematic application of disinfection methods, and (judging from personal experience) most engineers are far more disposed to employ the treatment, than to publish their results.

So far as I am aware, the chemicals, other than bleaching powder, used in sterilisation processes are hypochlorite of soda (producing chlorine), permanganate of potash (producing oxygen), and sulphur dioxide. The chemical action is probably essentially an oxidisation in every case.

(vi) *Neutralisation.*—Several preliminary treatments for special purposes such as the neutralisation of acid waters, have already been referred to. The principles are purely chemical, and the details of their application depend so exclusively upon local circumstances that no general rules can be given. Thus, the neutralisation of acid waters has been effected by the following means:

(*a*) By pouring each hour a weighed quantity of powdered chalk into the aqueduct.

(*b*) By passing the water through special filters of coarsely powdered limestone.

(*c*) By mixing powdered limstone with the sand of the filter beds.

(vii) *Straining.*—Similarly, the figures relating to the process of straining water through fine copper, or brass sieves, are very conflicting. Thus (*P.I.C.E.*, vol. 126, p. 8), in two cases in the North of England, sieves of 900 and 14,400 meshes per square inch were adopted. The circumstances under which the two types of sieves were used were very similar, and, so far as differences exist, the 900-mesh sieve is used under the less favourable conditions.

The effects (other than the removal of visible impurities) of straining through fine wire sieves are not well known. The process obviously prevents the entrance of fish spawn, or other large organisms, into the mains; but it is doubtful if it has any permanent effect in preventing the development of algæ or slime (see p. 438). In cases where it is the only treatment which is given to the water, such fine gauze as 14,400 meshes per square inch may be useful; but, if the water is later subjected to any other effective process of purification, even a 900 mesh per square inch screen seems to be unnecessarily fine.

REMOVAL OF VISIBLE IMPURITIES FROM WATER. — As a matter of historical record, the first attempts to purify water were entirely directed with a view to removing visible particles from the water. Even at the present date, engineers are accustomed to obtain a rough idea of the effectiveness of any purification process by pouring the purified water into long tubes, and examining the colour and transparency of a 2 feet, 3 feet, or 4 feet layer of water. The test is unscientific, but in experienced hands the relative results afford a very valuable preliminary indication of irregularities in the working of the process. Regarding the question from this point of view, the visible impurities (mud, slime, etc.) in water can be wholly or partially removed by straining or sedimentation.

Straining is effected by passing the water through orifices which are sufficiently small to retain the visible impurities. Such orifices are afforded by the pores of a sand layer, or by the meshes of wire gauze. The details have already been discussed.

Sedimentation is a convenient term for the slow deposition of the visible impurities which takes place under the action of gravity. This action can be assisted by various mechanical or chemical processes. The term coagulation may be applied to this assisted sedimentation.

As will appear later, sedimentation produces other, and generally, far more important effects on the water than are indicated by the mere diminution in turbidity produced by the process.

SEDIMENTATION.—This process is best applied to turbid waters, since the deposition of the suspended matter not only clears the water, but bacteria are mechanically entangled and are carried down to the bottom, thus producing a material purification.

The following figures show that at least 75 per cent. of bacterial purification may be obtained in suitable waters, merely by 24 hours' storage in a sedimentation tank.

At Louisville, Kentucky, in 24 hours, 75 per cent. reduction.
At Kansas City, in 24 hours, 83 per cent. reduction.

On the other hand, some waters contain very finely divided particles of clay, as small as 0·00001 inch in diameter (*i.e.* practically of bacterial size), and in such cases, unless a coagulant is added, sedimentation has comparatively little effect. For example, at New Orleans, we find the following results :

Time of Subsidence.	Turbidity in Parts per Million.	Percentage Reduction of Turbidity.
Initial	650	0
12 hours	435	33
24 "	360	45
48 "	300	54
72 "	265	59

This comparatively high rate of reduction, followed by a rapid decrease, may be considered as characteristic of all sedimentation. If in the average state of the water the reduction in turbidity is less than 60 to 70 per cent. after 24 hours' sedimentation, storage in a large and well-designed sedimentation basin for at least a week, or some coagulation process, is advisable, either regularly, or at certain seasons only.

The effect of sedimentation on bacterial content is not entirely mechanical. It appears that the mere fact of storage causes a reduction in the number of bacteria existing in an initially polluted water, even though this water is quite clear. The reasons are obscure, and it is not yet certain whether the bacteria are mechanically deposited and sink to the bottom of the water, or whether the available nutriment contained in the pollution existing in the water is exhausted, and the bacteria are starved, but the balance of evidence seems to favour the latter supposition (see p. 552).

In cases where a heavy and rapidly falling precipitate is produced in the water (*e.g.* in Clark's process for softening water, where a pulverulent precipitate of calcium carbonate, "precipitated chalk" is formed), the bacterial reduction may be so great as to render filtration unnecessary. As examples, the following reductions have been secured merely by applying Clark's process (see p. 590):

Reduction of bacteria from 322 to 4 per c.c.
Reduction of bacteria from 182 to 4 „ „

Where coagulants producing a gelatinous precipitate are used, as in the aluminium sulphate, or, better still, the ferrous sulphate and lime process, many American cities are content with the results obtained by such assisted sedimentation only. As for example, St. Louis, where the average results show a bacterial reduction from 65,100 to 200 per c.c.

Sedimentation is usually regarded as a preliminary to filtration, and from this point of view it has merely been considered as a means of removing the larger suspended particles, and so increasing the period during which the filters run without cleaning, the bactericidal effect, until lately, not being recognised. Hence, the period of sedimentation allowed in practice, is very variable.

Taking typical cases :—It has been proposed to allow a 56 days' storage for Thames water, not so much for sedimentation, as to avoid the use of turbid flood water. This may be regarded as a counsel of perfection ; and, actually, the periods of sedimentation (or storage) allowed, varied in 1894, from 15 days for the East London Water Co. (using Lea River water principally), and 12, for the Chelsea Water Co., down to 3·3 for the Grand Junction Co. The storage capacities have since been largely increased (roughly doubled), and in view of the "Reports on Bacterial Results" by Houston (*Reports to the London Water Board*) the expenditure appears to be justified.

The circumstances in London are peculiar. The raw water is heavily polluted, and, except in flood time, is not very turbid. Also the quantity supplied is now growing very close to the amount that may be drawn from the river in the months when its flow is least. An increase in storage capacity is therefore necessary, if only to ensure against shortage of supply. Houston's investigations have probably produced the construction and use of reservoirs as sedimentation basins but a few years previous to their inception for storage requirements only. Indeed, it may be doubted whether an engineer accustomed to such turbidity as occurs in America or India would consider the London Water Board reservoirs to be correctly termed sedimentation basins. Some sedimentation does occur in these reservoirs, and as far as possible no water is delivered to the filters which has not been passed through the reservoirs. A study of Houston's reports will show that the reduction in bacterial content which occurs in the reservoirs is attributed to storage rather than sedimentation. The distinction is not of great practical importance, as even should it be proved that the duration of the storage is the only factor effective in destroying the bacteria, it is highly improbable that any engineer will build reservoirs to store clear water for this purpose alone. Where, however, turbidity or shortage of supply enforces the use of sedimentation reservoirs, Houston's researches show that under favourable circumstances a marked diminution of the bacterial content may be expected even when the water is initially free from turbidity.

Houston ("Third Report on Storage of River Water antecedent to Filtration") suggests 30 days' storage previous to filtration, as the best period for Thames water, and it may be inferred that a shorter period would in most cases (other than the Thames) suffice to secure all the advantages anticipated from storage.

The usual figures in British practice are two to three days' sedimentation; or, where the rejection of the turbid water which comes down with floods is desired, 10 or 12 days' storage suffices. Thus, in ordinary circumstances, 10 or 12 days' sedimentation is procured. American engineers are usually satisfied with one day's sedimentation, but the more modern installations are provided with sedimentation basins of two or even three days' capacity.

German practice is usually satisfied with 24 hours' sedimentation, but it must be remembered that the working of filters in Germany is a matter of official regulation, and that the maximum permissible filtration velocity is somewhat low when compared with that usual in other countries. Thus, it is quite possible that, were greater latitude given to German engineers, they would find a longer period of sedimentation economically justified.

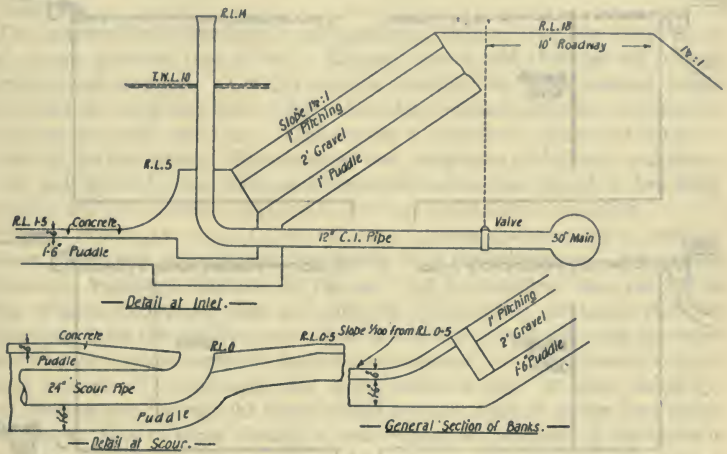
Where polluted or turbid water occurs at definite seasons, a storage reservoir of sufficient capacity to allow of the rejection of this bad water, is a very useful adjunct, and can be used as a sedimentation basin at other periods. In the Nile, for example, during the first rise of the flood each year the river is heavily charged with fæcal matter washed from the newly covered banks. Since 12 days' storage would permit an entire rejection of this polluted water, and since the normal Nile water is but slightly polluted, it is plain that this amount of storage would permit a far less efficient filtration plant than is now required to secure good results throughout the year.

A consideration of these figures shows that when coagulation processes are not applied about 24 hours' sedimentation may be taken as the minimum period, and that we may then expect to secure a deposition of 25 to 50 per cent. of the suspended matter in unfavourable, and 90 to 99 per cent. in favourable cases. The accompanying diminution in bacterial content is somewhat less variable, and may be taken as ranging between 60 and 95 per cent. if the water is markedly turbid. In clear waters, however, it is doubtful whether 24 hours' sedimentation has any noticeable effect on the number of bacteria.

An increased period of sedimentation is indicated in the case of muddy and heavily polluted waters, such as are found at Shanghai, where two days' sedimentation is allowed, and more would be advantageous. For very clear waters, particularly those that have already been stored, special sedimentation appears to be quite useless.

A scientific investigation of the subject would treat the construction of the filters and the period of sedimentation as mutually dependent, but the requisite statistics are not available. The general principles, however, are plain. The finer the filter sand, and the thicker the layers, the shorter the period of sedimentation required to produce good bacterial results, although it must always be understood that filters of fine sand need more frequent cleansing, and are therefore more easily worked with ample sedimentation.

The question is intimately connected with the value of land, and where land is cheap the present practice (which is the outcome of experience in localities where land is generally dear) should not be too closely followed, and at least twice the usual sedimentation can be given with advantage.



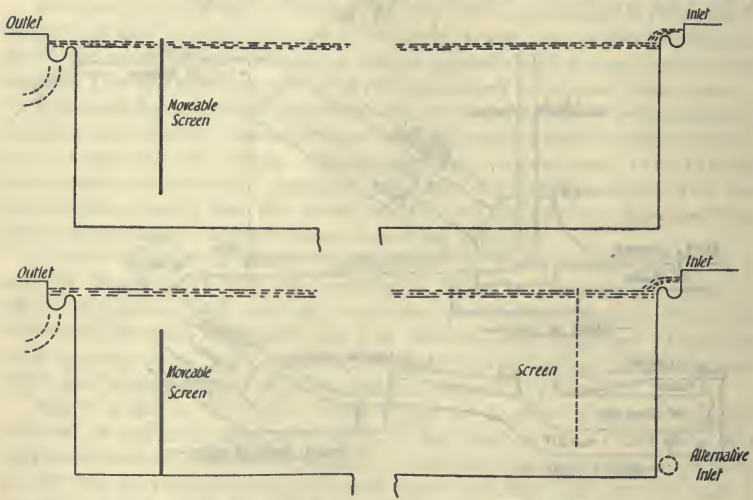
SKETCH NO. 145.—Albany Sedimentation Basin.

Construction of Sedimentation Basins.—Depth seems to have but little influence on the efficiency of a sedimentation basin. Thus, a deep basin is indicated. Since pollution by the subsoil water can usually be regarded with equanimity, careful precautions against leakage from the outside, such as are provided in filters, or service reservoirs, are unnecessary.

The best design is a tank with sloping sides, say 2 to 1, or 3 to 1, according to the character of the soil, paved with slabs of concrete of the type used at Staines (see p. 331).

The bottom of the basin should be covered with 6 inches to 1 foot of concrete, laid at a slope of 1 in 100, and smoothly rendered. It is then found that the deposited silt is very easily and rapidly removed with squeegees, assisted by a stream of water from a hose (Sketch No. 145).

The action on bacteria in sedimentation is entirely different from that produced by a long storage of fairly clear water, or slow-sand filtration. In the latter methods the bacteria are destroyed. In sedimentation, however, the bacteria accumulate in the silt deposit, and although individual species may die, the general result is a rapid increase in numbers, and, if the silt deposit is disturbed, they may again be distributed throughout the water. We are not, as yet, precisely aware how coagulation alone, or coagulation followed by rapid filtration, affects bacteria. In certain cases it is believed that they are partly destroyed, but, as a rule, we must assume that the bacteria are not destroyed in these processes, but accumulate in the wash water, or silt deposits. The matter is not so frightful as the idea of a concentrated deposit of bacteria might at first sight appear. Disease-producing germs are not highly resistant forms of life, probably the majority die very rapidly ; and, moreover, no one is likely to drink a semi-liquid mud. The real importance lies in the indication that systematic cleaning of the sedimentation basins is necessary, and that the present practice



SKETCH NO. 146.—Convection Currents in Sedimentation Basins.

of cleaning when you "must," should be superseded by routine cleanings at stated intervals.

The velocity with which the water passes through a sedimentation basin should not exceed four or five inches per minute when reckoned on the quantity of water passing per minute divided by the area of the cross-section of the basin normal to the general direction of flow. In practice these velocities are rarely exceeded, and about 1½ inch per minute represents the average velocity. Both the inlet and outlet to the basin should be of such a character that localised currents are prevented. Sketch No. 146 shows a typical method, and also the provision necessary to prevent differences in temperature from influencing the working of the basin.

In the hot weather, when the entering water is cooler than that in the sedimentation basin, the outlet screen is let down on to the floor of the basin, so that the outlet draws from the warmer upper layers of water which are those which have been longest in the basin. In the winter, the warmer incoming water tends to float on the colder layers, and sedimentation is therefore less efficient. This is partially cured by raising the outlet screen, so that the outlet draws from the bottom of the basin. The width between the screen and the wall should be so calculated that the velocity of the rising water is not sufficiently great to lift up the silt deposits.

Clay particles of equal size probably fall more slowly than quartz, and special experiments must be made before the final design of the basin can be determined. Wiley states that $d=0{\cdot}0314v^2$, where *d*, is the diameter of a particle in millimetres, and *v*, is its velocity of fall in millimetres per second. The experiments were made on particles of soil between 0·012 mm. (0·00048 inches) and 0·075 mm. (0·003 inches) diameter, *i.e.* particles which settle in a reasonable period. The formula is incorrect for markedly smaller particles. If **d**, be expressed in **hundredths of an inch** and **v**, in **inches per second** we get $\mathbf{d}=0{\cdot}969\mathbf{v}^{1\cdot1}$.

The constructional details of a sedimentation basin are very similar to those of a storage reservoir (see p. 618). Except in very cold climates the arched covering is not required. It might also be inferred that since leakage is less detrimental the walls and sides might with safety be made thinner. In practice this is not generally the case. The reason is intimately connected with the function of the two works. Service reservoirs are placed on hilltops, and therefore in well-drained soils, while sedimentation basins are placed in low-lying situations, and are therefore exposed to external ground water pressure.

Hazen (*Trans. Am. Soc. of C.E.*, vol. 53, p. 40) has endeavoured to treat the question of the size of sedimentation basins in a logical manner. The whole argument is rendered defective by the fact that he considers each particle as falling independently, and does not allow for the action of the larger particles in dragging down the smaller ones. For this reason I consider that his views concerning the advantages of shallow, as opposed to deep, basins are not entirely correct. The principle laid down, however, that the time which the water takes to pass through the basin should be some multiple of the time which the sediment takes to settle through a space equal to the depth of the basin is correct, provided that for the depth of the basin we substitute the space which the particles fall through before they have clotted together into relatively large masses. Allowing for this modification, it would appear that successful American practice agrees very fairly well with the rule: The capacity of the sedimentation basin=25 to 50 times the period of clotting together of the particles, as ascertained experimentally in tubes of 6 to 8 feet vertical height.

The rule is rough, as no particulars are given of the relative efficiency of the basins; but it has the practical advantage that if the turbidity is of such a character that a 6 or 8 foot fall through still water does not produce any marked clotting, plain sedimentation is almost useless, and coagulation, or some other method, must be employed to deal with the turbidity.

COAGULATION.—Sedimentation does not produce any very great improvement in a water unless it contains a sufficient proportion of rapidly falling particles to carry down a large proportion of the small particles which are detrimental to filters, and which fall very slowly by themselves. In waters where the larger particles are deficient, ordinary sedimentation has but little effect. Consequently, in such waters it is necessary to produce a rapid fall of the small particles by artificial means. The most usual method is to introduce into the water a gummy, adhesive substance, which collects in flocks, to which the particles adhere, and then falls more or less rapidly to the bottom of the sedimentation basin or coagulating tank. The process may, in fact, be regarded as passing a filter through the water, in place of passing the water through a filter. The gummy, adhesive substance is produced by chemical reactions which are set up in the water by the addition of various salts. The added salt is usually termed the coagulant, and the flocky precipitate, or falling substance, is referred to as the coagulating substance.

The coagulating substance is produced by the reaction of the coagulant with certain dissolved substances contained in the raw water, and the chemistry of this reaction requires consideration.

The chemical reactions are separately discussed under each coagulation process, and in each case it will be found that the "alkalinity" of the raw water is, chemically considered, the most important factor in the reaction.

In practice, coagulation is usually only a portion of the complete process of water purification, being a preliminary to filtration either by slow-sand or mechanical filters, and it is only occasionally that coagulation alone is considered to produce a satisfactory water.

Thus, the details of coagulation processes will be found to be largely influenced by the treatment which the water afterwards undergoes, and the consequent differences in the methods adopted will be indicated as exactly as our present knowledge permits.

Alkalinity.—The alkalinity of a water is a convenient term for that portion of the temporary hardness which reacts with a coagulant such as aluminium, or ferrous sulphate, or with slaked lime, in a period that is sufficiently short to be practically useful in the coagulating or water-softening process.

The chemical process (usually referred to as Hehner's process) generally employed by chemists in ascertaining the hardness of water determines the temporary hardness (reported as alkalinity, if this term is used by the chemist) as so many parts of calcium carbonate per 100,000. The alkalinity, however, may exist in the following forms:

(i) $CaCO_3$ (not more than three parts per 100,000).
(ii) $Ca(HCO_3)$, *i.e.* carbonate and "bicarbonate" of lime.
(iii) $MgCO_3$.
(iv) $Mg(HCO_3)_2$, *i.e.* carbonate and bicarbonate of magnesia.
(v) Na_2CO_3, carbonate of soda.

If this last exists, there can be no permanent hardness in the water.

From the point of view of an engineer engaged in coagulating or softening the water, the magnesia salts are unfavourable. The reactions producing water softening, or coagulation, do occur with magnesia alkalinity, but they take time, and in practice it may be stated that the alkalinity produced by magnesia is only about one-half as effective as the same weight of alkalinity produced by lime or soda. The statement is a rough one, as a great deal depends on the size of the coagulation and sedimentation basins.

Taking the case of lime alkalinity, we have as follows:

(*a*) With alumina sulphate:

$$Al_2(SO_4)_3 + 3Ca(HCO_3)_2 = Al_2(HO)_6 + 3CaSO_4 + 6CO_2$$

(*b*) With ferrous sulphate:

$$FeSO_4 + Ca(HCO_3)_2 = Fe(HO)_2 + CaSO_4 + 2CO_2$$

Similar reactions occur with magnesia alkalinity, but take time.

(*c*) With lime:

$$Ca(OH)_2 + Ca(HCO_3)_2 = 2CaCO_3 + 2H_2O$$

With magnesia alkalinity the above reaction occurs slowly, and then the reaction:

$$Mg(CO_3) + Ca(OH)_2 = CaCO_3 + Mg(OH)_2$$

occurs.

Thus, remembering that alkalinity is reported in parts per 100,000 of $CaCO_3$, we obtain the following table for lime alkalinity:

ONE PART OF LIME ALKALINITY PER 100,000 REACTS WITH PARTS PER 100,000 OF

Alumina Sulphate.		Ferrous Sulphate.		Lime.	
Dry Salt.	Crystallised Salt.	Dry Salt.	Crystallised Salt.	Caustic Lime.	Slaked Lime.
1·14	2·22	1·52	2·78	0·56	0·74
Grains per Cube Foot of Water.					
4·99	9·72	6·66	12·16	2·45	3·24

Similarly, if time were given, the magnesia alkalinity would react in the same proportions, except that the quantities of lime would be doubled.

More definite information concerning the complications produced by finely-divided turbid matter, etc., is given when the various processes are discussed.

As a rule, however, any further information must be obtained by special experiment on the water considered. If the problem is put before a chemist, the proportion of the magnesia alkalinity that reacts in 1 hour, 2 hours, 3 hours, etc., can be ascertained, but it is necessary to check these determinations by large scale (say 200 gallons of water) experiments.

Although the chemically determined alkalinity of a river usually varies from day to day, the ratio of the lime and magnesia salts does not vary to anything like so marked a degree.

Thus, let us assume that a water contains 10 parts per 100,000 of alkalinity, of which 4 parts are in reality produced by magnesia.

The water softening reaction (*c*) can be calculated as follows:

Temporary hardness, 10 parts, equivalent to .	5·60	parts of caustic lime	
Additional for 4 parts of magnesia hardness .	2·24	,,	,,
Total reaction equivalent . . .	7·84	,,	,,

Thus, the complete precipitation of the 10 parts of temporary hardness requires 7·84 parts per 100,000, or 34·3 grains of lime per cube foot, and the correct dose of caustic lime may be anything from $(6 \times 0{\cdot}56 \times 4{\cdot}375) = 14{\cdot}7$ grains, to 34·3 grains per cube foot, according to the size of the softening basin, the first figure corresponding to the removal of all the lime, and no magnesia, temporary hardness (which would be produced almost instantaneously), and the last figure to the total removal of all temporary hardness (which would probably not be attained in less than 48 hours). If the coagulation basin were of say 6 hours' capacity, it is probable that 23 to 26 grains per cube foot would be a correct dose, but once the exact figure, say 25 grains, is ascertained, we may rest assured that for this particular water, and for this particular size of coagulation basin, the rule, 2·5 grains of caustic lime per cube foot per part of alkalinity per 100,000, will never be very far wrong, and that variations in the temperature of the water will probably have more effect in altering the correct dose of lime, than any variation in the chemical composition of the alkalinity.

For the coagulation processes (*a*), and (*b*), the question is not so acute. We do not usually wish to remove all the alkalinity, but merely wish to employ a certain portion in order to produce a coagulating precipitate. Also, if the alkalinity is deficient, lime can always be added. Thus, the distinction between lime and magnesia alkalinity is by no means so important, and, judging by certain experiments of my own, the reactions of the coagulants with magnesia alkalinity usually proceed with sufficient rapidity for practical purposes. Exceptions do occur in very cold water (say under 40 degrees Fahr.) where only magnesia alkalinity is present. I was then unable to produce a satisfactory coagulating precipitate with alumina sulphate in less than two hours, but, as the lime alkalinity had been specially removed, the fact is not likely to cause trouble in practice.

Sulphate of Alumina Process.—The typical coagulant is sulphate of alumina. This chemical, when added to a water containing lime or magnesia alkalinity, breaks up into sulphuric acid (which unites with the lime) and aluminum hydrate. The hydrate collects in gelatinous, flocculent masses, and gathers up the particles suspended in the water. The sticky film thus formed adheres to the top layer of sand in the filter, and there makes an artificial Schmutzdecke.

The details of the reaction require consideration. In the first place, there must be sufficient alkalinity in the water to decompose the alumina sulphate, and form an adequate amount of precipitate. Secondly, clay particles appear to unite with a certain quantity of aluminum hydrate and deprive it of coagulating properties. Thus, in turbid waters, a larger quantity of coagulant is required to produce the same quantity of coagulating precipitate than in clear waters. In very turbid water, containing but little alkalinity, it may be necessary to add lime in order to provide a sufficient degree of "alkalinity," to decompose the total quantity of sulphate required both to unite with the clay particles and to furnish the quantity of coagulating precipitate required to carry down the turbidity.

The relation between the turbidity and quantity of sulphate of alumina required to produce effective coagulation depends considerably on the physical character of the turbidity, the amount of sedimentation previous to coagulation, and the method of filtration afterwards adopted.

As an example, I give the three methods experimented with at Cincinnati, and the two at New Orleans :

Turbidity Parts per Million.	Grains of Alumina Sulphate per Cube Foot.					
	Cincinnati.			New Orleans.		Kansas City.
				Rapid Filters.		
	Raw Water afterwards treated by Slow-Sand Filters.	Sedimented Water approximately 2 hours afterwards, Slow-Sand Filters.	10 Hours afterwards, Mechanical Filters.	Approximately 7 Hours' Sedimentation.	No Sedimentation.	No after-filtration.
10	...	...	5·6	...	...	...
25	...	...	9·4	...	...	...
50	...	...	11·2	12·8	...	...
75	...	9·8	14·6	13·9	...	...
100	11·2	12·0	16·5	15·7	...	3·75
125	12·0	13·5	18·4	17·2	...	...
150	12·8	15·0	19·9	18·4	22·5	7·5
175	13·5	15·8	21·4	19·5	23·6	...
200	14·6	16·5	22·5	20·2	24·8	11·2
300	16·9	18·4	28·5	25·5	29·7	18·0
400	18·8	20·6	33·0	30·7	34·9	25·5
500	21·0	...	...	...	42·4	32·2
600	22·9	...	...	...	47·2	39·7
750	25·5	...	...	...	52·9	...
1000	30·0	...	...	...	76·0	...
1200	35·6	...	...	...	...	...

Turbidity in Parts per Million.	Hazen's Values.	
	Ordinary Average.	Favourable.
50	7·6	3·7
100	10·0	5·8
150	12·1	7·6
200	13·7	8·7
300	16·3	11·2
400	17·6	13·1
500	18·6	14·7
600	20·0	16·0
750	22·5	18·0
1000	24·7	20·0

In view of these figures, the principles are fairly obvious. A mechanical filter needs more coagulant for proper working than a sand filter, and previous sedimentation produces a certain economy in coagulant, although, in some cases (when sedimentation has removed a portion of the turbidity) the water may require more coagulant than a raw water which has not been allowed to deposit any portion of its turbidity, and is as turbid as the sedimented water becomes after sedimentation.

The efficiency of pre-sedimentation is most marked in clay-bearing waters, containing a large proportion of their turbidity in the form of very small particles (say under 0·0005 inches in diameter). Such waters are capable of decomposing alumina sulphate, with, it is believed, the formation of alumina silicate, which is useless as a coagulant. The advantage gained by pre-sedimentation in such cases is well illustrated by the figures for New Orleans. In Cincinnati, the improvement is less marked, since, although the turbidity has been reduced by sedimentation, the portion which is still in suspension plainly decomposes the coagulant to a far greater degree.

It will also be clear that both the time allowed, and the precautions taken to ensure a good coagulation, are of great importance. The figures given may be taken as covering the most unfavourable cases that are likely to occur, and economy both in chemicals, and time required for coagulation, can be secured by adding the coagulant in two doses, as later explained.

The figures given in Hazen's schedule (Column No. 2) (*Engineering Record*, June 27, 1908) should be attained under ordinary working, when the operator becomes accustomed to local peculiarities; although on individual days variations of 30 to 40 per cent. may occur. Those figures given as "Favourable" (Column No. 3) should be considered as attainable by good working under average conditions (see p. 575).

It must be remembered that economy in chemicals can be carried so far as to cause the filters to require cleaning more frequently than is desirable, and a cautious increase in the quantity of coagulant added, up to as much as 10 per cent. in excess of the figures given by Hazen, will frequently cause a mechanical filter to run from 15 to 20 per cent. longer without requiring cleaning.

The above results are nearly all drawn from American practice.

The work of Schreiber at Berlin, and of Bitter at Alexandria, indicate that turbid waters can be treated with quantities of chemical equal to those given by Hazen's rule, the variations observed being well within the extreme results of good American practice.

When applied to clear waters, however, large increases may be necessary in order to obtain the best bacterial results, and no rule can be given other than that the presence of algæ is unfavourable to an economy in coagulant.

The maximum quantity of aluminium sulphate that can theoretically be usefully added in order to produce coagulation is determined by the alkalinity in the water; and when (which is very rarely the case) the above tables show a deficiency in alkalinity, lime must be specially added.

The reaction is represented by the equation:

$$Al_2(SO_4)_3 + 3Ca(HCO_3)_2 = 3CaSO_4 + Al_2(HO)_6 + 6CO_2$$

i.e. temporary hardness is changed into permanent hardness, and 342 parts of alumina sulphate react with 300 parts of carbonate of lime as bicarbonate, or, with 132 parts of semi-free carbonic acid.

As a matter of fact, commercial alumina sulphate contains nearly 50 per cent. of water, the theoretical formula for the crystallised salt being:

$$Al_2(SO_4)_3 + 18H_2O$$

Consequently, approximately 666 parts react with 300 parts of carbonate, as bicarbonate, etc., or roughly, 2·2 parts per part of alkalinity.

Hence, we find that, corresponding to an alkalinity of one part per 100,000 (assumed as entirely lime alkalinity), about 9·7 grains of alumina sulphate per cube foot is the maximum quantity that can be decomposed.

In practice, owing to decomposition by clay particles, the theoretical quantity is almost invariably exceeded.

Allowing for the fact that commercial alumina sulphate may contain slightly more than the theoretical quantity of the dry salt, it may be stated that even the very clearest water can decompose 9·0 grains of commercial salt per cube foot of water for every part of alkalinity per 100,000 which the water contains. The figure 9·0 becomes 10, or even 11 in more turbid waters. Practically, however, owing to difficulties in securing complete mixture and the time taken to complete the reaction, it does not appear advisable to reckon on decomposing much more than 80 per cent. of the quantity given by these figures. Therefore, if we wish to ascertain the amount of precipitate in a coagulating state, it is best to calculate it as corresponding to 8·0 grains of coagulant, or 1·8 to 1·9 grains of aluminium hydrate per cubic foot of water per 1 part of alkalinity per 100,000 in the water.

The final result of the process is that temporary hardness (or "alkalinity") is replaced by permanent hardness, and carbonic acid is set free in the water. The water produced is therefore relatively more pleasant to drink, and is more palatable than would be the case were the same natural water treated by a non-chemical process of purification (such as slow sand filtration), but is less suitable for use in boilers, or for most trade purposes. It is also more likely to attack and corrode unprotected metals, although this defect can be easily removed by treatment with lime, if considered advisable.

In any given case, the alkalinity should be measured, and the quantity of coagulant decomposed should be ascertained by experiment. Allowances for variations in the lime-magnesia ratio can then be made if necessary.

The necessity, or otherwise, for the addition of lime can thus be ascertained.

The coagulant being added, the process should be directed more with a view to the removal of bacteria, than to that of turbidity. In comparatively clear waters (as at Cincinnati), the period of coagulation appears to have little effect; while, in turbid waters, time is important, and increasing the period from a half to six hours, or even longer, promotes the removal both of bacteria and turbidity.

The crucial point of the method is not so much the time of subsidence or coagulation: these are quite minor matters compared with the care taken in correctly adjusting the quantity of coagulant.

Broadly speaking, it would appear that the following is the most efficient method. Add one-half to three-quarters of the amount of coagulant required; allow the precipitate to subside for a period varying from half an hour, for waters of say 10 parts per 1,000,000 turbidity, up to six hours for 500 parts per 1,000,000; then add the remainder of the coagulant, and filter.

It will therefore be evident that the consulting engineer has less control over the process than the supervisor. The best that the engineer can do when designing for a water the characteristics of which are not well known, is to give the man who works it as much power to alter conditions as is consistent with economy, and to insist on the services of a highly efficient supervisor.

The method in which the water is coagulated, *i.e.* the character of the motion forced on the water in the coagulating basins, is of importance. So far as my experience goes, the best results are not obtained by absolutely quiet water, nor by a turbulent motion over weirs or cross walls, but by a quiet swirling motion, just sufficient to bring the particles of precipitate into contact; but not sufficiently violent to destroy the flocks when formed. Motion of this character is best obtained by causing the water to move with a velocity of approximately 3 inches per minute. This should be quickened up to say 2 feet per minute about six times in the whole coagulation period, by means of cross walls with submerged apertures. Very good results are also obtained by circulation in spiral channels the depth of which increases towards the exit.

It will be noticed that the weak point in the alumina sulphate coagulation process is the possibility of the water not containing sufficient alkalinity to decompose the quantity of aluminium sulphate required to produce adequate precipitation, and so to properly coagulate, and collect the turbid matter. Such a deficiency of alkalinity should not give bad results, as a careful supervisor should be able to recognise the defect, and should supplement it by the addition of lime, or carbonate of soda.

The designer should, however, arrange the piping systems so as to permit the addition being made without special arrangements having to be resorted to. The necessity will probably arise without any warning, and the supervisor's attention should not be distracted by the provision of some temporary makeshift.

This consideration is even more urgent in cases where the primary object of coagulation is the removal of colour, rather than turbidity. Coloured waters are frequently deficient in alkalinity. The complete precipitation of the colouring matter by aluminium hydrate, may consequently require far more hydrate than the untreated water can produce. Thus, the addition of lime may become a normal feature of the treatment (in the case of the removal of turbidity it is doubtful whether the addition of lime will be required for more than 30 days in each year). In such cases aluminium chloride—which is stated to decompose into aluminium hydroxide more readily than the sulphate—has sometimes proved a useful coagulant. As a rule, however, ferrous sulphate and lime form the best coagulant when the natural alkalinity is deficient, although conditions exist where aluminium hydrate forms a better precipitant of colouring matter than ferrous hydrate; but it is believed that these are very rare. Usually the failures of either process in removing colouring matter may be considered to be due to the chemical reactions not being properly considered; either the necessity for extra lime is not realised, or the water is not sufficiently turbid to produce nuclei for the coagulant to adhere to, which can easily be provided against by the addition of powdered clay.

The chemical principles underlying the question of adding lime, or replacing aluminium sulphate by aluminium chloride, are very well illustrated by Whipple (*Use of non-basic alum in connection with Mechanical Filtration*) who has very clearly shown that an aluminium sulphate possessing theoretical constitution, $Al_2(SO_4)_3.18H_2O$, is inferior, as a coagulant, to what is commercially known as basic alum.

A salt possessing the above constitution contains 15·32 per cent. by weight of Al_2O_3, and 36 per cent. of SO_4.

In the "basic alum" the first quantity is increased, and the second is diminished, so that a certain amount of Al_2O_3, is not combined with the SO_3, thus justifying the term "basic."

Whipple recommends the following as a useful buying specification :

The basic sulphate shall contain at least 17 per cent. of Al_2O_3, soluble in water, and at least 5 per cent. of this shall be in excess of the quantity theoretically necessary to combine with the SO_3, present.

This specification defines a salt which will produce the quantity of aluminium hydrate required for coagulation, with the least possible consumption of alkalinity existing in the water.

The addition of lime, or the substitution of aluminium chloride for aluminium sulphate, are now plainly seen to merely be further applications of the same principle.

Practical Details.—Aluminium sulphate being easily soluble, its solution in water causes but little difficulty. It is usual to employ some mechanical stirring arrangement, in order to prevent variations in the concentration of the solution. Very good results are also obtained by permitting the water to enter the bottom of the solution chamber, and flow out at the top, thus rising through a layer of sulphate resting on a grating. In such cases, it is necessary to add (about twice every hour) the quantity of salt that will be consumed in the next half-hour, and the total quantity of undissolved salt in the chamber should not be less than about 2 hours' consumption. The water supply should therefore be adjusted so that the water leaving the chamber is almost a saturated solution of aluminium sulphate.

In very large plants, constant mechanical feeding of the sulphate into the dissolving tank is sometimes adopted, but the complication seems hardly necessary.

A combination of the upward flow method with a second mixing tank containing approximately the quantity of solution used in one hour's working, will evidently prevent any sudden variations in the amount of salt delivered. In practice, except in very large or very small (one filter) plants, the supervisor is usually able to keep the quantity of salt delivered to the water sufficiently under control by adjusting the taps regulating the supply of water to the dissolving tank, and any extra tanks or stirrers appear to be unnecessary.

Owing to the action of the solution on iron, all pipes and metal exposed to the concentrated solution are usually of brass.

It will be found in practice that the best place for the addition of the coagulating solution depends to a large extent upon the temperature of the water and air, and possibly on the amount of dissolved substances in the water and the solution. The final location of the delivery pipe should therefore be left for the supervisor to ascertain by experience. At first, it is usually sufficient to deliver the solution through a pipe opening into the centre of the entry channel of the coagulating basin.

In two very successful installations the centre of the coagulating basin was the location finally fixed upon. Distribution was effected by a fountain in one case, and by a small Barker's mill in the other.

In my opinion, most of the present installations are insufficiently provided with sedimentation tanks before the addition of coagulant, as also with coagulating basins in which the coagulating process occurs.

The general rules when clarification rather than bacterial purification is the main object of the process, are as follows: 24 hours' sedimentation before the addition of coagulant, and a coagulating basin of [5, to 6 hours' capacity.

Such an installation may be regarded as throwing an undue share of work on the filters. These, being patentable articles, have been developed and improved, so as to afford satisfactory results with even less capacity than that stated above. I believe that 4 hours' sedimentation and half an hour's coagulation is too hard for any filter, although 6 hours, and one hour appears to be quite a usual design. Obviously much depends on the raw water, and on the popular standard of a satisfactory filtrate.

Any economy in first outlay is plainly secured at the cost of an extra expenditure in washing the filters, and it is even possible that additional filters may be required in extreme cases. And, for this reason, the advice of filter makers, as opposed to consulting engineers, should not be too readily followed.

My own belief is that an engineer should consider 48 hours' pre-sedimentation, and 12, to 15 hours' coagulation (in which should be included not only the capacity of the coagulating basin, but also that of the water in the filters) as justifiable, in places where land is cheap, and the power and labour used in washing the filters is costly. From this high standard we can work down to say 8 hours' sedimentation, and 2 hours' coagulation, which forms a limit below which it is undesirable to go, unless unsatisfactory results may occasionally be permitted in order to secure a diminution in cost. It will be plain that where the filters are washed mechanically, and where the power for such washing is cheaply obtained, the 12 hours and 6 hours stated as common are probably justified by economical considerations.

Where slow sand filters, cleansed by hand, are employed, and the water is initially very turbid, the 48 hours and 15 hours may require to be exceeded. Engineers contemplating the combination of slow sand filters with coagulation, should consider all mechanical filter installations as insufficiently provided with sedimentation and coagulation capacity. It is also necessary when considering the adoption of any proprietary filters or processes to allow very carefully for the fact that, until late years, the success of all processes was judged almost entirely by the physical appearance of the filtrate. This method of judgment is by no means entirely obsolete, and while bacterial tests are probably more systematically employed in America than in England, there is little doubt that an engineer who copies even modern American practice blindly in an Asiatic installation will not obtain as great a decrease in water-borne death-rate as he expects. This statement must not be construed as adverse to the adoption of American methods when carefully adapted to local conditions. Three of the five Oriental cities that I consider have best solved the local filtration problems use more or less modified American processes, and none of the five relys exclusively on purely British methods.

Ferrous Sulphate Process.—This differs from the alumina sulphate process in that ferrous sulphate is employed instead of alumina. Except in very alkaline waters, such as are rarely used for human consumption, it is necessary to add lime or other substances, as well as the ferrous sulphate, in order to procure a coagulating action within a reasonable period.

The infrequent use of ferrous sulphate (as compared with alumina sulphate), may be attributed to the difficulty of handling the two chemicals. As already stated, coagulation by alumina sulphate requires considerable care. When therefore the supervisor of the coagulation process is generously allowed two chemicals (*i.e.* lime, and ferrous sulphate), in place of one, to make mistakes with, it is hardly a matter of astonishment if the one chemical process is found to work more easily and regularly, and is consequently preferred.

This is somewhat unfortunate; as, when the ferrous sulphate process is correctly worked, it produces a precipitate which is in every way better adapted for coagulation. Being heavier, it collects in larger flocks, and more rapidly. It also appears to be totally unaffected in its coagulating properties by clay, or other turbid substances. It is brown in colour, whereas aluminium hydrate is white, and while I do not profess to explain the reason, it is nevertheless a fact that a brown-coloured precipitate (probably since it resembles dirt) causes less fear of wholesale poisoning in Oriental and other unscientific minds than a white one.

So also, the medical profession appears to consider a "little iron and lime in the water" as less likely to affect public health than "an excess of alum."

In view of the fact that, when properly worked, neither process permits any undecomposed coagulant to pass the filters, and both usually produce an increase in permanent hardness, the reasoning of both parties is, to my mind, equally logical, but the waterworks' engineer should respect such prejudices.

Before this process can be satisfactorily applied to a water even more careful and systematic preliminary experiments must be made than are needed for an aluminium sulphate process. Our present knowledge hardly justifies its application to heavily polluted waters, except on so large a scale that the salary required to secure a first-class supervisor forms but a small portion of the running expenses.

The reaction occurring in the ferrous sulphate process is a very complex one. The chief practical difference between this reaction and the one which occurs when aluminium sulphate is used as a coagulant lies in the fact that the reactions which occur in the ferrous sulphate process are only completed after a certain time, while the aluminium sulphate reaction (see p. 556) is practically instantaneous.

Broadly speaking, if the ferrous sulphate is added to the water before the lime, we have:

$$FeSO_4 + CaH_2(CO_3)_2 = FeH_2(CO_3)_2 + CaSO_4 \quad . \quad . \quad . \quad (1)$$

The ferrous bicarbonate is soluble, and will only slowly decompose into carbonic acid gas and ferric hydroxide. Thus, lime is added to hasten the reaction, and the following reactions occur:

$$Fe(HCO_3)_2 + 2Ca(OH)_2 = Fe(OH)_2 + 2CaCO_3 + 2H_2O \quad . \quad . \quad (2)$$

$$FeSO_4 + Ca(OH)_2 = Fe(OH)_2 + CaSO_4. \quad . \quad . \quad . \quad . \quad (3)$$

That is to say, both the ferrous bicarbonate resulting from the first reaction, and any excess of ferrous sulphate, are decomposed, and produce ferrous hydroxide.

This last very rapidly oxidises to ferric hydroxide, which is the precipitate:

$$2Fe(OH)_2 + H_2O + O = Fe_2(OH)_6 \quad . \quad . \quad . \quad (4)$$

In addition, however, the lime reacts with any bicarbonates of lime, and magnesia, which have not been previously decomposed by the ferrous sulphate in the manner indicated in equation (1), and therefore the reactions above discussed occur simultaneously with the various reactions employed in Clark's water softening process, such as:

$$\left.\begin{array}{l} Ca(HO)_2+CO_2=CaCO_3+H_2O \\ Ca(HO)_2+Ca(HCO_3)_2=2CaCO_3+2H_2O \\ MgH_2(CO_3)_2+2Ca(OH_2)=Mg(OH)_2CaCO_3+2H_2O \end{array}\right\} \quad . \quad . \quad (5)$$

The normal calcium carbonate thus produced is insoluble, and rapidly precipitates. The magnesia hydrate is less rapidly precipitated, and re-dissolves as the water absorbs carbonic acid from the air. The magnesia hydrate is therefore only removed under favourable circumstances.

Thus, it will be evident that the two processes of coagulation and water softening cannot be separated, and calculations which are based on the coagulation process only do not indicate the whole circumstances.

Broadly speaking, when a water of ordinary composition is considered, the calculations are as follows:

(i) Ascertain the amount of temporary hardness, and calculate from equations (1), to (4), inclusive, the quantity of iron sulphate and lime necessary to cause the reactions. If this is likely to produce a sufficient quantity of precipitate to satisfactorily coagulate the turbidity, and form an efficient Schmutzdecke, we must experimentally ascertain whether the quantity of lime given by equation No. (2), is sufficient, or whether reactions of the type given by equation No. (5), also occur, with the corresponding amount of water softening.

(ii) If, however, the amount of precipitate thus produced is insufficient, we must in addition calculate from equation No. (3), the necessary quantity of lime and ferrous sulphate required to produce the extra amount of precipitate in accordance with reaction (No. 3), and also experimentally ascertain whether any excess of lime is required for reactions of type No. (5).

The crux of the process is whether reactions of type Nos. (2), and (3), occur before, or after those of type No. (5). This question can only be answered by experiment, as the results depend upon the relative concentrations (*i.e.* the number of parts per 100,000) of each of the salts considered, which occur in the water.

It will be evident that the process hitherto considered consists of a coagulation, combined with the transformation of a part, or, possibly, nearly all, the temporary hardness into permanent hardness, as per equation No. (1). This is the process most generally employed.

Let us, however, consider the reverse process, where the lime is first added. We then have reaction No. (5) occurring, and the temporary hardness is entirely, or, nearly entirely, precipitated, just as in Clark's process.

On adding ferrous sulphate, reactions No. (1), and No. (2), do not occur; but No. (3) does, if an excess of lime remains. By suitably proportioning this excess, we can obtain a sufficient quantity of coagulating precipitate.

This second method has the advantage that we not only coagulate the water, but can also partly soften it if the process is carefully conducted. Thus, the second method is preferable in waters containing a large amount of temporary hardness. It has, however, the disadvantage that the heavy precipitate of calcium carbonate (which in the first, or usual process, forms simultaneously with the precipitate of ferric hydrate, and loads the flocks and materially increases the rate of subsidence) is now formed, and falls separately. Hence, the subsidence is less efficient, and, broadly speaking, a larger subsiding and coagulating basin will be required. Also, the consumption of lime, will probably be largely increased. Where the turbidity is great, the first, or usual process, should be adopted. A careful supervisor will, of course, change over from one process to the other, according as temporary hardness or turbidity is the most important factor. It may be noted that since the more turbid waters are those drawn from the river when in high flood, they are usually less alkaline than the normal waters, so that the change in method has practical advantages.

It will be evident that any arithmetical rule, even if only as approximate as that given in considering the alumina sulphate process, is so liable to mislead as to be useless.

On the other hand, the above investigation shows that the process is a very powerful one. I believe that if the details are correctly planned, it is applicable to a greater number of varieties of water than the alumina sulphate process, and can be made to produce better results.

At St. Louis, Mo. (*Trans. Am. Soc. of C. E.*, vol. 60, p. 170), for instance, a fairly satisfactory water (from the bacteriological point of view) is produced by coagulation and sedimentation only, from a very turbid and somewhat polluted river water. Although I consider that the process, as at present adopted in the above-mentioned city, is merely educative, and will rapidly lead to a popular demand for an even better water, I doubt whether any such rough process with alumina sulphate as coagulant would prove satisfactory, even as a temporary measure.

Therefore, as knowledge advances, I expect that this process will largely supersede the aluminium sulphate process. Even at the present date, if softening by Clark's, or a similar process, is desired, and a coagulation process (other than the slight coagulating effect obtained by the deposition of the calcium carbonate precipitate on particles of suspended matter), is also required, ferrous sulphate should be adopted rather than aluminium sulphate, since, where lime is added for softening, the extra complication of two chemicals has to be faced in any case.

A modified ferrous sulphate process has lately been introduced, in which lime is not employed, live steam being injected into the water after the addition of the requisite quantity of ferrous sulphate. Details, however, have not as yet been published.

Certain small scale experiments of my own indicate that reaction No. (1) occurs, and that the steam then causes the ferrous bicarbonate to rapidly decompose into ferrous hydroxide, and free carbonic acid. It will consequently be plain that the natural water must contain enough alkalinity (as calculated by the first equation) to produce sufficient coagulating precipitate.

The modified process is therefore subject to the possibility of troubles caused by deficient alkalinity in just the same manner as the aluminium sulphate process. This defect, however, is known to be practically of little significance in turbid waters, and cases entailing the addition of some lime for the production of reaction No. (3), are likely to be almost entirely confined to coloured waters, where the object of coagulation is to remove colouring substances or odours.

The process certainly deserves the fullest consideration, since it will be plain that the addition of an excess of live steam has no such adverse effect on the quality of the water as that produced by an excess of lime.

I am unable to state definitely whether the live steam required is less costly than the lime which it replaces, but the general belief is that the lime process is the more economical. I therefore infer that the question really depends upon whether a supply of live steam is already available (*e.g.* from the pumping station attached to the filter plant). In small installations, where steam would have to be specially generated, owing to the extra superintendence thus required the lime process may prove cheaper.

My own experiments also lead me to believe that the pressure of the steam has little effect on the reaction, and that the best numerical results are obtained with 12 lbs. per square inch gauge pressure.

The water produced by the ordinary ferrous sulphate process is relatively less palatable, and less pleasing to the eye than that which is produced from the same raw water by an aluminium sulphate or a simple filtration process. A consideration of the chemical reactions will show that the lime deprives the water of the dissolved carbonic acid, which remains in the water in the aluminium sulphate process. It will, however, be plain that when the process is carefully worked, and advantage is taken of the alternatives produced by the two methods of adding the chemicals, a water can be produced which does not attack metals, and which is well suited for use in boilers. Thus, the advantages of the aluminium sulphate process can be over-rated, and in considering this question an engineer should remember that the American standard of a "palatable" water is higher than that which is usual in Europe. The fact is curious, although it in some respects explains the relatively lower standard of "healthy" waters which undoubtedly prevails in America.

The above discussion of the various reactions which occur when ferrous sulphate and lime are added to a water shows that a table of the quantities of chemicals per cube foot of water, similar to that given for aluminium sulphate, is useless. If attention is confined solely to the coagulation portion of the process, the following rule will be found to be approximately true provided that the water is not unusually alkaline.

Add of crystallised ferrous sulphate two-thirds, and of slaked lime one-fifth, of the weight of aluminium sulphate which is found to produce satisfactory coagulation. The coagulation thus induced will generally prove equally effective, and in the case of very turbid waters will probably be even more so. If the water contains a large amount of temporary hardness (alkalinity), some amount of softening will generally occur, and the lime should be increased. In such cases it is advisable to ascertain by experiment whether the dose of ferrous sulphate cannot be diminished, since the additional amount of calcium carbonate which is produced in the water-softening reaction will probably load the coagulating precipitate, and produce a more rapid fall of the turbidity. Ferrous sulphate is more costly than lime, and each grain added produces an increase in the permanent hardness, so that a blind adherence to any rule is unadvisable.

The following figures are the average of the results of each month as obtained in actual working at Cincinnati.

The water is sedimented for 40 to 48 hours, and is then treated with sulphate of iron. After being thoroughly mixed, lime is added; and coagulation for 1 to 8 hours is allowed. The filters are each 1400 square feet in area, and treat 4 million U.S. gallons per 24 hours, say 380 feet vertical, or, allowing for cleaning and rejection of effluents after cleaning, work at a rate of about 400 feet vertical per 24 hours.

The working head of a clean filter is 2 feet, and the filters are washed when the head is 12 feet. The wash water is used at a rate of 1·8 to 2·7 cubic feet per minute per square foot of filter, and the wash water is 3·8 per cent. of the total quantity filtered during the year.

The following table shows the monthly average results obtained during one year's working:

Average Turbidity in Parts per Million.		Average Grains applied per Cube Foot of Water.		Average Interval between Washings of Filters in Hours.
Raw Water.	Water after Sedimentation.	Ferrous Sulphate.	Lime.	
7	6	9·2	6·3	11·60
11	9	7·6	5·2	14·45
50	31	11·7	6·9	14·34
93	33	11·9	6·7	12·26
145	85	10·0	6·0	16·44
Not recorded.	85	8·5	5·0	15·09
190	90	8·5	5·9	13·00
255	150	15·0	8·0	15·16
270	125	13·1	7·0	21·39
410	220	17·3	9·0	11·11
430	230	15·2	8·6	11·72

It must be noted that where the water is coloured by vegetable matter, aluminium sulphate is usually found to give better results than ferrous sulphate. This cannot, however, be taken as an invariable rule. The chemical facts underlying the question appear to be that some varieties of vegetable colouring matter combine readily with iron, and in most cases this combination is less easily dealt with than the uncombined colour. I have, nevertheless, met with coloured water (usually accompanied by a history of pollution by animal matter, and therefore possibly entirely of animal origin), which, when treated with ferrous sulphate, produced a bulky, rapidly settling precipitate, very easily removed by sedimentation alone, and for such waters the ferrous sulphate process is invaluable.

Under this head it is as well to refer to the old method in which a dilute solution of mixed ferrous and ferric sulphates and sulphites is produced by passing the fumes of burning sulphur over metallic iron exposed to thin streams of falling water. The dilute solution thus obtained is added to the raw water.

It is plain that the process amounts to an unregulated coagulation, accompanied by a possible disinfection from such sulphur dioxide as remains dissolved in the water, and uncombined with iron. The principle appears to be a good one, but in view of the fact that the supervisor has absolutely no control over the relative quantities added, and very little over the total amount of mixed gases and salts, it is doubtful whether really regular working can be obtained.

Practical Details.—The practical details are very similar to those of the aluminium sulphate process. The capacities of the sedimentation and coagulating tanks are usually some 10 per cent. larger than in the case of the alum process, and, this addition being made, the reasoning there set forth applies.

The greatest difficulties, however, are connected with the addition of lime. If the lime is dissolved as lime water, the volume of lime solution is a very considerable fraction (as much as $\frac{1}{7}$th or $\frac{1}{8}$th in some cases) of the quantity of water treated. It is therefore customary to add the lime as milk of lime, containing about one-tenth its weight of hydrated lime. This milk of lime must be carefully strained, stirred, and kept in motion until added to the raw water. This is best effected by a screw propeller, working in the mixing tank, which should be placed as nearly as possible vertically over the point where the lime is delivered to the raw water.

Troubles are frequently caused by deposits and incrustations of lime salts formed in the pipes. Such obstructions occur at the point where the lime is added to the raw water, or in the pipe systems of the filters.

The first deposits seem to be almost entirely caused by the lime being added to the water when in violent motion, carrying air bubbles. This can usually be stopped by a change in the point where the lime is added. Some chemical action may also be suspected, since such incrustations rarely take place unless the lime is first added. The filter deposits appear to be mainly due to an excess of lime, and whenever the ratio :

$$\frac{\text{Weight of lime}}{\text{Weight of iron sulphate}}, \text{ much exceeds } 0{\cdot}4$$

such deposits may be considered as likely to occur. Occasional additions of lime, far in excess of the above value, have no ill effect, provided that such conditions are temporary, and are succeeded by periods during which the ratio is less than 0·4.

A quantity of lime in the treated water greatly exceeding 1·5 parts per 100,000 of normal carbonates, if continually present, should be considered as unsatisfactory, since incrustation will sooner or later occur.

Coagulation with Slow-Sand Filters.—The processes of coagulation already discussed are generally preparatory to mechanical filtration. There is, however, no difficulty in applying them as preliminaries to slow-sand filtration. The following discussion of the treatment adopted by Fuertes (*Trans. Am. Soc. of C.E.*, vol. 66, p. 135) in the case of the Steelton (Pa.) water excellently illustrates the strong and weak points of the process.

The water is frequently heavily polluted, is very turbid, and is also liable to become extremely acid owing to contamination by mine refuse. Altogether, the water is as variable as can well be imagined. The acidity is neutralised, and the necessary alkalinity is produced by the addition of lime in such quantities as to react with the required dose of alumina sulphate, and still leave about 0·6 parts per 100,000 of alkalinity. The dose of alumina sulphate is entirely determined by the turbidity, and, when graphically plotted, the relation is represented by straight lines which can be laid down from the following particulars :

Turbidity in Parts per Million.	Grains of Alumina Sulphate per Cube Foot of Water.		
	Falling Turbidity.	Rising Turbidity.	Summer Water.
100	4·7	3·1	2·6
700	13·3	10·7	8·2

Twelve minutes' "sedimentation" is then allowed, and the water is turned on to a roughing filter composed of 5 feet of anthracite coal screenings of an effective size of 1 mm. (0·04 inch), and with a uniformity coefficient of 2·4.

The filtration takes place at the rate of 85 million U.S. gallons per acre per 24 hours, and the filter is washed by compressed air and water at the rate of 1 cubic foot of air per square foot per minute, and 0·67 to 0·75 cubic foot of water per square foot per minute, as often as the filtration head becomes equal to 2·5 feet.

This roughing filter evidently acts like a dégroisseur, and the whole of the 5 feet retains turbid matter. A large bacterial reduction (on the average 60 to 70 per cent.) is also produced. Without any assistance from coagulation the roughing filter can reduce water containing turbidities of less then 25 parts per million to a state fit for the sand filters.

These sand filters work at the rate of 3·62 million U.S. gallons per acre per 24 hours, and consist of:

4 feet of sand, specified as follows:

Not more than 5 per cent. under 0·24 mm. in diameter
" " 10 " 0·29 " "
" less " 90 " 0·80 " "

The effective size is 0·30 to 0·33 mm., and the uniformity coefficient is 1·6 to 1·8.

The sand lies on 3 inches of crushed sandstone, between ½ to ¼ inch in size.

Then 3 inches of crushed sandstone, between ¾ and ¼ inch size.

Then 6 inches of crushed sandstone between 3 and ¾ inches in size.

The bacterial results at present obtained would not be considered satisfactory when the bacterial content of the raw water much exceeds 6000 per c.c., but there is a marked tendency towards increased bacterial efficiency, and after six months' work the limit is nearer 10,000 per c.c.

As is usual in American filters, the *B. coli* tests are relatively better than might be expected, so that the large number of bacteria in the raw water are not necessarily pathogenic (see p. 515). The filtrate is invariably clear.

To a British engineer the mere idea of using raw water of such a character is abhorrent. In fact, Mr. Fuertes has to perform not only filtration operations, but a process of purification which should rightly be conducted, at the expense of the people who produce the pollution, before the polluted water is discharged into the river. Setting this legal problem aside (which an engineer as a citizen ought not to do), the engineering problem is most excellently solved, and the only adverse remark that can be made is that the filtration velocities given above are only very rarely attained, and that it is doubtful if good bacterial results can be produced if the raw water is simultaneously highly polluted. The preliminary studies were very complete, and it is probable that a great demand for filtered water and heavy pollution will not occur simultaneously. The substitution of a small dégroisseur for a large sedimentation and coagulation basin is especially noteworthy.

Mechanical Filtration.—The various processes of mechanical filtration are entirely the invention of engineers in America, and the method has not as yet been systematically adopted in other countries. Consequently, anything but American experience on this particular subject is unreliable, and it may be doubted whether the full advantages of mechanical filtration have yet been attained except in the United States.

As a rule, American waters may be considered as less subject to pollution by pathogenic bacteria than are either German or English waters. On the other hand, turbidity of a troublesome character (combined with abnormal colorations, tastes, and odours) is more common than in Europe. At present, therefore, the mechanical filter is seen at its best when dealing with a turbid water, and its weak points are most manifest in the case of heavily polluted waters containing many bacteria.

Auxiliary processes, such as disinfection and coagulation, are more easily applied in conjunction with mechanical filters, than in the case of slow-sand filters. We may consequently consider the following suggestions as applicable in modern practice :

(*a*) For water with a turbidity after sedimentation frequently exceeding 50 parts per million, and which contains a fair proportion of particles less than 0·0005 inch approximate in diameter, mechanical filters are indicated.

(*b*) For heavily polluted non-turbid water, slow-sand filters are better than mechanical filters.

(*c*) If a water is of such a character that it possesses two objectionable properties (*e.g.* colour and turbidity, or odours and extreme hardness) each requiring a separate treatment, mechanical filters are usually desirable. Sand filters should be adopted if the water is heavily polluted, and the treatment for the other objectionable constituent does not assist in removing bacteria, so that the water as delivered to the filters frequently (say more than 20 days in the year) contains more than 2000 bacteria per cubic centimetre as ascertained by Koch's test.

This last statement is derived from a consideration of the bacterial results generally obtained by mechanical filtration. There is no doubt that in many cases skilled bacteriologists can treat a water containing more than the above number of bacteria per cubic centimetre by mechanical filtration, and can regularly produce a filtrate containing less than 100 bacteria per c.c. The average supervisor (whose working tests are based on the clearness of the filtrate), however, cannot with certainty reduce the bacterial count below this number by mechanical filters if the raw water contains a number of bacteria considerably greater than 1000 to 1500 per c.c. Thus, the above rule is a practical one, and, if anything, is too favourable to mechanical filters as they are usually handled. It must not, however, be regarded as an expression of opinion that systematic preliminary tests of mechanical filters when preparing a large scale project for filtering polluted waters are useless, since it is believed that their adoption may prove economically advantageous when systematic supervision by a good bacteriologist can be obtained. A truly scientific solution of the problem could only be given if it were possible to state the conditions in terms of the number of pathogenic bacteria in a given volume of the raw water (see pp. 515 and 569).

The above rules may be regarded as correct, provided that no consideration is given to the available capital and labour, and other local conditions.

Mechanical filters are cheaper in first cost, but are more expensive to maintain and to supervise than the slow-sand type. As usually installed, they contain a good deal of metal work, but the masonry construction employed at Baroda shows that this can be avoided where iron and ironworkers are costly.

On the other hand, machinery with its expensive supervision is always necessary, except in cases where a 40 to 50 feet head of water is available. In modern types, however, special machinery, other than pumps, is but a minor item. Also, the area occupied is small in comparison with slow-sand filters.

The development of the rapid, or mechanical filter, arose from efforts to clarify very turbid water, and when the bacterial rationale of slow-sand filtration first became generally known, rapid filters were viewed with suspicion.

The modern aspect of the matter may be summed up as follows: Rapid filters require more care, and a higher grade of supervision, in order that biological results equal to those given by slow-sand filters may be obtained. Their use, therefore, is at present mainly advisable in new and rapidly developing countries, where labour is dear, but intelligent, so that skilled supervision is relatively cheaper than manual labour. It is also as well to use mechanical filters where labour is markedly inefficient, and of uncleanly habits; since, the cleaning being done by mechanical means, such outrages as defecation on the filter beds are prevented.

In principle, the mechanical filter consists of about 30 inches of sand, resting on 6 inches of gravel, drained by a system of closely spaced strainers. Great care was taken in the early days of such filters to obtain a sand of large effective size (up to 0·60 mm.), and small coefficient of uniformity (1·5, or even as low as 1·2). Experience has shown that the expense thus entailed is unnecessary, and at present any sand suitable for slow-sand filters is used. It is also found that the numerous washings which the sand undergoes rapidly produce a suitable grade of filtering material.

The rate of filtration is high, generally 80 to 110 million imperial gallons, or 100 to 125 million U.S. gallons, per acre per day of 24 hours. This entails washing at intervals, which rarely, if ever, exceed two days. The formation of a biological Schmutzdecke can therefore hardly occur, even though favoured by other circumstances. Its place is taken by an artificial Schmutzdecke, formed by the addition of a coagulant to the raw water. A coagulating basin and apparatus (consisting of valves and pipes for regulating not only the addition of the coagulant, but also the proportion of the mixture of coagulating precipitate and turbid matter that reaches the filter) forms an essential portion of every installation; and, if the water is not naturally turbid, the addition of artificial turbidity, usually in the form of powdered clay, may be found necessary.

The principles of operation are simple. The addition of a coagulant to the raw water, combined with the sojourn of the water in the coagulating tank, produces a flocky suspension of coagulant, and turbid matter in the water. On entering the filter, these flocks pack together on the surface of the sand, and form an effective barrier to the passage of bacteria. The biological action of the Schmutzdecke is greatly reduced, if not entirely arrested, so that the filtration is mechanical in a far more essential sense than that used by the inventor of the term "mechanical filter."

To use a somewhat broad simile,—a mechanical filter arrests bacteria mainly as close-mesh wire netting arrests the flight of birds. Whereas, the Schmutzdecke and sand of a slow-sand filter arrest bacteria much as a loose heap of bird-limed twigs catches birds.

The velocity of filtration is large (roughly, 400 feet in 24 hours, say 109 million imperial gallons, or 131 million U.S. gallons per acre). Thus, the head forcing the water through the filter is high, and the deposit of coagulating precipitate and turbid matter that forms on the sand rapidly clogs the filter. Hence, the filter must be washed at frequent intervals, and in order to save time and labour these washings are effected mechanically.

Since a rapid filter passes through a cycle of operation in 20 to 30 hours, it is obvious that we have to form a satisfactory Schmutzdecke about once a day, so that we should test the working of a rapid filter every hour in order to have the effluent under as close a supervision as that obtaining in a slow-sand filter subjected to daily tests. This is obviously impossible, and forms the greatest defect in rapid filters; which, from a bacteriological point of view, are liable to be markedly irregular in their working. While great improvements have been made of late years in this respect, rapid filters are still largely inferior to the slow-sand type, simply because the filtrate of the period immediately after cleaning cannot be so accurately classed as either good or bad, from a bacterial point of view, as is the case with sand filters during the first day or two after cleaning.

Therefore in considering the installation of a system of rapid filtration, an engineer must be prepared for very irregular bacterial results, especially during, say, the first six months of working, while the supervising staff are acquiring local experience.

It is doubtful whether this defect will ever be entirely overcome, although the rapid improvements of late years are encouraging.

As this defect is probably the factor which is most adverse to the adoption of mechanical filters by British engineers, it is as well to consider the manner in which it may be most rapidly minimised.

The method is fairly simple. Reasoning on the analogy of slow-sand filters, we require to form our artificial Schmutzdecke as rapidly as possible. Once, however, it is formed, the thickness should not be allowed to increase at the same rate, since this would produce a rate of increase in the working head of the filter which would cause the filter to need cleaning far too frequently.

It is consequently obvious that during, roughly, the first hour after washing the filter should be worked in a manner which differs from that which is advisable when it is running normally. This change in method does not, as yet, appear to have been explicitly recognised, although there is little doubt that supervisors of filters are well aware of its existence.

The general outlines are clear. Until a satisfactory Schmutzdecke is formed, the filter should be abundantly supplied with the precipitate produced by the coagulant. The Schmutzdecke being formed, as may be recognised by the working head of the filter increasing, say, two or three feet above the initial head, the working becomes normal, and as little of the precipitate as possible should be passed on to the filter. The rate of increase of the thickness of the Schmutzdecke is thus kept low during the period in which the filter delivers a satisfactory effluent.

In practice, the effluent from a mechanical filter is usually considered to be satisfactory when it is free from turbidity. In waters of the American type, where a large proportion of the particles of turbidity are of bacterial size, this test is probably fairly effective in discriminating between effluents which are bacterially safe and unsafe, although it is believed that the filtrate becomes free from turbidity somewhat more rapidly than it becomes bacterially safe, if less than 100 bacteria per c.c. is taken as an absolute criterion of safety. The matter has been systematically investigated by bacterial methods at Cincinnati, Berlin, Alexandria (Egypt), and elsewhere, and it may be inferred that the effluent is usually bacterially safe when the working head of the filter exceeds 3 feet. When, however, mechanical filters are used to treat waters which do not carry a large quantity of particles which are of, or near to, bacterial size, systematic bacterial tests are required; and it is quite plain that a great economy in working can be obtained by ascertaining the minimum head at which a bacterially satisfactory effluent is delivered. The supervisor can then systematically alter the method of working as above indicated, and is thus enabled to largely diminish the frequency of the filter washings.

The importance of these preliminary bacterial investigations is very great, and, if they are systematically carried out, the mechanical filter can probably be made quite as effective in removing bacteria as the slow-sand filter. Even under such circumstances it will usually be found that when the water which reaches the filter contains more than 2000 bacteria per c.c. (as estimated by Koch's test), the working head required to produce a satisfactory effluent becomes so great that slow-sand filters are generally preferable to mechanical filters.

In practice, the above theory is more or less roughly followed. The following methods have been adopted:

(*a*) The water for a newly washed filter is taken from the bottom of the sedimentation tank, and, when the head exceeds a certain value (details are not given), the supply is drawn from the top of the tank.

(*b*) An extra dosing of coagulant is given after washing. It will be obvious that this method cannot always be applied to the aluminium sulphate process, for, unless the water is sufficiently alkaline to decompose the extra amount in addition to that usually added, unchanged aluminium sulphate will pass through the filter. It is therefore most frequently adopted in the iron and lime process where the necessary alkalinity can always be obtained by the addition of lime.

(*c*) An artificial Schmutzdecke is formed by adding powdered clay to the water, either before, or simultaneously with the addition of the coagulant.

It is somewhat doubtful whether this last process has been designedly adopted for the purpose at present discussed. It is usually referred to in reports as being resorted to in cases where, for a few days at a time, the raw water was abnormally clear, and coagulation slow, or entirely arrested for want (apparently) of nuclei for the precipitate to form on.

(*d*) In many cases it has been found advantageous to add one-half the necessary amount of coagulant,—to allow the reaction to complete itself,—and later to add the remaining quantity; and after an interval (generally shorter than that permitted between the two dosings of coagulant) to pass the water to the filters. I am unable to state the exact rationale of this process, or the advantages derived, but it is quite plain that such a method lends itself admirably to the rapid formation of a Schmutzdecke, by the addition, just after washing, of the whole of the coagulant at the point where the second half usually enters, and thus securing (for say the first hour) that the whole of the precipitate reaches the filter. Later, the filtration head having attained a satisfactory value, the double addition process can be carried out.

The methods here suggested must be regarded as tentative, and it is to be hoped that systematic investigations may be undertaken, and that the results will be published.

In any case, no consideration need be given to any Filter Company's tender which does not explicitly deal with this question of irregular working, by guaranteeing that the daily results of bacterial counts will not exceed some maximum, more than a definite number of times per annum. A tenderer might reasonably request three to six months preliminary operation, before rigidly complying with his guarantee.

Bacterial Tests of Coagulation as applied to Mechanical Filters.—The doses of aluminium sulphate or iron sulphate which produce the best effect in reducing turbidity have been discussed on pages 558 and 568. The figures there given are those which are usually adopted previous to mechanical filtration. It is, however, very doubtful whether they are those which are best adapted to cause the maximum possible reduction in the number of bacteria during the whole process of coagulation and mechanical filtration. The only available information is furnished by the experiments of Schreiber at Berlin (*Mitt. Kong. Preuss. Pruefungsanstalt fuer Wasserversorgung*, 1906).

If the reduction in turbidity were alone employed to judge the efficiency of the process, Schreiber found that 10 grains of coagulant per cubic foot (23 grammes per cubic metre) should be employed, while the best bacterial results were obtained with 18 grains per cubic foot (43 grammes per cubic metre). Thus, with aluminium sulphate dosing at a rate of 23 grammes per cubic metre, one hour usually elapsed between the washing of a filter and the delivery of a bacterially satisfactory (50 bacteria per c.c.) effluent. With a dose of 43 grammes per cubic metre, the period never exceeded 30 minutes, and was usually far less.

As the coagulating tank held only one hour's supply, it is doubtful whether the advantage is inherent in the larger dose, or is only due to the fact that the larger dose causes the filtering film, or Schmutzdecke, to attain the requisite thickness more rapidly. If this supposition be true, the methods of working detailed on page 575 are probably advantageous bacterially, as well as in reducing turbidity. As a preliminary rule it may be inferred that the filtrate should be wasted until the increase in the working head over that initially required to force the water through the clean filter is twice that which is necessary to produce a non-turbid effluent. The water Schreiber used would hardly be considered very turbid in America, so that this rule is possibly over-stringent for markedly turbid raw waters.

Cleaning Filters.—The frequency of the cleanings of a mechanical filter is affected not only by the dose of coagulant, but (contrary to what is found to hold good in slow-sand filters) also by the rate of filtration. Wernicke (*ut supra*, 1907) finds the following results in the case of Posen water coagulated by aluminium sulphate :

Rate of Filtration in Feet per 24 Hours.	Cubic Feet of Water per Square Foot of Filter Area Filtered between Two Cleanings.	
	13·2 Grains of Sulphate per Cubic Foot of Water.	22 Grains of Sulphate per Cubic Foot of Water.
393	66	98
360	92	124
328	118	154
296	148	181
263	174	206
230	200	236
197	230	262

These experiments of Schreiber and Wernicke were carried out on far clearer and, probably, more dangerously polluted waters than are usually dealt with by mechanical filtration. They form, however, almost the only series of experiments on mechanical filtration in which bacteriological standards were systematically employed to indicate the efficiency of the filtration. The figures are probably applicable only to the local circumstances under which they were obtained, but they are more interesting to the British engineer than the far more extensive American experiments (reports of Water Boards of Cincinnati, Louisville, New Orleans, etc.). In these, bacterial tests were usually considered as of secondary importance in comparison with the removal of turbidity. The American method is obviously the easiest, and, when applied to American waters, it probably secures a bacterially satisfactory effluent. When mechanical filtration is applied to waters which are highly polluted and are not turbid, bacterial tests must be used to investigate the conditions of working, and in view of the great field for mechanical filtration in tropical countries, should soon become standard.

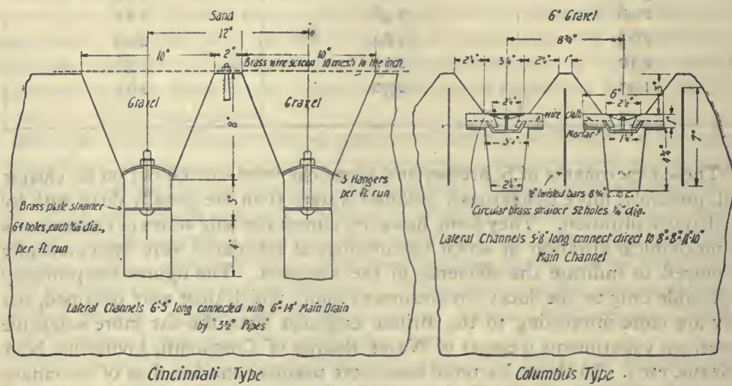
Practical Details.—The design of mechanical filters is mainly in the hands of patentees. The following matters require consideration by engineers when selecting filters:

The drainage system consists of pipes and strainers, and the difficulties caused, in slow-sand filters, by unequal resistances to flow, occur in an accentuated form.

As a rule, the total head lost in the filter when the sand is clean, is equivalent to about two-thirds of the depth of the sand (approximately 1·6 to 2 feet head), and the loss in the strainers (the area of which is about 0·4 square inches per square foot) should not exceed one-half this quantity (say 0·8 foot to 1 foot head at most, and usually 0·5 to 0·6 foot). So also, differences in frictional head in the pipe system (due to unequal lengths of piping) should not exceed about 0·2 foot. This condition generally leads to the area commanded by one system of pipes being from 12 feet square, to 18 × 15 feet, and the individual filter is made up of four such units, *i.e.*, varies from about 600, to 1100 square feet. The area is arrived at by a compromise between expenditure in the brass pipe work, caused by the large pipes, required to minimise the differences of frictional resistance, and a desire for as large a filter unit as possible, in order to economise in the valves and piping connecting the individual filters. When the conditions affecting the rejection of the effluent after washing are better understood, satisfactory working may be hoped for when the individual filters and the pressure differences in the pipes are larger than are now usual.

The design of the strainers is of the utmost importance. Any dead water spaces, such as existed in the earlier types, are objectionable, and should (if possible) be filled in with concrete. But the tendency of design is towards continuous lines of perforated brass plates, the upper surfaces of which are slightly sunk below the concrete bottom of the filters.

As an example, at Cincinnati 3-inch wide plates are spaced at 12 inches from



SKETCH No. 147.—Strainer Systems for Mechanical Filters.

centre to centre, and are perforated with 64 holes, $\frac{3}{32}$ of an inch in diameter, per foot run. (Sketch No. 147.)

At Columbus, the strainers are $2\frac{1}{4}$-inch circular brass plates, with $\frac{1}{16}$-inch holes, spaced $8\frac{3}{4}$ inches from centre to centre, in both directions, and hence the distribution of the water flow is obviously less perfect. (Sketch No. 147.)

The old-fashioned rose, slit cup, or perforated pipe (Sketch No. 148) strainers screwed into pipes, must be regarded as decidedly inferior. These were usually spaced about four to the square foot, although such figures as $2\frac{1}{2}$ per square foot occur, and as dead water spaces are then more easily filled up the wider spacing should be adopted if such strainers are used.

All strainers (even if not so designed) should be sunk at least 2, to 3 inches (better, if possible, 5, or 6 inches) below the general surface of the concrete, and the pits thus formed should be filled in with gravel, which should extend at least 3 inches (and preferably, if the strainer distribution is poor, 6 inches) above the top of the concrete.

The gravel is usually sized into two layers, the lower one $\frac{1}{4}$, to $\frac{1}{2}$ an inch average diameter, and the upper one 0·05, to 0·1 inch.

The sand is generally 0·30 to 0·40 mm. in effective size, and the uniformity coefficient should be low, preferably not more than 2 (after, say, three months it will be reduced to 1·5, and the loss should be provided for). In cases where the washing is not effected by raking, a brass wire screen is required to retain the gravel, 10 meshes per linear inch appearing usual.

The Cincinnati filters may be regarded as good practice, although the advance in design is so rapid that any specification is liable to be obsolete. The sand layer is 31 inches thick, and, on the average, the effective size is 0·33 mm., and the uniformity coefficient is 2·0, although the figures for the top layer are 0·29 mm., and 1·35.

The sand rests on 7 inches of gravel, which is kept in place by means of a brass screen with apertures 0·063 inch square. The gravel layers are as follows :

4 inches of $\frac{1}{10}$th inch to $\frac{1}{8}$ inch in diameter.
2 inches of $\frac{1}{8}$th inch to $\frac{1}{4}$ inch in diameter.
1 inch of $\frac{1}{4}$ inch to $\frac{1}{2}$ inch in diameter.
1 inch of $\frac{1}{2}$ inch to 1 inch in diameter.

It will be seen that no figures have been given concerning the loss of head in the filters.

In practice, the working head varies from 1, to 1·5 foot at the commencement, and becomes at least 2·5 feet before a satisfactory effluent is delivered. If bacterial results alone are considered, the head when the filtrate is turned into the mains should be at least twice the initial head. The filters are usually washed as soon as the head exceeds 12 feet, although cases where 20, or even 26 feet is employed are not infrequent.

It appears that the depth of water above the sand and the vertical height of the clear water pipes must always be so adjusted that the absolute pressure at the strainers is at least equal to the atmospheric pressure. No reason can be given, but if this condition is not observed the results are less satisfactory.

Broadly speaking, the rate of filtration is about 300 to 400 feet vertical (say 100 to 130 million U.S. gallons, or 80 to 100 million imperial gallons per acre per 24 hours). The results of bacterial tests generally indicate that the lower rate should be preferred. The matter is influenced by the effective size and uniformity coefficient of the sand. As an illustration, the case of Exeter (Mass.) may be mentioned, where the effective size of the sand being 0·24 mm., and the uniformity coefficient 1·52, the "usual rate" of "125 million U.S. gallons is reduced to 75 million gallons per acre per day."

It may be inferred that the effect of an abnormal effective size or uniformity coefficient of the sand is very similar in character to that produced in a slow-sand filter. In view of the relatively small quantity of sand required for all but the very largest installations of mechanical filters, it appears advisable to procure a sand of normal character. So far as our present knowledge goes, an effective size of 0·35 to 0·45 mm. and a uniformity coefficient of 1·7 to 2·2 specifies a good working sand, but each firm of filter makers has its own particular ideas, and the effect of a badly sized sand is so marked that the expense of procuring the size required by the makers appears to be justifiable.

WASHING OF SAND IN MECHANICAL FILTERS.—Washing is effected by reversing the flow of the water. Pressure water enters by the strainers, and passes through the gravel, agitating the sand.

In the earlier filters, the quantity of water forced through the sand was only sufficient to loosen it, and the dirt was therefore loosened and the sand stirred up by rakes hung from rotating arms. The water was mainly relied on to carry away the dirt and impurities collected on the sand, and the raking loosened the dirt, and secured an even distribution of water.

Of late, it has become usual to obtain a superior cleaning by forcing so much water through the sand that the whole bed is lifted bodily. The advantages over the old system are not very obvious, and appear mainly to consist in the suppression of the small, and therefore inefficient machinery, used to work the rakes. The rapid adoption of the practice in existing plants shows that it possesses appreciable advantages.

If water alone is introduced through the older types of strainers, it is found that passages are readily formed through the sand, and the washing will then be incomplete. This action is prevented, and an even and regular distribution of water is secured over the whole sand bed by the introduction of air under pressure in order to loosen the dirt, and afterwards removing the loosened dirt by water.

At Cincinnati, where washing by water alone is employed, the escape level is 31 inches above the top of the sand, when at rest; the effective size of the sand being 0·33 mm. Of the sand that was initially carried over, about 96 per cent. was found to be less than 0·45 mm. in diameter, so that any smaller difference in elevation would, in the long run, cause the effective size of the sand to be increased through loss of the smaller grains. It may be noted that the sand used had been selected and graded to such an extent that 7000 cubic yards of material were worked over in order to secure 4000 cubic yards of filter sand.

The best results are obtained at Cincinnati by applying the wash water in the following manner:

For the first minute, at a rate of 0·50, to 0·75 of a cubic foot per minute per square foot of filter area. This loosens the particles of dirt and mud, which are "stored up" in the whole depth of the sand.

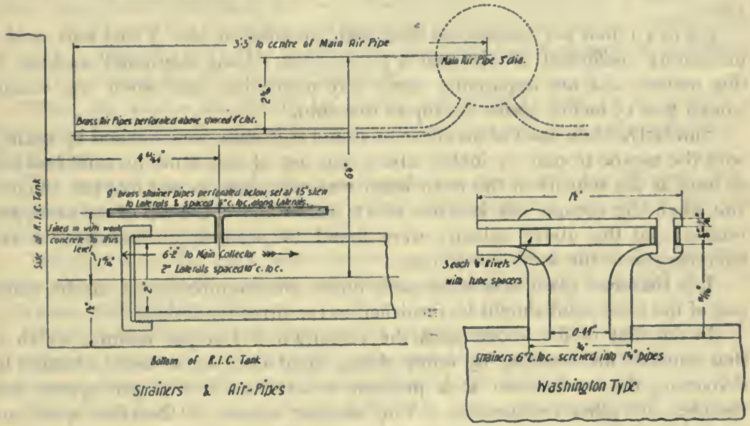
For two or three minutes, at a rate of 2, to 2·25 cubic feet per minute per square foot. Under this rate of washing it is found that a few grains of sand are carried up as high as 29 inches, but only a small quantity rises as high as 11·5 inches, and the main body of the sand only rises 10·25 inches. The exact details are interesting, although (since they are obtained with different filters) we cannot assume that the sands are identical in size and grading.

Cubic Feet per Square Foot.	A Few Grains Rise to a Height of	A Small Quantity Rises	The Main Body Rises
2·00	12·25 to 29 inches.	11·50 inches.	10·25 inches.
2·05	13·25 to 29 ,,	11·50 ,,	10·25 ,,
2·08	14·0 to 29 ,,	13·25 ,,	12·25 ,,
2·18	15·0 to 30 ,,	14·00 ,,	13·25 ,,
2·37	23·75 to 30 ,,	21·50 ,,	18·00 ,,

The results are somewhat irregular, but it seems fair to infer that a rate much in excess of 2·6 cubic feet per minute would cause the whole body of the sand to pass over. It is also as well to realise that each filter apparently possesses a definite rate of washing which produces the best results ; although, in practice, the valves controlling the wash water are arranged so as to prevent a greater delivery than 2 cubic feet per square foot per minute. The total pressure of the wash water is equivalent to a head of approximately 40 feet. This system necessitates a special design of strainers, otherwise the pressure at which the wash water is delivered becomes too high for economy (see p. 582).

The figures which relate to the systems in which compressed air or rakes are employed to loosen the dirt are somewhat discordant, as the wash water merely removes the dirt after loosening by air or rakes.

The major portion of the work is not effected by the water ; and, con-



SKETCH No. 148.—Strainer and Air Pipes for Mechanical Filters.

sequently, the strainers are not usually designed so as to produce an efficient distribution of the wash water. As already shown, when the strainers are properly designed, air injection and raking are not required. Thus, the figures now quoted are in reality greatly dependent upon the design of the strainers. Better results could probably be obtained were the strainers designed so as to secure a more equable distribution of the water over the whole area of the filter.

Compare the perforated pipe strainers in Sketch No. 148, with the Washington strainer or those in Sketch No. 147.

The following figures may be noted as affording partial answers to questions that should be considered when selecting one or other of the various patented filters and strainers.

The velocity of wash water which just raises the sand, and fits it for cleaning by rakes, is about 0·7 foot per minute reckoned on the gross filter area, for sand of the following effective size :

0·42 mm., and a uniformity coefficient of 1·8; and 0·5 foot per minute for:

0·22 mm., and a uniformity coefficient of 1·5: where no air is used. In the latter case, no appreciable loss of sand occurs when the escape is 10, to 12 inches above the surface of the sand, which is about 4 feet deep. A velocity of 0·625 foot causes some of the finer sand to be lost.

The loss of head in the strainers when the flow is reversed is somewhat high in actual filters, being about 12 feet for 1·1 foot per minute in the above cases, and therefore about 5 feet and 2·5 feet at the velocities used in washing. During filtration at a rate of 100 million U.S. gallons per acre per day (*i.e.* about 0·2 foot per minute velocity through the sand), the loss is approximately 0·80, to 0·85 foot. The strainers experimented on were the old cup type, and the losses in the case of plate strainers of the type used at Cincinnati, are apparently about 60 per cent. of the above values.

Similarly, the velocities employed in two other cases for washing by raking, were:

0·9 to 1·1 foot per minute, on sand with an effective size of 0·63 mm., and a uniformity coefficient of 1·1, and 2·3 feet deep. Long-continued washing in this manner did not apparently cause any noticeable loss when the escape trough was 15 inches above the top of the sand.

Similarly, when sand of an effective size of 0·36 mm. was washed by raking, with the escape trough 25 inches above the top of the sand, no sand escaped so long as the velocity of the wash water was restricted to one foot per minute. But when the escape was lowered to 15 inches above the top of the sand, 100 washings at the above velocity were found to produce an increase in the effective size of the sand to 0·40 mm.

It is therefore plain that when such filters are enquired for, a careful sizing test of the local sand should be furnished to the firms tendering.

In the first of the above cases the velocity of 2 feet per minute, which is that suitable for washing by water alone, could only have been attained by delivering the wash water at a pressure of at least 17 lbs. per square inch (besides all other resistances). The strainer system is therefore quite unsuited for this method of washing. Thus, when either water washing, or water plus compressed air washing is contemplated, the tenderers should be asked to specify the necessary pressure of the wash water, or of the air and the water.

Where the washing is by water alone, effective work is only possible when the strainers are of the perforated brass plate type, and are buried in pits with sloping sides as shown in Sketch No. 147. The head required to cause the sand to begin to lift is usually approximately equal to the depth of the sand. Russell (*Journ. of Ass. of Eng. Soc.*, vol. 42, p. 323) gives the following:

$$H = \frac{(100 - W)\ (\rho - 1)}{100} d \text{ feet}$$

where H, is the head of water required to lift d feet of sand, of a specific gravity ρ, containing W per cent. of voids. The formula is only valid when applied to sands ranging from 0·30 to 0·50 mm. in effective size. Russell also finds that when the sand has been lifted, the following figures occur for the relation between H, and v, the velocity of the wash water in feet per minute.

Thirty-one inches depth of sand of an effective size of 0·33 mm., and a uniformity coefficient of 2·0.

H = 1·58 feet.	v = 2·06 feet per minute.
H = 2·61 ,,	v = 2·54 ,, ,,
H = 2·20 ,,	v = 2·96 ,, ,,

With 30 inches of sand of an effective size of 0·37 to 0·40 mm., and a uniformity coefficient of 1·4 to 1·5 :

H = 2·05 feet.	v = 2·08 feet per minute.
H = 2·54 ,,	v = 2·52 ,, ,,
H = 2·33 ,,	v = 2·89 ,, ,,

The last figure in each case shows the relation when the sand is thoroughly loosened. Since the head lost in the strainers in reversed flow is from 0·5 to 0·6 foot, when v = 2·00 feet per minute, it is plain that the total head given by Russell's equation will suffice :

(*a*) To start motion in the sand when no water flows through the system.

(*b*) To pass the water through the lifted sand and the strainer system, at a rate of 2 to 2·5 feet per minute, according to the amount of loosening which the sand has experienced.

Thus, the supervisor should be able to adjust the valves so as to reduce the pressure during the stage (*b*) slightly below that which is required to start the motion.

The use of compressed air is evidently an additional complication, and appears to be inadvisable, unless some corresponding advantage in the economy of wash water is guaranteed.

The velocity of the wash water (when compressed air is employed) appears to be about 1·5 foot per minute, with sand of an effective size of 0·28 mm. to 0·36 mm., and 1·8 foot deep.

The area of the exit holes from the air pipes must be carefully proportioned, in order to secure an equable distribution, and about 0·02 to 0·03 square inch per square foot of the filter area is usual (Sketch No. 148).

The pressure of the air is usually from 3, to 5 lbs. per square inch, and can apparently be calculated from the static head of the water over the orifices (*i.e.* up to the escape level), together with an allowance of 0·25 lb. per square inch, which apparently represents the friction in the sand.

The figures are clearly not in agreement, although the observations are accurate. It may be inferred that the design of the strainers is capable of improvement in this connection, and that the older types are unsuited for washing otherwise than by raking.

In every case the size of the escape troughs requires careful consideration, since the flow in these must be sufficiently rapid to remove the dirt, while the whole volume of clean water in the troughs is wasted as far as cleaning or filtration is concerned.

The volume of wash water used is by no means immaterial. Local circumstances have a great effect; but, on the average, it is about 5 per cent. of the total water filtered, falling to 2, or 3 per cent. in very favourable circumstances. Values as high as 10 per cent. are reported, but with experience it is believed that 7 per cent. should not be exceeded.

During periods of bad water, however, these values may occasionally be doubled.

Let us assume a bad day, with 7 per cent. of wash water used. The rate of washing with rakes is about three times that of filtration, so that unless we have a storage tank, the pump supplying pressure water must be capable of delivering at a rate of one-fifth of the volume dealt with by one filter. In the case of washing by suspension, the figure is 60 per cent.

Thus, for a large filter delivering about three million U.S. gallons per 24 hours, a pressure water supply at the rate of two millions U.S. gallons, or $1\frac{2}{3}$ million imperial gallons, per 24 hours, is required for washing alone, and the gross pressure may be considered as approximately 20 lbs. per square inch. For rake washing, the figures are about one-third of those given above, and a pressure of 12, to 15 lbs. suffices. This means 17 horse-power in one case, and 5 horse-power in the other, for washing alone; and, in a plant containing few filters, an elevated storage tank may prove economical.

The details as to the quantity of water that should be allowed to run to waste after each washing, before a satisfactory effluent is obtained, are not very well known. The question was not considered in the earlier types, a non-turbid effluent being held to be satisfactory. Later bacterial tests on such filters gave such values as 3, or 5 per cent. with a test by no means as exacting as Koch's; but a mere comparison of these designs, and modern types, shows that these figures would now be overestimates.

I am inclined to believe that a skilfully managed filter (every care being taken to establish a Schmutzdecke as rapidly as possible) should pass Koch's tests with not more than 2 per cent. of waste.

It was also customary to disinfect filters at intervals of six months, by soda lye, or steaming. This appears to be unnecessary in modern types, where dead water space does not exist.

Deferrisation, or Enteisenung.—Many ground waters contain iron in various forms. Such waters, when exposed to air, usually deposit this iron as a precipitate, giving rise to turbidity in the originally clear water.

In other cases, the iron causes trouble by depositing as iron mould on clothes, or interfering with cooking processes, or by encouraging the growth of slime in the water mains (see p. 437).

Similar remarks apply to waters containing manganese.

Processes for the amelioration of such conditions are nearly all of German origin, and the literature is almost exclusively so. The best information in English is found in a paper by Weston (*Trans. Am. Soc. of C.E.*, vol. 64, p. 112).

We may divide ground waters containing iron into three classes:

(1) Those which begin to precipitate the iron as soon as aerated, and which generally contain iron in the form of ferrous hydrate.
(2) Those which will hold the iron in solution indefinitely, even when aerated. In these cases the iron is usually combined with some vegetable acid, and appears to be in a colloidal form.
(3) Waters which contain iron in both the above forms, and therefore deposit part, but not all, of the iron content after aeration.

The amount of iron that will cause trouble cannot be definitely stated.

In waters of the first class, 0·3 parts per million usually appear to be safe, and over 0·5 give trouble, although Weston states that in certain cases 0·1 part per million causes difficulty.

In water of the second class, quantities up to 0·9 parts per million do not generally require special treatment. It must, however, be remembered that in the process of time the water yielded by a well which is steadily pumped from, frequently changes its properties; and experience indicates that, while hardness is likely to diminish, the liability to iron troubles increases.

In waters of the first class, it may be said that the only trouble is to ensure that the iron is precipitated and removed before it enters the supply mains. The weight of oxygen required to deposit the iron is only one-seventh that of the iron, and this is generally so small a quantity that special precautions would be required in order to prevent the water from becoming sufficiently aerated by pumping only. Difficulties arise from the fact that the precipitate of ferric hydrate in some cases remains in a colloidal form, and the water cannot be filtered in a reasonable period. The time required after aeration, before the water is fit for filtration, in the sense that a coarse sand or fine gravel filter will remove all the iron, depends on the chemical constitution of the water. Usually, the presence of sulphates is favourable, and little trouble is likely to occur when they are present. Chlorides and nitrates are unfavourable, but they also indicate a polluted water. The presence of carbonic acid is also unfavourable; but this effect may be greatly minimised by passing the water during, or after aeration, through fine sand, or gravel.

We may generally treat ferruginous waters of the first class by aeration,—either that obtained by leaky glands in pumps,—or, where carbonic acid or other detrimental substances are present, by trickling the water over a rough filter of gravel, sand, coke, or broken bricks, followed by a rough filtration in order to remove the precipitated iron.

It will be evident that where the precipitation is slow, storage (after aeration) improves the efficiency of the process; but I am not aware of any cases where special arrangements for storage have been found useful. In Germany, nearly all the waters contain sulphates, and precipitate readily. In America, those waters which precipitate more slowly are usually treated by chemical methods, as will be later explained.

According to Weston, in the case of 21 German plants, 12 have brick or coke aerators, with sand filters. In 5 others, the aerators are wooden slats, which may be considered as easily cleaned substitutes for a coke or brick aerator (subject to the disadvantage of possibly fostering bacterial growths).

In the four other cases, various special aerators are used, composed of wood shavings impregnated with tin oxide.

The aerators appear to be worked so as to pass water at a velocity varying from 15, to 48 feet per hour, and there are indications that a speed much above 48 feet per hour would require the installation of a "sedimentation" tank between the aerator and filters. So far as can be ascertained, the rate at which the water passes through the aerator bears no relation to the content of iron in the water, and is probably far more influenced by the other salts present.

With 4 parts per million of iron in the water, coke aerators apparently require cleaning after passing about 200,000 cubic feet per square foot.

The final filters are of gravel, the mean diameter of the finest layer being about 0·20 inch, and work at a rate varying from 12 feet to 70 feet per day, the mean being about 50 feet daily.

It will be evident that these filters are in the nature of dégroisseurs, rather than sand filters, and that the iron deposits not only on the top, but also in the interstices. A thickness of about 65 inches of material, graded from 0·20 inch to 2 inches mean size, is usual.

If the constitution of the water is such that the iron remains in a colloidal state after aeration, we have (to all intents and purposes) a water of the second class. Such waters, as also those of the third class, should, as far as our present knowledge goes, be treated in the following manner after aeration.

The methods are all founded on the principle of encouraging the transformation of the colloids into a precipitate, by forming a second precipitate in the water. Thus, where the factor preventing precipitation is an excess of carbonic acid, a precipitate of calcium carbonate is produced by the addition of lime.

Where the disturbing factor is of vegetable origin, the presence of iron is usually combined with a coloured water. The Anderson process (see p. 547) has been successfully applied, but the most practical method appears to be the addition of sulphate of alumina and clay; the clay with the alum hydrate forming a rapidly settling precipitate, and removing the colour and iron.

In such cases, special provision for aeration is probably unnecessary, and there is a certain amount of evidence to show that aeration previous to coagulation is in some cases positively harmful as regards reduction of colour.

The water at Reading (Mass.) may be taken as an example. It contains iron, and is coloured by vegetable humus, the quantity of iron and the coloration being variable between the limits 0·3 and 10·3 parts per million of iron and at least from 40 to 70 on the colour scale adopted.

The treatment consists of the addition of about 7·5 grains of alumina sulphate and 4 grains of powdered clay per cubic foot of water. So far as can be judged, neither the iron content nor the colour in any way influence the quantities of sulphate and clay required. The action is probably entirely mechanical. This statement is also confirmed by the experience in certain other cases recorded by Weston, where the formation of a precipitate of calcium carbonate with the primary object of softening the water (see p. 590) is found to cause the iron and colouring matter to assume a filtrable form.

The treatment after the iron has been rendered filtrable requires special experiments. As an example, coagulation with aluminium sulphate, as above, followed by 1½ hours' sedimentation, followed by aeration in a trickling filter, has proved satisfactory, and has also removed odours which previously existed.

The waters now considered are generally fairly free from bacteria. Consequently, the filters employed are usually of the mechanical type, with large-grained sand, but where clay is added to destroy the colloids by precipitation sand of an effective size less than 0·40 mm. should be used. Where slow-sand filters are adopted, 7·5 to 10 million gallons per acre per day is found to give satisfaction, the filtration being a rough straining rather than typical filtration.

A peculiar example exists at Posen, which may serve as a useful hint. The town is supplied with water from two sources, from one of which an initially clear water is obtained which deposits iron on standing. The other water is dark coloured, contains humic acid, and, judging by the description, is heavily stained with peat. These waters are very difficult to treat separately, but when mixed are easily filtered. It appears that one part of iron reacts with the colouring substance in any ratio between 2·2 and 7 parts, so that any reasonable mixture of similar waters may be expected to react in a favourable manner.

COLOURED WATERS.—The occurrence of colour in water is usually produced by prolonged contact with decaying vegetation. The chemistry is therefore somewhat complicated. When accompanied by iron salts, the removal of the iron should be the first thing attempted; although, even in such cases, the possibility that the addition of more iron (either as ferrous sulphate, or by Anderson's process) may upset the chemical balance, should not be forgotten.

In Great Britain, the colour produced by peat is well known, although of late years, owing to the custom of cutting deep catch-water drains through all the peat-bogs existing on the catchment area, it has become rarer. But, even at the worst, such discoloration usually causes but little trouble, since the consumers do not generally regard it as a matter for complaint.

Slight peat discoloration is easily removed by sand filtration,—or by aeration, followed by a rough filtration. When organic chemical standards of purity are seriously regarded, peat may give rise to indications which may possibly suggest pollution.

In hot climates, where the life and decay of vegetation are more intense than in Temperate countries, the colour problem becomes more urgent; and (unlike peat staining) waters so affected are usually injurious to health. Details of successful processes vary in nearly every case, but the general principles are mainly the same. It should be remembered that while colour is the obvious source of offence, the real success of the treatment depends upon the effect which the treated water has upon the health of the consumers.

In the first place, the source of the colour is usually evident, and where this can be removed at small cost, it should be done. Such steps as denuding the banks of the reservoir of turf and vegetation (for say 20 feet below high-water level), and removing all dead trees, are obvious. The expense, however, may be too great, and such excessive caution as was displayed in the case of some of the New England reservoirs by stripping the whole of the reservoir site of all vegetable earth appears to be a waste of money, because a proper process of purification, including its running expenses, would have proved less costly.

The methods of purification may be stated as, aeration followed by treatment with metallic iron, or other coagulant, followed by filtration. It is only rarely that the coloration is so obstinate as to require all three processes.

I shall therefore describe the most complex method that I am acquainted with, and would remark that one or other of these three alone usually suffices for the removal of all coloration.

Chadwick and Blount (*P.I.C.E.*, vol. 156, p. 18) dealt with a Mauritius reservoir water of the following composition,

Total solids	7·60	parts per	100,000
Chlorine	1·81	,,	,,
Free ammonia	0·002	,,	,,
Albuminoid ammonia	0·064	,,	,,
Oxygen absorbed	0·42	,,	,,
Nitrogen as nitrates	...		
Nitrogen as nitrites	...		
Hardness	2·5 degrees		

and proceed as follows;

The reservoir was drawn down as far as possible, and all the trees and vegetation were removed from the low-water line to 10 feet above high water. The roots and stumps of all submerged vegetation were removed by a grab dredger.

The compensation sluice was also lowered, so that the compensation water was drawn from the deepest portion of the reservoir, instead of being taken from the top layers as previously; and a regular system was adopted of discharging the excess water, as far as possible, through this lowered sluice, in place of over the escape weir.

The water intended for town supply is first treated with iron in an Anderson revolving purifier, where it takes up about 20 lbs. of iron per million gallons. It is then aerated by being passed into a series of trays, 2 feet × 1 foot × 1 foot, with bottoms of delta metal plates, ⅜ths of an inch thick, pierced with 4584 holes per tray, each hole being 0·04 inch in diameter, and discharging 0·87 gallon per hour, under 8 inches head. Thus, one tray deals with 3988 gallons per hour and fifteen with 1,000,000 imperial gallons per 24 hours (with allowance for cleaning). The water falls 6 feet, and is thus aerated to saturation, and is then filtered by slow-sand filters of the usual type.

This question of aeration by small orifices is interesting. The authors state that unless the head on the orifice exceeds a certain value, which they term the "critical head," the small threads of water tend to coalesce, and good aeration is not secured. Above this value, jets that have actually coalesced, automatically separate when allowed to do so.

They further give the following table:

Diameter of Hole in Inches.	Critical Head in Feet.	Delivery per Hour under Critical Head.	
		Cubic Feet.	Imperial Gallons.
0·040	0·67	0·14	0·87
0·036	0·92	0·13	0·83
0·032	1·00	0·11	0·67
0·028	1·33	0·083	0·52
0·024	1·83	0·083	0·52

A similar process has been applied at Nairobi (Uganda), except that here, after aeration, aluminium sulphate (about 6·25 grains per cubic foot), and approximately 3 grains of lime per cubic foot are added, and the coagulation thus obtained permits mechanical filtration to be employed.

These processes are obviously somewhat complicated, and, while quantities of 1,000,000 gallons per 24 hours are successfully handled, the method employed by Tomlinson at Singapore (*P.I.C.E.*, vol. 156, p. 42) seems more practical when really large volumes are dealt with. At Singapore the water is somewhat unsystematically aerated by turning the filter supply pipes upwards, and the filters, which work at a rate of about 3·2 feet per 24 hours of actual work (say 900,000 imperial gallons, or 1,080,000 U.S. gallons per acre per 24 hours), are cleaned every 4, to 14 days, according to the season, and are aerated for 12 hours after each cleaning. The raw water is very bad, and the rate of filtration (in view of the frequent cleanings and the time lost in aeration) is slow. A far greater speed could be attained by really systematic aeration of the water by means of fountains, or sprays; and the time required for aeration of the filters could be reduced by embedding a layer (say 6 inches) of porous carbon, or charcoal, in the filter sand, as is done in the case of at least one English town, where the raw water is turbid and discoloured by peat, after rain.

Colour in water occasionally assumes a colloidal form. The question has been discussed under the removal of iron (see p. 586); and, in addition to the methods already described, the coagulation processes there discussed may be employed. The most effective process when applied only to remove colour, appears to be an addition of powdered clay followed by coagulation with ferrous sulphate. The after filtration cannot usually be satisfactorily effected at such high rates as are used when precipitated colloidal iron is strained out. The question depends entirely upon the bacterial content of the water, and must be settled by a bacterial examination.

The removal of colour either by ferrous sulphate, or by aluminium sulphate, may occasionally be found to proceed badly, even although the colour itself rapidly combines with the hydrates of iron or aluminium (*i.e.* the coagulating precipitate). The causes are obscure, but good practical results have been obtained by producing the hydrates in a concentrated form by precipitating a solution of the coagulant with lime or sodium carbonate in a special vessel, and then adding the precipitate of hydrates to the coloured water. Such cases require careful investigation. Sometimes the precipitate and the mother liquor can be thrown into the coagulating basin as soon as formed, but occasionally I have found that this method is useless, and that success was only attained when the coagulating precipitate was freed from the mother liquor by decantation before adding it to the coloured water.

Odours and Tastes in Waters.—These are usually caused by the decay of plants or animals inhabiting the water. They are therefore most frequent in hot climates, and in more Temperate countries usually occur in the summer and autumn.

Treatment generally consists in the destruction of the organisms, the decay of which gives rise to the odours and tastes.

A very common method is the application of copper sulphate. This salt (in a dilute solution of one part per million or less) usually effects the destruction of algæ. It is best applied by placing the requisite quantity in a bag, which is moved about the reservoir so that the salt dissolves slowly and uniformly. It will be found that two consecutive applications of copper sulphate at, say, a week's interval are more effective than one of the same total quantity. The process requires care, since the presence of an abundant growth of algæ or other organisms generally produces growths of living plants in the mains, which live on the products of the original organisms. If these are killed by copper sulphate, or other treatment, the growths in the pipes will die, and may temporarily give rise to worse conditions than those originally prevailing.

As processes less specially adapted to the removal of taste and odours, all those described under Colour in Water, are effective. Also, in the case of slow-sand filters, merely diminishing the usual rate of filtration is in most instances beneficial. So far as I am aware, all difficulties on record occurring when sand filters are used, have been successfully dealt with by aeration and double filtration. This process is no doubt simple, but obviously must be frequently quite impracticable, for want of filters. From personal experience, I have found that aeration and a dosing with alumina sulphate and powdered clay (the water was too clear to give a good fall of coagulated matter without this addition) was very effective. Towards the end of the season of bad water, aeration was not required. I considered the water extremely offensive, but in these matters there is no fixed standard to appeal to.

The most complicated process I am aware of is that in use at Charleston (S.C.), where a very shallow reservoir water receives the following treatment:

(1) Copper sulphate in the reservoir.
(2) Aeration.
(3) Coagulation and two days' sedimentation.
(4) Aeration.
(5) Mechanical filtration.
(6) Aeration.

The process is complicated, and I believe that the following defects exist:

(1) No clay, or other finely divided matter is added to "help the coagulation down."
(2) Mechanical filtration is certainly less suited to deal with tastes, and odours, than slow-sand filtration.

It should be realised that these matters are essentially biological in character, and that the correct manner of dealing with them, after their first occurrence, is to study the life-history of the plants and animals. The remedies should also be applied not at the moment when the odours or tastes manifest themselves, but rather when the first generation of the organisms appears. Considerable assistance is afforded by encouraging frogs and fish in reservoirs, and where the water is filtered before consumption, the presence of such animals is quite unobjectionable.

Cases arising from the presence of dissolved gases (*e.g.* sulphuretted hydrogen) should be the subject of special investigation. As a general principle, however, these gases usually occur in ground waters, and it is only very rarely that any undesirable factor in a ground water is not greatly improved by aeration.

Softening Processes.—In principle, these processes are founded on the method introduced by Dr. Clark in 1841.

The hardness of water is due to two causes, and may be divided into two classes, temporary, and permanent. Temporary hardness consists of bicarbonates of lime and magnesia, which may be considered as simple carbonates of these elements held in solution by carbonic acid gas dissolved in water. When this carbonic acid gas is expelled (as by boiling), the carbonates become insoluble, and are precipitated.

Clark's process consists in adding to the water a sufficient amount of lime to combine with the dissolved carbonic acid, and thus produce a precipitate not only of the carbonates already existing in the water, but also of those formed by the combination of the added lime and the carbonic acid in the water.

The reaction is expressed by:

$$Ca(HCO_3)_2+Ca(HO)_2=2CaCO_3+2H_2O$$

The water presumably being already saturated with all the calcium carbonate it can dissolve, when no carbonic acid is present, the whole of the temporary hardness is removed.

This equation indicates that for every 100 parts of calcium carbonate existing as bicarbonate, or for every 44 parts of carbonic acid, 56 parts of freshly burnt unslaked lime are required, and 200 parts of calcium carbonate are precipitated.

If, however, magnesium also exists in the water, in the form of magnesium bicarbonate, the results are more complicated. In the first place, the equation similar to that given above, shows that:

Eighty-four parts of magnesium carbonate, or 44 parts of carbonic acid as a bicarbonate, consume 56 parts of lime, and 184 parts of the mixed carbonates are deposited. But magnesium carbonate is fairly soluble, so that it is usually necessary to double the quantity of lime, producing the reaction:

$$MgCO_3 + Ca(OH)_2 = Mg(OH)_2 . CaCO_3$$

so that the final result is:

Eighty-four parts of magnesium carbonate combine with 112 parts of lime, producing 258 parts of a mixed precipitate.

Thus, a priori, any calculation from the quantity of the carbonic acid, or the total weight of the combined carbonates, is impossible. In a natural water, however, although the alkalinity may vary from day to day, it is unlikely that the ratio of the lime and magnesia salts will greatly alter. Thus, if we are aware that, as a rule, one part of alkalinity per 100,000 requires—let us say—three grains of lime per cubic foot, for proper softening, it is unlikely that the ratio will alter greatly over the whole year, and one or two estimates of the weight required (covering as far as possbile the whole variation in alkalinity) will usually suffice to enable a calculation of the yearly weight of lime to be made.

Permanent hardness principally consists of lime and magnesia salts in combinations other than those above discussed. The effect which an addition of softening chemical will have can only be stated in general terms. The usual reactions are:

$$\text{(i)} \quad CaSO_4 + Na_2CO_3 = Na_2SO_4 + CaCO_3$$

That is to say, soluble sulphate of lime is thrown down in the form of chalk, by the addition of carbonate of soda, and the less objectionable sulphate of soda remains in solution.

$$\text{(ii)} \quad CaCl_2 + NaCO_3 = 2NaCl + CaCO_3$$

This is the same principle, chalk being deposited, and common salt remaining in solution.

Similar reactions occur with magnesium salts. These processes (and many others) are often used for the preparation of waters intended for use in steam boilers. Their application on a large scale to the purification of water intended for human consumption is unusual, but is growing more common. I cannot but consider that their systematic use must generally (excluding very arid regions) be considered as indicating a fundamental error in the selection of the source from which the water is drawn.

The bacterial results of a water softening process are merely accidental, and resemble those of a long sedimentation (see p. 551).

If any suspended matter is present in the water, the precipitate will generally form round the suspended particles, and these will be rapidly carried down.

On the other hand, the precipitates obtained consist mainly of calcium carbonate, which is a granular, pulverulent substance, has but poor coagulating properties, and does not form an efficient Schmutzdecke on a filter. However, practical experience has shown that water softening processes do not materially interfere with coagulation, and the two processes may, where necessary, be carried out simultaneously.

Such combined processes have been largely introduced in America of late years. In England, the Porter-Clark process is frequently employed for softening waters drawn from chalk wells. Since such wells generally yield clear waters of great bacterial purity, coagulation, or sand filtration, is rarely necessary.

Practical Details.—These have been discussed in connection with the ferrous sulphate process. The main difficulties (other than the mechanical ones connected with the addition of milk of lime) lie in the time required for the reactions given above to be fully completed.

Delay is chiefly caused by the presence of magnesia, since the reaction expressed by the last equation is not only slow in itself, but appears to exercise a prejudicial effect on the previous reactions.

The determination of the size of the precipitation basin is consequently a somewhat complicated matter. If the basin is too small, incrustations in the filter drains and pipes will occur, or the full benefit of the reactions cannot be attained, and the effluent will contain a certain amount of hardness that could be removed.

The problem is really therefore intimately connected with local opinion as to the amount of hardness that is permissible. Waters exist in which at least 12, to 15 hours would be required in order to remove the whole amount of hardness that it is possible to deposit. A settling basin of this size solely for the purpose of ameliorating the water is usually more expensive than the benefits gained justify (see p. 564). If the water also contains a large amount of permanent hardness, and either the entire removal of the temporary hardness, or the partial removal both of temporary and permanent hardness, is necessary to produce a satisfactory water, the cost of a large settling basin may be justified in view of the fact that the permanent hardness can only be removed by means of the relatively costly carbonate of soda.

The largest basin in practical use appears to be at Winnipeg, and holds eight hours' supply.

The chalk waters of southern England contain comparatively little permanent hardness (on the average 2 or 3 degrees only), and a very large proportion (28 degrees is reduced to 3½ degrees, and 18 degrees to 6 degrees) of the temporary hardness is removed by one, or at the most three hours' settling. But these must be regarded as favourable cases, since the magnesia content is small.

We may sum up the facts by stating that six hours may be considered as normal (although probably somewhat in excess of present-day practice) for waters containing about 20 per cent. of their temporary hardness in the form of magnesia, and accompanied by an amount of permanent hardness such that a reduction of the temporary portion to 5 degrees gives a satisfactory water.

The size of the reaction basin may be increased in the case of waters containing more magnesia temporary hardness, or permanent hardness, and may be diminished in favourable examples of less magnesia and permanent hardness.

Careful laboratory experiments should be made in any particular instance, and should be checked by a fairly large scale test—say 1500 gallons—before the final designs are drawn up.

A certain decrease in sedimentation capacity can be obtained by re-carbonating the treated water by the injection of carbonic acid, usually produced by the combustion of coke. After-deposits in the filters or mains are thus prevented, and the water is rendered more pleasant as a beverage. So far as my experience goes, the advantages over the original Clark process are most marked in the case of waters that contain but little magnesia.

The precipitate, in so far as it consists of calcium carbonate, is very readily removed, and the cloth screens introduced by Atkins give great satisfaction in the treatment of chalk water. These screens are washed by reversing the flow, and it will therefore be plain that they are less well adapted to deal with precipitates containing a large proportion of somewhat glutinous magnesia hydrate. Difficulties, however, are unlikely, provided that the above rules are followed in the determination of the sedimentation capacity.

In cases where magnesia hardness is of importance, the completion of the reaction can be somewhat hastened by stirring up (usually by compressed air) the old precipitate, and mixing it with the newly dosed water. The falling precipitate in some way encourages the completion of the reaction. The sedimentation capacity can then be reduced to about one-half of that stated above. This is not the only advantage. If a magnesia water is treated by lime a small quantity of magnesia hydrate (?) always remains in solution, and is only deposited on heating. This hydrate (?) is a gummy substance, and may rapidly clog the feed valves of a water heating apparatus. The precipitate stirring process removes this hydrate, and is therefore an almost indispensable addition to softening processes when applied to waters containing magnesia which are largely used for boiler feeding or other purposes entailing heating.

Regulating Apparatus Employed in Filtration.—The necessity for some method of regulating the quantity of water filtered is obvious. The head required to force the water through a filter, at a given rate, varies according to the construction of the filter; and also more markedly, from day to day, according to the condition of the Schmutzdecke. This factor alone is responsible for variations ranging from a few inches, up to 5, or 6 feet; or, in mechanical filters, from 2 feet, up to 15 or 20 feet.

The original regulating apparatus was a valve in the discharge main from the under-drains, which was adjusted by hand to pass water at the required rate, and gradually opened as the head necessary to pass the desired quantity increased.

This was a rough method, and entirely depended upon the care and judgment of the operator. Later, a weir was added in order to measure the quantity delivered accurately, and with careful operation perfectly satisfactory results can be obtained.

The more usual method, however, consists of a telescopic tube raised or lowered by a screw, and provided with a graduated scale to show the quantity of water taken in over the circular weir thus formed. Adjustment by means of the screw permits a given quantity to be delivered daily, and is superior to the method of valve and weir, but the necessity for constant supervision is equally great.

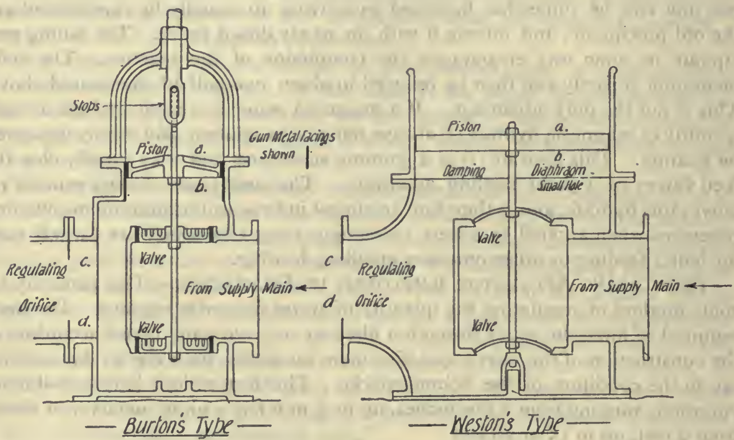
The discharge can be automatically regulated if the telescopic tube be fixed to a float so that the top of the tube remains at a constant depth below the water surface. The Philadelphia modification is probably the most reliable form, since the discharge is through a series of deep slots, instead of over a long, circular weir. As is shown in the general discussion on Weirs, an error in the position of the sliding tube (*e.g.* due to sudden change in the water level, accompanied by a sticking of the tube in its guides) produces far less variation in discharge under these circumstances.

Burton designed the regulator shown in Sketch No. 149, which is in effect a balanced valve, governed by a piston, and the difference of the pressure on the sides *a*, and *b*, of this piston is plainly the head required to force the quantity of water delivered through the orifice *cd*. We thus see that:

If *w*, is the weight of the piston, valves, and their connecting stem, when weighed in water, and *a*, is the area of the piston in square inches:

The pressure required to lift the valve is:

$$p=\frac{w}{a} \text{ lbs. per square inch} = 2\cdot3\,\frac{w}{a} \text{ feet head of water.}$$



SKETCH No. 149.—Automatic Diaphragm Regulating Valves.

Now, if *q*, is the quantity of water passing in cubic feet per second, and A, is the area in square feet of the hole in the diaphragm *cd*, we have $q=cA\sqrt{2gp}$, where *c*, is the coefficient of discharge for the circular orifice; and *p*, is expressed in feet head of water. Hence we get:

$$q=cA\sqrt{2g\,\frac{w}{a}\,2\cdot3} = 7\cdot91\,A\sqrt{\frac{w}{a}} \text{ cusecs}$$

since *c*, can be taken as 0·65, in view of the suppression of contraction.

The details of the non-coned valve and valve seats given by Burton (*Water Supply of Towns*, Fig. 114) should be carefully adhered to. Where (as in Weston's design applied to the regulation of mechanical filters) coned valves and valve seats are desirable for practical reasons, oscillations and hammering of the valve will occur. Weston largely, but not entirely, prevents this effect by the insertion of a stilling device in the form of a diaphragm with a small hole in it, between the valve and the balancing piston. (Sketch No. 149.)

Fuertes (*Water Filtration Works*, p. 171) gives a sketch of a regulator consisting essentially of a weir closed by a slide, which is governed by a float actuated by the water level in the clear water well. This seems less liable to give trouble through sticking, than sliding tubes, although I cannot say that my own experience with sliding tubes has been unfortunate in this respect, and I have rarely heard them complained of.

On examining the working drawings of actual examples of such sliding tube regulators, it will appear that a good deal of ingenuity has been devoted to securing water-tightness at the sliding joint, where the telescopic tube enters its holder. This seems somewhat unnecessary, since leakage at this point (unless large) matters but little, for what we are concerned with is not the absolute magnitude, but rather the amount of variation in the quantity of water passing, as the water surface in the clear water well rises and falls with the variation in working head.

As an example, let us assume a variation in working head of 5 feet (which is greater than that usually permitted), and that when the filter starts working, the effective head producing leakage through the joint is 1 foot.

Assuming a joint 4 feet long, and $\frac{1}{64}$th of an inch wide, we have an area of 0·75 square inch and the leakage does not exceed 0·025 to 0·03, or as a mean 0·027 cusec, or about 15,000 gallons per 24 hours, and under a 6-foot head the leakage in the same period will be about 37,000 gallons.

Such a regulator may be assumed as intended to pass at least 2,000,000 gallons per 24 hours, so that even if no adjustment of the regulating screw is made during working, it would actually pass about 2,015,000 gallons when first started, and about 2,037,000 gallons when working at maximum head. The variation, therefore, is at the most a little over 1 per cent. Thus, it would appear that a plain metal joint, such as can be constructed by any workman provided with a lathe, will suffice for quite as accurate regulation as is necessary.

If, however, for any reason such variation is not permissible, it will be quite evident that any ordinary hat leather packing will give all the accuracy we can possibly require. Personally speaking, in view of the fact that if the tube sticks, the Schmutzdecke may be ruptured, and unfiltered water may be delivered into the mains, I consider that a tight joint or complicated packing, should be avoided.

A pitfall exists in the design of the upper portion of a sliding tube. Let us consider the case already sketched out. A 4-foot weir without end contractions, with its sill 0·50 foot below the water surface, will discharge about $3{\cdot}33 \times 4 \times 0{\cdot}36 = 4{\cdot}8$ cusecs, or roughly 2,600,000 gallons daily.

The area of a circular pipe 4 feet in circumference, is about 1·29 square foot, or the mean velocity of the water when it enters the pipe should be about 3·7 feet per second. The mean velocity over the weir, even if no shock occurs, is actually somewhat less than 2·4 feet per second, so that the upper end of the pipe will barely carry the weir discharge. Thus, for safety, a bellmouth, of the form indicated, is necessary. The extra length of weir thus obtained reduces the head required to discharge 2,600,000 gallons per day, and in this particular case we may enter the region of low heads, where Francis' formula ceases to be accurate (see p. 106).

In practice, it is simpler to design the weir so as to pass the required quantity under a fairly low head, say 0·40 foot, and to adjust it accurately by observing the rise in the clear water reservoir as soon as operations begin.

INFLUENCE OF CLIMATE ON PROCESSES FOR WATER PURIFICATION.—The effect of hot or cold weather on various processes has been referred to on several occasions (pp. 520, 532, 554, and 587).

Generally speaking, the hotter the weather the more effective all chemical processes will be. It is often difficult to effect a satisfactory coagulation in very cold water (*e.g.* water drawn from rivers covered with thick ice).

The difficulties are not entirely due to the temperature, as the worst cases occur when fairly clear and very cold water has to be coagulated. Success can usually be obtained by adding sufficient powdered clay to form nuclei on which the coagulating precipitate can begin to form.

The processes which are distinctly more biological than chemical in character, such as sand filtration and natural sedimentation or storage, are most effective at temperatures which range from 55 to 75 degrees Fahr. In colder waters the processes are not very markedly less effective, but a sand filter, or other biological machine, requires a far longer period to get into proper working order. In hotter climates the processes are (as a rule) less efficient, and while a sand filter gets into the best possible working order very rapidly, it is less efficient than at a lower temperature, and becomes useless (*i.e.* requires cleaning) far more rapidly.

Certain exceptions occur. The action of a dégroisseur is probably essentially biological, but, nevertheless, in very cold waters a dégroisseur works badly, and is probably far less effective than coagulation when applied to very cold waters which contain turbidity of the same character as that which occurs in the southern United States.

The general principles are now fairly obvious. Slow-sand filters are most suitable for insular climates, and best of all for Temperate insular climates. Dégroisseurs should probably be covered when used in conjunction with covered slow-sand filters, and are probably very efficient in Tropical climates.

Chemical processes are less affected by cold weather than are filters or dégroisseurs, but are most efficient in hot climates.

In Tropical climates, therefore, it is advisable to effect the major portion of the purification by chemical methods, and (unless repairs are difficult owing to scarcity of skilled mechanics) mechanical filtration is preferable to the slow-sand process.

CHAPTER XI

PROBLEMS CONNECTED WITH TOWN WATER SUPPLY

CONSUMPTION OF WATER.—Average daily consumption over the whole year—Maximum daily consumption—Maximum hourly consumption—Effect of waste—Values of the ratios

$$\frac{\text{Maximum daily supply}}{\text{Mean daily supply}}, \quad \frac{\text{Maximum hourly supply}}{\text{Mean hourly supply}}.$$

Temperate climates—Eight hours main—Baths—Gardens—Hotter climates—Future variations of the ratios.

AVERAGE ABSOLUTE QUANTITY OF WATER USED. — Minimum possible — British values—German values—American values—Australian values—Values obtained for domestic use exclusively—European figures—Indian figures—House to house *versus* hydrant supplies—Chinese and Japanese figures—South African figures.

TRADE SUPPLIES.—Effect of rates—Private trade supplies.

Prevention of Waste of Water.—Inspection—Water meters—Preliminary work—Leakage from mains—From house fittings—Necessity for repeated measurements—Standard fittings—Reducing valves—Sale by meter.

WATER METERS—Requirements of accuracy.

Town Water Supply.—Special pipes, special valves, air, scour, etc.

SPECIAL PROBLEMS RELATING TO MAINS.—Examples—Calculation of the pressure at any point.

SERVICE RESERVOIRS.—Capacity required — Practical conditions introduced by the various objects of a service reservoir.

DETAILS OF CONSTRUCTION OF A SERVICE RESERVOIR.—Masonry or Concrete Service Reservoir—Puddle lined type—Cement rendered type—Bitumen or asphalte sheeted type—Roofing.

POPULATION STATISTICS.

CONSUMPTION OF WATER.—**Units : gallons per head per day.**—The average daily consumption over the whole year is important in the design of large works, such as storage reservoirs, or supply mains. For works of the second order, such as pumping stations, or filter beds, the chief factor is the maximum daily consumption, which usually occurs in the hottest season of the year. Works of the third order, such as town mains, and their minor reticulations, are designed for the maximum hourly requirements, plus an allowance for the extra demand that may be caused by fires. These quantities are usually expressed in **gallons per head per day.**

The ratios of these figures depend on the climate, the habits of the population, its standard of living, and above all, on the waste of water.

The importance of this last factor has frequently been overlooked, chiefly for two reasons:—Firstly, the amount is usually not accurately known, and when not approximately ascertained, is invariably underestimated. Secondly, since waste occurs every hour of the day at a fairly constant rate, its effect is to minimise the percentage variations of the total consumption, and to produce an apparent constancy in demand. Consequently, I believe that many of the rough rules at present employed for dimensioning the minor works of a town supply are incorrect; and by way of indicating my personal views on the importance of the matter, I propose to deal with it first of all.

As an example, I select the figures for a German city in the month of August. These are very accurately ascertained, and refer to a period of maximum consumption, to a very well constructed system, the habits of the people also being such as to ensure that very little of the waste was wilful,—in fact, without wishing to depreciate the skill of the supervising engineers, they, and not the householders, must be held responsible for any waste. I consider that the figures do both parties great credit, as they were obtained without any special efforts being taken to prevent waste.

We have (Sketch No. 150) in percentages of the total 24 hours' supply:—

A.M.		A.M.		P.M.		P.M.	
12–1 .	1·85	6–7 .	5·28	12–1 .	5·83	6–7 .	5·04
1–2 .	1·79	7–8 .	5·25	1–2 .	5·77	7–8 .	4·69
2–3 .	1·80	8–9 .	6·00	2–3 .	5·48	8–9 .	3·59
3–4 .	1·78	9–10 .	6·31	3–4 .	5·55	9–10 .	2·93
4–5 .	1·83	10–11 .	5·95	4–5 .	5·16	10–11 .	2·38
5–6 .	2·75	11–12 .	6·04	5–6 .	5·18	11–12 .	1·76

We see at once that the minimum, mean, and maximum hourly consumptions are:

Per cent.		Per cent.		Per cent.	
1·76	:	4·17	:	6·31	or, as 0·42 : 1 : 1·51.

Now, in this city there is very little consumption for hydraulic power, and the habits of the people are such as to render any great demand for water during the dead hours of the night improbable; nevertheless, for the six hours 11 p.m. to 5 a.m. we find an average consumption of 1·80 per cent.

The period is too long for us to assume that the pump counters registered water that was actually consumed later, and there are few service reservoirs in which the excess could be stored. It is therefore very hard to avoid the conclusion that a large portion of this consumption is waste, and later (see Prevention of Waste) I shall produce evidence to confirm this view.

Let us merely assume that in this city two-thirds is waste. It consequently appears that $24 \times 1{\cdot}20$, or over 28 per cent. of the water pumped is wasted. This is a somewhat unfair estimate, since, during these dead hours the pressure in the mains is higher than during the period of intense consumption (although it must be remembered that being a pumping supply, large variations of pressure due to changes in demand such as occur in a gravity supply, are improbable). It seems just, therefore, to conclude that the loss during the other 18 hours is not less than 1 per cent. per hour. We may thus assume that if no waste took place, the correct figures would be (as percentages of the present supply) 0·56 : 3·17 : 5·30, or as 0·18 : 1 : 1·68, and the whole 24 hours' supply would be about 74 to 80 per cent. of the present consumption.

These figures are at first sight somewhat astonishing, but they are amply confirmed by other examples; and, as I have already stated, they were (water meters not being used) very creditable to the engineers responsible for the service. The real lesson to be drawn is that in a well-maintained system supplying a careful, and—according to other than German ideals—an over-regulated population, 25 per cent. of the maximum day's delivery is wasted through leaks, and defective fittings. Consequently, no engineer who has not as yet measured the waste from house fittings and the leakage of his mains is entitled to assume a smaller quantity.

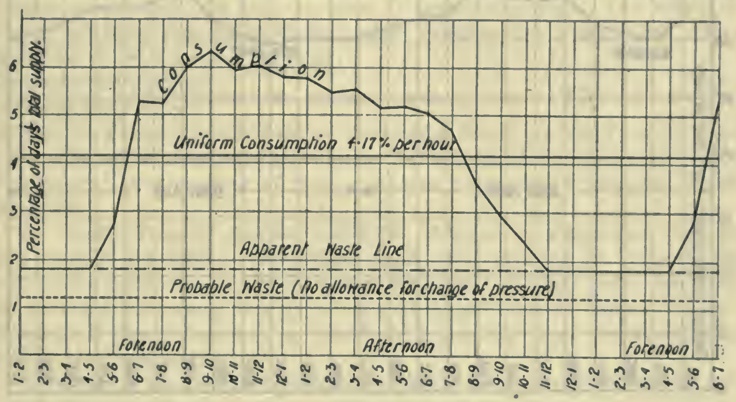
It is therefore plain that the ratios:

Maximum daily supply : Mean daily supply throughout the year,

and

Maximum hourly supply : Mean hourly supply throughout the 24 hours,

are to a large extent dependent on the waste of water; and that all figures given should be considered as liable to modification if waste is systematically checked.



SKETCH No. 150.—Diagram of Hourly Variations of Supply.

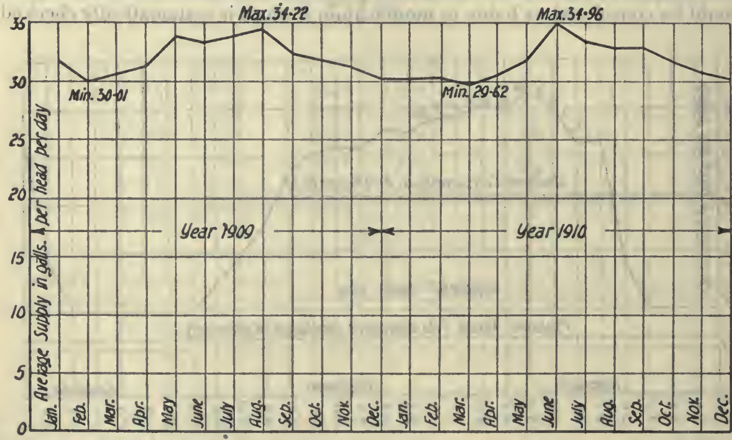
Thus, it appears that in cases where the systematic prevention of waste is contemplated, a logical design will take into account the alteration produced in the usual values of the ratios under consideration. The method is obvious:—Having obtained the ratios as affected by leakage waste, and the leakage waste by measurement of the consumption during the dead hours, we can assume that the leakage waste in other hours varies as the square root of the pressure in the service mains; and the new ratios can then be calculated. The general effects are apparent, since all major works will be diminished, some of them by as much as 33 per cent. or more. The town mains and reticulations will probably not be very greatly affected, except in the case of the larger mains, where a diminution of 15 per cent. may be attained.

On the other hand, the dimensions of all works which have the equalisation of supply as their purpose, such as service reservoirs, will certainly be increased relatively to the smaller supply for which they are to be calculated; and it may happen that they will prove insufficient in size even when designed according to rules based on the absolute value of the uncorrected supply.

Subject to these remarks, it may be stated that in Temperate climates, the maximum day's consumption is about 1·5 of the average daily consumption, and that the maximum hour's demand is also about 1·5 of the average hourly consumption. We thus deduce that the maximum consumption of any hour of the year is about 2·25 of the average hourly consumption. These rules are obtained from experience of carefully maintained and well-constructed waterworks.

"Eight Hours Main."—A very common practical rule is thus arrived at. The supply main in towns where large service reservoirs do not exist should be proportioned on an eight-hour basis. That is to say, the pumps and the supply main should be capable of delivering the average consumption of 24 hours in a period of 8 hours only. The assumption amounts to:

Maximum consumption during 1 hour = 3 × average hourly consumption during the year.



SKETCH No. 151.—Diagram of Monthly Variations of Supply for Years 1909–1910 at London.

The rule plainly affords a certain excess capacity which is useful in case of fire or other abnormal demands.

In cases where the greatest possible care is taken to prevent waste, it is probable that the ratios are as high as:

Mean daily	:	Maximum daily	::	1 : 1·7
Mean hourly	:	Maximum hourly	::	1 : 3

but such values are only obtained by systematic and unremitting efforts to minimise waste. (Sketch No. 151 shows the variation in consumption by months and is therefore more applicable to such questions as the draught from storage reservoirs than to the variation of the daily ratios.)

Where the waste is considerable, the ratios are greatly diminished, but actual figures are not available for such cases, since great losses by waste are almost invariably accompanied by a systematic neglect of all measurements.

Besides waste, the habits of the people influence the above ratios. A population accustomed to a daily bath will evidently draw heavily from the mains in the early morning hours, and the standard of living being high, the house fittings may be expected to be of good quality, so that waste is consequently minimised, and both causes tend to produce a large hourly consumption ratio.

On the other hand, where the town authorities permit the water supply to be utilised for gardening purposes, although we may expect a very high daily ratio (*e.g.* at Rochester in Victoria where the average supply is 54 gallons per head per day, the maximum daily supply reaches 215 gallons per head per day, being mainly employed in watering fruit gardens), yet the hourly ratio is not likely to be large, as watering is usually done in the evening, when domestic consumption is small.

If a considerable portion of the supply is utilised for trade purposes, we may expect both the hourly and daily variations to be reduced.

In hotter climates, while the hourly variation is much the same as in Temperate countries, the daily variation is largely increased. The values for a series of Australian cities where garden watering is not permitted show a maximum value of 2·25 for the ratio ;

$\frac{\text{Maximum daily supply}}{\text{Average daily supply}}$, and an average value of 1·90. While, for climates such as the Punjab, the dry zone of the United States, or South Africa, where a hot summer succeeds a cool winter, the average daily ratio may rise as high as 2·25, and maxima of 2·40, and 2·50 occur where irrigation is not permitted.

In Insular Tropical climates where the changes in temperature are not very marked, the ratio is lower than usual. For instance, at Colombo the value of the daily ratio oscillates between 1·10 and 1·46 ; and since this city is not provided with water-closets, and is well looked after as regards waste, it appears that these values may be considered higher than those usually obtaining in Equable Tropical climates.

In respect to the probable direction in which future variations of the ratio may be anticipated,—any increase in the standard of living, or in the trade of a town may be expected to reduce the daily variations, but will probably increase the hourly variations.

AVERAGE ABSOLUTE QUANTITY OF WATER USED.—In a civilised community, there is an absolute minimum which has been fairly closely attained in many different countries. In several cities in the Midlands of England it is about 18 gallons (say 22 U.S. gallons) per head per day. This has only been reached by constant inspection, and has in a sense been forced on the population by the paucity of the available supply. While no discontent is expressed, and no inconvenience is felt, I cannot say that I think such a consumption is an ideal to aim at. Such cities are by no means popular residential places, and any community which contents itself with so small a provision may be considered unfortunate, in that it deliberately adopts a less high standard of life in reference to its water supply.

I believe that a city is well supplied for the requirements of the British climate, which actually uses (abnormal waste apart) about 22, to 25 gallons (*i.e.* 26 to 30 U.S. gallons) per head per day ; this, allowing for trade consumption, means that each citizen enjoys approximately 15 to 18 gallons per day. With a normal allowance for waste, this would work out at 30 to 35 gallons (36 to 42 U.S. gallons) per day, which is slightly less than the consumption of well-inspected residential quarters in London.

It may be objected that I am here advising a larger supply than has frequently sufficed, but to this I would reply that in such cases the scarcity of water has induced a sparing use, and a standard of living (in the matter of water) has been produced which custom alone renders tolerable.

I consider it an engineer's duty, in so far as it lies in his power, to foster an advance in the general standard of living, and if I personally have any doubt as to whether the above standard is correct, it is only in regard to future sufficiency.

It must also be remembered that an abundant supply of good water is no mean factor in a city's equipment, from the point of view of competition in trade.

Starting with this figure as a minimum, we may say that British cities are rarely, if ever, supplied with more than 50 gallons (60 U.S. gallons) per head per day.

In German cities the present practice is to allow a somewhat smaller quantity than in Great Britain, but there is little doubt that an increase will be found advisable should the present advance in prosperity continue. The limits are 12, to 30 gallons (say 14, to 35 U.S. gallons), and the mean appears to be about 22 gallons (say 27 U.S. gallons) per head per day. These figures are for partly metered supplies, and for unmetered water an increase of 20 per cent. is recommended.

In American cities the supply of water per person is on the average far larger than in Europe. Whilst it is a matter of notoriety that the pumps which form the water meters of many American cities are exceedingly leaky, and that waste from house fittings is in many cases regarded as of no importance; there is no doubt that the population does use more water for legitimate purposes than a similarly situated European population. I am quite unable to consider that this greater consumption is in any way undesirable. The standard of living amongst Americans is high, the daily bath is a widely spread habit; and, while I fully agree that waste should be avoided, the large consumption per head in many cities, where waste has been cut down to a quantity comparing favourably with European practice, can only be regarded as a sign of a high civilisation. The larger figures, however, can only be explained on the assumption that continued and obvious waste is permitted.

The statistics for 111 cities with populations of over 25,000 are as follows:

Maximum consumption	270 gallons	(324 U.S. gallons)	per head per day.
Average	,,	88 ,,	(105 ,,)	,,
Minimum	,,	26 ,,	(31 ,,)	,,

For 76 cities, with a population less than 25,000, similar figures were:

Maximum consumption	124 gallons	(149 U.S. gallons)	per head per day.
Average	,,	51 ,,	(61 ,,)	,,
Minimum	,,	8·5 ,,	(10 ,,)	,,

There is, of course, little doubt that in the higher figures waste, or extensive watering of gardens (frequently both), must occur; but in view of the habits of the people, it appears undesirable to design works for an American city to supply less than 50 gallons (60 U.S. gallons) per head.

I am the more convinced of this, since in Australian cities, inhabited by a people as prosperous as the Americans, and where continued and obvious waste is as infrequent as in Europe, the average consumption of all towns where the water supply is plentiful is 48 gallons (58 U.S. gallons) per head. In smaller towns (some of which are not well supplied) the average is 30 gallons (36 U.S. gallons), and in these places trade consumption is almost negligible.

The following information is recorded not for any value attached to the absolute figures, but as a comparison enabling an engineer to test his own values.

(i) *Absolute Consumption of Water in purely Domestic Use.*—Whitney at Newton, Mass., found that:

(*a*) The consumption at the *kitchen tap* (average of five persons per house) was 5·5 imperial gallons (6·67 U.S. gallons) per head per day.

Where a second tap existed, it consumed 1·1 imperial gallon (1·3 U.S. gallon) per head per day.

(*b*) *Water-closets.*

First closet.	5	imperial gallons	(6 U.S. gallons)	per head per day.
Second „	2·2	„	(2·6 „)	„

(*c*) *Baths.*

First bath.	4·1	imperial gallons	(4·8 U.S. gallons)	per head per day.
Second „	0·8	„	(1·0 „)	„

Thus, we arrive at 14 to 15 imperial gallons (17 to 18 U.S. gallons) as a fair minimum value for a population leading what may be considered as a comparatively comfortable existence, with no waste.

This result is confirmed by Cooper (*Trans. Am. Soc. of C.E.*, vol. 55, p. 430), who found that a small educated population, too addicted to the luxury of baths (from a popular point of view), consumed 20 imperial gallons (24 U.S. gallons) per head daily, and he believed that the purely domestic use was 16·7 imperial gallons (20 U.S. gallons) per day.

Hunter (*P.I.C.E.*, vol. 137, p. 44) gives for domestic purposes, solely, in London (waste included) figures ranging from 55 imperial gallons (66 U.S. gallons), which includes stables and carriage washing, down to 14·3 imperial gallons (17 U.S. gallons) in a locality where daily baths are less frequent.

Data for requirements, exclusive of trade supplies, are more easily obtained, and the following, which refer to the period about the years 1895–97, are given by Griffith (*P.I.C.E.*, vol. 117, p. 190) subject to the annexed remarks:

(*a*) The towns are notorious as possessing a large percentage of working-class people.

(*b*) To my personal knowledge, the standard of comfort in several cases is below that of England as a whole, and the figures for London, although swollen to some extent by waste (I believe by about 15, to 20 per cent. on the average), may be considered as more applicable to residential cities.

Derby	13 imperial gallons	(15·6 U.S. gallons)	per head per day.
Nottingham	13·5 " "	(16·2 ")	" "
Liverpool	15 " "	(18 ")	" "
Leicester	14 " "	(16·8 ")	" "
Manchester	13 " "	(15·6 ")	" "
Norwich	10·5 " "	(12·6 ")	" "

London Water Companies (now merged in the Water Board):

New River	20·6 imperial gallons	(24·7 U.S. gallons)	per head per day.
Lambeth	22·3 " "	(26·8 ")	" "
Kent	22·9 " "	(27·5 ")	" "
West Middlesex	24·8 " "	(29·8 ")	" "
Southwark and Vauxhall	24·4 " "	(29·3 ")	" "
East London	25·5 " "	(30·6 ")	" " (waste occurs).
Chelsea	28·8 " "	(34·5 ")	per head per day. (more residential than others).
Grand Junction	31·2 " "	(37·5 ")	per head per day.

(ii) *Total Consumption.*—The following figures are not easily comparable, since they include trade consumption as well as domestic use.

For British cities, for all purposes, the figures range from:

	Consumption per Head per Day.	Remarks.
Leicester	16 imperial gallons (19 U.S. gallons)	These must be considered too low.
Wigan	17 " " (20 ")	
Sheffield	22 " " (26 ")	These are sufficient, but leave little margin for an enhanced standard of consumption.
Carlisle	23 " " (27 ")	
Nottingham	24 " " (29 ")	
Birmingham	24 " " (29 ")	
Liverpool	25 " " (30 ")	
Leeds	43 " " (52 ")	These are high, but two include supplies to shipping.
Brighton	43 " " (52 ")	
Aberdeen	43 " " (52 ")	
Plymouth	43 " " (52 ")	
Perth	50 " " (60 ")	

The French figures range from:

11 imperial gallons (13 U.S. gallons) to 170 imperial gallons (202 U.S. gallons),

but the consumption is apparently on the average somewhat higher than that in Great Britain, the difference probably being due to more systematic discouragement of private trade supplies.

Other Continental cities range from :

9 imperial gallons (11 U.S. gallons) at Venice, readily explained by the situation.
17 ,, (20 ,,) at Amsterdam, also explained by the situation.

to :

50 imperial gallons (60 U.S. gallons) in the lake towns of Zurich and Geneva,
and 200 imperial gallons (250 U.S. gallons) in Rome,

which is really a legacy from the Imperial City, although the restoration of the aqueducts is a modern achievement.

For Indian conditions the facts are more complicated. As a matter of experience, 6 imperial gallons (7·2 U.S. gallons) per head per day appear to be sufficient. Harriet (*P.I.C.E.*, vol. 143, p. 276) gives the following figures for Raipur :

1893–4	Average	4·4	imperial gallons	per head	per day.
1894–5	,,	5·8	,,	,,	,,
1895–6	,,	6·9	,,	,,	,,
1896–7	,,	7·4	,,	,,	,,
1897–8	,,	8·0	,,	,,	,,

The steady increase cannot be entirely regarded as extra consumption, and is probably more attributable to increased leakage.

The general rule, however, is to design with a view to 8 or 10 imperial gallons (10 to 12 U.S. gallons) per head per day; and this may be considered as the maximum day's consumption, since the water of most Indian towns is pumped from wells or rivers.

As examples, Simla with about 12 per cent. European population is supplied with 8 gallons. Amballa with 7 gallons, with an extra allowance for the gaol and barracks.

The above figures refer to towns in which the water is not delivered from house to house, but drawn from street hydrants, and public fountains. Whereas, in Bombay, and less markedly so in Calcutta, the supply is from house to house. In the former town we find 35 imperial gallons (42 U.S. gallons), some of which is waste ; and in the latter 25 imperial gallons (30 U.S. gallons) almost entirely for domestic use.

We may therefore assume that the 8 to 10 gallons usual in India is due to the poverty of the population. Nevertheless, the advantages gained by the introduction of a pure water, if acceptable to the population, are so great that I should personally be prepared to advocate even such small supplies as 3 or 4 imperial gallons (say 4 to 5 U.S. gallons) per head per day, when money was scarce, as preferable to the usual city well with its abnormal pollution and consequent infection resulting in outbreaks of cholera.

Following Indian experience, the present practice of English-speaking engineers is to consider 8 to 10 gallons as sufficient in China. But facts do not confirm this, since Hong-Kong is supplied with 17 to 18 imperial gallons (20 to

23 U.S. gallons) (*P.I.C.E.*, vol. 100, p. 247), and even this amount is apparently inadequate; which, in the case of a town with no water-closets would be peculiar, were it not that the requirements of the shipping are relatively large.

At Shanghai, with about 22 per cent. of Europeans, the figures are from 50 to 70 imperial gallons (60 to 84 U.S. gallons) per head per day (Johnson, *Trans. Am. Waterworks' Engineers Assn.*, 1907, p. 252).

So also, Moore (*P.I.C.E.*, vol. 180, p. 297) designed for 10 imperial gallons at Hankow, and was requested to provide 20 imperial gallons (24 U.S. gallons). Making every allowance for "Chinese face," I cannot but agree with the Mandarins.

The Japanese figures given by Johnson (*ut supra*) also seem to confirm the above. They are:

Tokio	Average	14	imperial gallons	(17	U.S. gallons)	per head	per day.
	Maximum	19	,, ,,	(23	,,)	,,	,,
	Minimum	9	,, ,,	(11	,,)	,,	,,
Yokohama	Average	20	,, ,,	(24	,,)	,,	,,
	Maximum	24	,, ,,	(29	,,)	,,	,,
	Minimum	16	,, ,,	(19	,,)	,,	,,
Kobe	Average	25	,, ,,	(30	,,)	,,	,,
	Maximum	33	,, ,,	(40	,,)	,,	,,
	Minimum	17	,, ,,	(20	,,)	,,	,,
Hiroshima	Average	23	,, ,,	(28	,,)	,,	,,
	Maximum	33	,, ,,	(40	,,)	,,	,,
	Minimum	12·5	,, ,,	(15	,,)	,,	,,

We may therefore conclude that Oriental communities should (where the money is available) be supplied with water on a scale but little inferior to a German population. Where funds are scarce, I consider that it is the engineer's business to boldly ignore all rules, and to give the best possible supply, without regard to previous experience.

South Africa.—The following figures, tabulated by Lindesay (*P.I.C.E.*, vol. 158, p. 420), will make it perfectly plain that the principles deduced above have already been applied in South Africa. They may be regarded as very creditable to all concerned, whether engineers or town councillors.

AVERAGE DAILY SUPPLY.

Port Elizabeth	9·5	imperial gallons	(11·5	U.S. gallons)	per head	per day.
Johannesburg	10	,, ,,	(12·7	,, ,,)	,,	,,
Cape Town	31	,, ,,	(37	,, ,,)	,,	,,
Pietermaritzburg	50	,, ,,	(60	,, ,,)	,,	,,
Durban	62·5	,, ,,	(75	,, ,,)	,,	,,
Pretoria	87	,, ,,	(101	,, ,,)	,,	,, (includes gardens).

Trade Supplies.—Except where otherwise stated the preceding figures include the consumption of water for manufacturing purposes. As already remarked, this fact renders any very close comparison of the figures impossible, and it will also partially explain the large variations in the consumptions *per capita* that appear in any tabulation of statistics of water supplies.

The actual ratio between trade and domestic consumption must vary not only in different towns, but also in different years in the same town. The experience of the London Water Board when a uniform system of water rates and charges was introduced all over the London Water Supply area, to replace the varying charges of the old Water Companies, shows that a very slight rise in the charge for water suffices to cause large consumers (both for trade and domestic purposes) to instal private water supplies wherever local conditions are favourable. Statistics are very hard to obtain, but two cases are frequently quoted. The private (well) supplies in a certain district in South London are capable of yielding 5 per cent. more than the gross capacity of the Waterworks Authority's installation. At Liverpool a similar excess used to occur, but this has diminished since the introduction of a better water supply.

In projects for town water supplies the engineer usually has some trade statistics to guide him. Nevertheless, a careful census of trade supplies should be taken, and in drawing up the final design the engineer should carefully consider whether it is advisable to endeavour to secure the custom of the trade consumers by low rates, or to let them shift for themselves. While the first course is obviously more preferable, the solution actually adopted must entirely depend upon local circumstances.

Prevention of Waste of Water.—I have previously taken occasion to express my views on the great importance of waste from leakage, and have referred to its possible prevention.

The principal source of leakage is found in defective house fittings, such as taps and service-pipes. In good modern practice, leakage in mains laid down in the streets is infrequent, and, except in gravelly soils, is usually manifest soon after occurrence by subsidences in the roads, or by the appearance of water. An engineer who takes charge of a system laid previously to 1880, say, (even though under the best supervision of its date) will, however, be well advised to institute systematic search for leaky mains and open joints.

The methods of detecting house leakage are two:

(i) *House to house inspection*, combined with a systematic replacement of defective fittings. I consider this to be most undesirable. In the first place, the Waterworks' Authorities cause trouble to all householders (whether careless or otherwise). Secondly, although this method when energetically carried out will reduce waste to its lowest point, the engineer is never exactly informed of the results obtained, and apart from his own personal curiosity on the matter, is unprovided with any accurate figures to justify his action when discontent is excited.

It is also evident to any observant individual that the engineer is taking no steps to enquire whether "his own fittings" (*i.e.* the service mains) are leaky; and since this must be regarded as a very practical objection to the method, I cannot recommend it except as a temporary expedient preliminary to the introduction of the second plan.

(ii) *The Water Meter System.*—This necessitates more preliminary work, and also a fairly large outlay on plant. A sketch of the mains and piping of the district to be dealt with, showing all stop-cocks, is first prepared. The approximate supply is then estimated, and the locality is divided into isolated sub-districts (which may render some alteration of the mains necessary), so that the total water entering each can be measured by a meter of such a size that the maximum supply does not exceed the capacity of the instrument which it is proposed to use.

All stop-cocks must then be made easily accessible, which, if the system is not laid out with a view to meter work, is often somewhat difficult.

The meter is then fixed, and diagrams are taken to ascertain the initial consumption.

Thereafter, the process is as follows :

About midnight the meter is set to record the flow, and inspectors visit each stop-cock and listen with a stethoscope for the sound of flowing water, which is best heard when the cock is only partially open. Each "sounding cock" is noted, and after testing, all are shut off, and remain so for half an hour. The record of this half-hour gives the leakage from the mains, and the houses supplied by all stop-cocks which gave a sound are inspected next day for defective fittings.

Diagrams are taken off the meter daily until the district is in good order, and thereafter, say three times each month, so that any fresh leakage can be detected, and, if large enough, sought for.

In well-constructed mains the leakage outside the houses is small, *e.g.* in three cases recorded by Stewart (*P.I.C.E.*, vol. 66, p. 348) the minimum flows were 1,500 gallons, 2,000 gallons, and 2,000 gallons per hour, and the leakages from the mains 0, 50, and 150 gallons. So that defective house-fittings were responsible for more than 90 per cent. of the leakage, even when the mains were in relatively bad order.

In view of this fact, it is plain that no effective measures for the prevention of waste are possible, unless legal powers are obtained either to enforce the repair and renewal of faulty fittings, or to cut off supplies in cases of proven and continued waste.

The sale of water by bulk through individual meters fixed in the pipes supplying each consumer is an obvious alternative, and is the most equitable method for trade supplies. The application of meters to domestic supplies is most objectionable, since any stint in the domestic use (not waste) of water is sooner or later likely to be visited by natural penalties far outweighing any private advantage that may possibly be derived. The legal provisions against sale by meter which occur in several American cities may be regarded (if rigidly restricted to domestic supplies) as founded on a high, even if merely instinctive, sense of rightful sacrifice for the advancement of public welfare. Opposition to the infliction of penalties for wilful or continuous waste appears less likely to be entirely caused by public spirit. Also, as a matter of finance, small meters are relatively costly and the interest on the investment entailed may easily exceed the value of the water which their use prevents being wasted.

To those unacquainted with the usual amount of waste the results are surprising. In a city where fittings had been subject to regulation and inspection for some two years, the following figures are typical for areas served by a single meter (*ut supra*).

SUPPLY IN IMPERIAL GALLONS PER HEAD PER DAY.

Before Starting the System.		After Three Meter Inspections.	
Total Daily Rate.	Night Rate.	Total Daily Rate.	Night Rate.
81·8	64·0	34·1	9·9
68·3	45·7	31·9	15·0
41·6	27·4	27·4	8·3
61·0	55·0	47·2	23·8

And, in the district as a whole, the consumption for all purposes was reduced from :

Daily.	Night Rate.		Daily.	Night Rate.
49	37·7	to	32	17·5

It must be remembered that the improvement thus obtained is essentially of a temporary nature, and that if inspection is neglected, the old rate will recur after a period measured only by months.

The cheaper classes of house fittings, such as are usually supplied by builders, deteriorate very rapidly, and become leaky. In some cases, therefore, the waste inspectors are instructed to perform such minor repairs as the insertion of a washer, without troubling the householders.

Where the pressure in the mains is great, some authorities have obtained legal powers either to enforce the installation of only such fittings as satisfy a certain standard of water-tightness, or to instal their own fittings at a fixed charge.

Local conditions, and the views of the community on a proposal which may have obvious objections, must be considered before adopting either of these methods.

There is also another method of minimising waste which deserves attention, especially in gravity supply systems. This consists in the introduction of reducing valves, so that the pressure in the mains can never exceed a fixed amount.

These valves prove useful, especially in cases where the area supplied varies markedly in level. They then not only diminish waste in the lower lying districts, but also increase the available pressure in the higher, which frequently suffer from lack of water.

The final results of measures for the prevention of waste, or metering of supplies, are very difficult to predict. The methods adopted in selecting the supplies to be metered or tested for waste have great influence on the results obtained. If consumers are metered or inspected at their special request only, no particular effect on the consumption can be expected until perhaps more than 50 per cent. of all householders have adopted the system. If, on the other hand, houses are first inspected, or metered, which are notoriously badly maintained, a considerable decrease in consumption may be anticipated almost immediately ; but the fixing of a meter is regarded in the light of a penalty, and the authorities become as popular as detectives.

The best system is to endeavour to meter all trade supplies, and to sell this water by bulk, while water for domestic requirements is supplied at a fixed rate, independent of the quantity actually used, waste being checked by street meters measuring the total consumption of, say, 200 to 500 houses, as already described.

Under such circumstances, the daily consumption per head may be decreased by 50 per cent. (even when the initial consumption was not very large, *e.g.* 32 to 16 gallons, or 42 to 21 gallons), when the system is in full swing, and about 20 per cent. of the total supply will then be unaccounted for. If every supply is metered, a further decrease of 10, to 15 per cent. may be anticipated, and not much more than 10 per cent. of the water will remain unaccounted for.

The most modern information on the subject is found in reports on "Waste of Water in New York, and its reduction by Meter and Inspection," principally the work of Fuertes, where the influence of conditions and variations in local policy is most ably discussed.

Mr. Fuertes states that even in America waste through leaky street mains must be considered as unproved. I had previously come to the conclusion tha such wastage was non-existent in Europe, and it therefore appears that the enormous consumption recorded from many American cities should be regarded as arising almost entirely from house waste, and not from any appreciable leakage from street mains.

The question of the accuracy of the records remains uninvestigated, although it may be remarked that the very large consumptions per head are usually recorded in cases where the supply is pumped; and if a badly maintained pump is used as a water meter, it is quite possible that the recorded figures bear no particular relation to the quantity actually pumped.

Water Meters.—It is quite impossible to describe all the types in existence, and their design is best left to specialists. What is really desired, from the point of view of a householder, is a meter which works noiselessly (it must be remembered that water pipes are excellent conductors of sound).

The requirements concerning accuracy of registration are variable. If the meter is used for selling water by bulk to private customers, a carefully calibrated and correctly registering instrument is necessary. If it is used solely for the detection of water waste, unimpeachable registration of the maximum flow is not very essential (since this only lasts a few minutes), but the meter must record the total quantity passed by a long continued small flow with certainty, accuracy in the rate of flow being less material.

For example, in a meter for the measurement of the total water supplied to a large town, correct registration of even so large a flow as 10 gallons per minute is immaterial; whereas, a waste detection meter should register the total quantity passed by a flow of 0·1 gallon per minute, over a period of even ten minutes.

SYMBOLS.

Problems connected with town water supply. The ordinary units are used.

d, with an appropriate suffix, denotes the diameter in feet of any pipe, d' is used when the diameter is measured in **inches.**

D, is a general symbol for the total demand by domestic consumption, fires, etc., during any individual hour, expressed in cusecs. D_{max} and D_e (see p. 616).

h, is used for the total loss of head, in feet, in a main of varying diameter conveying a constant quantity of water.

H, is used for the similar loss of head, when the main supplies water at various points along its length.

h_1, h , h_3 (see p. 615).

K, is a contraction for $\frac{1}{1000d^5}$.

k, (see p. 616).

l, is the length of a main, in feet, when the diameter is uniform throughout the length l.

L, is used for l when we consider different portions of the length l.

Q_o, is the quantity of water, in cusecs, entering the length l or L.

Q_1, or Q_e is the quantity leaving the length l or L.

q, is the quantity, in cusecs, drawn off from the main, per foot run.

Thus $ql = Q_o - Q_1$, and $qL = Q_o - Q_e$.

q_1, (see p. 617).

x, x_1, y, z (see p. 617).

Σ is the symbol for summation.

SUMMARY OF EQUATIONS.

Uniform volume of water.

$$h = \Sigma\frac{64Q^2l}{\pi^2C^2d^5} = \frac{Q^2}{1000}\Sigma\frac{l}{d^5}$$

$$= 250Q^2\Sigma\frac{l}{d^5} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{[Inches]}$$

More accurate formula. $h = \frac{Q^2}{1500}\Sigma\frac{l}{d^{5.33}}$.

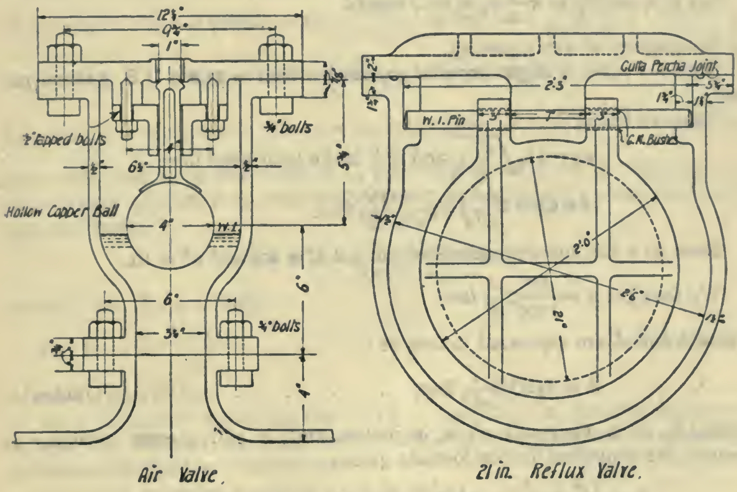
Variable volume of water.

$$h = \frac{L}{1000d^5}\left\{\frac{Q_o^2 + Q_oQ_e + Q_e^2}{3}\right\},$$

$$= \frac{L}{3000d^5}Q_o^2, \quad \text{if } Q_e = 0.$$

$$H = \frac{1}{1000}\Sigma\frac{Q_o l Q_1}{d_1^5}$$

It is not proposed to enter into the details of construction and maintenance that form the principal duties of a town waterworks manager. Local



SKETCH NO. 152.—Air Valve and Reflux Valve.

conditions are so important that any general rules are almost useless. The designs of street water pipes and mains that are used in London are suited to London conditions, but in my opinion have frequently been adopted, even in British cities, where a cheaper construction would have sufficed.

Modern practice, however, tends towards leaving these matters mainly in the hands of the local manager, and since I consider this is advisable, I do not propose to obtrude my own (equally local) ideas.

The dimensions of bends, tees, and other "specials" used in street mains may be ascertained from any pipe founder's catalogue. Similarly, such matters as air valves, reflux valves, scour valves and automatic cut-off valves (to provide for possible breaks in the mains) are usually procured from and designed by specialists.

The installation of these valves is advisable and the general principles of their location are obvious. Air valves are placed at the top of each crest in a large main and wherever a site near the crests of a small main which affords no possibility of contamination can be secured. Scour valves are placed at the bottom of valleys in those places where opportunities for discharging the surplus water into a drain or stream are easily secured.

Personally I distrust automatic cut-off valves: breaks rarely occur in well-laid mains and therefore the "automatic" cut-off valve usually sticks when it should act.

Special Problems Relating to Mains.—The most convenient method of treating problems relating to pipe mains of variable diameter, or those which convey variable quantities of water, is to consider the total loss of head.

Let the given main be made up of:

A length of l_1 feet, of diameter d_1 feet; a length l_2, of diameter d_2 feet; a length l_3, of diameter d_3 feet.

Let $Q = \pi d_1^2 \frac{v_1}{4} = \pi \frac{d_2^2}{4} v_2 =$ etc., cusecs

be the quantity of water passing.

Note.—1 cusec = 22,500 imperial gallons per hour = 27,000 U.S. gallons per hour.

Then we have:

$$v_1 = C\sqrt{\frac{d_1}{4}s_1}, \text{ and if } h, \text{ is the total head lost:}$$

$$h = \Sigma sl = \Sigma \frac{lv^2 4}{C^2 d} = \Sigma \frac{64 Q^2 l}{\pi^2 C^2 d^5} \text{ feet.}$$

Now, as a fair approximation, we can put $C = 80$, and $\pi^2 = 10$.

We thus get $h = \frac{Q^2}{1000} \Sigma \frac{l}{d^5}$ feet,

where l, and d, are expressed in feet, or:

$$h = 250 Q^2 \Sigma \frac{l}{d'^5} \text{ feet; } \quad \text{[Inches]}$$

where h, is in feet, and d', is in inches, and, if still greater accuracy is desired, the simplified Tutton formula gives:

$$h = \frac{Q^2}{1500} \Sigma \frac{l}{d^{5 \cdot 33}}, \text{ with } C_1 = 100 \text{ (i.e. badly incrusted pipes),}$$

and one-half this value may be assumed for clean pipes; where $C_1 = 140$.

Let us now consider the problem of a main of uniform diameter d, and length L, where Q_o, cube feet per second enter at the initial point, and a supply q, per foot run is drawn off by service pipes, so that the quantity passing at a distance x, from the initial point is $Q - qx$, and the velocity at this distance is $\frac{(Q_o - qx)}{\frac{\pi}{4}d^2}$.

Thus, the element of head dh, lost in the length dx, is:

$$dh = \frac{(Q_o - qx)^2 64}{\pi^2 d^5 C^2} dx$$

Integrating, since $h=0$, when $x=0$, the head lost in the length L, is:

$$h=\frac{64L}{C^2d^5\pi^2}\left(Q_o^2-qQ_oC+\frac{q^2L^2}{3}\right)$$

$$=\frac{L}{1000d^5}\left(Q_o^2-Q_oqL+\frac{q^2L^2}{3}\right)$$

or, if we put $Q_o-qL=Q_e$,

We get, $$h=\frac{L}{1000d^5}\left(\frac{Q_o^2+Q_oQ_e+Q_e^2}{3}\right)$$

and if $Q_e=0$, $$h=\frac{L}{1000d^5}\,\frac{(Q_o^2)}{3}$$

From a combination of the two results given above, we get the following fairly accurate and easy method of obtaining the loss in a main of varying diameter, with branch mains leaving it at different points.

Let a quantity Q_o, enter the main, and let the first length be l_1, and its diameter d_1, also let Q_1, be the quantity flowing in the main when the diameter changes to d_2.

Let us assume that $Q_1=Q_o(1-p)$, where p, is small. Then the head lost in the length l_1 is not greater than $\frac{Q_o^2l_1}{1000\,d_1^5}$, nor less than $\frac{Q_o^2l_1}{1000\,d_1^5}\left(1-p+\frac{p^2}{3}\right)$.

$$\text{Thus,}\quad H_1=\frac{Q_o^2l_1}{1000\,d_1^5}(1-p),\quad\text{or,}\quad\frac{Q_oQ_1l_1}{1000\,d_1^5}$$

is a minimum value which does not probably differ more than $\frac{pH_1}{3}$ from the true value, and since the value of C, may vary as much as 10 per cent. in a short length of pipe, it is sufficiently correct to say that the total head lost is obtained from ;

$$H=\Sigma\,H_1=\frac{1}{1000}\Sigma\frac{Q_ol_1Q_1}{d_1^5}\text{ feet,}$$

where l_1 and d_1 are in feet.

Or, $H=250\Sigma\frac{Q_ol_1Q_1}{d'^5_1}$ . . . . [Inches]

where d'_1 is in inches.

Q_o is the quantity entering the length l_1, and Q_1, the quantity that leaves it, and the order of the quantities in the summand suggests a regular and systematic method of calculation.

If greater accuracy is required, the formulæ:

$$H=\Sigma\frac{Q_o^2+Q_1Q_o+Q_1^2}{4500}\,\frac{l_1}{d_1^{5.33}}\text{ for incrusted pipes,}$$

$$\text{and } H=\Sigma\frac{Q_o^2+Q_1Q_o+Q_1^2}{9000}\,\frac{l_1}{d_1^{5.33}}\text{ for clean pipes,}$$

may be used.

These formulæ may be applied in order to calculate the pressure that exists at any point of the main system.

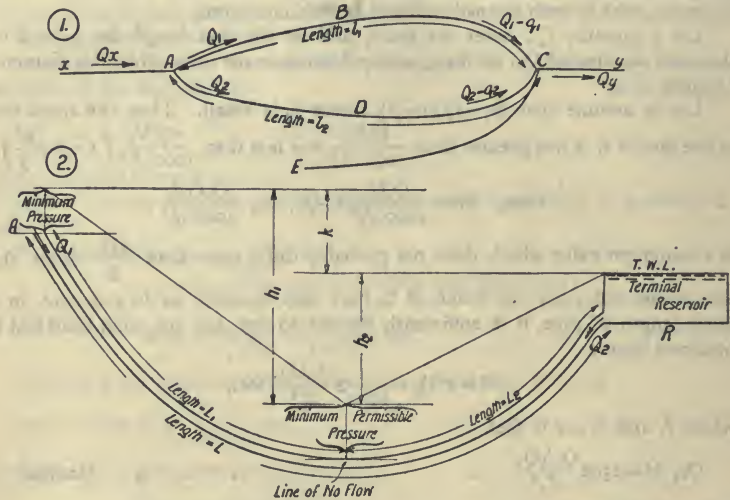
In actual practice, it would appear advisable to use the larger values of H, not so much because of incrustation in the pipes (which should rarely be allowed to reach a magnitude such as is indicated by $C_1=100$, in Tutton's formulæ), but to allow for the intense and localised draught that may occur during fires.

Very elaborate investigations of the pressures obtained at various points in a network of mains are given by Lueger (*Die Wasserversorgung der Städte*). The formulæ are too complex to be of practical use, even if the possibility of an outbreak of fire at any point in the network did not render the fundamental assumptions highly doubtful.

In actual practice, such questions are best solved by trial and error.

Thus, (Sketch No. 153) assume that a loop exists in the mains, which is represented by ABCD. Let Q_x, reach A, by the pipe xA, and Q_y, leave C, by the pipe C_y. Also let a quantity q_1, be drawn off from the main ABC of a length equal to l_1, between A, and C. Also a quantity q_2, from the main ADC, the length of which is l_2.

Thus, $$Q_x - Q_y = q_1 + q_2.$$



SKETCH NO. 153.—Diagrams for Branched Mains and Terminal Reservoirs.

Assume that Q_x, divides at A, into Q_1, flowing in the main ABC, and into Q_2, flowing in the main ADC.

Then, assuming that h, is the difference in pressure between A, and B, we have;

$$h = \frac{Q_1(Q_1 - q_1)}{1000}\ \frac{l_1}{d_1^5} = \frac{Q_2(Q_2 - q_2)}{1000}\ \frac{l_2}{d_2^5}$$

Now putting $Q_1 = zQ_x$ and $Q_2 = (1-z)Q_x$ we obtain a quadratic equation for z which is independent of h and which enables us to determine the ratio $\frac{Q_1}{Q_2}$ when d_1 and d_2 are given. So also if Q_1, Q_2 and h are given we can determine d_1 and d_2.

If, later, a third main of a length l_3, is laid to C, from another point E, the excess of pressure at which, relative to the pressure at C, is h_3, its discharge

Q_3, say, (under the assumption that the pressure at C, is unaltered) can be obtained by the usual equations.

The most probable supposition, however, is that the delivery Q_y, remains constant. We can then determine the new pressure at C, as follows:

The total delivery to C, under pressure differences h, and h_3, is $Q_1-q_1+Q_2-q_2+Q_3-q_3$; where q_3 is the supply drawn off from the main EC.

Let us assume that the new pressure differences are h', between A and C and h'_3, between E and C, where $h'_3=h_3-(h-h')$ so that the pressure at C exceeds the former pressure by $h-h'$, and calculate the new deliveries at C, which will be respectively, $(Q_1-q_1)\sqrt{\frac{h'}{h}}$, $(Q_2-q_2)\sqrt{\frac{h'}{h}}$, $(Q_3-q_3)\sqrt{\frac{h'_3}{h_3}}$ where q_1, q_2 and q_3 are assumed to be small in comparison with Q_1, Q_2 and Q_3.

The sum of these should be equal to Q_y. We thus get on substituting for h'_3 a quadratic equation for h', and the increase in pressure at C, is $h-h'$.

The above example is taken from an actual case where it was desired to erect a public fountain at the point C, and serve *en route* (by means of the third main) a newly erected building.

I suspect that more complicated problems require such detailed statistical information that it is but rarely that the equations will prove useful.

SERVICE RESERVOIRS.—Service reservoirs are required for two purposes. When placed between the storage reservoir and the town, at the lower end of a long supply main, they permit of the main being designed so as to deliver a full day's supply in 24 hours, whereas if no such equalising reservoir existed, the main would have to be capable of delivering water at a rate corresponding with that of the maximum demand; which, if only hours are considered, is about 170 per cent. of the average demand per hour taken over the whole 24 hours of the day on which the maximum demand occurs.

Similarly, in a pumping scheme, a service reservoir permits the pumps and connecting main to be designed for a uniform rate of delivery, in place of one varying with the demand, and if convenient (as in small schemes) we may arrange to run the pumps for say 8, 10, or 12 hours only per day, the reservoir storing up the surplus delivery for discharge during the hours when the pumps are idle.

When used for this purpose, the theoretical volume of the reservoir is evidently that required to equalise the draught over the day on which maximum consumption occurs (or more accurately, the day of maximum variability of consumption). In some actual examples this would amount to about 60 per cent. of the average day's consumption, or say 35 per cent. of the maximum day's consumption. The precise calculations are not of great importance, however, as the condition which really determines the size of a service reservoir is not variability of draught, but rather the time during which the supply main is likely to be out of order, and this evidently depends on such local considerations as its length, construction, whether duplicate or not, etc.

The question is determined by experience, and no accurate solution can be given, but the capacity of such equalisation of supply reservoirs rarely exceeds three days' supply, unless the supply main crosses a river or other obstruction rendering repairs more than usually tedious.

Similarly, in a pumping scheme we must not only consider the repairs to the mains, but also those to the pumps, so that the type of pump installed and the surplus power must also be taken into account.

It is evident that the most economical solution will largely depend on local circumstances, and in a flat country where no easily accessible sites for service reservoirs exist, it may be advisable to duplicate the main, and rest content with a very small service reservoir, if this has to take the form of a water tower or elevated tank, (see p. 944).

Where favourable sites exist, service reservoirs are frequently placed at the side of the town farthest removed from the point where the water supply first enters the town. The ultimate object is as before, namely, to secure a saving in the cost of mains ; but here it is not only in the supply mains to the town that the saving is effected, but also in the larger distribution pipes in the town.

It will be evident that in times of acute demand all mains near the service reservoir draw water from it, and in times of small demand the surplus water passes through the mains and refills the supply reservoir.

To accurately determine the size of such a supply reservoir (which I propose to call a terminal supply reservoir), is by no means easy, and the necessary information is also not always available.

Let us assume that the supply main is of constant diameter and that the draught per unit length from it is constant along its length. At the time of maximum demand, we wish to have a certain minimum pressure at every point of the main. This minimum pressure may be taken as about 40 lbs. per square inch as this permits a fire jet being delivered at about 60 to 70 feet above the main. Local conditions must fix the exact value and, personally, if funds permit it, the manager of a big Fire Insurance Co. is the best person to consult provided the corresponding reduction in fire insurance rates is offered.

Let D_{max} represent the demand from the whole length of the main during the hour of maximum demand, expressed in cusecs. Let L, be the total length of the main in feet. Then, referring to Sketch No. 153, let Q_1, cusecs enter at A, and supply a length L_1, feet of the main. Thus, $Q_1 = qL_1$. So also, Q_2 cusecs are drawn from the terminal supply reservoir, and $Q_2 = qL_2$, where $L_1 + L_2 = L$.

Now, if the supply be "constant," as should be the case in all good town supplies, the pressure at every point in the main AR, must not fall below a certain value, determined as above. For preliminary calculations we may take 100 feet of water.

Setting up lines to a height equal to this minimum, we get :

$$Q_1^2 = \frac{3h_1}{KL_1} \text{ and } Q_2^2 = \frac{3h_2}{KL_2}$$

where K, is written for $\frac{1}{1000d^5}$, d, being the diameter of the main in feet.

Thus, $$Q_1^3 + Q_2^3 = \frac{3h_1}{KL_1} qL_1 + \frac{3h_2}{KL_2} qL_2 = \frac{3q}{K}(h_1 + h_2)$$

and d, is to be a minimum subject to this equation, while $qL = D_{max}$.

Thus, $Q_1 = Q_2 = \frac{D_{max}}{2}$, and thus d, can be determined by :

$$d^5 = \frac{D^3_{max}}{12000q(h_1 + h_2)}$$

Now, put $D_e^2 = \frac{3k}{KL}$, where $k = h_1 - h_2$.

During all the hours in which the demand in cusecs is less than D_e, the quantity entering the main at A, is not only adequate for the demand, but also permits the delivery of a quantity equal to x, cusecs into the reservoir where:

$$x^2 + Dx + \frac{D^2}{3} = \frac{D_e^2}{3}$$

where D, represents the demand in cusecs during the hour considered.

x, can thus be calculated, and a volume of $3{,}600x$ cubic feet is stored up in the reservoir R. The minimum possible volume of the reservoir is thus given by $3{,}600\Sigma x$, where the summation includes all the hours during which D, is less than D_e.

Similarly, for the hours during which D, is greater than D_e, but less than D_{max}, we have:

y, cusecs enter the main at A, and supply a length of l_y, feet of the main, and x_1, cusecs leave the reservoir and supply l_x, feet of the main. The equations are of the same form as those given when considering the demand D_{max}, but K, is now known, and q, is no longer equal to $\frac{D_{max}}{L}$, but is, say, $q_1 = \frac{D}{L}$:

We thus get:

$$y^3 - x_1^3 = \frac{3kq_1}{K}, \text{ and } y + x_1 = D.$$

Put $\frac{x_1}{D} = z$, and we have:

$$(1-z)^3 - z^3 = \frac{3kq_1}{K}D^3$$

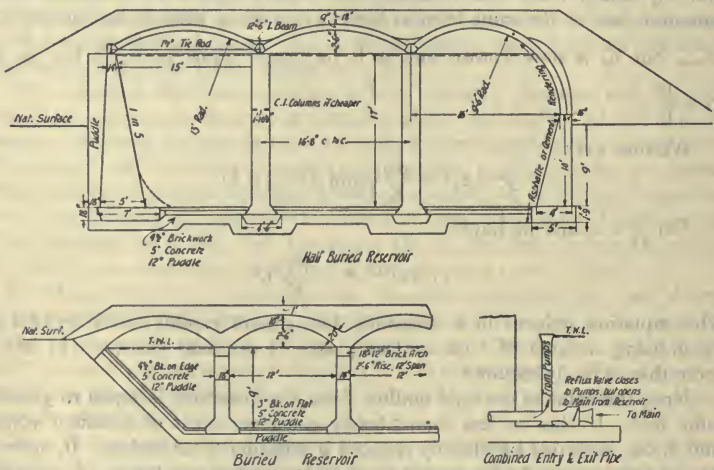
This equation reduces to a quadratic, but is more rapidly solved by trial and error, using a table of cubes. Two places of decimals (*i.e.* $z = 0{\cdot}81$ say), is more than is really required.

Now, the value of the total outflow from the reservoir is equal to $3{,}600\Sigma x_1$, cube feet. If this be less than $3{,}600\Sigma x$, the reservoir is of sufficient volume, and d, can probably be slightly reduced if considered advisable. If, however, Σx_1, is greater than Σx the supply main is not sufficiently large. A somewhat greater value of d, must therefore be selected, and the calculations of x, and x_1, must be repeated with new values of K, and D_e, until a balance between Σx, and Σx_1, is obtained.

The above calculations are probably not very useful in determining the size of the supply reservoir, but they afford a good deal of valuable information when questions concerning the necessity for reducing valves, or small service reservoirs, on isolated hills, are considered; and it is with that object in view that the subject has been considered in detail.

In practical cases the problem is by no means so simple, but the principles to be followed are the same. The necessity of affording sufficient pressure at the hour of greatest demand permits us to determine the supply pipe, which is usually of uniform diameter. Then, for the hours of small demand, we can calculate the supply which reaches the terminal reservoir, and for the hours of more intense demand, the draught from the terminal reservoir, so that the supply pipe must if necessary be enlarged until the total influx is greater than the total draught. It is advisable to secure a greater influx than the calculations of draught indicate, as such an excess may form a very useful reserve in case of accidents to the main, or when fires occur during the hours of intense supply.

In actual practice, it will usually be found that a terminal supply reservoir serves other purposes besides those which have been indicated here. Indeed, of eight cases where I am fully informed as to the reasons for their construction, only one is a purely balancing reservoir; two are in addition employed to store a quantity of water as a reserve against fires occurring in a very valuable district; three are also break pressure reservoirs; while the last two are principally considered as reserve reservoirs to permit repairs to mains crossing a river. It will therefore be plain that the above calculations cannot as a rule be permitted to solely determine the size of a reservoir. On the other hand, they should be used as a check, since the more the reservoir exceeds the volume indicated by these calculations, the more the water contained in it is



SKETCH No. 154.—Puddled Service Reservoir and Pipe Arrangements.

likely to stagnate. The arrangement of pipes shown in Sketch No. 154 is advisable, since it partly prevents stagnation, and in a pumping scheme eliminates sudden variations of the head pumped against.

DETAILS OF CONSTRUCTION OF A SERVICE RESERVOIR.—The necessary volume being determined as already described, we have to select the material. As a rule, service reservoirs are either made of masonry, or of concrete, and are located at the top of a convenient hill, or are built of steel plates, being then usually in the form of a water tower, or elevated tank (see p. 944).

Masonry, or Concrete Type.—Usually this type is the cheapest, if a suitable site is obtainable. The proportions are very much a matter of experience, and it is as well to remember that typical examples are generally situated on sites where land is valuable, so that a shallower reservoir than usual may prove economical when land is cheap.

The depths generally selected vary between 13 and 20 feet, although examples as shallow as 8 feet and as deep as 40 feet exist; but the latter are liable to prove unsatisfactory unless circumstances are exceptionally favourable.

The determining factors in design are: The means adopted to render the reservoir water-tight, and the quality of the foundations.

The early examples are nearly all made water-tight by an outer skin of puddle. I consider that this is an undesirable method, and is a relic of the days when good Portland cement and hydraulic lime were not so readily procurable, as at present. There are, however, cases where the foundations at the only available site are of such a quality as to render a layer of puddle advisable as a precaution against possible cracking of the masonry, owing to settlement. It may also happen that good puddle clay is found at, or close to, the site, when considerations of cost indicate its use.

When puddle is used the real difficulty is to prevent the angles of the masonry from fracturing the puddle lining, as it settles. This is best guarded against by having no portion of the puddle lining vertical, but laying it all either horizontally, or on a slope, and correspondingly sloping these portions of the foundations which are usually vertical.

I append designs for a covered reservoir, which seem best suited for such a case (see Sketch No. 154).

The other important points in the design are:

(i) Vertical side walls (of the type shown in the second design) are to be avoided, since they generally give trouble, probably owing to the unequal intensity of pressure which must exist across their base, due to horizontal earth or water pressure.

(ii) The pier foundations should be so proportioned as to produce the same intensity of pressure on the puddle as the water load and brick flooring combined, thus securing that all portions of the reservoir shall compress the puddle equally.

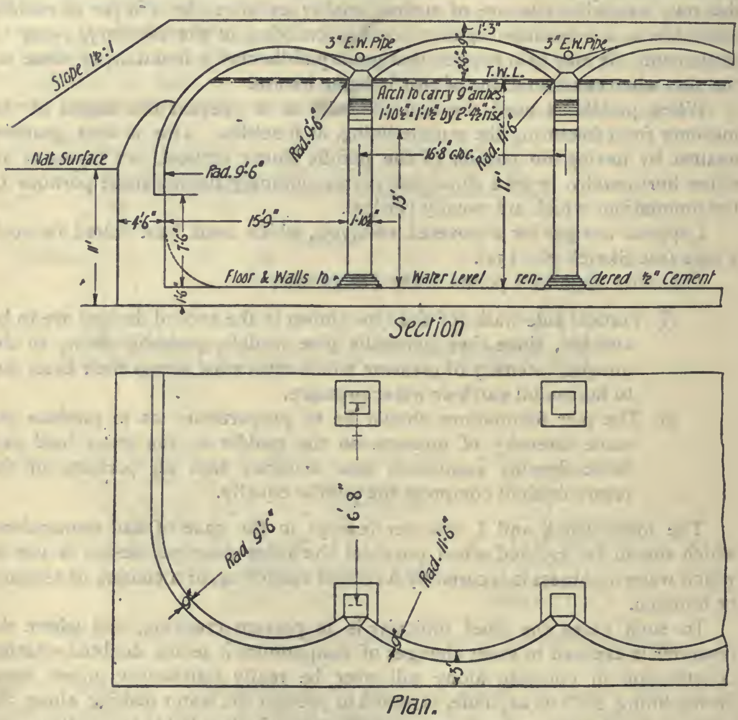
The more usual, and I consider (except in the case of bad foundations, which should be avoided when possible) the more practical design is one in which water-tightness is secured by a cement rendering, or a coating of asphalte or bitumen.

In such cases the chief difficulty is to prevent cracking, and where the reservoir is exposed to great changes of temperature it seems doubtful whether construction in concrete alone will ever be really satisfactory unless some elastic lining, such as asphalte, is added to prevent the water leaking along the cracks. Brickwork or stone masonry appears to be less liable to cracking, and in many cases water-tightness has been secured merely by a facing of hard bricks (*e.g.* those known in England as blue bricks) laid in cement mortar.

On the other hand, concrete has certain advantages, in that it can be rammed close up against the undisturbed sides of the excavation.

The design (Sketch No. 155), which includes a ½-inch rendering of Portland cement mortar, floated over with neat cement, seems to me to be the cheapest and most advantageous in good foundations. Where the foundations are not good it would appear better to thicken the walls 6 inches at the bottom, and to batter them at 1 in 8, where the earth seems likely to slip; while in cases where settlement is apprehended, an internal layer of asphalte or bitumen sheeting will prevent leakage through cracks even ¼-inch wide. In the very worst cases a rounding of the internal corner as dotted has been found to add strength to the work.

Roofing.—As shown in both designs service reservoirs are usually roofed over, and in the case of a filtered water, roofing of some sort may be considered imperative. The roofing is generally of brick arches, and should be kept as light as possible, as all it has usually to carry is about 12 inches of earth, (although in hot, dry climates it is often impossible to get plants to grow on so thin a layer of earth). The usual dimensions for such arches are 9 inches thick, up to 15 feet span. The drainage of the hollows above the springings of these



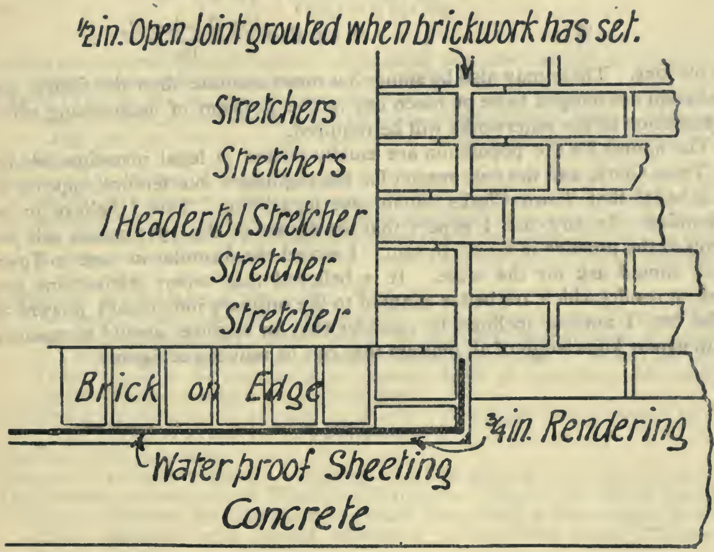
SKETCH No. 155.—Concrete Service Reservoir.

arches should be carefully looked to, and for this reason I cannot recommend the adoption of vaulted arches, since although, qua strength they possess certain advantages, an undrained pocket is formed over the top of each column.

It will be evident that reinforced concrete is well adapted to the construction of service reservoirs, and in view of the fact that reinforcement not only prevents cracking due to mechanical stress, but also a great deal of that caused by changes in temperature, it may with advantage be adopted where the changes in temperature are great, or the foundations are unusually bad. I must, however, state that in ordinarily good earth the extra expense entailed by reinforcement appears (in such cases as I have personally investigated) to outweigh the economy secured by thinner walls.

I have noted many cases where a local reinforcement has proved advantageous in dealing with a patch of bad foundation ; and I also believe that where, owing to local conditions, reservoirs of irregular form or of unusual depth have to be designed, it will prove a cheap method of construction.

While the majority of existing service reservoirs are square or oblong, a circular form is evidently economical, as the outer walls are reduced in length (for the same surface area) and it will be found that both designs are adaptable to a circular form, the only disadvantage being that the centerings for the arches and the arches themselves are somewhat more complicated.



SKETCH NO. 156.—Junction between Bitumen Sheeting and Brickwork.

In considering the application of these designs to unusual conditions it is well to remember that service reservoirs are usually placed on top of a hill ; thus ground water pressure or even wet soil pressure does not exist. I should expect all these designs to fail or, at the best, to crack if located in a valley.

POPULATION STATISTICS.—Engineers when drawing up reports concerning the water supply of towns frequently prepare tables showing the expected future increase in the population. In any examples I have seen these are prepared on the following basis :

Let P_1 be the total population say in 1881.
" P_2 " " 1891.
" P_3 " " 1901.
" P_4 " " 1911.

Then put $(1+p_1) = \frac{P_2}{P_1} \quad (1+p_2) = \frac{P_3}{P_2} \quad (1+p_3) = \frac{P_4}{P_3}$

and put $$p = \frac{p_1+p_2+p_3}{3}$$

Then P_5, the expected population in 1921, is given by $P_5 = (1+p)P_4$, or in some cases $P_5 = (1+p_3)P_4$.

The deductions such as:

$$\text{Population in 1912} = (1+p)^{\frac{1}{10}}P_4 = (1+p_3)^{\frac{1}{10}}P_4$$

or, in logarithms:

$$\log(\text{population in 1912}) = \log P_4 + \tfrac{1}{10}\log(1+p)$$

$$\text{or} \qquad = \log P_4 + \frac{\log P_4 - \log P_3}{10}$$

are obvious. These may also be somewhat more accurate than the figures for P_5, but do not happen to be of much use in the problem of determining when an extension of the waterworks will be required.

The figures for the population are usually recited in legal investigations by the Town Clerk, and the only reason for the engineer's interference appears to be a belief that Town Clerks cannot use logarithms. This I believe to be unfounded. In any case I expect that the figures of the 1911 census will put a stop to the practice in Great Britain. I record the formulæ in case a Town Clerk should ask for the table. It is believed that expert statisticians can produce results which are better adapted to the ordinary individual's powers of belief, but, I am not inclined to consider that an engineer should necessarily claim expert knowledge of all matters that can be reduced to figures.

CHAPTER XII

IRRIGATION

IRRIGATION.—Definition—Influence of agricultural methods—General description of an irrigation canal—Other irrigation systems—Effect of silt.

TERMS USED IN IRRIGATION.—Perennial and flood, or basin irrigation—Flow and lift irrigation—Canal, well and reservoir irrigation—Duties of an engineer—Bibliography—Hot and cold weather crops—Quantity of water available—Importance of local knowledge.

QUANTITY OF WATER APPLIED DURING THE GROWTH OF A CROP.—Depth of water used—DUTY of water—Base of the duty—Place of measurement—Complete specification of duty—Conversion of duty figures into depth of water—Losses in the canal and branches—Capacity of channels in terms of duty—Factors affecting the value of the duty—Excessive irrigation—Effect of cultivation—Quality of soil—Climate—Prediction of duty—Example.

Variations in the Value of the Duty.—Table of values—Values of duty as affected by the species of crop—Estimation of the duty—Number of waterings—Interval between waterings—Waterings of unusual depth—Experimental determination of the depth of a watering—Program of experiments—Seasonal variation in demand for water—Effect of area of the irrigated plots—Normal depth of a watering—Allowance for rainfall—Excessive irrigation—Rice—Marcite.

INFLUENCE OF THE RATE AT WHICH WATER IS APPLIED TO A FIELD ON THE QUANTITY OF WATER USED IN IRRIGATION.

INUNDATION IRRIGATION.

DESIGN OF IRRIGATION WORKS.—Enumeration of the structures on a large canal.

RELATIONSHIP BETWEEN THE DESIGN OF HYDRAULIC WORKS AND THEIR MAINTENANCE.—Vortices—Smooth surfaces—Punjab fall—Overflow dam—Ogee fall—Bell's dykes—Kanthak's wide crested submerged weirs—Silting tanks—Profiles—Silt berms—Scour.

REGIME.—REGULATION.

HEADWORKS.—General—Regulator across canal head—Bar across river—Low dam, or weir across river—Typical headworks—Storage dams—Undersluices—"Afflux," or flood conditions—Low-water conditions—Training works.

Working of a River for Irrigation Purposes.—Disposal of surplus water and silt.

Working of a River Carrying Sand only.—Sutlej, at Rupar.

Working of Rivers Carrying Boulders, or Gravel.—Jumna, at Tajewala—Ravi, at Madhupur—Suggestions for future working—Site for a canal head—Constricted channel above a weir—Bell's dykes.

WORKING OF INUNDATION CANALS.—Selection of site for canal head—Temporary gates—Stone pitching.

WEIRS.—Types—Description.

TYPE A.—American and Indian designs.

TYPE B.—Kistna weir maintenance—Godaveri weir—Sand or clay foundations.

TYPE C.—Description.

Design of Weirs.—Errors of record plans—Aprons—Curtain walls—Talus—Impermeable and permeable portions of a weir—Breadth of impermeable portions—Breadth of downstream apron and talus—Breadth of downstream apron—Thickness of apron—Thickness of dam wall—Reversed filter.

IRRIGATION.—The term irrigation is used to describe the processes connected with the artificial application of water to land for agricultural purposes.

There are probably few, if any, countries in which agriculture is practised where irrigation is entirely unknown. In this book, however, the practice is considered only under circumstances such that irrigation on a large scale forms a routine portion of the agricultural operations. The expression "routine" is employed in place of "essential," since modern investigations have shown that satisfactory crops can be obtained by special methods of cultivation under circumstances which are usually held to render irrigation absolutely necessary. Similar processes are well known to Indian agriculturists, but are employed only when local circumstances are highly favourable.

The boundary between irrigation operations and the purely agricultural processes employed in cultivating the soil, is very indefinite. It may indeed be said that the purely agricultural processes have far more influence in determining the best methods of irrigation than the climate of the locality, or the character of the soil. Similarly, the method actually adopted in irrigating an area of land affects both the crops grown and the necessary agricultural operations to a considerable extent.

A complete treatise on irrigation can therefore hardly be produced without the collaboration of an agriculturist, an engineer, and a lawyer. Such a treatise has not yet appeared, and would obviously either refer to an ideal state of affairs, or would merely be a handbook of little more than local interest.

My object is to deal with the design of irrigation structures in general. I therefore refer to local conditions as rarely as is consistent with clearness. It will, however, be noticed that the effect of local conditions appears at every point.

My own experience was acquired in the Punjab (Northern India), but has been supplemented by a personal study of the conditions occurring in Egypt, Ceylon, and the Californian irrigation districts, with a less detailed study of irrigation as practised in Lombardy.

The general arrangement of the chapter corresponds to the conditions existing on a modern irrigation canal in the Punjab. These canals present the most complicated problems normally found in irrigation.

The works consist of the following structures :

(i) A low weir, or dam, thrown across a river, with the training works required to control the river, which is frequently torrential, and is always subject to heavy floods.

(ii) The head works of the canal, which are mainly designed with a view to excluding silt.

(iii) The main canal, which usually crosses several subsidiary drainage channels; which necessitate syphons under, or aqueducts over the canal, for the disposal of the flood discharge of such drainages. Escapes are also required in order to dispose of surplus water, or to remove silt deposits.

(iv) The distributary and minor canals from which irrigation is actually effected.

(v) The field watercourses, by which the water is conveyed to the individual fields in which the crops are grown.

The problems arising in the design of such a canal include those which occur in the design of any normal irrigation system. The only two exceptions are as follows:

(*a*) In Southern India (and of late years in the Western United States and Egypt) the flow of the river is not only diverted, but is stored up in large reservoirs for use during periods when the natural flow would not suffice for the requirements of the irrigation system. The design of such reservoirs and their outlet works is treated in Chapter VII.

(*b*) In Egypt, and less markedly in Southern India, some portion of the irrigation is effected by rapidly flooding the land, and after the water has thoroughly soaked in, the surplus water is drained off. While the actual details of the methods adopted in such cases are extraordinarily complicated, and can only be satisfactorily treated by a long study of the local conditions (see p. 649), the principles of the design of the works are amply covered by a study of the sections on Escapes, Canal Regulators, and Syphons.

As a rule, the problems are far more simple than those which exist in the Punjab. Where the water is directly derived from a clear water stream, or from a reservoir, silt problems do not exist, and the design of the canals is simpler.

Practical experience, however, leads me to believe that this is by no means an unqualified advantage. An engineer accustomed to silty waters can always effect certain economies in construction and maintenance by skilful design, and it is by no means certain that these economies do not amply cover the expenditure in silt clearance which is required in a well-designed system. A clear water canal need not necessarily be provided with a head regulator, but it is doubtful if this economy is not generally attained at the cost of very inefficient working of the whole system, unless escapes are provided which are sufficiently powerful to act as regulators.

Terms used in Irrigation.—It is absolutely impossible to give a glossary of the technical terms used by irrigators in various countries. The Urdu (Northern India) irrigation vocabulary contains more than 100 terms, each with a definite meaning, and it is still incomplete. The following English terms are selected from those which possess the most extended geographical currency, and are believed not to conflict with any local terminology.

Irrigation is said to be perennial when water is applied to the land under crops at a fairly equable rate during the whole season of crop growth. When a large proportion of the irrigation water is secured by deeply flooding the land, and the crop growth is afterwards wholly or partially sustained by the moisture thus stored up in the saturated soil, the term "flood, basin," or "inundation" irrigation, is employed. The terms "perennial," and "flood," are not necessarily mutually exclusive, although they are frequently so used, as in most cases flood irrigation is supplemented by minor waterings.

The terms "flow," and "lift" irrigation are used in order to specify whether the water level is such that it will naturally flow on to the land, or has to be lifted artificially by means of various types of machines. Flood irrigation must necessarily be by flow (*i.e.* from a river, or canal), although the supplementary waterings frequently assume the form of lift irrigation; whereas, perennial irrigation may either be by lift, or by flow, and the same source of supply may obviously be used for either method.

We may also divide perennial irrigation and the supplementary waterings which may or may not be required in flood irrigation, into classes specified by the source of the water.

From this point of view three main classes exist :

(i) Canal irrigation, where the water is drawn from a canal which takes out from a river.

(ii) Well, or subsoil water irrigation.

(iii) Reservoir, or tank irrigation, where the water is stored in reservoirs.

Canal irrigation may either be by flow or by lift, but is usually by flow.

Well irrigation is usually by lift, but irrigation by flow from artesian wells is sometimes possible.

Tank irrigation is nearly always by flow, but occasionally a modified form of flood irrigation is obtained by cultivating the banks of the tank as the water level falls. In such cases, the crops usually receive one or more waterings by lift from the tank.

The terminology is hopelessly confusing. This is illustrated by the present state of certain portions of Egypt which receive flood irrigation in August and September, supplemented by well irrigation until March. From March to July the land is perennially irrigated by lift from canals ; these, from March to April, are supplied by the natural flow of the Nile, which is supplemented during May, June and July in steadily increasing proportions by water which has been stored in the Assouan reservoir. In practice, however, there is usually but little doubt as to the manner in which any definite area of land is irrigated, and it will be found that the terms employed have a real application, since the various conditions thus specified produce very different methods of cultivation.

In flood irrigation, the engineer is chiefly concerned with the clearing out of the canals before the flood, and with the prevention of breaches in the flood banks during the flood. He has but little responsibility in regard to the quantity of water supplied, as that principally depends upon the height to which the flood rises. After the flood has subsided his duties become very similar to those of an engineer in charge of perennnial irrigation. The Egyptian methods may be taken as standard, and reference may be made to Willcocks' *Egyptian Irrigation*, and to Barrois' *Les Irrigations en Egypte.*

In perennial irrigation, the engineer is most concerned with the design and maintenance of the canals which distribute the water over the land, and he is responsible during the whole year for the provision of a supply of water sufficient to meet the demands of the agriculturists. The Indian methods may be taken as the standard in this case, and reference may be made to Buckley's *Irrigation Works in India,* and also to Mullins' *Irrigation Manual.*

Italian methods are also good, but the available literature is small. Baird Smith's *Italian Irrigation*, although fifty years old, is still quoted in Milan as a reliable book of reference.

The works of Willcocks and Barrois (which is of later date) give a good idea of Egyptian methods of perennial irrigation ; but it must be remembered that Egyptian perennial irrigation is as yet young, and we can hardly be certain that the differences which exist between Egyptian and Indian methods will prove permanently advantageous.

In reservoir irrigation, the engineer has to consider the design and maintenance of reservoirs. Here, the actual methods of irrigation differ but slightly from those obtaining under perennial irrigation, and the books referred to above may be consulted with advantage.

The main problems in lift irrigation are the mechanical difficulties connected with the design of pumps, and their maintenance in a high state of efficiency. The civil engineer is chiefly concerned with the location of the channels and the methods of preventing leakage from them. This portion of the process has been sadly neglected, and, since every drop of water has to be lifted at a certain cost, the question deserves more attention than it has as yet received.

The Californian methods of thus economising water are standard, and the publications of the State Irrigation engineer, of the University of California, and of the United States Geological Survey, may be consulted.

In the majority of countries where irrigation is practised, the climate is sufficiently hot to produce crops all the year round. Such a condition favours good financial returns, as the money invested in irrigation works can then earn interest during the major portion of the year; whereas, if only one crop can be grown in the year, interest for twelve months has to be earned by six or perhaps eight months of cultivation.

Putting aside special cultures such as fruit trees, meadows, or sugar cane, which occupy the land for the whole, or a very large portion of the year, and intermediate crops which are sown and harvested at special seasons, it will usually be found that two very different classes of crops are grown in a year. These may be called the cold weather crops (which consist of staples such as wheat, barley, vetches, and other temperate zone cultures), and the hot weather crops (which consist of the more pronouncedly sub-tropical cultures such as cotton, millet, maize, and rice).

The varieties of crops vary from country to country, and localities exist where wheat and rice, or flax and cotton, are grown simultaneously. As a general rule, however, the distinction is well marked, and owing principally to the hotter temperature (the species of the crop does not appear to influence the question to any great degree, although rice and sugar cane always consume more water than the typical cold weather crops), the hot weather crops as a group consume far more water (roughly twice, or even three times the depth) than those grown during the cold weather. This fact gives the engineer ample opportunity to exercise his energies in adjusting the available supply of water so as to produce the best financial results. General rules cannot be given. In some cases, the problem is solved by cultivating a greater area during the cold weather than during the hot. In other cases (especially where the rivers rise in flood during the hot weather), the hot weather culture is taken as standard, and the canals are proportioned for the hot weather demand, the cold weather cultivation being fixed by the available supply in the river. In considering these problems the engineer must be chiefly guided by local experience, and the cases in which he is in a position to determine *a priori* the exact area of crops, and the method of culture, are infrequent. This is probably a blessing in disguise, for (except in newly settled countries) local experience has usually effected a very accurate adjustment of the methods of cultivation to local necessities. The work of an engineer is then usually best confined to the improvement of existing small scale practice in a scientific manner. Very careful investigations will usually show that while improvements in small details (such as the introduction of new manures, or varieties of plants) can be effected; yet, broadly speaking, local customs are the result of long ages of evolution, and possess all the adaptions and fitness necessary to secure their survival that characterise a well-established natural species of plant or animal. This statement must in no wise be considered as adverse to agricultural experimental stations. Such experimental stations form an integral portion of any large irrigation scheme, but the duty of the engineer is to modify his channels and works so as to suit the alterations in cultivation as they are adopted, rather than to hurry forward their adoption by means of premature modifications of the irrigation system. It is but small consolation to know that the system is admirably suited to a certain mode of cultivation, when the crops on the ground would be better served by channels designed on different principles.

It must however be borne in mind that a cultivator who enjoys an unlimited supply of water, will almost invariably be found to be using at least twice the quantity he really requires to mature as good a crop as he obtains with excessive water. The question should always be investigated, and the methods given on pages 642 and 647 will usually provide ample proof and indicate the remedies. The introduction and enforcement of these remedies is another and much broader question which I do not presume to answer. Personally I believe that rapid improvement is only possible in cases of lift irrigation, where the fuel bill forms a powerful and effective argument.

Quantity of Water applied during the growth of a Crop.— A certain amount of water is required for the development of any plant. This quantity varies considerably from an absolute minimum which will just keep the plant alive, up to an absolute maximum, which renders the ground so wet that the plant is ultimately killed. Putting aside such considerations, let us confine our attention to the quantity of water which is actually used in practice by skilled agriculturists. This quantity can be expressed in the same manner as rainfall, *i.e.* as a depth over the whole area covered by the crop.

Thus, let an area of a, square feet be occupied by a crop, and let the total quantity of irrigation water applied to mature this crop be v, cubic feet.

Then, the depth of irrigation water used is plainly $d=\frac{v}{a}$ feet.

The precise definition of the period during which the volume v, is applied, must of course depend upon local circumstances. As a rule, v, includes all the irrigation water used during the period between the first preparation for the crop, and its final removal from the ground. Water used in preparing the land preliminary to ploughing and harrowing, prior to sowing, should be included in the above irrigation water.

Irrigation engineers define the "duty" of a given quantity of water, as the area of cropped ground of which the agricultural requirements are satisfied by that quantity of water. That is, when the duty of 1 cusec of water during a period of say October to April, is 180 acres, the agricultural requirements of 180 acres during this period are satisfied by a continuous flow of 1 cusec.

In its primary sense duty has no connection with the maturing of crops, or with the quantity of water required for this purpose. Thus, if we state that in the Punjab the duty for the month of November is 170 cusecs, we merely mean that 1 cusec flowing continuously during that month suffices to provide a satisfactory water supply for the agricultural requirements of 170 acres under crop; and it must not be inferred that any individual acre of these 170 necessarily receives any, or still less, a proportionate, volume of water during the month.

If we wish to ascertain the depth of water supplied to the land (not necessarily applied to the cropped area, unless the cusec spoken of above is measured not far distant from the cropped area), we must also define the base, *i.e.* the period over which the duty is reckoned. Thus, the month of November having 30 days, converting 1 cusec for 30 days into cubic feet, and acres into square feet, the depth of water applied is :

$$\frac{30\times 24\times 60\times 60}{170\times 9\times 4840}=\frac{2\times 30}{170}=0{\cdot}35 \text{ feet,}$$

since

$$\frac{24\times 60\times 60}{9\times 4840}=\frac{1200}{605}=1{\cdot}983=2 \text{ approx.}$$

Since the agricultural operations in November usually consist of a single watering, as a preliminary to ploughing, it appears that this "ploughing watering" (which is usually heavier than the waterings which are applied later on in the season), is about 4 inches in depth. But, as it will be shown later, this does not necessarily mean that the cropped area is covered with water to a depth of 4 inches in the sense that a 4-inch rainfall would "cover" the area.

So also, if we are told that the duty of 1 cusec for the cold weather crops (wheat, etc.) is 202 acres, we cannot determine the actual depth of water used unless we know the number of days during which the canal supplied this 1 cusec. For example, let the "base" be 170 days. The depth of the water consumed is $\frac{2\times 170}{202} = 1{\cdot}67$ feet, say 20 inches.

Taking another example, the basin canals of Egypt flow for 40 days, and the water covers the basin to a depth of approximately 5 feet. Consequently, the duty is :

$$\frac{2\times 40}{5} = 16 \text{ acres per cusec.}$$

As another example. Willcocks (*Egyptian Irrigation*) gives figures relating to the perennial irrigation of cotton, maize, etc., which permit the following table to be drawn up :

Year.	Areas under Crop in Acres.	Discharge of Canals in Cusecs.	Duty, Acres per Cusec.	Depth of Water applied in Feet.
		For the month of May.		
1889 . .	1,200,000	8,120	148	0·42
1892 . .	1,500,000	12,360	121	0·51
		For the month of June.		
1889 . .	1,200,000	7,060	170	0·35
1892 . .	1,500,000	10,590	142	0·42
		For the month of July. (From the 1st to 15th only.)		
1889 . .	1,200,000	10,590	113	0·27
1892 . .	1,500,000	13,060	115	0·26

The crops cultivated consisted principally of cotton and similar "dry" crops, with about 10 per cent. of rice and "wet crops." In 1889 a great part of the rice perished, and the cotton crop was inferior. In 1892 the supply was inadequate. Thus, the figures 0·51 foot and 0·42 foot represent bare minimal "depths" for an area covered by the above combination of crops in a climate such as that of Egypt in May or June. So far as the circumstances are known to me, I believe the cropped area actually received about 75 per cent. of the above depths (see pp. 632 and 641).

The usual duties in this locality (Egyptian Delta) are 113 acres per cusec for cotton and dry crops, and 63 acres per cusec for rice. At this rate, the duty requisite for the satisfactory irrigation of a combination of the above crops in the ratio given should be 102 acres per cusec; or about 0·60 foot depth of water per month of 30 days; and a small pumping scheme should probably be designed to lift about 0·45 foot depth of water per month.

The place where the "duty" is measured must also be stated; for it is plain that if the 1 cusec is considered as measured at the head of the canal, the total corresponding volume of water will not reach the cropped area, as a portion will be lost by evaporation, and an even greater fraction by leakage from the canal during passage from its head to the irrigated land. The term duty can therefore only be held to be completely defined when the following are stated:

(i) Its base, or the number of days over which it is measured.

(ii) The place where the water is measured, and what allowances (if any) are made for losses during the passage of the water from the place of measurement to the irrigated area.

If Δ represent the duty expressed in acres per cusec, and B, the number of days in the base period, we see that the depth of water "used" during the base period is given by:

$$d = \frac{2B}{\Delta} \text{ feet}$$

although it must clearly be understood that unless the duty is measured at the field, d, in no way represents the actual depth of water which the crop receives.

As an example, it may be stated that on the Bari Doab Canal Kennedy found as follows:

One cusec entering the canal is, on the average, reduced by absorption and evaporation to 0·80 cusec when it reaches the head of a distributary, and to 0·74 cusec when it reaches the outlet from the distributary to the field watercourse, and to 0·53 cusec when it reaches the field. Thus, on the assumption that $\Delta = 170$ acres per cusec, when measured at the head of the canal, and $B = 170$ days, which is fairly close to the average results for the cold weather crops, we find that:

The depth of water which matures a crop if measured at the canal head is 2 feet;

The depth of water which matures a crop if measured at the distributary head is 1·60 foot;

The depth of water which matures a crop if measured at the outlet is 1·48 foot;

and the depth actually applied to the land is 1·06 foot only.

These figures are surprising, but are confirmed by Ivens' results on the

Ganges canal, where one cusec becomes 0·85 at the distributary head, 0·78 at the outlet, and 0·56 at the field.

The results refer to very long canals, and if comparison is desired with American or Egyptian conditions, it would probably be fairest to consider the heads of these canals as corresponding to the heads of distributaries in Indian canals. Nevertheless, even under these circumstances, the depth "used," if calculated on the duty of water passed into the canal, may exceed the actual depth which is received at the field by 33 to 40 per cent.

It must, nevertheless, be remarked that the loss from village watercourses occurs in the cultivated area, and can hardly be regarded as without influence on the maturing of the crop. Hence, we may consider that the figure "Duty at outlets," or "At head of village watercourse," although no doubt in excess of what is required, represents the water which matures the crop.

The figure is peculiarly important to an engineer, as it represents the quantity of water which must be lifted when it is desired to irrigate an estate of an area of 2000 to 5000 acres by pumping water from a canal or river.

The meaning of such terms as "cold weather duty at head of canal," and "hot weather duty at head of village watercourses" now become plain. When the number of days on which the canal is open during the "hot" or "cold" weather season is stated, the depth of water applied to the land can be calculated. It must be remembered that while the depth calculated from "duty at head of village watercourses" is very nearly equal to the depth actually applied to the land (the only losses being leakage in the short village water channels), the depth calculated on the "duty at head of canal" is far greater than the actual. The losses in addition to those in the village watercourses arise from leakage from the canal and its branches, plus the loss caused by any water that is passed out of the canal by the escapes. In regard to this latter loss, since it is visible, and is easily measured, due allowance can be made if necessary.

The above remarks show that the term duty needs to be clearly defined, but for that very reason its investigation forms the most practical method of determining the size of each channel of a canal. Thus, given that a canal is required to irrigate A_c, acres of crops, and that the duty during a certain period at the head of the canal is Δ_H, the average discharge of the canal during this period must be :

$$\frac{A_c}{\Delta_H} \text{ cusecs.}$$

So also, if the duty during the period in which the consumption of water is a maximum be Δ_M (*e.g.* in the Punjab the period is approximately from October 15th to November 15th) the maximum supply for which the canal must be proportioned is $\frac{A_c}{\Delta_M}$ cusecs ; the base being the interval between successive waterings.

Next, taking a distributary, or branch canal, irrigating A_d acres, the average supply is $\frac{A_d}{\delta_d}$, where δ_d is the average duty during the season considered, measured at the head of the distributary, and the maximum supply is $\frac{A_d}{\delta_m}$. Similarly, the sizes of the smaller channels can be determined, when the duties measured at their heads during the various periods of the year are known.

The quantity of water required to mature a crop varies considerably. The most important factors affecting the question are :

(i) The skill of the agriculturist, which is shown not only in the actual application of water, but also in the whole process of cultivation.
(ii) The species of crop.
(iii) The character of the soil, in which must be included not only its geological constituents, but also the depth down to subsoil water, and the amount of manuring and previous irrigation it has undergone.
(iv) The rain-fall, temperature, and other characteristics of the weather during the period for which the crop is on the ground.

In the long run, the first factor is more important than all the others. As a general principle, it may be stated that there is a certain minimum quantity of water which will mature a good crop. This quantity varies from year to year under the influences enumerated above, but any application of unnecessary water is injurious, and the ultimate result of excessive irrigation is a permanent diminution of the fertility of the soil. Unfortunately, an excess of water allows a good crop to be raised (in the early years of the practice) with far less labour than is necessary if the minimum quantity is used. Therefore, the personal interests of an irrigator (especially in newly developed countries) are adverse to the permanent fertility of the soil. Consequently, a skilled agriculturist who adapts himself to circumstances, will employ methods in a thickly settled country which differ widely from those which would be adopted in a newly developed area.

For instance, in India, wheat is generally raised with about 2 feet depth of water ; while in America, 4 feet is not considered abnormal. Both these quantities greatly exceed the possible minimum, and yet I consider that both are correct, and that both should be adopted in their respective countries if individual interests alone are considered. It will be evident that Indian practice is far better adapted for preserving the soil in a state of permanent fertility. Broadly speaking, any great excess over the minimum indicates an incorrect method of applying water, but the question will be discussed at length later.

For the present, we can generally state that a skilled irrigator (especially in countries where he cannot readily obtain new land) will raise a good crop with far less water than an unskilled man, but the labour entailed is greater. So also, careful cultivation of the soil during the growth of a crop, by manuring, mulching, and hoeing, diminishes the amount of water required.

The species of crop grown also largely influences the quantity of water required to mature it. Typical figures are given for Indian conditions, and it will be seen that the variations thus produced are very large (p. 640).

Again, the character of the soil affects the quantity of water used. It will be evident that a loose, sandy soil absorbs more water than a heavy clay, and if in addition the subsoil water level is close to the surface of the clay, and is far below that of the sand, these differences will become even more marked. In one particular case, I found that a crop of wheat grown on sandy soil required three times the depth of water that sufficed to mature a similar crop grown by the same agriculturist on an adjacent patch of clay soil.

In actual practice, however, the effect of the quality of the soil is somewhat obscured by the fact that crops requiring but little water are usually grown on soils which absorb water readily, and *vice versa*. Consequently, the extra water absorbed by the soil is partly balanced by a saving in that necessary for crop growth.

The effect of climate is important in the following cases :

(i) If it is proposed to introduce a new crop into a district, a study of the water requirements of that crop in other localities, combined with the differences in rain-fall and mean temperature during the crop season in the two localities, gives valuable results.

Thus, in considering the cultivation of cotton in Mesopotamia (see Willcocks, *Irrigation of Mesopotamia*) in ordinary years it is found that a rain-fall of about 2, to $2\frac{1}{2}$ inches occurs during the early portion of the season of cotton cultivation. Consequently, one less watering is required than is necessary during a similar period in Egypt where no rain occurs. Towards the end of the cotton-growing season, however, the mean temperatures are considerably higher than in the corresponding period in Egypt. It is therefore believed that during this period one, or even two waterings more than are given in Egypt will be necessary. The total depth of water required is therefore but little more than in Egypt, but its distribution during the period when the crop is on the ground differs greatly from that prevailing in Egypt.

(ii) The effect of the variations in rain-fall and mean temperature, from year to year, on the quantity of water required to raise a crop, are by no means so marked as might at first sight be expected. This is due to the fact that unless the crop season is continuously rainy, evaporation from the wetted ground after each rain storm is (in climates where irrigation is usually practised) so intense that the soil rapidly becomes baked, and its surface caked hard. Consequently, a fresh wetting, either by irrigation, or by rain, is needed very soon afterwards.

If the season is continuously wet, agriculturists are encouraged to plough and sow land which is not usually cultivated. This land is likely to absorb more water than land which is regularly irrigated, and crops needing more water than those usually grown will be raised on some portion of the area that is regularly irrigated. Hence, in a country which is well adapted for irrigation, the effect of a continuously wet year is not so much to increase the duty of the water, as to permit irrigation of a bigger area, and the raising of an unusually large proportion of crops requiring a great amount of water.

There is, however, another and financially very important aspect of this question. There are many localities where the rain-fall is of such a magnitude that the staple crops can be grown without irrigation during about one-half of a long series of years, but require more or less irrigation during the remaining years of the period. In such cases, it is quite probable that un-irrigated crops will fail for five, six, or even ten consecutive years. A demand for irrigation is thus produced ; and it frequently happens in cases where careful preliminary investigations have not been undertaken, that irrigation works are completed just at the beginning of a spell of wet years. In such cases, any discussion of the duty of water is futile, and owing to the almost certain financial failure of the scheme, unnecessary.

The above will, I hope, render it plain that the duty of water is a matter that cannot be readily predicted. In actual practice, it will be found that not only each canal, but also each estate supplied by a canal, has its own peculiar duty. These differences will ultimately (and under careful and progressive management, very rapidly) adjust themselves by slight and almost unperceived modifications in the sizes of the distribution channels. In the design of a canal, however, some average figure must be assumed, and it may at once be said that no matter deserves more careful preliminary study.

The principles are fairly plain. We must, as far as possible, ascertain the duties actually obtained in similar localities with crops such as will be grown on the area which it is proposed to irrigate. A careful estimate should also be made of the areas likely to be irrigated, and local opinion canvassed as to the crops proposed. It will then be possible to discover whether differences exist in the area to be irrigated which are likely to cause any portions to demand more or less water in proportion to their extent.

By such means, we can arrive at a figure for the average duty over the whole area, and can broadly determine whether any portions require special treatment.

The matter is further complicated by the fact that in most irrigated countries the climate is such that two, or even three crops, can be raised in the year, and these crops will usually require very different quantities of water. The complete solution will therefore take into consideration not only the crops which are likely to be raised, but the water which is available at different seasons of the year, and the relative values of the possible crops.

It is necessary to remember that any large error in selection may lead to financial failure by forcing the agriculturists to grow unprofitable crops. Fortunately, the engineer is assisted by favourable circumstances; he can afford to "guess high," in doubtful cases, and can justify his attitude by two undoubted facts,—Firstly, in the early years the agriculturists will be more or less inexperienced, and will use more water than in subsequent years. Secondly, during the early years of irrigation better crops are obtained from virgin soils by heavier watering than is later required.

Hence, the logical attitude is to design the canal for a duty which is somewhat less than that usually obtained. Later on, as matters approach their normal state, measures may be taken to increase the duty, and the water thus saved can be utilised in extending the irrigation to fresh land. Indeed, the most marked feature of a really successful canal is its steady growth; and the channels are, in the long run, more likely to prove too small, than too large.

The above considerations are best exemplified by actual examples. I select the following:

The first is Willcocks' discussion of the possibilities of irrigation in Mesopotamia (*Irrigation of Mesopotamia*, 1905), which may be considered as a reconnaissance in an almost unstudied country.

From local observation it is found that the crops grown are very similar to those of Egypt, but that the summer temperatures are somewhat higher, while the winter temperatures are slightly lower. The exact figures are tabulated by Willcocks (*ut supra*, p. 31), who assumes that the cold weather crops will be those of Egypt, and will not require more water. For the hot weather, he finds that maize and similar crops are grown during the season May, June, and July; while in Egypt similar crops and similar temperatures occur in August and September. Thus, the duty during May and June is taken as equal to that of the period August and September under perennial irrigation in Egypt. In the months of August and September, cotton alone remains on the land, and consequently only one-third of the area is occupied.

The assumptions produce a state of affairs under which the water requirements of the area agree very fairly well with the supply in the river, although the demand is but small during the months of March and April when the river supply is apparently at its maximum.

The assumptions are probably justified. They really mean that from November to March the whole irrigated area is under such crops as wheat and pulses. From May to July the whole area is under maize and cotton, and in August and September one-third of the area is under cotton, the maize and other crops having been harvested.

The duties are calculated as 86 acres in the hot weather, and as 244 in the cold, reckoned on the gross area. This latter figure is high, and it is assumed that a development of well irrigation equal to that now prevailing in Egypt will take place, otherwise a smaller duty would have to be assumed; probably approximately 160 acres if no well irrigation is adopted. The assumption can be relied upon, for should well irrigation not develop, it will be possible to give the extra supply required in order to provide for deficiencies in well irrigation, since the canal has three times the capacity required for the assumed cold weather duty. Therefore, trouble need not be anticipated until the irrigation of the whole country is so far developed that the cold weather supply of the river is insufficient for the requirements of the total irrigated area. So far as can be gathered, the river would suffice to irrigate about five times the area for which the project is drawn up.

The crux of the whole question lies in the excessive reliance placed on the cotton crop; for should this not become an important staple, the demand is not likely to fall from the calculated value of 7000 cusecs in July, to 2800 cusecs in August; and if this does not occur, the supply in the river may ultimately be deficient in August and September.

As a contrast to Willcocks' reconnaissance, the method employed in the project for the triple Punjab canals, namely, the Upper Jhelum, Upper Chenab, and Lower Bari Doab, may be given. Here, the country is accurately surveyed, and the present state of cultivation is recorded almost too minutely. The gross area "commanded" (*i.e.* that which is irrigable if only the relative levels of the water and the land are considered) by the Upper Chenab canal is 1,608,618 acres. Of this, 5 per cent. is assumed to be occupied by roads and buildings, or otherwise unculturable. 75 per cent. of the remainder is assumed to be the combined quota of canal and well irrigation. The remaining 20 per cent. is either low-lying land, flooded by small streams, or which is otherwise capable of producing a crop without the assistance of irrigation, or forms a reserve against possible water-logging (see p. 747).

The irrigable area is therefore . . .	1,146,141	acres.
Existing well irrigation is	497,773	,,
So that the canal irrigation will be . . .	648,368	,,

In the actual project, the canal irrigation area is given by districts, and the percentages 5 per cent. and 20 per cent. are by no means assumptions, having been arrived at after careful enquiry and a study of the existing population and methods of cultivation.

Half of the above area is assumed to be irrigated in the hot weather (May to September,—Kharif), and half in the cold weather (October to April,—Rabi).

The Kharif duty is taken as 100 acres per cusec, this being a figure obtained from experience on well-maintained and skilfully cultivated canal systems some 40 and 60 miles away. The gross discharge is therefore 3242 cusecs, plus an allowance for absorption (see p. 738) in the main canals. Experience has also shown that it is advisable to be able to give an extra supply of 25 per cent. in cases of emergency. We thus get the following table, where the capacity of each branch has been obtained by a similar calculation :—

Channel.	Cusecs Discharged.		
	Ordinary Supply.	Extraordinary Supply, $\frac{5}{4}$ Ordinary.	Absorption at 8 Cusecs per million square feet of wetted surface when the Extraordinary Supply is running.
Main line, upper .	No irrigation	No irrigation	253
Main line, lower .	1156	1442	679
Raya branch . .	1050	1314	130
Nokhar branch .	435	546	65
Khadir branch .	600	750	34
Total . . .	3242	4052	1161

So that the Upper Chenab canal has to carry 5213 cusecs for its own requirements. It also carries the whole supply of the Lower Bari Doab canal, which is syphoned under the river Ravi. This canal irrigates what is practically a desert. It is assumed (from previous experience in similar canals) that 85 per cent. of the gross area will be culturable. The commanded area is divided into two well-marked divisions, namely, the Bar, or high lands, where the subsoil water is 60 to 70 feet below the ground surface ; and the Bet, or low lands, where this depth is 33 feet in the Ravi Bet, and 45 feet in the Beas Bet.

Thus, the percentages of irrigation permitted are assumed as :—

	Per Cent.			Per Cent.		Acres.
In the Bar,	75	of culturable area	=	63·75	of gross area	= 547,373
In the Bet,	50	,,	=	42·5	,,	= 335,155
						882,528

The difference is due to the possibilities of water-logging. It must be remembered that these percentages will probably be exceeded in certain districts in the course of years, as the likelihood or otherwise of water-logging becomes better known.

It is realised that in this district since the Monsoon rains (*i.e.* Kharif season) are not sufficiently heavy to permit any cultivation on other than irrigated areas, it would be advisable for one-third of the above area to be cropped in the Kharif season, and two-thirds in the Rabi season. The supply in the river does not permit this; and the assumption that half the area should be Kharif cropped, and the remainder Rabi cropped, although financially less desirable, has to be made. The canal is therefore proportioned at 1 cusec per 100 acres for 441,264 acres, with an allowance of 25 per cent. extra for emergency supplies, and an allowance for absorption in the main line and large branches calculated at 8 cusecs per million square feet of wetted area (see p. 738).

We thus get:

Channel.	Cusecs Discharge.		
	Ordinary, *i.e.* $\frac{\text{Kharif Area}}{100}$.	Extraordinary: $\frac{5}{4}$ Ordinary.	Absorption.
Main line . . .	No irrigation	No irrigation	314
Montgomery branch .	3549	4436	544
Gugera branch . .	698	872	107
Shergarh . . .	166	208	This is a distributary, so no absorption is allowed.

The total capacity at the head is therefore . .	6481 cusecs
Add the quantity used and absorbed in the Upper Chenab	5213 „
Total capacity of Upper Chenab canal at its head	11694 „

The above is probably the most complete and carefully investigated project which has ever been made. It is, consequently, as well to state that in some respects it is not precisely a model one. If immediate financial returns were alone regarded, there is little doubt but that a greater percentage of area would be irrigated, and that this area would be afterwards reduced in those districts where water-logging became manifest. The policy followed is to allow such a percentage of irrigation that it will never be necessary to deprive any land which has once been irrigated of canal water, and therefore large extensions may be hoped for in most districts. If financial returns are alone considered, the ratio of Kharif and Rabi irrigation now adopted would be somewhat changed.

The Rabi duty is 200 acres per cusec, so that were the irrigation divided into two-thirds of the total area Rabi, and one-third Kharif, the supply in each season would be the same, and the total capacity of the two canals (all allowances being decreased in the same ratio) would be about 7463 cusecs. Consequently, a notable economy in first cost could be secured.

The question of the methods of regulation of the supply which are entailed by the fact that the maximum demand in the Rabi season is appreciably less than the maximum Kharif demand is discussed later (see p. 741). While no difficulty in the actual irrigation is experienced, once the system is in good working order, it will be plain that if the seasonal demands were rendered approximately equal, many of the masonry works for regulating the water level and distributing the supply into the various channels in turn could be dispensed with. The extra cost entailed by the larger Kharif supply is therefore not fully expressed by the extra capacity required in the earthen channels.

A study of the river discharges indicates that a supply which would permit this ratio of Kharif and Rabi irrigation, could probably be obtained in nine years out of ten, and that the deficiency in the tenth year would be small. The improvident habits of the Punjabi, combined with the fact that the crops during the Kharif season form the staple food of the population, fully justify the course adopted. But if the population were better able to cope with a bad season, the Rabi irrigation could be largely increased, and the Rabi crops are at present the more profitable. My own experience on a canal which was similarly situated to the Lower Bari Doab leads me to believe that the Rabi area will probably increase in a greater ratio than the Kharif; and we may expect that the final result will be something more like 700,000 acres of Rabi, and 500,000 acres of Kharif, in place of the 441,000 acres of Rabi, and 441,000 acres of Kharif, provided for.

Regarding the matter in this light, the real meaning of the allowance of 25 per cent. of extra supply becomes somewhat more plain. An engineer designing such a system for profit alone would consequently increase the Rabi area to the maximum permitted by the supply in the river, and would irrigate only such Kharif area as the capacity of the canal that will serve the Rabi area (with allowances for extra supplies) will permit.

Variations in the Value of the Duty.—The considerations detailed above will render it plain that the "duty" of water in a given locality is almost as variable as the rain-fall. In Egypt the rain-fall is practically nil, and therefore the "duty" of water used in perennial irrigation might be considered as likely to be fairly constant. As a matter of fact, putting aside all years when the water supply is deficient, and fairly good crops (in fact in some years of moderately deficient water supply the term "good crops" is applicable) are raised on a limited supply of water, we find that the duty varies, although not so greatly as in India, from year to year, owing to such influences as extra manuring, the substitution of cotton for sugar cane, etc. The Indian figures are even more variable, principally owing to variations in the rain-fall. As the Indian conditions are more closely akin to those which generally prevail in irrigated countries, rather than to those of Egypt, I tabulate a selection of the figures given in the official reports on the subject. These figures are calculated on the average discharge at the heads of the canals, and if it is necessary to compare them with Egyptian or American values, they should be increased by about 25 per cent. If the figures are used to calculate the water which is actually applied to the fields, a base of 180 days can be taken, and the depths thus obtained should be multiplied by 0·55 or 0·60.

The Sirhind canal irrigates an area which is somewhat more sandy than the land commanded by the Bari Doab. The higher Rabi duties of the Sirhind, as compared with the Bari Doab, during the years 1894–1900 are due to the silt troubles in the Sirhind, which are known to have caused a considerable economy in water during this period. It is for this reason that I have selected these two canals as examples, since the figures form a proof of the principle that strenuous efforts in economising water will enable an agriculturist to overcome such handicaps as sandy soil, etc.

Year.	Duties in Acres per Cusec.				
	Bari Doab Canal.			Sirhind Canal.	
	Kharif Duty.	Rabi Duty.	Month of November.	Kharif Duty.	Rabi Duty.
1893–1894 .	...	...	111	...	...
1894–1895 .	70	112	112	20	77
1895–1896 .	68	135	144	44	156
1896–1897 .	73	139	172	78	197
1897–1898 .	82	170	177	93	164
1898–1899 .	77	162	209	66	182
1899–1900 .	74	164	201	87	198
1900–1901 .	99	149	148	101	110
1901–1902 .	86	221	198	55	177
1902–1903 .	...	...	202	...	...
Base . .	180 days approx.	160 days approx.	30	180 days approx.	160 days approx.

While the absolute volume of water used is highly variable, a fairly close connection exists between the relative depths of water consumed by different crops in the same locality, and during the same year.

Using the depth of water consumed by a crop of wheat as unit, we find in India as follows:

Crop.	Depth Ratio.	Duration of Crop Season.
Wheat	1	5 months
Barley	0·8	5 "
Vegetables	1·5	4 to 6 months
Tobacco	2·5	6 months
Lucern and other permanent forage .	3·0	6 "
Gardens	2·0	6 "
Sugar cane	4·0	10 to 11 months

It so happens that the irrigation water applied to the wheat crops was, on the average, equivalent to a depth of 10 inches, and that 2 inches more was received as rain, but the figures for the absolute depth of water received by any crop are far more variable than the ratios tabulated above. It must also be remembered that the relative profits obtainable from an acre of crop have a great influence (*e.g.* compare tobacco with vegetables) on the quantity of water an agriculturist will apply. Thus in localities where the prices differ from Indian rates this factor alone may produce marked variations. In Egypt the influence of the extra manuring given to valuable crops frequently causes the water consumption of a well-manured field to be relatively much less than the species or value of the crop would indicate.

Estimation of the Duty of Water for any Crop, or Locality.—A study of the duties recorded on page 640 will show that they are so variable that any enumeration of the figures which are accessible on the subject would be as useless to an engineer who is preparing an estimate for an unstudied locality, as tables of observed rain-falls would be if they were used to estimate the rain-fall of a locality where observations were deficient. Consequently, the following discussion deals with the accumulation of facts by which local ideas on the water requirements of a crop can be checked, and its chief object is to enable the effect of the changes in the methods of applying water to the land on the water requirements of the crops to be estimated.

It will be found that while an agriculturist has, as a rule, the vaguest (and often most erroneous) ideas of the quantity (or total depth) of water which is applied to a crop, he has always a very definite knowledge of the number of waterings which a crop receives, and of the intervals between the successive applications of water. His knowledge on these two points may be regarded as incapable of improvement; and it will also be found that he not only knows the number of waterings given in a normal year, but has a very fair idea of the effect of an excess above, or deficiency below the normal rain-fall, and is acutely conscious of the longest period that can elapse between waterings without damage to the crop. This knowledge should not be taken as final, for the two following reasons:

(i) The small pioneer irrigation systems from which this knowledge is frequently derived are generally devoted to raising crops such as garden produce (*i.e.* melons and green vegetables, rarely potatoes, or vegetables which are able to stand transport), or fodder (such as green oats), and the more valuable crops such as opium, or condiments. These crops usually require more water than the less valuable staples which will be grown when a well-developed system of irrigation is introduced. The error thus produced is but small, as in the early years of irrigation the agriculturist will apply his experience of the above crops to the staple crops, and if the engineer does not give him sufficient water to permit this, the development of irrigation will probably be slow.

(ii) If the development is sufficiently advanced to permit of staple crops being raised under irrigation, it will usually be found that these staples all require water at the same time; and a Professor of Agriculture will be able to show that, in theory at any rate, far larger areas of mixed crops could be irrigated with the available water. This opinion must be neglected in practice. We are concerned not with "things as they ought to be," but with "things as they are." If, by a judicious adjustment of the water rates, and by scientific trials of new agricultural products, the agriculturist can eventually be persuaded to economise water, fresh tracts of land can be irrigated.

The practical result therefore is to accept the local custom as fixing the number of waterings which we are prepared to apply, and to lay out the system so as to permit of its extension. This is most easily effected by stopping each small distributing canal somewhat short of its maximum possible length; and the ideas of scientifically trained agriculturists on the subject of new crops and possible economies in water may be accepted in this matter. On the other hand, when considering the supply which is to be given to the area, which it is proposed to irrigate at the start, local custom should be held as binding.

The number of waterings being thus determined, the depth of each watering has to be considered. This is best obtained by an actual measurement of the water used (the best method is a triangular notch), and of the area covered. The expense is not heavy. A careful study of the depths of five waterings covering an area of seven acres (including a complete record of the weight of the crop) can be made (professional salaries and travelling expenses apart) for £50 to £60, and if the co-operation of an intelligent agriculturist can be secured, this expense can be decreased. I strongly recommend that at least three such studies should be made, on areas selected so as to permit the influence of the quality of the soil being determined. If, in addition, the rate at which the water is applied is varied, still more valuable results will be secured. Consequently, the fullest information would be secured if the depths of water applied under the nine following conditions were measured:

I. Sandy Soil.
- (*a*) Water applied at the rate of $\frac{3}{4}$ cusec.
- (*b*) " " 1·5 "
- (*c*) " " 3 "

II. Medium (loamy) Soil.
- (*a*) Water applied at the rate of $\frac{1}{2}$ cusec.
- (*b*) " " 1 "
- (*c*) " " 2 "

III. Heavy (clayey) Soil.
- (*a*) Water applied at the rate of $\frac{1}{3}$ cusec.
- (*b*) " " $\frac{2}{3}$ "
- (*c*) " " 1 "

The conditions specified above are selected in order to obtain information as to the rate at which water can best be delivered. The larger quantities are therefore suggested in sandy soil. The absolute quantities are selected from the experience of agriculturists using manual labour only, and in a flat country. Where machinery is employed, larger rates of flow might be advantageously applied. In a steep country, smaller rates of flow must usually be adopted. It will also be plain that if the cost of obtaining information were alone considered, it would be better to apply the larger rates of flow in the clayey soil, for a flow exceeding two cusecs is somewhat difficult to handle in sandy soil.

The depth of a watering as thus determined will usually be found to be fairly constant, except that the depth of the first watering (usually termed the "ploughing watering," although it may either precede or succeed the actual operation of ploughing), is some 25 per cent. greater than those given when the crop is in the ground. The second watering is, as a rule, a little deeper than the others; but the difference is only detected by weir measurements. It will, however, be found in certain crops that deeper waterings are desired at particular stages of the crop growth. In countries where irrigation has been practised for many years, any watering which is dignified by a special name must, as a rule, be suspected to be deeper than the average. The question is usually not an important one, but the matter must be recorded; for, should one of these deeper waterings occur simultaneously with a low-water stage, in the river or other source of supply, its depth may prove to be the factor which fixes the total possible area of irrigation.

Thus, in the Punjab, the depth of the ploughing watering combined with the possibility of a simultaneous low stage of the rivers during the period from the 15th October to the 15th December, and more acutely from the 1st November to the 30th November, will ultimately fix the total irrigable area.

In Bengal, a similar sudden demand for water from all rice cultivators, just before the rice harvest, taxes all channels to their utmost; and if the cultivators could be induced to grow varieties of rice which did not require this last drenching, the channels would probably irrigate an area greater by 50, or 60 per cent.

In Egypt, on the other hand, the acute demand (in cotton cultivation at any rate) is not produced by any abnormal requirements of the crop, but occurs in May, June, and the early part of July, when the crop requirements are normal, but the supply in the Nile is low. In this country, when the present reservoir and reclamation schemes are fully carried out, it is probable that the requirements of the maize crop during the ploughing watering will finally prove to be the factor limiting the extension of irrigation.

Such studies (when properly carried out) will permit the duty of water as measured at the field to be determined. The losses which occur between the river and the field can be estimated by the figures given on page 738, or better still, systematic gaugings can be made on existing channels by the methods already discussed. The triple canal project quoted on page 637 shows the method of allowance for the losses in the major canals, and a similar calculation can be applied to the minor canals and watercourses.

In many instances, however, the expense of such studies is grudged; and in any case an engineer must be prepared to give preliminary figures which will justify an expenditure of even £60 on "mere theory."

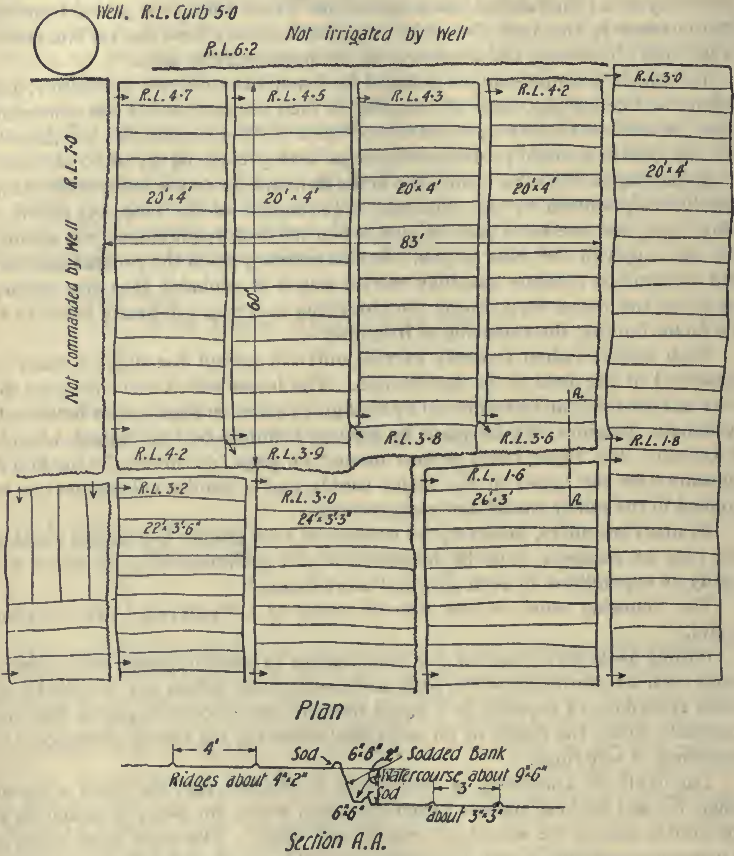
The following notes on the normal depth of a "watering" are therefore useful.

Putting aside such crops as rice, which grows in standing water, and valuable crops such as garden produce, fruit, and drug plants, which are supplied with water regardless of expense, it is found that the species of crop grown does not materially affect the depth of an individual watering, but merely influences the frequency of waterings.

The depth of a watering is determined by the fact that the water is cut off when the soil farthest removed from the point where the water is turned on to the land is seen to be wetted, or covered with water. The extra depth given in a ploughing watering is probably largely caused by the fact that the agriculturist does not cut off the water until the clods have fallen in pieces through being soaked in water. Possibly a desire to saturate the ground to a certain depth, and thus to render it more easily ploughed, may also have some influence.

The depth of a watering is therefore fixed principally by the time which the water takes to cover the whole area of each plot into which the field is divided. During this period the portions of the plot nearest to the point where the water enters are covered with water which is absorbed by the soil. Thus, the least depth of a watering is secured when the field is divided into very small plots, each of which is rapidly covered with water. Finally, the smaller the individual plots, and the greater the rate of flow of the water, the smaller is the depth which is needed over and above the minimum requirements of the crop.

In the case of irrigation from wells where the cost of lifting the water is the chief expense, and the rate of flow is but small, these individual plots are small, and resemble flower beds rather than fields, being, say, 16 feet × 4 feet as a



SKETCH No. 157.—Typical Field, irrigated from a Well.

minimum, and 50 feet × 10 feet as a maximum. (See Sketch No. 157 and contrast with Nos. 220 and 221.) Such small areas are not advisable in canal irrigation, and in India areas of 110 feet × 55 feet are standard, the water being applied at a rate of 0·3 to 1 cusec, or even more in some cases. In fact, practical experience has taught all agriculturists experienced in irrigation to make a very equable balance of the advantages between an economy in water secured by dividing the fields into small plots, and the extra labour entailed in constructing these plots. Thus, on the average (putting aside special methods employed for valuable crops), the normal size of a plot in any district bears a very close relation to the normal rate at which water is applied in that district. The irrigation engineer will probably consider that the plots are too large, but this conclusion will only be arrived at by underestimating the value of the labour expended in making the small banks surrounding the plots. I consider that the standard Indian area is a very fair balance between an engineer's ideal and an agriculturist's wish for rest. In countries where labour is more valuable, larger plots will be desirable; but the question is very intimately connected with the value of the crop. The introduction of machine cultivation is at present a factor which tends to increase the size of the plots, but this is only a transient phase of the question. When harvesters are adapted to traverse the irregularities formed by the ridges surrounding the plots, and when machinery is employed for making these ridges, it will certainly be found advantageous to decrease the size of the plots.

The fact that the greater the rate at which the water is applied the larger is the average area of the plots, causes the depth of a watering to vary but little, and actual measurements indicate the following:

Well, or lift irrigation. 0·15 foot to 0·25 foot.
Canal, or flow irrigation. 0·25 foot to 0·40 foot.

As a general rule, we may assume ploughing or other heavy waterings to be 0·33 foot. Succeeding less heavy waterings should be taken as 0·25 foot. These figures assume the presence of skilled irrigators. Where the water is carefully economised, such figures as 0·25 foot for the heavy waterings, and 0·20 foot for the lighter ones, are attained. In careless irrigation, such figures as 0·50 foot for the heavy waterings, and 0·40 foot for the lighter ones, may be expected, and the waterings usually succeed each other at much closer intervals than a skilled irrigator finds requisite.

The temperature and other climatic conditions seem to have very little influence on the depth of a watering, and the effect of all such conditions will be found to be amply allowed for by the variations in the intervals between successive waterings.

In careless irrigation, however, the fields are not usually divided into plots, and the water is allowed to flow over a large area, with the result that the portions near entry of the water are deeply saturated, while the further boundaries of the field suffer from a partial drought. Such methods are advantageous to no one; and, unless the custom is extremely firmly rooted, an engineer will hardly be well advised to design his channels with dimensions which tend to indefinitely prolong the practice.

In areas where a marked slope, or irregularities in the surface of the soil, exist, an extra quantity of water must be applied in order to fill up irregularities in the natural surface. This extra amount can be estimated by levelling a few sample fields, allowance being made for the fact that the above depths of water are sufficient to submerge small irregularities such as clods, and furrows, and that the extra depth now considered is required merely to fill up those depressions and irregularities that exist independently of agricultural operations.

The above principles permit the local duty of water for staple crops to be estimated with sufficient accuracy. Variations will occur from year to year, owing to abnormal rain-fall or temperatures, but these are incidental to all agricultural operations. These irregularities are amply covered by the extra capacity of 25 per cent. which is given to the channels in the triple canal project (see p. 637). In selecting the exact figure which is to be assumed as the duty of the water, we must of course consider not so much the duty for the whole period for which the crop occupies the ground, as the greatest intensity of demand that occurs during the life of the crop. The dimensions of the channel are therefore finally calculated so as to supply the average demand when running nearly full bore, and the most intense demand when carrying a little less than the extraordinary supply.

For example, consider the month of November, in the Punjab. Nearly all the area irrigated during the cold weather receives a ploughing watering. One cusec therefore at the fields has a duty of $\frac{2 \times 30}{0 \cdot 33} = 180$ acres. Allowing for absorption in the smaller branches of the canal, one cusec at the head of the distributary has a duty of $\frac{180}{1 \cdot 2} = 150$ acres, say. Thus, the extraordinary supply will just permit $150 \times 1 \cdot 25 = 187 \cdot 5$ acres to be irrigated in the thirty days, from a channel the full supply capacity of which is one cusec. The period of intense demand having thus been tided over, the crops (unless assisted by showers of rain) will require an ordinary watering every thirty days. Thus, one cusec at the distributary heads will water $\frac{2 \times 30}{0 \cdot 25 \times 1 \cdot 2}$ or 200 acres, and the real meaning of the statements "Rabi duty is equal to 200 acres per cusec," and "It is desirable to be able to give an extraordinary supply of $1 \cdot 25 \times$ ordinary supply," become obvious.

Allowance for abnormal rain-fall in the year of observation may be made by the following rule :

On the average, every 2 inches of extra rain-fall occurring in casual showers replaces 1 inch depth of water, as applied to the fields. The figure is the result of an examination of fifteen years of statistics of the duty and rain-fall of eight small areas in the Punjab, by the method of correlation coefficients. The rule is only an average one, although the values of the eight correlation coefficients cluster very closely (0·45 to 0·57) round the value ½. The rule evidently does not express the relative efficiency of rain and irrigation water in supporting the life of crops (I believe that rain is far more efficient than irrigation water); but expresses the fact that waterings are only given when required, while rain is equally likely to fall immediately after a watering, when not required, as at the moment at which a watering would otherwise have been given. In countries where rain falls at fixed, or nearly fixed dates, or when the weather continues "rainy" for periods comparable to the normal interval between successive waterings, the principle laid down by Willcocks (see p. 634) may be considered as correct; and in the Punjab I have frequently heard old farmers speak of the "*regular*" Christmas rains of their youth, which "were as good as an extra watering." Nevertheless, the expression *laudator temporis acti* must not be forgotten in this connection.

There remains one matter to discuss, namely,—How far, and to what degree should an engineer in preparing his project allow for methods of cultivation which he knows are wasteful of water?

The question is a very important one, for such methods not only waste water, but tend to ruin the land by the seepage water and alkaline salts which they produce. Wasteful irrigation is consequently injurious not only to the land, but also to the corporation supplying the water, and to the whole community and its future interests. Local conditions must decide the question, but I have no hesitation in saying that the smaller the latitude which is permitted, the better; and that an engineer who allows such conditions to become permanent will, sooner or later, find his employment gone, merely because irrigation will have ceased to be profitable, and extensions are no longer required.

The worst cases (and also the most skilful irrigation in the world) occur in America. The publications of the Department of Agriculture (Bureau of Soils), and of the Geological Survey (Reclamation Service) will provide a surfeit of evidence. The matter, put in a nutshell, resolves itself into the statement that if wasteful methods are permitted, the irrigation channels must be doubled, and the drainage channels trebled in size.

These remarks must not be taken as referring to the cultivation of such crops as rice, or the winter meadows of Lombardy. Rice needs from five to six times as much water as other crops grown in the same climate. It also requires drains in order to carry off this water, and its cultivation is therefore unlikely to produce alkalinity unless such drainage is neglected. Rice is not a crop to be encouraged, and in Lombardy it is not permissible to grow it within 1100 feet of a town. The flight of the Anopheles (malaria-carrying mosquito) being about 200 to 300 yards, the object of this old regulation now becomes plain. Winter meadows, or marcite culture, are in reality methods of keeping grass warm during the winter, with a view to fostering growth which would otherwise be arrested. Such methods dispose of water which would otherwise run to waste, and since they require a well-drained gravel subsoil, they are not likely to cause water-logging; but, nevertheless, the first signs of water-logging must be carefully looked for. Details will be found in Baird Smith's *Italian Irrigation* and in Scott Moncrieff's *Irrigation in Southern Europe.*

Influence of the Rate at which Water is Applied to a Field on the Quantity of Water used in Irrigation.—When water is applied to irrigate an area of land, what actually happens is that the water covers the land to a certain depth (say y, feet), and then flows on to cover more land. Now, over the whole wetted area water is percolating into the soil. We can therefore say that if water is applied at the rate of Q, cusecs; then, at a time t, when an area of A, square feet is covered with water, we have the following:

$$Q\,dt = y\,dA + pA\,dt$$

where p, is the rate at which water percolates into the soil from the area which is already wetted by the water. Thus:

$$t = \frac{y}{p} \log_e \frac{Q}{Q - pA}.$$

Now, the irrigation of any plot of land is not considered to be complete until all the plot is wetted. Thus, the time taken to irrigate a plot of an area A_o, is

$$t_o = \frac{y}{p} \log_e \frac{Q}{Q - pA_o}.$$

Now, in actual practice p, may be taken as $\frac{15}{1,000,000}$, for water is absorbed by sandy fields at an average rate of 21 cusecs per million square feet, and by loamy fields at a rate of 8 cusecs per million square feet (see p. 739).

Let us assume that $y=2$ inches $=\frac{1}{6}$th foot.

Thus $t_0=11{,}111 \log_e \frac{Q}{Q-pA_0}$.

Now, consider a plot of 6000 square feet in area, which is that which is usual in India. $pA_0=0{\cdot}09$ cusec.

Now, if there were no absorption :

$Qt_0 = 1000$, or $t_0=\frac{1000}{Q}$ seconds, and the quantity used is 1000 cubic feet.

Q, in Cusecs.	t_0, when Absorption is 15 Cusecs per million square feet. In Seconds.	t_0, when there is no Absorption. In Seconds.	Actual Quantity used when Absorption is taken into Account.
2	510	500	1020 cubic feet
1·5	688	666	1032 ,,
1	1044	1000	1044 ,,
0·75	1411	1333	1058 ,,
0·50	2200	2000	1100 ,,
0·25	5065	4000	1266 ,,

The great increase in the quantity of water used as Q, falls below 0·50 cusec must be noticed, and it is plain that if 2 inches are assumed to be an adequate depth, an irrigator who applies water at the rate of 0·25 cusec only must give his crop, on the average, a watering equal to 2·5 inches. Under such circumstances, the crop near to the entrance of the water must receive far more than it needs. The case becomes still worse if the irrigator is lazy, and endeavours to save labour in constructing watercourses by doubling the area of his plots. We then have, $t_0 = 11{,}111 \log_e \frac{Q}{Q-0{\cdot}18}$, and with :

$Q = 1$, $t_0 = 2200$; or, the volume used is 2200 cubic feet in place of 2000 cubic feet.
$Q = 0{\cdot}5$, $t_0 = 5066$; or, the volume used is 2533 cubic feet in place of 2000 cubic feet.

The circumstances may at first sight appear to be abnormal, as it is plain that if $y = 1$ inch, the advantages of the greater rates of flow are less marked. The value $y=2$ inches agrees fairly well with observation, and the quantities arrived at by these calculations are confirmed by accurate gaugings over triangular notches of the volumes actually applied to measured areas by skilled irrigators, who knew the object of the experiments.

Certain tests of the quantity of water used by agriculturists in ordinary work appear to agree better with $y = 2{\cdot}5$ to 3 inches. Under these circumstances the advantages gained by a rapid application of the water are of course still more marked. The experiments were mainly conducted on virgin soil, where heavy waterings are known to be advantageous. It is therefore believed that the figures for $y = 2$ inches best represent the circumstances when measures are taken to secure an economical utilisation of water, and irrigation is well established.

Kennedy and Ivens working independently on the Bari Doab and Ganges canals came to the experimental conclusion that irrigators waste a large proportion of the water which they receive. The figures are startling. Out of 1 cusec taken in at the head of the canal Kennedy found that the irrigator received 0·53 cusec, and "utilised" 0·28 cusec. Ivens' figures were 0·56, and 0·29 cusec per 1 cusec taken into the canal (see p. 631). The efficiency of Indian canals is probably higher at the present date, since owing to the systematic enforcement of rules concerning the sizes of individual plots (see p. 732) the irrigator "wastes" less water. It may, however, be doubted whether the difference is all "waste." The water saturates the soil, and is probably to a certain degree utilised by the crop, especially if this is a deep-rooted plant such as wheat or cotton. Nevertheless, it has been amply proved that an engineer working on methods deduced from a study of absorption can raise a better crop with a less depth of water than an ordinary irrigator under similar circumstances.

The fact that when an irrigator chooses to adopt the engineer's methods, and in addition utilises his agriculturist's knowledge, he produces a far better crop with less water than the engineer, is a very fair proof that the methods are correct, however disappointing the result may be to the engineer (see *Punjab Irrigation Branch Paper No.* 12).

Hence, we may finally deduce the following rules.

The engineer should arrange to deliver the water to irrigators in as large a stream as they can handle. This rate will depend upon the available labour, the character of the soil, and on the general slope of the country.

The irrigator can economise water by dividing his fields into plots of an appropriate area, and irrigating each separately from watercourses so arranged as to supply each plot individually as in Sketch No. 157, which also indicates the extreme care a skilled irrigator takes to avoid "uphill" irrigation of the individual plots (see also Sketch No. 220).

The best rate at which water should be delivered, and also the appropriate size of the plot, can be investigated mathematically; but the plan of experiments sketched on page 642 will not only settle these matters practically, but will also afford the engineer actual figures to justify his proposals.

The irrigator can also economise water by carefully smoothing the ground previous to irrigation. This is effected in India by drawing a flat beam of wood across the newly sown ground, thus crushing down the clods and irregularities.

The special methods adopted in the irrigation of tobacco, opium, gardens, or orchards, are hardly sufficiently important to affect the average consumption of water over a large area. Such special methods usually produce an economy in water, as only a fraction of the area is wetted, and hence the absorption during irrigation is reduced.

Inundation Irrigation.—The figures connected with inundation irrigation are purely local in their application, and cannot be determined by the methods which have been previously discussed.

The success or failure of inundation irrigation is settled by the two following conditions:

(*a*) The amount of water absorbed by the soil.

(*b*) The mean temperature of the season succeeding the inundation.

The conditions prevailing in Egypt may be considered as typical. The land should be submerged for fifty days, but in bad years a submergence of six or seven days only may be obtained. A crop can then be raised, but not a good one.

The flood water is drained off the land from about the 10th to the 30th October, and the crops are sown at once.

The mean temperature at Cairo is as follows :

October	73·5	degrees Fahr.
November	65·3	,, ,,
December	58·1	,, ,,
January	55·1	,, ,,
February	56·7	,, ,,
March	72·4	,, ,,

The crops grown during these months are those cultivated in Temperate climates. It is quite plain that if the mean temperature were materially less than the above figures, no crop could be grown on account of the cold. So also, if the temperature were such as to favour the cultivation of Tropical crops, it is open to doubt as to whether the saturation produced by one inundation would suffice to mature such crops unless the soil was unusually retentive, and the flood was of long duration. Thus, pure flood irrigation requires a somewhat nicely adjusted relationship between the duration and depth of the flood, and the mean temperature of the season which succeeds the flood. If the question is ever considered in practice the publications of the U.S. Department of Agriculture give useful information concerning the mean monthly temperatures required for healthy growth of cultivated plants.

The average depth of flooding in Egypt is 3·30 feet (1 metre). Depths exceeding 2 metres do not occur, apparently owing to the cost of the banks required to retain the water ; and a flooding of less than 1·6 feet (0·5 metre) is considered injurious. The figures are considerably affected by such questions as the quantity of silt in the water, and the permeability and retentiveness of the soil.

In some Indian examples, where the banks of a reservoir are cultivated as the water is drawn off, good crops are raised if the land is only covered to a depth of 6 inches for ten days. In these cases, the subsoil water level is but slightly below the surface, and the crops usually receive one or even two waterings by lift from the reservoir, or from the subsoil water, towards the end of their growth.

DESIGN OF IRRIGATION WORKS.—As will later appear, the logical method of designing an irrigation system is from the tail upwards. The preparation of a large project is, however, only likely to be undertaken by engineers who have some experience of irrigation work.

The usual work of an engineer, or designer, is concerned with the design of structures of which the main dimensions are already determined. Thus, it seems best to describe and consider the rules for the design of irrigation works in the order in which the water flows down the channel. I shall therefore describe :

(i) The diversion weir and headworks of a canal.
(ii) The escapes.
(iii) The aqueducts, syphons, and other works required to dispose of drainage water.

(iv) Bridges and other accessory works.
(v) The head regulators of branch canals.
(vi) Falls and rapids.
(vii) The preliminary design of a branch canal.
(viii) The final design of a distributary.

The question of silt arises in nearly all these structures, and is treated incidentally; but a special section is devoted to silt problems.

Buckley's *Irrigation Works* and Wilson's *Irrigation Engineering* contain numerous detailed plans of individual works. Bligh's *Design of Irrigation Works* (second edition) is almost indispensable. I have been forced to criticise some of Bligh's ideas rather closely, but am in full agreement with at least 95 per cent. of the book. I would, however, state that I believe some of the hydraulic coefficients adopted are rather high if applied to the small works that a young engineer will usually design. For large works such as occur in India they are excellent.

The sections of works that I give must be considered as illustrating principles rather than the best available examples. I have also endeavoured as far as possible to select only works I have personally examined. Engineers unacquainted with local conditions should bear in mind that Indian materials, especially bricks and mortar, are at the best only second class material when judged by British standards. Egyptian materials are better, but Egyptian irrigation is still directed by Indian-trained engineers, and I believe this influences the designs very materially. As an endeavour to sum up a very complex matter I would state that I believe the area of most of the designs shown is not excessive, but that given good material some of the thicknesses might be reduced. American designs if carefully tested for weak details are probably safe, and such material as I have seen is usually good.

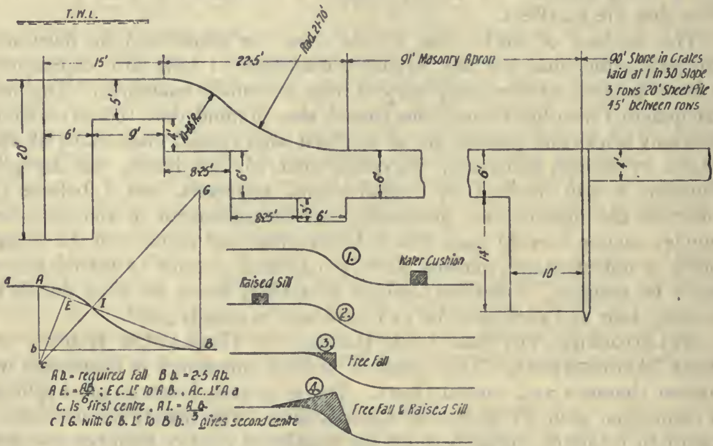
Relationship between the Design of Hydraulic Works, and their Maintenance.—This subject is of most importance in connection with earthen channels and natural rivers. The principles, however, are applicable in connection with all hydraulic constructions. The matter is discussed in regard to irrigation works, because the penalty of neglect then becomes most noticeable. The general principles may be summed up in two sentences, as follows:

The formation of vortices, or irregular currents, should be avoided. Preventive methods are both cheaper and more effective than repairs.

The first principle is very well illustrated by the facts given on page 719 when discussing the pitching of rapids. It is there shown that those rapids which cause the least trouble have smooth pitching, and that this smoothness has been attained by intelligent maintenance. So also, the Punjab type of fall (if carefully considered) is plainly designed with a view to preventing the formation of vortices, or whirls. These illustrations are the more striking when the history of the development of the types now adopted is known.

The present designs are then seen to be the outcome of long experience by engineers who were obliged to study the cost of maintenance very carefully, and whose improvements were severely criticised in the light of theories based on the idea that "the energy of the falling water should be dissipated in boils and whirls." It is almost amusing to read the struggles made by various chief and superintending engineers in order to obtain a "boil" somewhere, and the equally strenuous endeavours on the part of the men who have inspected the damage to get these "boil"-producing angles filled in with concrete or cheap rubbish. It may be said that the theory is still alive, and that the present designs are accepted only because they do their work so well that the theorists are not often given an opportunity to criticise.

The present American designs for overflow dams betray the same errors. The usual idea is that the "rough masonry of the ogee, or other curve, breaks the falling water into foam." The statement is correct, and the breaking up process does no one any harm, because the face stones of a well-made dam are very large, and are very firmly fixed in comparison with the thin sheets of water that they break up. The falling sheet of water, however, usually arrives at the bottom of the dam not as foam (unless it is a thin sheet), but as a mass of whirling water, which can and does scour out pebbles and sand. The water cushions which are sooner or later constructed below every "overflow" dam



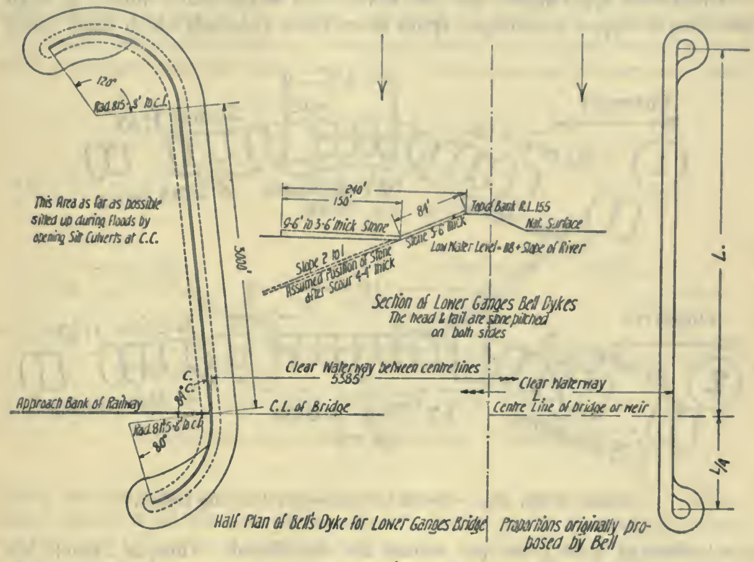
SKETCH No. 158.—9-Feet Ogee Fall.

that normally acts as an overflow, show the correct method of dissipating the energy of the fall (*i.e.* not by friction of water on stone or earth, but by friction of water on water). It is quite probable that better results could be obtained by carefully dressing the masonry to a smooth surface, were it not that this class of work is expensive.

Thus, we may consider that the ordinary rough-surfaced overflow dam is successful only because the dam is strong enough to resist the shocks caused by the falling water without material damage. It is also doubtful whether the usual overflow dam, even when constructed of first class material, would prove satisfactory if it were continually acting as an overflow. A proof of this statement is difficult, but my list of dam failures includes what is apparently an excessive proportion of overflow dams which retained volumes of water which were small in comparison with the annual discharge of the stream supplying the reservoir. The list does not pretend to be a complete one, and therefore the suggestion is put forward with diffidence. The Austin dam failure (see pp. 346 and 394) may be taken as an example. In this case the foundations appear to have been naturally weak, and although the failure can be explained without any assumption of damage by the action of the overflowing water, it is known that some damage had occurred.

Similarly, the Habra dam (see p. 372) was also greatly subject to overflow, and though its working stresses were no doubt high, they were not so abnormally high as to necessarily produce failure, independently of overflow action.

The old ogee falls (see Sketch No. 158), which were constructed of second-rate brickwork, had to be made smooth to prevent their destruction. In consequence, the energy was dissipated downstream of the masonry, and the scour holes thus produced sooner or later undermined the masonry work of the



SKETCH NO. 159.—Plan of Bell's Training Dykes.

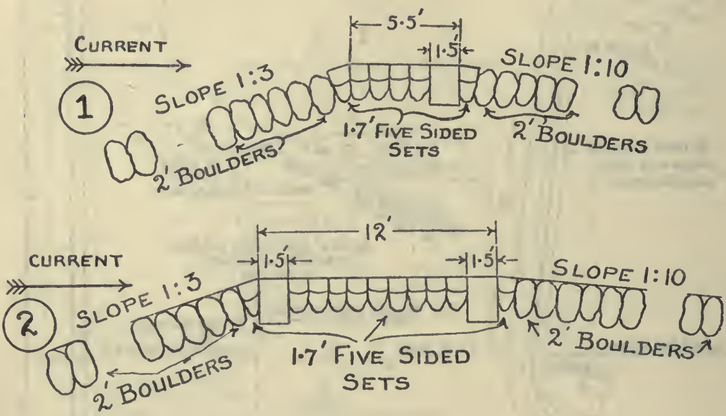
fall. The raised sills and steps indicated in the sketch show various methods that were tried in the hope of producing a better design. These are evidently first steps towards the modern water cushion, but the angles are evidently inadvisable as tending to produce boils.

Bell's training dykes (see p. 667) show a very logical development of the correct principles. The large scale plans of such training works (*e.g.* Sketch No. 159) show the manner in which the principle is applied. Nevertheless, it is found that the principle can be carried still further with advantage. The banks are faced with two or three feet of rubble stone, and, according to the usual theories, this should be laid so as to present a rough surface to the river. In practice, however, experience shows that a smooth face is advantageous, and Bell recommends that not only should the rubble be hand-packed to a smooth face, but that where possible the stone should be selected so as to assist in obtaining a markedly smooth face. When it is realised that some of these training works consume rubble stone in quantities of 10, to 20 million cubic feet and more, it will be plain that the advantages secured must be very considerable in practice.

Kanthak applied the principle at Madhupur. The conditions prevailing at Madhupur are discussed on page 662. When submerged during floods the training dykes are exposed to an intense erosion from water carrying boulders with velocities exceeding 20 feet per second.

The old design for the narrow crests and rough slopes of such dykes is shown in Fig. 1, Sketch No. 160. Fig. 2 shows the broad crest with smooth slopes introduced by Kanthak. The improvement has produced a material saving in maintenance costs.

The above applications are concerned with water which moves at high velocities, but equal advantages result when earth channels which carry water



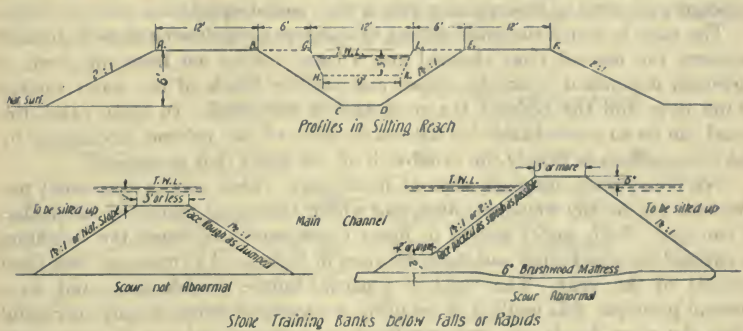
SKETCH No. 160.—Crests of Submerged Training Dykes.

at velocities of 2, or 3 feet per second are considered. Thus, in Sketch No. 161, ABCDEF represents the banks of a canal as constructed, and GHKL, the section of the channel as calculated from the designed bed slope, and the discharge of the channel.

At first sight it would appear legitimate to leave matters to Nature, and to expect that the extra area will silt up more or less rapidly. As a matter of fact, a reach of this character is usually a source of trouble, as scour very often sets up in the normal sized reaches, either above, or more generally below the enlarged reach, or tank. In some cases (usually in canals which have curved reaches near this "tank") no silt is deposited in the tank. The troubles are easily cured by a series of brushwood profiles, outlining the section GHKL (at say every 1000 feet length of the tank, for a tank 50 feet wide, and in proportion for narrower tanks). The expenditure is well repaid by the deposit of silt which occurs, and strengthens the banks. Once silting has begun, any signs of the channel meandering between the banks AB, and EF, should be carefully watched for; and should be stopped by the removal of excrescences, and by placing small, partial profiles of brushwood across any marked bends that may tend to form (see Sketch No. 219). The matter is of practical importance, as in many cases the natural soil is so tender that breaches continually occur until a good berm of silt has been deposited.

The two lower figures of No. 161 show the application of similar principles to stone training banks (or channel profiles) below falls and rapids (see p. 721).

On well-maintained canals (say ten or fifteen years old), all the channels will be found to be built up of silt, the water flowing on beds and between berms of silt. The future existence of such conditions should be kept in mind when designing a canal. In fact, the existence of silt in a water may be regarded as a favourable circumstance, provided that the laws of the disposition of silt and the proper proportions of the channels are carefully studied, and that the results are utilised in schemes for maintenance. A good engineer should be able to modify the grades and levels of the distributary system at a very small cost by such means as temporary dams which slowly raise the water level in reaches where the banks are weak. Thus, in one particular case I found



SKETCH No. 161.—Silting Profiles and Training Banks.

that it was possible to silt up and raise the bed and water level of a channel which proved to be too low, at the rate of 2 feet a year. So also, the careful working of a fall, or of a regulator, will permit a permanent local drop in the water surface to be obtained by scouring. This is often advisable where drainage questions have become acute, but careful arrangements should be made for disposing of the silt which is thus set in motion.

RÉGIME.—The term régime of a river or canal is used as a convenient expression for the adjustment which exists between the size and cross-section of the channel and its mean discharge. Thus, a reach of a river, or of a canal, is said to be in good or permanent régime when the river does not decisively attack its banks, or does not scour out or deposit silt in its bed, even although it is known that the individual particles which form the bed and banks of the river are in rapid motion downstream. The régime is considered to be bad if the banks are continually attacked, or if large quantities of silt are deposited in, or are scoured out of the channel.

The terms good and bad are obviously relative. A canal, say, 50 feet in width, can hardly be considered to be in first class régime unless its actual cross-section agrees with the designed cross-section to within a foot or so; and reaches which are designed to be straight should not diverge more than two or three feet from a straight line. In a large torrential river, however, the régime is considered to be permanent unless the services of a special pilot are required when navigating the river.

The regulation of an irrigation system, or of a river, is a convenient expression for the control which the engineer in charge has over the water level, and for the methods by which this control is obtained. Thus, in the Punjab the water level in the remotest branches of a large system can be predicted (accidents apart) to a tenth of a foot, and alterations of three or four feet can be effected in a period not greatly exceeding the time required for the water to travel from the head of the canal to the desired point. In a well-regulated river (flood time excepted), the water levels can generally be adjusted nearly as accurately. Whereas, in natural unregulated streams, the water level entirely depends upon the quantity of water which may be flowing down the stream channel.

HEADWORKS.—The term headworks forms a convenient expression for the structures required to divert water from a river into a canal.

The ratio in which the total volume of water passing down a river is divided between the natural river channel and a canal taking out from the river, is obviously dependent upon the slopes and relative levels of the water surface in the river and the beds of the river and of the canal. In some cases, the canal can be so graded that this natural division of the volume discharged by the river suffices to supply the canal with all the water that is required.

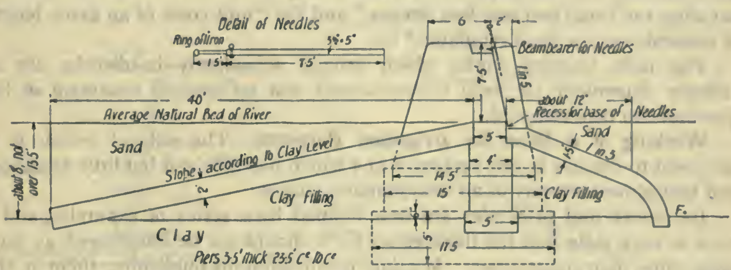
We thus get the simplest class of headworks, where neither the canal nor the river are in any way controlled, and where the canal takes the water that it can get. Such works suffice in many cases, especially when the irrigation is effected by inundation, and the river rises in high flood at more or less fixed periods of the year. The risks of a partial failure are obvious; and, as a general principle, this method of securing a supply of water is only successful when the river floods are very regular in occurrence, and are in most cases produced by melting snow. Typical examples are found in the basin irrigation of the Nile, and in the inundation canals of Northern India.

The natural method of securing a more certain supply of water is to grade the canal at so low a level that the desired supply can be secured at all seasons of the year. It is then usually found that the natural division of the river discharge will generally give the canal more water than is required. Thus, a head regulator, or line of movable gates, is constructed across the canal; and by partially closing these gates the excess of water can be diverted down the river. This method is frequently found to be entirely satisfactory and will then probably prove to be the cheapest. If, however, the river flows in alluvial soil, or if the bed is liable to shift to any extent, trouble may occur in the course of time, arising from the "retrogression of levels." This term expresses a slow alteration in the level of the river bed, which may lead to the formation either of a "deep" in front of the canal head, and a consequent difficulty in securing the required supply; or less frequently to a "shallow" or bank which may choke the canal with deposits of sand or gravel. These difficulties can usually be surmounted by forming a bar across the river. Typical sections of such bars are shown in Sketches No. 162 and Fig. 1 No. 180. The river works are comparatively simple, and the method should be adopted in all cases where the excavation of the low-level canal does not prove too costly (see p. 684).

In the majority of cases, however, especially when the full supply discharge of the canal is of about the same magnitude as the low water discharge of the river, such a design would necessitate a very costly canal, and an enormous canal head regulator, in order to keep the river out of the canal during floods. The canal is therefore graded at a relatively higher level, and it becomes necessary to raise the water level of the river by a weir or dam, in order to force the desired supply into the canal during low water seasons. This is the typical form assumed by headworks, and will be discussed at length.

The following general principles are obvious. The height of the dam may vary from a small quantity, such as 1, or 2 feet, up to 150, or 200 feet; and the higher dams are plainly not only diversion works, but permit a certain volume of water to be stored up. We are thus led to regard a tank or reservoir irrigation system merely as a development of the more usual river diversion system of irrigation.

The obstruction produced by the dam or weir encourages the deposit of silt at or near to the canal head. Therefore, unless the river carries very clear water, the weir must be provided with scouring or undersluices for the removal of silt. This question will be treated in detail later on.



SKETCH No. 162.—Sidhnai Bar.

The level of the canal bed or sill of the head regulator must be sufficiently below the highest level to which the river surface can be raised to permit the full supply of the canal to be forced into it when the discharge of the river is at its minimum. On the other hand, any marked elevation of the river level during floods is unnecessary, and necessitates a larger head regulator, and may cause damage to the weir. Thus, a certain portion of the rise in the river level should be produced by movable dams, or shutters. The calculations are simple, but the requisite information is usually deficient.

Let us first consider the flood discharge Q_f.

The term "afflux" is used to denote the difference between the high flood levels upstream and downstream of the weir.

The fixed portion of the weir must be at such a level and of such a length that when the weir and undersluices are passing a discharge Q_f, the afflux is not so great as to cause serious flooding, or to overtop the canal regulator, or the training banks upstream of the weir. The appropriate coefficients of discharge of the weir and the undersluices are discussed on pages 132 and 168. As a rule, the various upstream works are designed so as to be 4, or 5 feet above the calculated afflux level when the downstream high flood level is assumed to be the maximum observed before the weir was constructed.

Consider the low water season, and let Q_l, represent the maximum capacity of the canal.

Then, the relative levels of the sill of the head regulator and of the top of the movable portion of the dam must be such as to permit Q_l, to be forced into the canal. In this calculation an allowance for possible obstruction of the canal by silt deposits must be made. The sill level and the discharge capacity of the undersluices are determined by their object in scouring out silt deposits. The question therefore entirely depends on the average silt content in the water. General rules cannot be given, but the principles are discussed in detail on pages 695 and 700.

The accurate determination of the proportions and relative levels of the weir, the undersluices, and the canal head regulator, can therefore only be arrived at by a series of trial estimates of the cost of the whole headworks, and of the first reach of the canal (so far as the cost of this work is affected by its bed level). It is doubtful whether the preliminary studies of the discharge of the river are ever sufficiently extensive to permit a really accurate solution to be arrived at. The designer should, however, in all cases know the relative costs entailed by such operations as "raising the weir level one foot," "excavating the canal bed one foot deeper," and the "unit costs of an extra length of undersluices, or head regulator," etc.

The river training works which usually accompany headworks are so entirely dependent on local circumstances that no general treatment of the question can be attempted.

Working of a River for Irrigation Purposes.—The subject which it is proposed to discuss in this section is one which has received but little attention, and that almost entirely of an unsystematic nature.

Its nature and principles are best defined by a series of examples, and I must at once state that the theories set forth should not be considered as anything more than suggestions. My only justification for publishing them is the feeling that some one must take the first step; and the more searching the criticism aroused, the more satisfied I shall feel.

The leading principle is as follows:

When a river is utilised for irrigation, large volumes of water are drawn off, and the whole régime is altered, usually for the worse. It is only very rarely, however, that the demands for irrigation or power are so intense that the entire flow of the river is diverted. Thus, if the canal is provided with a regulator, or head sluices, and if the river is blocked by a weir with a movable crest or sluices (preferably both), it is possible to exercise a certain amount of judgment in selecting the water which is taken into the canal, and in disposing of the surplus water passed down the river.

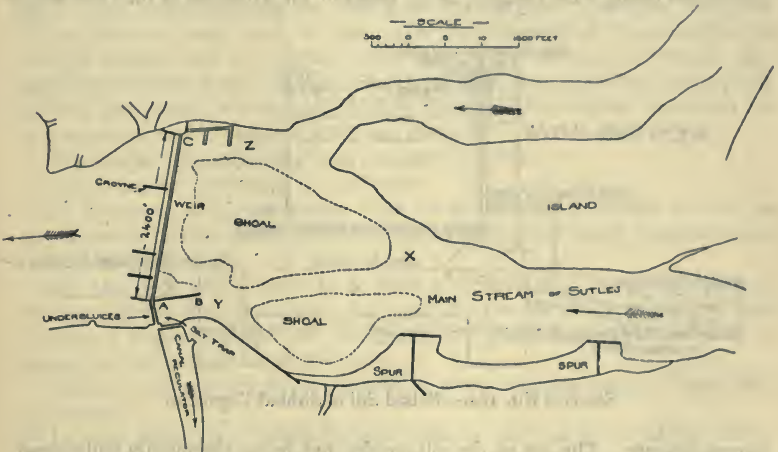
The primary object of this selection is to keep silt out of the canal as far as consistent with the correct performance of the duty for which the canal is constructed. This is of course best effected by the rejection of all water which is heavily silted, and by taking an excess of clear water into the canal for scouring purposes whenever possible. This side of the question has been frequently discussed, and is fairly well understood. The advantages obtained by a correct system for disposal of the surplus water in the river after the requirements of the canal have been satisfied are less well known, and it is proposed to discuss the principles of "working" a river, and its canal, with this object.

The methods at present employed are best illustrated by examples. It is believed that a study of these methods will prove useful not only in the working of existing canals, but also in the design of the weirs and the head regulators of new canals.

Working of a River carrying Sand only.—I select the Sirhind canal as an example. This canal takes out from the Sutlej, at Rupar (Punjab). The river at this point has a slope of $\frac{1}{2600}$, the maximum discharge is 130,000 cusecs approximately, and the minimum recorded is 2818 cusecs.

The canal can carry 6000 cusecs and has a slope of $\frac{1}{5000}$; and, as a general rule, it may be said that the river is completely diverted into the canal for about two months (usually January and February) each year. Sketch No. 163 shows the general arrangement of the weir, undersluices, and regulator.

From 1893 to 1898 large quantities (amounting to as much as 19·6 million cubic feet in five months in the year 1899, while deposits of 4 million cubic feet



SKETCH No. 163.—Plan of Sirhind Headworks.

in 10 days were not unusual) of sandy silt were deposited in the head reaches of the canal, especially in the first twelve miles (the above figures referring to deposits in this length only). Matters became so bad that in 1893 a total failure of irrigation was apprehended, and it was only by taking in quantities of water greatly in excess of what was really required for irrigation, and wasting these at an escape of twelve miles below the head, that the canal was kept in a more or less workable condition (see Sketch No. 198).

A study of the available records leads me to believe that during the early portion of this period it was thought that the path of salvation lay in taking in all the water available up to the full capacity of the canal, and rejecting the surplus which was not required for irrigation, by means of the escape.

After long and systematic studies of the size of the silt (see p. 758) it was realised that the trouble was caused by the coarser sand, and the sill of the regulator was consequently raised (see Sketch No. 164).

Thus, in 1894, the following remedies were adopted (see Kennedy, *Punjab Irrigation Papers*, No. 9) ;

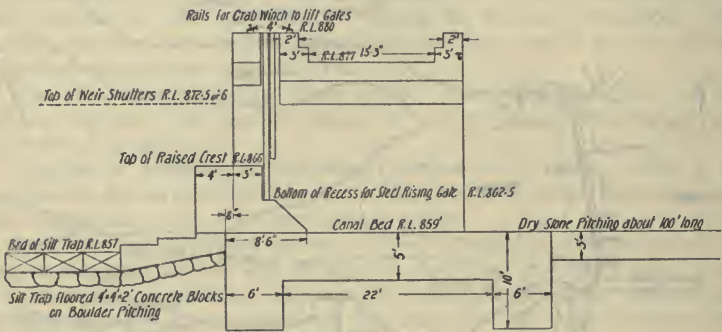
I. The capacity of the escape was increased from 2000 to about 4000 cusecs.

II. The canal was closed down during heavy floods.

III. The divide wall AB (see Sketch No. 163) was constructed so as to form a pond or silt trap, which was occasionally scoured out through the undersluices.

IV. The regulator sill was permanently raised to the extent of 7 feet above the floor of this pond, and was provided with a movable sill which could be raised still higher when the level of the water in the river permitted.

The first two ideas are excellent, and should be adopted where possible. In this particular case clear water is a rarity, and could only occasionally be passed out through the escapes, as the demand for irrigation in the clear water



SKETCH No. 164.—Raised Sill of Sirhind Regulator.

season is acute. The use of the silt grader not being thoroughly understood, the canal was often unnecessarily shut off simply because the water looked muddy (see p. 763).

The third idea is also good, but its full advantages were not obtained because the surplus water in the river was passed off through the undersluices, *i.e.* close to the canal head or regulator. The silt trapped in front of the regulator was thus stirred up, and fresh silt was brought into and deposited in the silt trap. Consequently, the raised sill had but little effect, for the silt grading experiments clearly show that the disturbance produced by the water entering the regulator was sufficient to lift silt of all grades up to $\frac{0{\cdot}00}{0{\cdot}40}$, over the raised sill, if such silt lay on the river bed outside and near to the sill. This being about the coarsest grade of silt which the river carries, the raised sill (although excluding a certain fraction of the coarser silt) was not capable of keeping detrimental bed silt outside the canal unless all silt which was sufficiently coarse to deposit in the canal was dropped in the river bed before it reached the pond or "silt trap" in front of the regulator.

The above-mentioned methods somewhat ameliorated conditions, but it was not until 1901 that the final and most important remedy was introduced. This was simple, but it forms the keynote of the working of all rivers which carry sand only.

The surplus water was systematically diverted as far as possible from the regulator, and was passed off by dropping the shutters at the far end C, of the weir (see Sketch No. 163).

The principle is to prevent any sand (mud, since the canal velocity will carry it forward, does not matter) from reaching the vicinity of the regulator. At first sight it will appear that this method must finally cause the area ABY to silt up solid. This is avoided by systematically sounding this area; and, when necessary (usually about once a month in the low water season), closing the canal completely for 24 hours, and passing the whole flow of the river through the undersluices. The regulator and the undersluices are never opened simultaneously. The effect is excellent. In the eight years from 1893 to 1900, the mean yearly quantity of silt deposited in the canal during the silting periods and removed during the clear water periods was 13·7 million cubic feet. In the first four years (1901 to 1904), after the adoption of this principle, the average deposit was 3·9 million cubic feet. The amount of silt that permanently remains in the canal is now about 1·6 million cubic feet, corresponding to a mean depth of 0·12 foot, which is immaterial in a canal which can run 13·7 feet deep, and is designed for 8 feet depth.

The problem therefore has been completely solved.

The necessary conditions are as follows:

(i) The river in its low stages must be entirely under control, so that the surplus water can be easily and certainly diverted to the bank opposite to the canal head.

(ii) The undersluices must be of such a capacity that, when fully opened, the flow through them is sufficient to scour out the silt trap and also to form a channel connecting the silt trap with the deep water channel which is created on the farther side of the river.

These conditions are easily secured in rivers carrying nothing larger than coarse sand.

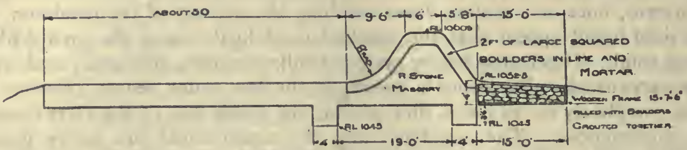
In some of the later canals the weir crest is laid not at a level, but with a fall of two or three feet towards the end corresponding to C, in Sketch No. 163. Personally, I doubt whether this is advisable, as it appears likely to cause the flood channel to work over towards the farther bank at C. This seems unnecessary, and if the action became too acute, the undersluices might not be able to scour out the connecting channel when required. The Rupar (Sirhind Canal) weir is level, and under the present conditions it serves its purposes admirably.

The floods scour out the deposits splendidly; and, as will be noticed, the two deep water channels XY, and XZ (see Sketch No. 163) which the method requires, have become permanent features of the river bed.

Working of Rivers carrying Boulders or Gravel.—These rivers are less easily controlled than sand bearing streams. Their floods are more intense, and the low water supply will hardly suffice to scour out and maintain the channel corresponding to that denoted by XY, in Sketch No. 163. The examples which are best known to me are the river Ravi, at Madhupur (supplying the Bari Doab canal), and the Jumna at Tajewala (supplying the Eastern and Western Jumna canals).

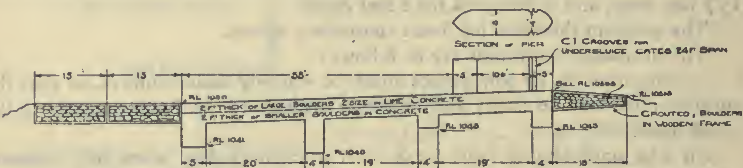
In the above cases, any attempt to divert the surplus water to a distance from the canal headworks would probably produce so great a deposit of shingle in front of the canal head that the undersluices could not scour it away.

The rivers being torrential, it is impossible to guarantee the maintenance of a weir with falling shutters across the river. At Tajewala the weir is usually in fair working order, while at Madhupur the weir is generally buried



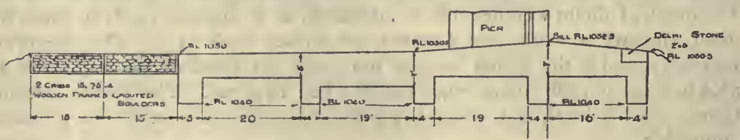
SKETCH NO. 165.—Tajewala Bar, or Undersluices, previous to 1876.

in shingle, and receives but scant consideration when the problem of regulation is dealt with. At both places the engineer in charge has to "get the river under control" at the beginning of each low water season. This is effected by blocking such channels as divert water from the canal head by means of more



SKETCH NO. 166.—Tajewala Undersluices, from 1876 to 1896.

or less (usually less) permanent dams of boulders and brushwood, or occasionally of rubble masonry in lime mortar. These headworks are now generally held to be wrongly located, and it is thought that sites farther down the river, where sand alone would have to be considered, would be preferable. This statement

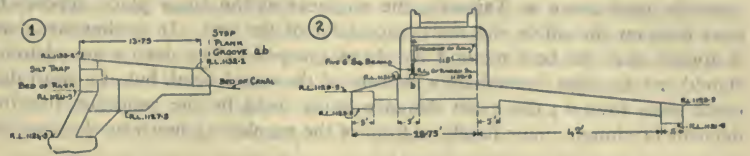


SKETCH NO. 167.—Tajewala Undersluices, as partly reconstructed in 1896.

may be correct as regards Madhupur, but I am not convinced that it applies to Tajewala. In any case, the problem must be faced in other countries; and the fact that the above rivers have not as yet been permanently controlled does not materially affect the method of working them, as the shingle deposit would certainly be increased if the river were permanently weired, or barred in any way.

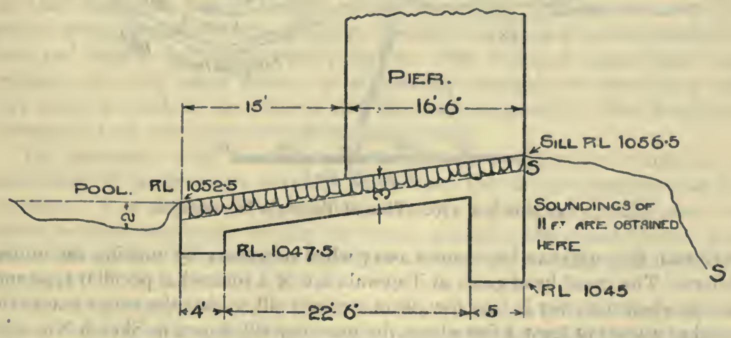
The exigencies of the situation are best explained by Sketches No. 165, No. 166, and No. 167, which show three sections of the combined weir and undersluice used at Tajewala. It is sufficient to state that the first two were destroyed in the years noted in the sketches, and that it may be anticipated that the present design will sooner or later be destroyed. No masonry on earth can be expected to stand the impact of boulders ranging up to 5 feet in diameter, under river velocities of 20 to 25 feet per second.

The regulation of these rivers in their low water stage is entirely effected by the undersluices, and the low water channel is kept close to the canal head. (This produces the rather confusing terminology shown in Sketch No. 170



SKETCH No. 168.—Madhupur Raised Sill and Regulator.

where what is usually termed undersluices is noted as a regulator, and the term head is used for the canal regulator.) Thus shingle and boulders would enter the canal were it not that the canal head is provided with a raised sill. Sketch No. 168 shows the design adopted at Madhupur, and the raised sill added in 1906. The Rupar design (see Sketch No. 164) is also suited to these conditions. Sketch No. 169 shows the Tajewala canal head. This, at first sight, has no raised sill; but, as a matter of fact, the whole regulator is raised, as a per-

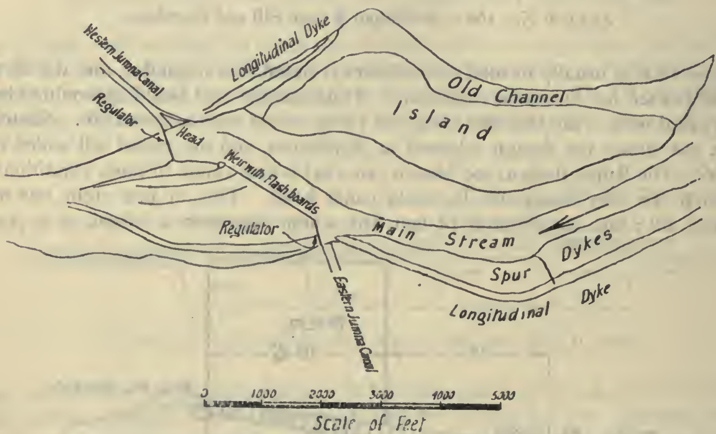


SKETCH No. 169.—Tajewala Regulator, or Canal Head.

manent deep water channel has formed in front of the work, soundings of 11 feet being regularly obtained within 2 feet of the face wall. This is a favourable condition for keeping silt out of the canal, but it cannot be considered as good practice. The error made is obvious. The opening of the sluices ("Regulator" in sketch) nearest to the regulator ("Head" in sketch, see Sketch No. 170) is always the one which is first opened. If the problem were better understood, there is no doubt that if this opening and the next one were kept closed, the deep water channel could be slightly diverted by passing off surplus water through the third and fourth openings. We might then hope to make the cross-section of the river in front of the canal head, of the form SS, shown in Sketch

No. 169. Erosion in front of the regulator would then be minimised, but more gravel would enter the canal.

At Madhupur matters are in a better state, as it has been found possible to maintain a level floor in front of the canal head or regulator. These statements reflect no discredit upon the officer in charge of Tajewala, since silt troubles are far less acute on the Western Jumna canal than on the Bari Doab. Madhupur is worked with too great a consideration for erosion, and too little regard to silt. At Tajewala the reverse is the case, and as the demand for water is more acute at Tajewala, the engineer at the latter place appears to have been on the whole the more successful of the two. In neither case does it appear that the best method has been grasped. The deep water channel should not be kept immediately in front of the canal head, but at a short distance away from it; the exact distance being fixed by the condition that the deposits of shingle immediately in front of the regulator (shown by the line SS,



SKETCH No. 170.—Plan of Tajewala Headworks.

in Sketch No. 169) can be scoured away when necessary by opening the undersluices. The canal head gates at Tajewala are of a somewhat peculiar type and, except when the river is very low, form a raised sill so that the water enters the canal at a level at least 3 feet above the masonry sill shown in Sketch No. 169.

It will be plain that sand (at any rate) will be drawn into the canal. At Tajewala sand alone enters the canal, but at Madhupur pebbles and occasional boulders are carried in. These two canals irrigate flat land in their lower reaches, and it is impossible to obtain sufficient velocity in the tail channels to carry this sand forward. Thus, scouring escapes form an integral portion of the canal, and must be provided.

The headworks of the irrigation canals of Lombardy are situated on rivers which are only slightly less torrential than the Ravi, or the Jumna. The canals, however, do not irrigate flat land, and therefore sandy silt causes but little trouble. The canals date from mediæval times, and their headworks are usually unprovided with regulators. Thus, pebbles and also boulders enter the canal. These are scoured out by escapes and sand traps.

We may therefore assume that the escapes found necessary on the Western Jumna are only required because the tail portions of the canal irrigate flat land. Were the slopes of the whole irrigated area of the same character as those of Northern Italy, it is probable that the precautions taken at Tajewala would suffice to exclude all detrimental silt.

The circumstances at Madhupur are somewhat less favourable; and the escapes and the double head reach now adopted would probably be required, however steep the tail channels might be, since at present silt deposits mainly occur in the head reaches of the canal. The Madhupur conditions, however, are not likely to occur in carefully designed systems. The headworks date back to 1864, and the modern troubles are mainly attributable to the fact that the canal carries about 50 per cent. more water than it was designed for.

In considering the design of headworks of canals under the above conditions the following rules appear to be advisable:

(*a*) The regulator or canal head should be relatively larger than is usual in the present designs. We wish to get the water into the canal with as little disturbance as possible. The Tajewala regulator could with advantage be made 25 to 30 per cent. larger than it is now, while an enlargement of the Madhupur regulator is known to be urgently required. The double head reach channel employed is described on page 766, and it is quite probable that if this system of working were abandoned, and if the entire length of the regulator was used to admit water, the sand deposits occurring in the lower reaches would be decreased.

(*b*) The undersluices should also be increased, not necessarily in length, but they should be placed deeper in relation to the regulator, so as to give greater scouring power. The question, however, needs very careful consideration, and should be experimentally tested. The Tajewala undersluices are probably too deep when worked as at present, although not necessarily so if they were worked in the manner that has been suggested. The Madhupur undersluices are certainly not sufficiently powerful.

The principle to be borne in mind is that the ordinary low water flow is insufficient to produce any material effect upon the shingle deposits near the regulator. The undersluices should therefore be capable of passing away the minor freshets that occur in the low water season. The canal should then be shut down, and these freshets should be utilised for scouring in front of the regulator.

It will be obvious that the root of all the troubles is the fact that the river is not under control during the periods when the major part of the motion of the shingle and boulders occurs. The engineer is expected to scour out deposits that are produced when the mean velocity of the river is about 15 to 20 feet per second, by manipulating low water flows which rarely, if ever, give a mean velocity in excess of 5, or 6 feet per second. The marked extension of the subsidiary weirs and training banks upstream of the canal head (p. 666) is in essence an endeavour to secure a fall sufficient to permit the low water flow to be used as a jet to scour away boulders. The difficulties are great, the maximum flood of the Ravi being about 170,000 to 190,000 cusecs, and its minimum low water flow only 1200 cusecs, while very few years pass without flows of 1500 to 2000 cusecs being recorded. The figures for the Jumna are slightly less variable. In view of the size of the rivers, it is doubtful whether any other solution can be found. In a smaller river of equally variable flow, however, it would appear advisable to erect a relatively high dam (say 20 or 30 feet) with undersluices at as low a level as possible, and thus provide a means of working the river as is done at Rupar.

The first cost is no doubt heavy, but the dam will provide storage capacity, and the present cost of maintenance and repairs at Madhupur and Tajewala forms a very heavy drain on the profits of what are even under present circumstances abnormally prosperous irrigation systems.

General Remarks.—The above discussion is confined solely to the working of existing canals. Where a choice is possible, a site similar to that at Rupar should be selected. The maintenance of headworks resembling those at Tajewala or Madhupur is an abominable business. At Tajewala, the system of spur dykes, training walls, etc., extends at least 15 miles upstream of the canal head, and at Madhupur 8 miles is probably the usual limit. The financial returns of an irrigation canal are so excellent that these enormous works are justifiable. Nevertheless, there is little doubt that if the headworks were situated on a less torrential portion of the river, an equally good use could be made of the flow of the river.

As a general principle, rivers in the torrential portion of their course should be utilised only for power, and the irrigation development should be effected in the lower reaches of the river where silt of a sandy nature alone occurs. This division will be seen to be logical. In torrential reaches of a river, the head requisite for a power development can generally be secured by relatively short head and tail channels. The general slope of the country being steep, the head channel can be graded so as to produce velocities which will carry forward all sand, and possibly even small gravel. These particles must be removed from the water before it reaches the turbines, by means of sand traps, or silting basins; but since the general slope of the ground is steep, sites for such traps or basins can be easily found.

When the site for headworks is selected in the sand-carrying reaches of a river the following principles should be borne in mind.

It appears that (contrary to accepted practice, which is based on the supposition that hard clay foundations exist on the bank towards which the river travels) the regulator of a canal should, if silt problems alone are considered, be situated on that bank which the stream as a whole tends to quit. Local variations due to the existence of marked and permanent bends may render it possible to discover a favourable site on either bank; but, as a rule, it will be found (in the Northern Hemisphere at any rate) that rivers flowing towards the north attack the eastern bank, and that rivers flowing towards the south attack the western. As an example, it may be noted that the Nile flows north, in the Northern Hemisphere; and, as a matter of historical record, the western bank was the one to be first reclaimed. Similarly, the Punjab rivers flow south, attack their western banks; and, with one exception (the Western Jumna canal), all canal heads are situated on the eastern banks.

The regulation of the river should be worked so as to produce still water on the canal side, keeping the main stream during low water on the other bank. The scouring out of the local deposits in the silt pocket should be effected during short closures of the canal, while floods are depended on to scour away the general accumulation of silt which occurs above the weir.

In cases where a site on the bank which the river tends to attack must be selected, the problem is more complicated. In actual examples, the solution is usually one of brute force, combined with expensive maintenance. The canal is systematically cleared of the silt deposits produced by the scour of the river banks, and large quantities of stone are thrown in front of the regulator and undersluices during each low water season, so as to preserve them from erosion in the succeeding floods. The work is costly, and in large rivers may entirely swallow up the whole profits of the undertaking. In small, and placid rivers, the disadvantages are less acute ; but, even in such cases, a site which the river tends to quit is preferable. If the worst comes to the worst, dredging a channel from the deep water stream to the canal head is cheaper than protecting several miles of bank. It therefore appears preferable in all such cases to train the river for some distance above the headworks, so as to produce a slight tendency to quit the canal head. The following suggestions for the design of such training works are put forward with the remark that the principles are fully proved, and have been applied for many years past in connection with bridges over rivers flowing in beds which are easily eroded.

Broadly speaking, the problem is to direct the main stream of the river away from the canal head, and to provide a place in the river bed where silt can be deposited during low water periods, and swept forward during floods. This has been completely solved by Bell's system of "bunds," or dykes (see Spring : *Indian Rivers . . . Guide Bank System*).

Consider a river which is normally 5000 feet broad. It will be found that such a river, when flowing in a sandy bed, is capable of eroding this bed to a certain depth (let us say 60 feet). This depth will depend upon the velocity of the river, the size of the sand grains, and their angle of repose when wet ; and is obviously governed by the rate at which the water can pick up the sand, the maximum depth of a scour hole being that at which the sand falls in from the surrounding portions of the river bed as fast as the water can pick it up.

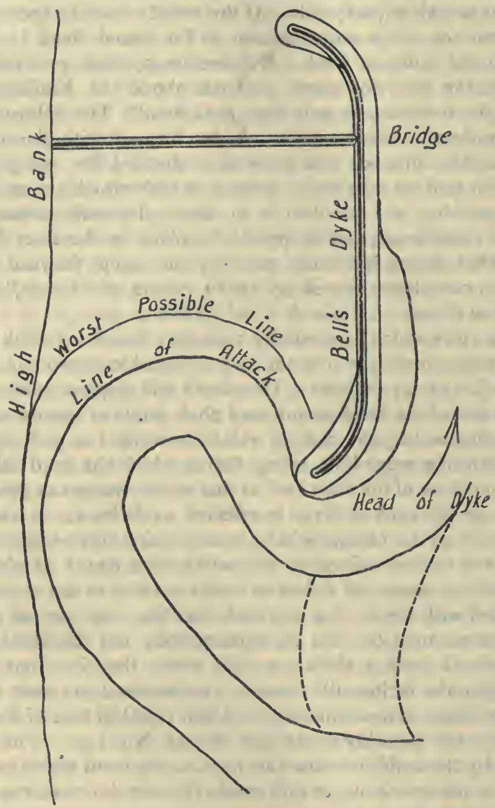
The training of such a river is effected as follows :—A certain breadth, somewhat less than the normal width (usually about three-fifths) of the river is selected, and the river is trained to this width for a length of about one to one and a half mile, by means of dykes or banks parallel to the main stream of the river, and faced with stone of a size such that the river current cannot carry it away. The stone may be, and is, undermined ; but the stone facing of the banks is made of such a thickness that when the river has eroded to its maximum depth, the facing still forms a continuous layer over the face of the bank when its slopes are continued to a depth equal to that of the deepest hole which the river can possibly erode (see Sketch No. 159). The length of the bank is fixed by the condition that the most acute bend which naturally occurs in the river cannot reach up to and erode the canal, or railway embankment, as shown in Sketch No. 171.

The river is thus forced to flow through a constricted bed, and can (within limits) be directed as required. Sketch No. 159 shows typical designs ; and the plan of the dykes should be such as to give as little opportunity as possible for whirlpools to be set up, and the stone facing should be packed by hand to as smooth an external face as possible (see p. 653).

The most severe attack will obviously be made on the heads of the dykes, and these are provided with an extra protection, as noted in Sketch No. 159.

Let us now consider what happens in the constricted channel between the dykes during a flood. The channel will evidently scour out to a depth such that the cross-section of the constricted channel is very approximately equal to the average cross-section of the natural river, the slight difference being caused by the increase in the mean velocity as determined by Kennedy's law (see p. 754).

Thus, during each flood we have a heavy scour in the constricted channel, and the area thus scoured out is filled up with fresh silt during the low water season, *pro tanto* reducing the amount of silt reaching the weir or canal head.



SKETCH No. 171.—Severe attack on a Bell Dyke.

These general principles of river training and control being clear, their application to a canal headworks is, I think, obvious.

A constricted channel should be established some distance above the canal head, and should be arranged so as to direct the flow of the river to about the middle of the weir ; *i.e.* where the normal breadth is 5000 feet, the prolongation of the nearer training bank should cut the weir about 2000 feet away from the canal head. This being done, the river is properly directed ; and, what is probably still more important, the deep scoured channel between the training banks forms a very effective silt trap during the low water season.

A complete design would of course specify the distance between the weir and the tail of the banks, and between the canal head and the line of the nearer bank measured perpendicular to the direction of the river. These are obviously very important; since, if the second is too great, the river may quit the canal head, and costly dredging may become necessary; while if too little, the river may attack the headworks.

Similarly, if the first is too big, heavy deposits may form in front of the weir, and canal head, during the floods; and if too small, the weir may be attacked. In actual practice, however, such works are not started until a weir and headworks have been in existence for some years, so that the necessary information should be on record. At Khanki (Sketch No. 172) the railway training works are obviously much too far above the canal head to be useful.

SKETCH NO. 172.—Training Works at Khanki.

A logical treatment of this subject would include the design of the whole plan of the headworks of a canal. Unfortunately, the practical aspect of the matter renders this treatment useless. In every case the headworks of a canal must be designed with incomplete information, since, even if really systematic preliminary studies were made for a generation, they could only acquaint us with the régime of the river before the diversion of its waters for the canal; and no amount of previous knowledge would allow us to predict the régime as altered by the almost total diversion of the low water flow. Thus, the practical view of the matter is that we have a headworks, and (possibly after minor modifications) we should accept this as a natural feature, and work the river to the best advantage of the canal.

WORKING OF INUNDATION CANALS.—In many countries it is usual to irrigate by canals which take out from the river at a level which enables them to draw water during flood time only.

The design of such works is usually somewhat empirical. A canal head is cut, and is expected to supply water during one season. If, at the end of that season it is not absolutely silted up, it is held to be a successful head, and is used until it becomes less expensive to cut a new head than to dispose of the deposits accumulated on the banks of the old.

The old principles followed by Arab engineers in Egypt are stated on page 752, and allowance being made for local conditions, they are found to be those used in all places where flood irrigation is effected. I believe that it is possible to obtain somewhat better results by a careful consideration of the effects on the canal itself. The most heavily silted waters are those which come down with the rise of the flood, and a shallow canal head will draw a greater proportion of such waters, and will therefore be more likely to fail through silting. The matter needs careful consideration, for although the waters of the highest floods carry most silt, the surface water of a river, stage for stage, carries least silt. An ideal design therefore, would be one which would permit the canal to be closed off during the rise of a flood, and would yet allow only the top layers of the river water to enter the canal when it was open. The ordinary flood or inundation canal has no regulator or head gates, and the two conditions are consequently conflicting.

The logical solution would appear to be as follows :—If a favourable site for a canal head is found, as indicated by the principles discussed on page 666, and is confirmed by the fact that the canal does not markedly silt during its first season, it would appear advisable to lay stone pitching across the head, and to put in temporary wooden gates, which can be shut down when the river water contains an abnormally large proportion of silt. Such a process is not recommended in the case of canal heads situated at places where the river is likely to attack its banks, as the mere fact that stone pitching is put in would probably produce a severe attack on the canal head. The risk is justifiable in cases where the banks are known to be fairly stable, and such reaches are probably the only places where satisfactory sites for a canal head are likely to occur. The advantages of locating the head on a permanent ana-branch of the river, if such can be found, are too plain to need discussion.

The matter is also of interest as it explains the rapid cessation of irrigation which follows once the maintenance of the canals is neglected.

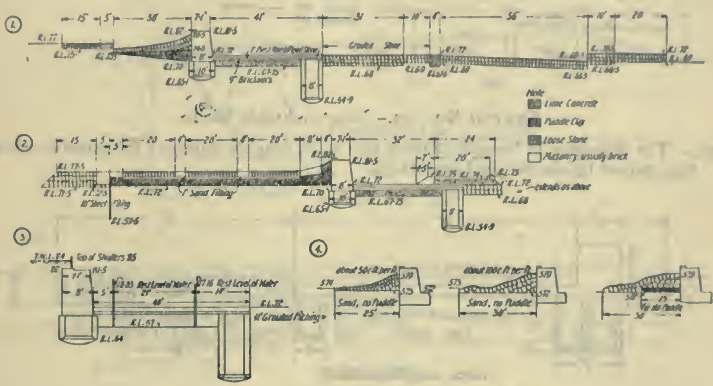
The process is as follows : Maintenance being neglected, the canals begin to silt. This small initial silting causes the next year's supply to be drawn from waters carrying a greater proportion of silt, as the canal bed is now so high that it only draws water when the river is in high flood; and the silt is so rapidly deposited that the clearer water of the falling flood is not taken in, or is taken in in a far smaller quantity than is requisite for scouring out the silt deposit. In the third year this action is still more marked, and in the fourth the canal probably ceases to flow.

WEIRS.—The term weir is frequently employed to denote a low dam across a river. The distinction between a weir and a dam of the overfall type lies in the fact that water is usually passing over a weir, and it is only rarely that some portion of the weir is not submerged. In an overflow dam, on the other hand, the dam is not usually submerged for more than three or four weeks during the year. Thus, the primary object of a weir is to raise the level of the water in a stream, permitting a portion of the normal discharge to escape ; while the primary object of a dam of the overflow type is to store up the normal flow of the river, flood water alone being lost. This difference in object is accompanied by a decided contrast in the construction and design of the two species of dams. The term weir is therefore restricted to dams which are so frequently submerged that protection against erosion by water passing over them forms an integral portion of their design.

Weirs may be divided into three types:

(*A*) A weir consisting of a dam with a vertical drop wall to raise the water level, and a horizontal floor at or about tail water level, to prevent erosion, as per Sketch No. 173.

(*B*) In this type the vertical wall also exists, but it is backed on its downstream side by a long slope of packed rubble over which the water flows. The rubble is retained in position by core walls of masonry, which form square or oblong cells into which the rubble is packed. (Sketch No. 174.)

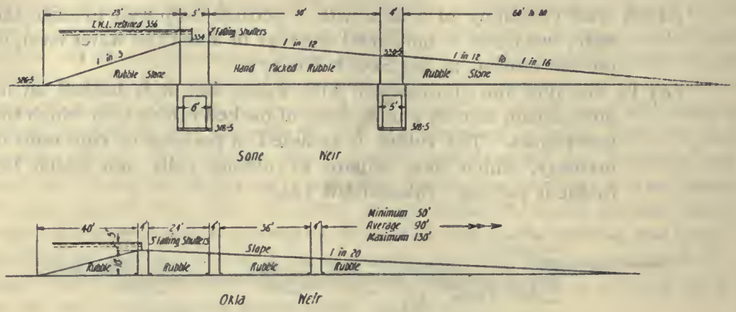


SKETCH NO. 173.—Narora Weir.

Fig. 1 shows the original construction; fig. 2 the alterations after the failure; fig. 3 the pressures under the weir apron shortly before the failure; and fig. 4 typical sections showing the damage that had occurred upstream of the drop wall before the apron cracked and failed.

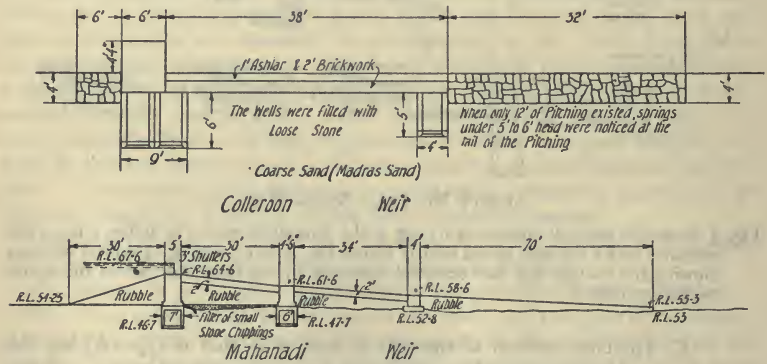
(*C*) The cross-section of the weir is similar to that of type (*B*), but the slope consists of smooth masonry of a definite thickness. Core, or drop walls, reaching to a deeper level than the masonry platform, exist, but do not form a necessary portion of the weir, their function being mainly to prevent local erosion. (Sketches Nos. 179, 181, 182.)

TYPE (*A*).—This type is best suited for localities where a firm and not markedly permeable foundation can be secured, *e.g.* clay, hard-pan, or firm gravel. The design of the dam itself follows the usual rules for dams, allowance being made for the depth of water passing over the dam, and for the diminution of weight caused by submergence, when selecting the least favourable case. Sketch No. 173 shows the section which is generally adopted in India, and Sketch No. 176, Fig. 1, the type usual in America, the section given being that of the Granite Reef weir. The two forms of the overflow face of the dam differ widely. In making a selection it should be realised that the Indian type is more likely to sustain damage in the floor; while, in the American type, the dam itself forms the weaker portion of the work. Thus, the quality of the available material and the character of the foundations must decide the question.



SKETCH No. 174.—Sone and Okla Weirs.

Note.—The difference in breadth of these two weirs roughly indicates the possible advantage gained by the deeper foundations.



SKETCH No. 175.—Colleroon and Mahanadi Weirs.

Note.—The Colleroon Weir is probably about as narrow as is consistent with safety in coarse sand. The Mahanadi Weir shows the transition from type B to type C. The reversed filter seems to me excellently located.

The American type may be recommended when first-class materials and a good foundation can be secured (say masonry in Portland cement, founded on weak rock, shale, or firm gravel).

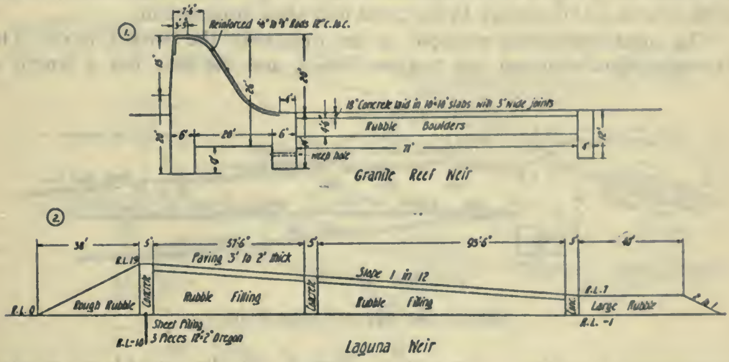
A dam of the Indian type can be erected on clay, or even on sand foundations (although this is not the type which is best adapted to such conditions), and can be constructed of brickwork, but will require constant maintenance and repairs, especially in the apron.

On the other hand, in the American type of dam failures are serious when they do occur; while the Indian type (if well looked after) has never failed so rapidly as to cause a disaster.

The section of type (*A*), possesses certain theoretical advantages over the section afforded by types (*B*), and (*C*).

I am inclined to believe that these advantages are somewhat over-estimated: but it will be plain that if the quantity of water passing over the weir crest is the same in both cases, the velocity at the section just below the vertical drop, in type (*A*), will be far less than the velocity at the same distance downstream of the weir crest in types (*B*), and (*C*); and erosion and wave action on the downstream apron is therefore not so much to be feared. Records of actual maintenance costs confirm this view.

On the other hand, the pressures on the foundations are far more localised in type (*A*), and unless the limits of pressure usually adopted (see p. 682) are greatly exceeded, it will usually be found impossible to obtain an adequate



SKETCH No. 176.—Granite Reef and Laguna Weirs.

base for the dam except in rock, shale, or extremely hard clay. Messrs. Pearson and Atcherley's theory regarding the vertical sections of dams (see p. 374) must be carefully considered, for neither clay nor shale foundations can be considered as capable of assisting the dam to any great degree against horizontal tensions.

A study of existing weirs of this type leads me to believe that the results of the above theory may be advantageously applied, and will prove valuable in indicating when type (*A*) can be used.

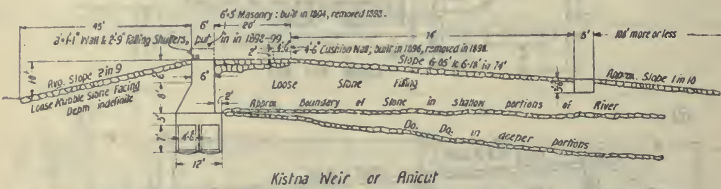
TYPE (*B*).—Often called the Anicut type. This is probably the oldest type of dam in existence, some of the Madras anicuts being more than 1500 years old. Both type (*B*), and type (*C*), can be maintained on foundations of the finest and most permeable sand. The choice between the two types is really determined by the available material and labour.

Type (*B*), contains but little cut stone, or mortar, and can consequently be erected where supplies of material and skilled labour are hard to obtain. It is, however, costly, requiring continual and unremitting maintenance. When fine sand forms the foundation, it is found economical to grout the rubble stone heap, or to face it with cut stone, or concrete blocks, so producing a bulky form of type (*C*) (*e.g.* the Mahanadi weir, Sketch No. 175). Where clay foundations exist, a weir of type (*A*), is usually adopted, and proves advantageous as being more cheaply maintained. Thus, type (*B*) (as is indicated by a study of the existing examples) should be restricted to coarse sand foundations, and even in these cases it is doubtful whether it would not be advisable to employ type (*C*).

Since type (*B*), requires but little skilled labour, while well sinkers and masons must be employed in type (*C*), the choice between the two types is usually determined in India by the character of the available labour. I consider that type (*B*), should only be employed in other countries when large and cheap supplies of rough stone are accessible.

Cases may occur in which rough rubble can be cheaply deposited in large quantities over the whole of the weir site by means of modern transporting machinery. The following notes concerning two rubble stone weirs existing in Madras are a digest of the information given by Welch (*Engineering Works of the Kistna and Godaveri Deltas*), and may then prove useful.

The most interesting example is the weir over the Kistna river. The maximum flood recorded was 770,000 cusecs, and the weir has a length of



SKETCH No. 177.—Kistna Weir.

approximately 3400 feet. The section adopted, with the actual levels existing at three points after thirty years of careful maintenance, is shown in Sketch No. 177.

This section may be considered as the minimum possible, for in 1894 (thirty-nine years after completion), an attempt was made to raise it three feet, as shown by the dotted lines. The weir at once began to give trouble through the formation of deep scour holes (750 feet × 20 feet × 12 feet average depth, and 250 feet × 20 feet × 12 feet average depth), in the 74 feet wide apron; and the masonry wall of the weir cracked.

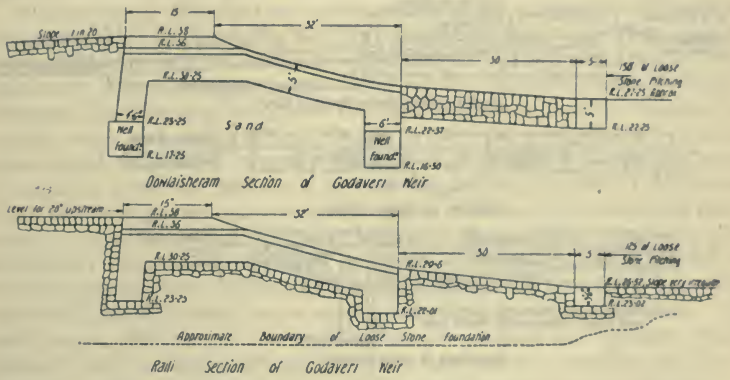
The usual inter-departmental discussion then followed as to the possible effect of a newly constructed railway bridge some 3000 feet below the weir, and its training works. The discussion is naturally only of local interest. The useful deduction is that the energy generated by the 3-foot drop on to the apron was sufficient to remove the stone work. This was recognised, and the lower wall (shown in dotted lines in Sketch No. 177) was built so as to obtain a water cushion.

In 1896, after the second largest flood on record, the apron was again damaged, and the talus was badly scoured.

The 3-foot wall was therefore removed, and was replaced by falling shutters of the usual design. The real lesson is that the weir was very accurately proportioned for the original drop of 14 feet, and was not capable of resisting the extra action thrown on it by increasing this drop to 17 feet. It will be fairly plain that a far narrower weir could be made to retain the 14 or 17 feet drop if water cushions were used. This, however, requires a large quantity of cut stone, and Cotton (the designer of the weir), having considered such a design, finally abandoned it because the present design, although calling for a far greater bulk of rough stone, was really cheaper in view of the extra cost of cut stone.

Under modern conditions, however, water cushions appear to be advisable; and if put in during construction, before the deposition of the rubble renders excavation difficult, they will not entail any great extra cost.

As a contrast, I give the sections of the Godaveri weir (see Sketch No. 178). This has to pass 1,500,000 cusecs, and is 11,945 feet in length. *A priori*, it would therefore appear that of the two weirs this one should be far more easily



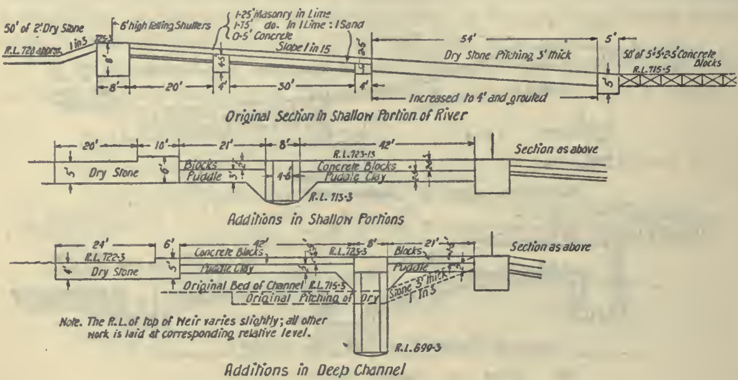
SKETCH NO. 178.—Godaveri Weirs.

maintained, especially since the difference of levels is, on the average, somewhat less than the 14 feet existing on the Kistna river.

As a matter of fact, the weir has only been maintained by a yearly expenditure of large volumes of rough stone (the records, which are known to be incomplete, indicate that at least 290,000 cube yards were expended in the first 20 years of the weir's existence). In the light of present experience, the reason for this comparative failure is fairly obvious:—The concave curve, is a very efficient means of directing the overflowing water against the talus, and the damage is probably caused not so much by actual transport of stone down the river, as by burying it in the deep holes which are excavated by the water, and afterwards filled with sand.

The lesson may be useful, as many overflow dams are still designed with tail aprons composed of loose rubble. These are unadvisable unless they rest on clay or shale; and in sand they should be replaced by a pavement made of closely fitted blocks of concrete. This aspect of the question has been realised by the designer of the Granite Reef floor (see Sketch No. 176), and his practice should be followed if the ogee type of dam is adopted.

The difference in design between the Dowlaisheram weir (which is typical of the major portion of the weir) and the short length known as the Ralli weir, should be noted. The masonry of the latter is laid on a pile of rough stone, and most engineers would consider that it was the more favourably situated of the two; but, as a matter of fact, it causes the most trouble. This is easily explained in the light of experience of Punjab weirs. The river carries coarse sand only, and what would be called fine silt in Egypt, or the Punjab, is almost entirely absent. Thus, it is doubtful whether the interstices in the rubble are even now stanched, and in the early years of the weir's existence the masonry during the flood season was exposed to an uplifting pressure, probably very nearly equal to the afflux over the weir. The masonry being thus strained and simultaneously exposed to shock from the overflowing water, it may be expected to crack, and each crack forms a point of attack. So, quite apart from any possible settlement produced by the removal of sand from under the rough



Sketch No. 179.—Lower Chenab Weir.

Note.—The upper figure shows the original design, which partially failed three years after construction, by uplifting pressures on the apron masonry (see p. 685). The two lower figures show the additions upstream of the crest wall, which have secured satisfactory working for over thirteen years.

stone (which is more likely to occur in seasons of low water than during floods), the damage to the Ralli weir could have been predicted. Experience of such weirs leads me to conclude that sand forms a better foundation than clay for flat portions of masonry, and that clay is superior to loose stone. This is fairly evident if we consider that in rivers which do not carry fine silt a certain amount of percolation under the masonry must be expected. If a definite channel is formed, it must be expected that the masonry will crack either from the uplifting pressure thus brought to bear, or, owing to lack of support; and although such channels are less easily formed in clay than in sand, once formed they are far more likely to remain open and to increase in size. In rivers carrying much fine silt, it is probable that rough rubble under small differences of pressure will soon become almost as impermeable as masonry.

It would therefore appear that all the weirs which are considered in this section could be greatly strengthened by a puddle coating upstream, in the position shown in Sketch No. 179 of the Lower Chenab weir. I believe that this device has been adopted in the new Madras weirs.

TYPE (*C*).—This is the type now usual in the Punjab and Northern India generally, and may be considered as the form which is best adapted for all cases where the foundations are not sufficiently good to permit the adoption of type (*A*), and where local conditions do not cause type (*B*) to be cheaper (see Sketches Nos. 179, 181 and 182).

I shall therefore discuss at length the rules which are at present adopted in the design of weirs which belong to this type, and shall merely refer incidentally to the rules for types (*A*) and (*B*).

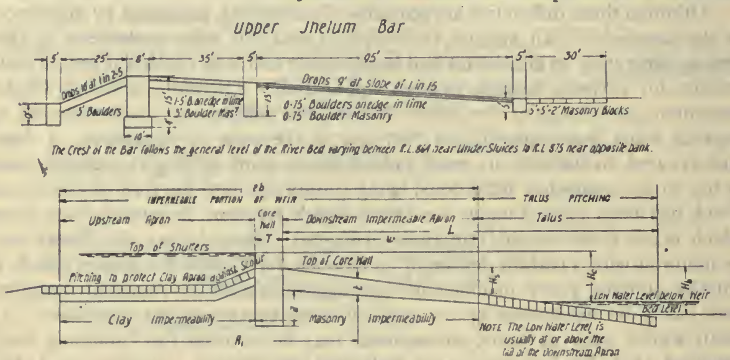
The rules are mainly empirical, and I doubt whether any general agreement on the matter yet exists amongst engineers. It must also be noted that while the results of the theory now put forward agree very fairly with the general practice in weirs, considerable differences will be found when the cognate question of undersluices and regulators is discussed.

Although these difficulties are partially, if not wholly, explained by differences in the intensity of tail erosion, the very fact that our rules endeavour to take into account such an accidental and incalculable factor as erosion is ample justification for extreme caution in their application. Therefore, although Bligh's treatment (see *Design of Irrigation Works*, 2nd edition) is closely followed as regards weirs and undersluices (but not in the case of regulators), I have endeavoured to indicate its weak points, since many of Bligh's designs seem to me to be somewhat hazardous, being (in my opinion) the fruit of experience which has been gained under conditions which are more favourable than those which occur either in the Punjab or in Egypt. Nevertheless, the theory may be followed with a certain degree of confidence, for should the design which is initially adopted prove insufficiently strong, the necessary remedial measures may easily be discovered by an application of the principles laid down. In costly works, such as weirs, an engineer may be excused for "guessing low," provided that he has previously indicated the correct remedies, and has "guessed sufficiently high" to avoid a really troublesome failure or disaster.

Taking matters at their worst, none of the designs shown (which include all known cases of bad failures) have ever resulted in so sudden a collapse that temporary remedies could not be applied, and the principle of the upstream apron introduced by Gordon and Clibborn provides a very excellent means of strengthening a weak weir.

Design of Weirs.—The design of all types of weir is intimately connected with the possibilities of percolation under the weir structure. The rules now given were arrived at after a study of existing works. It must, however, be stated that close personal acquaintance with seven such weirs leads me to believe that published drawings never properly represent the exact circumstances. The drawings show the initial construction of the weir; and, in some cases, where large modifications have been made after construction, these are more or less accurately recorded. Large sums, however, are annually spent in repairs and improvements on nearly every weir. These are generally patchwork additions to the aprons and talus, and vary from point to point along the weir. So also, large quantities of stone or concrete blocks are thrown in yearly, in order to stop local erosion. In particular the drawings usually show the talus as a horizontal layer of loose stone or blocks. Any inspection which is conducted under favourable circumstances will generally reveal that a pellmell arrangement of loose stone exists at the tail of a weir; and, in some cases, this is really a loose stone facing sloping down at an angle of 45 degrees, and more or less buried in sand. Some statistics of such repairs are given on page 675, and it will be plain that in an old weir (especially one belonging to type *B*), twice the volume of loose stone shown on the drawings is distributed somewhere in the neighbourhood of the weir site. Thus, the rules given below may be considered to represent a structure which can be maintained without undue risk. The final and permanent structure is quite another matter, and is probably arrived at after 20, or even 30 years' of careful maintenance.

A comparison of the designs of weirs and undersluices, as contrasted with the designs which are found sufficient for head regulators, has led me to believe that the value of the constant c (see p. 679), is considerably influenced by erosion on the downstream side of a work. Where erosion is absent (as in regulators), c, has a value which is approximately only one half, or even one third, of that which is found requisite in cases where deep holes or channels



SKETCH No. 180.—Bar across Jhelum (Type *C.*), and Diagrammatic Sketch of Type *C.*

may form at the tail of the apron. Record plans rarely, if ever, give any information regarding the size or depth of these holes even in the low water season.

Subject to these remarks, we may consider that all weirs may be divided into the following portions (see Sketch No. 180, lower Fig.):

(i) The upstream apron, which may, or may not, include one or more drop walls, or cut-offs.

(ii) The curtain, or core, wall, or dam wall proper.

(iii) The downstream apron, with its drop walls and cut-offs.

(iv) The downstream talus, or pitching.

Percolation is checked by the impermeable portion of the weir. This, in any typical case, is composed of the two aprons, and the core wall.

The theory of percolation under an impermeable dam or coating has already been given (see p. 292). The practical aspect of the question is obscure, and although the necessary data exist in a few cases, the results obtained are conflicting. Putting aside a few newly constructed weirs, it is plain that the cut-off walls are rarely, if ever, carried down to a depth which is sufficient to have much effect in stopping percolation.

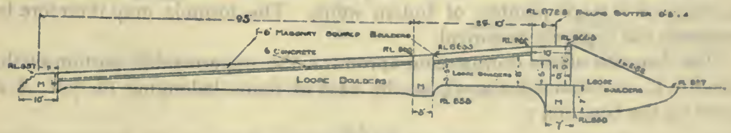
Thus, for the type of weir that is now generally employed, we have merely to consider $2b$, (see Sketch No. 180) the total breadth of the impermeable portions of the weir (clay or masonry, or grouted rubble). $2b$, should obviously be some multiple, of the maximum total head of water which is retained by the weir. The maximum total head usually occurs when there is no water flowing over the weir, and is therefore the difference between the top of the shutters on the weir, and the low water level downstream of the weir, say H_c.

Bligh finds that the relation is expressed by

$$2b = cH_c,$$ and :

I. $2b = 5$ to $9H_c$.	In clay, shale, or shingle.
II. $2b = 12H_c$.	In coarse sand. (This is the usual type.)
III. $2b = 15H_c$.	In fine sand, *e.g.* Punjab sand.
IV. $2b = 18H_c$.	In mud and silt, such as in the Nile.

I consider that any attempt to define the terms coarse sand, fine sand and silt is at present futile. We require considerably more knowledge of the relation between the size of the grains and the conditions producing the failures I have endeavoured to describe by the terms fountaining and piping (see p. 299). The sands used in the experiments there described were selected as being typical of Classes II. and III., *i.e.* "Madras" and "Punjab" sands. Spring (*ut supra*, p. 667), when discussing the scouring of rivers, gives a large



SKETCH No. 181.—Rupar Weir.

number of sand-sifting analyses, and it would appear from these that, broadly speaking 80 per cent. of the grains of a Madras sand are retained on a 40 mesh sieve, while 80 per cent. of Punjab sand grains pass through a 75 mesh sieve but are nearly all retained on a 100 mesh sieve. Similarly, about 60 per cent. of Egyptian grains pass through a 100 mesh sieve. These figures are given as rough guides to engineers who have no knowledge of the localities referred to. In practice I consider that systematic experiments are necessary, even when a successful weir already exists on the river it is proposed to deal with.

Bligh also considers that where the curtain wall or cut-offs are deep, twice the sum of their depth below the aprons should be added to the breadth ; and the total quantity thus obtained should be put equal to $2b$.

The Laguna weir (Sketch No. 176) shows an existing design, where sheet piling is used in order to form a cut-off. A definite statement cannot be made on this point. No doubt the very deep (20 to 30 feet) cut-off walls of steel piling recommended by Bligh check percolation ; although, according to theory (p. 296) they are not as efficient as the same length of horizontal apron ; but the rules are obtained from a study of existing weirs, where no such deep cut-offs exist. Thus, Bligh's extension of his rules must be regarded with caution. The matter is extremely important, for if deep cut-off walls are even only half as efficient in stopping percolation as Bligh considers them to be, their adoption will certainly enable considerable economies in design to be effected. For the present, and until further evidence is available, I think that it is wise to regard the shallow cut-off walls in most of the existing weirs as constructed merely as stops against localised percolation. Their main function, however, is to prevent any undermining of the weir masonry by pot-holes, which may form at the up and downstream edges of the masonry apron.

Thus, in my opinion, shallow cut-off walls act as an aid to the loose block talus, and cannot be regarded as in any way equivalent to an extra length of impermeable stratum of masonry or clay. The question of the precise function of deep cut-off walls of sheet piling must be left open until practical experience has accumulated.

The total breadth of downstream apron and talus is fixed by Bligh as follows:

$$L = 10c\sqrt{\frac{H_b}{10}}\sqrt{\frac{q}{75}} \qquad \text{for sand foundations,}$$

where c, is the constant in the equation $2b = cH_c$, H_b, is the height of the fall over the weir, *i.e.* the difference of level between the masonry crest of the weir, and the low-water level downstream of the weir, or the normal bed level below the weir, whichever happens to be the greater, and q, is the number of cusecs that pass over each foot length of the weir during the maximum flood.

In clay, or in weak rock, it is sufficient to put $L = 6H_b$.

The results of this formula agree very well with the figures obtained from drawings of a large number of Indian weirs. The formula may therefore be accepted, but is purely empirical.

The breadth of the downstream apron (*i.e.* the impermeable portion of the breadth L, as obtained above), in the case of dams belonging to type *A*, is given by the equation:

$$w = 4c\sqrt{\frac{H_s}{13}}$$

where H_s, is the fall from the top of the shutter to the top of this apron. In dams belonging to type (*B*), aprons do not occur, and in those belonging to type (*C*), Bligh puts:

$$w = 4c\sqrt{\frac{H_c}{13}}$$

The formulæ are empirical, and do not agree closely with practice, except in the case of type (*C*).

In the design of weirs belonging to type (*C*), as existing in the Punjab, w, is usually fixed by the condition that the masonry apron must extend at least as far down the weir as the place where the standing wave is likely to occur. In actual practice, the whole breadth of the downstream slope is sooner or later either grouted up, or surface pointed with mortar, with the object of reducing the cost of repairs. The latest designs (see Sketches No. 180 and No. 182) therefore show the whole slope as constructed of masonry, and in view of the fact that grouting up the loose rubble tends to increase the uplifting pressure on the masonry, this provision appears to be correct.

In clay, or weak rock, $w = 3H_s$, is found to be sufficient.

The thickness of the downstream apron is obtained by estimating the static pressure which exists on its lower surface, and which tends to blow it up.

Let A_1, be the length of the line of creep to any point in the downstream apron, *i.e.* :

> A_1 = the breadth of the impermeable portion upstream of this point, plus (according to Bligh) twice the depth of all the intervening drop walls.

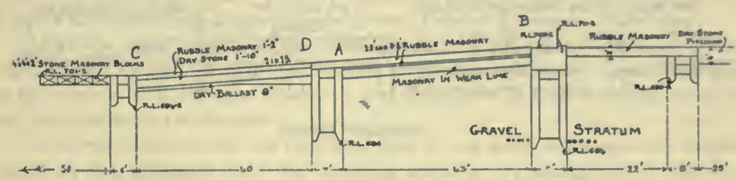
Then *t*, the thickness required, is given by the equation :

$$t=\frac{4}{3}\,\frac{H-h}{\rho-1}$$

where the apron is submerged at low water, and

$$t=\frac{4}{3}\,\frac{H-h}{\rho}$$

where the apron is not submerged, where ρ represents the specific gravity of the masonry or clay, whichever is employed, and $h=\frac{A_1}{c}$, and H, is the difference of level between the top of the shutters and the bottom of the apron at the point considered. This obviously secures an excess of 33 per cent. of dead



SKETCH NO. 182.—Lower Jhelum Weir.

weight against the probable upward hydrostatic pressure. The theory already given might be employed in order to calculate *h*, in the following form :

$$h=\frac{H}{\pi}\cos^{-1}\frac{x}{c} \qquad \text{(for notation see p. 292)}$$

but the designs of existing works are such that no great difference occurs whatever formula is used ; and, as already stated, sufficient information does not exist to justify any very wide departure from the lines of existing design.

The dam wall itself is usually proportioned as a dam to resist a head equal to H_c. Thus, if rectangular, its thickness would be represented by :

$$T=\frac{H_c}{\sqrt{\rho-1}}$$

the factor $\rho-1$, being used, since the wall may be considered as submerged.

The top width, however, is usually 1 foot, and, better still, 2 feet, greater than the height of the shutters ; and where these are high, this often determines the width of the wall.

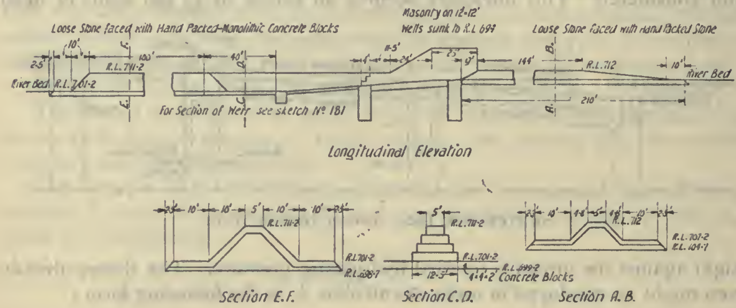
The formula has a theoretical justification in weirs belonging to type (*A*). In types (*B*), and (*C*), the formula appears to give unnecessarily great width, but the advantage of having a heavy, stable wall so as to resist the stresses produced by the water pressure on the shutters is obvious, and any decrease appears inadvisable, although in a few weirs belonging to type (*B*), slightly thinner walls have been found satisfactory (Sketch No. 174).

The pressure of the dam wall on its foundations should be calculated. The maximum values found in practice are usually those produced by the piers of the undersluices, and may be taken as:

One ton per square foot, for fine silt, as in the Nile.
Two tons per square foot, for coarse sand.
Four tons per square foot, for clay.

The value given for clay agrees very well with ordinary practice, but the values for sand and silt are lower than those which occur in similar soils in the case of such structures as bridge piers. It is therefore possible that pressures as high as 2 tons per square foot for silt, and 4 tons per square foot for coarse sand, might be employed; especially if the sand is prevented from moving under erosion by a coffer dam of sheet piling. The question is not of great importance, as the dimensions of the curtain walls are generally fixed by other considerations.

A study of Sketches No. 181 and No. 182 will show that these formulæ



SKETCH No. 183.—Groynes of Lower Jhelum Weir.

lead to results which agree fairly well with present practice. The sudden diminution in thickness at the second drop wall in the case of the Jhelum weir (see Sketch No. 182) is justified if the drop wall is relied upon to largely stop percolation. Qua damage by erosion, the lengths AB, and CD, are units; and if the theory is too closely followed the thinner portions of these floors might fail, and thus start the destruction of the thicker parts. Whereas, by keeping the thickness uniform, each length has a uniform strength to resist erosion, and is likely to fail independently. A complete breach of the weir is thus rendered less probable.

In all types of weir the masonry apron should be split up into cells by cross masonry walls parallel to the stream flow, carried down as deep as the cut-off walls and spaced say every 200 feet along the length of the weir. The object is plainly to localise and prevent the extension of any hole that may form in the apron.

The layer of spalls and fine stone shown in Sketch No. 182 and in No. 175, Fig. 2, may also be noted. This is a new idea, and is termed a reversed filter. The object is to provide stone which it is hoped will fall into and fill up any pipes tending to form in the sand, and so stop the further removal of the sand in the same way as the gravel bed retains the sand of a slow sand filter. The idea is the fruit of a discussion of certain experiments made by Clibborn (*Experiments on the Percolation of Water through Sand*). The principle appears to be sound, and it is obvious that if it works well nothing further will be heard of the matter. The position appears to be well selected, for should piping occur, the thin masonry over the stones will probably crack, and will permit the water to escape in an upward direction, which is exactly that which is most favourable for the action of a reversed filter.

Spalls and run of the breaker stone are cheap, and the principle might be extended with advantage by depositing similar masses of stone on the downstream side of each drop wall. The proposal was discussed during the design of the Jhelum weir, and was abandoned on the ground that the stone might provide an easy path along the cut-off walls for the water. In my opinion this is unlikely, and in any case the mere fact that a cut-off wall has been sunk secures that the sand near its face has been disturbed, and, consequently, in all probability already provides the easy path.

CURTAIN WALLS, OR CUT-OFFS.—The idea at present held in the Punjab is that curtain walls merely prevent erosion. This is very well illustrated by Floyd's statement (see *Upper Chenab Canal Project Estimate*), as follows :

"For the crest wall (*i.e.* the dam itself) it is proposed to sink a line of wells 12 feet × 8 feet (in plan) to a depth of 18 feet below the mean bed level of the river, which will take them to 5 feet above the bed of boulders found in the borings made on the site."

This really means that the lime plugging of the wells will reach the boulder bed, and the inference that the boulders indicate a stratum which the river has never yet eroded is confirmed by their appearance. This selection contrasts strongly with Bligh's idea that curtain walls stop percolation, as the design plainly forces the percolation towards a stratum which is favourable to percolation.

Personally, I am inclined to believe that a line of wells has but little influence, and, as will be seen from the Okla weir (Sketch No. 174), weirs of type (*B*), stand very well without curtain walls, or cut-offs of any description. A really well grouted (see p. 980), and perfectly impermeable cut-off formed of sheet piling has never yet been thoroughly tried. Such walls exist in Egyptian barrages (*e.g.* at Esneh), but they are shallow, and their function is probably only to retain the sand under the heavy pressure of the barrage piers.

The Lower Jhelum weir (see Sketch No. 182) is amply provided with deep cut-off walls. Nevertheless, it partially failed under the action of a strong cross stream current running parallel to the length of the weir. As a contrast, the Lower Chenab weir (see Sketch No. 179) has no deep walls, and although it did fail, its failure is amply explained by the insufficient length of impermeable aprons. The actual failure, moreover, occurred on the site of an old, deep channel. Consequently, although it is possible that had a curtain wall been carried down through the newly deposited silt into the firmer old bed of the river, matters would have been improved, it is not certain that deep walls were necessary.

Hence, I must confess that I am incapable of deciding the question, although I consider that Bligh's theories are very attractive, and deserve testing.

THE UPSTREAM APRON.—A certain, and perfectly safe economy can be secured by taking advantage of the principle of the upstream apron. Sketch

No. 179 shows this apron as adopted in the repairs which were effected on the Lower Chenab weir. The wells are generally unnecessary, as the correct level for such an apron is at, or near to, the bed level, where it is not exposed to erosion. The design adopted in such cases consists of 3 feet of puddle clay, covered by 2 feet of concrete blocks. The clay should be well joined to the masonry of the dam wall by the methods already discussed (see p. 323). The advantages are obvious. We wish to secure a certain length of impermeable coating. Downstream of the weir all structures are exposed to wave action, and erosion; and an impermeable downstream apron must therefore be formed of costly masonry. Upstream of the weir any structure situated 3, or 4 feet below the level of the fixed crest of the weir is but slightly exposed to erosion, and the upstream apron can therefore be made of the cheaper and equally effective (qua impermeability) clay. The 2 feet thickness of rubble, or concrete blocks, (blocks are better as affording less shelter for watersnakes and crayfish, or other animals, which might bore through the clay), secures the clay against erosion.

GROYNES.—Erosion by cross currents has to be provided against in all weirs that cross wide rivers. This is effected by means of groynes of the type shown in Sketch No. 183. The angle between the groyne head and the weir should be pitched with concrete blocks, so as to prevent erosion of the sand by the vortex formed a little downstream (with regard to the cross current) of the head of the groyne.

Bars.—So far we have assumed that the weir raises the water level of the river to a certain extent. Thus, take a river with a bed slope of $\frac{1}{2000}$, feeding a canal with a bed slope of $\frac{1}{8000}$. The weir is supposed to raise the water level 13 feet. The saving in canal length is consequently:

$$\frac{13}{\frac{1}{2000}-\frac{1}{8000}} \text{ feet} = 34{,}700 \text{ feet.}$$

This alone may pay for the whole weir.

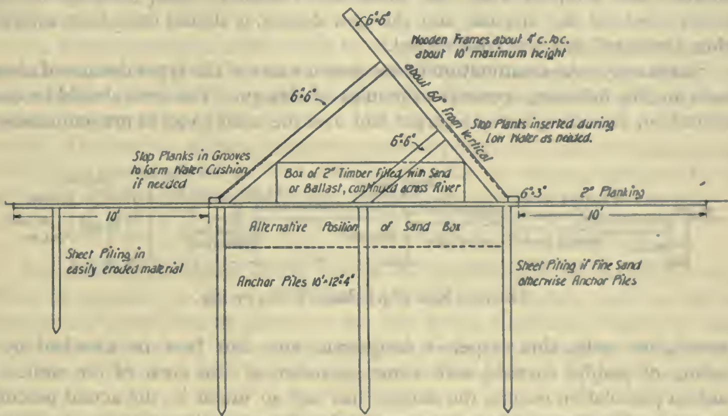
In a torrential river, however, the saving in canal length may be inappreciable; and in such cases, a bar or weir the crest of which is at, or very close to the normal bed level of the river, may suffice to secure a complete control of the river. Wherever possible, a bar of this description should be adopted in preference to a weir, for the river being torrential, the velocities over the weir in flood are likely to be very high if the weir obstructs the natural waterway to any extent. As will be seen when discussing the Bara failure, velocities exceeding 30 feet per second will probably destroy any raised weir.

Sketch No. 180, Fig. 1, shows a bar erected on the Jhelum river, in an extremely torrential reach (probably as bad a case as is ever likely to be dealt with). The section is very large, but it is quite impossible to unwater to a depth greater than that which is shown by the foundations, and piling is unprocurable.

In cases where sufficient pumping power is available, a far slighter design in sheet piling and masonry about 3 feet below bed level would suffice.

Sketch No. 184 shows a timber section which has performed very good work in America. The Sidhnai weir (see Sketch No. 162) crosses a markedly non-torrential river, and shows a good type where percolation (rather than erosion) has to be prevented. The Bengal gates (Sketch No. 190) may also be used in sand bearing rivers.

Failure of Weirs.—Three cases are selected.—Firstly, the Narora weir. The original design is shown in Sketch No. 173, and the large additions (which are probably unduly extensive) are also shown. The floor was blown up for a length of 350 feet, and it is probable that the failure was due to the floor not being sufficiently thick to resist the upward water pressure. The two pipes show the pressures which were actually observed by Beresford shortly before the failure. The values of the pressures, $(H - h)$, thus obtained seem to indicate that the upstream clay apron was that portion of the impermeable stratum which proved most efficient in diminishing the pressure arising from percolation. So far as the observations go, Bligh's theory regarding the efficiency of drop walls is confirmed. To my mind, the lesson is mainly that weirs belonging to type (*A*), are unsuited for sand foundations.



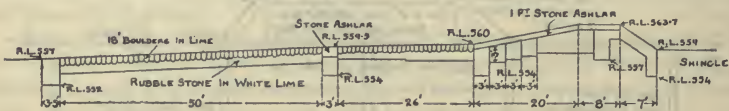
SKETCH No. 184.—Timber Bar.

The second example is the Chenab weir at Khanki. Sketch No. 179 shows the original design. The ratio $\frac{2b}{H_c}$, is approximately equal to 8·3, in place of 12 or 15. So also, the upward pressure downstream of the core wall is about 9 feet of water, which is slightly in excess of the weight of the 4 feet of stone masonry in lime mortar. The apron, however, did not blow up, as was the case with the Narora weir; and the circumstances attending the failure rather suggest that localised percolation occurred, of an intensity which was sufficient to undermine and finally crack the apron. The circumstances are obscure; the failure occurred at a point where the sand is known to be less well consolidated than is usually the case. The weir also appears to have sustained rather rough handling by the sudden dropping of long lengths of crest shutters, followed by an equally rapid raising. Nevertheless, the section is obviously weaker than usual, and the manœuvres above referred to are required in the systematic regulation of any river which is subject to sudden freshets.

The failure of the Bara weir (Peshawar district), which I take as the third example, was of an unusual character. Sketch No. 185 shows the cross-section. The weir was a small work, only 124 feet in length over all, and crossed an extremely torrential river with vertical banks 30 feet high. The circumstances are peculiar, the river section suddenly narrows to a width of 82 feet at the tail of the weir. Thus, the afflux is probably small, but the velocity over the weir is very great. The weir withstood several floods, of which one at least passed over at a mean velocity of 24 feet per second. It finally failed, and was completely destroyed by a flood which, if the weir stood until the maximum level was attained, must have had a mean velocity of at least 32 feet per second.

The conditions are plainly unsuited to a weir. The bed slope of the river is about $\frac{1}{100}$, and the water level could be maintained at the elevation 563·7 by means of a bar situated 500 feet upstream of the present site. This solution was adopted when the work was repaired; and, judging by the relative costs of the original and the new design, it should have been selected when the canal was first constructed.

SUMMARY.—An examination of successful weirs of the types discussed above leads to the following general principles of design. The weir should be considered not as a dam, but as a carpet laid over the sand so as to prevent erosion.



SKETCH NO. 185.—Bara Weir, or Bar.

Percolation under this carpet is dangerous, and can best be checked by a coating of puddle covered with stone upstream of the crest of the weir. If marked percolation occurs, the danger lies not so much in the actual percolation, as in the removal of sand from under the weir, thus forming a definite channel. Consequently, reversed filter beds at the tail of a weir are a very valuable means not so much of checking percolation (for they probably increase it), as of preventing the dangerous effects of percolation. From this point of view, rigid masonry in a weir is a mistake, as it prevents the stone carpet from falling in and filling up any defined channels that may form. On the other hand, if there is no covering of large blocks or rigid masonry, the upper stones of the carpet are likely to be displaced and carried away by floods. Thus, the masonry portion of the weir should be supported by shallow drop walls at frequent intervals, and should be as thin as is consistent with resisting the wave action of the overflowing water, and the upward pressure caused by percolation.

Even the masonry of the Lower Chenab weir is thicker than is requisite if an upstream apron is put in; although, until this upstream puddle coating was laid, the uplifting pressures were sufficient to severely tax the masonry, and a smaller thickness would probably have failed.

A masonry core wall at the tail of the downstream apron is of course necessary in order to prevent erosion, should the talus blocks be removed. Such walls are not intended to sustain water pressure in the sense that the dam or curtain wall does. The smaller drop walls serve two purposes, they assist in localising any damage done to the apron, and also act as chases to prevent the formation of percolation channels immediately beneath the stonework. It would therefore appear that they should be numerous and shallow, rather than few and deep. The upper surface of the downstream apron should be of cut stone in hydraulic mortar, pointed with cement, and should be carefully inspected during each period of low water.

If these ideas are followed to their logical conclusion, the proportioning of a weir would probably be as follows :

(i) The dam wall would be made rectangular, and of a thickness such that :

$$T = \frac{H_c}{\sqrt{\rho - 1}}$$

where ρ is the specific gravity of the masonry. The foundation level of the dam wall would then be fixed by a consideration of the pressure produced on the sand ; or, in practice, would probably be laid as deep as the subsoil water level permitted.

(ii) The downstream slope would be of good masonry, of sufficient thickness to resist the action of the water passing over it (say 2 feet in ordinary cases, and 3 feet where severe action was anticipated). These remarks only apply to a weir belonging to type (*C*).

(iii) The length of the upstream apron of puddle clay and concrete blocks would then be calculated by the condition that the percolation pressure $H - h$, acting on the downstream apron was not sufficient to blow it up, or say :

$H - h = 2$ feet when the downstream apron was 2 feet thick.

The downstream apron would provide the remainder of the length $2b = cH_c$, which is required to prevent localised percolation. A drop wall, or line of sheet piles, would then be placed at the tail of the downstream apron in order to check local erosion. The downstream talus of square concrete blocks would then be continued so as to provide the required total length L, given by Bligh's rules.

The general agreement with the Chenab weir as originally constructed is obvious, provided that the fatal defect caused by the absence of the upstream apron is neglected.

The above proposals lead to a design which contains the minimum possible quantity of cut stone masonry. Although probably more bulky than the present designs, it will prove cheaper, as the extra bulk consists of clay and concrete blocks (*i.e.* hydraulic lime concrete), and these are cheap. Also the larger portion of the impermeable coating is found in the upstream apron, where it is subject to but slight erosion.

When this design is sketched out, it will probably be found that the difference of level between the top of the weir and the normal bed level is such that the downstream apron is obviously somewhat short (it certainly will be so if tested by Bligh's empirical rules), and the tail cut-off wall is therefore somewhat higher than usual. A modified design with a somewhat longer apron, and consequently a somewhat lower tail cut-off wall, will therefore prove cheaper. The final design can best be arrived at by trial estimates of cost, and will greatly depend upon the unit prices of masonry, concrete blocks, and the material of which the cut-off wall is composed (which, in India, is usually masonry wells; and in other countries is probably steel sheet piling). Finally, the location of the standing wave must be considered, and estimates of the relative costs of an extension of the masonry apron, or the provision of an extra thickness of concrete blocks where the wave occurs, must be made.

So far the question of the relative cost of work done in the dry (*e.g.* the masonry tail apron), and of work done in the wet (*e.g.* the foundations of the dam wall, and possibly the puddle clay upstream apron) has not been considered, but this will also have some influence upon the final design.

Maintenance of Weirs.—The maintenance of a weir requires the careful attention of a skilled engineer during each low water season. It is so much a question of local knowledge that, contrary to the usual practice in Public Services, the officer in charge is rarely transferred to any other post.

The work usually consists of the following repairs:

(i) On the masonry work. Re-pointing all eroded joints, the renewal of all displaced stone, and the careful smoothing off of all irregularities and filling up of all hollows.

(ii) Concrete blocks. These are examined and replaced. The advantage of being able to inspect as much of the work in the dry as is possible is obvious, and forms the principal objection to the upstream apron. For this reason, a design in which the upstream apron is not at bed level, but is say from 4 to 5 feet below the weir crest, is preferable; for ring banks of sand can then be cheaply constructed so as to permit an examination of the weak spots which are disclosed by soundings.

The maintenance of the talus is less difficult, as a bare patch is not necessarily dangerous. In some rivers it is possible to expose the major portion of the talus during low water seasons. Where this, is not possible, as at Khanki (Chenab), the places which require repairs are found by soundings, or by wading over the talus. A small island of loose sand is then made at the spot discovered. Concrete blocks, previously prepared and seasoned in the block yard which is attached to each headworks, are laid on this island; and the weir shutters above the island are manipulated so as to direct a current of water against the sand. The sand scours away; and, when the process is skilfully managed, the block can be dropped into place to a nicety. It must, however, be remembered that the regular horizontal talus where blocks are carefully arranged in order, rarely exists in practice.

Talus repairs are occasionally effected in Egypt by filling the interstices with ballast, and then grouting up. The talus is more regular than is usual in the Punjab, and this process may assist in obtaining the result. The Nile floods, however, are believed to be far less severe in their action than those of Indian rivers, and in any case it is a matter of doubt whether the orderly arrangement shown in the drawings is best for the weir.

Special Precautions to be adopted in the Construction of Weirs.—In all hydraulic constructions founded on permeable soil or sand, springs or "boils" may be expected to occur in the area exposed when laying their foundation. In fact, the absence of such springs may generally be regarded as a sign that the foundations are too shallow, and should be deepened.

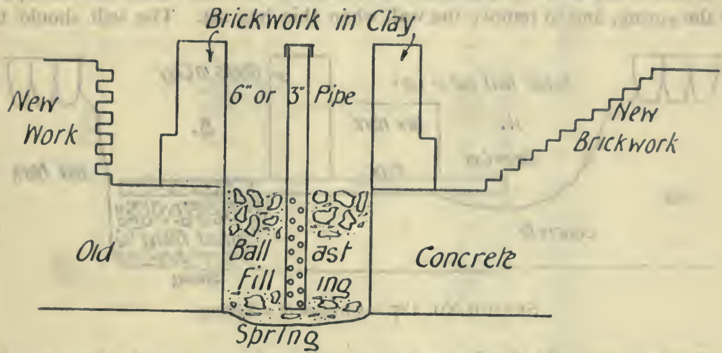
If properly treated, these springs give but little trouble. On the other hand, if they are either disregarded, or if their flow is checked before proper preparations have been made, they will inevitably burst out in some other locality, usually just where their presence is least desired.

In the first place, it should be realised that the ring banks which enclose the working area must be set well back from the work, and it will be found that a far larger area than that of the work itself can advantageously be unwatered. In my own practice, I have usually located these banks by the rule that a line drawn from the bottom of the foundation excavation to the water level in front of these banks shall in no case have a greater slope than 1 in 10.

This may be regarded as a minimum in small works where the extra area thus taken in forms a large portion of the whole area surrounded by the bank.

Even flatter slopes may be considered advisable in large works, as the extra width secured is useful for such purposes as temporary railway lines, and the storage of material.

Each spring, or boil, that occurs must be separately dealt with. The ruling principle is that the spring must not be covered with masonry, or in any way sealed up until it is surrounded by a ring of firmly set masonry of so large an area that the spring when sealed up will find it more easy to burst up outside the area covered by the masonry, than in any portion of this area. Thus, if a spring is *l* feet distant from the nearest boundary of the masonry, it should not

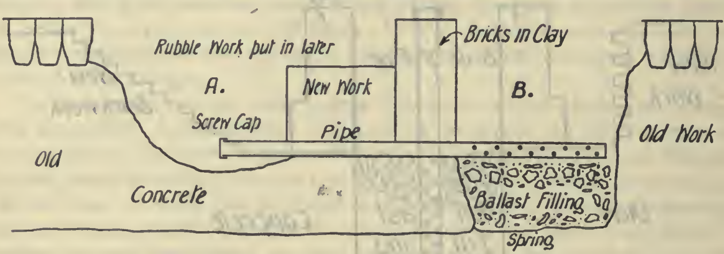


SKETCH No. 186.—Closure of a Spring.

be sealed until the ring of well set masonry round it has a radius of at least *l* feet.

Sketches Nos. 186 and 187 show two methods described by Hanbury Brown (*Irrigation as a Branch of Engineering*). Here, in each case, masonry, or concrete, is deposited all round, and as close to the spring as is possible consistent with the water of the spring being unable to wash away mortar or cement while setting. A ring of brickwork in clay is then built up round the spring, and the water is allowed to rise up inside this ring until it can be conducted away over the set masonry by means of pipes or launders. The masonry is then carried across from the scar end of the set masonry to this temporary ring of bricks in clay, and the sand through which the spring bubbles is carefully dug out (say 12 inches below foundation level), and the space is filled in with ballast so as to form a reversed filter. This masonry having set, a tube perforated with holes is placed so as to carry away the water. The brickwork in clay is then removed, and the whole vacant space is built in with cement masonry, or concrete, the tube being left open so as to permit the water to pass away freely. When this last material is thoroughly set, the tube is closed off by a screw cap. The second arrangement where the draining tube is horizontal and discharges into the small well A (see Sketch No. 187), which is kept free from water by means of a hand pump, is preferable. In my own practice, where a vertical tube had to be used, I have been accustomed to screw on a second length of tube (say 10 feet high), and to pour in cement grout. Then, unless the spring water issues under a pressure exceeding 20 feet head, the grout is injected into the ballast and fills all the void spaces which may have been produced under the set masonry while excavating round the spring.

This method must be used in springs under a pressure such that the water rises well over the top of the completed masonry. It obviously entails a good deal of costly work in cement, masonry, or concrete. In India the springs do not usually issue at a very high pressure, and it is generally possible to build a small wall of brick in clay, or sand bags, round the spring, to such a height that the spring ceases to flow, the pressure of the water ponded up inside this wall being adequate to stop the flow. In such cases it is sufficient to build a wall, and to deposit rich hydraulic lime concrete over the whole area occupied by the spring, and to remove the wall when this has set. The wall should be



Sketch No. 187.—Closure of a Spring.

absolutely water-tight, for if any water is allowed to escape the lime may be removed from the concrete, and setting will consequently be prevented.

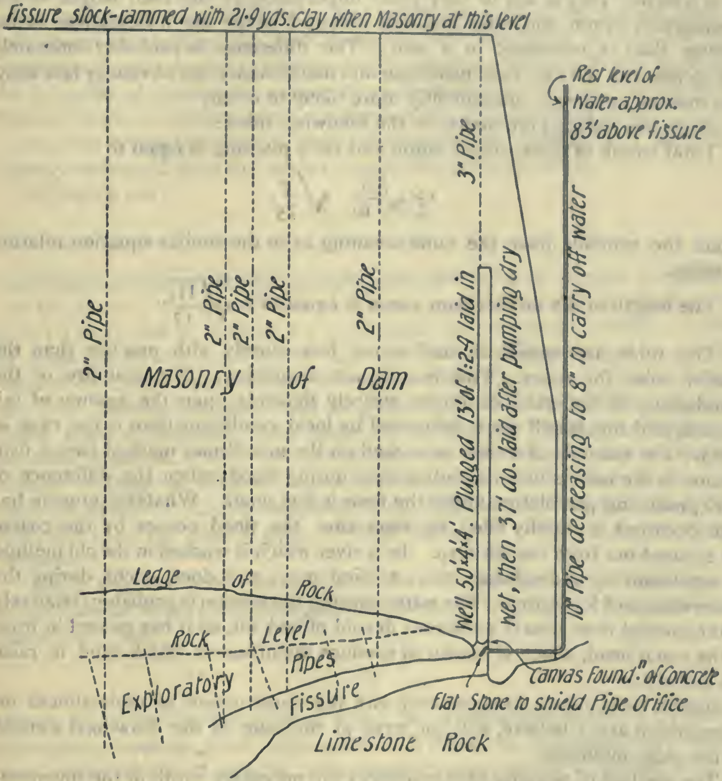
In certain cases, none of the above methods are sufficient, and the spring is then probably not of local origin. For example, there are at least two springs in the Delta barrage (Egypt) which, judging from their temperature, are probably not directly derived from the Nile. The best that can be done in such cases is to guide the spring away from the work by building masonry over it, and injecting grout over its original site, after the spring has made its appearance outside the work. A thick reversed filter covered with loose blocks of concrete or stone should then be laid over the final exit.

Well, or Pile Junctions.—A line of wells or metal sheet piling is frequently used as a cut-off wall. Examples exist in the Esneh barrage (cast-iron piling), and in the Jhelum, and other Indian weirs.

The section of the Esneh piles is shown in Sketch No. 82 (p. 320). After the piles have been driven, the junctions are made watertight by cleaning out the space between the piles, with a water jet, and then injecting cement grout.

In most of the Indian weirs the line of wells is assumed to be sufficiently watertight, the wells being square in section, and sunk very close to each other. The Indian well sinkers are very skilful, and probably watertightness is eventually secured through the stanching by silt that occurs as the weir grows older. In one case, where such a line of walls was momentarily exposed, the interstices appeared to be filled with a material resembling good puddle. In Egypt, the sand being finer, and the well sinkers less skilful than in India, more systematic measures have been adopted. Hanbury Brown (*ut supra*) describes the work at Shubra as follows :

The wells were sunk 6 inches apart, and piles of half-inch steel plate, stiffened with T irons, were driven upstream of, and close to, the wells.



SKETCH NO. 188.—Stockramming of a Fissure under a Dam.

A pipe was then water-jetted down to the level of the bottom of the wells in each angle between the piles and the wells, and was filled with sand in order to exclude the grout from the floor. The floor being completed (in this particular case the floor was of loose rubble grouted with cement), the pipes were cleared of sand and were run in with grout. The above is an exact description ; but, in my opinion, the pipes should have been grouted prior to the floor being grouted, as there is no assurance that the cleaning of the pipes did not produce a void under the rigid grouted floor.

UNDERSLUICES, OR SCOURING SLUICES.—The principles governing the design of the masonry portion of undersluices are the same as those employed in weir design. Although the water does not fall over the undersluices, it rushes through them when opened for scouring purposes, with a velocity which often exceeds the flood velocity over the weir. Thus, the masonry aprons must be quite as thick as in the weir, and tail erosion is probably more intense. The upstream apron of an undersluice is more liable to erosion than in a weir, and must therefore be made thicker, and cannot be relied upon to the same extent as in a weir. This is not of very great importance, as the total length of the downstream apron and talus pitching required below the undersluices is far greater than is necessary in a weir. The difference is probably intimately connected with the fact that inspection and maintenance are obviously less easy, and marked erosion is consequently more likely to occur.

Bligh has reduced the matter to the following rules :

Total length of downstream apron and talus pitching is equal to

$$15c\sqrt{\frac{H}{10}}\ \sqrt{\frac{q}{75}},$$

where the symbols have the same meaning as in the similar equation relating to weirs.

The length of the downstream apron is equal to $7c\sqrt{\frac{H_s}{13}}$.

The rules are empirical, and agree less closely with practice than the similar rules for weirs. This fact is not surprising. The stability of the foundations of undersluices almost entirely depends upon the amount of tail erosion, and this is still more influenced by local conditions than is the case in weirs. For example, if a river is worked on the new Rupar method (see p. 659) erosion at the undersluice tail only occurs during floods when the difference of level producing percolation under the floor is but small. Whatever erosion has then occurred is rapidly filled up soon after the flood ceases by the coarse silt scoured out from the silt trap. In a river which is worked in the old method of regulation by undersluices only, erosion may, and does, occur during the entire season of low water. The water causing the erosion is probably (relatively to the normal river water) somewhat devoid of bed silt, as it has passed in front of the canal head, which is known to produce disturbances which tend to raise the bed silt.

Bligh's rules appear to agree best with the construction of undersluices on rivers which are, I believe, still (or were at the date of the drawings) worked on the older methods.

The method of working also influences the necessary width of the impermeable masonry apron. In undersluices worked on the new principles, the width of impermeable masonry shown in the drawings is only about one-half to two-thirds that given by Bligh's rules. The explanation is obvious. The masonry bed of the upstream silt trap forms a fairly efficient impermeable apron, as the interstices are stanched by fine silt. The design is also affected by the fact that the talus is considerably wider than is the case in weirs ; so that under the new principles of river regulation downstream erosion is less likely to occur below the undersluices than below the weir.

The foundations of the piers of undersluices need careful consideration. The pressure on their base is great. In Indian practice, the piers are usually founded on lines of wells sunk to a depth which is at least equal to the total height of the masonry pier above the foundation level.

Under the conditions prevailing in India, this is probably the cheapest solution.

In Egypt, and in countries where well sinkers are less expert, and piling is more easily obtained than in India, the pier foundations are carried down as far as the subsoil water level permits, and the whole area covered by each individual pier foundation is surrounded by steel or cast iron piling, well grouted at all interstices (see p. 690). The normal depth of the bottom of this piling appears to be about 15 or 20 feet below the subsoil water level; and as this apparently suffices, the Indian type of well foundation is probably (considering the coarser grade of Indian sand) somewhat deeper than is necessary. As a matter of experience, I have never been able to find any record of prejudicial cracks or weaknesses in the piers of any Indian undersluices.

A study of the thickness of floors of undersluices in the light of the theory already developed for weirs leads to somewhat puzzling results.

If Bligh's rule:

$$t = \frac{4}{3}\frac{H-h}{\rho-1},$$

is accepted as correct, nearly all undersluice floors (as at present constructed) are found to be too thin. Thus, at Khanki, $t = 4$ feet, where the theory would give 7·5 feet; and at Narora, $t = 5$ feet, and theory requires 10 feet. Neither of these works has ever shown signs of failure. Equilibrium is possibly secured by the load produced by the weight of the piers, and if this is the case, the floor between the piers acts as an arch, which it is certainly capable of doing when its thickness and span are considered. A more probable supposition, however, is that the paved silt trap upstream of the undersluices rapidly becomes so stanched with silt as to act as an upstream apron. The question deserves careful consideration in view of the fact that the modern tendency is to use Stoney gates of large span, and to put in as few piers as possible.

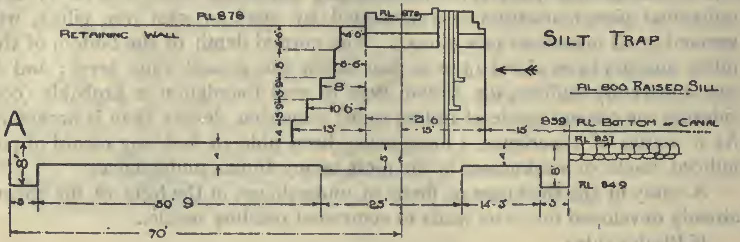
Bligh suggests that the action of the out-rushing water should be considered as the factor determining the thickness of these apron floors, and finds empirically that

$$t = \sqrt{\frac{3H}{2}},$$

where H, is the maximum head which the undersluices sustain. A similar divergence between theory and practice is found in regulators, and the fact that neither of these works is continuously exposed to the maximum head H, may explain the matter. Nevertheless, the question cannot be considered as settled, and deserves further investigation.

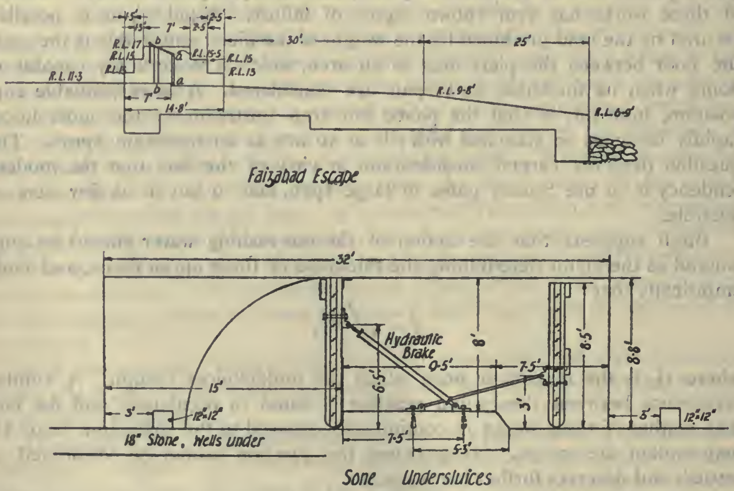
The proportions of piers must be determined as in the case of a dam. The thrust produced by the water pressure acting on the pier, and the two halves of the sluice gates on either side of it, can be calculated. This can be compared with the weight of the pier and arches and roadway (deductions being made for all portions of the masonry upstream of the gates which are submerged in water). The resultant must fall within the middle third.

The discharge of undersluices is usually fixed by a consideration of the area of the obstruction of the natural river channel produced by the weir. The afflux, or rise in the water level upstream of the weir compared with that downstream of the weir (which may be regarded as the natural level of the highest floods in the river) is calculated. The formulæ employed for the weir discharge are given on page 133, and for the undersluices on page 164. The calculation is important in determining the height to which the undersluice gates should be raised, or when damage to land or to the canal works is considered.



SKETCH NO. 189.—Rupar Undersluices.

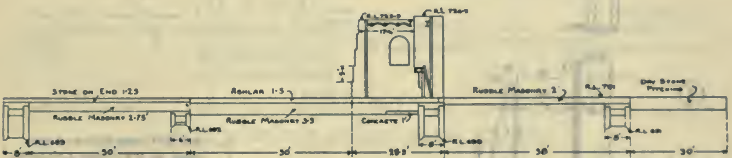
There was originally 124 feet of boulder pitching downstream of A. This is no longer visible, and a deep hole exists from 50 to 100 feet downstream of A. As, however, a very few concrete blocks every year keep the retaining wall foundations secure it is probable that the pitching exists beneath the sand surface.



SKETCH NO. 190.—Bengal Gates.

The method of determining the capacity of the undersluices appears to be illogical. Undersluices are not provided for flood disposal purposes, although they are used for passing flood water. Their essential function is to regulate the river, and above all to scour out silt deposits in front of the canal head. Thus, the discharge capacity of undersluices should be a function of the low-water discharge. I suggest that the undersluices should be capable of passing the average discharge of the three low-water months when the water is headed up to the top of the masonry of the weir, and the discharge of the maximum ordinary low-water freshet when the water is headed up to the top of the shutters. It is believed that the capacities thus calculated agree fairly well with those provided in existing works. Since undersluices are scouring machines, it hardly appears necessary to state that the openings should be as wide as possible (say 20 to 25 or even 30 feet in span), and that they should be provided with Stoney gates. The old Bengal type of gate (see Sketch No. 190) is cheap, and relatively obstructs floods less than the arched type, and may also be employed in clear water streams where silt is not feared, and where the river is regulated by means of undersluices. It is useless in cases where the regulation is effected by means of the weir shutters.

The above treatment of undersluices is obviously deficient. The facts, I believe, are as follows, the theory is incomplete, and owing to the rapid introduction of the new methods of river regulation (p. 661) a study of existing designs is not likely to be very useful. A large number of sections have been collected by Buckley (*Irrigation Works of India*). I believe these to be strong enough for the old methods of regulation, and if so they are over strong when the newer methods are employed. With one marked exception, the undersluice is usually that portion of the headworks that gives least trouble in



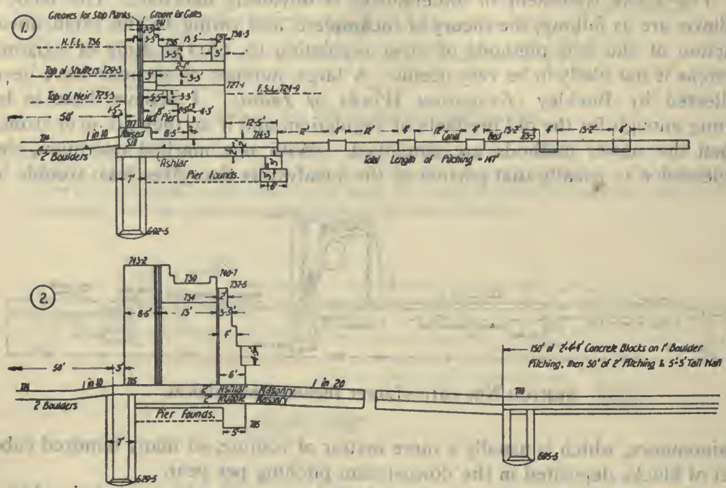
Sketch No. 191.—Lower Jhelum Undersluices.

maintenance, which is usually a mere matter of routine, so many hundred cube feet of blocks deposited in the downstream pitching per year.

Canal Regulators.—The control of the quantity of water admitted into a canal or any one of its branches is usually effected by means of a series of sluice gates erected in a wooden or masonry structure across the canal. The term regulator is a convenient description of the whole structure. The typical regulator consists of a series of piers which carry the sluice gates; and, for convenience, the openings thus afforded are usually arched over so as to provide a platform for the machinery which is used to raise or lower the gates, and this platform generally forms a passage for traffic. I do not propose to consider the design of the platform, as this depends upon local conditions.

Head Regulators.—The regulator at the head of the main canal which controls the quantity of water admitted to the whole system is one of the most important works connected with a canal, and its design largely influences the whole state of the canal, and above all its silt regime.

Canals for flood irrigation, or inundation canals, are not usually provided with head regulators. This defect (for it must be considered as such from a scientific point of view) must be accepted in many cases, for the floods of rivers which rise to a sufficient height to inundate the surrounding country are usually so violent as to cause large and widespread erosion of their banks, which would sooner or later destroy any regulator. The water brought down by the floods usually contains sufficient silt to render the choking of the head reach of the canal in one year quite possible under unfavourable circumstances. Thus, the annual silt clearances of such canals may be partly regarded as the price which is paid for dispensing with a regulator. Nevertheless, once a river has been got under control, it will frequently be found advisable to provide one large regulator, and to admit through it the water which is required for several inundation canals. The Jamrao canal in Sind (*P.I.C.E.*, vol. 157, p. 278), and the Ibrahimiya canal in Egypt are cases where this improvement has been carried out, and these examples illustrate the substitution of perennial for flood irrigation which usually takes place when these regulation works are carried out.



SKETCH No. 192.—Lower Chenab Regulator (1) and Undersluices (2).

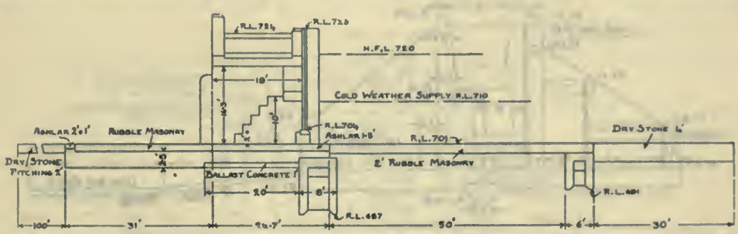
The design of a head regulator largely depends upon the following conditions:

(*a*) The difference between the bed level of the canal (or the lowest water level in the canal in cases where the canal is never closed down), and the highest flood level (corrected for the afflux produced by the weir if one exists) in the river.

(*b*) The quality of the silt carried by the river, and its liability to deposit in the canal.

The difference of level specified above is the maximum head of water which the regulator is required to sustain. The circumstances differ widely from those which exist in weirs and undersluices; for, although the work is usually founded upon soil which is as permeable as that which supports the weir and undersluices, tail erosion does not occur. Thus, the theory already given on page 292 indicates that the depth of the core or cut-off walls is of more importance than the breadth of the impermeable apron. At first sight this statement does not seem to be confirmed by a study of the designs of existing works. Except at Khanki (Sketch No. 192), Okla, and possibly some of the Madras weirs, the core walls of the regulators are carried but little, if at all, deeper than those of the weir and undersluices (compare Sketch No. 193 with Nos. 191 and 168). In the case of some Egyptian works the core walls of the regulators are considerably more shallow than those of the weirs and undersluices.

It must be remembered that in India, at any rate, the core walls are almost invariably carried down to as great a depth as local conditions permit. There is no doubt that in many places the constructional engineers, taking advantage of exceptionally favourable local conditions at the regulator site, have carried the regulator core walls to a greater depth than that shown in the designs. In some of the new projects this procedure has been sanctioned for adoption wherever possible. As a general rule, using the notation given on page 293, we find that $a = H$, or that the depth of the core wall is equal to the head of water retained. The rule is not closely followed owing to the circumstances



SKETCH NO. 193.—Lower Jhelum Regulator.

explained above. The breadth of the impermeable apron is usually given by the following rules of Bligh :

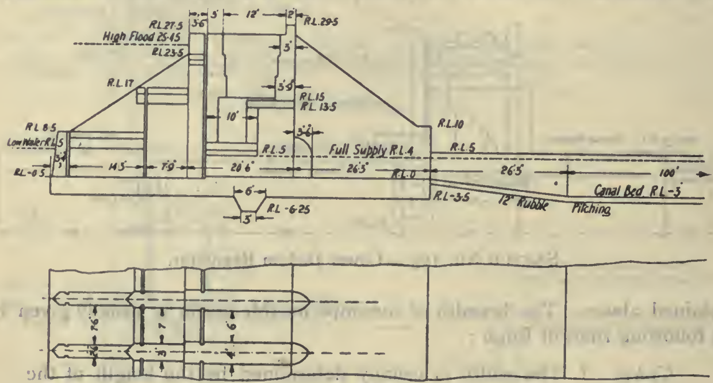
CLASS I. The width is usually determined by the length of the piers required to sustain the water pressure.
CLASS II. For coarse sands, as in Madras, or for the usual type of sand . . . $2b = 2$, to $3H$
CLASS III. For fine (Punjab), or Bengal sand . . $2b = 4$, or $5H$
CLASS IV. For fine silty sand, as in Egypt . . $2b = 8$, or $9H$

It will be noticed that these figures do not bear a constant ratio to the similar figures given for weirs or undersluices on page 679.

The explanation is probably to be found in the fact that the finer the sand the more easily it is eroded ; for it is unlikely that the relation between the head necessary to produce fountain failure and that required to induce piping failure through an equal length of sand, is materially affected by the size of the sand grains. It is also probable that the deeper foundations of the piers of both regulators and undersluices produce the same effect as a deep core wall across the whole breadth of the work.

The dimensions of the piers and their foundations are proportioned by the rules given for undersluices. The thickness of the masonry floor is usually fairly close to $\sqrt{H}$; but, as will be seen when the Trebeni design is discussed, local conditions may cause this to be insufficient.

The clear water way through the regulator should be relatively large, in order to minimise any disturbance in the entering water, and thus reduce silt troubles. A velocity of 3 feet per second should not be exceeded in waters which carry sandy silt. In rivers which carry boulders and gravel, the usual rule is 5 feet per second; and 7 feet per second is certainly too high. In my opinion, 3·5 to 4 feet per second should be adopted in these cases wherever possible. The advantages of large sluice gates are obvious, and spans of 30 feet are usual. It must, however, be realised that until quite recently the 20 to 30 feet spans of regulators were usually divided up into three small spans by jack piers (see Sketches Nos. 192 and 193 and p. 701) and that our experience of 30 feet spans is limited. Nevertheless, I see no reason why spans of 60 or even 80 feet should not be adopted with advantage; for, unless the retrograde step of using spans of 10 or 15 feet is considered advisable, Stoney or other "frictionless" gates must be used. The total cost of a fixed length of such gates largely depends upon their number, as



SKETCH No. 194.—Trebeni Regulator.

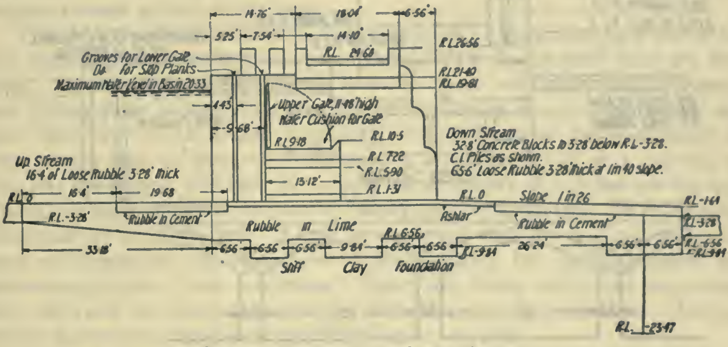
the frictionless gear is far more expensive than the steel I beams, and plating which form the gates.

In silt-bearing rivers, it is desirable to design the regulator so that water can be drawn off from the surface of the river at whatever level this may be. The arrangements adopted entirely depend upon the variation in river level, which may be taken as the difference between the highest flood level and the top of the weir, or its shutters. Sketch No. 194 shows the Trebeni (Bengal) canal head, which is admirably adapted for cases where this variation is great. The projecting double arches can be blocked by baulks of timber, so that surface water alone is taken into the canal. The design is an excellent one, as the energy of the water which falls over the arches is dissipated in the water cushion that can be maintained by the manipulation of the draw gate; and if damage does occur from this cause, it will be found on the upstream side of the sluice gates, where it can do least harm.

The front arches are only 7·5 feet in span, measured parallel to the length of the regulator; and, in view of the desirability of splitting up the falling stream into small bodies (as discussed under Falls, see p. 724), this may be considered to be the proper size. In the existing work, the sluice, or draw gate channel, is only 6 feet in span; but this might with advantage be increased to 20, or even 25 feet, if Stoney, or other modern gates were used.

The sketch of the Qushesha, Egypt, escape (No. 195) shows a very interesting example of another (and in my opinion far more costly) method of dealing with the problem. This work is not a regulator, but is an escape for draining a large series of flood irrigation basins. My criticism only refers to the adoption of a similar design for a regulator, as I am not sufficiently conversant with the local requirements to state whether the upper falling gates are required in the actual work.

The cross-sections of the Khanki and Rasul head regulators (*i.e.*, Sketches No. 192 and No. 193) show the application of the principle of the "raised sill" (see p. 660), for the exclusion of silt in rivers where the variation in level is less than at Trebeni. Sketches of the elevation of these works are not given, as the designs were made prior to the introduction of the Stoney sluice, and



SKETCH NO. 195.—Qushesha Escape.

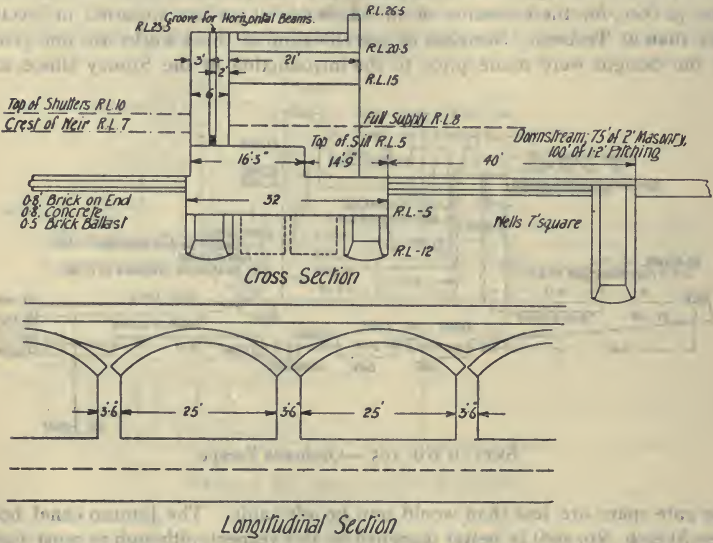
the gate spans are less than would now be advisable. The Jamrao canal head (see Sketch No. 196) is better designed in this respect, although in most cases sluice gates would have to be substituted for the regulating beams shown in the sketch.

The design of head regulators in coarse sand follows the same principles.

The cross-sections of the Tajewala regulator and of the Madhupur regulator show the forms of raised sill adopted in rivers which carry boulders (see Sketches Nos. 168 and 169).

As already stated, neither of these works can be considered as correct solutions of the problem of excluding gravel and sand from the canal. The rivers are not properly under control. If they are ever got under proper control there is no doubt that a raised sill of the Rupar type will soon be built across the regulator, so as to skim the surface water from the comparatively still pool that will then exist in front of the regulator. At present, as stated on page 662, the engineers construct dams across the river as it falls, after the flood season; and were the sill level any higher than it now is, it would probably be found impossible to secure a continuous supply of water in the canal during the interval between the last flood and the completion of the temporary regulating works. The adjustable gates used at Tajewala permit a temporary raised sill to be produced during the low water season. While the gates themselves and the 6 feet wide arches are almost prehistoric, the principle is a good one.

The length of the waterway of the head regulator is probably one of the most important factors in the whole design of a headworks; and there is no doubt that the penalties of insufficient waterway are very forcibly brought under the notice of all concerned with the working of the canal. The rules already given enable the nett area of the waterway to be calculated, and if any divergence from these rules is contemplated, it should certainly be in the direction of increasing rather than reducing the area of the waterway.



SKETCH No. 196.—Jamrao Regulator.

The principles of the calculations concerning the length of free waterway and the level of the sill of a canal head regulator are illustrated by the figures given in the "Project Report of the Upper Chenab Canal."

The nett waterway of the canal head regulator is 253·5 feet long. The raised sill will be 3·5 feet above the floor of the silt pocket, and 2·98 feet above the canal bed level. The full supply depth in the canal being 11·56 feet, the depth over the sill, considered as an orifice, is 8·58 feet, or the nett area is 2177 square feet. The maximum discharge of the canal is 11,694 cusecs. Thus, the head required to produce this discharge, assuming a coefficient of 0·80 (see p. 168), is given by the equation:

$$0{\cdot}80 \sqrt{2gh} = \frac{11694}{2177} = 5{\cdot}40\,; \qquad \text{or, } h = 0{\cdot}93 \text{ foot.}$$

The actual difference of level between full supply in the canal and the top of the shutters is 1·08 foot, so that a margin of 0·15 foot head remains available in case the canal becomes somewhat silted.

This method of calculation evidently affords a somewhat greater margin than stated, for the discharge is calculated as though it occurred through a submerged orifice, in place of over a drowned weir, and the piers of the regulator being nicely tapered off, the coefficient 0·80 is probably somewhat low.

If the accurate drowned weir formula is used, it will be found that the available margin of head is 0·20 foot. It is, however, plain that it is inadvisable to force the engineer in charge of the weir to head up the river to the level of the top of the shutters, except on rare occasions, lest he be caught by a sudden freshet. The whole design is intended to keep the velocity at entry low, and it will be found that the ordinary maximum supply, 9320 cusecs ($=\frac{4}{5}\times 11694$ cusecs), can be passed in under a head of about 0·78 foot, and that the velocity will not exceed 4·33 feet per second. This value is probably somewhat high, and a waterway proportioned for a velocity of 3, to 3·5 feet per second would be preferable, were it not for the extra cost entailed in constructing an additional length of about 50 feet of regulator.

The design considered above is composed of 13 bays, each 24·5 feet wide, and each separated into three openings 6·5 feet wide, by jack piers 2·5 feet thick. In the work as constructed, however, these jack piers were abolished, and Stoney gates 25 feet wide, span the whole opening of each bay. Thus, the clear waterway is 317·5 feet long, and the nett area as finally adopted is 2700 square feet, so that the extraordinary supply passes in at a velocity of 4·34 feet per second, and the "full supply" at a velocity of 3·47 feet per second. The advantages gained by the use of Stoney gates of large span are excellently illustrated by these figures. It may be remarked that the original designers were well aware of the advantages of Stoney gates, but were hampered by somewhat peculiar local conditions. Therefore, being competent engineers, they designed the work so that little difficulty was experienced in inserting Stoney gates when these local conditions ceased to exist.

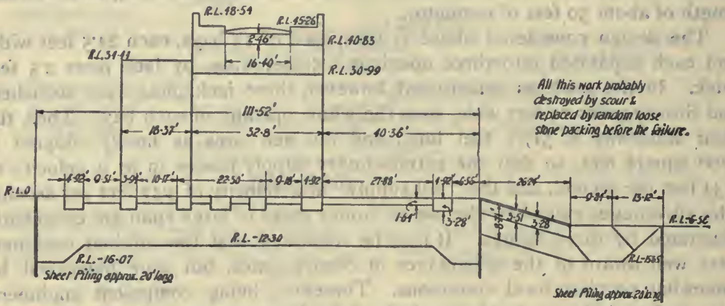
Failures of Regulators.—The most instructive example is the failure of the Menufiah regulator (a portion of the Delta barrage works in Egypt), which occurred on the 26th December 1909. The failure was very rapid, the regulator being reduced to a mass of masonry fragments within an hour of the first signs of failure.

Sketch No. 197 shows Mougel Bey's original design, which does not appear to have been added to or repaired in any way. The regulator was the only portion of the Delta barrage which was held not to require the grouting operations described on page 981. The failure is stated to have been caused by piping, and probably only one defined channel existed when the failure commenced. Personal inspection of the sheet piling permits me to say that the downstream line of piling was imperfect, since gaps an inch or more in width existed in many places between adjacent piles. Otherwise, the construction was first rate in quality, and it is believed that the whole of the foundations were laid in the dry. In my opinion, the work is amply safe against piping in the restricted sense in which I consider that the term is best employed. The failure probably occurred in the following manner:

Fountain failure was set up opposite to some unusually defective portion of the sheet piling, and this gradually removed the sand under the foundations from between the lines of sheet piling. A void space was finally formed under the work. Then piping began. Once sand removal started, the masses of stone which were thrown in downstream of the work in order to fill up the scour holes produced by each flood were of but little assistance, as the sand was swept through the interstices between the stones, so that the work was practically in the same condition as if tail erosion had occurred. If my theory is correct, the second line of sheet piling upstream was positively detrimental. The fact that the regulator failed when it retained only 3 metres (say 1·0 feet) of water, and had previously on frequent occasions retained 4 metres (say 13 feet) without showing any signs of failure, is of course not surprising; for once a passage extended partly across the breadth of the foundations, a smaller pressure would suffice to complete the failure.

The circumstances cannot be considered as typical of regulators in general, as tail erosion is known to have occurred to such a degree that systematic soundings were taken after each flood with a view to ascertaining its extent.

The work would probably have been saved had it been possible either to



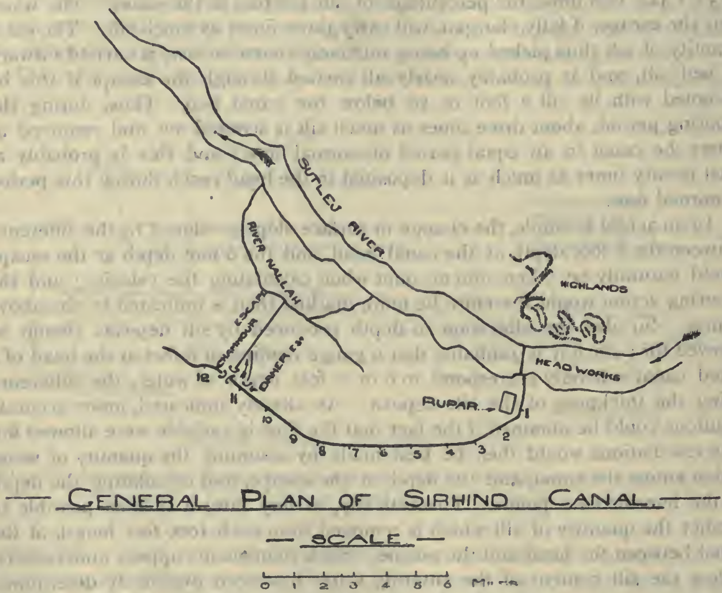
SKETCH No. 197.—Menufiah Regulator.

Note.—The upstream and downstream levels vary, but it is believed that the difference never exceeded 13·12 feet (4 m.), and was usually less than 10 feet.

prevent this erosion, or to construct a satisfactory and sand-tight cut-off wall to supplement the defective sheet piling. The clearest lesson to be drawn from the failure is that engineers should distrust wooden sheet piling. So far as I was able to observe, the piles were originally driven very closely together, but were not jointed in any way; aud it will be obvious that while a line of wells 10 feet in thickness may be said to be practically sand-tight, even when the spaces between the wells are 6 inches or 1 foot in width, the effect of a 1 inch gap in a line of sheet piling which is 6 inches thick at the most, is far more detrimental. It will also be plain that if my theories concerning tail erosion are accepted the drop from the regulator floor to the canal bed is a mistake.

Scouring action of Escapes.—The broad principles of the action of an escape are best illustrated by an example. Let us consider a channel which in steady flow carries 6 feet depth of water. Let us suppose that the head regulator is opened so as to produce a depth of 8 feet; but that the escape being opened the depth at the escape still remains 6 feet, part of the water being passed out by the escape, and part going on down the canal. The flow is obviously variable, and the depths and velocities at any point between the head and the escapes can be calculated by the rules given on page 485. For the general treatment of the subject it will suffice to note that the velocity at the head is at least $1{\cdot}15 \left(=\sqrt{\frac{8}{6}}\right)$ times that during normal flow, and just above the escape the velocity is $1{\cdot}54$ times the normal value. Kennedy's rules show that the quantity of silt carried forward per foot width of the channel is very approximately proportional to $v^{2.56}$. Now, $1{\cdot}15^{2.56} = 1{\cdot}44$ approximately, and $1{\cdot}54^{2.56} = 3{\cdot}00$ approximately. Thus, if the normal velocity is that which is just sufficient to carry forward the silt which exists in the water as it enters the canal, the water near the head is capable of carrying forward $1{\cdot}44$ times this quantity of silt or allowing for the extra quantity of water entering about $0{\cdot}75 \times 1{\cdot}44 = 1{\cdot}08$ times the percentage of silt existing in the water. The water near the escape, if fully charged, will carry three times as much silt. The extra quantity of silt thus picked up being relatively coarse in size, is carried forward as bed silt, and is probably nearly all carried through the escape if this be designed with its sill a foot or so below the canal bed. Thus, during the scouring period, about three times as much silt is scoured out and removed as enters the canal in an equal period of normal flow; and this is probably at least twenty times as much as is deposited in the head reach during this period of normal flow.

In an actual example, the change in surface slope produced by the difference between the 8 foot depth at the canal head, and the 6 foot depth at the escape would naturally be taken into account when calculating the velocity; and the scouring action would therefore be more marked than is indicated by the above figures. So also, the alteration in depth produced by silt deposits should be allowed for; since it is probable that a gauge reading of 8 feet at the head of a silted canal will only correspond to 6 or 7 feet depth of water, the difference being the thickness of the silt deposit. As already indicated, more accurate solutions could be obtained if the fact that the flow is variable were allowed for. The calculations would then be best made by assuming the quantity of water which enters the canal, and the depth at the escape, and calculating the depth at the intermediate points. Theoretically, at any rate, it is then possible to predict the quantity of silt which is removed from each 1000 feet length of the canal between the head and the escape. Such refinements appear unnecessary, unless the silt content of the entering water has been previously determined with greater accuracy than is at present customary. A skilled engineer when considering a canal which has been under observation for some years can probably select values which will agree fairly well with the results of observation, but such data are not likely to be available when it is desired to design an escape for a new canal. Thus, the more accurate solution appears to be unnecessarily complex. It may be observed that the value of C, in the equation $v = C\sqrt{rs}$, is somewhat influenced by silt deposits. As a general rule, C, has a higher value in a reach in which fine silt is uniformly and regularly deposited than is the case for a similar reach in which silting has not occurred. I have personally observed values of C, which are 20 per cent. greater than those which were observed on apparently similar channels in which silt had not deposited; and reliable observers have found an excess of 30 or even 35 per cent. The silt deposited in the head reach of a canal, however, is coarser than that in the reaches above referred to. Accurate experiments are difficult, because of the irregular manner in which the silt is dropped. It is generally believed that after heavy silting C, decreases to about 90 per cent. of its normal value. The capacity of an escape is best fixed by calculations which proceed upon the above lines. Thus, let us assume that the normal discharge of the canal is 2000 cusecs, and that about one-seventh of the silt taken into the canal is deposited in the head reach. The above calculation shows that an escape with a capacity of 1100 cusecs should suffice to keep the canal clear, if an extra 1100 cusecs of moderately clear water can be run to waste during 24 hours at intervals of about 20 days. The assumption is favourable ; and, as a general rule, escapes are not so powerful relatively to the canal discharge. The usual rule is that an escape can pass off a quantity of water which is not greatly in excess of one-third of the normal flow of the canal. This is probably a small capacity, as trial



SKETCH NO. 198.—Plan of Sirhind Canal.

calculations show that scouring is better effected by taking in large quantities of water for a short time, rather than an equal total quantity distributed over a longer period.

Also the capacity of the escape and of the escape channel needs to be considered, as the head reach of the canal is usually in deep digging.

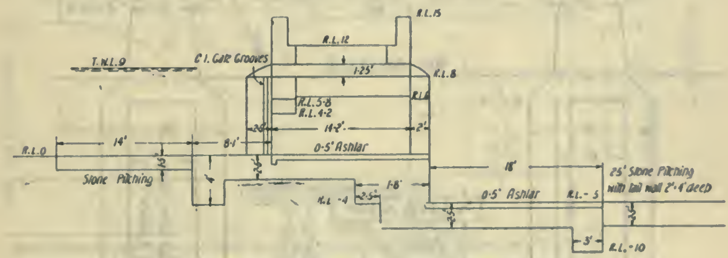
The design of an escape proceeds on the same lines as that of a regulator, with the following exceptions :

(*a*) The gates should open at the bottom. Raised sills are of course useless.

(*b*) The velocity through the escape should be as high as possible, 15 to 20 feet per second being probably the maximum.

For this reason, pitching of the bed and sides of the canal at and near to the escape is necessary. The size and length of this pitching is best determined by experience, although the velocity calculations made when the capacity is determined form a valuable guide.

In all calculations concerning the capacity of scouring escapes it must be carefully borne in mind that the permanent discharge capacity is usually fixed not by the size of the masonry work (*i.e.* the escape itself), but by the size and silt-carrying capacity of the escape channel. This is admirably illustrated by the Chamkour and Daher escapes on the Sirhind canal (see p. 704). The calculated discharge capacity of the original escape was about 2000 to 3000 cusecs. This was found to be insufficient (in reality due to an erroneous system of working), and was increased to a calculated capacity of about 5000 to 6000 cusecs. It will be evident that as the capacity depends on the level of the water in the canal, accurate calculations are impossible when a badly silted canal is considered; but it is known that the sluices could discharge all the water that the canal could carry when badly silted. As soon, however, as the scouring action of the enlarged escape became really efficient, the escape channel was obstructed by deposits, and about two years after the enlargement it was found that if more than 2000 cusecs was passed through the escapes valuable low-



SKETCH NO. 199.—Escape.

lying land was flooded. The escape channel has since been carefully attended to, and owing to the improved working of the canal is now able to perform its duties efficiently, and can probably carry 4000 cusecs if required.

Similar figures could be given for other escapes, and as a general principle it may be stated that nearly all scouring escapes which work efficiently are sooner or later found to be capable of flooding the banks of their tail channels if opened so as to pass their maximum discharge. Thus, the best location for an escape is that which has the shortest possible escape channel.

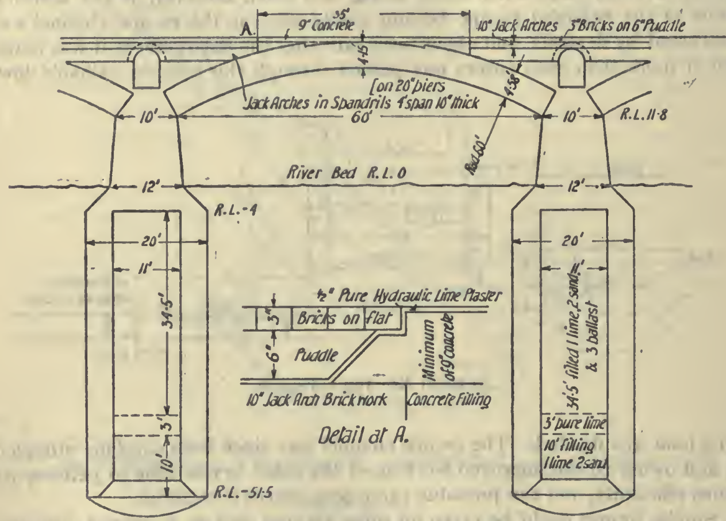
AUXILIARY ESCAPES.—In countries where rain falls during the irrigation season, skilled irrigators frequently refuse to take water when rain threatens; and the occurrence of a rainstorm produces a sudden and general cessation of demand for water.

In large canals water frequently takes 2 or 3 days to travel from the head to the tail of the canal. Thus, even with the best system of regulation, auxiliary escapes must be provided in order to carry off the excess of water which is thus permitted to enter the canal.

No rules can be given for such escapes, as they entirely depend upon the existence of depressions, or rivers, into which the water can be turned. Undue haste in constructing such escapes during the early years of the canal's existence may be deprecated. The behaviour of the agriculturists is unlikely to render the use of such escapes indispensable until their skill in irrigation has increased. Also, the existence of unnecessary escapes is conducive to careless methods of regulation. Thus, auxiliary escapes should be provided for under the head of "possible and additional works to be constructed, if and when found necessary."

Escape Reservoirs.—It is sometimes advisable to construct small reservoirs at favourable sites, into which surplus water may be diverted. The water is then not lost, as is the case when it is turned into an escape channel. In silt-bearing canals, these reservoirs rapidly silt up; and sometimes the value of the land which is thus reclaimed renders this method of disposing of surplus water financially profitable.

CANAL DRAINAGE WORKS.—I do not propose to enter into the rules which



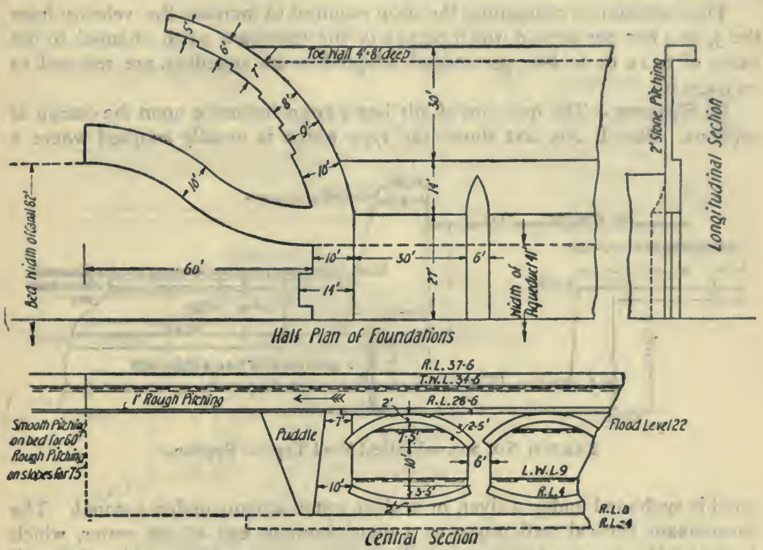
SKETCH No. 200.—Kali Nadi Aqueduct Foundations.

are adopted for proportioning the thickness of the masonry of the walls, piers, or arches of canal drainage works. The rules generally followed are those employed in other branches of engineering; but, as a matter of fact, an exact theoretical investigation usually leads to far slighter dimensions than those which are employed in practice.

Experience is the real basis of the proportions generally adopted; and in all probability the factor which determines the thicknesses adopted is not strength, but rather the prevention of leakage.

My object is to illustrate the correct outlines of the large scale plan of the works now considered, as this determines the general hydraulic efficiency.

(*a*) **Aqueducts.**—Aqueducts are required either for carrying a canal over a river, or for carrying a river over a canal. As a rule, there is rarely any alternative, as the relative levels of the two bodies of water determine the design. Where choice exists, it is usually best to carry the smaller body of water under the larger one, the maximum flood discharge of the river, of course fixing its size. When the canal is carried by an aqueduct, a certain economy can generally be effected by increasing the velocity of the water. The matter needs careful calculation. Where silt exists, it is not advisable to make the depth of water in the aqueduct much greater than in the canal, unless the aqueduct is arranged as a silt trap, and is provided with orifices for discharging silt. In any case, the whole length of the earth channels leading up to and away from the constricted section should be carefully pitched with brickwork, or other resistant facing. The alteration in cross-section should proceed gradually. In silt-bearing canals length of tapered section $= 25 \times$ difference in bed width, and in clear water $= 5 \times$ difference usually produce good



SKETCH No. 201.—Inverted Aqueduct Foundation.

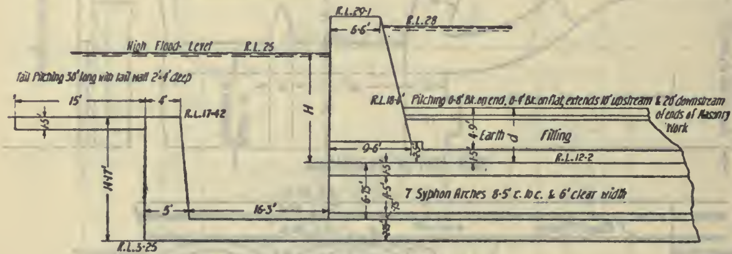
results (see Sketch No. 206). It will usually be found advantageous to pitch the upstream portion of the receding channel with roughened pitching of the form shown in Sketch No. 214; but the tail portion of the pitching should be smooth, so that the water reaches the unlined earth section devoid of all turbulent, scour producing eddies.

The foundations of an aqueduct may either be deep, and formed of wells, or of piling under each separate pier, as is indicated in Sketch No. 200 of the Kali Nadi (Ganges Canal) pier foundation; or the whole stream bed may be pitched, and the piers supported on relatively shallow foundations (Sketch No. 201, compare also No. 206). The deep type is usually adopted in fine sand, and the shallow type in coarse sand or clay. There seems to be no real reason for this selection, and considerations of cost may be relied upon to settle the question.

As a rule, the mean velocity of canal water in an aqueduct does not exceed 14 feet per second, and 8 or 10 feet per second is more usual. The real difficulties are caused not by damage to, or waves in the aqueduct, but by reducing the velocity of the water after it leaves the aqueduct to the 3 or 4 feet per second, which is permissible in earthen channels. The value of the velocity finally adopted will consequently to a certain degree depend upon the length of the masonry channel. The longer this is, the greater will be the saving produced by an increase in the mean velocity, and therefore, the greater the expenditure on roughened pitching and water cushions downstream of the aqueduct that is economically justified. There appears to be no valid reason why 20 feet per second should not be adopted in a long aqueduct if the required fall is available.

The calculations concerning the drop required to increase the velocity from the 3, or 4 feet per second which occurs in the upstream earth channel, to the value of 8, 14, or 20 feet per second adopted in the aqueduct, are referred to on page 16.

(*b*) **Syphons.**—The question of silt has a large influence upon the design of syphons. Sketch No. 202 shows the type which is usually adopted where a



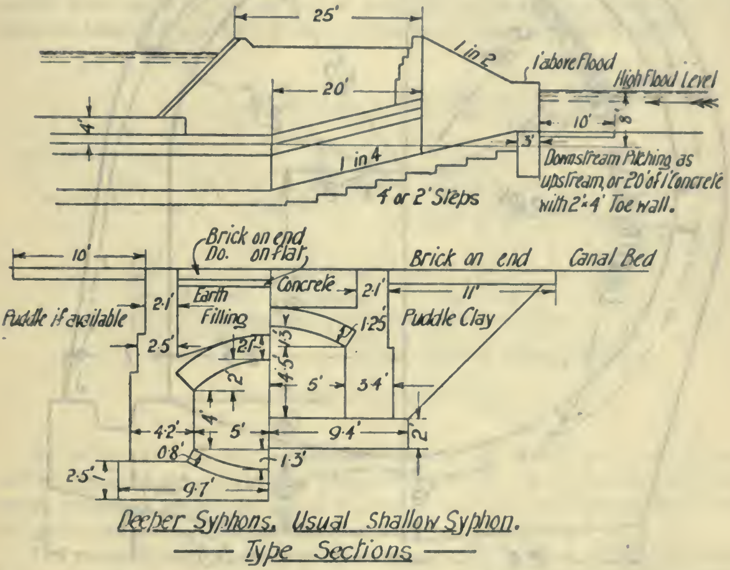
SKETCH NO. 202.—Vertical Well Type of Syphon.

canal is syphoned under a river, or a clear water stream under a canal. The downstream vertical well produces a quasi-fountain exit of the water, which dissipates the energy of the increased velocity in the syphon, and but little tail pitching is required. The deposition of silt is unlikely, as the canal flows steadily for say 300 days in the year.

The mean velocity in the syphon should be fairly large, in order to prevent any silt deposits. The usual rule is about 8 feet per second for maximum supply, or 10 feet per second in extraordinary supplies. No difficulties from tail erosion or scour have occurred in cases where the velocity was 16 feet per second, even though the earthen channels eroded badly at 6 feet per second. It is therefore believed that when the vertical well is adopted, and the fall is available, a velocity in the syphon of 12 feet per second for maximum supplies may be adopted in tender soil. This velocity may be increased to 16 or 20 feet per second in firm soil. Provision should be made for downstream pitching "if required."

Where the canal is a flood canal, carrying large quantities of silt, and only flows intermittently, the type shown in Sketch No. 203 should be employed, and should always be used for syphons which carry silt-bearing rivers. The flat upstream and downstream slopes are no doubt more expensive than the vertical drop walls, but the form thus obtained prevents any risk of silt depositing. Such syphons are employed in India for carrying off the water discharged by rivers which run but once or twice every five years, and are then extremely heavily charged with silt. As silt deposits never give trouble, the type may be adopted with confidence whenever choking of the syphon by silt is to be apprehended. Downstream erosion, however, is likely to occur, as the energy of the issuing water is but slightly dissipated. The drop wall shown in Sketch No. 203 is usually sufficient in the case of flood channels; but in a canal or river which flows constantly, pitching with brickwork, or concrete blocks, may be necessary.

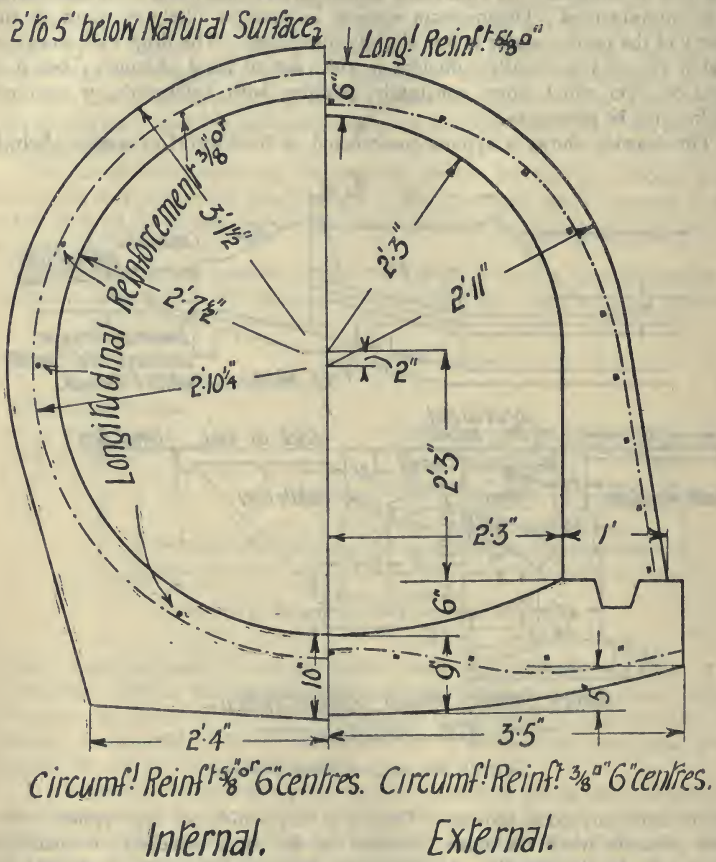
The drawing shows a syphon constructed of brickwork or masonry, which



SKETCH No. 203.—Chenab Syphon.

cannot resist any great tension. Thus, if it is possible for the syphon to be under pressure when the upper channel is dry, it is necessary to place a sufficient load of brickwork and earth on top of the syphon arches to equilibrate the upward water pressure Thus, the crown of the arches should be at least one half of the head of water in the syphon, below the canal or river bed, *i.e.*, $d=\frac{H}{2}$ (Sketch No. 202). The canal or river bed should also be pitched so as to prevent any possibility of the earth load being removed by scour. In modern construction syphons are frequently made of reinforced concrete (Sketch No. 204), steel pipes (Sketch No. 205), or other material capable of resisting tension. The saving in excavation and masonry then produced by the fact that d, need only be as large as considerations of strength render necessary, is material; and it is doubtful whether the old masonry design should now be adopted. The general form, however, should not be altered.

Sketch No. 205 shows a very neat design employed by the Kashmir State in crossing the Tawi river. A steel pipe 5 feet 6 inches in diameter carries

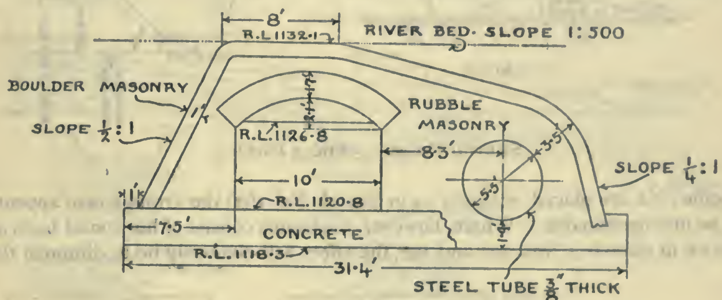


SKETCH No. 204.—Reinforced Concrete Syphon.

110 cusecs for the irrigation of the higher lands on the farther bank of the river. The larger discharge of 480 cusecs which irrigates the lower lands is carried across in the 10 feet by 7 feet 7 inches masonry arched opening, the bed level of the canal being dropped by a fall just above the river crossing, in order to prevent internal pressure being brought to bear on the arch. Were steel plating relatively cheaper, there is little doubt but that the whole work would be constructed of steel piping. The ample dimensions of the masonry

are necessary owing to the character of the river, which is extremely torrential, with floods amounting to 150,000 cusecs.

Level Crossings.—The levels of a river and canal are sometimes almost identical. In such cases (especially if the river and canal are approximately equal in size), a level crossing is adopted. This consists of two regulators, one across the river downstream of the crossing, and one across the canal downstream of the river. The Bengal type of regulator (Sketch No. 190) is usually employed. The river is usually dry, and ample warning of the approach of floods is possible. In cases where the river flows for long periods, and where it is necessary to take its water into the canal, the river regulator may be provided with falling shutters, and can be worked in conjunction with the canal regulator, just as the weir and regulator at the headworks of a canal are handled. The extra cost entailed in the construction of the regulator so as to enable it to retain 13 or 14 feet of water, in place of 6 or 7 feet, must be carefully balanced against the value of the water thus rendered available.



Sketch No. 205.—Tawi Syphon.

Bridges over Canals.—The construction of bridge work, or masonry arches, does not come within the scope of this book. In canals (and especially where the soil is tender and silt is carried in the water) the proportions of the bridges require consideration.

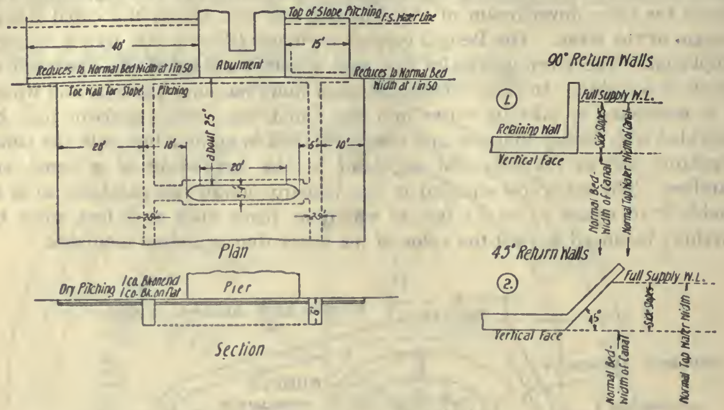
The rule :—Clear waterway is equal to the bed width of the canal plus the full supply depth, has been found advantageous in such cases ; and any marked obstruction of the waterway tends to induce scour, and a consequent deposition of silt lower down the canal. The circumstances thus produced are liable to become permanent in character. Thus, pitching of the character indicated in Sketch No. 206 is employed. The sketch refers to a case where the velocity is about 5 feet per second. At smaller velocities, a shallower toe wall and a smaller breadth of pitching (say 5 feet in width) are advisable in all cases where silt and scour are determining factors.

Regulators for Branch Canals.—The design of these works follows the same rules as for head regulators. The head of water sustained does not usually exceed the full supply depth in the main canal.

As a rule, it is desirable to disturb the proportions of silt and water existing in the main canal as little as possible. Hence the regulator should have ample waterway, and the rules given under Bridges may be adopted with advantage.

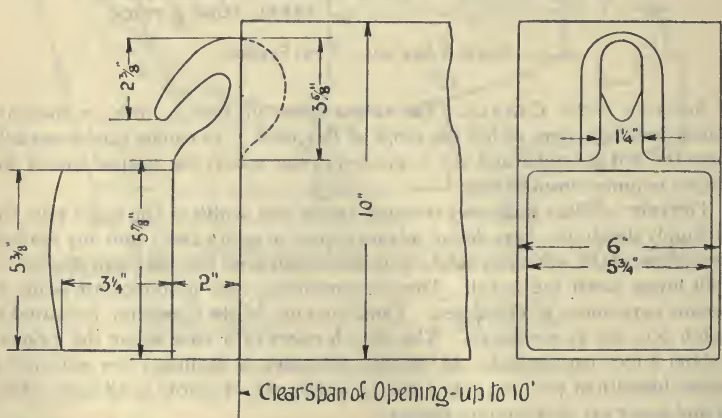
Raised sills and arrangements for drawing off surface water are not usually required.

In many cases the openings are closed not by sluice gates, but by means of wooden beams. The economy is obvious, and when the beams consist of



SKETCH NO. 206.—Bridge Pitching.

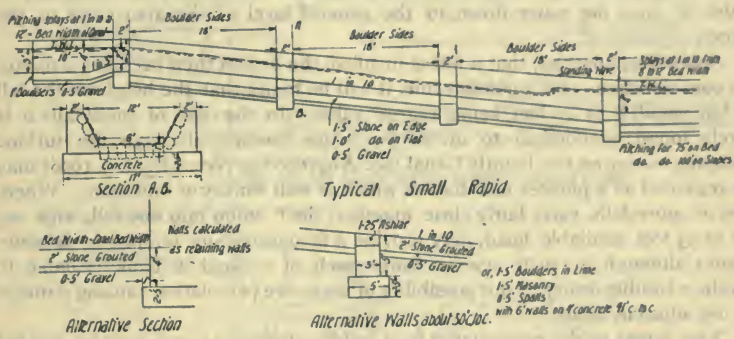
needles (*i.e.* are placed vertically as in Sketch No. 162) the arrangement appears to be unobjectionable. Where, however, the beams consist of horizontal bars, as shown in Sketches Nos. 207 and 196, the effect will obviously be to diminish the



SKETCH NO. 207.—Stop Planks.

proportion of bed silt drawn into the branch canal whenever any beams are in place, and the branch is not taking its full supply. This is hardly fair (qua silting troubles) to the portion of the main canal below the bifurcation where the regulator is situated. Horizontal stop planks are therefore inadvisable, unless the slope of the branch canal is far flatter than that of the main canal. If this is the case, it may be possible to partially sort out the silt, and to turn the major portion into whichever branch is found to be most capable of carrying it forward to the fields. In some cases, regulators are built across a canal for the purpose of raising the water level locally from time to time, in order to be able to supply water to a patch of high land. The advantages thus gained are very dearly purchased if the canal carries any silt, as the reach above the regulator is alternately a silt trap and a scouring reach. Such works may be regarded as radically bad in principle, and should never be constructed where any other solution of the difficulty can be found.

Falls and Rapids.—In the design of an earthen channel it will often be found impossible to allow the slope of the channel to be equal to the general slope of the ground surface. In the first place, as was discovered during the construction of the Ganges Canal, if a large earthen channel be graded at a uniform slope equal to the difference of the desired water levels at its head and tail, divided by its total length, the velocity is frequently so great that severe



Sketch No. 208.—Rapid.

scouring action, or even total destruction of the banks of the channel, is produced. Secondly, isolated patches of relatively higher land nearly always exist, which would not be commanded by a channel graded in this manner. In the final design of the channel a certain amount of adjustment can generally be secured by grading the channel so that the velocity is approximately constant. Thus, in place of a channel at a uniform slope, one which is graded in a series of short reaches will be obtained, the slope of each reach being approximately inversely proportionate to the hydraulic mean radius of the channel, and therefore increasing as the size and discharge of the channel decreases.

It will, however, frequently be found that even this adjustment, or the somewhat more complicated method introduced by Kennedy (see p. 753), does not produce a channel which commands the country satisfactorily, and is not likely to give trouble by scouring its bed.

The obvious method of improving the conditions is to grade the reaches at a smaller slope, and to obtain the total drop required by means of falls, or rapids, inserted at favourable points.

Since a fall or rapid, when judiciously inserted, will often permit long reaches of channel in embankment to be replaced by a channel at or near to the balancing depth, the cost of the required masonry is frequently recouped by a saving in earthwork.

Nevertheless, falls or rapids should be avoided if possible, especially in large canals.

A consideration of the conditions existing in earthen channels has led me to believe that a proposal for a canal which entails many falls or rapids in its earlier reaches should be carefully examined, and is probably fundamentally incorrect. The most probable error is that the site of the headworks of the canal has been selected too high up the river from which it takes out.

It may, however, be the case that headworks lower down the river are not advisable, either because no favourable site for a headworks exists, or (what is more likely to prove the case) because any line starting from the lower site crosses too many minor drainage channels.

When a canal, or better still, its minor branches, are approaching their ends, the slope of the ground often changes suddenly, especially when nearing a river or a drainage. In such cases, a series of falls may be necessary in order to drop the water down to the general level of the area close to the river.

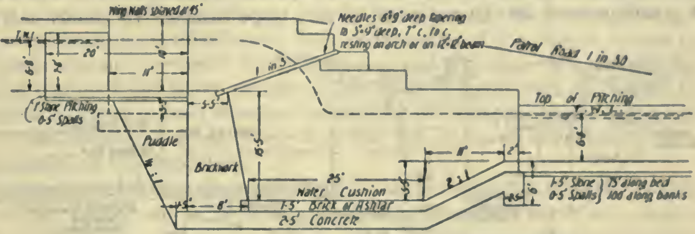
Assuming, however, that a fall is justified, the design then becomes a matter for consideration. As a general rule, it will be found that the height of the fall is but small, 8 or 10 feet being a large value. In the case of small falls it is rarely found economical to utilise the water power; although the turbine pumping station on the Huntly Canal (see *Engineering News*, Sept. 3, 1908) may be regarded as a pioneer installation, which is well worthy of imitation. Where two or more falls exist fairly close together, their union into one fall, with say 20 to 25 feet available head, may provide a favourable site for power development; although in such cases, a long reach of a canal in bank is liable to produce trouble owing to the possibility of excessive percolation causing damage to the adjacent lands.

The actual works necessitated by a sudden drop in a canal may be divided into two classes—*i.e.* falls and rapids. These are very different in their properties.

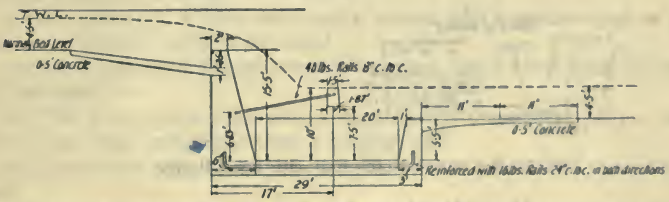
Speaking generally, a fall should be adopted if any irrigation is effected from the reach above its site, as it will hereafter be shown that a fall can be designed so as to permit accurate regulation of the water level in the canal. This the usual type of rapid does not permit, and rapids are therefore hardly advisable except where regulation is not a matter of importance. It is fortunate that this discrepancy in the functions of falls and rapids is paralleled by the difference existing in the materials requisite for their respective construction.

As will be seen from Sketch No. 208, a rapid contains an unusual proportion of large stone; while, in the case of a fall (No. 209 or 210), stone forms only a small portion of the material required. Now, as a rule, irrigation hardly begins until the canal has left the upper portion of the river valley, where stone is most easily obtained. Consequently, it can be stated in general terms that rapids are probably advisable on the upper reaches of the canal, from which no irrigation is effected. Falls, on the other hand, are required when the main canal has begun to split up into branches, and irrigation has commenced.

It will be found in practice that even soft bricks do not suffer serious erosion by water carrying sand, unless the velocity of the water exceeds 20 feet per second, or the "silt" contains a large proportion of stones which are greater than a pea in size. Thus, when the height of the drop in the water surface is restricted to values as small as 8 or 10 feet, actual damage to the masonry portion of the work need not be greatly feared. The principal difficulty is to prevent the irregular motion of the water as it leaves the rapid or fall from causing erosion of the earth banks of the canal below the fall. This side of the question was not at first understood, and the old ogee fall (see Sketch No. 158) was designed on the assumption that damage to the masonry works was likely to occur. Intense erosion of the banks took place; which, in some cases, became so acute that the masonry was undermined, and the fall was damaged. The design must be regarded as very badly adapted to small drops in the water surface. It is still used for drops of 30 or 40 feet or more, such as occur in dams of the overflow type. Here it is successful, because the flow



Typical Punjab Notch fall with Needles in place of Notches



Typical Uncompaghre 10' Needle Fall

Sketch No. 209.—Needle Falls.

is not continuous, and the banks of the downstream channel are usually of rock; or, at the worst, of hard gravelly earth.

For soils such as those in which the majority of irrigation channels are constructed, the fall should be designed so as to destroy the energy of the falling water as far as possible before it quits the masonry. This is best effected by utilising the internal friction of the water. The principle is as follows:

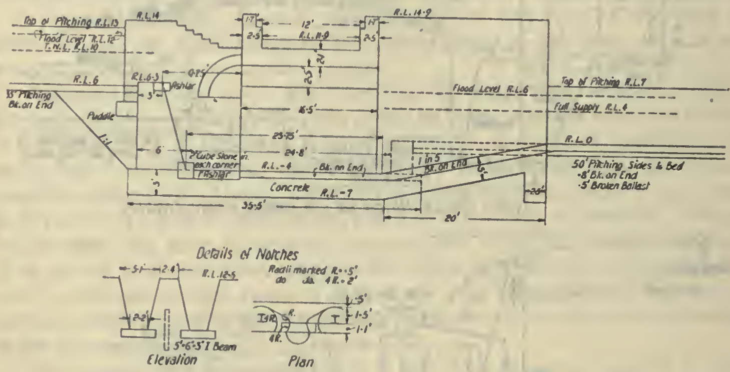
(*a*) The falling sheet of water is divided into a number of smaller stream which interfere with each other.

(*b*) A pool, or a water cushion, is formed below the fall, in which the falling water eddies and dissipates its energy.

Where no floating matter occurs in the water a very excellent solution is a fall obstructed by needles, as shown in Sketch No. 209. Here, the falling waters are split up into thin streams, and their mutual interference thoroughly destroys the energy generated by the fall. A similar expedient may be applied to a rapid with equal success. Where, however, any floating matter occurs, obstructions collect on the needles; and these, if not removed, may block the fall, and so cause a breach of the canal. The size of the apertures between the needles is such that weeds are quite as effective as branches of trees, or large masses of drift.

As a rule, therefore, such obstructed falls are not advisable; and Sketch No. 210 shows a fall, and Sketch No. 208 a rapid, so constructed that any normal drift will readily pass over the crest.

The details deserve careful consideration. Firstly, taking the fall, let us assume that the earth below the fall is very easily eroded (*e.g.* as in the conditions existing in the Punjab, or on the Nile, although falls rarely occur in Egypt, and even then are only small). In the first place, it is found that the falling water should be divided into separate streams, each of which does not greatly exceed 200 cusecs even in very large canals. In canals carrying



SKETCH NO. 210.—Notch Fall with Road Bridge.

less than 400 cusecs, there should be at least two, preferably four, notches; and one notch is rarely advisable unless the discharge of the canal is less than 50, or 60 cusecs. A very good rule is that the top width of the notch (see p. 724) should not exceed three-quarters of the depth of the channel.

In the second place, the breadth of the water cushion should be only slightly less than the bed width of the downstream channel. A large volume of water is thus secured in which the falling water can expend its energy. The depth below the lower bed level δ, and length l of this water cushion are usually fixed by the formulæ:

$$\delta = \frac{h+d}{3}, \qquad l = 3\delta,$$

where h is the height of the fall and d the fall supply depth of the canal.

Dyas found experimentally that a bottle was not smashed when passing over the fall when

$$\delta \text{ was greater than } \sqrt[3]{d}\,\sqrt{h};$$

and, in consequence, he adopted this value with:

$$l = 2\sqrt{hd}.$$

I do not know the reason why these formulæ were abandoned, as the value of δ agrees very fairly well with the depth of the holes which are frequently found below natural waterfalls, but the cisterns of the newer falls proportioned by the first rule are floored with ashlar in places where the action of the falling water is most intense. An inspection of the dry cisterns of falls which are so proportioned causes me to believe that, for materials such as are found in the Punjab (which are not good), the rule is close to the minimum safe depth (it is usually less than the depth as given by Dyas' rule), and should not be decreased. With better materials, and better soil (as will be seen later), the cushion may be diminished in depth, or may even be entirely dispensed with. The slope up at 1 in 2 of the downstream wall of the cushion is important. A vertical wall not only sets up waves in the canal and thus favours erosion, but prevents the escape of any small stones that may be swept into the cistern. If allowed to remain in the cistern, the stones will rapidly produce pot holes, and even a piece of broken brick may cause damage under such circumstances. For similar reasons, a raised crest wall downstream of the cistern cannot be permitted, although it would save excavation and masonry by increasing the effective depth of the water cushion. The irregularity in velocity created by the raised wall makes itself manifest in the form of intense erosion at a distance downstream of the wall equal to one half the bed width of the channel. The other details are less important.

In more resistant soils these proportions may be somewhat reduced. I had, however, an opportunity of repairing five falls in the usual Punjab soil, each of which carried about 500 cusecs, and in which the following divergences from the above design occurred :

(i) The water fell in one mass into a cistern with a width equal to three-quarters of the bed width of the canal.

(ii) The water fell in two masses into a cistern with a width equal to one-half the bed width.

(iii) The proportions of the cistern agreed with the typical design, but the downstream slope was replaced by a vertical wall.

(iv) As in case No. (iii), but here the wall was raised 1 foot above the bed level.

(v) A fall resembling Sketch No. 210 in all respects, except that the sill of the fall had a raised crest (see p. 722) and that the cistern width was three-fourths the bed width.

In the first two cases the downstream erosion had undermined, and produced cracks in, the masonry of the fall within less than three years.

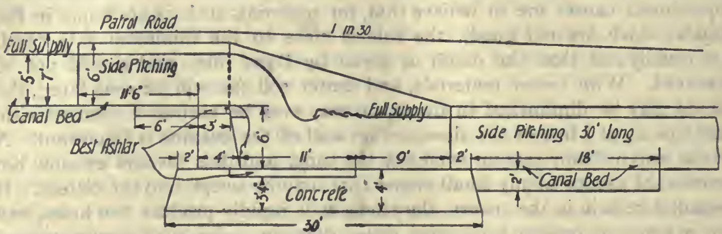
In case No. (iii), the bed pitching immediately downstream of the cistern was destroyed, and blocks of stone 2 cubic feet in size (which had been thrown in with a view to stopping erosion) showed signs of having been violently moved.

In case (iv), this action was even more violent, two stone blocks being broken. Pot holes 1 foot and 18 inches deep occurred in each cistern.

In case (v) the downstream damage was small ; in fact, not more than that which occurred in a similar properly designed fall. On the other hand, upstream erosion was very marked ; and the silt thus produced was deposited some 5000 feet below the fall, and gave great trouble by blocking the head reach of a branch canal.

I consequently believe that any divergence from these proportions should be avoided. When the proportions are adhered to, it will be found that a pitching of brick on edge, and brick on the flat suffices to prevent any noticeable erosion.

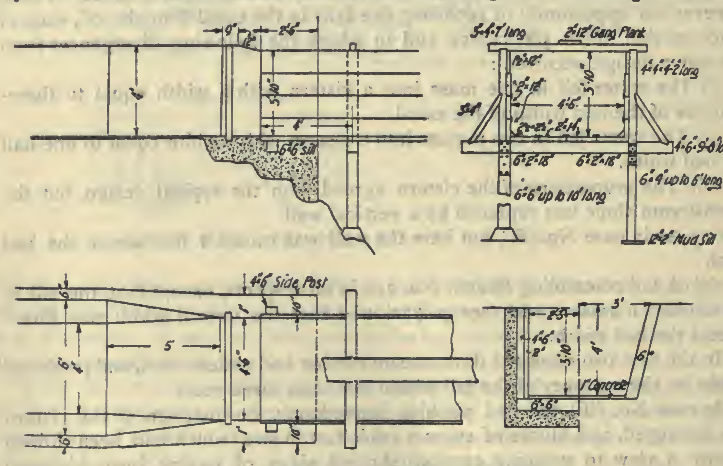
Where the soil is somewhat firmer, Bligh's rules (see *Design of Irrigation Works*), may be followed. Bligh makes the width of the fall seven-eighths that



SKETCH NO. 211.—Madras Type of Fall.

of the bed width of the downstream channel, and gives no water cushion, the thickness of the masonry bottom of the fall being represented by $\sqrt{h+d}$, and its length by $2(h+d)$.

Bligh criticises the Punjab design somewhat severely. The theoretical objections which he advances have but little weight. His designs are no



SKETCH NO. 212.—Timber Aqueduct.

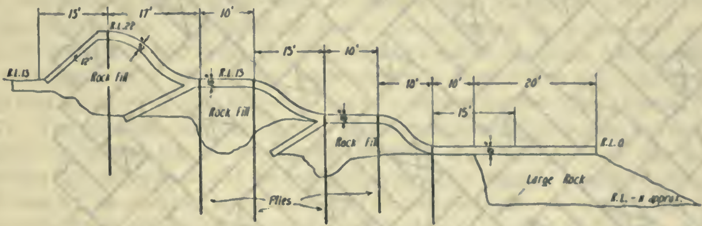
doubt cheaper than those which are adopted in the Punjab, but they are suitable for firmer soil and better material (see Sketch No. 211).

In really firm soil, falls are but rarely required, since the bed slope can be made sufficiently steep to dispense with falls without fear of erosion arising from the velocity thus produced.

In no case have I found good results obtained when the width of the fall is considerably less than Bligh's rule of seven-eighths of the bed width. Any saving that could be secured by adopting a smaller width would be but small; and the risk of downstream erosion is very great, even in the firmest soils.

The standard design (see Sketch No. 208) of a rapid calls for but little remark. The slope 1 : 10 has been arrived at by experience, and (in the soils of the Punjab at any rate) a steeper slope is accompanied by excessive erosion downstream. The slope is pitched with boulders, or with blocks of lime concrete. The drawings usually show the boulder pitching left rough, and this roughness appears to be considered advantageous. Actual inspection of a well maintained rapid in which erosion has been satisfactorily prevented will invariably show that all marked irregularities have been removed, and that great care has been taken to smooth the slope by filling all joints and hollow places with mortar.

The actual facts appear to be that the water flowing down the rapid is in a state of pulsation, and any stones which project into the stream are sooner or later set shaking. This shaking displaces the sand under the stone, and the stone sinks slightly; and when repairs are made the stone is smoothed up and brought into line and level by a coating of mortar. If the action is more intense, the stone may finally be displaced. Thus, after a few years, the slope



SKETCH NO. 213.—Debris Dam on Yuba River.

pitching becomes so smooth that it is frequently difficult to walk on it even when dry. This does not seem to render the rapid less efficient, as the standing wave that forms at its lower end is quite sufficient to dissipate the energy of the falling water. Some of the most satisfactory rapids are found to possess a pitching which is more than usually smooth. Thus, any reliance on the roughness of the pitching appears unnecessary. Such designs as rapids with broken slopes (see Sketch No. 213), or water-cushions, have been frequently and exhaustively tested in practice, and all such devices have now been abandoned.

It will be noticed that the typical rapid, like the typical fall, has a width equal to the bed width of the downstream channel. A wide rapid is less liable to produce downstream erosion than a narrow one, but the condition:

Width = bed width of channel,

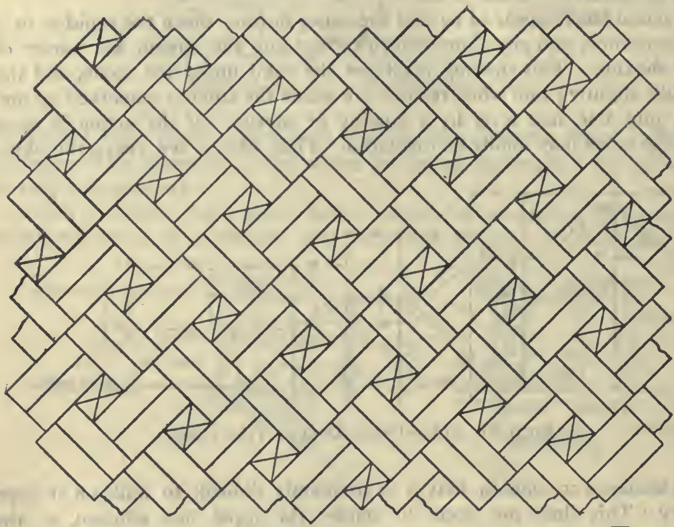
is by no means so important as in a fall. Erosion below rapids is less regular, and is apparently less dependent upon the design than is the case with falls.

The rules:

In tender soil, width of rapid = seven-eighths bed width, and
In firmer soil, width of rapid = three-fourths bed width,

may be adopted, and produce rapids which are slightly narrower than falls in similar soil. When these rules are adopted an additional amount of downstream pitching must be provided, but it is believed that economy in the total cost of the work will be secured provided that the rapids are carefully maintained.

Rapids are frequently required when it is necessary to increase the velocity of the water in a channel suddenly. Examples occur when an earthen channel is followed by a masonry aqueduct, or wooden flume. The general practice in the Punjab is to design the aqueduct with a full supply depth equal to that in the approach channel, and to provide the increase in velocity necessitated by the reduction in breadth that occurs in the aqueduct by means of a rapid.



Sketch No. 214.—Roughened Pitching.

The uncertainties affecting the calculations are referred to on page 16, but the design of the rapid follows the ordinary rules.

Prevention of Erosion below Falls and Rapids.—If a fall or rapid is left to itself, a pool forms just below the work, and the final plan of the channel somewhat resembles the section of a soda water bottle, the fall or rapid forming the neck. This form was considered to be correct in former days, and the banks of the pool were frequently pitched, or were even provided with retaining walls. The method is still adopted where the water (*e.g.* as in a canal escape, or in the outlet escape of one of the basins that are used in flood irrigation) is discharged into a river or natural water-course. Where, however, the water after quitting the fall flows in a canal which has to be maintained and regulated, this method is inadvisable, as it is found that a succession of pools and constricted necks are formed all the way down the canal. In some cases, a series of seven or eight such pools extending for a mile or so below the fall has been formed.

Consequently, the modern practice in canals (and lately even in escapes) is to pitch the banks of the canal so as to preserve a section which is just sufficient for the normal flow. The pitching is rough, and Sketch No. 214 shows a very ingenious chequer work of projecting bricks, which has been found extremely effective.

Bed scour is prevented by cross walls where required. The above sketches show the length of pitching. These should be considered as minima lengths. No fixed rules can be given; as, in practice, pitching and cross walls are added when considered advisable by the engineer in charge. On an average, it may be assumed that about twice the length of pitching shown will eventually be required; some works needing little or no additional pitching, whilst cases exist where 300 or 400 feet of brick pitching is ultimately found to be necessary. A great deal depends upon the care which is devoted to the maintenance of the channel and pitching, and measures should be taken to encourage the deposit of silt below the downstream end of the bank pitching by means of fences of brushwood, and stakes.

If a pool of the soda water bottle type has formed below a fall, the fault is best remedied by running out banks of rubble stone, so as to define a channel of the required size. These banks should not be carried up to the water level, as it is desired to encourage the deposition of silt in the pools behind them. Thus, banks with a crest 1 foot below the water level will at first suffice, and they can be raised as the deposit of silt accumulates behind them (Sketch No. 161). The principles are now plain. In place of permitting erosion to take its natural course, the size of the channel required to carry the supply that flows over the fall or rapid, should be calculated, and this channel should be defined and shaped by training walls consisting of loose rubble. If erosion has not occurred, the banks may be pitched; and profiles of the bed and banks of the correct channel section should be constructed of masonry or stakes and brushwood, so as to train the channel to its correct form. When these precautions are adopted, silt will rapidly deposit; and where the silt is of a clayey nature the roots of plants will bind the silt into a firm mass.

THE NOTCHED FALL.—When a fall, or rapid, is designed according to the preceding rules, it will be plain that there will be a draw down of the water surface just above the fall. In consequence, the mean velocity just above the fall will be greater than the mean velocity in the portions of the canal that are more distant from the fall. This increase in velocity may cause local erosion, and should therefore be avoided if possible. The drop down and the consequent increase in velocity can obviously be prevented either by raising the crest of the fall above the bed level of the canal, or by making the width of the fall less than the bed width of the canal.

The first solution is that usually adopted in the case of a rapid. The second solution has been found inadvisable both in falls and rapids, as the banks of the canal below falls or rapids which are considerably constricted are found to be badly eroded, and the ratio, width of fall is equal to seven-eighths of the bed width of channel, is found to be approximately the least that can be adopted. Either solution possesses the disadvantage that the depth of the water just above the fall is equal to the depth in the canal in uniform motion, for one discharge only.

The equations are as follows:

Let Q, be the number of cusecs passing over the fall, or rapid.

Let d, be the corresponding depth in the upstream canal in uniform motion, and w, the mean width of the channel for depth d.

Then, if l, be the length of the crest of the fall, or rapid, the head over the fall is given by the equation:

$$Q = C_1 l H^{1.5}$$

where, $C_1 = 3.40$ to 3.50 in the case of a rapid; and in a fall may be as small as 2.50 if the sill is flat, and H, is equal to 1 or 2 feet.

Thus, the crest must be raised to a height $x = d - H$, above the canal bed; and Q, is very approximately equal to $Cw\sqrt{s}d^{1.5}$, where $v = C\sqrt{rs}$, is the friction equation for the canal, so that:

$$x = d\left\{1 - \left(\frac{Cw\sqrt{s}}{C_1 l}\right)^{\frac{2}{3}}\right\}$$

Consequently, the crest height will be correct for one value of d, only. The variations in C, and w, as d, alters, slightly modify the above equation. If an actual case is calculated, it will be found that scour can be entirely prevented by a raised crest, but that the difference between $x + H$, and d, as Q, varies, is quite sufficient to cause trouble in the working of a distributary which takes out just above the fall. For, if we calculate x, so that $x + H = d$, when Q, is the full supply discharge of the canal, $x + H$ will be considerably greater than d, when Q, is say two-thirds of the full supply discharge, and silt deposits are likely. If we calculate x, so that $x + H = d$, when Q, is one-half the maximum discharge of the canal, then, when the full supply is passed down the canal, scour may occur; and it will be found difficult to give the distributary its full supply.

As an example, consider a channel of a bed width of 20 feet, with side slopes of $\frac{1}{2} : 1$, and a bed slope equal to $\frac{1}{4444}$. Let us assume that the channel is of earth, and that it is in fair order (*i.e.* Bazin's $\gamma = 1.54$, or Kütter's $n = 0.0225$ approx.); and let us calculate d, the depth in feet of the water in the channel when the motion is uniform, and the discharge is equal to Q, cusecs. We also calculate H, the head over the sill of a weir 20 feet long, discharging Q, cusecs, from the equation $Q = 3.50 \times 20H^{1.5}$, and H_1, a similar quantity for a weir 17 feet wide.

We can then calculate x, and x_1, the various heights of the sill required to produce a depth just above the fall or rapid, equal to the depth in the canal during uniform motion, from the equation:

$$x = d - H.$$

We thus obtain:

Discharge in Cusecs.	Depth in uniform Motion.	20-Foot Sill.		17-Foot Sill.	
		H	x	H_1	x_1
	Feet.	Feet.	Feet.	Feet.	Feet.
200	4·2	2·01	2·19	2·24	1·96
140	3·4	1·59	1·81	1·77	1·63
70	2·2	1·00	1·20	1·11	1·09

Consider the case where Q = 200 cusecs, and l = 20 feet. Assume that a sill 2·19 feet high is erected. Put D, for the depth that now occurs just above the fall, *i.e.* for any discharge we have :

$$D = 2{\cdot}19 + H.$$

We then get :

Q, in Cusecs.	*d*, Depth in Canal in uniform Motion, in Feet.	D, Depth just above the Fall, in Feet.
200	4·2	4·20
140	3·4	3·78
70	2·2	3·19

It is plain that the water level is headed up whenever the discharge is less than 200 cusecs. Silt troubles are therefore to be feared.

Similarly, if we consider that Q = 70 cusecs, and that l = 17 feet, we get the following table, where $D = H_1 + 1{\cdot}09$.

Q, in Cusecs.	*d*, Depth in uniform Motion, in Feet.	D, Depth just above the Fall, in Feet.
200	4·2	3·33
140	3·4	2·86
70	2·2	2·20

So that a drop down exists whenever Q, is greater than 70 cusecs, and scour is likely to occur.

The case selected is purposely somewhat unfavourable, although it is quite a practical example. In actual practice, the height of the sill can usually be selected so that these local silt and scour troubles do not produce very alarming results, and a rapid with a sill about 1·7 feet high would suffice to secure a satisfactory solution in this particular case. If silt troubles are intense, it is obviously desirable to give the canal no occasion for offending.

If, however, a distributary takes out just above the fall, the matter at once becomes serious. Whatever height is given to the raised sill, the distributary is always fed with water which is abnormally charged with silt. For if scour occurs, the distributary receives water charged with freshly eroded matter ; and if silt deposits occur, the distributary draws from a silt trap which is full of old silt. The results are astonishing. Channels 20 feet in width, and originally 3 feet deep, have been known to silt up to a depth of 10 inches for a length of 3000 feet in three weeks, and such cases are by no means rare. As just above a fall is otherwise a very favourable location for a distributary head, such abnormal silting should be avoided wherever possible.

These difficulties are obviated by the notch fall (see Sketch No. 210), which should always be adopted in falls on canals which carry silted water where a distributary takes out just above the fall.

The calculation of these notches is described by Reid (*Punjab Irrigation Branch Papers*, No. 2).

Put Q, as the quantity of water passing over the fall. Let l, be the length of the sill of the notch in feet, and $l+nx$, be the width of the notch at a height x, above the sill. Let y_m, be the full supply depth in the channel downstream of the fall, and y_o, be the corresponding depth for the least supply. Also let d_m, be the full supply depth in the channel above the fall, *i.e.* the depth corresponding to the maximum supply to be passed over the notch, plus the full supply of the distributary.

Then, as a rule, it is found that good regulation is secured by taking :

Q_1, the supply corresponding to the depth,

$$y_o+\tfrac{1}{4}(y_m-y_o)=y_1$$

and Q_2, corresponding to the depth,

$$y_o+\tfrac{3}{4}(y_m-y_o)=y_2$$

and proportioning the notch so that it passes the supply Q_1, when the depth in the canal above the notch is $d_1=\frac{d_m y_1}{y_m}$, and the supply, Q_2, when the depth is $d_2=\frac{d_m y_2}{y_m}$.

Two cases occur :

(i) The notch has a complete fall, and is never drowned.

We have : $$Q_1=5{\cdot}35cd_1^{1{\cdot}5}(l+0{\cdot}4nd_1)$$

$$Q_2=5{\cdot}35cd_2^{1{\cdot}5}(l+0{\cdot}4nd_2)$$

It has been found in actual practice that on large canals we may put $c=0{\cdot}78$. On small canals, where a slight excess over the calculated depth is advantageous, c, may be taken as 0·70.

We can thus calculate l, and n.

(ii) The notch is partially drowned.

Here, let f, be the difference between the water levels of the two reaches, and put,

$$e_1=y_1-f, \text{ and } e_2=y_2-f.$$

Then, $$Q_1=8{\cdot}02c\sqrt{d_1-e_1}\left(l+\frac{ne_1}{2}\right)e_1$$

$$+5{\cdot}35c\{(l+ne_1)(d_1-e_1)^{1{\cdot}5}+0{\cdot}4n(d_1-e_1)^{2{\cdot}5}\}$$

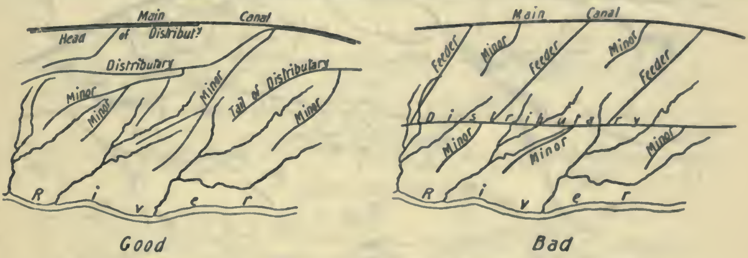
A similar equation holds good for Q_2.

Here also, as above, c, can be taken as 0·70, or 0·78 ; and the equations are obvious for the case where the notch has a free fall when Q_1, is passing, and is drowned when Q_2, is passing.

The above equations give the dimensions of a notch which will pass the total full supply of the canal. In practice, the notch is usually split up into a number of smaller notches, each passing about 100, to 200 cusecs. A very usual rule is :—The top width of each notch should be less than $0{\cdot}8d_m$. The selection of the discharges Q_1, and Q_2, gives scope for a study of the local conditions. As a general rule, it is better to make the notch too large, rather than too small, for should the water surface be found to be too low in practice, the sill can be slightly raised; whereas, if the water is unduly headed up, adjustment is more difficult.

LOCATION OF IRRIGATION CHANNELS.—An ordinary map of an area in which irrigation is well developed bears a great resemblance to the map of a natural river and its tributaries; but if the contour lines are inserted a marked difference is noticed. The river and its tributaries flow in valleys, whereas the canal and its distributaries, or small branches, flow along ridge lines. Just as each valley (except the very smallest examples) possesses a stream which drains the side slopes of the valley, so each ridge line should be covered by a distributary which irrigates the side slopes of the ridge. And while in general the valley of the main river is the lowest valley, the main canal is usually found at or near the highest ridge line.

Thus, broadly speaking, irrigation systems consist of two portions,—the main line and main branches which form the channels by which the water is led from the source of supply to the main ridge line, or ridge lines, of the area



SKETCH NO. 215.—Good and Bad Location of Distributaries (after Mackenzie).

Note.—These sketches are ideal, but the "bad" represents fairly closely the Indian practice of 1860–70. The long distributary and its feeders interfere with the natural drainage, and therefore water-logging is likely to occur. On the other hand, these feeders permit water to be very rapidly delivered to any point, and the canal staff does not need to be experienced.

The "good" represents modern practice, and most of the older canals have been systematically remodelled to this type. Such a system interferes with drainage as little as possible. On the other hand, the distribution of water needs systematic care.

When, in 1905, with considerable experience of waterworks, but with no knowledge of Indian irrigation, I received charge of about 200 miles of distributary, I came to the conclusion that had the native subordinate staff not been thoroughly experienced it would have taken me at least six months to get a real grasp of the methods, and I had the advantage of telegraphic communication with, and very careful instructions from, my immediate superior.

to be irrigated; and the distributary system, in which the canal splits up into a series of branches ramifying along all the ridge lines of the irrigated area. As a general rule, little or no irrigation is effected from the main line until it reaches a ridge line, as the water level is usually not sufficiently above the natural surface to permit it to cover the land. This is briefly expressed by the statement that the main line "commands" only a small area of land, or none at all, as the case may be. The primary object in the location of the main line is therefore to reach the main ridge line as soon as possible; and any irrigation that is effected from the sidelong reaches of the main canal is usually incidental only.

As secondary conditions, we must as far as possible avoid deep cuttings, or high embankments, or crossing drainage lines and stream channels. It will be plain that these objects are also (abnormal local conditions apart) best attained when a ridge line is rapidly reached. Thus, the longitudinal section of an irrigation canal bears a marked resemblance to the longitudinal section of most rivers, the bed slope being flat near the head of the canal (or mouth of the river), and growing more and more accentuated as the tails of the distributary canals (sources of the tributary streams) are approached.

No general rule can be given ; but, as a matter of experience, the head reach of a canal usually has a slope which is approximately half that of the general surface of the land. The design of the distributaries is really a matter of trial and error. The area irrigated will determine the volume of water carried at each point, and the level of the water in the canal feeding the dis-

Sketch No. 216.—Relation between Distributaries and Contour Lines.

Note.—This sketch shows a portion of the Upper Bari Doab Canal. I had in 1909 selected it as showing what are probably the best laid out distributaries in the Punjab; and Mackenzie, who had maps of all the Indian canals at his disposal, has also published the complete field map as a good example.

Personal acquaintance with the locality, and a knowledge of the working conditions, inclines me to believe that the regulation is somewhat more difficult than is usual. In fact, the principle has been overdone. This is not very important, as it is usually impracticable to follow the theoretical location so closely.

tributary, and that required to command the area near the tail of the channel will fix its mean bed slope.

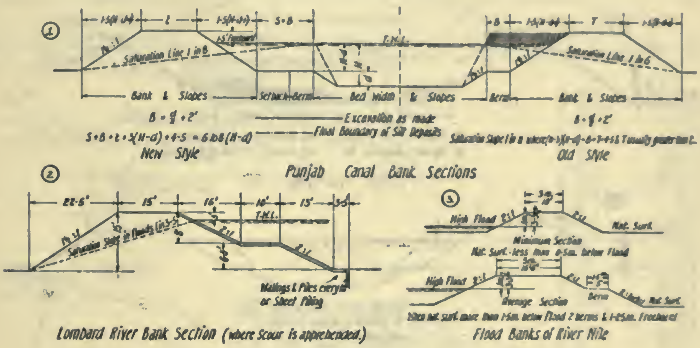
Having fixed the gradient, we have next to inquire whether the channel will silt, or scour. Kennedy's principles (when properly applied) enable us to answer this question. If it should happen that the ruling gradient is so steep as to cause scour to be apprehended, it is necessary to reduce the surface slope of the water by putting in falls (see p. 713). The cross-section of each reach of the channel and its bed width being thus calculated, we can determine the balancing depth, *i.e.* the depth of excavation, such that the earth excavated to form the canal is just sufficient to make the banks.

It is evident that the canal should be located so that the depth of the excavation will, as far as possible, be the balancing depth ; and it is plain that where the choice exists it is cheaper to excavate less deeply than the balancing depth, and to secure the necessary earth by extra excavation in the canal bed ;

breaches, on the other hand, are less likely to occur if the canal is kept below the balancing depth.

The design of the banks is also important. These are generally constructed without any puddle or core wall, and the canal water therefore percolates through, especially in the early years before the interstices in the banks have been stanched by fine silt. Thus, the quality of the earth must be carefully considered. The matter is somewhat more simple than it looks, for in most cases it is proposed to carry the water of a river across the alluvial deposits which this same river has laid down, so that there is a fairly close relation between the quality of the silt carried in the river water and the earth through which the channels are constructed. Thus, Kennedy found that the earth through which the Bari Doab Canals were constructed never gave trouble by scouring, unless the mean velocity exceeded $1 \cdot 30 v_0$, and a similar relation holds good in nearly all cases (see p. 754).

So also, if the river carries a coarse silt, we may expect that the canal banks will, on the average, be constructed of fairly coarse earth. The Punjab silt is



SKETCH No. 217.—Bank Sections.

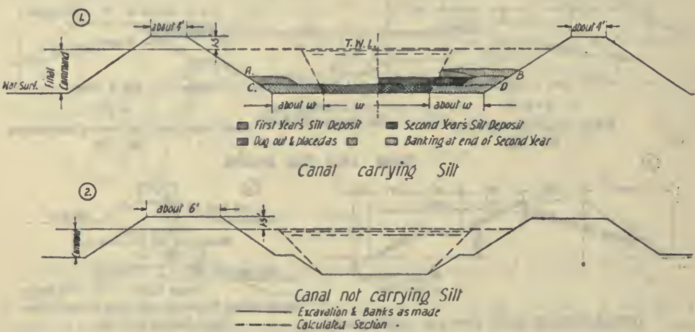
very fine, whilst the earth is nearly as finely grained as is possible. Practical experience shows that a bank consisting only of such earth is not safe against breaches until the gradient through the bank is approximately equal to 1 in 7. Sketch No. 217, Fig. 1, shows the old and new sections adopted in such cases.

The Nile banks are of the section shown in Fig. 3, and it will be evident that the hydraulic gradient is about 1 in 8; while, as a contrast, the embankments of the River Po in Northern Italy are as per Fig. 2, and the hydraulic gradient is about 1 in 4.

So long as the branches exceed a size (say 150 cusecs) such that a breach would be a serious disaster, it appears advisable to excavate them once for all to the cross-section determined by Kennedy's rules, or whatever local modification is found advisable. Sketch No. 219 shows the modern Punjab practice, and it will be seen that by skilful maintenance the otherwise detrimental silt is deposited against the banks and finally produces a very safe section. The two insets show how carelessness may produce deposits of silt that do not much strengthen the banks or curved banks that increase the roughness of the channel.

In the case of minor branches of a size such that a breach in the first years of the canal's existence may be considered as unlikely to cause much damage (and it must be remembered that the flooding of desert land when water is plentiful, is by no means a disaster), the design is somewhat different.

A large amount of capital expenditure, and considerable after outlay on maintenance can then be avoided by constructing these channels as shown in Sketch No. 218, Fig. 1. Such banks are weak, and may breach; they are, however, cheap, and scour is unlikely to occur. In the early years of a canal's existence the required supply will probably be far less than the designed full supply, and the water will probably not rise far above the level AB, and the whole area up to CD, will silt up with fine silt, forming a highly impervious bank. During the first clearance of silt, the silt in the area occupied by the designed section can be dug out, and can be thrown on the water face of the banks, as shown. The next season's work will silt solid up to a level about AB, and the final outcome (after say four or five seasons) will be the channel shown in chain dots. This is flowing in a self-formed bed of highly impervious silt,



SKETCH No. 218.—Canal Sections.

matted together by grass roots, and it will be found that breaches are almost unknown in such a channel.

The advantages of skilled silting are well illustrated by comparison with Fig. 2, which shows a channel carrying the same discharge of water not containing silt.

In this case the banks cannot very well be formed of earth taken from outside the channel, and in consequence the bed of the channel is sunk somewhat below the natural surface, and the "command" is therefore far inferior to that in Fig. 1, where matters can always be arranged so that the bed of the final channel is level with or, if necessary, above the natural surface.

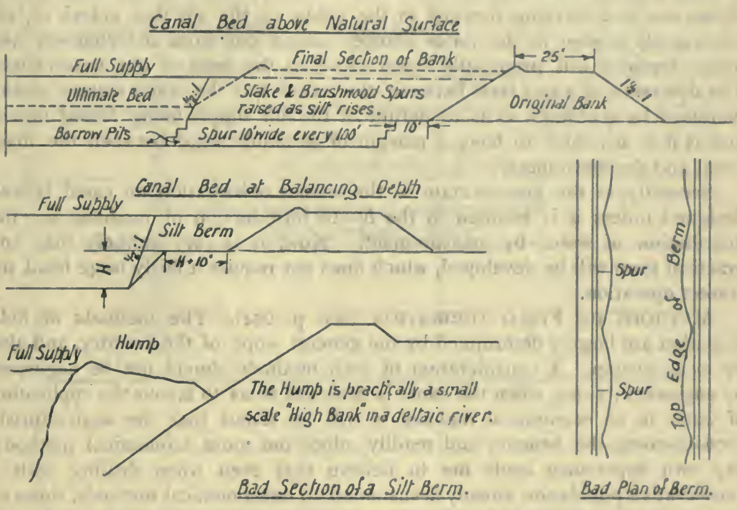
The early maintenance of channels of the type indicated in Fig. 1, is troublesome, owing to numerous small breaches, but after the third or fourth year (indeed, in some cases after the first year) the maintenance cost is considerably less than that of a channel as shown in Fig. 2, even although the command of the latter may be less.

The financial advantages are obvious. The channel costs very little until it begins to earn a revenue. Again, the first settlers are cultivating virgin soil, do not require silt to enrich their fields, and are not diverted from their agricultural development by the labour of silt clearance. Such breaches as do occur merely saturate the soil, which requires water, and are usually regarded as beneficial. In fact, the one disadvantage of the system is that the cultivators are likely to make breaches in order to secure pasturage for their cattle; and I cannot see that this is altogether an evil.

A Government may consider such damage as an encouragement to settlers, whilst a private company should remember that water is cheap in the early years, and that rapid settlement is an advantage to all concerned.

The above discussion assumes that the Kennedy type of channel is exclusively used.

It must, nevertheless, be acknowledged that the adoption of Kennedy channels does not tend towards economy in construction. Kennedy channels



SKETCH NO. 219.—Maintenance of Silt-bearing Canals.

are (comparatively speaking), wide and shallow in form. Hence, even in the very largest sizes their construction entails extra expense in excavation; for it will be found that the balancing depth, even in very large canals, rarely exceeds 5 feet, and consequently mechanical excavation is seldom profitable, while the extra width of the canal causes the lead of the excavation to be great, and entails a larger expenditure on land.

Hence, unless the water is known to deposit large quantities of silt, it will sometimes be found that the increased capital expenditure is greater than the capitalised cost of annual silt clearances, although it must always be remembered that the expenditure on such clearances does not fully represent the damage done by silt, and that the value of the crops which may be lost by a bad supply of water must also be taken into account.

COMMAND.—The question of the relative level of the water in the minor canals, and the adjacent land, is of importance. Silt deposits are liable to occur at the head of each minor channel, owing to the disturbance of the water caused by the bifurcation of the main channel, and also by the regulator controlling the supply to the minor channel. These local head reach deposits will probably be increased by designing the major channel according to Kennedy's principles. It is therefore advisable to calculate the regulator carefully as a drowned orifice (the coefficient of discharge may be taken as 0·70 to 0·75), and to allow a margin of 6 inches in small, and 1 foot in large channels, over the calculated head, so that the full supply can be forced into the channel, even when the head reach is badly silted.

The general slope of the channel should be taken so that the level of full water supply may be about 18 inches to 2 feet above the level of the adjacent land. This, in view of the fact that the channel is presumed to be on the highest adjacent land, may at first sight appear unnecessarily large, but it will be found advisable for two reasons. Firstly, the channel being a non-silting one (*i.e.* carrying forward to the fields all the silt that enters it), silt will rapidly deposit in the water courses taking out from the channel, and these deposits will principally occur close to the head of the watercourse. The difference of water level between the canal and the water course should therefore be kept high, so as to permit of the full supply being forced into it, just as it is advisable to have a margin of available head between the main canal and the distributary.

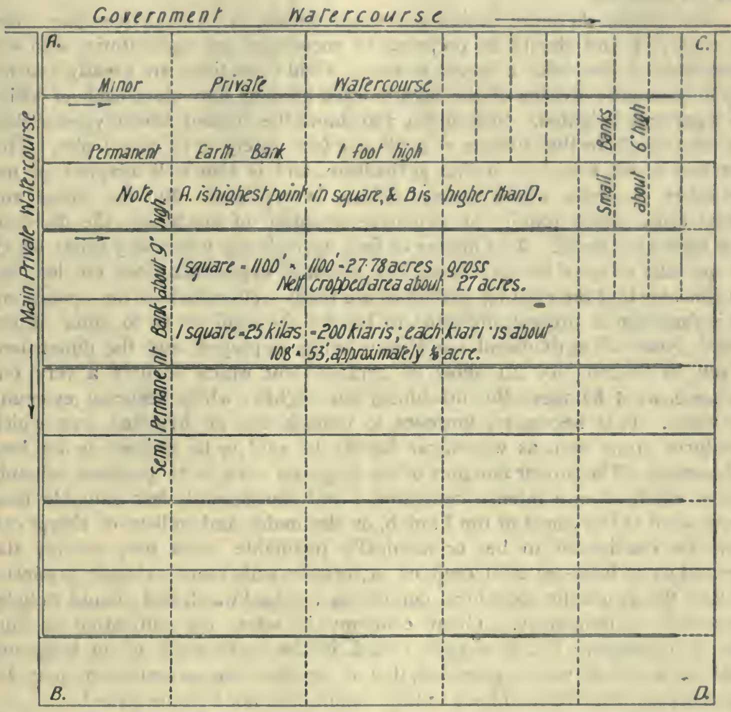
Secondly, in the present state of the science of irrigation no canal is well designed unless it is adapted to the future introduction of modules for the distribution of water by measurement. Now, it is very unlikely that any practical form will be developed, which does not require a fairly large head for correct operation.

METHODS OF FIELD IRRIGATION (see p. 644).—The methods of field irrigation are largely determined by the general slope of the country, and also by local custom. A consideration of such methods should not be neglected by engineers; since, when the canal is designed so as to favour the application of water in an economical manner, it will be found that the agriculturists soon discover the benefits and readily adopt the more economical methods. My own experience leads me to believe that even when dealing with a conservative population already accustomed to uneconomical methods, three or four years' experience will suffice in order to produce an almost universal adoption of better methods.

The methods adopted by agriculturists when left to their own devices are wasteful of water, their weakness principally lying in the lack of attention paid to the design of the small channels conveying the water from the engineer's canals to the fields (*i.e.* the watercourses). An agriculturist considers these channels as necessary evils, and is inclined to make them as small as possible. The Punjab Irrigation Department has carried out very systematic experiments on the subject, and I have investigated the results mathematically, in order to discover the relative advantages of the methods of procedure of both parties. Broadly speaking, an engineer can always raise a better crop with less water than an agriculturist; while, on the other hand, he fails lamentably in taking advantage of local rainstorms, cloudy days, and other favourable weather conditions. The table given on page 733 is used in the Punjab, and corresponds roughly to Kütter's $n = 0{\cdot}035$, but smaller channels are found to be uneconomical.

It will therefore be evident that the proper function of an engineer is to deliver water to the fields in a manner such as to encourage (in fact, force might be considered the more correct word) the agriculturist to utilise it to the best advantage. The engineer, however, oversteps his proper functions if he in any way endeavours to control the periods or quantity of watering. His concern is always with the "how," and never with the "when."

The area of land which each agriculturist tills is relatively small, being about 28 acres in India, and 40 acres in America, and these holdings are large



SKETCH No. 220.—Division of a Square of Land. See also No. 157.

compared with those of other irrigated countries. We are therefore justified in considering that in a well designed system of irrigation water should be delivered to at least one point per 30 to 40 acres irrigated. It is plain that this point should be the highest spot on the area considered, and even where the slope of land is so great that water delivered by a channel at, or about, the level of the natural surface of the ground, will flow over the whole area, it will still be found economical to insist on delivering water at the highest point, and to arrange so that the water level at that point is at least 6 inches above the ground level. On the other hand, anything much above 1 foot requires somewhat costly banks.

If we were solely concerned with economy in water, there is little doubt but that the best results would be obtained by flooding the whole area as rapidly as possible, and I have personally obtained very excellent results with a flow of 2·5 cusecs (second feet). This volume of water requires a somewhat large force of labour to handle it properly, and although a big landowner might deal with this volume, and would find it advantageous to do so, the ordinary man with 40 acres can hardly be expected to handle much more than one cusec; and, if his watercourses are not in good order, even this quantity will tax his energies.

We should therefore design our watercourses so as to deliver one cusec to each plot, and should be prepared to encourage any agriculturist who will undertake to deal with a larger stream. Field operations are greatly assisted by a systematic division of the area of each holding into plots, each of which is separately irrigated. Sketch No. 220 shows the method officially prescribed in the Punjab for the division of a 28-acre (accurately 27·77 acres) plot. This method is well suited to Indian agriculture, and is also well adapted for use in other countries where intense culture is practised. But for wheat and those crops which require an extensive adoption of machinery, the divisions are somewhat small. As a matter of fact, agricultural machinery exists which is specially adapted for such cases, but personal experience does not lead me to consider that the existing machines are really well suited to the conditions. It is therefore at present advisable to lay out the land so as to allow of the usual types of agricultural machinery being employed, and the dimensions given in Sketch No. 221 show an arrangement which secures a very fair balance, as it hampers the machinery but slightly, whilst securing economy in water. It is necessary, however, to remark that an irrigated area which produces crops such as wheat, can hardly be said to be utilised to its best advantage. The proper function of an irrigated area is to produce valuable crops which require intense cultivation; and consequently less valuable food crops such as the wheat of the Punjab, or the maize and millets of Egypt can only be considered to be economically justifiable when they occupy the ground as an inter-seasonal crop, or in rotation with more valuable products. Where this is not the case local conditions are backward, and should only be regarded as temporary. Great economy in water for cultivation is thus hardly necessary; for, as already stated, in the early years of an irrigation scheme waste of water, provided that it benefits the agriculturist, may be regarded as justifiable, so long as the custom does not become rooted.

Design of a Distributary.—The final design of a distributary requires a knowledge of the arrangement and position of the field watercourses. Similarly, the final design of a branch assumes that the sizes and positions of the distributaries are previously fixed. In fact, the final design of the whole canal must be regarded as proceeding from the tail upwards.

The example given below adheres somewhat closely to the conditions which are usually found in India. The individual watercourses are perhaps slightly larger in capacity than the average of those found in India, but in this respect the design is typical of the more advanced Indian practice. The question has already been discussed, and there is little doubt that were the Indian agriculturists more accustomed to mutual co-operation, even larger watercourses might be constructed with advantage, since the capacity of the watercourse fixes the rate at which water is applied to each individual field division or plot.

Thus, the larger the discharge of the watercourse, the smaller is the waste which occurs by absorption during the actual application of the water to the land. The Indian rule on the matter is as follows :

"All watercourses which exceed three miles in length are to be considered as distributaries, and are to be maintained by the Government."

FINAL DETERMINATION OF THE DIMENSIONS OF A DISTRIBUTARY.

Reduced Distance of Watercourse Outlets in Feet.	Commanded Area in Acres	Allowance of Water.	Discharge of Watercourse in Cusecs.	Nett Discharge of Distributary in Cusecs.	Bed Slope.	Preliminary Cross-section of Distributary.	Ratio $\frac{\text{Mean Velocity}}{v_0}$.	Absorption at 8 Cusecs per Million Square Feet.	Gross Discharge, including Absorption.	Final Section of Distributary.	Ratio $\frac{\text{Mean Velocity}}{v_0}$.
0	...	3·1 cusecs per 1000 acres	...	43·1	$\frac{1}{4000}$ = 0·25 foot per 1000 feet	10 × 2·6	1·00	...	47·0	12 × 2·4	1·03
3000 left .	1070		3·3	39·8		10 × 2·4	1·00	0·4	43·3	11 × 2·4	1·02
7000 right .	700		2·2	35·7		9 × 2·4	0·99	0·6	38·6	10 × 2·3	1·01
and left .	600		1·9	...		...	...	...	...	...	...
10000 right	1290		3·9	31·8		9 × 2·3	0·99	0·3	34·4	10 × 2·2	1·01
14000 left .	1000	2·4 cusecs per 1000 acres	2·4	29·4		9 × 2·3	0·99	0·4	31·6	9·5 × 2·2	1·01
17000 right	1330		3·2	26·2		7 × 2·2	1·00	0·5	27·9	9·5 × 2	1·01
21000 left .	2000		4·8	21·4		6 × 2·2	0·97	0·4	22·7	7·5 × 2	1·02
27000 right	1500		3·6	17·8	$\frac{1}{3636}$ = 0·275 foot per 1000 feet	6 × 2·1	0·97	0·5	18·6	6·5 × 1·8	1·01
29000 right	1330		3·2	9·9		5 × 1·6	0·97	0·1	10·6	5 × 1·6	0·97
and left .	1960		4·7	...		...	...	...	...	...	...
38000 right	1800		4·3	5·6		3 × 1·4	0·95	0·6	5·7	4 × 1·4	0·93
40000 tail .	2330		5·6	5·6		...	...	0·1	(5·6)	...	...

The filling of the table is obvious. The first two columns are obtained from the survey of the irrigated area, and the watercourses must be previously laid down on this plan. The third column "Cusecs per 1000 acres irrigated," is determined by the depth of subsoil water below the natural surface in the areas served by the various watercourses, and is usually constant for the whole distributary; although, in the example given, the allowance is varied in order to emphasise the necessity for bearing this condition in mind. The fifth column is the sum of these supplies for all watercourses below that Reduced distance, and consequently determines the Nett discharge (without allowance for absorption) of the distributary between the point considered and the next watercourse downstream. The sections of the distributary channel are then determined (in the example for slopes of $\frac{1}{3636}$ and $\frac{1}{4000}$) by Kennedy's Graphic Hydraulic Diagrams. The selection is so made that the mean velocity is near to (preferably slightly in excess of) v_0, which is the non-silting velocity determined by Kennedy (see p. 754).

In another locality where Kennedy's rules are not applicable, the selection will usually depend upon economy in earth work.

The table is purposely made somewhat more complicated than usual. As a rule, both the allowance of water per 1000 acres, and the bed slope, are constant throughout the distributary. The circumstances selected are, however, favourable. Relatively speaking the slope is steep, and hence it is possible to obtain Kennedy channels without making the width an unduly large multiple of the depth. With flatter slopes, however, it would be found necessary as the discharge of the distributary becomes small to reduce the ratio $\frac{\text{Mean velocity}}{v_0}$ to 0·95, 0·90, or even to 0·85. No definite rules can then be given, but if the fact that the channel carries silt and water is borne in mind, it will be plain that such ratios are permissible towards the tail, provided that the outlets of the upper watercourses are situated close to bed level; so that the upper watercourses draw more than their fair share of silt from the channel. In one particular instance, where I was forced to make the ratio equal to 0·73 for the last quarter of the channel, this disproportionate allowance was sufficiently marked to cause complaints. In all cases it is plain that no very sudden change should be permitted, and I believe that it is far better to grade down gradually through say: $\frac{\text{Mean velocity}}{v_0} = 1{\cdot}00, 0{\cdot}97, 0{\cdot}93, 0{\cdot}90$, etc., etc., than to produce a sudden drop in the ratio, such as: 1·00, 1·00, 1·00, 0·90, etc.

The preparation of a table showing the bed level and the water surface level at each watercourse outlet is now easily effected. It is, however, necessary to observe that the changes in depth must be assumed to be produced not by a rise in the bed but by a drop in the water surface. Thus, the difference in bed levels between the points R.D. 0, and R.D. 14,000 is $14 \times 0{\cdot}25 = 3{\cdot}5$ feet, but the difference in water surface levels is:

$$14 \times 0{\cdot}25 + (2{\cdot}4 - 2{\cdot}2) = 3{\cdot}7 \text{ feet.}$$

Similarly, the total fall in the bed in the distance of 40,000 feet is:

$$14 \times 0{\cdot}25 + 26 \times 0{\cdot}275 = 10{\cdot}65 \text{ feet,}$$

but the drop in water surface is:

$$10{\cdot}65 + (2{\cdot}4 - 1{\cdot}4) = 11{\cdot}65 \text{ feet.}$$

It will be noticed that the absorption is calculated on the basis of eight cusecs per million square feet of wetted perimeter, and the gross discharge of the distributary as tabulated in Column No. 10 is greater than the sum of the watercourse discharges. The corrected section is shown in Column No. 11, and is obviously sufficiently near to the truth to require no further corrections for absorption. The total absorption is 3·9 cusecs in a total discharge of 43·1 cusecs; and, as a general rule, it will be found that if an extra allowance of 10 per cent. be made in the discharge of each watercourse sufficient accuracy is obtained. The calculation of a branch canal proceeds on exactly the same lines, but it is usual to allow a slightly larger supply per 1000 acres commanded in calculating the distributary discharges in order to avoid the necessity of previously determining the watercourses. Thus, we find rules such as the following:

When the supply of individual watercourses is calculated as 2·8 cusecs per 1000 acres, the gross distributary discharge is taken as 3·1 cusecs per 1000 acres; and, until local knowledge has been accumulated, these rough rules must suffice. It will be plain that the above methods of calculation will usually in practice be applied only when "remodelling" existing canals so as to obtain the closest possible adjustment between the canal discharge and the local conditions. In project work it is usual to design the main branches by treating the distributaries in the manner in which watercourses are treated when remodelling existing canals.

The final plans of a distributary, or branch, include a tabulation of bed levels, natural surface levels and full supply levels at say every 1000 feet.

The quantities of excavation and banking can then be taken out, and the irrigation facilities (*e.g.* the command and the difference between the full supply level in the distributary and the various watercourses) can be calculated.

The work is laborious, but commonplace. The form adopted depends upon local conditions, and my own opinion is that it should include all matters connected with the distributary.

For example: At every 1000 feet interval the bed level should be tabulated, together with the following:

(1) The full supply level.
(2) The water surface level at any supply which is less than the full, and which is frequently delivered.
(3) The area of excavation.
(4) The area of the banks.
(5) The command.

At every masonry work, or outlet, the following should be recorded:

(1) The full supply level.
(2) The command.
(3) The head through the outlet of the watercourse as defined below.
(4) The particulars of the work (*e.g.* bridge of 2 spans, each 8 feet wide, piers 14 feet by 2 feet over all; pitched 3 feet above piers, and 5 feet below ditto).

Similar particulars should be recorded at each outlet, including the area of the orifice, and the discharge. If the work is a regulator, the head for which it is designed should be given; and if a fall, the fall and the dimensions of the notch should be stated.

The ruling principle to be borne in mind is that distributaries are usually at least 10 miles in length, and that the engineer should inspect on horseback, or by bicycle; and that the single sheet which he carries with him should provide all the information necessary for a decision to be made on the spot.

The one matter which calls for remark is that a decrease in depth must be regarded as producing a drop in the water surface. For example, at R.D. 3000 (see Table p. 733) the depth is decreased from 2·4, to 2·3 feet, and this must be held to produce a drop of 0·1 foot in the water level at, or near R.D. 3000 feet. This fall is probably caused by the change in velocity which occurs, but its existence is undeniable, and if neglected, difficulties will arise.

The supply to each watercourse is at present measured by an orifice in a stone slab. The orifice area is determined by the equation:

$$Q = 5\sqrt{h} \text{ area of orifice,}$$

where Q, is the supply to the watercourse in cusecs, and h, is the difference between the full supply level in the distributary and the full supply level in the watercourse.

In actual practice, these orifices are not fixed in place until irrigation has been carried out for some two or three years. The delay is in some respects a disadvantage, but in actual work it is found that the opportunity given for local investigations regarding the necessity for an increase or decrease of the supply (by such factors as unusually absorbent soils, or peculiar methods of irrigation) justifies the delay. I do not give standard drawings of the outlets which are employed, because the selection of the design and the materials used must entirely depend upon local conditions. It must, however, be realised that these outlets are constructed by dozens, if not by hundreds, so that their design is a matter which deserves careful consideration.

MODULES.—These are instruments for the supply of water by bulk, and are analogous to town water meters. The advantages to all concerned are obvious, an agriculturist who pays a rate of so much per acre irrigated has no personal interest in economising water, whereas if he pays a rate per 1000 cubic feet of water delivered, each economy effected in applying the water to the cropped area is his own gain.

The obvious methods are a recording gauge and a weir, or a vaned wheel, fixed in a channel of known dimensions, whose revolutions are recorded. The practical difficulties are: (*a*) any such instruments are too costly; (*b*) except in North Italy and parts of America the average agriculturist will steal water, tamper with instruments, and has public sentiment to support him in this attitude; (*c*) in general the mean slope of the ground is so small that any device entailing a large loss of head is unpractical.

Thus the problem will not be solved until some instrument is discovered whose records cannot be stolen or falsified, which, if tampered with, invariably delivers less water than when in proper order, and which works under a head of say 0·5 foot as a maximum.

I was employed by the Punjab Government to experiment on two forms of module, the Kennedy and the Gibb. Both, I believe, fulfil these conditions. Being proprietary articles I do not describe them. The Kennedy I consider the better, but this statement refers only to silt-bearing waters. The Gibb retains nearly all the silt in the water in the canal, and therefore alters the régime for the worse. This may be an advantage in clear water canals, but I have no practical experience of the case. When standardised I believe the Gibb will be the cheaper; at present its accurate erection and calculation is very tedious, while the Kennedy can be ordered in fixed sizes and put in place by an untrained mason. Also the Kennedy can be examined and seen to be working correctly merely by reading a gauge.

SURVEYS PRELIMINARY TO IRRIGATION.—The quantity of water which it is desired to supply to the land can be estimated from the observations which have already been given. The following figures (which, when possible, should be supplemented by direct observation) permit an estimate of the losses in transit from the river, or other source of supply, to be made. The capacity of each individual channel of the system can thus be estimated when an accurate survey of the irrigable area has been made. It is not proposed to discuss the methods adopted in such surveys. Broadly speaking, the country must be mapped with an accuracy comparable with that attained by such bodies as the British Ordnance Survey, or by other Government services. Practical experience in checking the levels given by Ordnance surveys and good maps of irrigated districts enables me to state that the levels (although not the topography) are probably more accurately determined by irrigation engineers. As rough rules it may be stated that the level of the ground surface should be determined to a tenth of a foot at every corner of a series of a fifth of a mile squares, in plain country; and that the accumulated errors must be carefully balanced. I have twice had to work across a boundary of levelling operations where the reference datum on one side of the boundary was 30 miles distant from the reference datum on the other side. The difference was not very great, and the original levelling work must have been very accurate, but the trouble entailed was enormous, and in one case led to what might have proved a serious defect in the supply. Wherever possible it is therefore advisable to select ridge lines as the boundaries of the operations of the various levelling parties.

In a country with a more accentuated topography, such difficulties are less likely to arise, but the ground level will then have to be determined at more closely spaced points.

Absorption, or Leakage from Channels or Reservoirs.—The leakage from an unlined channel, or reservoir, is so entirely dependent upon local circumstances, that any general figures for its value must be regarded as mere indications of its possible magnitude.

It is of course possible to obtain a theoretical formula, by the process employed in the investigation of percolation under a dam or weir. But this seems useless, since the leakage occurs over so large an area that it would be impossible to ascertain the average permeability of the soil accurately. The same difficulty occurs in the original investigation, but in that case we need not consider the permeability at a distance from the work, for although this may largely influence the total leakage, it cannot materially affect the stability of the dam. The statement is illustrated by certain early British reservoirs, where the dam is constructed according to the best British practice, with puddle walls carried deep into impermeable strata, and is therefore probably as little liable to leakage as any human work of an equal area can be. The reservoir sites, however, are crossed by a geological fault, and the reservoir is consequently rendered practically useless by local leakage along the line of the fault.

Some writers on this subject have given sketches showing the theoretical paths of the leakage water. Reasoning on these lines, some investigators have arrived at rules such as the following:—

"A canal on the top of an embankment is less leaky than one in a cutting," etc.

I cannot quarrel with their mathematical reasoning. If the slopes of a bank are to be considered as impermeable boundaries because they do not visibly sweat, or leak water, the proposition hardly needs mathematical demonstration. I do object to the idea that water does not leak across an earth surface because it never appears in a visible form on such a surface. Actual calculation shows that, quite apart from the rank growth of vegetation which is often found on the outer slopes of canal embankments or earthen dams, evaporation from a bare earth surface is quite sufficient to dispose of all the leakage water that reaches it from the canal.

It is generally found that canals in bank do leak less than those in cutting, owing probably to the fact that the rolled earth forming the banks is more water-tight than the unrolled earth which usually forms the banks of a canal in cutting.

The only factors that are sufficiently powerful to overcome local differences, such as care in construction, height of the bed above subsoil water level, etc., are the character of the soil, and the amount of silt deposit that forms on the bottom of the channel or reservoir.

The following figures represent the observed leakage (including evaporation from the water surface) from certain French navigation canals, reduced to cusecs per million square feet of wetted perimeter. The figures usually refer to the summit level of these canals and are therefore probably greater than would usually occur. This opinion is confirmed by the fact that most of the figures are obtained from reports concerning proposals for remedial measures. It is also believed that nearly all these canals are supplied with water which has been stored in reservoirs, and therefore contains very little silt. It is quite certain that (except for two doubtful cases where the canal takes out from the Rhone, and may therefore receive silted water) the water is always extremely clear when compared with the water usually occurring in irrigation canals.

Fissured rock .	maximum 270;	minimum 28;	average 90
Chalk . .	„ 270;	„ 8;	„ 48
Gravel . .	„ 60;	„ 5;	„ 40
Alluvial soil .	„ 44;	„ 1·5;	„ 12

For very fine soils such as occur in the Punjab, we have the following values (Kennedy, *Indian Irrigation Congress*, 1905), which refer to water carrying much silt:

Main line of Bari Doab. 4000 cusecs discharge. 6 feet deep, constructed in shingle and sandy soil.	Average, 9·7 cusecs per million square feet.
Do., Sirhind. 7 feet deep, constructed in sandy soil, but the subsoil water level is nearer to the bed level than in the case of the Bari Doab.	Average, 9·0 cusecs per million square feet.
Branches, 1000–3000 cusecs. In good loamy soil. No sandy soil. Side slopes silted, but no bed silt.	Average, 2·2 cusecs per million square feet.
Do., Sirhind. All sandy soil. No side silt, or bed silt.	Average, 5·2 cusecs per million square feet.
Distributaries of Bari Doab. 300–1000 cusecs. Conditions as in branches, but some fine silt in bed in a few cases.	Minimum, 2·3 cusecs per million square feet. Average, 3·3 „ Maximum, 4·4 „
Do., Sirhind. Sandy soil.	Minimum, 5·0 cusecs per million square feet. Average, 8·0 „ Maximum, 12·0 „

Distributary Branches, 0·5 to 3 cusecs discharge. All conditions. Some quite new, with no silt. All in loamy soil.	Minimum,	3·3	cusecs per million square feet.
	Average,	9·4	"
	Maximum,	30	"
Do., Sirhind. All conditions. Sandy soil.	Minimum,	7	cusecs per million square feet.
	Average,	22	"
	Maximum,	80	"
Fields. First watering. In loamy soil.	Minimum,	5·5	cusecs per million square feet.
	Average,	8	"
	Maximum,	16	"
Do. Dry or moist. In sandy soil	Minimum,	12	cusecs per million square feet.
	Average,	21	"
	Maximum,	60	"

Judging from the comments passed upon Kennedy's statement, these values are lower than is usually the case in India; but it must be remembered that the subsoil water level in the Punjab is generally far deeper below the canal bed than elsewhere in India, while the soils are finer in texture. Hence, it is probable that the silt had stanched the beds in many cases. Kennedy's figures are, however, confirmed by the value of 7·15 cusecs per million square feet lately obtained on the Ibrahimiya Canal (Egypt), and in this case the canal would not be considered well silted in the Punjab sense of the term.

The figures for the Punjab may be taken as fair averages, and do not, like the French figures, refer to abnormal cases; since the discharge of all the channels is systematically measured.

There is of course little doubt that the depth influences the leakage, and the formula:

Cusecs percolation per million square feet $= C\sqrt{\text{depth in feet}}$, where for the Punjab C is 3·5, has been proposed by Dyas. This formula is useful as showing the effect of marked variation in depth when other conditions are unaltered.

The general results may be summed up as follows:

(i) Guillemin states that at the first filling of a new canal a volume equal to that of the canal should be provided for initial leakage.

(ii) The average loss in those sections of a canal which are not unduly leaky may be considered as about 4 to 8 cusecs per million square feet, when all possible care is taken to minimize losses. As the canal deteriorates by age (silt being assumed not to be deposited), this quantity increases to double after some five to ten years.

(iii) For an old canal where silt has never been deposited, or where it has been removed, such values as 20 to 40 cusecs per million square feet are possible.

In channels in rocky and fissured soil, these losses may be doubled, or trebled; but it must be remembered that such abnormally leaky sections cause trouble, and are therefore more frequently studied, and the figures relating to them are more frequently quoted.

In some channels the water table of the surrounding ground is sufficiently high to cause the channel to act as a drain. Here, no loss occurs, and ground seepage replenishes the canal. For example, in the first eight miles of the Sirhind Canal it is believed that seepage into the canal occurs at the rate of about 5 cusecs per million square feet.

(iv) In canals carrying muddy water some stanching by deposition of this mud in the cracks and fissures of the soil always occurs, even though silt is not deposited in the channel. Kennedy's values may be taken as correct, and the usual allowance in the Punjab for leakage at the rate of 8 cusecs per million square feet of wetted perimeter is ample. The canal will probably become less leaky as it ages, but any scour may cause the leakage to increase suddenly.

Lining of Irrigation Channels.—The figures given above show that in a large canal system the absorption losses, between the canal head and the fields, may reach 50 per cent. of the total volume entering the canal, and rarely fall below 20 per cent. This loss, as it tends to raise the subsoil water level and encourage waterlogging and alkaline soils, is directly detrimental.

Thus linings to stop percolation have been frequently proposed. Earthen channels lined with concrete or rendered with cement, and iron pipes form the standard systems of distribution in Southern California. In my own practice, I have usually found that wherever the water is lifted more than 20 or 30 feet similar linings (of the main channels at any rate) will always earn interest on their cost by the diminution secured in the fuel bill.

In gravitation and large pumping schemes, however, such linings are too costly.

The University of California and the Punjab Irrigation Department have carried out many experiments. The linings tried include concrete (lime and cement), cement rendering, oiled paper, mixtures of light and heavy oils with gravel and dry earth, puddle clay, etc.

The only effective linings appear to be the concretes, rendering, puddle clay and oiled paper. The last deteriorates with age, and the last two, unless made too thick to be economical, are very easily perforated and rendered less efficient by animals trespassing on the channels to drink.

The following figures may be useful:

Loss in unlined Californian channels	= 9·4	cusecs per million square feet of water surface area.	
Loss in channels lined with:			
$2\frac{1}{2}$-inch cement concrete	= 1·3	,,	,,
$2\frac{1}{2}$-inch lime concrete	= 3·1	,,	,,
1-inch cement mortar	= 3·4	,,	,,
$3\frac{1}{2}$-inch puddle clay	= 5·2	,,	,,
oiled paper	= 3 to 5	,,	,,

The puddle clay does not appear to have been good puddle, but there are very few irrigation districts where good puddle is available.

Leakage of Reservoirs.—One favourable circumstance exists in the case of a leaky reservoir. There being no current to remove silt, stanching of the percolation passages takes place; and, unlike a flowing canal, the leakage of a reservoir will decrease with age.

Further than this, it appears impossible to make any statements. Modern studies show that the evaporation from the water surface of a large reservoir is considerably less than (usually about ⅔) that which is observed in ordinary evaporation pans.

Now, engineers have been accustomed to apply the results of evaporation pans in order to obtain the loss from large reservoirs, and no discrepancy which is so extensive as to arouse suspicion has yet been noticed. It would therefore appear that the leakage from reservoirs is generally small, and that it does not probably exceed a depth of 1 foot annually in the case of a good reservoir.

It must be remembered that in dry years when the reservoir is drawn down below its normal level, percolation occurs from the ground into the reservoir. Herschell and Ftely have estimated this at as much as 10 per cent. of the volume of the reservoir.

A safe and practical rule appears to be as follows :

Allow for evaporation and leakage combined the quantity reported as evaporated from a free water surface, *i.e.* from a small evaporation pan.

The real difficulty is that the total leakage is probably less than 2 per cent. of the volume received by the reservoir ; and usually we cannot accurately measure such volumes to 2 per cent., the probable error being about 5 per cent.

The following figures represent the allowances for leakage plus evaporation which are usually made in practice :

Allowances are rarely made in Great Britain, but 2 feet per annum seems to be a maximum.

In Germany, 1·64 feet (0·5 metre), or occasionally 2·46 feet is allowed.

In India, from 6 to 7 feet ; or where reservoirs are known to leak, 10 feet has been allowed, and 13 to 15 feet has been observed under unfavourable circumstances. These allowances are usually stated to refer not to the year, but to the interval between the end and the beginning of two successive wet seasons.

In Australia, 6 feet is allowed, but this is insufficient in the hotter districts.

In South Africa, the usual allowance appears to be 7 feet per annum; and this refers to valley reservoirs in mountainous country, so that the conditions are favourable.

In the Eastern United States, 40 inches used to be allowed, and in the Western arid tracts estimates are based on a value of 8 to 10 feet per annum.

Regulation of an Irrigation System.—The figures given during the discussion of the irrigation duty of water will render it plain that the agricultural requirements of a cropped area are variable. Quite apart from such climatic conditions as falls of rain or unusually hot spells of weather, land invariably requires water to be supplied at a greater rate during the ploughing season than later on when the crops are growing.

Similarly, the available supply in the river, or other source from which the water is drawn, may vary from day to day.

The adjustment of supply to demand therefore forms a very intricate problem, which deserves the best efforts of all concerned, since the success or failure of the system largely depends on the efficiency of this adjustment. Putting local conditions aside, it is plain that in a season when the demand or the maximum available supply was, say, three-quarters of the maximum quantity which the canal can carry, it would be futile to distribute this diminished quantity of water *pro rata* among the individual branch canals, so that each carried about three-quarters of its maximum supply.

The correct distribution is plainly obtained by shutting down certain branch canals, the combined capacity of which is about one-quarter of that of the whole system entirely, and running the remaining channels full bore. Thus, any difficulties caused by variations in the water level are prevented. As a general principle, a channel from which irrigation is effected should always carry something between 0·9 and 1·1 of its designed maximum supply, or else be bone dry.

For this reason, the areas which are commanded by the larger channels (*i.e.* the main canal and the three or four largest branches) should not be irrigated directly from these channels, but from small branch canals taking off from the main channels, and in some cases running for long lengths not more than 100 feet distant from the parent channel. (Sketch No. 216.)

The preparation of a systematic table showing the periods during which each individual channel receives its full supply is a matter that can only be arrived at by long experience. The principles are obvious :—The preliminary local investigations give full information regarding the permissible interval between successive waterings, and this interval must form the basis of the table, since each individual irrigation channel must receive its full supply at least once during this interval. During the early years of the system careful studies of the available supply and of the normal demand must be made ; and finally, after some five or six years have elapsed, a fair working schedule can be laid down. This schedule will require modification from time to time, as the irrigated area increases, or as the staple crops change. The existence of such changing conditions forms the only real reason for the services of an expert engineer on an old and well maintained canal, and it is therefore plain that irrigation engineers who aspire to be more than supervisors of small repairs must consider their position mainly dependent on the efficiency with which they contrive to utilise the available water to the best advantage.

The watercourses taking out from each small distributary of a canal system must also be regulated on the same principles during the period over which it receives water, and a table of the times during which each individual farmer is entitled to use the full supply of the watercourse which irrigates his fields must be drawn up. The days of the week, or the hours of the day form convenient and unmistakable divisions. As a general rule, so long as the distributary receives water, each watercourse should be open, and should be permitted to take its full supply. Any attempt to close off individual watercourses is usually considered unjust, and invariably leads to "unauthorised irrigation," and the manufacture of "legal crimes" is an unprofitable affair.

ALKALINE SOILS.—A soil may be considered to be alkaline when it contains a sufficient quantity of soluble salts to interfere decidedly with the growth of crops under circumstances which favour this interference. Consequently, the term covers a very extensive range of conditions, varying from soils covered with layers of common salt, some 2 or 3 inches in depth (capable under favourable circumstances of being utilised commercially), to those which contain distributed throughout their top 3 or 4 feet a quantity of soluble salts which is just sufficient to damage delicate plants when concentrated into the top 2 or 3 inches of the soil.

The salts which usually render a soil alkaline are sodium chloride, sodium sulphate, and sodium carbonate, but damage has also been traced to magnesium sulphate, and chloride, and to calcium chloride. The most dangerous "alkali" is sodium carbonate (the so-called black alkali of the United States), since a far smaller quantity of this salt suffices to render a soil alkaline, than any of the others (with the possible exception of calcium chloride). The practical problems relating to the removal of all these salts, however, are more or less identical. It must be realised that once water is applied to an alkaline soil, reactions transforming the original sodium chloride into say sodium carbonate, and calcium chloride, are not only possible, but actually take place under favourable circumstances. At present the exact conditions are not well known, except that the dangerous black alkali, sodium carbonate, can be more or less rapidly transformed to the less harmful sodium sulphate by the action of powdered gypsum in the presence of water. The chemical reaction is expressed as follows:

$$Na_2CO_3 + CaSO_4 = CaCO_3 + Na_2SO_4$$

As calcium carbonate is very insoluble under all conditions, the reaction is fairly certain. On the other hand, such reactions as:

$$2NaCl + CaCO_3 = CaCl_2 + Na_2CO_3$$

although theoretically possible, depend (under practical conditions) on such matters as the temperature and the amount of water present, and are therefore by no means certain to occur.

Practically, therefore, an engineer can be satisfied with regarding all alkaline soils as alkaline without drawing any distinctions as to the chemical composition of the salts present, and will find that the practical methods either for removing alkalinity, or for preventing its occurrence, are not markedly affected by the chemistry of the alkali. The chemical composition of the alkali is known to have a decided influence upon the time and quantity of water required to remove it. As yet the matter has not been exhaustively studied, and only relative values can be given.

Soils which are rendered alkaline by sodium chloride (and probably all other chlorides) are relatively easily reclaimed. Sulphate alkalis are somewhat less easily removed, but cannot be regarded as difficult to deal with. Sodium carbonate is a far more difficult proposition. It appears to render the soil less permeable, as may easily be observed by adding a few drops of a solution of sodium carbonate to a cubic foot of sand and testing the permeability both before and after the addition of the salt. In consequence, the soil appears to be capable of choking drain pipes and entering into gravel beds in a manner which leads me to believe (although I have been unable to detect the effect microscopically) that the individual grains of the soil are broken up. While soils which are heavily charged with sodium carbonate have been reclaimed and rendered fit for cropping, I consider that the process is rarely, if ever, economically profitable. Dressing with gypsum (as already indicated) appears to be a necessary preliminary.

The engineer's classification of alkaline soils is as follows:

(*a*) Soils which are naturally alkaline.

(*b*) Soils which are artificially alkaline.

Soils belonging to the first class are those which, prior to irrigation, contain sufficient alkali to prevent growth. It is of course quite possible that this alkali is a relic of former irrigation.

The soils of the second class are at first capable of supporting the growth of crops ; but they contain (either equally distributed throughout the depth of the soil, or obtained from the water used for irrigation), a sufficient quantity of alkaline salts to prevent crop growth if concentrated in the upper layer of the soil, *i.e.* the upper 1, to 3 inches depth.

The problem of irrigation in connection with alkaline soils therefore consists in :

(i) The reclamation of soils of class (*a*).

(ii) The treatment of soils of class (*b*), in such a manner as to prevent their becoming alkaline.

The reclamation of alkaline soils is a very simple process in principle. The soil is covered with water which dissolves the soluble salts, and this water is drained off, carrying away the salts. The details are by no means simple. Alkaline soils rarely contain much above 2 per cent. by weight of alkali, and were it possible to obtain a solution even approximately saturated, a watering of about 4 inches depth should be sufficient to remove the alkali in the top 3 feet of soil. As a matter of fact, the last remnants of the alkaline salts are very difficult to wash away. The process actually consists in removing sufficient salts from the top layers of the soil to permit the growth of some plant (e.g. *Panicum crus galli*, or some varieties of clover), which is more than usually resistant to the action of alkaline salts. Having thus obtained a covering to the soil, so as to prevent abnormal evaporation, the remainder of the alkali is washed down by the water used for irrigating the first crop, as it percolates to the drains.

The most easily reclaimed land is that which is alkaline by nature, *e.g.* the beds of dried-up salt lakes, such as occur in the Western United States, India, Egypt, and elsewhere. In such cases, it is sufficient to dig drains about 66 feet apart, and of such a depth that the drainage water stands about 1 foot 3 inches to 1 foot 6 inches below the soil level. The earth excavated from the drains is thrown out on either side so as to form banks to retain the washing water, as is shown in section in Sketch No. 221.

The great difficulty usually found in such work is to obtain a sufficient fall to ensure good drainage. The problem is that of irrigating low-lying land with large quantities of water, and yet preventing waterlogging. The two conditions are to a certain degree antagonistic.

The natural slope of the soil is rarely sufficient ; and, in consequence, the drainage water has frequently to be pumped out artificially. The larger drains must therefore be very deep close to the pump, in order to obtain a correct fall. It will therefore be found that one pump cannot economically drain much more than 2500 acres. The pumps may be proportioned so as to deal with 1 cusec per 150 acres during reclamation, although 1 cusec per 250 acres will suffice to keep the reclaimed land properly drained. Unless ample water is available, the extra pump capacity only permits a slightly more rapid reclamation.

The rate at which reclamation proceeds depends almost entirely upon the permeability of the soil. It will be found that where previous to reclamation, the soil has been trodden on, or has otherwise been rendered hard (*e.g.* footpaths, or places where materials have been deposited) it remains alkaline long after the surrounding soil has been reclaimed.

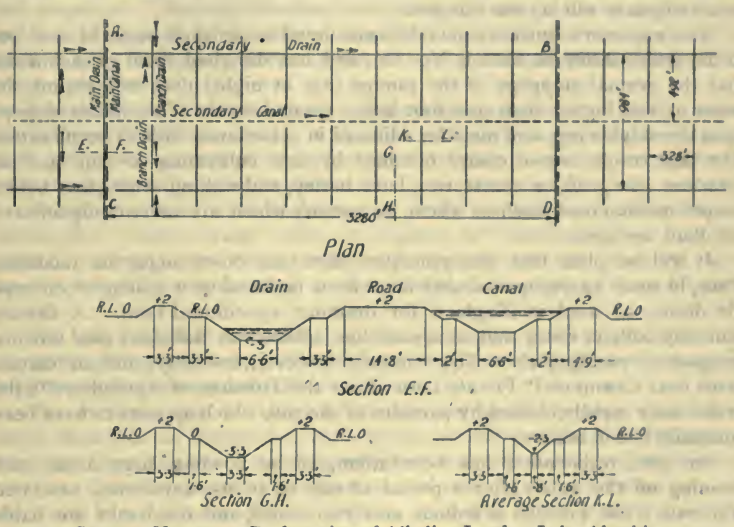
The following sketch (No. 221) shows the methods developed by Anderson and Sheppard (see *P.I.C.E.*, vol. 101, p. 189) at Lake Aboukir.

The large fields A B C D are 3280 feet long (1000 metres) by 984 feet wide (300 metres). The main drains and irrigation canals run along A C, and B D, as shown in section at E. Along A B and C D, run the secondary drains, as shown at F; and an irrigation canal of equal cross-section is dug down the middle of the plot.

The minor drains are usually about 2·3 feet deep, with a bottom width of 0·83 foot, but vary with the nature of the soil. Consequently, the plot is cut up into twenty smaller plots, each 328 feet wide by 492 feet, drained by a minor drain, and surrounded by small banks which permit them to be flooded to a depth of 1 foot, to 18 inches.

The slope of the drains is usually less than $\frac{1}{200000}$, but where the slope of the ground permits, steeper slopes are adopted.

The general conditions are favourable. The soil consists of about 2, to 3



SKETCH NO. 221.—Reclamation of Alkaline Land at Lake Aboukir.

feet of loamy clay, underlain by a stratum of sand, which affords a very efficient subsoil drainage. The plan was successful on the more easily reclaimed portions of the soil, but at the present date (1910), when the last remnants of the area are being reclaimed, a far closer spacing of the minor drains is found requisite, and the average size of a plot may be taken as about 170 feet by 400 feet, instead of 330 feet by 500 feet.

The average quantity of salt removed in the early days was about 8 lbs. per cubic foot of water pumped. This may be considered as very good work. At present, similar details cannot be given; but it is probable that owing to the great amelioration affected in the whole area, not much more than 2 lbs. of salt per cubic foot of water is removed when the quantity which is used for reclamation is alone considered.

The above methods, combined with wider or closer spacing of drains (according to local circumstances), will usually suffice for the reclamation of any soil.

A very fair estimate of the necessary spacing of the drains, together with their depth, can be made by observing the effect of two parallel drains on the slope of the subsoil water between them.

The drains must be sufficiently deep and close together to keep the subsoil water level say 1, or 2 feet below the natural surface at the points farthest removed from them, and when cotton and other delicate crops are put in this depth must be increased to 3 or even 4 feet by deepening the drains or constructing extra drains.

A more rapid reclamation can usually be effected by means of shallow cross drains, which split up the plots into smaller areas. I have noticed that practical agriculturists in Egypt and India are quite capable of making these cross drains themselves; and from their intimate acquaintance with each field they obtain better, and more economical results, than any regular spacing such as an engineer will lay out can give.

The engineer's business may be considered as finished once he has put in the drains shown in Sketch No. 221, and has designed them of such a size that the normal stoppage of the pumps (*e.g.* at night) does not permit the water to rise higher than one foot below ground level. The details depend upon the staple crop, and must be adjusted in accordance with its peculiarities. The best results are of course obtained by first cultivating a crop such as *Panicum crus galli*, or coarse rice, later barley, and waiting some years before deeper rooted crops (such as wheat, or cotton), which are more easily affected by alkali, are sown.

It will be plain that the principles here laid down might be modified. Thus, in some cases, open drains have been replaced by a system of covered tile drains, as used in England for draining agricultural land. A certain economy both in water and time was thus effected at Sakalous (see Barrois, *Irrigations en Egypte*). The method, however, is costly, and in certain cases near Cawnpore ("Private Letter from the Director of Agriculture"), the drains were rapidly choked by particles of the soil, which appears to have been unusually fine in texture.

So also, reclamation has been attempted by flooding large areas, and draining off the water, after a period of ten, or twenty days, when saturated with salt. The process is tedious, costly in water, and the banks are liable to damage by wave action. So far as I am aware, permanent success has never been attained, although the method may prove useful provided that systematic drainage is carried out as soon as crops can be grown on the land.

In all reclamation work on soils of this class the engineer must bear in mind that the land should be handed over to the agriculturist as rapidly as possible. As soon as a crop of even the coarsest quality can be persuaded to grow over the whole surface of a plot, the further progress of reclamation entirely depends on the standard of the cultivation which the land receives. The engineer can damage the land by neglecting the drainage channels, or by giving a bad water supply. But once a plot of land has been reclaimed sufficiently to support a crop which is capable of resisting a little alkali, the most careful drainage and washing will produce but little progress unless supplemented by careful cultivation. The further improvement is apparently entirely due to diminished evaporation produced by the shade afforded by the crop. Such operations as mulching, surface-hoeing, and green manuring, produce rapid improvement, and should be adopted in bad cases, even if not necessary for the crop actually on the ground.

In Egypt and in India little difficulty is experienced in persuading agriculturists to rent and cultivate a plot which has produced a crop of coarse rice, beerseem (Egyptian clover), or *Panicum crus galli.* The leases should of course contain provisions preventing the obstruction of drains, or other interference with the drainage works.

Soils belonging to the second class become alkaline through the salt contained either in the soil, or in the irrigation water, being carried up to the surface of the land, and concentrated there by evaporation.

Although the natural content of alkaline salts in the soil has a certain effect, all water for irrigation contains sufficient salts to render the soil alkaline, if continually applied in excess, and allowed to stagnate on the soil and evaporate. We may therefore consider that if more water is applied to a soil than is required for vegetation, combined with the quantity which can be drained away through the lowest layers of the soil, the soil will sooner or later become alkaline.

Where the soil itself contains an appreciable proportion of alkaline salts, the action will be more rapid. If the soil is naturally very alkaline to start with, one or two seasons of irrigation may finish the process. Over-irrigation will render any soil alkaline in time.

It will be plain that the process is as follows :—Capillary action raises the excess water with its quota of dissolved salts to the surface. Evaporation then removes the water, and the dissolved salts are very rapidly concentrated at the surface, and stop, or impede plant growth.

The best method of preventing such action consists in drainage with a view to removing the excess water. Prevention of evaporation, although of secondary importance, is also very helpful.

The worst damage is caused by the application of a greater quantity of water than can be disposed of by vegetation and percolation. Therefore, accidents apart, the less permeable soils are those which most rapidly become alkaline. For this reason, the reclamation of soils rendered alkaline by over-irrigation, is a far more difficult problem than the reclamation of naturally alkaline land; since drainage difficulties are increased by the greater impermeability of the soil.

Precautions against alkalinity are consequently far more important than the reclamation of land which has become alkaline.

As a rough index, the depth at which the subsoil water level lies below the natural surface of the ground is most easily ascertained ; although, as already stated, this does not allow for the effect of variations in the permeability of the soil. Orders exist in the Punjab to the effect that when the depth of water below soil level is less than 25 feet, only 25 per cent. of the land is to receive canal water each year, the remaining irrigation being effected from wells. When the water is more than 25 feet below soil level, and less than 40 feet, the permissible quota of canal irrigation rises to 40 per cent. When more than 40 feet below soil level, the quota reaches 75 per cent.

I am not aware that these rules are ever enforced, and it may be stated that any rigid insistence would be inadvisable. They may be taken as representing the percentage of area which it is desirable to irrigate when canal water is first introduced. Even so, it must be understood that the limitations apply to very large areas. Consequently, zones where the water is less than 25 feet below soil level, at the initiation of canal irrigation, may be considered as exposed to the danger of waterlogging not only by local irrigation, but also by percolation from large areas higher up the canal, where the subsoil water is more than 25 feet below the ground level.

As a matter of fact, good crops are raised, and irrigation up to 75 per cent., and even 110 per cent. of the area of the land (allowing for double crops) goes on in places where the subsoil water level is only 6 feet below soil level, provided that :

(*a*) There is a good drainage, provided by natural stream channels or artificial drains.

(*b*) This intense over-irrigation, and high subsoil water, are local, and extend only over a small area, so that the subsoil water slopes are steep, and subsoil drainage is therefore good.

It can generally be stated that such limited irrigation as is indicated above, under Indian conditions (*i.e.* corresponding to 3·1, 2·4, and 1·6 cusecs per 1000 acres of gross area according to the depth to subsoil water) does not give rise to any very marked and continuous increase of the subsoil water level year after year. The alkaline soil question in the Punjab is (comparatively speaking) a very minor one, thanks not so much to the limitations of the percentage of the area irrigated, as to the economy in water exercised by the irrigators. While local limitations are badly required, no universal waterlogging has occurred when the water is supplied in quantities equal to those stated above, even although occasionally, and on isolated estates, a larger quantity has been allowed, and the subsoil level is close to the natural surface.

The question is far more acute in Egypt. Here, in place of quantities of water as above mentioned, the supply for "dry" crops (corresponding to Indian conditions), is about 8 cusecs per 1000 acres ; and for rice about 16 cusecs per 1000 acres. The slope of the country is much flatter, and, in consequence, the natural drainage channels which generally suffice in the Punjab are inadequate, and artificial drains with a capacity of 3, to 5, cusecs per 1000 acres have to be constructed.

Taking the case of the early American irrigators, who used 25, to 30 cusecs per 1000 acres, it is not surprising to find that virgin land was very rapidly rendered alkaline. Two years usually sufficed in unfavourable cases, and ten, to twelve years was almost the maximum period of fertility. On the other hand, some of the very carefully irrigated orchards of California, although undrained, and lying in hollows, such that the subsoil water is but 8, or 9 feet below the natural surface, show no signs of alkalinity, in spite of this accumulation of unfavourable circumstances. Salvation here is probably entirely due to the lining of the channels (see p. 740).

The principles are therefore obvious. When the first signs of alkalinity appear, a drainage system should be laid out, and should be extended as necessary. Every possible means for securing economy in water used for irrigation should also be employed.

As a matter of practice, when land has once become thoroughly alkaline through over-irrigation, it is usually found less costly to abandon the land and irrigate new tracts. This is not a desirable solution of the difficulty, but the statement serves to emphasise the importance of taking preventive measures as soon as any alkalinity becomes apparent.

There is a fair amount of evidence to show that if land becomes alkaline, and is at once reclaimed by washing, it will be found to have deteriorated in fertility, the salts essential for plant life having apparently been removed with the alkali. Thus, in Egypt, such lands are often treated by warping with silt, in the hope of restoring their fertility by the manurial value of the silt.

Lands which have been alkaline for a long period, and are then reclaimed, do not appear to be thus prejudicially affected to any very marked degree.

This is not easy to explain, as very little is known about the chemistry of such soils, but the matter must be borne in mind when such questions are dealt with.

SILT.—The term silt is a convenient expression for all non-floating substances carried forward by water flowing in rivers and canals, and may be contrasted with drift, which includes all floating substances.

From an engineer's point of view, two classes of silt exist, in all rivers which carry silt. These are:

(i) Bed silt, which is rolled along the bottom of the river, rather than carried forward.

(ii) Lighter particles of silt, which are carried forward in suspension in the water, as turbid matter, or suspended silt.

If the journeyings of individual particles could be traced, it would probably be found that there were very few which were not sometimes included in the bed silt, and at other periods were suspended in the water; yet, the general characteristics of a sample of bed and suspended silt are widely different.

The factor which determines whether a particle performs the greater part of its travels in the form of bed or suspended silt is evidently the relation between its size and the velocity of the water near the bottom of the channel. If this velocity (as discussed on p. 488) is large enough to lift and suspend the particle, it will (except for certain short periods between jumps) be lifted off the bottom of the channel and will be carried forward. Whereas, a somewhat larger particle will be rolled forward as described in the above reference. Hence, the larger and coarser particles of the silt form the bed silt, and the finer particles are carried forward in suspension. Consequently, bed silt is coarser and more gritty than suspended silt, and may be said to be a sandy material; while the suspended silt is (comparatively speaking) clayey. Rivers exist where the term gravelly might be substituted for sandy, and then of course the suspended silt would be mainly sandy in nature; but the relative distinction remains, and once this distinction has been pointed out, there is very little doubt as to the class to which a deposit should be assigned.

The term "rolling," can hardly be considered to correctly describe the manner in which bed silt travels. When a channel is examined which is heavily charged with silt, it will be found that the bottom usually forms a series of steps rising about 1′ in 20′ and then dropping on the downstream side at 1 : 1, much like a flat sand dune. It would therefore appear that the motion of the particles of bed silt is probably as follows; the entire upper layer moves up the long slope, and then falls down to rest at the bottom of the short slope, so that each wave moves forward by means of an internal motion of particles from back to front of the wave, in a manner similar to sand dunes under the influence of air currents.

The silt content of the water of a river is produced by the erosion of the catchment area by the river and its tributaries. In a canal or irrigation channel, which does not receive small tributaries, it will be plain that the silt content is entirely due to the two following sources:

(*a*) Silt derived from the river or reservoir supplying the canal; which I shall call "original silt."

(*b*) Silt which the water has eroded from the banks or bed while in the canal; which I shall call "derived silt."

Precautions against silt troubles consequently have the two following objects:

(i) To pass forward or remove the original silt contained in the water as drawn from the source of supply of the canal.

(ii) To prevent the occurrence of derived silt, by stopping erosion of the bed or banks of the canal.

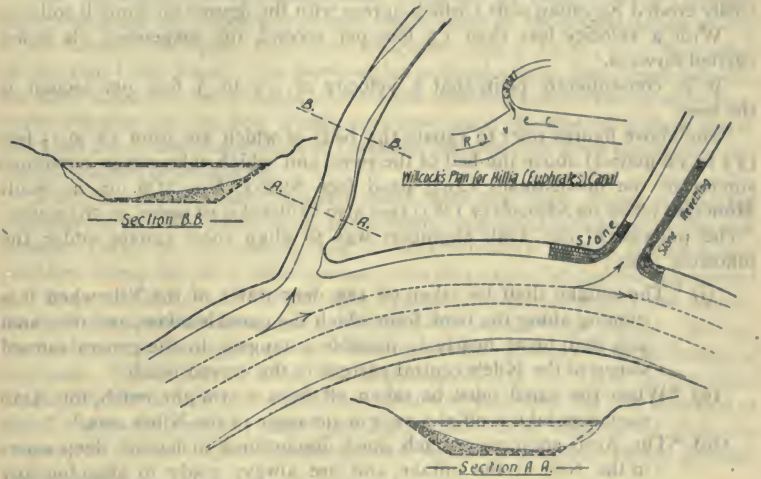
If the headworks of the canal are properly designed, the original silt will mainly consist of suspended silt. A considerable proportion of the small amount of bed silt which is taken into the canal may be removed by silt traps, or escapes. On the other hand, derived silt contains a larger proportion of bed silt; and, since erosion may occur at any point, it is difficult to provide effective silt traps and escapes. Consequently, the prevention of erosion is very important, since, so far as my experience goes, bed silt invariably gives trouble. Suspended silt, on the other hand, may be regarded as a favourable factor, as it rarely produces trouble, and can be caused to deposit itself so as to diminish leakage and strengthen weak places in the canal banks.

It is well known that erosion occurs if water travels at velocities which exceed certain limits. It is quite impossible that such erosion should continually prevail in an earthen channel, as it would ultimately destroy the whole channel. Thus, the erosion which is detrimental to the maintenance of earthen channels is accomplished at velocities which are less than the limits spoken of on page 495, and is principally conditioned not by the absolute magnitude of the velocities, but by irregularities and sudden changes in velocity. It is therefore extremely important that the channels should be uniformly graded, and that they should be correctly proportioned to the quantity of water which they are required to carry. The systematic design of a channel on these lines is detailed on page 733. It may be assumed that such a channel (provided that it is not required to carry quantities varying greatly from those for which it is designed) will, in virtue of its correct proportioning, be liable to but slight erosion.

The principle is best illustrated by the case of the Ibrahimiah Canal. In former years this canal silted very badly. The silt was not originally contained in the water as it entered the canal from the Nile during the low water season, but was derived silt, produced by the erosion of the banks in the upper length of the canal where the floods of the Nile annually deposited a fresh supply. The annual quantity of silt removed has now been reduced from 600,000 cubic metres, to 100,000 cubic metres, by the construction of stone groynes which prevent the high flood waters from eroding the banks to a section which would be too large for the normal discharge of the canal during the low water periods.

So also, Sketch No. 222 (adapted from Buckley's *Irrigation Works of India*) shows upstream, a canal head taking off at too large an angle from the river. Consequently, the flow of the canal is set oscillating from bank to bank, and alternate patches of scour and silt (analogous to the deeps and shallows of a natural river) occur. In the downstream canal head, the water enters the canal in a fair curve, and the revetment of stones round the head guides the water in so regular a fashion that scour below the revetment is improbable. The flow remains tranquil; and, as a result, the banks are not attacked.

The large scale plan of the Rupar Headworks (see Sketches Nos. 163 and 200) illustrates a somewhat similar theory. In this case the head reach is actually located so as to be directed somewhat upstream. An even more marked upstream direction has been proposed by Willcocks for the head reach of the Hindan (Mesopotamia) Canal (see *Engineering*, May 31, 1910). The advantages gained by designs such as these last two are somewhat doubtful. The Sirhind Canal silted badly (see p. 659). The principle should be kept in mind, especially when an inundation canal, or a canal unprovided with a



SKETCH NO. 222.—Location of Canal Heads.

"raised sill" head regulator is being designed, but it is probably inadvisable to adopt such locations if they entail deep cuttings.

If a river carries but little silt, and that in a suspended state, and if the bed of the canal is also well above the bed of the river, such precautions combined with careful proportioning of the cross-sections of the individual reaches of the canal will banish silt entirely. Thus, in cases where the initial content is not large, and erosion does not occur, the rules are simple, and are as follows:

(i) The mean velocity must not exceed a certain value; which in the case of the most easily eroded alluvial soil, may be taken as 3·30 feet per second, and for more tenacious soils rises to 3·50, 3·70, or even 4·00 feet per second.

(ii) The mean velocity must not fall below certain other values, which are those necessary to carry forward the major portion of the silt found in the river. As preliminary rules we may say that the canal should carry all the suspended silt, and about three-quarters of the bed silt found in the river, the proportion obviously depending on the care taken in locating the head reach.

For example, Willcocks (*Egyptian Irrigation*) gives the following rules for Egyptian canals, the headworks of which are located in accordance with the principles now discussed.

Mean Velocities.

2·3 to 3·3 feet per second (0·70 to 1 metre per second):—No silt is deposited.

1·97 feet per second (0·60 metre per second):—0·5 metre depth of silt is deposited, *i.e.* about one-sixth or one-eighth of the area of the canal section is silted up during the flood season.

1·64 feet per second (0·50 metre per second):—1 metre of silt is deposited.

1·33 feet per second (0·40 metre per second):—mud is deposited.

Hence, a velocity exceeding 3·3 feet per second produces erosion of the most easily eroded Egyptian soils; which agrees with the figures for Punjab soils.

With a velocity less than 1·3 foot per second, the suspended silt is not carried forward.

It is consequently plain that a velocity of 2·5 to 3 feet per second is the best.

The above figures refer to canals the beds of which are from 12 to 15 feet (3·5 to 4·6 metres) above the bed of the river, and which take out in situations similar to the downstream canal head (see Sketch No. 222), or, as Scott Moncrieff (*Note on Sharaki in* 1882) (see also Willcocks, *ut supra*, p. 76) states: "The practice of the Arab engineers was to align their canals under the following rules:

(i) "The offtake shall be taken off the deep water of the Nile when it is running along the bank from which the canal is taken, and the canal axis shall be as nearly as possible a tangent to the general curved sweep of the Nile's central current in the curved reach."

(ii) "When the canal must be taken off from a straight reach, the Arab engineers take it off at a very acute angle to the Nile's axis."

(iii) "The Arab engineers attach much importance to having deep water in the Nile at the offtake, and are always ready to abandon any canal head that takes off at a point in the Nile where a sand bank is forming. I consider that they find from experience that coarse sand which rolls along the bottom of the Nile bed does not enter the canal unless the Nile bed has become silted up by a sand bank to nearly the level of the canal bed."

The above rules can be summed up as follows:

(i) Keep the Nile bed silt out of the canal.

(ii) Prevent the formation of bed silt in the canal by stopping erosion of the banks either by groynes, or by pitching; or better still, by keeping the mean velocities sufficiently low to prevent erosion, and designing the canals so that sudden changes in velocity which produce eddies do not occur.

The bed silt being but a minor factor, the rules for the lower limit of the velocity can be summed up in the single statement:

(iii) Keep the velocity sufficiently high so as to move forward the suspended silt, together with the little bed silt which has entered.

The conditions are somewhat different in the Punjab. At least three out of the six rivers are of the same order of magnitude as the Nile, but their bed slopes near the canal headworks are from $\frac{1}{2000}$, to $\frac{1}{4000}$, in place of $\frac{1}{11000}$, in the Nile. The rivers are consequently shallower. Thus, in spite of all the provisions for regulating the rivers, and skimming the clearer water (see p. 660), it is found impossible to prevent a certain amount of the river bed silt from entering the canal. In fact, personal observations lead me to believe that every Punjab canal receives initial silt in quantities which are comparable to those occurring in a very badly situated canal on the Nile, where Scott Moncrieff's condition No. (iii) is entirely disregarded. In such cases, the bed silt is the most important factor, and merely to prevent erosion in the canal itself is not sufficient to obviate silting. We must so proportion the canal that the initial bed silt is not deposited, but is passed on to the fields.

Our knowledge of the precautions necessary in order to keep channels fairly free from silt deposit is almost entirely due to the investigations of Mr. R. G. Kennedy (*Graphic Hydraulic Diagrams*, and *P.I.C.E.*, vol. 119, p. 282) on the Bari Doab Canal. As, however, his principles have been found valuable on other canals in the Punjab, where local conditions differ very greatly from those existing on the Bari Doab, and have stood the test of sixteen years' practical use, it seems advisable to carefully describe both these conditions, and Mr. Kennedy's observations.

The Bari Doab Canal receives water from the Ravi River, a snow-fed Himalayan stream, which is in flood during the greater portion of the hot weather (May to September), and is then intensely turbid, rolling heavy boulders along its bed ; while surface velocities exceeding 20 feet per second are frequently observed. In the cold weather (October to April), the river,—except for freshets of infrequent occurrence,—is a clear water stream. During the hot weather the supply of water in the river far exceeds the requirements of the canal. Nevertheless, irrigation continues during the whole of the flood season, and it is only rarely that any closure of the canal for the rejection of silted water is possible ; whilst the headworks are so constructed that the engineer in charge has but little chance to select the less heavily loaded portions of the water.

We are therefore entitled to assume that the Bari Doab Canal is fed for six months with water which is more heavily charged with silt than that which is usually admitted into Punjab canals. On the other hand, a clear water season lasting six months, is unusual on other Punjab canals.

It is, however, very probable that the average (over the whole year) volume of silt per unit volume of water is very much the same as it would be were the headworks of the Bari Doab (like those of most other Punjab canals) situated somewhat lower down the river, where the stream is less torrential in régime. It would then be possible to reject a certain quantity of the more heavily silted water during the hot weather, as the relatively less torrential river would be under better control. On the other hand, during the cold weather the river would carry more silt than higher up its course at the same season, as the originally clear water of the cold weather season picks up the silt which is deposited in the intermediate course during the hot weather floods.

Mr. Kennedy's observations were mostly made on branches and minor distributaries of the Bari Doab, situated at least 50 miles below the headworks. The turbid and clear water periods are by no means clearly defined in these cases, as some silt is deposited in the intermediate channels during each period of turbid water, and is picked up and swept forward during the period of clear water. In a general sense the action is similar to that which has been described as occurring in the natural river channels of the Punjab.

Records of the character of the water do not exist; but, judging from my own observations (taken after Mr. Kennedy's principles had been systematically applied), it may be assumed that the period of turbid water in these channels lasted some eight months, and was followed by a period of four months of more or less clear water. The nett effect, therefore, was to produce in every channel under Mr. Kennedy's control, a period of about eight months during which silt was deposited in the channels, and a succeeding period of approximately four months during which the channels were scoured. The circumstances were consequently very similar to those prevailing close to the headworks in other Punjab canals.

In most cases, the result was a constant increase in silt deposits, and had not the channels been cleared each year, they would have been more or less rapidly silted up.

Mr. Kennedy discovered twenty-two channels in which, year in, year out, the deposits during the silting period were scoured out, and were swept forward during the corresponding clear water period; and as a nett result these twenty-two channels were unaffected by silt deposits; eight other cases in which the nett silting or scour was small were also found. After very exhaustive investigation of the whole circumstances, Mr. Kennedy concluded that these channels were characterised by the fact that the mean velocity of the water bore a certain relation to the depth. Hence, he deduced the following law:

$$v_0 = 0{\cdot}84 d^{0{\cdot}64}$$

Whence, if

$d=$	1	2	3	4	5	6	7	8	9	10	12 feet.
$v_0=$	0·84	1·30	1·70	2·04	2·35	2·64	2·92	3·18	3·43	3·67	4·12 feet per second.

Where: v_0 = the mean velocity in feet per second in a channel that neither silts nor scours (a Kennedy channel) (see p. 727).
d = the depth of channel in feet.

Mr. Bellasis has observed that the expression

$$v_0 = 1{\cdot}05\sqrt{d}$$

represents the actual observations nearly equally well.

This description of Mr. Kennedy's observations is purposely intended to disclose the apparently slight foundations of the rule. When the rule became well known, they were systematically applied to determine the form of the cross-section of the channels on the whole of the Lower Chenab Canal (approximately 2,000,000 acres of irrigated land), and to a large portion of the Lower Jhelum Canal (approximately 600,000 acres of irrigated land). The conditions affecting the silt deposits in these two canals differ greatly from those which prevail on the Bari Doab.

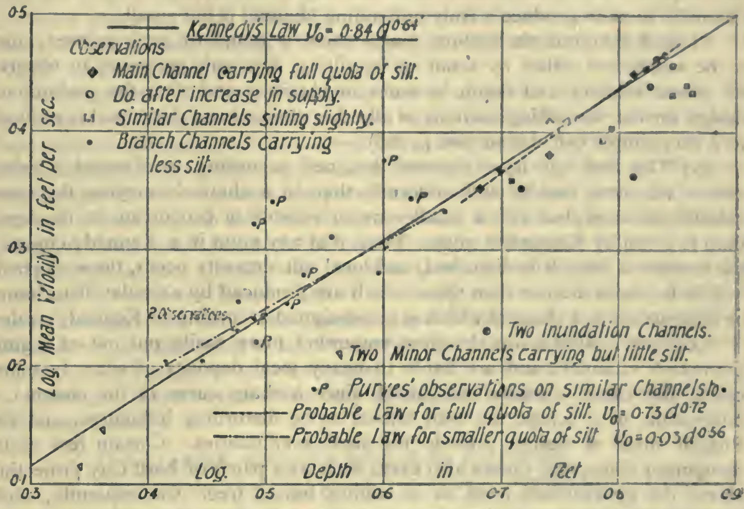
The Jhelum silt is finer than that of the Bari Doab, but the clear water periods rarely last more than a month.

The Chenab silt is somewhat coarser than that of the Bari Doab, but the clear water period lasts about two months.

The general effect of adopting Mr. Kennedy's rules has been a large decrease in silt clearances on all three canals. A similar decrease in silt clearances has also been effected on other Punjab canals.

These rules have been thoroughly tested, as they have formed the basis for nearly all remodellings of the Punjab canals for the last eight years. Following the usual custom of a Government service, it may be said that they are now applied somewhat blindly. While this method is not so advantageous to the interests of the Government as a more elastic system, it has at any rate secured a very interesting large scale test of the laws affecting such channels.

A consideration of all the information which I have been able to obtain, combined with the detailed results of my own experience in preventing silt



SKETCH No. 223.—Kennedy's Observations.

deposits in about 200 miles of small channels (which had been designed with no attention to Kennedy's rules), has led to the following conclusions :

(i) The duration of the clear and turbid water periods has but little influence on silt deposits when the rules are properly applied, and a channel can be designed which will carry turbid water the whole year round, without silting.

(ii) On the Punjab canals, bed silt is in motion all the year round, whether the water is clear, or turbid. While the motion is more rapid in periods of clear water, there is little, if any, difference in the volume carried forward. Probably, therefore, in periods of turbid water, the bed silt is in motion over a greater depth.

(iii) Mr Kennedy's rule, that the least velocity at which silt is not deposited is given by the equation :

$$v_0 = cd^{0.64},$$

is correct for all classes of silt that occur in the Punjab.

The value $c = 0.84$, however, is peculiar to the Bari Doab Canal ; and slight, but quite noticeable differences may be observed not only on the different canals of the Punjab, but also on the various reaches of the same canal as we go down the canal. If a sample of silt is collected from the bottom of a canal, and is tested by the grading tube described on page 758, it will be found that c, is very approximately proportional to the percentage of silt of which the velocity of fall in still water is greater than 0·10 foot per second. The value $c=0·84$ corresponds very fairly well with a silt in which 40 per cent. of the grains fall more rapidly than 0·10 foot per second. This last rule is only approximate, but the differences which I have observed (although quite apparent in careful experiments), are never sufficiently great to cause a channel designed by these rules to silt up to such an extent that any noticeable deposit is produced in a period of one year. The further clearance necessary in the second year to shape the channels so as to produce a truly non-silting channel is but small.

Thus, if even one channel on a new canal is found to work properly, and to be unaffected either by scour or by silting, it is only necessary to observe its mean velocity and depth in order to obtain c, and then the preliminary design for the non-silting sections of all other channels on that canal is reduced to a very simple calculation (see p. 768).

(iv) The bed silt in a channel designed according to Kennedy's rules, moves far more rapidly and uniformly than in a channel carrying the same quantity of water, but with a smaller mean velocity in proportion to its depth than is given by Kennedy's rules. Thus, if at any point in a Kennedy channel, the motion of the silt is disturbed, and local silt deposits occur, these deposits will be far more intense than those which are produced by a similar disturbance or obstruction in a channel which is not designed according to Kennedy's rules.

Kennedy channels are therefore somewhat more easily put out of régime than other channels, and are liable to heavy local deposits of silt. In many cases, the cause is easily recognised, since a sharp curve in the channel, a bifurcation, or a bridge, are well known to be disturbing influences, and the Punjab rules of design provide for such eventualities. Certain less easily recognised disturbing causes also exist, such as a patch of hard clay projecting above the general bed level, or an almost buried tree. Consequently, while the volume of silt that is deposited in a Kennedy channel is relatively small, the channel must be very carefully inspected, and Kennedy's rules are often considered useless by engineers whose ideas of careful inspection are derived from experience of ordinary channels.

I must remark that many engineers in the Punjab who have given quite as much consideration to the question, and who possess greater experience than myself, disagree with me as to rule (i), while other engineers may consider rule (iii) as an unnecessary refinement. These gentlemen, however, usually have experience of only one canal, and very few of them have enjoyed the opportunity for systematic observation which was my lot for nearly three years.

It appears to me that the application of Kennedy's principles to all canals carrying silted water must become universal, and it seems necessary to explain why they have not as yet been discovered and applied outside the Punjab.

In the first place, it must be recognised that Kennedy's rules do not in any way minimise silt deposits, but rather the reverse: they merely alter the place where the deposits occur. The silt is no longer dropped in the channels, and removed during the yearly clearance, but is carried forward and deposited on the fields, or in the small field watercourses. This is no doubt a far easier place to deal with silt; and under the conditions obtaining in the Punjab, the result is to shift the labour of silt removal from the staff responsible for the maintenance of the canals, on to the agriculturists, who find that their water-courses silt up more rapidly than was previously the case.

The silt of the Punjab canals frequently possesses fertilising properties, and is very rarely absolutely injurious to crops. Thus, the Punjab agriculturist is well content to undertake the extra labour entailed, fully realising that he thereby obtains a more certain supply of water.

In Southern India, however, the silt is of a more sandy nature, and the general size of the grains is far greater than in the Punjab. No agriculturist can therefore be expected to view the prospect of the continual deposition of such silt upon his fields with equanimity. Consequently, in Southern India and in similar cases (*e.g.* most of the United States' silt-bearing waters, other than those of the Colorado River), a servile application of methods adopted in the Punjab would be inadvisable.

Nevertheless, I believe that many advantages can be obtained by the application of Kennedy's principles so as to secure that the silt is mainly deposited in selected channels where land suitable for the disposal of such deposits is available. Such selected channels might be regarded as silt traps, and could either be systematically and continually cleared, or could be constructed in duplicate, so that the channels could be alternately closed off and cleared without interfering with the regular supply of water.

In Egypt, as has already been explained, owing to the flatter slope of the river, and its relatively deeper bed, it is usually possible to exclude nearly all the troublesome bed silt. The general slope of the land is also so flat that, if bed silt enters the canal in large quantities, it is almost impossible to obtain a mean velocity which is sufficiently great to carry it forward.

General Principles.—We may divide silt-bearing rivers into the two following classes:

(*a*) The Egyptian, where the slope of the river is flat.
(*b*) The Punjab, where the slope is (comparatively speaking) steep.

The dividing line may be roughly taken as slopes which are flatter or steeper than $\frac{1}{7500}$.

In the first case, coarse silt (*i.e.* silt which falls in still water at a velocity greater than 0·10 foot per second), is not very abundant. It will be found that it is usually possible to prevent any large quantity from entering the canal. Such coarse silt as enters the canal will usually (the canal being obviously graded at a slope which is at most only one-half that of the river, say at the steepest $\frac{1}{10000}$), be deposited close to the head, and can be removed. The canal water being thus deprived of the initial silt, any further silting is entirely prevented, provided that erosion does not occur.

The Egyptain rules have already been given.

So far as I am aware, the figures for other localities of a similar character, differ very slightly from those for Egypt, but an observation of existing channels will easily disclose any small differences.

The second, or Punjab class, can be recognised by the fact that samples of silt taken from deep channels of the river contain an appreciable percentage (say over 20 per cent.) of grains which fall in still water with a velocity exceeding 0·10 foot per second. The slope of the river varies from $\frac{1}{2000}$, to $\frac{1}{8000}$, larger percentages of coarse silt usually occurring in those rivers which possess the steeper slopes. It is then impossible to prevent an appreciable quantity of coarse silt from entering the canal. The slope of the canal being about one-half that of the river, it will be possible to pass this silt forward to the fields, provided that the canal is proportioned according to Kennedy's law. Consequently, the important factor is the value of c, in the equation:

$$v_0 = cd^{0.64}$$

This is best ascertained by a direct observation of the mean velocity and depth of a channel which is known not to silt. In default of such experience, samples of bed silt may be taken, and the percentage of coarse silt, as above defined, can be observed, and c, can be calculated by the rule:

$$c = \frac{0{\cdot}84p}{40}$$

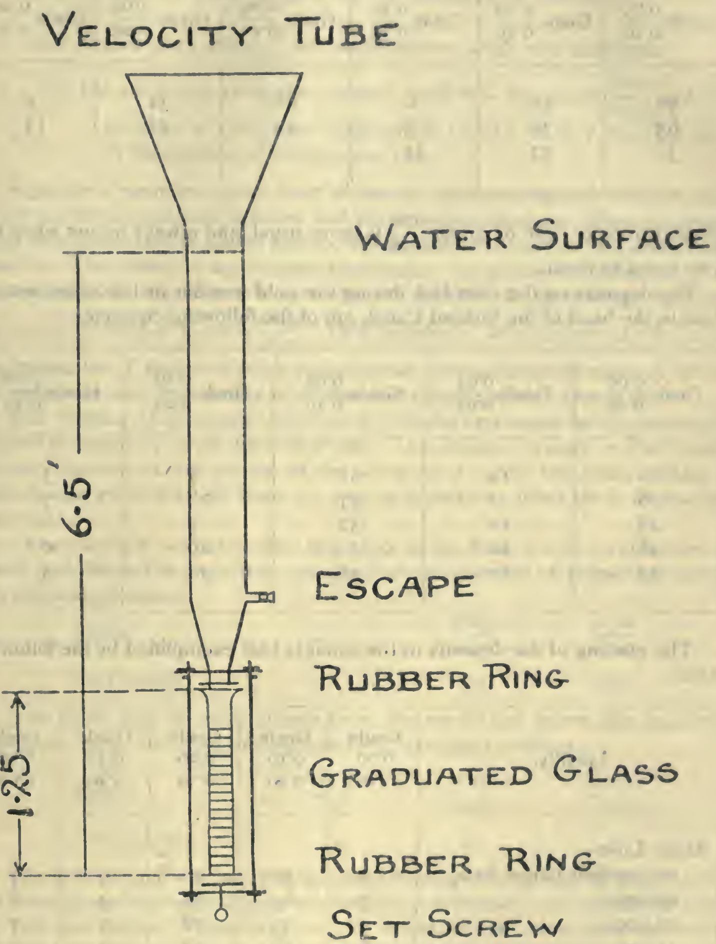
where p, is the percentage of coarse silt (see p. 768).

Either method may lead to erroneous results, for the following reasons. A channel which does not silt may be so situated that the conditions are unusually favourable (*e.g.* its head may be very well placed on a deep channel of the river), and may carry less silt than is the case with the channels of the proposed system. Thus, the silt deposited in the bed of such a channel should be carefully classed and compared with the silt which is normally found in the rest of the channels which take out from the river. The rule $c = \frac{0{\cdot}84p}{40}$ is founded solely upon experience gained in the Punjab, and may be too high for rivers in which the average content of silt per unit volume of water is less than that in the Punjab rivers, or too low for rivers which carry a larger proportion of silt. My own experience, however, leads me to believe that if Kennedy's principles are intelligently followed, the first designs for the cross-section of a channel will be so close to the required form that the silt deposit during the first year will give but little trouble. Consequently, the true form can be obtained, and the channel can be constructed during the clearances which are necessary at the end of each season owing to the growth of weeds.

Grading of Silt.—Sketch No. 224 shows the silt classifier used in the Punjab (see *Punjab Irrigation Branch Papers*, No. 9). The glass is graduated into divisions, having a volume of 0·001 cubic foot. One-tenth of a cubic foot of silt is thrown into the water, and the intervals are noted at which the falling sand fills the tube up to the first, second, third, etc. marks. Let us assume that the first grains reach the bottom of the tube in 26 seconds, and that the tube is filled to mark No. 10 in 65 seconds. Then, the heaviest grains fall 6·5 feet in 26 seconds, or, have a velocity of 0·25 foot per second, and 10 per cent. by volume of the individual grains fall with a velocity greater than 0·10 foot per second. This fact is expressed by the notation 10 per cent. of the silt is of the grade $\frac{0{\cdot}10}{0{\cdot}25}$.

The apparatus is simple, and for that very reason the results obtained by its use do not possess the accuracy that can be obtained by the use of upward flow graders. The observations, however, can be taken by comparatively unskilled men, and the apparatus consequently deserves to be used in practice. It will be seen that by a systematic use of such an apparatus the engineer may expect to improve his designs. In our present state of ignorance concerning silt deposits any further refinements are useless, and even detrimental if their adoption decreases the number of observations.

The following figures are taken from the above report on the River Sutlej



SKETCH No. 224.—Silt Classifier.

and the Sirhind Canal, which is fed from this river, but they may be considered as typical of all Punjab rivers.

By actual experiment it is found that suspended silt (even in a canal which is silting heavily) rarely contains any grains that fall with a greater velocity than 0·20 foot per second ; and, for the most part, the velocity of fall is less than 0·10 foot per second. The following gradings occur in the Punjab rivers when in flood :

PERCENTAGES.

Grade $\frac{0·05}{0·10}$	Grade $\frac{0·10}{0·20}$	Grade $\frac{0·20}{0·30}$	Grade $\frac{0·05}{0·10}$	Grade $\frac{0·10}{0·20}$	Grade $\frac{0·20}{0·30}$
50	50	0	66	34	0
65	30	5	44	42	14
38	37	25	...	...	...

although 100 per cent. of grade $\frac{0·05}{0·10}$ is more usual, and always occurs when the river is not in flood.

The deposits on the river bed during the cold weather or low water season, close to the head of the Sirhind Canal, are of the following character :

Grade $\frac{0·00}{0·03}$	Grade $\frac{0·03}{0·05}$	Grade $\frac{0·05}{0·10}$	Grade $\frac{0·10}{0·20}$	Grade $\frac{0·20}{0·30}$
26	14	50	7	3
8	2	57	28	5
48	10	37	5	0
48	10	32	10	0
30	10	46	11	3

The grading of the deposits in the canal is best exemplified by the following table :

Locality.	Grade $\frac{0·00}{0·10}$	Grade $\frac{0·10}{0·20}$	Grade $\frac{0·20}{0·30}$	Grade $\frac{0·30}{0·40}$	Grade $\frac{0·40}{0·60}$
Main Line—					
10,000 feet below head .	4	50	30	9	7
15,000 ,, ,, .	7	65	21	4	3
20,000 ,, ,, .	5	53	28	7	7
25,000 ,, ,, .	6	52	28	7	7
Branch Line—					
150,000 feet below head	28	56	12	4	0
170,000 ,, ,, .	29	57	11	3	0

The above figures are confirmed by many other results. A certain proportion of the grains are of local origin, being derived from the adjacent banks and bed of the canal, which was silting badly at the time at which these observations were taken.

Hence, the following points are fairly obvious :

(*a*) Grains of a grade $\frac{0\cdot00}{0\cdot10}$ are rapidly moved along the canal.

(*b*) Grains of a grade $\frac{0\cdot30}{0\cdot60}$ are hardly moved at all, and therefore :

(*c*) The silt which gives the greatest trouble is the grade $\frac{0\cdot10}{0\cdot30}$, and the quantity of this grade entering the canal is a very close measure of the amount of troublesome silt.

A study of samples taken from Kennedy channels suggests that, in such channels the demarcation between bed silt and suspended silt differs somewhat from that given by the above figures, which refer to a canal which was silting heavily. The matter is not of great importance, but my own experiments show that some Kennedy channels pass forward appreciable percentages (5 to 10 per cent.) of sand of a grade $\frac{0\cdot15}{0\cdot25}$. This property is advantageous to the canal engineers, but I doubt whether it will prove permanently satisfactory to the agriculturists who have finally to receive and dispose of such coarse silt.

Silt Traps.—In all canals it is found that the first reach below the head is subject to relatively large deposits of silt. The cause is obvious :—The change of the direction of the motion of the water as it enters the canal creates a disturbance which lifts silt from the river bed, and this lifted silt is drawn into the canal.

As an example :—On the 28th July 1896, in the River Sutlej, 10 cubic feet of water just below the regulator contained 0·002 cubic foot of suspended silt, of the following grades :

Grade	$\frac{0\cdot05}{0\cdot10}$	$\frac{0\cdot10}{0\cdot20}$
Percentage . . .	50	50

Ten cubic feet of water drawn from the canal just below the regulator contained 0·004 cubic foot of suspended silt, grading as follows :

Grade . . .	$\frac{0\cdot05}{0\cdot10}$	$\frac{0\cdot10}{0\cdot20}$	$\frac{0.20}{0\cdot30}$
Percentage .	74	21	5

The first sample was taken from water flowing at a velocity of about 3 feet per second, and the second from water which was flowing at a velocity of about 2·5 feet per second. Thus, a priori, we might expect the second sample to contain a smaller quantity and a finer quality of silt. The difference is due to the fact that the water in the second sample had been passed through the openings of the canal regulator, and had there attained a velocity of 3·5 to 4 feet per second. This, *per se*, would not entirely explain the difference, but in addition this increase and decrease in velocity had produced swirls and vortices in the water, so that silt was picked up from the bed of the river in front of the regulator.

The action can be observed at and around any bridge pier. The nett result is that the first sample in reality represents the suspended silt in the river, while the second sample represents a mixture of the suspended silt and the bed silt in the river. Consequently, the vital importance of preventing the river bed silt from ever approaching the canal head is obvious; and it should be remembered that these observations were obtained when the water was drawn in over a sill raised 3 feet above the river bed.

The nett result, therefore, is that before the water in the canal can have the same silt content as the river, each 10 cubic feet of water must drop about 0·002 cubic foot of silt of the grade $\frac{0·05}{0·10}$, and about 0·0002 cubic foot of the grade $\frac{0·20}{0·30}$. Similar figures could be obtained for all rivers, and the above example is by no means an uncommon one; although the extra amount of silt is probably greater than that which is found in the more favourably situated canals.

Putting aside the temporary extra loading of silt caused by the disturbance produced by the change in direction which occurs at the canal head (and also by the regulator if one exists) it is probable that most canals are not capable of carrying so large a proportion of silt as the river from which they take out. In the head reach of the canal the quantity of silt in the water is changed from that which prevails in the river, to that which the slope and dimensions of the canal permit it to carry. Thus, not only is all the silt which has been temporarily picked up dropped in the head reach of the canal, but also a certain portion of the silt which is normally carried forward by the water of the river.

The quantities of silt thus deposited may be enormous. For example, in the month of July 1896, the deposition in the first 14 miles of the Sirhind Canal was at the rate of 0·55 cubic foot of silt per 1000 cubic feet of water passed down the canal, and far larger figures occur (see *Punjab Irrigation Papers*, No. 9). It will, however, be found in every case that a well defined distance from the canal head exists beyond which this silt of adjustment is not at first deposited in marked quantities. I say at first,—because, if the silt is permitted to accumulate at the head of the canal, sooner or later this head deposit will begin to move down the canal. I am not therefore definitely prepared to state whether the limited length of canal inside which the main deposit occurs is caused by some physical law of silt deposit (*e.g.* is determined by the relation existing between the mean velocity of water in the canal, and the rate at which silt particles fall in water moving with this velocity) or, is in reality dependent upon the length of time during which the silt has been accumulating, and is therefore essentially an artificial matter, more or less under human control. Personally, I incline to the former view, for in cases where the canal has been neglected and more than the normal quantity of silt is deposited, the deposit of silt does not usually extend farther down the line; but the whole mass moves bodily forward, and a wave of deposited silt travels slowly down the canal.

This observation, however, cannot be regarded as conclusive; since, when such waves are observed, there has usually been a change in the method of drawing water into the canal, and it is uncertain whether the wave is not caused by the endeavours to clear the canal.

The distance within which the major portion of the silt is deposited is of great practical importance, since this knowledge enables us to fix the most suitable sites for silt traps and scouring escapes. I regret that I am unable to give any definite rules for determining the distance. In rivers which carry so fine a silt as that which is found in the canals of the Punjab, it is measured in miles; being about 14 miles in the case of the Sirhind, and about 10 in the case of the Jhelum Canal. While, in rivers carrying coarser sand, as in Madras, it appears to be about four miles; and in the case of gravel or boulders, about half a mile.

Broadly speaking, the finer the silt, and the wider the canal, the greater is the distance, and the above figures refer to canals 100 to 200 feet wide.

In certain cases, as in inundation canals, or flood water canals, and in rivers such as the Nile, where the canal head has no regulator to set up disturbances in the water, and where the first reach of the canal takes off nicely from the river (so that no unnecessary disturbance occurs), and above all where the bed of the canal is situated well above the river bed, it is possible to prevent a large quantity of harmful silt from entering the canal. In these cases, the canals usually run dry after the flood is over, and the small deposit of silt that has occurred can be dug out. Usually, however, some, or all of the three conditions given above must be violated, and disturbances occur in the motion of the water. Silt is then inevitably drawn into the canal, and the means of removing it, or minimising its effects must be considered.

The most efficient method of clearing away this deposit of "adjustment" silt, is to admit (in seasons when the river water carries less silt than is usual) water in excess of that which is required for irrigation, and to pass off this excess by means of a special escape. The effect is obvious:—The cleaner water, which moves at a higher velocity than usual, picks up the bed silt; and this temporarily suspended matter is carried away with the escaping water (see p. 702).

The details of the process require some consideration. In the first place, it is the bed silt, or rolling silt, that causes such deposits. Thus, although a very turbid water usually carries a large proportion of bed silt, the turbidity of water as judged by the eye is by no means an infallible index of the quantity of bed silt, especially as relatively clear water often carries a certain quantity of unusually coarse grained bed silt, and may therefore be extremely undesirable for scouring purposes. Consequently, every opportunity should be taken to secure samples of the bottom layers of water, and the engineer in charge should accustom himself to rely upon the results of such samplings, rather than on an inspection of the upper layers (for that is what judging a water by its visible turbidity really amounts to).

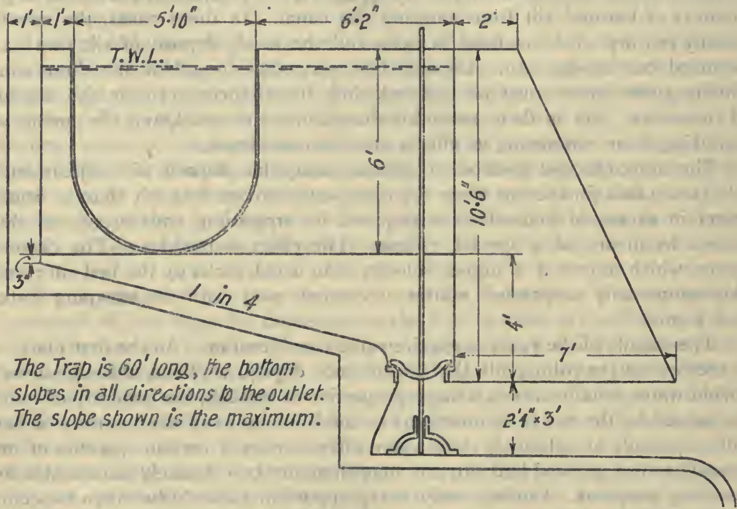
Secondly, the escape sluices should be designed so as to draw water from the bottom of the canal, rather than from the top; and raised sills and similar kindred devices such as are used in regulators are out of place in escapes.

It is also obvious that unless the canal is completely closed below the escape, a silt wave of the character already described may be set up, and part of the temporarily suspended silt may be carried into the canal below the escape, and dropped there.

Clear water periods in a river obviously coincide with low water periods. If the capacity of the canal, as compared with the low water flow of the river, is large, it may be impossible to spare clear water for scouring purposes, since the more urgent demands of irrigation absorb all the available supply.

For these reasons, escapes have now grown somewhat out of fashion, and the newer Punjab canals, when compared with the older ones, are badly provided with escapes. In Egypt also, scouring escapes are not employed. Nevertheless, wherever an escape exists, and clear water is available, the engineers in charge are very glad to make use of it. I therefore consider that escapes should be provided wherever the canal crosses a drainage or small stream at such a relative elevation that the water can be rapidly run off. The floods of the stream can then be relied upon to carry away the silt. On the other hand, escapes discharging into long, artificial channels, only change the location of trouble. Such escapes are rapidly rendered useless by the escape channel silting up, while any further discharge of silt and water merely floods and deteriorates the low lying land near the escape.

The usual mistake made in the location of escapes is that they are placed too close to the head of the canal. Escapes have proved most successful in the



SKETCH NO. 225.—"Sand" Trap (American).

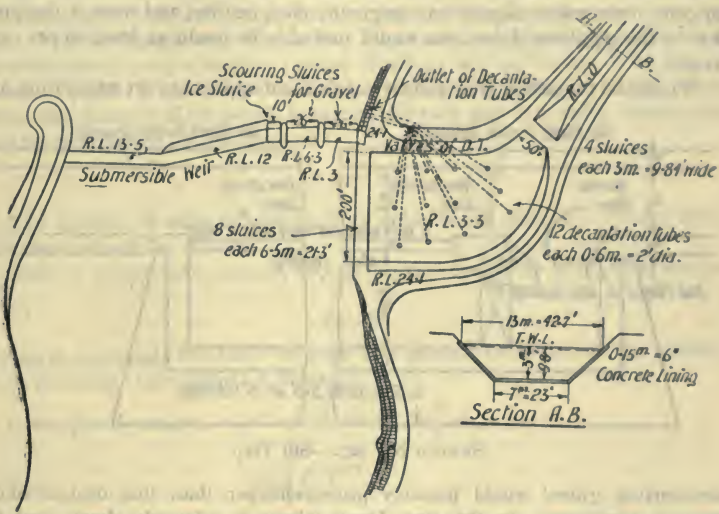
Sirhind Canal (referred to above), and the most effective escape is situated 12 miles below the head.

As a general rule, no escape which is not removed from the headworks at least three-quarters of the total length of the adjustment section (in which silt is deposited) will keep the canal free from silt. The ideal design would be one escape at half, and another at $1\frac{1}{2}$ times the length of this section below the canal head; and if a choice must be made between the two, the lower escape is the better one to select.

For the above reasons, the sand trap has been largely adopted; and proves most effective in the case of silt which is somewhat coarser than that which is usually found either in the Punjab, or in Egypt. In small canals (say not more than 40 feet in bed width) the sand trap shown in Sketch No. 225 will catch most of the sand as it rolls along the bottom of the canal; and, if correctly designed, a small extra quantity of water may be regularly admitted into the canal, and can be as regularly passed out with the entrained sand, by means of a sand trap.

Careful observation of the working of such sand traps permits me to say that the farther apart they are spaced the more effective they will prove. An ideal design would provide about ten, spaced 500 feet apart, over the second mile of the canal, in cases where the natural length in which deposits of silt occur is four miles.

The necessity for discharging the mixture of sand and water into some natural watercourse, where floods can carry the accumulation away, usually renders such a distribution of traps impracticable. Nevertheless, this plan should be adhered to as closely as possible, and traps in the first quarter of the adjustment length will not prove highly efficient unless there is an unusual proportion of large grains in the sand.



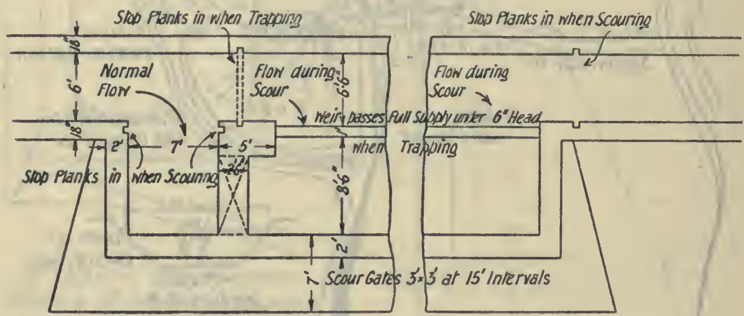
SKETCH No. 226.—Canal Head at Ventavon.

These upper traps should therefore be designed for coarser silt, and the proportions shown in Sketch No. 225 are best suited for coarse sands, while those of No. 227 will remove finer sands and silt if necessary.

In wider canals, it is usually difficult to draw the sand which falls into the middle of the trap, to the sides. I therefore consider that it will be advisable to take advantage of all places where streams are syphoned under the canal, and to provide central orifices so as to draw off the sand from the centre. The installation at Ventavon, on the Durance (see *Genie Civil*, Dec. 31, 1910), is a good example of this method. The river is extremely torrential, carrying gravel and boulders, and these are drawn off from the decantation basin by means of 12 orifices discharging into tubes 1·97 foot (0·6 metre) in diameter. The design is interesting, but it must be realised that the second and smaller regulator (see Sketch No. 226) would be a mistake, were it not that the canal below it is lined with concrete, and the mean velocity of the water exceeds 6 feet per second, which is amply sufficient to carry forward any sand suspended by the eddies caused by this regulator. This sand is later removed by an ordinary sand trap at the lower end of the canal.

In cases where the canal is unlined, and where the velocity in consequence cannot greatly exceed that normally prevailing in the river, the best method appears to be that practised on the Bari Doab Canal, where the first 3000 feet of the canal are double, and the water is alternately passed down each channel, while the other is being cleared. I am, however, inclined to believe that the entry of gravel into a canal must usually be regarded as evidence that the area of the waterway through the regulator at the canal head is insufficient. In the Bari Doab Canal, measures have been taken to provide a less turbulent method of entry by means of bellmouth orifices in front of the sluices. The present insufficiency of waterway is due to the fact that the canal now supplies some 60 per cent. more water than it was originally designed for, and were it designed *de novo* the regulator sluice area would probably be made at least 60 per cent. greater.

It must be remarked that proper mechanical appliances for excavating and



SKETCH No. 227.—Silt Trap.

transporting gravel would possibly prove cheaper than this design, under ordinary conditions. In the Bari Doab labour is relatively cheap, and the gravel and sand excavated in the clearances is required for making concrete, and other repairs. Consequently, these materials are more cheaply procured than if special excavations were made in the river bed.

In cases where most of the large gravel can thus be kept out of the canal, it will usually be found that the smaller rounded gravel which enters the canal can be removed by one or two sand traps. Since such gravel is rounded, and rolls easily, central escapes are usually not required.

On the Bari Doab, the sand which enters the canal and is not caught in the first 3000 feet is dealt with by means of scouring escapes. The fact that the canal head is situated on a torrential river, while the land irrigated is very flat, causes the silt deposits on this canal to be extremely complex in character The real lesson to be learnt is that the head is too high up the river, and should have been located lower down, where only coarse sand and clay are carried by the stream. As a rule (as is the case at Ventavon), when the river is torrential, the canal can be given such slopes and mean velocities that coarse sand is carried to the fields without difficulty. The canals of Lombardy which take out from rivers with a steep bed slope, and are provided with gravel and sand traps, illustrate this principle; and I was surprised to find how little attention is here paid to silt problems, once such traps have been set to work.

When designing escapes, or silt traps, it must always be remembered that it is necessary to catch the bed silt alone. Turbidity of water is mainly produced by clayey silt, and I am not aware of any case where clayey silt is not wanted; in fact, personally speaking, the more clayey turbidity existing in the water, the better I am pleased, as it is required to stanch leaks, to form berms, and to fertilise fields; and provided that these objects are secured, the slight deposits produced are amply compensated for by the counterbalancing advantages.

Physical Basis of Kennedy's Rule.—The physical meaning of Kennedy's rule seems to have been somewhat misunderstood, and doubts have frequently been expressed as to whether such a "peculiar" law can have any true physical foundation. The actual facts are that Kennedy channels are channels which carry a certain amount of silt as well as water. If we refer to Deacon's studies of the laws of the transport of sand by water (see p. 488), we see that q, the quantity of silt which is carried forward per foot width of the canal, may be represented by the equation:

$$q = kv^n \text{ say.}$$

The quantity of water carried per foot width of the canal is:—$Q = vd$, where d, is the depth of the channel.

Now, in a Kennedy channel, taking the average of the year's flow, we find that:

$$q = pQ$$

Where p, represents the ratio between the quantity of water and the quantity of silt that enter the channel in a year.

Consequently we get:—$kv^n = pvd$, or $v^{n-1} = \frac{p}{k} d.$

Thus, $$v = \left(\frac{p}{k}\right)^{\frac{1}{n-1}} d^{\frac{1}{n-1}}$$

Thus, Kennedy's form of the relation between v, and d, might be theoretically deduced. If we accept Kennedy's figures we find that:

$$n = 3, \quad \text{if } v_0 = 1{\cdot}05 d^{0{\cdot}5};$$
$$\text{or, } n = 2{\cdot}56, \quad \text{if } v_0 = 0{\cdot}84 d^{0{\cdot}64};$$

are respectively taken as the algebraic expressions of Kennedy's observations. The matter can be still further tested. Kennedy (see *P.I.B.*, Paper No. 9, pp. ii and v) believes that a Kennedy channel carries a quantity of silt in suspension ranging from $\frac{1}{3300}$ to $\frac{1}{9000}$ of the volume of its water discharge. In addition, a quantity varying from $\frac{1}{18000}$ to $\frac{1}{90000}$ of the volume of the water is rolled along the bottom in the form of bed silt. The larger ratios seem to occur when the water is clear, and the smaller ratios when the water is turbid, and heavily charged with mud.

We may thus assume that $q = 0{\cdot}00016Q$, is a fairly probable average relation between the silt and the water volumes.

Thus, since a cubic foot of silt weighs about 125 lbs. (the figure is doubtful, as values ranging from 140 to 80 lbs. have been observed), we may say that q_s is about 0·02 lb. per cubic foot of water ; or that in a 100-cusec channel, 2 lbs. of silt are swept forward every second (the exact figures are absolutely immaterial, for the present discussion is concerned with relative quantities only). Now, picking out the non-silting channels, as given by the intersection of the lines representing v_o, and 100 cusecs, from Kennedy's Graphic Diagrams, we find that :

Slope.	Bed Width in Feet.	Depth in Feet.	Mean Velocity $=v_o$, in Feet per Second.	Lbs. of Silt swept forward per Second per Foot Width of the Bed.
0·0002 .	22·5	2·65	1·57	0·098
0·000225 .	15·3	3·35	1·85	0·130
0·00025 .	12·3	3·73	1·95	0·162
0·000275 .	10·1	4·05	2·11	0·198
0·0003 .	8·7	4·35	2·16	0·230
0·00035 .	7·0	4·80	2·29	0·284
0·00040 .	5·7	5·15	2·40	0·350

The last two columns permit of a curve (Sketch No. 228) being plotted for the Bari Doab silt, similar in character to that given by Deacon for Liverpool sand. Other points on this curve can be constructed by taking any other discharge (say 50, or 1000 cusecs), and tabulating in a similar manner. In this way the plot of v_s and S_s is prepared, where S_s represents

$$\frac{0·02 \text{ discharge}}{\text{bed width}}$$

and is consequently approximately equal to the silt discharge per foot width of the channel, measured in pounds per second, and $v=v_o$, is the mean velocity in a Kennedy channel. The points do not fall accurately on a continuous curve, but seem rather to be included in a narrow zone. Nevertheless, the general resemblance to Deacon's results is quite evident. When the values of log v_s and log S_s are plotted certain irregularities manifest themselves. The wider channels (say 50 to 100 feet bed width) are apparently slightly more efficient carriers of silt than the narrow (say 5 to 20 feet bed width) examples. This probably arises from the fact that the wider channels observed by Kennedy did actually carry somewhat more silt per cubic foot of water than the narrower ones.

Buckley (*Irrigation Channels*) gives certain additional information concerning non-silting velocities.

In Sind the rule, $v_o=\frac{3}{4}$ (Punjab v_o)$=0·63d^{0·64}$ has been officially adopted.

In Burma a table of non-silting velocities which is very close to,

$$v_o=0·91d^{0·57},$$

has been used with success.

While in Egypt it is stated that,

$$v_o=\tfrac{2}{3}\text{ (Punjab } v_o)=0·56d^{0·64}.$$

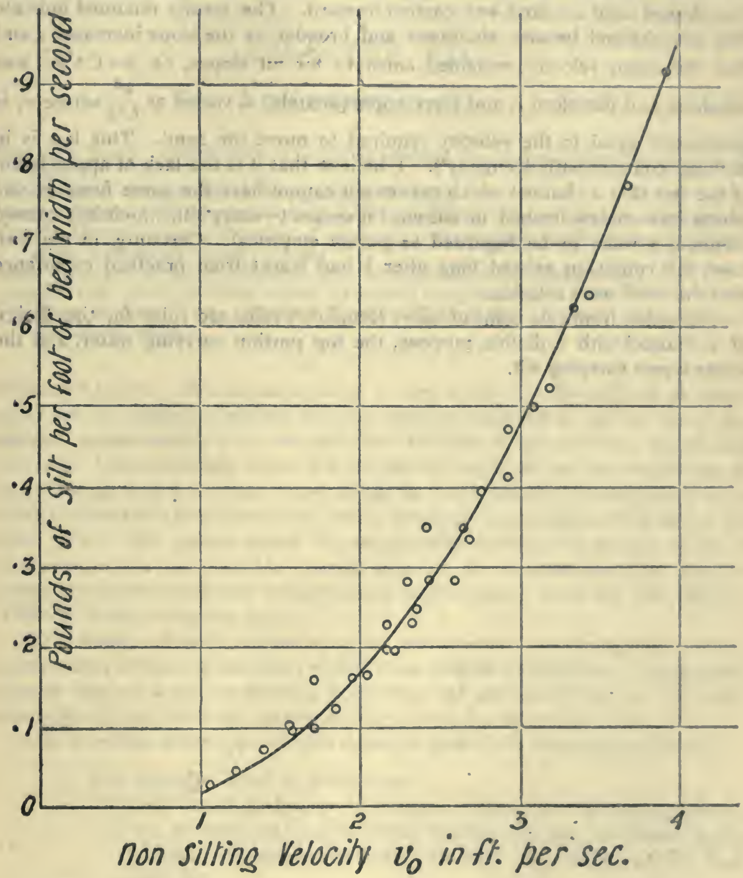
My own short experience is rather adverse to this last rule.

On summarising these rules and Deacon's figures and Thrupp's curves, it is quite plain that some relation of the form

$$v_0 = cd^n$$

always obtains, and I believe it is also true that:

The coarser the silt, the less the value of n, and that the value of c, increases in proportion to the volume of silt per unit volume of water.



SKETCH NO. 228.—Relation between Mean Velocity and Quantity of ilt swept forward in Kennedy Channels.

The physical facts underlying the rules given for the velocities of rivers which carry pebbles or boulders (see p. 492) are now fairly obvious. The mean velocity must increase with the depth, not because the increase in depth in any way prevents scour or prevents detritus from being swept forward, but because the increase in depth causes the quantity of detritus swept forward per foot width of the channel to increase in the same ratio as the depth (in actual fact, rather faster than the depth, owing to the increase in mean velocity produced by the greater depth), because the ratio of silt carried forward to water discharge must remain constant.

In this connection, the very interesting small scale experiments by Seddon (*Trans. Assoc. of Eng. Soc.*, 1886, p. 127) deserve notice. Here, the channels did not carry silt, but were merely permitted to erode the sand bed in which they flowed until no sand was carried forward. The results obtained indicate that the channel became shallower and broader as the slope increased; and that the mean velocity remained constant for all slopes, *i.e.* $v = C\sqrt{rs}$, was constant, and therefore r, and (very approximately) d, varied as $\frac{v^2}{C^2 s}$ where v, is practically equal to the velocity required to move the sand. This law is in striking contrast with Kennedy's. I believe that it is the lack of appreciation of the fact that a channel which carries silt cannot have the same form as one which has eroded its bed in silt until it ceases to carry silt, which has caused Kennedy's rules to be regarded as purely empirical. Certainly, in my own case, this confusion existed long after I had learnt from practical experience that the rules were reliable.

Regarded from this point of view, Kennedy's rules are rules for the design of a channel with a double purpose, the top portion carrying water, and the lower layers carrying silt.

CHAPTER XIII

MOVABLE DAMS

MOVABLE DAMS.—Advantages—Conditions affecting the design—Selection of types of dam.
Flashboards.—Applicable in the regulation of rivers.
Shutter Dams.—Not adapted to rivers carrying boulders—Dangerous when over-topped.
Trestle Dams.—Limits of height of water retained—Vertical needles versus horizontal gates—Difficulties caused by drift.
BEAR TRAP AND OTHER MECHANICAL DAMS.—Necessity for careful design.
CALCULATION OF BEAR TRAP DAMS.—Faults of older examples.
Old Type of Bear Trap.—Parker Type of Bear Trap.—Stresses in the leaves—Stiffness of leaves.

MOVABLE DAMS.—The advantages of a dam which can be utilised to retain water for an indefinite period, and yet rapidly removed so as to leave the channel across which it is erected free for the escape of flood water, are obvious. Thus, movable dams are frequently employed in the regulation of rivers during their low water stage, either in the interests of navigation, or in order to divert the low water flow into a canal, or into another branch of the river. They also prove useful for temporarily closing the escape weirs of reservoirs, since the available storage capacity of the reservoir can thus be greatly increased, while the escape weir is rapidly made ready for the passage of floods by removing the dam.

The design of such dams is in a very chaotic condition, and a mere enumeration of the various types would be as tedious as useless. I have given a great deal of study to existing examples, and am unable to see any valid reason for the adoption of more than three, or at the most four types.

The selection of the type of dam depends upon the following conditions:

(i) The height of water to be retained.

(ii) Whether the dam has to be erected when water is flowing over its base, or when its base is deeply immersed in the backwater below the dam; or merely lifted after the flood has ceased and the base is left dry.

The details of design are largely influenced by the quantity of silt carried by the river, by the frequency of floods, and by the rapidity with which the river rises.

As will be shown later, when the river carries boulders or is subject to frequent and sudden rises, certain types of dam which would otherwise be suitable, cannot be employed.

The four types are as follows:

(i) Flashboards.
(ii) Shutters.
(iii) Trestle dams, either with needles, or sluice gates.
(iv) Bear trap, and other mechanical dams.

(i) The ordinary flashboard type is suitable for all cases where the head held up does not exceed 5 feet, although, if the height is more than 4 feet, shutters will probably be more efficient. This type is simple in construction, and can be used in all cases, for although boulders may damage the flashboards and their supports, these are cheap and are easily replaced. Flashboards are, however, difficult to replace until the water has fallen to the level of their base, and are therefore best adapted to high dams or escape weirs, over which the water flows but rarely.

(ii) The shutter, or flashboard on fixed hinges, type, as developed in India. This is suitable for heads up to 8 feet (although if the head greatly exceeds 6 feet difficulties arise in working), and may be considered as applicable to very large rivers, provided that they do not carry much gravel.

Shutters can be lifted and put into place when the backwater level is 2, or even 3 feet above their base. They are therefore suitable for the crowns of low dams which are frequently submerged.

(iii) The trestle type of dam. This type is but little affected by gravel or boulders, unless these are of such a size that the masonry and angle irons are destroyed by impact. Trestle dams are suitable for heads up to 11 feet, and under favourable circumstances 15, or 16 feet of water can be retained. If machinery is employed for lifting the trestles and placing the needles, greater depths of water can be retained, although I am unaware whether a greater depth than 20 feet has yet been dealt with.

Trestles can be lifted and the needles put into place wherever the water level stands. They are therefore suitable for bars or dams which are permanently under water.

(iv) The bear trap, or other mechanical dam. These are suitable for conditions similar to those under which the trestle type is used. They are more costly, and are at present not capable of retaining a greater depth of water. Consequently, it is unlikely that they will be adopted except in rivers which are subject to such sudden floods that a shutter or trestle dam could not be dropped with sufficient rapidity. In such cases, a mechanical dam appears to be necessary, although previous to erection it is advisable to consider whether timely warning of approaching floods cannot be obtained by means of regular reports of gauge readings on the upper portion of the river. In countries where labour is scarce or inefficient, a mechanical dam may prove advantageous, as it dispenses with the more or less numerous staff necessary to work the non-mechanical types. A study of existing examples of movable dams does not, however, favour this idea, and it will usually be found that the mechanically operated portion is but a small fraction of the total length of movable dam in any installation, so that the necessary staff is not greatly diminished.

Thus, unless local conditions are peculiar, a short length of mechanical dam is usually employed as a relief valve for dealing with small and sudden fluctuations of the river flow. Longer lengths of trestles and flashboards are relied upon to pass the larger and slower variations.

Regarded in this light, a short length of bear trap dam permits a long length of trestle, or shutter dam, to be employed with the best efficiency, since the water level can with safety be kept just a few inches below the top of this portion of the dam, the bear trap being relied upon to pass off any sudden rise during a period of sufficient duration to permit the trestles or flashboards to be dropped.

Flashboards.—These have been discussed on page 402. They are equally applicable to the partial regulation of rivers. It will be plain that in river regulation flashboards must be supplemented by sluices, or movable dams, because once they have fallen, re-erection cannot take place until the height of the river has abated some 3 or 4 feet.

Consequently, flashboards are best adapted for cases where the river has a marked and well-defined flood season, followed by an equally well-defined low water period. They are then used to block the flood channels, and the water level in the low season is kept a little below their top. All regulation of the river in the low water season is effected by a manipulation of the sluices or movable dams, and the flashboards are only moved when unusual floods occur. Broadly speaking, we must provide a sufficient area of sluices and hinged dams to deal with the annual maximum low water flow (or at least three-quarters of this maximum). The large excess of the maximum flood over the annual maximum of the low water season can then be dealt with by the flashboards.

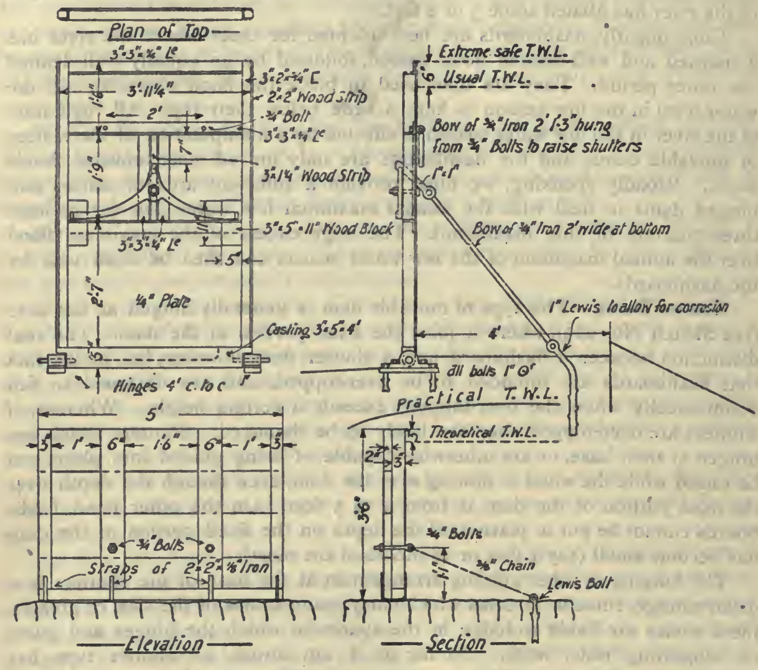
Shutter Dams.—This type of movable dam is generally hinged at the base (see Sketch No. 229), where it joins the fixed portion of the dam. The real distinction between a flashboard and a shutter dam, however, lies in the fact that flashboards are intended to be over-topped, and are designed to fall automatically when the over-topping exceeds a certain height. Whereas, if shutters are over-topped, they are likely to be damaged. Shutters which are hinged at their base, or are otherwise capable of being guided into place, can be raised while the water is flowing over the dam, even though the depth over the fixed portion of the dam is from 4 to 5 feet. On the other hand, flashboards cannot be put in place until the depth on the fixed portion of the dam has become small (say 1 foot or 18 inches at the most).

The hinging or other guiding arrangement at the base of the shutters is a disadvantage, since in streams which carry many stones of fist size, or greater, these stones are liable to lodge in the spaces in which the hinges and guide or supporting rods, work. So far as I am aware, no shutter type has yet proved really successful in such cases. Otherwise, this type has shown itself capable of dealing with far more exacting conditions than any other movable dam.

Several dams exist in India which are more than 4000 feet in length, and which hold up water to a height of 16 feet, of which 6 or 7 feet is retained by the shutters. The few troubles which occur have never been attributed to any defect in the shutters.

Sketch No. 229 shows a typical Indian shutter, which is dropped by releasing the curved horizontal lever. The raising of the shutter is effected by a hand crane, which hooks on to the smaller (hanging) loop on the upstream face of the shutter. Trained men can raise and set these shutters with ease in 5, 6, or even 7 feet of water, provided that the backwater below the dam is not so high as to interfere with the working of the crane.

The dropping of these shutters is perfectly easy, provided that they are not over-topped. If once over-topped, it is almost impossible to get them down, although this has been effected with some risk when the river did not rise rapidly after over-topping the shutters. If the shutters are over-topped, and the rise of the water continues, the dam will probably be destroyed, not so much by the impact of the water falling over the shutters, as by cross-currents induced along the dam from the portion where the shutters are up, towards that where they are down. These currents set up eddies, which rapidly undermine and destroy the dam.



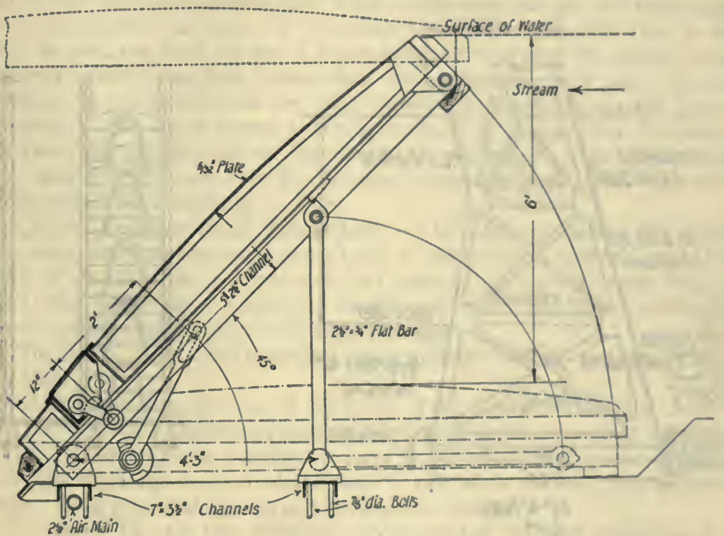
SKETCH No. 229.—Indian Steel Shutters and Wooden Flashboards.

Consequently, many designs have been prepared for automatically dropping the shutters. As a rule, these designs consist of a projecting bevelled arm, which is forced down by the fall of one shutter, and sets the next one free. Sketch No. 230 shows a non-automatic shutter that can be opened even when over-topped.

Automatically falling shutters are at present in the experimental stage, and are consequently not given in detail. It must be remembered that the shock produced by the simultaneous fall of a long length of shutters is a severe trial for a weir, or dam.

Experience in India suggests that over-topping of shutters is rarely, if ever, due to any cause except carelessness, and it may therefore be inferred that a staff which has become lax enough to permit the shutters to be over-topped would also sufficiently neglect any automatic falling gear to preclude the possibility of its working when required.

Trestle Dams.—Trestle dams are especially indicated when the dam has to be closed when there is an appreciable backwater below it. In such cases, the problem is best solved by a series of trestles, or horses, hinged at their base to axes parallel to the flow of the river. These trestles are raised to a vertical position, and are then used to support a series of vertical needles, or horizontal sluice boards. The total height of water that can be retained is fixed by the weight of one of these needles. It is found by experience that wooden needles of sufficient strength, and say 4 inches wide, are too heavy for a man to handle if the depth of water retained greatly exceeds 9 or 10 feet.



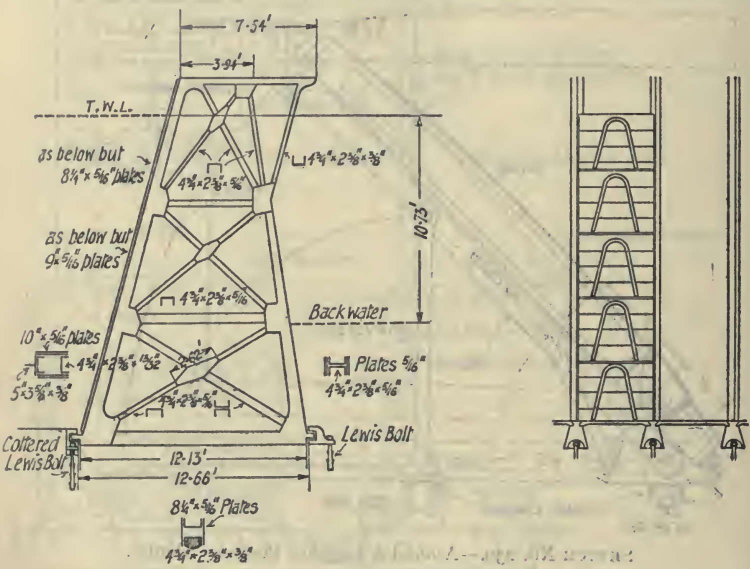
SKETCH NO. 230.—Ashford & Leggett's Buoyancy Shutter.

Eleven feet may be regarded as the maximum depth of water retained, unless machinery is used to remove the needles. The weight of the needles may be somewhat reduced by providing an intermediate supporting bar hung on chains. This additional complication has not found much favour, except in cases where the needles are but rarely moved, say once a year at the most.

Where horizontal sluice gates are employed, differences in water level greater than 11 feet can be maintained, but as the head increases these sluices become very heavy, unless the trestles are spaced close together. The trestle dam at Suresnes (*A.P.C.*, vol. 18, 1889, p. 49) retains 17·2 feet (5·27 metres) of water, and this is nearly the maximum that can be dealt with. Sketch No. 231 shows the dimensions.

A study of Sketch No. 231 will show that owing to the close spacing (1 metre = 3·28 feet) of the trestles, the sill has to be raised somewhat above the base of the trestles. In a stream like the Seine, which normally carries clear water, this is of little importance, but in a silt-bearing stream the sill should be as low as possible. Hence, when down the trestles should not lie over each other, and their horizontal spacing should therefore be very nearly equal to their height. When this is the case, it will be plain that the pressure on a horizontal sluice gate of a span equal to the height of water retained, is so great that unless the head is very small (say 6 or 7 feet) a man cannot move it. For silt-bearing waters, therefore, vertical needles are generally employed. These are made of wood, and if tapered off at the ends, a trained man can handle them provided that the head does not exceed 13 feet, each needle being about 4 inches wide (Pontoise dam, see *Trans. Am. Soc. of C.E.*, vol. 39, p. 459).

For greater heads, machinery must be employed in order to place the needles. In these cases, the needles are usually 2 to 3 feet wide, and made



SKETCH NO. 231.—Suresnes Trestles.

of wood, stiffened by steel angles. Heads up to 20 feet are thus dealt with, and there is no reason to suppose that this is the maximum possible (see *ibid.*, p. 490).

It must, however, be remembered that accumulations of drift are unhealthy, and must be avoided in thickly populated localities. Needles are very badly adapted to such cases, for, although it is perfectly easy to let off small excesses of water by propping a few needles forward of the general line of the dam, drift will not readily pass through the narrow orifices thus formed. With sluices, on the contrary, the water and the drift are easily passed over the top of the dam by opening a few of the upper sluices. For this reason sluice gates mounted on wheels have been proposed. The difficulties are great, for if the bearings are of the ball-bearing type, they will rapidly rust; and roller bearings are liable to jam if the trestles move relatively to each other in any degree.

Bear Traps and other Mechanical Dams.—It is quite impossible to enumerate the various forms of these dams which have either been adopted, or proposed. In essence all these dams consist of two leaves fixed to hinges in the bed of the river. These leaves (with connecting leaves or "idlers" in some types) form a closed chamber, into which water can be admitted under pressure. This pressure water opens out the leaves, and the dam is thus raised. When it is desired to let the dam down, the pressure water is withdrawn, and the dam falls by its own weight.

The outlines of chief types are shown in Sketch No. 232 and are the original "Bear Trap," and the Parker and Lang modifications. The Lang type is obviously best adapted for rivers carrying silt and drift, as the idler leaf prevents these being caught between the leaves. Our present knowledge does not permit us to state whether the Parker modification has any real advantages over the original bear trap. It is probably used more frequently, but, as will later be seen, the facts are that a badly designed bear trap will refuse to rise, whereas a badly designed Parker dam refuses to fall. It is plain that this difficulty is easily obviated by such expedients as weighting the leaves. Whereas, a dam which refuses to rise is not easily lifted up. For this reason, therefore, the adoption of the Parker type in preference to the simpler bear trap may only indicate that the ordinary designs of both types are badly proportioned fundamentally.

Calculation of Bear Trap Dams.—The older examples of this type of dam were badly proportioned, and a study of the excellent results obtained by some (if not all) of the newly erected and scientifically designed dams built by the United States Army Engineers, leads me to believe that the capabilities of these dams are at present greatly underestimated.

The failure of the older examples was principally caused by the two following faults :

(i) The relative proportions of the leaves of the dam were such that the dam was liable to stick, and consequently failed to rise, or fall as the case might be.

(ii) The leaves were insufficiently rigid in a longitudinal direction. Hence, it was possible for one end of the dam to rise, while the other end remained down. This produced strains, and consequent leakage.

It cannot be said that sufficient experience has yet been accumulated to enable these difficulties to be entirely overcome. The following analysis must be regarded as a preliminary sketch, but so far as I am aware, all dams to which similar principles have been applied work fairly well.

All of the older dams which have proved notorious failures have been found to be defective when tested by these rules.

The proportioning of the leaves has been investigated by Powell (*Journ. of Assoc. of Eng. Soc.*, vol. 16, p. 177), who deduces the following results :

Old type of bear trap (see Sketch No. 232, Figs. 1 and 2).

Let X, be the length of the upstream leaf.

Let Y, be the length of the downstream leaf, and let the distance between the hinges be represented by Q.

Let $Z = X + Y - Q$, be the overlap of the upstream leaf on the downstream leaf when both are lying flat.

The critical positions are as follows :

(i) When the dam lies flat, and begins to rise under a head h, say.

(ii) When the dam is raised to its greatest height, and begins to fall.

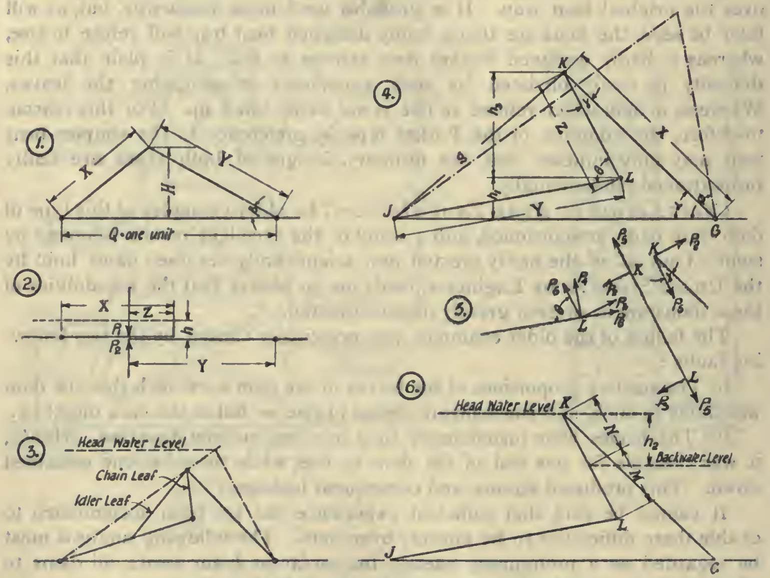
(i) Let us consider one foot length of the dam. The head h, may be considered as produced by the difference in level of the water surfaces above and below the dam. The most unfavourable assumption, therefore, is that the pressure caused by h, feet of water acts downwards on the whole of the leaf X, and upwards on the whole of the leaf Y, and on that portion of X (*i.e.* a width $X-Z$) which is not covered by Y.

Let P_1, be the downward pressure at the end of the leaf Y.

Then, taking moments about the upstream hinge ; $P_1(X-Z)=62{\cdot}5hZ\left(X-\frac{Z}{2}\right)$.

The upward pressure on the leaf Y is : $P_2=\frac{1}{2}Yh\ 62{\cdot}5$.

Therefore : $P_1=nP_2$, if $Y=\frac{2XZ-Z^2}{n(X-Z)}$, and in order to have a reserve of force to overcome friction, it is plain that n, must be less than 1.



SKETCH NO. 232.—Diagrams for Movable Dams.

Thus, we can obtain Z, in terms of X, and Y, and putting $Q=1$, so that X, is now equal to :

$$\frac{\text{Length of upper leaf}}{\text{Distance between the hinges}}$$

we find X, and Y, in the forms :

$$X=\sqrt{(1-n)Y^2-2(1-\tfrac{1}{2}n)Y+1}$$

$$Y=\frac{1-\frac{1}{2}n}{1-n}-\sqrt{\frac{X^2}{1-n}+\frac{1}{4}\left(\frac{n}{1-n}\right)^2}$$

For example : $n=1$; $Y=1-X^2$

$n=0{\cdot}8$; $Y=3-\sqrt{5X^2+4}$: etc.

Thus, for an assumed X, we can calculate Y.

The height to which the top of the upper leaf rises (*i.e.* the head of water which the dam can hold up) is given by the equation :

$$H^2 = Y^2 \sin^2 \beta, \qquad \text{where } \cos \beta = \tfrac{1}{2} n Y + (1 - \tfrac{1}{2} n).$$

Finally, if we make H, a maximum for a given value of *n*, by expressing H, in terms of Y, and differentiating, we obtain the following equation :

$$Y = \frac{1}{n}\sqrt{\tfrac{1}{4}\left(1-\frac{n}{2}\right)^2 + 2} - \frac{3}{2n}\left(1-\frac{n}{2}\right)$$

whence we can express X, Y, and H, in terms of *n*.

Thus, when H, is a maximum, we obtain :

When $n = 0{\cdot}8$	$Y = 0{\cdot}6821$	$X = 0{\cdot}5239$	$H = 0{\cdot}333$,
When $n = 0{\cdot}75$	$Y = 0{\cdot}6811$	$X = 0{\cdot}5144$	$H = 0{\cdot}323$,

and so on.

(ii) In order that the gate may fall, the angle between the leaves should be greater than 90 degrees, say 100 degrees, or more. If *n*, be less than 1, this condition is always satisfied.

In some of the earlier examples *n*, was greater than 1, that is to say, the gate could only rise when the pressure below the leaves was greater than that on the upper side of the upstream leaf. Such designs should be avoided.

In successful gates it is found that : $n = 0{\cdot}80$, to $0{\cdot}85$, at the most. It is doubtful whether *n*, should be less than $0{\cdot}75$, lest the dam should be run up too rapidly, and a shock should be produced. The amount of friction between the leaves and at the hinges is evidently important, and the more silt in the water the less should be the value of *n*.

The Parker Type of Bear Trap Dam.—In this case let the distance between the hinges be equal to unity, and in this unit let (Fig. 4) :

X, be the length of the downstream leaf (which is now the upper leaf when the dam lies flat).

Y, be the length of the lower upstream leaf, and Z, the length of the intermediate two-hinged leaf.

Then : $X + Y - Z = 1$.

When the dam is raised to its greatest height :

$$\cos \phi = \frac{X^2 + 1 - (Y+Z)^2}{2X}$$

where ϕ is the angle which the leaf X, then makes with the horizontal.

$$\text{Also } Z = \tfrac{1}{2}\left\{\sqrt{1 + X^2 - 2X\cos\phi} - (1 - X)\right\}.$$
$$Y = \tfrac{1}{2}\left\{\sqrt{1 + X^2 - 2X\cos\phi} + (1 - X)\right\}.$$

$$\text{Therefore, } YZ = \tfrac{1}{2}X\,(1 - \cos\phi).$$

Also, *g*, the distance between the top of X, and the upstream hinge when the dam is partially opened, and X, makes an angle a with the horizontal, is given by :

$$g^2 = 1 + X^2 - 2X\cos a$$

and if θ be the angle between Y, and Z :

$$\cos\theta = \frac{Z^2 + Y^2 - g^2}{2YZ}.$$

Hence, $(1-\cos\theta)(1-\cos\phi)=2(1-\cos a)$,

so that θ can be calculated when a and ϕ are given.

The critical positions of this type of dam occur when it is falling; so that the pressure inside the dam is atmospheric, or at the most that due to the backwater, while the upstream faces of the leaves Y, and Z, are exposed to pressure caused by water which may rise as high as the top of the dam.

Resolve the forces caused by these pressures, as shown; where P_5, and P_6, act in the direction of Y, and Z, respectively, and the other forces are perpendicular to the respective leaves.

Taking moments about G, we get the following equation for the equilibrium of the leaf X:

$$P_2 \cos\gamma - P_5 \sin\gamma = 0.$$

Similarly, since P_3 and P_5 acting at L are equivalent to P_4 and P_6 acting at the same point; we have resolving along P_4,

$$P_5 = \frac{P_4}{\sin\theta} - P_3 \cot\theta.$$

Or, substituting for P_5,

$$P_4 - P_3\cos\theta - P_2\cot\gamma\sin\theta = 0.$$

If there be no backwater, denoting the various depths as shown in Sketch No. 232,

$$P_2 = \tfrac{1}{6}h_3 Z \qquad P_3 = \tfrac{1}{3}h_3 Z \qquad P_4 = \tfrac{1}{2}h_3 Y + \tfrac{1}{6}h_1 Y.$$

Therefore:

$$\left(\frac{h_1}{h_3}+1\right)Y + 2(Y - Z\cot\theta) - Z\sin\theta\cot\gamma = 0.$$

In practice γ is best obtained graphically, but if necessary we can calculate the angles χ, and ψ from:

$$\sin\chi = \frac{\sin a}{g}, \quad \text{and} \sin\psi = \frac{Y\sin\theta}{g}; \quad \text{and } \gamma = \chi - \psi.$$

The critical position will be found to occur when the dam is falling, and is just about to reach the horizontal position.

In actual practice $\frac{h_1}{h_3}$ will then have a certain limiting value depending upon the area of the passages available to pass water around the dam. Powell, however, assumes that h_1, and h_3, then vanish simultaneously, which is a less favourable case. Consequently, his assumption that:

$$P_2\cos\gamma - P_5\sin\gamma = 0,$$

in place of $P_2\cos\gamma - nP_5\sin\gamma = 0$, which is similar to the equation used to investigate the old type, will lead to a satisfactory dam.

In the final calculations it would nevertheless appear advisable to make certain that when $a = 5$ degrees or 10 degrees say, there is sufficient overplus of downward pressures to overcome friction. Bearing in mind that initial friction is always greater than moving friction, and that silt deposits have a certain sucking power which prevents the dam from starting, but is not very active in stopping motion when it has once begun, we may believe that a dam is more likely to refuse to start when nearly down than to cease closing up just as it gets flat when motion has once started.

Evaluating the indeterminate fractions $\frac{h_1}{h_3}$, and $\sin\theta\cot\gamma$ for the case $\theta=0$, we get :

$$\frac{h_1}{h_3}+1 = \frac{1-X}{\sqrt{\tfrac{1}{2}(1-\cos\phi)}-Z}$$

$$\text{and,}\qquad \sin\theta\cot\gamma = \frac{1-X}{Y-\sqrt{\tfrac{1}{2}(1-\cos\phi)}}.$$

Substituting these values in the above equation, we finally obtain :

$$(1-X)^3-2\tfrac{1}{2}(1-\cos\phi)(1-X)+(1-\cos\phi)^2=0.$$

This equation can be solved by the ordinary rules, and X, having thus been determined, Y, and Z, can be calculated.

Where the dam is exposed to a backwater, the critical position will be found to occur when the dam is falling, and its crest is just level with the backwater.

In the general case, when the leaf Z, is only partially immersed in the backwater, we have :

$$P_2=\tfrac{1}{2}h_2M+\frac{h_2}{6}\ \frac{N^2}{Z} \qquad\qquad P_3=\tfrac{1}{2}h_2Z-\frac{h_3}{6}\ \frac{N^2}{Z}$$

$$P_4=\tfrac{1}{2}h_2Y.$$

In the critical position it is found that $M=Z$, and $N=0$.
So that: $Y-Z\cos\theta-Z\sin\theta\cot\gamma=0$, which, expressed geometrically gives,

$$\text{angle KJG}=\theta-a.$$

$$\text{Thus,}\qquad X=\frac{\sin(\theta-a)}{\sin\theta},\qquad\text{and,}\qquad \sin a=\frac{h}{H}\sin\phi.$$

The best solution is most easily found by calculating a series of values of X, Y, Z, and H, for assumed values of ϕ and $\frac{h}{H}$.

The above investigations can only be regarded as a preliminary sketch.

Before the proportions of a dam can be finally determined, the local conditions must be investigated, and the possible value of the pressure liable to arise inside the dam must be determined. It is fairly plain that if an artificial head can be produced (*e.g.* by accessory stop planks, or reservoirs) which is somewhat greater than that retained by the dam when just about to rise, the leaves of the dam may be shortened. The economy thus secured may justify the expense.

The effect of leakage through the hinges in diminishing this internal head must also be investigated.

The friction of the hinges and possible deposits of silt also require consideration.

The dimensioning of the leaves is evidently a problem which is somewhat akin to that of a bridge under moving loads. The forces producing the stresses for given values of θ and a have been written down. The maxima bending moments and shears can be determined, but the algebraic expressions are complicated, and it is best to draw the position of the leaves for, say every 10 degrees of increase in a, and measure h_1, h_2, and h_3. The bending movements and shears can then be calculated by the usual formulæ, and their maxima values can be selected, and the sections of the leaves dimensione accordingly.

In this connection, however, the stiffness of the leaves along the length of the dam deserves investigation. As already stated, in many existing dams the end where the pressure is applied may rise, and the other end remain down, owing to the diminution in pressure produced by leakage as the water travels along the inside of the dam.

The matter has been investigated by Bowman (*Trans. of Am. Soc. of C.E.*, vol. 39, p. 609). By a calculation similar in principle to that made by Powell, but which takes into account the weight of the gates (but not the friction at the hinges, for which reason I do not give it) Bowman finds that a pressure of about 35 lbs. per square foot, say 7 inches head, is required to cause a certain bear trap dam (old type) to begin to rise.

It is found from actual experience that if such a dam be more than 50 feet long, and insufficiently stiff in a longitudinal direction, one end is likely to rise and the other end to stay down. Bowman therefore infers that in actual practice the 7-inches head occurs at one end of the leaf, but gradually dies out to 0, at the other end. He therefore investigated the deflection of a cantilever under a load of p lbs. per inch run at its end, which gradually diminishes to 0, at the inner end. The equation is :

$$\frac{d^2y}{dx^2} = \frac{p}{2\mathrm{EI}}\left(x^2 - \frac{x^3}{3l}\right)$$

where p in this case is equal to :

$$\frac{35 \times \text{width of leaf in feet}}{12} \text{ lbs. per inch run of the beam,}$$

$$\text{and } l = 50 \times 12 = 600 \text{ inches.}$$

Integrating, since $\frac{dy}{dx} = 0$, when $x = l$ and $y = 0$, when $x = 0$; we get the maximum deflection, $\delta = \frac{19}{120}\frac{pl^4}{\mathrm{EI}}$, where I, is the moment of inertia of a section of the leaf by a plane parallel to the direction of flow of the river. For the actual case considered $\delta = 1{\cdot}84$ inch, so that the leaf is obviously sufficiently stiff.

Bowman's paper forms a very valuable example of the detailed calculation of the forces acting on a dam as it rises. The general proportions agree very fairly well with Powell's rules. The omission of all friction renders the figures less accurate than could be wished, but there is little doubt that certain of the assumptions partially compensate for this error. Reference may also be made to Willard's (*ibid.*, p. 573) investigation of the Lang type of dam. This is very complex, and a comparison with Powell's results, indicates that the Parker type is preferable in all cases. Consequently, the calculations are not given.

CHAPTER XIV

HYDRAULIC MACHINERY OTHER THAN TURBINES

THE following discussions must not be considered as intended to provide all the information required for the complete design of any hydraulic machine. The constructional problems are almost entirely ignored; and the most important of all hydraulic machines—the piston pump—receives no discussion. The circumstances under which hydraulic engineers generally work justify this action. Assuming that a hydraulic engineer possessed the requisite knowledge and experience of the working properties of metals to permit him to produce a first-class design, it is extremely improbable that he would have access to the tools necessary for its construction. Thus, in nearly all cases, a trained mechanical engineer will be associated with the design. Under these circumstances, any hydraulic engineer who interferes with the mechanical details merely creates friction and assumes an unnecessary responsibility.

I have personally worked with excellent mechanical engineers who stated that: "Such large pipes are always exposed to intense pressures, and therefore the thickness of plating should be increased by 50 per cent." These large pipes were actually exposed to less than one-half of the pressure sustained by pipes of equal thickness and smaller diameter which the mechanical engineers had already installed. The facts and theory were therefore hopelessly erroneous. Nevertheless, on investigation the pipes as designed were found to be somewhat less rigid than they might be, and the extra weight of metal was finally applied, not in increased thickness, but in the form of stiffeners.

A knowledge of the theory of hydraulic machinery is really useful under the following circumstances. An existing machine works satisfactorily under a certain head and when utilising a certain volume of water. It is desired to use this machine under a different head, and when utilising a different volume of water. A little consideration will show that a knowledge of the friction losses in the machine, as installed, will permit the efficiency of the machine to be predicted under the new circumstances with a fair degree of accuracy. The pressure and velocity at any point in the machine under the new circumstances can then be calculated; and the stresses produced in its various members can be estimated, as also the necessity for alterations in the loading of the valves, or other details.

This work is undoubtedly best performed by a hydraulic engineer, and should also be carried out when purchasing hydraulic machinery. The following chapters are therefore devoted solely to this problem, and as a rule it is assumed that the coefficients of skin friction, which, in practice, will also include losses of head at bends and obstructions, are determined by previous observation. The problem is therefore somewhat more simple than that which occurs when designing a new machine.

The first two sections, however, are devoted to a consideration of the hydraulic properties of enlargements and contractions in pipes, and the loading and motion of valves. Similar questions concerning bends have been discussed on page 28, and the uncertainties there disclosed form the greatest defect in the following theories.

SYMBOLS CONNECTED WITH VALVES AND ENLARGEMENTS.

A and a (see p. 795).
A_o is the area in square feet left vacant by a valve or other obstruction in a pipe.
A_p is the area in square feet of the unobstructed pipe.
A_1, A_2 (see p. 793).
c (see p. 790).
c_c is the coefficient of contraction of the area A_o considered as an orifice subject to suppression of contraction by the upstream portion of the pipe.
C is the coefficient of discharge of the same orifice.
d_1, d_2 (see Sketch No. 236).
h_o is used for the loss of head in feet when the velocities at the points between which h_o is measured are the same.
h_1, h_2 (see p. 793).
H_o is used for the loss of head in feet when the velocities at the points between which the loss occurs differ and this difference is taken into account.
h_m is the pressure in feet of water at the smallest cross-section of a diverging cone (see p. 797).
$H=\frac{v_m^2-v^2}{2g}$ (see p. 797), h_f and h_d (see p. 799).
K (see p. 797).
l (see Sketch No. 236).
m (see p. 787).
$n=\frac{A_o}{A_p}$ (see p. 789). On p. 796 n is a number.
Q, the quantity of water flowing through the valve or other orifice in cusecs.
$r=\sqrt{n}$ (see p. 790).
s (see p. 796).
v_p is the velocity in feet per second in the unobstructed section A_p of the pipe.
v_s is used for the velocity in the smaller pipe where there are two pipes.
v_m is the velocity at the smallest cross-section of the path of the water or the diverging cone considered.
These and all other v's are defined by the relation, $Q=vA$, with appropriate suffix.
δ is the vertical angle of a conical enlargement (see Sketch No. 236).
ζ a coefficient in the equation $h_o=\zeta\frac{v^2}{2g}$.
η and η_d (see p. 799).
ρ (see Sketch No. 236).

SUMMARY OF FORMULÆ

Theoretical formula for head lost at an obstruction:

$$h_o = \frac{(v_m - v_p)^2}{2g} = \frac{v_p^2}{2g}\left(\frac{A_p}{c_c A_o} - 1\right)^2 = \zeta\frac{v_p^2}{2g}$$

Relation between C and ζ:

$$C = \frac{1}{n\sqrt{\zeta}}$$

Head lost at a valve:

$$h_o = \frac{Q^2}{C^2 A_o^2 2g} = \frac{\zeta v_p^2}{2g}$$ (see p. 787).

Sudden contraction of a pipe:

$$h_o = \left(\frac{1}{c} - 1\right)^2 \frac{v_s^2}{2g}, \qquad c = 0{\cdot}582 + \frac{0{\cdot}0418}{1{\cdot}1 - r}.$$

Fire nozzles:

$$C = 0{\cdot}571 + \frac{0{\cdot}0429}{1{\cdot}1 - r}.$$

Sudden enlargements in a pipe:

$$H_o = \frac{(v_1 - v_2)^2}{2g} \quad \text{or} \quad \frac{v_1^2 - v_2^2}{2g}$$ (but see p. 795).

St. Venant's rule:

$$H_o = \frac{(v_1 - v_2)^2}{2g} + \frac{1}{9}\frac{v_2^2}{2g}.$$

Labyrinth packings, with n enlargements:

$$h_o = \frac{Q^2}{2g}\frac{n+1}{a^2}.$$

Gradual enlargements:

(*a*) Corrected for friction only:

$$h - h_m = \frac{v_m^2 - v^2}{2g}(1 - K)$$ (see p. 797).

$$K = \frac{4g}{C^2 \tan\frac{\delta}{2}}, \qquad \text{where } v = C\sqrt{rs}.$$

(*b*) Correction for friction and divergence loss:

$$h_1 = \frac{v_m^2 - v^2}{2g}\eta = \frac{v_m^2 - v^2}{2g}(1 - K - \eta_d)$$

Table of η and η_d (see pp. 799 and 800).

VALVES AND OTHER OBSTRUCTIONS IN PIPES.—Our knowledge of the head lost at these is mainly due to Weisbach (*Versuche über der Ausfluss des Wassers*). His experiments were small scale, and effected by observing the time taken to pass a given quantity of water through the system of pipes and valves experimented on. The values are therefore mean values for a head varying from about 3 feet downwards. Despite the fact that the observations were very carefully conducted, comparison with such modern experiments as exist on a larger scale leads me to believe that the numerical results are by no means exactly applicable to larger pipes, although it is highly probable that they form a guide to the general effect of the obstruction.

There is a certain theory which allows us to test some of Weisbach's experimental results, and which throws some light on the probable applicability of his observations to large pipes, and higher heads.

Let us consider the effect of an obstruction leaving an area A_o, vacant in a pipe of area A_p (Sketch No. 233). The water, which flows with a velocity v_p, in the pipe, passes the obstruction with a velocity v_o, given by $A_o v_o = A_p v_p$, and issues in a jet the smallest area of which is $A_o c_c$, where c_c, is the appropriate coefficient of contraction. The velocity at the "vena contracta" is then:

$$v_m = \frac{A_p}{c_c A_o} v_p$$

When the water has travelled some little distance along the pipe the jet expands, and fills the pipe (the jet being previously surrounded by eddying water, or air, according as there is a sufficient vacuum at the vena contracta to set free air or not). The velocity then becomes v_p.

The theory given by Borda for the loss of head caused by a sudden enlargement in a pipe probably applies to the above described motion with far more accuracy than to the case actually considered by Borda (see p. 793). The loss of head caused by the obstruction is therefore:

$$h_o = \frac{(v_m - v_p)^2}{2g} = \frac{v_p^2}{2g}\left(\frac{A_p}{c_c A_o} - 1\right)^2 = \zeta \frac{v_p^2}{2g}, \text{ say.}$$

The "lost head" h_o, can be directly observed as the difference of the pressures indicated by the two pressure gauges.

The case should be carefully distinguished from that shown in the lower Figure, where the final velocity v_q, is not the same as the initial velocity v_p. In this case, even if no obstruction existed, and the change of velocity was accomplished without any loss of energy, Bernouilli's equation shows that:

$$h_p + \frac{v_p^2}{2g} = h_q + \frac{v_q^2}{2g}$$

Hence, H_i, the difference in pressure indicated by the gauges, is composed of:

(i) A change in pressure equal to $h_p - h_q = \frac{v_q^2 - v_p^2}{2g}$ which is not necessarily accompanied by a loss of energy, since the velocity head is increased or decreased, as the case may be, by the same amount.

(ii) A loss of pressure equal to $H_o = \frac{(v_m - v_q)^2}{2g}$ which, so far as the practical requirements of engineers are concerned, is accompanied by a loss of energy.

In the case shown, v_p is greater than v_q, so that putting aside the effect of the obstruction the pressure at Q, should be greater than the pressure at P. Hence, the observed difference H_i, does not fully represent the loss of energy.

The application of this theory to the values of ζ experimentally obtained by Weisbach leads to values of c_c, which are very close to those experimentally obtained on orifices of similar size and under like hydraulic circumstances to those formed by the valves and cocks used by Weisbach.

When it is desired to obtain the value of ζ for an obstruction in a large pipe, or under a head which greatly exceeds 3 feet, it is consequently probable that a value of ζ which is more applicable to the actual circumstances than that given by Weisbach, can be obtained by selecting (in default of special experiments) the value of $c_c = \frac{C}{0{\cdot}98 \text{ or } 0{\cdot}99}$ appropriate to the size of the orifice left free by the obstruction, and the head under which it works, and calculating from the above equation.

The method is only recommended when direct experiment is not available, but it is more likely to lead to correct results than a blind application of Weisbach's figures for a head of 3 feet on an obstruction in a 1-inch pipe to a similar obstruction under 50 feet head in a pipe 3 feet in diameter.

Valve in Circular Pipe.—Weisbach (*ut supra*) experimented on a closely fitting, thin slide. Kuichling (*Trans. Am. Soc. of C.E.* vol. 26, p. 439) on a commercial 24-inch stop valve, and Smith (*Trans. Am. Soc. of C.E.*, vol. 34, p. 235) on a commercial 30-inch stop valve. Kuichling has discussed the results (*Trans. Am. Soc. of C.E.*, vol. 34, p. 243), and I have followed his method, which is to consider the valve opening as an orifice of an area A_0, equal to n times the area of the pipe, and to calculate C its coefficient of discharge under the head h_0, lost at the valve, *i.e.* :

$$Q = C \times A_0 \sqrt{2gh_0} = Cn \text{ area of pipe } \sqrt{2gh_0}$$

Weisbach calculates ζ given by :

$$\zeta v_p^2 = 2gh_0$$

where v_p, is the mean velocity in the pipe so that :

$$C = \frac{\text{Area of pipe}}{A_0 \sqrt{\zeta}} = \frac{1}{n\sqrt{\zeta}}$$

In commercial valves the valve has to be lifted slightly before any passage is opened. Reckoning the lift from this point, we have the following table, where :

$$\frac{\text{Lift}}{\text{Diam. of pipe}} = m$$

Smith. 30-inch Pipe.			Kuichling. 24-inch Pipe.		Weisbach. 1·57-inch Pipe = 0·04 Metre.		
m	*n*	C	*n*	C	*n*	C	ζ
0·1		0·92	0·106	0·98			
0·125	0·125	0·88	0·138	0·88	0·159	0·64	97·8
0·2		0·84	0·233	0·73			
0·25	0·287	0·82	0·296	0·70	0·315	0·77	16·97
0·3		0·83	0·359	0·71			
0·375	0·443	0·84	0·451	0·75	0·466	0·91	5·52
0·4		0·85	0·482	0·77			
0·5	0·593	0·90	0·598	0·92	0·609	1·14	2·06
0·6		1·04	0·708	1·19			
0·625	0·729	1·09	0·733	1·28	0·740	1·50	0·81
0·7		1·34	0·807	1·61			
0·75	0·851	1·60	0·853	not observed	0·856	2·29	0·26
0·8		2·08	0·893				
0·875	0·946	not observed	0·948		0·948	4·00	0·07
0·9			0·961				

The results obtained with the 24-inch pipe are the most accurate, the difference of head on either side of the valve being directly observed; while Smith observed the total head consumed by the valve, and a certain length of pipe, and assumed that the head lost by friction in this length of pipe was proportional to the square of the quantity of water flowing, which is somewhat doubtful when n is less than 0·50. Neither the theory of Kuichling, nor that of Weisbach is absolutely satisfactory, and some of the differences may be explained by this fact.

The losses of head vary from 11·6 feet in Smith's observations, and 9·3 feet in those of Kuichling, at small openings, down to practically nothing when the valve is nearly full open. Duane (*Trans. Am. Soc. of C.E.*, vol. 26, p. 464), observed the time taken to fill a certain length of pipe through a partially opened valve, and thus obtained the mean value of C, under heads varying from a certain maximum down to nothing.

He gives—C = 0·75 for a 6-inch pipe, with valve 1 inch open, i.e. $m = 0{\cdot}166$, and $n = 0{\cdot}124$, the maximum head being 12·1 feet, and C = 0·75 for a 12-inch pipe, with valve 1 inch open, i.e. $m = 0{\cdot}083$, and $n = 0{\cdot}074$, the maximum head being 52 feet.

Graeff (*Traité d'Hydraulique*, Tables, p. 28), gives details of experiments with a pipe 0·40 metre in diameter (say 16 inches), with openings ranging from 0·4 inch (0·011 metre) to 1·54 inch (0·0385 metre), under heads varying from 52 feet to 131 feet, C, was found to vary between 0·795, and 0·820.

These last experiments seem to show that under very high heads such matters as the form of the orifice, which plainly possess a certain influence under low heads, no longer affect the coefficient of discharge; which becomes what would be predicted for a circular orifice in a thick wall.

The valve used by Graeff being some 6 inches thick, it is plain that for the orifices experimented on, the "wall" can be considered as thick. It is doubtful whether in these experiments the jet issuing from beneath the valve ever expanded so as to completely fill the pipe. The circumstances of the orifice are shown in Sketch No. 233, as the experiments are almost classic and have formed the basis of many assumptions concerning coefficients of discharge under great heads.

The value C = 0·75 to 0·80, may be assumed, and used in all calculations where the question is of practical importance, as it is only rarely that we require to accurately predict the discharge of a valve except for small openings under high heads.

Sluice in a Rectangular Pipe.—Weisbach worked on a pipe 1·98 inch × 0·96 inch (5·02 cms. × 2·48 cms.), and the opening was always 1·98 inch wide. He gives $h_o = \zeta \frac{v_p^2}{2g}$, where v_p, is the velocity in the pipe.

$\frac{\text{Area of orifice}}{\text{Area of pipe}}$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·00
ζ . . .	193	44·5	17·8	8·12	4·02	2·08	0·95	0·39	0·09	0·00

The coefficients of contraction indicated are:

c_c . . .	0·67	0·65	0·64	0·65	0·67	0·68	0·72	0·77	0·85

and these variations may be explained in general terms by the effect of the wider orifice in the earlier stages, followed by that of the partial suppression of contraction at the upper edge in the later stages.

The theory may therefore be applied, and it would appear that for a similar sluice on a larger scale, the coefficients of contraction being smaller, we may expect to get somewhat larger values of ζ.

Cock or Throttle Valve.—The pipes experimented on by Weisbach were:

(i) Circular, 1·58 inch in diameter (4 cms.); and

(ii) Rectangular, 1·98 inch × 0·96 inch (5·02 cms. × 2·48 cms.). The coefficients in terms of the angle θ through which the cock or valve is turned, are given below; $\theta = 0$, corresponding to full on:

θ	Cock.				Throttle Valve.			
	Circular Pipe.		Rectangular Pipe.		Circular Pipe.		Rectangular Pipe.	
Degrees.	n	ζ	n	ζ	n	ζ	n	ζ
0	1·000	0·00	1·000	0·00	1·000	0·17	1·000	0·26
5	0·926	0·06	0·926	0·05	0·913	0·24	0·913	0·28
10	0·850	0·29	0·849	0·31	0·826	0·52	0·826	0·43
15	0·772	0·75	0·769	0·88	0·741	0·90	0·741	0·77
20	0·692	1·56	0·687	1·84	0·658	1·54	0·658	1·34
25	0·613	3·10	0·604	3·45	0·577	2·51	0·577	2·16
30	0·535	5·47	0·520	6·12	0·500	3·91	0·500	3·54
35	0·458	9·68	0·436	11·2	0·426	6·22	0·426	5·72
40	0·385	17·3	0·352	20·7	0·357	10·8	0·357	9·25
45	0·315	31·2	0·269	41·0	0·293	18·7	0·293	15·3
50	0·250	52·6	0·188	94·5	0·234	32·6	0·234	24·9
55	0·190	106	0·116	309	0·181	58·8	0·190	42·7
60	0·137	206			0·134	118	0·137	77·4
65	0·091	486			0·094	256	0·091	159
70					0·060	751	0·052	369

The original experiments indicate that when n, the ratio:

$$\frac{\text{Area of free passage}}{\text{Area of pipe}} = \frac{A_o}{A_p}$$

is small, the value of ζ is influenced by the circumstances of the discharge: *e.g.* whether a long pipe succeeds the valve or not, and whether the discharge is into air or into water. To judge from Weisbach's remarks, these differences are mainly, if not entirely, due to the flow not being regular under small heads such as occurred towards the end of the discharge. It is therefore unlikely that these differences will occur when the head through the valve is greater than 3 or 4 inches, as will probably be the case in practical applications. The values tabulated have been selected from those observations in which this irregularity of flow was least marked.

It would appear that the deflection produced by the throttle valve has very little influence on the value of ζ compared with the contraction in the area of the passage. No deductions as to applicability to larger valves can be given.

Circular Diaphragm in a Circular Pipe.—Weisbach found experimentally as follows:

Ratio of areas $A_o = nA_p$	$n=$	0·1	0·2	0·3	0·4	0·5
Approach channel much larger than pipe . . .	$\zeta=$	231·7	51·0	19·8	9·61	5·26
	$c_c=$	0·616	0·614	0·612	0·610	0·607
Diaphragm in a cylindrical pipe . .	$\zeta=$	225·9	47·8	30·8	7·80	1·75
	$c_c=$	0·624	0·632	0·643	0·659	0·681
Ratio of areas $A_o = nA_p$	$n=$	0·6	0·7	0·8	0·9	1·0
Approach channel much larger than pipe . . .	$\zeta=$	3·08	1·88	1·17	0·73	0·48
	$c_c=$	0·605	0·603	0·601	0·598	0·596
Diaphragm in a cylindrical pipe . .	$\zeta=$	1·80	0·80	0·29	0·06	0·00
	$c_c=$	0·712	0·755	0·813	0·892	1·00

Note.—In the first case the correction for change of velocities explained on page 786 was applied before calculating the lost head.

The values of c_c, agree fairly well with what might be expected, since we know that partially guiding a jet after exit increases the coefficient of contraction. Hence it is probable that the values of ζ hold for larger pipes, but are slightly increased. As a matter of fact, we know that the value $\zeta = 0·48$ (which corresponds to the head lost at entry in the ordinary case of pipe flow) is increased to 0·505 in large pipes, and this would indicate that $c_c = 0·584$. This is about 2 per cent. below the theoretical value, and the difference is probably explained by skin friction, as has already been pointed out in the case of cylindrical orifices (p. 149). The values of ζ for larger pipes are probably some 3 or 4 per cent. in excess of those given above.

Sudden Contraction in a Pipe.—Merriman (*Treatise on Hydraulics*, p. 177) suggests that the loss of head may be calculated from the formula:

$$\zeta\frac{v_s^2}{2g} = \left(\frac{1}{c} - 1\right)^2 \frac{v_s^2}{2g}$$

where v_s is the velocity in the smaller pipe, and:

$$c = 0·582 + \frac{0·0418}{1·1 - r}$$

where $r\ (= \sqrt{n})$, is the ratio of the diameters of the two pipes. He gives:

$r=$0·0	0·4	0·6	0·7	0·8	0·9	0·95	1·0
$c=$0·62	0·64	0·67	0·69	0·72	0·79	0·86	1·00
$\zeta=$0·375	0·317	0·242	0·202	0·151	0·071	0·026	0·00

The rule is founded on the collation of several series of experiments, and is therefore quoted. The actual loss, however, appears to be largely affected by the circumstances of the prior motion, and the character of the contraction.

Merriman's rule applies best to cases where the velocity is high, and the contraction sharp, such as small pipes, coupled up by metallic joints, and conveying water under high pressure. This is probably the case which occurs most frequently in practice.

As a contrast, Brightmore (*P.I.C.E.*, vol. 169, p. 323) experimented on 6-inch pipes, contracting to 4 inches and 3 inches in diameter.

The experiments on the contraction of a 6-inch to a 3-inch pipe show that the loss of head is only very slightly less than $\frac{(v_s-v_p)^2}{2g}$, so that we may believe that, due to the pipes being rusted, the orifice formed by the contraction was not "sharp," and that the loss is therefore that caused by the reduction in velocity from that in the 6-inch pipe, to that in the 3-inch pipe. The results obtained in the 6-inch to 4-inch contraction are markedly irregular, and cannot be represented by any formula. If we put:

v_p = velocity in the 6-inch pipe,
and v_s = velocity in the 4-inch pipe,

we find, as the mean of several observations, that:

when $v_p = 4\cdot2$ feet per second. The head lost $= \frac{0\cdot57(v_s-v_p)^2}{2g}$

when $v_p = 4$ feet per second. The head lost $= \frac{0\cdot73(v_s-v_p)^2}{2g}$

when $v_p = 2\cdot5$ feet per second. The head lost $= \frac{0\cdot65(v_s-v_p)^2}{2g}$

The matter deserves closer study. For the present, it would appear that when the head lost is a quantity measured in feet, Merriman's value is probably the best; but when the head lost is small, and can best be measured in inches, the loss is best represented by the expression $h = \frac{0\cdot7(v_s-v_p)^2}{2g}$. The change of law will probably prove to be more or less intimately connected with the critical head phenomena discovered by Bilton (see p. 142). For the present it appears advisable to use Merriman's formula only when the head given by his equation exceeds Bilton's critical value for an orifice of the size of the smaller pipe, as we are then safe against an underestimation of the loss.

Freeman (*Trans. Am. Soc. of C.E.*, vol. 21, p. 463) experimented on fire nozzles. These are orifices at the end of a channel of an area comparable to that of the orifice. Putting r, as the ratio $\frac{\text{Diameter of orifice}}{\text{Diameter of pipe}} = \sqrt{\frac{A_o}{A_p}}$, Merriman (*ut supra*) finds that the experiments agree very fairly with $C = 0\cdot571 + \frac{0\cdot0429}{1\cdot1 - r}$ where C, represents the coefficient of discharge of the nozzle. The tabulation of the experiments is as follows:

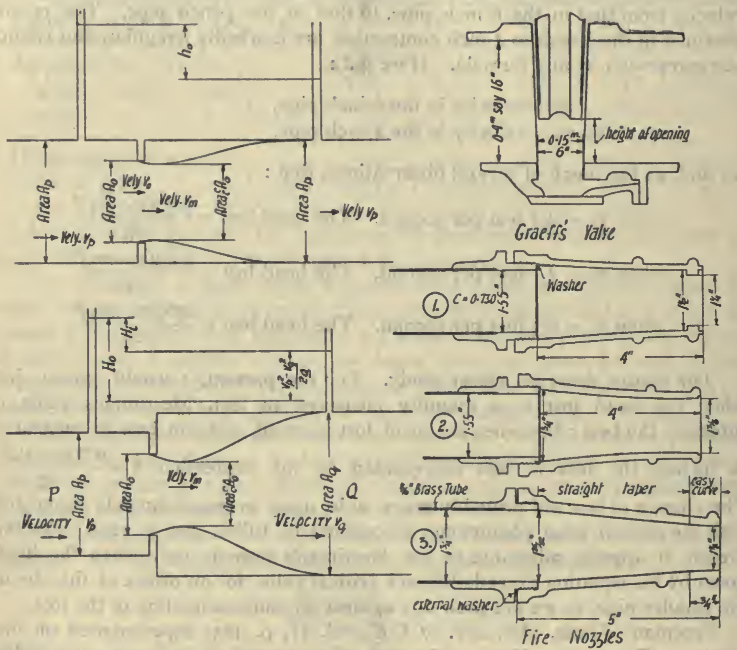
r	0·5	0·833	0·848	0·886	0·95	1·000
C (experimental) .	0·634	0·736	0·729	0·742	0·866	0·975
C (calculated) . .	0·643	0·732	0·741	0·771	0·857	1·000

These values are applicable to nozzles of the form shown in Fig. No. 1, Sketch No. 233. Fig. No. 2 shows a more favourable case, and No. 3 Freeman's standard design which produces C=0·95 to 0·97.

In large scale experiments the difficulties discussed under Enlargements (see p. 795) occur, but by no means to so marked a degree. If the edge of the contraction is sharp, a vena contracta occurs beyond the contraction, and the rules given on page 786 are applicable. (See Sketch No. 234.)

In practical experiments on a large scale the edge must usually be regarded as rounded, and the value

$$H_o = \frac{(v_s - v_p)^2}{2g}$$



SKETCH No. 233.—Contractions and Enlargements in Pipes.

probably overestimates the loss, provided that the motion in the larger pipe has become steady. If, however, the motion is unsteady, the rule $\frac{1\cdot5(v_s - v_p)^2}{2g}$ is probably more correct, but accurate experiments on a large scale do not exist. In some experiments smaller values have been found, but it may be suspected that these are due to incorrect positions of the pressure gauges, or to the motion being stream line (see p. 17). Some experiments of my own, where A_p, was 8 inches in diameter, and A_s, was 6 inches in diameter, give values varying between 1·2, and 1·4 $\left\{\frac{(v_s - v_p)^2}{2g}\right\}$; and as in Brightmore's work the values are apparently accidental, in that duplicate experiments rarely give identical results, and the differences are greater than can be explained by errors of observation.

Sudden Enlargements in a Pipe.—Let A_1, be the area of a pipe which conveys a quantity of water $A_1 v_1$, and let the area of the pipe section suddenly alter to A_2, where A_2, is greater than A_1.

If the water fills the enlargement, and the pipe continues to flow full bore, the new velocity is v_2, where :

$$v_2 A_2 = v_1 A_1$$

If h_1, and h_2, represent the pressures at points in the areas A_1, and A_2, which are at the same level, we have :

$$h_1 + \frac{v_1^2}{2g} = h_2 + \frac{v_2^2}{2g} + H_o, \text{ say,}$$

where H_o, represents the "head lost" at the enlargement.

Borda has given a theoretical investigation, which shows that there is reason to believe that

$$H_o = \frac{(v_1 - v_2)^2}{2g}$$

As a matter of experiment, this value of H_o, is usually slightly exceeded.

Baer (*Dingler's Journal*, March 23, 1907) experimented on pipes where A_1, was a circle of 2 inches (0·05 metre) in diameter, and the ratio $\frac{A_2}{A_1}$, was successively 3, 6, and 11·55.

The values of v_1, ranged from 1·05, to 9·28 feet per second in each case. Putting $H = \frac{(v_1 - v_2)^2}{2g}$ and $H' = \frac{v_1^2 - v_2^2}{2g}$ we find that :

(i) For $\frac{A_2}{A_1} = 3$. The loss of head is slightly greater than H, and is practically, H + a constant, for all values of v_1.

(ii) For $\frac{A_2}{A_1} = 6$. The law is as for $\frac{A_2}{A_1} = 3$, but the constant is slightly greater.

(iii) For $\frac{A_2}{A_1} = 11·55$. The loss is more nearly represented by H′ − a constant.

We may therefore state that :

When $\frac{A_2}{A_1}$ is small (say not greater than 3), the loss is very close to H, but slightly exceeds H, and this excess increases as $\frac{A_2}{A_1}$ increases.

When $\frac{A_2}{A_1}$, is greater than 10, the loss is better represented by H′ − a constant, and the constant decreases as $\frac{A_2}{A_1}$, increases.

St. Venant states that :

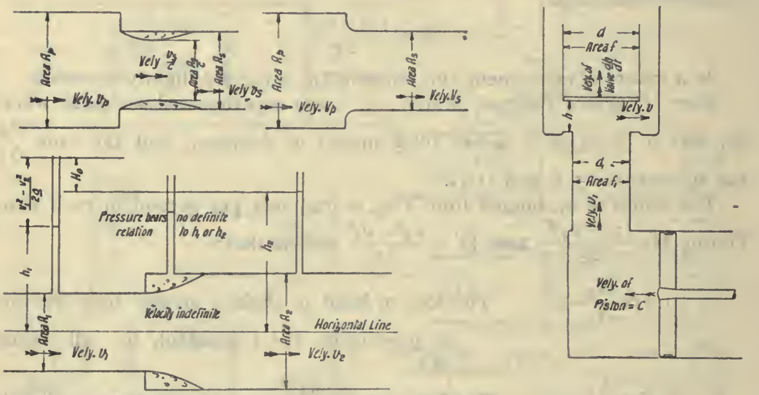
$$\text{Loss of head} = \frac{(v_1 - v_2)^2}{2g} + \frac{1}{9} \frac{v_2^2}{2g}$$

This rule agrees very closely with Baer's results for $\frac{A_2}{A_1}=3$, although the differences are greater than can reasonably be explained by errors in measurement. The rule, however, is quite sufficiently accurate for all practical purposes.

I suggest the following :

If $\frac{A_2}{A_1}$ is less than 2.	Then the head lost = H.
If $\frac{A_2}{A_1}$ is between 2 and 4.	Then St. Venant's rule may be applied.
If $\frac{A_2}{A_1}$ is between 4 and 10.	Then the head lost $=\frac{3H+H'}{4}$.
If $\frac{A_2}{A_1}$ is over 10.	Then the head lost = H'.

These rules are intended to slightly overestimate the loss.



SKETCH No. 234.—Pipe Contractions and Motion of a Pump Valve.

Brightmore (*P.I.C.E.*, vol. 169, p. 323) worked with a 3-inch pipe, enlarging to 6 inches in diameter; and a 4-inch pipe, enlarging to 6 inches in diameter. After correcting for friction and change of velocity in the length of pipe between the points where the pressure was observed, he found that :

If $\frac{A_2}{A_1}=4$. Then the head lost = H for all velocities up to $v_2=3{\cdot}5$ feet per second.

If $\frac{A_2}{A_1}=2{\cdot}25$. Then the head lost = H − 0·04 to 0·10 foot, for velocities up to $v_2=6{\cdot}5$ feet per second.

I consider that the differences are explicable by uncertainties as to the exact value of the friction.

The whole question is also probably greatly influenced by the roughness of the pipes.

In actual experiments, if the ratio $\frac{A_2}{A_1}$, is large, the stream issuing from the smaller pipe may move as a jet without mixing with the mass of eddying water which lies in the corners of the enlargement. If only a short length of pipe, of an area A_2, occurs, it may then be found that the loss bears no fixed relation to $\frac{v_1^2}{2g}$. In such cases, the amount of head lost (or rather, the length of the path which must be traversed before the loss is complete), is not determined in any way, and although St. Venant's value, or the value $\frac{v_1^2-v_2^2}{2g}$, may be finally attained when the jet has filled the pipe, the distance beyond the enlargement where this occurs, depends on accidental circumstances. Full bore flow without eddies may be attained in a length equal to the diameter of the pipe, or may not occur for 40 diameters or more. In many cases, where the enlargement is but temporary, the moving water may flow through the enlargement between "banks" of eddying water, with but slight diminution of velocity, and consequently with but small loss of head. (Sketch No. 234

In *Cassier's Magazine* (March 1907), the head lost in a pipe 9 feet in diameter, and 29 feet in length, which then enlarged to 12 feet in diameter by a diverging cone 18 feet long, and was followed by 43 feet of piping, 12 feet in diameter, is given for values of v_2, up to 8 feet per second. The results are puzzling, and some error in the friction coefficients may be suspected. There is, however, not the slightest doubt that the excess of the observed loss of head over that which is given by the usual friction rules is very close to:

$$\frac{v_1^2-v_2^2}{2g}$$

and that no reasonable assumption concerning the values of the friction coefficients will permit the value $\frac{(v_1-v_2)^2}{2g}$ to be obtained.

The lengths of the pipe are extremely short in proportion to the diameters, but until further evidence is available, it is advisable to regard the larger value $\frac{v_1^2-v_2^2}{2g}$ as nearer the truth than the usual rule $\frac{(v_1-v_2)^2}{2g}$ in such cases.

Labyrinth Packings.—These packings are well known, and although they are more employed in small apparatus than in large machinery, of late years they have been used with great success on a large scale.

If a, be the area of a narrow passage, and A, be the area of the enlargements, the theory developed above indicates that for a leakage Q, the head lost at each enlargement will be:

$$\frac{Q^2}{2g}\left(\frac{1}{a}-\frac{1}{A}\right)^2$$

and the energy of the exit velocity being also lost, we see that for a packing consisting of n enlargements, and $n+1$ constrictions, the total head, or pressure difference, is:

$$\frac{Q^2}{2g}\left\{n\left(\frac{1}{a}-\frac{1}{A}\right)^2+\frac{1}{a^2}\right\}=\frac{Q^2}{2g}\,\frac{n+1}{a^2} \text{ approx.}$$

whence the leakage can be calculated.

Actually, an allowance should be made for skin friction.

The experiments of Becken (*Ztschr. D.I.V.*, July 20, 1907), on a packing as per Sketch No. 235, where the circumstances, owing to the zigzag path, are far more favourable to loss by shock than in the ordinary packing, show that the above theory is fairly close to the truth, but that in practical designs the effect of friction is by no means negligible. In actual examples, the friction can be taken as proportional to v^2 (although under small differences of pressure capillary motion with resistance proportional to v, does occur); and if l, be the length, and s, the height of the constricted passages, we can write:

$$\text{Difference in pressure} = \frac{Q^2}{2ga^2}\left(n + 1 + 0{\cdot}02\frac{nl}{s}\right)$$

where, of course, both l, and s, may be variable.

In ordinary cases, s, varies between 9, and 3 thousandths of an inch.

SKETCH No. 235.—Labyrinth Packings.

Losses of Head at Gradual Enlargements, or Contractions.—The usual treatment of this somewhat important subject is defective. The application of the theoretical principles to the case of Venturi meters has been considered on page 79. The draft tubes of turbines, and Herschell's fall intensifier form other practical examples.

Unfortunately, all the precise experiments upon the subject are on a small scale, and very many of them are useless. In the first place, Andres (*Ztchr. D.I.V.*, Sept. 17, 1910) has plainly shown that the manner in which the water enters the enlargement (experiments on contractions do not exist, but the effect is probably very small in such cases) greatly influences the loss. Thus, if the water immediately before reaching the enlargement is passed through a fine mesh sieve, the loss differs considerably from that which occurs when the sieve is replaced by a diaphragm containing an orifice, or by an apparatus which causes the water to rotate round the axis of the diverging tube. Secondly, unless the pressures are so arranged that the vacuum existing at the contraction does not exceed a certain value (usually 20 to 24 feet head of water, or 10 to 14 feet absolute, but which is obviously dependent upon the temperature and the amount of air contained in the water), air will be released from the water, which will not therefore entirely fill the tube. Hence, the velocities cannot be calculated from the geometrical size of the pipes, and the experiments are useless.

Thirdly, in small scale experiments, the quantity of water passing is frequently such that at some point of its passage the velocity falls below Osborne Reynolds' upper critical velocity (see p. 20). The law of skin friction consequently changes, and the experiments are useless.

The necessity of bearing these conditions in mind is shown by the fact that 206 out of the 254 experiments conducted by Fliegner are certainly affected by one or other of these causes, and possibly even some of the remainder as well.

The theoretical treatment is simple. Let the velocity at the throat (or cross-section of minimum area) of a diverging tube be equal to v_m, feet per second, and let h_m, be the pressure in feet of water at this point. Then, h, the pressure at a point where the velocity is equal to v, is given by the following equation:

$$h - h_m = \frac{v_m^2 - v^2}{2g} = \text{H, say,}$$

and v_m, and v, can be calculated from the geometrical form of the mouthpiece, when the quantity of water passing is known.

The corrections for skin friction are uncertain. If the mouthpiece is circular in cross-section, and conical in longitudinal section, we find that:

$$h - h_m = \frac{v_m^2 - v^2}{2g}(1 - \text{K}) \quad . . . (\text{A})$$

where,

$$\text{K} = \frac{g}{\text{C}^2 \tan\frac{\delta}{2}}$$

where δ is the vertical angle of the cone, and $v = \text{C}\sqrt{rs}$, is the frictional equation of the pipe. An equation of similar form, differing only in the fact that K, is multiplied by a constant, can be derived when the frictional equation is:

$$v = \text{A}r^n\sqrt{s}$$

It is usually stated that C, should be selected so as to correspond with the velocity v_m, and a size of pipe equal to that of the throat of the mouthpiece. This statement does not appear to possess any very reliable experimental basis, and I doubt if it can be verified.

As will later be seen, this detail is not of much practical importance.

In actual practice, however, a very important distinction exists. If the motion is directed from the wider end of the mouthpiece towards the throat, the corrected equation is found to represent the experimental facts with sufficient accuracy. The fall in pressure indicated in equation No. A occurs, and any difference between theory and experiment is quite sufficiently explained by uncertainties as to the exact value of C.

If, however, the motion is reversed, and is from the throat towards the wider end, the observed increase in pressure is less than the value calculated from the above equation, and the difference is usually greater than can be explained by any reasonable value of the friction coefficient. As Andres states, the transformation of velocity into pressure is imperfect, and an extra loss (which he terms the divergence loss) of head over and above that due to friction occurs.

The experiments carried out by Andres were on twenty-two forms of diverging tube, with circular, square, and rectangular cross-sections. Of these, nine were ordinary conical mouthpieces, and in seven the longitudinal sections were so calculated that if Bernouilli's equation held exactly, the pressure head h, would have varied uniformly from one end of the mouthpiece to the other. In the other six, the pressure head would have varied in a parabolic manner.

Andres defines :

$$\eta = \frac{2gh_1}{v_m^2 - v^2} = \frac{h_1}{h - h_m},$$

where h_1 is the observed difference between the pressure at the wider portion of the tube, where the velocity is equal to v, and the pressure at the throat of the tube, where the velocity is v_m.

Thus, $\eta = 1$, would mean that Bernouilli's equation was accurately true. Of course the influence of skin friction prevents $\eta = 1$, from ever being attained. The maximum possible value of h_1, is $\frac{v_m^2}{2g} - \frac{v^2}{2g}$ — head lost in skin friction, and then, $\eta = 1 - K$.

The best results are invariably obtained with the nine conical tubes, for which the mean value of η in normal motion is 0·720. For the seven straight line pressure tubes the mean is 0·658, and for the six parabolic tubes the mean is 0·673. I shall therefore merely consider the conical tubes, although variations in η of a similar nature are produced in all forms of tube by alterations in the character of the water motion. The character of the water motion is fixed as follows :

The water before entering the diverging tube is passed :

I. Through twenty fine wire sieves.
II. Through one similar sieve.
III. The normal case where there is no obstruction in the pipe.
IV. The water is passed through three small circular holes in a diaphragm.
V. The water is passed through a channel obstructed by spirally twisted vanes, and thus enters the tube with a rotary motion round its axis.

The effect of these preliminary operations is best illustrated by taking the mean values of η for all the tubes (conical or otherwise) experimented upon.

(A) The mean value of η for seven tubes, when treated according to the method described in No. I, is 0·696, and the mean value of η for these same seven, in normal motion, is 0·761.

(B) For six tubes, treated according to case No. II, we get a mean $\eta = 0·656$, and the mean for the same six tubes in normal motion is 0·727.

(C) For twelve tubes, treated as in case No. IV, we get a mean $\eta = 0·807$, and the mean for the same twelve tubes in normal motion is 0·776.

(D) For eight tubes, treated as in case No. V., the mean value of η is 0·881, and for the same eight in normal motion it is 0·810.

Thus, we see that the more disturbed and turbulent the motion, the better is the value of η obtained. The high values obtained for a rotating motion suggest that the real reason why η does not always reach its theoretical value as corrected for friction is to be found in the fact that the moving water does not naturally fill the tube completely, but that an annulus of dead, or eddying water, forms between the walls of the tube, and the jet of moving water which issues from the throat.

The following tabulation shows the results for the six circular conical tubes, the symbols being shown in Sketch No. 236.

d_1	d_2	l	$\rho=\left(\frac{d_1}{d_2}\right)^2$	$\tan\frac{\delta}{2} = \frac{d_1-d_2}{2l}$	η for Case No.					Remarks.
In Inches.					I.	II.	III.	IV.	V.	
										Surface—
2·86	0·62	10·62	21·07	0·105			0·744	0·824	0·892	Unpolished.
1·74	0·60	8·46	8·47	0·068	0·865	0·871	0·883	0·925	0·989	Polished.
1·78	0·60	8·46	8·70	0·068			0·854	0·861	0·964	Unpolished.
1·77	0·67	10·23	6·86	0·054		0·806	0·838	0·903	0·920	,,
1·20	0·60	8·46	4·00	0·034	0·890		0·890	0·892	0·928	Polished.
1·20	0·61	8·08	3·88	0·037			0·812	0·855	0·857	Unpolished.

The final deductions by Andres for the case of a conical enlargement of circular cross-section are founded on a discussion of his own experiments, and of those of Francis and Banninger, and are as follows :

The theoretical increase in pressure being $h-h_m = H = \frac{v_m - v^2}{2g}$ that actually observed in normal motion (Case III.) is h_1. We can account for a certain portion of the difference $H-h_1$, by skin friction, say $h_f = \frac{gH}{C^2\tan\frac{\delta}{2}}$. Andres puts $h_1 = H-(h_a+h_f)$, and finds that $h_a = \eta_a H$, where η_a is a function of δ only, being independent of C, and of v_m, provided that the pressures are so adjusted that air is not disengaged at the throat, and that Reynolds' critical velocity is not approached at any point in the enlargement (see p. 20).

The points representing the values of η_a are somewhat irregularly distributed, although not more so than can be explained by the difficulties already enumerated. The following table may be used :

$\delta =$	12°	11°	10°	9°	8°	7°	6°	5°	4°	3°	2°
$\eta_a =$	0·20	0·13	0·09	0·066	0·055	0·050	0·046	0·042	0·039	0·036	0·033

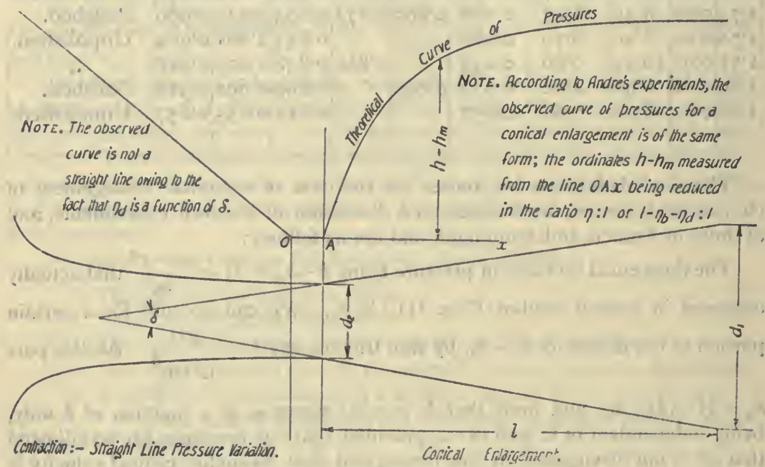
It will be plain that the smaller C is, the less will be the value of δ which will give the largest value of $\eta = \frac{h_1}{H}$, and the best design is found by making $\eta_a + \frac{h_f}{H}$ a minimum, or η a maximum.

It would also appear that a similar equation may be applied to enlargements which are not conical. A mean value of δ or of η_a is selected for each short length. This short length is then considered as conical, and η is calculated. The value of $h'_1 = \eta H'$, where H′ is the theoretical increase in pressure in the short length, can thus be obtained, and by repeating the process the pressure at any point in the enlargement can be ascertained. Andres states that the results of this process agree fairly well with observation.

A similar law probably holds good for enlargements which are square or rectangular in section; but as these, in practical cases (*e.g.* the wheel or guide vane passages of turbines, or centrifugal pumps), generally have a curved axis further and more detailed experiments are necessary before any useful rule can be given.

Gibson (*Proc. Roy. Soc.*, vol. 83, p. 368) has also investigated this subject, using tubes of varnished wood in which: $d_1 = 3$ inches, $d_2 = 1{\cdot}5$ inch. The arrangements were by no means so perfect as in Andres' experiments, but for that very reason they resemble those which are likely to be employed in engineering practice.

The results obtained by Andres for normal motion are generally confirmed, more especially the most important one that with tubes of circular cross-section,



SKETCH No. 236.—Andres' Experiments.

at any rate, a conical enlargement is more efficient than any other form. So also, η_d is found to depend upon δ only, provided that "the velocity exceeds 5 feet per second."

The following tabulation shows the values of δ and η_d for the pipes experimented upon. The differences between this table and that given by Andres (which I consider to be the more practical inside its own range) are easily explicable by the fact that Gibson used the formula:

$$h_f = \mu\int\frac{v^{1\cdot78}}{d^{1\cdot20}}dl$$

in order to determine the head lost in friction:

δ . .	90°	60°	50°	40°	30°	20°	17½°
η_d . .	0·67	0·72	0·67	0·60	0·49	0·24	0·19
δ . .	15°	12½°	10°	7½°	5°	4°	3°
η_d . .	0·14	0·10	0·073	0·052	0·028	0·022	0·013

SYMBOLS FOR PUMP-VALVES

$a = \frac{f\pi n}{30\, lv}$ (see p. 803).

$b = \frac{d-d_1}{2}$ (see also p. 805). B (see p. 804).

$c = \frac{dh}{dt}$, is the velocity of the valve cover in feet per second.

c_s, is the velocity with which the valve cover strikes its seat.

$C = U \frac{\sin \pi nt}{30}$, is the velocity of the pump piston in feet per second.

C_s, is the valve of C at the instant when $c = c_s$.
d, is the diameter of the valve cover in feet.
d_1, is the diameter of the orifice in the valve seat in feet.
E (see p. 804).

$f = \frac{\pi}{4} d^2$, is the area of the valve cover in square feet.

$f_1 = \frac{\pi}{4} d_1^2$, is the area of the orifice in the valve seat.

F, is the area of the pump piston in square feet.
h, is the lift in feet of the valve cover above its seat.
h_{max}, is the maximum value of h.
h_o, is the value of h when $C = o$.

$H = \zeta \frac{v_1^2}{2g}$, is the head lost at the valve in feet.

i, is the number of guide ribs and s is the thickness of one rib, thus $l = \pi d - is$.
l, is the nett circumference of the cylinder traced out by the edge of the valve cover which is free for the passage of water. $l = \pi d$ or $\pi d - is$.
l_p (see p. 804).
$m = \frac{F}{f}$. M_p, M_v (see p. 804).
n, is the number of revolutions the pump makes per minute.
P, is the load on the valve in lbs.
Q, is the delivery of the pump in cusecs.
S, is the stroke of the pump in feet.
v, is the velocity of the water through the area lh in feet per second.

v_1, is the velocity of the water through the area f_1, i.e. $v_1 = \frac{Q}{f_1}$, but v is not equal to $\frac{Q}{lh}$ (see p. 802).

α, β, ν (see p. 805).
κ (see p. 805).
χ (see p. 805).

SUMMARY OF EQUATIONS

$$\tan \chi = -a$$

$$= \frac{mfU}{lv\sqrt{1+a^2}} \sin\left(\frac{\pi nt}{30} + \chi\right); \quad h_{max} = \frac{mfU}{lv\sqrt{1+a^2}}$$

$$h_o = -\frac{mfU}{lv}\,\frac{a}{1+a^2}$$

$$c = \frac{mUa}{\sqrt{1+a^2}}\left(\cos\frac{\pi nt}{30} + \chi\right). \quad c_s = \frac{mUa}{\sqrt{1+a^2}}$$

$$C_s = \frac{Ua}{\sqrt{1+a^2}}. \quad c_s = mC_s = \frac{\pi n h_{max}}{30} = -\frac{lv}{f}h_o$$

$$c_s = 0{\cdot}0055\,\frac{FSn^2}{lv} = 0{\cdot}0055\,\frac{Qn}{lv}$$

VALVES.—It may at first sight appear that the treatment of this subject by a civil engineer is somewhat presumptuous. My excuse is that the necessity for some special and definite knowledge on the matter has been forcibly brought to my notice by failures in my own designs. I have also found that the troubles which occur in pumping machinery may generally be attributed to incorrect proportioning of the valves, and an engineer in charge of pumps in a locality which is far distant from a regular workshop can at any rate assure himself that if his troubles (exclusive of those caused by neglect or careless repairs), are not due to errors in valve design, it is extremely improbable that his mechanical resources will be sufficiently extensive to permit any remedy.

I believe that the literature on the subject is exclusively of German origin, and that it has not been translated.

Motion of a Pump Valve.—The ideal case of the motion of a valve through which water is being pumped by an ordinary piston pump can be investigated as follows:

Let F, denote the area of the pump piston, and $C=U \sin \frac{\pi nt}{30}$, its velocity at any time t.

Let f, be the area of the cover (or moving portion) of the valve; and let h, represent the height of the cover above the valve seat; or, for shortness, the height of the valve opening.

Then $c=\frac{dh}{dt}$, is the velocity of the valve cover, or, for shortness, the velocity of the valve.

Let l, be a length such that lh, represents the area which is free for the passage of water between the valve cover and its seat, when h, is the height of the valve. That is to say, in a simple, circular valve of d feet diameter, $l=\pi d$, where $f=\frac{\pi d^2}{4}$.

Let v, be the velocity of the water through the space between the valve cover and its seat, so that lhv, represents the quantity of water entering the rising main. Then plainly:

$$FC=lhv+fc$$

where fc, represents a volume of water that passes through the orifice in the valve seat, but does not at once get through the valve itself, being temporarily stored up underneath the valve cover.

Now, $C=U \sin \frac{\pi nt}{30}$, where n, is the number of revolutions of the pump per minute ($2n$=the number of strokes in a double acting pump), and therefore:

$$lvh=f\left\{mU \sin \frac{\pi nt}{30}-\frac{dh}{dt}\right\}$$

where $F=mf$. Thus, $h=\dfrac{mfU}{lv\sqrt{1+\left(\frac{f}{lv}\frac{\pi n}{30}\right)^2}} \sin\left(\frac{\pi nt}{30}+\chi\right)$

and $c=\dfrac{dh}{dt}=\dfrac{mfU\frac{\pi n}{30}}{lv\sqrt{1+\left(\frac{f}{lv}\frac{\pi n}{30}\right)^2}} \cos\left(\frac{\pi nt}{30}+\chi\right)$

where $\tan \chi = -\frac{f}{lv}\frac{\pi n}{30}$, as is easily proved by substituting the values in the original equation.

Let a, denote the small quantity $\frac{fn\pi}{30lv} = -\tan \chi$.

Now, the maximum value of h, occurs when $\frac{\pi nt}{30} + \chi = \frac{\pi}{2}$;

and $h_{max} = \frac{mfU}{lv\sqrt{1+a^2}} = \frac{mfU}{lv}$ approximately.

When $t=0$, or $\frac{30}{n}$ (*i.e.* at the dead points of the piston stroke), h, is not zero, but has a certain value, h_0, say, and :

$$h_0 = \frac{mfU}{lv\sqrt{1+a^2}} \sin \chi = -\frac{mfU}{lv}\frac{a}{1+a^2} = -\frac{mf^2U\pi n}{(lv)^2 30}, \text{ approximately,}$$

and when $t = \frac{30}{n}$, $h = -h_0$, and is positive. Thus when the piston arrives at its dead point the delivery valve is still slightly open, while the suction valve (on the other side of the piston) has closed a little before this instant.

So also, $h=0$, when $\frac{\pi nt}{30} + \chi = 0$; or, since χ is a small angle, we can put $\tan \chi = \chi$, so that $h=0$, when $t = \frac{f}{lv}$.

Thus, the interval between the arrival of the piston at a dead point and the closure of the valve is independent of the rate at which the pump runs, and of the length of its stroke. This has been experimentally confirmed by Bach (see Mueller, *Das Pumpen Ventil*).

The velocity with which the valve closes down on its seat is obtained by putting :

$$\cos\left(\frac{nt\pi}{30} + \chi\right) = 1, \quad \text{and is } c_s = \frac{mfU\pi n}{30lv\sqrt{1+a^2}} = \frac{mUa}{\sqrt{1+a^2}}.$$

The simultaneous velocity of the piston is:

$$C_s = -U \sin \chi = \frac{Ua}{\sqrt{1+a^2}}.$$

Therefore, $c_s = mC_s = h_{max}\frac{\pi n}{30} = -h_0\frac{lv}{f}$.

Now, put S, for the stroke of the pump. Then, $S = \frac{60U}{n\pi}$.

Thus, $c_s = \frac{FS\pi^2 n^2}{60 \times 30 lv} = 0{\cdot}0055\frac{FSn^2}{lv} = 0{\cdot}0055\frac{Qn}{lv}$,

where $Q = FSn$, is the delivery through the valve in cubic feet per second, that is to say, is one half of the delivery of the pump in the case of a double acting pump.

Consequently, we may at once deduce that if a pump with a stroke equal to S, is known to work without shock at n, revolutions per minute, when delivering Q, cusecs, a pump with valves of the same design will work without shock, provided that its stroke, speed, and delivery are such that :

$$n_1^2 S_1 = n^2 S, \quad \text{or } Q_1 n_1 = Qn.$$

The great importance of the velocity v, which expresses the speed of the water through the valve opening, is also evident.

The above theory cannot be considered as complete. In many cases we should take into account the acceleration of the valve. The height to which the valve can rise is frequently limited.

It is also quite certain that in many cases (other than piston pumps) v, is not constant, but is greatly influenced by the value of h. Nevertheless, it is plain that if we merely consider the motion of the valve just before its closure, we can assume that v, is constant; and can thus determine the magnitude of the shock produced by closure of the valve, with a very fair degree of accuracy.

It will be seen that some shock must occur, and that the limit at which the shock becomes detrimental is somewhat a matter of opinion.

Lindner (*Ztschr. D.I.V.*, Aug. 29, 1908) states that c_s, should not exceed 0·33 foot per second. Berg (*Ztschr. D.I.V.*, Aug. 6, 1904), states that there is no "audible shock," provided that h_o, does not exceed $\frac{d}{250}$, in a circular valve, the diameter of the valve cover being d, or $\frac{B_1}{250}$ in the case of an annular valve, where B, is the radial breadth of the valve cover.

This mathematical investigation may at first sight appear to be somewhat artificial. Berg (*ut supra*) gives several graphic comparisons between the path of a valve cover as actually observed, and as calculated by this theory. The agreement is satisfactory, more especially during the interval just before the closure of the valve. When applying the theory to practical problems the difficulties which arise are mainly caused by the fact that the experiments were not well adapted to discover what are the conditions under which the shock at closure proves detrimental. Berg, Bach, and Westphal, all appear to consider that the valve works satisfactorily, provided that it does not give rise to an audible sound when closing. If a metal valve "sounds," it will rapidly become leaky, but valve covers made of leather or rubber may work well under circumstances which would cause a metal valve to sound.

Thus the above opinions are only approximations, for the harm done by the shock depends upon the total energy of the moving water at the instant of closure. At this moment the piston is moving with a velocity equal to C_s; and the valve being closed, the water in any passage in communication with the piston is moving with a velocity equal to c_p, where $c_p F_p = C_s F$, the area of the passage being F_p. If l_p, be the length of this passage, the mass of water moving with a velocity c_p is $\frac{62 \cdot 5}{g} F_p l_p = M_p$, say.

$$\text{Thus, the total energy is } \Sigma M_p \frac{c_p^2}{2} + \frac{M_v c_s^2}{2} = E, \text{ say.}$$

Or $E = \frac{62 \cdot 5}{2g} C_s^2 F^2 \Sigma \frac{l_p}{F_p} + \frac{M_v c_s^2}{2}$, where M_v, is the mass of the valve, and the summation is extended over all the water which is not separated from the piston by a closed valve, or by an air chamber.

Thus, if the pressure valve is considered, we must take into account all the water in the cylinder, on the valve side of the piston, and also the water in the suction main down to the foot valve, unless there is an air vessel on the suction main, in which case we have only to consider the portion of the main between the cylinder and the air chamber.

The Value of v.—The value of v, has been shown to be of primary importance in this connection, whatever opinion may be adopted as to the precise conditions for avoiding detrimental shock.

Bach (*Versuche über Ventilbelastigung und Ventilwiderstand*, and other papers, a complete list of which is given in Hutte, vol. 1, p. 884) states as follows :

Let P, be the load on the valve in lbs. including its own weight, and any spring, or other mechanically produced loading.

$f_1 = \frac{\pi d_1^2}{4}$ is the area of the opening in the seat of the valve (not the valve cover), in square feet.

v_1, is the velocity of the water through the area f_1, i.e. $v_1 f_1 = vlh + fc = \mathrm{FC}$.

Let $b = \frac{d - d_1}{2}$ be the breadth of the valve seat.

The head lost in the motion through the valve, is given by $\mathrm{H} = \zeta \frac{v_1^2}{2g}$.

(*a*) Then, for valves without guiding ribs, *i.e.* in which $l = \pi d$:

$$\mathrm{P} = 62{\cdot}5 f_1 \frac{v_1^2}{2g}\left\{\lambda + \left(\frac{d_1}{4\mu h}\right)^2\right\}.$$

(*b*) If the valve has i guide ribs, of a thickness represented by s, so that $l = \pi d - is$:

$$\mathrm{P} = 62{\cdot}5 f_1 \frac{v_1^2}{2g}\left\{\lambda + \left(\frac{f_1}{\mu(\pi d_1 - is)h}\right)^2\right\}.$$

The values of λ and μ are variable.

I. For flat valves with no ribs (see Sketch No. 237, Fig. 1):
Case (*a*), with :

b, lying between $0{\cdot}10 d_1$, and $0{\cdot}25 d_1$,
and h, lying between $0{\cdot}10 d_1$, and $0{\cdot}25 d_1$.

$$\lambda = 2{\cdot}5 + 19\frac{b - 0{\cdot}1 d_1}{d_1}, \quad \mu = 0{\cdot}60 \text{ to } 0{\cdot}62\,;$$

$$\zeta = a + \beta\left(\frac{d_1}{h}\right)^2, \quad \text{where } a = 0{\cdot}55 + 4\frac{b - 0{\cdot}1 a_1}{d_1},$$

$$\beta = 0{\cdot}16 \text{ to } 0{\cdot}15,$$

the first values of μ and β occurring when b, is large.

II. For flat valves with ribs (see Fig. No. 2) :
Case (*b*) :

λ and μ are 10 per cent. less than in Case (*a*),

$$\zeta = a + \beta\left\{\frac{d_1^2}{(\pi d_1 - is)h}\right\}^2$$

a is 1·8 to 2·6 times the value of a in Case (*a*), and
$\beta = 1{\cdot}70$ to $1{\cdot}75$.

III. For conical edged valves, with $b = 0{\cdot}1 d_1$, $h = 0{\cdot}1 d_1$ to $0{\cdot}15 d_1$ (see Fig. No. 3), as in Case (*a*) :

$$\lambda = -1{\cdot}05, \quad \mu = 0{\cdot}89.$$

$$\zeta = a + \beta\left(\frac{d_1}{h}\right) + \gamma\left(\frac{d_1}{h}\right)^2\,; \quad a = 2{\cdot}60, \quad \beta = -0{\cdot}8, \quad \gamma = 0{\cdot}14.$$

IV. For valves with a conical bottom and with conical seat. With $h = 0{\cdot}125 d_1$ to $0{\cdot}25 d_1$ (see Fig. No. 4), as in Case (a):

$$\lambda = 0{\cdot}38, \qquad \mu = 0{\cdot}68.$$

$$\zeta = a + \beta\left(\frac{d_1}{h}\right)^2: \qquad a = 0{\cdot}60, \qquad \beta = 0{\cdot}15.$$

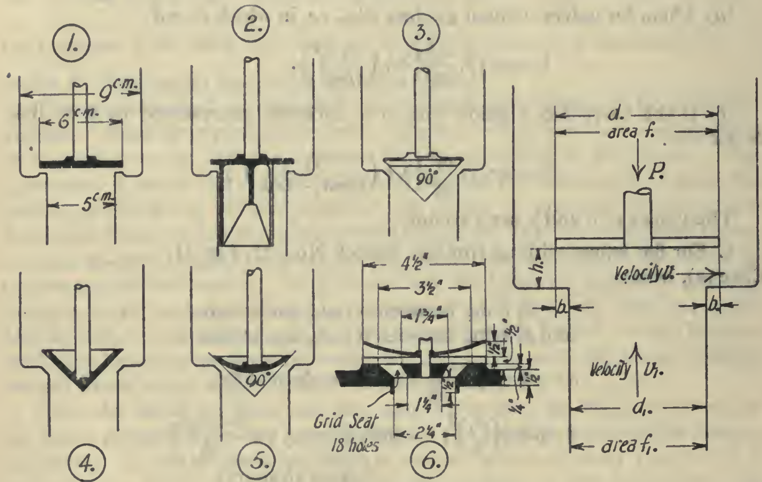
V. For valves with a spherical bottom and conical seat. With $h = 0{\cdot}1 d_1$ to $0{\cdot}25 d_1$ (see Fig. No. 5), as in Case (a):

$$\lambda = 0{\cdot}96, \qquad \mu = 1{\cdot}15.$$

$$\zeta = a + \beta\left(\frac{d_1}{h}\right) + \gamma\left(\frac{d_1}{h}\right)^2:$$

$$a = 2{\cdot}70, \qquad \beta = -0{\cdot}80, \qquad \gamma = 0{\cdot}14.$$

The values given above cannot be considered as universally applicable. With one exception they were all obtained from valves of the dimensions



SKETCH No. 237.—Pump Valves.

shown in Sketch No. 237, and apparently not more than one valve of each type was experimented on.

It will also be observed that the equations given refer to cases when the valve is fairly wide open. Bach (*Ztschr. D.I.V.*, May 15, 1886) has given equations for P, and ζ for two valves of Type I. which hold from $h = \frac{d}{50}$, to $h = \frac{d}{2}$. Berg (*ut supra*) states for flat valves (Type I.) that:

$$v_1 = \kappa\sqrt{2g\frac{\mathrm{P}}{f}}$$

and tabulates κ as follows, when $d = 60$ mm.

h	0	0·1	0·2	0·3	0·4	0·5	0·6	0·8	1·0	1·5 mm.
κ	0·65	0·71	0·78	0·845	0·89	0·911	0·913	0·902	0·870	0·788

For larger values of h, the equations already given apply.

It is probable that these values apply equally well to all valves except perhaps those included in Type II.

The load P, is usually produced by a spring. Putting $P = w + S$, where w, is the weight in water of the moving portions of the valve, and S, is the spring load, S, can be calculated as follows (see Unwin, *Machine Design*, p. 99):

Inch Units are used.—Let r, be the radius in inches of the cylinder on which the axis of the wire forming the spiral spring lies. Let d, be the diameter of the wire in inches, and n, the number of turns in the spiral spring.

Then, for steel wire, less than $\frac{3}{8}$ of an inch in diameter, a load of S pounds shortens the spring by δ inches, and:

$$\delta = \frac{Snr^3}{180{,}000d^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

S, must not exceed $\frac{12{,}000d^3}{r}$ lbs. . . . . . . [Inches]

The general equations are:

$$\delta = \frac{64r^3nS}{Cd^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad [\text{Inches}]$$

where C, is the coefficient of rigidity of the wire, and:

S, should not exceed $\frac{f\pi d^3}{16r}$ lbs. . . . . [Inches].

where f, is the permissible shearing stress in lbs. per square inch.

The calculation of v, is now obvious. We calculate the value of v_1, from the known value of P, and by substitution arrive at the value of v. Since we are mostly concerned with a valve which is very nearly closed, we can, for a first approximation, consider P, as the load corresponding to a closed valve (*i.e.* when P, is produced by a spring, the spring is extended as far as the valve seating permits). So also, if we think that it is desirable to take into account the acceleration of the valve, we can consider that P, is decreased by a term $M^1_v \frac{d^2h}{dt^2}$.

These figures are also useful when we are dealing with the valves of a hydraulic ram. Here P, is given, and also $H = \zeta\frac{v_1^2}{2g} = h_1$, in the original investigation (see p. 845). Consequently, we can calculate v_1, and so determine h, the opening of the valve. In practice, the question is best investigated by tabulating P, in terms of h, and h_1, or $\zeta\frac{v_1^2}{2g}$, in terms of v_1, and then selecting the correct values.

CHAPTER XIV.—(SECTION B)

WATER HAMMER

WATER HAMMER.—Pressure produced by a change in the velocity of water in a pipe—Effect of the time occupied in producing the change—Corrections for the elasticity of the pipe metal—Practical rules—Comparison with observation.

GRADUAL STOPPAGE OF MOTION IN A PIPE.—Gibson's investigation—Criticism—**General Investigation.**

Resonance.—Period of valve closure—Period of the pressure waves in the main—PRACTICAL APPLICATIONS—Values of η_2 and η_3.

. . . .

SYMBOLS

A, is the area of the pipe, in square feet.
a, is the area of the vena contracta near the valve (see p. 814).
a_1 (see p. 814).
$C = \frac{v}{\sqrt{rs}}$, is the coefficient of skin friction for the pipe.
d, is used for the sign of differentiation.
E (see p. 811).
f, is the diameter of the pipe, in feet.
H, is the total head, in feet, producing motion through the pipe and valve.
h, is the head producing motion through the valve.
K (see p. 811).
l, is the length of the pipe, in feet.
l_1 (see p. 815).
p_1, is the pressure near the valve when the motion of the water is uniform, and its velocity is v_1, feet per second.
p_i, is the maximum impulsive pressure.
p_m, is the mean value of the impulsive pressure.
p_s, is the statical pressure, *i.e.* the head H, expressed in pounds per square inch.
The p's, are all measured in pounds per square inch.
q (see p. 815).
Q (see p. 815).
t, is the symbol for time in general.
t_1 (see p. 811).
$T = \frac{2l}{\lambda}$ or $\frac{2l}{\lambda_e}$ (see p. 810).
T_o (see p. 814).
u, is the thickness of the pipe walls, in inches.
v_1, is the initial velocity of the water, in feet per second.
v_2, is the final velocity of the water in the pipe, in feet per second.
Δv (see p. 816).
η (see p. 816).
λ, and λ_e (see p. 811).
κv^2, represents the head lost in the pipe when the velocity is uniform and equal to v, feet per second.

SUMMARY OF EQUATIONS

$p_m=\frac{62{\cdot}5l}{144gt}(v_1-v_2)$ lbs. per square inch.

$p_t=\frac{2\times 62{\cdot}5l}{144gt}(v_1-v_2)+p_1-p_2$ lbs. per square inch.

Practical Formulæ.

(*a*) t, less than $\frac{2l}{4700}$ seconds (see p. 811).

$p_t=63{\cdot}4(v_1-v_2)+p_1-p_2$ lbs. per square inch maximum value.
Corrected for alteration in wave velocity produced by pipe thickness:

$p_t=60(v_1-v_2)+p_1-p_2$ for small pipes
$p_t=50(v_1-v_2)+p_1-p_2$ for large pipes } lbs. per square inch.

(*b*) $p_t=0$ if t, is greater than $\frac{0{\cdot}027(v_1-v_2)}{p_2-p_1}$.

WATER HAMMER.—When the motion of water in a pipe is altered in any way, the change in the velocity of flow is attended by a certain alteration in the momentum of the whole mass of water in the pipe. This can only be effected by a definite force.

Let l, be the length of the pipe in feet, and A be the area of its cross-section in square feet. The mass of water in the pipe is expressed by:

$$\frac{62{\cdot}5lA}{g}$$

and if v_1, be the velocity of the water, in feet per second, the momentum is:

$$\frac{62{\cdot}5lAv_1}{g}$$

If the velocity be reduced to v_2, in a period of t, seconds, the rate of change in momentum is:

$$\frac{62{\cdot}5lA}{gt}(v_1-v_2)$$

and the force required to produce this rate of change is:

$$F=\frac{62{\cdot}5lA}{gt}(v_1-v_2)$$

Thus p_m, the mean impulsive pressure required to produce the change in velocity is given by:

$$144Ap_m=F$$

$$\text{or, } p_m=\frac{62{\cdot}5l}{144gt}(v_1-v_2) \text{ lbs. per square inch.}$$

Now, let p_1, be the pressure in pounds per square inch, measured at the valve, the closure of which produces the alteration in velocity, when the velocity is v_1, and is uniform.

Let p_2, be the similar pressure when the velocity is v_2, and is uniform. Then, approximately $p_1=p_s-kv_1^2$, and $p_2=p_s-\kappa v_2^2$, where p_s, is the statical pressure.

Sketch No. 238 shows the general course of events during the time that elapses while the uniform velocity v_1, is changed to the uniform velocity v_2. The diagram indicates that the change in the pressure from p_1, to p_2, does not proceed uniformly, but that the value of the pressure oscillates backwards and forwards, and only attains a steady value equal to p_2, after a certain time has elapsed. The maximum impulsive pressure generated by the alteration of velocity is given by $p_i+p_1-p_2$, and experiment shows that this is very approximately equal to $2p_m$.

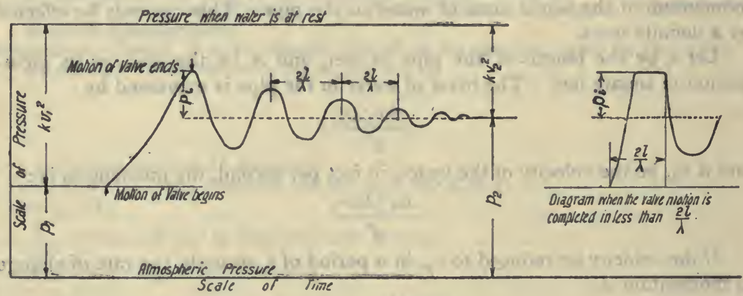
We therefore denote this maximum impulsive pressure by p_i, and find that:

$$p_i=2p_m+p_1-p_2=\frac{2\times 62{\cdot}5l}{144gt}(v_1-v_2)+p_1-p_2 \quad \text{lbs. per square inch}$$

represents the maximum pressure tending to rupture or otherwise damage the pipe.

It will be plain that the equation could be obtained by assuming that the relation between pressure and time is of an approximately triangular shape, as shown in Sketch No. 238. If the matter is thus regarded, the experimental results merely indicate that the minor waves in the time and pressure diagram do not materially affect the maximum value of the pressure.

So far we have implicitly assumed that water is incompressible, so that the velocity of the whole volume of lA, cube feet of water is changed from v_1, to v_2,



SKETCH NO. 238.—Impulsive Pressure in a Pipe.

in the interval t. This is not the case, and the diagram of pressures already referred to shows that the pressure jumps up and down much as a weight hung on a spring would do. Mathematical investigations, which have been confirmed experimentally, show that if t, be less than a certain value, T, the pressure does not increase beyond the value given by putting $t=T$, in the above equation. The value of T, can be calculated from the equation $T=\frac{2l}{\lambda}$ where λ is the velocity with which a wave of compression travels in the water and the pipe. We thus have two cases to consider:

(i) Where t, is less than $\frac{2l}{\lambda}$. Put $t=\frac{2l}{\lambda}$ in the equation for p_i, and we get:

$$p_i=\frac{62{\cdot}5\lambda}{144g}(v_1-v_2)+p_1-p_2.$$

(ii) Where t, is greater than $\frac{2l}{\lambda}$:

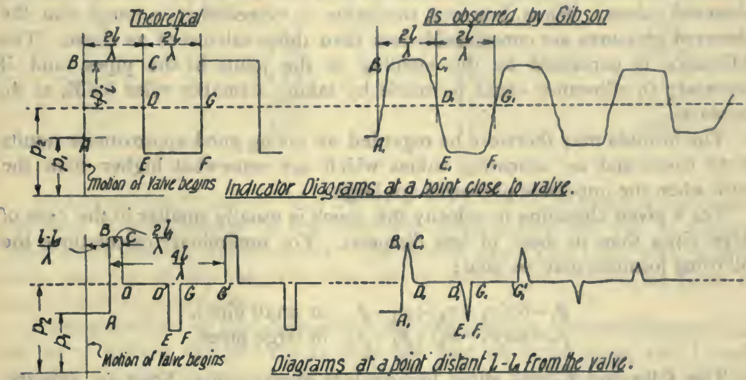
$$p_i=0{\cdot}027\frac{l}{t}(v_1-v_2)+p_1-p_2.$$

The distinction between the two cases is at first sight somewhat artificial. Its physical basis is best realised by considering that if t, be less than $\frac{2l}{\lambda}$ the whole change of velocity near the valve is completed before the wave of compression can travel up to and return from the upper end of the pipe. Thus, the momentum of the water at and near to the upper end of the pipe cannot be transmitted with sufficient rapidity to produce an effect in increasing p_i, and what really happens is that the value p_i, is attained not for one instant only, as is shown in Sketch No. 238, but for a certain space of time as is indicated in the inset to the Sketch. The real basis of the theory is the fact that it can be experimentally confirmed.

Considering the first case, we have as follows:

Theoretically λ is equal to the velocity of sound in water, and is equal to about 4700 feet per second. Thus:

$$p_i = 63{\cdot}4(v_1 - v_2) + p_1 - p_2 \quad \text{lbs. per square inch.}$$



SKETCH No. 239.—Gradual Change of Velocity in a Pipe.

In actual practice, we must take into account the fact that not only is water compressible, but that the metal of which the pipe is composed is extensible. This fact somewhat reduces the value of λ.

Let K, be the bulk modulus of water, $=300{,}000$ lbs. per square inch.

Let E, be Young's modulus for the metal of which the pipe is made, *i.e.* $E=30{,}000{,}000$ lbs. per square inch for steel, and $E=15{,}000{,}000$ lbs. per square inch for cast iron.

Let R, be the mean radius of the walls of the pipe, and t_1, their thickness in inches. Then:

(i) If the pipe is fixed so that it can only expand circumferentially, the effective value of λ, say λ_e, is:

$$\lambda_e = \frac{\lambda}{\sqrt{1+\frac{2K}{E}\frac{R}{t_1}}} = \frac{\lambda}{\sqrt{1+\frac{R}{50t_1}}} \quad \text{for steel,}$$

$$= \frac{\lambda}{\sqrt{1+\frac{R}{25t_1}}} \quad \text{for cast iron.}$$

(ii) The pipe can expand both longitudinally and circumferentially:

$$\lambda_e = \frac{\lambda}{\sqrt{1+\frac{5}{2}\frac{KR}{Et_1}}} = \frac{\lambda}{\sqrt{1+\frac{R}{40t_1}}} \text{ for steel,}$$

$$= \frac{\lambda}{\sqrt{1+\frac{R}{20t_1}}} \text{ for cast iron.}$$

The first case is that which most closely resembles the conditions which are usually met with in practice.

The experimental verification of these formulæ has been effected by Joukowsky, Gibson, and others. When the value of $63 \cdot 4(v_1-v_2)$, does not greatly exceed 120 to 150 lbs. per square inch, or v_1-v_2, does not greatly exceed 2, to 2·5 feet per second, it is found that the calculated values of p_i, corrected for the difference between λ and λ_e, agree very exactly with the observed values. When, however, this value is exceeded it is found that the observed pressures are considerably less than those calculated as above. The difference is explained by the yielding of the joints in the pipes, and if necessary an allowance could be made by taking a smaller value for E, as p_i, increases.

The formula may therefore be regarded as giving good approximate results in all cases, and as indicating values which are somewhat higher than the truth when the impulsive pressures are large.

For a given alteration in velocity the shock is usually smaller in the case of large pipes than in those of less diameter. For preliminary calculations the following formulæ may be used:

$$p_i = 60(v_1-v_2)+p_1-p_2 \quad \text{in small pipes.}$$
$$p_i = 50(v_1-v_2)+p_1-p_2 \quad \text{in large pipes.}$$

The following figures show Joukowsky's observations (*Stoss in Wasserleitungsrohren*) on pipes 4 and 6 inches in diameter, with $l=1050$ feet, and 1066 feet, and $t=0 \cdot 03$ seconds. Consequently, t, is less than $\frac{2l}{\lambda_e}$. The calculated values are obtained from the formulæ:

$$p_i = 63 \cdot 7v, \text{ corresponding to inextensible pipes;}$$
$$\text{and } p_i = 56 \cdot 6v, \text{ corresponding to a cast iron pipe, with}$$
$$\frac{R}{t_1} = 8, \quad \text{and } E = 15{,}000{,}000 \text{ lbs. per square inch,}$$

where v, now represents the change in velocity, and in this particular case the valve being entirely closed:

$$v_2 = 0, \text{ so that } v = v_1.$$

The second case presents but few difficulties, and it is sufficient to observe that $p_i=0$, or that there is no shock provided that:

$$t, \text{ is equal to, or greater than } \frac{0 \cdot 027 l(v_1-v_2)}{p_2-p_1}.$$

FOUR-INCH PIPE.

v, in Feet per Second.	p_i observed in Lbs. per Square Inch.	p_i calculated. $p_i = 63{\cdot}7\,v$ in Lbs. per Square Inch.	From the Formula $p_i = 56{\cdot}6v$ in Lbs. per Square Inch.
0·5	31	31	27·9
1·9	115	118	106·0
2·9	168	183	162·0
4·1	232	258	228·0
9·2	519	580	512·0
SIX-INCH PIPE.			
0·6	43	38	33·5
1·9	106	118	106·0
3·0	173	189	167·0
5·6	369	353	312·0
7·5	426	472	429·0

GRADUAL STOPPAGE OF MOTION IN A PIPE.—This problem is extremely complex. The principles of the following investigation are due to Gibson (*Hydraulics and its Applications*). The results agree very fairly with experiment, and the theory may therefore be considered as correct, for pipes of small diameter at any rate.

A consideration of the terms which are neglected in the investigation has led me to believe that the theory may not hold for large pipes, and I shall therefore discuss a more exact theory later on. I have not, however, been able to discover any case where Gibson's theory does not agree very fairly well with experiment, and the accurate theory may therefore be regarded merely as a subject for investigation in cases where Gibson's theory does not entirely agree with the results which are experimentally obtained.

Referring to the theory already laid down, we find that the important instant of time is $\frac{2l}{\lambda}$, seconds before the valve closes down on its seat.

Let v_1, be the velocity of water in the pipe close to the valve at this moment.

Let $\left(\frac{dv}{dt}\right)_1$, represent the rate of change of the velocity at this moment.

Then, since λ is the rate at which a wave of compression travels along the pipe, the velocity at the upper end of the pipe is :

$$v_1 + \frac{l}{\lambda}\left(\frac{dv}{dt}\right)_1$$

and the mean velocity of the whole body of water in the pipe is :

$$v_1 + \frac{l}{2\lambda}\left(\frac{dv}{dt}\right)_1$$

This velocity is reduced to o, in a time $\frac{2l}{\lambda}$, and also at the instant considered the water is being retarded at a rate represented by $\left(\frac{dv}{dt}\right)_1$. Thus, it is necessary to apply the results of the two cases which have been previously discussed and the pressure produced by the closure is:

$$p_i = 63{\cdot}4\left\{v_1+\frac{l}{2\lambda}\left(\frac{dv}{dt}\right)_1\right\}+\frac{62{\cdot}5l}{144g}\left(\frac{dv}{dt}\right)_1+p_1-p_2$$

where p_1, is the pressure corresponding to uniform velocity v_1, and p_2, is the pressure when the valve is shut, and the water is at rest (i.e. $p_2 = p_s$).

The determination of v_1, and $\left(\frac{dv}{dt}\right)_1$ is effected by Gibson as follows:

Let H, be the total head in feet causing motion through the pipe, *i.e.* the head corresponding to the pressure $p_s = p_2$.

Then, if f, be the diameter of the pipe, and h, be the portion of H, expended in forcing the water through the valve, then:

$$H = \frac{v^2 4l}{C^2 f}+h$$

where $v = C\sqrt{rs}$, is the friction equation of the pipe. Also, if a, be the area of the vena contracta of the jet issuing from the valve, and v_v, the velocity of the jet at this point, then:

$$av_v = \frac{\pi f^2}{4}v = Av, \text{ say,}$$

$$\text{and } h = \frac{v_v^2}{2g} = \frac{v^2}{2g}\left(\frac{A}{a}\right)^2.$$

$$\text{Thus, } H = v^2\left(\frac{4l}{C^2 f}+\frac{A^2}{2ga^2}\right) = \kappa v^2, \text{ say.}$$

Now, Gibson assumes that the valve closes so that $a = \frac{At}{T_o}$, where t, is the time before the complete closure of the valve, and T_o, is the time before closure when a, was equal to A, assuming that the valve was closed uniformly.

Thus, putting $a_1 = \frac{A2l}{\lambda_e T_o}$, so that a_1, is the area of the vena contracta at a time $\frac{2l}{\lambda_e}$ before the complete closure of the valve, we get:

$$v_1 = \sqrt{\frac{H}{\frac{4l}{C^2 f}+\frac{A^2}{2ga_1^2}}}\,;\quad \left(\frac{dv}{dt}\right)_1 = \frac{\sqrt{H}}{\left\{\frac{4l}{C^2 f}+\frac{A^2}{2ga_1^2}\right\}^{1.5}}\ \frac{A^3}{2ga_1^3 T_o}.$$

From these values p_1, and p_i, can be calculated.

The theoretical difficulties are obvious, and Gibson has since (see *Water Hammer in Pipes*) treated the problem by a different method. In particular, the value for $\left(\frac{dv}{dt}\right)_1$, is plainly obtained in a somewhat peculiar manner, and the equation:

$$\left(\frac{dv}{dt}\right) = \frac{g}{l}(H-\kappa v^2)$$

might be employed with advantage.

Nevertheless, the results calculated by Gibson's method agree very well with observation, and the discrepancies which occur when the pressure is large are probably amply explained by yielding of the lead joints of the pipes, as has been previously mentioned.

The experiments refer to pipes 3·6 inches in diameter, and a careful examination of the theory leads me to doubt if it will be found equally accurate when applied to larger pipes. The general investigation now given renders it likely that Gibson's equation does not lead to results which are less than the truth, unless the motion of the valve is purposely adjusted so as to produce resonance effects. I therefore believe that Gibson's method is valuable, for if its application shows that the calculated pressures do not exceed values which the pipes can sustain, we are entitled to consider that failure by shock is improbable.

General Investigation.—Consider Sketch No. 239. We see that if the velocity is altered by an amount represented by dv, waves of compression and rarefication are set up in the water, and in the metal of the pipe. The observed values of the pressure can be represented by $q = \frac{62{\cdot}5}{144g} dv\phi(\tau)$ lbs. per square inch, where τ represents the time reckoned from the moment when the valve was first moved, *i.e.* the point A, and q, is measured from the dotted line, which shows the value of the pressure when the oscillations have completely died out. Now, putting λ for the effective velocity of these waves (*i.e.* their velocity when corrected for the elasticity of the pipe), theoretically if l, be the length of the pipe, and l_1, be the distance of the point considered from the upper end of the pipe, we have:

Where, measuring from the line DG, or $p=p_2$:

$\frac{62{\cdot}5}{144g} dv \quad \phi(\tau) = -p_2+p_1$, so long as τ is less than $\frac{l-l_1}{\lambda}$

$\phi(\tau) = \quad 1$, when τ lies between $\frac{l-l_1}{\lambda}$ and $\frac{l+l_1}{\lambda}$

$\phi(\tau) = \quad 0$, ,, ,, $\frac{l+l_1}{\lambda}$,, $\frac{3l-l_1}{\lambda}$

$\phi(\tau) = -1$, ,, ,, $\frac{3l-l_1}{\lambda}$,, $\frac{3l+l_1}{\lambda}$

$\phi(\tau) = \quad 0$, ,, ,, $\frac{3l+l_1}{\lambda}$,, $\frac{5l-l_1}{\lambda}$

and thereafter the last four values of $\phi(\tau)$ repeat with a period of $\frac{4l}{\lambda}$.

The observed values do not agree with the theoretical ones, as is evident from Sketch No. 239, and we get the wavy curves $A_1B_1C_1D_1E_1$ . . . in place of the crenellated diagrams ABCDE . . . The difference is possibly largely explained by defects in the indicating apparatus, and as the diagrams given in Sketch No. 239 were taken from a pipe which was only four inches in diameter, where friction has a relatively large influence, it is possible that good diagrams from say a 4-foot pipe would resemble the theoretical curve far more closely.

Putting this aside for the moment, it is plain that the total impulsive pressure produced at any time t, by a continuous movement of the valve, is given by the equation :

$$Q=\frac{62{\cdot}5}{144g}\int_0^{\tau}\left(\frac{dv}{dt}\right)_1 \phi(t-t_1)\, d(t-t_1),$$

where t, is reckoned from the commencement of the motion of the valve and $\left(\frac{dv}{dt}\right)_1$ is the value of $\left(\frac{dv}{dt}\right)$, when $t=t_1$, and $\phi(t-t_1)$ or $\phi(\tau)$ is a function of t, l and l_1 of the character shown.

This equation is almost useless. But let us suppose that we have experimentally obtained a diagram of the type shown in Sketch No. 239, and let this diagram be reduced so as to correspond to $dv=1$ foot per second. Then if:

The area of the hump $A_1B_1C_1D_1=\eta_1$.
The area of the hump $D_1E_1F_1G_1=\eta_2$, where η_2 is negative.
The area of the third hump $=\eta_3$, and so on.

Then theoretically:

If $l_1=l$,	If l_1 is not equal to l,
$\eta_1=\frac{2l}{\lambda}=\mathrm{T}$	$\eta_1=\frac{2l_1}{\lambda}$
$\eta_2=\frac{-2l}{\lambda}=-\mathrm{T}$	$\eta_2=-\frac{2l_1}{\lambda}$, and so on.

Now, let Δv_1, represent the change in v, during the interval

$$t=0 \text{ to } t=\mathrm{T}=\frac{2l}{\lambda}.$$

Let Δv_2, represent the change in v, during the interval

$$t=\mathrm{T} \text{ to } t=2\mathrm{T}=\frac{4l}{\lambda}.$$

Let Δv_n, represent the change in v, during the interval

$$t=(n-1)\mathrm{T} \text{ to } t=n\mathrm{T}.$$

Then, when $t=n\mathrm{T}$, we have approximately:

$$\mathrm{Q}=\eta_1\Delta v_n+\eta_2\Delta v_{n-1}+\eta_3\Delta v_{n-2}+\text{etc. }+\eta_n\Delta v_1.$$

The form lends itself to solution by the method of arithmetical integration. Starting at $t=\mathrm{T}$ with uniform velocity v_1, we can ascertain Δv, by considering that the total available head H, is consumed as follows:

(i) In the friction head required to maintain a velocity equal to $v_1-\frac{1}{2}\Delta v_1$.

(ii) In the head required to produce a discharge equal to $\mathrm{A}\{v_1-\frac{1}{2}\Delta v_1\}$ through the variable opening formed by the valve.

(iii) In accelerating the velocity from v_1, to $v_1-\Delta v_1$ in the time T. This of course in the usual case of a closing valve has a negative sign.

The mathematical aspect of the question is fully treated under the head of Water Towers, and in actual practice I have found that it is simplest to use the two following equations:

(*a*) H − Q = The friction head + The retardation head.
(*b*) Q = The head producing discharge through the valve.

The experimental difficulties are very great. The time and pressure diagrams are not easily obtained, and are probably affected by instrumental errors in very much the same manner as indicator diagrams of gas engines. I should not therefore have considered that this discussion (which is defective in many ways) was worth the space it occupies were it not that it enables the principles affecting "resonance" to be intelligibly dealt with.

Resonance.—It will be obvious that while η_1, η_3, etc. are positive, η_2, η_4, etc.

are negative. If therefore for any reason $(\Delta v)_{n1}$, $(\Delta v)_{n-3}$, etc. are equal to 0, or are small in comparison with the terms $(\Delta v)_n$, $(\Delta v)_{n-2}$, etc. the value of Q, may become very large. Thus, any method of valve closure which is likely to produce such results is extremely prejudicial. As examples, let us assume that T=0·5 second, and that the valve is closed by a machine running at 120, or 60 revolutions per minute. It is obviously possible for the machine to fall into step with the pressure oscillations of the water (which must be carefully distinguished from the far slower visible oscillations which the whole mass of water in the pipes performs, and which produce surges in the surface of the water in any vessel which is in communication with the pipe), and thus produce abnormal pressures. A similar action may occur if the natural period of the governor which regulates the admission of water to a turbine is a small multiple of T. The matter cannot be reduced to calculation, as the pressures induced are so great that safety is generally secured by the machinery being forcibly put out of step. But for this very reason, the effects (when they do occur) are far reaching. Thus, a pumping engine which naturally ran at or near to a prejudicial rate would probably be forced to run at a slightly different period, although cases have occurred where the pipes were damaged before the period was changed. A far smaller machine, however, which worked on a balanced valve would not be materially affected by the induced pressures, and there is little doubt that a machine thus situated could be designed which would infallibly break any long pipe carrying water at sufficient velocity, when adjusted so as to close down the valve in the appropriate manner.

The question has not as yet become acute, although most hydraulic engineers have experienced "inexplicable fractures," and are well aware that a repetition of these fractures is likely unless some small and apparently unimportant modification is introduced when the break is repaired.

Designers of turbine governors are, however, fairly well aware that the relation between the period of the governing machinery and the length of the main conveying water to the turbines requires consideration; and in these cases the question is likely to become more acute when the deflecting nozzle (see p. 929) is abandoned in favour of regulators which do not secure safety by wasting water. Indeed, I have been privately informed by several engineers that trouble has already been experienced in certain cases, and regret that owing to the difficulties which attend accurate observation none of these gentlemen were able to furnish numerical data which would permit the matter to be theoretically tested. In one case, however, I am fairly certain that resonance was produced by the coincidence of the period of a three-phase alternator with that of the pressure main.

PRACTICAL APPLICATIONS.—At present we are without any definite information for large pipes, but it may be inferred that the larger the pipe, the less rapidly the values of η decrease.

It would therefore appear that very fair results will be obtained in small pipes by considering only the first term. In the case of larger pipes diagrams of a simple alteration, v_o, to v_1, say, must be taken and studied.

When investigating a question of this character, which related to a pipe 12 inches in diameter, I was led to take the following as values of the ratio η : $\frac{2l}{\lambda}$

for l $\eta_1 = 0·9$ $\eta_2 = -0·8$ $\eta_3 = 0·7$
$\eta_4 = -0·6$ $\eta_5 = 0·2$ and $\eta_6 = \eta_7 = 0$

I do not pretend that these values have any pretensions to accuracy. They appeared reasonable when compared with the best procurable diagram, and the results obtained permitted me to discover that a small fracture which had occurred, was probably not due to resonance, but to an accidental stoppage in a branch main.

Trial being made after the obstruction had been removed, the fracture was not repeated.

The values of η_2, η_3, etc., depend upon the reflection of the wave at the upper end of the pipe. The subject has been investigated by Rayleigh (*Theory of Sound*) and by Gibson.

The following circumstances appear to favour a good reflection :

(i) The upper end of the pipe opens well below the water surface in the forebay, or feeding reservoir.

(ii) The pipe has neither alterations in section, nor a bellmouth entry.

Our object is to minimize η_2, η_3, etc., as far as possible.

Thus, the upper end of the pipe should be bellmouthed, and should enter the reservoir at as high a level as possible (allowance for unavoidable variations in water level being made). Also a wall, or other obstruction in front of the upper end of the pipe, should be avoided if possible. The best solution, therefore, is a bellmouth entrance in a horizontal plane, as little below the lowest water level as is possible.

CHAPTER XIV.—(SECTION C)

EJECTORS AND SYPHONS

JET PUMP, OR WATER EJECTOR.—General theory.
SYPHONS.—General theory—Preliminary design of a syphon—Syphons carrying air and water—Theory—Examples—Practical applications.

SYMBOLS CONNECTED WITH JET PUMPS

a, is the area of the orifice of the jet.

a_2, is the area of the cross-section of the ejector cone at the point where the mixture is complete, which is assumed to be the throat, or minimum section of the cone.

a_3, is the area of the cross-section of the pipe through which the mixture is delivered.

$A = av + cu$. $A_1 = av^2 + cu^2$ (see p. 821).

$B = H + h_b + \frac{v_3^2}{2g}$ (see p. 822).

c, is the area of the orifice through which the substance lifted is delivered previous to mixture.

h, is the gauge pressure, and $H_0 = h + h_b$, the absolute pressure, in feet of water, of the jet or pressure water just before it passes through the orifice a.

h_b, is the height of the water barometer, in feet (see p. 6).

H, is the geometrical lift, in feet, from the ejector to the reservoir into which the mixture is delivered.

H_1, is the absolute pressure, in feet of water, existing at the point where the pressure water and the lifted substance first come into contact.

H_2, is the absolute pressure, in feet of water, at the point where complete mixture takes place, *i.e.* at the cross-section a_2.

H_3, is the absolute pressure in feet of water at the cross-section a_3. $H_3 = B$, approximately, or accurately, B + friction terms.

k, is the gauge pressure, and $K_0 = k + h_b$, the absolute pressure in feet of water, at which the lifted substance is delivered just before it passes the orifice c.

$q = \frac{av + cu}{av} = \frac{\text{Total discharge of ejector}}{\text{Discharge of jet}}$ (see p. 538).

u, is a velocity such that cu, is the total quantity in cusecs of the substance lifted by the ejector.

v, is a velocity such that av, is the total quantity in cusecs of pressure water passing through the jet.

$v_2 = \frac{av + cu}{a_2}$, is the mean velocity in feet per second of the mixed substances just after mixture is complete.

v_3, is the velocity, in feet per second of the mixed substances in the rising main.

w, is the weight, in pounds per cube foot of the pressure water.

w_1, is the weight, in pounds per cube foot of the substance lifted.

ζ, is a coefficient expressing the frictional losses.

Thus, $\zeta\frac{v_2^2}{2g}$ = frictional loss of head in the converging cone of the ejector. Theoretically,

$\zeta = \frac{8gl}{C^2 d}$, where g, is the acceleration of gravity, l, the length of the pipe considered, and d, its diameter in feet. Practically this equation does not hold as C, is altered from its usual values by turbulence in the flow, and the mixture of air or sand with the water. Therefore, ζ is used to indicate this altered value.

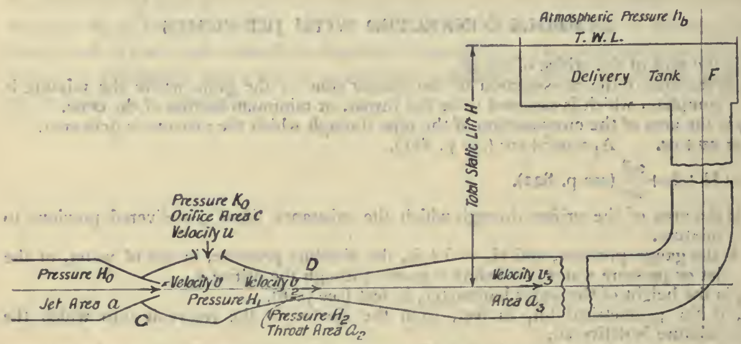
$\eta = \frac{H_o - h_b}{H_3 - h_b}$ (see p. 538).

JET PUMP, OR WATER EJECTOR.—The theory of this apparatus is in a very unsatisfactory state. The results developed below require experimental confirmation, and the most that can be said is that this theory is the least open to objection of any that have as yet been put forward.

Let a jet with an orifice of an area denoted by a, deliver a quantity of water av, at the point C.

Let the absolute pressure at which this water reaches the orifice be:

$H_o = h + h_b$ say, where h_b, is the height of the water barometer.



SKETCH NO. 240.—Jet Pump Diagram.

Let a quantity of water cu, be delivered at the point C, through an orifice the area of which is c, and let this water reach the orifice under an absolute pressure $K_o = k + h_b$ say.

Let the two bodies of water move along the pipe CD, and arrive at D, in a state that can, for practical purposes, be represented by a volume $av + cu$, moving with a uniform velocity v_2, so that the area at D, is $a_2 = \frac{av + cu}{v_2}$.

The mathematical difficulties of the investigation are entirely due to our ignorance of the processes involved in the complete mixture of the two bodies of water which is assumed to take place in the pipe or converging cone CD. The definition of complete mixture is plain,—the velocities at individual points in the area a_2, must not differ materially from those which prevail in a pipe of an area a_2, carrying a quantity of water equal to $a_2 v_2$, in steady motion.

So far as can be inferred from the practical details of ejectors (which are, however, known to be designed by rule of thumb only), the length CD, is usually about five or six times the diameter of a circle of an area a_2, and the best results are certainly obtained when CD, is a converging cone. There are also certain indications that H_1, as later defined, should be greater than h_1, if rapid mixture is desired.

Let the mixed stream pass along the diverging cone, and finally enter a pipe with an area represented by a_3, with a velocity $v_3 = \frac{a_2 v_2}{a_3}$. The stream then flows through this pipe and is delivered into a tank at F, say.

Let H_1, be the absolute pressure at C, *i.e.* just beyond the orifices.

Let H_2, be the absolute pressure at D, *i.e.* just after complete mixture has occurred, and assume that C, and D, are at the same level.

The motion between C, and D, may be considered as follows:

There enters at C:

(i) A quantity of water av, with an energy proportional to $H_1 + \frac{v^2}{2g}$.

(ii) A quantity of water cu, with an energy proportional to $H_1 + \frac{u^2}{2g}$.

During the motion from C, to D, the first quantity changes its velocity from v, to v_2, so that energy proportional to $\frac{(v-v_2)^2}{2g}$ is lost. Similarly, the second quantity loses energy proportional to $\frac{(u-v_2)^2}{2g}$.

In addition, some energy is lost by both quantities of water in frictional and other resistances to motion, and this is assumed to be proportional to $\zeta\frac{v_2^2}{2g}$.

Finally, the quantity $av+cu$, leaves D, with an energy proportional to $H_2 + \frac{v_2^2}{2g}$. In practice, v_2 is always greater than u or v.

The whole reasoning is a mass of assumptions, and experimental proof is difficult. Quite apart from any questions as to the validity of Borda's equation (which Zeuner and other good authorities consider to be inapplicable to the conditions prevailing) the value of ζ almost certainly bears no relation to C, in the equation $v = C\sqrt{rs}$, which would be the frictional equation for the pipe CD under ordinary circumstances.

Thus, all that can really be stated is that the form of the equation is probably correct, and therefore the investigation forms a starting point for comparison between ejectors which do not differ very greatly in size and general proportions.

We thus arrive at the following equation:

$$av\left(H_1+\frac{v^2}{2g}\right)+cu\left(H_1+\frac{u^2}{2g}\right)=(av+cu)\left\{H_2+\frac{v_2^2}{2g}(1+\zeta)\right\}+av\frac{(v-v_2)^2}{2g}+cu\frac{(u-v_2)^2}{2g}$$

Simplifying, and putting $A = av+cu$. $A_1 = av^2+cu^2$

$$A(H_1-H_2)=A\frac{v_2^2}{2g}(2+\zeta)-\frac{A_1 v_2}{g}$$

$$\text{or,}\qquad H_1-H_2=\frac{v_2^2}{2g}(2+\zeta)-\frac{A_1}{A}\frac{v_2}{g}$$

If in place of water lifting water, a fluid with a weight equal to w, lbs. per cube foot lifts a fluid of a weight equal to w_1, lbs. per cube foot, the equation is of precisely the same form, but:

$$A = wav + w_1cu, \qquad \text{and } A_1 = wav^2 + w_1cu^2\,;$$

and the heads H_1, and H_2, are expressed as feet of a fluid of a weight equal to: $\frac{wav+w_1cu}{av+cu}$ lbs. per cube foot, *i.e.* the weight of the mixed fluid that flows in the rising main.

The mixture has now to be lifted to F, at a height H, above the point D. Thus, for the motion from D, to F, we have:

$$H_2 + \frac{v_2^2}{2g} = H + h_b + \frac{v_3^2}{2g}(1+\zeta_2) = B + \zeta_2\frac{v_3^2}{2g} = H_3 \text{ say};$$

where $\zeta_2\frac{v_3^2}{2g}$ represents the head lost by friction in the rising main, so that if this be of a length equal to l, and of a diameter d, then $\zeta_2 = \frac{8gl}{C^2d}$ approximately.

Adding these equations, we get:

$$H_1 = B + \frac{v_2^2}{2g}\left\{1+\zeta+\zeta_2\left(\frac{a_2}{a_3}\right)^2\right\} - \frac{A_1v_2}{Ag}$$

$$= B + \frac{v_2^2}{2}\left\{1+\zeta+\zeta_3\right\} - \frac{A_1v_2}{Ag}$$

where, $\quad \zeta_3 = \zeta_2\left(\frac{a_2}{a_3}\right)^2.$

The corrections for friction and other losses in the diverging cone are obvious, but do not alter the form of the equation, and in view of the uncertainties existing as to the correct values of ζ and ζ_3 it hardly seems necessary to express them by special symbols.

Solving this equation, we have:

$$v_2 = \frac{A_1}{A(1+\zeta+\zeta_3)} \pm \sqrt{\frac{2g(H_1-B)}{1+\zeta+\zeta_3} + \left\{\frac{A_1}{A(1+\zeta+\zeta_3)}\right\}^2}$$

The negative sign alone is significant, and v_2, attains its maximum value $\frac{A_1}{A(1+\zeta+\zeta_3)}$ when the terms under the radical sign vanish. Thus, we have:

$$v_2 = \sqrt{\frac{2g(B-H_1)}{1+\zeta+\zeta_3}} = \frac{A_1}{A(1+\zeta+\zeta_3)} = \frac{av^2+cu^2}{(av+cu)(1+\zeta+\zeta_3)}\,;$$

or, $\quad v_2 = \frac{wav^2+w_1cu^2}{(wav+w_1cu)(1+\zeta+\zeta_3)}$; when the jet and lifted fluid differ in density.

This equation for v_2 is arrived at by all investigators, but as it represents a maximum value of v_2, only, this fact is no proof that their methods are all equally correct.

We can now determine H_1, in feet head of the mixture, for a given value of v_2, and thence we calculate v, and u, from the ordinary formulæ:

$$v = c\sqrt{2g(H_0-H_1)} \qquad u = c_1\sqrt{2g(K_0-H_1)}$$

where c, and c_1, are coefficients of velocity, with an allowance for pipe friction if required, and where H_1, in each equation must be calculated in feet head of the fluid considered if the weight per cube foot of the fluids differ. The proportions of a practical ejector are plainly best obtained by trial and error.

The value of ζ is not well known, but certain measurements of my own suggest that it varies from twice to three times the value calculated from the values for C, found by experiment in cases where the mixture does not occur. Jet pumps, however, usually lift a mixture of water and air, or water and earth. The first case has been investigated by Gibson, and rules for the value of $\zeta+\zeta_3$ are given on page 833. In the second case the experiments of Hazen and Blatch (see p. 540) determine $\zeta+\zeta_3$ with all necessary accuracy.

SYMBOLS CONNECTED WITH SYPHONS

$C=\frac{v}{\sqrt{rs}}$, is the skin friction equation for the syphon.

d, is the diameter of the syphon pipe, in feet.
h_1, is the height, in feet, of the water surface in the upper reservoir above a fixed plane.
h_2, is the height, in feet, of water surface in the lower reservoir above the fixed plane.
h_3, is the height, in feet, of the crest of syphon above the fixed plane.
$h_4=h_3-h_1$.
$h_5=h_3-h_2$.

$K_1=\frac{4l_1}{C^2d}$. $K_2=\frac{4l_2}{C^2d}$.

l_1, is the length, in feet, between the upper reservoir and the crest of the syphon.
l_2, is the length, in feet, between the lower reservoir and the crest of the syphon.
q', is the total volume of air carried by the syphon, in cube feet per second.
q, is the volume of q', measured at atmospheric pressure.
v, is the velocity of the water, in feet, per second.
v_1, and v_2, are velocities in the lengths l_1, and l_2, if these differ.
$w=33-x$.
x, is the absolute pressure existing at the crest, in feet of water.
μ, is the number of cube feet of air carried per cube foot of water (see p. 826).

SUMMARY OF EQUATIONS

Syphon carrying water:

$$v_1^2=\frac{w+h_1-h_3}{\frac{4l_1}{C^2d}+\frac{1\cdot5}{2g}},\qquad v_2^2=\frac{h_3-h_2-w}{\frac{4l_2}{C^2d}}$$

Limit of $$\frac{l_1}{l_2}=\frac{w_1+h_1-h_3}{h_3-h_2-w_1}=\sigma,\text{ say.}$$

Syphon carrying air and water:

$$33-x-h_4=v^2\left(K_1+\frac{1\cdot5}{2g}\right)$$

$$h_5(1-\mu)+x-33=\frac{v^2}{(1-\mu)^2}K_2$$

$$q'=\frac{\pi}{4}d^2\mu\left(\frac{v}{1-\mu}-5\cdot5\sqrt{d}\right)$$

$$q=q'\frac{33-\frac{x}{2}}{33}$$

Syphons.—In its simplest form a syphon consists of an inverted U tube, both legs being full of water.

In Sketch No. 241, Fig. 1, let l_1 be the length of tube from C, at entry in the upper reservoir, to the crest D; and l_2, the length of tube from D to E, the exit in the lower reservoir.

Let the friction equation for water moving in the pipe CDE be, $v = C\sqrt{rs}$.

Then, we have the usual equation for motion through the whole length of the tube.

$$h_1 - h_2 = v^2\left\{\frac{4(l_1+l_2)}{C^2 d} + \frac{1\cdot 5}{2g}\right\} \quad \ldots \text{1.}$$

where $\frac{1\cdot 5 v^2}{2g}$, represents the loss at entry by shock and generation of velocity.

Now, consider motion through the length CD. We have atmospheric pressure at C, of absolute value say 33 feet of water (*i.e.* the height of the water barometer); at D we have an absolute pressure of x, feet say (*i.e.* a vacuum of $w = 33 - x$, feet).

We thus get :

$$h_1 + 33 - (h_2 + x) = v^2\left\{\frac{4l_1}{C^2 d} + \frac{1\cdot 5}{2g}\right\} = w + h_1 - h_3. \quad \ldots \text{2.}$$

For the flow from D to E, since there is atmospheric pressure at E,

$$h_3 + x - (h_2 + 33) = v^2\frac{4l_2}{C^2 d} = h_3 - (h_2 + w). \quad \ldots \text{3.}$$

Now, these two equations permit us to determine w, and for a first approximation we may neglect $\frac{1\cdot 5 v^2}{2g}$, and get :

$$w = h_3 - \frac{l_2 h_1 + l_1 h_2}{l_1 + l_2}$$

which shows w, is equal to DG, the height the crest of the syphon lies above the line joining the water surfaces above its ends.

But plainly, w cannot exceed a certain value, which is theoretically, 33 to 34 feet, and practically depends on the amount of air entrained in the water, and the temperature of the water, since this determines its vapour pressure.

Take w_1, as this maximum value (roughly about 28 feet), then the corresponding values of v_1, the velocity in the inlet leg, and v_2, the velocity in the outlet leg, are given by :

$$v_1^2 = \frac{w_1 + h_1 - h_3}{\frac{4l_1}{C^2 d} + \frac{1\cdot 5}{2g}}, \quad \text{and} \quad v_2^2 = \frac{h_3 - h_2 - w_1}{\frac{4l_2}{C^2 d}},$$

and plainly, the syphon will not run full if v_2, is greater than v_1. This gives us an equation for the ratio $\frac{l_1}{l_2}$, which is, neglecting $\frac{1\cdot 5}{2g}$:

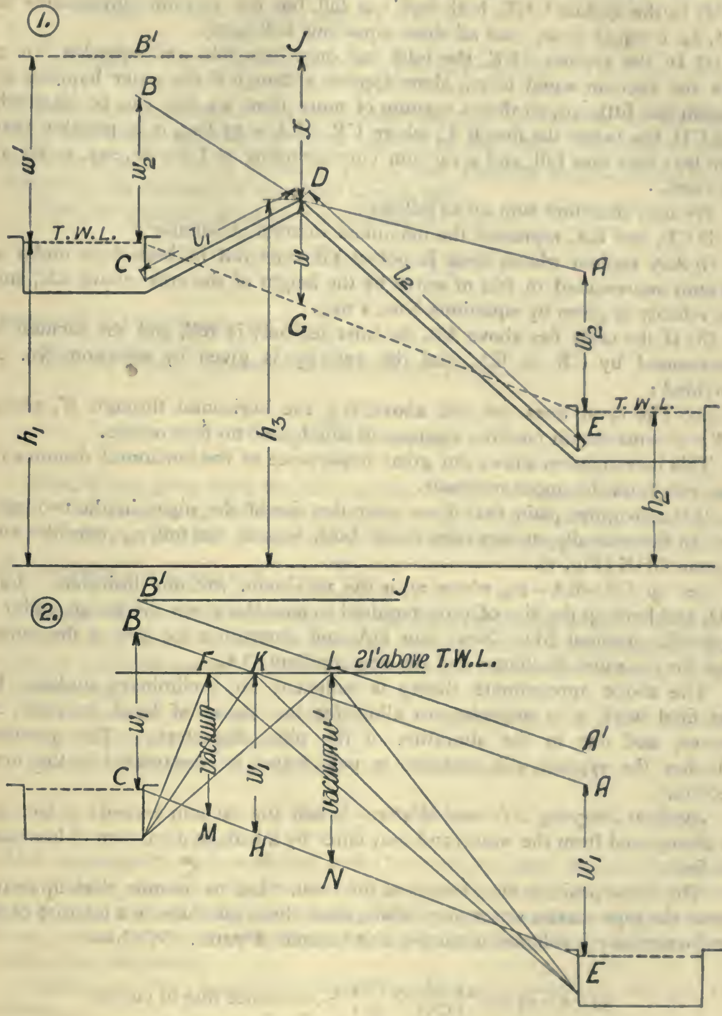
$$\frac{l_1}{l_2} = \frac{w_1 + h_1 - h_3}{h_3 - h_2 - w_1} = \sigma, \text{ say,}$$

and plainly if $\frac{l_1}{l_2}$ exceeds this value the vacuum necessary to raise water to the crest as fast as it is carried away cannot be attained, and the outlet leg does not flow full

In general, we have $h_3 - h_1$, and $h_3 - h_2$ given, and wish to work with as small a vacuum as possible. The minimum possible value, $(h_3 - h_1)$ is given by $l_1 = 0$, *i.e.* the crest should be as close to the upper reservoir as possible.

The graphical construction is evident :

Set up (Fig. 2) above the two reservoirs heights CB, and EA, equal to $w_1 = 27$ or 28 feet at most. Then join AB. The crest of the syphon should lie below, or on the line AB, where the line CE, is assumed to be the developed longitudinal section of the syphon. For barely possible flow, put $w_1 = 33$ feet, corresponding to a complete vacuum at the crest of the syphon.



SKETCH No. 241.—Diagram of Syphon Flow.

As examples: Take three syphons, whose crests are all at the same evel.

(*a*) In the syphon CFE, the velocity is given by either of the three equations

Nos. 1 to 3, both legs run full, and the maximum vacuum corresponds to the height FM, or, if great accuracy is desired and the term $\frac{1\cdot5}{2g}v^2$ is taken into account a slightly larger value will be obtained.

(*b*) In the syphon CKE, both legs run full, but the vacuum corresponds to KH, *i.e.* is equal to w_1, and all three equations still apply.

(*c*) In the syphon CLE, the inlet leg only runs full, and equation No. 2 with the vacuum equal to w_1, alone applies, although if the water happens to contain but little air, so that a vacuum of more than w_1 feet can be attained, and CD, lies below the line B′A′, where CB′ = EA′ = 33 feet, it is possible that both legs may flow full, and a vacuum corresponding to LN = w', say, exists at the crest.

We may therefore sum up as follows:

If CB, and EA, represent the maximum vacuum obtainable:

(i) Any syphon whose crest is below AB runs full in both legs under a vacuum represented in feet of water by the height of the crest above CE, and the velocity is given by equations Nos. 1 to 3.

(ii) If the crest lies above AB, the inlet leg only is full, and the vacuum is represented by CB or EA, and the velocity is given by equation No. 2, provided:

(iii) The crest does not rise above B′J, the horizontal through B′, where CB′ = w' = maximum possible vacuum, in which case no flow occurs.

This investigation shows the great importance of the horizontal distance of the crest from the upper reservoir.

It is also quite plain that if we alter the size of the pipes in the two legs, we can theoretically, at any rate, cause both legs to run full, *e.g.* consider the syphon CDE (Fig. 1).

Set up CB = EA = w_2, where w_2 is the maximum vacuum desirable. Join BD, and look up the size of pipe required to pass the given discharge under a hydraulic gradient BD. Next, join DA, and determine the size of the outlet pipe for the same discharge and the new gradient DA.

The above approximate theory is sufficient for preliminary studies. In the final work it is necessary to allow for the losses of head at entry, at curves, and due to the alteration of the pipe diameters. The question whether the syphon will continue to pass water is investigated in the next section.

Syphons carrying Air and Water.—When the vacuum exceeds 20 feet, air is disengaged from the water, and may enter by a leak, at a vacuum of less than 20 feet.

The worst position for a leak is at the crest. Let us assume that, up to the crest, the pipe carries water only, while after the crest there is a mixture of air and water, say μ volumes of air per unit volume of water. We have:

$$33 - x - h_4 = v^2\left\{\frac{4l_1}{C^2d} + \frac{1\cdot5}{2g}\right\} + \text{resistance due to curves}$$

$$= v^2\left(K_1 + \frac{1\cdot5}{2g}\right), \text{ say;} \qquad \ldots 4.$$

where $h_4 = h_3 - h_1$.

Also since the mixture of air and water has a specific gravity equal to $(1-\mu)$, and for want of better information, we assume the skin friction and other resistances are not altered by the presence of air, except in so far as the velocity is altered, we get :

$$h_5(1-\mu)+x-33 = \frac{v^2}{(1-\mu)^2}K_2, \qquad \ldots 5.$$

where $h_5 = h_3 - h_2$; and $K_2 = \frac{4l_2}{C^2 d}$.

Thus,
$$v^2\left\{K_1 + \frac{1 \cdot 5}{2g} + \frac{K_2}{(1-\mu)^2}\right\} = h_5(1-\mu) - h_4 \qquad \ldots 6.$$

Now, experimentally it has been found that the air bubbles do not travel along the outlet leg with the same velocity as the water, but tend to travel upwards with a velocity equal to $5 \cdot 5\sqrt{d}$ (see Herzog, *A.P.C.*, 1904, p. 19), where d is the diameter of the outlet pipe in feet.

Thus, the quantity of air conveyed away is :

$$q' = \frac{\pi d^2}{4}\mu\left(\frac{v}{1-\mu} - 5 \cdot 5\sqrt{d}\right)$$

measured at a mean pressure $33 - \frac{x}{2}$.

Thus, the volume measured at atmospheric pressure is :

$$q'\frac{33-\frac{x}{2}}{33} = q, \text{ say.}$$

Now, in any given case, we can assume a series of values of μ. Equation No. 6 then gives us v, and from Equations Nos. 4 or 5 we can calculate x, and then q' and q. We find that q has a maximum which corresponds to a value of μ between $\mu = 0$, and $\mu = 1 - \frac{v}{5 \cdot 5\sqrt{d}}$.

Hence, we calculate what volume of air the syphon can possibly dispose of without being sooner or later stopped by the accumulation of air, either from leaks, or dissolved air set free from the water by the vacuum existing at and near the crest.

As an example of the above calculations, consider a pipe of 4 feet diameter, with $h_5 = 20$ feet, $h_4 = 14$ feet.

Let us assume that the preliminary calculations have shown :

$$K_1 + \frac{1 \cdot 5}{2g} = 0 \cdot 032$$

which corresponds to $l_1 = 86$ feet, with $C = 100$, *i.e.* a very incrusted pipe with many bends. $K_2 = 0 \cdot 016$, corresponding to $l_2 = 160$ feet. It must be noted that the question of both legs running full has not been gone into, although, in any actual calculation, this should be determined before computations of air removal are attempted.

Substituting the numerical values, and putting $(1-\mu)^2 = 1$, we get the following :

$$v^2(0 \cdot 048) = 20(1-\mu) - 14, \qquad \text{or } v^2 = 125 - 416 \cdot 7\mu$$

$$33 - x = 14 + 0 \cdot 032v^2$$

$$q' = 12 \cdot 56\mu\left(\frac{v}{1-\mu} - 11\right)$$

The tabulation as effected by a slide rule is :

μ	$1-\mu$	v^2	v	$33-x$	$\frac{v}{1-\mu}$	q'	q
0·005	0·995	122·92	11·08	17·94	11·14	0·00879	0·00678
0·010	0·990	120·84	10·99	17·86	11·10	0·01256	0·00969
0·015	0·985	118·75	10·89	17·80	11·05	0·00942	0·00725
0·020	0·980	116·67	10·80	17·73	11·02	0·00502	0·00386

We thus see that this syphon can clear away about 0·0097 cube foot of air at atmospheric pressure per second, or as the volume of water passing is 138·1 cube feet per second, the percentage is about 0·0070, which, as will be seen from the figures, given below, is probably too small for satisfactory working.

Let us now try a 1 foot pipe under similar circumstances.

$$K_1+\frac{1{\cdot}5}{2g}=0{\cdot}074 \qquad K_2=0{\cdot}100 \text{ approximately.}$$

$$\text{or,}\quad v^2(0{\cdot}174)=6-20\mu \qquad v^2=34{\cdot}48-114{\cdot}9\mu$$

$$33-x=14+0{\cdot}074v^2 \qquad q'=0{\cdot}785\mu\left(\frac{v}{1-\mu}-5{\cdot}5\right)$$

The tabulation is :

μ	$1-\mu$	v^2	v	$33-x$	$\frac{v}{1-\mu}$	q'	q
0·01	0·99	33·33	5·77	16·46	5·83	0·00259	0·00194
0·02	0·98	32·18	5·67	16·38	5·78	0·00439	0·00329
0·03	0·97	31·03	5·57	16·29	5·73	0·00542	0·00405
0·04	0·96	29·89	5·47	16·22	5·69	0·00596	0·00444
0·05	0·95	28·74	5·36	16·13	5·64	0·00550	0·00409

The smaller pipe is thus able to carry off about 0·0044 cube foot of air per second, and is then carrying about 4·3 cube feet of water per second, so that the percentage is approximately 0·1, or over fifteen times that of the other pipe.

This figure, I believe, is above that required for satisfactory working.

The above figures for air removal may be considered as minima, since, in syphons with short outlet legs, the air bubbles do not become sufficiently large to move upwards with the velocity attained in a vertical tube, whereas, the outlet legs in the above examples, if straight, are inclined at 1 in 8, to the horizontal. On the other hand, in most practical cases, the outlet leg dips 3 to 4 feet below the surface of the reservoir into which it delivers, and this will produce an extra retardation in the equation for v^2, *e.g.* if the submersion is 4 feet, the equation becomes $v^2(0{\cdot}174)=6-24\mu$.

The relative values are however, probably very fairly correct, and introduce the principle of sucking syphons, used in several French ports.

Here, the smaller pipe (*e.g.* the 1 foot pipe in the above example) is started, and allowed to suck air by a small auxiliary tube from the larger.

The figures in the above examples are roughly comparable with those of the Tréport syphons, described by Herzog (*ut supra*), but the general practice is to use a small smooth tube (say 2-inch lead pipe), for the starter. In any given case, the best diameter is easily obtained by two or three trial calculations.

This device has not often been employed in English speaking countries, and the usual practice is to remove air either by pouring water into a reservoir at the crest of the syphon, or by a steam jet, or water jet pump. These, as requiring attention, or mechanical appliances, seem to me less practical, and the sucking syphon device, with the small syphon so calculated as to be amply capable of keeping both syphons clear, seems to me preferable. In this connection it must be noted that the above examples are exceedingly unfavourable, as the pipes are assumed to be very heavily incrusted.

Actual figures as to the accumulation of air in syphons (*i.e.* the difference between the air given off by the water at the crest and the quantity removed as above), are very rare. The only one I have been able to find is given by Anthony (*Trans. Am. Soc. of C.E.*, vol. 59, p. 64) who states it to be 0·0038 per. cent. of the volume of the water for a 12-inch pipe, under a vacuum ranging from 8 to 13 feet head of water. By scaling his diagrams, I estimate (very roughly, as the scale is small) that the syphon could remove 0·021 per cent., so that we may estimate the air given out as 0·0248 per cent. and equally roughly believe that 0·028 to 0·030 per cent. removal will permit continuous working under such a vacuum.

We may therefore consider that a removal of 0·050 per cent. will be generally satisfactory, but in Anthony's case, the climate (South African) is hot, so that in temperate countries less removal might suffice. I estimate that if a smooth 4-inch pipe had been laid to suck air from his syphons, or those of my example, continuous working can be secured.

In general, we find that a "small steam jet" is considered sufficient.

The Brusio syphon of 3 meters diameter, under a vacuum of 16 feet is, however, provided with a small centrifugal pump, for filling with water. As it is unprovided with non-return valves, the pump seems absolutely necessary.

The theory developed above appears to agree fairly well with experience of short syphons of large diameter, *i.e.* say 2 to 4 feet tubes, 120 to 150 feet long; the general principles being that a velocity somewhat exceeding the value $5{\cdot}5\sqrt{d}$ should be attained under as small a vacuum as possible.

In smaller pipes the question is complicated by incrustation, but a 2-inch lead pipe seems capable of removing air from very large syphons, and of keeping them in regular work.

For the ordinary cast iron pipe, some 6 to 8 inches in diameter, accidental leaks are not only more likely, in view of the greater number of joints, but of relatively greater importance, owing to the smaller cross-section of the pipes; hence, air leakage, or accumulations of dissolved air sooner or later stop the working. All such syphons, therefore, should be provided with some arrangement for refilling with water.

CHAPTER XIV.—(SECTION D)

AIR LIFT AND HYDRAULIC COMPRESSOR

AIR LIFT PUMPS.—Theory—Correction for friction and velocity head—Rules for friction —Approximate rule for ratio of volume of air to volume of water—Example—Efficiency—Cyclic flow—Minimum value of velocity of entry—Influence of size of the pipe—Values of efficiency—Pipes with cross-section increasing as the air expands.

THE HYDRAULIC AIR COMPRESSOR.—Theory—Extra losses not occurring in the air lift—Effect of size on efficiency—Isothermal compression of the air—Practical details—References—Injection of air into water—Orifice area—Velocity of the water and air bubbles—Efficiency—Summary—Webber's experiments.

SYMBOLS—AIR LIFT AND HYDRAULIC COMPRESSOR

a, is the area of the cross-section of the pipe conveying the mixture of air and water, in square feet.

$C = \dfrac{v}{\sqrt{rs}}$, is the skin friction constant.

d, is the diameter in feet of the pipe, the area of which is a square feet.

δ, is the distance in feet below the level from which H is measured at which air is delivered into (air lift), or leaves the water (compressor).

η, is the observed mechanical efficiency of the process.

H, is the height in feet through which the water is lifted (air lift), or falls (compressor).

h_f, is the head lost by skin friction, curves, etc. (see p. 832).

h_v, is the head lost by exit velocity (see p. 832).

$K = \dfrac{np_a}{p_1 - p_a} \log_e \dfrac{p_1}{p_a}$ (see p. 832).

n, is the ratio $\dfrac{\text{Volume of air delivered}}{\text{Volume of water delivered}}$.

p_a, is the pressure of the atmosphere.

p_1, is the absolute pressure (*i.e.* the pressure as measured by a pressure gauge $+p_a$) of the air when delivered into (air lift), or set free from (compressor) the water.

p, is the pressure at any point.

q, is the volume of water passing through the machine, in cusecs.

v_a, is the velocity in feet per second with which the mixture of air and water quits the rising main (air lift), or the down shaft (compressor).

v_e, is the velocity with which the mixture enters the rising main (air lift), or the down shaft (compressor).

v, is the velocity at any point.

x, is the volume occupied at pressure p, by the mass of air which occupies q cube feet at a pressure p_a.

SUMMARY OF FORMULÆ

Air Lift.

$$\delta - \frac{H + \delta}{1 + K} = h_f + h_v, \text{ in feet of water}$$

$$K\delta = H + h_f + h_v, \text{ in feet of mixture.}$$

$$v_a = \frac{q}{a}(1 + n) \qquad h_v = \frac{v_a^2}{2g}(1 + K) \text{ feet of mixture.}$$

$$h_f = \frac{24(\delta + H)}{C^2 d} \frac{q^2}{a^2}\left(1 + \frac{n}{2}\right)^2 \text{ feet of mixture (see p. 833).}$$

Hydraulic Compressor.

$$\frac{H+\delta}{1+K}-\delta=h_f+h_v, \text{ in feet of water.}$$

$$h_v=\frac{v_e^2}{2g}+\frac{v_u^2}{2g}(1+a) \text{ (see p. 424)}$$

h_f = as air lift + friction loss for pure water in the rising shaft.

There is also an additional uncalculable loss of head owing to the energy expended in mixing the air with the water (see p. 839).

TABLE OF VALUES OF $\frac{K}{n}=\frac{p_a}{p_1-p_a}\log_e\frac{p_1}{p_a}$.

Absolute Pressure of the Air in Atmospheres, when delivered at the Bottom of the Lift Tube $=\frac{p_1}{p_a}$.	Approximate Gauge Pressure in Pounds per Square Inch. (1 Atmosphere = 15 Lbs. per Square Inch.)	$\frac{K}{n}$
2	15	0·69
3	30	0·55
4	45	0·46
5	60	0·40
6	75	0·36
7	90	0·32
8	105	0·29
9	120	0·27
10	135	0·26
13	180	0·21
15	210	0·19

Air Lift Pumps.—Air lift pumps form the simplest means of lifting water from wells, or other narrow reservoirs.

A pipe is placed in the well, and air under pressure is blown into it. The air bubbles up, and when the lift works properly forms an emulsion, or intimate mixture, with the water, lifting it to the top of the well (see also p. 834).

Let H, be the height the water is lifted. Let the air enter the pipe at a depth δ, hereafter termed the "dip," below the water surface in the well.

Let the mixture of air and water that is lifted in one second consist of q, cube feet of water, and a quantity of air the volume of which measured at atmospheric pressure is qn, cube feet ($p_a=34$ feet of water approximately). Further, assume (as is probably very nearly the case) that the proportion of air and water is the same throughout the tube. Let p_1, be the absolute pressure of the air at the bottom of the tube. Then p_1, corresponds to the pressure produced by a column of water $\delta+34$ feet high. For, if p_1, be less than this, the air cannot leave the pipe, and if p_1, be much greater than this, air will bubble out from the bottom of the pipe. Now, as the air rises, it expands, and the air being in intimate mixture with the water, the expansion must be isothermal (*i.e.* no heating or cooling occurs).

Thus, if x, be the volume of air used per second, measured at an absolute pressure p, we have:

$$x=\frac{qnp_a}{p}$$

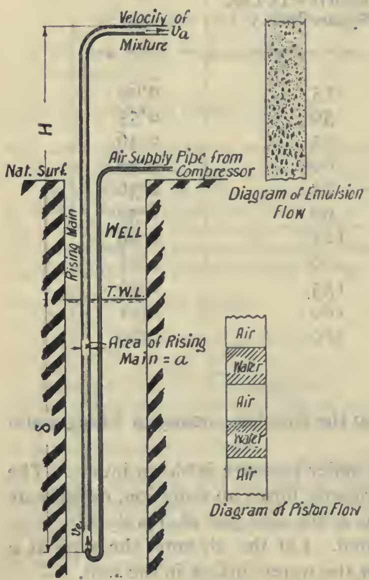
since the volume when measured at atmospheric pressure is equal to qn, cube feet per second. Therefore the mean volume of a cube foot of the air during its passage through the tube is given by:

$$\frac{p_a}{p_1-p_a}\log_e\frac{p_1}{p_a}$$

The mean specific gravity of the mixture of air and water in the rising main is therefore given by:

$$\frac{q}{q\left\{1+\frac{np_a}{p_1-p_a}\log_e\frac{p_1}{p_a}\right\}}=\frac{1}{1+\frac{np_a}{p_1-p_a}\log_e\frac{p_1}{p_a}}=\frac{1}{1+K},\text{ say.}$$

Thus, the head producing flow is given by the difference of the pressures produced by a column of water δ feet high, and a column $(H+\delta)$ feet high of mixture of a specific gravity $\frac{1}{1+K}$.



SKETCH No. 242.—Diagram of Air Lift Pump.

Thus,

$$\delta-\frac{H+\delta}{1+K}=h_f+h_v$$

Where:

h_f, is the friction head lost in the pipes, etc.

h_v, is the velocity head of the water escaping from the pipes.

At present, these quantities are expressed in feet of water, but it is convenient to express everything in feet of mixture.

We get:

$$K\delta-H=h_f+h_v \quad \ldots \text{(i)}.$$

where h_f, and h_v, are now expressed as feet head of the mixture.

Now, let a, be the area of the pipe carrying the mixture, and d, its diameter.

The velocity of the mixture at entry into the pipe is:

$$v_e=\frac{q}{a}\left(1+\frac{np_a}{p_1}\right)=\frac{q}{a}\left(n+1\frac{34}{34+\delta}\right)\text{ approx.}$$

The velocity of exit from the pipe is $v_a=\frac{q}{a}(1+n)$ and in feet of water $h_v=\frac{v_a^2}{2g}$, or:

$$h_v=(1+K)\frac{q^2}{2ga^2}(1+n)^2\text{ feet of mixture.}$$

h_f, is less easy to calculate. Gibson (*Hydraulics and its Applications*), states that $h_f = 6 \times \frac{4(\delta + H)}{C^2 d} \frac{q^2}{a^2} \left(1 + \frac{n}{2}\right)^2$ feet of mixture ;
where C, is the coefficient in the ordinary equation, $v = C\sqrt{rs}$, a value corresponding to the motion of water with velocity $\frac{q}{a}\left(1 + \frac{n}{2}\right)$, in a pipe of diameter d, being selected.

This equation leads to results which agree fairly well with experiment, and in default of better information, it must be used in calculations.

The calculations in any given case are now obvious. We must assume a value of n, calculate K, h_f, and h_v, and see whether equation No. (i) is satisfied. If not, another value of n, must be tried.

In actual practice, we find that :

When . .	H = 10	20	30	50	100 feet
n, is approximately	1·0	1·5	2·0	2·5	3·0

and these values can be used in preliminary work.

It will be found that h_f, and h_v, are appreciable fractions of H. Thus, take Kelly's experiment No. 1, Table V. (*P.I.C.E.*, vol. 163, p. 372).

Here, we have, $q = 0·74$ cusec, $n = 6·4$, $a = 0·136$ square feet, $d = 0·42$ feet, $\delta = 176·7$ feet, H = 132·8 feet, $p_1 = (73 + 15)$ lbs. per square inch.

Thus, $K = 6·4 \times 0·206 \log_e 5·87 = 2·33$.
$v_e = 11·3$ feet per second. $v_a = 40·3$ feet per second.

We have, $h_v = \frac{1624}{64·4} = 25·3$ feet of water $= 84·3$ feet of mixture ;

$$h_f = 6 \times \frac{4 \times 309·5}{10000 \times 0·42} \times 22·9^2 = \begin{cases} 929·5 \text{ feet of mixture,} \\ 278·9 \text{ feet of water.} \end{cases}$$

Thus, the efficiency of the air lift is about :

$$\frac{H}{H + h_v + h_f} = \frac{132·8}{132·8 + 25·3 + 278·9} = 0·305.$$

Experimentally, we find that the efficiency calculated from :

$$\frac{\text{H.P. of water raised}}{\text{I.H.P. of air in compression cylinder}} = 0·293\ ;$$

so that the efficiency as above defined, which is :

$$\frac{\text{H.P. of water raised}}{\text{H.P. of air as delivered at bottom of lift pipe}}$$

is probably about 0·33.

The important point, however, is that h_f, is but slightly less than twice H, even if we assume the efficiency of the air lift to be as great as 0·40 (which, in view of the experimental result, would require the air compressor to be in very bad order), and calculate h_f, from this suppositious value. Consequently, if good efficiency is desired, the losses by friction, and exit velocity, must as far as possible be minimised. Thus v_e, and v_a, must be kept as small as possible, and in order to minimise friction the air pipe should be separated from the rising main, and should not (as is frequently the case) be placed centrally inside this pipe, which produces extra skin friction in the rising main.

In actual practice, if v_e, be less than a certain value, the air and water do not thoroughly mix (as is assumed in the above theory), and the flow becomes variable, the air accumulating in large masses, filling the whole bore of the tube, and driving blocks of water before it, very much as was the case with the pistons of the old fashioned chain pump. The action is illustrated by Kelly's experiments (*ut supra*). Here, we find that:

If v_e, be less than 12 feet, per second, the flow in a 5-inch pipe is distinctly cyclic.

The results are as follows:

v_e	Period of the Cycle.	Time per Cycle during which Water was lifted.	n	Efficiency. $\frac{\text{H.P. Water raised}}{\text{I.H.P. in Air Cylinder}}$
Ft. per Sec.				
5·76	2 mins. 20 secs.	50 secs.	8·98	0·141
7·60	2 mins.	1 min. 15 secs.	6·88	0·176
9·30	1 min. 15 secs.	1 min.	6·36	0·197
12·80	Flow is continuous, but the volume delivered varies momentarily		4·95	0·275
14·40			5·47	0·249

Thus, in the first four (certainly in the first three) experiments, the air acted like a series of pistons. In the fifth, the air and water issued thoroughly mixed.

The maximum efficiency occurs at, or near, the velocity at which the mixture first becomes complete. Thus, the determination of the value of v_e, at which piston action ceases, and emulsion delivery begins, is very important, and the size of the pipe should be determined so that the desired delivery is then obtained.

So far as our present knowledge extends, the value of v_e, at which the change occurs depends only upon the size of the rising main.

The following figures are obtained from:

Jossé's experiments with $H+\delta=120$ feet (approx.):

Piston action ceases when v_e, is less than 5·60 feet per second, in a pipe of $2\frac{3}{4}$ inches diameter.

Piston action ceases when v_e, is less than 7·30 feet per second, in a pipe of 3 inches diameter.

Kelly's experiments with $H+\delta=310$ feet (approx.) give:

Piston action ceases when v_e, is less than 11·8 feet per second, in a pipe of 4 inches diameter.

Piston action ceases when v_e, is less than 12·7 feet per second, in a pipe of 5 inches diameter.

Further details cannot be given, but these values are believed not to be greatly in excess of the velocities at which the change from piston to emulsion motion occurs.

The above figures show that if high efficiency is desired, the volume of water delivered by an air lift pump is not very great.

For example, take a 5-inch pipe, and assume that:

$H=100$ feet, or $p_1=3$ atmospheres gauge, or 60 lbs. per square inch absolute, n, is about 3. Thus, the mixture which enters the pipe is composed of equal volumes of air and water, and v_e, should be about 12 to 13 feet per second. Thus, the volume of water delivered per second is about:

$$\frac{12\cdot5}{2}\times\text{area of a 5-inch pipe}=0\cdot875 \text{ cube foot}=5\cdot5 \text{ gallons.}$$

This method of proportioning with accurately obtained values of n, will be found to produce a very efficient pump. In practice, however, air lift pumps are usually employed in deep wells, where space is limited, and the pump is expected to lift all the water which the well can safely yield. Thus, the efficiencies recorded are usually smaller than could be obtained if more space were available.

In actual work the recorded efficiency is usually calculated from:

$$\eta=\frac{\text{Water Horse Power}}{\text{I.H.P. of the cylinder of the air compressor}}$$

which is probably about 0·90, of the true efficiency of the air lift alone.

Jossé found when $\delta+H=119\cdot7$ feet, with a pipe $2\frac{3}{4}$ inches in diameter, that:

When $\frac{\delta}{H}=\frac{3}{2}$ $\eta=0\cdot329$ to $0\cdot256$ $n=2\cdot45$ to $3\cdot48$.

When $\frac{\delta}{H}=\frac{4}{3}$ $\eta=0\cdot445$ to $0\cdot279$ $n=2\cdot49$ to $3\cdot78$.

When $\frac{\delta}{H}=1$ $\eta=0\cdot423$ to $0\cdot272$ $n=3\cdot30$ to $4\cdot67$.

With a pipe 3 inches in diameter;

When $\frac{\delta}{H}=\frac{4}{3}$ $\eta=0\cdot397$ to $0\cdot307$ $n=2\cdot58$ to $3\cdot20$.

When $\frac{\delta}{H}=1$ $\eta=0\cdot372$ to $0\cdot311$ $n=3\cdot66$ to $2\cdot17$.

Kelly, with $\delta+H=433$ feet, and a pipe 4 inches in diameter, found that:

When $\frac{\delta}{H}=1\cdot77$ $\eta=0\cdot193$ to $0\cdot099$ $n=10\cdot35$ to $17\cdot95$.

With $\delta+H=335$ feet, and a pipe 4 inches in diameter;

When $\frac{\delta}{H}=1\cdot63$ $\eta=0\cdot243$ to $0\cdot218$ $n=7\cdot8$ to $8\cdot1$.

When $\frac{\delta}{H}=1\cdot51$ $\eta=0\cdot139$ to $0\cdot123$ $n=11\cdot8$ to $13\cdot0$.

When $\frac{\delta}{H}=1\cdot39$ $\eta=0\cdot242$ to $0\cdot175$ $n=8\cdot2$ to $10\cdot5$.

With $\delta+H=309\cdot5$ feet, and a pipe 5 inches in diameter;

When $\frac{\delta}{H}=1\cdot33$ $\eta=0\cdot293$ $n=6\cdot4$.

When $\frac{\delta}{H}=1\cdot06$ $\eta=0\cdot300$ to $0\cdot198$ $n=7\cdot3$ to $10\cdot0$.

When $\frac{\delta}{H}=1{\cdot}45 \quad \eta=0{\cdot}386$ to $0{\cdot}234 \quad n=3{\cdot}8$ to $7{\cdot}16$.

When $\frac{\delta}{H}=1{\cdot}51 \quad \eta=0{\cdot}343$ to $0{\cdot}321 \quad n=5{\cdot}2$ to $5{\cdot}68$.

With $\delta+H=347$ feet, and a pipe 4 inches in diameter:

When $\frac{\delta}{H}=1{\cdot}56 \quad \eta=0{\cdot}304$ to $0{\cdot}149 \quad n=5{\cdot}77$ to $11{\cdot}83$.

With $\delta+H=313$ feet:

When $\frac{\delta}{H}=1{\cdot}40 \quad \eta=0{\cdot}309$ to $0{\cdot}175 \quad n=5{\cdot}80$ to $12{\cdot}60$.

The whole of the available results indicate that n, should be kept as small as possible for high efficiency. In any given case, the best value of the dip δ is that which makes n, least. This condition can be obtained by trial and error from the equations already given.

The above theory makes it evident that when v_e, is determined by the emulsion flow condition, both h_v, and h_f, can be considerably diminished by decreasing v_a.

If we increase the cross-section of the pipe from the bottom upwards so that the velocity $v=\dfrac{q\left(1+n\,\dfrac{p}{p_a}\right)}{\text{Area of pipe}}$ is kept constant where p, is the pressure at the level considered, we plainly secure that the velocity at every point is equal to v_e. Thus, without diminishing v_e, below the minimum value already referred to, we can considerably diminish h_f, and h_v, and so increase the efficiency. As an example, take the conditions already calculated on page 833.

$$\text{We have } h_v= 25{\cdot}3\times\left(\frac{11{\cdot}3}{40{\cdot}3}\right)^2=2{\cdot}00 \text{ feet of water,}$$

$$h_f=278{\cdot}9\times\left(\frac{11{\cdot}3}{22{\cdot}9}\right)^2=88{\cdot}2 \quad \text{,,}$$

$$\text{Efficiency}=\frac{132{\cdot}8}{132{\cdot}8+2+88{\cdot}2}=0{\cdot}60$$

and the area of the top of the pipe is $0{\cdot}136\times\frac{40{\cdot}3}{11{\cdot}3}=0{\cdot}43$ square foot, or say, 9 inches (accurately 0·74 foot) in diameter.

The principle is understood to be the subject of patents, and considerable increases in efficiency have been reported. It is doubtful whether the increase is as large as is indicated by theory, since the practical difficulties attending an exact fulfilment of the conditions are obvious.

THE HYDRAULIC AIR COMPRESSOR.—From a mathematical standpoint, the Hydraulic Air Compressor may be regarded as a reversed air lift. Having a head H, available, we sink a shaft to a depth δ measured below tail water level. The head H, is employed in keeping in motion a downward moving column of a mixture of water and air, of a length equal to $H+\delta$, and an upward moving column of water of a length δ. At the bottom of the shaft the water enters a large chamber, where the air bubbles rise, and are collected under a pressure of $0{\cdot}433\delta$ lb. per square inch (above atmospheric pressure).

The relation between H, and δ is therefore given by the equation (see p. 832):

$$\frac{H+\delta}{1+K}-\delta=h_f+h_v$$

where h_f, is the head lost by friction; and h_v, is the head lost by change of velocities, and the velocity of exit; both h_f and h_v being measured in feet of water.

$$K, \text{ is equal to } \frac{34n}{\delta}\log_e\frac{34+\delta}{34}$$

where n, is the volume of air carried down per cube foot of water entering the compressor, measured in cube feet at atmospheric pressure.

This equation can be treated as indicated on page 833, and is best solved by trial. The expressions for h_f, and h_v, however, differ slightly from those given for the case of an air lift, since we have now to consider not only the downward flow of the water, but also the losses of head that occur during its upward motion in the upcast, or rising shaft.

Thus, h_f, consists of two parts, as follows:

(i) The friction head for the downward motion of the mixed air and water. This can be calculated from the formulæ given for an Air Lift Pump.

(ii) The friction head for the upward motion of the water after it has been freed from air. This is calculated by the ordinary rules, and is a loss which does not occur in the air lift.

So also, h_v, is made up of two portions, as follows:

(i) The mixture of air and water reaches the bottom of the shaft with a velocity v_e, and (putting aside such bell mouth, or draft tube arrangements as are shown in Sketch No. 243) a head equal to $\frac{v_e^2}{2g}$, is consequently lost.

(ii) The usual methods show that a head equal to $\frac{v_w^2}{2g}$ (where v_w, is the velocity of exit at the top of the upper shaft) is lost. Also, that a certain fraction of $\frac{v_w^2}{2g}$, is expended in setting the water in motion at the bottom of this shaft. These last two losses do not occur in the air lift.

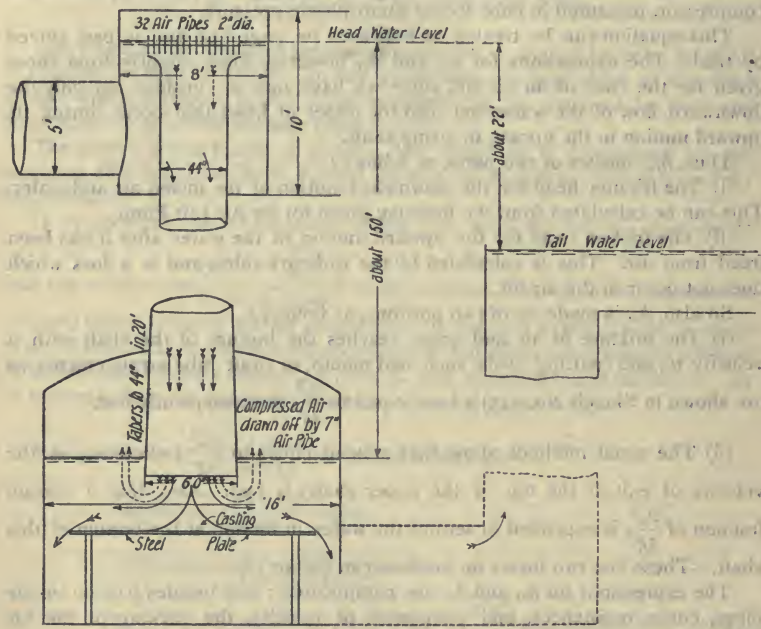
The expressions for h_f, and h_v, are complicated; and besides friction in the pipes, curve resistances, and generation of velocity, the sucking of the air bubbles into the water column consumes a certain amount of head.

Regarded merely as a hydraulic machine, the compressor has more sources of loss to strive against than the air lift. For, on entering the air chamber, the velocity of the water and air must be reduced to a low value, in order to secure the liberation of the entrained air, and thereafter the velocity must again be speeded up when the air-free water enters the up-shaft.

On the other hand, all existing hydraulic compressors are large pieces of machinery, an 18-inch column of air and water being a small compressor. Whereas, an air lift with a 6-inch column of air and water is a comparatively large air lift. Thus, the efficiencies actually obtained are larger than those usually observed in air lifts.

Also, if we consider the two cycles—from free air, back to free air, in each process—the overall efficiency of the compressor is as a whole by far the greater of the two. For in the air lift, the compression of the air in the air pump is not an efficient process, as owing to mechanical difficulties the ideal isothermal compression can never be attained. In the hydraulic air compressor, the air is very intimately mixed with a large volume of water, and is consequently compressed isothermally. The efficiency obtained in the process of compression is probably as near to unity as is ever attained in practice. The utilisation of the compressed air is no doubt subject to the practical difficulties arising from possible freezing which compressed air machinery (except the air lift) always labours under, but these are easily obviated by the use of preheaters.

Details of practical application of the process are not obtainable. It



Sketch No. 243.—Hydraulic Compressor. (After Webber.)

appears that many of the methods in use are at present covered by patents. Sketch No. 243 shows the diagrammatic scheme usually made public, but this must merely be regarded as a diagram.

Literature containing complete numerical data is rare. I believe that the papers by Frizell (*Journal of the Franklin Institution*, 1880), and Webber (*Trans. Am. Soc. of Mech. Eng.*, vol. 22 p. 599), contain all that is of scientific value. Tests showing larger efficiencies than Webber's results have been partially published by various patentees. So far as the results of these tests can be checked, they appear to be reliable; and the improvement obtained would justify the use of the patent, or more accurately, of obtaining the assistance of the patentees' special knowledge by use of their patent. Nevertheless, an engineer who advises his clients to pay a royalty will be justified in obtaining guarantees of efficiency with penalties for non-success.

Let us now consider the process more in detail.

The sucking of the air into mixture with the water requires a certain amount of pressure difference. Frizell (*ut supra*) merely allowed the water to fall freely through a certain height, and states that one foot head was lost thereby. His method was crude, and the figures given by Webber (*ut supra*) show that the head required to force the air through the orifices varied from 0·58 foot to 1·03 foot of water. So far as can be gathered, this head is obtained by providing the entry tube with a constriction near its upper end, thus obtaining a pressure slightly below atmospheric; the arrangement being similar in principle to a Venturi meter, or a jet pump (see p. 80).

The area of the constriction appears to be about 0·60, to 0·75 of the entry area; but, in any actual case, calculations by the rules given under Jet Pumps are indicated, and allowance must be made for the volume of the air sucked in.

When the vacuum produced by the constriction is determined, the gross area of the air orifices is easily calculated by the ordinary rules.

The design of the orifices is likely to cause trouble. It is plainly important that the air should be sucked in in bubbles, of not too large a size. Thus, there is a size which each individual orifice must not exceed. As will be shown later, Webber uses 34 holes, 2 inches in diameter, and it may be inferred that this is about as large a hole as is advisable.

It is, however, evident from Webber's values that n, the proportion of air sucked in, greatly influences the efficiency; and it would appear that the greater the area of the air orifice, the less the volume of water which produces the best efficiency.

The best efficiency in Webber's experiments was attained when $n=\frac{1}{4}$, or was slightly in excess of $\frac{1}{4}$. Judging from the indications thus obtained, I am inclined to believe that the head required for the injection of air was obtained by an arrangement resembling a Venturi cone, with a throat area about 0·50, to 0·53 times the entrance area. It is also evident that the total area of the air orifices must be capable of adjustment, so as to provide for variations in the water supply, if good efficiency is desired.

The mixed air and water travels down a pipe with a velocity reckoned on the water alone of 6 to 9 feet per second in Webber's case; and the mean velocity of the mixture when the best efficiency was obtained appears to have been about 7 feet per second. Frizell's best results were obtained when the velocity was about 4 feet per second, and although it is perfectly evident that he never got enough air entrained to secure the best efficiency, the difference from Webber's best velocity is very great, and it is probable that the dimensions of the shaft largely influence the value of the velocity required in order to obtain the best results. Frizell states that the air bubbles rise against the water current at a velocity of about 0·75 foot per second, and that this should be allowed for in calculation. The friction losses in the downward pipe are unknown. As already stated, Gibson believes that (when expressed in feet head of the mixture) they are about six times the feet head of water lost in a pipe of the same size conveying pure water with a velocity equal to that of the mixture. Frizell's results agree fairly well with this rule. His apparatus, however, had bends and constrictions that might equally well account for the extra loss. The bottom of the down shaft is coned out, after the manner of a draft tube, in order to minimise the loss of head due to this velocity of 6 or 7 feet per second as far as possible. The water-way through the air chamber must of course be large, in order to reduce the velocity sufficiently to allow of the bubbles rising up from the water. The cone at the bottom is evidently intended to assist this disengagement of the air from the water.

Webber's apparatus is 16 feet in diameter, so that the velocity is only reduced to 3 or 4 feet per second. It would therefore appear that a reduction to Frizell's value of 0·75 foot per second is not required, probably owing to the agitation produced by the cone.

I tabulate the results of Webber's experiments, and the dimensions shown in Sketch No. 243 are those given in his paper. If we take these results as reliable (and the work appears to have been quite as good as is likely to be attained in experiments of this magnitude) we may roughly state that for about 60 cusecs the best efficiency is attained with an air inlet area of 120 square inches. For 70 cusecs the best inlet is 113 square inches, and for 80 cusecs an area of 106 square inches would probably give a better result than that obtained with larger areas. Now, if the area of the air orifice be kept constant, n, the quantity of air sucked in per cube foot of water is also constant. Consequently, we may infer that the greater the volume of water used, the less n, should be. This, of course, is merely an indication that the terms h_f and h_v (which depend on n), increase as the volume of water used increases, and that, just as in the case of air lifts, the best efficiency is obtained when the velocity of the water is just sufficient to produce an intimate mixture of air and water and carry the air down to the bottom of the shaft. We may also suspect that somewhat better results could have been obtained with a more perfect method of injecting air into the water. I should consequently be inclined to design the quasi-Venturi meter with a constriction of 0·40, rather than 0·50, as it appears to have been made. In default of fuller details, this criticism must be regarded as a bold one.

Frizell's results are less illuminating, as we are aware that the arrangements for the supply of air were bad. They are also irregular, and it can only be said that had he been able to inject air in ratios equal to Webber's (*i.e.* 1 cube foot of free air per 4 cube feet of water) it is likely that his efficiencies would have been as large, or possibly larger, than those obtained by Webber. As it was, the best efficiency obtained was 0·52, for 1 cube foot of free air per 8·8 cube feet of water. Frizell's other good results cluster around 0·52 to 0·45, for 1 cube foot of free air, per 9 to 10 cube feet of water, and descend as low as 0·40, when 1 cube foot of air per 20 to 25 cube feet of water is compressed.

The practical aspect of the process seems to be as follows:

It affords a very excellent method of obtaining power from large volumes of water, under a low head; and in such cases the efficiency of the power plant is probably higher than that which could be obtained with the low speed turbines that are alone permissible, since these cannot be employed for driving the machinery direct. On the other hand, compressed air is not a practical method of distributing large quantities of power over any large area, the theoretical efficiencies being good, but the leakage losses increasing rapidly as the mains grow old. The practical application of the method therefore appears to be limited to cases where the demand for power is concentrated over a small area such as a factory, or a pumping station. The shaft is deep (*e.g.* 230 feet, for 100 lbs. per square inch pressure), and the first cost is probably equal to that of a turbine installation, but the maintenance of the shaft and pipes should be almost negligible.

I am inclined to believe that the method is not as frequently applied as it might be. The reasons are obvious. The process is not greatly studied, and the power is obtained in a form which is very little employed. It may really be said that the installation would require a specialist to design it and the utilisation of the power also requires somewhat special, and rare experience.

Webber's installation was as follows:

The head available varied from 18·7 to 20·5 feet, the shaft was 150 feet deep below the normal head water, so that the air pressure varied from 52, to 54 lbs. per square inch gauge.

We have:

I. Experiments with Thirty-four 2-inch Pipes as Air Orifice, *i.e.* 106·8 Square Inches.

Head in feet	20·54	20·35	19·93
Volume of water in cusecs	62·9	67·8	73·5
Cube feet of water per cube foot of free air	4·37	4·20	4·04
Air pressure, lbs. per square inch	51·9	53·2	53·7
Efficiency, *i.e.* $\frac{\text{Air HP.}}{\text{Water HP.}}$	56·8	60·3	64·5

II. Orifice as above, with Fifteen ¾-inch Pipes in addition, *i.e.* 113·4 Square Inches.

Head in feet	20·12	19·51	19·31	18·75
Volume of water in cusecs	58·5	71·5	78·3	84·3
Cube feet of water per cube foot of free air	4·58	3·74	4·26	4·34
Air pressure, lbs. per square inch	51·9	53·3	52·9	53·3
Efficiency	55·5	70·7	62·2	63·3

III. Orifice as No. I., with Thirty ¾-inch Pipes in addition, *i.e.* 120 Square Inches.

Head in feet	20·0	19·58	19·41	19·31
Volume of water in cusecs	60·5	71·8	76·7	89·1
Cube feet of water per cube foot of free air	4·03	4·32	4·30	4·79
Air pressure lbs. per square inch	53·7	53·7	53·6	52·7
Efficiency	64·4	61·3	62·0	55·4

CHAPTER XIV.—(Section E)

HYDRAULIC RAM

Hydraulic Ram.—Description.

Theoretical Treatment.—Cycle of operation—Period of escape—Closure of escape valve—Period of delivery—Shock loss—"Indicator diagram"—Observations—Approximate indicator diagram—Efficiency—Loading of the escape valve.

Practical Rules.—Diameter of ram and delivery pipes—Air vessel—Observed values of the efficiency.

SYMBOLS

A, is the area in square feet of the cross-section of the ram pipe.

a_0, is the area in square feet of the opening of the escape valve.

a_v, is the area in square feet of the cover of the escape valve.

$a=e-\text{D}$ (see p. 848).

$b=e+\text{D}$ (see p. 848).

c_d, is the coefficient of discharge of the area a_0.

$cv'+mv'^2$, is the resistance of the ram pipe and delivery valve (see p. 846). For c', and m see p. 846.

$\text{C}=\frac{v}{\sqrt{rs}}$, is the coefficient of frictional resistance of the ram pipe.

d, is the diameter in feet of the ram pipe.

$ev+nv^2$, is the alternative value of $c'v+m'v^2$ (see p. 846).

D (see p. 848). D′ (see p. 848).

H, is the difference of level in feet between the surface of the water in the working reservoir, and the escape valve.

h, the geometrical difference in level between the water level in the reservoir into which the water is lifted and the water level in the working reservoir plus an allowance for skin friction and other resistances in the rising main.

h_1 (see p. 845).

$\text{J}=\frac{2g\sqrt{m\text{H}}}{l}$ (see p. 845).

J_1 (see p. 846).

$\text{K}v^2$, is the loss of head in the ram pipe by friction, bends and entry head (see p. 845).

l, is the length of ram pipe in feet.

mv^2, is the total loss of head in the ram pipe and escape valve (see p. 845).

$\text{M}=\frac{2nv'+b}{2nv'+a}$ (see p. 848).

nv^2, is the value of mv^2, when the delivery valve is opened, and the escape valve is shut (see p. 847).

p (see p. 848).

Q, is the total quantity of water used in cube feet, per cycle of the ram.

Q_l, is the total quantity of water lifted by the ram per cycle.

Q_e, is the total quantity of water escaping through the escape valve per cycle.

r, is the hydraulic mean radius of the ram pipe.

s_v, is the maximum height in feet which the escape valve cover rises above its seat.

S_c, and S_o (see p. 850).
T_o, T_s, T_d, T_c (see p. 845).
t, is the general symbol for time, in seconds.
v, is the velocity of water in the ram pipe, in feet per second.
V, is a velocity defined by $mV^2 = H$ (see p. 846).
v_x, is the value of v, when the escape valve begins to close, *i.e.* at a time T_o.
V_m, is the value of v, just before the escape valve closes, *i.e.* at a time $T_o + T_s$.
v', is the value of v, just after the closure of the escape valve (see p. 846).
V_{max}, is the maximum value of v (see Sketch No. 245).
W, is the weight of moving portion of the escape valve in pounds.
a and β (see p. 848).
η, is the mechanical efficiency of the ram (see p. 849).
η_v, is the volumetric efficiency of the ram (see p. 849).
λ (see p. 846).

SUMMARY OF FORMULÆ

(i) Escape valve full open:

$$\frac{dv}{dt} = \frac{g}{l}(H - mv^2) \qquad \text{Thus, } v = \sqrt{\frac{H}{m}} \tanh \frac{Jt}{2}$$

$$h_1 = \frac{A^2v^2}{2ga_o^2c_d^2} \qquad J = \frac{2g\sqrt{mH}}{l}.$$

(ii) Closure of escape valve:

$$T_s = 2\sqrt{\frac{Ws_v}{gS_o}} \text{ approximately.}$$

(iii) Delivery valve open:

$$v' = \frac{\frac{\lambda}{g}V_m - h}{\frac{\lambda}{g} + c'}, \qquad \text{approximately, } v' = \frac{146V_m - h}{146 + c'}.$$

$$\frac{dv}{dt} = -\frac{g}{l}(h + ev + nv^2)$$

$$\text{Thus, } (a)\ v = \frac{1}{2n}\,\frac{aMe^{pt} - b}{1 - Me^{pt}}, \qquad \text{and } T_d = \frac{1}{p}\log_e \frac{b}{aM}.$$

$$\text{or, } (b)\ v = \frac{D'}{2n}\,\frac{a - \tan\beta t}{1 + a\tan\beta t} - \frac{e}{2n}, \qquad \text{and } T_d = \frac{1}{\beta}\tan\frac{2nv'}{D' + e}.$$

$$\text{or, } (c)\ t = \frac{l}{g\sqrt{nh}}\left\{\tan^{-1}v'\sqrt{\frac{n}{h}} - \tan^{-1}v\sqrt{\frac{n}{h}}\right\}.$$

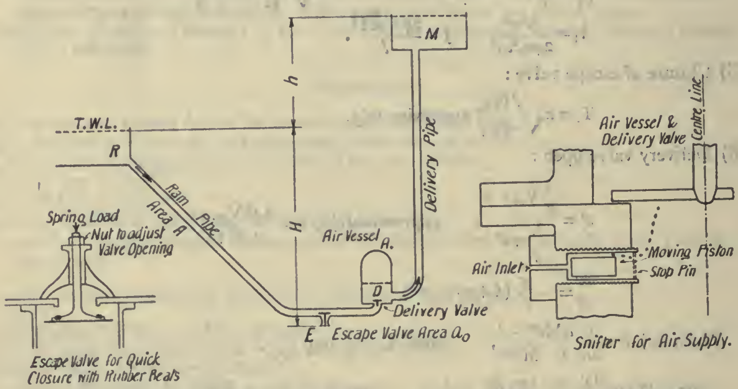
HYDRAULIC RAMS.—This form of pumping machine is old. The following may seem to be unduly lengthy if the present practical importance of the ram is alone considered, but since the principles laid down permit an approximately exact and highly efficient design to be obtained with but little trouble, the discussion is not without value.

Sketch No. 244 shows the essential portions of a hydraulic ram diagrammatically. The power is obtained by water falling from the reservoir R, through the ram pipe RE, and escaping by the escape valve E. Putting aside for future consideration the conditions requisite to obtain proper working, it is assumed that the escape valve opens suddenly. The water starts to flow down the ram pipe, and its velocity rapidly increases until the escape valve suddenly closes. This sudden closure produces water hammer, or shock, and the rise of pressure thus induced opens the delivery valve D, and forces a certain volume of water through this valve. This volume accumulates in the air chamber A, and the pressure existing in the air chamber forces this water up through the delivery pipe, or rising main AM, into the reservoir M. The delivery valve closes after a certain interval, and the escape valve E, again opens. The cycle is automatically repeated indefinitely.

Using the symbols which are explained later, a quantity of water represented by:

$$Q = Q_e + Q_l$$

leaves the reservoir R, during each working cycle. A fraction Q_e, escapes through the escape valve, and energy equivalent to 62·5 Q_eH foot lbs. is thus expended. The remaining fraction Q_l is lifted through the height h, and energy equivalent to 62·5 Q_lh foot lbs. is stored up in the reservoir M. The levels between which H, and h, are measured should be carefully noted. It is hoped that the discussion which follows will clear up any uncertainties regard-



SKETCH No. 244.—Hydraulic Ram, Escape Valve, and Snifter.

ing the conditions required to produce this somewhat mysterious cycle of operations.

Hydraulic rams are at present the monopoly of three or four firms of specialists, who work by "practical experience," and by the rule of thumb. It is hoped that the following discussion will enable practical engineers to adapt commercial rams to varying circumstances. My own experience on the subject is that a few careful experiments and a set of spiral springs of graduated strength (see p. 807) are all that are necessary.

THEORETICAL TREATMENT.—The exact theory of the Hydraulic Ram is unknown, and is probably so complicated as to be useless for any practical purposes.

The following approximate theory has been experimentally investigated with great care by Harza (*Bull. of Univ. of Wisconsin*, March 1908). His results show that it is sufficiently close to the actual working of the machine to form a useful guide when discussing modifications of an existing ram.

The working cycle of a ram may be divided into four portions:

1. The escape valve opens, water flows down the ram tube, and gradually attains a maximum velocity at a time T_o, from the commencement of the cycle.
2. The valve continues to close, and is completely shut at a time T_o+T_s.
3. The delivery valve opens, and water is forced up the rising main for a time T_d, the delivery valve closing at $T_o+T_s+T_d$, after the commencement of the cycle.
4. The escape valve opens at a time T_e, after the delivery valve closes. The whole period of the cycle is therefore $T_o+T_s+T_d+T_e$.

The important portions are (1), and (3), and it is as well to note that both T_s, and T_e, are very small if the ram works well.

Period 1.—Let H, be the total difference of level between the surface of the water in the reservoir, and the escape valve. Let v, be the velocity of water in the ram pipe, the area of which is A. Let a_o, be the area of the discharge orifice of the escape valve, and c_d, its coefficient of discharge, which may be considered as constant during the time T_o, but varies during T_s.

Then we have:

h_1, the head required to force the water through the escape valve is given by:

$$c_d a_o \sqrt{2gh_1} = Av$$

and the head available to accelerate the velocity of the water is:

$$H-h_1-Kv^2$$

where Kv^2, represents the head lost in the ram pipe by friction, curves, bends, and the velocity of entry; so that:

$$Kv^2 = v^2\frac{4l}{C^2 d} + \frac{1\cdot5}{2g}v^2 + \text{Head lost at curves and elbows,}$$

where l, is the length, d, the diameter, and $v = C\sqrt{rs}$, the friction equation of the ram pipe.

Hence, we have as the equation of motion:

$$\frac{dv}{dt} = (H-mv^2)\frac{g}{l} \quad . \quad . \quad . \quad . \quad . \quad \text{(i)}$$

where, $mv^2 = h_1 + Kv^2 = Kv^2 + \dfrac{A^2v^2}{2gc_d^2a_o^2}$

Now, this can be integrated as:

$$t = \frac{l}{2g\sqrt{mH}}\log_e\frac{\sqrt{H}+v\sqrt{m}}{\sqrt{H}-v\sqrt{m}}$$

$$\text{or:} \quad v = \sqrt{\frac{H}{m}}\,\frac{e^{Jt}-1}{e^{Jt}+1} = \sqrt{\frac{H}{m}}\tanh\frac{Jt}{2} \quad . \quad . \quad . \quad \text{(ii)}$$

$$\text{where} \quad J = \frac{2g\sqrt{mH}}{l}$$

Now, this curve of v, in terms of t, can be plotted either from the first form, or more easily with the help of a table of hyperbolic tangents. We can also plot a curve showing :

$$h_1 = \frac{A^2 v^2}{2g a_o^2 c_d^2} \text{ at each instant of time.}$$

As an aid in studying the general problem, it may be noted that if J, is varied, the form of the curve is not varied. For example, if we design a ram with J_1, in place of J, the value of $v\sqrt{\frac{m}{H}}$ at the time t_1, is the same as that in the original at the time $t = \frac{J_1 t_1}{J}$; so that one curve permits of a study of several rams by properly adjusting the time and velocity scales. So also, if $\frac{A}{c_d a_o}$, is not changed, the curve for h_1, is unaltered.

Now, in theory, this motion continues until $\frac{dv}{dt} = 0$, or $H = mV^2$, say.

Actually, however, the escape valve is caused to begin to close when v, has a certain value, v_x, which is best obtained experimentally.

We can thus obtain the time at which closure occurs, $T_o + T_s$, and if we assume (as is very nearly the case), that the valve closes instantaneously, we can measure T_o, on the time scale with a fair degree of accuracy. Let V_m, represent the value of v, just before the valve closes, which is not necessarily the maximum value of v, unless the closure of the valve is instantaneous.

Period 2.—We neglect, as a first approximation.

Period 3.—A shock occurs, and the delivery valve is forced open. The water in the ram pipe now has a velocity v', less than V_m.

Let λ be the velocity of a wave of compression in the water in the ram pipe, *i.e.* λ is approximately the velocity of sound in water (equal to 4700 feet per second), and can be calculated more accurately from the rules given on page 811.

$$\text{Then} \qquad \frac{\lambda}{g}(V_m - v') = h + c'v' + m'v'^2$$
$$= 146(V_m - v') \text{ approximately,}$$

where h, is the head pumped against, including friction, and curve and other losses in the rising main; and $c'v' + m'v'^2$ are the frictional and other losses up to the beginning of the rising main, including those in the delivery valve.

The resistance of the delivery valve now takes the place of that of the escape valve. In addition, the resistance of the small length of pipe and bends between the escape valve and the air chamber also contributes to the term $c'v' + m'v'^2$.

Now, according to Harza's experiments, it would appear that $m' = m$, and that the term $c'v'$, is sufficiently large to require consideration. Harza apparently experimented on one form of valve only, and I am inclined to believe that in consequence his results apply solely to a rather special case. So far as can be judged from other experiments on valves (see p. 805), while the condition $m = m'$, is likely (and can certainly be secured by good design), it is not very probable that $c'v'$ is at all large in a well constructed valve. The delivery valve used by Harza is shown in Fig. 6, Sketch No. 237.

Nevertheless, as a general rule, it is best to follow Harza's investigation, since it is quite easy to put $c' = 0$, when applying his formulæ to any given case.

We have as a first approximation for the initial velocity after the shock:

$$v' = \frac{\frac{\lambda}{g}V_m - h}{\frac{\lambda}{g} + c'} = \frac{146V_m - h}{146 + c'}$$

and as a second approximation:

$$v' = \frac{146V_m - h}{146 + c'} - \frac{m'(146V_m - h)^2}{(146 + c')^3}.$$

which permits us to set off the height $Z'W = v'$, as the initial point for the curve representing the period (3), and it will be plain that the shock has diminished the velocity in the ram pipe by a quantity represented graphically by $ZW = V_m - v'$.

The difficulties attending an experimental determination of this loss of velocity by shock are very great. The theoretical equations neglect the pressure required to open the delivery valve. No rules can be given for calculating this pressure; but it is evident that the delivery valve should be designed so as to open easily. Thus, this valve should be light, and the breadth of its seat (denoted by $\frac{d-d_1}{2}$ on p. 805) should be small, and the load necessary to secure its closure during the period when the escape valve is open should be produced by a spring. In actual practice, the loading of the delivery valve appears to have little effect on the efficiency, and losses are mainly attributable to the seat being too wide. It is, however, quite possible that the apparently excessive values of $d-d_1$, found in practice are required in order to secure that the valve does not leak when it is old and worn.

The most important practical condition is that the delivery valve shoul be fully exposed to the pressure produced by the alteration of velocity.

Consequently, a proper design of the approach passages is probably far more important than the actual load on the valve.

The equation of motion thereafter is plainly:

$$\frac{dv}{dt} = -\frac{g}{l}(h + ev + nv^2) \quad \ldots . \text{(iii)}$$

where Harza takes $e = c'$, and $n = m' = m$, which is probably not absolutely correct, since the resistance between the delivery valve and the air chamber should be included in these coefficients. Harza's notation has therefore been abandoned.

The fact that h, expresses not only the geometrical height through which the water is lifted, but the frictional and other resistances in the rising main, should be borne in mind. These resistances, if required, can be estimated on the basis that the velocity in the rising main is uniform and has such a value that Q_l cube feet are delivered per cycle, *i.e.* at the rate $\frac{Q_l}{T_o + T_s + T_d + T_e}$.

The solution of this equation has two forms, according as e^2, is greater, or less than $4nh$.

(*a*) Corresponding to relatively small delivery heads: *i.e.* e^2, greater than $4nh$.

$$\text{Put } D = \sqrt{e^2-4nh}, \quad a = e-D, \quad b = e+D, \quad p = \frac{gD}{l},$$

$$M = \frac{2nv'+b}{2nv'+a}.$$

Since $v = v'$, when $t = 0$, we get:

$$v = \frac{1}{2n}\,\frac{aMe^{pt}-b}{1-Me^{pt}}, \quad \text{and } s = \frac{D}{np}\,log_e\,\frac{1-Me^{pt}}{1-M} - \frac{bt}{2n}; \quad \ldots \text{(iv)}$$

and $v = 0$, *i.e.* the curve cuts the time axis when:

$$t = \frac{1}{p}\log_e\frac{b}{aM} = T_d, \text{ approximately.}$$

This curve can be set off, just as the curve of v, and t, was set off for the first period.

(*b*) For relatively high heads, when e^2 is less than $4nh$,

$$\text{put } D' = \sqrt{4nh-e^2}, \text{ and } \beta = \frac{gD'}{2l}, \text{ and } a = \frac{2nv'+e}{D'},$$

$$v = \frac{1}{2n}\,\frac{2nv'\cos\beta t-(D'+ae)\sin\beta t}{\cos\beta t+a\sin\beta t} \quad \ldots \text{(v)}$$

$$\text{and } s = \frac{l}{ng}\log_e(\cos\beta t+a\sin\beta t) - \frac{et}{2n}.$$

This curve cuts the time axis when:

$$t = \frac{1}{\beta}\tan^{-1}\frac{2nv'}{D'+ae} = T_d \text{ approximately.}$$

(*c*) The most useful form is that obtained when $e=0$. We get:

$$t = \frac{l}{g\sqrt{nh}}\left\{\tan^{-1}v'\sqrt{\frac{n}{h}} - \tan^{-1}v\sqrt{\frac{n}{h}}\right\}. \quad \ldots \text{(vi)}$$

In any particular case we can draw the various curves obtained by the above theory, and obtain a diagram such as OXYZWO′ (see Sketch No. 245, Fig. 1). In this diagram the abscissæ represent time intervals, and the ordinates the values of v, where Av, represents the volume of water in cube feet per second passing through the ram pipe RE. Thus, if we consider the area B′BCC′, we see that the total volume of water that enters the ram pipe in the interval B′C′, is given by A × area B′BCC′, cube feet.

The diagram can thus be considered as a species of indicator diagram for one cycle of the ram. Hence we have as follows:

OX′ = T_0, XX′ = v_x = velocity in the ram pipe when the escape valve begins to close. Similarly YY′ = V_{max} = maximum value of v, attained during the valve closure. X′Z′ = T_s and ZZ′ = V_m = value of v, at the closure of valve; and A × area OXYZZ′ = Q_e. During the lifting portion of the cycle we have:

WZ = $V_m - v'$ = shock loss. Z′W = v' = value of v, when delivery valve opens, and Z′O′ = T_d; so that A × area Z′WO′ = Q_l.

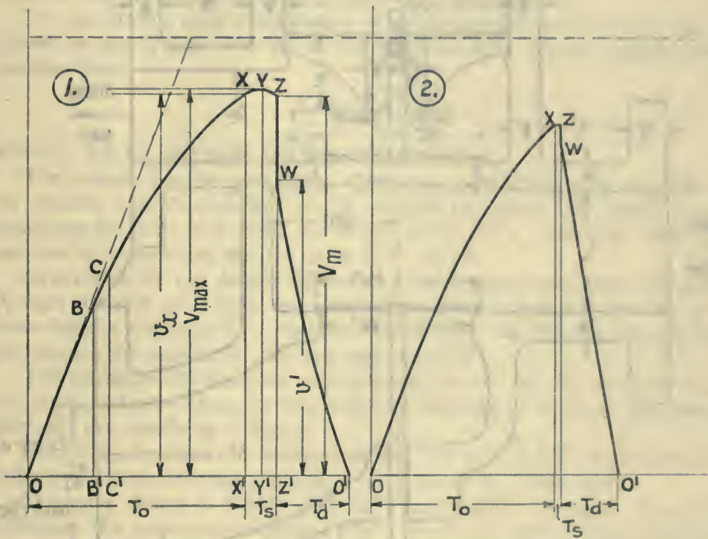
Thus, if the "indicator" diagram be completely determined, we can calculate:

$$\text{The volumetric efficiency, } \eta_v = \frac{Q_l}{Q_e} = \frac{\text{Area } Z'WO'}{\text{Area } OXYZZ'};$$

$$\text{and the mechanical efficiency, } \eta = \frac{Q_l h}{Q_e H} = \frac{h}{H}\eta_v.$$

This last definition is that known as Rankine's, and d'Aubuisson considers the mechanical efficiency, $\eta_A = \frac{Q_l(H+h)}{(Q_e+Q_l)H}$.

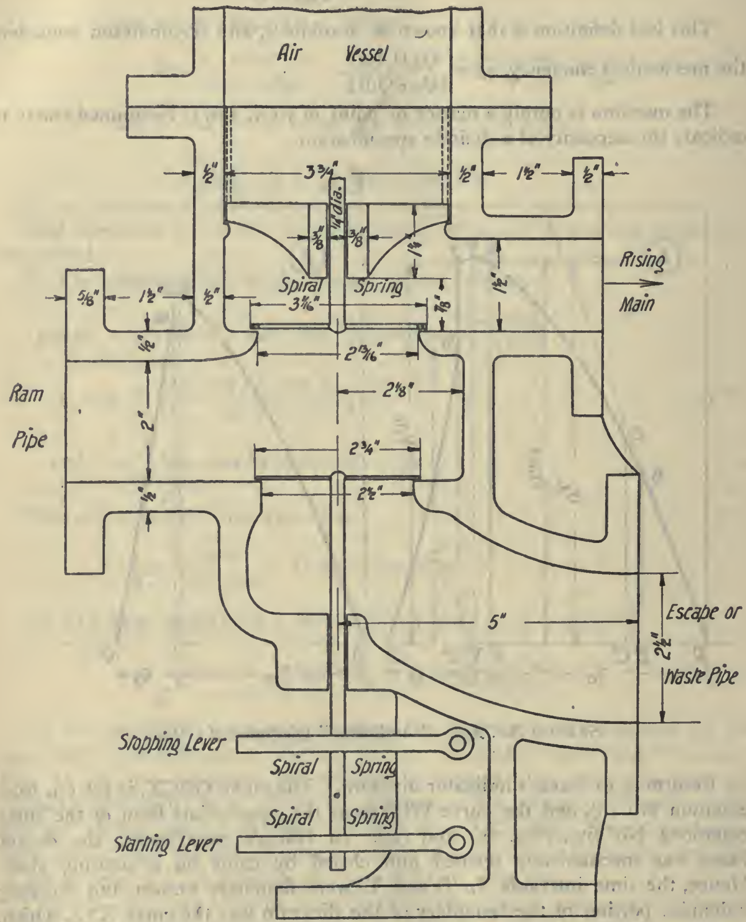
The question is purely a matter of point of view, and is mentioned solely to indicate the necessity of a definite specification.



SKETCH NO. 245.—"Indicator" Diagrams of a Ram.

Returning to Harza's indicator diagram. The curve OBCX, is set out from equation No. (ii), and the curve WO′, from the appropriate form of the three equations No. (iv), Nos. (v) and (vi). In Harza's experiments the escape valve was mechanically opened and closed by cams on a rotating shaft. Hence, the time intervals T_o, T_s and T_d were definitely known, and the only indefinite portion of the boundary of the diagram was the curve XYZ, which, since v_x, V_{max}, and V_m, are all nearly equal, cannot differ much from a horizontal straight line. Under these circumstances Harza's experiments show that if the various constants c_d, m, c, e, n, are determined and their average values used to calculate the curves OX, and WO′, the calculated values of the volumetric and mechanical efficiencies agree very fairly with the values obtained by measuring Q_e and Q_l.

The theory can therefore be practically applied to such cases as Pearsall's ram, where the valves are opened and closed by cams, worked by a heavy pendulum which receives an impulse at each swing by the escape of a small quantity of air from the air reservoir. In many other cases the escape valve is controlled by a swinging weight, and while the times T_o, T_s, and T_d, may not be accurately determined, T_o+T_s, and T_d+T_e, can be calculated.



SKETCH No. 246.—Hydraulic Ram with Spring Loaded Valves.

In general, however, the valves are controlled by springs, and the theory is more complicated. I have investigated the matter experimentally, and find that the following process permits a trial value for the loading of the valve to be obtained.

Calculate S_c, the spring load on the escape valve in pounds, when it is closed. S_o, the spring load on the escape valve in pounds when it is open to its fullest extent. W, the weight of the valve in pounds.

Then, taking the case shown in Sketch No. 246, where the escape valve opens upwards, we see that,

$$S_c - W = 62{\cdot}5 a_o H:$$

gives a minimum value of S_c, as if S_c, be less than this value the escape valve never opens.

$$\text{Again, } S_o - W = 62{\cdot}5\, a_v h_1 \quad \text{and } h_1 = \frac{A^2 v_x^2}{2g a_o^2 c_d^2}.$$

gives the value of v_x. Thus, we can determine XX′, and so fix X. If the valve is of one of the types discussed on page 805 a more accurate value for v_x, could of course be obtained by using the figures there given.

No experiments exist which enable us to calculate T_s, but it is plain that T_s, should be small, and thus W, should be decreased. In my experiments I assumed that:

$$T_s = 2\sqrt{\frac{W}{g}\frac{s_v}{S_o}}$$

where s_v, was the maximum lift of the valve cover.

The formula has no pretensions to accuracy, but the results obtained did not conflict with observations of the quantity Q_e, which will plainly depend greatly on the value of T_s, if T_s, is a large fraction of T_o. The shock loss ZW, can now be determined, and the curve WO′, set off.

In practice, we can determine m, and J, by observing the steady discharge through the ram pipe when the escape valve is held open, and similarly n, by observing the discharge from the reservoir M, when the delivery valve is open, the escape valve closed, and the ram pipe removed.

The process is obviously not as accurate as Harza's, but the practical results are good, and after three or four trials an increase of 10 to 15 per cent. in efficiency can usually be obtained.

Also, if Q_e, be observed, a check on the value of $T_o + T_s$, is obtained, and similarly if Q_l be observed, a check on the value of T_d, is obtained.

The valve shown in Sketch No. 244 is probably too heavy, but excellently illustrates the principles of good design, since the rubber beats produce a certain increase in S_c, without any increase in S_o, and the form of the valve cover secures a large value of the ratio $\frac{a_v}{a_o}$. Similarly, in the valve shown in Sketch No. 246, S_c, can be increased as necessary by lifting the starting lever, while S_o does not depend upon S_c. So far as my experiments go good efficiencies are usually obtained with S_c, only a little greater than the minimum value, and $S_o - W$, about 75 per cent. of $S_c - W$, but the ratio $\frac{a_v}{a_o}$ is more important. The delivery valve load should be as small as is consistent with preventing water from leaking back through it.

In preliminary calculations we may take the values given by Eytelwein.

$T_o = 0{\cdot}65$ period of complete cycle.
$T_s = 0{\cdot}10$ „ „
$T_d = 0{\cdot}20$ „ „
$T_e = 0{\cdot}05$ „ „

Fig. 2, Sketch No. 245, shows a diagram calculated for a case of spring loaded valves, by the methods detailed above; the only assumption made was that $v_x = V_{max} = V_m$, and the calculated and experimental results agreed within 2 per cent. The average error was found to be about 8 per cent., but the efficiency could usually be predicted within 5 per cent. after the first experiment.

Practical Rules.—The function of the air chamber is merely to absorb shocks, and to keep the water moving steadily forward in the rising main. Since the ram is in essence a shock producing machine, the air chamber is a most important factor in producing smooth working; and great care must be taken to keep it well charged with air. See Snifter in Sketch No. 244.

The size of the ram pipe is usually calculated so as to pass at least three times the available quantity of water under the working head H.

The length of the ram pipe is often taken as:

Length of ram pipe = Total vertical height between the escape valve and the reservoir $M = H + h$.

If we modify this by including an allowance for friction in the vertical height, the rule agrees very fairly with successful practice.

For small values of H, however, this rule gives somewhat shorter ram pipes than are found necessary.

Clarke suggests as follows (*Hydraulic Rams*, p. 54):

H . .	2 ft.	3 ft.	4 ft.	5 ft.	6 ft.	7 ft.	8 ft.	9 ft.	10 ft.
$l = h \times$ . .	3	2·8	2·65	2·45	2·25	2·0	1·85	1·65	1·5

if h, exceeds 100 feet;

$l = h \times$ . .	3·5	3·25	3·0	2·8	2·6	2·5	2·25	2·1	2·0

and states that while shorter lengths work successfully, trouble is experienced in adjusting the valves.

The delivery pipe, or rising main, should be proportioned according to the ordinary rules. A rule of thumb is:

For long distances:

$$\text{Diameter} = \frac{\text{Diameter of ram pipe}}{2}$$

For short distances:

$$\text{Diameter} = \frac{3 \times \text{diameter of ram pipe}}{8}$$

The air chamber is usually proportioned by the rule:

$$\text{Volume} = 2 \times \text{area of delivery pipe} \times \text{length of rising main.}$$

It is plain that if the air chamber can be proportioned so that its period of oscillation is a fraction of the period of the ram cycle, so much the better.

It seems needless to enter into such details as the precautions necessary when impure water is used to lift the pure article for human consumption. So far as I can ascertain, no appreciable loss of efficiency occurs in such cases.

The theoretical investigation has, I think, made it obvious that the escape and delivery valves should be as light as possible, and that the easier the delivery valve opens, the better. I append a very excellent design (Sketch No. 246).

The efficiency of a well designed ram is high, *e.g.* the Bollée ram at St. Julien le Vaublanc, where $h=45·9$ feet, $H=11·5$ feet, $\eta_v=16·1$ per cent. The mechanical efficiency is 80·5; or, if friction in the rising main is allowed for, 81·7 per cent.

At Vernon, $h=47·5$ feet, $H=39·4$ feet. The mechanical efficiency $=66·9$ per cent. when allowance is made for friction.

Harza's results show that a ram with a ram pipe two inches in diameter can be adjusted so as to have an efficiency exceeding 60 per cent. over a very wide range of head. Considering the size of the machine, it may be inferred that a carefully adjusted ram is probably one of the most efficient hydraulic machines in existence.

CHAPTER XIV.—(SECTION F)

RESISTANCE TO MOTION OF SOLID BODIES IN WATER

RESISTANCE TO BODIES MOVING IN WATER.—Friction of a flat body moving in water—Froude's experiments.
RESISTANCE TO A BODY MOVING IN A PIPE.
Friction of Rotating Discs.—Unwin's experiments at low velocities—Table—Gibson and Ryan's experiments at high velocities—Table—Influence of temperature—Average values.

SUMMARY OF FORMULÆ

Board moving in water:

$$F = B\left(\frac{v}{10}\right)^n \text{ pounds per square foot (see p. 855).}$$

Body moving in a pipe:

$$F = \frac{\pi d_1^2}{4} \xi \frac{v^2}{2g} \text{ pounds (see p. 856).}$$

Friction of a rotating disc (both sides):

$$M = \frac{4\pi\omega^n}{n+3} f R^{n+3} \text{ foot pounds (see p. 859).}$$

$$\text{Horse power} = \frac{4\pi\omega^{n+1}}{550(n+3)} f R^{n+3} \text{ horse power.}$$

Friction of a Body moving in Water.—The laws of friction between a body moving through, or past still water seem to be very similar to those between a body at rest and water moving past it.

When a long, straight board is towed in still water, it is found that the first three or four feet experience a greater resistance per square foot than the remainder of the length. This is obviously due to the friction of the first three or four feet having set the particles of water near the board in motion, so that the velocity of the remainder of the board relative to the water surrounding it is diminished. This effect is not likely to occur if the water is moving sufficiently fast to have a turbulent motion, unless the surface of the board is abnormally rough.

The following figures are given by Froude (*Report on Frictional Resistance of Water on a Surface*, see also *Encyc. Brit.*, article "Hydromechanics"), for boards with sharp ends, towed in water:

The resistance in pounds per square foot of area is:

$$F = fv^n = B\left(\frac{v}{10}\right)^n$$ where B is the value of F, when $v = 10$ feet per second.

Character of Surface.	Length of Surface, or Distance from Cutwater in Feet.					
	Two Feet.			Eight Feet.		
	n	B	C	n	B	C
Varnish	2·00	0·41	0·39	1·85	0·325	0·264
Paraffin	...	0·38	0·37	1·94	0·314	0·260
Tinfoil	2·16	0·30	0·295	1·99	0·278	0·263
Calico	1·93	0·87	0·725	1·92	0·626	0·504
Fine sand	2·00	0·81	0·690	2·00	0·583	0·450
Medium sand	2·00	0·90	0·730	2·00	0·625	0·488
Coarse sand	2·00	1·10	0·880	2·00	0·714	0·520
	Twenty Feet.			Fifty Feet.		
	n	B	C	n	B	C
Varnish	1·85	0·278	0·240	1·83	0·250	0·226
Paraffin	1·99	0·271	0·237	...	...	...
Tinfoil	1·90	0·262	0·244	1·83	0·246	0·232
Calico	1·89	0·531	0·447	1·87	0·474	0·423
Fine sand	2·00	0·480	0·384	2·06	0·405	0·337
Medium sand	2·00	0·534	0·465	2·00	0·488	0·456
Coarse sand	2·00	0·588	0·490	...	...	...

The values of B, give the average resistance over the whole area of the board of the given length, in pounds per square foot, at 10 feet per second.

The values of C, give the resistance under the same circumstances of one square foot at a distance from the cutwater equal to the length given at the head of the column for any length of board.

RESISTANCE OF A BODY MOVING IN A PIPE.—Our knowledge of the motion of bodies the cross-section of which is of a size comparable to that of the pipe through a pipe filled with water, is limited.

Sorge (*Gluckauf*, December 14, 1907) gives as follows:

The force required to move a cylinder of diameter d_1, and 0·3 m. (say 10 inches) long, with a velocity v feet per second, through a pipe of diameter d=0·026 m. (say 1 inch) is given by the equation:

$$F=\frac{\pi}{4} d_1^2 \xi \frac{v^2}{2g} \text{ pounds,}$$

where ξ, is as follows:

$\left(\frac{d_1}{d}\right)^2$	ξ	c	$\left(\frac{d_1}{d}\right)^2$: as	ξ	c
0·25	1·07	0·93	0·74	45·6	0·56
0·45	3·77	0·83	0·76	52·1	0·56
0·64	15·7	0·67	...	...	...

Sorge also states that these figures are very fairly represented by:

$$\xi+1=\frac{1}{(1-\eta)^{2\cdot8}}, \quad \text{where } \eta=\left(\frac{d_1}{d}\right)^2$$

The theoretical value would be:

$$+1=\frac{1}{c^2(1-\eta)^2}$$

where c, is the coefficient of contraction for the orifice formed by the cylinder, and the walls of the pipe; and the values of c, thus obtained, are tabulated above.

No experiments exist which would enable us to test these values of c, but we may expect that:

(*a*) In larger pipes, with cylinders the area of which bears the same ratio to that of the pipe, the values of ξ will be somewhat increased, at any rate for the larger values of η.

(*b*) For a thin disc, or a sphere, the values of ξ will be somewhat less than those for a cylinder of the same relative cross-section.

Friction of Rotating Discs.—For circular discs rotating about an axis we have:

If afv^n, be the friction on a small area of a, square feet, moving with a velocity of v, feet per second, the frictional moment for both sides of the disc is plainly:

$$M=\frac{4\pi\omega^n}{n+3} f R^{n+3} \quad \text{foot lbs.}$$

where ω is the angular velocity of the disc about its axis, and R, is its radius in feet.

Unwin (*P.I.C.E.*, vol. 80, p. 221) gives the following figures for a disc with a radius of 0·85 foot rotating in a cylindrical chamber.

Character of Surface.	Value of n.	Value of $f \times 10^n$ when the distance between the Sides of the Disc and the Ends of the Chamber is			
		1½ in.	3 ins.	6 ins.	3·5 ins.
Clean polished brass . .	1·85	0·202	0·209	0·230	...
Clean polished brass, chamber coated with coarse sand .	1·95	...	0·244	...	...
Painted cast iron . . .	1·86	0·218	0·232	0·247	...
Painted and varnished cast iron	1·94	...	0·220	0·233	...
Tallowed brass . . .	2·06	...	0·218	...	...
Cast iron	2·00	0·213	0·227	0·243	...
Cast iron covered with fine sand	2·05	...	0·340	...	...
Cast iron covered with coarse sand	1·91	0·587	0·638	0·715	...
Cast iron covered with coarse sand, chamber coated with coarse sand	2·17	...	...	...	0·799

Apparently f, is independent of the radius, but its value increases as the distance between the edges of the disc and the sides of the chamber is increased.

These values may be employed in the calculation of the effect of dead water friction on the wheels of centrifugal pumps and turbines; and it should be remembered that in most cases the distance between the wheel and the fixed casing being less than 1½ inch, they may be considered as high.

While Unwin's experiments cover the greatest variety of surface, they were made at speeds ranging from 67, to 350 revolutions per minute (*i.e.* about 10 feet per second mean velocity). Those of Gibson and Ryan (*P.I.C.E.*, vol. 179, p. 313) being made at speeds of 450 to 2200 R.P.M., agree better with the conditions usually found in modern centrifugal pumps.

Messrs. Gibson and Ryan use the formula given above, except that the thickness of the outer end of the disc is held to influence the friction; which, although correct for the particular experiments under consideration, does not usually apply in practical examples.

A tabulation of Messrs. Gibson and Ryan's results is shown in table on page 858, the term "clearance" being used to indicate the distance between the sides of the disc, and the sides of the casing.

Messrs. Gibson and Ryan also investigated the influence of the temperature of the water on the frictional resistance.

Let P_t, denote the quantity known as Poiseuille's ratio (see p. 19) where:

$$P_t = \frac{1}{1+0·0337T+0·000221T^2}$$

if T, be expressed in degrees Centigrade,

$$\text{or } P_t = \frac{1}{0·474+0·0144t+0·000682t^2}$$

if t, be expressed in degrees Fahr.

Character of Disc.	Character of Casing.	n	Value of $f \times 10^2$ when the Clearance is ⅛ in.	⅝ in.	1⅛ in.	1⅝ in.	2⅛ ins.
Polished brass, 12 inches in diameter.	Rough cast iron.	1·8 to 1·81	0·409	0·422	0·414	0·432	0·474
...	Painted ,,	1·79 to 1·80	0·360	0·359	0·356	0·359	...
...	Smooth ,,	1·77 to 1·82	0·346	0·373	0·438	0·421	0·471
Do., 9 inches in diameter.	Painted ,,	1·83	...	0·342	...	...	...
Rough cast iron, 12 inches in diameter.	Rough ,,	1·91	...	0·300	0·301	0·297	0·302
	Do., with 2 annular baffles ½ inch deep.	1·88	...	0·337	0·337	...	...
	Painted cast iron.	1·80 to 1·81	...	0·428	0·426	0·424	0·414
Do., 9 inches in diameter.	,, ,,	1·85	...	0·372	...	...	...
Painted and varnished, 12 inches in diameter, cast iron.	Rough ,,	1·85	...	0·353	...	...	...
	Painted ,,	1·80	...	0·361	0·374	...	...
Painted and varnished cast iron, 9 inches in diameter.	,, ,,	1·83	...	0·340	...	...	...
Brass, 12 inches in diameter, with 4 radial vanes in each face.	Painted cast iron, vanes ½ inch deep	1·91	0·903	1·067	...	...	...
	Do.		Clearance measured over the Vanes.				
...	Vanes, ¼ inch deep.	1·95		$\frac{3}{32}$ inch. 0·668		⅝ inch. 0·687	

Let w_t, denote the weight of a cube foot of water at a temperature t.

Then putting R_t for the resistance at a temperature equal to t degrees and R_{65} for the resistance at 65 degree Fahr.:

$$R_t = R_{65}\left(\frac{P_t}{P_{65}}\right)^{2-n}\left(\frac{w_t}{w_{65}}\right)^{n-1}$$

and the figures tabulated above are those appropriate to a temperature of 65 degrees Fahr.

The increase in resistance per degree Fahrenheit at a temperature near to 65 degrees Fahr. is about ½ per cent. when $n = 1{\cdot}80$, and is inappreciable when $n = 2{\cdot}00$.

In none of Gibson and Ryan's experiments does this increase amount to 2·5 per cent. It is therefore not proved that the correction will hold accurately for such temperatures as 120 or 150 degrees Fahr.

A study of their own results, and those obtained by Unwin, has led Gibson and Ryan to propose the following table of average values of n, and f:

Casing.	Mean Velocity of Disc in Feet per Second.	Disc. Polished Brass.		Disc. Painted or Varnished Metal.		Disc. Rough Cast Iron.	
		n	$f \times 10^2$	n	$f \times 10^2$	n	$f \times 10^2$
Smooth, *i.e.* machined or painted metal.	10	1·85	0·31	1·94	0·26	2·00	0·23
	20	1·84	0·33	1·91	0·29	1·96	0·27
	30	1·83	0·35	1·88	0·32	1·91	0·32
	40	1·82	0·37	1·84	0·35	1·86	0·37
	50	1·80	0·39	1·80	0·37	1·81	0·42
Rough cast iron.	10	1·92	0·29	1·97	0·27	2·00	0·26
	20	1·89	0·33	1·94	0·29	1·98	0·27
	30	1·86	0·37	1·91	0·31	1·96	0·28
	40	1·83	0·41	1·88	0·33	1·93	0·29
	50	1·80	0·44	1·85	0·35	1·91	0·30

These results cover all practical cases, and serve to show the extreme importance of keeping the clearances small.

I believe that the suggestions made by the authors for design (*vide ut supra*) are erroneous, since their sketches seem likely to permit more leakage to take place than is really permissible. Their principles, however, are quite sound, and are decidedly neglected in many modern designs.

CHAPTER XIV.—(SECTION G)

IMPACT OF WATER ON MOVING BODIES

GENERAL EQUATION OF THE MOTION OF WATER IN A TUBE THAT IS MOVING IN A GIVEN MANNER.—General equation—Equation for pressure at a point in the tube—Application to turbine wheel—General equation of turbines—Practical rules for selection of points of entry and exit.

IMPACT OF A STREAM OF WATER ON A MOVING BODY.—Theoretical equations—Deflection angles—Loss of head by shock—Practical rules—Pelton wheels—Loss by friction on the surface of the body—Francis turbine—Loss by change of velocity.

SYMBOLS

a, is the area of the jet of water before it strikes the vane.
A, is the area of the wetted surface of the vane (see p. 867).
a_i, is the area in square feet of the closed channel at the point of complete entry (see p. 862).
h_l (see p. 862). h_i (see p. 862). h_s (see p. 862).
H, is the total head in feet under which the turbine works.
A suffix notation, is employed in connection with the following symbols:
Suffix a, refers to any point intermediate between i, and e.
Suffix e, refers to the point of exit from the vane, or moving channel.
Suffix i, refers to the inlet or point of entry into the vane or moving channel.
p, is the pressure in feet of water.
Q, is the quantity of water delivered in cubic feet per second.
u, is the absolute velocity of any point in the moving body in feet per second.
v, is the velocity of the water relative to a point moving with velocity u.
w, is the absolute velocity of the water.
V_1, is the observed velocity of the water immediately after the impact is complete, relative to a point moving with velocity u_i.

v'_e, u'_e, w'_e (see p. 865). $v'_i=\dfrac{Q}{a_i}$ (see p. 865).

$V_o=\dfrac{v_i+v_e}{2}$ (see p. 867).

β, is the angle between the positive directions of u, and v.
δ, is the angle between the positive directions of u, and w.
ϵ, is the hydraulic efficiency of the turbine.
θ_i, is the angle between the direction of v_i as geometrically obtained, and the observed direction of V_1, *i.e.* the shock angle.
κ_i, is the angle between PD, and PE, the actual direction of V_1.
λ_i, is the space angle between v_i and V_1.

SUMMARY OF FORMULÆ

Energy imparted to the vane or pipe:

$$\frac{62{\cdot}5}{g}(u_i w_i \cos\delta_i - u_e w_e \cos\delta_e) \text{ foot lbs. per cusec.}$$

Relative velocities:

$$v_i^2 = u_i^2 + w_i^2 - 2u_i w_i \cos\delta_i$$

$$\frac{w_i}{\sin\beta_i}=\frac{v_i}{\sin\delta_i}$$

Pressure equation:

$$p_i - p_a - h_i = \frac{u_i^2 - u_a^2}{2g} - \frac{v_i^2 - v_a^2}{2g}$$

General equation of a turbine:

$$g\epsilon H = w_2 u_2 \cos \delta_2 - w_4 u_4 \cos \delta_4$$

$$\epsilon H = \frac{w_2^2 - w_4^2}{2g} + \frac{u_2^2 - u_4^2}{2g} + \frac{v_4^2 - v_2^2}{2g}$$ (see p. 880).

Loss by impact at entry:

Theoretical, $\frac{v_i^2 \sin^2 \theta_i}{2g}$ or, $\frac{v_i^2 \sin^2 \lambda_i}{2g}$

Practical, 0·5 to 0·7 $\frac{v_i^2 \sin^2 \theta_i \text{ or } \lambda_i}{2g}$.

Observational, $\frac{v^2 - V_1^2}{2g}$ or, $\frac{v_i^2 - v_i'^2}{2g}$.

Loss during motion over the vane:

Observational, $\frac{V_1^2 - v_e^2}{2g}$ or, $\frac{v_a - v_e^2}{2g}$ $v_i'^2 = \frac{Q}{a_i}$.

Farmer's rule for all losses: $\frac{v_i^2 - v_e^2}{2g}$ with $\frac{v_i - v_e}{V_0} = 0·0266 \frac{A}{a} \frac{1}{V_0^{0.3}}$.

General Equation of the Motion of Water in a Tube that is moving in a given Manner.—The motion of a fluid in a space the boundaries of which are themselves in motion is a somewhat complex problem.

Since the surfaces bounding the fluid are themselves in motion, the pressure may be a discontinuous function of the co-ordinates, and the general hydrodynamical equations given by Euler require modification. No very good investigation exists in English, although Mise's *Theorie der Wasserräder* gives a comparatively simple presentation in German of the practical case of a turbine wheel in which the space considered is rotating round a fixed axis.

At best, the proofs are long, and require greater mathematical equipment than engineers usually possess.

The following method of investigation is simple, and is subject only to the same uncertainties as affect the use of Bernouilli's equation in ordinary hydraulic problems. In the only practical application that has yet been made the results are known to be quite as accurate as are those of any hydraulic calculations, and they are believed to be applicable to all possible cases.

Consider first the simplest case of motion in one plane, and measure all velocities in feet per second.

Let Q, cusecs of water enter the pipe or moving space, with an absolute velocity w_i, at a point the velocity of which is u_i. Then, Q, cusecs of water must leave the pipe with a certain absolute velocity, say w_e, at a point the absolute velocity of which is u_e.

Then, assuming that there are no losses by shock at entry, or exit, the energy imparted to the pipe is:

$$\frac{62·5Q}{g}(u_i w_i \cos \delta_i - u_e w_e \cos \delta_e) \text{ foot lbs. per second};$$

where δ_i and δ_e are the angles between the directions of u_i, and w_i, and u_e and w_e.

This equation can be easily verified for such simple cases as a pipe moving with uniform velocity (i.e., $u_i = u_e$), by actually calculating the pressures on the pipe. A general proof for three dimensional motion is given by Thomson and Tait (*Natural Philosophy*, Part 1, p. 208). Thus, the water loses the same amount of energy, and:

$$p_i+\frac{w_i^2}{2g}=p_e+\frac{w_e^2}{2g}+\frac{w_i u_i \cos\delta_i-w_e u_e \cos\delta_e}{g}.$$

The equation can also be transformed by considering the relative velocities v_i, and v_e. We have, see Sketch No. 247:

$$v_i^2 = u_i^2+w_i^2-2u_i w_i \cos\delta_i,$$

and $$v_e^2 = u_e^2+w_e^2-2u_e w_e \cos\delta_e.$$

Thus, $$p_i-p_e = \frac{u_i^2-u_e^2}{2g}-\frac{v_i^2-v_e^2}{2g}.$$

This equation must of course be corrected for frictional and other losses in the pipe, and also for any difference in elevation between the points represented by i and e (see below). The results of such experiments as exist (which, for practical reasons, are mainly confined to pipes moving uniformly or rotating round a fixed axis) confirm the equation, and do not indicate that the laws of friction are in any way altered, provided that the velocity of the water, relative to the pipe, is used when calculating the frictional losses.

The questions concerning losses by shock at entry or exit are discussed in detail later. For the present it suffices to state that if h_l, represent the total loss of head by friction, curvature, and shock, reduced to feet of water by the usual rules, then:

$$p_i-p_e-h_l=\frac{u_i^2-u_e^2}{2g}-\frac{v_i^2-v_e^2}{2g}.$$

So also, if it be desired to ascertain the motion at any point a, intermediate between i, and e, we have:

$$p_i-p_a=\frac{u_i^2-u_a^2}{2g}-\frac{v_i^2-v_a^2}{2g};$$

and if h'_l, represent the friction and other losses, and h_s, the height of the point represented by a, above the point represented by i, then:

$$p_i-p_a-h'_l-h_s=\frac{u_i^2-u_a^2}{2g}-\frac{v_i^2-v_a^2}{2g}.$$

In applying these equations to practical problems, two cases occur, which should be distinguished before the frictional and other losses are calculated.

(i) Open Vane.—The water while on the vane has a free surface exposed to constant pressure. Then $p_i = p_e = p_a$, so that v_e, or v_a, can be calculated. This is the case of a Pelton wheel, or free deviation turbine. We have at once:

$$u_i^2-u_a^2-(v_i^2-v_a^2) = -2g(h'_l+h_s)$$

As a particular case, we frequently have $u_i = u_a = u_e$.

Then, if we assume h'_l, and $h_l = 0$,

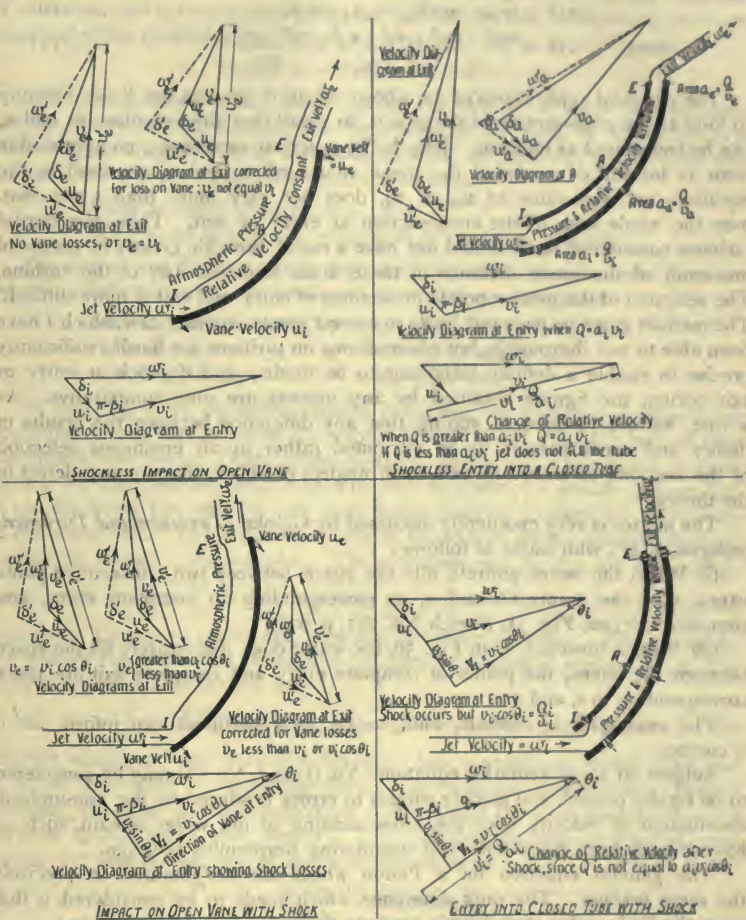
$$v_i^2-v_a^2 = 2gh_s, \text{ and if } h_s = 0,\ v_i = v_a = v_e.$$

(ii) Closed Pipe.—The water moves in a closed passage:

Then, $v_i = \frac{Q}{a_i}$, $v_e = \frac{Q}{a_e}$, and $v_a = \frac{Q}{a_a}$.

Thus, p_e, or p_a, can be calculated.

These general equations can be applied to any problem. In practice, however, the fundamental equation of a turbine is more easily arrived at by the following process. For distinction's sake (see p. 907) use suffixes 2, and 4, foı the entry and exit sections (see p. 877) of a turbine. Then, the change



SKETCH No. 247.—Diagrams for Motion on Vanes and Pipes.

in the moment of momentum of the water round the axis of the turbine is $\frac{62 \cdot 5}{g}(w_2 r_2 \cos \delta_2 - w_4 r_4 \cos \delta_4)$ foot pounds per cusec.

Thus, if ω be the angular velocity of the wheel, work is done at a rate of:

$$\frac{62 \cdot 5\omega}{g}(w_2 r_2 \cos \delta_2 - w_4 r_4 \cos \delta_4) = \frac{62 \cdot 5}{g}(w_2 u_2 \cos \delta_2 - w_4 u_4 \cos \delta_4)$$

ft. lbs. per cusec.

The work done by one cusec of water is :

$$62{\cdot}5\epsilon H \text{ foot lbs.}$$

where ϵ (see p. 880) is the hydraulic efficiency of the turbine. We thus get :

$$g\epsilon H = w_2 u_2 \cos \delta_2 - w_4 u_4 \cos \delta_4, \quad \ldots \text{(i)}$$

and, $$\epsilon H = \frac{w_2^2 - w_4^2}{2g} + \frac{u_2^2 - u_4^2}{2g} + \frac{v_4^2 - v_2^2}{2g} \quad \ldots \text{(ii)}$$

The practical applications of the above equation present but little difficulty so long as the cross-section of the pipe is so small that the velocities w, and u, can be considered as uniform. Judging by practical experience, no appreciable error is introduced provided the mean values of w, and u, are used in the equation, and the value of u_2, or u_4, does not vary more than 5 per cent. over the whole area of the cross-section at entry or exit. Thus, the partial turbines considered later should not have a radial breadth greatly in excess of one-tenth of the mean distance of these areas from the axis of the turbine. The selection of the precise points or sections of entry and exit is more difficult. The method given on page 907 leads to correct results in every case which I have been able to test thoroughly, but observations on turbines are hardly sufficiently precise to enable a definite statement to be made; and if shock at entry or exit occurs, the figures obtained by any process are only comparative. As a rule, we are justified in stating that any difference between the results of theory and experiment is to be attributed rather to an erroneous selection of the sections of entry and exit, or to neglect of shock, than to any defect in the theory.

The matter is very excellently discussed by Gelpke (*Turbinen und Turbinenanlagen*, p. 57), who states as follows :

(i) When the water entirely fills the space between two consecutive guide vanes, take the points O, and *a*, as corresponding to complete entry, and complete exit (see Fig. 3A, Sketch No. 261, p. 926).

(ii) Where, however, as in Fig. 3B, the water does not entirely fill the space between the vanes, the points of complete entry and complete exit are those corresponding to *e*, and *a*.

The reasoning is obvious, and, so far as experiment can inform us, it is correct.

Subject to these remarks, equations No. (i) and No. (ii) may be considered to be rigidly proved, and as only subject to errors produced by the non-uniform distribution of velocity over the cross-sections of the water stream, such as have already been discussed when considering Bernouilli's equation.

The general equation for a Pelton wheel can be obtained in precisely the same manner. The only difference which needs to be considered is that since the jet is under atmospheric pressure throughout its whole motion :

$$p_i = p_e = \text{atmospheric pressure}$$

Consequently :

$$v_e^2 = v_i^2 - (u_i^2 - u_e^2) - 2g(h_l + h_s);$$

which permits us to calculate v_e when h_l has been obtained.

SHOCK OR IMPACT LOSSES.—In the above discussion we have assumed that no shock or impact losses occur at entry. The geometrical condition for this is plainly that the direction of v_i shall be the same as the direction of the tangent to the vane or pipe at entrance.

Let us now assume that v_i as determined by the velocity triangle makes an angle θ_i with this direction. As will later appear, θ_i may be a space angle, *i.e.* determinable by three dimensional geometry only.

The best method of ascertaining v (i.e. v_i, v_n or v_e) is the ordinary diagram of velocities, and is best expressed by stating that the line representing w is the diagonal of the parallelogram formed by u and v. Or in vector notation,

$$\text{Vector } w = \text{Vector } u + \text{Vector } v.$$

The circumstances which occur during the motion of water over an open vane (*e.g.* a Pelton wheel bucket) are shown on the left hand of Sketch No. 247. The full line diagrams, with undashed symbols for velocities, show how exit occurs when $u_e = u_i$. Theoretically, we then have:

With no shock at entry, $v_e = v_i$.
With shock at entry, $v_e = v_i \cos\theta_i = V_1$, say in practice.

The dotted line diagrams, with dashed symbols for velocities, refer to a case where u_e, differs from u_i. According to the theory we then have:

With no shock at entry, $v_e^2 = v_i^2 + (u_e^2 - u_i^2)$.
With shock at entry, $v_e^2 = v_i^2 \cos^2\theta_i + (u_e^2 - u_i^2)$
or, in practice, $= V_1^2 + (u_e^2 - u_i^2)$.

In practice, these equations require correction for friction and other losses occurring during the motion on the vane. Thus, v_e, will generally be less than the theoretical value. Observation, however, shows that when shock at entry occurs, V_1, the observed relative velocity just after entry, is usually greater than $v_i \cos\theta_i$; hence the effect of the vane losses may be masked by a decrease in the shock loss. The right hand side of the Sketch shows similar diagrams for motion through a closed tube. The circumstances at exit are less complicated, since the ratio $\frac{v_e}{v_i}$, is determined by the cross-sections of the tube at entry and exit. The entry conditions, however, are more complex. Neglecting for the present the difficulties introduced by three dimensional motion, let us consider geometrically shockless entry.

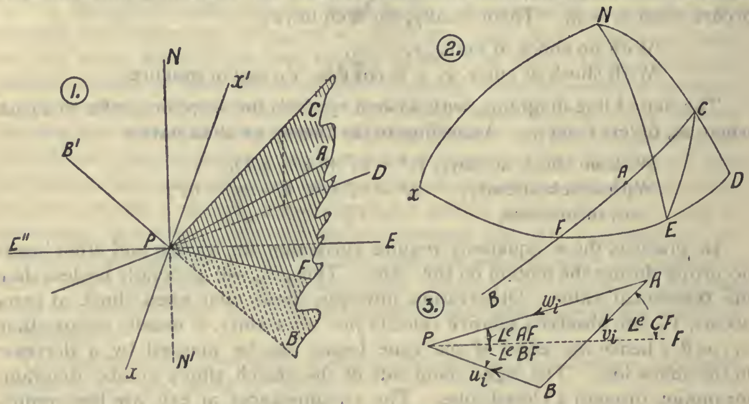
Put $v'_i = \frac{Q}{a_i}$, and let v_i, denote the geometrically obtained value of the relative velocity at entry. If v_i, be greater than v'_i, the tube is not filled, and if v_i, be less than v'_i, splashing occurs. In practice, however (*e.g.* Francis turbines), the tube (or wheel cell) is not isolated, and the jet is bounded by rigid surfaces (guide vanes and crowns). Thus usually, the tube is filled at entry, and an increase or decrease of pressure equivalent to

$$\frac{v'^2_i - v_i^2}{2g} \text{ feet occurs.}$$

Thus, in reality, the entry, although geometrically shockless, is probably attended by a certain loss of head (compare p. 798) caused by incomplete conversion of velocity into pressure, or *vice versa*. A similar loss is possible when shock at entry occurs; the equations are as above, except that $v_i \cos\theta_i$ or V_1, must be substituted for v_i.

It should also be realised that while the above discussion makes a distinction between an open vane and a tube, a large jet impinging on an open vane may produce a very fair imitation of the geometrical (as distinct from the pressure) conditions of impact into a tube, by the interference of individual portions of the jet.

Sketch No. 248 is intended to illustrate this case, which is of importance when Pelton wheels with large jets (say over two inches in diameter) are considered. Let a jet moving with absolute velocity w_i, along AP, strike a body which is moving with absolute velocity u_i, along PB′, at P. Then (see Fig. 3) AB = v_i, is the relative velocity, and is in the direction CP. But (see Fig. 1) the surface of the body being represented by the plane E″x′Ex, shock occurs, and the water will move off along DP, the projection of CP on this plane, with a relative velocity which is theoretically equal to $v_i \cos \lambda_i$, where λ_i is the angle CPD. Now, assume that either due to mutual interference of the various portions of the jet, or due to the body being really a closed tube, the water is



SKETCH No. 248.—Three Dimensional Impact.

(1) Perspective Sketch. (2) Spherical Diagram. (3) Velocity Diagram.

constrained to move off along the direction EP, a further shock occurs, and theoretically the final relative velocity is $v_i \cos \theta_i$, where

$$\cos \theta_i = \cos \lambda_i \cos \kappa_i ;$$

the angle DPE being denoted by κ_i; so that θ_i is the space angle CE. The spherical diagram (see Fig. 2, which has been turned through 90 degrees around the normal NP, relative to Fig. 1) shows that if DPE = k_i, we have:

$$v_i \cos \theta_i = v_i \cos \lambda_i \cos \kappa_i$$

and the shock loss is, $\dfrac{v_i^2 \sin^2 \theta_i}{2g}$.

I have endeavoured to test the above theories experimentally. The difficulties are very great, and the results were by no means concordant. In every case, however, I was able to assure myself that a loss of the order of magnitude indicated by the theory occurred. Thus, while I believe that the numerical rules now given are extremely unreliable, I have no hesitation in stating that the theory forms a good basis for design, and that the various methods of producing loss by shock which are here indicated should be avoided whenever possible. Considering the losses in detail:

(*i*) Loss by shock. The usual theory states that:

$$\text{Loss} = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

Observational results suggest that this value is not attained until θ_i is at least 40 to 50 degrees.

For smaller values of θ_i it appears that some water adheres to the vane near the point of impact, and this (probably eddying) mass of water forms a curved excrescence, so that "shock" probably does not occur, the observed loss being more of the nature of curve loss.

The tests carried out on Francis turbines suggest that an average value is;

$$\text{Loss by shock} = 0{\cdot}5 \text{ to } 0{\cdot}7 \frac{v_i^2 \sin^2 \theta_i}{2g}$$

and it is probable that the coefficient the average value of which is 0·5 to 0·7 is really a function of θ_i, which increases from 0, when $\theta_i = 0$, to 1 when $\theta_i = 45$ degrees approximately.

Tests of Pelton wheels and Francis turbines which receive water over a portion of their circumference only indicate that the full value of the theoretical loss may occur, but I have never found a case where more than the full value was obtained.

Thus, generally speaking, we can assume that V_1, the observed velocity just after entry, is greater than $v_i \cos \theta_i$, but is less than v_i.

(*ii*) Loss by impact of water on water.

This is peculiar to closed tubes, and is caused by v_i, not being equal to v'_i.

Theoretically, if v'_i, is greater than v_i, and splashing is prevented, there is no loss, but merely a drop of pressure equal to $\frac{v'^2_i - v_i^2}{2g}$.

Similarly, if v'_i, is less than v_i, Andres' experiments lead us to expect that a rise of pressure will occur, but that the observed rise will be about 70 to 90 per cent. of the theoretical value $\frac{v_i^2 - v'^2_i}{2g}$.

I have observed both the rise and fall, but in no case did the observed values come within 20 per cent. of the theoretical.

(*iii*) Losses in the vane or tube.

These are analogous to skin friction and curve losses in fixed pipes.

Reference is made to Finkle's (see p. 933), and Eckhart's (see p. 937) experiments, also to page 888 for Lang's rules.

Farmer (*Trans. of Canadian Soc. of C.E's.* 1897, p. 275) found experimentally that

$$\frac{v_i - v_e}{V_o} = 0{\cdot}0266 \frac{A}{a} \frac{1}{V_o^{0\cdot3}}$$

where A, is the area of the wetted surface of the vane, a is the area of the cross-section of the jet, and $V_o = \frac{v_i + v_e}{2}$.

The form of the equation has a certain rather insecure theoretical basis. Details are not given, but it is known that the jet used did not greatly exceed three-quarters of an inch in diameter, and also that the velocity of the jet did not greatly exceed 120 ft. per second. The values thus obtained when $\frac{A}{a} = 6{\cdot}6$, range from 0·955 to 0·947, and compare so favourably with the experiments of Eckart and Finkle that I am inclined to believe that there can have been no loss by shock at entry (*i.e.* the plane was adjusted until $\theta_i = 0$), and that the stream cannot have traversed a curved path during its motion on the vane.

CHAPTER XV

TURBINES AND CENTRIFUGAL PUMPS

TURBINES.—"Francis Turbines"—"Pelton Wheels"—Method of developing the theory—**Partial turbines**—Mathematical methods employed.

DESCRIPTION OF A FRANCIS TURBINE.—Guide vanes and crowns—Wheel vanes and crowns—Motion of the water—"**Flow lines,**" or boundaries of partial turbines—Arrangement of turbines relative to the turbine house—References—Horizontal and vertical turbines—Multiple turbines with two, three, or more wheels on one shaft.

NOTATION.—List of symbols—"Space" angles and lengths.

THEORY OF THE IDEAL TURBINE.—Losses in the various portions of the water path—Pressure at the various sections of the water path—**General equation**—Expression in absolute velocities only—Hydraulic efficiency—Mechanical efficiency.

PRACTICAL DESIGN OF TURBINES.—Ordinary equations relating to pipe friction are probably inapplicable—Values of the individual losses—Methods of reducing these losses—Loss by leakage—Allowance for curve losses in the wheel—Draft tube losses—Loss due to residual velocity—Relation between η and ϵ—Table—Estimation of ϵ when the loss due to residual velocity is given—Example.

PRELIMINARY SKETCH DESIGN OF A TURBINE.—Experimental basis of the fundamental equation—Calculation of the losses due to skin friction.

DETERMINATION OF THE NECESSARY PROPORTIONS AND SIZE OF A TURBINE.—Relation between the head and the angular speed and horse-power of a turbine—Classification of turbines by values of C—Gelpke's eight types—Table—Comments—"Specific speed"—Extension of table—Moody's values—Examples—Variation of the head—Modification of the standard types—Rôle of turbine designers.

TABULATION OF VELOCITIES AND DIMENSIONS IN THE VARIOUS TYPES.—Quantities which mainly depend on the speed of the turbine—Dimensions which mainly depend on the horse-power of the turbine—Comments—Estimation of the hydraulic efficiency.

NUMBER OF GUIDE VANES AND WHEEL VANES.—Table.

PRELIMINARY ESTIMATION OF THE EFFICIENCY OF A TURBINE.—Influence of the quantity of water passing through the turbine—Form of the efficiency curve—Influence of the speed of the turbine—Detailed estimation of the losses in the guide vanes—Allowance for curvature losses in the wheel.

SYSTEMATIC ESTIMATION OF THE VARIOUS LOSSES.—Losses in the draft tube—Entry into the turbine—Losses in the guide passages—Regulation—Length of guide vanes.

CIRCUMSTANCES IN THE WHEEL.—Leakage losses—Entry into the wheel—Determination of the shock loss by selection of the point of entry—Comments—Passage through the wheel—Partial turbines—Initial and final points on the turbine boundaries—Determination of the intermediate portions of the boundaries—Kaplan's method—Criticism—Number of partial turbines—Conditions at exit—"Radial exit".—Definitions—Determination of the exit angles—Summary of the exit losses—Approximate estimation of τ_3 and τ_4.

ACCURATE METHOD OF GELPKE.

GEOMETRY OF WHEEL VANES.

APPLICATION TO AN EXISTING TURBINE.—Practical rules.

MECHANICAL DESIGN OF TURBINES.—End pressures—Balancing piston—Shafts—Vanes and crowns—Calculation of thickness of wheel vane—Wheel crowns—Wheel boss—Guide vanes—Clearance space—Bearings—Lubrication.

METHODS OF REGULATION.—Fink's method—Cylinder gates—Valve regulation—Governor specification.
THE FALL INTENSIFIER.—Herschell's theory—Comments.
PELTON AND SPOON WHEELS.—Description—Methods of regulation—Sphere of utility—Size of jet—Practical rules.

(i) *TYPICAL PELTON WHEELS.*—Dimensions—Ratio of area of jet to area of bucket—Number of buckets—Direction of jet relative to buckets—Angle of impact—Angle of exit.
(ii) *SPOON WHEELS.*—Description—Dimensions—Direction of jet relative to buckets—Number of buckets—Theoretical advantage over the typical Pelton wheel—Criticism—Probable sphere of utility—Probable value of the efficiency of Pelton and Spoon wheels.

NOTATION.
INVESTIGATION OF THE EFFICIENCY OF A PELTON WHEEL.—Finkle's method—Eckart's values for the energy of the jet—Deviation of the water at various points of the jet—Impact losses—"Foam and friction" losses—Loss due to residual or exit velocity—Final value of the hydraulic efficiency—Eckart's values of the various losses.
CENTRIFUGAL PUMPS.—Theory—Estimation of hydraulic efficiency—Method of ascertaining the type and preliminary dimensions of a centrifugal pump—Design of housing.
LOSSES BY SHOCK.—Design of guide vanes—Practical corrections—Variation of H as Q is altered—Values of efficiency—Preliminary rules—Divergences from turbine design.
GOVERNING OF TURBINES.
Water Tower.—General theory—Solution by the method of arithmetical integration—Example—Water tower of variable cross-section—Preliminary approximations—Harza's method—Preliminary determination of the size of the water tower—Larner's method.
Differential Water Tower.—Theory—Johnson's investigation.

THE treatment of turbines given in this chapter is almost exclusively regarded from the point of view of a buyer. The customs of turbine manufacturers are referred to on several occasions, and are almost invariably contrasted with the results of calculations. It is therefore advisable, once for all, to state that I consider that these customs are logical and practically desirable. Attention is drawn to these customs, for the very practical reason that ignorance or forgetfulness of such matters on the part of the buyer, leads to delay and mutual dissatisfaction. I would strongly advise that every engineer when inquiring for turbines should obtain the makers' values for the maximum volume of water that the turbine can pass, and also the horse-power then developed. Even if the information is not required, the attention paid to the subject in the reply will form a very excellent preliminary means of discriminating between the agent whose information is derived from a study of catalogues, and the trained engineer who can provide valuable practical advice, in addition to turbines delivered F.O.B.

I must at once acknowledge that my methods of investigation are largely founded on those laid down by Gelpke (*Turbinen und Turbinenanlagen*, lately translated as Gelpke and van Cleeve, *Turbines and Turbine Installations*). Either this book or Mead's (*Water Power Engineering*) gives a more complete presentation of the subject than exigencies of space permit me to do. Indeed, were the turbine designer's or maker's the only possible point of view this chapter would be incomplete, and in a certain degree misleading. My object, however, is to treat the subject from a civil engineer's point of view, which is solely that of a turbine buyer.

Thus the treatment of page 888 *et seq.* is intended to fix the main dimensions, and approximate outlines of the turbines required. The accompanying hydraulic works can be then designed, and tenders for the turbines invited. When the maker's drawings are obtained, the methods of page 901 *et seq.* can be used to investigate the designs in detail. The relative values of the divergences from the generally accepted theories can then be estimated from the theory detailed on page 916 *et seq.*

Two defects must be carefully borne in mind. No treatment of the mechanical, as distinct from the water tower (see p. 944), methods of turbine regulation is attempted. A mathematical theory, very closely resembling that employed in connection with alternate currents in electrical design, can be developed, and I intend to publish the same shortly. In practical applications, however, the various constants cannot be estimated from the drawings, and the makers have not at present realised their commercial value. Thus the investigation would form no check on the maker's guarantees.

The sketches of this chapter must be regarded mainly as diagrams. I have endeavoured to avoid obviously unpractical design, but in practical work, the drawings to be useful should be on a large scale (usually natural size). Thus, when illustrating principles, it has frequently been necessary to exaggerate defects to a degree that should never occur in modern practice.

The mechanical details of turbine design are not considered. In practice the strength and stiffness of the shafts, and the pressures on the bearings and footsteps should be calculated. These are usually correct. The minor details of nearly all turbines are far less well designed than is usually the case with engines or pumps, but I doubt if a civil engineer can with advantage insist on alterations, and in practice the best turbine hydraulically is usually also the best designed mechanically.

TURBINES.—It is not proposed to enter into the question of the various types of turbine that have been employed. Modern turbines, almost without exception, fall into the two following classes :

The inward flow, central discharge turbine, generally known as Francis' turbine (see Sketches Nos. 249 and 250); and the class in which a free jet impinges on an open bucket, usually termed a Pelton wheel (see Sketch No. 262). If the history of the development of the machine is alone considered, these names are somewhat misleading. The type of turbine used by Francis was a very special form of the far larger class now termed Francis turbines, and was probably invented by Boyden, while the theory of the machine (as used by engineers) was first developed by Poncelet. Francis' investigations (*Lowell Hydraulic Experiments*) were, however, the foundation of all really practical rules for design, and his methods might, even after more than sixty years have elapsed, be generally imitated in reports on turbine tests with great advantage; thus, the use of his name in connection with turbines of this class is a well deserved compliment. The term Pelton wheel is less justifiable, as it is doubtful whether Pelton had anything other than a commercial connection with the development of the class. The term is none the less well understood by engineers, and saves repetition.

The theory of the design of a turbine can be simply expressed if we assume that the cross-sections of all the channels traversed by the water are very small. This assumption is not correct in practice, and the variations in the velocities, both of the water and of the turbine, that actually exist complicate the calculations. In order to apply the results of the theory to commercial turbines, we are therefore obliged to consider the turbine as split up into several "partial" turbines, of such a size that the cross-sections of the channels can be considered sufficiently small to permit the theory to be applied to each partial turbine. Thus, the practical process for the design of a turbine really consists in designing four or five, or more, partial turbines by theoretical rules, and then combining these into a practical machine. So also, the mathematical testing of the proportions of an existing turbine is effected by splitting it up into several partial turbines, and ascertaining how far these partial turbines depart from the theoretical rules. This book being intended for the use of civil engineers, the second process is the more important. I therefore proceed as follows. The theory of an ideal turbine of small cross-section is developed, and the methods of selecting the practical type which most closely conforms to local conditions are given. I then assume that the first draft design of such a turbine is selected in accordance with practical experience, and show how this rough design can be split up into partial turbines. These are then designed so as to accurately conform to the special requirements of the case, according to theoretical rule.

We can thus obtain the direction and shape of the edges of the guide and wheel vanes of the turbine at any point which may be selected. The intermediate portions of the vanes are not in any way defined, and the designer must form them so as to produce a smooth, continuous passage for the water, avoiding any sudden enlargements, or unduly lengthy passages, in order to reduce the losses of head that would thus be produced.

The final form of the vanes therefore largely depends upon the skill of the designer, and it may be necessary to depart to a certain extent from the theoretical results in order to obtain a well "formed" vane.

No rules for effecting this adjustment between the claims of theory and practical necessities can be given, and in practical work the final design depends very largely upon the skill of the moulders, and upon the methods used in constructing the turbine. I therefore prefer to consider at length the practical methods for ascertaining the degree to which an existing turbine conforms to the theoretical rules, and the amount of discrepancy between practice and theory which is usually permissible.

The hydraulic calculations connected with the design of turbines are comparatively simple, although the five- or six-fold repetition necessitated by the use of partial turbines is tedious. The fact that the motion of the water is in three dimensions, however, introduces many geometrical difficulties, since very few of the dimensions required in the calculations can be measured direct from drawings such as are usually employed by engineers.

The selection of the best method for dealing with the geometrical problems that thus arise has been very carefully investigated. The ordinary methods of plane trigonometry, or geometrical diagrams, are insufficient. After many trials, I have decided to employ spherical trigonometry exclusively. The selection has been made for the following reason. The method of geometrical projection from plan and elevation diagrams, is probably clearer, but actual trial shows it to be more tedious. Very few civil engineers employ the method in ordinary practice; and, judging by my own experience, they would require to re-learn a method which had probably been laid aside on leaving the Technical College. On the other hand, most civil engineers are accustomed to use spherical trigonometry at intervals when dealing with surveying problems, and although they may have to recall the methods, it is probable that this will entail less labour than would be required if the projection method were employed. A skilled draughtsman accustomed to the methods of solid projection will, however, find it advisable to employ them, and the additional clearness of conception thus gained is very great. I am of the opinion that it forms the only satisfactory method for practically laying out the forms of a turbine vane, and I not only employ it myself, but believe that it is used by all practical turbine designers. The practice in vogue some years ago in Technical Schools of making the students design and draw turbines as though they were flat machines is probably largely responsible for the depraved designs which were then common in England.

Description of a Francis Turbine.—A Francis turbine in its simplest form consists of two portions, the guide crowns, and the turbine wheel (see Sketches No. 252, etc.). The guide crowns are two fixed circular rings, usually flat and parallel to each other, and the space thus enclosed is cut up by metal sheets which are termed guide vanes. The crowns and vanes form a series of passages through which the water passes, and their function is to direct and guide the motion of the water so as to cause it to arrive at the wheel with a definite velocity in a definite direction.

The turbine wheel consists of two crowns, connected by correctly shaped sheets of metal (termed the wheel vanes), and the crown and vanes rotate round a fixed axis.

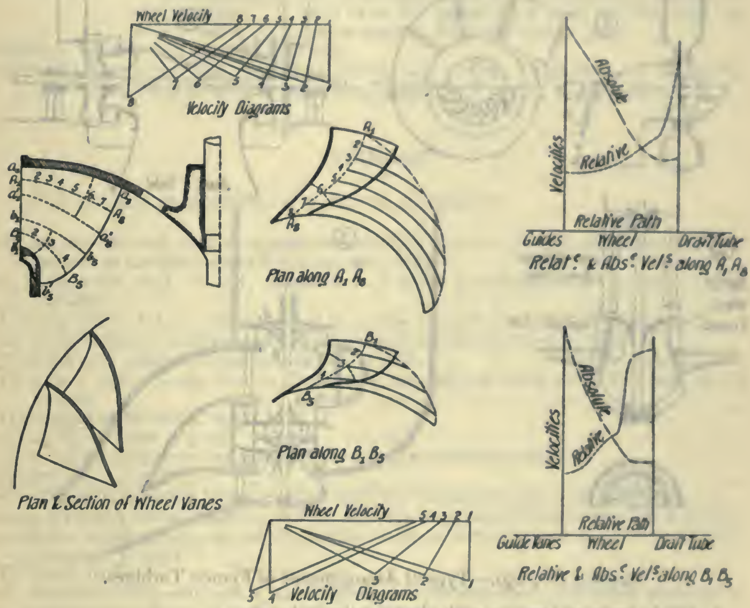
A typical turbine in which the axis is vertical is shown in Sketch No. 252, and it will be noticed that while the guide crowns and vanes are flat pieces of metal, the wheel crowns and vanes are curved in a somewhat complicated manner, so that if the vertical projection of the motion is considered the water enters the wheel in a horizontal direction, and leaves it in an approximately vertical direction. Similarly, if the horizontal projection is considered, the water leaves the guide passages with a motion of rotation round the axis, and enters the wheel with a velocity relative to the wheel, which is more or less radial, finally quitting the wheel with a velocity relative to the wheel which is nearly the reverse of the velocity with which it left the guide vanes, but which, when the absolute velocity in space is considered, is practically radial.

This somewhat complicated deflection of the water causes the water to perform work upon the wheel. It will be evident that while the horizontal projection of the motion is important in producing work, the whole of the vertical deflection is relatively unimportant, and is merely an unfortunate necessity due to the fact that the water must get away from the wheel somewhere.

The motion of the water through the guide crowns and wheel is plainly in three dimensions, and is extremely complex.

The lines a_1a_8, A_1A_8, $a'_1a'_8$, etc. (see Sketch No. 249) show the approximate projections of the motion of the water on the vertical section, and the dotted lines A_1A_8, B_1B_5, similarly indicate the approximate projections of the motion of the water relative to the wheel vanes (not the absolute motion in space, which is a very different matter, when the motion through the wheel is considered, and is shown by the chain dotted lines). These projected relative paths will hereafter be referred to as flow lines, or partial turbine boundaries (see p. 908), and, unless otherwise stated, the term "flow lines" is reserved for the projections on the vertical section. When the projections on the horizontal section are referred to, the "horizontal flow lines" will be used. From a hydraulic point of view, the subsidiary portions of a turbine are the approach passages, or mains, and the egress passages. For reasons which will later appear, the egress passages usually take the form of a diverging conical tube, which is termed the draft tube.

The arrangements adopted in practice are very various. A selection is shown in sketch No. 250. This book, however, is mainly concerned with the design rather than with the arrangement of turbines. The works of Gelpke (*Turbinen und Turbinenanlagen*), Wagenbach (*Neuere Turbinenanlagen*), and Pfarr (*Turbinen*) may be consulted with advantage. Thurso (*Modern Turbine Practice*) gives a less complete presentment in English, but the three German



SKETCH NO. 249.—Motion of Water through a Turbine Wheel.

books are so excellently illustrated that a knowledge of German is hardly essential if facts and suggestions regarding the arrangement of power houses and turbines only are wanted, and it is hoped that the problems of design are sufficiently dealt with in this book.

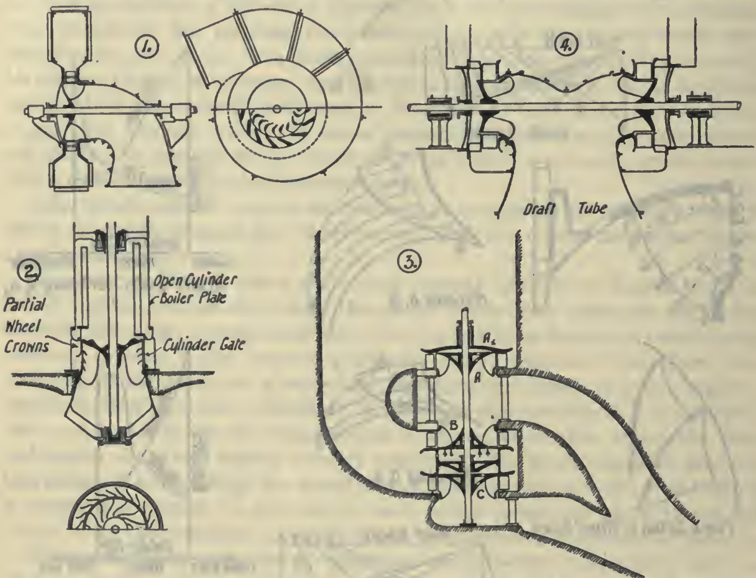
For present purposes we may state as follows (Sketch No. 250):—

(i) The axis of the turbine may be horizontal (Figs. Nos. 1 and 4), or vertical (Figs. Nos. 2 and 3), and the water may reach the guide vanes by an open shaft (see Fig. No. 2), or by a closed shaft (see Figs. Nos. 3 and 4), or through a spiral housing (see Fig. No. 1).

(ii) As many as eight separate wheels (*i.e.* hydraulically considered, eight separate turbines) have been fixed on one shaft, so as to produce a machine of eight times the power that one wheel would generate (see Fig. 1, Sketch

No. 261). Double and triple turbines (*i.e.* two and three wheels on one shaft, Figs. Nos. 3 and 4) are common, indeed are probably quite as common as the simple turbine consisting of one wheel per shaft.

(iii) While in theoretical discussions the wheel is assumed to receive water all round its circumference, cases exist in which the guide crowns and vanes extend only over a portion of the circumference of the wheel, so that the water enters along say one-half, or one-quarter, or an even smaller fraction, of the wheel. Thus, each cell or wheel passage runs empty for a portion of the time during which it rotates round the axis.



SKETCH No. 250.—Typical Arrangements of Francis Turbines.

1. Single horizontal turbine with plate metal spiral casing.
2. Single vertical American turbine with cylinder gate regulation and partial wheel crowns.
3. Triple vertical turbine. The wheels A and C are provided with leakage holes and covers. The wheel B is solid and uncovered, thus the water pressure on this wheel partially balances the weight of the turbine.
4. Double horizontal turbine with bearings in the dry.

These sketches are founded on actual installations by the firms of Rieter, Sampson, Bell, and Escher Wyss, but do not represent the details accurately.

NOTATION

The various cross-sections of the path of the water through the turbine are defined by a suffix notation (see p. 879).

Suffix 0, refers to entry into the guide vanes.
Suffix 1, refers to exit from the guide vanes.
Suffix 2, refers to the beginning of entry into the wheel.
Suffix e, refers to the definite entry section of the wheel vanes, as selected on page 907.
Suffix 3, refers to the completion of entry into the wheel vanes.
Suffix 4, refers to exit from the wheel, or entrance into the draft tube.
In considering partial turbines these points occasionally need to be distinguished. In such cases the prefix m, is used for exit from the wheel, and k, for entrance into the draft tube (see p. 912).
Suffix 5, refers to exit from the draft tube.
Suffix 6, refers to the loss by residual velocity (see p. 883). That is to say, losses after the cross-section 5, has been passed.

SYMBOLS

The symbols in alphabetical order are as follows:

A, denotes the nett area in square feet, available for the passage of water, measured normal to the direction of V (for A_c, see p. 883).
a, is the width of A, in feet, measured in a plane perpendicular to the axis of the turbine.
b, is the width of A, in feet, measured in a plane through the axis of the turbine.
B, is the common value of b_o, b_1, b_2, when these are all equal (see p. 895).
C, is the type constant of the turbine (see p. 888).
D, with a suffix, is the double distance in feet, of any point from the axis of the turbine.
D, without suffix (see p. 895).
d_o, d_1, d_2, etc. (see p. 910).
d_4, is the diameter between crowns of the wheel at exit (see p. 895).
d_t, and D_m (see p. 886).
e, is the suffix referring to the point of entry into the turbine wheel (see p. 907).
e_l, is the width of the clearance, in feet, between the wheel and its casing (see p. 905).
e_x, is the length of the edge of the turbine vane, in feet, intercepted between two flow lines (see p. 917).
E_x (see p. 918).
f, is the ratio of the nett area A, to the gross area obtained from geometrical calculations, when the thickness of vanes, etc. are neglected.
g, is the acceleration of gravity = 32·2 feet per second per second, in units now used.
H, is the total head, in feet, under which the turbine works.
h, is the depth of any point below head water level in feet.
K, is the symbol referring to the efficiency of the draft tube (see pp. 797 and 902).
k, as a prefix, is used to distinguish the point at which entrance into the draft tube occurs from a near point on the area of exit from the wheel (see p. 912).
k (see p. 912).

$k_2 = \frac{u_2}{\sqrt{2gH}}$ (see p. 897).

L, is the length of the boss by which the wheel is keyed on to the shaft.
l, is the length, in feet, of the exit edge of the wheel vane intercepted between two consecutive flow lines.
m, is a prefix used in distinction to k (see p. 912).

$m = \frac{n}{\sqrt{H}}$ (see p. 888).

m, is also used on page 912, for b_m.

n, is the number of revolutions which the turbine wheel makes per minute. For n_r, n_s, n_e, see page 899.
N, is the horse-power generated by the turbine.
o, is the suffix referring to entry into the guide vanes.

$P=\frac{N}{H^{1\cdot5}}$ (see p. 888).

p, with a suffix, is the pressure at any point, in feet of water.
p, is the number of partial turbines.
p_w, is the pressure producing leakage between the wheel and its casing (see p. 883).
p_g (see p. 920).
Q, is the number of cusecs of water passing through the whole turbine. For Q_r, Q_s, Q_e, Q_m, see page 898.
Q_t, is the number of cusecs of water doing work in the wheel.
q_l, is the leakage between the wheel and its casing.

$$Q=Q_t+q$$

$q=\frac{Q}{\sqrt{H}}$ (see p. 889).

r, is the distance of any point from the axis of the turbine.
r_c (see Sketch No. 253).
s_1, is the thickness, in feet, of a guide vane.
s_2, is the thickness, in feet, of a wheel vane.
t, is the distance, in feet, between corresponding points on two consecutive vanes.

$$t=\frac{2\pi r}{z}$$

u, is the velocity, in feet per second, of any point moving with the turbine wheel.
v, is the velocity, in feet per second, of the water relative to a point moving with a velocity u. For ${}_mV_4$, see page 915.
V is the symbol used to include v, and w, in formulæ which apply to either velocity (see p. 888).
w, is the absolute velocity, in feet per second, of the water in space. For ${}_mW_4$, see page 915.
x, is a suffix referring to any point in the wheel.
y, is the prefix referring to a definite partial turbine.
y_o, y_c (see Sketch No. 253).
z_1, is the number of guide vanes.
z_2, is the number of wheel vanes.

In order to save continual repetition, whenever an angle or length is referred to which can only be obtained by considering the three dimensions in space, the word "space" is placed in brackets before the word angle, or length. Thus, the statement β, is the (space) angle between u, and v, is merely a hint that β, cannot be measured directly from the general drawing of the turbine, but must be taken from a special diagram, or must be calculated by spherical trigonometry. Whereas, if the statement were β_3, is the angle between u_3, and v_3, the angle can be measured from the general drawing, or can be calculated by the ordinary rules of plane trigonometry.

In all the definitions of direction given below, it is assumed that the shaft of the turbine is vertical, as shown in the sketches.

β, is the angle between the positive directions of v, and u.
γ, is the angle between the direction of u, and the projection of u on the plane of the wheel vane (see p. 917).
δ, is the angle between the positive directions of w, and u.
ϵ, is the hydraulic efficiency of the turbine. For ϵ_r, see page 900. For ϵ_{max}, see page 898.
ζ_x, is the angle between the direction of u, and the intersection of a vertical plane through the axis and the vane plane (see p. 919).
η, is the mechanical efficiency of the turbine.
θ_x, is the angle between the direction of u, and the intersection of a horizontal plane and the vane plane.

θ, without suffix is used for the impact angle (see p. 866).
κ, is the angle between the projection of u, on the vane plane, and the intersection of the vane plane and a horizontal plane (see p. 919).
λ, is the angle between the line of flow and the intersection of the vane plane and a horizontal plane (see p. 919).
μ_l, is the coefficient of discharge of the space between the wheel and its casing.
μ_x (see p. 918).
ν, is the ratio of the losses in the wheel, as calculated for skin friction only, to the same losses as obtained experimentally (see p. 901)
$\rho = 1 - \epsilon - \frac{w^2_4}{2g\text{H}}$ (see p. 885).
σ, is a coefficient expressing the ratio of the head lost by shock to the total available head (see pp. 908 and 914).
τ, is a coefficient expressing the ratio of the losses of head by skin friction, etc., to the total available head (see p. 879).
ϕ, is the angle between the line of flow and the projection of v, on the vane plane (see p. 917).
Φ (see p. 918).
ψ, is the angle between a vertical plane through the exit edge of the wheel vane, and a vertical plane through the radius (see p. 918).
χ (see p. 900).
ω (see Sketch No. 253).

THEORY OF THE IDEAL TURBINE.—The following theory refers to a turbine in which the velocity of the water is supposed to be completely specified at each point. Thus, the dimensions of the cross-sections of the turbine, and other channels traversed by the water, must, in theory, be indefinitely small in comparison with the diameter of the rotating portion of the turbine. In practice, the relative size of the actual turbine and the ideal channel for which the equations are correct, is best illustrated by a study of the equations, and of the "partial" turbines used in designing work. For the present it is sufficient to state that no very great error is introduced so long as the maximum dimension of any moving channel does not exceed one-tenth of its mean distance from the axis round which it rotates.

Let us consider a turbine and all its connected channels (see Sketch No. 251), which works under a total head of H feet, *i.e.* let H be the difference between the head and tail water levels, so that H includes all losses by friction in the pressure main and generation of the velocity in the tail water channel.

All velocities are measured in feet per second.

All lengths and areas are measured in feet and square feet.

All pressures are measured in feet head of water.

The expression "relative velocity" means the velocity of the water relative to that point of the wheel with which the particle of water considered happens to coincide at the moment referred to.

We divide the path of the water passing through an ideal turbine into six sections (Sketch No. 251):

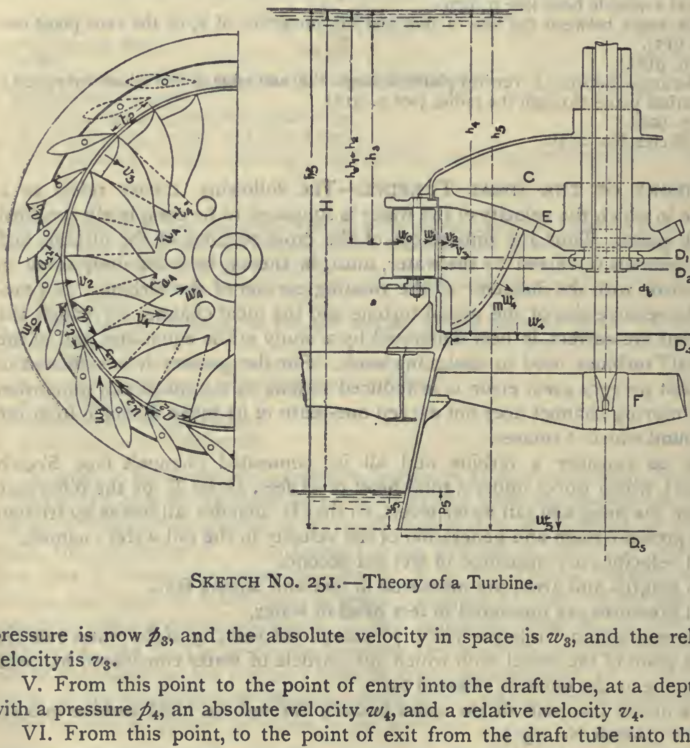
I. From the point where the water enters the case, or masonry housing, of the turbine, to the point where it completely enters the guide vanes with a velocity w_0, and a pressure p_0; the depth of this point below the head water level being h_0.

The expression "complete entry" is ambiguous, but for the present we may consider that it defines the first cross-section of the water stream that is completely surrounded by the guide vanes and their housing, so that the area of the stream can be determined by actual measurement. The matter is more completely discussed on page 907, where complete entry into the turbine wheel is defined.

II. From the point defined as above, to the point where the exit from the guide vanes begins, with a velocity w_1, and a pressure p_1, the depth below head water level being h_1. The beginning of exit is defined in the same way as the completion of entry.

III. From this point to the point where the water reaches the first portion of the wheel vanes, with an absolute velocity w_2, a relative velocity v_2, a pressure p_2, and at a depth h_2, below the head water level.

IV. From this point, until complete entry (as defined for guide vanes under I.) into the wheel vanes at a depth h_3, below head water level. The



SKETCH NO. 251.—Theory of a Turbine.

pressure is now p_3, and the absolute velocity in space is w_3, and the relative velocity is v_3.

V. From this point to the point of entry into the draft tube, at a depth h_4, with a pressure p_4, an absolute velocity w_4, and a relative velocity v_4.

VI. From this point, to the point of exit from the draft tube into the tail race, at a depth h_5, below the head water level, with a pressure p_5, and an absolute velocity w_5.

Strictly speaking, we should divide Section V. into two portions :

V. From complete entry into the wheel vanes, to complete exit from the wheel vanes.

V*a*. From complete exit from the wheel vanes, until the last portion of the wheel vanes has been left behind and entry into the draft tube at a depth h_4, etc. occurs.

The real reason for not regarding exit from the wheel vanes as an equally complex matter with entry into the wheel vanes, is that the losses that occur in Section V*a*. are by no means as important as those that may occur in Section IV. Also, unlike those that occur in Section IV., the losses in Section V*a*. will be found to be equally easily investigated when the whole of Section V. is considered as a unit.

If necessary, the equations can be easily written down, and the investigation for a partial turbine given on page 912 will suffice to clear up any difficulties that arise.

We also define :

u_3, and u_4, as the velocity of the wheel vanes at the points defined under Sections IV. and V.

Now, for all these portions, except IV. and V., Bernouilli's equation $p-h+\frac{w^2}{2g}=\text{constant}$, holds theoretically ($h$ being positive when measured downwards), and this equation corrected for pipe friction, curves, and irregularities in water motion due to sudden enlargements, and shock, is applicable to the actual motion.

For Sections IV. and V., however, the matter is less simple. The question is investigated on page 862, and the equations there proved are assumed to hold. Reference is also made to that section and page 907 for a discussion of the questions concerning complete entry and complete exit.

It will be noticed that the suffix notation employed in the first of the above sections differs from that now used, suffix i being used for entry and suffix e for exit. The object is to firmly impress the principle that the points of complete entry into, and exit from the wheel, are not fixed, but must be calculated afresh whenever the quantity of water passing through the wheel, or the angular velocity, are altered. Were this book intended for teaching purposes, a simpler and absolutely concordant notation could have been employed.

In applying the equations to the general theory, it is convenient to express the losses by friction in each portion of the motion, not in terms of the velocities w_0, w_1, . . . etc., but as fractions of the total head H.

We thus get as follows :

In the first portion we have :

$$p_0+\frac{w_0^2}{2g}=h_0-r_0\text{H}$$

where $r_0\text{H}$, represents the loss in friction, etc., in the channels up to the point represented by the suffix o.

In the second portion :

$$p_1+\frac{w_1^2}{2g}=p_0+\frac{w_0^2}{2g}+h_1-h_0-r_1\text{H}$$

In the third portion :

$$p_2+\frac{w_2^2}{2g}=p_1+\frac{w_1^2}{2g}+h_2-h_1-r_2\text{H}$$

In the fourth portion :

$$p_3+\frac{v_3^2-u_3^2}{2g}=p_2+\frac{v_2^2-u_2^2}{2g}+h_3-h_2-r_3\text{H}$$

In the fifth portion :

$$p_4+\frac{v_4^2-u_4^2}{2g}=p_3+\frac{v_3^2-u_3^2}{2g}+h_4-h_3-\tau_4 H$$

In the sixth portion :

$$p_5+\frac{w_5^2}{2g}=p_4+\frac{w_4^2}{2g}+h_5-h_4-\tau_5 H$$

Adding the first three equations, we get :

$$p_2+\frac{w_2^2}{2g}=h_2-(\tau_0+\tau_1+\tau_2)H \quad \ldots \text{(i)}$$

This permits us to calculate the pressure at exit from the guide vanes, when w_2 (which can be obtained by measuring the exit area when the quantity of water passing through the turbine is given) is known.

The fourth and fifth equations :

$$p_4+\frac{v_4^2}{2g}=p_2+\frac{v_2^2}{2g}+h_4-h_2-\frac{u_2^2-u_4^2}{2g}-(\tau_3+\tau_4)H \quad \ldots \text{(ii)}$$

This equation permits us to determine p_4, when v_2, v_4, u_2, and u_4 (which are determinable by measurement when the angular speed of the turbine and the quantity of water passing through it are given) are known.

Consider the sixth equation :

p_5, is plainly the pressure equivalent to the depth below the tail water level, so that $h_5-p_5=H$, and $\frac{w_5^2}{2g}$, can be expressed as a fraction of H, say :

$$\frac{w_5^2}{2g}=\tau_6 H$$

Thus, $$p_4+\frac{w_4^2}{2g}+H-h_4-(\tau_5+\tau_6)H=0$$

Adding this equation to the last two, and putting,

$$1-(\tau_0+\tau_1+\tau_2+\tau_3+\tau_4+\tau_5+\tau_6)=\epsilon, \text{ we get :}$$

$$\epsilon H=\frac{u_2^2-u_4^2}{2g}+\frac{w_2^2-w_4^2}{2g}+\frac{v_4^2-v_2^2}{2g} \quad \ldots \text{(iii)}$$

As already stated, the last equation should be considered as fundamental, and if the results do not agree with observational data, the points 2, and 4, may be considered to be wrongly selected. In practice, if uncertainties as to the value of τ_3 and τ_4 do not explain the difference, the turbine is not well designed. Such a case has actually been observed, and the error appears to have been attributable to the fact that the turbine had too few wheel vanes, so that the values of v_2, and v_4, were probably very inaccurate.

The equation can now be transformed by substituting for v_2, and v_4, from the equations (Sketch No. 252) :

$$v_2^2=u_2^2+w_2^2-2u_2w_2\cos\delta_2$$
$$v_4^2=u_4^2+w_4^2-2u_4w_4\cos\delta_4$$

We thus get :

$$g\epsilon H=u_2w_2\cos\delta_2-u_4w_4\cos\delta_4 \quad \ldots \text{(iv)}$$

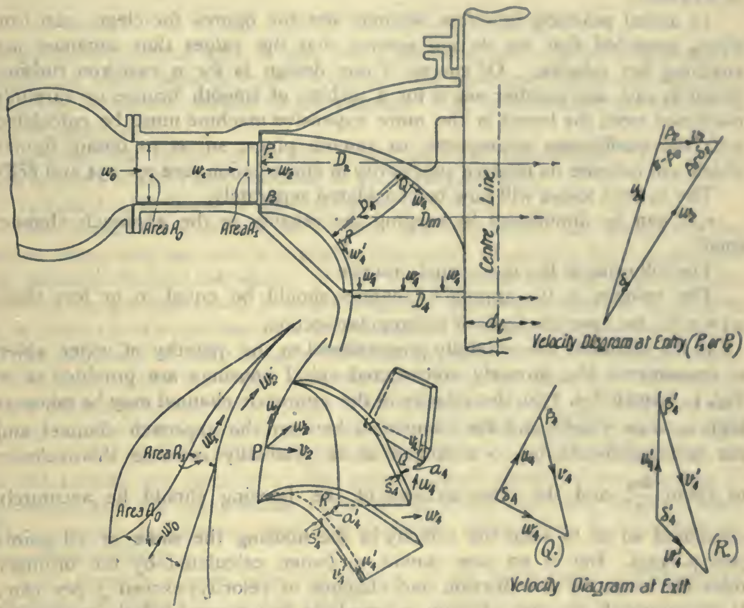
which is the general equation of turbine motion, and has already been proved.

The fraction ϵ thus arrived at is termed the Hydraulic efficiency of the turbine. Its relationship to the somewhat smaller fraction η, the mechanical efficiency of the turbine, is discussed on page 884.

It should also be noticed that the transformation now arrived at is only justifiable when no losses by shock occur at entry into or exit from the wheel.

In actual practice, the equation is usually assumed to hold good when shock occurs, the only difference being that the value of ϵ is somewhat diminished in order to allow for shock losses. Under these circumstances, the foundation of the equation is experimental only.

Practical Design of Turbines.—While the above equations strictly apply only to ideal or partial turbines such as have already been defined, it is plain that an equation which will approximately define the motion of



Sketch No. 252.—Preliminary Design of a Turbine. The Guide Vanes and Wheel have been slightly separated in Plan so as to secure clearness.

water in any turbine can be obtained by inserting mean values of the velocities u, v, and w. This method enables a preliminary study to be made of the various sources of loss which occur in a turbine.

The values of the various losses τ_0H, . . . τ_6H, are dependent upon the design of the turbine.

Theoretically speaking, each coefficient can be calculated by the usual rules for loss of head by:

(*a*) Skin friction and divergence (see p. 799).
(*b*) Change in velocity.
(*c*) Shock by sudden changes of velocity.
(*d*) Curve loss.

In a good design losses similar to those in class (*c*) should not occur, and, except in the wheel, losses of class (*d*) can usually be neglected.

Our experimental knowledge of the exact values of the coefficients is very vague. Consider skin friction; the available observations mostly refer to circular pipes of cast iron, and the velocities do not greatly exceed 7 to 10 feet per second. In a turbine the passages are square, or rectangular in section, and are usually of very smooth metal, while the velocities may considerably exceed 20 feet per second. Thus the calculated values are likely to differ from the truth. The calculations, however, are useful, as they afford the only means we possess of comparing different designs of turbines, but the calculated values of τ_0, etc. τ_8 should be considered as giving comparative figures only.

In actual practice, therefore, we may use the figures for clean, cast-iron pipes, provided that we do not assume that the values thus obtained are anything but relative. Of course, if one design is for a cast-iron turbine, rough as cast, and another one is for a turbine of smooth bronze, or carefully machined steel, the losses in the more expensive machine must be calculated with the coefficients appropriate to smooth pipes, so as to obtain figures which will indicate its relative superiority in construction (see pp. 434 and 888).

The various losses will now be considered separately.

$\tau_0 H$, can be diminished by keeping the velocity in the approach channel small.

The following is the most usual practice:

The velocity in the approach channel should be equal to, or less than, $0{\cdot}1\sqrt{2gH}$, for open channels of rectangular section.

If the channels are carefully proportioned to the quantity of water which is transmitted (*i.e.* properly constructed spiral housings are provided as in Fig. 1, Sketch No. 250), the velocity in the approach channel may be taken as high as $0{\cdot}20\sqrt{2gH}$, and the connection between the approach channel and the housing should be so arranged as to gradually increase this velocity to about $\frac{w_0}{2}$, and the cross-sections of the housing should be accurately calculated so as to keep the velocity in the housing the same at all points (see p. 939). But in no case should τ_0 (when calculated by the ordinary rules for losses by skin friction and changes in velocity) exceed 3 per cent. If the approach channel, or main, is very long (say several miles), economical considerations may require it to be of such a size that the loss of head by friction is a large fraction of the total head. In these cases, the nett head available at the lower end of the approach channel should be considered as equal to H, and the above rule should be followed in determining the losses in the turbine casing. Since the velocity in the approach channel will probably be large, spiral housings should be provided.

Similarly, $\tau_1 H$, can be decreased by sharpening the outer ends, and smoothing the surfaces of the guide vanes, and shaping them so that the change from w_0, to w_1, occurs gradually, and as far as possible uniformly. Their number should be kept as small as is consistent with properly guiding the water at entry into the wheel, so as to avoid losses by shock (see p. 907).

τ_1, as calculated by the ordinary rules, should not exceed 2·5 per cent.

$\tau_2 H$, should be treated similarly to $\tau_1 H$.

Certain losses occur, due to leakage through the clearances between the wheel and its casing.

In preliminary designs it is usual to assume that this leakage loss q_l, is about 3 to 5 per cent. of the total volume of water passing through the turbine. Sketch No. 259 shows rubbing strips, and Sketch No. 260 labyrinth packings (see p. 795), arranged so as to diminish the leakage; as a rule, careful fitting of the wheel and the casing is considered sufficient, but in actual designs the leakage pressure (see p. 905) should be calculated. An approximate value of q_l, can be obtained as follows:

If p_w, be the pressure producing leakage, which is approximately equal to $p_2 - p_4$ (see p. 880), the leakage is represented by:

$$q_l = \mu_l A_c \sqrt{2gp_w} \qquad \text{cusecs,}$$

where A_c, is the total area of the two clearances, and μ_l is the coefficient of discharge, which depends upon the width of the clearance space, and upon the length of the narrow portion of the passage. For approximate calculations μ_l can be taken as 0·5.

τ_2 should not exceed 2 per cent. when the entry is shockless. The losses due to shock at entry are discussed in detail on page 906.

$\tau_3 H$, and $\tau_4 H$, should be treated similarly to $\tau_1 H$.

The actual value of $\tau_3 H$, is largely dependent upon the design of the wheel vanes, and the loss is probably principally caused by the curvature of the passages through the wheel. If this be neglected, and the usual rules for skin friction are alone used in calculating $\tau_3 H$, it will generally be found that τ_3 is about 1·5 to 2 per cent. in a well designed turbine. The observed values of the efficiency of well designed turbines indicate that τ_3 is probably somewhere near double the value thus obtained. We may therefore believe that when the wheel vanes, and the form of the wheel crowns, are carefully designed, $\tau_3 + \tau_4$, should not exceed 4 per cent. provided that the entry into and the exit from the turbine occur without shock. When, owing to bad design, or to the turbine not running at the proper speed, shock occurs, the loss is increased, and the approximate value of the shock loss must be calculated. The question is obscure, and the available information is discussed on pages 907 and 913.

$\tau_5 H$, depends entirely upon the design of the draft tube, and upon the efficiency with which it converts velocity into pressure.

This question is probably one of the most obscure that exist in turbine design, and Andres' experiments (see p. 799) merely serve to show how much remains to be discovered. The only lesson that can be drawn from them is that the condition of "radial" exit, as usually laid down in theoretical investigations (although by no means always adopted in practice), is probably less important than was believed to be the case.

The component of velocity along the axis of the draft tube at entry usually varies between $0·1\sqrt{2gH}$, and $0·3\sqrt{2gH}$, and τ_5, varies from 2 to 6 per cent. according to the value of this velocity, and the length and form of the draft tube.

$\tau_6 H$. This loss is evidently dependent upon the velocity w_5. As a general rule, with a vertical draft tube $\tau_6 H = \frac{w_5^2}{2g}$. If the draft tube is so arranged that w_5, is equal to the velocity of the water in the tail channel, and is in the same direction, the loss may be only a small fraction of $\frac{w_5^2}{2g}$. This diminution, however, is obtained by curving the draft tube, and is therefore attended by an increase in τ_5. The general result is that $\tau_5+\tau_6$, may be taken as about 5 per cent. on the average. As will be seen later, the assumed value of $\tau_5+\tau_6$, or of w_4, forms a starting-point for the preliminary design of the turbine.

Finally, it must be noted that all these figures depend somewhat upon the size of the turbine, as is evident once the fact that they principally represent losses by pipe friction is realised.

Summing up the values of τ_0, etc. τ_6, enumerated above, we get $1-\epsilon=0{\cdot}165$, or $\epsilon=0{\cdot}835$.

We term ϵ, the hydraulic efficiency of the turbine, and it must be remembered that ϵ, is a function of Q, and n; but that, unless specially stated, it is assumed that ϵ, denotes the value of the efficiency when Q, and n, are so adjusted that the efficiency has its greatest value.

The corresponding maximum mechanical efficiency, as obtained by brake tests of the turbine, will be denoted by $\eta=\frac{550\ \text{N}}{62{\cdot}5\ \text{QH}}$, where N, is the horse-power given out by the turbine shaft.

Plainly η, is slightly less than the corresponding value of ϵ, as η, includes the losses by friction of the turbine shaft in its bearings, and also the friction of the outer sides of the wheel crowns against the surrounding water, and the power expended to produce the automatic regulation, and the forced lubrication of the turbine in cases where these are used.

Thus, we may say that: $\eta=\epsilon-0{\cdot}015$ in a large turbine, and that $\eta=\epsilon-0{\cdot}03$ in a small turbine, without forced lubrication, etc. If the power expended in such devices is also deducted, Gelpke states that:

$$\eta=\epsilon-0{\cdot}02\text{, in a large turbine, and } \eta=\epsilon-0{\cdot}04\text{, in a small turbine.}$$

Thus, Gelpke (*ut supra*, p. 44), gives for well-designed turbines:

	ϵ	η	η With Forced Lubrication and Automatic Regulation.
30 HP. turbine . .	0·78	0·750	0·740
100 ,, ,, . .	0·81	0·785	0·777
1000 ,, ,, . .	0·84	0·820	0·813
10000 ,, ,, . .	0·87	0·855	0·850

These values may be employed in preliminary calculations, and an even closer approximation may be made in certain cases. The above values of ϵ, are obtained on the assumption that the exit loss $(\tau_5+\tau_6)$H, is 0·04 to 0·05H. Now, the exit loss can be easily calculated by measuring the size of the draft tube. Thus, if we find that $\frac{w_4^2}{2g}$, differs materially from 0·04, or 0·05H, we may at once modify ϵ accordingly.

Thus, suppose we take a turbine of 100 horse-power and find that

$\frac{w_4^2}{2g}=0{\cdot}07\text{H}$, the appropriate value of ϵ, is $0{\cdot}81+0{\cdot}05-0{\cdot}07$, say $0{\cdot}79$ or $0{\cdot}78$.

Similarly, if $\frac{w_4^2}{2g}=0{\cdot}02\text{H}$, it would be fair to assume that $\epsilon=0{\cdot}83$ to $0{\cdot}84$.

The first turbine is evidently small, cheap, and relatively inefficient; while the second is large, costly, and highly efficient,—indeed, the figure given is probably better than could be obtained in practice.

We may also say that if $\epsilon=1-\rho-\frac{w_4^2}{2g}$, then $\rho=0{\cdot}10$, in a large turbine where no cost is spared to secure high efficiency; and $\rho=0{\cdot}17$ in a small turbine, where the design is good, but cheapness, rather than efficiency, has been aimed at.

Finally, ϵ, may be made as high as 0·88, or even 0·90 (though this figure is somewhat doubtful), and may descend as low as $0{\cdot}73=1-0{\cdot}17-0{\cdot}10$ (exit loss), without the design being in the least discreditable to the makers.

As an example, the 10,000 horse-power turbines of the Canadian Niagara Company may be considered. For this size $\epsilon=0{\cdot}87$, but the value of C (see p. 889) is high (2600), so that $\epsilon=0{\cdot}84$ probably represents the maximum attainable at the date of the design. The turbines are erected in a deep pit, dug out of the rock, and space is limited; the draft tube is therefore cylindrical, and $w_4=w_5=21$ feet per second. Thus:

$$\frac{w_5^2}{2g}=6{\cdot}9 \text{ feet, and } \text{H}=134{\cdot}6 \text{ feet}$$

Therefore, $r_6=0{\cdot}052$. The draft tube is long, and the friction loss in it is approximately $0{\cdot}03\text{H}=r_5\text{H}$.

Now, in a turbine of this size, we might expect that:

$$r_5+r_6=0{\cdot}01, \text{ or } 0{\cdot}02.$$

Thus, the small size of the draft tube causes a decrease of 6 per cent. in the efficiency, and for this particular turbine we find that:

$$\epsilon=0{\cdot}84-0{\cdot}06=0{\cdot}78, \text{ say.}$$

Experimentally, van Cleve (*Trans. Am. Soc. of C.E.*, vol. 62, p. 199), found that $\eta=0{\cdot}727$, or probably $\epsilon=0{\cdot}757$.

The difference 0·023 is explained by the fact that the draft tube is curved, and the loss actually observed at and near this curve amounted to 0·029H.

The design is therefore very excellently adapted to the circumstances, and although the nett efficiency is low, it is probable that 0·84 could be obtained with these turbines if space could be found for a straight and well-proportioned draft tube of a size sufficient to reduce w_5, to 8 or 10 feet per second.

Preliminary Sketch Design of a Turbine.—Assume that he velocities w_1, w_2, w_4, v_2, v_4, u_2, and u_4, are the same for all portions of the cross-sections of the turbine passages denoted by the suffixes 1, 2, and 4. This amounts to selecting average values of D_2, D_4, etc., and calculating average values of the angles β_2, β_4, etc.

Then, $w_4 \sin \delta_4 \, \text{D}_m \pi b_4 \frac{a_4}{a_4+s_4}=\text{Q}$.

Now, for a first approximation, $\delta_4=90$ degrees, and $\frac{a_4}{a_4+s_4}=\frac{1}{1{\cdot}1}$.

Thus, $w_4 = \frac{1 \cdot 1\, Q}{\pi D_m b_4}$, where D_m, is a mean value of the diameter at exit, say approximately $b_4 D_m = D_4^2 - d_i^2$ (see Sketch No. 252), and, since the exit is assumed to be radial, the velocity triangle at exit gives :

$$\tan \beta_4 = -\frac{w_4}{u_4}, \text{ where } u_4 = \frac{\pi D_m n}{60},$$

and the negative sign indicates that β_4 is greater than 90 degrees.

Also, $g\epsilon H = w_2 u_2 \cos \delta_2 - w_4 u_4 \cos \delta_4$.

Since the exit from the wheel is radial, $\cos \delta_4 = 0$.

So that, $g\epsilon H = w_2 u_2 \cos \delta_2$.

Also, the velocity triangle at entry gives :

$$\frac{w_2}{\sin \beta_2} = \frac{v_2}{\sin \delta_2} = \frac{u_2}{\sin (\beta_2 - \delta_2)}$$

$$u_2 = \sqrt{g\epsilon H \left(1 - \frac{\tan \delta_2}{\tan \beta_2}\right)} = \frac{\pi D_2 n}{60}.$$

Also, $w_o = w_1 = w_2$ approximately.

We can thus determine the mean velocities and the size of the turbine, if we assume values for β_2 and δ_2, and the ratio $\frac{w_4}{\sqrt{2gH}}$.

The method is logical, since the ratio $\frac{w_4}{\sqrt{2gH}}$ is a fairly accurate measure of the cost and efficiency of the turbine, while the angles β_2 and δ_2 (or β_1) define the general lay out of the wheel and guide vanes.

In practice, however, it is easier to determine the type and appropriate size of the turbine as shown on page 894, and to use the values of D_2, D_4, and of β_2 and δ_2, which are there tabulated, in order to sketch out the rough design. The above equations can then be employed to discover the effect of any slight alterations which may be considered necessary. Having thus determined the modified values of the velocities, we can estimate the various losses by skin friction, and ascertain whether the small alterations are likely to cause the efficiency to differ materially from the desired value. Sketch No. 252 shows how the velocity diagrams form a check on the vane outlines. These are correct near Q, but require modification near R if radial exit is essential. The diagrams are drawn with allowance for the varying directions of the velocity u produced by the wheel vane not being radial. In practice, errors are minimised by taking the direction of u, the same in all diagrams.

When applied to turbines belonging to Type VIII. to V. (see p. 889), this average method produces results which are quite sufficiently accurate to form a basis for preliminary estimates.

For turbines belonging to Types IV. to I., however, the results are less accurate, and it is usually advisable (even in preliminary work) to consider the turbine as made up of two partial turbines, and to apply the above equations separately to each turbine, as is done in the Sketch. In practice, the mid path method illustrated in Sketch No. 249, is probably preferable to considering the extreme outlines of the vanes (P_1Q and P_2R) as is done in sketch No. 252.

Experimental Basis of the Fundamental Equation.—It is evident that we can, with a certain degree of accuracy, calculate each of the quantities $\tau_0 H \ldots \tau_6 H$, in terms of w's, or v's. These can be expressed in terms of Q, the quantity of water passing through the turbine; and u_2, and u_4, can be expressed in terms of the number of revolutions of the turbine. Hence, equation No. (iii) page 880 can be transformed into:

$$2gH = \mathbf{A}Q^2 + \mathbf{B}nQ + \mathbf{C}n^2$$

where **A**, **B**, and **C**, are expressions depending upon the areas of the turbine passages at the various points where the velocities are measured, the angles the vanes make with the radii, and the various experimental coefficients for pipe friction, curve resistance, and shock losses.

We can also investigate the matter experimentally by measuring a series of simultaneous values of Q, n, and H (where it is plain that the turbine must not be regulated in any way during the observations, as otherwise the areas A_0, and A_1, where $w_0 = \frac{Q}{A_0}$, $w_1 = \frac{Q}{A_1}$ will be altered). We can then determine, by the method of mean squares, an equation:

$$2gH = \mathbf{a}Q^2 + \mathbf{b}Qn + \mathbf{c}n^2$$

and can ascertain how closely this represents the actual observations.

The values of **a**, **A**; **b**, **B**; **c**, **C**; can also be compared, and thus the agreement between theory and observation can be tested.

When this work is performed it will usually be found that the experimental and theoretical values agree fairly well, although there is a certain amount of evidence to indicate that the experimental relation includes a term in n. That is to say;

$$2gH = \mathbf{a}_1 Q^2 + \mathbf{b}_1 Qn + \mathbf{c}_1 n^2 + \mathbf{d}_1 n$$

In view of the fact that the losses due to skin friction are probably more closely expressed by:

$h_f = \mathbf{k}V^2 + \mathbf{l}V$ or, $h_f = \mathbf{m}V^{1 \cdot 8}$, than by the formula $h_f = \mathbf{m}_1 V^2$;

this is not very surprising, and it may be very fairly inferred that when the experimental coefficients for losses by friction and curvature are better known, the agreement between theory and experiment will become even closer.

We may therefore consider ourselves justified in assuming that the general equation of turbine motion as given by the equation:

$$g\epsilon H = u_2 w_2 \cos \delta_2 - u_4 w_4 \cos \delta_4$$

can always be transformed by the substitution indicated on page 880. Thus, the equation:

$$g\epsilon H = \frac{u_2^2 - u_4^2}{2g} + \frac{w_2^2 - w_4^2}{2g} + \frac{v_4^2 - v_2^2}{2g}$$

and the other forms employed on page 880 can be regarded as sufficiently accurate for practical calculations even when shock occurs at entry to, or exit from the wheel, provided the value of ϵ is modified to allow for these losses. The differences between theoretical and experimental results which are sometimes observed may therefore be attributed rather to an improper selection of the points represented by 2, and 4, than to any defect in the mathematical reasoning.

Calculation of the Skin Friction Losses.—In calculating the losses by skin friction in guide and wheel vane passages of turbines, it is advisable to remember that the velocities are large. The only formula founded on experiments which include velocities of the magnitude occurring in turbine passages is that given by Lang (see Hütte, vol. 1, p. 271).

This formula when transformed to English measure gives :

$$h_f = \frac{lV^2}{r} \frac{a + \frac{0 \cdot 0059}{\sqrt{Vd}}}{8g}$$

where **r**, is the hydraulic mean radius, and **d**, is the diameter of the pipe, and **l** its length in feet. V, is the velocity in feet per second, and in moving passages $V = v$, while in fixed passages $V = w$.

For smooth pipes $\mathbf{a} = 0 \cdot 012$. For clean cast iron $\mathbf{a} = 0 \cdot 020$.

For smooth turbine passages, we can take $h_f = \frac{5}{8} \frac{lV^2}{10000r}$.

For cast-iron passages, we can take $h_f = \frac{lV^2}{10000r}$.

The figures are approximate, and probably lead to results which are somewhat in excess of the truth.

THE DETERMINATION OF THE NECESSARY PROPORTIONS AND SIZE OF A TURBINE.—The difficulties underlying the design of a satisfactory turbine are entirely due to practical necessities. The problem of designing a turbine so as to utilise the energy of falling water with a very high degree of efficiency is a simple one, provided that the proportions of the turbine are so selected that the quantity of water and the speed of rotation are related so as to produce a type of machine which favours high efficiency.

It is plain that such a machine will not, as a general rule, be a good practical solution of the problem of the generation of power from falling water.

The cost of the turbines in a modern power station is but a small fraction of the total investment, and it is only, so to say, accidentally that the conditions are such as to permit turbines of this favourable type to be installed.

To state the problem more precisely.

Let H, be the fall available in feet, and assume that the general conditions are such that a turbine of a horse-power represented by N, running at n, revolutions per minute, is required.

Put $m = \frac{n}{\sqrt{H}}$, and $P = \frac{N}{H^{1 \cdot 5}}$, and $C = m^2 P = \frac{n^2 N}{H^{2 \cdot 5}}$.

Then, a Francis, or inward flow turbine, can be designed so that ;

$C = 100$ to 4900, roughly speaking (the exact limits and the possibility of their increase or diminution will be discussed later on), and for the most favourable type referred to above, $C = 400$ to 900. While for Pelton wheels, C, should not exceed 60.

These figures refer to Francis turbines with one wheel, or to Pelton wheels with one nozzle only, and it is evident that if two wheels per shaft, or two nozzles per wheel, be used, the horse-power (and therefore also C) is doubled. So also, if the turbine receives water over only a part of the circumference, C, is proportionately modified.

Nevertheless, the possibilities of turbines are by no means limitless, and at the present date skill in design is mainly shown by a careful selection of the type which is best adapted to the actual conditions. The proportions of the various types are now (thanks to the very careful series of designs published by Gelpke in his work, *Turbinen und Turbinenanlagen*) well established.

The principles underlying the selection of the appropriate type of turbine are probably more important to civil engineers than the actual design of a turbine. At the present date the design and construction of a turbine are matters which concern specialists attached to manufacturing firms, and the work of hydraulic engineers is mainly confined to designing the general arrangement of the power station, so as to permit the turbines to obtain the best results.

Confining our attention to Francis turbines, Gelpke (who was Escher Wyss' designer until about 1905), has laid down eight standard types, and the information given permits us to determine the following table :

Type	mD	$\frac{q}{D^2}$	$\frac{P'}{D^2}$ if $\eta=0·80$.	$C=m^2P$	$\sqrt{C}$	Approximate Maximum Value of η.
VIII .	77	0·222	0·020	118	10·8	0·83
VII .	80	0·302	0·027	173	13·1	0·84
VI .	83	0·441	0·040	276	16·6	0·845
V .	89	0·685	0·062	491	22·2	0·86
IV .	96	1·03	0·094	866	29·4	0·87
III .	107	1·58	0·144	1649	40·6	0·87
II .	121	2·23	0·203	2972	54·5	0·83
I .	138	2·87	0·260	4951	70·4	0·77

The symbols are as follows :

$m=\frac{n}{\sqrt{H}}$, $P=\frac{N}{H^{1·5}}$, where H, is measured in feet.

D, is the overall diameter of the turbine wheel, measured in feet, *i.e.* $D=D_2$ approximately.

Q, is the number of cusecs passing the wheel when N horse-power is developed under a head of H feet.

$$q=\frac{Q}{\sqrt{H}}$$

Thus, $N=\frac{62·5\ QH\eta}{550}$, or $P=\frac{N}{H^{1·5}}=\frac{62·5\eta q}{550}=\frac{q}{11}$ if $\eta=0·80$.

The entries in Column 4 are purposely somewhat inaccurate. The quantities fixed by the design of the turbine are $m=\frac{n}{\sqrt{H}}$ and $q=\frac{Q}{\sqrt{H}}$.

The commercial requirements generally determine the values of n, and N, the speed and the horse-power of the turbine. Thus, we must assume a relation between N, and Q, or between P, and q, *i.e.* the value of η must be assumed.

Column 4 is calculated on the assumption that $\eta=0·80$.

Reference to Column 7 will, however, show that better values of η can be attained for all types except No. VIII. In the final calculations this must be allowed for; and in large turbines under favourable circumstances, the value of D, may be reduced by 5, or even by 10 per cent.

The practical application of the table is obvious.

Calculate P (and if double or triple turbines are used, P, is the value corresponding to one wheel only), *m*, and C.

The value of C, fixes the type of the turbine, and then from the figures in Columns II. and IV. the diameter of the turbine can be calculated. As a general rule, the two values of D thus obtained do not agree. Thus, a typical turbine running at the given speed will develop less, or more, power than is required. Hence, the practical requirements must determine whether a typical turbine of approximately the required speed and power can be utilised, or whether a special non-typical turbine must be designed. The matter will be considered in detail later on (see p. 892).

The values of $\sqrt{C}$, are tabulated in Column 6, as this quantity is frequently used by designers under the somewhat misleading description of "specific speed." It may be noted that $\sqrt{C}$, in metric measurement, is equal to $4{\cdot}45 \times \sqrt{C}$, expressed in English measure. A similar series of types is given by Kaplan (*Bau Rationeller Francisturbinen Laufrader*).

The classification of turbines by values of C, may be somewhat extended. Thus, according to Graf and Thomas (*Ztschr. D.I.V.*, June 29, 1907):

Values of C, from 0, to 32, indicate that Pelton wheels are advisable, and their best mechanical efficiency is about 0·83 to 0·85.

Between $C = 32$, and $C = 61$, turbines with free deviation are indicated, and $\eta = 0{\cdot}80$.

From 61 to 5800, typical Francis turbines are used.

The appropriate values of η, are those given in the above table, the extreme values being:

$$C = 61 \qquad \eta = 0{\cdot}82$$
$$C = 5800 \qquad \eta = 0{\cdot}67$$

These values do not indicate the extreme possible values of C, as of late years such designs as those of Wagenbach, Moody, and Larner have greatly extended the possible range. The information is tabulated by Moody (*Trans. Am. Soc. of C.E.*, vol. 66, p. 306, *et seq.*), and the following shows the present possibilities both in American and German work:

C	η	C	η	C	η
45	0·88	2000	0·915	6100	0·86
250	0·90	3150	0·905	7100	0·85
500	0·915	3400	0·90	8100	0·83
1100	0·92	4500	0·88	...	...

These values of η are maxima, and are not likely to be attained at present, except with very well designed and well constructed turbines under the most favourable circumstances.

Returning to the table (p. 889). In view of the cheapness of the turbine in comparison with the rest of the machinery installed, it will usually be found that N, and n, are fairly well indicated by a consideration of the usual speed of the dynamo, or of whatever other machinery it is proposed to drive. The table, however, at once permits us to determine whether a turbine can be designed to drive the dynamo directly, and also what horse-power this turbine can give.

As an example, consider the following conditions :

H = 50 feet, $n = 300$ revolutions per minute, and 500 horse-power per wheel is required.

$$\text{Thus, } m = \frac{300}{\sqrt{H}} = \frac{300}{7{\cdot}07} = 42{\cdot}5$$

$$P = \frac{500}{H^{1{\cdot}5}} = \frac{500}{354} = 1{\cdot}41$$

Thus, $C = 42{\cdot}5^2 \times 1{\cdot}41 = 2550$, or a type intermediate between II. and III. is indicated.

If Type II. be selected, it is found that a turbine 2·85 feet in diameter will run at 300 revolutions per minute, and will develop 580 horse-power. A turbine 2·63 feet in diameter will develop 500 horse-power, and will run at 325 revolutions per minute. The first solution is probably that which is best adapted to practical requirements, and, unless these are very exacting, a stock turbine of this type and 2 feet 9 inches in diameter, running at slightly less than its theoretical speed, can be utilised.

Next, consider the same requirements of power and speed, but with H = 40 feet.

We get, $m = 47{\cdot}5$ $P = 1{\cdot}98$ $C = 4480$, and it will be found that a turbine of Type I. 2·90 feet in diameter, will run at 300 revolutions per minute, and will develop nearly 550 horse-power.

Such a turbine, under a 50-foot head, would normally run at 335 revolutions per minute, and would develop :

$$0{\cdot}260 \times 2{\cdot}90^2 \times 354 = 770 \text{ horse-power.}$$

Thus, if it is desired to develop 500 horse-power continuously, at a speed of 300 revolutions, under heads varying from 50 down to 40 feet, the turbine described above will satisfy the conditions, and it is plain that the fact that the speed is supposed to be kept absolutely constant may entail a certain decrease in the efficiency when the head is 50 feet.

In power plants working under a variable head water usually is least abundant when the head is high. Modern turbines are generally so designed that the efficiency is greatest when the guide vanes are adjusted so as to pass about three-quarters of the quantity of water which the machine could pass under the same head when the vanes were fully open. Thus, the turbine considered above should be designed in this manner, and would be run as follows, so that even when designed as indicated above, the maximum efficiency would be slightly less than could be attained by a non-typical turbine. Under a 50-foot head, the guide vanes would be opened so as to pass two-thirds of the maximum quantity of water (under a 50-foot head), and the shockless entry speed would be about 336 revolutions per minute. Under a 40-foot head the quantity passed would be about 0·9 of the maximum possible quantity (under a 40-foot head). These maxima can be calculated from the tabulated figures.

If the speed is not rigidly fixed at 300 revolutions per minute, a somewhat smaller turbine (where $D=2\cdot75$ feet approximately) would develop 500 horse-power at a 40-foot head, taking its maximum quantity of water. Under a 50-foot head, the same horse-power would be developed with about 0·7 of the maximum quantity that the turbine could pass, and the shockless entry speed would be 350 revolutions per minute. The speed, however, would be about 318 revolutions per minute under a 40-foot head, and if the turbine were run at a slower speed less than 500 horse-power would be generated.

The losses in efficiency caused by the speed of shockless entry differing from the speed at which the turbine is run must not be considered as at all serious. The efficiency curve of a well-designed modern turbine is very flat near the maximum value, and in the 2·90-foot turbine the differences would probably be well within the errors of observation. In the 2·75-foot turbine, a diminution of 2 per cent. in ϵ will probably amply allow for any difference. The diminution in power at 40-foot head, however, is serious, and, in cases where the speed is of importance, would suffice to cause the rejection of the smaller turbine.

Summing up, and remembering that for this type ϵ is somewhat less than 0·80, it appears that a stock turbine 3 feet in diameter will certainly suffice, and that a guarantee from a good firm for a turbine 2 feet 9 inches in diameter might be accepted. Also, the starting point for a special design would be found by taking the vane angles appropriate to Type I. with a diameter of 2·90 feet, and somewhat decreasing the dimension B. Roughly speaking, B_1 should be about 0·9 $\left(\text{equal to } \frac{500}{550}\right)$ of the value given by the ordinary rules, *i.e.*

$B=0\cdot9\times0\cdot35\times2\cdot90=0\cdot315\times2\cdot90=0\cdot91$ foot. (See Sketch No. 253 and p. 895.)

The lower boundary will therefore be slightly below the line N_{11} which is that adopted for Type II. (in which $B=0\cdot30D$); and D_4, in place of being $1\cdot23\times2\cdot90=3\cdot89$ feet, will be about $3\cdot89\sqrt{\frac{500}{550}}=3\cdot61$ feet.

Any further calculations are premature. The leading dimensions are now sufficiently closely determined to permit a preliminary value of the efficiency to be estimated, and the losses in the draft tube and approach mains must be approximately determined before an exact solution can be obtained. It is, however, obvious that the final solution cannot differ materially from a turbine 3 feet in diameter, and approximately 21 inches deep overall, with a draft tube about 3 feet 9 inches in diameter at the top. The guide crowns will be about 8 inches in radial breadth, and the total space occupied need not exceed 3 feet 9 inches$+2\times8+2\times12$ inches$=6$ feet 4 inches in diameter, although a slightly better efficiency can be obtained by increasing the last dimension to 14, or even 18 inches.

The circumstances assumed in this example illustrate the usual manner in which the problem of the selection of a type arises. They are in one sense peculiar. The appropriate type closely resembles Type I. It is practically impossible to design a turbine to revolve at a speed greater than that attained by this type. Thus, in the design of the special turbine, the angles and other quantities dependent on the speed of rotation were those appropriate to Type I. If, however, the value of C_1 were such as to indicate that the special turbine was intermediate between say Types III. and IV., it will be plain that the range of choice open is somewhat greater. The special design can then be selected from among the following alternatives.

(*a*) The turbine possesses the diameter, angles, and speed characteristics of a turbine belonging to Type III., with the required speed, but B, is adjusted so that the requisite horse-power is developed.

(*b*) The diameter and dimensions are those of a turbine belonging to Type IV. which develops the required horse-power, and the vane angles are adjusted to produce the required speed.

(*c*) Neither the dimensions nor the vane angles are typical, but each is adjusted so as to produce the required speed and horse-power. The latitude of choice thus obtained is so great that special designs are but rarely absolutely necessary, except under low and variable heads. The circumstances requiring special designs are fairly obvious. The average head should be considered, and if the required speed and horse-power can be attained with a machine belonging to any type except Type I. the head must be more variable than is usually the case before a special design is required.

Unfortunately, high head water powers are now not frequently developed, as the available sites are already occupied, and the problem is usually of the character already discussed. Thus, the design of Type I. and Type II. and intermediate machines is really the most important problem.

The above discussion should suffice to indicate the difficulties which arise, and the necessity for the services of designers who are specialists on the subject. The hydraulic engineer can, however, greatly assist or hamper their endeavours. The following principles may be laid down :

(*a*) Large and well-formed draft tubes should be provided. This is not usually difficult. A draft tube is a space which is not filled with concrete, or masonry, and is consequently not costly.

(*b*) The head being small, the pressure main should, if possible, be an open shaft. Spiral housings and similar devices are costly, and should be avoided.

Thus, the hydraulic engineer will be well advised to procure from the turbine makers the following machinery only :

(i) The turbine wheel, shaft and bearings.
(ii) The guide crowns and vanes.
(iii) The regulating apparatus.

If the turbine makers also supply steel casings, pressure mains, or curved metal draft tubes, it may usually be inferred (high head developments and abnormal conditions being excluded) that the general design of the hydraulic works is not that which is best adapted to secure a highly efficient utilisation of the water power.

Tabulation of Dimensions and Velocities in the Various Types.—The type of the turbine being selected, and the overall diameter D having been determined, the following tables permit the leading dimensions of Gelpke's types to be stated. So far as my experience goes these dimensions are very closely adhered to by all first class designers, and a tenderer who proposes relative dimensions (taking D as unit) which materially differ should be prepared to justify his figures either by local conditions or reference to independent experiments. A value of D, much less than the calculated value is (in Types I. to III.) suspicious, a greater value of D, is less material but requires investigation, though, since it is usually accompanied by a higher price, the case is not very important.

Quantities which depend mainly on the Speed of the Turbine.—The following table gives the values of the various velocities and the angles β_1 and β_2 (so far as these are determined by the construction of the wheel and guide vanes) for each type as given by Gelpke (*Turbinen und Turbinenanlagen*).

These values should not be considered as in any way rigidly binding. The proportions of a turbine are determined by experience. The conditions of no shock at entry, combined with "radial exit," theoretically permit all the angles and velocities to be rigidly determined when any two have been selected (*e.g.* u_2, and β_1). In practice, we have to consider not only the conditions under full load, but also when the machine is over or underloaded.

The figures tabulated below compare very well with modern German practice, and may be considered as applicable to cases where high efficiency is desired, when the turbine utilises a quantity of water which is not greatly removed from the maximum quantity which the turbine can pass under the given head, and where the turbine is frequently required to develop about three-quarters of its maximum horse-power.

In small pioneer installations, where ample water is always available, and where the turbines constantly take the maximum quantity of water that they can pass (*i.e.* develop their full horse-power always), the values of β_1 tabulated in the column headed "Values for full load" may be selected when the lowest possible cost is desired. As a general rule, under such circumstances manufacturers are inclined to advise the installation of turbines of which the catalogued "full load" is about three-quarters of that which would be calculated by the figures given in Table on page 889. A high efficiency is consequently secured with turbines of comparatively rough construction. Where the cost of transport is not high, this practice proves economical. Under modern conditions, however, ample water power (compared with the commercial requirements of the district) is only likely to be available in isolated localities where the cost of transport is high. Thus, when the cost of transport, as well as the manufacturers' price, is considered, a smaller and accurately constructed (and therefore possibly more costly) turbine, which is designed to attain its best efficiency at full load, appears to provide a cheaper solution of the problem.

Type Number.	w_1 or w_2 / $\sqrt{2gH}$	u_2 / $\sqrt{2gH}$	$w_2 \sin \delta_2$ / $\sqrt{2gH}$ or $v_2 \sin \beta_2$ / $\sqrt{2gH}$	δ_2 or β_1, if no Shock occurs at Full Load.	δ_2 or β_1 for Full Load only.	β_2	Mean value of u_4 / $\sqrt{2gH}$.	$w_4 \sin \delta_4$ / $\sqrt{2gH}$.
I	0·55	0·90	0·325	39° 30′	36° 30′	138°	0·79	0·325
II	0·58	0·79	0·295	32° 20′	29° 30′	130°	0·62	0·295
III	0·62	0·70	0·25	24° 50′	23° 40′	115°	0·50	0·25
IV	0·67	0·63	0·205	18° 30′	18° 0′	90°	0·42	0·205
V	0·71	0·58	0·17	14° 10′	13° 50′	60°	0·37	0·17
VI	0·75	0·545	0·14	11° 0′	10° 50′	40°	0·34	0·14
VII	0·78	0·52	0·12	9° 0′	8° 50′	30°	0·32	0·12
VIII	0·80	0·505	0·11	8° 10′	8° 0′	25°	0·31	0·11

Dimensions which depend mainly on the Horse-power of the Turbine.—The quantity of water passed by a turbine is approximately expressed by the equation:

$$\pi BD_2 w_2 \sin \delta_2 = Q, \text{ cusecs.}$$

In a typical turbine all the factors in this expression except B, are dependent upon the speed of rotation. Thus, the quantity of water used by, and therefore the horse-power, of the turbine, can only be materially changed by altering B. The typical turbines are designed on the assumption that :

$w_2 \sin \delta_2$ is approximately equal to $w_4 \sin \delta_4$, *i.e.* constant radial velocity (see Sketch No. 249).

Thus, if the alteration of the entrance area πBD_2, be accompanied by a proportional alteration of the exit area NN (see Sketch No. 253), we arrive at a modified turbine developing a slightly different horse-power, but running at the same speed as the typical turbine. An example has already been given.

Subject to the above remarks, the following table gives the geometrical proportions of the turbine wheel for Gelpke's eight types. These figures deserve to be even less rigidly adhered to by the designer than those already given concerning the velocities and the vane angles.

If the proportions are widely departed from in either direction, the design becomes more difficult, and an inexperienced designer may find it impossible to attain a high efficiency. The ratios $\frac{B}{D}$, and $\frac{d_4}{D}$, may, however, be varied proportionately within fairly wide limits, without introducing any great difficulty.

The notation needs some consideration.

D, is the overall diameter of the turbine wheel in feet. For all practical purposes, so long as shock calculations (see page 907) are not considered, $D = D_2$.

d_4, is the diameter between the crowns of the turbine wheel at exit, and, as shown in Sketch No. 251, is very nearly equal to D_4, the top diameter of the draft tube.

The area NN, is the area measured normal to the lines of flow, just outside the wheel vanes. In the sketch the area appropriate to each type is indicated by suffixes.

As a first approximation we may take $\frac{Q}{\text{Area NN}} = w_4 \sin \delta_4$ all along the exit from the wheel.

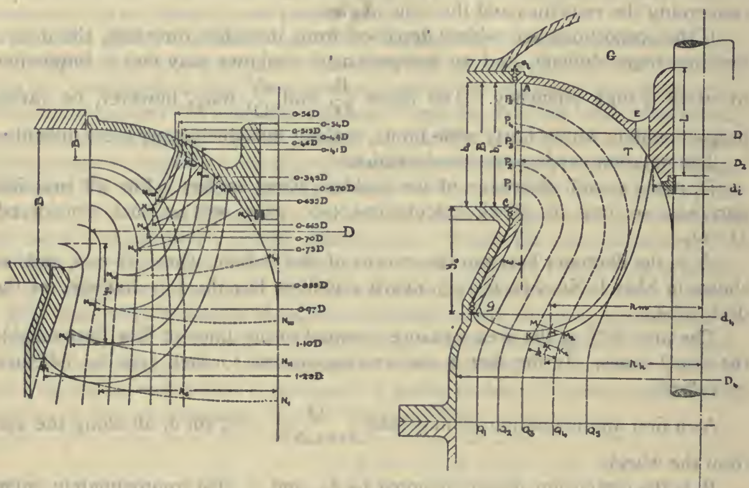
B, is the dimension usually denoted by b_2, and is also approximately equal to b_1, and b_3.

The other symbols are shown in Sketch No. 253.

Type	$\frac{B}{D}$	$\frac{d_4}{D}$	$\frac{d_i}{D}$	$\frac{y_o}{D}$	$\frac{NN}{D^2}$	$\frac{y_c}{D}$	$\frac{r_c}{D}$
I . .	0·35	1·23	0·27	0·24	1·225	0·31	0·475
II . .	0·30	1·10	0·35	0·22	1·005	0·26	0·44
III . .	0·25	0·97	0·41	0·20	0·755	0·22	0·42
IV . .	0·20	0·86	0·46	0·18	0·565	...	...
V . .	0·16	0·75	0·49	0·15	0·440	...	...
VI . .	0·125	0·70	0·51	0·125	0·345	...	...
VII . .	0·10	0·67	0·54	0·10	0·270	...	...
VIII . .	0·08	0·66	0·56	0·08	0·210	...	...

The cross-sections shown in Sketch No. 253 are known to produce efficient turbines, but they are by no means the only possible forms. In fact, they are those which are appropriate to cases where there is ample space for a draft tube. Where this space is limited, the diameter d_4 (especially in Types I. to III.) is often increased far above that given by the tabulated ratios $\frac{d_4}{D}$. In no case, however, should the angle ω exceed 45 degrees.

Many designers also make the curve of the upper crown far flatter than is indicated here, and, so far as can be judged from Larner's statements (*Trans. Am. Soc. of C.E.*, vol. 66, p. 383), there is some experimental evidence to justify this view of the matter. At present, however, the whole matter is determined by vague opinion. Certain firms of turbine builders are known to



SKETCH NO. 253.—Gelpke's Eight Type Outlines (*Turbinen*, p. 79), and Kaplan's Outline for Types near to I. and II.

have carried out systematic experiments, and the best form of wheel section for any particular case can usually be selected from published drawings, provided that the circumstances of exit (*i.e.* length, flare, and curvature of the draft tube) are carefully considered. The wide divergences which at first sight exist between the practice of German and American firms are in reality quite justified if it is remembered that the typical American turbine is a cheaply constructed machine, and is therefore larger than the better constructed German article of the same "catalogued horse-power." Owing to this cheapness of construction, American turbines are usually less efficient than German ones, although there is but little to choose between the best productions of either country. The term German in this connection may be extended to include Swiss, Swedish, and a few French firms.

Until lately, turbines of British design could be regarded as not worth consideration. At present two or three firms are prepared to supply scientifically designed machines, and, judging by their centrifugal pumps, can justify their claims. When high efficiency is desired, good workmanship is of great importance. I believe, therefore, that these British firms will soon produce evidence that they supply the best possible turbines, as very few machines at present in existence can be regarded as of first-class workmanship, according to the standards of modern machine shop practice.

The general outlines of the proposed turbine being thus selected, the final design requires a knowledge of the hydraulic efficiency of the machine.

A rule for roughly estimating this under all circumstances is given on page 900.

Usually we know u_2, and can express it in the form $k_2\sqrt{2gH}$.

Then, with all the required accuracy, we find that :

$$w_2=\frac{g\epsilon H}{k_2\sqrt{2gH}\cos\delta_2},$$

and the quantity of water passing the turbine is given by the equation:

$$Q=f_2\pi D_2 b_2 w_2 \sin\delta_2,$$

where f_2, is a factor allowing for the obstruction of the area $\pi D_2 b_2$, by the wheel vanes.

The efficiency under the assumed circumstances can now be investigated, and if the value thus obtained does not agree with the assumed value, the calculations can be repeated. The labour is not great, as (under the usual assumptions) all the losses, except the shock losses σ_2, and σ_4, are proportional to Q^2. Thus, for example, let ϵ be taken as 0·75, and, when calculated in detail, let the losses be found to be as follows:

$$\tau_0+\text{etc.}+\tau_6=0{\cdot}20$$

$$\sigma_2=0{\cdot}06 \quad \sigma_4=0{\cdot}03.$$

So that ϵ, is in reality about 0·71. Q, has plainly been over-estimated by some 3 to 5 per cent. Thus, the partially corrected value of

$\tau_0+\text{etc.}+\tau_6$ is about 0·18 or 0·19.

Unless the alteration in the values of w_2, and w_4, greatly alters σ_2 and σ_4, which can easily be estimated from the velocity diagrams, the true value of ϵ, is about 0·72, or 0·73, and the quantity of water used is about 0·97 of that assumed, and the nett horse-power is about $\frac{0{\cdot}97\times 0{\cdot}725}{0{\cdot}75}=0{\cdot}94$ of that assumed.

The results are quite as close as is practically required, especially when the fact that errors amounting to 10 per cent. of the quantity $\tau_0+\text{etc.}+\tau_6$, and 20 per cent. of the quantity $\sigma_2+\sigma_4$, are not at all improbable unless the calculations are founded on special experiments.

Number of Guide Vanes and Wheel Vanes.—The number of vanes is entirely determined by practical experience. The only binding condition is that the number of guide vanes must not be equal to the number of wheel vanes.

Gelpke (*ut supra*) states as follows:

For a turbine D, feet in diameter, z_1, and z_2, are as follows:

D	Number of Guide Vanes $= z_1$		
	β_1 equal to or less than 20 Degrees	β_1 between 20 and 33 Degrees	β_1 over 33 Degrees
Less than 2 feet	10	12	16
2 feet to 3 feet	12	16	20
3 feet to 4 feet 6 inches	16	20	24
4 feet 6 inches to 7 feet	20	24	28
7 feet to 9 feet	24	28	32
Over 9 feet 6 inches	...	32	36

D	Number of Wheel Vanes $= z_2$				
	β_2 equal to or less than 40 Degrees	β_2 about 60 Degrees	β_2 about 90 Degrees	β_2 about 115 Degrees	β_2 about 135 Degrees
Less than 2 feet	15 to 17	15	13	11	9
2 feet to 3 feet	19 „ 21	19	15	13	9
3 feet to 4 feet 6 inches	23 „ 25	21	17	15	11
4 feet 6 inches to 7 feet	27 „ 29	25	19	15	11
7 feet to 9 feet	31 „ 33	29	23	17	13
About 10 feet	...	...	25	19	13

These values are not very closely adhered to in practice, and my personal opinion is that they may be diminished without detriment. I have not, however, had much experience of regulation problems, and believe that these form the real basis for the determination of the required values.

Preliminary Estimation of the Efficiency of a Turbine.—In practice, turbines are generally so regulated that the angular speed of the machine round its axis remains constant under all circumstances.

Consider a turbine running at a constant speed of n, revolutions per minute, and suppose that the quantity of water passing is gradually increased from 0, to Q_m, cusecs, where Q_m, is the maximum quantity of water which the turbine can pass under the given head.

There are three values of Q, which deserve consideration:

(i) Q_r, the value when the exit from the wheel is "radial" and "shockless," as defined on pages 911 and 913.

(ii) Q_s, the value when the entry into the wheel is shockless.

(iii) Q_e, the value when the greatest value of the hydraulic efficiency, say ϵ_{max}, is observed.

The values Q_r, and Q_s, can be calculated by the methods given on pages 913, and 907. Q_e, however, must be obtained by observation.

In a well designed turbine regulated by Fink's (or other rotating guide vane) method, it will be found that the entry is shockless over a fairly wide range of values of Q, say from Q_{s1}, to Q_{s2}, and Q_s, may be defined as the value at which entry is shockless when the tips of the wheel vanes are selected as defining the point of entry, and the guide vanes are open to their fullest extent.

As a matter of observation, it can be stated that if $Q_r = Q_s$, as above defined, then Q_e, is very nearly equal to Q_r, or Q_s, and under these circumstances the greatest possible value of ϵ_{max} is obtained, but the values of ϵ fall off somewhat rapidly as Q, departs from Q_e. Thus, in a turbine where high efficiency at one load only, say Q_l, is desired, it is advisable to make $Q_r = Q_s = Q_l$. If, however, a fairly constant value of ϵ over a wide range of Q, say from Q_l, to $\frac{3}{4}Q_l$, be desired, it is found best to design so that $Q_s = Q_l$, and $Q_r = \frac{3}{4}Q_l$.

The maximum efficiency then never attains quite so high a value as in the previous case, but does not diminish so rapidly as the values of Q, depart from Q_e. The exact figures, of course, greatly depend upon the size of the turbine and upon its type, but it may be stated that if

$Q_r = Q_s = Q_l$, then we might expect to find that,

$$\epsilon = \epsilon_{max} = 0{\cdot}84 \text{ when } Q = Q_l = Q_e,$$
$$\text{and } \epsilon = 0{\cdot}75 \text{ when } Q = \tfrac{3}{4}Q_l.$$

Whereas if $Q_s = Q_l$ and $Q_r = \frac{3}{4}Q_l$,

$$\text{then,} \quad \epsilon = 0{\cdot}81 \text{ when } Q = Q_l.$$
$$\epsilon = \epsilon_{max} = 0{\cdot}83 \text{ when } Q = \tfrac{7}{8}Q_l,$$
$$\epsilon = 0{\cdot}80 \text{ when } Q = \tfrac{3}{4}Q_l.$$

The figures are merely illustrative, and refer to favourable cases ; but they serve to indicate the necessity for considering the matter in designing or specifying for turbines. The subject could be entered into more in detail, and in some very excellent turbines (and even more frequently in centrifugal pumps) practically constant values of ϵ are secured over a very wide range of values of Q, by making the entry "shockless" for $Q = \frac{4}{3}Q_e$, and the exit shockless at say $Q = Q_e$, and the exit radial at say $Q = \frac{2}{3}Q_e$.

The methods by which a detailed investigation of the efficiency is obtained are discussed later on.

It will also be plain that if the speed n, be supposed to vary, we could investigate the speeds n_r, n_s, n_e, at which radial exit, shockless entry, and the best efficiency, occur ; when Q, the quantity of water passing, is constant. The practical importance of such results rarely justifies the labour, although the question becomes of importance when steam-driven centrifugal pumps are considered.

German stock turbines are usually designed so that,

$$Q_s = Q_m \text{ and } Q_r = \tfrac{3}{4}Q_m. \qquad \text{Hence, } Q_e, \text{ is about } \tfrac{7}{8}Q_m.$$

American and French stock turbines are usually designed so that $Q_s = Q_r = \frac{3}{4}Q_m$, and the horse-power corresponding to $\frac{3}{4}Q_m$, is that given in catalogues. Hence, American turbines can take a markedly greater load than that at which they are catalogued ; whereas German turbines cannot.

Subject to the above remarks, let ϵ be the efficiency of the turbine when running at a speed n, and when passing Q, cusecs of water.

We have $\epsilon = 1 - (\tau_0 + \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \sigma_4)$.

Calculate $\tau_0 + \tau_1$ by the ordinary rules, and put $\epsilon + \tau_0 + \tau_1 + \sigma_4 = \chi$.

When the turbine runs at n', revolutions, and passes Q′, cusecs, the regulating apparatus (whether hand or automatic in design) moves the guide vanes so as to alter δ, the angle of exit from the guide vanes, and a_0, and a_1, the distance between two consecutive guide vanes at entry and exit. Let a'_0, and a'_1, be the new values. Hence, the new value of w_0, is $w'_0, = \frac{Q'}{z_1 a'_0 b_0}$, and $w'_1 = \frac{Q'}{z_1 a'_1 b_1}$.

Thus, τ'_0 and τ'_1 can be calculated, and very approximately:

$$\tau'_0 = \tau_0 \left(\frac{w'_0}{w_0}\right)^2, \text{ and } \tau'_1 = \tau_1 \left(\frac{w'_1}{w_1}\right)^2.$$

If the regulation is effected by a cylinder gate (Sketch No. 250, Fig. 1), then

$$w'_0 = \left(\frac{Q'}{Q}\right) w_0, \text{ and } w'_1 = \left(\frac{Q'}{Q}\right)\left(\frac{b_1}{b'_1}\right) w_1$$

where b'_1, is the new value of b_1, and in this case there is also a loss of head caused by the sudden decrease of velocity from w'_1 to w_2, which should be included in τ_2.

Then $v'_2 = \frac{Q'}{z_2 a_2 b_2}$ and $u'_2 = \frac{2\pi r_2 n'}{60}$ can be calculated, and the loss by shock at entry can be graphically obtained as indicated on page 907. Let this be σ'_2H. So also, $(\sigma'_4 - \sigma_4)$H, the difference in loss due to residual rotational velocity and shock at exit, can be calculated. It cannot be assumed that ϵ was obtained when the exit was purely radial, since, as a matter of experiment, the best efficiency does not usually occur when the exit is radial.

If shock at entry occurs under the original conditions, σ'_2H, is of course the difference between the two shock losses.

The reasoning is not rigorous, and is probably erroneous if n', differs greatly from n. In practice, however, n', is usually equal to n, and the formula

$$1 - \epsilon' = (1 - \chi)\left(\frac{Q'}{Q}\right)^2 + \tau'_0 + \tau'_1 - \tau_0 - \tau_1 + \sigma'_2 + \sigma'_4 - \sigma_4$$

can then be proved, under the assumption that all losses (other than shock losses) are proportional to the square of the velocities.

When the matter is tested by actual observation, the results given by the formula agree quite as accurately with observations as is necessary for practical purposes.

The chief source of error in the above investigation arises from the fact that the actual value of $(\tau_3 + \tau_4)$ is by no means accurately obtainable by calculations of skin friction, owing to the neglect of the effect of the curvature in the wheel passages. The following method of allowing for this error has been found useful in practice, and is therefore put forward.

Let us assume that η has been observed, and that ϵ has been calculated by allowing for bearing friction, friction of the wheel against dead water, and the power consumed in lubrication, etc., as already detailed.

Let us also calculate:

$$1 - \epsilon_1 = \tau_0 + \tau_1 + \tau_2 + \tau_3 + \tau_4 + \tau_5 + \tau_6 + \sigma_2 + \sigma_4$$

In theory $\epsilon_1 = \epsilon$. In practice, a slight difference will be found to exist. This may arise from an erroneous assumption of the value of the skin friction losses, but these can usually be ascertained for all portions of the machine, except the wheel, from pressure gauge observations.

We may therefore assume that if ${}_c\tau_3$ and ${}_c\tau_4$ represent the values of τ_3 and τ_4 that actually occur (*i.e.* the values corrected for curvature), then :

$$ {}_c\tau_3 + {}_c\tau_4 - \tau_3 - \tau_4 = \epsilon - \epsilon_1 $$

Thus, ${}_c\tau_3 + {}_c\tau_4 = \nu(\tau_3 + \tau_4)$, where ν is a coefficient which in practice is usually about 1·5 to 2.

It is now fairly obvious that the expression

$$ 1 - \epsilon' = \tau'_0 + \tau'_1 + \tau'_2 + \nu(\tau'_3 + \tau'_4) + \tau'_5 + \tau'_6 + \sigma'_3 + \sigma'_4 $$

where all the coefficients are obtained by calculation, will probably give a more accurate value of ϵ' than that obtained by the preceding method. The method is merely suggestive. In a case of actual observation, however, this method was applied as soon as the three-quarter load value of ϵ had been obtained, and seven predicted values of ϵ' over the range $0{\cdot}4Q_m$, to Q_m, agreed within 0·005 of the observed values, and the errors were well within the errors of observation.

In this case, six pressure gauges were read to obtain values for the skin friction. The pressure gauges had been fixed for some days before the brake tests were started, and the departures in τ_0, τ_1, τ_2 and τ_5, τ_6 from the law $h_f = \mathbf{m}_1 V^2$ had been accurately ascertained.

When the method was applied to cases where the skin friction had not been determined experimentally, differences of 0·015 occurred, but these are probably explicable by erroneous assumptions concerning the skin friction.

It will be seen that in practice the process mainly enables the number of brake tests to be decreased.

The first method forms a very excellent check on the makers' guarantees of efficiency, and the consultant's requirements. A reputable firm's guarantees of efficiency under ordinary loads (say three-quarter, and full load) are usually extremely accurate, but if they are asked to predict the efficiency at small loads, or at variable speeds, the values stated are frequently found either to be impossible, or are obtained by installing a far larger turbine than would otherwise be required. The last result is unsatisfactory, and the consultant deserves most blame when it occurs, since he could avoid this eventuality by demanding smaller efficiencies under these abnormal circumstances, after calculating the values that can possibly be attained with a turbine which is not too large for its work.

SYSTEMATIC ESTIMATION OF THE VARIOUS LOSSES.—The order in which the losses are estimated may at first sight appear peculiar. It is that which is found most convenient in practical work, as large errors, or departures from the specified conditions, can be detected early in the calculations.

Losses in Draft Tube, τ_5 *and* τ_6.—Let Q, be the total quantity of water which passes through the turbine, in cubic feet per second.

Let H, be the gross available head under which the turbine works.

The gross area of the upper end of the draft tube is given by $\frac{\pi}{4} D_4^2$, and the nett area is $A_4 = f_4 \frac{\pi}{4} D_4^2$, where f_4, is a coefficient expressing the total fraction of the gross area that is not obstructed by the shaft and its bearings.

In preliminary work we can take $f_4 = 0{\cdot}97$, for cases where the bearings of the shaft are outside the draft tube, and $f_4 = 0{\cdot}93$, in cases where the shaft bearing and supports are inside the draft tube.

Thus, $$w_4 = \frac{Q}{f_4 \frac{\pi}{4} D_4^2},$$

Similarly, $$w_5 = \frac{Q}{f_5 \frac{\pi}{4} D_5^2}.$$

where f_5, is a coefficient similar to f_4, but which refers to the exit area of the draft tube. We thus obtain :

$$p_5 - h_5 - p_4 + h_4 = \frac{w_4^2 - w_5^2}{2g} (1 - K)$$

where K, is an allowance for the head lost in friction, and divergence losses in the draft tube. We can usually assume that, $1 - K = 0{\cdot}80$, but the form of the draft tube and its curvature must be considered. If Andres' results (see p. 799) can be assumed to hold good for large tubes, such as are now considered, it would appear that divergence losses may be entirely neglected, as there is no doubt that the fact that the water has passed through the turbine renders the circumstances extremely favourable.

Now, p_5, is plainly equal to y_5, the depth of the centre of the exit area of the draft tube below the tail water level.

Thus, y_5, and $(h_5 - h_4)$, the vertical height of the draft tube, being measurable, we can determine p_4; and, as a rule, p_4, is less than the atmospheric pressure.

Also, the exit loss can be expressed in the form ;

$$\frac{w_5^2}{2g} = \tau_6 H, \text{ say (see p. 883)},$$

and the loss in the sixth section of the turbine is given by the equation :

$$\frac{K (w_4^2 - w_5^2)}{2g} = \tau_5 H, \text{ say.}$$

In preliminary designs it is frequently customary to assume that

$$(\tau_5 + \tau_6) H = \frac{w_4^2}{2g}.$$

The assumption has a practical advantage, for the value of D_4 fixes (for a given type of turbine) all the other dimensions of the turbine within very narrow limits. Thus, if we decide to make $\frac{w_4^2}{2g} = 0{\cdot}02H$, we obtain a relatively large, but efficient turbine. Whereas, if we go to the other extreme, and make $\frac{w_4^2}{2g} = 0{\cdot}12H$, we obtain a small and less efficient machine. Thus, the value of $\frac{w_4^2}{2g}$ ("the wheel exit loss") forms a short and concise method of specifying the hydraulic qualities of the turbine. For example, if a designer is told that the wheel exit loss should have a value of 0·08H, he is practically informed that cost and efficiency should both be considered. Whereas, a wheel exit loss equal to 0·04H, or to 0·05H, is a clear indication that a good efficiency rather than cheapness is desired.

Entry into the Turbine, τ_0.—We have now to consider the earlier sections of the water path.

The friction in the approach channel, or pressure main, can be calculated by the ordinary rules. If the guide crowns are situated at the bottom of an open shaft no other losses occur. If, however, the water enters the casing of the turbine at one side, the casing must be shaped into a spiral form in plan (Sketch No. 250, Fig. 1), otherwise losses due to change in velocity occur similar to those investigated for a centrifugal pump on page 939. The subject is not considered in detail, as such losses should not occur in a well designed turbine. Any consideration of this question will show that badly shaped, or roughly constructed metal casings, are undesirable. Modern practice tends towards properly shaped casings of reinforced concrete, forming an integral portion of the power house. If metal casings are used, these should be of first class workmanship, and Goldmark's specification for pipes (see p. 459) may be studied with advantage. It must, however, be realised that the conditions are somewhat more exacting than in steel pipes, as any deformation of the casing may produce shocks which will destroy all the gain in efficiency that it is desired to attain. The rivets should certainly be countersunk on the inside, and stiffeners should be placed wherever requisite. German practice appears to employ cast-iron casings exclusively.

Losses in the Guide Passages, τ_1 *and* τ_2.—By measurement from the drawings we can calculate:

$$w_0 = \frac{Q}{z_1 a_0 b_0}, \qquad w_1 = \frac{Q}{z_1 a_1 b_1} \text{ (Sketch No. 254).}$$

z_1, represents the number of guide vane passages which receive water, and in a turbine which receives water all round its circumference z_1, is equal to the number of guide vanes (see p. 898).

a, denotes the breadth of a passage measured between two consecutive vanes perpendicular to the velocity w; and b, denotes the distance between the guide crowns; so that ab, represents the nett area of a single passage measured perpendicular to the direction of w.

Both a_0, and a_1, vary when the guide vanes are moved in order to regulate the turbine. (See Sketches Nos. 259, 260.)

Sketch No. 254 shows the conditions occurring. The velocity at the cross-section denoted by A, is w_0, and at the cross-section denoted by C, is w_1. The guide vane outlines should be constructed so that the change from w_0, to w_1, occurs gradually, and the form of the passage should resemble that of a jet. In the left hand sketch, the links for moving the guide vanes occupy so much space that the section at B, is smaller than at A, or C. This should be avoided.

The losses are best estimated as follows:

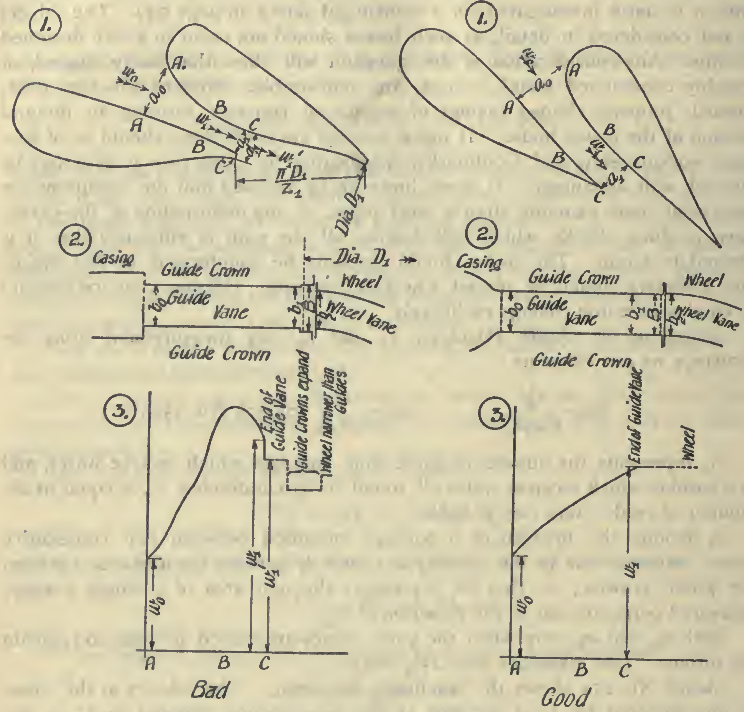
(*a*) Up to A, the circumstances resemble a bell mouth orifice, and the head lost is about 0·02 or 0·03 $\frac{w_0^2}{2g}$.

(*b*) From A, to C, the usual friction laws hold. The formulæ given on page 888 may be employed.

(*c*) On exit from the guide vanes at C, a certain loss of head may occur by sudden expansion. The right hand case is favourable, as the guide vanes end in a sharp edge.

In the left hand case the ends of the guide vanes are blunt. Then $w'_1 = \frac{Q}{\pi D_1 B_1 \sin \delta_1}$ should be calculated, where D_1, is the diameter measured to the ends of the guide vanes, and a loss $\frac{w_1{}^2 - w'_1{}^2}{2g}$ probably occurs.

In a well designed turbine, in good order, $B = b_1 = b_2$.



SKETCH NO. 254.—Losses in the Guide Vanes.

D_1, varies when the turbine is regulated, owing to the motion of the vanes. The shock losses at exit from the guide vanes, can be expressed by :

$$\frac{(w_1 - w'_1)^2}{2g} = \tau_2 H,$$

but this is probably greater than the true value.

w'_1, however, should be used in place of w_1, in estimating the circumstances at entry into the wheel.

The calculations concerning this loss are unreliable, and are referred to in order to illustrate the importance of regularly examining the condition of the guide vanes, and repairing them if blunted by wear or erosion (from sand or occluded gases). The sketch shows the possibility of other losses caused by bad fitting of the vanes and crowns, or the crowns and the wheel.

The losses typified by τ_1H, and τ_2H, are least in Type I., and increase rapidly as Type VIII. is approached.

No definite rules for the length of the guide vanes can be given. Theory indicates that a length equal to three or four times the width a_0, or a_1, should suffice. Four or five times a_0, or a_1, seems to be more usual in practice.

The case where b_0, b_1, and b_2, are not equal is not discussed. It occurs in turbines regulated by cylinder gates, and in cases where the wheel is not properly adjusted relatively to guide crowns. The loss in efficiency is obvious, and can be calculated if desired.

The values of τ_0, τ_1, and τ_2 can thus be estimated.

A little consideration will show that while τ_0 and τ_1 are easily calculated (when Q, is known) from the readings on pressure gauges screwed into the guide crowns near the entry and exit sections, τ_2 is not very easily observed. Certain tests of my own gave very peculiar results, and, in practice, it appears advisable to consider τ_2 as included in σ_2.

Circumstances in the Wheel.—The design of the rotating portion of the turbine must now be considered. When Q, cusecs of water pass out through the draft tube, the turbine does not utilise the whole of this quantity, as a certain leakage takes place through the spaces between the rotating wheel and its casing.

Let this leakage loss be equal to q_l cusecs. Then, if r_l, be the radius of the rubbing strip between the turbine wheel and its casing, and e_l, is the "play" allowed (Sketch No. 253), we have :

$$q_l = 4\pi r_l e_l \mu_l \sqrt{2g(p_2 - h_2 - p_4 + h_4)}$$

since there are two paths for leakage.

The value of μ_l can be taken as 0·5 to 0·6, and $p_2 - h_2 - p_4 + h_4$ can be calculated from equation No. (ii), page 880.

For preliminary calculations, we may assume that

$$2g(p_2 - h_2 - p_4 + h_4) = 2g(\epsilon - \tau_5 - \tau_6)\text{H} - w_1^2.$$

The value thus obtained is probably somewhat in excess of the truth ; for if the turbine fits its casing closely, the water in the space G, is probably set rotating, and thus a more or less efficient centrifugal pump is working against the pressure.

It may therefore be assumed that the leakage through the upper rubbing strips takes place under a pressure equivalent to,

$$p_2 - h_2 - p_4 - h_4 - \frac{u_2^2 - u_4^2}{8g}.$$

In the turbine which is now being considered (Sketch No. 253), the leakage through the lower space can hardly be supposed to be restricted in this manner, but two rubbing strips are shown, so that the formula for a labyrinth packing (see p. 795) might be applied.

The value of $p_2 - h_2$ can be calculated from equation No. (i) on page 880, but the independence of ϵ thus obtained is only apparent. $h_4 - h_2$ is plainly the difference of level between the top of the draft tube and the rubbing strips ; and in accurate calculations the value differs for the two paths of leakage.

Entry into the Wheel, σ_2.—In the typical Francis turbine this problem is one of two dimensional geometry ; in the case of the cone turbine or in an axial turbine, three dimensions must be considered.

Let z_1, be the number of guide vanes.

Let b_1, be the height between the guide crowns, and a_1, the width between two consecutive vanes at exit from the guide vane, measured perpendicular to the direction in which the velocity of the water at exit is directed.

In the modern types of turbine which are regulated by moving the guide vanes, a_1, is variable, and in the older and cheaper types which were regulated by means of a cylinder gate, b_1 is variable.

$$\text{Then,} \qquad w_1 = \frac{Q}{z_1 a_1 b_1}$$

and for preliminary designs we can put :

$$z_1 a_1 b_1 = b_1(\pi D_1 \sin \delta_1 - z_1 s_1)$$
$$= \frac{b_1 \pi D_1 \sin \delta_1}{1 \cdot 1} \quad \text{approximately};$$

where s_1, is the thickness of the guide vanes at exit, and δ_1 is the angle which their direction at exit makes with the tangent to a circle of diameter D_1 ; or more accurately, which the direction of the velocity w_1, makes with this tangent.

Now, let z_2, a_2, b_2, s_2 be similar quantities for the entry into the wheel vanes, and put $Q - q_l = Q_t$.

Then $v_2 = \dfrac{Q_t}{z_2 a_2 b_2}$.

Similarly, in preliminary calculations we may put :

$$z_2 a_2 b_2 = b_2(\pi D_2 \sin \beta_2 - z_2 s_2).$$

$$\text{Also,} \qquad u_2 = \frac{\pi D_2 n}{60},$$

where n, is the number of revolutions which the turbine makes per minute.

Now, set off $AB_2 = u_2$, on any convenient scale, and in a direction parallel to u_2, and $B_2C_2 = v_2$, in a direction parallel to v_2, and $AD = w_1$, in a direction parallel to w_1.

Thus, the angle $AB_2C_2 = \pi - \beta_2$, and the angle $B_2AD = \delta_1$.

Now, if the points D, and C_2, coincide, the entry is without shock.

The case is then fairly simple. We need not in practice draw any very delicate distinction between the points 2, and 3, and no questions regarding the loss of head at entry will be found to arise.

The numerical conditions are obvious :

$$w_1{}^2 = w_2{}^2 = u_2{}^2 + v_2{}^2 + 2u_2 v_2 \cos \beta_2,$$

$$\text{and } \delta_1 = \beta_1 = \delta_2, \text{ where } \frac{\sin \delta_2}{v_2} = \frac{\sin \beta_2}{w_2}.$$

The fact that β_2 is measured between the positive directions of u_2, and v_2, must be remembered.

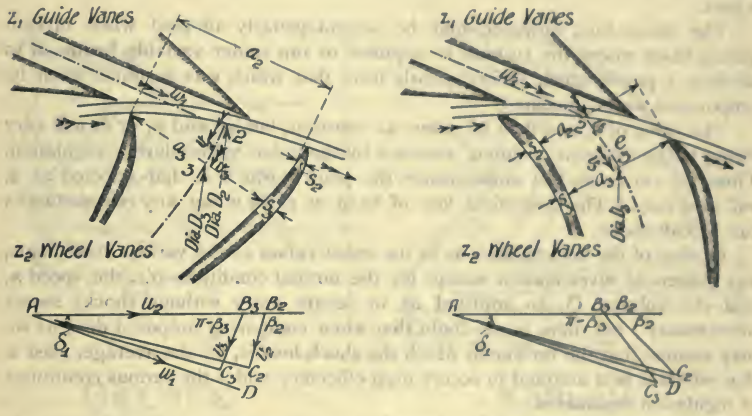
If, however, as shown in Sketch No. 255, the points C_2, and D, do not coincide, the theoretical " velocity of shock " is represented by the line DC_2, and some shock may occur at entry. The determination of the precise point of entry into the turbine now becomes uncertain. Visual study of many glass models, and mathematical investigations of recorded experiments, lead me to believe that the velocities are spontaneously adjusted as far as is possible, so that the shock may be minimised. The question can be investigated by considering the form of the wheel vanes between the point 2, and the point 3, where the water is completely surrounded by the wheel vanes and crowns, and ascertaining whether an intermediate point *e*, can be selected which will render the shock less than at the points 2, or 3.

The matter can be investigated more definitely by calculating

$$u_3 = \frac{2\pi r_3 n}{60} = \frac{\pi D_3 n}{60}, \quad v_3 = \frac{Q_t}{z_2 a_3 b_3},$$

and setting off $AB_3 = u_3$, and $B_3C_3 = v_3$, where the angle $AB_3C_3 = \pi - \beta_3$, and AB_3, represents u_3.

In Sketch No. 255, left-hand figure, the circumstances have been purposely selected so that w_1, is greater than w_2, or w_3. Thus, whatever point intermediate between 2, and 3, be selected as the point of entry, some shock occurs, and the



SKETCH NO. 255.—Shock at Wheel Entry.

minimum shock is represented by DC_2. These circumstances indicate that the design is bad as regards shock at entry.

Sketch No. 255, right-hand figure, however, shows a more favourable case, where D lies between C_2 and C_3, so that we can select a point *e*, say, between 2, and 3, where the velocities u_e, and v_e, would compound into a resultant w_e, which very nearly coincides with AD. The shock velocity DC_e, is now far smaller than :

DC_2, which corresponds to the point 2, as "point of entry," or DC_3, which corresponds to the point 3, as "point of entry."

The circumstances at the point *e*, and the velocity diagram thus determined, are believed to most nearly represent the actual occurrences at entry into the wheel.

We may now assume that there is a loss of head represented by $\frac{DC_e^2}{2g}$ at entry into the wheel.

The matter is best investigated geometrically, as has already been indicated, but if desired, the formula:

$$DC_e^2 = w_1^2 + w_e^2 - 2w_1 w_e \cos(\beta_1 - \delta_e) = w_s^2$$

may be used, and we find that:

$$\text{the head lost by shock} = \sigma_2 H = \frac{w_s^2}{2g}.$$

If our knowledge of the experimental coefficients determining the values of the various loses of head were sufficiently precise, it would now be advisable to regard the point denoted by suffix e, as being the point 2, in equation page 880, and we should then regard the motion from the guide vanes to the wheel vanes as from 1, to e (2 or 3), and then from e, to the point denoted by 4. In practice, the refinement is unnecessary; and, as will be noticed, the distinction drawn between the points 2, and 3, is referred to in this connection only. Hereafter, the suffix 2, is employed without distinction for all points at entry into the wheel.

The distinction, however, may be advantageously adopted when investigating cases where the turbine is required to run under variable heads, or to develop a power which differs greatly from that which was assumed when its proportions were calculated.

The value of n, can then be taken as constant, but a_1, and β_1 or δ_1 will vary owing to the different positions assumed by the guide vanes during regulation. Thus, w_1, varies, and in consequence the point e, which is that selected as 2, will also vary. The theoretical loss of head at entry under any circumstances can be calculated.

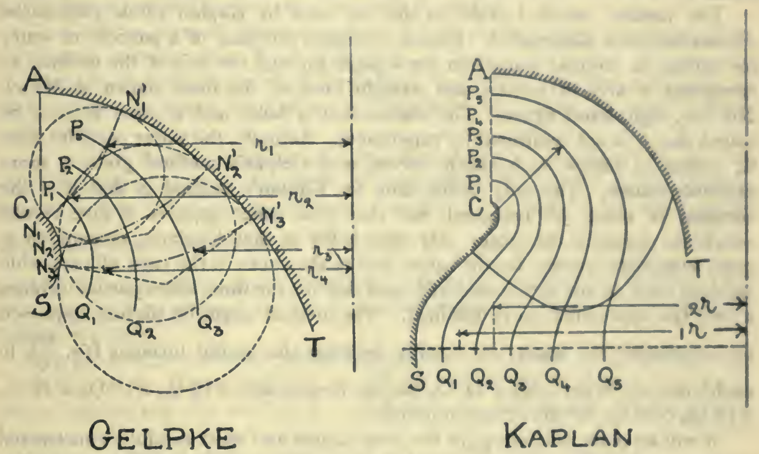
In view of the uncertainties as to the exact values of the various coefficients, any numerical investigation except for the normal conditions (*i.e.* the speed n, and the volume Q, so adjusted as to secure entry without shock) seems unnecessary; but there is no doubt that when comparing proposed designs we may assume that the turbine in which the shock loss is, on the average, least is that which is best adapted to secure high efficiency under the various conditions of regulation considered.

The precise commercial value of the properties investigated above is doubtful. As an expression of personal opinion only, I am inclined to consider that good workmanship and solid construction are far more important. In practice, however, good design and workmanship as a rule occur together, so that the selection of a turbine is not usually complicated by such considerations.

Passage through the Wheel, τ_3 *and* τ_4.—The division of the turbine wheel into a series of partial turbines has now to be considered. The assumptions upon which the process is founded are somewhat flimsy. Such experiments as have been made indicate that the velocities of the water in turbine wheels differ materially from those obtained by the methods at present adopted. The justification for the assumptions therefore lies in the fact that turbines designed in this manner are highly efficient. The principles may consequently be considered to be correct, and when sufficient experimental data are available the details can be modified in the manner indicated on page 910, so as to secure additional accuracy. A well-designed turbine is already so efficient that it is open to doubt whether any great practical advantage can thus be secured. The present method of turbine calculation is probably quite as close to physical truth as are any other engineering calculations. Additional experimental data rather than more refined mathematical methods form the real needs of the designers of turbines.

The practical difficulty is that each designer appears to have his own particular method of determining the flow lines which bound the partial turbines, and by actual trial I have found that it is consequently possible to somewhat under-estimate the qualities of a proposed design by using a method which differs from that employed by its designer. A reference to Sketch No. 256, which shows partial turbines as laid out by the methods employed by Gelpke and Kaplan, will illustrate this point. It is fairly evident that the forms of the wheel boundaries AT, and CS, have been materially influenced by the difference in method, and that if a turbine is scientifically designed, it is not very easy to draw by any sound method satisfactory partial turbines which materially differ from those obtained if the designer's own methods were used.

Gelpke's method is as follows: describe a circle to touch the crown sections



SKETCH NO. 256.—Gelpke's and Kaplan's Methods of Drawing Partial Turbines.

at N_1 and N'_1 and sketch in the line $N_1N'_1$ which is perpendicular to both crowns. Now take P_2, so that

$$r_1 N'_1 P_2 = r_2 N_1 P_2$$

Then P_2 is a point on the middle flow line P_2Q_2. Similarly we can select P_1 and P_3 and determine corresponding points on the lines $N_2N'_2$ and $N_3N'_3$. Thus 3 or 7 flow lines and 4 or 8 partial turbines can be sketched out. The process has no foundation in theoretical hydrodynamics, and plainly amounts to making the areas of the partial turbines equal at each level line NN′. Since Gelpke distinctly states that his method is a preliminary one, it appears equally logical and less laborious to adopt Kaplan's method, which consists in taking AC, the entry to the wheel, as a level line and dividing it into equal parts CP_1, P_1P_2, and P_5A. The top of the draft tube SQ_1Q_2, etc., is also taken as a level line and divided at Q_1, Q_2, so that

$${}_1r.Q_1Q_2 = {}_2r.Q_2Q_3 = \text{etc.}$$

thus producing equal areas at the top of the draft tube.

Nearly all designers assume that the velocities of the water are uniform both across the wheel entry and across the entry into the draft tube.

Thus, (Sketch No. 257) divide up the wheel height b_2, into say four equal parts AP_1, P_1P_2, etc., and split up the diameter D_4, into four sections BQ_1, Q_1Q_2, etc. such that the areas generated by the sections are equal, *i.e.*:

$$d_1^2 - d_o^2 = d_2^2 - d_1^2 = d_3^2 - d_2^2 = d_4^2 - d_3^2 = \frac{d_4^2 - d_o^2}{4}$$

where d_o, is the diameter of the shaft, and in this particular case since there are only 4 partial turbines, $d_4 = D_4$.

The initial and final points on the boundaries of the partial turbines pass through P_1, and Q_1, P_2, and Q_2, respectively.

The determination of the remainder of the boundaries is a matter of judgment.

The method which I prefer is the one used by Kaplan (*Bau rationaller Francisturbinen Laufrader*). Kaplan considers the path of a particle of water (or rather its circular projection on a plane through the axis of the turbine) as composed of arcs of circles, and straight lines of the form shown in Sketch No. 256, right-hand figure. The assumption is bold, and it must at once be stated that it is not confirmed by experiment. Actually, the water particles near C, refuse to follow such sharp curves, and Gelpke's method gives a more probable course. The only justification for Kaplan's method is that it is the simplest of those yet proposed, and that none other appears to give results which are closer to the truth. My own belief is that Kaplan's assumption is quite sufficiently correct for the upper half of the vanes in the case of very wide turbines such as are now considered, and that for the three lower partial turbines a very fair approximation is obtained. The method might be slightly improved by distributing the water, not equally between the partial turbines (*i.e.* $\frac{Q_t}{6}$, to each), but say in the ratio 0·14 Q_t, for the lowest, and 0·15 Q_t, 0·17 Q_t, 0·18 Q_t, 0·18 Q_t, 0·18 Q_t, for the others in order.

It will be plain that except in the very largest and most carefully constructed turbines such refinements would produce variations in the calculated angles, which are hardly capable of reproduction in the forms of the vanes as actually constructed. Even the most skilled designers are rarely able both to satisfy rigidly all the geometrical conditions obtained by calculation, and to produce a nicely formed and easily constructed vane.

The final design of a vane is a compromise between theoretical requirements and practical considerations, and many firms of turbine builders have also expended considerable sums of money in experimental research. Thus, except when working up the results of refined experiments, the assumption that the water is equally distributed between the partial turbines appears to be justifiable.

Each partial turbine thus sketched out is now considered as a separate machine passing a quantity of water equal to $\frac{Q_t}{p}$ cusecs, under a head H. Where p denotes the number of partial turbines.

We have already determined the pressures at entry into and exit from the wheel, so that the assumed efficiency will (theoretically speaking) be obtained.

Thus, we have merely to satisfy the geometrical conditions at exit.

The value of p, depends upon the type of turbine adopted.

As a rule, it will be found that:

For Types I. or II., $p=6$ or 8.

For Types III. to V., $p=4$ or 6.

For Types VI. to VIII. there appears to be but slight reason for taking more than two partial turbines.

Conditions at Exit, σ_4.—As a general rule, it is assumed that the absolute velocity of the water in space at exit from the wheel is "radial" in direction. This term is a relic of the old types of turbines where the wheel was so narrow in comparison with its diameter at exit, that the problem could be (or rather was in practice somewhat unjustifiably) assumed to be one of motion in two dimensions.

Sketch No. 257 shows clearly that this assumption is erroneous in the case of modern turbines. The water at exit from the wheel has an axial velocity along the flow line, and the condition which is really considered is the value of δ_4, the (space) angle between the directions u_4, and w_4.

The exit is termed "radial" when $\delta_4=90$ degrees, and the exit is at 10 degrees forward when $\delta_4=100$ degrees.

There is no very great reason to believe that radial exit has any virtue in itself, especially where the turbine is provided with a long, and well formed draft tube.

In modern designs we generally find that if entry into the wheel without shock (for the case where the point 2, is selected as the point of entry as on p. 907) occurs when Q, cusecs, pass through the turbine, radial exit usually takes place when approximately $\frac{3Q}{4}$ cusecs pass through the turbine. Consequently, when Q, cusecs pass through the turbine, δ_4 is about 100 degrees, and, as a rule, Q, is so selected that it is the maximum quantity of water that the turbine can pass.

The process for determining the exit angles of the wheel vanes is as follows.

Calculate the area available for the passage of water at exit from each partial turbine, say ${}_yA_4$, where y, is a prefix denoting the number of the partial turbine.

Thus, ${}_yv_4$, the relative velocity of exit at the point Y, is given by:

$$\frac{Q_t}{{}_yA_4 p} = {}_yv_4$$

We can also calculate, ${}_yu_4=\frac{2\pi {}_yr_4 n}{60}$, the velocity of rotation of the point of exit.

We thus determine the (space) velocity triangle $A_yB_yC_y$, where $A_yB_y={}_yu_4$, and $B_yC_y={}_yv_4$, and the condition which must be satisfied is that the angle $B_yA_yC_y={}_y\delta_4$ (where ${}_y\delta_4$, is written for δ_4 in order to show that δ_4 may vary from point to point along the exit edge of the guide vane) should be either 90 or 100 degrees, or whatever value has been selected.

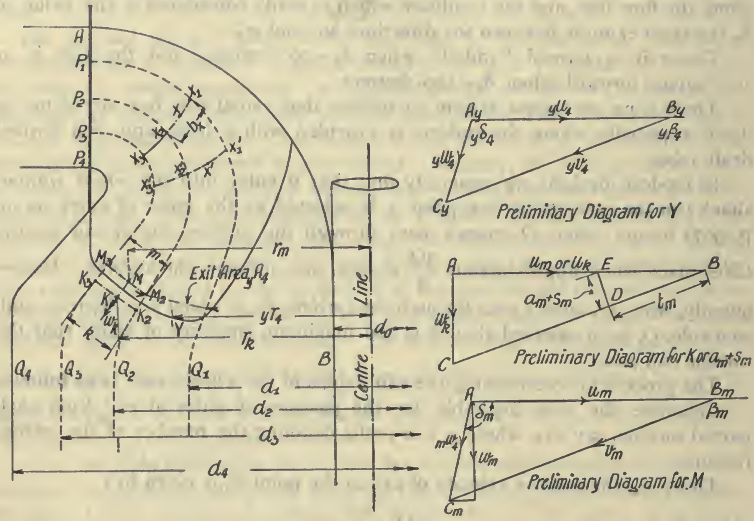
We thus determine the angle ${}_y\beta_4=\pi-A_yB_yC_y$, (the exit angle of the vane) for as many points on the exit edge of the guide vane as there are partial turbines. It must be carefully borne in mind that the angles thus determined are the angles between ${}_yv_4$, and ${}_yu_4$, and are therefore (space) angles measured in a certain plane which is fixed by the fact that the projection of ${}_yv_4$ on the vertical projection drawing is the flow line through the point Y, so that the direction in which the angle is measured varies at every point along the exit edge of the wheel vane.

In practice we must obtain the angle ${}_y\beta_4$ by successive approximation; since any alteration in ${}_y\beta_4$ also alters ${}_yA_4$, and therefore ${}_yv_4$.

Consider any one of the partial turbines, say P_2Q_2 and P_3Q_3, and let M, denote the middle point of the exit area (*i.e.* M is the projection of the mass centre of ${}_yA_4$). Let w_m, represent the component of ${}_mw_4$ or ${}_mv_4$, perpendicular to ${}_mu_4$ (i.e. $w_m = {}_mw_4 \sin {}_m\delta_4$). Put $M_2MM_3 = m$.

Let K, be a point close to M, just outside the wheel, such that MK, is a flow line, and let w_k, be the velocity of the water at K, in a direction parallel to MK.

Then w_k, is calculated by measuring the length $K_2KK_3 = k$, say, for $w_k = \frac{Q_t}{p2\pi r_k k}$.



SKETCH No. 257.—Determination of the Approximate Exit Angle.

We also know that w_m, should not differ materially from w_k, in order to avoid detrimental shock at exit.

Thus, construct the triangle ABC, where $AB = u_m$ or u_k (if these differ materially) and $AC = w_k$, and BAC is a right angle. Then the angle ABC is approximately equal to $\pi - \beta_m$, where β_m is the required value of β_4 at the point M.

Set off along BC, the length $BD = \frac{2\pi r_m}{z_2} = t_m$ say, on any convenient scale.

Then draw DE perpendicular to BC.

Then DE represents $a_m + s_m$, the width of the passage between two consecutive wheel vanes when the exit angle is ABC, and when the wheel vanes have no thickness. If s_m, be the thickness of a vane, the nett exit area is $z_2 b_m a_m$, where $b_m = m = M_2MM_3$, and $v_m = \frac{Q_t}{pz_2 b_m a_m}$, is approximately the value of v_4 at the point M.

The velocity diagram AB_mC_m, can now be drawn with $AB_m = u_m = \frac{2\pi r_m n}{60}$ and $B_mC_m = v_m$, as above obtained.

A more correct value of the exit angle ${}_m\beta_4 = \pi - AB_mC_m$, and the exit velocity can thus be calculated, and the condition $\delta_4 = 90$ or 100 degrees, as the case may be, can be taken into account. *E.g.* in the sketch it is plain that if radial exit be required β_m must be decreased; thus a_m will be increased and the corrected value of v_m will therefore be decreased. Thus the final design will make w_m more nearly equal to w_k, and δ_m more nearly 90 degrees, than is shown in the sketch.

The whole of the quantities considered are measurable from the drawings, so that ${}_mv_4 = v_m$ is determined.

This is of course only a first approximation to v_m, but we can now ascertain whether w_m, differs materially from w_k, and can adjust the angle AB_mC_m, so as to cause ${}_yw_4$, to have the correct direction (*i.e.* AC_m, should be perpendicular to AB_m, if "radial exit" is desired).

The value of a_m, can now be corrected for the alteration caused by the angle AB_mC_m, not being equal to the angle ABC, and a new (and presumably sufficiently accurate) value of ${}_mv_4$ can be calculated.

The value of the angle $\beta_m = \pi - AB_mC_m$, which a line traced on the vane plane in a direction the projection of which on a plane through the axis of the turbine is MK, makes with the direction u_m, or ${}_yu_4$, is thus obtained under the assumption that the "exit is in a given direction" (*e.g.* radial or at 10 degrees forward) when the turbine runs at n, revolutions per minute, and Q_t, cusecs of water pass through the wheel.

In this manner we can determine the angle of inclination of lines traced in definite directions on the wheel vanes at p, points; and the complete design of the vanes consists in, so to say, stretching a fair formed skin over a skeleton thus obtained.

The actual design of a wheel vane of types such as I. to IV., is a problem for the skilled draughtsman, and the forms adopted by most firms have been arrived at by some such mathematical method as that sketched above checked by careful experimenting (in the case of American types of turbines, I believe, almost wholly by experiment). The above process enables us to check the dimensions of an existing turbine, and to detect any gross errors. The method, mathematically considered, is a weak one, and only leads to accurate results when carefully employed. The system of drawing the partial turbines used by Gelpke (*ut supra*) is in some ways more satisfactory from a theoretical point of view. In actual practice, however, Gelpke's system is so complicated that unless regularly employed confusion is bound to occur. When tested by high efficiency under working conditions, no appreciable difference exists between turbines designed by either method.

The method of testing a given design is obvious: ${}_mv_4$, and ${}_mu_4$, can be calculated, and the value and direction of ${}_mw_4$, can be geometrically determined. This can be compared with w_k, as determined by measurement.

The losses at exit are then as follows:

(i) Loss by non-radial exit is equal to $\frac{{}_mw_4^2\cos^2\delta_4}{2g}$.

(ii) Loss by shock of water on water at exit is equal to $\frac{({}_mw_4\sin\delta_4 - w_k)^2}{2g}$.

The methods of determining these losses are apparent.

The average of the sum of the above losses when expressed as fractions of H, for all the partial turbines is denoted by σ_4H.

It is difficult to estimate $(\tau_3+\tau_4)$H, the loss during passage through the wheel.

Our present information hardly justifies more than the following process.

Calculate $_yv_x$, the velocity at various points intermediate between entry and exit, by measuring $b_x = X_1XX_2$, and calculating a_x, by the method illustrated in Sketch No. 249 (p. 873). Then any marked difference between $_yv_x$, and $_yv_x'$, where X, and X′, are points close to one another on the same flow line, is detrimental. So also, any marked difference between $_yv_x$, and $_{y1}v_{x3}$, where XX_3, is a line perpendicular to the flow line, and y, and y_1, denote two adjacent partial turbines, is probably caused by incorrect drawing of the flow lines, and should be investigated. The more accurate methods will be discussed later.

No numerical rules can be given, but an investigation of the efficiencies of well-designed turbines (the proportions of badly designed turbines are never published) renders it probable that if the above conditions are fulfilled $(\tau_3+\tau_4)$H, is about one and a half times to twice as large as would be indicated by applying the usual rules for skin friction.

Accurate Method of Gelpke.—The above method is probably the best when it is desired to investigate the efficiency of an existing turbine. Where, however, it is desired to design a turbine of a previously determined efficiency ϵ, the following method of Gelpke (*ut supra*, pp. 62 and 94) is employed :

Consider the general equation :

$$\epsilon H = \frac{w_2^2 - w_4^2}{2g} + \frac{v_4^2 - v_2^2}{2g} + \frac{u_2^2 - u_4^2}{2g}$$

This refers to a case where there is no shock loss at entry, and where the exit is "radial," and without shock. If losses of these kinds occur, we have, considering the partial turbine M :

$$\frac{_mv_4^2}{2g} = \epsilon H - \sigma_2 H - \frac{w_2^2}{2g} + \frac{v_2^2}{2g} - \frac{u_2^2 - {_mu_4^2}}{2g} + \frac{_kw_4^2 - w_5^2}{2g}$$

where, for the sake of clearness, $_kw_4$ is written for the velocity of the water, just outside the wheel vanes, and $_mu_4$ represents the velocity of the corresponding portion of the vane edge. The first four terms on the right-hand side of the equation are the same for all the partial turbines ; and $\frac{u_2^2 - {_mu_4^2}}{2g}$ can be determined by measuring $_mr_4$, or r_m.

Also, $\frac{_kw_4^2 - w_5^2}{2g} = (\tau_5+\tau_6)$H approximately, and is given as one of the principal conditions of the design of the turbine.

We thus arrive at the formula :

$$\frac{_mv_4^2}{2g} = \psi H - \frac{_mu_4^2}{2g}$$

where ψH, represents,

$$\epsilon H - \sigma_2 H - \frac{w_2^2}{2g} + \frac{v_2^2}{2g} - \frac{u_2^2}{2g} + \frac{_kw_4^2 - w_5^2}{2g} ;$$

all converted into fractions of H.

We thus calculate $_mv_4$ for each partial turbine.

Also $_m w_4 \sin {_m\delta_4} = \frac{Q_t}{p f_4 2\pi r_4 b_m}$

where $b_m = M_2MM_3$ (see Sketch No. 257), and f_4 is a coefficient depending on the thickness of the wheel vanes, which may be taken as 0·85, to 0·90 for preliminary work.

We can now correct the preliminary value of $_m v_4$, by the equation :

$$\frac{_mV_4^2}{2g} = \frac{_mv_4^2}{2g} + \frac{_mw_4^2 \sin^2 {_m\delta_4}}{2g} - \frac{_1w_4^2 - w_3^2}{2g} \quad \ldots \text{(i)}$$

and a preliminary value of $_m\beta_4$ can now be obtained by setting off $_m w_4 \sin {_m\delta_4}$, $_mV_4$ and $_mu_4$, in the proper directions.

We can now calculate a more accurate value:

$$_mW_4 \sin {_m\delta_4} = \frac{\frac{Q_t}{pb_m}}{2\pi r_m - z_2 s \frac{\sqrt{1 - \sin^2 \phi_m \sin^2 {_m\beta_4}}}{\sin {_m\beta_4} \cos \phi_m}}$$

where ϕ_m is the angle between M_2M_3, and E_2E_3, the vane edges (see p. 917).

A still more accurate value of $_mV_4$ can now be secured by putting $_mW_4$, for $_mw_4$ in Equation No. (i), and thus the final corrected value of $_m\beta_4$ can be obtained.

The process is essentially an endeavour to allow for the shock losses σ_3 and σ_4 (which is variable as M alters) and to make each partial turbine *per se* of an efficiency equal to ϵ.

In practice the distribution of the velocities at the exit edge of the vane derived by this method differs somewhat from that obtained when the simpler method previously given is employed to design the turbine. Neither method produces a distribution entirely resembling that which experiments lead us to believe occurs, but Gelpke's method is closest to the results of such experiments as have yet been made. It is consequently probable (quite apart from the authority which is attached to any statement made by Gelpke) that this method will lead to better results than any other. Nevertheless, it does not appear advisable to consider a turbine designed on these lines as necessarily more efficient than any other carefully designed machine.

Geometry of Wheel Vanes (see Sketch No. 258).—The formulæ and geometrical constructions developed on page 912 and in Sketch No. 249, afford a means of obtaining the area of the cross-section of the tube formed by two consecutive wheel vanes with sufficient accuracy for preliminary designs.

More accurate methods are, however, desirable when testing the lay out of an existing turbine, especially when, as is frequently the case in Types I., II., and III., the area occupied by the metal of the wheel vanes forms an appreciable portion of the gross cross-section.

For the sake of clearness let us assume that the axis of the turbine wheel is vertical, and that the water is discharged downwards.

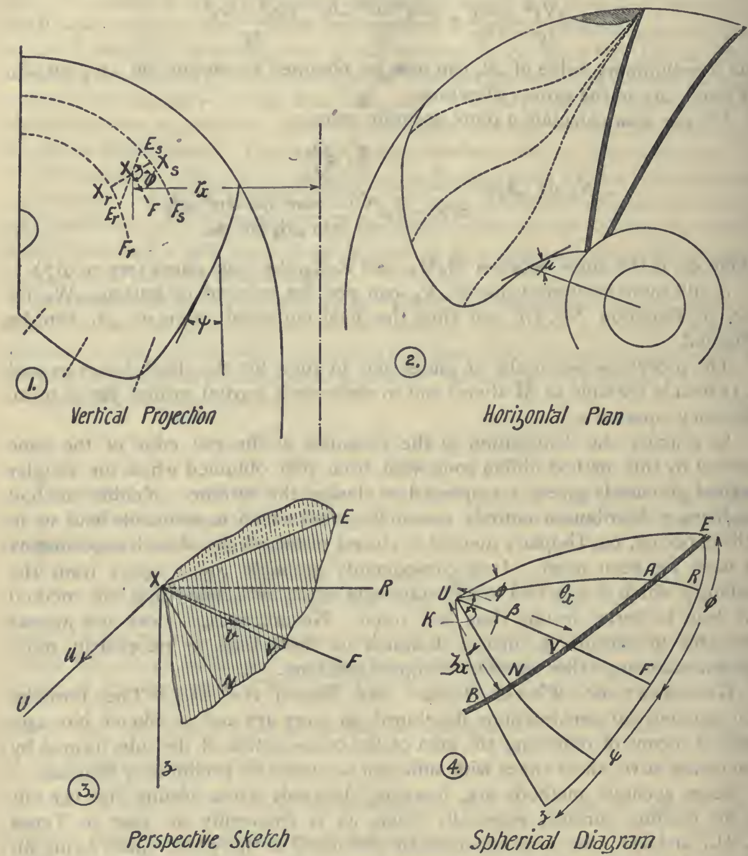
The following is a list of the makers' drawings ;

(i) A circular projection of the wheel crowns, and vane edges, on a vertical plane through the turbine axis. This will be referred to as the Vertical Projection. (Fig. 1.)

(ii) A plan, or vertical projection of the wheel vane edges on a horizontal plane. This we call the Horizontal Plan. (Fig. 2.)

It is assumed that the flow lines, or partial turbine boundaries, have been traced on the vertical projection, and if necessary corrected in accordance with the results of the preliminary investigations.

Consider any point X, on a wheel vane. Draw XF, the flow line through X, and X_rXX_s (where $X_rX = XX_s$) perpendicular to XF, in Fig. 1, of any convenient length. Through X_r, and X_s, draw flow lines X_rF_r, and X_sF_s. In



SKETCH NO. 258.—Determination of the Relative Velocities and Accurate Exit Angle.

practice X_rF_r, and X_sF_s are conveniently taken as partial turbine boundaries. Let $X'_rX'_s$, $F'_rF'_s$, etc. denote corresponding points on the next wheel vane. Denote all quantities referring to X, by the suffix x.

We wish to ascertain the nett area (allowance being made for the space occupied by the metal of the wheel vane) available for the passage of water between the two vanes, and the two cylindrical surfaces generated by X_rF_r, and

X_sF_s. This area must obviously be measured normal to the direction of v_s, the relative velocity of the water at X.

Consider either Fig. 3, a "perspective" sketch, or Fig. 4, a spherical diagram.

Draw XR, in the direction r_x, Xz vertically downwards, and XU, perpendicular to XR and Xz, and therefore in the direction of u_r. Let XNVE, be the vane plane. In this plane draw XN the projection of XU on the plane, XV the direction of v_x, and XE the intersection of the plane with the plane XRz. In Fig. 1, draw E_rXE_s parallel to XE, which is at present not accurately determined.

Now, we know the following quantities approximately:

> β_x = angle UXV = arc UV, the angle between u_x and v_r.
> $m_x = X_rX_s$, the perpendicular distance between the flow lines X_rF_r, and X_sF_s, measured in a vertical plane.

The following quantities are at present unknown:

> γ_x = angle UXN = arc UN.
> $e_x = E_rE_s$, the length intercepted between two flow lines on the intersection of the vane plane and a vertical plane.
> ϕ_x = angle FXE = arc FE = angle FUE, the angle between XF and XE_s.

Also $XX' = \frac{2\pi r_x}{z_2} = 2r_x \sin \frac{\pi}{z_2}$, if $\frac{\pi}{z_2}$ be a large angle; and XX′ lies along XU,

and therefore makes an angle $\frac{\pi}{2} - \gamma_x$ with the normal to the vane face.

The gross area available for water passage measured normal to XV, with no deductions for the thickness of vanes, is given by

$$E_rE_s \sin VXE.XX' \sin UXV = e_x \sin VXE \frac{2\pi r_x}{z_2} \sin \gamma_x.$$

The nett area A_x, is obtained by putting $\frac{2\pi r_x}{z} \sin \gamma_x - s_x$, for $\frac{2\pi r_x}{z_2} \sin \gamma_x$,

where s_x, is the thickness of the vane at X, measured normal to the vane face.

Now, since VFE, is a right angle, we have:

$$\sin VXE = \frac{\sin VE}{\sin \frac{\pi}{2}} = \frac{\sin FE}{\sin FVE} = \frac{\sin \phi_x}{\sin UVN}$$

and since UNV, is a right angle, we have:

$$\frac{\sin UVN}{\sin UN} = \frac{\sin UVN}{\sin \gamma_x} = \frac{\sin \frac{\pi}{2}}{\sin UV} = \frac{1}{\sin \beta_x}.$$

$$\text{Thus, } \sin VXE = \frac{\sin \phi_x \sin \beta_x}{\sin \gamma_x}.$$

$$\text{Thus, } A_x = e_x \sin \phi_x \frac{\sin \beta_x}{\sin \gamma_x} \left\{ \frac{2\pi r_x}{z_2} \sin \gamma_x - s_x \right\}$$

$$= e_x \sin \phi_x \left\{ \frac{2\pi r_x}{z_2} \sin \beta_x - s_x \frac{\sin \beta_x}{\sin \gamma_x} \right\}$$

$$= m_x \left\{ \frac{2\pi r_x}{z_2} \sin \beta_x - s_x \frac{\sin \beta_x}{\sin \gamma_x} \right\}; \text{ since plainly, } E_rE_s \sin \phi_x = X_rX_s.$$

Thus, except for the relatively small correction term in s_x, the area A_x, is expressed in terms of easily measurable quantities. Also $\frac{\sin \beta_x}{\sin \gamma_x}$ is always greater than 1, except when N_x and V_x coincide, and then $\sin \beta_x = \sin \gamma_x$, and $\phi_x = \frac{\pi}{2}$. Thus, the obstruction produced by the guide vane metal is least when the direction of v_x, and the projection of u_x on the vane plane coincide; and as a general principle the flow lines should be as nearly as possible perpendicular to the lines of intersection of the vane plane and the vertical plane through the radius.

Returning to the general problem:

EUN_x is a right angle. Thus, $NUV = \frac{\pi}{2} - \phi_x$, and from the right angled triangle VNU, we get:

$$\tan \beta_x = \tan UV = \frac{\tan UN}{\cos NUV} = \frac{\tan \gamma_x}{\sin \phi_x}$$

$$\text{Therefore } \sin \gamma_x = \frac{\sin \beta_x \sin \phi_x}{\sqrt{\cos^2 \beta_x + \sin^2 \beta_x \sin^2 \phi_x}} = \frac{\sin \beta_x \sin \phi_x}{\sqrt{1 - \sin^2 \beta_x \cos^2 \phi_x}}.$$

Substituting, we get:

$$A_x = m_x \left\{ \frac{2\pi r_x}{z_2} \sin \beta_x - s_x \right\} \frac{\sqrt{1 - \sin^2 \beta_x \cos^2 \phi_x}}{\sin \phi_x}$$

Thus, once the direction $E_r E_s$ is laid off the nett area can be determined in terms of β_x, which is approximately known, and ϕ_x, which is measurable.

The most important case is when X_x is a point at exit from the wheel vane. In the majority of cases (*e.g.* in Sketch No. 249) the exit edge of the wheel vane is a radial plane, thus the wheel vane edge as shown in the vertical projection is the direction $E_r E_s$, and the angle ${}_{v}\phi_4 = \phi_x$ can be measured at once.

In Sketch No. 252, however, the wheel vane edge is a rather complicated curve. The question has been considered by Gelpke (p. 33). The correction obviously only affects the term s_x, and at the exit edge s_x, owing to the vanes being sharpened, is usually small. Thus, the following correction seems practically unnecessary.

Put Φ_x for the measured angle between the flow line and the vane edge in the vertical projection.

ψ_x for the measured angle between the vertical and the vane edge in the vertical projection.

μ_x for the measured angle between the vane edge and the radius in the horizontal plan.

Then, the measured vane edge length, between two flow lines, is E_x, say, and,

$$e_x = E_x \sqrt{1 + \sin^2 \psi_x \tan^2 \mu_x \left(1 + \frac{1}{\frac{4 r_x^2}{E_x^2 \sin^2 \psi_x}(-1)} \right)}$$

and approximately, $e_x = E_x \sqrt{1 + \sin^2 \psi_x \tan^2 \mu_x}$.

Now, we have $e_x \sin \phi_x = m_x = E_x \sin \Phi_x$.

Thus, the correction factor is obtained by using the calculated angle ϕ_x in place of the measured angle Φ_x.

I have applied this formula to check the vane shown in Sketch No. 252, in which the exit edge departs greatly from a radial plane. The differences disclosed are relatively large, but the effect on the nett area A_x, is so small that I consider the extra calculations useless, unless the workmanship of the turbine is far more accurate than is usually the case. The sketch shows a very large and well designed turbine, and is the only example that I have ever considered would repay the labour entailed.

The area A_x, being determined, we have:

$$v_x = \frac{Q_t}{p z_2 A_x}$$

where p, is the number of partial turbines.

Thus, the whole circumstances of the motion are determinable.

Practical Testing of Wheel Vanes.—The above process allows us to test the drawings of a turbine.

The determination of similar quantities in an existing turbine is somewhat laborious. I have employed the following methods in several cases. Assume that flat strips of thin lead about ½, to 1 inch wide are moulded along horizontal lines traced on the vanes, *i.e.* intersections of the vane plane with horizontal planes. These strips can be laid down edgewise on a drawing board, and the following angles determined. (See Sketch No. 258, Fig. 4.)

The angle $\theta_x = UXA$, which the tangent to the trace of the strip on the board makes with the perpendicular to the radius.

The angle $\frac{\pi}{2} - \zeta_x = zXB$, which the line perpendicular to the board in the plane of the strip near X, makes with the vertical; so that ζ_x, is the angle between this line and the direction of u_x.

Thus, arc $UA = \theta_x$, and arc $UB = \zeta_x$.

Denote the angle NUA by κ_x.

Then, $\tan \gamma_x = \tan \theta_x \cos \kappa_x = \tan \zeta_x \sin \kappa_x$.

Thus, $\tan \gamma_x = \frac{\tan \theta_x \tan \zeta_x}{\sqrt{\tan^2\theta_x + \tan^2\zeta_x}}$,

$$\text{and } \cos \kappa_x = \frac{\tan \gamma_x}{\tan \theta_x}.$$

Thus, the point N, is determined.

Also, we know from the drawings, or from the preliminary calculations, β_x, and ϕ_x, and can calculate another value of γ_x, say λ'_x, from:

$$\tan \lambda'_x = \tan \beta_x \sin \phi_x$$

The values of γ and λ, can be compared, or the value of ϕ_x, say ϕ'_x, calculated from:

$$\tan \gamma_x = \tan \beta_x \sin \phi'_x$$

compared with that obtained from the drawings.

As already indicated, at points removed from the vane edges the best condition, if obtainable, is that given by $\phi_x = \frac{\pi}{2}$, or $\beta_x = \gamma_x$.

The process is only a rough one, as neither θ_x, nor ζ_x, can be accurately measured, but it affords a very fair insight into the workmanship and accuracy of the vanes. I believe that some turbine designers employ similar methods, as in one very efficient low head (Type I.) turbine thus studied it was quite plain that the condition $\phi=\frac{\pi}{2}$, had been very carefully followed, even when it entailed a markedly non-radial exit. A study of this particular turbine has led me to consider that if the draft tube is well proportioned, and tapers widely, the loss by non-radial exit (see p. 913) is regained by greater draft tube efficiency.

Mechanical Design of Turbines.—(*a*) *End Pressures.*—If Sketch No. 251 is considered, and the hole E, is assumed not to exist, it will be plain that the space C, would soon be filled with water at a pressure p_2. The pressure at any point on the underside of the upper crown of the wheel depends upon the distance from the axis, but is plainly less than p_2, and, as has been shown, may be less than atmospheric near the axis. Thus, the wheel would be exposed to a very heavy endways pressure, and the load on the footstep bearing F, would be far greater than that caused by the weight of the wheel.

Owing to the existence of the hole E, the pressure on the upper side is but small, and if the area of the hole be large in comparison with the leakage area $2\pi r_1 e_1$, the resultant pressure is probably upwards, and tends to decrease the pressure on the bearing F, the weight of the turbine being wholly or partially water-borne.

The question can be mathematically investigated as follows:

Let p_g, be the difference between the pressure in the space G, and the pressure at the top of the draft tube. p_g, can be calculated when $p_4-p_2-h_4+h_2$ is known, if the areas and coefficients of discharge of the leakage strip $2\pi r_1 e_1$, and the hole E, are determined; as a first approximation, $p_g=0$.

The resultant upward pressure of the water inside the turbine on both crowns is equal to $\frac{\pi D_4^2}{4}\frac{w_4^2}{g}$, as this expresses the vertical momentum generated in one second (the correction in cases such as cone turbines where the velocity at entry into the wheel is not in a horizontal plane, is obvious).

Thus, if W_t, be the weight of the wheel, shaft, etc., the resultant force in an upward direction is:

$$\frac{\pi}{4}D_4^2\frac{w_4^2}{g}-\frac{\pi D^2}{4}p_g-W_t.$$

In turbines of Types I. or II., especially if the shaft is horizontal, so that W_t, is 0, this force may be small, or it may even be advisable to make $p_g=p_4-p_2-h_4+h_2$, *i.e.* to abolish the hole E.

In these cases, and in double turbines, the force is not large, and a thrust bearing capable of taking a small end pressure suffices. The thrust bearing can never be entirely omitted, as the loss in efficiency by the wheel getting even slightly out of position with reference to the guide crowns is too great to allow any risk to be taken.

As a rule, however, the resultant force is downwardly directed, and when H, is large, the magnitude of the force thus produced is great, and a balancing piston and cylinder of the character indicated in Fig. 3, Sketch No. 250, is required. An exact calculation of the size of the piston and of the pressure required to produce balance is impossible, although the formula given permits a close approximation to be made.

From a practical point of view this is not important, as a very exact adjustment of the pressure difference on the two sides of the piston can be secured by manipulating the valve which controls the supply of pressure water to the underside of the piston. In the sketch, water taken from the supply to the turbine is used, and no regulating valve exists, so that adjustment is not possible. Large modern turbines are usually provided with a system of lubrication by which oil under pressure is pumped through the bearings by means of a small pump driven off the shaft of the turbine. In such cases the oil is usually pumped under the balancing piston, and is also employed to work the regulating machinery. In turbines which only receive water over a portion of the circumference of the wheel (see p. 864) sideways pressures may obviously occur. Such pressures are usually eliminated by supplying water at two places symmetrically situated with respect to the turbine axis.

In view of the losses disclosed by the discussion on page 904, it is necessary, especially in high speed turbines, to investigate the balancing of the wheel, its shaft, and all rigidly connected bodies around the axis of the wheel shaft. If W, be the total weight of the rotating bodies in lbs., and their mass centre be situated at a distance of r, feet from the geometrical axis, the "centrifugal force" is given by $F=\frac{W}{g}\, r\left(\frac{2\pi n}{60}\right)^2$ lbs.

and the shaft deflection produced with say $r=0{\cdot}01'=\frac{1}{8}''$ should be investigated. Uncertainties as to the exact value of F, are avoided by remembering that if $n=60$, *i.e.* 1 revolution per second, $F=W$ lbs., when $r=0{\cdot}817$ feet,

or $r=10$ inches approximately.

Proportions of Turbine Shafts.—The shafts of turbines are unusually favourably situated as regards liability to shock.

The twisting moment is given by the equation :

$$T=63{,}000\ \frac{N}{n} \quad \text{inch-lbs.}$$

Horizontal turbine shafts are exposed to bending moments produced by the weight of the wheel. The shafts of Pelton wheels and other turbines which receive water only over a portion of their circumference are also exposed to bending moments produced by the sideways pressures of the water jet or jets.

Let M, represent the bending moment thus produced. Put $M=kT$.

The usual theory states that the diameter of the shaft in inches is :

$$d=C\sqrt[3]{\frac{N}{n}}\ \sqrt[3]{k+\sqrt{k^2+1}}$$

where Unwin (*Theory of Machine Design*, vol. 1) gives the following table of values for C.

Material.	C.
Cast iron	4·32
Wrought iron	3·50
Mild steel	3·18
Medium steel	2·99
Steel castings	3·58

The figure given for steel castings is probably high, as cast steel has greatly improved of late. On the other hand, so far as I understand a matter which is still the subject of dispute, the theory which produces the factor $\sqrt[3]{k+\sqrt{k^2+1}}$, is now held to be erroneous and the factor should be altered to $\sqrt[3]{1+k^2}$.

In large turbines (especially where e_l, the width of the clearance space, is small) it appears advisable to investigate the deflection of the shaft under the bending moment M, and to ascertain if any rubbing is likely to occur.

Vanes and Crowns.—The advantages of thin wheel vanes are obvious. The thinnest vanes are obtained by using steel plates, and are $\frac{3}{16}$ths, to $\frac{1}{2}$ an inch in thickness. Their strength only needs to be considered when the turbine wheel is both large and wide. Each vane transmits a moment equal to 63,000 $\frac{N}{nz_2}$ inch-lbs. to the upper crown. If this is assumed to be produced by a pressure which is equally distributed over the whole of the projected area of the vane, and if the vane is then considered as a cantilever supporting this uniform pressure, the calculated stress is plainly somewhat in excess of the truth.

The vanes, however, should not be too thin, as erosion by sand and corrosion by entrained gases are likely to occur. Plate vanes are usually pressed into expanding grooves in the crowns by hydraulic pressure. In small, cheap turbines they are sometimes fixed in the moulds, and the crowns are cast round them. The plates should extend at least half an inch, and better still three-quarters of an inch, into the metal of the crowns.

In Types I. to V., however, the form of the vanes is so complex that a steel plate cannot be shaped so as to make a satisfactory vane. In these cases, the whole wheel (crown and vanes together) is usually cast in one piece. The minimum thickness varies from a half to three-quarters of an inch in the lower portions of the vanes, to three-quarters to an inch in the upper portions. The material is usually cast iron, but steel and bronze are growing more common, especially in cases where high heads are met with.

The wheel crowns are usually cast of iron, steel, or bronze, and are from 1 to 2½ inches in thickness, according to the size and speed of the turbine. Certain recent accidents suggest that it would be advisable to investigate whether the wheel can resist the stresses produced if (owing to the failure of the regulating apparatus) the wheel "runs away," and attains a speed which is approximately equal to twice that for which it is designed (see p. 943). A large factor of safety is of course unnecessary when provision is made against such abnormal occurrences.

The whole wheel must be firmly fixed to the shaft. Pfarr (see Hutte, vol. 2, p. 35) states that $L = 0{\cdot}6 + \frac{D}{8}$ inches approximately, where L, is the length of the key boss.

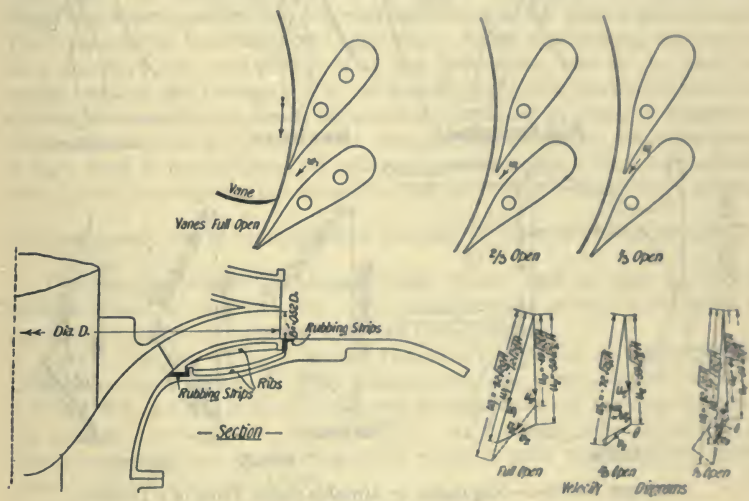
The guide crowns and vanes need less careful consideration. If the vanes are fixed, steel plates will suffice. If the vanes are movable for purposes of regulation, they are usually made of cast iron. Where bronze is specified, bronze bushes and turning pins must also be provided.

Clearance Space.—The space between the wheel and its housing largely influences the loss due to leakage (see p. 905). The exact value depends entirely upon the accuracy of the construction.

We may assume that $e_l = \frac{3}{16}$ths of an inch in small, and $\frac{3}{8}$ths of an inch in large turbines.

Bearings.—The design of turbine bearings is not a difficult matter once the magnitude of the forces involved has been realised.

In small turbines, the end pressure produced by the combined weight of the wheel and shaft and the water pressures, is usually carried on a submerged footstep bearing. Lubrication is then difficult, but if the bearing is bushed with lignum vitæ the water provides all requisite lubrication. Typical drawings are given in Unwin's *Machine Design*. In somewhat larger turbines the bearing is enclosed, and is lubricated by oil delivered under pressure. This method appears to be less satisfactory than that employed in many modern turbines, where, as already stated, the whole of the end thrust is taken up by oil or water pressure on the under surface of a balancing piston. The under surface of the piston and the upper surface of the bottom of the cylinder in which it works should be provided with grooves shaped in plan somewhat like



SKETCH NO. 259.—Regulation by Movable Guide Vanes of a Turbine narrower than Type VIII.

the vanes of a centrifugal pump. The object of such grooves is to encourage a rapid flow of oil, or water, underneath the piston. Details are given by Unwin (*ut supra*), and also by Van Cleeve (*Trans. Am. Soc. of C.E.*, vol. 62, p. 199). The inflowing oil should be cooled by passing it through small, thin walled pipes, which are exposed to a stream of water.

METHODS OF REGULATION.—The methods of regulating the horse-power and the speed of a turbine are very various. Gelpke enumerates seven, and more could be collected.

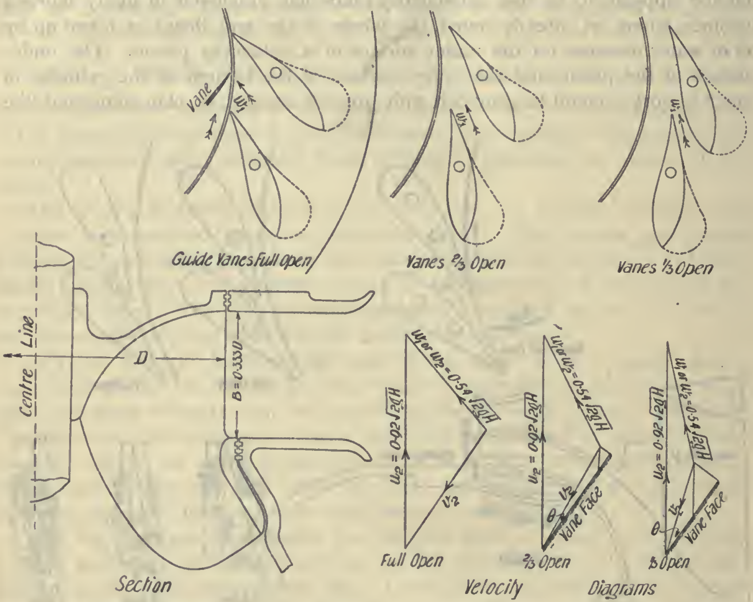
The following classification may be adopted :

(*a*) Regulation by moving the guide vanes.
(*b*) Regulation by obstructing the entry into, or exit from, the guide vanes.
(*c*) Regulation by valves, or sluice gates in the approach channels, or pressure mains.

Under (*a*) we need merely consider the method of Fink, where the vanes are rotated round fixed axes. The variants where the ends of the vanes are shifted sideways, or where a portion only of the vane moves, are equally expensive in construction, and produce shock losses of the character already discussed.

The only objection to the method is its cost.

The calculation of the power requisite to move the vanes can be theoretically effected by investigating the pressures on either side of a guide vane, and



SKETCH No. 260.—Regulation by Movable Guide Vanes of a Turbine near to Type I.

These sketches show the difficulties introduced by any departure beyond the limits of Gelpke's types, when the turbines are regulated. The underlying assumption is that w_1 remains constant and equal to w_2, however much the vanes be moved. Even under this favourable assumption the shock losses in No. 259 are very great.

If in addition the value of w_2 is corrected for:

(*a*) Decrease in w_1 owing to greater friction in the guide vanes;
(*b*) Shock loss on quitting the guide vanes;
(*c*) Loss caused by decreased hydraulic efficiency, owing to shock losses at entry to wheel;

it is obvious that w_2 will decrease rapidly as the guide vanes close up and the shock losses will become still larger.

taking the moments of the unbalanced pressures round the fixed axis. As a matter of practice, however, the friction of the various bearings in the link work connecting the piston of the regulating apparatus with the vanes influences the result to such an extent that it is advisable to assume that the full pressure caused by the head H, acts on one side of the vanes only, and to proportion the links and regulating apparatus for the forces thus produced. A certain excess of power is thus obtained, which will be useful if the lubrication of the pins or link bearings becomes defective.

(*b*) Until lately the standard American method of regulation was a cylinder gate working between the wheel and guide crowns. The loss of efficiency caused by the sudden expansion of the water stream as it issues from beneath the gate is obvious. In some cases, partial crowns were fixed between the vanes, and the loss was thus materially reduced (see Sketch No. 250). The method is cheap, and the gate requires little power to move it. It may therefore be adopted in cases where the first cost must be kept low.

(*c*) These methods practically amount to reducing the effective head. It is doubtful whether they are ever advisable.

THE FALL INTENSIFIER.—The fall intensifier, like the Venturi meter, is an application of the principle of the diverging tube, and is due to Clemens Herschell. The circumstances favouring its practical application are best illustrated by a description of the proposed installation at La Plaine, near Geneva. This power station is worked under a low head. At low water seasons (approximately 100 days per year) a head of 43 feet is available, and all the water is passed through the turbines. For the remaining 265 days the flow of the river exceeds the quantity that can be economically employed for power development, and owing to a rise in the tail water level, the available head diminishes, and in high flood is only 26 feet. It will consequently be plain that the turbines which suffice to utilise the low water flow, and under such circumstances develop N, horse-power, will in flood time only produce $\frac{26^{\frac{3}{2}}}{43^{\frac{3}{2}}}$ N = 0·48 N, approximately.

Thus, under ordinary conditions, some device such as cone, or double turbines, would be required; and even so the questions relating to the speed of the turbines would need somewhat careful consideration.

Sketch No. 261 (Fig. 2) shows Herschell's proposal diagrammatically. During low-water periods the water passes through the turbines by the channels ABCD. The circular gate at D, is closed during floods, and the water entering the turbines travels by the route ABCPG. At the same time the gate Q, is opened, and the whole, or a portion of the excess of water now available, passes through the fall intensifier RQPG.

The theory of the process is as follows:

Let H, be the visible head, *i.e.* the difference between the upstream and downstream flood water levels.

Let us assume that a vacuum of h, feet of water exists at p.

Then, the turbine works under an effective head equal to $H+h$, and when the form of the passages and the design of the turbine are known, we can calculate Q_t, the quantity of water used by the turbine.

So also, the passage RP, works under a head $H+h$, and discharges a quantity of water Q_i, say. The diverging passage PG, discharges a quantity of water equal to Q_t+Q_i. Thus, the velocities at P, and G, say v_p, and v_g, can be calculated when the dimensions of the intensifier are known.

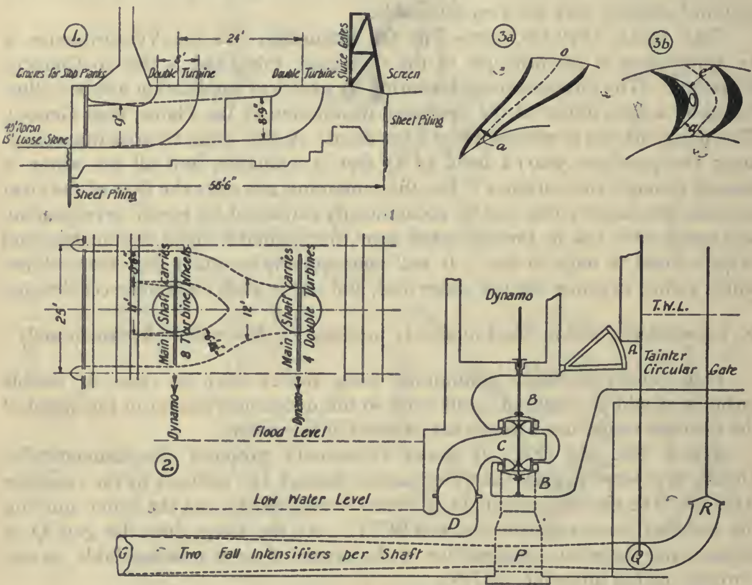
Then, theoretically, we have:

$$\frac{v_p^2}{2g}-h=\frac{v_g^2}{2g}+h_2, \qquad \text{or, } h_2+h=\frac{v_p^2-v_g^2}{2g},$$

where h_2 is the depth of the centre of G below tail water level.

Practically, h_2+h, is say 0·70, or 0·80 of the theoretical value.

At La Plaine it would be advisable to make h, equal to about $43-26=17$ feet, and probably if we assume that at low water the turbines are run so as to develop 0·80 of the maximum possible power (this being the point where the greatest efficiency is secured), and in floods so as to develop the maximum possible power under the smaller head, the effective head during floods might be reduced to x feet, where $x^{\frac{3}{2}}=0·80\times 43^{\frac{3}{2}}$, or $x=37$ feet say. Thus, efficient working with well designed turbines might be secured if h, was only 10, or 11 feet, although it will be evident that if turbines of Types I. or II. are used, the speed must also be investigated.



SKETCH NO. 261.

(1) Low Head Turbine Installation at Berrien Springs (after *Engineering Record*).
(2) Herschell's proposal for Fall Intensifier Installation.
(3) Wheel Vanes of Francis Turbine (*a*) and Free Deviation Turbine (*b*).

If Sketch No. 261 were a scale drawing, it is plain that G, should be at, or close to, tail water level so as to keep h_2, small. The necessary data regarding the efficiency of diverging tubes of the size now contemplated, are not available. Experience with draft tubes of turbines suggests that v_p, should not exceed 2·5 to 3 times v_g, and, consequently, if $h_2+h=15$ say, we get:

$$\frac{v_g^2}{2g}(2·7^2-1)\times 0·70=15, \qquad \text{or } v_g=13 \text{ or } 14 \text{ feet per second.}$$

Thus, the velocities contemplated are not larger than those which are frequently employed in turbine work.

Herschell (*The Fall Intensifier*) has published the results of certain tests of the principle, which prove that it works satisfactorily in practice. The details of the channels near P, are far more important than any tests, and I find it hard to believe that the experimental arrangements give any indication of the methods which are employed in practice. Further details of practical installations must be secured before a really intelligent design can be made. It will, however, be obvious that when the real efficiency of the arrangement can be approximately predicted, the method will be largely adopted. At present, in considering its application to any given case, we are quite unaware as to whether we shall have sufficient surplus water to produce the required vacuum at P. A certain amount of safety can be secured if the turbines are designed to work most efficiently at a low fraction of their full load (say 0·70 full load). Whether these conditions can be secured in any given case depends upon local conditions, and full records of the variations in head, and of the quantity of water available, are required.

The fall intensifier is a cheap solution, and really amounts to a substitution of the two large gates at D, and Q (see Sketch No. 261), for a certain number of turbines. The intensifier itself is a hole in the foundations of the power house, and is therefore not a very expensive piece of work.

PELTON AND SPOON WHEELS.—The Pelton type of turbine may be regarded as a turbine in which the guide vanes are replaced by one or more (not usually more than three) jets which are circular or rectangular in section. The water issuing from these jets impinges upon a series of moving buckets which are analogous to the wheel vanes and wheel passages of the ordinary Francis turbine.

In typical Pelton wheels the water does not pass through the wheel, but escapes on either side of the buckets at approximately the same distance from the axis of the wheel as it entered the buckets. In the Loffel wheel, or turbine with free deviation, the water passes through the wheel and escapes axially in the same manner as in a Francis turbine. Pelton and Loffel wheels are more easily constructed than Francis turbines, as the open buckets can be cast and machined separately, and can afterwards be bolted to the wheel rim. The regulating mechanism is cheaper, as at most three orifices have to be dealt with. As a general rule, the size of the jet is altered either by a slide, or better still by a central spear (see Sketch No. 262). In some cases regulation is effected by turning the jet slightly away from the wheel, so that only a part of the jet strikes the buckets, and the remaining portion is discharged into the tail race without doing work. Water is consequently wasted, but if the supply main is long it is frequently inadvisable to shut off the water suddenly (see p. 808).

The values of C, which indicate that Pelton or Loffel wheels are desirable are given on page 888. The efficiency of these wheels is slightly less than that of a Francis turbine. Consequently, even in California (the original home of the Pelton wheel) a Francis turbine is now frequently employed where five or ten years ago a Pelton wheel would have been installed.

The symbols employed in the calculations of Pelton and Spoon wheels are similar to those used for turbines, but are detailed in order to prevent obscurity.

b, is the axial breath of the bucket, in feet.
d, is the diameter of the jet orifice, in feet.
d_j, is the diameter of the jet at its vena contracta, or section of minimum area.
D, is the diameter of the wheel, in feet, measured to the tip of the buckets.
e_J, represents the total energy of the jet.
e, is the energy of a small portion of the jet at a distance r, from the axis of the jet in the cross-section at the vena contracta.

h, is the radial height of the buckets.

H, is the pressure in the supply main, in feet of water (see p. 882). Practically H = head utilised by the wheel.

Q, is the quantity of water passed by the jet, in cusecs.

s_n, is used for the efficiency of the nozzle, etc. (see p. 935).

u_2, is the velocity of the bucket at the point where the jet strikes it.

v, is the velocity of the water relative to the bucket, in feet per second; v_3 is used to denote the relative velocity just after complete entry, and therefore $v_3 = v_2$ less impact losses.

w_j, is the velocity of the jet at the vena contracta, in feet per second.

$w_2 = w_1 = w_j$, approximately, is the velocity with which the jet arrives at the bucket.

δ_2, is the angle between u_2 and w_2 or w_j.

ϵ, is the hydraulic, and η, the mechanical efficiency of the wheel.

θ, is the impact angle; theoretically, θ, is a space angle, but Finkle uses θ for the impact angle measured in a plane perpendicular to the axis of the wheel.

ϕ, is the foam and friction loss angle; as a definition we may put $\frac{v_4}{v_3} = \cos\phi$. (See p. 935.)

Size of the Jet.—Let H, be the pressure in the supply main, as measured at a point close to the wheel, where the velocity is small in comparison with $\sqrt{2gH}$, say not over 0·05, to 0·1 $\sqrt{2gH}$; so that H, is approximately equal to the geometrical head from the water level in the forebay to the jet orifice as corrected for the friction losses in the supply main. The velocity of the water in the jet is equal to:

$$w_1 = 0{\cdot}95 \text{ to } 0{\cdot}98\sqrt{2gH}.$$

The velocity of the bucket should be about one-half this, say:

$$u_2 = 0{\cdot}45 \text{ to } 0{\cdot}50\sqrt{2gH}.$$

Thus, D_2, the diameter of the wheel at the point where the jet strikes the bucket, is given by

$$\frac{\pi D_2 n}{60} = u_2.$$

If the value of u_2, when the efficiency is greatest is experimentally investigated, it will generally be found that $\frac{u_2}{\sqrt{2gH}}$ varies from 0·40 to 0·48. The higher values occur in the more efficient wheels in which the design of the buckets when tested according to Finkle's rules, is good. The lower values usually occur in less efficient machines where Finkle's process indicates that the buckets are badly designed. The rule is not without exceptions, but it will frequently permit the best speed of the wheel to be selected. So far as theory can be applied to the question, this experimental fact seems to indicate not so much that the buckets are badly designed, as that bad designers crowd the buckets too closely together.

The size of the jet orifice is best determined as follows:

> Let $H + h_f$ be the total static head from top water level to the nozzle, so that h_f represents the head lost in the supply main when Q cusecs are passing.
>
> Let l be the length and D′ the diameter of the supply main, and $v = C\sqrt{rs}$ its skin friction equation.

Then,
$$w_j = \sqrt{\frac{2g(H + h_f)}{\frac{4l}{C^2 D'}\left(\frac{d}{D'}\right)^4 + \frac{1}{c_v^2}}}$$

where c_v is the coefficient of velocity for the nozzle. The energy of the jet is ;

$$e_j = 62{\cdot}5\frac{\pi d^2}{4}\frac{w_j^3}{2g} = \frac{\pi 62{\cdot}5}{8g}\left\{\frac{2gc_v^2 D'^5 d^{\frac{4}{3}}(H+h_f)}{\frac{4l}{C^2}c_v^2 d^4 + D'^5}\right\}^{\frac{3}{2}}$$

and this is a maximum when :

$$d = \left\{\frac{D'^5}{\frac{8l}{C^2}c_v^2}\right\}^{\frac{1}{4}}.$$

This gives $w_j = 0{\cdot}816 c_v \sqrt{2g(H+h_f)}$.

This solution is that appropriate when the cost of the supply main is a large fraction of the total cost of the installation.

Gelpke, apparently considering only the efficiency of the wheel, gives a formula which is approximately,

$$d = \frac{D_2}{9{\cdot}5}.$$

The horse-power of the wheel, if $\eta = 0{\cdot}80$, is given by $\frac{QH}{11}$, and if d_j be the diameter of the jet at its minimum section,

$$Q = \frac{\pi}{4}w_j d_j^2.$$

Values of the ratio $\left(\frac{d}{d_j}\right)^2$ are given on page 937.

We can thus determine whether one, or two, or more nozzles are required in order to supply the requisite volume of water ; and D_2, and n, should then be adjusted in order to as far as possible conform to the two relations given above.

Square or rectangular nozzles are not common, but their area is made equal to $\frac{\pi}{4}d_j^2$, where d_j is obtained as above.

The jet nozzle should be so directed that the directions of w_j and u_2, the velocity of the wheel at the point where the centre of the jet meets the bucket tip circle make an angle of about 30 degrees.

If regulation is effected by deflecting the nozzle away from the wheel this should be allowed for by somewhat increasing this angle. Gelpke gives a table showing that this angle δ_2 is least in large wheels, and increases to about 40 degrees in small wheels, regulated by deflecting the nozzle. His figures appear to represent German practice well, but American designers usually use smaller values of δ_2.

(i) *Pelton Wheels.*—Let h be the height of the buckets measured radially, and b their breadth measured axially, then :

Gelpke states that approximately $h = 1{\cdot}7d_j$, $b = 3{\cdot}3d_j$, where d_j is the diameter of the jet.

His rule gives :

$$\frac{\text{The area of contracted section of jet}}{\text{Projected area of bucket}} = 0{\cdot}13, \text{ approximately.}$$

Le Conte states that the best efficiency in eleven Californian machines occurred when this ratio was 0·0979.

The maximum value of the ratio was 0·1417.
The minimum value of the ratio was 0·0861.

So also, in the above eleven installations, the ratio,

$\frac{d}{b}=\frac{\text{Diameter of jet}}{\text{Width of bucket}}$, ranged from 0·253 to 0·346, the average being 0·282.

The number of buckets depends on the ratio of d_j or d to D_2, and on the angle δ_2. It is best determined by drawing the jet and the bucket circle, and spacing the buckets so that the outer edge of the jet is neither obstructed by the tip of the bucket behind that on which it impinges, nor does the inner edge impinge so deeply into the bucket as to produce too short or too rapidly curved a path across the bucket. In general this produces a bucket with a tip somewhat recessed so as to prevent obstruction, and yet enables a close spacing of the buckets to be secured.

Gelpke gives a table of z_2 in terms of D_2 ranging from $z_2=16$, when D_2 is 12 inches or less, up to $z_2=24$ when D_2 exceeds 60 inches.

On trial I found that its value is largely dependent on Gelpke's ratio for $\frac{d}{D_2}$ being adhered to. Since I believe this ratio to be only adapted to favourable developments (*i.e.* where the ratio $\frac{l}{H+h_f}$ is small, see p. 929), it is always necessary to check results either by a drawing as above, or by Finkle's more elaborate method. When Gelpke's ratio is adhered to the spacing produces very excellent designs, but when applied to such ratios of $\frac{d}{h}$, as occur in Le Conte's examples, a wider spacing is desirable.

The breadth of the housing of the wheel should be approximately equal to $3b$, or $10d$.

The entry diagram can now be drawn, and this should be investigated for say three elements of the jet and three positions of the bucket as it moves over the selected spacing (see Sketch No. 263). Also, though in preliminary calculations it may be neglected, we should finally determine the impact angles as space angles, *i.e.* take into account the axial deviation of the jet as it is split by the central knife edge of the bucket.

It will be plain that there must be a slight residual velocity $v_4 \sin \delta_4$ at exit, producing an exit loss of head represented by $h_4=\frac{v_4^2 \sin^2 \delta_4}{2g}$. Usually $h_4=0{\cdot}05H$.

As a rule, however, the detailed geometrical construction given by Finkle should be adopted.

(ii) "*LÖFFEL*," OR *SPOON WHEELS*.—In this type the form of the bucket resembles a spoon, and the water escapes axially.

The Löffel wheel is less frequently used than the Pelton wheel, and appears to be slightly less efficient, owing to the fact that the escaping water is liable to drop back on to the rim of the wheel.

Gelpke gives the following:

$$b=3d \qquad h=1{\cdot}25d. \text{ approx.}$$

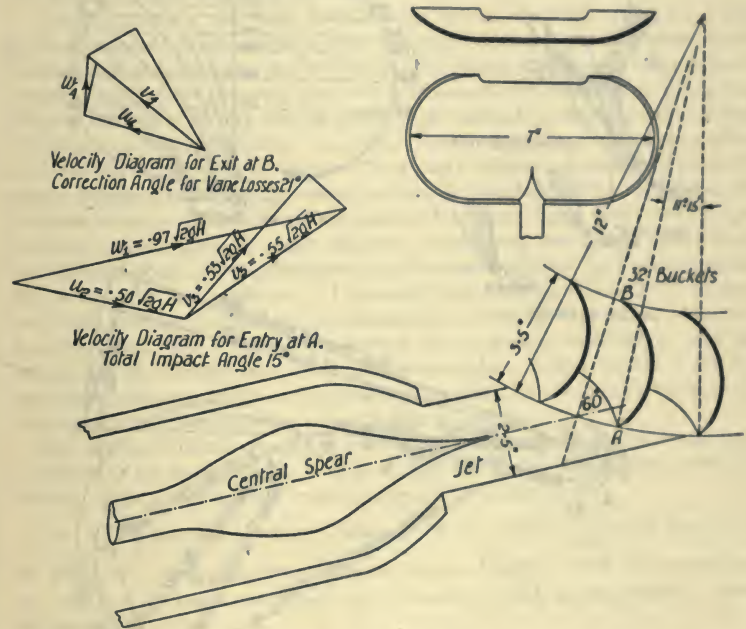
and the distance between the entry points on two consecutive buckets is given by;

$$t=0{\cdot}40 \text{ to } 0{\cdot}45 \frac{d}{\sin\delta_1}.$$

so that the number of buckets is $\frac{\pi D_2}{t}$, and is roughly twice that of a Pelton wheel of the same diameter; see, however, page 930.

The conditions for shockless entry and radial exit are theoretically somewhat more easily satisfied in a Löffel wheel than in a Pelton wheel, and this theoretical advantage appears to account for its adoption in Germany.

It is at present impossible to state whether the Löffel wheel really deserves adoption when the value of C_s is such as to permit a Pelton wheel to be built. At present, in the form of the inside feed turbine with free deviation (see Sketch No. 261) it fills up the gap between $C=32$, which corresponds to the

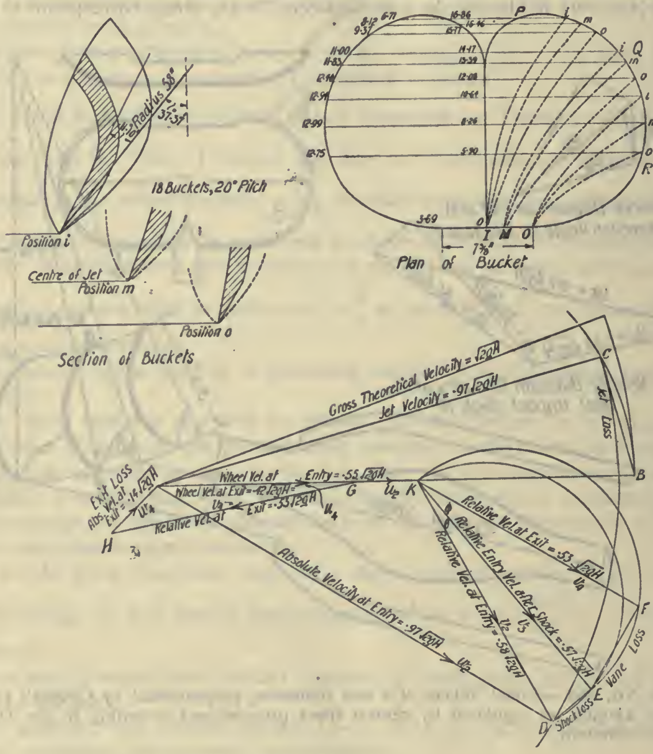


SKETCH No. 262.—Löffel Wheel of 2 feet diameter, proportioned by Gelpke's rules, with $2\frac{1}{2}$-inch jet regulated by central spear proportioned according to the Doble Co.'s standard.

Pelton wheel of the largest capacity, and $C=61$, which corresponds to a Francis turbine of very small capacity. So far as can be judged, even this limited sphere of utility is being rapidly encroached upon as designers acquire experience.

The circumstances of any given wheel can be investigated by Finkle's construction.

Detailed Designs of Pelton Wheels.—The formulæ and figures detailed above were the only information that I possessed in the period during which I enjoyed opportunities for testing Pelton wheels (I have never had personal experience of a Spoon wheel). I then came to the conclusion that the efficiencies usually stated were probably but rarely attained, but my experience being solely concerned with small wheels, I did not feel justified in making any definite statement. The following investigation of Finkle's, in my opinion, forms the first real step towards a logical design of Pelton wheels of high efficiency. When the methods are applied to drawings of existing wheels the hydraulic efficiency obtained is usually 1 to 3 per cent. lower than that calculated from actual observations. The results, however, convince me that efficiencies of 0·80 are far less commonly attained than is generally stated to



SKETCH NO. 263 —Pelton Wheel Buckets and Finkle's Detailed Velocity Diagrams (after Finkle).

be the case, and that the difference between the efficiencies of a well designed Pelton wheel and a stock piece of machinery is probably considerably greater than that which exists in Francis turbines constructed by reputable firms. It will also be plain that a highly efficient Pelton wheel can only be obtained with certainty by utilising the results of similar experiments.

So far as I can judge, Mr Eckart's experimental methods should be followed, but his mathematical investigations are less powerful than those undertaken by Mr. Finkle.

It is not proposed to discuss the mechanical design of a Pelton or Spoon wheel. The constructional form of the wheel, unlike that of the usual (*i.e.* Types I. to IV.) Francis turbine, is admirably adapted for securing rigidity and strength with but little trouble. The small size Pelton wheel is manufactured in considerable numbers by firms who are specialists, and, although I have frequently found reason to criticise the hydraulic design, the mechanical proportions are always good. Even when the strength is excessive, the fly-wheel effect produced by a heavy rim is always beneficial. Larger and specially constructed Pelton wheels are usually built on the tension spoke (bicycle wheel) principle, and no case of trouble arising from insufficient strength, or, what is probably equally important, insufficient rigidity, has been recorded.

Investigation of the Efficiency of a Pelton Wheel.—The general methods of investigating the influence of the forms and sizes of the buckets and the jet upon the efficiency of a Pelton wheel, and other types of impulse turbines, are precisely the same as those used in similar work concerning Francis turbines. A very elegant geometrical method has been given by Finkle (*Engineering News*, December 24, 1908). This is a development of the method which is usually adopted in German Technical Colleges. In some respects the theory is open to objection, and the experimental data required for a full discussion are most defective. Nevertheless, the results of the method give a very clear insight into the various ways in which efficiency is lost, and there is no doubt that a bucket form which gives satisfactory results when examined by Finkle's process will prove to be highly efficient. I therefore give an explanation of the method, and shall criticise certain of the assumptions made by Finkle, not because I discredit them (for I realise the defects of our present knowledge, and am aware that Mr. Finkle had more precise data upon Pelton wheels of the size considered than any which are generally accessible), but as an indication of the lines upon which experimental research should proceed.

Let AB (see Sketch No. 263), represent the velocity due to the total head available at the nozzle of the wheel, *i.e.* the total geometrical head from the forebay to the nozzle less all losses by pipe friction, bends, etc.

On AB, describe a semicircle, and set off BC, where $\frac{BC^2}{AB^2}$ = the fraction of the head lost in the nozzle = 0·06, according to Finkle. Then AC, represents the velocity of the issuing jet, and $AC = 0{\cdot}97\sqrt{2gH}$ approximately. More accurately:

$$\frac{AC^2}{AB^2} = \frac{\text{Energy of jet}}{\text{Discharge} \times \text{Nett available head}},$$

and Eckart (*Inst. of Mech. Eng.*, 1910) finds experimentally that:

Diameter of jet in inches	= $4\frac{23}{64}$	$5\frac{39}{64}$	$6\frac{37}{64}$	$6\frac{61}{64}$
Ratio $\frac{AC}{AB}$	= 0·958	0·968	0·982	0·986

so that Finkle's value which refers to a large and very well designed nozzle running full bore, is probably slightly low.

Now, along AB set off AK equal to the velocity of the bucket, and draw AD = AC in a direction making the same angle with AK, as the velocity of the bucket makes with the velocity of the jet (angle $DAK = \delta_2$). Since the cross-section of the jet bears a finite ratio to the size of the bucket, in a detailed study of a large wheel it will be necessary to select a certain number (three in Finkle's example) of points, or more acurately representative elements of the jet, and to obtain the angle DAK, and construct a separate diagram for each of these elementary jets. This is, of course, best done by drawing the jet and bucket to a convenient size, and where the necessary data exist, the variation in AD, or AC, that occurs over the cross-section of the jet, as well as the easily calculated variation in AK, might be taken into account. As a general rule, however, AD, is taken as constant.

Thus KD, represents the velocity of the jet relative to the bucket. Now, the bucket cannot be so correctly designed that the direction of its surface at the point of impact of the elementary jet will accurately coincide with KD. We therefore calculate or measure the impact angle $\theta = \text{DKE}$, which represents the (space) angle between KD, the ideal direction of the relative velocity, and KE, the path which the water actually follows on the bucket. This evidently requires certain assumptions to be made regarding the manner in which the water leaves the bucket, and Finkle sketches out (see Sketch No. 263) three paths corresponding to three positions of the bucket for each of the three elementary jets. The assumption made appears to be that the discharge is uniform over each element of the discharge edge PQR, of the bucket. This may be accepted as true, and in view of the high velocity of the water is probably more accurate than the similar assumption made regarding partial turbines in a Francis turbine. Now, describe a semicircle on KD, cutting KE, in E. Then KE, represents v_3 the velocity with which the water starts to travel along the bucket, and in theory $v_3 = v_2 \cos \theta$.

The question of loss by impact at entrance has already been discussed. We have no direct evidence upon the subject in the case of Pelton wheels, and such evidence as is afforded by the characteristic curves of turbines tends to show that the theory followed by Finkle is (for small values of θ such as occur in a well-designed bucket) erroneous, and overestimates the loss. Nevertheless, if the drawing shows that $\frac{\text{DE}^2}{\text{AB}^2}$ (the impact fraction of the hydraulic loss) is large, the bucket must be considered as badly designed, and a better shape must be sketched out. It is hardly necessary to state that θ must be calculated by the spherical trigonometrical methods already indicated.

Next, set off the angle $\text{EKF} = \phi$ and describe a semicircle on KE, cutting KF in F. If ϕ be properly selected, KF, represents the relative velocity of exit of the water from the bucket. Finkle takes $\phi = 21$ degrees, in all the nine cases, and this is evidently an experimental value. As a matter of fact, it is fairly plain that ϕ largely depends upon the form and curvature of the path in which the water travels across the bucket. In a well-designed bucket each elementary jet is deflected through approximately the same total angle while crossing the bucket, so that the water particles which lose the least energy from friction probably lose more energy owing to the sharper curvature of the path which they traverse. Thus, the assumption that $\phi =$ a constant, is a rational one.

Hence (subject to the "error" discussed later), KF, represents v_4, the relative exit velocity of the water, and $\frac{\text{EF}^2}{\text{AB}^2}$, represents the fraction of the energy lost by "foam and friction." Now draw AG, along AB, to represent the velocity of the bucket at the point where the water (the velocity of which is represented by KF) leaves the bucket; and set off GH = KF, so that the angle AGH, represents the angle between the velocity of the bucket and the velocity KF. This must be measured from the drawing of the bucket, and the bucket is usually designed so that $\pi - \beta_4 = AGH = 10$ degrees approximately. Then AH, represents the absolute velocity of the water when it quits the bucket, and $\frac{AH^2}{AB^2}$, represents the fraction of energy lost by exit velocity. We may therefore utilise the result of this construction in order to determine the final value of the exit angle AGH.

The overall hydraulic efficiency of the Pelton wheel can now be estimated. The fractions of the energy available just before the nozzle, which are afterwards dissipated without doing work, are as follows:

(i) Lost in the nozzle . . . . $s_n = \frac{BC^2}{AB^2}$

(ii) Impact loss at entry . . . . $s_s = \frac{DE^2}{AB^2}$

(iii) Foam and friction loss . . . . $s_r = \frac{EF^2}{AB^2}$

(iv) Exit velocity loss . . . . $s_e = \frac{AH^2}{AB^2}$

Thus, the hydraulic efficiency is:

$$1 - \frac{BC^2 + DE^2 + EF^2 + AH^2}{AB^2} = \epsilon$$

The sketch shows one of Finkle's nine diagrams, but is not a representative diagram, since I desired to show the various losses clearly and therefore selected a case with relatively large values of s_s and s_r.

Finkle does not explain his methods, and the following criticisms may therefore be due to misconceptions. In the first place, the impact angle θ is apparently obtained as though the impact were in two dimensions only. Since the various paths I*i*, . . ., O*o*, are apparently freely sketched out, this is not very important. The angle obtained may not refer to the precise portion of the jet considered by Finkle, but some stream does experience the assumed deviation. Secondly, since u_4 or AG is not equal to u_2 or AK; v_3 or KE is not equal to v_4 or KF even if no foam and friction loss occurs. This appears likely to produce a certain error for the real meaning of Finkle's assumption, $\phi = 21$ degrees, is that when a stream of water enters a bucket with a certain measured (after shock has occurred) relative entry velocity, the relative exit velocity is observed to be 0·933 of the entry velocity. Now, these observations were almost certainly taken on a fixed bucket, so that the fact that the ratio was obtained by observation does not entirely justify the neglect of the theoretical alteration of the relative velocities (see p. 865).

Finkle's original paper is well worth consultation, but as he does not state what experimental foundation his figures possess, I do not quote them.

Eckart (*Inst. of Mech. Eng.*, 1910) has investigated the matter experimentally, by means of a special Pitot tube (see p. 72). He measures the velocity of the jet at several points, and puts:

$$W_j = \frac{2\pi \int_0^R wrdr}{a_j}$$

where a_j, is the area of the jet at the point where the measurements are taken, and w, is the velocity at a distance r, from the centre of the jet. Where R is the radius of the jet, the energy of the jet is :

$$e_j = \frac{2\pi \int_0^R erdr}{a_j}$$

where $e = \frac{w^2}{2g} 62{\cdot}5$ foot-lbs.

Thus, approximately $e_j = \frac{62{\cdot}5}{2g\ a_j} W_j^3$.

Eckart states that in his tests the accurate value of e_j, was always somewhat less than that given by the approximate formula, the difference being as follows :

1·44 per cent.	when the diameter of the minimum section of the jet was	$4\frac{23}{64}$ inches.
1·68 "	" " " "	$5\frac{39}{64}$ "
1·24 "	" " " "	$6\frac{37}{64}$ "
0·81 "	" " " "	$6\frac{52}{64}$ "

We thus obtain (see p. 933) the accurate values of the ratio $\frac{AC^2}{AB^2}$ already given.

Then, putting $u_2 = AK =$ the velocity of the bucket at the point of contact, and $v_2 = KD =$ the velocity of the water at entrance, relative to the bucket, $\delta_2 =$ angle KAD :

$$v_2^2 = W_j^2 + u_2^2 - 2W_j u_2 \cos \delta_2$$

which is the algebraic expression of Finkle's geometrical construction.

Eckart states that $\delta_2 = 4$ degrees 41 minutes, or $\cos \delta_2 = 0{\cdot}9968$.

The power of the wheel is obviously :

$$P = \frac{62{\cdot}5}{g} u_2(W_j \cos \delta_2 - u_2 - v_2 \cos \beta_2)$$

where $\beta_2 =$ the angle AKD, and we assume $u_4 = u_2$.

Eckart now measures $v_4 = KF$, or GH, the velocity of the water when leaving the bucket, and states that the loss of head due to friction and eddies in the bucket is represented by :

$$h_1 = \frac{v_2^2}{2g} - \frac{v_4^2}{2g} = s_r H$$

The underlying assumption is that $\theta = 0$ in a construction similar to Finkle's, and theoretically, therefore, the total loss is slightly underestimated. Practically, the error is slight, and the figures in the column "Other Hydraulic Losses" show that even if the whole amount entered in the column is due to this cause (which is not probable), the value of θ need only be considered when very careful measurements are made.

Next, put $AG = u_4 =$ the velocity of the bucket at the point of exit, and let the angle of exit $= AGH = \pi - \beta_4$.

The absolute velocity at exit, or the residual velocity is, $w_4 = AH$, and $w_4^2 = (u_4 - v_4 \cos \beta_4)^2 + v_4^2 \sin^2 \beta_4$, which expresses Finkle's geometrical construction. The head lost is $\frac{w_4^2}{2g}$.

Eckart's value of β_4 is 14 degrees, or $\cos \beta_4 = 0·9703$.

The experimental results are as follows :

Diameter of minimum section of jet (inches)	$4\frac{23}{64}$	$5\frac{39}{64}$	$5\frac{57}{64}$	$6\frac{23}{64}$
Coefficient of contraction of jet	0·994	0·951	0·891	0·847
Coefficient of velocity	0·971	0·976	0·984	0·989
Coefficient of discharge	0·965	0·929	0·877	0·838
Efficiency of nozzle, equal to $\frac{W_j}{\sqrt{2gH}}$	0·958	0·968	0·982	0·986
Loss in eddies and friction in bucket (per cent.)	23	23·2	27·7	29·2
Loss in residual velocity (per cent.)	1·1	1·0	1·8	1·9
Other hydraulic losses (per cent.)	1·5	1·6	1·1	0·8
Hydraulic efficiency (per cent.)	74·4	74·2	69·4	68·1

The results are not as good as those which Finkle believes that he has attained, but they are actual measurements, while Finkle's results are calculations, though apparently founded on observations on other Pelton wheels.

CENTRIFUGAL PUMPS.—*SYMBOLS.*—The symbols used in discussing centrifugal pumps are precisely those used in discussing Francis turbines (see p. 875). When it is desired to distinguish between the hydraulic efficiency of a pump and that of a turbine, the symbols ϵ_p and ϵ_t are employed. Also, since the water passes through the pump in the reverse direction to the flow of water in a turbine, the suffix 2 refers to exit from the pump and 4 to entry into. The notation has not been altered, as suffix 2 will be found to refer to the outer circumstances of the wheel both in pumps and turbines.

The centrifugal pump is a reversed turbine. The water flows through the pump in the opposite direction (from the centre to the circumference), and the wheel rotates in the opposite direction, doing work on the water ; while in the case of a turbine the water does work on the wheel.

Using suffix 4 to denote entry into (*i.e.* the inner portion of) the wheel, and suffix 2 for exit from (*i.e.* the outer portion of) the wheel ; we have :

$$\frac{H}{\epsilon} = \frac{u_2^2 - u_4^2}{2g} + \frac{w_2^2 - w_4^2}{2g} + \frac{v_4^2 - v_2^2}{2g}$$

where ϵ is the hydraulic efficiency of the pump. This may be expressed in words as follows :

The gross head pumped against (allowance being made for pump efficiency), is equal to the sum of :

(i) The pressure produced by the centrifugal force.

(ii) The pressure required to produce the change in the absolute velocities at entry and exit.

(iii) The diminution of pressure caused by the change in the relative velocities at entry and exit.

The statement is not a proof, but the form is easy to remember.

The value of ϵ, however, differs considerably from that obtained in turbines. This difference is believed to be solely due to the exigencies of practical design. A turbine is assumed to work under a constant head, and if the head varies greatly, the quantity of water passed through the turbine is adjusted so as to secure the best efficiency. A centrifugal pump is expected to pump against a head that varies over a far greater range than is usual in turbine work, and to deliver the maximum possible quantity of water at all heads. Thus, the pump must be compared with a turbine which is often run at an unsuitable speed, and is systematically overloaded. The design of the pump must therefore be adapted to these unfavourable circumstances. Thus, $\epsilon=0{\cdot}67$ is a mean value, in the same sense as $\epsilon=0{\cdot}80$ for a turbine. The efficiency of a well designed centrifugal pump may be estimated as follows:

(*a*) For the ordinary type of portable centrifugal pump which is expected to pump against heads varying from 10 to 30 feet, take $\epsilon_p=\epsilon_t-0{\cdot}20$, where ϵ_p is the efficiency of the pump, and ϵ_t is the efficiency of a turbine of the same size, when running under a head equal to the mean head pumped against.

(*b*) For a fixed centrifugal pump, working against a large, and not very variable head, take:

$$\epsilon_p=\epsilon_t-0{\cdot}10$$

The rules are rough, and it is probable that ϵ_p does not exceed 0·40, to 0·45 in the case of contractors' pumps working under ordinary conditions; while most firms will guarantee $\epsilon=0{\cdot}75$ to 0·78 (not including pipe friction) against a steady head.

The efficiency of the high pressure centrifugal pumps, which are now employed for draining mines, is greatly in excess of that given by the above rule; and the difference between ϵ_p and ϵ_t in these cases is amply explained by the fact that the velocity of the water in the long rising mains is far greater than the velocity of the water in the supply mains of turbines.

The best method of sketching out the preliminary design of a pump is as follows:

Let H_1 be the maximum head pumped against.
Let H_2 be the minimum head pumped against.
Select a value of ϵ and calculate:

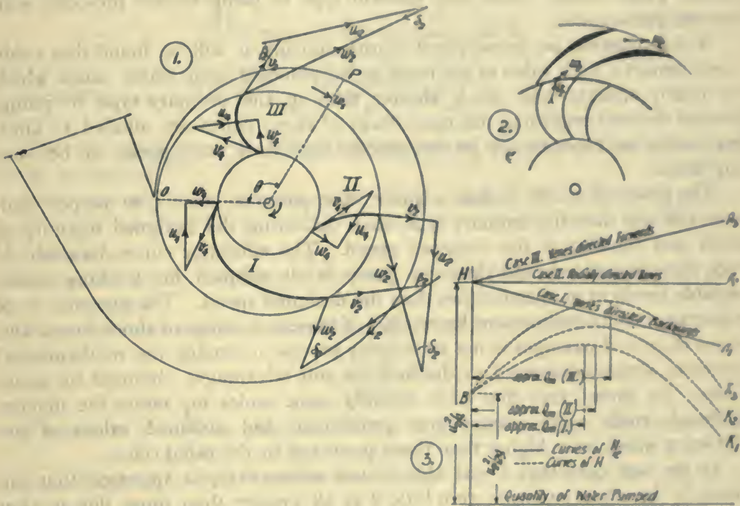
$$1{\cdot}25\,\frac{H_1}{\epsilon}=K_1, \qquad \text{and } 1{\cdot}25\,\frac{H_2}{\epsilon}=K_2.$$

Now, let Q_1, and n_1, be the quantity of water delivered, and the speed of the pump, when the head is H_1, and Q_2, and n_2, be the values of Q_1, and n_1, when the head is H_2. Determine the size and type of a turbine that will use Q_1, cusecs, and run at n_1, revolutions under a head K_1, and also of a turbine that will use Q_2, cusecs, and run at n_2, revolutions under a head K_2. The two turbines thus obtained probably differ radically.

Let us assume that the K_1, turbine is the larger of the two. Calculate the type and size of a turbine that uses $0{\cdot}80Q_1$ (say) cusecs at n_1, revolutions. We thus usually get a smaller turbine which more closely resembles the K_2 turbine in type. Similarly, calculate a turbine which passes $1{\cdot}20Q_2$ (say) cusecs, at n_2, revolutions under a head K_2. These two turbines should not differ very greatly, and the size and type of the pump can now be selected. The velocities u_2, u_4, v_2, v_4, and w_2, w_4, can now be estimated, and we can determine whether the general equation is satisfied for the two circumstances H_1, Q_1, n_1, and H_2, Q_2, n_2. Slight modifications may be required, but, as a general rule, we can proceed to estimate the losses due to friction, and shock, and can sketch out the forms of the vanes.

The process is empirical, and it is quite possible to select values of H_1, Q_1, n_1, which are entirely incompatible with the conditions H_2, Q_2, n_2. If such a case occurs, in practice, we must assume that the pump will be extremely inefficient under one or other of the conditions, and should accordingly design on that assumption.

Sketch No. 264 shows three typical forms of vane with radial entry, and a spiral pump housing, or body. The spiral housing must plainly be so designed



SKETCH No. 264.—Centrifugal Pump Vanes.

that the velocity of the water at every point is equal to $w_2 \cos \delta_2$. Thus, when the pump is delivering Q, cusecs the area at a point P, distant θ degrees from the point O, is plainly $\dfrac{Q\theta}{2\pi w_2 \cos \delta_2}$

LOSSES BY SHOCK.—In Sketch No. 264 the loss by shock at entry is nil, as the vanes are so directed that the entry is radial. If the quantity of water delivered is altered, and the speed of the pump is not altered, w_4, is altered and shock occurs, and its value can be calculated by the velocity diagram.

The conditions at the point of exit are somewhat different. There are no guide vanes. Thus, a loss of head equal to

$$\frac{w_2^2 \sin^2 \delta_2}{2g}$$

always occurs.

Also a loss $\left(\dfrac{w_2 \cos \delta_2 - w_1}{2g}\right)^2$ occurs, where $w_1 = \dfrac{Q\theta}{2\pi \text{ area at P}}$.

This last loss varies over the whole exit circumference of the pump, but can be calculated by taking the average values at three or four points.

In some modern high-pressure centrifugal pumps guide vanes analogous to those of a turbine are provided outside the wheel. Sketch No. 264, Fig. 2, shows the plan of a typical example.

The tangents to the inner end of these vanes should be parallel to the direction of w_2, and their outer tangents should be as nearly as possible perpendicular to the line joining the outer end to the centre of the wheel. The losses are obtained by putting w_e the velocity of exit from the guide vanes for w_2, and δ_1 the angle the end tangent makes with a perpendicular to the radius for δ_2 in the equations given above. There is also a certain extra loss by skin friction on the guide vanes. One very efficient type of pump is also provided with internal guide vanes.

If the question be investigated mathematically, it will be found that guide vanes permit a high value of the head to be obtained with wheel vanes which are nearly straight and much shorter than in the ordinary type of pump. Several theories regarding the exact form of these vanes are alluded to later, but so far as experiments go the precise form does not appear to be very important.

The practical result is that a guide vane pump is some 5 to 10 per cent. more efficient than the ordinary type when delivering the designed quantity of water and running at the designed speed. The efficiency curve, however, is very sharply pointed, and thus the pump is ill adapted for working under variable heads or at speeds other than the designed speed. The question is of importance, since our present knowledge of the exact values of shock losses and losses in curved passages is not sufficiently precise to enable the mathematical processes already discussed to disclose the full advantages obtained by guide vanes. In three cases that have recently come under my notice the makers tendered guide vane pumps and guaranteed and obtained values of the efficiency which were higher than those predicted by the usual rules.

In the one case that I was able to test exhaustively it appeared that the losses in the wheel passages were little if at all greater than those due to skin friction. The wheel vanes were nearly straight, and the workmanship of the pump was excellent. The one doubt that still remains is whether the efficiency will not decrease very rapidly when the pump becomes worn. I have not as yet felt justified in recommending a guide vane pump for silted water.

Neumann (*Theorie der Zentrifugal Pumpen*) has proposed to make these guide vanes conform to an evolute of a circle, the radius of the generating circle being fixed by the initial angle of inclination of the guide vane, as calculated from the equations of relative velocity.

Bergeron (*Revue de Mechanique*, 31st October 1910) states that this form is not so good as the equiangular spiral which produces a very efficient shape when tested experimentally. Bergeron also considers the correction caused by the fact that both the exit channels from the wheel and the entrance channels into the guide chamber are of finite size. Both his own experiments and those of Wagenbach (*Ztschr. des Gesamte Turbinenwesen*, 30th June 1908) show that δ_2 is in practice somewhat less than the value obtained by drawing the triangle of velocities.

Thus, H, the height to which the pump can lift the water, is somewhat less than the theoretical value in vanes which are directed either backward or radially, and is somewhat greater than the theoretical value when the vanes are directed forward.

In general, when high efficiency is desired it is advisable to use backwardly directed vanes, but the required lift can be attained (even if the lesser value of the efficiency be taken into account) at the least speed of pump rotation by using forwardly directed vanes.

Variation of H as Q is altered.—Consider the equation:

$$\frac{gH}{\epsilon} = w_2 u_2 \cos \delta_2 - w_4 u_4 \cos \delta_4$$

When the entry is radial $\cos \delta_4 = 0$, and $\cos \delta_4$ is always small. Thus, for speeds which are not very different from that of radial entry:

$$\frac{gH}{\epsilon} = w_2 u_2 \cos \delta_2 = u_2^2 + u_2 v_2 \cos \beta_2$$

$$\text{Therefore } u_2 = -\frac{v_2 \cos \beta_2}{2} + \sqrt{\frac{v_2^2 \cos^2 \beta_2}{4} + \frac{gH}{\epsilon}}$$

Now, for vanes which are directed backwards at exit (see I, Sketch No 264), $\cos \beta_2$ is negative.

For vanes which are radial at exit (see II, Sketch No. 264), $\cos \beta_2 = 0$, or $u_2 = \sqrt{\frac{gH}{\epsilon}}$.

For vanes which are directed forward at entry (see III, Sketch No. 264), $\cos \beta_2$ is positive.

Now, v_2 can be expressed in terms of Q:

$$v_2 = \frac{Q}{z_2 a_2 b_2}.$$

Thus, if we plot $\frac{H}{\epsilon}$ as ordinate, and Q, as abscissa, we get the three lines A_1H, A_2H and A_3H, which represent the values of $\frac{H}{\epsilon}$ in terms of Q, when u_2, or n, is constant.

HA_1, refers to a vane which is directed backwards at exit ($\cos \beta_2$ being negative), and therefore slopes downwards as Q, increases.

HA_2, refers to a vane which is radial at exit ($\cos \beta_2 = 0$), and is therefore horizontal.

HA_3, refers to a vane which is directed forwards at exit ($\cos \beta_2$ being positive), and therefore slopes upwards as Q, increases.

The value of OH, *i.e.* $\frac{H}{\epsilon}$ when $Q = 0$, is plainly $\frac{u_2^2}{g}$.

Now, as a matter of experiment, ϵ varies as Q alters, owing to shock, and losses due to the fact that entry is only radial for one particular value of Q. The values of H, as actually observed, therefore lie not on the straight lines HA_1, HA_2, and HA_3, but on the approximately parabolic curves B_1K_1, B_2K_2, and B_3K_3. It is also an experimental fact that, very approximately, OB_1, OB_2, and OB_3 are each equal to $\frac{u_2^2}{2g}$. A general idea of the relation between H, and Q, can thus be obtained. The results can be rendered quite sufficiently accurate for ordinary practical purposes by calculating the theoretical curves of $\frac{H}{\epsilon}$, as indicated, and calculating ϵ for any given value of Q, as follows:

Put ϵ_m for the observed value of ϵ when the pump passes the quantity of water, say Q_m, which produces the best efficiency at the given speed, or for the calculated value of ϵ under these circumstances.

Estimate the shock losses at entry and exit for the new value of Q, say Q_1.

Then the investigation of turbine efficiencies given on page 900 applies, and the new value of ϵ becomes:

$$\epsilon_m\left(\frac{Q_1}{Q_m}\right)^2-\sigma_2-\sigma_4=\epsilon_1 \text{ say.}$$

The value of the head produced is given by $\epsilon_1 H_1$, where H_1, is the value of $\frac{H}{\epsilon}$, as calculated by the theoretical straight line equations already given.

The results of this process are quite sufficiently accurate, provided that Q_1, is not less than say $0{\cdot}6Q_m$, and is not greater than say $1{\cdot}4Q_m$. If, however, the method is applied to calculate H, when $Q_1=0{\cdot}1Q_m$, or when $Q_1=2Q_m$, errors may be expected, and the head which will be obtained will usually be less than the truth when Q_1 is a small fraction of Q_m. The head is greater than the truth if Q_1, considerably exceeds Q_m.

For preliminary estimates the following rule will be found to agree well with the makers' catalogues

$$u_2=8{\cdot}5 \text{ to } 9{\cdot}2\sqrt{H}$$

the lower values evidently refer to more efficient pumps.

In very well designed pumps with guide vanes such values as:

$$u_2=7{\cdot}5 \text{ to } 8{\cdot}2\sqrt{H} \text{ are attained.}$$

The theory given above includes all points in which the design of a centrifugal pump differs from that of a turbine. It may, however, be noted that in cases where the pump is below water level, so that troubles caused by a vacuum existing at the entrance to the pump need not be feared, a certain gain in efficiency can be secured by making the ratio $\frac{D_5}{d_4}$ considerably larger than is usual to turbines. The theory is obvious, since divergence losses do not occur, and K (see p. 902) depends only on frictional losses. The size of the pump can thus be materially reduced. The extra losses caused by the increase in the velocity of the water in the pump, and (unless a diverging mouthpiece be provided) in the rising main, are easily calculable.

The question of multiple stage pumps is not considered, as the difficulties in design caused by the double and triple wheels are entirely mechanical. The efficiency of multiple stage pumps is high, but is probably not higher than that of a carefully designed single stage pump working against a practically constant head.

GOVERNING OF TURBINES.—Our present knowledge does not permit any very definite statements to be made on this subject. The requirements of a governor are:

(i) Sensitiveness, *i.e.* it should change the position of the regulating apparatus with as small an alteration of the number of revolutions of the shaft as possible.

(ii) Rapidity of action, *i.e.* the governor should be powerful enough to move the regulating apparatus rapidly. This condition must be carefully considered. While an instantaneous opening of the apparatus is not objectional, an instantaneous closure would produce water-hammer, thus the time of closure can with advantage be greater than the time of opening.

(iii) Non-hunting properties, *i.e.* the governor should bring the turbine to its proper speed rapidly, with as few oscillations about this speed as possible.

The following investigation is given by Pfarr (*Turbinen*).

> Let aN represent the horse-power initially developed by the turbine, and let this be suddenly diminished, or increased, to bN ; let aM and bM represent the corresponding turning moments given out by the turbine shaft, in foot-pounds.
>
> Let n_a represent the speed of the shaft when the governor holds the regulating apparatus at the position corresponding to the supply of water required for aN horse-power.
>
> Let n_b be the corresponding speed for bN horse-power. The more sensitive the governor the less $n_a \sim n_b$.
>
> Let T represent the line required to close off or open out the regulating apparatus completely, *i.e.* from $a = 1$ to $b = 0$. T is smaller the more rapid the governor.
>
> Let s represent the time between the alteration of load and the first motion of the governor, *i.e.* s is smaller the more sensitive the governor.
>
> Let I represent the moment of inertia of the shaft and all rigidly attached masses, *i.e.* wheels and dynamo if this is direct driven, about its axis, in lbs.-feet^4.

Then if we assume :

(i) No change in the effective head H by waves or oscillations in the pressure main ;

(ii) That the governing apparatus once in action works at its maximum speed, and shuts off or admits the water uniformly :

> The maximum (for a diminution of load) or the minimum (for an increase of load) speed is given by

$$n_{\max} = n_a + \frac{30\text{M}}{\pi \text{I}}\left\{(a-b)s + (a-b)^2\frac{\text{T}}{2}\right\}$$

$$n_{\min} = n_a - \frac{30\text{M}}{\pi \text{I}}\left\{(b-a)s + (b-a)^2\frac{\text{T}}{2}\right\}$$

and, as already stated, T usually differs for closure and opening.

The assumptions in my opinion are so far removed from the practical condition that the equation is only useful for comparative purposes. In particular, nearly all good governing apparatus do not work uniformly, but at a speed approximately proportional to the change from the required speed n_b. This could be allowed for in the mathematical investigation were it not that most makers cannot supply the values of s, T, n_a or n_b, with any accuracy.

In addition, the formula in no way discloses the hunting possibilities of the governing apparatus. These Pfarr has investigated graphically. So far as I am aware the results do not agree with those obtained by tacheometric studies of the speed oscillations, but the difficulties regarding s, T, etc. will amply explain this.

The present position, therefore, is that the civil engineer must accept makers' guarantees, and is frequently obliged to buy a turbine he would not otherwise select, in order to obtain a reliable governor. His difficulties can be greatly minimised by careful design of the supply mains and providing relief valves.

The best method, however, is the water tower, which I therefore investigate in detail.

SYMBOLS

A suffix notation is employed for β, C, Q, v and y during the arithmetical integration. v_n represents the value of v_n, $10n$ seconds after the change of load occurs, and $\Delta v_n = v_{n+1} - v_n$.

$C = \frac{v}{\sqrt{r_s}}$ is the coefficient of skin friction for the main, C^n (see p. 946).

d, is the diameter of the main in feet.
F, is the area of the cross-section of the main in square feet.
F_w, is the area of the cross-section of the water tower in square feet.
H, is the total head in feet, measured from forebay to the tail water channel of the turbine.
h, is the head in feet, expended in producing a uniform velocity v through the length l of the main.
k (see p. 952).
l, is the length of the main in feet from forebay to water tower.
m, as a suffix indicates a minimax value.
P (see p. 953).
Q, is the quantity of water received by the turbine in cusecs.
$Q_b = Fv_b$. $Q_b = Fv_b$. Q_2 (see p. 954).
R (see p. 954).
t, is the symbol for time; dT, indicates the period of time, usually 10 seconds, during which a change Δv or Δy occurs.
v_b and v_b are the initial and final velocities of the water in the main.
V_1 and V_2 (see p. 954).
x (see p. 954).
y, is the height in feet of the water surface in the water tower above the water level in the forebay.
z_m (see p. 952).
$\beta v^2 = h = v^2 \left(\frac{1}{2g} + \frac{4l}{C^2 d}\right)$ (see p. 946). β_n (see p. 946).

Problem of a Water Tower or Vessel at the Lower End of a long Pressure Main.—Consider a pressure main d, feet in diameter, and l, feet in length, and of an area represented by $F = \frac{\pi d^2}{4}$, which is delivering (*e.g.* to a power station), a quantity of water equal to Q_b, cubic feet per second, where $Fv_b = Q_b$. Suppose (one turbine having been shut off, or set to work) that the demand suddenly changes to $Q_f = Fv_f$. It is quite evident that the acceleration or retardation of a mass of $62{\cdot}5Fl$, pounds of water from velocity v_b, to velocity v_f, will take time, and that there will be a sudden fall, or rise of pressure, producing shocks and irregular running of the turbines.

In order to avoid such irregularities, it is usual to construct a water tower, or some other device close to the lower end of the main, in order to equalise the supply. The change will then occur gradually.

The theory is rather complex, and I propose to develop it, and incidentally to give some idea of the amount of irregularity that still remains uncompensated for.

Let F_w, be the surface area of the equalising device. Let y, be the height of the water surface in this device, measured from the level corresponding to

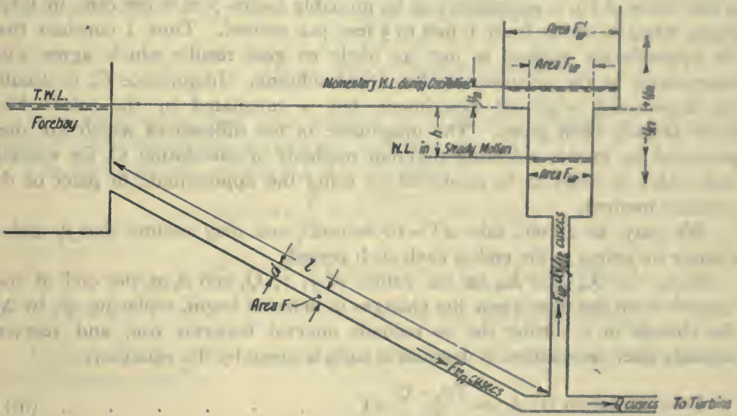
$Q=0$, *i.e.* the level of the water in the forebay at the upper end of the main. Thus, when the velocity, is steady, y, is always negative.

Let v, be the velocity of the water in the main at any time t, after the change in demand occurs.

Then, assuming that a quantity Q, is furnished to the turbines, we have:

$$Q\,dt = F v\,dt - F_w\,dy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (i)$$

and $h=\beta v^2 = v^2\left(\frac{1}{2g}+\frac{4l}{C^2 d}\right)$ is the head required to maintain a uniform velocity v, in the main. The force accelerating the mass of water in the main is consequently represented by $62{\cdot}5F(-y-h)$, since y, is positive when the water surface in the water tower is above the water level in the forebay.



SKETCH No. 265.—Water Tower.

The mass accelerated is: $\frac{62{\cdot}5}{g}Fl$ pounds.

Thus, we get:

$$\frac{dv}{dt}=\frac{g}{l}(-y-h) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (ii)$$

Eliminating v, between equations (i) and (ii), we can get a differential equation for y. The solution of this equation can only be obtained on the assumption that Q, and β, are constant. Now, if $Q=Q_f=$ a constant, the equalising reservoir, or chamber, works satisfactorily. Provided that this can be relied upon, there is no particular reason for an engineer to waste his time on mathematical gymnastics.

Thus, for practical purposes, some process is required which enables us to see what actually takes place, and to determine how much Q (the supply to the turbine) really does vary.

The following approximate process is open to mathematical objections, and, if carelessly handled, may lead to results which differ materially from the truth. The method amounts to tracing a curve on the assumption that a small arc of the curve can always be replaced by its tangent, provided that we calculate the inclination of the tangent at sufficiently frequent intervals. Errors are thus accumulative, but can be sufficiently minimised for practical purposes by shortening the time intervals whenever the tangent is steep, *i.e.* Δy, or Δv, is large. I have calculated several cases which permit of accurate solutions, by the approximate process, and find that errors in the absolute magnitude of y, or v, are not very great (4, to 6 per cent. at most), although the time at which they occur is frequently some 10 per cent. in error. Now, we are really only concerned with the minimax values of y, and the time at which they occur is not highly material. In the accurate solutions, however, we must assume that Q, is constant. As a matter of fact, we really wish to find whether the tower is sufficiently large to cause Q to remain constant. We must also assume that β is constant, which (even if we have experimented on the pipe, and know the actual value of C), is erroneous; as C, probably varies 7 or 8 per cent. in large pipes, when v, varies from 1 foot to 5 feet per second. Thus, I consider that the approximate method is just as likely to give results which agree with experiment, as the accurate mathematical solution. In practice C, is usually not determined by actual experiment, but is calculated by the rules which have already been given. The magnitude of the differences which are then produced by merely selecting different methods of calculating C, far exceeds that which is likely to be produced by using the approximate in place of the accurate method.

We may, as a rule, take $\Delta T = 10$ seconds, and may assume that y, and v, change by jumps at the end of each such period.

If y_n, v_n, Q_n, and β_n, be the values of y, v, Q, and β, at the end of $10n$, seconds from the time when the changes in demand begin, replacing dy, by Δy, the change in y, during the 10 seconds interval between $10n$, and $10n+10$ seconds after the change in demand occurs, is given by the equation:

From equation (i) $\Delta y_n = \frac{Fv_n - Q}{F_w} \Delta T$ . . . . (iii)

From equation (ii) $\Delta v_n = -\frac{g}{l}(y_n + \beta_n v_n^2) \Delta T$ . . . . . . (iv)

and $\left.\begin{array}{l} y_{n+1} = y_n + \Delta y_n \\ v_{n+1} = v_n + \Delta v_n \end{array}\right\}$ are the values of y, and v, at the end of $10n+10$ seconds.

Then, by substituting these values in equations (iii), and (iv), we can obtain y_{n+2}, v_{n+2}, and so work on for as long a period as is requisite.

The calculation of β_n, is obvious:

$\beta_n = \frac{1}{2g} + \frac{4l}{C_n^2 d}$ where C_n is the variable value of C, appropriate to d, and v_n,

and if the entry to the pipe is not bell-mouthed, the term $\frac{1}{2g}$, may become $\frac{1 \cdot 5}{2g}$, as already discussed (see p. 414).

It will also be plain that if $h = \kappa v^n$, better represents the friction equation of the main, this form can be introduced into the calculations.

The method will be rendered clearer by the following example, which is founded on the design for the Baden State Railways Power Scheme in the Murgtal (*Die Wasserkraft Anlage im Murgtal*, 1910). Slight alterations have purposely been made, as the original object of the work was to estimate the effect of such variations by comparison with the results given in the treatise referred to.

The supply main is of the following dimensions:

Length $l=10{,}040$ feet. Cross-section $F=88{\cdot}23$ square feet. Cross-section of the water tower, $F_w=1217{\cdot}5$ square feet. The full supply $Q_b=494$ cusecs, or $v_b=5{\cdot}61$ feet per second. The power house is suddenly shut down, so that $Q_f=0$, and $v_f=0$.

The value of β, is given as 0·709, or $y_b=-22{\cdot}30$ feet.

The time of vibration of the system (when unaffected by friction) is: $\pi\sqrt{\frac{l}{g}\frac{F_w}{F}}$. Or in this case:

$$\pi\sqrt{\frac{10040}{32{\cdot}2}\,\frac{1217{\cdot}5}{88{\cdot}23}}=210 \text{ seconds.}$$

According to Johnson (*Proc. Am. Inst. of Mech. Engr.*, vol. 30, p. 442), the time as influenced by friction is not very far removed from:

$$\pi\sqrt{\frac{l}{g}\frac{F_w}{F}+\frac{F_w^2}{F^2}\beta^2(v_b+v_f)^2}=272 \text{ seconds.}$$

It is therefore evident that intervals of 10 seconds should be sufficiently close to introduce no very great error in the results.

The equations are, putting $\Delta T=10$ seconds:

$$\Delta y_n=10\times\frac{F}{F_w}v_n=\frac{882{\cdot}3}{1217{\cdot}5}v_n=0{\cdot}725v_n$$

$$\Delta v_n=-\frac{10g}{l}(y_n+\beta v_n^2)=-\frac{32{\cdot}2}{1004}(y_n+0{\cdot}709v_n^2)$$

$$=-0{\cdot}0321y_n-0{\cdot}0228v_n^2.$$

The actual calculation requires great care, and is best effected by means of a table ruled as below:

Time	y feet	Δy feet	$0{\cdot}725v$	v feet p. s.	Δv feet p. s.	$0{\cdot}0321y$	$0{\cdot}0228v^2$
0	−22·30		4·07	5·61		0·716	0·716
		4·07			0		
10	−18·23		4·07	5·61		0·585	0·716
		4·07			−0·131		
20	−14·16			5·48			

In the first line, the given values of y_0, and v_0 (*i.e.* y_b and v_b), are inserted in columns 2 and 5. Column 4 is obtained by multiplying v_0 by 0·725. Since $Q_f=0$, there is no term in Δy of the form $-\frac{Q_f}{F_w}$, so that column 4 is not needed in this case, and the result $\Delta y=4{\cdot}07$, might be written at once in column 3 of line 2.

Columns 7 and 8 of line 1 are now filled in as shown.

The determination of the signs of these terms is difficult. The term in v^2, always has a sign opposite to that of v, and the term in y (as shown by the equation for Δy), is positive if v, and y, have different signs, and negative if they have the same sign. Thus, in this particular case $\Delta v_0=0$, and line 2 is filled in with $\Delta y_0=4{\cdot}07$, $\Delta v_0=0$.

T	y feet	Δy	v feet per Sec.	Δv	Remarks
0	−22·30	4·07	5·61	0	
10	−18·23	4·07	5·61	−0·136	
20	−14·16	3·96	5·47	−0·227	The signs of the terms of Δv are: $0·0228v^2$ negative, $0·0321y$ positive.
30	−10·20	3·81	5·25	−0·302	
40	− 6·39	3·59	4·95	−0·353	
50	− 2·80	2·81	4·60	−0·330	
58·4	0·0	0·28	4·27	−0·060	
60	+ 0·28	1·98	4·21	−0·404	Change of section occurs.
70	+ 2·26	1·77	3·81	−0·404	
80	+ 4·03	1·58	3·41	−0·394	
90	+ 5·61	1·40	3·02	−0·388	
100	+ 7·01	1·22	2·63	−0·382	
110	+ 8·23	1·02	2·25	−0·375	The term $0·0228v^2$ is negative, so is the term $0·0321y$.
120	+ 9·25	0·87	1·88	−0·377	
130	+10·12	0·70	1·50	−0·376	
140	+10·82	0·52	1·12	−0·375	
150	+11·34	+0·35	0·75	−0·372	
160	+11·69	+0·18	0·38	−0·379	
170	+11·87	0·0	0·0	−0·380	
180	+11·87	−0·18	−0·38	−0·376	The term $0·0228v^2$ becomes positive, since v is now negative, the term $0·0321y$ remains negative.
190	+11·69	−0·35	−0·76	−0·362	
200	+11·34	−0·52	−1·12	−0·336	
210	+10·82	−0·68	−1·46	−0·298	
220	+10·14	−0·82	−1·76	−0·255	
230	+ 9·32	−0·94	−2·02	−0·202	
240	+ 8·38	−1·03	−2·22	−0·157	
250	+ 7·35	−1·10	−2·38	−0·107	
260	+ 6·25	−1·15	−2·48	−0·061	
270	+ 5·10	−1·18	−2·54	−0·017	
280	+ 3·92	−1·19	−2·56	+0·023	
290	+ 2·73	−1·18	−2·54	+0·061	
300	+ 1·55	−1·16	−2·48	+0·094	
310	+ 0·39	−0·39	−2·39	+0·040	For 3·5 seconds.
313·5	0·0	−1·11	−2·35	+0·080	For 6·5 seconds.
320	− 1·11	−1·64	−2·27	+0·153	Change of section occurs as before.
330	− 2·75	−1·54	−2·12	+0·190	
340	− 4·29	−1·40	−1·93	+0·223	The term $0·0228v^2$ is still positive, but $0·0321y$ is also positive, since y has changed sign.
350	− 5·69	−1·24	−1·71	+0·249	
360	− 6·93	−1·06	−1·46	+0·273	
370	− 7·99	−0·86	−1·19	+0·288	
380	− 8·85	−0·66	−0·90	+0·306	
390	− 9·51	−0·43	−0·59	+0·313	
400	− 9·94	−0·20	−0·28	+0·321	
410	−10·14	+0·03	+0·04	...	The term $0·0228v^2$ becomes negative, since v is positive.
420	−10·11	...	...	...	

The calculations were made with a 50 cm. slide rule, and the third place figures in Δv are plainly liable to error. The agreement with calculations effected on an arithmometer is, however, very close, and the slide rule appears to be sufficiently accurate for practical purposes.

The columns headed y, and v, can now be filled in in line 3, which refers to a time 10 seconds after the alteration in demand.

From the above values we can calculate the entries in columns 4, 7, and 8, of line 3, and, determining the signs as already indicated, we fill in columns 3 and 6 of line 4 with

$$\Delta y_1 = 4{\cdot}07, \qquad \text{and } \Delta v_1 = -0{\cdot}131.$$

For 20 seconds (line 5) we consequently find that :

$$y_2 = -14{\cdot}16 \qquad \text{and } v_2 = 5{\cdot}48$$

and the work can proceed as far as is required.

In the actual problem the water tower is stepped, and its area for positive values of y, is $F'_{10} = 1901$. Consequently, when y, is positive, we have $\Delta y = 0{\cdot}464v$. The change occurs at $T = 58{\cdot}4$ seconds, and again at $313{\cdot}5$ seconds. The table shows the solution for the first 410 seconds, and the maximum and minimum values of y, and v, are found to be as follows :

Maximum of $y = +11{\cdot}87$ feet when $T = 180$ seconds.
Minimum of $y = -10{\cdot}14$ feet when $T = 410$ seconds.
Maximum of $v = +5{\cdot}61$ feet per second when $T = 0$ seconds.
Minimum of $v = -2{\cdot}56$ „ „ „ 280 seconds.

The case might be further investigated, but it is fairly plain that no greater oscillations can occur.

Let us now consider a case where the demand is suddenly increased from 247 cusecs to 494 cusecs. We have :

$$-y_0 = \frac{22{\cdot}3}{4} = 5{\cdot}57 \text{ feet} \qquad v_0 = 2{\cdot}80 \text{ feet per second.}$$

Assuming that 494 cusecs is actually delivered to the turbine by the combined contributions of the pipe and the water tower, the equations for 10 seconds interval are as follows :

$$\Delta y_n = -4{\cdot}05 + 0{\cdot}725v_n$$
$$\Delta v_n = -0{\cdot}0312y_n - 0{\cdot}0228v_n^2$$

The sample table for the first 40 seconds is as follows :

T	y	$0{\cdot}725v_n$	Δy_n	v_n	Δv_n	$0{\cdot}0321y_n$	$0{\cdot}0228v_n^2$
0	−5·57	2·03		2·80			
			−2·02		0		
10	−7·60			2·80		0·244	0·179
			−2·02		+0·065		
20	−9·62	2·08		2·87		0·309	0·188
			−1·97		+0·121		
30	−11·59	2·17		2·99		0·372	0·204
			−1·88		+0·168		
40	−13·47	2·29		3·16		0·432	0·228
			−1·76		+0·204		

Thereafter the tabulation is :

T	y	Δy	v	Δy
50	−15·23	−1·62	3·36	0·232
60	−16·85	−1·45	3·59	0·247
70	−18·30	−1·27	3·84	0·251
80	−19·57	−1·09	4·09	0·247
90	−20·66	−0·90	4·34	0·234
100	−21·56	−0·73	4·57	0·218
110	−22·29	−0·57	4·79	0·193
120	−22·86	−0·44	4·98	0·168
130	−23·30	−0·32	5·15	0·144
140	−23·62	−0·22	5·29	0·120
150	−23·84	−0·13	5·41	0·095
160	−23·97	−0·06	5·50	0·080
170	−24·03	−0·03	5·58	0·068
180	−24·06	+0·05	5·65	0·051
190	−24·01	+0·08	5·70	0·031
200	−23·93	+0·10	5·73	0·021
210	−23·83	+0·12	5·75	0·011
220	−23·71	+0·13	5·76	0·004
230	−23·58	+0·13	5·76	−0·002

In view of the small difference between these values, and the steady motion values $y = -22·3$ feet, $v = 5·60$ feet per second, it is unnecessary to carry the work any further.

The total head in the case to which these examples refer exceeds 300 feet, and the maximum oscillation being only about 33 feet (Example No. 1) the variation in the head is only 11 per cent. Thus, even if the governor did not move during the first 180 seconds, the variation in Q (the quantity passed on to the turbine) would at the worst be but 5 per cent., so that we may consider that this regulation is very satisfactory.

Let us, however, assume that the water tower is not widened out at $y=0$, but has a cross-section of 1217·5 square feet all the way up. Let us also assume that the regulation is by hand, and that when the full demand corresponding to a load represented by $v_b = 5·60$ feet per second ceases, the admission valve of the turbine is adjusted so as to pass only 49·4 cusecs, under an effective head of 287·7 feet. We find that the maximum effective head is 317·8 feet, and that the minimum head is 272·3 feet. The maximum delivery to the turbine is consequently $49·4\sqrt{\frac{318}{288}} = 52·0$ cusecs, and the minimum is 48·4 cusecs.

This variation is sufficient to materially influence the calculation, and in such cases a 9th and 10th column should be introduced in the arithmetical work, in which we calculate Q, from the formula :

$$Q = Q_f\sqrt{\frac{H+y}{H}}$$

Then, the equation for Δy, in place of being :

$$\Delta y = \left\{ \frac{F}{F_w} v - \frac{Q_f}{F_w} \right\} \Delta T$$

where $\frac{Q_f}{F_w}$, is a constant, is represented by

$$\Delta y = \left\{ \frac{F}{F_w} v - \frac{Q_f}{F_w} \sqrt{\frac{H+y}{H}} \right\} \Delta T$$

The question is of most importance when H, is small (say 50 to 60 feet). In such cases, it is usually an easy matter to make the ratio $\frac{F_w}{F}$, large, so that y, is only a relatively small fraction of H.

Returning to the general problem. It will be plain that the process given above permits us to determine the rise and fall of the water level in the water tower, or surge tank, when F_w, is determined. The problem met with in practice, however, is more usually as follows. We assume a certain value for $v_b - v_f$, corresponding to the fraction of the total load of the power station that is likely to be suddenly thrown off, or put on, say, $v_b - v_f = kv_m$, where v_m, is the velocity corresponding to full load, and k, is a fraction determined by the variability of the load.

The practical problem for a given k, is then to determine F_w, so that the extreme oscillations of the water level may be such as to produce the cheapest solution.

The question has been very carefully investigated by Johnson (*Trans. Am. Soc. of Mech. Eng.*, vols. 30 and 31). Certain very useful equations are also given by Johnson, Harza, and Larner, in the paper and its discussion. Having very carefully compared these with the results of the arithmetical methods developed above, I consider that Harza's methods are most suitable for practical design. Harza considers the equations :

$$\frac{dv}{dt} = -\frac{g}{l}(y + \beta v^2), \quad \text{and} \quad \frac{dy}{dt} = \frac{Fv - Q}{F_w}$$

and in addition assumes that the governor of the turbine acts instantaneously, so as to keep the power delivered to the turbines constant, and thus arrives at a third equation :

$$Q(H - y) = Q_f(H - y_f)$$

where y_f, is the value corresponding to a steady velocity v_f, and H, is the total head measured from the forebay above the main to tail-water below the turbine.

An accurate solution of these three equations is impossible, and Harza's method of attacking the problem is not in accordance with the mathematical process of successive approximation. He obtains approximate solutions by neglecting the friction term (βv^2), and the governor motion (*i.e.* the third of the above equations). These solutions are used later on for substitution in certain terms of the accurate equations, but these substitutions are not effected in a logical manner.

A very careful examination of the method leads me to believe that it is satisfactory, provided that βv^2, does not exceed $0{\cdot}10$H, or $0{\cdot}15$H. If, however, βv^2 (where v, represents the greater of the two velocities) be a large fraction of

H, the development is illogical. This opinion is justified by the fact that cases can be constructed where Harza's equations lead to results which are hopelessly erroneous. In such examples $\frac{\beta v^2}{H}$, is always a large fraction.

Harza's equation is consequently applicable to all practical cases. In view of our present small experience of the problems concerning water towers, the final results must be checked by the arithmetical process already given. This checking is the more necessary, the greater the value of $\frac{\beta v^2}{H}$.

Subject to these remarks, let us assume that:

y_m, is the first minimax value of y, *i.e.* the value of y, at the crest of the first surge (in the case of a decrease in load), or at the bottom of the first suck down (in the case of an increase in load) that occurs after the change of demand.

Let z_m, be the alteration in water level produced by this first surge, or first suck down. So that:

$$z_m = y_m + \beta v_b^2 \text{ for a surge,}$$
$$z_m = -y_m - \beta v_b^2 \text{ for a suck down.}$$

Then, Harza (*Trans. Am. Soc. of Mech. Eng.*, vol. 30, p. 478), gives the following:

$$z_m^2 - 2Hz_m = -\frac{2F}{F_w}\left\{\frac{l}{2g}(v_b^2 \backsim v_f^2) + H\sqrt{\frac{l}{g}\frac{F_w}{F}}(v_b \backsim v_f)\right\}$$

where (as is indicated by the sign $\backsim$), the right-hand side of the equation is always to be made negative. That is to say:

$v_b^2 - v_f^2$, and $v_b - v_f$, are to be taken if v_b, be greater than v_f,

and $v_f^2 - v_b^2$, and $v_f - v_b$, if v_f, be greater than v_b.

Now, this equation can be used to determine the extreme oscillations of the water surface in the water tower, as follows:

(i) Ascertain the greatest height to which the water rises in the water tower by assuming that the water in the forebay is at its maximum level, and considering a fraction k of the load as shut off. Thus, take v_b, successively equal to v_m, $0{\cdot}9v_m$, $0{\cdot}8v_m$, etc., and v_f, therefore, as equal to $(1-k)v_m$, $(0{\cdot}9-k)v_m$, etc., and $v_f = 0$, when v_b, is equal to, or less than kv_m. Determine the values of z_m, the first surge up for each case, and the absolute height of the water level which is then attained from the equation: $y_m = z_m - \beta v_b^2$.

It is an easy matter to determine the absolute maximum of y_m.

(ii) Take the water level in the forebay at its lowest possible, and similarly calculate the sucks down, and the minimum absolute water level. The only change is that the power is now switched on, instead of being cut off, so that v_f, is greater than v_b. Thus, e, the total possible oscillation of the water level produced in a tower with an area equal to F_w, is determined.

The cost of a tower of an area F_w, and a height e, can be estimated, and a new area F_w, may be assumed, and the new e, determined. Consequently, the dimensions of the most advantageous tower can be selected, and more accurate values of y_m, may be determined for this tower only by the arithmetical process.

As an example, take:

$H = 50$ feet, $l = 500$ feet, $F = 50{\cdot}3$ square feet, $F_w = 402{\cdot}4$ square feet, and

$v_m = 7$ feet per second, with $kv'_m = 3$ feet per second, and $\beta = 0{\cdot}1$. Consider the case of a shut down. We have:

$$z^2 - 100z = -1{\cdot}94\{v_b^2 - (v_b - 3)^2\} - 139(v_b - v_b + 3) = -P$$

say, where $v_b - 3$, is never negative.

The tabulation is:

v_b Feet per Second	P	z_m	βv_b^2	$y_m = z_m - \beta v_b^2$
7	481	5·07	− 4·9	+ 0·17
6	469	4·93	− 3·6	+ 1·33
5	458	4·81	− 2·5	+ 2·31
4	446	4·68	− 1·6	+ 3·08
3	434	4·45	− 0·9	+ 3·55
2	286	2·96	− 0·4	+ 2·56
1	141	1·47	− 0·1	+ 1·37

Thus, the maximum possible level of the water is about 3·55 feet above that in the forebay, and occurs when the load is completely shut off and three-sevenths of the power was previously being delivered.

If, on the other hand, we assume a complete shut down, *i.e.* $v_b = 7$, and $v_f = 0$, we get:

$$z = 12{\cdot}15 \text{ feet}, \quad \text{and } y = 7{\cdot}25 \text{ feet}$$

The amount of the suck downs produced by sudden increases in demand is calculated in the same way:

v_b	P	z_m	βv_b^2	y_m
0	434	4·45	0	− 4·45
1	446	4·68	0·1	− 4·78
2	458	4·81	0·4	− 5·21
3	469	4·93	0·9	− 5·83
4	481	3·07	1·6	− 6·67
5	326	3·38	2·5	- 5·88
6	164	1·67	3·6	− 5·27

So that here the interval near $v_b = 4$ feet per sec., must be examined more closely.

According to Larner (*Trans. Am. Soc. of Mech. Eng.*, vol. 31, p. 117), the value of y_{max} obtained by Harza's equation may differ as much as 19, or even 27 per cent. from the truth, and is usually less (on the average 5 per cent. less) than the truth. It therefore becomes necessary to find some method of obtaining a value which z, does not exceed.

This is done as follows:

Calculate the quantity:

$$V_1=\frac{v_f(H-\beta v_f^2)}{H-\beta v_b^2-z_m}$$

where z_m, is derived from Harza's equation, and v_f, is assumed to be greater than v_b.

Then, Johnson and Larner (*ut supra*) find that:

$$z_1^2-\frac{l}{g}\frac{F}{F_w}(V_1-v_b)^2-\beta^2(V_1^2-v_b^2)^2$$

is always greater (on an average 12 per cent. greater) than z_m as obtained by the arithmetical method.

If v_f, be substituted for V_1, we obtain a value z_2, which is always less than z_m.

Larner has further developed the matter, and shows that if we substitute $V_2=v_b-R(V_1-v_b)$ for V_1, in the above equation, we get a value of z, which rarely departs more than 3 per cent. from the truth. R, is determined as follows;

$$\text{Put } x=\frac{Fl}{F_w 1000}$$

$$\text{Then,} \quad x^2-60x+7023{\cdot}7\,(R-R^2)=-1872$$

These last three expressions are purely empirical, and consequently have no such claims to reliability as Harza's rules.

In actual practice, k, the fraction of the load which is suddenly shut off or switched on, varies from 0·05 to 0·20 in American examples. The Murgtal towers, on the other hand, are designed for $k=0{\cdot}50$, when the load increases, and $k=1{\cdot}00$ in the case of a shut down.

Differential Water Tower.—On referring to the first of the tabulated examples, it will be noticed that the oscillations have been considerably reduced by the widening which occurs at the level $y=0$.

Mathematically speaking, when $y=0$, $\frac{dy}{dt}$, or $\frac{\Delta y}{\Delta T}$, is materially reduced, and in the case under consideration $y=0$, happens to be the equilibrium value of y.

The principle thus disclosed may be applied in a more general manner by feeding the water tower with a properly adjusted auxiliary supply equal to Q_2, cusecs. We then have the following equations:

$$\frac{dy}{dt}=\frac{Fv}{F_w}-\frac{Q_f+Q_2}{F_w}$$

$$\frac{dv}{dt}=-\frac{g}{l}(y+\beta v^2)$$

Now, if we could adjust Q_2 so that when $y=y_f$, $\frac{dy}{dt}=0$ and $\frac{dv}{dt}=0$, simultaneously, the oscillation would end at once; for both v, and y, would then reach their equilibrium values, and, being momentarily steady, would remain so.

The above adjustment, however, cannot be effected, and the best that can be done is to endeavour to make $\frac{dy}{dt}$ small, when $\frac{dv}{dt}$, is also small. That is to say, the first time that y, is close to $-\beta v_f^2$, $\frac{dy}{dt}$ or $\frac{\Delta y}{\Delta T}$ should be made as small as possible.

Johnson (*ut supra*) has endeavoured to apply this principle. He separates the water tower into two portions, as follows. One, the ordinary water tower communicating directly with the pipe, and the other a riser communicating with the ordinary tower by means of constricted orifices. These ports, or orifices, are designed so that they can deliver a quantity of water represented by :

$$Q_2 = F(v_f - v_0) \text{ cusecs}$$

under a head x_m, which is the maximum alteration in level of the water in the ordinary water tower (*i.e.* is equivalent to z_m, in Harza's approximate theory). This he assumes as occurring when $v = v_f$.

So far as I understand the matter, Johnson's mathematical assumptions are incorrect, although the equations are correct if the assumptions be true. His actual practice, however, is founded on the results of arithmetical work, and is consequently far less likely to be erroneous.

The practical development is fairly obvious. Consider the second example (p. 949):

When $y = -22{\cdot}3$ feet, v, the velocity in the pipe, is only 4·79 feet per second, and in consequence y, continues to decrease. Thus, when v, has its correct value, y, is too great negatively, and v, and y, continue to recede from their equilibrium values. Let us, however, assume that when $y = -22{\cdot}3$ feet $= y_f$, a properly adjusted quantity of water is delivered so as to keep y steady. Then, when v, attains the value v_f, y, still retains the value appropriate to steady motion with $v = v_f$, and equilibrium is secured without any further oscillations.

The mathematical development of the motion under these assumptions is as follows :

$$\frac{dv}{dt} = -\frac{g}{l}(y + \beta v^2) = \frac{g}{l}\beta(v_F^2 - v^2)$$

$$\frac{dy}{dt} = \frac{Fv - Q_F + Q_2}{F_w} = 0, \text{ so that } Q_2 = F(v \ -v)$$

since y, is assumed to remain constant, and equal to βv_F^2.

$$\text{Hence, } \frac{2v_F g\beta}{l} t = \log_e \frac{v_F + v}{v_F - v} + \text{a constant}$$

and the total quantity of water that must be supplied to the tower from the riser is given by the equation :

$$A = \int Q_2 dt = \frac{Fl}{g\beta}\int \frac{dv}{v_F + v} = \frac{Fl}{\beta g} \log_e (v_F + v)$$

The limits for both integrations are given by :
$v = v_x$, and $v = v_F$, where v_x, is the value of v, when y, first becomes equal to v_F, e.g. $v_x = 4{\cdot}79$ feet per second in the example above referred to.

Thus, the time before equilibrium is actually attained is theoretically infinite. In practice, however, we cannot arrange so as to deliver the variable quantity Q_2. We must therefore select the value of y, which differs slightly from $-\beta v_F^2$, say :

$$y_f = -\beta v_F^2 + \text{one foot, say}$$
$$= -\beta u_F^2, \text{ say.}$$

We can then calculate T, the time between the limits $v=v_b$, and $v=u_F$ and A, the corresponding total quantity of water which is delivered. We can thus arrive at the size of the riser chamber, and estimate the head under which the delivery orifices work, which will be slightly less than $\beta(u_F^2-v_b^2)$. The size of these ports can thus be determined, and a preliminary design can be arrived at for the riser. The exact circumstances of the motion can then be arithmetically investigated, the levels in the water tower and in the riser being determined, and the accurate value of Q_2, calculated for intervals of say ten seconds. The final design can only be arrived at by ascertaining the circumstances in this manner for the cases which produce the greatest oscillations in the water level in the tower. These are probably :

(i) The rejection of all the load, when the load is k, of the full load.

(ii) The switching on of a similar fraction of the load when $(1-k)$ of the full load is already on.

The practical results obtained by this method have not as yet been published. Johnson speaks very highly of its efficiency, and appears to consider it advisable in all cases. He also states that F_w, can be reduced to at least one half of the required area in an ordinary water tower. Trial calculations of my own confirm this statement, but they also indicate that the oscillations, while never so great as in an ordinary water tower, are by no means so small as is assumed in the mathematical theory. For this reason I do not give any examples, and I consider that the mathematical theory is merely a rough approximation. More exact rules for the preliminary design of the riser, are greatly to be desired, for at present its proper proportioning is so laborious as to render the principle almost useless.

CHAPTER XVI

CONCRETE, IRONWORK AND ALLIED HYDRAULIC CONSTRUCTION

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General Design.—Deflection of metallic structures—Rules for Design—Strength of structures—Working stresses—Bearing pressures—Roller bearings—Ball bearings, not advisable.
Special Cases.—Worm gearing—Stanching rods—Plating—Rules for joint rivetting—Deflection and strength—Deflection of a trussed frame.
Water Towers.—General investigation of stresses in the external plating—Supporting girder.

CONCRETE.—The following discussion is almost entirely concerned with impermeable concrete. The question of obtaining the strongest concrete (under either tensile or compressive stresses) does not in reality greatly concern the hydraulic engineer. So far as our knowledge goes, the mixture which produces the most impermeable concrete does not greatly differ from that which produces the strongest concrete. If in any particular case the difference should prove to be marked, it is doubtful whether the adoption of the strongest mixture can be justified, for it is uncertain whether concrete which is markedly permeable by water will not rapidly lose strength through the removal of the cementing material by solution in the percolating water.

I do not propose to enter into such questions as the manufacture, composition, or specification of Portland cement. These matters have now been reduced to standards, and, except in the case of very large orders, an engineer cannot (at any reasonable price) force his own particular ideas on the manufacturer. Speaking as one who saw a great deal of the final results of the older method, where the consultant specified, and the manufacturers produced a more or less close approximation, I regard the present practice as more likely to give good results, even when (as was the case in several works I was employed on) the consultant had a thorough practical knowledge of the manufacture of cement, and knew both what he really required and how to manufacture it. When contrasted with the usual circumstances of a "scissors and paste" specification, accompanied by hearsay knowledge of manufacturing processes, there can be no comparison whatsoever.

The matters which the engineer should control are the treatment of cement after its reception, the proportioning of the quantities of cement, sand, and gravel that go to form the concrete, and the specification and enforcement of the processes comprised in the term "mixing of concrete."

Treatment of Cement.—This depends greatly on the quality of the cement. The cements of the period 1890–1902, contained "particles of free lime," or at any rate were improved by being exposed (under cover from rain), in layers 6 inches to 9 inches deep, and turned over twice or thrice in a period of a month to six weeks. This process was termed "air slaking," and is still practised in many cases. I believe that air slaking is less necessary now that cements are ground so much more finely than in former years. It is also extremely doubtful whether air slaking may not be injurious to a very finely ground rotary kiln cement.

At present the practical effect is that air slaking should be considered for cement which is made close to the place where it is used, and the advice of the makers (or better still, of the analyst and tester, if such are retained) should be taken, and confirmed by tests of its expansion after setting. If cement is used after a sea voyage (imported cement) air slaking is not usually necessary, but samples should be tested. I may state that I have had experience of three cases of sea-borne cement which appeared to have been deteriorated by air slaking. Each sample, however, was the product of a newly started rotary kiln, and such occurrences are consequently less probable nowadays. The whole question has been very carefully investigated by Bamber (*P.I.C.E.*, vol. 183, p. 85), and it would appear that the above opinions concerning the inutility of air slaking are, if anything, not sufficiently severe. If tensile tests alone are relied upon, the process always appears useless, and is sometimes detrimental. So far as my experience goes, modern cements, when they fail to conform to the Institution of Civil Engineers or British Standard Specification, usually fail only by not being sufficiently finely ground, and this test should always be applied first.

Proportioning of Concrete.—Concrete consists of Portland cement, sand, and aggregate, by which last term is meant the larger non-cementing material, stones, broken bricks, gravel, cinders, slag, etc.

We may consider sand as comprising the non-cementing material that passes a sieve of four or eight meshes per lineal inch, according to the quality of the available raw material. Aggregate is the material which is larger than this size.

In practice, it is frequently found that sand and aggregate occur in Nature mixed together, and it is more convenient to make the concrete by adding cement to the unseparated mixture. In the following discussions of the proportioning of concrete mixtures, I therefore use the term sand for whatever the engineers propose to use as sand, and aggregate for whatever it is proposed to use as aggregate. A very varied experience has led me to believe that there are very few, if any, circumstances where the ratio of the cost of cement to that of properly separating the materials, is such that a considerable economy in cost cannot be obtained by separating the raw excavated materials and carefully proportioning the mixture. The only exceptions are cases where it is desired to fill a small cavity with firm material, very weak concrete (made with material excavated from the cavity) being used; and, even under such conditions, it is only the cost of bringing the sieves and grading apparatus to the site that turns the scale.

The correct method of proportioning concrete entirely depends upon the voids existing in the sand, the aggregate, and the cement itself. These are best determined as follows:

(*a*) *Cement.*—Take a measured bulk of cement, mix with water so as to form a paste, and measure the bulk of the paste. This will usually be between 0·80, and 0·90 of the bulk of the cement. I propose to assume 0·85 as a mean value. In practice, the engineer should consider whether he proposes to employ a very wet or very dry mixture for concrete, and should proportion the amount of water added accordingly. He should remember that if a very wet concrete mixture is employed, the figure obtained for the bulk of mortar in a small scale experiment will probably be slightly below the truth, when contrasted with that obtained on the quantity of cement used in making a batch of concrete. For example, in my own experiments I have obtained 0·84, using 1000 c.c. (say 60 cube inches) of cement, and afterwards found from 0·85 to 0·86 when using 7 cube feet of cement.

The following table shows the effect of varying percentages of water on the final volume of the resulting compacted paste, as found by Fuller and Thompson (*Trans. Am. Soc. of C.E.*, vol. 59, p. 67) in small scale experiments, on 300 grammes (say 12 cube inches) of pure cement:

Percentage of Water mixed with the Cement.	Percentage $\frac{\text{Volume of Paste}}{\text{Volume of dry Cement Powder}}$.
20	87
23	90
26	92
32	96
50	119
100	114

Thus, in small scale experiments, the wetness of the paste has a considerable influence on the results obtained. The effect is less marked in practical trials on a large scale, since the permissible variations in the percentage of water are not so marked, but it must nevertheless be allowed for.

It is also advisable to note that if small measuring vessels, such as test tubes, are used, the cement or sand may pack abnormally, and appear more bulky than when tested in larger vessels. It is therefore as well to work with at least 50 cube inches of material, and to use vessels at least 3 inches in diameter. These difficulties are avoided by working by weight (as Fuller and Thompson did) and not by volume. I recommend volume working for the trial proportioning, simply because the cement, sand, and aggregate, will be measured by volume when the concrete mixture is deposited. Thus, in practice, it is advisable to weigh a cube foot each of cement, sand, and aggregate, before determining the voids, so that any error caused by abnormal packing can be detected.

(*b*) *Sand.*—Take the sand as it will be used (not artificially dried), pour a known bulk into a water-tight vessel, and fill in water from a graduated glass, until the water just appears over the top of the sand. The shrinkage of the sand which occurs in wetting should be disregarded, and the volume of water used should be taken as the voids in the original bulk. The figure obtained varies considerably. I have found figures as low as 0·23 on extremely fine and somewhat dirty sands, such as occur in the Punjab. Sands freshly dredged from the sea, or from rivers (where the finer particles have been removed by the action of currents), show figures as high as 0·48 or 0·50, the usual value being between 0·35 and 0·45. I propose to assume 0·40 as an average.

(*c*) *Aggregate.*—A sand of which the individual grains are markedly porous should be considered as unfit for making good concrete. Porous aggregates are often employed, but I doubt whether they ever make really first class impermeable concrete. Their use, however, is quite justified in circumstances where bulk, rather than strength, is desirable, (*e.g.* for partition walls, or for filling up cavities where percolation can be disregarded). Therefore, if the aggregate is porous, it should first be wetted and allowed to absorb all the water possible; then freed from the visible water, and tested for voids, just as the sand was. Aggregate being an artificial material (in a sense that sand is not), the voids are even more variable than in the case of sand, and the method of filling employed in practice should be closely followed. For example, a difference of 3, or 4 per cent. in the voids may be caused merely by carting over a rough road, or by carriage by rail. What we wish to ascertain is the voids as they exist in the aggregate, when measured by the workmen in preparing the concrete. I assume 0·35 for calculations.

A very close approximation to the mixture which produces the densest, and therefore the least permeable concrete, can be obtained by the following method.

The cement paste should fill the voids in the sand, and the mortar thus obtained should fill the voids in the aggregate. As a rule, an excess of 10 per cent. is allowed in each case, in order to compensate for irregularities in mixing.

Thus, take the assumed 0·40 of voids in sand. We require $0{\cdot}40(1{\cdot}10) = 0{\cdot}44$ of cement paste. Thus, 1 part of cement (producing 0·85 parts of paste) fills the voids in $\frac{0{\cdot}85}{0{\cdot}44} = 1{\cdot}93$, say 2 parts, of sand, and presumably makes $2(1 - 0{\cdot}40) + 0{\cdot}85 = 2{\cdot}05$ parts of mortar.

The voids in 1 part of aggregate are 0·35, or, adding 10 per cent., 0·38, so that 2·05 parts of mortar fill the voids in $\frac{2\cdot05}{0\cdot38} = 5\cdot04$, say 5 parts of aggregate.

The proportion which just produces "no voids" is, therefore:

1 cement : 1·93 sand : 5·04 aggregate;

or close enough to 1 cement : 2 sand : 5 aggregate.

The following figures indicate the bulk of the ingredients, and of the wet concrete when proportioned by these principles. The cement passed a specification of not more than 7 per cent. residue, on a sieve of 5776 holes per square inch, and passed entirely through a sieve of 1600 holes per square inch. The aggregate and sand were procured from good Thames ballast by sifting through a sieve of 4 meshes per linear inch, *i.e.* say 0·15 inch size. All above being aggregate, and all below, sand.

Proportioning .	Cement.	Sand.	Cube Feet. Aggregate.	Total.	Wet Concrete.
Excess of cement	9·37	10·64	28·00	48·01	36·36
Properly proportioned	7	11·10	28·40	46·50	32·30
	7	10·50	28·00	45·50	32·25
	7	10·00	28·00	45·00	30·80
	7	10·80	28·00	45·80	34·10
	7	10·80	28·00	45·80	33·80
	7	10·80	28·00	45·80	34·06
Deficiency of cement .	7	13·3	35·90	56·2	41·2
	3·5	10·8	28·00	42·3	33·3

The contrast between a properly proportioned mixture, and one containing an excess or deficiency of cement, or sand, is very marked. In the first class the total volume of wet concrete exceeds that of the aggregate by about 17 per cent. on the average, while in the others the excess is rarely less than 20 per cent. The contrast would be even more marked were it not that the volume of sand in each of these mixtures had been experimentally proportioned so as to secure as small a percentage of voids as was possible consistent with the general ratio of cement to aggregate. Also, the last three cases of the properly proportioned concrete are in reality somewhat misleading, since the properties of the sand had changed to such a degree as to necessitate a slight alteration in the proportions. In practice, the new proportions were ordered wherever 28 cube feet of aggregate (with the sand and cement) produced a volume of wet concrete in excess of 33·60 cube feet, (*i.e.* 20 per cent. excess). The figures, however, show the results attained under somewhat careful supervision, and are therefore more useful than an enumeration of laboratory tests.

The following figures are the average results of some 100 large scale tests on "bankers," of roughly 1 cube yard capacity. Since they include many cases where the mixtures are known to be only roughly proportioned for minimum voids, it is desirable to check them at the first opportunity. I find the figures useful in preliminary estimates, and if confined to such purposes they will not prove misleading.

A Mixture by Volume of			Produces Set Concrete Parts.
Cement Parts.	Sand Parts.	Aggregate Parts.	
1	1½	4	4·6
1	2	4	4·5
1	3	6	6·6
1	4	8	8·9
1	5	10	11·25

The method seems to deserve careful criticism. In the first place, the experience upon which it was founded is largely based on cement which would now be considered as extremely coarsely ground. Thus, in 1880, Grant (*P.I.C.E.*, vol. 62, p. 101), states that 15 to 27 per cent. of good English Portland cement was retained in a sieve of 2500 holes per square inch.

In 1891, Carey (*P.I.C.E.*, vol. 107, p. 47), whose ideas on this subject were very advanced, used cement giving a residue of 9 per cent. on such a sieve.

In 1897, Butler (*P.I.C.E.*, vol. 132, p. 346), gave the following percentages or the residues found in actual samples :

Sample.	Number of Holes per Square Inch.		
	32,400	5776	2500
F.	33	16	4
G.	35	20	8
H.	28	11	4
I.	39	15	2·5

The British Standard Specification (1910) is as follows :

"Residue on a sieve of 5776 holes per square inch not to exceed 3 per cent., and on a 32,400 hole sieve not to exceed 18 per cent.

American Portland cement is probably equally finely ground. Typical figures in 1905, for the residue on a sieve with 40,000 holes per square inch, are from 21 to 36 per cent., with an average of 28·7 per cent. These figures cannot be considered as indicating the whole difference that has occurred in the last twenty years, since even more finely ground cements are now procurable and at no very great increase in cost.

The sands usually employed in Great Britain and the Eastern United States are coarse in comparison with those commonly found in the vast alluvial plains characteristic of India, and the Middle United States. Consequently, I consider that the assumption that 0·85 of cement paste, and 2(1 − 0·40) of sand, always works out at 2·05 of mortar, needs experimental checking.

As a matter of fact, the relation does hold to about ± 2 per cent. of difference (which is approximately equal to the possible error in the large scale measurements) with finely ground cement and coarse sand.

I have found an increase in bulk of about 4 per cent. in the case of a somewhat coarsely ground cement and fine sand, which, I believe, is beyond any possibility of error. With very fine sand (30 per cent. passing a 100 mesh sieve), and British Standard cement, I have sometimes found a shrinkage of 5 per cent. I therefore consider that at present it is well worth while to test the mortar carefully, and to make sure that its bulk does not differ from that shown by the calculation. Should the resulting bulk differ materially, the discrepancy must be allowed for, and a series of trial mixtures of sand and cement should be made up, in order to select that which produces the densest mortar.

The fact that such differences may occur renders it somewhat doubtful whether the process given above is applicable in every case. It would appear that where the larger particles of the cement are of approximately the same size as the smaller particles of sand, the resulting mixture is very close to the densest possible. If there is a marked gap in the sizes of particles contained in the mixture of sand and cement, so that very few particles of, let us say, $\frac{1}{300}$th of an inch in diameter, exist, the theory may require modification. At any rate, it is always as well to investigate the bulk of mortar produced, and, if necessary, to modify the proportions of cement and sand, so as to get a denser mortar mixture.

It will also be plain that this method may lead to a somewhat peculiar proportioning of the materials. For instance, let us assume that the aggregate, instead of being almost entirely composed of particles exceeding a quarter of an inch in size, is "run of the breaker" stone, containing an appreciable proportion of stone dust. If this does not materially exceed say 10 to 15 per cent. of the whole volume of aggregate, the effect will be to diminish the percentage of voids in the aggregate, and therefore the amount of mortar used; whereas, if this stone dust was screened out, and was added to the sand, an additional quantity of cement would be required to fill the voids in the screened dust. It is therefore desirable to consider the problem apart from any practical classification of the raw materials.

Mixing of Granular Substances.—The rules now discussed are at first sight somewhat meaningless, but their physical basis becomes clearer when it is realised that they merely express the methods by which the closest possible packing of the individual particles of the mixture of cement, sand, and aggregate, which makes up the concrete, may be obtained.

For simplicity, consider a granular substance composed of rigid spherical particles only. If all the particles are of the same size, we obtain a material which (absolute dimensions being neglected) may be compared to a number of billiard balls. If these balls are piled together in cubical order, so that the centres of each eight adjacent balls lie at the angles of a cube, it will be found that the volume occupied by the ivory of the balls is $\frac{4\pi}{24} = 0{\cdot}52$ of the total space, and 0·48 of the total space is occupied only by air. A little consideration will show that since the length of the diagonal of a cube is:

$$\sqrt{3} x \text{ length of side,}$$

a smaller ball, of a diameter equal to $\frac{\sqrt{3}-1}{2} = 0{\cdot}366$ diameter of the original balls, might be fitted into the void space existing between each eight adjacent balls. There being one such void space for each one of the original balls, an extra portion of the original space, amounting to $(0{\cdot}366)^3$, can be filled up with ivory; so that a packing of balls of unit diameter, with an equal number of balls the diameter of which is 0·366 units, will produce a space containing $0{\cdot}52 + 0{\cdot}049 = 0{\cdot}57$ of ivory, and 0·43 of air. And, plainly, if the smaller balls are of greater diameter than 0·366, or are more numerous than the original balls, the packing cannot be made as close; and if smaller than 0·365, or less numerous, some voids which could be filled will remain unfilled.

Thus, adopting a regular cubical packing, we can take a bulk of 100 cube feet of billiard balls, containing 52 cube feet of ivory, and 48 cube feet of air; and also a bulk of about 9·4 cube feet of smaller balls, each with a radius equal to 0·366 of the radius of the larger balls, and containing 4·9 cube feet of ivory and 4·5 cube feet of air. On packing the mixed balls regularly in order we obtain a bulk of 100 cube feet of the mixture, containing 56·9 cube feet of ivory. The process can be described as mixing 10 volumes of grains of unit diameter with 1 volume of grains, the diameter of which is approximately one-third of a unit, and obtaining 10 volumes of a denser mixture with 0·43 voids, in place of 11 volumes of less dense unmixed substances containing 0·48 voids.

The example is selected because a model is easily constructed, and the purely geometrical difficulties are not great. It is otherwise somewhat misleading. As a matter of fact, the closest packing of a set of spheres of equal size is obtained when a regular tetrahedron is taken as the basis of arrangement. I am unable to prove, or, to discover a proof which shows that this is absolutely the closest possible packing; but, assuming that the statement is correct, we find that:

The volume of ivory $= \frac{\pi \sqrt{2}}{6} = 0{\cdot}74$ of the total volume.

The volume of air $= 0{\cdot}26$ of the total volume.

The diameter of the smaller spheres which can just be fitted into the interstices $= 0{\cdot}225$ of the diameter of the original spheres.

The extra volume of ivory thus filled in $= 0{\cdot}011$ of the original volume.

The volume of the space occupied by these smaller balls before mixture $= 0{\cdot}015$ of the space occupied by the larger balls.

Thus, the process is expressed by mixing 60 volumes of spheres of unit diameter with slightly less than 1 volume of spheres of about 0·22 diameter, in order to obtain 60 volumes of mixture with 24·9 per cent. of voids, in place of 61 volumes of unmixed material with 26 per cent. of voids.

The granular substances used in practice are not made up of spherical grains, nor are the individual grains of uniform diameter, but the figures already given show that void spaces exist. The practical test of pouring dry sand into a bucket "full" of road metal, will at once dispel any doubts concerning the possibility of making a denser substance by mixing two granular substances of different size of grain. The figures arrived at in the two examples considered above will also serve to show the general principles. The mean sizes of the two substances must not be too close together; and any admixture of grains, say one half the diameter of the larger grains, is certainly useless, and may be detrimental.

A practical example is afforded by the tests of Feret (*A.P.C.*, July 1892).

The dust produced by crushing Cherbourg quartzite was sized into three classes.

G, containing grains which passed a sieve of 4 meshes per square cm., (*i.e.* roughly 10 mesh per lineal inch), and which were retained by a sieve of 36 meshes per square cm., (15 mesh per lineal inch) (approximately from 0·20 to 0·06 inch in diameter).

M, passing a 36 mesh sieve, and retained on a 324 mesh (or 36 meshes per lineal inch) sieve (approximately from 0·06 to 0·02 inch in diameter).

F, passing a 324 mesh sieve (approximately all grains less than 0·02 inch in diameter).

Each of these "sands" contained about 50 per cent. of voids.

The three substances were then systematically mixed in various proportions.

The densest mixture was 0·6 G, to 0·4 F, with about 36 per cent. of voids. No mixture of F, and M, only, had less than 42 per cent. of voids; and no mixture of G, and M, only, had less than 47 per cent. of voids.

Similar information could be collected from many sources; and, as will be seen later, the exact figures concerning the proportions which produce the densest mixture, and the voids in the individual materials, and the various mixtures, are very variable.

The general results are, however, quite in accordance with the hints afforded by the mathematical theory. Roughly speaking, taking the average size of a cement grain as unit, the densest mixture is obtained by selecting sand grains of sizes, say, 4 and 16 times this unit size; and the stones of sizes, say, 64 and 256 times this unit size. It is obvious that this sizing (if the bulk proportions are properly selected) secures a dense "sand," and a dense "stone"; and that the cement should pack nicely inside the sand, and the sand inside the stone.

This aspect of the question has been investigated by Fuller and Thompson (*Trans. Am. Soc. of C.E.*, vol. 59, p. 67), who regard the cement, sand, and aggregate as graduating one into the other, so that when they speak of particles less than 0·003 inch in diameter, both particles of sand and cement are included. Similarly, a few of the larger grains of cement may be included in the size exceeding 0·01 ins. in diameter.

The principles now set forth refer to all concrete mixtures, but the actual figures are probably only applicable to sands and aggregates that do not differ greatly as regards the general size and shape of grains from those used in the experiments. The experiments show that a substitution of angular aggregate and angular sand, for round aggregate and round grained sand, can alter the proportions of the best mixtures to the extent of 5, or even 10 per cent.

The following conclusions are, however, generally confirmed by the experiments of Feret (although the details are possibly subject to certain exceptions), and may be considered as a basis for special tests in any given case.

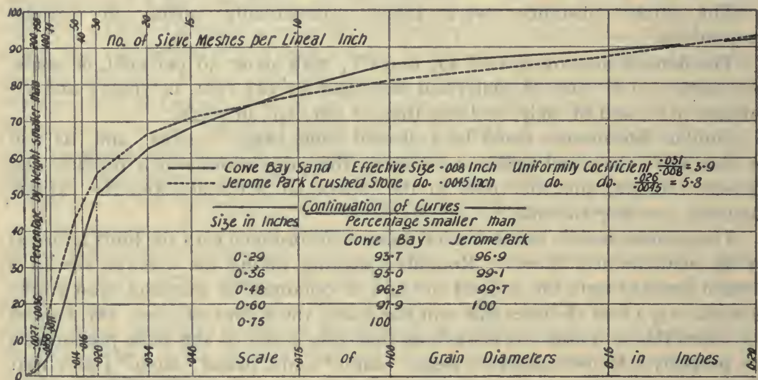
(i) A mixture in which the particles have been graded so as to give a concrete of great density when water and cement are added, produces a concrete of greater compressive strength than a mixture containing the same proportions of cement and natural materials, (*i.e.* sand and aggregate), but which yields a less dense concrete, owing to a less favourable grading of the sizes of the particles.

(ii) The density (and usually the strength) of concrete is affected but slightly, if at all, by decreasing the quantity of medium size stone in the aggregate, and increasing the quantity of coarsest size stone. An excess of medium size stone appreciably decreases the density and strength of the concrete.

(iii) Variations in the size of the sand particles have more effect on the strength and density of the concrete, than variations in the size of the stone particles.

(iv) An excess of fine, or medium sand, decreases the density, and also the strength of the concrete; and, where the proportion of cement is small, a deficiency in sand of fine size has the same effect.

(v) The substitution of cement for fine sand does not affect the density of the



SKETCH NO. 266.—Fuller and Thompson's Ideal Sizing Curves.

mixture, but increases the strength, although in a slightly smaller ratio than the increase in the proportion of cement.

(vi) The correct proportioning of concrete as regards impermeability consists in finding (with any given percentage of cement) the concrete mixture possessing a maximum density. The requisite strength is secured by an increase or decrease of the cement (thus, in effect, substituting cement for the finer particles of the sand).

(vii) With a given sand and stone, and a given percentage of cement, the densest and strongest mixture is attained when the volume of the mixture of sand, cement, and water, is such that it just fills the voids in the stone. In other words, in practical construction as small a proportion of sand, and as large a proportion of stone, should be used, as is possible without producing visible voids in the concrete.

In the above definitions the size of separation between "sand" and "stone" is taken as one-tenth the size of the largest stone. For example, for stone running up to 2¼ inches all material below 0·22 inch in diameter is sand, (*i.e.*

all passing say a ¼ inch mesh), and for ½ inch stone, all material below 0·05 ins. in diameter, (*i.e.* all passing a sieve of 15 meshes to the linear inch).

The terms "excess" and "deficiency" are relative, the standard being the "ideal" sizing curves given by Fuller and Thompson. These curves are probably applicable (with a fair degree of accuracy) to most mixtures of broken stone, and sand, or gravel, but further trials are greatly to be desired. The exact form of the curve is affected by the shape of the individual grains; and when angular "sand," procured by crushing stone, is mixed with rounded aggregate, other and larger modifications probably occur. According to Fuller and Thompson, the best mixture is one in which the cement and sand together form about 34, to 38 per cent. of the whole volume, and about 7 per cent. of this is less than 0·003 ins. in diameter, say 200 meshes to the linear inch.

The ideal grading is shown in Sketch No. 266, where the sizing or grading curve starts with 7 per cent. of the whole mixture less than 0·0027 ins., and runs as an ellipse to a percentage of $30 + 2{\cdot}2\,D$, for a size $\frac{D}{10}$, where D, is the diameter of the largest stone, in inches. The stone is then graded uniformly, as is shown by the straight line; and it will be noticed that slight differences exist, according as the "sand" and "stone" are angular or rounded.

Now, these definite figures, and the elliptical and straight line laws, are both probably only of limited applicability. The generally applicable deductions (not only from these investigations, but also from those of Feret) are as follows:

Taking D, as the maximum diameter, sizes of stone between 0·6 D, and 0·3 D, are unfavourable, and should be partially removed by sieving.

So also, 0·1 D, being the maximum diameter of sand, sizes between 0·06 D, and 0·03 D, require diminution. For example, when the maximum stone is 2½ ins. in diameter, stones of a size between 1·5 ins. and 0·75 ins. may be found to be in excess when compared with the curve. Sand grains of a size between 0·15 ins. and 0·075 ins. will probably be in excess when compared with the curve, and should consequently be removed as far as possible.

As an example, let us take Sketch No. 267, which shows a sizing curve for Thames ballast. Here the sizing of the mixture, as dug from the excavation, is as follows:

About 73 per cent. of the whole material is less than 0·20 ins. in diameter (Fine), and about 14 per cent. is between 0·20 ins. and 0·48 ins. in diameter (Medium). About 13 per cent. is above 0·48 ins. in diameter (Coarse).

A close approximation to the ideal curve is obtained by a mixture of the following proportions, namely:

Containing 38 per cent. of Fine, 14 per cent. of Medium, and 48 per cent. of Coarse.

Now, D, being 1·5 ins., the effect of the proposed grading is to diminish the percentage of the particles of sand less than 0·20 ins. in size (Fine) to about one half of that which occurs in Nature, the desired object being the diminution of the medium sized grains of sand.

The percentage of gravel over 0·48 ins. (Coarse) is about four times that which occurs in Nature, and this causes the natural proportion of small gravel (Medium) to fit the curve very nicely.

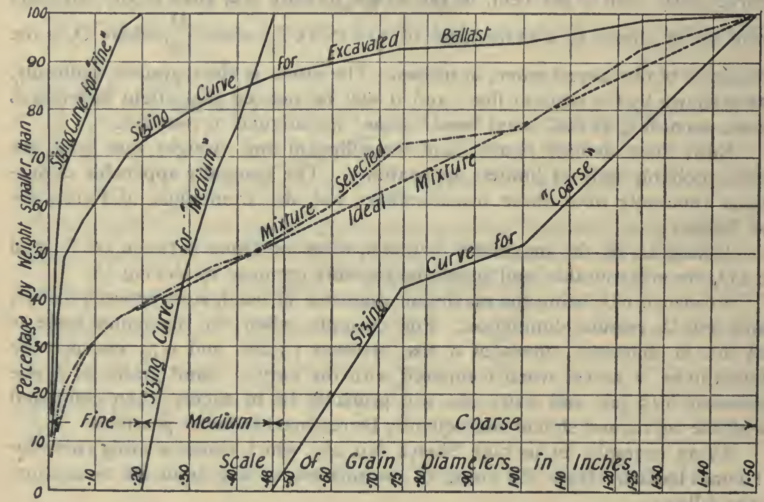
If the matter is carefully studied, it will be found that this division of the natural "ballast" into three classes, and a proper selection of their proportions, will permit a large rejection of medium sand and medium gravel particles to be made.

These rules do not always produce the densest mixtures. The variations likely to occur are illustrated by Feret's experiments. Here the material was separated into:

G, containing material between 2·36 ins. and 1·57 ins.
M, „ „ 1·57 ins. and 0·79 ins.
F, „ „ 0·79 ins. and 0·39 ins.

These substances separately contain about 52 per cent. of voids when the material consists of broken stone, and 40 per cent. of voids when it is rounded gravel.

The mixture possessing the smallest percentage of voids (and, in consequence, presumably the best mixture) is composed of 1 G, to 1 F, in broken



SKETCH No. 267.—Sizing of Thames Ballast.

stone, and contains 47 per cent. of voids. In the case of gravel, the proportions are 3·5 G, to 1 F, and the voids fall to 34 per cent.

The results differ from Fuller's proportions in that the class M, is totally eliminated, and are obviously less adapted for practical mixtures. The large difference, both in the proportions and the voids in the densest mixtures, which is produced by the substitution of gravel for broken stone, indicates that neither of the experimenters has succeeded in obtaining a rule which is universally applicable. Consequently, any blind application of Fuller's curve is undesirable. It must not be forgotten that both experimenters worked with an artificially dried material. The absolute values of the voids therefore differ considerably from those existing in a "dry material," as used in practical work.

I have carried out special tests on the subject, and find that if we call the mixture containing the smallest percentage of voids, the "optimum" mixture, then the proportions of the "optimum" mixture for artificially dried materials differ very slightly from those of the "optimum" mixture for "dry" materials, as found in practice, although the absolute percentages of voids in the two mixtures may be very different.

It will be found that a fair approximation to the best mixture (as obtained either by the methods of Fuller or Feret), can in many cases be secured by passing the natural material through a trommel with holes about 0·20 ins. in diameter, and taking from 2, to 5 measures of coarse, to 1 of fine material, in place of the natural mixture, which usually consists of 1 of coarse material to 3 or 4 of fine. The exact proportions are best arrived at by a series of trial mixtures. When the densest mixture has thus been discovered the result should be checked by tensile tests of the mortar thus obtained. In practice, I find that a comparison of the sizing curves of the available material with Fuller and Thompson's ideal sizing curve, as plotted in Sketch No. 268, will afford valuable preliminary indications. This is, of course, the usual sand and stone classification.

The more complicated "Fine, Medium, Coarse" process indicated in Sketch No. 267 is of practical importance. When the usual sand and aggregate are thus divided into three classes, it is often possible to obtain a good "voidless" (*i.e.* practically impermeable) concrete with a mixture of—1 cement : 9 or 10 of a scientifically graded mixture of the three classes. Whereas, the usual practical mixture, obtained by taking only two classes (*i.e.* sand and aggregate as they are generally used), is—1 : 2 : 5, or 1 : 7 of the combined material.

When considering the sizing of materials it is often desirable to know what size of particle will pass a screen of say 40 meshes per linear inch. The question is very easily solved in meshes down to say 10 per inch, by actual test and measurement. Below 10 meshes per linear inch I have found that the following rule is very close to commercial practice :

The maximum diameter of a particle which passes a commercial sieve of n, meshes per linear inch, is not far removed from $\frac{62}{n}$ inches, and is still more closely represented by $\frac{66}{n}$, if n, is less than 40, and $\frac{56}{n}$, if n is greater than 40.

Sand.—The usual specification for sand is as follows :

The sand to be clean, sharp, angular pit sand, free from clay or mud, or other impurities which adhere to the grains. The practical tests employed in judging sand are :

(i) If it stains the fingers when rubbed, it is considered to be dirty, and is consequently washed.

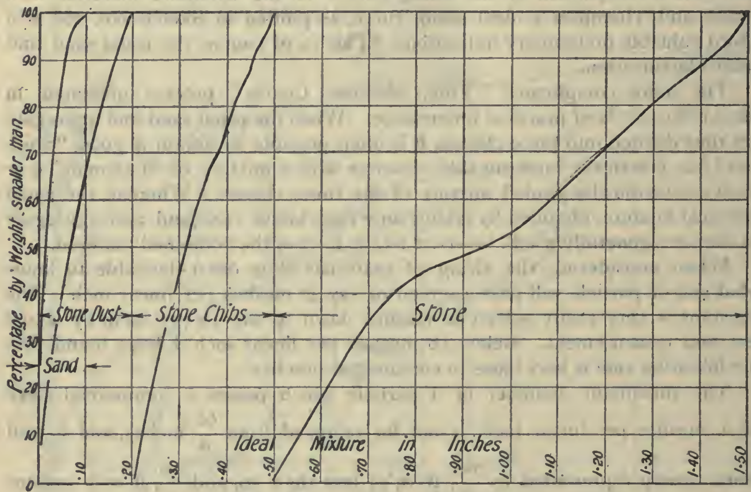
(ii) The angularity of the grains is judged by examination by the eye, or by a lens.

The specification is really one which secures a sand which is well adapted for making good mortar for bricklaying purposes, and is founded on the properties of clean pit sand, as distinguished from river sand, which is generally less angular, and contains fewer fine grains. Sea sand is usually even more rounded and more devoid of fine particles than river sand, and, being coated with salt, is unsuitable for mixing with cement.

The specification therefore usually enables the sand which is best adapted for concrete work to be selected from the two or three classes of sand which occur in Nature. It is, however, quite useless for selecting the best pit sand when three or four sources are accessible, and it in no way indicates the best method of preparing the raw sand.

The specification does not describe an ideal sand for concrete, although a sand passing this specification will no doubt produce a good mixture, and forms a good raw material from which a still better sand can be selected.

The sand required for concrete is a hard, non-porous substance, containing grains of all sizes, roughly in the proportions indicated by the sizing curves already given. This may be accurately specified by sizing tests; but, as a rule, the smaller the percentage of voids the more closely the sand conforms to the sizing curves, and the better it is fitted for concrete. The material should not contain any free lime, or salts, which may act injuriously on the cement.



SKETCH No. 268.—Sizing of Fine Sand and Stone Dust.

Thus, a good, up-to-date specification can only be obtained by systematic preliminary tests on the local sand; and the following form is probably necessary in cases where large quantities of sand are used:

The raw material for sand to be procured from approved pits; and, if directed, to be washed and graded into two sizes by passage through screens of (half an inch) and ($\frac{1}{8}$ of an inch) mesh.

The required proportions of cement, fine sand, coarse sand, and aggregate, will be selected by the engineers, according to the results of sizing tests conducted, as shown in the annexed schedule; and payment for the cement will be made according to the weight actually used.

The difficulties are obvious, and a similar specification of the aggregate is also required. The concrete mixtures at Staines were proportioned on these principles, although in a less scientific manner (the date being 1899); and, since it was my duty as engineer's assistant to conduct the tests, I can state that a specification similar to the above, if properly used, is not unjust to the contractor.

The whole of the above experimental work may at first sight appear to be somewhat unnecessary. The real justification for the discussion is that with care from 6d. to 1s. can be saved in the cost of each cube yard of concrete, even under British conditions; and, in localities where cement is really costly, the saving is frequently sufficient to enable cement to be used in place of less satisfactory local lime or other mortars.

Similar investigations for hydraulic lime, and lime and clay cements (Natural Portland) are badly needed.

The information at present available almost entirely consists of tension tests of various mixtures, and is not sufficient to enable any definite statements to be made. The principles above developed should be followed when investigations are made; but, as a rule, the requisite strength is obtained with difficulty, and, if it is obtained, and the cementing material is finely ground, it is probable that the resulting concrete will be very nearly "voidless."

In this connection I would observe that no modern engineer can afford to neglect such cementing materials as hydraulic lime and trass, or the weak hydraulic limes produced by burning kankar in India and hardpan in Australia. I do not, however, consider that I have sufficient chemical knowledge to discuss the principles involved in the selection of these materials and the production of concrete made by their use.

I therefore merely state that certain French firms supply a hydraulic lime with which concrete—as reliable as a Portland cement concrete—can be produced; and that certain German firms sell trass, from Andemach, which is equally reliable.

The resulting concrete is not as strong as Portland cement concrete, and at present I merely use these materials when a cheaper result can be produced. It must also be remembered that the above firms are by no means the only possible sources (the grey lias lime of England being equally useful). The only reason for specially mentioning these firms is that they market a properly tested material of uniform quality, and therefore form as good a source of supply as any reliable manufacturer of Portland cement.

The following details may be useful in drawing up a logical specification:

In the case of modern cements, it has been fairly well established that a round sand gives the strongest concrete when tested in compression, the angular sands giving better results in tension tests only. Now, concrete structures are designed for exposure to compressive stresses; thus, to insist on an angular sand seems somewhat unnecessary. In some cases (*e.g.* retaining walls, and broad foundations), we know that the concrete does actually sustain tensile stresses, and then angular sand may be advantageous. I do not consider that it is permissible to reject an otherwise satisfactory sand merely because it is not angular, since a want of angularity really means that the sand is not very fine, for it will generally be found that the finer the sand, the more angular are the individual grains.

The following tests of sands by Feret show the influence of the shape of the grains on the percentage of voids:

Sand with laminated grains contains 34·6 per cent. of voids.
Sand with flat grains (crushed shells), 31·8 per cent. of voids.
Sand with angular grains (crushed stone), 27·4 per cent. of voids.
Sand with rounded grains, 25·6 per cent. of voids.

These results are only comparative, and in Nature the percentages of voids are usually larger, but an angular sand may be expected to have from 2 to 5 per cent. more voids than similar rounded sands.

Hazen states that a sand with a uniformity coefficient of less than 2, as a rule contains 45 per cent. of voids; and if the uniformity coefficient is between 2, and 3, the voids fall to 40 per cent.; while with a coefficient of 6, or 8, the voids fall to 30 per cent. The figures are interesting, as showing the extreme importance of removing the medium sized grains. Hazen probably refers to angular sands.

My own experience is distinctly adverse to the washing of sand in order to remove clayey particles, unless such particles exceed say 5 per cent. in bulk, or are collected in comparatively large masses.

In practical tests of sand (such as is met with in the Thames valley), the unwashed material generally produces a stronger mortar when tested by 1 cement : 3 sand, tensile tests (probably because the finer and more angular particles are not removed), and the mortar is usually less permeable by water. An engineer desirous of making the best use of the available material should consequently specify that washing may be ordered if necessary; but should definitely state that sand which tests well (either in tension or in compression, or for permeability, according to the purpose for which it is required), need not be washed simply because it contains clay.

The best method of preparation has been much discussed. So far as I can sum up a matter which obviously depends largely upon local conditions, I am inclined to believe that while vegetable mould should invariably be removed, the presence of a small proportion of clay (say 2 to 5 per cent., or even in some cases 10 per cent.), is usually beneficial, especially where impermeability, rather than strength, is the main requirement. I attribute those cases where sand is improved by the removal of small quantities of clay, not so much to the removal of the clay, as to the washing away of salts coating the grains of sand. And there is little doubt that a washing which is sufficiently violent to remove the smaller grains of the sand, almost always results in a sand which produces a weaker (in tension) mortar.

In this connection, it is necessary to refer to the alkaline salts (such as sodium carbonate, sulphate, or chloride), which occasionally occur, either in sand, or in water used for mixing concrete. The presence of such salts necessitates great care in proportioning and mixing concrete.

In some localities I have found it absolutely impossible to make a good concrete with local sand and well water. Such extreme cases are easily recognised, since the cement either sets very rapidly, or refuses to set within a reasonable period. The more dangerous cases, however, are those where the concrete works properly, and is deposited without arousing any suspicion, and then rapidly disintegrates.

So far as can be judged, the conditions are very similar to those producing the disintegration of concrete by sea water, so much discussed ten or fifteen years ago. The solution of the problem is—I believe—the same. Concrete which is rapidly destroyed by the action of alkaline salts always appears to have been markedly permeable, and its destruction is probably due to the crystallisation of salts in the interstices. If this be so, it is plain that a properly proportioned impermeable concrete will probably disintegrate at the surface only, and the practical effect of the presence of alkaline salts will be to prohibit the use of other than impermeable concrete. Where impermeable concrete would be too expensive, coatings, or layers of asphalte, or bitumen sheeting, are indicated, in order to prevent percolation. I have also found a mere coating of tar to be satisfactory for a period of at least three or four years.

In this connection it is advisable to refer to the "working qualities" of mortar. These are not easily defined, but are fully appreciated by bricklayers and masons. A mortar "works well" when it fills the joints readily, and adheres nicely to the bricks or stones. Now, as a matter of experience, if the ordinary specification of "clean angular sand" is rigidly adhered to, and this sand is mixed with a good modern Portland cement (or even in some cases with hydraulic lime), the resulting mortar, especially if rich in cement, does not work well.

As a practical matter, therefore, it is frequently advisable to secure a good working mortar for brickwork or masonry, even if it is not quite so strong in tension or otherwise satisfactory under test, in order to obtain good workmanship. The quality of the inspection and the control over the labour will, of course, be important factors in arriving at the final decision.

Similarly, the rejection of odd lots of mortar left over at meal intervals, or even over night, is an unending source of friction, especially with quick setting cements. While I do not consider the practice an ideal one, I have frequently found that if this half-set mortar be mixed in small quantities with freshly made mortar, the resulting mixture works very well. In one case, where the results of tensile tests were favourable, and the normal mortar worked very badly, I found better results were obtained by systematically mixing about the last tenth of each batch with the new batch.

Aggregate.—The most important point is to be certain that the aggregate does not contain any chemical calculated to act on, or disintegrate, the cement.

The deleterious substances most commonly found are :—Soluble sulphates in cinders or slag; or, where the cinders are not properly cleaned, in unconsumed coal. Such salts sometimes occur in brick bats, which, when otherwise satisfactory, form a good concrete aggregate if broken to size. So also, lime in the old mortar adhering to the bats may cause trouble.

The only other case commonly found is that of a limestone aggregate, which is occasionally destroyed by acid moorland waters. (See Barrett, *P.I.C.E.*, vol. 167, p. 153.)

The qualities required in a good aggregate are very much akin to those of a good sand, the size of the individual grains being the only marked difference.

The ideal aggregate is a hard, angular stone, with rough surfaces affording a good grip for the mortar. A granite road metal may be taken as typical.

Hard water-worn pebbles are generally held to be less favourable substances, but they nevertheless form a good concrete, and usually contain a smaller proportion of voids. The fact that a water-worn pebble remains hard is a very fair indication that it will not disintegrate when used for aggregate. In actual practice, any hard material can be used. Very good results have been obtained with burnt clay, although this is probably better adapted for a concrete with a matrix of hydraulic lime, rather than Portland cement.

CONCRETE.—As a rule, on all works of sufficient size to justify the cost of a machine and its driving power, machine mixing alone is employed in concrete work ; though I do not consider its advantages justify an engineer in specifying it exclusively.

Mixing machines are of many types. The cubical box rotating round a diagonal possesses certain theoretical advantages, but well mixed concrete can be turned out by nearly all existing types of machine. A very good test consists in placing the sand, aggregate, and cement, separately in the mixer, mixing the cement with about 10 per cent. of some easily recognisable colouring matter, insoluble in water (I usually employ brick dust, sifted so as to be of the same fineness as the cement), and then seeing how this colour has spread over the mixed mass, testing both dry and with water added.

The most logical test is to take samples of the mortar, or concrete, from 6 or 8 different portions of a batch, and to make up separate tests from each sample (either briquettes for tension, or cubes for compression). Then thoroughly mix the whole batch by hand, and take the same number of samples from the mixed heap for testing purposes. If the first set of tests either vary materially *inter se*, or are markedly (say more than 5, to 10 per cent.) weaker than those of the hand mixed set of tests, the machine may be considered as defective. When applying this method, it is as well to bear in mind that a batch mixer gives very similar results, however it may be worked, provided that the number of turns is the same ; while a continuous mixer is far more sensitive to careless working. Thus, not only are the above tests more easily faked for special purposes, but a continuous mixer always needs more rigid inspection.

The usual specification for hand mixing is as follows. The concrete to be mixed on a close boarded, or impervious floor, and when dry to be turned over at least twice ; then three times after water is added, until thoroughly and satisfactorily mixed.

I consider that concrete should be deposited with as little disturbance as possible. I am aware that it was formerly the practice to drop the concrete from stages (sometimes specially erected) through a height of 20, or 30 feet. Engineers who persist in this practice do not realise that modern cement is a substance composed of particles which are far finer than any ordinary sand, and are therefore easily sized out, and separated from the sand and aggregate. Personally, I am so adverse to any undue disturbance that I view the practice of running concrete down a shoot with disfavour, and prefer to deposit it in tipping buckets, or better still, in flap-bottomed grabs. It must, however, be realised that a hard, coherent filling is all that is required in certain cases, and under such circumstances any undue restrictions merely delay progress.

The question of very dry, versus very wet concrete, has been the subject of much discussion (see p. 396). I believe that a very dry concrete, rammed until it quakes, and slightly sweats water, is the ideal mixture. But I am equally convinced that competitive contractors cannot be expected to sacrifice their own interests, and that administrative working does not justify useless extravagance. Thus, since a very dry concrete, insufficiently rammed, produces worse results than a wet concrete slapped into place, I believe that the best results are obtained by considering the permissible degree of wetness of the concrete as a function of the thickness of the work, and its irregularity. For thin walls of irregular section, encumbered by metallic reinforcement or expansion joints, the use of dry concrete will lead to friction with the workmen, slow progress, displacement of the reinforcement, and (despite the best supervision) possibilities of unperceived cavities always remain. Such cavities are most likely to occur between the reinforcing rods, and are positively dangerous in such a position, since they give rise to corrosion and a lack of adhesion between the concrete and the steel. Consequently, the concrete should be wet, and the wetness cannot be considered to be excessive unless clear water is found to rise to the top of the concrete. The above degree of wetness is only permissible when the shuttering is cement-tight, which is best secured by the use of tongued and grooved boarding.

At the other extreme lie wide walls, such as a dam, where the workmen can stand and ram, or trample all over the surface of the concrete.

In such cases the concrete should be mixed fairly dry, and should be rammed into place.

The methods adopted by Deacon (*P.I.C.E.*, vol. 126, p. 24), may be taken as a model, subject to the remarks that they are expensive, and if carelessly carried out will produce worse results than concrete which is too wet.

The cement was slow setting, the specification stipulating that a thermometer when inserted into a jam jar containing freshly made cement plaster should not rise more than 2 degrees Fahr. in the first 15 minutes, and 3 degrees Fahr. in the first 60 minutes, after making the plaster. This cement was mixed with three times its volume of a mixture consisting of 1 part of sand, and 2 parts of stone dust from the breakers, which passed through a sieve of 8 meshes per inch; and about 6 to 8 per cent. of water was added. The mortar thus obtained resembled putty in consistence, but (the amounts are not given, but the mortar and the concrete were proportioned in the manner indicated on p. 961 so as to be "voidless") when rammed became "jelly like," and resembled a "quicksand," and was almost impermeable to water. The concrete was rammed with beaters made of ½-inch iron plates, 1 foot square, with the edges turned up ⅜ths of an inch all round.

With the cement of the present day a greater quantity of water would probably be required, but the consistency aimed at is that employed by a skilful cement tester when making briquettes for testing purposes. The results are good, and with careful supervision 1 foot thickness of concrete made by these methods is water-tight under a head of 30 feet, but such results are unlikely to be attained by a contractor.

As a rule, wide walls of the character now considered are constructed of concrete, with stone plums or displacers. These are large blocks of sound stone (Deacon used blocks up to 6 tons in weight, and his average size was about 2 tons; but, in bridge piers, and such work, 2 to 6 cubic feet is more usual) roughly trimmed so as to remove concavities, and set in the concrete, generally point end downwards.

Deacon beat the blocks into place with fifty or more blows from two-handed wooden mallets, 6 to 7 inches in diameter, and 14 inches long; but such blocks are usually wriggled or levered into a good bedding in the concrete by means of crowbars (see p. 404). As a rule, these plums are separated from each other by at least 4 inches of concrete, so that the plums rarely form more than 40 per cent. of the volume. Deacon, however, having made a plastic concrete by the method described, found that the plums could be packed far more closely, and consequently adopted 1 inch as the minimum distance. Blunt tools ("swords" and "slicers") were used for ramming the concrete into the narrow joints between the blocks.

It is pure hypocrisy to treat wet concrete as though it were dry, and *vice versa*. The only effect of ramming wet concrete is to bring more "laitance" to the surface, and to cover it with what is really dead and useless cement, which has been drowned by the excess of water. If the concrete is not rammed, a smaller quantity of this laitance is produced, and that which is produced remains distributed throughout the entire mass. All that wet concrete really requires is to be worked with a spade, or blunt tool, in order to force it into corners of the shuttering, or between the reinforcing bars. Similarly, dry concrete, if not rammed, is liable to form arches, and void cavities, are left; also it appears that ramming is actually necessary in order to bring the water into contact with the cement, and start the setting of the concrete.

SHUTTERING.—The design of the forms, or moulds for retaining the concrete while setting, deserves consideration. The forms are exposed to the pressure of the unset concrete, the pressure, of course, varying not only with the wetness, but with the height of fresh material deposited at a time. The usual practice is to calculate the stresses as though the concrete were a liquid weighing 70 to 80 lbs. per cube foot. General experience therefore leads to the following rules, which are introduced mainly to prevent excessive deflection of the shuttering.

A board, 1 inch thick before planing, should be supported by a stud piece every 2 feet. If 1½ inches thick, every 4 feet, and, in the case of 2 inch boards, every 5 feet.

The experiments of Ashley (*Engineering News*, 30th June 1910) show that a wet concrete in walls 3 feet thick produces pressures corresponding to a liquid weighing from 63, to 79 lbs. per cube foot, and that these pressures increase according to the hydrostatic law (*i.e.* vary as the depth of wet concrete), until at least 6 feet of concrete has been deposited, so that the lower boards are exposed to a pressure of 470 to 480 lbs. per square foot some five hours after the start of the work.

In very wide walls (say 5, to 10 feet wide), with extremely wet concrete, pressures equivalent to a liquid weighing 150 lbs. per cube foot may be attained.

Schunk (*Engineering News*, 9th September 1909) says that if R, be the number of feet depth of concrete deposited per hour, the pressure increases at this rate until a time $T=C+\frac{150}{R}$, minutes has elapsed. Therefore, the maximum pressure may amount to $2{\cdot}5\ CR+375$ lbs. per square foot, and, if t, be the temperature of the concrete, we have as follows:

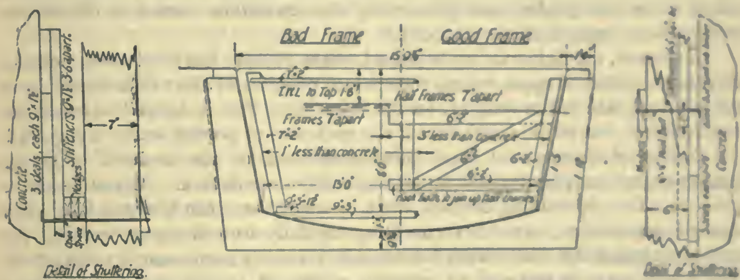
t =	80	70	60	55	50	40	degrees Fahr.
C =	20	25	35	42	50	70	

so that pressures of 1500 lbs. per square foot may occur at low temperatures. The circumstances appear to have been abnormal, and it is but rarely that R, exceeds 2 feet per hour, but the matter must be provided for when the work has to be pressed forward.

In grouting work, pressures equivalent to a fluid weighing 125 lbs. per cube foot must be provided for, but in such cases the pressure is necessary, and is not likely to lead to inconvenience.

It must be remembered that not only can the individual planks of the shuttering deflect, or break, under these stresses, but that the whole form must be supported on rigid frames or struts. Sketch No. 269 shows a good, and a bad type for the lining of a canal. The frame on the left-hand side is deformable, and even in so small a canal as is here illustrated, cases of an excess thickness of 1½ inches of concrete occurred, while half an inch was the general value. The details for making frames cement tight need consideration; in general, tongued and grooved boarding suffices, but if the boards are firmly braced together and well covered with soft soap good results are obtained with plain edges, especially after the shuttering has been used once or twice, and has absorbed moisture, and has had the interstices filled with old concrete.

The precautions required when concrete is deposited on a surface of old, set concrete, have been discussed under the heading of Puddle Walls (p. 316). While these are effective when properly carried out, all designs in concrete should be carefully considered with a view to preventing the occurrence of permeable joints. I am fully persuaded that concrete can (with care) be made perfectly water-tight. Where cement is expensive, it is highly probable that economy can be secured by trusting to walls of a thickness which is not



SKETCH No. 269.—Shuttering of Staines Aqueduct.

greater than the stresses demand, combined with a sandwiched layer of asphalte, or bitumen sheeting. It is, of course, understood that the layer outside the sheeting cannot be relied upon to take any portion of the stresses, and may therefore be of weaker, and, if necessary, even of permeable concrete, just to form a filling.

Rendering.—Concrete linings to canals and channels are frequently rendered with a mortar of cement and sand.

The following specification is typical:

The concrete walls when well set to be carefully picked over, and rendered with two coats of Portland cement, and clean sharp sand, in equal proportions. Each coat to be well worked in, and rubbed down with a trowel. The first coat to be approximately ⅜ths of an inch thick, and the contractor to remove all projections in the concrete where these would cause the coat to be less than ¼th of an inch, and to fill all deficiencies so as to bring the surface to a true line and level. The second coat to be ⅜ths of an inch thick.

The phrase "sharp sand" is merely an interesting relic of the past, since impermeability is now the true desideratum. Sharp sand does secure a nice-looking coat, much admired by local builders, which a rounded sand will probably fail to do. In practice, it is unusual to insist upon angular grains, even when specified.

My own experience of such work is unsatisfactory. The facing is good, but it is of course costly. Its hydraulic advantages are over-rated. The extra smoothness of channel is easily procured with the usual concrete specification, by employing planed shuttering, covered with soft soap, and working the concrete against the shuttering during deposition, with a spade or fireman's slicer. The rendering is also supposed to make the wall quite impermeable. Properly laid rendering is certainly impermeable, but if there is a really open joint in the concrete wall, so that the rendering is exposed to any degree of hydraulic pressure, it does not possess the necessary strength, and is consequently fractured.

In cases where impermeability is not absolutely essential a facing of extra rich concrete can be deposited simultaneously with the poorer backing, by means of a temporary shuttering formed of a piece of plank lifted up as the two mixtures are deposited. The fact that this temporary internal shuttering does not entirely prevent a mixture of the two grades of concrete, is advantageous, as a graduation from the poorer to the richer quality finally exists, which secures a union of the two grades, thus preventing any separation caused by differences in expansion under heat, or elasticity.

In general, the richer facing is placed where visible; but from every point of view (except that of appearance), the correct position of the richer facing is on the pressure side of the wall, *i.e.* on the water face if it is desired to retain the visible water, but on the earth face if the exclusion of ground water is required.

Facing of Concrete.—I have already referred to the methods of giving concrete what may be termed an hydraulically smooth face. Any attempts to produce a face resembling dressed stone in smoothness are likely to fail. The desired appearance can be secured by carefully trowelling, or working the concrete against the shutterings, but the result is rarely permanently satisfactory. The smoothness is attained by bringing a skin of cement and fine grains of sand to the surface. This almost invariably develops hair cracks, and after a year or so the surface resembles a badly disintegrated stone. These cracks rarely penetrate below the fine skin, but they effectually destroy the desired appearance. If such a facing is demanded, it is best obtained by rendering the surface, as already explained. If the appearance of concrete is disliked, two better methods exist. Either the whole mass of concrete may be faced with masonry, or brickwork, well bonded to the concrete by headers (a useful specification being one course of headers to three of stretchers in brickwork, and an equivalent if masonry is used). The cost is not excessive, for the facing work can be raised, 1 to 2 feet according to its thickness, above the concrete before each batch is deposited, thus rendering shuttering unnecessary. The more logical method, however, is to produce a rough cast stone face by removing the cement and sand to an average depth of about $\frac{1}{8}$th of an inch, by means of washes of acid, and brushing over with wire brushes, thus leaving the aggregate exposed. In such cases, if a facing layer of concrete with a nice-looking aggregate (*e.g.* broken syenite, or granite) is used, a very excellent appearance is obtained.

Expansion Joints.—It will be found that all "monolithic concrete" (mass concrete) structures crack under the influence of temperature stresses. These cracks usually occur at intervals of about 40 feet along the length of the work. Two remedies suggest themselves.

The most usual is to place expansion joints at every 30 feet of length along the work. The design of expansion joints is simple; or rather, the simpler their design, the better they work. The concrete mass is divided by chase joints at every 30 feet in length, and near every marked change in section. These joints are run in with asphalte, bitumen, or other elastic material. The trade terms for such substances are very varied, and in many cases are apparently meaningless. It is best to specify the use, and to ask for a guarantee from the sellers.

The following is a useful specification:

The expansion joints shall be 1½ inch wide, filled in with asphalte, or other approved substance. The filling shall be sufficiently elastic to give and take with the expansion and contraction of the concrete, without rupturing. Samples shall not be decomposed by soaking in water in 2 inch square pats, ½ inch thick, for three months. Such pats shall not crack when exposed to a temperature of   for   days, or flow under pressures of   at a temperature of   .

The filling in of the blanks is obvious, and similar tests can be specified in order to cover any peculiar circumstances. Thin steel plates, or lead flashing, built into the concrete on either side of the open joint (say ½-inch wide joints) have also been found to give satisfactory results.

The second method consists in reinforcing the concrete longitudinally with sufficient steel to take up the tensile stresses (see Sketch No. 204). The usual calculation is as follows:

The "elastic limit" of steel rods being 60,000 lbs. per square inch, and the tensile strength of concrete 200 lbs. per square inch, the area of steel should be $\frac{1}{300}$th of the area of the concrete.

I do not feel very much impressed by a calculation which departs so far from the ordinary practical rules, but mass concrete reinforced with this proportion of steel does not usually crack, and the cracks that do occur can be explained by sudden alterations in the section of the work.

A mass of concrete reinforced in this manner at all sudden variations of section and in all thin portions, especially those which are exposed to changes of temperature (*e.g.* the top of a dam, or the parapets of an arch), will rarely be found to crack badly.

Where the character of the work permits, very good results are obtained by splitting the concrete up into blocks which do not greatly exceed 30 feet in any dimension, laying alternate blocks, and filling in the void spaces later on. The joints between the blocks are then in reality cracks of a regular nature, which will be found to open and close as the temperature changes, so that the method merely regularises the cracks, and does not in any way prevent them. If the concrete is faced with brickwork, or ashlar, cracks do not usually appear on the face, but in many cases they can be discovered running through the concrete, so that the method is one of concealment rather than of prevention.

The figures relating to the expansion of mass concrete are discussed under dams (see p. 398). The circumstances occurring in dams are probably less favourable than is the case with mass concrete buried in the earth, so that the details there given can be applied to buried masses of concrete with satisfactory results.

In the case of thin walls of concrete which are exposed to the atmosphere on both sides, it is doubtful whether any method other than systematic reinforcement, or efficient expansion joints at every 20 to 30 feet, will entirely prevent cracking.

Grouting with Cement.—The theory of this process is very simple. If Portland cement is stirred with not too great a proportion of water it mixes with the water, forming "grout," or liquid paste, which contains approximately equal proportions of cement and water, and has a specific gravity of about 2.

This paste flows like a viscid liquid; and, provided that it is not exposed to an excess of water, it sets hard under water, and binds together any stones, ballast, or sand, the interstices of which it may fill, into a solid mass of rich concrete. It must be carefully noted that if Portland cement is shaken up with an excess of water (say more than 1 part of cement to 5 parts of water), the resulting mass will not set, and the cement is "killed" by the excess of water. Consequently, if we can secure that the grout is not unduly diluted by water, we can inject it into the interstices of a mass of rubble, stone, or gravel, lying under water, and rest assured that it will produce a hard, continuous mass of good concrete.

The above statements must be taken as referring to the majority of commercial Portland cements. The published data mainly refer to the comparatively coarsely ground cements of the period 1890–1900. My own tests on the newer, more finely ground, rotary kiln cements indicate that these are less easily killed by water. So far as I am aware, the results of large grouting operations with such cements have never been published, although a considerable degree of success was obtained under extremely difficult conditions by Walker on the Seiswan super-passage of the Sirhind Canal in 1905–07. I therefore believe that modern Portland cement is well adapted for grouting operations, and that a certain admixture of fine sand (to replace the coarse grains that existed in the older cements), is probably not only permissible, but is actually advantageous. The best, and only reliable test, is of course the practical one of grouting up a small mass of rubble, or stone, and sand, under water. For specification purposes, a small bulk of cement may be shaken up with say three or four times its volume of water, the resulting grout carefully deposited under water, and its setting properties noted.

The advantages are obvious. A hole can be dredged in a river bed and loose stone deposited. The grout can then be injected into the stone, and, when it has set, a continuous uniform bed of concrete will result, which forms an excellent foundation for any work it may be desired to erect. Thus, if it is necessary to unwater an area in permeable soil, the area may be dredged out to the required depth; rubble may then be deposited and the whole grouted up. When this has set, walls, or coffer dams, can be erected on the concrete, and the surrounded area can be pumped dry. If the concrete bottom is sufficiently thick, springs and boils are entirely prevented, and the saving in pumping plant may amply justify the expense in cement.

The process is not, however, infallible; and certain conditions must necessarily be fulfilled. In the first place, grout, like any other liquid, will find its own level, and it must be prevented from leaking away. In the second place, if water is injected into the grout during its setting the cement may be more or less "killed," and some or all of the grout will refuse to set. The French term "laitance" is a good expression for this dead cement.

The principles are best illustrated by two successful and one unsuccessful examples.

The Delta Barrage (Egypt) was known to stand on a mixture of fine sand, broken bricks, and rubble stone, the latter being the remains of the concrete which Mougel Bey was obliged to deposit in running water, with the natural result that the cement was swept away. It was also discovered (see Hanbury Brown, *P.I.CE.*, vol. 158, p. 1) that in other places where masonry or concrete had been successfully laid, layers of silt (in some cases more than 2 feet 6 inches thick) existed, sandwiched in between the masonry. In other cases, this silt had been removed, leaving void spaces in the masonry. The work was rendered fit to sustain water pressure by systematically drilling 4-inch holes through the masonry and concrete, and also as far below the foundation level as these holes could be kept open. Grout was then poured down the holes until no more was taken in. The top of the holes was some 20 to 30 feet above the water level in the river. There was no difference between the water levels above and below the barrage. Thus, not only was the grout injected under a pressure of at least 20 to 30 lbs. per square inch (the maximum value being 37 lbs. per square inch), but there was nothing to cause further water to mix with the grout once it had displaced the water originally existing in the void spaces and interstices between the rubble and sand.

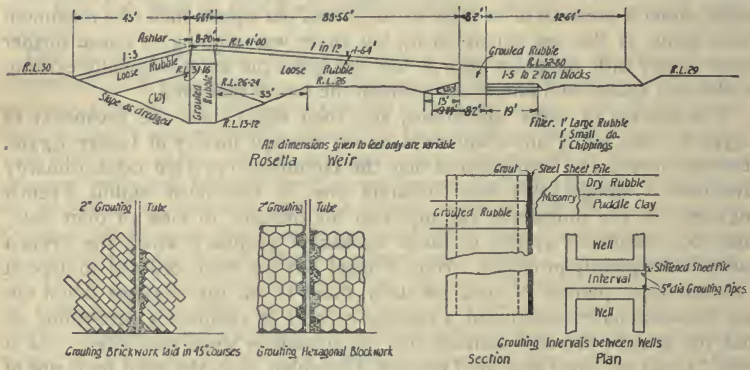
The success of these operations, and their effect upon the prosperity of Egypt, are the really important facts in the modern history of Lower Egypt. It must, however, be remembered that the circumstances were extraordinarily favourable. Mougel Bey was certainly one of the most skilful French engineers of the nineteenth century, and his designs (in view of their date) must be considered as one of those strokes of prophecy which the French nation so frequently produces. Had Mougel Bey not been ordered to deposit a certain fixed quantity of concrete daily, there is not the slightest doubt but that he would have constructed a barrage capable of adequately performing all that the present repaired barrage and its subsidiary weirs now effect. As a proof, it may be stated that until the 26th December 1909, the head regulator of the Menufiah Canal, which forms part of the Delta Barrage, was as Mougel Bey built it, and performed its portion of the work of the barrage satisfactorily. We may therefore consider that the grouting merely repaired certain accidental failures in construction; and, had the original design been radically bad, no amount of grouting would have produced good results.

The discoveries made during these grouting operations produced a certain distrust (in my opinion unwarranted) of the barrage. Mougel Bey had designed the barrage to produce a difference of 13 feet 1 inch in the water levels; and after grouting it actually had sustained 14 feet 3 inches. It was, however, decided to produce only 9 feet 10 inches difference in water level at the barrage, and an additional 10 feet 6 inches by a subsidiary weir some 3000 feet downstream. An additional "command" of 6 feet or more was thus secured.

Sketch No. 270 shows the design of the weir, which is only economically justifiable by the fact that the financial rewards of success were so enormous that any extravagance was permissible, provided that a real success was obtained without delay.

The core wall was constructed as follows (*P.I.C.E.*, vol. 158, p. 17). A wooden frame (resembling a box with neither top nor bottom) 32 feet 9 inches by 9 feet 10 inches, and from 19 to 26 feet high, was formed by driving groups of sheet piles 3 inches thick and 4 feet 11 inches (1·5 metres) wide, into the river bed. This box was lined internally with sacking, held in position by match boards nailed on at intervals of 1 metre. Four perforated pipes, 5 inches in diameter, were then fixed vertically at intervals of 8 feet 2 inches along the centre of the box. The box was then filled to the desired depth with a mixture of rubble stone, with 20 per cent. of road metal, and 15 per cent. of pebbles. The proportion of pebbles was possibly somewhat too great to yield the best results ; but, per contra, the wall is certainly thicker than required.

An unperforated 3-inch pipe was then run down inside two of the 5-inch pipes, and floats adjusted so as to sink in water, but to float on grout, were placed in the other two. Grout was then poured steadily, and, as far as possible, continuously down the 3-inch pipes, until the floats indicated that there was about 1 foot 8 inches (0·5 metre) in the other 5-inch pipes. The 3-inch filling pipes and floats then changed places, and grouting from alternate ends was



SKETCH No. 270.—Weir below Delta Barrage.

continued in this manner until from 5 to 7 feet (1·5 to 2 metres) of grout had been poured in. Work was then stopped until this bottom layer had set solid. Thereafter grouting was carried on steadily until the desired height had been attained.

The pouring was thus regulated so as to cause the grout to rise uniformly all over the area of the box. In addition, the box was, as far as possible, made grout-tight. It will consequently be evident that the necessity for preventing the grout from mixing with the water was fully realised ; and, laitance being lighter than grout, it will be plain that any laitance produced floated up, and finally appeared as a scum at the top of the box.

The work was quite successful, although it is hard to see how a wall of such magnificent dimensions could possibly fail, unless the construction was radically bad, and almost vile in quality.

Let us now consider an unsuccessful piece of work. The Staines reservoir supply channel was of the section shown in Sketch No. 269. The concrete side considered as a retaining wall was avowedly weak, and the design is perfectly justified once it is realised that the channel is only the first of a series, and that it will later be supported on either side by additional channels. In one place the channel crossed a bed of peaty soil overlying gravel, carrying water which was flowing somewhat rapidly at right angles to the line of the channel. The probability of failure had been recognised, and special designs for strengthening the wall were prepared as soon as the soil was exposed in the excavation. As a matter of fact, the wall did not fail; but open horizontal joints were formed in the concrete, due to the cement having been washed out before it had set. It was therefore decided to inject grout, and to see whether this would suffice. Grout was consequently pumped into the wall and soil, under a pressure of about 40 to 50 lbs per square inch. The results were unsatisfactory; and the wall was repaired with brickwork, and was strengthened against the abnormal thrust of the peat by jack arches. The failure of the grouting process had been predicted, and the reasons are obvious.

In the first place, it was impossible to prevent water flowing from the gravel through the open joints into the channel, while the grout was setting. The gravel was apparently traversed by a stream of water under a pressure which was greater than that of the water in the channel.

Secondly, the joints which had to be grouted were thin, and were horizontally directed; and, in consequence, laitance accumulated in a layer at the top of each joint. Union between the grout and the horizontal under side of each block of concrete which roofed a joint was thus prevented. The peat also formed a very efficient filter for retaining the particles of cement.

The principles of grouting work are now plain. A viscid liquid is dealt with, which does not readily fill thin cracks (especially if they are horizontal), and which carries on its surface a deposit of useless scum ("laitance"). Thus, grouting is only likely to prove successful when the interstices which have to be filled are fairly large (although not so large as to cause the grout to be drowned), are not traversed by flowing water, and are not roofed by flat or concave surfaces, which will either catch the laitance, or will prevent the grout from expelling air or water.

When rough rubble stone is grouted up in the manner described above, the volume of cement used is about 35 to 40 per cent. of the volume occupied by the stone. In coarse sand the expenditure of cement is generally slightly less (say 30 to 35 per cent.); but it must be realised that to satisfactorily grout any material which is finer grained than fine gravel is a difficult matter, and should only be attempted under very favourable circumstances.

If regular blocks of stone are fitted together, so that horizontal joints do not occur (*e.g.* brickwork laid dry, with the courses inclined at 45 degrees to the horizontal, or hexagonal blocks as shown in Sketch No. 270), the proportion of cement may be reduced to 10, or even 6 per cent. It is, however, obvious that the greatest advantage of subaqueous grouting work is then lost, as the blocks must be laid by a diver.

Certain small scale operations (100 to 150 cube feet) of my own enable me to state that this procedure will produce far more satisfactory results than the usual method of depositing 3 to 1 concrete (actually 1 cement to 4 of sand and aggregate) in bags. The dry brickwork is not easily laid so as to fill up irregular cavities, and the method is best adapted for plugging square wells.

Artificial Methods of producing Impermeable Concrete or Mortar.—The principle underlying these methods consists in mixing some extremely fine grained substance with the cement, or mortar, in order to fill the void spaces which probably exist, either between the cement grains, or due to the concrete not being voidlessly proportioned. Theoretically, I believe the substance should be a colloid, but this term is at present employed somewhat loosely.

The dangers are obvious. No doubt the substance can fill the voids, but there is no certainty that it will not go further and insert itself between the particles of cement, and prevent their union. Thus, any local excess of the "filler" may produce weakness, and tests of samples mixed under laboratory conditions can hardly be regarded as showing what may happen if the large scale mixing is carried out with ordinary commercial care.

The typical process is Sylvester's. This consists in producing a filler by mixing about 1 per cent. of powdered Castile soap, and 1 per cent. of alum (reckoned on the volume of the cement) with the cement. The mortar produced is water-tight, and works nicely. I have been accustomed to use the process regularly when building brickwork walls in hydraulic lime mortar for fixing gauging notches, and these walls were invariably "drop tight," even under the most refined tests. The walls, however, were always proportioned so as to be capable of resisting the water pressure even if the mortar possessed no tensile strength, and samples of the mortar taken from the workmen's pans were invariably 5, or 10 per cent. weaker under tensile tests than similar samples which were not treated by Sylvester's process. In careful laboratory work, however, this difference did not occur. The standard Indian practice is to apply the Sylvester process to a 1 inch, or 2 inch facing layer of the mortar only, or to lay the face course only of a brickwork wall in Sylvestered mortar. Even under this restricted application, the process has not found much favour, and is usually regarded as a substitute for careful workmanship in the whole wall.

Gaines (*Trans. Am. Soc. of C.E.*, vol. 59, p. 165) has proposed the following:

(*a*) About 1, to 2 per cent. by weight of alum in the water used for mixing concrete, or,

(*b*) To substitute finely ground powdered clay for from 5, to 10 per cent. of the cement.

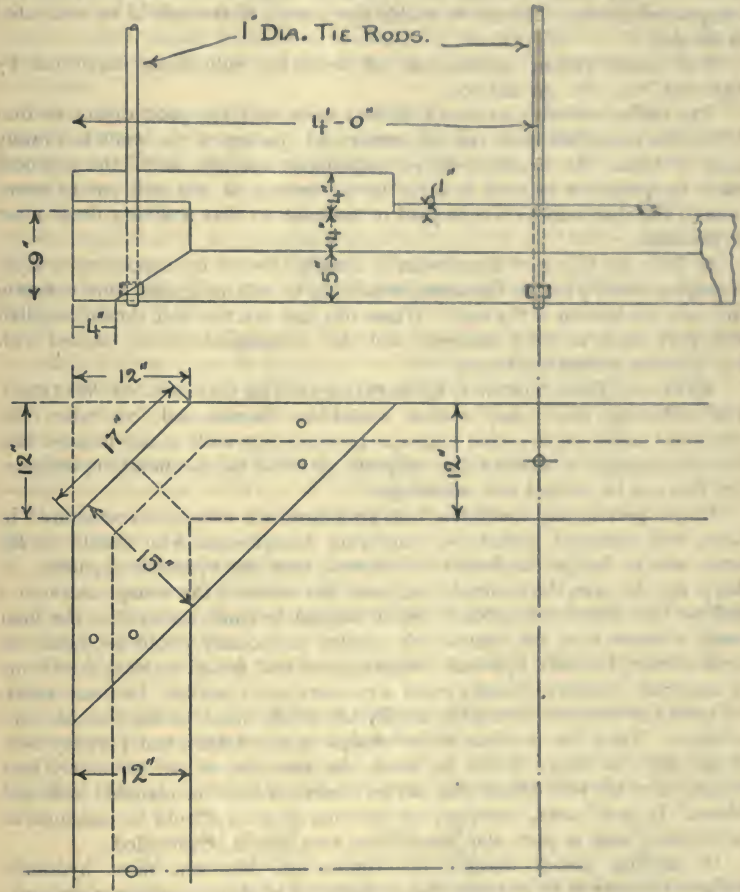
(*c*) A combination of both processes.

The mortar produced is far more impermeable than untreated mortar of the same proportions, and the 7, to 28 days' tensile tests show a gain in strength. This cannot, however, be considered as ensuring that the concrete thus made will be permanently stronger, since high initial tensile strength is often found to be accompanied by a comparative decrease in strength two or three years after making.

The present state of the question appears to be that the impermeability of all concretes, or mortars, can be increased by an admixture of inert fillers of the character described above. In small works which are carefully inspected, where impermeability is the great desideratum, the processes may be employed with advantage. In large work, until further experience has been accumulated, the risk of a decrease in strength is too great to be lightly undertaken. Reference is made to the discussion concerning clayey sands on page 972.

Wells.—The methods employed in sinking shafts, or tunnelling, under water, by means of compressed air, can only be advantageously treated by a specialist. The following section is therefore devoted solely to the consideration of the open brickwork well used in India.

Sketch No. 271 shows a typical well curb. The brickwork is built on this curb to about 10 feet high; and, when set, the well is sunk about 10 feet. A fresh height of brickwork is then built, allowed to set, and the well is again sunk. The object being to produce a cut-off wall, or stop against percolation, the well is square in section, and the distance between adjacent wells should



Sketch No. 271.—Indian Well Curb.

not exceed 6 inches. The methods adopted in order to grout up, or close the space between two adjacent wells are discussed on page 983.

A well of this design can be sunk in sand, if fairly free from boulders or tree trunks, to depths of 30 or 40 feet below subsoil water level by removing the sand from its inside with a Bell's grab, or any usual type of small dredger. Bell's grab being adapted to work close up to the corners of the well is more efficient than the "orange peel" shaped grabs. The volume of sand removed is roughly about twice the volume of the well, and this fact is in no way disadvantageous, since the sand forms a very cheap loading for the well, and the quantity removed from the well usually suffices to produce the required load.

For greater depths, or for gravel, or clayey sand, and especially if tree trunks or large boulders occur, the services of a diver are usually required, or compressed air must be used to enable the bottom of the well to be excavated in the dry.

The usual Indian applications of wells are sufficiently illustrated by Sketches Nos. 182, 192 and 200.

The Indian well-sinkers form a distinct trade, and are good divers, so that difficulties connected with the alignment and spacing of the wells but rarely arise in India. In countries where well-sinkers are less skilful the engineer must be prepared to send a diver to the bottom of one well out of every twenty, and then remove a cube yard of material, so that a diving dress must be provided.

In India the filling of these wells is usually effected by depositing a layer averaging about 3 feet in thickness, consisting of rich hydraulic lime concrete laid over the bottom of the well. When this has set, the well should be filled with pure sand, or weak concrete, and this covering should be capped with 2 or 3 feet of arched brickwork.

METALLIC CONSTRUCTION AS APPLIED TO THE CONTROL OF WATER.—The following notes may appear somewhat disconnected, but when the conditions under which a civil engineer procures ironwork at the present date are considered, it is believed that they will be found to contain all the information that can be utilised with advantage.

Under present-day conditions iron structures are usually manufactured in large, well equipped workshops, employing draughtsmen who should be far better able to design the details of ironwork than any hydraulic engineer. If this is not the case, the hydraulic engineer has selected the wrong contractor; and his best efforts will probably be so hashed in construction that the final result is worse than the contractor's unaided inefficiency would produce. In certain cases, however, hydraulic engineers are still forced by local conditions to construct ironwork (usually small structures only) locally. In these cases, the tools and workmen available usually set certain limits to the possible construction. Thus, the problem of the design of the details, and possibly even of the large members, is not so much the selection of the absolutely best design, as of the best design that can be constructed by the available tools and labour. In such cases, therefore, low working stresses should be assumed in calculation; and, in particular, ample rivet area should be provided.

In making sketch designs for contractors, however, most hydraulic engineers appear to be unaware of the cheapness of planing, or surfacing work, when effected in a well equipped machine shop. As a general rule, whether steel or cast iron work (and even more so in the case of brass, or bronze work) is considered, wood, lead, or felt packing is nowadays quite unnecessary, and a good faced metal to metalwork joint is usually quite as cheap, and is almost invariably well worth its extra cost. This remark does not apply to grouted joints, or to injections of grout for filling up cavities. Cement grout is a good preservative of metal work, and can be renewed, if necessary, during ordinary maintenance. It is, however, obviously inadvisable to design a joint for thin lead packing, or metal to metal, and to afterwards expect an injection of grout to compensate for defective workmanship. Thus, joints which are intended to be grouted should be designed with that object in view.

The following peculiarities of hydraulic construction are, however, frequently not entirely realised by the designers of ironwork, and are therefore set forth. The notes are divided into the two following classes :—

Class I.—I believe that these peculiarities are absolutely certain facts, and would consider myself justified in insisting on the modification of any design that, in important portions of the structure, was not in accordance with these conditions.

Class II.—I consider that these peculiarities are important, but I realise that the general agreement of engineering opinion concerning such matters is by no means well established. Some of them may therefore appear to be merely personal fads. In my own practice I have been accustomed to consider all of them as of only slight importance. In most cases they are merely matters to be discussed with the draughtsman, and his views may usually be accepted if he can show any reason for their adoption beyond : "That was how I happened to put it on the paper." Subject to these remarks we have as follows :

Class I. **(General Design).**—Most of the metallic structures employed by hydraulic engineers are of such a size in relation to the stresses produced that they are more likely to be deficient in stiffness than in strength. Thus, the first test of a structure is concerned with its deflection, rather than the stress in pounds per square inch.

Using Inches as the Unit of Length.—*Deflection.*—Consider a structure loaded in a manner similar to an ordinary beam.

Let δ, represent the central deflection, in inches, in the direction in which the load acts.

Let i, represent the angle of slope of the beam at its end.

Let l, represent the span in inches.

Let I, represent the moment of inertia of the cross-section of the beam, considered in $(\text{inches})^4$.

Let M, represent the maximum bending moment which the beam has to sustain, in inch-pounds.

Let E, represent the modulus of elasticity, in pounds per square inch. E = 28 to 30 million pounds per square inch for steel.

The case of a simply supported beam of uniform cross-section, under a uniform load of w, pounds per inch run, will be considered in detail. We have :

$$\delta = \frac{5}{384}\,\frac{wl^4}{EI} = \frac{5}{48}\,\frac{Ml^2}{EI}, \qquad i = \frac{wl^3}{24EI} = \frac{Ml}{3EI}; \qquad \text{since } M = \frac{wl^2}{8}$$

Now, the beam is assumed to support plating, or other water-tight material, and leakage only occurs where the plating is not continuous, *i.e.* at the supports of the beam, where presumably the plating is keyed or jointed into some other water-tight material. The strain that is produced at these joints is proportional to iS ; where S, is the length in inches of the seating of the beam on the wall or support. The value of iS, that will permit troublesome leakage evidently depends on :—

(*a*) The design of the joint.

(*b*) The workmanship of the joint.

but, obviously, if these factors are assumed to be constant, and if S, is assumed to be some fraction of l, and the volume of the leakage that produces trouble is also assumed to be proportional to l, the value of $\frac{\delta}{l}$ forms a fair measure of the powers of the structure to resist leakage.

The value usually found in existing hydraulic structures is :

$$\frac{\delta}{l}=\frac{1}{2000} \text{ to } \frac{1}{2500}$$

Assuming that :

$$\frac{\delta}{l}=\frac{1}{2308}=\frac{1}{7\times 384}, \qquad \text{we get :}$$

$$\frac{8Ml5}{384\,EI}=\frac{1}{7\times 384}, \qquad \text{or, } Ml=\frac{EI}{56\times 5}=100{,}000\,I.$$

Thus, for simply supported beams, $I=\frac{Ml}{100{,}000}$.

Similarly, for a beam the ends of which are built in so as to make $i=0$, we find that $I=\frac{Ml}{500{,}000}$.

The formulæ for a simply supported beam under a central load of W, pounds are :

$$\delta=\frac{Wl^3}{48\,EI}=\frac{Ml^2}{12\,EI}, \qquad i=\frac{Wl^2}{16\,EI}=\frac{Ml}{4\,EI},$$

since in this case $M=\frac{Wl}{4}$.

We thus get, $I=\frac{Ml}{125{,}000}$.

These formulæ must be applied with judgment, but the process of checking a design in the manner thus indicated may be applied with advantage not only to the whole beam spanning an opening, but also to those component members of a trussed girder which support plating, or other water-tight skins.

Where the designer's details are well thought out, as for instance in a case where stanching rods are used, or the seats of the beams are supported on spherical bearings (see Sketch No. 272), I have passed such values as $I=\frac{Ml}{70{,}000}$, in a simply supported beam under uniform load.

When cast iron is used, $E=14{,}000{,}000$ approximately.

Thus, $I=\frac{Ml}{200{,}000}$ for a simply supported beam, and $I=\frac{Ml}{1{,}000{,}000}$ for a built-in beam, and this last formula can rarely be applied, owing to the weakness of cast iron in tension.

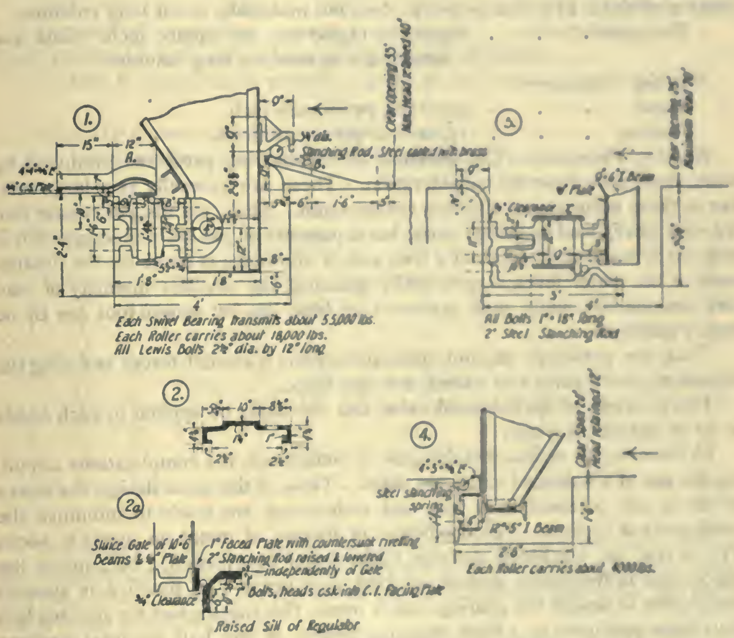
Strength of Structure.—The above rules usually lead to a structure which is of ample strength. In addition, the following rules require consideration :

No metal exposed to water should be less than $\frac{1}{4}$th of an inch thick, and, unless the metal can be painted and examined at very frequent intervals, this thickness should be increased to $\frac{3}{8}$ths of an inch. Also, most hydraulic structures contain bolts, and any bolt less than $\frac{5}{8}$ths of an inch in diameter is liable to be strained by unduly vigorous tightening. Thus, all members which contain bolt holes must be at least $\frac{3}{8}$ths of an inch thick, and 3 inches wide. Consequently, quite apart from stress calculations, anything smaller than a $2\frac{1}{2}$ inch $\times\, 2\frac{1}{2}$ inch $\times\, \frac{3}{8}$ inch angle, or a 3 inch $\times\, \frac{3}{8}$ inch flat, is unusual.

If, however, stress calculations become of importance, the values of the working stresses may be taken from the table given below, subject to the following remarks:

The absolute values may be objected to, and for good material and first class workmanship I have myself used values 33 per cent. in excess; and for important and badly inspected structures values 20 per cent. in defect, when I considered that general circumstances justified their adoption, but I believe that the following principles should always be borne in mind:

(i) Owing to the fact that water loads (excluding water hammer, wave action, and shocks by floating bodies) produce no impact stresses, members in



SKETCH No. 272.—Stoney Sluice Bearings.

compression require a larger factor of safety than members in tension (provided that the rivet holes in these latter are either drilled or reamed out after punching).

(ii) In order to prevent leakage, the shearing area of the rivets should be relatively greater than the bearing area when hydraulic work is compared with bridge work.

(*a*) *Tensile Stresses*—

Steel . . . 11,000 to 13,500 lbs. per square inch,—the larger value being used when shock is entirely absent.

Cast iron . . . 1850 lbs. per square inch.

(*b*) *Compressive Stresses*—

Steel . . . Pin ends, $11500-44\frac{l}{r}$ lbs. per square inch.

" . . . Flat ends, $11500-30\frac{l}{r}$ lbs. per square inch.

In neither case should the stress exceed 11,000 lbs per square inch, and $\frac{l}{r}$ should never exceed 150, where l is the length, and r the radius of gyration $=\sqrt{\frac{I}{\text{area}}}$ of the member considered, in inches. This value may appear much less than the corresponding tensile stresses, but it must be remembered that the absence of shock in hydraulic works does not materially assist long columns.

Cast iron . . . 10,000 to 11,000 lbs. per square inch. Cast iron should not be used for long columns.

Riveting Stresses—

Shear . . . 9,000 lbs. per square inch.

Bearing . . . 15,000 lbs. per square inch.

Bearing Pressures.—The question of the bearing pressures produced by water loads now deserves consideration. These are generally very large relative to those occurring in ordinary construction. Thus, 300 lbs. per square foot is a very heavy load in bridge work, but a pressure of 300 lbs. per square foot is attained in water at a depth of 5 feet, and, if allowance is made for the fluctuations of the water level, is, practically speaking, the smallest intensity of load ever considered; while such pressures as 6000 lbs. per square foot are by no means unknown.

Thus, the pressures on, and consequently the frictional forces resisting the motion of, sluice gates and valves, are very large.

The principle of the balanced valve can obviously be applied to such heads as 50 or 100 feet of water.

If, however, an ordinary sluice gate is considered, the complications attending the use of a balanced valve are plain. Thus, in the usual design the heavy pressures are accepted as a fact, and endeavours are made to minimise the coefficients of friction. The coefficient of friction of metal on metal is about 0·15 to 0·20 for machined surfaces under water (not rusted); but, unless the sluice gate is frequently moved, sticking may occur. In practice it appears inadvisable to design the gearing which opens the sluice gates for stresses less than those produced by a force resisting motion of one half the total pressure on the gate (*i.e.* coefficient of friction = 0·50). The ordinary power required, however, will not greatly exceed two-fifths of that calculated on this assumption.

As a rule, the gate is mounted on wheels. Since the greatest force is required to start the motion, the initial friction should be diminished as far as possible. *Roller bearings* form the best solution.

Ball bearings have also been used, but invariably give trouble by corrosion, not at the point of contact of the balls and the ball races, but at a small distance away from this point. As this action may occur even in ball bearings which are filled full of vaseline and are not exposed to water, it appears impossible to prevent it.

Owing to the slow speed at which sluice gates and other hydraulic machinery move, the important value of the coefficient of friction is that which occurs when starting motion. According to Stribech (*Ztschr. D.I.V.*, 1902, p. 1464):

Let z be the total number of rollers, b inches in length, and d inches in diameter, which carry a total load of P lbs., from a shaft of r inches radius. Then, the maximum pressure is given by

$$p = \frac{5P}{zbd} \text{ lbs. per square inch,}$$

and the moment resisting motion is given by:

$$M = 1{\cdot}2\, Pf\frac{D_0}{d} \text{ inch-lbs.}$$

$$\text{where } D_0 = 2r + \frac{d}{z},$$

and if:

$p=$	42	70	106	142	213 lbs. per sq. inch.
$f=$	0·0018	0·0013	0·0011	0·0009	0·0007 inches,

these values also hold very approximately for initial motion.

The safe working load in pounds is given by Le Guern:

$$\{\overline{D+K} \times 10\}^2 \times 14$$

When D = diameter of roller in inches.

K = coefficient (given in the following table):

Dia.	K	Dia.	K	Dia.	K
$\frac{1}{4}$	0·1	$\frac{11}{16}$	0·625	$1\frac{1}{8}$	1·15
$\frac{5}{16}$	0·175	$\frac{3}{4}$	0·7	$1\frac{3}{16}$	1·225
$\frac{3}{8}$	0·25	$\frac{13}{16}$	0·775	$1\frac{1}{4}$	1·3
$\frac{7}{16}$	0·325	$\frac{7}{8}$	0·85	$1\frac{5}{16}$	1·375
$\frac{1}{2}$	0·4	$\frac{15}{16}$	0·925	$1\frac{3}{8}$	1·45
$\frac{9}{16}$	0·475	1	1·0	$1\frac{7}{16}$	1·525
$\frac{5}{8}$	0·55	$1\frac{1}{16}$	1·075	$1\frac{1}{2}$	1·6

The Stoney sluice is a well-known device. The live roller train reduces the total friction either initial or during motion to about 1 per cent. of the water and dead load. Sketch No. 272 shows the details of stanching rods and roller trains found in modern work.

The contrast between Fig. 1, which is adapted to a gravel-bearing river, and Fig. 3, which shows modern Indian practice in rivers carrying fine sand only, is instructive, and should be followed in future designs.

Class II. (**Special Cases**).—In all gearing work, worms seems to be preferable to toothed wheels. It would appear that the slight sliding motion that occurs in a worm and wheel is more effective in preventing rust than the rolling of one tooth over another, such as occurs in toothed wheels.

For securing water-tightness I have been accustomed to rely mainly on *stanching rods* (Sketch No. 272), as used in Stoney gates. A wooden stanching rod, weighted with lead or iron at the lower end, fitting roughly into a brickwork and wooden groove, will be found very effective, provided the wooden rod is not too rigid.

In cases where stanching rods are not advisable, a metal-to-metal fitting of the sluice gate to a rigid cast iron bearing plate appears to be the best design. Spherical seated bearing plates rapidly become useless when submerged.

Plating.—The joints of plating are not generally exposed to tensile stresses, and should be designed according to the rules for boiler work.

Leakage has been successfully prevented with far less rivet work by packing each joint before riveting up with tape, well covered with red lead. The practice has been permanently successful in the case of gas holders, where the conditions, apart from the pressures sustained, are quite as exacting (possibly more so) as in sluice gates. In sluice gates the process is apparently new, and one gate thus caulked certainly leaked badly ten years after installation. The plating of sluice gates may be proportioned either for strength or for deflection.

Plating of Gates.—The following investigation, in so far as it concerns the thickness of plating, may be regarded as applicable to such sluice gates as occur in irrigation and power canals. These gates are worked under the following conditions:

(*a*) Should the plating become entirely corroded, temporary repairs can be made with wood or cloth packings; and opportunities for permanent repairs occur, at the very worst, at least twice a year.

(*b*) The gates can be overhauled and inspected in the dry at least once a year. Under circumstances such as occur in sluice gates, closing the draw-off tunnels of reservoirs, or the filling gates of locks, the question of possible failure by corrosion is important, and an ample margin in excess of the thickness indicated below must be allowed.

The plating of a sluice gate may be considered as continuous over two spans, between three stiffeners, at least; and as subjected (except in the plating close to the water surface) to a uniform load of 0·43 D, lbs. per square inch, where D, is the depth in feet below the water surface, measured either to the centre or bottom of the interval between the three stiffeners considered.

Let t, represent the thickness of plating, in inches.

l, the span of the plate between the stiffeners, or cross girders, in inches.

The deflection δ is represented by:

$$\delta = \frac{2{\cdot}04\, l^4 \times 0{\cdot}43\, D \times 12}{E t^3}$$

where, E, is the elastic modulus for steel; that is to say, 28 to 30 million pounds per square inch.

Hence, if $\delta = \frac{l}{n}$, we get: $t = \frac{\sqrt[8]{D}\, l}{K}$;

where we have:

$n=$	500	1000	1500	2000	2500	3000
$K=$	129	102	89	82	75	71

Similarly, if the plating is not continuous over two spans, but is fixed at each support, we obtain a similar equation:

$$t = \frac{\sqrt[8]{D}\, l}{K_1}, \text{ where } K_1 = 1{\cdot}26\, K$$

So also, for strength, we have, if f, be the stress on the plating in pounds per square inch, reckoned without deductions for rivet holes:—

(i) Plating continuous over two spans:

$$t = \frac{l\sqrt{D}}{N}$$

(ii) Plating, not continuous, but fixed at the ends of each span:

$$l = \frac{t\sqrt{D}}{N_1}, \text{ where } N_1 = 1{\cdot}23\, N$$

and the values of N_1 are:

f =	10000	12500	15000	17500	20000 lbs. per square inch,
N =	175	196	215	232	248

As an example, take D = 12 feet, n = 1000, and $t = \frac{5}{16}$ inches.

Thus, we get $l = \frac{102 \times \frac{5}{16}}{\sqrt[8]{12}}$ inches $= \frac{32}{2{\cdot}29} = 14$ inches,

and for this value of l, $N = \frac{14 \times 16\sqrt{12}}{5} = 44{\cdot}8 \times 3{\cdot}46 = 155$, or, $f = 8850$ lbs. per square inch, approximately.

Allowing for rivet holes, the stress in the plates at the line of rivets over the centre girder is probably about 10500 lbs. to 11000 lbs. per square inch.

If we take $f = 12500$, we get $l = \frac{62{\cdot}5}{3{\cdot}46} = 17{\cdot}5$ inches,

and $K = \frac{17{\cdot}5 \times 16}{5} \times 2{\cdot}29 = 56 \times 2{\cdot}29 = 129$, corresponding to a deflection of $\frac{l}{500}$, or 0·035 inches.

For the portion of the plating near the top of the tank, or sluice gate, where the variation in water load as the depth increases must be taken into account, we have a somewhat different set of equations:

$$\delta = \frac{2{\cdot}55}{384} l^4 \frac{0{\cdot}43\, D \times 12}{t^3 E} \text{ and } f\frac{t^2}{6} = 0{\cdot}128\, l^2 \times 0{\cdot}43\, D$$

where D is now measured to the centre of the span of plating, the upper end of which is assumed to lie at the water surface.

It will be found that the stiffness entirely fixes the thickness of the plating, until:

$$\frac{\sqrt{D}}{\sqrt[3]{D}} = D^{0{\cdot}166} = \frac{N}{K}$$

which, even under such abnormal conditions as $f = 10000$ lbs. per square inch, $n = 500$, gives:

$$D^{0{\cdot}166} = \frac{175}{129} = 1{\cdot}35, \text{ or } D = 6{\cdot}08 \text{ feet},$$

while the more reasonable conditions, $f = 12500$ lbs. per square inch, $n = 1000$, give:

$$D^{0{\cdot}166} = 1{\cdot}94, \text{ or } D = 53 \text{ feet}.$$

Also, the thickness of the plating at the water surface should be sufficient to sustain shocks from floating bodies and ice-thrust, if these are likely to occur.

Summing up:—It appears advisable to proportion the plating wholly by considerations of stiffness, and to make the absolute deflection under the water load about $\frac{l}{1000}$ at the most.

Framing of Gates.—The plating which forms the outer skin of sluice gates and water tanks is usually supported on stiffeners. In general, these are made of I or channel beams, and in hydraulic work are usually best proportioned by stiffness. The rules given on page 988 may be followed for sluice gates. In tanks, or gates provided with stanching rods, less stiffness will usually be found sufficient; and the rule $Ml = 150{,}000\,I$ will generally give good results, provided the strength is sufficient.

Framed Stiffeners.—The stresses in a trussed frame stiffener can be ascertained by the ordinary rules. The stiffness, however, is important.

Let δ, represent the deflection in any given direction, at any point R, in inches.

Let l, be the length of any member of the truss, in inches.

Let p, be the stress in pounds per square inch existing in this member, under the load that produces the deflection at the point R.

Let u, be the stress produced in this member by a force of one pound, applied at R, and acting in the direction in which δ is measured.

Then, we have
$$\delta = \frac{\Sigma pul}{E}$$

where the summation extends to every member of the truss, and the signs of p, and u, must be taken into account. That is to say, on the usual convention, p, and u, are positive when they represent compressive stresses, and negative when they represent tensile stresses. Thus, for example, p, compressive, and u, compressive, gives a positive term, and also p, tensile, and u, tensile; while p, tensile, and u, compressive, gives a negative term.

As a rule, the central deflection does not exceed $\frac{\text{Span}}{2000}$ in successful examples, but the connection between the central deflection and the liability to leakage is plainly not so close as in simple beams under uniform load.

Water Towers.—The use of elevated tanks as service reservoirs is quite common. The principles concerning the hydraulic design of these tanks are treated under Service Reservoirs (see p. 612). The following notes are concerned solely with the general form of the tank; and although the actual wording refers to steel plate tanks, the stresses found and the principles laid down are equally applicable to tanks built of reinforced concrete.

Sketch No. 273 shows the ordinary forms. The circumferential stress on the cylindrical portion is:

$$T = 0{\cdot}43DR \text{ lbs. per lineal inch.} \quad \textbf{(Inches.)}$$

where D is the depth below the top water level, in feet, and R is the radius of the tank, in inches. The details concerning the usual stresses, minimum thickness of plating, and workmanship in general, have already been considered.

The stresses in the bottom of the tank are, however, somewhat more complicated.

Consider any point P (see Sketch No. 273). Let the tangent to the bottom plating at P, make an angle a with the horizontal. Let r, be the distance of P, from the central axis of the tank, in inches. Let W, be the total weight of water and metal which lies on the farther side (from the supports) of P.

Then, the total stress per lineal inch in the plating in a radial direction (relative to the axis of the tank) is given by:

$$\sigma = \frac{W}{2\pi r \sin a} \text{ lbs. per lineal inch.}$$

In addition, a circumferential stress τ exists in the plating.

Put ρ for the radius of curvature of the diametral section of the bottom of the tank, in inches.

Then,
$$\pm\frac{\sigma}{\rho}\pm\frac{\tau}{r\cot a}=\pm 0{\cdot}43\mathrm{D}$$
where D is the depth in feet below top water level.

In the only two cases which it is proposed to consider we have:

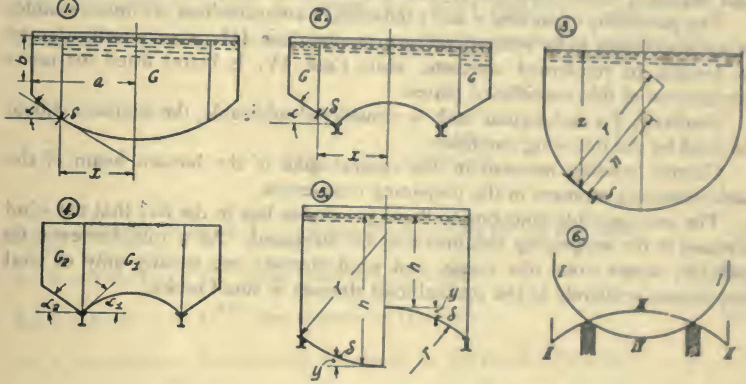
(*a*) Tanks with spherical bottoms. ρ = radius of the sphere.

(*b*) Tanks with conical bottoms. ρ = infinity.

The signs require careful consideration. If we denote compression by a positive sign, and tension by a negative sign, the sign of $\sigma 43\mathrm{D}$ is always the same as that of $\frac{\tau}{r\cot a}$.

In Sketch No. 273 the following four cases are shown:

	I.	II.	III.	IV.
σ . .	+	−	+	−
τ . .	−	+	+	−



SKETCH NO. 273.—Stresses in Water Tower Plating.

(See Hutte, vol. iii. p. 231. The sign conventions are different, but the various cases are solved in detail for a spherical bottom.)

We have for:

(*a*) Tanks with spherical bottoms. $\rho=r\cot a.$

(*b*) Tanks with conical bottoms. $\frac{1}{\rho}=0.$

Thus, σ and τ can be obtained.

The portion of the bottom near the supports also requires consideration.

This position is exposed to a radially directed load, which produces a circumferential compression:

$$\mathrm{P}=\frac{\mathrm{W}_1\cot a_1}{2\pi}\text{ lbs. (see Fig. No. 1)}$$

or,
$$\mathrm{P}=\frac{\mathrm{W}_2\cot a_2-\mathrm{W}_1\cot a_1}{2\pi}\text{ lbs. (see Fig. No. 4)}$$

according as the supporting ring is at the outer circumference of the tank, or at some distance less removed from the axis of the tank.

According to Halphen's investigation (*Fonctions Élliptiques*) the ring will preserve its form if:

$$P, \text{ be less than } \frac{3}{f}\frac{EI}{a^2}$$

where $a=r_1 \cot a_1$, or, $r_1 \cot a_2$, whichever is the greater. f is a factor of safety. I is the moment of inertia of the diametral section of the ring in $(\text{inches})^4$, and $E=30{,}000{,}000$ lbs. per square inch.

The entire sufficiency of this last formula is open to doubt. The dimensions are usually determined by the fact that the whole ring has to afford sufficient bearing area on the masonry for the load W_1, or W_1+W_2, and in good examples f is usually about 4, although it is improbable that the designer used the above formula.

The principle of supporting a water tower, not at its outer edge, but somewhere about the middle of each radius, is, however, a good one; and Intze's designs, which make the inner portion spherical and the outer portion conical, and $W_1 \cot a_1$ nearly equal to $W_2 \cot a_2$, produce a very pretty result.

The possibility of making σ and τ tensions, or compressions, at choice, enables other advantages to be secured. For example, Case III. is very well adapted to designs in reinforced concrete, while Case IV. is better fitted for tanks constructed of thin unstiffened plates.

Similarly, if a rectangular tank is considered advisable, the supports should be fixed by the following condition:

Central bending moment in the central span of the bottom beam of the tank = bending moment in the projecting cantilevers.

The only possible objection to these principles lies in the fact that the wind stresses in the supporting columns may be increased. As a rule, however, no difficulty arises from this cause, and wind stresses are usually only of vital importance relatively to the statical load stresses in small tanks.

TABLES

THE following Tables, if used in conjunction with Diagrams Nos. 1 to 10 and the ordinary tables of powers and areas employed by engineers, will be found to give all the information required in hydraulic calculations.

LIST OF TABLES.

TABLE No. 1.—VELOCITY HEAD FOR USE IN WEIR CALCULATIONS

VALUES OF $H=\frac{v^2}{2g}$, OR HEADS DUE TO VELOCITIES FROM 0 TO 4·99 FEET PER SECOND.

Velocity in Feet per Second.	Head in Feet.	Velocity in Feet per Second.	Head in Feet.
0·0	0·0000	2·5	0·0972
0·1	0·0002	2·6	0·1051
0·2	0·0006	2·7	0·1133
0·3	0·0014	2·8	0·1219
0·4	0·0025	2·9	0·1307
0·5	0·0039		
0·6	0·0056	3·0	0·1399
0·7	0·0076	3·1	0·1494
0·8	0·0099	3·2	0·1592
0·9	0·0126	3·3	0·1693
		3·4	0·1797
		3·5	0·1904
1·0	0·0155	3·6	0·2015
1·1	0·0188	3·7	0·2128
1·2	0·0224	3·8	0·2245
1·3	0·0263	3·9	0·2365
1·4	0·0305		
1·5	0·0350	4·0	0·2487
1·6	0·0398	4·1	0·2613
1·7	0·0449	4·2	0·2742
1·8	0·0504	4·3	0·2875
1·9	0·0561	4·4	0·3010
		4·5	0·3148
		4·6	0·3290
2·0	0·0622	4·7	0·3434
2·1	0·0686	4·8	0·3582
2·2	0·0752	4·9	0·3733
2·3	0·0822		
2·4	0·0895	5·0	0·3885

This table is based on $2g=64·32$; and, if the usual $2g=64·4$ be used, the fourth place of decimals will be found in error, being about one unit too high from $v=2·6$ to $v=3·6$, two units up to $v=4·4$, and three units beyond.

A more complete table by hundredths of a foot in v is given by Horton (*Weir Experiments*) and others. Reference is made to page 157 of Horton's treatise for a complete bibliography concerning weir tables.

TABLE No. 2.—VALUES OF $H^{1·5}$

H	$H^{1·5}$	H	$H^{1·5}$
0·00	0.0000	0·40	0·2530
0·01	0·0010	0·41	0·2625
0·02	0·0028	0·42	0·2722
0·03	0·0052	0·43	0·2820
0·04	0·0080	0·44	0·2919
0·05	0·0112	0·45	0·3019
0·06	0·0147	0·46	0·3120
0·07	0·0185	0·47	0·3222
0·08	0·0226	0·48	0·3325
0·09	0·0270	0·49	0·3430
0·10	0·0316	0·50	0·3536
0·11	0·0365	0·51	0·3642
0·12	0·0416	0·52	0·3750
0·13	0·0469	0·53	0·3858
0·14	0·0524	0·54	0·3968
0·15	0·0581	0·55	0·4079
0·16	0·0640	0·56	0·4191
0·17	0·0701	0·57	0·4303
0·18	0·0764	0·58	0·4417
0·19	0·0828	0·59	0·4532
0·20	0·0894	0·60	0·4648
0·21	0·0962	0·61	0·4764
0·22	0.1032	0·62	0·4882
0·23	0·1103	0·63	0·5000
0·24	0.1176	0·64	0·5120
0·25	0·1250	0·65	0·5240
0·26	0·1326	0·66	0·5362
0·27	0·1403	0·67	0·5484
0·28	0·1482	0·68	0·5607
0·29	0·1562	0·69	0·5732
0·30	0·1643	0·70	0·5857
0·31	0·1726	0·71	0·5983
0·32	0·1810	0·72	0·6109
0·33	0·1896	0·73	0·6237
0·34	0·1983	0·74	0·6366
0·35	0·2071	0·75	0·6495
0·36	0·2160	0·76	0·6626
0·37	0·2251	0·77	0·6757
0·38	0·2342	0·78	0·6889
0·39	0·2436	0·79	0·7022

TABLE NO. 2—*continued.*

H	$H^{1.5}$	H	$H^{1.5}$
0·80	0·7155	1·20	1·3145
0·81	0·7290	1·21	1·3310
0·82	0·7425	1·22	1·3475
0·83	0·7562	1·23	1·3641
0·84	0·7699	1·24	1·3808
0·85	0·7837	1·25	1·3975
0·86	0·7975	1·26	1·4144
0·87	0·8115	1·27	1·4312
0·88	0·8255	1·28	1·4482
0·89	0·8396	1·29	1·4652
0·90	0·8538	1·30	1·4822
0·91	0·8681	1·31	1·4994
0·92	0·8824	1·32	1·5166
0·93	0·8969	1·33	1·5338
0·94	0·9114	1·34	1·5512
0·95	0·9259	1·35	1·5686
0·96	0·9406	1·36	1·5860
0·97	0·9553	1·37	1·6035
0·98	0·9702	1·38	1·6211
0·99	0·9850	1·39	1·6388
1·00	1·0000	1·40	1·6565
1·01	1·0150	1·41	1.6743
1·02	1·0302	1·42	1·6921
1·03	1·0453	1·43	1·7100
1·04	1·0606	1·44	1·7280
1·05	1·0759	1·45	1·7460
1·06	1·0913	1·46	1·7641
1·07	1·1068	1·47	1·7823
1·08	1·1224	1·48	1·8005
1·09	1·1380	1·49	1·8188
1·10	1·1537	1·50	1·8371
1·11	1·1695	1·51	1·8555
1·12	1·1853	1·52	1·8740
1·13	1·2012	1·53	1·8925
1·14	1·2172	1·54	1·9111
1·15	1·2332	1·55	1·9297
1·16	1·2494	1·56	1·9484
1·17	1·2656	1·57	1·9672
1·18	1·2818	1·58	1·9860
1·19	1·2981	1·59	2·0049

TABLE No. 2—*continued.*

H	$H^{1.5}$	H	$H^{1.5}$
1·60	2·0238	1·80	2·4150
1·61	2·0429	1·81	2·4351
1·62	2·0619	1·82	2·4553
1·63	2·0810	1·83	2·4756
1·64	2·1002	1·84	2·4959
1·65	2·1195	1·85	2·5163
1·66	2·1388	1·86	2·5367
1·67	2·1581	1·87	2·5572
1·68	2·1775	1·88	2·5777
1·69	2·1970	1·89	2·5983
1·70	2·2165	1·90	2·6190
1·71	2·2361	1·91	2·6397
1·72	2·2558	1·92	2·6604
1·73	2·2755	1·93	2·6812
1·74	2·2952	1·94	2·7021
1·75	2·3150	1·95	1·7230
1·76	2·3349	1·96	2·7440
1·77	2·3548	1·97	2·7650
1·78	2·3748	1·98	2·7861
1·79	2·3949	1·99	2·8072
		2·00	2·8284

The original authority for the tables of $H^{1.5}$ is believed to be Francis (*Lowell Hydraulic Experiments*). The most complete set, covering thousandths of a foot from 0 to 1·49, and hundredths of a foot up to 12 feet, is given by Horton (*Weir Experiments*). No tables which permit quick calculation when the head is measured in fractions of an inch are known to me, and conversion into decimals of a foot on the basis

Inches .	1	2	3	4	5	6
Feet . .	0·08	0·17	0·25	0·33	0·42	0·50

7	8	9	10	11
0·58	0·67	0·75	0·83	0·92

is usually sufficiently accurate. The American Well Works publish a very excellent table of discharges based on Francis' formula, by sixteenths of an inch, but the results are given in U.S. gallons per minute.

TABLE No. 3.—VALUES OF $H^{2 \cdot 5}$ FOR USE IN CALCULATING DISCHARGE OF TRIANGULAR NOTCHES

H	$H^{2 \cdot 5}$	H	$H^{\cdot 25}$
0·40	0·1012	0·75	0·4871
0·41	0·1076	0·76	0·5036
0·42	0·1143	0·77	0·5203
0·43	0·1213	0·78	0·5373
0·44	0·1285	0·79	0·5547
0·45	0·1359	0·80	0·5724
0·46	0·1435	0·81	0·5905
0·47	0·1514	0·82	0·6089
0·48	0·1596	0·83	0·6276
0·49	0·1681	0·84	0·6467
0·50	0·1768	0·85	0·6661
0·51	0·1857	0·86	0·6859
0·52	0·1950	0·87	0·7060
0·53	0·2045	0·88	0·7264
0·54	0·2143	0·89	0·7472
0·55	0·2244	0·90	0·7684
0·56	0·2347	0·91	0·7900
0·57	0·2453	0·92	0·8118
0·58	0·2562	0·93	0·8341
0·59	0·2674	0·94	0·8567
0·60	0·2789	0·95	0·8796
0·61	0·2906	0·96	0·9030
0·62	0·3027	0·97	0·9266
0·63	0·3150	0·98	0·9508
0·64	0·3277	0·99	0·9752
0·65	0·3406	1·00	1·0000
0·66	0·3539	1·01	1·0252
0·67	0·3674	1·02	1·0508
0·68	0·3813	1·03	1·0767
0·69	0·3955	1·04	1·1030
0·70	0·4100	1·05	1·1297
0·71	0·4248	1·06	1·1568
0·72	0·4399	1·07	1·1843
0·73	0·4553	1·08	1·2122
0·74	0·4711	1·09	1·2404

Table No. 3—*continued.*

H	$H^{2·5}$	H	$H^{2·5}$
1·10	1·2691	1·40	2·3191
1·11	1·2982	1·41	2·3608
1·12	1·3275	1·42	2·4028
1·13	1·3573	1·43	2·4453
1·14	1·3875	1·44	2·4883
1·15	1·4182	1·45	2·5317
1·16	1·4492	1·46	1·5756
1·17	1·4807	1·47	2·6200
1·18	1·5126	1·48	2·6647
1·19	1·5448	1·49	2·7100
1·20	1·5774	1·50	2·7557
1·21	1·6105	1·51	2·8109
1·22	1·6440	1·52	2·8485
1·23	1·6778	1·53	2·8956
1·24	1·7122	1·54	2·9431
1·25	1·7469	1·55	2·9910
1·26	1·7821	1·56	3·0395
1·27	1·8176	1·57	3·0885
1·28	1·8538	1·58	3·1379
1·29	1·8901	1·59	3·1877
1·30	1·9269	1·60	3·2381
1·31	1·9642	1·61	3·2890
1·32	2·0019	1·62	3·3402
1·33	2·0400	1·63	3·3920
1·34	2·0786	1·64	3·4443
1·35	2·1176	1·65	3·4972
1·36	2·1570	1·66	3·5504
1·37	2·1968	1·67	3·6040
1·38	2·2371	1·68	3·6582
1·39	2·2779	1·69	3·7129
		1·70	3·7681

TABLE No. 4.—AUXILIARY VALUES FOR SOLUTION OF KÜTTER'S FORMULA. (See p. 472.)

Kütter's formula for the value of C in the equation

$$v = C\sqrt{rs}$$

is—

$$C = \frac{\frac{1·811}{n} + \left(41·6 + \frac{0·00281}{s}\right)}{1 + \frac{n}{\sqrt{r}}\left(41·6 + \frac{0·00281}{s}\right)}$$

If we put $\frac{1·811}{n} = a$, and $41·6 + \frac{0·00281}{s} = b$, the formula becomes

$$C = \frac{a+b}{1 + \frac{nb}{\sqrt{r}}}$$

TABLE OF *a*.

n	*a*
0·010	181·1
0·011	164·6
0·012	150·1
0·013	139·3
0·014	129·4
0·015	120·7
0·016	113·2
0·017	106·5
0·018	100·6
0·019	95·3
0·020	90·6
0·021	86·2
0·022	82·3
0·0225	80·5
0·023	78·7
0·024	75·5
0·025	72·4
0·026	69.7
0·027	67·1
0·0275	65·9
0·028	64·7
0·029	62·4
0·030	60·4
0·0325	55·7
0·035	51·7
0·0375	48·3
0·040	45·3

and a depends merely on the value of n selected, while b is a function of the slope only.

The values of a and b are tabulated below.

In practice it is usually simplest to calculate $C\sqrt{r}$ directly without obtaining the value of C.

We have
$$C\sqrt{r}=\frac{(a+b)r}{\sqrt{r}+nb}$$
and in this form the calculation requires

One addition : $a+b$.
One multiplication : $(a+b)r$.
One multiplication : bn.
One addition : $\sqrt{r}+bn$.
One division : $C\sqrt{r}=\dfrac{(a+b)r}{\sqrt{r}+bn}$.

TABLE OF b.

Slope.		b	Slope.		b
Absolute.	1 in		Absolute.	1 in	
0·01000	100	41·9	0·00009	11000	72·5
0·00500	200	42·2		12000	75·3
0·00250	400	42·7		13000	78·1
0·00200	500	43·0		14000	80·9
0·00100	1000	44·4		15000	83·8
0·00067	1500	45·8		16000	86·6
0·00050	2000	47·2		17000	89·4
0·00040	2500	48·6		18000	92·2
0·00033	3000	50·0		19000	95·0
0·00029	3500	51·4		20000	97·8
0·00025	4000	52·8		Beyond this limit the formula rests on slender, and probably erroneous evidence.	
0·00022	4500	54·2			
0·00020	5000	55·7			
0·00018	5500	57·1			
0·00017	6000	58·5			
0·00015	6500	59·9		22000	103·4
0·00014	7000	61·3		24000	109·0
0·00013	7500	62·7		26000	114·7
0·000125	8000	64·1		28000	120·3
0·00012	8500	65·5		30000	125·9
0·00011	9000	66·9		Formula is probably quite inapplicable beyond this limit.	
0·000105	9500	68·3			
0·00010	10000	69·7			

TABLES Nos. 5 AND 6.—BACKWATER FUNCTION $\phi(x)$, DROPDOWN FUNCTION $\chi(x)$. (See p. 483.)

As already pointed out, the backwater functions $\phi(x)$ are tabulated under the argument $\frac{1}{x}$. The dropdown functions $\chi(x)$ are tabulated under argument x.

x or $\frac{1}{x}$	$\phi(x)$	$\chi(x)$
0·0	0·0000	−0·6046
0·1	0·0050	−0·5046
0·2	0·0201	−0·4042
0·30	0·0455	−0·3025
0·31	0·0486	−0·2922
0·32	0·0519	−0·2819
0·33	0·0553	−0·2716
0·34	0·0587	−0·2612
0·35	0·0623	−0·2508
0·36	0·0660	−0·2403
0·37	0·0699	−0·2298
0·38	0·0738	−0·2192
0·39	0·0779	−0·2086
0·40	0·0821	−0·1980
0·41	0·0865	−0·1872
0·42	0·0909	−0·1765
0·43	0·0955	−0·1656
0·44	0·1003	−0·1547
0·45	0·1052	−0·1438
0·46	0·1102	−0·1327
0·47	0·1154	−0·1216
0·48	0·1207	−0·1104
0·49	0·1262	−0·0991
0·50	0·1318	−0·0878
0·51	0·1376	−0·0763
0·52	0·1435	−0·0647
0·53	0·1497	−0·0530
0·54	0·1560	−0·0412
0·55	0·1625	−0·0293
0·56	0·1692	−0·0172
0·57	0·1761	−0·0050
0·58	0·1832	+0·0074
0·59	0·1905	+0·0199

TABLES NOS. 5 AND 6—*continued.*

x or $\frac{1}{x}$	$\phi(x)$	$\chi(x)$
0·60	0·1980	0·0325
0·61	0·2058	0·0454
0·62	0·2138	0·0584
0·63	0·2221	0·0716
0·64	0·2306	0·0851
0·65	0·2395	0·0987
0·64	0·2486	0·1127
0·67	0·2580	0·1268
0·68	0·2677	0·1413
0·69	0·2778	0·1560
0·70	0·2883	0·1711
0·71	0·2991	0·1864
0·72	0·3104	0·2022
0·73	0·3221	0·2184
0·74	0·3343	0·2350
0·75	0·3470	0·2520
0·76	0·3603	0·2696
0·77	0·3741	0·2877
0·78	0·3886	0·3064
0·79	0·4039	0·3258
0·80	0·4198	0·3459
0·81	0·4367	0·3668
0·82	0·4544	0·3886
0·83	0·4733	0·4144
0·84	0·4932	0·4353
0·85	0·5146	0·4605
0·86	0·5374	0·4872
0·87	0·5619	0·5156
0·88	0·5884	0·5459
0·89	0·6173	0·5785
0·900	0·6489	0·6138
0·901	0·6522	0·6175
0·902	0·6556	0·6213
0·903	0·6590	0·6251
0·904	0·6625	0·6289
0·905	0·6660	0·6327
0·906	0·6695	0·6366
0·907	0·6730	0·6405
0·908	0·6766	0·6445
0·909	0·6802	0·6485

TABLES NOS. 5 AND 6—*continued.*

x or $\frac{1}{x}$	$\phi(x)$	$\chi(x)$
0·910	0·6839	0·6525
0·911	0·6876	0·6566
0·912	0·6914	0·6607
0·913	0·6952	0·6649
0·914	0·6990	0·6691
0·915	0·7029	0·6733
0·916	0·7069	0·6776
0·917	0·7109	0·6820
0·918	0·7149	0·6864
0·919	0·7190	0·6908
0·920	0·7231	0·6953
0.921	0·7273	0·6999
0·922	0·7315	0·7045
0·923	0·7358	0·7081
0·924	0·7401	0·7138
0·925	0·7445	0·7186
0·926	0·7490	0·7234
0·927	0·7535	0·7283
0·928	0·7581	0·7332
0·929	0·7628	0·7382
0·930	0·7675	0·7433
0·931	0·7723	0·7485
0·932	0·7772	0·7537
0·933	0·7821	0·7589
0·934	0·7871	0·7643
0·935	0·7922	0·7698
0·936	0·7974	0·7753
0·937	0·8026	0·7809
0·938	0·8079	0·7866
0·939	0·8133	0·7923
0·940	0·8188	0·7982
0·941	0·8244	0·8041
0·942	0·8301	0·8102
0·943	0·8359	0·8164
0·944	0·8418	0·8226
0·945	0·8478	0·8289
0·946	0·8539	0·8354
0·947	0·8602	0·8430
0·948	0·8665	0·8487
0·949	0·8730	0·8554

TABLES NOS. 5 AND 6—*continued.*

x or $\frac{1}{x}$	$\phi(x)$	$\chi(x)$
0·950	0·8795	0·8624
0·951	0·8862	0·8694
0·952	0·8931	0·8767
0·953	0·9002	0·8840
0·954	0·9073	0·8916
0·955	0·9147	0·8992
0·956	0·9221	0·9071
0·957	0·9298	0·9151
0·958	0·9376	0·9233
0·959	0·9457	0·9317
0·960	0·9539	0·9402
0·961	0·9624	0·9489
0·962	0·9709	0·9580
0·963	0·9799	0·9672
0·964	0·9890	0·9767
0·965	0·9985	0·9865
0·966	1·0080	0·9965
0·967	1·0181	1·0068
0·968	1·0282	1·0174
0·969	1·0389	1·0283
0·970	1·0497	1·0396
0·971	1·0610	1·0512
0·972	1·0727	1·0632
0·973	1·0848	1·0757
0·974	1·0974	1·0886
0·975	1·1105	1·1020
0·976	1·1241	1·1160
0·977	1·1383	1·1305
0·978	1·1531	1·1457
0·979	1·1686	1·1615
0·980	1·1848	1·1781
0·981	1·2019	1·1955
0·982	1·2199	1·2139
0·983	1·2390	1·2323
0·984	1·2592	1·2538
0·985	1·2807	1·2757
0·986	1·3037	1·2990
0·987	1·3284	1·3241
0·988	1·3551	1·3511
0·989	1·3841	1·3804

TABLES NOS. 5 AND 6—*continued.*

x or $\frac{1}{x}$	$\phi(x)$	$\chi(x)$
0·990	1·4159	1·4125
0·991	1·4510	1·4486
0·992	1·4902	1·4876
0·993	1·5348	1·5324
0·994	1·5861	1·5841
0·995	1·6469	1·6452
0·996	1·7213	1·7206
0·997	1·8172	1·8162
0·998	1·9523	1·9517
0·999	2·1834	2·1831
1·000		

Several discrepancies exist between the various published copies of this table. A systematic check could not be undertaken, but all values occurring in several years' work have been checked, and any suspicious changes examined. It is believed that the third place of decimals is always reliable ; and it is known that the fourth place is frequently two, or even three, units in error.

TABLE No. 7.—PUNJAB WATERCOURSES.

Table of Discharges for Watercourses.

Bed Slope	1/300	1/500	1/750	1/1000	1/1500	1/2200	1/2857	1/3636	1/4400	1/5500
Full Supply Depth in Feet for 1·0 Cusec Discharge.										
Bed Width. Feet.										
1·0	0·6	0·7	0·8	0·9	0·95	1·0	1·1	1·2	1·25	1·3
1·5	...	...	...	...	...	0·85	0·95	1·0	1·05	1·15
2·0	...	...	...	...	...	0·7	0·8	0·85	0·90	1·0
2·5	...	...	...	...	...	...	...	0·75	0·80	0·90
Full Supply Depth in Feet for 1·5 Cusec Discharge.										
1·0	0·7	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6
1·5	...	...	...	0·85	0·9	1·0	1·1	1·2	1·3	1·4
2·0	...	...	...	...	...	0·85	0·95	1·0	1·1	1·2
2·5	...	...	...	...	...	...	0·8	0·9	1·0	1·1
3·0	...	...	...	...	...	...	...	0·85	0·95	1·05
3·5	...	...	...	...	...	...	...	0·8	0·9	1·0
4·0	...	...	...	...	...	...	...	...	...	0·9
Full Supply Depth in Feet for 2·0 Cusecs Discharge.										
1·0	0·8	0·9	1·0	1·1	1·3	1·4	1·6	1·7	1·8	2·0
1·5	0·7	0·75	0·8	0·9	1·05	1·2	1·3	1·4	1·5	1·7
2·0	...	...	...	...	0·9	1·0	1·1	1·2	1·3	1·4
2·5	...	...	...	...	...	0·9	0·95	1·05	1·1	1·2
3·0	...	...	...	...	..	0·8	0·85	0·9	1·0	1·1
3·5	...	...	...	...	...	...	...	...	0·9	1·0
4·0	...	...	...	...	...	...	...	...	...	0·9
Full Supply Depth in Feet for 2·5 Cusecs Discharge.										
1·0	0·9	1·0	1·1	1·25	1·4	1·6	1·75	1·9	2·0	2·2
1·5	0·8	0·9	1·0	1·1	1·2	1·35	1·5	1·6	1·7	1·9
2·0	0·75	0·8	0·85	0·95	1·0	1·1	1·25	1·4	1·5	1·7
2·5	...	...	...	...	...	0·95	1·1	1·2	1·3	1·5
3·0	...	...	...	...	...	0·85	1·0	1·1	1·2	1·3
3·5	...	...	...	...	...	...	...	1·0	1·1	1·2
4·0	...	...	...	...	...	...	...	0·9	1·0	1·1

TABLE NO. 7—*continued.*

Bed Slope	1/300	1/500	1/750	1/1000	1/1500	1/2200	1/2857	1/3636	1/4400	1/5500
FULL SUPPLY DEPTH IN FEET FOR 3·0 CUSECS DISCHARGE.										
Bed Width. Feet.										
1·0	1·0	1·1	1·25	1·4	1·6	1·8	1·9	2·0	2·1	2·2
1·5	0·9	0·95	1·05	1·2	1·4	1·6	1·7	1·9	2·0	2·1
2·0	0·8	0·85	0·9	1·0	1·2	1·4	1·5	1·6	1·7	1·8
2·5	0·7	0·75	0·8	0·9	1·0	1·2	1·3	1·4	1·5	1·6
3·0	...	...	...	0·8	0·9	1·0	1·1	1·2	1·3	1·4
3·5	...	...	...	...	0·8	0·9	1·0	1·1	1·2	1·3
4·0	...	...	...	...	...	0·8	0·9	1·0	1·1	1·2
FULL SUPPLY DEPTH IN FEET FOR 3·5 CUSECS DISCHARGE.										
1·0	1·1	1·2	1·4	1·5	1·7	1·9	2·0	2·1	2·3	2·5
1·5	0·95	1·05	1·2	1·3	1·5	1·7	1·8	1·9	2·0	2·2
2·0	0·8	0·9	1·0	1·1	1·3	1·5	1·6	1·7	1·8	2·0
2·5	...	0·8	0·9	1·0	1·1	1·3	1·4	1·5	1·6	1·8
3·0	...	...	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·6
3·5	...	...	...	0·8	0·9	1·0	1·1	1·2	1·3	1·45
4·0	...	...	...	...	0·8	0·9	1·0	1·1	1·2	1·3
FULL SUPPLY DEPTH IN FEET FOR 4·0 CUSECS DISCHARGE.										
1·0	1·2	1·35	1·5	1·7	1·9	2·1	2·2	2·3	2·5	2·7
1·5	1·0	1·15	1·3	1·45	1·6	1·8	1·9	2·05	2·2	2·45
2·0	0·9	1·0	1·1	1·2	1·4	1·5	1·6	1·8	2·0	2·2
2·5	...	0·9	1·0	1·1	1·2	1·3	1·4	1·6	1·8	2·0
3·0	...	...	0·9	1·0	1·1	1 2	1·3	1·4	1·6	1·8
3·5	...	...	0·8	0·9	1·0	1·1	1·2	1·3	1·45	1·6
4·0	...	...	...	0·85	0·9	1·0	1·1	1·2	1·3	1·5
FULL SUPPLY DEPTH IN FEET FOR 4·5 CUSECS DISCHARGE.										
1·0	1·3	1·5	1·7	1·9	2·1	2·3	2·4	2·5	2·7	2·9
1·5	1·1	1·3	1·5	1·6	1·8	2·0	2·1	2·2	2·4	2·6
2·0	1·0	1·1	1·3	1·4	1·6	1·7	1·8	1·9	2·1	2·35
2·5	0·9	1·0	1·1	1·2	1·4	1·5	1·6	1·7	1·9	2·1
3·0	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·7	1·9
3·5	...	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·55	1·7
4·0	...	...	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·6

TABLE NO. 7—*continued.*

Bed Slope	1/300	1/500	1/750	1/1000	1/1500	1/2200	1/2857	1/3636	1/4400	1/5500
FULL SUPPLY DEPTH IN FEET FOR 5·0 CUSECS DISCHARGE.										
Bed Width. Feet.										
1·0	1·4	1·6	1·8	2·0	2·2	2·4	2·5	2·65	2·8	3·0
1·5	1·2	1·4	1·6	1·7	1·9	2·1	2·2	2·35	2·5	2·7
2·0	1·1	1·2	1·4	1·5	1·6	1·8	1·9	2·1	2·2	2·45
2·5	1·0	1·1	1·2	1·3	1·4	1·6	1·7	1·9	2·0	2·2
3·0	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·7	1·8	2·0
3·5	...	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·8
4·0	...	0·8	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·65
FULL SUPPLY DEPTH IN FEET FOR 5·5 CUSECS DISCHARGE.										
1·0	1·5	1·7	1·9	2·1	2·3	2·5	2·7	2·8	2·9	3·1
1·5	1·3	1·5	1·7	1·8	2·0	2·2	2·4	2·5	2·6	2·8
2·0	1·2	1·3	1·5	1·6	1·7	1·9	2·1	2·2	2·3	2·6
2·5	1·1	1·2	1·3	1·4	1·5	1·7	1·9	2·0	2·1	2·3
3·0	1·0	1·1	1·2	1·3	1·4	1·5	1·7	1·8	1·9	2·1
3·5	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·9
4·0	...	0·9	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·7

These tables are based on the assumption that the side slopes of the excavation and banking are dressed to 1 : 1; and that 0·5′ = 6″ freeboard above full supply water level is given to the banks. The figures are the result of experience, rather than of definite calculation, and produce a very practical set of channels. Given the ordinary maintenance, breaches, or deficiencies in supply, rarely occur; and, if tested by calculation, it will be found that the larger channels, which receive more careful attention, are, relatively to the smaller channels, proportioned on this basis. Thus, no unduly excessive factor of safety is employed. Where labour is dearer, or less efficient than in India, I have been accustomed to employ these tables, but provide a greater freeboard than 6 inches.

GRAPHIC DIAGRAMS

THESE diagrams are prepared according to the principles laid down by M. d'Ocagne (*Traité de Nomographie*).

The present applications are believed to be original, although at least one hydraulic diagram based on these methods has been published (Hülte, vol. 3, p. 245).

Since the principle is a powerful aid in graphing nearly every hydraulic formula of general application, the following details may prove of utility.

Consider the line joining the points $A(-x_1, y_1)$ and $B(x_2, y_2)$.

The equation of this line is :

$$\frac{x+x_1}{x_2+x_1}=\frac{y-y_1}{y_2-y_1}$$

Putting $x=0$, we find that the intercept OC on the axis of y is given by, $y_0=\frac{x_1 y_2+x_2 y_1}{x_1+x_2}$; which determines the distance OC.

Thus, any formula of the type :

$$q=\frac{a_1 b_2+a_2 b_1}{a_1+a_2};$$

can be reduced to a graph, once the a's and b's are properly plotted.

The typical hydraulic formula is usually :

$$Q=El^n h^m,$$

or, taking logarithms,

$$\log Q-\log E=n \log l+m \log h;$$

and n and m are generally constants.

Thus, if we plot logarithmic scales of h and l, and insert a properly divided logarithmic scale (usually with a different unit for base) at the proper intermediate distance between the scales of log h and log l, the readings on this intermediate scale will represent log Q − log E, and the scale can therefore be graduated so as to read Q directly.

The real difficulties in preparing workable diagrams arise merely from the necessity of avoiding unduly acute intersections of the line AB which joins the points representing the given values of l and h, with the scale of log Q − log E on which the corresponding value of Q is read off.

Thus, in employing the following diagrams, we merely select the points corresponding to the two known quantities typified by l and h on the appropriate scales, and lay a straight edge across the diagram to join these points; the value corresponding to the graduation at which the line of the straight edge crosses the third scale will be found to represent the third (unknown) quantity typified by Q.

The diagrams have all been tested, not only by careful arithmetical checking, but by a period (in some cases, three or four years) of office use. They are drawn and reproduced on a scale which permits an accuracy of 1 per cent. to be obtained. This is amply, in fact over, accurate for all practical purposes except the comparison of the more refined observations that are rarely undertaken save under laboratory conditions.

It should, however, be noted that if these drawings are enlarged to more than double the size, arithmetical checking will disclose errors, which, in No. 1 amount to as much as 0·6 per cent.

LIST OF DIAGRAMS.

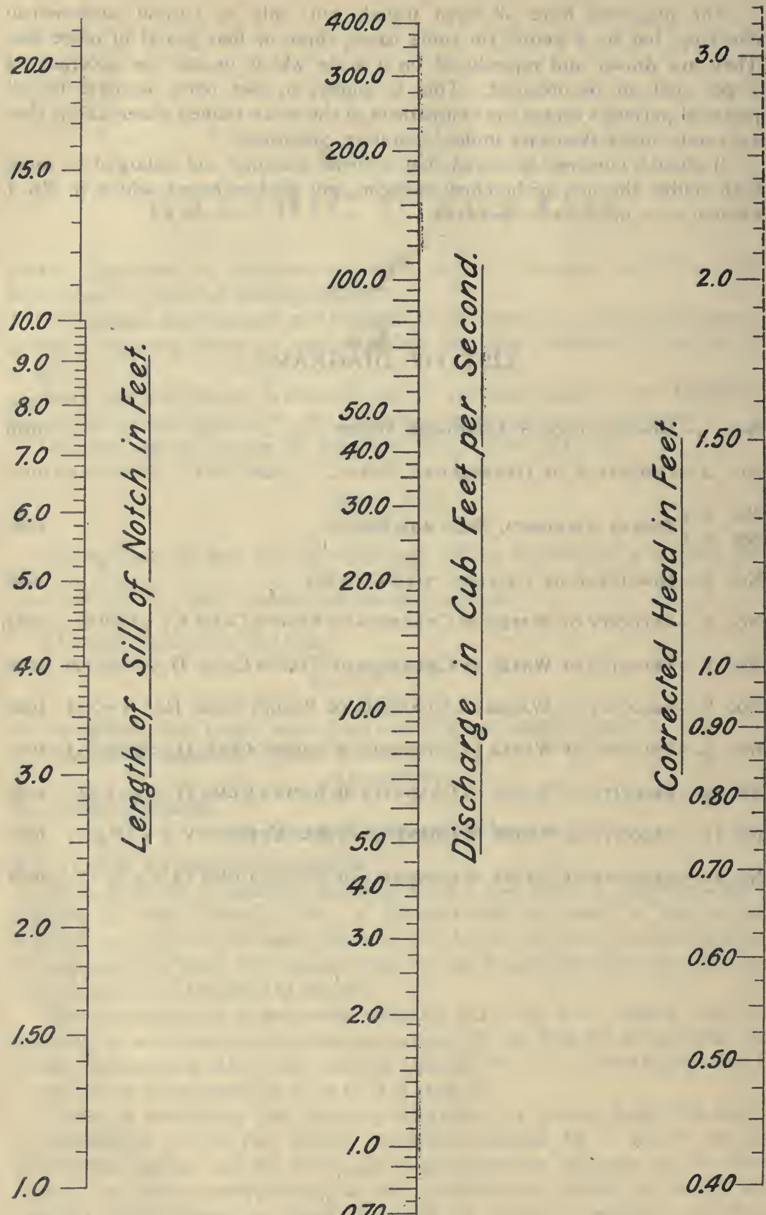


Diagram No. 1.—DISCHARGE OF SHARP-EDGED WEIRS.

DISCHARGE OF SHARP-EDGED WEIRS.

THE diagram on the opposite page represents the Discharge of Water over Sharp-Edged Weirs, as detailed on p. 112.

Head over 0·40 feet :

$$Q = 3{\cdot}110\ L^{1{\cdot}02}\ H^{1{\cdot}465};\ \text{with L less than 4 feet.}$$

$$Q = 3{\cdot}122\ L^{1{\cdot}016}\ H^{1{\cdot}475};\ \text{with L between 4 and 10 feet.}$$

$$Q = 3{\cdot}148\ L^{1{\cdot}013}\ H^{1{\cdot}485};\ \text{with L greater than 10 feet.}$$

$$\text{Where, } H = D + 1{\cdot}4h = D + 1{\cdot}4\frac{v^2}{2g}.$$

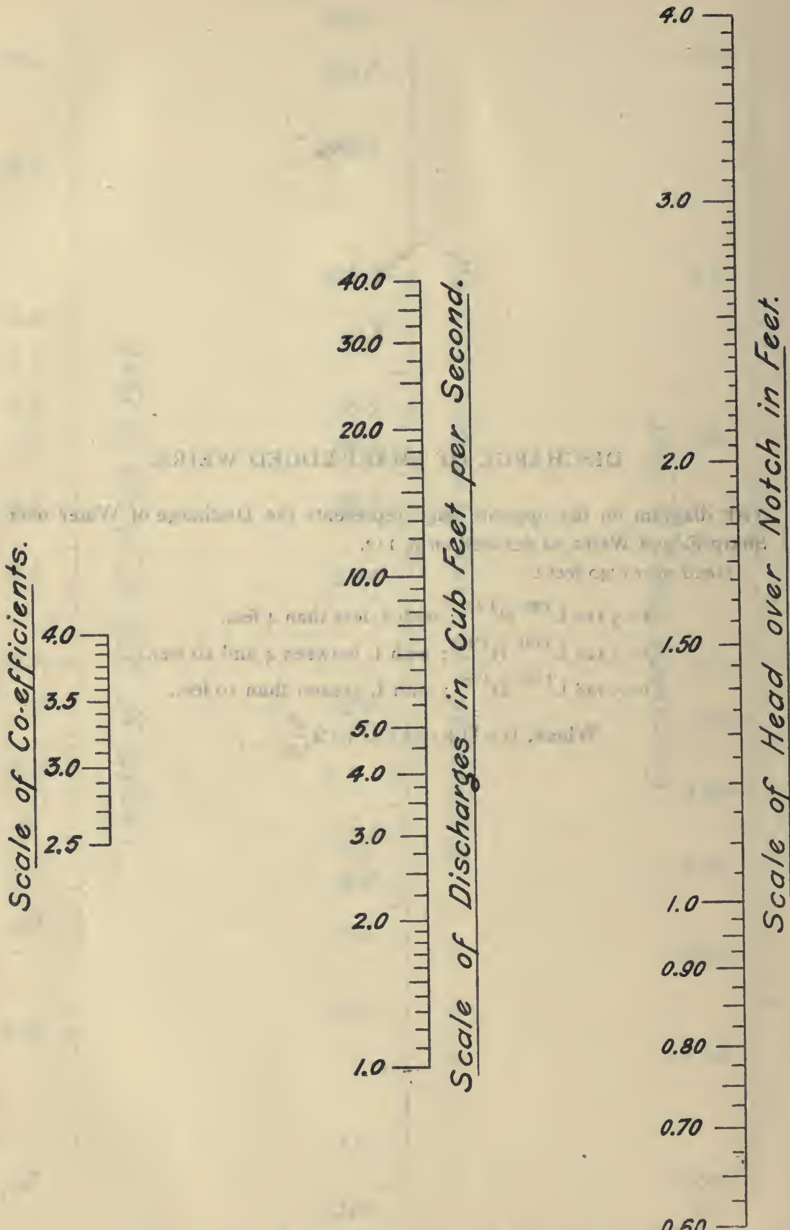


Diagram No. 2.—Discharge of Generalised Weir.

DISCHARGE OF GENERALISED WEIR.

THE diagram on the opposite page represents the general formula :

$$Q = CLH^{1 \cdot 5}$$

which has been found to be more or less applicable to all weirs, whatever their section may be.

The table on page 1020 shows the sections of all weirs for which it is known that

the value of C is constant,

and gives the range of H over which C remains constant.

Similarly, the table on page 1021 shows the sections of all weirs for which the expression

$$C = a + bH$$

holds ; and the upper and lower limits of H between which this expression holds good.

In every case, slight variations in C, not exceeding 1 per cent. either way, from the constant value or the straight line law have been disregarded.

The experiments on which these tables are founded are mainly those of Bazin ; but, for their conversion into English measure, and for a series of graphic plots of C, which have materially helped me in arriving at the tables, I am indebted to Horton (*Weir Experiments*).

Following Horton, in cases where the accuracy of the observation requires it, I generally put

$$H = D + h = D + \frac{v^2}{2g}$$

I may state that these tables have been in steady practical use for some five years, and that resort has been made to every accessible published observation in order to check them. It is believed that no result obtained by their use is likely to be more than 3 per cent. in error, so far as the value of C affects the result (*i.e.* errors in observing H are neglected). In most cases, the observed errors are less than 2 per cent., and this statement applies also to some 80 small scale experiments specially undertaken for the purpose of checking the tables.

Value of C	Section of Weir	Value of p	H exceeds	Authority
3·87	2:1 2:1	1·64	0·75	B
3·85	1:1	2·46	0·3	B
3·83	1:1 2:1	1·64	0·70	B
3·74	1·5 3 rad. 3	11·25	2·4	C
3·74	2:1 ·67	4·9	2·0	C
3·72	1:2 2:1	1·64	0·55	B
3·70	1:1 ·67	4·90	1·8	C
3·68	1:3 2:1	1·64	0·55	B
3·66	28·75 13·1 10 −2·1 25:1 7·9 8·05	7·9	0·2	C
3·62	as above but wire cloth on crest	7·9	0·2	C
3·58	2:1	1·64	0·5	B
3·58	3:1 ·67	4·9	2·0	C
3·52	1:1 5:1	1·64	0·5	B
3·50	2:1	2·46	0·25	B
3·49	4:1 ·67	4·9	1·7	C
3·43	4·7:1 ·67	4·9	2·0	C
3·42	1·5 1:12 7'	4·65	2·5	C
3·41	3:1	1·64	0·75	B

Value of C	Section of Weir	Value of p	H exceeds	Authority
3·39	5:1 ·67	4·90	1·8	C
3·33	Francis	Sharp-edged		Weir
3·32	·48	11·25	0·8	G
3·31	1·65	11·25	2·6	G
3·31	·93	11·25	1·6	G
3·30	1 6	11·25	2·5	G
3·22	46·04 10 6 7·8 15 6 2·8 1·8 −7·15 −6·75 −4·5 3·7 5·25 r=10 −8·06	7·75	0·6	C
3·21	as above but with smooth planks	7·75	0·4	C
3·22	6:1 0·2 0·75 6:1 10·58 12	4·53	1·8	C
3·15	as C = 3·21 but rough slope & wire cloth on crest	7·75	0·8	C
3·14	5:1	2·46	0·95	B
3·09	Theoretical value for broad sloping crested weir			
2·93	10:1	2·46	0·70	B
2·81	6·56 0·33 rad.	4·56	1·0	C
2·64	Flat Top 12·21 wide	11·25	1·8	G
2·64	Do. 5·88 wide	11·25	1·0	G
2·63	Do. 16·30 wide	11·25	1·0	G
2·62	Do. 8·98 wide	11·25	1·0	G

Diagram No. 3.—WEIRS WITH CONSTANT COEFFICIENTS.

Section of Weir	Value of C	Upper Limit of H	Section of Weir	Value of C	Upper Limit of H
W = 0·328 p = 2·46	2·76 + 1·80(H − 0·15)	0·55 F.D	m = 4	2·82 + 0·32(H − 0·6)	1·5 F.S.
W = 0·656 p = 2·46	2·81 + 1·05(H − 0·45)	1·05 D	m = 6	2·80 + 0·27(H − 0·5)	1·5 F.S.
W = 1·315 p = 2·46	2·64 + 0·51(H − 0·6)	1·55 F.S	l = 2	3·11 + 0·54(H − 0·45)	1·5 D.
W = 2·62 p = 2·46 p = 4·56	2·58 + 0·11(H − 0·5) 2·77 + 0·175(H − 2·0)	1·4 F.S. 5·5 F.S.	l = 1	3·06 + 0·67(H − 0·5)	1·5 D
W = 6·56 p = 2·46 p = 4·57	2·52 + 0·11(H − 0·5) 2·40 + 0·05(H − 2·0)	1·5 F.S. 5·2 F.S.	l = ½	2·78 + 0·01(H − 0·4)	1·15 D
W = 0·48 p = 11·25	2·50 + 1·40(H − 0·2)	0·8 D	Upstream Face Vertical	2·55 + 0·96(H − 0·25)	1·1 D
W = 0·93 p = 11·25	2·57 + 0·62(H − 0·3)	1·4 D	p = 5·28 Top Width = 5′	3·23 + 0·10(H − 0·8)	4·0 F
W = 1·65 p = 11·25	2·52 + 0·42(H − 0·6)	2·4 D	16 — 13 — 10	3·60 − 0·16(H − 0·4)	2·3 F
W = 2·17 p = 11·25	2·64 + 0·17(H − 1·5)	5·0 F	p = 11·25 l = 0·75	3·40 + ·03(H − 0·7)	3·2 F
W = 2·62 p = 2·46 p = 4·57	2·93 + 0·11(H − 0·4) 2·84 + 0·17(H − 1·0)	1·55 F.S. 5·2 F.S.	p = 11·25 l = 1·5	3·52 + 0·13(H − 1·0)	2·7 D
W = 6·56 p = 2·46	2·83 + 0·13(H − 0·6)	1·35 F.D.	p = 11·25 l = 2·86	3·56 + 0·20(H − 1·0)	3·1 F
n = 1	2·72 + 0·06(H − 0·2)	1·2 D	p = 11·25 l = 0·75 One end contraction	3·24 + ·00 (H − 0·6)	4·2 R
n = 2	2·60 + 0·85(H − 0·2)	1·15 D	p = 11·25 l = 1·5 One end contraction	3·37 + 0·12 (H − 0·8)	4·7 R
n = 3	2·60 + 0·70(H − 0·2)	1·2 D	4·5	3·35 + ·07 (H − 1·1)	3·8 F
n = 4	3·04 + 0·40(H − 0·65)	1·5 D	4·5	3·17 + 0·16(H − 0·2)	2·8 F
n = 5	2·73 + 0·54(H − 0·2)	1·15 D	8·04 — 0·65 — 3·85	3·07 + 0·11(H − 1·3)	4·1 F
1·317 2·46 2:1	2·80 + 0·47(H − 0·6)	1·5 F.S.			

Diagram No. 4.—Weirs with Linear Coefficients.

Diagram No. 5.—DISCHARGE OF ORIFICES.

DISCHARGE OF ORIFICES.

THE diagram on the opposite page represents the usual formula for orifice discharge :

$$Q = CA\sqrt{2gh}$$

The value of C selected is 0·625, so that the formula shown in the graph is :

$$Q = 5{\cdot}01A\sqrt{h}$$

Corrections for special values of C are easily made by remembering that, a change of 0·025 in C produces a variation of 4 per cent. in Q.

Thus :

C	Discharge.
0·575	0·92 Q
0·600	0·96 Q
0·625	1·00 Q
0·650	1·04 Q
0·675	1·08 Q
0·700	1·12 Q
0·725	1·16 Q
0·750	1·20 Q
0·775	1·24 Q
0·800	1·28 Q

For values of A and *h* outside the limits of the diagram it suffices to remember that :

(i) If A be doubled or halved, Q is doubled or halved ; and if A be multiplied by 10, Q is multiplied by 10.

(ii) If *h* be multiplied or divided by 4, Q is doubled or halved.
If *h* be multiplied by 100, Q is multiplied by 10.

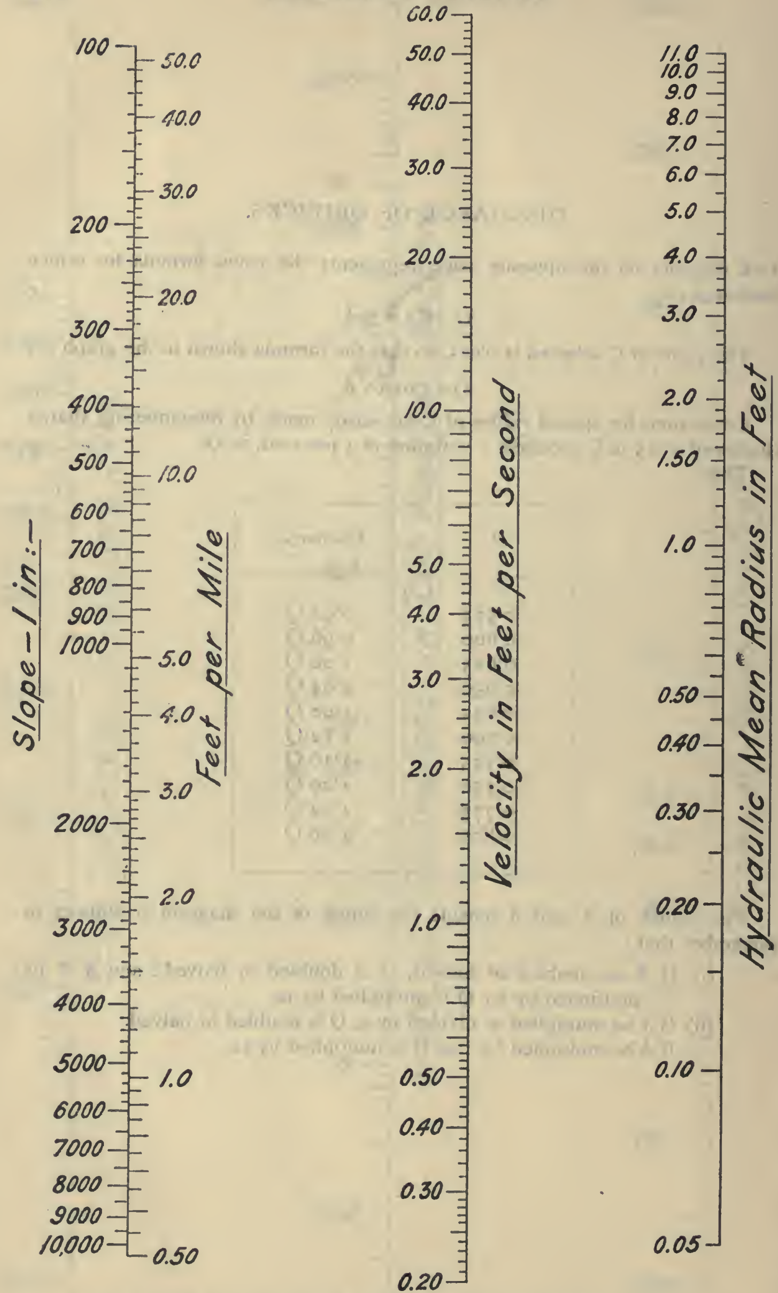


Diagram No. 6.—VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS I.; $\gamma = 0{\cdot}109$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS I.; $\gamma=0{\cdot}109$.

THE diagram on the opposite page, together with the five succeeding diagrams, represent the formulæ:

$$v=C\sqrt{rs};$$

$$\text{with, } C=\frac{157{\cdot}6}{1+\frac{\gamma}{\sqrt{r}}}:$$

as laid down by Bazin in 1897 (see p. 474). The present diagram corresponds to Class I., $\gamma=0{\cdot}109$: and Bazin assigns channels of smoothed cement and planed wood to this class.

In practical commercial engineering I am inclined to believe that very few channels are sufficiently carefully constructed to fall within this class.

I have succeeded in constructing a planed wooden flume which was smooth enough to give $\gamma=0{\cdot}100$, but did not feel sufficiently certain of it remaining in this smooth state to design the flume on this assumption, and therefore based my calculations on $\gamma=0{\cdot}13$.

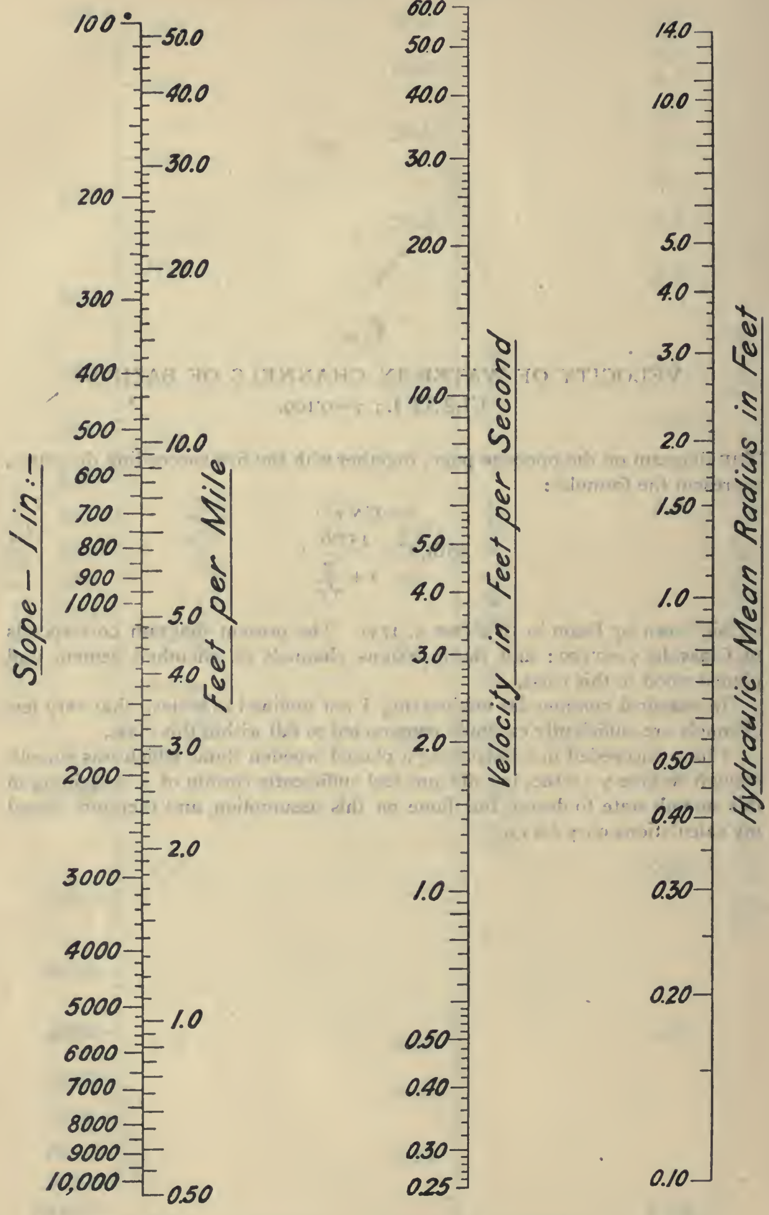


Diagram No. 7.—VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS II.; $\gamma=0{\cdot}290$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS II.; $\gamma = 0 \cdot 290$.

THE diagram on the opposite page corresponds to Class II.; or $\gamma = 0 \cdot 290$.

Bazin assigns all channels of planks, bricks, and cut stone to this class.

The remarks on pages 474 and 475 should, however, be consulted.

I do not usually design channels of this class in practical work.

The requisite smoothness of surface is easily obtained; but, under normal conditions, the extra cost of the more careful workmanship required usually exceeds the saving produced by the smaller dimensions of the channel. The exceptions generally prove to be large channels, with r exceeding say 3 feet.

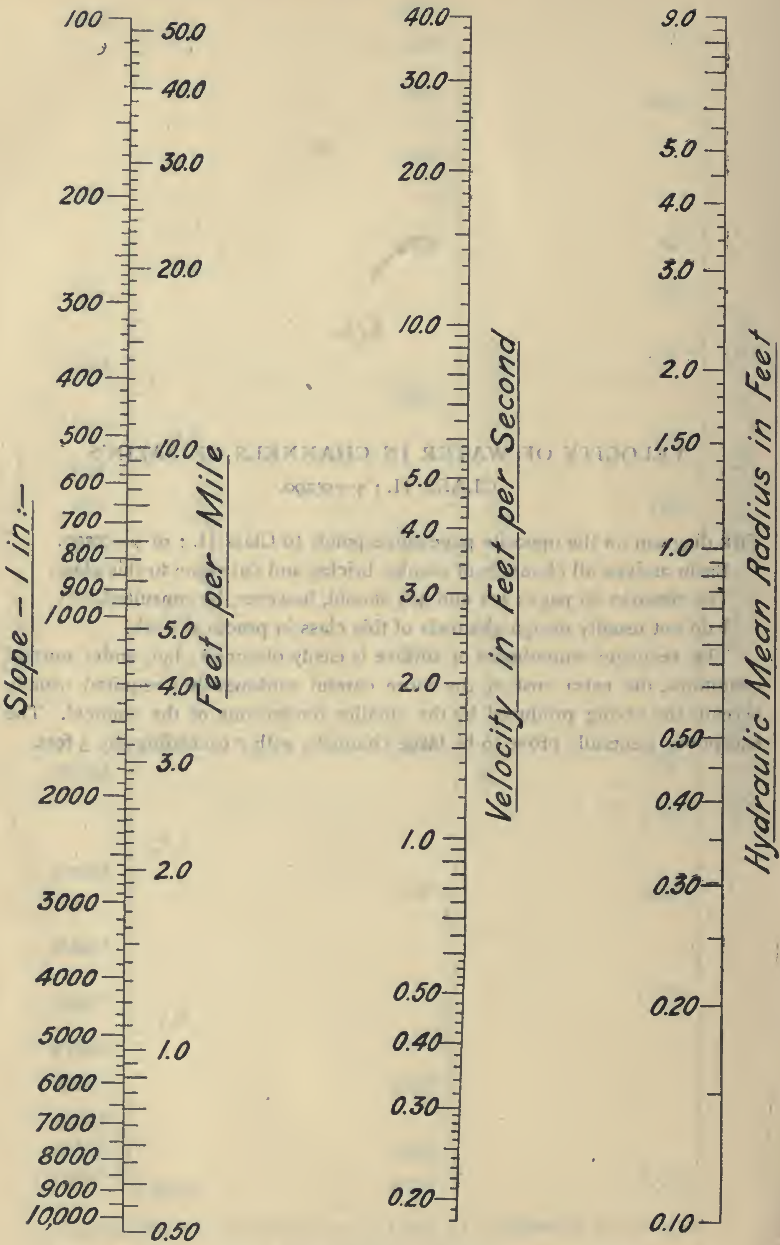


Diagram No. 8.—Velocity of Water in Channels of Bazin's Class IIa.; $\gamma = 0{\cdot}55$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS IIA.; $\gamma=0·55$.

THE diagram on the opposite page corresponds to a class determined by Bazin's $\gamma=0·55$.

This class was not specified by Bazin.

In practice, it is probably that most frequently used in the design of artificially lined channels. All slime encrusted channels, such as occur in sewerage and other works dealing with polluted water, rapidly pass into this class as they age, whether the original channel was smoother or rougher than is indicated by $\gamma=0·55$. The only exceptions appear to be those channels which are initially rougher to the eye than rough cast plaster work or brickwork enclosure walls,—*i.e.* relatively speaking, carelessly laid masonry or brickwork. Ordinary unrendered concrete which has not been deposited with excessive care also agrees very fairly with $\gamma=0·50$; and all slimed concrete rapidly reaches the state where $\gamma=55$.

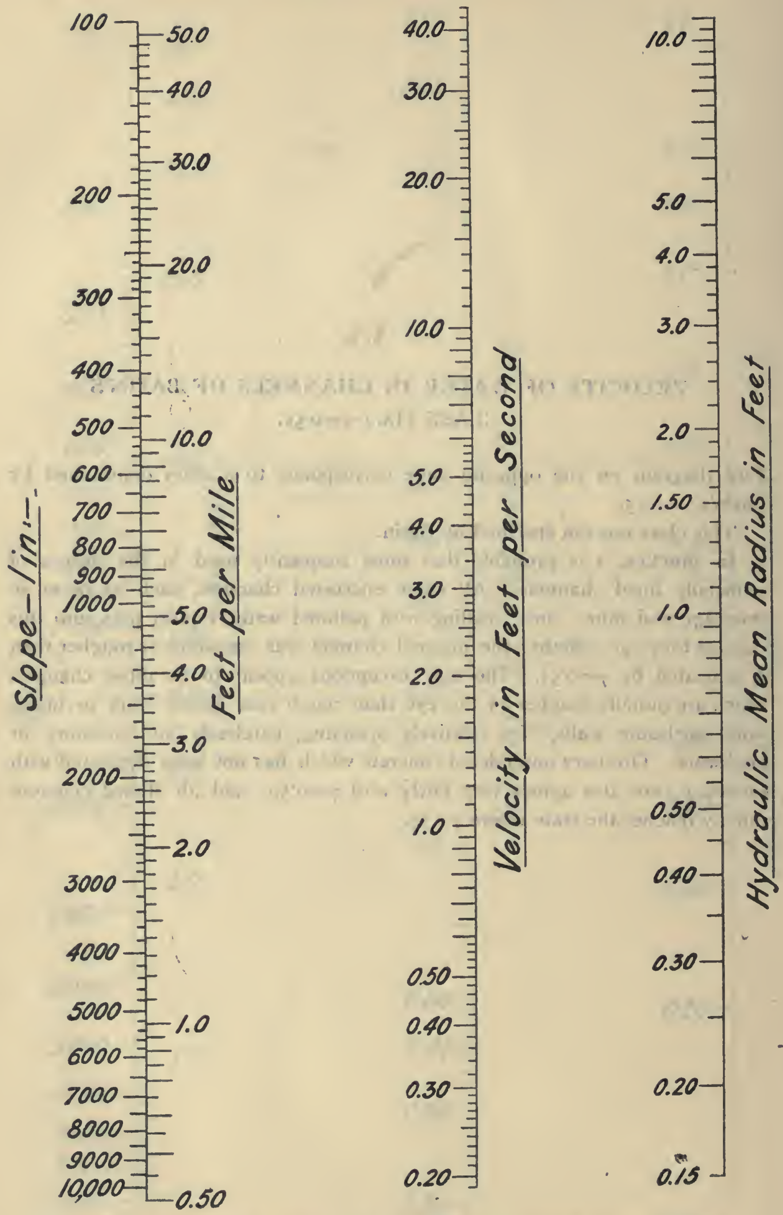


Diagram No. 9.—VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS III.; $\gamma = 0{\cdot}833$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS III.; $\gamma = 0{\cdot}833$.

THE diagram on the opposite page corresponds to Bazin's Class III.; or $\gamma = 0{\cdot}833$, which Bazin specifies for rubble masonry.

In practice, I consider that it is advisable to provide for some extra work in roughly dressing the faces of the stones, and filling hollows with mortar, in order to secure a smoothness corresponding to $\gamma = 0{\cdot}55$. The cost, except in small channels, does not exceed the advantages secured.

If, however, either concrete, masonry, or brickwork facings are laid on newly excavated earth slopes, and especially on artificial banks of earth, the adoption of $\gamma = 0{\cdot}833$ is advisable, since bad settlement will produce a state of affairs corresponding to this class.

By careful maintenance it is also possible to change a channel of this character from $\gamma = 0{\cdot}833$, to $\gamma = 0{\cdot}60$, or $0{\cdot}65$ approximately; and in many cases considerable increases in discharge may be secured without heavy capital expenditure.

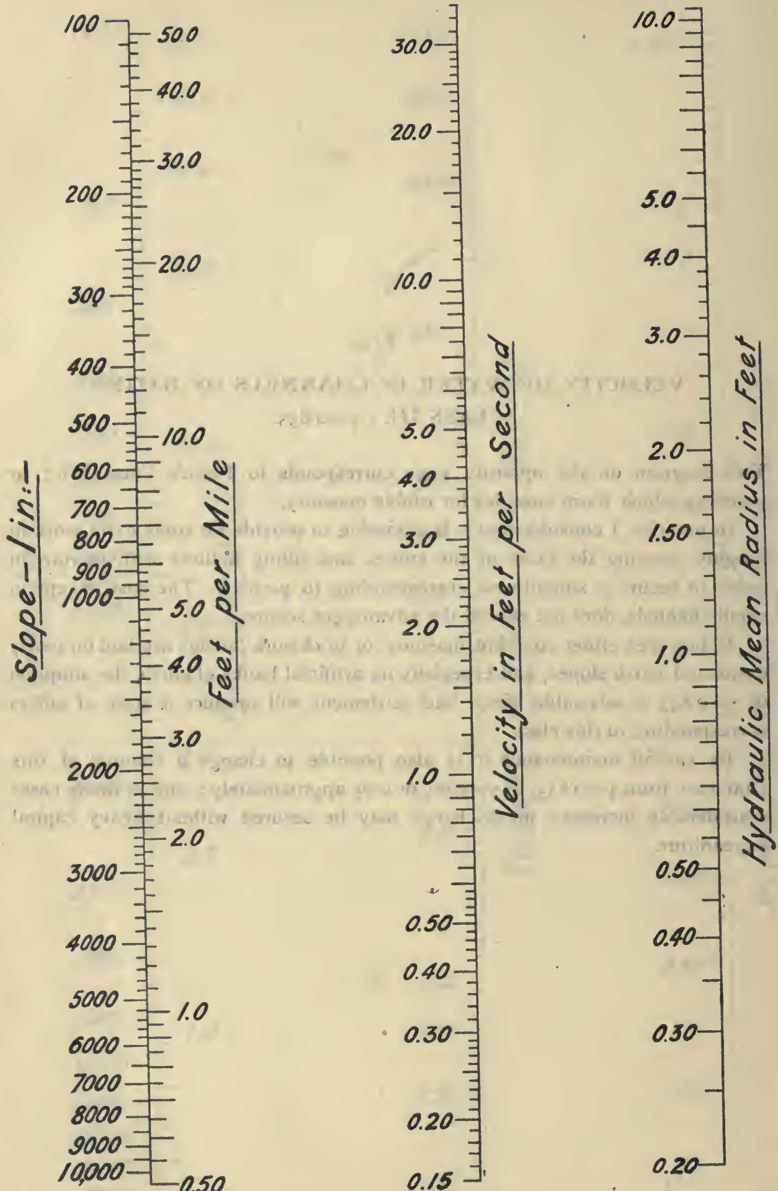


Diagram No. 10.—Velocity of Water in Channels of Bazin's Class IV.; $\gamma = 1{\cdot}54$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS IV.; $\gamma = 1{\cdot}54$.

THE diagram on the opposite page corresponds to Bazin's Class IV.; or $\gamma = 1{\cdot}54$.

Bazin specifies this for earth channels of very regular surface, or reveted with stone.

In actual practice I have found that only very carefully maintained channels in earth fall under this class. The presence of fine silt permits of $\gamma = 1{\cdot}54$ being attained fairly rapidly, provided that the maintenance is intelligently directed. Likewise, coarse sandy silt is unfavourable, and such channels should not be designed with γ much under $2{\cdot}00$. My experience of stone revetments is not very extensive; but, so far as it goes, in cases where pains are taken to secure a smooth surface, any carefully constructed revetment is smoother than $\gamma = 1{\cdot}54$, and corresponds more nearly to $\gamma = 1{\cdot}20$ to $1{\cdot}30$. A rough revetment of loose stone of large size is usually rougher than $\gamma = 1{\cdot}54$, at any rate in small channels.

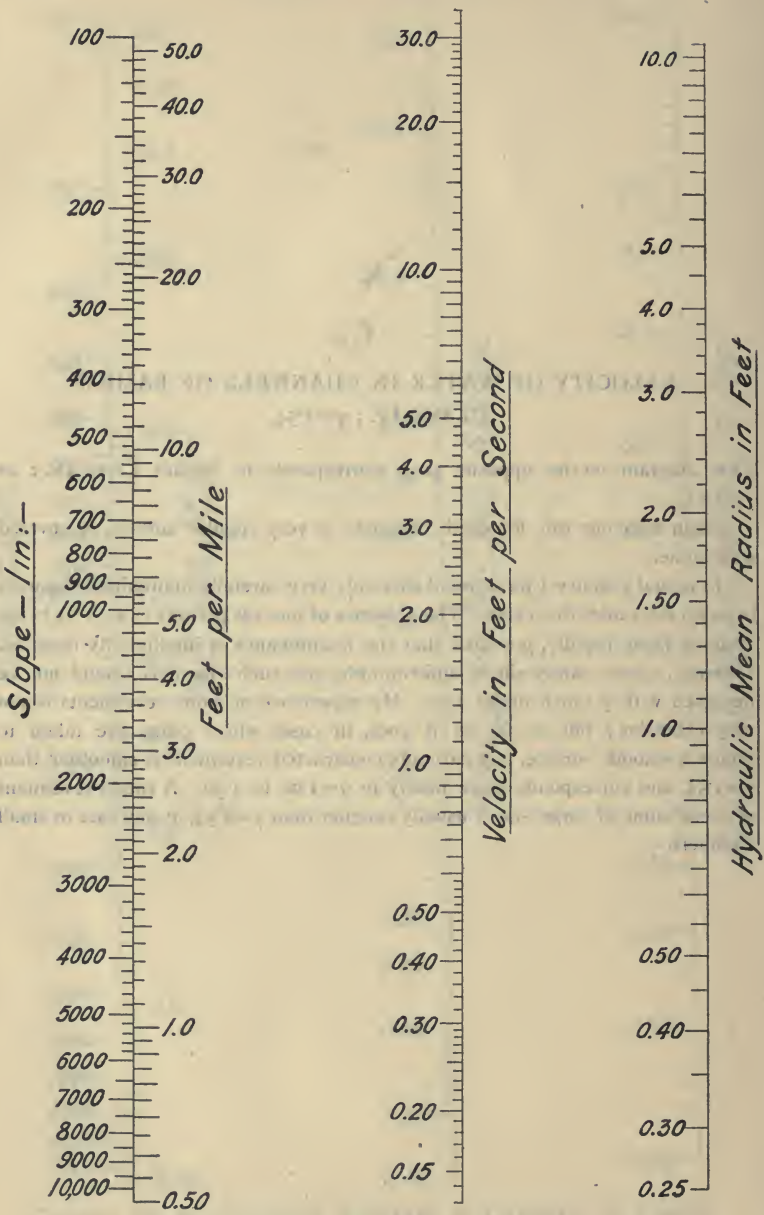


Diagram No. 11.—Velocity of Water in Channels of Bazin's Class V.; $\gamma = 2{\cdot}35$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS V.; $\gamma=2\cdot35$.

THE diagram on the opposite page corresponds to Bazin's Class V.; or $\gamma=2\cdot35$.

Bazin specifies this for ordinary earth channels.

I believe that designs for artificial channels should only be based on this value when the channel is markedly crooked in plan, or when maintenance is not well attended to; but no advanced hydraulic engineer will allow an important channel to remain in such a state for a lengthy period.

Silt and scour are, of course, unfavourable conditions, and when present, the value $\gamma=2\cdot35$ may temporarily prove too low.

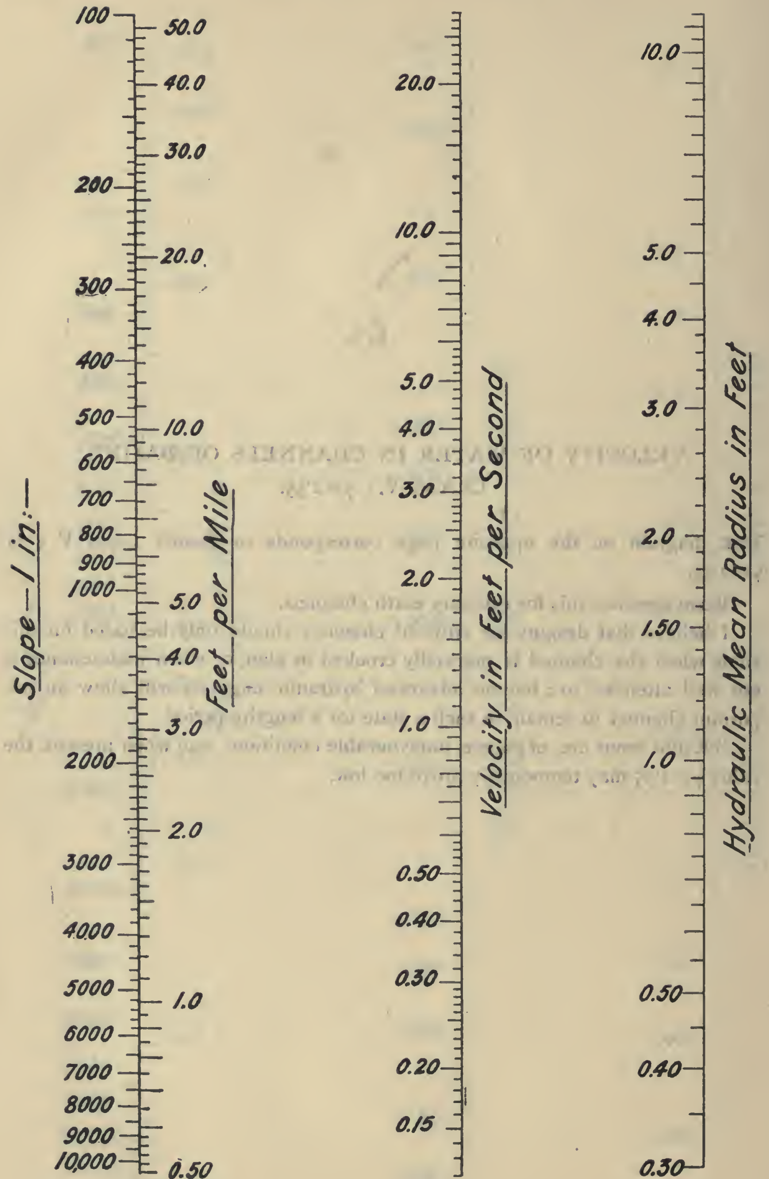


Diagram No. 12.—Velocity of Water in Channels of Bazin's Class VI.; $\gamma = 3{\cdot}17$.

VELOCITY OF WATER IN CHANNELS OF BAZIN'S CLASS VI.; $\gamma = 3{\cdot}17$.

THE diagram on the opposite page corresponds to Bazin's Class VI.; or $\gamma = 3{\cdot}17$.

Bazin specifies this for exceptionally rough earthen channels (bed covered with boulders), or weed-grown sides.

Judging by my own experience, such values of γ usually occur only in flood gaugings.

I have employed $\gamma = 3{\cdot}17$, in the design of a channel which was likely to be largely obstructed by weed growth, and in which it was undesirable to cut or destroy the weeds more frequently than once a year.

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