THE CONSTRUCTION

OF

WROUGHT IRON BRIDGES.

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THE CONSTRUCTION

OF

WROUGHT IRON BRIDGES:

EMBRACING

THE PRACTICAL APPLICATION

OF THE

PRINCIPLES OF MECHANICS

TO

WROUGHT IRON GIRDER WORK.

BY

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WITH NUMEROUS DETAIL PLATES.

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PREFACE.

As this work is analytical throughout, I would begin by saying a few words upon *Analysis*.

Analysis (separation) applied to any subject, always indicates a distinct classification of what we know of the subject, in such a way as to make that knowledge effective and useful. The usefulness of unarranged knowledge is very imperfect; a "general impression" can be formed from it, but no argument: for argument implies some arrangement. Our first knowledge, whether practical or scientific, is necessarily obtained from experience of some kind: if that experience be first separated into distinct and classified facts, and then good reasoning be brought to bear upon those facts, our primary knowledge is improved, and becomes perfected.

Thus analysis may be described as of two kinds. One is the separation of observed facts—of events actually passing before the eyes,—so that those parts of them which bear upon a particular object are detected, and drawn out with distinctness and connection:—excellence in this branch of analysis constitutes the "practical" man. The other is the separation of the various bearings which the facts thus drawn out have, so that the bearings which they have *on the object in view* may be connectedly stated in sentences whose logic is easily seen; and, at last, the valuable information which can be drawn from those facts, be written separately and clearly. Excellence in this branch of analysis constitutes the "theoretical" man.

There is great difficulty in combining, to any useful effect, the *practical* with the *theoretical* analysis of bridges, and other works of construction; so much so, that the *experience* of the workshop and the *reasoning* of mathematics are rarely considered as in union, but rather as opposed to one another. Yet it is only by their union that perfection can be obtained. It is a matter of fact that these two branches of effective knowledge are often esteemed so distinct, that a man of whom it is only known that he excels in one, is at once pictured as deficient in the other: whereas reason tells us that *he* will most effectively analyze his practical experience so as to deduce from it useful facts, who sees best what facts are of a kind to give a basis for reasoning. And, again, that *he* will reason best upon facts already obtained, who is best acquainted with the practice to which his conclusions are to bring assistance.

This estrangement of reason and observation—of theory and practice—is very well illustrated in our present literature. The axioms adopted by our engineers during their experience have, with their experiments and results, (generally) been as little bent towards immediate usefulness for mechanical reasoning, as this, again, has been directed towards usefulness in construction. Of late years, the properties of iron have become more and more known and defined; and the resources and accuracy of our workmanship render a very close analysis of the strains upon a structure capable of being carried out both *soundly* and *economically*. It is therefore especially satisfactory to see the chasm between the literature of our highest experience and highest reasoning really closing[1]. But how great a gap still exists between the abstract theorems and rudimentary figures in the highest mathematical books of our universities, and elsewhere; and the explanations of our practice, the elaborated repetition of experiments, and expensive plates in the most practical engineering books! The object of the present treatise is to exhibit the application of mechanical theory, in *as simple working forms* as possible, to those points of girder-work in which that application is proved *practically valuable*.

In hopes of accomplishing this, I have put together forms of calculation, such as I have actually applied when called upon by the late J. M. Rendel, Esq., M.I.C.E., to assist in the detail of some of the iron bridges constructed by him. I so assisted in allotting the proportions to the plate-girders of the swing-bridge over the river Nene at Wisbech, in Cambridgeshire, of 90 feet clear opening; and to the lattice-girders now constructed for erection over the river Soane, for the East Indian railway; which carry a rail- and foot-way, in spans of 150 feet, for a total length of about 3000 feet. Subsequently, I have been employed on a girder-bridge of larger span, designed by Messrs. M. and G. Rendel for the same railway, as well as on small girders from 28 feet span

[1] The writings of Messrs. E. Clark, W. Fairbairn, and Tredgold, and the papers of Messrs. Hodgkinson, Willis, Pole, and others might be mentioned.

(of which 160 have been sent to India) upwards. In all these cases complete working drawings were made: and since I believe that no calculations should stop short of this, I have taken care that, in the present treatise, all plates and woodcuts which relate to iron-work, should be described and dimensioned in the way sufficient and necessary for a contract drawing.

To the above is added a careful, but free, investigation of the comparative *merits* and *weights* of the suspension bridge, suspended girder, and hanging girder.

I shall be content if these calculations, after having already done service, should prove of any assistance to my readers, and help in the slightest degree to cement that union between theory and practice, which, though it exists even now theoretically, is far from being yet practically consummated.

The unwillingness of our Universities to admit into their system of education and examinations any natural philosophy but such as can be put into an abstract form—unencumbered by numbers, and defined so as to exclude contingencies,—is founded upon just grounds, though, perhaps, carried too far. I allow that it entails apparently a kind of inefficiency; for *no* philosophy can ever be applied directly to human benefit, except through the medium of numbers, and subject to the contingencies of workmanship. Natural philosophy, as studied at the University, is therefore, strictly speaking, in an inapplicable form; and so, a mind formed upon that study is, so far, but imperfectly educated for practical life of any kind whatever.

Thus, as a rule, a student, after his Senate-house honours are won, is by no means an experienced or practical philosopher. The theoretical element of his mind is too abstruse and metaphysical; his method of judgment too ideal and irresponsible; and, above all, his habits may be entirely innocent of acuteness and patience in balancing the truth and the value of unclassified data. He has had no *practice* in the drudgery of amassing facts, before their value is yet known; nor in the skill, which earliest selects the most valuable, and the patience which relinquishes the pursuit of the useless ones; nor, again, in the providence which renders impossibilities possible by an early detected method of classification. He has a high appreciation of the value of true principles of natural philosophy when elicited, but has not even realized that others have required the above-mentioned qualities in order to elicit those principles. Moreover, the above form of mind, first allowed, or fostered, by its prolonged abstraction upon standard theorems of philosophy, is apt gradually to extend to all the questions of social life; unless immediately after the student's University course his mind be stirred, as it were, by some external force, and a more workable and less angular internal structure be imparted to it.

But in this lies no great argument against the present University education. Everything cannot be learned at once. A father who sends his son to the University to acquire in its colleges intimate experience and facility in the details of practice, or of the tempers and occupations of practical men of various ranks, is not one whose objection at any result should have much weight. Of theory and practice it seems right that, in the way of education, sound theory should be the first lesson inculcated. After that the practical efficiency will be more quickly obtained, and is likely to be more true and serviceable, as being obtained with more ability and zest. To unite the two educations into one, except as each man can do so in himself, has again and again proved a failure.

No body of *practical* statesmen or philosophers have ever advocated an uniform tenor of life, but rather one of alternate business and seclusion,—experience and study. The commencing life by a theoretical education in philosophy and history, best carried on in a temporary seclusion, was, and still should remain, the acknowledged leading idea in University education.

Still, the teaching of the Universities should not be unguided by what is going on in the practical world. An University should arrange its course of study so as to bear as immediately as possible on the practical sciences which are benefitting mankind; and even, if possible, so select its education and examinations as to keep the mind of the student prepared for a labour to come; and give him intimation of, and a zest for, after-progress to be made in *practical* knowledge (theological, political, or philosophical, as the case may be). In a letter of the Astronomer Royal to the Vice-Chancellor of Cambridge, on proposed changes regarding the Smith's Prizes, Dec. 5, 1857, he expresses himself very strongly upon the duty of maintaining this connection with "the science of the world," as far as regards mathematical studies. In advocating the introduction into the examinations of such theories as The Figure of the

Heterogeneous Earth (which were absolutely forbidden by the University) he says:

"The results of this theory are known in the scientific "world, not only to mathematical men, but also to every "military or naval officer of moderate acquirements, who "has been employed in national surveys or pendulum "experiments. May it not be fairly said that a University, "more particularly devoted to mathematics, which sends "out its most highly honoured graduates at the average "age of twenty-two years, without the slightest knowledge "of such a subject, has already sunk to the position of a "second-rate academy? I think so."

If Professor Airy sees so strongly the importance of embracing in the studies of the first mathematical University of England, the Theory of the *Figure of the Heterogeneous Earth*, and the use of Laplace's coefficients; how much more the importance of including, and the disgrace of neglecting, theorems bearing upon engineering construction!

It is quite true that first mechanical notions are best argued upon extravagant suppositions (such as "Let the line AB be a rigid lever without weight," &c.), but a man should not leave the University with high honours, unless he have learned to adapt mathematics to *natural* difficulties. There are theorems regarding the strains and deflection of girders, or the strength of joints; which might supply in the simpler Statics that confidence in the *adaptation* and *power* of calculation, whenever it should be required for an actual case; which I myself only imbibed from the Lunar and Planetary theories, and the Wave theory of Light.

The investigations extending from page 47 to page 61, and

Lemmas X, XI, upon the strains of girders; and the chapter on Deflection, supply interesting mechanical exercises in the forms ordinarily used in the University.

As I have said so much upon this interesting subject, I may here further venture to add my conviction, that the way to make a *theoretical education* bear most directly on the practical business of life, is to extend it, equally with the spread of practical science; but contentedly, and within its own proper limits. Extend it, in fact, wherever it can be extended *soundly* and *completely*. If theory can be soundly extended, in the strict academical forms, to problems in girder-work and deflection; then it is the bounden duty of the University to extend its education so as to embrace so beneficial a subject. Has not the time already come for doing this? I must answer in the words of Professor Airy—"I think so."

We cannot look to *professorships* as a means of ensuring vitality and practical efficiency in an University education, with half so much hope as to a sound extension of theoretical science. At all events, the latter is essential to practical efficiency: as to the former, though a few professorships seem to give a most valuable fillip to other modes of education, amongst which they may have been exercised as exceptions to the general rule of education; yet the "professorial system" has always seemed to fail. The Scotch professors complain:—in London, where the professors are more successful, the less talented of the pupils murmur; for it is a matter of experience that those who have passed through the "professorial system" and examinations in Divinity, of

King's College, London, are not all certain to pass the more practical test of a Bishop's examination. And men have passed through the German "Professorial systems," including special examinations in Algebra and the Differential Calculus, without having acquired the practical management of more than the simpler operations of arithmetic. The professorial system makes a few excellent. The English University system should not rest there: its first exertion should be to make all sound.

This book, then, has been written under a firm conviction that the greatest excellence either in Theory or Practice can never be attained until the two are more united than I have yet seen them. This conviction is not only based upon my own experience, but also upon plain reasoning. To take as an instance my own profession: an engineer who confidently attempts to carry out a design formed upon *true* theory, if it turn out to have been based on data unwittingly inferior, or even false, will cause an evident and heavy loss. But an engineer also, who with time and skill analyzes evidence, without a good, and perhaps with a false, notion of the conclusions to which it should tend, and whose final decision is cramped by inability to reason upon principal truths, may be reasonably expected to cause perhaps the abandonment of beneficial works, more likely their failure, and at least undetected but burdensome expense. Thus neither theory nor practice can be reasonably advocated alone. We cannot believe in the existence of a *good* theorist, who is not also a very fair practical analyst; nor of a man of proved practical sagacity who is not a very fair theorist.

There *have* been men, who sought, in all their endeavours, to *combine* theory with practice; some of whom (as Watt and Stephenson), though labouring under great difficulties perhaps, have ultimately succeeded, and produced results as beneficial as magnificent: there have also been men, who have seemed to unite in them the danger which attaches to the pure ideal "theorist," with the old fashioned ponderousness of the ideal "practical man;" who, with the excessive original expenditure and grandeur of the latter, have combined as much final unforeseen failure and loss as ever dogged the steps of the most chimerical theorist. Most men of eminence have excelled *more* in one than in the other of the two branches; and every man is inclined to distrust and depreciate that in which he excels least, since he feels in it insecurity and difficulty: yet we have never heard of an eminent practical man, who professed himself actually enervated by what theory he possessed; nor of a man excelling chiefly in theory, who did not desire to increase his knowledge of practice. The application of the two in combination is daily more and more acknowledged to be the most powerful science, and the only true economy; and if this treatise prove of value in shewing their united application to Girder-work, my labours will be repaid.

LITTLE EATON, DERBY,
November, 1858.

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THE PRINCIPLES

OF

MECHANICAL PHILOSOPHY

APPLIED TO

WROUGHT IRON GIRDER WORK.

CHAPTER I.

DEFINITIONS.

In writing dimensions, ′ is used for *feet*, most properly for lineal feet. For square or cubic feet the abbreviations, sq. ft. or ft. super. and cu. ft., are properly used. So ″ is used in lineal measure to denote *inches;* but in square or cubic measure it properly denotes the 12th part of a square or cubic foot. Thus 13′.64 duodecimal notation is written sometimes 13′.6″.4‴, and means 13 feet 6 twelfths and 4 144ths of a foot. But if one met with the symbol 1270″ in what one knew must be cubic measure, it would mean 1270 cubic inches, and should be written 1270 cu. in.

The following definition will be new to some who are likely to be my readers. It is the definition of the use of symbols of denomination in practical analysis.

All symbols, whether of length, size, weight, time, or any other kind, have in practical analysis an irregular use. Thus the symbol *tons* is often used for the expression (*being the number of tons in the weight*); *sq. ft.* for (*being the number of square feet in the section*), and so on; the subject to which this weight or section &c. belongs being left to be determined by the acuteness of the reader.

Thus, suppose we have twelve pieces of bar iron 4 inches by $1\frac{1}{2}$ inches, and a yard long; and it be granted that a bar one inch square and a yard long weighs 10 lbs.; and we require the weight of the bars. The following is the strict solution of the problem.

The number of pounds in the whole weight is equal to the product of the number of inches in the breadth of the bar × the number of inches in the depth × the number of yards in the length × the number of pounds, which a bar 1 inch square and 1 yard long weighs × the number of the bars

$$= 4 \times 1\tfrac{1}{2} \times 1 \times 10 \times 12 = 720;$$

∴ the weight of the bars is 720 lbs.

This is the strictly mathematical way: a more manageable, and to one acquainted with the general method of working out the weight of bars, an equally lucid one would be to state the problem thus:

The weight in lbs. = 4 (inches) × $1\frac{1}{2}$ (inches) × 1 (yard) × 10 (lbs.) × 12 (no.) = 720,

or the weight = 720 lbs.:

where the denominations in parentheses give the reader a clear reference to the data from which its prefixed number is taken, and an easily understood hint as to the theory by which the result is arrived at.

Still we want a shorter statement for practice; and this is obtained by leaving out the parentheses and stating the problem thus:

The weight = 4″ × $1\frac{1}{2}$″ × 1 yd. × 10 lbs. × 12 no. in lbs.

= 720 lbs.

In speaking of the section of iron in a bar, or collection of bars or plates, subject to compressive or tensile strain, I shall use the terms *gross section* and *net section*. The net section is the section on which we can rely for resisting strain, after making allowance for bolt-holes, rivet-holes, and extra iron. The gross section is the actual average section, including stiffening irons, covers, &c.

Thus from the net section we know the strength of the combination: from the gross section we know its rigidity, or may deduce its weight.

I will add that the symbol $\frac{3}{4}\frac{1}{16}''$ is often written for $\frac{13}{16}''$: and that in working out equations which are to lead us to a numerical result, very small fractions are often disregarded; and if the result be the thickness for a plate, for instance, or the diameter of a rivet, it is of no use working it out more accurately than to give us the fraction with denominator 16, which is next largest or next under the expression we are reducing. In every case care is taken to be on the safe side. I may mention that I have avoided using tables to assist in getting bearing and sectional areas, or square roots, in order to avoid the confusion caused by them; and that the theory of continued fractions will furnish the key to most of the arithmetical steps.

We may now pass from the drudgery of ideal definitions to the quaintness of practical ones.

A *bar* of iron is either round in section, square, or rectangular, in which last case it is called *flat.* The ordinary bar is rolled at once from the bloom, which is passed between grooved rollers accurately defining the shape of the bar. Thus a bloom to form a square bar, would be passed between rollers in each of which was cut a rectangular groove of equal sides, with its angle nearest the axis of the roller, and of course in a plane perpendicular to that axis. The rollers when in their frame, and adapted for rolling, would have their surfaces in contact, and their grooves coinciding so as to leave a square opening between the two rollers with one angle uppermost. A number of grooves are made thus in each pair of rollers so as to create, side by side between the rollers, square openings of continually diminishing size. The heated bloom is passed first through one of the large openings, and then successively through smaller and smaller ones until it is reduced to the exact size required.

If square or round, bar varies from $\frac{5''}{8}$ to 3″ or $3\frac{1}{2}''$ in the side or diameter: if flat, from 1″ to 6″ or 7″ broad. If the dimensions fall without these the bars bear an extra price. Bar cannot be rolled more than 9″ broad.

A bar 10′0 long, 5″ broad, and $1\frac{1}{8}''$ thick, is written

$$10' \times 5'' \times 1\tfrac{1}{8}''\ \textit{bar}.$$

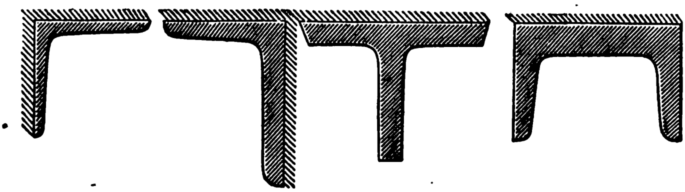
A *plate* of iron is a physical plane of iron of uniform thickness. The ordinary plates are rolled from faggots, which are bundles of flat bar iron, generally from the first rolling, cut all to rectangles of the same size, laid one upon the other, and bound together with a withe of iron wire to hold them together while heating. The alternate layers are of different quality, as hard and soft, unrefined and refined iron. This faggot is rolled to make as nearly as possible a rectangular figure, and the edges are finally sheared (for boilers all the better if it be to a gauge) to make it strictly rectangular.

Plate varies from $\frac{1''}{4}$ to $\frac{3''}{4}$ in thickness, in differences of $\frac{1''}{16}$; but plates may be rolled thicker than $\frac{3''}{4}$. It has an extra price per cwt. put upon it if it weigh above 3 or 4 cwt., and sometimes if it contain more than about 25 or 30 sq. ft., or is of peculiar shape. If less than $\frac{1''}{4}$ thick the plate is called *sheet* iron, is more expensive, and generally of better quality.

A plate 3′6″ wide, $\frac{9''}{16}$ thick, and 7′0 long, is written

$$7'0 \times 3'6'' \times \frac{9''}{16}\ \textit{pl.}$$

Angle iron (∠ i.) is rolled of various shapes. If of equal sides it is passed through rollers so cut as to roll the iron with the corner upwards; thus when the angle iron (see figure) passes through the last pair of rollers the thickness of *both* sides can be equally regulated by one movement of the rollers, nearer or farther from each other. On the other hand, in unequal angle



$2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{1}{4}'' \angle$ i. $3\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{7}{16}'' \angle$ i. $4'' \times 3'' \times \frac{1}{2}''$ T. i. $3\frac{1}{2}'' \times 2\frac{1}{2}'' \sqcup$ i.

The shaded side in each shews the part generally rolled by the upper roller, the rest is rolled against the lower one. The scale is 3″ to the foot.

iron, and in tee and channel irons, the thickness of the broad side only can be altered, since this is then rolled uppermost; the projecting rib must always be of the same thickness with the same rollers.

To crank an angle or T iron is to bend it by heating. If cranking be necessary it is done by cutting out a portion of the projecting edge, bending it together and welding it up again. Similarly a piece may be added, if necessary.

Tee iron (T i.) and *channel iron* ($\sqcup$ i.) are other forms of rolled iron.

If angle or tee iron exceed 7 or 8″ in the sum of its extreme breadth and depth, or weigh more than 3 or 4 cwt., then an extra price per cwt. is affixed to them. It can be rolled 16′0 long with ease. In ordinary use it is not well to have a $3'' \times 3'' \angle$ i. less than $\frac{3''}{8}$ thick; and a $3\frac{1}{2}'' \times 3\frac{1}{2}'' \angle$ i. should hardly be less than $\frac{1''}{2}$ thick.

Angle iron, whose section is $3\frac{1}{2}''$ on the outside of one leg, and $2\frac{1}{2}''$ on the outside of the other, and averaging $\frac{7''}{16}$ thick is written

$3\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{7''}{16} \angle$ *i.*; or often, $\angle$ *i.* $3\frac{1}{2}'' \times 2\frac{1}{2}''$ *at* $8\frac{1}{8}$ *lbs. per ft. run.*

Tee iron, the base of whose section is 6″, and extreme height 3″, and $\frac{1''}{2}$ thick, would be written $6'' \times 3'' \times \frac{1''}{2}$ *T. i.*, or often *T. i.* $6'' \times 3''$ *at* $14\frac{1}{8}$ *lbs. per foot run.*

The angle iron in the above instance is $3\frac{1}{2}'' + 2\frac{1}{2}'' = 6''$ in the sum of the sides, and would therefore have no extra price.

The tee iron, however, is $6'' + 3'' = 9''$ in the sum of the sides, and at works where extra price is charged above 7″, will probably be charged £1 a ton more than their ordinary price for tee iron.

Generally the price of tee iron is about £1. 10*s.* a ton above that of bar; and the price of angle iron and plate about £1 a ton above the same.

The causes of an extra price having to be put upon these kinds of iron when their weight or dimensions differ from the ordinary amount are threefold.

1. If the weight exceeds 3 or 4 cwt., the number of men required to handle the mass of iron during heating and rolling is increased beyond the most economical number. The manufacturer therefore finds that with plates of above a certain weight, a given sum of money spent in wages does not produce him the proportional number of cwt. of the heavy plates: he has therefore to charge an extra price upon heavy plates.

2. If the dimensions differ from the ordinary dimensions, e.g. if a plate is to be rolled of great size though of ordinary thickness, or very thin though of ordinary size, or if a bar be above 9″ broad, then such a plate or bar will require an additional heating besides the ordinary ones, and the expense incurred in extra coals, time and labour, necessitate a higher price per cwt.

3. If the dimensions of ∠, T, or channel irons are peculiar, special rollers will have to be made in order to roll them, the expense of which must be wholly covered by the iron ordered to be so rolled; this is done by a special price per cwt., unless the amount ordered be very large, in which case the advantages of a large uniform series of work compensate for the expense of the rollers.

Judging from the progress of iron work in late years, it may be safely predicted that other forms of iron will soon be rolled by our iron manufacturers, for girder work.

As the forms, which would be most generally useful, in addition to the tee and angle irons, become more certainly determined, the manufacturers of most foresight, energy, and scientific interest, will begin to roll them; and much will thus be done to facilitate the adoption of wrought iron to the national use.

The channel iron, first suggested by the author as a particularly suitable, and at the same time handsome, form for the compression bars of a lattice girder, has been already extensively used. Other more difficult forms, but which must come into extensive use for lattice or triangular girders of from 30 to about 80 feet span, (could they be obtained at moderate prices,) might be here suggested.

When speaking roughly of iron I may include plate and sheet iron under the generic term plate iron; angle, T and channel and bar iron, under that of ∠ iron.

In this treatise I shall consider all iron, if reduced to an uniform section, to weigh 10 lbs. per yard for every square inch in the section. Or, which is the same thing, if it be reduced to an uniform $\frac{1''}{4}$ plate, to weigh 10 lbs. for every square foot of plate.

This is about 2 thousandths below the weight of wrought iron, and a little above the weight of cast iron; but in taking out the weight of wrought iron work, allowances are made for rivet-heads and contingencies, and added to the weight calculated on the above assumption. These are founded upon experience, and may be reasonably supposed to cover this difference, even were it more considerable. In the case of cast iron, the calculated weight would be excessive, were it not that the fillets or brackets, in the angles of the castings, the bosses, and some other irregularities, are in practice neglected in the calculation, and will generally be assumed to make up the deficiency.

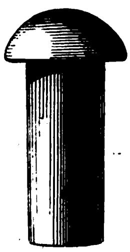
The reader will therefore perceive, that if, in taking out the weight of a piece of iron work under his consideration, he adopt a different rule for the weights of iron, in plate and ∠ iron, than that above enunciated, though it be in itself more accurate; then still his final result will be more inaccurate, unless he also have a new scale of percentages to add for rivet-heads, joints, &c. got out by experience to suit his new rule of weights.

A good engineer must know how his work, as designed on paper, is likely to be carried out in the workshops. He must know what casualties are *inseparable* from the ordinary workmanship; otherwise he will not allow for them, and will design a structure which absolutely cannot be carried out; or one which, though carried out in appearance, will be in reality deficient. This is especially the case in rivetted work, where any error in the plates as punched, or in the rivet inserted through the holes in the plates, is equally dangerous: and it is also workmanship that limits the safe use of welds, planed bearing surfaces, bolted bars, &c. Again, the engineer must know the expense and supervision necessary to secure good workmanship in *difficult* cases; otherwise the safe carrying out of his design may require more of the former than he anticipates, or more of the latter than can be given to it without great trouble and cost.

Now an experience of this sort cannot be conveyed on paper, but can only be obtained by actual observation of each kind of workmanship joined contemporaneously with facilities for experiment and investigation. Still much may be done on paper, where at least suggestion and direction may be given which shall awaken inquiry, and render most valuable its exercise. The reader will see the connection and force of the following descriptions better, when he peruses them a second time, after having some knowledge of their application to work.

With these observations I pass to the more interesting definitions of workmanship.

A *rivet* is formed out of a rod of round iron of the diameter of the intended rivet; one end is raised to a welding heat; a length is then cut off by gauge from the red-hot end, of a proper length to form the rivet, and is immediately formed at one end into a head, either by machine or hand. The rivet when cold is accurately its nominal diameter from the head to a point which will be about midway between its heads when rivetted into the work; it then tapers in order that it may enter the hole when swelled by heat. At many works they will

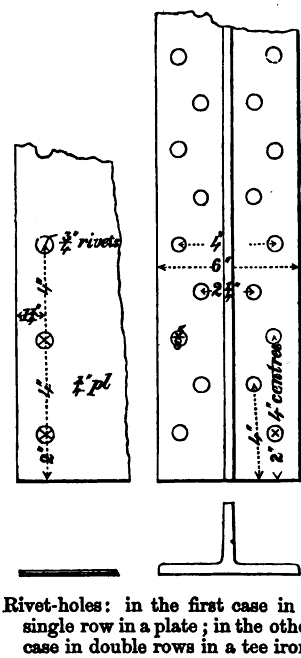


A ¾" rivet, ½ size.

be under their nominal size from $\frac{1''}{32}$ in smaller to $\frac{1''}{16}$ in larger rivets.

Rivet-holes are made by means of a punch, which punches them out of the iron plate, bar, ∠ or T iron while cold. The *punch* is a piece of cylindrical steel of the diameter of the intended rivet-hole, moved by the punching machine, to which it is attached, upwards and downwards, over the block or matrix. The matrix is a cast-iron block, under the punch, secured to the punching machine. It has through it a slightly conical hole increasing downwards, whose axis is in a line with that of the punch, and its upper diameter nearly $\frac{1''}{16}$ larger than that of the punch; the punch descends so low that at its lowest point it enters this hole.

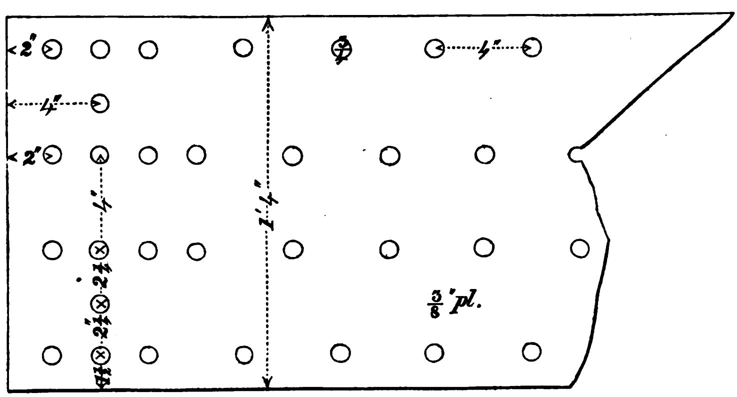
When a piece of plate or angle iron requires to be punched, the exact spot in which the hole is required is brought under the punch immediately after its ascent; then the punch on its next descent punches a hole through it, slightly larger at the bottom than the top; and which may be taken as averaging $\frac{1''}{32}$ to $\frac{1''}{16}$ larger than its nominal size.



Rivet-holes: in the first case in a single row in a plate; in the other case in double rows in a tee iron. The figuring gives the description of their size and position, in each case, necessary and sufficient for a contract drawing. 1½″ scale.

In some cases the machine itself moves the plate on, after each hole is punched. This plan may be adopted for punching a number of equal holes equidistant in a straight line. Some machines are even made to punch two rows of rivet-holes at a time, by means of two punches working isochronously, and at each stroke punching a hole in each row. In this last case accurate adjustment of the punches and machine is first necessary; and then the correct placing of each plate upon the machine, alone requires the skill of the workman who superintends it. After it is once fixed, he has nothing more to do than to put it into gear with the motive machinery, and the holes are all punched in succession by the machine.

But in perhaps the majority of cases the plate is placed by hand under the punch for each individual hole to be punched; and this must necessarily be the case where there is irregularity in the rivetting. To prepare the plate or angle iron for this process a template, or false plate, of thin sheet iron, is prepared, of the exact size of the plate, or of the back of the angle iron. On this the centres of the required holes are marked by rule and compass, and the holes then drilled out. This template is laid successively upon each plate ∠ or T iron requiring to be punched to its pattern; the two are clamped together; and the plate is marked with white paint through the holes of the template, so as, on its removal, to leave under each hole a white ring of the diameter of the hole; which will of course shew the exact spots upon which rivet-holes are required. If the patterns



One end of a plate containing 4 rows of rivets at 4" centres; but which has at the end some extra rivets inserted, in order to admit of firm connection to a cover plate. 1½" scale.

of two plates are alike, except that one has a few extra holes besides those in the other, the same template is used for both; and has the extra holes drilled in it, but marked by a circle of white paint or otherwise, in order that they may be omitted in the one case when the rest are marked through. Wooden templates may be often used.

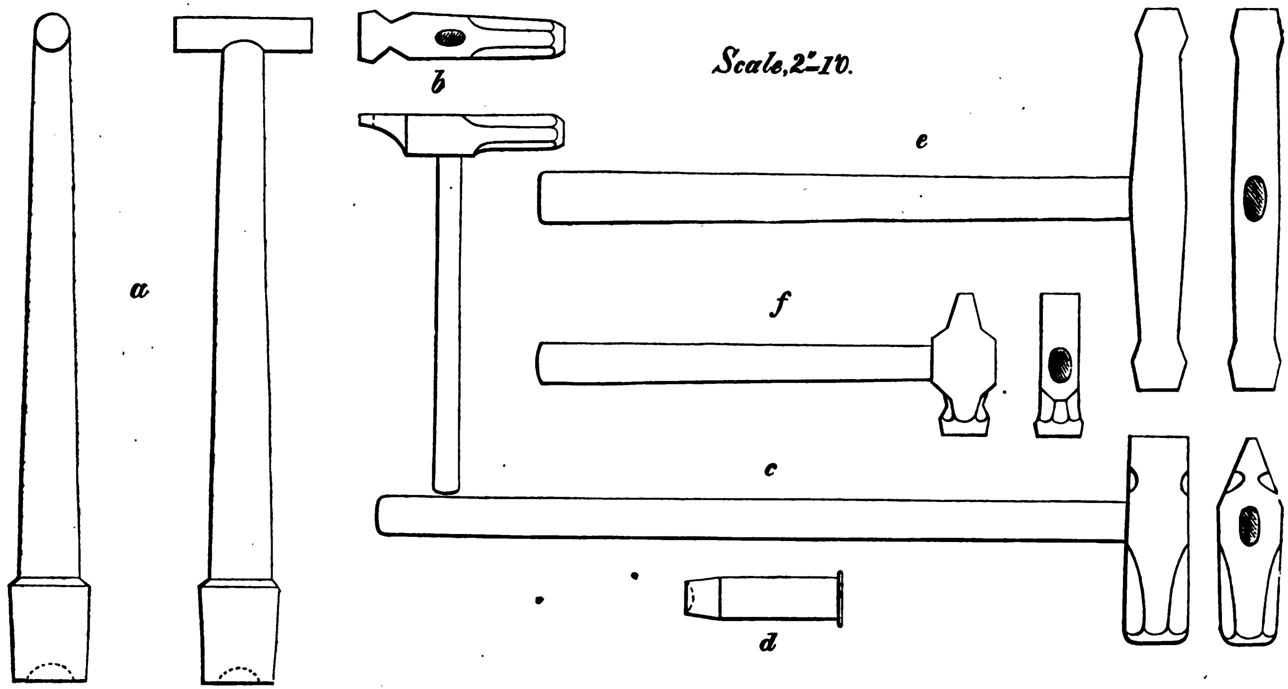
The workmen who subsequently punch out these holes at the punching machine, bring each of these white rings successively under the punch, so that at each stroke a hole may be punched out exactly through the white ring; an operation requiring great skill and readiness when the plate weighs 3 or 4 cwt., and requires a gang of 5 or 6 men to move and control it; the operation is generally attended by complete silence, and consummated with success, except that a stroke or two may be lost at the corners.

A rivet-hole cannot be punched with its edge nearer the edge of the plate than its own diameter without "extra" risk of its bursting through; to this it is safe to add from $\frac{1''}{8}$ to $\frac{1''}{4}$ as the plate and rivet get thicker and large. The edges of two neighbouring rivet-holes cannot be nearer than from 1 to $1\frac{1}{2}$ diameters, without risk of the two holes punching into one.

The *rivetting* of two plates together forms a *joint*. When two or more plates are to be rivetted, they are placed together in the proper position, having their holes (if properly punched) exactly over one another, and are screwed together by temporary screw-bolts inserted through some of the holes.

If two or more holes that should fall over one another, cannot be brought to do so, nor opened by a steel punch sufficiently to admit the proper rivet, they may be *rimed out* by the insertion and forced revolution of a square sharp-cornered steel rod, or *rimer*, made slightly tapering; after this has been worked a sufficient length of time the holes will be made to coincide, but will require a rivet, say $\frac{1''}{8}$ larger than was originally designed.

A rivetting gang consists of 3 men and 2 boys. The two latter have the control of the furnace; their implements consist



Tools of a gang of rivetters. *a* holding-up iron. *b* rivetting-hammer. *c* cupping-hammer. *d* cup. *e* for rivetting where *b* is inapplicable. *f* hand-hammer for chiselling, caulking, &c.

of a wooden box full of rivets, a portable smith's forge and bellows, 2 pairs of peculiarly shaped pincers adapted for holding the rivets, a trough of water, some small iron plates about 6″ or 8″ square pierced with holes $\frac{1''}{8}$ larger in diameter than the rivets, and an iron for stirring the fire; their duty is to serve to the men, at any moment, a rivet brought to a welding heat from the end nearly up to the head. To accomplish this one of the iron plates is placed upon a clear part of the fire, and 5 or 6 rivets are inserted therein by one boy while the other keeps at work with the bellows. The rivet-ends are in the heart of the fire, their heads only being kept out by being too large to pass through the holes in the plate. When the men require a rivet, the hottest is served out to them, and another put in its place. The chief management required is to keep the rivets from getting overheated and burnt, during any temporary delay in the rivetting.

Of the men one, the holder-up, holds an iron or lever, with a heavy head the top of which is cupped so as to fit the head of the rivet; the handle is either in one piece with it if of iron, or fixed into it if of wood, and is of the shape most convenient for holding up in the particular cases for which it is made. He also has a pair of short pincers. His duty when engaged in the rivetting is to hold his iron against the plate close by the rivet-hole about to be rivetted up, while his comrades drive a steel punch hard into it. When he has knocked this punch back again to them he has to pick up the hot rivet (thrown him by the boy) with his pincers, and put it either directly into the hole, or, if that be impracticable, into the cup in the iron head, by which he can convey it to the hole. With his iron he hammers it into the hole, and presses against it while it is being hammered from the opposite side.

His two comrades have each a rivetting-hammer, whose head is elongated to enable them to hammer the rivet when it is sheltered behind a projecting ∠ or T iron. They also generally have a cup with a hazel twig twisted round it, and cupping-hammer, and a slightly tapering punch or two whose greatest diameter just exceeds that of the holes.

The punch is first driven into a hole by means of the cupping-hammer. This is useful for opening out a passage for the rivet, where it has to pass through several plates whose rivet-holes do not accurately coincide. When the holder-up has knocked the punch out again, inserted the rivet from behind the plate, and got his iron pressing upon its head, then the rivetters first hammer the iron in the immediate vicinity of the rivet, to bring the plates tight and the rivet-head close up to them. They then hammer down the end of the rivet to form the head; and unless this be neatly done it cannot be afterwards cupped to a smooth surface. The hammering down of a rivet is inefficiently done unless it have the centre of its head immediately over the centre of the rivet. The best rivetting is when the rivet is not only flattened at the end, but so squeezed through its whole length as to fill the rivet-holes firmly, even though they be rather irregular. The rivet in cooling "draws" the plates firmly together.

To assist this last process, and also to finish off the exposed heads neatly, they are often cupped. After the rivet has been hammered by the rivetting-hammers till losing its red heat, and its head being then neatly formed, the cup is held upon it by one rivetter, and hammered by the other with the cupping-hammer, the holder-up still applying his iron to the head; this process both makes the head even and spherical, and cuts off any surplus metal, so as to leave all the heads of an uniform size; by the time this is finished the rivet has lost all visible heat: rivetting thus with a cup is often called snap-rivetting.

If plates be large, as in the sides of a tank, or web of a girder, and at the same time thin, they will not preserve their flatness during the rivetting of their edges, but will curve or bulge, in a way depending on the position of the plates and the method of stiffening them with angle or tee irons. The amount depends on the workmanship, material, and precautions, and may make them very ugly and even unsafe.

The preceding definitions contain as much as can well be given, where the subject is too boundless to allow of their being complete, and the writer unwishing to be prolix, and diffident lest he should confuse or overburden the time or attention of any.

I now pass on to treat of the *Efficiency of Iron*. This will depend of course chiefly on the way in which the above workmanship, and much more unmentioned, is executed (much previous to, and much contemporaneous with that described above). And that again depends upon where the iron is got, and how far its preparation is in good hands or under good inspection. But I now proceed to give the measure and amount of efficiency which may safely be reckoned on in "good" workshops; which is true *comparatively;* and which I adopt hereinafter.

Efficiency of a Plate, &c.

The *Efficiency* of a plate, &c., is the strain that may safely be put upon it, without any danger or injury either by immediate dislocation of its parts, or by gradual crystallisation, buckling, or other causes.

The *Line of Fracture* of a plate, &c., is a line drawn through it (on its surface), crossing all the lines of strain in such a way that the section of the plate along it is a minimum. It is found by experiments on ordinary fibrous plate, that the plate is more likely to break in such a line, even though it be irregular and zigzag, than in any other.

The *Breaking Area* is the section of the plate in sq. inches along the line of fracture.

The efficiency of a plate in tension will be considered to be $4\frac{1}{2}$ tons to the square inch of section along the line of fracture.

In fig. 1, which represents a plate $8'' \times \frac{1''}{2}$,

At *A*, where there are no rivet-holes, the line of fracture is *Aa* or 8″.

$$\text{Efficiency} = 8'' \times \frac{1''}{2} \times 4\tfrac{1}{2} \text{ tons},$$

$$= 18 \text{ tons}.$$

At B, where there are $\frac{3''}{4}$ rivet-holes $1\frac{1}{2}''$ from edge at $4''$ centres, placed alternately,

$$Bb = 8'' - \frac{3''}{4} = 7\tfrac{1}{4}'',$$

$$\begin{aligned} Bb' &= 3 + \sqrt{25 + 4} - 1\tfrac{1}{2}, \\ &= 3 + 5\tfrac{3}{8} - 1, \text{ nearly}, \\ &= 8\tfrac{3}{8} - 1\tfrac{1}{2} = 6\tfrac{7}{8}'', \end{aligned}$$

which is less than Bb, and therefore the line of fracture;

therefore line of fracture $= 6\frac{7}{8}''$.

$$\begin{aligned} \text{Efficiency} &= 6\tfrac{7}{8}'' \times \frac{1''}{2} \times \frac{9}{2} \text{ tons}, \\ &= 15\tfrac{1}{2} \text{ tons nearly.} \end{aligned}$$

At C, again,

$$\begin{aligned} Cc &= 8 - 1 = 7, \\ Cc' &= 4 + \sqrt{16 + 9} - 2, \\ &= 4 + 5 - 2, \\ &= 7''; \end{aligned}$$

therefore the plate will break indifferently through Cc or Cc'.

$$\begin{aligned} \text{Efficiency} &= 7'' \times \frac{1''}{2} \times \frac{9}{2} \text{ tons}, \\ &= 15\tfrac{3}{4} \text{ tons.} \end{aligned}$$

Thus, whether the plate be in tension in the direction of, or at any inclination across, the fibre, its efficiency will be taken at $4\frac{1}{2}$ tons to the square inch.

The efficiency of a plate in compression, if rivetted in the method usual for the top and bottom of a girder, would, if the rivet-holes were quite filled up by the rivets, be unaffected by their existence, in which case it would be taken as 4 tons to the square inch in the full gross section (wrought iron being weaker in compression than in tension), but in practice that is hardly the case; and yet the rivets do not weaken the plates to the full extent due to their holes.

The efficiency then of a plate in compression, if rivetted in the top or bottom of a girder, will be considered to be 4½ tons to the square inch section along the line of fracture.

In the same way the efficiency of ∠, T, or ⊔ iron will be considered to be 4½ tons to each square inch of section along the line of fracture, whether in tension or compression.

In cases where the metal subject to compression is under disadvantages of form or otherwise, a less efficiency must be reckoned upon.

Efficiency of a bearing surface.

The *efficiency* of the bearing of two surfaces in contact under pressure, is the pressure that may safely be put upon them without any danger of injury, either by immediate rupture or by gradual crystallization. Crystallization is caused by continual motion in combination with strain, but how is not yet exactly agreed upon. Annealing, at proper intervals (i. e. raising it to a red heat and allowing it to cool slowly), is sufficient to prevent the tendency to crystallize in iron.

When a small plane surface (but not very minute) bears upon a large one, for many years, the efficiency is in proportion to the area of the smaller surface, and the hardness of the softer of the two materials. I have said that the surfaces should bear for many years, in order to eliminate the assistance given to the surface, in the larger plane, by the neighbouring parts. This assistance, though it will always act, will probably be much decreased in course of time, and I do not intend to reckon upon it at all. We may say more generally: a failure of two plane, or other, surfaces in contact under pressure, must be caused by dislocation of a certain amount of the softer metal; *every part of which resists dislocation.* Therefore if we have surfaces of the same metals, but of various areas, pressed together in pairs, then the pressure necessary to cause failure in any pair is in proportion to the amount of metal which is *liable* to be dislocated in that pair; or which *must be* dislocated in that pair, in order to allow of a given minute failure (such as the thousandth of an inch or less). This is equally evident whether the surfaces be flat or curved. And the efficiency is also in proportion to the crushing weight of the metals; and hence we arrive at the following definition, applying equally to flat, curved, or slanting surfaces.

The *effective bearing area* of a surface bearing upon another, is obtained by dividing the cubic amount of bearing material which must be displaced in order that any very small failure may take place, by the lineal extent of such failure.

The efficiency of the bearing surface of wrought iron upon any other metal will be considered equal to the number of square inches in its effective bearing area multiplied by 5 tons; (5 tons per square inch may therefore be considered the effective *hardness* of plate).

The *effective line of bearing* is a term I shall also use when a rivet or bolt has its bearing upon the thickness of a plate; it expresses the breadth of the same plate which would be required to give a sectional area equal to the effective bearing area of the rivet or bolt; it therefore corresponds to the line of fracture of a plate in tension or compression.

It will therefore appear that a line of bearing represents the same degree of efficiency in the same plate as a line of fracture (or line of section) of $\frac{10}{9}$ its length; since it will bear 5 tons where the latter will only be safe for $4\frac{1}{2}$; and conversely, 10″ length of section of plate is equivalent to 9″ length of rivet bearing in the same plate.

It will also be noticed, that any crack in the plate caused by punching, any riming necessary, or any excess in the size of the punch used, will operate to diminish the strength of the plate without affecting that of the rivet, or its bearing. Hence if this rule of efficiency be a good one, it shews that the value of a true section of plate is really worth *more* than $\frac{9}{10}$ the same area of bearing surface, and may be equal to it.

Example, and application to rivets. Thus, in order that a rivet, Fig. 2, of a diameter AD in a $\frac{1''}{2}$ plate may cut a length AB through the plate in the direction of the arrow, it must crush out the iron contained beneath the figure $ABFCDE$, where $AB = CD$. Join BG, HC; then the area of the semicircle $BFC = AED$.

Add to each the areas ABG, DHC, and subtract the area EGH, and we have the figure $ABFCDE =$ the rectangle $ABCD$.

Therefore the quantity of material in the plate under these figures will be equal.

And the effective bearing area as above defined

$= $ the quantity of iron under $ABFCDE \div AB$,

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; ABCD \div AB$,

$= \frac{1''}{2} \times AB'' \times BC'' \div AB''$, $\frac{1''}{2}$ being the thickness of plate,

$= \frac{1''}{2} \times BC''$,

$=$ the diameter of the rivet into the thickness of the plate. And therefore the effective line of bearing is equal to the diameter of the rivet. By a similar process it will be found that if the rivet fail instead of the plate, for say the thousandth of an inch, the dislocation will be just the same as if the plate failed as much, and the rivet remained sound.

In practice the rivet is generally the softer in girder work, and if the bearing surface be too weak, is the first to get marked by the superior hardness of the plate: but it is seldom abraded; and therefore if the rivet be sufficiently stout in diameter, in proportion to the thickness of the plate, its outer surface may be hardened by the compression of its particles until it can make an impression upon the plate. But this matter seems one of interest rather than profit.

Ex. The efficiency then of the bearing of a $\frac{7''}{8}$ rivet in a $\frac{3''}{4}$ plate is

$$\frac{7''}{8} \times \frac{3''}{4} \times 5 \text{ tons} = 3.3 \text{ tons nearly.}$$

It may be observed that if we chose to assume the actual area pressing against the rivet, i. e. the length of circumference

$$AED \times \frac{1''}{2},$$

as the effective bearing area of the rivet, such an assumption could only give us a true result if the friction of the two surfaces together were so great as to prevent any slipping, except under a tangential force equal to the normal force necessary to crush them. Such is not the case; and it is not safe to reckon on any friction at all, except in particular cases. In the case of rivets, if it be available, it is already reckoned in the estimate of 5 tons for the harness of iron, which is founded on experience on rivets.

The Efficiency of a rivet.

The efficiency of a rivet is the strain that may be put upon it when in the work, either to tear or to shear it, without danger of injury either, &c.

It is well to describe the duties of a rivet, and what amount of efficiency in the rivet will be considered necessary to ensure their fulfilment.

I. The rivet must be so strong, that when the plates which it holds are subject to a force tending to make them slide upon one another, it shall neither be shorn off (i. e. cut off by the plates acting like a pair of scissors), nor shall its internal structure be so tried by the tendency to shear it, as to endanger crystallization or alteration of fibre.

The rivet's capability in this particular is, as might be foreseen, in proportion to the section tending to be shorn, which is its sectional area; and therefore varies as the *square* of the diameter.

II. The rivet must be strong enough *at the same time* to be drawing the plates so firmly together as to prevent any buckling or looseness. Its capability in this particular is also in proportion to its sectional area.

It will be observed that the force which requires these duties in the rivet must, in the first case, be impressed upon the rivet by the bearing surface of the plate against it; which varies as the thickness of the plate and the *diameter* of the rivet; and, in the second case, will depend upon the thickness and stiffness of the plates which the rivet has to draw into close contact, and upon the frequency of the rivets. Hence it is that every thickness of plate has a particular diameter of rivet assigned to it by practical rivetters as the best for that plate, and which is not in direct proportion to the thickness of the plate. If the diameter of the rivet be much greater than this, the plate will be too weak to call its full strength into play; if much less, the rivet, if it ever succeed in drawing the plates tight together at all, will at all events bend or even shear before it has called forth the full strength of its hold on the plate. The distance of the rivets from each other is also to be considered in determining their diameter.

The efficiency of a rivet in united shearing and drawing strength is unconnected, except indirectly, with the strain on the plates; but in ordinary work it may be considered that 3½ tons strain may be put on the plate for every square inch section in the rivets, if otherwise well proportioned.

I may here state that this is quite an empirical rule founded upon the ordinary practice in girder work. Further experiments are wanted before any trustworthy rule can be made. (I have not used this rule in my subsequent calculations, and that it will admit of alteration will appear probable from the following, founded upon plain experiments.)

The efficiency of a rivet in tension only, will be considered equal to 4 tons for every square inch in its section.

This assumes the heads to be formed of a length of the rivet-rod equal to 1½ of its diameter to each head; if formed of less metal than this, the head will break off before the rivet tears asunder.

Rivets, being of tougher iron than plate, might be safely trusted with 5½ tons to the square inch; were it not that the danger of weak heads, or imperfection in the rivet, and the completeness and suddenness of a failure if it should occur, together with the initial strain put on them by cooling, make it better not to trust them with more than 4 tons on the square inch, especially where it is necessary that they should still draw the plate while under load.

The efficiency of a rivet in shearing will be taken equal to $4\frac{1}{2}$ to 5 tons to the square inch of section, according to the advantage or disadvantage of its position.

The Efficiency of the friction of two rivetted plates.

It is certain that in a good joint the friction of the two plates rivetted together, aided by the insensible hollows and inequalities of their surface, is so great, that for a time it takes the whole strain; and that even if the rivet did not fill the rivet-holes, the plates would not always stir nor move upon one another so as to bring the rivet in contact with the sides of the rivet-holes.

But this only applies to a freshly rivetted joint, in which all the rivets draw the plates with their *full* tensile power. About 15^0 ($14\frac{1}{2}^0$) of Fahr. cause the same expansion of wrought iron as 1 ton per square inch tension would do. A tension of 24 tons per square inch would therefore be induced in the rivet by its cooling 24×15^0 or 360^0 after rivetting, more than the plate in its immediate vicinity. If then the rivet were on the average 360^0 hotter than the neighbouring plate, when it became a dull red, there would be on it a tension of 24 tons per square inch of its section when the rivet and plate had reached the same temperature on cooling. The coefficient of friction for wrought iron upon wrought iron is about $\frac{1}{4\frac{1}{8}}$. Therefore supposing, as is probable, that the rivets of a joint when cool are at their full tension of 24 tons per square inch in their section, the friction which has to be overcome to make the plates slide, is equal to $\frac{1}{4\frac{1}{8}}$ of such tensile power $= \frac{1}{4\frac{1}{8}}$ of 24 tons $= 5\frac{1}{2}$ tons per square inch section of the rivets, which may be called the breaking weight of the friction, and which is enough to hold the plates even of a bad joint firmly together.

Now supposing the rivet to sustain its full tensile power permanently, one of two things might happen with regard to the friction as affecting the strength of the joint.

I. If the joint be undisturbed, or even if never subjected to more than $\frac{1}{4}$ of the ultimate strength of the friction, the two plates continually pressed together with great force by the rivets will continually obtain a closer frictional contact, and might perhaps, if sufficiently clean, in time actually amalgamate, as two plates of glass will do if packed tightly together for a length of time.

II. If the joint be at frequent intervals subjected from the first to tension very nearly approaching the ultimate power of the friction, the molecules in the surfaces of the plates in contact may become so stretched, altered, or crystallized, as to make the friction weaker and weaker, until at last the surfaces gradually shift. This will take place more rapidly, if the molecules, while under strain, be subjected to violent vibrations of the class producing sound. The shifting may go on until the full force of the tension, calculated upon from a given load, comes upon the bearing surfaces of the rivets; while on each temporary discontinuation of the tension and of the vibrations simultaneously, the friction may even hold the plate close against the bearing surfaces with a great proportion of their maximum pressure. This will produce no apprehension, if you have made the bearing area of the rivets sufficient to maintain permanently the maximum load; a requirement which, in this treatise, I wish to fulfil in proportioning every part of the structure.

But again.

III. The original tensile force of the rivets may gradually be eased by continual alterations of temperature, and gradual alteration of fibre; and the maximum power of friction decreased in the same proportion.

These three considerations leave the mind disposed to withdraw any firm confidence from joints depending for their strength mainly on the friction.

The efficiency of the friction of two surfaces rivetted together will be considered valueless.

With regard to the sustaining power of rivets in tension, (as well as of the friction of joints,) the peculiarity is, that it is "unstable." If a rivet compress in bearing, or bend in shearing, its strength is by no means diminished, until the yielding has gone on to a sensible extent; but when a number of rivets have to sustain a load by their tension, if any one insensibly yields in the fibre it is so much weaker, and the more strain is thrown on the rest; this is particularly dangerous in girder work, which, unlike boiler work, is exposed to violent vibrations while under strain. This kind of connexion is much, perhaps too much, avoided by engineers in the construction of girders.

Of cold rivetting the author has had no experience.

The efficiency of iron in the preceding rules has been selected as most trustworthy and economical for girders of large span. For small railway girders a lower standard of efficiency is better.

The reader will observe that I have fixed the above as the measures of the efficiency of iron in each case, for use in this treatise. In practice it is well to consider them afresh in each work; and they may even be varied with advantage in the same work. Their amount depends, among other things, on the following considerations, viz. the quality of iron, the quality of workmanship, the facilities of inspection during or after construction, and of preservation and restoration after construction. It is found to have been affected also by the measure of control the engineer may have possessed over the manufacturing department, his own boldness or experience, the manufacturer's straightforwardness; and lastly, more, sometimes, than has been imagined, on a spirit of honour and of ingenuous circumspection caught from the proprietors. These last points, though all will agree that they are *immaterial*, no practical man will have found to be unimportant.

The preceding measures of efficiency are about $\frac{1}{4}$ of the average strain capable of causing immediate failure. The following particulars may be interesting in their reference to this subject. All weights mentioned have reference to *an area of a square inch.*

The cohesive strength of English bar or plate iron averages 25 tons to 20 tons.

The compressive strength of English bar or plate iron averages 20 tons.

If 1″ rivets in $\frac{1''}{2}$ plate in a girder be subjected to a calculated average testing strain of 7 tons to the square inch of effective bearing area, some will be found marked by the plate's cutting them in a few days.

The drawing power of a rivet in a girder need not obviously be so strong either as in a boiler, which has to be water-tight under pressure, or in a ship's skin, where the joints should be water-tight, and strong enough to stand when caulked. But the following Table will shew the practice in rivetting for boilers and ships.

Rivets in steam or water-tight joints; (dimensions in inches).

Thickness of plate.	Diameter of rivet.	Length of rivet from head.	Central distance of rivets.	Lap in single joints.	Lap in double joints.	Equivalent length of head.
$\frac{3}{16}$	$\frac{3}{8}$	$\frac{7}{8}$	$1\frac{1}{4}$	$1\frac{1}{4}$	$2\frac{1}{16}$	$\frac{1}{2}$
$\frac{1}{4}$	$\frac{1}{2}$	$1\frac{1}{8}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$\frac{5}{8}$
$\frac{5}{16}$	$\frac{5}{8}$	$1\frac{3}{8}$	$1\frac{5}{8}$	$1\frac{7}{8}$	$3\frac{1}{8}$	$\frac{3}{4}$
$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{5}{8}$	$1\frac{3}{4}$	2	$3\frac{3}{8}$	$\frac{7}{8}$
$\frac{1}{2}$	$\frac{13}{16}$	$2\frac{1}{4}$	2	$2\frac{1}{4}$	$3\frac{3}{4}$	$1\frac{1}{4}$
$\frac{9}{16}$	$\frac{7}{8}$	$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{2}$	$4\frac{1}{8}$	$1\frac{3}{8}$
$\frac{5}{8}$	$\frac{15}{16}$	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{3}{4}$	$4\frac{5}{8}$	$1\frac{1}{2}$
$\frac{11}{16}$	1″	3″	$2\frac{3}{4}$	3	5	$1\frac{5}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$3\frac{1}{4}$	3	$3\frac{1}{4}$	$5\frac{7}{8}$	$1\frac{3}{4}$

When two plates are rivetted together, and the rivetting proportioned so that the strength of the rivets and of the plate are equal to one another for a *permanent* strain; there is no doubt that if the plates be torn asunder by the abrupt violence of an experiment, then the rivets will be sure to hold, and the plate to break. This may be because rivets will yield, enough to spoil a permanent joint, before they will all "bear" as they would in the experiment. Other reasons might be mentioned. The friction alone may increase the temporary strength of the rivets by $\frac{1}{3}$.

CHAPTER II.

COMBINATION.

The strength of a joint.

THE application of the definitions of Chapter I. to the determination of the strength of joints forms one of the most intricate items in girder calculations.

A common boiler, or *lap-joint.*

Let A, B, Fig. 3, be two plates rivetted by a common boiler-joint. Then the joint may be faulty in the following ways:

I. The plate in either A or B may be too weak along the line of fracture.

The line of fracture will be straight through all the rivet-holes in either plate,

or Line of Fracture, (L. F.) $= 9'' - 4 \text{ no.} \times \frac{1''}{2} = 7''$.

Breaking area $= 7'' \times \frac{1''}{4} = 1\frac{3}{4}''$.

Efficiency $= \frac{7''}{4} \times \frac{9}{2}$ tons $=$ say 8 tons.........(1).

Hence with more than 8 tons on the joint, the plate would be liable to break away through all the rivet-holes.

II. The bearing surfaces may crush.

The bearing area of a $\frac{1''}{2}$ rivet in $\frac{1''}{4}$ plate $= \frac{1}{8}$ sq. in.;

therefore, the bearing area of the rivets in this joint

$$= \frac{1''}{8} \times 4 \text{ no.} = \frac{1}{2} \text{ sq. in.}$$

$$\text{and may bear } \frac{1''}{2} \times 5 \text{ tons} = 2\tfrac{1}{2} \text{ tons} \ldots\ldots\ldots\ldots (2).$$

Hence with more than $2\frac{1}{2}$ tons the rivets would be liable to cut through the plate.

III. The rivets may all shear or lengthen so as to make a loose joint.

$$\text{The area of a } \frac{1''}{2} \text{ rivet} = \frac{1''}{5} \text{ nearly;}$$

$$\text{therefore, rivet area} = \frac{1''}{5} \times 4 \text{ no.} = \frac{4}{5},$$

$$\text{and may bear } \frac{4''}{5} \times \frac{7}{2} \text{ tons} = 2\tfrac{4}{5} \text{ tons say} \ldots\ldots\ldots (3).$$

Hence with more than $2\frac{4}{5}$ tons the rivets are liable to shear or slacken.

Summary. It therefore appears that the value of the joint is determined by the bearing surface of the rivets and is $2\frac{1}{2}$ tons.

In practice the above analysis will be much abbreviated, since it would be impossible to use so cumbrous a form in calculating the value of every joint, tried, as well as adopted, in even a small girder. I will, therefore, now give the analysis of the same joint in an abbreviated form. The third head, relative to the shearing and drawing power of the rivet, will be always omitted: for it will be assumed that the engineer considers the rivets he is using strong enough for the thickness of the plate: and it is obvious that, if the diameter, and therefore the section, of one rivet be in due proportion to its bearing area on the plate, then the united sections of all the rivets in a joint must also be in the same due proportion to the united bearing area.

Let A, B be two plates rivetted by a common boiler-joint.

Line of fracture of either plate right through the rivet-holes

$$= 9'' - 4 \text{ no.} \times \frac{1''}{2} = 7''.$$

Efficiency $= 7 \times \frac{1}{4} \times \frac{9}{2} = \text{say } 8 \text{ tons.}$

Efficiency of bearing area of 4 no. rivets

$$= 4 \text{ no.} \times \frac{1''}{2} \times \frac{1''}{4} \times 5 = 2\tfrac{1}{2} \text{ tons.}$$

∴ The efficiency (or value) of joint is $2\frac{1}{2}$ tons.

Fig. 3*a* is the same case, except that another rivet is inserted; it may be supposed to be required by some connexion or other cause; but is a case that could not well occur in practice, and is used merely as an example.

The joint would go by the failure of the *plate* between *b* and *d*, and of the bearing of the outer rivets; for we have

1. Outside *b*, *d*. Value of rivets *a* and *e* and half of *b* and *d*

$$= 1\tfrac{1}{2}'' \text{ bearing}$$

$$= \frac{3''}{2} \times \frac{1''}{4} \times 5 \text{ tons} = 1\tfrac{7}{8} \text{ tons.}$$

2. Inside *b*, *d*. Value of rivets *c* and half of *a* and *e*

$$= 1'' \text{ bearing}$$

$$= 1 \times \frac{1}{4} \times 5 \text{ tons} = 1\tfrac{1}{4} \text{ tons.}$$

Value of plate between *b* and *d*

$$= 2'' - 1'' = 1''$$

$$= 1 \times \frac{1}{4} \times \frac{9}{2} \text{ tons} = 1\tfrac{1}{8} \text{ tons.}$$

Between *b* and *d* we must therefore take the value of the plate as limiting the value of the joint; which is, therefore,

$$1\tfrac{7}{8} + 1\tfrac{1}{8} = 3 \text{ tons.}$$

If such a joint did occur in practice it might safely be trusted with the $3\frac{1}{8}$ tons due to the rivets, since the strength of the plate between *a* and *b* would be sufficient to account for the whole strength of the rivet *b*. I have given this example to shew the proper application of the method of calculation here adopted to such a case; and also the sort of modification required, if necessity require an unworkmanlike joint, for which this method of calculation is liable to fail.

Fig. 4 shews a boiler-joint, as usually constructed when with $\frac{1''}{2}$ rivets: see Table, page 25.

Line of fracture of either plate, through the rivet-holes,

$$= 9'' - 6 \text{ no.} \times \frac{1''}{2} = 6''.$$

Effective line of bearing $= 6 \text{ no.} \times \frac{1''}{2} = 3''$.

The efficiency of the bearing therefore limits that of the joint, which is

$$3'' \times \frac{1''}{4} \times 5 \text{ tons} = 3\tfrac{3}{4} \text{ tons},$$

or $\frac{3''}{9''} \times \frac{10}{9} = \frac{10}{27} = \frac{1}{2.7}$ *that of the entire plate.*

(There is no doubt it might be really more efficient if exposed to the rust and regular strain of a boiler.)

A common butt joint.

Let the plates *A* and *B* in figure 5 be jointed by means of the cover plate *C*.

1. The plate *A* may break through the rivets *a*, *c*, *e*.

$$\text{L. F.} = 9'' - 3 \times \frac{3''}{4} = 9 - 2\tfrac{1}{4} = 6\tfrac{3}{4}''.$$

2. The plate *A* or cover *C* may break through the rivets *a*, *b*, *c*, *d*, *e*.

$$\text{L. F.} = 1\tfrac{1}{2} + 1\tfrac{1}{2}\sqrt{2} \times 4 \text{ no.} + 1\tfrac{1}{2} - 5 \text{ no.} \times \frac{3''}{4} = 3 + 8\tfrac{1}{2}'' - 3\tfrac{3}{4} = 7\tfrac{3}{4}''.$$

3. The cover may break through a, b, d, e.

$$\text{L. F.} = 1\tfrac{1}{2} + 1\tfrac{1}{2}\sqrt{2} + 3 + 1\tfrac{1}{2}\sqrt{2} + 1\tfrac{1}{2} - 4 \times \frac{3''}{4} = 6 + 4\tfrac{1}{4} - 3 = 7\tfrac{1}{4}''.$$

(The same remarks that apply to A, obviously apply to B.)

Of these three ways we see that the first is the most likely; therefore the value of the plate as determined by its fracture right through a, c, e, is

$$6\tfrac{3}{4}'' \times \frac{3''}{8} \times \frac{9}{2}\text{ tons} = 11\tfrac{1}{2}\text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A).$$

Again the rivets may all cut the plate.

Efficiency of bearing

$$= \frac{3''}{4} \times \frac{3''}{8} \times 5\text{ no.} \times 5\text{ tons} = \frac{45 \times 5}{32} = 7\text{ tons} \ldots\ldots\ldots\ldots (B).$$

Hence the value of the joint is determined by the strength of the bearing surface, and is 7 tons.

When a joint has occurred in a part of a plate liable to compression only, it has often been attempted to plane the ends of the plates A and B which butt together, so that they should fit against one another with perfect accuracy. A very slight cover has then been used, or the plates have been left without any cover, if retained in their places by the contiguous parts of the structure. But, however accurately the ends may be planed and polished, and however closely they may be pressed together in the work before being finally rivetted, it is doubtful whether such a joint can sustain a due proportion of the strength of the plate, without danger of violence to the neighbouring parts in order to allow of closer mechanical contact to the butting ends. Also, though a cover be thus saved, the expense of the extra care required by such a joint has by many engineers been considered condemnatory. With every care, the difficulty of getting them unfailingly put together in practice is, in fact, insuperable, and leads to the wretched expedient of using cements or rust to fill up the space. Plates jointed by a butt joint will, in this treatise, be considered not in contact.

A joint might be formed by a weld of the two plates together. This may be done with safety in some cases with angle irons; and has been adopted in particular cases in plate subject to compression. But in the present state of workmanship it is considered to require too refined a care for constant use; and after all is only applicable to small portions of a girder, which can be moved about and turned over while being welded.

It may be noticed that the last joint, Fig. 5, is evidently deficient in rivets. It may be improved by increasing their number and lengthening the cover proportionally, Fig. 6.

In this case we see, as before, that the weakest part of either plate is in a straight line through the first row of rivets; and its efficiency to be

$$11\tfrac{1}{4} \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (A).$$

The efficiency of the bearing of all the rivets will be

$$8 \text{ no.} \times \frac{3''}{4} \times \frac{3''}{8} \times 5 \text{ tons} = \frac{45}{4} \text{ tons} = 11\tfrac{1}{4} \text{ tons} \ldots\ldots\ldots (B).$$

This is obtained by a *cover* $10\tfrac{1}{2}''$ *long.*

We thus get the joint $= 8 \times \frac{3}{4} \times \frac{10}{9} \times \frac{1}{9} = \frac{20}{27} = \frac{3}{4}$, or rather less, of the *strength of the whole plate.*

To avoid cutting so much out of the plate by the outer lines of rivets they might be made of smaller diameter, and the inner rows of greater diameter; this will lengthen the line of fracture, and allow a greater bearing surface to be got, also, by the same number of rows of rivets, disposed according to the old method. This would be practically inapplicable unless the cover is thicker than the plate. Thus in Fig. 7 each plate has two rows of rivets, the one row of larger diameter than the other. Both rows might be punched at once by a machine, such as that described on page 10, without change of punches or ambiguity.

To give an instance of such a joint. It might be made of 2 strips of $6'' \times \frac{3}{16}''$ iron covering the $\frac{1}{4}''$ plate; it is, however, here represented with a broad T iron on one side, as if for a web-joint. (Fig. 7.)

Here the $\frac{7}{8}''$ rivets have 2″ between their centres, therefore the line of plate between 2 rivets is $1\frac{1}{8}$, and the equivalent line of bearing is $\frac{9}{8}'' \times \frac{9}{10} = 1''$.

Hence the strength of the plate between the $\frac{7}{8}''$ rivets is in excess of the strength of the bearing surfaces of those rivets...(1).

Again, the sectional line of plate between any two $\frac{7}{16}''$ rivets and a $\frac{7}{8}''$, as a, b, c, is

$$2\sqrt{\left|\frac{9}{8}\right|^2 + 1^2} - \frac{7}{8}'' - \frac{7}{16}'' = 2\sqrt{\frac{145}{64}} - 1\tfrac{5}{16}'' = 2 \times \frac{12}{8} - 1\tfrac{5}{16} \text{ nearly}$$

$$= 1\tfrac{11}{16},$$

equivalent to a line of bearing of

$$\frac{27''}{16} \times \frac{9}{10} = \frac{24\cdot 3}{16} = 1''.52.$$

Now the amount of bearing acting on this section of plate is that due to two rivets

$$= \frac{7}{8} + \frac{7}{16} = 1\tfrac{5}{16}'',$$

therefore the strength of the $\frac{1}{4}''$ plate is enough to cut all the rivets..(2).

Therefore the efficiency of the joint as far as the $\frac{1}{4}''$ plate is concerned is determined by its hold upon the rivets, and is, therefore, per foot run of joint

$$\frac{21''}{16} \times \frac{1''}{4} \times 6 \text{ no.} \times 5 \text{ tons} = \frac{630}{16 \times 4} \text{ tons} = 9\cdot 84 \text{ tons} .... (A)$$

Again, the efficiency of the T iron between the $\frac{7''}{8}$ rivets is per foot run

$$\left(12'' - 6 \text{ no.} \times \frac{7''}{8}\right) \times \frac{3''}{8} \times \frac{9}{2} \text{ tons}$$

$$= 6\tfrac{3}{4} \times \frac{27}{16} > \frac{27}{4} \times \frac{27}{18} > \frac{81}{8} > 10 \text{ tons} \ldots\ldots. (B).$$

Hence the efficiency of the joint per foot run is determined by (A), and is

9.84 tons.

That of 1′0 run of plate is

$$\frac{1''}{4} \times 12'' \times \frac{9}{2} \text{ tons} = 13\tfrac{1}{2} \text{ tons};$$

therefore the proportion of the *strength of the entire plate* obtained by *two rows* of rivets is, (A),

$$\frac{630}{16 \times 4} \times \frac{2}{27} = \frac{35}{48}, \text{ or 73 per cent.}$$

Another joint which has more interest, because likely to be more used than the other, is Fig. 8.

I. The left-hand plate may break across through the rivet a,

$$\text{line of fracture} = 10'' - \frac{3}{4}\frac{1}{16} = 9\tfrac{3}{16} \ldots\ldots\ldots\ldots (1).$$

The plate will not break from a to b rather than straight across, since

distance from a to edge $= 5''$,

$$\ldots\ldots \text{ through } b \ldots\ldots\ldots = 2\tfrac{1}{2} + \sqrt{9 + \frac{25}{4}} - \frac{3}{4}\frac{1}{16},$$

$$= 2\tfrac{1}{2} + \frac{1}{2} \text{ of } 7\tfrac{3}{4} - \frac{3}{4}\frac{1}{16},$$

$$= 1\tfrac{5}{8}\frac{1}{16} + 3\tfrac{7}{8},$$

$$= 5\tfrac{9}{16},$$

giving a total line of fracture through *bac* of

$$2 \times 5\tfrac{9}{16} - \frac{3}{4}\ \frac{1}{16} = 10\tfrac{5}{16} \ldots\ldots\ldots\ldots\ldots (2),$$

nor through *b* and from *b* through *d*, since

distance of *b* from edge $= 2\tfrac{1}{2}''$,

.... through *d* to edge

$$= 1\tfrac{1}{4}'' + \sqrt{\left[\frac{5}{4}\right]^2 + \left[\frac{6}{4}\right]^2} - \frac{3}{4}\ \frac{1}{16} = \frac{5}{4} + \frac{8}{4} - \frac{3}{4}\ \frac{1}{16},$$

$$= 3\tfrac{1}{4}'' - \frac{3}{4}\ \frac{1}{16} = 2\tfrac{7}{16}'',$$

which is only $\frac{1}{16}$ less.

The total distance then from *a* through *b* and *d* to edge is

$$3\tfrac{7}{8} + 2\tfrac{7}{16} - \frac{3}{4}\ \frac{1}{16} = 6\tfrac{5}{16} - \frac{3}{4}\ \frac{1}{16} = 5\tfrac{1}{2}'',$$

giving a total line of fracture of $10 - \frac{3}{4}\ \frac{1}{16} = 10\tfrac{3}{16} \ldots (3)$;

therefore if the plate goes without tearing a rivet, it will go directly across rivet *a*.

II. The plate will not tear *a* and go through *b* and *c*, for the

section of plate on this line $= 10'' - 1\tfrac{5}{8} = 8\tfrac{3}{8}''$

section equivalent to *a*'s bearing $= \frac{3}{4}\ \frac{1}{16} \times \frac{10}{9} = \frac{7}{8}\ \frac{1''}{32}$

Total $9\tfrac{1}{4}''\ \frac{1}{32} \ldots (1)$.

But the plate will tear *a* and go through *dbefcg*, since the line of this section

$$= 5'' + 4 \text{ no.} \times \sqrt{\left[\frac{5}{4}\right]^2 + \left[\frac{3}{2}\right]^2} - 6 \text{ no.} \times \frac{13''}{16},$$

$$= 5 + 4 \times \frac{1}{4} \times \sqrt{61} - \frac{39}{8},$$

$$=5+8-\frac{1}{32}-4\tfrac{7}{8}=8\tfrac{1}{8}-\frac{1}{32},$$

$$=8\frac{1}{16}\,\frac{1}{32},$$

which, adding a length of plate equivalent to *a*'s bearing, gives a total of

$$8\frac{1}{16}\,\frac{1}{32}+\frac{7}{8}\,\frac{1}{32}=9'' \text{..................} (2).$$

If the plate were to be cut by the rivets *abc* and break through *defg*; then the

length of the plate on this line

$$=10''-4\times\frac{3}{4}\,\frac{1}{16}=10-3\tfrac{1}{4}=6\tfrac{3}{4}'',$$

length equivalent to bearing of *abc*

$$=3\text{ no.}\times\frac{13''}{16}\times\frac{10}{9}=\frac{130}{16\times3}=\frac{21\frac{2}{3}}{8}=2\tfrac{3}{4}''$$

$$\text{Total } 9\tfrac{1}{2}'' \text{............} (3),$$

or through *dhefig* the section will be nearly the same.

If the plate be strong enough not to break at the rivets *bcdefg*, it will certainly not break at any point beyond them.. (4).

III. All the rivets may cut their bearings; the line of section equivalent to this will be

$$10\text{ no.}\times\frac{13''}{16}\times\frac{10}{9}=9.02'' \text{.....................} (1).$$

Hence, in this joint the way this plate (and the others are similar) will be most strained, will be nearly equally

on the bearing of all the rivets: III. (1).

.... *a*, and the section zigzagging between the next two rows: II. (2), and the value of the joint is

$$\frac{1''}{2} \times 9'' \times \frac{9}{2} \text{ tons} = 20\tfrac{1}{4} \text{ tons},$$

and therefore $= \frac{9}{10}$ *of the value of the entire plate.*

Such a joint is often applicable in the lower member of a girder; and if it be used as an overlap joint in rivetted suspension-chains, it will save at least $\frac{1}{8}$ the weight of the iron in an ordinary chain.

If two or more plates require to be jointed together, much will be saved by having the same cover-plate to include them all, and jointing them successively, as in figs. 9—11.

In fig. 9, the breaking line of any plate through the rivet-holes (taking them as liable to be 1″ diameter; for where many plates have to fit one on another, riming may be expected to be sometimes rendered necessary) is

$$2'0 - 6 \text{ no.} \times 1'' = 15''.$$

The bearing line on the rivets

$$18 \text{ no.} \times \frac{15}{16} = \frac{135}{8} = 17,$$

equivalent to 18.9″ breaking line, which is therefore amply sufficient. The joint therefore is $\frac{3}{4}$ the *strength of the entire plate.*

If we suppose the plates *ABC* to be loaded with 3 tons per square inch of section (that is, equal to 4 tons on the square inch along the line of fracture), the plates will be stretched a certain distance x*, say, between every two rows of rivets; and

* The value of x may be easily determined, since iron expands $\frac{\cdot 84}{10,000}$ of its length for every ton tension per square inch of its section: hence

$$x = 3 \text{ tons} \times \frac{\cdot 84}{10,000} \times 3'' \text{ in inches.}$$

$$= \cdot 0007.56 = \text{about } \tfrac{3}{4} \text{ of } \tfrac{1}{1000} \text{ of an inch.}$$

the only force exerted by the rivets beyond the cover-plate will be to hold the plates close together.

We shall, then, have the rows *ab*, $3''+x$ apart; and so *gh* would, as far as plates *B*, *C* are concerned, tend to be $3''+x''$ apart. The break in the plate *A* at the joint between *g* and *h*, would however bring additional strain upon them; were it not that plate *A* must stretch plate *D* to a length $3''+x$ between *g* and *h*, before *B* and *C* can be left free to expand, even so much as they would without additional strain. This *A* does, and no more. And it is also probable that if the plates have, previously to use, been proved by a *test* load, then very nearly $\frac{1}{3}$ of the tension will pass up from *A* by each row of rivets *e*, *f*, *g* into *D*; i. e. at the rate of 1 ton per square inch of the whole section will pass up each row. The plate *E* is, similarly, stretched between *j* and *k* by the pressure which the break in the plate *C* between *j* and *k* brings upon the bearing surfaces of the rivets in *h*, *i*, and *j*. These act at a disadvantage *through* the plate *B*; still if they do not, immediately after rivetting, bear sufficiently on *C* and *E* to pass *E*'s strain through *B*; but would leave some of it to be supported by *B* between *g* and *h*, and so to be taken up by *C* through the rivets *e*, *f*, *g*; then *B* would be so first stretched by the test load as to have a greater permanent elongation in the parts overstrained: and it would thence be probable that *subsequently*, for ordinary loads, the strain would be evenly distributed over the three continuous plates at each discontinuity of the fourth in this joint. Two plates may certainly be jointed together with one cover-plate with full efficiency; and three, as in fig. 9, or more, as in fig. 10, thus jointed, will be as efficient if jointed with one cover-plate as with more.

Fig. 11, Nos. 1 and 2, shews the same plate, with ∠ i. below it. The following is the analysis for the joints.

As in fig. 9, the line of fracture of the plate or cover, requires the bearing of 18 rivets.

The line of fracture of an ∠ i. with one rivet hole in it is

$$(4\tfrac{1}{2}''+4'')-1''=7\tfrac{1}{2}''=\frac{15}{2},$$

and no. of rivets required for it

$$=\frac{15''}{2}\times\frac{9}{10}\times\frac{16''}{15}=7.2, \text{ say 8 rivets.}$$

The line of fracture through 2 rivets (see development) is

$$=4''+\sqrt{\overline{4\tfrac{1}{2}''}^2+\overline{1\tfrac{1}{2}''}^2}-2\text{ no.}\times 1''=2+\frac{1}{2}\sqrt{81+9}$$

$$=2+\frac{3}{2}\sqrt{10}=2+\frac{3}{2}\times\frac{19}{6},$$

$$=2+4\tfrac{3}{4}=6\tfrac{3}{4},$$

and requires a number of rivets

$$=\frac{27}{4}\times\frac{9}{10}\times\frac{16}{15}=6.48, \text{ say 7 rivets.}$$

The line of fracture of the wrapper through two rivets is

$$4+\sqrt{\overline{3\tfrac{1}{2}}^2+\overline{1\tfrac{1}{2}}^2}-2\text{ no.}\times 1''=2+\frac{1}{2}\sqrt{49+9}$$

$$=2+\tfrac{1}{2}\sqrt{58}=\text{say } 5\tfrac{2}{3},$$

and will take rivets in number

$$=\frac{17}{3}\times\frac{9}{10}\times\frac{16}{15}=5.44 \text{ rivets.}$$

If we give the ∠ i. a cover of 1 rivet in its full strength, and of 6 rivets in the wrapper which are worth 5.44 only, (since its breaking area will not take more), we get a virtual total of 6.44 rivets, which, as the holes have been taken full size, will be enough for the double-rivetted angle iron, which has been shewn to require 6.48 rivets.

Let us now suppose the plate *A* and angle irons *B* to be under an uniform strain, before covered by *C*; and follow out the action of the rivets by which the strain is passed safely into *D* and *E*.

In fig. 11, No. 1, the cover-plate *C* covers 18 rivets in the plate *A*, and thus takes up the whole strain from it, leaving the strain of the ∠ i. *B* unaltered up to the rivet *d*. The plate *D* receives the strain of 12 rivets in *def* out of the cover-plate; but of the 6 outer rivets in *def*, from the ∠ i. *B*, which is beginning to contract from *k* towards *d*, owing to the inability of the rivets *g* to *k* to stretch it fully.

The rest of the arrangement is obvious; the strain thrown upon the rivets *g* to *k* by the ∠ i. *B* is by them transferred to the plate *D*; and the strain upon *C* is transferred by the next 8 rivets to the ∠ i. *E*.

Now the above arrangement has many objections; as, first mechanically, we should have a large stiff plate *C* rivetted to the ∠ i.'s *B* and *E* of comparatively light section, through the plate *D*. Now, supposing this joint to have been just rivetted and then subject to a strain in tension, then the parts of *D* and *E* beyond the cover would stretch; when they come first under the cover this will be prevented by the stiff cover-plate, and the rivets *p* to *s* will have a great strain brought on them by the bearing surfaces of both plate *D* and ∠ i. *E*; which will *each* bring a crushing strain on *s*, *r*..., but little on *m*, *n*.... For the cover-plate will resist the endeavours of the rivets *s*, *r*, *q*, and perhaps *p*, to stretch it equably with the other plate *D* and the angle iron *E*; until those rivets have transferred to it as much strain as would be properly allotted to about a dozen rivets. Now this arrangement will not have attained theoretically a stable condition under this strain, until the rivets *m* to *s* have cut their way through the cover-plate, enough to release them from all bearing on the plate *D*; and till they thus take each only its due proportion of strain from the ∠ i. *E* to transfer it into the cover *C*. And so of other parts it may be seen that they would not be disposed at first to stretch together, as uniformly as is desirable in a good joint.

It may be here observed, that supposing the rivets *s*, *r*, *q*, &c. to *l*, to have thus cut their way under successive repetitions of a test, or other, load till they all take under it a uniform strain; then, on the removal of the load, the angle iron will contract between the joint and *s* much more than the cover-plate; which was stretched less. This will cause the rivets *s* and *r* and *q* and *p*, to be successively taken from bearing on the cover-plate at all; while the rivets *l*, *m*, &c. will have still as much strain upon them as ever. Thus if the whole load on *D* and *E* were diminished one half; then theoretically the tension of the angle iron *E* between *o* and *s* would be constant, and none of those rivets would bear; while the tension of *E* at every point between *o* and the end of it, will be exactly the same as if it were under the full load, and the rivets will be bearing exactly the same way, as under full load. Actually the action is not so well defined as this, but we see that bad jointing by over stiff covers is as bad in a permanent structure as that by considerably inefficient covers. For, in the former case, many of the rivets, as the bridge wears to true bearings under its heaviest loads, will be permanently saddled with their maximum strain, or thereabouts; and if under this they weaken and their molecules become overstrained and inefficient, the evil will be transferred to those next them, as they yield more and more under successive maximum loads; until at last the joint must give way.

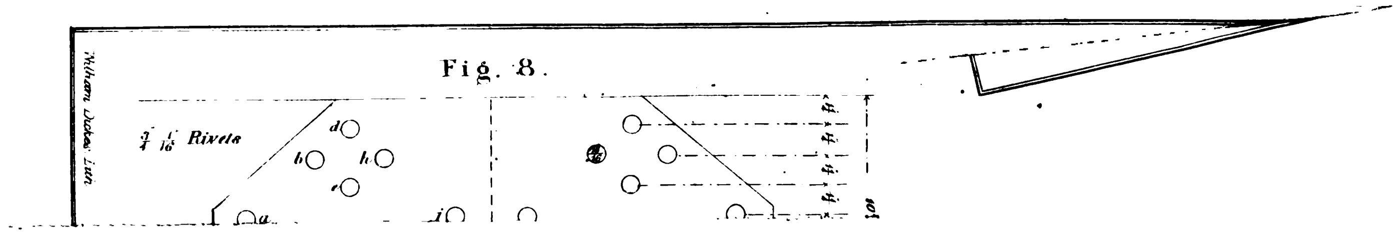
Also, the cover-plate will not be *fairly* stretched by rivets in rows along its two edges only; like those from *l* to *s* on each margin.

Again, thirdly, *supposing* the rivets to have worked in the course of 25 years to such suitable bearings as to distribute the strain in the most even manner over the different plates and bearings. Yet, economically considered, the cover-plate is large and heavy, and will thus cost more, and will also add more to the weight on the bridge, than if a joint could be constructed able to convey in a shorter space the strain from *B* into a cover, and then back to *E*.

Now these objections may be avoided in various ways, but often by incurring others; for instance, as follows:

1. By cutting out of the cover-plate all the metal as shewn within the dotted lines; each cross section would then be in proportion to the strain which the rivets *l* to *s* pass into it, and the whole would therefore expand uniformly, be equally strained and be light: but this would be a very expensive business.

Fig. 8.



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2. Partially, by adding rivets so as to make every row from g to s like those from a to f; or rather like those beyond the cover-plate in figure 9; this would make the parts expand together throughout the joint, but would make the joint for 4′0 length very stiff in comparison to any other part of the structure with which it might be incorporated, and this might be very objectionable; this extra rivetting would be an additional expense and the joint still heavy.

But the best way is to adopt another kind of jointing; by inserting rivets through the other flanges of the ∠ irons (and, if necessary, increasing their thickness throughout to compensate for the loss of section by the additional rivet-hole), and by adding a cover, or wrapper, to take up the strain through these additional rivets.

3. A $4\frac{1}{2}'' \times \frac{5''}{8}$ plate might be put at the back of each angle iron extending far enough on each side of its joint to take 4 rivets: for its breaking area being, with 1″ rivet hole,

$$3\tfrac{1}{2}'' \times \frac{5''}{8} = \frac{35}{16} \text{ sq. in.}$$

would be able to take 4 no. $\frac{15''}{16}$ rivets in $\frac{1''}{2}$ plate if

$$\frac{35''}{16} \times \frac{9}{10} > 4 \text{ no.} \times \frac{15''}{16} \times \frac{1''}{2},$$

or if $$21 > 20,$$

which is the case.

The cover-plate C need then only take three more rivets, and would thus be diminished from 4′9″, which it is at present, to 2′3″, while the covers at the back of the ∠ irons would have to be 2′3″ long.

4. But the best method is that adopted in fig. 11, No. 2, in which a "wrapper" F is adopted; which is a piece of iron rolled exactly as an ∠ i. is rolled, and of a form to fit truly in the hollow of the ∠ i.: the arms of its section will therefore be

about $\frac{1''}{2}$ less than those of the angle iron. It may be rolled so much thicker than the ∠ i., if desired, as to have a breaking section equal to it: here I suppose it $\frac{1''}{2}$ thick, the same as the ∠ i. Its efficiency has been analyzed above; and the improved joint will be readily understood from the figure, and from the analysis referred to.

If my reader have made himself master of the examples contained in this chapter, his time has not been wasted. My object has been, not to make him acquainted with a few empirical forms of joint; but to guide him, through all the ordinary resources available for rivetted joints, to skill in judging of their respective advantages. Nothing is more important than good *firm* jointing. A bad joint can never be detected by the deflection of the girder, while it is by its weakness cheating, and silently destroying other iron associated with it in bearing the strain: it cannot, at least, until great damage has been incurred, and much risk run.

CHAPTER III.

THE BRIDGE. A TRIANGULAR GIRDER.

A BRIDGE consists of girders laid across an open space, near the edges of which it bears; and of a roadway fixed to, or upon them.

The weight of the roadway is generally large in proportion to the weight of the girders; and is uniform in weight throughout the whole length: the girder of an ordinary bridge (though not so in a swing-bridge) is also pretty nearly of a uniform weight throughout. Hence as a close approximation the weight of a girder, with the roadway upon it, is considered uniform; and equal in weight to so much per foot run. Hereinafter this will be called the *dead weight*. The weight of the heaviest load to which the girder is liable, is also reckoned as so much per foot run; this load may cover the whole or any part of the bridge. Hereinafter it will be called the *live weight*. The most accurate investigation and most reiterated endeavour could not obtain for calculation a value either of the dead weight or live weight, which on the completion of the bridge should turn out to be rigidly correct. As to the latter this is evident; and as to the dead weight, it is quite probable that the timber may turn out heavier than was anticipated; the scantlings may be very full, and half-an-inch or more excess of ballast may be put on; and, in fact, a thousand and one accidents (many of which I allow could only result from imperfect, yet hardly faulty, management) may cause the bridge to be heavier than designed; if not on its first construction, at least in the course of its existence. When, in addition, the structure is thoroughly saturated with 2 or 3 days' rain, it is obvious that it may cause a much greater strain on its girders than was at first hoped. These considerations will shew the danger of trying to cut down the dead weight for calculation, too much; and the advantage of having set the defined efficiency of iron safely within its true powers. At the same time, it may be relied upon, that, even if the actual weights only equal but never exceed those taken as the basis of these calculations, there will be no *waste* of iron caused by an excessive margin, in the present state of the manufacture.

It is not uncommon in practice, to have *one* diagram made of a bridge and its strains, so as to be used for *all* bridges, in the following way.

First: you must determine the span of your bridge; and the *greatest* strain upon the horizontal member composing the top and bottom, due to the weight of the bridge (as conjectured) and the whole load. Also the *greatest* strain upon the web-plate, or bars (as the case may be), due to the weight of the bridge; and the same due to the weight of the load. Then: on your diagram you will find

A line whose length must represent the span of the bridge, along which to measure distances.

A line to represent the maximum horizontal strain; and a curve, the ordinates to which, at any distance, give you the proportionate strain due to a uniform load throughout the bridge.

Two more lines to represent the maximum strains on the web; and two curves whose ordinates give you the proportionate strains on the web at any distance along the span; assuming the weight of the bridge uniform, but the load to vary.

If such a plan be used, it can furnish, very approximately for small bridges, the strains of any part. Such a diagram can be formed by means of any of the following propositions. Such a method of determining the proportions of a bridge, by rote, is very useful in an office, for forming estimates; but is not sufficiently close in approximation to the strains, for a detail design of a large bridge; nor is it *safe* to be used for such a purpose, unless under the superintendence of those who are acquainted with the theory by which the diagram is formed; none other can really know what the diagram gives and what it does not give.

Axioms.

1. If a bridge be made strong enough to bear a load, defined in weight and position, in any particular method or way; then it will be able to bear it.

The lightest bridge, however, will support the given load by the easiest method.

2. But unless you can prove that the bridge can support the given load safely, by some one method or other, no reliance can be placed on the bridge.

A TRIANGULAR GIRDER,

Of one of which Fig. 12 is a skeleton-drawing.

This girder is supported at A_0B_0; the roadway and weight is sometimes fixed to it along the bottom A_0B_0, and sometimes laid on the top along C_1D_1.

In accordance with the first Axiom, I shall make the members of the bridge so strong that, if the connexions at the points A_0C_1, A_1C_2, &c. were merely pins, they would be strong enough to sustain the necessary strain. It will then be easy to see that if any other connexion equally firm is used at these points, the strength of the bridge will not be impaired. Each line in the figure represents a bar, or equivalent combination of iron. A_0 and B_0 are supposed to rest on rollers on the supporting piers.

*Statement of the strains upon this girder**.

With an uniform load upon the *top* of the girder, Fig. 12, the bars A_0C_1, A_1C_2, A_2E, B_0D_1, B_1D_2, B_2E will be compressed, the bars A_1C_1, A_2C_2, B_1D_1, B_2D_2 extended.

Hence A_0C_1 will be pushing the pin C_1; A_1C_1 pulling it; therefore the pin C_1 must press against C_1C_2 in the direction of its length, as well as sustain the weight upon it.

And so for the other points; the bars forming the sides of the triangles push the pins C_1 C_2, D_1D_2 upwards and towards E.

Hence the bars C_1C_2, D_1D_2, which prevent the pins C_1, D_1 approaching the centre, are compressed by those pins, and therefore press the pins C_2, D_2 towards the centre E; these latter pins, again, compress the bars C_2E and D_2E, not only with the pressure received from the bars C_1C_2, D_1D_2, but with more, since they have on them the additional thrust of another triangle.

So the bars A_0A_1, B_0B_1 are stretched by the thrust of A_0C_1, B_0D_1 against the pins A_0, B_0.

But A_1A_2 and B_1B_2 are stretched more severely, since they have also the draw of the bars A_1C_1, B_1D_1 and the thrust of A_1C_2, B_1C_2.

* This is a *statement*, not a *proof*.

A_2B_2 has the greatest tensile strain upon it, since it has to hold together the feet of all the triangles on each side.

How these strains are proportioned, and how they support the load, will appear hereafter.

Now, as will hereafter be seen, it is the slanting bars of this girder that support the load; and the horizontal bars merely keep the slanting ones stretched to their original position. In treating of this and other girders, the general term which we shall give to the bars or plate forming the bearing portion of the girder will be *web:* this name is not new. For the top and bottom horizontal bars or boxes, no general name has yet been found; but as the want of one will be a serious inconvenience and obscurity in a treatise of this kind, I shall use for them the term *boom*, as implying a member that keeps the web stretched, in the same way that a yard or boom stretches a sail. The vertical bars immediately over the bearings upon the pier, and which form the end of the girder, and often support its whole weight, I shall call the *end pillars*, or *pillars* of the girder. Hence in a girder, the weight of itself and load is transferred by the web to the end pillars, by which it is sustained; the web being maintained in an extended condition by the booms, which, as such, bring no strain upon the pillars.

To investigate, analytically, the strains on each member of a triangular girder, loaded at the top: Fig. 13.

(This investigation is analytical, and may be omitted by any who is unequal to it, without affecting the rest; provided he read the *summary* of the corollary.)

Let W tons be the weight of the bridge and road; i. e. the total dead weight,

W' tons be the weight of the load; i. e. the total live weight.

Since the greater proportion of the dead, and all the live weight lies on the top of the girder, I shall consider *all* the weight as if it lay solely on the top of the girder; and that the bars C_1C_2, C_2C_3, &c. (or better if it be other longitudinal girders forming part of the roadway) suffice to transmit all such weight to the pins C_1, C_2...

Suppose the length of the girder to be l,
its depth ...... b,
the no. of triangles ...... n,
the $\angle$ of the bars with horizon ...... α.

Then the weight produced by the dead weight upon each pin C_1, C_2, &c. will be $\frac{W}{n}$.

Consider first only the weight $\frac{W}{n}$ which comes upon C_1: it will by the laws of the lever produce a pressure on the pier

$$B_0 = \frac{\text{horizontal distance of } C_1 \text{ from } A_0}{\text{span of girder}} \times \frac{W}{n}$$

$$= \frac{\frac{1}{2}}{n} \times \frac{W}{n} = \frac{\frac{1}{2}}{n^2} W \text{ tons},$$

and on $$A_0 = \frac{\text{horizontal distance of } C_1 \text{ from } B_0}{\text{span of girder}} \times \frac{W}{n}$$

$$= \frac{n - \frac{1}{2}}{n} \frac{W}{n} = \frac{n - \frac{1}{2}}{n^2} W \text{ tons}:$$

now we have to consider how $\frac{W}{n}$ must affect the bars in order to do this.

The vertical pressure, then, on A_0 must be the pressure down A_0C_1 resolved vertically; for there is no other bar which *can* produce any vertical pressure whatever upon A_0; in other words,

$$\text{the compression of } A_0C_1 \times \sin\alpha = \frac{n - \frac{1}{2}}{n} W \text{ tons};$$

$$\therefore \frac{n - \frac{1}{2}}{n} \frac{W}{\sin\alpha} = \text{compression of } A_0C_1 \text{ due to the weight on } C_1 .. (1).$$

Now consider the weight $\frac{W}{n}$ on C_2 only.

This will produce, by the law of the lever, a pressure $\frac{3}{2n} \times \frac{W}{n}$ at B_0, and a pressure $\frac{n - \frac{3}{2}}{n^2} W$ on A_0, which last must be produced as above by

$$\frac{n - \frac{3}{2}}{n^2} \frac{W}{\sin \alpha} = \text{compressive force in } A_0C_1 \text{ due to weight on } C_2 \ldots (2);$$

and so on, however many points they are, till you get to D_1, where by similar reasoning we get

$$\frac{\frac{1}{2}}{n^2} \frac{W}{\sin \alpha} = \text{compression on } A_0C_1 \text{ due to weight on } D_1 \ldots\ldots\ldots (n).$$

Therefore the whole pressure on $A_0C_1 = (1) + (2) + \ldots + (n)$.

$$= \frac{W}{n^2 \sin \alpha} \left\{\left(n - \frac{1}{2}\right) + \left(n - \frac{3}{2}\right) + \ldots + \frac{3}{2} + \frac{1}{2}\right\} = \frac{W}{n^2 \sin \alpha} \times \frac{n^2}{2}$$

$$= \frac{W}{2 \sin \alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

Next to consider A_1C_2.

Pressure at B_0 due to the weight $\frac{W}{n}$ on $C_1 = \frac{W}{2n^2}$, which must be caused by a thrust down B_0D_1, which, when resolved vertically, shall be equal to $\frac{W}{2n^2}$, i. e. by a thrust $= \frac{W}{2n^2 \sin \alpha}$ in B_0D_1; and this is necessarily producing a vertical thrust on the pin D_1, equal to the vertical thrust it is producing on B_0, i. e. equal to $\frac{W}{2n^2}$.

The only bar which can be counteracting this upward thrust of B_0D_1 on pin D_1, is the bar B_1D_1, which, since it is inclined at the same angle as B_0D_1, must, by the same reasoning as before, produce the downward effect $\frac{W}{2n^2}$ on the pin D_1 by a tension of $\frac{W}{2n^2 \sin\alpha}$. This is equal to the compression of B_0D_1, as might have been expected.

So it will be seen that the weight $\frac{W}{n}$ on C_1 produces a compression $\frac{W}{2n^2 \sin\alpha}$ in all the bars B_0D_1, B_1D_2, &c. ... A_1C_1, and a tension $\frac{W}{2n^2 \sin\alpha}$ in all the bars B_1D_1, B_2D_2 ... A_1C_2; therefore strain on bar A_1C_2 due to the weight $\frac{W}{n}$ on C_1 is a tension of

$\frac{1}{2n^2}\frac{W}{\sin\alpha}$ tons ....(1).

Again, the weight $\frac{W}{n}$ on C_2 produces a pressure

$$\frac{n-\frac{3}{2}}{2n^2} W \text{ at } A_0,$$

which must be caused by thrusts

$$\frac{n-\frac{3}{2}}{2n^2}\frac{W}{\sin\alpha}, \text{ in } A_0C_1, \text{ and } A_1C_2;$$

therefore strain on bar A_1C_2, due to the weight $\frac{W}{n}$ on C_2 is a compression of

$$\frac{n-\frac{3}{2}}{n^2}\frac{W}{\sin\alpha} \text{ tons............ (2).}$$

And extending the process as before, we get the total effect of the dead weight upon A_1C_2 to be a compression

$$\frac{W}{n^2 \sin\alpha}\left\{\left(n-\frac{3}{2}\right)+\left(n-\frac{5}{2}\right)+\ldots\ldots+\frac{3}{2}+\frac{1}{2}-\frac{1}{2}\right\}$$

$$=\frac{W}{n^2 \sin\alpha}\times\frac{(n-2)\times n}{2}$$

$$=\frac{W}{2\sin\alpha}\times\frac{n-2}{n} \text{ ....(2).}$$

Similarly the total thrust caused by the dead weight on A_2C_3

$$=\frac{W}{2\sin\alpha}\times\frac{n-4}{n}\;\ldots\ldots\ldots\ldots(3);$$

and so on for other bars. In each series, giving the amount of compression on the corresponding bar, we shall observe an increasing number of negative terms, caused by the fact that an increasing amount of the weight causes a tensile instead of compressive force on the bar. In fact, as we go on, all the weight upon the apices of triangles behind the bar under consideration cause a tension on it, which appears as a negative item in the series. Consequently when at last the negative element preponderates we do right in calling the tons to which it is affixed, tons tension.

Finally the total strain on B_1D_1

$$=\frac{W}{n^2\sin\alpha}\left\{\frac{1}{2}-\frac{1}{2}-\frac{3}{2}-\ldots.-\left(n-\frac{3}{2}\right)\right\}$$

$$=\frac{W}{n^2\sin\alpha}\times\left\{-\frac{(n-2)\times n}{2}\right\}$$

$$=-\frac{W}{2\sin\alpha}\times\frac{n-2}{n}\;\ldots\ldots\ldots\ldots\ldots\ldots(n).$$

The negative sign must plainly be taken to indicate that the strain is tensile.

This series (1), (2) ... (*n*) gives the strain in all the bars inclined from right to left; and the bars inclined the other way being similar in every respect, have the like strains.

We thus see that the bar A_1C_1, which has by similarity the same strain as the n^{th} B_1D_1, has a tension of

$$\frac{n-2}{n}\times\frac{W}{2\sin\alpha}$$

equal to the compression of A_1C_2. This might have been seen at once, for A_1C_1 is the only bar which can cause a vertical upward pressure on the pin A_1, equal to the vertical downward pressure caused by the pressure of C_2 on A_1C_2. And as it is similarly inclined, and there is no other weight on pin A_1, it must therefore have a tension equal to the compression of A_1C_2.

Corollary. To find the most favourable angle of inclination of the bars.

The weight of all the bars as depending on the angle of inclination

$\propto$ the sum of the section of each bar $\times$ its length,

$\propto$ .... strain on each (neglecting its sign), $\times$ the length of any one (since all are of equal length);

or since the length of any one bar is $\frac{b}{\sin\alpha}$, summing the series (1), (2), (3)...(n), when altered to suit an even number of triangles, and multiplying by 2 to get the whole number of bars,

$$\propto \frac{W}{n\sin\alpha}\{n+(n-2)+(n-4)+\dots$$
$$+2+0+2+\dots+(n-2)\}\times\frac{b}{\sin\alpha},$$

if n be even, or

$$\propto \frac{W}{n\sin\alpha}\{n+(n-2)+(n-4)+\dots$$
$$+3+1+1+3+\dots+(n-2)\}\times\frac{b}{\sin\alpha},$$

if n be odd;

$$\propto \frac{Wb}{n\sin^2\alpha}[n+2\{(n-2)+(n-4)+\dots+2\}], \text{ if } n \text{ be even,}$$

or $$\frac{Wb}{n\sin^2\alpha}[n+2\{(n-2)+(n-4)+\dots+1\}], \text{ if } n \text{ be odd}$$

$$\propto \frac{Wb}{n\sin^2\alpha}\left\{n+2\frac{n(n-2)}{4}\right\}, \; n \text{ even,}$$

or $$\frac{Wb}{n\sin^2\alpha}\left\{n+2\frac{(n-1)^2}{4}\right\}, \; n \text{ odd;}$$

$$\propto \frac{Wb}{n\sin^2\alpha}\times\frac{n^2}{2}, \; n \text{ even,}$$

or $$\frac{Wb}{n \sin^2 \alpha} \times \frac{n^2+1}{2},\ n \text{ odd};$$

$$\propto \frac{Wb}{2 \sin^2 \alpha} \times n,\ n \text{ even},$$

or $$\frac{Wb}{2 \sin^2 \alpha} \times \frac{n^2+1}{n},\ n \text{ odd}.$$

Now the length of the base of any triangle is $2b \times \cot \alpha$, therefore $l = n \times 2b \cot \alpha$,

or $$n = \frac{l}{2b} \tan \alpha;$$

therefore total weight of struts and ties, taking the case of n being even,

$$\propto \frac{Wb}{2 \sin^2 \alpha} \times \frac{l \tan \alpha}{2b},$$

$$\propto \frac{Wl}{4} \frac{1}{\sin \alpha \cos \alpha} \ldots\ldots\ldots\ldots (\alpha).$$

$$\propto \frac{1}{\sin 2\alpha}.$$

The weight is therefore a minimum when $\sin 2\alpha$ is a maximum or equal to unity,

or when $\alpha = 45^\circ$.

Again, taking the case of n being odd,

total weight $$\propto \frac{Wb}{2 \sin^2 \alpha} \left(n + \frac{1}{n}\right),$$

$$\propto \frac{Wb}{2 \sin^2 \alpha} \left(\frac{l \tan \alpha}{2b} + \frac{2b}{l \tan \alpha}\right),$$

$$\propto \frac{Wl}{4 \sin \alpha \cos \alpha} + \frac{Wb^2}{l} \cot \alpha \ldots\ldots (\beta).$$

or $$\frac{l}{\sin 2\alpha} + \frac{2b^2}{l} \times \frac{1}{\sin^2 \alpha \tan \alpha}.$$

The 1st member is a minimum as before when $\alpha = 45°$, but the 2nd, which has its constant part smaller than the 1st term in the proportion of $2b^2 : l^2$, i. e. in ordinary bridges of from 1 : 50 to 1 : 130, has its minimum value when α is $90°$, and gets larger; but is still only from $\frac{1}{25}$ to $\frac{1}{70}$ of the other term when α is $45°$. If α be smaller than $45°$, both terms increase very rapidly.

Results. For a dead weight for a given span, the bars composing the whole *web* of the girder will be lightest if the number of triangles be even, and their sides inclined at an angle of $45°$ to the horizon; in which case their weight will not depend on the depth or number of triangles.

But if you have an odd number of triangles, you add to their weight to the extent of $\frac{1}{25}$ to $\frac{1}{70}$.

Reasons of construction may, or rather will, modify these results in any bridge designed to be actually constructed.

In what follows it will be seen that the weight of the whole girder will (as mechanically required) be very much affected by the weights of the top and bottom horizontal members; whose weight manifestly decreases as the angle of the triangulation increases without limit.

Now to find the weight of the top and bottom horizontal members.

We see that the strain on A_0A_1 will be equal to that on A_0C_1 resolved horizontally, or, in other words, multiplied by $\cos\alpha$, and therefore tension of $A_0A_1 = \frac{W}{2n\tan\alpha} \times n$..................(1);

so compression of $C_1C_2 =$ the sum of the compression of A_0C_1 and tension of C_1A_1, which both affect it in the same direction resolved horizontally, or strain on $C_1C_2 = \frac{W}{2n \tan \alpha} \{n + (n-2)\}$......................(1)',

so, ... $A_1A_2 = \frac{W}{2n \tan \alpha} \{n + 2(n-2)\}$....................(2),

... $C_2C_3 = \frac{W}{2n \tan \alpha} \{n + 2(n-2) + (n-4)\}$.........(2)';

....

... on centre of top, that is,

if n be even on $C_{\frac{n}{2}}D_{\frac{n}{2}}$

$$= \frac{W}{2n \tan \alpha} \{n + 2(n-2) + 2(n-4) + \ldots + 2 \times 2\} \ldots \left(\frac{n}{2}\right)';$$

if n be odd on $C_{\frac{n-1}{2}}E$

$$= \frac{W}{2n \tan \alpha} \{n + 2(n-2) + 2(n-4) + \ldots + 2 \times 3 + 1\} \ldots \left(\frac{n-1}{2}\right)',$$

and strain on the centre of the bottom, i.e.

if n be even on $A_{\frac{n}{2}-1}F$.

$$= \frac{W}{2n \tan \alpha} \{n + 2(n-2) + 2(n-4) + \ldots + 2 \times 2\} \ldots \left(\frac{n}{2}\right),$$

if n be odd on $A_{\frac{n-1}{2}}B_{\frac{n-1}{2}}$

$$= \frac{W}{2n \tan \alpha} \{n + 2(n-2) + 2(n-4) + \ldots + 2 \times 1\} \ldots \left(\frac{n+1}{2}\right).$$

Hence (I.) the sum of the top strains if n be odd

$$= 2 \times (1)' + 2 \times (2)' + \ldots + 2\left(\frac{n-1}{2}\right)' = 2 \times \left\{(1)' + (2)' + \ldots + \left(\frac{n-1}{2}\right)'\right\};$$

adding these series term by term as they stand, we get a general factor $\frac{W}{n \tan \alpha}$ outside; and inside the bracket a series, the first term of which is $\frac{n(n-1)}{2}$, which we will divide into

$$-\frac{n(n-1)}{2} + n(n-1);$$

each of the next rows of terms we add up into one, except the top term of each, which we put at the end. And thus we have the aggregate strain on the top

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n(n-1)}{2} + n(n-1) + (n-2)(n-3) + (n-4)(n-5) \right.$$

$$\left. + \ldots + 5 \times 4 + 3 \times 2, + (n-2) + (n-4) + \ldots + 1 \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n(n-1)}{2} + \overline{n-1}|^2 + (n-1) + \overline{n-3}|^2 + (n-3) + \overline{n-5}|^2 \right.$$

$$\left. + (n-5) + \ldots 4^2 + 4 + 2^2 + 2, + (n-2) + (n-4) + \ldots + 1 \right\},$$

$$= \frac{W}{n \tan \alpha} \left[-\frac{n(n-1)}{2} + 4 \left\{ \overline{\frac{n-1}{2}}\Big|^2 + \overline{\frac{n-1}{2} - 1}\Big|^2 + \ldots + 2^2 + 1^2 \right\} \right.$$

$$\left. + (n-1) + (n-2) + (n-3) + \ldots + 3 + 2 + 1 \right],$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n(n-1)}{2} + 4 \frac{(n-1)(n+1)n}{24} + \frac{n(n-1)}{2} \right\},$$

$$= \frac{W(n+1)(n-1)}{6 \tan \alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots (A).$$

II. If n be even, the aggregate of the top strain, similarly,

$$= 2 \left\{ (1)' + (2)' + \ldots + \left(\frac{n}{2} - 1\right)' + \left(\frac{n}{2}\right)' \right\} - \left(\frac{n}{2}\right)',$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^2}{2} + n^2 + \overline{n-2}|^2 + \overline{n-4}|^2 + \ldots + 4 \right.$$

$$+ (n-2) + (n-4) + \ldots + 2 + 0$$

$$\left. - \left(\frac{n}{2} + (n-2) + (n-4) + \ldots + 2 \right) \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^2}{2} + 4 \left(\overline{\frac{n}{2}}\Big|^2 + \overline{\frac{n}{2} - 1}\Big|^2 + \overline{\frac{n}{2} + 1}\Big|^2 + \ldots + 1 \right) - \frac{n}{2} \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^3}{2} + 4 \left(\frac{n^3}{24} + \frac{n^2}{8} + \frac{n}{12} \right) + -\frac{n}{2} \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ \frac{n^3}{6} - \frac{n}{6} \right\} = \frac{W}{n \tan \alpha} \frac{n(n-1)(n+1)}{6},$$

$$= \frac{W(n+1)(n-1)}{6 \tan \alpha} \quad \ldots\ldots\ldots\ldots (B).$$

The above shews that the expression for the weight of the top horizontal bars is unaffected by n being even or odd.

III. The aggregate of the bottom strain if n be even

$$= 2(1) + 2(2) + \ldots + 2\left(\frac{n}{2}\right),$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^2}{2} + n^2 + \overline{n-2}|^2 + \overline{n-4}|^2 + \ldots + 2^2 \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^2}{2} + 4\left(\overline{\frac{n}{2}}\Big|^2 + \overline{\frac{n}{2} - 1}\Big|^2 + \ldots + 1^2 \right) \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n^2}{2} + 4\left(\frac{n^3}{24} + \frac{n^2}{8} + \frac{n}{12} \right) \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ \frac{n^3}{6} + \frac{n}{3} \right\},$$

$$= \frac{W(n^2 + 2)}{6 \tan \alpha} \quad \ldots\ldots\ldots\ldots (C).$$

IV. The aggregate of the bottom bars if n be odd

$$= 2(1) + 2(2) + \ldots + 2\left(\frac{n-1}{2}\right) + \left(\frac{n+1}{2}\right),$$

$$= \frac{W}{n \tan \alpha} \left\{ \frac{n(n-1)}{2} + n(n-1) + (n-2)(n-3) + \ldots + 3 \times 2, - \frac{n}{2} \right.$$
$$\left. + n + (n-2) + \ldots + 1 \right\},$$

$$= \frac{W}{n \tan \alpha} \left\{ -\frac{n(n-1)}{2} + \overline{n-1}|^2 + \overline{n-2}|^2 + \ldots + \overline{2}|^2 - \frac{n}{2} \right.$$
$$\left. + (n + n - 1 + n - 2 + \ldots + 1) \right\},$$

$$=\frac{W}{n\tan\alpha}\left(-\frac{n(n-1)}{2}+4\left\{\frac{(n-1)(n+1)n}{24}\right\}-\frac{n}{2}+\frac{(n+1)n}{2}\right),$$

$$=\frac{W}{n\tan\alpha}\left\{\frac{n}{2}+\frac{1}{6}\{n^3-n\}\right\}-\frac{W}{(n-1)n\tan\alpha}\left\{\frac{n^3}{6}+\frac{n}{3}\right\},$$

$$=\frac{W(n^2+2)}{6\tan\alpha}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(D);$$

the expression for the weight of the lower horizontal bars is therefore also unaffected by the fact of n being even or odd.

Hence, for the weight, the sum of the strains of the horizontal bars, multiplied by the length of any one, which is $2b\cot\alpha$,

$$=\left\{\frac{W(n^2-1)}{6\tan\alpha}+\frac{W(n^2+2)}{6\tan\alpha}\right\}\times 2b\cot\alpha,$$

$$=\frac{Wb}{3}\cot^2\alpha\,(n^2-1+n^2+2),$$

$$=(2n^2+1)\,\frac{Wb}{3}\cot^2\alpha;$$

but $\qquad n=\frac{l}{2b}\tan\alpha;$

therefore this expression may be written,

$$\left(\frac{l^2\tan^2\alpha}{2b^2}+1\right)\frac{Wb}{3}\cot^2\alpha,$$

$$=\frac{Wl^2}{6b}+\frac{Wb}{3\tan^2\alpha}\ \ldots\ldots\ldots\ldots\ldots\ldots(\gamma).$$

Uniting (α) and (γ), and (β) and (γ), we get, that the whole weight of the girder, length l and depth b under a load W, spread uniformly over the top when n is even,

$$\propto\frac{Wl}{4\sin\alpha\cos\alpha}+\frac{Wl^2}{6b}+\frac{Wb}{3\tan^2\alpha}\ \ldots\ldots\ldots\ldots(\delta),$$

and when n is odd,

$$\propto\frac{Wl}{4\sin\alpha\cos\alpha}+\frac{Wb^2}{2l}\times\frac{\cos\alpha}{\sin^3\alpha}+\frac{Wl^2}{6b}+\frac{Wb}{3}\cot^2\alpha\ldots(\epsilon).$$

I. n even (δ) gives that the whole weight

$$\propto \frac{l}{2\sin 2\alpha} + \frac{l^2}{6b} + \frac{b}{3\tan^2\alpha};$$

therefore when this is a minimum, differentiating

$$-\frac{l\cos 2\alpha}{\sin^2 2\alpha} - \frac{2b}{3}\cot\alpha\,\mathrm{cosec}^2\alpha = 0,$$

or

$$l\frac{\cos^2\alpha - \sin^2\alpha}{4\cos^2\alpha\sin^2\alpha} + \frac{2b}{3}\frac{\cos\alpha}{\sin^3\alpha} = 0;$$

$$\therefore\ 3l\left(\frac{1}{\sin^2\alpha} - \frac{1}{\cos^2\alpha}\right) + 8b\frac{\cos\alpha}{\sin^3\alpha} = 0,$$

or

$$3l(1 - \tan^2\alpha) + 8b\cot\alpha = 0,$$

or

$$3l\tan^3\alpha - 3l\tan\alpha - 8b = 0;$$

writing x for $\tan\alpha$ and reducing

$$x^3 - x - \frac{8}{3}\frac{b}{l} = 0.$$

This equation gives us x, or $\tan\alpha$, for any given value of $\frac{b}{l}$.

If $\frac{b}{l} = \frac{1}{16}$, as in Newark Dyke-bridge, and generally in those designed by C. H. Wyld, Esq., we get

$$x^3 - x - \frac{1}{6} = 0,$$

whose roots are (Hymers' *Theory of Equations*, p. 97,)

$$\frac{2}{\sqrt{3}}\cos\frac{\lambda\pi \pm \theta}{3},$$

if

$$\cos\theta = \sqrt{\frac{3}{16}}, \text{ and } \lambda \text{ be any integer.}$$

This gives

$$\theta = 64^\circ\,26'.$$

Hence the positive root is

$$\tan\alpha, \text{ or } x = 1.07448,$$

and

$$\alpha = 47^\circ\,3\tfrac{1}{2}', \text{ nearly.}$$

Again, if $\frac{b}{l} = \frac{1}{12}$, as in bridges designed by M. and G. Rendel, Esqrs., our equation becomes

$$x^3 - x - \frac{2}{9} = 0,$$

whose roots are

$$\frac{2}{\sqrt{3}} \cos \frac{\lambda\pi \pm \theta}{3},$$

if

$$\cos \theta = \frac{1}{\sqrt{3}}.$$

Hence the positive root is

$$\tan \alpha, \text{ or } x = 1.0964,$$

and

$$\alpha = 47^\circ\, 38', \text{ nearly.}$$

II. When n is odd, then the weight of the whole girder, see (ϵ),

$$\propto \frac{l}{2 \sin 2\alpha} + \frac{b^2}{2l} \cot \alpha\, (1 + \cot^2\alpha) + \frac{l^2}{2b} + \frac{b}{3} \cot^2\alpha.$$

For a minimum value of which we must have its differential coefficient, viz.

$$-l \frac{\cos 2\alpha}{\sin^2 2\alpha} - \frac{b^2}{2l} (\operatorname{cosec}^2\alpha + 3 \cot^2\alpha \operatorname{cosec}^2\alpha)$$

$$- \frac{2b}{3} \cot \alpha \operatorname{cosec}^2\alpha = 0;$$

therefore, since the first term may be written

$$l \frac{\sin^2\alpha - \cos^2\alpha}{4 \sin^2\alpha \cos^2\alpha},$$

we have, multiplying by $\sin^2\alpha$,

$$\frac{l}{4} (\tan^2\alpha - 1) - \frac{b^2}{2l} (1 + 3 \cot^2\alpha) - \frac{2b}{3} \cot \alpha = 0;$$

multiply by $\frac{4}{l} \tan^2\alpha$, and write x for $\tan \alpha$, then

$$x^4 - x^2 - 2 \frac{b^2}{l^2} x^2 - 2 \frac{b^2}{l^2} - \frac{8}{3} \frac{b}{l} x = 0,$$

or $$x^4 - \left(1 + 2\frac{b^2}{l^2}\right)x^2 - \frac{8}{3}\frac{b}{l}x - 2\frac{b^2}{l^2} = 0.$$

If $x = \frac{1}{16}$,

$$x^4 - \left(1 + 2\frac{1}{128}\right)x^2 - \frac{1}{6}x - \frac{1}{128} = 0;$$

for a first approximation omit the small terms, then since we expect x to be nearly equal to unity, we are sure that

$$x^4 - x^2 - \frac{1}{6}x = 0, \text{ very nearly;}$$

or $$x^3 - x - \frac{1}{6} = 0.$$

This equation gives us $x = 1.07448$ nearly, as in case I.

For a second approximation divide the first 3 terms of our equation by x, and the last by 1.07448, and we get

$$x^3 - \frac{129}{128}x - \frac{1}{6}\left(1 + \frac{1}{22.9}\right) = 0, \text{ very closely.}$$

We shall not work this equation out, but it will evidently give a larger value of x than before, and may possibly give the value of α 48 instead of 47 degrees.

If $x = \frac{1}{12}$, we should by exactly similar reasoning get an angle very near, but slightly larger than $47\frac{1}{2}^\circ$.

Summary of Corollary.

If the sections of each part of a triangular girder under an uniform load be constructed in an uniform proportion to the strains to which each is subjected,—

1. The whole weight of the *web* will be least if the number of triangles be even, and the inclination of the bars 45°.

2. The *whole* weight of such girder will be least if the number of triangles be even and the inclination lie between 47° and 48° according to the depth of the girder.

REMARK.—Simplicity in constructing the work requires an angle defined by some measure which a workman can understand and test, and which is not complicated of attainment; this condition is amply fulfilled by the angle of 45° or half a right angle; which is very close to the angle of minimum weight.

Now, a bar or combination of bars formed into a strut or tie cannot be fined down or diminished below a certain scantling*. A bar acting as a *tie* must be kept large enough to resist the continued action of weather, and the occasional action of blows or contingencies; and it is often economical to keep otherwise waste iron in it in order to admit of room for bolts and fastenings. A bar acting as a *strut*, i. e. under compression, has all the above causes to prevent its being fined down, with the addition of the following two, which are very much more difficult to deal with. If a long strut is under pressure it may need a great quantity of extra iron to stiffen it, so as to prevent flexure. Thus if a strut 10'0 long have 9 tons to bear, the section will be strong enough to resist crushing if of 2 square inches; but the section considered necessary to make it safe against bending might be, say, that of 4 no. ∠is. $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{3''}{8}$ $= 7$ sq. inches, or $3\frac{1}{2}$ times the former. Again, if a long strut be abutting against another, as in the horizontal top of a girder, it may be necessary, either for simplicity, or appearance, or even for possibility, to have it of the same general shape as its fellows; as, for instance, of a section in the form of a rectangular box with angle irons at the corners. Now the plates and angle irons composing such box or other shape cannot safely be fined down below $\frac{3''}{8}$ to $\frac{1''}{4}$ thick (according to circumstances), and

* I believe *scantling* means "*reduced dimensions* with reference to the sectional area."

may thus be many times the weight calculated on in the above analysis.

It may be feared then that that arrangement of iron given by the analysis, as of lightest weight, may be such as to demand a great number of struts and ties, but to apportion to each strut or tie a small amount of strain, and a proportionately small sectional area and weight.

If this were the case the analysis might prove in practice useless; because in practice every such strut, and many such ties, would have to have their sectional area *dis*proportionately large compared with the strain upon them; and that to such an extent as to make the assumptions on which the analysis is based no approximation to the truth; while the number of the "connexions" would cause increased expense.

The above analysis, then, in order to be useful, must shew that the proportions which make the girder lightest if imagined constructed according to the analytical assumptions; also cause the members composing that girder to be few in number, but heavy in section, *as compared* with other practicable designs; for then the girder will be capable of being constructed in the closest conformity to the assumptions of the analysis. If this be the case the weight of the girder constructed on the basis of the above analysis will be a minimum, *both* as regards the requirements of construction, *and* as regards mechanical requirement, founded on theoretical assumptions closely approximating to the truth.

Now, first of all, it is clear that an angle of 45^0 divides the girder into fewer parts than would be the case if the angle were *greater* than 45^0. And they will also be heavier.

Consider, for instance, a girder made with an equilateral triangulation.

The no. of bars will be greater, their strains less, and their lengths shorter than in a rectangular triangulation.

Since the no. of bars is greater, the no. of joints will be greater, which is in favour of the rectangular system.

Since the strains are less, the number of bars requiring extra stiffening metal will be greater, and the no. of bars requiring, *mechanically*, less than the minimum *practicable* section, will be greater. Hence the waste is greater in the equilateral system.

Since the bars are shorter in the proportion of 2 : 2·45, the minimum practical section of a strut, which shall be safe against flexure, will be less than in the rectangular system; this restores a little favour to the equilateral system.

Since the strain on the bars is less, the strain upon the joints will be less; this is in favour of the equilateral system in cases where the joints are single pins, but not otherwise.

Again, in the equilateral girder the length of the top and bottom booms is less, and the strain on those nearest the piers less.

In the bottom of the girder this may be no disadvantage except as regards the no. of joints, which is greater on this account.

But in the top, the size of the top boom being large enough to enable it to act as a pillar from pin to pin at the centre of the girder, will generally be so large for the ends that the section *must* be much heavier there than is required, in the case of either kind of triangulation. It is, therefore, a pity not to save in the weight of the web bars, by laying them at the angle of 45°, and thus also in the number of the joints; since this can be done by putting greater strain upon the ends of the top boom, which must in any case be strong enough to bear the strain whether put on it or not.

On the side of the equilateral system is the shorter distance between the pins, upon which the weight of the roadway has to be brought. If, by the use of the equilateral system, the roadway can be made to sustain itself and the load between two consecutive apices of the triangles, without laying weight upon the bars composing the boom, in a case where this cannot be done if the rectangular system be used; it may be well to use the former.

The reader may perhaps have expected a decision as to what angle is *the best;* but that of course can only be given when the particular requirements of the bridge are known. It must not be understood that the inadvisability of answering the general question as regards the angle, implies any impossibility of selecting the best angle in a particular case.

It is needless to repeat all the above investigation for the case of a lattice-webbed girder. That is simply a more complicated case, but involving the same functions of α, and consequently giving the weight of the bars, and practically of the girder, a minimum for the same value of α, viz. 45^0.

This remark also applies to live load. The same functions of α being involved, we may expect the same value of α to give a minimum, whether in triangular or lattice-girders.

Practical Arithmetical Computation of the Strains on Triangular Girders.

To lead us to this computation we need to insert here a few lemmas, or preliminary propositions. In all of them we shall use the same figure, which it will be well to explain first; in order that an idea of its dimensions, and of the weight upon it, may be clearly formed.

Let fig. 14 be a triangular girder 3′0 deep and 33′0 span with its bars inclined at an angle of 45^0 to the horizon.

Then, in any triangle $A_1B_1A_2$, $A_1A_2 = \sqrt{2} \times A_1B_1$, and $A_1B_1A_2$ is a right angle.

Suppose the connexions at A_1, A_2, A_3 ..., B_1, B_2, B_3, ... to act only in the most disadvantageous way for creating stiffness, viz. only as single pins: that is, suppose that the bars AA_1, BA, BA_1, BB_1, &c. can only exert force in the directions of their length.

The straight lines in the figure will then represent bars or combinations of iron, pinned together at the ends by through pins perpendicular to the paper.

Suppose the roadway to be fastened to the *bottom* of the girder and to weigh 10½ cwt. per foot run; which, as we will suppose two girders, will give 5¼ cwt. on each girder. The girder will weigh about 1 cwt. per foot run, which we shall reckon in with the roadway to make 6¼ cwt.; presuming that there will be enough surplus strength in the bars to admit of our taking this slight liberty. The live load we will take as 1¼ tons per foot run, or 12½ cwt. per foot run per girder;

therefore

dead load =	$6\frac{1}{4}$ cwt. per ft. =	.3125 tons,	both supposed acting at
live load =	$12\frac{1}{2}$ .... =	.625 ...	the bottom of girder;
	$18\frac{3}{4}$	.9375	

	cwt.	tons.
therefore the dead load on the base of a triangle =	$37\frac{1}{2}$ =	1.875
live load .... =	75 =	3.75
	$112\frac{1}{2}$	5.625

(The load on the pin A_1 will, exactly, be $\frac{3}{4}$ of $112\frac{1}{2}$ cwt.; but we shall suppose the weight brought upon *every* pin by the loads to be that due to the base of a triangle, in order to save complicity.)

LEMMA I. *To find accurately the strains produced by a weight of* 1.875 *tons resting on any one of the pins* A_1, A_2, ... *as for instance* A_1.

(By the principle of superposition of forces we are entitled to consider the effect of one weight at a time; and then the sum of the effects of each weight, will be the whole effect. We consider then the girder ABB_6A_6 as a lever without weight, and suppose only one weight of 1.875 tons to be hung on the pin A_1.) The proportions borne by the two piers A and A_6, will be as A_1A_6 to AA_1 respectively, by the principle of the lever; or the weight borne by A = ten times that borne by A_6.

Or, the weight of 1.875 tons on A_1 is borne by the pier A to the extent of $\frac{10}{11}$ of its weight; and by pier A_6 to the extent of $\frac{1}{11}$ of its weight;

$\therefore$ pier A resists a vertical pressure downward of $\frac{10}{11}$ of 1.875 tons, on account of the weight on the pin A_1.

Now this cannot be brought upon the pier by the bars AA_1, which can only act horizontally; therefore it must come down AB, and therefore from the pin B.

By the same reasoning this vertical force of $\frac{10}{11} \times 1.875$ tons from the pin B, must be brought upon it by the bar BA_1, and therefore from the pin A_1.

Now to produce a *vertical* force of $\frac{10}{11} \times 1.875$ tons upon B, A_1B must be exerting a force so great that, when it is resolved along BA and BB_1, the former is $\frac{10}{11} \times 1.875$ tons;

$$\text{i. e. it must be } \sqrt{2} \times \frac{10}{11} \times 1.875 \text{ tons*};$$

and *this* will bring a pressure upon the bar BB_1, by means of the pin B, equal to its resolved part along BB_1,

$$\text{i.e.} = \frac{1}{\sqrt{2}} \times \sqrt{2} \times \frac{10}{11} \times 1.875 \text{ tons}\dagger$$

$$= \frac{10}{11} \times 1.875 \text{ tons.}$$

Now to avoid the needless repetition of figures,

$$\frac{1.875}{11} \text{ which} = .17045, \text{ we shall call } W,$$

$$\frac{1.875}{11}\sqrt{2} \text{ which} = .24102, \text{ .... } w\ddagger.$$

* And, generally, $= \frac{1}{\sin \alpha} \times \frac{10}{11} \times 1.875$ tons; if α = the angle of the bars with the horizon.

† And, generally, $= \frac{\cos \alpha}{\sin \alpha} \times \frac{10}{11} \times 1.875$.

‡ It is often convenient to make use of the following classification of symbols in these girders:

W = the total weight of girder and load.

$W = \frac{\text{the dead weight on each space of a triangle or lattice}}{\text{no. of spaces between the bearings on piers}}$.

$w = \frac{W}{\sin \alpha}$ is the strain required in a strut or tie, at an inclination of α with the horizon, in order to support W.

And so for the live weight,

$$\frac{1}{11} \text{ of } 3.75 \text{ we shall call } W',$$

$$\frac{1}{11} \text{ of } 3.75 \times \sqrt{2} \text{ we shall call } w'.$$

In this case then,

$$W = .17045,\ w = .24106,\ W' = .34091,\ w' = .48212.$$

Again, we have shewn that the weight 1.875 tons, $= 11\ W$, on A_1, produces a compression 10 W on AB and BB_1 and a tension of 10 w on A_1B. Now this latter must, by similar reasoning, be producing a pressure on the pin A_1, equivalent to pressing it upward with 10 W tons and horizontally towards A with 10 W tons also: the vertical pressure supports $\frac{10}{11}$ of the weight of 11 W tons on the pin A_1; but we are left having still to account for a weight 1 W unsupported by BA_1, and for 10 W horizontal pressure, both by BA_1 on the pin A_1 towards A, and by BB_1 on the pin B_1 towards B_6.

Leaving these for the present, we will recur to the pressure of the pier A_6. This supports a vertical thrust of W tons, which

ω (ōmĕga) $= \frac{W}{\tan \alpha}$ is the strain caused in the booms by such strut or tie; viz. $\frac{W}{\sin \alpha}$ resolved horizontally, $= \frac{W}{\sin \alpha} \times \cos \alpha = \frac{W}{\tan \alpha}$. (If $\alpha = 45^0$, and $\therefore \tan \alpha = 1$, we use W instead of ω).

$W'w'$ and ω' (read great W dashed, little w dashed, and ōmĕga dashed), denote the same as W, w, and ω only for the live, instead of the dead, load; so that $\omega' = \frac{W'}{\tan \alpha}$, $w' = \frac{W'}{\sin \alpha}$.

If it be desirable to divide the load into two, one part acting on the top of the girder, and the other part on the bottom; exactly the same symbols are used, with a 1 subscribed for the top weights, and a 2 subscribed for the bottom weights; thus, W_1, W_2, w_1, w_2, (read great W one, great W two, &c.)

Still in lattice and plate-webbed girders, W may be used to express the total weight, live and dead, upon the whole bridge; for in those girders the ratios for which W, W' stand here, are not useful.

must come upon it down the bar B_5A_6 acting with a thrust of $\sqrt{2} \times W = w$.

The thrust w tons of B_5A_6 on the pier A_6 being taken in a vertical direction only by the pier, there remains the horizontal resolved part of this w tons, viz. W tons, to be taken by the horizontal bar A_5A_6, in tension.

And the thrust w of B_5A_6 on the pin B_5 is equivalent to two, one W vertically upwards and the other W horizontally towards B. The latter must be taken by B_4B_5 in compression; since the bar A_5B_5 *must* be in tension in order to take the former, and to do this must exert a tension $W\sqrt{2} = w$, and so add an additional horizontal pressure (towards B), $= W$, on the pin B_5, and therefore on B_4B_5.

It will be easily seen that this, and similar reasoning, brings us to the following conclusion:

That in order to produce the pressure W, caused on the pier A_6 by the weight 1·875 on pin A_1, there exists,

	in	tons		tension	in		compression	in
compression	B_5A_6	of w	producing	W	A_5A_6	and	W	B_4B_5,
tension	B_5A_5	of w	{causing an increase to}	$2W$	A_4A_5	and	$2W$	B_4B_5,
compression	B_4B_5	of w	...	$3W$	A_4A_5	and	$3W$	B_3B_4,
tension	B_4A_4	of w	...	$4W$	A_3A_4	and	$4W$	...
compression	B_3A_4	of w	...	$5W$	...	and	$5W$	B_2B_3,
tension	B_3A_3	of w	...	$6W$	A_2A_3	and	$6W$	...
compression	B_2A_3	of w	...	$7W$	...	and	$7W$	B_1B_2,
tension	B_2A_2	of w	...	$8W$	A_1A_2	and	$8W$	...
compression	B_1A_2	of w	...	$9W$	...	and	$9W$	BB_1,
tension	B_1A_1	of w	...	$10W$	on A_1	and	$10W$	...

It will be at once seen that these last strains are the counterpart of those caused by the pier A, which meet and consist with them; the only unbalanced strains being the upward one of $10W$ on the pin A_1 caused by BA_1, and of $1. W$ on the same pin caused by B_1A_1; and these are exactly met by the weight $11\ W$, which is on the pin A_1.

The above table then gives us, at one view, the whole scheme of strains due solely to a weight 1.875 on A_1; and it will be well to consider it attentively: the strain in each bar is produced by the action of the pins at its extremities; and by means of them the force of the weight at A_1 is equilibrated, or supported, by the *distant* reactions of the piers A and A_6.

So, a weight of 1·875 *tons on pin* A_2 produces a pressure

$$\frac{3}{11} \text{ of } 1.875 = 3W \text{ on pier } A_6$$

$$\frac{8}{11} \text{ of } 1.875 = 8W \text{ on pier } A,$$

and subjects BA_1, B_1A_2 to tensions of $8W\sqrt{2} = 8w$ tons,

A_1B_1 to compression $8w$,

A_2B_2, A_3B_3, A_4B_4, A_5B_5 to tension $3w$,

A_3B_2, A_4B_3, A_5B_4, A_6B_5 to compression $3w$,

AA_1 to a tension and B_5B_6 to compression $= 0$,

A_1A_2 ... $16W$,

A_5A_6 ... $3W$,

A_4A_5 ... $9W$,

A_3A_4 ... $15W$,

A_2A_3 ... $21W$,

BB_1 to a compression $8W$,

B_1B_2 ... $24W$,

B_4B_5 ... $6W$,

B_3B_4 ... $12W$,

B_2B_3 ... $18W$.

The student will do well to draw figures, marking these strains upon them; and making himself quite at home with the way in which the different *bars* act, and assist to hold the *pins* in the same relative position, notwithstanding the weights laid upon the latter.

LEMMA II. *To find the whole strain produced upon one bar of the web, by the whole weight of the bridge, dead load.*

First for BA_1. By Lemma I.,

The weight of 1.875 tons on the pin				A_1 creates a tension			10w tons	on A_1B	
...	...	...	...	A_2	...	...	...	$8w$	...
...	...	...	...	A_3	...	...	...	$6w$	...
...	...	...	...	A_4	...	...	...	$4w$	...
...	...	...	...	A_5	...	...	...	$2w$	...

Therefore the total tension on BA_1

$$= (10 + 8 + 6 + 4 + 2)\,w = \frac{12 \times 5}{2}\,w$$

$$= 30\,w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1).$$

So total tension on $B_1A_2 = (8 + 6 + 4 + 2)\,w$,
the total compression $= 1.w$ due to weight on A_1,
or, strain in tension on B_1A_2

$$= (-1 + 8 + 6 + 4 + 2)\,w = \left(\frac{10 \times 4}{2} - 1\right) w$$

$$= 19w \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

So strain in tension on B_2A_3

$$= (-1 - 3 + 6 + 4 + 2)\,w = 8w \ldots\ldots\ldots\ldots\ldots\ldots (3),$$

and strain in tension on B_3A_4

$$= (-1 - 3 - 5 + 4 + 2)\,w = -3w \ldots\ldots\ldots\ldots\ldots (4),$$

or we might state this result,

strain in compression on $B_3A_4 = (1 + 3 + 5 - 4 - 2)\,w = 3w$:

... ... $B_4A_5 = (1 + \ldots + 7 - 2)\,w = 14w \ldots (5)$,

... ... $B_5A_6 = (1 + \ldots + 9)\,w = 25\,w \ldots\ldots (6)$.

Here it may be observed that the series for (2) differs from the series for (1) in having a 10 left out and a -1 added, making a difference in its value of 11.

So the series (3) differs from (2) in having an 8 left out and − 3 added, making an equal difference of 11.

And so throughout, so that if the first two series are given we see at once how to get at the whole list of strains; since we should see that the first strain was a tension of $30w$ tons, and that each succeeding one was $11w$ tons less, i.e. more compressive.

Again (Lemma I.), as the weight of the bridge on each pin produces the same strain on each side of a triangle, having its base towards the roadway; it is obvious that whatever be the number of such triangles, the total strain upon each side, produced by the aggregate weights on the pins, must be equal. We can, then, at once write down the strain on the other bars, from those already found. We will write compression positive and tension negative, and the whole list of bars will be (weights in tons),

	tons.		tons.		tons.
A_1B under	$-30w$;	A_2B_1 under	$-19w$;	A_3B_2 under	$-8w$;
		A_1B_1 ...	$19w$;	A_2B_2 ...	$8w$;
A_4B_3 ...	$3w$;	A_5B_4 ...	$14w$;	A_6B_5 ...	$25w$;
A_3B_3 ...	$-3w$;	A_4B_4 ...	$-14w$;	A_5B_5 ...	$-25w$.

LEMMA III. *To find the maximum strain that can be produced upon any one bar of the web, by the live load.*

Consider any one bar A_3B_3.

The greatest compression will be produced upon this bar, when the live load covers the bridge so far as to bring the full weight of 3.75 tons upon each of the pins A_4 and A_5,—these being the pins which when loaded create a compressive strain on A_3B_3; but not so far as to bring any weight upon any of the pins $A_1A_2A_3$,—since weight upon these pins would diminish the compression on A_3B_3.

Suppose this condition to be accurately complied with. This is an unfavourable view to take, since (whatever connexion there be at A_4,) it cannot be *accurately* complied with in the case of both A_3 and A_4, under the most common forms of the load. Then by computing the strain upon A_3B_3 as in Lemma II., we find that,

The maximum compression due to live load on A_3B_3,

$$= w'(2+4) = 6w'.$$

So the maximum tension due to live load on A_3B_3, is when the load is on $A_1A_2A_3$, and

$$= w'(1+3+5) = 9w'.$$

COROLLARY.

			tons compression.
Since	the dead strain on A_3B_3 is	$-3w=-$	.72318,
and the max. live strain positive	....	$6w'=$	2.89272,
... ...	negative	$-9w'=-$	4.33908.

It follows that the greatest compression to which A_3B_3 can be subjected in the bridge is

$$2.89272 - .72318 = 2.16954 \text{ tons},$$

and the greatest tension is

$$4.33908 + .72318 = 5.06226 \text{ tons}.$$

It is well for the reader to form a very perfect notion of the theory contained in these three Lemmas. He should be able to go through the same reasoning, also, for other values of $\sin\alpha$ and $\cos\alpha$; such as in Prop. II.; or if $\alpha = 60^\circ$ or 30°. He should be able, without labour, to see at once, when a weight is put upon any pin, how its strain upon any bar is arrived at. And he should also see, when any number of pins are weighted, how to arrive at the strain on any bar; in this last he will be assisted by Lemma IV., which now follows:

LEMMA IV. *To find the whole strain upon any, top or bottom, bar of the booms.*

It has been noticed in Lemma I., that the strain due to the load, both dead and live, upon any one bar, B_3B_4, is equal to the strain brought upon it by the tensions in A_1B, A_2B_1, A_3B_2,

A_4B_3, and also by the compressions in A_1B_1, A_2B_2, A_3B_3; such being the bars operating to produce strain along BB_4 on the left of B_3B_4. And the same would be true of those bars operating to produce it on the right of B_3B_4, along B_3B_6.

Now in the case of the dead load, this gives a fixed strain to any bar, such as B_3B_4.

And in the case of the live load, where we want the maximum strain, we see by Lemma I., that a load on any pin produces a compression in all the top bars, and a tension in all the bottom bars. We therefore get the greatest compression on any top bar, or greatest tension on any bottom bar, by supposing the live load to cover the whole bridge, and therefore to load every pin.

From Lemma II. we get the total strain on every inclined bar, produced by the dead and live weight together on the girder, by substituting $w+w'$ for w in (1), (2), (3), ... (6); and we then get the resolved horizontal or vertical strain of each bar upon its pin, by writing $W+W'$ for $w+w'$. This gives us, for the fully weighted bridge, the

horizontal strain from $BA_1 = 30\ (W+W')$

.... B_1A_1, or $B_1A_2 = 19\ (W+W')$

.... B_2A_2 ... $B_2A_3 = 8\ (W+W')$

.... B_3A_3 ... $B_3A_4 = 3\ (W+W')$

.... B_4A_4 ... $B_4A_5 = 14\ (W+W')$

.... B_5A_5 ... $B_5A_6 = 25\ (W+W')$

Adding these horizontal effects, according to the figure; the strain on

compression.		tension.
BB_1 is $30\ (W+W')$	and on	$A_1A_2 = 49\ (W+W')$
$B_1B_2 = 68\ (W+W')$	...	$A_2A_3 = 76\ (W+W')$
$B_2B_3 = 84\ (W+W')$	...	$A_3A_4 = 81\ (W+W')$
$B_3B_4 = 78\ (W+W')$	...	$A_4A_5 = 64\ (W+W')$
$B_4B_5 = 50\ (W+W')$	...	$A_5A_6 = 25\ (W+W')$

The object of the next Lemma will be more clearly seen when the subsequent proposition is arrived at. This object is, to enable the calculator to write down all the strains, when he has calculated two or three of them from the series.

LEMMA V. *On First and Second Differences.*

The way to form rapidly a complete table of a series of quantities like those in the preceding Lemmas, is this:—

I. We ascertain, by inspection of the figure, which bar in the side of a triangle is bearing the least strain*; and write down the series for the strain upon it, but (Lemma IV.) with $W+W'$ in place of w. Thus, we notice that the triangle nearest the centre (with its base to the roadway of course) is $A_3B_3A_4$; in which A_3 is 5 spaces, and A_4 4 spaces from the piers.

The horizontal strain, then, caused by either of the bars A_3B_3 or B_3A_4, under full load, will be on both the bottom and top booms,

$$(W+W')\,(5+3+1-4-2)$$

$=.51136 \times 3 = 1.53408$ tons, tension in A_3A_4, compression in B_2B_3.

II. The series for the effect of the next pair of bars A_4B_4 and B_4A_5 will be

$$(W+W')\,(7+5+3+1-2);$$

therefore the difference from the former pair is $11 \times (W+W')$

$$= 5.62496 = D_2.$$

We call this D_2, because it will form the 2nd difference in our future table.

I will here digress, to shew my readers how we can now by inspection (assisted, if there appear any difficulty, by writing down a number of the series in full) determine the following

* This way I prefer as safest in practice, though the method most direct and simple in theory, viz. of beginning at one end of the girder and working by subtraction, might be used. The objectionableness to the latter consists in its requiring subtraction, which is more laborious and insecure than addition. I have used the latter in Prop. I.

table; in which we, now, know the first strain and difference, and that the latter is constant throughout.

Strain caused in horizontal bars by

Bar.	Tons strain caused.	Difference.
A_3B_3 or A_4B_3	1.53408	
		5.62496
A_2B_2 or A_3B_2	*4.09088	
		5.62496
A_1B_1 or A_2B_1	*9.71584	
		5.62496
A_1B	*15.34080	

where the figures with stars ([5 6 7] of next table) are filled in, after all the rest are written, by successive additions of the difference.

To return to our main object, which this digression will make more easy of attainment: we have got, that the *difference* of the horizontal strain caused by each of the bars of two consecutive triangles (omitting signs where useless)

$$= 5.62496 = D_2 \text{ suppose.}$$

We have also got, that the *amount* of horizontal strain caused by each of the bars of the most central triangle is

$$1.53408 = D_1 \text{ suppose;}$$

and is of course tension in the bottom, and compression in the top horizontal bars.

We can then, with help from the figure, write the following table:

No. of bar.	Tons.	D_1	D_2
		(15.34080)	
BB_1	15.34080[7]	9.71584[6]	5.62496[1]
A_1A_2	25.05664[8]	,,	,,
B_1B_2	34.77248[9]	4.09088[5]	,,
A_2A_3	38.86336[10]	,,	,,
B_2B_3 Max.	42.95424[11]	1.53408[2]	,,
A_3A_4	41.42016[16]	,,	,,
B_3B_4	39.88608[15]	7.15904[3]	,,
A_4A_5	32.72704[14]	,,	,,
B_4B_5	25.56800[13]	12.78400[4]	,,
A_5A_6	12.78400[12]		,,
		,,	

In which table; we first write down the names of the bars, marking by the affix of max., for maximum, that bar which we see by inspection to have the greatest strain. We then write down the other numbers, in the order indicated by the small figures; i.e. we first fill up the second differences; and this, since they are all alike, is easily done: and we next put down the amount of horizontal strain caused by each of the central bars of the web, in the first differences, and between the bars of the boom, which it influences. (Thus A_8B_8 has an influence in tension of 1.53408 tons, which will operate between B_2B_8 and A_8A_4; and A_4B_8 has a like influence in compression, which it exerts between A_8A_4 and B_8B_4). We then complete the column of first differences by successive addition of the second difference; and then beginning first at one end and then at the other and working towards the maximum, we fill in the desired strains; which are caused by accumulation of the horizontal thrusts of the inclined bars in the column D_1. This method obviates the need of any subtraction; except in one case, where the horizontal thrusts change direction nearest the bar marked Max.

Proposition I.

To construct a wrought iron triangular girder of a span of 33′0, *and a vertical depth of about* $\frac{1}{10}$ *of that span. Given*

Dead load on the top = $6\frac{1}{4}$ cwt. = .3125 tons *per foot run.*
Live load on the top = $12\frac{1}{2}$ cwt. = .625 tons ....

From the data, the number of triangles must be 11; the length of each base will be 6′0.

The depth between pins will thus be 3′0, and the girder will be full 3′4″ deep when complete.

(The live load appears to be double the dead weight)

∴ dead load on each pin = .3125 × 6 = 1.875 tons,
live load on each pin = double this = 3.75 tons;

5.625

$$\left.\begin{aligned} \therefore\ W &= \frac{1.875}{11} = .17045 \text{ tons} \\ w &= W\sqrt{2} = .24106 \text{ tons} \\ W' &= \ 2W \ = .34091 \text{ tons} \end{aligned}\right\} W + W' = .51136.$$

$$w' = \ 2w \ = .48212 \text{ tons.}$$

Number the struts in order of the weight upon them in the way done in Figure 14; then

maximum compression on strut 9, due to dead and to live load

$$= .24106\,(2 - 7 - 5 - 3 - 1) + .48212\,(2)$$

$$= .24106 \times \left(6 - \frac{8 \times 4}{2}\right) = .24106 \times (6 - 16)$$

$$= -\,2.41060.$$

Maximum compression on strut No. 8,

$$= .24106\,(3 + 1 - 6 - 4 - 2) + .48212\,(3 + 1)\,;$$

$$\therefore\ D_1 = .24106 \times (6) + .48212 \times (2) = .48212 \times 5 = 2.41060.$$

Maximum compression on strut No. 7,

$$= .24106\,(4 + 2 - 5 - 3 - 1) + .48212\,(4 + 2)\,;$$

$$\therefore\ D_1 = .24106\,(5) + .48212 \times (2)\,;$$

$$\therefore\ D_2 = -\,.24106,$$

and we can see that D_2 will be caused by alternations of the 1st term of D_1 from $.24106 \times 6$ to $.24106 \times 5$, and back again; *and* by a regular addition of .48212 in every alternate case in the second term: i.e. D_2 will be $-.24106$ and $.24106 + .48212$ alternately.

We can now write the following table:

TABLE I.

No. of bar in Compression.	No. of bar in Tension.	Amount of maximum strain in tons.	D_1	D_2
9	4	−2.41060		
			2.41060	
8	5	0.00000		−.24106
			2.16954	
7	6	2.16954		+.72318
			2.89272	
6	7	5.06226		−.24106
			2.65166	
5	8	7.71392		+.72318
			3.37484	
4	9	11.08876		−.24106
			3.13378	
3	10	14.22254		+.72318
			3.85696	
2	11	18.07950		−.24106
			3.61590	
1		21.69540		

The above table is composed by

I. Writing down the first column, the 1st term in the 3rd and 4th columns, and all the 5th column; this is but transcribing from our previous work.

II. Completing the 5th column by successive addition from the 4th, and by addition from the 4th column completing the third.

III. Notice in the figure the bar which has the greatest tension, No. 11. It will be seen that its tension *must* be caused solely by the strut No. 2, and must be equal to the compression of strut No. 2; enter it in the 2nd column opposite strut No. 2, and fill up the whole column above it; as is done in the table.

The use of the table is this: Given any bar, say No. 5, and that you want to find the greatest compression, and greatest tension, which can possibly come upon it in the bridge. For its greatest compression you look for it in the first column, and find that the maximum strain upon it in compression is 7.71392 tons; for the greatest tension you look for it the second column, and find that the maximum strain upon it in tension is nothing.

In the above table we see that we should have begun our investigations with bar No. 7 instead of bar No. 9, as might have been guessed at starting; but I have supposed the beginner to have selected bar No. 9. Its greatest compression has been found to be negative, or in other words to be tension; and its meaning is this, that 2.41060 tons is the least amount of tension which can ever be on the bar 9; and the least amount of compression which can ever be on bar No. 4. To resume,

---

Again, horizontal thrust of bar 11 under full load

$= (W + W')(10 + 8 + 6 + 4 + 2) = .51136 \times 30 = 15.34080$ tons,

horizontal thrust of 3 (or 10 either)

$$= (W + W')(8 + 6 + 4 + 2 - 2);$$

$$\therefore D_2 = -(W + W') \times 11 = -.51136 \times 11 = -5.62496 \text{ tons.}$$

We can now, with help of the figure, write down this table:

TABLE II.

No. of bar in Compression.	No. of bar in Tension	Strain in tons.	D_1	D_2
A_5A_6		15.34080	15.34080	−5.62496
	B_4B_5	25.05664	9.71584	
A_4A_5		34.77248	"	
	B_3B_4	38.86336	4.09088	
A_3A_4		42.95424	"	
	B_2B_3	41.42016	−1.53408	
A_2A_3		39.88608	"	
	B_1B_2	32.72704	−7.15904	
A_1A_2		25.56800	"	
	BB_1	12.78400	−12.78400	

This table completes the calculation of strains; which in practice are better drawn out, as in fig. 14*a*.

But if the bars of the triangles, when they meet the horizontal bars, are intended to be pierced through it by a single pin; as, for instance, the bars B_2B_4, A_8B_8, and A_4B_8, where they meet at B_8; it is necessary to find the total effect of the greatest united strains along A_8B_8, A_4B_8 upon this pin, in order to know with what security it must be attached to the horizontal bar through which it passes.

Now the greatest strain on A_4B_8 is 7.71392 tons compression; part of this resolved has to support the weight of 5.625 tons on the pin; the rest to support the tension of A_8B_8.

If x be the tension in A_8B_8 coexistent with 7.71392 compression in A_4B_8, and R the resultant total pressure on the pin, we know then that

$$7.71392 - x = 5.625 \times \sqrt{2} = 7.95495;$$

$\therefore \; x = -.24103$, shewing that A_8B_3 is in compression, not tension;

$$\text{and } \therefore \; R^2 = (.24103)^2 + (7.71392)^2$$

$$= \text{nearly } \left(\frac{1}{4}\right)^2 + (7\tfrac{3}{4})^2 = (7\tfrac{3}{4})^2 \text{ nearly,}$$

$$R = 7\tfrac{3}{4} \text{ tons pretty nearly.}$$

Again, for pin B_4 (since the difference 7.9495 will be constant throughout) tension in A_4B_4, coexisting with compression 14.22254 in A_5B_4

$$= 14.22254 - 7.95495 = 6.26759;$$

$$\text{and } R^2 = (6.26759)^2 + (14.22254)^2$$

$$= (6\tfrac{1}{4})^2 + (14\tfrac{1}{4})^2 \text{ pretty nearly}$$

$$= \frac{1}{16}(625 + 3249) = \frac{1}{16}(3874);$$

$$\therefore \; R = \frac{1}{4}(62\tfrac{1}{4}) \text{ tons}$$

$$= 15\tfrac{1}{2} \text{ tons nearly,}$$

and so on. The total strain on *each* pin will never have to be calculated, as it will only be some pins towards the ends of the girder which will be found to be in danger of weakness.

Illustrations of the strains on a triangular Girder.

In fig. 15, the dark lines represent a triangular girder; in which we will suppose

the weight upon each pin A_1, A_2, A_3, &c. to be W tons,

and w to be equal to $\frac{W}{\sin \alpha}$.

Then

the strain on $AA_1 = \frac{w}{12}(11+10+9+8+7+6+5+4+3+2+1)$

$$= \frac{w}{12}\left(\frac{12 \times 11}{2}\right) = 5\tfrac{1}{2}w \ldots\ldots\ldots\ldots\ldots (1)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots A_1A_2 = \frac{w}{12}(10+9+8+\ldots+3+2+1-1).$$

Now the $+1$, due to the effect which the weight B_1 has on A_1A_2, is balanced by the -1 due to the effect which the weight A_1 has on A_1A_2: and the result is that the strain on A_1A_2 is the same as if the weights A_1 and B_1 were both taken off the girder, in which case

the strain on $A_1A_2 = \frac{w}{12}(10+9+\ldots+3+2)$, involving 9 terms,

$$= \frac{w}{12}\left(\frac{12 \times 9}{2}\right) = 4\tfrac{1}{2}w \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= \frac{w}{10}\left(\frac{10 \times 9}{2}\right)$$

$$= \frac{w}{10}(9+8+\ldots+2+1),$$

which is the strain on A_1A_2, supposing the girder to terminate at A_1, B_1 and to be supported at those points.

So strain on A_2A_3

$$=\frac{w}{12}(9+8+7+\ldots+3+2+1-1-2), \text{ by ordinary rule,}$$

$$=\frac{w}{12}(9+8+\ldots+4+3), \text{ which is as if } A_1A_2B_1B_2 \text{ were unloaded,}$$

$$=\frac{w}{8}(7+6+\ldots+2+1), \begin{cases}\text{which is as if the girder were sup-}\\ \text{ported, and ended, at } A_2B_2;\end{cases}$$

and so on.

It is also clear that the strains on AA_1, A_1A_2, ... are the same as if the girder and weight C were cut in two through the centre line of the girder, and the half girder fixed at the points A and E only. This fact illustrates directly the method by which the Table I., Prop. I. is formed, in taking out the strains on these girders, for a dead load only. Each of the inclined bars has to support all the dead weight between itself and the centre of the bridge; and their strains due to the dead weight increase from the centre of the bridge with a constant difference.

Again, for the moving load of the same amount on each pin.

The strain upon any strut A_4A_5 with load from A_5 to B_1

$$=\frac{w}{12}(7+6+\ldots+2+1)=\frac{w}{12}\times\frac{8\times 7}{2}$$

$$=2\tfrac{1}{3}\times w.$$

This may be obtained otherwise, thus

The load extended from A_5 to B_1 inclusive includes 7 pins, and weighs, therefore, $7W$; its centre of gravity is above B_4.

Hence, the pressure caused by it at A

$$=7W\times(BB_4\div AB)$$

$$=7W\times\frac{4}{12}$$

$$=\frac{7}{3}W;$$

and the strain on strut AA_1, and therefore on any strut or tie from AA_1 to A_4A_5 inclusive, must therefore be

$$= \frac{7}{3} w$$

$$= 2\tfrac{1}{3} w.$$

There is an analogy between the strains on a triangular girder and those on a roof, which it will be interesting here to notice.

In fig. 15, the lines with capital figures represent the triangular girder; the dotted lines and small figures represent the projection of the bars forming the triangulations, so as to form a figure AcB, which may be supposed to be a truss in a roof, $A_1a_2a_3$.... $B_1b_2b_3$.... being the points of attachment of the purlins.

Let W be both the weight of roof brought on this truss by each purlin, and therefore we may suppose hung upon the points $A_1a_1a_2$... $B_1b_2b_3$... in the roof; and also the weight of roadway brought upon each pin of the girder, and we may therefore suppose hung upon the pins $A_1A_2A_3$... $B_1B_2B_3$...

Let $w = \dfrac{W}{\sin \alpha}$, $\quad \omega = \dfrac{W}{\tan \alpha}$,

then in the truss the beams Ac, Bc, supposed rigidly stiff, being similar and similarly loaded, will exert a horizontal pressure upon one another at C.

Therefore equating the vertical forces acting upon any cross-section of AA_1, we have,

compression of $AA_1 \times \sin \alpha =$ the sum of the weights $A_1a_2a_3$... and $\frac{1}{2}$ of the weight at c.

Therefore compression of $AA_1 = 5\tfrac{1}{2} W \times \dfrac{1}{\sin \alpha}$,

$$= 5\tfrac{1}{2} w \ldots\ldots\ldots\ldots (1).$$

so compression of $A_1a_2 \times \sin \alpha =$ sum of weights $a_2a_3a_4$..., and $\frac{1}{2}$ of that at c.

Therefore compression of $A_1a_2 = 4\frac{1}{2}\,w$ ....(2),

so .... $a_2a_3 = 3\frac{1}{2}\,w$ ....(3),

&c. = &c.,

and .... $a_5c = \frac{1}{2}\,w$ ....(6).

Thus the strains of the truss-bars are the same as those of the girder, found above.

Also the strain of AA_2 in the girder is that due to the thrust of AA_1, and is, therefore, the same as the strain of AB if supposed to act as a tie-bar to the roof-truss AcB.

The strain of A_1A_3 is that due to the thrust of AA_1, and A_1A_2; and equal the sum of the strains due to AA_1 and A_1a_2 in two roofs AcB, A_1cB_1. And so, if a_5b_5, a_4b_4, &c., be made strong enough to act severally as tie-bars to the whole roof above them; then,

Section of AA_2 in girder = section of AB in truss; and

$\therefore$ metal in AA_2 of girder = metal in AA_2 of truss,

so ...... A_1A_3 ...... = ... A_2A_4 and A_1A_3 ......

...... A_2A_4 ...... = ... A_4C, A_3A_5, a_2G ......

&c. = &c.,

...... A_5B_5 ...... = ... $B_2B, B_3B_1 \ldots a_5b_5$ ......

Also the metal of AB in the girder is just the same as would be got by superimposing all the bars AB, A_1B_3, $a_2b_2, \ldots a_4b_4$, of the truss.

Again, if we suppose the weights taken off A_1a_2 only, and $a_3\,b_3$ secured by a bar leaving a joint at a_3 in Ac, and AB also secured by a tie whose tension is T suppose, while the reaction of the support at A suppose to be R; then for the equilibrium of the whole truss AcB, since it does not turn round B,

$$R \times 12 = W\,(9 + 8 + \ldots + 2 + 1),$$

and for the equilibrium of Aa_3, since it does not turn about a_3,

$$R \times 3 \cos\alpha = T \times 3 \sin\alpha,$$

or

$$T = R \cot\alpha;$$

therefore the strain in Aa_3

$$= R \sin \alpha + T \cos \alpha$$

$$= R \sin \alpha + R \frac{\cos^2 \alpha}{\sin \alpha}$$

$$= \frac{R}{\sin \alpha}$$

$$= \frac{W}{12 \sin \alpha} (9 + 8 + \ldots + 2 + 1)$$

$$= \frac{w}{12} (9 + 8 + 2 + \ldots + 2 + 1)$$

equal the maximum strain of A_2A_3 under a load upon the girder extending from B to A_3.

The above illustrations are useful as supplying continual checks to whatever method may be chosen as the main one for calculation.

CHAPTER IV.

A COMPOUND TRIANGULAR GIRDER WITH VERTICAL STRUTS.

(Plate II. Fig. 34.)*

PROPOSITION II.

To construct a wrought iron triangular girder of a clear span of 200′0, and a vertical depth of about $\frac{1}{12}$th of that span; of the kind shewn in Fig. 34.

Given

weight of bridge and roadway

(2 girders) approximately $1\frac{1}{2}$ tons per ft. run.

live load on lower boom .... $\frac{1}{2}$ ....

.... upper .... 1 ....

Make the centres of the booms 15′0 apart, since 16′8″ is to be about the depth of the girder; and the struts 10′0 apart; each triangle in the skeleton elevation will therefore form a right-angled triangle with its sides in the proportion of 3, 4, and 5, viz. the upright strut 15′0 long, the tie 25′0 long, and the base on the boom 20′0 long.

* This proposition is not explained as elaborately as it perhaps ought to be, occurring as it does so early in the book. I am anxious to insert one proposition just in the form I have in practice worked it out; so as to present a clear view of the theoretical analysis requisite for a new design: and this proposition is in that form; with the exception of the part in smaller type.

In figure 34, the *angles* marked B and d should have been marked β and α; the reader had better correct this mistake at once.

The *number of spaces* between the struts must therefore be 21; the bridge will then bear 5′0 upon the piers, and its virtual span between bearings be 210′0.

We will take out the strains of the struts accurately as if the whole of the weights were on the top of the bridge, and add a subsequent correction. We shall consider the weight to be at the top with regard to all the other strains, since they are unaffected by the fact of the weights being on the top or bottom.

The *weight per space* of the bridge will be

$$\begin{aligned} \text{dead} &= 15 \text{ tons} = 7\tfrac{1}{2} \text{ tons per girder,} \\ \text{live} &= 15 \text{ tons} = 7\tfrac{1}{2} \ldots\ldots\ldots\ldots\ldots\ldots \\ \text{total } &30 \text{ tons} \quad 15 \text{ tons} \end{aligned}$$

of which dead and live weights, about $\frac{2}{3}$ are on the top and $\frac{1}{3}$ on the bottom boom.

In this girder we see from the figure that

$$\sin\alpha = \frac{\text{strut}}{\text{tie}} = \frac{3}{5}, \qquad \tan\alpha = \frac{\text{strut}}{\text{base}} = \frac{3}{4};$$

$$\left.\begin{aligned} \text{and } \therefore\ W = W' &= \frac{7\frac{1}{2} \text{ tons}}{21 \text{ no.}} = \frac{15}{42} = \frac{5}{14} \text{ tons} \\ w = w' &= W \times \frac{5}{3} = \frac{25}{42} \text{ tons} \\ \omega = \omega' &= W \times \frac{4}{3} = \frac{10}{21} \text{ tons} \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots \text{(I.)}$$

While

$$\sin\beta = \frac{15}{\sqrt{\overline{15}|^2 + \overline{10}|^2}} = \frac{3}{\sqrt{3^2+2^2}} = \frac{3}{\sqrt{13}} = \frac{5}{6} \text{ nearly,}$$

$$\tan\beta = \frac{15}{10} = \frac{3}{2}.$$

In the figure the letters *a*, *b*, *c* in each panel refer, each, to the vertical *strut* on the left of that panel, to the *tie* which meets its right-hand lower corner, and to the upper and lower booms which form its top and bottom respectively.

I. *Struts.*

Maximum strain on strut l = weight of full load = 15 tons,

on strut $k = W(2+4+6+8+10+12-7-5-3-1)$
$+ W'(2+4+6+8+10+12)$,

.... $i = W(1+3+5+7+9+11+13-6-4-2)$
$+ W'(1+3+......+13)$; $\therefore D_1 = W \times 11 + W' \times 7$,

.... $h = W(2+4+6+8+10+12+14-5-3-1)$
$+ W'(2+......+14)$; $\therefore D_1 = W \times 10 + W' \times 7$,

.... $g = W(1+ .... +15-4-2)$
$+ W'(1+......+15)$; $\therefore D_1 = W \times 11 + W' \times 8$,

and so on; so that, since $W' = W$, we have by summation that the strain on strut k

$$= W \times \left(\frac{14 \times 6}{2} \times 2 - \frac{8 \times 4}{2}\right) = W \times (84 - 16) = W \times 68 \text{ tons},$$

and the first difference $= W \times 18$, and is diminished by W and increased by $2W$ alternately:

$$\text{i.e. on strut } k \text{ we have } \frac{5}{14} \text{ tons} \times 68 = \frac{170}{7} = 24\tfrac{2}{7} \text{ tons},$$

$$\text{and the first } D_1 = \frac{5}{14} \text{ tons} \times 18 = 6\tfrac{3}{7} \text{ tons},$$

and can write the following table:

TABLE I.

No of strut.	Reduced strain in tons.	Strain in tons.	D_1	D_2
l	10	15		
k	$20\frac{1}{2}$	$24\frac{2}{7}$		
			$6\frac{3}{7}$	
i	27	$30\frac{5}{7}$		$-\frac{5}{14}$
			$6+\frac{1}{14}$	
h	33	$36\frac{5}{7}+$		$\frac{5}{7}$
			$6\frac{5}{7}+$	
g	$39\frac{1}{2}$	$43\frac{4}{7}$		$-\frac{5}{14}$
			$6\frac{3}{7}$	
f	46	50		$\frac{5}{7}$
			$7\frac{1}{7}$	
e	$52\frac{1}{2}$	$57\frac{1}{7}$		$-\frac{5}{14}$
			$6\frac{5}{7}+$	
d	$59\frac{1}{2}$	$63\frac{6}{7}+$		$\frac{5}{7}$
			$7\frac{3}{7}+$	
c	$66\frac{1}{2}$	$71\frac{3}{7}$		$-\frac{5}{14}$
			$7\frac{1}{7}$	
b	$73\frac{1}{2}$	$78\frac{4}{7}$		
a	$147\frac{1}{2}$			

As before, we write first the whole of the column D_2, and the first terms of 'strain in tons' and 'D_1'; we then complete the column D_1 by addition, and then that headed strain in tons.

The reduced strains we write roughly from the following considerations:

1. We have reckoned 15 tons on the top of strut *l* where there will be 10 only, due to the upper road-way and upper load.

2. If we take the excessive $2\frac{1}{2}$ tons dead weight from the top of any strut and reckon it at the bottom, we make a difference of $2\frac{1}{2}$ tons on the strain of that strut.

3. If we take the excessive $2\frac{1}{2}$ tons of live weight from the top of any strut, but do not place it at the bottom, we make a difference of about $1\frac{1}{4}$ tons to the strain, if it be on *k*, increasing to $2\frac{1}{2}$ tons in the case of *a*.

In the case of strut a we observe, that were the whole weight at the top of the girder, it would have to sustain half of it, viz. $\frac{1}{2}$ of 21×15 tons $= \frac{1}{2}$ of 315 tons $= 157\frac{1}{2}$; deducting the 5 tons we have a's strain as above.

The greatest tension upon the struts k or i it is needless to obtain, since those struts are sure to be strong enough for it.

We might now obtain the greatest tensions on the ties from i to b, by multiplying the column of unreduced strains of struts from k to c by $\frac{5}{3}\left(=\frac{1}{\sin \alpha}\right)$. And the unreduced strain on strut b by $\frac{5}{6}\left(=\frac{1}{\sin \beta}\right)$ will be the tension on tie a. For the unreduced strain is the vertical weight, which must be supported by that tie which meets the bottom of the strut to which the unreduced strain is affixed. This might be the easiest method in a girder where the strength of the four centre ties (which this method would not give) is intended to be made so excessive, that it become needless to know the strains upon them. But where it is desirable to get the greatest strains in compression or tension on the centre ties more accurately, we proceed as follows:

II. *Ties.*

Maximum tensions on tie l

$$= w(10+8+6+4+2-9-7-5-3-1)$$
$$+ w'(10+8+6+4+2) = w\left(\frac{12\times 5}{2}\times 2 - \frac{10\times 5}{2}\right) = 35w,$$

on $k = w(11+9+7+5+3+1-8-6-4-2)$

$$+ w'(11 + \ldots\ldots + 1);\ \therefore D_1 = 11w + 6w' = 17w,$$

and so on, D_1 will be diminished by w and increased by $2w$ alternately.

Now
$$35w = \frac{35 \times 25}{42} = \frac{125}{6} = 20.833,$$
$$17w = 17 \times \frac{25}{42} = \frac{425}{6 \times 7} = \frac{70.833}{7} = 10.129,$$
$$w = \frac{25}{42} = \frac{4.166}{7} = .595,$$

and we may write the following table:

TABLE II.

No. of tie.	Maximum tension.	D_1	D_2	Greatest compression.	D_1	D_2
l	20.833	8.929	1.190	11.904	− 8.929	−.595
k	30.952	10.119	−.595	2.380	− 9.524	
i	40.476	9.524	1.190			
h	51.190	10.714	−.595			
g	61.309	10.119	1.190			
f	72.718	11.309	−.595			
e	83.431	10.714	1.190			
d	95.336	11.904	−.595			
c	106.645	11.309	1.190			
b	119.144	12.499				
a	$94.\frac{2}{7}$.					

For a we have, that, since the maximum load at bottom of strut b (Table I.) $= 78\frac{4}{7}$ tons;

$$\therefore \text{ maximum tension of } a = 78\tfrac{4}{7} \times \frac{6}{5} = 15\tfrac{5}{7} \times 6 = 94\tfrac{2}{7} \text{ tons.}$$

Strain on bottom of strut $c = 71\frac{3}{7}$;

$$\text{and} \quad 71\tfrac{3}{7} \times \frac{5}{3} = \frac{357}{3} = 119 \text{ tons,}$$

which agrees with the strain on tie ~~b~~ l

~~*Check on Table II.*~~

The greatest compression on tie ~~b~~ l

$$= w(9 + 7 + \ldots\ldots - 2) + w'(9 + \ldots\ldots + 1),$$

which would be the series got for a tension of a tie *m*, had it existed; and we therefore only produce the tension column of Table II. backwards, as though to a tie *m*, *n*, &c., in order to get the maximum compressions on those ties which are subject to compression.

III. *Booms.*

Under a full load, horizontal tension due to tie *l*

$$= (\omega + \omega')(10 + 8 + 6 + 4 + 2 - 9 - 7 - 5 - 3 - 1)$$

$$= \omega(12 \times 5 - 10 \times 5) = 10\omega,$$

to tie $k = (\omega + \omega')(11 + 9 + 7 + 5 + 3 + 1 - 8 - 6 - 4 - 2)$;

$$\therefore D_2 = 22\omega, \text{ and } 20\omega \text{ alternately.}$$

$$\text{Now } 10\omega = \frac{100}{21} = \frac{33.33}{7} = 4.762,$$

$$20\omega = 9.524,$$

$$22\omega = 9.524 + .9524 = 10.476.$$

And we have the following table, in which the commencement of the column of strains (that is the bottom) is founded on the horizontal strain due to tie *a*, which must be (Table I.)

$$78\tfrac{4}{7} \times \frac{1}{\tan\beta} = 78\tfrac{4}{7} \times \frac{2}{3} = 26\tfrac{4}{21} \times 2 = 52.4.$$

TABLE III.

No. of boom. Upper.	No. of boom. Lower.	Strain in tons.	D_1	D_2
k and *l*		551.400		
			4.762	
i		546.638		10.476
			15.238	
h	*k*	531.400		9.524
			24.762	
g	*i*	506.638		10.476
			35.238	
f	*h*	471.400		9.524
			44.726	
e	*g*	426.638		&c.
			55.238	
d	*f*	371.400		
			64.762	
c	*e*	306.638		
			75.238	
b	*d*	231.400		
			84.762	
a	*c*	146.638		
			94.238	
	b	52.400		
	a	0		

l has a strain equal to that on the upper boom *i*, minus the horizontal pull of tie $l = 546.638 - 4.762 = 541.876$.

Check on Table III.

The whole load on a girder

$$= \mathrm{W} = \frac{3}{2} \text{ tons} \times 210 \text{ feet} = 315 \text{ tons},$$

whence, Lemma VIII, the strain on the centre of the boom

$$= \frac{315}{8} \times \frac{210'0}{15'0} = .315 \times \frac{7}{4}.$$

$$= 551.25,$$

which agrees with the strains on *k* and *l*.

CHAPTER V.

THE LATTICE-GIRDER.

THE principle of construction of a lattice-girder will be understood from figures 33 and 35. Its web is composed of bars of iron, closer together than in a triangular webbed-girder; and which, by the passing of a rivet through the points where they cross each other, are made to support each other: so that a strut does not, as in a triangular girder, act the part of an unsupported pillar; since it is held rigidly firm, *in* the plane of the web; and is, as it were, bound into that plane, with respect to motion at right angles to it, by the lattice-bars crossing it, which are in tension.

Statement of the strains upon the lattice-girder, under an uniform load upon its top and bottom booms.

In figure 33, the bars numbered, slope from right downward to left. Under an uniform load they will be in tension from 1 to about 40, and are called *ties;* in contradistinction to those crossing them, which will be in compression, and are called *struts.* And they will be in compression from about 40 to 80, while those crossing them will be ties. The strains upon different bars are lighter towards the middle of the girder, and increase towards the piers.

Supposing the bars were unrivetted to each other, their action would require no previous explanation, but be readily understood from that of the bars of a triangular girder. As it is, I shall defer the consideration of the effect of rivetting the lattice-bars together till the end of this chapter. For the present they must be supposed to act independently of one another. It must also be supposed that the *vertical* strain, brought by any bar of the web upon the boom, is not resisted by the boom; and is therefore borne by the bar which meets it upon the boom. It must also be supposed, that each lattice-bar at the ends of the girder, is secured to a *fixed* point vertically over the bearings. The practical difficulties in the way of making good these suppositions, or means of fulfilling them, will be found treated of at the end of the chapter.

LEMMA VI. *To find the whole strain produced upon any one bar of a lattice-girder.*

The lattice-girder which we shall take for investigation, is the one, fig. 33, of 150′0 span; with lattice-bars 2′0 apart horizontal distance, inclined at an angle of 45°; the extreme depth of the lattice between its outer crossings being 12′0, which we call the depth of the lattice.

We observe that each bar forms one of a series of triangulation; each triangle of the series having its apex and base in the upper or lower boom. Thus the bar No. 37 is in triangulation with 25, 13, 1, and with the bars joining their extremities; also with 49, 61 and 73.

Now the top of bar 37 is distant 31 spaces of 2′0 from the end A of the span, and consequently $75 - 31$ or 44 spaces from the other end B. A weight, say of 6 tons, on the bridge at the top of bar 37 would be therefore borne, $\frac{31}{75}$ at pier B, and $\frac{44}{75}$ at pier A; the strain would be transferred to those pins (supposing the stiffness of the booms to be unavailable) by means of the bars in triangulation with No. 37, and would cause a strain of

$$\frac{44}{75} \times \sqrt{2} \times 6 \text{ tons}$$

on the bars amongst which are Nos. 25, 13, 1 towards A,

$$\text{and of } \frac{31}{75} \times \sqrt{2} \times 6 \text{ tons}$$

on the bars amongst which are Nos. 49, 61, 73 towards B.

Thus with 6 tons on the top of *every* lattice-bar in the bridge and 4 tons on the bottom of it, we shall have the strain on the bar 37 a tension in tons of

$$\frac{6\sqrt{2}}{75}(32+20+8-31-19-7)$$

$$+\frac{4\sqrt{2}}{75}(44+32+20+8-19-7).$$

But the top load per space ÷ no. of spaces is represented by W_1;

∴ in this case

$$6 \text{ tons} \div 75 = W_1, \text{ and so} \qquad 4 \text{ tons} \div 75 = W_2,$$

and $\sqrt{2}\times 6 \text{ tons} \div 75 = w_1$ ... $\sqrt{2}\times 4 \text{ tons} \div 75 = w_2$;

∴ the strain on 37, if put in a form applicable to *any* load, is a tension in tons of

$$w_1(32+20+8-31-19-7)+w_2(44+32+20+8-19-7).$$

LEMMA VII. *To find the strain on any part of the boom of a lattice-girder.*

Suppose *ABCD*, fig. 19, to represent the two booms and end pillars of a lattice-girder; *AC*, *BD* being vertical lines through the centres of bearings on the pier; *AB*, *CD* horizontal lines through the top and bottom crossings of the lattice, which should also pass through the centre of gravity of every cross section of the booms. *EF* is the centre line; *PQ* shews a point at which the strain on the booms is desired.

As before, it is clear, that since a strain is produced on the booms by every weight laid on either top or bottom of the girder, therefore the greatest strain on any part of a boom is caused by a full load on the girder. Call the whole weight upon the girder of bridge and load, W tons.

The action of each pier at *A* and *B* will therefore be such as to produce a pressure on the girder of $\frac{W}{2}$ tons.

Call the length of the span l feet, yards, or other measure;
.... depth (BD) .... d of the same measure;
.... variable distance, EP, x ....

then the whole weight of $PDBQ$ is $\frac{\frac{1}{2}l - x}{l} \times W$.

Let us consider the forces acting upon the part $PDBQ$ of the girder.

These are,

(1) a pressure $\frac{W}{2}$ tons vertically upwards at B;

(2) the earth's attraction $\frac{\frac{1}{2}l - x}{l}W$ tons vertically downwards; which will act as though at the centre of gravity, as regards the equilibrium of the whole mass of $BDPQ$; i.e. as though pressing on $BDPQ$ down the dotted line half way between BD and PQ, or $\frac{\frac{1}{2}l - x}{2}$ feet, yards, or whatever the measure is, from P.

Also (3), the action of the lattice; half of which crosses PQ upwards from left to right, pulling the parts towards P in $BDPQ$ down, and to the left; while half crosses PQ downwards from left to right, and presses the parts towards Q in the piece $BDPQ$ down, and towards the right. We shall calculate as if these two halves acted with equal force (an accurate correction might easily be applied afterwards), and they will therefore in result act as a single force downwards, in the line PQ, which passes through the centre of them;

(4) and lastly, a horizontal pressure at $P = P$ tons suppose,

(5) .... tension at $Q = T$ ....

We will consider why the piece $BDPQ$ does not turn in the plane of the paper round P.

The forces tending to turn it in the direction of the hands of a watch are,

$$\frac{1}{l}\left(\frac{l}{2}-x\right) \text{W tons at a distance } \tfrac{1}{2}\left(\frac{l}{2}-x\right) \text{ from } P,$$

$$T \quad \text{tons} \ \ldots\ldots\ldots\ldots\ldots\ldots \ d \quad \ldots\ldots\ldots$$

and that tending to turn it in an opposite direction is

$$\frac{W}{2} \text{ tons at a distance } \left(\frac{l}{2}-x\right);$$

no other forces tend to turn it round P as a centre; therefore these must balance one another; or,

$$\frac{l-2x}{2l}W \times \frac{l-2x}{4} + T \times d = \frac{W}{2}\left(\frac{l-2x}{2}\right);$$

$$\therefore \ T = \frac{W}{4d}\left\{l - 2x - \frac{(l-2x)^2}{2l}\right\},$$

$$= \frac{W}{8dl}\{2l^2 - 4lx - l^2 + 4lx - 4x^2\},$$

$$= \frac{W}{8dl}\{l^2 - 4x^2\} = \frac{W}{2dl}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}$$

$$= \frac{W}{2dl}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}+x\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

= half the total load and weight ÷ the area of the elevation of the girder, and × the product of $CP \times PD$ .... (2).

Again, that $P = T$ is evident either by considering the fixity of $BDPQ$ about Q instead of P, in which case all that you have to do is to substitute P for T throughout the above analysis; or from seeing that it is by their being equal and in opposite directions, that the piece $BDPQ$ is kept from moving bodily horizontally.

COROLLARY 1. At the centre of the bridge we have $x = 0$,

and $$P \text{ or } T = \frac{Wl}{8d} = \frac{W}{4} \times \frac{l}{2} \div d.$$

Now if we imagine the girder loaded only by a weight $\frac{W}{2}$ at the centre of it, the weight on either pier would be $\frac{W}{4}$. Considering then the tendency of either half to turn round the centre of the top or bottom boom, we should have, if P and T were the strains due to this load at those points,

$$P \text{ or } T \times d = \frac{W}{4} \times \tfrac{1}{2} l,$$

and $$\therefore P \text{ or } T = \frac{W}{4} \times \frac{l}{2} \div d.$$

Hence the tension or compression produced at the *centres of the booms* by a load uniformly distributed over the girder is equal to that produced by half that load placed at the centre of the girder.

Proposition III.

To construct a wrought iron double lattice-girder of a span of 150′0 *between bearings, and a vertical depth of about* $\frac{1}{10}$ *of that span; given upon two girders*

dead load on top	= 70 *tons,*	*on bottom*	60 *tons.*
live	= 1 *ton per ft.*	....	$\frac{1}{2}$ *ton per ft.*

I. *Lattice.*

As suitable to the data, we will take the horizontal distance of the lattice-bars apart, as 2′0; the depth of lattice as 12′0; the angle of lattice 45°;

	tons per bridge.	tons per lattice.
∴ dead load on each pin at top	$=\frac{70}{75}=\frac{28}{30}$	$=\frac{7}{30}$
.... bottom	$=\frac{60}{75}=\frac{4}{5}$	$=\frac{1}{5}$
live load on each pin at top	$=2$	$=\frac{1}{2}$
.... bottom	$=1$	$=\frac{1}{4}$

since there are 2 girders and 4 lattices.

Dividing by 75, the number of spaces in the length of the web, and multiplying by $\sqrt{2}$ $\left(\text{which} = \frac{1}{\cos 45^\circ}\right)$, we get

$$w_1 = \frac{7}{30} \times \frac{\sqrt{2}}{75}, \qquad w_2 = \frac{1}{5} \times \frac{\sqrt{2}}{75},$$

$$w_1' = \frac{1}{2} \times \frac{\sqrt{2}}{75}, \qquad w_2' = \frac{1}{4} \times \frac{\sqrt{2}}{75},$$

$$\left.\begin{aligned} \text{Now } \log\ 7 &= .845098 \\ \tfrac{1}{2}\log\ 2 &= .150515 \\ -\log 75 &= \bar{2}.124939 \\ -\log 30 &= \bar{2}.522879 \end{aligned}\right\} \qquad \text{hence } \log \frac{\sqrt{2}}{75} = \bar{2}.275454 = \log .0188565.$$

$$\log .004{,}399{,}7 = \bar{3}.643424$$

Hence,

$$\left.\begin{aligned} &\left.\begin{aligned} w_1 &= .0043997 \\ w_1' &= .009428 \end{aligned}\right\} \text{total } .013828 \\ &\left.\begin{aligned} w_2 &= .0037712 \\ w_2' &= .004714 \end{aligned}\right\} \text{total } .008485 \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots \text{(I.)}$$

Maximum compression of bar No. 32

$$\left.\begin{aligned} &= w_1\,(26+14+2-37-25-13-1) \\ &+ w_2\,(20+8-43-31-19-7) \\ &+ w_1'\,(26+14+2) \\ &+ w_2'\,(20+8) \end{aligned}\right\},$$

$$\left.\begin{aligned} &= w_1\left(\frac{28\times 3}{2} - \frac{38\times 4}{2}\right) \\ &+ w_2\left(28 - \frac{50\times 4}{2}\right) \\ &+ w_1'\,\frac{28\times 3}{2} \\ &+ w_2'\times 28 \end{aligned}\right\} \qquad \left.\begin{aligned} &= w_1 \times (42-76) \\ &+ w_2 \times (28-100) \\ &+ w_1' \times 42 \\ &+ w_2' \times 28, \end{aligned}\right\}$$

$$\text{or, (I.),} \left.\begin{aligned} &= .0043997 \times (-34) \\ &+ .0037712 \times (-72) \\ &+ .009428 \times 42 \\ &+ .004714 \times 28 \end{aligned}\right\} \qquad \begin{aligned} &\left.\begin{aligned} &= -.149590 \\ &\ \ -.271526 \end{aligned}\right\} = -.421116 \\ &\left.\begin{aligned} &+ .395976 \\ &+ .131992 \end{aligned}\right\} + .527968 \end{aligned}$$

$= .106852$ tons...(1)

Maximum compression on bar 33,

$$= w_1 (27 + \ldots + 3 - 36 - \ldots - 0) + w_2 (21 + 9 - 42 - \ldots - 6)$$
$$+ w_1' (27 + \ldots 3) \qquad + w_2' (21 + 9);$$

$$\therefore D_1 = w_1 \times 7 + w_2 \times 6 + w_1' \times 3 + w_2' \times 2 = \left.\begin{matrix} .030798 \\ +.022627 \\ +.028284 \\ +.009428 \end{matrix}\right\} = \overset{\text{tons.}}{.091137} \ldots (2)$$

Maximum compression on bar*

$$\text{No. } 34 = w_1 (28 + \ldots + 4 - 35 - \ldots - 11) + w_2 (22 + 10 - 41 - \ldots - 5)$$
$$+ w_1' (28 + \ldots + 4) \qquad + w_2' (22 + 10); \therefore D_2 = -w_1.$$

$$\text{No. } 35 = w_1 (29 + \ldots + 5 - 34 - \ldots - 10) + w_2 (23 + 11 - 40 - \ldots - 4)$$
$$+ w_1' (29 + \ldots + 5) \qquad + w_2' (23 + 11); \therefore D_2 = 0.$$

$$\text{No. } 36 = w_1 (30 + \ldots + 6 - 33 - \ldots - 9) + w_2 (24 + 12 - 39 - \ldots - 4)$$
$$+ w_1' (30 + \ldots + 6) \qquad + w_2' (24 + 12); \therefore D_2 = 0.$$

$$\text{No. } 37 = w_1 (31 + \ldots + 7 - 32 - \ldots - 8) + w_2 (25 + \ldots + 1 - 38 - \ldots - 3)$$
$$+ w_1' (31 + \ldots + 7) \qquad + w_2' (25 + \ldots + 1);$$
$$\therefore D_2 = w_2 + w_2'.$$

$$\text{No. } 38 = w_1 (32 + \ldots + 8 - 31 - \ldots - 7) + w_2 (26 + \ldots + 2 - 37 - \ldots - 2)$$
$$+ w_1' (32 + \ldots + 8) \qquad + w_2' (26 + \ldots + 2); \therefore D_2 = 0.$$

and writing only the distinctive figures,

No. 39 $= w_1(\ldots 9 \ldots 6) + w_2(\ldots 3 \ldots 0) + w_1'(\ldots 9) + w_2'(\ldots 3)$; $\therefore D_2 = 0.$

No. 40 = ... 10...5 ....4...11 ...... 10 .... 4; $\therefore D_1 = -w_2.$

No. 41 = ... 11...4 ....5...10 ...... 11 .... 5; $\therefore D_2 = 0.$

No. 42 = ... 12...3 ....6... 9 ...... 12 .... 6; $\therefore D_2 = 0.$

No. 43 = ... 1...2 ....7... 8 ...... 1 .... 7; $\therefore D_2 = w_1 + w_2.$

No. 44 = ... 2...1 ....8... 7 ...... 2 .... 8; $\therefore D_2 = 0.$

No. 45 = ... 3...0 ....9... 6 ...... 3 .... 9; $\therefore D_2 = 0.$

* After a very little practice the reader will be able to detect where the difference changes, without writing down all these series.

The characteristic terms of this last series are just like those for bar 33, and the values of D_2 will come regularly over again. For the numerical values of w_1 w_2 &c. see (I.).

We can now therefore write the following table:

Table of Lattice-Bars under Compression.

No. of bar.	Strain in tons.	D_1	D_2
31	.015715		
		.091137	
32	.106852		0
33	.197989	„	−.004400
		.086737	
34	.284726		0
35	.371463	„	0
36	.458200	„	.008485
		.095222	
37	.553422		0
38	.648644	„	0
39	.743866	„	−.003771
		.091451	
40	.835317		0
41	.926768	„	0
42	1.018219	„	.013828
		.105279	
43	1.123498		0
44	1.228777	„	0
45	1.334056	„	−.004400
		.100879	
46	1.434935		0
47	1.535814	„	0
48	1.636693	„	.008485
		.109364	
49	1.746057		0
50	1.855421	„	0
51	1.964782	„	−.003771
		.105593	
52	2.070378		0
53	2.175971	„	0
54	2.281564	„	.013828
		.119421	
55	2.400985		0
56	2.520406	„	0
57	2.639827	„	−.004400
		.115021	

No. of bar.	Strain in tons.	D_1	D_2
58	2.754848		−.004400
		.115021	
59	2.869869		0
60	2.984890	"	.008485
		.123506	
61	3.108396		0
62	3.231902	"	0
63	3.355408	"	−.003771
		.119735	
64	3.475143		0
65	3.594878	"	0
66	3.714613	"	.013828
		.133563	
67	3.848176		0
68	3.981739	"	0
69	4.115302	"	−.004400
		.129163	
70	4.244465		0
71	4.373628	"	0
72	4.502791	"	.008485
		.137648	
73	4.640439		0
74	4.778087	"	0
75	4.915735	"	−.003771
		.133877	
76	5.049612		0
77	5.183489	"	0
78	5.317366	"	.013828
		.147705	
79	5.464071		0
80	5.611776	"	0
		"	
75 to 80 =	31.542049		

This table is formed exactly as is the one on p. 79.

The sum of the strains on the bars bearing on the end pillar is taken as a very desirable check upon the accuracy of such a lengthened system of addition.

Thus, we know that the whole weight of the bridge and load is

$$70 + 60 + 150 + 75 = 355 \text{ tons};$$

this gives for each end of each lattice 44.375 tons to support; which, multiplied by $\sqrt{2}$, gives 62.746 tons as the total strain of the lattice-bars of one web which meet the pillar, which is about double the total in our table; but of this hereafter.

We might find the strain in tension on the bars just in the same way as the above; but the following is less liable to error.

Again, the maximum tension on bar No. 43

$$= w_1 (26 + \ldots + 2 - 37 - \ldots - 1) + w_2 (32 + 20 + 8 - 31 - \ldots - 7)$$
$$+ w_1' (26 + \ldots + 2) \qquad\qquad + w_2' (32 + \ldots + 8),$$

which, we observe, = maximum compression on bar No. 32

$$+ 75w_2 + 32w_2',$$

so tension on bar No 42 = compression on No. 33

$$+ 75w_2 + 33w_2',$$

and so on.

Now, $\left.\begin{array}{r} 75w_2 = .282840 \\ 32w_2' = .150848 \end{array}\right\}$ total .433688 tons.

Whence we form the following table, observing that in practice we do not want more than 3 places of decimals at the outside.

It is formed thus: for bar 43, as we have seen, we are to add .433688 tons (which is accordingly placed in column D_1') to the maximum compressive strain on bar 32 in the former table. So for bar 42 we must add to the maximum compression on bar 33, .433688 tons $+ w_2'$, which is .004714 tons, and so on; bar 41 corresponds with 34, 40 with 35, &c.

TABLE OF LATTICE-BARS UNDER TENSION.

No. of bar.	Strain.	D_1'	D_2'
44	.446	.428974	.004714
43	.530	.433688	"
42	.636	.438402	"
41	.728	.443116	"
40	.819	.447830	"
39	.911	.452544	"
38	1.011	.457258	"
37	1.110	.461972	"
36	1.210	.466686	"
35	1.307	.471400	"
34	1.403	.476114	"
33	1.499	.480828	"
32	1.609	.485542	"
31	1.719	.490256	"
30	1.829	.494970	"
29	1.934	.499684	"
28	2.040	.504398	"
27	2.146	.509112	"
26	2.260	.513826	"
25	2.374	.518540	"
24	2.488	.523254	"
23	2.597	.527968	"
22	2.708	.532682	"
21	2.819	.537396	"
20	2.943	.542110	"
19	3.067	.546824	"
18	3.191	.551538	"
17	3.311	.556252	"
16	3.431	.560966	"
15	3.550	.565680	"
14	3.678	.570394	"
13	3.807	.575108	"

No. of bar.	Strain.	D_1'	D_2'
12	3.935	.579822	.004714
11	4.060	.584536	"
10	4.184	.589250	"
9	4.308	.593964	"
8	4.447	.598678	"
7	4.585	.603392	"
6	4.723408	.608106	"
5	4.857285	.612820	"
4	4.991162	.617534	"
3	5.125039	.622248	"
2	5.267401	.626962	"
1	5.409763	.631676	"
1—6	30.374058		

Check.—Total strain of struts against pillar = 31.542049
.... ties .... = 30.374058

.... bars of one lattice .... = 61.916107

Again, total weight of bridge and load = 355 tons, of which all but 2′0 run, (viz. all but half a space at each end,) is supported by the lattice-bars. That is, about 350¼ tons are supported by the lattice, which gives for each end of each lattice 43.79 tons; and $43.79 \times \sqrt{2} = 61.928$ tons strain; which is near enough to shew the correctness of our calculation.

II. *Booms.*

Our data give us:

total weight of upper half of dead load = 70 tons,
.... lower .... = 60 tons,
.... upper live load = 150′ at 1 ton = 150 tons,
.... lower .... ½ ton = 75 tons,

∴ total weight .... = 355 tons,
or, per girder, W = ½ × 355 tons.

Our equation for giving the strain on the centre of the booms is, page 100,

$$P \text{ or } T = \frac{Wl}{8d}$$

$$= \frac{355 \times 150}{2 \times 8 \times 12} = \frac{106500}{8 \times 8 \times 6} = \frac{13312}{8 \times 6} = \frac{2220}{8} \text{ nearly}$$

$$= 277.5 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Our equation for giving the strain on the booms at x feet distance from centre is

$$P \text{ or } T = \frac{W}{2dl}\left(\frac{l}{2} - x\right)\left(\frac{l}{2} - x\right)$$

$$= \frac{355}{4 \times 12 \times 150}(75 - x)(75 + x)$$

$$= \frac{71}{1440}(75 - x)(75 + x) \text{ tons.} \ldots\ldots\ldots\ldots (4)$$

COROLLARY. But this requires a correction due to the action of the lattice. In fig. 17 suppose the dotted line PQ to be the place where we wish to find the strain on the booms. Consider the tendency of the piece $AGPQ$ to turn round P.

First, we know from above that *apart* from the action of the lattice it requires a tension of $\frac{71}{1440}(75 - x)(75 + x)$ tons at Q, to prevent the piece $AGPQ$ turning round P in a direction opposite to the hands of a watch; we now will find out how the *lattice* tends to turn it.

From the series on page 102 we notice that as far as dead load only is concerned, the difference between the strains on any two parallel bars 12 spaces apart, (as, for instance, fig. 17, bars a and n, bars 4 and 16, &c.) is caused by adding

$$w_1 (1 + 1 + 1 + 1 + 1 + 1 + 1) = 7w_1 \text{ for 3 times,}$$

and $\quad w_1\,(1+1+1+1+1+1) = 6w_1$ for the other 9 times, and also the same for w_2.

If then we make the full load extend over the whole bridge; i. e. if we write $w_1 + w_1'$ for w_1, and $w_2 + w_2'$ for w_2, we get, that the difference of strain in bars 12 spaces apart, is

$$3 \times 7\,(w_1 + w_1') + 6 \times 9\,(w_1 + w_1')$$

$$3 \times 7\,(w_2 + w_2') + 6 \times 9\,(w_2 + w_2'),$$

which gives a total of $(21 + 54) \times (.013828 + .008485)$.

Prop. III. (I.).

$$\text{or } 75 \times .022313 \text{ tons}$$

$$= 1.6735 \text{ tons},$$

which gives an average difference in the strain of two consecutive bars of .14 tons very nearly.

Also the difference between the strain on any two bars meeting on the lower boom, as No. 7 and n, is the weight on one space of the lower boom $\times \sqrt{2}$ or (page 100)

$$= \left(\frac{1}{5} + \frac{1}{4}\right)\frac{7}{5}\text{ tons} = \frac{9 \times 7}{100}$$

$$= .63 \text{ tons}.$$

Now the perpendicular distance of P from the bars 8 and n, is the same; and equal to the distance of P from their point of crossing each other $\div \sqrt{2} = 11' \times \frac{5}{7}$.

And the tension of bar 8 at the line PQ tends to turn $AGPQ$ round P like the hands of a watch; and the compression of n at the line PQ tends to turn it in the other direction, with the same leverage, viz. the perpendicular distance of P from either bar, but with a ~~greater~~ force. Therefore the influence of 8 and n united is equal to the excess of the strain on n over that on 8 $\times$ the distance $\left(11 \times \frac{5}{7}\right)'$, tending to turn $AGPQ$ contrarily to the hands of a watch.

Now the excess of the strain on 7 over n is .63; and therefore of the strain on n over that on 8, $-.63+.14=.49$ tons;

∴ influence (contrary to hands of a watch) of the pair of bars

$$8 \text{ and } n \text{ on } AGPQ \text{ is } (-.63+.14)\times 11\times\frac{5'}{7}$$

so of 9 and m	...	$(-.63+3\times .14)\times 9\times\frac{5'}{7}$
10 and l	...	$(-.63+5\times .14)\times 7\times\frac{5'}{7}$
11 and k	...	$(-.63+7\times .14)\times 5\times\frac{5'}{7}$
12 and i	...	$(-.63+9\times .14)\times 3\times\frac{5'}{7}$
13 and h	...	$(-.63+11\times .14)\times 1\times\frac{5'}{7}.$

$$\text{Total } -.63\times 36\times\frac{5}{7}+.14\times\frac{5}{7}\times 2\,(11+27+35),$$

$$=-16.20+.2\times 73=-16.20+14.6,$$

$$=-1.6 \text{ tons at 1 foot distance,}$$

$$=-.133 \text{ tons at point } Q \text{ 12'0 distant.}$$

This shews that T at Q must really be .133 tons less than the above equation (4) would give it.

Again, the whole horizontal effect of the lattice crossing PQ = the sum of the compressions from h to n − the sum of the tensions from 8 to 13, all $\div\sqrt{2}$

$$=6\times\text{ the difference of strains in bars 8 and } h\div\sqrt{2}$$

$$=6\times(-.63+6\times .14)\times\frac{5}{7}$$

$$=30\times(-.09+.12)$$

$$=.9 \text{ tons, pushing } AGPQ \text{ bodily from } PQ.$$

Hence T must exceed P by .9 tons, and we have the following equations:

$$T = \frac{71}{1440}(75 - x)(75 + x) - .133 \text{ tons} \quad \ldots\ldots\ldots (5)$$

and $$T = P + 6.3 \text{ tons};$$

therefore $$P = \frac{71}{1440}(75 - x)(75 + x) - 1.033 \text{ tons} \ldots\ldots\ldots (6).$$

Hence we may get the section required at distances of 12′0 beginning at the centre, viz.

	top	bottom
At the centre of the girder	277 tons	and 278 tons,
12′0 from the centre	270	and 271,
24′0	242	and 243,
36′0	213	and 214,
48	163	and 164,
60	99	and 100,
72	21	and 22.

I here leave this girder; merely worked out to the approximation which the data are calculated to give. If it be found, upon getting out the weight of the girder, as proportioned from these calculations, that its *average* weight has been correctly assumed; then any further adjustment of its proportions made in order to allow for the weight of the girder being really greatest in the centre (and not uniform), will in so small a girder be best done by artifice; and not by going all through the method I have applied to the larger lattice-girder, Chap. VII.

The calculations for a girder though necessarily lengthy, are at the same time *less* lengthy and *more* interesting than many calculations required to ascertain the tidal capacity of rivers or the excavation for railroads. There is no doubt that the reasoning is much more complex, and the working less capable of assistance from tables; but not in a higher ratio than the value of the material in one case to that in the other.

LEMMA VIII. *On the practical termination of the lattice-bars on the pillar.*

In the fig. 17, if the upright line AG represented a number of bars AB, BC, &c., FG, pinned together without lateral stiffness, it is clear that the strains down any two bars of the web, which meet upon it, as b and 4, must be equal; one being compression and the other tension, and their horizontal effect equal and opposite. In that case, any calculation which allowed the bars b and 4 to bear unequal strains must be inapplicable. Hence, the value of the whole theory depends on the construction of the end pillar represented by AG.

This upright line AG, fig. 17, will be taken in any subsequent investigation as an ideal line, typifying the deep plate-pillar shewn in fig. 33. The horizontal section of this will be such as is shewn in fig. 21; and is so strong, as a girder set on end, that the line AG, fig. 17, representing it, will be assumed incapable of deflection. The strain therefore coming down any bar b, and which will operate at E, will be at once supported in a vertical direction by the pillar below E; and resisted in a horizontal direction, mainly by the tension of bar 4, and the surplus by the lateral stiffness of the end pillar AG; which will transfer all horizontal strains brought upon it at E, to A and G, where they are taken by the booms, and accounted for. Hence no part of the compression of b acting on E will increase the tension of bar 4, beyond what it may have independently.

But the use of this pillar may be obviated, and supports, capable of vertical resistance only, used, if the lattice-web be produced 3′0 farther over the pier, as indicated by the dotted lines. The lower boom is here supposed so stiffened between H and W, as to form a rigid bearing for all vertical strains brought upon it; itself being supported by a bed-plate or bearing about A.

The lines HX, AG, VY, WZ, indicate ∠ or T irons, rivetted on each side of the lattice-web, and rigidly connected to the booms at H, A, V, W, X, G, Y, Z; they will of course have longitudinal, but *not* lateral, stiffness. WZ must be by far the strongest of the four pillars.

It will be seen that in this system, any strain coming along any lattice-bar, is conducted to a rigid bearing, either *H*, *A*, *V*, *W*, *X*, *G*, *V* or *Z*; without drawing for any support from any other lattice-bar to the left of *HX*.

Thus bar 7 is rigidly supported by *HX*, and the top boom *PX*. So for bars 6 and 5. Bar 4 is supported vertically, only, by *WZ*; that pillar not resisting horizontal influence, which is therefore borne at *Y*; since *Y* is held firm by *VY* and *PY*. So the strains of 2 and 3 are borne at *X* and *G* respectively; bar 1 sustains the strain of *b*, which it conveys to *H*, and so on.

It may be still felt, although these strains are thus satisfactorily accounted for, either by a stiff pillar of plate, or by a repetition of lighter pillars; that still the amount of deflection and "give," which will occur at the end of the lattice-web, before its strains are conveyed to the bed-plate on the pier, may be extensive, and cause the calculated strains on the bars to be so increased or diminished, in practice, as only to prove a rough approximation to the real strains.

Now they may be relied upon as very close approximations, (and we will not assert more), for

I. A practical man will admit that, if the load be placed, say, at the top only of the girder typified by fig. 17; then the compression down *a*, *b*, will be much greater than the tensions in 1 and 2; in fact he cannot stop short of saying, that they will have just the average difference that would be shewn in the calculation.

II. The sum of all the strains down the bars 1 to 6 and *a* to *f* must be accurately that calculated; as it has to support the load, which is correctly known.

III. Any derangement of the real from the calculated strains, caused by sudden or gradual changes in the stiffness of the bars, is soon spread evenly over the breadth of the lattice, and their strains are thus again left in close subjection to the theoretical laws of differences according to which they support the load.

In proportioning the bars, however, the following care may be taken; if (for instance) bar 12, fig. 17, is of a section but just sufficient for its maximum strain, and therefore bar 11 be made of a heavier section, and consequently surplus strength; then the bar *e*, meeting 11 at the top, and which is a strut, should have some surplus strength also; for the bar 11 being more rigid than 12, takes off, through the stiffness of the bottom boom, some of the weight which would otherwise fall upon 12, and brings upon *e* more than its due share of strain.

The lattice-web of a girder being united by rivetting, is in perfect bearing upon all the rivets; and before the weight of the bridge is thrown on it, is entirely free from strain. This would not be the case if bolts and not rivets were used in the web; for the bolts would not all bear when first put together, and therefore on the weight's being first thrown on the lattice, some bars might be over strained before others got a bearing on their bolts; and again, it might require some straining of the bars to get the bolts originally into the holes. When the weight then is thrown upon the *rivetted* lattice the strains produced are as unmixed with unknown disturbances—as theoretically pure—as they can be in any human structure, and each bar exerts its influence without violence or uncertainty. And the lattice-girder, as we shall see, has a tendency to destroy, and never to accumulate, any irregularity, occasioned by any cause, in the strain of the bars as rivetted.

The remainder of this chapter will be occupied by the *general argument upon the rivetting together of the bars of a lattice-web; as affecting the strains on those bars and on the booms.*

(Some of the following argument requires a knowledge of the principles explained in Chapter VIII.; i.e. it requires some elementary notions of curvature. The reader may, if he like, pass on for the present, to the summing up; and then to the next chapter.)

First, we shall consider what violence is caused by the fact of the lattice being rivetted; when a deflection takes place due to an alteration in the booms only.

Suppose a deflection to be caused by a contraction of the upper boom, and expansion of the lower one, called into being by some extraneous force; or at all events without at all affecting the lengths of the lattice-bars.

We will assume the extent of this contraction, or expansion, to be $\frac{1}{2000}$; or about what would be due to a test load of 6 tons to the square inch of gross section.

The general effect of this will be, a contracting of the lattice spaces of the top, and a widening of them at the bottom, of the web; the distance of the booms apart being left to be adjusted at the will of the lattice.

Under these circumstances, all the squares formed by the lattice towards the bottom of the web, will be widened laterally, and shortened vertically; those towards the top will be narrowed laterally and lengthened vertically; those nearer the middle will be less and less altered; and the booms will retain their former distance, measured radially from the centre round which they form arcs (except indeed an utterly immeasurable decrease due to the shortening of the lattice-bars by their mere curvature).

Fig. 16 will shew this effect as caused by bending to excess a girder of this kind but 4'.0" deep only.

In this figure the top boom is shewn compressed $1\frac{1}{2}''$ for every 1'0 run, and the bottom boom extended to the same amount. This causes a very violent curvature; the supposed effect of the strain being here $\frac{1}{8}$th instead of $\frac{1}{2000}$, the extreme in practice; i.e. 250 times that due to a proof load in practice. The dotted line shews the original position of the booms, and the small circles the original position of the lattice-pins upon them, before flexure.

In order to give this problem a general form we will take literal, not numerical, dimensions for the depth of the girder, and the amount of the contraction of the booms; and in terms of them obtain the set off from the tangent to the upper boom 1'0 from the point of contact. Then we will apply the result to the girders fig. 16, and also to fig. 33, Pl. II.

Let r be the radius of the arc of the upper boom, in feet,

d the depth of the girder,

δ the contraction, or expansion, in a length of a foot;

Then $$r : 1-\delta :: r+d : 1+\delta,$$

$$\therefore\ r : 1-\delta :: d : 2\delta,$$

$$r=\frac{d}{2\delta}(1-\delta).$$

Again, denoting the set off from the tangent, 1′0 from the point of contact, to meet the arc, by Δ;

$$\Delta : 1 = 1 : 2r;$$

$$\therefore\ \Delta=\frac{1}{2r}=\frac{\delta}{d(1-\delta)}$$

$$=\frac{\delta}{d}\text{ very nearly.}$$

In our exaggerated example $\delta=\frac{1}{8}$, $b=4$, therefore,

$$\Delta=\frac{1}{32}.$$

In fig. 33, $\delta=\frac{1}{2000}$, $b=12$,

$$\therefore\ \Delta=\frac{1}{24{,}000};$$

Or the curvature in one case is

$$\frac{24{,}000}{32}=750 \text{ times as great as in the other.}$$

Yet, even under this exaggeration, we see that, in fig. 16, the lattice is bent into arcs of a single curvature nearly uniform; not into S curves, in which case it would require great violence to make the bars bend backwards and forwards to form them. It is seen, therefore, that if the lattice be pinned together before the flexure of the girder, the result is that the bars must bend laterally into an uniform curve; and that, that done, no further violence is needed.

Now, in this example, if we follow the bar ABC, fig. 16, we see that the top, A, is *raised* $4 \times \Delta$ feet $= \frac{1'}{8}$ from its original position; and by the contraction is brought to the *left* $2 \times \frac{1}{8} = \frac{1'}{4}$ from its original position; that the bottom, B, is raised also $\frac{1'}{8}$, and brought to the left $\frac{1'}{4}$ also. Hence, the original bar could lie straight and unstretched between A and B: and, if it were not rivetted to the bars crossing it, would lie unstretched upon the dotted line. But the point C is prevented, by the crossing bars to which it is rivetted, from remaining thus midway between A and B; but is thrust downward, and to the right again, by the approximation of the upper and distension of the lower parts of the lattice. We will not calculate the new position of C; but by drawing the dotted chord of the arc assumed by the bar, we find, by measurement, that its versed sine is about $\frac{3}{8}$ of a foot.

If this be altered to suit fig. 33, with its curvature $\frac{1}{750}$ times, and the length of its bars and its depth, $\frac{12}{4}$, or 3 times that of the example; we have, observing that the versed sine of a bar varies as its curvature and the square of its length, and that the curvature of the bar is in proportion to that of the girder,

versine of a bar in girder, fig. 33, under test load

$$= \frac{3'}{8} \times 3^2 \times \frac{1}{750} = \frac{9}{8 \times 250} = \frac{1}{222.2} \text{ feet*}$$

* This process may be thus made clearer:

In fig. 16 the deflection of $ACB = \frac{3}{8}$ feet; the curvature call 1.

If we suppose the scale of fig. 16 reduced $\frac{1}{3}$, then all the dimensions including the radius of curvature are increased 3 fold, and therefore, while the depth becomes .12′o,

$= \frac{1}{18\frac{1}{2}}$ inches in the bar 12′0 long.

Hence, in our girder, fig. 33, we may expect a test load so to affect the booms, as to impart a curve to each lattice-bar in the plane of the web, of a little less than $\frac{1''}{16}$ in their 12′0 length; in which position they are rigidly secured by the bars crossing them.

That this is not a dangerous deflection under a test load is clear, and has been proved amply by experiment: and that it will not much affect the strains on any individual bar may also be very safely supposed.

Secondly. We shall consider the deflection caused by the expansion and contraction of the lattice-bars, supposing the booms to be unaltered. We shall thus gain a true idea of how far the contraction and expansion of each bar affects the others rivetted to it.

Suppose the whole bridge uniformly loaded, and that we are noticing the effect on the bars acting as ties and struts in the webs, without considering the change caused in the booms.

We first *remark* that, as the alteration in the length of the booms caused an uniform curvature in the girder without altering its radial depth; so the alteration in length of the lattice-bars would, if they were duly proportioned for an uniform load, cause the girder to form an obtuse angle at the centre of the span; each half being straight, and its *vertical* depth unchanged.

In a girder for traffic, however, the centre bars are much stronger than would be necessary were the load fixed; and this causes the deflection due to lattice to diminish towards the

the deflection of ACB becomes $\frac{3}{8} \times 3$ feet, the curvature becomes $\frac{1}{3}$.

If we suppose the curvature then altered to $\frac{1}{750}$

the deflection of ACB becomes $\frac{3}{8} \times 3 \times \frac{3}{750}$, and the curvature $\frac{1}{750}$.

centre, and the bridge to assume a curved shape. This curvature, since it is owing to, is therefore proportionate to the *excess* of metal in the bars; and is greatest in the middle of the girder, and diminishes to nothing at the ends.

To shew the truth of the above, we will first consider the bars of one end of the girder, fig. 33, as enlarged in fig. 17.

To begin with the triangle AHB, fig. 17. The alteration of AH has been already considered, in treating of the curvature of the booms; and, therefore, AH must be here considered of constant length. AG, which represents part of the supporting pillar of the girder, is generally so heavy in section, that for the present it will be considered constant in length, since it is nearly so in practice. But HB will be increased $\frac{1}{2000}$ of its length, supposing it to be subject to a strain of about 6 tons to the square inch of its section.

Referring to figure 17*a*, which is an enlarged view of this triangle, we will denote the increase of 17″ (the length of AK, HK and BK) by δ; and therefore the increase of 2′0 (the length of AB and AH) by $\sqrt{2}\,\delta$. δ will then be

$$\frac{17''}{2000} = .0085'' \text{ and } \sqrt{2}\,\delta = .012''.$$

By glancing at fig. 17*a*, in which the increments of the bars are immensely exaggerated, we see that if BH increases $\frac{1}{2000}$th; i. e. 2δ, $= Hs$ suppose, then H will come to H'; where $AH' = AH$, and also

$$\text{where } HH' = \sqrt{2} \times Hs = 2\sqrt{2}\,\delta,$$

and K if unrivetted would come to K', where

$$KK' = \frac{1}{2} \text{ of } HH' = \sqrt{2}\,\delta.$$

Now the bar AK will diminish δ inches, and the question arises, Will this require any curvature of the bar BH?

The answer is, No; for $AK' = Ar$;

therefore $$AK - AK' = Kr = KK' \times \frac{1}{\sqrt{2}} = \delta.$$

Hence it is plain that, if the pillar AG be made very strong, then as far as regards the effect of a load on the lattice, the rivetting of HB and AK together at K, will make no difference in the position of, and therefore none in the strain upon, those bars.

By a similar figure we might see, that if CL, fig. 17, were free

L would fall $4\sqrt{2}\,\delta$ vertically,
M ... $3\sqrt{2}\,\delta$ ...
N ... $2\sqrt{2}\,\delta$ ...
O ... $\sqrt{2}\,\delta$ ...

If we get to M along $BKHM$, we find that H falls to $2\sqrt{2}\,\delta$, and that the contraction of HM allows to M a further vertical fall of $\sqrt{2}\,\delta$; which agrees with M's total fall given above.

So the contraction of AN, 2δ will admit of N coming down $2\sqrt{2}\,\delta$; and this will bring the new position of N to the same point as was assigned it above, and keep it in a straight line with AK.

And so on: it will be seen that were the strength of every lattice-bar in proportion to the strain brought upon it by any load; then, considering the effect of the load on the lattice-bars only, *no* additional strain would be caused by rivetting them together.

It must now be observed, that if the lattice were adapted for an uniform load only (distributed on the top and bottom of the girder), there might be some bars left out entirely at the centre; viz. all those that cross the centre line; including Nos. 38 to 43, fig. 33, and the corresponding ones crossing in the other direction: in that case the girder would without difficulty, as far as the lattice is concerned, form the obtuse angle into which the lattice would deflect: the lower rivet of each bar being .012″

Grea
Compr

Great
Tensi

A

Willia

above the next rivet towards the centre line, and as much below the next rivet towards the pin.

But as this kind of construction would be liable to injure the booms, and, moreover, is impossible with a moving load, the bars are therefore continued through the centre, and, at the same time, made of a section highly disproportionate to the strain of the uniform load. Their refusal, therefore, to contract and extend,—more and more decided as they approach the centre,—causes them to raise the booms gradually above the slant line of the theoretical deflection, until at the centre they make the booms meet upon a common horizontal tangent. It is evident from this description, that the extra strength of the central bars cannot cause any extra longitudinal strain on bars which have no extra strength: and that by putting the extra metal in the centre bars ease may be produced, but no violence.

Again, it may be remarked with reference to the *sudden alteration* in the section of bars, necessitated in practice, that in fig. 17

I. If the bars 1...12 and $a...n$ are kept of uniform section, though the strain on them decreases, then the bars $a...n$ will be curved with the concave side uppermost, since the drop of each of the lower angles H, L, &c. below the last will get less and less.

II. If the bars 1...8 say are of a certain section, but 9 onwards of a lighter section; then, as far as the lattice is concerned, they, with the bars $a...n...$, would curve the booms upward till they came to the bar 8; and there make a slight angle in them, so as to make them more horizontal again, but resuming, at once, their curve upwards. In practice what prevents this angle is the stiffness of the booms.

The above is a fair statement of the effect caused on the rivetted lattice-bars, by the deflection of the booms, and of themselves. I wish I could state them more plainly. I could do so, if I were willing to force them to any conclusion, instead of wishing to leave them in that necessarily undecided form, in which they should rightly come before the practical engineer, as long as our knowledge and practice remain as they are. My own conclusions follow the next paragraph, which relates to the effects of the booms being made in long lengths, and not flexible at every junction of a lattice-bar with them.

Then now as to the stiffness of the booms. It is certain that if the girder were perfectly proportioned, and all the joints in boom and lattice were mere pins, then the booms would lie in exactly the same curves under a full load, as they will when each is continuously rivetted from end to end (excepting only, that a slight difference will be due to the stiffness of the rivetted boom itself, as a girder, if it be made continuous throughout, which will be imperceptible though favourable). In such a bridge, if the boom gradually became continuous, the strains upon the lattice would be the same as if, and might, therefore, be *calculated* as if, the boom were still in pieces.

In *summing up* we may say, that in a lattice-girder, speaking of it generally under fixed and moving loads:

1. The deflection due to the booms will not in practice strain the lattice, but will only curve them all alike. Except towards the ends of the girder, there the necessarily excessive stiffness of the booms leaves the lattice-bars straighter: and provided too violent drops be not made in the section of the booms.

2. The deflection due to the lattice is normally rectilinear. But if the lattice gradually get out of proportion to the strains, a curvature is produced in them, in the same direction as that caused by the deflection due to the booms, but of a very much less amount, since it is due not to the amount of strain, but to the ratio of the difference of successive strains to the strains: and again, if the lattice suddenly drop in its proportions, there is a tendency to make a more abrupt bend downwards at such a point.

3. That if the boom be calculated upon as jointed, when in fact it is stiff, the result is twofold. If a moveable load come on

the bridge, say 60′0 long at a ton to the foot, then the stiffness of the boom will spread it to, say, 80′0; and thus make 80 feet the virtual length of the 60 ton load, by the time its effect reaches the lattice. It will thus put the lattice-bars in a more easy condition, than if they had to bear the load, as they are *calculated* to do, on the 60′0 length only. Add to this, that where our practice in workmanship obliges us to make a sudden considerable alteration (say 20 per cent.) in the strength of the lattice-bars, we may trust that the stiffness of the booms will help to preserve evenness of strain, which might otherwise be endangered.

The rivetting together of the lattice, then, can never add to the longitudinal strain upon it; but, by calling into play the resistance of the bars to curvature, adds to the stiffness of the bridge. The stiffness of the boom has no other effect than to disperse any temporary derangement of the calculated strains.

CHAPTER VI.

A PLATE-WEBBED GIRDER.

THIS girder, fig. 18 and Pl. I. 3, consists of two end pillars, supporting the weight of the girder and load, which is brought upon them by the web; which web is of plate iron rivetted by regular rows of rivets to the pillars. The web is kept stretched by the upper and lower booms, to which it is also rivetted continuously; the joints of the web are butt joints; made, for convenience, at equal distances apart, and extending vertically, if possible, from the top to the bottom of the web; thus all the bearing of the web plates, at every margin, is upon rivets. The booms are also of plate iron, rivetted together throughout their whole length.

We may pass, in theory, from a lattice to a plate web, as follows:

If we supposed a lattice-girder, with its bars at a horizontal distance equal to that between the rivets which attach the plate-web to the booms (4″ for instance), we should *then* be able—*first*, to calculate closely the maximum strains brought upon each rivet by the two lattice-bars attached by it to the boom; and, consequently, the resultant of those strains.

If we *next* suppose every bar of this lattice, which slopes (say) down from left to right, to be widened out to $4'' \times \frac{1}{\sqrt{2}}$ (i. e. about 3″) width, and made proportionately thin; then the sides of two contiguous bars will be in contact; and we may further imagine them joined so as to form a web-plate. They will form a plate strained by parallel strains down from left to right, at an angle of 45° with the horizon; the strains increasing towards the pier and diminishing towards the centre. We may *now* suppose the other lattice-bars, which crossed the former at right angles, to be removed, for the strains thus superimposed on the new web-plate will be at right angles to, and therefore will not interfere with, those already acting in it: and it only remains to make the thickness of the plate, and the diameter of each rivet connecting it to the boom, so large, as to have together a bearing surface sufficient to pass the resultant of the strains, which, as above, would have been brought upon each rivet in the lattice-girder, from the new web-plate to the rivet, to be by it transferred to the boom.

This reasoning leads us to notice the following peculiarities in which a plate-girder differs from a lattice-girder; the reason of them is manifest.

1. The theoretical weight of a plate-web, that is, the net section required by theory, is only half that required in a lattice-girder.

2. While in a plate you can hardly hope to secure a bearing on its rivets of above $\frac{1}{4}$ or $\frac{5}{16}$ of its gross section along the line of rivets; in a lattice, on the other hand, a bearing can as readily be obtained equivalent to from a half or $\frac{5}{8}$ (in the ties), to from $\frac{5}{8}$ to the whole (in struts), of the section of each bar.

3. In a plate-girder, since the web forms a continuous rigid vertical communication between the top and bottom booms; it is, therefore, theoretically immaterial whether the load be placed on the top or bottom boom, or any part of the web.

4. There is an extra weight in the plate-web due to covers and stiffeners; and there is more rivetting.

5. The plate-web resists curvature of the booms, incomparably more than the lattice-web. If the plate were already taxed to its full power, as a supporting web, the curvature caused by deflection would probably split it: but (see 2) the plate-web is generally 4 times above its required strength.

LEMMA IX. *To find the maximum strain upon any point of a plate-web, acting purely as web.*

Let *ABCD*, fig. 19, represent a plate-girder, *PQ* a section of the web, at a distance x feet from the centre line, on which the strain is required.

The only strain upon the web at *PQ*, which we have now to consider, is the having to *support* the vertical strain brought upon it.

Firstly. Suppose the girder uniformly loaded throughout; whether at top or bottom is immaterial in a plate-girder, as the plate makes a continuous rigid vertical communication between the top and bottom, and divides the weight placed on either top or bottom evenly along its whole depth. Let the load be W tons per foot run: consider why the part *BQPD* does not ascend bodily; it is because

$$\text{The pressure at } B = \begin{cases} \text{the downward action of the weight of } BDPQ \\ + \text{ the downward pressure of } AQPC \text{ on } PQ. \end{cases}$$

$$\text{The pressure at } B = \frac{lW}{2}.$$

$$\text{The weight of } BDPQ = \left(\frac{l}{2} - x\right)W;$$

$\therefore$ the downward strain on *PQ*

$$= \text{the pressure at } B - \text{the weight of } BDPQ$$

$$= \frac{l}{2}W - \left(\frac{l}{2} - x\right)W$$

$$= Wx \quad \text{.............................. (1).}$$

Secondly. For the maximum strain that can be produced by a load; we see at once that it will be when there is no weight to be subtracted from the pressure at B: for by taking weight off any part of *BDPQ*, though we certainly diminish the pressure at B by a portion of it, yet we prevent the subtraction of the whole of it. With the weight then extending over $ACPQ$ we get

the strain on PQ = pressure at B

$$= \left(\frac{l}{2}+x\right)W' \times \frac{\frac{1}{2}\left(\frac{l}{2}+x\right)}{l}$$

$$= \left(\frac{l}{2}+x\right)^2 \frac{W'}{2l} \ldots\ldots\ldots\ldots\ldots (2).$$

And generally, the maximum strain at any distance x feet from centre, and including both loads,

$$= \left(\frac{l}{2}+x\right)^2 \frac{W'}{2l} + Wx \ldots\ldots\ldots (3).$$

Cor. At the pillar the maximum strain is greatest, and

$$= \frac{1}{2}\frac{W'}{l}.$$

At the centre the maximum strain is least, and

$$= \left(\frac{l}{2}\right)^2 \frac{W'}{2l} = \frac{1}{8}W'l.$$

We will begin by a large example of a plate-girder. Plate I.

Proposition IV. A.

To construct a wrought iron plate-webbed girder, of a clear span of 200′0, *and a vertical depth of about* $\frac{1}{12}$ *of that span: given*

weight of bridge and roadway (2 *girders*) *approximately* $1\frac{1}{2}$ *ton per ft. run,*

.... *live load* $1\frac{1}{2}$ ....

Plate I. Make the centres of the booms 15′0″ apart, since 16′8″ is to be about the depth of the girder. Make each girder a box-girder, of the section fig. 4, with two webs; and allow 5′0, on each side of the span, to the centre of bearings.

Then the weight of the dead *or* live loads per foot run per web

$$=\frac{3}{2}\times\frac{1}{4}\text{ tons}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(I.)}$$

The total weight of both loads is, as affecting

$$\text{each boom} = \frac{3}{2}\text{ tons}\times 2\text{ no.}\times 210'\div 2 = 315\text{ tons}\ldots\text{(II.)}$$

$$\text{each web} = \quad\text{half this, viz.}\quad = 157\tfrac{1}{2}\text{ tons}.\text{(III.)}$$

The only points of interest before beginning to practically design the plate-girder, are,

1. The greatest strain on either boom,
2. the general strain on either boom,
3. the greatest strain to which any part of the web is liable,
4. the general maximum strain to which any part of the web is liable,
5. the least maximum strain to which any part of the web is liable.

1. The maximum strain on a boom is at E and F, under a full load of 315 tons distributed, fig. 20,

$$= \text{that due to } \frac{315}{2}\text{ tons collected at the centre } F.$$

And $\frac{315}{2}$ tons at F would produce $\frac{315}{4}$ tons pressure on A or B;

$$\therefore\ \text{tension at } E\times EF = \frac{315}{4}\text{ tons (at } A)\times CF;$$

∴ strains on E and F, which are equal,

$$=\frac{315}{4}\text{ tons}\times\frac{105'}{15'}$$

$$=\frac{315}{4}\times 7\text{ tons}=\frac{2205}{4}\text{ tons}$$

$$=551\tfrac{1}{4}\text{ tons}\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

and requires 122.5 inches of metal.

2. Our general formula for strain on the boom at a distance x from the centre point, viz.

$$T=P=\frac{W}{2dl}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}+x\right)$$

becomes by proper substitution

$$T=P=\frac{315}{2\times 15\times 210}(105-x)(105+x)$$

$$=\frac{1}{20}(105-x)(105+x)\text{ tons}\ldots\ldots\ldots\ldots\ldots(2).$$

In so large a bridge it will be useful to get at once the strain at points 8′0 apart along the booms. (It will be seen hereafter that 8′0 will be the length of the plates, and therefore it is chosen in preference to any other distance).

TABLE I.

		Tons strain.	sq. in. required.
At the centre we have, from above		$551\frac{1}{4}$ or	$122\frac{1}{2}$
8′0 from centre	$T=\frac{1}{20}\times 97\times 113=548$		$121\frac{3}{4}$
16′0	$T=\frac{1}{20}\times 89\times 121=538.5$		$119\frac{3}{4}$
24′0	$T=\frac{1}{20}\times 81\times 129=522.5$		$116\frac{1}{8}$

			Tons strain.	sq. in. required.
32′0	from centre	$T = \frac{1}{20} \times 73 \times 137 = 500$	or	$111\frac{1}{8}$
40′0	....	$T = \frac{1}{20} \times 65 \times 145 = 471\frac{1}{4}$		$104\frac{3}{4}$
48′0	....	$T = \frac{1}{20} \times 57 \times 153 = 436$		97
56′0	....	$T = \frac{1}{20} \times 49 \times 161 = 394.5$		$87\frac{5}{8}$
64′0	....	$T = \frac{1}{20} \times 41 \times 169 = 346.5$		77
72′0	....	$T = \frac{1}{20} \times 33 \times 177 = 292$		65
80′0	....	$T = \frac{1}{20} \times 25 \times 185 = 231$		$51\frac{1}{2}$
88′0	....	$T = \frac{1}{20} \times 17 \times 193 = 164$		$36\frac{1}{2}$
96′0	....	$T = \frac{1}{20} \times 9 \times 201 = 90.5$		$20\frac{1}{8}$
104′0	....	$T = \frac{1}{20} \times 1 \times 209 = 10.5$		$2\frac{3}{8}$

·3. The maximum strain on the web is at its attachment to the pillar, when fully loaded, *i.e.* along *AC* or *BD*, in fig. 20; and, Lemma IX. Cor. and (III.),

$$= 157\tfrac{1}{2} \times \frac{1}{2} \text{ on each end, 15′0 high, of each web}$$

$$= \frac{315}{4 \times 15'} \text{ tons per foot run of end}$$

$$= \frac{21}{4} \text{ tons} = 5\tfrac{1}{4} \text{ tons} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3).}$$

4. And generally the maximum strain on the web x feet distance from the centre, (Lemma IX.)

$$= \left(\frac{l}{2} + x\right)^2 \frac{W'}{2l} + Wx, \text{ where } W = \frac{3}{8} = W' \text{ from (I.)}$$

$$= (105 + x)^2 \frac{3}{16 \times 210} + \frac{3}{8} x$$

$$= (105 + x)^2 \frac{1}{16 \times 70} + \frac{3}{8}(105 + x) - \frac{315}{8} \ldots\ldots (4).$$

5. The least maximum strain $= \frac{1}{8} \times 210 \times \frac{3}{8}$

$$= \frac{315}{8} \text{ tons} \times \frac{1}{4}$$

$$= \frac{315}{15 \times 8 \times 4} \text{ per vertical foot run of web}$$

$$= \frac{21}{32}$$

$$= \frac{5}{8} \text{ and } \frac{1}{32} \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

This completes the calculation.

PROPOSITION IV. B.

To construct a wrought iron plate-girder of a span between bearings of 40'0, *and a vertical depth of about* $\frac{1}{12}$ *of that span; given*

weight of bridge and roadway, on 2 girders,

12 *cwt. per ft. run,*

.... *live load* 24 ....

Total ..... 36 ....

This gives the total load on one girder to be 18 cwt. per ft. run,

$$= 18 \times 40 \times \frac{1}{20} \text{ tons} = 36 \text{ tons.}$$

Suppose the girder to be 3′6 deep between the centre of gravity of the booms; with a single plate-web, Fig. 18.

In this smaller girder the points of interest will be:

1. The maximum strain on a boom, $\left(\frac{Wl}{8d}\right)$

$$= \frac{36 \times 40 \times 2}{8 \times 7} = \frac{360}{7}$$

$$= 51\tfrac{3}{7} \text{ tons and requires } 11\tfrac{3}{8}'' \text{ section.}$$

2. The general expression for the strain on a boom

$$\left\{\frac{W}{2dl}\left(\frac{l}{2} - x\right)\left(\frac{l}{2} + x\right)\right\} = \frac{36}{7 \times 40}(20 - x)(20 + x)$$

$$= \frac{9}{70}(20 - x)(20 + x).$$

And the differential coefficient of this

$$= \frac{9}{35}x,$$

or multiplying by $\frac{2}{9}$, this shews us that the decrease of section required for a boom at x feet from the centre is $\frac{2}{35}x$ square inches per foot run. A fact useful to keep in mind in jointing the girder.

Lemmas X. and XI. treat of the relation between a boom, as theoretically proportioned (and whose section is therefore at each point in a fixed proportion to the strain caused by an uniform load on the bridge); and a boom whose section should be imagined uniform, and equal to the *average* section of the theoretical boom. I insert them here as being of continual service, in the practical construction of girders.

LEMMA X. *To find the average section of a Theoretically Proportioned Boom, from its central section.*

Take the same girder as in Lemma VII. If κ be the weight per foot run of a section of iron capable of resisting a strain of 1 ton (either compression or tension as the case may be); then, Lemma VII, (1).

The strain on a boom at distance x from centre

$$= \frac{W}{2dl}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\},$$

and the weight per foot run, at the same point, therefore,

$$= \kappa\frac{W}{2dl}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\};$$

therefore the total weight of the boom

$$= 2\kappa\int_0^{\frac{l}{2}}\frac{W}{2dl}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}dx$$

$$= \kappa\frac{W}{dl}\left\{\left(\frac{l}{2}\right)^2 x - \frac{1}{3}x^3\right\} + C$$

$$= \kappa\frac{Wl^2}{d}\left(\frac{1}{8} - \frac{1}{3}\times\frac{1}{8}\right)$$

$$= \kappa\frac{Wl^2}{12d}.$$

Therefore the average weight of the boom per foot run

$$= \kappa\frac{Wl}{12d} \text{..........................(1).}$$

But the strain of the boom at the centre is, Lemma VII. Cor.

$$\frac{Wl}{8d},$$

and therefore weighs per foot run

$$\kappa\frac{Wl}{8d}.$$

Hence the average weight of the boom per foot run : the central weight

$$=\kappa\frac{Wl}{12}d : \kappa\frac{Wl}{8d}$$

$$=2 : 3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Or the average section $=\frac{2}{3}$ the central section.

COR. Hence, if the weight of the boom be B tons,

then its average weight will be $\frac{B}{l}$ tons per foot,

and its central weight therefore $=\frac{3}{2}\frac{B}{l}$ ....

LEMMA XI. *To find the mechanical effect at the centre of the Span, of a Theoretical Boom whose* average *weight per foot run is given, in terms of its effect if of the same average weight but uniformly distributed.*

Taking the same girder. The weight of the whole boom being B, the strain due to the uniform boom, weight B, at the centre $=\frac{Bl}{8d}$.

Now the weight of the theoretical boom at x feet from the centre will, Lemma VII, (1),

$$=X\times\left\{\left(\frac{l}{2}\right)^2-x^2\right\},$$

X some constant;

where if $x=0$, $$X \times \left(\frac{l}{2}\right)^2 = \frac{3}{2}\frac{B}{l},$$

viz. its central weight per foot, Lemma x. cor.;

and therefore $$X = \frac{6B}{l^3};$$

or the weight of the theoretical boom at any distance x,

$$= \frac{6B}{l^3}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}.$$

Now the strain caused on the booms at the centre of the span by the weight of an element δx feet run of this,

$$= \frac{6B}{l^3}\left\{\left(\frac{l}{2}\right)^2 - x^2\right\}\delta x \times \frac{\frac{l}{2} - x}{l} \times \frac{l}{2d},$$

$$= \frac{3B}{l^3 d}\left\{\left(\frac{l}{2}\right)^3 - \left(\frac{l}{2}\right)^2 x - \frac{l}{2}x^2 + x^3\right\}\delta x.$$

Integrating this from

$$x = 0 \text{ to } x = \frac{l}{2},$$

and doubling it, we get the strain upon the centre of a boom due to the weight of a whole theoretical boom,

$$= 2 \times \frac{3B}{l^3 d}\left\{\left(\frac{l}{2}\right)^4 - \frac{1}{2}\left(\frac{l}{2}\right)^4 - \frac{1}{3}\left(\frac{l}{2}\right)^4 + \frac{1}{4}\left(\frac{l}{2}\right)^4\right\},$$

$$= \frac{5Bl}{32d} \text{..} (1),$$

which bears to the strain it would cause if uniform the ratio of

$$\frac{5Bl}{32d} : \frac{Bl}{8d},$$

$$= 5 : 4;$$

therefore the effect, at the centre of the span, of the theoretical boom is $\frac{5}{4}$ that of the same weight spread uniformly...(2).

Example of a kind of investigation often useful.

Given that the boom of a girder is to be divided into three equal parts; each part to be made of the same scantling throughout. Required the most economical position for the joints.

Let x be the distance of the joints from the centre of the girder;

A the weight of the central, B of the side, portions of boom per foot run.

Then the weight A = that due to the central strain,

$$=\kappa\frac{Wl}{8d};$$

therefore the weight of the central portion of boom,

$$=\kappa\frac{Wl}{8d}\times 2x.$$

So, the side portions have to be strong enough to sustain the strain at x; and therefore

$$B=\kappa\frac{W}{2dl}\left(\frac{l^2}{4}-x^2\right),$$

and their total weight,

$$2B\times\left(\frac{l}{2}-x\right)=\kappa\frac{W}{2dl}\left(\frac{l^2}{u}-x^2\right)\times 2\left(\frac{l}{2}-x\right).$$

And we must have the whole weight a minimum; therefore dividing out $\kappa\frac{W}{d}$,

$$\frac{1}{4}lx+\frac{1}{8l}(l^3-2l^2x-4lx^2+8x^3)\text{ is a minimum};$$

or $$\frac{1}{8}l^2-\frac{1}{2}x^2+\frac{1}{l}x^3\text{ is a minimum};$$

therefore differentiating,

$$-x+\frac{3}{l}x^2=0;$$

$$\therefore\ x=\frac{1}{3}l.$$

Therefore the joints should, so far, be one-third of the span distant from the centre. In practice this might make the centre plate too heavy; and it should be made as long as is possible without extra cost.

A great majority of iron girders do not require their calculation to be carried to a second approximation: but the first, which treats their weight as *uniform*, suffices; either since their whole weight is small, or because any excess in one part due (say to a heavy section of boom) is balanced in another (as by a heavier web, or bracings). In such cases their practical construction is made upon the first approximation, corrected a little perhaps by some artifices which may present themselves. The web plate girder, however, on which we are here treating, of 200′0 span, would *require* a second approximation; owing to its size, and the consequent great weights of the booms near their centres. My reasons for inserting here its practical construction, without a second approximation, are, 1. It is *necessary* for the reader to see one girder constructed on the first approximation in order that he may realize the necessity of a second. 2. It is necessary to give him fixed ideas upon practical construction, before he can usefully consider a second approximation. 3. The *principle* and *method* of proportioning the plates to the strain, is as perfectly shown if the strains are inaccurate, as if they were formally accurate. 4. The 2nd approximation of plate and other girders can be inferred from that of the lattice-girder hereafter.

Practical construction of the plate-webbed Girder of Prop. IV. A. Plate I.

As best for illustration, and not disadvantageous mechanically, I select a rather cramped section for the booms, Plate I. fig. 1. This it will be seen will cause us to use large angle irons, involving some extra expense.

The maximum strain on the booms, see Prop. IV. A, requires a net section of 122.5 inches. This may be thus obtained, fig. 1, where (see fig. 4 for distinctive letters)

		net.		gross.	lbs. per ft.
*Pls. a,	$30'' \times 1''$ give section	$24\frac{3}{4}''$	and	$30''$	$= 100$
c,	$30'' \times 1''$ ...	$24\frac{3}{4}$	...	$30''$	$= 100$
Bars b,	$9'' \times \frac{7''}{8}$ each ...	$12\frac{5}{8}$	...	$15\frac{3}{4}$	$= 52\frac{1}{2}$
∠is. A, B, C, D, E,	$4\frac{1}{2}'' \times 4\frac{1}{2}'' \times \frac{11''}{16}$ ...	$48\frac{3}{4}$	...	$60\frac{1}{2}$	$= 202$
∠is. F,	$5\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{11''}{16}$ ...	11	...	12	$= 40\frac{1}{4}$
	Totals	$121\frac{7}{8}$ sq. in.			$494\frac{3}{4}$
	5 per cent., covers and rivets, say				$22\frac{3}{4}$
	Total weight per foot,				$517\frac{1}{2}$

The two booms will therefore weigh 1035 lbs. per foot run; and the web will be two plates 14′0 deep, averaging together $\frac{1}{2}''$ thick, say; and will therefore weigh 280 lbs. per foot run per girder: making, in all, the girder to weigh 1315 lbs., or 11 cwt. 3 qrs. per foot run at the centre.

Now we have estimated the dead load per girder at 15 cwt. per foot run; and the roadway therefore must weigh but 6 cwt. 2 qr. per foot run at the centre of the bridge, in order to keep the whole dead weight below that assumed. This opens a question which must be here settled, regarding the weight of the roadway; also that of any bracing between the two girders, and of stiffening and diaphragms within the web of each girder.

This should be entered into in the following way.

We see from above that we may reckon a boom to weigh about $\frac{13}{14}$ lb. per foot, for every ton strain upon it. We can see at once then from Table I. how much per foot the booms of the girder will weigh approximately at every 8′0 from the centre.

* This table is supposed to be an approximation, merely: its details are gone into further on.

Add to each of these weights, the average weight per foot of the double web, with its covers, diaphragms, &c. at each 8′0 distance; also add the weight per foot of the roadway, and consider where the weight of any bracing used to connect the two girders will be laid on the bridge; this last may probably be divided by 8, and placed to the credit of the nearest 8′0 distance, as a weight of so much per foot for that 8′0.

When this is done, you know the dead weight with great accuracy for every part of the girder; and must, beginning again, form a new calculation of the maximum strains brought upon every part of the booms and web, under these dead weights, and the live load previously given.

I must avoid a detailed description of a roadway, bracing, &c. which is quite unnecessary in this work; and wish here merely to shew how to construct practically a girder capable of resisting given calculated strains.

I shall therefore suppose the strains already ascertained on the data given in Prop. IV. to be settled as those for which we have to provide in our girder. Plate I. is a drawing shewing every part of the girder, separate from all accessories.

I. *To investigate the thickness of plate necessary for the web.*

The strain on the web-plate at its attachment to the pillars is $5\frac{1}{4}$ tons per foot run, and therefore requires (page 18) 1.05 inches bearing per foot run.

With the double rivetting shewn in the web of fig. 2, Pl. I. we should have the line of bearing per foot run of joint, which would contain 6 rivets,

$$\frac{5''}{8} \times 6 \text{ no.} = \frac{15}{4} \text{ inches lineal};$$

which will therefore give the required bearing area in a plate

$$1.05 \times \frac{4}{15} = \frac{42}{15} = .28'' \text{ thick};$$

and $\therefore \frac{5}{16}$ plate, which is .3″ thick, will suffice.

We now want to find where we may drop the web-plates to $\frac{1}{4}''$ thick, which will only give 6 no. $\times \frac{5''}{8} \times \frac{1''}{4}$ bearing per foot run

$$= \frac{15''}{16} \text{ bearing per foot run,}$$

$$= \frac{15''}{16} \times 5 \text{ tons} \times 15'0 \text{ tons per web-plate virtually } 15'0 \text{ deep.}$$

Let x feet be the distance from the centre at which this is the strain on the web; then the maximum strain to which the web is liable at x must be equal to, or under, the above; therefore, Prop. IV. A. equation (4),

$$\frac{15}{16} \times 5 \times 15 = (105 + x)^2 \frac{1}{16 \times 70} + \frac{3}{8}(105 + x) - \frac{315}{8},$$

multiplying by 16×70 and transposing;

$$\therefore (105 + x)^2 + 420(105 + x) + (210)^2 = (15)^2 \times 5 \times 70 + 420 \times 105 + (210)^2$$

$$= 25(9 \times 5 \times 70 + 2 \times 42 \times 42)$$

$$= 25 \times 9 \times 7(50 + 4 \times 14)$$

$$= (15)^2 \times 7 \times 106 = 15^2 \times 742;$$

$$\therefore 105 + x = 15\sqrt{(742)} - 210$$

$$= 15 \times 27.2\left(1 + \frac{1}{739}\right) - 210$$

$$= 408\tfrac{1}{2} - 210, \text{ nearly, } = 198\tfrac{1}{2},$$

$$\text{and } x = 93\tfrac{1}{2} \text{ feet.}$$

Now if we use plates 2′0 broad, and commence 2′0 beyond the centre line of the bearings of the girder on the pier, fig. 3; we shall have 107 plates in each web, or 53 on each side of the centre plate. Number the plates, as in the figure 3, from each end to the centre. Then we must, in practice, make the alteration of plate at the joint next within the distance x; *i.e.* at 93′0 distance from the centre, which is 46 plates from the centre one, and therefore 7 from the end;

$$\therefore \text{ plates Nos. } 1\text{—}7 \text{ must be } \frac{5''}{16} \text{ thick.}$$

The web-plates may drop to $\frac{3''}{16}$, giving 6 no. $\times \frac{5''}{8} \times \frac{3''}{16}$ bearing per foot run; *i.e.* $\frac{5 \times 9}{8 \times 8}$ bearing per foot run, and $\frac{5 \times 9 \times 5 \times 15}{8 \times 8}$ tons per web-plate 15′0 deep at a distance x, if

$$\frac{25 \times 9 \times 15}{8 \times 8} = (105 + x)^2 \frac{1}{16 \times 70} + \frac{3}{8}(105 + x) - \frac{315}{8};$$

and ∴ if

$$\begin{aligned}(105 + x)^2 + 420\,(105 + x) + (210)^2 &= \frac{25 \times 9 \times 1050}{4} + 210 \times 210 + (210)^2 \\ &= \frac{1050}{4}(25 \times 9 + 8 \times 21 \times 2) \\ &= \frac{350 \times 9}{4}(25 \times 3 + 8 \times 14) \\ &= \left(\frac{3}{2}\right)^2 \times 350\,(75 + 112) \\ &= \left(\frac{3}{2}\right)^2 \times 350 \times 187 \\ &= \left(\frac{3}{2}\right)^2 \times 65450;\end{aligned}$$

$$\therefore\ 105 + x = \frac{3}{2} \times 255.83 - 210 = 383\tfrac{3}{4} - 210$$

$$= 173\tfrac{3}{4},$$

$$\text{and } x = 68\tfrac{3}{4}.$$

And the change can be made at the joint 67′0 from the centre, *i.e.* 33 plates from the centre one, and $53 - 33 = 20$ from the end;

$$\therefore \text{ plates Nos. 8—20 must be } \tfrac{1}{4}'' \text{ thick.}$$

Lighter joints may be used (were this advisable on other accounts), giving a bearing with single rivets at 3″ centres, fig. 1,

$$\text{of } 4 \text{ no.} \times \frac{5}{8} \times \frac{3}{16} \text{ sq. in. per foot run,}$$

$$\text{or } \frac{15}{32} \times 5 \times 15 \text{ tons per whole depth,}$$

at a distance x feet from the centre, if

$$\frac{15}{32} \times 5 \times 15 = (105 + x)^2 \frac{1}{16 \times 70} + \frac{3}{8}(105 + x) - \frac{315}{8},$$

$$\text{or } (105 + x)^2 + 420\,(105 + x) + (210)^2 = \tfrac{1}{2}(15)^2 \times 5 \times 70 + 210 \times 210 + (210)^2$$

$$= (15)^2 \left(\frac{5}{2} \times 70 + 14 \times 14 \times 2\right)$$

$$= (15)^2 \times 7\,(25 + 4 \times 14)$$

$$= (15)^2 \times 7 \times 81$$

$$= (135)^2 \times 7;$$

$$\text{and } \therefore \text{ if } 105 + x = 135 \times 2.646 - 210 = 357 - 210$$

$$= 147;$$

$$\text{and } \therefore\ x = 42, \text{ say 41 feet off,}$$

i.e. 20 plates distant from the centre, and therefore 33 from the end.

Therefore the joints within plate No. 33 may be single rivetted.

The top and bottom of each plate of course needs the same bearing area per foot run as the sides, and may therefore be rivetted to the same pattern as the sides.

But since double rivetting will be inconvenient, in the attachment of the web to the booms, we shall discontinue it as soon as possible.

The double rivetting of the web may be discontinued in the top and bottom angle irons connecting it to the booms, and the ordinary pattern of boom-rivetting substituted, viz. $\frac{7}{8}''$ rivets at 5 to the foot in a single row, giving a bearing

of $5 \text{ no.} \times \frac{7''}{8} \times \frac{3''}{16}$ sq. in. per foot run,

or, $\frac{15}{16} \times \frac{7}{8} \times 15' \times 5$ tons per whole depth at a distance x feet from the centre if

$$\frac{15 \times 7 \times 15 \times 5}{16 \times 8} = (105 + x)^2 \frac{1}{16 \times 70} + \frac{3}{8}(105 + x) - \frac{315}{8},$$

$$\text{or } (105 + x)^2 + 420\,(105 + x) + (210)^2 = \frac{1}{8} \times 70 \times (15)^2 \times 7 \times 5 + (210)^2 + (210)^2$$

$$= \frac{1}{4} \times 7^2 \times 3^2 \times 5^2\,(25 + 32);$$

$$\therefore\ 105 + x = \frac{1}{2} \times 7 \times 3 \times 5 \sqrt{(57)} - 210 = \frac{1}{2} \times 105 \times 7.55 - 210$$

$$= 105\,(3.775 - 2) = 105 \times 1.775$$

$$= 186.35;$$

and $\therefore x = 81$,

i.e. 40 plates from the centre, and therefore 13 from the end.

The change will however be made 17 plates from the end, since that is next before the angle iron C thickens, so as to make single rivetting in it necessary.

II. For the booms we will suppose that the general section, figure (4), is selected for trial.

It has the following advantages and capabilities:

1. It requires no bending of plates, and consequent expense and risk of weakening from careless bending or from neglect in annealing, nor any bending of angle iron.

2. It can be made very heavy, if all the plates and angle irons are thickened; and yet very light, if they be all fined down; as will be seen hereafter.

3. It can be put together without difficulty, in that the men can get to both sides of every plate to rivet it, whether at the middle of the plates or at the joints, without having to use long handles for holding up against the rivetters*.

(a) *In making the bottom.*

1. The plates a_1, a_2 will be laid down upon chocks of wood, set across the longitudinal balk of timber on which the girder is to be built; and the cover-plates a_0 bolted on, together with the angle irons A, D, and F, the plates b, and wrappers G. The horizontal rivets through AbD will be first rivetted, and then the others; the chocks being readily moved to allow the rivetting, and finally removed to allow the plate to descend to the balk; or, in any case, finally fixed; so as to give the plates of the girder an uniform level, or cambre.

2. The angle irons B and E will be attached to b.

* I had intended to keep D unattached to b and A by the horizontal rivet; in the present arrangement it must be attached, and the rivets β ϵ must be rivetted by means of a holding-up iron with a long handle, after the plate a, &c. is bolted into its place. The first method is the best.

3. The plates c_0, c_1, c_2, d, being rivetted together by κ and λ upon chocks, and great care being taken to keep their rivet holes to fit those of B and E, will be let down upon B and E, and the rivets ι and μ driven; C and I are then bolted on, and the rivets θ and ν also driven.

4. The web-plates are then bolted up, with their covers and diagonal plates complete; and then rivetted to C and together, the holder up being between them.

(b) *In putting on the Top*

just a reverse order is followed; C is bolted on to the web first; then c and d jointed, so that the rivet holes shall agree with those of C; then κ and λ are rivetted, and the plates c raised off C while BEb are set on and $\eta\,\iota\,\mu$ rivetted; the whole is then let down upon C and θ, ν rivetted: and so on.

The greatest section required is 122½ sq. inches net.

A trial will soon shew that the only way this can be accomplished is by placing angle irons upon the plates forming the *sides*, as well as at the *corners* of the box*; and by using very heavy angle irons.

This difficulty at once raises the question, whether it would not be well to alter the section of the boom; since

1. Such a section might require too much care in templates, punching and putting together.

2. The joints will require more skill in designing than can perhaps be relied upon.

3. Such heavy angle irons may, at many works, involve too much extra cost.

4. It may be feared that the full strength of some of the extra angle irons will never come into play.

* A *box* in wrought iron plate work, is an arrangement of plates whose cross section shews two plates parallel to one another, and at right angles to two more; all united by angle irons at each corner where the plates (produced if necessary) meet. In this girder both booms, web, and end pillars have a box section.

We will suppose these objections, of which the third is most serious, overruled, and the section still adhered to.

To arrive at the strength available, in section figure 4, with different thicknesses of plate, we must consider the *effect of the rivet-holes in weakening* the plates and angle irons contained in that section.

The line of fracture of each of the two horizontal plates is

$$30'' - 6 \text{ no.} \times \frac{7''}{8} = 24\tfrac{3}{4}'',$$

the line of fracture of each of the two vertical plates is

$$9'' - 2 \text{ no.} \times \frac{7''}{8} = 7\tfrac{1}{4}''.$$

The amount virtually cut out of each angle iron, if double rivetted, will be seen by drawing a development of one; which will be $8\frac{3}{4}''$ broad, with $\frac{7}{8}''$ holes $2''$ from the edge, and 5 to the foot, (as they are in Pl. I. figs. 7 and 8); and will be nearly the same as if the two rivet-holes were exactly opposite one another: in which case they would cut out $1\frac{3}{4}''$ breadth of the angle iron.

More accurately,

the line of fracture of the ∠ i. is $4'' + \sqrt{\{(8\frac{3}{4} - 4)^2 + (1\,.\,2)^2\}} - 1\frac{3}{4}''$

$$= 4'' + \sqrt{(22.56 + 1.44)} - 1\tfrac{3}{4}$$
$$= 4'' + 5'' - 1\tfrac{3}{4} = 7\tfrac{1}{4}'';$$

therefore, throughout, in double-rivetted angle iron we may suppose the rivets to cut out $1\frac{1}{2}''$ breadth; leaving, virtually, a $4'' \times 4'' \times \frac{11''}{16}$ angle iron. (It must be noticed that the lighter angle iron will measure so much less in each limb, as it is thinner than the heavy ones.)

This more accurate refinement we had better discard; since, in so thick angle irons, the punching will be rather coarse, and it is well to allow for some riming either in the angle irons or plates; since riming is very likely to be resorted to a little, especially at the joints, where are the most critical points.

The actual area of an angle iron $4\frac{3}{4}'' \times 4\frac{3}{4}'' \times \frac{11''}{16}$ will therefore be = 6.09 sq. inches (see *Table of* ∠ *i.* at end of book), if rivetted in both flanges then its virtual size becomes

$$3\tfrac{7}{8}'' \times 3\tfrac{7}{8}'' \times \frac{11''}{16} = 4.853, \text{ say } 4\tfrac{7}{8},$$

if rivetted in one flange, then its virtual size becomes

$$4\tfrac{5}{16} \times 4\tfrac{5}{16} \times \frac{11}{16} = 5.455, \text{ say } 5\tfrac{1}{2}.$$

Hence, applying this to our section figure 2, we find that

					sq. inches.
No. 2	plates	a_1, a_2	give a breaking area of	$24\frac{3}{4} \times \frac{1''}{2}$	$= 24\frac{3}{4}$
2	...	c_1, c_2	....	same	$= 24\frac{3}{4}$
2	bars	b	....	$7\frac{1}{4} \times \frac{7''}{8}$	$= 12\frac{5}{8}''$
10	∠i.	$A, B, C, D, E,$	double rivetted,	$4\frac{3}{4} \times 4\frac{3}{4} \times \frac{11''}{16}$	$= 48\frac{3}{4}$
2	...	F	single rivetted,	$6'' \times 3\frac{1}{2}'' \times \frac{11''}{16}$	$= 11$
				Total	$121\frac{7}{8}$

Which is $\frac{5}{8}''$ less than we want, and therefore quite safe.

We have also, shewn in the same figure, for the purpose of jointing,

No. 1 cover a_0 to the top plate 30″ broad,
1 .... c_0 .... bottom .... 20″ .....
1 bar d .... 9″ .....

And wrappers *G, H, I* may be used, which will be 4″ × 4″.

And it becomes necessary, next, to ascertain what *length of each plate will be required to be covered* for a joint. This will be of course independent of their thickness, provided the plate covered, and plate covering, are of the same relative thickness.

The rivets are in general centred at 5 to the two feet. But in a cover plate or wrapper, intermediate rivets will be inserted; so that the rivets will then be 5 to the foot.

The 30″ plates have a line of fracture of $24\frac{3}{4}''$ lineal;

equivalent to a bearing of $24\frac{3}{4} \times \frac{9}{10}$, or $22\frac{1}{2}''$ ...

and requiring $22\frac{1}{2} \times \frac{8}{7}$, or $25\frac{3}{4}$ no. of $\frac{7''}{8}$ rivets.

Now the rivets cannot be put more than 6 in a row, as in the figs. 7 and 8; and as the plates require more than 4 rows of rivets, we must cover 5 rows; *i.e.* a total of 30 rivets. And the cover will be 1′0 long over the joint.

The double rivetted $\angle$ i. has above 4″.85 breaking area,

$$\text{equivalent to } 4.85 \times \frac{9}{10} = 4.4 \text{ bearing area,}$$

$$\text{which will require } 4.4 \times \frac{16}{11} \times \frac{8}{7} = 7\tfrac{3}{8} \text{ no. } \frac{7''}{8} \text{ rivets in } \frac{11''}{16} \text{ metal,}$$

say, 8 no. rivets;

which, again, requires $4 \times 2''.4 + 1''.2 = 10\frac{7}{8}''$ length covered; since the rivets in the angle iron are alternate.

The single rivetted $\angle$ i. has 5.49 breaking area,

requiring 4.94 bearing ....

$$\text{and } \therefore\ 4.94 \times \frac{16}{11} \times \frac{8}{7} = 8\tfrac{1}{4} \text{ no. rivets,}$$

say, 9 no. rivets,

and $9 \times 2''.4 = 21\frac{5}{8}''$ length covered.

The side bars have $7\frac{1}{4}$ line of fracture,

requiring 6.5 .... bearing;

and $\therefore$ $7\frac{3}{7}$, say, 8 no. rivets,

and $4 \times 2.4 = 9\frac{5}{8}''$ length covered.

The wrappers, taking their development at $7\frac{1}{2}''$ have

$$4'' \times \sqrt{\{(3.5)^2 + (1.2)^2\}} - 2 \times \frac{7}{8} = 2\frac{1}{4} + \sqrt{(12.69)} = 2\frac{1}{4} + 3\frac{1}{2}$$

$$= 5\frac{3}{4}'' \text{ line of fracture};$$

and $\therefore$, at $\frac{9}{16}''$ thick, $\frac{23}{4} \times \frac{9}{16} = 3\frac{1}{4}''$ breaking area,

requiring $\frac{23}{4} \times \frac{8}{7} \times \frac{9}{10} = 6$ no. rivets;

and will take half the strain of a bar *b*, whose section is

$$7\frac{1}{4} \times \frac{7}{8} = 6\frac{3}{8} \text{ sq. in.}$$

The lower cover-plate 20″ broad, $\frac{9}{16}''$ thick, with 4 no. rivet-holes has

$$20'' - 4 \text{ no.} \times \frac{7''}{8} = 16\frac{1}{2} \text{ line of fracture};$$

and $\therefore$ can support 14.85 .... bearing,

equivalent to $14.85 \times \frac{9}{8}$ .... in $\frac{1}{2}''$ plate,

and will take $14.85 \times \frac{9}{8} \times \frac{8}{7} = 19.09$ no. $\frac{7}{8}''$ rivets in $\frac{1}{2}''$ plate,

while it will have $16\frac{1}{2} \times \frac{9}{16} = 9\frac{1}{4}''$ breaking area.

The covering bar *d*, 9″ broad, with 2 no. $\frac{7}{8}''$ rivets, is similar to the other bars *b*; and takes $7\frac{3}{7}$ rivets, and 9″.6 length of cover.

Now as to the length of the plates and angle irons which we had better use.

The 30″ plates $\frac{1}{2}$″ thick, will have 15″ section, and weigh 150 lbs. per yard; and would bear ordinary price up to 8′0 long.

The angle irons have a section of 6.09 inches, and therefore weigh 61 lbs. a yard; and may be rolled 20′0 long, but better 16′0.

It will be noticed that, towards the centre of the boom, the strain continually in-coming from the web is so small, that we need take no precaution against breaking joint immediately over a web joint.

We can now try to arrange the joints for a 16′0 length of the boom: one arrangement is shewn in figures 7 and 8.

It is desirable to make the edge of a cover or wrapper always fall free of a rivet. In Plate I. figs. 7 and 8, this is not accomplished, in one case; for the right-hand end of the wrappers which cover the joint in *b*, falls upon a rivet, whose head, therefore, would interfere with the wrapper, and consequently has to be countersunk, and does not shew in the drawings. *Countersinking* is done by widening out the rivet hole in the outer of the plates, before they are rivetted, so as to give it a conical form (or dove-tail in longitudinal section). The rivet is put in from the back as usual, but the red hot end, instead of being hammered to form a head, is hammered down in the widened hole, so as to make a flat surface with the outer plate, and any excess of metal is pared off with a chisel. The conical form is given to the hole in the outer plate by a drilling machine or by hand.

This countersinking causes extra trouble, and also cuts out more section from the angle iron than we have reckoned upon. In practice the wrapper would be better cut away so much as to allow of its being put in place after the rivet head had been formed, without interfering with it. The cutting away of the angle iron, to allow the head to be countersunk into it, though it might be tolerated if it occurred at the other end of the joint in figure 7 (for there the amount of general section would be excessive), cannot be tolerated as it is.

A good point in the arrangement here shewn is that the plates and angle irons forming one-half of the boom, viz. a_1 a_2 A D_1 are jointed together by the same cover-plate, thus saving much metal: the same of the other portion, viz. c_1 c_2 B C E. The jointing of the first half falls halfway between two joints of the second half; and this arrangement provides that when one half is jointed the other half has generally an excess of metal, (since it was proportioned to the strain at its last joint) and, in any case, the two halves are so firmly connected together by the two short thick bars b, and the 8 angle irons A B D E, as to assist one another, if they be alternately weakened by joints, or otherwise.

The only difference in the rivetting which occurs at a joint, is caused by the *adding* of rivets only; viz. a rivet is, at the joints, placed halfway between the regular ones. The regular rivetting is at 5 to the two feet, at the joint they are 5 to the foot. It has been before observed how important it is to keep the "regular" system of rivets unbroken through the joints; if this be not done, the plates are in great danger of getting punched where the holes are intended to be left out, and, at least, the time of the workmen must be lost in extra caution.

The only alteration in the character of the rivetting is in that which connects the web to the booms, which changes from double to single rivetting, between plates 17 and 18 of the web. This is arranged so as to cause no change in rivetting in any one web-plate, and takes place in the angle iron C just as that angle iron is fined down to $\frac{9''}{16}$ thick, and close by its joint. It might, if found awkward, be obviated by thickening the web-plates near the end of the girder, to give them more hold on the rivets through C, and continuing the $\frac{7''}{8}$ single rivetting.

For jointing the boom (figs. 7 and 8), that member is divided into two parts; one portion consisting of the plates, angle irons, and bars, shewn in fig. 7; viz. $a_1 a_2 b$, ADF, and the other portion as in fig. 8. To take the strain as part of the first portion, whilst its members are successively jointed, we employ the cover-plate a_0, which gives a breaking area equal to that of the largest plate in that portion; and the wrappers G and H to form a cover for the bar b. And to act as part of the second portion, we employ the cover-plate c_0 and bar d, which, at $\frac{9''}{16}$ thickness, have together a breaking area equal to that of four angle irons, and larger than either plate $c_1 c_2$.

For jointing the first portion of the boom, Plate I. figs. 4 and 7. (The reader should try to keep to fig. 4 while reading the following.) We have found (pp. 148–9) that we must have covered, for efficient jointing, in

	Breaking area.	No. of rivets.			
Pls. $a_0 a_1 a_2$	of $12\frac{3}{8}''$	$25\frac{3}{4}$	or say,	30, i.e.	1′0 covered;
Bars b	... $6\frac{5}{16}''$	$7\frac{3}{4}$	......	8 ...	9″.6 ...
∠i. AD	... $4\frac{7}{8}$	$7\frac{3}{8}$	......	8 ...	10″.8 ...
∠i. F	... $5\frac{1}{2}$	$8\frac{1}{4}$	......	9 ...	1′9″.6 ...

Looking at the general section (and considering, *pro tem.*, the two portions 7 and 8 as independent), we will now follow the plates through the joints, which take place in the following order, FAa_1a_2, bDa_1a_2 F &c.

Taking the zero line, from which to measure distances, as in fig. 7.

1. at 0′.2″.4 — the cover a_0 begins, and F can at once begin to transmit strain to a_1 and a_2, since they can pass it through the same, and other rivets into a_0. When the cover has 4 rows of rivets only, it has got enough rivet hold to take the whole strain of both angle irons F; which have

each a breaking area of 5.5 sq. in., and therefore require of $\frac{7''}{8}$ rivets, in $\frac{1''}{2}$ plate only,

$$2 \times \frac{11''}{2} \times \frac{9}{10} \times \frac{8}{7} \times 2 \text{ no.} = 23 \text{ no.}$$

which is under 24, the no. in 4 rows. But we must have F covered more than this; for we want to prevent it transmitting any strain upon a_1 and a_2, except when they have the cover upon them to take some of their own strain, and thus prevent their being overburdened. Each rivet γ through F, bears $\frac{11''}{16}$ on F, and will therefore require more than a $\frac{1''}{2}$ plate to bear against, in order to give its bearing on F full effect: this we see it has; for it may bear if it likes on all 3 plates a_0, a_1, a_2, giving a total length of bearing of $1\frac{1}{2}''$ deep; and a_1 and a_2 will at once throw strain into a_0 by the other rivets $\alpha\beta$ which connect them.

2. at 2′0, or 21.6″ beyond, F has its joint; its full strain has by this time been safely taken up through 9 rivets by the other plates; and the general section will on the whole be under less strain than before reaching the cover; since the cover has added $12\frac{3}{8}''$ net section, and F has subtracted only 11″, that is, $1\frac{3}{8}''$ less.

Immediately after this joint, the angle iron A begins to pass its strain into the rest of the system; at just double the rate F is taking strain up from it, so that, when

3. at 3′0, or 1′0 beyond, A are jointed; A have thrown strain from 9¾ sq. in. of section into the system; F have taken up that represented by

$$2 \text{ no.} \times 5 \text{ no.} \times \frac{7''}{8} + \frac{10}{9} \times \frac{11''}{16}$$

$$= 6\tfrac{3}{4} \text{ sq. in. of section,}$$

the difference 3 sq. in. diminishes the net section to 1⅝″ less than it was outside the cover *.

F and A beyond the joint rapidly take up their full strain thrown upon them by the approach of a joint in a_1: thus, by the time

4. ..... 4′0 ..... 1′0 beyond, a_1 is jointed, F has had 2′0 length wherein to take up its share of strain; for its first foot, as fast as the rivets would convey it, and for the second foot, with a rivet to spare; and A has had 1′0 length in which it has taken up its full strain.

5. ..... 5′0 ..... 1′0 beyond, the joint of a_2 follows, and then

6. ..... 6′0 ..... 1′0 beyond, or 5′9⅝″ from its commencement, the cover ends.

7. ..... 9′0 ..... 3′0 beyond, the wrappers G, H commence on the outer angle irons of the box of the boom, and take by means of 5 rivets the strain from b.

* There ought therefore to be more space between the joints of F and A, 1′2″.4 would suffice. This mistake has arisen from a clerical error not discovered till it was too late to alter the figure.

8. at 9'10.8", or 10.8" beyond, is the joint of b.

9. ... 10'4.8" ... 6" beyond, the cover c_0 recommences.

10. ... 10'19.6"... 4"8 beyond, the wrappers G and H stop.

11. ... 11'2.4" ... 4.8" beyond, the angle irons D joint.

12. ... 12'0 ... 9.6" .... plate a_2 joints.

13. ... 13'0 ... 1'0 ... ... a_2 joints.

14. ... 14'9 ... 1'0 .... the cover-plate c_0 ends, 3'7".2 from its commencement. Other joints occurring where the boom is fined down are similar, or less complicated.

For jointing the second portion of the boom we have the following data (pp. 148–9). We must have covered, in

		breaking area.					covered.
Pls. c_1c_2	of	12⅜"	25½,	or say	30,	no. of rivets, i.e.	1'0
Pl. c_0	...	9"	19⅛	......	20	... ...	1'0
Bar d	...	3⅝"	7$\frac{3}{7}$	......	8	... ...	9".6
∠is. BE	...	4⅞"	7⅛	......	8	... ...	10".8
Wrapper	...	3¼"	6	......	6	... ...	8".4

Looking at the general section, fig. 4, and again considering this portion as unconnected with the former one, we will follow the joints, which take place in the following order, $EBc_2c_1\,Cc_2c_1$, &c.

We will take our distances from the last joint of c_2; then, Plate I. fig. 4, *suppose* that

1. at 5'0 the cover c_0 begins and receives strain, thrown by E, into *it* c_1 c_2 and B, through 7 of the rivets

η, ι, thus taking $\frac{7}{9}$ of the whole strain in E; and

2. at 5′.4.8, or 4.8″ beyond, the cover bar d commences and receives the rest of E's strain through 2 rivets; and also 4 rivets' value of B's, by the 6 rivets which pass through it before B joints.

3. at 6′0, or 7.2″ beyond, E having had 5 rivets covered which pass through the plates, and 4 which pass through the $\angle$ i. B is jointed; thus, withdrawing $9\frac{3}{4}$ sq. inches from the general section: B is also approaching a joint, and has thrown the strain of 4 rivets, i.e. of $2\frac{1}{2}$ sq. in. into the general section. Thus, $9\frac{3}{4} + 2\frac{1}{2}$, or $12\frac{1}{4}$ extra square inches are required by the general section, and supplied by the cover-plate and bar.

4. at 6′5″, or 5″ beyond, B would joint.

Now upon looking at the section of the boom, we see that in the above arrangement we have supposed E to be passing its full strain into $c_0\, c_1\, c_2$ through 5 rivets ι; and its remaining strain, which could be taken by 3 rivets if at full advantage, to be passed into B by 4 horizontal rivets η, and thence diffused and thrown upon the cover-plates c_0 and d_1 by means of the vertical rivets θ through $c_1\, c_2$. We have, also, supposed B to pass its strain, simultaneously, through both η and θ; and this is manifestly impossible.

We must therefore change our arrangement; and we will also change our zero for distances to the same as we have already been using for fig. 7.

It may be remarked, that, in thus arranging a joint for the first time; having only the general section, fig. 4, we cannot glance continually (as the reader now can) to the figures 7 and 8 already drawn out. The designer has to remember the distance between the rivets, and to consult perpetually the pages 146—9:

with these aids the distances between every change of plate &c. are first written in the margin, and thence the total distances from the zero line filled in subsequently; and from these again the figures 7 and 8 are readily drawn.

1\. at 13′.4″.8 — the cover c_0 begins and receives strain thrown by E into it c_1c_2; E passes strain into $c_0c_1c_2$ and B through 6 rivets, 3 of ι and 3 of η, which will be $\frac{3}{4}$ of the whole strain in E, when

2\. ... 14′00, or 7.2 — the cover-bar d begins and receives the remaining strain of E (for which c_0 is too weak) by 2 more rivets. Of these 8 rivets of E, if we consider the 4 ι to have full value; then the 4 η need only be worth a little more than $\frac{3}{4}$ their full value.

3\. ... 14′.3″6 ... 3″.6 — E joints, having had 2 rivets since d commenced. E then begins (as well as d) to take up strain from B through both η and by ι through the rivet θ, and plates c_1c_2.

4\. ... 15′.1″.2 ... 9″.6 — B joints having had 4 rivets η directly into E, and 4 of θ ($3\frac{1}{8}$ is the further number theoretically required).

Now before B joints, c_2 may pass the strain of $2\frac{7}{8}''$ into the system; for c_0 and d together contain $12\frac{5}{8}''$; while the joint in B only taxes them with $9\frac{3}{4}''$. But the joint of B runs through one rivet η, and it will therefore

		require 10″.8 covering in order to receive its full strain, and thus entirely relieve c_2.
5. at 16′0,	or 10″.8	c_2 joints.
6.... 17′0	... 1′0	c_1 .....
7.... 18′0	... 1′0	c_0 stops; having delivered by each of 20 rivets, 1st the full strain due to their bearing on c_1, and 2nd, a strain to c_2, which is delivering into c_1 through the medium of rivets θ and ν.
8.... 18′9.6	... 9′.6	d ends, having been continued far enough to deliver its full strain into c_1 by 8 rivets after c_0 has ended.
9.... 6′.2″.4	... 3′.4″.8	d recommences.
10.... 6′.3″.6	... 0.1″.2	The wrappers I commence.
11.... 7′0	... 0.8″.4	c_0 commences.
12.... 7′.2″.4	... 0.2″.4	C joints; having passed its strain by 4 rivets η into its wrapper or d, the latter of which will at any rate take any to which the wrapper yields; and by 4 rivets into the wrapper only, which being $\frac{9''}{16}$ thick only, gives the rivets only $\frac{9}{11}$ of their full bearing on C. These latter 4 rivets, then, are only worth $\frac{9}{11}$ of the former, or $= 3\frac{3}{11}$ such as the former; giving C virtually a cover to $7\frac{3}{11}$ rivets, which is sufficient. The web-plate, therefore, is not called upon to assist as a cover.

C's joint withdrew $9\frac{3}{4}''$ from the system, of which its wrapper took $6\frac{1}{2}$, leaving $3\frac{1}{4}''$ for c_0 and d; this $3\frac{1}{4}''$ will be retaken by C from the plate c_2 (not from c_0 and d), and will require

$$3\tfrac{1}{4} \times \frac{16}{11} \times \frac{8}{7} \times \frac{9}{10}$$

number of rivets to connect C and c_2; that is, for each of C,

$$\frac{1}{2} \times \frac{13}{4} \times \frac{16}{11} \times \frac{8}{7} \times \frac{9}{10} = 2.4,$$

or 3 rivets, which will require a length of 7.2″ between the joints of C and c_2; in which space C will receive strain from I only by the lower rivets.

13. at 8′0,	or 9″.6	c_2 joints; c_0 has been able by this time to take up its full strain without interfering with other joints.
14. ... 8′.2.6	... 2″.6	The wrapper, having had 6 rivets (independent of the 3 above mentioned) in order to take the strain independently from the wrapper, ends.
15. ... 9′0	... 9″.4	c_1 joints.
16. ... 10′0	... 1′0	c_0 ends.
17. ... 10′9″.6	... 9″.6	d ends.

We can now draw a length of each portion in detail, figs. 7 and 8.

Next, by drawing fig. 5, and simultaneously composing Table II., we work out the thicknesses, and ascertain the

TABLE II.

No. of Sect. Pl. 1, fig. 5.	14		13		12		11		10		9		8		7		6		5		4		3		2	
Sq. in. Sect. required.		122 1/2		121 3/4		119 3/4		116 1/8		111 1/8		104 3/4		97		87 5/8		77		65		51 1/2		36 1/2		20 1/8
	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.	"	sq. in.
a_1	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	7/16	10 3/4	3/8	9 1/4	3/8	9 1/4	3/8	9 1/4	1/4	6 1/4						
a_2	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	3/8	9 1/4	1/4	6 1/4	1/4	6 1/4
F	11/16	11	11/16	11	11/16	11	9/16	9	9/16	9	1/2	7 3/4	1/2	7 3/4												
A	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	5/8	9	5/8	9	5/8	9	5/8	9	5/8	9	5/8	9	9/16	8 1/8	9/16	8 1/8	7/16	6		
D	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	5/8	9	5/8	9	1/2	6 3/4	1/2	6 3/4										
b	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	7/8	12 5/8	5/8	9	1/2	7 1/4
E	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	9/16	8 1/8	9/16	8 1/8	1/2	6 3/4	1/2	6 3/4	1/2	6 3/4								
B	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	5/8	9	5/8	9	5/8	9	5/8	9	1/2	6 3/4	1/2	6 3/4	7/16	6
c_1	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	1/2	12 3/8	7/16	10 3/4	9/16	13 7/8	1/2	12 7/8	7/16	10 3/4	3/8	9 1/4	5/16	7 3/4	1/4	6 1/4
c_2	1/2	12 3/8	7/16	10 3/4	3/8	9 1/4	5/16	7 3/4	1/4	6 1/4	3/16	4 5/8	3/16	4 5/8												
C	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	11/16	9 3/4	9/16	8 1/8	9/16	8 1/8	9/16	8 1/8	9/16	8 1/8	7/16	6	7/16	6	7/16	6	7/16	6
Section obtained		121 7/8		120 1/4		119 3/4		116 1/2		110 5/8		104 1/2		97		87 3/4		79 1/2		65 1/8		52		41 3/4		31 3/4

lengths of every plate or angle iron; taking care to make the section never less than that required by calculation. As an assistance in this, where the strains rapidly fall off, we observe; that when, for instance, the difference between the areas of two adjacent sections (as between 100″ and 109″, or between 109″ and 115½″), is about 8″, that is at the rate of 1″ additional per foot run; that between 75½″ and 89″ is about 1½″ per foot run, and so on.

No drawings are given in Plate I. of the joints in the lighter parts of the boom, since they are supposed to be exactly similar to the others except in length only. After the supplementary angle irons leave off, however, the joints might be altered so as to cut out less section, (or at all events shortened,) by the insertion of some more rivets amongst the others in the cover.

The cambre considered necessary for a plate-webbed girder may be given by constructing it upon a platform, to the upper surface of which that cambre has been given. A cambre of 2 or 3 inches in girders 150′0 to 200′0 span, will not be felt seriously in the rivetting of the girder.

Practical Construction of the Plate-webbed Girder of Prop. IV. B. fig. 18.

We shall make the girder of the section shewn in the 'section' fig. 18, 3′8″ deep, and 42′0 long.

The rivetting adopted will be 4 no. to the foot, and $\frac{3''}{4}$ diameter.

I. Web.

The thickness of web-plate required at the end pillar to support half of 36 tons, on $3\frac{1}{2}' \times 4 = 14$ rivets

$$= 18 \text{ tons} \times \frac{4}{3} \times \frac{1}{14} \times \frac{1}{5} \text{ inches}$$

$$= \frac{12}{35}, \text{ which is under } \frac{3''}{8}.$$

Now $\frac{3''}{8}$ plate, 3′6″ deep, weighs 15 lbs. per square foot, and, therefore, 52½ lbs. per foot run. It would be well, therefore, to take the plates about 7′0 long. And we may evidently make the whole web of six plates, each 7′0 long; the two outer ones $\frac{3''}{8}$; the next $\frac{5''}{16}$, and the centre ones $\frac{1''}{4}$, covering the joints with a cover 4½″ broad, single-rivetted. If these covers be $\frac{3''}{16}$ or $\frac{1''}{4}$ thick, with a 4″ × 3″ × $\frac{3''}{8}$ T iron on the other side, the joints will be very good.

II. Booms.

Make the angle irons at the centre 3″ × 3″ × $\frac{9''}{16}$. Then the $\frac{3''}{4}$ rivetting, 1¼″ from their edge, will be found, if alternated, to cut out 1⅜″ lineal, leaving them virtually each $2\frac{5}{16} \times 2\frac{5}{16} \times \frac{9''}{16}$, and, therefore, together equal to $4\frac{5}{8} \times 4\frac{5}{8} \times \frac{9''}{16}$, or 4.88 sq. inches, say 4⅞.

There remain then 11⅜″ − 4⅞″, or 6½ sq. inches, which we shall get by making the plate 14½″ broad and $\frac{1''}{2}$ thick. It is well to have as broad a plate as possible, to keep the girder stiff laterally.

The longer this centre plate the better (p. 136). It will weigh 72.5 lbs. per yard, and may, therefore, be 12′0 long.

Now, 8′0 from the centre the strain on the boom (see Prop. IV. B)

$$=\frac{9}{70}\times 12\times 28 \text{ tons} = 43.2 \text{ tons},$$

and requires 9.6 sq. in. section.

This may be got by

2 no. $3\times 3\times\frac{1''}{2}$ angle irons giving $4\frac{3}{4}''$ breaking area,

1 plate $14\frac{1}{2}''\times\frac{3''}{8}$, giving $13\times\frac{3}{8}=4\frac{7}{8}$ ....

We may diminish the angle irons to $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{7}{16}$, giving a breaking area together, virtually, of $3\frac{5}{8}\times 3\frac{5}{8}\times\frac{7}{16}$, or above $2\frac{7}{8}$ sq. inches; when the total section required is less than $4\frac{7}{8}''+2\frac{7}{8}''=7\frac{3}{4}$ sq. inches,

and, therefore, when the strain = 35 tons, say 36;

or at x feet from centre, if

$$36=\frac{9}{70}(20-x)(20+x),$$

$$\text{or } 280=400-x^2,$$

$$x^2=\sqrt{120},$$

$$\text{or } x=11 \text{ feet}.$$

We are, therefore, amply safe in dividing the angle iron on each side of the web into 8 portions:

2 no. 16′0 long, $3\times 3\times\frac{9''}{16}$ in the centre;

4 no. 8′0 long, $3\times 3\times\frac{1''}{2}$ next to these;

2 no. about 13′6″ long, $2\frac{1}{2}\times 2\frac{1}{2}\times\frac{3''}{8}$ cranked round the ends of the girder.

Joints of the above.

The plate requires a bearing line $= 13'' \times \frac{9}{10}$ inches, in order to take its strength; this it will have if it bear on $\frac{4}{3} \times 13 \times \frac{9}{10}$ no. $\frac{3''}{4}$ rivets $= 15.6$ or 16 rivets.

If the top (or bottom) plate of the girder have to have holes in it at intervals, for putting bolts through in order to fasten the roadway on, it must be made wider than $14\frac{1}{2}''$; enough to make up for the amount cut out by these holes.

In this case the cover-plate may have 4 no. rivets in the last row upon the $\frac{3''}{8}$ plate, since the rivet-holes may cut out less than the bolt-holes. But if the plate $14\frac{1}{2}''$ wide be used, as above, the cover-plate over the $\frac{3''}{8}$ plate must have no more than two rivets (the two passing through the angle irons) in the last row. It may have 4 (if properly disposed in the next row), as we have explained in the chapter on joints. In this case the corners of the cover-plate would be cut off at the end covering the $\frac{3''}{8}$ plate; but not at the other end, at which there might be 4 or 6 in the last row without danger to the $\frac{1''}{2}$ plate, since the section of *that* plate near the joint is more than enough.

The wrappers for the larger $\angle$ is. require $1\frac{3}{4}'' \times \frac{4}{3} \times \frac{9}{10} \times \frac{16}{9}$ no. of $\frac{3''}{4}$ rivets in $\frac{9''}{16}$ plate $= 4$ no.

A wrapper $\frac{9''}{16}$ thick, on the $3''$ angle irons, will have a breaking area of $1\frac{3}{4}''$. Two of these, and the web between the angle irons which is $3'' \times \frac{5''}{16}$, make a total of $4\frac{1}{2}''$, and will be barely enough to joint the 3″ angle irons; which had better be made to break joint 6″.

The figure 18 shews the girder, with a couple of cast iron brackets at the ends, which may be made in one piece or not. If the girder be not braced at intervals to its partner, T irons should be put on both sides of two of the joints in the web, and continued to meet the boom-plates, and then cranked round and rivetted to them.

CHAPTER VII.

THE LATTICE-GIRDER—SECOND APPROXIMATION.

(Figs. 21 and 35.)

PROPOSITION V.

To construct a wrought iron double-lattice girder, of 200'0 *clear span, and of* 15'0 *vertical depth between the centres of gravity of the booms; with the lattice* 2'0 *apart horizontal distance* (Fig. 35).

Given, per foot run, for 2 *girders.*

	tons.		tons.
Weight of roadway on top =	.45,	*of live load on top* =	1
.... *bottom* =	.15	.... *bottom* =	$\frac{1}{2}$
Total	.6	*Total*	$1\frac{1}{2}$

Figure 35 will shew the general dimensions of the given girder.

FIRST *Approximation.* Assume the whole weight of the girders to be 192.6 tons for a length of 214'0; this is nearly what we might expect from a knowledge of the weights of previous bridges; and gives the average weight per foot run .9 tons.

We have then for our 1st approximation:

Total dead weight per foot run = 1.5 tons = $\frac{3}{4}$ tons per girder,
...... live weight .... = 1.5 ... = $\frac{3}{4}$ ....
Total 3 ... = $1\frac{1}{2}$.................

1. *Approximation to the weight of the lattice.* At the centre of the girder the lattice-bars will have their minimum section, shewn in the upper "section of channel iron," fig. 21. That section has an area of $1\frac{5}{8}''$, and therefore weighs

$$\frac{13}{8} \times \frac{10}{3} \text{ lbs. per foot;}$$

there will be two lattice-bars per foot run of each girder, of a length $= \sqrt{2} \times 15'0 = 36'9''$ each; these will therefore weigh 195 lbs. per foot run of girder.

At the pillars 2 lattices will have to sustain half the weight on one girder, or

$$\tfrac{1}{2} \text{ of } 210'0 \times 1\tfrac{1}{2} \text{ tons} = \frac{315}{2} \text{ tons.}$$

Dividing this equally between the $7\frac{1}{2}$ no. struts, and the $7\frac{1}{2}$ no. ties, which theoretically form the end of each lattice, we find that at the end of one lattice

	tons.		tons per sq. in.	sq. in.		
$7\frac{1}{2}$ ties sustain	$\frac{315}{8}$,	which requires at	$4\frac{1}{2}$	$8\frac{3}{4}$	or $1\frac{1}{8}\,\frac{1}{32}''$	each,
$7\frac{1}{2}$ struts.......	$\frac{315}{8}$	....	4	10	... $1\frac{3}{8}''$	...

Now a tie-bar $3\frac{1}{2}'' \times \frac{1}{2}''$ gives $1\frac{3}{16}''$ section, even if $1\frac{1}{8}''$ were punched out or damaged for the $1''$ rivet by which it is secured to the plate; and will have a gross section of $1\frac{3}{4}''$.

A strut $3\frac{1}{2}''$ broad and $\frac{5}{8}''$ thick channel iron, shewn in the lower "section of channel iron," will give $1\frac{1}{2}''$ safe section, besides the projecting parts, after a $\frac{5}{8}''$ rivet-hole has been punched out for rivetting it to the crossing lattice-bars*; and will have a gross section of 3″.

Hence the average gross section of the bars at the pillar is

$$\tfrac{1}{2}(3+1\tfrac{3}{4})=2\tfrac{3}{8}'';$$

nearly half as much again as at the centre.

Now if we take the double lattice as averaging with its rivet-heads 224 lbs. = 2 cwt. = .1 tons per foot run, we shall get as near to the weight of all the central portion of the girder (up to within a few feet of the pillars) as we can.

2. *Approximation to the weight of the booms.*

Our formula gives the strain at any point of a boom, Lemma VII.

$$=\frac{W}{2dl}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}+x\right),$$

where $W=\frac{3}{2}$ tons $\times\, l$, $d=15$, $\frac{l}{2}=105$;

$$=\frac{3}{2\times 2\times 15}(105-x)(105+x),$$

$$=\frac{1}{20}(105-x)(105+x).$$

Now the booms will be made in 12′0 lengths, the centre of the middle length being the centre of the bridge. By substituting for x, we are able to write down the greatest strains upon each portion of the boom, and thus form the first two columns of the following table.

* In allowing for weakness caused by rivet-holes in a bar or plate subject to compression, we have only to consider the holes caused by *intermediate* rivets. The rivets against which the bar bears at its extremities, furnish a bearing equal to whatever section is cut by their holes.

TABLE I.

1. Point in question.	2. Strain on boom, tons.	3. Weight of 12′0 of both booms.	4. Corrected 2 per cent.	5. All other weight in 12′0.	6. Total live and dead weight per 12′0 length.	7. $\frac{4}{5}$ of total weight.	8. Sum of the last column from the bottom.
centre	551	5.65	5.76	13.8	19.56	15.648	
6′0 from do.	549	5.63	5.74	13.8	19.54	15.632	118.000
18′0 ...	535	5.485	5.59	13.8	19.39	15.512	102.368
30′0 ...	506	5.185	5.28	13.8	19.08	15.264	86.856
42′0 ...	463	4.745	4.84	13.8	18.64	14.912	71.592
54′0 ...	405	4.160	4.24	13.8	18.04	14.432	56.680
66 0 ...	333	3.415	3.48	13.8	17.28	13.824	42.248
78′0 ...	247	2.530	2.58	13.8	16.38	13.104	28.424
90′0 ...	146	1.495	1.52	13.8	15.32	12.256	15.320
102′0 ...	(30) (for 3′0)	.374	.38	3.45	3.83	3.064	
Totals		(with $\frac{1}{2}$ centre)	36.53...	(without centre)	147.50		

The 3rd &c. columns of this table will be deduced hereafter.

Now we want to find the weight of the booms necessary to resist the above strains. Suppose the section of the *upper boom* to be as represented in fig. 21. Then the heaviest section will be

$$\left.\begin{array}{l} \left.\begin{array}{lr} \text{No. 2 plates } 26'' \times 1'' = & 52 \\ \quad \text{2 plates } 16'' \times 1'' = & 32 \\ \quad \text{2 plates } 1'0 \times \frac{5''}{8} = & 15 \\ \quad 8 \angle \text{ i. } 3\frac{1}{2} \times 3\frac{1}{2} \times \frac{5''}{8} = 4 \text{ sq.in.ea.} = & 32 \end{array}\right\} \text{gross} = 131 \\ \left.\begin{array}{lr} \text{rivets 6 no.} \times 1'' \times \frac{7''}{8} = & 5\frac{1}{4} \\ \quad \text{8 no.} \times \frac{5''}{8} \times \frac{7''}{8} = & 4\frac{3}{8} \end{array}\right\} \quad = 9\frac{5}{8} \end{array}\right\} \text{net} = 121\tfrac{3}{8}''.$$

This will be a sufficient section in compression for 551 tons, which at $4\frac{1}{2}$ tons per sq. inch requires $122\frac{1}{2}$ sq. inches. Now the average weight of this section in a 12 foot length will be per foot run, upper boom,

Gross weight of section

$$= 131'' \times \frac{10}{3} \text{ lbs. per foot} = \ldots\ldots\ldots\ 440 \text{ lbs. per foot.}$$

Joints, of a $\frac{1}{2}''$ sandwich plate* and $\angle$ is., av. 14 ....

Heads of rivets 72 no. to the foot

$$= 72 \text{ no.} \times 1\tfrac{1}{8}'', \text{ or } 81'' \text{ of } \tfrac{7}{8} \text{ rod} = \ldots\ 14\ \ldots\ldots\ldots\ldots$$

Total 468 lbs = .209 tons.

The *lower boom* will consist of bars 8″ deep, by 1″ or $\frac{1}{2}''$. Of the 8″, $1\frac{1}{2}''$ will be cut away by $1\frac{1}{2}''$ bolts; therefore each $8'' \times 1''$ bar will be good for $6\frac{1}{2}'' \times 4\frac{1}{2}$ tons $= 29\frac{1}{4}$ tons.

* The sandwich plate is not shewn in the figure. It is a half-inch plate, inserted between each of the 12′0 lengths of the upper boom, and rivetted through, by all the rivets whose heads are shewn in plan in the "heaviest section" figure 21. By thus making each length equal to $12'0\frac{1}{8}''$ instead of 12′0 a cambre is given to the whole bridge as explained hereafter.

Each bar, of 12′0 net length, may be taken to weigh, including overlap for bolting, and nuts and heads of bolts, &c., as much as if it were 14′0 long: i.e.

$$8'' \times 14' \times \frac{10}{3} \text{lbs.} = \frac{1120}{3} \text{lbs.} = \tfrac{1}{6} \text{ ton};$$

therefore the weight of a bar, one inch thick, per foot $= \frac{1}{72}$ tons.

This is the weight of the lower boom for every $29\frac{1}{4}$ tons strain upon it; therefore the weight per foot at the centre of the girder

$$= 551 \text{ tons} \div 29\tfrac{1}{4} \div 72, \text{ tons} = .262 \text{ tons per foot run.}$$

Hence the *average weight of both booms* at the centre is

$$.209 + .262 = .471 \text{ tons per foot.}$$

(Hence, if we divide the strain on one boom by 2, and subtract from the quotient its 7th part; the result will represent the weight of the boom per foot run, if taken as a decimal. By this rule, column 3 of Table III. is formed.)

We will now see how far the above weight of the centre booms and lattice agrees with our assumed approximate weight.

The average weight of the double lattice, as affecting strain, has been shewn to be throughout

$$.100 \text{ tons per foot.}$$

Also the average weight of both booms, as affecting strain, will be (Lemma XI.) $\frac{1}{5}$ less than .471 tons

$$= 471 - .094 = .377 \text{ tons per foot.}$$

Making an average weight for both girders, as creating strain,

$$=2\,(.100+.377)=.954 \text{ tons.}$$

We assumed it .9 tons; and therefore the error amounts to .054 tons per foot in a total weight and load of 3 tons per foot; = 1.8 per cent. too little. If then a consideration of the way this result is obtained make it seem advisable, the sections of the booms and lattice, obtained on the assumption of .9 tons, and also their weights, must all be increased 2 per cent., in order to make the approximation as near as possible.

The weight of both booms at the centre for a 12′0 length, from above, is $12' \times .471 = 5.652$ tons.

Hence a 12′0 length of both booms under a strain of 551 tons weighs 5.652 tons. We shall assume that this proportion between strain and weight holds throughout the booms. If we take the 551 as a decimal, multiply it by 10 and divide it by 4, and add the results, we get 5.648; which is nearly 5.652. And we may accordingly treat all the 2nd column of Table I. in order to get the 3rd column.

To each term of the 3rd column we add 2 per cent., and thus obtain the 4th column. The other columns are filled up hereafter.

SECOND *Approximation.* This calculation takes into account not only the weight of the girder itself, but also the position of its weight. The data for this will be,

	tons per bridge.	tons per girder.
On the top, dead weight of roadway	= .45	= .225
... ... lattice		.050
... live load	= 1	.500

	tons per bridge.	tons per girder.
On the bottom, dead weight of roadway	= .15	.075
... ... lattice		.050
... live load	= $\frac{1}{2}$	.250

And the dead weight of the booms, as in Table I., which we shall divide equally between the top and bottom (as being near enough).

Hence we have uniform per girder,

TABLE II.

		Tons per foot per girder, *or* tons per space per lattice.		
Dead weight on	top	= .275	for which	$w = .003704$
....	bottom	= .125	......	$w = .001683\frac{5}{8}$
live load on	top	= .5	......	$w = .006734\frac{1}{2}$
....	bottom	= .25	......	$w = .003367\frac{1}{4}$
		1.15		

where, for each weight, w = weight per space $\times \sqrt{2} \div 105$.

3. *Second Approximation to the strains on the lattice-bars.*

Our second approximation to the strains on the *lattice* bars will be calculated as though the booms were divided into 15′0 lengths, instead of into 12′0 lengths. The difference of weight thus introduced will be very trifling, and on the safe side; and the difference of labour required, very great*.

* It will be seen, from the result of this proposition, that it will seldom be worth while to form a second approximation to the strains on the lattice of a girder, in our present state of knowledge with respect to the strength of the long bars in compression: for if the first approximation be a good one, it will give the strains on the lattice-bars as closely, both in relation and amount, as we

Our equation for the strain on a boom at $x'0$ from the centre

$$=\frac{W}{2dl}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}+x\right),$$

where $\frac{W}{l}=1.53$ (adding 2 per cent.), $d=15$, $\frac{l}{2}=105$,

$$=\frac{1.53}{2\times 15}\times(105-x)(105+x)$$

$$=.051\times(105-x)(105+x).$$

Putting $x=0$, 15, 30, &c., we get the following Table.

TABLE III.

1. Point in question.	2. Strain on boom.	3. Weight per foot of boom.	4. Weight $\times\sqrt{2}\div 105$.
Centre.	562¼	.241	.003245
15'0 from do.	551	.236	.003179
30'0 ...	516	.221	.002980
45'0 ...	459	.196	.002649
60'0 ...	378	.161	.002185
75'0 ...	276	.117	.001588
90'0 ...	149	.063	.000859
Totals.................		1.235	.016685

The second column is got from the above equations; the third as explained near the beginning of the proposition, (p. 171); the fourth by the use of logarithms from the third.

Now there are 105 spaces, 2'0 long each, and two lattices in each girder, therefore the weight on each lattice is per

can possibly adapt the scantling of our bars. Again, it will appear that a second approximation to the strain on the booms *is* essential in a large girder. However a method of second approximation to the strains on the lattice is worked out here, as satisfactory and interesting, both in itself, and from the close analogy a *lattice* bears to a *plate* web.

space what that on the girder is per foot; therefore the column 3, Table III., is the weight of one boom per space per lattice; and the last column of Tables II. and III. are the numbers for which w and w' were written in the former lattice-girder analysis.

Hence the maximum compression on bar No. 46,

$$
\begin{array}{ll}
=.003704 \times (39\tfrac{1}{2}+24\tfrac{1}{2}+9\tfrac{1}{2}-50\tfrac{1}{2}-35\tfrac{1}{2}-20\tfrac{1}{2}-5\tfrac{1}{2}) & i.e. \times -38\tfrac{1}{2} = -.142104 \\
+.001683\tfrac{5}{8} \times (32+17+2-58-43-28-13) & i.e. \times -91 = -.153210 \\
+.006734\tfrac{1}{2} \times (39\tfrac{1}{2}+24\tfrac{1}{2}+9\tfrac{1}{2}) & i.e. \times 73\tfrac{1}{2} = +.473986 \\
+.003367\tfrac{1}{4} \times (32+17+2) & i.e. \times 51 = +.171730
\end{array}
$$

$$
\left.\begin{array}{l}
+.000859 \times (-5\tfrac{1}{2}+2) \\
+.001588 \times (9\tfrac{1}{2}-13) \\
+.002185 \times (-20\tfrac{1}{2}+17) \\
+.002649 \times (24\tfrac{1}{2}-28) \\
+.002980 \times (-35\tfrac{1}{2}+32) \\
+.003179 \times (39\tfrac{1}{2}-43) \\
+.003245 \times (-50\tfrac{1}{2}-58)
\end{array}\right\}
\begin{array}{l}
i.e.\ .016685 \times -3\tfrac{1}{2} = -.058398 \\
+.003245 \times -105 = -.340725
\end{array}
$$

$$= .645716 - .694437 = -.048721 \text{ tons.}$$

Writing down the series for the bar No. 48, we should find that the difference

$$
\left.\begin{array}{l}
D_1 = .003704 \times 14 = .051856 \\
\quad +.001683 \times 14 = .023569 \\
\quad +.006734\tfrac{1}{2} \times 6 = .040407 \\
\quad +.003367\tfrac{1}{4} \times 6 = .020204 \\
\quad +.016685 \times 4 = .066740
\end{array}\right\} = .202776 \text{ tons.}
$$

The only terms which alter D_1 are:—First, the third and fourth, for which we can easily write down D_2 in the Table IV. by inspection of the series from which those terms are obtained. Do this then at once. Secondly, the last term, due to the weight of the booms. Now we may by trial

see, that this last term of D_1 goes on unaltered, until the bar, whose strain is sought, be the first on a 15′0 length of boom: and that then this term is diminished by 15×.016685, but increased by 105 × the value of w for that upper and lower boom, between which the bar lies across.

Thus, as an example, the terms in the series for the greatest strain on bar

No. 88 are	No. 90 are	Their difference may be written
.003245 × ($51\frac{1}{2}$ + 59)	.003245 × ($53\frac{1}{2}$ + 46)	.003245 × (4 − 15)
.003179 × ($66\frac{1}{2}$ + 44)	as before × ($38\frac{1}{2}$ + 61)	as before × (4 − 15)
.002980 × ($36\frac{1}{2}$ + 74)	„ × ($68\frac{1}{2}$ + 31)	„ × (4 − 15)
.002649 × ($81\frac{1}{2}$ + 29)	„ × ($23\frac{1}{2}$ + 76)	„ × (4 − 15)
.002185 × ($21\frac{1}{2}$ − 16)	„ × ($83\frac{1}{2}$ + 16)	„ ×(4−15+105)
.001588 × ($-8\frac{1}{2}$+14)	„ × ($8\frac{1}{2}$ − 14)	„ × (4 − 15)
.000859 × ($6\frac{1}{2}$ − 1)	„ × ($-6\frac{1}{2}$ + 1)	„ × (4 − 15)

Therefore

the difference = .016685 × (4 − 15) + .002185 × 105;

whereof .016685 × 4 is the ordinary difference; and the rest, which answers to the description above given, is D_2.

Now,

15 × .016685 =	.250275	
105 × .003245	.340725 and subtracting .250275 leaves	.090450
... × .003179	.333795	.083520
... × .002980	.312900	.062625
... × .002649	.278145	.027870
... × .002185	.229425	−.020850
... × .001588	.166740	−.083535
... × .000859	.090195	−.160080

All we have now to do is to insert the terms in this last column, into the column D_2 of Table IV. as nearly as possible

opposite the last bar, whose lower end is on the corresponding 15′0 length. Of course to do this we must pencil the 15′0 divisions of the boom upon fig. 35.

It will be observed that in the Table IV. we have written down the second differences opposite the bars *nearest* which they occur, in such a way as to get no *accumulative* error; though it will cause a temporary one of a few pounds.

Table IV.

No. of bar.	Max. compression in tons.	D_1	D_2 For live load.	D_2 For booms.	D_2 Total.
46	−.048721				
		.202776			
48	.154054				
50	.356831	”		.090450	
		.293226			
52	.650057		.013469	−.090450	
		.216245			
54	.866302				
56	1.082547	”			
58	1.298792	”	.003367	+.090450 =	.093817
		.310062			
60	1.608354		.003368	−.090725	
		.222980			
62	1.831834				
64	2.054814	”			
66	2.277794	”	.013469	+.083520 =	.096989
		.319969			
68	2.596763			−.083520	
		.236449			
70	2.833212				
72	3.069661	”		+.031313	
		.267762			
74	3.337423		.006734		
		.274496			
76	3.611919			−.031313	
		.243183			
78	3.855102				
80	4.098285	”		+.027870	
		.271053			
82	4.369338		.013469	−.027870	
		.256652			
84	4.625990				
86	4.882642	”			
		”			

No. of bar.	Max. compression in tons.	D_1	D_2 For live load.	D_2 For booms.	D_2 Total.
88	5.139294		.003367	−.020850	=−.017483
		.239169			
90	5.378463		.003367	+.020850	
		.263386			
92	5.641849				
		"			
94	5.905235				
		"			
96	6.168621		.013469	−.083535	=−.070066
		.193320			
98	6.361941			+.083535	
		.276855			
100	6.638796				
		"			
102	6.915651			−.160080	
		.116775			
104	7.032426		.006734	+.160080	
		.282589			
106	7.315015				
		"			
108	7.597604				
		"			
110	7.880193				
104 to 110	29.825238				
	2				

59.650476 = Total strain on the struts meeting the end pillar.

So the maximum tension on bar No. 48 (which is the first not liable to compression)

$$
\begin{array}{lll}
= .003704 & \times(\ 48\tfrac{1}{2}+33\tfrac{1}{2}+18\tfrac{1}{2}+3\tfrac{1}{2}-41\tfrac{1}{2}-26\tfrac{1}{2}-11\tfrac{1}{2}) & = .090748 \\
+.001683\tfrac{5}{8} & \times(\ 56+41+26+11-34-19-4) & .206591 \\
+.006734\tfrac{1}{2} & \times(\ 48\tfrac{1}{2}+33\tfrac{1}{2}+18\tfrac{1}{2}+3\tfrac{1}{2}) & .700388 \\
+.003367\tfrac{1}{4} & \times(\ 56+41+26+11) & .451178 \\
+.000859 & \times(\ 3\tfrac{1}{2}-4) & -.000429 \\
+.001588 & \times(-11\tfrac{1}{2}+11) & -.000794 \\
+.002185 & \times(\ 18\tfrac{1}{2}-19) & -.001093 \\
+.002649 & \times(-26\tfrac{1}{2}+26) & -.001324 \\
+.002980 & \times(\ 33\tfrac{1}{2}-34) & -.001490 \\
+.003179 & \times(-41\tfrac{1}{2}+41) & -.001590 \\
+.003245 & \times(\ 48\tfrac{1}{2}+56) & +.339102
\end{array}
$$

$=1.788007 - .006720 = 1.781287.$

$$D_1 = \left.\begin{array}{r} .003704 \times 14 = .051856 \\ + .001683 \times 14 = .023569 \\ + .006734 \times 8 = .053876 \\ + .003367 \times 8 = .026938 \\ + .016685 \times 4 = .066740 \end{array}\right\} = .222979.$$

We write the following table by referring to fig. 35 (on which we have pencilled the imaginary joints 15′0 apart), and from it seeing where the No. of the bars pass from one section of boom to another: we then know where to apply the second differences, which are the same in amount as in the Table IV.

TABLE V.

No. of bar.	Max. strain.	D_1	D_2	
48	1.781287			
46	2.004266	.222979		
44	2.227245	”	.013469 + .083520 =	.096989
42	2.547213	.319968	−.083520	
40	2.783661	.236448		
38	3.020109	”		
36	3.256556	”	.006734 + .062625 =	.069359
34	3.562364	.305807	−.062625	
32	3.805546	.243182		
30	4.048728	”	.006735 + .027870 =	.034605
28	4.326515	.277787	.006734 −.27870	
26	4.583166	.256651		
24	4.839817	”		
22	5.096468	”	.006735 − .020850 =	−.014115
20	5.339004	.242536	+.020850	
18	5.602390	.263386		
16	5.865776	”		
14	6.129162	”	.013469 − .083535 =	−.070066
		.193320		

No. of bar.	Max. strain.	D_1	D_2
12	6.322482		+.083535
10	6.599337	.276855	
8	6.876192	"	
6	7.153047	"	.006734 −.160080 = −.153346
4	7.276556	.123509	+.160080
2	7.560145	.283589	
2 to 6	21.989748		
	2		
	43.979496		
add 7 viz.	7.014919		

50.994115 = total strain on ties meeting end pillar,

also 59.650476 = ... struts ....

∴ 110.644591 = total strain on the bars at the end of a lattice.

General check upon the accuracy of the lattice calculations.

We find that the total strain on the bars at one end of one lattice is 110.644591, as given by the Tables IV. and V.

Now we can easily find this strain, independently, from the weight those bars have to support: for the bars whose strains form this total are supporting one-fourth the weight of the whole girder fully loaded, except 1′0 length of each end, which is borne by the pillar.

Now, our calculations are based upon Table III.

Weight of two booms 208′0 long = (1.235 tons × 15′ − .063 × 2) × 4 = 74.110 tons

Weight of 208′0 of lattices, and half-loads and -roads = 1.15 × 208 = 239.2

313.310

This gives at each end of each lattice 78.33 tons; and the total strain necessary to support this is

$$78.33 \times \sqrt{2} = 110.69 \text{ tons},$$

shewing an excess of .05 tons in 15 bars. Thus the strains obtained by calculation may be considered to be rightly added up.

Now the girder is *built*, divided as in Table I., from which weight of two booms = (36.53 − .38) × 2 no. = 71.1 tons, which is on the right side of the weight on which we have *calculated*, viz. 74.110 tons.

4. *Second Approximation to the strain on the Booms.*

It is more important to get this than to carry the lattice calculation on to a second approximation.

If we add the column 6 of Table I. together, except the top term, we find it to amount to 147.50. If we double this and add the top term the result is:

The total weight of one loaded girder between its end pillars

$$= 295 + 19.56 \text{ tons} = 314.56 \text{ tons};$$

therefore the total weight of one loaded girder upon one pier (omitting the weight of the end pillar itself)

$$= \text{half this} = 157.73 \text{ tons};$$

also weight of the 15′0 of loaded girder which is nearest the bearings, Table I. col. 6,

$$= 15.32 + 3.83 = 19.15.$$

Therefore the strain on a boom, at 90′0 from the centre, (and therefore 15′0 from the bearings on the pier)

$$= (15' \times 157.73 - 7\tfrac{1}{2}' \times 19.15) \div 15'0 \text{ depth},$$

$$= 157.73 - 9.58 = 148.15 \text{ tons}.$$

The strain 78′0 from the centre, or 12′0 farther from the piers than the last,

$$= \{157.73 \times (15' + 12') - 19.15 \times (7\tfrac{1}{2}' + 12') - 16.38 \times 6\} \div 15;$$

$$\therefore D = \frac{12}{15} \text{ of } 157.73 - \frac{12}{15} \text{ of } 19.15 - \frac{6}{15} \text{ of } 16.38,$$

and generally,

$$D = \frac{12}{15} \text{ of } 157.73 - \frac{12}{15}$$

of the weight of the girder up to the *last* division $- \frac{1}{2}\ \frac{12}{15}$ of the weight of the division considered,

$$= 126.184 - \frac{4}{5} \text{ of the weight up to the last division } - \frac{2}{5}$$

of the weight of the division.

All this can be written down from Table I., if columns 7 and 8 be completed.

Table VI.

No. of feet from centre.	Corrected strain		Strain on boom in tons.	D		
	top (P)	bottom (T)		Pier.	Intermediate weight.	Total.
90	$146\frac{5}{8}$	$147\frac{5}{8}$	148.150	126.184	$\left.\begin{matrix}-15.320\\-6.552\end{matrix}\right\}=-21.872$	105.312
78	252	253	253.462	”	$\left.\begin{matrix}-28.424\\6.912\end{matrix}\right\}=-35.336$	91.848
66	$343\frac{3}{4}$	$344\frac{3}{4}$	345.310	”	$\left.\begin{matrix}-42.248\\7.216\end{matrix}\right\}=-49.464$	76.720
54	$420\frac{1}{2}$	$421\frac{1}{2}$	422.030	”	$\left.\begin{matrix}-56.680\\7.456\end{matrix}\right\}=-64.136$	62.048
42	$482\frac{1}{2}$	$483\frac{1}{2}$	484.078	”	$\left.\begin{matrix}-71.592\\7.632\end{matrix}\right\}=-79.224$	46.960
30	$529\frac{1}{2}$	$530\frac{1}{2}$	531.038	”	$\left.\begin{matrix}-86.856\\7.756\end{matrix}\right\}=-94.612$	21.572
18	$551\frac{1}{8}$	$552\frac{1}{8}$	552.610	”	$\left.\begin{matrix}-102.368\\7.816\end{matrix}\right\}=-110.184$	16.000
6	$567\frac{1}{8}$	$568\frac{1}{8}$	568.610	63.092	$\left.\begin{matrix}-59.000\\1.956\end{matrix}\right\}=-60.956$	2.136
Centre	$569\frac{1}{4}$	$570\frac{1}{4}$	570.746			

In this table, under the head *D*, is placed, first, the term expressing the pressure of the pier, and which is common to every difference (except that between the centre and 6'0 from the centre, when it requires halving). Next, the two terms shewing the diminution of the effect of the pier's pressure, owing to so much of the weight of the girder lying between the point in question and the pier. And, in the last column, the difference of the other two, and, therefore, the desired difference between the strains.

The column containing the "strain on boom in tons" would be correct for both booms in a plate-girder, or in a lattice-girder stiffened by upright T or ∠ irons at close intervals; but requires correction in our present girder, for the difference of weight on the top and bottom booms. To this we now pass.

We shall slightly condense the same method as was pursued, pp. 108—111, for the smaller girder.

The whole weight and load per space per lattice = 1.5 tons approximately.

That on the bottom boom = .55 tons approximately.

Hence, first, the difference of the strain of two bars 15 spaces apart, under full load

$$= 1.5 \times \sqrt{2}.$$

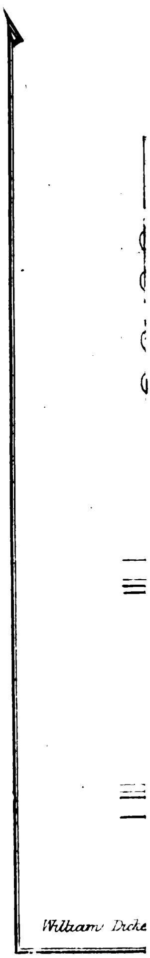
And consequently the difference of the strain of two consecutive bars

$$= 1.5 \times \sqrt{2} \div 15 = \frac{1}{10}\sqrt{2}.$$

Hence, also, the difference of the strain of two bars which meet on the lower boom

$$= .55 \text{ tons} \times \sqrt{2}.$$

Take a vertical section of the lattice through the point of intersection of any two bars upon the upper boom. Then


William Dicke

the excess of the strain of the strut over the tie in the two lowest bars in the section

$$= -55\sqrt{2} + \frac{1}{10}\sqrt{2}.$$

And the moment of that difference, tending to increase T,

$$= \left(-.55\sqrt{2} + \frac{1}{10}\sqrt{2}\right) \times 14'0 \frac{1}{\sqrt{2}}$$

$$= \left(-.55 + \frac{1}{10}\right) \times 14.$$

So the moment of the difference of strain in the two bars above the last

$$= \left(-.55 + 3 \times \frac{1}{10}\right) \times 12,$$

and of the next $= \left(-.55 + 5 \times \frac{1}{10}\right) \times 10$

...... $= \left(-.55 + 7 \times \frac{1}{10}\right) \times 8$

...... $= \left(-.55 + 9 \times \frac{1}{10}\right) \times 6$

...... $= \left(-.55 + 11 \times \frac{1}{10}\right) \times 4$

...... $= \left(-.55 + 11 \times \frac{1}{10}\right) \times 2.$

Total, $-.55 \times \frac{16 \times 7}{2} + \frac{1}{10}(14 + 36 + 50 + 56 + 54 + 44 + 26)$

$= -30.8 + 28 = -2.8$ tons at distance $1'0$ to increase T,

$= .453$ at same distance as T, viz. $15'0$, to decrease T.

And the sum of the horizontal push (towards the pier) of the bars cut by the above cross section, by a similar reasoning to that in the smaller girder,

$$= 7 \times \left(-.55\sqrt{2} + \frac{1}{10} \times 7\sqrt{2}\right)\frac{1}{\sqrt{2}} = -3.85 + 4.9$$

$$= 1.05 \text{ tons.}$$

Hence we have to subtract (on the average) about .5 from the column strain in tons, Table VI. in order to get T, and to subtract $1.05 + .45 = 1.5$ tons, in order to get P.

Fig. 21 shews the chief parts of this girder in its approximate state. This we now have to alter to conformity with our more accurate tables of strains, IV., V., and VI. The heaviest section of both booms will require to be increased. The figure explains itself: and if the joints of the lower boom, either those by rivetting or by bolts, be analyzed, they will be found to have a bearing surface upon the $\frac{7''}{8}$ rivets, or upon the $1\frac{1}{2}''$ bolts, in due proportion to the breaking area of the plate or bar. In the "heaviest section" of the lower boom the ends of the 8″ bars composing the next 12′0 length of the boom, are, of course, seen in elevation between those of the length whose section is taken. The ends of the boxes composing the upper boom are (after the angle irons are rivetted on) planed so as to make a perfectly true surface, and then bolted together, as in the elevation of the upper boom, or with a plate inserted between them.

Suppose that $\frac{3''}{16}$ be planed off each end of the open box forming one division of the upper boom; this, if the ends be well constructed, will be enough to secure a good planed surface, so that the whole of the section shall bear upon the sandwich plate. Let this be done in the planing machine, so as to leave each portion with a gauged length of $11'.\ 11\frac{5}{8}''$. Then if a $\frac{1''}{2}$ sandwich plate be inserted at each joint, the total length of each division is raised to $12'0\frac{1}{8}''$. The points of intersection of the lattice-bars will thus be made $2'0\frac{1}{48}''$ apart instead of $2'0$; for they must be set off by compass (or gauge) upon the boom so as to get 6 of them into the length of $12'0\frac{1}{8}''$ of a division.

Under these circumstances the cambre given to the girder will be that due to an increase of $\frac{1}{8 \times 12 \times 12}$ th in the upper boom,

$$= \frac{l^2 (e + e')}{8d} \text{ in feet, Lemma XII. (3),}$$

$$= \frac{40000}{8 \times 15 \times 8 \times 12} \text{ inches}$$

$$= 3\tfrac{1}{2}'' \text{ (all but } \tfrac{1}{16}).$$

No difficulty will be experienced in rivetting the $\frac{5''}{8}$ rivets in the web, on account of the slight disturbance caused by thus separating the lattice at the top.

The above method of obtaining a cambre, is by far the most simple that can be used, wherever it is practicable. In all girders whose webs are open work, it is preferable to the method of creating cambre by alteration in the lengths of the bars of the web; both as regards ease of adjustment and gauging in the workshops, and ease of erection on the site.

CHAPTER VIII.

ON DEFLECTION.

WHEN any structure, such as a girder, is subjected to a strain caused by its own weight, or by any other load placed upon it, the material composing it gives way a little under the strain. If a tension is produced by the weight or load upon any part of it, that part will elongate in the direction of the tension; and any part on which a compression acts will be reduced in the direction of the pressure. In the case of iron girder-work we shall divide "deflection" into two parts, the "permanent set" and the deflection proper of the girder.

I. That portion of the deflection which will not disappear on the removal of the cause which first produced it, is called the permanent set of the girder. This portion must also be divided into two parts, one due to workmanship and one to a property of iron.

First, to illustrate the permanent set due to workmanship, we will suppose that a girder is to be built in the position which it is to occupy as part of the bridge; it will be probably done in the following way. A connected series of balks of timber is laid down across the opening, exactly under the intended site of the girder: the top of them is intended, during erection, to be in contact, throughout, with the lower surface of the girder. This line of balk, by which the weight of the girder will be borne during erection, is firmly supported upon piles or other available means; it is secured firmly to a parallel line under the sister girder; and it is also constituted capable of vertical adjustment, by being supported throughout at close intervals upon wedges. Now we will suppose these balks to be laid down so that their upper surface shall in elevation form the arc of a circle; the highest point being at the centre of the span, and 3″ higher than the lowest point at the piers. Upon this the girder will be built, and will be said to be built with 3″ cambre.

When the girder is finished, the wedges are evenly and gradually knocked outwards. This brings no strain necessarily upon the iron, nor does it alter the form of the girder, until the bearings of the girder over the piers reach the bed-plates upon which they are designed to rest; any further loosening of the wedges will now alter the form of the girder by diminishing its cambre. If the girder have a plate-web its whole substance will form virtually one solid mass; and any further loosening of the wedges will begin to bring a portion of its weight, *throughout*, upon the end pillars and piers: so that all the decrease of cambre, caused by knocking away wedges after the girder has reached its bearings, will be the *result*, and therefore shew the amount of the actual straining of the iron in the girder.

But if the girder have some of its connexions formed by bolts, which cannot fit so accurately into their holes as rivets do, or by planed surfaces of any kind; then the decrease of cambre upon the further loosening of the wedges will, at first, be owing to the movement in the connexions necessary to bring these bolts and surfaces into contact; and when they are in contact, the contact will not at first be perfect, but their projecting parts and irregularities will for a time prevent the pressure being borne by the whole surface.

Any such decrease of cambre, then, is due to defect in workmanship, and not to a stretching of iron. Its amount is less than one unacquainted with the accuracy of our modern workmanship would be likely to guess.

After all the wedges are knocked away, and the girder is therefore left unsupported except at the ends, it is usual to place on it a test-load of about twice the amount that could ever in practice come upon it. The result of this is, as far as workmanship is concerned, to draw the connexions tight up; bruising down any roughness, projections or irregularity in the tooled bearings, and thus causing a further permanent set due to workmanship.

The second portion of the permanent set is that due to a quality of iron. If a bar of iron be extended forcibly, it is lengthened, by the tension produced in it, in direct proportion to the force employed. If it be now released it will return towards its original length, but will never reach it: and the same holds for compression. Thus, if a bar of cast iron be subjected to 12 tons on the inch, compression, and contract 1″ under it, then a pressure of 6 tons only *would have* contracted it only ½″: also a pressure of 6 tons more would contract it ½″ more, and 24 tons pressure would contract it 2″. But if the first pressure of 12 tons be removed, the bar will return only about ½″ back, for the permanent set due to 12 tons per square inch will be about ½ an inch. If we *now* put 6 tons pressure on the bar it will extend only ¼″, 12 tons will of course extend it ½″; and in fact for all weights under 12 tons the bar is twice as stiff as before, but permanently ½ an inch shorter. But when the 12 tons pressure is on, its elasticity for any further weight is as at first, and 6 tons addition would compress the bar ½″ again. The permanent set of wrought iron is believed to be *very small*, compared with that of cast iron.

Hence it will be seen, that when our girder is subjected to its own weight and that of a test-load, doubling any load that will in any probability come upon it in practice, then, besides that all the fitting parts in the bearings and connexions of the girder get ground down, as it were, to a true contact, the iron of the girder also is itself subject to a permanent alteration of form, so as in future to shew a permanent deflection from its original cambre, but to be proportionally stiffer as regards passing loads.

II. And this deflection, caused by passing loads, and from which the girder will recover immediately after the load has passed, is the true deflection of the bridge when in position and in daily use.

It has as yet proved impossible to determine where the permanent set of iron begins, and its true elasticity ends. There is in all probability a continual addition going on to the permanent set of a bridge which is in constant use, but one which never can go beyond a certain limit, and is very soon completely insensible even after long intervals of time. It soon becomes, like the decay of a granite bridge, theoretically certain, but practically nonexisting.

To illustrate the state of deflection in our present iron-work structures, I will here insert the real and calculated deflection of a lattice-girder bridge of 150'0 span, consisting of two girders, the weight of which with two roadways (one railway and one foot-road) was 130 tons, and calculated to sustain a load of $1\frac{1}{2}$ tons per foot run of the bridge.

Under a test-load of 350 tons it deflected at the centre 2.90 inches below the position it occupied when under its own weight only. After the removal of the 350 tons it returned 2.10 inches. The calculated deflection at $\frac{.84}{10{,}000}$ ths per ton per square inch, is 2.15 inches.

Supposititious division of the set in this bridge.

Permanent set due to workmanship in inches	.75
... ... iron	.05
...... deflection to be expected on repeating the test	2.10
	2.90

The object of the succeeding analysis on deflection is to determine if possible,

I. At what cambre a girder should be built, in order that, after testing, its upper or lower edge, as the case may be, on which the roadway rests, may when covered by an ordinary load be a straight line.

II. How to apportion the members of the girder so as to obtain this cambre.

Thus we have seen that the permanent set in the joints, caused by putting a load of 350 tons upon the above bridge, was $\frac{3''}{4}$. Now the whole permanent set in the joints, *including* that needed to support the weight of the bridge, was 1.9 inches.

Hence, the best cambre for traffic at which this bridge could have been built is not more than 3 inches, which would be divided thus:

	inches.
Permanent set of joints due to weight and test-load	1.90
... ... iron, inappreciable, say	.05
Deflection due to a very heavy load of 175 tons	1.05
Total......	3.00

If iron, occupying an important position in a structure, be subject to alternate compression and tension, it is probable that much more violence is done to its fibre, than if it be always subjected to the one or the other. This probability, first suggested by practical experience, is borne out by the consideration of the permanent set, which shews that there is a permanent movement of the fibre in the direction of the last strain which has operated upon the iron. Hence the unwillingness of engineers to adopt any class of structure, in which iron in an important position must be subjected to *both* kinds of strain.

LEMMA XII. *To find the deflection of a girder, due to an uniform contraction and expansion in the upper and lower booms, respectively.*

Let $ABCD$, fig. 22, represent the girder; the lines indicating the booms and pillars.

Let l be its length between bearings in feet,

... d ...... depth .... the centres of gravity of its booms.

Let the top boom be decreased by a regular contraction throughout its whole length, at the rate of e feet per foot run; and the bottom boom lengthened in the same manner e' feet per foot run.

Let pRq be the position into which the line PQ is thrown by this change; the lines pL, RM, qN having been straight and horizontal originally; and $RM =$ the original distance of PQ from the centre line $= x$.

Then if pq meet the centre line produced in O, the ratios

$$\begin{aligned} Op : OR : Oq &= pL : MR : qN, \text{ by Euclid,} \\ &= x(1-e) : x : x(1+e') \\ &= 1-e : 1 : 1+e'; \end{aligned}$$

and therefore are constant throughout the whole bridge. Hence all lines, including AC, BD, originally vertical, now radiate from O; and the line XY, originally straight (which is still of unaltered length), and also CD and AB form arcs of circles round the same point O.

Hence we may justly complete the figure 22.

Let the lines CD, AB assume the positions cd, ab; the dotted line being that which does not alter its length,

then $$Cc = \tfrac{1}{2}l - \tfrac{1}{2}l(1-e) = \tfrac{1}{2}le,$$

$$Aa = \tfrac{1}{2}l(1+e') - \tfrac{1}{2}l = \tfrac{1}{2}le';$$

$$\therefore\ CX : AX \text{ (which} = Cc : Aa) = e : e';$$

$$\therefore\ CX = \frac{e}{e+e'}d, \quad AX = \frac{e'}{e+e'}d \ldots\ldots\ldots\ldots (1).$$

Then if $OX = r$:

Since $$Oa : OX : Oc = ab : XY : cd,$$

or, $$r + \frac{e'}{e+e'}d : r : r - \frac{e}{e+e'}d = l(1+e') : l : l(1-e');$$

$$\therefore\ \frac{e'}{e+e'}d : r : \frac{e}{e+e'}d = le' : l : le$$

$$= e' : 1 : e,$$

$$\therefore\ r = \frac{d}{e+e'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2).$$

Now if Δ feet be the deflection of the girder, $= GL$, we have (Euclid, III. 35),

$$\Delta \times (2 \times OM - \Delta) = XM^2;$$

or $$\Delta \times (2r - \Delta) = \left(\frac{l}{2}\right)^2;$$

or $$\Delta = \frac{l^2}{4} \times \frac{1}{2r}, \text{ very nearly}$$

$$= \frac{l^2}{8r}$$

$$= \frac{l^2}{8d}(e + e') \text{ in feet} \ldots\ldots\ldots\ldots (3).$$

COR. 1. In a theoretically proportioned girder, of which a large plate-web girder may be considered an instance sufficiently close, the deflection due to the booms in feet (3)

$$= \frac{l^2}{8d}(e + e'),$$

where l is the length between bearings in feet,

... d ... depth ...... centres of gravity of booms,

... e is $\frac{.84}{10{,}000} \times$ average tonnage per sq. in. on top boom,

... e' is $\frac{.84}{10{,}000} \times$ ... ... bottom boom.

COR. 2. From Lemma XII. (3) it appears that if the deflection under a given load is to be fixed, and we double the length of the bridge, we must make it 4 times the depth; *or* twice the depth, and of half the average tonnage per square inch of boom under the whole weight and load. If the deflection be fixed as so much per foot run of girder, then, if we double the bridge, we have only to double the depth, adopting the same average tonnage per square inch of booms. If by making better *joints* we are able to diminish the amount of metal in the booms, and therefore increase the *average* tonnage per square inch, then the deflection will be increased, unless we increase the depth in the same proportion.

Obs. It may here be remarked, that the *deflection* of a bridge depends on the *average* strength of the booms; the *strength* of a bridge, on the *minimum* strength of the booms, i. e. their strength in the weakest part. Hence the deflection of a girder gives *no clue whatever* to its strength.

Cor. 3. If e and e' be the contraction and expansion of the upper and lower booms at any point of the bridge, then

if e and e' continued for one foot run of the girder, the radius of curvature of that foot length

$$= \frac{d}{e + e'};$$

therefore the alteration of direction, or angle caused by the curvature of this foot (circular measure)

$$= 1'0 : \text{radius of curvature},$$

$$= \frac{e + e'}{d}.$$

In a lattice-girder, we have seen that there is a double deflection of the web: one, due to the shortening of the bars, takes place in a vertical direction; and one, due to the curvature of the bars, creates in them only inappreciable resistance, and was therefore not considered in the calculation. This curvature of the lattice-bars was shewn to be due to the horizontal contraction of the web at the top, caused by the contraction of the upper boom, and to the simultaneous horizontal extension of the web at the bottom caused by the extension of the lower boom. In other words, it corresponds to the alteration in the lengths of the booms; and it is only from the fact of the resistance offered by the bars to the curvature being inappreciable, that we may consider the deflection in a lattice-girder due to the booms and to the lattice as independent of one another.

The deflection of a plate-webbed girder, on the other hand, is sensibly resisted by the plate-web; which not only resists deflection in a way similar to the lattice-web, but also as offering resistance to the contraction and extension of the booms.

The amount of resistance offered by the plate-web, in each character, will be shewn in the following Lemmas.

LEMMA XIII. *To find the deflection of a plate-girder, due to a uniform yield of its web, apart from the alteration in the length of its booms.*

Let AB, fig. 23, be the pillar of a plate-girder, GH its centre line, $PQRS$ any narrow portion of the web-plate.

Let W be the weight of the bridge towards GH, which has to be supported by the web at PQ;

e be the extension, per foot run, which would result if the vertical weight W acted horizontally on the web PQ (or RS), either in compression or tension;

κ be the area of the section of the web along PQ.

Divide $PQRS$ into an indefinite number of strips, inclined at an angle of 45° to the horizon, which we will suppose, during our investigation, free to move upon one another; i. e. separated along their edges, but attached at their ends along the lines PQ and RS. Then the sum of the cross sections of the strips is $\frac{1}{\sqrt{2}}\kappa$.

Now the weight W hanging upon RS is equivalent to two, each equal to $\frac{1}{\sqrt{2}}W$, spread in their action uniformly along RS, like W, but inclined downwards, to the right and to the left respectively, at an angle of 45°.

Of these, we will suppose the latter, since it acts in the direction of the strips shewn in the figure, to be sustained by those strips; then the aggregate tension of $\frac{1}{\sqrt{2}}W$ will be acting on an aggregate area of $\frac{1}{\sqrt{2}}\kappa$.

And since e is the extension of an area κ under a strain W tons,

therefore, e .... $\frac{1}{\sqrt{2}}\kappa$ .... $\frac{1}{\sqrt{2}}W$ ...;

therefore the actual extension of each strip is $e \times$ its length, which is $PS \times \sqrt{2}$; i.e. equals $\sqrt{2} . e . PS$. And RS will, therefore, move bodily downwards and to the left, in a direction inclined at 45° to the horizon, to an extent $\sqrt{2} . e . PS$. Or, in other words, RS will move so as to be $e . PS$ feet lower than it was originally, and ePS more to the left also.

Now the strips have not slid upon each other at all, and therefore we may suppose them to unite again. And then we may cut the plate $PQRS$, still extended as above, into strips at right angles to the former. If we now consider the other portion $\frac{1}{\sqrt{2}} W$ of the weight W, which portion acts in a line with the new set of strips, and is to be resisted by them, similar reasoning will shew that the line RS will be brought *downwards* another distance equal to $e . PS$ and moved also to the *right*, back again to its original line, without causing any sliding of one part on another.

We have thus accounted for all the movements in the particles of iron in any element $PQRS$, which would be caused by the action of a weight W in RS, resolved in two directions at right angles to one another. The artifice of imagining the iron temporarily cut into strips in the direction of each force which that iron had to resist, is simply used in order to assist the thoughts, by enabling them to exclude all those resistances in the iron which can be of no use in resisting that force. The result is shewn to be that RS descends $2e \times PS$, and that all other elements, like $PQRS$, in the web $ABGH$ do the same.

Now if the web yield uniformly, as being always of a thickness proportionate to the vertical strain at any point, then e is constant throughout the web, and

$$\text{the descent of } GH = \begin{cases} \text{the sum of the descent of all the points } S \\ \text{below the points } P, \text{ in all the elements,} \end{cases}$$

$$= 2e \text{ (sum of all the distances } PS)$$

$$= 2e \times AH.$$

Since no curvature nor alteration of length is caused in the booms by this deflection of the web, it is the just deflection to be ascribed to the web.

COR. 1. If e' be the extension per ton per square inch of the iron in the web, e the contraction per ton per square inch, then it follows that

$$\text{the descent per foot run} = \frac{W}{\kappa}(e + e'),$$

at any point; where W tons and κ square inches are the weight and section at that point of the web.

COR. 2. And in any case, if the thickness of the web be not proportionate to the vertical strain, but vary gradually from that proportion, still, considering e constant within the element $PQRS$,

$$\text{the descent of } GH = \begin{cases}\text{the sum of the descent of all the points } S \\ \text{below the points } P, \text{ in all the elements,}\end{cases}$$

$$= 2 \times (\text{sum of the products } e \times PS)$$

$$= \begin{cases} 2 \times (\text{average value of } e \text{ between } A \text{ and} \\ H) \times AH \end{cases}$$

$$= El \ldots\ldots\ldots\ldots\ldots\ldots (1),$$

if E be the average value of e, and l the span.

In this case the deflection due to the web is curved, and, therefore, is modified, insensibly, by the resistance of the booms to flexure. It will always be so curved near the centre of the girder.

COR. 3. In the case of a lattice-web.

If E be the average extension or compression of the lattice-bars per foot run, l the span of the bridge;

Then, the deflection of any part x feet from the pier

$$= 2Ex;$$

And the centre of deflection due to lattice

$$= 2E \times \frac{l}{2} = El \ldots\ldots\ldots\ldots\ldots (2).$$

LEMMA XIII. A. *To find the relation between the section of a plate-web and its resistance to the contraction and expansion of the booms.*

Let e be the contraction of the top boom,

e' ...... expansion ...... bottom ......

Then, Lemma XII. (1), the distance of the neutral line on the web, (XY fig. 22,) from the centres of gravity of the top and bottom booms, is, respectively,

$$\frac{e}{e+e'}\,d, \text{ and } \frac{e'}{e+e'}\,d.$$

Let b be the thickness of the web, z the distance of an element, $b\,dz$ in area, from the neutral axis; t the strain per square inch in the boom, necessary to produce the contraction e.

Then the strain on our element z feet above the neutral axis

$$= b\,dz \times \frac{z(e+e')}{de} \times t = \frac{bt}{d}\,\frac{e+e'}{e}\,z\,dz;$$

and the moment about the neutral axis

$$= \frac{bt}{d}\,\frac{e+e'}{e}\,z^2dz.$$

Integrating from $z = 0$ to $z = \frac{e}{e+e'}\,d$, we get the aggregate moment of the strain on the section of web lying above the neutral axis

$$= \tfrac{1}{3}bt\,\frac{e^2}{(e+e')^2}\,d^2.$$

Now the section of the web above the neutral axis is $\frac{e}{e+e'}\,bd$; and if this were put into the upper boom, it would support a total strain

$$\frac{e}{e+e'}, btd,$$

and a total moment about the neutral axis

$$\frac{e^2}{(e+e')^2} btd^2.$$

Hence the strain taken by the web above the neutral axis, since it is one-third of this, is one-third of the strain it would take if in the boom.

So the web below the neutral axis takes one-third the strain it would if in the lower boom.

COR. If $e = e'$, as is generally the case in a plate-girder, the neutral line divides the girder in half. In this case we have only to increase the gross sectional area of each boom by one-sixth the gross section of the web, in any cross section of the girder, and then we may calculate the deflection, as if the booms were actually of this increased sectional area; and the result will include the accurate allowance for the resistance of the web to the deflection caused by the booms.

LEMMA XIV. *To find the deflection at any point of a girder caused by a value of* e, e' *differing at different points of the girder, considering the booms only.*

If e e' be the contraction and expansion per foot run at a distance x from either pier, A or B;

α be the alteration of direction in the girder, per foot run, consequent thereon; then,

First, an angle α at C, fig. 24 (α very small), will bring any straight line AB, originally drawn horizontally on the girder, to the position ACB*. Suppose P to be the point, z feet from A†,

* There will not in *practice* be this angle at c, but a curve a foot long, to which AC, BC are tangents; but this fact would produce no appreciable difference in any result, since α is very small; and, in fact, the following reasoning only applies, if α be as small as it really is in practice.

† A point x (or z, &c.) feet from the pier will hereafter be called 'the point x', (z, &c.), in investigations of this kind, provided no ambiguity can result.

at which we want to ascertain the amount of deflection caused by this angle α at C, which is x feet from A. Produce BC to D, then

$$AD = AC \times \alpha = x\alpha;$$

$$\therefore\ Pp = \frac{BP}{AB} \times AD$$

$$= \frac{l-z}{l} x\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now $\alpha = \frac{e+e'}{d}$ (Lemma XII. Cor. 3),

$$\therefore\ Pp = \frac{(l-z)x}{l} \times \frac{e+e'}{d}.$$

This is true for all values of x from 0 to z.

Therefore, similarly, the deflection due to an angle α at x' feet from B, where $x' < l - z$

$$= \frac{z}{l} x'\alpha \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= \frac{x'z}{l} \times \frac{e+e'}{d}.$$

Again, the total deflection at z

= { sum of the deflections caused by all the alteration of direction between A and P
+ sum of all caused between B and P in exactly similar method }

$$= \int_0^z \alpha \frac{l-z}{l} x\,dx + \int_0^{l-z} \alpha \frac{z}{l} x'\,dx'$$

$$= \frac{l-z}{l}\int_0^z \alpha x\,dx + \frac{z}{l}\int_0^{l-z} \alpha x'dx'$$

$$= \frac{l-z}{ld}\int_0^z (e+e')\,x\,dx + \frac{z}{ld}\int_0^{l-z} (e+e')\,x'dx' \ldots\ldots\ldots (3),$$

where α, e, e' are the same functions of x as of x'.

COR. 1. The deflection at a point z, caused by an uniform alteration e, e' in the booms between two points distant a and b from A, where a and b are both less than z,

$$=\frac{l-z}{ld}\int_a^b (e+e')\,xdx = \frac{1}{2}(e+e')\,\frac{l-z}{ld}\,(b^2-a^2).$$

Next, if a and b are both measured from B, and are less than $l-z$, then the deflection at z

$$=\frac{z}{ld}\int_a^b (e+e')\,x'dx' = \frac{1}{2}(e+e')\,\frac{z}{ld}\,(b^2-a^2).$$

Hence, if e, e' be constant throughout the girder, the lemma gives us the total deflection at z, equal to the sum of the deflections caused on each side of z,

$$=\frac{e+e'}{2ld}\{(l-z)z^2+z(l-z)^2\}$$

$$=\frac{1}{2}(e+e')\,\frac{(l-z)z}{ld}\,(l-z+z)$$

$$=\frac{1}{2d}(e+e')(l-z)z \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4):$$

and if e, e' be constant from a to b only, then the deflection at z

$$=\frac{1}{2}(e+e')\,\frac{l-z}{ld}\,(b^2-a^2),\ \text{ or }\ \frac{1}{2}(e+e')\,\frac{z}{ld}\,(b^2-a^2)\ldots(5),$$

according as a, b are measured from the same or the opposite pier, as z.

COR. 2. The alteration of direction of the booms, which takes place between a and b feet from A,

$$=(b-a)\,\frac{e+e'}{d}\quad \text{Lemma XII. Cor. 3};$$

and deflects the point z feet from A, to an extent (Cor. 1)

$$\frac{1}{2}(e+e')\,\frac{l-z}{ld}\,(b^2-a^2).$$

But if we suppose the above alteration of direction of the booms, instead of taking place gradually between a and b, to take place suddenly at a point halfway between them, i.e. $\frac{a+b}{2}$ feet from the pier A, then the deflection at z feet from A (Lemma XIV. 1.)

$$=\frac{l-z}{l}\frac{a+b}{2}\times\frac{e+e'}{d}(b-a),$$

which is exactly the same.

Or, in other words; if we be considering the effect of an uniform deflection between two points, upon a point in the girder outside them, then we may consider the deflection of the girder between the two points to take place suddenly halfway between them, in the form of an angle, instead of gradually, in the form of a curve .... (6).

LEMMA XV. *To find the deflection due to booms, of a girder as actually constructed, and subjected to any given load.*

Divide the girder into lengths (chosen so as to give the truest result), throughout each of which we may consider the deflection, *i.e.* the value of e, e', as uniform, with approximate truth. In this lemma we will take for illustration a girder 80′0 long divided into 10 parts of 8′0 each.

Write down at each division the strain in tons under the given load.

Also write down by each length the mean of the strains, which you have written, as above, at each end of that length; and the average gross section of that length.

Dividing one by the other you will get the average tonnage per square inch of section of the booms, for each length: the sum of the tonnage of both booms of a division multiplied by $\frac{.84}{10,000}$ will give the sum of the contraction and extension per foot run, $(e+e')$, for each division.

Then, this last, divided by the depth of the girder between the centres of gravities of the booms, gives the change of direction per foot run of the booms in that division, (Lemma XII. Cor. 3).

And this multiplied by the length of each division will therefore give the whole change of direction in one division: call this $\alpha_1, \alpha_2 \ldots\ldots \alpha_5$ in the five divisions on either side of the centre line of the girder we are considering.

Now Lemma XIV. Cor. 2. The deflection z feet from one end A of our girder, due to the change of direction α_1 between 0 and 8′0,

$$= \text{that due to a change of direction } \alpha_1 \textit{ at } 4'0$$

$$= \frac{l-z}{l} x\alpha, \text{ Lemma XIV. (1)}$$

$$= \frac{80' - z}{80'} \times 4\alpha_1,$$

so the deflection at z feet due to α_2

$$= \frac{80 - z}{80} \times 12\alpha_2,$$

and so on. The positions at which the deflection is required should correspond with divisions of the girder if possible; and when you have got to the division next to z from the side A, begin at the side B. Thus the deflection z feet from A caused by α_1 in the first division from B

$$= \frac{z}{80} 4\alpha_1,$$

and so on. The total deflection at z will be the sum of these.

Cor. If $z = 40'0$ we get the central deflection due to each angle α_1

$$= \tfrac{1}{2} \times 4\alpha_1;$$

and therefore to both angles α_1 in the two outer divisions

$$= 4\alpha_1.$$

So for $\alpha_2 \alpha_3 \alpha_4 \alpha_5$; so that we find the total central deflection

$$= 4\alpha_1 + 12\alpha_2 + 20\alpha_3 + 28\alpha_4 + 36\alpha_5 \ldots\ldots\ldots (1);$$

and generally,—To find the central deflection of a girder due to the booms, divide one half the girder into portions; add together the average tonnage of both booms for each portion; multiply each sum by the length of the portion $\times \dfrac{.84}{10{,}000} \times$ the distance of its centre from the nearest pier, $\div$ the depth of the girder. Add together the amount thus arrived at for every portion, and the result is the central deflection in feet.

If the length of each portion be the same, and the depth of the girder uniform, the multiplication by the former, and division by the latter, may be performed for all the deflections at once, since it will be the same for all.

This centre deflection may be illustrated as in figure 26.

There the curvature of $A1$ and $B9$, if operating alone, would bring a line drawn previously straight upon the girder, to the form $AabB$; where the deflection at the centre

$$= A1 \times \angle aA1 = 8'0 \times \tfrac{1}{2}\alpha_1 = 4\alpha_1, \text{ as above.}$$

Now if a fresh line AB be drawn on the girder, between the same points A and B, and then the curvature of **12, 89** permitted, that will bring this line to the form $AcdEB$, where the deflection at the centre

$$= A1 \times \angle 1Ac + \mathbf{12} \times \angle xcd$$

$$= 8 \times \alpha_2 + 8 \times \tfrac{1}{2}\alpha_2 = 12\alpha_2, \text{ as before;}$$

and so, the deflection at centre due to the curvature of **23** and **87**

$$= 16\alpha_3 + 8 \times \tfrac{1}{2}\alpha_3,$$

and so on; so that the total deflection at centre equal the sum of

$$\begin{aligned} CD, CE, \ldots CH = &\quad 4\alpha_1 \\ &+ 8\alpha_2 + 4\alpha_2 \\ &+ 16\alpha_3 + 4\alpha_3 \\ &+ \ldots\ldots\ldots\ldots \end{aligned}$$

which agrees with the rule given above.

LEMMA XVI. *To find the deflection of a bridge, due to booms, at the front of an uniform load as it progresses from one pier of a theoretically proportioned bridge till it covers the whole bridge.*

Let W tons per foot run, be the load in question.

- α, a function of x (or x'), be the angle caused per foot run in a girder at the point x (or x') feet from the piers, when the load has come on a length z feet from pier A (fig. 25).
- e, a function of x (or x'), as well as of z, be the sum of the contraction and expansion of the booms per foot run at x or x', under the load Wz.
- E, a constant, be the same at every point of the bridge under a full load Wl tons.
- δ (variable) the deflection at front of the load Wz.
- Δ (constant) the deflection at centre under the full load Wl.

Then E, which is uniform throughout the girder, is due at the point x (or x') to a strain on the booms

$$= \frac{W}{2d}(l-x)\,x, \text{ Lemma VII. (1),}$$

e at point x is due to a strain (see figure),

$$Wz \times \frac{l-\frac{1}{2}z}{l} \times \frac{x}{d} - Wx \times \tfrac{1}{2}\frac{x}{d} = \frac{W}{2ld} \times z\,(2l-z)\,x - W\frac{x^2}{2d}$$

$$= \frac{W}{2ld}(2lz - z^2 - lx)\,x:$$

e at point x' is due to a strain (see figure),

$$Wz \times \frac{\frac{1}{2}z}{l} \times \frac{x'}{d} = \frac{W}{2ld}z^2x';$$

therefore at the point x,

$$\frac{e}{E}, \text{ which} = \frac{\text{strain at } x \text{ due to } Wz}{\text{strain at } x \text{ due to } Wl}.$$

$$= \frac{\frac{W}{2ld}(2lz - z^2 - lx)\,x}{\frac{W}{2d}(l-x)\,x} = \frac{2lz - z^2 - lx}{l\,(l-x)}.$$

So at point x',

$$\frac{e}{E}=\frac{z^2}{l(l-x')}.$$

Therefore the total deflection at z (Lemma XIV. 3), or

$$\delta=\frac{l-z}{ld}\int_0^z\frac{2lz-z^2-lx}{l(l-x)}Ex\,dx+\frac{z}{ld}\int_0^{l-z}\frac{z^2}{l(l-x')}Ex'\,dx';$$

and (Lemma XII.),

$$\Delta=\frac{El^2}{8d};$$

therefore $$E=\frac{8d}{l^2}\Delta;$$

if we substitute this, we have

$$\delta=\frac{8\Delta}{l^4}\left\{(l-z)\int_0^z\frac{2lz-z^2-lx}{l-x}x\,dx+z^3\int_0^{l-z}\frac{x'\,dx'}{l-x'}\right\}.$$

Integrating this between the limits, we get the total deflection at the front of the train in any position z, to be

$$\delta=4\Delta\left\{2\frac{z}{l}-5\left(\frac{z}{l}\right)^2+3\left(\frac{z}{l}\right)^3-2\left(1-\frac{z}{l}\right)^3\log\frac{l}{l-z}\right.$$
$$\left.+2\left(\frac{z}{l}\right)^3\log\frac{l}{z}\right\}\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Hence, if

$\frac{z}{l}=.1,\quad .2,\quad .3,\quad .4,\quad .5,\quad .6,\quad .7,\quad .8,\quad .9$

$\delta=.016\Delta,\ .084\Delta,\ .204\Delta,\ .353\Delta,\ .5\Delta,\ .604\Delta,\ .636\Delta,\ .556\Delta,\ .344\Delta$

Hence, fig. 27 shews the path of the front wheel of a load as it travels from 0 to 10; which load, if spread evenly over the bridge, would bring it down to the point marked as the deflection under full load. The vertical scale for ordinary cases would be about 60 times the horizontal. The line of rails is supposed to be level before the train reaches the bridge; and the deflection considered, is that due to the booms of the girders only.

COR. Differentiating the value of δ, we get

$$\frac{d\delta}{dz} = 4\frac{\Delta}{l}\left\{2 - 10\left(\frac{z}{l}\right) + 9\left(\frac{z}{l}\right)^2 + 6\left(1+\frac{z}{l}\right)^2 \log\frac{l}{l-z}\right.$$

$$\left. + 6\left(\frac{z}{l}\right)^2 \log\frac{l}{z} - 2\left(1-\frac{z}{l}\right)^2 - 2\left(\frac{z}{l}\right)^2\right\}$$

$$= \frac{4\Delta}{l}\left\{-6\frac{z}{l} + 5\left(\frac{z}{l}\right)^2 + 6\left(1-\frac{z}{l}\right)^2 \log\frac{l}{l-z}\right.$$

$$\left. + 6\left(\frac{z}{l}\right)^2 \log\frac{l}{z}\right\} \text{..............(2).}$$

$$\frac{d^2\delta}{dz^2} = \frac{4\Delta}{l^2}\left\{-6 + 10\frac{z}{l} - 12\left(1-\frac{z}{l}\right)\log\frac{l}{l-z} + 12\frac{z}{l}\log\frac{l}{z}\right.$$

$$\left. + 6\left(1-\frac{z}{l}\right) - 6\frac{z}{l}\right\}$$

$$= \frac{4\Delta}{l^2}\left\{-2\frac{z}{l} - 12\left(1-\frac{z}{l}\right)\log\frac{l}{l-z}\right.$$

$$\left. + 12\frac{z}{l}\log\frac{l}{z}\right\} \text{........................ (3).}$$

Now let us find the curvature when $\frac{z}{l} = \frac{5}{6}$, i. e. just beyond 8, in fig. 27.

This value of $\frac{z}{l}$ is selected by eye as likely to give us the greatest curvature.

In that case,

$$\text{radius of curvature} = \pm \frac{\left\{1 + \left(\frac{d\delta}{dz}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2\delta}{dz^2}};$$

Also, $\quad l = 1, \ \frac{z}{l} = \frac{5}{6};$

$$\therefore \frac{l}{l-z} = 6, \quad \frac{l}{z} = \frac{6}{5},$$

$$\left(\frac{z}{l}\right)^2 = .6944', \quad 1 - \frac{z}{l} = \frac{1}{6};$$

and therefore (2),

$$\frac{d\delta}{dz} = 4\Delta\left(-5 + 3.4722... + \frac{1}{6}\log 6 + \frac{25}{6}\log\frac{6}{5}\right)$$

$$= 4\Delta\,(-5 + 3.4722... + .1297 + .3299) = 4\Delta\,(-5 + 3.9318)$$

$$= -4.2728 \times \Delta;$$

$$\therefore 1 + \left(\frac{d\delta}{dz}\right)^2 = 1 + 18.2568\Delta^2.$$

Again,

$$\frac{d^2\delta}{dz^2} = 4\Delta\,(-1.6666 - 1.5563 + .7918) = 4\Delta\,(-3.2230 + .7918)$$

$$= -9.7248\Delta.$$

Now suppose the deflection Δ to be at the rate of $\frac{1''}{40}$ to one foot run

$$= \frac{1}{480} \text{ of the halfspan, } = \frac{1}{960};$$

therefore $$\frac{d\delta}{dz} = -.00445,$$

and $\therefore$ $$\left\{1 + \left(\frac{d\delta}{dz}\right)^2\right\}^{\frac{3}{2}} = (1 + .0002)^{\frac{3}{2}} = 1.0003:$$

and $$\frac{d^2\delta}{dz^2} = .01013;$$

therefore radius of curvature

$$= \frac{1.0003}{.01013} = 98.7 \text{ times the span.}$$

Now the radius of curvature under the full load is

$$\tfrac{1}{4} \times 960 = 240 \text{ times the span.}$$

Hence the curve described by the *front* of the load is more than double $\left(\frac{5}{2}\right)$ as sharp as that described by any point in the following part of the load, which traverses the bridge when entirely loaded.

I purpose to add a few words *upon the action which takes place between a bridge and a train as it passes over it.*

We will suppose a bridge, length l feet, to form part of a *perfectly* straight and level line of road; and that a train (say of engines) approaches, which is of uniform weight throughout, and is travelling at a regular speed u feet per second. Suppose Δ feet to be the deflection which that train would produce on the bridge, if standing upon it, and covering it; then if it covered it only from one end to the centre the deflection at the centre would be, necessarily, $\frac{1}{2}\,\Delta$ feet.

So much for the bridge; now for the train. Suppose the weight of the train to ride upon springs; which keep it from resting dead upon the wheels, by acting (as springs) to keep the carriage and axle separate. If the condition of the spring be looked at, it will be seen to be merely pressing *equally* downwards upon the axle-box (and so on the wheel and rail), and upwards upon the carriage or weight; the pressure with which it presses, both downwards and upwards, being equal to the weight of the carriage. If a sudden rise occur in the road the axle-box rises, the spring immediately separates the axle-box and carriage with greater force than before, and the latter, whose weight was before exactly equal to the pressure of the springs, and is now therefore less than that pressure, has an *acting* upward force applied to it equal to the *difference* between the pressure of its springs, and its weight: by acting force I mean such as is available to make the carriage rise in conformity to the rise in the road.

So, if a sudden drop, of one inch say, occur in the rail. Let us in this case consider each wheel separately; and suppose that it has to support 6 tons of carriage, by means of a spring which exerts a ton pressure for every inch it is compressed. Also suppose the spring to have had originally 10″ cambre. Then, were the carriage gradually placed upon the spring, gravity acting upon the mass of the carriage would bring it down with 6 tons force upon the spring; which it would compress, inch after inch, until, when it had compressed it 6″, the pressure of the spring would be 6 tons acting upwards and downwards, and would support the action of gravity on the mass of the carriage.

Then the wheel rests on the rail by its own weight due to gravity, and *also* by the pressure of the spring. The action of gravity upon its mass may be one third of a ton; in that case the spring exerts 18 times as much force upon it to keep it to the rail.

Now if this drop of 1″ in the rails suddenly occur, the wheel will not *fall* over it; it will drop the inch (if free from the rail as is supposed) under a force of 18 to 19 times its own weight; and will therefore drop it in $\frac{1}{\sqrt{18}}$ the time it would merely *fall*, or in less than one-fourth the time.

The spring is thus opened one inch, and therefore exerts one ton less pressure on the carriage; and therefore one ton of the action of gravity on the mass of the carriage is unbalanced; the carriage then will drop, but under one-sixth only of the force which it would have, if dropping under gravity alone in free air. The equation under which it commences motion would be, if free,

$$\frac{d^2x}{dt^2} = 32.2\,;$$

but as it is,

$$\frac{d^2x}{dt^2} = \tfrac{1}{6} \times 32.2,$$

and its velocity of dropping, or the jolt, will be eased to $\frac{1}{\sqrt{6}}$, say two-fifths, its natural amount; and the time of its fall to $\frac{1}{\sqrt{6}}$ the time it would take to fall freely*.

* The reader will here see that we are not as yet speaking accurately, since the ton force will not act equably while the carriage drops; and it will in fact drop more slowly than we here state. And the oscillation is much modified by friction, &c.

Still following the motion of the carriage, as it compresses the spring, it is clear that it will descend beyond its stable position, in fact so much as to compress the spring 7″ from its original cambre; and it will then be inclined to go on oscillating until the friction of the spring bring it to rest. But if, just as the carriage reaches its lowest point, the rail rise 1″ again suddenly, the pressure on the rail immediately afterwards will be neither 6 nor 7, but 8 tons, for the spring will be compressed 8″.

To apply this to the bridge. As an express train flies on to the bridge, the wheels of the engine press upon the rails with the full effect of the springs, and the bridge deflects accordingly. The engines drop more leisurely, for they do not fall under unresisted gravity, but only under the excess of the effect of gravity beyond that of the extended springs. The engine falling upon the springs at last compresses them to their original amount; but it will not stop there; it will go on until the excessive action of the springs above the effect of gravity can also stop the momentum generated by its fall; and it is then that the bridge will be burdened by the greatest load, and the springs will be acting with more violence upon the bridge than they would do on the ordinary road: if this greatest action occur just beyond the centre of the bridge, while the wheels are beginning to run up the opposite side of the bridge (a process which would alone cause extra pressure in the springs), the amount of violence called forth in the springs, and which must be endured by the bridge, may become serious.

I propose to investigate the amount of strain added to the natural weight of a train, under the most disadvantageous conditions; viz. with a high speed, and with a length of bridge such as to be most disadvantageous at that speed; and with a bridge whose deflection for a full load is one-eightieth of the span.

It is obviously a question of interest; since much care has been taken, in this country, to make railway bridges deep and stiff.

It is, unfortunately, impossible to investigate this question except upon very forced assumptions. The case I shall investigate will be an unreal one,—of an engine running off a level road on to an inclined plane whose length is half the span, and fall half the deflection of the bridge under a full load. We shall then be able to infer the effect of the road's rising, at once, by a similar inclined plane, to its former level. We see, from fig. 27, and from Lemma XVI., that such an inclined plane could be chosen which should pretty closely coincide with the downward path of the front wheel; and from the Corollary, that when the wheel has reached its lowest point, at about three-fourths of the way over, the alteration of its direction will be very rapidly changed, so as to make it rise rapidly against the spring of the carriage. The effect on the *wheel*, by impact or otherwise, is not that to which we wish to approximate; we are only desiring to approximate to the more gradual effect of the deflection on the *carriage*.

Let a train travelling at the rate of u feet per second be chosen for consideration, and let W tons be the weight upon each spring.

When the spring is already weighted with the load of the carriage, let w tons be the additional force requisite to depress, or to ease, the spring to the extent of 1.′0″, supposing the law of its movement to remain constant. We shall assume the spring so constructed that, when weighted, its variable depression is in proportion to the additional force employed to cause it; and its elevation also in the same proportion to the relief which allows it.

Let x be the number of feet which the carriage is, at any moment t, *higher* than its position of equilibrium upon the spring. Then the force tending to return the carriage $= wx$ tons; or, more properly $-wx$ tons.

It will simplify language and description if we take a point in the carriage at a *fixed height above the rails*, and call it O, and a point coincident with O when the carriage is in equilibrium, but *fixed to the carriage*, as P. Then O may be considered as an origin from which to measure the position and acceleration of P; and the height of P above O at time t will be x feet, and we shall have our equation of motion of the form

$$\frac{d^2x}{dt^2} = -\mu x.$$

But when $wx = -W$ the spring would cease to act; for the force to return P to O would then equal the weight of the carriage; and therefore $\frac{d^2x}{dt^2}$ would $= -g$. Hence,

$$-g = -\mu \frac{W}{w},$$

and $$\mu = \frac{wg}{W}, \quad \text{where } g = 32.2.$$

But the time of an oscillation $= \frac{\pi}{\sqrt{\mu}}$, and therefore is

$$\pi \sqrt{\frac{W}{wg}} \text{ seconds.}$$

Now when the carriage comes to the bridge, then O suddenly falls with an uniform velocity $u \times \frac{1''}{80'0}$ (since the declivity is half of 1″ in 40′0) $= \frac{1}{960} u$. This is equivalent, as far as relative motion is concerned, to O's remaining stationary, and P's being projected upwards with a velocity $\frac{1}{960} u$ feet per second.

The greatest strain will come upon the bridge when P has reached the bottom of its oscillation; that is, in three-fourths of the periodic time of oscillation (for the spring must first open, and then contract just as much farther as it opened). Hence the greatest strain comes upon the bridge in

$$\frac{3}{4} \times \pi \sqrt{\frac{W}{wg}} \text{ seconds,}$$

or at a distance from the top of the inclined plane of

$$\frac{3}{4} u\pi \sqrt{\frac{W}{wg}} \text{ feet} \ldots\ldots\ldots\ldots (1).$$

Also integrating the equation of motion we get

$$\left(\frac{dx}{dt}\right)^2 = C - \mu x^2.$$

Now when $x = 0$, $\frac{dx}{dt} = \frac{u}{960}$, the velocity of projection;

$$\therefore \left(\frac{dx}{dt}\right)^2 = \left(\frac{u}{960}\right)^2 - \mu x^2;$$

and therefore when $\frac{dx}{dt} = 0$, at the end of an oscillation,

$$x = \frac{1}{\sqrt{\mu}} \times \frac{u}{960} = \sqrt{\left(\frac{W}{wg}\right)} \times \frac{u}{960};$$

and therefore the force of the spring in excess of the weight of the carriage, which

$$= wx,$$

$$= \sqrt{\left(\frac{Ww}{g}\right)} \times \frac{u}{960} \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots (2).$$

From (1) we see that the *distance of the most critical point* from the top of the inclined plane,

for the same train $\propto u$, the velocity of the train:

for the same velocity $\propto \sqrt{\frac{1}{w}}$, *i.e.* $\propto$ inversely as the square root of the stiffness of the springs,

$\propto \sqrt{\frac{W}{w}}$, *i.e.* inversely as the square root of the stiffness of the springs under their load;

and is wholly independent of the amount of deflection.

While from (2) we see that the *shock* at the distance given by (1)

$\propto$ the square root of the stiffness of the springs directly,

$\propto$ the velocity directly,

$\propto$ the deflection per foot run directly.

Hence if the springs be four times as pliable in one case as another, the *greatest* pressure takes place twice as far from the pier, but is only half as great; and therefore on the whole is half as dangerous in the first case as the other: for the danger depends only on the increase of the load per cent.

Example. If

$u = 88$, giving the train a velocity of 60 miles per hour,

$w = 24$, giving the springs a stiffness of 1 ton per half-inch,

$W = 6$ tons on each spring.

Then the critical length for the bridge is about *twice* the distance given by (1)

$$= 2 \times \frac{3}{4} \times 88 \times \pi \times \sqrt{\frac{6}{24g}}$$

$$= 66\pi\sqrt{\frac{1}{g}} = 207.34 \times \frac{3}{17}$$

$$= 36.6 \text{ feet,}$$

and the greatest additional strain upon it is about equal to (2)

$$= \sqrt{\left(\frac{6 \times 24}{g}\right)} \times \frac{88}{960}$$

$$= \frac{3}{17} \times \frac{12 \times 88}{960} = \frac{33}{170}$$

$$= \tfrac{1}{5} \text{ nearly.}$$

Now this effect would be very much increased in fact; for by the time the weight has descended upon the spring so as to cause the greatest compression, the direction of the rails may have very rapidly altered (Lemma XVI. Cor.), so as to be thrusting the wheel up again, in an incline of 1″ in 40′0, caused by the depression of the bridge under nearly the whole load. This last effect, if operating alone, would cause the wheel to press with $\frac{3}{5}$ more weight than is due to the weight of the carriage merely.

Also this violence due to the rising of the bridge when the front wheel is more than halfway across, which must take place in $\frac{1}{4}$ of an oscillation, i. e. 6′0 beyond the point where the rise *begins*, will clearly be more developed than that caused by the descent of the wheel down the first half of the bridge; for that descent begins so gradually (fig. 27) that the full oscillation, due to the fall, is never actually called forth.

Thus, though the front wheel really descends, between **2** and **6**, fig. 27, $.60 - .08 = .52$ of the deflection, in a length .4 of the span; equivalent to .65 of the deflection in a length of half the bridge; yet this seems not more than equivalent, in its effect on the carriage, to our imaginary descent of .5 of the deflection down a plane of the length of half the span.

Summary.

We may then suppose, that if a train pass rapidly over a small bridge (*e. g.* an express at 60 miles per hour over a bridge 36 or 38 feet long; but 26′0 long if the springs of the train compress only $\frac{1''}{4}$ to 1 ton) on which the rails are originally level, and which deflects under that train, when covering it, $\frac{1}{80}$ of an inch to every foot in the span; then the action of the weight of the engine falling gradually upon the springs, as the wheels run down the deflected bridge, may cause so great a resistance to their rising again in order to run up the other side of the bridge, that the pressure actually upon the roadway at a point from $\frac{3}{4}$ to $\frac{7}{8}$ of the distance over the bridge, may be, at least, half as much again as the actual load.

If the train run 30 miles an hour, the most dangerous length of bridge would be half the above, and the heaviest pressure $\frac{1}{4}$ as much again as the actual load.

The same thing would happen with any carriage which passed over the bridge in the *middle* of the train; the one-fifth extra pressure caused (in our Example) by the descent would become two-fifths theoretically from the inclination being steeper; it would also be practically more surely obtained, since the descent begins more abruptly; but the violence caused by ascending the opposite side of the bridge would be less, for the curvature is more than twice as gradual as that under the front wheel (Lemma XVI. Cor.).

The above considerations (Lemma XVI. Cor. &c.) seem to have received a practical exemplification in the case of the bridge over the Dee at Chester. When that bridge broke, the engine was so far advanced that it actually went safely over, dragging the tender after it, though the latter was thrown completely off the rails. The carriages went into the river. Little reliance, however, can be placed on an instance of this kind.

The dynamical effect of a train's passing over a bridge, can be much diminished by giving a cambre to the rails which run over it. But since this cannot be made to suit all trains at all velocities, nor in fact all parts of the same train, and practically can only be rather roughly adjusted, it is well to allow a greater strength to bridges below 40′0 span. In wrought iron bridges the efficiency of iron in such bridges may be taken at 4 tons instead of 4½; and that will give a sufficient margin of strength. It must be remembered that the deflection, and therefore the shock, is diminished in proportion as the stiffness of the girder is increased; and therefore, in proportion as its depth is increased.

Also in designing bridges under 40′0 in span, it is decidedly well to avoid the collecting of a number of weak joints in the booms, just at the points on which the engine of a passing train may act with most violence. It is not unusual to see girders in which this error has been committed.

CHAPTER IX.

THE CONTINUOUS GIRDER.

IF a roadway have to be carried over a bridge consisting of a number of spans, then it may either be carried upon pairs of detached girders, each girder lying across a single span, and unconnected with those consecutive to it, over the adjacent spans; or it may be carried upon a pair of girders each of which is continuous through the whole bridge, and reaches from one end to the other.

In the latter case each continuous girder must be fixed at one pier, but must lie upon rollers on every other pier, in order that it may expand and contract freely under change of temperature.

Let the figure on the next page represent a continuous girder, of which AB is one of the central, and PQ the terminal span.

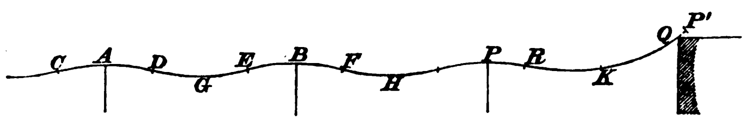
Then if AB be uniformly loaded, whether the adjacent spans be loaded or not, an equal portion of the weight of the load is borne by each pier A and B; and the girder is also symmetrical on each side of its centre G. Hence it follows that:

I. The strain on any part of the *web* between A and B is the same as if the girder were discontinuous at A and B. This is most readily seen in the case of a triangular girder. The reasoning about the web of a triangular (or any other) girder is the same, whether it be continuous or discontinuous.

Hence it follows that the action of the web upon the booms is the same, whether the girder be continuous or discontinuous, or the neighbouring spans loaded or not.

II. As to the booms. Suppose the strains upon the web to be already calculated as for a discontinuous girder, and also the strains on the booms as for a discontinuous girder. These latter will represent the *action* of the web upon the booms, whether the girder be continuous or discontinuous; whether they be *calculated* from consideration of that action, as in the triangular girder, or from independent considerations, as in the lattice- and plate-girders.

In the discontinuous girder, all the action of the web upon the boom has to be sustained by the portion of the boom between the point of such action and the centre of the span. Thus an action of the web upon the top boom at *A* and *B*, causing a strain of fifty tons upon the boom at those points, has to be borne by a pillar of metal, of about 12″ section, extending all the way from *A* to *B*. In the continuous girder this is not the case: for in this girder two points as *D* and *E* are chosen; and all the strains brought from the web upon the booms between *A* and *D*, *B* and *E* are referred to the points *A* and *B* respectively; all such strains between *D* and *E*, to the centre *G*. Thus the average distance to which strains are referred is only about one-fourth of that in the common girder; and, if the points *D* and *E* were constant for all kinds of load, the booms of the girder would be eased about one-fourth.



Elevation of a line upon an uniformly loaded continuous girder, which was drawn horizontally upon it when unloaded. The vertical lines at *A*, *B*, *P* are drawn through the centres of the bearings of the girder upon each pier. Vertical scale = about 150 times the horizontal scale.

LEMMA XVII. *To find the most economical positions for the points of inflection* (*as* D, E) *in a continuous girder, uniformly loaded.*

I. If the whole girder be loaded. In this case the best place for *D*, in order to produce lightness, is that which gives the products of every strain brought by the web upon the boom, multiplied by the distance it has to be carried along the boom before it is counteracted, a minimum; since each strain requires a proportionate section of metal, and that section has to be continued on, until it is met by the counter strain.

Now the strain which is being *added by* the web to the strain on the booms, at a distance y from the centre G, $\propto$ the strain on the web $\propto y = \kappa y$ say.

And we must have the sum of the products of the strains added by the web along AD, multiplied by their distance from A, added to the sum of the products of the strains added along DG, multiplied by their distance from G, a minimum.

In algebraic language this may be stated (calling the distance of an element from A or G respectively z, and the distance AD, which we are seeking, x)

$$\int_0^x \kappa\left(\frac{l}{2} - z\right) \times z\, dz + \int_0^{\left(\frac{l}{2} - x\right)} \kappa z \times z\, dz \text{ a minimum};$$

$$\text{or } \frac{l}{4}x^2 - \frac{1}{3}x^3 - \frac{1}{3}\left(\frac{l}{2} - x\right)^3 \text{ a minimum};$$

differentiating, $\frac{l}{2}x - x^2 - \left(\frac{l}{2} - x\right)^2 = 0,$

or
$$x^2 - \frac{3}{4}lx + \left(\frac{3}{8}l\right)^2 = -\frac{l^2}{8} + \frac{9}{64}l^2 = \frac{1}{64}l^2;$$

$$\therefore\ x = \left(\frac{3}{8} \pm \frac{1}{8}\right)l$$

$$= \frac{1}{2}l, \text{ or } \frac{1}{4}l. \ \ldots\ldots\ldots\ldots\ldots(1).$$

The first gives the maximum, the latter the minimum weight. Therefore the girder should be divided into quarters by the points D, G, E.

COR. 1. Hence all we have to do is to take the strain which we have found for the similar disconnected girder, at the point D one-fourth of the span from the pier, and to subtract that from the strain upon any other point of the boom of the similar disconnected girder. The result will be the strain in the corresponding point of the continuous girder; and if its sign be altered by the subtraction, we know that the character of the strain at such a point is altered from compression to tension, or *vice versa*.

Now, in an ordinary girder the strain on the booms at any point is in proportion to the products of the distances of the point from each end of the girder, Lemma VII.

Therefore, in the discontinuous girder,

the strain at $A = 0$,

$$\text{at } D = C \times \frac{l}{4} \times \frac{3}{4} l, \ C \text{ some constant,}$$

$$= C \times \frac{3}{16} l^2;$$

$$\text{at } G = C \times \frac{l^2}{4}$$

$$= C \times \frac{1}{16} l^2 \text{ more than at } D.$$

Therefore, in the continuous girder the

$$\text{strain at } A : \text{strain at } G = \frac{3}{16} : \frac{1}{16} = 3 : 1 \ldots\ldots (2).$$

This result might have been obtained by considering *DE* as a girder suspended at *D* and *E* from the cantilever girders *CAD* and *FBE*, and the whole loaded uniformly.

II. If the load be upon *AB*, and upon every *alternate* span only, then the loaded spans deflect downwards, and the unloaded spans upwards; the girders lying freely on the rollers upon the piers. There is a strain on the boom at *B*, which assists that at *G* in supporting this action: but this strain at *B* is not so easily called into play as in the first case; since it is not counteracted by a load upon *BH*, but only as the action of the web in BH, under the dead weight, is more and more referred towards B as the girder deflects, instead of partly towards H; or, even, if deflection still go on, by the resistance to curvature of the girder at H, which will be bent upwards. Hence, in order that deflection be stopped before it has gone too far, the booms at G must be stronger.

If the boom actually had no strength at C and F, no tension could be passed from the web beyond, towards A and B. But it is evident, that both strength must be given at the points C, D, E, F, and also additional strength given at $G H$, &c. The case is too complicated for investigation; and, if investigated theoretically, would be useless in practice, since the theory must depend, with great nicety, on the deflection of the various parts; and must assume a nicety in the proportions of the booms, unattainable in practice. If a bridge be designed, however, we may easily ensure its having sufficient strength.

If the web be made strong enough to sustain the weight and moving load, as for an ordinary girder, and the booms be made strong enough to sustain the dead weight as a continuous girder, and the live weight in addition, *either* as though the whole of it were on the continuous bridge, *or*, as though a load

$$= \frac{\text{live weight}}{\text{dead and live}} \times \text{live weight},$$

were on an ordinary girder, the bridge might be considered safe; but the points C, D, E, &c. should not be too weak. This approximation would be sufficiently close for an estimate.

III. *To find the proportions of the last span PQ*, we have only to remember that the strain on the booms at P must be the same, whether we consider it as a point in PQ, or in PB.

Now, in the case of the span AB (which is similar to BP) we have a girder DE, $\frac{l}{2}$ feet long, supported on the end of the cantilever BE; which also has to sustain its own weight, and is $\frac{l}{4}$ feet long;

$$\therefore \text{ the strain at } B = C\left(\frac{l}{4}\times\frac{l}{4}+\frac{l}{4}\times\frac{l}{8}\right), \ (C \text{ a constant})$$

$$= C\times\frac{3}{32}l^2.$$

Now, in the case of the span PQ, if $PR = x$, we have a girder QR, $l - x$ feet long, supported on the end of PR x feet long, which also has to sustain its own weight; therefore, with the same constant as the above, we have

$$\text{the strain at } P = C\cdot\left(\frac{l-x}{2}x + x\times\frac{x}{2}\right)$$

$$= C\times\frac{l}{2}x,$$

and these are equal, therefore

$$x = \frac{3}{16}l. \ \dots\dots\dots\dots\dots\dots\dots\dots (3).$$

On the same scale, the strain at K

$$= C\frac{(l-x)^2}{8} = C\frac{169}{8\times 16\times 16}l^2 = C\frac{4}{3}\times\frac{l^2}{16} \text{ nearly},$$

and the strain of a disconnected girder at centre of span

$$= C\frac{1}{8}l^2.$$

Hence (and from Lemma XVII. 2), the strains of

a disconnected girder at centre : strain at P : at H : at K

$$= 4 : 3 : 1 : 2\tfrac{2}{3} \dots (4).$$

IV. If the bridge, Lemma XVII., have two spans only, of which QP may represent one, then make

$$PR = x; \text{ and } \therefore \ PK = \frac{l+x}{2}, \text{ and } RK = \frac{l-x}{2};$$

and, in order that the weight under a full load be a minimum, we must have

$$\int_0^x \kappa\left(\frac{l+x}{2}-z\right) z\,dz + 2\int_0^{\frac{l-x}{2}} \kappa z^2 dz, \text{ a minimum};$$

$$\text{or, integrating, } \frac{l+x}{4}x^2 - \frac{1}{3}x^3 + \frac{(l-x)^3}{12}, \text{ a minimum};$$

$$\text{or, } \frac{1}{4}lx^2 - \frac{1}{12}x^3 + \frac{1}{12}(l^3 - 3l^2x + 3lx^2 - x^3), \text{ a minimum};$$

$$\text{or, } -\frac{1}{6}x^3 + \frac{1}{2}lx^2 - \frac{1}{4}l^2x + \frac{1}{12}l^3, \text{ a minimum};$$

$$\therefore \text{ differentiating, } \quad -\frac{1}{2}x^2 + lx - \frac{1}{4}l^2 = 0;$$

$$\text{or, } x^2 - 2lx + l^2 = -\frac{1}{2}l^2 + l^2 = \frac{1}{2}l^2,$$

$$\text{and } x = l\left(1 \pm \frac{5}{7}\right), \text{ nearly},$$

$$= \frac{2}{7}l, \text{ nearly}.$$

And the strains at P and K are then readily found.

Remarks on different forms of Girder Bridges.

A tubular girder is a girder with the boom made 16 or 18 feet broad, and with a plate-web, or pair of plate-webs, on each side, connecting the booms together, and having sufficient space between these webs for the roadway. The roadway is laid between the webs (if they be deep enough, as is supposed to be the case): if it be carried on the top of the tube, the girder is not strictly a tubular girder.

The characteristics of the tubular girder are as follows:

1. A great section of boom can be obtained without loading the plates with supplementary angle irons, such as F in fig. 4, Plate I. The boom of a single tubular girder (doing the work of two ordinary girders) may be as much as 16′0 wide. It

may be constructed in the form of a number of longitudinal cells or boxes, rivetted side by side; as is the case in the Menai tubular bridge. Or again, it may be formed merely of several thicknesses of plate, like the Brotherton bridge on the York and North Midland railway.

The *same* section can be obtained as easily by using two double plate-webbed girders of 8′0 width.

2. The *minimum* section of boom is very large. The boom of a girder whose webs are 14′0 or 16′0 apart needs great self-rigidity, otherwise the webs will act only upon the portions of it immediately contiguous to them, and the boom will be inclined to buckle, and double up. Hence, when fined down as much as is safe, the section of the boom is still very large, and a great length near the end is heavier than need be.

3. The webs are so distant in the tubular bridge, that much of the section of the boom, even in bridges of 450 feet span, must be taxed more than the rest; and this produces an indefiniteness, which can only be overruled by putting in *extra* metal so as to make a large and safe *margin* in the actual section, beyond the section required by calculation.

4. In any case, the outer edges of the booms, which are immediately fixed to the webs, are certain to yield to the strain, brought so immediately upon them by the webs, more than the middle portion of the booms, which is remote from the webs. And this inequality would increase to a sensible amount in a long bridge.

This inequality of compression and extension, in both top and bottom booms, must be prevented ever becoming large enough to cause bulging of the web, or other disturbance. To that end large and frequent gusset-plates are necessary in the angles between the booms and webs; and also very frequent and heavy stiffening T irons, and occasional diaphragms. All this iron is used for stiffness merely, and is additional, beyond the amount of iron required by calculation; and much exceeds the extra iron used for stiffening a couple of double plate-webbed girders of less than half the breadth.

5. The tubular girder, with roadway inside, is dark, and therefore difficult to inspect. An accident from an engine running off the rails is more possible in a tubular girder than in an open bridge.

These qualities tell against the tubular bridge; there are others which tell in its favour, as:

6. Since one of the booms is used for a roadway, supporting diaphragms are constructed through it (which may be rather lighter than the cross girders of an open bridge). The booms, if made merely to take strain as booms, would be hollow from end to end of the girder; but in combination with these cross diaphragms, which intersect and can support them, they form a solid floor, capable of sustaining the weight of the engine; so that if the engine do get off the rails, the accident, at a slow speed, is not necessarily destructive. Some engineers would consider the guard-rails used in an open bridge equally efficient against such an accident at a slow speed.

7. The booms form a horizontal bracing of great power; and this I believe to be the main advantage of this kind of bridge. No horizontal diagonal bracing, either above or below the roadway, can ever be formed of rods, which shall be as firm as the horizontal solid plates of the tubular bridge.

This remark always holds in comparing all plate with open work. The web of the plate-girder is much stiffer than any open triangulation, or even lattice, is likely to be. Its efficiency is determined by the rivetting along its margin, which does not by any means make effective the whole metal in the plate. Hence a plate-web is always much stronger, as regards deflection, than as regards its efficient strength: in fact as regards deflection, as a web only, it is doubly as rigid as a lattice of the same weight; and its resistance to curvature may increase its rigidity one-third more.

Advantage was taken of this property of a plate-web in the design for horizontally bracing the bridge for the river Soane, on the East Indian railway.

The roadway there lies on the top of the girders; a horizontal web of plate was formed on the top of the girders, extending from the top boom of one girder to that of the other. This web both formed a platform safe from fire, and also, with the two booms, formed a complete horizontal girder, capable of resisting any oscillation, and requiring only to be firmly secured at the piers of the bridge.

In a bridge of large span composed of two plate-webbed girders, the best position for the cross-girders (which support the road) is perhaps close to the top or the bottom of the main girders.

The angle irons would run all round the cross-girders, and be rivetted at their ends to the webs of the main girders. In this case the cross-girders and internal bracing of the main girders brace the latter in a vertical plane; and to prevent the bridge oscillating horizontally, the booms should be of a sufficient breadth; and any additional horizontal bracing must be in the form of diagonal ties lying close over or under the cross-girders. If the cross-girders be not fixed close to one boom or the other, diagonal bracing in the roadway is the more necessary.

A comparison has been made in the *Quarterly Review* (July, 1858), between the Newark Dyke Bridge, a triangular girder bridge erected on Warren's principle, of equilateral triangulation and turned pin connexions, and the Victoria tubular bridge of Robert Stephenson.

Comparisons of this kind belong to a more practical work than mine; but it will hardly be a digression if I hang a few remarks upon this case of comparison. It is impossible to give a *general* sketch of the relative merits of girders, without previously explaining at length their economic and practical construction; but I am unwilling to leave the subject without hazarding a few remarks.

It appears, in the above comparison, that this triangular bridge is 34 tons heavier than the tubular bridge of 1'0 less span.

Now nearly half the iron-work of the Newark Dyke bridge is cast iron (as Mr Humber carefully states in his book, which is being reviewed), and this weighs much more, but costs less, than wrought iron; this fact destroys the comparison of weight as an evidence of cost. Now, similar reasoning would often apply to structures entirely of wrought iron: their cost is by no means necessarily proportionate to the weight, but may vary above £5 per ton, or above 30 per cent.

It appears that the triangular bridge is more ricketty than the tubular bridge. This must be the case in all bridges whose connexions are made by pins, which, however well turned, do not fit their holes nor bind the plates as rivets do; and especially if the connexions be made by single pins, as in the Newark Dyke bridge. A rivetted bridge may require horizontal bracing to *keep* it straight; a bridge whose main connexions are wholly unrivetted, will require horizontal bracing to force it into straightness at all.

Again, for a girder bridge, whether secured by pins or by rivets, the designer may first give the bridge the weight required by calculation; and then iron may be added for stiffening and bracing, until it be made as firm as to him may seem *desirable.*

If he think that an open bridge will need so much bracing, before it be safe for the particular position which it is to hold, as to make its weight equal to the tube, it will be a question whether it be not better to use the tube. For my part, I believe that in order to apply a given weight of iron so as to give any desirable rigidity to a bridge, already sufficiently strong otherwise, there are plain practical and scientific rules of bracing, which are not all of them complied with in using the complete tube so much as they may be if you put the iron into the bridge in other forms.

As a guide to those who may desire some comparison to be made of the relative value of different forms of girder, I may, pretty safely, state the following.

If, in a particular bridge, it be doubtful whether *any* amount of rigidity in every direction will be sufficient to resist the violence of weather, or of accumulated oscillation under passing loads, then the tube offers the most efficient girder.

Next, in order of rigidity, comes the plate-webbed girder: if necessary the bridge may have a horizontal web between two sister booms, on a level with the roadway, and even an open diagonal horizontal bracing, also, over the roadway (or under, as the case may be), between the two other booms.

If the girders have to carry two roadways, a plate-web is inapplicable, unless the lower road may be left in darkness.

Generally, for girders of above 100′0 span either plate, lattice, or compound triangular girders may be used. Lattice can often be made as stiff as, and is generally cheaper than, plate. Open triangles are durable; but the struts should never be constructed so as to be for more than 6 or 7 feet of their length without a support.

For smaller bridges a simple triangular girder may also be used.

For small bridges there are seldom motives for not using plate, which may then be made to look as well as any other.

As to capability of resisting weather, the general arguments are about even for all kinds, if properly constructed.

CHAPTER X.

THE SUSPENSION BRIDGE.

LEMMA XVIII. *To find the tension of any point of the chain of a common suspension bridge.*

FIG. 29 represents a common suspension bridge. AB being the chain hung in the most desirable arc. (If left to itself, as may be the case, approximately, in large bridges, this arc will be nearly that of the catenary.)

If W be the weight of the chain and roadway between Q and C; and W' the weight of the live load per foot run;

and if y = horizontal distance QC,

α = the angle of the links at Q with the vertical,

T the tension at Q,

P be the point equidistant with Q from the centre line Cx;

it is clear that the weight to be supported by the equal tensions T, at P and Q, acting at an angle α with the vertical, is

$$2W + 2W'y;$$

$$\text{and} \quad \therefore \quad T \times \cos\alpha = W + W'y,$$

$$T = \frac{1}{\cos\alpha}(W + W'y).$$

LEMMA XIX. *To find approximate equations to the curve of a common suspension bridge; with the tension at any point.*

In the figure 29:

Let ACB be the chain,

C its lowest point,

P any point in it,

PT a tangent,

Cx, Cy axes of co-ordinates.

We shall consider all the supporting chains in one, since that one may be divided into two or four afterwards.

Now the strength of the chain at any point must be so great that its net section (with allowance for bolt-holes, &c.) will just sustain efficiently the tension at that point. It will be heaviest towards A, and lightest towards C; the weight of the roadway and live load however, which is horizontal, will be uniform. It will be a sufficient approximation to consider that, if the weight of each point of the roadway and load were transferred and added to the point in the chain directly above it, the result would be a chain of uniform weight. (If this approximation prove not near enough, the result must be modified*.) Suppose this to have been done in the figure.

Let the constants

t = the tension at C, in tons;

w = the weight, in tons, of roadway, load, and chains, per foot run, at C, and therefore, virtually, by our assumption, the weight of the chain throughout;

(κ = the weight, in tons, of a foot run of chain, strong enough to sustain 1 ton efficiently;)

c denote the length, in feet, of chain weighing w tons per foot, whose weight = t, = the tension at the lowest

* If this assumption be, for any particular bridge, very far from the truth, then some other empirical law must be chosen by trial, which shall be truer, and from it the equations to the chain and the tension at any point deduced.

point of the chain. This constant seems to burden us by adding a needless idea $\left(\text{since } c = \frac{t}{w}\right)$; but it will be seen that the form of the analysis forces the idea upon us, and it is, therefore, a relief to have a constant to express it.

And the variables, in feet or tons:

x, y be the abscissa and ordinate of the point P;

s ...... arc CP;

T ...... tension at P;

α ...... angle of the tangent at P with the vertical $= PTM$.

Then the forces which act upon the arc CP are

T, at P in direction PT;

t, at C .... parallel to PM;

ws, which we may suppose to act at the centre of gravity of the arc PC, and which acts in a vertical direction.

As the resultant of this weight ws passes through the centre of gravity of the arc CP, it follows that the directions of T and t must also pass through its centre of gravity, otherwise the arc could not rest in equilibrium. Hence we have, virtually acting through the centre of gravity of CP, the above three forces, respectively parallel to the sides of the triangle PTM and in equilibrium; they must, therefore, be proportionate to those sides.

$$\text{Hence, } \frac{t}{ws}, \text{ or } \frac{c}{s}, = \frac{MP}{MT} = \frac{dy}{dx};$$

$$\therefore \frac{dx}{ds} = \frac{1}{\sqrt{\left(1 + \frac{dy}{dx}\right)^2}} = \frac{s}{\sqrt{(s^2 + c^2)}}.$$

Integrating, since $s = 0$ when $x = 0$,

$$x + c = \sqrt{(s^2 + c^2)};$$

$$\text{or } s^2 = x^2 + 2cx \quad \text{.................}(1)$$

Hence, $\frac{dy}{dx}$, from above,

$$=\frac{c}{\sqrt{(x^2+2cx)}}.$$

Integrating, since $y=0$ when $x=0$,

$$y=c\log\left\{\frac{x+c+\sqrt{(x^2+2cx)}}{c}\right\}\text{.........}(2).$$

Again, from the figure,

$$\frac{T}{ws}=\frac{PT}{PM}=\frac{ds}{dx};$$

$$\text{or}\quad T=w\,.\,s\frac{ds}{dx}:$$

(1) differentiated gives us

$$s\frac{ds}{dx}=x+c;$$

$$\text{and}\ \therefore\ T=w(x+c)\text{..................}(3).$$

A form, more handy in many cases, for the equations (1) and (2), may be got thus:

(2) may be written

$$c\,.\,\epsilon^{\frac{y}{c}}=x+c+\sqrt{(x^2+2cx)};$$

taking over $x+c$, and squaring, we get

$$c^2\epsilon^{2\frac{y}{c}}-2c(x+c)\epsilon^{\frac{y}{c}}=-c^2;$$

$$\therefore\ x+c=\frac{c}{2}(\epsilon^{\frac{y}{c}}+\epsilon^{-\frac{y}{c}})\text{.............}(2)'.$$

And from (1) $s=\sqrt{\{(x+c)^2-c^2\}}$

$$=\frac{c}{2}(\epsilon^{\frac{y}{c}}-\epsilon^{-\frac{y}{c}})\text{........................}(1)'.$$

So also, from above,

$$\tan\alpha,\ \text{which}=\frac{dy}{dx},=\frac{c}{s}\text{..............}(4).$$

PROPOSITION VI.

To construct a suspension bring of 400'0 *span, with the height of the piers* $\frac{1}{10}$ *of that span.*

Given

weight of roadway, without the vertical rods, $\frac{1}{2}$ *ton per ft. run*

... *live load* 1 ....

(We shall consider both chains in one, as they can be separated into any number afterwards.)

To sustain $4\frac{1}{2}$ tons tension we need 1" square of iron. But we shall suppose $\frac{7}{8}\,\frac{1}{16}''$ out of every 7" to be lost at the joints, as in fig. 28, which joints will be 10'0 apart, and involve covers, and head and nut of the bolt, &c. equivalent in weight to 1'6" of the bar, at each end; for though a connexion with a pin *might* be made lighter, it would then involve extraordinary workmanship (possibly patent right) and expense.

Hence to sustain 1 ton tension, we need

$$\text{net section} = \frac{1}{4\frac{1}{2}} = .2222 \text{ sq. inches},$$

lost by rivet $\frac{7}{8}\,\frac{1}{16}'' : 6\frac{1}{16}'' = \frac{15}{97} = \frac{1}{6\frac{1}{2}} = .0342$ {which has to be added.

Total....... .2564

Add 3'0 in 10'0 for joints .0769

.3333 = $\frac{1}{3}$ sq. in. nearly,

which would weigh $\frac{1}{3} \times \frac{10}{3} \times \frac{1}{2240}$ tons per foot run

$$= \frac{1}{9 \times 224};$$

$$\therefore \ \kappa = \frac{1}{9 \times 224} = \frac{1}{2016} = .0005 \text{ nearly.}$$

Upon the countersinking required in fig. 28 it may be remarked, that the widening of the rivet-hole in the outer plate, page 150, into which to hammer down the end of the rivet, can be sufficiently accomplished without drilling, where the plates are so thick. If the hole, in the block upon which the plate is *punched*, be considerably larger than the punch, then the rivet-hole will be at once punched with a sufficient taper for many kinds of work.

The value of κ here obtained will apply with great closeness to any of the following methods of jointing the links of the "chain"; viz. either

1. By rivetting to each end of each bar forming a link, and on one or both sides of it, a short length of bar of the same breadth and of any required thickness, so as virtually to thicken the original bar. Through these thickened ends are then passed one or more pins, through the whole, to connect the joints. These pins, passing through the thickened ends of all the bars which meet at a joint of the chain, will connect them together there.

2. By rivetting one bar, or plate, forming a link, directly to the next by an overlap joint, or indirectly by means of a cover-plate.

3. By swelling, or widening the ends of the bars, by a patented, or other, process, and passing a pin through the swelled ends; (but this last method might perhaps be safely made lighter).

Now the weight of the bridge per foot run at C, the lowest point,

$$= w = \tfrac{1}{2} \text{ ton for roadway,}$$
$$+ 1 \text{ ton for load,}$$
$$+ t\kappa \text{ for chain, where } \kappa = .0005,$$
$$= 1\tfrac{1}{2} + t\kappa \text{ tons} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(I.)}$$

Then at the top of the pier we have

$$y = 200, \qquad x = 40.$$

These values of x and y at the pier would give us c from our equation (2): but as the labour of getting c at once from the equation is great, and may be saved by consulting some such tables as those of Sir Davies Gilbert in the *Philosophical Transactions* for 1826, we shall suppose that we have those tables before us. We extract that

if $y = 100$ and $x = 19.468993$ then $s = 102.483745$ and $c = 260$,

if $y = 100$ and $x = 21.126437$ ... $s = 102.893326$ and $c = 240$,

whence we easily gather by proportion, that

if $y = 100$ and $x = 20$, then $s = 102.6$ and $c = 253.6$;

and therefore, as in our case,

if $y = 200$ and $x = 40$, then $s = 205.2$ and $c = 507.2$....(II.)

Now, $$c = \frac{t}{w} = \text{, from above, } \frac{t}{1.5 + \frac{1}{2000}t}$$
$$= \frac{2000t}{3000 + t};$$

$$\therefore\ 2000t = (3000 + t) \times 507.2,$$

and $$t = \frac{1521600}{1492.8}$$
$$= 1019\tfrac{1}{4} \text{ tons (the tension at the centre)}\ldots(1).$$

Hence we get w; for

$$w = 1.5 + \kappa t = 1.5 + \frac{1019.2}{2000}$$

$$= 2.0096 \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2);$$

hence the weight of the chain, merely, at the bottom = .5096 tons per foot.

Again, we find that at the pier

$$T = w(x + c) = wx + t$$

$$= 2.0096 \times 40 + 1019.2$$

$$= 80.4 + 1019.2$$

$$= 1099.6 \text{ tons (at the pier)} \ldots\ldots\ldots\ldots\ldots\ldots (3).$$

At any other point the tension may be got by combining the equations (3) and (2)′, Lemma XIX.; whence

$$\left.\begin{aligned} T &= \frac{wc}{2}\left(\epsilon^{\frac{y}{c}} + \epsilon^{-\frac{y}{c}}\right), \text{ where } wc = t = 1019.2, \\ &= 509.6 \times \left(\epsilon^{\frac{y}{c}} + \epsilon^{-\frac{y}{c}}\right) \\ \text{also } \log_{10}\epsilon^{\frac{1}{c}} &= .08563; \text{ and therefore} \\ \log_{10}\epsilon^{\frac{y}{c}} &= y \times .08563 \quad \log_{10}\epsilon^{-\frac{y}{c}} = -\, y \times .08563 \end{aligned}\right\} \ldots (4).$$

Now the length of half the chain is 205.2 (II.); and taking its average tension as half the sum of that at the centre and at the pier, (1) and (3), we have the weight of the chain

$$= 205.2 \times (1019.2 + 1099.6) \times \frac{1}{2000}$$

$$= 205.2 \times 2119 \times \frac{1}{2000}$$

$$= 216\tfrac{1}{2} \text{ tons} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

But this weight does not include that of the chain beyond the piers. But supposing the most favourable case of what can be called a single span bridge; viz. when it has, in combination with it, a half-span on each side: then the moorings cannot consume less than 120′0 virtually of the heaviest section of the chain. Now if we add half of this, as though it were the proportion for the centre span, it amounts to

$$60 \times \frac{1100}{2000} = 33 \text{ tons};$$

and we shall be safely under-estimating it.

The suspending rods we will take as averaging 20′0 long, and sustaining the 1.5 × 400′ tons of the road and load, at 3 tons to the inch; this gives their weight

$$20'0 \times 600 \text{ tons} \times \frac{1}{3} \times \frac{10}{3} \text{ lbs.} = 13333 \text{ lbs.}$$

$$= 5\tfrac{1}{2} \text{ tons.}$$

∴ total weight of iron in suspension bridge, as sustaining 600 tons burden,

$$= 216\tfrac{1}{2} + 33 + 5\tfrac{1}{2} = 255 \text{ tons}$$

$$= .425 \text{ tons per ton suspended,}$$

adding then, chain to support the suspending rods, hitherto omitted, $5\frac{1}{2}$ tons × .425 = $2\frac{1}{3}$ tons, we have

Total weight of iron necessary to suspend roadway of $\frac{1}{2}$ ton per foot, and load of 1 ton per foot,

$$= 257\tfrac{1}{3} \text{ tons.}$$

We will now roughly compare this weight of a suspension bridge with the weights, I. of a girder of the same depth as the suspension bridge, and (to carry out the likeness further) having the same value of κ, viz. $\frac{1}{2000}$; II. of a girder of half that depth.

LEMMA XX. *Method of approximation to the weights of two girders, for the same span, viz.* 400′0.

I. Assuming the girder to be 40′0 deep, and to weigh an average of $2\frac{1}{2}$ tons to the foot run, this gives us with load and platform ($1\frac{1}{2}$ tons),

total distributed weight

$$= 400'0 \times (1\tfrac{1}{2} \text{ tons} + 2\tfrac{1}{2} \text{ tons}) = 1600 \text{ tons};$$

therefore central strain on top or bottom boom

$$= \frac{1600}{4} \times \frac{200'}{40'} = \frac{160000}{80} = 2000 \text{ tons};$$

therefore one boom weighs per foot at centre

$$\frac{2000}{2000} = 1 \text{ ton};$$

therefore one boom weighs on the average (Lemma X. 2)

$$= \frac{2}{3} \text{ tons per foot};$$

therefore both booms weigh on the average

$$\frac{4}{3} = 1.333 \text{ tons per foot};$$

therefore both booms have an effective weight (Lemma XI.)

$$= \frac{5}{4} \times \frac{4}{3} = 1.666 \text{ tons per foot.}$$

Now the web we will suppose to be equivalent to 4 no. plate-webs, averaging $\frac{5''}{16}$ thick each, or weighing 50 lbs. per foot sq. in all; their total weight then is

$$50 \text{ lbs.} \times 40' \times 400' = 800{,}000 = 357 \text{ tons},$$

or averaging .89 tons per foot run;

therefore the whole girder has a weight telling upon the centre section, as of

$$1.666\ldots + .89 = 2.56 \text{ tons per foot},$$

very nearly the assumed rate,

And the iron work will actually weigh

$$\text{average per foot run } 1.333 + .89 = 2.23,$$

$$\text{or total } = 2.23 \times 408'0,$$

say, allowing 4'0 bearings,

$$= 908 \text{ tons.}$$

The end pillars will have to support a weight of about 454 tons each at the bottom, and but little weight at the top. Suppose each to have an average horizontal section sufficient to support 227 tons, at two tons to the square inch; then for them κ will be about $\frac{1}{1000}$, and the two will weigh about

$$40'0 \times 2 \text{ no.} \times 227 \times \frac{1}{1000} = 18.16 \text{ tons.}$$

And the whole girder will weigh

$$908 + 18 = 926 \text{ tons} \ldots\ldots\ldots\ldots (1).$$

II. If the girder be constructed of half that depth, assume it to weigh $10\frac{1}{2}$ tons per foot run, then,

total distributed weight

$$= 400' \times 12 \text{ tons} = 4800 \text{ tons};$$

therefore central strain on top, or bottom, booms

$$= \frac{4800}{4} \times \frac{200}{20} = 12000;$$

therefore the top, or bottom, booms weigh per foot at centre

$$\frac{12000}{2000} = 6 \text{ tons};$$

therefore top and bottom together weigh per foot at centre

$$= 12 \text{ tons};$$

therefore top and bottom together weigh on the average

$$\frac{2}{3} \times 12 = 8 \text{ tons},$$

and their weight tells as if of an average weight $= 10$ tons.

The web we will take as equivalent to 4 no. of $\frac{3}{8}''$ thick each, therefore weighing together 60 lbs. per square foot; 480,000 in all, $= 214$ tons, or .53 tons per foot run.

Therefore the total effective weight of the girders is, as it were, about $10\frac{1}{2}$ tons per foot run; as we assumed:

and the girders weigh

$$408 \text{ feet} \times (8 + .53) \text{ tons}$$

$$= 3480 \text{ tons.}$$

The end pillars will weigh

$$20'0 \times 2 \text{ no.} \times 870 \times \frac{1}{1000} = 34.8 \text{ tons.}$$

And the girder complete will weigh

$$3480 + 35 = 3515 \text{ tons} \ldots\ldots\ldots\ldots (2).$$

Lemma XXI. *To illustrate the effect of heat upon a suspension bridge.*

We see from the tables quoted in Proposition VI., that the corresponding increments of c, x, and s for the same value of y, viz. 200'0 are

$$ds = .409481, \quad dc = -20, \quad dx = 1.657446.$$

Hence, for our suspension bridge,

$$\frac{dx}{ds} = \frac{1.657446}{.409481} = 4.048.$$

But if the length of the chain s differ as much as would be caused by a difference of 100° Fahrenheit, i.e. by $\frac{7}{10,000}$ of its length (being .00126 for 180°),

$$ds = 205.2 \times \frac{7}{10,000} = \frac{1436.4}{100,000} = .14364 \text{ feet,}$$

and $\therefore \quad dx = .14364 \times 4.048 = .5814 \text{ feet} = 6.977''$, say $7''$.

But two things exist to affect this deflection of 7″ to the 100 degrees Fahr.

One will diminish it, more or less. If the piers be made of open cast iron work they will expand under 100° Fahr. $\frac{1}{1620}$ of their height (being $\frac{1}{900}$ for 180° Fahr.)

or
$$40' \times 12 \times \frac{1}{1620} \text{ inches}$$
$$= \frac{8''}{27} = \text{say } \tfrac{3}{8} \text{ of an inch.}$$

This is the corresponding expansion of the piers if open, and affected by the change of temperature due to the atmosphere or sun *as much* as the chain. Of course this is not the case if the piers are of stone; but the elimination of the deflection due to this cause is so slight, that it may be neglected, in any case, in practice.

The other circumstance affecting the deflection, is the movement of the point of suspension of the chains; and tells most in a bridge, like the Hungerford or Menai, of one large span and two half side ones. The chain is generally supported on the top of the pier, on a cast iron block, moveable horizontally on rollers. When, then, the side chains expand, since they are fastened to the ground at their lowest point, and therefore cannot deflect there, their extra length allows the cast iron block to run towards the central span so much as to virtually let out as much to the central chain, as it takes up from the side ones. In such a case the deflection would be about doubled, and become 14″ for 100° Fahr.

CHAPTER XI.

HANGING BRIDGES;

INCLUDING THE MIXED FORMS WHICH CONSIST OF A COMBINATION OF A SUSPENSION BRIDGE AND GIRDERS.

THE characteristics of the common suspension bridge, as compared with the girder bridge, are as follows:

I. The weight. We find that with a bridge defined as 400′0 span between its bearings, and 40′0 deep between the centres of the highest and lowest sections of the chains, or boom (as the case may be),

1. The weight of a *chain* capable of sustaining itself and the roadway and load is about 257 tons, or .64 tons per foot run.

2. The weight of a *girder* capable of sustaining itself and the roadway and load is about 908 tons, or 2.27 tons per foot run.

II. The deflection under loads fixed, or moving so slowly as to cause no undulation.

1. Under an uniform load covering the whole bridge the *girder* will deflect the least. We may easily form an approximate comparison of the deflections—(A), of a girder 400′0 by 40 under an uniform alteration of iron of $\frac{1}{4000}$th feet per foot run: and (B), of a suspension bridge chain of 400′0 span and 40′0 versine, under the same violence, and therefore under about one-third of the *load*.

(A) The deflection of the girder due to the alteration of e per foot in both booms (Lemma XII. 3)

$$= \frac{l^2 e}{4d} = \frac{400 \times 400}{4 \times 40 \times 4000}$$

$$= \frac{1}{4} \text{ a foot} = 3'':$$

and that due to the web

$$= le = 400 \times \frac{1}{4000}$$

$$= \frac{1}{10} \text{foot} = 1''.2\,;$$

∴ total deflection of girder $= 4''.2$.......................(A).

(B) As in Lemma XXI, for the suspension chain

$$\frac{dx}{ds} = 4.048.$$

Therefore, if s increase $\frac{1}{4000}$th $= 205.2 \times \frac{1}{4000}$,

then x increases

$$\frac{4.048 \times 205.2}{4000} = .2076 \text{ feet}$$

$$= 2\tfrac{1}{2}'' \text{ nearly};$$

∴ the deflection of the suspension bridge $= 2\frac{1}{2}''$.........(B).

Hence the girder and suspension bridge would deflect about as 3 to 2 under live loads bearing the same proportion to the weight of the bridges loaded. Under the same load, then, the latter will deflect about twice as much as the former.

2. In a flexible suspension bridge a partial load will have to be supported by the chain by means of those vertical rods, only, which attach the *loaded* part of the roadway to the chain.

Hence, if a load reach (for instance) from one pier, over half the bridge; the roadway will deflect, first of all, from the extension of the chain by the extra weight, which *alone* would cause about three times the deflection of the girder of equal depth. Secondly, a deflection will take place, *apart from* any extension of iron, from the chain's being drawn on one side by the load, (which in the case we are considering will render one-half of the bridge double the weight of the other half). This deflection is so considerable that it has sometimes been measured (in actual cases) by *feet* instead of inches; and causes an equal *rise* in the other half of the chain, and therefore of the roadway.

3. When a load, of any extent, comes upon a girder, the weight of the load operates, from the first, throughout the whole girder in the same direction as it will when it covers or leaves it. (There is an exception in a small portion of the web, but it is imperceptible as affecting this argument). Thus any part of the girder, and especially of the booms, which affect deflection the most, is, from the moment the load reaches the bridge until it leaves it, gradually affected in a compression or tension; and each gradually increases to a maximum, and then again decreases to nothing, without ever altering in character. So, therefore, the deflection at every point of the girder gradually increases as the train comes on, and again gradually decreases as the train moves off, but never alters its direction.

This characteristic of a girder, besides being very favourable to the molecular action of iron, and the preservation of its fibre, results also in there being no deflection in the bridge except what is caused by actual extension or contraction of iron. That much more extensive and dangerous kind of deflection caused by *mere* alteration of form cannot possibly take place.

III. The oscillation caused by the sudden taking off, or placing on, (or both) of a partial load.

In this kind of oscillation, the element of *time* enters. The shock caused by an oscillation acting with a given fixed time for each oscillation, at any point, is in proportion to the square of the extreme distance that point is moved from its position of rest. This may be practically shewn thus: If one point in the bridge move through an oscillation of 3″ in a second, and another point oscillate through 6″, also in a second; then the latter vibration is twice the magnitude of the former, and (as each 3″ in the 6 has to be done in half a second) is performed with twice the velocity, it will then, on the whole, be 4 times as violent.

Hence, if a load, as for instance a heavy train, come rapidly upon a bridge of given span, so as to be T seconds in going the length of the span,

1. If it be a *girder* bridge, the maximum deflection at any point will be that due to a train the full length of the bridge, and will not be caused until the whole train is on the bridge, i. e. in T seconds.

Also the amount of deflection depends only upon the alteration of length of iron.

2. If it be a common *suspension* bridge, the maximum deflection, at a point one-fourth of the way across, will be when the front of a train, of half the length of the bridge, has reached the centre. This will be vastly greater than in the case of the girder, and be caused in half the time, viz. $\frac{T}{2}$ seconds. But in the next $\frac{T}{2}$ seconds a still more violent change will have taken place, in the removal of the train from one half of the bridge to the other. In this case the points one-fourth the span distant from the piers have been transferred from their *highest* to their *lowest* positions in $\frac{T}{2}$ seconds; thus quadrupling the violence of the vibration which was caused by the load's coming on to the first half of the bridge.

Thus when we read of a wave 2'0 high being observed upon a suspension bridge, traversed by a train; it is certain that the shock caused to the structure by the passage of the train, as compared with that to a girder bridge whose deflection is 3″ only, (which ∝ velocity × space,) was

$$\left(\frac{2'0}{3''}\right)^2 \times \frac{T}{\frac{1}{2}T} = 2 \times 8^2 = 128 \text{ times as great.}$$

And this is supposing no accumulation of oscillation; though it is certain that considerable accumulation may occur. It is then necessary for a train to traverse such a suspension bridge very slowly. It must not be inferred that the violence done to a suspension bridge is the same as that done to a girder by the same train travelling 128 times as fast. We have only here considered one element which causes violence of deflection; whereas there are many. A very large item in comparison, at high speeds, being due to curvature (in a vertical longitudinal section) of the path which the engine has to describe; and which, being a double curvature on the suspension bridge, promotes violence and accumulation much more on it than on a girder bridge.

We see, then, from the above remarks, what an enormous amount of violence is done to a suspension bridge by the rapid passage of a heavy train, as compared with the violence done by a similar train to a girder bridge. The consideration prepares us for another point of comparison, viz.

IV. The tendency to accumulate oscillations under a continual repetition of the exciting cause.

Hitherto each consideration has shewn us a more and more dangerous characteristic of the suspension bridge. The one on which we now enter is no exception to the rule; the facility with which it accumulates oscillation has proved more dangerous than every other liability of the suspension bridge; and the actual amount of calamity inflicted by bridges which have given way from this cause, prepares us for understanding how great a violence can be called forth in such structures by the regular accumulation of oscillations, from slight disturbances isochronously repeated.

If soldiers are marching upon a suspension bridge, as they come to a point one-fourth span distant from the pier, the chain and bridge may be liable to oscillate under their measured tread, in such a way as to have a node at each pier and in the centre. The momentum to resist oscillation is therefore only a quarter of that which would operate if the whole bridge formed (as a girder bridge does) a single wave. If more nodes were produced the momentum would be still less: or in other words, a less amount of force would be required in order to produce oscillations of a given magnitude. As we have seen above, observation leads us to suppose that unaccumulated oscillation is produced upwards of 100 times more easily in a suspension bridge than in a girder; and if we also consider how readily accumulation of oscillations has, by many accidents, been proved to take place, we begin to see that the locomotive, with her appendages of oscillating pistons, connecting rods and cranks, with her whole weight heaving under the force with which she thrusts the rails, is the last thing to travel safely over a suspension bridge.

V. We now come to compare the effect of heat on the two bridges.

1. It has been seen that in an ordinary *suspension* bridge of 400'0 span, the chain is liable to alter at different temperatures, varying by 100° Fahr., in such a way as to change the height of its lowest point by from 7 to 14 inches. This is worth no consideration for a roadway; and for a railway, if the bridge were so arranged as to make the rails level at a mean temperature; then the fall or rise of 7" in 200'0 would be of small moment.

2. The effect of temperature on a *girder* is simply to lengthen it, and does not cause any deflection whatever. To allow of this lengthening, one end may be supported upon a cast iron block, moveable horizontally on rollers, and thus allowing play to the expansion and contraction.

The above five points will be considered to give a rough comparison of the properties of a suspension and girder bridge; as near, perhaps, as can be exemplified without going at length into details of the different forms of construction, and of the various methods of constructing. If this comparison do not seem, in itself, perfectly fair; at all events I hope that it will not seem to be abused by the inferences I am about to draw from it.

The flexibility of a suspension bridge having been proved by experience to involve danger, as well as precaution; various ways have been started for partially, or wholly, doing away with the latter, and totally setting aside the former. The light weight of the ordinary chain, as compared with a girder capable of sustaining an equal load, gives the suspension principle a great start; inasmuch as much iron may be used, *merely* to stiffen the suspension bridge, so as yet to leave it lighter than a girder bridge would be.

I will now give a sketch of three of the various expedients by which it has been sought to obtain a stiff suspended bridge. In the first which I shall mention (Dredge's), the flexibility of the chain is partially destroyed, and the roadway partially stiffened: in the next (Ordish's) the flexibility of the *chain* is entirely removed: in the third the flexibility of the *roadway* is removed to any required extent. I shall then investigate the proportions of a hanging girder, in which the flexibility of the *chain* can be destroyed to any required extent, without departing from its most simple form; nor, as a consequence, relinquishing some of the advantages of the simple suspension chain. The figures 30—32 and 36 shew the most elementary forms of each kind of bridge.

I. A suspension bridge, which consumes, comparatively, little more iron than the common one, was patented by Mr Dredge, fig. 30. It has, under a union of calculation, modification, and trial, been made by him a very useful and light bridge, stiff enough for road traffic for small spans. It is not adapted for spans of above 200'0* owing to the difficulty of keeping the longer suspending chains straight.

Its principle is to make the bearers of the roadway, which are girders running *longitudinally* under it, as much as possible perform the part also of the chain; this they may do entirely at the centre of the bridge. If, as is sometimes the case, the roadway be made to abut on the piers, these bearers act in compression, diminishing towards the centre. It must obviously

* Even for that span the form shewn in the figure requires such modification, as to encroach somewhat on the principle of the bridge.

require but little more metal to be added to the bearers, when already strong enough to endure the strain of the inclined suspending rods, in order to make them also strong enough to endure, and transmit to these rods, the weight of passing traffic.

In this bridge, then, both the suspending rods are made straighter and stiffer than the ordinary suspension chain, and also a stiffened roadway is obtained with very little extra weight. The whole bridge will generally, however, be heavier than the common suspension bridge with a flexible roadway.

Again, in Dredge's bridge (taking its most simple form, fig. 30) the tendency to form, and accumulate, oscillation is much subdued. For it is clear that in an ordinary suspension bridge the smallest weight placed on the point Q, fig. 29, will draw the chain over Q down, and raise that over P. The chain hangs in an equilibrium in such a way, as to be (as far as it alone is concerned) susceptible of oscillation from the *slightest* cause; and that of a kind which can be *increased* with the same ease, were the cause repeated at proper intervals. In Dredge's bridge this excessive susceptibility is avoided. A weight at G is sustained by the inclined suspension rods adjacent to G, whose tension tends to draw the roadway horizontally towards the nearest pier to G. This tendency is met either by the attachment of the roadway to the pier; or by the tension induced in the suspending rods, in the half of the bridge H. In the latter case these rods will draw up the roadway, and originate an oscillation capable of being increased by a repetition of the pressure at G, just as in the common suspension bridge. But the stiffness of the roadway, and difference of inclination of the rods, prevent the possibility of the different parts of the bridge oscillating either independently or isochronously; no vibration can be in harmony with all.

The difference between an ordinary suspension bridge, and any of Dredge's, in which the roadway is in tension, is in fact like that between an ordinary pendulum swinging upon a knife edge, and the same pendulum oscillating upon two parallel knife edges in a horizontal plane equidistant from its axis. The one bridge is like a plank balanced across a beam; the other like the same plank lying across two parallel beams of equal height.

The above are good points in Dredge's suspension bridge, which make it very suitable for, what in suspension bridges must be called, small spans and for ordinary traffic. But since the evils of the common bridge are only in part modified, and in part unchanged, it is quite inapplicable for locomotive traffic.

It must be noticed, that the advantage of getting iron to act as a stiffening as well as chain, might be secured without making the chain coincide with the roadway platform at all. In an ordinary suspension bridge the links of the chains might be secured by three or four small pins instead of large ones, or even rivetted together; whereby the stiffness would be obtained. Or the chain might by a simple and easy construction be made in the form of a plate or other girder of very shallow web.

II. Ordish's patented suspension bridge, the chief principle of which is illustrated in fig. 31, includes an ordinary suspension chain, in order to sustain, by vertical rods, the dead weight of the inclined suspension chains; which latter sustain the weight of the bridge and load.

In this bridge inclined suspension chains can be used of any length, and yet retained in straight lines, without encroaching on the principle of the bridge. The roadway can be supported, at any number of points, completely by the inclined chains, so that the deflection at each point is only that due to the expansion of the chains, and does not tend to raise any other part of the bridge. Thus a weight at *G*, if the inclined chains supporting that point were proportioned to their strain, would tend to descend; and also to approach the nearer pier in consequence of the greater expansion of the longer rod. Here disturbance might end; since the roadway need not be stiff throughout, but only so between the points suspended. If simple means be taken to prevent longitudinal horizontal motion, by securing the platform at one pier, and hanging the roadway to the inclined chains by short vertical connecting links, then there is no more danger of oscillation than in the girder; by a contemplation of the strains in which, the bridge might have been suggested to the inventor.

In this bridge, however, there are the following weak points if it be applied to locomotive traffic.

1. A rise of temperature of 100° will cause the curved suspension chain to drop, relatively, more than the roadway.

Thus if the bridge have a span of 400′0,
and from the suspending points a depth of 40′0,
and the curved chain have a depth of 14′0, i.e. about one-third of 40′0;

then, if the chains be all made straight at the lowest temperature, and the central ones then have an inclination of α with the horizon, it follows that

$$200'0 \times \frac{1}{\cos\alpha} = \begin{cases}\text{length of the chain supporting the centre} \\ \text{of the roadway,}\end{cases}$$

also $$\frac{7}{10{,}000} = \text{the expansion for } 100^\circ \text{ Fahr.,}$$

and $$\tan\alpha = \frac{40}{200} = \frac{1}{5}.$$

Therefore for an increase of 100° of temperature, the centre of the bridge will fall

$$\frac{200}{\cos\alpha} \times \frac{7}{10{,}000} \times \frac{1}{\sin\alpha} = 200 \times \frac{7}{10{,}000} \times \frac{\sec^2\alpha}{\tan\alpha}$$

$$= \frac{14}{100} \times \frac{26}{25} \times 5 = \frac{182}{250} = .728 \text{ feet}$$

$$= 8\tfrac{1}{2}''.$$

Referring to Gilbert's tables, we find that with a half span of 100 and versed sine of 6.955 in a suspension-chain, the increment of .018 in the length of the half chain causes an increment

.2 in the versed sine; and that half the chain is $100\frac{1}{3}$ long. All these dimensions are just half those of our curved chain.

Hence our half chain $200\frac{2}{3}$ feet long expands $200\frac{2}{3} \times \frac{7}{10,000}$, or say $200 \times \frac{7}{10,000}$ feet; and relatively to the points of support drops at the centre

$$\frac{.2}{.018} \times 200 \times \frac{7}{10,000} \text{ feet}$$

$$= \frac{100}{9} \times \frac{14}{100} = \frac{14}{9} \text{ feet}$$

$$= \frac{56}{3}, \text{ or } 18\tfrac{2}{3} \text{ inches.}$$

The difference 10″, between the two deflections, will give a clue to the sag of the inclined chains; which we may suppose at some points to reach six or seven inches. Thus the deflection which a load will cause, will no longer depend solely on extension of metal, but will depend partly on the straightening of the chains.

2. If a heavy train proceed rapidly on to this bridge, though it will not cause the part in front of it to rise, yet it will not cause the parts in front to sink, nor put a *gradually* increasing tension upon the inclined chains supporting them. Thus at the centre, the train will reach the point of suspension next to the central one without having placed any strain upon the inclined chains supporting the centre. Suppose the distance between the points of suspension in the platform to be 44′0, and the train be going 30 miles per hour; then in one second the engine will have traversed that distance of 44′0, and its full weight will be thrown upon the inclined chains supporting the centre, which are above 200′0 long, and which were previously unstretched. These may deflect, say $\frac{1.5}{10,000}$ of their length (if their average tonnage be 3 tons to the inch under full load, and the live load weigh as much as the dead weight, or half the the full load). This alone would cause the centre of the bridge to

deflect $1\frac{7}{8}''$ in one second, even if the suspending chains were previously quite straight, and their points of support at the piers fixed. (The engine would of course not fall that depth in the second, but would move nearly on a level.) I do not know by experience any great objection to this; but it forms one great distinction between this mode of suspending a roadway, and the girder. But when we remember that the inclined chains may in hot weather be sagging owing to their weight, and to the great deflection of the curved chain, we see an element of vibration and violence existing in this bridge, which trial only can prove to be compatible with security even at moderate speeds. It is *certain*, however, that a speed far higher than the three or four miles an hour at which the suspension bridge across the Niagara is habitually traversed, would be safe on Ordish's bridge.

This bridge, then, is strictly a suspension bridge; yet it bears well a rigid scrutiny, with a view to railway traffic. It is heavier than a common suspension bridge, but lighter than a girder. It has many modifications besides the elementary form shewn in the figure.

III. The unsatisfactoriness of any of the above designs, in which the lightness of the suspending principle is partly relinquished, and in which the simplest and cheapest form of suspension in the ordinary bridge is given up for forms in which the iron chains can convey the strains of a passing weight more directly to the points of suspension, has made engineers again look for resource to another method long ago exercised, viz. of putting ironwork into the platform *merely* to stiffen it. This method is exemplified in wrought iron in the Inverness and Chelsea bridges, by the carrying a light wrought iron girder along both sides of the roadway platform, and suspending the whole by wrought iron suspension chains of the ordinary kind. This trussing is found to make the bridge vastly more rigid and stable.

But I shall pass on to the more complete union of the suspension and girder principle, fig. 32, in which the girder as well as the chain is made to support some of the weight. For it is certain that a girder heavy enough to make a large bridge passable for locomotive traffic would be a condemning weight, unless it were employed in helping to support some of its own, or other, weight.

It is clear that the reason for, and object of, the combination would be this. It is found that an ordinary suspension chain is a safe, and the cheapest, method of supporting dead weight. But it is thoroughly unsafe for a live load; and that

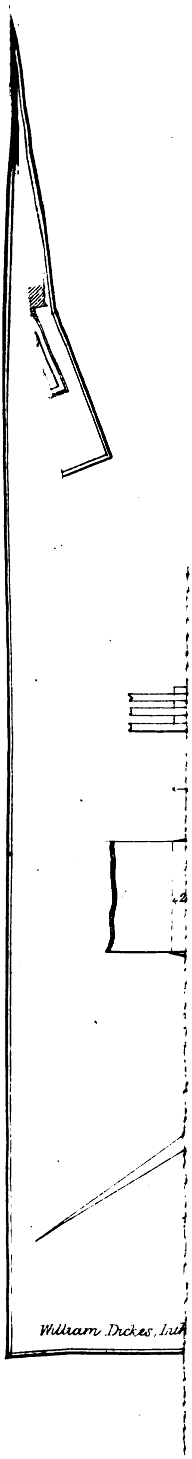
A girder is the only approved method of sustaining a live load in its transit over a bridge.

Therefore, if we reason on the subject, we should say, Why not make an union of the two in a bridge, and employ a chain to support the dead weight, of itself, the roadway and girder; and a girder strong enough to sustain the live load only, in every position, but not proportioned with any reference to its own weight?

If this be once done, and be found rigid and stable enough in practice, then, in the next design, the chain may be made to support a *portion* of live load when distributed,—say a fourth, more or less according to successive trials: this will, of course, enable the girder to be made so much weaker, and will lighten the bridge; since less metal is required in a chain than in a girder, in order to support a given weight.

Thus, by trial, might soon be solved—what is essentially a problem to be solved by trial only—the relative amount of the load to be borne by the girder and chain, in order to make the lightest effective bridge for given railway requirements. Now mechanical principles unfortunately cast a certain amount of chill upon this scheme at the very outset. For, taking our former proportions for the suspension chain, and the same weight per foot run for load and roadway as in Prop. VI., and also the same value of κ, viz. $\frac{1}{2000}$; we will endeavour—

Lemma XXII. *To estimate the weight of a suspended girder bridge, of* 400′0 *span.*


William Dickes, Lit

Our bridge will therefore have

A span between supports of 400'0,
A versed sine to chain of 40'0,
A deflection for range of temperature of 7"

(that is, Lemma XXI., if it be one of a great number of spans it will deflect 7" only, if of a fewer number, more; and if of one span with two side half spans, 14").

Now, if e be the extension of the one boom, and contraction of the other, which will cause a deflection Δ in a girder of a span between supports l and depth d:

$$\text{then (Lemma XII. 3) } \Delta = \frac{l^2}{4d} \times e, \quad \text{or } d = \frac{l^2}{4\Delta} \times e.$$

Hence, in order that our girder should deflect 7" without producing more strain on the booms than 2 tons to the inch, we must have $\left(\text{taking the extension per ton as } \frac{1}{10,000}\right)$

$$l = 400, \qquad e = \frac{2}{10,000}, \qquad \Delta = 7'' = \frac{7}{12} \text{ feet};$$

$$\text{and } \therefore \ d = \frac{(400)^2 \times 12 \times 2}{4 \times 7 \times 10,000}$$

$$= \frac{160,000 \times 6}{7 \times 10,000} = \frac{16 \times 6}{7} = 13'9''.$$

And its greatest section, in order that it may support the live load of 1 ton per foot, will have to carry

$$\frac{400 \text{ tons}}{4} \times \frac{200'0}{13'9} = \frac{80,000}{55} \text{ tons};$$

$$\text{and will weigh } \frac{80,000}{55 \times 2000} = \frac{40}{55} \text{ tons per foot run.}$$

The whole booms of the girder will therefore weigh on the average, Lemma X.,

$$2 \times \frac{2}{3} \times \frac{40}{55} = 1 \text{ ton per foot run} = 400 \text{ tons total.}$$

The webs 400′0 by 13′9, taken at an aggregate of 56 lbs. to the square foot ($= 1\frac{3}{8}''$ thick), i. e. at $\frac{1}{40}$ tons per square foot, will weigh

$$\frac{5500}{40} = 138 \text{ tons};$$

therefore the girder weighs, as affects its suspension, 538 tons.

The chain has to support the girder = 538 tons, the roadway = 200 tons, and itself. We have seen that a chain of 255 tons will sustain a load of 600; therefore *this* chain will weigh

$$\frac{738}{600} \times 255 \text{ tons} = 314 \text{ tons}.$$

This chain then supports

	Tons.	
Itself . . .	314	
The girders .	538	738 tons carried by chain.
The roadway	200	
Total	1052	

Now the girder deflects 7″ without being strained more than 2 tons to the inch, that is, we will suppose, under half its load (if 4 tons per inch, gross section, be the efficiency of the boom). It therefore deflects 14″ under its full load of 400 tons,

$$= \frac{7''}{200} \text{ per ton of load distributed.}$$

And the chain deflects 7″ under an extension of $\frac{7}{10,000}$; such as would be caused by about $\frac{100}{12}$ tons per inch of section: for, while we estimate the deflection of the girder as if the boom extended $\frac{1}{10,000}$ per ton per square inch, in order to include approximately the deflection of the webs, we must only allow $\frac{.85}{10,000}$ extension per ton per inch to the chain. Since, then, the chain is constructed so as to have a tension of 4 tons to the inch under a load of 1052 tons, it will therefore deflect

$$4 \times \frac{12}{100} \times 7'' = 3.36 \text{ inches,}$$

under that load.

$$\text{Or, say, } \frac{7''}{2200} \text{ per ton of load distributed.}$$

Suppose, at the warmest temperature, that the girder, covered by the load, is so adjusted to the chain that both are at full strain.

Then the chain sustains a weight equal to the dead weight, and the girder a weight equal to the live load. On the removal of the live load, 400 tons out of 1452 will be taken from the bridge, and the centre will therefore rise x'' if

$$\frac{2200}{7} x + \frac{200}{7} x = 400;$$

$$\text{or } x = \frac{7}{24} \times 4 = 1\tfrac{1}{6} \text{ inches.}$$

Now suppose extreme cold to have come on, and we will again consider the condition of the bridge.

The centre (taking a case unusually favourable, i. e. supposing the chains *fixed* to the towers at the points of suspension) would now rise 7″ were the chain free; i. e. just as if

$$7'' \div \frac{7}{2200} = 2200 \text{ tons}$$

were taken from the dead weight of the bridge. It will, therefore, actually rise above its position when warm and *loaded*, as if 2600 tons had been then taken off the whole bridge, i. e. x'', if

$$\frac{2200}{7} x + \frac{200}{7} x = 2600;$$

$$\text{or } x = \frac{7}{24} \times 26 = 7\tfrac{7}{12} \text{ inches;}$$

and now, therefore, the girder is carrying weight equal to a deflection of $14'' - 7\frac{7}{12}'' = 6\frac{5}{12}$ inches,

$$= 6\tfrac{5}{12} \times \frac{200}{7} = 183\tfrac{3}{8} \text{ tons.}$$

And when the train has come on, the height above its position when warm and loaded is x'', if

$$\frac{2200}{7}x + \frac{200}{7}x = 2200;$$

$$\text{or } x = \frac{7}{24} \times 22 = 6\tfrac{5}{12} \text{ inches.}$$

And the girder therefore carries only

$$(14 - 6\tfrac{5}{12}) \times \frac{200}{7} = 216\tfrac{5}{8} \text{ tons.}$$

Thus nearly half the live load must be borne by the chain; and to make that safe we must increase the weight of the chain in the ratio 200 tons : 738 tons, of 314 tons (its present weight), = 85 tons. The chain will therefore weigh 400 tons.

Now, if the full load came on when the bridge was cold, a trial might shew that, though half only of the load would be carried by the girder, still the girder would stiffen the bridge enough to prevent objectionable oscillation and vibration.

The girder would, when the load was half on only, be supported in the centre by the chain, and thus have its unloaded end canted up, in which movement the chain would assist. Still I think trial would shew that this double objection would not proceed to any serious extent. And we get our bridge at the following expense of metal.

The chains weigh 314 + 85, say 400 tons; therefore we have that

	tons			tons
The chains weigh........	400	and can support		
		Themselves	=	400
		The girders	=	538
		The roadway	=	200
		200 tons' load....	=	200
		Total......		1338

The girders weigh...... 538 and can support 1 ton per foot in all positions,

The vertical rods weigh 10 and support 938 tons; being 25′6 long, and carrying 3 tons to the sq. in.

total 948 tons of wrought iron work;

which is not less than simple girders might be constructed for, (Lemma XIX.).

If the girder were deeper, its stiffness would throw still more weight on the chain in cold weather, and would oblige us to make the chain still heavier. Were the girder lighter the bridge approaches yet nearer to the ordinary trussed suspension bridges, now so well known.

This great weight is due to the balance of the following difference between this and the ordinary suspension.

In the ordinary suspension bridge,

The chain supports itself, the load, and roadway.

In the mixed bridge,

1. A girder is introduced capable of sustaining the load. This girder weighs 538 tons, while the chain capable of carrying the same weight would weigh 170 only.

2. The chain is strengthened in order to support this girder, but released from about half the weight of the load; thus it is burdened with 538 and saved 200 tons; the balance of which 338 tons increases the weight of the chain.

Hence, not only is the chain heavier instead of lighter than in the ordinary suspension bridge, but a girder is added to the iron work.

The above considerations upon this bridge may be extended, with caution, to similar bridges of other dimensions.

If the chains' deflection were greater under change of temperature, *cæteris paribus*, the girder must be shallower, and therefore heavier, (and therefore the chain also must be heavier), or else the girder, under cold temperature, would be more useless. If the versed sine of the chains were increased, the deflection might be decreased considerably and the chains lightened.

In fact, the above girder has been taken, as of good proportions, and as being a fair example, upon which to base our conception and reasoning. It also forms a link of comparison with the other methods which are got out on the same, or half the same, size.

Still it must not be considered as of the very best proportions in order to obtain *lightness*, any more than the proportions of the other bridges have been chosen with the sole view of making them *light*. The proportions of this, and of the other bridges, are such as could not be found fault with in practice: where all the qualifications which make a good bridge have to be attended to, and weighed against one another.

We now leave this method of mixed bridge, as consisting merely of a combination of two kinds of bridge before treated upon. It is of the less interest on account of the clumsiness and want of harmony between the two kinds of bridges when joined, the deflection being very widely different in each kind under change of temperature, and also very different under both dead and live weight; and we pass on to a more complete and promising union of the two principles.

If the chains of an ordinary suspension bridge be divided into four, two pairs; and one pair be put on one side the roadway and one on the other; if in each pair, fig. 36, the two chains be hung one over the other, and the two braced by diagonals between the joints of the chains, so as to make in appearance a curved triangular-webbed girder; then the two pairs of chains become what we will call two hanging girders, and a roadway suspended from them will be subject to deflection, at all events, at first sight, much less than with an ordinary chain.

The curves of the chains may be made of any shape; the easiest to be put together would probably be the catenary, that giving most ease in preparing the several pieces, the circular arc. We shall suppose them to hang in catenaries, (or their natural curves, rather,) in the following investigation.

Let fig. 36, Plate II. be a skeleton elevation of such a bridge; each girder will be a hanging triangular girder 12′0 deep, and having its bars inclined at an average of 45° to the radii of the chains, and we shall suppose 16½ triangles to go to the whole girder.

In this bridge we have, really and physically, what we found practically in the last, an impossibility of combining the suspension with what can be called strictly the girder system.

For, in fact, *ABCD* is not in the position of an ordinary girder (but is more analogous to the form known as the continuous girder), in that it is fixed at *A B C D*, four points. In an ordinary girder, the end pillar is free to assume an inclined position, coinciding (in a theoretically accurate girder) with the radius of the circle of which the alteration of the booms causes the girder to become an arc. Here that is impossible, since the booms are so much stronger (about five times) as chains than as booms, that the positions of *A B C D* are determined chiefly by the lengths of the booms as chains, and by the lengths of the bars of the web as affecting the curves in which those chains hang; but the strains caused by the web in the booms, as such, are comparatively insignificant in determining the position of *A B C D*, as will hereafter be seen.

Statement of the Strains on the hanging-girder Bridge.

I shall state the strains as if the arc were a catenary, and the temperature suitable.

If the bridge have no live load upon it, then the chains *AECBFD* will bear the dead weight just as in an ordinary suspension bridge, and the web will have no strain upon it.

If the bridge be covered with its full load, the same is the case.

I. If the bridge be half covered with live load, from *M* to *E*, the greatest possible amount of side disturbance takes place.

The load tends to draw the chains, and web, down from *AB* to *EF*, and the chains tend to draw themselves, and the web, up, between *EF* and *CD*, in exactly an equal and similar degree. This is evident, since another half load over *EN* would destroy this tendency of the chains.

If the chains were free, *any* load laid over *ME* would draw down the chains, causing a deflection from *A* to *E* besides that created by the extension or contraction of the iron, and depending only on alteration of position. We wish the web to prevent *any* deflection taking place, unless it be accompanied by extension and contraction of the iron in it.

The *web* must therefore be employed in transferring a portion (about a quarter) of the load which is on *ME*, to the points *A* and *B*: and it must transfer another quarter to the chains between *EF* and *H*, in order to help to keep them down; and thus not only transfer to them that quarter, but, with the assistance of the downward pressure (equal to a quarter also) of the web between *H* and *CD*, make the chains able to sustain the other half load over *ME* untransferred.

Hence we may venture to say approximately, that

bars 1—8, 26—33, (10)—(25) are in compression,

bars 10—25, (1)—(8), (26)—(33) are in tension.

The result of this will be seen to be, that the booms, besides having to support the weight as *suspension chains*, will be taxed with extra strain in performing the office of booms to the lattice. For instance, considering the upper boom in the figure, and at the same time glancing at the above list of web bars, we see that from *D* to the point over *H*, that boom's tension is increased by the web; from thence to the point over *G* decreased, (or perhaps altered to compression), from thence to *B* increased again.

II. If the bridge be covered with a live load from *G* to *H*, over the centre half, then the greatest possible amount of central depression takes place.

This load tends to increase the curvature between G and H, and has an *equally* strong tendency to flatten it beyond G and H; for if the other half load were put on, this tendency would be just counteracted.

In this case then the bars of the web transfer half the weight tending to draw down the booms between G and H, to the parts of the booms beyond G and H.

LEMMA XXIII. *To estimate the strain which it is necessary to provide for in any bar of the web of a hanging double-triangle girder*, fig. 36.

There are various simple ways of considering the strains on the bars. In the first case we will employ two successively, in order to shew how they agree with one another.

I. With a load from E to M, of 1 ton per foot.

1. Suppose the load, of 1 ton per foot, to cover the whole girder. Then the way it is sustained is the following:

The vertical weight of the load is transferred (by means of the vertical rods) to the pins in the corners of the triangles, as in the straight triangular girder.

The tension of the girder's booms, acting as chains* caused by the weight of the load, reacts with such force in the direction nearly normal to the arcs in which they hang, tending to straighten them, as alone resists all the above described pull upon the pins; so that the 1 ton per foot along the whole girder just suffices, by means of its vertical action on the pins, to keep the booms in their curved position, leaving the web unstrained.

Now if we take off the load upon EN, we may suppose at once the tension of the chains diminished one half throughout.

* In this investigation the chains and booms are the same thing in fact; but I shall call them *chains* when referring to their action as chains, and *booms* when referring to their action as booms. Thus I refer to the same member in the girder whether I write boom or chain; but in the former case I shall be speaking of it as keeping the web bars stretched, and in the latter as supporting or creating normal action.

(If the chains were free, they would alter their position at once, and a readjustment of their curves would take place.) The tension then of the chains along *AE*, *BF* only suffices to produce a normal action sufficient to support half the weight on the pins; while the tension in *CE*, *DF*, will straighten those chains until a normal action, of equal amount, be induced on the pins in *CE*, *DF*.

And if we can shew that before the curves of the chains have altered sensibly, the web may cause an action upon the chains of the unloaded half *EN*, exactly equal to that which half a ton per foot on *ME* is causing upon the chains over *ME*; we have only to make the web strong enough to produce this action, in order to secure rigidity to the bridge.

Suppose then half of the 1 ton per foot upon *ME*, to be supported by the chains, we will see how the other half will be supported by the web, which in so supporting it will press down the other unloaded half of the chain, with a force of half a ton per foot.

From *A* to *G* the extra weight of half a ton per foot is supported by the pier *M*, and therefore by the web fixed at *AB*.

The upward action caused by the tension of the chains over *HE* supports the half ton per foot upon *EG*. The upward action between *NH* is resisted by the web *CH*, acting against *CD*.

What will prevent the girder *HG* turning over to the right will be the compression in the upper boom, and tension in the lower over *G*, and the reverse over *H*, induced, as will be hereafter seen, by the web.

We thus see that the bars (taking both girders of the bridge as one)

1, (17), 33 have a compression

$$=\frac{1}{2}200 \text{ tons} \times \frac{1}{4} \times \sqrt{2} = 25 \times \frac{7}{5} = 35\tfrac{1}{3} \text{ tons},$$

(1), 17, (33) have a tension = also $35\frac{1}{3}$ tons.

2. Suppose no load on the girder.

Then by putting a pull upon any pin in the direction of a normal to the chain (approximately, but, correctly speaking, in a vertical direction), we induce a tension in the chain, which it requires an equal normal action at every other pin to balance. Suppose we put, then, a weight upon a pin in *ABEF*, such as would be produced by a load of half a ton per foot on *EM* (neglecting the disarrangement caused by the obliquity of the vertical weight to the slightly inclined normals, as being of trifling amount); we see that, if the chains retain very approximately their former shape, an equal weight must be laid upon all the other pins in the girder. That weight may be laid anywhere upon the roadway, provided a strength can be given to the web and added to the booms, in such a way that a girder shall be superimposed, as it were, on the existing material of the bridge, capable of transmitting such weight to the proper pins in the chains. We may, then, put the weight of half a ton per foot *directly* upon all the pins from *A* to *E*. If we put another half a ton per foot upon ~~it~~ ME, half of this will be borne by the pins *A B*, and half carried by *E F* to the pins of the unloaded half of the girder; the remaining weight upon which must come from the points *CD*.

Thus in either way we get to the same thing; viz. that the chains have a tension enough to bear half a ton load per foot, or 200 tons total, and therefore draw *A* and *B* with a force whose vertical effect is 100 tons, and *C* and *D* with a similar force.

Of this last 100 tons, half is supported by the bars (1) and 33, which rest upon the chains between *CD* and *H*; leaving 50 tons only to be supported by the *pier N*.

The pier *M* has to support 150 tons, 50 tons being laid on it by the bars 1 and (33).

II. If the load cover *GH*, then half is borne by the chains between *G* and *H*, and half transferred to them over *GM*, *HN*.

In this case then

9 and (25) have a compression of about $35\frac{1}{8}$ tons,

(9) and 25 ...... tension .... $35\frac{1}{8}$ ....

III. If the load cover *MG*, *HN*, these last bars have the same strain, but the compression and tension is reversed.

IV. So if the load cover *EN*, the case I. is reversed.

Hence we must make

1, 9, 17, 25, 33,

and the bars of the same numbers within parentheses, capable of sustaining a tension or compression of $35\frac{1}{8}$ tons. And we can easily see that no greater strain can be brought on any of the bars of the web; for if, for instance, the load cover three-fourths of the bridge *MH*, the lattice has to keep down, between *HN*, only half the number of points as when the load covered *ME* only; though each point requires half as much power again to keep it down.

And this may be shewn algebraically; for if the bridge be loaded with the full load per foot run, extending a length of x feet only; then the greatest strain on the bars is at the extremities of the length x, also the tension on the suspension chains

$= $ that due to a load $\frac{x}{l}x$ over the whole bridge;

therefore the weight unsupported on x

$$= \frac{l-x}{l}x;$$

therefore the whole strain on the bars at either end of x

$$= \frac{1}{2}\frac{l-x}{l}x \times \sqrt{2} = \frac{1}{\sqrt{2}.l}(lx - x^2),$$

and is a maximum if

$$l - 2x = 0,$$

or $$x = \frac{l}{2}.$$

In order that the strain upon the bars of the web may be equable, in the way in which we have taken them, we must everywhere suppose that an equal part of the live load is supported by the pins of each boom, as such.

And since the tensions of the upper and lower chains at the points *A B* and *C D* must be, respectively, equal; we must also suppose the weight supported by the chains, as such, to be supported equally by the pins in each. Any disturbance caused in the tension of the booms by the action of the web, must be self-contained, since it must not affect their tensions at the points *A B C D*.

LEMMA XXIV. *To estimate the greatest amount of strain upon the booms of a hanging girder.*

This strain is caused thus: when a load of 1 ton per foot, and x feet long, covers a part only of a girder; then a portion of it equal to $\frac{l-x}{l}$ tons per foot, has to be transferred by the web either to the points of support *A B C D*, or to the chains outside the loaded portion. The web in effecting this transference calls the booms into action.

I. When the load extends from *M*, or *N* by similarity, x feet over the girder, then the strain in the chains at *A B C D*, and at the points x feet from *M* (over the end of the load), must be the same as if the whole bridge were covered with $\frac{x}{l}$ tons per foot. The strains on the *pins* at *A B C D* will not be the same as if the whole span were so covered, because the bars of the web will influence them; as was shewn in our reasoning in the previous Lemma. The strain in the chains over the front of the load will be the same as at either pier; since the strains of the web in each, loaded and unloaded, division of the bridge are necessarily symmetrical with respect to the vertical (or, more correctly, normal) lines bisecting that division.

1. Hence the strain on the booms over the loaded division is equal to that in a girder x feet long, 12′0 deep, and loaded

with $\frac{l-x}{l}$ tons per foot (such being the amount that has to be transferred to the unloaded part of the chain)

$$= \tfrac{1}{4} \cdot \frac{l-x}{l} x \times \frac{x}{24}$$

$$= \frac{1}{96l}(lx^2 - x^3), \text{ or } \frac{1}{96}\frac{l-x}{l}x^2,$$

and is therefore a maximum, when

$$2lx - 3x^2 = 0,$$

or
$$x = \frac{2}{3}l.$$

In that case the strain on the boom $\frac{1}{3}l$ distant from the pier

$$= \frac{1}{96} \times \frac{1}{3} \times \frac{4}{9}(400)^2$$

$$= 247 \text{ tons},$$

which must be provided for by an increase in the strength of the *lower* chain, in which the strain is tensile. Of course the compressive strain only relieves the chain in which it occurs.

2. And the strain on the booms over the unloaded division is equal to that in a girder $l-x$ feet long, 12′0 deep, and loaded with $\frac{x}{l}$ tons per foot; and by similarity to case 1, is a maximum (transposing $l-x$ and x), when

$$l - x = \frac{2}{3}l,$$

or
$$x = \frac{1}{3}l,$$

in which case the maximum strain

$$= \frac{1}{96}\left(\frac{x}{l}\right)(l-x)^2,$$

and is the same as before, but requiring an increase of strength in the *upper* chain (in which the tension is evidently caused) exactly over the points where the lower boom was previously shewn to want a like strengthening.

II. 1. With a central load of 1 ton to the foot, and x feet long, the points where the chains have the tension due to an uniform load $\frac{x}{l}$ per foot run, will be at A, B, C, D. And the web has to spread $\frac{l-x}{l}$ tons per foot of the load on x, over the unloaded portions of the chains towards the piers. Thus the case becomes similar to that of a girder reaching from the centre of one unloaded portion to the centre of the other (such being the centres of bearing), and loaded with $\frac{l-x}{l}$ tons per foot over x feet in the centre. Its total length then is

$$x + \frac{l-x}{2} = \frac{l+x}{2};$$

therefore its central strain

$$= \tfrac{1}{2}\frac{l-x}{l} x \text{ tons} \times \frac{l+x}{4 \times 12} - \tfrac{1}{2}\frac{l-x}{l} x \text{ tons} \times \frac{x}{4 \times 12}$$

$$= \frac{1}{48}\frac{l-x}{l} x \left(\frac{l+x}{2} - \frac{x}{2}\right)$$

$$= \frac{1}{96}(l-x)\,x.$$

And is a maximum when

$$l - 2x = 0,$$

or

$$x = \frac{1}{2} l.$$

In which case the strain at the centre of the booms

$$= \frac{1}{96}(200)^2$$

$$= 417 \text{ tons}.$$

This requires an increase of strength in the lower boom.

2. So a load of $\frac{1}{4}l$ long extending from each pier M and N, would require an equal increase of tension in the top boom at the centre.

3. Also generally, in this case, we shall have a strain in the booms at y feet from the centre, certainly under

$$417\left(1 - \frac{4y^2}{l^2}\right) \text{ tons,}$$

which, Lemma VII. (1), is the strain y feet from the centre of an uniformly loaded girder, whose central strain is 417 tons; i.e.

$$< \frac{4}{l^2} \times 417 \left(\frac{l}{2} + y\right)\left(\frac{l}{2} - y\right);$$

and, if $y = \frac{l}{6}$; then the strain at the point of maximum strain in case I., given by this equation,

$$= 4 \times 417 \times \frac{2}{9} = 371 \text{ tons};$$

which is so far in excess of the maximum strain caused by a partial load, reaching from the pier (as in case I.), that we can certainly conclude, both that the strain of the booms due to this load of half the span long upon the central portion of the bridge, is the maximum strain at the centre of the booms; and also that if the strength of the booms be thence diminished by the same law as in an ordinary girder, then their strength will be more than sufficient for the maximum strains to which they can be exposed at any other point.

LEMMA XXV.—*On the effect which the extension of the chains by load or heat has upon the web.*

We have hitherto considered extension or contraction of the chains by an uniform load to produce no action on the web: this is evidently not the case; for an alteration in the length of the chains must alter the curve in which they hang, and may do violence to the web, which is much weaker than the chains.

Thus, a full load, spread uniformly over the bridge, will lengthen both chains: it would lengthen both to the same amount were it not that the web will tend to draw them a little closer together as they expand in length, (an effect which we may neglect, as inducing but little irregularity,) and will also resist the increase of curvature, caused by the chains being lengthened while the distance between the points of suspension remains unchanged.

Now the strain of the web caused by this increased curvature will be lessened one half, by the use of a bearing girder (shewn by the dark lines in the figure) to support the points AB: the bearing girder itself being supported on an axis at its centre (shewn by the dark circle). The girder AB, and similarly CD, will be thus able to assume an inclined position, just as the end pillar of a common girder is able to do so; and our case becomes similar to that of a common girder.

To find the strain caused by a fall of any number of inches in the centre of the bridge, due to extension of the chains, we may, then, consider the girder straight, and forced to deflect that distance at the centre. As we know the relative strength of the booms and web, we may calculate how much such a deflection will affect the booms, and how much the web; and we must provide extra strength in order to meet the extra strain.

Proposition VII.

To construct a hanging girder of double triangles. Plate II. fig. 36.

Given

Span = 400 *feet between supports.*

Weight of platform = ½ *ton per foot* = 200 *tons total.*

..... *live load* = 1 ...... = 400 .....

Assume the weight 1200 tons; take its depth 12′0; and its proportions as in the figure. There will be two girders and four chains; but we will consider them for the time as one girder and two chains.

I. In Prop. VI. we see that a chain supporting 600 tons and itself, weighs 255 tons; therefore our chains will weigh together

$$\frac{1200}{855} \times 255 = 357 \text{ tons} \ldots\ldots\ldots\ldots\ldots (1).$$

And will together have a tension, Prop. VI. equations (1) and (3),

$$\text{at the pier} = \frac{1200}{855} \times 1100 = 1544 \text{ tons} = 772 \text{ per chain,}$$

$$\text{at the centre} = \frac{1200}{855} \times 1020 = 1432 \text{ tons} = 716 \text{ per chain,}$$

or an average of 744 tons.

II. The strength required in the web, in order that it may carry the variable load.

The total length of the bars of the web is $2 \text{ no.} \times 400'0 \times \sqrt{2}$, and they must bear a strain of $35\frac{1}{8}$ tons each on an unsupported length of $6'0 \times \sqrt{2} = 8'4''$. We will take them, then, of an uniform area of 12″ in section, and they will weigh 40 lbs. per foot, or total

$$2 \times 400 \times \frac{7}{5} \times 40 \div 2240 = 25 \text{ tons} \ldots\ldots\ldots (2).$$

III. The extra strength required in the chains, in order to enable them to act as booms.

We have seen above, that the average tension of one chain, necessary to support 1200 tons on the bridge, is 744 tons;

therefore that necessary to support 100 tons on the bridge

$$= 62 \text{ tons,}$$

and requires an increase of weight in one chain, (1)

$$=357 \times \frac{1}{24} = 15 \text{ tons.}$$

Hence the tension induced in a chain by a load of

$$\tfrac{1}{2} \times 400 \text{ tons} = 2 \times 62 = 124 \text{ tons.}$$

But, Lemma XXIV., if we take off half the load, thus easing the chains of 124 tons strain throughout, we can, by properly placing the other half of the load, induce a tension of 417 tons in the booms at E and F. The difference

$$417 - 124 = 293 \text{ tons}$$

is the *additional strain* to which the chains at E and F are liable, beyond what they have when acting as a chain, merely, under a full load.

It will be observed, that we have never found strictly the maximum amount of this additional strain. We have only found when the positive term in it is a maximum; but I think safety will be completely secured by the following general proportions for the chains. First, let the chains be sufficiently strong to sustain the whole load; that is, for a tension of

772 tons each at the piers,

716 .... the centre.

Next, increase them by a strength sufficient for a strain of 300 tons at the centre, making a total of ~~1072~~ 1416 tons, and increase the rest of the chains by as much section as would be sufficient for a girder of 400 feet span, whose central strain is 300 tons.

Hence Lemma x., the total additional weight in each chain

$$= \frac{2}{3} \times 300 \times \frac{1}{2000} \text{ tons} = \frac{1}{10} \text{ tons per foot,}$$

$$\doteq 40 \text{ tons, total} \ldots\ldots\ldots\ldots (3);$$

and it is not available for carrying its own weight.

Hence, so far our bridge weighs

Platform...........	200 tons
Load	400 ....
Chain	357 ....
Extra in chains...	80 ....
Web	25 ....
Total........	1062 tons,

or above 100 tons short of the assumed weight.

IV. Extra iron required to take the strains induced by the required uniform alteration in the length of the chains.

We know that an expansion due to 100° Fahrenheit will lengthen the chains $\frac{7}{10,000}$, and cause a deflection of 7″; our full load will weigh one-third of the total weight on the chains, and will therefore cause a tension in them of about $\frac{4}{3}$ tons per square inch, and therefore an extension in them of $\frac{4}{3} \times \frac{.84}{10,000}$; or of $\frac{4}{3} \times .84 \times \frac{1}{7} = .16$ times as much as is due to 100° Fahrenheit, *i.e.* about 1″. The chains, then, are liable to vary in height at the centre point to an extent of 8″, owing to a variation in their length of $\frac{8}{10,000}$.

When the chains hang at their lowest deflection, their curvature will be considerably greater than when hanging at their highest point; and this curvature does violence to the web, and induces in it, and therefore in the chains, a considerable initial strain, independent of what the load itself has been calculated to produce.

The main part of this violence could be got rid of by carrying a sufficiently strong iron bar across the span, from one pier to the other; so constructed that its ends should be secured, over the piers, to iron blocks moveable horizontally on rollers; to which blocks the centres of the end girders, in fig. 36, would likewise be secured. In this method the temperature of the atmosphere will affect the bar nearly equally with the chains; and the bar by separating the points of support of the chains, in the same proportion as the chains themselves expand, will constitute with the chains a figure (in elevation) always remaining similar to itself. This bar might be constructed in the form of a tube; which is the form of the one erected by I. K. Brunel, Esq. in the Saltash Bridge. To make it self-supporting it should form an arch. It should be of such strength of section as to resist the pull of the suspension chains upon their bearing blocks, when the full load is on the bridge: and so arched, as to have a tendency to rise from the pull of the suspension chains, when the bridge is unloaded. To prevent it rising it must be connected with the chains, or roadway, at frequent intervals by vertical rods. When a load comes upon the bridge it will depress the tube as much as the chains, and force the bearing blocks farther apart; and so nearly prevent the change of curvature which we are now considering.

This addition of such a tube would add so much to the weight of the simple suspension chain, both in itself and in the bracings it would require, that we will suppose it not used for the bridge we are considering. The addition of the tube alters the character of the bridge from that of a suspension bridge, to a combination of the suspension bridge and arch.

Suppose the chains to hang unrestrained, with the web free; and then suppose the chains to be lengthened or shortened, so as to cause a deflection of $3\frac{1}{2}''$, making the total range $7''$. The inch due to load may be neglected, since under full load we do not care though the web be a little strained.

1. If the bars of the web were united to the chains by pins, some amount of play might be allowed in their joints. A little play would not make the bridge seriously *ricketty* when the load came upon it, and it would ease the bridge triflingly under the alteration of form caused by temperature.

Thus suppose that a play of $\frac{1}{100}''$ exists in each joint of a bar of the web, (I select $\frac{1}{100}''$, since that is the limit of error generally allowed in contracts, in boring the holes for single pins for joints), this would allow a play of $.02''$ in each bar, or a total deflection in $.200'0$ length, in which there are 17 bars,

$$= 17 \times .02 = .34'';$$

we cannot then rely much upon this for ease under changes of temperature.

2. Our hanging girder has chains of an average weight per foot run

$$= \frac{744}{2000} \text{ tons} + \frac{40}{400} = .472 \text{ tons},$$

or an average section $= .472 \times 2240 \times \frac{3}{10}$ sq. in.

$$= 318 \text{ sq. inches};$$

and the bars in its web have $12''$ section.

Now there are 33 bars on each side the centre; which have, therefore, a total section of 396 sq. inches.

Hence a force tending to curve the girder uniformly, and so bringing an equal strain on all the bars, of w tons say, will strain the booms at the centre

$$\frac{396}{\sqrt{2}}w = 283w;$$

and therefore, if the extension (and compression) with which the bars are affected be e, then that with which the centre of the booms will be affected will be

$$\frac{283}{318}e.$$

Now the deflection of the girder caused by e in the *bars*

$$= 2e \times 200'0$$

$$= 400e \text{ .... } (1),$$

and the deflection caused by the expansion and contraction $\frac{283}{318}e$ in the *booms* would be, if uniform,

$$\left(=\frac{l^2e}{4d}\right) = \frac{160,000}{48} \times \frac{8}{9}\text{(nearly)} \times e$$

$$= 3000e \text{ nearly .... } (2).$$

Now the sum of (1) and (2) must be $3\frac{1}{2}''$ (or half of the $7''$, forming the total variation);

$$\therefore\ 400e + 3000e = \frac{7}{2} \times \frac{1}{12},$$

and $\therefore$
$$e = \frac{7}{24} \times \frac{1}{3400}$$

$$= \frac{.86}{10000},$$

causing a strain in the bars of

$$\frac{86}{84} = 1 \text{ ton per square inch, nearly.}$$

Now as they are efficient for 3 tons on the square inch only, this leaves only 2 tons per square inch disposable, or only $\frac{2}{3}$ of what we assumed before.

Therefore we must increase their weight $\frac{1}{2}$, *i.e.*

$$\text{to } \frac{3}{2} \text{ of } 25 = 37\tfrac{1}{2} \text{ tons.}$$

And this will render the chain liable at the centre to an additional strain, which will render it necessary to increase the weight of the "extra metal in the chain."

We have added half as much again to the strength of the web in order to resist uniform *tendency* to curvature in the chains, *i.e.* 6″ section apiece.

Therefore the total section added on half the span

$$= 33 \text{ no.} \times 6'' = 198 \text{ sq. in. at 3 tons per sq. in.}$$

and will require an extra section in the booms at the centre of the span

$$= 198 \times \frac{1}{\sqrt{2}} = 140 \text{ sq. in. at 3 tons per sq. in.}$$

i.e. a section capable of sustaining 420 tons; or an increased weight of

$$\frac{420}{2000} = .21 \text{ tons per foot.}$$

The additional strength required is in inverse proportion to the distance from the centre of the span, and therefore the whole additional weight required in each boom

$$= \tfrac{1}{2} \times 400'0 \times .21 = 42 \text{ tons.}$$

Hence the total additional weight in the chains

$$= 80 \text{ tons} + 2 \text{ no.} \times 42$$
$$= ~~162~~ \; 164 \text{ tons,}$$

and the total weight of our bridge becomes

	tons.
Platform = $\frac{1}{2}$ a ton per foot =	200
Load = 1	400
Chains to carry 1200 tons.	357
Extra in chains	~~162~~ 164
Web	$37\frac{1}{2}$
Total...........	$1156\frac{1}{2}$

Since our chains are heavy enough to support 1200 tons, we may safely weaken the chains to the extent of 50 tons, this will reduce their own weight by $\frac{1}{24}$th, that is, by 15 tons.

Then our table of weights will stand thus,

	tons.
Platform = $\frac{1}{2}$ a ton per foot =	200
Load = 1	400
Chains to carry 1150 tons.	342
Extra in chains............	~~162~~ 164
Web.........................	$37\frac{1}{2}$
Vertical rods...............	$8\frac{1}{2}$
Total...........	1150

It must be remembered that in this estimate, the weight of the land chains is included. Though land chains would probably not be used for a bridge of this description, yet it is well to include in our estimate the weight we have allowed for them, in order to cover the weight of the end girders, and fastenings to the pier. The weight of the iron work will be

	tons.
Chains	504
Web	$37\frac{1}{2}$
Rods (say)	$8\frac{1}{2}$
Total.........	550

or about three-fifths that required by a girder; half that required by a suspended girder; and as much again as a simple suspension bridge would require.

This bridge would require very careful hanging in order to have the strain on the web as nearly as possible zero, at a medium deflection. If the right point were nearly, but not exactly, hit, no harm would follow; since the bars would adapt themselves more and more to their mean length.

The only ways in which this bridge is inferior to a girder are these. Mechanically, that when a load first comes upon it, *every* part does not deflect in the direction in which its deflection will continue. This defect in so long a bridge experience may shew to be not worth consideration; provided (as is the case in the hanging girder bridge) the stiffness of the bridge be such, that every point within 100'0 of the engine *does* deflect in its final direction. Secondly; practically the hanging girder does not itself give any assistance nor stiffness to the roadway, which has to be made independently of it. And there is also generally more difficulty in forming the piers, and in firmly fixing the bed plates at an elevation of 40 or 50 feet above the roadway, than, as in a girder, at the level of a roadway. But the erection of the ironwork of a hanging girder would be easier, and in many cases cheaper, than that of a girder.

For a wider span, the advantages of the hanging-bridge would be even more marked.

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QUAL-SIDED ANGLE-IRONS.

of iron 36 cu. in. of which weigh 10 lbs.

5/8		11/16		3/4		13/16		7/8	
W. lbs.	*A.* sq. in.	*W.* lbs.	*A.* sq. in.	*W.* lbs.	*A.* sq. in.	*W.* lbs.	*A.* sq. in.	*W.* lbs.	*A.* sq. in.
4.95	1.483								
5.99	1.795	6.44	1.931						
7.03	2.108	7.58	2.275						
8.07	2.420	8.72	2.619	9.37	2.810				
9.11	2.733	9.87	2.962	10.62	3.185				
0.15	3.046	11.02	3.306	11.87	3.560				
1.19	3.358	12.17	3.650	13.12	3.935	14.05	4.215		
2.23	3.670	13.31	3.994	14.37	4.310	15.41	4.621		
3.28	3.983	14.46	4.337	15.62	4.685	16.76	5.028		
4.32	4.295	15.60	4.681	16.87	5.060	18.12	5.434		
5.36	4.608	16.75	5.025	18.12	5.435	19.47	5.840	20.78	6.234
6.40	4.920	17.89	5.369	19.37	5.810	20.82	6.246	22.24	6.671
7.45	5.233	19.04	5.712	20.62	6.185	22.18	6.653	23.69	7.109
8.49	5.545	20.18	6.056	21.87	6.560	23.53	7.059	25.15	7.546
9.53	5.858	21.33	6.400	23.12	6.935	24.89	7.465	26.61	7.984
0.57	6.170	22.47	6.744	24.37	6.310	26.24	7.871	28.07	8.421
1.62	6.483	23.62	7.086	25.62	6.685	27.60	8.277	29.53	8.859
2.66	6.795	24.76	7.431	26.87	7.060	28.95	8.684	30.99	9.297
2.69	7.008	25.92	7.775	28.12	8.435	30.30	9.090	32.45	9.734
1.04	.312	1.14	.344	1.25	.375	1.36	.406	1.46	.438