simplifying

the

slide rule

# WHAT IS THE UNIFACT METHOD?

The Unifact method is a teaching machine in book form that maximizes learning efficiency by presenting one fact at a time, then fixes this fact in memory by immediate use of examples, repetition, feedback, and quiz.

SIMPLIFYING THE SLIDE RULE through the Unifact method makes all basic slide-rule operations easy. One hour after beginning this book, a student who has never before seen a slide rule can use one to multiply, divide, and solve problems in ratio and proportion, rates and percentages.

# HOW ABOUT ADVANCED OPERATIONS?

In about another hour, he can use the slide rule to find squares and square roots, cubes and cube roots, logarithms, and also trigonometric functions.

THE MOST EFFECTIVE METHOD YET DEVISED FOR LEARNING THE SLIDE RULE!

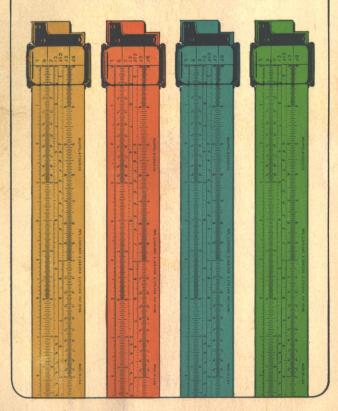


Introducing the Unifact Method

The easiest and most effective way yet devised for learning the slide rule.

By Robert W. Marks

# simplifying the slide rule



### **ACKNOWLEDGMENTS**

For review of the manuscript and the resulting contribution to the competence of the book, the author is grateful to Mr. Raymond Walsh, head of the mathematics department of Westport High School, Westport, Connecticut; and to the Educational Division of Keuffel & Esser Co., Hoboken, New Jersey.

The author acknowledges also the cooperation of Keuffel & Esser Co. in making available the slide rules that were used in the preparation of the text and cover illustrations, and the copyrighted material used in the practice exercises.

# HOW TO USE THIS UNIFACT TEXT

i. Where should we begin?

With Frame No. 1.

ii. But Frame No. 1 is obvious. Can't we skip to some place where the going gets tough?

No.

iii. Why not?

Because if we read every frame carefully, the going will never get tough.

iv. But isn't it tiresome to read what we already know?

The purpose of this book is not to entertain, but to teach the use of the slide rule—simply, directly, and painlessly.

v. After reading the first frame, what next?

Continue to the first Spot Check. Don't skip.

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SIMPLIFYING THE SLIDE RULE Bantam Matrix edition published September 1964

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Practice Exercises © Copyright, 1938, 1944, 1945, 1947, 1954, 1955,
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Division

Percentage

**Four-Digit Numbers** 

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### TO THE READER

Learning is easiest when it is acquired in small increments . . . and reinforced by repetition.

This text develops the essentials of slide-rule theory in small, concise steps. Each frame contains a single idea. Each new idea is followed by an example. Each example is reinforced by quiz question.

Explanations are simple and compact, enabling you to keep the key idea in sharp focus. Key ideas are purposely repeated. Each repetition, in turn, develops an expansion of the initial concept.

Learning is further reinforced by the text's self-quiz structure. Spot Checks, following each new discussion, repeat the text material and give the reader a quick report on his progress. To the reader who has followed the text, step by step, the answers to the Spot Check questions are obvious. This obviousness is an indication of successful learning . . . of new information and concepts acquired without conscious effort.

The Spot Checks are also progress reports. Each question is keyed to the frame which explains the idea the question checks. Thus, any weakness the quiz discloses can be instantly remedied.

Practice Exercises, spotted throughout the book, cover in sequence every major idea developed earlier in the text. If you can answer correctly every question in the Practice Exercises, you can be sure you have learned everything this book has to teach.

Answers to the Spot Checks and Practice Exercises will always be found on the following left hand page.

### **ACKNOWLEDGMENTS**

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The purpose of this book is not to entertain, but to teach the use of the slide rule—simply, directly, and painlessly.

v. After reading the first frame, what next?

Continue to the first Spot Check. Don't skip.

# vi. How are the Spot Checks to be used?

Try to answer every question correctly. Fill in the blanks (or write answers on a separate sheet of paper)... then check all answers against the text answers (which are printed on the next left hand page).

# vii. What if some of our answers are wrong?

Return to the frame indicated opposite each Spot Check question missed. The frame will tell us what we did wrong.

# viii. When can we move on to a new section?

When we have been able to answer every question correctly in the preceding Spot Check.

# ix. Is it necessary to progress so slowly?

Learning is easy if we learn one fact at a time—and learn it thoroughly—before moving on to the next. We make the greatest progress slowly . . . with constant repetition of key ideas. This is the psychological basis of the UNIFACT method.

# x. Doesn't the text contain many repetitions?

It does. The repetitions have been carefully planned and tested.

# xi. Why?

Because repetitions emphasize a fact and fix it in the mind. Repetitions are the key to efficient learning.

# xii. How about the Practice Exercises. If we understand the principle they illustrate, can't we skip them and go on to the next subject?

Absolutely not. The exercises are both a check on our progress and an effective device to deepen learning. Repetition of operations reinforces what we have already learned.

# xiii. What is an average score for the exercises?

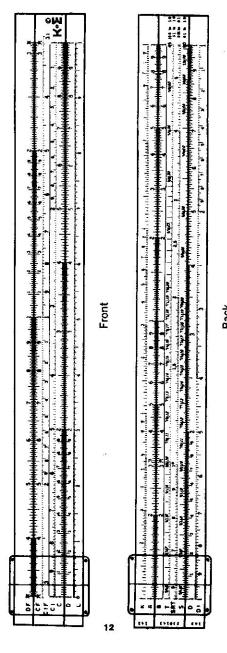
100%. If our score is less than 100%, in any section, we should turn back a few pages and reread the section.

# xiv. Summarizing: What are the basic rules for using this text?

- 1. Don't skip.
- 2. Read every frame.
- 3. Master every question in every Spot Check.
- 4. Work every problem in every Practice Exercise.

# xv. What can we expect as a result?

To acquire easily—with minimum effort—a thorough understanding of basic operations of the slide rule.



# INTRODUCING THE SLIDE RULE

1. What is a slide rule?

A device to make calculating easy.

2. What kind of calculations is a slide rule good for?

Multiplication, division, finding roots and powers, and the sines, cosines, tangents, and cotangents of angles . . . to name a few.

3. What other operations can a slide rule perform?

It can solve problems in ratio and proportion, and find the reciprocals and logarithms of numbers. There are special slide rules for other operations.

4. What does a slide rule look like?

There are several types of slide rules. One of the rules most commonly used is shown at the left.

5. How does a slide rule work?

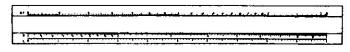
A slide rule has a number of scales. When these are set in the proper relation to one another, the answer to a given problem can be read directly on one of the scales.

# 6. What are the main parts of a slide rule?

1. A frame (or body) containing fixed scales. 2. A slide containing other scales; it can be moved to right or left. 3. A transparent runner containing a hairline.

# 7. What does the frame look like?

Like this:



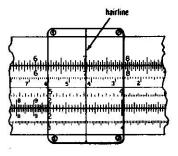
# 8. What does the slide look like?

Like this:



# 9. What does the runner look like?

Like this:



# 10. What is the main purpose of the hairline?

To show when a number mark on one scale is directly above or below a number mark on another scale.

# 11. Does the hairline have any other use?

Yes. To hold a place when a number found in one operation is to be used in a second operation.

# 12. What is an index?

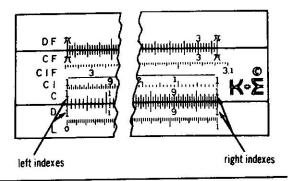
The vertical mark above (or below) each large "1" on any scale.

# 13. How many indexes are there on a scale?

The C and D scales have an index at the left end of the scale (the *left index*) and one at the right end of the scale (the *right index*). Others have one, three, or more.

# 14. What do the indexes on the C and D scales look like?

Like this:



| SPOT CHECK 1   |
|--|
| Why is a slide rule useful?  |
| What are some of the operations that can be carried out on a slide rule? (Name three.)   |
| 1.   |
| 2.   |
| 3.   |
| [2-3]  |
| Can trigonometric functions of angles be found on a slide rule?  |
| Can logarithms be found on a slide rule?   |
| [3]  |
| What is this part of a slide rule called?  |
| and the state of t |
|  |
| [7]  |
| What is this part called?  |
|  |
| [8]  |
| Which of these parts is the runner?  |
|  |
| A  |
|  |
| B [6-9]  |
|  |

|             | number      |          |          |          |          |
|-------------|-------------|----------|----------|----------|----------|
| scale.      | or _        |          | а        | number   | on anot  |
|             |             |          |          |          |          |
| The verti   | cal mark th | at begir | ns or en | ds every | scale on |
| slide is ca | lled the    |          |          |          |          |

### ✓ ANSWERS: SPOT CHECK 1

- 1. To make calculating easy.
- Any three of the following: multiplication, division, finding roots, finding powers, finding logarithms, ratio, proportion, finding reprocals, finding the sines, cosines, tangents, cotangents of angles.
- Yes. [Except on some of the most inexpensive rules.]
- 4. Yes. [Except on some of the most inexpensive rules.]
- Frame.
   Slide.
- 7. c.
- 8. Hairline.
- 9. Hairline; above; below.
- 10. Index.

# READING NUMBERS ON SLIDE-RULE SCALES

# 15. How are numbers indicated on a slide rule?

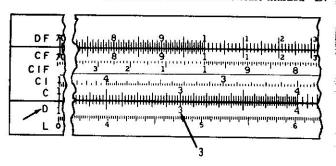
As on a foot ruler or a yardstick—by the markings on any of the scales.

# 16. How is the number 3 indicated?

By a "3" on any scale.

# 17. Show this on a specific scale.

Here is the number 3 shown on the scale marked "D."



### 18. How is 30 indicated on the slide rule?

Also by 3.

## 19. How is 300 indicated on the slide rule?

Also by 3. "3" stands for 0.3, 3, 30, 300, 3000, and all other place values of 3.

# 20. How can one figure (such as 3) stand for all the place values of that figure (0.3, 30, 300, etc.)?

Because the place value is given by the position of the decimal point, not by the digit (or digits) indicated on the scales.

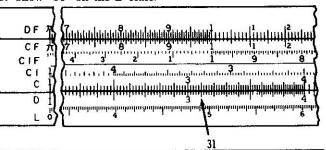
# 21. Does the slide rule indicate where the decimal point is to be placed?

No. The slide rule indicates only the non-zero digits. (Rules for locating the decimal point are given on pp. 44, 51, 55, 76, 78-79, and 88.)

# 22. How is the number 31 indicated on a scale?

By the first major subdivision to the right of "3."

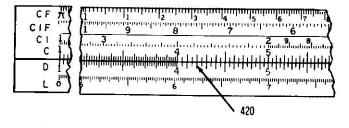
# 23. Show "31" on the D scale.



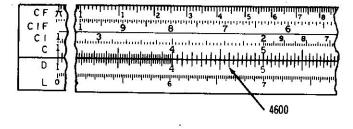
# 24. What other values can "31" represent?

All numbers that have 31 as their significant figures . . . such as 0.031, 0.31, 3.1, 310, 3100, and 31,000.

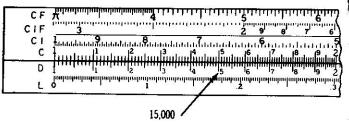
# 25. Find 420 on the D scale.



# 26. Find 4600 on the D scale.

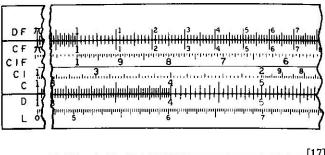


# 27. Find 15,000 on the D scale.



# ✓ SPOT CHECK 2

1. Find 4 on the D scale in the diagram.



- 2. How is 40 indicated on the slide rule?
- 3. How is 400 indicated on the slide rule?
- \_\_\_\_\_[19]
- 4. Name five numbers represented by 4 on the D scale.
- 5. Are decimal points shown on a slide rule?
  - \_\_\_\_
- 6. Find 41 on the D scale in the diagram.

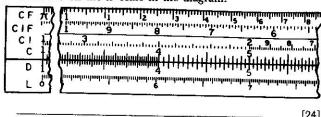


...[22–23]

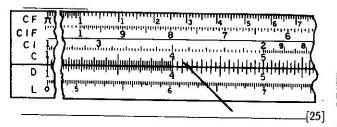
\_[18]

[21]

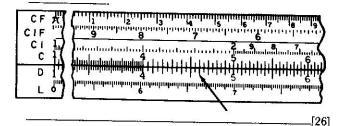
7. Find 410 on the D scale in the diagram.



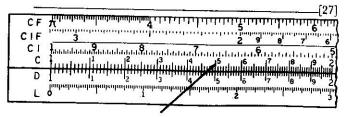
8. The three-digit whole number indicated on the scale is



9. The four-digit whole number indicated on the scale is



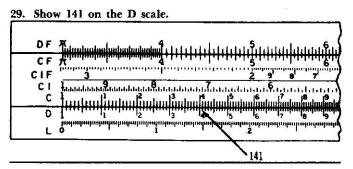
10. The five-digit whole number indicated on the scale is



# **READING THREE SIGNIFICANT DIGITS**

28. The number 140 is represented on the D scale of the slide rule as "14"—that is, four large intervals to the right of "1." How is 141 represented on the D scale?

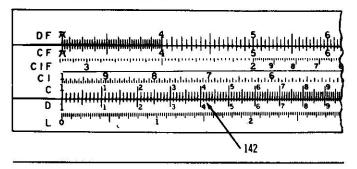
By the first small interval beyond (to the right of) 14.



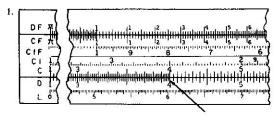
30. How is 142 represented?

By the second small interval beyond (to the right of) 14.

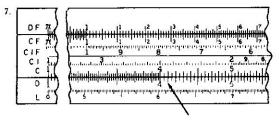
31. Show 142 on the D scale.







- As a 4. Also as a 4.
- ... 0.04; 0.4; 40; 400; 4000;
- 40.000. . . .
  - No. DF N CIF CI C



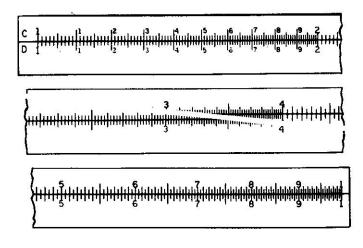
- 410. 4600.
- 15,000.

# 32. Are the intervals on slide-rule scales all the same size?

No. On most of the scales the intervals grow smaller as we move from the lower to the higher digits.

## 33. Show this on the rule.

Here is a diagram showing the C and D scales. Notice how the intervals contract as we move from left to right.

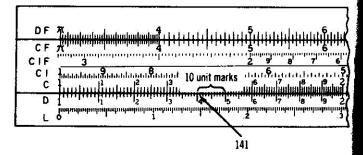


# 34. Can as many digits be read in the small intervals as in the large intervals?

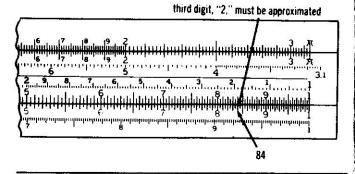
No. In the interval between 1 and 2 on the C and D scales, three digits can be read exactly . . . and a fourth approximated. But at the other end of these scales, where there are fewer marks, the third digit is not always indicated by a mark. In such cases it must be approximated.

# 35. Give an example.

As we have seen, 141 can be indicated exactly, because there is space between 1 and 2 for all necessary unit marks.



To indicate 842, however, we have to approximate the 2, because the space between 8 and 9 has fewer subdivisions. Each of the subdivisions is divided into 2 parts instead of 10 parts.



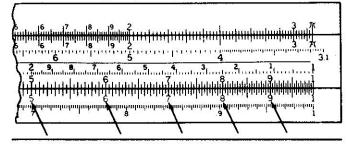
# 36. Doesn't this fact limit the accuracy of a slide rule?

It does. A slide rule has limited accuracy. In the interval between 1 and 2, three digits can be read accurately (and a fourth can be approximated). In the higher parts of a scale, only the first two digits are given precisely. The third must be approximated.

# 37. What are the marks called that are printed above or below the large numerals?

Primary marks.

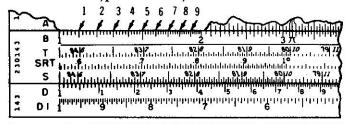
# 38. Show some typical primary marks.



# 39. What is the name for the larger marks between primary marks?

Secondary marks.

# 40. Show some typical secondary marks.



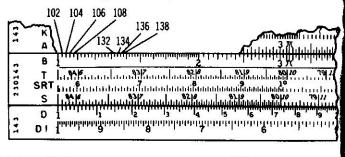
# 41. How many secondary marks are there between primary marks?

Between primary marks on the C and D scales there are always nine secondary marks. These are not numbered, however, except in the interval between primary marks 1 and 2.

42. Between the secondary marks there are other marks, still smaller. What are these called?

Tertiary marks.

43. Show some typical tertiary marks.



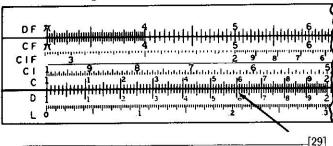
44. Using these terms, state a general rule for representing a three-digit number on a slide-rule scale.

The first digit of the number is represented by a primary mark; the second, by a secondary mark; the third, by an indicated or approximated tertiary mark.

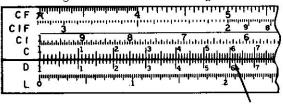
# **▶** SPOT CHECK 3

| vv hat | three | digits | are r | epresente | a on | tne | υ | scare | bу | tne | nrst |
|--------|-------|--------|-------|-----------|------|-----|---|-------|----|-----|------|
| small  | mark  | to the | righ  | t of 14?  |      |     |   |       |    |     |      |
|        |       | - 22   |       |           |      |     |   |       |    |     | [28] |

2. What three-digit whole number is shown on the scale?



3. What digits are indicated in the diagram?



4. What digits are indicated here?

| :IF | 3                      | ENGLISH TOURSET OF THE SERVICE | <u> </u>            |             | - 2            | 9'                  | 6            | 7'      |
|-----|------------------------|--------------------------------|---------------------|-------------|----------------|---------------------|--------------|---------|
| CI  | 100000                 | indatata (                     | nananalah           | 444744      | nigdad         | 6                   | ուսեւ        |         |
| C   | <del>իսոկավա</del>     | լում <mark>ա</mark> յ          | mijmi               | ունաև       | របៀបរៀបប       | 6<br>10111111       | ,<br>ndomina | min     |
| D   | Վուտակու               | ավետ                           | ավելու              | ակար        | ակափա          | 6 100               |              | ահա     |
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[30]

| 5.          | What do you notice about the intervals as you move from lower to higher numerals on the slide rule? |
|-------------|---|
|             | [32]  |
| 6.          | How is the third digit determined in the higher parts of most slide-rule scales?                    |
|             | [34]  |
| 7.          | How many digits can be read accurately on the part of the D scale shown below?                      |
|             | K   A   B   T   T   T   T   T   T   T   T   T   |
|             | D 111111111111111111111111111111111111  |
|             | DI โดยเดิดเลยเดินเกลเดินเกลเดินกาลเปลากาลเรื่อนการปการเกลียนการปลาการประการกร้า                     |
|             | [35]  |
| 8.          | What are "primary" marks on a scale?  |
|             | [37]  |
| 9.          | What is the name for the long marks between the primary marks?                                      |
|             | [39]  |
| i0.         | What are tertiary marks?  |
|             | [42]  |
| ļ1 <b>.</b> | In a three-digit numeral, what digit is given by  |
|             | a. The secondary mark?  |
|             | b. The primary mark?  |
| •           | c. The tertiary mark?   |
|             | <u>[41]</u>   |

# **MULTIPLICATION**

45. What scales are used for multiplication?

The most commonly used scales are the C and D scales.

46. Where is the C scale?

On the slide.

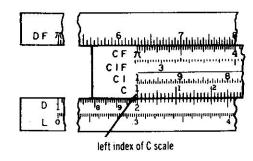
47. Where is the D scale?

On the body of the rule, directly below the C scale.

48. What do we do if we want to multiply some number by 2?

Set the left index of the C scale over 2 on the D scale.

49. Show this on the rule.



### ✓ ANSWERS: SPOT CHECK 3

- 141. 161.
- 162.
- 4. 164.

Three.

- The intervals become smaller. 6. Usually it must be approximated.
- The marks between the secondary marks.

large numerals.

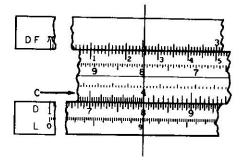
Secondary marks.

The marks above or below the

- 11. a. The second digit. b. The first digit. c. The third digit.
- 50. Suppose the number to be multiplied is 4. What is the next step?

Move the hairline of the runner to 4 on the C scale.

51. Show this on the rule.

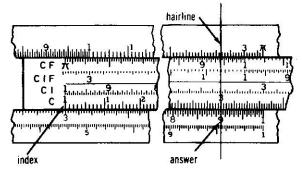


# 52. What is the answer?

Read the answer under the hairline on the D scale. The answer is 8.

# 53. Now multiply $3 \times 3$ .

Set the left index on the C scale over 3 on the D scale.

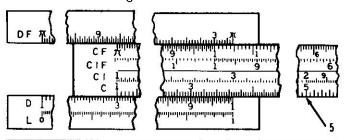


Move the hairline to 3 on the C scale . . . and read 9 directly below (on the D scale).

# 54. What happens if, in multiplying $3 \times 5$ , we again set the left index of the C scale over 3 on the D scale?

The 5 on the C scale extends beyond the frame of the rule.

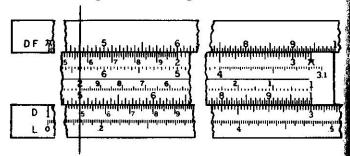
# 55. Show this in a diagram.



56. How, then, is 5 to be multiplied by 3?

By using the right index of the C scale.

57. Show this operation in a diagram.



In the diagram, the right index of the C scale is set over 3 on the D scale. The answer, 15, is read on the D scale when the hairline is moved to 5 on the C scale.

# SPOT CHECK 4

|   | Name two slide-rule scales that can be used for multiplicat        |
|---|--|
|   | Which of these scales is found on the slide?                       |
|   |  |
|   | Which is on the body of the rule?                                  |
|   |  |
|   | If we want to multiply $2 \times 4$ , what is the first operation  |
|   |  |
| 1 | What is the next operation?  |
|   |  |
|   | Where do we find the answer?                                       |
|   |  |
|   | In multiplying $3 \times 3$ , where is the left index of the C sc. |
|   |  |
|   | Where is the answer?   |
|   |  |

[54]

# MANSWERS: SPOT

- The C scale and the D scale.
- The C scale.
- The D scale.
- Set index of C scale over 2 on the scale.
- Move hairline to 4 on the C scale. On the D scale, under the hairline. [The answer is 8.]
- Over 3 on the D scale.
- On the D scale, under hairling set at 3 on C scale. The answer
- 9. It extends beyond the body of the slide rule.

# **MULTIPLICATION** (Continued)

58. If we want to multiply  $1.5 \times 3.5$ , what do we do about the decimal point?

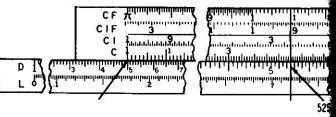
> Ignore it until we finish our operation on the rule. The slide rule does not indicate decimal points.

59. What numbers do we read on the slide-rule scales? 15 and 35.

60. Show the whole operation.

Set the index of the C scale over 15 on the D scale. Move the hairline of runner to 35 on the C scale.

Under the hairline, on the D scale, read 525.



61. How do we know where to put the decimal point in the answer (525)?

By inspection. Think of 1.5 as 2; 3.5 as 4.  $2 \times 4 = 8$ .

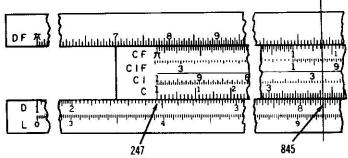
The largest whole number in the answer is a one-digit number.

What is the complete answer?

5.25.

63. Multiply  $2.47 \times 34.2$ .

We repeat the procedure. We multiply 247 by 342 and locate the decimal point by inspection.



- 1. Set the C index to 247 on D.
- Move hairline to 342 on C.
- 3. Read 845, under the hairline, on D.

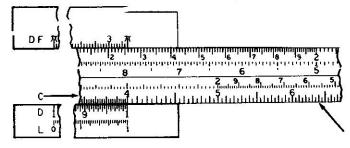
# 64. Now locate the decimal point.

By inspection we see that  $2 \times 34 = 68$ . The answer, consequently, will be a number less than 100. Thus 845 is pointed off as 84.5. (Note: This method of approximation will not work in certain marginal cases. It is useful, however, as a rule of thumb.)

# 65. Multiply $25 \times 6.4$ .

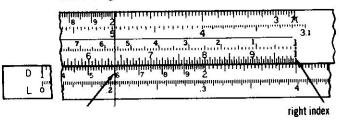
We can estimate the place of the decimal point without any trouble. We know that  $25 \times 6$  is 150. The answer, therefore, will have three figures to the left of the decimal point.

But the problem involves the operation we first encountered in multiplying  $3 \times 5$ . When we put the left index of the C scale over 25 on the D scale, 64 on the C scale extends beyond the body of the rule . . . as



# 66. What then?

Use the right index of the C scale.



Set the right index of the C scale over 25 on the D scale.

Move the hairline to 64 on the C scale . . . and below, on the D scale, read the result: 16.

We know from our estimate that  $25 \times 6.4$  is a three-place number. Our answer, therefore, is 160.

# **▶** SPOT CHECK 5

1. What do we do about the decimal point when locating num-

|             | bers on the slide rule?  |
|-------------|--|
|             | [58]   |
| 2.          | How are 1.5 and 3.5 indicated on the C and D scales?   |
|             | [59]   |
| 3.          | What mental operation do we perform to locate the decimal point in the product of $1.5 \times 3.5$ ?                             |
|             | [61]   |
| 4.          | In the diagram, read the indicated figure as a number having<br>one place to the left of the decimal point. The number is        |
|             | [62]   |
| (12<br>(11) | B 3 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4  |
| (m)         |  |
| 5.          | If we multiply $2.47 \times 34.2$ on the slide rule, the two numbers   |
|             | we indicate on the C and D scales are  |
|             | [63]   |
| 6.          | When we multiply $2.47 \times 34.2$ on the slide rule, we expect   |
|             | our answer to have digits to the left of the decimal point.  |
|             | [64]   |
| 7.          | If, in multiplying $25 \times 64$ , we set the left index of the C scale over 25 on the D scale, where is the 64 on the C scale? |
|             | [65]   |
| 8.          | In a case of this sort, what is the correct operation?   |

**[661** 

### ANSWERS: SPOT CHECK 5

- 1. Ignore it.
- As 15 and 35.
   Multiply 2 × 4.
- 3. Multiply 2 X
- 4. 5.25. 5. 247 and 342.

- 6. Two.
- 7. Extending beyond the body of the rule.
- 8. Set the right index of C over 25 on D.

# FOUR-DIGIT NUMBERS

67. Can a fourth digit be read on slide-rule scales?

Only by approximation.

68. Why only by approximation?

Because only three significant digits are shown by the printed marks. In the upper parts of most scales even the third digit sometimes has to be approximated.

69. Can the fourth digit be approximated on all parts of the scale?

No. On an ordinary slide rule, the fourth significant digit can only be approximated in the low parts of the C and D scales.

70. Is it necessary to use four significant digits?

No. In most shop and laboratory work, the solution to a problem is satisfactory when it is accurate to three significant digits. 71. Can the number 1675 be shown on the slide rule?

Yes (approximately).

72. Locate 1675 on the C scale.

In the diagram, 1675 is approximated as a point half-way between 1670 and 1680.



73. What is the best way of handling four-digit numerals in slide-rule calculations?

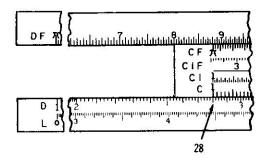
Because the average slide rule is accurate only to three figures, numerals with four (or more) significant digits should be rounded off to three figures.

74. Give an example.

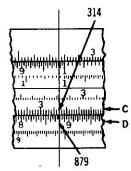
Suppose (using the C and D scales) we want to find the circumference of a wheel 28 inches in diameter. The problem is to multiply 28 by  $\pi$ , which is 3.14159. . . . Obviously, we can't set 3.14159 . . . on the C scale or the D scale. Consequently, we round off 3.14159 . . . to 3.14.

# 75. Perform the indicated operation.

We set the index of the C scale to 28 on the D scale.



Under 314 on the C scale, we read 879 on the D scale



We find the decimal point mentally. Since  $3 \times 30 = 90$ , the answer must have two figures to the left of the decimal point. Thus we point off 879 as 87.9.

# SPOT CHECK 6

|         | low can an additional digit sometimes be found?   |
|---------|---|
|         | n the upper parts of the C and D scales, is the third ways accurately indicated?  |
| W<br>lo | That advantage in indicating numbers is to be found in wer parts of some scales?  |
| H       | ow many significant digits are important in most labored shop work?   |
| fir     | egardless of the position of the decimal point, what an<br>st four non-zero digits of all numbers associated wit<br>ark indicated on the diagram? |
| m       |   |

8. How can  $(\pi)$  3.14159 . . . be represented on the D scale?

[73]

easier on the slide rule?

### ANSWERS: SPOT CHECK 6

- Three at most. (In the upper parts of most scales the third digit may have to be approximated.)
- 2. By approximation.
- 3. No. The third digit may lie between tertiary marks.
- A fourth digit can be approximated.
- 5. Three.
- 6. 1245.
- 7. Round off to three figures.
- 8. As 314.

# MATH REFRESHER

# How to Find the Decimal Point in Your Answer WHEN MULTIPLYING DECIMALS

Rule: The number of decimal places to the right of the decimal point in the product of two numbers is the sum of the number of places to the right of the decimal point in the multiplier and multiplicand.

# Example 1. Multiply 3.13 by 0.04.

| 3.13  | Two places to right of decimal point                    |
|-------|---|
| .04   | Two places to right of decimal point                    |
| .1252 | Point off four places (2 + 2) to right of decimal point |

# Example 2. Multiply 0.012 by 99.

| 0.012 | Three places to right of decimal point                     |
|-------|--|
| 99.   | No places to right of decimal point                        |
| 108   |  |
| 108   |  |
| 1.188 | Point off three places $(3 + 0)$ to right of decimal point |

# Practice Exercise 1: Multiplication

By now you should be able to

- 1. Read three-figure numbers on the C and D scales.
- 2. Approximate some four-figure numbers.
- 3. Place decimal points mentally.
- 4. Multiply numbers containing four (or more) figures, after rounding off to three figures.

Given below are sixteen Practice Exercises. Work each problem, and check your answer against the correct answer given on page 46.

If you get at least 12 out of the 16 right, you can be sure that you have mastered the basic principles of multiplication on the slide rule. In this case, start at once on "Division," the next section of this book.

If you get fewer than 12 correct answers, turn back to Frame 14... and once more read through to page 45. Perform the indicated multiplications:

|    | 0 0              | ^   | 4 = 2 2 2 2 2 2       |
|----|------------------|-----|-----------------------|
| 1. | $3 \times 2$     | 9.  | $1.536 \times 30.6$   |
| 2. | $3.5 \times 2$   | 10. | $0.0756 \times 1.093$ |
| 3. | $5 \times 2$     | 11. | $1.047 \times 3080$   |
| 4. | $2 \times 4.55$  | 12. | $0.00205 \times 408$  |
| 5. | $4.5 \times 1.5$ | 13. | $(3.142)^2$           |

6. 
$$1.75 \times 5.5$$
 14.  $(1.756)^2$  7.  $4.33 \times 11.5$  15.  $(83.0)^2$  8.  $2.03 \times 167.3$  16.  $4.98 \times 576$ 

### ANSWERS: PRACTICE EXERCISE 1

| 1. | 6    | 9.  | 47.0   |
|----|------|-----|--------|
| 2. | 7    | 10. | 0.0826 |
| 3. | 10   | 11. | 3220   |
| 4. | 9.1  | 12. | 0.836  |
| 5. | 6.75 | 13. | 9.87   |
| 6. | 9.62 | 14. | 3.08   |
| 7. | 49.8 | 15. | 6890   |
| 8. | 340  | 16. | 2870   |

# DIVISION

# 76. How is division performed on the slide rule?

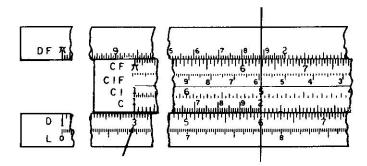
Like multiplication, but with the operations in reverse order.

# 77. Give an example.

To multiply  $3\times 2$ , we set the index of the C scale over 3 on the D scale; and under 2 on the C scale we read 6 on the D scale.

The same setting, read in reverse order, gives the answer to  $6 \div 2$ .

78. Show the division of 6 by 2 on the rule.



To divide 6 by 2, we set 2 of the C scale directly over 6 on the D scale.

The answer, 3, is shown on the D scale under the index of the C scale.

# 79. What conclusion can we draw from this fact?

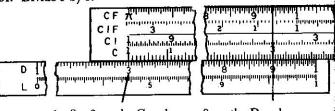
Division is the reverse of multiplication.

# 80. Restate this fact as a rule for division.

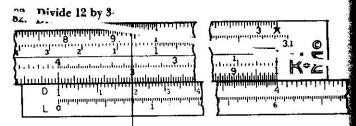
Rule: To divide one number by another, set the divisor on the C scale directly over the dividend on the D scale.

Read your quotient on the D scale, directly below the C index.

81. Divide 9 by 3.



- 1. Set 3 on the C scale over 9 on the D scale.
- Read the quotient 3 on the D scale under the left index of the C scale.



- 1. Set 3 on the C scale over 12 on the D scale.
- Under the right index of the C scale read the answer . . . 4 on the D scale.

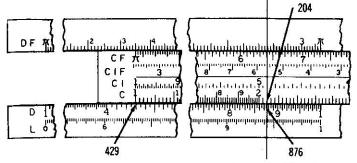
# 83. How do we know where to put the decimal point?

As in multiplication on the slide rule . . . by rough calculation, using rounded numbers.

84. If we divide 876 by 20.4, where do we put the decimal point?

Think of 876 as 800. Think of 20.4 as 20. We find, by inspection, that  $\frac{800}{20} = 40$ . This tells us that the required quotient will have two places to the left of the decimal point.

85. Divide 876 by 20.4.



- 1. Set 204 on the C scale over 876 on the D scale.
- 2. Read the quotient, 429, on the D scale under the left index of the C scale.

We know the required quotient must have two places to the left of the decimal point. Our answer, therefore, is 42.9.

# SPOT CHECK 7

| iultipli<br>rder. | cation, but the operations are performed in  | 1000 |
|-------------------|--|------|
|                   |  | 3/2  |
| he se             | ting of the slide rule for $6 \div 2$ is the same                                  |      |
| ×                 |  |      |
|                   |  | -    |
| o divi            | de 6 by 2, we set on the C scale direc   | tl   |
| on the            | D scale.   |      |
| <del></del>       |  | -    |
| n this            | operation, the quotient can be found on the rectly below the index on the C scale. |      |
| care, u           | rectly below the index on the o sealer   |      |
| r 31              | de 9 by 3, we set 3 on the C scale directly ove                                    | r    |
|                   | scale.   | •    |
| m me -            | state.   |      |
|                   | 10 the Decade directly under 3 o   | n    |
|                   | we set 12 on the D scale directly under 3 o  |      |
| scale, v          | ve have set the rule to divide by  |      |
|                   |  |      |
|                   | o we find out how many digits on our answer<br>of the decimal point?               | r    |
|                   |  |      |
| If we             | divided 626 by 21.3, the quotient would ha   | av   |
| many p            | laces to the left of the decimal point?  |      |
|                   |  |      |

# MATH REFRESHER.

# When Dividing Decimals, What Do You Do ABOUT THE DECIMAL POINTS?

Rule: Make the divisor a whole number . . . by moving the decimal point all the way to the right. Move the decimal point in the dividend (the number to be divided) the same number of places to the right . . . adding zeros if necessary.

Example 1. Divide 12.5 by 0.5.

Move the decimal point one place right:

0.5 becomes 5. 12.5 becomes 125.

5)125

25 Ans.

Example 2. Divide 12.5 by 0.005.

Move the decimal point three places right:

0.005 becomes 5.

12.5 becomes 12,500.

5)12,500 2500 Ans.

## ANSWERS: SPOT CHECK 7

- Reverse.
- 2.
- 3. 4. D.
- 9; D.

- 6. 12; 3.
  - calculation By rough rounded numbers.
- Two.

# **Practice Exercise 2: Division**

# Perform the indicated operations:

- 1.  $87.5 \div 37.7$
- $2. \quad 3.75 \div 0.0227$
- 3.  $0.685 \div 8.93$
- $1029 \div 9.70$
- $0.00377 \div 5.29$
- $2875 \div 37.1$
- $871 \div 0.468$  $0.0385 \div 0.001462$

- 9.  $3.14 \div 2.72$
- 10.  $3.42 \div 81.7$
- 11.  $529 \div 565$
- 12.  $0.0341 \div 0.0222$ 13.  $396 \div 0.643$
- 14.  $0.0592 \div 1.983$
- 15.  $0.378 \div 0.0762$
- 16.  $10.05 \div 30.3$

# **PERCENTAGE**

86. What is 1% of a number?

1/100 of that number.

87. What is 2% of a number?

2400 of that number.

88. What is 7% of a number?

7400 of that number.

89. How are fractions indicated on a slide rule?

As decimal fractions.

90. Write 1/100 as a decimal fraction.

0.01.

91. Write 1/100 as a decimal fraction.

0.02.

92. Write 7/100 as a decimal fraction.

0.07.

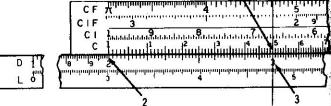
### ANSWERS: PRACTICE EXERCISE 2

| 1. | 2.32     | 9. 1.154   |
|----|----------|------------|
| 2. | 165.2    | 10. 0.0419 |
| 3. | 0.0767   | 11. 0.936  |
|    | 106.1    | 12, 1.535  |
|    | 0.000713 | 13. 616    |
| 6. | 77.5     | 14. 0.0298 |
| 7. | 1861     | 15. 4.96   |
| 8. | 26.3     | 16. 0.332  |
|    |          |            |

93. How can 2% of a number be found on the slide rule?

By multiplying that number by 0.02.

94. Using the slide rule, find 2% of 150. 150

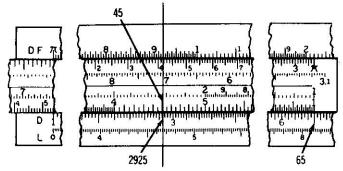


- 1. Set the index of the C scale over 2 on the D scale.
- 2. Under 150 on the C scale read 3 on the D scale.

- 1. Leave the index of the C scale over 2 on the D scale.
- 2. Under 155 on the C scale read 31 on the D scale.

# 96. Find 61/2% of \$4500.

(Hint: Think of 61/2% as 0.065.)



- Set the right index of the C scale over 65 on the D scale.
- 2. Under 45 on the C scale read 2925 on the D scale.

A rough calculation tells us that  $0.06 \times 5000$  is 300. Therefore our expected answer should have three places to the left of the decimal point. Thus "2925" is read: \$292.50.

# MATH REFRESHER

In finding a per cent of a number, we locate the decimal point by the same rule used in multiplying decimals. The number of places to the right of the decimal point in the product is found by adding the number of places to the right of the decimal point in both multiplier and number multiplied. 2% of  $1550 = 0.02 \times 1550 = 31.00$ . There are two places to the right of the decimal point in our answer, but since these are zeros, we can ignore them.

# SPOT CHECK 8

| 1. | 1/100 of a number is what per cent of that number?     |      |
|----|--|------|
|    |  | 86]  |
| 2. | 3/100 of a number is what per cent of that number?     |      |
|    |  | 87]  |
| 3. | 5% of a number is hundredths of that number.           |      |
|    |  | 88]  |
| 4. | Fractions are represented on the slide rule as         |      |
|    |  | 89]  |
| 5. | 1/100 written as a decimal fraction is                 | - 3  |
|    |  | 90]  |
| 6. | 0.02 can be written as                                 | 91]  |
| 7. | 0.007 can be written as                                | 92]  |
| 8. | If a number is multiplied by 0.02, the product will    | be   |
|    | % of that number.                                      | 93]  |
| 9. | To find 2% of 150 we set the index of the C scale of   | ver  |
|    | on the D scale, and look for the answer                | on   |
|    | the D scale under on the C scale.                      |      |
| 0. | To find 61/2% of \$4500, we set the right index of the | C    |
|    | scale over, and look for the answer on                 |      |
|    | D scale under on the C scale.                          | [96] |

# Practice Exercise 3: Percentage

# Find:

 1.
 86.3% of 1826
 6.
 19% of 18

 2.
 75.2% of 3.46
 7.
 22% of 23

 3.
 18.3% of 28.7
 8.
 4½% of \$3,580

 4.
 0.95% of 483
 9.
 21% of 2100

 5.
 6½% of \$2700
 10.
 210% of 21

### ANSWERS: SPOT CHECK 8

2/100. 7/100. 3%. 9. 2; 15 (or 15; 2). Decimal fractions. Decimal; 0.01.

65; 45 (or 45; 65).

### ANSWERS: PRACTICE EXERCISE 3

1576 5.06 2.60 8. \$152.15\* 5.25 4.59 \$175.50

\*Read 152.1.

# HOW TO FIND WHAT PER CENT ONE NUMBER IS OF ANOTHER

# 97. How do you find what per cent 4 is of 8?

You divide 4 by 8.  $\frac{4}{8}$  = 0.50, or 50%.

98. How do you find what per cent 4 is of 10?

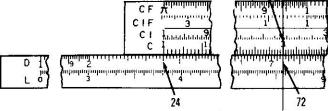
You divide 4 by 10.  $\frac{\pi}{10}$  = 0.40, or 40%.

# 99. How do you find what per cent 40 is of 50?

You divide 40 by 50.  $\frac{10}{50}$  = 0.80, or 80%.

100. Using the slide rule, find what per cent 72 is of 300.

The problem is to divide 72 by 300. CIF

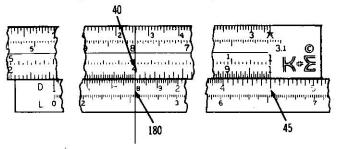


- 1. Set 300 on the C scale directly over 72 on the D scale.
- 2. Under the left index of the C scale read 24 on the D scale.

The answer is 0.24, or 24%.

101. A security which costs \$40 pays \$1.80 as an annual dividend. Using the slide rule, find the dividend rate.

The problem is to divide 1.80 by 40.



- 1. Set 40 on the C scale directly over 180 on the D
- 2. Under the right index of the C scale read 45 on the D scale.

The answer is 0.045, or 4.5%.

# SPOT CHECK 9

| To find what per cent 3 is of 6, you divide by                 |
|--|
| To find what per cent 3 is of 9, you divide by                 |
| If you divide 40 by 50, you can find what per cent             |
| is of  |
| · [9   |
| In using the slide rule to find what per cent 72 is of 300, yo |
| set on the C scale over on the D scale.                        |
| [100   |
| To find the rate of dividend of a security which costs \$4     |
| and pays \$1.90 per annum, we set on the C scale d             |
| rectly over on the D scale.                                    |
| [10]   |

# Practice Exercise 4: Finding What Per Cent One Number Is of Another

- 1. 18 is what per cent of 69?
- 2. 85 is what per cent of 132?
- 3. 192.8 is what per cent of 87.6?4. 28 is what per cent of 1027?

### ✓ ANSWERS: SPOT CHECK 9

1. 3; 6.

4. 300; 72.

2. 3; 9.

5. 45; 190.

### ANSWERS: PRACTICE EXERCISE 4

1. 26.1%

3. 220% 4. 2.73%

# **PROPORTION**

102. How can ratios be shown on a slide rule?

As fractions. The first step is to express ratios in fractional form. Thus to indicate that 2 is to 4 as 4 is to 8, we indicate  $\frac{2}{4} = \frac{4}{8}$ .

103. If a photograph 8 inches high and 10 inches wide is to be reduced to fit a space 4 inches wide, what fact can we use to find the required height?

The fact that the ratio of height to width is the same in both cases.

104. What do we call the operation to be performed?

Finding the proportionate height.

105. How, then, can we define a proportion?

An expression containing two equal ratios.

106. How are proportions shown on the slide rule?

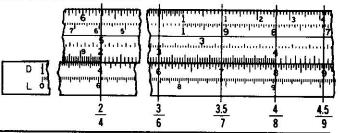
By the relations between the numbers on any two similar scales, such as the C and D scales.

107. Give a specific example.

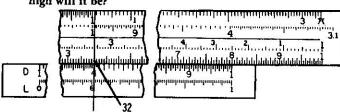
If 2 on the C scale is set over 4 on the D scale, all the remaining numbers on the C scale will have the same ratio to all the remaining numbers on the D scale. . . .

Thus 3 will be over 6, 3.5 over 7, 4 over 8, etc.

108. Show this on the rule.



109. Now, use the slide rule to solve the problem in Frame 102. If a photograph 8 inches high and 10 inches wide is to be reduced to fit in a space 4 inches wide, how high will it be?



- 1. Set 8 on the C scale over 10 on the D scale.
- Above 4 on the D scale, read 32 on the C scale. Answer: 3.2 inches.

110. How do we know on which scale to look for the answer?

Use the C scale for all numerators; the D scale for all denominators. If the number to be found is a numerator, look for it on the C scale. If a denominator, look for it on the D scale.

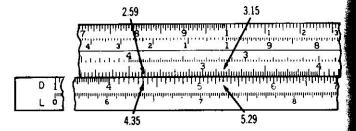
The problem above could be written

$$\frac{8}{10} = \frac{x}{4} \quad \frac{\text{(C scale)}}{\text{(D scale)}}$$

The answer, therefore, would be on the C scale.

111. Find the value of x in the proportion  $\frac{3.15}{5.29} = \frac{x}{4.35}$ .

Remember: Use the C scale for numerators. Use the D scale for denominators.



- Set 3.15 on the C scale directly over 5.29 on the D scale.
- Directly above 4.35 on the D scale read 2.59 on the C scale.

112. What do we do if one of our numbers on the C scale lies to the right of the body of the rule?

Set the hairline to the left index of the C scale. Then move the slide to the left until the right index is under the hairline.

113. What do we do if one of our numbers on the C scale lies to the left of the body of the rule?

Interchange the indexes of the C scale. To do this set hairline to the right index of the C scale. Then move the slide right until the left index is under the hairline.

# ✓ SPOT CHECK 10

| -        | [102–105]   |
|----------|---|
| Ex       | apress in words the ratios $\frac{2}{4} = \frac{4}{8}$ .  |
|          | [10   |
| W        | hat is meant by a proportion?   |
|          | [10   |
| C        | an the C and D scales be used to show a proportion?   |
| If<br>ex | 2 on the C scale is set over 4 on the D scale, what can we pect of all the other numbers on the respective two scales |
|          | 2 on the C scale is set over 4 on the D scale, what numer on the D scale will correspond to 3 on the C scale?         |
|          | a photograph 4 inches high and 5 inches wide is enlarge<br>til it is 8 inches high, how wide will it be?              |
| _        | in a proportion, both numerators are indicated on the   |
|          | ale, the will be indicated on the D scal  |
|          |   |

| 10. | If, in solving a proportion on the slide rule, one of the numbers on the C scale should lie to the right of the body of |  |  |
|-----|---|--|--|
|     | the rule, we set the hairline to the index of   |  |  |
|     | the C scale, then move the slide to the   |  |  |
|     | until the index is under the hairline.  |  |  |
|     | [112]   |  |  |

## MANSWERS: SPOT CHECK 10

The ratio of height to width.

2 is to 4 as 4 is to 8.

An equality of ratios.

They will be in the same ratio

7. 10 inches.

8. Denominators.

9. 3.15. 10. Left; left; right.

# **Practice Exercise 5: Proportion**

In each of the following proportions, find the value of the unknowns (the x's, y's, and z's):

1. 
$$\frac{x}{5} = \frac{78}{9}$$

2. 
$$\frac{x}{120} = \frac{240}{170}$$

3. 
$$\frac{7}{8} = \frac{249}{x}$$

4. 
$$\frac{2}{3} = \frac{x}{7.83}$$

$$5. \quad \frac{x}{1.804} = \frac{y}{25} = \frac{1}{0.785}$$

6. 
$$\frac{x}{709} = \frac{246}{y} = \frac{28}{384}$$

7. 
$$\frac{17}{x} = \frac{1.365}{8.53} = \frac{4.86}{y}$$

8. 
$$\frac{8.51}{1.5} = \frac{9}{x} = \frac{235}{y}$$
  
9.  $\frac{x}{2.07} = \frac{3}{61.3} = \frac{y}{1.571}$ 

10. 
$$\frac{x}{0.204} = \frac{y}{0.0506} = \frac{5.28}{z}$$
$$= \frac{2.01}{z}$$

# **SQUARES AND SQUARE ROOTS**

114. If  $2 \times 2 = 4$ , what is 4 in relation to 2?

The square of 2.

115. And what is 2 in relation to 4?

The square root of 4.

116. Indicate the square of 2 in mathematical symbols.

22.

117. Indicate the square root of 4 in mathematical symbols.

 $\sqrt{4}$ .

118. If  $3 \times 3 = 9$ , what is  $3^{2}$ ?

9.

119. What number is represented by  $\sqrt{9}$ ?

±3. Note: a square root of a number can be either a positive or a negative number (Ex.  $3 \times 3 = 9$ ;  $-3 \times -3 = 9$ ). In the examples that follow, however, only the positive roots are indicated.

## ANSWERS: PRACTICE EXERCISE 5

x = 169.4x = 285x = 5.22

x = 2.30, y = 31.8

6. x = 51.7, y = 3370x = 106.2, y = 30.4

x = 1.586, y = 41.4

x = 0.1013, y = 0.0769

10. x = 3.97, y = 0.984, z = 0.272

# 120. What number is represented by 62?

36 (that is,  $6 \times 6$ ).

# 121. What number is represented by $\sqrt{36}$ ?

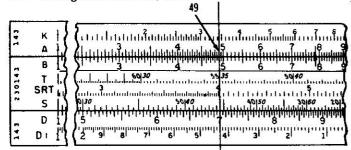
6.

# 122. How can the square of a number be found on the slide rule?

By locating the number on the D scale, then reading the number directly above it on the A scale.

(Every number on the A scale is the square of the number below it on the D scale.)

123. Making use of this fact, find 72.



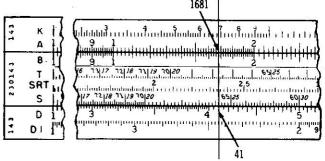
- 1. Move the hairline to 7 on the D scale.
- 2. On the A scale, under the hairline, read 49.

# 124. Now find $\sqrt{49}$ .

Simply reverse the process.

- 1. Set the hairline over 49 on the A scale.
- 2. On the D scale, under the hairline, read 7.

# 125. Using the A and D scales, find 412.



- 1. Move the hairline to 41 on the D scale.
- 2. On the A scale, under the hairline, read 1681. (Note: The last digit in a 4-digit product may be found mentally as the last digit of the product of the last digits of the factors.)

#### SPOT CHECK 11

| 1.  | What number is the square of 2?  |
|-----|--|
|     |  |
| 2.  | What positive number is a square root of 4?  |
|     | [115]  |
| 3.  | Express in words 5 <sup>2</sup> .  |
| -80 | [116]  |
| 4.  | Express in words $\sqrt{25}$ .   |
|     | [117]  |
| 5.  | What number is represented by 52?  |
|     | [118]  |
| 6.  | A STATE OF THE STA |
|     | [120]  |
| 7.  | $7 = \sqrt{?}.$  |
|     | [121]  |
| 8.  | On the slide rule, every number on the scale is  |
|     | the square of the number directly below it on thescale.  |
|     | [122]  |
| 9.  | Using the slide rule (or the illustration in Frame 123) we   |
|     | find that $8^2 = $   |
|     | [123]  |
| .0. | If 64 is under the hairline on the A scale, is under the hairline on the D scale.  |
|     |  |

# SQUARES AND SQUARE ROOTS (Continued): Right and Left Parts of the A Scale

126. The A scale is divided into two parts. In finding a square root, how do we know which part to use?

If the number is greater than 1, simply count the number of digits to the left of the decimal point.

- If the number of digits is odd, use the left scale.
   If even, use the right scale.
- 127. To find  $\sqrt{6}$ , which scale do we use?

There is one digit to the left of the decimal point. 1 is odd. Accordingly, we use the left scale.

128. To find  $\sqrt{60}$ , which scale do we use?

There are two digits to the left of the decimal point. 2 is even. We use the right scale.

129. Which scale do we use to find  $\sqrt{600}$ ?

There are three digits to the left of the decimal point. 3 is odd. We use the left scale.

130. How do we know which scale to use when we have a number such as 0.06, which is less than 1?

Count the number of zeros to the right of the decimal point (to the first significant digit). If the number of zeros is odd, use the left scale; if even, the right. 0.06 has one zero to the right of the decimal point. Use the left scale.

| 1. | 4.                             | 6.  | 49.  |
|----|--------------------------------|-----|------|
| 2. | 2.                             | 7.  | 49.  |
| 3. | 5 squared, or the square of 5. | 8.  | A; D |
|    | The square root of 25.         | 9.  | 64.  |
| 5. |                                | 10. | 8.   |

# 131. Suppose the number is 0.6, in which there are no zeros to the right of the decimal point?

When there are no zeros, the count is regarded as even. Use the right scale.

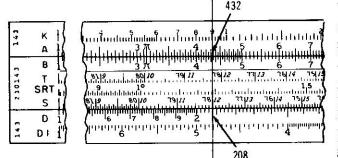
### 132. How about a number like 0.6035?

We are only interested in the number of zeros between the decimal point and the first significant digit. Ignore the zero that follows the 6.

When there are no zeros, the count is regarded as even. Use the right scale.

#### 133. Find $\sqrt{432}$ .

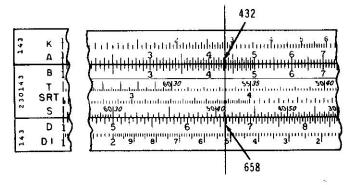
There are three digits to the left of the decimal point. 3 is odd. Therefore we use the left A scale.



- 1. Move hairline to 432 on the left A scale.
- 2. Below, under the hairline on D, read 20.8.

#### 134. Find $\sqrt{0.432}$ .

There are no zeros between the decimal point and 4, the first significant digit. Therefore the decimal is regarded as even, and we use the right A scale.



- 1. Move hairline to 432 on the right A scale.
- 2. Below, under the hairline on D, read 0.658.

# 135. Find the length of the side of a square whose area is 6400 sq. ft.

We know from geometry that the area of a square is found by squaring one of its sides. It follows that the length of any side is found by taking the square root of the area.

- 1. 6400 has an even number of digits to the left of the decimal point. Therefore we use the right A scale. Set the hairline over 6400 on the right A scale.
- 2. Below, under hairline on the D scale, read 80. Answer: 80 ft.

# 136. In finding square roots, how do we know where to put the decimal point in our answer?

As in multiplication, or division, but making rough calculations mentally . . . or on scrap paper.

In Frame 135, the significant number found on the D scale was 8. But 8 could not be the answer, because  $8 \times 8$  equals 64, not 6400. We then tried a two-place number. Obviously 800, a three-place number, would have been too large.

### **▶** SPOT CHECK 12

| Into how many parts is the A scale divided?   |
|---|
| [126]   |
| If we wish to find the square root of a number (greater than 1) which contains an odd number of digits to the left of         |
| the decimal point, we indicate the number on the  |
| If the number of digits to the left of the decimal point is   |
| even, we use the part of the A scale. [126]   |
| To find $\sqrt{7}$ , we use the part of the A scale.  |
| [127]   |
| To find the square root of a two-digit whole number (such   |
| as 60), which part of the A scale do we use?  |
| [128]   |
| We use the left scale to find $\sqrt{0.06}$ , because the number of zeros between the decimal point and the first significant |
| number is (Odd or even?) [130]  |
| A number such as 0.6 has no zeros between the decimal point and the first significant digit. This is considered as            |
| an count. (Odd or even?) [131]  |
| Therefore, for $\sqrt{0.6}$ , use the part of the A scale. [132]  |
| To find $\sqrt{432}$ , we use the A scale. [133]  |
| To find $\sqrt{0.432}$ we use the A scale. [134]  |
|   |

| 1. | Two.   | 6.  | Odd.  |
|----|--------|-----|-------|
| 2. | Left.  | 7.  | Even. |
| 3. | Right. | 8.  | Right |
| 4. | Left.  |     | Left. |
| 5. | Right. | 10. | Right |

# MORE ABOUT THE DECIMAL POINT IN SQUARE ROOTS

# 137. Is there any rule that determines where the decimal point is to be placed . . . in a square root?

The rule is this: Before looking for the square root of a number on the slide rule, move the decimal point right or left an *even* number of places (if necessary)... until you obtain a number between 1 and 100.

The decimal point is moved left if the original number is greater than 100. (Example:  $\sqrt{895}$  becomes  $\sqrt{8.95}$ .) The decimal point is moved right if the original number is less than 1. (Example:  $\sqrt{0.000895}$  becomes  $\sqrt{8.95}$ .)

In the answer (the reading on the D scale) the decimal point is moved back (in the reverse direction) one-half the number of places it was originally moved.

### 138. Using the decimal-point rule, find $\sqrt{23,400}$ .

- We move the decimal point four places left and obtain √2.34.
- 2. Setting 2.34 under the hairline on the left A scale, we read on the D scale, under the hairline, 1.53.
- 3. Now . . . moving the decimal point back two places right (half the distance it was moved left), we have 153, which is the required answer.

#### 139. Try this method with $\sqrt{3850}$ .

- 1. Point off two places to the left, to obtain  $\sqrt{38.50}$ .
- 2. Set hairline to 38.50 on the right A scale, and read 6.20 on D.
- 3. Move decimal point back *one* place to the right. The answer, then, is 62.0.

# 140. How do you point off the square root of a number less than 1?

The same procedure applies.

### 141. Show this with $\sqrt{0.000585}$ .

- Move the decimal point an even number of places right (in this case four) to obtain √5.85.
- 2. Using the A and D scales, find  $\sqrt{5.85} = 2.42$ .
- 3. Move the decimal point back two places to the left. The answer is 0.0242.

### **▶** SPOT CHECK 13

| 1. | To determine where to put the decimal point in the square root of a number, it is useful to move the decimal point |
|----|--|
|    | an number of places thus obtaining a number between 1 and 100. [137]   |
| 2. | In our answer (the square root), we move the decimal point   |
|    | in the reverse direction the number of places it was originally moved. [137]                                       |
| 3. | Using the above procedure to find the square root of 34,500,   |
|    | we first move the decimal point place(s) left.   |
|    | [138]  |
| 4. | In our answer (the square root), we then move the decimal  |
|    | point back place(s) right. [138]   |
| 5. | To find $\sqrt{4961}$ , we first move the decimal point place(s) left. [139]                                       |
| 6. | In reading the answer to the problem above, we move the  |
|    | decimal point back place(s) right. [139]   |
| 7. | Do we use the same procedure in finding the square root of a number less than 1?                                   |
|    | [140]  |
| 8. | In this connection, how would we point off $\sqrt{0.000585}$ ?   |
|    | [141]  |
| 9. | In reading the answer (on the A scale), we move the decimal  |
|    | point back place(s) left. [141]  |

# Practice Exercise 6: Square Roots

Find the square roots of each of the following numbers:

1. 8 7. 0.89

2. 12 8. 7280

 3. 17
 9. 0.0635

 4. 89
 10. 0.0000635

5. 8.90 11. 63,500 6. 890 12. 100,000

Find the length of the side of a square whose area is:

13. 53,500 sq. ft. 15. 3.27 × 10,000,000

14. 0.0776 sq. ft. sq. ft.

1. Even.

6. One.

One-half.
 Four.
 Two.

Two.

- 7. Yes. 8. 5.85. 9. Two.

#### ANSWERS: PRACTICE EXERCISE 6

- 1. 2.83 2. 3.46 3. 4.12
- 9. 0.252 10. 0.00797 11. 252
- 4. 9.43 5. 2.98
- 5. 2.96 6. 29.8 7. 0.943

- 10. 0.00797 11. 252 12. 316 13. 231 ft.
- 14. 0.279 ft. 15. 5720 ft.

# CUBES AND CUBE ROOTS

142. If  $3 \times 3 \times 3 = 27$ , what is 27 in relation to 3?

The cube of 3.

143. And what is 3 in relation to 27?

The cube root of 27.

144. Using mathematical symbols, indicate the cube of 3.

33.

145. What number corresponds to 33?

27.

146. Using mathematical symbols, indicate the cube root of 27.

 $\sqrt[3]{27}$ .

147. What number corresponds to  $\sqrt[3]{27}$ ?

3.

148. What is meant by the expression 43?

The product of  $4 \times 4 \times 4$ .

149. What number corresponds to 43?

64.

150. What is meant by the expression <sup>3</sup>√64?

The number that, when taken as a factor three times (raised to the third power), gives 64.

151. What number corresponds to  $\sqrt[3]{64}$ ?

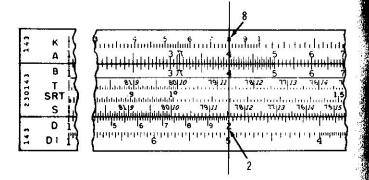
4.

# 152. What scale on the slide rule indicates the cubes of numbers?

The K scale. When the hairline is set to a number on the D scale, the cube of that number is under the hairline on the K scale.

Note: On most slide rules the K scale is located on the body of the rule. Some rules, however, have the K scale on the slide. In this case, the C scale is used in combination with the K scale.

### 153. Using the D and K scales, find 23.

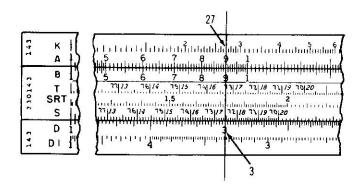


Set hairline over 2 on the D scale. Under hairline read 8 on the K scale.

#### 154. How are cube roots found on the slide rule?

By reversing this operation.

#### 155. Using the D and K scales, find $\sqrt[3]{27}$ .



Set hairline over 27 on the K scale. Under hairline read 3 on the D scale.

#### 156. But there are three K scales: one on the left of the rule one in the middle, and one on the right. How do you know which K scale to use?

To find the cube root of a number between 1 and 10, use the left K scale.

To find the cube root of a number between 10 and 100, use the middle K scale.

To find the cube root of a number between 100 and 1000, use the right K scale.

### SPOT CHECK 14

1. The operation of using 3 as a factor three times is called

|    | finding the of 3. [142]  |
|----|--|
| 2. | What number is the product of $3 \times 3 \times 3$ ?[142]   |
| 3. | What number is the cube root of 27?[143]   |
| 4. | Express 3 <sup>3</sup> in words[144]   |
| 5. | Express $\sqrt[3]{27}$ in words[146]   |
| 6. | The expression 4 <sup>3</sup> means × ×[148]   |
| 7. | 64 = (?)3,[149]  |
| 8. | What number corresponds to $\sqrt[3]{64}$ ?[151]   |
| 9. | When the hairline is set over a number on the D scale, the   |
|    | of that number is under the hairline on the K scale. [152]   |
| 0. | When the hairline is set over 2 on the D scale, is under the hairline on the K scale. [153]                              |
| 1. | We reverse this operation to find the of a number. [154]   |
| 2. | The K scale is divided into a left part, a middle part, and a right part (each called a scale); to find the cube root of |
|    | a number between 1 and 10, we use the K  |
|    | scale; of a number between 10 and 100, the K   |
|    | scale; of a number between 100 and 1000, the [156]   |
|    |  |

# WORKING WITH THE DECIMAL POINT IN CUBE ROOTS

157. Why do we use the left K scale to find  $\sqrt[3]{7}$ ?

Because 7 is a number between 1 and 10.

158. Which K scale do we use in finding <sup>3</sup>√64?

The middle K scale . . . because 64 is between 10 and 100.

159. Do we use the middle K scale to find  $\sqrt[3]{164?}$ 

No. We use the right K scale . . . because 164 is between 100 and 1000.

160. What K scale do we use to find  $\sqrt[3]{64.3}$ ?

The middle K scale . . . because 64.3, like 64, is between 10 and 100.

161. What K scale do we use to find <sup>3</sup>√64.376?

The middle K scale . . . because 64.376 lies between 10 and 100.

162. How is the position of the decimal point determined in a cube root?

The position can be found by approximation. It can also be determined by a special rule.

 1. Cube.
 7. 4.

 2. 27.
 8. 4.

 3. 3.
 9. Cube.

 4. The cube of 3 (or 3 cubed).
 10. 8.

 5. The cube root of 27.
 11. Cube root.

4; 4; 4. 12. Left; middle; right.

# 163. Is there any helpful rule of thumb to find the decimal point in a cube root?

Yes. In the cube root of any number between 1 and 1000, the decimal point is placed immediately after the first digit.

Examples:  $\sqrt[3]{9} = 2.08$  $\sqrt[3]{99} = 4.63$  $\sqrt[3]{990} = 9.97$ 

# 164. What is the special rule for locating the decimal point in the cube root of any number?

- 1. In a given number, move the decimal point right (for a number greater than 1) or left (for a number less than 1) three places at a time until a number between 1 and 1000 is obtained.
- 2. In the answer, move the decimal point back (in the reverse direction) one-third as many places as it was moved originally.

# 165. Using the special rule for locating the decimal point, find 3/23,400,000.

- 1. Moving the decimal point six places left, we obtain  $\sqrt[3]{23.4}$ .
- 23.4 is a number between 10 and 100, therefore we use the middle K scale.
- 3. When the hairline is at 23.4 on the middle K scale we read 2.86 (at the hairline) on the D scale.
- We move the decimal point back two places to the right (one-third of six) and obtain 286, which is the required answer.

### 166. Now, using the special rule for locating the decimal point, find √0.000585.

- Moving the decimal point six places right, we obtain <sup>3</sup>√585.
- 2. 585 is a number between 100 and 1000, therefore we set the hairline over 585 on the right K scale.
- 3. Under the hairline on the D scale we read 8.36.
- Moving the decimal point back two places left (one-third of six), we obtain 0.0836, the required answer.

89

# SPOT CHECK 15

| We use the left K scale to find $\sqrt[3]{5}$ , $\sqrt[3]{6}$ , and $\sqrt[3]{7}$ because the numbers lie between and    |
|--|
| To find a number between 10 and 100, we use theK scale.  |
| [158]  |
| The right K scale is used to find \$\sqrt{164}\$ because   |
| [159]  |
| Is the middle K scale the proper scale to use in finding   |
| <del>√√74.4</del> ?  |
| [160]  |
| What scale should be used to find $\sqrt[3]{74.439}$ ?   |
| [161]  |
| The cube root of any number between 1 and 1000 is a number represented by a numeral in which the decimal                 |
| point immediately follows the digit.   |
| [163]  |
| The special rule for locating the decimal point in the cube<br>root of a number requires that (if necessary) the decimal |
| point of the given number be moved (right or left)places at a time until a number between 1 and 1000 is obtained.        |
| [164]  |
| Using the special rule, move the decimal point in $\sqrt[3]{45,600,000}$ to obtain a number (under the cube-root sign)   |
| between 1 and 1000: <u>%</u> .   |
|  |
|  |

| 9.  | In the problem above, how many places and in what di-  |
|-----|--|
|     | rection was the decimal point moved?,  |
|     | [165]  |
| 10. | When the cube root of 45,600,000 is read on the D scale,   |
|     | the decimal point will be moved back places to   |
|     | the,   |
|     | [165]  |
| 11. | Again using the special rule, point off $\sqrt[4]{0.000697}$ to obtain a number (under the cube-root sign) between 1 and 1000: |
|     | <b>3</b>   |
|     | [166]  |
| 12. | When the cube root of 0.000697 is read on the D scale,   |
|     | the decimal point will be moved back places to   |
|     | the  |
|     | [166]  |
|     | •  |
|     |  |
|     |  |
|     |  |

1; 10.

Middle.

164 is between 100 and 1000.

The middle K scale. First.

Three.

45.6.

Six; left. Two; right.

697.

12. Two; left.

### Practice Exercise 7: Cubes and Cube Roots

#### Cube each of the following numbers:

2.1

3.2 62

89 5. 733 7. 0.452 8. 3.08

9. 1.753

10. 0.0334

11. 0.943 12. 5270

Find the cube root of each of the following numbers:

13. 8.72

30

15. 729 850

17. 7630 18. 0.00763

19. 0.0763 20. 0.763

21. 89,600

22. 0.625

# COMBINED **OPERATIONS:** MULTIPLICATION AND DIVISION

167. In many formulas in geometry, physics, and chemistry it is necessary to multiply three or more factors. (Example: The lateral surface of a cylinder =  $\pi \times$  diameter × height.) Can this be done on the slide rule?

> Yes. Multiplying three or more factors is just as easy as multiplying two.

#### 168. Multiply $642 \times 3.5 \times 0.0164$ .

- 1. Set the right index of the C scale above 642 on the D scale.
- 2. Move hairline to 35 on the C scale. Directly below we could read the intermediate product (225) on the D scale. However, it is not necessary to take this reading . . . we are only interested in the final product.
- 3. Move the left index of the C scale to the hairline (still over 225 on D).
- 4. Now . . . under the third factor (164 on C), read the final product, 369, on D.
- 5. Determine the position of the decimal point by approximation ( $600 \times 4 \times 0.01 = 24$ ).
- 6. The answer, thus, is 36.9.

#### ANSWERS: PRACTICE EXERCISE 7

| 1.  | 9.25        | 12. 146,400,000,000 | ) |
|-----|-------------|---------------------|---|
| 2.  | 32.8        | 13. 2.06            |   |
| 3.  | 238,000     | 14. 3.11            |   |
| 4.  | 422,000     | 15. 9.00            |   |
| 5.  | 705,000     | 16. 9.47·           |   |
| 6.  | 394,000,000 | 17. 19.69           |   |
| 7.  | 0.0925      | 18. 0.1969          |   |
| 8.  | 29.2        | 19. 0.424           |   |
| 9.  | 5.39        | 20. 0.914           |   |
| 10. | 0.0000373   | 21. 44.7            |   |
| 11. | 0.84        | 22. 0. <b>855</b>   |   |

169. Can the same method be used to multiply more than three factors?

Yes. You can multiply any number of factors this way. The position of the decimal point is easy to determine by approximation.

170. How about formulas requiring both multiplication and division?

On the slide rule, multiplication and division can be performed together.

171. What is the easiest way to combine multiplication and division?

The easiest way is to perform one division first, then one multiplication . . . and (if necessary) to continue alternating until you come to the end of the problem.

172. Find the value of  $\frac{7.36 \times 8.44}{92}$ .

Procedure: First divide 7.36 by 92, then multiply the result by 8.44.

- 1. Set 92 on C over 736 on D.
- 2. Opposite 844 on C, read 0.675 on D.

173. In this problem, how was the decimal point located?

By approximation.  $\frac{7 \times 9}{92} = \frac{63}{92}$ , which lies between 0.5 and 1.

174. Find the value of  $\frac{18 \times 45 \times 37}{23 \times 29}$ .

Procedure: (a) Divide 18 by 23. (b) Multiply the product by 45. (c) Divide this product by 29. (d) Multiply this third result by 37.

- 1. Set the hairline to 18 on D.
- 2. Pull 23 on C under the hairline.
- 3. Push hairline to 45 on C.
- 4. Set 29 on C under the hairline.
- 5. Push hairline to 37 on C.
- 6. On D, under hairline, read 449.7. Point off 449 by approximation.

$$\frac{20 \times 40 \times 40}{20 \times 30} = \text{about } 50$$

8. The answer, therefore, is 44.9.

### SPOT CHECK 16

|           | ntinued multiplication on the slide rule is just as easy  | <b>Y</b> |
|-----------|---|----------|
|           |   | 16       |
|           |   |          |
| In<br>rea | multiplying $642 \times 3.5 \times 0.0164$ , it is not necessary d the intermediate product because we are only interest. | st       |
| in        | the product.  |          |
|           |   | 16       |
|           | ow many factors can be multiplied by a continuing slee operation?   | ić       |
| 500       |   | 16       |
| In        | slide-rule operations division can be combined  |          |
|           | ,   | 1.       |
| _         |   | 1 /      |
| In        | operations of this type, it is best to perform one divi   | si       |
| fir       | st, then one  |          |
|           |   | 17       |
|           |   |          |
|           | ter this, and are<br>til the end of the problem is reached.   | _        |
| un        |   | 1 '      |
| _         |   |          |
|           | ow do you locate the decimal point in the answer oblem such as $\frac{7.36 \times 8.44}{92}$ ?                            | to       |
|           |   | 1        |
|           |   |          |
|           | here does the decimal point fall?   |          |

# Practice Exercise 8: Combined Multiplication and Division

- 1.  $\frac{7 \times 8}{5}$  9.  $9.21 \times 0.1795 \times 0.0672$ 2.  $\frac{11 \times 12}{7 \times 8}$  10.  $37.7 \times 4.82 \times 830$
- 3.  $\frac{9 \times 7}{8 \times \frac{1}{4}}$  11.  $\frac{65.7 \times 0.835}{3.58}$
- 4.  $\frac{1375 \times 0.0642}{76,400}$  12.  $\frac{362}{3.86 \times 9.61}$
- 5.  $\frac{45 \ 2 \times 11.24}{336}$  13.  $\frac{24.1}{261 \times 32.1}$ 6.  $\frac{218}{4.23 \times 50.8}$  14.  $\frac{75.5 \times 63.4 \times 95}{3.14}$
- 7.  $\frac{235}{3.86 \times 3.54}$  15.  $\frac{3.97}{51.2 \times 0.925 \times 3.14}$
- 8.  $2.84 \times 6.52 \times 5.19$  16.  $\frac{47.3 \times 3.14}{32.5 \times 16.4}$
- 17.  $\frac{3.82 \times 6.95 \times 7.85 \times 436}{79.8 \times 0.0317 \times 870}$
- 18.  $187 \times 0.00236 \times 0.0768 \times 1047 \times 3.14$
- 19.  $\frac{0.917 \times 8.65 \times 1076 \times 3152}{7840}$
- \*20.  $\frac{45.2 \times 11.24\pi}{336}$
- \*21.  $\frac{45.2 \times 11.24}{336\pi}$
- \* The DF and CF scales have  $\pi$  (pi = 3.14159 . . .) at the left end, and are convenient for multiplications and divisions involving  $\pi$ .

- Two.
- Final.
- Any number. Multiplication.
- Multiplication.

- 6. Division; multiplication; alter-
- By approximation. Before the first digit.

#### ANSWERS: PRACTICE EXERCISE 8

- 11.2
- 2.36 39.4
- 0.001155
- 1.512 6. 1.015
- 7. 17.2
- 96.1 0.1111
- 150.800 11, 15.32

- 12. 9.76 13. 0.00288
  - 14. 144.800 15. 0.0267
  - 16. 0.279
  - 17 41.3 18. 111.4
- 19. 3430 20. 4.75
- 21. 0.481

# RECIPROCALS

175. What is the relation of ½ to 2?

½ is the reciprocal of 2.

176. What is the relation of 2 to 1/2?

2 is the reciprocal of ½.

177. What do we mean by the reciprocal of a number?

I divided by that number.

178. What is the reciprocal of 3?

1/3.

179. What is the reciprocal of 1/3?

3 (because 
$$\frac{1}{\frac{1}{3}} = 1 \times \frac{3}{1} = 3$$
).

180. What is the reciprocal of 3?

$$\frac{3}{2}$$
 (because  $\frac{1}{24} = 1 \times \frac{3}{2} = \frac{3}{2}$ ).

181. How can reciprocals be found on a slide rule?

By using the C scale together with the CI scale (which is directly above the C scale).

When the hairline is set to a number on the C scale. the reciprocal of the number is under the hairline on the CI scale (which is read from right to left).

Conversely, when the hairline is set to a number on the CI scale, its reciprocal is under the hairline on the C scale.

182. But the reciprocals of whole numbers shown in the examples above were common fractions. (Example: the reciprocal of 2 was given as 1/2.) How are common fractions indicated on slide-rule scales?

By their decimal equivalents. Examples:

$$\frac{1}{2} = 0.5$$

$$\frac{1}{3} = 0.333$$

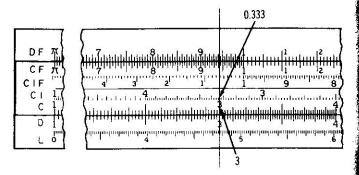
$$\frac{73}{4} = 0.667$$

$$\frac{1}{16} = 0.2$$

$$\frac{26}{26} = 0.4$$

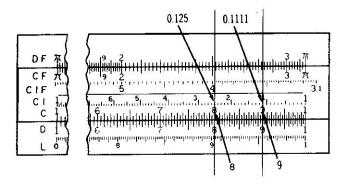
$$\frac{2}{3} = 0.4$$

183. Using the C and CI scales, find the reciprocal of 3.



- 1. Move the hairline to 3 on C.
- 2. Under the hairline, on CI, read 0.333.

184. Using the C and CI scales, find the reciprocals of 8 and 9.



- Set the hairline at 8 on C; under hairline on CI, read 0.125.
- Move the hairline to 9 on C; under hairline on CI, read 0.1111. (Note: The final digit is approximated.)

185. Can reciprocals be found on scales other than C and CI?

Yes. When working on the reverse side of the rule, reciprocals can be found with the D and DI pair. The DI scale gives the reciprocals of numbers on the D scale.

186. Have the reciprocal scales any other use in slide-rule operations?

Multiplication and division can be performed on either pair of reciprocal scales.

187. If we can multiply and divide using only the C and D scales, why should we bother with the reciprocal scales (C and CI, or D and DI)?

To save time and increase accuracy. When reciprocal scales are used (in combination with the C and D scales), it is possible to multiply three factors without resetting the slide.

188. Using the CI scale, multiply  $19 \times 6$ .

Think:  $19 \times 6 = 19 \div \frac{1}{6}$ .

CF Annulum manulum m

- 1. Set 6 on CI directly over 19 on D.
- 2. Under the left index, read 114 on D.

189. Using the CI scale multiply  $19 \times 6 \times 1.71$ .

Think:  $19 \times 6 \times 1.71 = \left(19 \div \frac{1}{6}\right) \times 1.71$ .

- 1. As before, set 6 on CI directly over 19 on D.
- 2. Move hairline to 171 on C.
- 3. On D, under hairline, read 195.

Note that the use of the CI scale has made it possible to multiply three factors with only one setting of the slide.

190. Using the D and CI scales, divide 732, in turn, by 14, 23, 32, 41, and 50.

- 1. Set the right index of the CI scale to 732 on D.

  2. Move harding to 14 on the CI scale. Under
- Move hairline to 14 on the CI scale. . . . Under hairline on D, read 52.3.
- Move hairline to 23 on the CI scale. . . . Under hairline on D, read 31.8.
- 4. Move hairline to 32 on the CI scale. . . . Under hairline on D, read 22.87.
- Move hairline to 41 on the CI scale. . . . Under hairline on D, read 17.85.
- 6. Move hairline to 50 on the CI scale. . . . Under hairline on D, read 14.64.

#### **✓** SPOT CHECK 17

- 1. The reciprocal of 5 is \_\_\_\_\_\_\_[175]
  2. The reciprocal of ½ is \_\_\_\_\_\_\_[176]
  3. We obtain the reciprocal of a number when we divide \_\_\_\_\_\_ by that number. [177]
  4. What is the reciprocal of ½? \_\_\_\_\_\_\_\_[180]
- On the slide rule, the reciprocal of any number on the C scale is shown, directly above, on the \_\_\_\_\_ scale. [181]
- On slide-rule scales, common fractions are represented by their \_\_\_\_\_ equivalents. [182]
- 7. On slide-rule scales, ½ is represented as \_\_\_\_\_\_[182]
  8. What is the reciprocal of 0.333? \_\_\_\_\_\_\_[183]
- 9. The reciprocal of a number under the hairline on the C scale can be found under the hairline on the CI scale; similarly the reciprocal of a number under the hairline on the
  - larly, the reciprocal of a number under the hairline on the D scale can be found under the hairline on the scale. [185]
- 10. Finding reciprocals is not the only function of the reciprocal scales; these scales can also be used for \_\_\_\_\_\_\_ and \_\_\_\_\_\_ [186]
- 11. When the reciprocal scales are used in combination with the C and D scales, it is possible to \_\_\_\_\_\_ three factors without resetting the slide. [187]

2. 7. 3. 1. 4. 7/2.

5. CI.

Decimal.

5 (representing 0.5)

10. Multiplication; division.

11. Multiply.

# Practice Exercise 9: Reciprocals and the **Reciprocal Scales**

- 1. Use the DI scale to find the reciprocals of 16, 260, 0.72, 0.065, 17.4, 18.5, 67.1
- 2. Find  $18.2 \times 21.7$  in the usual way and then read  $1/(18.2 \times 21.7)$  on DI opposite the first answer on D. Similarly find the values of  $1/(2.87 \times 623)$ , and  $1/(0.324 \times 0.497)$
- 3. Using the D scale and the CI scale, multiply 18 by 1/6 and divide 18 by 1/6
- 4. Using the D scale and the CI scale multiply 28.5 by 1/0.385 and divide 28.5 by 1/0.385. Also find 28.5/0.385 and  $28.5 \times 0.385$  by using the C scale and the D scale
- 5. Using the D scale and the CI scale multiply 41.3 by 1/0.207 and divide 41.3 by 1/0.207

# LOGARITHMS

191. In the statement  $10^2 = 100$ , does the exponent 2 have any special significance?

> Yes. It means that 2 is the logarithm of 100 to the base 10. In other words, 10 raised to the second power is 100.

192. What, then, is a logarithm?

The power to which a given base must be raised to produce some specified number.

193. What is the base of the system of common logarithms?

10.

194. Why are logarithms useful?

They simplify many mathematical operations.

195. Give some examples.

By using logarithms, multiplication is reduced to addition, division to subtraction; finding the root of a number is reduced to simple division . . . and raising a number to any desired power is made a matter of simple multiplication.

#### ANSWERS: PRACTICE EXERCISE 9

1. 0.0625; 0.00385; 1.389; 15.39; 3. 2; 162 0.0575; 0.0541; 0.01490 4. 74.0; 10.97 2. 0.00253; 0.000559; 6.21 5. 199.5; 8.55

196. If logarithms can be used to simplify the raising of a number to a power . . . does it follow that logarithms can simplify the finding of roots?

Yes. In finding the power of a number, using logarithms, we multiply the logarithm of the number by the exponent of the power. . . .

Similarly . . . in finding a root of a number, we divide the logarithm of the number by the index of the root.

### 197. Are most logarithms whole numbers?

No. Logarithms consist of two parts: an integral part, and a fractional part. (Example: the logarithm of 50 is 1.699.... 1 is the integral part, .699... is the fractional part.)

### 198. What is the fractional part called?

The mantissa.

# 199. Give an example of a mantissa.

In the logarithm 2.699, the mantissa is .699.

#### 200. What is the integral part called?

The characteristic.

#### 201. Give an example of a characteristic.

In the logarithm 2.699, the characteristic is 2.

### 202. How are common logarithms found on the slide rule?

When the hairline is set over a number on the D scale, the hairline is also over the mantissa of the common logarithm of that number on the L scale.

Thus . . . mantissas can be read directly from the rule. Characteristics must be found by inspection.

#### 203. What determines the characteristic?

The number of places to the left or right of the decimal point.

The characteristic of the logarithm of a number greater than 1 is one less than the number of digits to the left of the decimal point. . . .

The characteristic of the logarithm of a number less than 1 is negative. Its absolute value is one more than the number of zeros immediately to the right of the decimal point.

### 204. What is the characteristic of the logarithm of 155?

There are three digits to the left of the decimal point; the characteristic is one less than 3. . . .

The answer, therefore, is 2.

| 205. | What | is | the | abbreviation | for | "the | logarithm | of | 155"? |  |
|------|------|----|-----|--------------|-----|------|-----------|----|-------|--|
|------|------|----|-----|--------------|-----|------|-----------|----|-------|--|

The abbreviation is "log 155."

### 206. What is the characteristic of log 155.32?

The characteristic is 2, because there are still only three digits to the left of the decimal point.

# 207. What is the characteristic of the logarithm of 0.00015?

The number is less than 1, therefore the characteristic is negative. There are three digits to the right of the decimal point, so the absolute value of the characteristic is one more than three. The characteristic is -4.

# 208. Now, using the D and L scales, find the logarithm of 50.

- 1. To find the mantissa of log 50, push hairline to 50 on D. . . .
- 2. On L, under hairline, read .699. Hence, the mantissa is .699.
- 3. The number 50 has two digits to the left of the decimal point, thus its characteristic is 1.
- 4. Therefore,  $\log 50 = 1.699$ .

#### 209. Find the logarithm of 1.6.

- 1. To find the mantissa of log 1.6, push hairline to 16 on D. . . .
- 2. On L, under hairline, read .204.
- 3. 1.6 has only one digit to the left of the decimal point, thus its characteristic is 0.
- 4. Therefore,  $\log 1.6 = 0.204$ .

### **▶** SPOT CHECK 18

|   | the base[191   |
|---|--|
| • | A logarithm is the exponent of the to which a given base must be raised to produce some specified number           |
|   | [192   |
|   | or the system of common logarithms is  |
|   | [193]  |
|   | Logarithms are useful because they many mathematical operations.   |
|   |  |
|   | Logarithms consist of two parts: a(n) part (a  |
|   | whole number), and a(n) part.  |
|   | The mantissa of a logarithm is the part.   |
|   | [198]  |
|   | In the logarithm 2.699, the mantissa is  |
|   | [199]  |
|   | What is the characteristic of a logarithm?   |
|   | [200]  |
|   | In the logarithm 2.699, the characteristic is  |
|   | [201]  |
|   | When the hairline is over a number on the D scale, the mantissa of the logarithm of that number is under the hair- |
|   | line on the scale.   |
|   | [202]  |

| 1. | The characteristic of a logarithm of a number greater than                                     |
|----|--|
|    | 1 is always one than the number of digits to the left of the decimal point.                    |
|    | [203]  |
| 2. | The absolute value of the characteristic of a logarithm of a                                   |
|    | number less than 1 is always one than the number   |
|    | of zeros immediately to the of the decimal point.  Is the characteristic positive or negative? |
|    | [203]  |
| 3. | The characteristic of the logarithm of 366 is  |
|    | [204]  |
| 4. | The characteristic of the logarithm of 0.00036 is  |
|    | [206]  |

# **WORKING WITH LOGARITHMS**

### 210. What is an antilogarithm?

The number that corresponds to a logarithm. Thus, if 0.30103 is the logarithm of 2, then 2 is the anti-logarithm of 0.30103.

### 211. When do we want to find antilogarithms?

When we look for the answer to a problem in which logarithms are used. We don't want a logarithm for an answer . . . we want the number that corresponds to this logarithm. The answer, thus, is an antilogarithm.

# 212. Using the slide rule, find the antilogarithm of 0.204.

- 1. Set the hairline to 204 on the L scale.
- 2. Under the hairline, on the D scale, read 16.
- The characteristic, 0, tells us that the number we are seeking has one digit left of the decimal point.
- 4. Therefore the antilogarithm of 0.204 is 1.6.

# 213. Find the antilogarithm of 1.204.

- Proceed as above, and again read 16 on the D scale.
- 2. The characteristic, 1, tells us that the number we want has two digits to the left of the decimal point.
- 3. The antilogarithm of 1.204, therefore, is 16.

10. Power. 8. The integral part. 9. 2.

3. 10. Simplify.

> Fractional. .699.

- 5. Integral: fractional.
- 10. L. 11. Less. 12. More; right; negative.
- 13. 2. 14. -4.
- 214. How can we use logarithms to find a given power of a number?
  - 1. We first find the logarithm of the given number. 2. We then multiply this logarithm by the exponent of the given power.
  - 3. Finally, we find the antilogarithm of the product.

### 215. Using logarithms, find 25.

- 1. The characteristic of the log of 2 is 0.
- 2. The mantissa of the log of 2 is .301. 3. The complete log of 2, thus, is 0.301.
- 4. Now multiply by 5:  $5 \times 0.301 = 1.505$ .
- 5. The mantissa .505 (on the L scale) corresponds to 32 (on the D scale).
- 6. The characteristic, 1, tells us that the answer (the required antilogarithm) has two digits to the left of the decimal point.
- 7. Therefore  $2^{5} = 32$ .

#### 216. How can we use logarithms to find any given root of a number?

Simply reverse the operations used in finding the power of a number.

- 1. Find the logarithm of the given number.
- 2. Divide this by the exponent of the required root.
- 3. Find the antilogarithm of this quotient.

# 217. Using logarithms, find √32.

- 1. The logarithm of 32 is 1.505.
- 2. Divide this by 5:  $1.505 \div 5 = 0.301$ .
- 3. The antilogarithm of 0.301 is 2.

### SPOT CHECK 19

| _        |  |
|----------|--|
| V        | When a problem is solved by the use of logarithms, the                         |
| sv<br>nı | wer we want is not a logarithm but a(n)umber that corresponds to a logarithm). |
|          |  |
| I        | f the characteristic of a certain logarithm is 1, the nur                      |
|          | nat corresponds to this logarithm will have of the left of the decimal point.  |
|          |  |
| Ί        | o raise a number to a certain power, using logarithms                          |
| W        | re have to do is the logarithm of the r  |
|          | er by the index of the given power, then find the f this product.              |
|          |  |
| 7        | To raise 2 to the fifth power, multiply the logarithm                          |
| b        | y Then find the antilogarithm of the produ                                     |
|          | ,  |
|          | Γο find any given root of a number, we simplyhe above operation.               |
| _        |  |
| ~        | To find the fifth root of 32, the logar  |

# Practice Exercise 10: Logarithms

Using the D and L scales, find the logarithms of the following numbers:

- 1. 1.34 4. 735 2. 0.312 5. 53,000
- 3. 0.067 6. 0.00053

Using the D and L scales, find the numbers whose logarithms are given below:

7. 2.924 9. 8.380 - 10 (or  $\overline{2}.380$ ) 8. 1.544 10. 0.845

Using the D and L scales to find logarithms and antilogarithms, and the C and D scales for multiplication and division, find to three significant digits the values of the following:

- 11. (3.2)5 14. **∜**286 12. 4254 15.  $\sqrt[7]{1430}$
- 13. <del>₹3.46</del>

| 1. | Antilogarithm. |
|----|----------------|
| 2  | Antilogogithm  |

5. 5.

6. Reverse. 7. Divide.

Multiply; antilogarithm.

ANSWERS: PRACTICE EXERCISE 10

| 1. | 0.127 |
|----|-------|
| •  | T 404 |

8. 35 9. 0.024

T.494 or 9.494 - 10 7.828 or 8.828 - 10 2.866

11. 335

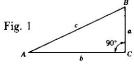
4.724 4.724 or 6.724 - 10 12. 32,600,000,000 13. 1.521 14. 3.10 15. 2.825

MATH REFRESHER

#### TRIGONOMETRY

In the section that follows, it is assumed that the reader is familiar with the elements of trigonometry. The text frames, consequently, are concerned only with the use of the slide rule in simple trigonometric operations-not with an explanation of the trigonometric functions. For those who may have forgotten some definitions and formulas, however, a concise listing is provided:

In the right triangle ABC of Fig. 1, the side opposite the angle A is designated by a, the side opposite B by b, and the hypotenuse by c. Referring to this figure, we write the following definitions and relations.



Definitions of the sine, cosine, and tangent:

sine 
$$A$$
 (sin  $A$ ) =  $\frac{a}{c}$  =  $\frac{\text{opposite side}}{\text{hypotenuse}}$   
cosine  $A$  (cos  $A$ ) =  $\frac{a}{c}$  =  $\frac{\text{adjacent side}}{\text{hypotenuse}}$   
tangent  $A$  (tan  $A$ ) =  $\frac{a}{b}$  =  $\frac{\text{opposite side}}{\text{adjacent side}}$ 

Reciprocal relations:

cosecant 
$$A$$
 (csc  $A$ ) =  $\frac{c}{a} = \frac{1}{\sin A}$   
secant  $A$  (sec  $A$ ) =  $\frac{c}{b} = \frac{1}{\cos A}$   
cotangent  $A$  (cot  $A$ ) =  $\frac{b}{a} = \frac{1}{\tan A}$ 

Relations between complementary angles:

$$\sin A = \cos (90^{\circ} - A)$$

$$\cos A = \sin (90^{\circ} - A)$$

$$\tan A = \cot (90^{\circ} - A)$$

$$\cot A = \tan (90^{\circ} - A)$$

Relations between supplementary angles:

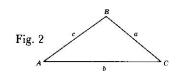
$$\sin (180^{\circ} - A) = \sin A$$
  
 $\cos (180^{\circ} - A) = -\cos A$   
 $\tan (180^{\circ} - A) = -\tan A$ 

Relation between angles in a right triangle:

$$A + B = 90^{\circ}$$

If in any triangle such as ABC of Fig. 2, A, B, and C represent the angles and a, b, and c, represent, respectively, the lengths of the sides opposite these angles, the following relations hold true:

Law of sines: 
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



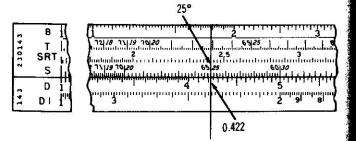
# TRIGONOMETRY

#### 218. How are sines of angles found on the slide rule?

When the hairline is set to an angle on the S scale, the sine of the angle is shown under the hairline on the C scale (or on the D scale when the rule is closed). On rules which have the C scale on the reverse side of the slide (from the S scale), it is usually convenient to operate with the slide closed, and to read the sines on the D scale.\*

\* Note: Although the general principles of slide-rule operations relating to trigonometric functions are more or less the same for all slide rules, the number of scales provided, and their relative positions on the rule, vary with the style and type of rule. The following descriptions apply to the Keuffel & Esser Polyphase rule.

#### 219. Show this in an actual operation: find sin 25°.



- 1. Make sure the rule is closed (index of S scale aligned with index of D scale).
- 2. Set hairline over 25° (blue or black) on the S scale.
- 3. On the D scale, under hairline, read 0.422.

# 220. Why was sin 25° read as 0.422—with the decimal point in front of the first digit?

Because all sines corresponding to the S scale are between 0.1 and 1.0. (On most modern slide rules, this fact is indicated in small numerals at the right of the S scale.)

# 221. What is the purpose of the numerals printed in red on the S scale?

To give the values of cosines. The red numerals increase from right to left.

#### 222. Find cos 65°.

Set the hairline in the same position as for sin 25°, and read on the D scale the same value as read for sin 25°, i.e., 0.422.

Sin 25° and cos 65° occupy the same place on the S scale, because for any angle A, sin  $A = \cos (90^{\circ} - A)$ .

# 223. The smallest angle shown on the S scale is a little over 5.5°. How do we find the sine of smaller angles?

By using the SRT scale, which is immediately above the S scale.

### 224. What is the range of the SRT scale?

The SRT scale is used to find the approximate sines and tangents of angles from a little over 30' to a little under 6°.

# 225. When the sines of angles represented on the SRT scale are found, is the decimal point in the same place as in sines of angles found from the S scale?

No. The range of the sines of angles on the SRT scale, is from 0.01 to 0.1. Therefore, in reading the number under the hairline on the D scale, the decimal point should be placed as in 0.01 (one zero to the right of the decimal point always precedes the first significant digit). Examples:  $\sin 2^{\circ} = 0.0349$ ,  $\sin 3^{\circ} = 0.0523$ , and  $\sin 5^{\circ} = 0.0871$ .

#### 226. Find sin 3.4°.

- 1. Close the rule.
- 2. Set hairline at 3.4° on the SRT scale.
- 3. Under hairline on the D scale, read 0.0593.

#### 227. Find sin 36.4°.

- 1. Close the rule.
- 2. Set hairline at 36.4 (blue or black) on the S scale.
- 3. Under hairline on the D scale, read 0.593.

#### 228. Find cos 36.4°.

- 1. Close the rule.
- Reading right to left red numerals on S scale, set hairline at 36.4.
- 3. Under the hairline on the D scale read 0.805.

### SPOT CHECK 20

| 1. | If the hairline is set over any angle on thescale, the sine of that angle will be under the hairline on              |
|----|--|
|    | the scale. When the rule is closed, the sine   |
|    | can also be read on the scale.   |
|    | [218]  |
| 2. | All sines specified by the S scale have a value between  |
|    | and  |
|    | [220]  |
| 3. | What numbers on the S scale are used to find cosines?  |
|    | [221]  |
| 4. | On the S scale, the numbers specifying cosines are to be read in what direction?                                     |
|    | [221]  |
| 5. | About 5.5° is the smallest angle shown on thescale.  |
|    | [223]  |
| 6. | To find the sine of 4° we would look for the angle on the  |
|    | scale.   |
|    | [224]  |
| 7. | In finding the sine of 4° we read the number below on the D scale, then point off by putting in front of this number |
|    | a(n), a(n), and a(n)   |
|    | [225]  |
| 8. | In finding sin 3.40°, we read 593 on the D scale. When the   |

decimal point is properly located, we obtain the number

| To find sin 36.4°, we set the hairline at 36.4° on the S sca<br>and read the digits 593 on the D scale under the hairline |
|---|
| We then locate the decimal point and obtain   |
| Cos 36.4° is found by using the S scale, reading the r  |
| numbers from to   |
| [29   |

# COSINES OF VERY LARGE AND VERY SMALL ANGLES

229. How can cosines be expressed in terms of sines?

The cosine of any angle is the sine of  $(90^{\circ} - \text{that angle})$ .

230. Express this idea in mathematical notation.

As follows:  $\cos A = \sin (90^{\circ} - A)$ .

231. Find cos 88°.

Since  $\cos 88^{\circ}$  is not on the S scale, we subtract  $88^{\circ}$  from  $90^{\circ}$ , then find the sine of the difference . . . that is,  $\sin 2^{\circ}$ .

- 1. Set hairline at 2° on the SRT scale.
- 2. Under hairline on D scale, read 349.
- 3. Fix decimal point to obtain 0.0349.

232. On the S scale of the slide rule, no cosines are shown for angles larger than about 84.3°. How would you find cos 87°?

By subtracting 87° from 90°, thus obtaining 3° as a difference; then find sin 3° from the SRT and D scales.

233. How do we find cosines of very small angles—angles smaller than about 5.7°?

Cosines of angles smaller than 5.7° all have an approximate value of 1.0. On some modern slide rules the S scale has been extended to include the cosines of small angles.

|                           | ANSWERS: SPOT CHECK 20  |
|---------------------------|---|
| 2.<br>3.<br>4.            | S; C; D. 6. SRT. 7. Zero; decimal point; zero. The numbers printed in red. 8. 0.0593. From right to left. 9. 0.593. S. 10. Right; left. |
| -<br> <br> <br> <br> <br> | SPOT CHECK 21   |
| 1.                        | The cosine of any angle is the of (90° - that angle).   |
|                           | [229]   |
| 2.                        | $Sin (90^{\circ} - A) = \underline{\qquad} A.$  |
|                           | [230]   |
| 3.                        | Is cos 88° indicated directly on the S scale?   |
|                           | [231]   |
| 4.                        | To find cos 87° on the S scale, we subtract 87° from  |
|                           | , obtaining as a difference; then   |
|                           | find sin  |
|                           | [232]   |
| 5.                        | Cosines of angles smaller than about 5.7° have an approxi-  |

### Practice Exercise 11: Sines and Cosines

By examination of the slide rule verify the fact that on the S scale from the left index to 10° the smallest subdivision represents 0.05°; from 10° to 20° it represents 0.1°; from 20° to 30° it represents 0.2°; from 30° to 60° it represents 0.5°; from 60° to 80° it represents 1°; and from 80° to 90° it represents 5°.

7 1 5029

Find the sine of each of the following angles:

| ۷. | 30"   | /.  | 1.505  |  |
|----|-------|-----|--------|--|
| 3. | 38°   | 8.  | 14.63° |  |
| 4. | 3.33° | 9.  | 22.4°  |  |
| 5. | 90°   | 10. | 11.80° |  |
| 6. | 88°   | 11. | 51.5°  |  |

Find the cosine of each of the following angles:

| 12. | 30°   | 17. | 1.583° |
|-----|-------|-----|--------|
| 13. | 38°   | 18. | 14.63° |
| 14. | 3.33° | 19. | 22.4°  |
| 15. | 90°   | 20. | 11.80° |
| 16. | 88*   | 21. | 51.5°  |

Find angle A in each of the following equations:

| 1.  | I find angle 11 in each of the following equations: |     |                  |  |  |  |
|-----|---|-----|------------------|--|--|--|
| 22. | $\sin A = 0.5$                                      | 28. | $\cos A = 0.5$   |  |  |  |
| 23. | $\sin A = 0.875$                                    | 29. | $\cos A = 0.875$ |  |  |  |
| 24. | $\sin A = 0.375$                                    | 30. | $\cos A = 0.375$ |  |  |  |
| 25. | $\sin A = 0.1$                                      | 31. | $\cos A = 0.1$   |  |  |  |
| 26. | $\sin A = 0.015$                                    | 32. | $\cos A = 0.015$ |  |  |  |
| 27. | $\sin A = 0.62$                                     | 33. | $\cos A = 0.62$  |  |  |  |

[233]

mate value of \_\_\_\_\_\_.

Sine.
 Cos.
 No.

1.00

4. 90°; 3°; 3°. 5. 1.0.

33. 51.7°

#### ANSWERS: PRACTICE EXERCISE 11

| 2.  | 0.5    | 18. | 0.968  |
|-----|--------|-----|--------|
| 3.  | 0.616  | 19. | 0.925  |
| 4.  | 0.0581 | 20. | 0.979  |
| 5.  | 1      | 21. | 0.623  |
| 6.  | 0.999  | 22. | 30°    |
| 7.  | 0.0267 | 23. | 61°    |
| 8.  | 0.253  | 24. | 22°    |
| 9.  | 0.381  | 25. | 5.74°  |
| 10. | 0.204  | 26. | 0.86°  |
| 11. | 0.783  | 27. | 38.3°  |
| 12. | 0.866  | 28. | 60°    |
| 13. | 0.788  | 29. | 29°    |
| 14. | 0.988  | 30. | 68°    |
| 15. | 0      | 31. | 84.26° |
| 16. | 0.0349 | 32. | 89.14° |

# HOW TO FIND TANGENTS AND COTANGENTS ON THE SLIDE RULE

#### 234. Does the slide rule have a scale for tangents?

Yes... the T (tangent) scale. The tangents of angles indicated under the hairline on the T scale can be read under the hairline on the C or CI scales.

#### 235. Where is the T scale?

On the slide.

#### 236. Where are the C and CI scales?

On the reverse side of the slide. (With the rule closed, the tangent can be read directly on the D and DI scales.)

# 237. On the T scale, why are some of angles indicated in blue or black type . . . and some in red?

The blue or black numbers on the T scale represent angles from 5.71° to 45°... the red numbers represent angles from 45° to 84.29°.

# 238. What is the relation of the angles on the T scale to the numbers indicated on the C, CI, D, and DI scales?

When the hairline is set to an angle A on T blue or black, tan A is under the hairline on the C scale (or on the D scale when the rule is closed).

When the hairline is set to an angle A on T red, tan A is under the hairline on the CI scale (or on the DI scale when the rule is closed).

# 239. In reading tangents on the slide rule, where is the decimal point placed?

If the tangent is to be read on the C (or D) scale, the decimal point precedes the first significant digit. (Example: Opposite 26° on T (blue or black) read 0.488 on C.)

If the tangent is to be read on the CI (or DI) scale, the decimal point follows the first significant digit. (Example: Opposite 64° on T (red), read 2.05 on CI.)

| 240 | How | can | cotangents | be | found | on | the | slide | rule? |
|-----|-----|-----|------------|----|-------|----|-----|-------|-------|
|     |     |     |            |    |       |    |     |       |       |

The easiest way is to think of the cotangent of an angle as the tangent of  $(90^{\circ} - \text{that angle}) \cdot ...$  in short, as the tangent of the complement of the given angle.

#### 241. Find cot 26°.

Cot  $26^{\circ} = \tan (90^{\circ} - 26^{\circ}) = \tan 64^{\circ}$ .

- 1. Set hairline at 64° on T.
- Read 2.05 under the hairline on CI (or on DI when the rule is closed).

# 242. How can we find the tangents of angles smaller than 5.71°?

The sine and the tangent of any angle smaller than 5.71° are so nearly equal that for all practical purposes they may be considered the same. Thus . . . to find the tangent of an angle smaller than 5.71°:

- 1. Set hairline over angle on the SRT scale.
- 2. Read the significant digits on the C scale (or on the D scale, when rule is closed).
- 3. Point off by placing a zero, a decimal point, and another zero in front of the first significant digit.

#### 243. Find tan 2.25°.

- 1. Set hairline over 2.25° on the SRT scale.
- 2. Under hairline on the C scale (or on the D scale when rule is closed), read the significant digits 393.
- Putting a zero, a decimal point, and a zero in front of the first significant digit, we have: tan 2.25° = 0.0393.

#### **✓** SPOT CHECK 22

| 1. | The tangents of angles represented by numbers on the T                                  |
|----|---|
|    | scale are to be read on the or  |
|    | scales; or with the rule closed, on the or  |
|    | scales.   |
|    | [234, 236]  |
| 2. | On the T scale, angles from 5.71° to 45° are represented by                             |
|    | numbers; the numbers  |
|    | represent angles from to 84.29°.  |
|    | [237]   |
| 3. | The tangents of angles printed in blue or black on T scale are to be read on the scale. |
|    | [238]   |
| 4. | The tangents of angles printed in red on the T scale are to be read on the scale.       |
|    | [238]   |
|    | [230]   |
| 5. | In pointing off a tangent read on the C (or D) scale, the                               |
|    | decimal point the significant digit.  |
|    | [239]   |
| 6. | In pointing off a tangent read on the CI (or DI) scale, the                             |
|    | decimal point the significant digit.  |
|    |   |

|     | [2-   |
|-----|---|
| То  | find cot 26°, we look up tan  |
|     | [2·   |
|     | find the tangents of angles smaller than 5.71°, we sSRT scale, which is the same table we used to find  |
|     | of small angles.  |
|     | [2-   |
|     | nen an angle is represented by a number on the SRT sca<br>sine or tangent read on the C scale (or D scale, wh<br>rule is closed) is a number whose first significant di |
|     |   |
| the | preceded by a(n), a(n)  |

# Practice Exercise 12: Tangents and Cotangents

1. Fill out the following table:

|  |  |  | Angl |
|--|--|--|------|
|  |  |  |      |

|     | 8.1° | 27.25° | 62.32° | 1.017° | 74.25° | 87° | 47.47° |
|-----|------|--------|--------|--------|--------|-----|--------|
| tan |      |        |        |        |        |     |        |
| cot |      |        |        |        |        |     |        |

The following numbers are tangents of angles. Find the angles.

2. 0.24

8. 0.432

0.785
 0.92

9. 0.043 10. 0.0149

5. 0.54

11. 0.374 12. 3.72

6. 0.0597. 0.082

The following numbers are cotangents of angles. Find the angles.

- 13. 0.24
- 14. 0.785 15. 0.92
- 20. 0.043 21. 0.0149

16. 0.54 17. 0.059 22. 0.374

19. 0.432

18. 0.082

23. 3.72

1. C; CI; D; DI.

2. Blue or black; red; 45°.

3. C (or D. when rule is closed).

4. CI (or DI, when rule is closed).

Precedes; first.
 Follows; first.

90° — the angle (or the complement of the angle).

9. Sines.

10. Zero; decimal point; zero.

#### ANSWERS: PRACTICE EXERCISE 12

| 1.  | 0.142  | 0.515 | 1.907 | 0.0177 | 3.55    | 19.1   | 1.09  |
|-----|--------|-------|-------|--------|---------|--------|-------|
|     | 7.03   | 1.94  | 0.524 | 56.40  | 0.282   | 0.0524 | 0.918 |
| 2.  | 13.50° |       | 707   |        | 13. 76. | 5°     |       |
| 3.  | 38.15° |       |       |        | 14. 51. | 85°    |       |
| 4.  | 42.60° |       |       |        | 15. 47. | 40°    |       |
| 5.  | 28.37° |       |       |        | 16. 61. | 63°    |       |
| 6.  | 3.38°  |       |       |        | 17. 86. | 6°     |       |
| 7.  | 4.69°  |       |       |        | 18. 85. | 3°     |       |
| 8.  | 23.36° |       |       |        | 19. 66. | 6°     |       |
| 9.  | 2.465° |       |       |        | 20. 87. | 54°    |       |
| 10. | 0.855° |       |       |        | 21. 89. | 145°   |       |
| 11. | 20.5°  |       |       |        | 22. 69. | 5°     |       |
| 12. | 74.95° |       |       |        | 23. 15. | 05°    |       |

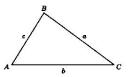
# **SOLUTION OF TRIANGLES**

### 244. What is meant by "solving" a triangle?

Finding the unknown parts.

#### 245. What are the "parts" of a triangle?

The three sides . . . and the angles opposite these sides.



The parts are usually indicated as shown in the diagram. The angles are designated by the capital letters A, B, and C. The side opposite A is designated by a small a, the side opposite B by a small b, the side opposite C by a small c.

#### 247. To solve a triangle, what information must we have?

In most cases it is enough to know the values of two sides and an angle, of two angles and a side, or three sides.

# 248. What is the easiest way to solve a triangle in which one side, the opposite angle, and another angle are given?

By applying the law of sines.

#### 249. What is the law of sines?

The rule which asserts that in any triangle the sides are respectively proportionate to the sines of the opposite angles.

That is:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

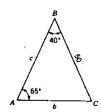
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### 250. Why is the law of sines important?

Because when it can be used, it reduces the operation of solving a triangle to the simple matter of finding the missing parts in a proportion. (See Frames 102–113.)

### 251. Solve a triangle by applying the law of sines to sliderule operations.

Given a triangle in which a = 50,  $A = 65^{\circ}$ , and  $B = 40^{\circ}$ . Find b, c, and C.



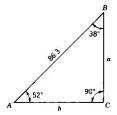
First determine C. We know, from geometry, that  $A+B+C=180^\circ$ , hence  $C=180-(A+B)=180^\circ-105^\circ=75^\circ$ .

With C found, it is easy to find b and c by using the proportion

$$\frac{\sin 65^{\circ}}{50} = \frac{\sin 40^{\circ}}{b} = \frac{\sin 75^{\circ}}{c}$$

- 1. Opposite 50 on D, set 65° on S.
- Push hairline to 40° on S, and on D, under hairline, read: b = 35.5.
- Push hairline to 75° on S, and on D, under hairline, read: c = 53.3.

### 252. Find the unknown parts in this right-angled triangle:



Note that 90° is opposite side 86.3.\*

- 1. Opposite 863 on D, set 90° on S.
- 2. Opposite 52° on S read a = 68.0 on D.
- 3. Opposite 38° on S read b = 53.1 on D.

\*Observe that in solving a triangle it is not necessary to write out the law of sines. We merely apply the principle of the law of sines by representing opposite parts of a triangle as opposite settings on the S and D scales.

The parts to be set opposite can be read directly from the diagram.

# 253. The law of sines can be used only when we know at least one pair of opposite parts. How are other triangles solved?

By applying one or more of the standard trigonometric formulas. (See review list of formulas, pp. 116–117.) The law of sines, when it applies, is a convenient short cut.

#### **✓ SPOT CHECK 23**

| 1. | Finding the unknown parts of a triangle is calledthe triangle.  |
|----|---|
|    | [244]   |
| 2. | The three sides, and the angles opposite these sides, are called the of a triangle.   |
|    | [245]   |
| 3. | The angles of a triangle are usually designated by capital letters. The same letters are used for the sides opposite these  |
|    | angles. But the sides are designated by letters.  |
|    | [246]   |
| 4. | If we are to solve a triangle, what is the minimum number of parts whose values must be known?  |
|    | [247]   |
| 5. | If among the parts of a triangle that are given, we have two angles and the side opposite one of them, the easiest way to solve the triangle (using a slide rule) is to apply what law? |
|    | [248]   |
| 6. | State this law in mathematical notation.  |
|    | [249]   |
| 7. | This law reduces the operation of solving a triangle to the   |
|    | simple matter of finding the missing parts in a   |
|    | [250]   |
| 8. | How are triangles solved when the given parts do not include a pair of opposite parts?  |

# **Practice Exercise 13: Solution of Triangles**

Solve the triangle having the given parts:

- 10. a = 83.4\*19. a = 501. a = 50 $A = 72.12^{\circ}$ c = 66 $A = 65^{\circ}$  $C = 90^{\circ}$  $A = 123.2^{\circ}$  $B = 40^{\circ}$ 11. a = 6020. a = 8.782. c = 60c = 10c = 100 $A = 50^{\circ}$  $C = 90^{\circ}$  $A = 61.4^{\circ}$  $B = 75^{\circ}$ 21. b = 0.23412. a = 0.6243. a = 550 $A = 10.2^{\circ}$ c = 0.91c = 0.198 $B = 109^{\circ}$  $C = 90^{\circ}$  $B = 46.6^{\circ}$ 13. b = 4250 $†22. \quad a = 21$ 4. a = 795 $A = 52.68^{\circ}$  $A = 79.98^{\circ}$  $A = 4.17^{\circ}$  $C = 90^{\circ}$  $B = 75^{\circ}$  $B = 44.68^{\circ}$ 14. b = 2.89123. b = 85. a = 50.6 $A = 38.67^{\circ}$ c = 5.11a = 120 $A = 60^{\circ}$  $C = 90^{\circ}$  $C = 90^{\circ}$ 24. a = 4015. b = 5126. a = 729
  - $B = 68.83^{\circ}$  c = 900 b = 3  $C = 90^{\circ}$   $A = 75^{\circ}$ 7. b = 200 16. a = 52 25. c = 35.7  $A = 64^{\circ}$  c = 60  $A = 58.65^{\circ}$

 $C = 90^{\circ}$ 

17. a = 120

 $A = 43.5^{\circ}$  b = 80  $B = 10.85^{\circ}$   $C = 90^{\circ}$   $A = 60^{\circ}$   $C = 90^{\circ}$  b = 47.7 18 b = 91.1 27. a = 0.821

 $C = 90^{\circ}$ 

26. c = 0.726

- 9. b = 47.7 18. b = 91.1 27. a = 0.821  $B = 62.93^{\circ}$  c = 77  $B = 21.57^{\circ}$   $C = 90^{\circ}$   $B = 51.1^{\circ}$   $C = 90^{\circ}$
- 28. The length of a kite string is 250 yds., and the angle of elevation of the kite is 40°. If the line of the kite is straight, find the height of the kite.
- 29. A vector is directed due northeast and its magnitude is 10. Find the component in the direction of north,
  - \*  $\sin 123.2^{\circ} = \sin (180^{\circ} 123.2^{\circ}) = \sin 56.8^{\circ}$
  - † The SRT scale must be used for 4.17°.

 $C = 90^{\circ}$ 

8. c = 11.2

† The SRT scale must be used for 4.17.

† The SRT scale must be used for angle B.

[253]

1. Solving.

2. Parts.

Small (or lower-case).

Three.

5. The law of sines.

 $B = 46.7^{\circ}$ 

b = 0.662

a = 5570

c = 7010

13.  $B = 37.32^{\circ}$ 

14.  $B = 34.4^{\circ}$  $A = 55.6^{\circ}$ a = 4.22

 $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ 

7. Proportion.

 $A = 55.3^{\circ}$ 

 $B = 34.7^{\circ}$ a = 740

 $A = 60.1^{\circ}$  $B = 29.9^{\circ}$ b = 29.9 $B = 35.3^{\circ}$  $C = 84.7^{\circ}$ c = 138 $A = 87.8^{\circ}$  $C = 41.1^{\circ}$ a = 117Impossible  $B = 28.6^{\circ}$  $C = 90^{\circ}$ b = 4.79 $A = 17.9^{\circ}$  $C = 53.1^{\circ}$ a = 0.076b = 279c = 284 $C = 100.83^{\circ}$ c = 123.8 $B = 3.31^{\circ}$  $C = 116.69^{\circ}$  $B = 4.15^{\circ}$  $C = 100.85^{\circ}$ c = 40.7 $B = 31.35^{\circ}$ a = 30.5b = 18.57 $A = 79.15^{\circ}$ a = 0.713b = 0.1367

8. By applying one or more of the standard trigonometric formulas.

#### EXERCISE 13

|       |                                | ANSWERS: PRACTICE E |
|-------|--------------------------------|---------------------|
| ı.    | $C = 75^{\circ}$               | 15.                 |
|       | b = 35.45                      |                     |
|       | c = 53.3                       |                     |
| 2.    | $C = 55^{\circ}$               | 16.                 |
|       | b = 70.7                       |                     |
| 10000 | a = 56.1                       | 90000               |
| 3.    | $C = 123.2^{\circ}$            | 17.                 |
|       | b = 2255                       |                     |
|       | c = 2600                       |                     |
| 4.    | $C = 55.34^{\circ}$            | 18.                 |
|       | b = 568                        |                     |
| 5.    | c = 664<br>$B = 51.33^{\circ}$ | 19.                 |
| J,    | c = 81.0                       | 20.                 |
|       | c = 61.0<br>b = 63.2           | 20.                 |
| 6.    | $A = 21.17^{\circ}$            |                     |
| υ.    | b = 1884                       | 21.                 |
|       | c = 2020                       |                     |
| 7.    | $B=26^{\circ}$                 |                     |
| -     | a = 410                        | 22.                 |
|       | c = 456                        |                     |
| 8.    | $B = 46.5^{\circ}$             |                     |
|       | a = 7.71                       | 23.                 |
|       | b = 8.12                       |                     |
| 9.    | $A = 27.07^{\circ}$            |                     |
|       | a=24.4                         | 24.                 |
| _     | c = 53.6                       |                     |
| 0.    | $B = 17.88^{\circ}$            |                     |
|       | b = 26.9                       | 25.                 |
|       | c = 87.6                       |                     |
| 1.    | $A = 36.9^{\circ}$             | oc                  |
|       | $B = 53.1^{\circ}$<br>b = 80   | 26.                 |
| 2.    | $b = b0$ $A = 43.3^{\circ}$    |                     |
| 4.    | A 73.3                         |                     |

# HOW TO CONVERT MEASUREMENTS EASILY

The following tables show a simplified slide-rule method of conversion from various units of measurement to others.

For instance: 1 inch = 2.54 centimeters. To convert inches to centimeters, set the index of the C scale to 2.54 on the D scale. Then all readings on the C scale will represent inches, and the corresponding readings on the D scale will show the equivalents in centimeters. The conversion factors given are correct to four significant digits.

#### **Conversion Factors**

| *   | Set index<br>of C scale<br>to D<br>scale at:       | On C<br>scale<br>read<br>measure-<br>ment in:                  | On D<br>scale<br>read<br>equiva-<br>lent in:  |
|---|--|--|---|
| LINEAR MEASURE  |  |  |   |
| 1 inch = 2.54 cm<br>1 foot = 0.3048 m<br>1 yard = 0.9144 m<br>1 mile = 1.609 km<br>1 mile = 5280 ft.<br>1 naut. mile = 1.152 mi.  | 2.54<br>0.3048<br>0.9144<br>1.609<br>5280<br>1.152 | in.<br>ft.<br>yds.<br>mi.<br>mi.<br>naut. mi.                  | cm<br>m<br>m<br>km<br>ft.<br>mi.  |
|   |  |  |   |
| AREA MEASURE  1 sq. inch = 6.452 cm <sup>2</sup> 1 sq. foot = 0.0929 m <sup>2</sup> 1 sq. yard = 0.8361 m <sup>2</sup> 1 sq. mile = 2.59 km <sup>2</sup> 1 sq. mile = 640 acres 1 acre = 43,560 sq. ft.  VOLUME MEASURE 1 cu. inch = 16.39 cm <sup>3</sup> 1 cu. foot = 0.0283 m <sup>3</sup> | 6.452<br>0.0929<br>0.8361<br>2.59<br>640<br>43,560 | sq. in. sq. ft. sq. yds. sq. mi. sq. mi. acres cu. in. cu. ft. | cm <sup>2</sup><br>m <sup>2</sup><br>m <sup>2</sup><br>km <sup>2</sup><br>acres<br>sq. ft.<br>cm <sup>3</sup> |
| 1 cu. yard = $0.7646 \text{ m}^3$   | 0.7646   | çu. yds.   | $m_3$   |
| MEASURE OF CAPACITY  1 U.S. gallon = 3.785 liters  1 U.S. gallon = 231 cu. in.  1 cubic foot = 28.32 liters   | 3.785<br>231<br>28.32                              | U.S. gal.<br>U.S. gal.<br>cu. ft.                              | liters<br>cu. in.<br>liters   |
| WEIGHT  |  |  |   |
| 1 pound == 0.4536 kg<br>1 grain == 0.0648 g<br>1 U.S. gallon == 8.345 lbs.*<br>1 cu. ft. of water == 62.43 lbs.*  | 0.4536<br>0.0648<br>8.345<br>62.43                 | lbs.<br>grains<br>U.S. gal.<br>cu. ft.                         | kg.<br>grams<br>lbs.<br>lbs.  |

<sup>\*</sup> Pure water at maximum density, 39.1° F.

27.  $A = 68.43^{\circ}$ 

28. 160.7 yds.

29. 7.07

b = 0.325

c = 0.883

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2BC-8-64