SIMPLIFIED
REINFORCED CONCRETE MATHEMATICS

DERIVATION OF SIMPLE, UNIVERSAL FORMULAS AND APPLICATION OF SAME TO BEAMS, COLUMNS AND ARCHES

WITH NOMOGRAPHIC COMPUTING DEVICE

BY

MELVIN D. CASLER, B.E.
Assoc. M. Am. Soc. C.E.

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PREFACE

THE main purpose of this booklet is to provide practical working formulas for the design and investigation of reinforced concrete members, and means for applying these formulas with a minimum of laborious computations.

Furthermore, the proposed formulas are derived for general application to beams subjected to direct longitudinal stress in conjunction with transverse moment, to eccentrically loaded columns and to arches. The paper also presents some labor-saving devices for use in proportioning members, and the application of the formulas to various beams, columns and arches is demonstrated by definite examples.

The multitudinous curves and tables which have been offered for the use of the designing engineer since the advent of reinforced concrete would form a unique and voluminous collection if all could be gathered together; and of course the only excuse for their existence is the complex and cumbersome nature of the theoretically evolved formulas. One of the objects of this paper is to so simplify the formulas, without in the least detracting from their mathematical accuracy, as to make it entirely unnecessary to resort to special curves and tables for the various assumptions as to properties, stresses, dimensions, percentage of reinforcement, manner of loading and supporting, etc., etc., in an effort to avoid the laborious operations involved by the present formulas. Such a set of special tables and curves, covering all the various assumptions met with in current practice under varying conditions and circumstances, attempts too much to be easily handled, and often leads to confusion and erroneous results if followed blindly without a true conception of their limitations and without first checking the accuracy of their construction. Moreover, much time is always lost in finding the curves or tables applicable to any special case, with the possibility that the ones selected might not after all be the proper ones to apply under the circumstances.

For work of a varied nature, the methods hereinafter developed effect a large saving of time over prevalent methods of computation, even when the latter are aided by the tables and diagrams offered for that purpose. M. D. C.

YONKERS, N. Y.,
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SIMPLIFIED REINFORCED CONCRETE MATHEMATICS

CHAPTER I

DERIVATION OF FORMULAS

Fig. 1 represents a portion of a reinforced concrete beam or arch before applying load. Fig. 2 represents the same member as deflected by bend-

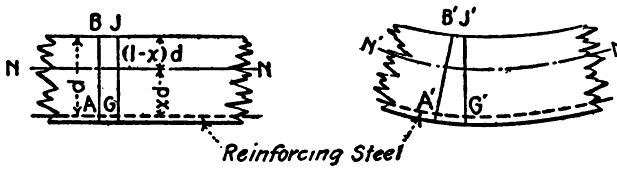


FIG. 1 FIG. 2

ing moment; the sections AB and GJ, which are perpendicular to the neutral surface, NN, taking the positions $A'B'$ and $G'J'$ respectively, assuming $A'B'$ and $G'J'$ to remain plane sections after deflection. Let us assume, for simplicity, that the sections AB and GJ are 1 inch apart, so that the deformations indicated by $A'G'$ and $B'J'$ will be unit deformations. Also, said deformations may be considered as being produced by the stresses developed at section AB; that is, the stresses are considered as being constant through the distances $A'G'$ and $B'J'$. A is on the tension side of the member and B is on the compression side.

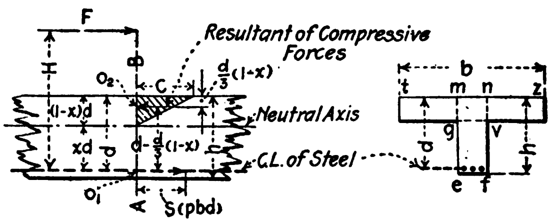


FIG. 3 FIG. 4

In order to make the following discussion general, it is first applied to a reinforced concrete member subjected to a direct longitudinal stress in conjunction with bending moment. The bending moment may be produced by a transverse loading or by the eccentricity of the longitudinal force, F, acting at a distance, H, from the center line of the reinforcing steel as indicated in Fig. 3, or by a combination of such loadings.

Nomenclature

D_c = unit deformation of concrete, $= BJ - B'J'$.

D_s = unit deformation of steel, $= A'G' - AG$.

C = maximum unit compressive stress in concrete at section AB in pounds per square inch.

S = unit tensile stress is steel at section AB in pounds per square inch.

M = transverse bending moment at section AB in inch-pounds.

E_c = coefficient of elasticity of concrete in compression.

E_s = coefficient of elasticity of steel in tension.

$r = \frac{E_s}{E_c}$.

b = width of member in inches.

d = depth of member to center line of steel in inches, as indicated in Fig. 3.

h = total depth of member in inches, as indicated in Fig. 3.

p = percentage of steel reinforcement = area of steel in tension in section AB in square inches $\div bd$.

a = cross-sectional area of steel in tension in section AB in square inches $= pbd$.

F = total direct stress normal to section AB in pounds.

H = perpendicular distance in inches between the force F and the intersection of center line of reinforcing steel with section AB as indicated in Fig. 3.

x is a decimal coefficient as indicated in Fig. 3.

Signs

Observe the following rules as to signs:

F is + when compression and − when tension.

External moments tending to produce tension in the steel are considered as + moments; opposite moments are −.

We will then give H the proper sign to make the sign of the moment FH satisfy the above assumptions. Referring to Fig. 3, it is seen that these conditions give H the + sign when measured from O_1 towards the compression side of the member and − when measured in the opposite direction.

Assumptions

1. Concrete tensile resistance below the neutral axis not considered.

2. Plane cross-sections before deflection remain planes after deflection.

3. E_c and E_s remain constant for the various working values of C and S respectively.

4. Assumptions 2 and 3 lead to the assumption that the unit concrete stress decreases uniformly from the maximum value, C, at the compression side of the member to zero at the neutral axis.

Fundamental Stress Relations:

Let us first review the fundamental relations existing between the compressive stress in the concrete and the tension in the embedded steel.

From the definition of the coefficient of elasticity,

$$E_c=\frac{C}{D_c} \quad \text{and} \quad E_s=\frac{S}{D_s},$$

whence,

$$D_c=\frac{C}{E_c} \quad \text{and} \quad D_s=\frac{S}{E_s}.$$

$$\therefore \quad \frac{D_c}{D_s}=\frac{E_sC}{E_cS}=r\frac{C}{S}.$$

Referring to Figs. 1 and 2, NN and $N'N'$ represent the neutral axis, on which $A'B'$ and $G'J'$ are the same distance apart as AB and GJ. Hence it is seen that

$$\frac{D_c}{D_s}=\frac{(1-x)d}{xd}=\frac{1-x}{x}.$$

Then, since we have proven $\frac{D_c}{D_s}=r\frac{C}{S}$, we have

$$\frac{1-x}{x}=r\frac{C}{S}. \qquad \ldots\ldots \quad (a)$$

This relation holds whatever the condition of loading.

Attention is called to the fact that whenever the conditions of the problem are such as to fix the value of the quantity, x, then for a given r the quantities C and S bear a definite ratio to each other; that is, we cannot under such conditions assume C and S independently of each other.

Necessary Conditions for Equilibrium:

To produce equilibrium on section AB we have the following three necessary conditions:

The algebraic sum of the vertical forces acting on section AB (those lying in the plane AB) must $=$ zero.

The algebraic sum of the horizontal forces (those normal to AB) must = zero.

The algebraic sum of the moments about any point, of the forces acting on section AB must = zero.

We will refer to these equilibrium conditions as conditions A, B and C, respectively.

Equilibrium Condition A

Equilibrium condition A is taken care of in the consideration of the shear on the section. The total resultant shear of the external forces in the plane AB must be resisted by the shearing stresses in the concrete and steel cut by section AB.

Equilibrium Condition B

Referring to Fig. 3, the total compression in the concrete on section AB =

$$b\frac{C(1-x)d}{2}=Cbd\frac{1-x}{2}.$$

Total tension in steel on section $AB = Sa = Spbd$.

Then, making F + when compression and − when tension, we have

$$F=Cbd\frac{1-x}{2}-Spbd.$$

Substituting for C in the above its value in terms of S as given by Eq. (a) we have,

$$F=\frac{S}{r}\,\frac{1-x}{x}\,bd\,\frac{1-x}{2}-Spbd.$$

$$Sbd\left[\frac{(1-x)^2}{2rx}-p\right]=F,$$

$$\frac{(1-x)^2}{2rx}-p=\frac{F}{Sbd},$$

$$\frac{(1-x)^2}{2x}=rp+\frac{rF}{Sbd}. \quad . \quad . \quad . \quad . \quad (b)$$

Equilibrium Condition C

Taking moments about O_1, Fig. 3, and ignoring all possible tensile stress in the concrete, it is seen that the resisting moment = the total concrete compression times the arm

$$\left[d-\frac{d}{3}(1-x)\right]=Cbd\frac{1-x}{2}\left[d-\frac{d}{3}(1-x)\right]$$

$$=Cbd^2\frac{1-x}{2}\left[1-\frac{1-x}{3}\right]$$

$$=Cbd^2\frac{(1-x)(2+x)}{6}$$

As stated before, the total external moment about the point O_1 may consist of the transverse bending moment M, or the moment FH, or a combination of M and FH.

Observing the rules given under the table of assumptions as to moment signs, we may equate the external moment and the resisting moment as follows:

$$Cbd^2\frac{(1-x)(2+x)}{6}=M+FH,$$

from which

$$\frac{(1-x)(2+x)}{6}=\frac{M+FH}{Cbd^2}. \quad . \quad . \quad . \quad (c)$$

If $M+FH$ should be negative, the steel must be placed in the opposite side of the member, all moment signs must be reversed, and the distance H must be correspondingly corrected by the amount $(2d-h)$.

We now have the three fundamental formulas as follows:

$$\frac{1-x}{x}=r\frac{C}{S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

resulting from conditions of relative elasticity of concrete and steel.

$$\frac{(1-x)^2}{2x}=rp+\frac{rF}{Sbd}, \quad . \quad . \quad . \quad (b)$$

resulting from equilibrium condition *B*.

$$\frac{(1-x)(2+x)}{6}=\frac{M+FH}{Cbd^2}, \quad . \quad . \quad . \quad (c)$$

resulting from equilibrium condition *C*. (In terms of concrete stress.)

Every design must satisfy all three of these equations.

Multiplying (*a*) by (*c*) we have

$$\frac{(1-x)^2(2+x)}{6x}=\frac{r(M+FH)}{Sbd^2}, \quad . \quad . \quad (d)$$

resulting from equilibrium condition *C*. (In terms of steel stress.)

Eq. (*d*) can also be derived directly from equilibrium condition *C* by taking moments about O_2 instead of O_1.

For simplicity, let us represent the above four functions of x as follows:

$$\frac{1-x}{x}=D,$$

$$\frac{(1-x)^2}{2x}=T,$$

$$\frac{(1-x)(2+x)}{6}=Y,$$

$$\frac{(1-x)^2(2+x)}{6x}=Z.$$

Then from (*a*) we have

$$D=\frac{rC}{S}; \quad \ldots\ldots\ldots \quad (1)$$

from (*b*) we have

$$T=rp+\frac{rF}{Sbd}; \quad \ldots\ldots \quad (2)$$

from (*c*) we have

$$Y=\frac{M+FH}{Cbd^2}; \quad \ldots\ldots \quad (3)$$

from (*d*) we have

$$Z=\frac{r(M+FH)}{Sbd^2}. \quad \ldots\ldots \quad (4)$$

In these four equations D, T, Y and Z are different functions of the same variable, x, and tables or curves or parallel scales may be prepared so that when the value of any one of these functions is known, the corresponding values of the other three can be read off directly without further computations.

From the above Eqs. (1), (2), (3) and (4) we arrange the following in shape for immediate use:

$$C=\frac{DS}{r}=\frac{M+FH}{Ybd^2}; \quad \dots\dots \quad (5)$$

$$S=\frac{rC}{D}=\frac{r(M+FH)}{Zbd^2}; \quad \dots\dots \quad (6)$$

$$M+FH=YCbd^2=\frac{ZSbd^2}{r}; \quad \dots\dots \quad (7)$$

$$bd^2=\frac{M+FH}{YC}=\frac{r(M+FH)}{ZS}; \quad \dots \quad (8)$$

$$p=\frac{T}{r}-\frac{F}{Sbd}=\frac{a}{bd}; \quad \dots\dots \quad (9)$$

$$a=pbd=\frac{Tbd}{r}-\frac{F}{S}. \quad \dots\dots \quad (10)$$

The above formulas are the general equations applying to a reinforced concrete member subjected to direct stress in conjunction with bending moment. They may be applied to arches, to eccentrically loaded columns, to rectangular beams, and also to T-beams if the neutral axis of the latter lies in the compression flange, b being the width of flange. If the neutral axis of the T-beam lies in the web of same, the above formulas must be altered accordingly for a rigid T-beam analysis. If the neutral axis lies in the web but a short distance below the lower surface of the flange, the error resulting from the application of the above formulas will be correspondingly slight, consisting of two small triangular prisms of compression, each of a length $=\frac{1}{2}(b-gv)$ (see Fig. 4), erroneously assumed to be acting between the lower surface of the wings and the neutral axis of the beam.

In a reinforced concrete floor system composed of slabs supported by beams and placed monolithically with them, those portions of the slabs immediately adjacent to the beams are sometimes assumed as acting in conjunction with the latter, forming T-beams. In such cases, however, b should not exceed certain limits. Some authorities give rules for determining the width, b, which it is safe to assume as acting in conjunction with the stem. This maximum allowable value of b is variously given in terms of one or more of the following elements: the width, ef, of the stem; the thickness, mg, of the slab; the spacing, center to center, of the stems; the span of the stems. Some authorities suggest certain empirical formulas for the design of T-beams. Others work out complicated formulas for their theoretical consideration.

Some of the theoretical formulas neglect the concrete compression below *gv*, thus assuming the compression diagram to be a trapezoid. Others assume the unit concrete compression to decrease from a maximum at *m* and *n* to zero at *t* and *z*.

In any case, if the beam is designed as a T-beam, care must be taken to keep the unit horizontal shear on sections *gv*, *gm*, and *vn* below safe limits and to make sure there is sufficient width of concrete in the stem to provide bond for the steel and to enable provision to be made for the web stresses.

In view of the uncertainties involved and the complications introduced by the theoretical formulas, the writer is of the opinion that the proper place to give consideration to the T action is in the adoption of moment formulas and unit stresses; and to design the beam *mnfe* by the formulas herein set forth (if *mnfe* is a monolithic section) and to design the slabs in the same manner with their spans normal to the beams.

If there is no transverse moment, the term M in Eqs. (1)–(10) vanishes.

If F acts in line with the resultant of the compressive forces,

$$H = d - \frac{d}{3}(1-x) = d\left(1 - \frac{1-x}{3}\right) = d\frac{2+x}{3}.$$

If F acts in line with the neutral axis, $H=xd$.

If F acts at mid depth of the total beam,

$$H=d-\frac{h}{2}=\frac{2d-h}{2}.$$

If F is a general end reaction of the beam, acting longitudinally and distributed over the cross-section of the beam in proportion to the coefficients of elasticity of the materials, the steel takes r times the total stress which an equal cross-sectional area of concrete would take and F would be located in line with the resultant of the resisting forces.

If desired, the value of H resulting from any of the above assumptions as to the location of F may be substituted directly in place of H in the formulas.

If H = zero, we have the direct stress acting in line with the reinforcing steel and the term FH vanishes.

If there is no direct stress F = zero and Eqs. (1)–(10) reduce to the following:

$$D=\frac{rC}{S}; \quad \ldots \ldots \quad \text{(I)}$$

$$T=rp; \quad \ldots \ldots \quad \text{(II)}$$

$$Y=\frac{M}{Cbd^2}; \quad . \ . \ . \ . \ . \ . \ . \quad \text{(III)}$$

$$Z=\frac{rM}{Sbd^2}; \quad . \ . \ . \ . \ . \ . \ . \quad \text{(IV)}$$

$$C=\frac{DS}{r}=\frac{M}{Ybd^2}; \quad . \ . \ . \ . \ . \quad \text{(V)}$$

$$S=\frac{rC}{D}=\frac{rM}{Zbd^2}; \quad . \ . \ . \ . \ . \quad \text{(VI)}$$

$$M=YCbd^2=\frac{ZSbd^2}{r}; \quad . \ . \quad \text{(VII)}$$

$$bd^2=\frac{M}{YC}=\frac{rM}{ZS}; \quad . \ . \ . \ . \ . \quad \text{(VIII)}$$

$$p=\frac{T}{r}=\frac{a}{bd}; \quad . \ . \ . \ . \ . \ . \quad \text{(IX)}$$

$$a=pbd=\frac{Tbd}{r}. \quad . \ . \ . \ . \ . \quad \text{(X)}$$

The above equations are seen to involve operations of multiplication and division only, thereby enabling the work to be rapidly performed on the slide rule.

In Eq. (VIII)$_1$, $bd^2=\frac{M}{YC}$, it is to be borne in mind that M = the total transverse bending moment in inch-pounds to which the member is subjected, including the moment resulting from the dead weight of the member itself. The size of a beam or slab is sometimes computed tentatively for the live load only, after which the dead weight is added and the size recomputed: or the dead weight may be taken account of in the first computation as follows:

Let $M=M_L+M_D$; in which M_L is the live load moment and M_D is the moment resulting from the weight of the member, both acting transversely. Let w_D = uniformly distributed dead load in pounds per running foot of beam, or in pounds per square foot of slab area; slabs to be considered as a series of beams side by side, each 12 inches wide.

Then

$$w_D=\left[\frac{bh}{144}\times 150\right]+\left[\frac{bd}{144}\times .01\times(490-150)\right];$$

assuming $p=1$ per cent, concrete = 150 lbs. per cubic foot, steel = 490 lbs. per cubic foot. Hence

$$w_D=1.04\ bh+.0236\ bd.$$

In designing beams, we will assume that $h=1.1d$ and that for slabs $h=1.3d$.

Then for beams,

$$w_D=1.144bd+.0236bd=1.1676bd.$$

For slabs,

$$w_D=1.352bd+.0236bd=1.3756bd.$$

In computing the weight of a beam, varying ratios between b and d may affect this dead weight by more than 50 per cent for members of equal strength. We may put the dead weight in the designing formulas for beams by assuming a ratio between b and d, which will conform to usual practice, and revising this ratio and the assumed dead weight, if necessary, after determining the tentative dimensions of the beam.

Let us assume that for beams, $b=0.7d$.

For slabs $b=12$.

Hence we have for beams,

$$w_D=.817d^2. \quad . \quad . \quad . \quad . \quad . \quad \text{(XI)}$$

(XI) is to be used only when $b=0.7d$.

For slabs,

$$w_D=16.51d. \quad . \quad . \quad . \quad . \quad . \quad \text{(XII)}$$

For horizontal slabs and beams w_D acts transversely and $M_D=Kw_Dl^2$; in which K is a coefficient depending upon the manner in which the member is supported and l = span of beam or slab in feet.

Then for beams,

$$M_D = .817Kl^2d^2. \quad . \quad . \quad . \qquad \text{(XIII)}$$

(XIII) is to be used only when $b = 0.7d$.

For slabs,

$$M_D = 16.51Kl^2d. \quad . \quad . \quad . \qquad \text{(XIV)}$$

Hence, from Eq. $(\text{VIII})_1$ we have for beams

$$0.7d^3 = \frac{M_L}{YC} + \frac{0.817Kl^2d^2}{YC},$$

for slabs,

$$12d^2 = \frac{M_L}{YC} + \frac{16.51Kl^2d}{YC},$$

whence for beams

$$d^3 - \frac{1.167Kl^2d^2}{YC} = \frac{M_L}{0.7YC},$$

$$d^2\left(d - \frac{K_Bl^2}{YC}\right) = \frac{M_L}{0.7YC}, \quad . \quad . \quad . \qquad \text{(XV)}$$

in which

$$K_B = 1.167K.$$

For slabs,

$$d^2 - \frac{1.376Kl^2d}{YC} = \frac{M_L}{12YC},$$

$$d\left(d - \frac{K_Sl^2}{YC}\right) = \frac{M_L}{12YC}, \quad . \quad . \quad . \qquad \text{(XVI)}$$

in which

$$K_S = 1.376K.$$

Various values of K_B and K_S are given by Table I.

In (XV) M_L = the total live-load moment sustained by the beam.

In (XVI) M_L = the moment resulting from the live load on a portion of the slab 12 inches wide and l feet long.

Eq. (XVI) may be expressed in the form of a quadratic and solved for d, but the resulting equation is more cumbersome than (XVI) and no more easily solved.

Pertaining to Table I:

K = numerical coefficient of wl^2;

W = concentrated live load at middle of span in pounds;

w = uniformly distributed live load in pounds per running foot of beam, or in pounds per square foot of slab area;

l = span of beam or slab in feet;

+moments produce compression at loaded side of member;

−moments produce tension at loaded side of member.

TABLE I

DESIGNING DATA FOR BEAMS AND SLABS

Case.	Description.	Location of Maximum Moment.	Maximum Live Load Moment in In.-lbs. $=M_L$. Concentrated Load. Ft.-lbs.	Concentrated Load. In.-lbs.	Uniform Load. Ft.-lbs.	Uniform Load. In.-lbs.	Dead Load Constants. (See Eqs. (XV) and (XVI).) K	$K_B=1.167K$	$K_s=1.376K$
1	Cantilever beam.	At support.	$-\frac{Wl}{2}$	$-6Wl$	$-\frac{wl^2}{2}$	$-6wl^2$	6	7.002	8.256
2	Simple beam resting on two supports.	At middle of span.	$+\frac{Wl}{4}$	$+3Wl$	$+\frac{wl^2}{8}$	$+1.5wl^2$	1.5	1.75	2.064
3	Beam fixed at one end, supported at other.	At fixed end.	$-\frac{3}{16}Wl$	$-2.25Wl$	$-\frac{wl^2}{8}$	$-1.5wl^2$	1.5	1.75	2.064
4	Beam fixed at both ends.	At supports.	$-\frac{Wl}{8}$	$-1.5Wl$	$-\frac{wl^2}{12}$	$-wl^2$	1.0	1.167	1.376
5	Continuous beams of uniform span, l.	At supports.			$-\frac{wl^2}{10}$	$-1.2wl^2$	1.2	1.4	1.651
6	Square slab supported on four sides.	At middle of span.			$+\frac{wl^2}{16}$	$+0.75wl^2$	0.75	0.875	1.032
7	Square slab fixed at four sides.	At supports.			$-\frac{wl^2}{20}$	$-0.6wl^2$	0.6	0.7	0.826

If L = uniform live load in pounds per square foot of supported floor space and s = spacing of floor beams in feet, center to center, then $w = Ls$.

Slabs to be considered as beams 12 inches wide spaced 1 foot center to center.

For beams continuous over several supports precise expressions may be derived for the bending moment at each support, though these moments are sometimes assumed as all being $\frac{wl^2}{10}$ ft.-lbs., as indicated by Case 5 in Table I.

If a slab is a rectangle whose length in plan is twice its width or more than twice its width, the above moment for Case 6 becomes about the same as that for Case 2, while the moment for Case 7 becomes the same as that for Case 4 or Case 5, the span, l, in the case of rectangular slabs always being the short side of the rectangle. The main reinforcing bars of the slabs are placed parallel to the shorter span, though some steel is placed parallel to the longer span to resist shrinkage and temperature stresses.

CHAPTER II

LABOR-SAVING DEVICES—REINFORCED CONCRETE SLIDE RULE

FIG. 5 represents the application of a scheme devised by the author for the purpose of facilitating the mathematical operations involved in reinforced concrete design. The device is nothing more than a series of four concentric circular scales, the graduations of which represent values of the four functions of x denoted in formulas (1) to (10) by the letters D, T, Y, and Z. The scales of Fig. 5 are lettered to indicate the function represented by each and are so graduated that the corresponding values of the functions represented are radially opposite each other; that is, any radial line drawn from the center, O, will cover corresponding values of these functions. In practice, a straight line scratched on a piece of transparent sheet celluloid serves very nicely to define this radial line. Or the reader may mount Fig. 5 or a copy of it on a piece of cardboard, piercing a hole through the cardboard at the center, O, and drawing through this hole a black thread for defining the radial lines. It is seen from Fig. 5 that upon evaluating any one of the functions, *D*, *T*, *Y*, *Z*,

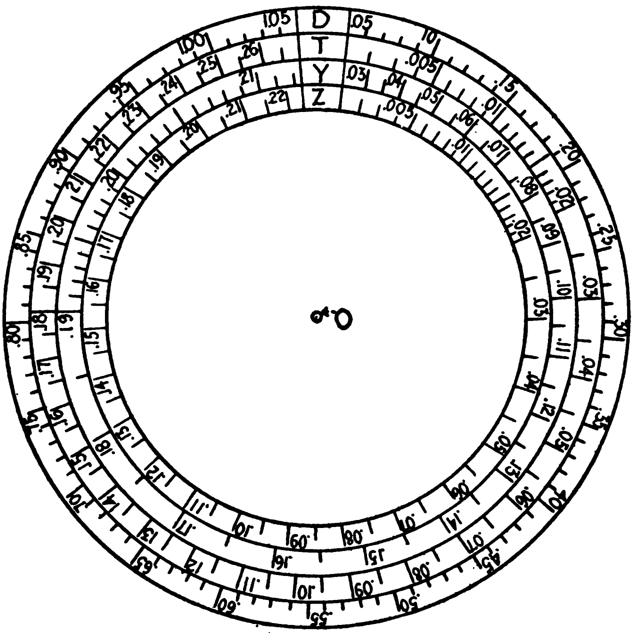
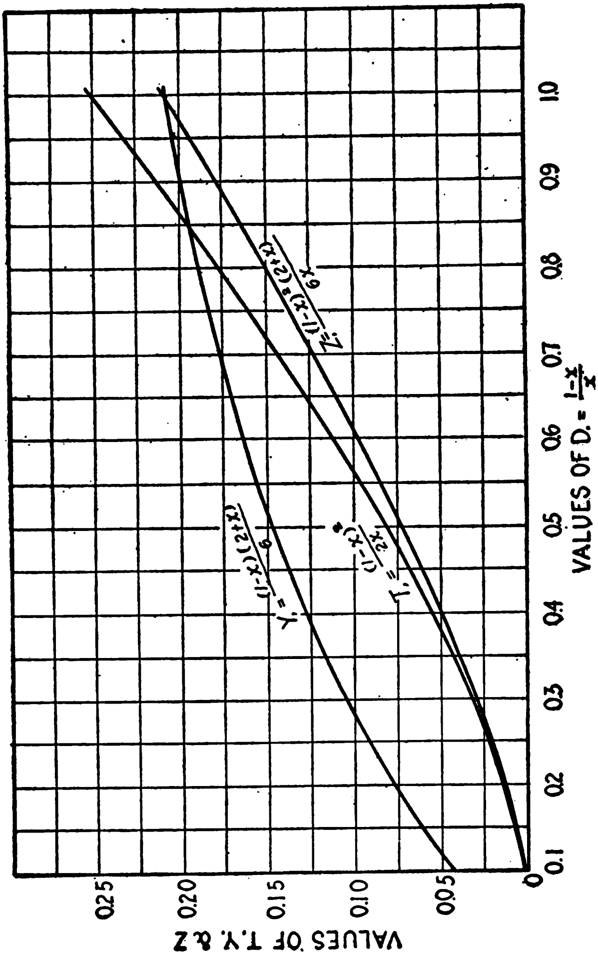


FIG. 5

the other three become known at once. The graduations of scale *D* of Fig. 5 are uniformly spaced. By letting the graduations of scale *D* represent values of the function, *D*, the correct locations of



VALUES OF T, Y & Z
0.25
0.20
0.15
0.10
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
VALUES OF D. = $\frac{1-x}{x}$
Z = $\frac{(1-x)^2(2+x)}{6x}$
Y = $\frac{(1-x)(2+x)}{6}$
T = $\frac{(1-x)^2}{2x}$

FIG. 6

the graduations of scales, T, Y and Z are determined by preparing a table of the corresponding values of the four functions

$$\frac{1-x}{x}, \quad \frac{(1-x)^2}{2x}, \quad \frac{(1-x)(2+x)}{6} \quad \text{and} \quad \frac{(1-x)^2(2+x)}{6x},$$

represented by D, T, Y, and Z respectively.

Fig. 6 pictures the relations of these four functions graphically.

The foregoing formulas are mathematically correct for the assumptions given and are so simple and so easily applied, with the aid of Fig. 5, as to justify their use in all practical work. Fig. 5 is perfectly general in its application, and may be used for any desired stresses in the concrete and steel and for any coefficients of elasticity, without the aid of special tables or diagrams for special assumptions, and without performing any mathematical operations other than those which may be rapidly read from a slide rule.

The principle of the parallel scales of Fig. 5 may be carried still further by utilizing one of the scales of a logarithmic slide rule in place of one of the said parallel scales, resulting in a reinforced concrete slide rule; which is simply the familiar logarithmic

$D=\frac{1-x}{x}$	$T=\frac{(1-x)^2}{2x}$	$Y=\frac{(1-x)(2+x)}{6}$	$Z=\frac{(1-x)^2(2+x)}{6x}$
.04	.00077	.01898	.00076
.05	.00119	.02343	.00117
.06	.00170	.02777	.00167
.07	.00229	.03200	.00224
.08	.00296	.03612	.00289
.09	.00372	.04015	.00361
.10	.00455	.04408	.00441
.11	.00545	.04791	.00527
.12	.00643	.05166	.00620
.13	.00748	.05531	.00719
.14	.00860	.05889	.00824
.15	.00978	.06238	.00936
.16	.01103	.06579	.01053
.17	.01235	.06913	.01175
.18	.01373	.07239	.01303
.19	.01517	.07558	.01436
.20	.01667	.07871	.01574
.21	.01822	.08176	.01717
.22	.01984	.08475	.01865
.23	.02150	.08767	.02016
.24	.02323	.09053	.02173
.25	.02500	.09333	.02333
.26	.02683	.09608	.02498
.27	.02870	.09877	.02667
.28	.03062	.10140	.02839
.29	.03260	.10398	.03015
.30	.03462	.10651	.03195
.31	.03668	.10899	.03379
.32	.03879	.11142	.03565
.33	.04094	.11380	.03755
.34	.04313	.11614	.03949
.35	.04537	.11843	.04145
.36	.04765	.12068	.04344
.37	.04996	.12288	.04547

$D=\frac{1\quad x}{x}$	$T=\frac{(1-x)^2}{2x}$	$Y=\frac{(1-x)(2+x)}{6}$	$Z=\frac{(1-x)^2(2+x)}{6x}$
.38	.05232	.12504	.04752
.39	.05471	.12717	.04960
.40	.05714	.12925	.05170
.41	.05961	.13130	.05383
.42	.06211	.13331	.05599
.43	.06465	.13528	.05817
.44	.06722	.13722	.06038
.45	.06983	.13912	.06260
.46	.07247	.14099	.06486
.47	.07514	.14283	.06713
.48	.07784	.14463	.06942
.49	.08057	.14640	.07174
.50	.08333	.14815	.07407
.51	.08613	.14986	.07643
.52	.08895	.15155	.07881
.53	.09180	.15321	.08120
.54	.09468	.15483	.08361
.55	.09758	.15644	.08604
.56	.10051	.15801	.08849
.57	.10347	.15956	.09095
.58	.10646	.16109	.09343
.59	.10947	.16259	.09593
.60	.11250	.16406	.09844
.61	.11556	.16552	.10097
.62	.11864	.16695	.10351
.63	.12175	.16835	.10606
.64	.12488	.16974	.10863
.65	.12803	.17110	.11122
.66	.13120	.17245	.11382
.67	.13440	.17377	.11643
.68	.13762	.17508	.11905
.69	.14086	.17636	.12169
.70	.14412	.17762	.12433
.71	.14740	.17887	.12700

$D=\frac{1-x}{x}$	$T=\frac{(1-x)^2}{2x}$	$Y=\frac{(1-x)(2+x)}{6}$	$Z=\frac{(1-x)^2(2+x)}{6x}$
.72	.15070	.18010	.12967
.73	.15402	.18131	.13236
.74	.15736	.18250	.13505
.75	.16071	.18367	.13775
.76	.16409	.18483	.14047
.77	.16749	.18597	.14320
.78	.17090	.18710	.14594
.79	.17433	.18821	.14869
.80	.17778	.18930	.15144
.81	.18124	.19038	.15421
.82	.18473	.19144	.15698
.83	.18822	.19249	.15977
.84	.19174	.19353	.16257
.85	.19527	.19455	.16537
.86	.19882	.19555	.16817
.87	.20238	.19654	.17099
.88	.20596	.19753	.17383
.89	.20955	.19849	.17666
.90	.21316	.19944	.17950
.91	.21678	.20039	.18235
.92	.22042	.20132	.18521
.93	.22407	.20224	.18808
.94	.22773	.20314	.19095
.95	.23141	.20403	.19383
.96	.23510	.20492	.19672
.97	.23881	.20579	.19962
.98	.24253	.20665	.20252
.99	.24626	.20750	.20543
1.00	.25000	.20833	.20833
1.01	.25376	.20916	.21125
1.02	.25752	.20998	.21418
1.03	.26131	.21079	.21711
1.04	.26510	.21158	.22004

slide rule with the addition of three auxiliary scales on the face of same as illustrated by Fig. 7. Scales 1, 2, 3 and *D* are the usual slide-rule scales, and scales *T*, *Y* and *Z* the added scales for reinforced concrete work. This device enables us to evaluate the four functions *D*, *T*, *Y*, and *Z* simultaneously; the lettered scales of Fig. 7 representing the same functions of x as the similarly lettered scales of Fig. 5. Scales *T*, *Y* and *Z* are so constructed that when the index line (i) on the movable rider covers a particular value of the function *D* on scale *D*, it covers at the same time the corresponding values of the functions *T*, *Y* and *Z* on scales *T*, *Y* and *Z* respectively. Hence, the only difference between the four lettered scales in Figs. 5 and 7 is that in the former the graduations of scale *D* are uniformly spaced and in the latter they form a logarithmic scale; while the accompanying scales *T*, *Y* and *Z* are graduated accordingly.

The principle of the reinforced concrete slide rule is apparent from its construction. In the following references to the slide rule, the term "index" as applied to the scales refers to the graduation "1" on those scales; while the rider index is the line (i) on the movable rider.

Suppose we wish to design a reinforced concrete beam with the slide rule illustrated in Fig. 7. We must first, of course, adopt the values of C, S, and r upon which the design is to be based.

Let $C=600$ lbs. per sq.in.;
$S=12,000$ lbs. per sq.in.;
$r=15$;
$M=40,000$ inch-lbs.

Then from Eq. (I)

$$D=\frac{rC}{S}=\frac{15\times 600}{12,000}.$$

To perform this operation on the slide rule, we set the left index of scale 3, Fig. 7, opposite 15 on scale D; place the rider index over 600 on scale 3; leaving the rider in this position, run scale 3 under the same until 12,000 on scale 3 is under the rider index. Then the correct value of D will be that reading on scale D which is opposite the index of scale 3. Hence if the rider index is placed over the index of scale 3 in the final position of the latter it will cover the correct values of D, T, Y, and Z on scales D, T, Y and Z respectively. Having these values, they may be substituted for the symbols D, T, Y, Z in formulas (I) to (X) or (1) to (10). The slide and rider of Fig. 7 are set for the above operation, giving

$$D=0.75, \quad T=0.161, \quad Y=0.184, \quad Z=0.138.$$

From (VIII) $bd^2=\frac{M}{YC}=\frac{40,000}{.184\times 600}=362.$

Knowing $bd^2=362$, we can utilize the slide rule to find different combinations of b and d to satisfy this value of bd^2 as follows: Place the rider index over 362 on either portion of scale 1. With the rider in this position, the rider index will cover the value of d on scale 3 which corresponds to that value of b found on scale 1 opposite the indices of scale 2; care being taken properly to place the decimal points. Hence, by running the slide under the rider index with the latter set as stated, the corresponding values of b and d are read directly from scales 1 and 3 respectively, as follows:

$b=$ 10	6	4
$d=$ 6	7.8	9.5

In this way, suitable values of b and d are easily selected; for instance, $b=6''$, $d=8''$.

Fig. 5 used in conjunction with an ordinary slide rule will prove nearly as great a time saver as a reinforced concrete slide rule.

CHAPTER III

ILLUSTRATIVE EXAMPLES

Problems

THE ease and rapidity with which reinforced concrete computations may be performed with the aid of Fig. 5 is demonstrated in the solutions of the following specimen problems. The four typical problems stated in Table II will cover the more common cases of reinforced concrete beams subjected to transverse bending moment only.

The following four problems, numbers 1, 2, 3, and 4, illustrate the four typical cases covered by Table II.

PROBLEM 1. To design a beam.

Assume a reinforced-concrete floor system as follows, uniformly loaded with 350 lbs. per square foot exclusive of weight of floor: beams 6 ft. apart, center to center; span of beams 15 ft.; beams constructed monolithically over supports; spaces between beams covered by slabs of uniform thickness resting on the beams but not cast monolithic-

TABLE II

Problem.	Known Conditions.	Required Information.	Use Following Formulas in Order.
(1) To design a beam.	M, C, S, r	b, d, a	(I), $(VIII)_1$, or $(VIII)_1$ as revised by (XV) and (XVI), $(X)_2$.
(2) To investigate the stresses in a given beam.	M, b, d, a, r	C, S	$(IX)_2$ (II), $(V)_2$, $(VI)_1$.
(3) To compute the safe load on a given beam.	b, d, a, r Maximum C Maximum S	M	$(IX)_2$, (II), $(VII)_1$ or $(IX)_2$, (II), $(VII)_2$, whichever series gives the smaller value of M.
(4) To compute the necessary reinforcement in a beam of known dimensions.	M, b, d, r Maximum C Maximum S	a	(III), $(X)_2$ or (IV), $(X)_2$ whichever series gives the greater value of a.

ally with them; slabs cast separately between joints on center lines of beams.

Let $C=500$, $S=16{,}000$, $r=15$.

From (I)

$$D=\frac{15\times 500}{16{,}000}=.469.$$

From Fig. 5,

$$T = .075, \quad Y = .143, \quad Z = .067.$$

Design of Slabs. Call span of slabs 6 ft. Use Case 2, Table I.

$$M_L = 1.5wl^2 = 1.5 \times 350 \times 36 = 18{,}900;$$

$$K_S = 2.064.$$

From (XVI),

$$d\left(d - \frac{2.064 \times 36}{.143 \times 500}\right) = \frac{18{,}900}{12 \times .143 \times 500},$$

$$d(d - 1.04) = 22;$$

$$d = 5.3 \quad h = 1.3d = 6.9,$$

$h - d = 1.6$ ins. concrete below steel.

Make $h = 7$ ins., $d = 5.5$ ins.

From $(X)_2$

$$a = \frac{.075 \times 12 \times 5.5}{15} = 0.33 \text{ sq.in. steel}$$

for portion of slab 12 ins. wide, $= .0275$ sq.in. steel per inch width of slab. Hence we can use ½-in. bars 9 ins. apart or ⅜-in. bars 5 ins. apart.

In designing the beams we must add the dead weight of the slabs per square foot to 350 to obtain the total load carried by beams.

From (XII) w_D for slabs $=16.51\times5.5=90.8$, say 91.

Then for beams

$$w=(350+91)\times6=2646 \text{ lbs.}$$

For the beams use Case 5, Table I.

$$M_L=1.2wl^2=1.2\times2646\times225=714,000;$$

$$K_B=1.4.$$

From (XV)

$$d^2\left(d-\frac{1.4\times225}{.143\times500}\right)=\frac{714,000}{.7\times.143\times500};$$

$$d^2(d-4.41)=14,270;$$

$$d=25.8, \text{ say } 26 \qquad b=0.7d=18.1, \text{ say } 18.$$

From $(X)_2$,

$$a=\frac{.075\times18\times26}{15}=2.34 \text{ sq.ins. steel.}$$

Suppose we decide to use six $\frac{3}{4}$-in. bars $=3.375$ sq.ins.

Suppose further that we do not wish d to be greater than 24 ins. Then, since bd^2 must approximate $18(26)^2$ we have

$$b=\frac{18(26)^2}{(24)^2}=21.1.$$

Then assume the new beam as follows:

$$b=21, \quad d=24, \quad h=26, \quad a=3.375, \quad r=15,$$

and investigate this beam as

PROBLEM 2. To investigate the stresses in a given beam.

$$M_D=Kw_Dl^2.$$

From Case 5, Table I,

$$K=1.2;$$

$$w_D=\left[\frac{bh}{144}\times 150\right]+\left[\frac{a}{144}\times(490-150)\right]$$

$$=569+8=577;$$

$$\therefore \quad M_D=1.2\times 577\times 225=155{,}800;$$

$$M=M_L+M_D=714{,}000+155{,}800=869{,}800;$$

From $(IX)_2$,

$$p=\frac{3.375}{21\times 24}=.0067;$$

From (II),

$$T = 15 \times .0067 = .1005;$$

From Fig. 5,

$$D = .561, \quad Y = .158, \quad Z = .088;$$

From $(V)_2$,

$$C = \frac{869,800}{.158 \times 21 \times (24)^2} = 455;$$

From $(VI)_1$,

$$S = \frac{15 \times 455}{.561} = 12,170.$$

PROBLEM 3. To compute the safe load on a given beam.

Let $b = 9$, $d = 11$, $a = 1$ sq.in, $r = 12$.

Maximum allowable $C = 600$,
" " $S = 14,000$.

From $(IX)_2$,

$$p = \frac{1}{9 \times 11} = .0101;$$

From (II),

$$T = 12 \times .0101 = .1212;$$

From Fig. 5,

$$D = .629, \quad Y = .168, \quad Z = .106;$$

From $(VII)_1$,

$$M = .168 \times 600 \times 9 \times 121 = 109{,}800;$$

From $(VII)_2$,

$$M = \frac{.106 \times 14{,}000 \times 9 \times 121}{12} = 134{,}700.$$

Hence

$$M = 109{,}800.$$

From $(VI)_1$,

$$S = \frac{12 \times 600}{.629} = 11{,}450.$$

Hence we cannot exceed 11,450 for S under the present circumstances without exceeding 600 for C.

From this moment of 109,800 in.-lbs. we must subtract the moment produced by the dead weight of the beam itself to obtain the safe live-load moment, M_L.

$$M_D = K w_D l^2.$$

Assume the beam to be a cantilever 7 ft. long. Then from Case 1, Table I, $K = 6$.

Let $h=12.5$. Then

$$w_D=\left[\frac{bh}{144}\times 150\right]+\left[\frac{a}{144}\times(490-150)\right]$$

$$=117.2+2.36=119.56.$$

Then

$$M_D=6\times 119.56\times 49=35,150.$$

$$\therefore\ M_L=109,800-35,150=74,650.$$

If the live load is uniformly distributed we have from Case 1, Table I, $M_L=6wl^2$.

Hence $w=\frac{M_L}{6l^2}=\frac{74,650}{294}=254$ lbs. per running foot of beam.

PROBLEM 4. To compute the necessary reinforcement in a beam of known dimensions.

Let $b=12$, $d=15$, $r=16$.

Maximum $C=400$. Maximum $S=12,000$.

Compute M from the following loading.

Assume a uniform live load of 1500 lbs. per running foot on a fixed beam 10 ft. long and an additional concentrated live load of 1000 lbs. at middle of span. Then from Case 4, Table I,

$$M_L=(1500\times 100)+(1.5\times 1000\times 10)=165,000;$$

$$M_D=Kw_Dl^2.$$

From Case 4, Table I, $K=1.0$.

Assuming $p=1$ per cent and $h=17$, we have

$$w_D=\left[\frac{bh}{144}\times 150\right]+\left[\frac{bd}{144}\times .01\times(490-150)\right]$$

$$=212.5+4.25=216.75.$$

Hence

$$M_D=1\times 216.75\times 100=21{,}675.$$

$$M=M_L+M_D=165{,}000+21{,}675=186{,}675.$$

From (III),

$$Y=\frac{186{,}675}{400\times 12\times 225}=.173.$$

From Fig. 5,

$$D=.665,\quad T=.133,\quad Z=.116.$$

From (IV),

$$Z=\frac{16\times 186{,}675}{12{,}000\times 12\times 225}=.092.$$

From Fig. 5,

$$D=.573,\quad T=.105,\quad Y=.160.$$

It is apparent from Eq. $(X)_2$ that the larger of the above values of T will result in the greater value of a.

Hence from $(X)_2$,

$$a = \frac{.133 \times 12 \times 15}{16} = 1.496 \text{ sq.ins.}$$

Since $T = .133$, $C = 400$ and $D = .665$.

Then from $(VI)_1$,

$$S = \frac{16 \times 400}{.665} = 9620.$$

The conclusion is, of course, that we cannot reinforce the beam in question under the conditions imposed so as to stress the steel to more than 9620 lbs. per square inch tension without compressing the concrete to more than 400 lbs. per square inch.

PROBLEM 5. To compute the required dimensions of a beam containing a known area of reinforcement.

Known conditions:

M, a, r;

Maximum allowable value of C;

Maximum allowable value of S;

Required information:

b, d.

In this problem, although the value of a is given, p cannot be computed until we have b and d. Furthermore, we do not know the actual values of C and S.

Hence none of the functions D, T, Y, Z can be computed directly and the simplest manner of approaching this problem is to assume a value for p which, in the light of previous experience, appears reasonable. Then compute bd from $(X)_1$ and choose values of b and d to satisfy the same. If these dimensions produce values of either C or S above the allowable maximum, we must assume new values for b and d.

Let $M=260{,}000$, $a=2$ sq.in., $r=12$.

Maximum $C=650$;

Maximum $S=18{,}000$;

Assume $p=.01$. Then from $(X)_1$, $bd=\frac{2}{.01}=200$.

Assume $b=0.7d$. Then $.7\ d^2=200$.

$$d^2=286,\quad d=17,\quad b=.7d=12,\quad bd^2=3468.$$

From (II),

$$T=12\times.01=.12.$$

From Fig. 5,

$$D=.624,\quad Y=.167,\quad Z=.105.$$

From (V)2,

$$C=\frac{260,000}{.167\times 3468}=449.$$

From (VI)1,

$$S=\frac{12\times 449}{.624}=8630.$$

Second Trial. Let $b=10$, $d=15$. Then

$$bd^2=2250.$$

From (IX)2,

$$p=\frac{2}{150}=.0133.$$

From (II),

$$T=12\times .0133=.160.$$

From Fig. 5,

$$D=.748,\quad Y=.183,\quad Z=.1375.$$

From (V)2,

$$C=\frac{260,000}{.183\times 2250}=631.$$

From (VI)1,

$$S=\frac{12\times 631}{.748}=10,120.$$

Hence $b=10$ and $d=15$, will satisfy the conditions.

The results show that the dimensions are determined by the limiting value of 650 for C since otherwise S could have been greatly increased.

PROBLEM 6. Let us apply to the beam of Problem 1, in addition to the loading there given, a direct tension of 5000 lbs. applied on the compression side of the beam at a distance of 18 ins. from the center line of the steel.

From the rules for signs already established for F and H, we note that $F=-5000$, $H=+18$ and $FH=-90,000$ in.lbs. Hence

$$M_L=714,000-90,000=624,000.$$

From (1)

$$D=\frac{15\times 500}{16,000}=.469.$$

Hence

$$T=.075, \quad Y=.143, \quad Z=.067, \text{ as before,}$$

also

$$K_B=1.4 \text{ as before.}$$

Then from (XV)

$$d^2\left(d-\frac{1.4\times 225}{.143\times 500}\right)=\frac{624,000}{.7\times .143\times 500};$$

$$d^2(d-4.41)=12,470;$$

$$d = 24.8, \quad \text{say } 25;$$

$$b = 0.7d = 17.4, \quad \text{say } 17.5.$$

From $(10)_2$

$$a = \frac{.075 \times 17.5 \times 25}{15} + \frac{5000}{16,000} = 2.5,$$

the result being to decrease the concrete section and increase the steel of the beam computed in Problem 1.

PROBLEM 7. Investigate the stresses in a horizontal section of a short reinforced concrete column 18 ins. square, with 5 vertical reinforcing bars, each ½ in. square, along each side of the column 2 ins. from face of concrete. The load on the section is 50,000 lbs. applied 4 ins. outside of one of the vertical faces by means of a bracket. The above arrangement gives 16 vertical bars in the column. The bars between the neutral axis and the load will take r times as much compression as the concrete replaced by same and all the bars on the other side of the neutral axis will be stressed in tension proportionally to their distances from the neutral axis.

We will assume in our investigation, however, that the steel in compression takes the same stress as the replaced concrete and that the tension due to the eccentric loading is all taken by the five bars furthest from the load.

$$b=18,\quad d=16,\quad a=5\times(\tfrac{1}{2})^2=1.25,$$

$$M=0,\quad F=+50{,}000,\quad H=+20,\quad FH=+1{,}000{,}000.$$

Let $r=12$. Then from $(9)_2$

$$p=\frac{1.25}{18\times 16}=.00434.$$

From (2) it is seen that the expression for T involves S, which is unknown.

Substitute in (2) the value of S as given by $(6)_2$ $=\frac{rFH}{Zbd^2}$. Then from (2)

$$T=rp+\frac{rF}{bd}\,\frac{Zbd^2}{rFH}=rp+\frac{Zd}{H}$$

$$=12\times .00434+\frac{16}{20}Z$$

$$=.0521+0.8Z.$$

$$T-0.8Z=.0521.$$

Then we must find values of T and Z which will satisfy this equation and also the relation represented by Fig. 5.

$T = .165$ and $Z = .141$ satisfy these conditions.

Whence, from Fig. 5,

$$D = .763, \quad Y = .185.$$

From $(5)_2$

$$C = \frac{1,000,000}{.185 \times 18 \times (16)^2} = 1173.$$

From $(6)_1$

$$S = \frac{12 \times 1173}{.763} = 18,450,$$

the results indicating that the assumed loading is excessive, in view of its eccentricity.

Assume the same load, 50,000 lbs., applied at the axis of the column. Then all the concrete and steel is in compression.

The unit compressive stress in the steel

$$= rC.$$

Total stress taken by steel

$$= 16 \times (\tfrac{1}{2})^2 \times 12 \times C = 48C.$$

(Assuming r for compression $= r$ for tension.)

Total stress taken by concrete =

$$[(18)^2 - 16 \times (\tfrac{1}{2})^2]C = 320C.$$

Hence $$50{,}000 = 48C + 320C,$$

$$368C = 50{,}000 \quad C = 136.$$

Whence compressive stress in steel $= 12 \times 136 = 1632$ lbs. per square inch.

The Design of Arches

Figs. 8 and 9 represent a reinforced concrete conduit for carrying water under pressure. The dotted lines thereon represent the pressure lines of same, while the heavy solid lines represent the theoretical location of the steel reinforcement as dictated by the assumed pressure lines. We will not here go into the question of the proper location of these pressure lines, but will illustrate the application to such a problem of the foregoing formulas in connection with Fig. 5.

The pressure line in Fig. 8 is constructed from the following loading: External ground water pressure; dead load of earth covering; weight of masonry; no internal water pressure.

The pressure line of Fig. 9 is constructed from the following loading: Internal water pressure; weight of masonry; no external ground water; no earth cover.

These loadings result in the usual arch compression line in the case of Fig. 8, but in Fig. 9 the pressure line represents tensile forces as indicated by the arrows at crown and invert.

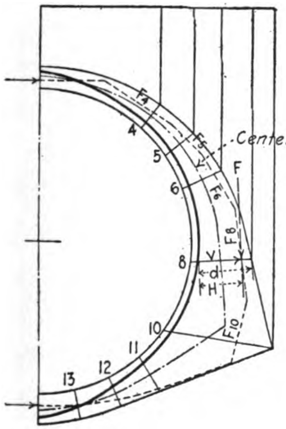


FIG. 8

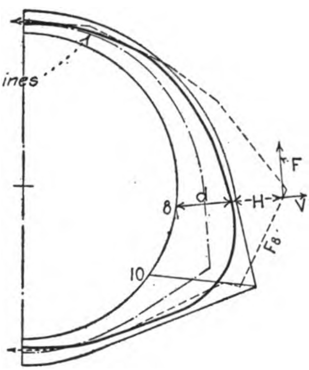


FIG. 9

In designing conduits for carrying water under pressure the question of leakage becomes a matter of importance and in some instances it becomes advisable to limit the unit steel stress to a figure that will avoid tension cracks in the concrete. The concrete in immediate contact with the steel on the tension side of a member is subjected to a unit tensile stress equal to the unit tensile stress in the steel divided by r. Let C_t represent the unit tensile stress in the concrete immediately surrounding the steel. Then $C_t = \frac{S}{r}$, or $S = C_t r$.

For the problem in hand, let us allow $C_t = 200$ and assume $r = 15$.

Then maximum allowable $S = 200 \times 15 = 3000$.

Let maximum allowable $C = 500$, $r = 15$, $b = 12$.

Let us investigate the imaginary joint, 8, with reference to the amount of steel reinforcement needed therein for each of the above loadings, keeping the concrete compression below 500 lbs. per square inch, and the tension in the steel below 3000 lbs. per square inch. d at joint $8 = 34$ ins.

The force acting on joint 8 is F_8. This problem falls under (4) of Table II. Care must be taken to give F and H their proper signs as called for by the table of assumptions.

Suppose it is found from the force diagram for Fig. 8 that $F_8 = 14{,}020$ lbs. for a portion of the conduit 1 ft. long. Then F_8 is resolved into F, $= +14{,}000$ lbs., normal to joint 8 and producing bending moment about the arm H; and V, $= 510$ lbs., producing shear on joint 8. H in Fig. $8 = +28$ ins.

In like manner, supposing that F_8 in Fig. 9 = 7000 lbs., F for this case = −6000 lbs., while V = 3600 lbs., H = −35 ins.

There being no transverse bending moment to be considered, M in formulas (1) to (10) disappears.

Let us first compute the steel called for by the assumptions of Fig. 8.

It seems reasonable from the conditions of the problem that the steel stress will be the determining factor in the design. Hence we will start out with Eq. (4) rather than (3):

From (4)

$$Z=\frac{rFH}{Sbd^2}=\frac{15\times 14{,}000\times 28}{3000\times 12\times (34)^2}=.141$$

From Fig. 5,

$$D=.762,\quad T=.165,\quad Y=.185$$

From $(10)_2$

$$a=\frac{.165\times 12\times 34}{15}-\frac{14{,}000}{3000}$$

$$=4.488-4.667=-0.179$$

From $(5)_1$

$$C=\frac{.762\times 3000}{15}=152 \text{ lbs. per sq.in.}$$

Hence it is seen that we cannot reinforce joint 8 under the given conditions so as to stress the concrete to more than 152 lbs. per square inch compression without exceeding 3000 lbs. per square inch tension in the steel, and hence exceeding 200 lbs. per square inch tension in the concrete surrounding the steel.

The above solution of joint 8 is seen to give a negative value for a. Hence no steel is required at that joint for the present loading. This result is brought about by the compressive effect of the direct stress. If there is no steel in the section the above computations based on its presence are erroneous. To get the concrete stress if there is no steel present, we may proceed upon two assumptions: first that the concrete is capable of taking both tension and compression; second, that the concrete can take compression only.

In the first case, the tension and compression are computed directly from the bending moment rerulting from the eccentric loading; the maximum unit compression being $\dfrac{6F\left(\dfrac{h}{2}-d+H\right)}{bh^2}+\dfrac{F}{bh}=95.5$ (assuming $h=d+3$), while the maximum unit tension $=\dfrac{6F\left(\frac{h}{2}-d+H\right)}{bh^2}-\dfrac{F}{bh}$, $=32.5$. $\left(\frac{h}{2}-d+H\right)$ is the eccentricity of the force F.

In the second case, the center of gravity of the concrete compression diagram must always be opposite the force F. Since we are ignoring the tension which is shown by Case 1 to be present, the compression diagram will be a triangle and the total compression will equal F.

$$\text{Area of compression triangle}=\frac{C\times 3(d-H)}{2}.$$

Total compression

$$=\frac{b\times C\times 3(d-H)}{2}=12\frac{18C}{2}=108C.$$

$$\therefore \quad 108C=F \quad \text{and} \quad C=\frac{F}{108}=129.6 \text{ lbs. per sq.in.}$$

While steel reinforcement is seen to be unnecessary at the inner side of this joint for the present loading as a means of strength, it may nevertheless be advisable for the purpose of preventing the formation of leakage cracks in the concrete.

The steel required at joint 8 for the loading assumed in Fig. 9 is computed as follows:

$S = 3000$ or less;
$C = 500$ or less;
$F = -6000$;
$H = -35$;
$b = 12$;
$d = 34$;
$r = 15$.

From (4)

$$Z = \frac{rFH}{Sbd^2} = \frac{15 \times 6000 \times 35}{3000 \times 12 \times 34 \times 34} = .076.$$

From Fig. 5,

$$D = .508, \quad T = .085, \quad Y = .149.$$

From $(5)_1$,

$$C = \frac{.508 \times 3000}{15} = 102.$$

From $(10)_2$,

$$a = \frac{.085 \times 12 \times 34}{15} + \frac{6000}{3000}$$

$$= 2.312 + 2.000 = 4.312 \text{ sq.ins.};$$

say three $1\frac{1}{4}$-in. square twisted rods per lineal foot of conduit.

CHAPTER IV

GENERAL NOTES ON REINFORCED CONCRETE DESIGN

The author does not wish by any means to infer that the subject of reinforced concrete design is wholly a question of mathematics. Mathematics, however, is the tool by which the members are to be proportioned after the various constants and assumptions are decided upon and, as such, should be made as efficient and simple a tool as possible.

The following discussion of some of the more important general factors and principles, as affecting the mathematical aspects of the question, touches upon those matters more or less superficially, and may seem in parts rather elementary, but is included as a caution against a tendency, apparent in the work of some designers, to regard the mathematical formulas as the essence of the design; whereas the formulas should be considered as merely vehicles by which to express and apply the theoretically correct mathematical factors as modified by practical conditions and by proper consideration of the more general non-mathematical factors.

As stated in the table of assumptions, the foregoing reinforced concrete formulas assume no tension to be carried by the concrete. This is actually the case when shrinkage or temperature cracks occur across the lower portion of a reinforced concrete beam. Such a crack, however, would not necessarily interfere with the compressive functions of the beam. If the concrete below the neutral axis of a beam is intact, the embedded tension steel will be stressed less severely under a given loading than the foregoing formulas would indicate until the loading is increased to a point where the concrete in maximum tension is ruptured. Then the said formulas would give theoretically correct results at the point of rupture. Hence, so long as the concrete is taking tension, the formulas here set forth give results erring somewhat on the side of safety; but the assumptions upon which they are based are used quite generally, since the designer can never feel certain that the concrete in his beam will be capable of resisting tension.

One possible combination of loadings may call for steel in one side of a reinforced concrete member at some particular section, as AB, while another possible loading may call for steel at the other side of the section; but in the investigation of the section for either of these loadings the presence of steel due to the other loading is usually ignored, since it is on the compression side of the member as regards the loading in question and hence can act only in compression as far as that loading is concerned. The only error involved in this treatment is a slight one on the side of safety, since the steel on the compression side, instead of taking the same amount of compressive stress as an equal cross-sectional area of concrete, takes r times that stress; but the area of steel in compression is such a small fraction of that of the concrete that the error is negligible. If it is desired to take this compression steel into consideration, let a' represent the cross-sectional area of the steel in compression. Then assuming that the unit concrete stress immediately about the compression steel $= C$ and that E_c and E_s are constant for both tension and compression, we have rCa' as the total stress taken by the compression steel. $Ca' =$ compression taken by an equal area of concrete. Then $Ca'(r-1) =$ compression added by the steel. Hence, equilibrium condition B becomes,

$$F = Cbd\frac{1-x}{2} + Ca'(r-1) - Spbd$$

and equilibrium condition C (taking moments about O_1, Fig. 3), becomes

$$Cbd\frac{1-x}{2}\left[d - \frac{d}{3}(1-x)\right] + Ca'(r-1)(2d-h) = M + FH;$$

thus changing Eqs. (2), (3) and (4) accordingly, and all equations derived therefrom.

The complications introduced by taking account in the preparation of formulas, of all the slightly modifying conditions, many of which are illogical and of insignificant influence, are fully shown by illustrative formulas worked out on such assumptions in the standard reinforced concrete text books.

All students of reinforced concrete should of course study the standard text books on the subject, so as to thoroughly understand the limitations and peculiar characteristics of the combined materials known as reinforced concrete. Such matters as the study of published tests on reinforced concrete specimens, temperature stresses, permeability to water, bond between concrete and steel, proper time for removing forms, shear, web reinforcement, etc., etc., and any special considerations called for by the particular case in hand, such as the composition and proportions of concrete aggregates, methods of mixing and placing the concrete, capacity of plant for one day's work as affecting the location of construction joints, sizes of reinforcing bars, quality of steel, methods of loading the finished structure, vibrations, etc., are all questions to be carefully looked into by the designer as affecting the assumptions used in proportioning members, and the writing of the specifications under which the work is to be carried out.

In reinforced concrete design, it is particularly true that " a little knowledge is a dangerous thing," and no one should undertake the responsibility for the design of a reinforced concrete structure without first availing himself of the published experiences and researches of others in that line of work. Such a grounding is of first importance as affecting the general features of the design. When it comes to detailing the dimensions of the various beams, slabs, columns, arches, etc., the designer should appreciate the limitations of actual construction work under practical conditions as affecting the refinement to which theoretical formulas should carried. For instance, anyone observing the operation of placing reinforced concrete under ordinary conditions cannot reflect upon the matter of the straight-line compression diagram versus the parabolic diagram resulting from a consideration of the slight variation in the value of E_c for different unit stresses, without being struck with the absurdity of introducing into the practical formulas for determining dimensions of members the complications called for by the parabolic assumption; especially in view of the slight gain in theoretical accuracy thereby attained within the usual limits of the working stresses. The difference in the actual strengths of two average beams constructed under everyday practical conditions, if subjected to actual tests, would far overshadow the effect of many of the proposed refinements in the designing formulas. The practical conditions governing the construction and operation of any particular piece of work are of first importance in writing the specifications and in determining the justifiable working stresses to be used in the design, the value of r, etc., and on these points the judgment of the designer comes into play. The proper working stresses and the correct maximum bending moments for members connected and loaded in various possible manners are something the reader will not find in the formulas, which take up the matter of detailing the members from the point where the working stresses and the bending moment assumptions have been decided upon.

Web Reinforcement

The importance of a mechanical bond between the steel and concrete is now generally conceded; the simplest means of accomplishing this being to use square reinforcing bars and have them twisted to guard against slip of the steel in the concrete and to assure the proper co-operation of the materials.

Another point to be taken care of is the matter of shear and diagonal tension in the web of the beam; that is, the stresses occurring in that part of the beam between the steel reinforcement and the compressed concrete, and through the medium of which the steel and concrete co-operate to form a united beam. It is customary to provide for these stresses by placing vertical or inclined bars at intervals in the web of the beam. The theoretical position of these web bars is at an inclination of 45 degrees, inclining away from the center of the beam. Various rules, some empirical and some theoretical, are put forth for obtaining the size and spacing of this web reinforcement. A theoretical consideration of this matter leads to an assumption of truss action in the interior of a reinforced concrete beam; the concrete in compression acting as the compression chord, the steel forming the tension chord and the central portion of the beam playing the part of the web members of the truss.

A common form of failure of reinforced concrete beams is by horizontal shearing just above the plane of the tension reinforcement. The horizontal shear in this plane between any two points of the beam may be determined as follows: Choose the points, A_1 and A_2, at a distance, Z_1, apart longitudinally of the beam; compute the moments and flange stresses at these two points, and the difference between the total flange stresses at A_1 and A_2 will be the increment of stress, or the total longitudinal shear between these points. This longitudinal shear must be taken care of by the web concrete in the said distance, Z_1, aided, if need be, by web reinforcement. The reader will find this matter of web reinforcement fully discussed in any of the standard reinforced concrete text books.

The matter of most advantageously distributing the reinforcing metal in the beam so as to bring it all into play in its proper place always calls for consideration in detailing the reinforcement. For instance, consider a continuous beam running over several supports. If the beam is uniformly loaded there will of course be a maximum positive bending moment at the middle of each beam and a maximum negative bending moment over each support, and between these maximum moment points will be points of contraflexure where the bending moment is zero, but where the shear must receive proper consideration. Some of the reinforcing steel of each beam is ordinarily bent up at properly selected points between the middle of the beam and its support and thence run over the support in the top of the beam.

In such an arrangement, the inclined portions of the steel bars act as web reinforcement. The location of the bends in the steel must be determined by a study of the progressive change in bending moments. For a strictly economic design, the steel would be distributed according to the magnitudes of the various positive and negative moments.

The thickness of concrete to be provided between the reinforcing steel and the nearest surface of the beam will be influenced by the purpose to be fulfilled by the structure, sizes of aggregate, method of placing, etc., 1 to 3 ins. being the usual limits of this dimension.