Optics for photographers

Hans Harting

OPTICS FOR PHOTOGRAPHERS

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Translated by

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PREFACE

THIS book was written for amateur and professional photographers. It is intended to explain the fundamental laws of geometrical optics, on which depends the construction of photographic objectives. In order not to make the book too bulky, I have been obliged to refrain from introducing too many details and from treating some theories *in extenso*, for instance, that of Abbe on the limitation of rays. For the same reason I have not treated stereoscopy. If lovers of photography are led by it to a more thorough study of their most important instrument, the objective, the purpose of this book will be achieved.

H. HARTING.

Berlin, 1909.

Most of these chapters were originally translated for and published serially in AMERICAN PHOTOGRAPHY. Numerous calls for the numbers containing them could not be satisfied. It has, therefore, seemed advisable to revise and complete the translation for book publication. All the tables in English measures in the last chapter have been calculated by the translator, and some explanatory matter pertaining to the enlarging table has been added.

FRANK R. FRAPRIE.

Boston, 1918.

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Optics for Photographers

CHAPTER I

Sources and Propagation of Light

Luminous Bodies. — We distinguish between *luminous* and *illuminated* bodies. Among the former are included the sun, by far the most important non-terrestrial source of light, and also every body heated above a certain temperature, and, therefore, luminous. To this group belong all the ordinary artificial sources of light used in photography, *e.g.*, magnesium light, the electric arc, the Nernst lamp, etc. Phosphorescent bodies may also be included, such as luminous paint.

Illuminated Bodies.— When light emitted by a luminous body falls on a non-luminous one, a part of the radiation may reach the eye by reflection from its surface, and the body then appears to be *illuminated*. Thus the moon, itself dark, reflects the sun's light to us. How much light the illuminated body reflects, depends on the texture of its surface, and the position of the luminous and illuminated bodies with respect to the eye of the observer. The numerical value of the first of these two factors may be determined by experiment and varies within very wide limits. While, for instance, new-fallen snow reflects irregularly or as *diffused* light a very large part of the incident rays, soot or platinum black appears to us almost black, because almost all the rays are *absorbed*.

THE SPEED OF LIGHT.— If a source of light exists in an *isotropic medium*, *i.e.*, one whose physical properties are alike in all directions, the radiation is propagated *uniformly*, with a velocity dependent on the nature of the medium. According to Michelson, this velocity in a vacuum is about 299,850 km (186,200 miles) per second and is independent of the intensity of the radiation. This number corresponds exactly with the determinations of the speed of propagation of electric waves, and this agreement may be assumed as a proof of the identity of electrical and luminous waves. How far the speed of light depends on its color we shall see later. Here we will merely remark that in a vacuum the velocity appears to be independent of the color.

DEFINITIONS.— If from the rays of light emitted in all directions by an infinitely small luminous body, *i.e.*, a point, we separate a cone whose apex lies in the point, we obtain a *light-pencil*, which is *homocentric* with all others emanating from the same point. Its *axis* is the straight line about which the pencil is symmetrical. If we follow it away from the luminous point, we call it *divergent;* if in the reverse direction, towards its point of convergence, *convergent*.

REVERSIBILITY OF THE PATH OF LIGHT.— We may consider all the phenomena of the propagation of light in accordance with the principle of the *reversibility* of the path of the rays, since every ray can traverse its path, no matter in what way this may be bent, as well backwards as forwards.

THE RELATION BETWEEN BRIGHTNESS AND DISTANCE.— In the case of rays emitted from a light source in an isotropic medium, all points of equal brightness lie on the surface of a sphere whose center is the source of light. The total area of the surface of a sphere of any given radius is, therefore, a measure of the amount of the whole radiation when the amount for a unit of surface is known. If we now consider a second concentric sphere of *twice* the radius, then, as the radiation is spread over its surface, in case no absorption of light has taken place between the two spherical surfaces, *four* times as great a surface must be covered by an equal number of rays. On a unit of surface of the larger sphere, therefore, only one quarter as much of the emitted light falls. Since the surfaces of two spheres are proportional to the squares of the radii, the quantity of light falling upon a unit of surface must be *inversely proportional to the squares of the distances* from the source of light. At three times the distance, the luminosity diminishes to one ninth, at four times the distance to one sixteenth, etc., provided that the surface of the radiant body is so small, compared with the distance to the eye or the receptive plane, that we can speak of the source of light as a *point*.

Apparent Deviations from the Law of Inverse Squares.— This basic law finds widespread application in photography. We may instance the various gradations which may be obtained when making transparencies by contact, using burning magnesium wire as a source of light, by varying the distance between the negative and the source of light. It must be remembered, however, that the blackening of the sensitive film does not increase in the same proportion as the brightness of the incident light, in case this varies within very wide limits. We must, therefore, when applying the basic photometric law of the inverse proportionality of the brightness to the square of the distance, make certain how far the photochemical process to be set in motion is dependent on the strength of the illumination.

We will not here go into details in regard to the processes for comparing the relative brightnesses of two sources of light, but merely remark that the character of the radiation from every source of light must be carefully investigated with the spectroscope, in order to determine its use in photographic work. The difference between luminous and illuminated bodies is without photographic significance, if the amount of light emitted is otherwise equal.

CHAPTER II

The Pinhole Camera

The Pinhole Camera, the Simplest Optical Apparatus.— On the law of the rectilinear propagation of light depends the simplest optical apparatus by means of which a picture can be obtained, the pinhole camera. This is in reality the prototype of all photographic exposure apparatus, and in recent years has again become popular on account of the soft character of the pictures obtained thereby. It will, therefore, repay us to go fully into its theory in beginning our consideration of the progress of photographic optics.

If we darken a room by placing an opaque screen in the window frame, and through this bore a fine hole, we will see the exterior world reproduced in an inverted position, on a screen placed within the room, exactly as we do on the ground glass of a camera. The oldest notice of this phenomenon, as one already well known, occurs in the writings of Leonardo da Vinci, who died in 1519; it was again discovered in 1553 by Giambattista della Porta.

Image Formation by an Aperture.— How this image is formed is shown by Fig. 1. S is an opaque plate which is pierced by a circular aperture with the diameter s_1s_2. If there exists on the left of S and in the axis of the aperture a luminous point P, the rays from this pass in a straight line through the aperture s_1s_2 into the space to the right of the plate S, in which the screen E stands perpendicular to the axis of the pencil of rays Ps_1s_2. There is, therefore, formed on E a luminous circle with the diameter t_1t_2 and the center P'. Since this spot of light is produced *only* by rays proceeding from P, it must be considered as the image of this point. The farther P is from the screen S the less the circle t_1t_2 surpasses in size the circle s_1s_2; if P lies at infinity $s_1s_2 = t_1t_2$.

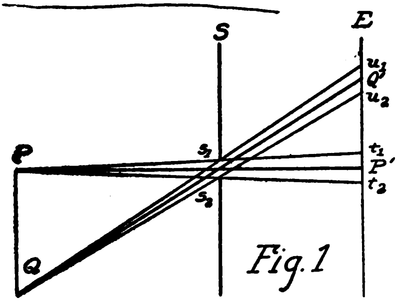

Fig. 1

THE CIRCLE OF CONFUSION; FORMATION OF AN IMAGE.— If there is a second luminous point Q which lies to one side of the axis PP' of the pinhole camera, there will be formed on the screen E, as a result of the rectilinear propagation of light, another image of the same kind, the spot of light u_1u_2. It is bounded, however, by an ellipse, since the cone of rays emitted from Q is not cut perpendicular to its axis QQ' by the screen E. If Q lies below P the image will be above the axis; consequently the image is inverted as to right and left, top and bottom, as compared with the object. We see, therefore, that an *image* of the *object* space, which in the drawing lies to the left of the plate S, is found in the *image* space, which lies on the other side. To every point in the object space there corresponds a spot of light in the picture plane, which we call a *circle of confusion*, although this designation as a circle is exact only for the spot lying exactly on the axis of the pinhole camera.

SHAPE OF THE SPOT OF LIGHT.— It is further evident that the *shape* of the spot of light must correspond to that of the aperture s_1s_2. If this is a square, the spot of light t_1t_2 must also be a square. If instead of the point P a luminous surface is imaged, the single image spots overlap each other, and as a result it is immaterial what shape the aperture, and consequently every image spot, has. Only on the edge of the image can the actual shape of the aperture become visible on the screen.

INFLUENCE OF THE DISTANCE OF THE OBJECT.— If the luminous object in the object space is relatively *distant* from the aperture, the circles of confusion remain approximately the same size, even if the screen E is moved farther back. The diameter of the circle of confusion is, however, a measure of the sharpness of the image, since this appears sharper as t_1 and t_2, u_1 and u_2, approach each other. The sharpness, therefore, remains essentially unchanged in this case, however great the distance of the screen E, or of the light-sensitive film used in its place, from the plate S. This fact can be clothed in a form which is familiar to every photographer, if we introduce the conception of *depth* into our consideration.

DEPTH.— The term *depth* designates the distance in the object space within which objects at different points all appear sharp to the eye at any given position of the screen E or of the ground glass. [This definition is that of depth of *field*. Depth of *focus* is the distance within which the screen may be similarly moved while the object remains stationary.— *Translator*.] The depth varies greatly according to the character of the instrument; although it is extremely small in the case of high-power microscopic systems, photographic wide-angle objectives are possessed of considerable depth. On what this depends we shall see later. As far as we may speak at all of sharp images in connection with the pinhole camera, we may call it a system of extraordinarily great depth, but we must also call attention to the fact that with this considerable depth goes a very small *luminosity* of the image.

ORTHOSCOPY.— We have hitherto assumed that the image screen E stands perpendicular to the axis PP' of the apparatus. From this follows an important property of the pinhole camera, which is of great significance, the *exact similarity* of object and image. If the screen E is as far from the aperture as the luminous surface PQ, object and image must be equally large. In every other position of the screen, as long as it remains perpendicular to the axis PP', object and image are unequal in size, but on account of the similarity of the two triangles whose bases are object and image and whose common angle lies in the center of the aperture, they are similar to each other in every detail. Such a completely similar delineation of an object by an optical instrument is called *orthoscopic*. The opposite of *orthoscopy* is named *distortion*.

APPLICATION OF THE PINHOLE CAMERA.— On account of its orthoscopy the pinhole camera is often used in cases in which we require a reproduction with absolutely true angles of *architectural objects*, the image of which covers a wide angle on the photographic plate.

As soon, however, as we incline the screen to the axis, the similarity of object and image ceases and we obtain an image showing *distortion of convergence*. This sort of distortion should not be confused with the previously named form.

BRIGHTNESS.— As a glance at the screen shows, the *brightness* of the image is very slight compared with that of the image produced by a photographic lens. To be sure, there is a means for heightening the brightness, and thereby shortening the exposure, *i.e.*, enlarging the aperture. If we make the circle s_1s_2 (Fig. 1) twice as large, the surface of the aperture is increased four-fold, and correspondingly the time of exposure becomes one fourth. Simultaneously, however, the diffusion increases so much that one would probably prefer the smaller opening in spite of the longer exposure.

INFLUENCE OF DIAMETER OF APERTURE ON SHARPNESS.— In photographing immovable objects one naturally thinks of increasing the sharpness by diminishing the aperture. Here also we soon come to a limit which we may not pass. It appears that the picture gains in sharpness only to a certain point, when we diminish the diameter of the aperture. If we go below this point in diameter, the diffusion of the image again increases, and that very rapidly when the diameter of the opening becomes less than 0.1 millimeter (1-250 inch). If we could go indefinitely far in diminishing the aperture, we would find a point where the circle of confusion t_1t_2 would completely cover the screen E, so that the whole space between S and E would be filled with light, and the formation of an image would cease. It therefore becomes evident that the lines Ps_1t_1 and Ps_2t_2 are not necessarily straight, but that below a certain size of the aperture s_1s_2 these lines are distinctly bent at s_1 and s_2. The light is no longer propagated in a straight line.

DIFFRACTION.— This phenomenon, which we shall often have the opportunity of observing, and the study of which gives us the first key to a knowledge of the action of all optical instruments, is the *bending* or *diffraction* of light. If we assume that light is a wave motion of the ether, we can satisfactorily explain all phenomena observed when an opaque plane is placed in the path of the rays of a pencil of light. Since we must assume that light is so propagated that every particle which is met by a ray itself becomes a center of radiation, a complication can obviously enter if the propagation of the wave trains in all directions is interrupted in one direction: for instance, through the introduction of a perforated screen. In this case there occurs a change in the distribution of the light on the edges of the cone of light passing through the opening, *i.e.*, along the lines s_1t_1 and s_2t_2. If the opening is small enough, these changes, which appear as differences of brightness, may become apparent even in the neighborhood of the axis PP', and with extremely small apertures they completely overshadow the usual phenomena of the rectilinear propagation of light.

From the manifest occurrence of diffraction in the pinhole camera are drawn some very important conclusions, which we cannot leave unnoticed. The first is that the law of rectilinear propagation of light requires considerable limitation. The same is true of the apparently obvious law as to the coincident effect of two luminous rays, namely, that if light is added to light, only light can result. A careful study of the phenomena of diffraction has shown that places of diminished brightness may occur on the image plane, which can only be explained by the fact that wave trains of light proceeding from different points partly or wholly extinguish each other. We may thus produce a weakening of the light or even complete darkness.

THE FOUNDATIONS OF GEOMETRICAL OPTICS.— If, in spite of this, we proceed to build on the foundation of *geometrical optics* and base our arguments on the law of rectilinear propagation of light in an isotropic medium and on that of the independent action of two pencils of light on each other, in accordance with which darkness can never exist from the union of light and light, we do so because the inferences drawn from these laws correspond very well with experience. Only when it is a question of a pencil of light rays of infinitesimal diameter — and such pencils do not come into consideration for us — must we resort to the absolute theory of light. In practice, calculations based on the laws of geometrical optics are fully sufficient for the construction of optical instruments. We have only to remember that we always have to do with a pencil of rays and *never with a single ray of light*, since the attempt to isolate such a ray of light from a pencil of finite diameter, by the introduction of a screen with a minute aperture, results in marked diffraction. Therefore, we can speak of a single ray of light only in a geometrical sense.

THE MOST USEFUL APERTURE.— As we have seen, there is a specific size of aperture for which the circle of confusion is smallest. The size of this most favorable aperture is dependent on the distance of the screen *E* from the aperture, and increases with the distance of the screen. Practical experiments have proved that for a screen distance of 10 cm (4 in.), the diameter of the *most useful* aperture is 0.3 mm (1-80 in.); for 20 cm (7⅞ in.), 0.5 mm (1-50 in.); for 30 cm (11¾ in.), 0.6 mm (1-40 in.).

TIME OF EXPOSURE.— As to the brightness of the image, it depends not only on the size of the pinhole, but also on the distance of the screen. If, for instance, we move the screen back to twice its distance, the image covers four times the surface, and if the aperture remains the same, the luminosity sinks to one fourth. If, however, the diameter of the pinhole increases in the same degree as the distance of the screen, the luminosity remains unchanged. Here, as in the case of photographic lenses, the relative aperture, that is, the proportion of the actual aperture to the screen distance (which here corresponds to the focus), determines the brightness, which is proportional to the square of the relative aperture.

Numerical Data.— For a pinhole 0.3 mm (1-80 in.) in diameter and a plate distance of 10 cm (4 in.), the proper exposure, for a rapid plate, in full sunlight, in summer, on an open landscape without too near foreground, has been found to be one minute. Correspondingly, with an aperture of 0.5 mm (1-50 in.) and 20 cm (7⅞ in.) distance, the exposure should be one and one half minutes; with 0.6 mm (1-40 in.) aperture and 30 cm (11¾ in.) distance, two and one quarter minutes. Only, we must not forget that all pinhole exposures suffer from lack of details in the deepest shadows, because the effect of light is here too weak to produce any photochemical change of the silver.

Distance of the Screen.— In the choice of the *distance of the screen*, the same points are to be considered as in deciding the focal length of an objective, with which we shall deal later. In addition we must remember that an object will not be recognizable in its image on the screen unless this is at least twice as large as the circle of confusion; only then will the overlapping of the single scale-like spots of light which form the image become unimportant. If we desire to render details better, we must use a greater screen distance. If the circles of confusion are to appear as points, we must so arrange matters that they subtend an angle less than that which the eye can perceive as a surface, *i.e.*, less than one minute of arc. Therefore, the eye must be placed at a distance about thirty-five hundred times the diameter of the circle of confusion if the picture is to appear sharp. For example, with a pinhole whose diameter is 0.5 mm (1-50 in.) the distance of vision must be 1.75 m (69¼ in.).

Exaggerated Perspective.— That the *exaggeration of perspective* caused by the use of too short focal lengths, which is sometimes falsely called distortion, has long been known in the case of the pinhole camera and the camera obscura is apparent from the following remark of Johann Georg Büsch, written in the year 1775: "The camera obscura, that exceptionally useful instrument, may betray the painter who blindly follows it, so that he actually paints worse and does not delude the vision as he hoped; for it sketches objects solely in accordance with the proportion of the angle of vision, while the human mind judges size not alone in accordance with the angle of vision, but has other grounds in addition."

Preparation of the Pinhole; Vignetting.— If one desires to make exposures with a pinhole camera on a large plate, it is necessary to take care that the aperture is properly made. If a thick sheet of metal is taken and a cylindrical hole of the most useful diameter is bored through it, we observe the phenomenon known as *vignetting*. In the tubular aperture, long in proportion to its breadth, the rays which enter from the sides are partly cut off, with a loss of light. Only those light rays can get through which penetrate the tubular opening within its diagonals, and outside of this is darkness. In order to avoid this vignetting a circular sheet of metal, fitted with a screw thread for the camera front, is rotated on its axis under a very dull conical point until the point appears through the back side of the metal in an opening of the desired diameter, from which naturally the rough edges must be removed. Independent of the vignetting, a *natural diminution of brightness* occurs in pinhole pictures from the center toward the edge. This phenomenon is apparent also in the case of every other optical system, as we shall see later.

CHAPTER III

The Course of the Light Rays on Passing into Another Medium

Reflection and Refraction.— When we find in the rectilinear path of a light ray, in an isotropic medium, a second, likewise isotropic medium, we perceive characteristic phenomena which may be explained by the laws of *reflection* and *refraction*. We know that light is always reflected in these circumstances. We must, however, in the first place eliminate a case, which does not come in question in our consideration of the formation of an image.

Diffuse Reflection.— We may find, for instance, that the surface of the new medium is irregular in texture, and consists of infinitely small surfaces which lie promiscuously in all possible directions in regard to each other. An example of this is the surface of a glass lens coarsely ground with emery. By reflection on these individual tiny surfaces of the glass, an incident light pencil is so broken up that its rays scatter in all directions. In this case we speak of a *diffuse reflection* and likewise, since part of the light also penetrates into the glass, of a *diffuse refraction*. There is no longer any relation between rays thus affected and the source of light from which they had their origin.

Polishing of a Glass Surface.— If, however, we work the glass surface with emery of finer grain and finally with a substance whose single particles have a very small diameter, such as rouge, we succeed — and this last stage of the working is called *polishing* — in so arranging the single particles of the surface in relation to each other that they touch one another in a regular order. As a result of this the incident light pencil is so affected that it can be used for the production of an image. We shall consider only such surfaces.

TRANSPARENCY.— We must next investigate how the medium behaves toward rays of light which penetrate it, and must distinguish between *transparent* and *opaque* materials. As a matter of fact these designations are only to be considered as relative, for there are neither completely transparent nor completely opaque bodies. *Every* substance has the property of weakening or *absorbing* light which passes through it; still we must take a great thickness of a very transparent substance, such as air or water, if we desire to make the absorption perceptible, while on the other hand we are able to prepare the metals, which we ordinarily consider as completely opaque, in such thin sheets that they are transparent. Therefore, light penetrates even into opaque media, although only to an infinitely small distance.

There are many gradations between transparent and opaque bodies. Thus, many substances absorb only rays of certain colors, while they pass others with hardly any weakening; for instance, solutions of the dyes used for the preparation of color filters. Finally we must mention *translucent* substances, which are partly transparent, but because they are filled with opaque particles, split up an incident pencil of light in an irregular manner. To this category belong colloidal solutions, opal glass, etc. These bodies are unimportant for our consideration.

Laws of Reflection and Refraction.—The laws, in accordance with which the reflection and refraction of a ray take place at a smooth surface of a medium, together with those of the rectilinear and independent propagation of all light rays, form the foundations on which the structure of geometrical optics is erected.

A ray E (Fig. 2), which moves in the medium I, may be assumed to meet at the point P, the surface TO separating this from the medium II. Since we may assume the ray E to be a line, the surface TO needs to be plane only in

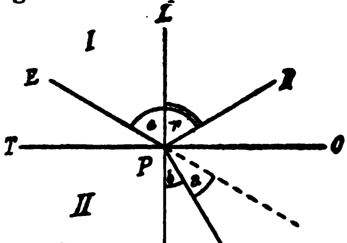

Fig. 2

a very minute extent. Therefore, our results will hold good also for all curved surfaces, since we may always assume these to be composed of an infinite number of very small facets, each of which corresponds to the plane TO. The ray E is divided into two rays, one reflected, R, and one refracted, B. Both rays obey the same law, that is, both R and B, together with the incident ray E and the perpendicular line LL', which is erected on the plane surface OT of the medium II at the point of incidence P, lie in *one* plane EPL, or abbreviated:

I. *The reflected and the refracted ray both lie in the plane of the incident ray.*

As to the direction of the reflected ray R, it forms with the normal at the point of incidence LL' the same angle that this forms with the incident ray E, except that the two rays lie on different sides of the normal. Therefore, the angle e equals the angle r, or:

II. *The angle of incidence is equal to the angle of reflection.*

If a light ray falls perpendicularly on the surface of an optical medium, the reflected ray coincides with it. We may easily observe the truth of the law of reflection in daily life. If a billiard ball rolls against the edge of the table, it rebounds — in case it is not affected by English or side — at the same angle which its direction of incidence makes with the normal to the cushion. Electrical rays are also reflected in the same way. The law of the reflection of light rays was known before the Christian era.

Optical Density.— The direction of the refracted ray *B* depends on the properties of the two media I and II, and is determined by the ratio of the velocities with which light is propagated in these media. We call one medium *optically denser* than another, if it opposes a greater resistance to the motion of light, if, therefore, the velocity of propagation of light in it is smaller. The concepts, optical density and mechanical density, must not be confused. If a ray enters an optically denser medium, it is refracted toward the normal, and correspondingly, since the process is reversible, it is bent away from the normal on passing into an optically rarer medium.

We can picture this process in the following manner: If an extended line of soldiers marches over a smooth field, in the direction of the line *E* (Fig. 2), and must continue the march with the same expenditure of energy over very difficult ground, for instance the ploughed field II, the right flank involuntarily slows its pace as soon as it passes the line of separation *TO* between the smooth grass and the ploughed ground.

Since, however, the left flank continues to march with undiminished speed over the smooth field, the direction of march of the whole line turns toward the right. On the contrary a deviation toward the left will occur if the march is accomplished along the line *B* from the rough to the smooth ground.

THE LAW OF REFRACTION OF SNELL.— The law of refraction discovered by Snell asserts that the angle of incidence *e* and the angle of refraction *b* are so related that the ratio of their sines: $\frac{\sin e}{\sin b}$ is equal to that of the velocities of light in the two isotropic media I and II, and is, therefore, constant, or:

III. *The sine of the angle of refraction is equal to the sine of the angle of incidence, divided by a constant number.*

INDEX OF REFRACTION.— These constants, that is, the ratios of the velocities of light in the media in front of and beyond the refracting surface, is called the relative *indices of refraction.* As a rule we assume, in order to characterize the optical properties of a medium, that the ray proceeds from a vacuum or, the only case occurring in practice, from air, into the optically denser medium. This is then defined by its index of refraction *n*, which is equal to the velocity of light in air, divided by the velocity in the given medium, whence $n = \frac{\sin e}{\sin b}$. This law is also followed by electrical waves.

DEVIATION OF THE REFRACTED RAY.— From the size of the angles *e* and *b* we can easily determine the amount *a* by which the refracted ray deviates from the direction of the incident ray. It is expressed by the equation $a = e - b$. The following table gives an idea how the *deviation* a increases with the change of the angle of incidence e. The index of refraction is assumed to be 1.5, and, therefore, corresponds approximately to that of a crown glass of low refraction.

e	b	a	e	b	a
0°	0° 0′	0° 0′	50°	30° 43′	19° 17′
10	6 39	3 21	60	35 16	24 44
20	13 11	6 49	70	38 47	31 13
30	19 28	10 32	80	41 2	38 58
40	25 22	14 38	90	41 49	48 11

From this table we deduce another important phenomenon. Since an angle of incidence of 90° corresponds to a ray falling in the direction of the surface itself, we obtain a pencil of rays incident at the point P from every direction of space in the medium I if we ascribe to the angle e all values from 0° to 90°, in all planes which contain the normal LP. If we construct the refracted ray for each one, we find on comparison that the angle of refraction increases only from 0° to 41° 49′; that, therefore, to a hemisphere filled with light described about P in the medium I, *i.e.*, air, there corresponds a cone of light in medium II, half of whose apical angle is 41° 49′, and whose apex is likewise at P. The remainder of the medium II remains dark.

TOTAL REFLECTION.— Let us now assume that a luminous point P (Fig. 3) existing in the optically denser medium II, whose index of refraction $n = 1.5$, radiates light in all directions. The ray Ps_1, falling perpendicularly on the surface of separation, is divided into two rays, a refracted one s_1b_1, which proceeds in the same direction in the medium I, air, and a reflected one s_1r_1, which returns through P. A second ray Ps_2, inclined at a small angle to the normal, divides into the rays s_2b_2 and s_2r_2, which are refracted and reflected according to the law as stated. More inclined rays also pass into the air until we come to a ray Ps_0 whose angle with Ps_1 amounts to 41° 49′. To this, however, corresponds in the medium I an angle, between normal and refracted ray, of 90°, so that this proceeds in the direction of the surface s_0b_0; in addition a part of the light is reflected toward s_0r_0. What happens, however, to the rays of light proceeding from P which make an angle greater than 41° 49′ with Ps_1? If we try to determine for them the angle of refraction in air, according to the law of Snell, we find a sine which is greater than 1, an impossibility. Hence, they cannot leave the medium II and, therefore, they are *totally* — unlike the partly reflected rays Ps_1 and Ps_2 — reflected at the surface, which forms for them an impenetrable mirror. We, therefore, name the angle of incidence to which corresponds an angle of refraction in air of 90°, the *limiting angle of total reflection*. If n is the index of refraction of a given medium, this limiting angle is determined by the equation $sin\ b_0 = \frac{1}{n}$. The larger n is, the smaller is b_0. Thus water, with an index of refraction of 1.34, has

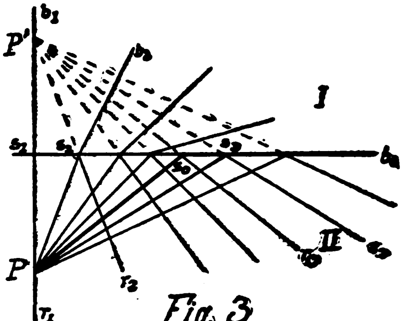

Fig. 3

a limiting angle of 48° 16′, while the diamond, with a refractive index of 2.47, has an angle of total reflection of 23° 53′. Since the possibility of a ray being able to escape from a faceted body into the air must be greater, in proportion as its limiting angle of total reflection is smaller, a body of high refractive index like the diamond must show a greater number of reflections than, for instance, glass, which makes itself evident in the well-known sparkle of the gem. If we must determine the limiting angle for the passage of light from an optically denser to a rarer medium, other than air, we use in place of the absolute the relative index of refraction, that is the ratio of the absolute indices of refraction of the two media.

Homocentric Union of Rays in the Case of Reflection from a Plane.— By reference to these laws we are now able to determine the position of the image which is derived from a luminous point through reflection or refraction at a *plane* surface. If we imagine the rays s_1r_1, s_2r_2, etc., which are reflected at the surface s_1b_0 (Fig. 3), projected backwards, we find that they all appear to proceed from *one* point P', which appears to lie at the same perpendicular distance from the mirror as the object point, on the opposite side. We apply this to any object in the object space and find that the image produced by a plane mirror is of the same size as the object, and is symmetrically placed with reference to the mirror. Since *all* rays appear to proceed from the mirror image P', this relation of position and size holds for any angle of aperture, no matter how large, of the pencil of light which forms the image.

The Caustic.— A different relation occurs when an image is formed by *refraction* at a plane surface. If we imagine that the rays s_2b_2 to s_0b_0 are prolonged backwards to their intersections with Ps_1, which, to avoid confusion, has not been done in the drawing, we observe that every refracted ray has a different intersection with its neighboring ray, and that the image of the point P is not a point, but a curved line. This is called the *caustic*. Only when the pencil of light proceeding from P has a very small section does this caustic line shrink together so far, that one can speak of a point image. A similar limitation is also necessary in regard to the size of the object. While an image of an object of any size can be formed by reflection from a plane mirror, closer investigation shows that by refraction at a plane surface the image only of an object of very small dimensions can be obtained.

Principle of Fermat and Helmholtz.— At the close of this chapter we may very cursorily remark that the laws of reflection and refraction may be deduced from a general principle, according to which every ray of light which proceeds from one point to another by means of reflection or refraction, takes the shortest path to its goal. This law was first propounded by Lachambre and Fermat and received its definite proof from Helmholtz.

CHAPTER IV

Refraction of Light at Several Planes

If after the passage of a ray of light through a refracting plane surface, a third refracting medium also with a plane surface stands in its path, the position of this surface is determined by the angle which the two refracting planes form with each other.

The Plate with Plane Parallel Faces.— Let us now assume that this angle is zero, therefore, that the two planes are *parallel* to one another; also that the third refracting medium is identical with the first. We thereby arrive at a case which is particularly important in practice, and which is represented in Fig. 4. The plate with plane parallel faces of the substance II lies imbedded in the medium I. The incident ray E, which lies in a plane perpendicular to both refracting planes, makes with the first plane the angle of incidence e and the angle of refraction b. As on account of the parallelism of the planes TO and $T'O'$, the two angles b and e' are equal, the angles b' and e must also be equal and, therefore, the emerging ray E' is parallel to the incident ray E.

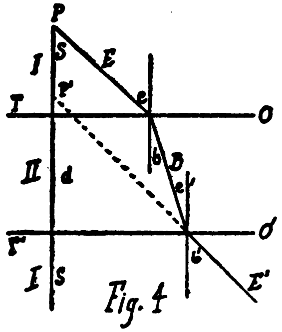

Fig. 4

Formation of the Image of a Point by the Plate with Plane Parallel Faces.— If we investigate how the image of a luminous point P, which emits the ray E, is formed by the plate with plane parallel faces, we must determine the path of a second ray proceeding from P. The best ray to choose for this purpose is S, perpendicular to the two surfaces TO and $T'O'$, as its path is not changed by the introduction of the plate. The point of intersection of S and the ray E' prolonged backwards is the image point P'. Therefore, the luminous point P is apparently brought nearer to the plate by the distance PP'.

Calculation now shows that the position of the point P' is dependent on the size of the angle of incidence e. Consequently here also the image of a luminous point appears as a caustic line which shrinks to a point only when the angle of divergence of the pencil of rays proceeding from P is very small. Under this assumption, the displacement PP' equals $d\,\frac{n-1}{n}$ where d is the thickness of the plate and n is its index of refraction. For crown glass the factor $\frac{n-1}{n} = \frac{1}{3}$. We, therefore, have the law:

The introduction of a plate with plane parallel faces in the path of a pencil of light rays emanating from a point does not change the direction of the single rays. In the case of a pencil of small divergence the object point appears to be displaced by a distance equal to about one third of the thickness of the plate if this consists of ordinary crown glass, and this displacement is toward the plate when the pencil is divergent, and away from the plate when it is convergent. This result is not changed when we have to do with several such plates. The total displacement is then determined by adding the various values of $d\frac{n-1}{n}$.

COLOR FILTERS.— We have to do with such an arrangement of refracting surfaces in photography when we use *color filters*. In general use we have to deal only with those which are placed just before or behind the lens, as filters placed close to the plate are used only in reproduction. That the surfaces of the filter must be as plane as possible, is the first requirement, since otherwise the rays of light passing through the objective are irregularly refracted. When we use filters consisting of a colored liquid between two glass plates, there are three plane parallel plates to be considered, the influence of which can be easily determined. If, for instance, the thickness of each glass plate is 1.3 mm and that of the colored layer 1.4 mm, the rays appear to come from an object point about 1.3 mm nearer, or to reach a focus about 1.3 mm farther from the lens, according as the filter is placed before or behind the lens. If, on the other hand, the surfaces of the filter are not plane, it acts as a lens, and may seriously affect the focusing and the quality of the picture. The displacement of the object or the image is much greater in exposures for reproduction with cellular filters which have a considerable thickness.

COVER GLASSES FOR MICROSCOPIC PREPARATIONS.— On the other hand, considerable errors may also exist when one, in the case of very large angles of divergence, such as those of microscopic objectives, does not allow for the slight thickness of the cover glass over the preparation, which also forms a plane parallel plate, even though only a few tenths of a millimeter in thickness.

In the case of the comparatively small angle of the pencil of rays which passes through a photographic objective, the broadening of the image of a point into a caustic line through the introduction of a color filter is not noticeable.

HALATION.— Since with every refraction reflection must also occur, a part of the light is reflected within a plane parallel plate at its second surface and must return through the first surface into the external medium. This causes *halation*, which is always to be feared in photographic exposures on objects full of contrast. The light penetrates through the sensitive film into the glass, is reflected at its farther side and then acts on the back of the film in the neighborhood of the lightest portions of the image. The thicker the glass the greater must be the diameter of the halation around a high light. Films, being thin, are little affected by this defect. In the Isolar plates this defect, which is especially apparent in interior exposures, is avoided by the introduction of a layer of red dye between the gelatine film and the glass plate. The occurrence of halation has nothing to do with *solarization* or *reversal*, which is to be ascribed to the properties of the gelatine emulsion.

ABSORPTION IN A PLATE WITH PLANE PARALLEL FACES.— Finally we must mention that the paths of the single rays of a pencil in a plane parallel plate have different lengths. While the ray (Fig. 4) which passes through perpendicularly covers a distance equal to the thickness d of the plate, the path of every other ray is longer, and may be expressed by the formula $d: \cos b$. Therefore, the single rays are absorbed in different degrees, so that toward the edge of the image on the photographic plate, there must be a falling off of luminosity, if a strongly absorbing medium like the color film of a filter is introduced in the path of the rays. Since we generally have to do with an angle of view of at the most 60° to 65°, a loss of light of only a few per cent compared with the middle of the image is produced, as a result of the oblique passage of the outside rays through the filter.

THE PRISM.— We will now consider the general case of the refraction of a pencil of rays at two planes which form any given angle with one another. The space which lies between two such planes is called *prismatic*, and a body thus bounded is named a *prism*. We represent the prism diagrammatically by a section in the plane of the paper, perpendicular to the common line of both planes, *the edge*, *i.e.*, as the angle w between the sides SA and SA' (Fig. 5). Since the course of rays passing through the prism is dependent on the size of this angle w, it is also called the *refracting angle*. In the construction of a prism for optical instruments, it is usually bounded by a third plane cutting the first two, whose section is the base AA'. We assume in the following consideration that the rays follow exclusively a path in the *principal section*, perpendicular to the edge, therefore, in the plane of the paper in the drawing.

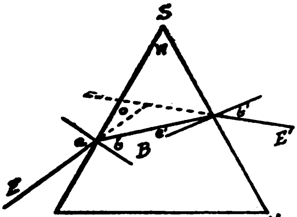

FIG. 5

COURSE OF A RAY IN THE PRINCIPAL SECTION.—According to the law of refraction, $\sin b = \frac{1}{n} \sin e$, the angle of refraction b corresponds to the angle of incidence e of the ray E reaching the surface SA from the air. An easy geometrical proof shows that the angle of incidence e' of the refracted ray B within the prism on the second surface SA' is connected with b by the equation $e' + b = w$. The ray B now passes out into the air, and the second angle of refraction b' is determined by the equation: $\sin b' = n \sin e'$. The ray E' is refracted from the path of the original ray E by the angle $o = e + b' - w$.

MINIMUM DEVIATION.— A more thorough consideration shows that the *deviation* of a ray by a prism is *least* when the prism is placed *symmetrically* to the entering and emergent rays, therefore, when e and b' are equal. If we measure the angle o in this position, we can from this and the angle of the prism w easily calculate the coefficient of refraction n of the prism.

TOTAL REFLECTION WITHIN A PRISM.— We reach a very important conclusion by an analysis of the equation, $e + b' = w$. As we have already seen, to the angle of incidence $e = 90°$, or the so-called grazing incidence, corresponds the limiting angle b_0 of total reflection and *vice versa;* therefore, $\sin b_0 = 1 : n$. This is naturally true also for refraction at the second surface SA'. Therefore, if the ray is to go completely through the prism, the prism angle w can never be larger than twice the limiting angle of total reflection for the material of which the prism is constructed. If the angle of the prism is larger, the ray B cannot emerge from the prism but is totally reflected at the surface SA'. If it then impinges for the second time on the first surface SA, it can again emerge into the air. We must also remember here that a reflection occurs at every refraction. Even when the ray E pursues the course shown in Fig. 5, a weakening of its brightness by reflection must occur at both refractions.

LIMITS FOR THE ANGLE OF THE PRISM.— From the data already given as to the magnitude of the limiting angle of total reflection, it follows that the angle of the prism for water cannot be larger than 96° 32′, in case the light is to be allowed to emerge. The larger the index of refraction of the substance of the prism is, the smaller must the angle *w* remain. For crown glass with the refractive index 1.5, it amounts to 83° 38′, for diamond 47° 46′.

THE RIGHT-ANGLED ISOSCELES PRISM.— We use the fact of total reflection in certain prisms to direct a pencil of light into a photographic objective. Most useful for this purpose is a *right-angled isosceles prism* (Fig. 6), constructed of the most transparent possible crown glass. If a ray E falls perpendicularly on one face, SA, of the prism, its two angles before and after total reflection at the hypotenuse AA' must equal 45° to the perpendicular. This is larger than the limiting angle, and, therefore, the light is totally reflected. The ray E' emerges perpendicularly to the other face SA' and is bent 90° from its path on entering. Prisms of this kind are used particularly in photographing for reproduction, when it is necessary to reproduce the object as a mirror image, that is, with right and left reversed. For prisms of large measurements we need, to be sure, very heavy masses of glass, the fabrication of which involves considerable difficulty. On the other hand,

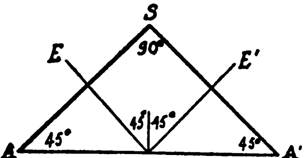

FIG. 6

no light is lost in the actual reflection at the hypotenuse of the prism, although some disappears through partial reflection accompanying refraction at the two faces, and through absorption by the glass.

When we attempt to replace totally reflecting prisms by plane mirrors, we must take into account the fact that polished metallic mirrors absorb light in reflecting it, to a considerable extent. The loss of light, however, is small when using glass mirrors with external silvered surfaces, but the fine silver layer is easily scratched and is destroyed by small amounts of certain chemical fumes.

The Trough Prism.— Another totally reflecting prism depends on the action of a mirror including an angle of 90°, *i.e.*, two reflecting surfaces which form a right angle with each other. If such a *trough* is placed in the path of a pencil of light, the image is reversed as to right and left, top and bottom, compared with the object. Such a trough prism was first proposed by Amici; in using it, the end surfaces are placed perpendicularly to the axis of the pencil of rays, so that this is refracted through a right angle. If a trough prism is placed before each lens of a stereoscopic camera, as was first proposed by H. Fricke, the prints do not need to be cut apart and transposed. Direct prints from the negative give properly placed images.

Equivalence of a Total Reflecting Prism and a Plate with Plane Parallel Faces.— As far as the optical effect of these totally reflecting prisms is concerned, it corresponds exactly to that of a plane parallel plate of the same material and of a thickness equal to the length of the path of the rays within the prism. Here also, therefore, a point image is obtained only under certain conditions.

CHAPTER V

Reflection of Light at a Curved Surface

Transition from Plane to Curved Surfaces.— After we have investigated the behavior of a light ray at a plane surface separating two optical media, we must consider the general case when this separating surface is curved. As we have already mentioned, we can imagine that in this case the surface is composed of infinitesimal plane facets, so that the corresponding laws are applicable to the course of a light ray at each of these planes. Mathematically speaking, therefore, we consider in place of such a plane, the tangent plane at the point of contact of the ray and the curved surface, and instead of the perpendicular, the normal to the curve.

Spherical Surfaces.— In photographic optics, the only curved surface to be considered is the *surface of a sphere*. Even although the optician is equally well able to grind other surfaces, their utility would not be proportionate to the expense of their production, since a combination of spherical surfaces meets all requirements which are practically demanded of a good lens. It would, therefore, be useless to employ surfaces whose construction is not so easy.

Reflection at a Concave Spherical Surface.— The simplest case of image formation by media limited by spherical surfaces is offered us by reflection at a *concave spherical surface*. If we cut a circular segment from a carefully polished hollow sphere we obtain a concave mirror, whose axial section (Fig. 7) is the arc of a circle. If we desire to construct the image of a luminous point, we must follow two rays diverging from this and find their intersection after reflection. This is the desired image point.

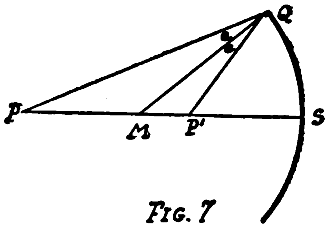

FIG. 7

Let us first assume that the luminous point *P* lies on the axis of the mirror, which goes through the center *M* and the vertex *S*. If a ray goes in the direction of the axis *MS* it is reflected back upon itself, since the radius is perpendicular to the tangent. The image of the point *P* must, therefore, lie on the axis of the mirror. Another ray *PQ* forms with the radius *MQ* the angle *e*; therefore, after reflection, it must make with the perpendicular — that is, with the spherical radius *MQ* — the same angle *e*. The ray *QP′* thus defined cuts the axis of the mirror in *P′*; therefore *P′* must be the image of the luminous point *P* and *P* the image of *P′*.

Focus and Focal Length.— Of especial importance is the position of the image point which corresponds to an infinitely distant luminous point. In this case the line *PQ* is parallel to the axis, and a short calculation shows that the corresponding image point lies equally distant from the vertex *S* and the center *M*. This especial image point is called the focus, because man early learned that all sun rays could be collected to one point, the *focus* of the mirror. The smaller the radius of curvature the nearer the focus lies to the mirror, and the smaller is the image of an object produced by the mirror. We use as the measure of the optical effect of a mirror, not its radius, but its *focal length;* that is, the distance between the focus and the vertex of the mirror, which, as just stated, is equal to half the radius MS.

Construction of the Mirror Image.— The process for the construction of the image produced by a concave mirror is as follows: If the object OP (Fig. 8), perpendicular to the axis, is to be imaged, we follow two rays emanating from O. Of the one which goes through the center, M, we know that it falls perpendicularly on the surface of the mirror, and, therefore, it is reflected backwards on itself from T. The other ray OQ, parallel to the axis, after reflection goes through the focal point F, which is located by the relation $MF = FS$. The intersection of the two rays QF and TM is the image of O and the perpendicular $O'P'$ is the desired image of the object OP.

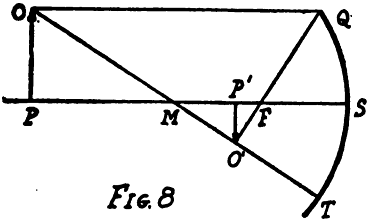

Fig. 8

Relation between Object and Image.— Calculation now shows further that a very simple relation exists between the distances of the object plane and the image plane from the plane of focus, measured along the axis. Even although the formation of an image by means of mirrors is of very little importance in photography, it appears valuable to investigate the size and position of the images at this place, because, as we shall see, these relations are of a very general nature, and their exact understanding cannot be dispensed with. Naturally, we cannot thoroughly investigate the mathematical principles, but must satisfy ourselves with proceeding from formulas known to be correct.

BASIC EQUATIONS FOR THE POSITION AND SIZE OF AN IMAGE.— We will measure the distance of the object and its image, not, as is usually the case in elementary textbooks of photographic optics, from the vertex S, but from the focus F. We shall designate here, as later, the distance of the object plane from the focus by x, that of the image plane by x', the focal length, equal to half the mirror radius, by f. Then the first basic equation for the *position* of object and image is:

(A) $$xx' = f^2.$$

Let us further call the size of the object y and that of the image y'; then the second basic equation for the relation of the *sizes* of the object and the image is:

(B) $$y' : y = f : x = x' : f.$$

NEGATIVE AND POSITIVE IMAGE FORMATION.— From the first equation it follows that x and x' are always simultaneously either positive or negative; we will call x or x' positive when the object or the image lies between the focus and the vertex. We can, therefore, state the law thus: *object and image always lie on the same side of the focus of a concave mirror*. Further, since the product xx' is a constant equal to f^2, a small image distance x' must correspond to a large object distance x; and *vice versa*, if x increases, x' diminishes. Such an image formation, in which the object and image act reciprocally, we call *negative*, in opposition to *positive* image formation, which occurs in all cases of refraction.

THE SCALE OF THE IMAGE.— In the second equation the quotient $y' : y$ may have a negative value. The image is then inverted, while for a positive value of $y' : y$ it stands upright. If $y' : y$ is greater than unity, the image is enlarged; if it is less than unity, the image is reduced. If the focal length f increases, y' becomes larger. Therefore, the scale of the image is determined by the focal length.

If we give to the object distance x all values which it can assume, from minus infinity to f, we obtain three sharply defined spaces, the first from minus infinity to the center of curvature, the second from the center to the focus, and the third from the focus to the vertex.

THE THREE DIVISIONS OF THE OBJECT AND IMAGE SPACE.— If the object lies in the first space, between minus infinity and the center of curvature, the image lies between the focus and the center; it is inverted, reduced, and real, *i.e.*, it may be seen upon a screen. If the object and the image are both at the center of curvature, they are equally large, but one is inverted with respect to the other.

If the object is in the second space, between the center of curvature and the focus, the image lies between the center and minus infinity. It is inverted, enlarged, and real.

If the object lies in the third space between the focus and the vertex, the image lies between plus infinity and the vertex (therefore in Fig. 8 to the right of S); it is upright, enlarged, and since it lies behind the mirror, virtual, *i.e.*, it cannot be received upon a screen. These changes in the position and size of object and image are the same here as in systems of lenses, except that in the latter case two focal points have to be considered, while here there is only one.

THE REGION OF THE FORMATION OF POINT IMAGES BY PARAXIAL RAYS.— Underlying the formulas (A) and (B) which we have just given, there is a condition which is of the greatest importance for our further consideration. As we have already seen, we can speak only of the formation of a point image in the case of refraction of a homocentric pencil of rays at a plane, if the angle of aperture is made so small by stopping down that the caustic curve contracts to a point. This is also true of the formation of an image by reflection at a spherical surface. A necessary condition for the formation of a point image in this case is that the rays of light should make a very small angle with the axis of the mirror. Such rays, the path of which is very close to the axis, are called *paraxial*, and the very small cylindrical region penetrated by them and surrounding the axis is called the *Gaussian space*. Such a Gaussian image-formation by means of paraxial rays is nearly approached, if we give the mirror a very small diameter.

SPHERICAL ABERRATION.— If, however, the concave mirror is fitted with a stop, as is often necessary in practice, and we determine, according to the law of reflection, the point of intersection with the axis of every ray of a homocentric pencil emitted by a point on the axis, we find that, even with small angles of aperture, this point of intersection departs from that of a paraxial ray. This deviation, measured along the axis, is called *spherical deviation or aberration*. It is the larger the more the rays incline to the axis.

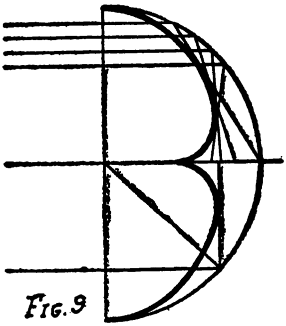

FIG. 9

Fig. 9 represents the path of a pencil of large dimensions, and for the sake of simplicity the luminous axial point is assumed to be infinitely distant, so that all its rays are parallel to the axis. In this case the image point of the paraxial rays lies in the focus, while the axial points of intersection of the other rays approach more nearly to the vertex of the mirror the greater the distance from the axis at which they reach the mirror. If we join the intersections of the various rays with each other, we obtain a characteristic curved line (heavily drawn in Fig. 9) which, on account of the symmetry of the mirror about the axis, is likewise symmetrical to this and has an apex in the focus point. This curve is the previously mentioned *caustic or focal line*, from whose shape we are able to draw a conclusion as to the distribution of brightness in the different planes of focus between the focal point and the vertex of the mirror. When this curve is produced by reflection alone it is also called a *catacaustic*. For the special case of an object-point at infinite distance, the focal line is an *epicycloid*. If we rotate the caustic curve about the axis of the mirror, we obtain the *caustic or focal surface* for the whole pencil of light falling on the mirror.

DEVIATIONS FROM THE SPHERICAL SHAPE IN A CONCAVE MIRROR.— We must, therefore, always expect the occurrence of spherical aberration in the formation of the image of a luminous point on the axis by reflection at a concave spherical surface. Only when the point is at the center of curvature are all rays emitted by it again united in *one* point, *i.e.*, in the luminous point itself. If, however, we use a mirror of another shape, it is possible homocentrically to unite all rays coming from a point in another point at any desired place. The reflecting surface in this case is an ellipsoid of rotation, and object and image lie at its two foci. If the luminous point is infinitely distant, *i.e.*, if all rays fall parallel to the axis, this surface becomes a paraboloid of rotation; and *vice versa*, a parabolic concave mirror sends forth a bundle of rays parallel to the axis if the source of light is placed at its focus. Thus are constructed searchlights, which emit a beam of light which even at great distances has the same diameter, and, illuminates an object, save for loss of light through absorption by the air, with almost the same brightness no matter what the distance.

APPLICATION OF CONCAVE MIRRORS IN PHOTOGRAPHY. — Although the use of a concave mirror for photographic purposes has often been proposed, apparatus built in this way has found no commercial use. The principal reason for this is the peculiar position of the image, which requires the interposition of other mirrors, as do astronomical reflecting telescopes. If the photographic plate is placed directly in the plane of the image, as in the case of astro-photographic exposures, too great a portion of the incident light is intercepted. The useful image is also small, so that the concave mirror cannot compete with photographic objectives in this respect. It might possibly be available for cinematographic exposures, especially in three-color work. On the other hand, the concave mirror finds a favorable reception for astronomical exposures, in which it is necessary to reproduce the fine details of difficult objects, such as nebulae. The loss of light is very small, about six per cent in the case of surface-silvered glass mirrors. Since the mirror can be given a very large relative aperture, up to f: 2, the exposures are relatively small. Even if the size of the useful image with such large openings is very small, the quality of the images of axial objects is so perfect because of the complete lack of the chromatic aberrations unavoidable in lenses, that the finest details are perfectly reproduced on the photographic plate, including those which the best combinations of lenses fail to reproduce.

THE CONVEX MIRROR.— The complement of the concave or collecting mirror is the *convex* or *dispersing mirror*. In this the reflection takes place at the outer surface of a sphere. As Fig. 10 shows, the ray parallel to the axis emitted from *O* is reflected at *Q* as if it came from the point *F*, which lies halfway between *S* and *M*. Our earlier assumptions are also valid here; only the focus *F* and the focal length *SF* are no longer real, but virtual. As the corresponding formulas (A) and (B) when applied to a convex mirror show, the image formation here is also negative, and there corresponds to an object *OP* moving from minus infinity to the vertex *S*, a virtual image, erect and reduced, which simultaneously moves between the focal point *F* and the vertex *S*. The construction of this is shown in Fig. 10, the letters of which are the same as for corresponding points in Fig. 8.

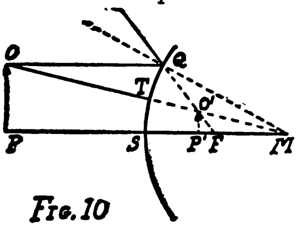

FIG. 10

We find here also that the imaging by a homocentric pencil ceases as soon as we leave the space of the paraxial rays, and that a spherical aberration occurs, which, however, is of opposite character to that of the concave mirror. Practical optics has used the convex mirror only in the construction of the Cassegrainian reflecting telescope. If it is required to focus an optical instrument on a luminous point, we often choose for this the virtual image of the sun produced by the surface of a thermometer bulb filled with quicksilver.

CHAPTER VI

The Simple Lens

Lenses, Their Elements and Nomenclature.—If we construct a solid body bounded by two spherical surfaces, we obtain a spherical *lens*. Although there are other kinds of lenses limited by other curved surfaces, which, however, are of little practical use, we commonly use the word "lens" alone to designate a spherical lens. It is not necessary in the case of a lens that the whole space between the two spherical surfaces should be filled with an optically active material; this is only necessary for the amount of this space which is penetrated by the rays necessary for the formation of an image. A straight line joining the two centers of curvature is called the *optical axis* or the *axis of the lens:* it is the axis of symmetry. The distance between the two spherical vertices measured along the axis is the *thickness* of the lens. The diameter of the lens is determined by the two spherical radii and the thickness, in case the lens is bounded by the circular line of intersection of the two spherical surfaces. In this case the lens is called *sharp*. If, however, the thickness is greater than is made necessary by the given diameter and curvature of the two spherical surfaces, we are accustomed to *center* the lens, *i.e.*, to bound it by a cylindrical surface parallel to the axis, which is without optical effect. To construct a lens, therefore, we must know five elements: the index of refraction of the material, the radii of the two surfaces, the thickness, and the diameter or actual opening.

If we desire to represent diagrammatically a lens and the course of the rays influenced by it, we content ourselves with a principal section through the axis. The intersections of this with the spherical surfaces then appear as circular arcs, while the cylindrical bounding surface is intersected in two straight lines parallel to the axis. Finally, we assume in all investigations that the lens is in contact with air on both sides.

Refraction at a Spherical Surface.— Let us first investigate the path of light through the first spherical surface (Fig. 11). Let PP' be the axis of the lens, which passes through the center of curvature M of this surface and its vertex S. Let P represent a luminous point, which emits a pencil of rays in the plane of the paper from which we will select two rays, PS and PQ.

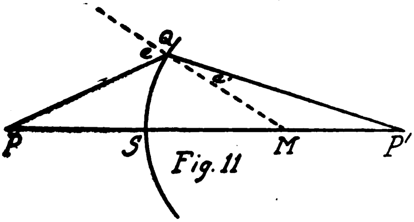

Fig. 11

As in our earlier investigations, the refracted pencil must proceed in the same plane. If we desire to consider the totality of all rays in space proceeding from P, we must rotate the whole figure about the axis PP', since the path of the rays in all directions of space is symmetrical to this.

The ray which falls upon the refracting spherical surface at the vertex S proceeds in the direction of the axis and the radius, and therefore strikes the surface perpendicularly; PSM is the normal to the surface. The image of the point P or the point *conjugate* to P

must, therefore, lie on the optical axis; since the line SP' in the image space corresponds to the line PS in the object space, the axis is conjugate to itself.

Character of the Surface.—The refracting surface may be either concave, or, as it is shown in Fig. 11, convex toward the point P in the air. We will distinguish between these two positions, by the sign which we give to the radius $r = QM$ of the surface; we call the surface *positive* when it is convex to the incident light, therefore if the center of the sphere is separated by the surface from the incident pencil of light. The radius and the surface are *negative* if this is concave to the incident light. We assume, in this connection, that the direction of the rays is always from left to right.

In order to determine the direction of the ray refracted at the positive surface QS (Fig. 11) which is conjugate to the ray PQ, we erect the perpendicular at the point of incidence Q, which coincides with the radius MQ. The angle e' of the refracted ray QP' with the radius QM is determined by the equation

$$\sin e' = \frac{1}{n} \sin e.$$

After we have thus found the straight lines conjugate to two lines emanating from the point P, their point of intersection must be conjugate to that of the lines in the object space. P' is therefore the image of P.

Limitation to Paraxial Rays.—This assumes, however, that all rays emitted from P intersect after refraction in P'. As calculation shows, and as is obvious from our previous demonstrations, this conclusion is valid only for the narrow cylindrical space which surrounds the axis and within which the image is formed by paraxial rays.

Focal Point and Focal Length.— If the ray PQ travels parallel to the axis, if, therefore, P', according to our notation, lies at minus infinity, the conjugate ray cuts the axis in a point whose distance from the vertex S of the refracting surface can be mathematically shown to be $r\frac{n-1}{n}$. We call this point the *back focal point* of the surface, its distance from the vertex of the surface the *back focal length*. On the other hand, we obtain the *front focal point* if we allow to reach the surface from the right a paraxial ray, which on its exit into the air deviates from the perpendicular at the point of incidence, and cuts the axis in a point conjugate to infinity. Its distance from the vertex is then $r(n-1)$ and equals the *front focal length*. We see, therefore, that the front and rear focal lengths of a refracting spherical surface are not equal.

It is, therefore, evident that in the passage of light from an optically rarer to a denser medium there occurs a collective action if the surface is convex, and this is likewise true for a concave surface when the light passes from the denser into the rarer medium. We speak in this case of a *collecting* surface.

On the other hand, a surface is *dispersive* if it is concave for the passage from an optically rarer to a denser medium, or convex for the passage from a denser to a rarer. In this case light rays parallel to the axis appear after refraction to come from a focal point on the other side of the surface, which we, therefore, call *virtual* as contrasted with the *real* focal point of the collecting surface.

Path of the Rays in a Lens.— The path of the rays of an axial pencil passing through a lens is immediately found by the application of the laws just stated to the two surfaces of the lens. The ray QP' (Fig. 12) meets the second lens surface at Q' and passes out into air after refraction. Since in this case the angle e''', according to the equation: $\sin e''' = n \sin e''$, is greater than the angle of incidence e'', the convergence of the ray PQQ' is increased by the collecting action of the surface $Q'S'$, so that the image P'' is brought nearer to the lens. We see also that the position of the point P'' is considerably influenced by the thickness of the lens d.

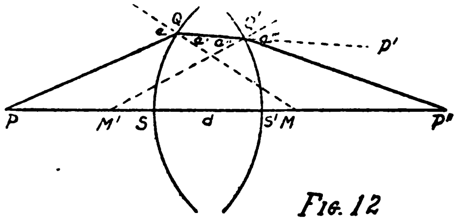

FIG. 12

INFINITELY THIN LENSES.— If this thickness is so small that the path of the rays is not affected by it, if the lens, therefore, as it is generally expressed, is *infinitely thin*, the distance of the focal point P'' (conjugate to the infinitely distant point P) from the coincident lens vertices S and S', is equal to the back focal length. The same is true of the position of the front focal point, in the case of an infinitely thin lens.

FOCAL LENGTHS.— It thus appears that the front and back focal length of a lens of any thickness which is placed in air are equal. This appears to the photographer as a self-evident fact, for if he has made an exposure he would, other things being equal, obtain the object on the plate in the same size, if he repeated the exposure after reversing the objective. If the thickness of the lens (Fig. 12) becomes so small that S and S' coincide, the distances of the two foci from the infinitely thin lens are equal. This, however, is no longer the case when we have to deal with a lens of finite thickness. In this general case we obtain the points from which we measure the focal length by setting these off on the axis in the direction toward the lens from the two focal points. These two points, whose position is dependent on the radii, the thickness, and the index of refraction of the lens, are usually called the *principal points*. We shall later define them more thoroughly; here we will only say that they have only a geometrical significance, not a physical one like that of the focal points.

ZERO LENSES.— If we had given the second lens surface in Fig. 12 an opposite curvature, the convergence of the pencil of rays emitted by P would have been decreased, if not actually changed to divergence. If the lens were infinitely thin, the collecting effect of the first surface would be neutralized by the dispersing effect of the second surface, if the two surfaces were equally curved. Such lenses, which have the form of a watch crystal, are called *zero lenses*. They behave in the paraxial space like a plate with plane parallel surfaces, and do not change the direction of a ray. Zero lenses are often used in photographic optics. If we desire so to apply one lens to a second that the light-rays pass directly from one to the other, we give the adjacent surfaces equal but opposed curvatures. A little Canada balsam is placed between these two surfaces, and the excess removed by pressing them together. The lenses are *cemented* by hardening of the Canada balsam, which assumes the form of a zero lens, and does not affect the course of the rays. Only in the case of very unskilful cementing does the presence of this brownish material produce any appreciable absorption.

Lens Forms.— By the combination of convex and concave surfaces, we obtain different *forms* of lenses. We include the special case of a surface with infinitely great radius of curvature, which is, therefore, a plane, and whose construction is optically very simple. Of the lenses shown in Fig. 13, *1*, *2*, and *3* are collecting; *1* is called bi-convex, *2* plano-convex, and *3* convexo-concave, having the radius of the first positive surface smaller than that of the second dispersing surface. *4*, *5*, 6 are dispersing lenses. *4* is bi-concave, *5* plano-concave, 6 convexo-concave. A convexo-concave lens, such as *3* or 6, is called a *meniscus*.

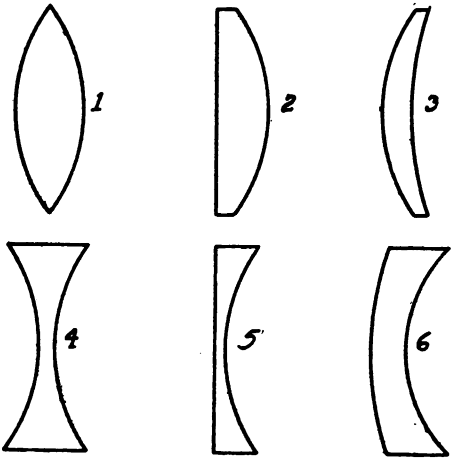

Fig. 13

Central and Marginal Thickness.— If we compare the six types of the drawing with respect to their *central thickness*, we find the law: collecting lenses become thinner toward the edge, dispersing lenses become thicker. This law holds for all lenses of any thickness with the exception of bi-convex lenses, and of menisci which increase in thickness toward the edge, but in the case of these forms is true only for lenses possessing only moderate thickness. These exceptional cases, however, do not need to be considered for our purposes.

RELATION OF THE FOCAL AND PRINCIPAL POINTS.— The position of the focal and principal points of collecting and dispersing lenses is important. If we assume that the light falls from the left, in the case of collecting lenses, the designated points occur in this order (Fig. 14, *a*): front focal point F, principal points H, H', rear focal point F'. In the case of a dispersing lens (Fig. 14, *b*): their relation is rear focal point F', principal points H, H', front focal point F. Always, however, $FH = H'F'$.

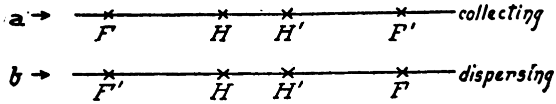

FIG. 14

FORMULA FOR THE FOCAL LENGTH OF AN INFINITELY THIN LENS.— It can be easily proved that a simple relation exists between the constants of an infinitely thin lens and its focal length f. If r_1 and r_2 are the radii of the two spherical surfaces, whose signs are determined by the curvature in respect to incident light, and if n is the index of refraction of the lens in air, then:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If we cannot neglect the thickness, another factor enters the equation. If we make r_1 and r_2 positive and negative in this equation, we obtain the forms of lenses shown in Fig. 13, and also the numerical value of the focal lengths, so that we can determine if these are positive, and the lens, therefore, collecting, or if they are negative and the lens dispersing.

EXAMPLE.— In order to elucidate the use of the formula for focal lengths by an example, let us assume that a convexo-concave meniscus is constructed of a glass with the index of refraction $n = 1.6$, and that the radii are 24 mm and 34 mm. Since both surfaces are convex to the incident light, we must make $r_1 = +24$, and $r_2 = +34$. Neglecting the thickness of the lens we find that $\frac{1}{f} = 0.6\left(\frac{1}{24} - \frac{1}{34}\right)$. Therefore, the focal length of the collecting meniscus is + 136 mm. The rear focal point lies 136 mm to the right of the principal point which coincides with the common vertex of the two surfaces, the front focus the same distance to the left.

If we have a bi-concave lens of the same glass with the radii 24 mm and 36 mm, we must make $r_1 = -24$ and $r_2 = +36$, and we find that the focal length is —24 mm, and thus the lens is dispersing. The rear focal point corresponding to a paraxial bundle incident from the left, therefore, lies on the *left* side of the dispersing lens at a distance of 24 mm from its vertex; correspondingly the front focus lies at the same distance on the right side of the lens. This peculiar position of the designated points in the case of negative lenses must be carefully considered in studying the action of a telephoto lens.

MEASURE OF CURVATURE AND STRENGTH.— If a lens surface is plane, r = infinity, and therefore $\frac{1}{r} = 0$. We call the reciprocal $\frac{1}{r} = \rho$ of the radius r the *measure of curvature* of the surface; the smaller the radius the greater the curvature and its measure. Correspondingly the influence which a lens, by the change of its convergence or divergence, exercises on a pencil of rays is expressed by the reciprocal $\frac{1}{f} = \phi$ of the focal length. While in the case of a plate with plane parallel surfaces, that is, of a lens with two infinitely great radii, there is no influence on the divergence of a bundle of rays, the collecting action of a lens is the greater, the smaller its focus and the smaller the image produced by it. This collecting or dispersing effect of a lens ϕ, equal to the reciprocal of the focus f, is, therefore, called the *strength of the lens*.

It would be useless to consider here more carefully the position and size of the images produced by a simple lens, since they form only a special case of what we are next to consider, the centered lens system consisting of any desired number of refracting surfaces.

CHAPTER VII

Centered Lens Systems

Limitation to Spherical Surfaces.— In the investigation of optical systems consisting of any desired number of surfaces, we shall limit ourselves to objectives which may find application as photographic systems. We shall, therefore, assume that we are dealing only with refracting surfaces of rotation, whose axes of symmetry coincide and form the common optical axis. In general, such surfaces are spherical, and consequently the axis of the system passes through the centers of the spheres. The whose system is, therefore, a plurality of co-axial or centered lenses. In accordance with photographic practice, the centered lens system is assumed to be in the air.

In previous chapters we have deduced certain phenomena of reflection and refraction from the laws governing these, following mainly earlier writers whose labors made possible the ascertainment of the position and size of images produced by centered systems, by their discovery of these laws. As first among these we must mention Gauss. These investigations refer only to the production of images only in the narrow cylindrical space surrounding the axis.

Optical Image Formation According to Abbe.— Collineation.— It was the great service of Abbe to have shown that all these stipulations as to the extent of the space involved, as well as to the form of the laws of reflection and refraction, which until his time investigators had imposed upon themselves in the investigation of optical images, are unnecessary limitations. In fact, we can deduce all the laws which give explanations of the size and position of optical images when we simply assume, without any stipulation as to a special law of reflection and refraction, that the formation of the image of one region of space in another region occurs by means of *rectilinear rays*. The particular way in which the optical imaging is accomplished is quite unimportant for this consideration. Such a relation between two regions of space, or as we commonly call them, the object and the image space, is usually designated as *collinear*.

When we determine that the imaging of the object space in the image space and *vice versa* shall occur by means of rectilinear rays, we only say something which is clearly evident to us in the practical use of a photographic objective and appears a matter of course. If we desire to limit ourselves to the actual case of a lens, we must make a second assumption. The greater the influence of an optical system, the more is a ray of a pencil parallel to the axis deviated and the more obliquely it reaches the image point on the axis, which is the focal point. This angle of inclination must be equal for either direction of incidence of the ray parallel to the axis, so that it is unimportant whether this first falls upon the front or the rear surface of an optical system, consisting of centered single lenses and limited by air on both sides.

From these two assumptions follow the laws of general optical imaging, without any assumption as to limited space, in the form in which we have learned them for reflection at a mirror with the limitation to paraxial space. In what manner, however, the formation of the image is actually to be effected can be determined only from the special laws of dioptrics.

POSITION OF OBJECT AND IMAGE.— In order to absolutely determine the position of object and image, we measure the distances x and x' of the object and image from the front and rear foci F and F' (Fig. 15). The light is supposed to move from left to right through the centered system.

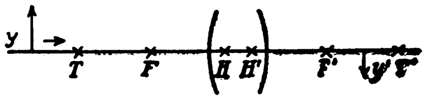

Fig. 15

Then the first general law of image formation for a centered lens system in the air is:

(A) $$xx' = -f^2$$

In this f is a constant of the system, the so-called *equivalent focus*, whose value, as we see, has nothing to do with a law of refraction, and in accordance with our hypothesis is independent of the direction of motion of the light.

SIZE OF OBJECT AND IMAGE.— If we further designate the size of the object by y, the size of the image by y', we obtain the second general equation:

(B) $$y' : y = f : x = x' : f$$

The greater the focal length f, the larger the images. *The focal length, therefore, determines the scale of enlargement or reduction.* If in equation (A) we give to the object distance x a large negative value, we obtain a small positive value for the image distance x'. If the object moves farther toward the right, then, because x decreases, the distance of the image from the rear focal point F' continually increases until it becomes equal to infinity, when $x = 0$ and the object stands at the front focal point F. If the object moves still nearer to the system, the image jumps from right infinity to left infinity, and then moves likewise from left to right toward the system. The directions of motion of object and image, are, therefore, always the same; such an image formation is called *positive* as opposed to the negative which we discovered in mirror reflection.

If we now note that to a positive value of $y' : y$ there corresponds an erect, to a negative value of $y' : y$ an inverted image, we can immediately draw the following conclusions from the two equations (A) and (B), which are rigidly true for any photographic objective of positive focus, that is a *collecting* system:

THE THREE PARTS OF THE OBJECT SPACE IN THE CASE OF A COLLECTING SYSTEM.— 1. The object y moves from left infinity to a point T which lies at a distance TF equal to the focal length f to the left of the front focal point F. At the same time the image y' moves from the rear focal point F' to a point T' whose distance $F'T'$ from the rear focal point F' is equal to f. The image is *real*, *inverted*, and *reduced;* the size of the image increases until when object and image are respectively at T and T' the image is of the same size as the object. Generally speaking, *photographic exposures* are made in this region.

2. The object moves on from T to the front focal point F. At the same time the image moves from T' to right infinity, and is *real*, *inverted*, but *enlarged*. This region is that of *photographic enlargements*.

3. The object moves from the front focal point F by the amount of the focal length f to H. The image jumps from right infinity to left infinity and then moves

until it reaches the point H', whose distance from the rear focal point F' is likewise equal to the focal length f. It is *virtual, erect,* and *enlarged,* as we have all sufficiently experienced when using a lens as a *magnifier.* When object and image are located at the two principal points H and H', they are equally large and both erect.

As we are dealing here only with the imaging of real objects, it is superfluous to investigate the size and position of the images when the object moves beyond the principal point H toward the right. The phenomena occurring in this case, can, however, be very easily deduced if we assume that the light is falling from the right, and, therefore, that the object and image space have changed places.

THE TWO PARTS OF THE OBJECT SPACE IN A DISPERSING SYSTEM.— If we assume that the rear focal length is negative, we have a dispersing system (Fig. 16), and will find that the following relations exist:

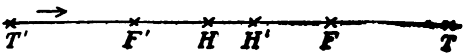

Fig. 16

1. The object moves from left infinity to the principal point H, meanwhile the image moves from the rear focal point F' to the principal point H', and is *erect* and *reduced.* At the principal points, H and H, the object and image are equally large and are both erect.

2. The object moves from the principal point H to the front focal point F. Meanwhile the image moves from the principal point H' to right infinity and is *erect* and *enlarged.*

Of especial importance for us is the introduction of a dispersing system in the path of a convergent pencil of rays, as is the case in the telephoto lens. We shall thoroughly investigate this case later, but will here remark, that negative lenses can sometimes give real images.

INFLUENCE OF THE MEDIUM SURROUNDING A LENS.— The fact is also worthy of remark, that a system which when surrounded by air is collecting becomes *dispersing* when we place it in a medium whose index of refraction is greater than its own. The reverse change is also experienced by a dispersing system placed in such a medium. Therefore, an air lens surrounded by glass, and having the shape of a bi-convex collecting lens, acts dispersively, and *vice versa*. We must remember this fact if we wish to understand the action of anastigmats composed of separated lenses.

SCALE OF THE IMAGE.— We will now apply the equations (A) and (B) to a series of examples for collecting systems, which occur very often in photographic practice. For this purpose we can advantageously introduce the conception of the scale of the image, which is very important, especially in reproduction work. Let us use the letter m to designate the relation of the object size to the image size, $y : y'$, terming it the *reduction* or *scale of the image*, then the equation (B) is transformed to:

$$x = mf,\ x' = f : m$$

When the size of object and image are the same, the distances x of the object and x' of the image from the focal points are, as we saw, equal to the focal length, and object and image lie near the lens at the principal points when the image is erect, and away from the lens when the image is inverted. If an object is to be photographed half the natural size, m equals 2, and correspondingly the object must be placed at a distance of two focal lengths from the front focal point, while the image moves half a focal length away from the focal point, etc.

Distance between Object and Image.— By help of the reduction number m we find a very simple expression for the distance E (Fig. 15), between object and image:

$$E = x + f + HH' + f + x'$$

If we substitute in this the values given above for x and x', it becomes:

$$E = 2f + mf + \frac{f}{m} + HH' = \frac{(m+1)^2}{m} f + HH'$$

Compared with the distance E the distance between the principal points is so small that we may disregard it, so that we finally get:

$$E = \frac{(m + 1)^2}{m} f.$$

Examples.— 1. With an objective of 36 cm focal length a square map 72 cm on the side is to be reduced to an image 12 cm square. Where must the map be placed, and what is the distance between map and image?

Here $f = 36$, $m = 6$; substituting, $E = \frac{7 \times 7 \times 36}{6} = 294$ cm, $x = 6 \times 36 = 216$ cm, $x' = 36 \div 6 = 6$ cm. The map must be placed at a distance $216 + 36 = 252$ cm from the front principal point, so that the image lies $6 + 36 = 42$ cm behind the rear principal point.

2. In a room 5 m long, ninefold enlargements have to be made. What is the maximum focal length of a lens?

If we leave 1 m free for manipulation we have left a distance E of 4 m between the plate and the enlarged image. Consequently:

$$400 = \frac{10 \times 10}{9} f \text{ and } f = 400 \frac{9}{10 \times 10} = 36 \text{ cm}.$$

The focal length can, therefore, not exceed 36 cm. If the available objective has only 27 cm focus, the distance of the plate from the rear principal point

$$= \frac{27}{9} + 27 = 30 \text{ cm},$$

that of the projection easel from the front principal point $= (27 \times 9) + 27 = 270$ cm.

3. A negative 1½ times natural size is to be made with an objective of 15 cm focal length. What is the distance of the object and the ground glass from the principal points?

The reduction number is $\frac{2}{3}$. Consequently

$$x = 15 \times \frac{2}{3} = 10 \text{ cm}. \quad x' = 15 \times \frac{3}{2} = 22.5 \text{ cm}.$$

Consequently the camera must be at a distance of $10 + 15 = 25$ cm from the front principal point, while the distance of the ground glass from the rear principal point is $22.5 + 15 = 37.5$ cm. The total distance of the ground glass from the object to be photographed is, therefore, $37.5 + 22.5 = 60$ cm, independent of the distance between the two principal points, which we neglect.

4. In a room of 4.5 m available length, enlargements are to be made with a lens of 15 cm focal length. How many times can one enlarge?

Here $E = 450$, $f = 15$ cm; the equation, therefore, is $E = 2f + mf + \frac{f}{m}$ or $450 = 30 + 15m + \frac{15}{m}$.

Clearing of fractions this becomes a quadratic equation: $15\ m^2 - 420\ m = -15$ or: $m^2 - 28\ m = -1$. Completing the equation, and extracting the square root, we get $m = 14 \pm \sqrt{195}$. Since the positive sign of the square root gives the value which we desire, and we can, with sufficient accuracy, say that the square root of 195 is equal to 14, the greatest enlargement m under the given conditions is twenty-eight times; consequently the negative is $\frac{15}{28} + 15 = 15.5$ cm, the easel $15 \times 28 + 15 = 435$ cm from the corresponding principal points. The sum of $435 + 15.5$ mm is sufficiently close to the number $E = 450$ cm.

5. The bellows of a hand camera measured from the rear principal point of the objective is 21 cm long, and the focal length of the lens 12 cm. What is the scale of the image in the extreme case?

The ground glass can be moved to a point $21 - 12 = 9$ cm from the rear focal point, therefore, $x' = 9$, and substituting in the equation $x' = \frac{f}{m}$, we find that $m = \frac{12}{9} = \frac{4}{3}$. Therefore, an object 4 cm long will appear 3 cm long on the ground glass when the bellows is fully drawn out. The distance of the object from the front principal point is then $12 \times \frac{4}{3} + 12 = 28$ cm. Object and ground glass are, therefore, $28 + 21 = 49$ cm apart.

6. In a picture taken with a lens of 36 cm focus a grown person appears to be 34 mm tall. At what distance was the camera from the man?

If the actual height of the person was 170 cm, which is about the average, the reduction number

$m = \frac{1700}{34} = 50$, consequently the person was $50 \times 36 = 1800$ cm from the front focal point, or 1836 cm from the front principal. Correspondingly, the ground glass must be moved about $\frac{360}{50} = 7.2$ mm farther away from the lens than the point of focus on distant objects.

VERTEX DISTANCE AND FOCAL LENGTH.— In these examples we have emphasized that object and image distance are always to be measured from the corresponding principal points, and we have done this in order to avoid the perplexing confusion of these distances with the corresponding measurements from the nearest lens surface. These latter distances, which are called *vertex distances*, depend on the special dioptric peculiarities of the lens system, and coincide with the distances from the principal points, only when these principal points lie on the surface of the lens. This is, for instance, the case with a plano-convex collecting lens; here the rear principal point coincides with the vertex of the curved surface.

The same thing is true in the case of all lens systems of infinitely small thickness; here the distance of the focal point from the nearest lens surface is equal to the equivalent focus, and the vertex distance is equal to the distance from the focal point to the principal point.

DATA ON FOCAL LENGTH AND PRINCIPAL POINTS.— As the thickness of the lens can never be neglected in the case of photographic objectives, it would be desirable if all optical establishments would furnish with their objectives exact data as to the equivalent focal length and the position of the principal points with respect to a permanent part of the objective, preferably the flange which screws into the camera. Every lens should be submitted to an exact test on the test chart before it leaves the factory, and it would, therefore, require little labor to determine these constants. If such data were given, the makers would avoid an enormous number of questions from purchasers of lenses.

PARALLELISM OF THE RAYS THROUGH PRINCIPAL POINTS.— The geometrical construction of the images produced by an optical system is not of great importance to us, and it is, therefore, unnecessary to go more deeply into this matter. It is well, however, to call attention to one property of the principal points of photographic lenses. If we draw a ray in the object space at any angle to the axis, and passing through the front principal point, the conjugate ray passing through the rear principal point is *parallel* to it. Therefore, object and image subtend the same angle when seen from the principal points.

THE RAY THROUGH THE CENTER OF AN INFINITELY THIN LENS.— This property is immediately explained if we assume the lens system to be infinitely thin, in which case the principal points coincide with the vertices of the refractive surfaces. The whole system is thus reduced to its central point, and the path of the rays corresponds to their path through a plate with plane parallel sides. Since the two media through which the ray passes before and after refraction are the same, the ray must leave the system at the same angle to the axis that it made before its entrance. Therefore, every ray goes unrefracted through the central point of an infinitely thin lens.

COMBINATIONS OF SEVERAL SYSTEMS.— We can easily find the general relations for the combination of several systems to a single one following the laws of image formation through a system. It will be sufficient if we go fully into two cases, which are of importance in the consideration of photographic lenses. One is the combination of an objective and a dispersing lens to form a *telephoto lens;* to this we shall give a special chapter in the sequel.

DOUBLE OBJECTIVES AND SUMMATION FORMULA.—The other case deals with the combination of two similarly constructed objectives to form a *symmetrical combination*, as we are accustomed to do with the components of lens sets. As it is only necessary for us to obtain an approximate value for the focal length of such a combined system, we will assume that the thickness of the two lenses I and II (Fig. 17) whose focal lengths are f_1 and f_2 and whose axial distance is d, can be neglected. In this case the general equation takes a very simple form. The focal length f of the double objective is found from the equation: $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$.

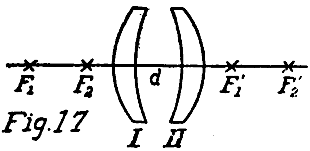

Fig. 17

In order to find the sum of the effects of two single lenses, we use, therefore, the reciprocal of the focus, that is, the strength, of each. If the distance d of the two lenses is zero, the strength of the whole system is equal to the sum of the single strengths. As the distance of the lenses increases, this sum is *diminished* by the product of the strengths and the distance. If, for instance, we take $f_1 = +20$, $f_2 = +10$, the focus f when $d = 0$, becomes $+6.67$, when $d = +10$, $+10$, when $d = 20$, $+20$, when $d = 30$, infinity, when $d = 40$, -20, etc.

THE TELESCOPIC SYSTEM.— Therefore, the farther the two lenses are separated from one another, the larger becomes the focal length of the double objective, until finally at a distance equal to the sum of the foci it reaches infinity. An optical system of infinite focus is called *telescopic*. In such a system, to an axial ray in the object space there always corresponds an axial ray in the image space. Telescopic systems are practically used in telescopes, of which the well-known Galilean telescope is of especial interest photographically because it forms one sort of telephoto lens.

ENLARGING SYSTEMS.— If the distance of the lenses is still further increased, the equivalent focus of the whole system becomes negative, that is, the real images produced by it are no longer inverted, but erect, and corresponding to the decrease of focal length become smaller as the two lenses are separated more and more. In certain astrophotographic work we use a second positive lens of this kind in order that the image produced by the objective proper may be photographed by the telescope itself on a considerably enlarged scale.

FOCUS OF A DOUBLE OBJECTIVE COMPARED WITH THAT OF ITS COMPONENTS.— In photographic practice this consideration is important, inasmuch as it shows that when we combine two similar lenses to form a double objective, its focal length is *larger* than half of the focus of each component, by an amount which increases with the separation of the lenses. Therefore, the more compact the construction of a double lens, the shorter is its focal length when its parts have the same foci. Compared with anastigmatic double objectives the aplanats, likewise composed of similar components, have very long mounts.

Equivalent Focus.— To the two lenses we may add still another, and the focal length of the whole system can always be calculated from the focal lengths and distances of the single lenses. This is true not only for infinitely thin but also for thick lenses of any size. We can, therefore, replace the action of any desired number of optical systems by that of a single infinitely thin lens, which as far as the size and position of the image is concerned, is exactly equivalent to all of the lenses used. This is what we mean when we speak of the *equivalent focus* of an optical system.

CHAPTER VIII

The Actual Realization of Optical Image Formation

Validity of the General Laws of Image Formation.— We have previously seen that from the assumption of image formation by rectilinear rays and of a uniform medium enclosing the optical system, we can draw conclusions of a quite general nature as to the position and size of the image. We made no assumptions of any kind as to the dimensions of the object and image as compared with the focal length. It appears, however, in accordance with our conclusions as to the path of the rays in a simple lens, that the formation of a point-image by a homocentric light pencil is only possible when it moves in the infinitely small cylindrical space surrounding the axis and when the image formation occurs exclusively by means of paraxial rays.

Examples of the Union of Rays in a Point.— If we were actually confined to this narrow *Gaussian space* surrounding the axis, and if only an absolute point-image were available for photographic purposes, the science could hardly exist. The limited value of an image of this character can be seen if we, for example, stop down a lens of 12 cm (5 in.) focal length so far that a free aperture of only 2 mm (1-12 in.) is used. With such an effective aperture of *f*: 60 very little can be done in photography, and this science must have remained in a very humble status if it had not fortunately been possible by thoughtful combinations of lens elements considerably to enlarge the region of image formation by a centered system. As a matter of fact, it became necessary to sacrifice the condition of *point-image* formation; this, however, can be safely done in the case of photographic objectives, because the images obtained by them are not usually subjected to the serious test of a considerable subsequent enlargement. Working on these lines the last decades have brought forth marvelous advances in the construction of photographic objectives which have wide, flat fields with proportionately large openings. We shall see that nevertheless we are able to obtain images consisting almost of points.

Difference between Photographic and Visual Systems.— Systems which are used as objectives in instruments intended for visual observation must satisfy far more severe conditions than photographic objectives, because the images produced by them undergo a strong ocular enlargement. This is the reason why, in order to produce a satisfactory collection of rays in the case of telescope and microscope objectives, we must confine ourselves to obtaining an image merely of an axial point and its immediate surroundings. We must also consider that the reproduction of the photographic image is not on a structureless surface, but that the light-sensitive film has a fine grain which enables us to dispense with a collection of rays as exact as in the case of the other systems just mentioned.

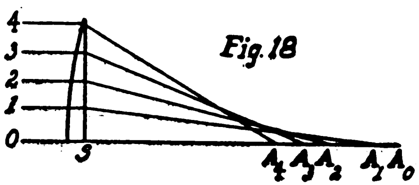

Fig. 18

In the following sections we shall investigate the phenomena apparent to us when we abandon the paraxial space and, instead of the ideal, consider an actual image formation by means of straight rays.

A.— Spherical Aberration of an Axial Point

As object let us assume a luminous ~~point~~ pencil which lies on the optical axis and produces a real image. Since in photographic exposures we are mostly concerned with distant objects, we will assume that the luminous point is infinitely distant, so that the pencil of rays emitted by it is parallel to the axis. Since this pencil is symmetrical about this, we will confine ourselves to the rays which travel in a single plane containing the axis.

Incidence-height.— Each of these rays falls upon the front surface of the lens (Fig. 18), at a different *incidence-height* h above the axis. While the paraxial ray with the incidence-height $h = 0$ is so refracted that after emergence from the lens system it cuts the axis at the focal point A_0, the incident rays at the finite heights h_1, h_2, h_3, have other distances of intersection SA_1, SA_2, SA_3, so that point-image formation by means of a homocentric pencil has ceased. The distance of the point of intersection of a ray incident at a finite height above the axis from the paraxial image point is called *spherical aberration*, or *aberration along the optical axis*. The actual relation of the points A_0, A_1, A_2, A_3 to each other cannot be stated in general terms.

Spherical Under-correction.— If, however, we assume that the optical system consists only of a collecting lens, we find that the distance of intersection becomes smaller the greater the incidence-height of the rays above the axis. Therefore, in accordance with our system of notation, the point of intersection A_2, of a ray with greater incidence-height h_2, lies at a negative distance A_2A_1, from the point of intersection A_1 of a ray with a smaller incidence-height h_1. The difference A_1A_0, A_2A_1, etc., is designated as *spherical under-correction*. It is a property of all collecting lenses. We can easily satisfy ourselves of this if we place such a lens in a camera. If we cover the greater part of the lens, exclusive of a small zone around the edge, with a round screen held in front of it, the ground glass must be brought nearer to the lens than when we focus with the same lens sharply stopped down.

MINIMUM DEVIATION.— We are not able, by changing the radii of a collecting lens, while retaining the same focal length, to remove the spherical under-correction. There is, however, a lens form with which the spherical aberration is at a minimum; that is, when the radii have the ratio to each other of one to six. This form is very near that of a plano-convex lens whose convex surface is turned toward the incident light. For this reason the two condenser lenses of a projection lantern are placed so that the plane surfaces are on the outside. The paths of the rays between the two plano-convex lenses are practically parallel, and thus the imaging of the source of light in the plane of the diaphragm of the projecting lens is produced with the least possible spherical aberration.

SPHERICAL OVER-CORRECTION.— The same observations can be applied to dispersing lenses. Here we find a *spherical over-correction;* that is, the axial points of intersection of the single rays follow each other in the direction of the incident light as the incidence-height above the axis increases.

REMOVAL OF SPHERICAL ABERRATION.— Hence it follows that we can produce a collecting system *free from spherical aberration* by combining a collecting and a dispersing lens. The forms of the lenses, however, as well as the materials from which they are made, must be so chosen that, in spite of the removal of the spherical aberration, a collecting action still remains.

Let us assume that a system has been so corrected, through the proper combination of collecting and dispersing lenses, that the axial point of intersection of a ray incident at the height h coincides with the focal point of the paraxial rays; the system is then indeed spherically corrected for the aperture $2h$, but on the other hand the position of the point of intersection of all other rays is undetermined. In most cases rays incident at lesser heights than h intersect the axis after refraction *nearer* the objective, while the points of intersection of rays incident at greater heights will have a more distant point of intersection than the actual focal point. How individual photographic objectives will behave in this particular only trigonometric calculations can show.

GRAPHICAL REPRESENTATION OF SPHERICAL ABERRATION.— The results of such calculations can be easily understood when they are graphically reproduced. We may usefully follow in the construction of the corresponding curves the method of M. von Rohr. If A_0 to A_4 (Fig. 19) are the image points corresponding to the incidence-heights 0 to 4 of an optical system, we represent their distance from the paraxial focal point A_0 as a function of the incidence-height h; in a rectangular co-ordinate system we take the spherical aberration along the axis as the abscissa and the corresponding height as ordinate. If we prolong the incident rays to their intersections with the perpendiculars erected on the axis at the focal points belonging to each ray, we obtain a curve by joining all these points of intersection.

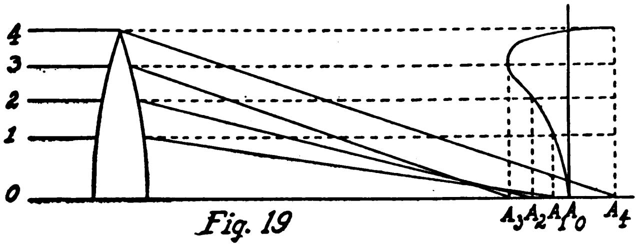

Fig. 19

The zero line or ordinate axis is the perpendicular to the axis at the focal point A_0, and the perpendicular distances from this, the abscissae, are the values of the spherical aberration. The part of the curve lying to the left of the ordinate axis represents the spherical under-correction and that to the right, the spherical over-correction. The scale on which M. von Rohr drew his curves is well chosen. Assuming a focal length of 100 mm, in his figures an incidence-height of 1 mm (ordinate) is plotted as 4 mm and a spherical aberration of 1 mm (abscissa) as 20 mm. It would be extremely desirable if this scale were preserved in all publications on the conditions of correction of photographic objectives.

Spherical Zones.— We perceive from the graphical representation that in an objective which is spherically corrected for a certain aperture, a point-image of the object point is exactly attained only when the curve coincides with the ordinate axis in its whole extent, and is, therefore, a straight line. In all cases which concern us, however, there is present a deviation which we call the *spherical zones* or *zonal error of spherical aberration*. The purpose of every satisfactory spherical correction must be to diminish these zonal errors as far as possible. Their influence can be perceived in different ways.

Circles of Confusion.— When the objective is spherically corrected for the incidence-height h (Fig. 20), the corresponding ray after refraction passes through the focal point A_0. If, however, we place the ground glass in the plane perpendicular to the axis at A_0, the rays incident at a lesser height h_1 intersect the plane of the ground glass in a *circle of confusion*, whose diameter is B_0C_0. The occurrence of this circle of light causes here, as in the case of the pinhole camera, a degradation of the image. If we move the ground glass to the plane perpendicular at A_1, the point of intersection of the inner ray with the axis, we again find a circle of confusion with the diameter B_1C_1 produced by the ray incident at the height h. In our drawing there lies between these two circles of confusion a third, BC, smaller than either, and consequently producing a lesser *indistinctness*.

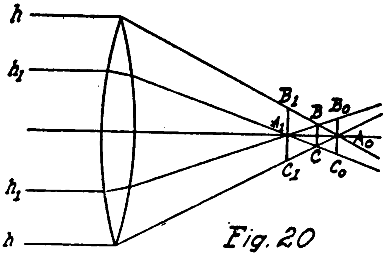

Fig. 20

Collection of Rays.— The point of most satisfactory *collection of rays*, is, therefore, to be sought not where the paraxial ray intersects the edge ray, but rather where the influence of the circles of confusion is smallest. The position of this most favorable location, however, cannot be deduced from the principles of geometrical optics. If we stop down a lens so that the exactly corrected edge ray and its neighboring rays are eliminated, the optical effect of the pencil is displaced toward the left in Fig. 19. We must, therefore, move the ground glass toward the lens in order to find the new position of sharpest focus; or if we allow the ground glass to remain where it was, suffer a loss in sharpness.

Focal Difference.— This is the well-known phenomenon of *focal difference*, which makes itself apparent when the zonal error of spherical aberration is considerable, so that the circles of confusion begin to approach the limit of perceptibility. The small size of the spherical zonal error in the newest anastigmats has led to a complete disappearance of this error in focusing. In older objectives, however, which are not free from it, one must take care always to focus with the stop with which the exposure is to be made.

Spherical Zones and Size of Image.— From Fig. 20 it is apparent that the circles of confusion are greater the wider the opening of the lens with the same focus. Since, however, the indistinctness of the image produced by spherical aberration, which still remains below the limit of perceptibility, and, therefore, may be overlooked, is independent of whether the actual aperture be large or small, we must take care, in objectives of large aperture, that the zonal error remains considerably smaller than in the case of objectives of smaller aperture. If we study the published curves of spherical aberration, we actually find that the zonal errors in wide-angle lenses are much larger than in large-aperture anastigmats.

Focal Length and Relative Aperture.— In general, therefore, the amount of the zonal error is dependent on the ratio of the actual aperture to the focal length; that is, on the relative aperture of the objective. As we likewise perceive from Fig. 20, the zonal error increases when we enlarge, while keeping the same relative aperture, the size of the image or the focal length; therefore, we cannot go above a certain focal length without overstepping the limit of permissible indistinctness. On the other hand, we can diminish the diameter of the circles of confusion by decreasing the angle at which the edge rays, after refraction, cut the axis; that is, by reducing the effective aperture. This explains the fact that in general the effective apertures of the objectives of a series *decrease* with their focal length; but this decrease is smaller, the better the collection of rays in the manner we have described.

SPHERICAL ABERRATION FOR OTHER DISTANCES OF THE OBJECT.—The previous statements are true in equal degree for the imaging of any point at any desired distance on the axis. When the spherical correction is carried out for any given distance, it remains practically as satisfactory when the object is moved along the optical axis within the limits which are practical for photographic exposures. The stability of the correction is greater, the smaller the relative aperture of the optical system. Since this, however, in the case of photographic objectives, lies far within the limit whose overstepping would influence the perfection of the image through changing the object distance, as is the case with microscopic objectives, we may ignore this factor and conclude that an objective corrected for great distances gives an equally good image for proportionately small distances.

B.—Aberrations in the Imaging of a Small Surface near the Axis

Transition from an Axial Point to a small Surface near the Axis.— By a considerable reduction of the spherical aberration it is possible to form the image of a point on the axis by means of a large pencil of rays with an approximation which is completely satisfactory for photographic purposes. This result by itself, however, is not of much value, for the greatest importance is placed, in photographic work, on having the largest possible field of view. We must, therefore, investigate and see if it is not possible to obtain wider limits of image formation.

A calculation shows that our hopes can be fulfilled only to a very modest degree. Even when the luminous axial point is imaged entirely without zonal aberration at the full opening of the objective, there is no possibility of a point-image formation even for points quite close to the axis in a plane perpendicular to it. The circles of confusion in some circumstances may be almost as large as the distance of the object point from the axis in this plane. Even although we are still in the paraxial space, as far as distance perpendicular to the axis is concerned, there is no longer a point-image formation. Only when the path of the rays from the spherically-corrected axial point fulfils a certain condition, will parts of the perpendicular plane infinitely near this point be imaged as points. This condition is, as was first shown by Abbe, that the infinitely small object must be imaged with the same enlargement by all parts of the objective, no matter at what distance from the axis they are.

Abbe's Sine Condition.— The mathematical expression for this requirement is the well-known *sine* *condition of Abbe.* If u is the angle of inclination to the axis of a ray emitted by the axial point, and u' that of the ray after its refraction, the ratio $\sin u' : \sin u$ must be constant for all rays which pass through the objective and must be equal to the linear enlargement. The sine condition assumes a special form for the special case of a point infinitely distant, which is almost always the case in photography. Then the quotient $h : \sin u'$ must be constant for all incident rays parallel to the axis at a given incidence-height h, and must be equal to the equivalent focus f. The differences of these quotients from f, therefore, represent the deviations from the sine condition.

Fulfilment of the Sine Condition with Spherical Zones.— As we have seen, the spherical aberration cannot be absolutely eliminated for all the rays from an axial point, but still the sine condition retains its importance. As experience has shown, it is in this case necessary to bring the curve of spherical aberration as nearly as possible into coincidence with that of the deviations $\frac{h}{\sin u'} - f$ from the sine condition. For perspicuity the two curves are usually plotted on one co-ordinate drawing, according to the process of M. von Rohr. It is, however, more useful to draw beside the curve of spherical aberration, the difference of this from the deviations from the sine condition. This new curve of differences must then fall as nearly as possible on the ordinate axis, or at least cut it in a point near the edge of the lens.

Brilliance.— If this is the case, and the spherical correction is good, the central point of the image and its immediate neighborhood are seen on the ground glass with the characteristic clearness and sharp contrast between adjacent light and dark portions of the object which we designate as *brilliance*. If the sine condition is not fulfilled with sufficient precision, the middle of the image appears dull and foggy, no matter how good the spherical correction may be.

APLANATIC POINTS.—Conjugate points for which the spherical correction is satisfactory and the sine condition is fulfilled, were called by Abbe *aplanatic*, and in such a case the photographic objective is aplanatic as regards this pair of points. It is necessary to avoid confusing this term with the name of the well-known double objective of Steinheil.

IMPOSSIBILITY OF IMAGING A PORTION OF SPACE.—The fact is of great importance to us, that it is impossible, by any combination of available elements, to produce by means of an optical system with finite aperture a sharp image of an *infinitely small portion of space*. We must content ourselves merely with being able to form a sharp image of an infinitely small portion of a flat surface by a pencil of rays of finite aperture.

C.— ASTIGMATISM

THE PRINCIPAL RAY.— We now come to the imaging of extended plane objects by means of pencils of moderate aperture,— as, for instance, in the photographing of a landscape with a very small diaphragm opening. While in the previous sections we accomplished ideal image-formation by assuming for the imaging pencil a finite aperture, we will now assume this to be infinitely small, but, on the other hand, give the luminous point a position at a finite distance from the axis. Thus, instead of a cone of light falling symmetrically upon the objective, we obtain one obliquely incident, whose base is determined by the diaphragm opening of the lens. The central line of the small cone of light, passing through the center of the aperture, is called its *principal ray*. While, however, in our previous considerations the principal ray of every pencil coincided with the optical axis and this was the line of symmetry of the pencil, such is no longer the case. Since the pencil falls obliquely on the refracting surfaces, the course of the rays in the different planes which contain the principal ray has become dissimilar. We must, therefore, abandon the limitation of the investigation of the path of the ray in only one plane, and consider its *position in space* in general.

THE TWO PRINCIPAL PLANES.— In the case of an oblique pencil we must distinguish two special directions. One is given by the plane which we pass through the principal ray of the oblique pencil and the optical axis, and choose as the plane of the drawing. This is designated as *the first, meridional, or tangential principal plane*. Perpendicular to this stands the second chosen direction, which is called *the second, equatorial, or sagittal principal plane*. The wedge-shaped sections of the refracted, oblique, infinitely small pencils which are formed by these two planes are filled by the rays in a peculiar manner which may be easily understood by study of the drawing made by M. von Rohr* (Fig. 21).

ASTIGMATIC DEFORMATION.— Let us assume that the light is propagated from left to right, and emerges from the last refracting surface in the shape of a pencil of circular cross-section $A'BAB'$, and that the two lines $A'A$ (solid) and $B'B$ (dotted) represent respectively

*"Theorie und Geschichte des photographischen Objectives," von M von Rohr. Berlin 1899, page 39.

the meridional and the equatorial principal planes. For the sake of clearness, below the actual construction of the path of the rays, the cross-sections perpendicular

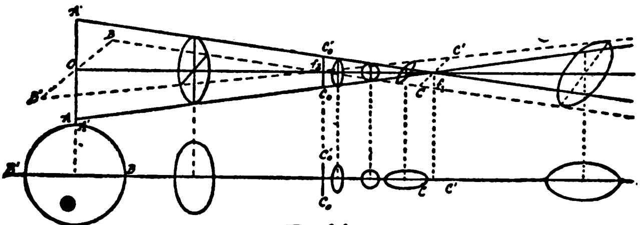

Fig 21

to the plane of the drawing are represented for the most important places in the course of the pencil. As we pass with the light toward the right, the pencil contracts more rapidly in its breadth than in its height, so that the cross-section assumes the form of an ellipse whose minor axis, in the equatorial principal plane, becomes shorter and shorter in proportion to the longer axis lying in the meridional principal plane. Finally, the short axis of the ellipse contracts to a point f_2, and all the rays of the infinitely narrow pencil pass through a small focal line $C_0'C_0$, lying in the meridional plane, which is produced by the rays of the equatorial pencil which cross each other in it. The pencil contracts still further. Its cross-section again becomes elliptical with the longer axis lying in the meridional plane. Then comes a place of minimum contraction in which the cross-section is a circle; beyond this the pencil is again flattened so that the longer axis of its elliptical cross-section lies in the equatorial plane. At the point f_1 all the meridional rays intersect, and thus a small focal line $C'C$ is formed, which lies in the equatorial plane. Then the pencil spreads again and attains an elliptical cross-section, the minor axis of which lies in the meridional plane. The two focal lines which are formed at the focal points, always stand perpendicular to each other.

This peculiar splitting of the infinitely small oblique pencil is called *astigmatic deformation.* It is characterized by the occurrence of two astigmatic focal lines in the place of a focal point, a phenomenon designated as *astigmatism.*

FOCUSING ON A CROSS.— We are now in the position to understand the phenomena which occur when we endeavor to focus sharply on an object composed of simple lines lying outside of the axis,— for instance, on a cross, whose two arms lie in the principal planes, or vertically and horizontally (Fig. 22). In the formation of an image of the vertical arm, each point in its whole length is represented by an image consisting of a straight line instead of a point.

Fig. 22

At f_2, the focal point of the equatorial rays, these small lines stand vertically, so that they partially cover each other and form a straight line, while at f_1, the focal point of the meridional rays, they have a horizontal position, and therefore form a blurred image. On the other hand, the horizontal arm of the cross is so imaged, that in f_2 the partial focal lines lie perpendicularly side by side and produce unsharpness, while through their horizontal overlapping at f_1 the image appears sharp.

If, opening the camera, we move the ground glass slowly away from the lens, at a certain distance the vertical arm of the cross appears sharp, while the horizontal remains blurred. This is the focal point of the equatorial rays. As the distance is increased, the vertical arm loses in sharpness, while the horizontal increases, until at the focal point of the meridional rays, the latter appears sharpest. Beyond this, both arms are blurred. The difference of the bellows length in these two positions is usually called the *astigmatic difference*. If we revolve the arms of the cross through an angle of 45°, none of them appear sharp in any position, since they no longer fall in the principal planes. Similar phenomena appear if we take as the object to be focused concentric circles and their radii.

Dependence of the Astigmatism on the Angle of the Principal Ray with the Axis.— If we move the luminous point nearer to the optic axis, the astigmatic image-points move closer together, until finally, when the point lies upon the axis, the two image-points coincide. In general, therefore, the astigmatism increases with the distance of the object from the axis, and coincidently we note an equal increase in the unsharpness on the ground glass placed at the axial focal point.

Astigmatic Curves and Image Surfaces.— Using the different angles which the principal ray in the object space may make with the axis, we obtain a series of astigmatic image-points for the meridional and equatorial rays, which, when they are joined to each other, give the two *astigmatic curves*. If we revolve these around the optical axis, we obtain the *astigmatic image surfaces*, which are symmetrical to the axis. At their point of intersection with the axis, these surfaces are perpendicular to it, and tangent to each other and the plane of the ground glass.

Graphic Representation.— In order to clearly represent the degree of astigmatic correction of a photographic objective, a graphic representation is desirable. For this purpose it is advantageous to choose an infinitely distant point emitting rays inclined to the axis, but parallel to the principal ray, which are incident in an infinitely small section. The influence of distance on the character of the astigmatic image surfaces is negligible. In accordance with the scheme of M. von Rohr, we take a plane passing through the principal ray as the projection plane, and plot on a sheet of co-ordinate paper the deviation of every astigmatic image-point obtained by measuring the movement of the ground glass as an abscissa, representing each difference of 1 mm by 4 mm on the drawing As ordinates we take the angle of inclination of the principal ray to the axis in the object space, and for each 10° of inclination we plot 24 mm.

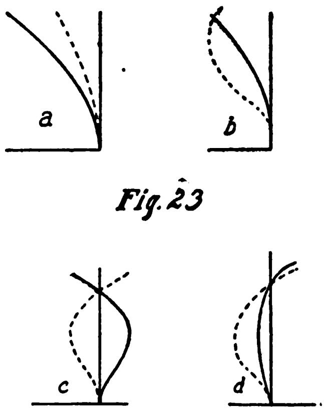

Fig. 23

The general position of the astigmatic curves of a collecting lens is represented in Fig. 23, a. The meridional image-points (dotted) lie farther from the lens than the equatorial, and the distance increases continually as the angle of inclination becomes greater than zero. In Fig. 23, b, the two curves intersect at a certain distance from the axis. At a certain angle of inclination, therefore, the astigmatism is corrected and the objective is *anastigmatic*. This is also true of the lenses represented in Fig. 23, c, and 23, d, but here the anastigmatic point falls on the plane of the ground glass.

ANASTIGMATIC ZONES.— Here also, when we have succeeded in bringing the two curves to intersection, and thus producing an astigmatic point, we must investigate the course of the curves between this point and their axial intersection, as well as beyond the anastigmatic point. These deviations of the two curves we call *anastigmatic zones or zonal errors*. The smaller they are, the less is the unsharpness caused by astigmatism. Their allowable amount is dependent on the relative aperture, since they are less important in proportion to the amount the objective is stopped down.

D.— COMA

Our consideration of astigmatism was based on the assumption that the pencil of rays surrounding the principal ray had a very small diameter. This pure astigmatic image-formation, if we can use this expression, actually exists in a manner similar to the Gaussian image-formation in the narrow cylindrical space surrounding the optical axis. The principal ray takes the place of the axis. This kind of image-formation actually occurs when we stop down the lens strongly. It does not help us practically, however, for we demand from the photographic objective, in opposition to the telescope objective, a wide angle of view with considerable brightness.

DIFFERENCE BETWEEN THE TWO PRINCIPAL PLANES IN THE CASE OF A FINITE CROSS-SECTION OF THE

OBLIQUE PENCIL.— If we now give to the pencil surrounding the principal ray a finite diameter, there occur deviations in the imaging of the two astigmatic image-lines, which we designate as *spherical aberrations of the extra-axial pencil*. We must treat separately the meridional and the equatorial rays corresponding to the two principal planes. The latter rays are unimportant. Since the course of the rays in the equatorial plane on the two sides of the principal ray is wholly symmetrical to this, the aberrations which are present are harmless in photographic lenses.

The course of the rays in the meridional section is essentially different and is represented in Fig. 24. In

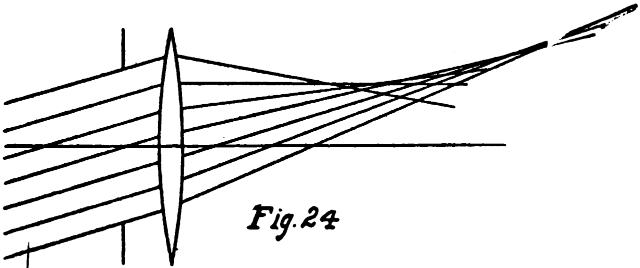

Fig. 24

this case there is no kind of symmetry. The ray falling on the upper edge of the objective is most strongly refracted and, in general, corresponding in character to a spherical under-correction, cuts its neighboring ray at a shorter distance than this its next neighbor. We thus obtain instead of an image point, a *caustic curve* which produces a very serious unsharpness on the ground glass.

COMA IN THE MERIDIONAL SECTION.— In contrast to the equatorial rays, therefore, the meridional rays give a completely unsymmetrical distribution of the light, which manifests itself by the occurrence of oval, cometlike defects. This is the so-called *coma*, the complete removal of which should be absolutely insisted on in every photographic objective, but which shows itself even in some fairly corrected systems by a lack of sharpness of the image.

Coma and the Sine Condition.— If it is a question only of small inclinations of the pencil to the axis, the coma is removed as soon as the sine condition is fulfilled. Therefore, for the same aperture at which axial spherical aberration is removed, the spherical aberration of pencils of small obliquity also disappears.

Also, for *any given inclination* of pencils of *finite aperture*, the spherical aberration and likewise coma appear to be completely removed in anastigmats, as soon as beside the axial correction the sine condition is also fulfilled as exactly as possible. At least the anastigmats calculated by the author have always shown, when practically tested, that if these two conditions were fulfilled, the image field in its whole extent was completely free from coma.

E.— Curvature of Field

By removal of astigmatism it has become possible to form a point-image of a plane object by means of infinitely narrow pencils within a certain image field whose aperture can be finite in case the spherical aberration remains unimportant in the two principal planes. We tacitly assume that the spherical zones are very small. In general, we shall then have to deal with a relation of the astigmatic curves similar to that shown in Fig. 23, b.

The Sharp Image must be Plane.— In order to obtain a sharp image in this case we would have to give the ground glass a concave form, as has been proposed and tried in the case of projection screens. That this is impossible for photographic purposes is apparent, because we always have to deal with plane surfaces for focusing the image and as a support for the light-sensitive film.

ANASTIGMATIC FLATTENING OF THE FIELD.—This deviation of the image surface from a plane is called *curvature of the field*. It shows itself, when we focus on the ground glass, by unsharpness increasing from the middle of the image to the edges. To remove this is the purpose of the *anastigmatic flattening of the field* which occurs when the anastigmatic point lies in the plane of the ground glass (Fig. 23, c). Before the introduction of objectives with anastigmatic flattening of the field it was the custom to obviate this defect in many cases, for instance, in group pictures, by arranging the parts of the object on the arc of a circle concave to the objective, and thus displacing their images on the ground glass. Truly, however, modern photography dates its beginning from the day when, for the first time, an objective with anastigmat flattening of the image, or briefly an anastigmat, was to be obtained.

ZONES IN THE ANASTIGMATIC FLATTENING OF THE FIELD.— If, however, we have brought the anastigmatic point into the plane perpendicular to the axis at the axial image-point, it is not yet necessarily true that the part of the image lying between these two points must appear sharp on the ground glass. It is instead probable, as Fig. 23, d, shows, that there will occur zonal errors in the flattening of the field which will give rise to the formation of circles of confusion within the limit of visibility. If care is not taken that the anastigmatic image lies sufficiently near to the ground glass in all parts of the field which are to be used for the exposure, there will appear unsharp zones in the image which can only be removed by stopping down. The greater the aperture of the objective, the nearer the plane of the ground glass must the anastigmatic curves lie. Unsharp zones can also appear even if the median line between the two curves corresponds approximately with the plane of the ground glass, but at the same time the astigmatism goes beyond permissible limits. This also must be avoided in every anastigmat.

Extent of the Flat Field.— For how wide a field astigmatism and curvature of field must be thoroughly corrected, depends in general on the relative aperture of the objective. Systems of wide aperture, as a rule, cover smaller plates than narrow angles, but this comparison can only be made between anastigmats. It is most useful to calculate the anastigmatic correction not for the edge of the field, but for a point of lesser inclination to the principal ray, in order to minimize the zones in the middle of the field as far as possible, so that by stopping down a considerably larger field can be covered.

That in the case of difficult exposures of a plane object perpendicular to the axis, one must not focus exclusively on the middle of the field, but seek to obtain a compromise between the different parts of the field, is well known.

CHAPTER IX

Limitation of the Rays

In the previous chapters we have often introduced the idea of stopping down. We will now consider, although briefly, the influence of the introduction of a diaphragm on the course of the rays. We cannot, however, attempt to include a comprehensive treatment of Abbe's theory of the limitation of rays.

The Diaphragm.— Every optical system must have some sort of physical bounds. In photographic objectives this occurs through the introduction of a circular aperture whose center lies in the optical axis. This is the *diaphragm*. By the introduction of this into the course of the light rays, a definite pencil is separated, which, according to its area of cross-section, produces a more or less bright image. In order to be able to vary the amount of illumination for various purposes, we use diaphragms of different diameters, which can be either gradually changed (iris diaphragms), or varied by definite steps (sets of stops, or in case the apertures are arranged in *one* piece of metal, the rotating stop). For certain special tasks in photo-engraving, there may be arranged in *one* diaphragm several apertures, whose position and shape must correspond to the structure of the screen. These are *Grebe's coincidence diaphragms*.

The *stop* (B, Fig. 25), as it is also called, is placed between the members of the objective in compound objectives. The principal rays of the pencils inclined to the optical axis go through its center, and therefore every principal ray, *before* its refraction by the front member of the objective, L_1, must tend toward a point which forms the center of the virtual image of the stop B produced by the lens L_1. In exactly the same way every principal ray, *after* its refraction by the rear lens L_2, appears to come from the center of the virtual image of the stop B produced by the lens L_2.

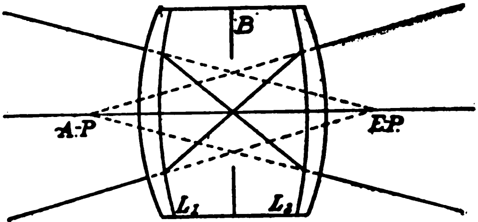

Fig. 25

Pupils.— These two centers of the virtual images are called by Abbe the *pupils;* the incident principal ray is directed toward the center of the *entrance-pupil* (E.-P., Fig. 26), while the principal ray, after its emergence, appears to come from the center of the *exit-pupil* (A.-P.). The entrance- and exit-pupils are, therefore, the bases of the incident and emergent light pencils whose apices lie at the object point and the image point. Instead of the word diaphragm, one often uses *iris* in comparison with the construction of the human eye.

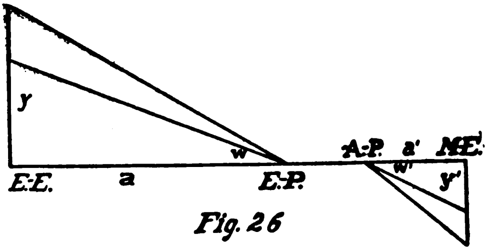

Fig. 26

Objectives with Front Stops.— In the case of objectives with stops in front of the lens (often called *landscape lenses*), as, for instance, the rear half of a symmetrical objective, the stop coincides with the entrance-pupil. The exit-pupil is here virtual. We shall later see how the external cylindrical surfaces of photographic objectives, which also form stops, act on the course of the ray.

Solid Objects.— In point-image formation an image plane corresponds to every object plane, and it is not possible to form a sharp image of even an infinitely small portion of space in *one* plane. We have, however, in scientific, technical, and artistic photography, almost always to deal with solid objects, whose single points, on account of their different distances from the camera, must produce images in various planes of the image space. If we should insist upon a strict point-image formation, and require that all aberrations must be completely removed, every photographic process would be out of the question, for we can actually use in the image space only a *single* plane upon which the point-image formation must occur, while, as a matter of fact, the images lie in different planes.

The Focusing Plane Conjugate to the Plane of the Ground Glass.— We can simplify the conception considerably if we translate all the space phenomena of the object space to a *single* plane, as proposed by M. von Rohr. This *focusing-plane* (E.-E.) has as its image the *plane of the ground glass*. If these two planes are determined by sharp focusing of the objective on a given point, the image formation proceeds as if the whole object space were replaced by this plane, upon which all objects in the image space are projected by means of the principal rays from all the object points to the center of the entrance-pupil. The principal rays, going from the exit-pupil in the image space, then intersect the ground glass plane (M.-E.) in the image points which are conjugate to the various points of the projection on the focusing plane.

PROJECTIVE PROPERTIES OF THE PUPILS.— From this it follows that if the figure produced by a projection of the solid objects on the focusing plane is viewed from the center of the entrance-pupil, or if the image is viewed from the center of the exit-pupil, they appear in *central projection*. The *centers of the pupils* are, therefore, the *centers of perspective*. If we assume that the double objective within which the diaphragm is placed becomes infinitely thin, the stop coincides with the two pupils, and becomes the center of projection, as the pin-hole does in its camera.

PRINCIPAL RAYS AND PRINCIPAL POINTS.— If the objective is symmetrical and the diaphragm is placed in the center of the air-space between the two halves, then the centers of the pupils are identical with the principal points for very small inclinations of the principal rays. Except in this single case, however, the points of intersection of the principal rays with the axis before and after refraction occupy somewhat different positions from the principal points.

VIEWING DISTANCE.— It is apparent that the purpose of every photographic exposure is the production of a picture which is in all its parts exactly similar to the projection of the external world upon the focusing plane. The viewing of this photographic picture, however, must, if it is to make a truthful impression upon the eye, include the same angle which the corresponding principal rays form at the center of the entrance-pupil. A simple mathematical calculation shows that this condition if fulfilled if we view the picture from a distance equal to the *focal length of the taking lens*. It is true that this rule is exactly valid only when the focusing plane was infinitely distant from the objective; however, the deviation of the viewing distance from this limiting case in hand-camera exposures, which give a considerably reduced image, is so slight that we can neglect it. If a picture, therefore, is to be viewed at the normal viewing distance of an average eye, the focal length of the objective must be about 25 cm (10 in.), if the impression is to be true to nature. If the focal length is less, the eye must be brought nearer to the picture.

Viewing Lenses for Photographic Prints.— Since, however, the accommodation of the eye renders it impossible to use much shorter distances for viewing such pictures, we must use, between the eye and the print, a *collecting lens* which must fulfil two conditions. In the first place the picture viewed through this lens must appear to be at the same distance which the object actually had when taken, and secondly it is absolutely necessary that the picture should appear in correct perspective.

Focal Length of the Viewing Lens.— If we state these conditions in the shape of a very simple mathematical formula, we immediately obtain the interesting law that every photographic picture should be viewed through a collecting lens whose focal length is equal to that of the taking objective, and from a distance equal to this focal length. One may easily convince himself of the truth of this law by viewing one of the well-known, wide-angled street pictures with a collecting lens of the same focal length; the apparent distortions of perspective are then completely removed. An especially constructed apparatus for this purpose is the *Verant* of M. von Rohr, the lenses of which are corrected for the conditions here stated.

PERSPECTIVE.— As we have seen, the centers of the pupils are the centers of perspective. The determining factors for this are, therefore, the distance of the entrance-pupil from the focusing plane, and the angle which includes the projection of the solid objects on the focusing plane when viewed from the center of the entrance pupil, that is the angle of the principal rays. The only requirement for the objective used for the taking is that it shall exactly reproduce the figure of projection on the focusing plane on the ground glass plane in its finest details. If we, therefore, in place of one objective, substitute another, such that the center of the entrance-pupil has the same distance from the focusing plane, the perspective of both pictures will be exactly the same. All pictures made from the same view-point, irrespective of the kind of lens, whether portrait lens or wide angle, therefore, show the same perspective, and it is incorrect to speak of the false perspective of short-focus lenses.

ANOMALIES OF PERSPECTIVE.— If, however, we change the distance of the entrance-pupil from the focusing plane, then the perspective becomes different, because of the simultaneous change of the angle of the principal rays. One often goes nearer to an object in order to get it as large as possible on the plate when taking it with a lens of short focus. In this case, it is true, the background appears unnaturally small as compared with a picture taken with a lens of longer focus made on the same size of plate at a greater distance from the object. If, however, we view these pictures with apparently false perspective from the proper distance and through the corresponding collecting lens, the anomalies of perspective disappear and the eye sees the object space as if it were placed at the center of the entrance pupil.

However interesting may be a discussion of natural perspective and the choice of photographic objectives as determined by this, a further consideration would be superfluous in this place, since the decisive factors are esthetic.

CHAPTER X

Orthoscopy

We have already discovered in the pin-hole camera an optical instrument whose images are similar in their smallest details to the object reproduced and have now to investigate how far a photographic objective can produce a picture free from distortion. With the conclusions of the previous chapter in mind, we can give a general answer to this question.

Condition for Freedom from Distortion.— Let us project every solid object in the object space which is to be pictured, upon the focusing plane (Fig. 25), which is conjugate to the plane of the ground glass. Let us assume that the finite pencils are replaced by their principal rays. Let us, as before, designate the size of the object and the size of the image by y and y', the angles of inclination of the principal rays with the axis in the pupils on the object and image sides by ω and ω', and let the distances of the pupils from the focusing plane and ground-glass plane equal a and a'.

If the objective is to be free from distortion, the reduction factor $m = y : y'$ must remain constant for all values of the angle ω. The mathematical expression for this is:

$$\frac{y}{y'} = \frac{a \tan \omega}{a' \tan \omega'} = m.$$

This condition is independent of the amount of correction of the objective, and assumes only that every oblique pencil can be represented by a principal ray.

BARREL-SHAPED AND CUSHION-SHAPED DISTORTION. — When the reduction factor m is not constant, we can distinguish two general cases. If the scale of reduction decreases with increasing size of the object, a square (Fig. 27, a) is imaged with its sides curved away from the center (Fig. 27, b). Such a distortion which is called *barrel-shaped*, is observed in the case of ordinary objectives with a front diaphragm. If, on the other hand, m increases simultaneously with y, the image of the square has its sides concave. We observe this *cushion-shaped* distortion when we place the stop behind the lens. These phenomena become especially striking if we move the stop away from the lens, and thus modify the path of the rays.

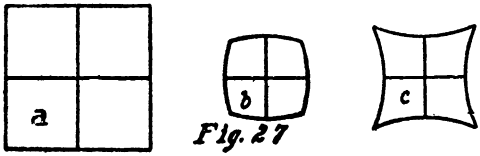

Fig. 27

BEHAVIOR OF THE SYMMETRICAL OBJECTIVE.— As we have already seen, the important class of symmetrical objectives is characterized by the fact that the principal rays before and after refraction form equal angles ω and ω' with the optical axis. The condition for orthoscopy then becomes $a : a' = m$. The relation of these distances of the focusing and ground-glass planes is, however, in general, not constant for any desired inclination of the principal rays, but only when $a = a'$. A symmetrical objective is, therefore, *absolutely* free from distortion only in the special case when the distances of the object and the image from the pupils are *equal*,— that is, when copying full size. In all other cases, the two pencils of principal rays intersecting in the pupils are not free from spherical aberration, so that in general we cannot say that the symmetrical objective is completely orthoscopic. Nevertheless, its deviations from orthoscopy are so slight that they do not become important, even for many tasks of reproduction. It is, however, very easily possible,— and, as a matter of fact, such corrections have been carried out for purposes of photogrammetry,— to give a symmetrical objective a higher degree of freedom from distortion by removing the aberrations in the pupils.

Zones of Distortion.— If the objective is not symmetrical, with a given reduction factor *m*, we must fulfil the general condition for a properly chosen angle of inclination of the principal ray.

If an objective is corrected for an inclination of the principal ray, ω, corresponding to the condition for orthoscopy, for smaller and larger angles there will occur in the usual way, zones, which, however, are practically unimportant. Zones also occur if we change the scale of reduction; thus, for example, if we move the object from a position near the objective to an infinite distance. These errors also are harmless in practice. We can assume in general that the modern types of photographic objectives in use at the present day are practically free from distortion, as long as they are used for the purposes intended by their designers and are not forced to cover too large a field. This is true even for the modern objectives with front diaphragms, which work at small apertures, and are used only for small sizes of plates; these are naturally never used for photogrammetry or reproduction.

Distortion of Convergence.— We assume in our consideration that the focusing plane, and, therefore, the plane of the ground glass, are perpendicular to the optical axis. It is well known, however, that characteristic distortions of convergence appear in the image when the ground glass is placed at an angle to the focusing plane; these, however, have nothing to do with the above-mentioned deviations from orthoscopy. In such pictures of parallel arrangements we find the unpleasantly converging lines which are, for instance, apparent in architectural pictures, if the photographic apparatus was not placed exactly vertical. This divergence of lines can be removed by an extremely tedious method of copying.

Anamorphosis.— We will refer only briefly to the *anamorphous* lens systems in which a distortion is purposely produced, so that, for instance, a circle is transformed into an ellipse or a square into a rectangle. Such apparatus is valuable technically, especially in the preparation of designs. These transformations may, as long as the scale of reductions in the two directions perpendicular to each other is not very divergent, be produced also by means of a strongly diaphragmed lens or a pin-hole camera.

CHAPTER XI

The Brightness of the Image

Diminution of Illumination towards the Edge. — When a luminous point occurs on the optical axis of a ~~photographic~~ lens, the cone of rays emanating from it is defined by the stop and converges again to an image point on the axis. Let the object point be now moved away from the axis so that its principal ray shall have the angles of inclination to the axis ω in the entrance pupil and ω' in the exit pupil. Calculation shows that for a practically corrected lens the brightness of the shifted image point is $\cos \omega \ \cos^3 \omega'$, if we assume that that of the axial image point is unity. We can assume with sufficient accuracy in the case of all unsymmetrical objectives and exactly for symmetrical objectives, that this expression equals $\cos^4 \omega'$, and we thus find that the diminution of brightness of the image of an evenly illuminated surface from the center to the edge, equals the *fourth* power of the cosine of the angle of inclination of the principal ray on the image side. This distribution of light is, therefore, completely *independent* of the character of the lens in respect to aperture and focal length, as well as its length between exterior surfaces.

Calculating the numerical value, we obtain the following values for the brightness H of the image for various angles of inclination ω' of the principal ray on the image side, equal to half the angle of view.

ω′	*H* in %	ω′	*H* in %	ω′	*H* in %
0°	100.0	30	56.2	60	6.2
5	98.5	35	45.0	65	3.2
10	94.1	40	34.4	70	1.4
15	87.1	45	25.0	75	0.4
20	78.0	50	17.1		
25	67.5	55	10.8		

We see that because of the serious loss in illumination, the time of exposure for the edge of the field of view increases considerably in the case of wide-angle exposures. If the field of view is 60°, the necessary exposure for the edge is 1.8 times that needed for the center; for 90° it is fourfold; for 120°, sixteenfold; and for 150°, 223 times as great.

Over-exposure in the Center or Under-exposure at the Edge.—If we do not become conscious of the weakening of illumination toward the edge of the plate in the case of exposures with normal objectives, it is because the under-exposure on the edge is taken care of by the latitude of the plate, if the exposure for the center is full. Nevertheless, in the case of rapid instantaneous exposures, especially of an evenly illuminated surface, it is easy to perceive the presence of the *unavoidable* diminution of luminosity in consequence of this law. In very wide-angle work, this becomes so evident that the pictures are useless, since either the edge is seriously under-exposed, or the center as strongly over-exposed.

Stolze's Star-stop.— In order to neutralize this disadvantage, although at the expense of general brightness, we may use the *star stop* (Fig. 28), first proposed by F. Stolze. If such a solid screen is placed centrally in front of the lens, the central rays are cut off and this part of the plate is held back while all of the edge rays affect the plate. In order to avoid a shadow of the stop, this is rapidly revolved by means of a blast of air, and we thus obtain a picture which is evenly illuminated to the edge.

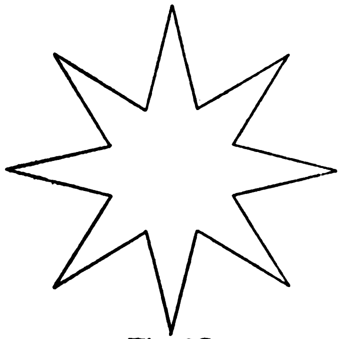

Fig. 28

We may also note that the falling off of illumination in a pin-hole camera follows closely the law of the fourth power of the cosine of the angle of inclination.

VIGNETTING.— Up to now we have assumed that the cross-section of the incident light pencil is affected solely by the size of the stop or the entrance pupil. From this it must follow that we could let the angles ω and ω' increase up to 90°, a limiting value to which would correspond an infinitely large angle of view. That this cannot happen, every photographer knows from experience. The diaphragm is, as a matter of fact, not the only stop which limits the rays; the physical construction of a photographic lens causes a further limitation of the light pencil. The lens tube cuts off, as may be seen from Fig. 29, a portion of the rays entering from the side. While the axial pencils from an infinitely distant object are not affected by the mounting, and this is also true for pencils but little inclined to the axis, a part of the pencil II is cut off by the upper part of the rear lens mount. The result is a corresponding diminution of brightness at the corresponding spot B_{II} on the ground glass. If we allow the inclination to increase,

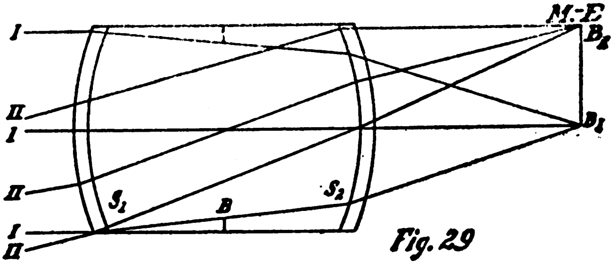

Fig. 29

we finally find an angle of inclination ω_0' on the image side, for which the ray incident below at the front edge of the mounting is entirely cut off above by the rear of the mounting. We, therefore, find darkness at the corresponding spot on the ground glass, so that the diameter of the whole *circle of illumination* of the image is $2\,\omega_0'$. This phenomenon, which appears in every lens, and the particular kind of diminution of brightness caused by it, are called *vignetting*. If several stops are present, the limitation of the image field is in general produced by the stop which subtends the smallest angle when seen from the point of crossing of the principal rays. This is called, by Abbe, *the field-of-view stop*. In the case of photographic lenses it is one end of the lens mounting.

THE NECESSITY OF SHORT-BARRELED OBJECTIVES.—Vignetting is, therefore, unavoidable, but becomes of less importance with the increase of the angle subtended by the mount when viewed from the center of the stop, and, therefore, in proportion to the shortness of the lens barrel. For this reason *compactness* of the objective is a very important condition which every modern lens must satisfy, and in which great progress has been made in recent times. While modern anastigmats exhibit very closely placed lenses, the older ones show a disproportionately long construction, since only thus was it possible for their designers to obtain a satisfactory extent of sharp image field. In the case of wide-angle lenses, the difference between old and new is insignificant, but it becomes more apparent the greater the relative aperture of the lens. Compared with the modern portrait lenses working at f: 4.5, the aplanats of the same aperture and the Petzval portrait lenses of great luminosity are more than twice as long in the barrel. This explains the small field of the old types.

In no case should the axial cross-section, even in the case of a lens of large aperture, be much longer than its width. Since it is entirely possible with modern anastigmats to extend the sharp imaging over a large angle of view, even an exceptionally well-corrected objective becomes worthless when, because of premature falling off of illumination, we encounter such a limitation of the size of plate as is customary in the case of the old objectives.

Increase of the Diameter of the Lens.— To avoid vignetting one seeks, especially in wide-angle objectives, to increase the free aperture as much as possible; as is, for instance, the case with the Hypergon double anastigmats. If one usually, in the case of objectives of medium aperture, makes the free aperture equal to the effective, this is done in order not to decrease their wieldiness. On the other hand it is necessary to take care that stopping down does not occur through the introduction of opaque parts in the course of the rays of a compact objective, which can easily occur when using filters, lens hoods, and shutters.

INFLUENCE OF STOPPING DOWN ON THE DECREASE OF LUMINOSITY NEAR THE EDGE.—From the course of the rays in Fig. 29 it is apparent that decrease in luminosity through stopping down, which is superimposed upon the vignetting, appears at a rather large angle of inclination of the principal rays on the image side. We, however, can draw no practical advantage from this fact, since the astigmatic portrait objectives, those which vignette most strongly, must be stopped down as little as possible.*

BRIGHTNESS OF A POINT-IMAGE.—We now have to investigate how the brightness of the axial image depends on the constants of the lens, and will first assume that there exists on the axis a luminous *point* from which is emitted a pencil parallel to the axis. The brightness of the image must then increase with the size of the entrance pupil which limits the rays, and according to the fundamental photometric law decrease proportionately to the square of the distance. If we call the diameter of the entrance pupil D, the distance a, then the brightness of the image point is proportional to $D^2 : a^2$. From this follows the important law, that in the imaging of point objects, the brightness of the image point is *independent* of *focal length* and *effective aperture*.

LUMMER'S AMPLIFICATION.—As a matter of fact, there is no such thing as a photographic point-image. As is well known, the photographic film consists of an enormous number of small, light-sensitive elements, of which each separates into a number of smaller particles. O. Lummer has shown that the law of brightness just

*If the stop is unfavorably placed, the circle of illumination can, as a matter of fact, be reduced by decreasing the stop.

mentioned is also valid in the case of the imaging of a small surface, in as far as the image of this small surface is not larger than one of the elements of the plate. As a matter of fact, neither artistic nor technical photography has to deal with such objects, but only a branch of scientific photography,— that is, *astrophotography.*

BRIGHTNESS OF THE IMAGE OF A PLANE.— As soon, however, as the size of the image surpasses the above-mentioned limit, its brightness must be determined by another rule. It is then proportional directly to the area of the entrance pupil and to the area of the luminous object, and inversely to the square of the distance. A simple transformation shows that the brightness of the image of a surface near the axis is proportional to

$$\left(\frac{x}{x+f}\right)^2 \left(\frac{D}{f}\right)^2.$$

Here, as usual, f designates the focal length of the lens, and x the distance of its front focal point from the luminous surface. If this is at a distance which is large in proportion to the focal length, the fraction $\frac{x}{x+f}$ is practically equal to unity, and we obtain the following rule important in photography:

The light-strength of an objective, provided that the object is a surface at a great distance, is equal to the square of the ratio of the aperture to the focal length.

Therefore, however great one may choose this latter, the brightness of the image and consequently the time of exposure always remain the same, provided only that the aperture of the objective is changed in the same relation as the focal length. If we decrease the aperture of the same lens, the brightness decreases in proportion to the squares of the apertures.

Entrance Pupil or Effective Aperture and Stop Diameter.— From what goes before, it will now be clear that to designate the light-strength of an objective, in addition to its focal length, the diameter of the effective aperture, that is, the *entrance pupil*, is requisite. The confusion of this image of the aperture-stop, produced by the portion of the objective in front of the stop, with the stop itself is, unfortunately, such a common error that it is worth while to devote a few words more to this subject. If parallel light falls on the front half of a symmetrical objective (Fig. 29) the pencil becomes convergent because of the collecting action of this portion. Its cross-section must, therefore, be smaller in the plane of the stop than before the refraction at the first surface. The ratio of these two cross-sections, in the case of anastigmats, is about 1 to 1.15, and in the case of aplanats, 1 to 1.3. It is, therefore, an error when one tries to determine the relative aperture by dividing the diameter of the stop by the focal length. Only when the number thus found is divided by the value just given, which depends solely on the type of the objective, does one obtain the relative aperture. Only when the stop is placed in front of the lens can we replace the diameter of the effective aperture by that of the stop.

Influence of Distance.— If in general we may assume that the object, and consequently the focusing plane, are far distant from the lens, there are cases in which we go close to the object to be taken with the camera. This is the case with all those zoological and botanical pictures which have become so popular in recent years and as to whose great value there can be no two opinions. In these the object is often shown in natural size or even enlarged. In this case the factor

$$\left(\frac{x}{x+f}\right)^2,$$

which we have assumed to be negligible, becomes much less than one. If we introduce again the reduction number m we obtain the law:

In the case of exposures on near objects we must increase the time of exposure as determined for distant exposures under the same conditions in the ratio of

$$1 : \left(1 + \frac{1}{m}\right)^2.$$

TIME OF EXPOSURE IN ENLARGING.— If one takes a picture natural size, for which $m = 1$, the times of exposure have the ratio of 1 : 4; we must, therefore, give four times the exposure. This increases very rapidly, as soon as we increase the enlargement. For instance, in a case of twofold enlargement, the exposure is nine times as great. It is well known what enormous exposures are necessary in microphotography. On the other hand we see that with a reduction $m = 20$ the increase in the time of exposure is only eleven per cent, — an amount which has no practical significance. This reduction corresponds in the case of a focus of 15 cm (6 in.) to a distance of 315 cm (10 ft., 4 in.) from the front principal point of the lens.

EQUIVALENT OF THE ENTRANCE PUPIL IN VIGNETTING. — We have previously assumed, in treating of brilliancy, that we were considering the image of a surface *near the axis*. The limiting of the rays is then caused only by the entrance pupil. If we, however, give the object surface a considerable extent sidewise, the influence of the field-of-view stop, in the shape of vignetting, becomes predominant, as we have already seen. In place of the entrance pupil we must now use the lenticular common portion of the two circles, shaded in Fig. 30, produced by the overlapping of the openings of the lens mount, when we incline the objective to the axis of the pencil of light. We can prove this by taking out the ground glass and looking through the objective obliquely from the rear, and at the same time discover if a diminution of the circle of illumination is not possibly caused by the intrusion into the light cone of some portion of the camera.

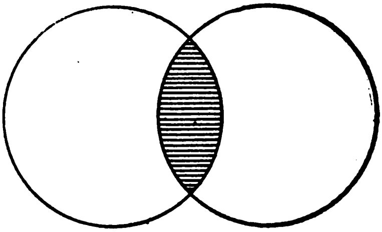

Fig 30

Shape of the Stop. — Although for general photography it is unimportant what shape we give the stop opening, on practical grounds, without exception, *circular* stops are used. Only in reproduction photography, when a ruled screen is placed before the light-sensitive film, do we need stops whose form is related to the structure of the screen. Of the unchangeable stops, the Waterhouse variety is at present used only for reproduction and portrait lenses of large diameters, while revolving stops are generally used for stereoscopy. For general work we, therefore, have to deal only with the iris diaphragm.

Scale of the Relative Apertures.— In order to be able to determine the relative aperture of a given objective, at any desired opening of the stop and the entrance pupil, without trouble, a series of numbers are placed on the diaphragm, shutter, or lens barrel, from which the proper time of exposure is to be determined. In spite of the numerous proposals for scales of stops, practically only *four* systems are in present-day use, of which we will speak briefly. One sort of figuring is common to these four systems; that is, that to two numbers following one another in a series, correspond exposure times in the ratio of 1 : 2. Two of these systems give as the numerical value for the relative aperture a fraction $f : p$, where p equals the focal length divided by the effective aperture; they differ, however, in the choice of unit stops.

The first system was proposed by F. Stolze. As a unit for the strength of light, he has assumed the ratio $f : \sqrt{10}$ so that he obtains the following scale:

Light-strength f:	3.16	4.5	6.3	7.7	9	12.5	18	25	36	50	71
Time of Exposure	1	2	4	6	8	16	32	64	128	256	512

Instead of this scale of relative apertures, there is in use, in England, and partially in the United States of America, the following, in which the relative aperture $f : 4$ forms the unit.

Light-strength f:	4	5.6	8	11.3	16	22.6	32	45	64	128
Time of Exposure	1	2	4	8	16	32	64	128	256	512

[The times of exposure given in the second line of figures are also the stop numbers according to the Uniform System used on most American cameras.—TRANSLATOR.]

Since the squares of the units of light strength of the two systems have the ratio $\sqrt{10}$: 4 we obtain the time of exposure in one system from the corresponding number of the other system by multiplying or dividing by 1.6. If, for instance, with a stop f: 6.3 in the upper system we expose five seconds, for *f*: 8 in the lower, we must expose 5 x 1.6 = 8 seconds.

In order to save the photographer the work of calculation, the objectives of the Carl Zeiss optical works are figured on the system of P. Rudolph, according to *brightness*, in which the brightness for the stop *f*: 50 is made equal to unity. We thus get the following scale for the stop numbers:

Brightness	1	2	4	8	16	32	64	128	256
Light-strength *f*:	50	36	25	18	1.25	9	6.3	4.5	3.16
Time of Exposure	256	128	64	32	16	8	4	2	1

In the fourth system the *time of exposure* is given as the stop number, so that the numbers increase as the stops decrease, while in Rudolph's system they decrease.

STOPS OF AN OBJECTIVE SET.— We may call attention in this place to the fact that the designation of the stops for a given light-strength undergoes some complication, when the same stop must be used for several objectives. This case occurs when several objectives with front stops are united in a set which also comprises their combinations. Thus, from a set of three halves, six objectives may be combined, and, therefore, there are as many stop divisions, different from each other, required. Placing all these scales on the lens tube beside the turning ring of the iris stop is in general not to be recommended because of the small space available. It is, therefore, preferable, as P. Rudolph has proposed, to divide the scale to correspond with the stop opening in millimeters, and to provide a table from which the relative aperture corresponding to a given stop diameter can be found for every combination of lenses.

CHAPTER XII

Depth of Focus

Abandonment of the Exact Union of Rays to a Point.— As we have learned from the law that image formation is completely independent of the kind of refraction, separate planes, in the object space, perpendicular to the axis, are imaged in the image space by similar separate planes perpendicular to the axis. It is, therefore, impossible to produce a sharp image of a body bounded by two such planes on *one* plane in the image space. Let us, however, abandon the requirement of an exact union of rays in a point on the ground-glass plane, and define the word "sharp" as "appearing sharp to the eye on the ground glass." Then there must correspond, to this latitude which we allow in the union of rays on the image side, a similar latitude on the object side in the position of the object to be imaged in respect to the focusing plane. This conception is expressed by the word *depth*.

Depth.— Much has been written on the dependence of depth on the circumstances in which the exposure is made, and in general with a display of formulas which does not encourage one to a study of this chapter of photographic optics, which is still unpleasantly mysterious to many. And yet, there is scarcely a theme of this subject which can be handled in so simple and satisfactory a way as the relation of depth to the ordinary conditions of picture taking, if we use the train of thought of M. von Rohr. Here, again, we shall see how advantageous is the conception of the focusing plane conjugate to the ground-glass plane. We assume only that the objective is anastigmatically corrected.

Unsharpness on the Focusing Plane.— In the consideration of orthoscopy we saw that a solid object in the object space may be replaced by a projection figure on the focusing plane, which is produced by the principal rays intersecting this focusing plane. It was also assumed that all the pencils are infinitely small. If, however, we give them a finite cross-section, there occurs, instead of a *point of intersection* on the focusing plane, a *circle of confusion*, if the object point is outside of the focusing plane. We see from Fig. 31 that one

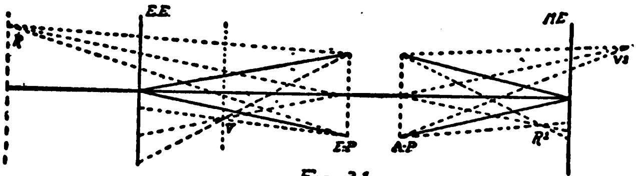

Fig 31

property is common to all these cones,— that is, that they have the same base in the entrance pupil. Let us now assume that the allowable amount of unsharpness on the focusing plane is expressed by a circle of confusion with the diameter *z* (understanding that *z* may vary for different kinds of pictures); then there must be two planes, one on each side of the focusing plane, in the points *R* and *V* of which lie the apices of the cones of light which are cut by the focusing plane in the circles of confusion *z*.

Depth Forward and Backward.— It immediately follows that with the given maximum unsharpness *z* on the focusing plane, the distance t_v of the plane V from the focusing plane, that is the depth forward, as well as the distance t_r of the plane R from the focusing plane, that is the depth backwards, and consequently also the distance $t_r + t_v$ of the two planes R and V from each other, that is, the total depth, depend solely on the size d of the entrance pupil or the effective aperture of the objective, and its distance a from the focusing plane. The smaller the entrance pupil, the farther from the focusing plane and from each other are the limiting planes of depth of field. Absolutely *without influence*, however, is the manner in which the image of the figure of projection on the focusing plane is produced, and, therefore, the scale of reduction, the focal length and the type of the anastigmatic objective.

TRANSFER OF THE FIGURE OF PROJECTION TO THE GROUND-GLASS PLANE, AND LIMIT OF SHARPNESS.— If an image is to be formed by an objective of focal length f, and the reduction number m is known, it is possible to determine the diameter z' of the circle of confusion on the image side in which the ground-glass plane is intersected by the cones of rays converging towards the image points R' and V'; it is expressed by the equation $z = mz'$. We can, however, set a maximum limit for the unsharpness of the image designated by z'. Taking for a basis the value given by Helmholtz, we may regard an angle of *one minute of arc* as the smallest amount possible of perception by a normal eye. But every photographic image should be viewed, after the introduction of a collecting lens the focal length of which is equal to that of the objective, from a distance a', which is equal to that of the exit pupil from the ground-glass plane during the exposure. Hence, we have a relation between the viewing distance a', the allowable circle of confusion z', and the limiting angle of perception of one minute. This is: $z' = 2a' \tan 30'' = 0.000291a'$

In most cases it is sufficient as in our previous examples, to use, instead of a', the focal length f of the objective.

Formula for Depth.— If we introduce, instead of t_v and t_r, the distances a_v and a_r from the entrance pupil, to which the necessary sharpness reaches, both forwards and backwards, and make the relative aperture of the objective $\frac{d}{f}$ equal to $\frac{1}{n}$, we obtain the following relations:

Sharpness forwards reaches to the distance

$$a_v = \frac{af}{f + 0.000291\,an},$$

Sharpness backwards reaches to the distance

$$a_r = \frac{af}{f - 0.000291\,an}.$$

Laws of Depth of Focus.— According as we change one of the quantities, a, f, or n, while the other two remain constant, we obtain three laws:

1. When effective aperture and focal length remain the same, the depth of focus increases as the distance of the focusing plane from the entrance pupil becomes greater;

2. With equal distances and equal apertures, the depth increases when the focal length is less;

3. With equal distances and equal focal lengths, the depth increases when the diameter of the stop is smaller.

Importance of the Stop.— The truth of these laws is proven in every photographic exposure, for only by use of the stops can the depth of an objective be increased in practice. At the same time, however, the sharpness of the image on the ground glass on and off the axis also increases, since the unavoidable circles of confusion due to uncorrected optical errors are decreased by stopping down. As long as photographers had no anastigmats the only way of enlarging the sharp field of image was through stopping down, by which the depth was simultaneously increased. It would thus seem as if one, by continued stopping down, might finally reach a practically sufficiently sharp imaging of the whole object space on a single plane in the image space.

LIMIT OF STOPPING DOWN.— Experiment shows, however, that the gain in sharpness ceases when we reach a certain aperture, and that the union of rays with very small stops finally becomes so poor that we can no longer speak of an image. The cause of this phenomenon, which we have already observed in the consideration of the course of the rays through a pinhole, is the *diffraction* of light. Since, on account of the increase of the time of exposure, it would be useless to go beyond the most favorable point of stopping down, we must content ourselves with the relative aperture f: 71 as the smallest possible stop, and this can be used *only* in telephotography. Usually, f: 50 is prescribed as the smallest stop. On the other hand, for esthetic reasons, every picture should be made with the largest possible opening.

DEPTH AND BRIGHTNESS ARE INCOMPATIBLE.—It is now evident, however, that *depth* and *brightness* are qualities which, in a certain sense, are *incompatible* in one objective: *to unite both is impossible and not to be* *attained through any choice of a special type of objective.* If one must have depth, he must sacrifice brightness, and *vice versa.* On the other hand it is possible, by the introduction, before a fast objective, of an absorptive medium which does not influence the course of the rays, to decrease brightness, and thus combine long exposure with slight depth. No one, however, will find much use for this combination.

SHORT FOCAL LENGTHS.— There is a method of obtaining greater depth with the same aperture,— the use of two objectives instead of one. We saw that the depth is dependent solely on the diameter of the entrance pupil, when the scale of reduction is fixed. If we now compare two objectives with different focal lengths, but equal actual apertures, we can considerably increase the depth of the short focus objective by stopping down to the relative opening of the objective of longer focus. If we enlarge the image made by this objective in the ratio of the two focal lengths, we obtain a picture whose dimensions are exactly equal to those of the original image. The sharpness is equal in both pictures, since this depends only on the focal length. The enlarged picture, however, shows *greater depth* than the unenlarged one. This is the reason why, in recent years, small cameras have become more and more popular, although the pictures require subsequent enlargement.

INFLUENCE OF THE GRAIN OF A PLATE.— We have tacitly assumed that the process of enlargement does not affect the sharpness of the image. This, however, is not true, since the size of the *grain of the plate* is not without influence. Since this is simultaneously enlarged, we lose so much in sharpness in great enlargements that we must confine ourselves to moderate dimensions. The finer the grain of the plate, therefore, the better results are obtained. For this reason, in many cases plates of this kind are used, despite their slowness.

EXAMPLE.— How objectives of short focal length surpass in depth objectives of longer focus, but the same speed, is apparent from the following numerical example. An objective with the relative aperture f: 4.5 and the focal length f = 90 mm (3.54 in.) is sharply focused on a plane at the distance a = 6 m (19 ft. 8 in.). Then n = 4.5 and we find that the sharp zone lies between 5.518 m (18 ft. 1 in.) and 6.574 m (21 ft. 7 in.). Its width or the total depth is, therefore, 1.056 m (3 ft. 6 in.). The diameter z' of the circle of confusion on the ground glass, which has no influence on the sharpness, is 0.026 mm (0.001 in.). If we now make the same picture with a lens of focal length f = 180 mm (7.08 in.) and the same relative aperture f: 4.5, the zone of sharpness lies between 5.749 m (18 ft. 10 in.) and 6.274 m (20 ft. 7 in.), and the total depth is 0.525 m (1 ft. 9 in.). If we enlarge the first picture to twice its size, its circles of confusion would be increased in the same ratio to a size of 0.052 mm (0.002 in.), therefore just as large as in the second picture; the depth is, however, practically twice as great.

DIFFERENT AMOUNTS OF DEPTH, FORWARD AND BACKWARD.— These numbers show also a second peculiarity which is noticeable in the projection upon the focusing plane of solid objects in front of or behind this. It is a fact, as the general equations show by inspection and practice confirms, that *the depth backwards is always greater than that forward.* While in the case of the short focus objective of our example we could remove the limiting plane of practically sharp imaging to a distance of 574 mm (22.6 in.) back from the focusing plane, the corresponding zone forward,— that is, toward the objective,— is only 482 mm (19 in.).

CONDITION FOR SHARPNESS TO INFINITY.— Especially interesting is the case when the depth reaches infinitely far backward. This is true when the denominator of the fraction in the equation for a_r equals zero; that is, when $f = 0.000291\ an$. If, for example, with an objective of 90 mm focal length focused for a distance of 6 m, the image of an infinitely distant point must be made practically sharp by stopping down, solving the equation for n, we get the expression $n = \frac{3518 f}{a}$. Therefore, the necessary stop to be used is f: 53. In all these cases the distance a_v of the point nearest to the objective which appears sharp is equal to *half* the distance of the entrance pupil from the focusing plane. In this example, therefore, the total depth extends from infinity to 3 m.

If we introduce, instead of the focal length and the relative aperture, the diameter d of the entrance pupil, or the effective aperture, we find between this and the distance a of the focusing plane, upon which we must focus if we desire to get infinity sharp, the simple relation $a = 3518\ d$. The nearest sharp point has a distance of 1759 d.

For objectives with the diaphragm in front, d is equal to the actual diameter of the aperture. As we have previously seen, in the case of modern anastigmats with the stop between the lenses, the diameter of the stop is smaller than the effective aperture in the ratio of about 1:1.15. Hence follows the rule that one must focus on a distance of about 4,000 times the actual diameter of the stop in the case of objectives with a between-lens stop, when it is necessary to have infinity sharp on the ground glass. The inner limit of sharpness is then about 2,000 times the diameter of the stop.

Depth in the Case of Pictures taken Full Size. — In natural history and technical photography it is often necessary to make pictures life size. Every photographer knows what difficulties he has to overcome in order to obtain the necessary amount of depth. A little reasoning shows that for this case, where the depth is assumed to be small, it must be equal forward and backward. As a matter of fact, it follows from our equations that the deviations are practically negligible, and we obtain the simple relation that the depth in mm $t = \frac{nf}{430}$; therefore, if we use a lens of 180 mm focal length working at f: 9, to make a picture full size, the width of the sharp zone equals 3.8 mm.

Different Opinions on the Allowable Unsharpness.— It is easy to see that we may get large values for the depth if we allow the diameter z' of the circle of confusion in the image to have a large value. We often find the amount of 0.1 mm (0.004 in.) given for this, irrespective of the focal length; it is possible to imagine cases in which one can go much farther with the allowable unsharpness. If, however, the picture is to be viewed with the unaided eye, we are rigorously restricted to the limits which we have just set forth. It may even be necessary to further limit z' if the picture is to be used for reproduction or measurements.

Focusing Rule of F. Stolze.— If in practice we desire to obtain the best distribution of sharpness over a certain depth of the object space, we must take into consideration the lesser depth toward the lens. From this fact is drawn the well-known *rule for focusing* of F. Stolze, according to which we focus first on the most distant point which is to appear sharp, then determine the farthest point forward which is also sharp, and finally make the exposure after focusing on this point. In every case we must avoid placing the sharpest point too far back, quite independent of the esthetic requirements which the picture must meet.

SACRIFICE OF SHARPNESS TO OBTAIN DEPTH.— It has been attempted, in artistic pictures, simultaneously to increase the effectiveness of the picture and the amount of depth by completely abandoning sharpness. If we give a photographic objective spherical aberrations, the transition from moderate to complete unsharpness is so gradual that in this sense we may, in fact, speak of an increase of depth. In addition, it has been proposed that the lens should be moved during the exposure in the direction of the axis, so that all planes of the object to be taken shall successively appear sharp on the ground glass and that the negative shall show a moderate and even unsharpness. It has also been attempted to make use of diffraction by the introduction of a fine-meshed wire netting in the course of the rays in order to eliminate microscopic sharpness. The same effect is given by shaking the camera during the exposure. All of these expedients, however, have but a limited value. If one desires to change the sharpness of the image in this direction, it is preferable to do it in the positive processes following the exposure, where means to this end are far more numerous than the expedients just mentioned.

CHAPTER XIII

Color Aberrations and Optical Glass

The Spectrum.— We have previously assumed that the influence which an optical medium exercises upon a light ray which enters it causes only a change of direction. As was first shown by Newton, however, a ray of white light, when refracted, is bent from its original direction and at the same time separated into a wedge of color, the single hues of which have the same order as in the rainbow. This band of color is the *spectrum;* it is obtained in its purest form by introducing a prism of a strongly dispersing material in the path of a narrow convergent pencil of white light. We perceive that the red end of the spectrum is bent the least from the original direction, and that the other rays are more strongly refracted in this order: orange, yellow, green, blue, indigo, and violet.

Speed of Propagation of Colored Light.— While our previous investigations concerned solely the path of monochromatic light,— that is, that which has *one* fixed coefficient of refraction,— it is now our task to consider the great variety of lights of different colors, characterized by a continuous change of the coefficients of refraction. These different coefficients of the different colors which form white light are caused by the different speeds of propagation of different-colored lights in optical media. Every colored ray which has a definite coefficient of refraction is indecomposable.

Fraunhofer's Lines.— We now have the problem of so defining each spot of the spectrum that we may be delivered from the uncertainty which, before Fraunhofer's investigations, reigned in the characterizing of definite colors, and hence in the measuring of the corresponding coefficients of refraction. While the light emitted by every glowing solid body produces a continuous spectrum, we know that sunlight produces a spectrum in which the color series is no longer uninterrupted. Certain colors are missing, so that the spectrum appears to be interrupted by dark lines. Fraunhofer's immortal renown depends on the fact that he first recognized the unchangeable position in the solar spectrum of these lines which are named after him, and used them for the characterization of the colors. In this way an unmistakeable connection of color and refractive index became possible and simultaneously a foundation for optical construction.

Line Spectra.— As is well known, Fraunhofer's lines are intimately connected with the *line spectra* produced by incandescent gases and vapors, so that by their observation it is possible to determine the occurrence of these substances in the sun and other heavenly bodies. The dark lines in the solar spectrum are so spaced that we can define by them the colors corresponding to similar bright lines in the spectra of luminous terrestrial bodies. As the Fraunhofer lines exist in very large numbers, it is desirable to be able to designate any particular portion of the spectrum on a continuous scale.

Wave Lengths.— For this purpose we use the *wave length*, λ, with which light of a given color vibrates in a vacuum. It is expressed in units of 0.000001 mm = $1\mu\mu$, while the Fraunhofer lines are designated by the letters of the Roman alphabet. The following table gives the most important regions of the spectrum for our purpose:

Fraunhofer Lines	Wave Length λ in $\mu\mu$	Color
A′	768	Dark Red
B	687	Red
C	656	Light Red
D	589	Yellow
E	527	Green
F	486	Prussian Blue
G′	434	Indigo
h	410	Violet

DISPERSION.— The index of refraction corresponding to a Fraunhofer line is designated by the addition of the proper Roman letter as subscript to n. The difference of two indices of refraction of the same optical medium is called its *dispersion*. The smaller this is, the less is the difference in the separaton of the various colored rays when refracted.

INDEPENDENCE OF DISPERSION AND INDEX OF REFRACTION.— We now discover a fact of the greatest importance,— that the color dispersion of optical media is *independent* of their refracting power. There are bodies, such as the diamond, which combine with a large coefficient of refraction, a relatively small dispersion, while flint glasses refract far less strongly, but have greater dispersion; on the other hand, fluorspar has very small refraction and dispersion. The error of Newton, who believed that the color dispersion of all bodies was proportional to their indices of refraction, retarded the development of practical optics for a century.

Optical Glass.— The sole material used in the preparation of photographic lenses is optical glass. The first investigator, who, in the beginning of the nineteenth century, sought to determine the influence of the chemical composition of a melt of glass on its powers of refraction and dispersion was Fraunhofer, who made careful investigations for this purpose. After his death the art of preparation of optical glass was practiced almost exclusively in France (E. Feil in Paris) and England (Chance Bros. of Birmingham), until, in the fall of 1884, the world-renowned Jena glass factory was opened and effected a complete revolution in constructive optics. Such a result was made possible only by the carefully planned researches of the two founders, E. Abbe and O. Schott, whose efforts were directed toward producing, by the addition of chemical substances not hitherto used, new kinds of glass of differing optical properties; that is, possessing various indices of refraction with the same dispersion, and *vice versa*.

Nomenclature.— In the price-lists published by the Schott glass works, every glass is designated by a name approximating its chemical composition, and data are furnished on the index of refraction for the D-line, n_D, and the dispersion. It appears, however, that it is desirable, in addition to the dispersion itself, — for instance, $n_F - n_C$, to give another constant whose significance we shall learn. This is the Abbe number $\nu = \frac{n_D - 1}{n_F - n_C}$. The stronger the dispersion of the glass, the smaller must be the number ν when the index of refraction n_D remains the same; and on the other hand, a small dispersion corresponds to a large value of ν. In certain cases it is necessary to give the character of the dispersions more exactly; therefore, Schott's lists contain also the dispersions $n_D - n_{A'}$, $n_F - n_D$, and $n_{G'} - n_F$ as well as their relations to the total dispersion $n_F - n_C$. These fractions, $\frac{n_D - n_{A'}}{n_F - n_C}$, $\frac{n_F - n_D}{n_F - n_C}$, and $\frac{n_{G'} - n_F}{n_F - n_C}$, are called by Abbe the *partial dispersions*, and by them and the corresponding n_D and ν he designates the optical character of the glass. As an example, we take the following data from Schott's catalogue:

Name	O. 144 Borosilicate Crown	O. 211 Heavy Barite Crown	O. 41 Heavy Silicate Flint
n_D	1.5100	1.5726	1.7174
$n_F - n_C$	0.00797	0.00995	0.02434
ν	64.0	57.5	29.5
$n_D - n_{A'}$	0.00519 0.651	0.00630 0.633	0.01439 0.591
$n_F - n_D$	0.00559 0.701	0.00702 0.706	0.01749 0.718
$n_{G'} - n_F$	0.00446 0.559	0.00568 0.571	0.01521 0.625

The numbers in the three lower compartments under the main lines, are the corresponding dispersions.

Preparation of Optical Glass.— To give a correct idea of the difficulties inseparable from the preparation of a melt of optical glass, we will briefly describe the process of fusion. The mixture of chemicals is fused in a clay glass pot. Before the mixture is introduced the pot must be slowly heated to bright redness, a process lasting several days, and then coated in the melting furnace with a layer of the same glass which has been left over from earlier melts. The mixture is then piled in in layers. The pot and its contents must now be kept at the temperature of fusion until it becomes liquid. This period is critical for the success of the melt, for if the temperature is too high, a reaction may take place between the pot and the melt by which the composition of the glass may be changed, and, on the other hand, the mass must become liquid enough to allow gas bubbles to escape. After the impurities have been skimmed from the surface, a clay stirring rod is introduced, and while the temperature slowly falls the fluid mass is stirred for several hours, until it becomes so stiff that this is no longer possible. The pot and its contents are then cooled slowly, in an annealing oven, for about four days. During this time the block of glass cracks into irregular pieces, which form the optical glass in its first stage of manufacture. Only a small part of such a mass of glass, however, can be used. The purpose of the stirring is to destroy the stratifications which are formed in the liquid mass by the evaporation of the liquid glass from the surface, and to make the mass homogeneous in all directions. As the mass is usually only semi-liquid, that this can never be completely obtained is self-evident.

STRIAE.— Every piece, therefore, after cooling must be tested to see if it contains portions of different indices of refraction or striae. If such spots remain in the glass, they would produce a course of the rays different from that calculated, and the result would be an imperfect image. All striae, therefore, as far as they are apparent, must be cut out of the blocks of glass. Since one cannot do much with irregular lumps in an ordinary optical establishment, and also cannot thoroughly test them, they are now formed in regular shapes. For this purpose they are laid in round or rectangular molds, which are heated with their contents until these soften. The glass takes the form of its container and is then slowly cooled, the process lasting about two weeks. It can then first be thoroughly tested; for this purpose it is ground and polished on two opposite surfaces, so that the striae, bubbles, and impurities become visible.

STRAINS.— But even plates which are proved to be free from these faults are not yet ready for use. If such a plate is examined by polarized light, we discover the presence of more or less strains. Even though a small strain would have very little influence on the quality of the image of small photographic lenses, larger lenses prepared from such a glass, and especially the reversal prisms used in photo-mechanical work, would be totally useless. The glass must, therefore, be annealed.

ANNEALING.— For this purpose the glass plates are placed in an oven at a temperature which, according to the composition of the glass, may be between 350° and 480° centigrade, so that the strains are released. In order, however, that no new ones may appear, the cooling must proceed very slowly and regularly. After about six weeks' cooling, an annealed glass is thus produced, from which the lens surfaces are worked in the usual way by chipping, grinding, and polishing.

The process of fusion which we have outlined varies in its details according to the chemical composition of the glass.

Difficulty of Preparation.— That the preparation of optical glass is a process of enormous difficulty is apparent from the previous description. This is, however, increased in the manufacture of the kinds of glass which are most important for photographic lenses, which, without exception, stand at the limit of possibility of preparation, and in the making of which a yield of twenty per cent of the weight of material used is called good. It is hard enough to prepare these difficult glasses without striae, and it would be impossible to produce them at all if they were required to be completely free of small bubbles. Such a requirement is wholly unnecessary, for the value of the lens is not influenced in the slightest by the presence of the bubbles. As O. Schott has proved, the loss of light through these bubbles amounts to at most 1-50th of one per cent, an absolutely unimportant amount in practice. In the final analysis, one uses photographic lenses to take pictures, and not to look through. The case may also occur that in the working of a lens a bubble is cut and appears as a black spot on the external surface. This likewise is without significance, as is also the occurrence of a tiny lump of dirt, or even a small flaw, which, on account of its small size as compared with the aperture of the lens, may be neglected.

Effect of Dispersion.— Chromatic Aberration of a Collecting Lens.— We have now to discover what effect the change of the index of refraction with the color of the light has on the imaging of an object by a photographic lens. Let us start with the simplest system, the infinitely thin collecting lens. Since the index of refraction of all glasses is greater for blue than for red, the decomposition of a white, incident paraxial ray coming from infinity must take place in such a way that instead of a focal point, there exists an axial band of focal points of spectral colors, whose red end is nearer the collecting lens.

UNDER-CORRECTION.— We speak in this case of a *chromatic aberration*, which is designated as an *under-correction*, the amount of which can be easily determined for any given dispersion from the usual formula for infinitely thin lenses:

$$\frac{1}{f} = (n - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

In this case also we find circles of confusion on the ground glass, but their color changes from edge to center through the overlapping of the circles of different colors.

OVER-CORRECTION.— The similar color effect in the case of an infinitely thin dispersing lens has the opposite character and is called a *chromatic over-correction*. It is thus possible to produce chromatic correction of an infinitely thin collecting lens by constructing it of two infinitely thin single lenses in contact with each other, one of which collects, the other disperses. In order, however, that the compound lens may continue to be a collecting lens, the two components must be made of different kinds of glass, and the collecting lens must have a smaller color dispersion than the dispersing lens.

THE ACHROMAT.— It thus became possible, more than one hundred and fifty years ago, to build an achromat in which the collecting lens was made of a weakly refracting and dispersing crown glass, the dispersing lens of a strongly refracting and dispersing flint glass. Such a combination is known as an *old achromat.* On the other hand, in the case of an achromatic dispersing lens, the collecting component must possess a greater color dispersion than the dispersing component. If we calculate the very simple mathematical conditions for the achromatizing of a combination of two single lenses in contact with each other, we find that it depends solely on the constant ν of the two glasses. This is the reason why this number has been chosen for defining the optical character of glass.

SECONDARY SPECTRUM.— The chromatic aberration would be completely removed if it were possible to unite all the colored images again to a single white focus. This is, however, only possible if the sorts of glass of which the two components are made possess a very special character of dispersion. The two spectra of these glasses, if reduced to equal length, must absolutely coincide with one another; that is, the partial dispersions must be equal throughout the whole spectrum for the two glasses. In the case of flint glasses, however, the relative color dispersions in the blue and violet parts of the spectrum are generally considerably *larger* than in the case of crown glasses, which have a stronger dispersion at the less refracted end of the spectrum. It was, therefore, not possible with the old glasses known before the founding of the Jena glass works to unite more than two colors, through a suitable choice of the focal lengths of the collecting and dispersing lenses, when one was limited to a two-lens achromat. The other colored rays form their foci at various distances from the main focus, a chromatic aberration which is called the *secondary spectrum.*

To make the secondary spectrum as small as possible, is of great importance to every constructor of optical instruments. The choice of two colors must be made according to the purpose for which the lens is to be used, so that the residual chromatic error shall be as harmless as possible. It is, therefore, first necessary to determine which rays are most valuable in the formation of an image. For photographic purposes, we must use the part of the spectrum which acts on the sensitive film.

Actinic Achromatism.— As every photographer knows from experience, only the more refracted portions of the spectrum, from the light blue on, have any effect on the ordinary non-color-sensitive plate, while red, yellow, and green light cause no blackening of silver bromide. Above all, the *ultra-violet* rays which are invisible to the human eye actively affect the ordinary plate. Taking into account this characteristic sensitiveness of undyed gelatino-bromide emulsion, it has been found that the chromatic aberration for all actinic light is the least when the colors corresponding to the Fraunhofer lines F and h are united. This is *actinic achromatism.*

Chemical Focus.— In such a union of colors, however, the visible colored rays near the red end of the spectrum must cause a very considerable chromatic aberration of the focal point, which causes an important focal difference. This is the so-called *chemical focus*, which occurs in all non-chromatic objectives. If, for example, one focuses with the naked eye on the ground glass, only the visible rays of the spectrum are used, of which the yellow-green appears brightest to the eye. The plane of the sharp image, however, by no means coincides with that of the ground glass. Such an achromatic objective is, therefore, not available for general photographic work.

Astronomical Photographs.— In *astronomical photography*, however, the chemical focus may be neglected, since in this case the point of best union of rays is not determined by focusing on the ground glass, but once for all by a series of experimental exposures on different planes at equal distances before and behind the focal plane, since in this case we are dealing only with infinitely distant objects.

Color Correction of Photographic Lenses.— For ordinary photographic work, an exact coincidence of the optical and the photographic image is required, and this is obtained by uniting the colored rays corresponding with the Fraunhofer lines D and G′. Generally speaking, however, we pay for this by an enlargement of the secondary spectrum, as compared with a lens achromatized only for the actinic rays. This sacrifice of good correction is without serious importance in practice, since in the case of modern objectives we do not use sorts of glass which, like the silicate flints, have especially large dispersion at the blue end of the spectrum.

Pairs of Glasses with Equal Partial Dispersions.— It is possible, however, and this was a point of Abbe and Schott's programme in the founding of the Jena glass works, to minimize the secondary spectrum by constructing achromats from kinds of glass, the partial dispersions of which are equal. In this case the common focal point of two colored rays is also that of the others, so that an achromatism of higher order is obtained. As a matter of fact, pairs of glasses have been prepared which, by themselves, completely remove the secondary spectrum.

THREE-COLOR WORK.— In practice, however, the application of objectives thus constructed is limited to the cases in which color separations with light filters are made, as in the three-color process. It is an absolutely essential condition in this work that the partial images must coincide, especially in the case of large work.

APOCHROMATS.— In all other cases, *apochromatism*, as this higher grade of achromatism in photographic lenses has been called, is unnecessary. Considering the difficulty of making these special Jena glasses with coincident partial dispersions available for the construction of anastigmats, it is advisable to investigate carefully any claim that an ordinary hand-camera lens is apochromatic.

COLOR-SENSITIVE PLATES.— If we use, instead of ordinary plates, color-sensitive emulsions, the region of effective rays is lengthened toward the red end of the spectrum. If we are working without a filter, the blue sensitiveness is nevertheless so predominant that even here the ordinary achromatizing for the D and G′ lines is the best.

ACHROMATISM IN VISUAL INSTRUMENTS.— If we eliminate by a yellow filter the ultra-violet part of the spectrum which is invisible to the human eye, we must theoretically achromatize just as is done in the case of telescope lenses for visual purposes, that is, by uniting the C and F lines. Apart from the impossibility of using the same objective in several constructions for different kinds of plates, such a pure optical achromatism is useless, because the photographic image is considerably less sharp than the visual image of the telescope, and is not subjected to nearly as much enlargement as this is. In this case, also, therefore, the secondary spectrum may be disregarded.

Achromatizing for Focal Length.— If we now consider the case of photographic objectives of finite thickness, we can apply to them our conclusions drawn from infinitely thin lenses. We must merely recognize that focal length and vertex distance are no longer the same. It is evident that it is not sufficient to effect achromatism only in respect to the vertex distance; that is, to produce the different colored images in the same plane. They must also be of the same size, so that the edge of the image field shall show no colored fringes. This necessitates that the *achromatism* shall exist for the *focal distance*. Only when these two conditions are fulfilled can a photographic objective be called completely achromatic. Generally speaking, the achromatizing for focal length demands a particular arrangement of the axial distances of the refracting surfaces. We shall see, however, that the uniting of two objectives to form a symmetrical or quasi-symmetrical double objective, of itself produces the necessary coincidence of the focal length.

Chromatic Difference of the Aberrations.— In the same way as the vertex and focal distances, the different aberrations are also affected by the change of the wave length and color of light, as a result of their dependence on the indices of refraction. They, therefore, have different values in different parts of the spectral region which produces the image on the plate. These chromatic differences of the aberrations are, however, of little significance for us, because they are reduced to very small amounts by the absolutely necessary axial achromatizing. We must always bear in mind that, because of the considerable depth with which, fortunately, an overwhelming majority of pictures are made, these differences fall within the latitude of permissible unsharpness. Even the chromatic difference of the spherical aberrations, which is evident in a spherical under-correction for the yellow-green rays, and an over-correction for the ultra-violet rays, could only become dangerous in the case of color separations on very large plates. Since, however, special lenses are always used for this purpose and stopped down sharply, this error, which in the case of microscope objectives requires most careful elimination, is unimportant in photographic lenses.

Attention to Color in Comparative Exposures. — We may finally remark, although it is only remotely related to the subject of this chapter, that when making comparative exposures for determining the working speed of an objective, which must be undertaken under exactly similar circumstances, we must always work with light of the same color. If we expose on one occasion on a green object, and the next time on a dark blue one, we would obtain different results for the speed of the lens, since the plate is not equally sensitive for all kinds of light.

CHAPTER XIV

The Principal Types of Lenses

We will now investigate how the commonest types of photographic lenses differ from each other, and on what principles they are constructed. On account of the large number of systems and the small space at our command, it is, of course, not possible to mention all types. We shall confine ourselves, therefore, to the best known, and refer those who wish to make a more thorough study to the book already mentioned, "Theorie und Geschichte des Photographischen Objektives," by Moritz von Rohr, as well as to the publications of the various optical manufacturers and their scientists.

The explanatory figures show the types of objectives only *diagrammatically*, and, therefore, no conclusion as to the elements of construction can be drawn from them.

We have made a general division of lenses into astigmats and anastigmats, according to the quality of their corrections for astigmatism and flatness of field.

PART I. ASTIGMATS

1. The Simple Collecting Lens

The Best Form for Photographic Work.— As we have already discovered, it is not possible to correct a collecting lens spherically. The most favorable form, for which the spherical aberration for parallel incident light is as small as possible, is a double convex lens with strongly convex front surface, the radii of which have about the relation of 1 : 6. Such a lens cannot be used for photographic purposes, since even when strongly stopped down the center alone appears sufficiently sharp. Far better results are obtained if the lens is so constructed that it is concave toward the incident light, and thus has the form of a *meniscus*. In this case, however, the spherical aberration on the axis increases considerably, and correspondingly the central sharpness diminishes. The position of the astigmatic image-surfaces is better, however, so that with considerable stopping down and proper placing of a front stop, we may attain a field of view which allows its use in portrait photography. This form of lens was introduced by Wollaston about a hundred years ago under the name "periscope," for projection in the camera obscura. In modern times it often finds application for artistic purposes (under the names "monocle" or "spectacle lens"). It is to be noted, however, that the simple meniscus is not chromatically corrected, and, therefore, has chemical focus. If we desire to obtain an image as sharp as possible, we must, therefore, after focusing on the ground glass, move this toward the objective by an amount equal to the focal length divided by the glass constant ν. As ν, in the case of the crown glasses from which spectacle lenses are cut, is to be determined from the color dispersion between the D and G′ lines, the chemical focus is about two per cent of the focal length, or in the case of an eight-inch lens about one-sixth of an inch. In addition, we must note that the position of the ground glass is also dependent on the amount of stopping down; the larger the aperture, the nearer to the objective must we bring the ground glass.

2. The Achromatic Collecting Lens

The Achromat with Convex Cemented Surface. — The chromatic errors of the collecting meniscus were avoided by constructing a lens, such as is shown in Fig. 32, of a dispersing bi-concave and a collecting bi-convex component. After the focal length had been determined for the available kinds of glass, the further requirement was made that the lenses were to be cemented together. The inner surfaces were thus given equal curvatures of opposite sign, and the first radius could now be so determined that the working of the lens should be as satisfactory as possible. This system is well known under the name *landscape lens*. It is advantageously used with a small stop, which should not exceed *f*: 18; these objectives can, therefore, not well be used for hand cameras. Today one returns to them only on account of the moderate price.

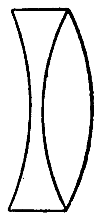

Fig 32

The Spherically Corrected Achromat.— It became evident that one could not accomplish much with the form of lens shown in Fig. 32, since the spherical correction was not sufficient. An essential advance occurred, however, with the introduction of a form in which, as Fig. 33 shows, the arrangement of the glasses is reversed. By adding to a collecting meniscus of crown glass of low refractive index a dispersing one of flint glass, it became possible, along with complete spherical correction, to increase the central sharpness and to obtain a fairly satisfactory disposition of the astigmatic surfaces for such a simple construction of the system. Such achromatic

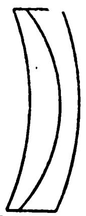

Fig. 33

lenses, working at f: 12.5, have a sharpness sufficient for many purposes, but must usually be stopped down in order that the pencils passing through the stop shall have so small a diameter that the circles of confusion shall remain below the limit of recognition. We shall later see what effect the small thickness of the lens has.

3. The Aplanat of A. Steinheil as Representative of the Symmetrical Objectives

The Symmetrical Objective.— It is apparent that one can obtain a better spherical correction by constructing the achromatic meniscus of three crown and flint glasses, instead of two. All objectives of this kind, as well as those just described, lack, in addition to astigmatic correction, those for coma, chromatic enlargement difference and distortion, which, on account of the position of the stop before the lens, is barrel-shaped. It was, therefore, a great advance when a new type was constructed through a very simple combination of well-known forms, which, without further complication, removed these three errors. This is the symmetrical objective.

Freedom from Distortion for Copying Full Size.— If, for instance, we place two similar achromats, such as are shown in Fig. 33, symmetrically to the front stop, the principal rays pursue parallel paths before and after refraction. Since, however, as we have previously seen, the aberration of the oblique pencils affects the positions of the pupils alike when the object and image lie symmetrically to the objective, the symmetrical objective is absolutely *free from distortion* when object and image are of the same size. The distortion is, however, practically eliminated for every distance of the object, since the zonal errors remain small. If the two halves of the double objective are not alike, but only similar, the distortion is absolutely corrected for a relation of size between object and image which depends on the constants (radii, thicknesses) of the two parts. How the halves are corrected in other ways is quite without significance for the removal of distortion.

COMA.— If the sizes of object and image in the case of a symmetrical, spherically corrected double objective are equal, the pencils of rays emanating from the axial point are parallel between the two halves of the objective (Fig. 34). If we now follow the course of all meridional pencils in the object and image space which

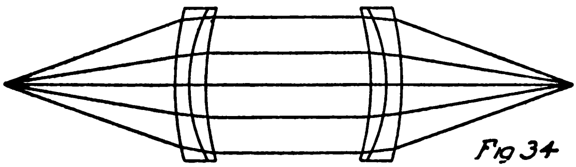

Fig 34

have parallel paths between the lenses, a little consideration shows that these pencils are also spherically corrected, and that the system is free from *coma* as far as these pencils are concerned. The law of freedom from coma of symmetrical double objectives is absolutely true only for the special case mentioned. If the object has another position, coma may be apparent when the halves of the objective have considerable comatic errors and the size of the aperture exceeds a certain limit. If the halves consist of cemented lenses, even at an aperture of f: 4.5 the double lens is practically free from coma for any object distance, while this is not true for the symmetrical anastigmat composed of four single lenses, which we shall later consider, the halves of which, on account of their strong coma, form a satisfactory image only with small stops. Thus it is that the astigmatic double objective of the aplanat type, even at large apertures, produces a very brilliant image on account of its practical freedom from coma.

ERRORS OF THE CHROMATIC ENLARGEMENT DIFFERENCE.— A further, likewise simple, consideration shows that the color dispersions of a pencil in the two halves of a symmetrical lens have opposite characters. If white light is incident, after its passage through the objective the colored rays are again united to white light, consequently the images of different colors have the same size, without this necessarily being true in the case of the halves. If, therefore, the objective is to be sufficiently achromatic for practical purposes, both for the vertex distance and the focal distance, it is only necessary to achromatize the halves for the vertex distance; in the case of the whole lens, the colored partial images then fall of themselves into coincidence, as well in their *position in space* as in *size*.

INCREASE OF THE WORKING APERTURE.— By thus constructing a lens of two like or similar halves, not only were coma, distortion, and chromatic enlargement differences, so to speak, automatically removed, if the halves only and thus also the whole were spherically corrected, and had equal vertex distances for the different colors, but, greatest advance of all, the *relative aperture* increases to almost twice that of the halves. If we could consider the halves as infinitely thin, and bring them infinitely near, the focal length of the whole objective would be half that of each part. In the case of finite dimensions and distances, however, the focal length is increased by a certain amount, so that the aperture is not quite double. The time of exposure is, therefore, somewhat less than fourfold for the halves.

Numerous Names for Steinheil's Aplanat.— These properties explain the tremendous enthusiasm with which the symmetrical double objective, which in its essentials we owe to A. Steinheil of Munich, was greeted by photographers in the second half of the sixties of the last century. Even today, under the most varied names, the Steinheil aplanat is a great favorite, because of its relatively great working possibilities. It would be well worth while for us, even today, to remember occasionally that good pictures were made even before the introduction of anastigmats, and that photo-engravers did first-class work in the eighties with the old wide-angle lenses.

Kinds of Aplanat.— The aplanat, or rapid rectilinear, consists of two similar halves placed symmetrically to the stop, each consisting of a collecting meniscus next to the stop, and a dispersing one on the outside. It is constructed in all possible gradations of working aperture, or so-called *series*. Roughly, we can separate four kinds which have proved themselves to be especially practicable.

The Portrait Aplanat.— In the series of greatest aperture, which are still used, but only for *portraits*, it is not practical to exceed the working aperture f: 4.5. The halves are widely separated, and because of this, the lens vignettes considerably.

The Group Aplanat.— Next comes the group aplanat, with the relative aperture f: 6.3, whose halves are placed nearer together, so that the axial cross-section of the aplanat approaches more nearly a square as the relative aperture decreases. This series is very often used in photographic studios for *group* pictures, in which a certain amount of stopping down must be done to obtain the necessary depth.

The Hand-camera Aplanat.— For amateurs the aplanat is constructed to work at $f: 8$ to $f: 9$ as a cheap *hand-camera lens*, and is much used. Whether it is really advisable for a beginner in his first photographic work, which is generally done with a hand camera, to use a lens *without* astigmatic flatness of field, we shall leave undecided; he will certainly accustom himself more quickly and satisfactorily to the technique of picture making if he begins with a modern lens. On the other hand, he must be warned that the designation of a photographic objective as an anastigmat is no proof that this actually possesses anastigmatic flatness of field, since this term is unfortunately only too often applied to astigmats.

Wide-angle Aplanat.— For *wide-angle* exposures there is offered the least efficient series of aplanats with a relative aperture of $f: 18$, which, however, has been completely displaced by the modern wide-angle anastigmat.

Varieties of Glass.— A. Steinheil constructed all of his aplanats of lead-silicate-flint glasses, as well the collecting as the dispersing lenses. He thus obtained a tolerably good correction of astigmatism. The larger the aperture the more carefully should the spherical correction be carried out, while in the wide-angle series, the flattening of the field was most important. The difference of the coefficients of refraction and the constant ν of the two single lenses had to be larger, in accordance with the perfection of central sharpness desired. After the introduction of the Jena glasses, it became possible to replace the somewhat yellowish lead-silicate-flints by colorless crown glasses, which increased the brilliancy of the lens. The central sharpness of the aplanats is good, and absolutely equal to that of the symmetrical anastigmats of the same aperture.

WORKING QUALITIES.— If we desire to properly appraise the working qualities of the aplanat, we must not forget that the practical correction for coma includes only the rays in the meridional planes. The whole system is, therefore, because of the construction of the halves, afflicted throughout with astigmatism and curvature of field, and is hence astigmatic. It is true that by increasing the distance between the halves we can somewhat stretch the astigmatic image surfaces and thus bring them nearer to the plane of the ground glass; in this process, however, the astigmatism increases, so that this practice, which is exhaustively treated in all the old text-books of photographic optics, is practically of small value. If we desire to have the sizes of plates given for aplanats in price-lists actually covered, we must stop down until the edge of the image has sufficient sharpness for the purpose desired. The data as to the size of field of astigmats should never be taken in any other way. Here and there we find the remark that the old objectives have more depth than the new anastigmats, which, as we have previously seen, is incorrect; this, however, may be easily explained, since one earlier always used small stops which carried with them increase in depth, while today we try to avail ourselves of the especial properties of the anastigmats at the largest possible apertures.

4. A. Steinheil's Periscope

The Non-achromatic Double Objective.— Even before the construction of his aplanat, Steinheil had placed upon the market a symmetrical objective, known as the *periscope*. It consists, as Fig. 35 shows, of two like menisci, symmetrical to the stop. It, therefore, shares with the aplanat freedom from distortion and greater working aperture than its halves, but is neither spherically nor chromatically corrected. It, therefore, has chemical focus, and consequently the plate-holder must be moved toward the lens after focusing, as in the case of the simple meniscus. The periscope, under innumerable names, is used as a cheap, hand-camera lens, as a rule with a fixed focus with allowance made for chemical focus, so that no ground glass is necessary. It is usually stopped down to about $f:12$, so that the sharpness and size of field, even in the case of short focal lengths, are in some measure satisfactory. Because of the small thickness of the glass, there is only an insignificant loss of light in passing through the lens, so that with good illumination one can count on sufficiently exposed images. In isolated cases lenses of this type of considerable focal length are used for artistic pictures, characterized by flatness and softness of image.

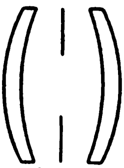

Fig 35

We will but mention the aplanat sets of Steinheil. We will return to a full consideration of the formation of sets of objectives in our consideration of the first anastigmats.

5. The Portrait Objective of J. Petzval

Great Brilliancy.— One of the most noteworthy achievements in the realm of photographic optics is the construction and marketing of the well-known portrait objective of J. Petzval, in the year 1840. While previously, a photographic portrait had only been possible in exceptional cases, because of the small aperture of the lenses in use, at a single step all doors were opened to photographic art because of the great effective aperture (*f*: 3.2) of this lens. The extraordinary progress caused by the introduction of this new instrument must be more highly esteemed by us when we consider that Petzval approached a completely unworked field, for the opening of which only the coarsest empirical methods had been previously applied. It was only because he based his constructions on the most solid mathematical foundations that he was able to solve this difficult problem in a way which, even today, after more than seventy years, must excite our greatest admiration.

Asymmetry.— The Petzval portrait objective (Fig. 36) consists of four lenses assembled in two groups. The front combination consists of a cemented pair, crown before flint, while the second contains two separated lenses, flint before crown. In order to obtain a better spherical correction, the objective is now constructed according to the proposal of J. H. Dallmeyer in 1866, with the crown before the flint in the rear combination, but still with the separated lenses. It is completely unsymmetrical in both cases. The very long con-

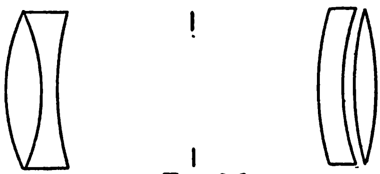

Fig 36

struction of the objective is characteristic, and in this it is only surpassed by the similarly constructed objective with the aperture $f: 2.3$ of H. Zincke-Sommer. By this construction, the astigmatic image surfaces were somewhat extended, so that a passably satisfactory extent of the sharp image field for portrait purposes was obtained.

Quality of Corrections.— The sharpness and brilliance of the image on and near the axis is very good, because of the excellent spherical correction and freedom from coma; the same is true of the removal of distortion and the chromatic enlargement differences. As far as it can be compared with the portrait aplanats, with their extremely different aperture relations, the Petzval objective surpasses these in central sharpness and in this respect is almost equal to the modern anastigmats of large aperture. Unfortunately, it is not suitable for ordinary work on several grounds besides the small size of the field. In the first place, it is clumsy; in addition, it vignettes seriously,— a necessary consequence of the great separation of the components. Nevertheless, this type of lens, which is constructed with different apertures between $f: 3$ and $f: 5$, is found in most studios because of its great aperture.

Application for Projection.— In addition, however, it is an extraordinarily good lens for *projection*, especially with large sources of light, and is also extremely good as a motion-picture lens. A peculiar property is the very small increase in effective size of field on stopping down; this has, as almost its only effect, an increase in depth.

We must not omit to call attention to the fact that the Petzval objective must be very carefully constructed if it is actually to show its whole efficiency. Unfortunately, this cannot be said of most of the so-called projection lenses made on the Petzval formula.

6. The Antiplanet of A. Steinheil

The Idea of the Antiplanet.— We will but briefly allude to this very interesting creation of A. Steinheil, the basic idea of which is unique. If we consider especially the portrait *antiplanet* of large aperture (Fig. 37), which was produced in the year 1881 and possesses an effective aperture of about $f:4$, we find in the front group, consisting of a cemented pair, crown before flint, a strong concentration of optical defects, which are removed by the defects of opposite sign occurring in the two rear and separated lenses, flint before crown. It was thus possible to increase the efficiency of the lens within certain limits, but only by sacrificing the possibility of increasing the size of field by stopping down.

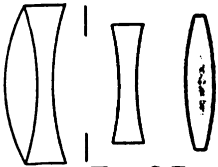

Fig 37

In addition to the portrait objective, we may still occasionally find in use the group antiplanet with relative aperture of about $f:6$, consisting of two pairs of cemented lenses, of which the rear combination is unusually thick. This also depends on the basic idea of the concentration of opposite aberrations in the two halves, and consequently is also unsymmetrical. Although the antiplanet type allowed a better correction of astigmatism and curvature of field than the older astigmats, it could make no progress against the anastigmatic objectives, the construction of which owed its beginning to the productions of the Jena glass factory.

PART II. ANASTIGMATS

On what Astigmatism Depends.— An important characteristic of the great group of photographic objectives just considered is a position of the astigmatic image surfaces such that a practically sharp image can be obtained only in case of a very moderate inclination of the principal rays to the àxis. As we saw in considering the Steinheil aplanat, the amount of the astigmatism and curvature of field depends on the particular varieties of glass from which the cemented crown and flint lenses are constructed. While, for instance, these two errors are very large in the case of the aplanats of large aperture, they can be considerably diminished in wide-angles. This is based on a general law that the amount of the astigmatism depends on the difference of the indices of refraction of the cemented crown and flint glasses. While this difference in the best aplanats must amount to about 0.8 in order that the spherical correction may be satisfactorily effected for the large aperture, it sinks to 0.35 in the case of the wide-angle, which has to be corrected only for astigmatism, and in which, because of the small aperture, large zonal errors of spherical aberration are allowable.

The Normal Glass-pair.— It is, therefore, not possible, with aplanatic halves, that is, spherically corrected achromatic objectives where the collecting lens refracts and disperses less than the dispersing lens, to obtain an astigmatic correction. These *normal glass-pairs* were all that opticians could obtain until the founding of the Jena glass factory. If it were possible, however, to make the difference of the coefficients of refraction of the crown and flint glasses negative, that is, to construct the collecting lens of a more refracting glass than the dispersing lens, it must be possible to cause an intersection of the astigmatic image-surfaces and even to carry them over to the opposite position.

ANOMALOUS GLASS-PAIRS.— OLD AND NEW ACHROMATS.— The great achievement of Abbe and Schott was actually to construct such pairs of glasses. As we have outlined, the crown glass must have a greater index of refraction, and a smaller color dispersion, than the flint glass. Such a pair is called *anomalous* and achromatic lenses constructed from them, *new achromats*, in distinction to the *old achromats*, names very satisfactorily chosen by O. Lummer. The typical anomalous glass-pair consists of the heaviest barite crown: $n_D = 1.61$, $\nu = 57$, and the highly dispersive crown: $n_D = 1.52$, $\nu = 52$, while a typical normal pair of glasses for an ordinary telescope objective consists of the hard crown: $n_D = 1.517$, $\nu = 60$, and the heavy flint $n_D = 1.621$, $\nu = 36$. We see from these numbers that the earlier designations, simply crown and flint glasses, for the materials of the collecting and dispersing lenses in an old achromat, are no longer tenable; instead, we must always designate as crown the glass whose optical constant ν is smaller than that belonging to the glass to be called flint.

COLLECTING CEMENTED SURFACE.— Corresponding to the use of an anomalous pair of glasses in a new achromat, the cemented surface is *collecting*. This form of achromats, therefore, unlike the old achromats, cannot be spherically corrected, while the old achromat does not allow of an astigmatic correction, which can be attained in the new achromat.

1. The Protars of P. Rudolph

Anastigmats.— It was P. Rudolph of Jena who first showed that by the combination of an old and a new achromat to form an objective, both spherical and astigmatic errors could be removed. By properly separating the two groups of lenses, it is, therefore, possible to locate the anastigmatic point in the plane of the ground glass, and thus also to get rid of curvature of field. Such objectives, which in other particulars show the same quality of correction as the astigmats, are called *anastigmats*.

Opposition.— If we recall that the cemented surface in an old achromat is dispersing, and in a new achromat collecting, we can also perceive the possibility of an anastigmatic flattening of the field of a spherically corrected objective, when there exists *an opposition* in the gradation of the coefficients of refraction or in the action of the cemented surfaces separating the refracting media. If Rudolph's law is stated in this form, it includes also a case which is scarcely of practical importance, which is that one half of an anastigmat may be a dispersing lens. In this case this achromat also can be constructed with a normal glass-pair, so that anastigmatic flatness of field can also be attained with two old achromats. This variety, however, does not exist in any commercial lens.

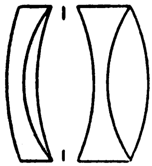
Fig 38

Protars.— The first fruits of Rudolph's work were the *Protars*, introduced by the optical establishment of Carl Zeiss in Jena in the year 1890. They are unsymmetrical doublets; that is, objectives consisting of two groups of cemented lenses with a central stop. According to their purpose the groups contain two or three lenses. At the present time Protars are still constructed in two series, IIIa and V. They show (Fig. 38) the typical form of the two opposed achromats: the first group consists of a normal glass-pair and has the form of half of an aplanat, while the rear half consists of a new achromat; the fourth lens is made of the heaviest barite crown, so important in the construction of anastigmats.

Series V working at f: 18 is the well-known wide-angle, which is surpassed in extent of field only by special objectives made for particular conditions. This lens is also suitable for certain kinds of reproduction work. Since we are always anxious to make use of the wide angle of field of this objective, it is used in focal lengths which are small in proportion to the size of the plate. On account of its small aperture, it is not used for artistic pictures. The Protar Series IIIa possesses a larger aperture, f: 9, but not sufficient to make it of great importance for general photographic work, although it may usefully be applied in cases where objectives are used with a small stop, especially in stereoscopic and architectural work. The longer focal lengths are used in reproduction work.

2. The Protar Set of P. Rudolph

Anastigmatic Lens Element with Front Stop.—The law of the opposed gradation of coefficients of refraction can, however, be also applied to the other form of a photographic objective, the single lens with front stop. If we undertake the task of freeing such a spherically corrected system from astigmatism and curvature of field, we find it can be attained by the combination of *three* lenses, which must have a certain relation to the parts of the anastigmat doublet, the old and the new achromat.

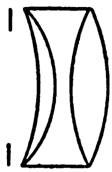

Fig 39

Three-glass Element.— For this purpose we replace (Fig. 39) the flint lens of the Steinheil aplanat by a new achromat, the front dispersing lens of which then forms an old achromat with the collecting meniscus placed next to the stop. Consequently the indices of refraction of the three lenses must increase as we pass from one to the next in the direction of the incident light. The first is a meniscus of silicate glass with low refraction and color dispersion; the middle one bi-concave of a light flint of medium refraction, but color dispersion greater than that of the two other lenses; and the third a bi-convex lens of the heaviest barite crown with the highest possible refraction and small color dispersion.

The Double Anastigmat.— This anastigmatically corrected objective with front stop was independently invented by two men soon after the introduction of Rudolph's doublet. While Rudolph had endeavored to increase the capabilities of this three-lens objective itself, as far as possible, and to use it as the basis for his anastigmat set, it was applied by E. von Höegh as half of an anastigmatic symmetrical double objective in order to increase the quality of correction of the whole. We will later investigate von Höegh's construction, known as the *double anastigmat*, in our study of the symmetrical anastigmats, and will now proceed further with Rudolph's single lens.

Four-glass Element.— This was essentially improved in 1895 by increasing its relative aperture to f: 12.5 and simultaneously completing its spherical and astigmatic corrections, and now is the basis of the Series VII Protars. In it, because of the increase of the number of lenses to *four*, the two forms of achromat are clearly apparent. The first two lenses (Fig. 40) the bi-concave and bi-convex, form the new achromat; the last two, corresponding in form to half of a Steinheil aplanat, form the old achromat.

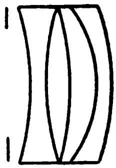
Fig 40

CONVERTIBLE SETS.— This single lens is the basis of the convertible Protar sets. We designate as a *set* any desired number of independently corrected objectives with front stops, similar to one another, by which we mean that their constants (radii and thicknesses) may be derived from one another by proportional changes. All the halves may for convenience be screwed into the two ends of a lens tube, so that we may use them singly or combined in pairs. The simplest set consists of two halves of different foci, which give three focal lengths. Sets of three elements give six focal lengths, and so on. As we have already seen, such a combination of like or similar halves automatically corrects distortion, coma, and chromatic enlargement differences. If, therefore, each half-lens is as well corrected as possible, the double objective must be equally well corrected and also have the advantage of a much larger aperture. The possibility of being able to photograph a given object from any point in different sizes is such an enormous advantage that we can easily understand the long-continued efforts of opticians to construct an objective set.

THE DOUBLE PROTAR.— Corresponding to the aperture *f*: 12.5, of the Series VII elements, the double objective Series VIIa, composed of two like lenses, has the aperture *f*: 6.3. To preserve this advantage, it is advisable not to exceed a focal relation of 2 : 3 between the elements. This limit corresponds to an equivalent focus for the double objective of 1.33 compared with a lens of equal halves, and to the aperture *f*: 7.7. Naturally this does not exclude the presence of shorter and longer foci in the whole set; the lenses, however, should be combined only in such a way that this relation of foci is not exceeded. We must take care that the lens of longer focus is always placed *in front of* that of shorter focus. It is also important that we should never use a single element in the front end of the tube, so that the convex external surface is in front and the stop behind the lens, because this is corrected for rays incident in the other direction. For the same reason the front lens of a double objective must always have its convex external surface turned toward the incident light, because the corrections are dependent on a nearly parallel course of rays between the lenses.

Stop Marking.—As we have already mentioned, Rudolph marked his objective sets with a millimeter scale giving the sizes of the apertures. It was thus possible to get along with a single scale, but in order to determine the ratio of the aperture, it was necessary to consult a table giving the relation between stop diameter and aperture ratio. For sets consisting of only three elements, it is advisable to engrave the lens barrel with six scales corresponding to six marks on the ring of the iris diaphragm; with a greater number of single lenses, even with the help of a movable tube, the legibility of the scales is difficult to preserve.

The Halves of the Double Objective in Con-

VERTIBLE SETS.— Because of the excellent correction of the Rudolph element for coma and chromatic enlargement difference together with its relatively great speed, it has not yet been equalled in performance by any other lens serving the same purpose. Its nearest competitors are the three-glass cemented elements of the objectives of the double-anastigmat and orthostigmat-collinear type; in combination these naturally perform much better than their elements. A large number of symmetrical objectives, however, cannot be used as convertible objectives. This is, for instance, the case with all double objectives, the elements of which are composed of two separated lenses. If the halves are not fairly well corrected for coma and chromatic enlargement difference, they can never be recommended as convertible lenses, since they must be stopped down too sharply to obtain even a moderately useful image. This naturally does not prevent completely or nearly symmetrical double objectives composed of such elements from performing well, as we have previously seen.

3. SYMMETRICAL OBJECTIVES WITH CEMENTED ELEMENTS

THE DOUBLE ANASTIGMAT.—As we have already mentioned, the efforts of E. von Höegh were directed to improving the quality of the symmetrical double objective. He produced the *Double Anastigmat* which was introduced by the optical firm of C. P. Goerz A.-G. in Friedenau, and which was very satisfactorily corrected when it first appeared. Its construction (Fig. 41) is that of a symmetrical objective

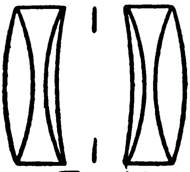

Fig. 41

composed of two like Rudolph anastigmat elements of the first type (Fig. 39), but according to the view of the discoverer, its corrections are better than those of this objective, even when the element is corrected as well as possible. The elements of the double anastigmat, however, do not equal the best corrected single lenses. The effective aperture is *f*: 6.8 for the shorter foci and less for the longer; we have seen that this has its origin in the necessary reduction of zonal errors when enlarging the objective constants for longer foci.

ARRANGEMENT OF THE CEMENTED SURFACES.— Von Höegh's double anastigmat possesses a characteristic arrangement of *cemented surfaces*. The dispersing cemented surface nearest the stop is concave to this and the next one is collecting and convex. It is thus possible to very effectively carry out the spherical and astigmatic corrections.

SECOND FORM OF THE DOUBLE ANASTIGMAT.— There is, however, a second arrangement of the three lenses in which the cemented surfaces have equal curvatures, as shown in Fig. 42. Here the arrangement of the two cemented surfaces is reversed, and the bi-convex lens of the heaviest barite crown is placed in the center. This objective was announced by von Höegh as a second form of his double anastigmat; it has been made for many years by Watson of London under the name *Convertible Lens*. The Zeiss Convertible Series IV also has this form.

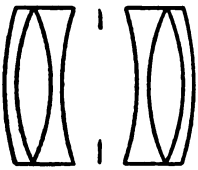

Fig. 42

To give full details of the construction of these, and other types to be later considered, in the different series, corresponding to varying apertures, would consume too much space. Full details as to the effectiveness of the various lenses may be obtained from the catalogs of the optical firms.

Orthostigmat and Collinear.— Some years after the introduction of the double anastigmat the optical firms of Voigtländer & Sohn in Brunswick and Steinheil in Munich produced a type which was simultaneously invented by H. Scheffler and R. Steinheil. The rear element of this symmetrical objective (Fig. 43) consists of a bi-concave lens of medium refraction but great color dispersion, to which is joined a collecting meniscus with small refraction and dispersion; the third component is the bi-convex lens of the heaviest barite crown. The anastigmatic flattening of field is here produced by the convexity of the collecting cemented surfaces toward the stop. If the meniscus is given a rather large central thickness, the correction can be very satisfactorily carried out, so that the elements may even be used as single lenses with some stopping down. This type is produced in several series by Steinheil under the name *Orthostigmat* and by Voigtländer under the name *Collinear*. We shall later have to refer to another variation of this type, when we consider the objectives in which the secondary spectrum is diminished.

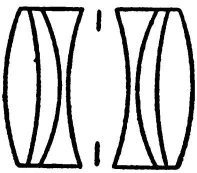

Fig. 43

Eight-lens Double Objectives.— A number of eight-lens systems have been evolved from these symmetrical objectives with three-glass elements, among which we may mention Rietzschel's *Linear*. We have already thoroughly described Rudolph's four-lens combination sets. All these objectives are composed of an old and a new achromat, and differ only in the arrangement of dispersing and collecting cemented surfaces. In practice they do not surpass the previously mentioned symmetrical anastigmats consisting of six lenses, while with the number of cemented lenses, the difficulty of centering, that is, the exact placing of all centers of curvature in the axis of the lens tube, increases. For this reason opticians have given up the construction of ten-lens symmetrical anastigmats, in which the absorption also becomes very noticeable. When it is a question not of a single element but of a whole symmetrical double objective, there is no advantage in increasing the number of lenses over six. How far one can go in increasing the aperture remains to be seen.

4. Von Höegh's Anastigmatic Meniscus

The Petzval Condition.— Pseudo-focus.— The construction of an objective of an old and a new achromat presents a particular case in which an anastigmatic flattening of field can be attained. It is the consequence of a general law announced by Petzval in 1843, according to which anastigmatic flattening of field is present when the sum $\frac{1}{f_1 n_1} + \frac{1}{f_2 n_2} + \ldots = 0$, where $f_1, f_2, \ldots$ are the focal lengths of the single lenses of the objectives, *neglecting the lens thicknesses*, and $n_1, n_2, \ldots$ are the corresponding indices of refraction. We must, therefore, calculate f_1 from the equation $\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$, and similar values for all the lenses; these values of f, which are, therefore, called also *pseudo-foci*, are different from the *actual* foci. It is remarkable that this law of Petzval is valid for groups of lenses of any thickness. Because this, however, was overlooked by Petzval himself, it was believed for half a century that, as a matter of fact, one could obtain an astigmatic flattening of field *only* through the use of highly refracting crown glasses, a possibility which can be easily calculated from Petzval's law, and which was confirmed by the construction of the first anastigmats after the production of the heavy barite crown in Jena. When we apply this to lenses of *finite* thickness, we arrive at curvatures of the refracting surfaces which make possible the construction of anastigmats even out of the old glasses, and produce entirely new forms.

THE ANASTIGMATIC MENISCUS.— The simplest form is the *anastigmatic meniscus* (Fig. 44), invented by von Höegh, which is reduced to practice in the *Hypergon double anastigmat* produced by Goerz. In accordance with the Petzval condition, the two surfaces of the meniscus must have equal curvatures in the same direction. In the case of an infinitely thin lens, this produces an infinitely large focal length, but as the thickness increases, the focal length finally becomes positive. The anastigmatic flattening for the meniscus and the double objective is very good; according to the arrangement of the anastigmatic point for any given angle of view, the sum of the Petzval equation has a small finite value.

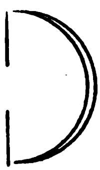

Fig 44

ABERRATIONS OF THE MENISCUS.— DEVIATION OF THE PRINCIPAL RAY.— The meniscus can naturally not be corrected for the other aberrations. If this is to be achieved, the lens must be composed of several

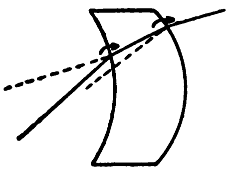

Fig 45

glasses, without, however, introducing a deviation from the Petzval condition. Thus we arrive at all forms of objective and, therefore, von Höegh could properly, in the year 1900, claim that his anastigmatic meniscus was the basic form of all anastigmats.* The only condition is that the deviation of the principal ray (Fig. 45) at the inner dispersing and the outer collecting surface shall each time be in the *same* direction, so that the astigmatic deformation of the infinitely small pencil at the inner surface is neutralized by the same action at the outer surface. In choosing lens constants, one must, therefore, fulfil the Petzval condition in order to obtain flatness of field. How the surfaces are arranged is unimportant as long as these conditions are fulfilled.

Division of the Meniscus.— If, as in the aplanat element, a dispersing surface is introduced into the meniscus, spherical correction is only possible by omitting to fulfil the Petzval condition, since otherwise the dispersive action of the two first surfaces must be eliminated by a stronger curvature of the outer surface, which is incompatible with spherical correction. We must, therefore, in order to again remove the introduced astigmatism, add a second surface which deviates the principal ray in the same direction as the dispersing surface and also collects. This can only be achieved by introducing a *collecting cemented surface*, concave to the incident light, and, therefore, bounding a bi-

* See Archiv für wissenschaftliche Photographie, Volume 2, 1911, pages 83, 134, and 167.

convex lens of high index of refraction. Now, the thus-formed three-lens objective actually consists of an old and a new achromat; the law of Rudolph must, therefore, be completed by stating that the collecting cemented surface to correct astigmatism must be *convex* to the incident light.

EXPOSURES WITH THE HYPERGON.— The Hypergon double anastigmat derived from von Höegh's meniscus was the first symmetrical anastigmat which could be constructed without using the heavy barite crown. In order to use it practically, it must be strongly stopped down and account taken of the chemical focus, which is limited as far as possible by the choice of a glass of small dispersion. It is sharply focused with the stop *f*: 22 and exposure made at *f*: 31. Since, on account of the extraordinary expansion of the angle of view to 140°, vignetting becomes very apparent, a Stolze star-stop (Fig. 28, p. 100) is placed before the objective during the exposure and set in rotation by a blast of air. It is thus possible, with a focal length of eight inches, to cover a plate 24 x 28 to the edges without noticeable dropping off of the illumination.

THE PANTOSCOPE.— It is notable that von Höegh's double meniscus had a predecessor in 1865, which is used even today. This is the *Pantoscope* of E. Busch (Fig. 46), whose flattening of field is almost astigmatic. It is a symmetrical objective constructed, of course, of old glasses, but already possesses the collecting cemented surface, which in this case must be concave to the stop. The order of the glasses is, therefore, the reverse of that of the Steinheil aplanat. Here also the

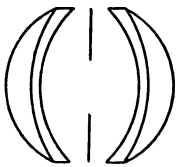

Fig 46

curvatures, as in the anastigmatic meniscus, are very great, so that the spherical aberration is very large, because of the absence of a dispersing cemented surface. This wide-angle lens, therefore, can only be used with a small stop, but then covers a field of 110°, which is surpassed only by the Hypergon. The chromatic correction of the Pantoscope is good, and thus a difference of chemical focus is avoided.

5. The Celor and Syntor of E. von Höegh

Development from the Orthostigmat.— The Orthostigmat or Collinear (Fig. 43) contains, between the bi-concave and bi-convex lenses, a collecting meniscus whose coefficient of refraction is smaller than that of its neighbors. If we assume this to be as small as possible, or unity, we obtain an objective (Fig. 47), whose halves consist of only two lenses and form a further development of the new achromat. The cemented surface has disappeared, and we have in its place an air-lens which has the form of a collecting meniscus, and therefore disperses. It thus becomes possible to eliminate spherical aberration, which is not possible in a new achromat consisting of two lenses cemented together. This type has since 1900 been constructed by Goerz in various series from calculations by von Höegh under the names of *Celor* for the more efficient and *Syntor* for the narrow angle series.

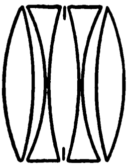

Fig 47

Quality of Corrections.— Noteworthy are the simple construction of this objective, consisting of only four lenses, and its compactness, because of the small vertex distance of the refracting surfaces. The anastigmatic flatness of field covers a large angle with small zonal errors, and this angle can be satisfactorily used because of the insignificant amount of vignetting. The zones of spherical aberration are also small; but here, for the first time in the case of a double objective, we find a not inconsiderable coma.

COMA OF THE HALVES.— The error of coma in the halves of a double objective consisting of two cemented groups of lenses is not considerable, even if it can not be wholly removed. On the other hand, the halves of a double objective, which consist of two separate lenses, can in general *not* be corrected for coma if they are required to have anastigmatic flatness of field; on the contrary, on account of their strong coma, they are useful to a certain extent only with small stops, and even then do not approach cemented anastigmat halves in capability. This is true even for the efficient *Celors*, whose aperture in medium focal lengths is about *f*: 4.5. In consequence of this the double objective also shows coma, in a greater degree as the aperture increases.

COMA OF THE DOUBLE OBJECTIVE.— While in the case of double objectives whose elements are cemented, we can discover no coma with a relative aperture of *f*: 5.4, and can even increase the effective aperture to *f*: 4.5 without detecting too serious a loss of brilliance in the image, in the case of a symmetrical objective composed of four single lenses coma is already noticeable at *f*: 6.3, and is so strong at *f*: 4.5 that these objectives cannot be compared for fineness of detail and resolving power with the unsymmetrical anastigmats which are corrected for coma. Whether the quality of the picture suffers from this lack, depends on the purpose for which this is intended. As we have already mentioned, anastigmatic elements consisting of two lenses not cemented together can in no case be used in objective sets.

6. The Symmetrical Objective of the Gaussian Type

Gaussian Telescope-objective.— The theory of the telescope-objective has given us knowledge of the type named after its discoverer, Gauss, which allows the removal of spherical aberration, not only for one, but also for any second portion of the spectrum, so that the chromatic difference of the spherical aberrations is thereby practically eliminated. Such an objective (Fig. 48) consists of two menisci with their concave sides facing the incident light, the first dispersing, the second collecting. The air-space between the two lenses has the form of a dispersing meniscus. This two-lens telescope-objective can also, without injuring its good spherical correction for two colors, be corrected equally well for astigmatism and curvature of field. For this purpose the second bending of the principal ray at the rear surface of the von Höegh meniscus, which occurs in the same direction as that at the front surface, is replaced by deflection through a separate rear lens. Nevertheless, the Gaussian objective, even when thus changed, possesses strong coma.

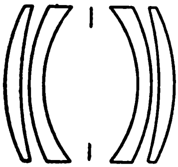

Fig. 48

The Double Objective.— The Planar.— If two thus spherically and astigmatically corrected objectives with front stops are united in the usual way to form a whole, we obtain a symmetrical objective, which in its simplest form contains four separate menisci. The idea of using the possibility of good anastigmatic correction inherent in this type in the way just described is due to P. Rudolph, and is incorporated in the *Planar* constructed by Zeiss in 1896 on Rudolph's calculations. In construction one more difficulty had to be overcome. It was found that with the choice of an index of refraction of about 1.57, which Rudolph believed most favorable, there could be found in the list of Jena glasses none whose color dispersion allowed the necessary chromatic correction. Therefore, each of the inner lenses was constructed of two glasses, whose refractive indices were equal while their dispersions were different. Each of the new double lenses behaves, in respect to all aberrations which are dependent only on the index of refraction for a given color, like a *single* lens, while their chromatic aberrations can be changed at will by change of the radius of the cemented surface, and thus be suited to the corresponding aberrations of the outer lenses. In the special case of the Planar, each *hyperchromatic* double lens, as Rudolph calls it, is composed of a bi-concave flint lens and a bi-convex lens of heavy barite crown, from which glass the external collecting meniscus is also made.

The Planar and the Petzval Lens.—The Planar has a relative aperture of about *f*: 3.5, and because of the strong coma of its halves, the whole symmetrical objective possesses a not inconsiderable coma at full opening, which can only be removed by stopping down, if the symmetry is preserved. From this objective we can perceive how fruitful was the impulse proceeding from the Jena glass factory. While the portrait-objective of Petzval, with the same aperture, has only a very small flat field, this is extraordinarily increased in the

Planar. The feat of obtaining that microscopic sharpness of image in the center of the field which the older objective possesses in a superlative degree, remained to be achieved, as we shall see, by the unsymmetrical anastigmat.

THE ARISTOSTIGMAT.— A few years after the invention of the Planar, H. Kollmorgen succeeded in proving, in spite of the belief of Rudolph, that it is possible to achromatize the Gaussian objective without losing the astigmatic corrections, or increasing the number of lenses. The necessary condition is that the two lenses of each element shall be constructed from an anomalous glass pair; therefore, that the coefficient of refraction of the collecting meniscus with smaller color dispersion must be at the least as large as that of the dispersing lens. The objective, thus characterized, consisting of four single lenses, was constructed by Meyer in Görlitz at the beginning of this century in several series under the name Aristostigmat, although with slight deviations from a strictly symmetrical form. Especially remarkable is the large flat field which this objective shares with that next to be described.

THE OMNAR.— The last and most remarkable step in the development of the simple Gaussian telescope-objective to a modern symmetrical anastigmat was made by K. Martin, some years after the appearance of Kollmorgen's lens. He showed that it was possible to eliminate the above-mentioned limitation in the arrangement of the indices of refraction of the halves, and that it was therefore easily possible to calculate, using the old glasses known before the existence of the Jena factory, an anastigmatic and achromatic objective with spherical correction for a finite aperture as large as desired. The form of this objective also is schematically represented by Fig. 48. That a limitation in the construction of the Gaussian objective to an anomalous glass pair is not necessary, follows, as a matter of fact, from the Petzval condition; for it is immaterial how we supply the effect of the external surface of the anastigmatic meniscus, if the bending of the principal ray at it is in the same direction as at the inner concave surface.

This discovery is the more remarkable because Alvan G. Clark had described as early as 1889 an objective which was similar to that of Martin, both in the shape of the lenses and the arrangement of the indices of refraction, without, however, possessing anastigmatic flatness of field.

From Martin's calculations came the *Omnar*, constructed by Busch of Rathenow and possessing the same quality of correction as the Aristostigmat. The coma, in the series of large aperture, is the same as in the other Gaussian lenses.

7. The Symmetrical Objectives with Cemented and Single Lenses

Derived Forms.— The fact that the halves of a symmetrical anastigmat consisting of three cemented lenses can be changed by the omission of one cemented surface opened a wide field for the activity of optical constructors. It would lead us too far to give details of the forms thus evolved, especially as little use has been made of them. This is because it appears that the slight gain in better correction, mostly for coma, stands in no practical relation to the considerably higher cost of manufacture. Makers have therefore generally preferred to stick to the three-glass cemented elements.

Von Höegh's Symmetrical Objective.— What results from separating the halves we may briefly see from an example given by von Höegh. If, in the elements of his double anastigmat, we open the first dispersing cemented surface, the objective separates into a solitary meniscus and a two-glass cemented lens whose cemented surface collects, and is convex to incident light. Since we can change the radii of the rear meniscus and the surface following it, without regard to one another, we have a free hand in the arrangement of the indices of refraction, and can make the middle bi-concave lens of a glass of low refractive index. Thus the difference of the indices of refraction at the collecting cemented surface becomes less, and hence the same quality of correction for astigmatism and curvature of field, as is usual in the case of cemented halves, indirectly leads to a better correction for coma. Nevertheless, this type has not been placed upon the market, probably because its advantage over the double anastigmats was too small.

8. The Unar of P. Rudolph

Reversed Properties of Air Lenses.— The idea of improving the quality of the corrections of an anastigmat by separating the cemented lenses led P. Rudolph, in the year 1900, to calculate his *Unar*, which was constructed by Zeiss. As we have seen, the idea on which the Protar was constructed was the production, by the introduction of a collecting and a dispersing cemented surface, of an opposition of properties which was necessary in order to produce anas-

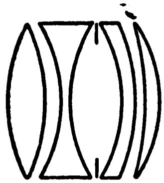
Fig 49

tigmatic flatness of field together with spherical correction for a finite aperture. If we give differing curvatures to the adjacent inner surfaces of the two halves, we obtain four single lenses (Fig. 49). If the objective is to be anastigmatic, we must again have an opposition of the air lenses in the two halves, one of which must have the form of a collecting, and the other that of a dispersing lens. It may easily be seen that the number of possible elements of construction (radii, thicknesses, and kinds of glass) is considerably increased, and, therefore, we are able to obtain better corrections in such a lens than are possible in the Protars, which consist of cemented lenses.

9. The Unofocal of R. Steinheil

The Petzval Condition for a System of Two Lenses.— A very ingenious application of the Petzval principle was made by R. Steinheil in the construction of a system of two lenses. If we assume that the coefficients of refraction of two lenses are equal, it follows from the Petzval equation that their pseudo-foci are equal and opposite in sign. The spherical correction is made possible, as previously mentioned, by two refractions in the same direction, with the assistance of suitable changes of thickness and distance, and the chromatic correction is effected by a suitable choice of glasses. If we unite two objectives thus calculated (Fig. 50), we obtain a four-lens symmetrical objective, which is corrected in the usual way for

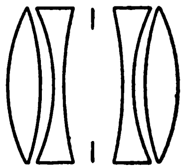
Fig 50

distortion and chromatic enlargement differences, and at moderate apertures is practically unaffected by coma. This lens is constructed from Steinheil's calculations in his establishment under the name of *Unofocal.*

10. The Cooke Lens of H. Dennis Taylor

A Coma-free Anastigmat.— In the year 1894 H. Dennis Taylor, the scientific director of the optical establishment of T. Cooke & Sons in York, made known the construction of an anastigmat the properties of which deserve careful study. Taylor attempted to calculate a lens with anastigmatic flatness of field which should be *free from coma*, independent of the correction for other aberrations. He found that the simplest form was an objective consisting of three separate lenses, the two outer glasses of which collect, while the inner disperses. As an example we may consider two series constructed by Voigtländer and Sohn under the names *Triple Anastigmat* and *Portrait Anastigmat.* In the first less efficient series with the relative apertures f: 6.8 to f: 9 (Fig. 51), the two collecting lenses are constructed of the heaviest barite crown and the central one of light flint, in which a greater coefficient of refraction is united with a lesser color dispersion. The two front lenses, which together have a very large focal length, are placed close together, while the third glass stands a proportionately great distance. In the faster series with the aperture ratio f: 4.5 (Fig. 52), the collecting lenses

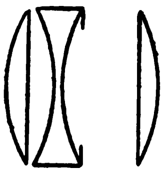

Fig. 51

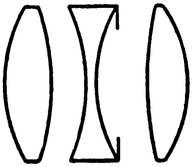

Fig. 52

are constructed of heavy barite crown, the central one of heavy flint; in contrast to the previous series, a larger coefficient of refraction here corresponds to a larger color dispersion. The distance between the first two lenses is somewhat less than that between the second and third.

EXCELLENT PERFORMANCE WITH A LIMITED NUMBER OF ELEMENTS OF CORRECTION.— Of the greatest importance is the good correction for coma, even in the most efficient series, which corresponds to an equally good fulfilment of the sine condition. In fact the Cooke lens is the *first* anastigmat which surpassed the anastigmats known at the time of its production in sharpness of definition over the usable angle of view. This was especially true for the series f: 4.5, the calculation of which must be regarded as an absolutely brilliant achievement, the more so because the constructor had at his disposal to meet the eight conditions for the construction of an anastigmat a very limited number of elements of construction. In both series we must call attention to the compact construction and the large increase of the effective angle of view on stopping down; the lens is extremely convenient in practice because of the light weight of the three thin lenses of the first-mentioned series especially when mounted in aluminum. These objectives are constructed in England by the firm of Taylor, Taylor, and Hobson of Leicester.

11. THE TRIPLETS WITH COLLECTING CEMENTED SURFACES OF H. HARTING AND P. RUDOLPH

INTRODUCTION OF COLLECTING CEMENTED SURFACES INTO THE COOKE LENS.— THE HELIAR.— The possibility of still further increasing the effectiveness of the Cooke lens by introducing collecting cemented surfaces, and thus diminishing the zonal errors, was first recognized by the present author and reduced to practice in the *Heliar* (Fig. 53), which was produced in 1902 by Voigtländer & Sohn from the author's calculations. As the zonal errors are very small, the effective aperture could be retained at *f*: 4.5 for all focal lengths up to 60 cm (24 in.). Because of the introduction of two collecting cemented surfaces the Heliar became a five-glass system, the first and third groups of which each consist of two lenses cemented together and constructed of an anomalous glass pair, while the central bi-concave dispersing lens is of silicate glass of low refraction. Excessive curvatures of the external surfaces were avoided by this construction. In this objective the sine condition is rigorously met over the whole aperture and consequently coma is absolutely removed over the whole field.

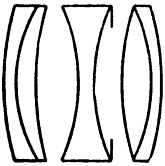

Fig 53

THE TESSAR.—The *Tessar*, *f*: 6.3 (Fig. 54), introduced in 1902 by Zeiss, from calculations by P. Rudolph, owes the progress in its quality of corrections, compared with those of earlier anastigmats, to the introduction of a cemented collecting surface in the last lens of a triplet. It thus consists of four glasses, of which the first two are separated by an air lens which has the form of a collecting lens and therefore disperses. Therefore, while in the Unar both cemented surfaces of the four-lens Protar are opened, the Tessar has lost only the dispers-

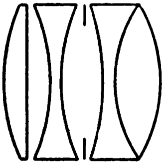

Fig 54

ing cemented surface, which is replaced by an air lens of the same character. The two types, with one and with two collecting cemented surfaces, are completely equivalent as far as performance is concerned. They were supplemented later by other series, and Tessars are at present constructed with the apertures *f*: 3.5, *f*: 4.5, and *f*: 10. The extent of the field of view in the three series corresponds to the apertures; the last mentioned is intended only for reproduction, and hence is produced only in large focal lengths.

THE TESSAR AND THE PETZVAL LENS.— The most efficient Tessars, with aperture *f*: 3.5, show clearly the enormous progress of photographic optics since 1840, the year when the Petzval lens was introduced. Like this, the Tessar consists of only four glasses, but in contrast to this, at the same aperture, together with equally good spherical, comatic, and orthoscopic corrections, it possesses freedom from astigmatism and curvature of field over a large angle of view.

THE DYNAR AND THE OXYN.— The Heliar type was not exactly followed in the construction of two new series produced by Voigtländer & Sohn from the author's calculations, but was somewhat changed. In the case of the *Dynar* working at *f*: 6 (Fig. 55) this change consists of the transposition of the dispersing and collecting lenses in the two cemented groups, so that here the three dispersing lenses are neighbors within the objective. In the case of the author's *Oxyn*, which is intended for reproduction, the lenses of the first group have the arrangement: dispersing, collecting; while those of the last group are reversed.

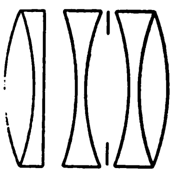
Fig. 55

12. Objectives corrected for the Secondary Spectrum

Apochromats.— As we have already seen, it is possible by the choice of suitable glasses to remove the secondary spectrum of a photographic objective. Such glasses were brought into commerce by the Jena factory, and thus we have obtained a series of types calculated for this higher degree of achromasy (p. 132). Here belong the apochromatic *Planars*, *Tessars*, *Collinears*, *Orthostigmats*, the *Alethar* of W. Zschokke, produced by Goerz, and the *Oxyn*. We will briefly remark that these objectives are used only in photo-engraving, and, generally speaking, only in color reproduction. In all other kinds of photographic work this higher degree of color correction is of no value. If three-color pictures of small size are to be made from nature, we endeavor to use lenses of the largest possible aperture, and therefore avoid apochromats, which are too slow for this purpose.

13. Conclusion

The Great Number of Types of Objectives.— That, besides the already mentioned objectives, a far greater number can be constructed by combining and separating cemented surfaces is clearly apparent from the previous sections. Experience, however, has shown that at best these objectives have no better quality than the principal types already described, while their construction very often involves greater difficulties. As far as the glass is concerned, improved varieties have been employed as they were made possible by progress in the manufacture of optical glass.

Progress of the Glass Works.— While, for instance, the heaviest barite crown, at the opening of the Jena factory, had a refractive index of about 1.61, in the course of time this has been raised to almost 1.62. There has also been great progress in the manufacture of crown glasses of low refraction. We can therefore at present avail ourselves of greater differences of refractive indices at the dispersing and collecting cemented surfaces or the corresponding air lenses, so that the objective can be better corrected than formerly for aberrations and their zonal errors. We shall again be able to see the same advance as often as the glass factories succeed in further advancing the present limits of refraction and color dispersion of the glasses necessary for the construction of anastigmats, since we already know how the corrections of every objective can be thus changed. We may also mention that the keeping qualities of the optical glasses have been *considerably* improved in the course of the last twenty years.

QUALITY OF CORRECTIONS AND PERFORMANCE IN THE MODERN UNSYMMETRICAL ANASTIGMATS.—As to the *performance* of modern anastigmats, it appears to have reached the high-water mark in the Tessar, Heliar, and Dynar. In these lenses we have attained such a high degree of freedom from aberration with large aperture and wide angle of view that practically no necessity exists for improvement. If we pass to the halves consisting of cemented lenses and the corresponding more rapid symmetrical objectives, which may possibly have a preference because with them we have the choice of a long and a short focus, we find that they will cover a plate 9 x 12 cm (3½ x 4¾ in.), with a focal length of 12 cm (4¾ in.) and working aperture *f*: 6.3, and that corresponding lenses of foci of 13½ cm (5¼ in.) and 15 cm (6 in.) will work at apertures of *f*: 5.4 and *f*: 4.5. This is done with a sharpness at the edge of the plate which can be called *absolute*, especially in comparison with the astigmats which can perform as well in the case of foci from 12 cm on, only at apertures of *f*: 12.5 at the most If we stop down to small apertures, the available field of view increases in the case of the symmetrical objective so that the whole illuminated field is sharp. Therefore, the modern *universal objectives*, as we name systems working at from *f*: 6 to *f*: 7, are really wide-angle lenses which perform just as well, but at a larger opening than the astigmatic wide-angle lenses. For pictures of extraordinary wide angle we may use the Hypergon double-anastigmat, which, on account of the serious falling off of illumination towards the edge of the image field, offers the greatest angle which constructive optics can hope to offer.

Efficient Lenses.— At the other extreme stand the *highly efficient* objectives working at *f*: 3.5, which is sufficient for any purpose. These objectives, however, can be used only in exceptional cases for general photographic purposes, especially for use on hand cameras, because of their large size and small depth.

Limit of Effective Aperture.—The aperture *f*: 4.5 appears to be the *most efficient* when it is necessary, as in hand-camera work, to unite rapidity of exposure with sufficient depth. It is highly necessary to use satisfactory finders in order completely to use the efficiency of such lenses. For this purpose, *reflecting cameras* offer the greatest advantages. Since great depth and high speed cannot be united, an increase of the working aperture has no real value. That in general an objective working at $f:4.5$ cannot give as great an extent of sharp field of view as one working at $f:9$, is apparent from what we have previously said. Even when the faster lens is stopped down to $f:9$, it is still surpassed by the slower lens, because in the latter case the zonal errors of anastigmatic correction can be much larger, so that the anastigmatic point lies farther from the axis than in the case of the faster lens.

Central Sharpness.— As far as central sharpness is concerned, there is *no* fundamental difference between astigmats and anastigmats. In both varieties there are systems with more or less sharpness in the center of the image, corresponding to the greater or smaller zonal errors of spherical aberration, as well as to the fulfilment of the sine condition. Since, however, the introduction of the collecting cemented surface or its equivalent air lens reduces the spherical zones, some anastigmats (and especially lenses of the type of the Tessar, Heliar, and Dynar) show a fineness of central sharpness which one seeks in vain among the astigmats.

CHAPTER XV

Reflection and Absorption in Photographic Lenses

Reflection when Light is Refracted.— If a ray falls upon the surface of a lens, it is divided by refraction and reflection into two rays, of which the refracted one takes the course which the optician has prescribed for it to produce an image. The reflected ray returns toward the object unless it comes in contact with a second surface, at which it is again separated. In this case the refracted ray is eliminated from our consideration, since it is directed out of the lens into the object space. The reflected portion, on the contrary, is sent back to the image space, and passes through the refracting surfaces of the lens toward the ground glass.

Twice-reflected Rays.— It therefore follows that, in addition to the image of the object produced by the refracted rays, there may appear others, produced by rays reflected an *even* number of times at the surfaces of the lenses. These *secondary images* may be most easily observed by placing a candle a few yards in front of the camera and after removing the ground glass looking at the lens from some little distance. We then see a number of sharp images of the candle, some upright and some inverted. If we now replace the ground glass and focus on the candle, we can see only *one* sharp image, since the secondary images are formed in different positions in space. However, each of the cones of rays emitted by them forms a large circle of confusion on the ground glass, varying in size according to the distance of the image from this, and this circle is called a *flare-spot.*

POSITION AND NUMBER OF THE SECONDARY IMAGES. — If we have to deal with a small object lying on the axis, the secondary images are also on the axis; if the object is situated to one side, however, the images occupy various positions outside the axis. It is clear that these are the more offensive in proportion to the number of lens surfaces between glass and air (cemented surfaces produce no flare-spots), and that they are worse the greater the diameter of the lens and especially the nearer they lie to the ground glass. An objective consisting of four single lenses produces twenty-eight secondary images, while with five single lenses there are forty-five.

DISADVANTAGE OF FLARE.— Fortunately the secondary images and the flare-spots produced by them are not especially detrimental in the case of the present-day objectives, unless the object contains very strong contrasts. Every photographer knows how dangerous it is to make exposures with the camera turned toward the sun, and even in the case of pictures including a sky with white clouds fogging is evident. If, however, we avoid such contrasty subjects, flare-spots are not particularly noticeable in modern objectives, especially when they are stopped down so that some of the rays converging toward the secondary images are eliminated.

AVOIDANCE IN PLANNING THE OBJECTIVE.— In the older text-books of photographic optics we find it stated that the position of the secondary images must be taken into consideration when calculating the lens.

This is not exactly true, because the calculator must determine his lens constants exclusively in accordance with the dioptric laws developed in the previous chapters, and has no free elements to use for the elimination of secondary images. If it appears, however, that these lie too near the ground glass when the calculations have been completed, the lens must be abandoned. These images always cause a small loss of luminosity in the image, but this is very small in, for example, an ordinary achromatic landscape lens.

Loss of Light by Reflection.— If the part of the incident light reflected toward the ground glass makes itself apparent in a definite though small fogging of the image, there must be a *loss of light*, because neither this part nor that reflected back into the object space takes part in the production of the image. As the influence of the cemented surfaces is nil, the amount of this loss depends on the number of surfaces between glass and air. Theory shows that the brightness of the reflected ray in air at a surface which bounds a medium with the coefficient of refraction n is $\left(\frac{n-1}{n+2}\right)^2$, if the brightness of the incident ray equals 1. This relation is absolutely true only for paraxial rays; but von Rohr has shown that we can assume it to be approximately true for all angles of inclination which come into question in the ordinary photographic objective.

In accordance with this, in the case of reflection at a surface where $n = 1.5$, 4% of the incident light is lost, and if $n = 1.6$ the loss is 5.3%. As the heaviest barite crown is much used in anastigmats, the average loss of light at a surface may be assumed as about 5%, and thus in the case of an objective with two air surfaces, 10 % of the light is lost; with four, 19%; with six, 26%; with eight, 34%. Since photographic objectives, except single combinations, have at least four air surfaces, at least a fifth of the incident light is lost.

LOSS OF LIGHT BY ABSORPTION.— Besides the loss in illumination by reflection, there is another through the *absorption* of light in the glass. To diminish this as much as possible is the endeavor of every factory manufacturing optical glass. Nevertheless, it is impossible to avoid a certain amount of absorption, because the manufacturer is forced, in order to produce a given optical quality, to adhere to a very narrowly limited composition of the glass mass, and because the most important glasses in modern photographic optics lie at the limit of possibility of preparation. Because of this, we must accept a certain amount of color, or do without objectives made with such glass.

DEPENDENCE OF THE ABSORPTION OF GLASS ON THE WAVE LENGTH.— The absorption of the glass depends on the wave length of the light, and, generally speaking, is greater the shorter the wave length. The photographic image is, consequently, more affected by absorption than the visual image, and it is, therefore, premature to draw conclusions as to the brightness of the actinic image from a consideration of the visual image. The absorption of optical glasses shows extraordinary variation. H. A. Krüss states that while, for instance, a plate 1 cm thick, made of the borosilicate crown used for totally reflecting prisms, absorbs about 2% of violet light of the wave length 410 $\mu\mu$, the absorption in the case of the heaviest barite crown is about 4% and in the case of the heavy silicate flint, $n_D = 1.67$, about 12%. If the thickness of the plate is doubled, the absorption increases in geometrical ratio.

Loss of Light in Different Objectives of the Same Series.— It is thus apparent that if an objective is proportionately changed to increase or diminish the focal length, the absorption changes in geometrical proportion and is, therefore, greater in longer focal lengths of the same series than in shorter. The loss in brightness by reflection is independent of the focal length of the type, since it depends solely on the number of surfaces in contact with air and the corresponding coefficients of refraction. If the lenses are improperly cemented, however, the Canada balsam used in excess may exercise a not inconsiderable absorptive effect. Short-focus lenses, therefore, have an advantage in brilliancy over longer ones of the same type.

Cemented and Uncemented Lenses.— The question of the relation of actual brightness of image in the case of cemented and uncemented lenses of the same relative opening is very often raised. From what we have said, it can be seen that the absorption depends on the kind of glass; as in the same melt various portions of the glass may vary in respect to absorption, the loss of light may vary somewhat even in two lenses of the same construction and focal length. For several years the author had almost uninterruptedly the opportunity of comparing for their photographic brilliancy a six-lens symmetrical objective, consisting of two halves of three cemented lenses (Collinear) and an objective consisting of three single lenses (Triple Anastigmat). Both anastigmats of medium focal lengths have the same relative aperture, either *f*: 6.8 or *f*: 7.7. He was never able to distinguish a photographically measurable difference in the actual light-passing capacity of those two kinds of objective representing the extremes of construction. It is true that the loss of light in the simple landscape lens is less than in the Zeiss double Protar consisting of eight lenses. But even this difference is unimportant in picture taking in comparison with the far greater deviations from the proper exposure in sunlight which even the most experienced photographer cannot avoid, but which are made absolutely imperceptible in the process of development.

CHAPTER XVI

THE TELEOBJECTIVE

COMBINATION OF INFINITELY THIN COLLECTING AND DISPERSING LENSES.— If we place an infinitely thin collecting lens of focal length f_1 and an infinitely thin dispersing lens of focal length f_2 at zero distance from each other, the combination disperses if the absolute value of f_2 is smaller than f_1. If we move the negative lens in the direction of the light rays, away from the collecting lens and toward the rear focal point of this, the negative focal length of the combination increases and becomes infinity when the focal points of the two lenses coincide.

GALILEAN TELESCOPE.— In this position the combination forms a Galilean telescope with the magnifying power $\gamma = f_1 : f_2$. If we move the dispersing lens still farther away beyond this *zero position*, the equivalent focal length f of the combination may attain any desired positive value from infinity to f_1. In the latter case the distance of the negative lens from the zero position, where f equals infinity, must be f_2, for this lens has no effect on the focal length of the collecting lens when it is placed at the focal point of the latter.

CONSTRUCTION OF THE TELEOBJECTIVE.— Such a combination of a collecting and a dispersing lens, whose thicknesses in practice must be assumed as finite, is called a *telephoto lens* or *teleobjective*. The purpose of such a construction is to utilize the large focal lengths f

which occur when two lens systems with focal lengths f_1 and f_2, which theoretically at least may be chosen at will, are placed at such distances from each other as are indicated by the preceding section. Aside from the long-known inclusion of a dispersing lens in the optical system of the astronomical telescope, we find that the first teleconstruction in the modern sense was due to J. Porro, who made exposures as early as 1851 with varying separations of the two parts of the teleobjective, the *telepositive* and *telenegative*. In the early nineties of the last century the construction was independently invented in several quarters. It is now placed upon the market by most optical firms.

CHANGEABLE FOCUS.—OPTICAL INTERVAL.—As we have seen, the collecting effect of the telecombination begins at the zero position, which is determined by the coincidence of the two focal points on the ground-glass side. If we designate the separation of these two focal points, also called the *optical interval*, by Δ, limiting ourselves to positive values of the focal length f of the whole system, that is, to values of Δ smaller than f_2, we find from the general law of image formation that $f = \frac{f_1 f_2}{\Delta}$. As Δ increases, the focal length f of the teleobjective decreases. Knowledge of the optical interval Δ is, therefore, of great importance if we desire to use the teleobjective at different focal lengths. In the possibility of obtaining focal lengths of *any desired magnitude* by *change of the separation* of the two combinations, lies one of the advantages of using the teleobjective. If we know the values of f_1 and f_2, f is easily figured for any value of Δ. According to the Zeiss practice, the tube of the teleobjective is for this reason provided with a millimeter scale starting from $\Delta = 0$, so that the optical interval can be read directly from the tube. In practice the divisions are carried only far enough to show proportionately large values of f as compared with f_1.

RELATION OF THE FOCI OF THE COMPONENTS.— It has proved useful also to introduce in the above equation the ratio $\gamma = f_1 : f_2$. We then get $f = \gamma \frac{f_2^2}{\Delta}$; here the quantity γ is greater than unity, since f_2 must be smaller than f_1.

POSITION OF THE PRINCIPAL POINTS.— BELLOWS LENGTH.— ENLARGEMENTS.— Another change takes place, however, because of the separation of the two parts. When the lenses f_1 and f_2, assumed as infinitely thin, are in contact, the rear principal point H coincides with the common lens vertex, but this travels toward the left as the components are separated from each other (Fig. 56). Therefore, with the teleobjective, and

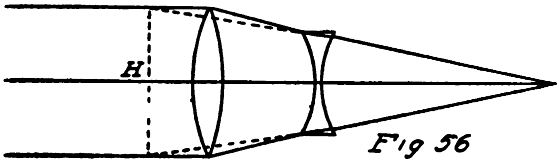

Fig 56

herein lies one of its great advantages compared with ordinary objectives, the distance of the ground glass from the lens is *less* than the equivalent focus of the whole teleobjective. With short cameras pictures can thus be taken on a larger scale than with ordinary lenses of the same focal length. The image of a distant object produced by the teleobjective f is, compared with that of the telepositive f_1, larger in the proportion $f_2 : \Delta$, while the corresponding ground-glass distances from the rear surface of the two optical systems to be compared are in the ratio $\gamma : 1 - \frac{f_1}{f}$.

LIMIT OF THE FOCAL LENGTH RATIO.— From this we see the importance of the quantity γ. The *smaller* the focal length f_2 of the telenegative in proportion to that of the telepositive f_1, the shorter may be the length of the bellows. We must, however, observe that we cannot make the number γ too large, else we obtain too deep curves in the lenses and too small a diameter of the telenegative, which would have an unfavorable effect on sharpness and angle of view. In practice it is found that γ can never be larger than four. Generally the negative focal length is chosen as about one third the positive focal length.

PERSPECTIVE.— While the distance of the ground glass from the lens with a teleobjective is considerably diminished, the opposite is the case with the object distance. This is, in the case of a teleobjective, *larger* than with an ordinary lens of the same focal length f, and the size equals $f(\gamma - 1) + f_1$. As we have already seen, the perspective produced by a photographic lens depends on the distance between the ground-glass plane and the entrance pupil. This distance is, assuming equal focal lengths, *larger* in the case of a teleobjective than with an ordinary system. Consequently the teleobjective produces images which seem to be in better perspective, and this difference becomes more marked the greater the size of the object upon the ground glass. In the case of landscapes this difference disappears. This advantage of the teleobjective properly has been praised for portrait work, since it is well known that in the case of ordinary lenses of too short focal length, in consequence of bringing the lens too close to the object, the picture produced is often in bad perspective.

ADVANTAGES AND DEFECTS OF THE TELEOBJECTIVE. — It thus appears that the teleobjective has three fundamental advantages which every photographer can appreciate: variable focal length and size of image, short bellows length, and less distorted perspective in portrait work. If now, in spite of the pains which lens-makers take to increase the use of the teleobjective, its application is still very limited, it must have very considerable disadvantages as well. In fact, its universal application is greatly hindered, and in many cases made impossible, by its small working aperture and the consequent *long exposure*.

SMALL EFFECTIVE APERTURE.— We know that the brightness of the image produced by a given objective, when the object is remote, is measured by the ratio of the effective aperture D to the focal length f. The effective aperture of a teleobjective is that of the positive front combination, while the equivalent focal length f equals $\frac{f_1 f_2}{\Delta}$. We thus obtain the relative aperture $\frac{D}{f}$ of a telesystem by dividing that of the front combination $\frac{D}{f_1}$ by $\frac{f_2}{\Delta}$, that is, by the enlargement produced by the introduction of a telenegative, compared with the size of the image produced by the front lens f_1 alone. If, for instance, the collecting part has the focal length $f_1 = 180$ mm (7¼ in.) and the aperture $f:6.3$, the telenegative the focal length

$-f_2 = -45$ mm (1 ¾ in.) and if the optical interval $\Delta = 9$ mm (11/32 in.), the focal length of the whole teleobjective $f = \frac{180 \times 45}{9} = 900$ mm (about 35½ in.) and its relative aperture is $\frac{1}{6.3 \times 5} = f:32$. We must, therefore, expose twenty-five times as long as with the telepositive alone.

Choice of the Telepositive.— If we desire to use a teleobjective to make pictures of very distant objects with clearly recognizable details, we must give very long exposures, if it is not possible considerably to increase the power of the telepositive. As a matter of fact, Rudolph endeavored to increase the use of the teleobjective by the introduction of a very fast model consisting of four cemented lenses. Such positives, of a type similar to telescope objectives, are, however, not suited for photographic work, as their available angle of field is too small. The same is also true of the use of a fast anastigmat as the collecting element, since this combined with a telenegative gives only a small field of view of the necessary sharpness, and this small field cannot be greatly increased by stopping down. If we further remember that there can be no question of a complete correction of the telenegative because of the fact that the distance of the two halves is varied within considerable limits, we see why it is not useful to employ fast anastigmats as telepositives. Hence it happens that in the usual construction of the telenegative, which cannot be allowed to injure the corrections of the telepositive, of three cemented lenses, we must be prepared to allow a small sacrifice of central sharpness, if the diminution of sharpness toward the edges is not to become excessive.

UNIVERSAL OBJECTIVES AS TELEPOSITIVES.— Therefore, we use, as telepositives, anastigmats working at not more than f: 5.4 because we must stop down a little to increase the definition. The focal length $-f_2$ of the telenegative must, as we have seen, be not less than one quarter of the focal length f_1 of the positive combination, while its diameter must be as large as possible to prevent the mount from acting as a stop. It is self-evident that the ratio of the aperture to the focal length of the telenegative has nothing to do with the working aperture of the teleobjective, since its diameter is far greater than the cross-section of the axial pencil.

LIMIT OF STOPPING DOWN.— Just as we have previously seen, that in the case of an ordinary objective the increase of sharpness and depth by stopping down reached a limit because of the appearance of diffraction phenomena, so we find the same true of the teleobjective. When the relative aperture becomes less than about f: 71 we notice a deterioration of definition. The iris diaphragm of the telepositive is the aperture stop and we therefore obtain the aperture ratio which forms the limit for the teleobjective if we desire maximum sharpness, when we multiply 1/71 by the enlargement compared with that given by the positive lens alone. With twelvefold enlargement we may, therefore, stop down only to f: 6, with fourfold to f: 18. This rule is often violated to the detriment of the picture.

DEFECTS IN EXPOSURE — With such long exposures it is very difficult to obtain a good picture. Independent of the fact that the camera must be very solidly built to avoid vibration, still air, the indispensable condition for sharp images on outdoor exposures, rarely occurs. There are almost always strata of different densities at whose boundaries currents are caused, so that, at least in the temperate zones, there are very few days in the year on which one can make teleëxposures successfully at certain hours. The greater the enlargement, the more noticeable are these effects. If we desire to make teleëxposures with a hand camera, the aperture of the teleobjective, even with favorable lighting, for instance at sea, may not be less than *f*: 12.5, so that with an aperture of the front combination of *f*: 5.4 we obtain an enlargement of only 2½ times.

TELEOBJECTIVES WITH FIXED FOCAL LENGTH.—THE BIS-TELAR OR COOKE TELAR.— Even such an enlargement, however, is a great advantage in many kinds of photographic work, for instance, in the photographing of moving ships, wild animals at large, views from aircraft, etc. If we are willing to surrender the variable enlargement and content ourselves with the advantages of the short bellows, the teleobjective may be greatly simplified. Petzval produced a dialytic objective, which is nothing more than a teleobjective of fixed focal length, consisting of two achromatic menisci, of which the first collects and the second disperses. A few years since such teleobjectives of *fixed focal length* were produced independently by various firms, including Zeiss and Busch. That of the latter firm consists (Fig. 57) of only four lenses; the concave surfaces of both menisci are placed toward the stop.

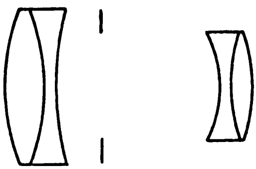

Fig. 57

This *Bis-telar*, or *Cooke Telar*, as it is called in America, designed by K. Martin, was first made with an aperture of *f*: 9, which more recently has been increased to *f*: 7. Such a lens of 27 cm (10¾ in.) focal length and 40 mm (1⅝ in.) aperture covers at full opening a plate 9 x 12 cm (3½ x 4½ in.); the focal point, however, has a distance of only 14 cm (5½ in.) from the last lens surface. Since the rays fall upon the refracting surfaces at large angles, the zonal errors of spherical aberration are much larger than in an anastigmat of the same aperture; this is, however, of no importance for the purposes for which they are designed.

Long Exposure in Portrait Work.— For portrait exposures the slowness of a teleobjective is very unfavorable. In addition, we must take account of the fact that in the case of near views the working aperture must be multiplied by the factor of reduction. The lens, therefore, requires times of exposure which the portrait photographer generally cannot give. This explains the rare use of the teleobjective for such purposes, in spite of its theoretical advantages.

The Adon.— In 1899 F. R. Dallmeyer of London produced an optical apparatus included in the group of teleobjectives. If a parallel pencil of rays falls upon a teleobjective in the zero position, it emerges parallel and can be united at the focus of an ordinary objective placed behind this. We can easily calculate that the picture thus produced is on a scale γ times as large as that of the picture taken with the objective alone, γ being again the ratio $f_1 : f_2$ of the focal lengths of the two telelenses. If the telenegative has the same aperture as the objective, there is no loss of speed, and even after the *Adon* has been added to the lens we work with the same relative aperture. This system can, however, be used advantageously only with objectives of moderate diameter and at moderate enlargements, since its deviations are otherwise too large.

CHAPTER XVII

CONCLUSION

THIS chapter gives some rules and facts which are of importance to photographers, and which depend on the principles given in the previous chapters. Strictly speaking, some of these do not belong in a book on optics, but as they are of importance in pictorial photography, they will doubtless prove useful in many cases.

(A) DETERMINATION OF EQUIVALENT FOCAL LENGTH

FOCUSING ON INFINITY.— It is not necessary to use any particular lens-testing apparatus for this purpose, for a simple tripod camera with a bellows as long as possible will suffice, especially if the bed is furnished with a scale graduated in millimeters or inches. The position of the *focal point* is determined by focusing on a very distant object. How far distant this must be depends on the focal length as well as on the exactness which is desired. If, for instance, it is desired to determine the position of the focal point to within 0.1 mm (.004 in.), the distance of the object from the focal point on the object side is to be determined from the equation $x = f^2 : x'$, where $x' = 0.1$ mm. If f equals 15 cm (6 in.), the object must be at a distance of at least 225 m (700 ft.). An object on the farther side of even a very broad street is entirely unsatisfactory.

Determination of the Focal Length by Focusing a Near Object Natural Size.— The focal length may be determined from the formula $y : y' = f : x'$, particularly in the special case when object and image are of *equal size*. Let us set up, perpendicular to the optical axis, a millimeter or inch scale, or a flat object with sharp outlines and perforations, so that the axis shall pass through its center, and adjust the focus so that the object on the ground glass shall coincide with a similar rule or object placed against the ground glass. Then the distance of the ground glass from the position of the focal plane, marked on or read from the camera bed, is equal to the equivalent focal length. The difference between the focal length and the distance from the focal plane to the front surface of the lens board gives the distance from this latter plane to the corresponding *principal point*. If the same measurements are repeated with the objective reversed, we may determine the position of the front focal point and principal point. Only in the case of objectives not spherically and chromatically corrected, is it necessary to stop down to obtain sufficient sharpness; otherwise, and especially with all modern objectives, these determinations are made at full aperture.

If we desire to determine the focal length as exactly as possible in the camera, it is advisable to make a number of measurements at various distances in the neighborhood of the point where the image appears full size, as may be seen in the following table, in which the size of the object is assumed to be 100 mm. As the mean of ten measurements, the focal length is found to be 152.2 mm.

Distance x' of the plane of the ground glass from the focal plane in mm	Observed size of image y' in mm	Focal length in mm $f = 100 \frac{x'}{y'}$
136	90	151
145	96	151
149	97	154
154	101	152
156	102	153
161	106	152
165	109	152
169	111	152
170	112	152
170	111	153

(B) Determination of Relative Aperture

Steinheil's Process.— To determine this, as we now know the focal length, we must determine the diameter of the *effective aperture*, for this corresponds with the diameter of the stop, as we have already seen, only when the stop is placed in front of the lens. In all other cases the size of the effective aperture can be determined as follows, by a process devised by A. Steinheil. The ground glass is brought into the rear focal plane by focusing on infinity, taken out of its frame, and replaced by an exactly fitting and equally thick piece of cardboard, which has in the optical axis a circular hole about 2 mm (say 1-16 in.) in diameter. In the darkroom a round piece of bromide or gaslight paper is placed in the lens cap and this is placed upon the lens. A white light is then held behind the hole in the cardboard and the bromide paper is developed. The diameter of the black circle is equal to that of the effective aperture, as may be seen by studying the course of the rays.

(C) Determination of the Size of the Circle of Illumination and Angle of View

To determine the diameter of the circle of illumination, we focus on infinity and expose a sufficiently large plate towards a white background, so that the whole circle of illumination appears on the plate. It is immediately apparent from the negative how much of the illuminated circle can be used.

PLATE SIZES.— What size of plate can be accommodated by the diameter found, may be determined from the following tables:

Plate size in cm	Diagonal in mm	Plate size in cm	Diagonal in mm
6 x 9	108	21 x 27	342
8.5 x 10	131	24 x 30	384
8.3 x 10.8	136	26 x 31	406
9 x 12	150	30 x 40	500
10.2 x 12.7	163	35 x 45	570
9 x 14	166	40 x 50	640
12 x 16	203	45 x 55	711
13 x 18	225	50 x 60	781
13 x 21	247	55 x 65	851
16 x 21	264	60 x 70	922
18 x 24	300	70 x 80	1,063

Plate size in in.	Diagonal in in.
$\frac{3}{4}$ x 1	$1\frac{1}{4}$ 1.25
$1\frac{1}{8}$ x $1\frac{1}{4}$	$1\frac{11}{16}$ 1.68
$1\frac{1}{4}$ x $1\frac{3}{4}$	$2\frac{1}{8}$ 2.15
$1\frac{1}{2}$ x 2	$2\frac{1}{2}$ 2.50
$1\frac{1}{2}$ x $2\frac{1}{4}$	$2\frac{5}{16}$ 2.30
$1\frac{5}{8}$ x $2\frac{3}{8}$	3 2.98
$1\frac{11}{16}$ x $2\frac{5}{8}$	$3\frac{1}{8}$ 3.12
$1\frac{3}{4}$ x $2\frac{5}{16}$	$2\frac{7}{8}$ 2.90
$2\frac{1}{4}$ x $2\frac{1}{4}$	$3\frac{3}{16}$ 3.18
$2\frac{1}{4}$ x $3\frac{1}{4}$	$3\frac{15}{16}$ 3.95
$2\frac{5}{16}$ x $3\frac{11}{16}$	$4\frac{3}{8}$ 4.35
$2\frac{1}{2}$ x $3\frac{1}{2}$	$4\frac{5}{16}$ 4.30
$2\frac{1}{2}$ x $4\frac{1}{4}$	$4\frac{15}{16}$ 4.93
$2\frac{7}{8}$ x $4\frac{7}{8}$	$5\frac{5}{8}$ 5.66
$3\frac{1}{8}$ x $4\frac{7}{16}$	$5\frac{3}{8}$ 5.37
$3\frac{1}{4}$ x $3\frac{1}{4}$	$4\frac{9}{16}$ 4.60
$3\frac{1}{4}$ x 4	$5\frac{1}{8}$ 5.15
$3\frac{1}{4}$ x $4\frac{1}{4}$	$5\frac{3}{8}$ 5.35
$3\frac{1}{4}$ x $5\frac{1}{2}$	$6\frac{3}{8}$ 6.39
$3\frac{1}{2}$ x $3\frac{1}{2}$	$4\frac{15}{16}$ 4.95
4 x 5	$6\frac{3}{8}$ 6.40
$4\frac{1}{4}$ x $6\frac{1}{2}$	$7\frac{3}{4}$ 7.77
$4\frac{3}{4}$ x $6\frac{1}{2}$	8 8.01
5 x 7	$8\frac{5}{8}$ 8.60
5 x 8	$9\frac{7}{16}$ 9.43
$6\frac{1}{2}$ x $8\frac{1}{2}$	$10\frac{11}{16}$ 10.70
8 x 10	$12\frac{13}{16}$ 12.81
10 x 12	$15\frac{5}{8}$ 15.62
11 x 14	$17\frac{13}{16}$ 17.80
12 x 15	$19\frac{3}{16}$ 19.21
14 x 17	22 22.02
16 x 20	$25\frac{5}{8}$ 25.61
17 x 20	$26\frac{1}{4}$ 26.25
18 x 22	$28\frac{7}{16}$ 28.43
20 x 24	$31\frac{1}{4}$ 31.24

In determining the actual size of the plate which may be covered, the area shielded by the rebates of the plate holder should be taken into consideration.

It must also be noted that the whole of the illuminated circle can be utilized only when the optical axis of the axis passes through the center of the plate. If the lens is moved up, down, or sidewise, vignetting may be apparent on one side of the plate.

Relation Between Diameter of Image, Focal Length, and Angle of View.— The following table serves for finding the *angle of view* for any diameter of image in the case of distant objects. To use, divide the diameter of the image by the focal length, and read the nearest angle from the table.

If the Quotient is	The Angle is	If the Quotient is	The Angle is	If the Quotient is	The Angle is
	Degrees		Degrees		Degrees
0.175	10	0.748	41	1.453	72
0.193	11	0.768	42	1.480	73
0.210	12	0.788	43	1.507	74
0.228	13	0.808	44	1.535	75
0.245	14	0.828	45	1.562	76
0.263	15	0.849	46	1.591	77
0.281	16	0.870	47	1.619	78
0.300	17	0.890	48	1.649	79
0.317	18	0.911	49	1.678	80
0.335	19	0.933	50	1.708	81
0.353	20	0.954	51	1.739	82
0.371	21	0.975	52	1.769	83
0.389	22	1.000	53	1.801	84
0.407	23	1.020	54	1.833	85
0.425	24	1.041	55	1.865	86
0.443	25	1.063	56	1.899	87
0.462	26	1.086	57	1.931	88
0.480	27	1.109	58	1.965	89
0.500	28	1.132	59	2.000	90
0.517	29	1.155	60	2.035	91
0.536	30	1.178	61	2.071	92
0.555	31	1.201	62	2.107	93
0.573	32	1.225	63	2.145	94
0.592	33	1.250	64	2.183	95
0.611	34	1.274	65	2.221	96
0.631	35	1.300	66	2.261	97
0.650	36	1.323	67	2.301	98
0.669	37	1.349	68	2.342	99
0.689	38	1.374	69	2.383	100
0.708	39	1.400	70	.....	..
0.728	40	1.427	71	.....	..

The following tables give the plate diameter covered by lenses of various focal lengths when used to include various angles of view. The sizes of plates most nearly approximating any given diameter may be found from the tables last given, or the table may be used to determine approximately the angle of view included by a lens of any given focal length on the plate which it is necessary to use.

Diameter of Image for Various Angles of View

Focal Length in Inches	10°	15°	20°	25°	30°	35°	40°	45°	50°
2	0.35	0.53	0.71	0.89	1.07	1.26	1.46	1.66	1.87
2½	0.44	0.66	0.88	1.11	1.34	1.58	1.82	2.07	2.33
3	0.52	0.79	1.06	1.33	1.61	1.89	2.18	2.49	2.80
3½	0.61	0.92	1.23	1.55	1.88	2.21	2.55	2.90	3.26
4	0.70	1.05	1.41	1.77	2.14	2.52	2.91	3.31	3.73
4½	0.79	1.18	1.59	2.00	2.41	2.84	3.28	3.73	4.20
5	0.87	1.32	1.76	2.22	2.68	3.15	3.64	4.14	4.66
5½	0.96	1.45	1.94	2.44	2.95	3.47	4.00	4.56	5.13
6	1.04	1.58	2.12	2.66	3.22	3.78	4.37	4.97	5.60
6½	1.14	1.71	2.29	2.88	3.48	4.10	4.73	5.38	6.06
7	1.22	1.84	2.47	3.10	3.75	4.41	5.10	5.80	6.53
7½	1.31	1.97	2.64	3.33	4.02	4.73	5.46	6.21	6.99
8	1.40	2.11	2.82	3.55	4.29	5.04	5.82	6.63	7.46
8½	1.49	2.24	3.00	3.77	4.56	5.36	6.19	7.04	7.93
9	1.57	2.37	3.17	3.99	4.82	5.68	6.55	7.46	8.39
9½	1.66	2.50	3.35	4.21	5.09	5.99	6.92	7.87	8.86
10	1.74	2.63	3.53	4.43	5.36	6.31	7.28	8.28	9.33
10½	1.84	2.76	3.70	4.66	5.63	6.62	7.64	8.70	9.79
11	1.92	2.90	3.88	4.88	5.89	6.94	8.01	9.11	10.26
11½	2.01	3.03	4.06	5.10	6.16	7.25	8.37	9.53	10.73
12	2.10	3.16	4.23	5.32	6.43	7.57	8.74	9.94	11.19
12½	2.19	3.29	4.41	5.54	6.70	7.88	9.10	10.36	11.66
13	2.27	3.42	4.58	5.76	6.97	8.20	9.46	10.77	12.12
13½	2.36	3.55	4.76	5.99	7.23	8.51	9.83	11.18	12.59
14	2.45	3.69	4.94	6.21	7.50	8.83	10.19	11.60	13.06
14½	2.54	3.82	5.11	6.43	7.77	9.14	10.56	12.01	13.52
15	2.62	3.95	5.29	6.65	8.04	9.46	10.92	12.43	13.99
15½	2.71	4.08	5.47	6.87	8.31	9.77	11.28	12.84	14.46
16	2.80	4.21	5.64	7.09	8.57	10.09	11.65	13.25	14.92
16½	2.89	4.34	5.82	7.32	8.84	10.40	12.01	13.67	15.39
17	2.97	4.48	6.00	7.54	9.11	10.72	12.37	14.08	15.85
18	3.15	4.74	6.35	7.98	9.65	11.35	13.10	14.91	16.79
19	3.32	5.00	6.70	8.42	10.18	11.98	13.83	15.74	17.72
20	3.50	5.27	7.05	8.87	10.72	12.61	14.56	16.57	18.65
21	3.67	5.53	7.41	9.31	11.25	13.24	15.29	17.40	19.59
22	3.85	5.79	7.76	9.75	11.79	13.87	16.01	18.23	20.52
23	4.02	6.06	8.11	10.20	12.33	14.50	16.74	19.05	21.45
24	4.20	6.32	8.46	10.64	12.86	15.13	17.47	19.88	22.38
25	4.37	6.58	8.82	11.08	13.40	15.77	18.20	20.71	23.32
26	4.54	6.85	9.17	11.53	13.93	16.40	18.93	21.54	24.25
27	4.72	7.11	9.52	11.97	14.47	17.03	19.65	22.37	25.18
28	4.90	7.37	9.87	12.41	15.01	17.67	20.38	23.20	26.11
29	5.07	7.64	10.23	12.86	15.54	18.29	21.11	24.02	27.05
30	5.25	7.90	10.60	13.30	16.08	18.92	21.84	24.85	27.98
31	5.42	8.16	10.93	13.74	16.61	19.55	22.57	25.68	28.91
32	5.60	8.43	11.29	14.19	17.15	20.18	23.29	26.51	29.84
33	5.77	8.69	11.64	14.63	17.68	20.81	24.02	27.34	30.78
34	5.95	8.95	11.99	15.07	18.22	21.44	24.75	28.17	31.71
35	6.12	9.22	12.34	15.52	18.76	22.07	25.48	28.99	32.64
36	6.30	9.49	12.70	15.96	19.29	22.70	26.21	29.82	33.57
37	6.47	9.74	13.05	16.41	19.83	23.33	26.93	30.65	34.51

Diameter of Image for Various Angles of View

55°	60°	65°	70°	75°	80°	85°	90°	95°	100°
2.08	2.31	2.55	2.80	3.06	3.36	3.67	4.00	4.37	4.77
2.60	2.89	3.19	3.50	3.84	4.20	4.58	5.00	5.46	5.96
3.12	3.46	3.82	4.20	4.60	5.03	5.50	6.00	6.55	7.15
3.64	4.04	4.46	4.90	5.37	5.87	6.41	7.00	7.64	8.34
4.16	4.62	5.10	5.60	6.14	6.71	7.33	8.00	8.73	9.53
4.69	5.20	5.73	6.30	6.91	7.55	8.25	9.00	9.82	10.73
5.21	5.77	6.37	7.00	7.67	8.39	9.16	10.00	10.91	11.92
5.73	6.35	7.01	7.70	8.44	9.23	10.08	11.00	12.00	13.11
6.25	6.93	7.64	8.40	9.21	10.07	11.00	12.00	13.10	14.30
6.77	7.51	8.28	9.10	9.98	10.91	11.91	13.00	14.19	15.49
7.29	8.08	8.92	9.80	10.74	11.75	12.83	14.00	15.28	16.68
7.81	8.66	9.56	10.50	11.51	12.59	13.74	15.00	16.37	17.88
8.33	9.24	10.19	11.20	12.28	13.43	14.66	16.00	17.46	19.07
8.85	9.81	10.83	11.90	13.04	14.26	15.58	17.00	18.55	20.26
9.37	10.39	11.47	12.60	13.81	15.10	16.49	18.00	19.64	21.45
9.89	10.97	12.10	13.30	14.58	15.94	17.41	19.00	20.73	22.64
10.41	11.55	12.74	14.00	15.35	16.78	18.33	20.00	21.83	23.84
10.93	12.12	13.38	14.70	16.11	17.62	19.24	21.00	22.92	25.03
11.45	12.70	14.02	15.40	16.88	18.46	20.16	22.00	24.01	26.22
11.97	13.30	14.65	16.10	17.65	19.30	21.08	23.00	25.10	27.41
12.49	13.86	15.29	16.81	18.42	20.14	21.99	24.00	26.19	28.60
13.01	14.43	15.93	17.51	19.18	20.98	22.91	25.00	27.28	29.79
13.53	15.01	16.56	18.21	19.95	21.82	23.82	26.00	28.37	30.99
14.06	15.59	17.20	18.91	20.72	22.66	24.74	27.00	29.47	32.18
14.58	16.17	17.84	19.61	21.49	23.49	25.66	28.00	30.56	33.37
15.10	16.74	18.48	20.31	22.25	24.33	26.57	29.00	31.65	34.56
15.62	17.32	19.11	21.01	23.02	25.17	27.49	30.00	32.74	35.75
16.14	17.90	19.75	21.71	23.79	26.01	28.41	31.00	33.83	36.94
16.66	18.48	20.39	22.41	24.55	26.85	29.32	32.00	34.92	38.14
17.18	19.05	21.02	23.11	25.32	27.69	30.24	33.00	36.01	39.33
17.70	19.63	21.66	23.81	26.09	28.53	31.16	34.00	37.10	40.52
18.74	20.78	22.93	25.21	27.62	30.21	32.99	36.00	39.29	42.90
19.78	21.94	24.21	26.61	29.16	31.89	34.82	38.00	41.47	45.29
20.82	23.09	25.48	28.01	30.69	33.56	36.65	40.00	43.65	47.67
21.86	24.25	26.76	29.41	32.23	35.24	38.49	42.00	45.84	50.05
22.91	25.40	28.03	30.81	33.76	36.92	40.32	44.00	48.02	52.44
23.95	26.59	29.31	32.21	35.30	38.60	42.15	46.00	50.20	54.82
24.99	27.71	30.58	33.61	36.83	40.28	43.98	48.00	52.38	57.20
26.03	28.87	31.85	35.01	38.37	41.96	45.82	50.00	54.57	59.59
27.07	30.02	33.13	36.41	39.90	43.63	47.65	52.00	56.75	61.97
28.11	31.18	34.40	37.81	41.44	45.31	49.48	54.00	58.93	64.35
29.15	32.33	35.68	39.21	42.97	46.99	51.31	56.00	61.11	66.74
30.19	33.49	36.95	40.61	44.51	48.67	53.15	58.00	63.30	69.12
31.23	34.64	38.22	42.01	46.04	50.35	54.98	60.00	65.48	71.51
32.28	35.80	39.50	43.41	47.57	52.02	56.81	62.00	67.66	73.89
33.32	36.95	40.77	44.81	49.11	53.70	58.65	64.00	69.84	76.27
34.36	38.11	42.05	46.21	50.64	55.38	60.48	66.00	72.03	78.66
35.40	39.26	43.32	47.61	52.18	57.06	62.31	68.00	74.21	81.04
36.44	40.41	44.59	49.01	53.71	58.74	64.14	70.00	76.39	83.42
37.48	41.57	45.87	50.42	55.25	60.42	65.98	72.00	78.57	85.81
38.52	42.72	47.14	51.82	56.78	62.09	67.81	74.00	80.76	88.19

Finite Distance.— If the object is not so far distant that we can assume that its image falls in the focal plane, we must use in connection with the foregoing tables instead of the focal length the distance of the image from the rear principal point, *i.e.*, the value $f + x' = f + \frac{f^2}{x}$. The data on angle of view and size of field in lens catalogues always refer to very distant objects.

It must further be mentioned that we are here dealing with the angles of inclination on the *image side* of the corresponding principal rays. We can replace them with sufficient accuracy by the angles in the *object side*, except in the case of lens with front stops, where, as previously shown this in not allowable.

(D) Determination of the Distance Relation from the Scale of Reduction

Table of A. Steinheil.— If we know the relation of the size of object and image for a given focal length, *the distance of the object and image* from the corresponding principal points, the positions of which are known, can be easily calculated. If the halves of an objective with central stop are close enough together, for the first approximation we may assume that the two principal points coincide with the center of the stop. If the focal length is made equal to unity, the following table calculated by A. Steinheil may be used advantageously.

Reduction	Image Distance	Object Distance	Reduction	Image Distance	Object Distance
1.0	2.00	2.0	6.5	1.15	7.5
1.1	1.91	2.1	7.0	1.14	9.0
1.2	1.83	2.2	7.5	1.13	8.5
1.3	1.77	2.3	8.0	1.12	9.0
1.4	1.72	2.4	8.5	1.12	9.5
1.5	1.67	2.5	9.0	1.11	10.0
1.6	1.62	2.6	9.5	1.10	10.5
1.7	1.59	2.7	10.0	1.10	11.0
1.8	1.56	2.8	11.0	1.09	12.0
1.9	1.53	2.9	12.0	1.08	13.0
2.0	1.50	3.0	13.0	1.08	14.0
2.1	1.48	3.1	14.0	1.07	15.0
2.2	1.45	3.2	15.0	1.07	16.0
2.3	1.43	3.3	16.0	1.06	17.0
2.4	1.42	3.4	18.0	1.06	19.0
2.5	1.40	3.5	20.0	1.05	21.0
2.6	1.38	3.6	22.0	1.04	23.0
2.7	1.37	3.7	24.0	1.04	25.0
2.8	1.36	3.8	26.0	1.04	27.0
2.9	1.34	3.9	28.0	1.04	29.0
3.0	1.33	4.0	30.0	1.03	31.0
3.2	1.31	4.2	35.0	1.03	36.0
3.4	1.29	4.4	40.0	1.02	41.0
3.6	1.28	4.6	45.0	1.02	46.0
3.8	1.26	4.8	50.0	1.02	51.0
4.0	1.25	5.0	60.0	1.02	61.0
4.5	1.22	5.5	70.0	1.01	71.0
5.0	1.20	6.0	80.0	1.01	81.0
5.5	1.18	6.5	90.0	1.01	91.0
6.0	1.17	7.0	100.0	1.01	101.0
Enlargement	**Object Distance**	**Image Distance**	**Enlargement**	**Object Distance**	**Image Distance**

EXAMPLES. 1. A print 741 mm in diameter is to be reproduced with an objective of 24 cm focal length to form an image 285 mm in diameter. The reduction is therefore $\frac{741}{285} = 2.6$. Consequently the image distance is 1.38 for focal length 1, and 1.38 x 240

mm = 331 mm, for the given focal length, corresponding to an object distance 3.6 x 240 mm = 864 mm.

2. With an objective of 180 mm focal length, we are to photograph an object at a distance of 153 cm. The object distance is therefore $\frac{1530}{180} = 8.5$ focal lengths, so that the reduction is 7.5 and the image distance 1.13 x 180 mm = 203 mm.

3. A plate 150 mm in diameter is to be projected to form an image 480 mm in diameter. The enlargement is $\frac{480}{150} = 3.2$. If we use the focal length of 135 mm, the plate distance is 1.31 x 135 mm = 173 mm, the image distance 4.2 x 135 mm = 567 mm.

While the table above given may be used for calculations in English measures without difficulty, we reproduce from *Practical Photography* No. 3, "How to Choose and Use a Lens," reducing and enlarging tables in English measures, with instructions for their use.

Reducing and Enlarging Tables

All figures in table are in ins.	Reductions											
	Same size	1/2 size	1/3 size	1/4 size	1/5 size	1/6 size	1/7 size	1/8 size	1/9 size	1/10 size	1/11 size	1/12 size
Focus of lens used	Enlargements											
	Same size	2 times	3 times	4 times	5 times	6 times	7 times	8 times	9 times	10 times	11 times	12 times
3	**6**	**9**	**12**	**15**	**18**	**21**	**24**	**27**	**30**	**33**	**36**	**39**
	6	4 1/2	4	3 3/4	3 3/5	3 1/2	3 3/7	3 3/8	3 3/9	3 3/10	3 3/11	3 3/12
3 1/2	**7**	**10 1/2**	**14**	**17 1/2**	**21**	**24 1/2**	**28**	**31 1/2**	**35**	**38 1/2**	**42**	**45 1/2**
	7	5 1/4	4 2/3	4 3/8	4 1/5	4 1/12	4	3 15/16	3 8/9	3 17/20	3 9/11	3 19/24
4	**8**	**12**	**16**	**20**	**24**	**28**	**32**	**36**	**40**	**44**	**48**	**52**
	8	6	5 1/3	5	4 4/5	4 2/3	4 4/7	4 1/2	4 4/9	4 4/10	4 4/11	4 1/3
4 1/2	**9**	**13 1/2**	**18**	**22 1/2**	**27**	**31 1/2**	**36**	**40 1/2**	**45**	**49 1/2**	**54**	**58 1/2**
	9	6 3/4	6	5 5/8	5 2/5	5 1/4	5 1/7	5 1/16	5	4 19/20	4 10/11	4 7/8
5	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**65**
	10	7 1/2	6 2/3	6 1/4	6	5 5/6	5 5/7	5 5/8	5 5/9	5 1/2	5 5/11	5 5/12
5 1/2	**11**	**16 1/2**	**22**	**27 1/2**	**33**	**38 1/2**	**44**	**49 1/2**	**55**	**60 1/2**	**66**	**71 1/2**
	11	8 1/4	7 1/3	6 7/8	6 3/5	6 5/12	6 2/7	6 3/16	6 1/9	6 1/20	6	5 23/24
6	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**	**60**	**66**	**72**	**78**
	12	9	8	7 1/2	7 1/5	7	6 6/7	6 3/4	6 6/9	6 6/10	6 6/11	6 1/2
6 1/2	**13**	**19 1/2**	**26**	**32 1/2**	**39**	**45 1/2**	**52**	**58 1/2**	**65**	**71 1/2**	**78**	**84 1/2**
	13	9 3/4	8 2/3	8 1/8	7 4/5	7 7/12	7 3/7	7 5/16	7 2/9	7 3/20	7 1/11	7 1/24
7	**14**	**21**	**28**	**35**	**42**	**49**	**56**	**63**	**70**	**77**	**84**	**91**
	14	10 1/2	9 1/3	8 3/4	8 2/5	8 1/6	8	7 7/8	7 7/9	7 7/10	7 7/11	7 7/12
8	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**	**88**	**96**	**104**
	16	12	10 2/3	10	9 3/5	9 1/3	9 1/7	9	8 8/9	8 4/5	8 8/11	8 2/3
9	**18**	**27**	**36**	**45**	**54**	**63**	**72**	**81**	**90**	**99**	**108**	**117**
	18	13 1/2	12	11 1/4	10 4/5	10 1/2	10 2/7	10 1/8	10	9 9/10	9 9/11	9 3/4
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**	**110**	**120**	**130**
	20	15	13 1/3	12 1/2	12	11 2/3	11 3/7	11 1/4	11 1/9	11	10 10/11	10 5/6
11	**22**	**33**	**44**	**55**	**66**	**77**	**88**	**99**	**110**	**121**	**132**	**143**
	22	16 1/2	14 2/3	13 3/4	13 1/5	12 5/6	12 4/7	12 3/8	12 2/9	12 1/10	12	11 11/12
12	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**132**	**144**	**156**
	24	18	16	15	14 2/5	14	13 5/7	13 1/2	13 1/3	13 1/5	13 1/11	13

Bold figures are distances of lens from easel in enlarging, or from lens to photograph being reduced in copying. Light figures are distances from lens to negative being enlarged or camera extension in case reduced size copies are being made. The outer end of lens (cap end) should face bromide paper in enlarging and in reducing should face object being copied. Distances are measured from principal points, not diaphragm of lens, and while measuring these distances from diaphragm will give satisfactory results in many cases, when enlarging with large apertures or at great distances final focusing should be done by inspection. Data not given in the table may be calculated as follows:

CONJUGATE FOCI.— Let u = distance of object from lens, v = distance of image from lens, F = focal length of lens.

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v}, \text{ for example, } \frac{1}{3} = \frac{1}{12} + \frac{1}{4},$$

or $\quad F(u + v) = uv$, for example, $3(12 + 4) = 12 \times 4$.

If object is reduced n times upon the focusing screen, u is $n + 1$ times the focal length of the lens, and v is the focal length plus $\frac{1}{n}$ of the focal length. Thus 12 in. photographed down to 1 in. with a 6-in. lens gives

$$u = 13 \times 6, \text{ and } v = 6 + \left(\frac{1}{12} \times 6\right) = 6\frac{1}{2}.$$

RULE FOR COPYING.— To find distance from lens to original, multiply focal length of lens by the number of times of reduction, and add one focal length thereto. To find camera extension, divide focal length by number of times of reduction, and add one focal length thereto (see tables).

RULE FOR ENLARGING.— To find distance from negative to lens, divide focal length by number of times of enlargement, and add one focal length thereto.

To find distance from lens to paper, multiply focal length by number of times of enlargement, and add one focal length thereto.

(E) Focal Length for a Given Size of Plate

Moderate Angles of View.— In landscape photography the focal length should be so chosen that foreground and distance appear to stand in proper relation in the picture. Experience shows that views including an angle of 45° are thoroughly harmonious. In this case the objective may be used at speed *f*: 4.5. The camera therefore must be equipped with a satisfactory focusing arrangement, if the depth is to be properly divided and the view well placed on the plate. Reflecting cameras are of great advantage in this case.

If some means of focusing on the ground glass does not exist, the angle of view should be increased to about 55°, and if only a small finder is available 60° is better. The speed of the lens in this case will be about *f*: 6.3, and therefore the depth will be considerably greater. The proper *focal lengths of the objective* for different sizes of plates are as follows:

Size of plate in cm	Focal length in cm with an angle of view of		
	45°	55°	60°
6 x 9	13	10.5	9
9 x 12	18	14.5	13
13 x 18	27	22	19
16 x 21	32	25	23
18 x 24	36	29	26
24 x 30	46	37	33

Size of plate in in.	Focal length in in. with an angle of view of		
	45°	55°	60°
2¼ x 3¼	4½	3½	3
3¼ x 4¼	6	5	4½
3¼ x 5½ / 4 x 5	7½	6	5½
5 x 7	10	8	7
6½ x 8½	12½	10	9
8 x 10	15	12	11

In order to avoid distorted perspective, the focal length for portrait exposures should be so chosen that the distance between face and objective is at least 2 m (7 ft.). Therefore the objective for busts in *carte de visite* size must have a focal length of at least 20 cm (8 in.); for cabinets, 30 cm (12 in.); for boudoir size, 40 cm (16 in.). Further information on this point may be found in the catalogue of optical firms.

(F) Determination of the Quality of Correction of a Lens

Modern anastigmats are so well corrected for the field of view which one can properly expect them to cover, that it is unnecessary for the practical photographer to institute a thorough investigation as to the amount of the existing aberrations, a subject which is thoroughly treated in most textbooks on photography. It is unnecessary to say that there are excellent methods for exactly determining uncorrected errors. We will describe here only the processes by which we can determine the presence of chemical focus, focal differences and zones of unsharpness.

Focusing on a Plane Test Object.— If we desire to investigate the quality of a lens, we will arrive at entirely false conclusions unless we use as the *test object exclusively a plane object placed perpendicular to the optical axis*. Only in this case do we exclude the consideration of depth of field, so that we can deal solely with the object plane. The test object should also be as *contrasty* as possible, and free from color, that is, *black and white*. It may advantageously consist of vertical and horizontal lines, concentric circles and fine shadings, as well as text matter in different sizes down to small type. Such symbols and figures are most satisfactory when they are printed by photo-lithography on dull white paper, which does not discolor. Even lighting is especially necessary. Dark objects which show against the sky at a great distance, such as lightning rods, chimneys, or weather vanes, can be used for the investigation. A single object of this kind may suffice, even if not at a great distance. By turning the camera on the tripod its image can be brought to the edge of the circle of illumination.

I. CHEMICAL FOCUS

DETERMINATION OF CHEMICAL FOCUS BY FOCUSING ON SEVERAL PLANES OF THE IMAGE SPACE.— A 3¼ x 4¼ plate is entirely sufficient for this test. The lens must be tested at its extreme aperture. We place at the rear of the camera, immediately in front of the ground-glass, an opaque piece of paper with a perpendicular slit about ⅞ of an inch wide, at the open end of the plate holder. The test object is brought to the middle of the slit, and focused as closely as possible by means of a focusing glass. The plate holder, which must bring the plate exactly to the ground-glass plane, is then pushed in far enough so that the first fifth of the plate is behind the slit, and the back of the camera is racked in about ⅛ of an inch toward the lens. The exposure is made in this position. For the second exposure the camera back is moved back about 1-16 of an inch, and the second fifth of the plate brought before the slit. We proceed in this way until, for the last exposure, the plate holder has been pushed completely in so that the last fifth of the plate may be exposed, and the camera back is ⅛ of an inch back of the plane of sharp focus. After development we find five identical images on the plate, which, however, are not equally sharp. If the central image is sharpest, the visual and the photographic image coincide. Otherwise chemical focus is present, and we have under- or over-correction, according as the first or the last image shows the greatest sharpness. In the case of universal objectives of medium speed and focal length, such as are used for hand cameras, the chemical focus should in no case exceed 1-32 of an inch.

This method of focusing on different planes of the *image space*, which can be changed at will, according to the objects available, deserves preference over that employing focusing on different planes of the *object space*, because we can obtain without calculation the value of the chemical focus by several exposures on *the same object*.

Chromatic aberration depends on the character of the light which is used for exposure. Under some circumstances, however, ray-filters may be introduced and appropriate plates used. With ordinary light, it is advisable to use for this and the following test exposures, fine-grained plates.

II. FOCAL DIFFERENCE

Repetition of the Exposures with Different Stops.— In order to determine the influence of stopping down on the position of the sharpest focus, we repeat the exposures just described with the objective stopped down. If the greatest sharpness lies in the same plane as in the case of exposures at full aperture, the objective is free from focal difference, because of the absence of zones in the spherical aberration. As a matter of fact, this is actually the case with most modern anastigmats.

III. THE EXTENT OF THE SHARP IMAGE

Exposures Showing the size of the Circle of Illumination, on Different Planes of the Image Space.— It is advisable to make three exposures on plates large enough to show the whole circle of illumination. One exposure is made in the plane of the greatest photographic central sharpness, while the planes of the two others lie 1/16 or ⅛ of an inch before and behind this. If we repeat this group of exposures with different apertures, we can discover the distribution of sharpness in the image plane, and obtain an idea of how it may be possible, by ignoring central sharpness, to increase the extent of the sharp image. If the test chart is regularly divided, the image can also simultaneously be investigated for *distortion*.

Recognition of Chromatic Enlargement Differences.— If the objective has *chromatic enlargement differences* they will be evidenced by colored edges, which are more apparent toward the edge of the image. For an exact test of the suitability of the objective for three-color photography, the test exposures must be made through appropriate filters; if the objective has no secondary spectrum, the partial images will be equally large and equally sharp.

Necessary Sharpness.— We may be allowed to repeat that the test exposures must be made on a plane object. Exposures on a landscape with foreground are *not suitable* for judgment of the qualities of a lens. What sharpness the lens shall give is determined by the requirements necessitated by the purpose for which the lens is to be used. It is well known that the greatest sharpness is required for reproduction work. In general, it is advisable to use, as much as possible, lenses which are free from coma, and therefore possess a high rendering power. To test this, it is sufficient to carefully examine the image on the ground glass with a magnifier which enlarges not more than ten times. If the edges of the black figures are sharply cut and the transition to the white surroundings is absolutely definite, no coma exists; if this is present, the boundaries are soft and the image gray.

Whoever desires to become thoroughly acquainted with his lens and its use will not fail to make these few test exposures with the test chart set up in a position absolutely perpendicular to the optical axis.

(G) Technical Defects of the Lens

Flare Spots.— In order to determine the presence and size of flare spots, we may stick a little piece of black paper somewhat away from the center on the matt side of the ground-glass, focus on infinity, and point the apparatus at the sun, so that its image falls upon the black paper. We then easily see the position of the flare spots, whose size and shape are determined by the position of the secondary images, and their distance from the ground-glass, and also by the fact that they are images of the stop and are seen in distorted perspective. If it is not desired to focus on the sun itself, its reflection in a thermometer bulb may be used.

Reflections from Bright Parts of the Mount.— *Reflections* often occur, which are caused by unblackened parts of the mount. For this reason, when we are working with half the objective it must always be screwed into the front of the mount. Incompletely blackened spots in teleobjectives are especially dangerous on account of the length of the lens mount, if the camera walls do not receive the reflections. In this case it is especially useful to have a protecting hood on the front of the lens tube. It is useful to construct this with several diaphragms, the size of which increases from within outward to correspond with the shape of the cone of entering rays; for ease of packing it is advisable to make this hood telescopic, so that it can be closed up when not in use.

DECENTERING.— It may occur that a lens may be decentered by an accident in transport, and so give poor images. To test the *centering* the objective must be set up in a lathe head so that its optical axis coincides with the axis of rotation. If the lenses are centered, the images of the surroundings, for instance of a light, produced by the reflecting surfaces, must be immovable during rotation. If one or more of them vibrate, this shows that not all of the centers of curvature of the lens surfaces coincide with the optical axis. This defect can be easily remedied by the maker of the lens.

(H) THE POSITION OF THE RAY FILTER WITH RELATION TO THE LENS

THE RAY FILTER AS A PLATE WITH PLANE PARALLEL SIDES.— A ray filter may be placed either between the object and the lens or between the lens and the image. *In no case* should it be placed within the lens, because as a plate with plane parallel sides it would effect a change in the light path between the lens elements which would influence the quality of any anastigmat.

Only in the case of lenses of the aplanat type, with great separation of the elements, is the lengthening of the distance between the halves caused by the filter without effect.

Position in Photographing Full Size.— If the object is so placed in relation to the lens that its image is produced full size, the displacement of the image produced by introducing a filter, is equally large whether it is used before or just behind the lens. If the distance of the lens is changed, however, the displacement of the image must be less if the filter is placed between the lens and the more distant of the two principal points. In landscape photography the image is therefore displaced only a small amount, if the filter is placed before the lens. Although it is recommended for this work to place the filter in this position, the construction of the camera or the position of the shutter may make it advisable to place the filter behind the lens.

If the quality of the image produced by the objective is not to be degraded, the filter must be prepared as an optical instrument of precision, and be absolutely *a plate with plane parallel sides*. If, however, it is placed in the image plane, the ill effects of possible deviations from plane-parallelism are avoided, but in this case the filter must be as large as the plate.

Sharp Focusing with a Filter.— In every case the image on the ground glass is moved away by introducing a filter, so that the bellows length must be increased. We must, therefore, always focus with the filter *in place:* in this way we also allow for a possible lens effect of the filter, if its sides are not exactly parallel.

In order to avoid a change in the position of the ground glass and the surface of the plate when working with screen-plates, as in the Autochrome process, the firm of Zeiss has prepared color filters which act as very weak dispersing lenses.

ERRATA

Page 19.	Line 18.	Change *is* to *are.*
Page 67.	Line 5.	Change *Point* to *Pencil.*
Page 98.	Line 2.	Delete *Photographic.*
Page 108.	Line 26.	Change $\sqrt{10:4}$ to $\sqrt{10:16}$.

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