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for

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MENSURATION

FOR

SHEET METAL WORKERS

AS APPLIED IN WORKING ORDINARY PROBLEMS IN SHOP PRACTICE

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CONTENTS.

Mensuration for Sheet Metal Workers.

Very little has been written about mensuration for the sheet metal worker, and although various collections of tables have been published, the rules are not generally explained so as to be easily understood or to enable one to proceed intelligently with practical problems which come up. In the pages which follow examples in computing the circumferences, areas and capacities for various shapes arising in practice are given in detail. The comprehension of these will enable the student to compute any ordinary problem in the shop.

Besides methods for finding the lengths, areas and volumes of the simpler geometrical forms, examples are given in computing the areas of heating and ventilating pipes of all ordinary shapes, making their areas equal to those of pipes of other profiles. The use of the prismoidal formula for obtaining the capacities of various shaped bodies is fully explained, as is a method of obtaining the hight of any solid to hold a given quantity when the diameter is known, or *vice versa*. A short rule is illustrated for finding the diameters of branch pipes taken from a given main pipe, so that the areas of the branches will equal the area of the main, and many other problems are treated.

One of the first problems arising in the shop is to find the true length of material required for round or other shaped pipes. The rule for obtaining the circumference of any circle is to multiply the diameter by 3.1416, or, as is sometimes used in practice, by 3 1-7.

In Fig 1, A represents a circle 4 inches in diameter. Then 4 inches × 3.1416 = 12.5664, or 12 9-16 inches

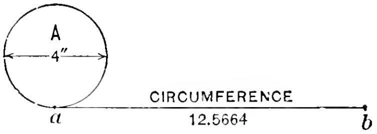


Fig. 1.—Finding Length of Material Required for a Round Pipe.

approximately, the circumference, as shown rolled out from *a* to *b*.

Referring to Fig. 2, let *a b c d* represent a 4-inch square. To obtain the perimeter or amount of material for this pipe, it is only necessary to multiply 4 inches by 4, which equals 16 inches. As shown from *a* to *a′*, the distance represents the length of the sum of the sides of the square figure.

In the same manner the length of the perimeter of the hexagon is obtained in Fig. 3. In this each side measures 3 inches. The figure has six sides, so we have 6 × 3 inches = 18 inches, the length shown from *a* to *a′*.

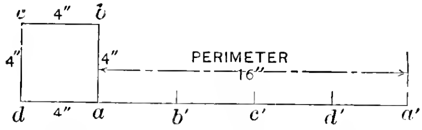


Fig. 2.—Finding Amount of Material for a Square Pipe.

Sometimes a round pipe is to be formed to a square section at the opposite end, using the same amount of material as in the round pipe. This makes it desirable to know what the length of the sides at the square end will be. Knowing that the diameter of the circle *a* in Fig. 4 is 4 inches, and that the circumference is 12.5664,

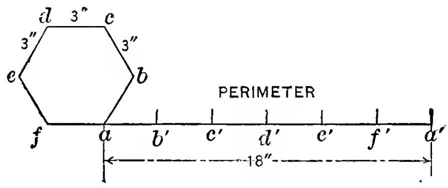
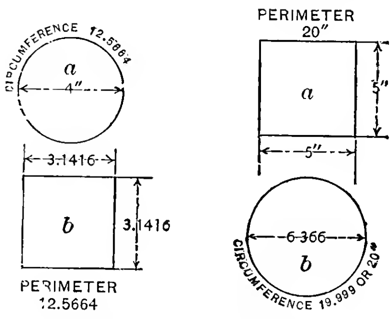


Fig. 3.—Finding Length of Perimeter of a Hexagon.

it is only necessary to divide by four, which will give 3.1416 inches, the length of the sides of the square *b*. If the circumference is not known, multiply the diameter by 0.7854; thus, 4 inches $\times$ 0.7854 $=$ 3.1416 inches, which



Figs. 4 and 5.—Finding Square and Circular Sections of Pipe of Given Dimensions.

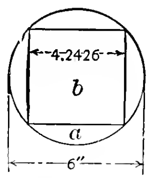


Fig. 6.—Finding Length of Side of a Square to Be Inclosed by a Given Circle.

multiplied by four sides $=$ 12.5664 inches, the perimeter for the square *b*, and is equal to the circumference of the circle *a*, proving the above rules.

If, however, the conditions were reversed, and each side of a given square measured 5 inches, as shown at *a* in Fig. 5, making the perimeter of the square 20 inches, and it is desired to know what diameter a circle would have whose circumference would be equal to that perimeter, to obtain this diameter multiply the length of one side, or 5 inches, by 1.2732, which equals 6.366 inches, the required diameter. Multiply this diameter, 6.366

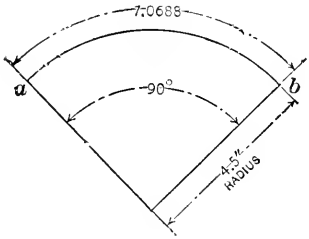


Fig. 7.—Finding Length of Arc When Angle and Radius Are Known.

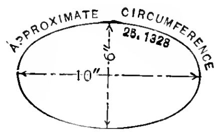


Fig. 8.—Finding Amount of Material for an Ellipse When Length and Width Are Given.

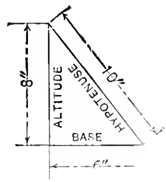


Fig. 9.—Finding Length of Hypotenuse in a Right Angle Triangle.

inches by 3.1416, and the product will be 20 inches, the perimeter of the square thus proving the rule.

When double ventilation pipes are constructed, as shown in Fig. 6, where the outer pipe is a true circle and the inner pipe a square, allowing an air space between the two pipes, and it is desired to know the length of the side of a square to pass inside of a given circle, it is only necessary to multiply the diameter of the given pipe by 0.7071. Suppose the round pipe *a* were 6 inches in diameter, then 6 inches × 0.7071 = 4.2426, or nearly 4¼ inches, for the side of the desired square.

When it is desired to find the length of an arc, when only the angle and radius are known, then multiply the number of degrees by the diameter and the product by 0.008727. In Fig. 7 it is desired to find the length of the 90-degree arc *a b*, whose radius is 4.5 inches, or diameter 9 inches. Following the above rule, we have 90 × 9 = 810. 810 × 0.008727 = 7.0688 inches, the length of *a b*.

When the length and width of an ellipse are given, and it is desired to know how much material is required for its circumference, multiply half the sum of the two diameters by 3.1416. Thus, in Fig. 8, we have an ellipse

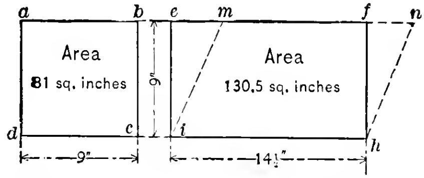


Fig. 10.—Areas in Square, Rectangle and Rhomboid.

the major axis of which is 10 inches and the minor axis 6 inches. Following the above rule, $\frac{10 + 6}{2} = 8 \times 3.1416 = 25.1328$. It should be understood that the circumference of an ellipse cannot be accurately determined, and the above rule is merely an approximation giving fairly close results.

When a large smoke stack is to be carried up at an angle on a building and the vertical hight and horizontal projection are known, the length of the slant can be obtained by the rule illustrated in Fig. 9, which shows how the length of the hypotenuse is found in a right angle triangle. Add the square of the base to the square of the altitude, and the square root of the sum will be the hypotenuse. The base being 6 inches and the altitude 8 inches, we have $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$. The square root of 100 is 10, because 10 is the number which, when multiplied by itself, will equal 100.

The area of any surface is the number of square inches or square feet within its outline. In connection with the illustrations, which immediately follow, the rules will be given for obtaining the areas of the various geometrical shapes, which will enable the student to proceed intelligently when computing areas and capacities of various forms arising in practice. While many sheet metal workers understand how to compute the areas of the more common geometrical shapes, there are some who do

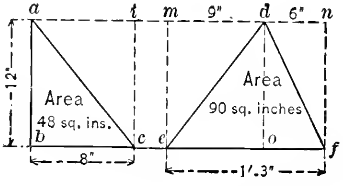


Fig. 11.—Areas in a Right and an Oblique Triangle.

not know how this branch of mathematics can be applied in practical work.

One of the most simple figures, the area of which the student usually masters first, is that of the square, shown by *a b c d* in Fig. 10, each side of which measures 9 inches. To obtain the area simply multiply the length of the side by itself; thus: $9 \times 9 = 81$ square inches.

To obtain the area of the rectangle *e f h i*, multiply the width by the length, thus: $9 \times 14.5 = 130.5$ square inches.

Suppose a surface had to be covered with sheet metal, the shape being that of a rhomboid, shown by *m n h i*, in which *m n* is parallel to and of the same length as *h i*; then knowing the perpendicular hight *c i* to be 9 inches, and the length to be 14½ inches, we would have the same area as that shown in the rectangle *e f h i*, because the triangle *f n h* equals the triangle *c m i*.

When the area of a triangle is to be found, whether right or oblique, the base and perpendicular hight being known, the rule is to multiply the perpendicular hight by the base, and half the product is the area. In Fig. 11, let *a b c* represent a right triangle, whose base is 8 inches and hight 12 inches; then $12 \times 8 = 96$. $96 \div$

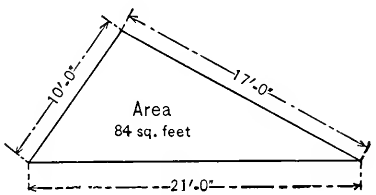


Fig. 12.—Area of Triangle When Three Sides Are Given.

$2 = 48$ square inches, the area. If the product 96 were not divided by two it would represent the area of the rectangle *a i c b*, but by drawing the diagonal *a c* we divide the rectangle into two right triangles, each equal to one-half of 96, or 48, as shown.

The diagram *e d f* represents an oblique triangle, whose base is 15 inches and perpendicular hight to the apex 12 inches. Now, following the above rule, we have $12 \times 15 = 180$. $180 \div 2 = 90$ square inches area of *d e f*. To prove this, construct the rectangle *e m n f*, and from *d* drop the vertical line *d o*. The distances *m d* and *d n* are 9 and 6 inches, respectively. We then have two rectangles, one 9 x 12 and the other 6 x 12 inches.

By drawing *d e* and *d f* we obtain two right triangles. *d o e* and *d o f*. Following the explanation given in connection with *a b c*, we have $\frac{9 \times 12}{2} = 54$, and $\frac{6 \times 12}{2} = 36$; $36 + 54 = 90$ square inches, the same as the area of *d e f*.

Sometimes an irregular shaped structure is to be covered and none of the angles and only the dimensions of the three sides are known or can be obtained. The rule to be followed in this case is as follows: From half the

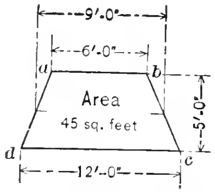


Fig. 13.—Area of Trapezoid.

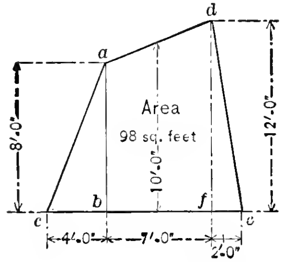


Fig. 14.—Area of Trapezium.

sum of the three sides subtract each side separately; find the continued product of the half sum of the sides and the three remainders, and the square root of this product is equal to the area of the triangle. In Fig. 12 is shown a portion of a surface, each side measuring respectively 21, 17 and 10 feet. The sum of the three sides is $21 + 17 + 10 = 48$. $48 \div 2 = 24$, the half sum. Subtracting each side separately from half the sum of the sides, we get the three remainders, $24 - 21 = 3$ feet; $24 - 17 = 7$ feet, and $24 - 10 = 14$ feet. Now, $24 \times 3 \times 7 \times 14 = 7056$. $\sqrt{7056} = 84$.

The method of obtaining the square root is as follows: Pointing off the number 7056 into periods of two figures each, we get 70′56, which shows that the complete part of the square root contains two figures. Then proceed as below:

Trial divisor.	Correct divisor.	Root.
160	164	70′56(84 ans.
		64
		656
		656

It will be observed that the greatest number whose square is contained in 70 is 8; and, therefore, 8 is the first root of the figure; $8 \times 8 = 64$. Subtracting 64 from 70 and bringing down the next period, 56, we get the first partial dividend, 656. The double of 8, the partial root already found, is 16, and annexing a cipher to this we get 160 as the first trial divisor. This trial divisor is contained in the partial dividend 656 four times, which suggests four as the second figure of the root. Adding 4 to 160 we obtain 164 as the correct divisor. When the product ($4 \times 164 = 656$) is subtracted from the partial dividend 656 there is no remainder. Eighty-four is the required square root, and $84 \times 84 = 7056$. The triangle in Fig. 12 contains 84 square feet area.

In Fig. 13 is shown a trapezoid, and in Fig. 14 a trapezium. The difference between the two lies in the fact that the trapezoid has two sides parallel to each other, while the trapezium has no sides parallel. When computing the area of an irregular surface having two parallel sides use the rule for obtaining the area of a trapezoid, shown in Fig. 13, which is as follows: Multiply half the sum of the parallel sides by the perpendicular hight. Half the sum of the parallel sides in Fig. 13 is $6 + 12 = 18$; $18 \div 2 = 9$, the mean distance. Then

9×5 (the perpendicular hight) $= 45$ square feet area in *a b c d*.

In computing the area of *a d c c*, in Fig. 14, we divide it into two triangles and a trapezoid, by drawing vertical lines from the angles *a* and *d*, at right angles to *c c*. The bases of the two triangles are 4 and 2 feet, respectively, and the altitudes 8 and 12 feet, respectively; the mean distance of the trapezoid is $\frac{8 + 12}{2} = \frac{20}{2} = 10$

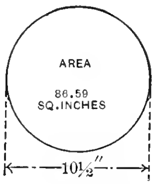


Fig. 15.—Area in Circle.

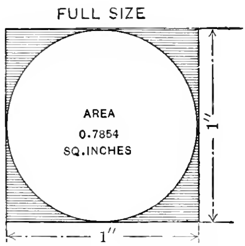


Fig. 16.—Area in Circle and Square.

Then $\frac{8 \times 4}{2} = \frac{32}{2} = 16$, area of triangle *a b c*; $\frac{12 \times 2}{2} = \frac{24}{2} = 12$, area of triangle *d e f*; $10 \times 7 = 70$, area of trapezoid *b a d f*. Then $16 + 12 + 70 = 98$ square feet area in *a d c c*.

To obtain the area of a circle, square the diameter and multiply by 0.7854. Fig. 15 shows a section of a pipe 10½ inches in diameter, of which it is desired to find the area. Then, $10.5 \times 10.5 = 110.25$; $110.25 \times 0.7854 = 86.59$ square inches. Fig. 16 shows a square containing 1 square inch. The inscribed circle is 1 inch in diameter. Although it is 1 inch in diameter, it contains only 0.7854 square inch.

When the area of a ring is to be determined, as shown in Fig. 17, in which the outside diameter is 12 inches and the inside diameter 5 inches, deduct the square of the small diameter from the square of the large diameter and multiply the remainder by 0.7854. Following this rule, we have $12^2 - 5^2 = 12 \times 12 - 5 \times 5 = 144 - 25 = 119$; $119 \times 0.7854 = 93.462$ square inches in the ring.

Sometimes a ventilating pipe is to be constructed, whose section is a sector of a circle, as shown by *a b c e* in Fig. 18, and it is necessary to know its area. In the

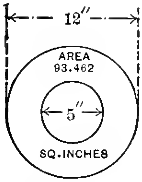


Fig. 17.—Area in a Ring.

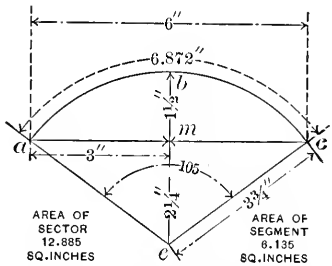


Fig. 18.—Areas in Sector and Segment of Circle.

illustration, the radius *e c* is 3¾ inches; the angle *a e c*, 105 degrees (which is determined by the use of the protractor); the chord *a c*, 6 inches, and the rise or middle ordinate *m b*, 1½ inches. Before obtaining the area, it will be necessary to know the length of the arc *a c*, which is obtained by following the rule given in connection with Fig. 7, Part I, as follows: In Fig. 18 we have $105 \times 7.5 \times 0.008727 = 6.872$ inches, the desired length of arc. Then to obtain the area of *a b c e*, multiply the length of the arc by half the radius. The radius is 3.75; $3.75 \div 2 = 1.875$. Then $1.875 \times 6.872 = 12.885$, the area of the sector.

If the area of the segment *a b c a* is desired, knowing the area of the sector, it is only necessary to deduct from this amount the area of the triangle *a c e*. Thus half the distance of the chord *a c* is 3 inches, and the hight from the chord *m* to the center *e* is $2\frac{1}{4}$ inches. Then $3 \times 2.25 = 6.75$ square inches, the area of *a c e*. Then 12.885 (the area of the sector) — 6.75 (the area of the triangle) = 6.135 square inches, the area of the segment.

If the chord *a c* and the rise *m b* of the segment were

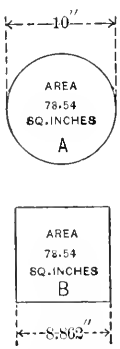


Fig. 19.—Square Whose Area Is Equal to Area of Given Circle.

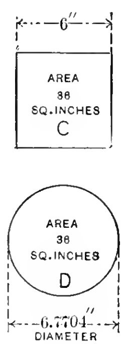


Fig. 20.—Circle Whose Area Is Equal to Area of Given Square.

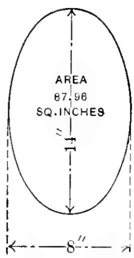


Fig. 21.—Area of Ellipse.

given, and it was desired to find the diameter of the circle of which the segment is a part, divide the sum of the squares of half the chord and the rise by the rise, and the quotient is the desired diameter. Half of the chord *a c* is 3 inches, and the rise *m b* is $1\frac{1}{2}$ inches. Following the above rule, we have $3^2 + 1.5^2 = 11.25$; $11.25 \div 1.5 = 7.5$ inches, the diameter. As twice the radius *e c* equals $7\frac{1}{2}$ inches, the above rule is proven.

It sometimes happens that a transition is to be made for a heating or ventilating pipe from round to square, and the square end is to have the same area as the given round end. Let A in Fig. 19 represent a section of a 10-inch heating pipe; then, following the rule given in connection with Fig. 15, the area of A in Fig. 19 is 78.54 square inches. To find the dimensions of a square of equal area, multiply the given diameter by 0.8862. Thus $10 \times 0.8862 = 8.862$, or the side of the square B.

If, however, the conditions were reversed and the square end of the transition piece were given, say 6 inches, as shown in C, Fig. 20, then multiply the given side by 1.1284, and the product will be the diameter of a

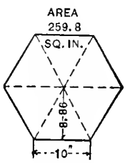


Fig. 22.—Area in Regular Polygon.

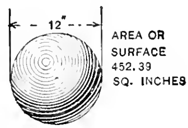


Fig. 23.—Area of Sphere.

circle of equal area. Thus $6 \times 1.1284 = 6.7704$ inches, diameter of circle D.

When the area of an ellipse is required, multiply the long diameter by the short diameter, and this product by 0.7854. In Fig. 21 is shown an ellipse, whose long diameter is 14 inches and short diameter 8 inches. Then $8 \times 14 = 112$; $112 \times 0.7854 = 87.96$ square inches area. The method for obtaining the dimensions of the opposite end of a transition piece, with one end a given ellipse and the opposite end to have similar area in either round, square or rectangular section, will be explained later under the head of Practical Examples for the Shop.

When a surface is to be covered with sheet metal whose shape is that of any regular polygon, the rule to be followed in obtaining the area is to multiply the sum of the sides by half the perpendicular distance from center to sides. For example, there is shown in Fig. 22 a regular polygon having six sides, called a hexagon. The one side, *c d*, equals 10 inches, and the sum of the six sides, 6 × 10, or 60 inches. The whole perpendicular distance *b a* is 8.66 inches, which divided by two gives 4.33, or half the perpendicular distance. Then 60 × 4.33 = 259.8 square inches area in the hexagon.

When a sphere is to be made of copper or sheet iron,

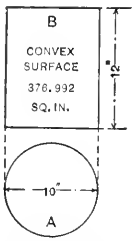


Fig. 24.—Convex Surface of Cylinder.

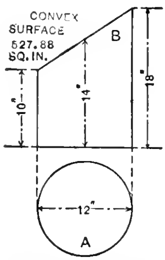


Fig. 25.—Convex Surface of Frustum of Cylinder.

and it is required to know the amount of material it will take, the area is obtained by squaring the diameter of the sphere and multiplying by 3.1416. The sphere shown in Fig. 23 is 12 inches in diameter. To find its area, we have 12 × 12 = 144; 144 × 3.1416 = 452.39 square inches area.

In Fig. 24, A is the plan of a cylinder 10 inches in diameter and B the elevation, 12 inches high. It is desired to find the convex surface of this cylinder. In this problem, as well as in others which will follow, the areas of the ends will not be considered, as this was explained in previous problems. The rule to be employed in obtaining the area of the convex surface of any cylinder is to multiply the circumference by the hight. As the cylinder is 10 inches in diameter and the hight 12 inches, then 10 × 3.1416 = 31.416, or circumference; 31.416 × 12 = 376.992 square inches in convex surface.

In Fig. 25, A and B show the plan and elevation of a frustum of a cylinder. To obtain this convex surface, multiply one-half the sum of the greatest and least hights by the circumference. The greatest hight is 18, the least 10; $10 + 18 = 28$; $\frac{28}{2} = 14$; $14 \times 3.1416 \times 12 =$ 527.78 square inches area.

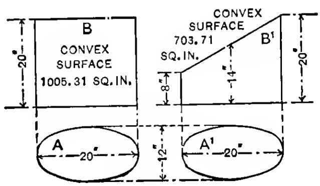


Fig. 26.—Convex Surface of Elliptical Cylinder and Frustum of Elliptical Cylinder.

Another problem often arising in the shop is to find the convex surface of an elliptical tank. The same rule is employed as in a cylinder—that is, multiply the circumference by the hight. In Fig. 26, A and B and A¹ and B¹ show, respectively, the plans and elevations of an elliptical cylinder and the frustum of an elliptical cylinder, whose long diameter is 20 inches and short diameter 12 inches. Following the rule given in Fig. 8, Part I, for obtaining the circumference, we have $\frac{20 + 12}{2} = \frac{32}{2}$ $= 16$; $16 \times 3.1416 = 50.2656$; $50.2656 \times 20 = 1005.31$ square inches in the convex surface B. For the frustum

B^1 we have $\frac{8 + 20}{2} = \frac{28}{2} = 14$; 14×50.2656 (the circumference) $= 703.71$ square inches.

The same rules applicable to the cylinder are also applicable to prisms whose bases are regular polygons, whether they be right prisms or frustums. In Fig. 27, A and B represent, respectively, the plan and elevation of a prism 18 inches high, each side of the polygon being

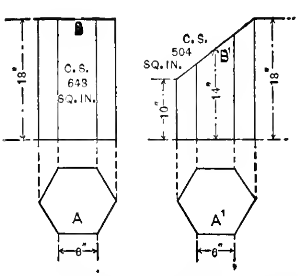


Fig. 27.—Convex Surface of Right Prism and of Frustum of Right Prism.

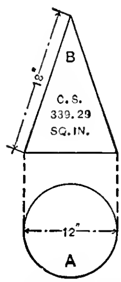


Fig. 28. — Convex Surface of Right Cone.

6 inches. The perimeter of the hexagon is $6 \times 6 = 36$ inches; $36 \times 18 = 648$ square inches in the convex surface B. For the convex surface of the frustum, we have $\frac{10 + 18}{2} = \frac{28}{2} = 14$; 14×36 (the perimeter) $= 504$ square inches.

When the convex surface of any right cone or pyramid is desired, then multiply the circumference, or periphery, of base by half the slant hight. In Fig. 28 A and B show the plan and elevation of a right cone, which will serve as an example. The diameter of the cone at its base is 12 inches; its circumference is therefore 12

$\times$ 3.1416 = 37.6992; half of the slant hight is 9; then 9 $\times$ 37.6992 = 339.29 square inches in the convex surface of the cone B.

Using the same rule for Fig. 29, whose base or plan A is a hexagon with 7-inch sides, we have 7 $\times$ 6 = 42; 9 $\times$ 42 = 378 square inches in the convex surface of the pyramid B.

Suppose the elevations in Figs. 28 and 29 were cut off parallel to the base, forming the frustums of a cone

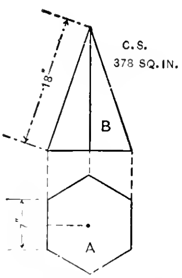


Fig. 29.—Convex Surface of Right Pyramid.

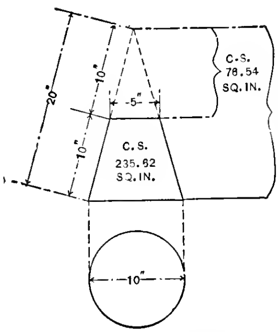


Fig. 30.—Convex Surface of Frustum of Right Cone.

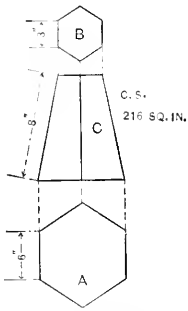


Fig. 31.—Convex Surface of Frustum of Right Pyramid.

and pyramid. The rule for obtaining the area of these convex surfaces is to multiply the circumferences or peripheries of the two ends by half the slant hight. An example of this problem is given in Fig. 30, in which the plan of the base A is 10 inches and the diameter at the top 5 inches. Then 5 $\times$ 3.1416 = 15.708, or circumference at top, and 10 $\times$ 3.1416 = 31.416, or circumference at base; 15.708 + 31.416 = 47.124; 47.124 $\times$ 5 (half the slant hight) = 235.62 square inches of convex surface. To prove this problem, we will assume that we have a right cone in Fig. 30, whose base is 10 inches and slant hight 20 inches. Then, following the rule given in connection with Fig. 28, we have $10 \times 3.1416 \times 10 = 314.16$ square inches convex surface in whole cone. The area of the upper half of the cone shown by dotted lines, whose base is 5 inches, is $5 \times 3.1416 \times 5 = 78.54$ square inches; $314.16 - 78.54 = 235.62$ square inches, the area of the frustum of the cone, proving the problem.

In Fig. 31, A is a regular polygon, with 6-inch sides, and the plan of the base of the frustum of a pyramid;

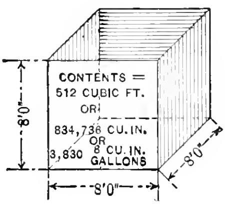


Fig. 32.—Contents of Cube.

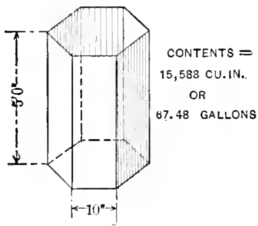


Fig. 33.—Contents of Hexagonal Prism.

B the plan of the top, whose sides are 3 inches; C, the elevation, has a slant hight of 8 inches. Then $6 \times 6 = 36$, or perimeter of base; $3 \times 6 = 18$, or perimeter of top; $36 + 18 = 54$; one-half the slant hight is 4; $4 \times 54 = 216$ square inches in C.

Finding Capacities of Tanks, Etc.

Some sheet metal workers would be at a loss to proceed if a customer came in the shop and required a tank constructed of No. 20 galvanized sheet iron to hold 63½ gallons, the tank to fit in a space 51 inches high. Knowing the number of gallons and the hight, it would be necessary to know the diameter before the tank could be laid out. While rules given in various publications are understood by those versed in mensuration, the less skilled do not know how to apply them practically. In computing the capacity of any vessel we deal with cubic and liquid measures, and therefore it may not be out of place to present the tables of cubic and liquid measures:

Cubic or Solid Measure.

1728 cubic inches = 1 cubic foot, or 12 x 12 x 12.
27 cubic feet = 1 cubic yard, or 3 x 3 x 3.
231 cubic inches = 1 United States gallon.
57.75 cubic inches = 1 United States quart.
28.875 cubic inches = 1 United States pint.
7.21875 cubic inches = 1 United States gill.

Liquid Measure.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.
31½ gallons = 1 barrel.
63 gallons, or 2 barrels = 1 hogshead.
1 gallon = 4 quarts = 8 pints = 32 gills.

In the problems that follow practical examples have been given as they are apt to arise in the shop, and the student should have no difficulty in learning to figure the capacity of any vessel which may arise in practice. One of the most simple forms to be computed is that of a cube or square tank, shown in Fig. 32. Here we have a tank 8 x 8 feet square and 8 feet high. The rule to be employed in finding the solidity, whether the base is a square or rectangle, is to multiply the length of any one side by its adjoining side and multiply the product obtained by the hight. Then we have $8 \times 8 = 64$; $64 \times 8 = 512$ cubic feet. As a cubic foot contains 1728 cubic inches, we have 1728×512, or 884,736 cubic inches. To find the number of gallons the tank will hold divide 884,736 by 231, the number of cubic inches in a gallon, and we get $884{,}736 \div 231 = 3830$ gallons, and 6 cubic inches over.

Suppose a tank is to be constructed whose base is any regular polygon, as shown in Fig. 33, where a tank is shown 5 feet high whose base is a hexagon, each side measuring 10 inches. The rule to be used is to multiply the area of the base by the hight. Referring to Fig. 22, we find that the area of a hexagon whose side is 10 inches is 259.8 square inches, which multiplied by the hight in Fig. 33 will give the cubic contents. The hight shown is 5 feet. As the area of the base is in inches, then reduce the hight to inches; then $60 \times 259.8 = 15{,}588$ cubic inches. For the number of gallons, $15{,}588 \div 231 = 67\frac{1}{2}$ gallons, scant.

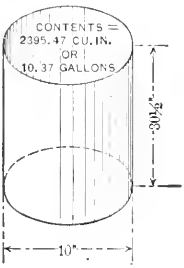


Fig. 34.—Contents of Cylinder.

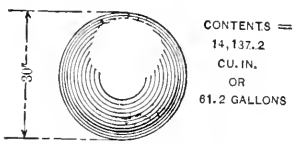


Fig. 35.—Contents of Sphere.

In Fig. 34 is shown a cyclinder or round tank, the contents of which is obtained by the same rule as in the preceding figure. The bottom of this round tank is 10 inches in diameter and its hight $30\frac{1}{2}$ inches. Then $10^2 \times 0.7854$ equals the area of the base; or $10 \times 10 = 100$; $100 \times 0.7854 = 78.54$ square inches; $30.5 \times 78.54 = 2395.47$ cubic inches. For the number of gallons, divide the above product by 231 and the quotient will be 10.37 gallons.

When a copper ball is made to use as a float in a large tank, it is sometimes desirable to know the number of cubic inches in same. Thus in Fig. 35 is shown a sphere 30 inches in diameter, whose capacity is formed by multiplying the cube of the diameter by 0.5236, or $30^3 = 27,$-000; $27,000 \times 0.5236 = 14,137.2$ cubic inches; $14,137.2 \div 231 = 61.2$ gallons capacity.

When the contents are required of a cone or pyramid, the rule to follow is to multiply the area of the base by one-third the perpendicular hight. In Fig. 36 is shown a

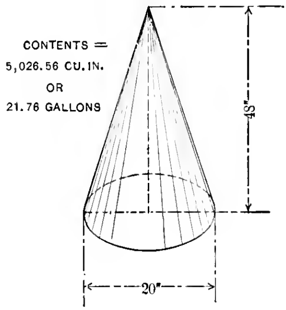


Fig. 36.—Contents of Cone.

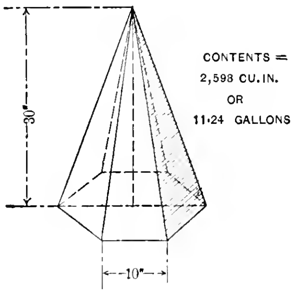


Fig. 37.—Contents of Pyramid.

cone, whose base is 20 inches and whose vertical hight is 48 inches. Following the above rule, we have $20 \times 20 = 400$; $400 \times 0.7854 = 314.16$ square inches, or area, multiplying which by one-third the vertical hight, 48, or 16, we have $314.16 \times 16 = 5026.56$ cubic inches. Dividing this by 231, the number of cubic inches in a gallon, we get 21.76 gallons.

The method of computing the solidity of a pyramid is shown in Fig. 37, which shows a pyramid whose base is a hexagon, each side being 10 inches, and whose vertical hight is 30 inches. As the area of a hexagon each side of which is 10 inches is 259.8 square inches, then multiply this amount by one-third the vertical hight (30), or 259.8 $\times$ 10 = 2598 cubic inches, which divide by 231 and we get 11.24 gallons.

The prismoidal formula can be used in calculating the volume or capacity of a prism, cylinder, cone, pyramid, frustum of a cone or pyramid, wedge, as well as many irregular shaped bodies. A prismoid is by definition a solid whose bases are polygons and lie in parallel plans and whose faces are quadrilaterals or tri-angles. To find

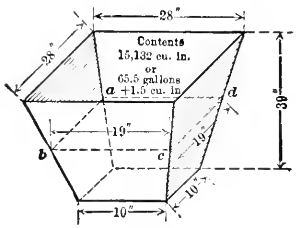


Fig. 38.—Contents of Hopper.

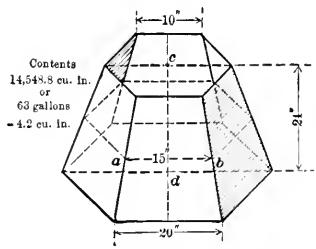


Fig. 39.—Contents of Frustum of Pyramid.

the contents of a prismoid, or any of the above mentioned solids, add together the areas of the two parallel planes and four times the area of a section taken midway between and parallel to them, and multiply the sum by one-sixth of the perpendicular distance between the parallel planes.

Applying this rule for obtaining the volume of a wedge, pyramid or cone, the area of the upper base is 0, because it runs to an apex. For prisms or cylinders the areas of the upper, lower and middle planes are equal. The prismoidal formula when applied to the frustum of a pyramid saves the labor of extracting the square root, as required under the old rule.

As an example let us apply the above formula in obtaining the solidity of the cone shown in Fig. 36. Following the above rule, we have as the area of the lower base 314.16 and the area of the upper base 0. Four times the area of the middle section, which is 10 inches in diameter, is $10 \times 10 \times 0.7854 = 78.54 \times 4 = 314.16$. The sum is $314.16 + 314.16 = 628.32$. One-sixth the perpendicular hight is $48 = 8$; $8 \times 628.32 = 5026.56$ cubic inches.

In Figs. 38 and 39 is shown the method of computing the volumes of the frustums of a square and hexagonal pyramid. Fig. 38 shows an inverted hopper, such as is usually made of heavy galvanized iron. Whether the shape is regular or irregular, the same method is used in determining the capacity. The top opening is 28 x 28 inches, the bottom 10 x 10 inches, and the sides, a section taken midway between the top and bottom, would equal $\frac{28 + 10,}{2}$ or 19; then 19 x 19 inches would be the size for the middle section, *a b c d*. The area of the upper plane equals 28 x 28 inches, or 784 square inches; of the lower plane 10 x 10 inches, or 100 square inches, and of the middle plane 19 x 19 inches, or 361 square inches. Four times the middle plane is 1444. Then, following the rule, we have upper plane, 784, + lower plane, 100, + four times middle plane, 1444, = 2328. This is multiplied by one sixth the vertical hight, 39. Then $2328 \times \frac{39}{6} = 15{,}132$ cubic inches. This divided by 231, the number of cubic inches in a gallon, gives 65½ gallons and 1½ cubic inches over.

The same rule is applied to Figs. 39 to 42, inclusive. Fig. 39 shows a hopper hexagonal in shape, each side of the lower base of which is 20 inches, the upper base 10 inches, and a section taken midway between the two $\frac{10 + 20}{2}$, or 15 inches, as *a b*. To obtain the area of these planes multiply the square of one of the sides of the regular poly-

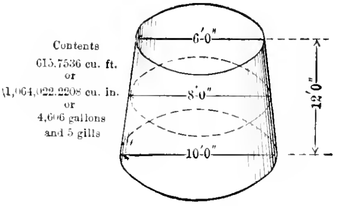


Fig. 40.—Contents of Frustum of Cone.

gon by the multiplier given in the following table for the proper polygon:

Name.	Sides to polygon.	Multiplier.
Triangle	3	0.433
Square	4	1.000
Pentagon	5	1.720
Hexagon	6	2.598
Heptagon	7	3.634
Octagon	8	4.828
Nonagon	9	6.182
Decagon	10	7.694

As the shape in question is a hexagon, we find the multiplier in the table to be 2.598. Then, following the above rule, the area of the lower plane is equal to $20 \times 20 = 400$; $400 \times 2.598 = 1039.2$. The area of the upper plane, $10 \times 10 \times 2.598 = 259.8$, and four times the area of the middle plane, $15 \times 15 \times 2.598 \times 4 = 2338.2$. Then $1039.2 + 259.8 + 2338.2 = 3637.2$. $3637.2 \times \frac{24}{6} =$ 14,548.8 cubic inches contents. This divided by $231 = 63$ gallons, or 2 barrels, less 4.2 cubic inches.

Fig. 40 shows the frustum of a cone. A tank of this form is usually made from ¼-inch metal, riveted and reinforced with angles and tees. The top diameter is 6 feet, bottom 10 feet and the middle diameter $\frac{6 + 10}{2}$, or 8 feet. The area of the top section is $6 \times 6 \times 0.7854 = 28.2744$ square feet; the bottom area, $10 \times 10 \times 0.7854 = 78.54$ square feet; the middle area, $8 \times 8 \times 0.7854 = 50.2656$. Then $28.2744 + 78.54 + (50.2656 \times 4) = 307.8768 \times \frac{12}{6} = 615.7536$ cubic feet capacity. Multiply this amount by 1728, the number of cubic inches in a foot, and divide

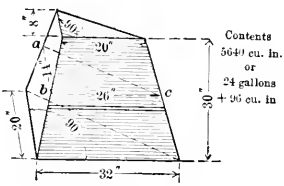


Fig. 41.—Contents of Prismoid.

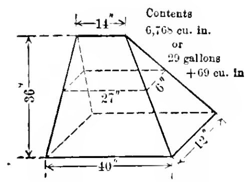


Fig. 42.—Contents of Wedge.

by 231, the number of cubic inches in a gallon, and we obtain the capacity in gallons. Thus, $615.7536 \times 1728 = 1{,}064{,}022.2208$ cubic inches, which divided by $231 = 4606$ gallons and 5 gills.

Fig. 41 shows an odd shaped vessel, whose parallel bases are right angle triangles. The bottom base is 20 x 32 and contains 320 square inches; the top 8 x 20, and contains 80 square inches. The size of the middle section is obtained by adding 20 and 32 and dividing by 2, which gives the length *b c*, or 26 inches. In similar manner $\frac{8 + 20}{2} = 14$, length of *a b*. Then the area of *a b c* is

$\frac{14 \times 26}{2} = 182$. Then $80 + (182 \times 4) + 320 = 1128$.

$1128 \times \frac{30}{6} = 5640$ cubic inches. Divide this amount by 231 and we get 24 gallons and 96 cubic inches over.

In Fig. 42 is given another example in obtaining the capacity of odd shape solids, this example being in the form of a wedge. The rectangular base is 12 x 40 inches and contains in arear 480 square inches. The top runs to an apex 14 inches wide, whose area is 0. A section taken midway between the top and bottom equals 6×27 inches and has an area of 162 square inches. Then $0 + (162 \times 4) + 480 = 1128$. $1128 \times \frac{36}{6} = 6768$ cubic inches, which, divided by $231 = 29$ gallons, 1 quart and 11.25 cubic inches over.

Practical Examples for the Shop.

The section of an 8 x 32 inch rectangular pipe is represented in Fig. 43 of the diagrams. If a transition is made to a perfectly square pipe, what must each side measure? In solving this problem proceed as follows: Extract the square root of the area of the rectangular pipe, 256 square inches, which is 16 inches, the size of the square pipe.

Supposing this 8 x 32 inch pipe was to form a transition to another rectangular pipe, the width of which was 12 inches, what must be its length to have the same area? Simply divide 256 by 12, and the quotient will be 21 1-3, making the size of the pipe 12 x 21 1-3 inches.

If this 8 x 32 inch pipe were to form a transition to an oblong pipe with semicircular ends, 8 inches in diameter, as shown at A, what must the length of the distance be, shown from *a* to *b*? As two semicircles make a full circle, then deduct the area of the 8-inch circle from 256, and divide the remainder by eight, as follows: Area of 8-inch circle = 50.26; 256 — 50.26 = 205.74; 205.74 ÷ 8 = 25.72, or 25¾ inches scant, the length from *a* to *b*.

In Fig. 44 is shown a fitting in hot air piping known as a boot. With it a 14-inch round horizontal pipe and a vertical rectangular pipe whose width is 7 inches are

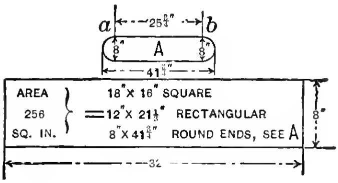


Fig. 43.—Pipes Equal to Rectangular Pipe.

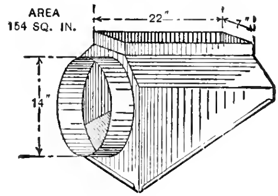


Fig. 44.—Boot.

joined. What must the length of this rectangular pipe be so that it will have the same area as the 14-inch round? The area of a 14-inch round pipe is 153.93, or approximately 154 square inches. Divide 154 by 7, and the quotient will be 22 inches, the desired length.

Suppose the size of the rectangular pipe is given, say 7 x 22, and it is desired to know what size round pipe will have similar area? Divide the area of the given pipe, or 154, by 0.7854, and extract the square root of the product. Thus 154 ÷ 0.7854 = 196. $\sqrt{196} = 14$. Then 14 inches is the diameter of the round pipe.

In Fig. 45 is shown a chimney cap, which measures 8 x 6¼ inches at the bottom. The cap is to form a transition from square to round. What must be the diameter at the top so that the area will be similar to the base?

The same rule could be used as given in connection with Fig. 44, but by using a table of areas and circumferences of circles, much labor in computing can be saved.

These tables are found in some text books, in engineers' pocket books and in the " Tinsmith's Helper and Pattern Book," published by the David Williams Company.

In the following examples the use of these tables will be explained when applied to sheet metal work. Referring to Fig. 46, the base measures 8 x 6¼ inches and equals 50 square inches area. Instead of computing, to

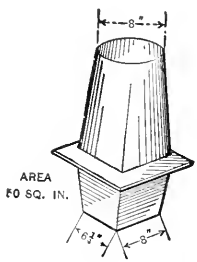


Fig. 45.—Chimney Top.

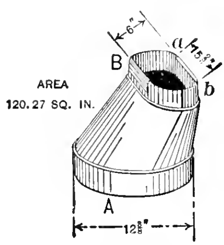


Fig. 46.—Boot.

obtain the diameter of the top of the cap, simply refer to the table of areas and circumferences of circles, following the column under areas until 50.2655 is reached (the nearest to the required number 50), when we find it is the area of an 8-inch circle, which will be the top diameter of the cap.

This method could have been used in Fig. 44, in which the area of the rectangular opening is 154 square inches. Follow the area column in the table until 153.9380 is reached, which suggests a 14-inch circle, as shown.

In Fig. 46 is shown another form of boot. In this case the inlet A is 12⅜ inches in diameter and equals

120.27 square inches. If the outlet B is to be oblong in shape with semicircular ends 6 inches in diameter, what must the length be of the straight side *a b?* Find the area of a 6-inch circle, which is 28.27 square inches. Deduct this from 120.27, which leaves 92. Divide this amount by six, the given width of the outlet, and the quotient will be 15 2-3 inches, the desired length of *a b*. Both sections then equal 120.27 square inches.

In Figs. 47 and 48 is shown how to compute the amount that a duct must be increased in size in proportion to the number and size of inlets connected. Fig. 47

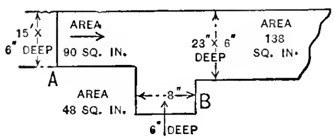


Fig. 47.—Pipe with Branches.

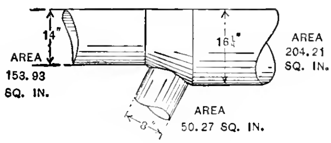


Fig. 48.—Pipe with Branches.

shows a rectangular duct, whose depth is 6 inches throughout. Ducts of this kind are usually employed in ventilating work, and are increased in size every time a register is connected. A shows the first inlet, 6 x 15 inches, having 90 square inches area. The second inlet, B, is 6 x 8 inches, and has 48 square inches area. The combined area of A and B is 138 square inches. Now what size must the duct be beyond B so as to have the area of the two inlets A and B? Divide 138 by 6 and the quotient will be 23, the width of the duct. This same rule is followed no matter how many inlets are taken up.

In Fig. 48 is shown the method employed when the pipe is round. The first inlet is 14 inches and the second 8. They contain, respectively, 153.93 and 50.27 square inches area. The combined area of the two inlets is 204.20

square inches. The circle that has this area is found to be 16⅛ inches in diameter. This is then the diameter of the large pipe.

A triangular ventilating pipe is shown in Fig. 49. The size of the pipe is 16 x 32, containing one-half of the product of its dimensions, or 256 square inches area. What must the size of the various pipes be if a transition is desired to square, to rectangular 8 inches wide, and round? To obtain the size of the square pipe, simply ex-

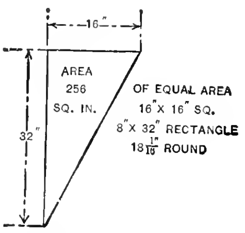


Fig. 49.—Triangular Pipe.

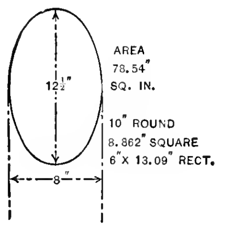


Fig. 50.—Elliptical Pipe.

tract the square root of 256. Then 16 x 16 is the size of the square pipe. The length of the section of the rectangular pipe, whose width is desired to be 8 inches, is obtained by dividing 256 by 8. The quotient will be 32, or the required length. For the size of the round pipe, which should have an area equal to the triangular, follow the column of areas in the table of circle areas until 256.2398 is reached, which will be found to equal the area of a circle 18 1-16 inches in diameter and is the desired round pipe.

In Fig. 50 is shown the section of an elliptical pipe measuring 8 x 12½ inches. The area of this ellipse is $12.5 \times 8 \times 0.7854 = 78.54$ square inches. Suppose a transition is to be made to round, square or rectangular pipe, 6 inches wide, whose areas must be similar to the ellipse, what must their sizes be? Following the column of areas in the table, we find the area 78.54 is for a 10-inch round pipe. For the size of the side of a square pipe we have $\sqrt{78.54} = 8.862 +$ inches, or about $8\frac{7}{8}$ inches. Thus the square pipe would be $8\frac{7}{8} \times 8\frac{7}{8}$ inches. For the length of the rectangular pipe, whose given width is 6 inches, divide 78.54 by 6, and the quotient will be 13.09. Then 6×13.09 inches will be the size of the rectangular pipe.

In putting up ventilating, blower and blast pipes, it is

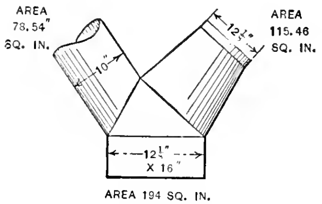


Fig. 51.—Two-Pronged Fork.

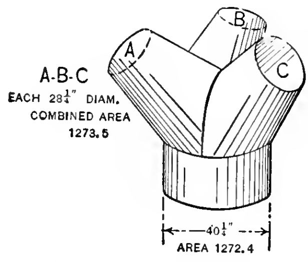


Fig. 52.—Three-Pronged Fork.

often the case that a number of branches are connected to one main, and the main pipe must have the combined area of the branches. An example of this is shown in Fig. 51, where two round branches are connected to a rectangular main pipe, whose width must be $12\frac{1}{8}$ inches. What must the length of the section of this main pipe be to have a combined area of the two branches? The area of the 10-inch pipe is 78.54 square inches. The area of the $12\frac{1}{8}$-inch pipe is 115.46 square inches. The sum of these two areas is 194 square inches; $194 \div 12.125$, the width of the main pipe, $= 16$. Therefore the size of the main pipe is $12\frac{1}{8} \times 16$ Fig. 52 shows three branches of round pipe, connecting in fork shape to a round main. Each of the branches, A, B and C, is 23¼ inches diameter, the combined area of which equals 1273.6 square inches. What must the size of the main be? Following the table of areas, we find the nearest number to be 1272.4, which is the area of a pipe 40¼ inches diameter. By using the tables of areas and circumference much time and labor are saved in computing. If these tables

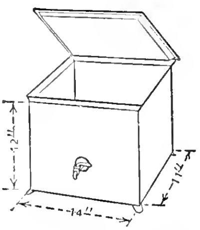


Fig. 53.—A Square Tank.

Fig. 54.—A Sphere.

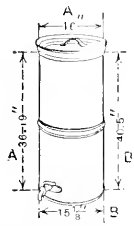


Fig. 55.—An Oil Tank

are not available, the calculations must be made by methods described in the earlier installments of this series.

Ascertaining Sizes of Articles.

What follows is devoted to explaining how to obtain the unknown size of an article when the hight and capacity are given, or *vice versa.* The first problem, shown in Fig. 53, represents a water tank. Assume that a customer has ordered an 8-gallon tank, whose base is to measure 11 x 14 inches. How high must it be to have the desired capacity? The rule to follow in any square or rectangular tank is: Reduce the gallons to cubic inches; divide this amount by the area of the base, the quotient being the desired hight. As there are 231 cubic inches in the United States gallon, then in 8 gallons there will be 8 × 231 or 1848 cubic inches. The base is 11 x 14 and contains 154 square inches area. Then 1848 ÷ 154 = 12 inches, the required hight of the tank.

Suppose the hight and length of one of the sides and the capacity are given. What will the size of the remaining side be? Assuming the capacity to be 8 gallons, the hight 12 inches and the given side 11 inches, then dividing the number of cubic inches in 8 gallons by 12 and the quotient by 11 the result will be the required side. Thus,

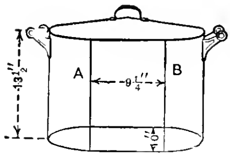


Fig. 56.—Wash Boiler.

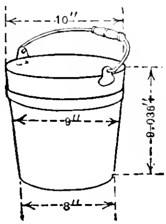


Fig. 57.—Flaring Pail.

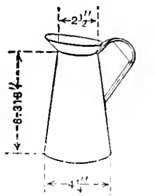


Fig. 58.—Flaring Measure.

1848 ÷ 12 = 154; 154 ÷ 11 = 14, the required length of side.

A copper ball Fig. 54, to be used as a float must contain 22,449 35-100 cubic inches. What must the diameter be to contain the above number of cubic inches? The rule to follow is to divide the number of cubic inches by 0.5236, and from the quotient thus obtained extract the cube root. As the sphere is to contain 22,449.35 cubic inches, then 22,449.35 ÷ 0.5236 = 42,875. The next step is to extract the cube root of this quotient.

When a number is multiplied by itself, as 5 × 5, the product, 25, is called the square of that number. When a number is multiplied by itself twice, as $5 \times 5 \times 5$, the product, 125, is called the cube of that number. Therefore the extraction of the cube root is nothing more than the finding of that number which, when multiplied by itself twice will result in the given number. To extract the cube root of 42,875, start at the decimal point and, counting to the left, separate the number into periods of three figures, as shown by 42'875.

Trial divisor.	Number.	Cube root.
	42'875.(	35
	27	
2700	15875	
	42875	

Find the greatest number whose cube is contained in the first or left hand period, 42; $4 \times 4 \times 4 = 64$, and is too great. Then take 3; $3 \times 3 \times 3 = 27$. Therefore 3 is the first figure of the root. Subtracting 27 from 42 we obtain 15. Bring down the next period, 875, obtaining the first partial dividend, 15,875. Take three times the square of the root already found, which is $3^2 \times 3 = 3 \times 3 \times 3 = 27$. Annex two ciphers to it, and we have 2700 for the trial divisor. Divide the trial divisor into 15,875, which suggests 5 as the second figure of the root. Prove this by multiplying $35 \times 35 \times 35$, which equals 42,875, as shown, and leaves no remainder. Then 35 is the cube root of 42,875. Therefore the sphere in question must be 35 inches in diameter.

The trial divisor is sometimes contained in the partial dividend, a higher number than required. Whether it is too high or not can be ascertained by cubing the root found, and if its product is higher than the partial dividend a lower number must be taken, whose cube will be equal to or smaller than the partial dividend. If there had been a remainder in the problem just shown, and it was desired to continue the root, periods of three ciphers each would have to be added to the whole number, 42,875, and continued as above described, so as to obtain the decimal part of the root.

In Fig. 55 is shown an oil tank. Assume a tank whose diameter is 16 inches that must hold one barrel: Then what must be the hight of the tank? Reduce the barrel to cubic inches, into which divide the area of the 16-inch circle. The quotient will be the required hight. One barrel equals 31½ gallons, or 7276.5 cubic inches. The area of a 16-inch circle is 201.062 square inches; $7276.5 \div 201.062 = 36.19 +$. Therefore 36 1-5 inches is the desired hight.

Suppose the tank is to hold the same quantity and the hight is to be 40½ inches. What must the diameter be? In this case multiply the number of cubic inches by the hight and divide the quotient by 0.7854. From the quotient thus obtained extract the square root, which will be the desired diameter. 7276.5 cubic inches, the capacity of the tank, $\div$ 40.5 inches, the hight, $= 179.6667$; $179.6667 \div 0.7854 = 228.758 +$; $\sqrt{228.758} = 15.124$, or 15⅛ inches, the desired diameter, as shown at B.

In Fig. 56 is shown a 10-gallon wash boiler, whose base is 10 inches wide, with semicircular ends, the length of the straight part of the side from A to B being 9¼ inches. What must be its hight? Reduce the capacity to cubic inches and divide it by the area of the bottom, the quotient giving the required hight. Ten gallons equal 2310 cubic inches. The area of the bottom equals the area of a 10-inch circle, which is 78.54, plus 92.5, the area of the rectangle $9\frac{1}{4} \times 10, = 171.04$ square inches. $2310 \div 171.04 = 13.5$ inches, the hight of the boiler.

If for any reason the hight of the boiler and the diameter of the semicircular ends are given, and it is desired to know the width of the straight side A B, then find the capacity in cubic inches and divide by the hight; from the quotient obtained subtract the area of the two semicircles and divide the remainder by the given width of the base, and the quotient will be the desired width. Thus: $2310 \div 13.5 = 171.11$; $171.11 - 78.54 = 92.57$; $92.57 \div 10 = 9.26$, the desired width of A B.

In Figs. 57 and 58 are shown a flaring pail and a measure whose capacities and top and bottom diameters are given, and it is required to find their hight. The rule applicable to any form of flaring ware whose section is round, no matter what its capacity may be, is to find the number of cubic inches in the given capacity, which divide by the sum of the areas of the top and bottom diameters and four times the area of the middle section and multiply the quotient by six. As one quart contains 57.75 cubic inches, a 10-quart pail, shown in Fig. 57, will contain 577.5 cubic inches. The area of the top diameter equals 78.54, the bottom diameter 50.26, and the middle section, whose diameter is 9 inches, 63.61 square inches; $63.61 \times 4 = 254.44$ square inches. Then $254.44 + 50.26 + 78.54 = 383.24$; $577.50 \div 383.24 = 1.506 \times 6 = 9.036$ inches, the required hight.

The measure shown in Fig. 58 is to hold one quart. Its top and bottom diameters are $2\frac{1}{2}$ and $4\frac{1}{4}$ inches, respectively. What must the hight be? Following the same rule as above we have: Area of top equals 4.90. Area of bottom equals 14.18. The middle diameter equals $\frac{2.5 + 4.25}{2} = 3.375$. Its area equals 8.94. $8.94 \times 4 = 35.76$.

Capacity of one quart equals 57.75 cubic inches; divided by combined areas, or 54.84, leaves a quotient 1.053, which multiplied by 6 equals 6.318 inches, or hight.

In Fig. 59 is shown the method of finding the hight in elliptical flaring ware when the top and bottom dimensions and capacity are given. The tub in this case is to hold 32 pints and 21⅜ cubic inches, the top dimensions to be 11 x 15½ and the bottom 8 x 12½ inches. What must the hight be? The rule to follow is the same as in the preceding problem. It should be remembered that

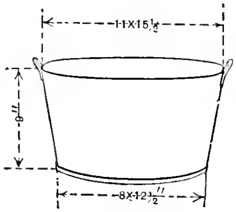


Fig. 59.—Flaring Elliptical Tub.

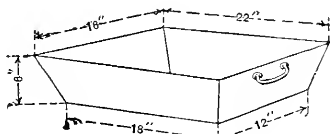


Fig. 60.—Flaring Pan.

the area of an ellipse is found by multiplying the short and long diameters together and their product by 0.7854. Working this out, it will be found that 9 inches is the desired hight of the tub.

Another problem where the same rule is employed is shown in Fig. 60, in which a drip pan to hold 29 quarts and 13¼ cubic inches is illustrated. The top is to be 16 x 22 and the bottom 12 x 18 inches. The middle section is found as follows: $\frac{16 + 12}{2} = 14$; $\frac{18 + 22}{2} = 20$, or 14 x 20 inches.

Following the rule as before we have 6, the number of inches in the hight of the pan.

Short Rules in Computation.

We now come to a point where a few short rules in computation may be of value to the sheet metal worker. In Fig. 61 let A B represent a wall with a rounded corner from *a* to *b*, on which a molding, gutter or cornice is to be placed, and it is desired to find the radius. To do this measure the distance from *a* to *b*, which is 5 feet. Bisect *a b*, obtaining *d*, and from *d* measure the distance, at

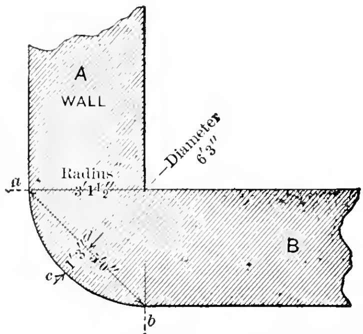


Fig. 61.—Finding Diameter of Circle.

right angles to *a b*, to the outside of the curve at *c*, which in this case measures 1 foot 3 inches. Divide the sum of the squares of one-half the chord and the rise by the rise, and one-half the quotient will be the desired radius. Thus, reducing to inches, we have,

$$\frac{30^2+15^2}{15\times 2}=\frac{900+225}{30}=\frac{1125}{30}=37\frac{1}{2} \text{ inches}$$

or 3 feet 1½ inches radius.

In Fig. 62 is shown how the area of a given object can be obtained, even though it is so far away that measurements cannot be taken. This is obtained by proportion of triangles. Let us assume in this case that the spire shown at A is to be covered with metal or slate. If we obtain the contract, we will erect scaffolding to do the work; but it will not pay to erect scaffolding to take the measurements and measurements must be obtained to estimate on the job. As we can get to the ridge of the roof, B C, the number of square feet in the four sides of the tower is obtained as follows: Measure a given dis-

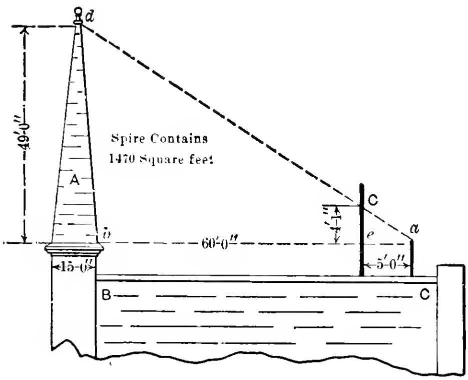


Fig. 62.—Finding Hight of Inaccessible Point.

tance from the bottom of the roof of the spire, *b*—in this case 60 feet—to *a*. At *a* place a rod so high as to make the imaginary line *b a* horizontal. Take another rod and place it vertically at a distance of about 5 feet from *a*, as shown at C. Then, with the eye at *a*, cast the line of sight from *a* to the top of the spire, at *d*, and mark the second rod where the line of sight crosses it at C, and measure the distance from C on *a d* to *e* on *a b*, which in this case is 4 feet 1 inch. Then, C *e* is to *d b* as *e a* is to *b a*. Then the vertical hight from *d* to *b* will be 49 feet. As the vertical hight is 49 feet the pitch *d b* will be a fraction of a foot more, but 49 will be safe enough for estimating purposes.

In Fig. 63 is shown the method of finding the diameters of two circles whose areas are to be equal to a given area, or *vice versa*, and Fig. 64 shows various examples solved by the steel square. Let A in Fig. 64 represent two branches of 3-inch pipes, which are to be

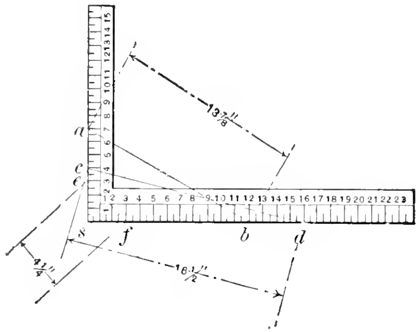


Fig. 63.—Use of Steel Square.

connected to a pipe whose area is equal to the areas of the two 3-inch pipes; what must the diameter of the main pipe be? As both the given pipes are 3 inches, then simply lay the 2-foot rule from 3 to 3 on the square, in Fig. 63, as shown by *c f*, which will measure 4¼ inches, the diameter of the main pipe to have twice the area of one 3-inch pipe.

Suppose the diameter of the main, or 4¼-inch, pipe in Fig. 64 were given, and it was desired to take two branch pipes of equal diameter from it. Lay the rule of 4¼ inches over the angle of the square in Fig. 63 so that the distances from *s* to *c* and *s* to *f* are equal, then each side will measure 3 inches, as shown, the diameter of the branches shown in Fig. 64.

In B are shown two branches, 7 and 12 inches, respectively; what must the diameter of the main be to equal their capacities? Set the rule on 7 and 12 in Fig. 63, as shown, from *a* to *b*, which will measure 13⅞ inches, the diameter of the main pipe in B in Fig. 64.

Again, suppose the main pipe of 13⅞ inches diameter were given and a 7-inch pipe were already in place, what diameter must the other branch be so that the two

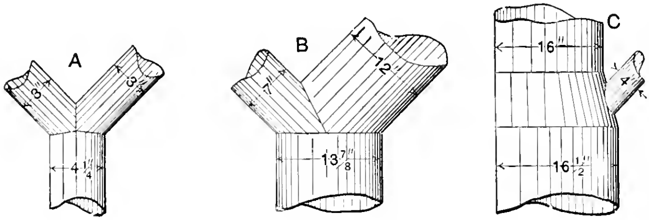


Fig. 64.—Problems for Steel Square Solution.

branches will have the capacity of the main? Place the 13⅞-inch mark of the rule on the 7-inch point, or *a*, and where the zero end intersects the lower edge of the square, which is at *b*, the 12-inch point on the square. This indicates that the diameter of the desired branch is 12 inches.

Suppose in C in Fig. 64 we have a 16-inch pipe, to which a 4-inch branch was added. How much must the pipe increase in size? Place the rule on the square in Fig. 63 from 4 to 16, or from *c* to *d*, and it will measure 16½ inches, the size of the increased main in Fig. 64. This rule can also be used to advantage when square pipes are to be used.

In Fig. 65 is shown the method of obtaining the length of radius by computation. This rule saves the time and labor of laying out the full size drawing of an article which has little flare and of obtaining the blank for a curved molding in cornice work, such as shown in the diagram A^1 by *a b*. Let us suppose that a flaring collar is required, whose base, D C, is 40 inches in diameter, and top, A B, is 36 inches. How can the length of the radius be found? The rule to follow is to multiply the large

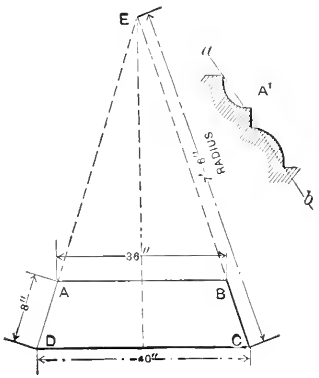


Fig. 65.—Obtaining Radius.

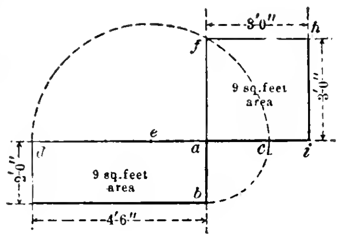


Fig. 66.—Rectangle to Square.

diameter by the slant hight and divide by the difference between the large and small diameters. The large diameter is 40 inches, the slant hight 8 inches, the small diameter 36 inches, and the difference between the large and the small is 4 inches. Then $8 \times \frac{40}{4} = 80$, or the length of the radius E C.

The five following problems are devoted to obtaining sections of pipes of similar area to given sections by means of the compass and steel square.

In Fig. 66 is shown how a rectangular section can be changed to a square section of equal area. Let *a b d* represent a section 2 x 4.5 feet, which contains 9 square feet of area. Extend *d a* as *d i*, and with *a* as center and *a b* as radius, draw the quarter circle *b c*. Bisect *c d* at *e*. With *e* as center draw the semicircle *d c*. Extend *b a* until it intersects the circle at *f*; then, from *f*, complete the square *f h i a*, which will be 3 x 3 feet and contain 9 square feet.

Fig. 67 shows how a circle is changed to a square of

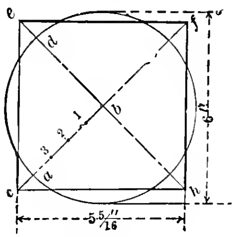


Fig. 67.—Circle to Square.

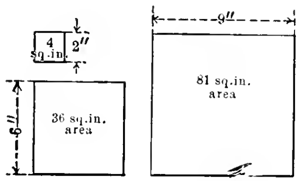


Fig. 68.—Three Squares.

equal area. Divide the radius *a b* into four equal parts and place one of the parts from *a* to *c* and from *d* to *e*. Draw *c e* and complete the square *c e f h*, which will measure, using the rule, 5 5-16 inches, the square of which will be its area and equal in area to the 6-inch circle.

Fig. 68 shows three separate square pipes, which are to be changed to one square pipe. One pipe is 2 x 2 inches; the second, 6 x 6 inches, and the third, 9 x 9 inches. Their total area is 121 square inches. In computation, we would simply extract the square root of 121 and obtain the side. In this case, draw, in Fig. 69, the right angle *a b c*, making *a b* 2 inches and *b c* 6 inches.

Draw *a c*, at right angles to which erect *c d*, 9 inches long. Draw *d a* and complete the square *a d e f*, which will contain 121 square inches.

In Fig. 70 is shown how to obtain a circle whose area is twice that of a given circle. Let A be the given circle; draw the diameter *a b*, at right angles to which draw *b c*, equal to *a b*. Draw *a c*; bisect the same and ob-

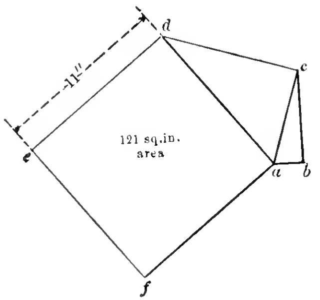


Fig. 69.—Method of Fig. 68.

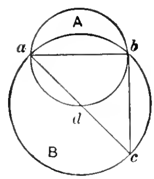


Fig. 70.—Circle of Four Times the Area.

tain *d*. With *d* as center and *d a* as radius draw the circle B, which will contain twice the area of A.

Finding Circumference of an Ellipse.

The following communication, brought out by the publication of the articles in The Metal Worker, Plumber and Steam Fitter, refers to the rule for finding the circumference of an ellipse:

From Engineer, New York.—In the article on "Mensuration for Sheet Metal Workers," the author gave a rule for finding the circumference of an ellipse that would only serve for the roughest kind of work. This rule—half the sum of the major and minor axes multiplied by

3.1416—is fairly accurate when the ellipse approaches a circle—*i. e.*, when the minor axis is nearly as long as the major axis. When the ellipse is very flat—*i. e.*, when the minor axis is small, compared with the major axis, the formula is very inaccurate, and could not be successfully used in practice. This rule produces a result too small; and for that reason should should not have been given for the use of the sheet metal worker, for it is a well-known principle in pattern cutting that the work, or the pattern, should not be made too small. If it cannot be made of the right size, it should be made too large, for it can always be cut down, but never can a small piece be cut larger.

When D (the major axis) and d (the minor axis) are equal, the formula works out accurately; when d is four-fifths of D, the error is — 0.42 per cent.; when d is three-fifths of D, the error is — 1.76 per cent.; when d is two-fifths of D, the error is — 4.60 per cent.; when d is one-fifth of D, the error is — 10.45 per cent.; and when d equals 0—*i. e.*, when the ellipse has flattened to a straight line, the error is — 21.45 per cent. Below is given a table showing the workings of four formulas. In calculating the results shown, the major axis in each case has been assumed as 10, and the minor axis as 10, 8, 6, 4, 2 and 0. The first formula used, designated by A, is that given by the author of " Mensuration for Sheet Metal Workers," and which has just been referred to. It is as follows:

$$\text{Circumference} = 3.1416 \frac{D + d}{2}$$

After it, is shown the percentage of error at the several points.

Formula B, the one next given, is:

$$\text{Circumference} = 3.1416 \sqrt{\left(\frac{D^2 + d^2}{2}\right)}$$

This is sometimes simplified into the form:

$$\text{Circumference} = 2.2215 \sqrt{(D^2 + d^2)}$$

while this is not as simple to work out as A, from the column of errors that follows it one can see that it is much more accurate and that the error is in giving a somewhat too large result.

Formula C, prepared by Trautwine for civil engineers, and which he claims is sufficiently accurate for ordinary principles, the error not exceeding more than 1 part in 1000, when D does not exceed 5 d, is as follows:

$$\text{Circumference} = 3.1416 \sqrt{\frac{D^2 + d^2}{2} - \frac{(D - d)^2}{8.8}}$$

If D exceeds five times d, then, instead of dividing $(D - d)^2$ by 8.8, divide it by the number in this table:

$D =$	Divide by	$D =$	Divide by
6 d	9.	20 d	9.8
7 d	9.2	25 d	9.87
8 d	9.3	30 d	9.92
9 d	9.35	40 d	9.98
10 d	9.4	50 d	10.04
12 d	9.5	60 d	10.10
14 d	9.6	70 d	10.17
16 d	9.68	80 d	10.23
18 d	9.75	100 d	10.35

Formula D is also quite accurate, and is somewhat neater to work than Formula C. It is as follows:

$$\text{Circumference} = 3.1416\ d + 2(D - d) - \frac{d(D - d)}{\sqrt{(D + d)(D + 2d)}}$$

In calculating the percentages of errors in Formulas A and C, it has been assumed that Formula D is correct, as it evidently is when $d = D$ and when $d = 0$.

The table comparing the formulas is as follows:

D = 10 d	A.	Error. Per cent.	B.	Error. Per cent.	C.	D.
10	31.42	0	31.42	0	31.42	31.42
8	28.27	− 0.42	28.45	+ 0.21	28.37	28.39
6	25.13	− 1.76	25.91	+ 1.29	25.57	25.58
4	21.99	− 4.60	23.93	+ 3.82	23.06	23.05
2	18.85	− 10.45	22.66	+ 7.65	21.02	21.05
0	15.71	− 21.45	22.22	+ 11.10	19.96	20.00

Sheet metal workers who have occasion to determine the circumference of the ellipse might do well to make note of these rules, because they will find it much quicker and cheaper to take a little more time in figuring out their work accurately, than to use a short formula with long error.