AMERICAN SOCIETY OF CIVIL ENGINEERS.

The Results of Investigations Relative to Formulas for the Flow of Water in Pipes.

By EDMUND B. WESTON.

WITH DISCUSSION

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THE RESULTS OF INVESTIGATIONS RELATIVE TO FORMULAS FOR THE FLOW OF WATER IN PIPES.

By EDMUND B. WESTON, M. Am. Soc. C. E.

WITH DISCUSSION.

INTRODUCTORY.

About thirteen years ago, when the writer first commenced to make a practical application of hydraulic formulas, he was unable to find one for calculating the loss of head due to the friction of water flowing in pipes, which he had not heard criticised more or less unfavorably by hydraulic engineers.

This unsatisfactory state of affairs led him to commence to make special investigations upon the subject, and to collect original data of an experimental nature relating to the same, in order to prove to his own satisfaction, if possible, if any of the formulas that he was familiar with were reliable for general use, and if not, to endeavor to construct one.

The results of these researches, which are illustrated by sketches and diagrams, are now presented in this paper.

The data of five hundred and twenty experiments were obtained, which are tabulated in a systematic manner in Table No. 1.

A careful study of the experimental data convinced the writer that the same formula would not apply to all cases, and that in order to make an intelligent investigation, it would be necessary to divide it into the three following classes, viz.: that which had been obtained with pipes having very smooth interior sides similar to brass and lead pipes, that which had been obtained with pipes having interior sides similar to new cast-iron pipes, and that which had been obtained with pipes having interior sides similar to old cast-iron pipes whose interior sides had become roughened by oxidation.

The result of this division was the construction by the writer of a new formula for the flow of water in pipes having very smooth interior sides, and his coming to the conclusion that two formulas constructed by the eminent French Civil Engineer, Henry Darcy, were very well adapted for pipes having interior sides similar to new cast-iron pipes.

A general formula was not found that would satisfactorily apply to old cast-iron pipes having interior sides that had become roughened by oxidation, and it was not possible to construct one from the limited data at hand, although a formula by Darcy, and several by the writer constructed for individual cases, agree very well with the experimental results from which they were derived.

The writer also briefly discusses the form of formulas the best adapted for the flow of water in pipes, and calls attention to formulas for computing the co-efficients of resistance, for enlargements, contractions, elbows and curves.

The investigations and subjects that this paper treats upon will hereafter be referred to, or described in detail in the following order:

NOMENCLATURE.

Unless distinctly stated to the contrary, the following characters will always represent English feet when they apply to linear distances, and have these significations :

h = the head. When the pipe is submerged, it is the difference in elevation between the surface of the water at the inlet and outlet; or when the pipe discharges into the open air, it is the difference in elevation between the surface of the water at the inlet, and the center of the outlet end of the pipe.

h_f = the loss of head due to friction, of the water flowing against the interior sides of the pipe, etc.

l = the length of the pipe.

d = the mean interior diameter of the pipe.

g = the acceleration of gravity.

v = the mean velocity, per second, of the water flowing in the pipe, or the velocity, per second, of the water issuing from the outlet end of the pipe.

ε = the co-efficient of influx or resistance of the entrance of the water into the pipe.

$\varDelta$ = the co-efficient of contraction of the entrance of the water into a pipe.

ζ = the co-efficient of friction of the water flowing against the interior sides of the pipe.

ϕ = the co-efficient of resistance of the water passing from a large to a small pipe.

μ = the co-efficient of resistance of the water passing from a small to a large pipe.

F = the area or cross section of a pipe.

ψ = the co-efficient of resistance of the water flowing in a curved pipe.

r = radius of curvature of a curved pipe.

τ = the co-efficient of resistance of the water flowing in an elbow pipe.

ρ = angle of an elbow pipe.

m = the total number of discharge pipes that are supplied by the single pipe under consideration.

α = an experimental derivation.

β = an experimental derivation.

z = a diametric co-efficient.

J = the sine of the inclination of the water surface, or fall in a length of 1 (used only in Kutter's formula).

n = the co-efficient of roughness dependent upon the surface of the material over which the water flows (used only in Kutter's formula).

ω = a variable co efficient.

$$\left.\begin{array}{l} b = \\ c = \\ e = \\ k = \\ o = \\ p = \\ x = \\ y = \end{array}\right\} \text{characters of Figures Nos. 1 and 2.}$$

THE FORM OF FORMULAS BEST ADAPTED FOR THE FLOW OF WATER IN PIPES.

The experience of quite a number of years in solving hydraulic problems, has led the writer to the conclusion that the forms of formulas most convenient for ascertaining the different results generally sought after, relating to the flow of water in straight pipes, are the following:

$$h_f = \zeta \frac{l}{d} \frac{v^2}{2g}.$$

$$v = \frac{\sqrt{2gh_f}}{\sqrt{\zeta \frac{l}{d}}}.$$

$$h = \frac{v^2}{2g} + \varepsilon \frac{v^2}{2g} + \zeta \frac{l}{d} \frac{v^2}{2g} = \left(1 + \varepsilon + \zeta \frac{l}{d}\right) \frac{v^2}{2g}.$$

$$v = \frac{\sqrt{2gh}}{\sqrt{1 + \varepsilon + \zeta \frac{l}{d}}}.$$

It is assumed in these forms of formulas that the loss of head due to friction increases at the same time with the square of the velocity (v^2), and with conditions that are included in the co-efficient of friction ζ, of the water flowing in the pipes. This co-efficient of friction, which is an experimental derivation, is not expressed individually in the majority of the formulas that have been published by different authorities who have made investigations upon the flow of water in pipes, nor is its value the same in these formulas, nor the laws upon which it is based. For instance, among the formulas that the writer is conversant with, in addition to being dependent upon the nature of the interior sides of the pipes, there are several in which it is dependent upon the velocity (v), one in which it is dependent upon the square root of the velocity ($\sqrt{v}$), one in which it is dependent upon the diameter (d) and one in which it is dependent upon both the velocity and diameter, etc.

The following are several of the different forms of formulas that are often met with in hydraulic text books:

$$h_f = \left(\alpha v + \beta v^2\right) \frac{l}{d}.$$

$$h_f = \frac{l}{\omega^2} \frac{v^2}{d}.$$

$$v = \omega \sqrt{\frac{h_f d}{l}}.$$

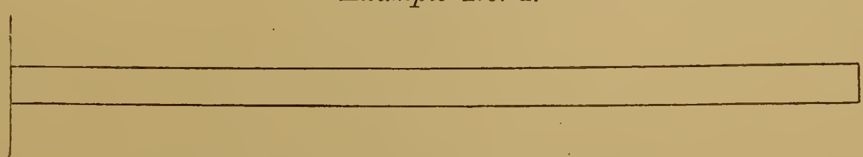
These forms of formulas are undoubtedly a little more convenient for solving very simple problems relating to straight pipes, than are those recommended by the writer; but they cannot be utilized without a considerable modification for solving the more complicated problems that sometimes arise in practice, such as the flow of water from a reservoir, through a straight pipe, a compound pipe, or a compound system of pipes; whereas the forms of formulas, recommended by the writer, can each be readily extended to cover a great variety of cases of this kind, as they contain all of the elements plainly set forth that are necessary for calculating the flow of water in pipes, with the exception of a little supplementary data, such as the co-efficients of resistance for contraction, enlargement, elbows and curves, which can easily be obtained and introduced when necessary. Then there is always a great advantage in having before you individually the co-efficient of friction ζ, which is practically the only important element in a formula, concerning the pipes themselves, that is determined by experiment, and upon which almost entirely depends the conversion of the theoretical head or velocity into the actual.

The co-efficient of friction ζ, as expressed in the formulas recommended by the writer, has been adopted for the reduction of all the experimental data contained in this paper, as well as for the basis of all formulas that have been constructed and all comparisons that have been made, relative to the flow of water in pipes.

The following equations illustrate the manner in which the formulas recommended can be extended to cover a number of cases. For facility of comprehension, the examples to which the different equations apply, are expressed by sketches (without regard to scale).

These sketches are intended to represent plans of different pipes, and combinations of pipes, all of which are supposed to be located below the hydraulic grade line.

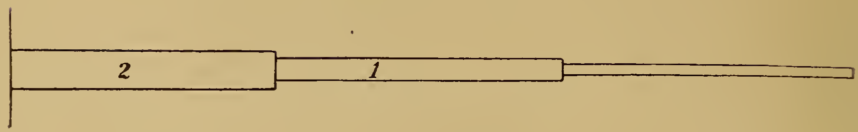
Example No. 1.



$$v = \frac{\sqrt{2\,g\,h}}{\sqrt{1 + \varepsilon + \zeta \dfrac{l}{d}}}$$

$$h_f = \left(\varepsilon + \zeta \frac{l}{d} \right) \frac{v^2}{2\,g}.$$

Example No. 2.



$$v = \frac{\sqrt{2\,g\,h}}{\sqrt{1 + \left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 + \phi + \zeta \frac{l}{d}}}.$$

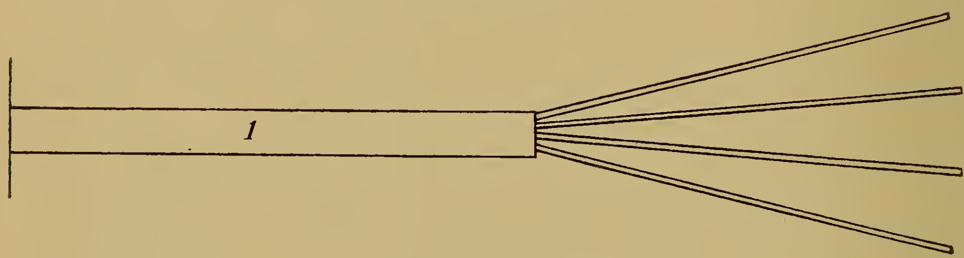
$$h_f = \left[\left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 + \phi + \zeta \frac{l}{d}\right]\frac{v^2}{2\,g}.$$

l, d, ζ, ϕ and v refer to the discharge pipe.

l_1, d_1, ϕ_1 and ζ_1 refer to pipe 1.

l_2, d_2, ε and ζ_2 refer to pipe 2.

Example No. 3.



$$v = \frac{\sqrt{2\,g\,h}}{\sqrt{1 + \left(\varepsilon + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}}}.$$

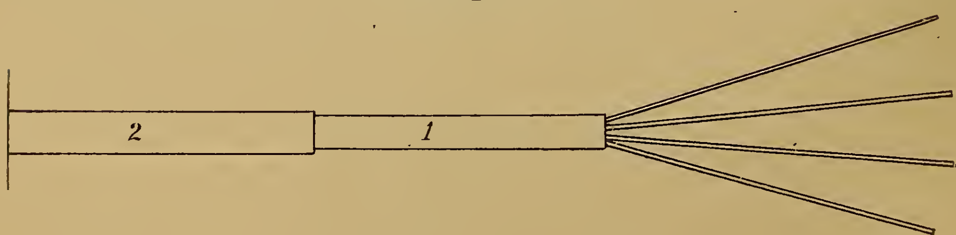
$$h_f = \left[\left(\varepsilon + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}\right]\frac{v^2}{2\,g}.$$

m = the total number of small discharge pipes.

l, d, ϕ, ζ and v refer to one of the small discharge pipes, their dimensions all being the same.

l_1, d_1, ε and ζ_1 refer to pipe 1.

Example No. 4.



$$v = \frac{\sqrt{2\,g\,h}}{\sqrt{1 + \left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 m^2 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}}}$$

$$h_f = \left[\left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 m^2 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}\right]\frac{v^2}{2\,g}$$

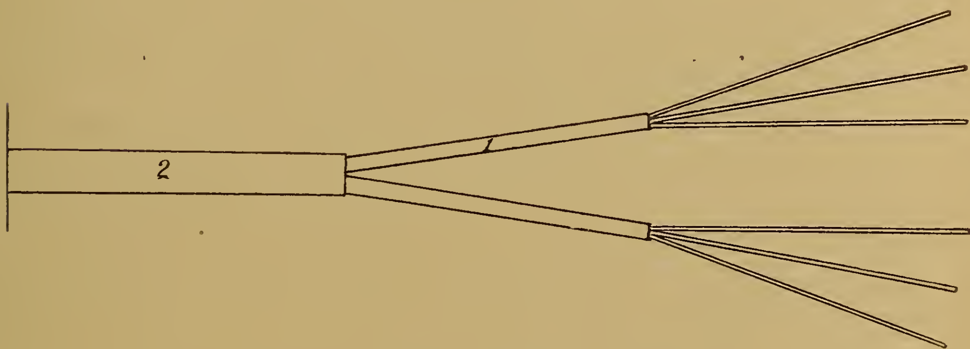
m = the total number of small discharge pipes.

l, d, ϕ, ζ and v refer to one of the small discharge pipes, their dimensions all being the same.

l_1, d_1, ϕ_1 and ζ_1 refer to pipe 1.

l_2, d_2, ε and ζ_2 refer to pipe 2.

Example No 5.



$$v = \frac{\sqrt{2\,g\,h}}{\sqrt{1 + \left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 m_1^2\, m^2 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}}}$$

$$h_f = \left[\left(\varepsilon + \zeta_2 \frac{l_2}{d_2}\right)\left(\frac{d}{d_2}\right)^4 m_1^2\, m^2 + \left(\phi_1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{d}{d_1}\right)^4 m^2 + \phi + \zeta \frac{l}{d}\right]\frac{v^2}{2\,g}$$

m = the number of small discharge pipes that are connected with one of the branch pipes.

m_1 = the number of branch pipes.

l, d, ϕ, ζ and v refer to one of the small discharge pipes, their dimensions all being the same.

l_1, d_1, ϕ_1 and ζ_1 refer to one of the branch pipes, the dimensions of both being the same.

l_2, d_2, ε and ζ_2 refer to pipe 2.

THE CO-EFFICIENT OF INFLUX.

As a co-efficient of influx, or of resistance, ε, to the entrance of the water into the inlet ends of cylindrical pipes, when the edges of these pipes are square and flush with the face of a wall or partition, was frequently used in computing the experimental co-efficients of friction (ζ), contained in Table No. 1, the writer will make a few remarks concerning it before proceeding to describe the table, viz.:

$$\varepsilon = \left(\frac{1}{\varDelta}\right)^2 - 1 \text{ and } h_f = \varepsilon \frac{v^2}{2g}.$$

0.505 is recommended by Weisbach, which is a result deduced from experiments made by himself.
0.513 is derived from the results of experiments made by Michelotte with tubes from 0.5 to 3 inches in diameter, under a head of water varying from 3 to 20 feet.
0.456 is derived from the results of five experiments made by Castel with tubes 0.61 inches in diameter.
0.539 is derived from the results of five experiments made by Bossut with tubes from 0.91 to 1.03 inches in diameter.
0.484 is derived from the result of an experiment made by Eytelwein with a tube 1.02 inches in diameter.
0.481 is derived from the result of an experiment made by Venturi with a tube 1.61 inches in diameter.
0.469 is derived from the results of approximate experiments made by Darcy with pipes from 1.42 to 11.81 inches in diameter.

The writer considers the co-efficient 0.505, recommended by Professor Weisbach, the most reliable. Therefore whenever the co-efficient of influx ε has been used in this paper, its value was considered as 0.505, unless otherwise distinctly specified.

DESCRIPTION OF TABLE No. 1.

This table contains the results of five hundred and twenty experiments, which have been made to ascertain the laws relating to the flow of water in pipes.

In making the experiments fifty-seven different sizes of pipe were used, ranging from 0.409 to 90 inches in diameter, and from 12 feet to 60,000 feet in length, which were constructed of tin, zinc, brass, glass, lead, sheet-iron, artificially coated and not coated, wrought-iron, new and old, cast-iron, new and old, artificially coated and not coated, wood, earthenware and brick.

The first column contains the tabular numbers of the experimental results.

The second column contains the mean diameter of the interior of the pipes.

The third column contains the heads, which are in all cases the total heads, unless they are indicated by *, when they are the frictional heads, due to the water flowing against the interior sides of the pipes.

The fourth column contains the lengths of the pipes. When the total head is given in the third column, opposite to it, in this column, is given the entire distance from the inlet to the outlet of the pipe; this is also the case if the frictional head is given, provided the loss of head due to the entrance of the water into the pipe, and the head required to generate the velocity, have simply been subtracted from the total head; but if manometric tubes, or substitutes for the same, were used in determining the head, the length given is the distance between the points where the tubes were connected to the pipe.

The fifth column contains the mean velocity of the water that was flowing in the pipes.

The sixth column contains the co-efficients of influx, or of resistance, to the entrance of the water into the pipes, if any were required, that were used in computing the co-efficients of friction (ζ), given opposite in the seventh column. When the frictional head is given in the third column, the co-efficient of influx does not enter into the problem. If, on the contrary, the total head is given, a co-efficient of influx was necessarily always employed, if the pipes did not have funnel shaped inlets, and in cases where one had not been determined by experimental investigation, 0.505 was used for reasons that have already been explained. Consequently if 0.505 is given in the sixth column, it is to be understood that it is an assumed value, and when other values are given it implies that they have been determined by experiment to meet the requirements of their respective cases.

The seventh column contains the co-efficients of friction (ζ), which have been computed from the results of the experiments contained in the table. When the frictional head was used, the equation employed for the determination of the co-efficient of friction (ζ), was

$$\zeta = \frac{2g\,h_f}{v^2}\;\frac{d}{l};$$

but when the total head was used, the head required to generate the velocity and the loss of head due to the entrance of the water into the pipe, had to be considered, and when the co-efficient of influx was assumed to be 0.505, the equation employed for a case of this kind was

$$\zeta = \left(\frac{2g\,h}{v^2} - 1.505\right)\frac{d}{l}.$$

The values of $2g$, that were used in the computations, have ranged between 64.326 and 64.4, depending upon the locality where the experiments were made. All of the experimental co-efficients of friction (ζ) given in this column may be used in the equation,

$$h_f = \zeta\;\frac{l}{d}\;\frac{v^2}{2g},$$

and in the other equations previously mentioned and recommended by the writer; so that should an instance arise in which any of the formulas that will be given in this paper, in the future, do not apply, the table can be referred to, and quite possibly an independent experimental co-efficient of friction (ζ) can be found, which will exactly fill the requirements of the case.

The eighth column contains the names of the authorities under whose direction the experiments were made or published, and the nature of the pipes that were used.

The ninth column contains the numbers of the diagrams upon which the co-efficients of friction (ζ) are platted that were used by the writer in determining his formula, and those upon which other co-efficients of friction (ζ) have been platted for comparison with formulas of different authorities.

There are quite a number of experimental results given in the table, which have not been platted on the diagrams nor mentioned in any form outside of the table, as they were not deemed sufficiently reliable, with the exception of those which were obtained from experiments that were made with wooden pipe, which were not required. These results can be ascertained by referring to the ninth column, as the space opposite to them in this column will not contain a diagram number.

TABLE No. 1.

EXPERIMENTAL VALUES of the Co-efficients of Friction (ζ) of Water in Pipes of Different Sizes, the Data from which they were Obtained, and the Number of the Diagram on which they are Platted.

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							WEISBACH.	
1	0.409	......	......	0.62	......	0.0527	Brass pipe................	1 and 7
2	"	......	......	0.63	......	0.0676	Brass pipe................	1 and 7
3	"	......	......	0.70	......	0.0498	Brass pipe................	1 and 7
4	"	......	......	1.87	......	0.0365	Brass pipe................	
5	"	......	......	6.63	......	0.0273	Brass pipe................	
6	"	......	......	27.92	......	0.0182	Glass pipe................	1 and 7
7	"	......	......	28.35	......	0.0187	Brass pipe................	1 and 7
8	"	......	......	40.42	......	0.0178	Brass pipe................	1 and 7
9	"	......	......	68.87	......	0.0169	Brass pipe................	1 and 7
10	0.563	......	......	0.73	......	0.0607	Glass pipe................	1 and 7
11	"	......	......	28.41	......	0.0172	Brass pipe................	1 and 7
12	"	......	......	33.40	......	0.0187	Glass pipe................	1 and 7
13	"	......	....	40.68	......	0.0174	Brass pipe................	1 and 7
14	"	......	......	70.84	......	0.0148	Brass pipe................	1 and 7
							DARCY.	
15	0.480	*0.28	328.09	0.11	......	0.1719		25
16	"	0.60	"	0.23	......	0.0869		25
17	"	1.00	"	0.38	......	0.0532		25
18	"	1.75	"	0.48	......	0.0590		25
19	"	2.47	"	0.55	......	0.0632		25
20	"	5.44	"	0.75	......	0.0751		25
21	"	8.47	"	0.94	......	0.0750	New wrought-iron pipe..	25
22	"	11.39	"	1.13	......	0.0706		25
23	"	14.43	"	1.29	......	0.0685		25
24	"	20.55	"	1.57	......	0.0656		25
25	"	28.07	"	1.88	......	0.0624		25
26	"	58.60	"	2.78	......	0.0597		25
27	"	112.95	"	3.92	......	0.0577		25
							RENNIE.	
28	0.500	1.00	15.00	2.35	0.505	0.0283	Lead pipe................	7
29	"	4.00	"	5.13	"	0.0230		7
							SMITH.	
30	0.502	0.82	11.13	2.08	0.488	0.0404		
31	"	1.27	"	2.72	"	0.0361	Glass pipe.....	
32	"	2.16	"	3.66	"	0.0335		
33	"	3.03	"	4.44	"	0.0316		
							DARCY.	
34	0.551	*0.10	164.05	0.13	......	0.1099		1
35	"	0.55	"	0.54	......	0.0339		1 and 7
36	"	1.41	"	0.81	......	0.0391		1 and 7
37	"	4.14	"	1.46	......	0.0349	New lead pipe..........	1 and 7
38	"	10.08	"	2.40	......	0.0315		1 and 7
39	"	18.76	"	3.44	......	0.0286		1 and 7
40	"	26.49	"	4.23	......	0.0267		1 and 7

* The heads of this authority are those due to friction only.

TABLE No. 1—*(Continued.)*

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							SMITH.	
41	0.628	0.81	60.13	1.03	0.469	0.4014		1 and 7
42	"	1.71	"	1.58	"	0.0368		1 and 7
43	"	3.31	"	2.30	"	0.0335		1 and 7
44	"	4.87	"	2.86	"	0.0318	New gas pipe............	1 and 7
45	"	6.59	"	3.39	"	0.0306		1 and 7
46	"	8.35	"	3.88	"	0.0296		1 and 7
							SMITH.	
47	0.746	0.67	34.94	1.40	0.488	0.0367		8
48	"	2.06	"	2.65	"	0.0309		8
49	"	3.66	"	3.67	"	0.0284	Glass pipe...............	8
50	"	5.07	"	4.37	"	0.0277		8
							WESTON.	
51	0.969	*121.10	781.80	6.03	......	0.0221	Wrought-iron pipe, lined with tin................	2 and 9
							WEISBACH.	
52	0.972	......	......	0.68	......	0.0432		2 and 9
53	"	......	......	0.79	......	0.0425		2 and 9
54	"	......	......	10.47	......	0.0196		2 and 9
55	"	......	......	15.52	......	0.0184	Zinc pipe................	2 and 9
56	"	......	......	20.47	......	0.0179		2 and 9
57	"	......	......	30.12	......	0.0167		2 and 9
							HODSON.	
58	1.000	150.00	100.00	21.70	0.352	0.0160	....	2 and 9
							NEVILLE.	
59	1.020	11.13	9.29	14.58	0.352	0.0185	....	2 and 9
60	"	20.80	19.20	15.67	"	0.0182	....	2 and 9
							DARCY.	
61	1.047	*0.07	328.09	0.12	......	0.0838		25
62	"	0.08	"	0.13	......	0.0836		25
63	"	0.08	"	0.13	......	0.0799		25
64	"	0.11	"	0.19	......	0.0515		25
65	"	0.18	"	0.28	......	0.0388		25
66	"	0.50	"	0.43	......	0.0462		25
67	"	1.60	"	0.81	......	0.0413		25
68	"	3.33	"	1.21	......	0.0391		25
69	"	6.36	"	1.71	......	0.0371	New wrought-iron pipe..	25
70	"	6.26	"	2.19	......	0.0367		25
71	"	14.27	"	2.61	......	0.0358		25
72	"	20.72	"	3.15	......	0.0357		25
73	"	22.88	"	4.05	......	0.0343		25
74	"	34.68	"	4.20	......	0.0336		25
75	"	58.49	"	5.52	......	0.0329		25
76	"	83.99	"	6.56	......	0.0335		25
77	"	101.55	"	7.17	......	0.0339		25

* The heads of this authority are those due to friction only.

TABLE No. 1.—*(Continued.)*

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							SMITH.	
78	1.054	0.47	60.17	0.96	0.563	0.0459		2 and 9
79	"	0.79	"	1.42	"	0.0345		2 and 9
80	"	1.66	"	2 15	"	0.0315		2 and 9
81	"	3.27	"	3.18	"	0.0282	New gas pipe, not tarred.	2 and 9
82	"	4.85	"	3.95	"	0.0269		2 and 9
83	"	6.59	"	4.67	"	0.0261		2 and 9
84	"	8.33	"	5.33	"	0.0253		2 and 9
							DARCY.	
85	1.055	*0.07	328.09	0.10	......	0.1285		
86	"	0.22	"	0.30	......	0.0416		2 and 9
87	"	0.74	"	0.51	......	0.0491		2 and 9
88	"	2.00	"	0.89	......	0.0436		2 and 9
89	"	3.72	"	1 26	......	0.0404		2 and 9
90	"	7.29	"	1.86	......	0.0363	New sheet-iron pipe, coated with bitumen.	2 and 9
91	"	9.96	"	2.22	......	0.0347		2 and 9
92	"	14.90	"	2.80	......	0.0328		2 and 9
93	"	38.87	"	4.81	......	0.0289		2 and 9
94	"	59.01	"	6.10	......	0.0274		2 and 9
95	"	80.12	"	7.23	......	0.0265		2 and 9
96	"	100.77	"	8.22	......	0.0258		2 and 9
97	1.063	0 07	164.05	0.21	......	0.0552		2 and 9
98	"	0.49	"	0.62	......	0.0450		2 and 9
99	"	1.34	"	1.09	......	0.0391		2 and 9
100	"	3.72	"	1.96	......	0.0337	New lead pipe...........	2 and 9
101	"	8.92	"	3.35	......	0.0276		2 and 9
102	"	17.23	"	4.72	......	0.0269		2 and 9
103	"	24.00	"	5.51	......	0.0275		2 and 9
							BOSSUT.	
104	1.066	0.36	53.27	1.09	0.505	0.0299	Lead pipe................	2 and 9
105	"	1.07	"	1.98	"	0.0267		2 and 9
							DUBUAT.	
106	1.066	0.01	65.45	0.14	0.505	0.0570		
107	"	0.40	65.45	0.32	"	0.0353		9
108	"	0.60	12.30	0.77	"	0.0376		2 and 9
109	"	0.37	12.30	0.93	"	0.1910		
110	"	0.37	12.30	0.95	"	0.1823		
111	"	0.53	65.45	1.18	"	0.0309		2 and 9
112	"	0.69	65.45	1.34	"	0.0315		2 and 9
113	"	0.80	65.45	1.45	"	0.0312		2 and 9
114	"	0.80	65.45	1.48	"	0.0300		2 and 9
115	"	1.09	65.45	1.78	"	0.0283	Tin pipe..................	2 and 9
116	"	1.22	65.45	1.86	"	0.0286		2 and 9
117	"	1.30	65.45	1.94	"	0.0280		2 and 9
118	"	2.10	65.45	2.55	"	0.0263		2 and 9
119	"	0.53	12.30	2.61	"	0.0256		2 and 9
120	"	1.60	10.39	5.18	"	0.0200		2 and 9
121	"	1.86	12.30	5.22	"	0.0209		2 and 9
122	"	2.37	10.39	6.33	"	0.0197		2 and 9
123	"	3.20	10.39	7.54	"	0.0181		2 and 9

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TABLE No. 1.—*(Continued.)*

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							SMEATON.	
124	1.25	0.17	100.00	0.54	0.505	0.0375	....	10
125	"	0.23	"	0.65	"	0.0350	....	10
126	"	0.32	"	0.73	"	0.0387	....	3 and 10
127	"	0.40	"	0.83	"	0.0374	....	3 and 10
128	"	0.52	"	0.96	"	0.0363	....	3 and 10
129	"	0.65	"	1.12	"	0.0332	....	3 and 10
130	"	0.89	"	1.29	"	0.0343	....	3 and 10
131	"	1.21	"	1.52	"	0.0336	....	3 and 10
132	"	1.51	"	1.75	"	0.0315	....	3 and 10
133	"	1.92	"	1.97	"	0.0316	....	3 and 10
134	"	2.30	"	2.25	"	0.0289	....	3 and 10
135	"	2.54	"	2.35	"	0.0293	....	3 and 10
136	"	2.86	"	2.48	"	0.0296	....	3 and 10
137	"	2.97	"	2.52	"	0.0298	....	3 and 10
138	"	3.92	"	2.96	"	0.0285	....	3 and 10
139	"	5.92	"	3.65	"	0.0282	....	3 and 10
							SMITH.	
140	1.26	1.57	62.05	1.65	0.563	0.0599	Wood pipe...............	
141	"	3.31	"	2.47	"	0.0564	"	
142	"	4.94	"	3.01	"	0.0567	"	
143	"	6.70	"	3.52	"	0.0562	"	
144	"	8.52	"	3.99	"	0.0556	"	
							WEISBACH.	
145	1.27	......	......	1.18	......	0.0343	Zinc pipe.................	3 and 10
							BOSSUT.	
146	1.42	1.07	191.78	1.12	0.505	0.0331	Tin pipe.................	3 and 11
147	"	1.07	159.82	1.25	"	0.0314	"	3 and 11
148	"	1.07	127.86	1.43	"	0.0296	"	3 and 11
149	"	2.13	191.78	1.68	"	0.0292	"	3 and 11
150	"	1.07	95.87	1.68	"	0.0281	"	3 and 11
151	"	2.13	159.82	1.87	"	0 0280	"	3 and 11
152	"	1.07	63.93	2.07	"	0.0267	"	3 and 11
153	"	2.13	127.86	2.13	"	0.0266	"	3 and 11
154	"	2.13	95.87	2.49	"	0.0254	"	3 and 11
155	"	1.07	31.96	2.94	"	0.0237	"	3 and 11
156	"	2.13	63.93	3.06	"	0.0243	"	3 and 11
157	"	2.13	31.96	4.31	"	0.0218	"	3 and 11
158	"	8.02	62.88	6.14	"	0.0228	"	3 and 11
159	"	15.16	125.76	6.15	"	0.0228	"	3 and 11
160	"	22.29	188.64	6.16	"	0.0228	"	3 and 11
							DARCY.	
161	1.43	* 0.08	328.09	0.15	......	......	Old cast-iron pipe, lined with deposit.	
162	"	0.23	"	0.26	......	0.0809	"	26
163	"	0.60	"	0.42	......	0.0813	"	26
164	"	2.20	"	0.81	......	0.0792	"	26
165	"	5.00	"	1.22	......	0.0793	"	26
166	"	10.63	"	1.76	......	0.0805	"	26
167	"	13.63	"	2.02	......	0.0782	"	26

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TABLE No. 1.—*(Continued.)*

No.	Diameter in inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							DARCY.	
168	1.43	*0.23	328.09	0.37	......	0.0396		27
169	"	0.59	"	0.62	......	0.0363		27
170	"	2.14	"	1.27	......	0.0309		27
171	"	4.73	"	1.97	......	0.0285	The same, cleaned.......	27
172	"	9.90	"	2.93	......	0.0271		27
173	"	13.01	"	3.39	......	0.0265		27
174	"	15.26	"	3.69	......	0.0262		27
							PROVIS.	
175	1.50	*1.85	80.00	2.70	......	0.0255		3 and 11
176	"	2.09	"	3.09	......	0.0220		3 and 11
177	"	1.15	60.00	2.46	......	0.0255		3 and 11
178	"	1.49	"	2.86	......	0.0244		3 and 11
179	"	1.65	"	3.00	......	0.0246		3 and 11
180	"	1.83	"	3.20	......	0.0240		3 and 11
181	"	1.98	"	3.34	......	0.0238		3 and 11
182	"	2.15	"	3.41	......	0.0248		3 and 11
183	"	2.16	"	3.52	......	0.0234		3 and 11
184	"	2.32	"	3.63	......	0.0236		3 and 11
185	"	2.44	"	3.77	......	0.0230		3 and 11
186	"	2.46	"	3.77	......	0.0232		3 and 11
187	"	2.64	"	3.95	......	0.0227		3 and 11
188	"	2.84	"	3.97	......	0.0242		3 and 11
189	"	2.92	"	4.18	......	0.0224		3 and 11
190	"	3.13	"	4.32	......	0.0225		3 and 11
191	"	3.40	"	4.56	......	0.0219		3 and 11
192	"	0.55	40.00	2.12	......	0.0246		3 and 11
193	"	0.87	"	2.66	......	0.0247	Lead pipe	3 and 11
194	"	0.90	"	2.68	......	0.0252		3 and 11
195	"	1.16	"	3.13	......	0.0238		3 and 11
196	"	1.18	"	3.16	......	0.0238		3 and 11
197	"	1.20	"	3.18	......	0.0239		3 and 11
198	"	1.45	"	3.52	......	0.0235		3 and 11
199	"	1.46	"	3.58	......	0.0229		3 and 11
200	"	1.49	"	3.58	......	0.0234		3 and 11
201	"	1.50	"	3.64	......	0.0228		3 and 11
202	"	1.66	"	3.90	......	0.0219		3 and 11
203	"	1.76	"	3.93	......	0.0229		3 and 11
204	"	1.77	"	3.98	......	0.0215		3 and 11
205	"	1.79	"	4.00	......	0.0225		3 and 11
206	"	1.98	"	4.25	......	0.0220		3 and 11
207	"	2.08	"	4.35	......	0.0221		3 and 11
208	"	2.03	"	4.37	......	0.0214		3 and 11
209	"	2.31	"	4.59	......	0.0220		3 and 11
210	"	2.38	"	4.66	......	0.0220		3 and 11
211	"	2.60	"	4.94	......	0.0214		3 and 11
							DARCY.	
212	1.55	*0.07	328.09	0.21	......	0.0435		25
213	"	0.26	"	0.36	......	0.0489		25
214	"	0.60	"	0.61	......	0.0413		25
215	"	1.10	"	0.86	......	0.0381	New wrought-iron pipe..	25
216	"	2.13	"	1.25	......	0.0346		25
217	"	4.22	"	1.84	......	0.0318		25
218	"	7.84	"	2.58	......	0.0298		25

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TABLE No. 1.—*(Continued.)*

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							DARCY.	
219	1·55	* 10.25	328.09	3.00	......	0.0289		25
220	"	14.27	"	3.59	......	0.0281		25
221	"	40.41	"	6.30	......	0.0259	New wrought-iron pipe..	25
222	"	57.59	"	7.56	......	0.0256		25
223	"	73.52	"	8.52	......	0.0257		
224	1.61	0.13	164.05	0.39	....	0.0458		3 and 11
225	"	0.59	"	0.91	......	0.0382		3 and 11
226	"	1.28	"	1.40	......	0.0342		3 and 11
227	"	3.79	"	2.60	......	0.0296	New lead pipe...........	3 and 11
228	"	9.19	"	4.32	......	0.0260		3 and 11
229	"	18.17	"	6.32	......	0.0240		3 and 11
230	"	24.97	"	7.56	......	0 0230		3 and 11
231	1.96	0.14	147.19	0.50	......	0.0399		4 and 12
232	"	0.51	"	1.02	......	0.0346		4 and 12
233	"	1.14	"	1.59	......	0.0320		4 and 12
234	"	3.41	"	2.93	....	0.0283	New glass pipe	4 and 12
235	"	8.48	"	4.85	......	0.0257		4 and 12
236	"	16.47	"	6.92	......	0.0245		4 and 12
							DR. ROBISON.	
237	2.00	12.75	3300.00	1.24	0.505	0.0277	....	4 and 12
							BOSSUT.	
238	2.14	1.07	191.78	1.45	0.505	0.0288		4 and 12
239	"	1.07	159.82	1.63	"	0.0273		4 and 12
240	"	1.07	127.86	1.84	"	0.0262		4 and 12
241	"	1.07	95.87	2.11	"	0.0258		4 and 12
242	"	2.13	191.78	2.20	"	0.0251		4 and 12
243	"	2.13	159.82	2.44	"	0.0241		4 and 12
244	"	1.07	63.93	2.59	"	0.0242	Tin pipe.................	4 and 12
245	"	2.13	127.86	2.74	"	0.0233		4 and 12
246	"	2.13	95.87	3.18	"	0.0225		4 and 12
247	"	1.07	31.96	3.58	"	0.0215		4 and 12
248	"	2.13	63.93	3.82	"	0 0221		4 and 12
249	"	2.13	31.96	5.23	"	0.0196		4 and 12
							LESLIE.	
250	2.50	0.46	1086.00	0.22	0.505	0.1212		
251	"	1.45	"	0.72	"	0.0347		4 and 13
252	"	2.78	"	1.09	"	0.0289		4 and 13
253	"	4.76	"	1.47	"	0.0271		4 and 13
254	"	7.01	"	1.73	"	0.0288		4 and 13
255	"	9.99	"	2.10	"	0.0278		4 and 13
256	"	1.01	540.00	0.58	"	0.0748		
257	"	1.78	"	0.79	"	0.0699		
258	"	2.50	"	1.07	"	0.0541	Lead pipe...............	4 and 13
259	"	5.50	"	2.21	"	0.0273		4 and 13
260	"	8.48	"	2.93	"	0.0239		4 and 13
261	"	9.99	"	3.35	"	0.0215		4 and 13
262	"	0.17	270.00	0.34	"	0.0700		
263	"	2.47	"	2.26	"	0.0229		4 and 13
264	"	5.44	"	3.26	"	0.0243		4 and 13
265	"	10.14	"	4.89	"	0.0199		4 and 13
266	"	0.19	100.00	0.86	"	0.0316		4 and 13

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TABLE No. 1.—*(Continued.)*

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per Second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							LESLIE.	
267	2.50	4.36	100.00	4.89	0.505	0.0213	Lead pipe..................	4 and 13
268	"	6.61	"	6.01	"	0.0214		4 and 13
269	"	8.25	"	6.90	"	0.0201		4 and 13
							DARCY.	
270	3.15	* 0.21	328.09	0.40	......	0.0693	Old cast-iron pipe, lined with deposit............	26
271	"	0.82	"	0.81	......	0.0641		26
272	"	2.38	"	1.44	......	0.0589		26
273	"	5.28	"	2.19	......	0.0568		26
274	"	10.17	"	3.01	......	0.0580		26
275	"	14.88	"	3.69	......	0.0564		26
276	"	0.28	"	0.63	......	0.0358	The same cleaned........	27
277	"	0.96	"	1.26	......	0.0314		27
278	"	2.37	"	2.01	......	0.0301		27
279	"	2.42	"	2.05	......	0.0299		27
280	"	5.11	"	2.83	......	0.0328		27
281	"	9.64	"	4.09	......	0.0299		27
282	"	14.68	"	5.01	......	0.0304		27
283	3.22	0.07	"	0.29	......	0.0418	New cast-iron pipe.......	15
284	"	0.27	"	0.56	......	0.0453		15
285	"	0.76	"	1.17	......	0.0291		15
286	"	1.74	"	1.84	......	0.0271		15
287	"	3.35	"	2.59	......	0.0262		15
288	"	7.40	"	3.89	......	0.0258		15
289	"	10.53	"	4.65	......	0.0256		15
290	"	13.26	"	5.15	......	0.0263		15
291	"	31.32	"	8.05	......	0.0255		15
292	"	32.49	"	8.16	......	0.0257		15
293	"	39.30	"	8.92	......	0.0260		15
294	"	55.14	"	10.62	......	0.0258		15
295	"	56.01	"	10.71	......	0.0257		15
296	3.25	0.09	"	0.33	......	0.0438	New sheet-iron pipe, coated with bitumen.......	5 and 14
297	"	0.22	"	0.58	......	0.0345		5 and 14
298	"	0.67	"	1.17	......	0.0258		5 and 14
299	"	2.06	"	2.18	......	0.0231		5 and 14
300	"	4.00	"	3.12	......	0.0219		5 and 14
301	"	7.50	"	4.44	......	0.0202		5 and 14
302	"	10.19	"	5.29	......	0.0194		5 and 14
303	"	13.35	"	6.15	......	0.0188		5 and 14
304	"	23.52	"	8.44	......	0.0176		5 and 14
305	"	34.96	"	10.53	......	0.0167		5 and 14
306	"	45.54	"	12.03	......	0.0167		5 and 14
307	"	51.20	"	12.79	......	0.0167		5 and 14
							JARDINE.	
308	4.50	51.00	14930.00	1.71	0.505	0.0282	Lead pipe.................	
							COUPLET.	
309	5.33	0.50	7481.66	0.18	0.505	0.0593	Stoneware and lead pipe.	16
310	"	1.01	"	0.28	"	0.0489		16
311	"	1.49	"	0.37	"	0.0422		16
312	"	1.87	"	0.43	"	0.0391		16
313	"	2.13	"	0.46	"	0.0379		16
314	"	2.22	"	0.47	"	0.0378		16

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TABLE No. 1—(*Continued.*)

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							DARCY.	
315	5.39	0.08	328.09	0.49	......	0.0292	New cast-iron pipe.......	16
316	"	0.29	"	0.98	......	0.0263		16
317	"	0.69	"	1.60	......	0.0236		16
318	"	1.56	"	2.50	......	0.0220		16
319	"	4.13	"	4.19	......	0.0208		16
320	"	7.30	"	5.62	......	0.0204		16
321	"	10.89	"	6.88	......	0.0203		16
322	"	12.81	"	7.48	......	0.0202		16
323	"	32.32	"	11.94	......	0.0200		16
324	"	54.98	"	15.39	......	0.0205		16
							WESTON.	
325	6.00	*21.19	1170.90	4.70	......	0.0264	Cast-iron pipe, coated with coal-tar, in service 4 years.................	16
326	"	47.64	"	7.25	......	0.0249		16
327	"	65.46	"	8.49	......	0.0249		16
328	"	73.16	"	9.26	......	0.0234		16
							DARCY.	
329	7.40	*0.09	328.09	0.65	......	0.0251	New cast-iron pipe	17
330	"	0.57	"	1.64	......	0.0259		17
331	"	1.21	"	2.50	......	0.0233		17
332	"	2.64	"	3.72	......	0.0231		17
333	"	4.40	"	4.90	......	0.0221		17
334	"	7.38	"	6.37	......	0.0220		17
335	"	12.50	"	8.25	......	0.0221		17
336	"	36.02	"	14.23	......	0.0215		17
337	"	47.87	"	16.23	......	0.0219		17
338	7.72	0.07	"	0.59	......	0.0237	New sheet-iron pipe, coated with bitumen.	23
339	"	0.16	"	0.91	......	0.0239		23
340	"	0.42	"	1.53	......	0.0228		23
341	"	1.08	"	2.56	......	0.0209		23
342	"	1.90	"	3.53	......	0.0193		23
343	"	3.90	"	5.44	......	0.0167		23
344	"	3.94	"	5.51	......	0.0164		23
345	"	6.89	"	7.41	......	0.0158		23
346	"	9.74	"	9.00	......	0.0152		23
347	"	11.94	"	10.01	......	0.0150		23
348	"	39.88	"	19.72	......	0.0129		23
349	9.63	0.31	"	1.00	......	0.0489	Old cast-iron pipe, lined with deposit	26
350	"	0.66	"	1.46	......	0.0487		26
351	"	1.55	"	2.29	......	0.0465		26
352	"	3.77	"	3.58	......	0.0463		26
353	"	7.51	"	5.01	......	0.0471		26
354	"	10.50	"	5.94	......	0.0468		26
355	"	13.47	"	6.72	......	0.0470		26
356	"	45.87	"	12.42	......	0.0468		26
357	"	0.17	"	0.91	.....	0.0316	The same cleaned........	27
358	"	0.54	"	1.76	......	0.0275		27
359	"	1.63	"	3.11	......	0.0266		27
360	"	3.79	"	4.66	......	0.0274		27
361	"	6.68	"	6.25	......	0.0269		27
362	"	8.97	"	7.24	......	0.0270		27
363	"	12.24	"	8.44	......	0.0271		27
364	"	37.22	"	14.75	......	0.0269		27

* The heads of this authority are those due to friction only.

TABLE No. 1—(*Continued.*)

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							SMITH.	
365	10.93	6.56	730.60	4.76	......	0.0219	Riveted sheet-iron pipe, coated with coal-tar and asphaltum.............	18
366	"	10.20	721.30	6.12	......	0.0209		18
367	"	12.85	713.90	6.95	......	0.0206		18
368	"	19.01	697.00	8.69	......	0.0198		18
369	"	24.22	684.80	10.05	......	0.0192		18
							DARCY.	
370	11.22	*0.23	328.09	1.30	......	0.0251	New sheet-iron pipe, coated with bitumen.	24
371	"	0.84	"	2.78	......	0.0198		24
372	"	1.42	"	3.87	......	0.0174		24
373	"	2.25	"	4.90	......	0.0172		24
374	"	3.90	"	6.67	......	0.0161		24
375	"	6.71	"	8.85	......	0.0157		24
376	"	9.21	"	10.52	......	0.0153		24
377	11.69	0.09	"	0.80	......	0.0275	Old cast-iron pipe, thoroughly cleaned........	18
378	"	0.39	"	1.76	......	0.0240		18
379	"	0.88	"	2.71	......	0.0229		18
380	"	1.76	"	3.79	......	0.0235		18
381	"	3.63	"	5.42	......	0.0238		18
382	"	7.56	"	7.84	......	0.0236		18
383	"	10.52	"	9.18	......	0.0239		18
384	"	13.47	"	10.37	......	0.0240		18
							SIMPSON.	
385	12.00	4.00	5200.00	1.45	0.505	0.0232	Cast-iron pipe, in service less than seven years...	18
386	"	19.00	"	2.91	"	0.0276		18
887	"	16.75	"	2.91	"	0.0243		18
388	"	40.00	"	4.35	"	0.0259		18
389	"	38.00	"	4.35	"	0.0246		18
390	"	5.50	8140.00	1.42	"	0.0213	Cast-iron pipe, no record of service	18
391	"	12.00	"	1.89	"	0·0264		18
392	"	18.00	"	2.21	"	0.0291		18
393	"	24.75	"	2.53	"	0.0305		18
394	"	27.00	"	2.64	"	0.0304		18
395	"	34.50	6600.00	3.57	"	0.0263	Cast-iron pipe, in service less than 4 years.......	18
396	"	46.00	"	4.11	"	0.0277		18
							BONN WATER WORKS.	
397	12.05	*21.30	17684.00	1.55	......	0.0322	New cast-iron pipe, coated with asphaltum	
398	"	21.30	"	1.57	......	0.0317		
399	"	34.40	"	2.10	......	0.0284		
400	"	32.80	"	2.12	......	0.0267		
401	"	47.60	"	2.60	......	0.0257		
402	"	45.90	"	2.62	......	0.0244		
403	"	64.00	"	3.10	......	0.0244		
404	"	60.70	"	3.13	......	0.0227		
							SMITH.	
405	12.67	5.13	718.40	4.61	......	0.0213	Riveted sheet-iron pipe, coated with coal-tar and asphaltum.............	18
406	"	10.89	709.20	6.98	......	0.0199		18
407	"	16.69	699.60	8.68	......	0.0200		18
408	"	24.51	684.90	10.76	......	0.0194		18

* The heads of this authority are those due to friction only.

TABLE No. 1—(*Continued.*)

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per Second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							SMITH.	
409	14.76	3.92	719.90	4.40	......	0.0205	Riveted sheet-iron pipe, coated with coal-tar and asphaltum.............	19
410	"	8.55	712.40	6.86	......	0.0184		19
411	"	9.55	710.70	7.33	......	0.0180		19
412	"	12.72	705.00	8.52	......	0.0179		19
413	"	18.93	695.60	10.75	......	0.0169		19
414	"	24.39	684.40	12.30	......	0.0168		19
							EDINBURGH WATER CO.	
415	16.00	230.00	25765.00	5.25	0.505	0.0277	Cast-iron pipe, in service 8 or 9 years............	19
416	"	420.00	29580.00	6.82	"	0.0261		19
417	"	184.00	3815.00	14.51	"	0.0191		19
							LAMPE.	
418	16.48	*15.09	25414.00	1.58	......	0.0210	New cast-iron pipe, coated with varnish........	19
419	"	43.65	31719.00	2.48	......	0.0197		19
420	"	51 69	31719.00	2.71	......	0.0196		19
421	"	52.39	26862.00	3.09	......	0.0180		19
							SMITH.	
422	16.99	303.60	4438.70	20.13	0.182	0.0150	Riveted sheet-iron pipe, coated with coal-tar, etc.	19
							SIMPSON.	
423	19.00	24.00	22444.00	2.06	0.505	0.0257	Cast-iron pipe, in service less than thirteen years.	20
424	"	27.50	"	2.26	"	0.0244		20
425	"	34.00	"	2.52	"	0.0242		20
426	"	41.00	"	2.73	"	0.0248		20
427	"	43.50	"	2.80	"	0.0251		20
							DARCY.	
428	19.69	*0.15	328.09	1.38	......	0.0252	New cast-iron pipe.......	20
429	"	0.15	"	1.47	......	0.0222		20
430	"	0·20	"	1.55	......	0.0263		20
431	"	0.39	"	2.60	......	0.0189		20
432	"	0.41	"	2.60	......	0.0195		20
433	"	0.69	"	3.42	......	0.0190		20
434	"	0.76	"	3.64	......	0.0183		20
435	"	0.85	"	3.66	......	0.0205		20
436	"	0.82	"	3.69	......	0.0194		20
							FANNING.	
437	20.00	......	8171.00	0.95	......	0.0271	Wrought-iron cement lined pipe..............	
438	"	......	"	1.49	......	0.0214		20
439	"	......	"	1.93	......	0.0210		20
440	"	......	"	2.33	......	0.0205		20
441	"	......	"	2·60	......	0.0213		20
442	"	......	"	2.87	......	0.0206		20
443	"	......	"	3.27	......	0.0200		20
444	"	......	"	3.44	......	0.0207		20
445	"	......	"	3.74	......	0.0209		20
446	"	......	"	3.92	......	0.0210		20

* The heads of this authority are those due to friction only.

TABLE No. 1—(*Continued.*)

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per Second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							FANNING.	
447	20.00	......	8171.00	4.00	......	0.0210	Wrought-iron cement lined pipe.............	20
448	"	......	"	4.04	......	0.0211		20
							BRUSH.	
449	20.00	* 0.729	1000.00	2.00	......	0.0197	Cast-iron pipe, coated with tar, in service five years..................	20
450	"	0.876	"	2.24	......	0.0188		20
451	"	1.029	"	2.36	......	0.0198		20
452	"	1.186	"	2.52	......	0.0200		20
453	"	1.337	"	2.68	......	0.0199		20
454	"	1.490	"	2.76	......	0.0210		20
455	"	1.645	"	2.92	......	0.0206		20
456	"	1.797	"	3.00	......	0·0214		20
							BAILEY.	
457	20.00	28.13	29715.00	1.44	0.505	0.0491	Cast-iron pipe, tuberculated, in service two years....................	26
							DARRACH.	
458	20.00	* 16.75	4320.00	2.71	......	0.0568	Cast-iron pipe, in service eleven years............	
459	"	19.64	"	3.01	......	0.0539		26
460	"	22.52	"	3.31	......	0.0508		26
461	"	25.42	"	3.61	......	0.0482		26
462	"	28.31	"	3.91	......	0.0459		26
463	"	31.19	"	4.21	......	0.0437		26
464	"	34.08	"	4.51	......	0.0413		26
465	"	36.96	"	4.81	......	0.0394		26
466	"	39.84	"	5.11	......	0.0376		26
								26
							SIMPSON.	
467	30.00	25.00	54120.00	1.77	0.505	0.0237	Cast-iron pipe, in service two or three years......	21
							DARRACH.	
468	30.00	* 1.55	4000.00	1.60	......	0.0242	New cast-iron pipe. The head due to friction, does not include the resistance of two check valves which were on this pipe...............	21
469	"	1.83	"	1.74	......	0 0242		21
470	"	2.11	"	1.87	......	0.0241		21
471	"	2.40	"	2.00	......	0.0240		21
472	"	2.68	"	2.14	......	0.0236		21
473	"	6.27	20200.00	1.47	......	0.0230	Cast-iron pipe, in service two years. The head due to friction, does not include the resistance of four check valves which were on this pipe...............	21
474	"	7.57	"	1.62	......	0.0228		21
475	"	8.87	"	1.76	......	0.0224		21
476	"	10.17	"	1.91	......	0.0221		21
477	"	11.44	"	2.06	......	0.0214		21
478	"	12.77	"	2.20	......	0.0209		21
479	"	14.07	"	2.35	......	0.0201		21
480	"	15.37	"	2.50	......	0.0185		21
481	"	16.67	"	2.64	......	0.0189		21
482	"	17.96	"	2.79	......	0.0181		21
483	"	19.26	"	2.94	......	0.0176		21
484	"	20.56	"	3.08	......	0.0172		21
485	"	21.86	"	3.23	......	0.0166		21

* The heads of this authority are those due to friction only.

TABLE No. 1—(*Continued.*)

No.	Diameter in Inches.	Head in Feet.	Length in Feet.	Velocity in Feet per Second.	Co-efficient of Influx.	ζ.	Name of the Experimenter, etc., and Nature of the Pipe.	No. of Diagram.
							DARRACH.	
486	30.00	* 4.04	4400.00	1.07	......	0.1283	Cast-iron pipe, in service nine years.............	
487	"	4.62	"	1.21	......	0.1160		
488	"	5.20	"	1.34	......	0.1053		
489	"	5.78	"	1.48	......	0.0966		
490	"	6.35	"	1.61	......	0.0892		
491	"	6.93	"	1.75	......	0 0828		
492	"	7.51	"	1.88	......	0.0772		
493	"	8.09	"	2.02	......	0.0724		
494	"	8.66	"	2.15	......	0.0681		
495	"	9.24	"	2.29	......	0.0644		
							GREENE.	
496	36.00	20.22	11217.00	3.00	0.505	0.0383	Cast-iron pipe, heavily tuberculated	28
							DARRACH.	
497	36.00	* 3.45	3700.00	1.58	......	0.0716	Cast-iron pipe, in service seven years	28
498	"	4.03	"	1.74	......	0.0687		28
499	"	4.61	"	1.89	......	0.0669		28
500	"	5.19	"	2.05	......	0.0640		28
501	"	5.77	"	2.21	......	0.0614		28
502	"	6 35	"	2.37	......	0.0586		28
503	"	13.00	12400.00	1.00	......	0.2013	Cast-iron pipe, in service seven years............	28
504	"	13.28	"	1.11	......	0.1668		28
505	"	13.57	"	1.22	......	0.1402		28
506	"	13.86	"	1.33	......	0.1208		28
507	"	14.15	"	1.44	......	0.1050		28
508	"	14.44	"	1.56	......	0.0926		28
509	"	14.73	"	1.67	......	0.0822		28
510	"	15.01	"	1.78	......	0.0739		28
511	"	15.30	"	1.89	......	0.0667		28
512	"	15.59	"	2.00	......	0.0604		28
							STEARNS.	
513	48.00	* 0.56	1747.20	2.62	......	0.0120	Cast-iron pipe, coated with coal-tar, in service three years	21
514	"	1.24	"	3.74	......	0.0131		21
515	"	2.13	"	4.97	......	0.0128		21
516	"	3.23	"	6.20	......	0.0124		
							GALE.	
517	48.00	* 5.00	5280.00	3.46	.....	0.0204	Cast-iron pipe, coated with coal-tar, in service eight years.......	21
							CLARKE.	
518	90.00	* 3.68	7166.00	3.77	......	0.0174	Brick tunnel.............	22
519	"	3.98	"	3.80	......	0.0186		22
520	"	4.17	"	3.93	......	0.0182		22

* The heads of this authority are those due to friction only.

DESCRIPTION OF THE EXPERIMENTS THE RESULTS OF WHICH ARE CONTAINED IN TABLE NO. 1.

All of the experiments that were made under the direction of one authority are described together, and each description is preceded by the title of the work or publication, from which the information upon which it is based, was obtained.

Experiments of Bossut.

"*Traité théorique et expérimental d'Hydro-dynamique*," *by the Abbé Bossut, Paris*, 1786.

The twenty-nine experiments were made under the direction of the Abbé Bossut, an eminent French geometer, Professor of Mathematics in the School of Engineers at Mezières, and Member of the Academy of Sciences, with lead pipes, 1.07, 1.42 and 2.14 inches in diameter, which in all cases discharged into the open air. Their tabular numbers are, 1.07 inch, 104 to 105; 1.42 inch, 146 to 160 inclusive; 2.14 inch, 238 to 249 inclusive.

Those numbered in Table No. 1, 104 and 105, 146 to 157, and 238 to 249 inclusive, were included in the fifty-one experiments that were used by Prony in determining his formula, and were made with straight pipes. These pipes were supplied with water from a closed metal box 1 foot square, that was connected by a pipe about 8 inches in diameter with a supply reservoir, in which the water was maintained at a constant elevation.

The other experiments that were made by M. Bossut, which are numbered in Table No. 1, 158 to 160 inclusive, were made with a pipe 1.42 inches in diameter that was laid on an incline of 6° 30′. The upper end of the pipe was connected with a supply reservoir, in which the water was kept at a constant head of about 0.89 feet above the center of the inlet end of the pipe; this head being equivalent to the head due to the velocity, and the loss arising from contraction at the inlet of the pipe. Three separate lengths of pipe were used in making these experiments, the inclination being the same in each instance.

Experiments of Bailey.

"*The Brooklyn Water Works and Sewers*," *by James P. Kirkwood. New York*, 1867.

This experiment was made by George H. Bailey, C. E., with a compound cast-iron main 29 715 feet long, belonging to the Jersey City Water Works. Its tabular number is 457.

"This main was 20 inches in diameter for a length of 29 587 feet, and 24 inches in diameter for a length of 128 feet. It connected the receiving with the distributing reservoir, and at the time the experiment was made is said to have been from one to two years old. The head was ascertained by leveling between the reservoir surfaces, and the flow was measured by the lowering of the surface of the water in the feeding reservoir.

Six experiments were really made, but their average only, as published by Mr. Kirkwood, is given in Table No. 1, and the diameter was used by the writer as 20 inches for the entire length of the pipe, in computing the co-efficient of friction ζ.

Experiments of Brush.

"*Transactions of the American Society of Civil Engineers,*" *Vol. XIX,* 1888.

The eight experiments were made under the direction of Charles B. Brush, M. Am. Soc. C. E., with a cast-iron pumping main, 20 inches in diameter and 75 000 feet long, belonging to the Hackensack Water Company. Their tabular numbers are 449 to 456 inclusive.

This main was laid along by the side of railroads, and in highways, with a large number of summits, angles and curves. Among the horizontal bends there were four right angles and ten quadrants of about 30 feet radius. It had been in service about five years when the experiments were made, and there was not any oxidization upon the interior surface. The pipes, when new, were coated in the usual way with tar. A pressure gauge was used in determining the head. The flow of water was measured by the displacement of the pump plungers of a Worthington pumping engine, 5 per cent. having been allowed for slip. The average delivery of the main was about 4 000 000 gallons per day, but 200 000 of this amount was supplied from the main to consumers, before the balance reached the reservoir, 100 000 gallons being taken at a point 13 500 feet from the pumping station, and the remaining 100 000 at a point 70 000 feet from the pumping station. The "effective" head ranged from 55 to 135 feet according to the quantity of water flowing in the main. The head given in Table No. 1 is the frictional head per 1 000 feet as given by Mr. Brush.

Bonn Water Works Experiments.

"*Hydraulics,*" *by Hamilton Smith, Jr., M. Am. Soc. C. E. New York,* 1886.

The experiments were made with a new cast-iron force main, coated with asphaltum, 12.48 inches in diameter and 17 684 feet long, belonging to the Bonn Water Works. Their tabular numbers are 397 to 404 inclusive.

The pipe was "supplied by pumps at the lower end;" it had "no obstructions of consequence, such as valves or sharp bends." The discharge "was determined by absolute measurement in a reservoir, and also by capacity of the plungers of the pumps; the two measurements agreed closely. Owing to the disturbed condition of the surface of the water in the reservoir" the quantity "was probably not very accurately determined. The head was determined by manometers at the pumps (probably Bourdon gauges attached to the air-chambers of the two pumps). There was a small amount of air in the pipe, which may have affected accuracy of results."

Experiments of Couplet.

"*Recherches sur le Mouvement des Eaux,*" *by M. Couplet, Memoirs of the Academy of Sciences*, 1732.

Of the fifteen experiments that were made at Versailles under the direction of M. Couplet, the writer has selected only six of the seven that were included in the fifty-one experiments that were used by Prony in determining his formula. The six experiments were made with a pipe 5.33 inches in diameter and 7481.7 feet long; for a distance of about 320 feet the pipe was of stone-ware, and for the remaining distance of lead. Their tabular numbers are 309 to 314 inclusive.

There were several slight turns, and one quite abrupt one, in the pipe. The difference between the elevation of the supply tank and the vertical outlet of the pipe was ascertained by connecting a temporary vertical pipe to the outlet, and allowing the quiescent water to take its hydrostatic level. The flow was determined by dividing the discharging jet of water into two parts, and measuring each one separately in a vessel having a capacity of about 0.63 cubic feet.

Experiments of Clarke.

"*Main Drainage Works of the City of Boston,*" *by Eliot C. Clarke, M. Am. Soc. C. E. Third Edition*, 1888.

The experiments were made under the direction of Mr. Clarke, with the Main Drainage Tunnel belonging to the City of Boston, which extends under the harbor from the mainland at Dorchester to Squantum Neck. Mr. Clarke mentions seven experiments, but the writer has selected only the three during which the flow seems to have been determined with accuracy. Their tabular numbers are 518 to 520 inclusive.

The tunnel is circular in form, 90 inches in diameter, 7 166 feet long, and lined with hard brick. It is flushed once in about two weeks with clean salt water, by utilizing the entire pumping capacity of the engines. At the time the experiments were made the tunnel was flowing full, and the volume consisted of about three-fourths salt water and one-fourth sewage.

The flow through the tunnel is generated by the difference in level of the elevation of the water at its two extremities. This difference, which is the frictional resistance, was carefully ascertained by knife-edged sliding gauges. The head was taken a short distance below the entrance of the tunnel during the three experiments, in order to avoid a correction for entrance. The flow was measured during Experiment No. 520, by the displacement of the pump plungers of the pumping engines, allowance having been made for a slip, which was ascertained by trial previous to the experiment in the reservoir, in which the flow during Experiments Nos. 518 and 519 were measured.

The section of the tunnel at its farther end increases gradually, similar to a diverging tube, which somewhat reduces the resistance, while on the other hand, the tunnel has a quarter turn of 9.75 feet radius and one angle of 23° 15′, which slightly increases the resistance.

Experiments of Darcy.

"*Recherches experimentales relatives au mouvement de l'Eau dans les Tuyaux,*" *by Henry Darcy, Paris,* 1857.

The experiments were made under the direction of Henry Darcy, an eminent French Civil Engineer, while he had charge of the water service of the City of Paris, during the years 1849, 1850 and 1851. They will be described at a considerable length, on account of the completeness of the apparatus that was used, and the thorough and elaborate manner in which they were conducted.

The synoptic table on page 27 gives the nomenclature, etc., of the different kinds of pipes that were used in making the 203 experiments.

At Chaillot, where the experiments were made, the natural facilities were very favorable. In a northerly direction from the building containing the pumping machinery, which was located on the north bank of the Seine, there was a circular sheet-iron tank, having a capacity of 105 677 gallons, elevated about 134.50 feet above the ground where the experimental plant was set up. In the same direction there were also four large reservoirs of different elevations, which could be connected with each other or isolated at will, each having a capacity of 873 589 gallons; their respective elevations were 77.10 feet, 81.70 feet, 83.99 feet, and 86.95 feet.

The circular tank was supplied by the pumping machinery through a conduit 9.84 inches in diameter, and the four reservoirs were supplied from the same source through a conduit 25.59 inches in diameter.

In the rear of the building containing the pumping machinery there was a cistern, suitable for measuring large quantities of water, which was ordinarily used to receive condensation water.

Number of experiments made.	Diameter in Inches.	Nature of the Pipes.	Total Length in Feet.	Length of each Pipe in Feet.	Nature of the Joints.	As numbered in Table No. 1.
13	0.480	New wrought-iron pipe.............	374.61	7.05	Screw..	15 to 27
17	1.047	" "	372.23	8.10	"	61 to 77
12	1.550	" "	371.92	12.63	"	212 to 223
7	0.551	New lead pipe......................	172.05	38.55	Solder.	34 to 40
7	1.063	" "	172.38	21.65	"	97 to 103
7	1.610	" "	172 41	17.06	"	224 to 230
12	1.055	New sheet-iron pipe, coated with bitumen.	371.86	9.15	Screw..	85 to 96
12	3.25	" "	365.07	9 51	"	296 to 307
11	7.72	" "	365 32	9.51	"	338 to 348
7	11.22	" "	365.47	9.51	"	370 to 376
6	1.96	New glass pipe......................	184.56	3.81	Flange.	231 to 236
13	3.22	New cast-iron pipe	366.10	8.20	Socket.	283 to 295
10	5.39	" "	365 74	8.20	"	315 to 324
9	7.40	" "	365.40	8.20	"	329 to 337
9	19.69	" "	365.34	8.20	"	428 to 436
14	1.43	Old cast-iron pipe, lined with deposit, and the same cleaned ..	374.94	4.27	Flange.	161 to 174
13	3.15	" "	366.31	8.20	Socket.	270 to 282
16	9.63	" "	365.35	8.20	Flange.	349 to 364
8	11.69	Old cast-iron pipe, thoroughly cleaned.	365.28	8.20	Socket.	377 to 384

The water used in making the experiments was taken from the 25.59 and 9.84-inch conduits, at a point about 325 feet north of the pump building. A vertical pipe of the same diameter as the conduit, and of suitable length, was connected to the 25.59-inch conduit; there was connected at a right angle, to the upper part of this vertical pipe, a conduit 11.81 inches in diameter; upon the 11.81-inch conduit there was a gate about 3 feet from the vertical pipe; two lead pipes 0.55 and 1.61 inches in diameter were arranged as connections between the vertical pipe and the 11.81-inch conduit, beyond the gate. The 9.84-inch conduit was also connected with the 11.81-inch conduit, beyond the gate, by a lead pipe 1.06 inches in diameter. Each of these small pipes was supplied with a stop-cock.

When the four large reservoirs were used, and small volumes required, the water was taken through the 0.55-inch pipe, its stop-cock having been opened more or less, dependent upon the requirements of the experiments; if the flow was not sufficient through this pipe it was shut off and the 1.61-inch pipe brought into service, which in due course was abandoned for the 11.81-inch conduit.

In this manner all the heads were utilized that could be furnished by the reservoir having the lowest elevation, which was maintained at a constant level by means of the water that was contained in the three others more elevated.

To obtain greater heads, communication was established with the sheet-iron circular tank, by opening the stop-cock on the 1.06-inch pipe, through which the water passed from the 9.84-inch conduit, which was supplied by this tank.

About 48 feet beyond the gate the 11.81-inch conduit was connected with a horizontal cast-iron cylinder 11.50 feet long, and 3.28 feet in diameter.

The horizontal cylinder formed an intermediate reservoir, and was intended to deaden the velocity of the water before it entered the experimental conduits. In the interior there was a vertical sheet-iron diaphragm perforated with holes, through which the water passed after its entrance from the 11.81-inch conduit. One end of the cylinder consisted of a semi-circular cap, and the other of a vertical plate to which the experimental conduits were connected.

The conduits used in making the experiments were laid upon blocks of masonry, on an incline inverse to the flow, in order to more easily free them from air.

The outlet end of each conduit discharged under water and was connected to the lower part of one of two vertical cast-iron cylinders. The bottoms of the cylinders were closed and the tops open. Each conduit had a gate or stop-cock located upon it at a short distance from where it entered its respective cylinder. These outlet cylinders were not of the same size, the larger was 5.30 feet in diameter and 10.93 feet high, and the smaller was 1.05 feet in diameter and 6.56 feet high. Only one of them was used at one time, the size depending upon the diameter of the conduit being experimented upon, and the volume of water flowing. A double communication was established between the large cylinder and the gauging basins; one of these communications was composed of a pipe 4 inches in diameter, which served to carry the water to a small gauging basin when small volumes were used; the other consisted of a notch 2.62 feet wide and 1 foot deep, at the top of the cylinder, into which was fitted a wooden channel which carried the water to the large gauging basins in the rear of the pump building; the 4-inch pipe was connected at a lower elevation than the channel, and when the capacity of the small gauging basin was insufficient, the inlet was stopped with a wooden plug, and the water rose in the cylinder and escaped by the channel. The water which entered the small cylinder was conducted to one of three small gauging basins, dependent upon the volume of water flowing, by means of a pipe 3.19 inches in diameter; this pipe had a movable elbow by which the water could be directed into either of the three, at will.

Seven gauging basins could be employed for measuring the water discharged by the experimental conduits. In order to be described more clearly, these basins will be numbered 1, 2, 3, 4, 5, 6 and 7. Basins Nos. 1, 2, 3 and 4 were formed by masonry division walls, which were located in the large rectangular cistern in the rear of the pump building; one wall divided the cistern into two equal parts, and another subdivided the one of these parts nearest to the pump building; by this arrangement there were available three basins, Nos. 1 and 2 each having an area about one-fourth of the total area of the cistern, and No. 3 having an area about one-half of the total area of the cistern; the wooden channel emptied into No. 1, which was located nearest to the vertical outlet cylinders; No. 4 was composed of the upper part of the cistern, from the level of the summit of the dividing walls to a point about 8.20 feet above these walls.

Two float gauges, located in Nos. 1 and 3, were used to measure the height of the water. Basins Nos. 1, 2 and 3 drained into the Seine, and could be connected or isolated at will; measurements were not made in Nos. 1, 2 and 3 until the water had covered their inverts, and not in No. 4, which was used only in making experiments with conduits of the largest diameter, until the water had reached a level above the division walls. Great care was taken in ascertaining the capacity of these basins, and in determining the amount of water that was lost by filtration and leakage from them; the amount lost in this way, however, was generally very insignificant.

The small gauging basins Nos. 5, 6 and 7, which were located quite near the outlet cylinders, were vertical cylinders with open tops; Nos. 5 and 6 were made of cast-iron, and No. 7 of lead; they all had stop-cocks or plugs, conveniently arranged for letting out the water after each experiment. On the outside of each there was adjusted a glass tube, which was connected with the interior, for measuring the elevation of the water, which was accomplished by placing a scale about 6.50 feet long against the tubes. Their respective diameters and heights were as follows:

	Diameter.	Height.
No. 5	3.15 feet	7.45 feet.
No. 6	1.08 "	8.20 "
No. 7	0.33 "	8.20 "

Five water and five mercurial manometers were used to determine the heads, and each set, to facilitate description, will be numbered 1, 2, 3, 4 and 5. (In reality, however, there were but four different manometric tubes, etc., as Nos. 3 and 4 consisted of one tube, etc., which was connected to the experimental conduits at two different points.) They were all set up against one post, which was located about half way between the horizontal inlet and vertical outlet cylinders, in order to be able to compare their indicated results without a possibility of error. The manometers were connected at different points to the experimental conduits, and to the horizontal cylinder, by lead pipes 0.55 inches in diameter, which were laid upon planks placed upon the masonry blocks that support the experimental conduits. A ladder was erected by the side of the post for convenience in observing the manometers.

Small closed reservoirs of a capacity of about 0.6 of a cubic foot were established upon the 0.55 inch manometer pipes near their junctions with the experimental conduits, with the exception of the pipe connecting manometer No. 5, the reservoir of which was located near the foot of the manometer post; in the upper part of these reservoirs, as well as at the highest points of the 0.55-inch pipes, the experimental conduits, the manometers, and the horizontal cylinder, air vents were established.

Water manometers were employed for all heads that did not exceed 19.69 feet, and mercurial manometers for heads above 19.69 feet.

The water manometers were adjusted upon a vertical plank graduated to millimeters, which was fastened to the front of the post facing the experimental conduits.

Each water manometer consisted of a vertical glass tube about 18 feet in length, formed by joining together in a copper fitting, with gum-lac, two glass tubes about 9 feet in length. They were all operated in the same manner, and had stop cocks at their junctions with the 0.55 inch connecting pipes.

The mercurial manometers were adjusted upon two vertical planks graduated to millimeters, which were erected behind the post; they were connected to the same 0.55 inch lead pipes that served as connections between the water manometers and the experimental conduits; and stop-cocks were so arranged that either the water or the mercurial manometers could be used at will.

Each mercurial manometer consisted of the following combination. A straight lead pipe rose to a height of about 6.30 feet above the bottom of the graduated plank; one end of a copper tube, about 4 inches long, was connected at a right angle with the upper end of the lead pipe by an elbow; a glass tube of the same length as the lead pipe was connected at a right angle with the other end of the copper tube by an elbow, and ran down in a direction parallel to the lead pipe to the bottom of the plank; one end of an iron tube, about 4 inches long, was connected at a right angle with the lower end of the glass tube; a second glass tube about 9 feet in length was connected at a right angle with the other end of the iron tube, by an elbow, and ran up parallel to the other glass tube and the lead pipe to the top of the plank. The iron tube had connected to its lower side a rubber pocket, containing mercury, which was introduced by pressure into the two glass tubes ; this pocket, which had a stop-cock upon its connection with the iron tube, also served to receive the mercury when it was necessary to discharge the manometer. The copper tube had a stop cock connected to its upper side over the shorter glass tube, to allow the introduction of a wire when it was necessary to free the tube of air. The joints of the lead pipe and glass and metal tubes were made respectively of solder and gum-lac.

Manometer No. 5 was connected to the horizontal cylinder, or intermediate reservoir, near its outlet end. Manometer No. 4 was connected to the experimental conduits quite near the horizontal cylinder. The difference between the readings of manometers Nos. 5 and 4, when small conduits were being used, gave the approximate loss of head due to the entrance of the water into the conduits from the horizontal cylinder, plus the head due to the velocity in the conduits. Manometer No. 3 was connected to the lead conduits 7.09 feet from the horizontal cylinder, and to the other conduits (with the exception of those of glass, the distance for which is not given by Mr. Darcy), from 13.68 to 19.29 feet from the horizontal cylinder. By an arrangement of cocks and connections, one 0.55 inch lead connection pipe served for both manometers Nos. 3 and 4. Manometer No. 2 was connected to all of the experimental conduits 164.05 feet from No. 3, with the exception of those of lead and glass, which were connected, respectively, 82.03 and 76.41 feet from No. 3. Manometer No. 1 was connected to all of the experimental conduits 164.05 feet from No. 2, with the exception of those of lead and glass, which were connected, respectively, 82.03 and 69.79 feet from No. 2. The distance from manometer No. 1 to the vertical outlet cylinders was upon the lead conduits about 16.41 feet, and upon the others, with the exception of the glass conduit, from 14.60 to 19.85 feet.

At first sight manometer No. 2 might not seem necessary, but it rendered great service during the experiments as a check upon Nos. 1 and 3, as well as in detecting, on several occasions, disturbances which would have materially interfered with the experiments, and which probably would not have been known but for this supplementary manometer. For instance, all things being equal, the difference in the observations of manometers Nos. 1 and 2 should be the same as the difference in the observation of Manometers Nos. 2 and 3. In actual practice, however, they were never exactly the same, as a slight variation in the mean diameter of the conduit, between either of these points, would cause a slight difference; but under these circumstances, as the difference was always in the same sense for all heads, the reason was readily understood. When these conditions were not fulfilled it was generally owing to leaks or accumulations of air.

There was also placed against the large outlet cylinder a graduated board with two glass tubes; one of these tubes was connected with the interior of the cylinder, and indicated the height of the water that it contained, and the other was connected to the experimental conduit near the cylinder, and indicated the head of the water moving in the conduit.

In all computations, the difference in the heights indicated by manometers Nos. 1 and 3, were always taken as the losses of head, and the lengths used were the distances between the points where these manometers were connected to the experimental conduits. The distance between Nos. 1 and 3 was for the glass conduit, 147.19 feet; for the lead conduits, 164.05 feet, and for all the other kinds of conduits, 328.09 feet. As the manometers were connected at the joints of the glass conduit, it was not possible to make the distances between Nos. 1 and 2 and Nos. 2 and 3 the same.

The absolute heads of the large reservoirs at Chaillot, or the circular sheet-iron tank, were used during the greater portion of the experiments. In order to vary the head for each experiment, while experimental conduits of large diameter were being used, the gate, or the stop cock of the branch conduits through which the water flowed that supplied the horizontal cylinder at the time, was throttled. This could not be done satisfactorily, however, when experiments were being made with small conduits. Two new apparatus were therefore constructed, for making the experiments with the glass and lead conduits, and with the wrought iron conduits 0.48 and 1.047 inches in diameter, by means of which reservoirs were formed of constant elevation, which furnished the different heads that were required.

The apparatus which was used with the wrought-iron experimental conduits consisted of a cast-iron column, composed of flanged pipes 9.84 inches in diameter. A lead pipe 0.55 inches in diameter, which was run up the side of the column and bent over the top, supplied the water while the experiments were being made; in the lower part of this lead pipe a cock served to shut off the water entirely, or to moderate the flow; the supply could also be regulated by means of a cock in the horizontal cylinder, to a branch of which the other end of the pipe was connected. Another lead pipe, 1.06 inches in diameter, was connected to a small branch at the bottom of the column, and ran up vertically by the side of the column; in this lead pipe, at convenient distances apart, there were branch openings, which were stopped with wooden plugs. A ladder was erected by the side of the column in order to manouver the plugs. When it was necessary to operate with the least head, the lower plug was removed, and by means of the cock in the lower part of the lead supply pipe, the arrival of the water was regulated in such a manner as to allow but a very small quantity to escape by the branch opening from which the plug had been removed, thereby maintaining a supply of water in the column at a constant level. In order to maintain a higher head, one of the upper plugs was removed and all the lower openings closed. The greatest head was obtained by closing all the branch openings and letting the water flow over the top of the column. The experimental conduits for which this apparatus was constructed, were connected to the lower part of the column and had a regulator cock near their origin. Manometer No. 5 was connected to the same branch as the 1.06-inch vertical lead pipe with the different branch openings.

The other apparatus, which was used with the lead and glass conduits, was constructed by taking the horizontal cylinder, 3.28 feet in diameter, and placing it vertically upon its semi-circular end, and connecting vertically to its other end a portion of the 9.84 inches cast-iron column that was used in making the experiments with the wrought-iron conduits. In order to create at will reservoirs of different levels, a sufficient length of the lead pipe with branch openings that had previously been connected to the cast iron column was connected to a branch at the lower end of the cylinder. Manometer No. 5 was connected to the same branch as the vertical lead pipe with branch openings. The supply of water was brought from the large conduits that formerly supplied the horizontal cylinder, by a lead pipe 1.61 inches in diameter, which was connected to a branch on the side of the cylinder opposite to the branch where manometer No. 5 and the vertical lead pipe with branch openings were connected; there was a stop cock on this supply pipe near its connection with the cylinder, to regulate the flow of water. The experimental conduits for which this apparatus was constructed were connected to a branch in the front of the cylinder, which was situated about half way between the branches where the supply pipe and vertical lead pipe were connected. Each experimental conduit had a stop cock near its junction with the cylinder to shut off the supply of water when necessary.

The experimental conduits of wrought-iron, lead and glass, for which the two apparatus that have just been described were constructed, discharged into the smallest of the two vertical outlet cylinders, and the general arrangement of the stop-cocks and manometers upon these conduits, with the exception of manometer No. 5, was the same as upon the other conduits.

General Observations Relating to Darcy's Experiments.

The pipes of each diameter were forced as close together as possible, in order to prevent the joints from interfering with the flow of the water.

Before commencing the experiments, the conduits were always tested under pressure, and if any leaks were discovered they were immediately repaired, even though they were of the most minute description.

After the conduits were in position the manometers were connected, and the locations of Nos. 1, 2 and 3 were determined with much care. After having connected manometer No. 2 at a convenient point near the manometer post, the proper distances were measured off above and below manometer No. 2, for fixing the positions of manometers Nos. 1 and 3. The measurements were made with an iron rod 16.41 feet in length.

Upon the cast-iron conduits 3.15 inches in diameter and above, the taps or stop-cocks of the 0.55-inch manometer connecting pipes were screwed into the cast-iron at right angles to the center of the conduits. These stop-cocks were filed at their lower ends to conform to the curvature of the interior of the conduits, beyond the surface of which they were not allowed to project.

Upon the conduits less than 3.15 inches in diameter, and upon the sheet-iron conduits coated with bitumen, the taps or stop-cocks of the manometer connecting pipes were soldered, above a hole from 0.08 to 0.12 inches in diameter.

The conduits, the stop-cocks, the joints, the air-cocks or vents, and the manometers, were all constantly under the most careful supervision; it was frequently necessary, however, notwithstanding the precautions taken, to make some of the experiments over a second time, on account of a small leak having been discovered at their conclusion, upon some portion of the apparatus.

Before commencing the experiments upon a conduit, the manometers were tested, by opening their stop-cocks and air vents, and putting the conduits under pressure Much difficulty was always experienced when the first experiment was commenced, in freeing the manometer pipes of air, and it was frequently necessary to let the water flow a long time under the greatest possible head in order to do so. During the experiments upon the conduit 0.48 inches in diameter, one entire day's work was entirely eliminated on this account, the difficulty being finally overcome by letting the water flow all night.

In order to be certain that the manometers were free from air, the difference of the readings of manometers Nos. 2 and 3, and Nos. 1 and 2, were compared. These differences, as has been previously mentioned, should be nearly equal, when the water was flowing in the conduits, if the manometers were working properly. Another test with the water at rest, was also made at times, during which the readings of all the manometers were exactly the same if they were entirely free from air.

Nearly all of the experiments were commenced with slight heads. As soon as the desired head was obtained, pins were put into the graduated plank to mark the heights of the manometers, and the water was allowed to flow long enough to acquire its normal regimen. When the water, or mercury, in the manometers became stationary, proceedings were taken to measure the discharge of the conduit.

The same person observed the heights of all the manometers when it was possible. While experiments were being made with low heads the observer was required to remain upon the manometer ladder the whole of the time, in order to have his eyes fixed constantly upon the manometers, and if a noticeable change in the heights, or an oscillation of extraordinary amplitude was observed, the experiment was commenced over again.

It was the duty of the person detailed to observe the heights of the manometers to watch carefully the conduits, the manometers and the other apparatus, in order to be assured that everything was working properly.

Considerable difficulty was experienced in making the experiments with high heads, as it was necessary to use mercurial manometers. Two consecutive experiments were never made without it being necessary to repair one or more joints, and frequently repairs had to be made several times during the same experiment.

In order to verify some of the calculations which necessitated the use of the mercurial manometers, the water manometer No. 1 was always left open, and No. 2, also, when the heads were not too great.

Heights of water were thus obtained, with which the heights indicated by the mercurial manometers could be compared.

The highest accuracy was always maintained in measuring the discharge of the conduits, and the same watch graduated to seconds was nearly always used.

The time consumed in making each experiment ranged from 135 to 3 240 seconds; it was rarely less than 240 seconds, however.

Extraordinary care was taken in determining the mean diameter of the conduits.

The diameters of the conduits 0.48, 1.05 and 1.56 inches in diameter, were ascertained by means of the quantity of water that they were able to hold. The water was drawn from a reservoir of well-determined section, that was placed in an elevated position, and under which the conduits were vertically arranged in fractions of length.

At each operation account was kept of the lowering of the water in the reservoir, and the product of the sum of the successive lowerings by the section of the reservoir gave a cube, which, divided by the lengths of the conduits, gave the mean section, from which was obtained the mean diameter.

The conduits 0.55, 1.06 and 1.61 inches in diameter, of drawn lead, had a diameter perfectly well established.

The sheet iron conduits, coated with bitumen of 1.06 and 3.25 inches in diameter, and the glass conduit of 1.96 inches in diameter, were determined by the first process described.

When the diameters of certain conduits were too large for the above process to be easily carried out, or when it was a question of conduits lined with deposit, the diameters were ascertained by means of the total capacities of the conduits, the latter being in place. This proceeding, which was followed in determining the diameters of the sheet-iron conduit coated with bitumen, 7.72 inches in diameter, and the cast-iron conduits 1.43, 3.15 and 9.63 inches in diameter, was carried out in the following manner:

First.—The extreme pipe was first dismounted (that is to say, the pipe contiguous to the feeding horizontal cylinder at the head of the conduit, and towards which the conduit sloped), thus allowing the water to flow entirely out of the conduit.

During this operation the gate at the other extremity of the conduit near the vertical outlet cylinder, was kept closed, the cylinder having been previously filled with water.

Second.—The extremity of the conduit which had been disconnected from the horizontal feeding cylinder was plugged. At the extremity of this plug a curved pipe with a stop-cock was connected, in order to allow the air to escape, while the conduit was being filled from the vertical cylinder by the opening of the gate.

Care was taken to note the height of the water in the vertical cylinder before and after the filling.

A stop-cock to facilitate the escape of air was placed at the middle of the conduit.

Third.—In order to fill the conduit, the gate near the vertical outlet cylinder was opened, and two observers, one of whom was placed at the middle of the conduit, and the other at the extremity by the plug, closed the air-cocks as soon as the water appeared. The gate was then closed, and from the lowering of the water in the vertical cylinder the mean section of the conduit was determined, from which was calculated the mean diameter.

The diameters of the sheet-iron conduit coated with bitumen, 11.22 inches in diameter, and the cast-iron conduits 3.22, 5.39, 7.40, 11.69 and 19.69 inches in diameter, were obtained by direct measurements.

The diameters of the conduits lined with deposit, given in Table No. 1, are the diameters of the conduits after they were cleaned, and not the diameters used by Mr. Darcy, which took into consideration the thickness of the deposit.

Experiments of Dubuat.

"*Principes d'Hydraulique,*" *by Chevalier Dubuat, Paris,* 1786.

Chevalier Dubuat, who was a celebrated French Civil Engineer, made quite a number of experiments with various sizes and lengths of pipe. The writer, however, has selected only the eighteen that were included in the fifty-one experiments used by Prony in determining his formula, which were made with a tin pipe 1.07 inches in diameter. Their tabular numbers are 106 to 123 inclusive.

The pipes used in making the eighteen experiments appear to have been laid straight, and discharged sometimes under water, and sometimes into the open air.

Experiments of Darrach.

"*Transactions of the American Society of Civil Engineers,*" *Vol. VII,* 1878.

The fifty-three experiments were made under the direction of C. G. Darrach, M. Am. Soc. C. E., with six cast-iron pumping mains, one of 20, three of 30, and two of 36 inches in diameter, belonging to the Philadelphia Water Works. The pipes were coated with coal tar in the usual way, before being laid.

The experiments that were made with each main are described separately, and each description is preceded by the tabular numbers of the experiments to which it refers.

Pressure gauges were used in determining the head.

Twenty-inch.—Nos. 458 to 466 inclusive. This main was eleven years old when the experiments were made. It had one quarter turn. The flow was measured by the displacement of the pump plungers of a Worthington pumping engine, 5 per cent. having been deducted for slip. The presure gauge was connected with the pump.

Thirty-inch.—Nos. 473 to 485 inclusive. This main was about two years old when the experiments were made. It had four check-valves, the weight of which produced a pressure equivalent to 1.8 pounds per square inch, for which allowance was made in the frictional head given in Table No. 1. The flow was measured by the displacement of the pump plungers of a Cramp pumping engine, 5 per cent. having been deducted for slip. The pressure gauge was connected with the air chamber.

Thirty-inch.—Nos. 468 to 472 inclusive. This main was laid the same year that the experiments were made. It had one quarter turn. All the other curves had a radius of 25 feet. It also had two check-valves, the weight of which produced a pressure equivalent to 1.3 pounds per square inch, for which allowance was made in the frictional head given in Table No. 1. The flow was measured by the displacement of the pump plungers of a Worthington pumping engine, 5 per cent. having been deducted for slip.

Thirty-inch.—Nos. 486 to 495 inclusive. This main was about nine years old when the experiments were made. It had one curve of short radius. The flow was measured by the displacement of the pump plungers of a Worthington pumping engine, 5 per cent. having been deducted for slip. The pressure gauge was connected with the main outside of the check-valve.

Thirty-six-inch.—Nos. 497 to 502 inclusive. This main was seven years old when the experiments were made. All the curves had a radius of 25 feet. The flow was measured by the displacement of the pump plungers of a Simpson pumping engine.

Thirty-six-inch.—Nos. 503 to 512 inclusive. This main was seven years old when the experiments were made. All the curves had a radius of 25 feet, with one exception, when 90 degrees were turned with a T-pipe. The flow was measured by the displacement of the pump plungers of a Worthington pumping engine, 5 per cent. having been deducted for slip. The pressure gauge was connected with the stand-pipe.

Experiments of the Edinburgh Water Company.

"*Excerpt Minutes of the Proceedings of the Institution of Civil Engineers,*" *Vol. XIV, Session* 1854–55.

The three experiments were mentioned by Mr. James Leslie, M. Inst. C. E., in a paper on the flow of water through pipes, conduits, etc. Their tabular numbers are 415 to 417 inclusive.

They were made with the Clointon cast-iron main between Clubbie-Dean Reservoir and Castle Hill. This main was 16 inches in diameter and eight or nine years old.

One experiment was made with the total length of the pipe, 29 580 feet; one with a length of 25 765 feet, between Torduff Cistern and Castle Hill; and one with a length of 3 815 feet, between Clubbie-Dean Reservoir and Torduff Cistern. In the first instance fifteen observations were made, in the second twenty-five, and in the third one.

Experiments of Fanning.

"*A Practical Treatise on Hydraulic and Water Supply Engineering,*" *by James T. Fanning, C. E. New York,* 1882.

The twelve experiments were made with a wrought-iron cement-lined pipe, 20 inches in diameter, and 8 171 feet long. Mr. Fanning does not mention the manner in which they were conducted. Their tabular numbers are 437 to 448, inclusive.

Experiments of Gale.

"*Transactions Institution of Engineers in Scotland,*" *Vol. XII,* 1869.

The experiment was made under the direction of James M. Gale, M. Inst. C. E., with a cast-iron main, 48 inches in diameter, and 5 280 feet long, that conducted a supply of water from Loch Katrine to the City of Glasgow. Its tabular number is 517.

The writer is indebted to Mr. James Forrest, Secretary of the Institution of Civil Engineers, for the following additional information: "Mr. Gale informs me that the pipes referred to in his paper had been at work for eight years prior to the date of the observations. Before being laid they were coated by the late Dr. R. A. Smith's patent process, and this coating had been very little affected by rust when the observations were made. He has recently seen some of these pipes" (August, 1884), "and finds that the tubercles inside are increasing in size and number, and he has no doubt that the discharge is falling off. The quantity of water delivered was measured over a weir 40 feet wide, in four bays of 10 feet wide each. The plates were of cast-iron brought to a thin edge. The formula used was cubic feet per minute $= 214\ A \sqrt{h}$."

Experiment of Greene.

"*The Brooklyn Water Works and Sewers,*" *by James P. Kirkwood. New York.* 1867.

This experiment was made by General George S. Greene, with a cast-iron main, 36 inches in diameter, and 11 217 feet long, belonging to the Brooklyn Water Works. Its tabular number is 496.

This main connected the receiving with the distributing reservoir. It was heavily tuberculated, and had three-quarter circle curves of 90 feet radius each.

The head was ascertained by leveling between the reservoir surfaces, and the flow was measured by the lowering of the water in the upper or feeding reservoir.

Experiment of Hodson.

"*Hydraulic Tables, Co-efficients and Formulas*," *by John Neville, C. E., London*, 1860–61.

Mr. Neville states that the results of this experiment, which was made with a pipe 1 inch in diameter and 100 feet long, was given to him by Mr. Hodson, of Lincoln.

As Mr. Neville used 0.352 as the co-efficient of influx in connection with this experiment, the writer has done the same.

Experiments of Jardine.

"*Brewster's Encyclopædia.*"

The experiment was made with a lead pipe 4½ inches in diameter, and 14 930 feet long, belonging to the Edinburgh Water Works. Its tabular number is 308.

Mr. Jardine states that the flow through the pipe was 11⅓ cubic feet per minute, for the years 1738–42. The velocity given in Table No. 1 is computed from these data.

Experiments of Leslie.

"*Excerpt Minutes of Proceedings of the Institution of Civil Engineers*," *Vol. XIV, Session*, 1854–55.

The experiments were made under the direction of James Leslie, M. Inst. C. E.

Mr. Leslie made sixty-four experiments with new lead pipes 2½, 1¾ and 1¼ inches in diameter. The writer has selected, and included in Table No. 1, only twenty of the thirty-eight experiments that were made with pipes 2½ inches in diameter, ranging in length from 100 to 1 086 feet; these experiments were simply selected so as to include velocities from 0.22 to 6.91 feet per second, the latter velocity being the highest that was obtained with these lengths of pipe. Their tabular numbers are 250 to 269 inclusive.

The experiments were commenced with a pipe 1 086 feet long; "the pipe was afterwards cut, successively, to the lengths of 540, 270, 100, 25 and 10 feet."

"The pipe was coiled nearly in a circle of 90 feet in diameter, and when of its integral or full length it made four revolutions.

Each upward curve was tapped by a small gimlet hole, into which a wooden plug was inserted; these were all taken out frequently to allow the air to escape, and were kept out until only water, free from air, issued from the orifice. The ends of the pipe, in each case, were inserted into the sides of two cisterns, each measuring 2 feet 6 inches by 1 foot 3 inches, and 2 feet deep, and were about 22 inches below the surface of the water. In order to be perfectly assured that there could be no mistake in the levels, the two cisterns were placed quite close together, and the head was measured by the difference of the respective surfaces of the water." The head could be varied at will by raising or lowering the upper cistern, which was supplied with water by a pipe that was carried up over the top, thence nearly to the bottom of the cistern. The water overflowed from the lower or outlet cistern, which was fixed, into a measuring box.

"The pipes were carefully joined and soldered, so as to have no internal obstruction, with the exception of one joint, where it was discovered, that by some accident, a little solder had got inside and caused a slight obstruction. The extent of it measured only about one twenty-fifth part of the area of the pipe, and would, it is imagined, be so trifling as not to appreciably affect the flow."

Experiments of Lampe.

"*Der Civilingenieur.*" *Vol. XIX*, 1873; *and* "*Hydraulics,*" *by Hamilton Smith, Jr., M. Am. Soc. C. E. New York*, 1886.

The four experiments were made with a 16-inch main, by Professor Dr. C. J. Lampe, an eminent German scientist. Their tabular numbers are 418 to 421 inclusive.

They are described by Mr. Smith as follows: "The pipe experimented upon was a cast-iron conduit, which conveyed by gravity a supply of spring water to the Town of Danzig.. It had a total length of 46 352 feet; the lower 9 040 feet in length had a considerably steeper inclination than the upper portion, so there were two hydraulic grade lines; the four experiments, however, were made upon the upper portion of the pipe. The pipe was laid in 1869; it had three curves, each of 10.3 feet radius, and a number of very easy curves as it followed the general contour of the ground. The pipe was coated with a patent varnish which did not appreciably diminish its section. Examination showed that from 1869 to 1871 the character of the inner surface had very slightly changed; the only material adhering to the surface in 1871 could be readily removed by rubbing with the finger, there being no signs of rust. The joints were 12 feet in length, united by lead and hemp packings. There were twenty-six air-cocks attached to the pipe along its course."

"The mean velocity was ascertained by measuring the discharge * * * in a masonry reservoir situated at the outlet end of the pipe.

"The pressures were determined by connecting a quicksilver manometer, first with one of the air-cocks, and then with another; these pressure determinations were not synchronous with the measurement; but, in some instances, several days apart. As in none of the four experiments was the pipe filled at its inlet, this lack of synchronism appears to form a dangerous source of error, the pipe being fed by the flow from springs, whose discharge must necessarily have been more or less irregular. Dr. Lampe, however, states to us in an explanatory letter, that he is satisfied no serious error could have arisen from this cause, there having been, directly preceding or during these intervals, no rains of consequence to notably affect the flow from the feeding springs; subsequent readings of the manometer, shortly after these intervals, also verified the constancy of flow. He considers that errors from this source will not change the given results more than one-half of 1 per cent.

"Another source of error in these experiments arises from the fact that the pipe had two hydraulic grade lines. Although the several pressures were all determined above the anticlinal point at which these lines united, still the condition of the pipe was more or less unfavorable to extreme accuracy of observation, as the sucking or siphon action as the water passed over the summit could not have been perfectly regular, as doubtless the amount of air accumulating at this summit varied slightly from time to time; the consequent intermittant sucking action of the water as it flowed below this summit, hence may have appreciably affected the nearest piezometers."

Experiments of Neville.

"*Hydraulic Tables Co-efficients and Formulas,*" *by John Neville, C. E. London*, 1860–61.

Mr. Neville states that the two experiments were the mean results of several experiments that were made by him with great care, with a pipe 1.02 inches in diameter and 9.29 and 19.20 feet in length. Their tabular numbers are 59 and 60.

Mr. Neville also ascertained that the co-efficient of contraction of the inlet end of the same pipe was 0.860, which corresponds to a co-efficient of influx of 0.352. This value was used by the writer in calculating the co-efficients of friction from these experiments.

Experiments of Provis.

"*Excerpt Minutes of Proceedings of the Institution of Civil Engineers.*" *Vol.* II, 1838.

The experiments were made under the direction of W. A. Provis, M. Inst. C. E., with lead pipes 1.5 inches in diameter. Thirty-seven of them have been selected by the writer. Their tabular numbers are 175 to 211 inclusive.

The pipes were drawn in 15 foot lengths, and soldered together with care, so as to avoid as far as possible any interior obstructions. Into the upper end of the pipe was inserted a stop-cock of similar bore, and from this cock the length of the pipe was measured ; the opposite end of the cock was inserted into a cistern in which the surface of the water was maintained at a constant elevation during each experiment. The flow of water through the pipes, which discharged into the open air, was measured in a tank having a capacity of four cubic feet.

Experiments were made with pipes 100, 80, 60, 40 and 20 feet long.

The co-efficient of influx was not ascertained by Mr. Provis, and the writer, after having calculated two independent sets of experimental co-efficients of friction from the experiments that were made with the pipes 100, 80 and 60 feet long, by using 0.97 as a co-efficient of influx in one instance, and 0.505 in the other, finally determined to work out co-efficients of influx and loss from the experiments themselves. This was done by taking the data of the experiments made with the pipe 20 feet long, and working out co-efficients of influx and loss for different velocities of flow for this length of pipe. The co-efficient of influx and loss thus determined were used in calculating the experimental co-efficients of friction of the experiments which were made with pipes 100, 80 and 60 feet long, as a slight variation in its value one way or the other would not materially affect the results obtained from these experiments, on account of the long lengths of pipe used, and their low velocities of flow ; with the experiments made with the pipe 40 feet long, however, there seems to be a question of doubt, consequently they were not considered by the writer. The values of the experimental co-efficients of friction obtained by using this latter co-efficient of influx and loss, were in the majority of cases more than was obtained by using as a co-efficient of influx 0.97, and less than was obtained by using 0.505. The lengths and heads of the pipes given in Table No. 1 are therefore less than those given by Mr. Provis ; the lengths by 20 feet, and the heads by an amount equivalent to the loss of head due to contraction at the entrance of the stop-cock, plus the loss of head due to friction in the stop-cock, and in a length of pipe 20 feet long.

Experiments of Robison.

"Encyclopædia Britannica."

The experiment is mentioned in an article on Rivers by Dr. John Robison, as having been made with a pipe 2 inches in diameter, and 3 300 feet long, which was part of a compound pipe that was used to convey water from a spring into the town of Haddington. The nature of the pipe and the manner in which the experiment was conducted is not given. Its tabular number is 237.

Experiments of Rennie.

"*Traité d'Hydraulique,*" *by J. F. D'Aubuisson de Voisins.*

The two experiments were made with a lead pipe 0.5 inch in diameter and fifteen feet long. The manner in which they were conducted is not mentioned. Their tabular numbers are 28 and 29.

Experiments of Simpson.

"*Excerpt Minutes of the Proceedings of the Institution of Civil Engineers.*" *Vol. XIV, Session* 1854–55.

The eighteen experiments were made under the direction of the late James Simpson, M. Inst. C. E., with pipes 12, 19 and 30 inches in diameter. Their tabular numbers are: 12 inch, 390 to 396 inclusive, 19 inch, 423 to 427 inclusive, 30 inch, 467.

Table No. I contains substantially all that Mr. Simpson mentioned in regard to the experiments. The following additional information, however, was furnished to the writer, in reply to some inquiries, by Mr. James Forrest, Secretary of the Institution of Civil Engineers:

"Mr. Arthur Telford Simpson, one of the sons of the late Mr. James Simpson, informs me that he cannot find the exact dates of the several experiments on the flow of water through pipes to which his father referred in the discussion at their institution in 1855, but he gives the following statement of the time when the mains were laid, which will serve to show that they could not have been in use a very long time."

Diameter in inches.	Place.	When laid.
12	Brixton to Streatham	1848
19	Belvedere Road to Brixton	1841
12	Liverpool	....
12	Carlisle	1850
30	Ditton to Brixton	1852

Experiments of Smeaton.

"*Practical and Experimental Researches in Hydraulics,*" *by R. A. Peacock, C.E.*

The sixteen experiments were made by John Smeaton, an eminent English Civil Engineer, with a pipe 1¼ inches in diameter, and 100 feet long. The manner in which they were conducted and the nature of the pipe that was used are not mentioned.

Their tabular numbers are 124 to 139 inclusive.

Experiments of Smith.

"*Transactions of the American Society of Civil Engineers.*" *Vol. XII,* 1883.

The experiments were made under the direction of Hamilton Smith, Jr., M. Am. Soc. C. E., with pipes of various kinds, lengths and diameters.

The writer has selected, and included in Table No. 1, only forty-two of the seventy-one experiments that were made by Mr. Smith. Twenty-seven of the remaining twenty-nine were omitted on account of having been made with small pipes, less than 64 feet in length, which had funnel shaped inlets, and two others on account of there being a question of doubt in regard to the discharge in one instance, and the diameter of the pipe used in the other.

The fourty-two experiments were made with pipes of 0.502, 0.628, 0.746, 1.054, 1.26, 10.93, 12.67, 14.76 and 16.99 inches in diameter.

The experiments are described in divisions, which are preceded by the tabular numbers of the experiments, to which they refer.

Nos. 365 to 369, 405 to 408 and 409 to 414, inclusive (10.93, 12.67 and 14.76 inches), "were made at North Bloomfield, California, with their pipes laid side by side across a ravine about 100 feet lower than the penstock. They were made of single-riveted No. 14 sheet-iron; the rivet heads had worn pretty smooth, and no deduction for them was made in computing the areas. The pipes had been carefully coated when first laid with a mixture of coal tar asphaltum, and their interior surfaces at the time of these measurements were quite smooth. They were put together in slightly conical joints, the greatest variation in diameter being about ¼ of an inch. The main joints were some 20 feet in length, put together stove-pipe fashion, as is always done in the California hydraulic mines. The various heads were obtained by adding to the length of the pipes at their outlets."

"The heads, lengths and mean diameters are given with exactness. The amount of discharge was measured by running the water over an iron weir, the co-efficients of whose discharge, with varying depths, had been carefully determined by absolute measurement, but at another point, and under conditions not exactly similar. This gauging was done with all the care practicable, the height of the water being measured with Boyden hook gauge, reading to 0.001 of an inch. The limit of possible error in these experiments is probably not over 2 per cent."

Nos. 407 and 408, "however, are somewhat unreliable, owing to the stoppage of a stone passing through the pipe."

"The co-efficient of contraction is assumed at 1 in these pipes, as each of them had a short funnel-shaped inlet, which was included in the measurement of their lengths."

In Nos. 369, 408 and 414, "the discharge was into a pool of water at the outlet box; in the other twelve the discharge was into the air."

"There were two angles in the line of these pipes, one of 9 degrees and the other of 11 degrees; for the remainder of the distance they were nearly straight; in the computations no deduction was made for these bends."

Nos. 30 to 33, 41 to 46, 47 to 50, 78 to 84 and 140 to 144, inclusive (0.502, 0.628, 0.746, 1.05 and 1.26 inch), were made at New Almaden, California. "All these experiments were with straight pipes, and were made with considerable accuracy."

"The mean diameters were computed by the weight of water contained in the pipes, taken by the sum of weights in the several joints, which varied slightly from the mean of the end diameters as directly measured."

"The discharge was determined by filling a tank of known size."

"The iron gas pipes were joined by their usual couplings. The ends of the glass pipes were smoothly ground, and a water-tight joint made by an outside rubber band."

"The wooden pipe was joined by usual plug couplings."

Nos. 78 to 84, inclusive, were made with a pipe that was new and in good order.

"The glass tubes were new and unscratched, of French manufacture, and somewhat conical."

"The measuring tank held 15.2 cubic feet, and the times were determinated to one-fifth of a second."

No. 422 (16.99 inch) "was with a new inverted siphon of double riveted sheet-iron pipe, with a pressure of about 800 feet at the lowest point, with a maximum tensile strain of 16 500 pounds per square inch, and with most of the joints made by sleeves with lead packing. It was coated with the usual mixture of coal-tar and asphaltum."

"No deduction was made for rivet heads, which for over one-half the length doubtless formed noteworthy obstructions, as the pipe was of comparatively small diameter—17 inches."

"The mean diameter head and length are given with exactness. The discharge was measured first by the flow over a weir, and afterwards through standard apertures."

"In all the experiments the actual or total head was either the difference in elevation between the surface of the water in the pen-stock and that in the outlet tank, or the difference between the water in the pen-stock and the center of the escaping jet, where the pipe discharged into the air." Experiments 30 to 33, 41 to 46, 47 to 50, 78 to 84 and 140 to 144, inclusive "were made under the latter condition."

Experiments of Stearns.

"*Transactions of the American Society of Civil Engineers*." *Vol. XIV*, 1885.

The four experiments were made by F. P. Stearns, M. Am. Soc. C. E., with a pipe 48 inches in diameter, and 17 472 feet long, belonging to the Boston Water Works. Their tabular numbers are 513 to 516, inclusive.

"The pipe was on the line of the Sudbury conduit. It was a cast-iron pipe coated with Dr. Angus Smith's coal-tar preparation. The changes in direction were made by two vertical curves, one of 500 and the other of 1 170 feet radius. The mean pressure on the pipes during the experiments was 41 feet. The pipes had been laid three years, and had been in use two, when the experiments were made, yet the tar-coating presented nearly as good a surface as when it was new. The volume flowing was measured at a weir about 10 miles distant up the conduit from the pipe, and to this measurment $\frac{3}{10}$ of a cubic foot per second was added for filtration into the conduit below the weir. The amount of the filtration was determined from fairly good data; but, even if somewhat inaccurate, it has little importance, as its whole effect was less than 1 per cent. The weir used was 19 feet long, and had previously been tested by the actual measurement of the water passing over it. The loss of head in a length about 60 feet shorter than the whole length of the pipe was measured. This method was adopted to avoid including in the measurement the loss of head at the entrance of the pipe, the gain or loss at the exit, the effect of two rather sharp curves (30 feet radius), near the ends of the pipe, and several other disturbing causes. The apparatus used for taking the heads was as follows: In each of the pipe chambers, at either end of the pipe, there was a float gauge designed for permanent use in measuring the height of the water in the chambers. This consisted of a vertical iron cylinder, 12 inches in diameter, plugged at the bottom; in this cylinder was a hollow brass float, with a suitably guided stem, carrying an index up and down the face of a graduated scale. Water was admitted to the float cylinder through a small pipe leading from the center of the pipe chamber, the flow being controlled by a stop-cock. Connected with the float cylinder was a small brass cup, having a level top of known elevation. By filling the cylinder until the cup was even full, the position of the index was adjusted. For the purpose of these experiments, the small pipes leading from the float cylinders were extended into the ends of the 48-inch pipe, along its bottom about 33 feet. The last 7 feet of each of these small pipes was a smooth, straight brass tube, with several holes drilled in its top, and with its end plugged."

The heights of the gauges were carefully tested, and precautions taken to expel all air from the small pipes leading to them.

Mr. Stearns was not able to conduct the experiments in person, but he gave detailed instructions as to the work to be done, to two experienced assistants.

Experiments of Weisbach.

"*The Mechanics of Engineering and of the Construction of Machines,*" *by Julius Weisbach, Ph.D. Translated from the German edition by Eckley B. Coxe, A.M. New York,* 1872.

The experiments were made with pipes 0.409, 0.563, 0.972 and 1.27 inches in diameter.

Professor Weisbach only mentions the experimental co-efficients of friction (ζ) computed from the twenty-one experiments, and the nature and diameters of the pipes used in making the experiments. Their tabular numbers are, 0.409-inch and 0.563-inch, 1 to 14, inclusive; 0.972-inch, 52 to 57, inclusive, and 1.27-inch, 145.

Experiments of Weston.

The five experiments were made by the writer during the years 1876 and 1877 with pipes 0.97 and 6 inches in diameter, belonging to the Providence Water Works.

The experiments that were made with each size of pipe are described separately, and each description is preceded by the tabular number or numbers of the experiment or experiments to which they refer.

Six-inch.—Nos. 325 to 328 inclusive. An ordinary straight cast-iron pipe with socket joints, which were calked with lead and yarn in the usual way, was used in making these experiments. The pipe had been coated when new by Dr. Angus Smith's process, and at the time the experiments were made it had been in service four years.

The supply of water was taken from a main 30 inches in diameter, which was connected with the upper end of the pipe.

Two very accurate hydraulic pressure gauges were used to determine the head ; the first was connected with the 6-inch pipe 500 feet from the 30-inch main, and the second 1 170 feet from the first, and 410 from the lower end of the pipe where the discharge took place. The total length of the pipe was 2 090 feet. The difference in the readings of the two gauges, allowance having been made for their elevation, was taken as the loss of head due to friction for a length of 1 170 feet.

The lower end of the pipe was connected with an 8-inch flush hydrant, which had a "chuck" with four lines of 2½-inch rubber hose about 100 feet long connected to it. The flow in the 6-inch pipe was regulated by opening and closing the valves of the "chuck." The first experiment was made with one valve open, the second with two, the third with three and the fourth with four. The discharge was ascertained by measuring the water flowing in the hose, which was done by connecting a hydraulic pressure gauge 1 foot from the hydrant to the line or lines of hose in use, and calculating the discharge from the pressure, a co-efficient of discharge having been previously determined by measuring the flow under different pressures, in a large tank arranged expressly for the purpose.*

Considerable time was devoted to making the observations, and a sufficient period was allowed for the water to get in train after each experiment, before commencing the next. The pipe had several side connections which were carefully closed before the experiments were begun.

0.969-*inch.*—No. 51. A new wrought-iron pipe, which had a layer of tin drawn through it for a lining, was used in making this experiment.

The lengths were coupled together like ordinary gas pipe, with the exception that sheet tin bushings were fitted into the pipe at the joints to prevent any corrosion from penetrating into the interior, that might take place at the ends of the lengths where the iron was not covered with tin.

The joints were carefully made, and the interior surface of the pipe was practically as smooth as the interior surface of an ordinary lead pipe.

The pipe was 781.83 feet long, was laid nearly straight and was connected by a $\frac{5}{8}$-inch tap with a main 36 inches in diameter, which was directly supplied with water from a large reservoir.

The diameter of the pipe was determined by measuring the diameters of a number of lengths.

The difference between the surface of the water in the reservoir, and the center of the outlet end of the pipe which discharged into the open air, was taken as the head.

The discharge was very accurately measured in a large tank.

Three independent experiments were actually made, but as their results are so nearly alike, their average only is given in Table No. 1.

The co-efficient of influx and loss of head in the $\frac{5}{8}$-inch tap were ascertained by experimenting with a short piece of pipe connected to the tap, previous to making the experiment with the long pipe.

A NEW FORMULA FOR THE FLOW OF WATER IN PIPES HAVING VERY SMOOTH INTERIOR SIDES FROM ½ TO 3½ INCHES IN DIAMETER.

Before proceeding any further, it may be well to repeat, that, for reasons already mentioned, the basis of investigation which was adopted by the writer at the commencement of his researches, is the co-efficient of friction ζ, which is a factor of the equation,

$$hf = \zeta \frac{l}{d} \frac{v^2}{2g}.$$

Consequently, the investigations which will now be described, as well as

*Transactions Am. Soc. C. E., Vol. XIII, 1884. Weston on the Flow of Water through Rubber Hose.

those which will be mentioned hereafter, have been entirely devoted to the determination of a formula for the co-efficient of friction ζ.

It was found, by consulting Table No. 1, that there were two hundred and thirty-eight experiments that had been made with twenty-seven different pipes having very smooth interior sides; one hundred and ninety-one of these experiments were made with pipes of lead, glass, zinc, tin, and sheet-iron coated with bitumen from 0.41 inch to 3.25 inches in diameter; eighteen that had been made with pipes of sheet-iron coated with bitumen 7.72 and 11.22 inches in diameter; thirteen that had been made with new gas pipe from 0.63 to 1.05 inches in diameter; and sixteen that had been made with a pipe 1.25 inches in diameter, the nature of which the writer has not been able to ascertain, although it evidently had smooth interior sides.

The last twenty-nine experiments mentioned, that were made with iron pipe, etc., were not given the same weight in the investigations as those that had been made with pipes which were known to have very smooth interior sides, but were more especially used for the purpose of substantiating the laws and results obtained with the latter.

Then, as pipes of 7.72 and 11.22 inches in diameter having very smooth interior sides are rarely used, and as the laws that seemed to apply to the results of the experiments that were made with the pipes of these diameters were slightly at variance with the laws that applied to the others, it was decided to consider only the experiments that were made with the pipes of the smaller sizes, in constructing the new formula, and to limit its application so as to include only pipes from 0.40 to 3.50 inches in diameter. (An independent formula was also determined by the writer for the two pipes, 7.72 and 11.22 inches in diameter, which will be mentioned hereafter.)

The diagrams which refer to this formula are numbered from 1 to 14, the first six being devoted to illustrating the method by which the formula was constructed, and the last eight to its comparison after completion.

For convenience of investigation, when it could be done without affecting the results sought after before reaching the fourth decimal place, the larger part of the experimental results that were obtained with pipes having diameters which were very nearly alike, were classified in each instance, under a common nominal diameter, as will be seen upon the diagrams.

The nominal diameters of the pipes that were used in making the experiments, the results of which were employed in constructing the formula, etc., the numbers that the experiments are given in Table No. 1, and the numbers of the diagrams upon which the results are platted, are as follows:

0.5-inch: Tabular numbers 1 to 14, 28 to 29, 34 to 46. Diagrams Nos. 1 and 7.

0.75-inch: Tabular numbers 47 to 50. Diagram No. 8.

1.0-inch: Tabular numbers 51 to 60, 78 to 123. Diagrams Nos. 2 and 9.

1.25-inch: Tabular numbers 124 to 139 and 145. Diagrams Nos. 3 and 10.

1.50-inch: Tabular numbers 146 to 160, 175 to 211, 224 to 230. Diagrams Nos. 3 and 11.

2.13-inch: Tabular numbers 231 to 249. Diagrams Nos. 4 and 12.

2.50-inch: Tabular numbers 250 to 269. Diagrams Nos. 4 and 13.

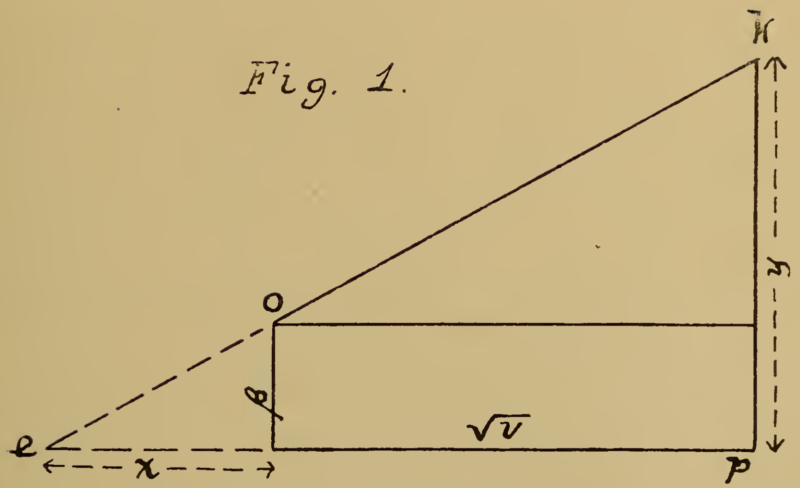
3.25-inch: Tabular numbers 296 to 307. Diagrams Nos. 5 and 14.

The results of any of the experiments that were made with velocities of less than 1 foot per second were not used in the actual construction of the formula, however, owing to the influence that minute errors of observation have upon results obtained with low velocities. For example, an error of 0.01 foot in observing the loss of head, in making an experiment with a pipe 1 inch in diameter and 100 feet long, when the velocity of flow was 0.25 feet per second, would affect the value of the co-efficient of friction to a greater extent than would an error of 1.5 feet in observing the loss of head, in making an experiment with the same pipe when the velocity of flow was 10 feet per second. The results shown on the diagrams from No. 1 to No. 5, that were obtained with velocities less than 1 foot per second, were platted after the formula was completed for the purpose of verification and comparison.

The first steps that were taken in the construction of the formula were to determine the conditions upon which the co-efficient of friction ζ was dependent, and after considerable preliminary platting, computation and study, it was found to be dependent, in this instance, upon both the diameter and the square root of the velocity.

The process by which the formula was finally developed, after the laws that applied to it had been ascertained by a method somewhat similar, is illustrated by Figs. 1 and 2, and upon the diagrams from No. 1 to No. 6.

Fig. 1.



An independent formula $\zeta = \alpha + \dfrac{\beta}{\sqrt{v}}$ was first determined for each diameter of pipe that the experimental data embraced.

The numerical values of α and β were deduced graphically in the following manner by the aid of the expression $\zeta\ \sqrt{v}$, which is the equation of the straight line $o\ k$, Fig. 1 (and the average lines shown upon the diagrams from No. 1 to No. 5), whose abscissæ are the values of $\sqrt{v}$, and whose ordinates are the values of $\zeta\ \sqrt{v}$.

A line was drawn upon the lower part of a large sheet of paper, with a sharp pointed lead pencil, as an axis of abscissæ, and another line drawn in the same way at right angles to it, at its left extremity, as an axis of ordinates.

Then at a scale of 1 inch to 4 feet the square roots of the velocities $(\sqrt{v})$ that were used in computing the experimental co-efficients of friction (ζ) were platted as abscissæ, and the experimental co-efficients of friction multiplied by the square roots of their respective velocities $(\zeta\ \sqrt{v})$ platted as ordinates, at such a scale that four decimal places could be easily read.

At the upper extremity of each ordinate a small circle or symbol was drawn in ink, as shown on the diagrams from No. 1 to No. 5, in order to render it conspicuous, and to designate the results obtained from the experiments of each authority.

If all of the minute errors could have been detected, that were, presumably, made in conducting the experiments from which the platted results were obtained, and if the conditions relative to the diameters and the interior sides of the different pipes that were used had been precisely the same, and if the laws upon which the formulas were based were absolutely correct, all of the circles or symbols upon each diagram from No. 1 to No. 5, would be in a true inclined line. As it is not possible to realize all of these theoretical conditions in actual practice, however, it was always necessary to draw an average line through the circles or symbols, as shown upon the diagrams from No. 1 to No. 5. This line was located by the aid of a piece of fine thread, and its position was generally such a very difficult matter to determine, that it was often not satisfactorily accomplished until after a number of trials.

When the average line was drawn, it was extended through the axis of ordinates to a prolongation of the axis of abscissæ, and the distance x, Fig. 1, scaled on this prolongation, from the point where the average line intersects it, to the axis of ordinates. A line was also drawn with a fine pointed lead pencil as an ordinate, from a point corresponding to the square root of some high velocity, at the extreme right of the axis of abscissæ, and the distance y, Fig. 1, between the axis of abscissæ and the average line, scaled upon it. Then the value of β was ascertained by scaling the distance on the axis of ordinates between the axis of abscissæ and the average line, Fig. 1, and the value of α, which is the tangent of the angle $k\,e\,p$, determined thus,

$$\alpha = \frac{y-\beta}{\sqrt{v}} = \frac{\beta}{x}$$

and the expression for the co-efficient of friction ζ, becomes for each diameter of pipe,

$$\zeta = \frac{y-\beta}{\sqrt{v}} + \frac{\beta}{\sqrt{v}} = \frac{\beta}{x} + \frac{\beta}{\sqrt{v}} = \alpha + \frac{\beta}{\sqrt{v}}.$$

For example, let

$$\beta = 0.0283,\ y = 0.0622,\ \sqrt{v} = 3,\ x = 1.347,\ \text{and}\ \zeta = 0.0113 + \frac{0.0283}{\sqrt{v}}.$$

When a formula for each diameter had been determined in the manner just described, it was found by taking the values of α and β, and platting them in a similar manner, and with the same degree of care, the diameters as abscissæ, and the corresponding values of α and β as ordinates, the extreme upper points of which were enclosed in circles, and drawing an average line from the axis of ordinates through the circles representing each as shown upon Diagram No. 6, that α was constant for all diameters and velocities, and that β decreased slightly as the diameters increased. The value of α, being constant, was ascertained by simply scaling the distance, which was found to be 0.0126, between the axis of abscissæ and the average line, they being parallel in this instance. The value of the diametric co-efficient, which we will call z, upon which β is dependent, was determined by measuring the distance b, Fig. 2, on the axis of ordinates between the axis of abscissæ

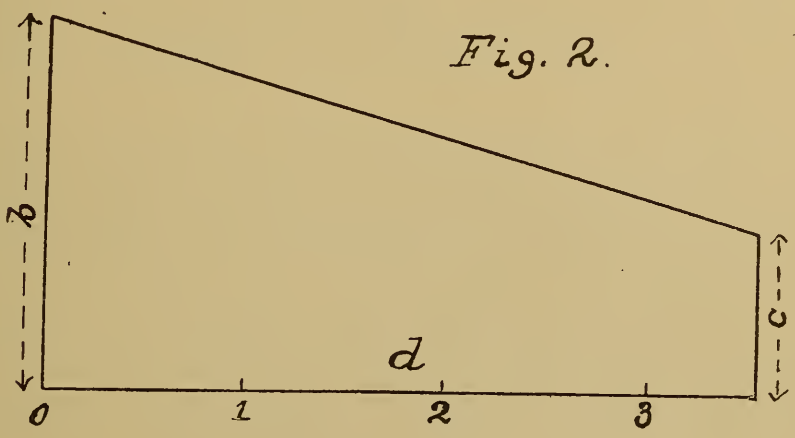


Fig. 2.

and the average line, and by drawing a line as an ordinate from the abscissa representing the greatest diameter, and scaling upon it the distance c, Fig. 2, between the axis of abscissæ and the average line, and using their values in this form,

$$z = \frac{b - c}{d} = \frac{0.0315 - 0.014}{3.5} = 0.005$$

and the expression for β becomes

$$\beta = b - d\,z,$$

and the formula for the co-efficient of friction ζ, for all diameters from 0.40 to 3.5 inches, and for all velocities

$$\zeta = 0.0126 + \frac{0.0315 - d\; 0.005}{\sqrt{v}}.$$

On account of the numerical values of the diameters having been used in inches in determining this formula, it can only be used in its present form when the diameter, d, is expressed in inches. (This is the only instance, with the exception of the cases when the same formula is shown upon the diagrams from No. 6 to No. 14, that any formula contained in this paper is given in measures other than feet.)

The formula therefore becomes, for measures in English feet,

$$\zeta = 0.0126 + \frac{0.0315 - d\,0.06}{\sqrt{v}}$$

$$\text{and } h_f = \left(0.0126 + \frac{0.0315 - d\,0.06}{\sqrt{v}}\right)\frac{l}{d}\frac{v^2}{2g}.$$

Diagrams from No. 7 to No. 14 show a comparison of this formula with the original experimental results from which it was derived, as well as with a number of other experimental results that were not used in constructing the formula.

Co-efficients of friction (ζ), that have been worked out from several well-known formulas for the flow of water through pipes, by different authorities are also platted for comparison on the diagrams from No. 7 to No. 15.

The equations for computing these last mentioned co-efficients of friction, as arranged by the writer, are as follows:

From the formula of Prony,

$$\zeta = 0.02733 + \frac{0.004464}{v}.$$

From the formula of Eytelwein,

$$\zeta = 0.0220 + \frac{0.005754}{v}.$$

From the formula of Darcy,

$$\zeta = 0.01989 + \frac{0.001666}{d}.$$

From the formula of Weisbach,

$$\zeta = 0.01439 + \frac{0.01704}{\sqrt{v}}.$$

Equations for the flow of water in pipes, in which the co-efficient of friction ζ, worked out by this formula, is used, will not admit of a direct solution, but the co-efficients should be first determined for different values of the velocity and tabulated, after which the true value of the velocity can be determined by finding an approximate value, and thence taking out the corresponding co-efficient from the table, which does not vary to any considerable extent for small changes of velocity.

The writer would have been much better satisfied had he been able to construct a formula that could be more easily solved than the one he has presented, but as his principal object was accuracy and not simplicity, he could not do this and conform satisfactorily to the experimental results.

The new formula will not be further discussed, as its merits or demerits are plainly set forth upon the diagrams upon which it is platted.

The following table gives the co-efficients of friction (ζ), for pipes having very smooth interior sides from 0.5 to 3.5 inches in diameter, and for velocities from 0.10 to 50 feet per second, calculated by the formula.

$$\zeta = 0.0126 + \frac{0.0315 - d\, 0.06}{\sqrt{v}}.$$

CO-EFFICIENT OF FRICTION ζ.

Velocity in feet per second.	DIAMETER OF PIPES IN FEET AND INCHES.									
	0.0417 ½	0.0521 ⅝	0.0625 ¾	0.0833 1	0.1042 1¼	0.1250 1½	0.1667 2	0.2083 2½	0.2500 3	0.2917 3½
0.10	0.1043	0.1023	0.1004	0.0964	0.0924	0.0885	0.0806	0.0727	0.0648	0.0569
0.50	0.0536	0.0527	0.0518	0.0501	0.0483	0.0465	0.0430	0.0395	0.0359	0.0324
1.00	0.0416	0.0410	0.0404	0.0391	0.0379	0.0366	0·0341	0.0316	0.0291	0.0266
1·20	0.0391	0.0385	0.0381	0.0367	0.0356	0.0343	0.0319	0.0296	0.0274	0.0250
1·40	0.0371	0.0365	0.0363	0.0347	0.0338	0.0324	0.0304	0.0283	0.0262	0.0239
1.60	0.0356	0.0348	0.0350	0.0332	0.0323	0.0312	0.0292	0.0272	0.0252	0.0232
1.80	0.0342	0.0336	0 0340	0.0321	0.0313	0.0302	0.0284	0.0265	0.0246	0.0226
2.00	0.0331	0.0327	0.0322	0 0313	0.0305	0.0296	0.0278	0.0260	0.0243	0.0225
2.50	0.0310	0.0306	0.0300	0.0294	0.0286	0.0278	0.0262	0.0246	0.0230	0.0214
3 00	0.0293	0.0290	0.0286	0.0279	0.0272	0.0265	0.0250	0.0236	0.0221	0.0207
3.50	0.0281	0.0278	0.0274	0.0268	0.0260	0.0254	0.0242	0.0228	0.0214	0.0201
4.00	0.0271	0.0268	0.0265	0.0259	0.0252	0.0246	0.0234	0.0221	0.0209	0.0196
4.50	0.0263	0.0260	0.0256	0.0251	0.0244	0.0238	0.0227	0.0216	0.0204	0.0192
5.00	0.0256	0.0253	0.0250	0.0244	0.0238	0.0233	0.0222	0.0211	0.0200	0.0189
5.50	0.0250	0.0246	0.0244	0.0238	0.0234	0.0228	0.0218	0.0207	0.0196	0.0186
6.00	0.0244	0.0242	0.0239	0 0234	0.0229	0.0224	0.0214	0.0204	0.0193	0.0183
6.50	0.0240	0.0236	0.0235	0.0230	0.0225	0.0220	0.0210	0.0200	0.0190	0.0181
7.00	0.0236	0.0233	0.0230	0.0226	0.0222	0.0217	0.0207	0.0198	0.0188	0.0179
7.50	0.0232	0.0229	0.0227	0.0222	0.0218	0.0214	0.0204	0.0195	0.0186	0.0177
8.00	0.0229	0.0226	0.0224	0.0220	0.0215	0.0211	0.0202	0.0193	0.0184	0.0175
8.50	0.0225	0.0223	0.0221	0.0216	0.0212	0.0208	0.0200	0.0191	0.0182	0.0174
9.00	0.0222	0.0220	0.0218	0.0214	0.0210	0.0206	0.0198	0.0189	0.0181	0.0173
9.50	0.0220	0.0218	0.0216	0.0212	0.0208	0.0204	0.0196	0.0188	0.0180	0.0172
10	0.0218	0.0216	0.0214	0.0210	0.0206	0.0202	0.0194	0.0186	0.0178	0.0170
11	0.0214	0 0212	0.0210	0.0206	0.0202	0.0198	0.0191	0.0184	0.0176	0.0168
12	0.0210	0 0208	0.0206	0.0203	0.0199	0.0195	0.0188	0.0181	0.0174	0.0166
13	0.0206	0.0205	0.0203	0.0199	0.0196	0.0193	0.0186	0.0179	0.0172	0.0165
14	0.0204	0.0202	0.020[illegible]	0.0197	0.0194	0.0190	0.0184	0.0177	0.0170	0.0164
15	0.0201	0.0200	0.0198	0.0195	0.0191	0.0188	0.0182	0.0175	0.0168	0.0162
16	0.0199	0.0197	0.0195	0.0192	0.0189	0.0186	0.0180	0.0174	0.0167	0.0161
17	0.0196	0.0195	0.0194	0.0190	0.0187	0.0184	0.0178	0.0172	0.0166	0.0160
18	0.0194	0.0193	0.0192	0.0188	0.0186	0.0183	0.0177	0.0171	0.0165	0.0159
19	0.0193	0.0191	0.0190	0.0187	0.0184	0.0181	0.0175	0.0169	0.0164	0.0158
20	0.0191	0.0189	0.0188	0.0185	0.0182	0.0180	0.0174	0.0168	0.0163	0.0157
25	0.0184	0.0183	0.0182	0.0179	0.0177	0.0174	0.0169	0.0164	0.0159	0.0154
30	0.0179	0.0178	0.0177	0.0174	0.0172	0.0170	0.0165	0.0161	0.0156	0.0152
35	0.0175	0.0174	0.0173	0.0171	0.0169	0.0167	0.0162	0.0158	0.0154	0.0150
40	0.0172	0.0171	0.0170	0.0168	0.0166	0.0164	0.0160	0.0156	0.0152	0.0148
50	0.0167	0.0166	0.0165	0.0163	0.0162	0.0160	0.0156	0.0153	0.0149	0.0146

The formula for a co-efficient of friction ζ for the two sheet-iron pipes, coated with bitumen, 7.72 and 11.22 inches in diameter, that has previously been mentioned, and which was constructed in a manner similar to the one that has just been described, is constant for both diameters, and is dependent only upon the square root of the velocity ($\sqrt{v}$), viz.:

$$\zeta = 0.0126 + \frac{0.0105}{\sqrt{v}}.$$

This formula is platted for comparison with the experimental results from which it was determined (which are numbered from 338 to 348, and from 370 to 376 in Table No. 1), upon Diagrams Nos. 23 and 24.

FORMULAS FOR THE FLOW OF WATER IN PIPES HAVING INTERIOR SIDES SIMILAR TO NEW CAST-IRON PIPES.

An examination of Table No. 1 led the writer to the conclusion that there were one hundred and eighty-eight experiments which it would be advisable to utilize in constructing a formula for the flow of water in pipes having interior sides similar to new cast-iron pipes, that had been made with twenty-eight different pipes from 0.48 to 90 inches in diameter.

It was very difficult in a number of instances to determine the number of years of service that should limit what would generally be called a new or clean pipe, especially if the experiments under consideration had been made with pipes that were not originally coated with coal tar, or some preparation of a similar nature, unless there was positive evidence of the exact condition of their interior sides at the time that the experiments were made, as it has been demonstrated by chemical analysis that the interior sides of cast-iron pipe oxidize much faster with slightly alkaline and aerated waters flowing through them than they do with other waters differently constituted. In the absence of more reliable data, the writer has generally been governed in cases of this kind by the values of the co-efficients of friction (ζ).

After carefully studying and platting the experimental results in a manner similar to that mentioned in the description of the construction of "A new formula for the flow of water in pipes having very smooth interior sides," the writer made up his mind that he could not construct a formula that would compare as satisfactorily with the experimental results, as two formulas by Henry Darcy, which are to be found on pages 254, 258 and 368 of his work entitled "Recherches expérimentales relatives au mouvement de l'eau dans les tuyaux."

The formulas for obtaining the co-efficients of friction (ζ), which is a factor of the equation $h_f = \zeta \frac{l}{d} \frac{v^2}{2g}$ as worked out from these formulas (Darcy's) by the writer, and reduced to English measures are as follows:

$$\zeta = 0.0198920 + \frac{0.00166573}{d}.$$

$$\zeta = 0.017379 + \frac{0.0015965}{d} + \frac{0.0040723 + \frac{0.000020816}{d^2}}{v}.$$

Darcy recommends the formula from which the first formula for the co-efficient of friction ζ was deduced, for general use, and it is ordinarily known as "Darcy's formula." As may be seen in the first formula, the value of the co-efficient ζ, is entirely dependent upon the diameter of the pipe. Darcy did not fail to recognize, however, that the velocity also entered into the problem, but considered (after analyzing the experimental results from which the formula was derived), that when the velocity is more than 0.33 feet per second, especially if the pipes have been in use a short time, the term relative to it, which would enter into the formula for the co-efficient of friction ζ, could be entirely eliminated, without the reliability of the formula being appreciably affected. The formula, from which the second formula for a co-efficient of friction ζ was deduced, was intended by Darcy to be used for all velocities, and to apply more especially to new pipes. (The same experimental results were used in constructing both of these formulas.)

The experimental co-efficients of friction (ζ) that were worked out from the one hundred and eighty-eight experiments, and co-efficients of friction worked out from Darcy's formulas, and several other well-known formulas, are platted for comparison on diagrams from No. 15 to No. 22, and on No. 25.

The formulas for obtaining the co-efficients of friction from the formulas last mentioned, as arranged by the writer (the first three of which have previously been referred to), are as follows:

From the formula of Prony,

$$\zeta = 0.02733 + \frac{0.004464}{v}$$

From the formula of Eytelwein,

$$\zeta = 0.0220 + \frac{0.005754}{v}.$$

From the formula of Weisbach,

$$\zeta = 0.01439 + \frac{0.01704}{\sqrt{v}}$$

From the formula of Kutter,

$$\zeta = \frac{2g}{\left(\dfrac{20.83 + \dfrac{0.9056}{n} + \dfrac{0.0014038}{J}}{1 + \left(41.66 + \dfrac{0.0028075}{J}\right)\dfrac{2\,n}{\sqrt{d}}}\right)^2}$$

For facility of investigation, when it could be done without affecting the results sought after before reaching the fourth decimal place, the greater part of the experimental results that were obtained with pipes having diameters very nearly alike were classified under a common nominal diameter, as will be seen upon the diagrams.

The nominal diameters of the pipes that were used in making the experiments, the results of which are platted on the diagrams, the numbers that the experiments are given in Table No. 1, and the numbers of the diagrams upon which the results are platted, are as follows:

0.48-inch, Tabular numbers 15 to 27. Diagram No. 25.
1.047-inch, Tabular numbers 61 to 77. Diagram No. 25.
1.55-inch, Tabular numbers 212 to 223. Diagram No. 25.
3.22-inch, Tabular numbers 283 to 295. Diagram No. 15.
5.39-inch, Tabular numbers 309 to 328. Diagram No. 16.
7.40-inch, Tabular numbers 329 to 337. Diagram No. 17.
11.69-inch, Tabular numbers 365 to 369, 377 to 396, 405 to 408. Diagram No. 18.
15.00-inch, Tabular numbers 409 to 422. Diagram No. 19.
19.69-inch, Tabular numbers 423 to 456. Diagram No. 20.
30.00-inch, Tabular numbers 467 to 485. Diagram No. 21.
48.00-inch, Tabular numbers 513 to 517. Diagram No. 21.
90.00-inch, Tabular numbers 518 to 520. Diagram No. 22.

There are platted upon diagrams Nos. 18, 19, 20 and 21, a number of experimental results that should probably not be given as much weight in making a comparison, as the other experimental results which are platted upon the same diagrams, viz.: Upon diagram No. 18, the results that were obtained from ten of the experiments made by Simpson, on account of old pipe having been used; upon diagrams Nos. 19 and 20, the results obtained from three of the experiments made by the Edinburgh Water Company, and five of the experiments made by Simpson, for the same reason; and upon diagram No. 21, the results obtained from eighteen experiments made by Darrach with pipes 30 inches in diameter, on account of allowances that had to be made, in computation, for the resistance of check-valves which were located upon the pipes.

As will be seen by an examination of the diagrams, the co-efficients of friction (ζ) worked out from Darcy's formulas agree very well in the majority of cases with the experimental results, and much better than any of the other formulas that are platted. Therefore, in the opinion of the writer, the formulas of Darcy are safe and reliable to use in general practice. The former, which is represented by a straight line on the diagrams, for velocities including and exceeding 0.33 feet per second, and the latter, which is represented by a curved line on the diagrams, for velocities of less than 0.33 feet per second. The former is also to be recommended for its simplicity, as well as for its accuracy, and also for the convenience with which it can be used in both the simple and extended forms of equations previously mentioned and recommended by the writer, for the flow of water in pipes.

Of the other co-efficients of friction (ζ), that are platted on the diagrams, which have been worked out from the formulas of different authorities, the writer will only mention in detail those that have been derived from the well-known and very valuable formula of Kutter, which during the last few years has been so much discussed and extensively used for computing the flow of water through masonry conduits. There seems to be a general impression among a number of engineers that this formula can be applied to iron pipes, although Kutter does not give any co-efficients of roughness, in connection with it, which were obtained from experiments that were made with iron pipes. In order to ascertain if this impression was correct, the writer used in computing the results for comparison for this formula, three co-efficients of roughness as given by Kutter, which in his opinion would be the most likely to apply to iron pipes, viz.: 0.010 which is for a surface of plaster in pure cement, 0.011 which is for a surface of plaster in cement with one-third sand, and 0.013 which is for a surface of brickwork or ashlar. The results obtained by using these three co-efficients of roughness are platted on the diagrams from No. 16 to No. 22, and in the writer's opinion neither of them compare as favorably with the experimental results as the formulas of Darcy; also the same co-efficient of roughness seems not to apply to all sizes of pipe, for instance, 0.010 applies best to pipes having nominal diameters of 5.39 and 7.40 inches, 0.011 to pipes having nominal diameters of 11.69, 16 and 19.69 inches, and 0.013 to pipes having diameters of 30 and 48 inches.

The results obtained from the experiments made by Clarke with a brick tunnel 90 inches in diameter are simply platted on Diagram No. 22, in order to show to what an extreme range of diameters the formulas of Darcy can be safely applied.

OLD CAST-IRON PIPES LINED WITH DEPOSIT, AND THE SAME CLEANED.

It was found by examining Table No. 1 that there were fifty-eight experiments that had been made with eight different old cast-iron pipes lined with deposit, from 1.43 to 36 inches in diameter, and twenty-two experiments that had been made with three of these pipes after they had been cleaned.

A preliminary study of the results obtained from the experiments that were made with the old cast-iron pipes lined with deposit convinced the writer that he could not satisfactorily construct a general formula from the data at hand. He was able, however, to construct three individual formulas for co-efficients of friction (ζ). In one instance with two sets of results obtained from experiments made with two different pipes of the same diameter, and in two instances with three sets of results obtained from experiments made with three different pipes of the same diameter, which are as follows:

For 20-inch pipe, $\zeta = 0.0156 + \dfrac{0.116}{v}$.

For 36-inch pipe, $\zeta = \dfrac{0.159}{v} - 0.0143$.

For 36-inch pipe, $\zeta = \dfrac{0.1835}{v} - 0.0225$.

(The same experimental results were used in constructing the two latter formulas.)

All of the co-efficients of friction (ζ) that have been derived from experiments that were made with old pipes lined with deposit (with the exception of ten that were obtained from experiments made with one pipe 30 inches in diameter), are platted upon diagrams Nos. 26 and 28, for comparison with these formulas, and with one worked out by the writer from a formula by Darcy for the flow of water in pipes slightly encrusted with deposit, viz.,

$$\zeta = 0.03978 + \frac{0.03332}{d}.$$

As the writer does not consider these formulas for old pipes lined with deposit of much value, they will not be further discussed.

The results that were obtained from experiments that were made with old cast-iron pipes after having been cleaned, are simply platted on diagram No. 27 for the purpose of comparing them with the formula for a co-efficient of friction ζ, worked out from a formula by Darcy for the flow of water in new cast-iron pipes, which has been previously mentioned, viz.,

$$\zeta = 0.019892 + \frac{0.00166573}{d}.$$

The diameters of the pipes that were used in making the experiments, the results of which have just been mentioned, the numbers that the experiments are given in Table No. 1, and the numbers of the diagrams upon which the results are platted, are as follows:

1.43-inch, Tabular numbers 161 to 174. Diagrams Nos. 26 and 27.
3.15-inch, Tabular numbers 270 to 282. Diagrams Nos. 26 and 27.
9.63-inch, Tabular numbers 349 to 364. Diagrams Nos. 26 and 27.
20.00-inch, Tabular numbers 457 to 466. Diagram No. 26.
30.00-inch, Tabular numbers 486 to 495. (Not platted.)
36.00-inch, Tabular numbers 496 to 512. Diagram No. 28.

THE CO-EFFICIENTS OF RESISTANCE OF THE FLOW OF WATER IN PIPES FOR ENLARGEMENTS, CONTRACTIONS, ELBOWS AND CURVES.

Should the forms of formulas that have been recommended by the writer for the flow of water in pipes, be used and extended to cover conditions other than those relating to straight pipes, one or more of the four co-efficients of resistance, for enlargements, contractions, elbows and curves, will probably be required, in order to do so. Therefore, as the writer has not anything original to present pertaining to these co-efficients, he has taken the following data relating to them, nearly entire, from the valuable work of Professor Weisbach, entitled, "A Manual of the Mechanics of Engineering and of the Construction of Machines," which was translated from the fourth German Edition, by E. B. Coxe, A. M., in 1872.

Sudden Enlargement.

$$\mu = \left(\frac{F}{F_1} - 1\right)^2, \; h_f = \mu \frac{v^2}{2g}.$$

Fig. 3.

The following table is calculated according to the formula.

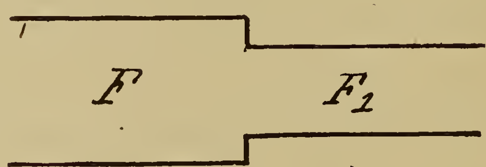
$\frac{F}{F_1}$	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
μ	.01	.04	.09	.16	.25	.36	.49	.64	.81

$\frac{F}{F_1}$	2.0	2.5	3.0	3.5	4.0	5.0	6.0	7.0	8.0
μ	1.00	2.25	4.00	6.25	9.00	16.00	25.00	36.00	49.00

CONTRACTION.

$$\phi = \left(\frac{1 - 0.64}{0.64}\right)^2 = 0.316, \; h_f = \phi \frac{v^2}{2g}.$$

Fig. 4.



but ϕ is increased by the resistance at the entrance into the pipe and by the friction of the water in the interior portion of the tube, to 0.505.

ELBOWS.

$$\tau = 0.9457 \sin \frac{2\rho}{2} + 2.047 \sin \frac{4\rho}{2}, \; h_f = \tau \frac{v^2}{2g}.$$

Fig. 5.

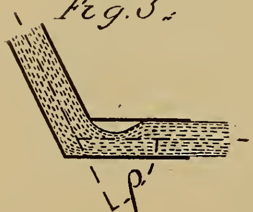


Fig. 6.

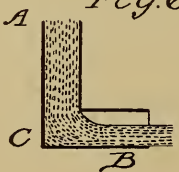


Fig. 7.

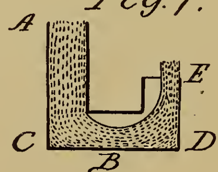
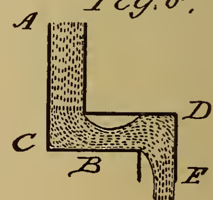


Fig. 8.



The following table contains a series of co-efficients of resistance calculated, according to the formula, for different angles of deviation :

$\frac{\rho}{2}$	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°
τ	.046	.139	.364	.740	.984	1.260	1.556	1.861	2.158	2.431

If to one elbow *ACB*, Fig. 6, another elbow is joined, as is shown in Figs. 7 and 8, a peculiar but at the same time easily explicable phenomenon of efflux is observed. The elbow *BDE*, Fig. 7, which turns the stream to the same side as the elbow *ACB*, Fig. 6, produces no further contraction of the stream, and, therefore, for efflux with full cross section, τ is no greater than for a simple elbow *ACB*. But if the other elbow *BDE*, Fig. 8, turns the stream to the opposite side, the contraction is a double one, and the co-efficient of resistance is consequently twice as great as for a single elbow. If, finally, *BDE* is so joined to *ACB* that *DE* stands at right angles to the plane *ABD*, τ then becomes about one and one-half times as great as for the single elbow *ACB*.

Curved Pipes.

For curved pipes with circular cross-sections:

$$\psi = 0.131 + 1.847 \left(\frac{d}{2r}\right)^{\frac{7}{2}}, \; h_f = \psi \frac{v^2}{2g}.$$

Fig. 9.

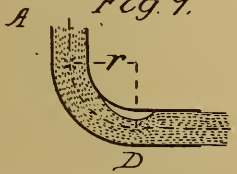


Fig. 10.

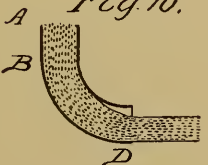


Fig. 11.

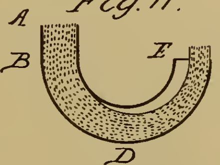
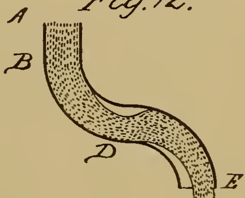


Fig. 12.



The following table is calculated according to the formula:

$\frac{d}{2r}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
ψ	.131	.138	.158	.206	.294	.440	.661	.977	1.408	1.978

If one curve BD, Fig. 10, is terminated by another, which turns the stream further in the same direction, if *eg*, the axis of the pipe forms a semicircle, like BDE, Fig. 11, the contraction is not changed and ψ has the same value as for the pipe in Fig. 10, which forms a quadrant. If, on the contrary, a curve DE, Fig. 12, which turns the stream in the opposite direction, is attached to the first one, an eddy is formed between the two, and a second contraction of the stream takes place by which the resistance (ψ) is nearly doubled.

PLATE I
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WESTON
ON
FLOW OF WATER IN PIPES.

Diagram No. 1.

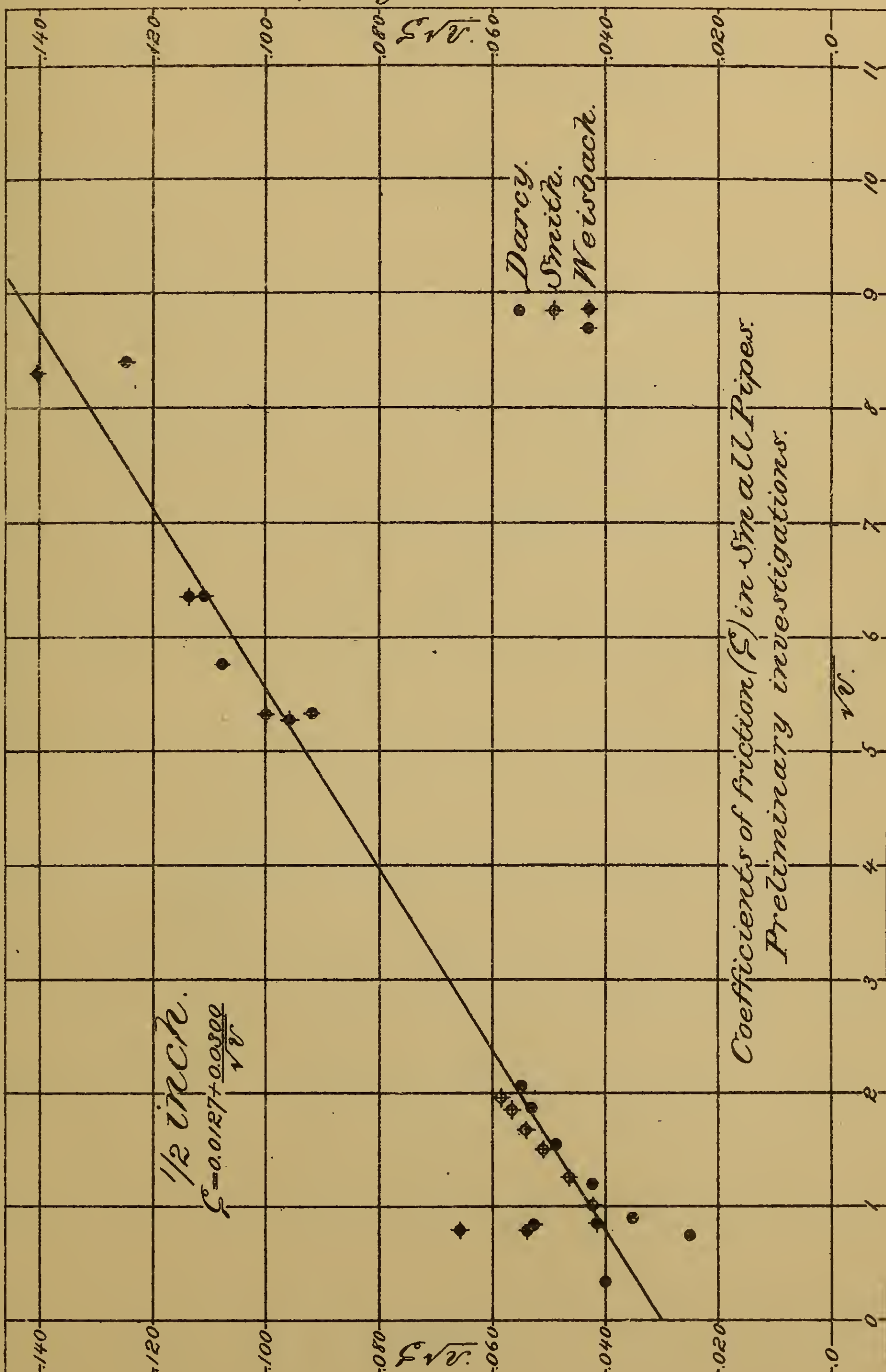

PLATE II
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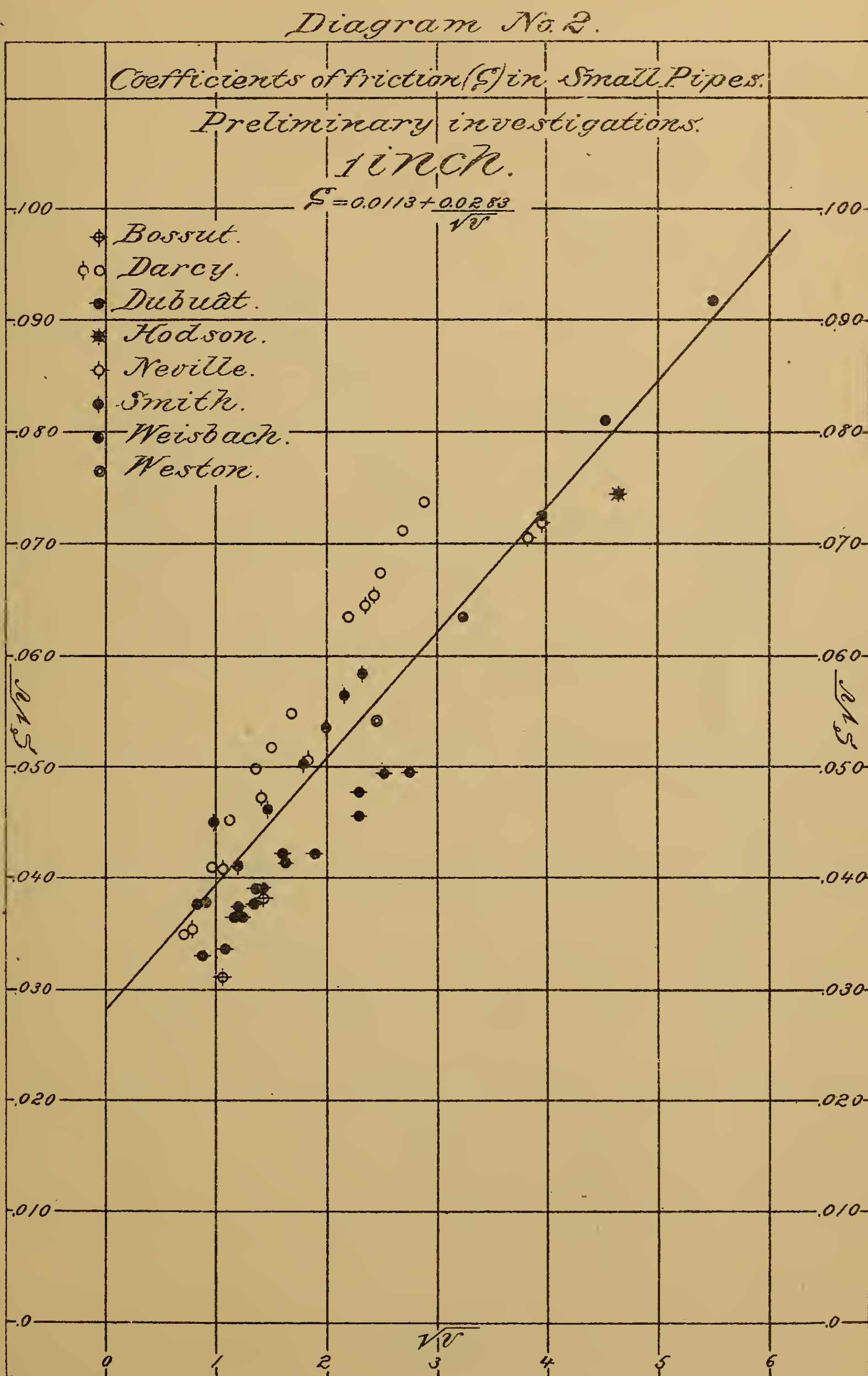

PLATE III.
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Diagram No. 3.

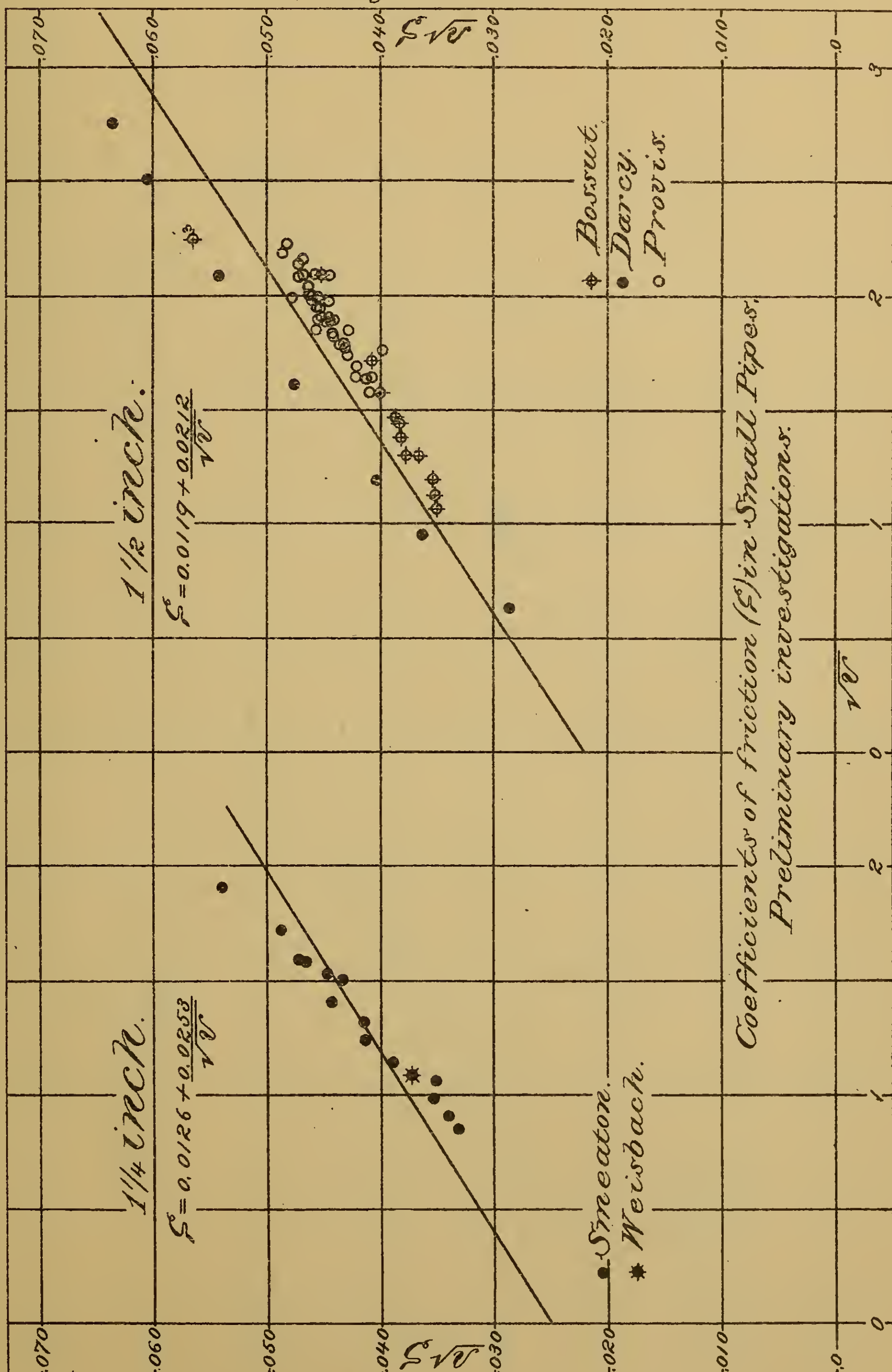

PLATE IV
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Diagram No. 4.

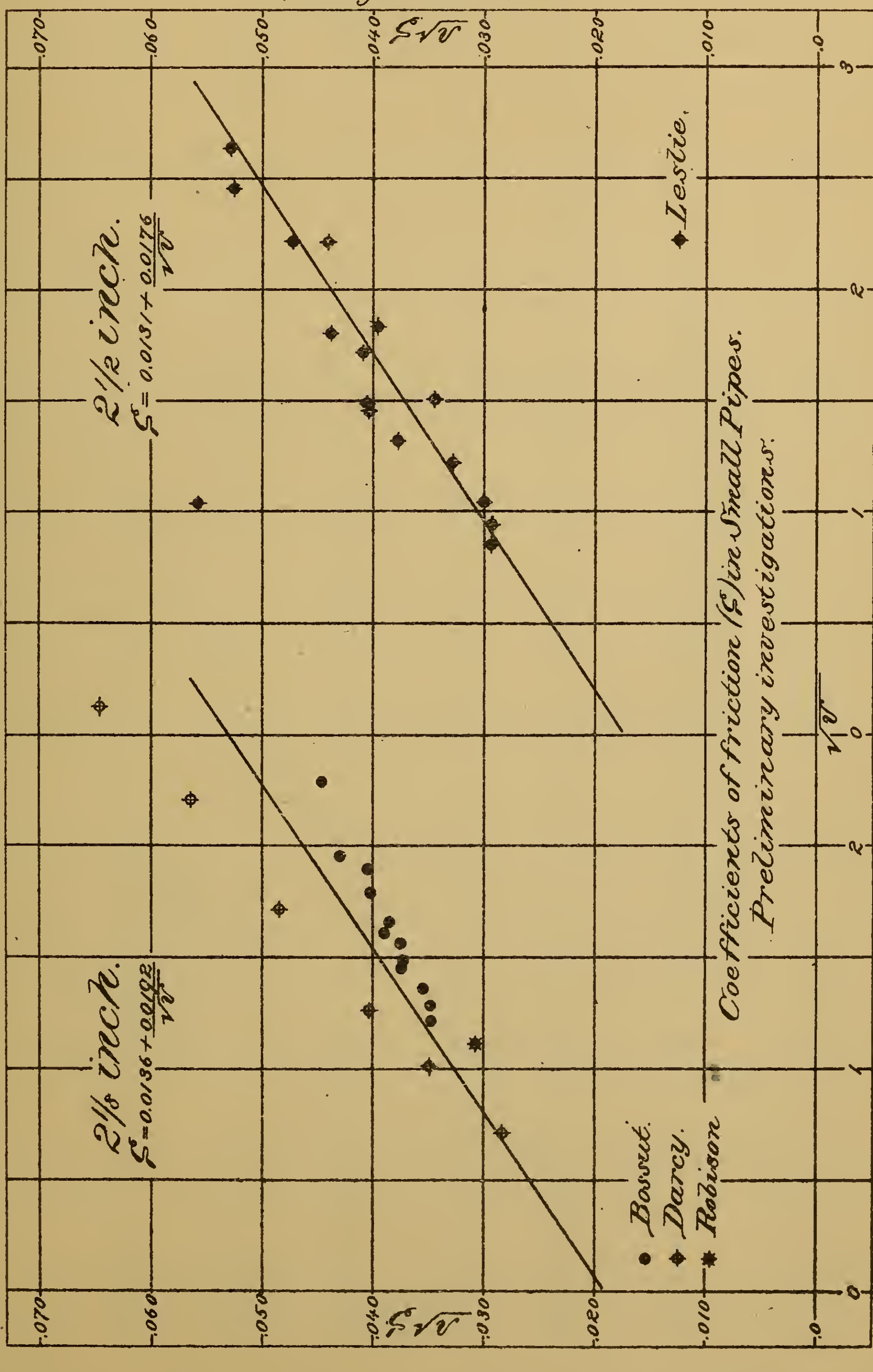

PLATE V.
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Diagram No. 5.

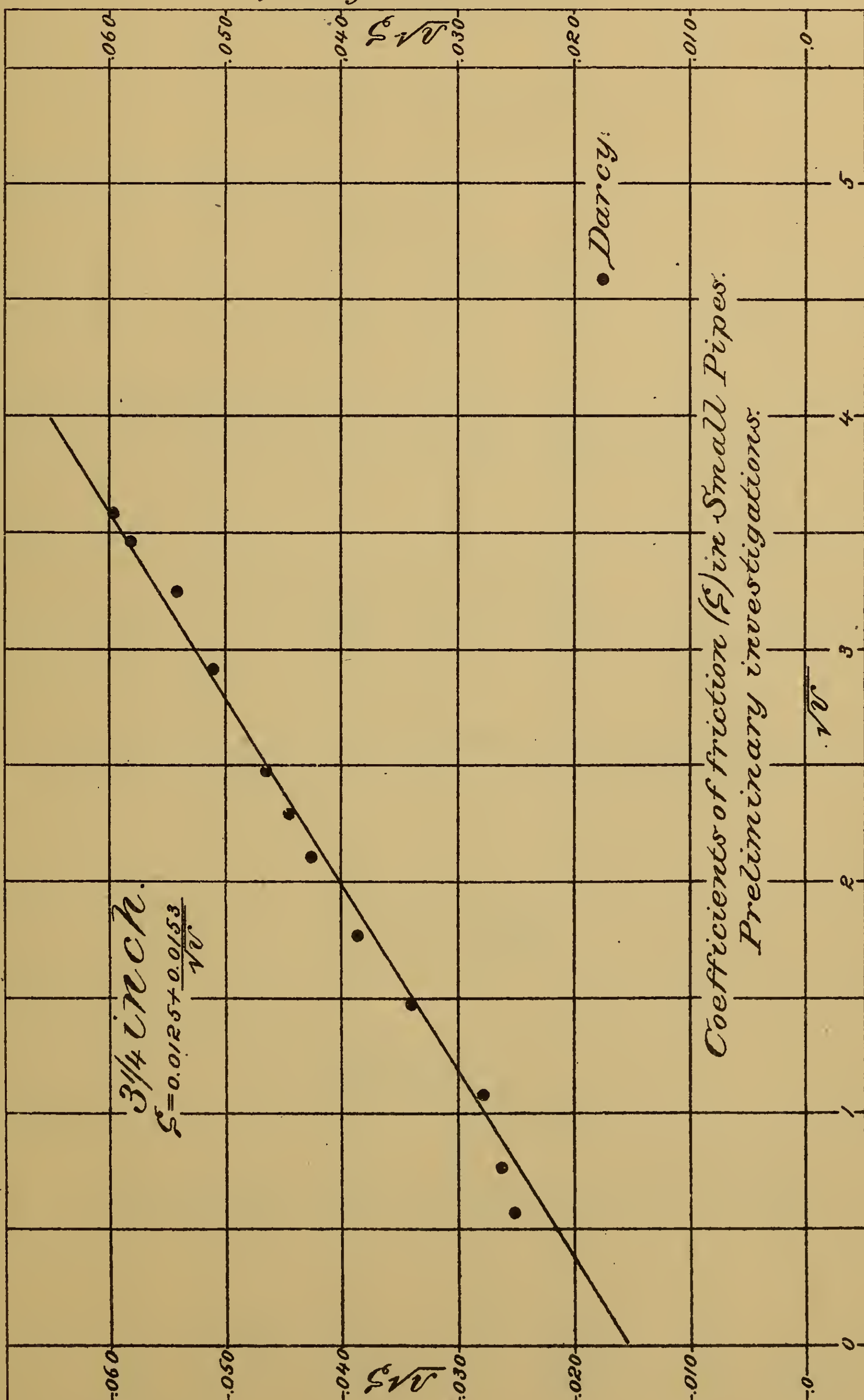

PLATE VI
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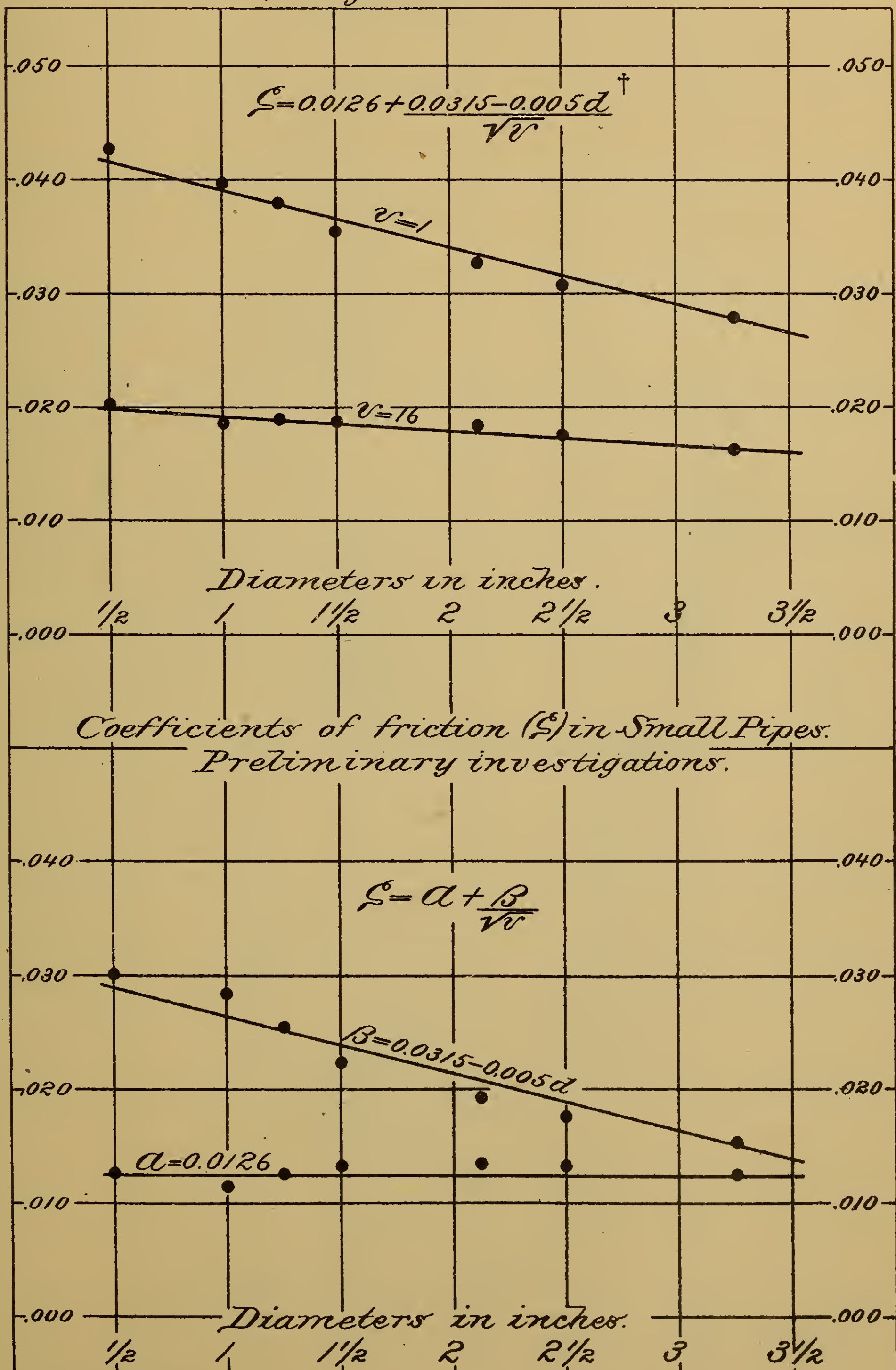
Diagram No. 6.



† d in inches.

PLATE VII
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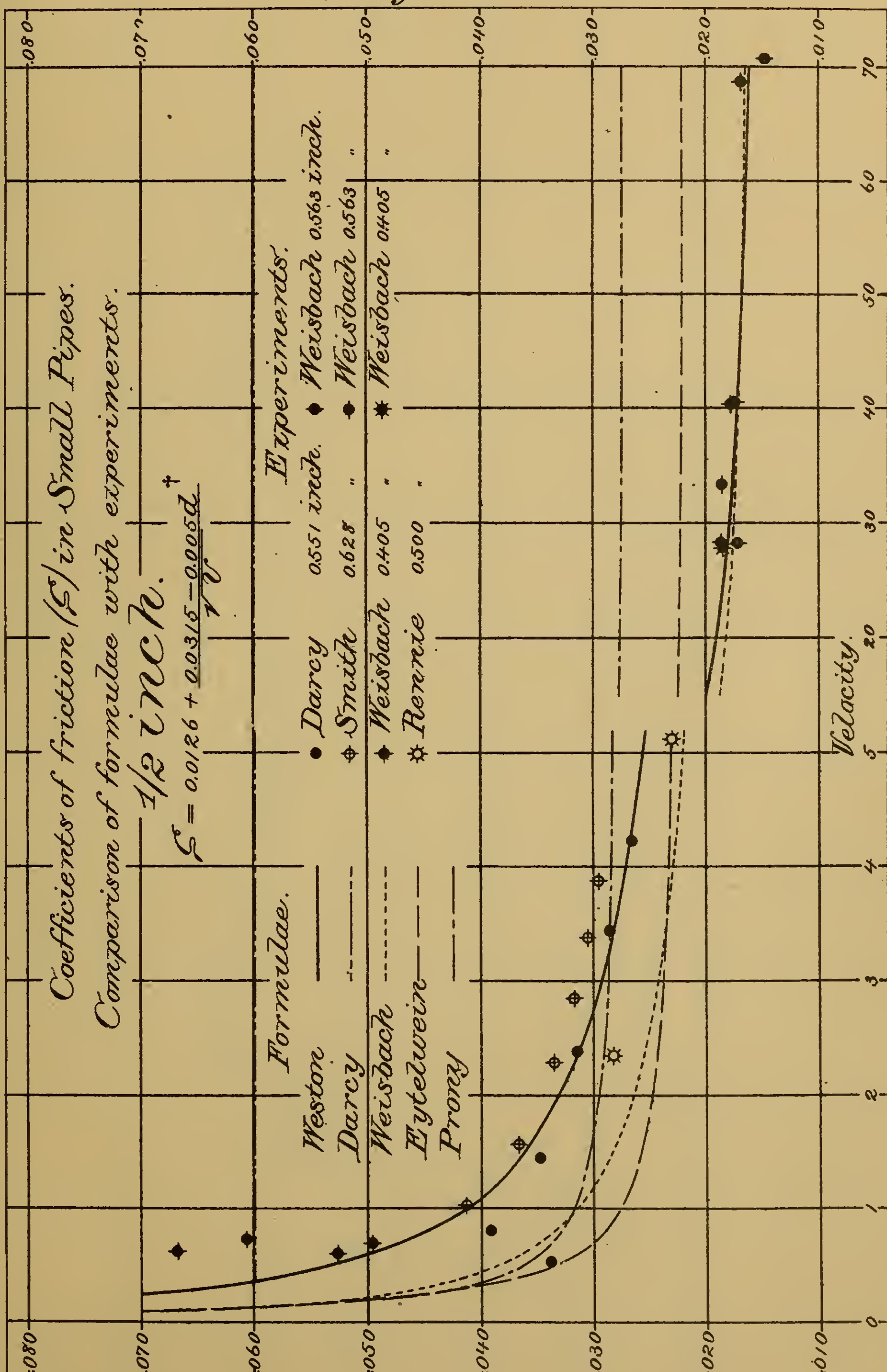
Diagram No. 7.



† d in inches.

PLATE VIII.
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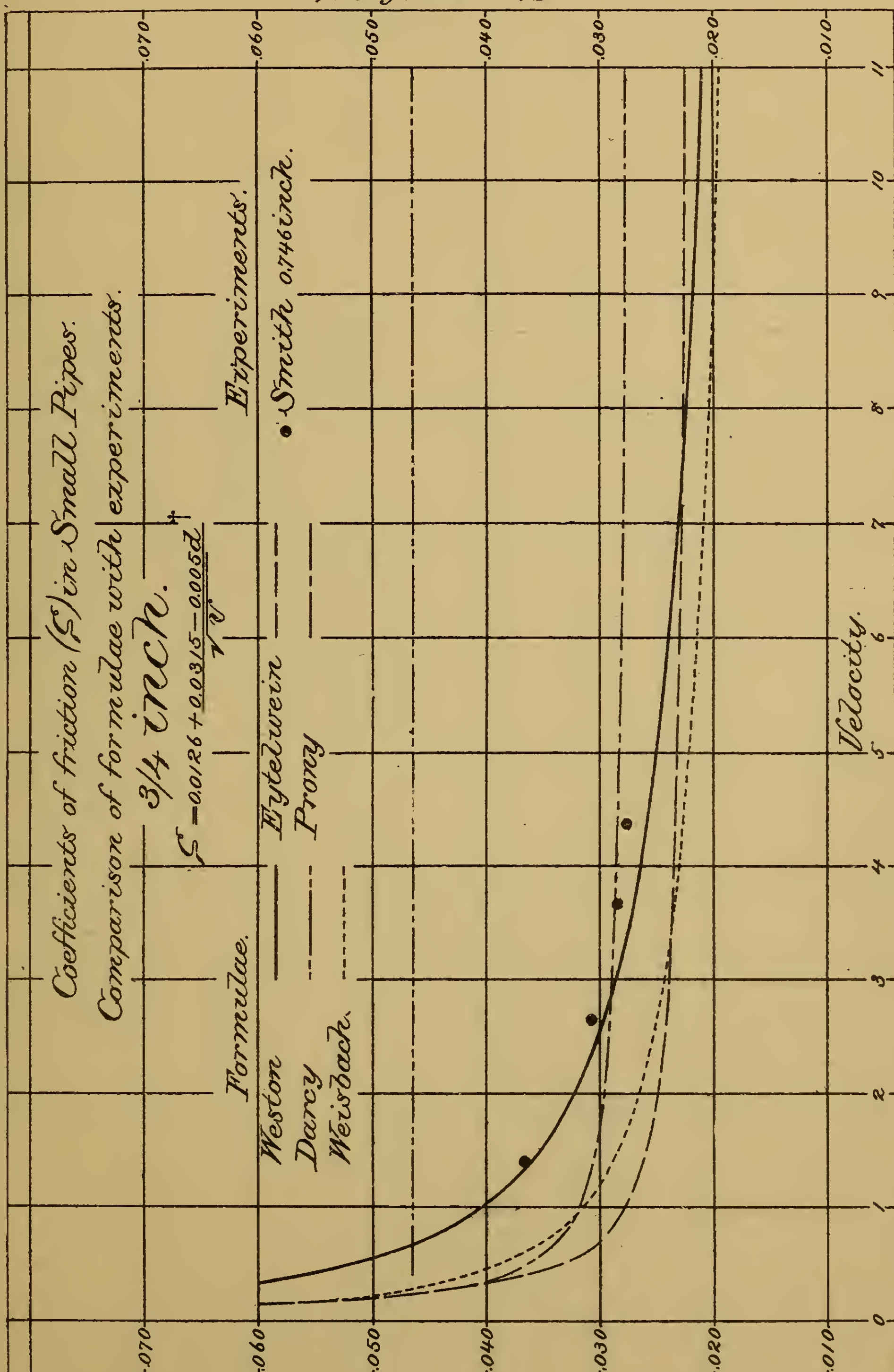
Diagram No. 8.



† d in inches.

PLATE IX.
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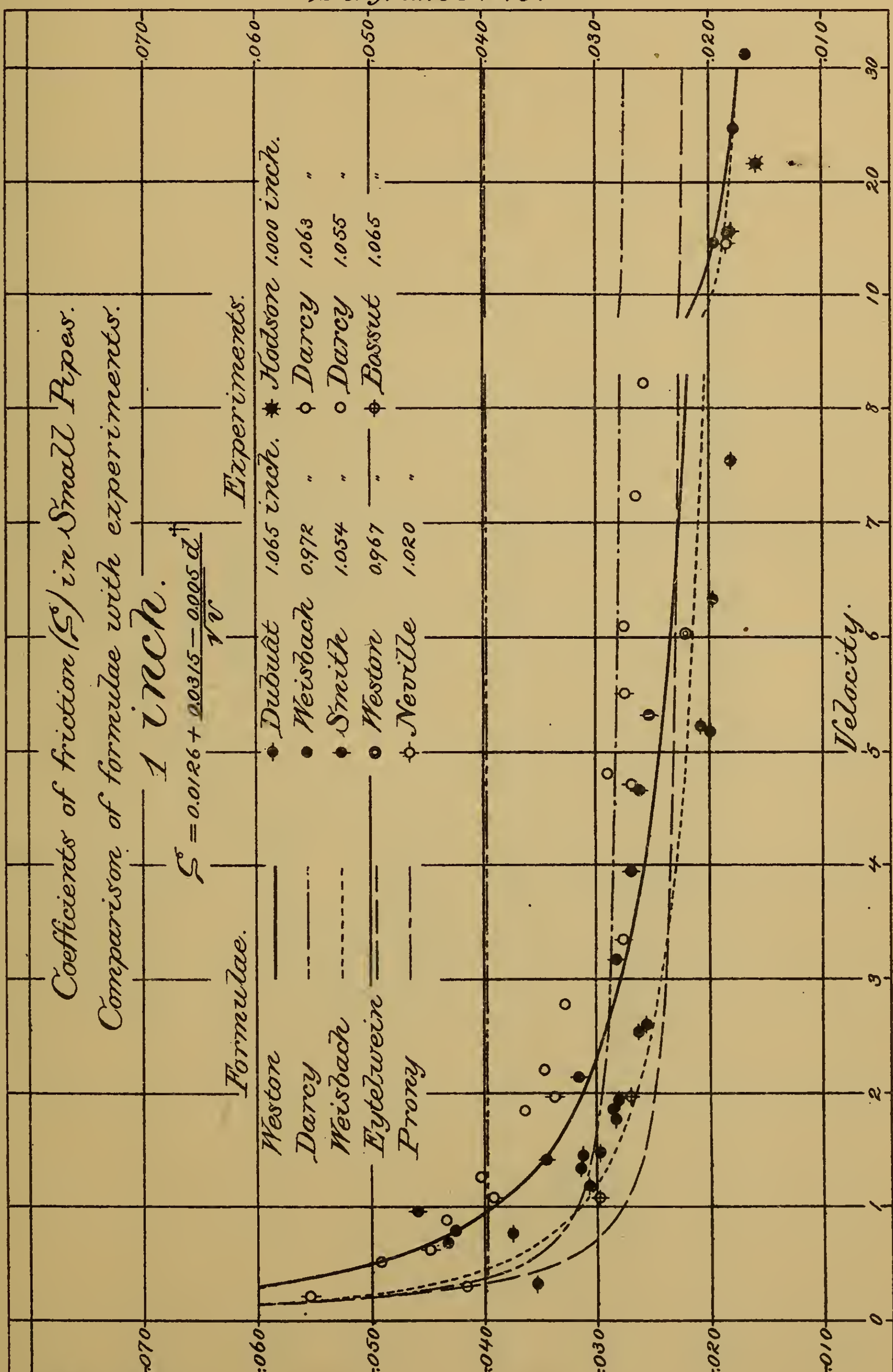
Diagram No. 9.



† d in inches.

PLATE X
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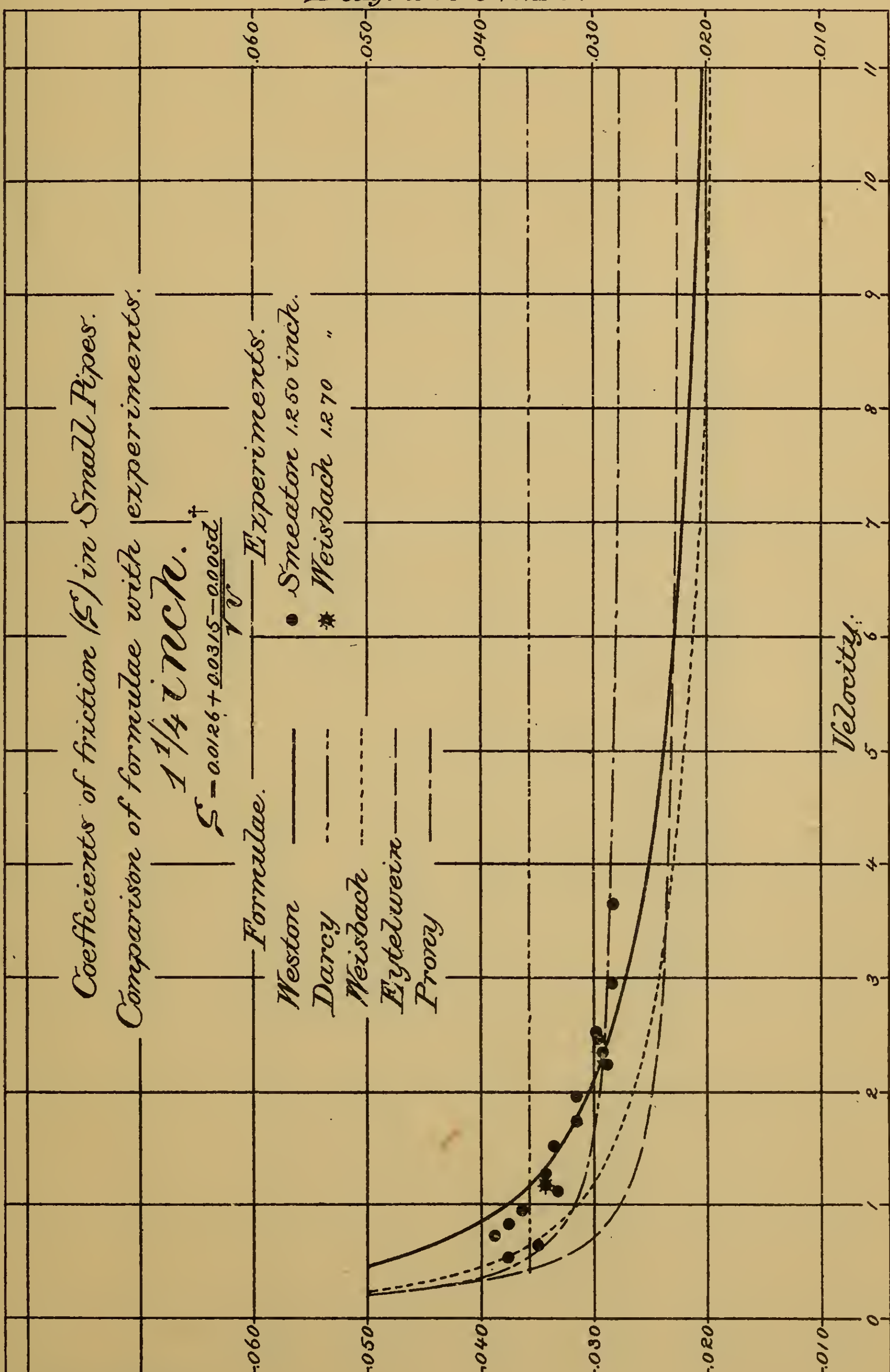
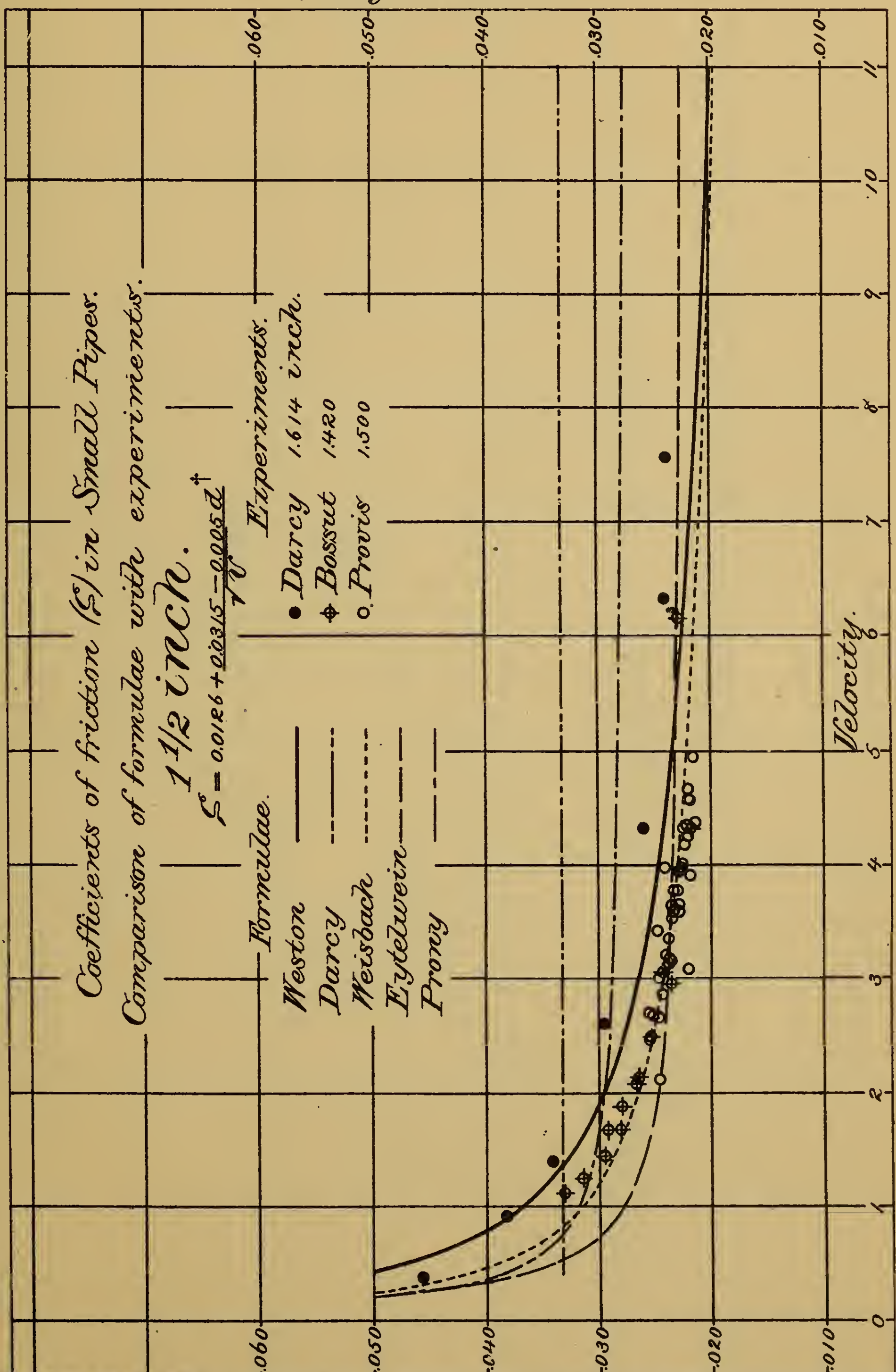
Diagram No. 10.



† *d in inches.*

Diagram No. 11.



† *d in inches.*

PLATE XII
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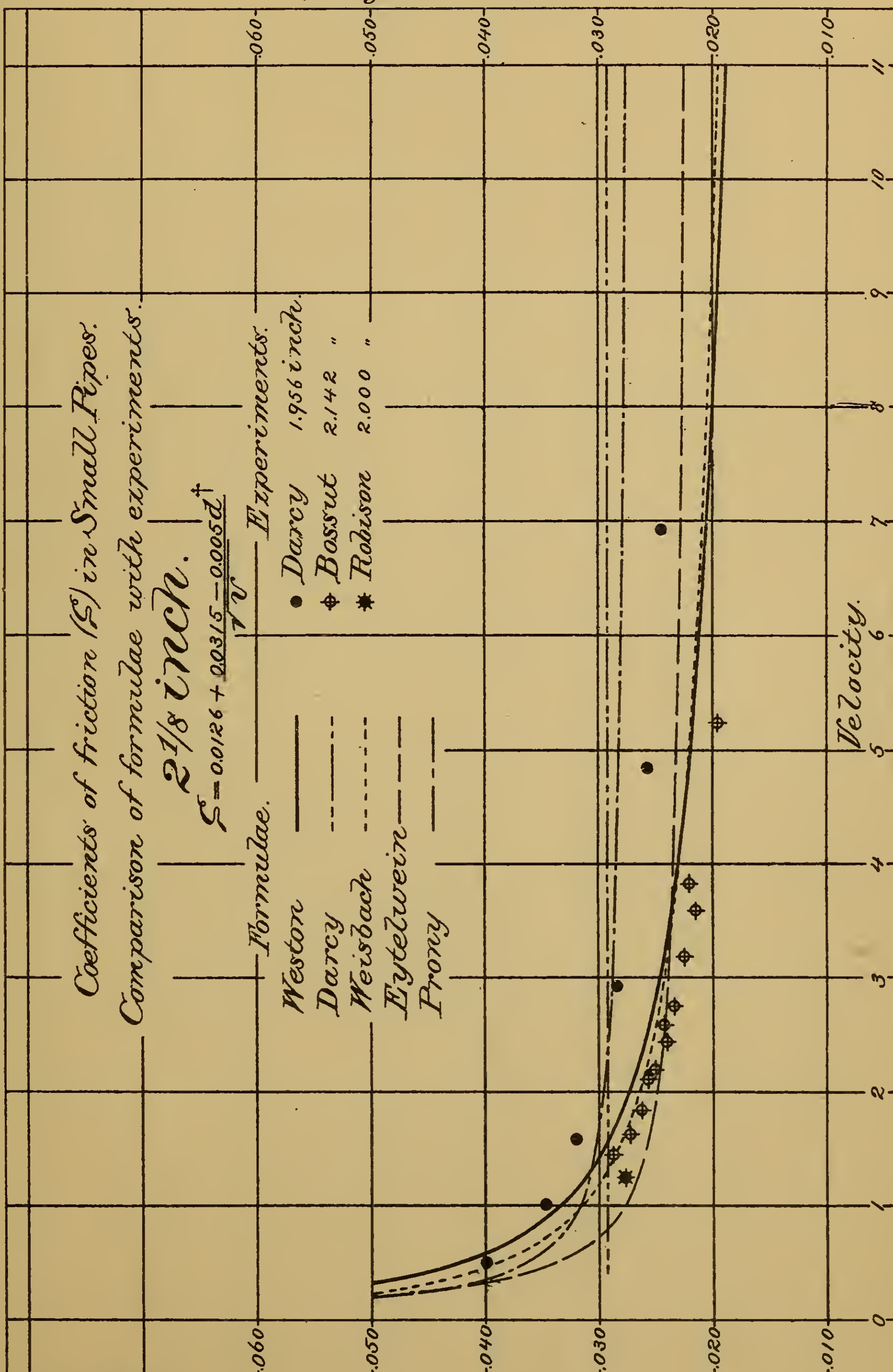
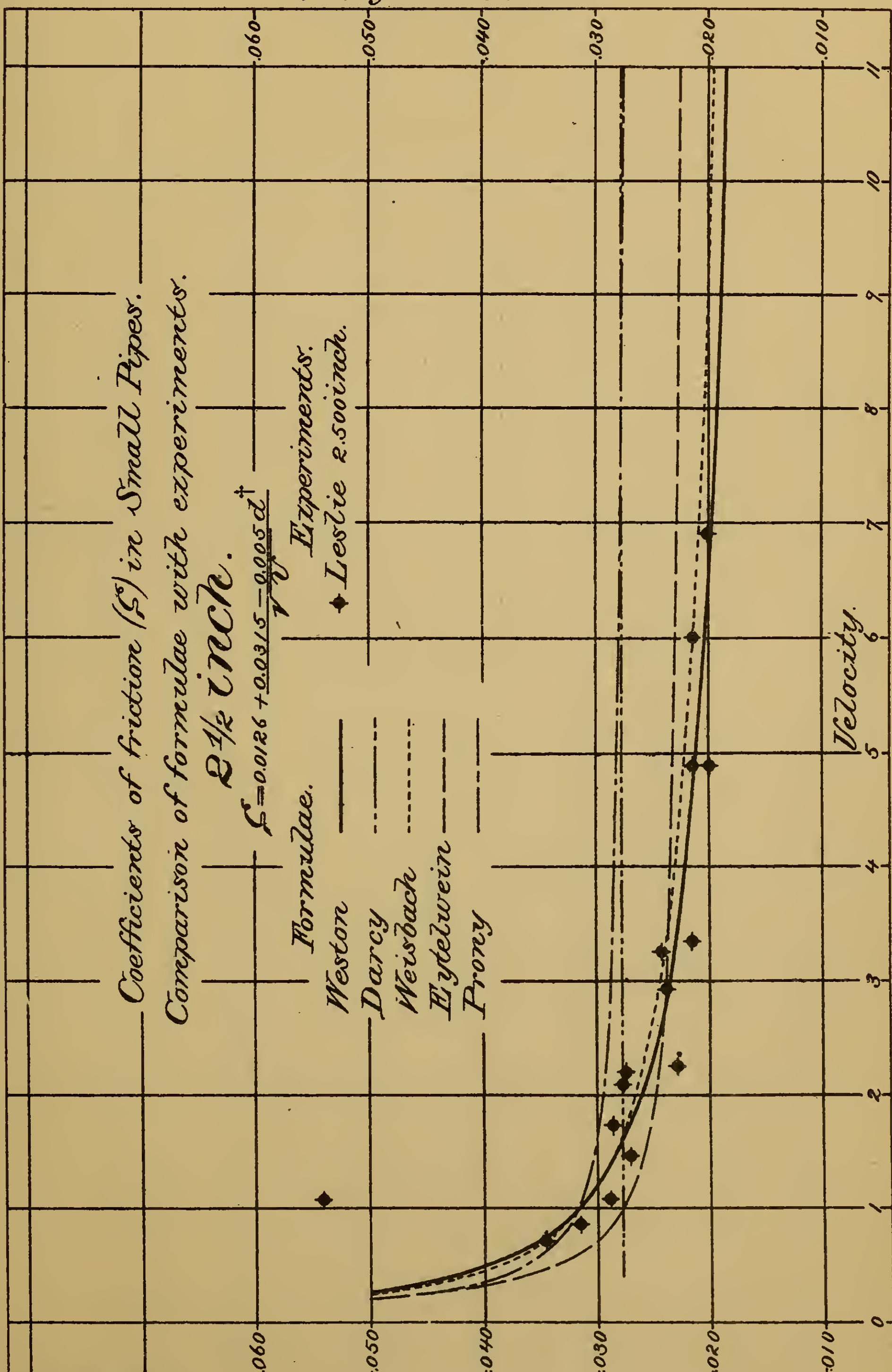
Diagram No. 12.



† *d in inches.*

Diagram No. 13.



† d in inches.

PLATE XIV
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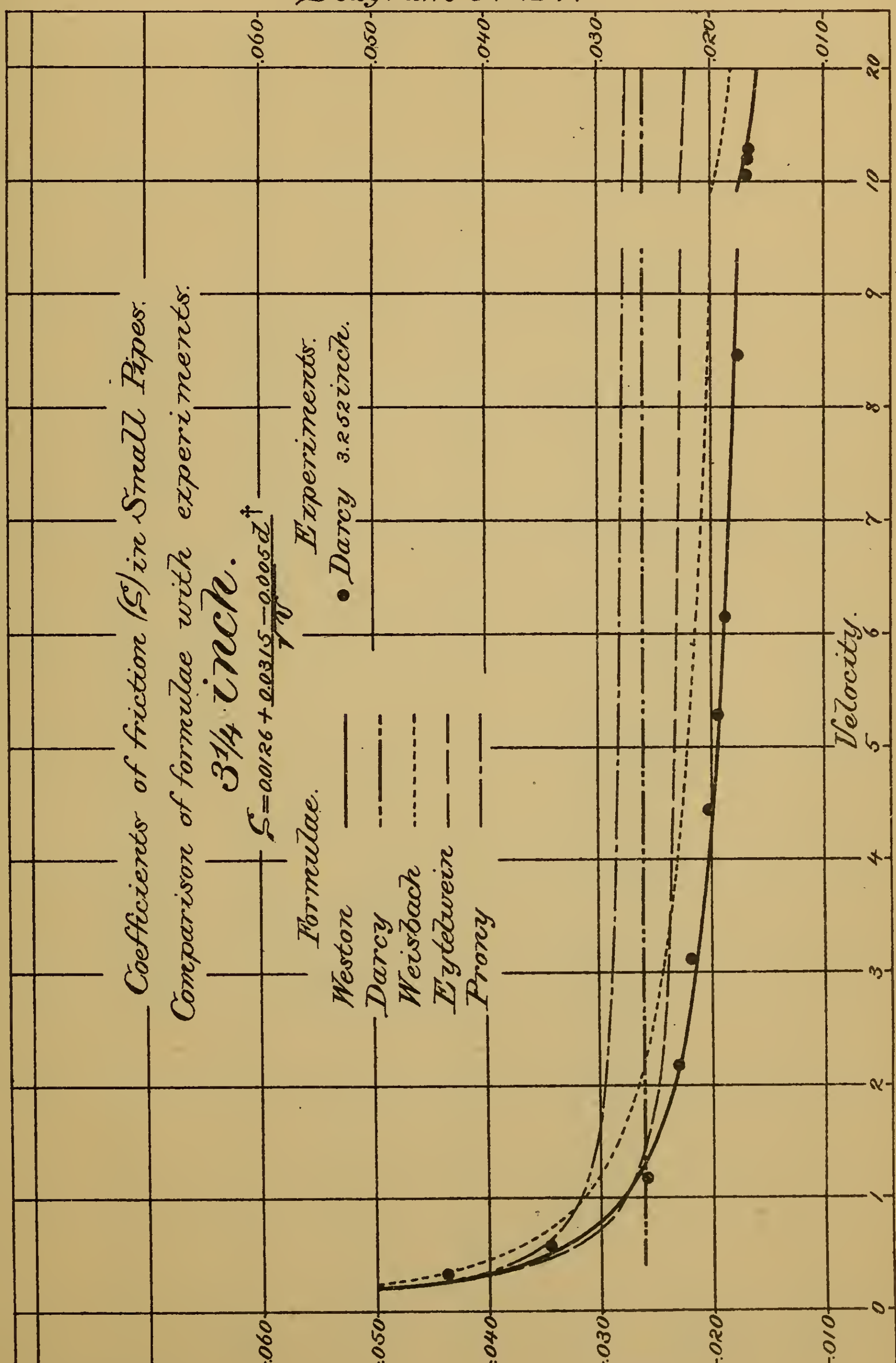
Diagram No. 14.



† d in inches.

PLATE XV
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Diagram No. 15.

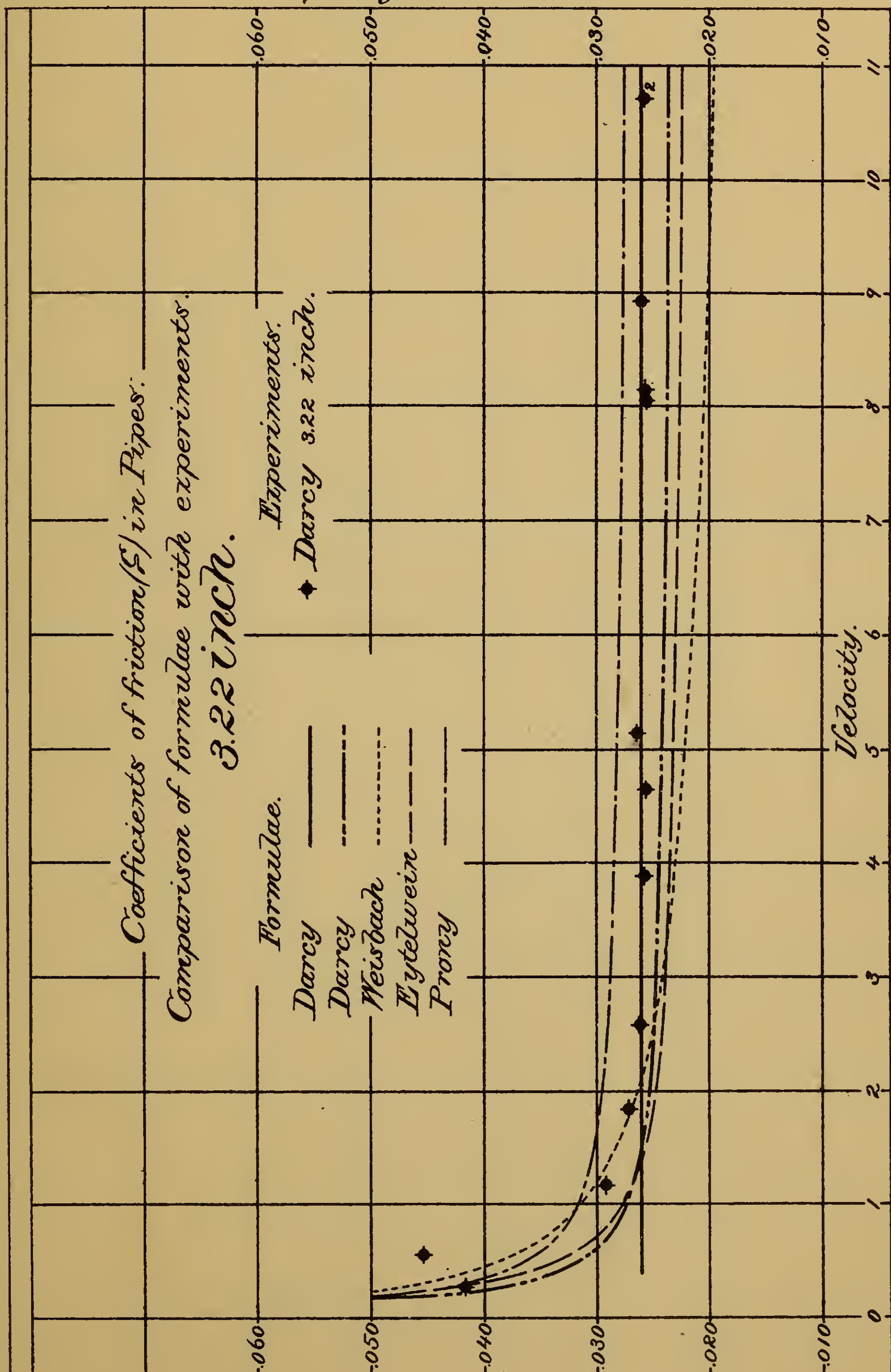

PLATE XVI
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Diagram No. 16.

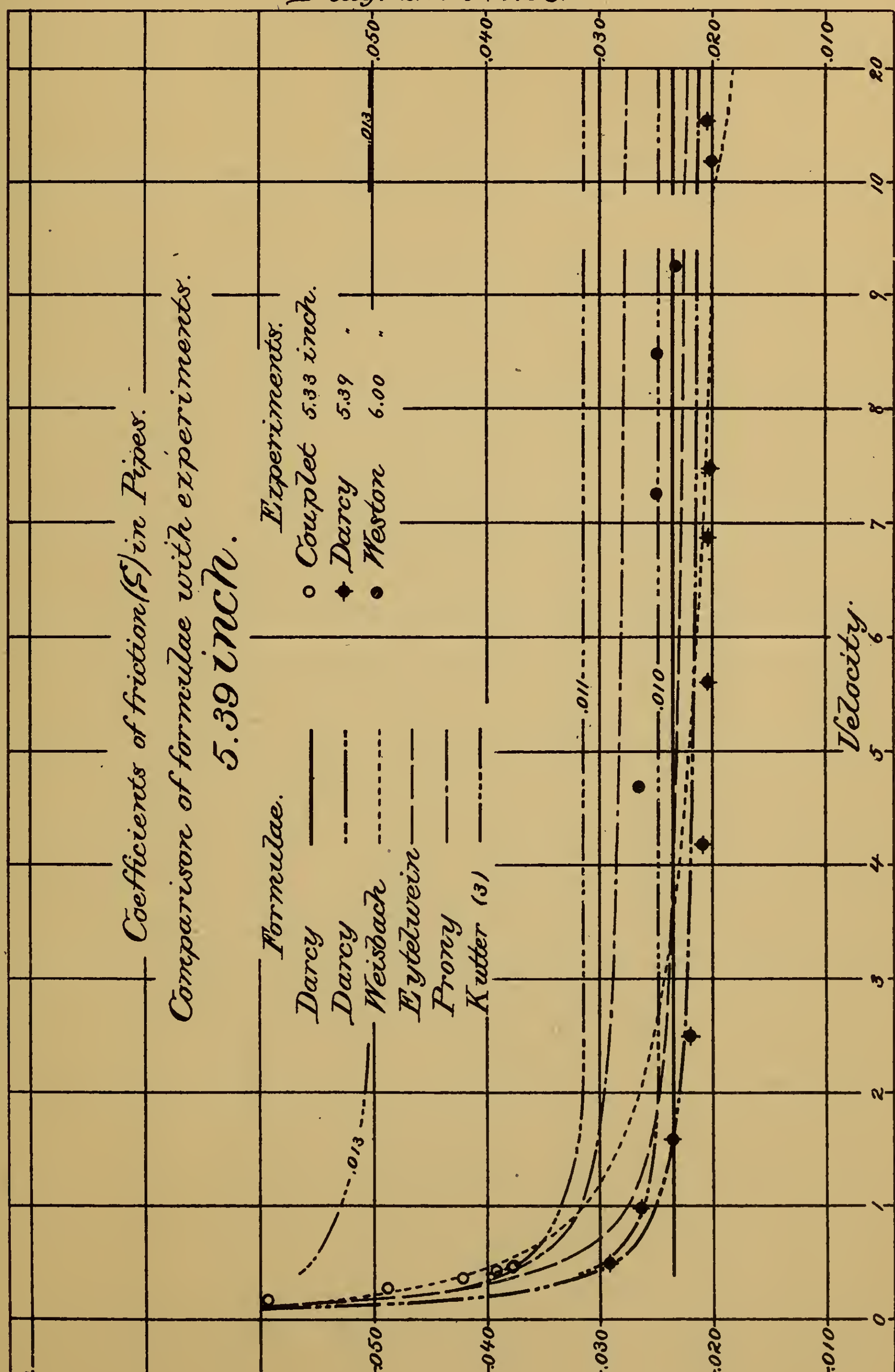

PLATE XVII
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Diagram No. 17.

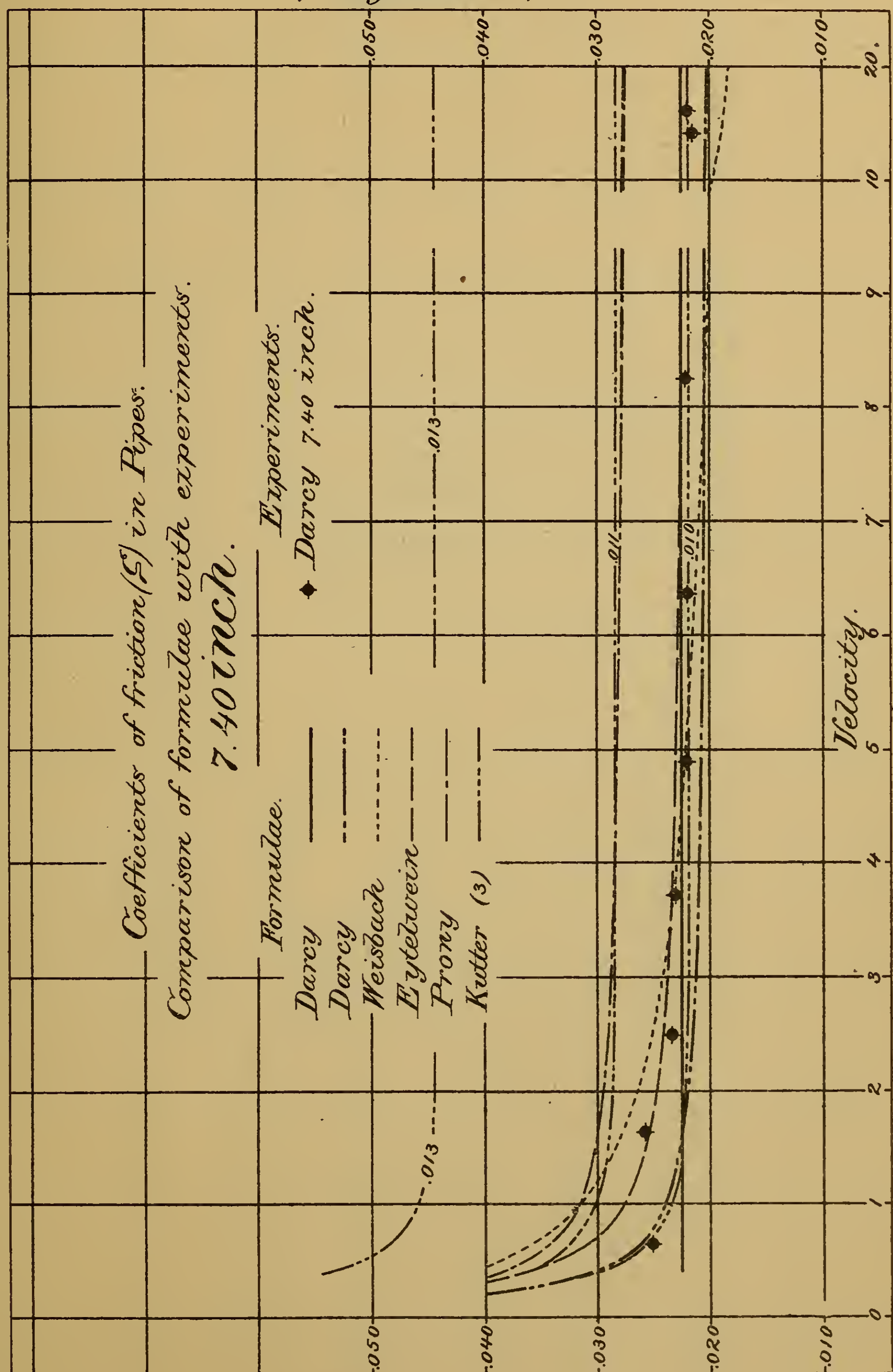

PLATE XVIII.
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Diagram No. 18.

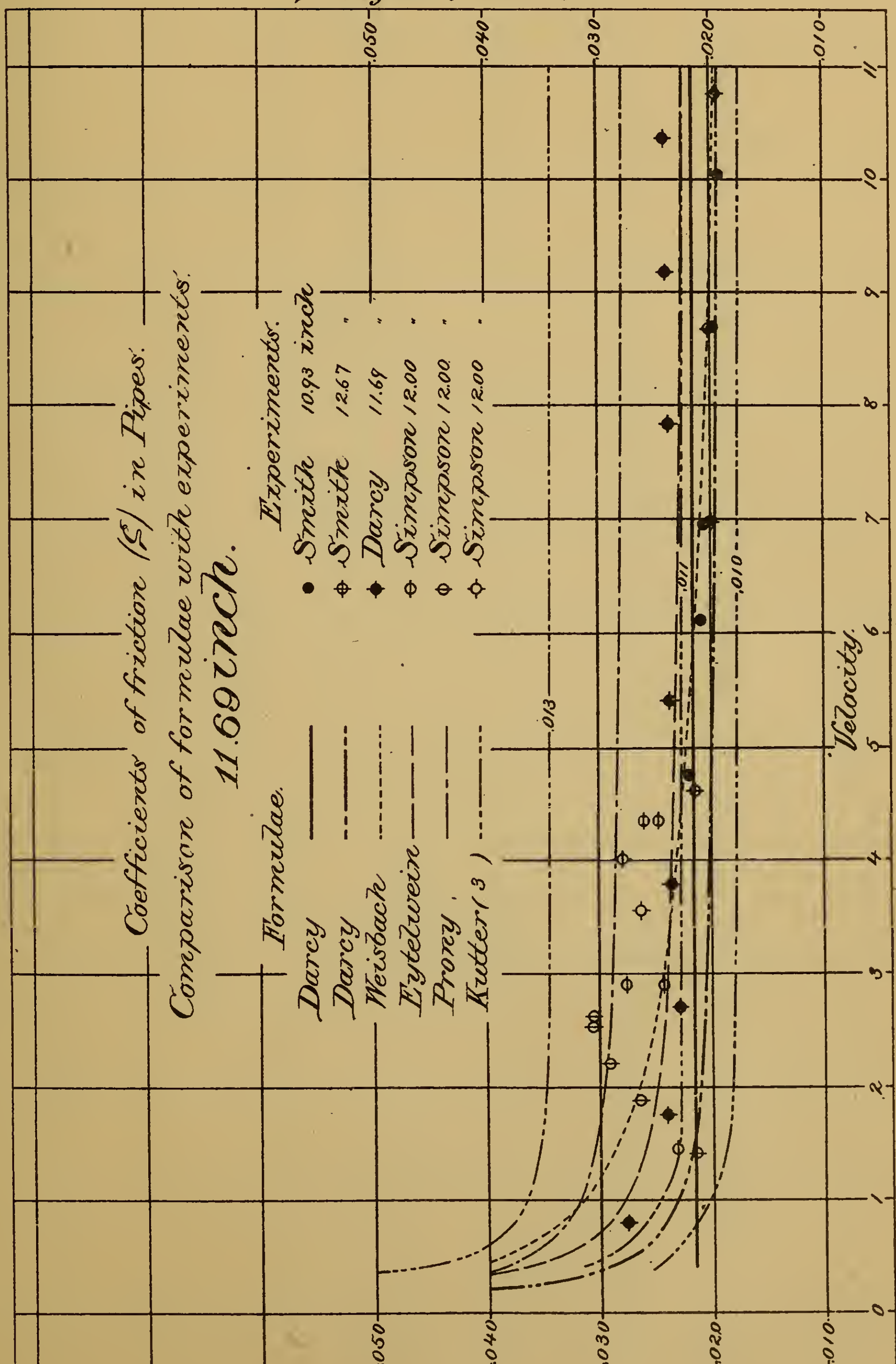

PLATE XIX
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Diagram No. 19.

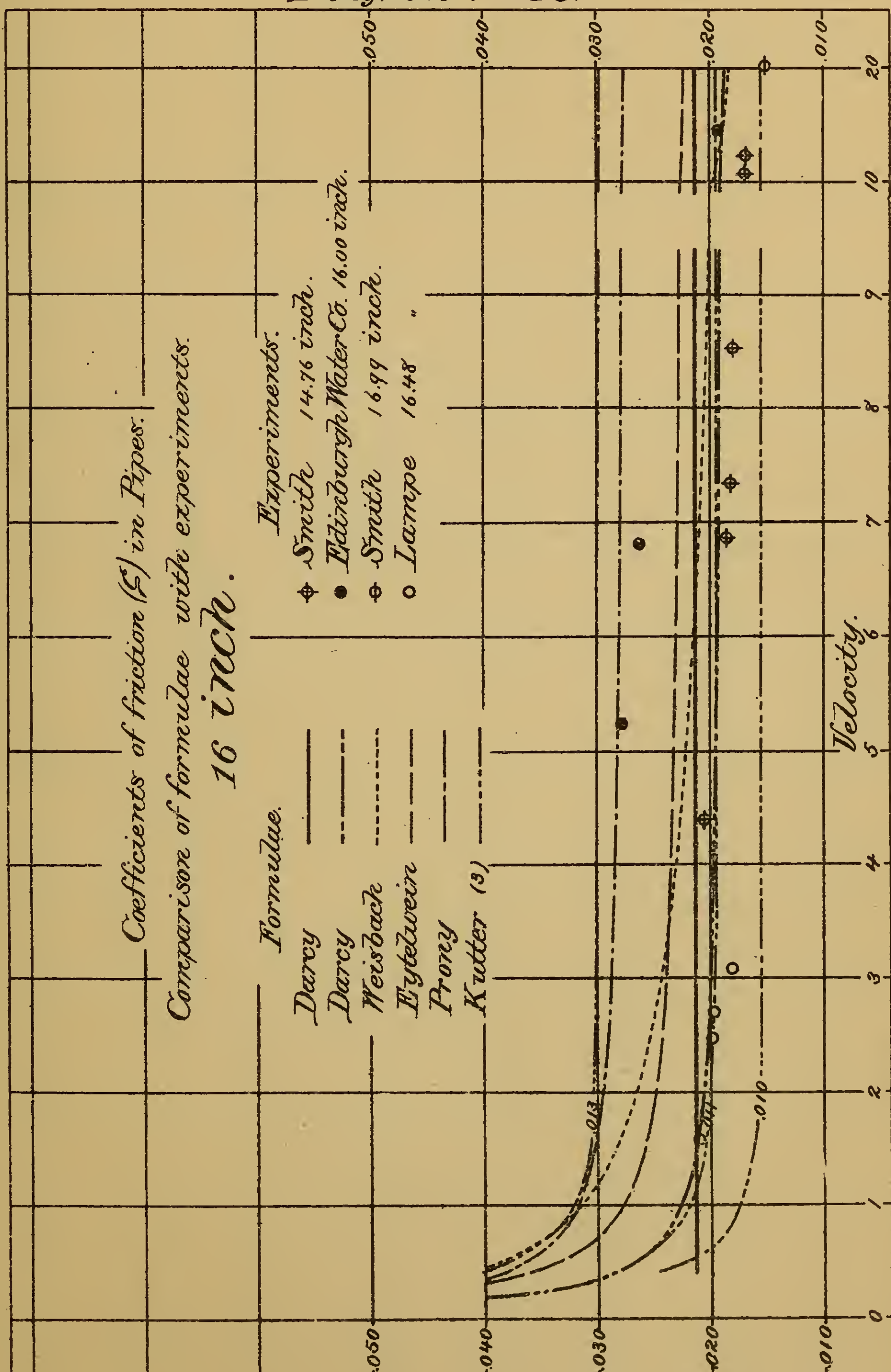

PLATE XX
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Diagram No. 20.

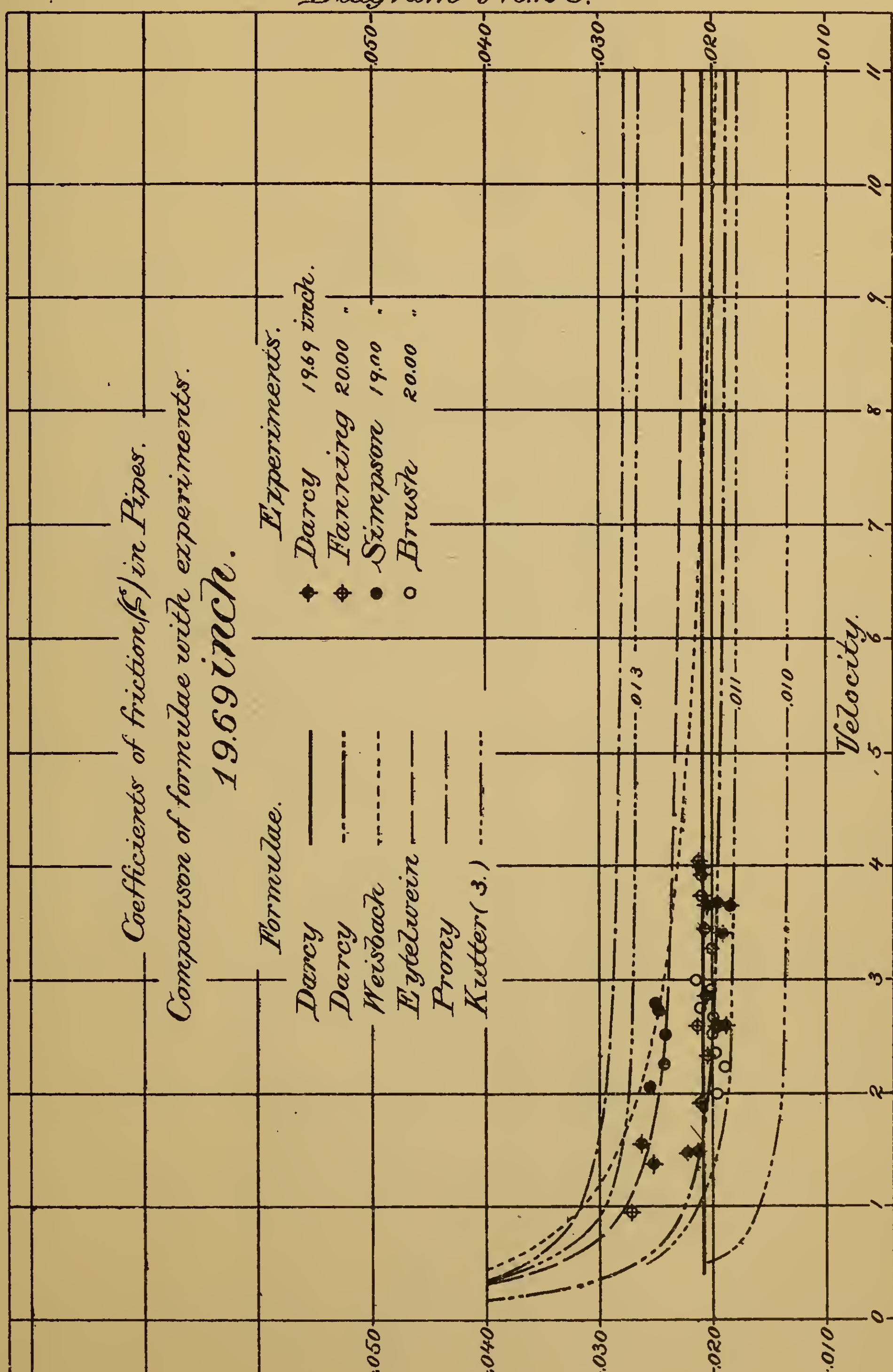

PLATE XXI
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Diagram No. 21.

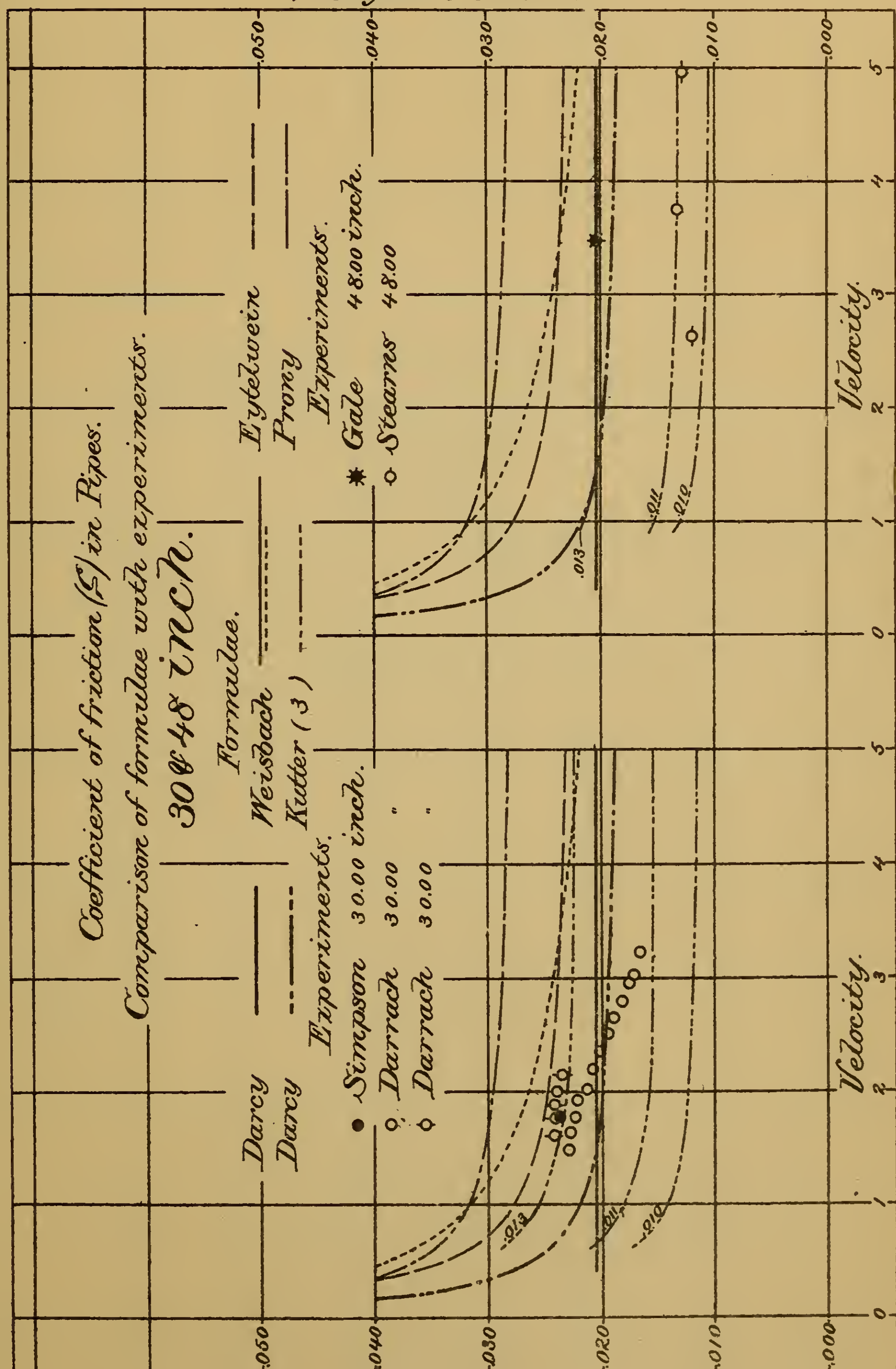

PLATE XXII.
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Diagram No. 22.

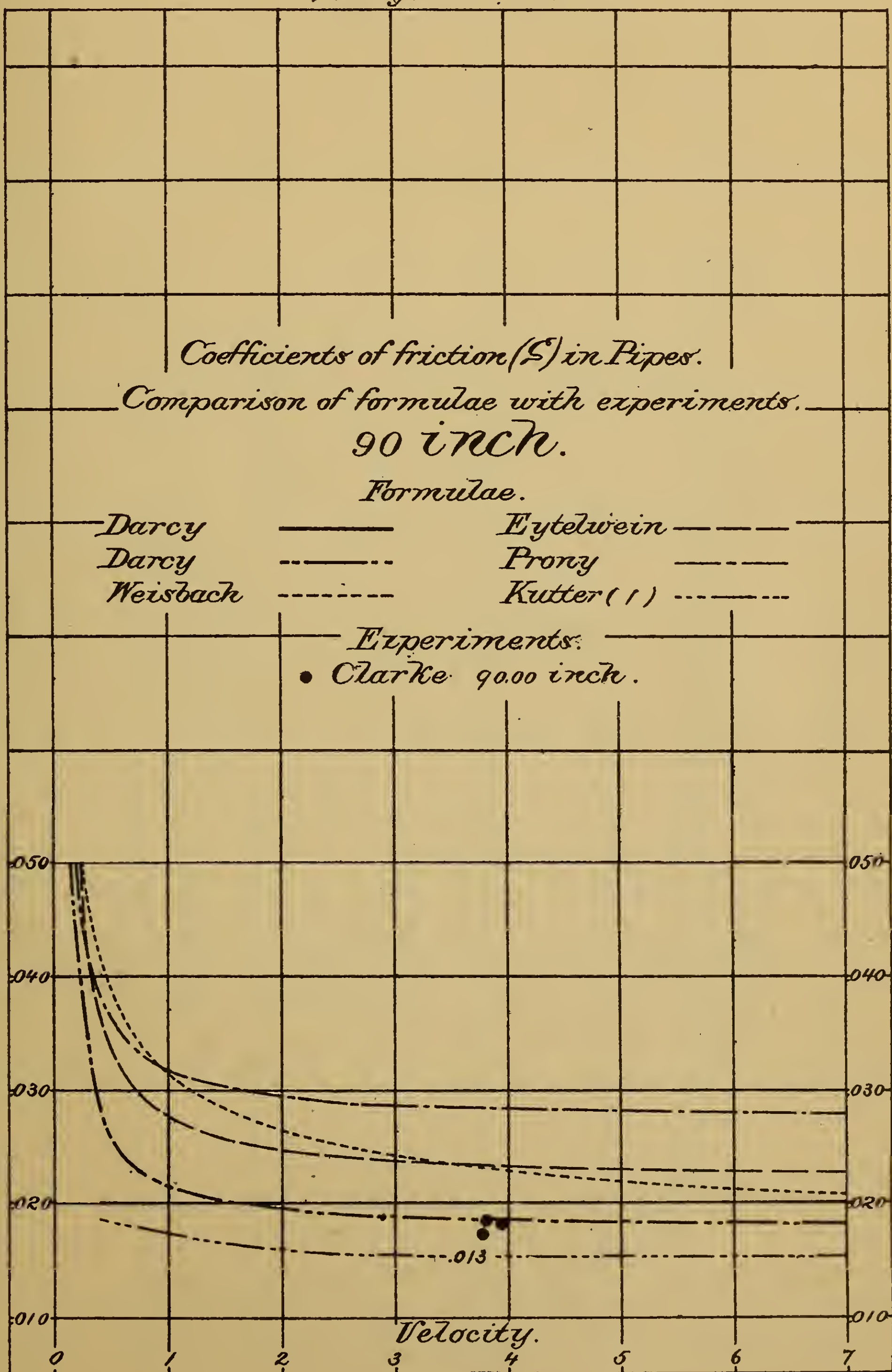

PLATE XXIII.
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Diagram No. 23.

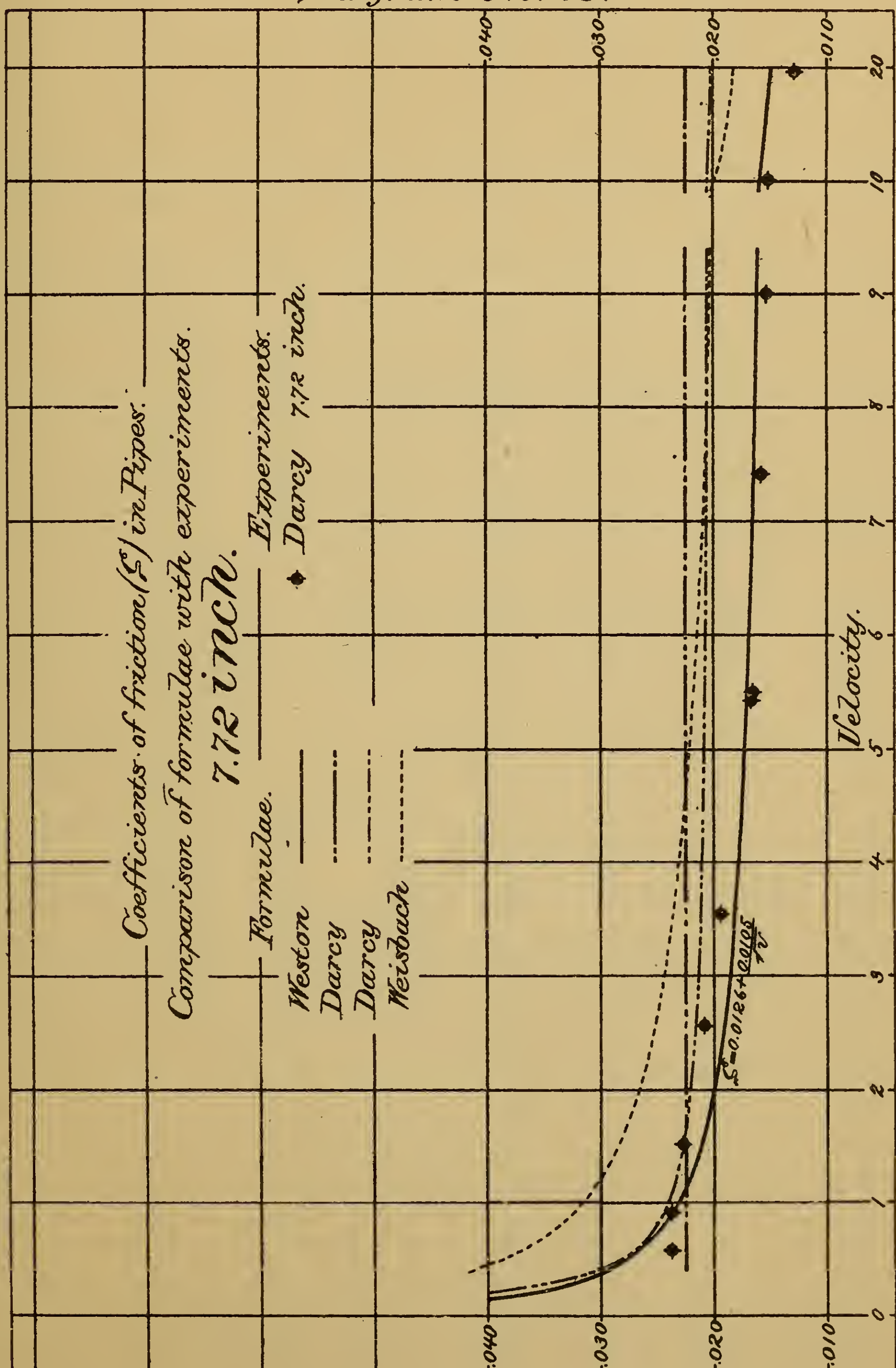

PLATE XXIV
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Diagram No. 24.

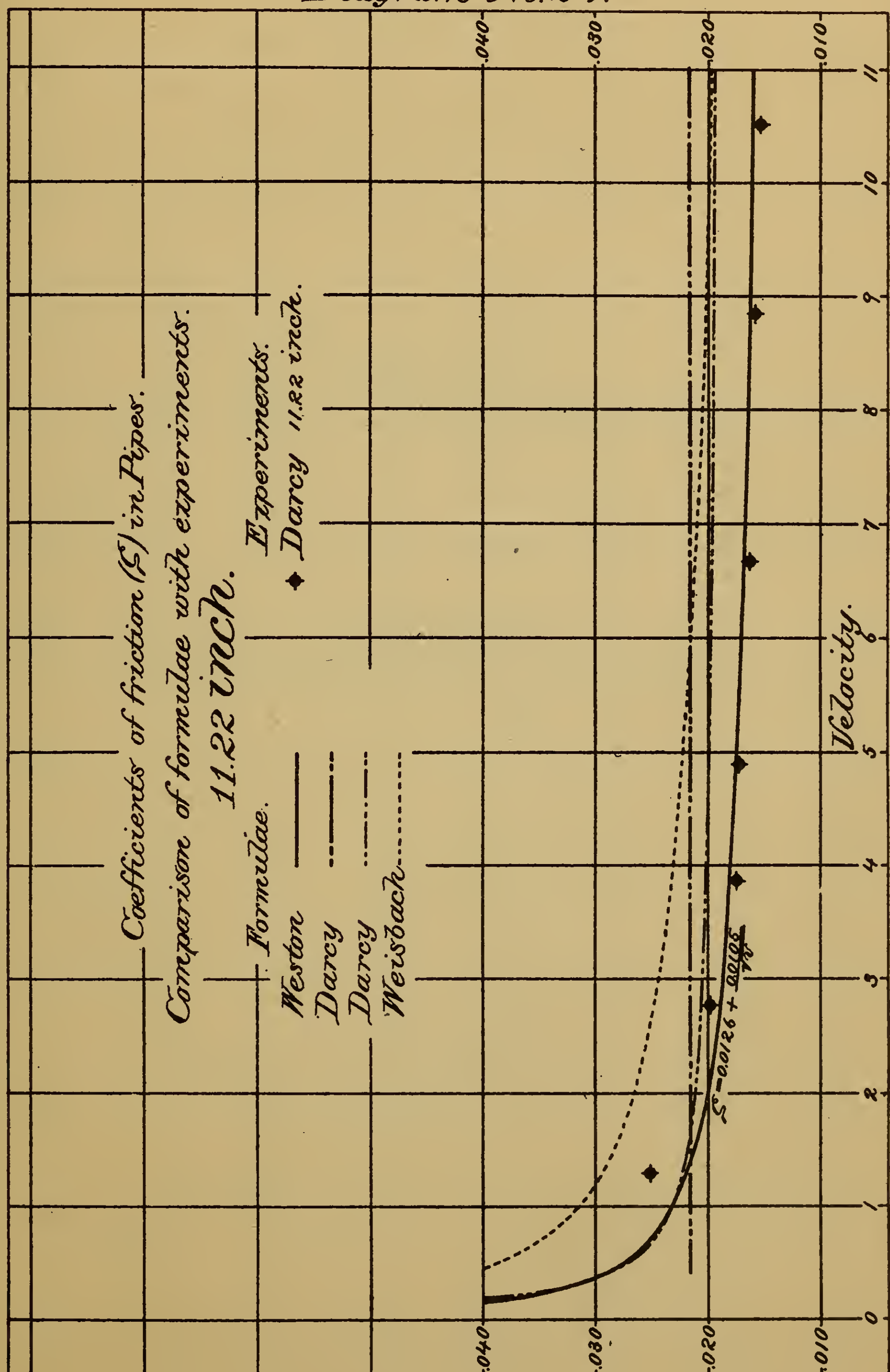

PLATE XXV.
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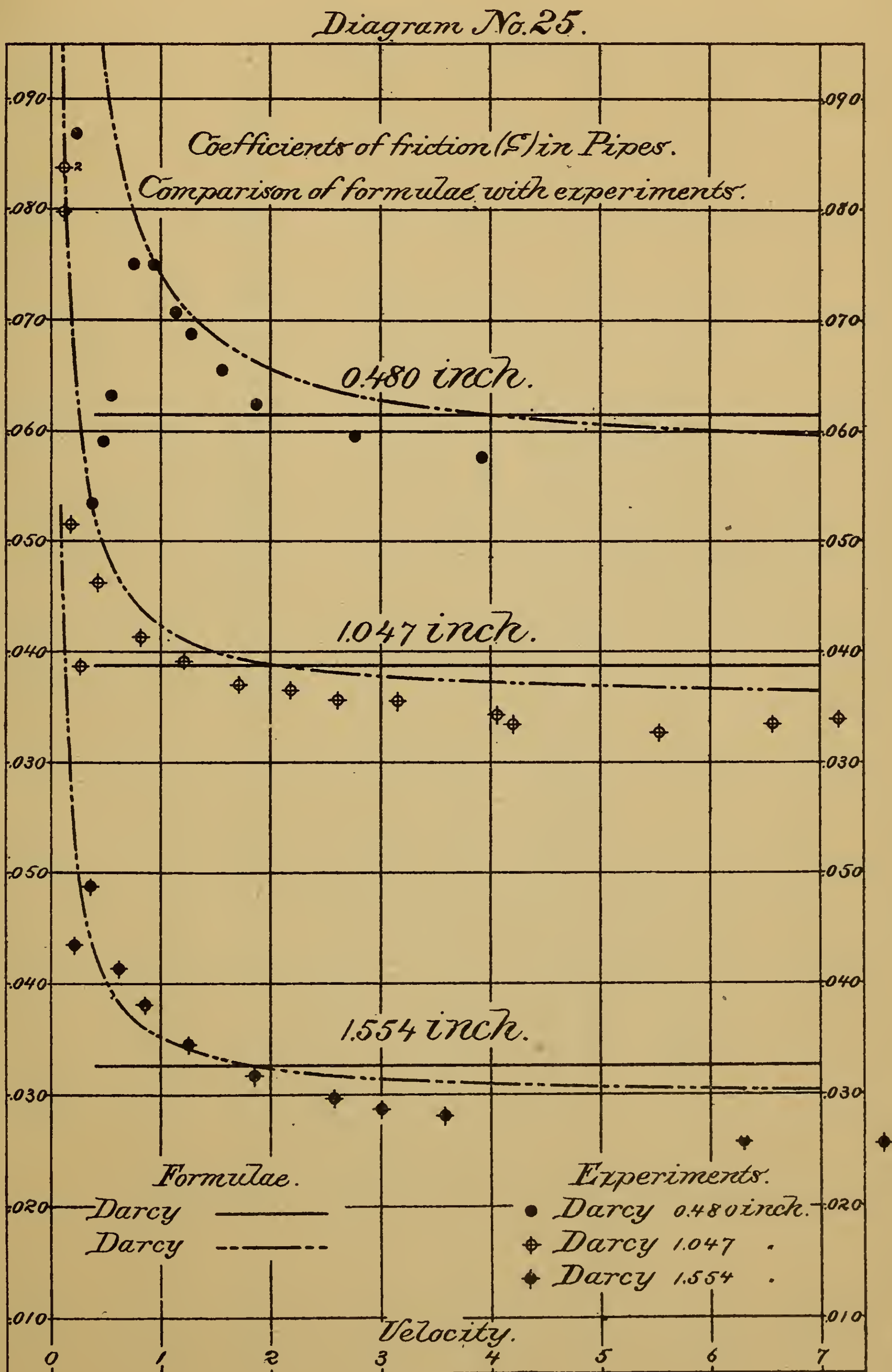

PLATE XXVI
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Diagram No. 26.

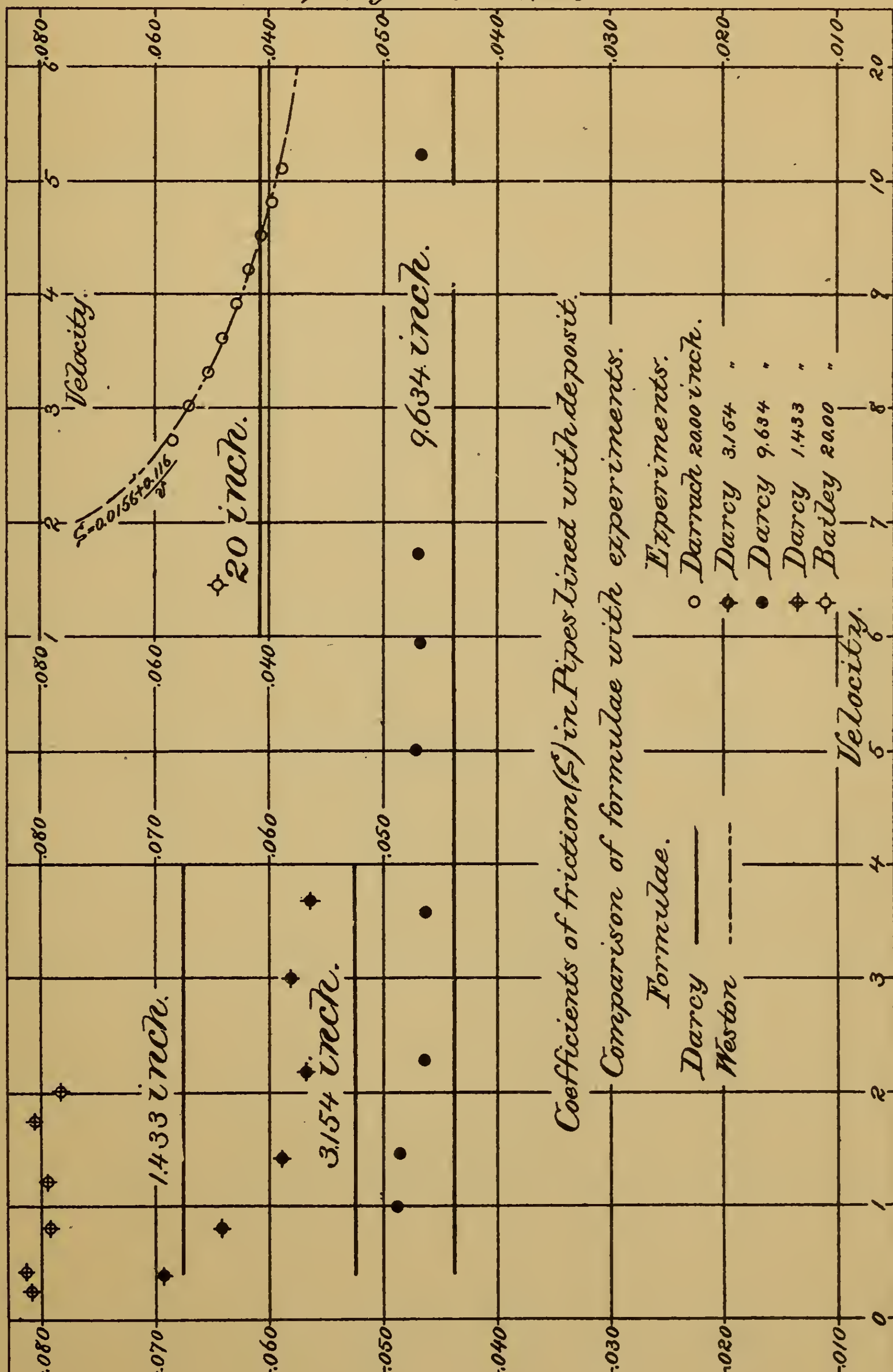

PLATE XXVII
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Diagram No. 27.

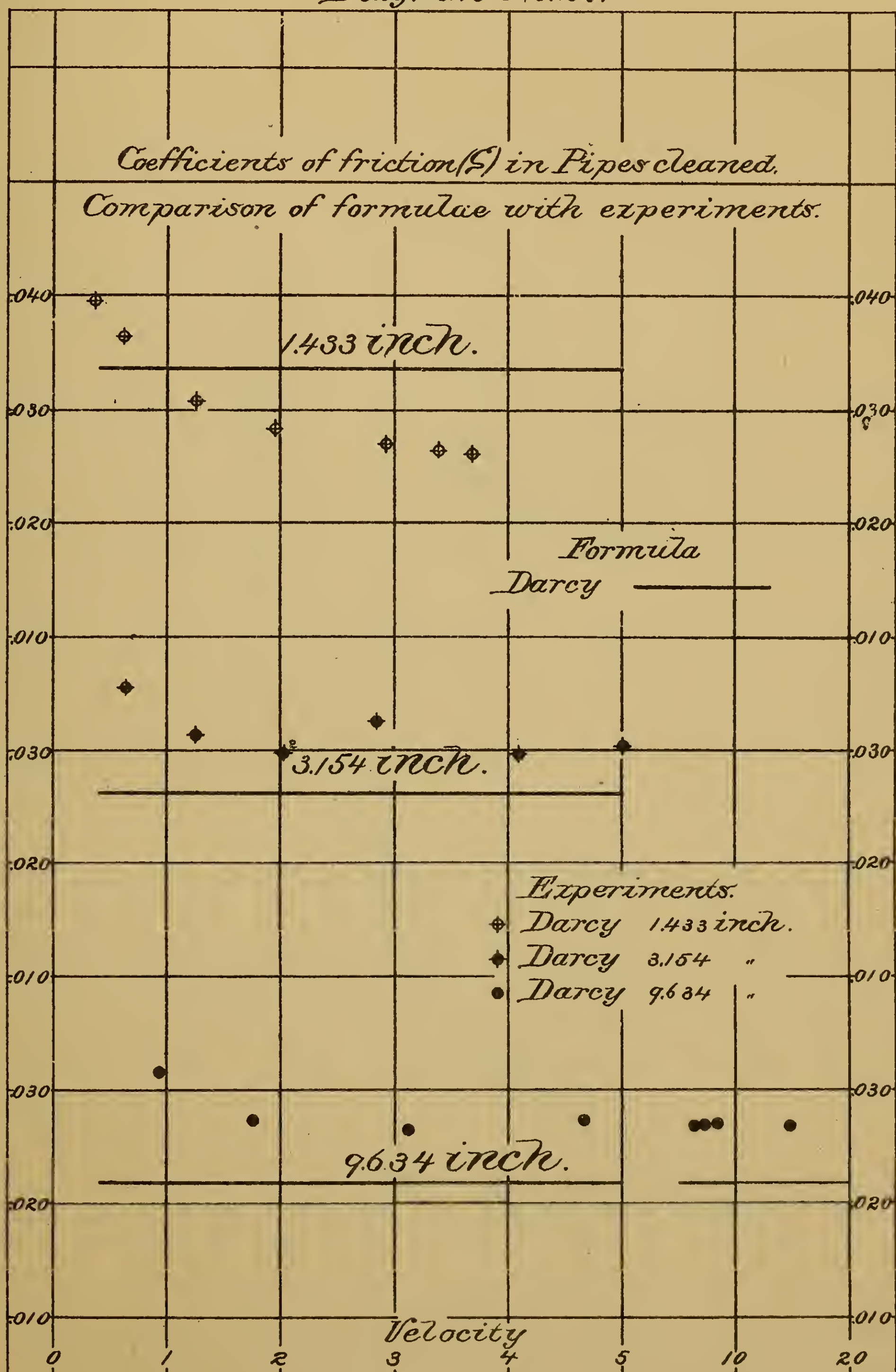

PLATE XXVIII
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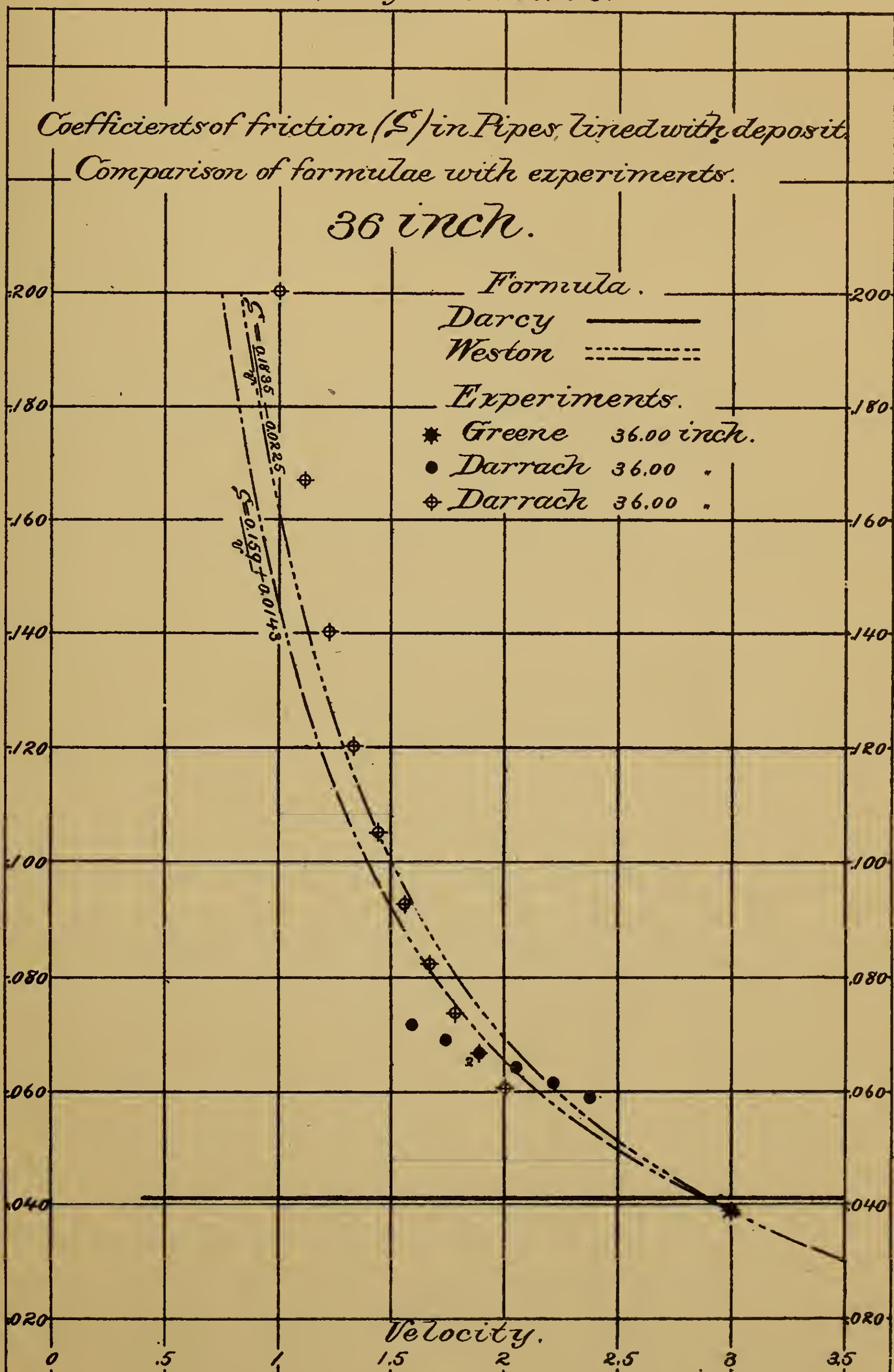
Diagram No. 28.



APPENDIX.

Loss of Head due to Friction of Water in Pipes having very Smooth Interior Sides similar to Brass and Lead Pipes, etc.

Table calculated from a formula by Edmund B. Weston, M. Am. Soc., C. E., for Pipes 100 Feet in Length, showing:

1st. The mean velocity of the water flowing in the pipe, in feet per second.
2d. The interior diameter of the pipe.
3d. The loss of head due to friction, in feet.
4th. The number of gallons of water delivered per minute.

Loss of Head due to Friction of Water flowing in Pipes 100 Feet in length, having very Smooth Interior Sides.

(*Weston's Formula.*)

Velocity in Feet per Second.	Internal Diameter of Pipe in Inches.									
	½		⅝		¾		1		1¼	
	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.
1	1.55	0.61	1.22	0.96	1.00	1.38	0.73	2.45	0.56	3.83
2	4.94	1.22	3.9[illegible]	1.91	3.21	2.75	2.34	4.90	1.82	7.65
3	9.85	1.84	7.79	2.87	6.41	4.13	4.68	7.34	3.65	11.48
4	16.18	2.45	12.79	3.83	10.54	5.51	7.72	9.79	6.02	15.30
6	32.83	3.67	25.99	5.74	21.43	8.26	15.73	14.69	12.31	22.95
8	54.57	4.90	43.24	7.65	35.68	11.02	26.23	19.58	20.56	30.60
10	81.23	6.12	64.39	9.56	53.17	13.77	39.14	24.48	30.72	38.25
13	130.17	7.96	103.26	12.43	85.32	17.90	62.90	31.82	49.44	49.73
16	189.60	9.79	150.49	15.30	124.41	22.03	91.82	39.17	72.26	61.20
20	284.81	12.24	226.19	19.13	187.10	27.54	138.24	48.96	108.92	76.50
25	429.08	15.30	340 93	23.91	282.17	34.43	208.71	61.20	164.64	95.63
30	600.90	18.36	477.65	28.69	395.48	41.31	292.79	73.44	231.16	114.75
35	799.95	21.42	636.09	33.47	526.84	48.20	390.32	85.68	308.39	133.88
40	1026.	24.48	816.02	38.26	676.08	55.08	501.16	97.92	396.21	153.00
45	1279.	27.54	1017.	43.04	843.03	61.97	625.23	110.16	494.54	172.13
50	1558.	30.60	1240.	47.82	1028.	68.85	762.44	122.40	603.35	191.25
55	1863.	33.66	1483.	52.60	1230.	75.74	912.71	134.64	722.58	210.38
60	2195.	36.72	1748.	57.38	1449.	82.62	1076.	146.88	852.11	229.50
65	2553.	39.78	2033.	62.17	1686.	89.51	1252.	159.12	992.01	248.63

Loss of Head due to Friction of Water flowing in Pipes 100 feet in length having very Smooth Interior Sides.

(Weston's Formula.)

Velocity in Feet per Second.	Internal Diameter of Pipe in Inches.									
	1½		2		2½		3		3½	
	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.	Loss of Head due to Friction.	Gallons per Minute.
1	0.46	5.51	0.32	9.79	0.24	15.30	0.18	22.03	0.14	30.00
2	1.47	11.02	1.04	19.58	0.78	30.60	0.60	44.06	0.48	60.00
3	2.96	16.52	2.10	29.38	1.58	45.90	1.24	66.10	0.99	90.00
4	4.89	22.03	3.48	39.17	2.64	61.20	2.07	88.13	1.67	120.00
6	10.03	33.05	7.18	58.75	5.47	91.80	4.33	132.19	3.51	180.00
8	16.78	44.06	12.06	78.34	9.23	122.40	7.34	176.26	5.99	240.00
10	25.11	55.08	18.09	97.92	13.89	153.00	11.08	220.32	9.08	300.00
13	40.47	71.60	29.26	127.30	22.54	198.90	18.05	286.42	14.85	390.00
16	59.22	88.13	42.92	156.67	33.14	244.80	26.63	352.51	21.97	480.00
20	89.38	110.16	64.95	195.84	50.29	306.00	40.52	440.64	33.54	600.00
25	135.25	137.70	98.53	244.80	76.49	382.50	61.80	550.80	51.30	750.00
30	190.08	165.24	138.73	293.76	107.92	459.00	87.38	660.96	72.71	900.00
35	253.77	192.78	185.50	342.72	144.54	535.50	117.23	771.12	97.72	1050.00
40	326.24	220.32	238.78	391.68	186.31	612.00	151.32	881.28	126.33	1200.00
45	407.44	247.86	298.54	440.64	233.20	688.50	189.64	991.44	158.53	1350.00
50	497.30	275.40	364.73	489.60	285.19	765.00	232.15	1101.60	194.28	1500.00
55	595.79	302.94	437.33	538.56	342.26	841.50	278.87	1211.76	233.60	1650.00
60	702.87	330.48	516.31	587.52	404.38	918.00	329.76	1321.92	276.46	1800.00
65	818.51	358.02	601.66	636.48	471.55	994.50	384.82	1432.08	322.86	1950.00

DISCUSSION.

Rudolph Hering, M. Am. Soc. C. E.—The present paper, like the former ones of Mr. Weston, contains matter of considerable value and interest and bears testimony to the painstaking and the accuracy in details with which its author has approached the problem. The completeness with which most of the known experiments on the flow of water in pipes have been given and arranged is, I think, not exceeded in any other single publication. The new formula which he has developed therefrom for the flow in very smooth pipes of ½ to 3½ inches in diameter is, as he clearly demonstrates, practically correct, and is possibly the best one that has up to the present time been suggested within the limits and under the qualifications given.

My object in sending this discussion is not to question the accuracy of this formula, but to express my doubts as to the propriety of advocating one having what I think is a faulty construction. Had the author confined himself to recommending the table on page 55, which gives the values of the co-efficient of friction within the indicated limits of the diameter of pipe and of the velocity in the same, or had he recommended a diagram exhibiting these values, I should have said that I believe the table or diagram gives the most reliable published average results for the purpose set forth in its heading. To suggest a new formula, however, particularly with so limited a range of applicability as in this case, is, I think, somewhat questionable wisdom. It is a case similar to that mentioned by Mr. F. P. Stearns, M. Am. Soc. C. E., in the discussion of Yonge's formula (see *Transactions*, Am. Soc. C. E., Volume XIV, page 15), which if applied not far outside of a given range may lead far from the truth. Could we always append to the formula the limits of its range as readily as we can indicate them in the integral calculus, this objection to the formula would not hold good. But as a rule this is not done, and we find one book after another, or one engineer after another, quoting formulas, but stripped of some of their essential qualifications, and then using them for what they were never intended. By naturally fixing the limitations in the form of tables or diagrams no one could unsuspectingly drop into an error.

Further, I am not able to agree with Mr. Weston on the fundamental construction of his formula. He has followed the precepts of Darcy in recommending a separate and distinct mathematical expression for different degrees of roughness of the interior surface, whereas we observe as much and even more continuity in its variation, as it changes from the smoothness of glass to the roughness of old cast-iron pipe, than we observe in the diameter of marketable pipes, and therefore might, with almost equal propriety, deduce a separate formula for each diameter. In fact, we see that often a barely appreciable difference in the character of the interior surface has a greater effect upon the co-efficient than a measurable difference of diameter.

It therefore seems to me essential to the proper construction of any formula used for the flow of water in pipes, that the degree of roughness should be embodied in it as a variable and determinable quantity just as much as the diameter, the length of pipe, or its hydraulic gradient. The introduction of such a variable, as is well known, was first attempted by Messrs. Ganguillet and Kutter. While I am convinced that their own formula can be improved both in construction and in the numerical values of its constants, yet it seems indisputable that the suggestion of a variable co-efficient of roughness is a step in advance which I think should no longer be ignored. Such a co-efficient gives the engineer an opportunity to exercise practical judgment in the selection of a definite value for it in accordance with the nature of the perimeter, which he is incapable of doing intelligently with a formula in which the co-efficient represents the combined effects of a number of causes, and conceals the separate effect of each one of them.

Nor do I think it logical, by following Weisbach's lead, to base the variation of the only co-efficient upon the velocity rather than upon the gradient or slope which produces it. There are good reasons for not doing this in large rivers where the slope is often inconsistent with the velocity, or cannot be determined with equal accuracy. But for pipe systems, where the hydraulic gradient can so readily be measured, I think formulas ought to have their co-efficient based upon the slope, irrespective of the other inconvenience in Weisbach's method of being first obliged to approximate the desired velocity in order to find the proper coefficient.

That Mr. Weston could not obtain a satisfactory formula for old cast-iron pipes is due to the fact that virtually he was trying to find for them a fixed degree of roughness. Had he introduced a coefficient permitting the exercise of some judgment regarding the relative degree of this roughness in a particular pipe, he would, I think, have found a formula in which he could have had more confidence.

Edmund B. Weston, M. Am. Soc. C. E.—I regret that I have not been able to construct a formula for the flow of water in pipes having very smooth interior sides, similar to lead and brass pipes, which I can recommend for larger diameters than 3.5 inches; but unfortunately I had not sufficient experimental data to do this satisfactorily. I therefore restricted myself to the experimental data that I had at hand, and constructed a formula which agrees remarkably well with experimental results, as can be seen by examining the diagrams from No. 7 to No. 14. I was well aware that there would be a possibility of the formula being applied to pipes having larger diameters than 3.5 inches, consequently I was very particular in my paper to emphasize, in addition to the limit of diameters, the nature of the pipe for which the formula was intended. And as the pipes whose interior sides are very smooth, which have diameters larger than 3.5 inches, are not used at the present time to convey water, unless possibly in very exceptional cases, it seems to me that with reasonable precautions there should not be any danger of the formula being applied beyond its limit. In fact there are few formulas but what are in reality more or less limited in their range, if the experiments from which they were constructed are criterions; even Ganguillet and Kutter state that their formula must not be expected to apply to cases beyond the range of the data from which it was derived.

I recognize how valuable a reliable general formula would be which could be applied to all kinds of pipes, and which would contain a co-efficient permitting the exercise of some judgment regarding the relative degree of roughness in a particular pipe; but had I been disposed to have attempted the construction of such a formula, it would not have been possible for me to have done so from the experimental data that I had accumulated, as a preliminary investigation that I made relative to a formula of this kind developed results from these data which were entirely too conflicting.

Mr. Hering's opinion that formulas ought to have their co-efficients based upon the hydraulic gradient rather than the velocity, does not entirely coincide with my views in regard to formulas for the flow of water in pipes under pressure, although I admit that such formulas would be very convenient at times, as in the majority of cases in water-pipe calculations the result sought after is the loss of head due to friction, and not the velocity, as is generally the case in calculations relating to rivers and open channels.

In regard to constructing a perfect and reliable general formula that could be applied to old cast-iron pipes, if such a thing were possible, especially if the age of the pipes was the basis upon which judgment was to be formed as to the condition of their interior surfaces, it seems to me that in addition to a co-efficient of roughness, etc., etc., it would be quite important to embody in the formula a co-efficient dependent upon the chemical constituents of the water that was flowing in the pipes, or that was to flow in the same, as it is well known that some waters cause cast-iron pipes to corrode much faster than other waters. For instance, it can be seen in Table No. 1 that co-efficients of friction (ζ) worked out from experiments that were made in Philadelphia with cast-iron pipes respectively seven and eleven years old, are about twice as large as co-efficients of friction (ζ) derived from experiments that were made in England with cast-iron pipes respectively seven and thirteen years old.

Chas. E. Emery, M. Am. Soc. C. E.—I had occasion to closely examine the general formulas on the subject of the flow of fluids in developing an expression for the flow of steam. The formula selected was based on that of Weisbach, and the simple form applicable for water was found to answer every purpose for small losses of pressure in transmission, although a differential formula was developed to cover all conditions. In a cursory examination of the various papers that have been read on the subject of the flow of water, it has occurred to me that the consideration of one important source of information has been omitted. Experiments have been made with great care on the skin resistance of vessels moving through the water. Froude published, in the Transactions of the British Association for the Advancement of Science, some ten years ago, a series of curves showing the resistance of a large model hull at different velocities and for different lengths of surface operated upon. He found that the unit resistance decreased rapidly as the length was increased, but became nearly constant after the length exceeded 100 feet. Necessarily the frictional resistance, caused by the interior surface of pipes, corresponds to the skin resistance of a vessel in the water, and the results from the two methods of experiment can be compared with advantage. The Froude results show that experiments with short pipes should show greater resistance.

I fully agree with Mr. Hering that all the results of experiments should be shown by one general formula. This can only be accomplished by introducing in such formula a suitable number of variables, usually called constants, which can be modified to suit the particular conditions and thereby cause the formula to produce substantially the experimental results. It has been stated again and again that the character of the interior surface should be considered in the formula, but it does not as yet appear to have been done satisfactorily. I have heard the proposed constant termed the co-efficient of rugosity, although the term does not appear in either of the papers of the Society. The skin resistance has been pretty well shown to be due to eddies produced by the roughness of the surface. Old navigators can tell pretty well the speed of a vessel by looking at the distance the foam produced by the eddies extends from the side. In a running stream the water striking large stones below is thrown upward to the surface, as shown clearly by Mr. Francis' experiments with white-wash in one of the races at Lowell. In a discussion of that paper I called attention to how well these experiments compared with the accepted theories of eddy resistance. The rougher the surface the more the particles are deflected from a direct course and the farther the eddy resistance penetrates the central stream. It would seem that sufficient experiments are now available with surfaces of different characters to enable a co-efficient to be introduced into the ordinary water formula which would allow accurately for all customary variations. It might be well for those interested in the subject to examine some discussions by Rankin and others on the laws of eddy resistance, so that the co-efficient of rugosity would be introduced at the proper power to produce variations corresponding with those shown by experiment.

Mr. Weston.—D'Aubuisson in his "Traité D'Hydraulique," published, I think, about 1850, in an article upon the action of water by its resistance, recognizes that the increase in the length of a body moving through the water very perceivably diminishes its unit of resistance.

A few preliminary experiments that I made in 1876, relative to the flow of water in pipes, owing to the peculiarity of the results obtained, led me to make an investigation with the experimental data of quite a number of authorities in order to ascertain if the co-efficient of friction (ζ) was in any way dependent upon the length of the pipes. This investigation was rather an unsatisfactory one in some respects, as in nearly the whole of the experimental data of the very short pipes that were used the co-efficient of influx (ε) entered into the problem, and in some instances it had to be assumed. The results, however, showed decidedly that the value of the co-efficient of friction (ζ) was not influenced by the length of the pipes.

I have already expressed my views in regard to a general formula in my reply to Mr. Hering's discussion.

E. Sherman Gould, M. Am. Soc. C. E.—This paper contains the results of some interesting experiments upon the flow of water in pipes, as well as a very comprehensive and valuable review of the labors of other investigators in the same line.

The great number of these experiments which have been made, and the somewhat discordant formulas deduced from them, naturally lead us to inquire into the cause of such disagreement.

This much must be said of all formulas founded upon experiment: granting the accuracy of the measurements and records, the formulas must be true for the cases from which they were derived, and any others which should give different results would, for these particular cases, be incorrect, for the results were known in advance, and the formulas made to fit them.

The value of such formulas depends wholly upon the extent, perfection and trustworthiness of their experimental data. In these respects the magnificent researches of Darcy seem to command the greatest confidence. No other experiments, unless conducted upon a still more perfect scale and guided by a surer and more discriminating judgment than his, should be considered as invalidating the results left us by this illustrious experimentalist. So far from being invalidated, it would appear that his formulas have been generally corroborated by subsequent investigations. Mr. Charles B. Brush's observations on a 20-inch main, which have been already laid before the Society, and quoted in this paper, agreed very closely with those obtained by using Darcy's formulas for the same data. Darcy's experiments do not include the larger diameters of pipe, but I understand that the actual volumes discharged by the 48-inch main of the Bronx River Water Supply agree within three per cent. of those given by his formula. I am glad to perceive, by the paper now under discussion, that Mr. Weston also finds his results to agree very well with those of Darcy.

From a practical point of view, the state of the question seems to be this: Given a certain number of trustworthy experiments it is easy to formulate an equation which shall return the observed results, and this equation will always be true for identical conditions of pipe. But any change in the conditions will vitiate the formula to a greater or less extent, and render a new one necessary. In order to apply the formula, we must know exactly the conditions upon which it was framed. These can only be described by saying that the interior surface of the pipe was "smooth," or "rough," or "slightly tuberculated," etc. All this is very vague, and yet the state of the inside surface is a controlling factor in the problem. Moreover, a system of pipes when first laid is, or should be, in a condition of smoothness. Later, it will become more or less rough or even tuberculated, which will not only change the character of the inside surface, but also reduce the diameter. It would seem idle, therefore, to go into a great refinement in the matter of a formula, particularly if the refinement leads to a shaving down of the diameter.

As regards the effect of the velocity itself as affecting the formula, although there seems to be no doubt that the co-efficient for the same pipe varies somewhat with the velocity—that is, with the sine of the angle of inclination—yet, it must be remembered that in practice no very great extremes of velocity, either way, are admissible, so that this factor is kept within comparatively narrow limits of variation, admitting of a mean value being assumed, for most cases.

I would not be understood as depreciating the value of such researches as those embodied in Mr. Weston's able and instructive paper. They lie at the very foundation of all sound science. I would wish simply to direct attention to the immediate bearing of the formulated results upon the actual practice of designing a line of water supply pipes, and to indicate what we can and what we cannot expect from a formula in this connection.

Mr. Weston.—That Mr. Gould has such a high opinion of the value of the investigation and formulas of the late Henry Darcy, I am very glad to learn, not only on account of my having recommended his formulas for new cast-iron pipes, but for the reason that a number of writers upon hydraulic subjects have seemingly taken great pains not only to depreciate the formulas of this eminent engineer, but also to question the value of his experimental results. I regret, however, that Mr. Darcy did not deduce a formula for pipes having very smooth interior sides, although his experiments with pipes of this class were somewhat limited.

Mansfield Merriman, M. Am. Soc. C. E.—The formula deduced by Mr. Weston for loss of head in friction has the disadvantage, in common with those presented by other authors, of being empirical and not rational. It is not easy to see, however, how a theoretical expression for this lost head can be established, and accordingly experiment must be the only guide for a long time to come. Mr. Weston's formula can be written thus:

$$h_f = 0.0126 \frac{l}{d} \cdot \frac{v^2}{2g} + 0.0315 \frac{l}{d} \cdot \frac{v^{\frac{3}{2}}}{2g} - 0.06\ l\ \frac{v^{\frac{3}{2}}}{2g}$$

in which only the first term of the second member is homogeneous with h_f, indicating that the numbers 0.0315 and 0.06 are functions of v or d, whose relations are not yet known. I do not like the presence of the minus sign before the last term of this expression, for if the friction head be due to several causes it is not easy to see why any cause would produce a negative effect.

The diagrams show that the formula agrees very well with the experiments, and hence the labors of Mr. Weston will be of value to all who wish to make computations on the discharge through pipes. It is my impression that other formulas might be devised which would exhibit an accordance nearly or quite as good, but it does not seem very important that further investigations should be undertaken until something is known regarding the theoretical laws which control the friction of flowing water. With such a complex empirical expression as that of Kutter's formula I have no sympathy whatever. The laws of nature are, of course, not simple, but it cannot be at all supposed that the true law of loss of energy in friction is represented by such algebra as Kutter's formula contains.

The loss of head due to sudden contraction of the diameter of a pipe is, I think, always less than that given by Mr. Weston. He states for all cases that the co-efficient ϕ is 0.316, and that this is increased by resistance and friction to 0.505. Now the number 0.505 applies to the case where the diameter d (Fig. 4),* is very large compared with d_1, as for instance, when water enters a pipe from a reservoir. It is clear that when $d = d_1$, the value of ϕ should be zero, for in this event no loss of head due to contraction occurs. A proper expression for the value of ϕ should hence depend upon the ratio of d to d_1, As a reasonable approximation I suggest the formula:

$$\phi = \tfrac{4}{3}\left(\frac{1}{c^1} - 1\right)^2$$

in which c^1 is the co-efficient of contraction of the stream in passing through the orifice whose diameter is d_1. Let r denote the ratio of d to d_1, then the value of c^1 in terms of r may be stated by the following expression, proposed in the second edition of my Treatise on Hydraulics:

$$c^1 = 0.582 + \frac{0.0418}{1.1 - r}$$

which is based upon the mean value $c^1 = 0.62$ when $r = 0$, and the known result $c^1 = 1.0$ when $r = 1$. The numerical values of ϕ for different values of r can now be ascertained, c^1 being first computed, and thus are found,

For $r = 0.0$,	0.2,	0.4,	0.6,	0.7,	0.8,	0.9,	1.0.
$\phi = 0.50$,	0.47,	0.42,	0.34,	0.28,	0.20,	0.09,	0.00.

These values of ϕ are given with only two decimals, for I regard it as useless to write the third decimal in cases where even the second is uncertain.

Mr. Weston.—I consider the ideas of Professor Merriman relative to the co-efficient of resistance (ϕ) of water passing from a large to a small pipe very reasonable. I gave in my paper, as I so stated, the co-efficient

* d and d_1 being diameters, and F and F_1 areas, as indicated on Fig. 4, page 62.

ϕ of Weisbach, as I had not made any experimental investigation based upon the idea of determining a co-efficient of this kind, considering that his co-efficient was a safe and simple one to use. The difference, however, in the loss of head by the formula $h_f = \phi \frac{v^2}{2g}$, would be only 0.11 feet for the greatest velocity that water should be allowed to flow through distribution pipes, by using either the smallest co-efficient 0.09 given by Professor Merriman, or the co-efficient 0.505 of Weisbach.

I have found recently in the second volume of Weisbach's work that a formula corresponding to $\phi = \left(1 - \frac{F_1}{F}\right)^2$ is mentioned (the co-efficient ϕ given in my paper being taken from the first volume).

JOHN R. FREEMAN, M. Am. Soc. C. E.—The present state of the art of computing the loss of head in an ordinary given pipe when delivering a given quantity, is about like figuring out a formula to four places of decimals and getting a result that may be relied upon as correct within 25 per cent. We are all interested in efforts to reduce this margin of uncertainty.

Occasional reports of Mr. Weston's work in the field of water supply have given me full confidence in the earnestness with which he seeks the truth and have led me to respect the generous diligence with which he gives the results of his labors to the craft.

I feel more free to discuss the matter at this time from the fact that the author has full opportunity to reply to all comments and criticisms in closing the discussion, and believe it for the best interests of both the Society and the authors that all papers like this should be subject to a searching examination from the different points of view of different minds while the whole matter is fresh and the author has opportunity to answer and explain.

It seems to me that Mr. Weston is off the true road towards a thoroughly satisfactory formula for the flow of water in pipes.

I would maintain as a foundation principle that a formula representing the law of flow in pipes, to be satisfactory, must contain as a very important and controlling factor, a term dependent upon the roughness of the interior surface of the pipe.

Mr. Weston's discussion of this question does this only in a crude and indirect manner, by adapting special formulas, each of narrow application, to some of the different classes of pipes. In one case his variable co-efficient is a function of only the diameter as a variable, in another of only the velocity, and in another of both the diameter and the velocity.

There is no reason to suppose that the actual law thus varies.

His investigation brings but little new data to light, and thus is not an experimental but an algebraic research, seeking the equation to a curve, which shall be as nearly as possible the mean of the plotted data for each of the several classes into which he divides certain data already well known and discussed.

Mr. Weston's form of investigation does not seek an expression for a law that is broad and general and covers the flow in all pipes, and in fairness to him we must, I think, admit that there is not on record the data with which he could have conveniently sought one.

I believe there is such a law, and that when we have more experiments it will be found. The missing link now needed to tie experiments on different pipes together and bring them exactly into one line, or explain the discrepancies in our data such as are so well shown in the convenient diagrams appended to Mr. Weston's paper, is an accurate and strictly definite description of the character of the surface.

It is surprising how this part of the description has been neglected in nearly all recorded experiments relating to the friction loss in iron pipes. Take even Darcy's experiments, in which he demonstrated what Hamilton Smith, Jr. calls the most brilliant hydraulic discovery of the century—the powerful influence of character of surface on the flow. The part of this research relating to effect of rough deposits was a qualitative analysis and not a quantitative analysis, for where, in his memoir, can you find it stated whether the tubercles or deposits which he found to double the friction loss were on an average as large as grains of wheat, or as large as cherries ? This is important information to have, since it might correspond to 50 per cent. or more difference in the loss of head.

While discussing the desirability of including a special co-efficient of roughness in our future formula, we may refer to the fact that that equation of wonderful range, the "Kutter formula," is applicable to express the law of flow in pipes, and that it contains a special co-efficient of roughness, and that in the very complete and convenient table of hydraulic data, given by Messrs. Hering and Trautwine in their appendix to Ganguillet and Kutter's treatise,* published about a year ago, nearly all of the five hundred experiments tabulated and described by Mr. Weston are there presented with the Kutter co-efficient of roughness computed for each.

This effort to describe the law of flow in all pipes by a single equation is highly commendable from a broad, general point of view, but it is not, however, altogether satisfactory.

I am inclined to think that some far simpler equation than that of the Kutter co-efficient may be found, which will express the law of flow in pipes so accurately that the error coming from approximations involved in its algebraic form will be le s than the actual variations in flow due to differences of roughness so small as to be altogether inappreciable in any ordinary description in character of surface.

* Flow of Water in Rivers and other Channels. Wiley & Sons. New York, 1889.

Turning, now, to the subject of loss of head in clean and new cast-iron pipes : These by no means all have the same character, and it is useless to expect one formula with one constant to apply to all. Thus, for instance, we, perhaps, explain in part the difference of 25 per cent. in co-efficient ζ found between Darcy's results and the two newly reported experiments of Mr. Weston on a 6-inch pipe. I have seen different lots of new 6-inch cast-iron pipe, which differed so much in character of surface that I have little doubt but the loss of head would be considerably more than 25 per cent. greater in one than in another. This difference in surface came from the degree of skill with which the tar coating was applied even more than from the difference in the casting as it came from the foundry sand. Thus some tar-coated cast-iron pipe is very smooth on the inside, while other pipe has "ribs" of tar projecting about $\frac{3}{16}$-inch, or even $\frac{1}{4}$-inch, running part way around the circumference where the tar has run while soft. On studying the collection of experiments on new cast-iron pipe now available as data and transcribed from one book to another, and given with fullness in Iben's, Smith's, Hering and Trautwine's and Weston's discussions, it will be seen that causes like this can easily obscure the true law or the refinements of a complicated formula.

Until we have new experiments on pipes in which great care is taken to accurately and definitely describe the exact degree of smoothness of the interior surface, it is almost hopeless to try and derive an accurate general formula. Meanwhile, had we not best try earnestly to avoid complicated algebraic expressions, and keep our formulas in the simplest possible form.

In other words, is not that simple formula of the last century, the Chezy formula, so called, when accompanied by a short and simple table or diagram of values for its co-efficient, better for practical use than any of the more complicated formulas? Until we get that extended series of quantitative experiments on effect of roughness is not this simple Chezy formula : $V = C\sqrt{RS}$.

In which $V =$ velocity in feet per second.
$C =$ co-efficient of flow dependent on diameter, velocity and roughness (Smith represents this by N).
$R =$ hydraulic radius $= \frac{1}{4}$ diameter of pipes of circular section.
$S =$ slope $=$ ratio of loss of head to length of pipe,

the best framework on which to display for comparison such new bits of experimental data as we may from time to time secure. Hamilton Smith, Jr., so used it in his most excellent treatise * (which is, perhaps, the most thorough of all in the care taken to exclude questionable data); and so did Hering and Trautwine in the appendix to their excellent

* Hydraulics, Hamilton Smith, Jr. Wiley & Sons. New York, 1886.

translation of the Ganguillet and Kutter work, given to the profession a year ago.

The valuable tables of data in the two recent books just mentioned give nearly all the experiments on flow in pipes recorded for a hundred years back, which have value as data and include nearly all those presented by Mr. Weston. When presenting new data by reducing it to this form, it is much more convenient for reference and comparison.

For the present, is it not better to observe experiment and secure new co-efficients for the old formula rather than re-thrash old, and in many cases blighted straw, to devise a new formula?

With regard to very rough or corroded pipes, Mr. Weston, like nearly every author who attempts to devise special formulas for the experiments now on record, or attempts to make experiments on one pipe fall in line with those on another, finally gives it up as a bad job.

The trouble in reconciling the experiments is, I think, in general, not that the loss of head has been inaccurately measured, not that the delivery was improperly determined, nor is it that the net diameter is not precisely known.

The uncertainty comes from vagueness in describing the character of the surface. The exact character of the surface is not easy to determine. In many cases there is no opportunity to inspect it, and even if opened to inspection a corroded surface is difficult to describe with precision.

In the present state of our experimental knowledge is it not almost useless to attempt a special form of equation for these very rough pipes? Is it not better to arrange the results of such experiments in the form of a comparison of the loss of head found in the rough pipe with that deduced from our simple formula for an average ordinarily smooth pipe, and thus, for instance, say that in the case of a certain pipe corroded as described, it was found that the loss of pressure was 2.4 times as great as the formula shows for a smooth pipe of same size and with same delivery? With each experiment expressed in this way, but condensed into tabular form along with the various hydraulic elements of the case, the present data is in most convenient shape for practical application, and is of very great value in conspicuously calling attention to the danger when designing works of computing the friction loss as though the pipes were always to remain new and clean.

With regard to the particular style for the fundamental formulas, I notice that Mr. Weston, on page 6 of his paper, presents the formulas which his own experience has shown most convenient. These are those sometimes known as Professor Weisbach's.* Is not the convenience of this type, as compared with the modified Chezy form as used by Smith, Hering and Trautwine and others, a question of personal habit more

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* Cox's Translation Weisbach's Mechanics, page 866 and page 870. Van Nostrand, 1872.

than anything else? The latter formulas include the value of $\sqrt{2\,g}$ in its constant, and thus gain slightly in simplicity. I was interested a while ago to look up this matter and see how much accuracy it was possible to lose in so doing.

If for the value of $\sqrt{2\,g}$ we always use 8.02, the greatest possible error that it will involve for any place in latitudes anywhere between the southern point of Greenland and Key West, and anywhere from sea level to 5 000 feet elevation will not be more than one-tenth of 1 per cent.

Thus, proper as it may seem to keep this value in sight in a general formula for text-book use, it seems needless to load a practical formula down with it, and we are justified in incorporating merely its average value into the constant of our formula for ready practical use where differences of surface so small they can hardly be noticed may cause 5 per cent. difference in the friction loss.*

On pages 6 and 7 of Mr. Weston's paper certain special formulas are presented for compound pipes and branching pipes. These are presented for practical use, and Mr. Weston uses their derivation as an argument for preferring the Weisbach arrangement of formula. On looking into these with a little care it appears that the formulas are less simple in their application than would appear from the first glance, since when seeking v for a given h in problem No. 5, for instance, the several values of ζ all depend on v, and the problem can therefore be solved only by successive approximations. Second, it seems to me that problem No. 5 especially presents a symmetry in the equal size and length of the pipes in the successive subdivisions, which would almost never occur in ordinary practice, and therefore I do not think the facility in deriving these formulas should be allowed to weigh too heavily in favor of the Weisbach form.

On pages 62 and 63 Mr. Weston copies from Weisbach without change or comment some formulas for effect of curves, which, though sad to tell, are the best yet on record, were derived from very small pipes, and may with good reason be distrusted, and ought not to be quoted in this manner without some limit or qualifications.

It may be noted that at the middle of page 62 the formula for effect of contraction as it stands is without meaning. A proper form is $\phi = \left(\frac{1}{c} - 1\right)^2$ in which c is co-efficient of contraction which depends on the relative area of F_1 and F, and thus needs a table of values to accompany the formula.

In other words, the formula as printed implies that ϕ is a constant whatever the amount of reduction in diameter; whereas, from the nature of the case, it must be a variable of considerable range.

* We might, indeed, in the effort to get things into the most convenient shape for practical use waive even a little more of the theoretical shape and write $Q = c' \sqrt{\frac{d^5 h}{l}}$.

With regard to the table of data and descriptions, occupying from the 8th to the 48th page, it appears that the experiments on two of the pipes there mentioned are original and published for the first time. Nearly all the others, which are reliable enough to found a formula upon, have been discussed in what seems to me a more thorough and searching manner by a distinguished member of our Society, Mr. Hamilton Smith, Jr., in his book published three years ago, and, as already intimated, all have their hydraulic elements and co-efficients adapted for both the Kutter and the Chezy formulas presented in the appendix to Hering and Trautwine's translation of Kutter. It is a matter to be regretted that the loss of flow deduced by Hamilton Smith, Jr., from substantially the same experiments discussed by Weston, was not represented along with the other curves upon the very excellent and convenient diagrams of Mr. Weston. This, prior to Mr. Weston, was the most recent study of the subject, and although Mr. Smith leaves the subject in rather an abrupt and incomplete shape, in that his values for $\triangle$ are not clearly defined, and that the gaps between his curves of values for practical use are rather wide, especially for pipes under 12 inches in diameter—(this was influenced, no doubt, by a lack of data, which still exists)—yet, his conclusions are of great value.

Mr. Smith used substantially the same data as that from which Mr. Weston satisfied himself that the Darcy formulas were satisfactory, and came to a different conclusion and presented a law of his own derivation, and one which, like Mr. Weston's new formula for smooth pipes, made the value of the co-efficient depend on both the diameter and the velocity. It is true Mr. Smith presented his law expressing the values of the co-efficient in graphical form rather than in an algebraic one, with perhaps the same view as expressed by a previous speaker, that a curve or a table is less liable to be used beyond its proper limits than a formula.

We may also refer to the very convenient diagram given in Mr. J. T. Fanning's comments on Mr. Brush's very interesting paper on Waste and Friction Loss in Water Mains,* as another illustration of the use of a diagram for expressing the various values for the co-efficient of flow in terms of both the diameter and velocity.

Finally, I may venture the comment that so far as I yet see Mr. Weston has left the main question just where he found it; that is the question concerning the flow in ordinary cast-iron pipes, which are those that the hydraulic engineer has to deal with in nineteen cases out of twenty. As I read the paper through, it seemed that the main new feature of value presented is that of the formula for very smooth pipes, from ½ to 3½ inches in diameter.

The presentation of the experiments by Mr. Weston upon the 1-inch

* *Transactions*, Vol. XIX, No, 395, p. 112, September, 1888.

tin-lined iron pipe and the 6-inch cast-iron pipe are to be welcomed ; but it is to be regretted that the possible limits of error are not investigated and stated, and the experiments described with the fullness so necessary to give high scientific value—such, for instance, as that with which Hamilton Smith, Jr., describes most of the new data on pipes which he presented to the Society in 1883, and described in his Hydraulics, page 290 *et seq.* As the record now stands we must regard these new experiments merely as approximations.

In the final revision of his paper, I hope Mr. Weston may explain a little more fully the magnitude of the obstruction formed by the sheet tin bushings at each joint, and will give more fully the method employed for determining the loss in the $\frac{5}{8}$ tap and in the main from the reservoir.

As to the 6-inch pipe it would add to the value of the experiments if Mr. Weston would kindly state the possible limits of error in the following directions :

(*a.*) In general even the best Bourdon gauges are not instruments of precision, and scales are liable to unaccountable derangements. Were these gauges tested by a mercury column immediately before and after the experiments, or was their position reversed ? An error of $\frac{1}{2}$ pound to each gauge alone might make experiment 325 11 per cent. in error.

(*b.*) Is it certain piezometer tubes were free of air ?

(*c.*) Were piezometric orifices exactly flush with inner surface and normal to axis of pipe ?

(*d.*) Was diameter of this particular pipe accurately measured, or is the 6 inches the "foundry size ?"

(*e.*) This pipe apparently was not laid with a view to the experiments or inspected after them. Is it absolutely certain it contained no obstruction like a bunch of lead from a bad joint, or that in the four years' use no tuberculation whatever had taken place ?

(*f.*) Were the lengths of hose the same ones that had been gauged, or were they other pieces subject to the ordinary commercial variations in diameter and character of surface of different lots ?

Eternal vigilance and everlasting patience are the price of scientific accuracy in this class of work.

These questions are of interest since Mr. Weston finds a friction loss about 20 or 25 per cent. greater than that found by Darcy for the cast-iron pipe nearest to this in diameter. Nevertheless, even if these questions cannot be answered, and if the experiments are less precise than Mr. Weston would have gladly made had circumstances favored, they are of value and interest as data, and all such are to be welcomed as a practical guide until the longed-for elaborate series of experiments are made by a second Darcy.

The experiments selected by Mr. Weston in developing his formula for smooth pipes are, in some respects, questionable, though undoubtedly they were the best data to be found, and it is no fault of his that better data were not in existence.

Thus: *First.*—Is it not a little doubtful to use experiments 1 to 14 as a basis for a formula, without some statement as to length of pipe and conditions under which experiments were made ? Mr. Smith and Messrs. Hering and Trautwine exclude these from their table of data.

Second.—In making use of experiments 28 and 29 by Rennie, upon a pipe only 15 feet long, and experiments 59 and 60 by Neville, there is liability to error from the fact that in such very short pipes the distribution of velocity and regimen of flow is not fully established at upper portion of pipe. This may be illustrated by reference to Series III and N, page 239, Smith's Hydraulics. Moreover, in each of these cases of Rennie's and Neville's experiments, the co-efficient of influx which has to be guessed at exercises a proportionally larger influence on the total loss of head than for longer pipes.

Third.—For exact scientific purposes it seems hardly fair to class on the ground of smoothness experiments on new wrought-iron lap-welded gas pipe, ⅝-inch and 1-inch diameter, along with the pipes of glass and lead. Is it not uncommon for iron pipe to come from the welding furnace with a skin as smooth as that on a lead pipe as it comes from the die? Moreover, why from description alone is it proper to make use of Smith's experiments on new uncoated 1-inch gas pipe and reject Darcy's experiments on new 1-inch gas pipe, even though results do not agree ?

Passing to the pipes of larger diameters, quoted by Mr. Weston, if the 3½-sheet-iron pipe and also the 7.40-inch and the 11.69-inch of Darcy had riveted seams, as stated by Hering and Trautwine, and as seems natural, it certainly would give a much greater friction loss than a smooth lead pipe, and cannot properly be included.

It is hardly following the true scientific method, but is arguing in a circle, to include experiment No. 58 by Hodson and No. 237 by Dr. Robinson, also Nos. 124 to 139 by Smeaton, as data for determining law of flow in very smooth pipes, merely because they happen to coincide approximately with the formula, while actually the smoothness of the pipe is not stated and the kind of pipe or manner of experiment is not known, and the diameter given by Mr. Weston to thousandths of an inch was apparently merely the nominal or commercial diameter.

Fourth.—For the tin pipe of experiments 146 to 160 we may ask, in order to gain an idea of the smoothness, just what kind of pipe was Bossut's "tin" pipe of one hundred years ago?

Fifth.—It may be of interest to note that the experiments of Leslie on 2½-inch lead pipe, No. 250 to 269, and of Provis, on 1½-inch lead pipe (Nos. 175 to 211), though referred to by Mr. Smith, are by him excluded from his results as unreliable.

Sixth.—Excluding the questionable data, we see that the largest smooth pipe left to furnish data for the new formula is the 2-inch glass pipe of Darcy.

Seventh.—To a person who is experienced in delicate and rigorously accurate hydraulic experimentation, the question will often occur, when trying to reconcile these early experiments, Did the experimenter clearly understand the disturbing influence of air bubbles, and were they driven out and excluded with positive certainty?

As we thus carefully examine the quality of the data we see how unsatisfactory much of it is as a foundation for a new formula for very smooth pipes, and how very few are the really reliable experiments.

If we wish accurate knowledge of the area of a sharply bounded field, is it not better to take a steel tape and a first-class modern transit and go out and survey it, making a clean, fresh, first-class job instead of hunting among the archives and averaging the more or less rough surveys of the past hundred years ?

If, for the time being, we desire working knowledge more than ancient history, and need accurate and convenient knowledge of the laws of flow in very smooth pipes like drawn lead, then if time is money, it would cost very little more to have some good lead pipes made ¼ inch, ½ inch, 1 inch, 1½ inch, 2 inch and 3 inch, a hundred feet of each, handle it delicately, keep it straight and perfect as it comes from the die, experiment on it with velocities from ½ foot to 25 feet per second, then try it bent around certain definite curves, let a plumber coil it up and straighten it out two or three times, crook it around half a dozen corners and then experiment on it again.

This is work which could, for instance, be very easily done in the new hydraulic laboratory now under construction for the Massachusetts Institute of Technology, and I sincerely hope it may be undertaken somewhere and the work done on a suitable scale and under conditions that may make the results unquestionable.

Then with the work once thoroughly well done, the results, with their own sharply defined limits, may stand unchallenged for a hundred years.

We all owe thanks to Mr. Weston for the diagrams appended to his paper, which show in an extremely convenient and interesting manner the way in which our available data agrees with various standard formulas.

To criticise the investigation or to detract from the just praise due Mr. Weston for his industry in presenting these matters is far from my purpose.

The main points I would make are:

First.—In securing the data now on record far too little attention has been paid by experimenters to ascertaining and describing the exact degree of smoothness of the wetted surface.

Second.—Investigators in attempting to derive formulas from the data have given too little attention to incorporating in their formulas a factor dependent on this roughness.

Third.—The profession greatly needs a new and extended series of experiments before satisfactory general laws or formulas can be derived.

Fourth.—Until we get these it is better to make use of very simple and familiar formulas like that of Chezy, and devote our opportunities to extending, tabulating and classifying values of the co-efficient.

In closing I may say that my own experience goes to show that a table on the plan of the excellent little table published by George A. Ellis, C.E., about ten years ago, but constructed with smaller gaps between quantities and diameters, is more convenient for practical use than any formulas or the tables in any of the engineering pocket reference books.

The plan of this is as below:

Gallons discharged per minute.	4-Inch Pipe.		6-Inch Pipe.		Etc.
	Mean velocity. Feet per second.	Loss of head per 100 feet.	Mean velocity. Feet per second.	Loss of head per 100 feet.	

If appended to this we could have compiled a table of factors by which to multiply these losses of head to allow for different degrees of roughness, the whole would be of the utmost convenience, and I hope some one who is ready to take his pay in the gratitude of the profession will prepare such a table based on some standard investigation.

Mr. Weston.—I think it is fortunate in many respects that all of the members of the human race have not the same opinions, for if such was the case, there would be but little I fear to stimulate progress in the direction of making new discoveries and improvements. Therefore, in my reply to the somewhat lengthy discussion of Mr. Freeman, I shall endeavor to confine myself mainly to answering the direct questions that he has propounded, and to replying to a number of his adverse criticisms that I consider were based upon misapprehensions, which, if I have surmised correctly, may very probably have been owing to his not having had a sufficient length of time at his disposal to make a thorough examination of my paper.

In reply to the remarks of Mr. Freeman (page 74), which commence thus: "It seems to me that Mr. Weston is off the true road toward a thoroughly satisfactory formula," * * * * and end thus: * * * * "In one case his variable co-efficient is a function of only the diameter as a variable, in another of only the velocity, and in another of both the diameter and the velocity "—I will say: *First*—What I have already intimated in my reply to the discussion of Mr. Hering, viz.: that I was governed by the experimental data that I possessed, and that the first preliminary investigations that I made relative to constructing a formula were to see if one could not be formed that would be sufficiently broad and general in its application to cover all kinds of pipes, and which would contain a factor dependent upon the roughness of the interior surface of the pipe. I was soon convinced, however, that a reliable formula of this kind could not be constructed. *Second*—That it strikes me the application of the formulas that I have recommended are just the reverse of narrow, as one of the formulas can be applied to all pipes having very smooth interior sides, from ½-inch to 3½ inches in diameter (pipes of this kind larger than 3½ inches being very rarely used for conveying water), and the other formula can be applied to all pipes having interior sides similar to new cast-iron pipes. *Third*—That a careful examination of my paper will show that I recommend the latter formula (Darcy's) on account of its conforming closely to experimental results, and for its simplicity and not for its particular form. *Fourth*—That the other formulas that I have constructed, that have been mentioned in my paper, were computed from a limited number of experimental results and were not recommended for general use.

In response to that portion of the discussion of Mr. Freeman (page 75), which commences thus: "While discussing the desirability of including a special co-efficient of roughness in our future formula," * * * * and ends thus: * * * * "Nearly all of the five hundred experiments tabulated and discussed by Mr. Weston are these tabulated with the Kutter co-efficient of roughness computed for each"—I will state that in my opinion there are reasons for thinking that it would be questionable to apply the "Kutter formula" to pipes in which water was flowing under pressure, as this formula was constructed from experiments that were made with water flowing in open channels. But assuming that in special cases "Kutter's formula" was adapted for the purpose, and that the tabulated experimental co-efficients of roughness mentioned by Mr. Freeman were available, the results that would be obtained by using these co-efficients in this complicated formula would not be more accurate, and the opportunities for exercising personal judgment as to the relative degree of roughness of the interior surface of the pipes more facilitated, than they would if Table No. 1 of my paper was referred to, in which each class of experiment is specified, and an experimental co-efficient of friction (ζ) selected and used in the simple formula,

$$v = \frac{\sqrt{2\,gh}}{\sqrt{\zeta\frac{e}{d}}}$$

In reply to Mr. Freeman's advocacy of the Chezy formula, I would call attention to pages 4, 5, 6 and 7 of my paper, where I discuss the form of formula the best adapted for the flow of water in pipes.

I will say in reply to the remarks of Mr. Freeman (page 78) relative to the forms of formulas that I have given on pages 6 and 7, that the values of the co-efficients of friction (ζ) that are included in these formulas do not necessarily depend upon the velocity (v), even in the first formula of each example which is arranged for computing the velocity for a given head, as co-efficients to friction ζ) derived from "Darcy's formula," an expression for which I have given on pages 57 and 61, can be used and the velocity determined without approximation; also when high velocities enter into the problem, there will be but little need of approximating when using co-efficients of friction (ζ) dependent upon the velocities that have been determined from the majority of known formulas, as their values change very slowly after reaching a velocity of 5 feet per second. Then the second formula in each example is arranged for computing the loss of head due to friction (h_f), and can be directly solved; even if co-efficients of friction (ζ), dependent upon the velocity, are used as in these cases, the velocity in the outlet pipe or pipes is one of the known quantites, from which, in cases of branch or compound pipes, the velocities in the other pipes can be readily determined and co-efficients of friction (ζ) selected corresponding to them.

In respect to the co-efficients of resistance due to contraction (ϕ), mentioned by Mr. Freeman (page 78), my reply to Professor Merriman's discussion expresses my ideas upon the subject.

The reason why I did not plat upon the diagrams of my paper the results of the conclusions of Mr. Smith, given in his work referred to by Mr. Freeman (page 79), is, that my investigations were relative to formulas for the flow of water in pipes, and Mr. Smith only gives graphical results, and not formulas.

In reply to the statement of Mr. Freeman (page 79), viz.: "Mr. Smith used substantially the same data as that from which Mr. Weston satisfied himself that the Darcy formula was satisfactory, and came to a different conclusion" * * * * — I would call attention to the diagrams from No. 14 to No. 22, inclusive, which express very plainly my reasons for recommending the formulas of Darcy for new cast-iron pipes.

In regard to my experiment with the 1-inch pipe mentioned by Mr. Freeman (page 80), I will remark: *First*—that the condition of the interior surface of the pipe, with the tin bushings in place, was about the same as the interior surface of an ordinary lead pipe, as I have already stated. I came to this conclusion by a careful examination of the different lengths as they were joined together while being laid. *Second*—that there is a possibility that the data given in Table No. 1, relating to this experiment, may be in error to the amount of 1 per cent. *Third*—that the loss of head in the $\frac{5}{8}$-inch tap was determined as follows: A short piece of lead pipe was soldered to the tap, before the 1-inch tin-lined pipe was connected, and a loss of head ascertained due to the discharge under the reservoir pressure, by measuring the flow. This loss of head was then compared with a diagram that had previously been constructed from the results of experiments, which covered a wide range of velocities, that had been made with other taps of the same kind connected to other main pipes, and as it agreed almost exactly with the law that had been developed upon the diagram, a loss of head was scaled from the diagram corresponding to a velocity of 6.03 feet per second, which was the velocity in the 1-inch tin-lined pipe during the experiment. The loss of head thus obtained was then added to the head required to generate the velocity and their sum subtracted from the total or reservoir head, the result of which is given in table No. 1 as the loss of head in the 1-inch tin-lined pipe. A considerable difference in judgment, however, one way or the other, in determining the loss of head in the $\frac{5}{8}$-inch tap, etc., would not have materially affected the results given in the table, owing to the small proportion that it would bear to the total loss of head, on account of the long length of the 1-inch tin-lined pipe; consequently, I did not consider the method followed in obtaining it of sufficient importance to be described in detail. There was not any measurable loss of head in the 36-inch main to which the $\frac{5}{8}$-inch was connected—about 5 000 feet from the reservoir—as the only water moving in it at the time (the main being full and under the reservoir pressure) was the small quantity that was used in making the experiments.

I will answer Mr. Freeman's questions relative to the possible error of my experiments, made with a 6-inch pipe, as follows:

(*a.*) The gauges, which were very reliable, were tested a short time before the experiments were commenced with a mercurial column, and directly before and after the experiments with a large test gauge that was kept especially for the purpose, and which had also been tested a short time before the experiments were commenced with a mercurial column. The dials of the gauges were 7 inches in diameter, graduated to $\frac{1}{2}$ pounds, and read from 0 to 100. The gauges were each observed at least twenty minutes during each experiment.

(*b.*) I do not think there is a question of doubt but that the piezometers were free from air, as great care was taken to blow them off before each experiment, and they were always full of water under pressure.

(*c.*) The piezometric orifices were exactly flush with the interior surface of the pipe, and normal to its axis.

(*d.*) The diameter of the pipe given in Table No. 1 is the "foundry size," as it was not possible under the circumstances, the pipe being under ground, to measure the diameter; but the pipe was exceedingly well made, it having been furnished by one of the best foundries in the country.

(*e.*) It is almost absolutely certain that the pipe did not contain any obstacles like bunches of lead, etc., as each length of pipe was carefully inspected while being laid by a careful and experienced inspector. From what I have seen by examining pieces of other pipe of the same kind, that have been cut out while making repairs or connections and which had been in service about an equal length of time, in Providence, where the experiments were made, I should say that it was probable that the interior surface of the pipe was not quite as smooth as when the pipe was laid, owing to tubercles that may have formed at odd places when the coal-tar coating may have been thin.

(*f.*) The lengths of hose were those that had been gauged.

I have long recognized what Mr. Freeman states, viz.: "that eternal vigilance and everlasting patience are the price of scientific accuracy in this class of work."

In reply to the comments of Mr. Freeman (page 81), which commence thus: "The experiments selected by Mr. Weston in developing his formula for very smooth pipes are in some respects questionable," * * * * and end thus: * * * * "Moreover, in each of these cases of Rennie's and Neville's experiments the co-efficient of influx which has to be guessed at exercises a proportionately larger influence on the total loss of head than for longer pipes"—I will say: *First*—that the data of the experiments, from No. 1 to No. 14, were considered reliable by such an eminent authority as Professor Weisbach, from one of whose works I obtained them, as I have before mentioned, and that I was not able to secure any other information concerning them than what I have given in Table No. 1. *Second*—that Mr. Freeman is in error when he insinuates that I made use of Rennie's experiments in developing my formula, as I only used these experiments for comparison after the formula was constructed, as may be seen in Table No. 1, and upon Diagrams Nos. 1 and 7. *Third*—that the co-efficient of influx was not guessed at that was used with Neville's experiments, as a co-efficient was determined by experiment by Neville especially for these cases, as I have already stated. *Fourth*—that I take exceptions to Mr. Freeman's idea relating to the regimen of flow not being fully established in the pipes that were used by Neville and Rennie in making their experiments. Such might have been the case if the velocities had been very low, but as the velocity of flow in the pipe used by Rennie was not less than 2.35 feet per second, and the velocity of flow in the pipe used by Neville not less than 14.58 feet per second, I do not think there is any question but that the regimen was fully established. As bearing upon the subject, I will state that I have found in experimenting with a $\frac{3}{4}$-inch pipe, 6 feet in length, that with as low a velocity as 0.90 feet per second, the outlet end of the pipe was flowing full and the water running clear.

In answer to Mr. Freeman's remarks (page 81), which commence thus: "For exact scientific purposes it seems hardly fair to class on the ground of smoothness experiments on new wrought-iron lap-welded gas pipe, $\frac{5}{8}$ inch and 1 inch in diameter, along with the pipes of glass and lead," * * * * and end thus, * * * * "Give a much greater friction loss than a smooth lead pipe, and cannot properly be included "—I wish to say: *First*—what I have previously stated on page 49, viz.: That the experiments that were made with the new wrought-iron gas pipe, mentioned by Mr. Freeman, were not given the same weight in the investigations as those that were known to have been made with pipes having very smooth interior sides, but were more especially used for the purpose of substantiating the laws and results obtained with the latter. *Second*—that I did not make use of Darcy's experiments which had been made with a new 1-inch wrought-iron pipe in the same manner that I did those of Smith, for the reason that I was well assured that wrought-iron pipes manufactured in France at the time Darcy made his experiments were not as well made, nor did they have nearly as smooth interior sides, as the gas pipe manufactured at the present time. *Third*—that I did not use in determining my general formula for pipes having very smooth interior sides, as I have stated on page 49, the experiments of Darcy that were made with riveted sheet-iron pipe, 7.40 inch and 11.69 inch in diameter.

In reply to the statements of Mr. Freeman (page 81), which commence thus: "It is hardly following the true scientific method, but is arguing in a circle," * * * * and end thus, * * * * "Was apparently merely the nominal or commercial diameter "—I will say: *First*—that as the nature of the pipe used in making the two experiments, Nos. 58 and 237, was questionable, I gave myself the benefit of the doubt, knowing that these two experiments would not appreciably influence the results which I should obtain, owing to the large number of other experiments that were used, which were made with pipes known to have very smooth interior sides. *Second*—that I have stated on page 49 that the data obtained from the experiments from No. 124 to No. 139 were not given the same weight in the investigations as the data derived from experiments that were made with pipes having very smooth interior sides:

I will say in reply to the following remarks of Mr. Freeman (page 81): "It may be of interest to note that the experiments of Leslie on 2½-inch lead pipe (Nos. 250 to 269), and of Provis on 1½-inch lead pipe (Nos. 175 to 211), though referred to by Mr. Smith, are by him excluded from his results as unreliable "—*First*—that I think I was justified in using the experiments of Leslie and Provis as I did, more especially the former, as I made particular inquiries of Mr. Leslie in regard to them. *Second*—that if the descriptions that I have given are not sufficiently convincing as to the value of these two sets of experiments, I would suggest to those who are specially interested in the subject that they examine the original papers that were written by these two experimenters.

In reply to this question of Mr. Freeman (page 82): * * * "Did the experimenter clearly understand the disturbing influence of air bubbles, and were they driven out and excluded with positive certainty?"—I will remark that the impression that I have derived by reading Darcy's description of his experiments is, that he decidedly knew what he was about.

To the following statement of Mr. Freeman (page 82): "As we thus carefully examine the quality of the data we see how unsatisfactory much of it is as a foundation for a new formula for very smooth pipes, and how very few are the really reliable experiments "—I will reply by saying that I think Mr. Freeman is decidedly wrong when he makes the assertion that very few of the experiments upon which I have based my new formula for very smooth pipes are really reliable, as I maintain that the reliability of the greater part of them should not be questioned, and I think a careful examination of my paper will prove that I am right in this respect, even if I have failed to show that the majority of Mr. Freeman's exceptions to my work, that he has mentioned in detail in his discussion and which I have previously replied to, are unwarranted.

I do not consider Mr. Freeman's comparison (page 82) of surveying a field with the making of hydraulic experiments, a fair one, and I am somewhat surprised after his recent experience in making extensive experiments with fire hose and nozzles,* that he should suggest such a comparison; and I can assure Mr. Freeman, from the results of my own experience, that if I could have made experiments relating to the flow of water in pipes as easily and with as little expense as I could have surveyed a field, I should not have given him or any other member of the Society an opportunity to criticise any experiments other than my own.

I will conclude by remarking, as Mr. Freeman has referred to tables relating to the flow of water in pipes (page 83), that I have appended to my paper in convenient form for platting, a short table that has been

* *Transactions*, Vol. XXI, No. 426, November, 1889; "Experiments Relating to Hydraulics of Fire Streams," by John R. Freeman.

calculated from my new formula for the flow of water in very smooth pipes. And that a table for the flow of water in new cast-iron pipes that was computed under my direction from Darcy's formulas, which is similar to the table mentioned by Mr. Freeman, though more complete, may be found in the appendix to the Report of the City Engineer of Providence, for the year 1888. I would also call attention to two other convenient tables relating to the same subject, one of which is calculated from Weisbach's formula, and is included in the discussion of a paper by James Leslie in the "Excerpt Minutes of Proceedings of the Institution of Civil Engineers, Vol. XIV, Session 1854–55," and the other is appended to a "Rudimentary Treatise on Civil Engineering, by Henry Law, C. E.," that was published in London, by John Weale.