THE SLIDE RULE.

(DUPLEX.)

THE

SLIDE RULE.

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BY WM. COX.

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CONTENTS.

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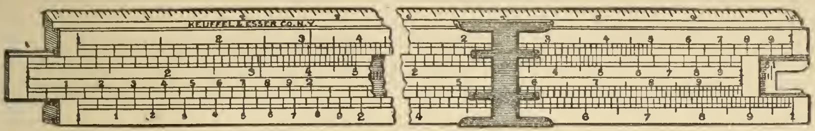

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THE SLIDE RULE.

BY WM. COX.

1.—INTRODUCTION.

As the usefulness of the Slide Rule for facilitating the working out of complicated calculations is not confined to the Engineer alone, but may be equally taken advantage of by the Merchant, Importer, Exporter, Spinner, Manufacturer, etc., we purpose in these introductory remarks to explain as briefly and simply as possible the principles upon whose basis this instrument is constructed. To those who are in the habit of using tables of logarithms, these principles will be self-evident. We therefore request them to excuse the introduction of this, to them, irrelevant matter.

There is a very general impression that the acquirement of a facile use of the Slide Rule is both tedious and difficult. This, however, is not the case. It may be easily learnt in spare moments, advantage being taken of these to attain to proficiency by frequent practice.

The construction of the Slide Rule is based upon logarithms, but the principles which require mastering are few and simple, and a little attention to the following observations will enable any one to understand them. Each step of the ladder once firmly gained will then become, not only an incentive, but an opening up of the way to mount higher, by showing the logical consequences of the knowledge already acquired.

Two general principles must at the outset be clearly understood, viz :

1st. A *unit* may be represented by a SPACE of any length measured off from a given point: and a *number* composed of two or more units may be represented by a DISTANCE composed of the same number of spaces. These distances may be increased by adding or joining on to them other spaces or distances, or decreased by taking from them some of the spaces or distances of which they are composed.

2d. Logarithms are a series of numbers in Arithmetical Progression (as 0, 1, 2, 3, 4, etc.), corresponding to another series of numbers in Geometrical Progression (as 1, 2, 4, 8, 16, etc.)

We will take two such series and place them together, thus :—

0	1	2	3	4	5	6	7	8	9	10
1	2	4	8	16	32	64	128	256	512	1024

Here the first line is a series of numbers in A. P. and they are the logarithms of the corresponding numbers in the second line, which is a series of numbers in G. P.

A little examination of the peculiar properties of these two series will enable us to understand how these properties have been taken advantage of and applied to the Slide Rule.

1st. If we add together any two numbers of the first line as 3 and 5, their *sum* 8 corresponds with 256 of the second line. Now 256 will be found to be the *product* of the two factors 8 and 32, which on the second line correspond with 3 and 5 on the first line.

2d. If we subtract one figure of the first line from any other figure of the same line, as 9 minus 6, their *difference* 3 corresponds with 8 of the second line, and 8 will be found to be the *quotient* of $512 \div 64$, which on the second line correspond with 9 and 6 on the first line.

3d. If we multiply any figure of the first line by 2, as 3×2, the *product* 6 corresponds with 64 of the second line, which is the *square* of 8, which on the second line corresponds with 3 on the first line.

4th. If we divide any figure of the first line by 2, as $8 \div 2$, the *quotient* 4 corresponds with 16 of the second line, and this 16 is the *square root* of 256, which on the second line corresponds with 8 of the first line.

5th. Likewise, if we multiply or divide by 3, 4, etc., we obtain the 3d, 4th, etc., power or root.

These are the peculiar properties of logarithms, which are of such immense advantage in the working out of abstruse calculations, and they have been applied to the Slide Rule by means of our first general principle.

A series of *equal* spaces are taken to represent the numbers of the series in arithmetic progression, and to these are affixed the corresponding numbers of the geometric series.

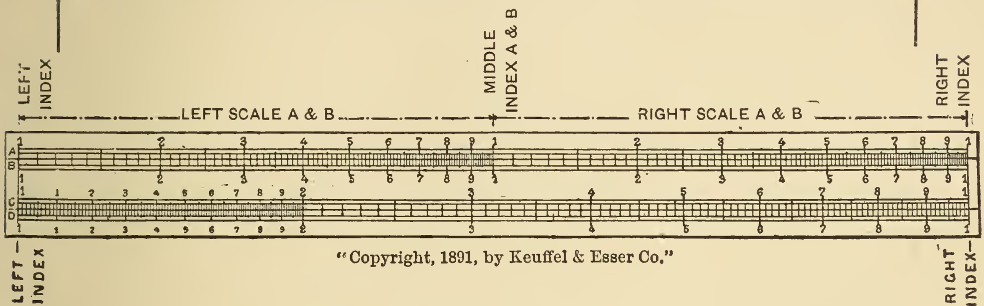
If we thus take any given length, say ten inches, and divide it into 1000 equal parts, and if against the starting point or zero we place the figure 1, then at the end of 301 parts we place the figure 2 ; at the end of 602 such parts we place the figure 4 ; and at the end of 903 parts we place the figure 8, we obtain a logarithmic scale, which, to complete it, merely requires assigning to it the exact position of the other and intermediate figures up to 10. Such a scale is found on the Slide Rule; the Slide, which is marked like the Rule, being for the purpose of adding or joining a given number of spaces on to any other given number of spaces taken on the Rule, just as if we marked off three inches on a sheet of paper, and taking two inches in the opening of a pair of compasses or on a slip of paper, joined these on to the former three, thus indicating on the sheet of paper the position of 5, or the *sum* of 3 and 2 ; but with this difference, that on the Slide Rule the adding of 2 spaces to 3 gives the *product* 6 and not the sum 5. In the same way, the Slide serves to take any number of spaces from any other given number on the Rule, the result of which, equivalent to subtraction in ordinary arithmetic, becomes division when a logarithmic scale is used.

The various operations therefore of Multiplication, Division, etc., are performed exactly as described in the case of two series of numbers in arithmetical and geometrical progression.

2.—DESCRIPTION OF THE MANNHEIM SLIDE RULE.

The Slide Rule, as recently perfected by Mannheim, an officer of Artillery at Metz, is generally made of well-seasoned box-wood, and is about 10 inches long, $1\frac{1}{2}$ inches broad, and $\frac{1}{4}$ to $\frac{3}{8}$ inches thick. In a later form introduced by us, and embodying the same improvement, it is made of mahogany and faced with *celluloid*, on which are engraved the graduations, the strong contrast between the white ground of the celluloid and the black lines rendering the "reading" easier than in the case of the less distinct graduations on wood. Along the centre a slip of the same material *slides* easily from left to right and right to left, in a groove to which it is accurately fitted, its face being perfectly level with the Rule. This is the Slide.

Reference to the accompanying figure will make the following explanations clear.



On the Rule will be seen, along its whole length, and close to the upper edge of the groove, a series of graduations, with an identically similar series along the upper edge of the Slide. These form the upper scales, called for easy reference A and B.

Another series of graduations will also be seen on the Rule along the lower edge of the groove, with a corresponding series on the Slide. These are called C and D, and are the lower scales. The starting or initial points of each scale on the left hand, marked 1, are the left indices, and the terminating points on the right, marked also 1, are called the right indices. It will be seen that the scales C and D are numbered 1, 2, 3, etc., from the left to . . . 9, 1 on the right. (The reason why this latter is not marked 10 will be explained shortly.) As would be gathered from our introductory remarks, it will be found that the space from 1 to 2 is the same length as that from 2 to 4, and also from 4 to 8. It is therefore a logarithmic scale, and joining on the distance 1 to 2 to itself by means of the slide, gives the product of 2×2, or 4. So also joining on the distance 1 to 2 to the distance 1 to 4, gives the product of 4×2, or 8.

That portion of the scales on C and D between 1 and 2 will be seen to be again divided into ten parts, also marked 1, 2, 3, etc., and each of these parts is again divided into ten other small parts. These subdivisions would be carried throughout if the spaces admitted of it, but on account of their decreasing size, they are afterwards divided into ten parts and subdivided into halves or fifths, which must be considered as decimal divisions and not fractional ones.

It must now be clearly understood that the figures as marked are arbitrary, and that the initial 1 or left index may have *any value* which is a decimal part or a multiple of 1, assigned to it, such as 10, 100, .1, .01, etc., but this same ratio must be observed throughout the whole scale, which then becomes 20, 30; 200, 300; .2, .3; .02, 03, etc. All the subdivisions are consequently decimal ones, and have their real value assigned to them by the fixing of the position of the decimal point, which depends upon the primary value given to the initial 1. If, for example, we give the left index the value of 100, the other figures will be 200, 300 . . .

900, 1000. The divisions between the left index and 2 will be 110, 120 . . . 190, 200, and the subdivisions between the initial 1 and the following small 1 will be 101, 102 . . . 109, 110. The subdivisions between 2 and 3 will be 202, 204 . . 208, 210, and the divisions will be 200, 210, 220 . . . 290, 300. From 4 onwards there are ten divisions between each two figures, which are again subdivided into halves. Assuming the left index to be still 100, these become 400, 405, 410, 415 . . . . 485, 490, 495, 500, and so on to the right index. It will therefore be seen that from 1 to 2 there are graduations which indicate the exact position of numbers composed of three figures; from 2 to 4, for numbers of two figures and fifths; and from 4 to the end of the scales, for numbers of two figures and halves. As we have stated, the value of the initial 1 may vary, so any of these numbers, such as 485, may have the value of 48.5; 4.85; .485; 48500, etc. This real value is entirely dependent upon the problem to be solved. If a number of more than three figures be needed, the position of the others must be "estimated" by the eye; and a little practice will soon render the operator proficient, so that he will be able without difficulty to read off all numbers of three and four figures.

If lines A and B of our illustration be now examined, it will be at once noticed that there are on the Rule, as also on the Slide, two distinct but similar scales, each marked exactly alike from the left index 1, 2, 3 . . . 9, 1, to the middle index, and then commencing again 1, 2, 3 . . . 9, 1, ending at the right index. If we give to the left index on the upper and lower scales the value of 1, we shall have on A and B, scales from 1 to 10, and from 10 to 100, each space of which is exactly half of the corresponding space on C and D. It will further be seen that 2 on the lower scales coincides with its square 4 on the upper scales, 3 with its square 9, and 10 on the lower scales with its square 100 on the upper scales. The scales A and B, being (numerically considered) twice as long as the scales C and D, are by reason of their graduations, and according to the third and fourth properties of logarithms, as explained in the introduction, scales of *squares* of all their coinciding numbers on C and D. In like manner scales C and D form *square roots* of all their coinciding numbers on A and B.

Each instrument is provided with a brass Runner, which enables coinciding points to be found on any of the scales, and also permits of extensive calculations being worked out without the necessity of "reading off" the intermediate results, thus securing a greater degree of accuracy in the final one.

Such is the complete Mannheim Slide Rule. Its successful use lies largely in the ability to read the graduations rapidly and correctly. This can only be acquired by actual practice. It must ever be borne in mind that whatever value is assigned to the left index, all the graduations are *decimal* ones whose real value depends upon that of the index; also that a fixed relationship exists between the upper and the lower scales, and that whatever be the initial value of C and D, the initial value of A and B *must* be its *square.*

3.—HOW TO USE THE SLIDE RULE.

Multiplication, Division, Proportion.

All calculations in multiplication, division and proportion are worked out on scales C and D, as by reason of the greater space allotted to each of the divisions, the results obtained are more accurate. We shall commence with proportion, as into it are resolved in reality, and with the Slide Rule, *in actual practice,* all sums of multiplication and division.

A leading principle of the Slide Rule is that, place the Slide as we will, *all* the numbers on the Slide bear the *same proportion* to their coinciding numbers on the Rule. Let us take the proportion or ratio of 2 to 4. We draw out the Slide towards the right until 2 on C is exactly over or coincides with 4 on D. It will then be seen that this ratio of 2 to 4 exists between every pair of coinciding numbers, as 1 : 2, 3 : 6, 42 : 84, 115 : 230, etc. So with any other ratio. It is evident, therefore, that if we set the two first terms of a proportion against each other on the Slide and Rule, we shall find the third term on the Slide coinciding with the fourth term on the Rule. We have, therefore, the following general statement:

$$\left.\begin{matrix}\text{Any number}\\ \text{on C}\end{matrix}\right\} : \left\{\begin{matrix}\text{The number}\\ \text{under it on D}\end{matrix}\right\} :: \left\{\begin{matrix}\text{Any other}\\ \text{number on C}\end{matrix}\right\} : \left\{\begin{matrix}\text{The number}\\ \text{under it on D}\end{matrix}\right.$$

We will now give the rule, and that in a diagrammatical form, which appeals to the eye, and simplifies the putting of it in practice, thus:

C	Set first term	Under third term
D	Over second term	Find fourth term;

the full meaning of which is, set the number corresponding to the first term on Slide C over the number corresponding to the second term on Rule D, and under the number corresponding to the third term on Slide C will be found the number corresponding to the fourth term on Rule D.

The reader is requested to take note of this formula or manner of stating a rule, as the same plan will be adopted throughout. It shows graphically the method of procedure, and is much simpler than a long statement in words.

Example: $12 : 21 :: 30 : X$, or $\frac{21 \times 30}{12} = 52.5$.

C	Set 12	Under 30
D	On 21	Find 52.5—Answer.

We shall not give here the rules for the position of the decimal point, or for ascertaining of how many figures the integral portion of the answer is composed, as they will be given in full further on. In most cases in actual practice it is self-evident, or easily determined by a glance at the figures of the problem.

This principle of similar proportions or ratios is one of the most important advantages of the Side Rule, as it enables a vast number of calculations to be made in the simplest manner possible. Thus, to find the circumference of a circle from its diameter,

C	Set 226	Under ANY diameter
D	On 710	Find corresponding circumference.

The figures 226 and 710 express the ratio which always exists between the diameter of a circle and its circumference, and is equal to 1 : 3.14159, being correct to six places of decimals. Such ratios are termed Equivalents or Gauge Points, and being given in this form are much easier to set on the Slide Rule than the ordinary form of reciprocals. These equivalents may be worked out for any class of trade, for the

Engineer, the Importer, the Exporter, etc. Thus a ratio or gauge point may be found to express in dollars per yard the price of French silks at so many francs per metre.

Example: What would be the price, in dollars per yard, of goods bought at 4 francs per metre, exchange being taken at $3.80 for 20 francs?

C	Set 800	Under 4
D	On 139	Find 695=69½ cents per yard.

A table of values in dollars per yard could thus be made out with the greatest facility and exactitude.

We have said that MULTIPLICATION and DIVISION resolve themselves into proportion, for what else is the proportion 1 : 4 : : 3 : 12 but a simple sum in multiplication, 4 and 3 being the two factors, and 12 being their product. With the Slide Rule this becomes

C	Set 1	Under 3
D	On 4	Find 12—the product.

The rule for MULTIPLICATION is therefore

C	Set 1	Under the other factor
D	On one factor	Find their product.

So of Division, which is merely the reverse of the above, and the proportion becomes 3 : 12 : : 1 : 4, and on the Slide Rule

C	Set 3	Under 1
D	On 12	Find 4—the quotient.

The rule for DIVISION is therefore

C	Set divisor	Under 1
D	On dividend	Find the quotient.

In these two examples the expressions "Set 1" and "Under 1" refer to either the left or the right index, according as the circumstances of the case require.

It will be found in some cases that the result of an operation lies beyond the rule either to the right or the left. For instance, the full working out of the example giving the price of French silks would be

C	Set 800	Bring Runner to right index	Draw slide to the right until left index is under Runner	Under 4
D	On 139			Find 695.

With the ordinary Slide Rule, the result of the first setting, which is the quotient of 139÷800, would have to be "read off," and the left index of the Slide set to it, so that the multiplication of this result by 4 might be effected. The use of the Runner does away with this, and consequently diminishes the chances of error which might be caused by an incorrect reading or re-setting of the Slide. The Runner is also of immense advantage when several factors have to be multiplied together, the final result being alone noted.

Example: $12 \times 4 \times 5 \times 3 = 720$ is worked thus:

C	Set 1	Runner to 4	1 to Runner	Runner to 5	1 to Runner	Under 3
D	On 12					Find 720—Answer.

In the same way several divisions can be easily performed, and also a combination of multiplications and divisions, as in the case of a train of wheels, as

$$\frac{71 \times 21.4 \times 35 \times 17}{8.5 \times 42 \times 5.8 \times 20} = 21.8, \text{ which is worked thus:}$$

C	Set 85	Runner to 214	42 to R	R to 35	58 to R	R to 1	20 to R	Under 17
D	On 71							21.8–Answer.

A little practice will enable similar problems to be worked out with ease. No notice has so far been taken of the real position of the decimal points, the finding of which has been left to inspection, which in many problems is all that is necessary. We now give the rules in full with examples and explanations.

4.—THE POSITION OF THE DECIMAL POINT.

The following are the rules for ascertaining the number of digits or figures of which the integral portion of any result is composed, or, in other words, the position of the decimal point. The number of figures in the integral part of a number is called its *characteristic*, thus:

2	is	the characteristic	of	24.135.
1	"	"	"	2.730.
0	"	"	"	0.583.
—1	"	"	"	0.079.

Multiplication.—If the product is obtained with the Slide projecting to the *left*, its characteristic is the *sum* of the characteristics of the two factors; but if the Slide projects to the *right*, the characteristic of the product is the sum of the characteristics of the two factors *less* 1.

Example: $45 \times 2.5 = 112.5$.

The sum of the characteristics of the two factors is 3, and the Slide projects to the left, therefore the integral portion of the product is composed of three figures.

Example: $3.3 \times 18 = 59.4$.

The sum of the characteristics of the two factors is 3, but to obtain the result, the Slide projects to the right. The product contains, therefore, 3—1, or two figures only in the integer.

Division.—If the quotient is obtained with the Slide projecting to the *left*, its characteristic is the characteristic of the dividend, minus the characteristic of the divisor; but if the Slide projects to the *right*, this difference must be *increased* by 1.

Example: $33 \div 7.5 = 4.4$.

The characteristic of the dividend, less the characteristic of the divisor, is 1, and as the Slide projects to the left, the integral part of the quotient contains one figure.

Example: $65.1 \div 21 = 3.1$.

The characteristic of the dividend, less the characteristic of the divisor, is 0, but as the Slide projects to the right, the quotient contains $0 + 1$, or one digit in its integral portion.

These four rules may be put into the following form, so that the eye may "catch" them at once.

When the SLIDE Projects to the

	LEFT	RIGHT
	the CHARACTERISTIC of the RESULT is the	
MULTIPLICATION	SUM of the Characteristics of the 2 factors.	SUM—1 of the Characteristics of the 2 factors.
DIVISION	Characteristic of DIVIDEND, less char'c of DIVISOR.	Characteristic +1 of DIVIDEND, less Char'c of DIVISOR.

PROPORTION.—Special rules might be given, but we believe the simplest plan is to adopt the preceding ones. The first operation being division, the characteristic of the quotient is easily obtained; this quotient being then one of the factors in the succeeding multiplication, the final characteristic is found by the rule already given.

Example: 12 : 21 : : 30 : 52.5.

1st operation, $21 \div 12$. Slide on right gives $(2 + 1) - 2 =$ characteristic 1.

2d operation, quotient $\times$ 30. Slide on right gives $1 + (2 - 1) =$ characteristic 2, that is, there are two figures in the integral part of the answer. In ordinary practice, and especially in the case of combinations of multiplications and divisions, the characteristics would be set down in the order in which they occur. Thus in the example of combinations, we have $1 + 2 - 2 + 2 - 1 - 2 + 1 + 1 = 7 - 5 = 2$ figures in the integral portion of the result, which becomes 21.8. A little study and practice will make these rules self-evident, and their application very easy.

5.—THE INVERTED SLIDE.

It frequently happens that in proportion *more* requires *less*, or *less* requires *more*. This is Inverse Proportion, and is most simply worked out by *inverting* the Slide, so that C shall lie along A, and B along D. The lines C and D then form throughout a series of inverse ratios, the "reading off" of which is effected by means of the Runner.

Example: If 6 men can build a wall in 4 days, how long will it take 12 men to build it?

This becomes 6 : 4 : : 12 : 2, or $\frac{6 \times 4}{12} = 2$, and on the Slide Rule

C Inverted	Set 6	Under 12
D	To 4	Find 2—Answer.

As proportion is composed of the two operations of multiplication and division, it naturally follows that these can be also worked out with the Slide inverted. The following are the formulæ:

MULTIPLICATION.	C Inverted	Set one factor	Under 1
	D	To other factor	Find product.

DIVISION.	C Inverted	Set 1	Under Divisor
	D	To Dividend	Find Quotient.

It will be noticed that the *product* of all the coinciding numbers on C and D is equal to the number found on D, under either index on C, thus:

C. Inv'd	Set 1	Find 9	8	6	4.8	4	3	2.4	2
D	To 144	16	18	24	30	36	48	60	72

In this way a series of factors is obtained whose product is always the same. A great many other calculations are worked out with advantage with the Slide inverted, illustrations of which are given further on. The rules for the position of the decimal point are just the reverse of those already given.

It may be interesting to some readers if we add a few words respecting the Inverted Slide. If the Slide is thus placed in the Rule with the right and left indices corresponding, it will be found that the product of every number on the Slide and its coinciding number on the Rule is always 10 or unity: whence it follows that the numbers on the Slide are the *reciprocals* of their coinciding numbers on the Rule, and vice-versa.

The rule for multiplication, with the Slide in its ordinary position, requiring one factor to be taken on the Slide, it follows that that factor, if the Slide be inverted, is the reciprocal of some other number, and multiplying by it in the ordinary way becomes consequently equivalent to dividing by that other number. Hence, the rules for multiplication, division and the decimal point are exactly reversed when the Slide is inverted.

6.—THE UPPER SCALES, AND SQUARES AND SQUARE ROOTS.

It has already been shown that all the numbers on scales A and B are the squares of their coinciding numbers on C and D, and also that the numbers on scales C and D are the square roots of their coinciding numbers on A and B. When a square or square root is sought, all that is necessary is to place the Runner to the given number on A or D, and the coinciding number on D or A will be the square root or the square. But combinations, such as $a^2 \times b$, $a^2 \div b$, etc., can also be solved with the utmost facility, thus:

$a^2 \times b = x$

Example: $8^2 \times 5$

A		Find x.
B	Set 1	Over b
C		
D	To a	

A		Find 320—Answer.
B	Set 1	Over 5
C		
D	To 8	

$a^2 \div b = x$

Example: $12^2 \div 16$

A		Find x.
B	Set b	Over 1
C		
D	To a	

A		Find 9—Answer.
B	Set 16	Over 1
C		
D	To 12	

The formula $a^2 \times b = x$ is the one which gives the area of a circle when its diameter is known. The following setting, which is rather different, gives a complete table of diameters and areas.

A	Under 205	
B	Set 161	Find corresponding area.
C		
D		Over any diameter

Here 205 : 161 is equal to the ratio 1 : .7854, but is easier to set on the Slide Rule. A blue line on scale B gives the position of .785, and another that of 3.1416.

In the same way the Cube of a number is easily found by means of the formula $a^2 \times a = a^3$.

Example: $5^3 = 125$

A		Find a^3.
B	Set 1	Over a
C		
D	To a	

A		Find 125.
B	Set 1	Over 5
C		
D	To 5	

The finding of Cube Roots is almost as simple. It may be done in two ways.

1st. Move the Slide from left to right, or from right to left, until the *same* figure is found, on B under the given number on A, and *also* on D under the left or right index of C. This figure is the cube root sought. This is clearly the inverse of the method for finding the cube of a number.

2d. Invert the Slide, set 1 on C under the given number on A, then look for the number on B which coincides exactly with the *same* number on D. This number is also the cube root sought.

Example: Find the cube root of 216.

A	Under 216	
B	Find 6	
C		and over 1
D		Find 6.

Or Inverted

A	Under 216	
C	Set 1	
B		Under 6
D		Find 6–Answer.

7.—TRIGONOMETRICAL COMPUTATIONS.

On the *under* side of the Slide three scales will be found, the upper one, marked S, being a scale of natural sines, and the lower one, marked T, a scale of natural tangents. Between these is a scale of equal parts which gives the logarithms corresponding to the series of numbers on scale D.

In order to use these, place the Slide in the groove with the under side uppermost, and the left and right indices coinciding. On A will then be found the SINES of the angles given on S, those on the left scale A having the characteristic —1, and those on the right scale A the characteristic 0; thus we find

Sine 3° = 0.0523, on left scale A.
Sine 15° 10″ = 0.262, on right scale A.

We have on D the TANGENTS of the angles given on T, the characteristic being always 0, thus:

Tangent 25° = 0.466 on scale D.

The scale gives the tangents as far as 45 degrees only, those for larger angles must be found by the formula

$$\tan a = \frac{1}{\tan (90 - a.)}$$

Multiplication and division of sines and tangents are executed in the same manner as for ordinary calculations, thus:

Sine 3° 40′ × 45.

A	To 45	Find 2.875—Answer.
S	Set 1	Over 3° 40′

565 ÷ Tan. 14° 30′.

T	Set 14° 30′	Under 1
D	To 565	Find 2183—Answer.

Tan 75° 15′ × 175. Here $\tan 75° 15' = \frac{1}{\tan (90 - 75° 15').} = \frac{1}{14° 45'}$, therefore

$$\tan 75^\circ 15' \times 175 = \frac{175}{\tan 14^\circ 45'}$$

T	Set 14° 45′	Under 1
D	To 175	Find 665—Answer.

The sines of angles may be found without reversing the Slide, by setting the given angle on scale S to the upper index on the under side of the Rule, and reading off the sine on B under the right index of A.

The Logarithms of numbers are found in a similar manner by setting the left index of C to the given number on D, and reading off the logarithm on the scale of equal parts against the lower index on the under side of the Rule.

Example: What is the logarithm of 5?

C	Set 1	Find on scale E. P. 699—Answer.
D	To 5	Against index on under side of Rule

With the scale of equal parts the cube and other roots or powers may be extracted, such as $X^{\frac{5}{2}}$, $X^{1 \cdot 7}$, etc.

Example: Find $4^{\frac{5}{2}}$ or $\sqrt{4^5}$.

By above method log 4 = .602, and log $4^{\frac{5}{2}} = .602 \times \frac{5}{2} = 1.505$. Now by placing 505 against the index on the under side of the Rule, we find 32 on scale D under the left index of C, which is therefore equal to $4^{\frac{5}{2}}$, the logarithmic index being 1.

The position of the decimal point in all these cases will be easily ascertained by those accustomed to this class of calculations. We do not give rules, as they would probably be found too tedious.

We now give, by kind permission of "ENGINEERING NEWS," a number of settings for complicated formulæ, and also a very useful and complete table of Equivalents or Gauge Points, specially adapted to Slide Rule practice.

These will probably enable any one to work out similar Equivalents for any special class of calculations.

FORMULÆ. SETTINGS.

$$\frac{a^2 \times b}{c} = X.$$

A		Find X.
B	Set c	Over b
C		
D	To a	

$$\frac{\sqrt{a \times b}}{c} = X.$$

	A	To b	
Inverted	C		Under c
Inverted	B	Set a	
	D		Find X.

$$a\sqrt{\frac{b}{c}} = X.$$

	A	Under b	
Inverted	C	Set a	Under c
Inverted	B		
	D		Find X.

$$c\sqrt{\frac{a \times b}{d}} = X.$$

	A	Under b			
Inverted	C			c to R	Under 1
Inverted	B	Set a	R to d		
	D				Find X.

or,

A	Under a			
B	Set d	R to b	1 to R	
C				Under c
D				Find X.

$\frac{\sqrt{a^3}}{c} = X.$

Inverted

A		
C		Under c
B	Set a	
D	To a	Find X.

$c\sqrt{b^3} = X.$

A				
B		R to b		
C	Set 1		1 to R	Under c
D	To b			Find X.

$\left(\sqrt{\frac{a \times b}{c}}\right)^2 = X.$

Inverted

A	Under a	Find X.
C	Set b	Over c
B		
D		

$\sqrt{\frac{a^2 \times b^2}{c}} = X.$

Inverted

A		
C	Set b	
B		Under c
D	To a	Find X.

To find the Geometric Mean, or Mean Proportional between two numbers, or $a : x :: x : b$

A		
B	Set less No. a	Below b
C		
D	To less No. a	Find X = G. M.

To reduce fractions to decimals.

C	Set numerator	Find equivalent decimal.
D	To denominator	Above 1

To reduce decimals to fractions.

C	Set decimal	Find equivalent numerators.
D	To 1	Find equivalent denominators.

EQUIVALENTS FOR SCALES C AND D.

GEOMETRICAL.

Set 226 = Diameters of circles.	C
To 710 = Circumferences of circles.	D
79 = Diameter of circle.	C
70 = Side of equal square.	D
99 = Diameter of circle.	C
70 = Side of inscribed square.	D
39 = Circumference of circle.	C
11 = Side of equal square.	D
40 = Circumference of circle.	C
9 = Side of inscribed square.	D
70 = Side of square.	C
99 = Diagonal of square.	D
205 = Area of square whose side = 1.	C
161 = Area of circle whose diameter = 1.	D
322 = Area of circle.	C
205 = Area of inscribed square.	D

ARITHMETICAL.

100 = Links.
___
66 = Feet.

12 = Links.
___
95 = Inches.

101 = Square links.
___
44 = Square feet.

6 = U. S. gallons.
___
5 = Imperial gallons.

1 = U. S. gallons.
___
231 = Cubic inches.

800 = U. S. gallons.
___
107 = Cubic feet.

22 = Imperial gallons.
___
6100 = Cubic inches.

430 = Imperial gallons.
___
69 = Cubic feet.

METRIC SYSTEM.

26 = Inches.
___
66 = Centimetres.

82 = Feet.
___
25 = Metres.

82 = Yards.
___
75 = Metres.

87 = Miles.
___
140 = Kilometres.

4300 = Links.
___
865 = Metres.

43 = Chains.
___
865 = Metres.

31 = Square inches.
___
200 = Square centimetres.

140 = Square feet.
___
13 = Square metres.

61 = Square yards.
___
51 = Square metres.

42 = Acres.
___
17 = Hectares.

22 = Square miles.
___
57 = Square kilometres.

5 = Cubic inches.
___
82 = Cubic centimetres.

600 = Cubic feet.
___
17 = Cubic metres.

85 = Cubic yards.
65 = Cubic metres.

6 = Cubic feet.
170 = Litres.

14 = U. S. gallons.
53 = Litres.

46 = Imperial gallons.
209 = Litres.

108 = Grains.
7 = Grammes.

75 = Pounds.
34 = Kilogrammes.

6 = Ounces.
170 = Grammes.

63 = Hundredweights.
3200 = Kilogrammes.

63 = English Tons.
64 = Metric Tonnes.

PRESSURES.

640 = Pounds per square inch.
45 = Kilogs per square centimetre.

51 = Pounds per square foot.
249 = Kilogs per square metre.

59 = Pounds per square yard.
32 = Kilogs per square metre.

57 = Inches of mercury.
28 = Pounds per square inch.

82 = Inches of mercury.
5800 = Pounds per square foot.

720 = Inches of water.
26 = Pounds per square inch.

74 = Inches of water.
385 = Pounds per square foot.

60 = Feet of water.
26 = Pounds per square inch.

5 = Feet of water.
312 = Pounds per square foot.

15 = Inches of mercury.
17 = Feet of water.

99 = Atmospheres.

2960 = Inches of mercury.
34 = Atmospheres.

500 = Pounds per square inch.
34 = Atmospheres.

7200 = Pounds per square foot.
30 = Atmospheres.

31 = Kilogs per square centimetre.
23 = Atmospheres.

780 = Feet of water.
3 = Atmospheres.

31 = Metres of water.
29 = Pounds per square inch.

67 = Feet of water.
1 = Kilogs per square centimetre.

10 = Metres of water.

COMBINATIONS.

43 = Pounds per foot.

64 = Kilogs per metre.
127 = Pounds per yard.

63 = Kilogs per metre.
46 = Pounds per square yard.

25 = Kilogs per square metre.
49 = Pounds per cubic foot.

785 = Kilogs per cubic metre.
27 = Pounds per cubic yard.

16 = Kilogs per cubic metre.
89 = Cubic feet per minute.

42 = Litres per second.
700 = Imperial gallons per minute.

53 = Litres per second.
840 = U. S. gallons per minute.

53 = Litres per second.
38 = Weight of fresh water.

39 = Weight of sea water.

5 = Cubic feet of water.

---

312 = Weight in pounds.

1 = Imperial gallons of water.

---

10 = Weight in pounds.

3 = U. S. gallons of water.

---

25 = Weight in pounds.

50 = Pounds per U. S. gallon.

---

6 = Kilogs per litre.

10 = Pounds per imperial gallon.

---

1 = Kilogs per litre.

30 = Pounds per U. S. gallon.

---

25 = Pounds per imperial gallon.

3 = Cubic feet of water.

---

85 = Weight in kilogs.

46 = Imperial gallons of water.

---

209 = Weight in kilogs.

14 = U. S. gallons of water.

---

53 = Weight in Kilogs.

44 = Feet per second.

---

30 = Miles per hour.

88 = Yards per minute.

---

3 = Miles per hour.

41 = Feet per second.

---

750 = Metres per minute.

82 = Feet per minute.

---

25 = Metres per minute.

340 = Footpounds.

---

47 = Kilogrammetres.

72 = British horse power.

---

73 = French horse power.

3700 = One cubic foot of water per minute under one foot of head.

---

7 = British horse power.

75 = One litre of water per second under one metre of head.

---

1 = French horse power.

In no case does the departure, in these equivalents, from the exact ratio attain 1 per thousand.

EXAMPLES.

What is the pressure in pounds per square inch equivalent to a head of 34 feet of water?

C	Set 60	Under 34
D	To 26	Find 14.75 pounds—Answer.

What head of water, in feet, is equivalent to a pressure of 18 pounds per square inch?

C	Set 26	Under 18
D	To 60	Find 41.5 feet—Answer.

How many horse power will 50 cubic feet of water per minute give under a head of 400 feet?

C	Set 3700	Runner to 400	1 to R	Under 50
D	To 7			Find 37.8 H. P.—Answer.

METHODS OF WORKING OUT MECHANICAL AND OTHER FORMULÆ.

1.—Diameters and Areas of Circles. $A = .7854\, D^2$

A	To 205			A	To 11	
B	Set 161	Find Areas.		B	Set 6	Find Areas in square feet.
C			or	C		
D		Above Diameters		D		Above Diameters in inches

2.—To Calculate Selling Prices of Goods, with Percentage of Profit on Cost Price.

C	Set 100	Below cost price.
D	To 100 plus percentage of profit	Find selling price.

3.—To Calculate Selling Prices of Goods, with Percentage of Profit on Selling Price.

C	Set 100 less percentage of profit	Below cost price
D	To 100	Find selling price.

Example: If goods cost 45 cents a yard, at what price must they be sold to realize 15 per cent. profit on the selling price?

C	Set 85 (=100—15)	Below 45
D	To 100	Find 53—Answer.

4.—To find the Area of a Ring. $A=\frac{(D+d)\times(D-d)}{1.2732}$

C	Set sum of the two diameters	Find area.
D	To 1.273	Above difference of the two diameters

5.—Compound Interest.

Set the left index of C, to 100 plus the rate of interest, on D, then take the corresponding number on the scale of Equal Parts, and multiply it by the number of years. Set this product on the scale of E. P. to the index on the under side of the Rule, then on D will be found the amount of any coinciding sum on C for the given years at the given rate.

Example: Find the amount of $150 at 5 % at the end of 10 years.

C	Set 100	E. P.=21.2×10=212	212 to I	C	Below $150
D	To 105	Under side of Rule and	Slide.	D	Find $244.35—Answer.

We thus obtain on D, below 1 on C, a gauge-point for 10 years at 5 %, and can obtain in like manner similar ones for any other number of years and rate of interest.

6.—Levers.

C	Set distance from fulcrum to power or weight applied	Find power or weight transmitted
D	To " " " " " transmitted	Above power or weight applied.

7.—Diameter of Pulleys or Teeth of Wheels.

In'd C	Set diameter or teeth of Driving	Diameter or teeth of Driven
D	To revolutions of Driving	Revolutions of Driven.

or,

C	Set diameter or teeth of Driving	Revolutions of Driven
D	To diameter or teeth of Driven	Revolutions of Driving.

8.—Diameter of two Wheels to work at given Velocities.

C	Set distance between their centres	Find diameter
D	To half Sum of their revolutions	Above revolutions of each.

Example: A shaft makes 21 revolutions, and is to drive another shaft which should make 35 revolutions. The distance between their centres is 48 inches. What should be the diameters of the gears?

C	Set 48	Find 36	Find 60.
D	To 28 (=21+35÷2)	Above 21	And above 35

The two wheels must therefore be 36 and 60 inches diameter.

9.—Strength of Teeth of Wheels.

$$P=\frac{\sqrt{H}}{0.6V}$$

A	To H. P. to be transmitted			
B				
C	Set gauge-point 0.6	R to 1	Velocity in ft. per second to R	Under 1
D				Pitch in inches.

10.—Diameter and Pitch of Wheels.

$$N=\frac{D\times\pi}{P}$$

C	Set 226	R to 1	Pitch to R	Under diameter of pitch circle
D	To 710			Find number of teeth.

11.—Strength of Wrought Iron Shafting.

$$D=\sqrt[3]{\frac{83\ H}{N}}$$ for crank shafts and prime movers.

$$D=\sqrt[3]{\frac{65\ H}{N}}$$ for ordinary shafting.

A	To 83 or 65		
B	Set revolutions per min.	R to Ind. H. P.	Number coinciding with R=diameter.
C			Under 1
D			Same coinciding number=diameter.

Note.—In this, as in other cases, the coefficients (83 and 65) may be altered to suit individual opinions, without in any way altering the methods of solution.

12.—To find the Change Wheel in a Screw-Cutting Lathe.

$$N=T\frac{S\times W}{M\times P}.\qquad W=N\frac{M\times P}{T\times S}$$

Where
- N =Number of threads per inch to be cut.
- T = " " " on traverse screw.
- M= " teeth in wheel on mandril.
- W= " " stud wheel (gearing in M).
- P = " " stud pinion (gearing in S).
- S = " " wheel on traverse screw.

C	Set T	R to P	S to R	Under M
D	To N			Find number of teeth in W or stud wheel.

13.—Rules for Good Leather Belting.

$$W=\frac{600 \text{ or } 375\ H.\ P.}{V \text{ ft. per min.}}$$

C	Set 600	Find width in inches	for Single Belts.
D	To velocity in feet per min.	Above actual horse power	

C	Set 375	Find width in inches	for Double Belts.
D	To velocity in feet per min.	Above actual horse power	

14.—Best Manilla Rope Driving.

A	To velocity in feet per min.	Find Actual Horse Power.
B	Set 307	
C		Above diameter in inches
D		

A	To 4	Find Strength in Tons.
B		
C	Set 1	Above diameter in inches
D		

A	To 107	Find Working Tension in Pounds.
B		
C	Set 1	Above diameter in inches
D		

A	To 0.28	Find Weight per Foot in Pounds.
B		
C	Set 1	Above diameter in inches
D		

15.—Weight of Iron Bars in Pounds per Foot length.

A	To 1	Weight of Square Bars.
B	Set 3	
C		Above width of side in inches.
D		

A	To 55	Weight of Round Bars.
B	Set 21	
C		Above diameter in inches
D		

C	Set 0.3	Below thickness in inches
D	Breadth in inches	Weight of Flat Bars.

16.—Weight of Iron Plates in Pounds per Square Foot.

C	Set 32	Below thickness in thirty-seconds of an inch.
D	To 40	Find weight in pounds per square foot.

17.—Weights of other Metals.

C	Set 1	Below G. P. for other metals
D	To weight in iron	Find weight in other metals.

Gauge-points of other metals, and weight per cubic foot.

	W. I.	C. I.	Cast Steel.	Steel Plates.	Copper.	Brass.	Lead.	Cast Zinc.
G. P.	1	.93	1.02	1.04	1.15	1.09	1.47	.92
Weight	480	450	490	500	550	525	710	440 pounds.

Example: What is the weight of a bar of copper, 1 foot long, 3 inches broad and 2 inches thick?

C	Set 0.3	R to 2 inches thick	1 to R	Below G. P. 1.15
D	To 3 inches broad			Find 23 pounds—Answer.

18.—Weight of Cast Iron Pipes.

C	Set .4075	Below Difference of inside and outside diameters in inches
D	To Sum of inside and outside diameters in inches	Find weight in pounds per lineal foot.

G. P. for other metals	Brass.	Copper.	Lead.	W. Iron.
	.355	.333	.259	.38

19.—Safe Load on Chains.

A		Safe load in tons.
B	Set 36	Above 1
C		
D	To diameter in sixteenths of an inch	

20.—Gravity.

C	Set 1	Below 32.2
D	To seconds	Velocity in feet per second.

A	Space fallen through in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second.

A		Space fallen through in feet.
B		Above 16.1
C	Set 1	
D	To seconds	

21.—Oscillations of Pendulums.

A		
B	Set length pendulum in inches	
C		Below 1
D	To 375	Number oscillations per minute.

22.—Comparison of Thermometers.

C	Set 5	Degrees Centigrade
D	To 9	Degrees+32=Fahrenheit.

C	Set 4	Degrees REAUMUR
D	To 9	Degrees+32=FAHRENHEIT.

C	Set 4	Degrees REAUMUR
	To 5	Degrees CENTIGRADE.

23.—FORCE OF WIND.

A		
B	Set 10	Find pressure in pounds per square foot.
C		
D	To 66	Velocity in FEET *per second.*

A		
B	Set 10	Find pressure in pounds per square foot.
C		
D	To 45	Velocity in MILES *per hour.*

24.—DISCHARGE FROM PUMPS.

A		Gallons delivered per stroke.
B	Set 29.4	Stroke in feet
C		
D	To diameter in inches	

25.—DIAMETER OF SINGLE-ACTING PUMPS.

A	To 29.4			
B	Set length stroke in ft.	R to gallons to be delivered per min.	No. strokes per min. to R	
C				Below 1
D				Diam. pump in inches.

26.—Horse Power required for Pumps.

C	Set G. P.	Height in feet to which the water is to be raised
D	To cubic feet or gallons to be raised per minute	Horse power required.

Gauge Points with different Percentages of Allowance.

Per cent.	None	10	20	30	40	50	60	70	80
For Gallons, Imp.	3300	3000	2750	2540	2360	2200	2060	1940	1835
" C. Feet	528	480	440	406	377	352	330	311	294
" U. S. Gallons	3960	3600	3300	3050	2830	2640	2470	2330	2200

27.—Theoretic Velocity of Water for any Head.

A	Head in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second.

28.—Theoretical Discharge from an Orifice 1 inch Square.

A		
B	Set 1	Under head in feet
C		
D	To G. P. 3.34	Discharge in cubic feet per minute.

If the hole is round and 1 inch diameter, the G. P. is 2.62.

29.—Real Discharge from Orifice in a Tank, 1 inch Square.

A		
B	Set 1	Under head in feet
C		
D	To 2.1 G. P.	Discharge in cubic feet per minute with coefficient .63.

If the hole is round and 1 inch diameter, the G. P. is 1.65.

Gauge Points for other coefficients.

Coefficient	.60	.66	.69	.72	.75	.78	.81	.84	.87	.90	.93	.96
G. P. Square,	2.	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.	3.1	3.2
" Round,	1.57	1.73	1.80	1.88	1.96	2.04	2.12	2.20	2.28	2.36	2.44	2.52

30.—Discharge from Pipes when real Velocity is known.

	A		Discharge in cubic feet per minute.
Inverted	C		Above 1.75
	B	Velocity in feet per second	
	D	Diameter in inches	

31.—Delivery of Water from Pipes.

$$W=4.71\sqrt{\frac{D^5 H}{L}}$$

Eytelwein's Rule.

C	Set 1	E P×5=x	x to I
D	To diameter in inches	Under side of Rule.	

(*Continue to next line.*)

A				To Z	
B				Set 1	
C	R to 1	Length in feet to R	Under head in feet		Under 4.71
D			Find Z		Cubic ft. per min.

This is worked out similarly to Formula 5, which is explained in full.

32.—Gauging Water with a Weir.

	A		
Inverted	C		
	B	Depth in inches	Under 4.3
	D	Depth in inches	Discharge in cubic feet per minute from each foot width of sill.

33.—Discharge of a Turbine.

$$\frac{\sqrt{H \times V}}{0.3}=D$$

	A		
Inverted	C		Under 0.3
	B	Head in feet	
	D	Square inches water vented	Cubic feet discharged per minute.

34.—Revolutions of a Turbine.

A	To head in feet			
B				
C	Set diameter in inches	R to 1840	1 to R	Under rate of peripheral velocity
D				Find revolutions per minute.

35.—Horse Power of a Turbine.

C	Set 530	R to discharge per c. ft. per min.	1 to R	Percentage useful effect.
D	Head in ft.			Horse power.

or,

A	Under head in ft.					
B						
C	Set 1	R to head in ft.	158 to R	R to vent in sq. in.	1 to R	Under useful effect
D						Horse power.

36.—Horse Power of a Steam Engine.

C	Set 21,000	R to diam.	1 to R	R to stroke in ft.	1 to R	R to rev. per min.	1 to R	Mean pressure per sq. inch.
D	To diam. in inches							Horse power.

or,

A						Horse power.
B	Set 21,000	R to stroke in ft.	1 to R	R to revolutions	1 to R	Mean pressure.
C						
D	To diam. in inches					

37.—Dynamometer; to Estimate the H. P. indicated by.

C	Set 5252	R to length of lever in feet from centre of shaft	1 to R	Under rev. of shaft per min.
D	Weight applied at end of lever in pounds, including weight of scale			Actual horse power.

THE

"DUPLEX"

SLIDE RULE.

BY WM. COX.

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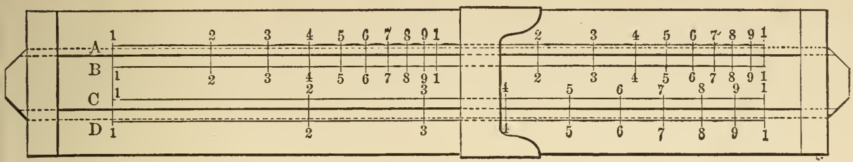
BRANCH : CHICAGO.

CONTENTS.

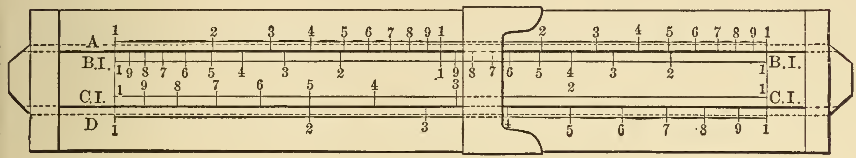
The following list of Formulæ of which detailed demonstrations are given, will enable the method of solving any problem to be at once found.

1. $a \times b = x$
2. $a \times b \times c = x$
3. $a \times a \times a = x = a^3$
4. $a^4 = x$
5. $a^5 = x$
6. $a^6 = x$
7. $\sqrt{a^5} = x$
8. $\frac{a}{b} = x$
9. $\frac{a}{x} = x = \sqrt{a}$
10. $\frac{a}{b \times c} = x$
11. $\frac{a}{x \times x} = x = \sqrt[3]{a}$
12. $\frac{b \times c}{a} = x$. Direct Proportion.
13. $\frac{a \times b}{c} = x$. Indirect Proportion.
14. $a \times b = c \times x$. Pulleys.
15. $\frac{a \times b \times c}{d} = x$
16. $\frac{a}{b \times c \times d} = x$
17. $\frac{a \times b}{c \times d} = x$
18. $\frac{a \times b \times c}{d \times e} = x$
19. $\frac{a \times b}{c \times d \times e} = x$
20. $(a \times b)^2 = x$
21. $\left(\frac{a}{b}\right)^2 = x$
22. $a^2 \times b = x$
23. $(a \times b \times c)^2 = x$
24. $(a \times b)^2 \times c = x$
25. $\frac{a^2}{b} = x$
26. $\frac{a^2 \times b^2 \times c}{d} = x$
27. $\frac{a}{b^2} = x$
28. $\frac{a^2}{b \times c} = x$
29. $\sqrt{a \times b} = x$
30. $\sqrt{\frac{a}{b}} = x$
31. $b\sqrt{a} = x$
32. $\sqrt{a \times b \times c} = x$
33. $\sqrt{a \times b} \times c = x$
34. $\frac{\sqrt{a}}{b} = x$
35. $\frac{a}{\sqrt{b}} = x$
36. $\frac{a}{\sqrt{b \times c}} = x$
37. $\frac{\sqrt{a}}{b \times c} = x$
38. $\sqrt{\frac{(a \times b)^2}{c}} = x$
39. $\sqrt{a^3} = x$
40. $b\sqrt{a^3} = x$
41. $\frac{\sqrt{a^3}}{b} = x$
42. Belt Speed.
43. Levers.
44. Pitch of Teeth of Wheels.
45. H. P. of a Fall of Water.
46. Velocity of Water for any Head.
47. Flow of Water in Channels or Pipes.
48. Weirs.
49. Average Pressure of Steam.
50. Horse power of a Steam Engine
51. Horse power of a Turbine.

Keuffel & Esser Co. New York and Chicago.



Upper Face.



Under Face.

Patented October 6th, 1891.

THE "DUPLEX" SLIDE RULE.

BY WM. COX.

1.—DESCRIPTION.

We referred briefly in chapter 5 of our Manual of the "Slide Rule," as also at greater length in Volume I of The Compass, to the Inverted Slide. This position of the slide is very much used in certain mechanical and engineering computations, and possesses great advantages, seeing that all the operations of multiplication, division and proportion, may be as easily effected in this manner as with the slide in its ordinary position. In some cases a calculation is entirely worked out with the slide inverted, whilst in others it is frequently necessary that the slide be first in the one position and subsequently placed in the other position, so that naturally these repeated movements not only materially detract from the useful simplicity of the instrument, but also expose it to greater risk of being damaged in the operation.

We have been consequently led to devote special attention to this point and have brought out our Patent "Duplex Slide Rule," which, we believe, overcomes the objections inherent in the ordinary slide rule, and largely increases its usefulness.

The "Duplex" Slide Rule is similar in most respects to an ordinary *Mannheim* slide rule, the distinguishing feature being that it is graduated on *both faces*, but the scales B and C of one side are in *reversed* order, that is, the initial indices are on the right hand, and the scales progress towards the left. These *reversed* lines of graduations are equivalent to inverting the slide itself, but possess considerable advantages which we shall enumerate. It will be noticed that the right and left indices of the different scales on the reversed face of the rule and slide coincide exactly with the indices of the ordinary face, so that if we set 1 of scale C to 4 of scale D, 1 of scale C.I. will be also found to coincide on the other side with 4 of scale D.

A metallic Runner encircles the *whole rule* and serves to indicate coinciding points on the four scales of each face, and also to perform extensive calculations without being compelled to read off intermediate results.

2.—ADVANTAGES.

1. The scale *B inverted* of the slide lies along scale A of the rule, and scale C *inverted* is contiguous to scale D of the rule, thus making the various operations much simpler than with the ordinary inverted slide, in which B.I. lies along D and C.I. along A, so that all settings have to be made, and results read off by means of the runner.

2. The figures denoting the values of the graduations are marked in their ordinary upright position, instead of being turned upside down as is the case when the ordinary slide is inverted.

3. As the indices on the two faces coincide, and as the scales correspond, we are enabled with the same setting of the slide to either multiply or divide any given number by any other number.

4. By using both faces of the slide rule, we are enabled to perform with one setting of the slide, two consecutive multiplications or divisions, so that formulæ like

$$\frac{a \times b \times c}{d} = x, \text{ or } \frac{a}{b \times c \times d} = x, \text{ etc.,}$$

can be much more promptly solved than with the ordinary slide rule.

5. It is often more convenient to read off the final result of a computation opposite one of the slide indices than to read it in coincidence with some number whose exact position may have to be estimated :—this can always be done by using the reversed face for multiplication, and the ordinary face for division.

6. If in the same computation the slide has to be used in its ordinary and in its inverted positions, the solution is liable to be erroneous, owing to the possibly incorrect setting of the slide after inversion, but with the "Duplex" this is impossible as one and the same setting applies to either position of the slide.

7. Among the minor advantages may be cited the diminished liability of the slide or rule to be broken by reason of the former being frequently drawn out and replaced after being turned round end for end.

3.—TRIGONOMETRICAL COMPUTATIONS.

These are very much simplified, as an interchangeable slide is supplied at a small extra cost, having on one side the usual scales B and C, and on the other side scales of sines, tangents and equal parts. The solution of a formula such as $\frac{b \times c \times \sin A}{2}$, giving the area of a triangle can thus be obtained without having to take the slide out, the arithmetical portion of the computation being effected on one side of the rule and the trigonometrical portion on the other side.

4.—HOW TO USE THE "DUPLEX."

As the principles of construction and the graduations of the "Duplex" are the same as those already described in our Manual of the Slide Rule, we do not enter into them here, but proceed at once to show by means of graphic demonstrations how various kinds of calculations are worked out with the "Duplex." We shall put them in the shape of formulæ which may then be easily applied to the solution of any particular problem whose terms are similarly composed. We shall then give a few practical examples of the working out of more extensive and complicated formulæ.

The face to be used in the case of each demonstration will be at once seen by the designation B or C for the slide scales on the *ordinary* face, and by B.I or C.I on the *reverse* face, while any change of face requiring to be made during the working out of a formula will be indicated by prefixing the scale designation, as shown in formula 2. When an operation involves neither powers nor roots, and can consequently be effected on scales C and D, these scales alone will appear in the demonstration; in other cases the four scales A, B, C and D will be given.

MULTIPLICATION.

1. $a \times b = x$

C.I.	Set a	Under 1
D	To b	Find x

2. $a \times b \times c = x$

C.I.	Set a	C	Under c
D	To b	D	Find x

This example of two succesive multiplications shows how the two faces of the rule and slide are used conjointly. The factors *a* and *b* are multiplied together on the reverse face, so that their product is found on scale D of the rule under one of the indices of the slide. As the scales D on both faces of the rule, and the indices on both faces of the slide coincide, this same product is likewise found on scale D of the ordinary face of the rule under the corresponding index of the slide. The product of the three factors is therefore then found on this face on scale D under the third factor on scale C.

Formula 2 is very useful for obtaining on scale D, with one single setting, the Cube of a number, by taking the same number to stand for the three factors a, b and c, thus:

CUBE OR 3D POWER.

3. $a \times a \times a = x = a^3$

C.I	Set a	C	Under a
D	To a	D	Find $x = a^3$

On account of the fuller graduations of scales C and D the result can be thus ascertained with more precision than if obtained in the usual way on scale A, thus:

A		Find $x = a^3$
B.I		Over a
C.I	Set a	
D	To a	

With slight modifications demonstration 3 may also be used to obtain with *one and the same setting* the *fourth*, *fifth* and *sixth* powers of numbers, as also the square root of the fifth power, thus:

4. $a^4 = x$

A		Find $x = a^4$
B.I		Over 1
C.I	Set a	
D	To a	

5. $a^5 = x$

A			Find $x = a^5$
B.I		B	Over a
C.I	Set a	C	
D	To a		

6. $a^6 = x$

A			Find $x = a^6$
B.I		B	
C.I	Set a	C	Over a
D	To a		

7. $\sqrt{a^5} = x$

A			
B.I		B	Under a
C.I	Set a	C	
D	To a		Find $x = \sqrt{a^5}$

It will be noticed that in formulæ 3 to 7, the setting is the same, the only difference in the working out being the positions of the third and fourth terms.

Division.

8. $\frac{a}{b} = x$

C.I	Set 1	Under b
D	To a	Find x

Formula 8 is the one used to obtain the Square Root of a number, with the simple difference that the terms b and x must be the same number, thus :

Square Root.

9. $\frac{a}{x} = x = \sqrt{a}$

C.I	Set 1	Under x
D	To a	Find $x = \sqrt{a}$

10. $\frac{a}{b \times c} = x$

C	Set b	C.I	Under c
D	To a	D	Find x

Formula 10, which is the reverse of No. 2, is used to find the Cube Root of a number, by making b and c equal to x, but the operation is somewhat altered to render it simpler, thus :

Cube Root.

11. $\frac{a}{x \times x} = x = \sqrt[3]{a}$

A	To a	
B.I	Set 1	Under x
C.I		
D		Find $x = \sqrt[3]{a}$

Proportion is direct or indirect. Both kinds may be performed with the slide inverted as with the slide in its ordinary position. In the case of *direct proportion* the operation is carried out as in arithmetic, the second and third terms being first multiplied together and their product then divided by the first term, thus:—

12. $a : b = c : x$ or $\frac{b \times c}{a} = x.$

C I	Set 2nd term, b	Under 1st term, a
D	To 3d term, c	Find 4th term, x

For direct proportion we naturally recommend the usual method without inverted slide, as it shows at a glance the given ratio and at the same time an indefinite number of equivalent ratios. It will be self-evident that the "Duplex" Slide Rule is as suitable for this method as is the ordinary form of slide rule.

In the case of *indirect proportion*,, if the terms are placed in the order of their sequence, the first and second terms are multiplied together and their product then divided by the third term, thus :—

13. $a \times b = c \times x$ or $\frac{a \times b}{c} = x$

C . I	Set 1st term, a	Under 3d term, c
D	To 2d term, b	Find 4th term x.

It will be noted in the above demonstrations of direct and indirect proportion, that the number on scale D under the slide index is always the same as the number which is found on scale C.I of the slide over the rule index; also that the fourth term may be taken on the rule under the first or third terms on the slide respectfully, as well as on the slide over the first or third terms on the rule, and this, because the product of all the coinciding numbers on the slide and rule is equal to each other, and also to the number on the rule under the slide index, or to the number on the slide over the rule index.

A very familiar and useful illustration of this equality of the products of coinciding numbers is furnished in the case of two pulleys or wheels, one transmitting power to the other, and in which their diameters or the number of their teeth are inversely proportionate to the number of their revolutions; and the products of their diameters or the number of their teeth with their respective number of revolutions are always equal. For instance, suppose a 40 inch pulley making 36 revolutions a minute drives another one 60 inches diameter, what will be the number of revolutions of the latter?

14. Diameter Driving × Revolutions Driving = Diameter Driven × Revolutions Driven.

C.I	To Diam. Driving, 40	Under Diam. Driven, 60
D	To Revs. Driving, 36	Find Revs. Driven, 24

The product of the two *driving* factors is thus equal to the product of the two *driven* factors, namely 1440, and any other coinciding numbers on C.I and D, as 48 and 30, 45 and 32, 72 and 20 or 80 and 18 would give the same result, their products being also 1440. The same formula applies equally to toothed wheels, the number of teeth taking the place of the diameters.

15. $\dfrac{a \times b \times c}{d} = x$

C.I	Set a	R to d	c to R	Under 1
D	To b			Find x.

or

C.I	Set a	C	R to c	d to R	Under 1
D	To b	D			Find x.

Note. In above and following examples R stands for Runner, but it will not always be found necessary to make such use of it, especially if an intermediate result falls upon a distinct graduation, to which the index or another distinct graduation can be easily set.

By using one or other of the above demonstrations the solution may be had either on the ordinary or on the reverse face. This formula, as well as many of the others given, may of course be worked out entirely on the ordinary face if preferred, although in many cases the number of operations would be thereby increased.

16. $\dfrac{a}{b \times c \times d} = x$

C	Set b	C.I	R to c	C	d to R	Under 1
D	To a	D		D		Find x

or

C	Set b	C.I	R to c	1 to R	Under d
D	To a	D			Find x

This formula, if worked out entirely on the ordinary face, would require five operations instead of three. With the first demonstration the answer is obtained under the slide index; and with the second, under the last factor of the dividend.

17. $\dfrac{a \times b}{c \times d} = x$

C.I	Set a	R to c	1 to R	Under d
D	To b			Find x.

18. $\dfrac{a \times b \times c}{d \times e} = x$

C.I.	Set a	R to d	c to R	Under e
D	To b			Find x

19. $\dfrac{a \times b}{c \times d \times e} = x$

C	Set c	C.I.	R to d or e	b to R	Under e or d.
D	To a	D			Find x.

The combination of the ordinary and reverse faces enables the numerator *a* to be divided by the two factors *c* and *d* of the denominator with one setting. This operation is the same as that of formula 10, $\dfrac{a}{b \times c} = x$, which is equivalent to $a = b \times c \times x$, so that the operation is in reality composed of two consecutive multiplications as in formula 2.

We have given as the second operation "Runner to *d* or *e*." When there are two or more factors, one may be more convenient for the position of the slide than the other, and it is immaterial which one is taken first. A little practice will soon enable the operator to perceive this and to select the most suitable one.

20. $(a \times b)^2 = x$

A		Find x
B. I		
C. I	Set a	Over 1.
D	To b	

21. $\left(\frac{a}{b}\right)^2 = x$

A		Find x
B. I		
C. I	Set 1	Over b
D	To a	

22. $a^2 \times b = x$

A		Find x
B. I	Set b	Over 1
C. I		
D	To a	

23. $(a \times b \times c)^2 = x$

A		A	Find x
B. I.		B	
C. I.	Set a	C	Over c
D	To b	D	

24. $(a \times b)^2 \times c = x$

A		A	Find x
B. I		B	Over c
C. I	Set a	C	
D	To b	D	

25. $\frac{a^2}{b} = x$

A		Find x
B. I		Over b
C. I	Set 1	
D	To a	

26. $\frac{a^2 \times b^2 \times c}{d} = x$

A		A			Find x
B. I.		B	R to c	d to R	Over 1
C. I.	Set a	C			
D	To b	D			

27. $\frac{a}{b^2} = x$

A	To a	Find x
B. I.	Set 1	
C. I		Over b
D		

28. $\frac{a^2}{b \times c} = x$

A		A	Find x
B	Set b	B.I.	Over c
C		C. I.	
D	To a	D	

29. $\sqrt{a \times b} = x$

A	To b	
B. I	Set a	
C. I.		Under 1
D		Find x

This formula gives the Geometric Mean of any two numbers. It will also be seen that *x* is the G. M. of *all* coinciding numbers on A and B. I.

30. $\sqrt{\frac{a}{b}} = x$

A	To a	
B. I.	Set 1	Under b
C. I.		
D		Find x

31. $b\sqrt{a} = x$

A	To a	
B. I.		
C. I.	Set b	Under 1
D		Find x

32. $\sqrt{a \times b \times c} = x$

A	To b	A	
B. I.	Set a	B	Under c
C. I.		C	
D		D	Find x

33. $\sqrt{a \times b} \times c = x$

A	To b	A	
B. I.	Set a	B	
C. I.		C	Under c
D		D	Find x

34. $\frac{\sqrt{a}}{b} = x$

A	To a	
B. I.	Set 1	
C. I		Under b
D		Find x

35. $\frac{a}{\sqrt{b}} = x$

A		
B. I.	Set 1	Under b
C. I.		
D	To a	Find x

36. $\frac{a}{\sqrt{b \times c}} = x$

A		A	
B	Set b	B. I.	Under c
C		C. I.	
D	To a	D	Find x

37. $\frac{\sqrt{a}}{b \times c} = x$

A	To a	A	
B		B. I.	
C	Set b	C. I.	Under c
D		D	Find x

38. $\sqrt{\frac{(a \times b)^2}{c}} = x$

A		
B. I.		Under c
C. I.	Set a	
D	To b	Find x

39. $\sqrt{a^3} = x$

A		
B. I.	Set a	Under 1
C. I.		
D	To a	Find x

40. $b\sqrt{a^3} = x$

A		A	
B. I.	Set a	B	
C. I.		C	Under b
D	To a	D	Find x

41. $\frac{\sqrt{a^3}}{b} = x$

A		
B. I.	Set a	
C. I.		Under b
D	To a	Find x

The following are a few examples, out of many which might be selected, of the application of these formulæ to the solution of practical problems. The case of pulleys and wheels has already been given, but the following modification may not be without interest.

42. To find the diameter of a wheel in inches and the number of revolutions per minute to give a required peripheral velocity or belt speed in feet per minute.

C. I.	Set Velocity	Under Diameter in inches.
D	To G. P. 3.82	Find Revolutions per minute.

Example. Required the diameter of a pulley and the number of revolutions per minute which will give a belt speed of 250 feet per minute.

C. I	Set 250 feet	Under 13 inches	or Under 18 inches.
D	To G. P. 3.82	Find $73\frac{1}{2}$ revs.	Find 53 revolutions.

and similarly, all other coinciding numbers on C. Inverted and on D give, the former, diameters, and the latter, revolutions; all giving the required belt speed of 250 feet per minute, and allowing choice of a suitable combination to be at once made.

If the belt speed is given in feet per second then the G. P. 229 must be used.

43. Levers are another instance of inverse proportion, the long leverage multiplied by the power applied, being equal to the short leverage multiplied by the power transmitted. We have therefore the following demonstration based on formula 14 :—

C. I	Set Long Arm	Under Short Arm
D	To Power applied	Find Power transmitted.

The superiority of the inverse method in the case of levers lies in the fact that any possible combination of arms and power producing the same result is seen by mere inspection.

44. Pitch of Teeth of Wheels. A formula often used is $P = \frac{\sqrt{H}}{0.6\ V}$ where P is the pitch of the teeth in inches, H the actual horse power to be transmitted by the wheel and V the velocity of the pitch line in feet per second. It is the same as demonstration 37, so that we have

A	To Horse Power.	A	
B		B. I.	
C	Set 0.6	C. I.	Under Velocity
D		D	Find Pitch in inches.

The advantage of this method is that we can see at a glance a suitable pitch for any velocity of the pitch line.

The pitch may also be found by means of $P = \frac{\sqrt{H} \times 382}{\text{Diam} \times \text{Revs}}$, where the diameter is given in inches, and the revolutions per minute. We have by demonstration 12 modified

A				
B. I.	Set H			
C. I.		R to Diam.	1 to R	Under Revs.
D	To 382			Find Pitch.

45. Horse Power of a Fall of Water. The formula is

$$\text{Theor : H. P.} = \frac{\text{Head} \times \text{Cub. ft. water}}{\text{Constant } 530}$$, for which we have by demonstration 12

C. I.	Set Head in feet	Under Constant 530
D.	To cub. ft. per min.	Find theoretic Horse Power.

This setting shows also at a glance all other combinations of Head and Volume of Water which will produce the same horse power. To find the real effective horse power, we have :—

$$\text{Effective H. P.} = \frac{\text{Head} \times \text{Cub. feet} \times \text{Efficiency}}{\text{Constant } 530}$$

which is solved by demonstration 15.

C. I.	Set Head	R to 530	Efficiency to R	Under 1
D	To Cub. ft.			Find H. P.

The above may be solved with one setting by replacing in the first demonstration the Constant 530 by the following Guage Points according to the percentage of efficiency.

Per Cent.	50	60	70	75	80	85	90	95
G. P.	1060	883	757	707	660	620	590	558

This demonstration also serves for obtaining the horse power of a Turbine or other Water Motor, when the Head, Discharge and rate of efficiency are known.

46. Velocity of Water for any Head. The theoretic Velocity is obtained by the well-known formula $V = 8.025 \sqrt{H}$, for which we have according to demonstration 31

A	To Head in feet	
B. I.		
C. I.	Set 8.025	Under 1.
D		Find Vel. in feet per second.

To find the *real* velocity of discharge with any coefficient, the formula becomes $V = C \times 8.025 \sqrt{H}$, for which we have the following demonstration.

A	To Head in feet	A	
B. I.		B	
C. I.	Set 8.025	C	Under Coefficient.
D		D	Find real Velocity.

47. Flow of Water in Channels or Pipes. The well-known formula is $V = C \sqrt{R \times S}$, the terms of which are the same as demonstration 33; we have therefore

A	To Sine of Slope	A	
B. I.	Set Hyd. mean Depth in feet	B	
C. I.		C	Under Coefficient
D		D	Find Vel. in feet per second.

The coefficient C varies according to different hydraulicians, but the above very simple method of obtaining the real velocity of the water cannot be equalled.

48. Weirs. Francis' formula is $D = 200 \sqrt{H^3}$ where D = Discharge in cubic feet per minute over each foot width of sill, H = Height of surface of water above sill in feet.

According to demonstration 40 we have

A		A	
B. I.	Set Height	B	
C. I.		C	Under 200
D	To Height	D	Find Discharge.

49. Average Pressure of Steam. The calculation of the average pressure of steam in an engine cylinder with any ratio of cut-off can be advantageously effected with the slide rule by means of the following original formula:—

$$A. P. = \frac{I. P. \times A. G. P.}{I. G. P.}$$

where A. P. = Mean Pressure during the stroke in lbs. per square inch, including atmosphere;
I. P. = Initial Pressure of steam in pounds per square inch including atmosphere;
A. G. P. = Average Gauge Point as per table;
I. G. P. = Initial Gauge Point as per table.

The demonstration is as follows: —

C. I.	Set I. P.	Under I. G. P.
D.	To A. G. P.	Find Average Pressure.

The following are the Gauge Points for the various portions of the stroke at which steam is cut off.

Cut off	7/8	3/4	5/8	1/2	3/8	1/4	1/8	5/6	2/3	1/3	1/6
I. G. P.	250	55	37	13	31	57	65	68	16	10	43
A. G. P.	248	53	34	11	23	34	25	67	15	7	20

Cut-off	9/10	8/10	7/10	6/10	4/10	3/10	2/10	1/10
I. G. P.	200	139	139	167	214	3	23	100
A. G. P.	199	136	132	151	164	2	12	33

Example. Given an initial pressure of 80 lbs. with cut-off at 5/8 of the stroke; find the average pressure for the whole stroke.

C. I.	Set 80 lbs.	Under 37.
D.	To 34	Find 73.5 lbs.

These various examples could be extended indefinitely; those given will show clearly how any kind of calculation may be worked out on the "Duplex" Slide Rule, We add two rather more complicated ones, which are, however solved very easily by the combined use of the ordinary and reverse faces of the "Duplex."

50. Horse Power of a Steam Engine. The formula is

$$\text{H. P.} = \frac{D^2 \times 0.7854 \times 2\,S \times R \times P}{33000}$$

where D = Diameter of the piston in inches;
S = Length of Stroke in feet;
R = Revolutions of fly-wheel per minute;
P = Average pressure of the steam.

We, however, reduce the formula to the simpler one :—

$$\text{H. P.} = \frac{D^2 \times S \times R \times P}{21008}$$

which we demonstrate thus :—

A				A	Find H. P.
B. I.	Set S	X to 21008	R to X	B	Over P
C. I.				C	
D	To D			D	

If we wish to ascertain the horse power with a given cut-off, we use the Gauge Points given in the previous table, and proceed as follows :—

A				A			Find H. P.
B. I.	Set S	X to 21008	R to X	B	X to P	I.G.P. to X	Over A.G.P.
C. I.				C			
D	To D			D			

Note. In these two demonstrations we use the letter X to signify the Runner, to avoid confusion with the Revolutions.

Example. Required the horse power of a steam engine whose piston is 20 inches in diameter, stroke 3 feet, initial pressure of steam 40 lbs., and cut-off at half stroke, and making 70 revolutions per minute.

A				A			Find 135.4 H. P.
B. I.	Set to 3	X to 21008	70 to X	B	X to 40	13 to X	Over 11
C. I.				C			
D.	To 20			D			

51. Horse Power of a Turbine, calculated from the square inches of water it vents. The formula is

$$\text{H. P.} = \frac{H \times \sqrt{H} \times 8\,025 \times 12 \times 60 \times 62\tfrac{1}{2} \times V \times E}{1728 \times 33000}$$

where H = the head or fall in feet;
V = the number of square inches water vented;
E = the percentage of useful effect.

We reduce this to the following simple form

$$\text{H. P.} = \frac{H \times \sqrt{H} \times V \times E}{158}$$

and operate thus

A	To H			A	
B. I.				B	
C. I.	Set H	R to 158	V to R	C	Under E
D				D	Find H. P.

Example. Head 23 feet; Vent 70 square inches; Efficiency 0.85; then we have

A	To 23			A	
B. I.				B	
C. I.	Set 23	R to 158	70 to R	C	Under 0.85
D				D	Find 41.5 H. P.

5. The Position of the Decimal Point.

The following are the rules for ascertaining the number of digits or figures of which the integral portion of any result is composed, or in other words, the position of the decimal point.

Multiplication. If the product is obtained with the slide projecting to the *right*, its characteristic is the *sum* of the characteristics of the two factors.

If the slide projects to the *left*, the characteristic of the product is the *sum* of the characteristics of the two factors *less* 1.

Division. If the quotient is obtained with the slide projecting to the *right*, its characteristic is the characteristic of the dividend, *minus* the characteristic of the divisor.

If the slide projects to the *left*, the characteristic of the quotient is the characteristic of the dividend *plus* 1, *minus* the characteristic of the divisor.

When a calculation is effected involving the use of both faces of the "Duplex," the characteristic of the result is a combination of the above rules and of those for the ordinary slide rule, which are the reverse of those for the inverted slide.

A little study and practice will make these rules self-evident, although in many cases the position of the decimal point is more easily determined by inspection of the figures of the problem.