Professor [illegible]
From the Author

INSTITUTION OF ENGINEERS IN SCOTLAND

WITH WHICH IS INCORPORATED THE

SCOTTISH SHIPBUILDERS' ASSOCIATION.

Chronometer ON THE

RATE OF A CLOCK OR CHRONOMETER

AS

INFLUENCED BY THE MODE OF SUSPENSION.

Baron

BY (SIR) WILLIAM THOMSON. Kelvin

READ BEFORE THE INSTITUTION, FEB. 27, 1867.

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ON THE

RATE OF A CLOCK OR CHRONOMETER

AS INFLUENCED BY THE MODE OF SUSPENSION.

By Professor Sir William Thomson.

(SEE PLATE VIII.)

It is well known that the rate of a chronometer, a clock, or a watch may be altered by altering its mode of support. On land, clocks ought to be fixed in as solid a manner as possible, so as to prevent vibration, either by their own action or from extraneous causes, from being communicated to the supports of the pendulum. Even the best astronomical clocks hitherto made are very badly arranged in this respect.

A marine chronometer or watch exhibits in a very striking manner the effects of varying the mode of support. A watch which keeps very good time when carried in the pocket, or laid on a soft pillow, will go at a different rate if laid on a marble slab, or on a hard board. These variations of rate are not due to any imperfections of the balance-wheel or mechanism of the watch or chronometer; but arise from reaction due to the motion of the moving parts. A well balanced watch will go equally well whether supported in a vertical or horizontal plane; and a well made watch will, I believe, not be subject to uncertainty of above a quarter of a second per day, if carried about in the pocket all day and put under the pillow at night. This I can testify from experience of a good pocket-watch which I have tried now for nearly two years; indeed, a good pocket-watch, if well treated, is comparable in its performances, with the best marine chronometer.

I was very much struck some time ago by a remark made to me by Mr. Archibald Smith, of Jordan Hill, regarding a demi-chronometer, with detached lever and compensated balance, presented to him by the Admiralty for the voluntary assistance he had given them in working out methods for adjusting the compasses of iron ships. Mr. Smith found that this watch was going well, until one day he observed it had gained 15 seconds, the reason of which he could not explain until he had recollected that instead of its having been put under the pillow as usual, it had been hung up in a suspended watch-case.

The question now arises, what is the cause of these variations, and how on dynamic principles are they to be explained. The dynamics of the subject are indeed very simple, and can be easily reduced to a well known general problem.

A simple pendulum when it vibrates through a very small arc, vibrates according to the law of simple harmonic motion. Take a spiral spring, with a heavy weight hanging by it, stretch it a little and let it go, and it vibrates according to the same law. The vibrations of a tuning-fork, or any other instrument giving a similar musical sound, are also according to the law of simple harmonic motion. Another case of simple harmonic motion we have when the piston moves to and fro in a cylinder, the head of the piston-rod being guided by a cross-head and slides, and the crank and fly-wheel making one revolution for every backward and forward movement of the piston. The balance-wheel of a watch, vibrating to and fro through a certain angle, is approximately a simple harmonic motion. The longer the hair-spring is, the more nearly it will approach to simple harmonic motion, and it will keep time the more accurately.

Now, against every change of motion of a body there is a certain reaction, and every motion to and fro of the balance-wheel of a watch or chronometer reacts upon the case of the watch or chronometer; and if the case is so suspended as to be free to vibrate, the motion of the balance-wheel will generate a vibration of the whole, so that we have two motions to consider—one, that of the balance-wheel inside the watch;

the other, that of the whole watch except the balance-wheel. Upon the mode of suspension of the watch or chronometer will depend the nature of the vibration which it takes up and the resultant effect upon the rate. The rate is accelerated or retarded according as the vibration of the case is in the opposite direction to that of the balance-wheel, or in the same direction; and the amount whether of acceleration or retardation, may be as much as a minute an hour, as I hope to demonstrate to you practically.

If a watch or chronometer, be allowed extreme freedom to move, it has always a faster rate than when the case is held quite fixed. Mr. Archibald Smith has made experiments on this point upon a pocket-watch, with chronometer escapement and compensated balance, and found that the moment of inertia of the frame was $\frac{1}{650}$th of that of the balance-wheel, from having observed that when hung horizontally by a long thread it had a gaining rate of some 67 seconds in the day.

Observations made by Daniel Bernoulli on the sympathy of vibrations* manifested by the pans hanging from the two ends of a common balance, and the solution by Euler of the particular problem thus presented, seem to have originated the great dynamic problem of the vibrations of stable systems.

When a system of particles displaced from a position of equilibrium experiences in consequence forces in simple proportion to the displacements of its different parts, its motion may be thoroughly investigated by a generalisation of this problem of Bernoulli and Euler. The solution involves an algebraic equation of the same degree as the number of independent motions which may be given to the system. When the roots of this equation, which are necessarily all real, are all positive, the equilibrium of the system is stable. It is convenient to confine our attention to this case; but it is interesting and important to remark that all the statements we make in reference to it are applicable by a proper mathematical extension of the language, to cases of unstable equilibrium. Each of the roots of this algebraic equation used in other formulæ belonging to the solution determines a particular

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* See a Paper "On the Sympathy of Pendulums," by Mr. Archibald Smith, in the Cambridge Mathematical Journal. No. IX. May, 1840.

proportion of different possible displacements, which, if made simultaneously, will give rise to *corresponding* forces of restitution, according to the following condition. The system, starting from rest in its displaced configuration, will, under the influence of these forces, move so as to diminish the displacements of all its parts in the same proportion. Thus all the displacements will come to zero simultaneously; and therefore the system will move precisely through its configuration of equilibrium. There being no frictional or other resistance, it will oscillate—each displacement varying from maximum positive to maximum negative according to the simple harmonic law; the system passing, twice in each period, through its configuration of equilibrium, and being twice for an instant at rest in the configuration of extreme displacement on either side. This is called a fundamental mode of vibration. There are as many such fundamental modes as the system has of degrees of freedom to move (independent variables). Every possible motion of the system may be resolved into simple harmonic vibrations according to these fundamental modes; or the superposition of simple harmonic vibrations, according to the fundamental modes, will give any possible motion of the system. The arbitrary circumstances of displacement and projection by which any possible motion of the system may be instituted are producible by giving proper values to the energies and proper times to the epochs of maximum displacement of the component fundamental modes. The squares of the periodic times of the fundamental modes are the roots of the algebraic equation referred to above. In particular cases, some of these periods may be equal to one another; or all may be commensurable. In general, however, the periodic times of the fundamental modes are all different and incommensurable; and then none of the compound motion—that is to say, no motion except one or other of the fundamental modes—is periodic. The mathematics of the problem, including proofs of these results, will be found in the first volume (now on the point of appearing) of Thomson and Tait's Elements of Natural Philosophy.

The theory is not limited to systems presenting a finite number of independent variables, such as two in the cases we are about to consider more particularly, but is applicable to flexible or elastic bodies and fluids; and to complex systems presenting a finite number of independent variables, on account of solid bodies or material particles, and infinite numbers of variables, due to flexible, elastic, or fluid matter, influenced by them. It includes, for example, the well-known dynamical theory of the vibrations of a stretched cord, of air in an organ pipe, or of water in an open basin of any shape. In the first two of these cases the periods are all sub-multiples of the gravest fundamental modes, whence the explanation of the harmonics of musical cords and of wind instruments; whence also the fact that a stretched cord struck or disturbed in any manner takes a perfectly periodic motion, and gives a true, although not a pure and simple, musical sound, with the peculiar character of the violin, pianoforte, or harp, depending on the way in which the vibration is excited. But the fundamental modes of vibration of an elastic solid—for instance, a stiff metal bar, or a stiff spiral wire (as the "bell" of an American clock), a sheet of metal, or a common bell, are incommensurable. Hence these bodies cannot give any true musical sound other than a pure and simple harmonic note. A large sheet of metal, or a gong, or a drum, when struck, produces an infinite number of discordant notes sounding simultaneously. But in the last-mentioned case, the gravest of the fundamental notes predominates more decidedly than does any one of the fundamental notes in the two other cases; and thus a drum gives a nearer approach to a true musical sound than a sheet of brass or a gong.

An excellent illustration of the general theory is presented by the double pendulum—one pendulum hung from the weight of another—Plate VIII., Fig. 1. If we admit only vibrations in one plane, the system has two degrees of freedom to move. The determinant equation becomes a quadratic with two roots, necessarily unequal. The mathematics need not be given here; but may be advantageously worked out as an exercise by the dynamical student. In the graver fundamental mode the two cords deviate always in the same direction from the vertical; the lower through a greater angle than the upper. In the quicker fundamental mode, the two deviate in opposite directions. The period of the graver fundamental mode is always longer than that of a simple pendulum, of length equal to that of the longer of the two cords; the period of the quicker fundamental mode is always shorter than that of the simple pendulum, equal in length to the shorter cord. If the upper mass is much greater than the one hung from it, and if the two strings be not approximately equal in length, the two fundamental periods differ but little from those of simple pendulums equal in length to the two cords respectively. The diagram—Figs. 1 to 5, Plate VIII.—illustrates the circumstances in the cases; first, when the upper cord is considerably longer than the lower; and second, when the lower cord is considerably longer than the upper. In each case OA is the length of the simple pendulum vibrating in the same period as that of the fundamental mode represented.

CASE I.

Figure 2 represents the first or graver fundamental mode; the period of the upper pendulum CP′ being made somewhat graver by the influence of the lower, which, in the course of the vibration, always exerts a force upon it *from* its middle position. Figure 3 represents the second or quicker fundamental mode; the vibration of the upper pendulum being in this instance excessively small in comparison with that of the lower, and forced, by the influence of the latter, to a period much smaller than its own would be if undisturbed.

CASE II.

Figure 4 represents the graver mode; the vibration of the upper pendulum through but a very small arc in comparison with that of the lower, being augmented by the influence of the lower, which, in the course of the vibrations, exerts a force upon it always *from* its middle position. Figure 5 represents the quicker mode; the vibrations of the upper pendulum being made somewhat faster by the influence of the lower, and the lower being influenced so as to vibrate as if it were shortened to the length O A, which is somewhat less than the length CP′. If P′ consisted of the frame and work of a spring clock, and P′ P were its pendulum, then, in Case I., the vibrations which would be maintained by the actions of the escapement wheel would be that represented by Figure 3, and the clock would go faster than if its frame were perfectly fixed. In Case II., the vibrations maintained by the escapement would be those represented by Figure 4, and the clock would go somewhat slower than its proper rate. Case I. could never occur in practice, but may be experimentally illustrated by hanging the works of a clock on a light stiff frame, movable round a horizontal axis. Case II., Fig. 4, with CP′ much shorter in proportion to P′P than shown in the diagram, represents the actual circumstances of an ordinary pendulum clock, which, owing to want of perfect rigidity of the frame, must experience a little of the influence of the pendulum in the manner there illustrated, causing the rate of the clock to be somewhat slower than it would be if the support of the pendulum were absolutely fixed. The clock cases of the best astronomical clocks are very ill adapted to give the steadiness necessary for good results; and it is wonderful that their performances are not even worse than they are found to be. The pendulum ought to be hung from a massive stone or metal support, attached to a stone pier, such as those used by astronomers for bearing their optical instruments. There can be no doubt but that the use of this simple precaution, and the making the pendulum many times heavier than has been hitherto used, would render the performances of an astronomical clock, even with a Graham's dead-beat escapement, not merely two or three times better than those of a good watch carried about in the pocket, but ten or twenty times better, which it certainly ought to be in its immensely more advantageous circumstances. A good marine chronometer is probably little less accurate than the best astronomical clocks of the present day. It seems strange that such a very great improvement on Graham's dead-beat escapement as either the chronometer escapement, or the detached lever, constitutes, should not yet have been applied to the astronomical clock. The mercury compensation pendulum, although very bad, cannot probably be blamed for the sudden variations of rate, amounting sometimes to as much as two-tenths of a second a-day, to which the best astronomical clocks at present in use are subject,

If a chronometer is suspended in the manner shown in Fig. 6, I find I can make it go fast or slow as I choose, by shifting the points of support nearer to, or farther from the centre. When the upper points of support are very near, the time of vibration of the chronometer as a whole, when turned a little round its vertical axis from the position in which it hangs in equilibrium and let go, is much longer than that of the balance-wheel. When left to itself, with the chronometer going, the reaction of the balance-wheel, through the spring, against the frame, gives rise to a vibration, illustrated by Fig. 3, in which the balance-wheel and the rest of the chronometer vibrate round a vertical axis always in opposite directions. The effect of suspension in this instance is to make the watch go faster than when its case is held perfectly fixed, but this effect is smaller the nearer the upper points of support are. The circumstances of the extreme case when they are as close as possible, are best realised by hanging the chronometer by a long single cord, from a fixed point, by means of a sling or two or three cords attached to the chronometer and so adjusted as to keep its face horizontal: thus giving the frame perfect freedom to move round a vertical axis. The permanent effect is then such, that the balance-wheel and the rest of the chronometer oscillate in opposite directions through ranges inversely as their moments of inertia. The period of this vibration is the same as that which the balance-wheel would have if the length of the hair-spring were diminished to the same proportion to its whole length that the moment of inertia of the chronometer with the balance-wheel free bears to the sum of this moment of inertia and the moment of inertia of the balance-wheel round its own axis. The period of vibration will be diminished according to square-root of this ratio. Thus, if the moment of inertia of the chronometer is 647 times that of the balance-wheel, the period will be $\sqrt{\frac{649}{650}}$, or less than $\frac{1299}{1300}$ of the proper rate; or the chronometer will gain one second in 1299, or about 67 seconds in the twenty-four hours. This was the result observed by Mr. Smith, from which he inferred the moment of inertia of the pocket-chronometer referred to above.

If, on the other hand, the upper points of support are put very wide apart, the vibration maintained is of the same character as that illustrated in Figure 4, and the watch goes slower than its proper rate. The farther apart the points of support are the less is this effect, as the circumstances approach more nearly to a perfect fixing of the frame.

If now, commencing with the upper points of support very close together, we gradually increase the distance between them, or, starting with them very wide apart, we gradually diminish the distance, a certain critical arrangement is approached from either direction, and the gaining rate in the former case, or the losing rate in the latter case, is augmented. This critical arrangement is such that the period of vibration of the suspended chronometer, when set to vibrate by an external disturbance, is approximately equal to the period of vibration of the balance-wheel. When the upper points of support are adjusted to produce it, and the chronometer, going, is left to itself, the action of the internal prime mover will bring the whole into a state of vibration, which may be either the first fundamental mode (balance-wheel and frame-work vibrating in the same direction), in which case the chronometer will have a losing rate, or the second fundamental mode (balance-wheel and frame vibrating in opposite directions), in which case the chronometer will have a gaining rate. The gain or loss may amount to as much as one second in sixty or eighty with an ordinary ship chronometer, taken off its gimbals, or a pocket detached lever watch. The amount of the effect will of course be much less for a marine chronometer, not removed from its gimbals, but suspended by cords attached to its outer case, on account of the great addition of moment of inertia due to the outer case. With a marine chronometer, or any watch having a chronometer escapement (Harrison's), or having a duplex escapement, the seconds hand jumps forward once, and one comparatively loud beat is heard, for each period of the balance-wheel; and thus it is easy to see whether the watch, when suspended, is vibrating according to the first fundamental mode (losing), or the second mode (gaining), by noticing in which direction the visible motion is at each beat of the escapement. With either of these kinds of escapement the experiments above described are liable to stop the watch when the upper points of support are adjusted for the critical arrangement. Thus, for instance, if the points of support have first been too close for the critical arrangement, and are gradually separated until the vibration of the frame becomes very large, a great gain of rate is produced; and if the distance is then a little farther increased, the watch will often stop: if then a slight impulse round the vertical axis is given to it to start it, it will commence vibrating according to the first fundamental mode, with a largely losing rate. The other corresponding result is obtained by commencing with the points of support too far asunder for the critical arrangement and bringing them gradually together.

Without exciting independent vibrations of the chronometer or watch as a whole, and counting them, it is easy to perceive whether the circumstances approach the critical condition, by applying the hand to steady the watch, and then observing the phenomena presented when it is left to itself. If the upper points of support are either much too wide apart or much too close together for the critical arrangement, the watch-case will not take any regular harmonic vibration, but will make a slight (perhaps scarcely perceptible) jump once every semi-period, or once every period of the balance-wheel, according to the character of the escapement. But if the upper points of support be set approximately to the critical arrangement, and the watch brought to rest and left to itself, it will be seen to commence vibrating through a gradually wider and wider arc until a maximum of vibration is attained. The amplitude of vibration will then diminish, but not to zero; will increase to a second maximum smaller than the first; will diminish to a second minimum not so small as the first minimum; increase to a third maximum smaller than the second; and so on, until, after several of these alternations, a sensibly steady state of vibration, very closely simple harmonic, is attained. How nearly the critical arrangement is approximated to, may be judged by counting the number of vibrations executed from starting to the first maximum, from the first maximum to the first minimum, and so on—the numbers being greater the nearer the adjustment is to the critical condition. I made these experiments first on board the *Great Eastern* during her last summer's cruise; and it was curious, as an illustration of the general principle of the superposition of motions, to watch the various phenomena of vibration of the suspended watch presented, quite independently of the swinging due to the rolling of the ship.

When the top points of support are arranged precisely to the critical condition, I find that the watch will, of itself, take sometimes one mode of vibration, sometimes the other. But a very slight deviation in either direction from the critical arrangement suffices to do away with this indifference, and to insure that, when the watch is steady and left to itself, it will take up either always the gaining or always the losing mode of vibration. But even then it may be compelled to take up either mode by properly-timed touches with the finger, and it continues vibrating accordingly when left to itself. Thus, when the top points of support are adjusted, either precisely, or somewhat approximately, to the critical condition, the watch may be made to go either faster or slower than its proper rate, by applying the hand to cause it to take up either mode of vibration at pleasure, and then leaving it to itself. This last experiment ought not, however, to be pushed too far with a valuable chronometer, as the effort to make it take up a mode of vibration opposite to that which it takes up of itself, is liable to make the escapement-wheel trip and run round rapidly, escaping from the control of the balance-wheel and the escapement—this disturbance not being produced by any violent action of the hand, but by very gentle touches properly timed. No such derangement can, I believe, ever take place when the watch is hung in the manner described, and left at rest to take up whatever mode of vibrating it will, and no damage to the most delicate chronometer can result.

The knowledge of those facts may be of advantage—first, in pointing out a simple plan for setting a chronometer without touching the hands; second, in showing how it ought to be supported, in regular use, so that it may go at a uniform rate and keep correct time. It is usual to place ship's chronometers on cushions, at sea, to guard against damage to the works, from tremors of the ship. If the cushion be moderately hard, the chronometer's rate does not (as I have found by trials on board the *Great Eastern*) differ sensibly from what it is when the chronometer is laid on a hard board, the instrument being of course always kept on its gimbals in its heavy outer case. If, however, the cushion is soft enough, the critical condition explained above may be reached or even passed; and great variations of rate in either direction may be produced. Thus, a certain degree of softness in the cushion may make the chronometer lose considerably; and a still softer cushion may make it gain considerably; and cushions softer yet would make the chronometer gain, although not so much. It is possible that an improvement in the practical performance of chronometers at sea may be attained by fixing the outer case of the instrument to a very heavily weighted base, this base being placed on an ordinary cushion.

At the conclusion of the paper, in answer to questions by the PRESIDENT, Mr. DAY, and Mr. DAVISON,

Sir WM. THOMSON said that the weight of the chronometer would influence the rate at which it would gain or lose by the oscillation, so that it was better to have a massive watch than a light one, as the former was more likely to go well. No doubt, the rate of an ordinary watch-chronometer is very much affected by railway travelling. His own pocket-watch gained from four to eight seconds in journeys to London and back. The railway carriage vibration affected as a prime mover the vibration of the balance-wheel, not merely as vibrations induced in the frame by the interior movement would do. If a chronometer case is well weighted, its performance will not be practically injured by the influence which has been described. If it were firmly attached to the middle of a four-feet-long plank, with heavy weights fixed near the ends, its rate would be sensibly the same as if its case were absolutely fixed, however this board is supported. To avoid damage from the tremors of the ship, this board should be placed on cushions, and strapped down, or lashed properly, for security.

If a watch be hung on a nail, it depended upon the dimensions of the watch and the time of the balance-wheel whether it will go faster or slower than its proper rate. If, when hung on a nail and set to swing, it vibrated more rapidly than the balance-wheel, then the effect of the hanging would be to induce a slower-rate; but if when set to swing it vibrates slower than the balance-wheel, then when left to itself it will go faster than when the case of the watch is held quite fixed. A watch regulated to go correctly when hanging on a nail, (according to a faulty practice, sometimes followed, he believed, in watchmakers' shops) cannot be expected to go at even approximately the same rate as when carried about in ordinary use.