THE FLOW OF WATER

A NEW THEORY

OF

THE MOTION OF WATER UNDER PRESSURE AND IN OPEN CONDUITS AND ITS PRACTICAL APPLICATION

BY

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PREFACE.

The present work is the outcome of a series of investigations begun several years ago with the object of finding a simple expression for the phenomenon of flow in irrigation channels.

The author hopes that his work will prove of interest and value to the student and useful to the practical engineer.

He also hopes that it will stimulate further research and thus tend to widen the field of hydraulic knowledge.

LOUIS SCHMEER.

LOS GATOS, CALIFORNIA, October, 1909

NOTATION.

$V_1\, v$ = Velocity in feet per second.

$R_1\, r$ = The mean hydraulic radius of a conduit.

$$\frac{\text{Area of cross section.}}{\text{Wet Perimeter.}}$$

$$\frac{\text{Diameter of circular or semicircular conduit.}}{4}$$

$D_1\, d$ = Diameter of a circular conduit.
Depth of Semi square or semi circle.
Depth of Water in a Channel.

$S_1\, s$ = Slope of Water Surface.

$$\frac{\text{Head in feet.}}{\text{Length of conduit in feet.}}$$

$$\frac{\text{Fall of surface in feet.}}{\text{Distance in feet.}}$$

c = The variable coefficient in the formula $v = c\sqrt{r\,.\,s}$.

f = The coefficient of friction, loss of head per unit area of surface at unit velocity.

z = A coefficient indicating the resistance of an impediment to flow.

$M_1 K$ = Coefficients indicating the degree of roughness of the wet perimeter.

a = A coefficient indicating the variation of the coefficient c with the velocity of flow.

$H_1\, h$ = A vertical distance, a head of water.

$L_1\, l$ = Length of a conduit, a horizontal distance.

$W_1 w$ = Width of surface of water.

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THE FLOW OF WATER.

INTRODUCTION.

THERE is no branch of the science of physics on which more has been written than on hydraulics. The master minds of the last four centuries have wrestled with the problem and thread by thread they have torn away the veil of mystery that enveloped the phenomenon of flow.

The universal mind of Leonardo da Vinci (1452–1519), painter, sculptor, scientist and engineer, was the first to pierce the darkness and although he did not give his thoughts on the flow of water mathematical expression, we are to-day, with all the knowledge and experience gained since his time, astounded at his clear and comprehensive reasoning.

The great Galileo (1564–1642) admitted that he had less trouble in finding the law of motion of the planets millions of miles away than in discerning any law in the motion of water in the stream flowing at his feet.

Torricelli (1608–1644), inventor of the barometer, investigated the laws of falling bodies and found that the velocities of bodies falling free vary with the square roots of the heights fallen through, or with $\sqrt{H}$.

Huygens (1629–1695) first found the numerical value of g, the acceleration due to gravity; and following him Bernoulli was (in 1738) able to write the fundamental formula for the velocities of bodies falling free,

$$v = \sqrt{2gH}.$$

On this general theoretical foundation our present system of hydraulics has gradually been built. Brahms (Dyke and other Hydraulic Constructions 1753) made the first step towards a practical application of the then existing theories of motion to the motion of water flowing in a channel. He found that the motion of water flowing in a channel is not like the motion of water falling free, or that of a body rolling down an inclined plane continually accelerated in speed, but moves with a uniform velocity, and that the resistance due to the friction of a fluid against the walls of the conduit depends on the relation of the wet perimeter to the area of the cross-section or on the mean hydraulic depth.

Chezy (in 1776), gave the ideas of Brahms an elegant mathematical expression by writing for the velocity of flow

$$v = c\sqrt{r \cdot s}$$

in which c is a coefficient, which Chezy assumed to be constant, and r the mean hydraulic depth. This simple formula found general application in practice and is still in use.

Subsequent writers occupied themselves chiefly with the definition of variations of the coefficient c in the formula proposed by Chezy.

Owing to the researches of Coulomb (1736–1806) on the resistance of fluids to slow motions, the variation of the coefficient c with the velocity of flow was the first to be recognized and Weisbach and others found expressions for this variation.

If Darcy was not the first to perceive the influence of the degree of roughness of the walls of a conduit on the velocity of flow, he at any rate was the first who thoroughly investigated the subject. (Mouvement de l'eau dans les tuyaux, Paris, 1851.) Beginning his investigations on flow in conduits under pressure he extended them to flow in open conduits and under the auspices of the government of France constructed a special test channel 596.5 meters (1956.5 feet) long and 2 meters wide. This channel was successively lined with materials possessing characteristic degrees of roughness, the cross-section was given various forms and the bottom various slopes. To regulate the discharge two reservoirs were constructed at the head of the channel and the water admitted through carefully tested sharp-edged orifices 20 centimeters square. The experiments were extended also to flow in channels lined with masonry and to flow in channels in earth.

Darcy's work was after his death completed by Bazin, his successor in the office of Chief Engineer of Bridges and Roads in France. Darcy-Bazin's experiments were made with the utmost care and precision and the tabulated data (Darcy-Bazin, Recherches Hydrauliques, Paris, 1856) bear the stamp of scientific exactness and truth; they are mines of reliable information on all matters relating to flow.

Darcy's experiments on flow in pipes have since his time been supplemented by many others. Hamilton Smith in California carefully gauged the discharge of sheet-iron riveted pipes under great pressures, and his data rank in reliability with those of Darcy. Clemens Herschel gauged the discharge of large steel-riveted pipes; Iben that of pipes coated with tar; Adams and Noble the discharge of circular pipes of planed boards.

Kutter, a Swiss engineer, extended the researches of Darcy-Bazin on flow in open conduits to channels of greater slopes and greater dimensions and published (in 1869) the results of his investigations under the title "Versuch zur Aufstellung einer allgemeinen Formel," etc.

Kutter and Ganguillet elaborated a general formula intended to define the variation of the coefficient c in the formula of Chezy with the mean hydraulic radius, the degree of roughness of the walls of the channel and also with the slope.

Despite its cumbrousness this formula found universal application. It has, however, many defects and is no longer regarded as embodying any true law of flow.

Bazin, in his memoir, "Etudes sur les mouvements des eaux dans les canaux decouverts" (Annales des Ponts et Chaussées, Paris, 1898), reviews the accumulated experimental data and proposes a formula of great simplicity. It does not, however, express the variation of c with the velocity or with the slope.

PRIMARY LAWS OF PRESSURE AND FALL.

A.

The physical laws relating to fluids at rest, which are of interest in their relation to fluid motion, are briefly as follows:

1. The pressure of water on a surface is proportional to the depth below the free surface.

Let H be the vertical distance of a horizontal plane below the free surface,

G the weight of one cubic foot of water = 62.37 pounds.

P the pressure in pounds per square foot,

then
$$P = GH = 62.37\ H$$

and the pressure per square inch

$$P = \frac{62.37}{144}\ H = 0.433\ H \text{ pounds.}$$

2. The pressure of water is the same at all points in a horizontal plane irrespective of the horizontal distance of any point in the plane from the free surface. No matter what the shape of the vessel or the length of the conduit may be the pressure at any point is always proportional to the vertical distance below the free surface.

At the bottom of a stand pipe 80 feet below the free surface of the water the pressure on the area of a circle 4 inches in diameter will be

$$0.433 \qquad 80.0 \qquad 4^2 \qquad 0.7854 = 435.2 \text{ pounds.}$$

Let a 4-inch pipe 5 miles long be connected with the standpipe at any point below the free surface, and the end of the pipe be placed in the same horizontal plane as the bottom of the standpipe, then, no matter how many curves or elbows there may be in the length of the conduit, the pressure will be as before, equal to 435.2 pound.

3. If a pressure be applied to the free surface of the water, this pressure is transmitted equally and undiminished in all directions, and to any distance, horizontal or vertical.

Into the upper end of a pipe 1 foot in diameter and filled with water let a piston be inserted and a pressure of 100 pounds applied. Then a pressure equal to $\frac{100}{0.7854} = 129.5$ pounds per square foot will be exerted on any square foot of the inner surface of the pipe, no matter how great the distance. Let the depth of the water below the surface be 20 feet. Then the total pressure per square foot will be

$$129.5 + (20\ 62.37) = 1403.9 \text{ pounds.}$$

If P_1 is the external pressure in pounds per square foot, the total pressure will be, for any distance H,

$$P = P_1 + GH.$$

The external pressure due to the atmosphere is equal to 14.7 pounds per square inch. It is consequently equal to that of a column of water $\frac{14.7}{0.433} = 33.9$ feet in height.

B

Torricelli's fundamental theorem for the velocity of bodies falling free is expressed by the equation:

1. $v = gt$
2. $v^2 = 2gh$
3. $h = \frac{1}{2}gt^2$

Or:

1. The speed of fall is proportional to the time of fall.
2. The square of the speed is proportional to the distance fallen through.
3. The distance fallen through is proportional to the square of the time of fall.

The velocity of fall in feet per second is consequently:

At the end of the first second of fall equal to $g = 32.2$ ft.
At the end of the second second of fall equal to $2g = 64.4$ ft.
At the end of the tenth second of fall equal to $10\ g = 322.0$ ft.

The velocity of fall in feet per second is equal.

At the end of the first foot of space fallen through to

$$\sqrt{2g} = 8.025.$$

At the end of the second foot of space fallen through to

$$\sqrt{4g} = 11.34.$$

At the end of the tenth foot of space fallen through to

$$\sqrt{20g} = 25.35.$$

The distance fallen through is equal:

At the end of the first second of the time of fall to $\frac{1}{2} g = 16.1$ ft.

At the end of the second second of the time of fall to $\frac{1}{2} g\, 2^2 = 64.4$ ft.

At the end of the tenth second of the time of fall to $\frac{1}{2} g\, 10^2 = 1610.0$ ft.

C.

The laws of fall thus stated apply to any body, solid or liquid falling free in vacuo.

For bodies falling in the atmosphere, the resistance of the air has to be considered. This resistance is proportionally the greater, the less the density of the body. Disregarding the resistance of the air, a jet of water issuing from a well-formed orifice has a velocity proportional to the square root of the height of the column of water above the centre of gravity of the orifice.

Let h be the head of water above the centre of gravity of the orifice.

b a coefficient of velocity differing with the nature of the orifice, and the velocity of the jet will be

$$v = b \sqrt{2gh}.$$

If the discharge is into free space the speed of the motion will continue to increase with the distance fallen through, and if h_1 be the vertical distance fallen through in the atmosphere, the water will have, at the end of its journey, acquired a velocity equal to

$$v = b \sqrt{2g(h + h_1)} \text{ nearly.}$$

D.

The motion of a rigid body descending in an inclined plane infinitely smooth is continually accelerated; the law of fall still holds, only with this difference, that in the equation

$$\frac{v^2}{2} = gh$$

g is replaced by g sin d, d being the angle which the inclined plane makes with the horizon. The kinetic energy or living force aquired by a body descending in a plane infinitely smooth is equal to

$$Wh \text{ or } \tfrac{1}{2}\, m\, v^2$$

in which $m = \frac{W}{g}$ = the mass of the body. The weight of the body W, divides into two components; one, equal to W sin d acts parallel to the plane and produces motion; the other, equal to W cos d, acts at right angles to the plane.

When the frictional resistance between the plane and the descending body is considered, the force that produces the motion or W sin d reduces to W sin d, $- zW$ cos d, z being a coefficient of friction.

The acceleration of motion continues as long as W sin d is greater than zW cos d. If they are equal, or if $\frac{\sin d}{\cos d}$, or tangent d is equal to z, the coefficient of friction, the motion will cease.

Following the laws of motion of a rigid body, the motion of a perfect fluid flowing down an inclined plane infinitely smooth would be continually accelerated. Owing, however, to internal friction, to its adhesive qualities, and the friction of the fluid against the surface of the channel in which it flows, water soon spends its accelerating force and the motion arrives at a state of steadiness more or less approaching uniformity.

The motion of water is said to be steady, when at a given point of the cross-section the fluid arrives with the same velocity and in the same direction.

The motion is said to be uniform, if in following a given course the mass of water has a constant velocity.

The motion is said to be varying, if in following a given course the velocity varies from point to point.

In our subsequent discussions of flow we always assume the motion to be uniform, or conditions to be such that there is no acceleration of velocity with increase of the distance fallen through, that the accelerating forces are equalized by frictional resistances and that the velocity of flow at any point in a given course remains constant as long as the slope remains constant.

PRIMARY LAWS OF FLUID FRICTION.

A plane surface moving in a still body of water is retarded in its motion by a resistance due to the friction of the fluid against the surface.

The subject of fluid friction was investigated by Coulomb by rotating disks of greater or lesser diameters and having surfaces of a greater or lesser degree of roughness with more or less speed in a still body of water, at greater or lesser depths, and ascertaining the work done under the various conditions.

The researches of Coulomb were extended by Froude in his investigations on the resistance of the surfaces of ships (1870–1874). For the rotating disks of Coulomb, Froude substituted sharp-edged planks or metal plates of greater or lesser length and coated with various substances. These he impelled to move in a still body of water and ascertained the resistance by a suitable device.

The laws deduced from experiments made by these investigators may be summed up as follows:

1. The pressure existing in any horizontal plane below the free surface or in any part of a conduit under pressure has no influence on the friction of the fluid against a solid surface. Though the pressure in pounds per unit area may be much greater in one part of a conduit than in another, the frictional resistance of the area is not thereby increased. This is demonstrated as follows:

A plank of suitable shape is immersed in a still body of water just below the surface, impelled to move at a certain constant speed, and the resistance to motion ascertained. If the plank is subsequently placed at a greater depth and impelled to move at the same constant speed, it is found that the resistance to motion has not been increased. If a pipe of constant dimensions is resting on an inclined plane, it can also be shown that the loss of head due to the frictional resistance is for equal lengths of the conduit the same in the lower part of the conduit where the pressure is greatest, as in the upper part, where it is least.

2. The resistance to motion, due to the friction of a fluid against a solid surface, is proportional to the area of the surface. This is demonstrated as follows: A plank of a certain length and width is impelled to move at a certain constant speed in a still body of water and the work done in foot pounds noted. If the width of the plank is subsequently doubled, thus doubling the area of its surface, and it is impelled to move at the same constant speed, it is found that the work done in foot pounds is also doubled.

If water flows in a pipe running full it is found that the amount of head consumed in overcoming the resistance of the walls is proportional to the length of the pipe.

Let A_0 be the area of a surface in square feet: W the weight in pounds required to move a plank in a still body of water at a velocity of one foot per second; f the frictional resistance in pounds per square foot of surface

then $$f = \frac{W}{A_0},$$

and the total resistance to motion in pounds at any velocity

$$W = fA_0v^x,$$

x being the variable exponent of the power of v, to which the resistance is proportional.

As the frictional resistance in pounds per square foot for a velocity of one foot per second corresponds to an equal pressure per square foot, the head corresponding to the resistance is

equal to $$h = \frac{f}{G}.$$

The head equal to the resistance or $\frac{f}{G}$, multiplied by $2g$, the acceleration due to gravity or

$$\frac{2\,gf}{G}$$

is termed the coefficient of friction and denoted by z. As $f = \frac{z\,G}{2\,g}$ the total resistance of a surface in pounds is equal to

$$W = z\,GA_0\frac{v^x}{2g}.$$

The velocity of flow remaining after the frictional resistance is equalized acts through a distance equal to v. The total work done in foot pounds in overcoming the frictional resistance of a surface is consequently:

$$f\,A_0\,v^{x+1} = \frac{zGA_0\,v^{x+1}}{2g}.$$

3. The resistance to motion due to the friction of a fluid against a solid surface is for equal areas of the surface greater for a short than for a long surface. This is demonstrated by impelling two planks of equal areas but different lengths to move at equal constant speeds in a still body of water. It will be found that more power is consumed in moving the shorter plank. There is a resistance due to the cutting edge of the plank, this resistance is proportionally more apparent the shorter the plank, because the total surface is proportionally less.

At the entrance of any kind of a conduit head is consumed by a resistance due to shock. For short conduits this head is an appreciable part of the total head consumed. With increasing length of the conduit the head thus consumed becomes proportionally less and less in comparison with the total loss of head and becomes insignificant for very long conduits.

4. The resistance to motion due to the friction of a fluid against a solid surface is increased by elbows, curves, etc.

Joessel, experimenting on the resistance of ships, found the resistance of oblique planes to be equal to

$$R = f\,\frac{\sin.^2 a}{0.39 + 0.61\sin. a}\,d\,A\,\frac{v^2}{2g}$$

in which f is a coefficient indicating the degree of roughness of the surface, varying between 1.1 and 1.7, d the density of the fluid, A the area of the surface, a the angle the plane makes with the line of motion.

The resistance to motion in conduits is proportional to the angle of deflection, the radius of a curve and its length.

5. The resistance to motion due to the friction of a fluid against a solid surface varies with the degree of roughness of the surface. It increases rapidly as the roughness of the surface increases. By impelling surfaces coated with different materials to move in a still body of water Coulomb found the following values of z, the coefficient of friction and f, the resistance in pounds per square ft.

Description of Surface.	z	f
For a varnished surface	00258	00250
For a planed and painted plank	00350	00339
For the surface of iron ships	00362	00351
For a new painted iron plate	00489	00443
For a surface coated with fine sand	00418	00405
For a surface coated with coarse sand	00503	00488

6. The power of the velocity to which the frictional resistance is proportional is not constant. It varies with the degree of roughness of the surface; with the length of the surface in the direction of motion; it is also influenced by angles, curves, etc., in the surface.

By impelling surfaces coated with various materials and of various lengths in the direction of motion to move in a still body of water Froude found the following values of x, the exponent of the power of v to which the resistance is proportional:

Description of Surface.	Length of Surface in Feet.			
	2	8	20	50
Varnished surface	2.0	1.85	1.85	1.83
Surface coated with paraffin		1.94	1.93	
Surface coated with tinfoil	2.16	1.99	1.90	1.83
Surface coated with sand	2.0	2.0	2.0	2.0

DISTRIBUTION OF HEAD.

Water issuing from a well-formed orifice flows with a velocity directly proportional to the square root of the vertical distance between the centre of gravity of the orifice and the free surface, and the velocity will continue to increase if the discharge is into free space.

A stream of water entering a conduit encounters various frictional resistance tending to equalize the accelerating forces and uniform motion ensues. The total head consumed in producing this uniform motion may be resolved into several components:

1. Head consumed in producing the velocity. This is always equal to

$$h = \frac{v^2}{2g}$$

and usually but a small fraction of the total head.

2. Head consumed in overcoming the frictional resistance due to the entrance of the conduit. Let z_0 be a coefficient indicating the resistance due to the entrance and the head consumed will be

$$h_0 = z_0 \frac{v^2}{2g}$$

3. Head consumed in overcoming the frictional resistance of the wet perimeter, or of the walls of the conduit.

We have previously seen that the energy expended in overcoming the resistance of a surface is

$$E = z_1 \, G A_0 \, \frac{v^3}{2g} \text{ foot pounds.}$$

Replacing A_0, the area of the surface by its equivalent P, the wet perimeter multiplied by L, the length of the conduit, this is

$$E = z_1 \, GPL \frac{v^3}{2g},$$

and since Q, the discharge, is equal to A_1, the area of the cross-section multiplied by v, the velocity,

$$E = z_1 \, QG \frac{P}{A_1} L \frac{v^2}{2g};$$

and as $$\frac{P}{A_1} = \frac{L}{R}$$

we have $$\frac{E}{AG} = z_1 \frac{L}{R} \frac{v^2}{2g}.$$

As E, the total force in foot pounds, is the product of height of fall, quantity and weight we have

$$\frac{E}{Q\,G} = h_1$$

and consequently

$$h_1 = z \frac{L}{R} \frac{v^2}{2g}.$$

4. Head consumed in overcoming the frictional resistances due to curves, elbows, changes of section, etc.

If z_n is a coefficient indicating the resistances due to these impediments to flow, the head consumed will be equal to

$$h_n = z_n \frac{v^2}{2g}.$$

Summing up all the components we have

$$H = h + h_0 + h_1 + h_n$$

or $$H = \frac{v^2}{2g} + z_0 \frac{v^n}{2g} + z \frac{Lv^2}{R\,2g} + z_n \frac{v^n}{2g}$$

or $$H = (1 + z_0 + z_1 \frac{L}{R} + z_n) \frac{v^2}{2g}$$

From this we have for the velocity

$$v = \left[\frac{2gH}{1 + z_0 + z_1 \frac{L}{R} + zn_n} \right]^{\frac{1}{2}}.$$

This is on the assumption that the resistance of a surface is proportional to the square of the speed. We have already observed, however, that this is not always the case; it is in fact an exception. But we are not yet in a position to give the true indexes of the powers of v to which the resistance is proportional.

DISTRIBUTION OF ENERGY.

A quantity of water, GQ, impounded at a vertical distance, H, above a horizontal plane, possesses with reference to that plane, a stored up or potential energy equal to

$$QGH.$$

If by means of a conduit of greater or lesser length the water is transported to the horizontal plane at the vertical distance H, below the free surface the stored-up energy is transformed into work. The total stored-up energy resolves into several components.

Let the difference of level between the free surface and the horizontal plane be 80 feet, the length of the asphalt-coated cast-iron conduit transporting the water 10,000 feet, and its diameter one foot.

Assuming for z_1 the average value 0.00489 we have for the velocity of flow from the data given

$$v = \left[\frac{64.4 \cdot 80}{1 + 0.505 + 0.00489 \dfrac{10000}{0.25}}\right]^{\frac{1}{2}},$$

or $v = 5.11$ feet per second.

The discharge in cubic feet per second will be

$$Q = 5.11\, d^2\, 0.7854 = 4.013 \text{ cubic feet.}$$

The total energy expended in transporting this quantity is equal to

$$E = 4.013 \,.\, 62.4 \,.\, 80 = 20{,}033 \text{ foot pounds.}$$

This total energy of 20,033 foot pounds is consumed as follows:

1. A quantity of work is done in producing the velocity of flow. This is equal to

$$QG \frac{v^2}{2g} = 4.013 \,.\, 62.4 \frac{26.14}{64.4} = 101.6 \text{ foot pounds.}$$

2. Another quantity of work is done in overcoming the resistance at the entrance. This is equal to

$$QGz_0 \frac{v^2}{2g} = 4.013 \,.\, 62.4 \,.\, 0.505 \frac{26.14}{2g} = 51.3$$

This is on the assumption that $z_0 = 0.505$.

3. The principal part of the work is done in overcoming the frictional resistance of the interior surface of the conduit. This is equal to

$$QGz_1 \frac{L}{R}\frac{v^2}{2g} = 4.013 \,.\, 62.4 \,.\, 0.00489 \frac{10{,}000}{0.25}\frac{26.14}{64.4} = 19{,}880$$

foot pounds.

The sum of the several quantities of work done in transporting 4.013 cubic feet of water a vertical distance of 80 and a horizontal distance of 10,000 feet is equal to

$$101.6 + 51.3 + 19{,}880 = 20{,}033 \text{ foot pounds,}$$

or

$$QGH = QG\frac{v^2}{2g} + QGz_0\frac{v^2}{2g} + QGz_1\frac{L}{R}\frac{v^2}{2g}$$

Dividing both sides of the equation by QG we have as before

$$H = \frac{v^2}{2g} + z_0\frac{v^2}{2g} + z_1\frac{L}{R}\frac{v^2}{2g}$$

$$= \left(1 + z_0 + z_1\frac{L}{R}\right)\frac{v^2}{2g}.$$

The Coefficient C in the Formula $v = C\sqrt{r.s}$.

Neglecting the loss of head due to the velocity, the loss of head due to the frictional resistance of the entrance, and the loss of head due to the resistance of other obstructions to flow, which severally or combined, form but a small part of the total head lost if the conduit is of a length of 4,000 times the mean hydraulic depth or the velocity not great, we have

$H = z_1\frac{L}{R}\frac{v^2}{2g}$ as the loss of head due to the frictional resistance of the walls of the conduit. From this we have

$$z_1\frac{v^2}{2g} = \frac{HR}{L} = r \cdot s,$$

and

$$v = \sqrt{\frac{2g}{z_1}}\sqrt{r \cdot s.}$$

The term $\sqrt{\frac{2g}{z_1}}$ is equal to the coefficient c first introduced into hydraulic calculations by Chezy, a French engineer (in 1776). On account of its brevity, this term is almost exclusively used to indicate the frictional resistance of long conduits of all descriptions.

As
$$z_1 = \frac{2\,gf}{G}$$

in which f = the frictional resistance in pounds per square foot of surface,

G = the weight of one cubic foot of water = 62.4 we may write

$$C = \left[\frac{2\,g}{\frac{2\,gf}{G}}\right]^{\frac{1}{2}},$$

and as $\frac{f}{G}$ = head lost per unit area of surface at unit velocity, we have finally

$$C = \sqrt{\frac{1}{\text{head lost per unit area at unit velocity.}}}$$

Chezy and many of his followers up to the middle of the last century considered the coefficient c to be a constant. The researches of Coulomb, the investigations of Prony, Eytelwein, Weisbach and others, however, revealed the fact, that it varies with the velocity of flow. Later researches by Darcy and Darcy-Bazin brought to light the astounding influence of the degree of roughness of the walls of a channel and of the value of the mean hydraulic radius on the value of c. The manifold variations of c render the problem of its exact valuation one of great difficulty. A mathematical expression embodying all variations will necessarily be very complex; to be of practical value, however, it should be as simple as possible. It is somewhat difficult to harmonize great exactness and great simplicity without making sacrifices at one end or the other. On this account two expressions are often found embodying the same idea and rendering it with great exactitude or great simplicity.

We will now proceed to investigate the laws on which the variation of c depends and to find suitable mathematical expressions embodying these laws.

I. Primary Determination of the Coefficient *c*.

Going back to first principles we may ask the question: To what power of R, the mean hydraulic radius, is the velocity of flow proportional? Using the exponential equation

$$\frac{v_1}{v_0} = \left(\frac{R_1}{R_0}\right)^x$$

which gives

$$x = \frac{\log v_1 - \log v_0}{\log R_1 - \log R_0}$$

we find that the value of x is, in the case of channels in earth, such as rivers and canals and with R varying between 1 and 50 feet in the majority of cases equal to

$$\frac{1}{1.333} \qquad \frac{2}{2.666} \qquad \text{or} \qquad \frac{3}{4}$$

For this class of conduits we may consequently write:

$$v = yR^{\frac{3}{4}} \sqrt{s},$$

in which y is variable, differing with the degree of roughness and with the slope of the conduit. As $v = c\sqrt{r.\ s.}$ and $R^{\frac{3}{4}} = \sqrt[4]{r}\ \sqrt{r}$ we have

$$C = y\ \sqrt[4]{r},$$

hence c increases directly with $\sqrt[4]{r}$.

Column 5, Table I, gives values of $y = \frac{c}{\sqrt[4]{r}}$ for conduits of several degrees of roughness. It will be observed that the formula gives fairly constant values of y only for large conduits, such as rivers and canals,

For small conduits however y increases with increase of R if the wet perimeter be smooth, but decreases with increase of R if the contrary is the case. Applying the exponential equation to other classes of conduits, the following values of x, the power of R, to which the velocity is proportional were found.

For a semi-circular channel of fine cement	$x = 0.67$
For a semi-circular channel of concrete	$x = 0.68$
For a rectangular channel of rough boards	$x = 0.69$
For a rectangular channel of rough masonry	$x = 0.75$
For a channel carrying coarse detritus	$x = 1.00$

The conclusions to be drawn from these data may be summed up as follows:

1. For rivers and canals the power of R, to which the velocity is proportional, is approximately equal to $\frac{3}{4}$.

2 For small channels the power varies with the degree of roughness of the perimeter and the form of the cross-section of the conduit.

3. For small channels the power of R increases with increase of roughness.

4. For the smoothest class of conduits the velocity is proportional to $R^{0.67}$ for the very roughest to $R^{1.0}$. Hence the rougher the wet perimeter, the more conditions are approached resembling those pertaining to flow in permeable strata, in which instance the velocity is proportional to the square of the diameter of the channel.

5. No formula, based on any one single power of R can give satisfactory results when applied to all classes of conduits.

TABLE I.

Description of Conduit.	R	1000 s	v	$y = \frac{C}{\sqrt[4]{r}}$
Sudbury Conduit. Smooth hard brick well pointed.	0.5	0.189	1.134	138
	0.6	0.189	1.371	135
	0.8	0.189	1.515	131
	1.0	0.189	1.754	127
	1.2	0.189	1.948	124
	1.4	0.189	2.148	121
	1.6	0.189	2.382	119
	1.8	0.189	2.514	119
	2.0	0.189	2.683	116
	2.2	0.189	2.843	114
	2.33	0.189	2.929	113
Semicircular channel lined with pebbles $\frac{3}{8}$ to $\frac{7}{8}$ inch diameter.	0.454	1.5	2.17	95.1
	0.546	1.5	2.50	95.3
	0.619	1.5	2.69	92.5
	0.681	1.5	2.93	92.3
	0.731	1.5	3.05	91.3
	0.784	1.5	3.22	90.4
	0.826	1.5	3.33	88.4
	0.900	1.5	3.54	87.6
	0.968	1.5	3.73	85.8
	1.012	1.5	3.95	87.9
Solani Embankment.	6.32	0.140	2.63	55.9
Jaoli Site.	6.53	0.144	2.70	55.0
Sides of brick set in mud, bottoms very rough.	6.79	0.145	2.80	54.9
	7.05	0.146	2.81	53.7
	7.46	0.160	2.94	51.5
Linth Canal, channel in earth, fairly regular.	5.14	0.29	3.414	58.6
	5.93	0.30	3.830	58.2
	6.48	0.31	4.152	58.2
	7.12	0.32	4.418	56.5
	7.52	0.33	4.753	57.4
	8.09	0.34	4.920	55.8
	8.28	0.34	5.058	56.1
	8.62	0.35	5.225	57.7
	8.87	0.36	5.392	55.5
	9.18	0.37	5.530	54.5
River Seine at Paris.	9.48	0.14	3.37	53.1
	10.92	0.14	3.74	52.8
	12.19	0.14	3.81	49.6
	14.50	0.14	4.23	49.4
	15.02	0.14	5.11	49.8
	15.93	0.14	4.68	49.5
	16.85	0.131	4.80	48.6
	18.39	0.103	4.69	51.8
Mill race at Pricbam, Hungary.	0.316	2.2	0.389	20.0
	0.336	2.2	0.588	28.4
Irregular channel lined with rubble masonry.	0.442	2.2	0.953	35.7
	0.548	2.2	1.135	37.7
	0.560	2.2	1.190	39.1
	0.566	2.2	1.270	41.3

Table II.

Description of Conduit.	v	c	a
I New straight asphalt-coated wrought-iron riveted pipe with screw joints. $m = 0.94$ $R = 0.0677$	0.328	76.7	0.80
	1.171	99.9	1.04
	3.117	108.4	1.13
	6.148	117.1	1.219
	10.535	124.0	1.289
	12.786	124.3	1.291
II Test pipe of clear cement. $m = 0.95$ $R = 0.658$	2.78	139.1	1.122
	3.65	139.2	1.139
	4.20	139.5	1.140
	4.72	140.4	1.141
	4.79	141.2	1.155
	4.92	141.4	1.157
	5.81	141.4	1.157
	6.58	142.5	1.166
III New cylinder joint asphalt-coated steel-riveted pipe with many curves. $m = 0.53$ $R = 1.0$	1.0	101.2	1.0
	2.0	108.3	1.09
	3.0	112.8	1.113
	3.5	113.4	1.119
	4.0	113.2	1.118
	5.0	112.0	1.105
	6.0	111.6	1.091
IV Old cast-iron pipe. $m = 0.45$ $R = 0.1995$	1.007	73.6	1.0
	2.32	75.5	1.023
	5.075	75.1	1.02
	6.801	75.2	1.02
	12.576	75.3	1.02
V Heavily incrusted cast-iron pipe. Twenty-five years in use. $m = 0.30$ $R = 0.416$	1.60	64.0	0.948
	2.70	60.0	0.900
	3.60	59.0	0.874
	4.50	58.0	0.858
VI Channel of dry rubble masonry of large stones, bed somewhat damaged. Six years old. $m = 0.30$ $R = 0.19$	8.442	57.4	0.890
	8.905	54.3	0.842
	9.181	50.7	0.784
	9.427	49.6	0.769
	10.145	47.2	0.731
VII Short conduit. Wrought-iron riveted pipe, somewhat rusty. Length 152.9 feet. Diameter 8.58 feet. $m = 0.54$ $R = 2.145$	0.50	126.9	1.089
	1.0	116.6	1.00
	1.5	111.9	0.959
	2.0	109.4	0.938
	2.5	109.0	0.934
	3.0	108.2	0.928
	3.5	107.0	0.917
	4.0	106.2	0.910
	4.5	105.6	0.905

Variation of the Coefficient C with the Roughness of the Wet Perimeter of a Conduit.

Although the primary formula $v = y\, R^{\frac{3}{4}} \sqrt{s}$ does not give satisfactory results when applied to all classes of conduits it may be made the basis of formulæ of general application.

Regarding $y \sqrt[4]{r}$ as an approximate value of c expressions may be found defining the variation of c with the roughness of the wet perimeter as depending on $\sqrt[4]{r}$. The primary value of c from which its variations with the slope or the velocity of flow must be derived is that value which corresponds to a velocity of one foot per second.

In order to retain if possible a straight line formula we may choose the expression

$$c = (y \sqrt[4]{r})\ 1 + m,$$

m indicating the condition of the wet perimeter of the conduit.

For a primary determination of y and m Darcy's values of c for clean iron pipes were selected. These data give $c = 112.0$ for $R = 1.0$ and $c = 80.4$ for $R = 0.0208$ (or a one-inch pipe).

These values of c are merely average values found by Darcy from a great number of experiments on clean pipes, which, however, did not include pipes of great diameters. Taking 50 as a trial value for y we find

$$\frac{112.0}{50 \sqrt[4]{r}} = 2.24, \text{ hence } 1 + m = 1 + 1.24$$

$$\frac{80.4}{50 \sqrt[4]{r}} = 4.21, \text{ hence } 1 + m = 1 + 3.21.$$

Dividing 1.24 by 3.21 that the quotient is 0.386. This is almost equal to 0.38, the fourth root of 0.0208, the value of R for the one-inch pipe. We have consequently in both instances

$$c = (50 \sqrt[4]{r})\ 1 + \frac{1.24}{\sqrt[4]{r}}$$

or in general $$c = (50 \sqrt[4]{r})\ 1 + \frac{m}{\sqrt[4]{r}}$$

Testing this formula by experimental data pertaining to flow in conduits differing widely in their degree of roughness it did not prove entirely satisfactory. As already stated, Darcy's experiments were made on conduits of comparatively small diameters and his coefficients for the larger conduits do not quite agree with those found by recent experiments. For the final determination of the value of y we choose the graphical method.

If
$$c = (y\sqrt[4]{r})\, 1 + \frac{m}{\sqrt[4]{r}}$$

then
$$\frac{c}{y\sqrt[4]{r}} = 1 + \frac{m}{\sqrt[4]{r}}$$

This is the equation of a straight line (Fig. 1) having for abscissae values of $\frac{1}{\sqrt[4]{r}}$, for ordinates values of $\frac{C}{y\sqrt[4]{r}}$ and having

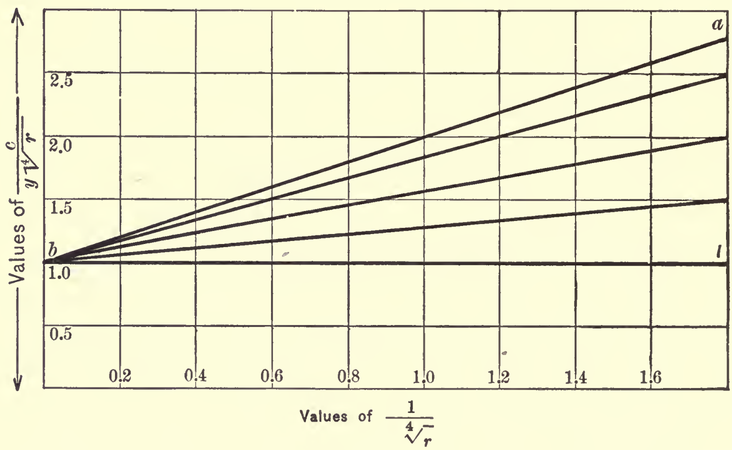

Fig. 1.

1.0 as the common distance from the axis of abscissae where all the lines intersect the axis of ordinates; the tangent

$$\frac{\frac{C}{y\sqrt[4]{r}} - 1.0}{\frac{1}{\sqrt[4]{r}}}$$

of the angle *a b c* will give the value of m. Identical values will be obtained by putting

$$\frac{V}{yR^{\frac{3}{4}}\sqrt{s}} = 1 + \frac{m}{\sqrt[4]{r}}$$

selecting data in which $v = 1.0$ foot per second.

Experimental data giving values of c corresponding to a velocity of one foot per second are not numerous while those giving values of c corresponding to a velocity of one metre per second are quite abundant, this coming nearer to being an average velocity. On this account data given in metric measure were chosen, taken chiefly from the writings of Darcy-Bazin.

After numerous trials, and using all the reliable material available, a constant value of y and corresponding values of m were found, producing a straight line in every instance. As our subsequent work depends much on the reliability of this constant, great pains were taken to find its exact value. In metric measure its value is equal to 50.0 for which in English measure we substitute 66.0. We have consequently for the value of c corresponding to a velocity of one foot per second

$$c = 66 \sqrt[4]{r}\ 1 + \frac{m}{\sqrt[4]{r}},$$

or, reducing to a straight line

$$c = 66\ (\sqrt[4]{r} + m).$$

As
$$66\ (\sqrt[4]{r} + m) = c = \sqrt{\frac{2g}{z}}$$

and
$$(66\ (\sqrt[4]{r} + m))^2 = c^2 = \frac{2g}{z}$$

we have
$$z = \frac{2g}{(66\ (\sqrt[4]{r} + m))^2}$$

or
$$z = \frac{0.01478}{(\sqrt[4]{r} + m)^2}.$$

As primary expressions for the velocity, in most instances true only when the velocity is equal to one foot per second we have now the formulæ

$$v = 66\ (\sqrt[4]{r} + m)\ \sqrt{r \,.\, s} \quad \ldots\ldots \quad (1)$$

$$v = \sqrt{\frac{2gH}{\frac{0.01478}{(\sqrt[4]{r} + m)^2}\,\frac{L}{R}}} \quad \ldots\ldots \quad (2)$$

In the formula $c = 66\ (\sqrt[4]{r} + m)$, when applied to calculations of flow in channels in earth of a great degree of roughness of the bed, the coefficient m, which indicates the degree of roughness will have a negative value and c will in consequence vanish for very small values of $\sqrt[4]{r}$. To avoid this defect the formula may be written, when applied to channels in earth, so that it reads

$$c = \frac{66\ (\sqrt[4]{r} + 1)}{1 + \frac{K}{\sqrt[4]{r}}} = \frac{66\ (\sqrt[4]{r} + \sqrt{r})}{\sqrt[4]{r} + K}$$

in which K is a coefficient increasing in value with increasing roughness of the wet perimeter. The relation between m and K is given by

$$K = \frac{2.0}{1 + m} - 1.0$$

Variation of the Coefficient *C* with the Velocity of Flow.

A.

The characteristics which distinguish water from a perfect fluid are its adhesive qualities, its viscosity. All fluids, including gases, have these qualities in a greater or lesser degree. It is even asserted that solids like ice become viscous under great pressures. The adhesive qualities of tar or crude oil are apparent to the eye, those of other fluids can only be inferred from their effects.

To its viscosity is due the fact, that water flowing in a channel perfectly smooth, is not, in accord with the law of falling bodies, continually increasing in speed. The retarding forces due to viscosity equalize the accelerating forces due to gravity and distance fallen through, the speed of the water shows no increase from point to point, in other words, the motion is uniform.

The layer of water immediately in contact with the walls of the channel in which it flows does not change except by diffusion;

it is held fast by surface adhesion. If the wall is perfectly smooth there is consequently no friction between it and the fluid directly in contact; the resistances to flow are entirely due to shearing stresses between the infinitely fine film coating the wall and the moving body of water.

Frictional resistances are always proportional to the areas of the surfaces in contact; surface areas near the periphery of a conduit are always greater than near the centre and the retardation will in consequence be greater and the velocity less.

This decrease of speed from the centre towards the periphery is in a measure counteracted by difference of pressure. Greater velocities are always accompanied by a corresponding fall of pressure and the pressure in the centre is in consequence less than near the wall. This difference of pressure continually tends to draw the water towards the centre and thus to equalize the speeds. When this equalizing tendency is for a moment interrupted, we suddenly perceive a wave or flash-like motion, clearly indicating the speed the water would acquire were it not for the resistances near the periphery. In conduits having smooth walls the equalization of velocities is performed so rapidly that a difference of speed between the centre and the periphery is scarcely perceptible. A wave-like rotation is set up and the water glides through the conduit very much like a bullet through a rifled channel.

Let R be the force required to keep up the flow of a liquid in two parallel planes past each other, let the surface area of each plane be A, let the respective distances of the two planes from a common plane of reference be D_1 and D_0, let the velocities be v_1 and v_0 and e a coefficient indicating the degree of viscosity of the liquid and we have:

$$R = \frac{eA\ (v_1 - v_0)}{D_1 - D_0}$$

or: the resistance is proportional to the degree of viscosity into the area and the relative velocity $v_1 - v_0$, the whole divided by the difference in the distance of the two layers from a common plane of reference.

For a circular conduit the total force required to set up motion in a stream line is given by

$$R = \frac{4el\,(v_1 - v_0)}{r_1^2 - r_0^2} + \frac{2flv}{r_1}$$

in which r_1 is the semi-diameter of the conduit, r_0 a distance from the axis of the conduit, l its length, and f the coefficient indicating the degree of roughness of the surface.

This indicates, that the resistance due to viscosity is least in the centre or the axis of the conduit where $r_0^2 = 0$ and greatest at the periphery where $r_1^2 - r_0^2 = 0$.

The last expression gives for the velocity of flow in a circular conduit

$$v = \sqrt{2gh + \frac{(4el)^2}{(r_1^2 - r_0^2 + 2\frac{e}{f}r_1)^2}} - \frac{4el}{r_1^2 - r_0^2 + 2\frac{e}{f}r_1}$$

from which, the coefficients e and f being known, the value of v and the discharge may be computed. The coefficient e depends for its value on the temperature of the liquid; its value diminishes rapidly with increase of temperature and is five times less for water at the boiling point than for water at the freezing point. According to Mayer its values are (in *c. g. s.* units):

at 0.6° Celsius $e = 0.0173$
at 10° Celsius $e = 0.0131$
at 20° Celsius $e = 0.010$
at 45° Celsius $e = 0.005833$
at 90° Celsius $e = 0.00339$.

The influence of the temperature on the flow of water through capillary tubes has been minutely studied by Poisseule. Slichter has demonstrated the immense influence of the temperature on the movement of water through permeable strata and Saph and Shoder have shown its influence on the discharge of pipes. A tube having a diameter of 0.5 millimetre (0.02 inch) or less is considered to be a capillary tube.

Poisseule's experiments demonstrated, that the velocity of flow in such tubes is equal to

$$v = gh\frac{r^2}{8eL}$$

and the discharge to

$$Q = \pi\, gh\frac{r^4}{8eL}.$$

This shows, that in capillary tubes the velocity is proportional to the head and not to the square root of the head, to the square of the radius and not to its square root.

In investigations on the movement of water through porous strata it has been found, that the velocity of flow is proportional to the square of the diameter of the soil grains through which the water percolates; from which it follows that it is also proportional to the square of the voids between the soil grains. The general equation for the movement of water in a permeable stratum may be written (v and Q per minute)

$$v = \frac{(h+z)^2 - h^2}{2L}\, 0.0189 d^2\, (0.7 + 0.03\, t'c)$$

$$Q = m\, v\, h\, b.$$

In these equations h is the elevation of the water table at the point of efflux, $h + z$ its elevation at the distance L, d the

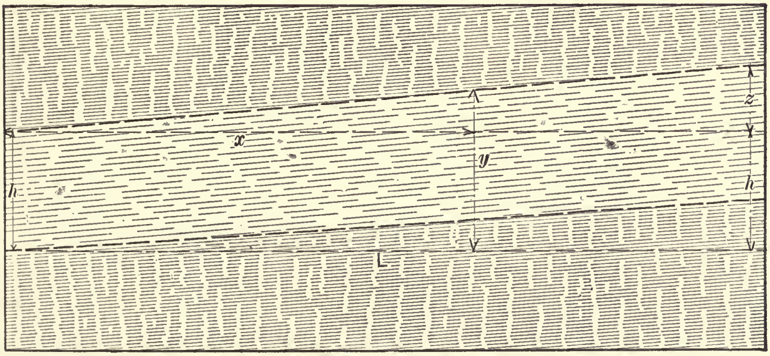

Fig. 2.

diameter of the soil grains in millimetres, $t°\,C$, the temperature of the water in degrees centigrade, m the percentage of the voids in the material, b the breadth of the stratum.

The elevation of the water table at the distance x from the point of efflux is equal to

$$y = \sqrt{h^2 + \frac{2Qx}{hm\, 0.0198\, d^2\, b}}.$$

The discharge of a well is given by

$$Q = \frac{(h + z)^2 - h^2}{\log L - \log R} \pi m\, 0.0189\, d^2\, (0.7 + 0.03\, t°)$$

in which R is the semi-diameter of the well, and the logarithms the Naperian

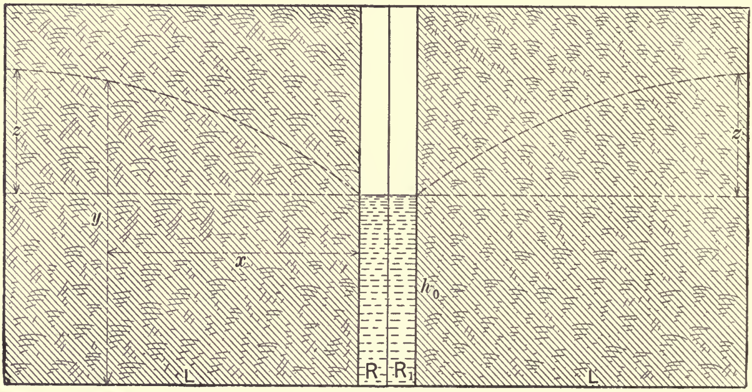

FIG. 3.

The elevation of the water table at a distance x from the well is given by

$$y = \sqrt{h^2 + \frac{Q}{\pi m d^2\, 0.0189} \log\left(\frac{x}{R}\right)}.$$

The surface is consequently a logarithmic curve. These equations serve to illustrate that between the movement in capillary tubes and in a porous stratum there is only this difference, that h is displaced by $\frac{h^2}{2}$, the velocity is not proportional to the head but to the square of the head.

On account of internal motions the phenomenon of flow in pipes and other channels is much more complex than in capillary tubes or porous strata.

The equation for the velocity of flow in a cylindrical conduit we have given above may be transformed so it will read

$$v^2 = 2gh + \frac{(4el)^2}{\left(r_1^2 - r_0^2 + 2\frac{e}{f}r_1\right)^2} - \frac{4el}{r_1^2 - r_0^2 + 2\frac{e}{f}r_1},$$

which shows that one of the terms above the line, denoting the internal resistance, is directly proportional to the velocity, the other to its square. This is also true of the two terms $2\frac{e}{f}r_1$ denoting the friction of the fluid against the walls of the conduit. Moreover, f, the coefficient denoting the surface friction, depends for its value on e, the coefficient denoting the internal friction; its value is consequently modified by temperature. Even in conduits having the smoothest walls there are always rotary and wave-like motions tending to equalize pressures to speeds. Wherever there are cross-currents there naturally is impact, one stream impinging on the other. To this impact and the attending shearing stresses between a streamline and its surroundings are due the increasing powers of the velocity to which the resistances are proportional. Furthermore, if the walls of the conduit are not perfectly smooth there are streamlines constantly impinging on projections, however small they may be.

Conditions existing at the entrance, curves, elbows, changes of section, etc., also affect the power of the speed to which the total resistance is proportional. It was formerly assumed that the resistances due to these impediments were proportional to the square of the velocity.

From experiments made by Hubbel and Fenkell (Detroit) to determine the resistances due to curves, the writer found, neglecting curves the radius R of which is less than 2.5 diameters of the conduit, that the resistance of a curve is equal to

$$4.9\, d^{\frac{3}{13}} \left(\frac{d}{R}\right)^{\frac{3}{13}}$$

times the resistance of a tangent of equal length, and the excess of frictional resistance in a curve equal to

$$\left(4.9\,d^{\frac{3}{13}}\left(\frac{d}{R}\right)^{\frac{3}{13}}\right) - 1.0$$

times the resistance in a tangent of equal length. The length of tangent equal in frictional resistance to the resistance in a curve of 90° is equal to

$$\frac{\pi}{2}R\left(4.9\,d^{\frac{3}{13}}\left(\frac{d}{R}\right)^{\frac{3}{13}}\right) - 1.0.$$

This evidently vanishes when $\frac{R}{d} = (4.9)^{\frac{13}{3}}\,d = 539.3$ and is a maximum when $\frac{R}{d} = (4.9)^{\frac{11}{3}}\,d.$

Hubbel and Fenkell's experiments were made on 30″, 16″ and 12″ conduits, and comparison showed that the influence of the diameter on the resistance depends on the value of $d^{\frac{3}{13}}$. The value of z, or f, the coefficient of friction is therefore for any curve.

$$z = \frac{n^0}{360}2\pi R\left(4.9\,d^{\frac{3}{13}}\left(\frac{d}{R}\right)^{\frac{3}{13}} - 1.0\,\frac{0.01478}{(\sqrt[4]{r} + m)^2 r}\right.$$

in which n is equal to the number of degrees in the curve and $d^{0.45}$ substituted for $d^{\frac{3}{13}}$ for diameter less than a foot.

Hubbel and Fenkell's experiments were supplemented by those of Saph and Schoder on 2-inch brass tubes and more recently by those of Alexander on a 1¼-inch wooden tube. Although we cannot accept the formulæ the latter deduced from his own experiments and those of Hubbel and Fenkell, Saph and Schoder, his experiments are valuable in indicating the powers of the velocity to which resistances in curves are proportional. While Alexander's experiments show that resistances in a curve are proportional to the same powers of the speed as resistances in a tangent, provided there is no shock, the experiments of Saph and Schoder indicate that the power of the speed increases rapidly with increasing values of $\frac{d}{R}\cdot$ Their data indicate that for $\frac{R}{d} = 10$ the resistance is proportional to $V^{1.8}$, for $\frac{R}{d} = 4$ to $V^{2.87}$. How far this holds good for diameters greater than 2 inches we are not prepared to say. It is probable, however, that with increasing diameter the force of the shock decreases and the powers of v with it.

It is probable that the resistances due to right-angled entrances right-angled elbows are also proportional to powers of v higher than 2.0.

The effect of the temperature on the variation of the power of v has so far not been determined with precision. Saph and Schoder, experimenting with a 2-inch brass pipe, found for a rise of 10° F. an increase in the discharge of 4%.

That the resistances to flow are not proportional to the square of the speed was recognized long before Darcy and Bazin demonstrated the great influence of the degree of roughness of the walls of a channel on its discharge.

The laws of fluid friction were first investigated by Coulomb. He states, that the total resistance to motion is a compound of two factors, one being proportional to v, the other to v^2. Dubois's experiments on flow confirmed this view and from his data Prony found for the resistance the expression (in metric measure)

$$Rv = 0.000044\, v + 0.000309\, v^2,$$

this corresponds to

$$H = \frac{z\, v^{1.77}}{2g} \frac{L}{r} .$$

Weisbach put Prony's formula into the form

$$H = 0.00741 \left(1 + \frac{0.0192}{v}\right) \frac{L}{r} v^2$$

which in our day is still used.

The relation of the power of the velocity, to which the resistance is proportional, to the variation of the coefficient c with the velocity is such, that c remains constant for all velocities if the resistance is proportional to v^2; it increases with increase of velocity if the resistance is proportional to v^{2-x}, and decreases if the resistance is proportional to v^{2+x}.

B.

If the value of the coefficient c corresponding to any velocity is divided by its value corresponding to a velocity of one foot per second; the quotient is a variable which we will call the coefficient of variation of c and denote by (a). Hence

$$a = \frac{c}{66\ (\sqrt[4]{r} + m)}.$$

While the term

$$\frac{1}{[66\ (\sqrt[4]{r} + m)]^2}$$

represents the frictional resistance per unit area of surface at unit velocity, the term

$$a = \frac{c}{66\ (\sqrt[4]{r} + m)}$$

indicates the power of the velocity to which the resistance is proportional. We shall presently see, that under normal conditions, that is if resistances proportional to different powers of v do not enter, the coefficient a is merely a root of v.

An analysis of the values of a found in Column 4, Table II, shows that its value does not entirely depend on the velocity, but is affected by the degree of roughness of the walls of the conduit, by its length and alignment, by conditions existing at the entrance, by changes of section, etc. According to the manner in which the coefficient a is affected we may classify conduits as follows:

1. Long straight conduits without internal obstructions and a great degree of smoothness of the wet perimeter.

2. Long conduits of a great degree of smoothness of the wet perimeter but with some easy curves or other impediments, also long straight conduits of a fair degree of smoothness of the wet perimeter.

3. Long conduits of a great or fair degree of smoothness of the wet perimeter but with sharp curves, angles or other impediments to flow.

4. Conduits whose walls are coated with rust, slimy or sticky substances.

5. Conduits of a great degree of roughness of the wet perimeter, badly tuberculated pipes, damaged masonry, channels in earth with sharp bends, bars or other obstructions.

6. Short conduits.

For classes 1 and 2 the resistances are proportional to powers of v less than 2.0 and the coefficients c and a continue to increase in value with increasing velocity. For the third class some resistances proportional to a power higher than 2.0 enter, a increases with increase of velocity and then decreases. For class 4 the resistance is proportional to v^2 or nearly so and a is constant. For classes 5 and 6 the resistances are proportional to powers of v higher than 2.0 and a continually decreases with increasing velocity.

C.

We have so far only found expressions for the value of c corresponding to a velocity of one foot per second. These give for the velocity

$$v = 66 \; (\sqrt[4]{r} + m) \sqrt{rs} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$v = \sqrt{\frac{2\,gH}{\dfrac{0.01478}{(\sqrt[4]{r} + m)^2}\dfrac{L}{R}}} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We will now proceed to find in what relation the value of v as found from the formula

$$v = 66 \; (\sqrt[4]{r} + m) \sqrt{r \, . \, s}$$

stands to the true mean velocity in all cases where v is more or less than unity or the value of a, the coefficient of variation, is affected by the conditions we have enumerated. Using the exponential equation

$$\frac{v_1}{v_0} = \left[\frac{(66 \; (\sqrt[4]{r} + m) \sqrt{rs})_1}{(66 \; (\sqrt[4]{r} + m) \sqrt{rs})_0}\right]^x$$

which gives

$$x = \frac{\log \; v_1 \qquad - \qquad \log \; v_0}{\log \;\; (66 \; (\sqrt[4]{r} + m) \sqrt{r \cdot s})_1 - \log \;\; (66 \; (\sqrt[4]{r} + m) \sqrt{rs})_0}$$

we find from the experimental data given by Darcy and Hamilton Smith for straight or nearly straight clean cast-iron, wrought-iron and sheet-iron riveted pipes of all diameters and for velocities up to 20 feet per second

$$x = \tfrac{9}{8},$$

in other words, from the data given by Darcy and Hamilton Smith we find, that the true mean velocity is equal to

$$v = (66\ (\sqrt[4]{r} + m)\ \sqrt{r \cdot s})^{\frac{9}{8}} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which may be written

$$v = 66\ (\sqrt[4]{r} + m)\ \sqrt{r \cdot s}\ \sqrt[8]{66\ (\sqrt[4]{r} + m)\ \sqrt{rs}};$$

hence the coefficient a, indicating the variation of the coefficient c with the velocity is equal to

$$a = \sqrt[8]{66\ (\sqrt[4]{r} + m)\ \sqrt{r\,.\,s}}.$$

From Formula 3 we have also

$$V^{\frac{8}{9}} = (66\ \sqrt[4]{r} + m)\ \sqrt{rs}$$

consequently

$$a = V^{\frac{1}{9}}$$

and

$$V^{\frac{1}{9}} = \sqrt[8]{(66\ \sqrt[4]{r} + m)\ \sqrt{r\,.\,s}}.$$

Table III contains a number of experimental data relating to flow in conduits under pressure. They are purposely selected in order to show the variation of the coefficient c as affected by various conditions of flow.

The values of the coefficient a found in columns 3, 6 and 9 show that for 1-inch pipes of tin and wrought iron, for sheet-iron riveted pipes up to 2.43 feet in diameter, for new cast-iron pipes up to 1.393 feet in diameter, for pipes of planed shares up to 4.5 feet in diameter the coefficient a is equal to $V^{\frac{1}{9}}$ or nearly so. The fact that $a = V^{\frac{1}{9}}$ holds good for a tin or wrought-iron pipe 1 inch in diameter, and also for a pipe of planed staves 54 inches in diameter allows us to conclude, that it holds good also, between these limits for other conduits having walls of a similar degree of roughness such as asphalt-coated, cast and wrought-iron or cement-lined pipes.

This, however, holds good only when the value of $\frac{L}{d}$, the ratio between the length of a pipe and its diameter, is at least 1,000. For lesser values of $\frac{L}{d}$ the value of (a) decreases with $\frac{L}{d}$. The experimental values given by Stearns and Fitzgerald relating to flow in four-foot cast-iron pipes indicate this plainly. In the case of the four-foot Sudbury conduit (Stearns) the ratio $\frac{L}{d}$ is equal to 439.

If the formula $v = (66 (\sqrt[4]{r} + m) \sqrt{r . s})^{\frac{9}{8}}$ is put into the form

$$v = \left[\frac{2g\ H}{\frac{0.01478}{(\sqrt[4]{r} + m)^2} \frac{L}{R}}\right]^{\frac{9}{16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 4$$

the term

$$\frac{0.01478}{(\sqrt[4]{r} + m)^2} \frac{L}{R}$$

includes all the resistances, those due to the velocity itself, those due to the entrance, and those due to the friction of the fluid against the walls of the conduit.

The loss of head due to the velocity itself is proportional to the square of the speed; resistances due to the entrance are proportional, according to the nature of the entrance all the way from the square of the speed up to its cube. An average value is probably 2.5.

The value of the coefficient of resistance due to the velocity itself is equal to 1.0; the value of z_0, the coefficient representing the resistance due to the entrance is, according to Weisbach, for a well rounded entrance, equal to 0.505; hence the value of z_1, the coefficient representing the resistance due to the walls of the conduit, is equal to

$$z_1 = \frac{0.01478}{\sqrt[4]{r} + m)^2} \frac{L}{R} - 1.505.$$

If the conduit is long, above 1,000 diameters in length, 1.505 is a quantity small in comparison with

$$\frac{0.01478}{\sqrt[4]{r} + m)^2} \frac{L}{R} - 1.505,$$

and does in consequence not appreciably affect the variation of c. With decreasing length of the conduit, however, the ratio between the two quantities changes at an increasing rate and more and more affects the variation of c. In the case of the four foot Sudbury conduit (Stearns), we have the following data, taking $m = 0.97$:

$$v = 3.738,\ H = 1.2421 \text{ ft.},\ L\ 1747 \text{ ft.}$$

$$\frac{0.01478}{(1 + 0.97)^2} \frac{L}{R} = 6.656$$

$$6.656 - 1.505 = 5.151 = z_1.$$

$\frac{(3.738)^2}{2g} \times 1.0 = 0.217 = h =$ loss of head due to velocity.

$\frac{(3.738)^{2.5}}{2g} \times 0.505 = 0.2119 = h_0 =$ loss of head due to entrance.

$0.217 + 0.2119 = 0.4289 = h + h_0.$

$1.2421 - 0.4289 = 0.8182 = h_1 =$ loss of head due to friction in the pipe itself.

Using the formula $\frac{2gh}{z} = v^x$ and inserting values we have

$$\frac{64.4 \times 0.8142}{5.151} = 10.17 = v^x.$$

Dividing log 10.17 = 1.0073209 by log 3.738 = 0.5736293 the quotient = 1.76 very near.

Consequently the frictional resistance in the pipe itself is proportional to $v^{1.76}$, corresponding closely to $V^{\frac{16}{9}}$, the value we have found for long pipes.

The data relating to flow in a riveted flume 8.58 feet in diameter and 152.9 feet long (Herschel, Holyoke Testing Flume) show the great influence of the length of the conduit on the variation of c most plainly.

TABLE III.

EXPERIMENTAL DATA SHOWING EXTENT OF VARIATION OF c WITH THE VELOCITY OF FLOW.

Tin pipe, straight. Dubuat. d = 0.0888 ft. L not given. m = 0.98 $a = V^{\frac{1}{9}}$			Wrought-iron pipe. Darcy. d = 0.1296 ft. L = 372 ft. m = 0.83 $a = V^{\frac{1}{9}}$			Asphalt-coated riveted pipe. Darcy. d = 0.271 ft. L = 365 ft. m = 0.94 $a = V^{\frac{1}{9}}$		
v	c	a	v	c	a	v	c	a
0.141	67.6	0.751	0.205	76.9	0.932	0.328	76.7	0.80
0.772	82.8	0.92	0.858	82.3	0.995	1.171	99.9	1.043
1.183	91.4	1.019	2.585	92.9	1.112	3.117	108.4	1.132
2.546	98.9	1.099	6.3	99.8	1.21	6.148	117.4	1.223
2.606	100.4	1.115	8.521	100.0	1.212	10.535	124.0	1.274
5.223	111.4	1.237				12.786	124.3	1.298

Asphalt-coated riveted pipe. Darcy. d = 0.643 ft. L = 365 ft. m = 0.92 $a = V^{\frac{1}{9}}$			Asphalt-coated riveted pipe. H. Smith. d = 0.911 ft. L = 700 ft. m = 0.68 $a = V^{\frac{1}{9}}$			Asphalt-coated riveted pipe. H. Smith. d = 1.229 ft. L = 700 ft. m = 0.69 $a = V^{\frac{1}{9}}$		
v	c	a	v	c	a	v	c	a
0.591	104.1	1.013	4.712	107.1	1.19	4.283	111.6	1.181
1.529	106.2	1.035	6.094	110.6	1.229	6.841	117.8	1.246
1.53	115.6	1.125	6.927	111.5	1.24	7.314	119.1	1.261
5.509	125.4	1.22	8.659	113.4	1.26	8.462	119.1	1.26
9.0	130.2	1.267	10.021	115.5	1.283	10.593	121.6	1.285
19.72	141.0	1.372				12.09	121.3	1.280

Table III. — *Continued.*

Asphalt-coated cast-iron pipe. Darcy. $d = 0.4495$ ft. $L = 366$ ft. $m = 0.90$ $a = V^{\frac{1}{9}}$			Asphalt-coated cast-iron pipe. Hubbel & Fenkell. $d = 1.0$ ft. L not given. $m = 0.83$ $a = V^{\frac{1}{9}}$			Asphalt-coated cast-iron pipe. Lampe. $d = 1.373$ ft. $L = 26{,}000$ ft. $m = 0.83$ $a = V^{\frac{1}{9}}$		
v	*c*	*a*	*v*	*c*	*a*	*v*	*c*	*a*
0.489	94.1	0.96	1.0	101.5	1.0	1.577	110.5	1.072
2.503	108.4	1.107	2.0	109.6	1.06	2.489	114.1	1.107
5.625	112.5	1.15	3.0	114.6	1.13	2.709	114.6	1.112
11.942	113.5	1.16	4.0	118.3	1.166	3.090	119.4	1.162
15.397	112.2	1.15	5.0	121.3	1.196			

Redwood Stave Pipes. A. L. Adams. $d = 1.166$ ft. $L = 80{,}006$ ft. $m = 0.93$ $a = V^{\frac{1}{9}}$			Cedar Stave Pipe. Th. A. Noble $d = 3.667$ ft. L not given. $m = 0.50$ $a = V^{\frac{1}{9}}$			Cedar Stave Pipe. Th. A. Noble. $d = 4.5$ ft. L not given. $m = 58$ $a = V^{\frac{1}{9}}$		
v	*c*	*a*	*v*	*c*	*a*	*v*	*c*	*a*
0.698	97	0.908	3.468	110.1	1.134	2.282	116.8	1.095
0.698	101	0.926	3.522	108.6	1.12	2.276	115.5	1.086
0.751	104	0.953	3.685	110.9	1.144	2.650	119.9	1.12
0.691	105	0.963	3.853	112.6	1.163	3.067	122.1	1.151
1.167	109	1.0	3.964	112.9	1.164	3.045	121.4	1.138
1.531	112	1.027	3.972	113.1	1.164	3.408	123.7	1.164
1.181	113	1.036	4.415	113.7	1.164	3.724	125.2	1.176
			4.635	114.9	1.183	3.929	126.2	1.179
			4.831	115.5	1.190	4.688	129.0	1.205

TABLE III. — *Continued.*

New steel-riveted pipe. Herschel. $d = 3.5$ ft. $L = 81{,}339$ ft. $m = 0.56$ $a = V^{\frac{1}{18}}$			New steel-riveted pipe. Herschel. $d = 4.0$ ft. $L = 24{,}648$ ft. $m = 0.47$ $a = V^{\frac{1}{18}}$			New steel-riveted pipe. Marx-Wing. $d = 6.0$ ft. $L = 4{,}367$ ft. $m = 0.50$ $a = V^{\frac{1}{18}}$		
v	*c*	*a*	*v*	*c*	*a*	*v*	*c*	*a*
1.0	101.0	1.0	1	97.1	1.0	1.07	103.5	1.0
2.0	104.3	1.032	2	101.3	1.043	1.67	108.0	1.024
3.0	106.4	1.053	3	102.2	1.052	2.14	113.0	1.091
4.0	107.8	1.067	4	104.2	1.073	2.50	108.0	1.024
5.0	108.4	1.073	5	105.1	1.083	3.0	112.0	1.082
6.0	108.5	1.074	6	105.2	1.084	3.84	113.0	1.091

Asphalt-coated cast-iron pipe. Stearns. $d = 4.0$ ft. $L = 1{,}747$ ft. $m = 0.97$ $a = V^{\frac{1}{18}}$			Cleaned cast-iron pipe. Fitzgerald. $d = 4.0$ ft. L not given. $m = 0.98$ $a = V^{\frac{1}{18}}$			Cedar stave pipe Marx-Wing. $d = 6.0$ ft. $L = 4{,}000$ ft. $m = 0.66$ $a = V^{\frac{1}{27}}$		
v	*c*	*a*	*v*	*c*	*a*	*v*	*c*	*a*
3.738	140.1	1.077	2.472	137.5	1.051	1.0	116.0	1.0
4.965	142.1	1.093	3.723	139.1	1.064	1.5	118.7	1.023
6.193	144.1	1.109	4.796	141.1	1.085	2.0	119.9	1.032
			6.141	143.6	1.100	3.0	121.4	1.046
						4.0	122.0	1.051
						5.0	122.4	1.055
						6.0	122.5	1.056

The experimental data relating to flow in riveted conduits show great diversities both in the values of m and a.

The coefficient m is equal to 0.94 for a riveted pipe 0.270 feet in diameter (Darcy) and equal to 0.51 for a butt-jointed riveted pipe six feet in diameter (Marx-Wing). This great difference in the values of m is mainly due to the size of the rivet heads. In pipes of small diameters the rivet heads, especially when coated with asphalt, do not offer an appreciable impediment to flow. In large conduits, however, their size is such, that they not only produce constriction of the section, but also vortex motions, thus reducing the discharge in a twofold manner. From data relating to flow in steel-riveted pipes exceeding three feet in diameter, given by Herschel, and by him considered the most reliable (see "Herschel" 115 Experiments), we find that the coefficient of variation of c for these conduits is fairly, though not precisely, equal to

$$a = \sqrt{V^{\frac{1}{9}}} = V^{\frac{1}{18}}$$

$$= \sqrt[17]{66\ (\sqrt[4]{r} + m)\ \sqrt{r\,.\,s.}}$$

Consequently

$$v = (66\ (\sqrt[4]{r} + m)\ \sqrt{r\,.\,s})^{\frac{18}{17}} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\text{or } v = \left[\frac{2gH}{\dfrac{0.01478}{(\sqrt[4]{r} + m)^2}\ \dfrac{L}{R}}\right]^{\frac{9}{17}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This value of $a = V^{\frac{1}{18}}$ we find to hold good also for flow in rectangular pipes (Darcy).

The experimental data relating to flow in old iron pipes, those not heavily incrusted or tuberculated show that the coefficient c does not vary to any extent with variations in the value of v. For this class of conduits we have consequently

$$a = 1.0.$$

The data relating to flow in badly incrusted or heavily tuberculated pipes indicate a decrease in the value of c with increasing velocity. Using as before the equation

$$x = \frac{\log v_1 \quad - \quad \log v_0}{\log\ (66\ (\sqrt[4]{r} + m)\ \sqrt{rs})_1 - (66\ (\sqrt[4]{r} + m)\ \sqrt{rs})_0}$$

we find for incrusted pipes:

$$x = \tfrac{18}{19}, \text{ hence } a = \frac{1}{V^{\frac{1}{18}}} = \frac{1}{\sqrt[19]{66\,(\sqrt[4]{r} + m)\sqrt{r\,.\,s}}},$$

and for very badly tuberculated pipes:

$$x = \tfrac{9}{10}, \text{ hence } a = \frac{1}{V^{\frac{1}{9}}} = \frac{1}{\sqrt[10]{66\,(\sqrt[4]{r} + m)\sqrt{r\,.\,s}}}.$$

The experimental data relating to flow in a 12-foot brick sewer at Milwaukee, a 7.5-foot brick sewer at Dorchester Bay, in a siphon aqueduct of 119 feet cross-section at the river Elvo all show a slight decrease in the value of c with increasing velocity. This decrease is due, in the first two cases, partly to the fact that these conduits are discharging under water against a hydraulic counterpressure, partly it is due to the greater viscosity of the sewerage and partly also to the relative shortness of these conduits. In the case of the siphon aqueduct, its length is so short comparatively, that it can only be considered as a short pipe, conditions being much the same as in the case of the Holyoke Testing Flume.

D.

The variation of the coefficient c as deduced from experimental data relating to flow in conduits under pressure may be summarized as follows:

1. For long, straight conduits fairly clean, such as pipes of glass, tin, lead, galvanized iron, cast and wrought iron, planed staves, cement, riveted pipes up to 3 feet in diameter, the coefficient of variation of c is equal to

$$a = V^{\frac{1}{9}}$$

and the frictional resistance is proportional to $V^{\frac{16}{9}}$.

2. For pipes rectangular in section, for riveted pipes exceeding 3 feet in diameter, for those enumerated under (1) between 300 and 1,000 diameters in length, the coefficient of variation of c is equal to

$$a = V^{\frac{1}{18}}$$

and the frictional resistance is proportional to $V^{\frac{17}{9}}$.

3. For the classes of pipes enumerated under (1) and (2) discharging against a hydraulic counterpressure, or between 100 and 300 diameters in length, for old pipes not incrusted or tuberculated the coefficient c does not appreciably vary with the velocity, and consequently

$$a = 1.0$$

and the frictional resistance is proportional to V^2.

4. For incrusted pipes and those enumerated under (1) and (2) less than 100 diameters in length the coefficient of variation of c is equal to

$$a = \frac{1}{V^{\frac{1}{18}}}$$

and the frictional resistance is proportional to $V^{\frac{19}{9}}$.

3. For very heavily tuberculated pipes the coefficient of variation of c is equal to

$$a = \frac{1}{V^{\frac{1}{9}}}$$

and the frictional resistance is proportional to $V^{\frac{20}{9}}$.

In our collection of experimental data we find many instances relating to flow in one and the same conduit which do not fit any of the values of (a) enumerated above and which indicate:

1. First an increase in the value of c with increasing velocity up to a certain critical velocity.

2. Then a decrease in the value of c with increasing velocity.

As instances of this kind we mention:

Two new steel riveted pipes at East Jersey, 3.5 and 4 feet in diameter (Herschel).

A cement lined pipe with elbows (Fanning).

This peculiar variation of c indicates the presence of resistances which are proportional to powers of the velocity greater than 2.0, that is resistances which produce shocks. In case of the steel pipes the shocks are no doubt due to the rivet heads, in the second to the elbows in the line of the conduit. This peculiar variation of the coefficient c is also very plainly indicated in the data relating to flow in channels of rough boards with cleats nailed crosswise to bottom and sides of the channel

(Darcy–Bazin, series 12–17). These cleats or laths were 1 centimetre thick and 2.5 centimetres wide. In one channel they were spaced apart 1 centimetre, in the other 5.0. The data relating to flow in the channel with the cleats spaced 1 centimetre indicate the highest values of both the coefficients m and a, plainly showing the effect of the shock due to the wider spacing of the cleats. In the first case the coefficients are $m = 0.41$, $a = V^{\frac{1}{36}}$, in the second $m = 0.03$ $a = \frac{1}{V^{\frac{1}{9}}}$, indicating that the frictional resistance was proportional in the first case to $V^{1.94}$, in the second to $V^{2.25}$.

Open Conduits.

E.

An analysis of experimental data relating to flow in open conduits of permanent cross-section, such as aqueducts, flumes, etc., indicates, that the coefficient c is affected in its variation with the velocity by the shape of the cross-section, or by the depth of the water in the channel.

For semicircular or well rounded channels, for the semi-square when flowing full, for all sections for which the mean hydraulic radius is equal to half the depth, for the triangle with sides inclined 45° the variation of the coefficient c with the velocity does not seem to be affected by slight variations in the value of r. For rectangular channels, however, and others having very steep side walls (excluding those mentioned above) the variation of c is affected by the depth of water in the conduit.

The coefficient a seems to have its normal value in all instances when the depth of water is equal to one-half the mean width of the channel, it increases in value as the depth of water decreases, and decreases in value as the depth of water increases.

This peculiar influence of the steepness of the walls of a conduit on the frictional resistance has been revealed by numerous current metre observations in rectangular flumes and aqueducts and other channels with steep side-walls. It has been found that in such channels the position of the thread of maximum velocity is situated at a greater distance from the surface than in channels having side walls more inclined; thus clearly indicating the retarding influence of the steepness of the walls. Experimental data relating to flow in rectangular flumes frequently indicate values of the coefficient (a) as high as $v^{\frac{2}{7}}$ for small depths, its value is generally equal to $v^{\frac{1}{9}}$ when the mean hydraulic radius is equal to one-fourth the width of the channel. Its value is less than the normal when the depth exceeds one-half the mean width of the channel. Applying the exponential equation

$$x = \frac{\log v_1 - \log v_0}{\log (66 (\sqrt[4]{r} + m) \sqrt{rs})_1 - \log (66 (\sqrt[4]{r} + m) \sqrt{r \cdot s})_0}$$

to data relating to flow in a semi-circular channel lined with neat cement (Darcy-Bazin, series 24) we find

$$x = \tfrac{11}{10} \text{ very near.}$$

Applying the same equation to data relating to flow in channels lined with rough boards, semicircular in section, we find (Darcy–Bazin, series 26)

$$x = \tfrac{13}{12},$$

thus indicating a slight decrease in the value of a with increasing roughness of the conduit's wet perimeter. As a mean between these two values and differing but slightly from either we may take

$$x = \tfrac{12}{11}$$

which corresponds to $a = V^{\frac{1}{12}}$

or $$a = \sqrt[11]{66 (\sqrt[4]{r} + m) \sqrt{rs}}$$

and the frictional resistance is proportional to $V^{\frac{11}{6}} = V^{1.83}$. This value of the power of the velocity we observe is the identical value Froude found for smooth plain surfaces in his investigations on the resistance of ships.

Besides the two series mentioned, given by Darcy-Bazin, we find that

$$x = \tfrac{12}{11}$$

holds also good for the following:

Darcy-Bazin, series 25, semicircular channel lined with smooth concrete.

McDougall, Provo Canal Flume, semicircular channel of planed staves.

Th. Horton, Conduit of North Metropolitan Sewage System of Massachusetts. Brickwork washed with cement. Diameter 9 feet. Values of R up to 2.31 feet.

F.

Applying the experimental equation as indicated above to data relating to flow in channels not semicircular in section and lined with cement or concrete, planed or rough boards, brickwork and good ashlar masonry we find

$$x = \frac{18}{17}$$

$$a = V^{\frac{1}{18}}$$

$$= \sqrt[17]{66\,(\sqrt[4]{r} + m)\sqrt{rs}}$$

and the frictional resistance is proportional to $V^{\frac{17}{9}}$. This we find to hold good for the following:

Darcy-Bazin, series 2, neat cement, section rectangular.

Darcy-Bazin, series 6, 7, 8, 9, 10, 11, 18, 19, 21, 22, and 23, sawed boards, section rectangular, triangular or trapezoidal.

Darcy-Bazin series 32, 33, 39, channels lined with good ashlar masonry, section trapezoidal.

Darcy-Bazin, series 3, rough brick work, section rectangular.

Darcy-Bazin, series 4, channel lined with pebbles up to $\frac{7}{8}$ inch in diameter, section rectangular.

Fteley and Stearns, Sudbury conduit, very good brickwork, sides of channel nearly vertical, bottom flat arch.

Fairlie Bruce, Aqueduct of Glasgow, smooth concrete, sides of channel nearly vertical, bottom flat arch.

Th. Horton, Conduit of North Metropolitan Sewage System of Massachusetts, brickwork washed with cement, covered with sewer slime, sides of conduit vertical, bottom flat arch.

Lippincott, San Bernardino Canal. Trapezoidal channels in earth, lined with concrete.

Kutter, Gontenbachschale, new and well built channel of dry rubble masonry.

Passini and Gioppi, Aqueduct of the Cervo, Canal Cavour. Floor of concrete, sides of brick, section rectangular. Values of R up to 7.2 feet.

G.

Applying the exponential equation as indicated to data relating to flow in channels having walls possessing a greater degree of roughness than those enumerated above we find

$$x = 1.0$$

$$a = 1.0$$

and the frictional resistance is proportional to v^2. This, amongst others, holds good for the following:

Darcy-Bazin, series 1, 34, 35, channels lined with roughly hammered stone masonry.

Darcy-Bazin, series 5, channel lined with pebbles $1\frac{1}{4}$ inch to $1\frac{1}{2}$ inch in diameter.

Kutter, numerous channels lined with dry rubble masonry.

Perrone, Torlonia drain tunnel, channel in rockwork, partly lined with rubble masonry.

We mention here also:

Cunningham, Aqueduct of the Solani, Ganges Canal. Floor of brick, laid flat, sides of masonry, length 920 feet.

In this case the fact that c does not vary with the velocity of flow is due to the shortness of the conduit. It has no independent slope and the movement of the water is influenced by the greater resistance in the rough channel in earth downstream. This is plainly indicated by the low value of the coefficient m.

Of open conduits, not channels in earth, there are few possessing a degree of roughness still greater than those enumerated, exceptional cases of old and damaged rubble masonry.

H.

The variation of the coefficient c with the velocity of flow as deduced from experimental data relating to flow in open conduits not channels in earth may be briefly summarized as follows:

1. For semicircular channels lined with cement, concrete, good brickwork, planed or rough boards, the value of the coefficient a is equal to $V^{\frac{1}{12}}$.

2. For rectangular, triangular or trapezoidal channels of the same description, for channels lined with rough brickwork, ashlar and very good rubble masonry, for channels lined with pebbles up to $\frac{7}{8}$ inch diameter the value of the coefficient a is equal to $V^{\frac{1}{18}}$.

3. For channels lined with roughly hammered stone or common rubble masonry, for channels lined with pebbles up to $1\frac{1}{2}$ inch in diameter, for channels in rockwork, for aqueducts of any description discharging into channels in earth and having no independent slopes, the value of the coefficient a is equal to 1.0.

4. For channels with obstructions producing shocks, such as channels with cleats nailed crosswise to retard the flow, for channels lined with old and damaged masonry the value of the coefficient a is equal to $\frac{1}{V^{\frac{1}{18}}} - \frac{1}{V^{\frac{1}{9}}}$.

Channels in Earth.

I.

When we scrutinize the data relating to flow in rivers and other channels in earth we perceive that these data contain many irregularities and contradictions which make them appear doubtful and untrustworthy. Even those given by the best authorities are not entirely free from anomalies. These irregularities and contradictions are occasionally the result of inaccurate measurements; more often, however, they must be attributed to the unstable character of the beds of these channels. This instability of the bed of the channels makes the phenomenon of flow a problem of great complexity. An exact valuation of all the facts entering is as yet, with the incomplete data at present available, out of the question. We here leave the path of exactitude and enter a labyrinth, satisfied if we come out with the gain of an increment of knowledge which may prove useful.

Natural and artificial channels in rock work or earth may be divided according to the stability of their beds, into three classes:

1. Channels having beds in a regime of stability at velocities exceeding the ordinary. Channels in rockwork, cemented gravel, channels in earth protected by riprap or masonry side walls.

2. Channels in a regime of stability at ordinary velocities. Channels in gravel, stiff clay, clayey loam, sandy soils with over 50 per cent clay.

3. Channels in a regime of instability at ordinary velocities. Channels in sand, sand with fine gravel, sandy loam with less than 50 per cent clay.

The beds of the second and third class are in a regime of stability until the velocity becomes sufficiently great to erode the bed.

The velocity at which erosion begins varies with the cohesion of the material. In channels in sand, sandy gravel, sandy soils with small percentages of clay, erosion begins at very low velocities; these channels are consequently very unstable. Omitting channels in firm rock or cemented gravel, the stability of the bed depends mainly on the percentages of clay in the material. According to W. A. Burr pure clay resists erosion up to a velocity of 7.35 feet per second. The following table, based chiefly on Burr's experiments, gives the mean velocities at which erosion begins:

Nature of Material Forming the Bed.	Mean Velocity.
Fine sand	0 .72
Coarse sand, sand with pebbles up to pea size	1 .10
Sandy soil 15 per cent clay	1 .20
Fine gravel up to ½ inch in diameter	1 .50
Sandy loam, 45 per cent clay	1 .80
Common loam, 65 per cent clay	3 .00
Gravel or pebbles from ½ to 1 inch in diameter	3 .15
Coarse gravel	4 .00
Clayey loam, 85 per cent clay	4 .80
Clay soil, 95 per cent clay, loose rock	6 .20
Stratified rock, slaty rock	7 .45
Hard rock	12 .00

In the process of erosion energy is consumed which varies with the specific gravity and the cohesion of the material.

The erosive power of a current is proportional to the square of its speed. Its transporting power, however, varies (according to Le Conte):

When the surface is constant with v^2.

When the velocity is constant with the surface of the object or with d^2.

When both vary the assistance is equal to $v^2 d^2$. But the weight of the object is proportional to d^3.

Hence, when the forces are in equilibrium or the weight equal to the energy $d^3 = v^2 d^2$.

Dividing by the surface or d^2 we have $d = v^6$.

Consequently when the forces are in equilibrium the resistance is proportional to v^6. In other words, the transporting power of a current is proportional to the sixth power of the speed. This indicates that powers of r ranging between 2 and 6 enter the problem of flow when erosion begins.

With the beginning of erosion the destruction of the bed will be the greater; the less the cohesion of material the greater the velocity. Changes and alterations in course and section generally continue till a channel is formed which, owing to its greater length, its deflections, curves and bars offers such resistances that the power of the current is reduced and course and section again become stable when force and resistance are in equilibrium. A stream will pick up material in a narrow, deep section of its course where the force of the current is great, and deposit it in a wide and shallow section where the current is feeble. At high water, the greater depth of the water in the shallow section will result in greater velocities, the material previously deposited will again be put in motion and carried to a place where the current is feeble.

The work done during these processes of building and rebuilding cannot be accurately measured, and on this account slope formulæ, when applied to flow in channels where erosion is going on, are always more or less deficient. They cannot be depended on in computing discharges; this falls into the province of the current metre and the rod float. They are useful, however, as a guide to the engineer in the design of new conduits, alterations in courses or sections, etc., etc.

The banks of channels having unstable beds are frequently protected by riprap or masonry walls. Frequently the bottoms of such channels are also protected by artificial bars made of boulders or masonry.

Rittinger, Borneman, Epper, Cunningham, and others, have given us data relating to flow in such channels. An analysis of these data gives surprising results. Using the exponential equation

$$x = \frac{\log\ v_1 - \log\ v_0}{\log\ r_1 - \log\ r_0}$$

we find the following values of x, the power of the mean hydraulic radius to which the velocity is proportional:

Rittinger, millrace of dry rubble side walls, bed very rough, depth of water 0.40 to 0.90, $x = 3.0$.

Rittinger, mill race, bed sand and gravel, side walls of masonry, depth 0.28 to 0.90 ft., $x = 1.77$.

Rittinger, Aqueduct in earth lined with dry rubble side walls, depth 0.61 to 1.27 feet, $x = 1.19$.

Cunningham, Solani Embankment, sides of masonry built in steps, bed of clay and boulders, with frequent artificial bars to prevent erosion. Main site, width, 150 to 170 ft.; depth of water, 1.7 to 4.1 ft., $x = 1.49$; depth of water, 5.6 to 9.34 ft., $x = 0.9$; Jaoli site, depth, 6.8 to 8.1 ft., $x = 0.93$.

Excluding extremes, the powers of R, to which the velocity is proportional as expressed in these data, may be given by the equation

$$x = 1.8 - 0.1R$$

so that for

$$R = 1.0\ x = 1.7$$

$$R = 2.0\ x = 1.6$$

$$R = 9.0\ x = 0.9.$$

A high value of x indicates a low value of the coefficient c, but a rapid increase in its value with increasing value of R; a low value of x indicates a high value of c and a slow increase in its value with increasing values of R.

The influence of the roughness of the bed is necessarily much greater when the water in the channel is shallow than when it is high; the diminishing values of x indicate a rapid decrease in the relative influence of the character of the bed. But, on the other hand, while the powers of R are abnormally high for shallow water in rough channels, the powers of the sine of the slope to which the velocity is proportional are abnormally low. This may be illustrated by data deduced from experimental values relating to flow in rough channels in earth. Amongst others we find:

Wampfler, Simme Canal, coarse gravel and detritus,
$R^{1.104}\ S^{0.23}$.

La Nicca, Rhine in the Forest, coarse gravel and detritus, depth 0.42 to 0.9 feet. $R^{0.9}\ S^{0.4}$.

La Nicca, Plessur River, coarse gravel to detritus, depth 1.25 to 4.58 feet. $R^{0.64}\ S^{0.4}$.

Darcy-Bazin, Grosbois Canal, Chazilly Canal, channels in earth, with stones and vegetation, depth 1.5 to 3.0 feet.
$R^{0.87}\ S^{0.43}$ to $R^{1.59}\ S^{0.4}$.

Reich, River Salzach, gravel and detritus, depth 3.53 to 7.39 ft.
$R^{0.8}\ S^{0.333}$.

Funk, Weser River, depth 4.5 to 11 ft. $R^{0.79}\ S^{0.5}$.

Villevert, River Seine, depth 5.66 to 18.39 ft. $R^{0.63}\ S^{0.443}$.

In general therefore, for shallow water in rough channels the power of the sine of the slope to which the velocity is proportional is equal to 0.4 and equal to 0.473 for depths exceeding 4 feet. The variations in the powers of both r and S with the depth of the water in the channel are chiefly due to the fact, that the bottoms of such channels are in most cases much rougher than the sides. In shallow water, the resistance due to the bottom preponderates, with increasing depth the influence of the less rough sides more and more reduces the mean resistance per unit area of surface.

The powers of r vary not only with the degree of roughness in general and with the depth of the water, but also with the value of a, the coefficient of variation of c.

For the same degree of roughness, the powers of r have their highest value for the highest value of a.

For $m = -0.33$ or $K = 2.0$ for instance,

and $a = 1.0 \quad R^x = R^{0.795}$.

But for $a = V^{\frac{1}{18}} \; R^x = R^{0.835}$

for $a = \dfrac{1}{V^{\frac{1}{18}}} \; R^x = R^{0.745}$

for $a = \dfrac{1}{V^{\frac{1}{9}}} \; R^x = R^{0.66}$.

This shows the great influence of bends, bars, or other impediments on the powers of R.

Our general equation expresses the variation of the powers of r with the depth with a fair degree of accuracy. Greater accuracy is obtained if the formula is put into the form $c = 66\left(\sqrt[4]{r} + \left(\frac{m}{2}\,1 + \sqrt[4]{r}\right)\right)$ and giving m a negative value, as for instance:

for $K = 1.20 \; m = -0.10$
for $K = 1.50 \; m = -0.20$
for $K = 2.0 \; m = -0.33$.

For values of R less than 1.0 foot the formula

$$c = \frac{66\,(\sqrt[4]{r} + \sqrt{r})}{\sqrt[4]{r} + K}$$

gives slightly excessive results.

Amongst the mass of experimental data accumulated during recent years those given by Fortier for irrigation channels are, considered from the practical standpoint, the most valuable. They relate to flow in channels possessing all possible degrees of roughness and a minute description of the nature of the bed is always given. Gaugings were, however, taken only for a single depth and a single slope at each section and on this account no deductions can be made in regard to the variation of the coefficient c with the velocity.

Besides these Dubuat, Darcy–Bazin, Legler, Cunningham, Rittinger and others have given valuable data relating to flow in canals to ditches; Funk, Villevert, Revy, Gordon and the U. S. Engineers, interesting data relating to flow in rivers. After a careful analysis of all the material available we come to the following conclusions in regard to the variation of the coefficient c with the velocity:

1. For channels of fairly regular cross-sections and courses having tolerably smooth beds, such as channels in firm clay, clayey loam, sandy soil with over 50 per cent clay, fine cemented gravel, the coefficient c increases at ordinary velocities with the velocity of flow. Under ordinary velocities in this sense we understand velocities which do not cause erosion.

The increase in the value of c with increasing velocities is equal to

$$a = V^{\frac{1}{18}}$$

for the smoothest down to

$$a = V^{\frac{1}{36}}$$

for the roughest channels of this class.

Examples:

S. Fortier, Bear River Canal Branch.
S. Fortier, Providence Canal.
S. Fortier, Solveron and Logan City Canals, Utah.
Darcy–Bazin, rectangular channel lined with pebbles up to $\frac{7}{8}$ inch diameter.
Epper, millrace, channel in earth, bottom covered with fine gravel.
Dubuat, Canal du Jard. Channel in earth.
Reich, River Salzach, reach very regular.

2. At velocities exceeding the ordinary, or when erosion begins, the coefficient c decreases in value for the classes of channels enumerated above. The decrease is usually such that

$$a = \frac{1}{V^{\frac{1}{18}}}.$$

Examples:

Legler, Linth Canal. The coefficient c increases until v is equal to 4.72 ft. per second, then decreases.

Gordon, Irrawaddi River. The coefficient c increases until v is equal to 2.62 ft. per second, then decreases.

In the first case the bed is firm earth, in the second sand.

3. For channels of fairly regular cross-section and course in rockwork, firm gravel up to 2 inches diameter, for channels in firm earth or sand, or sand with gravel, with stones or vegetation, the coefficient c does not appreciably vary with the velocity of flow. Consequently

$$a = 1.0.$$

Examples:

Perrone, Torlonia Drain tunnel, channel in rock work.

Darcy–Bazin, series 5, rectangular channel lined with pebbles up to $1\frac{1}{2}$-inches diameter.

Darcy–Bazin, series 36, 37, 38, 41, 43, 47, 48, 50, Grosbois and Chazilly Canals. Channels in earth of regular cross-section but with stones or weeds.

La Nicca, Moesa River, coarse gravel.

La Nicca, Plessur River, coarse gravel.

Funk, Weser River.

Passini and Gioppi, Canal Cavour, below the Syphon of the Sesia.

4. For the class of channels enumerated under (3) the coefficient c decreases in value whenever the velocity becomes sufficient to cause erosion. The decrease usually corresponds to

$$a = \frac{1}{V^{\frac{1}{18}}}.$$

5. For channels with very rough beds, channels with boulders, loose cobblestones, loose coarse gravel or detritus, for channels with artificial bars to prevent scour, the coefficient c decreases rapidly in value with increasing velocities. The decrease is equal to

$$a = \frac{1}{V^{\frac{1}{9}}}.$$

Example:

Cunningham, Solani Embankment, bed in clay and boulders with artificial bars to prevent erosion, sides of masonry.

Omitting the extremes, we may briefly sum up the variation of the coefficient c with the velocity as follows:

1. For channels of very regular cross-sections and courses in clay, clayey loam, sandy soils with large percentages of clay, cemented gravel up to one inch in diameter, the coefficient of variation of c is equal up to the eroding limit to

$$a = V^{\frac{1}{18}}.$$

2. For channels in rockwork or cemented gravel exceeding one inch in diameter, for ordinary channels in earth, channels with some stones or vegetation, the coefficient a is equal up to the eroding limit to

$$a = 1.0.$$

3. For channels in sand at any velocity and for all others at velocities exceeding the eroding limit, the coefficient c decreases in value with increasing velocities and the coefficient of variation is fairly equal to

$$a = \frac{1}{V^{\frac{1}{18}}}.$$

K.

In a preceding chapter we have mentioned the experiments made by Hubbel and Fenkell, Saph and Schoder to determine the loss of head due to the resistance in curves. From data given by them we computed, that, omitting values of $\frac{R}{d}$ less than 2.5, the friction per unit length of curve, in terms of the friction per unit length of tangent is equal to

$$4.9\, d^{\frac{3}{13}} \left(\frac{d}{R}\right)^{\frac{3}{13}};$$

and the excess of friction per unit length of curve in terms of tangent friction is equal to

$$\left(4.9\, d^{\frac{3}{13}} \left(\frac{d}{R}\right)^{\frac{3}{13}}\right) - 1.0,$$

and the length of tangent equal in the amount of frictional resistance to the frictional resistance in a curve of 90° equal to

$$0.5\, \pi\, R \left(4.9\, d^{\frac{3}{13}} \left(\frac{d}{R}\right)^{\frac{3}{13}}\right) - 1.0.$$

This vanishes when $\frac{R}{d} = 4.9^{\frac{13}{3}}\, d$, it is a maximum when $\frac{R}{d} = 4.9^{\frac{11}{3}}\, d$ and the total excess of friction is greatest. The loss of head due to any curve is consequently

$$h = \frac{n^\circ}{360}\, 2\, \pi\, R \left(4.9\, d^x \left(\frac{d}{R}\right)^y\right) - 1.0\, \frac{0.01478}{(\sqrt[4]{r} + m)^2 r}\, \frac{v^x}{2g}\cdot$$

Table IV.

Friction in Curves.

$\frac{R}{d}$	Values of $\left(4.9\, d^x \left(\frac{d}{R}\right)^y\right) - 1.0$. Diameters 1 to 72 Inches.											
	1″	2″	4″	6″	12″	18″	24″	30″	36″	48″	60″	72″
2.5	0.375	0.777	1.422	1.907	2.971	3.360	3.657	3.903	4.113	4.462	4.750	4.996
4	0.271	0.595	1.174	1.609	2.564	2.913	3.080	3.400	3.588	3.903	4.160	4.382
5	0.225	0.515	1.065	1.478	2.386	2.717	2.971	3.180	3.359	3.657	3.903	4.113
6	0.188	0.453	0.980	1.377	2.247	2.565	2.808	3.009	3.180	3.466	3.701	3.903
10	0.091	0.292	0.761	1.113	1.887	2.170	2.388	2.564	2.717	2.971	3.186	3.369
15	0.020	0.177	0.604	0.925	1.630	1.887	2.084	2.247	2.386	2.617	2.808	2.971
20	.....	0.101	0.501	0.802	1.461	1.703	1.887	2.039	2.168	2.386	2.564	2.717
25	.....	0.046	0.426	0.712	1.337	1.567	1.742	1.887	2.010	2.216	2.386	2.531
50	.....	.....	0.216	0.460	0.994	1.189	1.338	1.462	1.567	1.743	1.887	2.010
100	.....	.....	0.037	0.244	0.700	0.867	0.994	1.099	1.189	1.348	1.461	1.565
	Values of z. Curve of 90 degrees. $m = 0.95$.											
2.5	0.049	0.091	0.151	0.184	0.249	0.258	0.263	0.266	0.268	0.270	0.271	0.272
4	0.057	0.112	0.210	0.248	0.344	0.358	0.366	0.371	0.375	0.378	0.380	0.382
5	0.059	0.121	0.227	0.285	0.401	0.416	0.427	0.432	0.438	0.443	0.446	0.448
6	0.059	0.128	0.251	0.319	0.451	0.472	0.484	0.492	0.478	0.504	0.507	0.510
10	0.048	0.137	0.325	0.429	0.634	0.666	0.686	0.699	0.710	0.720	0.727	0.734
15	0.015	0.147	0.386	0.535	0.821	0.869	0.899	0.918	0.935	0.951	0.963	0.971
20	.....	0.095	0.427	0.619	0.982	1.046	1.085	1.111	1.133	1.157	1.173	1.184
25	.....	0.054	0.454	0.687	1.123	1.203	1.252	1.296	1.313	1.343	1.364	1.379
50	.....	.....	0.460	0.888	1.670	1.825	1.924	2.001	2.047	2.113	2.157	2.190
100	.....	.....	0.158	0.931	2.352	2.836	2.858	2.995	3.107	3.268	3.340	3.410

Table IV. A.

Weisbach's Coefficients for Resistances Due to Entrances, Elbows, Curves, Changes of Section, etc., etc.

Values of z.	Description of Resistance.
0.054	Funnel-shaped or bell-mouthed entrance not protruding into the reservoir.
0.505	Well rounded entrance not protruding into the reservoir.
0.505	Funnel-shaped or bell-mouthed entrance protruding into the reservoir.
1.957	Ordinary pipe protruding into the reservoir.
$0.9457 \text{ sine}^2 \frac{d}{2} +$ $2.047 \text{ sine}^4 \frac{d}{2}$	Elbows d = angle of deflection.
$0.131 + 1.849\left(\frac{d}{2R}\right)^{\frac{7}{2}}$	Curves. Section circular. d = diameter, R = radius of curve.
$0.124 + 3.104\left(\frac{d}{2R}\right)^{\frac{7}{2}}$	Curves. Section rectangular. d = Width of side parallel to R, the radius of the bend.
$\left(\frac{1}{am} - 1\right)^2$	Constrictions. $m = \frac{\text{Section contracted}}{\text{Section not contracted}}$. $a = 1.225 + 1.45\, m^2 - 1.675\, m$.
$\left(\frac{A_1}{A_2} - 1\right)^2$	Enlargements or Contractions. A_1 = Section not contracted, A_2 = Section contracted.
$0.12 \frac{n^\circ}{30}$	Bends of Rivers. n = Number of degrees in arc of bend.
$1.0874\left(\frac{1}{m^2} - 1\right)$	Obstructions in Rivers. m = Percentage not obstructed.

v^x being equal to $V^{\frac{1.6}{9}}$, $V^{\frac{1.7}{9}}$, $V^{\frac{1.8}{9}}$, etc., according to the degree of roughness of the conduit. The coefficient of frictional resistance is given by

$$z = \frac{n^\circ}{360} 2\,\pi R \left(4.9\, d \left(\frac{d}{R}\right)^y\right) - 1.0 \frac{0.01478}{(\sqrt[4]{r} + m)^2 r}$$

in these equations

n = number of degrees in curve.
π = 3.1416.
d = diameter of conduit in feet.
R = radius of curve in feet.
$x = \frac{3}{13}$ for diameters greater than 1 foot.
x = 0.45 for diameters less than 1 foot.
$y = \frac{1}{6}$ for a diameter of 1 inch.
$y = \frac{3}{13}$ for any other diameter.

From the foregoing we draw the conclusion, that the value of z depends:

1. On the value of $\frac{R}{d}$ and the value of d.
2. On the value of $\frac{n^\circ}{360}$.
3. On the value of m.

For any arc, multiply the values of z, found in the table, by the number of degrees and divide by 90.

For any degree of roughness multiply the values of z by the following:

m = 0.95, multiply by 1.0.
m = 0.83, multiply by 1.166.
m = 0.68, multiply by 1.436.
m = 0.53, multiply by 1.802.
m = 0.45, multiply by 2.060.
m = 0.30, multiply by 2.717.

If in the formula for the loss of head due to a curve we substitute

$$\frac{2\,grs}{V^2} \text{ for its equivalent } \frac{0.01478}{(\sqrt[4]{r} + m)^2}$$

and L for the length of the curve the formula will read, after reduction,

$$H = \left(4.9\, d^x \left(\frac{d}{R}\right)^y\right) - 1.0 \cdot L \cdot s,$$

which simply expresses the theory outlined at the beginning of this chapter that the excess loss of head due to a curve is

$$\left(4.9\, d^x \left(\frac{d}{R}\right)^y\right) - 1.0$$

times the loss due to an equal length of straight pipe; S being the sine of the slope to which velocities in the tangent are due.

Riveted Conduits.

L.

Riveted conduits form a class apart in so far as the degree of roughness varies with the diameter. Up to date the coefficients for such conduits have been fairly well determined for diameters up to 8.5 feet (Holyoke Testing Flumes); for larger sections they are as yet problematical.

Fairly reliable values of the coefficients for riveted conduits may be found by computing the losses of head due to the resistance of rivet heads, or to enlargements and contractions of the section as follows:

If in an 18-foot steel-riveted pipe we allow an internal pressure of 140 pounds per square inch, in the steel a tension of 20,000 pounds per square inch; and if we assume the efficiency of the riveted joints to be 70 per cent of the metal, we have for the thickness of the metal in inches

$$t = \frac{140 \times \text{diameter in inches},}{0.7 \times 40{,}000}$$

which gives $t = 1.08$ inches.

It is usual to take for the diameter of the rivet in inches

$$d = 0.15 + 1.5\, t,$$

and for the pitch of the rivets in a single row

$$s_1 = 0.375 + 2\, d,$$

and $s_2 = 0.75 + 3\, d$

for the pitch in a double row.

Hence in our case

$$d = 1.75,$$
$$s_1 = 3.875,$$
$$s_2 = 6.0.$$

The usual diameter of the rivet head is 1.8 d and its depth 0.6 d. This gives for the sectional area of the rivet at right angles to the line of flow

$$3.15 \times 1.05 = 3.3075 \text{ square inches nearly.}$$

As the circumference of the conduit is $12 \times 18 \times 3.14 = 678.25$ inches and the spacing 3.875 inches, there will be 175 rivets in the single circumferential row. The open space between the rivets will only be $3.875 - 3.25 = 0.725$ inches. The disturbance in the motion in this narrow space will be such, that it will be safe to consider the row of rivet heads as an unbroken line of a depth $0.6\,d = 1.05$ inches. Weisbach gives for the loss of head due to constrictions

$$h = \left(\frac{A_1}{aA_2} - 1\right)^2 \frac{v^2}{2g},$$

in which

$$A_1 = \text{section not constricted,}$$
$$A_2 = \text{section constricted,}$$
$$a = 1.225 + \left(\frac{A_2}{A_1}\right)^2 - 1.695\,\frac{A_2}{A_1}.$$

In our case $A_1 = 18^2 \times 0.7854 = 254.34,$

$$A_2 = (17.825)^2 \times 0.7854 = 249.5.$$

Inserting these values in Weisbach's formula we find

$$h = 00187489\,\frac{v^2}{2g}.$$

Assuming the metal sheets to be 10 feet each way there will be six sheets in the circumference, and as the pipe is double riveted longitudinally there will be twelve longitudinal rows of rivets, and allowing 1.6 d for the outside rim on each side there will be twenty circumferential rows, the pitch being six inches. The twelve rivets in each row will cause a constriction of $12 \times 3.3075 = 39.69$ square inches $= 0.275\ f^2$. According to Weisbach's formula this constriction causes a loss of head equal to

$$h = 00005936\ \frac{v^2}{2g},$$

and the twenty rows a loss equal to

$$h = 00011907\ \frac{v^2}{2g}.$$

Adding the resistances due to all the circumferential rows in a section of 9.5 feet we have

$$Z_1 = 00187489 + 00011907 = 00199396.$$

Assuming the conduit to be 20,000 feet long the total resistance due to the rivet heads will be

$$Z_1 = \frac{20000}{9.5} = 2105 \times 00199396 = 4.196985.$$

To this must be added the resistance due to the enlargement or contraction caused by the circumferential lap of the sheets. As the thickness of the metal is 1.08 inches the diameter is enlarged or contracted 2.16 inches at each lap. The loss of head due to enlargements or contractions is, according to Weisbach,

$$h = \left(\frac{A_1}{A_2} - 1\right)^2 \frac{v^2}{2g};$$

hence in our case $\left[\left(\frac{216''}{213.84''}\right)^2 - 1\right]^2 \frac{v^2}{2g} = 00041209\ \frac{v^2}{2g}.$

The total resistance due to all the enlargements or contractions is consequently

$$Z_2 = 2105 \times 00041209 = 0.86755.$$

If the conduit had no rivet heads or enlargements and contractions to increase the resistance, the value of the coefficient m

would be the same as for a cast-iron pipe, or equal to 0.83, and the frictional resistance per unit area of surface would be

$$f = \frac{0.01478}{(1.456 + 0.83)^2} = 002829$$

and the total resistance of the wet perimeter

$$Z_3 = 002829 \frac{20000}{4.5} = 12.473.$$

Adding, we have for the sum of all the resistances

$$Z_1 + z_2 + z_3 = 17.5375.$$

This gives for the total frictional resistance per unit area of surface

$$17.5325 \times \frac{4.5}{20000}$$

or $$f = 00394594;$$

hence the coefficient c is equal to $\sqrt{\frac{64.4}{00394597}} = 127.7$, and m is equal to $\frac{127.7}{66} - 1.456 = 0.48.$

Practical Applications of the Formulæ

M.

1. From the formula

$$v = (66\,(\sqrt[4]{r} + m)\,\sqrt{rs})^{\frac{18}{17}}$$

we have

$$V^{\frac{17}{18}} = \frac{V}{V^{\frac{1}{18}}} = 66\,(\sqrt[4]{r} + m)\,\sqrt{rs}$$

and $$s = \left[\frac{V}{V^{\frac{1}{18}}\,66\,(\sqrt[4]{r} + m)\,\sqrt{r}}\right]^2.$$

We have also $\frac{V}{V^{\frac{1}{18}}\,66\,\sqrt{s}} = (\sqrt{r} + m)\sqrt{r} = R^{\frac{3}{4}} + m\,\sqrt{r}.$

Putting $\sqrt[4]{r} = x$ and transposing we have

$$X^3 + mX^2 + 0 - \frac{V}{V^{\frac{1}{18}}\, 66 \sqrt{s}} = 0,$$

from which the value of $x = \sqrt[4]{r}$ is found by Horner's method.

We have also

$$m = \frac{V}{V^{\frac{1}{18}}\, 66 \sqrt{rs}} - \sqrt[4]{r}.$$

If the coefficient of variation of c is equal to $V^{\frac{1}{9}}$, $V^{\frac{1}{12}} \frac{1}{V^{\frac{1}{18}}}$ etc., these values are substituted in the given equations.

Values of $a = V^{\frac{1}{8}}$ $V^{\frac{1}{9}}$, $V^{\frac{1}{11}}$, $V^{\frac{1}{12}}$, $V^{\frac{1}{16}}$, $V^{\frac{1}{17}}$, $V^{\frac{1}{18}}$, $\frac{1}{V^{\frac{1}{18}}}$ $\frac{1}{V^{\frac{1}{19}}}$, are found in Table V.

Example: Let it be required to find the slope for a rectangular aqueduct of common brickwork or concrete 100 feet wide, 12.5 feet deep, the velocity to be 4 feet per second. The cross-section is 1,250 f^2, the wet perimeter 125 f, hence R = 10.0. In the table of roots of mean radii we find $\sqrt{10} = 3.163$ $\sqrt{10} = 1.78$. The value of m for common brickwork or concrete is 0.57. The value of $a = V^{\frac{1}{18}}$ for $v = 4.0$ is, according to Table V, equal to 1.08. Inserting these values into our formula we have for the slope

$$s = \left[\frac{4}{1.08 \times 66 \times (1.78 + 0.57) \times 3.163}\right]^2$$

$$= \left(\frac{4}{530}\right)^2$$

$$= 0.0000569.$$

Example: Let it be required to find the diameter of a semicircular channel lined with common ashlar or very good rubble masonry, the slope being 1 in 1,000, and the permissible velocity 10 feet per second.

In this case $m = 0.30$

$$\sqrt{s} = 0.0316$$

$$a = \sqrt[18]{10} = 1.137.$$

Solving by Horner's method and inserting values we have

$$X^3 + 0.3\,X^2 + 0.0 - \frac{10}{1.137 \times 66 \times 0.0316} = 0.0$$

$$\begin{array}{llll}
X^3 + & 0.3\,X^2 + 0.0 - 4.217 = 0.0 & \underline{|x = 1.521} & \\
 & 1.0 & + 1.3 & + 1.300 \\
\hline
 & 1.3 & + 1.3 & - 2.917 \\
 & 1.0 & + 2.3 & \\
\hline
 & 2.3 & + 3.6 & \\
 & 1.0 & & \\
\hline
 & 3.3 & + 3.6 & - 2.917 \\
 & 0.5 & + 1.9 & + 2.750 \\
\hline
 & 3.8 & + 5.5 & - 0.167 \\
 & 0.5 & + 2.15 & \\
\hline
 & 4.3 & + 7.65 & \\
 & 0.5 & & \\
\hline
 & 4.8 & + 7.65 & - 0.167 \\
 & 0.02 & + 0.096 & + 0.1544 \\
\hline
 & 4.82 & + 7.746 & - 0.0121 \\
 & 0.001 & + 0.005 & + 0.0077 \\
\hline
 & 4.821 & + 7.751 & - 0.0044.
\end{array}$$

This gives $x = 1.521$; hence the mean hydraulic radius $r = (1.521)^4 = 5.352$, and the diameter $= 21.408$ ft.

2. From the formula

$$v = \left[\frac{2\,gh}{\dfrac{0.01478}{(\sqrt[4]{r} + m)^2}\,\dfrac{L}{R}}\right]^{\frac{9}{16}}$$

we have

$$V^{\frac{16}{9}} = \frac{2\,gh}{\dfrac{0.01478}{(\sqrt[4]{r} + m)^2}\,\dfrac{L}{R}}$$

$$H = \frac{0.01478}{(\sqrt[4]{r} + m)^2}\,\frac{L}{R}\,\frac{V^{\frac{16}{9}}}{2\,g}$$

$$(\sqrt[4]{r} + m)^2\,R = 0.01478\,\frac{L}{H}\,\frac{V^{\frac{16}{9}}}{2\,g}$$

$$(\sqrt[4]{r} + m)\sqrt{r} = R^{\frac{3}{4}} + m\sqrt{r} = \sqrt{0.01478\,\frac{L}{H}\,\frac{V^{\frac{16}{9}}}{2\,g}}$$

and putting $x = \sqrt[4]{r}$ we have

$$X^3 + m X^2 + 0.0 - \sqrt{0.01478 \ \frac{L}{H} \ \frac{V^{\frac{16}{9}}}{2 g}} = 0.0,$$

which may be solved by Horner's method.

To facilitate calculations it is well to remember

that $V^{\frac{9}{16}} = V^{\frac{1}{2}} \times V^{\frac{1}{16}}$.

and $V^{\frac{16}{9}} = \left(\frac{v}{V^{\frac{1}{9}}}\right)^2$

Values of $V^{\frac{1}{9}}$ and $V^{\frac{1}{16}}$ are found in Table V.

Resistances due to entrance and the velocity itself are included in the term $\frac{0.01478}{(\sqrt[4]{r} + m)^2} \frac{L}{R}$ and need not be further considered unless the length of the conduit is less than 1,000 diameters. For pipes between 300 and 1,000 diameters in length (as also for riveted pipes exceeding 3 feet in diameter), the coefficient of variation is equal to $a = V^{\frac{1}{18}}$, and $\frac{9}{17}$ and $\frac{17}{9}$ are substituted in the given equations for $\frac{9}{16}$ and $\frac{16}{9}$. If the pipe is between 100 and 300 diameters in length (or an old pipe not very clean) the coefficient a is equal to 1.0, and $\frac{1}{2}$ and 2.0 are substituted in the given equations for $\frac{9}{16}$ and $\frac{16}{9}$. In case the conduit is less than 100 diameters in length the coefficient a is equal to $\frac{1}{V^{\frac{1}{18}}}$ and $\frac{9}{19}$ and $\frac{19}{9}$ are substituted for $\frac{9}{16}$ and $\frac{16}{9}$. Values of $\frac{0.01478}{(\sqrt[4]{r} + m)^2}$ are found in Table VI.

Example: Let it be required to find the velocity of flow in a new steel riveted conduit 6 feet in diameter, 10,000 feet long, the head to be 5 feet and the conduit to have 20 curves of 10° each and a radius of 30 feet. In this case $m = 0.53$, $R = 1.5$, $\sqrt[4]{1.5} = 1.107$. For the curves we have the relation $\frac{R}{d} = \frac{30}{6} = 5.0$. In Table IV we find the coefficient z_2 for $\frac{R}{d} = 5.0$ and a curve of 90° to be equal to 0.466. As there are 20 curves of 10 degrees we have $Z_2 = \frac{20 \times 10 \times 0.448}{90} = 0.995$. For $m = 0.53$

this is to be multiplied by 1.802, which gives for the total resistance due to curves $Z_2 = 1.782$.

Inserting values into our formula we have

$$v = \left[\frac{64.4 \times 5.0}{\dfrac{0.01478 \times 10000}{(1.107 + 0.53)^2 \times 1.5} + 1.782}\right]^{\frac{9}{17}}$$

$$= (8.312)^{\frac{9}{17}}$$

Remembering that $V^{\frac{9}{17}} = V^{\frac{1}{2}} \times V^{\frac{1}{34}}$ we first draw the square root out of the quotient and multiply this by the seventeenth root of the square root.

The quotient is 8.312, $\sqrt{8.312} = 2.884$. In the table of roots we find $\sqrt[17]{3} = 1.065$, $\sqrt[17]{2.75} = 1.059$. Interpolating we have for $\sqrt[17]{2.884} = 1.062$. Consequently $v = 2.884 \times 1.062 = 3.0628$ feet per second.

3. From the formula

$$v = (66\ (\sqrt[6]{r} + m)\ \sqrt{rs})^{\frac{9}{8}}$$

we have for the discharge of a circular conduit in cubic feet per second

$$Q = (66\ (\sqrt[4]{r} + m)\sqrt{r.s})^{\frac{9}{8}}\ d^2\ 0.7854.$$

From this we have for the head in feet

$$H = \left[\left(\frac{Q}{d^2\ 0.7854}\right)^{\frac{8}{9}} \frac{1}{66\ (\sqrt[4]{r} + m)}\right]^2 \frac{L}{R}$$

and for the diameter in feet

$$d = \left[\left(\frac{Q}{0.7854}\right)^{\frac{16}{9}} \frac{1}{[66\,(\sqrt[4]{r} + m)]^2} \frac{4\ L}{H}\right]^{\frac{9}{41}}$$

If $a = V^{\frac{1}{18}}$ the index $\frac{18}{17}$ is substituted for $\frac{9}{8}$, $\frac{17}{18}$ for $\frac{8}{9}$, $\frac{17}{9}$ for $\frac{16}{9}$ and $\frac{9}{43}$ for $\frac{9}{41}$. From this equation the value of d can only be found by trial, assuming a value of $\sqrt[4]{r}$ in the term $\sqrt[4]{r} + m$. For a first trial a value of $\sqrt[4]{r} = 1.0$ will give good results.

From the formula

$$v = \left[\frac{2\,gH}{\dfrac{0.01478}{(\sqrt[4]{r} + m)^2}\dfrac{L}{R} + Z_2 + Z_n}\right]^{\frac{9}{16}}$$

we have $$Q = \left[\frac{2\,gH}{\frac{0.01478}{(\sqrt[4]{r}+m)^2}\frac{L}{R} + Z_2 + Z_n}\right]^{\frac{9}{16}} d^2\, 0.7854$$

$$H = \left(\frac{Q}{d^2\, 0.7854}\right)^{\frac{16}{9}} \frac{\frac{0.01478}{(\sqrt[4]{r}+m)^2}\frac{L}{R} + Z_2 + Z_n}{2\,g}$$

$$d = \left[\left(\frac{Q}{0.7854}\right)^{\frac{16}{9}} \frac{1}{2\,gH}\, \frac{0.01478}{(\sqrt[4]{r}+m)^2}\, L + Z_1 R + Z_2 R\right]^{\frac{9}{41}}.$$

From this equation also d can only be found by a second or third trial, assuming a value of $\sqrt[4]{r}$ and R.

For $a = V^{\frac{1}{18}}$ the index $\frac{9}{17}$ is substituted for $\frac{9}{16}$, $\frac{17}{9}$ for $\frac{16}{9}$ and $\frac{9}{43}$ for $\frac{9}{41}$.

For $a = 1.0$ the index $\frac{1}{2}$ is substituted for $\frac{9}{16}$, 2 for $\frac{16}{9}$ and $\frac{9}{45} = \frac{1}{5}$ for $\frac{9}{41}$.

Example: What will be the loss of head corresponding to a discharge of 5 cubic feet per second, the conduit being a 2-foot riveted pipe 20,000 feet long and having 30 curves of 15° each and a radius of 20 feet.

In this case $m = 0.68$; $R = 0.5$; $\sqrt[4]{r} = 0.84$, consequently

$$\frac{0.01478}{(\sqrt[4]{r}+m)^2} = \frac{0.01478}{(0.84+0.68)^2} = 0.00638.$$

In Table IV we find for the resistance of a 90° curve for the relation $\frac{R}{d} = \frac{20}{2} = 10$ $Z_2 = 0.686$, consequently for 30 curves of 15° each $Z_2 = \frac{30 \times 15 \times 0.686}{90} = 3.430$. For $m = 0.68$ this is to be multiplied by 1.436 which gives $z_2 = 4.925$.

Inserting these values we have

$$H = \left[\frac{5}{4 \times 0.7854}\right]^{\frac{16}{9}} \frac{\left[0.00638 \times \frac{20000}{0.5}\right] + 4.925}{64.4},$$

or $H = 9.24$ feet.

5. The Kinetic energy or living force acquired by a body falling free or descending in a plane infinitely smooth is equal to

$$E = \tfrac{1}{2}\, m\, v^2 = Q.W.H.,$$

$$m = \text{the mass of a body} = \frac{\text{Weight}}{\text{Gravity}},$$

Q = the discharge in cubic feet per second,

W = the weight of one cubic foot,

H = the total fall in feet.

Expressed in horsepowers the energy is equal to

$$H.\ P. = \frac{Q.W.H.}{550},$$

or, in kilowatts, to

$$K.W. = \frac{Q.W.H.}{737}.$$

If a body of water is not falling free, the total head is reduced by an amount which depends on the velocity, the length of the conduit, its diameter and its degree of roughness.

The loss of head is equal to

$$h = \frac{V^{1\frac{6}{9}}}{2\,g},\quad \frac{V^{1\frac{7}{9}}}{2\,g},\quad \frac{V^2}{2\,g}\,\frac{0.01478}{(\sqrt[4]{r} + m)^2}\,\frac{L}{R},$$

as the case may be. For conduits of equal length the loss is evidently the least for the greatest diameter and for the lesser speed of flow.

For a given diameter the efficiency of a conduit as a transmitter of energy is greatest when the speed of flow is such, that one-third of the total available head is consumed in overcoming frictional resistances (see "Adams and Gummel," Eng. News, May 4, 1893).

Example: A new four foot steel riveted conduit 2,000 feet long, under a head of 300 feet is to deliver water to the generator at such a velocity that its efficiency will be a maximum. What will be the discharge and the horsepower transmitted?

Allowing one-third of the total head to be spent in overcoming frictional resistances we have $v = \frac{100}{2000} = 0.05$. For this value of v the velocity will be

$$v = (66\ (1 + 0.53)^{\bullet} \sqrt{1 \times 0.05})^{\frac{18}{17}} = 2.369 \text{ feet per second;}$$

$$\text{the discharge, } Q = 2.369 \times 16^2 \times \frac{11}{14} = 30.4 \text{ cubic feet per second;}$$

$$\text{the horsepower, } H\,P = \frac{30.4 \times 62.4 \times 200}{550} = 708.0.$$

TABLE V.

Table V contains roots of velocities or values of (a), the coefficient of variation of c.

To find the value of c corresponding to any velocity multiply the value of $66\ (\sqrt[4]{r} + m)$ by the value of $(a) = V^{\frac{1}{9}}$, $V^{\frac{1}{12}}$, $V^{\frac{1}{18}}$, $\frac{1}{V^{\frac{1}{18}}}$, as the case may be.

To find the velocity multiply the value of $66\ (\sqrt[4]{r} + m)\sqrt{r.s}$ by the value of

$(66\ (\sqrt[4]{r} + m)\sqrt{r.s})^{\frac{1}{8}}$ which in Table V is given as $V^{\frac{1}{8}}$,
$(66\ (\sqrt[4]{r} + m)\sqrt{r.s})^{\frac{1}{11}}$ which in Table V is given as $V^{\frac{1}{11}}$,
$(66\ (\sqrt[4]{r} + m)\sqrt{r.s})^{\frac{1}{17}}$ which in Table V is given as $V^{\frac{1}{17}}$,
$\frac{1}{(66\ (\sqrt[4]{r} + m)\sqrt{r.s})^{\frac{1}{19}}}$ which in Table V is given as $\frac{1}{V^{\frac{1}{19}}}$,
as the case may be.

Also, to find the velocity, multiply the value of

$$\sqrt{\frac{2\,gH}{f\,\frac{L}{R}}} \text{ by } \left[\frac{2\,gH}{f\,\frac{L}{R}}\right]^{\frac{1}{16}} \text{ which in Table V is given as } V^{\frac{1}{16}}.$$

Table V.

Roots of Velocities or Values of (*a*), the Coefficient of Variation of *c*.

V	$V^{\frac{1}{8}}$	$V^{\frac{1}{9}}$	$V^{\frac{1}{11}}$	$V^{\frac{1}{12}}$	$V^{\frac{1}{16}}$	$V^{\frac{1}{17}}$	$V^{\frac{1}{18}}$	$\frac{1}{V^{\frac{1}{17}}}$	$\frac{1}{V^{\frac{1}{19}}}$
0.25	0.841	0.857	0.882	0.891	0.917	0.925	0.925	1.081	1.075
0.50	0.918	0.925	0.939	0.944	0.958	0.959	0.961	1.040	1.037
0.75	0.964	0.964	0.974	0.976	0.982	0.981	0.982	1.018	1.015
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.25	1.026	1.025	1.0205	1.019	1.013	1.013	1.013	0.987	0.989
1.50	1.052	1.046	1.038	1.034	1.025	1.024	1.022	0.978	0.979
1.75	1.072	1.064	1.052	1.048	1.035	1.033	1.031	0.97	0.971
2.0	1.090	1.080	1.065	1.0594	1.044	1.042	1.039	0.962	0.964
2.25	1.107	1.095	1.077	1.070	1.052	1.049	1.046	0.956	0.959
2.50	1.121	1.107	1.087	1.080	1.059	1.056	1.052	0.95	0.953
2.75	1.135	1.118	1.096	1.086	1.065	1.061	1.058	0.945	0.948
3.0	1.147	1.130	1.105	1.096	1.071	1.067	1.063	0.940	0.943
3.25	1.159	1.140	1.113	1.103	1.076	1.072	1.068	0.936	0.940
3.50	1.170	1.150	1.121	1.110	1.081	1.077	1.072	0.932	0.936
3.75	1.18	1.158	1.128	1.116	1.086	1.081	1.076	0.929	0.933
4.0	1.189	1.166	1.134	1.123	1.090	1.085	1.080	0.926	0.930
4.25	1.198	1.174	1.141	1.128	1.094	1.088	1.083	0.923	0.927
4.50	1.207	1.182	1.146	1.133	1.098	1.093	1.087	0.919	0.924
4.75	1.215	1.189	1.152	1.139	1.102	1.096	1.090	0.917	0.921
5.0	1.223	1.196	1.158	1.144	1.106	1.10	1.093	0.914	0.919
5.25	1.231	1.203	1.163	1.149	1.109	1.103	1.098	0.912	0.917
5.50	1.237	1.209	1.168	1.153	1.112	1.106	1.099	0.910	0.914
5.75	1.244	1.215	1.173	1.159	1.115	1.108	1.102	0.907	0.912
6.0	1.251	1.220	1.177	1.161	1.118	1.110	1.104	0.905	0.910
6.25	1.258	1.226	1.181	1.165	1.121	1.114	1.107	0.903	0.908
6.50	1.262	1.231	1.186	1.169	1.123	1.116	1.109	0.901	0.906
6.75	1.269	1.237	1.190	1.173	1.126	1.119	1.112	0.899	0.904
7.0	1.275	1.241	1.194	1.176	1.129	1.121	1.114	0.897	0.902
7.25	1.281	1.246	1.197	1.18	1.131	1.124	1.116	0.895	0.901
7.50	1.286	1.251	1.201	1.183	1.134	1.126	1.118	0.894	0.899
7.75	1.292	1.256	1.205	1.186	1.137	1.128	1.12	0.893	0.898
8.0	1.297	1.260	1.208	1.189	1.139	1.130	1.122	0.891	0.896
8.25	1.302	1.264	1.212	1.192	1.141	1.132	1.124	0.889	0.895
8.50	1.307	1.268	1.215	1.195	1.143	1.134	1.126	0.888	0.893
8.75	1.311	1.272	1.218	1.198	1.145	1.136	1.128	0.886	0.892
9.0	1.316	1.277	1.221	1.201	1.147	1.138	1.130	0.885	0.891
9.25	1.320	1.280	1.224	1.204	1.149	1.140	1.131	0.884	0.889
9.50	1.325	1.284	1.227	1.207	1.151	1.142	1.133	0.882	0.888
9.75	1.329	1.288	1.230	1.209	1.153	1.143	1.135	0.881	0.887
10.0	1.333	1.292	1.233	1.212	1.155	1.145	1.137	0.880	0.886
10.5	1.341	1.298	1.238	1.216	1.158	1.149	1.139	0.877	0.884
11.0	1.348	1.305	1.244	1.221	1.161	1.152	1.142	0.875	0.881
11.5	1.357	1.310	1.249	1.226	1.165	1.155	1.145	0.873	0.879
12.0	1.364	1.318	1.254	1.230	1.169	1.158	1.148	0.871	0.877
14.0	1.39	1.340	1.271	1.246	1.179	1.168	1.158	0.863	0.870
16.0	1.414	1.365	1.286	1.260	1.189	1.178	1.169	0.855	0.864
18.0	1.435	1.38	1.301	1.272	1.199	1.185	1.175	0.851	0.859
20.0	1.450	1.395	1.311	1.288	1.204	1.193	1.182	0.846	0.854

Table VI.

Values of 66 ($\sqrt[4]{r} + m$) and Corresponding Values of f, the Coefficient of Friction. Conduits under Pressure.

Diameter in inches.	$m = 0.95$		$m = 0.83$		$m = 0.68$		$m = 0.53$		$m = 0.45$		$m = 0.30$	
	c	f	c	f	c	f	c	f	c	f	c	f
1	87.3	00839	79.9	01007	70.0	01313	60.1	01780	54.8	02142	44.9	03190
2	92.4	00750	84.5	00901	74.6	01156	64.7	01534	59.4	01823	49.5	02625
3	95.7	00699	87.8	00834	77.9	01060	68.0	01391	62.7	01636	52.8	02307
4	97.1	00679	90.2	00790	80.3	00984	70.4	01298	65.1	01528	55.2	02111
6	102.0	00615	94.1	00726	84.2	00907	74.3	01166	69.0	01351	59.1	01841
8	104.9	00581	97.0	00684	87.1	00848	77.2	01080	71.9	01244	62.0	01673
10	106.3	00566	99.4	00651	89.5	00803	79.6	01015	74.3	01165	64.4	01551
12	109.4	00535	101.5	00624	91.6	00767	81.7	00964	76.4	01102	66.5	01454
14	111.2	00517	103.3	00603	93.4	00737	83.5	00922	78.2	01052	68.3	01379
16	112.9	00502	105.0	00583	95.1	00711	85.2	00886	79.9	01007	70.0	01313
18	114.4	00489	106.5	00567	96.6	00689	86.7	00856	81.4	00971	71.5	01258
20	115.9	00498	107.8	00554	97.9	00671	88.0	00829	82.7	0094	72.8	01214
22	117.0	00467	109.1	00540	99.2	00654	89.3	00807	84.0	00912	74.1	01171
24	118.2	00458	110.3	00528	100.4	00638	90.5	00785	85.2	00886	75.3	01134
26	119.4	00449	111.5	00517	101.6	00623	91.7	00765	86.4	00862	76.5	0110
28	120.4	00441	112.5	00507	102.4	00611	92.8	00747	87.5	00840	77.6	01068
30	121.4	00434	113.5	00499	103.6	00599	93.7	00733	88.4	00823	78.5	01044
32	122.2	00428	114.3	00492	104.4	00590	94.5	00720	89.2	00808	79.3	01023
34	123.1	00422	115.2	00485	105.3	00580	95.4	00707	90.1	00792	80.2	00100
36	124.0	00416	116.1	00477	106.2	00570	96.3	00693	91.0	00777	81.1	00978
38	124.9	00410	117.0	00470	107.1	00560	97.2	0068	91.9	00762	82.0	00957
40	125.7	00405	117.8	00464	107.9	00552	98.0	0067	92.7	00748	82.8	00938
42	126.4	0040	118.5	00458	108.6	00545	98.7	0066	93.4	00737	83.5	00923
44	127.0	00396	119.1	00453	109.2	00539	99.3	00651	94.2	00728	84.1	00909
46	127.9	00391	120.0	00446	110.1	00531	100.2	00641	94.9	00714	85.0	00890
48	128.4	00386	120.8	00441	110.9	00523	101.0	00631	95.7	00702	85.8	00874
50	129.4	00382	121.5	00436	111.6	00516	101.7	00622	96.4	00692	86.5	0086
52	130.0	00379	122.1	00431	112.2	00511	102.3	00614	97.0	00684	87.1	00848
54	130.7	00375	122.8	00426	112.9	00504	103.0	00606	97.7	00674	87.8	00834
56	131.2	00372	123.3	00423	113.4	00500	103.5	00600	98.2	00667	88.3	00826
58	131.9	00368	124.0	00418	114.1	00494	104.2	00592	98.9	00657	89.0	00812
60	132.5	00364	124.6	00414	114.7	00489	104.8	00586	99.5	00650	89.6	00801
62	133.1	00361	125.2	00410	115.3	00484	105.4	00579	100.1	00642	90.2	00791
64	133.6	00358	125.7	00407	115.8	00480	105.9	00573	100.6	00636	90.7	00782
66	134.2	00355	126.3	00403	116.4	00475	106.5	00567	101.2	00628	91.3	00772
68	134.7	00352	126.8	00400	116.9	00471	107.0	00562	101.7	00622	91.8	00763
70	135.2	00350	127.3	00397	117.4	00467	107.5	00557	102.2	00616	92.3	00755
72	135.8	00347	127.9	00393	118.0	00461	108.1	00551	102.8	00609	92.9	00745
78	137.2	00340	129.3	00385	119.4	00451	109.5	00536	104.2	00592	94.3	00723
84	138.5	00333	130.6	00374	120.7	00441	110.8	00524	105.5	00574	95.6	00704
90	139.9	00327	132.0	00369	122.1	00431	112.2	00511	106.9	00563	97.0	00684
96	141.2	00321	133.3	00362	123.4	00423	113.5	00499	108.2	00549	98.3	00666
102	142.2	00316	134.3	00356	124.4	00416	114.5	00490	109.2	00539	99.3	00652
108	143.2	00312	135.3	00351	125.4	00409	115.5	00482	110.2	00530	100.3	00639
114	144.5	00306	136.6	00344	126.7	00401	116.8	00472	111.5	00518	101.6	00624
120	145.7	00301	137.8	00339	127.9	00393	118.0	00462	112.7	00506	102.8	00609
126	146.7	00297	138.8	00334	128.9	00387	119.0	00454	113.7	00497	103.8	00597
132	147.7	00293	139.8	00329	129.9	00381	120.0	00446	114.7	00489	104.8	00590
138	148.6	00290	140.7	00325	130.8	00376	120.9	00440	115.6	00481	105.7	00576
144	149.5	00286	141.6	00321	131.7	00371	121.8	00434	116.5	00474	106.6	00566
156	151.4	00279	143.5	00313	133.6	00360	123.7	0042	118.4	00459	108.5	00557
168	153.0	00273	145.1	00305	135.2	00352	125.3	00410	120.0	00447	110.1	00546
180	154.6	00268	146.7	00299	136.8	00343	126.9	00400	121.6	00435	111.7	00538

Table VI. A.

WELDED PIPES.

Tubes of Brass, Galvanized Iron, Sheet Iron, Steel, Etc.

Nominal Diameter in Inches.	Actual Diameter in feet.	Area = $d^2 0.7854$	Values of $66(\sqrt[4]{r} + m)\sqrt{r} = c\sqrt{r}$				Loss of Head in Feet per Unit Length of Conduit at Unit Velocity.			
			$m =$ 0.95	$m =$ 0.83	$m =$ 0.68	$m =$ 0.45	$m =$ 0.95	$m =$ 0.83	$m =$ 0.68	$m =$ 0.45
⅛	0.0225	0.00038	6.058	5.467	4.722	3.584	1.714	2.154	2.888	5.025
¼	0.0303	0.00072	7.111	6.423	5.56	4.239	1.274	1.561	2.083	3.585
⅜	0.0411	0.00133	8.483	7.665	6.677	5.139	0.8949	1.096	1.444	2.439
½	0.0516	0.00209	9.646	8.745	7.623	5.398	0.692	0.842	1.108	1.850
¾	0.0686	0.00370	11.34	10.31	9.006	7.018	0.5008	0.6063	0.794	1.308
1	0.0873	0.00599	13.02	11.85	10.38	8.142	0.3801	0.4587	0.597	0.9496
1¼	0.1150	0.01039	15.22	13.90	12.22	9.646	0.2781	0.3331	0.431	0.692
1½	0.1341	0.01412	16.65	15.20	13.39	10.61	0.2268	0.2790	0.359	0.572
2	0.1722	0.02339	19.24	17.59	15.54	12.40	0.1740	0.2081	0.267	0.4155
2½	0.2056	0.03320	21.34	18.23	17.30	13.85	0.1414	0.1936	0.215	0.3354
3	0.2556	0.05130	24.24	22.25	19.75	15.90	0.1095	0.1302	0.165	0.2548
3½	0.2956	0.06863	26.61	24.24	21.56	17.42	0.0910	0.1095	0.139	0.2122
4	0.3356	0.08840	28.44	26.15	23.29	18.89	0.0796	0.0941	0.119	0.1806
4½	0.3756	0.1168	30.40	27.98	24.94	20.39	0.0696	0.0823	0.103	0.1565
5	0.4204	0.1388	32.48	29.93	26.72	21.80	0.0610	0.0703	0.090	0.1355
6	0.5056	0.2008	36.26	33.45	29.94	24.54	0.0489	0.0575	0.0718	0.1069
7	0.5857	0.2694	39.61	36.60	33.05	27.00	0.0410	0.0481	0.0598	0.0883
8	0.6651	0.3474	42.73	39.53	35.49	29.30	0.0352	0.0412	0.0511	0.0750
9	0.7449	0.4356	45.74	42.25	38.08	31.52	0.0308	0.0351	0.0444	0.0648
10	0.8348	0.5473	48.97	45.38	40.85	33.91	0.0268	0.0303	0.0377	0.056
11	0.9166	0.6599	51.87	48.06	43.35	36.08	0.0239	0.0279	0.0343	0.0495
12	1.0	0.7854	54.70	50.70	45.77	38.18	0.0215	0.0250	0.0304	0.0442
13	1.1641	0.9531	57.99	53.84	49.76	40.66	0.0191	0.0222	0.0272	0.039
14	1.1875	1.1075	60.70	56.4	50.21	42.72	0.0174	0.0202	0.0248	0.0353
15	1.2708	1.2675	63.29	58.78	53.22	44.66	0.0160	0.0186	0.0227	0.0323

Table VII.

CIRCULAR CONDUITS.

Diameters, Internal Areas, Mean Hydraulic Radii and their Roots.

ERRATA

Page 72, Table VI A, for

Loss of Head in Feet per Unit Length of Conduit at Unit Velocity

substitute

$\frac{Z}{R'}$, or Loss of Head in Feet in a Length of $2\,g$ Feet at Unit Velocity.

Actual Internal Diameter in Inches.	Internal Area in Square Feet.	R	$\sqrt{r}$	$\sqrt[4]{r}$	Actual Internal Diameter in Inches.	Internal Area in Square Feet.	R	$\sqrt{r}$	$\sqrt[4]{r}$
							333	1 .0408	1 .0202
							25	1 .060	1 .030
							666	1 .0801	1 .0393
							083	1 .0993	1 .0485
							5	1 .118	1 .057
							966	1 .137	1 .066
							333	1 .415	1 .0684
							75	1 .173	1 .083
							166	1 .1903	1 .091
							583	1 .2076	1 .099
								1 .226	1 .107
							25	1 .275	1 .129
								1 .323	1 .150
							75	1 .369	1 .170
								1 .414	1 .189
							25	1 .457	1 .208
								1 .5	1 .224
							5	1 .541	1 .244
								1 .581	1 .257
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2 .625	1 .620	1 .273
16	1 .396	0 .3333	0 .5771	0 .759	132	95 .03	2 .75	1 .658	1 .288
18	1 .767	0 .3750	0 .6124	0 .782	138	103 .87	2 .875	1 .696	1 .302
20	2 .234	0 .4166	0 .645	0 .8031	144	113 .10	3 .0	1 .732	1 .316
22	2 .640	0 .4583	0 .6770	0 .8220	150	122 .72	3 .125	1 .768	1 .330
24	3 .142	0 .5	0 .7071	0 .841	156	132 .76	3 .25	1 .803	1 .343
26	3 .687	0 .5416	0 .7360	0 .859	162	143 .16	3 .375	1 .837	1 .355
28	4 .275	0 .5833	0 .7637	0 .874	168	153 .96	3 .5	1 .871	1 .368
30	4 .909	0 .625	0 .7906	0 .889	174	165 .17	3 .625	1 .904	1 .380
32	5 .585	0 .6666	0 .8165	0 .904	180	176 .70	3 .75	1 .937	1 .392
34	6 .305	0 .7084	0 .8416	0 .917	186	188 .70	3 .875	1 .968	1 .403
36	7 .069	0 .75	0 .866	0 .931	192	201 .03	4 .0	2 .0	1 .414
38	7 .876	0 .7916	0 .8898	0 .9435	198	213 .8	4 .125	2 .031	1 .425
40	8 .927	0 .8333	0 .9129	0 .9555	204	227 .0	4 .25	2 .062	1 .437
42	9 .621	0 .875	0 .9355	0 .967	210	240 .5	4 .375	2 .091	1 .446
44	10 .559	0 .9166	0 .9575	0 .9784	216	254 .5	4 .5	2 .121	1 .456
46	11 .509	0 .9584	0 .9983	0 .9991	222	268 .8	4 .625	2 .151	1 .466
48	12 .566	1 .0	1 .0	1 .0	228	283 .5	4 .75	2 .180	1 .476
50	13 .635	1 .0416	1 .0206	1 .0103	240	314 .2	5 .0	2 .236	1 .496

Table VI. A.

WELDED PIPES.

Tubes of Brass, Galvanized Iron, Sheet Iron, Steel, Etc.

Nominal Diameter in Inches.	Actual Diameter in feet.	Area = $d^2 0.7854$	Values of $66 (\sqrt[4]{r} + m)\sqrt{r}$				Loss of Head in Feet per Unit Length of Conduit			
			m (							
⅛	0 .0225	0 .00038	6							
¼	0 .0303	0 .00072	7							
⅜	0 .0411	0 .00133	8							
½	0 .0516	0 .00209	9							
¾	0 .0686	0 .00370	11							
1	0 .0873	0 .00599	13							
1¼	0 .1150	0 .01039	15							
1½	0 .1341	0 .01412	16							
2	0 .1722	0 .02339	19							
2½	0 .2056	0 .03320	21							
3	0 .2556	0 .05130	24							
3½	0 .2956	0 .06863	26							
4	0 .3356	0 .08840	28							
4½	0 .3756	0 .1168	30 .40	27 .98	24 .94	20 .39	0 .0696	0 .0823	0 .105	0 .1505
5	0 .4204	0 .1388	32 .48	29 .93	26 .72	21 .80	0 .0610	0 .0703	0 .090	0 .1355
6	0 .5056	0 .2008	36 .26	33 .45	29 .94	24 .54	0 .0489	0 .0575	0 .0718	0 .1069
7	0 .5857	0 .2694	39 .61	36 .60	33 .05	27 .00	0 .0410	0 .0481	0 .0598	0 .0883
8	0 .6651	0 .3474	42 .73	39 .53	35 .49	29 .30	0 .0352	0 .0412	0 .0511	0 .0750
9	0 .7449	0 .4356	45 .74	42 .25	38 .08	31 .52	0 .0308	0 .0351	0 .0444	0 .0648
10	0 .8348	0 .5473	48 .97	45 .38	40 .85	33 .91	0 .0268	0 .0303	0 .0377	0 .056
11	0 .9166	0 .6599	51 .87	48 .06	43 .35	36 .08	0 .0239	0 .0279	0 .0343	0 .0495
12	1 .0	0 .7854	54 .70	50 .70	45 .77	38 .18	0 .0215	0 .0250	0 .0304	0 .0442
13	1 .1641	0 .9531	57 .99	53 .84	49 .76	40 .66	0 .0191	0 .0222	0 .0272	0 .039
14	1 .1875	1 .1075	60 .70	56 .4	50 .21	42 .72	0 .0174	0 .0202	0 .0248	0 .0353
15	1 .2708	1 .2675	63 .29	58 .78	53 .22	44 .66	0 .0160	0 .0186	0 .0227	0 .0323

Table VII.

CIRCULAR CONDUITS.

Diameters, Internal Areas, Mean Hydraulic Radii and their Roots.

Actual Internal Diameter in Inches.	Internal Area in Square Feet.	R	$\sqrt{r}$	$\sqrt[4]{r}$	Actual Internal Diameter in Inches.	Internal Area in Square Feet.	R	$\sqrt{r}$	$\sqrt[4]{r}$
½	0 .001364	0 .010417	0 .10206	0 .3195	52	14 .750	1 .0833	1 .0408	1 .0202
1	0 .005454	0 .02083	0 .1444	0 .380	54	15 .904	1 .125	1 .060	1 .030
1½	0 .012272	0 .03125	0 .1768	0 .4302	56	17 .106	1 .1666	1 .0801	1 .0393
2	0 .02182	0 .04166	0 .2039	0 .4516	58	18 .347	1 .2083	1 .0993	1 .0485
2½	0 .03163	0 .0502	0 .2240	0 .4733	60	19 .635	1 .25	1 .118	1 .057
3	0 .04909	0 .0625	0 .25	0 .5	62	20 .964	1 .2966	1 .137	1 .066
3½	0 .06681	0 .07292	0 .270	0 .5196	64	22 .340	1 .3333	1 .415	1 .0684
4	0 .08726	0 .08333	0 .291	0 .5370	66	23 .758	1 .375	1 .173	1 .083
4½	0 .11045	0 .09375	0 .3062	0 .5533	68	25 .220	1 .4166	1 .1903	1 .091
5	0 .1364	0 .10416	0 .3227	0 .5681	70	26 .725	1 .4583	1 .2076	1 .099
6	0 .1963	0 .125	0 .3535	0 .594	72	28 .27	1 .5	1 .226	1 .107
7	0 .2672	0 .1458	0 .3819	0 .6180	78	33 .18	1 .625	1 .275	1 .129
8	0 .3490	0 .1666	0 .4082	0 .639	84	38 .48	1 .75	1 .323	1 .150
9	0 .4418	0 .1875	0 .433	0 .658	90	44 .18	1 .875	1 .369	1 .170
10	0 .5585	0 .2083	0 .4564	0 .675	96	50 .27	2 .0	1 .414	1 .189
11	0 .6599	0 .2297	0 .4787	0 .692	102	56 .75	2 .125	1 .457	1 .208
12	0 .7854	0 .25	0 .5	0 .7071	108	63 .62	2 .25	1 .5	1 .224
13	0 .9217	0 .2708	0 .5204	0 .721	114	70 .88	2 .375	1 .541	1 .244
14	1 .0689	0 .2916	0 .540	0 .735	120	78 .54	2 .5	1 .581	1 .257
15	1 .2272	0 .3125	0 .5339	0 .748	126	86 .59	2 .625	1 .620	1 .273
16	1 .396	0 .3333	0 .5771	0 .759	132	95 .03	2 .75	1 .658	1 .288
18	1 .767	0 .3750	0 .6124	0 .782	138	103 .87	2 .875	1 .696	1 .302
20	2 .234	0 .4166	0 .645	0 .8031	144	113 .10	3 .0	1 .732	1 .316
22	2 .640	0 .4583	0 .6770	0 .8220	150	122 .72	3 .125	1 .768	1 .330
24	3 .142	0 .5	0 .7071	0 .841	156	132 .76	3 .25	1 .803	1 .343
26	3 .687	0 .5416	0 .7360	0 .859	162	143 .16	3 .375	1 .837	1 .355
28	4 .275	0 .5833	0 .7637	0 .874	168	153 .96	3 .5	1 .871	1 .368
30	4 .909	0 .625	0 .7906	0 .889	174	165 .17	3 .625	1 .904	1 .380
32	5 .585	0 .6666	0 .8165	0 .904	180	176 .70	3 .75	1 .937	1 .392
34	6 .305	0 .7084	0 .8416	0 .917	186	188 .70	3 .875	1 .968	1 .403
36	7 .069	0 .75	0 .866	0 .931	192	201 .03	4 .0	2 .0	1 .414
38	7 .876	0 .7916	0 .8898	0 .9435	198	213 .8	4 .125	2 .031	1 .425
40	8 .927	0 .8333	0 .9129	0 .9555	204	227 .0	4 .25	2 .062	1 .437
42	9 .621	0 .875	0 .9355	0 .967	210	240 .5	4 .375	2 .091	1 .446
44	10 .559	0 .9166	0 .9575	0 .9784	216	254 .5	4 .5	2 .121	1 .456
46	11 .509	0 .9584	0 .9983	0 .9991	222	268 .8	4 .625	2 .151	1 .466
48	12 .566	1 .0	1 .0	1 .0	228	283 .5	4 .75	2 .180	1 .476
50	13 .635	1 .0416	1 .0206	1 .0103	240	314 .2	5 .0	2 .236	1 .496

Table VII. A.

Roots of Mean Hydraulic Radii.

R	$\sqrt{r}$	$\sqrt[4]{r}$	R	$\sqrt{r}$	$\sqrt[4]{r}$	R	$\sqrt{r}$	$\sqrt[4]{r}$
0.05	0.224	0.473	2.75	1.658	1.287	5.9	2.429	1.558
0.1	0.316	0.562	2.80	1.673	1.293	6.0	2.449	1.565
0.15	0.387	0.622	2.85	1.688	1.299	6.1	2.470	1.571
0.20	0.447	0.668	2.90	1.703	1.305	6.2	2.490	1.578
0.25	0.5	0.707	2.95	1.718	1.311	6.3	2.510	1.584
0.30	0.548	0.740	3.0	1.732	1.316	6.4	2.530	1.590
0.35	0.592	0.769	3.05	1.746	1.322	6.5	2.550	1.597
0.40	0.632	0.803	3.10	1.761	1.327	6.7	2.588	1.609
0.45	0.671	0.819	3.15	1.775	1.332	6.8	2.608	1.615
0.5	0.707	0.841	3.20	1.789	1.338	6.9	2.627	1.621
0.55	0.742	0.861	3.25	1.803	1.343	7.0	2.644	1.624
0.60	0.775	0.881	3.30	1.817	1.347	7.1	2.665	1.630
0.65	0.806	0.898	3.35	1.830	1.352	7.2	2.683	1.637
0.70	0.837	0.914	3.40	1.844	1.358	7.3	2.702	1.643
0.75	0.866	0.930	3.45	1.857	1.363	7.4	2.720	1.649
0.80	0.894	0.946	3.50	1.871	1.368	7.5	2.739	1.655
0.85	0.922	0.960	3.55	1.884	1.373	7.6	2.757	1.661
0.90	0.949	0.974	3.60	1.898	1.378	7.7	2.775	1.664
0.95	0.975	0.987	3.65	1.910	1.382	7.8	2.793	1.670
1.0	1.0	1.0	3.70	1.924	1.387	7.9	2.811	1.676
1.05	1.025	1.012	3.75	1.936	1.392	8.0	2.828	1.682
1.10	1.049	1.024	3.80	1.949	1.396	8.1	2.846	1.688
1.15	1.072	1.036	3.85	1.962	1.401	8.2	2.868	1.692
1.20	1.095	1.047	3.90	1.975	1.405	8.3	2.881	1.697
1.25	1.118	1.057	3.95	1.987	1.410	8.4	2.898	1.702
1.30	1.140	1.068	4.0	2.0	1.414	8.5	2.915	1.707
1.35	1.162	1.079	4.05	2.012	1.419	8.6	2.933	1.712
1.40	1.183	1.088	4.10	2.025	1.423	8.7	2.950	1.717
1.45	1.204	1.097	4.15	2.037	1.427	8.8	2.966	1.722
1.50	1.225	1.107	4.20	2.049	1.432	8.9	2.983	1.727
1.55	1.245	1.115	4.25	2.062	1.436	9.0	3.0	1.732
1.60	1.265	1.125	4.30	2.074	1.440	9.1	3.017	1.737
1.65	1.285	1.133	4.35	2.086	1.444	9.2	3.043	1.741
1.70	1.304	1.142	4.40	2.098	1.448	9.3	3.056	1.746
1.75	1.323	1.150	4.45	2.111	1.453	9.4	3.066	1.750
1.80	1.342	1.158	4.50	2.121	1.457	9.5	3.082	1.755
1.85	1.360	1.166	4.55	2.133	1.461	9.6	3.098	1.760
1.90	1.378	1.174	4.60	2.145	1.466	9.7	3.114	1.764
1.95	1.396	1.182	4.65	2.156	1.469	9.8	3.130	1.769
2.0	1.414	1.189	4.70	2.168	1.473	9.9	3.146	1.773
2.05	1.432	1.196	4.75	2.179	1.476	10.0	3.162	1.777
2.10	1.449	1.204	4.80	2.191	1.480	10.5	3.240	1.800
2.15	1.466	1.211	4.85	2.202	1.484	11.0	3.317	1.820
2.20	1.483	1.218	4.90	2.214	1.488	11.5	3.391	1.845
2.25	1.5	1.225	4.95	2.225	1.492	12.0	3.464	1.860
2.30	1.517	1.232	5.0	2.236	1.495	12.5	3.536	1.880
2.35	1.533	1.238	5.1	2.258	1.503	13.0	3.606	1.899
2.40	1.549	1.245	5.2	2.280	1.511	13.5	3.674	1.918
2.45	1.565	1.251	5.3	2.302	1.518	14.0	3.742	1.934
2.50	1.581	1.257	5.4	2.324	1.526	14.5	3.808	1.951
2.55	1.597	1.263	5.5	2.346	1.532	15.0	3.873	1.968
2.60	1.612	1.270	5.6	2.366	1.539	15.5	3.937	1.984
2.65	1.628	1.276	5.7	2.387	1.548	16.0	4.0	2.0
2.70	1.643	1.282	5.8	2.408	1.552			

TABLE VIII.

Table VIII contains the practically most useful coefficients indicating the degree of roughness of a conduit.

In the design of a new conduit it is well to remember, that the degree of roughness of a conduit is not a permanent quantity. Conduits lined with cement, smooth concrete, good brickwork, planed boards, metals, etc., gradually deteriorate and assume a degree of roughness which closely resembles that of a sawed board ($m = 0.68$), in case of sewers that of common brick work ($m = 0.57$), in case of large riveted pipes that of very rough brick work ($m = 0.45$).

If the velocity is feeble, or the flow often interrupted, cryptogamic plants sooner or later appear on the walls of open conduits and rust or calcareous matter coates the walls of pipes. In such a condition the degree of roughness corresponds to that of very rough brick work ($m = 0.45$).

If left to themselves, channels in earth of all descriptions likewise deteriorate and gradually assume a degree of roughness corresponding to that of a natural channel ($k = 1.93$ in most cases).

For artificial channels in earth Table VIII gives values of both m and k. Owing to the abnormally rapid decreases in the value of c with the decrease of the depth of the water in rough channels in earth a negative value of m gives better results than k.

The k formula, however, gives good results in all cases where R is greater than one foot.

The relation between K and m and the coefficient n of the formula of Kutter is given by

$$n = \frac{1 + K}{100} = \frac{0.02}{1 + m}.$$

Table VIII.

Values of m and k, the Coefficients Indicating the Degree of Roughness. A. Conduits under Pressure.

m	Description of Conduit.
1.0	New, straight tin or plated pipes.
0.95	Pipes of planed boards or clean cement, new. Very smooth new asphalt-coated cast and wrought-iron pipes. New asphalt-coated riveted pipes not exceeding 6 inches in diameter.
0.83	Ordinary new asphalt-coated cast and wrought-iron pipes. Wrought-iron pipes not coated, new. Glass and lead pipes. Pipes lined with smooth concrete or cement plaster.
0.68	Pipes lined with cement or smooth concrete, pipes of planed or rough boards, cast and wrought-iron pipes, coated or not coated, steel and wrought-iron riveted pipes not exceeding 3 feet in diameter (all some time in use but fairly clean).
0.57	Sewer pipe. Conduits lined with common brickwork or rough concrete.
0.53	New asphalt-coated steel-riveted pipe exceeding 3 feet in diameter.
0.45	Conduits lined with very rough brickwork or very rough concrete. Steel-riveted pipe exceeding 3 feet in diameter, some years in use. Old cast and wrought-iron pipes of all descriptions, not very clean.
0.30	Old steel-riveted pipe exceeding 3 feet in diameter.
0.20	Drain tile.

B. Open Conduits.

m	Description of Conduit.
1.0	Conduits lined with neat cement exceptionally smooth.
0.95	New conduits lined with neat cement or planed boards.
0.83	New brick conduits washed with cement, conduits smoothly dressed with cement mortar.
0.80	New conduits lined with smooth concrete or very good brick work.
0.70	Conduits lined with sawed boards or fairly good brick work. Aqueducts lined with neat cement, cement plaster, smooth concrete very good brickwork, planed boards (all some time in use).
0.57	Channels lined with common brickwork, rough concrete or smoothly dressed ashlar masonry. Sewers lined with neat cement, smooth concrete, brickwork washed with cement or plastered with cement mortar, fairly good and very good brickwork (all some time in use.)
0.45	Channels lined with very rough brickwork or concrete, fairly good ashlar masonry.
0.30	Channels lined with common ashlar or very good rubble masonry.
0.15	Channels lined with roughly hammered stone masonry.
0.0	Channels lined with common rubble masonry. Channels in rockwork.

TABLE VIII. — *Continued.*

C. CHANNELS IN EARTH.

m	*k*	Description of Conduit.
0.57	0.27	Channels of very regular cross-section in stiff clay or clayey loam.
0.15	0.74	Channels of fairly regular cross-section in fine cemented gravel.
0.0	1.0	Channels of fairly regular cross-section in coarse cemented gravel. Channels in rockwork.
−0.10	1.2	Fairly regular channels in sand or sand with gravel imbedded.
−0.20	1.5	Fairly regular channels in earth, tolerably free from stones and plants.
−0.32	1.93	Ordinary channels in earth or gravel. Channels with stones vegetation or other impediments to flow. Natural channels, creeks, rivers.

TABLE IX.

ALPHABETICAL LIST OF AUTHORITIES WHOSE EXPERIMENTAL DATA ARE GIVEN IN TABLE X, METHODS OF GAUGING AND PUBLICATION CONTAINING ORIGINAL RECORD OF EXPERIMENTS.

Author.	Description of Channel Gauged.	Method of Gauging.	Where Recorded.
Adams, A. L.	Wooden Stave pipes.	Discharge measured by rise in reservoir surface, loss of head by open standpipes.	Engineering News. Sept. 1898.
Baumgarten . .	Aqueduct. Bottom of cement, sides of brick.	Piezometer.	Darcy–Bazin. Recherches hydrauliques.
Benzenberg . .	Brick sewer.	Floats probably.	Trans. A. S. C. E.
Bossut	Tin and Lead pipes.	Discharge measured in tanks.	Hamilton Smith, Hydraulics, 1886.
Bruce	Aqueduct. Concrete.	Discharge measured by rise in reservoir surface.	Proceedings of Institute of C. E. London, 1896.
Brush	Cast-iron pipes.	Quantities measured at pumps.	Quoted by Kutter. "The Flow of Water."
Clarke	Brick sewer.	Discharge measured by rise in reservoir surface.	H. Smith. Hydraulics, 1886.

TABLE IX. — *Continued.*

Author.	Description of Channel Gauged.	Method of Gauging.	Where Recorded.
Cunningham . .	Aqueduct of masonry. Ganges Canal.	Velocities measured by one-inch tin rod floats.	Roorkee Hydr. Experiments, 1880.
Darcy	Pipes.	Discharge measured in tanks, loss of head by piezometer or mercury column.	Experiments sur le mouvement de l'eau dans les tuyaux.
Darcy-Bazin . .	Cement and concrete conduits.	Discharges measured by orifices previously tested.	Recherches hydr. Paris 1865.
Darcy-Bazin . .	Conduits of planed and rough boards, or lined with brick.	Discharges measured by orifices previously tested, 20 centimeter square.	Recherches hydr. Paris, 1865.
Darcy-Bazin . .	Canal lined with ashlar masonry.	Piezometer.	Recherches hydr. Paris, 1865.
Darcy-Bazin . .	Tailrace lined with ashlar masonry.	Discharge measured by orifice 50 centimeter square.	Recherches hydr. Paris, 1865.
Darcy-Bazin . .	Tunnel lined with ashlar masonry.	Current meter and reservoir.	Recherches hydr. Paris, 1865.
Darcy-Bazin . .	Section of Grosbois canal lined with masonry.	Current meter and reservoir.	Recherches hydr. Paris, 1865.
Darcy-Bazin .	Chazilly Canal.	Piezometer, current meter and reservoir.	Recherches hydr. Paris, 1865.
Darcy-Bazin .	Grosbois Canal.	Piezometer, current meter and reservoir.	Recherches hydr. Paris, 1865.
Dubuat	Canal du Jard.	Surface floats.	Principes hydr. Paris, 1786.
Ehman	Galvanized and cast-iron pipes.	Discharges measured by volumes.	Iben Druckhöhenverlust.
Fanning	Cement lined pipe.	Weir measurement probably.	Water Supply Engineering.

TABLE IX.—*Continued.*

Author.	Description of Channel Gauged.	Method of Gauging.	Where Recorded.
Fteley & Stearns	Sudbury Conduit. Brick coated with cement and not coated.	Weir measurement.	H. Smith. Hydraulics, 1886.
Fortier	Irrigation Channels.	Current meter.	U. S. Geol. Survey. Irr. Papers, 1901.
Hawks	Steel-riveted pipe.	Weir measurement.	Tr. A. Soc. C. E., 1899.
Herschel . . .	Steel-riveted pipes.	Discharges measured by Ventury meter, loss of head by Bourdon gauges.	115 Experiments.
Horton	Brick sewers washed with cement.	Weir measurement probably.	Eng. News.
Hubbel & Fenkell	Cast-iron ppes.		Tr. A. Soc. C. E., 1898.
Iben	Cast-iron pipes. Pipes coated with tar.	Discharges measured by volumes, loss of head by pressure gauges.	Druckhöhenverlust.
Kuichling . . .	Riveted pipes. Cast-iron pipes.	Quantities measured by rise in reservoir surface, loss of head by mercury gauges.	Marx-Wing & Hoskins, Tr. A. S. C. E., 1898–1899.
Kutter	Channels lined with rubble masonry.	Surface floats.	Die neue Theorie.
La Nicca . . .	Alpine Streams.	Surface floats.	Kutter, Die neue Theorie.
Lampe	Cast-iron pipes.	Discharges measured by reservoir contents, loss of head by pressure gauges.	Iben. Druckhöhenverlust.
Legler . . .	Canals.	Rodfloats.	Hydrotechnische Mittheilungen.

TABLE IX. — *Continued.*

Author.	Description of Channel Gauged.	Method of Gauging.	Where Recorded.
McDougall . .	Irrigation channels.	Current meter.	U. S. Geolog. Survey Irr. Papers.
Marx-Wing & Hoskins . . .	Riveted pipe. Stave pipe.	Discharges measured by Venturi meters, loss of head by mercury gauges.	Trans. A. S. C. E., 1898.
Noble	Stave pipes.		Trans. A. S. C. E., 1902.
Passini & Gioppi	Aqueduct. Bottom of concrete, sides of brick.	Current meter.	Giornale del. Genio Civile. Roma, 1893.
Passini & Gioppi	Syphon aqueduct of brick.	"	"
Passini & Gioppi	Canal Cavour.	"	"
Perrone	Aqueduct coated with clean cement.	"	Zoppi: Sul Volturno, Carte hydrographique d' Italie.
Perrone	Tunnel in rock-work.	"	
Rafter	Riveted pipes.	Discharge measured by rise in reservoir surface, loss of head by piezo-meter.	Tr. A. S. C. Eng., XXVI.
Revy	La Plata and Parana Rivers.	Current meter.	Hydraulics of great rivers, London, 1881.
Rittinger . . .	Channels lined with rubble masonry.	Discharges measured in tanks.	Bornemann: Der Civil Ingenieur.
Roff	Saalach River.	Piezometer.	Grebenau, Theorie der Bewegung des Wassers.
Rowland . . .	Wrought-iron pipes.	Discharge measured by volumes.	Brush.
Smith, H. . . .	Riveted pipes.	Velocities measured by weirs and Standard orifices.	Tr. A. S. C. E. "Hydraulics," N. Y., 1886.

TABLE IX. — *Concluded.*

Author.	Description of Channel Gauged.	Method of Gauging.	Where Recorded.
Smith, J. W. . .	Riveted pipes.	Discharge measured by weir, loss of head by piezometers.	Tr. A. S. C. E., Vol. XXVI.
Stearns	Brick aqueduct.	Current meter.	Report to the New Croton Aqueduct Com., 1895.
Schwartz . . .	Weser River.	Current meter.	Funk: Beitrag zur allgemeinen Wasserbaukunst Lemgo, 1808.
Wampfler . . .	Canal.	Surface floats.	Kutter: Die neue Theorie.

TABLE X.

Table X contains the most reliable experimental data from which the general formula is deduced. For conduits under pressure the numerical values of (a), the coefficient of variation of c is generally given. This is done in order to show the details of the variation of c. For open conduits the variation is generally indicated by giving the velocity roots. These roots are found from the formula

$$x = \frac{\log. v_1 \quad - \quad \log. v_0}{\log\left(66 \sqrt[4]{r} + m\right)\sqrt{rs}\,)_1 \quad - \quad \log.\left(66\left(\sqrt[4]{r} + m\right)\sqrt{rs}\,\right)_0}$$

The value of x being thus found the value of m is found by putting

$$\frac{V^{\frac{1}{x}}}{66\sqrt{r\,.\,s}} - \sqrt[4]{r} = m$$

or

$$\frac{c}{66\,a} - \sqrt[4]{r} = m$$

For artificial channels in earth the values of m have been given in addition to the values of K. Owing to the abnormally rapid decrease in the values of c with the decrease of the depth of the water in very rough channels a negative value of m gives better results than K. The K formula, however, gives good results in all cases where R is greater than one foot.

Table X.

EXPERIMENTAL DATA.

I. Riveted Pipes.

Description of Conduit.	*m*	*L*	*d*	*R*	1000 *s*	*v*	*c*	*a*
New straight asphalt-coated wrought-iron riveted pipe with screw joints. —Darcy.	0 .94	365	0 .271	0 .0677	0 .27	0 .328	76 .7	0 .80
	...	...	...	...	2 .028	1 .171	99 .9	1 .04
	...	...	...	...	12 .20	3 .117	108 .4	1 .130
	...	...	...	...	40 .70	6 .148	117 .1	1 .219
	...	...	...	...	106 .54	10 .535	124 .0	1 .287
	...	...	...	...	156 .05	12 .786	124 .3	1 .291
Do.	0 .92	365	0 .643	0 .1607	0 .20	0 .591	104 .1	1 .013
	...	...	...	...	1 .29	1 .529	106 .2	1 .035
	...	...	...	...	5 .80	3 .53	115 .6	1 .125
	...	...	...	...	12 .0	5 .509	125 .4	1 .220
	...	...	...	...	29 .7	9 .00	130 .2	1 .267
	...	...	...	...	121 .56	19 .72	141 .0	1 .372
Do.	0 .82	365	0 .935	0 .234	0 .70	1 .296	101 .3	1 .013
	...	...	...	...	4 .33	3 .868	121 .6	1 .216
	...	...	...	...	11 .90	6 .673	126 .5	1 .265
	...	...	...	...	28 .07	10 .522	129 .9	1 .299
Sheet-iron riveted pipe with funnel mouthpiece 7.8 ft. long. —Hamilton Smith.	0 .68	700	0 .911	0 .228	8 .50	4 .712	107 .1	1 .19
	...	...	...	...	13 .34	6 .094	110 .6	1 .229
	...	...	...	...	16 .95	6 .927	111 .5	1 .240
	...	...	...	...	25 .59	8 .659	113 .4	1 .260
	...	...	...	...	33 .09	10 .021	115 .5	1 .283
Do. Coated with asphalt. Funnel mouthpiece 12 ft. long.	0 .68	700	1 .056	0 .264	6 .68	4 .595	109 .4	1 .200
	...	...	...	...	14 .28	6 .962	113 .4	1 .242
	...	...	...	...	22 .19	8 .646	113 .0	1 .237
	...	...	...	...	33 .18	10 .706	114 .4	1 .253
Do. Funnel mouthpiece 14.8 ft. long.	0 .69	700	1 .229	0 .307	5 .02	4 .383	111 .6	1 .181
	...	...	...	...	10 .97	6 .841	119 .8	1 .246
	...	...	...	...	12 .27	7 .314	119 .1	1 .261
	...	...	...	...	16 .46	8 .462	119 .2	1 .260
	...	...	...	...	24 .70	10 .593	121 .6	1 .286
	...	...	...	...	32 .31	12 .09	121 .3	1 .285
Do. Double riveted pipe with some easy curves.	0 .65	4440	1 .416	0 .354	66 .72	20 .143	131 .1	1 .395
Do.	0 .69	1200	2 .154	0 .538	16 .41	12 .605	134 .1	1 .325
Do. Inverted syphon with 887 ft. depression.	0 .63	12800	2 .43	0 .607	11 .72	10 .78	127 .8	1 .30

TABLE X. — *Continued.*

Description of Conduit.	*m*	*L*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Wrought-iron riveted pipe with lap joints.	0 .54	152 .9	8 .58	2 .145	0 .0079	0 .50	126 .9	1 .089
	...	...	...	...	0 .032	1 .0	116 .6	1 .00
	...	...	...	...	0 .0837	1 .5	111 .9	0 .959
Paint coating worn off, somewhat rusty.	...	...	...	...	0 .1557	2 .0	109 .4	0 .938
	...	...	...	...	0 .2453	2 .5	109 .0	0 .934
— Clemens-Herschel.	..	...	...	...	0 .354	3 .0	108 .2	0 .928
	...	...	...	...	0 .4991	3 .5	107 .0	0 .917
	...	...	...	...	0 .6619	4 .0	106 .2	0 .910
	...	...	...	...	0 .8470	4 .5	105 .6	0 .905
Asphalt-coated steel-riveted pipe. — A. McL. Hawks.	0 .55	...	1 .166	0 .2915	0 .4550	0 .932	82 .2	0 .98
	...	...	...	...	0 .584	1 .136	86 .0	1 .026
Asphalt coated cylinder joint steel pipe.—A. L. Adams.	0 .65	16416	1 .33	0 .333	5 .0	4 .58	110 .0	1 .183
Asphalt-coated cylinder joint steel-riveted pipe with curves. — E. Kuichling.	0 .57	91641	3 .166	0 .7915	1 .01	3 .23	114 .0	1 .14
	...	...	...	...	0 .99	3 .27	116 .6	1 .166
Asphalt-coated taper joint steel-riveted pipe. New.—Clemens-Herschel.	0 .56	81139	3 .5	0 .875	0 .112	1 .0	101 .0	1 .0
	...	...	...	...	...	2 .0	104 .3	1 .032
	...	...	...	...	...	3 .0	106 .4	1 .053
	...	...	...	...	...	4 .0	107 .8	1 .067
	...	...	...	...	...	5 .0	108 .4	1 .073
	...	...	...	...	...	6 .0	108 .5	1 .074
Do.	0 .54	5574	3 .5	0 .875	0 .13	1 .0	96 .0	0 .96
	...	...	...	...	...	2 .0	107 .9	1 .08
	...	...	...	...	...	3 .0	112 .6	1 .128
	...	...	...	...	...	3 .5	113 .0	1 .132
	...	...	...	...	...	4 .0	112 .8	1 .130
	...	...	...	...	...	5 .0	110 .8	1 .111
	...	...	...	...	...	6 .0	110 .0	1 .102
Do.	0 .53	24000	4 .0	1 .0	0 .0976	1 .0	101 .2	1 .0
Cylinder joint, many curves.	...	...	...	...	...	2 .0	108 .3	1 07
	...	...	...	...	...	3 .0	112 .8	1 113
	...	...	...	...	...	3 .5	113 .4	1 .119
	...	...	...	...	...	4 .0	113 .2	1 .118
	...	...	...	...	...	5 .0	112 .0	1 .105
	...	...	...	...	...	6 .0	111 .6	1 .091
Asphalt-coated cylinder joint steel-riveted pipe.—J. W. Smith.	0 .80	39809	2 .916	0 .584	1 .31	3 .52	126 .8	1 .15
	0 .74	34176	2 .75	0 .55	1 .31	3 .96	123 .2	1 .166

TABLE X. — *Continued.*

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Asphalt-coated butt-jointed steel-riveted pipe with many curves.—Marx-Wing.	0 .50	4367	6 .00	1 .5	0 .07	1 .08	108 .0	1 .0
	...	...	...	...	0 .16	1 .57	114 .0	1 .055
	...	...	...	...	0 .24	2 .14	113 .0	1 .046
	...	...	...	...	0 .559	2 .59	110 .0	1 .018
	...	...	...	...	0 .495	3 .02	112 .0	1 .037
	...	...	...	...	0 .776	3 .84	113 .0	1 .046

II. OLD RIVETED PIPES.

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Cylinder joint asphalt-coated steel-riveted pipe. Fourteen years in use. —George W. Rafter.	0 .31	45400	2 .0	0 .50	3 .83	3 .32	76 .0	1 .0
	...	...	...	...	3 .58	3 .32	78 .0	1 .030
	...	...	...	...	3 .46	3 .35	80 .5	1 .06
Do.	0 .29	45400	3 .0	0 .75	0 .45	1 .47	80 .4	1 .0
	...	...	...	...	0 .43	1 .49	83 .0	...
Do. One year in use.— E. Kuichling	0 .59	45400	3 .17	0 .7915	1 .59	3 .88	109 .3	1 .08
	...	...	...	...	1 .61	3 .91	109 .3	1 .08
	...	...	...	...	1 .62	3 .90	109 .1	1 .08
Taper joint, steel riveted. Four years old.—Clemens-Herschel.	0 .26	24648	4 .0	1 .0	...	1 .0	78 .0	0 .94
	...	...	...	...	...	1 .5	84 .6	1 .019
	...	...	...	...	...	2 .0	89 .6	1 .080
	...	...	...	...	...	2 .5	92 .4	1 .113
	...	...	...	...	...	3 .0	93 .0	1 .120
	...	...	...	...	...	3 .5	93 .2	1 .121
	...	...	...	...	...	4 .0	94 .2	1 .135
	...	...	...	...	...	5 .0	94 .4	1 .137
	...	...	...	...	...	6 .0	94 .9	1 .143
Cylinder joint steel-riveted pipe, four years in use. — C. Herschel.	0 .47	24600	4	1 .0	...	1 .0	97 .2	1 .0
	...	...	...	...	...	1 .5	100 .8	1 .024
	...	...	...	...	...	2 .0	103 .3	1 .062
	...	...	...	...	...	2 .5	104 .9	1 .079
	...	...	...	...	...	3 .0	105 .3	1 .083
	...	...	...	...	...	3 .5	104 .8	1 .079
	...	...	...	...	...	4 .0	104 .0	1 .069
	...	...	...	...	...	5 .0	103 .9	1 .066
	...	...	...	...	...	6 .0	103 .7	1 .066

III. NEW WROUGHT-IRON PIPES, NOT COATED.

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Straight pipe. — Darcy.	0 .83	372	0 .0873	0 .0218	0 .33	0 .19	70 .7	0 .877
	...	...	...	...	10 .15	1 .207	81 .1	1 .006
	...	...	...	...	43 .48	2 .612	84 .8	1 .052
	...	...	...	...	105 .71	4 .203	87 .5	1 .098
	...	...	...	...	309 .52	7 .166	87 .2	1 .094

Table X. — *Continued.*

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Straight pipe.	0 .83	372	0 .1296	0 .0324	0 .22	0 .205	76 .9	0 .824
—Darcy.	...	...	...	...	3 .36	0 .858	82 .3	0 .989
	...	...	...	...	23 .89	2 .585	92 .9	1 .128
	...	...	...	...	123 .15	6 .300	99 .8	1 .212
	...	...	...	...	224 .08	8 .521	100 .0	1 .215
Do.	0 .83	31 .0	0 .0833	0 .0208	6258 .6	36 .1	100 .0	1 .240
Rowland.	...	31 .0	...	...	8935 .5	43 .4	100 .6	1 .248
	...	31 .0	...	...	10741 .9	48 .1	101 .7	1 .261
	...	97 .0	0 .0833	0 .0208	2000 .0	19 .9	97 .5	1 .209
	...	...	...	...	2855 .6	24 .5	100 .5	1 .247
	...	...	...	...	3432 .9	27 .2	101 .7	1 .261
IV. Pipes Coated with Tar.								
New cast-iron pipe.	0 .66	415	0 .335	0 .084	1 .98	1 .0	79 .0	1 .0
— Iben. 1876.	...	...	...	...	4 .11	1 .70	92 .0	1 .164
Quoted by Kutter.	...	...	...	...	6 .56	2 .10	90 .0	1 .139
	...	...	...	...	7 .83	2 .30	90 .0	1 .139
	...	...	...	...	11 .07	2 .80	91 .0	1 .152
Do.	0 .56	1093	0 .50	0 .125	4 .59	2 .00	82 .0	1 .080
	...	...	...	...	11 .62	3 .30	87 .0	1 .144
	...	...	...	...	16 .21	3 .90	88 .0	1 .156
	...	...	...	...	22 .32	4 .80	92 .0	1 .210
	...	...	...	...	30 .27	5 .30	87 .0	1 .144
Do.	0 .65	1795	1 .001	0 .25	1 .46	1 .60	85 .0	0 .944
	...	...	...	...	1 .830	2 .10	97 .0	1 .080
	...	...	...	...	2 .19	2 .60	112 .0	1 .244
	...	...	...	...	3 .84	3 .80	121 .0	1 .344
	...	...	...	...	6 .03	4 .80	125 .0	1 .388
Do.	0 .69	3514	1 .667	0 .417	0 .12	0 .70	105 .0	1 .077
	...	...	...	...	0 .48	1 .60	110 .0	1 .125
	...	...	...	...	0 .76	1 .90	109 .0	1 .115
	...	...	...	...	1 .21	2 .50	109 .0	1 .115
V. Asphalt-Coated, Wrought and Cast-Iron Pipe. New.								
Asphalt-coated	0 .89	60	0 .0875	0 .0218	26 .93	2 .22	91 .6	1 .095
wrought-iron pipe	...	...	...	...	52 .19	3 .224	95 .5	1 .140
with funnel mouth-	...	...	...	...	103 .38	4 .761	100 .2	1 .189
piece. — Hamilton	...	...	...	...	130 .64	5 .443	101 .9	1 .205
Smith.								
Asphalt-coated cast-	0 .90	366	0 .4495	0 .1124	0 .24	0 .489	94 .1	0 .960
iron pipe.—Darcy.	...	...	...	...	4 .25	2 .503	108 .4	1 .107
	...	...	...	...	22 .25	5 .623	112 .5	1 .150
	...	...	...	...	98 .52	11 .942	113 .5	1 .160
	...	...	...	...	167 .56	15 .397	112 .2	1 .150

Table X. — *Continued.*

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Asphalt-coated pipe five years old, in good condition. — Lampe.	0 .83	26000	1 .373	0 .343	0 .594	1 .577	110 .5	1 .072
	...	...	...	...	1 .376	2 .479	114 .1	1 .107
	...	...	...	...	1 .63	2 .709	114 .6	1 .112
	...	...	...	...	1 .95	3 .090	119 .4	1 .162
Cast-iron pipe. — Darcy.	0 .81	365	0 .6168	0 .1542	3 .68	2 .487	104 .4	1 .107
	...	...	...	...	22 .50	6 .342	107 .7	1 .142
	...	...	...	...	109 .80	14 .183	109 .0	1 .155
	...	...	...	...	145 .91	16 .168	107 .8	1 .142
Asphalt-coated pipe, four years in use. — Ehmann.	0 .80	810	0 .662	0 .166	0 .367	0 .73	92 .7	0 .984
	...	...	...	...	0 .850	1 .12	94 .7	1 .01
	...	...	...	...	1 .332	1 .45	97 .9	1 .039
	...	...	...	...	1 .883	1 .69	96 .0	1 .017
Asphalt-coated cast-iron pipe. — Hubbel and Fenkel.	0 .83	...	1 .0	0 .25	...	1 .0	101 .5	1 .0
	...	...	...	...	...	2 .0	109 .6	1 .08
	...	...	...	...	...	3 .0	114 .6	1 .13
	...	...	...	...	...	4 .0	118 .3	1 .166
	...	...	...	...	...	5 .0	121 .5	1 .196
Cast-iron pipe. — Darcy.	0 .77	365	1 .6404	0 .41	0 .45	1 .472	108 .4	1 .047
	...	...	...	...	1 .20	2 .602	117 .3	1 .135
	...	...	...	...	2 .10	3 .416	116 .4	1 .126
	...	...	...	...	2 .60	3 .674	112 .5	1 .090
Cast-Iron Force Main. Large number of summits, angles and curves, amongst which there are four right angles and ten quadrants of 30 ft. radius. — Brush.	0 .80	75000	1 .667	0 .417	0 .733	2 .0	114 .4	1 .08
	...	...	...	...	0 .880	2 .24	117 .0	1 .105
	...	...	...	...	1 .026	2 .36	114 .1	1 .080
	...	...	...	...	1 .187	2 .52	113 .3	1 .071
	...	...	...	...	1 .333	2 .68	113 .7	1 .071
	...	...	...	...	1 .493	2 .76	110 .6	1 .045
	...	...	...	...	1 .64	2 .92	111 .7	1 .055
	...	...	...	...	1 .800	3 .0	109 .5	1 .035
								$V^{\frac{1}{18}}$
Asphalt-coated cast-iron pipe. Some easy vertical curves. — Stearns.	0 .97	1747	4 .0	1 .0	0 .318	2 .616	146 .7	
	...	...	...	...	0 .711	3 .738	140 .1	1 .077
	...	...	...	...	1 .221	4 .965	142 .1	1 .093
	...	...	...	...	1 .849	6 .195	144 .1	1 .109

VI. Old Cast and Wrought-Iron Pipes.

Old cast-iron pipe. — Darcy.	0 .52	366	0 .2628	0 .0657	0 .84	0 .458	62 .0	0 .94
	...	...	...	...	7 .25	1 .463	67 .3	1 .04
	...	...	...	...	16 .10	2 .224	68 .7	1 .062
	...	...	...	...	45 .35	3 .777	68 .9	1 .065

TABLE X. — *Continued.*

Description of Conduit.	*m*	*L*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Old cast-iron pipe,	0.85	...	...	...	0.84	0.633	85.2	1.00
cleaned. — Darcy.	...	...	...	...	7.23	2.014	92.4	1.08
	...	...	...	...	15.57	2.835	88.6	1.026
	...	...	...	...	44.73	5.007	93.4	1.092
Old cast-iron pipe.	0.45	365	0.798	0.1995	0.94	1.007	73.6	1.0
— Darcy.	...	...	...	...	4.73	2.32	75.5	1.023
	...	...	...	...	22.90	5.095	75.1	1.02
	...	...	...	...	41.05	6.801	75.2	1.02
	...	...	...	...	139.81	12.576	75.3	1.02
Old cast-iron pipe,	0.45	541	1.0	0.25	2.24	1.79	75.7	1.0
twelve years in use.	...	...	...	...	2.84	2.03	76.2	1.01
Slightly tubercu-	...	...	...	...	...	...	...	...
lated. — Iben.	...	...	...	...	...	...	...	...
Do., two years in use,	0.56	2149	1.0	0.25	0.26	0.60	74.5	0.878
slightly incrusted.	...	...	...	...	0.41	0.80	81.0	0.962
	...	...	...	...	0.81	1.20	85.0	1.01
	...	...	...	...	1.28	1.60	92.0	1.092
	...	...	...	...	2.99	2.40	86.0	1.021
Do., fourteen years in	0.39	7179	1.00	0.25	0.42	0.70	71.0	0.979
use, slightly in-	...	...	...	...	1.65	1.60	78.0	1.075
crusted.	...	...	...	...	4.44	2.70	80.0	1.133
	...	...	...	...	9.43	3.90	80.0	1.133
Do., fifteen years in	0.30	1808	1.00	0.25	0.65	0.90	67	0.991
use. Heavily in-	...	...	...	...	3.76	1.80	58	0.860
crustated.	...	...	...	...	6.12	2.30	59	0.875
	...	...	...	...	7.13	2.60	58	0.860
Do., twenty-two years	0.05	1736	1.00	0.25	1.08	0.80	50	1.041
in use. Very heav-	...	...	...	...	4.29	1.50	44	0.179
ily incrustated.	...	...	...	...	10.91	2.40	45	0.937
	...	...	...	...	23.86	3.50	46	0.958
New asphalt-coated	0.83	...	3	0.75	1.38	4.204	129.4	$V^{\frac{1}{18}}$
cast-iron pipe.	...	...	...	...	1.50	4.234	125.25	...
Rochester, N.Y.—	...	...	...	...	1.50	4.234	125.25	...
E. Kuichling, 1895.	...	...	...	...	...	...	...	...
Do.	0.45	...	...	...	2.27	4.128	91.22	1.0
1897.	...	...	...	...	4.82	4.045	66.84	...
	...	...	...	...	4.85	4.022	66.24	...
Do.	0.13	...	...	...	4.34	4.034	70.23	1.0
1898.	...	...	...	...	3.76	4.026	75.32	...
Do.	0.28	...	...	...	3.44	4.084	79.79	1.0
1899.	...	...	...	...	3.25	4.079	81.93	...

TABLE X. — *Continued.*

Description of Conduit.	m	L	d	R	$1000\,s$	v	c	a
Old cast-iron pipe 16 years old, tubercles removed. — Fitzgerald.	0 .95	...	4	1 .0	0 .4167	3 .723	139 .1	$V^{\frac{1}{18}}$
	...	...	...	...	1 .241	4 .973	141 .1	...
	...	...	...	...	1 .8283	6 .141	143 .6	...
	...	...	...	...	...	...	...	...
Cast iron intake pipe at Erie, Pa., 8 years in use.	0 .48	8215	5	1 .25	...	0 .178	99 .8	...
	...	...	...	...	...	1 .088	102 .1	1 .0
	...	...	...	...	...	...	...	...
Old cast-iron pipe in good condition, some easy bends. — Jas. M. Gale.	0 .70	19600	4	1	0 .947	3 .458	112 .4	1 .0
	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...
VII. GALVANIZED PIPES, GLASS, TIN AND LEAD PIPES.								
New wrought-iron galvanized pipe, straight. — Ehmann.	0 .92	301 .8	0 .0842	0 .021	7 .61	1 .11	87 .1	1 .016
	...	...	...	...	29 .35	2 .13	85 .9	1 .00
	...	...	...	...	113 .04	3 .71	79 .2	1 .035
	...	...	...	...	225 .0	5 .80	84 .5	0 .984
	...	...	...	...	239 .13	5 .90	83 .2	0 .970
Glass pipe with funnel-mouthpiece. — Hamilton Smith.	0 .87	63 .9	0 .0764	0 .0191	25 .01	1 .955	89 .5	1 .078
	...	...	...	...	50 .77	2 .945	94 .6	1 .140
	...	...	...	...	75 .30	3 .685	92 .2	1 .171
	...	...	...	...	102 .6	4 .383	99 .3	1 .199
	...	...	...	...	129 .18	5 .009	100 .8	1 .219
Do., no funnel.	0 .84	63 .9	0 .0764	0 .0191	17 .97	1 .398	83 .6	1 .030
	...	...	...	...	132 .51	4 .373	96 .3	1 .185
Glass pipe, straight. — Darcy.	0 .84	147 .0	0 .163	0 .0407	0 .96	0 .502	80 .3	0 .930
	...	...	...	...	7 .71	1 .591	89 .8	1 .044
	...	...	...	...	57 .62	4 .849	100 .1	1 .164
	...	...	...	...	111 .91	6 .916	102 .4	1 .191
New lead pipe, straight. — Darcy.	0 .84	172	0 .0886	0 .0221	0 .44	0 .213	68 .3	0 .843
	...	...	...	...	8 .14	1 .089	81 .1	1 .00
	...	...	...	...	54 .36	3 .35	96 .5	1 .191
	...	...	...	...	146 .32	5 .509	96 .8	1 .195
New lead pipe, straight. — Darcy.	0 .84	172	0 .1345	0 .0336	0 .82	0 .394	75 .0	0 .90
	...	...	...	...	7 .48	1 .404	86 .8	1 .038
	...	...	...	...	56 .00	4 .318	99 .5	1 .178
	...	...	...	...	158 .82	7 .562	103 .5	1 .238
Tin pipe, straight. — Dubuat.	0 .98	...	0 .0888	0 .0222	0 .196	0 .141	67 .6	0 .746
	...	...	...	...	0 .641	0 .322	85 .3	0 .942
	...	...	...	...	3 .91	0 .772	82 .8	0 .914
	...	...	...	...	5 .39	0 .927	84 .8	6 .937
	...	...	...	...	7 .54	1 .183	91 .4	1 .009
	...	...	...	...	9 .91	1 .342	90 .5	1 .00
	...	...	...	...	13 .7	1 .476	92 .9	1 .015
	...	...	...	...	29 .82	2 .546	98 .9	1 .092
	...	...	...	...	30 .31	2 .606	100 .4	1 .109
	...	...	...	...	99 .01	5 .223	111 .4	1 .120

TABLE X. — *Continued.*

Description of Conduit.	m	L	d	R	$1000\ s$	v	t	a
Tin pipe. Straight. — Bossut.	0.95	192	0.1184	1.0296	5.40	1.116	88.2	1.00
	...	192	"	"	10.76	1.678	94.0	1.068
	...	64	"	"	15.08	2.075	98.2	1.116
	...	32	"	"	26.94	2.946	104.3	1.185
	...	32	"	"	52.98	4.31	108.8	1.234
Do.	0.94	63	0.1184	0.0296	113.4	6.143	106.0	1.220
	...	126	"	"	113.5	6.15	106.1	1.220
	...	189	"	"	113.4	6.157	106.2	1.220

VIII. PIPES AND OPEN CONDUITS OF PLANED OR ROUGH BOARDS.

Description of Conduit.	m	L	d	R	$1000\ s$	v	t	a
Redwood stave pipe. Los Angles, Cal. — A. L. Adams, 1898.	0.93	4–8000	1.166	0.292	0.17	0.698	99	0.908
	...	...	...	...	0.161	0.698	101	0.926
	...	...	...	...	0.178	0.751	104	0.953
	...	...	...	...	0.145	0.691	105	0.963
	...	...	...	...	0.391	1.167	109	1.01
	...	...	...	...	0.638	1.531	112	1.027
	...	...	...	...	1.355	1.181	113	1.043
Do. At Astoria, Ore. — A. L. Adams.	0.96	4188	1.5	0.375	2.07	3.605	132.9	$V^{\frac{1}{9}}$
	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...
Wooden stave pipe, at Cedar River, Wash. Long but easy curves. Several years in use. Some slight deposits.—Theron A. Noble.	0.50	...	3.67	0.917	1.067	3.468	110.1	$V^{\frac{1}{9}}$
	...	...	...	...	1.134	3.522	108.6	...
	...	...	...	...	1.191	3.685	110.9	...
	...	...	...	...	1.262	3.853	112.6	...
	...	...	...	...	1.33	3.964	112.9	...
	...	...	...	...	1.331	3.972	113.1	...
	...	...	...	...	1.401	4.072	112.9	..
	...	...	...	...	1.627	4.415	113.7	...
	...	...	...	...	1.757	4.595	113.8	...
	...	...	...	...	1.757	4.635	114.8	...
	...	...	...	...	1.888	4.831	115.6	...
Do.	0.58	...	4.5	1.125	0.342	2.282	116.8	$V^{\frac{1}{9}}$
	...	...	...	...	0.342	2.276	115.8	...
	...	...	...	...	0.436	2.65	119.4	...
	...	...	...	...	0.558	3.07	122.1	...
	...	...	...	...	0.557	3.05	121.4	...
	...	...	...	...	0.672	3.41	123.7	...
	...	...	...	...	0.783	3.724	125.2	...
	...	...	...	...	0.856	3.924	126.2	...
	...	...	...	...	0.983	4.215	126.5	...
	...	...	...	...	1.076	4.42	126.7	...
	...	...	...	...	1.162	4.69	129.2	...

TABLE X. — *Continued.*

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Wooden stave pipe	0.51	4.000	6.0	1.5	...	1.40	110.5	$V^{\frac{1}{9}}$
at Ogden, Utah.	...	...	...	...	...	1.68	112.5	...
Many easy curves,	...	...	...	...	...	2.14	115.0	...
two years in use.	...	...	...	...	...	2.43	119.5	...
— Marx-Wing and	...	...	...	...	...	2.96	122.5	..
Hoskins.	...	...	...	...	...	3.59	126.5	...
	...	...	...	...	...	3.63	124.5	...
Rectangular pipes	0.68	145.7	...	0.319	0.523	1.23	94.3	$V^{\frac{1}{18}}$
of unplaned board.	...	...	...	...	1.067	1.778	96.4	...
— Darcy.	...	...	...	...	1.933	2.267	96.8	...
	...	...	...	...	2.733	2.939	99.5	...
	...	...	...	...	3.867	3.529	100.5	...
	...	...	...	...	6.267	4.349	97.3	...
	...	...	...	...	7.267	4.625	96.1	...
	...	...	...	...	8.80	5.307	100.2	...
Rectanglar pipe of	0.73	230	...	0.505	0.475	1.67	107.6	$V^{\frac{1}{18}}$
unplaned boards.	...	...	...	...	1.076	2.52	108.1	..
—Darcy.	...	...	...	...	1.90	3.37	108.9	...
	...	...	...	...	2.91	4.23	110.2	...
	...	...	...	...	4.27	5.07	109.1	...
	...	...	...	...	5.06	5.52	109.3	...
	...	...	...	...	5.76	5.91	109.7	...
	...	...	...	...	6.61	6.37	110.3	...
Provo Canal Flume,	0.85	...	...	1.45	1.0	5.67	147.7	$V^{\frac{1}{12}}$
Utah. Semicircu-	0.81	...	...	1.46	1.0	5.37	141.8	$V^{\frac{1}{12}}$
lar conduit of	...	...	...	...	...	...	...	...
planed staves,	...	...	...	...	...	...	...	...
several years in	...	...	...	...	...	...	...	...
use.—W. B. Mc-	...	...	...	...	...	...	...	...
Dougall.	...	...	...	...	...	...	...	...
		W						
Wooden trough,	0.71	...	0.24	0.159	34.3	8.26	111.9	$V^{\frac{1}{8}}$
trapezoidal. Bot-	...	...	0.26	0.173	"	8.21	106.6	...
tom width 10.4	...	...	0.38	0.237	"	10.11	112.1	...
ft. Rittinger.	...	...	0.41	0.246	"	10.64	115.8	...
Rectangular test	0.94	0.328	0.04	0.029	4.7	0.90	76.5	$V^{\frac{1}{18}}$
channel of planed	...	...	0.08	0.052	"	1.30	83.0	...
boards. — Darcy-	...	...	0.11	0.066	"	1.58	89.4	...
Bazin. Series 28.	...	...	0.14	0.075	"	1.74	92.7	...
	...	...	0.17	0.084	"	1.94	97.6	...
	...	...	0.20	0.091	"	2.11	102.1	...
	...	...	0.22	0.093	"	2.16	103.2	...

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	$1000\,s$	v	c	a
Semicircular test	0.70	3.16	0.63	0.39	1.5	2.61	107.8	$V^{\frac{1}{12}}$
channel of un-	...	3.62	0.88	0.537	"	3.23	113.8	...
planed boards.	...	3.89	1.07	0.632	"	3.71	120.6	...
— Darcy-Bazin.	...	4.08	1.24	0.717	"	4.04	123.0	...
Series 26.	...	4.24	1.40	0.796	"	4.25	123.2	...
	...	4.33	1.55	0.856	"	4.51	125.8	...
	...	4.43	1.68	0.926	"	4.64	124.7	...
	...	4.48	1.79	0.964	"	4.87	128.2	...
	...	4.53	1.92	1.005	"	5.0	128.2	...
	...	4.56	2.02	1.054	"	5.18	130.3	...
	...	4.59	2.14	1.096	"	5.29	130.4	...
	...	4.59	2.24	1.129	"	5.45	132.3	...
	...	4.59	2.29	1.148	"	5.54	133.5	...
Rectangular test	0.66	6.53	0.26	0.24	2.08	2.08	93.2	$V^{\frac{1}{18}}$
channel of un-	...	...	0.41	0.363	"	2.69	97.8	...
planed boards. —	...	...	0.53	0.453	"	3.16	102.8	...
Darcy-Bazin.	...	...	0.63	0.528	"	3.53	106.5	...
Series 6.	...	...	0.73	0.601	"	3.78	106.9	...
	...	...	0.81	0.648	"	4.13	112.5	...
	...	...	0.90	0.704	"	4.34	113.5	...
	...	...	0.99	0.759	"	4.51	113.5	...
	...	...	1.06	0.801	"	4.72	115.8	...
	...	...	1.14	0.846	"	4.88	116.3	...
	...	...	1.20	0.880	"	5.09	119.0	...
	...	...	1.28	0.992	"	5.21	118.9	...
Do.	0.70	6.53	0.20	0.188	4.9	2.71	89.3	$V^{\frac{1}{18}}$
Series 7.	...	...	0.30	0.272	"	3.70	101.2	...
	...	...	0.38	0.342	"	4.35	106.2	...
	...	...	0.46	0.402	"	4.85	109.4	...
	...	...	0.53	0.453	"	5.29	112.2	...
	...	...	0.60	0.504	"	5.61	113.0	...
	...	...	0.66	0.547	"	5.93	114.5	...
	...	...	0.72	0.587	"	6.23	116.1	...
	...	...	0.78	0.628	"	6.45	116.4	...
	...	...	0.83	0.662	"	6.71	117.8	...
	...	...	0.89	0.698	"	6.90	117.9	...
	...	...	0.94	0.727	"	7.15	119.8	...
Rectangular test	0.71	6.53	0.15	0.147	8.24	3.52	100.4	$V^{\frac{1}{18}}$
channel of un-	...	...	0.25	0.231	"	4.42	101.4	...
planed boards.	...	...	0.32	0.289	"	5.23	107.1	...
Darcy-Bazin.	...	...	0.38	0.341	"	5.83	109.8	...
Series 8.	...	...	0.45	0.393	"	6.24	109.7	...
	...	...	0.50	0.431	"	6.74	113.1	...
	...	...	0.54	0.466	"	7.07	115.8	...
	...	...	0.60	0.506	"	7.44	115.2	...
	...	...	0.65	0.541	"	7.73	115.8	...
	...	...	0.69	0.572	"	8.03	116.9	...
	...	...	0.74	0.604	"	8.26	117.1	...
	...	...	0.78	0.630	"	8.57	119.0	...

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Triangular test channel of unplaned boards. — Darcy-Bazin. Series 23.	0.69	1.85	0.92	0.327	4.9	4.13	103.1	$V^{\frac{1}{18}}$
	...	2.39	1.19	0.422	"	5.02	110.4	...
	...	2.79	1.40	0.494	"	5.56	113.0	...
	...	3.10	1.55	0.549	"	6.03	116.2	...
	...	3.38	1.69	0.597	"	6.36	117.6	...
	...	3.64	1.82	0.643	"	6.59	117.3	...
	...	3.86	1.93	0.683	"	6.83	118.0	...
	...	4.07	2.03	0.719	"	7.03	118.4	...
	...	4.26	2.13	0.752	"	7.23	119.0	...
	...	4.43	2.22	0.783	"	7.40	119.5	...
	...	4.61	2.30	0.814	"	7.54	119.4	...
	...	4.75	2.37	0.839	"	7.75	120.9	...
Flume of Kern River Power Plant No. 1. Plain boards, seams covered with ½ inch battens. Sect. rect. Length 1029.6 feet. Tunnel lined with cement plaster, 1 cement to 2 sand of same section and slope below flume. — F. C. Finkle.	0.72	8.0	2.5	1.538	1.5	6.681	139.1	$V^{\frac{1}{18}}$
	...	...	3.0	1.714	"	6.968	137.4	...
	...	...	3.5	1.866	"	7.394	139.8	...
	...	...	4.0	2.00	"	7.920	144.8	...

IX. PIPES AND OPEN CONDUITS OF CEMENT OR CONCRETE.

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Cement lined pipe of of wrought iron. Three-stop valves and two large branches on line. — Fanning.	0.83	L 8171	1.667	0.416	0.23	0.949	97.4	0.92
	...	...	...	...	0.44	1.488	109.8	1.046
	...	...	...	...	0.73	1.925	110.7	1.054
	...	...	...	...	1.04	2.329	112.0	1.066
	...	...	...	...	1.34	2.598	110.1	1.046
	...	...	...	...	1.58	2.867	111.7	1.063
	...	...	...	...	1.99	3.271	113.5	1.081
	...	...	...	...	2.28	3.439	111.7	1.063
	...	...	...	...	2.72	3.741	111.1	1.056
	...	...	...	...	3.0	3.920	110.8	1.052
	...	...	...	...	3.20	4.040	110.6	1.051
Test pipe of clear cement. Diameter 0.8 meter. — Dijon. Quoted by Bazin.	0.95	...	2.624	0.656	0.625	2.78	137.1	$V^{\frac{1}{12}}$
	...	...	...	...	1.05	3.65	139.2	1.139
	...	...	...	...	1.375	4.20	139.5	1.140
	...	...	...	...	1.725	4.72	140.4	1.141
	...	...	...	...	1.75	4.79	141.2	1.155
	...	...	...	...	1.88	4.92	141.4	1.157
	...	...	...	...	2.57	5.81	141.4	1.157
	...	...	...	...	3.27	6.58	142.5	1.166

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	$1000\,s$	v	c	a
Semicircular test-channel of clear cement. — Darcy-Bazin. Series 24.	1.0	2.874	0.59	0.366	1.5	3.02	128.9	$V^{\frac{1}{12}}$
	...	3.294	0.83	0.503	"	3.82	135.6	...
	...	3.563	1.03	0.605	"	4.16	138.0	...
	...	3.707	1.18	0.682	"	4.60	143.7	...
	...	3.832	1.34	0.750	"	4.87	145.1	...
	...	3.924	1.47	0.809	"	5.12	147.1	...
	...	3.97	1.61	0.867	"	5.29	146.7	...
	...	4.05	1.72	0.915	"	5.51	148.8	...
	...	4.075	1.83	0.949	"	5.75	152.5	...
	...	4.095	1.94	0.992	"	5.91	153.3	...
	...	4.101	2.05	1.029	"	6.06	154.2	...
	...	4.16	2.08	1.034	"	6.11	155.1	...
Rectangular test channel of clear cement. — Darcy-Bazin. Series 2.	0.95	5.94	0.18	0.168	4.9	3.34	116.5	$V^{\frac{1}{18}}$
	...	...	0.28	0.251	"	4.39	125.1	...
	...	...	0.36	0.322	"	5.04	126.9	...
	...	...	0.43	0.375	"	5.68	132.4	...
	...	...	0.56	0.43	"	6.06	132.4	...
	...	...	0.56	0.475	"	6.51	135.1	...
	...	...	0.63	0.518	"	6.83	135.5	...
	...	...	0.69	0.558	"	7.12	136.2	...
	...	...	0.76	0.595	"	7.41	137.2	...
	...	...	0.80	0.632	"	7.63	137.2	...
	...	...	0.86	0.665	"	7.86	137.8	...
	...	...	0.91	0.696	"	8.07	138.2	...
Sudbury conduit. Plaster of pure cement over brick work. Sides nearly vertical, bottom flat segmental arch. — Fteley & Stearns.	0.94	8.6	3.071	1.863	0.1606	2.529	146.2	$V^{\frac{1}{18}}$
	...	...	3.574	2.048	0.1596	2.672	147.9	...
	...	...	3.768	2.111	0.1580	2.805	153.9	...
Aqueduct of the Serino, Naples. Pure cement, polished. Sides vertical, bottom ellyptical arch. — Perrone, 1896.	0.98	5.38	...	1.41	0.50	4.06	152.5	$V^{\frac{1}{12}}$

TABLE X. — *Continued.*

Description of Conduit.	*m*	*W*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Semicircular test channel of cement mortar. Two-thirds cement, one-third fine sand. — Darcy-Bazin. Series 25.	0 .85	2 .913	0 .61	0 .379	1 .5	2 .89	120 .5	$V^{\frac{1}{12}}$
	...	3 .36	0 .88	0 .529	“	3 .43	122 .0	...
	...	3 .616	1 .09	0 .635	“	3 .89	125 .2	...
	...	3 .760	1 .24	0 .706	“	4 .30	132 .1	...
	...	3 .891	1 .41	0 .787	“	4 .51	131 .3	...
	...	3 .963	1 .54	0 .839	“	4 .80	135 .3	...
	...	4 .029	1 .69	0 .900	“	4 .94	134 .5	...
	...	4 .068	1 .80	0 .941	“	5 .20	138 .3	...
	...	4 .088	1 .92	0 .983	“	5 .38	140 .1	...
	...	4 .095	1 .98	1 .006	“	5 .48	141 .0	...
	...	4 .095	2 .04	1 .022	“	5 .55	141 .7	...
	...	4 .095	2 .09	1 .038	“	5 .56	143 .5	...
Conduit of North Metropolitan Sewage System, East Boston section. Brickwork washed with cement. Section circular, diameter 9 ft.— Th. Horton. Experiments of 1896. 10 months in use.	0 .83	...	1 .02	0 .619	0 .333	1 .58	110	$V^{\frac{1}{12}}$
	...	...	1 .52	0 .928	“	2 .21	126	...
	...	...	2 .04	1 .208	“	2 .70	134	...
	...	...	2 .45	1 .408	“	3 .03	139	...
	...	...	3 .16	1 .830	“	3 .48	141	...
	...	...	3 .75	1 .999	“	3 .73	145	...
	...	...	4 .62	2 .31	“	4 .18	150	...
Do. Experiments of 1897.	0 .59	...	2 .15	1 .28	0 .333	2 .55	123	$V^{\frac{1}{12}}$
	...	...	2 .74	1 .56	“	2 .90	127	...
	...	...	3 .19	1 .76	“	3 .06	126	...
	...	...	3 .20	1 .97	“	3 .18	131	...
Do. Experiments of 1900.	0 .56	...	1 .99	1 .12	0 .333	2 .38	119	$V^{\frac{1}{12}}$
	...	...	2 .83	1 .61	“	2 .82	121	...
	...	...	3 .64	1 .95	“	3 .16	124	...
	...	...	4 .18	2 .13	“	3 .30	124	...
Do. Charlestown section. Sides vertical, bottom flat arch. Experiments of 1896. 10 months in use.	0 .65	6 .0	1 .02	0 .688	0 .50	1 .99	107	$V^{\frac{1}{18}}$
	...	...	1 .44	0 .958	“	2 .46	112	...
	...	...	1 .91	1 .187	“	2 .825	115	...
	...	...	2 .40	1 .387	“	3 .33	118	...
	...	...	2 .89	1 .539	“	3 .44	124	...
Do. Experiments of 1897.	0 .40	...	2 .91	1 .54	0 .5	2 .97	107	$V^{\frac{1}{18}}$
	...	...	3 .29	1 .64	“	3 .16	111	...

TABLE X. —*Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Conduit of North Metropolitan Sewage System, Charlestown section (continued). Experiments of 1900.	0.33	...	2.29	1.34	0.50	2.66	102	$V^{\frac{1}{18}}$
	...	...	2.78	1.51	"	2.86	104	...
	...	...	3.20	1.64	"	3.04	106	...
Aqueduct of Glasgow. Smooth concrete. Nearly rectangular, bottom flat arch.—Fairlie Bruce. 1896.	0.80	9.6	...	1.22	0.182	1.67	125.0	$V^{\frac{1}{18}}$
	...	...	...	1.47	"	2.07	126.3	...
	...	...	...	1.47	"	2.10	128.5	...
	...	...	...	1.49	"	2.21	134.5	...
	...	...	...	1.50	"	2.13	129.3	...
	...	...	...	1.50	"	2.15	130.3	...
	...	...	...	1.55	"	2.17	129.4	...
	...	...	...	1.60	"	2.20	129.3	...
	...	...	...	1.61	"	2.23	130.5	...
	...	...	...	1.61	"	2.22	129.7	...
	...	...	...	1.62	"	2.24	130.6	...
	...	...	...	1.63	"	2.25	130.8	...
	...	...	...	1.74	"	2.26	126.9	...
	...	...	...	1.81	"	2.41	135.4	...
Millrace at Idria, Hungary. Cement mortar on rubble masonry. — Rittinger.	0.65	...	2.04	0.977	0.5	2.523	114.1	$V^{\frac{1}{18}}$
Aqueduct of Roquefavour, canal of Marseilles. Bottom of clear cement, sides of good brick work. Rectangular.—Baumgarten.	0.72	6.88	...	1.504	3.72	10.26	137	$V^{\frac{1}{18}}$
Aqueduct of the Cervo, Canal Cavour. Bottom of good concrete, sides of brick. Rectangular. — Passini & Gioppi, 1892.	0.58	66.0	...	5.12	0.11	3.52	148.5	$V^{\frac{1}{18}}$
	...	...	...	5.76	"	3.76	149.3	...
	...	...	...	7.20	"	4.38	155.8	...

TABLE X. — *Continued.*

Description of Conduit.	*m*	*W*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Gage Canal. San Bernardino, Cal. Channels in earth, roughly coated with cement plaster, 1 part cement, 4 parts sand.— U.S. Geol. Survey.	0.49 0.48 0.46 0.48 0.44 0.47	9.25 10.25 14.0 12.25 16.0 17.0	3.5 3.5 3.5 5.5 3.5 3.5	1.82 1.94 2.13 2.13 2.38 2.38	0.4 0.4 0.478 0.382 0.520 0.413	3.14 3.28 3.78 3.38 4.24 3.78	117 117 119 119 120 121	$V^{\frac{1}{18}}$ ... ...
Canal of Verona. Channel lined with Beton masonry. Trapezoidal. Bottom width about 20 ft.	0.00	...	...	5.12	0.31	4.2	107.8	$V^{\frac{1}{18}}$
San Gabriel Tunnel No. 15. Coated with cement mortar, 1 cement to 3 sand. Section rect. L. 446 ft. — Lippincott.	0.89	4.5	4.0	1.31	0.96	5.01	141.3	$V^{\frac{1}{18}}$
San Gabriel Tunnel No. 23. L. 318 ft.	0.89	...	...	1.37	0.86	4.74	141.6	$V^{\frac{1}{18}}$
Old Aqueduct of Los Angeles. Coated with cement plaster on concrete. 4 years in use. Covered.	0.95 0.90	... ...	... ...	0.817 0.830	0.51 0.51	2.71 2.81	132.6 136.7	$V^{\frac{1}{18}}$ ...
Colton Canal. Channel lined with concrete. Bottom clean, sides lightly coated with moss.	0.26	...	...	0.98	20.70	2.27	86.7	$V^{\frac{1}{18}}$
Santa Ana Canal. Channel lined with concrete. Bottom covered with sand and gravel, sides coated with plants in places.	0.33	...	...	0.817	1.06	2.62	89.2	$V^{\frac{1}{18}}$

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Riverside canal. Open conduit coated with cement mortar on concrete. Bottom covered with fine sand to a depth of 1.5 to 2.5 ft. Trapezoidal.	–0.27	...	...	1.49	0.92	1.96	52.9	$\frac{1}{V^{\frac{1}{18}}}$
	–0.08	...	...	0.703	0.63	1.22	38.0	...
X. BRICK CONDUITS.								
Sudbury Conduit. Hard glazed brick, smoothly jointed, fairly clean. Sides nearly vertical, bottom flat arch. Fteley & Stearns.	0.80	9.0	4.672	2.359	0.0334	1.207	136.0	$V^{\frac{1}{18}}$
	...	...	4.972	2.417	0.0488	1.497	137.9	...
	...	...	3.319	1.963	0.0625	1.512	136.5	...
	...	...	2.561	1.648	0.0948	1.616	129.3	...
	...	...	2.998	1.838	0.1155	1.983	136.1	...
	...	...	3.369	1.981	0.1356	2.255	137.6	...
	...	...	2.192	1.468	0.1466	1.931	131.6	...
	...	...	4.602	2.343	0.1793	2.889	141.0	...
	...	...	3.878	2.151	0.2102	2.955	139.0	...
	...	...	3.266	1.943	0.2389	2.957	137.3	...
	...	...	1.799	1.251	0.2553	2.448	137.0	...
	...	...	2.245	1.495	0.2580	2.687	138.5	...
	...	...	2.707	1.714	0.2602	2.886	136.6	...
	...	...	2.881	1.789	0.4604	4.163	142.9	...
	...	...	3.437	2.005	0.4913	4.913	140.8	...
New Croton Aqueduct, New York. Good brickwork. Sides nearly vertical, bottom flat arch. — Fteley, 1895.	0.68	13.6	...	0.75	0.1326	1.1	110.4	$V^{\frac{1}{18}}$
	...	...	...	1.0	"	1.37	118.9	...
	...	...	...	1.25	"	1.59	123.0	...
	...	...	...	1.50	"	1.80	127.4	...
	...	...	...	1.75	"	1.94	128.3	...
	...	...	...	2.0	"	2.1	129.2	...
	...	...	...	2.25	"	2.27	131.2	...
	...	...	...	2.50	"	2.40	132.1	...
	...	...	...	2.75	"	2.52	132.1	...
	...	...	...	3.0	"	2.65	133.0	...
	...	...	...	3.50	"	2.89	134.0	...
	...	...	...	3.81	"	3.02	134.0	...
Rectangular test channel of common brickwork, rather rough. — Darcy-Bazin. Series 3.	0.57	6.27	0.20	0.192	4.9	2.75	89.7	$V^{\frac{1}{18}}$
	...	...	0.31	0.284	"	3.66	98.3	...
	...	...	0.41	0.365	"	4.18	98.8	...
	...	...	0.49	0.424	"	4.72	103.7	...
	...	...	0.57	0.481	"	5.10	105.1	...
	...	...	0.65	0.540	"	5.33	103.7	...
	...	...	0.71	0.580	"	5.68	106.3	...
	...	...	0.77	0.620	"	6.01	109.0	...
	...	...	0.85	0.668	"	6.15	107.4	...
	...	...	0.90	0.697	"	6.47	110.8	...
	...	...	0.97	0.739	"	6.60	109.7	...
	...	...	1.04	0.779	"	6.72	108.7	...

TABLE X. — *Continued.*

Description of Conduit.	m	L	d	R	1000 s	v	c	a
Brick sewer at Milwaukee, Wis. Smooth brick, well pointed. — G. H. Benzenberg.	0 .57	2534	12 .0	3 .0	0 .523	5 .083	128 .3	1 .0
	...	...	...	...	0 .547	5 .043	125 .1	...
	...	...	...	...	0 .814	5 .154	124 .5	...
	...	...	...	...	0 .821	6 .195	124 .9	...
	...	...	...	...	0 .793	6 .207	127 .9	...
	...	...	...	...	1 .046	6 .886	122 .9	...
	...	...	...	...	1 .046	6 .872	122 .7	...
	...	...	...	...	1 .040	6 .961	124 .3	...
	...	...	...	...	1 .010	6 .821	124 .2	...
Brick sewer, Dorchester Bay Tunnel. Inverted syphon. Hard brick, well pointed. Sewer slime. — Clarke, 1895.	0 .47	7166	7 .5	1 .875	0 .513	3 .769	121 .0	1 .0
	...	...	...	...	0 .554	3 .798	118 .0	...
	...	...	...	...	0 .581	3 .929	119 .0	...
Syphon Aqueduct of the Elvo, Canal Cavour. This conduit consists of 5 oval tubes, each having a cross section of 119.25 sq. ft. — Passini & Gioppi, 1892.	0 .40	581 .5	...	2 .78	0 .067	1 .61	117 .3	...
	...	...	...	...	0 .107	1 .95	112 .9	...
	...	...	...	...	0 .152	2 .31	112 .2	1 .0
	...	...	...	...	0 .208	2 .66	111 .6	...
	...	...	...	...	0 .276	3 .1	111 .8	...
	...	...	...	...	0 .361	3 .53	111 .6	...
	...	...	...	...	0 .462	4 .01	111 .8	...
	...	...	...	...	0 .586	4 .52	112 .0	...
	...	...	...	...	.733	5 .094	113 .1	...

XI. CHANNELS LINED WITH ASHLAR OR RUBBLE MASONRY.

		W						
Chazilly Canal. Ashlar masonry, smoothly dressed, section trapezoidal, very regular. — Darcy-Bazin. Series 39.	0 .57	4 .04	0 .50	0 .41	8 .1	5 .73	100 .0	$V^{\frac{1}{18}}$
	...	4 .10	0 .78	0 .57	"	7 .52	111 .0	...
	...	4 .14	1 .0	0 .68	"	8 .19	110 .0	...
	...	4 .18	1 .20	0 .77	"	8 .75	111 .0	...
Aqueduct of Crau, Canal of Craponne. Ashlar masonry, smoothly dressed. Section rectangular. — Darcy-Bazin.	0 .59	8 .5	3 .0	1 .774	0 .84	5 .55	125 .0	$V^{\frac{1}{18}}$

TABLE X. — *Continued.*

Description of Conduit.	*m*	*W*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Solani Aqueduct. Ganges Canal, India. Rectangular conduit consisting of two sections, separated by a central wall, length 920 ft. Floor of brick, laid flat, sides of masonry. Some deposits here and there. Right section. — Allan Cunningham, 1880.	0.44	85.0	2.66	2.52	0.151	2.20	112.8	1.0
	...	...	2.88	2.72	0.145	2.54	117.9	1.04
	...	...	3.13	2.94	0.20	2.51	103.5	0.90
	...	...	3.12	2.94	0.208	2.79	112.8	0.984
	...	...	3.18	2.99	0.253	3.20	116.4	1.016
	...	...	3.96	3.65	0.473	4.83	116.2	1.0
	...	...	4.60	4.20	0.025	1.24	121.0	0.99
New and well built channel of dry rubble masonry of large stones. Semicircular.— Kutter, 1867.	0.32	6.0	0.50	0.32	42.35	9.45	80.5	$V^{\frac{1}{18}}$
	...	...	...	0.32	46.42	10.49	85.4	...
	...	...	0.57	0.37	42.35	9.89	82.5	...
	...	...	...	6.37	46.42	10.97	83.6	...
Old channel of dry rubble masonry of large stones, bed somewhat damaged. Semicircular. — Kutter, 1867.	0.27	8.0	0.55	0.36	82.8	11.81	68.6	1.0
	...	8.0	0.55	0.38	99.3	13.32	68.4	"
	...	7.4	0.60	0.39	106.8	13.75	67.3	"
	...	10.6	0.90	0.58	82.8	15.54	70.6	"
	...	10.6	0.90	0.61	99.3	18.28	72.8	"
	...	9.0	1.0	0.65	106.8	19.17	72.8	0.97
Spillway of Grosbois Reservoir. Ashlar with cement joints, partly damaged, covered with a sticky slime. Rectangular. — Darcy-Bazin. Series 32.	0.15	5.98	0.36	0.324	101.0	12.29	67.9	$V^{\frac{1}{18}}$
	...	6.01	0.55	0.467	"	16.18	74.5	...
	...	6.05	0.71	0.580	"	18.68	77.2	...
	...	6.07	0.84	0.662	"	21.09	81.6	...
Do. Tailrace of Grosbois Reservoir. — Darcy-Bazin. Series 33.	0.16	6.0	0.49	0.424	37.0	9.04	72.2	$V^{\frac{1}{18}}$
	...	6.1	0.77	0.620	"	11.46	75.7	...
	...	6.1	0.97	0.745	"	13.55	81.6	...
	...	6.1	1.16	0.846	"	15.08	84.9	...
Tailrace of dry rubble masonry, paved, semicircular.— Rittinger, 1855. Quoted by Kutter.	–0.03	...	0.42	0.289	2.5	1.257	46.8	1.012
	...	...	0.56	0.359	"	1.491	49.8	1.015
	...	...	0.69	0.419	"	1.643	50.8	1.010

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Aqueduct of dry rubble masonry, paved. Rectangular. Rittinger.	–0 .03	...	0 .24	0 .213	4 .5	1 .324	42 .8	1 .012
	...	...	0 .65	0 .439	"	2 .396	53 .9	1 .056
	...	...	0 .81	0 .486	"	2 .432	52 .0	1 .0
Do. Tailrace of dry rubble masonry paved. Trapezoidal. Ritt.	–0 .02	...	0 .35	0 .278	3 .6	1 .502	47 .5	1 .022
	...	...	0 .47	0 .351	"	1 .928	54 .2	1 .08
	...	...	0 .56	0 .403	"	2 .104	55 .2	1 .041
Headrace of dry rubble masonry paved. Trapezoidal. — Rittinger.	0 .04	...	0 .26	0 .213	3 .8	1 .369	48 .1	1 .012
	...	...	0 .45	0 .344	"	1 .828	50 .6	0 .988
Grosbois Canal. Channel of roughly hammered stone masonry. — Darcy-Bazin. Series 1.	0 .15	3 .9	1 .6	0 .88	12 .1	9 .58	73 .5	1 .0
	...	3 .6	1 .5	0 .84	14 .0	8 .36	77 .3	1 .06
	...	3 .5	1 .2	0 .71	29 .0	11 .23	78 .4	1 .12
	...	3 .5	0 .9	0 .62	60 .0	13 .93	72 .5	1 .055
Do. Masonry in bad condition, mud and stones in bed. — Darcy-Bazin. Series 46.	–0 .02	6 .8	1 .5	0 .88	0 .648	1 .47	62	1 .02
	...	6 .9	2 .0	1 .23	0 .671	2 .02	70	1 .06
	...	6 .9	2 .4	1 .40	0 .683	2 .34	76	1 .02
	...	7 .0	2 .4	1 .42	0 .683	2 .78	87	1 .024
XII. CHANNELS LINED WITH PEBBLES HELD IN PLACE WITH CEMENT.								
Semicircular test-channel lined with pebbles ⅜ to ⅛-inch diameter, held in place with cement. — Darcy-Bazin. Series 27.	0 .38	3 .1	0 .7	0 .454	1 .5	2 .17	83 .1	$V^{\frac{1}{29}}$
	...	3 .4	0 .9	0 .546	"	2 .50	89 .4	...
	...	3 .5	1 .1	0 .619	"	2 .69	88 .2	...
	...	3 .7	1 .2	0 .681	"	2 .93	89 .5	...
	...	3 .8	1 .3	0 .731	"	3 .05	92 .1	...
	...	3 .8	1 .4	0 .784	"	3 .22	93 .9	...
	...	3 .9	1 .5	0 .826	"	3 .33	94 .6	
	...	4 .0	1 .7	0 .900	"	3 .54	96 .3	...
	...	4 .0	1 .9	0 .968	"	3 .73	97 .9	...
	...	4 .0	2 .0	1 .012	"	3 .95	102 .1	...
Do. Section rectangular. — Darcy-Bazin. Series 4.	0 .19	6 .0	0 .27	0 .25	4 .9	2 .16	61 .7	$V^{\frac{1}{18}}$
	...	...	0 .41	0 .357	"	2 .95	70 .5	...
	...	...	0 .53	0 .450	"	3 .40	72 .5	...
	...	...	0 .63	0 .520	"	3 .84	76 .1	...
	...	...	0 .73	0 .588	"	4 .14	77 .2	...
	...	...	0 .82	0 .644	"	4 .43	78 .8	...
	...	...	0 .91	0 .740	"	4 .64	79 .3	...
	...	...	0 .99	0 .746	"	4 .88	80 .7	...
	...	...	1 .06	0 .785	"	5 .12	82 .6	...
	...	...	1 .15	0 .832	"	5 .26	82 .4	...
	...	...	1 .23	0 .871	"	5 .43	83 .1	...
	...	...	1 .30	0 .910	"	5 .57	83 .4	...

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Rectangular test-	0 .00	6 .11	0 .32	0 .291	4 .9	1 .79	45 .7	$V^{\frac{1}{24}}$
channel lined with	...	...	0 .48	0 .417	"	2 .43	53 .8	...
pebbles 1¼ to 1½-	...	...	0 .61	0 .510	"	2 .90	58 .0	...
inch diameter,	...	...	0 .73	0 .587	"	3 .27	61 .1	...
held in place with	...	...	0 .84	0 .636	"	3 .56	62 .8	...
cement. — Darcy-	...	...	0 .93	0 .712	"	3 .85	65 .2	...
Bazin. Series 5.	...	...	1 .03	0 .772	"	4 .03	65 .5	...
	...	...	1 .13	0 .823	"	4 .23	66 .6	...
	...	...	1 .21	0 .867	"	4 .43	68 .0	...
	...	...	1 .29	0 .909	"	4 .60	69 .0	...
	...	...	1 .37	0 .946	"	4 .78	70 .3	...
	...	...	1 .46	0 .987	"	4 .90	70 .4	...

XIII. RECTANGULAR TEST CHANNELS WITH CLEATS NAILED CROSSWISE.

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Rectangular test-	0 .41	6 .43	0 .33	0 .302	1 .5	1 .65	77 .4	$V^{\frac{1}{36}}$
channel of boards,	...	...	0 .51	0 .442	"	2 .17	84 .5	...
with cleats 1 inch	...	...	0 .89	0 .634	"	2 .86	91 .0	...
by ⅝ inch nailed	...	...	1 .02	0 .775	"	3 .33	94 .0	...
crosswise on bot-	...	...	1 .23	0 .889	"	3 .68	97 .0	...
tom and sides, ⅜	...	...	1 .42	0 .986	"	3 .98	99 .0	...
inch apart. —	...	...	1 .62	1 .076	"	4 .19	99 .0	...
Darcy-Bazin.								
Series 12.								
Do.	...	6 .43	0 .22	0 .205	5 .9	2 .50	71 .8	$V^{\frac{1}{36}}$
Series 13.	...	...	0 .33	0 .302	"	3 .34	79 .0	...
	...	...	0 .51	0 .442	"	4 .40	86 .0	...
	...	...	0 .67	0 .552	"	5 .08	89 .0	...
	...	...	0 .80	0 .643	"	5 .63	91 .4	...
	...	...	0 .92	0 .716	"	6 .14	94 .5	...
	...	...	1 .05	0 .790	"	6 .48	94 .8	...
Do.	...	6 .40	0 .19	0 .182	8 .9	2 .85	70 .8	$V^{\frac{1}{36}}$
Series 14.	...	...	0 .30	0 .273	"	3 .75	76 .4	...
	...	...	0 .46	0 .403	"	4 .92	82 .4	...
	...	...	0 .59	0 .499	"	5 .77	86 .8	...
	...	...	0 .71	0 .582	"	6 .38	88 .9	...
	...	...	0 .83	0 .658	"	6 .86	89 .9	...
	...	...	0 .94	0 .726	"	7 .26	90 .5	...
Rectangular test-	0 .03	6 .43	0 .43	0 .378	1 .5	1 .28	53 .7	$\frac{1}{V^{\frac{1}{8}}}$
channel of boards	...	...	0 .66	0 .550	"	1 .68	58 .6	
with cleats nailed	...	...	1 .02	0 .777	"	2 .21	64 .8	...
crosswise to bot-	...	...	1 .33	0 .942	"	2 .55	67 .8	...
tom and sides,	...	...	1 .61	1 .073	"	2 .81	70 .1	...
cleats 1 by ⅝ inch,	...	...	1 .91	1 .197	"	2 .97	70 .0	...
2 inches apart. —	...	...	2 .18	1 .299	"	3 .11	70 .5	...
Darcy-Bazin.								...
Series 15.								

TABLE X. — *Continued.*

Description of Conduit.	m	W	d	R	1000 s	v	c	a
Do.	...	6 .43	0 .29	0 .264	5 .9	1 .91	48 .3	$\frac{1}{V^{1/8}}$
Series 16.	...	...	0 .44	0 .384	"	2 .56	53 .7	...
	...	...	0 .67	0 .553	"	3 .37	59 .0	...
	...	...	0 .87	0 .686	"	3 .88	61 .0	...
	...	...	1 .05	0 .791	"	4 .31	63 .1	...
	...	...	1 .21	0 .882	"	4 .65	64 .5	...
	...	...	1 .38	0 .965	"	4 .91	65 .1	...
Do.	...	6 .40	0 .25	0 .252	8 .86	2 .21	48 .7	$\frac{1}{V^{1/8}}$
Series 17.	...	...	0 .39	0 .35	"	2 .85	51 .2	...
	...	...	0 .60	0 .501	"	3 .75	55 .8	...
	...	...	0 .78	0 .628	"	4 .37	58 .6	...
	...	...	0 .94	0 .725	"	4 .85	60 .5	...
	...	...	1 .09	0 .812	"	5 .22	61 .5	...
	...	...	1 .22	0 .885	"	5 .57	62 .9	...

XIV. CHANNELS IN ROCKWORK.

Description of Conduit.	m	K	W	d	R	1000 s	v	c	a
Torlonia Drain Tunnel, Lake Fucino, Italy. Section oval, 13.12 ft. wide and 18.9 ft. high. Total length, 20,666 ft., of which ⅔ is in limestone rock, the rest is lined with free-stone masonry. —Perrone, 1894.	–0 .04	1 .07	13 .12	...	1 .93	1 .04	3 .25	75 .3	1 .0
	...	...	...	...	2 .07	"	3 .62	76 .2	1 .0
	...	...	...	...	2 .55	"	4 .24	82 .2	1 .027
	...	...	...	...	2 .67	"	4 .32	81 .7	1 .013
	...	...	...	...	3 .24	"	5 .05	86 .7	1 .027
	...	...	...	...	3 .43	"	4 .95	82 .9	0 .97
	...	...	...	...	3 .52	"	4 .92	80 .9	0 .94
	...	...	...	...	3 .75	"	5 .35	86 .2	0 .98
Beacon Street Tunnel, Sudbury Aqueduct. Length 4592 ft. Bottom lined with rough concrete, sides for the greater part unlined. Section oval. — Fteley & Stearns 1878.	–0 .02	1 .03	10 .0	...	2 .21	0 .281	1 .97	79 .2	1 .0

TABLE X.—*Continued.*

Description of Conduit.	m	K	W	d	R	$1000\,s$	v	c	a
Turlock Rock Canal. Excavated along the banks of Turlock River, California.	–0 .20	1 .50	50 .0	10 .0	5 .9	1 .5	7 .5	86 .0	1 .0
XV. ARTIFICIAL CHANNELS IN EARTH.									
Experiments by S. Fortier, U. S. Geol. Survey, 1901.									
Bear River Canal branch. Well rounded channel in clayey loam, coated with a fine sediment.	0 .58	0 .27	...	...	2 .49	0 .31	3 .62	130 .2	$V^{\frac{1}{18}}$
Do. Providence Canal.	0 .58	0 .27	...	...	0 .86	0 .12	1 .04	102 .6	$V^{\frac{1}{18}}$
Do. Logan & Hyde Park Canal.	0 .54	0 .27	...	...	0 .51	1 .88	2 .33	95 .3	$V^{\frac{1}{18}}$
Channel in fine hard gravel, ½ inch in diameter	0 .20	0 .66	...	...	1 .0	0 .32	1 .44	80 .8	$V^{\frac{1}{18}}$
Do.	0 .12	0 .77	...	...	1 .20	0 .83	2 .58	81 .5	$V^{\frac{1}{18}}$
Channel in clay with fragments of rock ½ inch in diameter imbedded.	0 .07	0 .86	...	...	1 .07	0 .62	1 .94	74 .6	$V^{\frac{1}{18}}$
Channel in sand some plants at edges.	0 .05	0 .90	...	...	1 .40	0 .15	1 .08	75 .4	1 .0
Channel in sand with small pebbles imbedded.	0 .05	0 .90	...	...	0 .52	0 .56	1 .01	59 .2	1 .0

TABLE X. — *Continued.*

Description of Conduit.	m	K	W	d	R	1000 s	v	c	a
Experiments of S. Fortier (continued). Channel in sand with small pebbles imbedded.	−0 .01	1 .04	...	...	0 .14	1 .35	0 .54	39 .3	1 .0
Channel in cemented gravel, 1, 2 and 3 inches in diameter.	0 .0	1 .0	...	...	1 .52	0 .77	2 .49	73 .0	1 .0
Channel in sand with gravel imbedded, no vegetation.	−0 .07	1 .17	...	...	0 .40	0 .40	0 .61	48 .0	1 .0
Do.	−0 .09	1 .22	...	...	1 .48	0 .43	0 .71	64 .9	1 .0
Do.	−0 .08	1 .18	...	...	0 .65	0 .75	1 .19	54 .0	1 .0
Do.	−0 .10	1 .24	...	...	0 .71	1 .75	1 .09	53 .8	1 .0
Channel in earth with gravel imbedded, size ½ to 2 inches.	−0 .10	1 .24	...	...	0 .55	1 .16	2 .93	50 .4	1 .0
Channel in earth with fragments of rock imbedded, size ½ to 2 inches.	−0 .15	1 .35	...	...	0 .65	1 .6	1 .61	50 .8	1 .0
Channel in gravel covered with sediment, gravel up to 2½ inches in diameter.	−0 .16	1 .37	...	...	1 .62	0 .60	1 .94	63 .2	1 .0
Channel in earth, bottom covered with fragments of rock.	−0 .18	1 .48	...	...	0 .35	1 .3	0 .84	39 .8	1 .0
Channel in cobbles covered with silt, edges irregular.	−0 .38	2 .52	...	...	0 .52	0 .35	0 .43	32 .0	1 .0

TABLE X. — *Continued.*

Description of Conduit.	m	K	W	d	R	1000 s	v	c	a
Channel in loose gravel up to $1\frac{1}{2}$ inches in diameter.	−0.48	2.44	...	...	0.27	9.91	1.35	25.9	1.0
Channel in earth, bottom and sides coated with fragments of rock up to 3 inches in diameter.	−0.42	2.90	...	...	0.20	12.2	1.02	20.9	1.0
Rough channel in coarse gravel and cobbles.	−0.40	2.96	...	...	0.23	17.1	1.33	21.1	1.0
Do.	−0.49	3.62	...	...	0.23	17.0	1.10	17.9	1.0
Experiments by Darcy-Bazin. Grosbois Canal.	−0.12	1.28	10.7	1.4	0.96	0.25	0.89	57	1.0
Trapezoidal	...	...	11.9	1.9	1.32	0.275	1.34	70	...
channel in	...	...	14.1	2.5	1.57	0.246	1.36	69	...
earth. No vegetation. Series 49.	...	...	15.7	2.9	1.78	0.275	1.49	66	0.98
Do.	−0.33	1.97	10.5	1.5	1.05	0.31	0.82	45	1.0
Some vegetation.	...	...	11.4	2.1	1.42	0.29	1.26	52	...
Series 50.	...	...	13.8	2.7	1.65	0.33	1.30	56	...
	...	...	15.5	3.1	1.85	0.33	1.41	57	...
Do.	−0.31	1.89	9.1	1.5	0.96	0.792	1.23	45	1.0
Stony earth, lit-	...	...	11.4	2.0	1.26	0.806	1.67	53	...
tle vegetation.	...	...	12.6	2.4	1.41	0.858	1.81	52	...
Series 37.	...	...	13.3	2.7	1.56	0.842	2.0	55	...
Do.	−0.33	1.97	10.1	1.6	1.04	0.445	0.96	45	1.0
Series 41.	...	...	12.0	2.3	1.38	0.450	1.27	51	...
	...	...	13.2	2.9	1.57	0.455	1.40	52	...
	...	...	14.3	3.0	1.71	0.441	1.51	55	...
Do.	−0.37	2.18	10.1	1.7	1.06	0.42	0.89	42	1.0
Covered with	...	...	12.3	2.4	1.41	0.47	1.18	46	...
vegetation at	...	...	13.5	2.8	1.60	0.43	1.31	49	...
many points. Series 43.	...	...	14.7	3.1	1.76	0.45	1.39	49	...

TABLE X. — *Continued.*

Description of Conduit.	m	K	W	d	R	$1000\,s$	v	c	a
Do. Bottom and sides coated with mud, little vegetation. Series 47.	-0.38	2.22	9.9	1.7	1.09	0.464	0.82	42	1.0
	...	...	11.1	2.2	1.38	0.450	1.32	53	...
	...	...	12.5	2.7	1.63	0.479	1.43	51	...
	...	...	14.0	2.9	1.71	0.493	1.68	58	...
Do. Section well rounded, little vegetation. Series 48.	-0.38	2.22	9.5	1.5	0.99	0.555	0.96	41	1.0
	...	...	10.7	2.1	1.30	0.555	1.48	55	...
	...	...	11.9	2.5	1.56	0.525	1.57	55	...
	...	...	13.5	2.9	1.71	0.515	1.75	59	...
Do. Trapezoidal channel in earth; vegetation at many points. Series 36.	-0.44	2.41	10.5	1.9	1.14	0.678	0.91	33	...
	...	...	12.9	2.5	1.42	0.633	1.28	43	1.0
	...	...	14.2	2.9	1.61	0.644	1.45	45	...
	...	...	15.2	3.2	1.74	0.622	1.65	50	...
Chazilly Canal. Trapezoidal channel in stony earth; little vegetation. Series 38.	-0.37	2.18	8.9	1.5	0.96	0.957	1.24	41	1.0
	...	...	11.0	2.0	1.18	0.929	1.70	51	...
	...	...	12.1	2.4	1.41	0.993	1.80	48	...
	...	...	13.0	2.6	1.54	0.986	1.96	50	...
Canal of the Jard. Channel in earth; no stones or plants. — Dubuat, 1779.	0.03	0.95	...	...	1.68	0.0362	0.449	57.6	$V^{\frac{1}{18}}$
	...	...	...	...	1.94	0.0362	0.479	57.0	...
	...	...	...	...	2.05	0.0458	0.607	62.6	...
	...	...	...	...	2.58	0.0651	1.069	82.5	1.0
Millrace of Kagiswyl. Very regular channel in earth, bottom covered with fine gravel. — Epper, 1885.	-0.07	1.04	...	...	1.04	1.754	2.819	65.8	$V^{\frac{1}{18}}$
	...	...	...	...	1.38	1.255	3.139	75.3	...
	...	...	...	...	1.41	1.20	3.221	78.3	...
Canal at Realtore. Trapezoidal channel in earth; bottom muddy. — Darcy-Bazin.	-0.18	1.55	19.7	4.5	2.87	0.43	2.54	72.2	1.0

TABLE X. — *Continued.*

Description of Conduit.	m	K	W	d	R	1000 s	v	c	a
Ganges Canal, Solani Embankment. Trapezoidal channel in earth; bed quite uniform. — A. Cunningham. Series 197.	−0 .22	1 .54	184 .2	9 .7	8 .35	0 .22	3 .98	92 .8	1 .0
Do. Bed somewhat rough. Series 222–225.	−0 .22	1 .52	64 .0	4 .6	4 .07	0 .306	2 .71	76 .8	1 .0
	...	...	64 .3	4 .8	4 .18	0 .304	2 .74	78 .3	...
	...	...	64 .8	5 .1	4 .37	0 .297	2 .79	77 .4	...
	...	...	65 .2	5 .3	4 .50	0 .291	2 .82	78 .8	...
Do. Bed uneven. Series 192.	−0 .27	1 .67	174 .9	10 .0	8 .64	0 .231	3 .98	89 .1	1 .0
Canal Cavour. Above the syphon of the Sesia. Bottom width, 65.8 ft.; side slopes, 1:1 — Passini & Gioppi, 1892.	−0 .24	1 .60	...	...	5 .16	0 .29	3 .10	80 .0	1 .0
	...	...	...	...	5 .83	"	3 .38	80 .2	...
	...	...	...	...	7 .32	"	3 .70	80 .7	...
Do. Below the syphon of the Sesia.	−0 .28	1 .80	...	...	4 .44	0 .33	3 .05	72 .2	1 .0
	...	...	...	...	5 .25	"	3 .08	78 .1	...
	...	...	...	...	5 .62	"	3 .40	79 .0	...
Escher Canal. Coarse gravel and detritus.— Legler.	−0 .33	1 .94	...	...	3 .76	3 .0	6 .986	65 .7	1 .0
	...	...	...	...	4 .42	"	8 .364	70 .6	...
Linth Canal at Grynau. Trapezoidal channel in earth; bottom slightly rounded. — Legler.	−0 .20	1 .50	123	...	5 .14	0 .29	3 .41	88 .4	1 .074
	...	...	...	...	5 .93	0 .30	3 .83	90 .8	1 .052
	...	...	...	...	6 .48	0 .31	4 .15	92 .6	...
	...	...	...	...	7 .12	0 .32	4 .42	92 .6	...
	...	...	...	...	7 .52	0 .33	4 .72	95 .4	...
	...	...	...	...	8 .09	0 .34	4 .92	93 .8	1 .0
	...	...	...	...	8 .28	0 .34	5 .06	95 .3	...
	...	...	...	...	8 .62	0 .35	5 .22	95 .1	...
	...	...	...	...	8 .87	0 .36	5 .39	95 .5	...
	...	...	...	...	9 .18	0 .37	5 .53	94 .9	0 .98

TABLE X. — *Continued.*

Description of Conduit.	*m*	*K*	*W*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Do.	-0.22	1.58	113.2	...	4.0	0.80	4.26	75.5	...
At Blaschen. Trapezoidal channel in gravel; bottom slightly rounded.	...	...	...	...	6.5	0.41	4.17	80.7	...
Simme Canal. Very coarse gravel and stones. — Wampfler, 1867.	-0.44	2.87	...	...	1.82	6.5	4.92	45.1	1.0
	...	...	...	...	1.87	7.0	5.37	46.9	...
	...	...	...	...	1.36	11.6	5.49	43.6	...
	...	...	...	...	1.32	17.0	5.99	39.8	...
Test channel in river sand. — Seddon, 1886.	0.48	...	0.57	0.065	0.0465	3.5	0.86	67.4	1.0
	...	...	...	...	...	3.7	0.87	67.8	...
	...	...	...	...	...	4.1	0.84	61.6	...
	...	...	...	...	...	5.2	0.83	54.0	...
	...	...	...	...	...	6.2	0.86	52.3	...
Do.	0.48	...	0.83	0.040	0.0316	7.9	0.93	58.9	1.0
	...	...	...	...	...	8.0	0.89	55.7	...
	...	...	...	...	...	8.6	0.88	53.2	...
	...	...	...	...	...	9.7	0.91	54.0	...
	...	...	...	...	...	11.3	0.91	51.1	...
Triangular channel in sand; very regular. — Fresno.	0.27	0.57	...	...	0.416	0.312	0.80	71.0	1.0.
Well rounded irrigation channel in sandy soil. — Fresno.	-0.03	1.04	...	...	1.0	0.06	0.50	64.5	1.0
XVI. ARTIFICIAL CHANNELS IN EARTH WITH SIDE WALLS OF MASONRY.									
Millrace at Pribram. Very regular channel in clay with side walls of masonry. Section trapezoidal. — Rittinger, 1855.	0.12	0.79	...	0.54	0.373	1.0	1.127	58.4	1.0
	...	...	...	0.66	0.425	"	1.254	60.8	...
Do.	-0.62	5.09	...	0.41	0.316	2.2	0.289	14.8	1.0
Trapezoidal channel in earth with dry rubble side walls. Very irregular.	...	...	...	0.44	0.336	"	0.588	21.6	...
	...	...	...	0.70	0.472	"	0.953	29.6	...
		...	...	0.80	0.548	"	1.135	32.7	...
	...	...	...	0.86	0.560	"	1.190	33.9	...
	-0.43	2.10	...	0.90	0.566	"	1.269	36.0	1.0

TABLE X. — *Continued.*

Description of Conduits.	m	K	W	d	R	1000 s	v	c	a
Millrace at Bezybanya. Trapezoidal channel in sand and gravel with side walls of masonry. — Rittinger.	−0.42	2.79	...	0.28	0.242	5.0	0.782	22.5	1.0
	...	...	...	0.35	0.282	"	1.191	31.7	...
	...	...	...	0.56	0.407	"	1.956	43.4	...
	...	...	...	0.73	0.483	"	2.134	43.4	...
	0.02	0.74	...	0.90	0.561	"	3.475	65.6	1.0
Millrace at Diosgyor. Rectangular channel in clay with side walls of dry rubble masonry. — Rittinger.	0.07	1.0	...	0.63	0.487	4.0	2.463	54.5	1.0
	...	...	...	1.14	0.736	...	2.750	50.7	...
	−0.28	1.38	...	1.66	0.924	...	3.323	54.8	1.0
Embankment of Solani Ganges Canal, India. Main Site.	...	...	150.0	1.7	1.69	0.090	0.44	35.7	...
	...	...	"	2.3	2.26	0.148	0.87	45.9	...
	...	...	"	3.9	3.86	0.088	1.35	73.2	...
Built up channel in clayey soil with many artificial bars of masonry and boulders. Side walls of masonry built in steps, bed very rough, masonry damaged in places. The first four gagings indicate the preponderating influence of the great roughness of the bed. — Allan Cunningham.	...	...	"	4.1	4.07	0.215	1.79	66.5	...
	−0.20	1.37	152.3	5.6	5.39	0.155	2.40	83.0	$\frac{1}{V^{\frac{1}{8}}}$
	...	...	157.0	6.8	6.18	0.171	3.05	93.8	...
	...	...	159.3	7.6	6.78	0.221	3.39	87.5	...
	...	...	161.3	8.2	7.26	0.214	3.22	81.7	...
	...	...	164.0	9.1	7.84	0.215	3.43	83.6	...
	...	...	166.3	9.9	8.42	0.227	3.58	83.6	...
	...	...	168.7	10.7	8.96	"	3.71	82.3	...
	...	...	170.1	11.0	9.34	"	4.02	87.3	...
Do. Jaoli Site. Side walls lined with brick set in clay; side slopes 1:2.	−0.17	1.32	190.9	6.8	6.32	0.140	2.63	88.4	$\frac{1}{V^{\frac{1}{8}}}$
	...	...	191.2	7.0	6.53	0.144	2.76	88.1	...
	...	...	191.5	7.3	6.79	0.145	2.80	89.2	...
	...	...	191.8	7.6	7.05	0.146	2.81	87.6	...
	...	...	192.3	8.1	7.46	0.166	2.94	85.1	...

Table X. — *Continued.*

Description of Conduit.	m	K	W	d	R	$1000\,s$	r	c	a
Do. Belra Site.	–0 .45	1 .97	187 .3	8 .6	7 .96	0 .208	3 .07	75 .4	$\frac{1}{V^{\frac{1}{18}}}$
	...	..	187 .5	8 .7	8 .21	0 .198	3 .01	74 .7	...
	...	...	188 .0	9 .5	8 .72	0 .200	3 .12	74 .7	...
	...	...	188 .4	9 .6	9 .02	0 .191	3 .17	76 .4	...

XVII. Natural Channels in Earth.

Description of Channel.	K	W	d	R	$1000\,s$	r	c	a
La Plata River, Catalina channel. Width of channel many miles. Bed fine sand. Slopes measured with great accuracy for a distance of 85 miles.—J. J. Revy.	1 .03	...	...	16 .22	0 .007	1 .391	128 .3	$\frac{1}{V^{\frac{1}{18}}}$
Parana de las Palmas.	1 .12	1222	...	50 .3	0 .007	3 .07	160 .0	$\frac{1}{V^{\frac{1}{18}}}$
Do.	...	...	...	49 .7	0 .0068	2 .95	160 .3	...
	...	...	...	49 .5	"	2 .87	156 .6	...
Parana. Rosario Section. Do.	1 .18	2460	...	44 .6	0 .0058	2 .63	152 .9	$\frac{1}{V^{\frac{1}{18}}}$
Seine River at Paris. Section between the bridges of Jena and "The Invalides." — Villevert.	1 .45	...	...	9 .48	0 .14	3 .37	92 .5	$\frac{1}{V^{\frac{1}{18}}}$
	...	...	...	10 .92	"	3 .74	95 .6	...
	...	...	...	12 .19	"	3 .80	92 .4	...
	...	...	...	14 .50	"	4 .23	94 .0	...
	...	...	...	15 .02	"	4 .51	98 .3	...
	...	...	...	15 .93	0 .173	4 .68	89 .5	...
	...	...	...	16 .85	0 .131	4 .80	102 .1	...
	...	...	...	18 .39	0 .103	4 .69	107 .6	...
River Po at Fossa d'Albero. Commission of Italian Engineers, 1878.	1 .96	...	...	9 .5	0 .119	2 .60	83 .5	$\frac{1}{V^{\frac{1}{18}}}$
	...	...	...	10 .1	0 .12	2 .92	88 .1	...
	...	...	...	11 .0	0 .12	3 .15	112 .0	...
	...	...	...	9 .8	0 .103	3 .25	96 .0	...
	...	...	...	11 .8	0 .104	3 .28	100 .0	...
Do. At Porto Morone.	...	...	...	12 .4	0 .093	3 .37	95 .0	...
	...	...	...	8 .5	0 .165	3 .04	81 .5	...

TABLE X. — *Continued.*

Description of Channel.	*K*	*W*	*d*	*R*	1000 *s*	*v*	*c*	*a*
Weser River near	1 .92	...	...	9 .4	0 .2499	4 .07	83 .8	1 .0
Minden.	...	...	...	9 .82	"	4 .37	87 .7	...
— Schwartz, 1808.	...	...	...	10 .57	"	4 .75	92 .8	...
	...	...	...	11 .18	"	4 .94	93 .2	...
	...	...	...	12 .07	"	5 .24	95 .0	..
	...	...	...	12 .64	"	5 .26	94 .2	...
	...	...	...	12 .66	"	5 .35	97 .0	...
	...	...	...	14 .13	"	5 .67	96 .0	0 .98
Do. near Minden.	1 .98	...	...	4 .50	0 .5032	3 .38	71 .0	1 .0
	...	...	...	5 .33	"	4 .0	78 .6	...
	...	...	...	6 .14	"	4 .88	88 .0	...
	...	...	...	6 .66	"	5 .24	90 .5	...
	...	...	...	7 .59	"	5 .75	93 .2	...
	...	...	...	8 .14	"	5 .95	93 .2	...
	...	...	...	8 .61	"	6 .15	93 .2	...
	...	...	...	10 .23	"	6 .66	93 .2	...
	...	...	...	10 .45	"	6 .56	90 .6	...
	...	...	...	10 .71	"	6 .56	94 .0	...
	...	...	...	11 .28	"	6 .94	92 .0	1 .01
Do. At Vlotow.	1 .51	...	...	6 .25	0 .5503	4 .95	81 .0	$\frac{1}{V^{\frac{1}{18}}}$
	...	...	...	7 .41	"	5 .32	84 .0	...
	...	...	...	8 .92	"	6 .30	90 .0	...
	...	...	...	9 .3	"	6 .52	91 .2	...
	...	...	...	9 .72	"	6 .65	91 .2	...
	...	...	...	11 .70	"	7 .53	93 .8	...
	...	...	...	13 .0	"	7 .92	93 .9	...
	...	...	...	13 .35	"	7 .90	92 .1	...
Plessur River. Some	1 .55	...	...	1 .25	9 .65	6 .6	54 .7	1 .0
stones and gravel.	...	...	...	2 .33	"	9 .99	66 .4	...
— La Nicca, 1839.	...	...	...	3 .48	"	10 .19	66 .4	...
	...	...	...	3 .58	"	13 .58	72 .4	...
	...	...	...	3 .59	"	13 .94	74 .8	...
Saalach River. Some	1 .61	...	...	1 .54	0 .875	2 .073	56 .5	1 .0
stones. — Roff,	...	...	...	1 .31	1 .100	2 .246	58 .8	...
1854.	...	...	...	1 .91	1 .242	3 .077	63 .0	...
	...	...	...	1 .98	1 .240	3 .385	68 .2	...
	...	...	...	2 .16	3 .660	5 .474	64 .3	...
River Rhine in Domleschger Valley.	1 .71	...	...	0 .25	5 .74	1 .25	32 .8	$\frac{1}{V^{\frac{1}{18}}}$
Gravel and detri-	...	...	...	1 .32	7 .73	4 .75	47 .0	...
tus. — La Nicca.	...	...	...	2 .95	7 .95	7 .42	48 .3	...

TABLE X. — *Concluded.*

Description of Channel.	K	W	d	R	1000 s	r	c	a
Salzach River. From Bergheim to Wildshut. Gravel and detritus. — Reich.	1.94	...	...	3.53	0.94	3.48	60.3	$\frac{1}{V^{\frac{1}{18}}}$
	...	...	...	4.20	0.94	4.03	63.9	...
	...	...	...	7.39	1.12	5.786	63.4	...
	...	...	...	3.51	1.55	4.10	55.4	...
	...	...	...	4.64	1.55	4.67	67.5	...
	...	...	...	3.87	1.79	4.45	53.4	...
	...	...	...	4.26	1.79	5.15	58.8	...
Zihl River near Gottstatt. Bed very irregular, covered with mud and detritus. — Trechsel, 1825.	1.98	...	...	3.52	0.4	2.296	61.0	$\frac{1}{V^{\frac{1}{18}}}$
	...	...	...	5.02	"	3.706	77.1	...
	...	...	...	5.53	"	4.625	69.1	....
Mississippi River at Columbus, Ky., at high water, 1895. Rep. of Miss. River Com. of 1896.	4.14	3122	...	64.9	0.08	6.415	91.9	$\frac{1}{V^{\frac{1}{18}}}$
Do. At Helena, Ark.	2.91	5100	...	40.5	0.07	5.207	97.8	...
Do. At Arkansas City	4.31	3453	...	65.2	0.064	5.807	89.7	"
Do. At Wilson Point, La.	2.68	3944	...	56.4	0.054	6.145	111.4	"
Do. At Natchez, Miss.	1.13	2173	...	69.5	0.0459	9.512	161.2	"
Do. At Red River Landing, La.	1.59	4044	...	57.2	0.0284	5.636	140.9	"
Do. At Carrollton, La., at high water. Bed sand.	1.22	2338	...	71.0	0.0219	6.494	162.3	"
	...	...	...	72.7	0.0254	6.254	161.8	...
Do. At low water.	1.51	2338	...	65.5	0.0021	1.842	158.0	"
Irrawaddi River at Saiktha, Burmah. Bed sand with stones, right bank rocky in places. — Gordon, 1873.	2.66	3395	35	16.28	0.0086	1.007	85.1	1.0
	...	...	39	18.49	0.0172	1.783	99.9	...
	...	...	43	19.99	0.0218	2.360	103.9	...
	...	...	47	21.13	0.0344	2.857	105.9	1.17
	...	...	53	26.42	0.0474	3.548	100.3	...
	...	...	57	29.80	0.0559	3.993	97.8	...
	...	...	63	35.44	0.0688	4.052	94.2	...
	...	...	69	41.01	0.0817	5.382	92.9	...
	...	...	73	44.47	0.0904	6.147	97.0	0.84

Forms of Sections of Conduits.

Relation of mean Hydraulic Radius to Wet Perimeter.

In the design of the form of cross-section of an artificial conduit two factors enter into consideration:

1. The material composing the walls of the channel.
2. The special purpose for which it is intended.

Conduits under pressure, whether constructed of metal, wood, earthenware, concrete or masonry are nearly always circular in section, because this form can best be given the strength to resist internal and external pressures. The thickness of the material forming the walls of a circular conduit is found from the formula:

$$t = \frac{PD}{T} + c$$

in which P is the pressure in pounds per square inch;
D the internal diameter in inches;
T the safe tensile strength of the material;
c a constant added to guard against defects in the casting or the welding.

For such conduits as are subject to water ram a pressure of 100 pounds per square inch is allowed in addition to the pressure due to the head which is equal to $P = 0.434\ h$. The stresses allowed in the material and the constants added are:

For cast iron	$T =$ 4,000,	$c = 0.33$
For wrought iron	$T =$ 17,000,	$c = 0.06$
For steel	$T =$ 20,000	
For lead	$T =$ 450,	$c =$ 0.3
For concrete, 2 per cent steel	$T =$ 480,	$c =$ 1.0

Since the advent of reinforced concrete, conduits constructed of this material are coming more and more into favor. Steel-concrete water pipes resisting pressures of heads exceeding 100 feet are now in use. The two aqueduct-syphons of Sosa have internal diameters of 12.46 feet, and resist the pressure of a head of 92 feet.

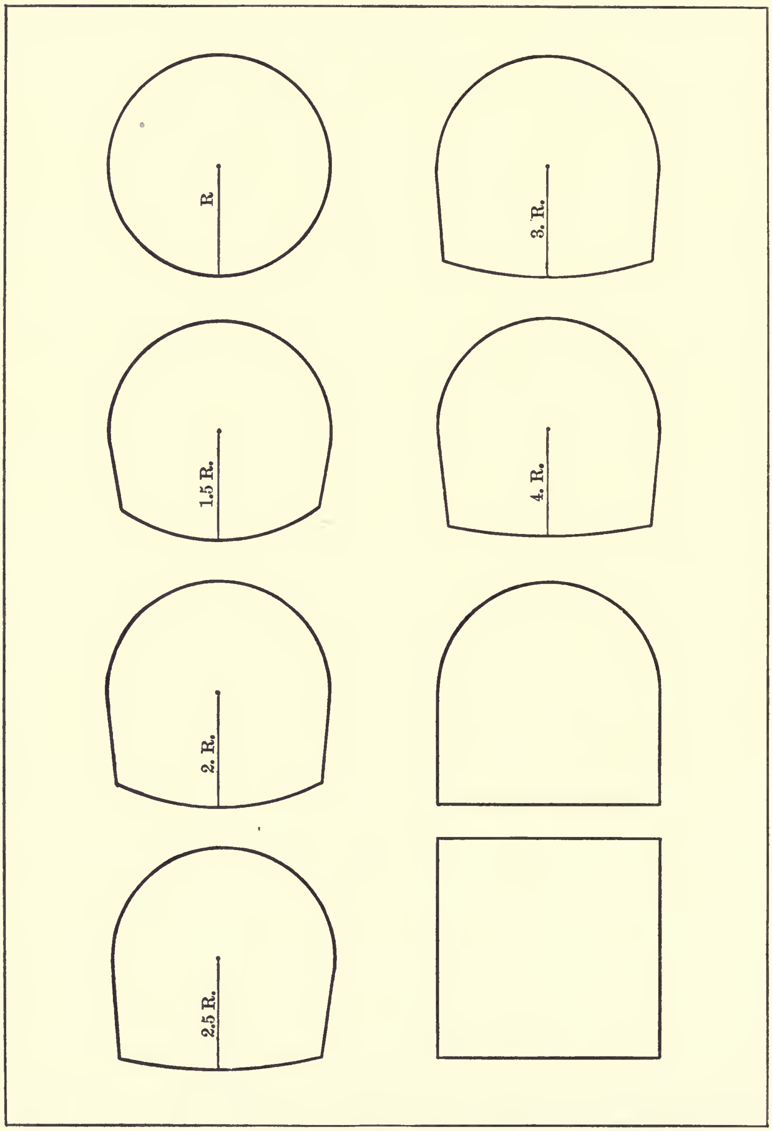

FIG. 4.

Forms of Sections of Masonry Conduits. The Numbers are Proportional.

Steel concrete sewer pipe is now made in diameters from 15 to 120 inches.

Open conduits lined with concrete are most frequently made semicircular in section. Wooden flumes which are acting simply as aqueducts are generally made semi-square in section, if they are intended to carry lumber or wood the triangular section is used. For aqueducts lined with masonry a section is generally preferred whose sides are vertical or nearly so, whose bottom is a flat segmental arch, and whose top (if covered) is a semicircle. Very large aqueducts, those crossing valleys, streams or other depressions are given a rectangular section.

In designing channels in earth the velocity enters into the problem. In those of some dimensions the bottoms are well rounded and the sides given slopes ranging from ½ to 1 for cemented gravel, to 3 to 1 for loose sand.

In a preceding chapter we have observed, that the form of the cross-section of a conduit has an appreciable influence on the power of the velocity to which the frictional resistance is proportional and that the circular or semicircular form is the one most favorable to flow. For rectangular conduits lined with boards, for instance, we have found the value of the coefficient a to be equal to $V^{\frac{1}{18}}$ and equal to $V^{\frac{1}{12}}$ for semicircular conduits lined with the same material. The circular form has the additional advantage of having a wet perimeter less in proportion to the area of the section than any other form.

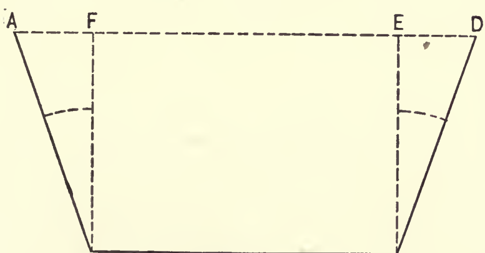

Fig. 5.

Let AD (Fig. 5) be the top width of a trapezoidal channel, BC its bottom width, and FB the depth.

The area of the cross-section will then be:

$$A = \frac{AD + BC}{2} FB,$$

and the wet circumference

$$P = AB + BC + CD.$$

Let

$$BC = b,$$
$$BF = d,$$
$$AD = t,$$
$$\frac{AF}{BF} = l.$$

Then the area $A = db + ld^2 = d(b + ld)$;

the wet perimeter $P = b + 2d\sqrt{1 + l^2}$;

the bottom width $b = \frac{A}{d} - ld$;

and the relation $\frac{P}{A}$, the reciprocal of R,

$$\frac{P}{A} = \frac{1}{d} + \frac{d}{A}(2\sqrt{l^2 + 1} - l) \cdot$$

Let the angle which the side of the conduit CD makes with the horizontal be denoted by a, and we have for the conditions most favorable to flow or $\frac{P}{A}$ a miminum, and $\frac{A}{P}$ a maximum,

the depth $d = \sqrt{\frac{A \sin a}{2 - \cos a}}$;

the top width $t = b + 2ld = \frac{A}{d} + d$ cotangent a;

the bottom width $b = \frac{A}{d} - d$ cotangent a;

the relation $\frac{\text{Wet Perimeter}}{\text{Area of Section}}$.

$$\frac{P}{A} = \frac{b}{A} + \frac{2d}{\sin a}$$
$$= \frac{1}{d} + \left(\frac{2 - \cos a}{A \sin a}\right) d$$
$$= \frac{2}{d} \text{ or } R = \frac{d}{2} \cdot$$

Consequently, for a given value of R and given side slopes the area of section is least if R is equal to one-half the actual depth of water.

For the semicircular section R is equal to one-half the radius, hence this forms fulfils the conditions best and other forms of section fulfil it the better the nearer they approach the semicircle. Table A contains values of R and areas of sections in terms of the radius or semi-diameter for semicircular conduits.

TABLE A.

Depth of Water in Terms of Radius.	Value of R in Terms of Radius.	Wetted Section in Terms of Radius.	Depth of Water in Terms of Radius.	Value of R in Terms of Radius.	Wetted Section in Terms of Radius.
0.05	0.0321	0.0211	0.55	0.320	0.709
0.10	0.0524	0.0598	0.60	0.343	0.795
0.15	0.0963	0.1067	0.65	0.365	0.885
0.20	0.1278	0.1651	0.70	0.387	0.979
0.25	0.1574	0.228	0.75	0.408	1.075
0.30	0.1852	0.298	0.80	0.429	1.175
0.35	0.2142	0.370	0.85	0.439	1.276
0.40	0.2424	0.450	0.90	0.446	1.371
0.45	0.2690	0.530	0.95	0.484	1.470
0.50	0.2930	0.614	1.00	0.500	1.571

Table B contains proportions of channels of maximum values of R, the mean hydraulic radius for a given area and given side slopes.

Half the top width is the length of each side slope and the wet perimeter is the sum of the top and bottom widths. The mean hydraulic radius is equal to one-half the depth of water.

TABLE B.

Description of Form of Section.	Inclination of Sides to Horizon.	Ratio of Side Slopes.	Area of Section in Terms of Depth.	Bottom Width in Terms of Depth of Water.	Top Width in Terms of Depth of Water.
Semi Circle	. . . . .	. . . . .	$1.571\,d^2$	. . . . .	. . . . .
Semi Hexagon . . .	60°	3 : 5	$1.732\,d^2$	$1.155\,d$	$2.31\,d$
Semi Square	90°	0 : 1	$2\,d^2$	$2\,d$	$2\,d$
Trapezoid	78° 58′	1 : 4	$1.812\,d^2$	$1.562\,d$	$2.062\,d$
Do.	63° 26′	1 : 2	$1.736\,d^2$	$1.236\,d$	$2.236\,d$
Do.	53° 8′	3 : 4	$1.750\,d^2$	d	$2.50\,d$
Do.	45.0	1 : 1	$1.828\,d^2$	$0.828\,d$	$2.828\,d$
Do.	38° 40′	1¼ : 1	$1.952\,d^2$	$0.702\,d$	$3.022\,d$
Do.	33° 42′	1½ : 1	$2.106\,d^2$	$0.606\,d$	$3.606\,d$
Do.	29° 44′	1¾ : 1	$2.282\,d^2$	$0.532\,d$	$4.032\,d$
Do.	26° 34′	2 : 1	$2.472\,d^2$	$0.472\,d$	$4.472\,d$
Do.	23° 58′	2¼ : 1	$2.674\,d^2$	$0.424\,d$	$4.924\,d$
Do.	21° 48′	2½ : 1	$2.885\,d^2$	$0.385\,d$	$5.385\,d$
Do.	19° 58′	2¾ : 1	$3.104\,d^2$	$0.354\,d$	$5.854\,d$
Do.	18° 26′	3 : 1	$3.325\,d^2$	$0.325\,d$	$6.325\,d$

Sewers.

The forms of the cross-section most frequently adopted for sewers are the circular and the oval or egg-shaped. Only sewers of very great dimensions are given a rectangular section, the roof being a flat segmental arch.

For sewers less than two feet in diameter glazed earthenware pipe is mostly used, less frequently concrete pipe. Sewers constructed of brick, masonry or concrete are, however, found with diameters down to two feet.

When the discharge of a sewer is estimated to be fairly constant the circular section is preferred, when it varies considerably, however, some form of an oval sewer is used.

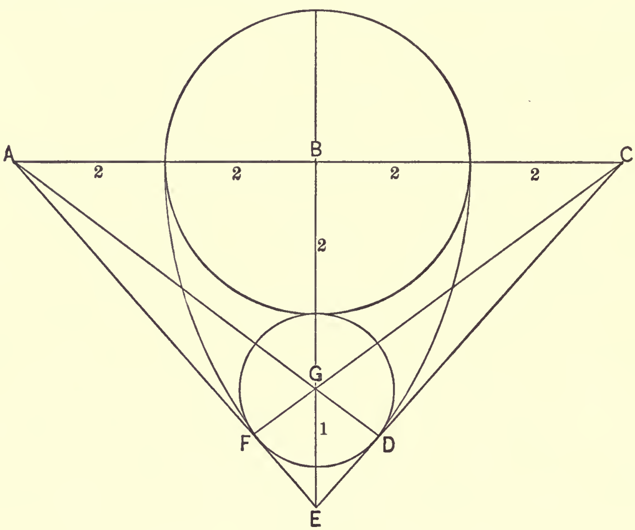

Fig. 6.

Two forms of egg-shaped sewers are in general use. The one most frequently adopted has the proportional parts as given in the annexed figure. In the other form the lower circle has a diameter of one-fourth the diameter of the upper circle only. This form is used for sewers of small dimensions and greatly varying in discharges.

The vertical diameter in both forms is always equal to $1\frac{1}{2}$ diameters of the upper circle.

In the figure $\quad$ Tangent $\frac{BG}{BC} = \frac{3}{4} = 0.75.$

Hence the angle $BCG = 36° 53'$,
and the angle $BGC = 180° - (90° + 36° 53') = 53° 7'$,
hence, the angle $FGD = 2 \times 53° 7' = 106° 14'$.

Using trigonometry, these data enable us to compute the different parts of the area and the circumference with precision.

For the proportional parts given in the figure we find by this method:

The area = 18.35 which is equal to 1.147 d^2,
the circumference = 15.8488 which is equal to 3.9622 d,
the mean hydraulic radius = 1.1584, which is equal to 0.2896 d,
d being the horizontal diameter.

By the same method we may compute the value of the mean hydraulic radius or the area for any depth of water in the sewer. The mean hydraulic radius has its greatest value when the conduit is about 0.85 full. It is equal, being

0.85 full to 0.345 the horizontal diameter;
$\frac{3}{4}$ full to 0.33 the horizontal diameter;
$\frac{1}{2}$ full to 0.28 the horizontal diameter;
$\frac{1}{4}$ full to 0.20 the horizontal diameter.

The speed of flow necessary to prevent a deposit of sewage is given, for all forms of the cross-section by the equation

$$v = 2 + \frac{0.125}{r},$$

or, in exceptional cases, when the sewer is very well constructed by

$$v = 2 + \frac{0.0625}{r},$$

r being the mean hydraulic radius.

Hence for the very greatest section the least permissible velocity is two feet per second. In order that the velocity should not fall below the permissible limit the value of

$$66\ (\sqrt[4]{r} + m)\sqrt{r},$$

on which for equal slopes the velocity depends should not, for any quantity of discharge vary greatly.

If we assume the horizontal diameter to be equal to 4 feet, the value of r is, for the sewer running full, equal to 1.1584 feet, for the sewer running $\frac{1}{4}$ full to 0.80 feet.

Taking $m = 0.57$ (corresponding to common brickwork) the value of $66\ (\sqrt[4]{r} + m)\ \sqrt{r}$ is in the first case equal to 113.6, in the second to 89.48.

For $r = 0.8$ the velocity necessary to prevent a deposit is equal to

$$2 + \frac{0.125}{0.8} = 2.156 \text{ ft. per second.}$$

For this velocity the slope will be

$$S = \left[\frac{V^{\frac{17}{18}}}{66\ (\sqrt[4]{r} + m)\ \sqrt{r}}\right]^2 = \left[\frac{2.156^{\frac{17}{18}}}{89.48}\right]^2 = 000053$$

For the same slope and the sewer running full the velocity will be

$v = (66\ (\sqrt[4]{r} + m)\ \sqrt{rs})^{\frac{18}{17}} = (113.6 \times 0.02304)^{\frac{18}{17}} = 2.575$ feet per second.

The cross-section is equal to $1.147\, d^2 = 18.864\, f^2$, hence the discharge, for the sewer running full

$$Q = 18.864 \times 2.575 = 48.57\, f^3$$

and

$$Q = \frac{18.864}{4} \times 2.516 = 9.09\, f^3$$

for the sewer running $\frac{1}{4}$ full. Thus, while the actual discharge in cubic feet per second for the sewer running full is 5.34 times the discharge of the sewer running $\frac{1}{4}$ full, the difference in the speed of flow is only $2.575 - 2.156 = 0.419$ feet per second.

EXPONENTIAL EQUATIONS.

General Relations between Diameters and Velocities or Quantities. General Relations between Slopes and Velocities or Quantities.

Long Circular Conduits Running Full.

A.

If in our general equation for the velocity of flow we substitute d, the diameter of a conduit in feet, for r, its mean hydraulic radius, the formula thus transformed will read:

$$v = 23.34 \ (\sqrt[4]{d} + 1.414\, m) \sqrt{ds}$$

and for $a = v^{\frac{1}{9}}$

$$v = (23.34 \ (\sqrt[4]{d} + 1.414\, m) \sqrt{ds})^{\frac{9}{8}}$$

which may be written

$$v = 34.607 \ (\sqrt[4]{d} + 1.414\, m)^{\frac{9}{8}} d^{\frac{9}{16}} s^{\frac{9}{16}}.$$

For the term $(\sqrt[4]{d} + 1.414\, m)$ and its variation with the velocity we have no adequate substitute, containing as it does two variables in an everchanging relation. This fact makes the problem of finding an exponential equation, giving values as exact as those found from the general formula an impossibility. The powers of the diameter (or the mean hydraulic radius) to which velocities and quantities are proportional are not constant, even for the same degree of roughness, but vary with the diameter (or the mean hydraulic radius) itself. On this account exponential equations with constant values of the powers of d or r are only approximations, fairly true between certain limits, but the more incorrect the farther outside of these limits. Such equations should only be considered as brief empirical expressions, valuable only on account of their brevity; they should never be substituted for the general formula when a great degree of accuracy is desired.

Computing the velocities and discharges of two long straight circular conduits of different diameters, but of the same degree of roughness and having the same slope from the general equation, we may by means of the data thus obtained find an expression for the relation between the diameter and the velocity or the discharge which holds good between the limits of the two values of d.

To find the exponents of the powers of d to which velocities and quantities are proportional we may put

$$x = \frac{\log V_1 - \log V_0}{\log d_1 - \log d_0}$$

$$y = \frac{\log Q_1 - \log Q_0}{\log d_1 - \log d_0}$$

By means of these equations and for values of d between 1 and 50 inches for $a = v^{\frac{1}{9}}$, and between 1 foot and 20 feet for other values of a we find the following values of x and y.

$a = v^{\frac{1}{9}}$	$m = 1.0$	$x = 0.67$	$y = 2.67$
$a = v^{\frac{1}{9}}$	$m = 0.95$	$x = 0.67$	$y = 2.67$
$a = v^{\frac{1}{9}}$	$m = 0.83$	$x = 0.68$	$y = 2.68$
$a = v^{\frac{1}{9}}$	$m = 0.68$	$x = 0.695$	$y = 2.695$
$a = v^{\frac{1}{18}}$	$m = 0.57$	$x = 0.70$	$y = 2.70$
$a = v^{\frac{1}{18}}$	$m = 0.53$	$x = 0.70$	$y = 2.70$
$a = 1.0$	$m = 0.57$	$x = 0.66$	$y = 2.66$
$a = 1.0$	$m = 0.45$	$x = 0.67$	$y = 2.67$
$a = 1.0$	$m = 0.30$	$x = 0.68$	$y = 2.68.$

Consequently, for $m = 0.68$ (pipes of planed staves, cast and wrought iron, etc., all some time in use), velocities are proportional to $d^{0.695}$, quantities to $d^{2.695}$ and we have between V and d the relation

$$\frac{v_1}{v_0} = \frac{d_1^{\,0.695}}{d_0^{\,0.695}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

$$v_1 = v_0 \frac{d_1^{\,0.695}}{d_0^{\,0.695}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (a)$$

$$d_1 = d_0 \left(\frac{v_1}{v_0}\right)^{1.453} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

and between Q and d

$$\frac{Q_1}{Q_0} = \frac{d_1^{\,2.695}}{d_0^{\,2.695}} \quad . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$Q_1 = Q_0 \left(\frac{d_1}{d_0}\right)^{2.695} \quad . \; . \; . \; . \; . \; . \; . \quad (a)$$

$$d_1 = d_0 \left(\frac{Q_1}{Q_0}\right)^{0.371} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (b)$$

$$\left[\frac{1}{0.695} = 1.453, \frac{1}{2.695} = 0.371\right].$$

Equations (a) 1 and 2 enable us to find velocities and quantities for a diameter d_1, provided velocities and quantities for a diameter d_0 are known.

From equations (b) 1 and 2 we may find the diameter d_1 for a velocity V_1 or a quantity Q_1, provided the diameter d_0 for the Velocity v_0 or the discharge Q_0 is known.

In the same manner we find for the relation between the slope and velocities and quantities:

$$\frac{v_1}{v_0} = \frac{S_1^{\frac{9}{16}}}{S_0^{\frac{9}{16}}} \quad \ldots\ldots\ldots \quad (3)$$

$$v_1 = v_0\left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad \ldots\ldots \quad (a)$$

$$S_1 = S_0\left(\frac{v_1}{v_0}\right)^{\frac{16}{9}} \quad \ldots\ldots \quad (b)$$

$$\frac{Q_1}{Q_0} = \frac{S_1^{\frac{9}{16}}}{S_0^{\frac{9}{16}}} \quad \ldots\ldots\ldots \quad (4)$$

$$Q_1 = Q_0\left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad \ldots\ldots \quad (a)$$

$$S_1 = S_0\left(\frac{Q_1}{Q_0}\right)^{\frac{16}{9}} \quad \ldots\ldots \quad (b)$$

Combining equation (1) and (3) we have:

$$\frac{v_1}{v^0} = \left(\frac{d_1}{d_0}\right)^{0.695} \left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad \ldots\ldots \quad (5)$$

$$v_1 = v_0\left(\frac{d_1}{d_0}\right)^{0.695} \left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad \ldots\ldots \quad (a)$$

$$d_1 = d_0\left(\frac{\frac{v_1}{v_0}}{\left(\frac{S_1}{S_0}\right)^{\frac{9}{16}}}\right)^{1.453} \quad \ldots\ldots \quad (b)$$

$$S_1 = S_0\left(\frac{\frac{v_1}{v_0}}{\left(\frac{d_1}{d_0}\right)^{0.695}}\right)^{\frac{16}{9}} \quad \ldots\ldots \quad (c)$$

Combining (2) and (4) we have:

$$\frac{Q_1}{Q_0} = \left(\frac{d_1}{d_0}\right)^{2.695} \left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$Q_1 = Q_0\left(\frac{d_1}{d_0}\right)^{2.695} \left(\frac{S_1}{S_0}\right)^{\frac{9}{16}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

$$d_1 = d_0\left(\frac{\frac{Q_1}{Q_0}}{\left(\frac{v_1}{v_0}\right)^{\frac{9}{16}}}\right)^{0.371} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

$$S_1 = S_0\left(\frac{\frac{Q_1}{Q_0}}{\left(\frac{d_1}{d_0}\right)^{2.695}}\right)^{\frac{16}{9}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

By means of these equations we may find:

The value of V_1 or Q_1 for any value of d_1 or S_1.

The value of d_1 for any value of V_1, Q_1, or S_1.

The value of S_1 for any value of V_1, Q_1 or d_1

provided values of V_0, Q_0, d_0, S_0 are known. In these equations $S^{\frac{9}{17}}$ is substituted for $S^{\frac{9}{16}}$ when $a = v^{\frac{1}{18}}$ and $S^{\frac{1}{2}}$ when $a = 1.0$.

Computing velocities and quantities of discharge for a conduit one foot in diameter and having different degrees of roughness from the general formula we find for three values of (a) the following equations:

1. For $a = v^{\frac{1}{9}}$,

$$v = 93.25 - 59.84\ (1 - m)\ d^x S^{\frac{9}{16}},$$
$$Q = 76.69 - 47.0\ \ (1 - m)\ d^y S^{\frac{9}{16}}.$$

2. For $a = v^{\frac{-1}{18}}$,

$$v = 70.44 - 41.85\ (1 - m)\ d^x S^{\frac{9}{17}},$$
$$Q = 53.33 - 32.87\ (1 - m)\ d^y S^{\frac{9}{17}}.$$

3. For $a = 1.0$,

$$v = 56.33 - 33.0\ \ (1 - m)\ d^x S^{\frac{1}{2}},$$
$$Q = 44.24 - 25.92\ (1 - m)\ d^y S^{\frac{1}{2}}.$$

In particular we have the following:

$m = 1.0$. New long straight conduits lined with clean cement very smooth. Tin pipes. Plated pipes.

$$v = 93.25\ d^{0.67}\ S^{\frac{9}{16}},$$
$$Q = 76.69\ d^{2.67}\ S^{\frac{9}{16}}.$$

$m = 0.95.$ Very smooth new asphalt-coated cast or wrought-iron pipes, also new asphalt-coated wrought-iron and steel riveted pipes not exceeding 6″ in diameter. New conduits of planed staves.

$$v = 90.17\, d^{0.68}\, S^{\frac{9}{16}},$$
$$Q = 70.82\, d^{2.68}\, S^{\frac{9}{16}}.$$

$m = 0.83.$ Ordinary new asphalt-coated cast and wrought-iron pipes. Wrought-iron pipes not coated. Glass and lead pipes. Pipes lined with smooth, concrete,

$$v = 82.7\, d^{0.68}\, S^{\frac{9}{16}},$$
$$Q = 65.01\, d^{2.68}\, S^{\frac{9}{16}}.$$

$m = 0.68.$ Pipes lined with cement or smooth concrete, pipes of planed or rough staves, cast and wrought-iron pipes, coated or not coated, wrought-iron and steel-riveted pipes not exceeding 36″ in diameter

$$v = 74.1\, d^{0.695}\, S^{\frac{9}{16}},$$
$$Q = 58.2\, d^{2.695}\, S^{\frac{9}{16}}.$$

(All some time in use but fairly clean.)

$m = 0.57.$ Sewer pipe. Conduits lined with common brick-work.

$$v = 52.5\, d^{0.70}\, S^{\frac{9}{17}},$$
$$Q = 41.27\, d^{2.7}\, S^{\frac{9}{17}}.$$

$m = 0.53.$ New asphalt-coated steel-riveted pipe exceeding 36″ in diameter.

$$v = 50.77\, d^{0.7}\, S^{\frac{9}{17}},$$
$$Q = 39.875\, d^{2.7}\, S^{\frac{9}{17}}.$$

$M = 0.45.$ Old cast and wrought-iron pipes of all descriptions, not very clean. Pipes of riveted steel exceeding 36″ in diameter, in use for several years.

$$v = 38.16\, d^{0.67}\, S^{\frac{1}{2}},$$
$$Q = 29.96\, d^{2.67}\, S^{\frac{1}{2}}.$$

$M = 0.30.$ Old pipes of riveted steel exceeding 36″ in diameter.

$$v = 33.23\, d^{0.68}\, S^{\frac{1}{2}},$$
$$Q = 26.10\, d^{2.68}\, S^{\frac{1}{2}}.$$

It will be observed that the powers of d to which Velocities and Quantities are proportional vary with (a) the coefficient of variation of c. The difference between the values of the powers of d is equal to 0.04 between the successive values of (a). For $m = 0.68$ we have for instance:

For
$$a = v^{\frac{1}{9}} \quad d^{0.695},\ d^{2.695},$$
$$a = v^{\frac{1}{18}} \quad d^{0.655},\ d^{2.655},$$
$$a = 1.0 \quad d^{0.615},\ d^{2.615},$$
$$a = \frac{1}{v^{\frac{1}{18}}} \quad d^{0.575},\ d^{2.575}.$$

Sewers.

B.

For sewers of circular section the general equations for velocity and quantity are:

for
$$a = V^{\frac{1}{18}},$$
$$v = 71.41 - 44.0 \quad (1 - m)\, d^x\, S^{\frac{9}{17}},$$
$$Q = 56.08 - 34.55\ (1 - m)\, d^y\, S^{\frac{9}{17}}.$$

and for the egg-shaped section
$$v = 78.68 - 47.7 \quad (1 - m)\, d^x\, S^{\frac{9}{17}},$$
$$Q = 92.76 - 56.24\ (1 - m)\, d^y\, S^{\frac{9}{17}},$$

d being the horizontal diameter.

The practically useful coefficients of roughness for sewers are as follows:

$m = 0.83$, $x = 0.68$, $y = 2.68$, smooth concrete, very good brickwork, brickwork washed with cement.

$m = 0.70$, $x = 0.69$, $y = 2.69$, good concrete, fairly good brickwork, very well laid sewer pipe.

$m = 0.57$ common concrete or brickwork, common sewer pipe.

Sewers of all descriptions become in the course of a few years, frequently in the course of a few months, coated with sewer slime. Sewage, moreover, does not, on account of its greater viscosity or stickiness, flow with the same velocity as pure water. The most reliable data pertaining to flow in sewers of all descriptions some time in use indicate, that a value of m greater than 0.57 cannot be safely taken.

For $m = 0.57$ we have in particular

$$a = v^{\frac{1}{18}},$$
$$v = 52.55\ d^{0.7}\ S^{\frac{9}{17}},$$
$$Q = 41.27\ d^{2.7}\ S^{\frac{9}{17}},$$

for the circular and

$$v = 57.43\ d^{0.7}\ S^{\frac{9}{17}},$$
$$Q = 67.70\ d^{2.7}\ S^{\frac{9}{17}},$$

for the egg-shaped section.

For $a = 1.0$ we have for the same value of m,

$$v = 42.19\ d^{0.66}\ S^{\frac{1}{2}},$$
$$Q = 33.14\ d^{2.66}\ S^{\frac{1}{2}},$$

for the circular and

$$v = 46.30\ d^{0.66}\ S^{\frac{1}{2}},$$
$$Q = 53.11\ d^{2.66}\ S^{\frac{1}{2}}$$ for the egg-shaped section.

In all these equations the horizontal diameter is assumed to be $\frac{2}{3}$ of the vertical diameter or the vertical diameter $1\frac{1}{2}$ the horizontal diameter. For long sewers with easy curves the equations corresponding to $a = v^{\frac{1}{18}}$ should be taken, for sewers with many sharp curves, sharp angles, etc., for sewers discharging under water against a hydraulic counterpressure the equations given under $a = 1.0$ give the best results.

Comparing the constants given for circular and egg-shaped sewers we find for equal horizontal diameters and $a = 1.0$ the relation

$$\frac{\text{Velocity egg-shaped section}}{\text{Velocity circular section}} = \frac{46.30}{42.19} = 1.097.$$
$$\frac{\text{Discharge egg-shaped section}}{\text{Discharge circular section}} = \frac{53.11}{33.14} = 1.602.$$

The velocity for the egg-shaped section is consequently 1.097 times and the discharge 1.602 times that of the circular section.

It would, however, be very erroneous to conclude that for an equal velocity an egg-shaped sewer should have a horizontal diameter of $\frac{1}{1.097} = 0.911$ and for an equal discharge a horizontal diameter of $\frac{1}{1.602} = 0.622$ times the diameter of the circular sewer.

For equal velocities the relation between the horizontal diameter of an egg-shaped section and the diameter of a circular section is given by:

Horizontal diameter egg-shaped section

$$= \left(\frac{42.19\, d^{0.66} \text{ circular section}}{46.30}\right)^{1.515}$$

$$\left(1.515 = \frac{1.00}{0.66}\right) = \text{Index},$$

and for equal discharges by:

Horizontal diameter egg-shaped section

$$= \left(\frac{33.14\, d^{2.66} \text{ circular section}}{53.11}\right)^{0.376}$$

$$\left(0.376 = \frac{1}{2.66}\right) = \text{Index}.$$

Assuming a circular sewer to have a diameter of 3 feet, an egg-shaped sewer will have, for the same velocity a horizontal diameter of

$$\left(\frac{42.19 \times 2.065}{46.3}\right)^{1.515} = 2.667 \text{ feet},$$

or 0.889 the diameter of the circular sewer.

For an equal discharge the diameter of the egg-shaped sewer will be

$$\left(\frac{33.14 \times 18.59}{53.11}\right)^{0.376} = 2.512 \text{ feet},$$

or 0.834 times the diameter of the circular section.

For a diameter of 12 feet we compute the relations, for equal velocities

$$\frac{d = 10.42 \text{ egg-shaped section}}{12} = 0.8701,$$

for equal quantities

$$\frac{d = 10.10 \text{ egg-shaped section}}{12} = 0.834.$$

Consequently we may say, that an egg-shaped sewer should have for an equal velocity a diameter of $0.88 = \frac{8}{9}$ times the diameter of the circular section and for an equal discharge a diameter of 0.834 = $\frac{5}{6}$ times the diameter of the circular section.

C. Open Conduits.

I. Form of the Cross-Section Most Favorable to Flow.

The form of cross-section most favorable to flow is one whose top width is equal to the two side slopes, whose top and bottom width together are equal to the wet perimeter, and whose mean hydraulic radius is equal to one half the depth. The semisquare is the simplest form fulfilling these conditions, and the areas of the trapezoids may be expressed in terms of the areas of this standard. Areas of trapezoids and the semi-circle thus expressed are found in the following table:

Form of Section.	Ratio of Side slopes	Proportional Areas.	Bottom width in terms of depth.
Semisquare	0:1	1.0	$2d$
Trapezoid	$\frac{1}{4}$:1	0.906	1.562d
"	$\frac{1}{2}$:1	0.868	1.236d
"	$\frac{3}{4}$:1	0.875	1.0d
"	1:1	0.914	0.828d
"	$1\frac{1}{4}$:1	0.976	0.702d
"	$1\frac{1}{2}$:1	1.053	0.606d
"	$1\frac{3}{4}$:1	1.141	0.532d
"	2:1	1.236	0.472d
"	$2\frac{1}{4}$:1	1.337	0.424d
"	$2\frac{1}{2}$:1	1.4425	0.385d
"	$2\frac{3}{4}$:1	1.525	0.354d
"	3:1	1.6625	0.325d
Semicircle	...	0.7854	...

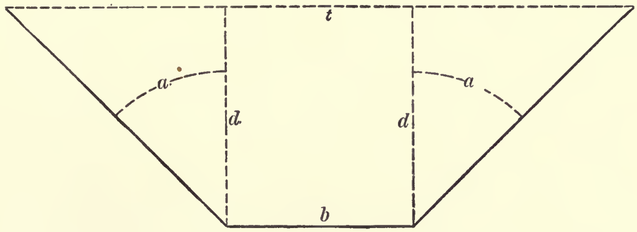

Fig. 7.

The area of a trapezoid is equal to

$$A = d(b + d \text{ tang } a);$$

the wet perimeter to

$$P = b + 2\,d\sqrt{1 + \text{tang}^2 a};$$

the mean hydraulic radius to

$$R = \frac{d(b + d \text{ tang } a)}{b + 2\,d\sqrt{1 + \text{tang}^2 a}};$$

the bottom width to

$$b = \frac{\text{Area}}{d} - d \text{ tang } a;$$

the top width to

$$T = \frac{\text{Area}}{d} + d \text{ tang } a.$$

General Relations between the Velocity, the Discharge and the Depth of Water in the Form of Section most Favorable to Flow.

Computing the velocities of flow for two conduits having mean hydraulic radii equal to 0.1 foot and 10.0 feet, respectively, from the general formula

$$v = [66(\sqrt[4]{r} + m)\sqrt{r \cdot s}]^x,$$

and using the values of v thus found in the equation

$$x \doteq \frac{\log v_1 - \log v_0}{\log R_1 - \log R_0}.$$

We find for the powers of R, to which the velocity is proportional, the values tabulated below:

Values of m or K.	Power of R or d.				
	$a = V^{\frac{1}{12}}$	$a = V^{\frac{1}{18}}$	$a = 1.0$	$a = \frac{1}{V^{\frac{1}{18}}}$	$a = \frac{1}{V^{\frac{1}{9}}}$
$m = 1.0 - 0.95$	0.69	0.67	...	...	...
$m = 0.83 - 0.80$	0.70	0.68	...	...	...
$m = 0.70 - 0.68$	0.71	0.69	...	...	...
$m = 0.57$	...	0.70	...	...	...
$m = 0.45$	...	0.715	...	...	...
$m = 0.30$	...	0.735	...	...	...
$m = 0.15$	...	0.75	...	...	...
$m = 0.0$ $K = 1.0$	...	...	0.75	0.71	...
$K = 1.25$	...	...	0.765	0.725	...
$K = 1.50$	...	...	0.775	0.735	0.70
$K = 2.0$	...	...	0.795	0.755	0.71

Computing the velocities of flow for semisquares one foot in depth from the general equation we find the following exponential equations:

For $a = V^{\frac{1}{12}}$,

$$v = 129 - 73\,(1 - m)\,d^x\,S^{\frac{6}{11}};$$

for $a = V^{\frac{1}{18}}$,

$$v = 111.6 - 63.3\,(1 - m)\,d^x\,S^{\frac{9}{17}};$$

for $a = 1.0$,

$$v = 85.86 - 46.61\,(1 - m)\,d^x\,S^{\frac{1}{2}};$$

for $a = \frac{1}{V^{\frac{1}{18}}}$,

$$v = 67.62 - 35.4\,(1 - m)d^x\,S^{\frac{9}{19}}.$$

In any of these equations the term $2 - \frac{2}{1 + k}$ may be substituted for its equivalent $1 - m$. In particular we have for the semicircle:

$m = 1.0$ Semicircular channels lined with clean cement,

$$v = 129\, d^{0.69}\, S^{\frac{6}{11}}.$$

$m = 0.83$ Semicircular channels of brickwork washed with cement,

$$v = 116\, d^{0.7}\, S^{\frac{6}{11}}.$$

$m = 0.70$ Semicircular channels lined with rough boards,

$$v = 104.5\, d^{0.71}\, S^{\frac{6}{11}}.$$

For the semisquare we have:

$$a = V^{\frac{1}{18}}.$$

$m = 0.95$ Channels lined with clean cement, planed boards,

$$v = 108.43\, d^{0.67} S^{\frac{9}{17}},$$
$$Q = 216.86\, d^{2.67}\, S^{\frac{9}{17}}.$$

$m = 0.80$ Channels lined with smooth concrete, very good brickwork,

$$v = 98.87\, d^{0.68}\, S^{\frac{9}{17}},$$
$$Q = 197.74\, d^{2.68}\, S^{\frac{9}{17}}.$$

$m = 0.70$ Channels lined with sawed boards or good brickwork,

$$v = 92.48\, d^{0.69}\, S^{\frac{9}{17}},$$
$$Q = 184.96\, d^{2.69}\, S^{\frac{9}{17}}.$$

$m = 0.57$ Channels lined with common brickwork, rough concrete or very good ashlar,

$$v = 84.23\, d^{0.70}\, S^{\frac{9}{17}},$$
$$Q = 168.46\, d^{2.70}\, S^{\frac{9}{17}}.$$

$m = 0.45$ Channels lined with rough brickwork, common ashlar or very rough concrete,

$$v = 76.67\, d^{0.715}\, S^{\frac{9}{17}},$$
$$Q = 153.34\, d^{2.715}\, S^{\frac{9}{17}}.$$

$m = 0.30$ Channels lined with good rubble masonry,

$$v = 67.27\, d^{0.735}\, S^{\frac{9}{17}}$$
$$Q = 134.54\, d^{2.735}\, S^{\frac{9}{17}}.$$

$m = 0.15$ Channels lined with roughly hammered masonry, channels in cemented gravel up to one inch in diameter,

$$v = 57.8\, d^{0.75}\, S^{\frac{9}{17}},$$
$$Q = 114.6\, d^{2.75}\, S^{\frac{9}{17}},$$
$$a = 1.0.$$

$m = 0.0$ Channels lined with common rubble masonry, tunnels
$K = 1.0$ in rockwork, channels in cemented gravel exceeding one inch in diameter,

$$v = 39.24\, d^{0.75}\, S^{\frac{1}{2}},$$
$$Q = 78.48\, d^{2.75}\, S^{\frac{1}{2}}.$$

$m = -0.1$ Fairly regular channels in loose sand, or sand with
$K = 1.2$ gravel imbedded

$$v = 35.39\, d^{0.765}\, S^{\frac{1}{2}},$$
$$Q = 70.78\, d^{2.765}\, S^{\frac{1}{2}}.$$

$m = -0.2$ Fairly regular channels in earth, free from debris or
$K = 1.5$ vegetation,

$$v = 30.86\, d^{0.775}\, S^{\frac{1}{2}},$$
$$Q = 61.72\, d^{2.775}\, S^{\frac{1}{2}}.$$

$m = -0.32$ Channels in earth with debris or vegetation,
$K = 1.93$

$$v = 26.1\, d^{0.795}\, S^{\frac{1}{2}},$$
$$Q = 52.2\, d^{2.795}\, S^{\frac{1}{2}}.$$

If the cross-section of the conduit is a trapezoid, the discharges are multiplied by the proportional areas found in the table given at the beginning of this chapter.

For a trapezoid having side slopes of 1:1, for instance, the discharges found from any of the equations given above are multiplied by 0.914, for side slopes of 2 : 1 by 1.236 etc. The depth being the same, the velocity is not affected by the side slope.

Values of the powers of d relating to Velocities are found in Table E; those relating to Quantities of discharge in Table F.

Values of the sines of the slope and their roots are found in Table C.

For channels in earth, in case the velocity exceeds the limit where erosion begins, the following equations may be used:

$$
\begin{aligned}
m &= -0.1 & v &= 28.68\ d^{0.725}\,S^{\frac{9}{19}},\\
K &= 1.20 & Q &= 57.36\ d^{2.725}\,S^{\frac{9}{19}},\\
m &= -0.20 & v &= 25.14\ d^{0.735}\,S^{\frac{9}{19}},\\
K &= 1.50 & Q &= 50.28\ d^{2.735}\,S^{\frac{9}{19}},\\
m &= -0.32 & v &= 20.90\ d^{0.755}\,S^{\frac{9}{19}},\\
K &= 1.93 & Q &= 41.80\ d^{2.755}\,S^{\frac{9}{19}}.
\end{aligned}
$$

It is, however, more convenient to use the equations previously given, for which the powers of d and S are found in the tables, and multiply the Velocities or Quantities found from the formulas by the coefficient of variation of C, which in these cases is equal to $a = \dfrac{1}{V^{\frac{1}{19}}}$. Values of $a = \dfrac{1}{V^{\frac{1}{19}}}$ are found in column 10, Table V.

II. General Equations.

In the design of cross-sections of channels it is not always possible to use the form of section most favorable to flow. Other forms are frequently required for special purposes, are constructed at less cost, or offer other advantages.

For wooden flumes, for instance, the triangular section is frequently adopted.

If the sideslopes of a triangle are 1 : 1, or the sides inclined 45° (which is the usual sideslope for a flume), the area of its cross-section, its mean hydraulic radius and consequently its velocity and discharge are equal to those of a semisquare when

(1) the depth $= \sqrt{\text{area of semisquare}}$.

(2) the top width $= \sqrt{4 \times \text{area of semisquare}}$.

In the design of channels in earth it is frequently necessary, in order to keep the velocity below the eroding limit, to make the sections wide and shallow, so that the frictional resistance may be increased and the flow retarded. In many cases a shallow section is also more easily constructed and at less cost.

The general exponential equations, derived as previously indicated, are as follows:

$$a = V^{\frac{1}{9}},$$

$$v = 243 - 131.6\,(1 - m)\,r^x\,S^{\frac{9}{16}},$$

$$a = V^{\frac{1}{12}},$$

$$v = 205.8 - 112\,(1 - m)\,r^x\,S^{\frac{6}{11}},$$

$$a = V^{\frac{1}{18}},$$

$$v = 176.3 - 93.5\,(1 - m)\,r^x\,S^{\frac{9}{17}},$$

$$a = 1.0,$$

$$v = 132 - 66\,(1 - m)\,r^x\,S^{\frac{1}{2}},$$

$$a = \frac{1}{V^{\frac{1}{18}}},$$

$$v = 100.6 - 48.3\,(1 - m)\,r^x\,S^{\frac{9}{19}}.$$

In any of these equations $2 - \dfrac{2}{1 + K}$ may be substituted for $(1 - m)$.

The practically most useful special equations are as follows:

$a = V^{\frac{1}{12}}$	$m = 1.0$	$v = 205.8$	$r^{0.69}$	$S^{\frac{6}{11}}$
$a = V^{\frac{1}{12}}$	$m = 0.83$	$v = 186.75$	$r^{0.7}$	$S^{\frac{6}{11}}$
$a = V^{\frac{1}{12}}$	$m = 0.70$	$v = 172.3$	$r^{0.71}$	$S^{\frac{6}{11}}$
$a = V^{\frac{1}{18}}$	$m = 0.95$	$v = 171.6$	$r^{0.67}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.80$	$v = 158.2$	$r^{0.68}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.70$	$v = 148.1$	$r^{0.69}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.57$	$v = 136.1$	$r^{0.7}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.45$	$v = 121.9$	$r^{0.715}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.30$	$v = 110.8$	$r^{0.725}$	$S^{\frac{9}{17}}$
$a = V^{\frac{1}{18}}$	$m = 0.15$	$v = 96.8$	$r^{0.75}$	$S^{\frac{9}{17}}$
$a = 1.0$	$m = 0.0$	$v = 66.0$	$r^{0.75}$	$S^{\frac{1}{2}}$
$a = 1.0$	$K = 1.2$	$v = 60.0$	$r^{0.765}$	$S^{\frac{1}{2}}$
$a = 1.0$	$K = 1.5$	$v = 52.8$	$r^{0.775}$	$S^{\frac{1}{2}}$
$a = 1.0$	$K = 1.93$	$v = 45.0$	$r^{0.795}$	$S^{\frac{1}{2}}$

It will be observed that the constants of these equations are equal to those given for the semisquare and the semicircle, multiplied by $2^x = 2^{0.67}$, $2^{0.68}$, $2^{0.69}$, etc., as the case may be.

Explanation of the Use of the Tables of Velocities and Quantities G, H, and I.

Table G contains the quantities of discharge in cubic feet per second of a conduit one foot in diameter, for seven different degrees of roughness and 174 slopes.

For the discharge in gallons per second multiply the quantities found in the table by 7.48052. For the velocity of flow multiply the quantities found in the table by $\frac{1}{0.7854} = \frac{14}{11}$ or, if the conduit is egg-shaped by $\frac{1}{1.147} = \frac{7}{8}$ nearly.

I.

To find the quantity of discharge for any diameter, and a given slope, multiply the value of Q found on line with the given slope under the value of m, which indicates the particular degree of roughness of the conduit, by the value of d^y found in Table D under the same value of m.

Example: What is the discharge of a cast-iron conduit same time in use; the diameter being 36 inches, and the slope 1 : 10,000?

In Table G, in column under $m = 0.68$, in line with $S = 0.0001$ we find $Q = 0.3349 f^3$ per second.

In Table D, under $d^y = d^{2.695}$, we find for $d = 36''$, $d^{2.695} = 19.31$. Hence $Q = 0.3349 \times 19.31 = 6.54 f^3$ per second.

II.

To find the loss of head or the slope corresponding to a given discharge and a given diameter, divide the given discharge by the value of d^y found in Table D as indicated above. The quotient then found will indicate, in Table G, under the proper value of m, the slope required to produce the given discharge.

Example: What is the loss of head corresponding to a discharge of 55 f^3 per second, the conduit being a new, asphalt-coated, steel-riveted pipe 6 feet in diameter?

In Table D, under $d^y = d^{2.7}$, we find for a diameter of $72''$ $d^{2.70} = 126.18$. Dividing 55 by 126.18 the quotient 0.436 is the quantity of discharge of a conduit one foot in diameter for the required slope.

In Table G, under $m = 0.53$, the value of Q coming nearest to 0.436 is 0.439. This stands in line with $S = 0.0002$, consequently $S = 0.0002$ is the slope required to produce the given discharge.

III.

To find the diameter corresponding to a given discharge and a given slope, divide the given discharge by the discharge of a conduit one foot in diameter for the given slope as found in Table G. The quotient is the value of d^y for which the diameter is found in Table D.

Example: What will be the horizontal diameter of an egg-shaped sewer, the discharge being 200 f^3 per second, and the slope 1 : 2500?

In Table G, in column headed "Egg-shaped section," and in line with $S = 0.0004$, we find $Q = 1.0759$. Dividing 200 by 1.0759 the quotient is 185.8.

In Table D, in column headed $d^{2.70}$ the nearest value above 185.8 is 191.3, which stands in line with $d = 7.0$ feet; hence 7 feet is the horizontal diameter required to produce the given discharge.

IV.

To find the loss of head or the slope required to produce a given velocity for a given diameter, divide the given velocity by the value of d^x found in Table D, as indicated above. The quotient thus found, multiplied by $0.7854 = \frac{11}{14}$ will indicate, in Table G, under the proper value of m, the slope required.

Example: What is the proper slope for an 8-inch sewer pipe?

For well constructed sewers the permissible velocity is

$$v = 2 + \frac{0.25}{d},$$

which gives for an 8-inch sewer $v = 2.375$ feet per second.

In Table D, in column headed $d^x = d^{0.7}$, in line with $d = 8$ inches, we find $d^x = 0.7529$. Dividing 2.375 by 0.7529 the quotient is 3.053, which multiplied by $\frac{11}{14}$ gives $Q = 2.477$.

In Table G, in column headed $m = 57$, we find the nearest value of Q above 2.477 to be 2.495, which is in line with $S = 0.005$, which is the slope required.

V.

In case the conduit is egg-shaped, proceed as before, but instead of multiplying by $0.7854 = \frac{11}{14}$ multiply by $1.147 = \frac{8}{7}$.

Example: What is the least permissible slope for an egg-shaped sewer having a horizontal diameter of 10 feet?

In this case: $v = 2 + \dfrac{0.25}{d} = 2.025$ feet per second.

In Table D, in column headed $d^x = d^{0.7}$ and in line with $d = 120$ inches, we find $d^{0.7} = 5.011$. Dividing 2.025 by 5.011 the quotient is 0.404, which multiplied by 1.147 gives $Q = 0.473$.

In Table G, in column headed "Egg-shaped section," we find the nearest value of Q above 0.473 to be 0.4738, which is in line with $S = 0.000085$, hence $S = 0.000085$ is the least permissible slope for a 10-foot egg-shaped sewer.

VI.

In Table H we find velocities of flow in a semisquare one foot in depth for the practically most useful values of m or K and 174 slopes.

Table H applies to any trapezoid having the form of section most favorable to flow.

To find the velocity of flow corresponding to any depth whatsoever, either in the semisquare, or the trapezoid having the form of section most favorable to flow, multiply the values found in the table by the values of d^x found in Table E.

Example: What is the velocity of flow in a channel in earth having the form of section most favorable to flow, the bed of the channel being covered with stones, the depth 10 feet and the slope 1 : 10,000?

In Table H under $K = 1.93$, and in line with $S = 0.0001$, we find $v = 0.261$.

In Table E, in column headed $K = 2.0$, and in line with $d = 10$, we find $d^{0.795} = 6.238$. Multiplying the two quantities, we find $v = 1.628$ feet per second.

VII.

Remembering that in the form of section most favorable to flow $2R = d$; or the mean hydraulic radius equal to one half the depth, it is plain that Table H can also be used to find the velocity of flow in channels not having the form of section most favorable to flow. It is only necessary always to consider $R = \frac{1}{2} d$, and multiply or divide by the value of d, which corresponds to the given value of R.

Example: What is the velocity of flow in a channel lined with common brickwork, the slope being 1 : 10,000 and the mean hydraulic radius 6 feet?

In Table H, in column headed $m = 0.57$, and in line with $S = 0.0001$, we find $v = 0.6424$.

$$R = 6 \text{ is equal to } d = 12.$$

In Table E, in column headed $m = 0.57$, and in line with $d = 12$, we find $D^{0.7} = 5.695$. Multiplying the two quantities we have $v = 3.658$ feet per second.

Example: What is the slope required for a velocity of 8 feet per second, the conduit being a triangular flume of sawed boards and the mean hydraulic radius equal to 2 feet?

$$R = 2 \text{ is equal to } d = 4.0.$$

In Table E, in column headed $m = 0.70$ and in line with $d = 4.0$, we find $d^{0.69} = 2.603$. Dividing 8 by 2.603, the quotient is 3.073, which is the value of v corresponding to a depth of one foot.

In Table H, in column headed $m = 0.70$, we find the value nearest to 3.073 to be 3.061, which is in line with $S = 0.0016$, which is the required slope.

Example: What is the value of the mean hydraulic radius required to produce a velocity of 2.8 feet per second, the slope being 1 : 10,000 and the conduit a channel in earth in good condition, free from stones and plants?

In Table H, in column headed $K = 1.5$ and in line with $S = 0.0001$ we find $v = 0.3086$. Dividing 2.8 by 0.3086 the quotient is 9.073.

In Table E, in column headed $K = 1.5$, we find the nearest value above 9.073, to be 9.191, which stand in line with $d = 17.5$. Hence R, the mean hydraulic radius required, is $\frac{17.5}{2} = 8.75$ feet.

VIII.

As Table H holds good for all conduits having the form of section most favorable to flow, it is evident that it may be used to find velocities of flow in circular conduits running full.

To find velocities corresponding to any diameter, it is necessary to keep in mind the fact, that the depth in a semicircle is one half the diameter, and multiply or divide by the value of d which corresponds to the semidiameter.

Example: What is the least permissible slope for a 6-inch sewer pipe?

Here the semidiameter or the depth is 3 inches. The permissible velocity is $v = 2 + \frac{0.25}{d} = 2.5$ feet per second.

In Table D, in column headed $d^x = d^{0.7}$, we find in line with $d = 3$ inches, $d^{0.7} = 0.3703$. Dividing 2.5 by 0.3703, the quotient is 6.751.

In Table H, under $m = 0.57$, the value of v coming nearest to 6.751 is 6.757, which is in line with $s = 0.0085$. Hence $s = 0.0085$ is the least permissible slope for a 6-inch sewer pipe.

IX.

For the classes of circular conduits whose degrees of roughness are indicated by the coefficient $m = 0.95$, $m = 0.83$, $m = 0.68$. Table H gives velocities also in case the conduit is between 300 and 1000 diameters in length, or has sharp elbows, such as city mains.

Example: What will be the velocity of flow in a city main 3 feet in diameter, the slope being 1 : 200?

Here the semidiameter is 1.5 feet.

The difference in the powers of d to which the velocity is proportional between $a = V^{\frac{1}{9}}$ and $a = V^{\frac{1}{18}}$ is equal to 0.04. For $m = 0.68$ or 0.70 and $a = V^{\frac{1}{9}}$, we have for the power of d, $x = 0.705$, consequently for $a = V^{\frac{1}{18}}$ $x = 0.665$.

In Table D, in line with $d = 1.5$, we find $d^{0.66} = 1.307$, $d^{0.67} = 1.312$, hence $d^{0.665} = 1.3095$.

In Table H, in column headed $m = 0.70$ (which is sufficiently equal to 0.68 to apply in such cases) and in line with $S = 0.005$, we find $v = 5.595$. Multiplying we have $1.3095 \times 5.595 = 7.3266$ feet per second. The discharge will be $Q = 7.3266 \times 9 \times 0.7854 = 51.79$ cubic feet per second.

X.

Table I gives the Quantities of discharge in cubic feet per second of a semisquare one foot deep for the practically most useful values of m or K and 174 slopes.

For the trapezoids or the semicircle the quantities given in the table are to be multiplied by their proportional areas.

Example: What is the discharge of a channel lined with dry rubble masonry, or a channel in rockwork, or a channel in coarse cemented gravel, the side slopes being one half to 1, the depth 12 feet, and the sine of the slope 0.0005?

In Table I, in column headed $m = 0.0$, and in line with $S = 0.0005\ S$, we find $Q = 1.755$.

In Table F, in column headed $m = 0.0$, and in line with $d = 12$, we find $d^{2.75} = 928.4$.

For a sideslope of $\frac{1}{2}:1$ the proportional area is 0.868. Multiplying the three quantities $1.755 \times 928.4 \times 0.868$, we find $Q = 1414.2\ f^3$ per second.

Example: What will be the dimensions of a channel in sand, the discharge being 200 cubic feet per second, the slope 1:10,000 and the sideslopes 3 : 1? For a sideslope of 3 : 1 the proportional area is 1.6625. Dividing 200 by 1.6625 we have for the discharge of a semisquare of equal depth $Q = 120.3$ feet.

In Table I, in column headed $K = 1.2$ and in line with $S = 0.0001$, we find $Q = 0.708$. Dividing 120.3 by 0.708 the quotient is 169.9.

In Table F, in column headed $K = 1.25$, we find the value nearest to 169.9 to be 169.5, which stands in line with 6.4 feet, which is the depth required.

The bottom width of the conduit will be $6.4 \times 0.325 = 2.08$ feet, the top width $6.4 \times 3 \times 2 + 2.08 = 40.48$ feet.

The cross-section will be $6.4 \times \frac{40.48 + 2.08}{2} = 136.2$ square feet, and the velocity $\frac{200}{136.2} = 1.46$ feet per second.

XI.

In the design of channels in earth, especially those in light soils, it is necessary to keep the velocities of flow within the eroding limits. For channels in light sandy soils a velocity exceeding 1.5 feet per second should not be allowed, for channels in earth with some clay 2.5 feet per second should be the limit.

To keep the velocities down two methods may be used:

(1) The slope may be reduced by means of weirs, dams and drops.

(2) The mean hydraulic radius may be reduced by making the channel wide and shallow.

Example: A channel in sandy soil is to carry 500 cubic feet per second, the velocity is not to exceed 1.5 feet per second, the sideslopes are to be 3:1 and the depth of the water 8 feet. What will be the slope of the channel?

The area of the cross-section will be $\frac{500}{1.5} = 333.3$ feet.

The mean width $\frac{333.3}{8} = 41.66$ feet.

The bottom width $41.66 - 8 \times 3 = 17.66$ feet.

The wet perimeter $17.66 + 2\sqrt{8^2 + (8 \times 3)^2} = 68.26$ feet.

The mean hydraulic radius $\frac{333.3}{68.26} = 4.883$ feet.

$R = 4.883$ corresponds to $d = 9.766$.

In Table E, in column headed $K = 1.25$, we find the value of $d^{0.765}$ for 9.8 to be 5.732 and 5.687 for 9.7, mean for $9.75 = 5.71$. Dividing 1.5 by 5.71 the quotient is 0.2627.

In Table H, in column headed $K = 1.20$, we find the value coming nearest to 0.2627 to be 0.2625, which stands in line with $S = 0.000055$. This is equal to a fall of 0.29 feet per mile.

Example: A channel in sandy soil is to carry 500 cubic feet per second at a velocity of 1.5 feet per second. The sideslopes are to be 3 : 1 and the slope of the channel 0.5 feet per mile, or

0.000095. What will be the depth of the channel and its bottom width?

In Table H, in column headed $K = 1.2$ and in line with $S = 0.000095$, we find $v = 0.345$. Dividing 1.5 by 0.345 the quotient is 4.347.

In Table E, under $K = 1.25$, we find the value of $d^{0.765}$ next above 4.347 to be 4.383, which stands in line with $d = 6.9$. Hence $R = 3.45$.

If the channel is given a depth of 4 feet, its mean width will be

$$\frac{333.3}{4} = 83.33 \text{ feet;}$$

its bottom width $83.33 - 4 \times 3 = 71.33$ feet;

its wet perimeter $71.33 + 2\sqrt{4^2 + (4 \times 3)^2} = 96.53$ feet;

its mean hydraulic radius $\dfrac{333.3}{96.53} = 3.45$ as above.

TABLE C.

SINES OF SLOPES AND ROOTS OF SINES OF SCOPES.

S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$	S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$
000025	002580	00309	00366	005	00050	01390	01583	01779	02236
000030	00284	00341	00402	00548	00055	01467	01668	01882	02346
000035	00311	00371	00437	00591	00060	01541	01748	01970	02450
000040	00336	00399	00469	00631	00065	01612	01826	02054	02550
000045	00359	00425	0050	00671	00070	01680	01902	02136	02645
000050	00381	00451	00528	00707	00075	01747	01975	02216	02739
000055	00402	00475	00556	00741	00080	01806	02045	02287	02830
000060	00422	00498	00582	00775	00085	01874	02114	02369	02916
000065	00441	00520	00607	00806	00090	01935	02181	02441	030
000070	00461	00541	00631	00836	00095	01995	02247	02511	03082
000075	00478	00562	00655	00866	001	02047	0231	02581	03163
000080	00496	00583	00677	00895	0011	02166	02424	02714	03316
000085	00513	00602	0070	00922	0012	02276	02552	02842	03464
000090	00530	00621	00721	00948	0013	02380	02665	02965	03605
000095	00544	00640	00742	00975	0014	02481	02775	03084	03742
0001	00562	00658	00762	010	0015	02579	02878	03199	03873
000125	00637	00743	008572	01118	0016	02647	02985	03310	040
00015	00706	00821	00945	01225	0017	02768	03077	03428	04121
000175	00770	00893	01003	01323	0018	02858	03183	03523	04243
00020	00831	00960	01123	01414	0019	02946	03278	03624	04359
000225	00887	01024	01172	015	0020	03033	03374	03725	04472
00025	00941	01085	01248	01581	0021	03117	03462	03822	04583
000275	00993	01140	01303	01658	0022	0320	03552	03918	04691
00030	01043	0120	01365	01732	0023	03280	03639	04011	04776
00035	01138	01301	01480	01871	0024	0336	03724	04103	04898
00040	01226	01442	01589	020	0025	03438	03808	04191	050
00045	01311	01474	01693	02122	0026	03515	03820	04280	05099

TABLE C—*Continued.*

S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$	S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$
0027	03590	03941	04367	05196	0080	06614	07182	07760	08944
0028	03665	04051	04452	05292	00825	06731	07303	07888	09083
0029	03737	04165	04535	05384	0085	06843	07423	08013	09193
0030	03810	04206	04617	05478	00875	06956	07542	08139	09357
0031	0388	04282	04698	05567	0090	07067	07659	0826	09413
0032	03951	04357	04779	05656	00925	07177	07774	0838	09618
0033	04019	04430	04856	05744	0095	07286	07888	08499	09747
0034	04087	04503	04934	05831	00975	07393	08001	08611	09805
0035	04155	04595	05010	05916	01	07499	08111	08733	10
0036	04227	04646	05092	060	01025	07604	08222	08848	10117
0037	04287	04716	05159	06083	0105	07708	08330	08962	10247
0038	04352	04785	05233	06164	01075	07818	08440	09032	10369
0039	04415	04853	05305	06245	0110	07912	08544	09185	10488
0040	04479	04921	05377	06310	01125	08012	08650	09295	10600
0041	04541	04987	05446	06402	01150	08113	08753	09361	10724
0042	04603	05053	05519	06481	01175	08211	08857	09512	10840
0043	04665	05109	05586	06557	0120	08309	08959	09620	10954
0044	04726	05187	05655	06633	01225	08406	09061	09722	11068
0045	04786	05248	05722	06709	01250	08502	09161	09828	1118
0046	04845	05314	05789	06782	01275	08597	09261	09939	11292
0047	04904	05374	05855	06855	0130	08691	09360	10034	11368
0048	04962	05435	05922	06928	01325	08785	09457	10136	11511
0049	05020	05497	05986	070	01350	08878	09554	10236	11619
0050	05078	05567	06051	07071	01375	08970	09650	10337	11726
0051	05135	05618	06115	07141	0140	09060	09745	10436	11832
0052	05191	05678	06178	07212	01425	09152	09840	10535	11937
0053	05247	05738	06241	07280	01450	09242	09934	10632	12042
0054	05303	05795	06301	07347	01475	09342	10026	10729	12145
0055	05357	05854	06364	07415	0150	09420	1012	10825	12248
0056	05412	05912	06425	07483	01525	09508	10211	10920	12349
0057	05466	05970	06486	07548	01550	09595	10302	11014	12450
0058	05520	06026	06545	07616	01575	09682	10392	11108	1255
0059	05574	06083	06605	07681	016	09767	10480	11199	1265
0060	05627	06139	06664	07746	01625	09854	10571	11293	1275
0061	05680	06193	06723	07810	01650	09939	10659	11385	1285
0062	05731	0625	06781	07874	01675	10024	10748	11476	1294
0063	05783	06304	06839	07937	0170	10107	10834	11566	1304
0064	05834	06359	06895	080	01725	10190	1092	11656	1313
0065	05886	06413	06953	08062	01750	10273	11007	11745	1323
0066	05936	06466	07009	08124	01775	10332	11092	11834	1331
0069	05986	0652	07064	08185	018	10437	11177	11921	1342
0068	06036	06573	07121	08246	01825	10519	11261	12009	1351
0069	06086	06625	07176	08307	01850	10600	11346	12096	1360
0070	06136	06677	07231	08367	01875	1068	11429	12182	1369
0071	06186	06739	07285	08427	0190	1076	11512	12268	1378
0072	06234	06781	07339	08485	01925	1088	11594	12353	1388
0073	06283	06832	07393	08544	01950	1092	11676	12437	1396
0074	06331	06883	07447	08602	01975	10946	11757	1252	1406
0075	06379	06933	07500	08660	020	11074	11812	1261	1414
0076	06426	06983	07552	08718	0205	11228	120	1277	1432
0077	06478	07033	07605	08775	021	11383	12157	1294	1450
0078	06521	07083	07657	08832	0215	11534	12315	1310	1466
0079	06568	07133	07709	08888	0220	11658	1247	1336	1483

TABLE C — *Concluded.*

S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$	S	$S^{\frac{9}{16}}$	$S^{\frac{6}{11}}$	$S^{\frac{9}{17}}$	$S^{\frac{1}{2}}$
0225	11842	1262	1343	150	075	2330	2434	2538	2739
0230	11980	1228	1357	1517	080	2415	2521	2626	2828
0235	12118	1293	1373	1533	085	2497	2606	2712	2916
0240	12270	1308	1388	1550	090	2581	2689	2795	30
0245	12414	1323	1404	1565	095	2644	2750	2859	3082
0250	12550	1337	1419	1581	10	2739	2848	2956	3163
030	1391	1478	1562	1732	20	4044	4157	4265	4583
035	1517	1606	1695	1871	30	5080	5185	5287	5478
040	1635	1728	1820	20	40	5973	6066	6157	6310
045	1748	1842	1931	2122	50	6771	6852	6928	7071
050	1854	1950	2048	2236	60	7503	7567	7631	7746
055	1956	2073	2153	2346	70	8182	8232	8279	8367
060	2008	2155	2255	2449	80	8820	8854	8899	8944
065	2155	2251	2353	2550	90	0 .7425	0 .9442	0 .9457	0 .9487
070	2240	2344	2447	2646	1 .00	1 .0	1 .0	1 .0	1 .0

TABLE D.

POWERS OF THE DIAMETERS OF CIRCULAR, SEMICIRCULAR AND EGG-SHAPED CONDUITS IN FEET.

Diameter in Inches.	$d^{0.66}$	$d^{2.66}$	$d^{0.67}$	$d^{2.67}$	$d^{0.68}$	$d^{2.68}$	$d^{0.695}$	$d^{2.695}$	$d^{0.7}$	$d^{2.7}$
1	0.0194	0.00135	0.0189	0.00134	0.01844	0.00128	0.0178	0.00123	0.0176	0.00192
2	0.3064	0.0085	0.3010	0.00835	0.2956	0.0082	0.2878	0.0080	0.2852	0.60772
3	0.4005	0.0150	0.3950	0.0246	0.3807	0.0243	0.3729	0.02385	0.3703	0.0235
4	0.4842	0.0538	0.4790	0.0532	0.4738	0.0526	0.4660	0.0518	0.4635	0.1511
5	0.5611	0.0974	0.5562	0.0965	0.5514	0.0957	0.5460	0.0941	0.5418	0.0949
6	0.6329	0.1582	0.6285	0.1571	0.6241	0.1561	0.6177	0.1544	0.6155	0.1534
7	0.7006	0.2584	0.6968	0.2371	0.6932	0.2358	0.6875	0.2339	0.6857	0.2335
8	0.7652	0.840	0.7620	0.3386	0.7590	0.3373	0.7543	0.3353	0.7529	0.3343
9	0.8271	0.4656	0.8247	0.4644	0.8223	0.4630	0.8187	0.4610	0.8175	0.4602
10	0.8866	0.6187	0.8810	0.6145	0.8834	0.6134	0.8809	0.6917	0.8801	0.6112
11	0.9442	0.7931	0.9433	0.7926	0.9424	0.7919	0.9413	0.7908	0.941	0.7904
12	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
13	1.0542	1.2370	1.055	1.238	1.056	1.239	1.057	1.2404	1.0576	1.241
14	1.107	1.507	1.1088	1.509	1.1105	1.511	1.1131	1.515	1.1139	1.516
15	1.159	1.810	1.1613	1.814	1.1639	1.819	1.1678	1.825	1.1691	1.827
16	1.209	2.150	1.2126	2.156	1.2161	2.162	1.2213	2.171	1.2239	2.175
18	1.307	2.941	1.312	2.952	1.317	2.964	1.325	2.983	1.328	2.988
20	1.401	3.892	1.408	3.912	1.415	3.931	1.426	3.961	1.430	3.971
22	1.492	5.012	1.501	5.045	1.511	5.075	1.524	5.122	1.528	5.138
24	1.580	6.321	1.591	6.364	1.602	6.409	1.619	6.475	1.625	6.498
26	1.666	7.820	1.679	7.881	1.692	7.942	1.711	8.035	1.718	8.066
28	1.749	9.524	1.764	9.605	1.780	9.687	1.802	9.810	1.810	9.852
30	1.831	11.443	1.848	11.547	1.866	11.552	1.890	11.815	1.899	11.870
32	1.910	13.585	1.929	13.720	1.948	13.854	1.977	14.06	1.987	14.128
34	1.988	15.96	2.009	16.13	2.030	16.30	2.062	16.56	2.073	16.64
36	2.065	18.59	2.088	18.79	2.111	19.0	2.146	19.31	2.158	19.42
38	2.140	21.46	2.165	21.71	2.190	21.97	2.214	22.36	2.240	22.46

TABLE D.—*Continued.*

Diameter in Inches.	$d^{0.66}$	$d^{2.66}$	$d^{0.67}$	$d^{2.67}$	$d^{0.68}$	$d^{2.68}$	$d^{0.695}$	$d^{2.695}$	$d^{0.7}$	$d^{2.7}$
40	2.214	24.60	2.241	24.90	2.268	25.20	2.316	25.67	2.324	25.81
42	2.286	28.0	2.315	28.36	2.344	28.71	2.388	29.26	2.404	29.44
44	2.358	31.63	2.388	32.11	2.419	32.53	2.467	33.17	2.483	33.39
46	2.427	35.67	2.460	36.15	2.493	36.63	2.544	37.38	2.561	37.63
48	2.496	39.25	2.531	40.51	2.567	41.07	2.621	41.93	2.038	42.22
50	2.565	44.53	2.612	45.17	2.639	45.82	2.696	46.87	2.795	47.15
52	2.632	49.43	2.671	50.16	2.711	50.89	2.771	52.03	2.791	52.41
54	2.698	59.65	2.739	55.47	2.781	56.32	2.844	57.60	2.866	58.03
56	2.764	60.19	2.807	61.13	2.851	62.08	2.917	63.53	2.940	64.02
58	2.829	66.08	2.874	67.13	2.919	68.20	2.989	69.83	3.013	70.38
60	2.893	72.32	2.940	73.50	2.988	74.69	3.060	76.51	3.085	77.13
62	2.956	78.91	3.005	80.22	3.055	81.55	3.131	83.58	3.157	84.26
64	3.019	82.0	3.070	83.38	3.121	84.80	3.201	86.95	3.228	87.68
66	3.081	91.07	3.124	94.79	3.187	96.42	3.270	98.92	3.298	99.76
68	3.143	100.92	3.198	102.78	3.253	104.48	3.339	107.23	3.368	108.16
70	3.203	108.98	3.260	110.91	3.318	112.89	3.408	115.92	3.438	116.94
72	3.262	117.46	3.322	119.58	3.382	121.75	3.444	125.05	3.505	126.18
78	3.440	145.3	3.505	148.0	3.571	150.9	3.673	155.1	3.707	186.6
84	3.612	177.0	3.683	180.5	3.755	184.0	3.867	189.5	3.904	191.3
90	3.780	212.7	3.857	217.0	3.936	221.4	4.057	228.2	4.098	230.5
96	3.945	252.5	4.028	258.0	4.112	263.2	4.246	271.5	4.287	274.4
102	4.106	296.7	4.195	303.1	4.286	309.6	4.425	319.7	4.473	323.2
108	4.264	345.3	4.359	353.1	4.455	360.9	4.604	393.0	4.655	377.1
114	4.318	399.7	4.519	407.9	4.622	417.1	4.781	431.5	4.835	436.4
120	4.571	457.1	4.678	467.8	4.786	478.6	4.955	495.5	5.011	501.1
126	4.720	520.4	4.832	532.8	4.947	545.5	5.125	565.0	5.186	571.8
132	4.868	589.0	4.986	604.7	5.107	619.3	5.294	642.0	5.356	648.2
138	5.012	662.9	5.136	679.0	5.263	696.1	5.460	722.1	5.527	730.9
144	5.155	742.4	5.265	761.0	5.418	780.0	5.624	809.8	5.694	819.6
150	5.296	827.5	5.432	848.7	5.576	870.4	5.786	904.0	5.859	915.5
156	5.423	916.4	5.564	940.0	5.708	964.6	5.931	1002.5	6.008	1005.4
162	5.572	1015.5	5.719	1042.3	5.870	1069.8	6.103	1112.5	6.184	1125.1
168	5.708	1118.7	5.861	1148.6	6.017	1179.3	6.260	1226.9	6.343	1243.2
174	5.842	1228.1	6.0	1261.0	6.162	1259.6	6.264	1348.6	6.501	1366.6
180	5.974	1344.0	6.137	1381.0	6.306	1419.6	6.567	1478.0	6.656	1497.8
186	6.104	1467	6.274	1507	6.448	1549	6.716	1614	6.808	1636
192	6.233	1596	6.409	1641	6.589	1687	6.869	1758	6.964	1783
198	6.361	1732	6.542	1781	6.728	1831	7.017	1910	7.116	1937
204	6.488	1875	6.675	1939	6.867	1985	7.164	2071	7.266	2100
210	6.613	2025	6.815	2084	7.003	2145	7.310	2238	7.415	2291
216	6.737	2183	6.935	2247	7.138	2313	7.454	2415	7.563	2450
222	6.86	2348	7.064	2417	7.272	2489	7.597	2600	7.710	2638
228	6.982	2579	7.191	2596	7.466	2673	7.740	2794	7.855	2836
234	7.103	2700	7.317	2782	7.538	2866	7.881	2997	7.999	3041
240	7.222	2889	7.442	2977	7.668	3067	8.021	3208	8.142	3256

TABLE E.

Powers of Depths of Water in the Form of Section most Favorable to Flow. Powers of Mean Hydraulic Radii in General.

d or *r* in Feet.	*m* = 0 .95	*m* = 0 .83	*m* = 0 .70	*m* = 0 .57	*m* = 0 .30	*m* = 0 .0	*K* = 1 .25	*K* = 1 .5	*K* = 2 .00
	$D^{0.67}$	$D^{0.68}$	$D^{0.69}$	$D^{0.70}$	$D^{0.735}$	$D^{0.75}$	$D^{0.765}$	$D^{0.775}$	$D^{0.795}$
	$R^{0.67}$	$R^{0.68}$	$R^{0.69}$	$R^{0.70}$	$R^{0.735}$	$R^{0.75}$	$R^{0.765}$	$R^{0.775}$	$R^{0.795}$
0 .05	0 .1344	0 .1304	0 .1265	0 .1228	0 .1123	0 .1057	0 .1010	0 .0981	0 .0924
0 .10	0 .2138	0 .2090	0 .2044	0 .1996	0 .1841	0 .1779	0 .1718	0 .1699	0 .1603
0 .15	0 .2805	0 .2752	0 .2713	0 .2650	0 .2480	0 .2410	0 .2343	0 .2298	0 .2163
0 .20	0 .3401	0 .3348	0 .3294	0 .3241	0 .3064	0 .2990	0 .2919	0 .2872	0 .2782
0 .25	0 .3951	0 .3896	0 .3842	0 .3789	0 .3609	0 .3535	0 .3463	0 .3415	0 .3322
0 .30	0 .4463	0 .4410	0 .4358	0 .4305	0 .4127	0 .4054	0 .3981	0 .3933	0 .3840
0 .35	0 .4949	0 .4897	0 .4847	0 .4796	0 .4622	0 .4550	0 .4480	0 .4432	0 .4340
0 .40	0 .5412	0 .5363	0 .5314	0 .5266	0 .510	0 .5030	0 .4961	0 .4915	0 .4827
0 .45	0 .5857	0 .5810	0 .5764	0 .5718	0 .5561	0 .5495	0 .5429	0 .5386	0 .5300
0 .50	0 .6285	0 .6241	0 .6198	0 .6156	0 .6008	0 .5946	0 .5885	0 .5844	0 .5763
0 .55	0 .670	0 .666	0 .662	0 .6580	0 .6444	0 .6389	0 .6330	0 .6292	0 .6218
0 .60	0 .7102	0 .7066	0 .7030	0 .6984	0 .6870	0 .6818	0 .6765	0 .6729	0 .6662
0 .65	0 .7493	0 .7461	0 .7429	0 .7397	0 .7286	0 .7239	0 .7192	0 .7162	0 .7100
0 .70	0 .7895	0 .7846	0 .7818	0 .7790	0 .7694	0 .7653	0 .7612	0 .7585	0 .7551
0 .75	0 .8247	0 .8223	0 .8199	0 .8176	0 .8094	0 .8059	0 .8025	0 .8001	0 .7956
0 .80	0 .8611	0 .8592	0 .8573	0 .8553	0 .8487	0 .8489	0 .8431	0 .8411	0 .8374
0 .85	0 .8968	0 .8955	0 .8939	0 .8924	0 .8874	0 .8853	0 .8831	0 .8815	0 .8788
0 .90	0 .9318	0 .9309	0 .930	0 .9289	0 .9254	0 .9246	0 .9226	0 .9216	0 .9196
0 .95	0 .9662	0 .9657	0 .9652	0 .9647	0 .9630	0 .9623	0 .9615	0 .9610	0 .9600
1 .0	1 .0	1 .0	1 .0	1 .0	1 .0	1 .0	1 .0	1 .0	1 .0
1 .05	1 .0332	1 .0337	1 .0342	1 .0347	1 .0365	1 .0373	1 .0380	1 .0385	1 .0396
1 .10	1 .0660	1 .067	1 .068	1 .0690	1 .0726	1 .0741	1 .0756	1 .0768	1 .0787
1 .15	1 .0982	1 .0997	1 .1012	1 .1028	1 .1082	1 .1105	1 .1128	1 .1144	1 .116
1 .20	1 .130	1 .132	1 .134	1 .1361	1 .1434	1 .1465	1 .1497	1 .1518	1 .156
1 .25	1 .1612	1 .1638	1 .1664	1 .1691	1 .1782	1 .1782	1 .1861	1 .1888	1 .1941
1 .30	1 .192	1 .195	1 .199	1 .2016	1 .2126	1 .2174	1 .2222	1 .225	1 .2319
1 .35	1 .223	1 .227	1 .230	1 .2338	1 .2468	1 .252	1 .258	1 .262	1 .269
1 .40	1 .244	1 .248	1 .253	1 .265	1 .281	1 .287	1 .294	1 .298	1 .307
1 .45	1 .283	1 .287	1 .292	1 .297	1 .314	1 .321	1 .329	1 .304	1 .343
1 .50	1 .312	1 .318	1 .323	1 .328	1 .347	1 .355	1 .364	1 .369	1 .380
1 .55	1 .341	1 .347	1 .353	1 .359	1 .380	1 .389	1 .398	1 .405	1 .417
1 .60	1 .370	1 .377	1 .383	1 .389	1 .413	1 .423	1 .433	1 .440	1 .453
1 .65	1 .399	1 .406	1 .413	1 .420	1 .445	1 .456	1 .467	1 .474	1 .489
1 .70	1 .427	1 .435	1 .442	1 .450	1 .477	1 .489	1 .501	1 .509	1 .525
1 .75	1 .455	1 .463	1 .471	1 .480	1 .509	1 .521	1 .534	1 .543	1 .561
1 .80	1 .483	1 .491	1 .500	1 .509	1 .540	1 .554	1 .568	1 .577	1 .595
1 .85	1 .510	1 .520	1 .529	1 .538	1 .572	1 .586	1 .601	1 .611	1 .631
1 .90	1 .537	1 .547	1 .557	1 .569	1 .603	1 .618	1 .634	1 .644	1 .665
1 .95	1 .564	1 .575	1 .586	1 .596	1 .634	1 .650	1 .667	1 .717	1 .699
2 .0	1 .589	1 .600	1 .611	1 .624	1 .664	1 .681	1 .700	1 .711	1 734
2 .05	1 .618	1 .629	1 .641	1 .653	1 .695	1 .713	1 .732	1 .744	1 .769
2 .10	1 .644	1 .657	1 .669	1 .681	1 .725	1 .745	1 .764	1 .777	1 .804
2 .15	1 .670	1 .683	1 .696	1 .709	1 .755	1 .776	1 .796	1 .810	1 .837
2 .20	1 .696	1 .710	1 .723	1 .736	1 .785	1 .809	1 .828	1 .842	1 .861

TABLE E. — *Continued.*

d or r in feet.	m = 0.95	m = 0.83	m = 0.70	m = 0.57	m = 0.30	m = 0.0	K = 1.25	K = 1.50	K = 2.00
	$D^{0.67}$	$D^{0.68}$	$D^{0.69}$	$D^{0.70}$	$D^{0.735}$	$D^{0.75}$	$D^{0.765}$	$D^{0.775}$	$D^{0.795}$
	$R^{0.67}$	$R^{0.68}$	$R^{0.69}$	$R^{0.70}$	$R^{0.735}$	$R^{0.75}$	$R^{0.765}$	$R^{0.775}$	$R^{0.795}$
2.25	1.718	1.732	1.746	1.764	1.815	1.836	1.860	1.875	1.905
2.30	1.747	1.762	1.777	1.792	1.844	1.867	1.891	1.907	1.939
2.35	1.773	1.788	1.803	1.818	1.874	1.898	1.922	1.939	1.973
2.40	1.798	1.814	1.830	1.846	1.903	1.928	1.954	1.971	2.006
2.45	1.826	1.839	1.856	1.872	1.932	1.958	1.985	2.003	2.038
2.50	1.848	1.864	1.882	1.899	1.961	1.960	2.016	2.034	2.069
2.55	1.872	1.890	1.908	1.926	1.990	2.018	2.046	2.065	2.104
2.60	1.897	1.915	1.934	1.952	2.019	2.047	2.077	2.097	2.137
2.65	1.921	1.940	1.959	1.978	2.048	2.077	2.108	2.138	2.169
2.70	1.946	1.965	1.984	2.004	2.075	2.106	2.136	2.160	2.203
2.75	1.970	1.990	2.010	2.030	2.103	2.135	2.168	2.190	2.212
2.80	1.994	2.014	2.035	2.056	2.131	2.165	2.198	2.221	2.267
2.85	2.017	2.038	2.060	2.081	2.159	2.194	2.228	2.252	2.306
2.90	2.041	2.063	2.085	2.107	2.187	2.223	2.258	2.282	2.331
2.95	2.064	2.089	2.110	2.132	2.215	2.251	2.288	2.313	2.363
3.0	2.088	2.111	2.134	2.157	2.242	2.279	2.317	2.343	2.395
3.05	2.111	2.135	2.159	2.183	2.270	2.308	2.347	2.373	2.426
3.10	2.134	2.159	2.183	2.208	2.297	2.337	2.376	2.403	2.458
3.15	2.159	2.182	2.207	2.233	2.324	2.364	2.406	2.433	2.488
3.20	2.180	2.205	2.231	2.258	2.351	2.393	2.435	2.463	2.521
3.25	2.201	2.229	2.255	2.282	2.378	2.421	2.464	2.493	2.551
3.30	2.225	2.252	2.279	2.307	2.421	2.448	2.493	2.522	2.582
3.35	2.248	2.275	2.303	2.331	2.432	2.476	2.521	2.552	2.614
3.40	2.270	2.298	2.327	2.355	2.458	2.504	2.550	2.580	2.645
3.45	2.293	2.323	2.354	2.379	2.485	2.531	2.579	2.611	2.677
3.50	2.315	2.344	2.374	2.404	2.511	2.559	2.608	2.640	2.707
3.55	2.337	2.367	2.397	2.428	2.538	2.586	2.636	2.666	2.738
3.60	2.359	2.389	2.420	2.451	2.564	2.613	2.664	2.699	2.769
3.65	2.381	2.412	2.443	2.476	2.590	2.641	2.693	2.725	2.799
3.70	2.403	2.434	2.467	2.499	2.616	2.668	2.720	2.755	2.830
3.75	2.424	2.457	2.489	2.522	2.642	2.695	2.749	2.785	2.858
3.80	2.446	2.479	2.512	2.546	2.668	2.722	2.777	2.814	2.890
3.85	2.468	2.501	2.536	2.590	2.693	2.748	2.804	2.843	2.920
3.90	2.489	2.523	2.558	2.593	2.719	2.775	2.833	2.871	2.951
3.95	2.510	2.545	2.580	2.616	2.745	2.802	2.860	2.900	2.980
4.0	2.531	2.567	2.603	2.639	2.770	2.828	2.888	2.928	3.01
4.05	2.552	2.588	2.626	2.662	2.796	2.855	2.915	2.956	3.04
4.10	2.573	2.610	2.647	2.685	2.821	2.881	2.943	2.985	3.07
4.15	2.595	2.632	2.670	2.708	2.846	2.908	2.970	3.013	3.10
4.20	2.616	2.653	2.692	2.731	2.871	2.934	2.998	3.041	3.13
4.25	2.637	2.675	2.714	2.753	2.897	2.960	3.025	3.070	3.16
4.30	2.657	2.696	2.736	2.776	2.921	2.986	3.052	3.097	3.19
4.35	2.678	2.717	2.758	2.798	2.946	3.012	3.080	3.125	3.219
4.40	2.698	2.739	2.780	2.821	2.971	3.038	3.107	3.153	3.247
4.45	2.719	2.760	2.801	2.844	2.997	3.064	3.134	3.180	3.277
4.50	2.740	2.781	2.823	2.866	3.021	3.090	3.160	3.208	3.306
4.55	2.760	2.802	2.845	2.888	3.045	3.115	3.187	3.235	3.335
4.60	2.780	2.823	2.866	2.910	3.070	3.140	3.214	3.263	3.364

TABLE E. — *Continued.*

d or r in feet.	$m =$ 0.95	$m =$ 0.83	$m =$ 0.70	$m =$ 0.57	$m =$ 0.30	$m =$ 0.0	$K =$ 1.25	$K =$ 1.50	$K =$ 20
	$D^{0.67}$	$D^{0.68}$	$D^{0.69}$	$D^{0.70}$	$D^{0.735}$	$D^{0.75}$	$D^{0.765}$	$D^{0.775}$	$D^{0.795}$
	$R^{0.67}$	$R^{0.68}$	$R^{0.69}$	$R^{0.70}$	$R^{0.735}$	$R^{0.75}$	$R^{0.765}$	$R^{0.775}$	$R^{0.795}$
4.65	2.800	2.844	2.888	2.932	3.094	3.166	3.240	3.290	3.393
4.70	2.820	2.864	2.909	2.954	3.119	3.192	3.267	3.318	3.422
4.75	2.840	2.885	2.930	2.976	3.143	3.218	3.294	3.345	3.451
4.80	2.860	2.906	2.952	2.998	3.163	3.243	3.320	3.373	3.480
4.85	2.880	2.926	2.973	3.020	3.192	3.268	3.346	3.404	3.509
4.90	2.900	2.947	2.994	3.042	3.216	3.295	3.373	3.427	3.538
4.95	2.920	2.967	3.015	3.064	3.240	3.318	3.397	3.454	3.567
5.00	2.940	2.987	3.036	3.085	3.264	3.344	3.425	3.481	3.595
5.1	2.980	3.028	3.078	3.128	3.312	3.393	3.478	3.534	3.652
5.2	3.018	3.068	3.119	3.171	3.360	3.444	3.530	3.589	3.709
5.3	3.057	3.108	3.160	3.214	3.407	3.493	3.582	3.642	3.765
5.4	3.095	3.148	3.201	3.256	3.454	3.542	3.633	3.695	3.822
5.5	3.134	3.188	3.243	3.298	3.501	3.591	3.684	3.748	3.878
5.6	3.171	3.227	3.283	3.340	3.548	3.640	3.735	3.818	3.933
5.7	3.210	3.266	3.323	3.382	3.594	3.689	3.787	3.853	3.990
5.8	3.249	3.307	3.365	3.423	3.640	3.737	3.837	3.908	4.045
5.9	3.284	3.343	3.403	3.464	3.686	3.786	3.888	3.958	4.101
6.0	3.322	3.382	3.443	3.505	3.732	3.834	3.938	4.009	4.156
6.1	3.358	3.420	3.482	3.546	3.778	3.882	3.988	4.061	4.211
6.2	3.396	3.458	3.521	3.586	3.823	3.929	4.038	4.113	4.265
6.3	3.432	3.496	3.561	3.627	3.868	3.977	4.087	4.164	4.320
6.4	3.469	3.534	3.600	3.667	3.913	4.604	4.138	4.215	4.375
6.5	3.505	3.571	3.638	3.707	3.958	4.071	4.187	4.265	4.429
6.6	3.541	3.608	3.677	3.747	4.003	4.118	4.236	4.317	4.483
6.7	3.576	3.645	3.715	3.787	4.047	4.164	4.285	4.367	4.537
6.8	3.612	3.682	3.754	3.826	4.091	4.211	4.334	4.417	4.590
6.9	3.648	3.719	3.792	3.865	4.136	4.257	4.383	4.468	4.644
7.0	3.683	3.755	3.829	3.905	4.180	4.304	4.431	4.518	4.698
7.1	3.718	3.792	3.867	3.944	4.223	4.350	4.480	4.567	4.751
7.2	3.753	3.828	3.904	3.982	4.267	4.396	4.528	4.618	4.804
7.3	3.788	3.864	3.942	4.021	4.311	4.441	4.576	4.667	4.857
7.4	3.822	3.900	3.979	4.069	4.354	4.487	4.623	4.717	4.910
7.5	3.857	3.936	4.016	4.097	4.397	4.532	4.671	4.766	4.962
7.6	3.892	3.972	4.053	4.136	4.439	4.577	4.719	4.816	5.014
7.7	3.926	4.007	4.090	4.174	4.483	4.622	4.766	4.865	5.066
7.8	3.960	4.042	4.126	4.212	4.526	4.667	4.813	4.913	5.119
7.9	3.994	4.077	4.163	4.250	4.568	4.710	4.860	4.962	5.171
8.0	4.028	4.113	4.199	4.278	4.611	4.754	4.908	5.010	5.223
8.1	4.061	4.147	4.235	4.325	4.653	4.801	4.954	5.059	5.275
8.2	4.095	4.182	4.271	4.362	4.696	4.847	5.002	5.108	5.328
8.3	4.128	4.217	4.306	4.399	4.737	4.890	5.047	5.155	5.379
8.4	4.162	4.251	4.343	4.436	4.779	4.934	5.094	5.204	5.430
8.5	4.195	4.286	4.378	4.473	4.821	4.978	5.141	5.251	5.481
8.6	4.228	4.320	4.414	4.516	4.862	5.022	5.184	5.300	5.532
8.7	4.261	4.354	4.449	4.547	4.904	5.066	5.233	5.348	5.584
8.8	4.294	4.388	4.484	4.583	4.945	5.109	5.279	5.395	5.635
8.9	4.326	4.422	4.519	4.619	4.987	5.153	5.324	5.442	5.686
9.0	4.359	4.455	4.555	4.656	5.028	5.196	5.370	5.490	5.736

TABLE E.—*Concluded.*

d or r in Feet.	$m =$ 0.95	$m =$ 0.83	$m =$ 0.70	$m =$ 0.57	$m =$ 0.30	$m =$ 0.0	$K =$ 1.25	$K =$ 1.50	$K =$ 2.00
	$D^{0.67}$	$D^{0.68}$	$D^{0.69}$	$D^{0.70}$	$D^{0.735}$	$D^{0.75}$	$D^{0.765}$	$D^{0.775}$	$D^{0.795}$
	$R^{0.67}$	$R^{0.68}$	$R^{0.69}$	$R^{0.70}$	$R^{0.735}$	$R^{0.75}$	$R^{0.765}$	$R^{0.775}$	$R^{0.795}$
9.1	4.391	4.489	4.589	4.692	5.069	5.239	5.416	5.537	5.786
9.2	4.423	4.522	4.624	4.728	5.110	5.282	5.461	5.584	5.837
9.3	4.455	4.556	4.659	4.764	5.150	5.325	5.506	5.631	5.888
9.4	4.487	4.589	4.693	4.799	5.191	5.368	5.552	5.678	5.938
9.5	4.519	4.622	4.728	4.835	5.232	5.411	5.597	5.725	5.989
9.6	4.551	4.655	4.762	4.871	5.272	5.454	5.642	5.772	6.039
9.7	4.583	4.688	4.796	4.906	5.312	5.496	5.687	5.818	6.089
9.8	4.615	4.721	4.831	4.942	5.352	5.539	5.732	5.864	6.138
9.9	4.646	4.754	4.865	4.977	5.393	5.582	5.779	5.910	6.188
10.0	4.678	4.787	4.898	5.012	5.433	5.624	5.821	5.957	6.238
10.5	4.833	4.948	5.065	5.186	5.631	5.833	6.042	6.186	6.484
11.0	4.986	5.107	5.231	5.358	5.827	6.040	6.261	6.413	6.728
11.5	5.137	5.263	5.394	5.527	6.020	6.245	6.478	6.638	6.969
12.0	5.285	5.418	5.554	5.695	6.211	6.447	6.692	6.861	7.210
12.5	5.432	5.570	5.713	5.859	6.401	6.648	6.905	7.082	7.437
13.0	5.576	5.721	5.870	6.022	6.588	6.846	7.115	7.300	7.684
13.5	5.719	5.870	6.025	6.183	6.773	7.043	7.323	7.516	7.907
14.0	5.860	6.017	6.178	6.343	6.957	7.238	7.530	7.731	8.150
14.5	6.000	6.162	6.329	6.501	7.139	7.430	7.735	7.945	8.380
15.0	6.137	6.306	6.479	6.657	7.319	7.622	7.938	8.156	8.610
15.5	6.274	6.448	6.627	6.812	7.497	7.812	8.140	8.369	8.837
16.0	6.409	6.589	6.777	6.964	7.674	8.000	8.340	8.574	9.063
16.5	6.542	6.728	7.016	7.116	7.850	8.187	8.538	8.782	9.287
17.0	6.674	6.866	7.064	7.266	8.024	8.372	8.736	8.986	9.511
17.5	6.805	7.003	7.206	7.415	8.197	8.556	8.932	9.191	9.732
18.0	6.935	7.138	7.347	7.563	8.368	8.739	9.125	9.395	9.953
18.5	7.063	7.272	7.488	7.709	8.538	8.920	9.320	9.596	10.172
19.0	7.191	7.405	7.626	7.855	8.707	9.101	9.512	9.796	10.390
19.5	7.317	7.538	7.765	7.999	8.875	9.280	9.702	9.996	10.610
20.0	7.442	7.668	7.902	8.142	9.042	9.410	9.892	10.192	10.830
21.0	7.689	7.927	8.172	8.425	9.372	9.810	10.268	10.585	11.25
22.0	7.806	8.182	8.439	8.704	9.698	10.168	10.640	10.974	11.674
23.0	8.173	8.433	8.702	8.979	10.020	10.502	11.008	11.359	12.094
24.0	8.409	8.680	8.961	9.250	10.338	10.843	11.373	11.739	12.510
25.0	8.642	8.925	9.217	9.518	10.653	11.180	11.734	12.117	12.920

TABLE F.

POWERS OF THE DEPTHS OF WATER IN THE FORM OF SECTION MOST FAVORABLE TO FLOW.

d in Feet.	m= 0.95	m= 0.83	m= 0.70	m= 0.57	m= 0.30	m= 0.0	K= 1.25	K= 1.5	K= 2.0
	$d^{2.67}$	$d^{2.68}$	$d^{2.69}$	$d^{2.70}$	$d^{2.735}$	$d^{2.75}$	$d^{2.765}$	$d^{2.775}$	$d^{2.795}$
0.05	.000336	.000326	.000316	.000307	.000281	.000264	.000252	.000245	.000231
0.10	.00214	.00209	.00204	.00200	.001841	.00178	.001919	.00168	.001604
0.15	.00631	.00619	.00610	.00596	.00558	.00542	.00527	.00517	.00428
0.20	.01361	.01339	.01311	.01292	.01226	.01196	.01168	.01150	.01113
0.25	.02469	.02435	.02401	.02368	.02256	.02210	.02164	.02134	.02076
0.30	.04017	.03969	.03921	.03875	.03715	.03648	.03583	.03540	.03456
0.35	.06063	.06000	.05936	.05875	.05663	.05574	.05488	.05430	.05317
0.40	.08660	.08581	.08502	.08425	.08159	.08048	.07939	.07866	.07723
0.45	.1186	.1176	.1167	.1158	.1126	.1113	.1099	.1091	.1093
0.50	.1571	.1560	.1550	.1534	.1502	.1487	.1471	.1461	.1441
0.55	.2026	.2015	.2003	.1990	.1949	.1932	.1914	.1903	.1881
0.60	.2557	.2544	.2531	.2518	.2473	.2454	.2435	.2421	.2398
0.65	.3163	.3152	.3139	.3125	.3078	.3058	.3039	.3026	.3000
0.70	.3859	.3845	.3831	.3817	.3770	.3750	.3730	.3717	.3690
0.75	.4639	.4626	.4613	.4600	.4553	.4533	.4514	.4501	.4475
0.80	.5511	.5428	.5487	.5475	.5432	.5413	.5396	.5383	.5360
0.85	.6480	.6469	.6459	.6449	.6412	.6396	.6380	.6370	.6345
0.90	.7552	.7542	.7532	.7522	.7496	.7484	.7472	.7465	.7450
0.95	.8720	.8716	.8711	.8707	.8691	.8684	.8678	.8672	.8664
1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
1.05	1.1391	1.1397	1.1402	1.1408	1.1428	1.1436	1.1444	1.1450	1.1461
1.10	1.290	1.291	1.292	1.294	1.298	1.300	1.302	1.303	1.305
1.15	1.452	1.454	1.456	1.458	1.465	1.469	1.472	1.474	1.478
1.20	1.627	1.630	1.633	1.636	1.647	1.651	1.656	1.659	1.665
1.25	1.814	1.819	1.823	1.827	1.841	1.847	1.853	1.858	1.866
1.30	2.015	2.020	2.025	2.030	2.050	2.059	2.066	2.071	2.082
1.35	2.228	2.235	2.242	2.249	2.272	2.283	2.293	2.30	2.314
1.40	2.456	2.464	2.472	2.481	2.510	2.523	2.535	2.544	2.561
1.45	2.697	2.707	2.717	2.727	2.763	2.778	2.794	2.804	2.825
1.50	2.952	2.964	2.976	2.988	3.031	3.050	3.068	3.081	3.106
1.55	3.222	3.236	3.251	3.265	3.316	3.338	3.359	3.377	3.404
1.60	3.508	3.524	3.540	3.557	3.616	3.641	6.668	3.685	3.720
1.65	3.808	3.827	3.846	3.866	3.934	3.954	3.994	4.014	4.054
1.70	4.124	4.146	4.168	4.190	4.268	4.303	4.337	4.360	4.407
1.75	4.456	4.481	4.506	4.531	4.621	4.660	4.699	4.726	4.778
1.80	4.804	4.832	4.860	4.889	4.991	5.035	5.080	5.110	5.170
1.85	5.168	5.200	5.232	5.265	5.379	5.429	5.479	5.513	5.582
1.90	5.550	5.585	5.621	5.658	5.786	5.842	5.899	5.937	6.013
1.95	5.962	6.002	6.042	6.068	6.212	6.274	6.338	6.380	6.466
2.0	6.356	6.400	6.444	6.498	6.643	6.727	6.797	6.845	6.940
2.05	6.798	6.847	6.896	6.947	7.123	7.198	7.278	7.330	7.436
2.10	7.250	7.304	7.358	7.413	7.608	7.693	7.780	7.837	7.955
2.15	7.720	7.779	7.839	7.899	8.114	8.208	8.302	8.366	8.495
2.20	8.209	8.273	8.339	8.405	8.640	8.743	8.847	8.917	9.059
2.25	8.696	8.767	8.838	8.931	9.188	9.301	9.415	9.464	9.646
2.30	9.243	9.321	9.398	9.477	9.757	9.880	10.004	10.111	10.256
2.35	9.790	9.873	9.958	10.044	10.348	10.482	10.616	10.708	10.892

TABLE F.— *Continued.*

d in Feet.	$m=$ 0.95	$m=$ 0.83	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.25	$K=$ 1.5	$K=$ 2.0
	$d^{2.67}$	$d^{2.68}$	$d^{2.69}$	$d^{2.70}$	$d^{2.735}$	$d^{2.75}$	$d^{2.765}$	$d^{2.775}$	$d^{2.795}$
2.40	10.355	10.446	10.538	10.631	10.952	11.106	11.253	11.352	11.553
2.45	10.141	11.040	11.139	11.265	11.597	11.754	11.913	12.021	12.276
2.50	11.548	11.654	11.761	11.870	12.256	12.426	12.600	12.710	12.950
2.55	12.175	12.289	12.405	12.52	12.92	13.12	13.21	13.43	13.69
2.60	12.82	12.95	13.07	13.20	13.34	13.84	14.04	14.18	14.45
2.65	13.49	13.63	13.76	13.89	14.37	14.59	14.80	14.94	15.27
2.70	14.18	14.32	14.47	14.63	15.13	15.35	15.59	15.74	16.06
2.75	14.90	15.05	15.20	15.35	15.91	16.15	16.40	16.56	16.90
2.80	15.63	15.79	15.95	16.12	16.71	16.97	17.23	17.41	17.77
2.85	16.88	16.56	16.73	16.91	17.54	17.82	18.10	18.29	18.68
2.90	17.16	17.35	17.53	17.72	18.39	18.69	18.99	19.19	19.61
2.95	17.97	18.16	18.36	18.56	19.27	19.59	19.91	20.12	20.56
3.0	18.79	19.00	19.21	19.42	20.18	20.51	20.86	21.09	21.56
3.05	19.64	19.86	20.08	20.31	21.11	21.47	21.83	22.08	22.57
3.10	20.51	20.75	20.98	21.22	22.09	22.45	22.84	23.10	23.62
3.15	21.40	21.65	21.90	22.15	23.06	23.46	23.87	24.14	24.70
3.20	22.32	22.58	22.85	23.12	24.08	24.50	24.93	25.22	25.81
3.25	23.27	23.54	23.82	24.10	25.11	25.56	26.02	26.33	26.96
3.30	24.24	24.53	24.82	25.11	26.19	26.67	27.14	27.47	28.14
3.35	25.23	25.53	25.84	26.16	27.29	27.79	28.30	28.64	29.34
3.40	26.26	26.59	26.98	27.23	28.42	28.95	29.48	29.84	30.58
3.45	27.29	27.65	28.02	28.32	29.58	30.13	30.69	31.08	31.86
3.50	28.36	28.72	29.00	29.44	30.76	31.34	31.94	32.34	33.16
3.55	29.45	29.83	30.21	30.59	31.98	32.60	33.22	33.60	34.50
3.60	30.57	30.97	31.37	31.77	33.23	33.87	34.53	34.97	35.88
3.65	31.72	32.13	32.55	32.97	34.50	35.18	35.87	36.34	37.29
3.70	32.89	33.33	33.76	34.21	35.81	36.52	37.25	37.74	38.74
3.75	34.09	34.55	35.01	35.47	37.15	37.89	38.65	39.17	40.22
3.80	35.32	35.80	36.26	36.80	38.53	39.30	40.10	40.63	41.73
3.85	36.57	37.07	37.57	38.08	39.93	40.74	40.62	42.14	43.29
3.90	39.86	38.37	38.90	39.43	41.36	42.21	43.08	43.67	44.88
3.95	39.17	39.71	40.26	40.82	42.82	43.42	44.62	45.24	46.50
4.0	40.51	41.07	41.64	42.22	44.34	45.25	46.20	46.85	48.17
4.05	41.87	42.46	43.06	43.67	45.85	46.82	47.82	48.49	49.87
4.10	43.27	43.88	44.50	45.14	47.42	48.43	49.47	50.17	51.61
4.15	44.89	45.54	46.19	46.64	49.02	50.08	51.15	52.13	53.39
4.20	46.14	46.81	47.48	48.17	50.65	51.76	52.88	53.65	55.20
4.25	47.62	48.38	49.02	49.74	52.32	53.46	54.64	55.46	57.06
4.30	49.00	49.85	50.59	51.33	54.02	55.21	56.44	57.26	58.96
4.35	50.67	51.42	52.19	53.06	55.95	57.00	58.27	59.12	60.89
4.40	52.24	53.02	53.81	54.62	57.52	58.81	60.14	61.03	62.87
4.45	53.84	54.66	55.47	56.31	59.33	60.68	62.05	62.98	64.89
4.50	55.47	56.31	57.17	58.03	61.17	62.57	63.99	64.96	66.25
4.55	57.13	58.01	58.89	59.79	63.05	64.50	65.98	66.98	69.05
4.60	58.83	59.73	60.65	61.58	64.96	66.46	68.00	69.05	71.19
4.65	60.44	61.37	62.32	63.41	66.91	68.47	70.07	71.02	73.37
4.70	62.30	63.28	64.26	65.26	68.89	70.51	72.17	73.29	75.60
4.75	64.09	65.09	66.12	67.15	70.92	72.76	74.31	75.48	77.87
4.80	65.91	66.95	68.00	69.08	72.98	74.72	76.49	77.70	80.18

TABLE F.—*Continued.*

d in Feet.	$m=$ 0.95	$m=$ 0.83	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.25	$K=$ 1.5	$K=$ 2.0
	$d^{2.67}$	$d^{2.68}$	$d^{2.69}$	$d^{2.70}$	$d^{2.735}$	$d^{2.75}$	$d^{2.765}$	$d^{2.775}$	$d^{2.795}$
4.85	67.75	68.83	69.93	71.04	75.08	76.88	78.72	80.06	82.54
4.90	69.65	70.75	71.88	73.05	77.21	79.08	80.98	82.28	84.94
4.95	71.54	72.70	73.87	75.07	79.39	81.31	83.29	84.63	87.38
5.0	73.50	74.69	75.90	77.00	81.60	83.59	85.63	87.03	89.87
5.0	77.49	78.77	80.06	81.37	86.14	88.27	90.46	91.92	94.99
5.2	81.61	82.96	84.34	85.25	90.85	93.12	95.44	96.85	100.3
5.3	85.87	87.31	88.78	90.27	95.69	98.12	100.61	102.30	105.77
5.4	90.26	91.80	93.36	94.94	100.71	103.29	105.94	107.89	111.44
5.5	94.79	96.43	98.09	99.76	105.90	108.67	111.46	113.37	117.31
5.6	99.46	101.20	102.95	104.74	111.25	114.16	116.65	119.74	123.34
5.7	104.08	105.80	107.79	109.87	116.77	119.85	123.02	125.2	129.6
5.8	109.50	111.44	113.42	115.15	122.45	125.7	129.1	121.7	126.1
5.9	114.3	116.4	118.5	120.6	128.3	131.8	135.5	137.8	142.7
6.0	119.6	121.7	123.9	126.2	134.4	138.0	141.8	144.3	149.6
6.1	125.0	127.2	129.6	132.0	140.6	144.4	148.4	150.9	156.7
6.2	130.5	132.9	135.3	137.9	147.0	151.0	155.2	158.1	164.0
6.3	136.2	128.8	141.7	144.0	153.5	157.8	162.3	165.3	171.5
6.4	142.0	144.7	147.4	150.2	160.3	164.8	169.5	172.6	179.2
6.5	148.1	150.9	153.7	156.7	167.2	172.0	176.9	180.2	187.1
6.6	154.2	157.2	160.1	163.2	174.4	179.4	184.1	188.0	195.2
6.7	160.5	163.5	166.8	170.0	181.7	187.0	192.4	195.9	203.6
6.8	167.0	170.3	173.6	176.9	189.2	194.7	200.4	204.3	212.2
6.9	173.7	177.1	180.4	184.1	196.9	202.7	213.5	212.7	221.1
7.0	180.5	184.0	187.6	191.3	204.8	210.9	217.1	226.5	230.2
7.1	187.4	191.1	194.9	198.8	212.9	219.2	225.8	230.2	239.5
7.2	193.7	197.5	202.4	206.5	221.2	227.9	234.7	239.4	249.0
7.3	201.9	205.9	210.0	214.2	229.7	236.7	243.8	248.7	258.8
7.4	209.3	213.6	217.9	222.3	238.4	245.7	253.2	258.3	268.9
7.5	217.0	221.4	225.9	230.5	247.3	254.9	262.8	268.1	279.1
7.6	224.8	229.4	234.1	238.9	256.5	264.4	272.5	278.1	289.8
7.7	232.8	237.6	242.4	247.5	265.8	274.1	282.6	288.5	300.4
7.8	240.9	245.9	251.0	256.3	275.3	284.0	292.7	298.9	311.5
7.9	249.3	254.5	259.8	265.2	285.1	294.1	303.4	309.7	322.7
8.0	257.8	263.2	268.7	274.4	295.1	304.2	314.1	321.1	334.3
8.1	266.5	272.1	277.9	283.7	305.3	315.0	325.1	331.9	346.1
8.2	275.3	281.2	287.2	290.8	315.8	325.9	330.6	343.4	358.3
8.3	284.4	290.4	296.7	303.1	326.3	336.9	347.7	355.2	370.5
8.4	293.6	300.0	306.4	313.0	337.2	348.2	359.5	367.2	383.1
8.5	303.0	309.6	316.3	323.2	348.3	359.6	371.4	379.4	396.0
8.6	312.7	319.5	326.4	333.5	359.6	371.4	383.6	391.1	409.2
8.7	322.5	329.6	336.8	344.1	371.2	383.4	396.0	404.7	422.6
8.8	332.5	339.8	347.3	354.9	383.0	395.7	408.8	417.8	436.4
8.9	341.9	349.4	358.0	365.9	395.0	408.2	421.7	431.1	450.3
9.0	353.	360.9	368.9	377.1	407.2	420.9	435.0	444.7	464.6
9.1	363.6	371.7	380.1	388.5	419.7	433.9	448.4	458.5	479.2
9.2	374.1	382.8	382.4	400.2	432.5	447.1	462.2	492.6	494.1
9.3	385.3	394.0	402.9	412.0	445.5	460.6	476.3	487.0	509.3
9.4	396.5	405.5	414.7	424.1	458.7	474.4	490.6	501.7	524.7
9.5	407.9	417.2	426.6	436.4	472.1	488.4	505.2	516.6	540.5
9.6	419.5	429.3	438.9	448.9	485.9	502.6	520.0	531.9	556.5

TABLE F.—*Concluded.*

d in Feet.	m= 0.95	m= 0.83	m= 0.70	m= 0.57	m= 0.30	m= 0.0	K= 1.25	K= 1.5	K= 2.0
	$d^{2.67}$	$d^{2.68}$	$d^{2.69}$	$d^{2.70}$	$d^{2.735}$	$d^{2.75}$	$d^{2.765}$	$d^{2.775}$	$d^{2.795}$
9.7	431.2	441.1	451.3	461.6	499.9	517.2	535.1	547.4	572.8
9.8	443.4	453.6	464.0	474.6	514.1	531.9	550.5	563.3	589.5
9.9	455.4	466.0	476.8	487.8	528.6	547.1	566.5	579.3	606.6
10.0	467.8	478.7	489.8	501.2	543.3	562.4	582.2	595.7	623.8
10.5	532.8	545.5	558.4	571.8	620.8	658.1	666.6	682.0	714.9
11.0	603.3	617.9	632.9	648.4	705.1	730.9	757.6	776.0	814.1
11.5	679.3	696.1	713.3	730.9	778.1	827.8	856.9	878.0	921.8
12.0	761.1	780.2	799.7	820.1	894.5	928.4	963.7	987.9	1038.3
12.5	848.7	870.4	892.6	915.5	1000.1	1038.7	1028.8	1106.4	1163.8
13.0	942.4	966.9	992.0	1017.8	1110.4	1157.0	1202.3	1233.7	1299.0
13.5	1042.3	1069.8	1098.0	1135.0	1234.4	1284.0	1335.0	1370.0	1443.0
14.0	1148.2	1179.3	1211.0	1243.0	1363.0	1419.0	1476.0	1516.0	1597.0
14.5	1261.0	1296.0	1330.0	1399.0	1501.0	1562.0	1626.0	1670.0	1762.0
15.0	1381.0	1419.0	1458.0	1498.0	1647.0	1715.0	1786.0	1835.0	1937.0
15.5	1509.0	1549.0	1592.0	1637.0	1801.0	1877.0	1956.0	2010.0	2125.0
16.0	1641.0	1687.0	1734.0	1783.0	1965.0	2048.0	2135.0	2195.0	2320.0
16.5	1781.0	1832.0	1884.0	1937.0	2137.0	2229.0	2325.0	2391.0	2528.0
17.0	1929.0	1971.0	2041.0	2100.0	2319.0	2420.0	2525.0	2597.0	2749.0
17.5	2084.0	2145.0	2207.0	2271.0	2569.0	2620.0	2135.0	2815.0	2980.0
18.0	2247.0	2313.0	2381.0	2450.0	2711.0	2832.0	2959.0	3044.0	3225.0
18.5	2417.0	2489.0	2563.0	2639.0	2922.0	3053.0	3264.0	3284.0	3481.0
19.0	2596.0	2673.0	2753.0	2836.0	3143.0	3285.0	3434.0	3536.0	3751.0
19.5	2782.0	2866.0	2952.0	3042.0	3375.0	3528.0	3689.0	3801.0	4033.0
20.0	2977.0	3067.0	3160.0	3257.0	3617.0	3871.0	3957.0	4079.0	4329.0
21.0	3391.0	3496.0	3604.0	3715.0	4133.0	4329.0	4528.0	4668.0	4962.0
22.0	3839.0	3960.0	4085.0	4213.0	4694.0	4917.0	5150.0	5312.0	5650.0
23.0	4323.0	4461.0	4603.0	4750.0	5300.0	5556.0	5823.0	6009.0	6395.0
24.0	4844.0	5000.0	5161.0	5328.0	5955.0	6245.0	6549.0	6762.0	7206.0
25.0	5401.0	5578.0	5761.0	5949.0	6658.0	6988.0	7334.0	7573.0	8077.0

TABLE G.

Quantities of Discharge in Cubic Feet per Second of a Conduit One Foot in Diameter.

Sine of the Slope.	Section Circular.			Section Egg-shaped	Section Circular.			
	$a = V^{\frac{1}{9}}$			$a = V^{\frac{1}{18}}$			$a = 1.0$	
	$m=$ 0.95	$m=$ 0.83	$m=$ 0.68	$m=$ 0.57	$m=$ 0.57	$m=$ 0.53	$m=$ 0.45	$m=$ 0.30
.000025	0.1826	0.1675	0.1501	0.2479	0.1501	0.1460	0.1460	0.1305
.000030	0.2023	0.1856	0.1663	0.2730	0.1663	0.1608	0.1608	0.1430
.000035	0.2207	0.2024	0.1813	0.2962	0.1804	0.1744	0.1744	0.1544
.000040	0.2378	0.2182	0.1955	0.3179	0.1941	0.1872	0.1872	0.1651
.000045	0.2541	0.2331	0.2088	0.3383	0.2061	0.1993	0.1993	0.1751
.000050	0.2697	0.2473	0.2216	0.3577	0.2179	0.2107	0.2107	0.1846
.000055	0.2845	0.2609	0.2338	0.3763	0.2292	0.2216	0.2216	0.1936
.000060	0.2988	0.2740	0.2455	0.3940	0.2400	0.2321	0.2321	0.2021
.000065	0.3126	0.2867	0.2582	0.4110	0.2504	0.2421	0.2416	0.2104
.000070	0.3259	0.2989	0.2678	0.4275	0.2604	0.2518	0.2507	0.2183
.000075	0.3387	0.3107	0.2784	0.4434	0.2701	0.2612	0.2656	0.2260
.000080	0.3513	0.3222	0.2887	0.4588	0.2725	0.2702	0.2681	0.2334
.000085	0.3635	0.3333	0.2987	0.4738	0.2886	0.2790	0.2763	0.2406
.000090	0.3753	0.3442	0.3084	0.4884	0.2975	0.2876	0.2843	0.2496
.000095	0.3869	0.3554	0.3180	0.5024	0.3060	0.2959	0.2855	0.2544
.0001	0.3983	0.3652	0.3349	0.5164	0.3145	0.3071	0.2997	0.2610
.000125	0.4516	0.4141	0.3711	0.5803	0.3535	0.3418	0.3351	0.2851
.00015	0.5003	0.4588	0.4111	0.6400	0.3898	0.3769	0.3671	0.3271
.000175	0.5456	0.8004	0.4484	0.6944	0.4133	0.4090	0.3965	0.3452
.0002	0.5881	0.5408	0.4833	0.7453	0.4539	0.4390	0.4239	0.3691
.000225	0.6284	0.5749	0.5165	0.7932	0.4831	0.4672	0.4444	0.4006
.00025	0.6668	0.6258	0.5480	0.8387	0.5108	0.4940	0.4739	0.4127
.000275	0.7035	0.6453	0.5782	0.8821	0.5373	0.5196	0.4970	0.4328
.0003	0.7388	0.6776	0.6071	0.9225	0.5629	0.5433	0.5191	0.4520
.00035	0.8057	0.7390	0.6630	1.0023	0.6104	0.5903	0.5607	0.4663
.00040	0.8686	0.7785	0.6817	1.0759	0.6553	0.6337	0.5994	0.5220
.00045	0.9281	0.8512	0.7627	1.1449	0.6973	0.6743	0.6358	0.5536
.00050	0.9848	0.9030	0.8092	1.2105	0.7373	0.7130	0.6701	0.5836
.00055	1.0390	0.9529	0.8538	1.273	0.7755	0.7329	0.7048	0.6138
.00060	1.0911	1.0007	0.8967	1.333	0.8120	0.7853	0.7341	0.6393
.00065	1.1414	1.0468	0.9369	1.391	0.8475	0.8195	0.7641	0.6654
.00070	1.1899	1.0913	0.9779	1.447	0.8610	0.8520	0.7930	0.6905
.00075	1.2370	1.1345	1.0166	1.501	0.9139	0.8837	0.8208	0.7149
.00080	1.283	1.1765	1.0541	1.553	0.9456	0.9144	0.8477	0.7382
.00085	1.327	1.2173	1.0907	1.603	0.9765	0.9447	0.8738	0.7609
.00090	1.371	1.257	1.1248	1.653	1.0065	0.9733	0.8991	0.7830
.00095	1.413	1.296	1.1611	1.70	1.0357	1.0015	0.9238	0.8044
.001	1.454	1.334	1.1951	1.747	1.0642	1.0291	0.9477	0.8253
.0011	1.535	1.407	1.261	1.838	1.1193	1.0823	0.9940	0.8656
.0012	1.611	1.478	1.324	1.924	1.1721	1.1334	1.0382	0.9041
.0013	1.686	1.546	1.385	2.007	1.2228	1.1824	1.0806	0.9410

TABLE G. — *Continued.*

Sine of the Slope.	Section Circular. $a = V^{\frac{1}{9}}$			Section Egg-shaped $a = V^{\frac{1}{18}}$	Section Circular. $a = V^{\frac{1}{18}}$		Section Circular. $a = 1.0$	
	$m =$ 0.95	$m =$ 0.83	$m =$ 0.68	$m =$ 0.57	$m =$ 0.57	$m =$ 0.53	$m =$ 0.45	$m =$ 0.30
.0014	1.757	1.612	1.444	2.088	1.272	1.2298	1.1214	0.9775
.0015	1.826	1.676	1.501	2.165	1.319	1.276	1.1608	1.0108
.0016	1.894	1.742	1.557	2.241	1.365	1.320	1.1988	1.0442
.0017	1.960	1.798	1.611	2.314	1.409	1.363	1.2357	1.0761
.0018	2.034	1.856	1.664	2.385	1.453	1.405	1.272	1.1083
.0019	2.089	1.914	1.715	2.454	1.495	1.446	1.306	1.1386
.0020	2.148	1.970	1.765	2.523	1.537	1.486	1.340	1.1672
.0021	2.208	2.025	1.814	2.588	1.576	1.524	1.373	1.1960
.0022	2.270	2.082	1.865	2.653	1.616	1.562	1.406	1.2242
.0023	2.323	2.131	1.909	2.715	1.654	1.599	1.437	1.252
.0024	2.385	2.183	1.955	2.777	1.692	1.636	1.468	1.279
.0025	2.435	2.234	2.001	2.838	1.729	1.672	1.498	1.305
.0026	2.489	2.283	2.046	2.898	1.765	1.707	1.528	1.331
.0027	2.543	2.332	2.085	2.956	1.801	1.741	1.557	1.356
.0028	2.595	2.381	2.133	3.014	1.835	1.775	1.586	1.381
.0029	2.647	2.428	2.175	3.070	1.870	1.808	1.614	1.405
.0030	2.698	2.474	2.217	3.126	1.904	1.841	1.642	1.430
.0031	2.748	2.520	2.259	3.180	1.937	1.873	1.669	1.453
.0032	2.798	2.566	2.299	3.234	1.970	1.905	1.695	1.476
.0033	2.847	2.610	2.339	3.288	2.002	1.936	1.722	1.500
.0034	2.895	2.655	2.379	3.340	2.034	1.967	1.748	1.522
.0035	2.942	2.699	2.418	3.391	2.066	1.996	1.773	1.544
.0036	2.989	2.742	2.457	3.442	2.097	2.028	1.798	1.566
.0037	3.036	2.784	2.495	3.493	2.127	2.057	1.823	1.588
.0038	3.082	2.826	2.532	3.542	2.158	2.086	1.848	1.608
.0039	3.126	2.867	2.569	3.591	2.187	2.115	1.872	1.628
.0040	3.172	2.909	2.606	3.640	2.217	2.144	1.896	1.651
.0041	3.216	2.950	2.643	3.688	2.246	2.172	1.919	1.671
.0042	3.260	2.990	2.679	3.735	2.275	2.200	1.943	1.691
.0043	3.304	3.030	2.715	3.782	2.304	2.227	1.965	1.711
.0044	3.346	3.069	2.750	3.829	2.332	2.255	1.988	1.731
.0045	3.389	3.109	2.786	3.875	2.360	2.282	2.011	1.751
.0046	3.431	3.147	2.820	3.920	2.387	2.308	2.033	1.770
.0047	3.473	3.185	2.854	3.964	2.415	2.335	2.055	1.789
.0048	3.514	3.223	2.888	4.009	2.442	2.361	2.076	1.808
.0049	3.555	3.261	2.922	4.053	2.468	2.387	2.098	1.827
.0050	3.596	3.298	2.955	4.096	2.495	2.413	2.119	1.845
.0051	3.636	3.335	2.988	4.140	2.521	2.438	2.140	1.864
.0052	3.676	3.372	3.021	4.182	2.547	2.463	2.161	1.882
.0053	3.716	3.408	3.053	4.225	2.573	2.488	2.182	1.900
.0054	3.755	3.444	3.086	4.265	2.598	2.517	2.202	1.917
.0055	3.794	3.480	3.118	4.308	2.623	2.538	2.223	1.936
.0056	3.833	3.515	3.150	4.350	2.649	2.562	2.243	1.953
.0057	3.871	3.550	3.181	4.391	2.674	2.586	2.262	1.970

TABLE G.—*Continued.*

Sine of the Slope.	Section Circular.			Section Egg-shaped	Section Circular.			
	$a = V^{\frac{1}{9}}$			$a = V^{\frac{1}{18}}$			$a = 1.0$	
	$m = 0.95$	$m = 0.83$	$m = 0.68$	$m = 0.57$	$m = 0.57$	$m = 0.53$	$m = 0.45$	$m = 0.30$
.0058	3.909	3.585	3.212	4.431	2.699	2.610	2.282	1.987
.0059	3.947	3.620	3.244	4.471	2.724	2.634	2.302	2.005
.0060	3.984	3.655	3.274	4.511	2.748	2.658	2.322	2.022
.0061	4.022	3.689	3.305	4.551	2.772	2.681	2.341	2.038
.0062	4.060	3.723	3.335	4.590	2.796	2.704	2.360	2.055
.0063	4.095	3.756	3.365	4.630	2.820	2.727	2.379	2.072
.0064	4.132	3.789	3.395	4.668	2.843	2.750	2.398	2.088
.0065	4.168	3.823	3.425	4.707	2.867	2.772	2.417	2.104
.0066	4.204	3.856	3.455	4.745	2.890	2.795	2.435	2.120
.0067	4.240	3.887	3.484	4.783	2.913	2.817	2.453	2.136
.0068	4.275	3.921	3.513	4.820	2.936	2.839	2.472	2.152
.0069	4.311	3.953	3.543	4.858	2.959	2.861	2.490	2.168
.0070	4.345	3.985	3.571	4.895	2.982	2.883	2.508	2.184
.0071	4.380	4.017	3.600	4.932	3.004	2.905	2.525	2.199
.0072	4.415	4.049	3.629	4.969	3.026	2.926	2.543	2.215
.0073	4.449	4.081	3.656	5.005	3.049	2.948	2.561	2.230
.0074	4.483	4.112	3.684	5.041	3.071	2.969	2.580	2.245
.0075	4.517	4.143	3.712	5.077	3.093	2.990	2.596	2.261
.0076	4.551	4.174	3.740	5.113	3.114	3.011	2.613	2.275
.0077	4.588	4.208	3.770	5.148	3.136	3.032	2.630	2.290
.0078	4.619	4.235	3.795	5.184	3.157	3.057	2.647	2.305
.0079	4.651	4.266	3.822	5.219	3.179	3.074	2.664	2.320
.0080	4.684	4.296	3.849	5.254	3.200	3.094	2.681	2.334
.00825	4.767	4.372	3.917	5.340	3.254	3.145	2.722	2.370
.0085	4.847	4.445	3.983	5.425	3.304	3.195	2.763	2.406
.00875	4.927	4.518	4.048	5.509	3.355	3.244	2.804	2.441
.0090	5.005	4.591	4.113	5.592	3.406	3.293	2.843	2.476
.00925	5.083	4.662	4.177	5.673	3.456	3.342	2.882	2.510
.0095	5.160	4.732	4.240	5.254	3.505	3.389	2.921	2.544
.00975	5.236	4.802	4.304	5.834	3.553	3.436	2.960	2.577
.010	5.311	4.871	4.366	5.913	3.601	3.482	2.997	2.610
.01025	5.885	4.939	4.425	5.990	3.649	3.528	3.032	2.640
.0105	5.458	5.006	4.486	6.067	3.696	3.573	3.071	2.674
.01075	5.531	5.073	4.545	6.143	3.742	3.618	3.107	2.706
.011	5.603	5.139	4.605	6.218	3.788	3.663	3.143	2.737
.01125	5.675	5.204	4.663	6.293	3.833	3.706	3.179	2.968
.01150	5.745	5.269	4.721	6.367	3.878	3.750	3.214	2.799
.01175	5.815	5.333	4.779	6.439	3.746	3.793	3.249	2.829
.012	5.886	5.397	4.836	6.511	3.966	3.835	3.283	2.859
.01225	5.953	5.460	4.892	6.583	4.010	3.877	3.317	2.888
.0125	6.021	5.523	4.949	6.653	4.053	3.919	3.351	2.918
.01275	6.089	5.584	5.004	6.726	4.096	3.960	3.384	2.947
.0130	6.155	5.645	5.059	6.793	4.138	4.001	3.330	2.967
.01325	6.222	5.706	5.113	6,862	4.180	4.042	3.450	3.004

TABLE G.—*Concluded.*

Sine of the Slope.	Section Circular.			Section Egg-shaped	Section Circular.			
	$a = V^{\frac{1}{9}}$			$a = V^{\frac{1}{18}}$			$a = 1.0$	
	$m=$ 0.95	$m=$ 0.83	$m=$ 0.68	$m=$ 0.57	$m=$ 0.57	$m=$ 0.53	$m=$ 0.45	$m=$ 0.30
.0135	6.287	5.767	5.167	6.931	4.221	4.082	3.482	3.032
.01375	6.353	5.826	5.220	6.998	4.263	4.122	3.514	3.060
.014	6.417	5.886	5.274	7.065	4.303	4.160	3.546	3.088
.01425	6.482	5.945	5.326	7.132	4.344	4.201	3.578	3.116
.0145	6.545	6.003	5.379	7.198	4.384	4.240	3.609	3.143
.01475	6.609	6.062	5.431	7.263	4.424	4.278	3.640	3.170
.015	6.671	6.119	5.482	7.328	4.464	4.316	3.671	3.196
.01525	6.734	6.176	5.534	7.392	4.503	4.354	3.700	3.222
.0155	6.795	6.238	5.584	7.456	4.542	4.392	3.731	3.249
.01575	6.857	6.289	5.635	7.520	4.580	4.429	3.761	3.275
.016	6.917	6.344	5.684	7.582	4.618	4.466	3.791	3.302
.01625	6.978	6.400	5.735	7.645	4.657	4.503	3.821	3.327
.0165	7.038	6.456	5.784	7.707	4.695	4.540	3.850	3.352
.01675	7.099	6.510	5.833	7.769	4.732	4.576	3.880	3.379
.0170	7.158	6.565	5.882	7.830	4.769	4.612	3.909	3.404
.01725	7.217	6.619	5.931	7.891	4.806	4.648	3.936	3.428
.0175	7.276	6.673	5.979	7.951	4.843	4.683	3.965	3.453
.01775	7.334	6.726	6.027	8.011	4.879	4.718	3.993	3.477
.018	7.392	6.779	6.074	8.071	4.916	4.754	4.021	3.501
.01825	7.450	6.832	6.122	8.130	4.952	4.788	4.049	3.526
.0185	7.507	6.885	6.169	8.189	4.987	4.823	4.076	3.549
.01875	7.564	6.937	6.215	8.247	5.023	4.858	4.104	3.574
.019	7.620	6.989	6.262	8.305	5.059	4.892	4.131	3.598
.01925	7.676	7.040	6.309	8.363	5.094	4.926	4.158	3.621
.0195	7.732	7.092	6.354	8.420	5.128	4.959	4.185	3.644
.01975	7.788	7.142	6.40	8.477	5.163	4.993	4.212	3.668
.02	7.843	7.193	6.445	8.534	5.198	5.026	4.239	3.691
.0205	7.952	7.293	6.535	8.645	5.266	5.092	4.291	3.737
.021	8.061	7.394	6.625	8.757	5.334	5.158	4.343	3.782
.0215	8.169	7.492	6.713	8.867	5.401	5.222	4.395	3.827
.022	8.275	7.590	6.801	8.977	5.469	5.286	4.445	3.871
.0225	8.389	7.692	6.892	9.089	5.536	5.353	4.496	3.915
.023	8.485	7.782	6.972	9.189	5.597	5.412	4.545	3.958
.0235	8.582	7.871	7.052	9.294	5.661	5.474	4.594	4.001
.024	8.690	7.970	7.141	9.398	5.724	5.536	4.643	4.043
.0245	8.792	8.063	7.225	9.501	5.787	5.596	4.691	4.085
.025	8.893	8.155	7.307	9.604	5.850	5.657	4.739	4.127
.030	9.852	9.036	8.096	10.577	6.442	6.230	5.191	4.485
.040	11.583	10.623	9.518	12.317	7.502	7.255	5.994	5.221
.050	13.13	12.052	10.791	13.86	8.443	8.164	6.701	5.836
.060	14.55	13.34	11.957	15.27	9.298	8.992	7.341	6.393
.070	15.90	14.55	13.40	16.57	10.089	9.756	7.929	6.904
.080	17.11	15.69	14.056	19.98	10.828	10.471	8.477	7.382
.090	18.28	16.76	15.020	18.92	11.525	11.144	8.992	7.831
0.10	19.39	17.79	15.936	20.0	12.186	11.784	9.478	8.253

TABLE H.

VELOCITIES OF FLOW IN A SEMISQUARE ONE FOOT IN DEPTH.

Sine of the Slope.	$a = V^{\frac{1}{18}}$					$a = 1.0$			
	$m=$ 0.95	$m=$ 0.80	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.2	$K=$ 1.5	$K=$ 1.93
.000025	0.397	0.362	0.3386	0.3084	0.2479	0.1962	0.1770	0.1543	0.1305
.000030	0.4372	0.3986	0.3730	0.3396	0.2730	0.2150	0.1939	0.1691	0.1430
.000035	0.4744	0.4326	0.4046	0.3685	0.2962	0.2322	0.2095	0.1826	0.1544
.000040	0.5092	0.4642	0.4342	0.3955	0.3179	0.2482	0.2239	0.1952	0.1651
.000045	0.5419	0.4941	0.4621	0.4192	0.3383	0.2642	0.2375	0.2070	0.1751
.000050	0.5863	0.5346	0.5001	0.4451	0.3577	0.2775	0.2503	0.2182	0.1846
.000055	0.6027	0.5495	0.5139	0.4681	0.3763	0.2911	0.2625	0.2289	0.1936
.000060	0.6311	0.5754	0.5382	0.4902	0.3940	0.3040	0.2742	0.2391	0.2021
.000065	0.6584	0.6003	0.5615	0.5114	0.4110	0.3164	0.2854	0.2488	0.2104
.000070	0.6847	0.6243	0.5839	0.5318	0.4275	0.3283	0.2962	0.2582	0.2183
.000075	0.7102	0.6475	0.6057	0.5516	0.4434	0.3399	0.3066	0.2673	0.2260
.000080	0.7349	0.6701	0.6267	0.5708	0.4588	0.3510	0.3166	0.2761	0.2334
.000085	0.7588	0.6919	0.6472	0.5894	0.4738	0.3618	0.3264	0.2845	0.2406
.000090	0.7822	0.7131	0.6671	0.6075	0.4884	0.3723	0.3358	0.2928	0.2476
.000095	0.8048	0.7338	0.6863	0.6251	0.5024	0.3825	0.3450	0.3008	0.2544
.0001	0.8720	0.7541	0.7053	0.6424	0.5164	0.3924	0.3540	0.3086	0.2610
.000125	0.9295	0.8475	0.7927	0.7220	0.5803	0.4388	0.3958	0.3450	0.2851
.00015	1.025	0.9346	0.8742	0.7961	0.646	0.4806	0.4437	0.3868	0.3271
.000175	1.1122	1.0141	0.9485	0.8638	0.6944	0.5191	0.4683	0.4083	0.3452
.0002	1.1937	1.0884	1.0180	0.9272	0.7453	0.555	0.5006	0.4365	0.3691
.000225	1.270	1.1584	1.0835	0.9869	0.7932	0.5885	0.5433	0.4737	0.4006
.00025	1.3404	1.2248	1.1457	1.0423	0.8387	0.6205	0.5597	0.4880	0.4127
.000275	1.413	1.289	1.2044	1.0975	0.8821	0.6508	0.5870	0.5118	0.4328
.0003	1.477	1.347	1.260	1.1476	0.9225	0.6797	0.6131	0.5345	0.4520
.00035	1.605	1.464	1.369	1.2444	1.0023	0.7342	0.6623	0.5774	0.4663
.00040	1.723	1.571	1.470	1.338	1.0759	0.7852	0.7080	0.6172	0.5220
.00045	1.834	1.672	1.564	1.424	1.1449	0.8325	0.7509	0.6547	0.5536
.0005	1.939	1.768	1.654	1.506	1.2125	0.8775	0.7915	0.6901	0.5836
.00055	2.039	1.859	1.739	1.584	1.273	0.9229	0.8325	0.7258	0.6138
.0006	2.135	1.947	1.821	1.659	1.333	0.9613	0.8671	0.7559	0.6393
.00065	2.229	2.032	1.901	1.731	1.391	1.0005	0.9025	0.7868	0.6654
.0007	2.317	2.113	1.976	1.800	1.447	1.0383	0.9366	0.8165	0.6905
.00075	2.403	2.191	2.050	1.867	1.501	1.0750	0.9697	0.8454	0.7149
.0008	2.487	2.269	2.121	1.932	1.553	1.110	1.0012	0.8729	0.7382
.00085	2.568	2.341	2.192	1.995	1.603	1.1441	1.032	0.8998	0.7609
.0009	2.647	2.413	2.257	2.056	1.653	1.1774	1.062	0.9259	0.7830
.00095	2.724	2.483	2.323	2.116	1.70	1.2096	1.091	0.9512	0.8044
.001	2.799	2.551	2.388	2.174	1.747	1.241	1.1194	0.9759	0.8253
.0011	2.943	2.684	2.510	2.286	1.838	1.302	1.174	1.0236	0.8656
.0012	3.082	2.810	2.629	2.394	1.924	1.360	1.2262	1.0691	0.9041
.0013	3.215	2.932	2.742	2.498	2.007	1.404	1.276	1.1127	0.9410
.0014	3.344	3.049	2.852	2.598	2.088	1.468	1.325	1.1548	0.9775
.0015	3.469	3.163	2.958	2.694	2.165	1.520	1.371	1.1953	1.0108
.0016	3.589	3.273	3.061	2.788	2.241	1.570	1.416	1.2347	1.0442
.0017	3.706	3.379	3.161	2.879	2.314	1.621	1.460	1.267	1.0761
.0018	3.820	3.483	3.258	2.967	2.385	1.665	1.502	1.309	1.1083
.0019	3.931	3.584	3.353	3.053	2.454	1.711	1.543	1.346	1.1386

TABLE H.—*Continued.*

Sine of the Slope.	$a = V^{\frac{1}{18}}$					$a = 1.0$			
	$m =$ 0.95	$m =$ 0.80	$m =$ 0.70	$m =$ 0.57	$m =$ 0.30	$m =$ 0.0	$K =$ 1.2	$K =$ 1.5	$K =$ 1.93
.0020	4.041	3.685	3.446	3.139	2.523	1.755	1.583	1.380	1.1672
.0021	4.145	3.779	3.535	3.220	2.588	1.798	1.622	1.414	1.1960
.0022	4.248	3.874	3.623	3.300	2.653	1.841	1.660	1.447	1.2242
.0023	4.349	3.966	3.709	3.378	2.715	1.882	1.698	1.480	1.252
.0024	4.449	4.056	3.794	3.456	2.777	1.923	1.734	1.521	1.279
.0025	4.546	4.145	3.877	3.532	2.838	1.962	1.770	1.543	1.305
.0026	4.641	4.232	3.958	3.605	2.898	2.001	1.805	1.574	1.331
.0027	4.735	4.317	4.038	3.678	2.956	2.039	1.839	1.604	1.356
.0028	4.827	4.401	4.117	3.749	3.014	2.076	1.873	1.633	1.381
.0029	4.918	4.484	4.194	3.820	3.070	2.114	1.906	1.662	1.405
.0030	5.008	4.566	4.271	3.890	3.126	2.149	1.939	1.690	1.430
.0031	5.094	4.645	4.345	3.957	3.180	2.185	1.971	1.718	1.453
.0032	5.180	4.724	4.418	4.024	3.234	2.220	2.003	1.746	1.476
.0033	5.265	4.801	4.491	4.090	3.288	2.254	2.034	1.773	1.500
.0034	5.350	4.898	4.562	4.155	3.340	2.288	2.064	1.800	1.522
.0035	5.432	4.953	4.633	4.219	3.391	2.322	2.094	1.826	1.544
.0036	5.514	5.027	4.703	4.283	3.442	2.355	2.124	1.852	1.566
.0037	5.594	5.101	4.771	4.346	3.493	2.389	2.153	1.877	1.588
.0038	5.674	5.173	4.839	4.407	3.542	2.419	2.182	1.902	1.608
.0039	5.753	5.245	4.906	4.468	3.591	2.451	2.211	1.928	1.628
.0040	5.830	5.316	4.972	4.528	3.640	2.482	2.239	1.952	1.651
.0041	5.907	5.386	5.037	4.588	3.688	2.513	2.267	1.976	1.671
.0042	5.982	5.455	5.102	4.647	3.735	2.543	2.294	2.000	1.691
.0043	6.057	5.523	5.166	4.705	3.782	2.573	2.321	2.024	1.711
.0044	6.132	5.591	5.229	4.763	3.829	2.603	2.348	2.047	1.731
.0045	6.206	5.659	5.293	4.820	3.875	2.633	2.375	2.070	1.751
.0046	6.278	5.724	5.354	4.876	3.920	2.662	2.401	2.093	1.770
.0047	6.350	5.790	5.415	4.932	3.964	2.691	2.427	2.116	1.789
.0048	6.421	5.854	5.476	4.987	4.009	2.719	2.452	2.138	1.808
.0049	6.491	5.919	5.536	5.042	4.053	2.747	2.448	2.160	1.827
.0050	6.561	5.982	5.595	5.096	4.096	2.775	2.503	2.182	1.845
.0051	6.630	6.045	5.654	5.150	4.140	2.802	2.528	2.204	1.864
.0052	6.698	6.108	5.713	5.203	4.182	2.830	2.553	2.226	1.882
.0053	6.767	6.170	5.771	5.256	4.225	2.857	2.577	2.247	1.900
.0054	6.832	6.230	5.827	5.307	4.265	2.884	2.601	2.268	1.917
.0055	6.901	6.292	5.885	5.360	4.308	2.910	2.625	2.289	1.936
.0056	6.967	6.352	5.941	5.412	4.350	2.937	2.649	2.310	1.953
.0057	7.032	6.412	5.997	5.462	4.391	2.962	2.672	2.330	1.970
.0058	7.097	6.471	6.053	5.513	4.431	2.989	2.695	2.350	1.987
.0059	7.162	6.530	6.108	5.563	4.471	3.014	2.719	2.370	2.005
.0060	7.226	6.588	6.162	5.613	4.511	3.040	2.742	2.390	2.022
.0061	7.289	6.646	6.217	5.662	4.551	3.065	2.765	2.410	2.038
.0062	7.352	6.704	6.270	5.711	4.590	3.090	2.787	2.430	2.055
.0063	7.415	6.761	6.324	5.759	4.630	3.115	2.810	2.450	2.072
.0064	7.477	6.818	6.377	5.808	4.668	3.140	2.832	2.469	2.088
.0065	7.539	6.873	6.429	5.856	4.707	3.164	2.854	2.488	2.104
.0066	7.600	6.930	6.481	5.903	4.745	3.188	2.876	2.507	2.120
.0067	7.660	6.985	6.533	5.951	4.783	3.212	2.897	2.526	2.136

TABLE H.— *Continued.*

Sine of the Slope.	$a = V^{\frac{1}{18}}$					$a = 1.0$			
	$m =$ 0.95	$m =$ 0.80	$m =$ 0.70	$m =$ 0.57	$m =$ 0.30	$m =$ 0.0	$K =$ 1.2	$K =$ 1.5	$K =$ 1.93
.0068	7.721	7.040	6.585	5.997	4.820	3.236	2.919	2.545	2.152
.0069	7.781	7.095	6.636	6.044	4.858	3.260	2.940	2.564	2.168
.0070	7.840	7.149	6.686	6.090	4.895	3.283	2.962	2.582	2.184
.0071	7.899	7.202	6.737	6.136	4.932	3.307	2.983	2.601	2.199
.0072	7.958	7.256	6.787	6.181	4.969	3.330	3.004	2.619	2.215
.0073	8.017	7.309	6.837	6.227	5.005	3.353	3.024	2.637	2.230
.0074	8.075	7.362	6.886	6.272	5.041	3.376	3.045	2.655	2.245
.0075	8.132	7.414	6.935	6.321	5.077	3.399	3.067	2.673	2.261
.0076	8.189	7.467	6.984	6.361	5.113	3.421	3.086	2.690	2.275
.0077	8.246	7.519	7.033	6.405	5.148	3.444	3.106	2.708	2.29
.0078	8.303	7.571	7.081	6.449	5.184	3.466	3.126	2.726	2.305
.0079	8.359	7.622	7.129	6.493	5.219	3.488	3.146	2.743	2.320
.0080	8.415	7.672	7.176	6.536	5.254	3.510	3.166	2.760	2.334
.00825	8.553	7.799	7.294	6.644	5.340	3.565	3.215	2.803	2.370
.0085	8.689	7.923	7.410	6.757	5.425	3.618	3.264	2.845	2.406
.00875	8.824	8.045	7.525	6.853	5.509	3.671	3.311	2.887	2.411
.0090	8.957	8.167	7.639	6.957	5.592	3.723	3.358	2.928	2.476
.00925	9.087	8.286	7.749	7.058	5.673	3.774	3.405	2.968	2.510
.0095	9.216	8.403	7.860	7.159	5.754	3.825	3.450	3.008	2.544
.00975	9.344	8.520	7.969	7.258	5.834	3.895	3.495	3.048	2.577
.01	9.476	8.635	8.076	7.356	5.913	3.924	3.540	3.086	2.610
.01025	9.594	8.748	8.182	7.453	5.990	3.970	3.581	3.122	2.640
.0105	9.718	8.860	8.288	7.548	6.067	4.021	3.627	3.163	2.674
.01075	9.839	8.971	8.391	7.643	6.143	4.069	3.670	3.200	2.706
.011	9.960	9.082	8.494	7.736	6.218	4.116	3.713	3.237	2.737
.01125	10.079	9.188	8.596	7.829	6.293	4.163	3.755	3.274	2.768
.0115	10.197	9.298	8.696	7.921	6.367	4.208	3.796	3.310	2.799
.01175	10.314	9.404	8.796	8.011	6.439	4.254	3.837	3.345	2.829
.012	10.430	9.510	8.895	8.101	6.511	4.289	3.869	3.381	2.859
.01225	10.544	9.614	8.992	8.190	6.583	4.343	3.918	3.415	2.888
.0125	10.658	9.717	9.089	8.278	6.653	4.388	3.958	3.450	2.918
.01275	10.772	9.822	9.187	8.367	6.726	4.431	3.997	3.485	2.947
.013	10.881	9.921	9.280	8.452	6.793	4.461	4.024	3.509	2.967
.01325	10.991	10.022	9.374	8.537	6.862	4.517	4.074	3.552	3.004
.0135	11.10	10.121	9.467	8.622	6.931	4.560	4.113	3.586	3.032
.01375	11.209	10.220	9.559	8.706	6.998	4.602	4.151	3.619	3.060
.014	11.316	10.318	9.652	8.790	7.065	4.643	4.188	3.652	3.088
.01425	11.423	10.415	9.742	8.873	7.132	4.685	4.226	3.684	3.116
.0145	11.528	10.511	9.832	8.954	7.198	4.726	4.263	3.716	3.143
.01475	11.633	10.607	9.921	9.036	7.263	4.766	4.299	3.748	3.170
.015	11.737	10.702	10.01	9.117	7.328	4.806	4.336	3.780	3.196
.01525	11.840	10.796	10.098	9.197	7.392	4.845	4.370	3.810	3.222
.0155	11.943	10.890	10.185	9.276	7.456	4.886	4.407	3.842	3.249
.01575	12.025	10.981	10.272	9.373	7.520	4.925	4.442	3.873	3.275
.016	12.230	11.073	10.357	9.433	7.582	4.965	4.479	3.905	3.302
.01625	12.274	11.165	10.443	9.512	7.645	5.003	4.513	3.934	3.327
.0165	12.345	11.256	10.522	9.589	7.707	5.041	4.547	3.964	3.352
.01675	12.443	11.346	10.612	9.665	7.769	5.080	4.582	3.995	3.376
.017	12.54	11.435	10.696	9.742	7.830	5.118	4.617	4.025	3.404

TABLE H.—*Concluded.*

Sine of the Slope	$a = V^{\frac{1}{18}}$					$a = 1.0$			
	$m =$ 0.95	$m =$ 0.80	$m =$ 0.70	$m =$ 0.57	$m =$ 0.30	$m =$ 0.0	$K =$ 1.2	$K =$ 1.5	$K =$ 1.93
.01725	12.64	11.524	10.779	9.817	7.891	5.154	4.649	4.053	3.428
.0175	12.74	11.612	10.861	9.892	7.951	5.191	4.683	4.083	3.453
.01775	12.83	11.700	10.943	9.967	8.011	5.228	4.716	4.112	3.477
.018	12.92	11.786	11.024	10.041	8.091	5.265	4.749	4.141	3.501
.01825	13.02	11.873	11.105	10.115	8.130	5.302	4.782	4.169	3.526
.0185	13.11	11.959	11.185	10.188	8.189	5.338	4.815	4.198	3.549
.01875	13.21	12.044	11.265	10.260	8.247	5.374	4.847	4.226	3.574
.019	13.33	12.157	11.344	10.332	8.305	5.409	4.879	4.254	3.598
.01925	13.40	12.213	11.423	10.404	8.363	5.445	4.911	4.282	3.621
.0195	13.48	12.297	11.502	10.475	8.420	5.480	4.943	4.310	3.644
.01975	13.58	12.380	11.579	10.546	8.477	5.515	4.975	4.337	3.668
.020	13.67	12.46	11.657	10.616	8.534	5.550	5.006	4.365	3.691
.0205	13.85	12.62	11.809	10.756	8.645	5.619	5.668	4.419	3.737
.021	14.03	12.79	11.962	10.895	8.757	5.687	5.129	4.472	3.782
.0215	14.20	12.95	12.112	11.031	8.867	5.754	5.191	4.525	3.827
.022	14.38	13.11	12.263	11.169	8.977	5.821	5.251	4.578	3.871
.0225	14.56	13.27	12.415	11.308	9.089	5.887	5.310	4.630	3.915
.023	14.72	13.42	12.55	11.432	9.189	5.952	5.369	4.680	3.958
.0235	14.89	13.57	12.70	11.563	9.294	6.016	5.427	4.731	4.001
.024	15.05	13.72	12.84	11.693	9.398	6.080	5.484	4.781	4.043
.0245	15.22	13.88	12.98	11.821	9.501	6.143	5.541	4.831	4.085
.025	15.38	14.03	13.12	11.948	9.604	6.205	5.597	4.880	4.127
.030	16.94	15.45	14.45	13.160	10.577	6.797	6.131	5.345	4.485
.040	19.73	17.99	16.83	15.32	12.317	7.851	7.082	6.174	5.221
.050	22.20	20.24	18.93	17.25	13.860	8.775	7.915	6.901	5.836
.060	24.45	22.30	20.85	18.99	15.27	9.613	8.671	7.559	6.393
.070	26.53	24.19	22.63	20.61	16.57	10.383	9.366	8.165	6.904
.080	28.48	25.96	24.28	22.12	17.78	11.10	10.012	8.729	7.382
.090	30.31	27.63	26.45	22.48	18.92	11.762	10.621	9.259	7.831
.100	32.04	29.22	27.33	24.89	20.00	12.41	11.194	9.760	8.253

TABLE I.

QUANTITIES OF DISCHARGE IN CUBIC FEET PER SECOND OF A SEMI-SQUARE ONE FOOT IN DEPTH.

Sine of the Slope	$a=V^{\frac{1}{18}}$					$a=1.0$			
	$m=$ 0.95	$m=$ 0.80	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.2	$K=$ 1.5	$K=$ 1.93
.000025	0.794	0.724	0.6772	0.6168	0.4958	0.3924	0.3540	0.3086	0.2610
.000030	0.8744	0.7972	0.7560	0.6792	0.5460	0.4300	0.3878	0.3382	0.2860
.000035	0.9488	0.8652	0.8092	0.7370	0.5924	0.4644	0.4190	0.3652	0.3088
.000040	1.0184	0.9284	0.8684	0.7910	0.6358	0.4964	0.4478	0.3904	0.3302
.000045	1.0838	0.9982	0.9242	0.8384	0.6766	0.5284	0.4750	0.4140	0.3502
.000050	1.1726	1.0692	1.0002	0.8902	0.7154	0.5550	0.5006	0.4364	0.3692
.000055	1.2054	1.0990	1.0278	0.9362	0.7526	0.5822	0.5250	0.4578	0.3892
.000060	1.2622	1.1508	1.0764	0.9804	0.7880	0.6080	0.5484	0.4782	0.4042
.000065	1.2168	1.2006	1.1230	1.0228	0.8220	0.6328	0.5708	0.4976	0.4208
.000070	1.3694	1.2486	1.1678	1.0636	0.8550	0.6566	0.5924	0.5164	0.4366
.000075	1.4204	1.3950	1.2114	1.1032	0.8868	0.6798	0.6132	0.5346	0.4520
.000080	1.4698	1.3402	1.2534	1.1416	0.9176	0.7020	0.6332	0.5522	0.4668
.000085	1.5176	1.3938	1.2944	1.1788	0.9476	0.7236	0.6528	0.5690	0.4812
.000090	1.5644	1.4262	1.3342	1.2150	0.9768	0.7446	0.6716	0.5856	0.4952
.000095	1.6096	1.4676	1.3726	1.2502	1.0048	0.7650	0.6900	0.6016	0.5088
.0001	1.6440	1.5082	1.4106	1.2848	1.0328	0.7848	0.7080	0.6172	0.5220
.000125	1.8590	1.6950	1.5854	1.4440	1.1606	0.8776	0.7916	0.6900	0.5702
.00015	2.050	1.8692	1.7484	1.5922	1.280	0.9612	0.8874	0.7736	0.6542
.000175	2.2244	2.0282	1.8970	1.7276	1.3888	1.0382	0.9366	0.8166	0.6904
.0002	2.3894	2.1768	2.0360	1.8544	1.4906	1.110	1.0012	0.8730	0.7382
.000225	2.540	2.3168	2.1670	1.9738	1.5864	1.1770	1.0866	0.9474	0.8002
.00025	2.6808	2.4496	2.2914	2.0866	1.6774	1.2410	1.1194	0.9760	0.8254
.000275	2.826	2.578	2.4088	2.1950	1.7642	1.3016	1.1740	1.0236	0.8656
.0003	2.954	2.694	2.520	2.2952	1.8450	1.3594	1.2262	1.0690	0.9040
.00035	3.210	2.928	2.738	2.4888	2.0046	1.4684	1.3246	1.1548	0.9326
.00040	3.446	3.142	2.940	2.676	2.1518	1.5704	1.416	1.2344	1.044
.00045	3.668	3.344	3.128	2.848	2.2898	1.6650	1.5018	1.3094	1.1072
.0005	3.878	3.536	3.308	3.012	2.4250	1.7550	1.5830	1.3802	1.1672
.00055	4.078	3.718	3.478	3.168	2.546	1.8458	1.6650	1.4516	1.2276
.0006	4.270	3.894	3.642	3.318	2.666	1.9226	1.7342	1.5118	1.2786
.00065	4.458	4.064	3.802	3.462	2.782	2.001	1.8050	1.5936	1.3308
.0007	4.634	4.226	3.952	3.600	2.894	2.0766	1.8732	1.6330	1.3810
.00075	4.806	4.382	4.100	3.734	3.002	2.150	1.9394	1.6908	1.4298
.0008	4.974	4.534	4.242	3.864	3.106	2.220	2.0024	1.7458	1.4764
.00085	5.136	4.682	4.394	3.990	3.206	2.2882	2.064	1.7996	1.5218
.0009	5.294	4.826	4.514	4.112	3.306	2.3548	2.124	1.8518	1.5660
.00095	5.448	4.966	4.646	4.232	3.40	2.4192	2.182	1.9024	1.6088
.001	5.598	5.102	4.776	4.348	3.494	2.482	2.2398	1.9518	1.6506
.0011	5.886	5.368	5.020	4.572	3.676	2.604	2.348	2.0472	1.7312
.0012	6.164	5.620	5.258	4.798	3.848	2.720	2.4524	2.1382	1.8082
.0013	6.430	5.864	5.484	4.996	4.014	2.808	2.552	2.2254	1.882
.0014	6.688	6.098	5.704	5.196	4.176	2.936	2.650	2.3096	1.9550
.0015	6.938	6.326	5.916	5.388	4.330	3.04	2.742	2.3906	2.0216
.0016	7.178	6.546	6.122	5.576	4.482	3.14	2.832	2.4694	2.0884
.0017	7.412	6.758	6.322	5.758	4.628	3.242	2.92	2.534	2.1562

TABLE I. — *Continued.*

Sine of the Slope	$a = V^{\frac{1}{18}}$					$a = 1.0$			
	$m=$ 0.95	$m=$ 0.80	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.2	$K=$ 1.5	$K=$ 1.93
.0018	7.640	6.966	6.516	5.934	4.770	3.330	3.004	2.618	2.2166
.0019	7.862	7.168	6.706	6.106	4.908	3.422	3.086	2.692	2.2772
.0020	8.082	7.370	6.892	6.278	5.046	3.510	3.166	2.760	2.3344
.0021	8.290	7.558	7.070	6.440	5.176	3.596	3.244	2.828	2.3920
.0022	8.496	7.948	7.246	6.600	5.306	3.682	3.320	2.894	2.4484
.0023	8.698	7.932	7.418	6.756	5.430	3.764	3.396	2.960	2.504
.0024	8.898	8.112	7.588	6.912	5.554	3.846	3.468	3.042	2.558
.0025	9.092	8.290	7.754	7.064	5.676	3.924	3.54	3.086	2.610
.0026	9.282	8.464	7.916	7.210	5.796	4.002	3.610	3.148	2.662
.0027	9.470	8.634	8.076	7.356	5.912	4.078	3.678	3.208	2.712
.0028	9.654	8.802	8.234	7.498	6.028	4.152	3.746	3.266	2.762
.0029	9.836	8.968	8.388	7.64	6.14	4.228	3.812	3.324	2.810
.0030	10.016	9.132	8.542	7.78	6.252	4.298	3.878	3.38	2.86
.0031	10.188	9.390	8.690	7.914	6.36	4.370	3.942	3.436	2.906
.0032	10.360	9.448	8.836	8.048	6.464	4.440	4.006	3.492	2.952
.0033	10.530	9.602	8.982	8.18	6.576	4.508	4.068	3.546	3.0
.0034	10.70	9.956	9.124	8.310	6.68	4.576	4.128	3.60	3.044
.0035	10.864	9.906	9.266	8.438	6.782	4.644	4.188	3.652	3.088
.0036	11.028	10.054	9.406	8.566	6.884	4.710	4.248	3.704	3.132
.0037	11.188	10.202	9.542	8.692	6.986	4.774	4.306	3.754	3.176
.0038	11.348	10.346	9.678	8.814	7.084	4.838	4.364	3.804	3.216
.0039	11.506	10.490	9.812	8.936	7.182	4.902	4.422	3.856	3.256
.0040	11.66	10.632	9.944	9.056	7.28	4.964	4.478	3.904	3.302
.0041	11.814	10.772	10.074	9.176	7.376	5.026	4.434	3.952	3.342
.0042	11.964	10.910	10.204	9.294	7.570	5.086	4.588	4.0	3.382
.0043	12.114	11.046	10.332	9.410	7.564	5.146	4.642	4.048	3.422
.0044	12.264	11.182	10.458	9.526	7.658	5.206	4.696	4.094	3.462
.0045	12.412	11.318	10.586	9.64	7.750	5.266	4.750	4.140	3.502
.0046	12.556	11.448	10.708	9.752	7.84	5.324	4.802	4.186	3.54
.0047	12.70	11.580	10.830	9.864	7.928	5.382	4.854	4.232	3.578
.0048	12.842	11.708	10.952	9.974	8.018	5.438	4.904	4.276	3.616
.0049	12.982	11.838	11.072	10.084	8.106	5.494	4.956	4.320	3.654
.005	13.122	11.964	11.190	10.192	8.192	5.550	5.006	4.364	3.690
.0051	13.26	12.090	11.308	10.300	8.280	5.604	5.056	4.408	3.628
.0052	13.396	12.216	11.426	10.406	8.364	5.660	5.106	4.452	3.764
.0053	13.534	12.34	11.542	10.512	8.450	5.714	5.154	4.494	3.80
.0054	13.664	12.46	11.654	10.614	8.530	5.768	5.202	4.536	3.834
.0055	13.802	12.584	11.770	10.720	8.616	5.82	5.250	4.578	3.872
.0056	13.934	12.704	11.882	10.824	8.700	5.874	5.298	4.620	3.906
.0057	14.064	12.824	11.994	10.924	8.782	5.924	5.344	4.660	3.94
.0058	14.194	12.942	12.106	11.026	8.862	5.978	5.390	4.70	3.974
.0059	14.324	13.060	12.216	11.126	8.942	6.028	5.438	4.74	4.010
.006	14.452	13.176	12.324	11.226	9.022	6.080	5.484	4.78	4.044
.0061	14.578	13.392	12.434	11.324	9.102	6.130	5.530	4.82	4.076
.0062	14.704	13.408	12.54	11.422	9.180	6.180	5.574	4.86	4.110
.0063	14.830	13.522	12.648	11.518	9.26	6.230	5.620	4.90	4.144
.0064	14.954	13.636	12.754	11.616	9.336	6.28	5.664	4.938	4.176
.0065	15.078	13.746	12.858	11.712	9.414	6.328	5.708	4.976	4.208
.0066	15.2	13.860	12.962	11.806	9.490	6.376	5.752	5.014	4.240

TABLE I.— *Continued.*

Sine of the Slope	$a=V^{\frac{1}{18}}$					$a=1.0$			
	$m=$ 0.95	$m=$ 0.80	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.2	$K=$ 1.5	$K=$ 1.93
.0067	15.32	13.970	13.066	11.902	9.566	6.424	5.794	5.052	4.272
.0068	15.442	14.080	13.170	11.994	9.64	6.472	5.838	5.090	4.304
.0069	15.562	14.190	13.272	12.088	9.716	6.520	5.88	5.128	4.336
.0070	15.680	14.298	13.372	12.18	9.790	6.566	5.924	5.164	4.368
.0071	15.798	14.404	13.474	12.272	9.864	6.614	5.966	5.202	4.398
.0072	15.916	14.512	13.574	12.362	9.938	6.660	6.008	5.238	4.430
.0073	16.034	14.618	13.674	12.454	10.010	6.706	6.048	5.274	4.460
.0074	16.150	14.724	13.772	12.542	10.082	6.752	6.090	5.310	4.490
.0075	16.264	14.828	13.870	12.654	10.154	6.798	6.134	5.346	4.522
.0076	16.378	14.934	13.968	12.722	10.226	6.842	6.172	5.380	4.550
.0077	16.492	15.038	14.066	12.810	10.296	6.888	6.212	5.416	4.580
.0078	16.606	15.142	14.162	12.898	10.368	6.932	6.252	5.452	4.610
.0079	16.718	15.244	14.258	12.986	10.438	6.976	6.292	5.486	4.640
.008	16.830	15.344	14.352	13.072	10.508	7.020	6.332	5.520	4.668
.00825	17.106	15.598	14.588	13.288	10.680	7.130	6.430	5.606	4.740
.0085	17.378	15.846	14.820	13.514	10.850	7.236	6.528	5.690	4.812
.00875	17.648	16.090	15.050	13.706	11.018	7.342	6.622	5.774	4.822
.009	17.914	16.334	15.278	13.914	11.184	7.446	6.716	5.856	4.952
.00925	18.174	16.572	15.498	14.116	11.346	7.548	6.810	5.936	5.020
.0095	18.432	16.806	15.720	14.318	11.508	7.650	6.900	6.016	5.088
.00975	18.688	17.04	15.938	14.516	11.668	7.750	6.990	6.096	5.154
.01	18.94	17.270	16.152	14.712	11.826	7.848	7.080	6.172	5.220
.01025	19.198	17.496	16.364	14.906	11.980	7.940	7.162	6.244	5.280
.0105	19.436	17.72	16.576	15.096	12.134	8.042	7.254	6.326	5.348
.01075	19.678	17.942	16.782	15.286	12.286	8.138	7.340	6.400	5.412
.011	19.920	18.164	16.988	15.472	12.436	8.232	7.426	6.474	5.474
.01125	20.158	18.376	17.192	15.658	12.586	8.326	7.510	6.548	5.536
.0115	20.394	18.596	17.392	15.842	12.734	8.416	7.592	6.620	5.598
.01175	20.628	18.408	17.592	16.022	12.878	8.508	7.674	6.690	5.658
.012	20.860	19.020	17.770	16.202	13.022	8.578	7.738	6.762	5.718
.01225	21.088	19.228	17.984	16.380	13.166	8.686	7.836	6.830	5.776
.0125	21.316	19.434	18.178	16.556	13.306	8.776	7.916	6.900	5.836
.01275	21.544	19.644	18.374	16.734	13.452	8.862	7.994	6.970	5.894
.013	21.762	19.842	18.560	16.904	13.586	8.922	8.048	7.018	5.934
.01325	21.982	20.044	18.748	17.074	13.724	9.034	8.148	7.104	6.008
.0135	22.20	20.242	18.934	17.244	13.862	9.120	8.226	7.172	6.064
.01375	22.418	20.440	19.118	17.412	13.996	9.204	8.302	7.238	6.120
.014	22.632	20.636	19.304	17.580	14.130	9.286	8.376	7.304	6.176
.01425	22.846	20.830	19.484	17.746	14.264	9.370	8.452	7.368	6.232
.0145	23.056	21.022	19.664	17.908	14.396	9.452	8.526	7.432	6.286
.01475	23.266	21.214	19.842	18.072	14.526	9.532	8.598	7.496	6.340
.015	23.474	21.404	20.02	18.234	14.656	9.612	8.672	7.560	6.392
.01525	23.680	21.592	20.196	18.394	14.784	9.690	8.740	7.620	6.444
.0155	23.886	21.780	20.370	18.552	14.912	9.772	8.814	7.684	6.498
.01575	24.050	21.962	20.544	18.746	15.040	9.850	8.884	7.746	6.550
.016	24.460	22.146	20.714	18.866	15.164	9.930	8.958	7.810	6.604
.01625	24.548	22.330	20.886	19.024	15.290	10.006	9.026	7.868	6.654
.0165	24.690	22.512	21.044	19.178	15.414	10.082	9.094	7.928	6.704
.01675	24.886	22.692	21.224	19.330	15.538	10.160	9.164	7.990	6.758

TABLE I.—*Concluded.*

Sine of the Slope	$a=V^{\frac{1}{18}}$					$a=1.0$			
	$m=$ 0.95	$m=$ 0.80	$m=$ 0.70	$m=$ 0.57	$m=$ 0.30	$m=$ 0.0	$K=$ 1.2	$K=$ 1.5	$K=$ 1.93
.017	25.08	22.870	21.392	19.484	15.660	10.236	9.234	8.050	6.808
.01725	25.28	23.048	21.558	19.634	15.782	10.308	9.298	8.106	6.856
.0175	25.48	23.224	21.722	19.784	15.902	10.382	9.366	8.166	6.906
.01775	25.66	23.400	21.886	19.934	16.022	10.456	9.432	8.224	6.954
.018	25.84	23.572	22.048	20.082	16.142	10.530	9.498	8.282	7.002
.01825	26.04	23.746	22.210	20.230	16.260	10.604	9.564	8.338	7.052
.0185	26.22	23.918	22.370	20.376	16.378	10.676	9.630	8.396	7.098
.01875	26.42	24.088	22.430	20.520	16.494	10.748	9.694	8.452	7.148
.019	26.66	24.314	22.688	20.664	16.610	10.818	9.758	8.508	7.196
.01925	26.80	24.426	22.846	20.808	16.726	10.890	9.822	8.564	7.242
.0195	26.96	24.594	23.004	20.950	16.840	10.960	9.886	8.620	7.288
.01975	27.16	24.760	23.158	21.092	16.954	11.030	9.950	8.674	7.336
.020	27.34	24.92	23.314	21.232	17.068	11.10	10.012	8.730	7.382
.0205	27.70	25.24	23.618	21.512	17.390	11.238	10.136	8.838	7.474
.021	28.06	25.58	23.924	21.790	17.514	11.374	10.258	8.944	7.564
.0215	28.40	25.90	24.224	22.062	17.734	11.508	10.382	9.050	7.654
.022	28.76	26.22	24.526	22.338	17.954	11.642	10.502	9.156	7.742
.0225	29.12	26.54	24.830	22.616	18.178	11.774	10.626	9.260	7.830
.023	29.44	26.84	25.10	22.864	18.378	11.904	10.738	9.360	7.916
.0235	29.78	27.14	25.40	23.126	18.588	12.032	10.854	9.462	8.002
.024	30.10	27.44	25.68	23.386	18.796	12.160	10.968	9.562	8.086
.0245	30.44	27.76	25.96	23.642	19.002	12.246	11.082	9.662	8.170
.025	30.76	28.06	26.24	23.896	19.208	12.410	11.194	9.760	8.254
.030	33.88	30.90	28.90	26.32	21.154	13.594	12.262	10.690	8.970
.040	39.46	35.98	33.66	30.64	24.634	15.702	14.164	12.348	10.442
.050	44.40	40.48	37.96	34.50	27.62	17.550	15.830	13.802	11.672
.060	48.90	44.60	41.70	37.98	30.54	19.226	17.342	15.118	12.786
.070	53.06	48.38	45.26	41.22	33.14	20.766	18.732	16.330	13.808
.080	56.96	51.92	48.56	44.24	35.56	22.20	20.024	17.458	14.764
.090	60.62	55.26	52.90	44.96	37.84	23.524	21.242	18.518	15.662
0.100	64.08	58.44	54.66	49.78	40.0	24.82	22.388	19.526	16.706

Weir Discharges.

Francis' Formula.

The discharge of a sharp-edged measuring weir is usually computed from Francis' formula, which reads:

$$Q = 3.33\ (b - n\,0.1H)\ (H + h)^{\frac{3}{2}} - h^{\frac{3}{2}},$$

in which Q = discharge in cubic feet per second;

b = breadth of weir in feet;

n = number of end contractions;

H = the vertical distance between the crest of the weir and the surface of the still water in the reservoir or the channel;

h = the head due to the velocity of approach.

The head due to the velocity of approach is found from the equation

$$h = \frac{\dfrac{Q^2}{A^2}}{2g}$$

in which Q = discharge found from the formula given above, neglecting the velocity of approach;

A = cross-section of the channel or reservoir parallel to the weir, at the point where the surface of the water begins to slope towards the weir.

If the discharge of the weir is small in comparison with the width and depth of the channel or the contents of the reservoir the velocity of approach and the head due to it may be neglected.

Table K contains values of $3.33\ H^{\frac{3}{2}}$.

The table is used as follows:

Let the depth of the water from the crest of the weir to the still surface be 3 feet.

Let the head due to the velocity of approach be 0.1 foot. Then $3.33\ (H + h)^{\frac{3}{2}} - 3.33\ h^{\frac{3}{2}} = 3.33\ (3.1)^{\frac{3}{2}} - 3.3\ (0.1)^{\frac{3}{2}} = 18.176 - 0.1053 = 18.0707$.

Let the breadth of the weir be 10 feet and we have:

$Q = (10 - 2 \times 0.3) \times 18.0707 = 169.86458$ cubic feet per second.

TABLE K.

H	$3.33\ H^{\frac{3}{2}}$	H	$3.33\ H^{\frac{3}{2}}$	H	$3.33\ H^{\frac{3}{2}}$	H	$3.33\ H^{\frac{3}{2}}$	H	$3.33\ H^{\frac{3}{2}}$
0.01	0.00333	0.30	0.5472	0.78	2.294	1.65	7.025	2.85	16.022
0.02	0.009406	0.32	0.6028	0.80	2.383	1.70	7.381	2.90	16.445
0.03	0.01722	0.34	0.6602	0.82	2.478	1.75	7.709	2.95	16.872
0.04	0.02664	0.36	0.7193	0.84	2.564	1.80	8.042	3.0	17.307
0.05	0.03898	0.38	0.7792	0.86	2.656	1.85	8.379	3.05	17.736
0.06	0.05125	0.40	0.8425	0.88	2.749	1.90	8.721	3.10	18.177
0.07	0.06167	0.42	0.9064	0.90	2.843	1.95	9.068	3.15	18.61
0.08	0.07535	0.44	0.9719	0.92	2.938	2.0	9.418	3.20	19.06
0.09	0.08991	0.46	1.0389	0.94	3.035	2.05	9.774	3.25	19.50
0.10	0.1053	0.48	1.1074	0.96	3.132	2.10	10.144	3.30	19.96
0.11	0.1215	0.50	1.1773	0.98	3.231	2.15	10.498	3.35	20.42
0.12	0.1383	0.52	1.2486	1.0	3.330	2.20	10.866	3.40	20.88
0.13	0.1561	0.54	1.3215	1.05	3.583	2.25	11.239	3.45	21.34
0.14	0.1744	0.56	1.3958	1.10	3.842	2.30	11.616	3.50	21.81
0.15	0.1934	0.58	1.4708	1.15	4.107	2.35	11.996	3.55	22.27
0.16	0.2133	0.60	1.5476	1.20	4.377	2.40	12.381	3.60	22.75
0.17	0.2333	0.62	1.6260	1.25	4.654	2.45	12.770	3.65	23.22
0.18	0.2543	0.64	1.7050	1.30	4.936	2.50	13.163	3.70	23.70
0.19	0.2758	0.66	1.7855	1.35	5.223	2.55	13.560	3.75	24.18
0.20	0.2978	0.68	1.8672	1.40	5.516	2.60	13.960	3.80	24.67
0.22	0.3436	0.70	1.7502	1.45	5.815	2.65	14.365	3.85	25.15
0.24	0.3915	0.72	2.0344	1.50	6.117	2.70	14.773	3.90	25.65
0.26	0.4413	0.74	2.1197	1.55	6.426	2.75	15.186	3.95	26.14
0.28	0.4938	0.76	2.206	1.60	6.739	2.80	15.602	4.0	26.64

THE FORMULA OF BAZIN.

The weir formula of Francis is based on experiments made with heads ranging between 5 and 19 inches and with weir crests up to 10 feet in length.

The accuracy of the formula when applied to flow over weirs having end contractions has been demonstrated; it also gives fairly good results when applied to flow over weirs whose sides are flush with the walls of the channel of approach. In that case $n = 0$. The difficulty in the application of the formula of

Francis consists in the fact, that it is frequently impossible to evaluate properly the head due to the velocity of approach.

If the formula of Bazin is used the head due to the velocity of approach does not enter directly into calculations; it is replaced by a coefficient which depends for its value on the relation between the head and the vertical distance between the crest and the floor of the channel of approach.

Bazin conducted his experiments with weirs 0.5, 1.0 and 2.0 meters wide, the heads ranging between 0.05 meters (2 inches) and 0.6 meters (24 inches). The crests of the weirs were raised to various heights above the floors of the channels of approach and the sides were flush with the walls of the channels. The formula of Bazin reads:

$$Q = \tfrac{2}{3}\, m \left[1 + 0.55 \left(\frac{h}{p + h}\right)^2\right] Lh\sqrt{2\,gh}$$

in which $m = 0.6075 + \dfrac{0.01418}{h}$,

h = the head above the crest to the surface of the still water;

p = the depth of the water below the crest to the floor of the channel of approach.

The formula as given holds good for any system of measure. For English measure it may be written:

$$Q = (3.2485\, h^{\frac{3}{2}} + 0.07914\sqrt{h})\, L\left[1 + 0.55\,\frac{h^2}{(p + h)^2}\right].$$

Values of $\left[1 + \dfrac{0.55\,h^2}{(p + h)^2}\right]$ are found in Table L.a. It will be observed, that the value of this factor diminishes rapidly as the relation $\dfrac{h}{p}$ diminishes in value.

It is equal to 1.2444 for $\dfrac{h}{p} = \dfrac{2}{1}$. 1.0220 for $\dfrac{h}{p} = \dfrac{1}{4}$.

1.1375 for $\dfrac{h}{p} = \dfrac{1}{1}$. 1.0068 for $\dfrac{h}{p} = \dfrac{1}{8}$.

1.0611 for $\dfrac{h}{p} = \dfrac{1}{2}$.

TABLE L.a.

$h:p$	$1+\frac{0.55\,h^2}{(p+h)^2}$	$h:p$	$1+\frac{0.55\,h^2}{(p+h)^2}$
1 : 25	1.0008	1 : 3	1.0344
1 : 20	1.0012	1 : 2.75	1.0391
1 : 15	1.0021	1 : 2.5	1.0449
1 : 10	1.0045	1 : 2.25	1.0529
1 : 9.5	1.0050	1 : 2	1.0611
1 : 9	1.0055	1 : 1.75	1.0727
1 : 8.5	1.0061	1 : 1.5	1.0880
1 : 8	1.0068	1 : 1.25	1.1086
1 : 7.5	1.0076	1 : 1	1.1375
1 : 7	1.0086	1.25 : 1	1.1698
1 : 6.5	1.0097	1.5 : 1	1.1979
1 : 6	1.0112	1.25 : 1	1.2228
1 : 5.5	1.0130	2 : 1	1.2444
1 : 5	1.0153	2.25 : 1	1.2638
1 : 4.75	1.0166	2.5 : 1	1.2806
1 : 4.5	1.0181	2.75 : 1	1.2960
1 : 4.25	1.0199	3 : 1	1.3095
1 : 4	1.0220	3.25 : 1	1.3220
1 : 375	1.0244	3.5 : 1	1.3331
1 : 3.5	1.0276	3.75 : 1	1.3431
1 : 3.25	1.0304	4 : 1	1.3520

TABLE L.b.

VALUES OF $Q = 3.2485\, h^{\frac{3}{2}} + 0.07914 \sqrt{h}$.

h	Q In Cu. Ft. per Sec.	h	Q In Cu. Ft. per Sec.	h	Q In Cu. Ft. per Sec.	h	Q In Cu. Ft. per Sec.
0.01	0.0111	0.82	2.4900	2.60	13.7481	4.65	32.7513
0.02	0.0204	0.84	2.5738	2.65	14.1393	4.70	33.2832
0.04	0.0418	0.86	2.6646	2.70	14.5405	4.75	33.8031
0.06	0.0695	0.88	2.7565	2.75	14.9417	4.80	34.3340
0.08	0.0960	0.90	2.8493	2.80	15.3529	4.85	34.8749
0.10	0.1279	0.92	2.9432	2.85	15.7641	4.90	35.4158
0.12	0.1625	0.94	3.0380	2.90	16.1753	4.95	35.9561
0.14	0.1998	0.96	3.1338	2.95	16.5864	5.0	36.4976
0.16	0.2304	0.98	3.2308	3.00	17.0176	5.10	37.5894
0.18	0.2819	1.0	3.3276	3.05	17.4387	5.2	38.7011
0.20	0.3261	1.05	3.5763	3.10	17.8698	5.3	39.8229
0.22	0.3724	1.10	3.8313	3.15	18.3010	5.4	40.9446
0.24	0.4208	1.15	4.0911	3.20	18.7421	5.5	42.0863
0.26	0.4712	1.20	4.3570	3.25	19.1732	5.6	43.2380
0.28	0.5238	1.25	4.6288	3.30	19.6143	5.7	44.3996
0.30	0.5773	1.30	4.9065	3.35	20.0654	5.8	45.5713
0.32	0.6330	1.35	5.1873	3.40	20.5165	5.9	46.7530
0.34	0.6904	1.40	5.4750	3.45	20.9675	6.0	47.9440
0.36	0.7493	1.45	5.7676	3.50	21.4186	6.2	50.3478
0.38	0.8090	1.50	6.0653	3.55	21.8797	6.4	52.8010
0.40	0.8720	1.55	6.3682	3.60	22.3407	6.6	55.2940
0.42	0.9356	1.60	6.6755	3.65	22.8018	6.8	57.8071
0.44	1.0080	1.65	6.9560	3.70	23.2728	7.0	60.3701
0.46	1.0673	1.70	7.3046	3.75	23.7438	7.2	62.9731
0.48	1.1353	1.75	7.6261	3.80	24.2248	7.4	65.6061
0.50	1.2047	1.80	7.9515	3.85	24.6959	7.6	68.2791
0.52	1.2753	1.85	8.2820	3.90	25.1769	7.8	70.9918
0.54	1.3473	1.90	8.6175	3.95	25.6598	8.0	73.7341
0.56	1.4204	1.95	8.9569	4.00	26.1463	8.2	.76.5075
0.58	1.4955	2.0	9.3023	4.05	26.6398	8.4	79.3202
0.60	1.5715	2.05	9.6483	4.10	27.1308	8.6	82.1629
0.62	1.6485	2.10	10.0011	4.15	27.6218	8.8	85.0157
0.64	1.7265	2.15	10.3575	4.20	28.1228	9.0	87.9483
0.66	1.8065	2.20	10.7188	4.25	28.6237	9.2	90.8909
0.68	1.8865	2.25	11.0830	4.30	29.1347	9.4	94.0335
0.70	1.9694	2.30	11.4524	4.35	29.6357	9.6	96.8661
0.72	2.0524	2.35	11.8247	4.40	30.1466	9.8	99.9187
0.74	2.1363	2.40	12.2010	4.45	30.6675	10.0	102.9812
0.76	2.2212	2.45	12.5823	4.50	31.1785		
0.78	2.3081	2.50	12.9656	4.55	31.6994		
0.80	2.3950	2.55	13.3569	4.60	32.2204		

If accurate results are desired from the application of this formula the depth p as well as the length L should never be less than $2h$, and the width of the channel of approach should increase up stream from the crest.

Heads are most conveniently and accurately ascertained by means of a plumb-bob, the string of which is hung over a nail driven horizontally and pulled horizontally along a board to which a graduated scale is attached. A datum reading is taken and laid off on the scale when the surface of the water is just flush with the crest and the point of the plumb-bob grazes the surface when it is made to swing to and fro.

Of weirs not originally constructed to be measuring devices those most frequently found are the sharp-crested triangular weir, the triangular weir with a quarter round crest and the rectangular, broad-crested weir. The factors of proportional discharge for these shapes, for which we are indebted to Bazin, the Cornell Engineers, G. W. Rafter and others, are as follows, the down stream face being in all cases vertical, air admitted under the descending sheet of water and the relation between h and p being the same as for the sharp edged measuring weir:

Description of Shape of Cross-Section.	Head on Sill in Feet.					
	0.5	1.0	1.5	2.0	3.0	4.0
Crest triangular, up stream face inclined 1 : 1.	1.06	1.079	1.092	1.094	1.082	1.072
Crest quarter round of circle, diameter 1 meter, up stream face inclined 1 : 1.	0.971	0.983	1.012	1.040	1.072	1.097
Crest rectangular, both faces vertical. Thickness of wall:						
0.5 feet	0.902	0.972	1.0	1.0	1.0	1.0
1.0 feet	0.830	0.904	0.957	0.989	1.0	1.0
1.5 feet	0.819	0.879	0.910	0.925	0.928	0.947
3.0 feet	0.797	0.812	0.821	0.821	0.813	0.808
6.0 feet	0.785	0.800	0.807	0.805	0.796	0.790
9.0 feet	0.783	0.798	0.803	0.800	0.797	0.783
16.0 feet	0.783	0.792	0.797	0.797	0.784	0.777

Weir Formulæ.

Weirs are constructed for the following purposes:

(1) To measure the discharge of a conduit.

(2) To regulate the discharge of a conduit.

(3) To serve as impounding and regulating dams for the storage of water.

(4) To raise the surface of the water at a certain point to a certain level.

According to the manner of outflow weirs are classified as follows:

(1) Complete overflow weirs, when the crest of the weir is above the surface of the run-off water.

(2) Incomplete weirs, when the crest of the weir is below the surface of the runoff water.

(3) Discontinuous weirs (wing dams, bridge piers, etc.) when the weir does not extend the whole width of the channel.

(4) Sluice weirs (water-gate, head-gate, regulating weir, needle weir, etc.), when the water flows out through an orifice.

Theoretically the discharge through a rectangular sharp-edged orifice is found as follows:

Let b be the breadth of a rectangular jet,
h_1 the depth of its upper,
h_2 the depth of its lower surface below the surface of the still water (Fig. 8).

An infinitesimal thin layer of the jet between its surface and an infinitesimal depth h has a section equal to bdh.

The velocity of flow in this infinitesimal layer bdh is equal to

$$v = \sqrt{2gh}.$$

The discharge will consequently be

$$q = \int_{h_1}^{h} b\sqrt{2gh}\,dh,$$

which integrated, gives for the discharge of the whole jet

$$q = \tfrac{2}{3}\,b\sqrt{2g}\,(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}}).$$

Let B be the breadth of the orifice,
H_1 the depth of its upper,
H_2 the depth of its lower edge below the free surface,

we then have for the coefficient of discharge

$$C = \frac{b\,(h_2^{\frac{3}{2}} - h_1^{\frac{3}{2}})}{B\,(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}})},$$

and for the discharge in terms of the orifice

$$Q = \tfrac{2}{3}\,CB\sqrt{2g}\,(H_2^{\frac{3}{2}} - H_1^{\frac{3}{2}}). \qquad (1)$$

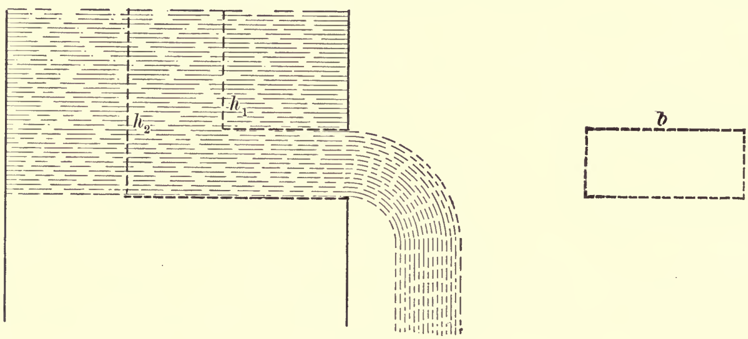

FIG. 8.

The value of C, the coefficient of discharge, differs with the nature of the orifice, and must be found by experiment. (For sharp-edged orifices and weirs $C = 0.622$, for broad-crested weirs $C = 0.577$).

For the discharge through a rectangular sharp-edged notch we have, since there is in this case no head H_1

$$Q = \tfrac{2}{3}\,CB\sqrt{2g}\,H_2^{\frac{3}{2}}. \qquad (2)$$

The discharge through rectangular notches and over sharp-crested weirs, has been minutely investigated by Bazin, Francis, and others. Bazin found for the discharge the expression,

$$Q = \frac{2}{3}\left[c\,1 + 0.55\left(\frac{h}{p+h}\right)^2\right]bh\sqrt{2gh} \qquad (3)$$

in which $c = 0.6075 + \dfrac{0.0148}{h},$

p = height of crest above bottom of channel.

Francis found that the loss of discharge due to end contraction is equal to $\frac{1}{10}$ the height of the submerged opening for each contraction. The discharge is consequently

$$Q = \tfrac{2}{3} c \, (b - 0.1 \, mH) \sqrt{2g} \, H^{\frac{3}{2}},$$

in which m is the number of end contractions.

If Francis' formula is used and the discharge is relatively large compared with the dimensions of the conduit, the head due to the velocity of approach must also be considered. This head is equal to

$$a = \frac{v^2}{2g},$$

the velocity of approach is equal to

$$v = \frac{Q}{b_0 h_0},$$

b_0 and h_0 being the breadth and depth of the channel at the point where the surface of the water begins to drop towards the crest of the opening. Making this correction for the head due to the velocity of approach, Francis' formula becomes

$$Q = \tfrac{2}{3} c (b - 0.1 H) \sqrt{2g} \, [(H + a)^{\frac{3}{2}} - a^{\frac{3}{2}}],$$

in which $c = 0.622$ for sharp-edged orifices.

Putting $\frac{2}{3} \, 0.622 \sqrt{2g} = 3.33$, Francis' formula reads

$$Q = 3.33 \, (b - m \, 0.1 \, H) \, [(H + a)^{\frac{3}{2}} - a^{\frac{3}{2}}]. \qquad (4)$$

Bazin's and Francis' formulæ give equally good results; the latter is the one most frequently used in this country.

To measure the discharge of a small stream (pipe line, flume, etc.), a temporary weir of planks is usually constructed. In order to arrive at accurate results care must be taken that the sill or the crest of the weir is perfectly level, that it is at right angles to the line of flow, and that it is above the surface of the run-off water. The head on the sill should not be less than one-half, nor more than 2 feet, and the depth of water in the channel should be at least three times the head on the crest. In order to measure the head on the sill a stake is driven in the bed of the stream a short distance above the weir. The top of the stake must be on a perfect level with the crest of the weir. A thin-edged graduated scale fastened vertically to the top of the stake is very convenient. On this gauge the height H, the head on the sill is read off to the surface of the still water.

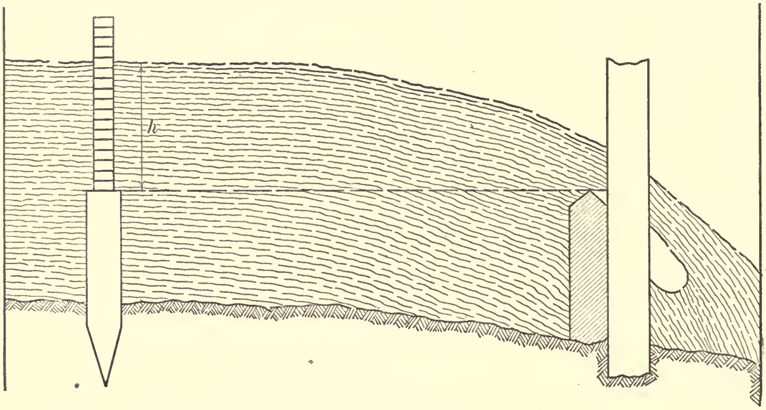

Fig. 9. Measuring Weir.

Weirs intended to regulate the discharge of a conduit, or to raise the surface of the water at a certain point to a certain level are constructed of various materials and in various forms. A complete overflow weir, when constructed of masonry in the bed of a stream, is usually of the form shown in Fig. 10.

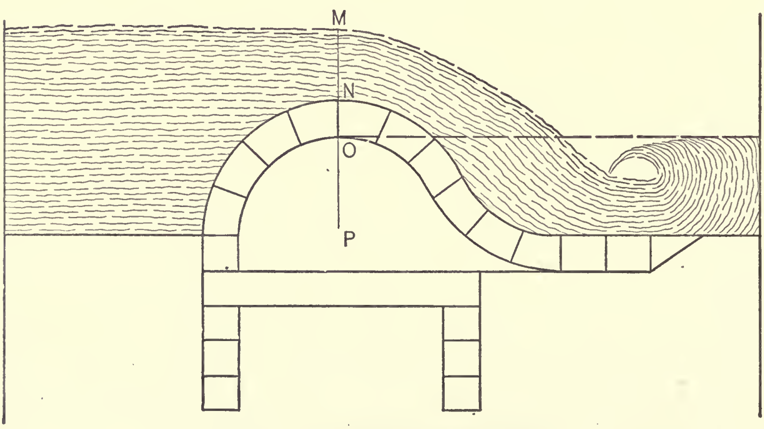

Fig. 10. Complete Overflow Weir.

The height of the weir necessary to raise the surface of the water to a given height h, is found as follows:

Let H be the head on the sill, measured to still water,
b the breadth of the channel,
a the height due to the velocity of approach,
h the difference of level between the surface of the water down stream and up stream, or the swell (MO, Fig. 10)

and the discharge is

$$Q = \tfrac{2}{3}\, cb\sqrt{2g}\,[(H + a)^{\frac{3}{2}} - a^{\frac{3}{2}}],$$

from which we find for the head on the sill

$$H = \left(\frac{\frac{3}{2}\,Q}{cb\sqrt{2\,g}} + a^{\frac{3}{2}}\right)^{\frac{2}{3}} - a$$

$$= \left(\frac{Q}{3.33\,b} + a^{\frac{3}{2}}\right)^{\frac{2}{3}} - a.$$

Denoting the height of the weir above the bottom of the channel by x and the depth of the run-off water down stream by f, we have

$$x + H = h + f,$$

hence $$x = (f + h) - H,$$

or $$x = f + h - \left(\frac{\frac{3}{2}\,Q}{cb\sqrt{2\,g}}\right)^{\frac{2}{3}}$$

when the velocity of approach is small.

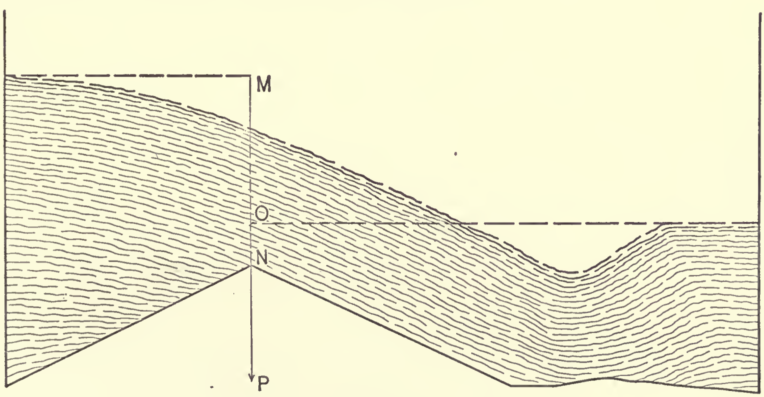

FIG. 11. Incomplete Weir.

For an incomplete weir the discharge and the height of crest necessary to raise the water to a given level are found as follows: The head on the sill (MN, Fig. 11) is greater than the swell-head (MO), therefore only the water above O flows off freely, while the water below O flows off under the head $(MO) = h$.

The discharge through (MO) is

$$q_1 = \tfrac{2}{3}\, cb \sqrt{2g}\, [(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}]$$

and that through $(ON) = H - h$ is

$$q_2 = cb\, (H - h) \sqrt{2g\, (h + a)};$$

hence the whole discharge:

$$Q = cb \sqrt{2g}\, \tfrac{2}{3}\, [(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}] + H - h \sqrt{(h + a)}.$$

From the discharge Q and the height h (MO) to which the water is raised, we find for the height of water above the crest, or the head on the sill,

$$H = h + \frac{Q}{cb\sqrt{2g\, (h + a)}} - \frac{2}{3}\, \frac{(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}}{\sqrt{(h + a)}};$$

hence we have for the necessary height of the crest of the weir above the bottom of the channel (NP),

$$(NP) = x = (f + h) - H = [(OP) + (MO)] - (MN).$$

Neglecting the velocity of approach we have for the height of the weir the simple expression

$$(NP) = x = f + \tfrac{2}{3}\, h - \frac{Q}{cb \sqrt{2gh}}\,.$$

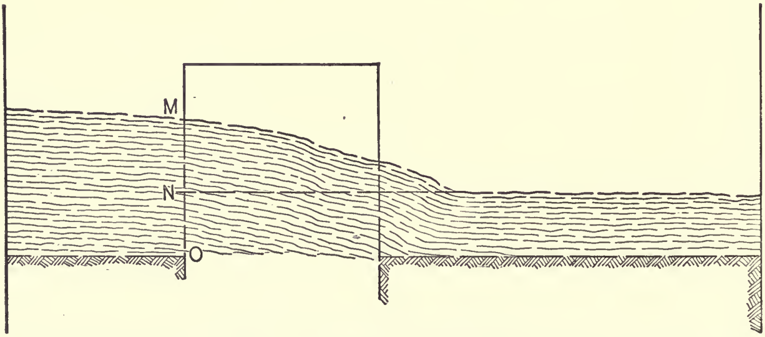

FIG. 12. Discontinuous Weir.

Wing dams are built whenever an obstruction extending the whole width of the stream is either on account of navigation not permissible, or on account of the form of cross-section of the channel not feasible or necessary.

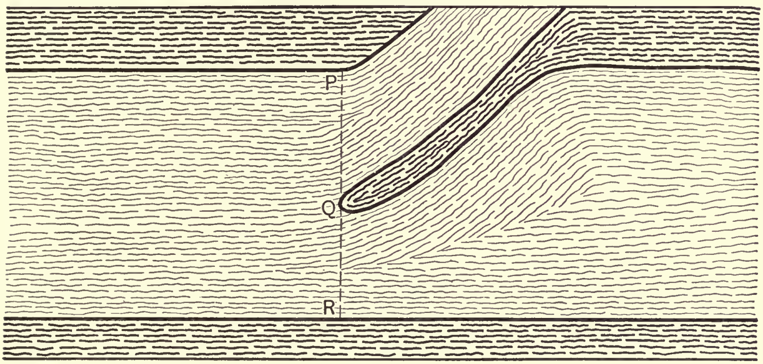

FIG. 13. Wing Dam.

While for overflow weirs the usual problem consists in finding the height of sill or crest necessary to raise the water to a given level, the problem for wing dams consists in finding the breadth of channel required to be closed in order to raise the surface of the water to the given level.

Let $QR = b =$ breadth of efflux (Fig. 13),
$MN = h =$ the height of swell,
$NO = f =$ the undercurrent,

and we have for the quantity of water flowing off freely above f the undercurrent

$$q_1 = \tfrac{2}{3}\, cb\sqrt{2\,gh^3},$$

and for the undercurrent f

$$q_2 = cbf\sqrt{2\,gh};$$

the whole discharge is consequently

$$Q = cb\sqrt{2\,gh}\,\left(\tfrac{2}{3}\,h + f\right),$$

from which we find, for the breadth of efflux,

$$b = \frac{Q}{c\,\left(\tfrac{2}{3}\,h + f\right)\sqrt{2\,gh}}.$$

If the velocity of flow in the stream is great, or the swell h comparatively small, it will be necessary to consider the velocity of approach. Denoting as before by a the head due to the velocity of approach, we have for the water flowing off freely

$$q_1 = \tfrac{2}{3}\, cb \sqrt{2\,g}\,[(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}]$$

and for the undercurrent

$$q_2 = cbf \sqrt{2\,g\;(h + a)},$$

for the whole discharge

$$Q = cb \sqrt{2\,g}\;(\tfrac{2}{3}\,[(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}] + f \sqrt{h + a},$$

and finally for the breadth of efflux

$$b = \frac{Q}{c \sqrt{2\,g}\;(\frac{2}{3}\,[(h + a)^{\frac{3}{2}} - a^{\frac{3}{2}}] + f \sqrt{h + a}}\,.$$

This formula may be applied to discontinuous weirs of any description, such as bridge piers, etc., etc. Denoting by b the

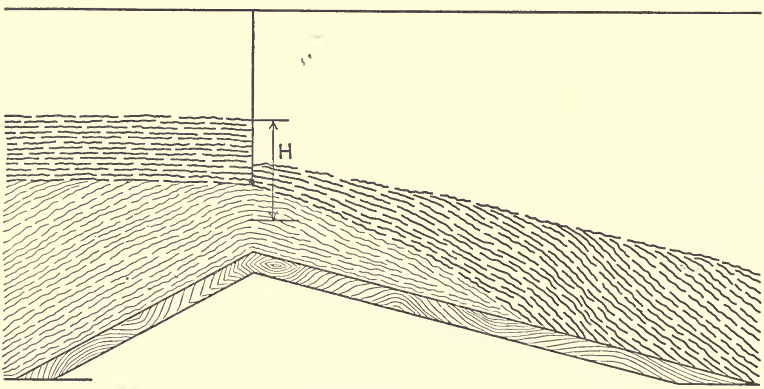

Fig. 14. Sluice Weir.

sum of the openings between bridge piers the swell may for instance be found by putting

$$\text{swell} = h = \left(\frac{\frac{3}{2}\,Q}{cb \sqrt{2\,gf}}\right)^{\frac{2}{3}}.$$

The coefficient of efflux for discontinuous weirs is very high, usually only the end contraction needs to be considered.

For wing dams $c = 0.98$ will give good results. For well-rounded bridge piers c may be taken equal to 0.90, for those forming acute angles $c = 0.95$, and for those of elliptical cross-section $c = 0.97$.

Sluice weirs are constructed to regulate the discharge of a conduit or reservoir as well as to raise the surface of the water to a given level.

In computations of the discharge of sluice weirs, the head H is measured from the free surface to the center of the opening.

If the water flows off freely we have for the discharge

$$Q = cfb\sqrt{2gH},$$

and

$$H = \frac{1}{2g}\left(\frac{Q}{cfb}\right)^2$$

in which f is the height of the opening,

b = the breadth

and $c = 0.60$.

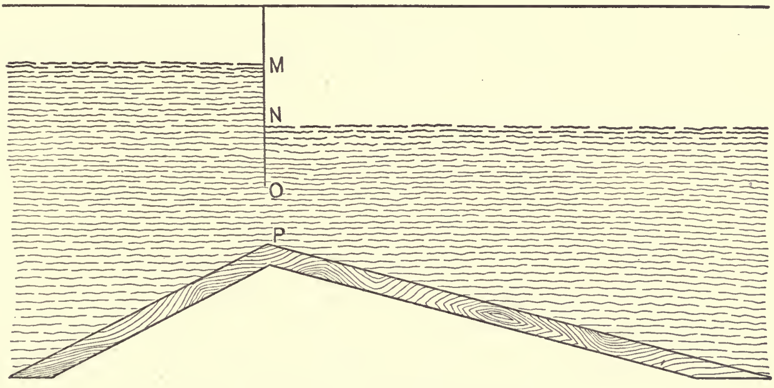

FIG. 15.

For a given discharge and a given head H the height of the opening is given by

$$f = \frac{Q}{cb\sqrt{2gH}}.$$

In case the surface of the run-off water down stream rises above the sluice opening, the effective head reduces to the distance MN (Fig. 15) and we have for the height of opening

$$f = \frac{Q}{cb\sqrt{2g\,(MN)}}.$$

If as in Fig. 16 the surface of the run-off water downstream lies somewhere within the opening, a part of the water runs off under water, while the rest flows off freely.

Let
$$MO = H$$
$$NO = f_1$$
$$OP = f_2,$$

and the discharge through NO is

$$q_1 = cf_1 b \sqrt{2\,gH - 0.5\,f_1},$$

and the discharge through OP

$$q_2 = cf_2 b \sqrt{2\,gH},$$

therefore the whole discharges through NP

$$Q = cb\sqrt{2\,g}\,(f_1\sqrt{H - 0.5\,f_1} + f_2\sqrt{h}).$$

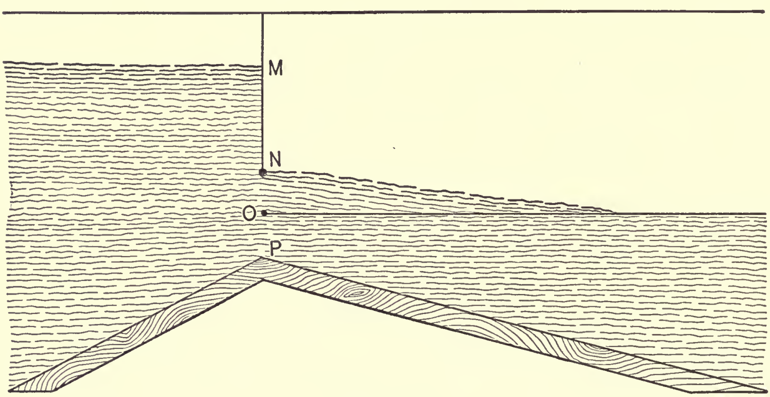

Fig. 16.

For a given discharge Q, a given effective head $H\,(MO)$ and a given height f_2 of the sill below the surface of the run-off water, the height f_1, or the distance of the lower edge of the sluice board above the surface of the runoff water may be found by putting

$$f_1 = \frac{\dfrac{Q}{cb\sqrt{2\,g}} - f_2\sqrt{H}}{\sqrt{H - 0.5\,f_1}}.$$

METHODS OF MEASUREMENT.

Loss of Head.

A.

When a conduit discharges into an open tank or reservoir, or into the open air, the loss of head is ascertained by levelling between the surface of the source of supply, and the surface of the discharge tank, reservoir or outflowing stream. When this is not the case, or when the loss in part of the conduit only is to be found, other methods must be employed. Where the pressures are not great, open stand pipes or piezometers are most convenient, otherwise the pressures are measured by means of manometers. A mercury manometer of the form generally used has the following essentials: A cast-iron mercury reservoir into one side of which a glass plate is fitted through which the height of the mercury within may be observed. A metal tube with a gate valve connects the top of the reservoir with the main pipe at the point at which the pressure is to be measured. At its highest point, this tube has an air valve. Into the mercury reservoir, which is about half filled with mercury, a vertical tube is placed, nearly reaching to the bottom. This tube, usually one quarter of an inch in diameter, is of brass or wrought iron in its lower part and of glass in its upper part. To the glass tube a graduated scale is attached. As mercury is very sensitive to changes of temperature the tube is surrounded by a water-jacket, in its upper parts also of glass. When the gauge is to be used the air valve in the connecting tube is opened and also, by degrees, the gate valve. When the air is wholly removed the air valve is closed and the gate valve fully opened. The pressure of the water in the reservoir depresses the surface of the mercury and causes it to rise in the tube. The height of the mercury column above the surface of the mercury in the reservoir is read on the graduated scale, both at times of discharge and times of no discharge.

If a is the difference of the heights of the mercury columns at two sections at times of no discharge, and A the difference at times of discharge, the loss of head between the two sections whose pressures are measured is equal to

$$H = 13.6 \quad A - a,$$

13.6 being the specific gravity of mercury.

When the conduit is of great length and the difference between the pressures at two sections considerable, a form of the manometer known as the Bourdon gauge, is used with good results. The essential parts of this instrument, universally used as a steam-gauge, consist of a hollow curved metal spring, one end of which is free to curve, while the other is fastened to the case of the instrument. A pipe connects the interior of the tube, which is oval in cross-section, with the main pipe at the point where the pressure is to be measured. The pressure of the liquid expands the spring, the free end moves and by a lever the movement is transmitted to a toothed bar lever, which again transmits the motion to a toothed wheel. The movement of the spring, thus converted into rotary motion, is, by a pointer, indicated upon a graduated circular scale. The pressure is indicated in pounds per square inch. This is converted into feet of pressure by dividing it by 0.434.

If A is the difference between the indicated pressures at two sections at times of discharge and a the difference at times of no discharge, the loss of head between the two sections is equal to

$$H = \frac{A - a}{0.434}.$$

Discharge of Conduits under Pressure.

B.

Discharges are measured by means of vessels, tanks, by the rise in the surface of a reservoir, or the overflowing stream is measured by a weir, an orifice or the current meter. When these methods are not feasible, some form of water meter is used. The best known devise of this kind is the Venturi meter, invented by Herschel and named for a celebrated Italian hydraulician.

The theory of the Venturi meter is based on the principles enunciated by Bernouilli:

"The fall of the free surface level between two sections of a conduit is equal to the difference of the heights due to the velocities at the sections."

If p_1 is the pressure at one section of a conduit and v_1 the velocity and p_2 and v_2 the pressure and velocity at another section and y and y_1 elevations above datum, then

$$\frac{p_1}{G} - \frac{p_2}{G} + y - y_1 = \frac{v_2^2 - v_1^2}{2g}.$$

In Fig. 17 the line p_1, p_2, p_3, shows the theoretical variation of the free surface level due to the contraction and subsequent enlargement of a conduit. The line p_1, p_4, p_5, shows the actual variation, the difference being due to the pressure expended in overcoming the frictional resistance of the walls of the conduit. It will be observed that this difference increases with the distance.

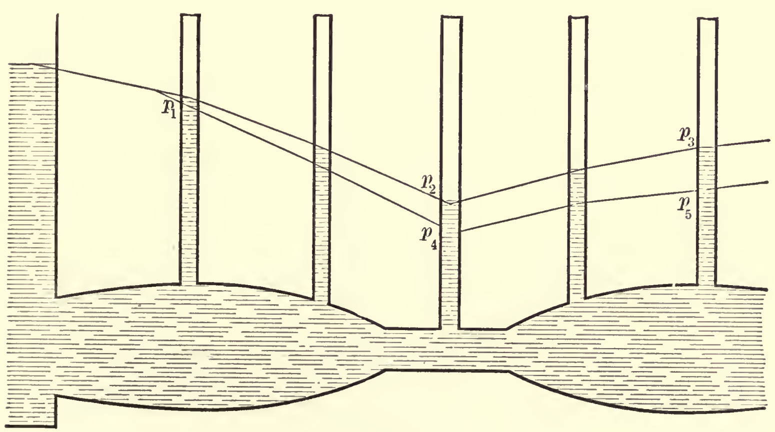

FIG. 17.

Differences of pressure in sections of conduits not far apart are most conveniently measured by mercury difference gauges. In Fig. 18 is shown a gauge of this kind connected to sections, the pressures at which are to be compared.

The bottom of the gauge is filled with mercury. When the gate valves are opened, the pressure of the water causes the mercury to rise or fall to heights which indicate the pressures at the points of the main to which the gauge is attached. A graduated scale allows a comparison of the pressures.

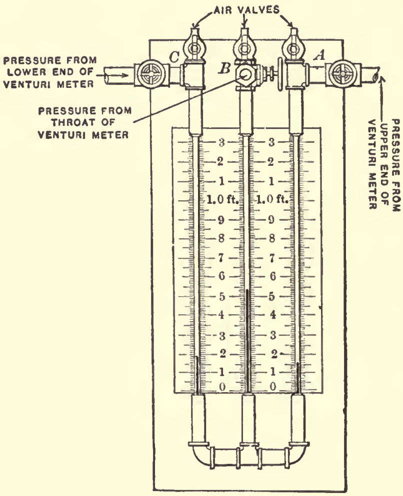

FIG. 18.

The difference between the pressures at the full section and the section most contracted indicates the difference between the velocities at the two points; the difference between the pressures at the full sections above and below the contraction corresponds to the loss of head between the two points.

Denoting the area of the section not contracted by A, the area of the section most contracted by a, and the difference between the pressures converted into feet of head of water by H, the theoretical quantity passing through the section most contracted per second is given by the equation

$$Q = \frac{A - a}{\sqrt{A^2 - a^2}} \sqrt{2 g H}.$$

For the actual discharge this is multiplied by a coefficient, which, however, differs little from unity. In the Venturi meter, as usually constructed, the area of the throat is contracted to one ninth the area of the full section of the main. Its length is from eight to sixteen times the diameter of the full section. It has a registering device which mechanically converts differences of pressure into corresponding velocities and these, for a given diameter and a certain interval of time (10 minutes), into gallons of discharge.

The meter is made in sizes from 2 up to 100 inches in diameter. The loss of head is insignificant and the condition of the water does not affect its working.

The discharge from vertical tubes was recently determined by Lawrence and Braunworth and formulæ deduced, which not only will prove to be of great value in computing the discharge of artesian wells, but furnish another method to determine the discharge of any conduit under pressure with a fair degree of accuracy. To do this, it will simply be necessary to give the end of the conduit a vertical direction and observe the elevation of the crest of the outflow above the rim of the conduit.

The investigators mentioned experimented with tubes ranging between 2 and 12 inches in diameter and 15 feet long and three conditions of out-flow were observed, depending on the pressure head. Under a feeble head the water flows simply over the rim of the conduit as it does over a sharp edged weir and the discharge is equal to

$$Q = 8.8\, d^{1.25}\ h^{1.35}.$$

When the issuing water forms a jet the discharge is equal to

$$Q = 5.57 d^{1.99}\ h^{0.53},$$

in which Q = cub. ft. per sec.

d = actual internal diameter in feet.

h = elevation of crest of water above the rim of the conduit, in feet, determined by sighting rod. For the condition intermediate between the weirflow and the jetflow no formula was deduced.

Discharge of Open Conduits.

C.

When the discharge of an open conduit cannot be measured by a weir or an orifice, it is necessary to find the mean velocity of flow.

The mean velocity in a vertical section is ascertained directly by means of rod-floats or by making measurements at the point where the thread of mean velocity is found, either with a current meter or with a double float. Indirectly the mean velocity is found by means of surface floats or by current meter observations at different points in the vertical section.

If the channel is narrow, measurements in one vertical section are generally sufficient, especially if a rod-float is used. With increasing width of the channel observations in two or more vertical sections are necessary.

When the mean velocity of flow in a river is to be ascertained, the channel is divided, at right angles to the line of flow, into sections 5, 10, 20 or more feet wide, the distance depending on the degree of accuracy desired.

The mean velocity at each section is found by means of rod-floats, by observations at the surface, at mid-depth, at the position of the thread of mean velocity or at points of proportional depth. The mean velocity for the whole channel is found by taking the mean of the mean velocities of all the sections.

For the discharge of the whole channel the mean velocity of each section is multiplied by its area and the discharges of all the sections summed up. If floats are used, the stretch over which the float is to pass should be carefully measured and staked off. If possible ropes or wires should be stretched across the stream, at right angles to the line of flow. The float should be started some distance above the rope and the time of its passage carefully observed.

The distance measured out may be 250 to 500 feet for swift streams; 50 feet will suffice if the current is feeble. The longer the stretch the more reliable the time observation. On the other hand, if the stretch is long it is often exceedingly difficult to keep the float in a position parallel to the axis of the stream. This is especially so near the banks. On this account it may be necessary to measure stretches as short as 20 feet.

A surface float may be a ball of wood or some other light material, or else a watertight metal cylinder, so loaded as to float flush with the surface of the stream. A small flag will render the float more visible.

Double floats are used to find the velocities at different depths below the surface. They consist of light surface floats connected by a fine strong cord, to a large sub-surface float. A ball of wood or a flat watertight metal box makes a good surface float, a watertight metal cylinder, heavy enough to keep the cord in tension, but not to drag it below the surface is an excellent sub-surface float. The speed of the surface float is identical with that of the larger float and observations of its passage will give the speed of the latter.

Usually the subsurface float is placed at the point where the thread of mean velocity is found. The use of double floats generally leads to trouble of one kind or another; they are rarely used, except to measure velocities in very deep channels.

A cylindrical wooden pole two inches in diameter and loaded at the bottom, so that it will float vertically, makes an excellent rod-float. It may be made in sections and screwed together. A brass cylinder screwed to the bottom makes an excellent weight. Into it shot may be placed to suit the weight to all requirements. Watertight tin tubes also make good rod-

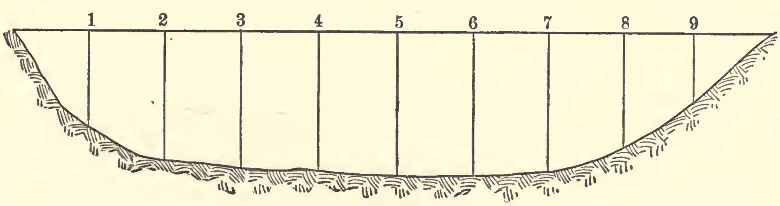

Fig. 19. Channel of River Divided into Vertical Sections.

floats. Rod floats should be loaded so that they nearly reach to the bottom of the channel, but never touch it. On the other hand they must not be too short, or else they will travel with a speed exceeding the mean velocity.

The rod-float is the ideal instrument to measure the velocity in a flume or aqueduct. The fact that it integrates the velocity of the whole section and thus indicates the mean velocity directly is an advantage not possessed by any other measuring device. If properly used it gives results whose accuracy cannot be questioned. However, if the bottom of the channel is very rough, covered with plants or else very deep, its use is not indicated.

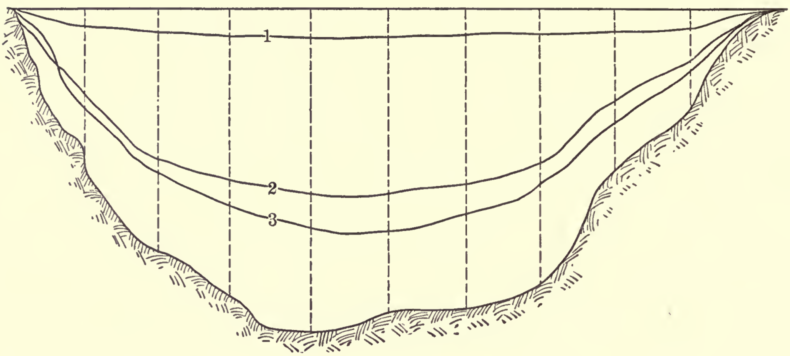

FIG. 20.

Velocity measurements are made in the centre of each section. Depths are taken by soundings.

Line (1) indicates the position of the thread of maximum velocity in each section, line (2) the position of the thread of mean velocity in each section, and line (3) the position of the thread of mean velocity for the whole section.

The current meter, like so many other hydraulic measuring devices, originated centuries ago in the Valley of the Po, Italy, the cradle of hydraulics. The earliest form consisted of a small paddle wheel mounted in a floating frame. It could only be used at the surface.

When Woltman, in 1790, added a recording device the instrument could be used at any depth. The recording mechanism consisted of an endless screw fitted to the horizontal axis; and a series of toothed wheels which transmitted the motion of the axis to a register. The recording mechanism was thrown in and out of gear by a string, attached to a lever. The instrument was fitted and clamped to a one-inch pole on which it could be slid up and down. To read the number of revolutions recorded the current meter had to be taken out of the water. The instrument was generally known as "Woltman's Tachometer."

Many modifications of this instrument appeared, mostly of the windmill pattern, with propellers and vanes. Some have the axis of the propeller horizontal, others vertical, and the shape of the propellers is variable. The general form of the instrument is, however, always the same. The present day current meter has an electrical signalling or registering device. The best known patterns are those of Harlacher in Europe, and those of Price and Ritchie-Haskell in the United States.

The Harlacher meter is of the windmill pattern; its propeller has four blades. A vane about 12 inches long and 5 inches wide is fitted to a prolongation of the axis of the wheel. This directing device keeps the face of the wheel at right angles to the line of flow. To the axis of the propeller is fitted an endless screw, operating a toothed wheel. A pin in the side of the wheel strikes an electric wire at each revolution, thus completing an electrical circuit. The battery with the registering or sounding device is kept at the surface. The meter slides up or down on a vertical rod. To move the meter up and down with a uniform speed an apparatus consisting of ropes, pulleys and weights is often used.

The propeller of the Price current meter has four cup-shaped wings; its axis is vertical and its revolutions are indicated by an electrical buzzer. The instrument is generally used without a rod; it is kept vertical by a weight attached to the frame and moved up and down by a cord. Its vane consists of two blades, one horizontal, the other vertical, intersecting in the middle at right angles. It is made in two sizes. The small meter measures velocities as low as 0.2 feet per second with a fair degree of accuracy; the large meter gives good results down to velocities of 0.5 foot per second.

The latest design in the line of meters is the Ritchie-Haskell so-called "direction current meter." Like the Harlacher and the Price this instrument has a device recording the number of revolutions of the propeller electrically. It has also a device indicating the direction of the current. The body of the instrument is a compass with a magnetic needle. An electrical circuit measures the angle between the direction of the needle and the direction in which the vane points and indicates the angle on a graduated dial.

Current meters must be rated; that is, the relation between the velocities and the number of revolutions of the propeller must be ascertained. This is done by pulling the meter at various constant speeds through a still body of water, and determining the relation between speeds and revolutions.

Current meters as furnished by the makers are always rated, but they must subsequently be rerated at frequent intervals, if good results are desired. As with floats, measurements with the current meter are made in various ways. The best method is no doubt the one adopted by Harlacher of sliding the instrument by means of a mechanism at a uniform speed up and down on a pole. By this process the velocity of the whole section is integrated and a very good mean value found. If no pole is used the instrument is most conveniently moved up or down by means of a cord thrown over a small pulley.

A good current meter, properly rated and carefully handled, surpasses any other instrument in the facility and extent of its application; it gives results nearly as trustworthy as the rod-float, and for average velocities nearly as accurate as a weir.

The Darcy gauge, an instrument formerly in great favor, is at present, owing to the great perfection of the current meter, but rarely used. The instrument consists of a combination of two Pitot tubes, fastened to a supporting frame.

A Pitot tube is a vertical glass tube with a right-angled bend. If such a tube is placed into a stream, with its mouth facing up-stream and at right angles to the line of motion, the water will ascend in the tube to a height which is equal to

$$h = \frac{v^2}{2g}, \text{ nearly.}$$

If the mouth of the tube faces the bank of the stream, and is in line with the line of motion, there will be no difference of level between the surface of the water in the tube and the surface of the stream.

If the mouth of the tube faces downstream and is at right angles to the line of motion, the surface of the water in the tube will be below the surface of the stream, the difference being equal to

$$h_1 = \frac{v_1^2}{2g}.$$

In this case the velocity is somewhat modified by the retarding influence of the tube. Darcy combined two tubes having their mouths at right angles, and provided their lower parts with stopcocks, which can be operated, when the instrument is in the water, by means of a string. If the cocks are open and the mouth of one of the tubes faces upstream at right angles to the line of motion the water will ascend in it while it will not ascend in the other tube. If the corks are then closed, the instrument may be lifted out of the water and the difference of level in the two tubes read off on a graduated scale.

Surface Mean and Bottom Velocities.

Position of Thread of Mean Velocity.

From 82 observations of flow in small channels Bazin deduces the following:

Mean Velocity = Maximum Velocity − 25.4 $\sqrt{r.s}$
Bottom Velocity ≐ Maximum Velocity − 36.3 $\sqrt{r.s}$
Bottom Velocity = Mean Velocity − 10.87 $\sqrt{r.s}$

From this we have

$$\frac{V \text{ mean} + 25.4\sqrt{r.s}}{V \text{ max.}} = 1.0,$$

$$\frac{V \text{ mean}}{V \text{ max.}} = \frac{1}{1 + 25.4\sqrt{r.s}},$$

and as

$$\frac{1}{\sqrt{\frac{r.s}{v^2}}} = c,$$

we have also

$$\frac{V \text{ mean}}{V \text{ max.}} = \frac{1}{1 + \frac{25.4}{c}},$$

and likewise

$$\frac{V \text{ bottom}}{V \text{ max.}} = \frac{1}{1 + \frac{36.3}{c}}.$$

Comparison of values of $\frac{V \text{ mean}}{V \text{ max.}} = \frac{1}{1 + \frac{25.4}{c}}$ with values of $\frac{V \text{ mean}}{V \text{ max.}}$ found by observations of flow in a great variety of channels shows that Bazin's formula is not of general application. It fails because the influence of the value of the total depth of the channel is not considered.

The following values of $\frac{V \text{ mean}}{V \text{ surface}}$ are given by the most reliable authorities:

	$\frac{V \text{ mean}}{V \text{ surface}}$
Revy, Parana de las Palmas, La Plata	0.835
Harlacher, Bohemian Rivers, 28 observations	0.838
Swiss Engineers, Swiss Rivers, 200 observations	0.835
Lippincott, Sacramento River, Cal., Depth, 3–5 feet	0.88
Lippincott, Tuolumne River, Cal., Depth, 1.12–1.84 feet	0.88
Lippincott, San Gabriel and Santa Anna, Rough channels, 10–20 feet wide. Depth, 0.25–1.0 feet	0.92
Pressey, Catskill Creek, partial section	0.82
Pressey, Fishkill Creek, partial section	0.93
Pressey, Mean of 28 observations of flow in rivers with rough bottoms, Average depth, 5.05 feet	0.80
Prony, Small wooden channels	0.8164
Prony and Destrem, Neva River, Russia	0.78
Boileau, Canals	0.82
Baumgartner, Garrone River, France	0.80
Cunningham, Solani Aqueduct	0.823
Humphreys & Abbot, Mississippi	0.79–0.82

From these and other data given by Murphy (Cornell testing flume) and others, the writer found that the relation between the surface velocity and the mean velocity may be expressed by the equation

$$\text{Mean velocity} = \frac{1}{1 + n\frac{\sqrt[4]{r}}{\sqrt[4]{c}}} \text{surface velocity} \qquad (1)$$

in which n is a coefficient ranging in value between 0.25 for the roughest and 0.35 for the smoothest classes of conduits.

Its value is

$$n = 0.32 \text{ for } K = 1.25$$
$$n = 0.30 \text{ for } K = 1.75$$
$$n = 0.27 \text{ for } K = 2.25$$

For the velocity at any point x, depth d, in the vertical section we found from data relating to flow in channels with rough bottoms, such as rivers with detritus or coarse gravel,

$$Vx = \frac{1}{1 + \left(\frac{dx}{D}\right)^{\frac{7}{2}}} \qquad (2)$$

in which D is the total depth. This is on the assumption that the bottom velocity is equal to one half the surface velocity, a relation which holds good only for channels with rough bottoms. Bazin found from observations of flow in small artificial channels that the difference between the surface and the bottom velocity ranges between 0.25 and 0.5 of the surface velocity, the difference increasing in value with the roughness of the walls. In canals and rivers with comparatively smooth bottoms the difference ranges between 0.3 and 0.4, the average difference being 0.35 of the surface velocity.

Combining the two equations (1) and (2), we have for the position of the thread of mean velocity in the vertical section of rivers and canals with somewhat rough bottoms and whose width is several times the depth

$$d = D\left(n \frac{\sqrt[4]{r}}{\sqrt[4]{c}}\right)^{\frac{2}{7}} \qquad (3)$$

as the depth below the surface at which the thread of mean velocity is found. The formula does not apply to flumes and other narrow, deep channels.

From equations (1) and (3) we find the following values of the relation $\frac{V \text{ mean}}{V \text{ surface}}$ and the relative position of the thread of mean velocity in a vertical section, assuming $K = 1.0$ and $v = 3$ feet.

R	$\frac{V \text{ mean}}{V \text{ surface}}$	Relative Depth.	R	$\frac{V \text{ mean}}{V \text{ surface}}$	Relative Depth.
1.0	0.898	0.538	10.0	0.854	0.604
2.5	0.881	0.563	15.0	0.842	0.616
5.0	0.369	0.583	25.0	0.832	0.631
7.5	0.859	0.596	30.0	0.813	0.656

APPENDIX I.

Variation of the Coefficient c with the Slope.

In the preceding chapters we have defined the variation of the coefficient c with the mean hydraulic radius, with the degree of roughness of the wet perimeter and with the velocity of flow. We will now proceed to investigate if it is possible to find a true expression for the variation of the coefficient c with the slope by the graphical method. From Formula III we have

$$66\left(\sqrt[4]{r}+m\right)V^{\frac{1}{9}}=c,$$

$$V^{\frac{1}{9}}=\frac{c}{66\left(\sqrt[4]{r}+m\right)},$$

$$v=\left(\frac{c}{66\left(\sqrt[4]{r}+m\right)}\right)^{9};$$

or substituting for v its equivalent

$$\left(66\left(\sqrt[4]{r}+m\right)\sqrt{r\,.\,s}\right)^{\frac{9}{8}}=\left(\frac{c}{66\left(\sqrt[4]{r}+m\right)}\right)^{9};$$

hence $$66\left(\sqrt[4]{r}+m\right)\sqrt{r\,.\,s}=\left(\frac{c}{66\left(\sqrt[4]{r}+m\right)}\right)^{8},$$

and $$\left(66\left(\sqrt[4]{r}+m\right)\right)^{9}\sqrt{r\,.\,s}=c^{8};$$

consequently

$$\left(66\left(\sqrt[4]{r}+m\right)\right)^{\frac{9}{8}}(r\,.\,s)^{\frac{1}{16}}=c;$$

or $$66\left(\sqrt[4]{r}+m\right)\left(66\left(\sqrt[4]{r}+m\right)\right)^{\frac{1}{8}}(r\,.\,s)^{\frac{1}{16}}=c.$$

This goes to show that c increases with $(rs)^{\frac{1}{16}}$; consequently the variation of the coefficient c with the slope depends on the value of R.

The variation of the coefficient c follows the law of the parabola. If values of the coefficients $a=V^{\frac{1}{9}}$ and $V^{\frac{1}{18}}$ are plotted as ordinates to values of v as abscissæ, the points so found lie in curves which are parabolas of the ninth or eighteenth order. A curve somewhat resembling a parabola is the equilateral hyperbola, and it is possible to draw a curve of this kind which nearly coincides with the parabola.

The equation of the equilateral hyperbola concave towards the axis of abscissæ may be put into the simple form

$$c = \frac{y}{1 + \frac{x}{R}} .$$

The curve in Fig. 1 represents the hyperbola of this equation. In the figure ZO is the vertical asymptote,

Zd the horizontal asymptote,
YK the axis of ordinates,
KX the axis of abscissæ,
Zg the axis of the hyperbola,
X the distance between the vertical asymptote ZO and the axis of ordinates YK,
c the ordinate of any point in the curve.

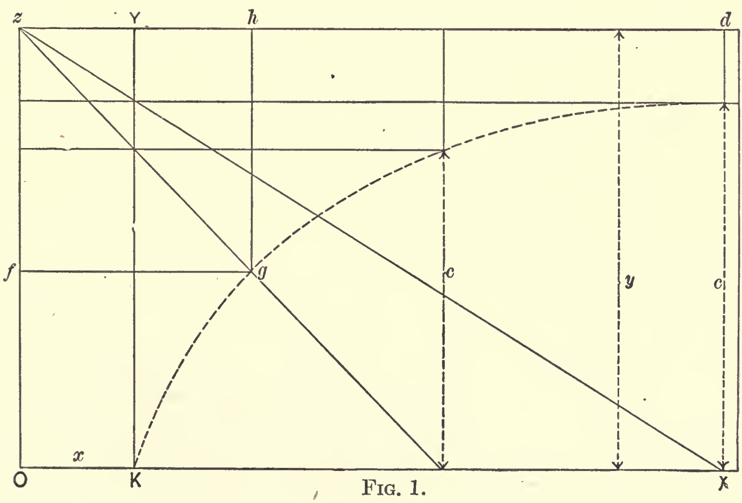

Fig. 1.

The area of the rectangle $ZOKY$ is the constant which determines the hyperbola. It is equal to the square $zfgh$ or the area of any rectangle comprised between the asymptotes and perpendiculars drawn to them from any point in the curve.

Consequently, if lines are drawn from the center z to points R on the axis of abscissæ, these lines will intersect the axis of ordinates in points which give the values of c corresponding to the values of R. In this way the hyperbola may be easily constructed.

Bazin in his paper, "Etude d'une nouvelle formule," etc., put the equation for the coefficient c into the form

$$c = \frac{y}{1 + \frac{g}{\sqrt{r}}}$$

in which y is constant and equal to 157.5 in English measure, and g, a variable, indicating the degree of roughness.

Dividing by y we have

$$c = \frac{1}{\frac{1}{y} + \frac{x}{y\sqrt{r}}} \quad \text{substituting } x \text{ for } g.$$

Transposing we have

$$\frac{1}{c} = \frac{1}{y} + \frac{x}{y}\frac{1}{\sqrt{r}}\,.$$

This is the equation of a straight line having values of $\frac{1}{\sqrt{r}}$ as abscissæ, values of $\frac{1}{c}$ as ordinates. If this equation would hold good, points of values of $\frac{1}{c}$ pertaining to one slope would lie in straight lines intersecting the axis of ordinates in a point $\frac{1}{y}$. If, however, values of $\frac{1}{c}$ and $\frac{1}{\sqrt{r}}$ are plotted as indicated it appears that only those points $\frac{1}{c}$ pertaining to data of flow in old pipes or fairly regular channels in earth lie in straight lines, while those pertaining to data of flow in very smooth conduits lie in curved lines convex towards the axis of abscissæ, and those pertaining to data of flow in very irregular channels lie in curved lines concave towards the axis of abscissæ.

If straight lines are drawn averaging between the points as much as possible, these lines will intersect the axis of ordinates

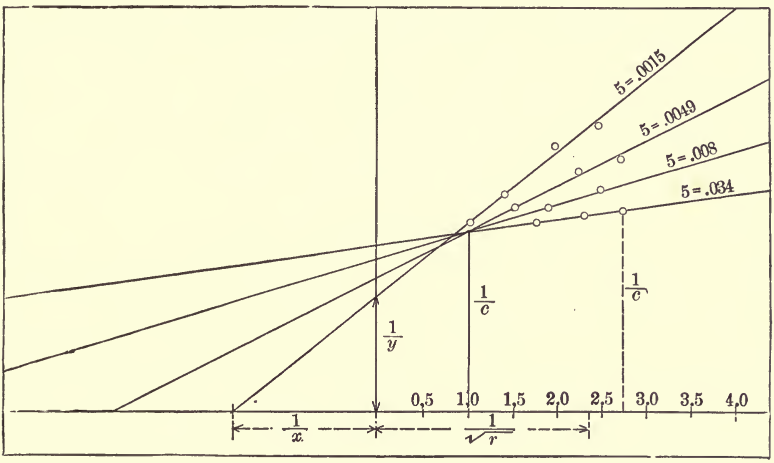

FIG. 2.

in points giving values of $\frac{1}{y}$ for the greatest value of v and the greatest value of R included in the series plotted. These lines will also intersect the axis of abscissæ in points which give the value of $\frac{1}{x}$ pertaining to each value of $\frac{1}{y}$. In Fig. 2 we thus plotted the experimental data of Darcy-Bazin, series 7, 8, 9 and one series given by Rittinger (s = 0.0343), all pertaining to flow in testing channels of rough boards.

It will be observed that the lines pertaining to the steeper slopes intersect each other in a point whose abscissa for $\frac{1}{\sqrt{r}}$ is 1.0. This is due to the fact that for the greater slopes s = 0.0049, 0.00824, and 0.0343, the velocity is so high that c varies but very little, while it varies much for the feebler slope s = 0.0015.

The highest value of $\frac{1}{c}$ corresponding to $\frac{1}{\sqrt{r}} = 1.0$ is 0.0084, the lowest 0.0080, average 0.0082. Denoting the abscissa of the point of intersection by a and the average ordinate by K we have

$$K = \frac{1}{y} + \frac{x}{y}\frac{1}{a},$$

and

$$x = Kay - a,$$

and considering $\frac{K}{\frac{1}{a}} = Ka$ as a tangent and denoting it by l, we have $x = ly - a$.

Consequently in our case

$$x = 0.0082\,y - 1.0,$$

which gives values of x very nearly equal to those found graphically. This formula will, however, only hold good for the values of R included in the series, the highest of which is 1.0.

In Fig. 3 the values of y found graphically from Fig. 2 are plotted as ordinates to values of $\frac{1}{s}$ as abscissæ. The points y are seen to lie in a curved line, intersecting the axis of ordinates at a point $B = 131.0$ nearly. If the line CD is produced, it will intersect the axis of ordinates in $y = 157.5$, which is the constant in the formula of Bazin mentioned above. The tangent of the angle CEF (in this instance 0.29) corresponds to Bazin's coefficient, g, indicating the degree of roughness. The value of m obtained from the given data is 0.70; hence the value of $\sqrt[4]{r} + m$, for the highest value of R is 1.70. Dividing 131.0 by 1.70 we have

$$y' = 77\ (\sqrt[4]{r} + m), \text{ nearly,}$$

as the value of y corresponding to the highest velocity included in the series plotted.

If from the point $B = 131.0 = 77\ (\sqrt[4]{r} + m)$ a line is drawn parallel to the axis of abscissæ, any increase in the value of

$77\ (\sqrt[4]{r} + m)$ due to any slope less than 0.0343 will appear as an ordinate above this line BG.

It will be observed that values of y'', y''', y^{iv}, etc., increase with the decrease of the slope or increasing values of $\frac{1}{s}$. We may therefore put y'', y''', etc., $= 77\ (\sqrt[4]{r} + m) + \frac{z}{s}$ in which z is a coefficient still to be determined.

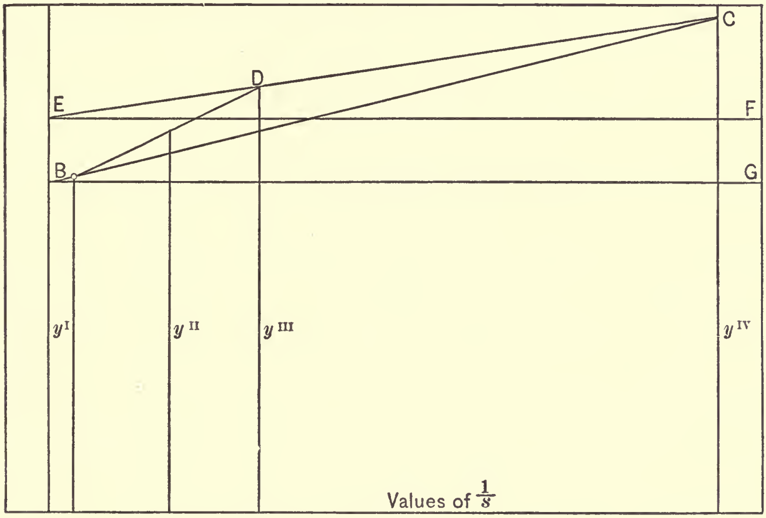

Fig. 3.

The line BDC is evidently a parabola. If a line is drawn from B to C the tangent of the angle CBG will be equal to z, for $s = 0.0015$. For this slope we have from the figure

$$y = 198.0$$
$$y' = 131.0;$$

therefore $\frac{z}{s} = 198 - 131 = 67$ and $z = 0.0015 \times 67$,

or $z = 0.1005.$

Hence $y = 77\ (\sqrt[4]{r} + m) + \dfrac{0.1}{s}$.

From experimental data pertaining to flow in small channels in earth, R ranging between 1 and 1.75 (Darcy-Bazin, Grosbois canal) which are, however, somewhat doubtful, we found $z = 0.0936$, while from data pertaining to flow in the La Plata and its tributaries we found $z = 0.00293$. From this it is evident that z is a variable and that its value depends on the value of R. Having found an expression for y, the value of x may be found from experimental data without resorting to graphical methods.

No. 12, series 6, Darcy-Bazin gives

$$R = 0.922$$
$$s = 0.00208$$
$$c = 118.9.$$

Hence $$y = 77\ (\sqrt[4]{0.922} + 0.68) + \frac{0.1}{0.00208}$$

or $$y = 127.87 + 47.6 = 175.47.$$

Dividing 175.47 by $c = 118.9$ we find $x = 1.475$, $= 1 + 0.475$. But 0.475 is equal to 0.01, $\frac{0.1}{0.00208}$ or 0.01, 47.6.

Denoting the term 0.01, which is variable, by l, we have from the given data for the variation of the coefficient c with the slope the expression

$$c = \frac{77\ (\sqrt[4]{r} + m) + \frac{0.1}{s}}{1 + \frac{0.1}{s}\frac{l}{\sqrt[4]{r}}},$$

which, within certain limits corresponds to

$$c = 66\ (\sqrt[4]{r} + m)\ V^{\frac{1}{18}}.$$

From data relating to flow in a semicircular channel of rough boards (Darcy-Bazin, series 26) we find

$$y + \frac{0.1}{s} = 210, \text{ hence } y = 210 - 67,$$

which is equal to

$$c = \frac{84\ (\sqrt[4]{r} + m) + \frac{0.1}{s}}{1 + \frac{0.1}{s}\frac{l}{\sqrt[4]{r}}},$$

and which corresponds within certain limits to

$$c = 66\ (\sqrt[4]{r} + m)\ V^{\frac{1}{12}}.$$

Dividing 84 by 66 the quotient 1.272 is the value of the coefficient of variation of c for $v = 18.0$. Hence $v = 18.0$ is the limit up to which the formula holds good.

The formula apparently gives good values of c up to the limit indicated. By trial we find, however, that it does not hold for values of R greater than 1.0; unless rs is substituted in the equation for s. Consequently the variation of c with the slope is dependent on the value of R, a fact we demonstrated at the beginning of this chapter.

The facts related plainly show that a formula derived in the manner indicated can only be of limited application. It holds good only within the range of values of R, s and m included in the series of data from which it is derived. In other words: We cannot get out of a formula what we do not put into it.

A general formula, like that of Ganguillet and Kutter, derived by the methods we have indicated, cannot embody true laws of flow, it naturally must be deficient in one respect or the other. The more so, if the data on which the formula is based are erroneous. The experimental data derived from observations of flow in the lower Mississippi by Humphreys and Abbot and embodied by Ganguillet and Kutter in their formula have been found to be incorrect, greatly at variance with those time and again found by the United States engineers. The contention of Ganguillet and Kutter, that, if values of $\frac{1}{c}$ are plotted as ordinates to values of $\frac{1}{\sqrt{r}}$ as abscissæ and lines drawn through all the points $\frac{1}{c}$ these lines will intersect each other in a point $\frac{1}{\sqrt{r}} = 1$ meter, and that therefore c will increase with increasing values of s if R is less than 1 meter, and decrease if R is greater than 1 meter, is also plainly a fallacy.

If values of $\frac{1}{c}$ and $\frac{1}{\sqrt{r}}$ derived from the numerous series given by Darcy-Bazin are plotted as indicated, it will be observed that for many of the series the lines intersect at $\frac{1}{\sqrt{r}} = 1$ foot. It would be absurd, however, to draw the conclusion therefrom that c will increase or decrease with increase of the slope if R is less or greater than 1 foot. The intersection of the lines at $\frac{1}{\sqrt{r}} = 1$ foot is due to the fact, that for the greater slopes values of c are nearly constant for values of R equal for 1 foot or more, because the value of $V^{\frac{1}{18}}$ increases slowly at high velocities.

APPENDIX II.

The Formula in Metric Measure.

THE general equation for the velocity of flow reads, for Metric measure,

$$V = 50\,(\sqrt[4]{r} + m)\,\sqrt{r\,.\,s}$$

$$= \left[\frac{2\,gH}{\dfrac{0.007844}{(\sqrt[4]{r}+m)^2}\;\dfrac{L}{R}}\right]^{\frac{1}{2}}$$

The coefficients of variation of c are equal, as for English measure, to

$$a = V^{\frac{1}{9}}\;\; V^{\frac{1}{18}},\; 1.0\; \frac{1}{V^{\frac{1}{9}}}, \frac{1}{V^{\frac{1}{18}}};$$

$a = V^{\frac{1}{9}}$ holds good also for semicircular open conduits.

Values of the coefficient m, indicating the degree of roughness, are found in the following table, mE signifying the English and mM the Metric values.

Exponential Equations.

The constants of the exponential equations which we have found for English measure are converted into Metric equivalents by putting

log constant Metric measure = log constant English measure
+ " 3.281x
− " 3.281.

x being the variable power of R or D.

The equations for conduits under pressure are as follows, diameters being in meters, velocities in meters per second and quantities in cubic meters (1000 liters) per second:

VALUES OF *mE* WHICH APPLY IN THE ENGLISH AND VALUES OF *mM* WHICH APPLY IN THE METRIC SYSTEM.

mE	*mM*	Description of Conduits.
1.0	0.85	Semicircular and circular conduits lined with pure cement. Long straight brass, tin, nickel and glass pipes.
0.95	0.80	Rectangular conduits lined with pure cement. New pipes of planed boards and very smooth asphalt-coated cast iron.
0.85	0.70	Semicircular conduits lined with cement plaster, 1 part cement, 2 parts sand.
0.83	0.75	Ordinary new straight asphalt-coated cast, wrought iron welded and wrought iron riveted pipes with screw joints, common lead, tin, glass, brass and galvanized pipes.
0.80	0.65	Rectangular conduits lined with cement plaster, smooth concrete or very good brickwork.
0.70	0.62	Semicircular channels lined with rough boards. Channels lined with fairly good brickwork or fairly smooth concrete.
0.68	0.60	Rectangular channels lined with rough boards. Sewer pipe very well laid.
		Pipes of planed boards, asphalt-coated cast and wrought iron, riveted wrought iron pipes of small diameters or with screw joints, pipes coated with tar or lined with cement or smooth concrete, all some time in use.
0.57	0.48	Common brickwork or concrete. Very good ashlar masonry. Ordinary sewer pipe.
0.53	0.45	Asphalt-coated riveted pipe above 3 feet in diameter.
0.50	0.47	Channels in earth roughly lined with cement mortar.
0.45	0.42	Old pipes of all descriptions, fairly clean. Channels lined with rough brickwork or rough concrete.
0.30	0.25	Old riveted pipes over 3 feet in diameter. Ordinary ashlar and very good rubble masonry.
0.20	0.20	Channels of regular cross-section in fine cemented gravel. Tile drains.
0	0	Channels of regular cross-section in coarse cemented gravel or rockwork.
−0.10	−0.1	Channels of fairly regular cross-section in firm sand or sand with pebbles, no vegetation.
−0.20	−0.2	Channels in earth somewhat above the average in regularity and condition, no stones or vegetation.
−0.27	−0.27	Ordinary channels in earth, with stones or vegetation here and there.
−0.32	−0.32	Channels of irregular cross-sections or channels of fairly regular cross-sections but with stones or plants.
		The values of K corresponding to $m = -0.1, -0.2, -0.27, -0.32$ are 1.2, 1.5, 1.75, 1.93.

mE	*mM*	*V* in Meters per Second.	*Q* in Cubic Meters per Second.
0.95	0.83	60.92 $D^{0.67}$ $S^{\frac{9}{16}}$	47.85 $D^{2.67}$ $S^{\frac{9}{16}}$
0.83	0.75	56.54 $D^{0.68}$ "	44.41 $D^{2.68}$ "
0.68	0.60	51.28 $D^{0.69}$ "	40.28 $D^{2.69}$ "
0.57	0.48	36.76 $D^{0.7}$ $S^{\frac{9}{17}}$	28.87 $D^{2.7}$ $S^{\frac{9}{17}}$
0.57	0.48	40.21 $D^{0.7}$ Egg-shaped	47.40 $D^{2.7}$ Egg-shaped
0.53	0.45	35.55 $D^{0.7}$ $S^{\frac{9}{17}}$	27.92 $D^{2.7}$ $S^{\frac{9}{17}}$
0.45	0.26	25.48 $D^{0.66}$ $S^{\frac{1}{2}}$	20.0 $D^{2.66}$ $S^{\frac{1}{2}}$
0.30	0.22	22.45 $D^{0.67}$ "	17.64 $D^{2.67}$ "

Values of $D^{0.67}$, etc., are found in Table E, values of $D^{2.67}$, etc., in Table F.

These tables give the values of the powers of diameters for diameters of 0.05, 0.10, 0.15, 0.20, 0.25 meters, etc. These correspond closely to 2, 4, 6, 8, 10 inches, one foot being 0.3048 meters, one meter 39.4 inches. In order that the powers of the diameters found in Tables E and F may apply to a greater range of diameters we shall find equations in which the unit is 1 decimeter = 0.1 meter, so that diameters must be taken in decimeters and fractions thereof. The results will be velocities in decimeters per second and quantities in cubic decimeters or liters per second.

The diameters found in the tables as 0.05, 0.10, 0.15, 0.20, 0.25, 0.50, 0.75 when taken as fractions of a decimeter correspond to 0.2, 0.4, 0.6, 0.8, 1.0, 2.0, 3.0 inches respectively.

The discharge of a new wrought iron pipe (m = 0.83) of one inch diameter (0.025 meter or 0.25 decimeter) for a slope of 1 : 100 is for instance:

$Q = 92.78\ (0.25)^{2.68}\ (0.01)^{\frac{9}{16}}$

$= 92.78 \,.\, 0.02435 \,.\, 0.075 = 0.1694$ liters per second or 10.164 liters per minute.

mE	*mM*	Velocity in Decimeters per Second.	Discharge in Liters per Second.
0.95	0.83	130.2 $D^{0.67}$ $S^{\frac{9}{16}}$	102.3 $D^{2.67}$ $S^{\frac{9}{16}}$
0.83	0.75	118.1 $D^{0.68}$ "	92.78 $D^{2.68}$ "
0.68	0.60	104.7 $D^{0.69}$ "	82.23 $D^{2.69}$ "
0.57	0.48	73.34 $D^{0.7}$ $S^{\frac{9}{17}}$	57.60 $D^{2.7}$ $S^{\frac{9}{17}}$
0.57	0.48	80.23 $D^{0.7}$ Egg-shaped	94.58 $D^{2.7}$ Egg-shaped
0.53	0.45	70.93 $D^{0.7}$ $S^{\frac{9}{17}}$	55.71 $D^{2.7}$ $S^{\frac{9}{17}}$
0.45	0.36	55.74 $D^{0.66}$ $S^{\frac{1}{2}}$	43.78 $D^{2.66}$ $S^{\frac{1}{2}}$
0.30	0.22	48.0 $D^{0.67}$ "	37.7 $D^{2.67}$ "

Exponential Equations Relating to Flow in Open Conduits.

Of the following sets of equations the first three relate to flow in the semicircle, the rest to flow in the semisquare, the depth being in meters, the velocities in meters per second, the discharges in cubic meters per second:

mE	mM	V in Meters per Second.	Q in Cubic Meters per Second.
1.0	0.85	$89.25\ D^{0.68}\ S^{\frac{6}{11}}$	$70.12\ D^{2.68}\ S^{\frac{6}{11}}$
0.85	0.70	$83.0\ D^{0.69}$ "	$65.1\ D^{2.69}$ "
0.70	0.62	$74.0\ D^{0.70}$ "	$58.2\ D^{2.70}$ "
0.95	0.80	$73.3\ D^{0.67}\ S^{\frac{9}{17}}$	$146.6\ D^{2.67}\ S^{\frac{9}{17}}$
0.80	0.65	$67.6\ D^{0.68}$ "	$135.2\ D^{2.68}$ "
0.70	0.62	$64.0\ D^{0.69}$ "	$128.0\ D^{2.69}$ "
0.57	0.48	$59.0\ D^{0.7}$ "	$118.0\ D^{2.7}$ "
0.50	0.49	$57.0\ D^{0.715}$ "	$114.0\ D^{2.715}$ "
0.45	0.42	$54.7\ D^{0.715}$ "	$109.4\ D^{2.715}$ "
0.30	0.25	$49.1\ D^{0.735}$ "	$98.2\ D^{2.735}$ "
0.20	0.20	$44.6\ D^{0.735}$ "	$89.2\ D^{2.735}$ "
0	0	$29.2\ D^{0.75}\ S^{\frac{1}{2}}$	$58.4\ D^{2.75}\ S^{\frac{1}{2}}$
K	K		
1.2	1.2	$26.75\ D^{0.765}$ "	$53.5\ D^{2.765}$ "
1.5	1.5	$23.6\ D^{0.775}$ "	$47.2\ D^{2.775}$ "
1.75	1.25	$21.6\ D^{0.785}$ "	$43.2\ D^{2.785}$ "
1.93	1.93	$20.5\ D^{0.795}$ "	$41.0\ D^{2.795}$ "

Of the following equations the first three apply to any depth of water in the semicircular section, the rest to any depth of water in any other form of section, R being in meters, velocities in meters per second.

mE	mM	Velocities in Meters per Second.	mE	mM	Velocities in Meters per Second.
1.0	0.85	$142.4\ R^{0.68}\ S^{\frac{6}{11}}$	0.30	0.25	$81.0\ R^{0.735}\ S^{\frac{9}{17}}$
0.85	0.70	$134.0\ R^{0.69}$ "	0.20	0.20	$74.0\ R^{0.735}$ "
0.70	0.62	$122.0\ R^{0.70}$ "	0	0	$49\ R^{0.75}\ S^{\frac{1}{2}}$
0.95	0.80	$116\ R^{0.67}\ S^{\frac{9}{17}}$	K	K	
0.80	0.65	$108\ R^{0.68}$ "	1.2	1.2	$45.4\ R^{0.765}$ "
0.70	0.62	$102.5\ R^{0.69}$ "	1.5	1.5	$40.4\ R^{0.775}$ "
0.57	0.48	$95.3\ R^{0.70}$ "	1.75	1.75	$37.2\ R^{0.785}$ "
0.50	0.47	$92.3\ R^{0.715}$ "	1.93	1.93	$35.3\ R^{0.795}$ "
0.45	0.42	$89.0\ R^{0.715}$ "			

English and Metric Equivalents.

The following relations between the units of the English and the Metric Systems of Measurements are of interest in their relation to the flow of water.

1 meter = 10 decimeters = 100 centimeters = 1000 millimeters.
1 sq. meter = 100 sq. decimeters = 10,000 sq. centimeters.
1 cu. meter = 10 hectoliters = 1000 liters.
1 liter of water at 4 degrees centigrade weighs 1 kilogram.
1 kilogram = 1000 grams.
1 meter = 3.280899 feet = 39.37079 inches.
1 foot = 0.304794 meter = 30.4794 centimeters.
1 inch = 25.3995 millimeters = 2.53995 centimeters.
= 0.253995 decimeter = 0.0253995 meter.
1 sq. meter = 10.7643 sq. feet = 1550 sq. inches.
1 sq. foot = 0.0928997 sq. meter = 928.997 sq. centimeters.
1 sq. inch = 6.451368 sq. centimeters.
1 cu. meter = 35.316585 cu. feet = 264.1863 gallons.
1 liter = 0.035316585 cu. feet; = 0.2641863 gallons.
1 cu. foot = 0.0283153 cu. meters, = 28.3153 liters.
1 cu. inch = 0.0163861 liters, = 16.38618 cu. centimeters.
1 gallon = 3.7852 liters.
1 liter weighs 2.204672 English pounds.
1 cu. foot weighs 62.425 English pounds.
1 gallon weighs 8.3448 English pounds.
1 gallon = 231 cubic inches.

The pressure of water in kilograms is equal
per square meter to $1000\,h$ (h in meters)
" " decimeter " $10\,h$
" " centimeter " $0.1\,h$
" " millimeter " $0.001\,h$.

A pressure of one pound per square inch is equal to
a pressure of 0.07031 kilo per square centimeter
" " " 0.0007031 " " " millimeter.

The tensile, shearing, or compressive strength of any material in pounds per square inch multiplied by 0.0007031 gives the value in kilos per square millimeter and multiplied by 0.07031, the value in kilos per square centimeter.

A pressure of 1 atmosphere = 14.7 pounds per square inch corresponds to a pressure of 1.03296 kilos per square centimeter or a head of 10.3296 meters. $2\,g = 19.61$.

Thickness of walls of conduits:

$$t = \frac{PD}{m} + C,$$

t, D, C and m in millimeters.

P in kilos per square millimeter $= 0.001\ h$.

Material.	m	C
Cast iron	2.8	7.6
Wrought iron	12.0	1.5
Steel	14.0	. . .
Lead	0.3	7.6

APPENDIX III.

Greatest Efficiency of a Conduit of a Given Diameter as a Transmitter of Energy.

Most Economical Diameter of a Conduit Transmitting Energy under Pressure.

I.

In a preceding chapter the ratio between the total head and the head lost in overcoming frictional resistances, which for a conduit of a given diameter under a given head corresponds to a maximum of efficiency, has been mentioned.

The potential energy of Qf^3 of water delivered per second at a vertical distance H above the generator is equal to

$$Q\, 62.4\, H \text{ foot-pounds,}$$

or

$$0.1134\, QH \text{ horsepowers.}$$

The discharge of a steel-riveted conduit in f^3 per second is equal to

$$Q = 40\, d^{2.7} S^{\frac{9}{17}},$$

which gives for the loss of head,

$$H = \frac{Q^{\frac{17}{9}} L}{1062\, d^{5.1}}.$$

Consequently the net energy transmitted to the generator is equal to

$$\text{H.P.} = 0.1134\, QH - \frac{Q^{\frac{17}{9}} L\, 0.1134\, Q}{1062\, d^{5.1}}$$

$$= 0.1134\, QH - \frac{Q^{\frac{26}{9}} L\, 0.1134}{1062\, d^{5.1}}.$$

This is to be a maximum.

Regarding Q as the variable and equating the first differential coefficient to zero we have

$$0.1134\, H - \frac{26\, Q^{\frac{17}{9}} L\, 0.1134}{9 \quad 1062 \quad d^{5.1}} = 0,$$

hence
$$0.1134\,H = \frac{26\,Q^{\frac{17}{9}}L\,0.1134}{9\;\;1062\;\;d^{5.1}}.$$

The root of this equation corresponds to a maximum. We have consequently, for the state of maximum efficiency

$$\frac{9\,H}{26} = \frac{Q^{\frac{17}{9}}L}{1062\,d^{5.1}};$$

or, $\frac{9\,H}{26}$ is the head sacrificed in overcoming frictional resistances when the conduit is in a state of maximum efficiency as a transmitter of energy.

We have also for the discharge which corresponds to $\frac{9\,H}{26}$,

$$Q = \left(1062\,d^{5.1}\,\frac{9\,H}{26\,L}\right)^{\frac{9}{17}}$$
$$= 40\,d^{2.7}\,0.57\,S^{\frac{9}{17}}.$$

Hence the efficiency of the conduit is greatest when the velocity and the discharge are 0.57 times the velocity and discharge corresponding to the total head H.

II.

Of much greater importance is the quest after the most economical diameter of a conduit for a given discharge and under a given head, a subject recently investigated by A. L. Adams.

The function of a pressure pipe is the transmission of energy with a minimum of loss; the usefulness of a power plant as a whole depends on several factors, chief amongst which is the amount of revenue derived from its operation.

In comparison with the power transmitted the cost of a conduit transmitting all or nearly all the energy would be excessive. The conduit having a diameter just sufficient to carry the given quantity of water under the given head delivers but a small percentage of the gross energy and its cost per horsepower transmitted is equally excessive as the cost of the conduit delivering all the energy. The diameter of a conduit just sufficient to carry a given quantity under a given head is equal to

$$d = \left(\frac{Q}{40\,S^{\frac{9}{17}}}\right)^{\frac{10}{27}}.$$

The diameter necessary to carry the same quantity with a loss of $\frac{1}{1000}$ of the gross energy is equal to

$$d = \left(\frac{Q}{40\,(0.001S)^{\frac{9}{17}}}\right)^{\frac{10}{27}}$$

$$= (1000)^{\frac{10}{51}} = 3.875 \text{ times the diameter,}$$

just sufficient to carry the given quantity.

A quantity of 100 f^3 of water delivered at an elevation of 1000 feet above the generator possesses a potential energy of

$$100 \times 1000 \times 0.1134 = 11{,}340 \text{ H.P.}$$

The diameter of a vertical steel-riveted conduit just sufficient to carry the given quantity,

$$d = \left(\frac{100}{40}\right)^{\frac{10}{27}} = 1.404 \text{ feet.}$$

The velocity corresponding to this diameter is equal to

$$V = 50.8 \times (1.404)^{0.7} = 64.38 \text{ feet per second.}$$

The energy transmitted is

$$\frac{(64.38)^2}{2\,g} \times 0.1134 \times 100 = 729.9 \text{ H.P.}$$

This is 6.43 per cent of the gross energy. The percentage transmitted by the conduit just sufficient is not constant but decreases with decreasing quantities and slopes.

For $Q = 10$, $H = 100$, $L = 1000$, for instance, the gross energy is 113.4 H.P. and the energy transmitted 3.635 H.P., which is 3.21 per cent.

The diameter corresponding to a loss of $\frac{1}{1000}$ of the gross energy is, for a vertical steel-riveted pipe carrying 100 f^3,

$$3.875 \times 1.404 = 5.437 \text{ feet.}$$

A diameter of 5.407 feet transmits

$$11{,}340 - \frac{(100)^{\frac{2.6}{9}} \times 1000 \times 0.1134}{1062 \times (5.437)^{5.1}}$$

$$= 11{,}328.7 \text{ H.P.}$$

The efficiency of the two conduits of 1.404 and 5.437 feet diameter is consequently as 729.9 to 11,328.7 or 1 to 15.52.

At n dollars per H.P. the value of the energy transmitted is equal to

$$D = 0.1134\, QHn - \frac{Q^{2\frac{6}{9}} L\, 0.1134\, n}{1062\, d^{5.1}}.$$

The thickness of the shell of riveted pipes is made equal to

$$t = \frac{0.434\, hd}{20{,}000} \text{ for } d \text{ in inches.}$$

Hence the cubic contents of a shell one foot long

$$f^3 = \frac{td\, 12\, \pi}{1728},$$

and its weight (specific gravity 7.854) per foot

$$w = \frac{0.434\, hd^2\, 12\, \pi\, 490}{20{,}000 \quad 1728},$$

which reduces to

$$w = 0.0334\, hd^2 \text{ for } d \text{ in feet.}$$

The weights of finished pipes indicate that the additional weight due to rivets, laps and straps is sensibly equal to

$$w = 0.00607\, hd^2,$$

so that the total weight of a finished pipe amounts to

$$w = 0.03947\, hd^2.$$

At m dollars a pound for steel the cost of a finished pipe will be

$$D_1 = 0.03947\, hd^2 m.$$

If we now compare the cost of the two conduits of 1.404 and 5.437 feet with the value of the power lost and the respective cost of the pipes per horsepower delivered we find, taking $n = 100$ and $m = 0.06$,

d	1.404 ft.	5.437 ft.
Cost	2318	34,803
Value of energy lost	1,061,000	1134
Cost of pipe per h.p.	3.171	3.072

Between the two extremes, the conduit delivering but a small percentage of the gross energy, and the conduit transmitting nearly the whole, both delivering the energy at an equally high expense per horsepower transmitted, there is evidently a condition more favorable to economy and it is evident that the greatest economy exists when

I. $\dfrac{\text{Cost of Conduit}}{\text{Value of energy transmitted}} = \text{a minimum.}$

II. Value of Energy lost + Cost of conduit = a minimum.

The value of the energy lost + cost of conduit is

$$\frac{0.1134\,Q^{\frac{26}{9}}Ln}{1062\quad d^{6.1}} + 0.03947\,hd^2Lm.$$

Equating the first differential coefficient with regard to d to zero we have

$$-\frac{5.1 \times 0.1134\,Q^{\frac{26}{9}}Ln}{1062\,d^{6.1}} + 2 \times 0.03947\,hdLm = 0,$$

which gives

$$d = \left(\frac{5.1 \times 0.1134\,Q^{\frac{26}{9}}n}{2 \times 1062 \times 0.03947\,mh}\right)^{\frac{10}{71}}$$

$$= 0.4962\,\frac{Q^{0.407}n^{\frac{10}{71}}}{H^{\frac{10}{71}}m^{\frac{10}{71}}}. \qquad (F\,\text{I})$$

Values of $Q^{0.407}$, $H^{\frac{10}{71}}$ and $n^{\frac{10}{71}}$ are found in the table below.

For steel at c cents per pound values of $\dfrac{0.4962}{m^{\frac{10}{71}}}$ and $0.4962\left(\dfrac{100}{m}\right)^{\frac{10}{71}}$ are as follows:

c.	$\dfrac{0.4962}{m^{\frac{10}{71}}}$.	$0.4962\left(\dfrac{100}{m}\right)^{\frac{10}{71}}$.
5.0	0.7566	1.447
5.5	0.7463	1.428
6.0	0.7381	1.410
6.5	0.7291	1.395
7.0	0.7213	1.381
7.5	0.7146	1.367
8.0	0.7079	1.354

If we again take the previous example of the vertical conduit 1000 feet long and carrying 100 f^3 per second, we find for $n = 100$, $m = 0.06$ and H a mean value of 500, from the tables,

$$d = 1.410\,\frac{6.516}{2.40} = 3.831 \text{ feet}$$

$$= 46 \text{ inches; very near.}$$

The loss of energy corresponding to this diameter is

$$\text{H.P.} = \left(\frac{100}{40\ (3.831)^{2.7}}\right)^{\frac{17}{9}} L\, 0.1134 \times 100$$
$$= 67.8$$

and the net horsepower = 11,340 − 69.8 = 11,272.2. The three conduits we have taken as examples compare as follows:

d in Feet.	Cost of Conduit.	Value of Energy Lost.	Cost of Conduit + Value of Energy Lost.	Cost of Conduit per net Horsepower.	Efficiency.
1.404	2,318	1,061,000	1,063,318	3.171	1.0
3.831	17,781	6,380	24,161	1.574	15.45
5.437	34,803	1,134	35,937	3.072	15.52

Formula I as given above gives best results when applied to conduits of riveted steel under high pressures.

The experiments of Darcy, Hamilton, Smith, C. Hershel indicate that for riveted pipes up to 4 feet in diameter the mean value of the coefficient c corresponding to a velocity of one foot per second is equal to 101.1, or nearly so. Taking the value of the coefficient of variation of c equal to $a = V^{\frac{1}{18}}$ the exponential equation corresponding to the given values of c and a reads,

$$V = 63.66\, d^{\frac{9}{17}} S^{\frac{9}{17}},$$

$$Q = 50\, d^{\frac{43}{17}} S^{\frac{9}{17}},$$

$$H = \frac{Q^{\frac{17}{9}} L}{1622\, d^{\frac{43}{9}}}\,.$$

This gives for the most economical diameter

$$d = 0.4465 \frac{Q^{\frac{26}{61}}}{H^{\frac{9}{61}}} \left(\frac{n}{m}\right)^{\frac{9}{61}}. \qquad (F\,\text{II})$$

For steel at c cents a pound and n = 100 dollars the values of $0.4465 \left(\frac{n}{m}\right)^{\frac{9}{61}}$ are as follows:

$c = 0.4465 \left(\frac{n}{m}\right)^{\frac{9}{61}}$	
5.0 = 1.371	6.5 = 1.319
5.5 = 1.351	7.0 = 1.304
6.0 = 1.334	7.5 = 1.291
	8.0 = 1.279

Formula II is best suited to small quantities of discharge and low heads. For our previous example, $Q = 100$, $H = 0.5 \times 1000$, the formula gives $d = 3.797$, hence 0.034 feet or 0.408 inch less than Formula I. The difference increases with the decrease of the quantity; for small diameters the difference amounts to as much as one inch. It will be observed that according to Formula I, the value of the energy lost is equal to $\frac{20}{51} = 0.392$, the cost of the pipe. Formula II gives $\frac{18}{43} = 0.418$.

VALUES OF Q AND $Q^{0.407}$.

Q	$Q^{0.407}$	Q	$Q^{0.407}$	Q	$Q^{0.407}$	Q	$Q^{0.407}$	Q	$Q^{0.407}$
0.5	0.737	15.5	3.051	36	4.300	82	6.011	210	8.813
1.0	1.000	16.0	3.091	37	4.348	84	6.071	220	8.982
1.5	1.179	16.5	3.130	38	4.395	86	6.128	230	9.146
2.0	1.326	17.0	3.168	39	4.442	88	6.185	240	9.305
2.5	1.452	17.5	3.206	40	4.488	90	6.243	250	9.462
3.0	1.564	18.0	3.242	41	4.533	92	6.299	260	9.614
3.5	1.665	18.5	3.279	42	4.578	94	6.354	270	9.763
4.0	1.758	19.0	3.315	43	4.622	96	6.408	280	9.907
4.5	1.844	19.5	3.350	44	4.665	98	6.463	290	10.063
5.0	1.912	20.0	3.385	45	4.708	100	6.516	300	10.190
5.5	2.001	20.5	3.419	46	4.751	105	6.647	310	10.327
6.0	2.072	21.0	3.453	47	4.792	110	6.772	320	10.413
6.5	2.142	21.5	3.486	48	4.834	115	6.898	330	10.594
7.0	2.202	22.0	3.519	49	4.874	120	7.018	340	10.723
7.5	2.211	22.5	3.551	50	4.914	125	7.136	350	10.850
8.0	2.331	23.0	3.575	52	4.994	130	7.251	360	10.978
8.5	2.389	23.5	3.614	54	5.098	135	7.363	370	11.098
9.0	2.446	24.0	3.645	56	5.146	140	7.472	380	11.219
9.5	2.500	24.5	3.676	58	5.221	145	7.580	390	11.339
10.0	2.553	25.0	3.707	60	5.293	150	7.684	400	11.455
10.5	2.604	26	3.766	62	5.364	155	7.789	410	11.572
11.0	2.654	27	3.824	64	5.434	160	7.890	420	11.686
11.5	2.702	28	3.881	66	5.502	165	7.990	430	11.799
12.0	2.759	29	3.937	68	5.570	170	8.087	440	11.909
12.5	2.795	30	3.992	70	5.636	175	8.184	450	12.018
13.0	2.840	31	4.046	72	5.701	180	8.277	460	12.127
13.5	2.885	32	4.098	74	5.765	185	8.370	470	12.223
14.0	2.929	33	4.150	76	5.828	190	8.463	480	12.388
14.5	2.969	34	4.201	78	5.887	195	8.551	490	12.443
15.0	3.011	35	4.251	80	5.951	200	8.640	500	12.545

VALUES OF H AND $H^{\frac{10}{71}}$, N AND $N^{\frac{10}{71}}$.

H	$H^{\frac{10}{71}}$	H	$H^{\frac{10}{71}}$	H	$H^{\frac{10}{71}}$	H	$H^{\frac{10}{71}}$	H	$H^{\frac{10}{71}}$
5	1.254	95	1.899	270	2.200	500	2.400	860	2.589
10	1.383	100	1.913	280	2.211	520	2.413	880	2.598
15	1.464	110	1.939	290	2.222	540	2.426	900	2.606
20	1.525	120	1.966	300	2.233	560	2.438	920	2.615
25	1.574	130	1.985	310	2.243	580	2.456	940	2.623
30	1.614	140	2.006	320	2.253	600	2.462	960	2.631
35	1.650	150	2.025	330	2.263	620	2.473	980	2.638
40	1.681	160	2.044	340	2.273	640	2.485	1000	2.646
45	1.709	170	2.061	350	2.282	660	2.495	1050	2.664
50	1.735	180	2.098	360	2.291	680	2.506	1100	2.681
55	1.758	190	2.094	370	2.300	700	2.516	1150	2.698
60	1.780	200	2.109	380	2.309	720	2.526	1200	2.717
65	1.800	210	2.123	390	2.319	740	2.536	1250	2.730
70	1.819	220	2.138	400	2.325	760	2.545	1300	2.745
75	1.837	230	2.151	420	2.341	780	2.555	1350	2.760
80	1.854	240	2.164	440	2.357	800	2.564	1400	2.774
85	1.870	250	2.179	460	2.371	820	2.573	1450	2.788
90	1.885	260	2.189	480	2.396	840	2.581	1500	2.801

III.

CONDUITS OF PLANED STAVES.

Circular conduits of planed staves are occasionally used for heads up to 200 feet. The thickness of the shell of such conduits is usually made equal to 2 or 2.5 inches. For these dimensions a conduit one foot in internal diameter will contain 9 or 12 feet board measure. Owing to the constant addition of 4 or 5 inches to the internal diameter the number of feet board measure does not increase with d but with $d^{\frac{8}{9}}$, very near. At l dollars a foot board measure the cost of the wooden shell put in place will be equal to

$$9 \text{ or } 12\, d^{\frac{8}{9}}Ll \text{ respectively.}$$

Likewise the length of the tension rods, usually $\frac{5}{8}$-inch steel, increases with $d^{\frac{8}{9}}$; their weight is therefore proportional to $d^{\frac{17}{9}}$. It is safe to allow a stress of 15,000 pounds per square inch in these rods and as the outside diameter of the one foot pipe is equal to 1.333 or 1.416 the inside diameter; the cost of the metal will be equal, at m dollars a pound, to

$$0.0338 \text{ or } 0.0359\, d^{\frac{17}{9}}\, hLm.$$

For pipes of planed staves a mean value of the coefficient c corresponding to a velocity of 1 foot per second is equal to 108, or nearly so. Taking the coefficient of variation of c equal to $V^{\frac{1}{18}}$ this gives the exponential equation

$$Q = 53.63\, d^{\frac{43}{17}} S^{\frac{9}{17}},$$

$$H = \frac{Q^{\frac{17}{9}} L}{1848\, d^{\frac{43}{9}}}.$$

Value of power lost at n dollars per horsepower,

$$\frac{Q^{\frac{26}{9}}\, L\, 0.1134\, n}{1848\, d^{\frac{43}{9}}}.$$

Equating the first differential coefficient of value of power lost plus cost of pipe to zero we have (for $t = 2$ inches)

$$-\frac{43}{9} \frac{Q^{\frac{26}{9}} L\, 0.1134\, n}{1848\, d^{\frac{52}{9}}} + \frac{17}{9}\, 0.0338\, d^{\frac{8}{9}} hLm + \frac{8}{9}\, 9\, d^{-\frac{1}{9}} Ll = 0;$$

hence

$$\frac{43}{9} \frac{Q^{\frac{26}{9}}\, 0.1134\, n}{1848} = 0.0638\, d^{\frac{60}{9}} hm + 8\, d^{\frac{51}{9}} l.$$

It is possible to solve this equation by Horner's, or some other method of approximation. Fairly good results are obtained by taking a mean of the exponents of d.

The formula will then read, after reduction,

$$d = \left(\frac{0.0002932\, Q^{\frac{26}{9}} n}{0.0638\, hm + 8\, l}\right)^{\frac{12}{73}} \qquad (F\ \text{III})$$

for $t = 2$ inches. For $t = 2.5$ inches 0.0678 is substituted for 0.0638 and 10.8 l for 8 l. This formula gives results sometimes above, sometimes below the true value. Where great accuracy is desired, the value of d obtained from the formula may be tested by putting its ninth root into the expression 0.0638, or 0.0678 X^{60} hm + 8, or 10.8 X^{51}, and increasing or diminishing the value of X till this expression is equal in value to

$$\frac{43}{9} \frac{Q^{\frac{26}{9}}\, 0.1134\, n}{1848}.$$

It is to be observed that for this class of conduits the ratio between the value of the power lost and the cost of the pipe which corresponds to a maximum of economy is not the same as for metal conduits. If the tension rods did not enter the problem the ratio would be as 8 to 43; as it is the ratio is variable but usually in the neighborhood of 11 or 12 to 43 or 0.25 to 0.28 to 1.0.

IV.

CONDUITS LINED WITH PLAIN OR ARMORED CONCRETE.

Concrete, plain or armored, is coming more and more into favor as a material forming the shells of conduits of all descriptions. Over metal and wood this substance possesses the great advantage not to be subject to corrosion and decay; it is practically indestructible. Experiments have brought to light the fact that plain concrete conduits under internal pressure fail when the stress in the material reaches 168 pounds per square inch or nearly so. Under external pressures, however, they fail only when the stress reaches 1500 pounds per square inch or nearly so. Plain concrete is therefore not economical where internal pressures enter the problem. But the material may be used to great advantage when great quantities of water are to be delivered under low heads.

The thickness of the shell of plain concrete conduits as commonly used for sewers and other conduits not subject to internal pressures is usually made equal to

2 inches for $d =$ 1 foot.
4 inches for $d =$ 3 feet.
8 inches for $d =$ 9 feet.
12 inches for $d =$ 18 feet.

These conduits will fail when the 1 foot pipe is under a head of 127 feet or the 18 foot pipe under a head of 43 feet.

The thickness of the shell in inches of such conduits is proportional to 2 $d^{0.63}$, the cubic contents of the shell to 0.611 $d^{1.63}f^3$. At c dollars per f^3 of concrete put into place the cost of such a conduit will consequently be

$$0.611\, d^{1.63}Lc.$$

Taking the coefficient of variation of c equal to $V^{\frac{1}{18}}$ the exponential equations which apply to flow in conduits lined with concrete are as follows:

$m = 0.95$, conduits smoothly dressed with neat cement,

$$Q = 54.3\, d^{2.66}S^{\frac{9}{17}},$$

$m = 0.83$, conduits lined with cement plaster, 1 part cement, 2 parts sand; plain concrete washed with neat cement,

$$Q = 50.2\, d^{2.67'} S^{\frac{9}{17}},$$

$m = 0.57$, conduits lined with plain concrete,

$$Q = 41.2\, d^{2.695} S^{\frac{9}{17}}.$$

For $m = 0.95$ the value of the power lost plus the cost of the conduit will be equal to

$$\frac{Q^{\frac{26}{9}} L\, 0.1134\, n}{1893\, d^{5.024}} + 0.611\, d^{1.63}\, Lc,$$

of which the first differential coefficient equated to zero

$$-\frac{5.024\, Q^{\frac{26}{9}} L\, 0.1134\, n}{1893\, d^{6.024}} + 1.63 \times 0.611\, d^{1.63} Lc = 0.$$

The most economical diameter will be equal

for $m = 0.95$ to $d = 0.2967\, Q^{\frac{13}{30}} \left(\frac{n}{c}\right)^{\frac{3}{20}}$,

for $m = 0.83$ to $d = 0.3044\, Q^{\frac{13}{30}} \left(\frac{n}{c}\right)^{\frac{3}{20}}$,

for $m = 0.57$ to $d = 0.3247\, Q^{\frac{13}{30}} \left(\frac{n}{c}\right)^{\frac{3}{20}}$.

It is to be observed that this class of conduits is in the state of greatest economy when the value of the power lost is equal (for $m = 0.83$) to $\frac{1.63}{5.043} = 0.323$ of the cost of the conduit.

Concrete beams armored with 1.75 to 2 per cent steel fail when the modulus of rupture equals 2400 pounds per square inch or nearly so. Taking 10 as a factor of safety the working stress for internal pressures will be 240 pounds per square inch and the thickness of the shell will be equal to

$$t = \frac{0.434\, hd}{480} + z,$$

z being equal to 1 inch for $h = 1$ to $h = 100$, and vanishing for $h = 1000$. Accordingly the thickness of the shell of a conduit 1 foot in internal diameter for $h = 1000$ will be 10.4 inches and the cubic contents of the shell for any diameter and any head will be

$$(0.611 + 0.0035\, h)\, d^{1.63} f^3.$$

Cracks in armored concrete begin to appear when the stress in the steel equals 12 to 15,000 pounds per square inch. A safe working stress will therefore be 10,000 pounds per square inch. Allowing one-sixth for the increase of the diameter where the armoring is placed in the 1 foot pipe and also one-sixth for the laps of the armoring, the weight of the metal in the 1 foot pipe will be for $h = 1$ equal to 0.0445 pounds.

But the thickness of the shell increases with h and consequently the length of the circumference where the armoring is placed. The necessary increase in the amount of the armoring is proportional to $\sqrt[17]{h}$ very near, so that for any head and any diameter the weight of the armoring will be

$$0.0445\, h^{\frac{18}{17}}\, d^{1.63}\, L.$$

Using these values we find for the most economical diameter, $m = 0.95$,

$$d = \left(\frac{0.0003031\, Q^{\frac{26}{9}} n}{0.0445\, h^{\frac{18}{17}} m + (0.611 + 0.0035\, h)\, c}\right)^{\frac{3}{20}},$$

$m = 0.83$,

$$d = \left(\frac{0.0003523\, Q^{\frac{26}{9}} n}{0.0445\, h \quad m + (0.611 + 0.0035\, h)\, c}\right)^{\frac{3}{20}},$$

$m = 0.57$,

$$d = \left(\frac{0.000514\, Q^{\frac{26}{9}} n}{0.0445\, h^{\frac{18}{17}} m + (0.611 + 0.0035\, h)\, c}\right)^{\frac{3}{20}}.$$

In these equations n = value of 1 horsepower,

m = value of 1 pound of steel,

c = value of 1 cubic foot of concrete.

V.

THE MOST ECONOMICAL DIAMETER FOR METRIC MEASURE.

The exponential equation for steel riveted pipes reads,

$$Q = 28\, d^{2.7} S^{\frac{9}{17}}.$$

$$H = \frac{Q^{\frac{17}{9}} L}{541.4\, d^{5.1}}.$$

Value of power lost at n dollars per kilowatt

$$\frac{9.81\, Q^{\frac{26}{9}} Ln}{541.4\, d^{5.1}}.$$

Allowing a tension of 14 kgm. per square millimeter in the steel the weights of the shell in kilograms will be

$$\frac{1000\, hd}{14{,}000{,}000}\, 3.1416\, d\, 7854 = 1.76246\, d^2 h,$$

and allowing for rivets, laps and straps, the cost of the conduit at m dollars per kilogram will be equal to

$$2.083\, d^2 hm,$$

which gives for the most economical diameter

$$d = \left(\frac{5.1 \times 9.81\, Q^{\frac{26}{9}} n}{541.4 \times 2 \times 2.083\, hm}\right)^{\frac{10}{71}}$$

$$= 0.5857\, Q^{0.407} \left(\frac{n}{Hm}\right)^{\frac{10}{71}}.$$

A mean value of the coefficient c corresponding to a velocity of 1 meter per second found from data relating to flow in 17 riveted conduits including the largest as well as the smallest is equal to 61.93. Taking $a = V^{\frac{1}{18}}$ this corresponds to the exponential equation,

$$Q = 29.76\, d^{\frac{43}{17}} S^{\frac{9}{17}},$$

from which we find for the most economical diameter

$$d = 0.5551\, Q^{\frac{26}{61}} \left(\frac{n}{hm}\right)^{\frac{9}{61}}.$$

In these equations d and h are in meters,
Q in m^3 per second,
n the value of 1 kilowatt,
m the value of 1 kilogram of steel.

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