ELEMENTS OF PHYSICS

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PREFACE

In preparing this edition of the *High School Physics* an effort has been made to bring it up to the requirements of the present day, and thus to merit a continuance of its wide use in the schools of Canada.

The revision has been thorough, many of the chapters being remodelled and rewritten. The greatest changes have been made in the part dealing with Electricity. Here the Electron Theory has been applied from the beginning, and an attempt has been made to give a modern, though brief, treatment of radio-communication. But in every portion of the book new subjects which have a special interest in our life to-day have been introduced.

Whenever it has been possible, references have been made to the applications of Physics in our everyday life. Other illustrations have been taken from industry and commerce, especially those seen in our own country.

Attention is directed to the diagrams and drawings, of which there is an exceptionally large number. Some of those in the former edition have been discarded, and a large number of new ones have been added. They have all been prepared specially for this work and great care has been taken to have them clear and easily understood.

The number of problems and questions has been greatly increased; and at the end of the chapters lists of books have been given where further information may be obtained. In general, these are elementary and can be consulted by the student as well as by the teacher.

The concise tables of physical constants appearing throughout the book have been taken from the *Smithsonian Physical Tables,* published by the Smithsonian Institution, Washington, D.C. This useful volume should be in every school.

The authors wish to acknowledge courteous assistance received from many firms and individuals regarding certain industrial applications of Physics. Among these may be specially mentioned the Hydro-Electric Power Commission of Ontario, whose work is referred to in several places in the book. Should any teacher or student desire further information regarding the operations of this great public service organization, a request addressed to the head office in Toronto will receive a ready reply.

In the preparation of the original edition the authors were greatly indebted to Prof. A. L. Clark, of Queen's University, Kingston, and to Prof. W. E. McElfresh, of Williams College, Williamstown, Mass., who read the proof sheets. In making the revision the authors utilized many valuable suggestions sent in by teachers actually using the book. But the revision would not have been so thorough or so worthy of confidence had it not been for the collaboration of Mr. George A. Cline, M.A., Instructor in Physics in the University of Toronto Schools, Toronto. To him are due many of the special features which have been introduced; and his wide experience, excellent scholarship and good judgment have been invaluable.

TORONTO, August, 1923.

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Part V—Sound

Part VI—Heat

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Part VIII—Electricity and Magnetism

TABLE OF EQUIVALENTS OF UNITS

Length

1 in. = 2.54 cm.	1 cm. = 0.3937 in.
1 ft. = 30.48 cm.	1 m. = 39.37 in. = 1.094 yd.
1 yd. = 91.44 cm.	1 km. = 0.6214 mi.
1 mi. = 1.609 km.	1 km. = 1000 m., 1 m. = 100 cm., 1 cm. = 10 mm.

Surface

1 sq. in. = 6.4514 sq. cm.	1 sq. cm. = 0.1550 sq. in.
1 sq. ft = 929.01 sq. cm.	1 sq. m. = 10.764 sq. ft.
1 sq. yd. = 8361.3 sq. cm. = 0.83613 sq. m.	1 sq. m. = 1.196 sq. yd.

Volume

1 c. in. = 16.387 c.c.	1 c.c. = 0.061 c. in.
1 c. ft. = 28317 c.c.	1 l. = 1000 c.c. = 61.024 c. in.
1 c. yd. = 0.7645 cu. m.	1 cu. m. = 1.308 c. yd.

1 Imperial gallon = 10 lb. water at 62° F.
= 277.274 c. in. = 4.546 l.
1 Imperial quart = 1.136 l.
1 U.S. gallon = 231 c. in. = 3.784 l.
1 l. = 1.7598 Imperial pints.

Mass

1 lb. av. (7000 gr.) = 453.59 g.	1 kg. = 2.205 lb. av.
1 oz. av. = 28.3495 g.	1 g. = 15.432 gr.
1 gr. = 0.0648 g.	

Abbreviations

in. = inch; ft. = foot; yd. = yard; mi. = mile; sq. = square; c. or cu. = cubic; m. = metre; mm. = millimetre; cm. = centimetre; km. = kilometre; c. cm. or c.c. = cubic centimetre; l. = litre; lb. av. = pound avoirdupois; gr. = grain; g. = gram; kg. = kilogram.

PART I—INTRODUCTION

CHAPTER I

Measurement

1. Physical Quantities. The various operations of nature are continually before our eyes, and by the time that we definitely enter upon the study of physics, we have gathered a store of observations and experiences.

We all know the great service which the waterfalls of our country give us. They grind our wheat and saw our lumber. They generate electricity which, after being transmitted over considerable distances, supplies motive power for our factories and street railways. We also know how steam drives the giant ships and railway trains, which carry the commerce of the nations. The automobile has now become an indispensable aid and the airplane, which was of outstanding value in war, is becoming an important means of transport in peace. Then we have the phonograph, the wireless telephone and the spectroscope, which reveals the nature of the distant stars. Ours is a wonderful age, indeed!

When asked to explain any of these things, we usually reply in vague terms. The study of physics is intended to enable us to state clearly the construction and the operation of these various contrivances. In order to do this we must understand the numerous phenomena observed in mechanics, heat, electricity, and other branches of physics; and our knowledge of these matters can hardly be considered satisfactory unless we are able actually to measure the various physical quantities involved.

2. Measuring a Quantity. In measuring a quantity we determine how many times a magnitude of the same kind, which we call a *unit*, is contained in the quantity to be measured.

Thus we speak of a length being 5 feet, the unit chosen being a *foot*, and 5 expressing the number of times the unit is contained in the given length.

3. Fundamental Units. There will be as many kinds of units as there are kinds of quantities to be measured, and the size of the units may be just what we choose. But there are three units which we speak of as *fundamental*, namely, the units of *length*, *mass* and *time*. These units are fundamental in the sense that each is independent of the others and cannot be derived from them; also we shall find that the measurement of any quantity—such as the power of a steam engine, the speed of a rifle-bullet or the strength of an electric current—can ultimately be reduced to measurement of length, mass and time. Hence these units are properly considered fundamental.

4. The English and the C. G. S. System. There are two widely used systems of units, namely, the English and the C.G.S. system. In the former the *foot*, the *pound* and the *second* are the units of length, mass and time, respectively. In the latter, which is used almost universally in purely scientific work, the units of length, mass and time are the *centimetre*, the *gram* and the *second*, respectively.

The former is sometimes called the F.P.S. system, the latter the C.G.S. system, the distinguishing letters being the initials of the units of the two cases.

5. Standards of Length—the Yard. There are two *standards* of length in use in English-speaking countries, namely, the *yard* and the *metre*.

The yard is said to have represented, originally, the length of the arm of King Henry I., but such a definition is not by any means accurate enough for present-day requirements. It is now defined as the distance between the centres of two transverse lines ruled on two gold plugs in a bronze bar, which is preserved in London, England, in the Standards Office of the Board of Trade of Great Britain.

The bronze bar is 38 inches long and has a cross-section one inch square (Fig. 1). At *a*, *a*, wells are sunk to the mid-depth of the bar, and at the bottom of each well is the gold plug or pin, about $\frac{1}{10}$ inch in diameter, on which the line defining the yard is engraved.

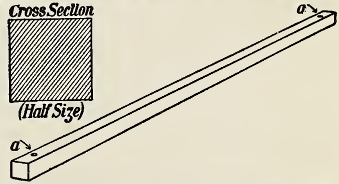

Fig. 1.—Bronze yard, 38 inches long, 1 in. sq. in section. *a*, *a* are small wells in the bar, sunk to mid-depth.

The other units of length in ordinary use, such as the inch, the foot, the rod, the mile, are derived from the yard, though the relations between them are not always simple.

6. The Metre. The metre came into existence through an effort made in France, at the end of the 18th century, to replace by one standard the many and confusing standards of length prevailing throughout the country. It was decided that the new standard should be called a metre, and that it should be one ten-millionth of the distance from the pole to the equator, measured through Paris. The standard bar representing the length was completed in 1799. It is of platinum, just a metre from end to end, 25 millimetres (about 1 inch) wide and 4 millimetres (about $\frac{1}{6}$ inch) thick.

As time passed, great difficulty was experienced in making exact copies of this platinum rod, and as the demand for such continually increased, it was decided to construct new standard bars.

The new bars are made of a hard and durable alloy composed of platinum 90 per cent. and iridium 10 per cent., and have the form shown in Fig. 2. The section illustrated here was chosen on account of its great rigidity, and also in order that the cross-lines which define the length of the metre might be placed on the face which is just mid-way between the upper and lower faces of the bar. The bars are 102 centimetres in length over all, and 20 millimetres square in section. Thus the lines which define the metre are one centimetre from each end of the bar.

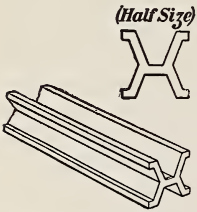

FIG. 2.—View of end and cross-section of the new standard metre bars. The line defining the end of the metre is a short mark on the surface midway between the top and bottom of the bar.

All the bars were completed in 1889. They were made as nearly as possible equal in length to the original platinum one of 1799, but of course minute differences existed between them—perhaps one part in one hundred million. So the one which appeared to agree most perfectly with the old standard was taken as the new International Standard. It is kept in a special vault, in the International Bureau of Weights and Measures at Sèvres, near Paris; and the other bars are distributed, as needed, to other nations.

Later investigations have shown that there are more than 10,000,000 of these standard metres in the earth-quadrant, and hence the metre is a little shorter than it was intended to be—about $\frac{1}{12}$ mm., or a hair-breadth, shorter.

The Canadian standards are kept in the Department of Inland Revenue, Ottawa.

7. Divisions and Multiples of the Metre. In the metric system the units are divided and multiplied decimally. The names of the sub-divisions are obtained by using the Latin prefixes, *deci* ($\frac{1}{10}$), *centi* ($\frac{1}{100}$), *milli* ($\frac{1}{1000}$); and the names of the multiples are formed with the Greek prefixes, *deca* (10), *hecto* (100), *kilo* (1000). Thus:—

10 millimetres (mm.) = 1 centimetre
10 centimetres (cm.) = 1 decimetre
10 decimetres (dm.) = 1 metre
10 metres (m.) = 1 decametre
10 decametres = 1 hectometre
10 hectometres = 1 kilometre (km).

The decametre and the hectometre are not often used.

8. Relation of Metres to Yards. In Great Britain the yard is the standard, and the relation between the metre and the inch is officially stated to be:—

1 metre = 39.370113 inches;

in the United States the metre is the fundamental standard, and by law

1 metre = 39.37 inches.

The difference between these two statements of length of the metre is only $\frac{1}{1000}$ inch, and the British and United States yards may be considered identical.

Approximately 1 in. = $2\frac{1}{2}$ cm.; 1 ft. = 30 cm.; 5 mi. = 8 km.

In Fig. 3 is shown a comparison of centimetres and inches.

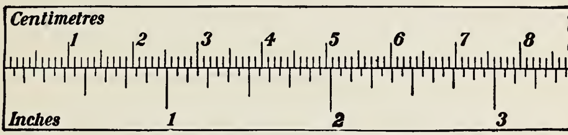

FIG. 3—Comparison of inches and centimetres.

9. Units of Area and of Volume. The ordinary units of surface and of volume are derived from the units of length. Thus, we have the *square metre* (sq. m.), the *square centimetre* (sq. cm.), the *square foot* (sq. ft.), the *square yard* (sq. yd.), etc.; also, the *cubic metre* (cu. m.), the *cubic centimetre* (c.c.), the *cubic inch* (cu. in.), the *cubic foot* (cu. ft.), etc. The *cubic decimetre*, which contains 1,000 c.c., is called a *litre*.

The *imperial gallon*, which is the legal standard of capacity in Canada, is defined as the volume of 10 pounds of water at 62° F., and it is equal to 277.274 cu. in. The Winchester gallon or wine-gallon, which is the common United States gallon, contains 231 cu. in. It is roughly four-fifths of the imperial gallon.

10. Measurement of Length. A dry-goods merchant unrolls his cloth, and, placing it alongside his yard-stick, measures off the quantity ordered by the customer. Now the yard-stick is intended to be an accurate copy of the standard yard kept at the capital of the country, and this latter we know is an accurate copy of the original preserved in London, England. In order to ensure the accuracy of the merchant's yard-stick a government official periodically inspects it, comparing it with a standard which he carries with him.

Suppose, next, that we require to know accurately the diameter of a wire, or of a sphere, or the distance between two marks on a photographic plate. We choose the most suitable instrument for the purpose in view. For the wire or the sphere a screw gauge would be very convenient. One of these is illustrated in Fig. 4. *A* is the end of a screw which works in a nut inside of *D*. The screw can be moved back and forth by turning the cap *C* to which it is attached, and which slips over *D*. Upon *D* is a scale, and the end of the cap *C* is divided into a number of equal parts. By turning the cap the end *A* moves forward until it reaches the stop *B*. When this is the case the graduations on *D* and *C* both read zero.

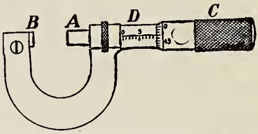

FIG. 4—Micrometer wire gauge.

In order to measure the diameter of a wire, the end *A* is brought back until the wire just slips between *A* and *B*. Then the scales on *D* and *C* indicate the whole number of turns made by the screw and also the fraction of a turn. Hence if we know the pitch of the screw, which is usually $\frac{1}{50}$ inch, we can at once calculate the diameter of the wire.

To measure the photographic plate the most convenient instrument is a microscope which can be moved back and forth over the plate, or one in which the stage which carries the plate can be moved by screws with graduated heads, much as in the wire gauge.

There are other devices for accurate measurement of lengths, but in every case the scale, or the screw, or whatever is the essential part of the instrument, must be carefully compared with a good standard before our measurements can be of real value.

PROBLEMS

(For table of values see opposite page 1)

1. How many millimetres are there in $2\frac{1}{2}$ kilometres?

2. Light travels 186,330 miles in a second; express this in kilometres.

3. The floor of a building is 13 x 30 metres; how many square centimetres are there in it?

4. If the mercury in a barometer is 760 mm. high, what will be the height in inches?

5. Reduce 1 cubic metre to litres and to cubic centimetres.

6. Lake Superior is 602 feet above sea-level. Express this in metres.

7. Dredging is done at 50 cents per cubic yard. Find the cost per cubic metre.

8. Air weighs 1.293 grams per litre. Find the weight of the air in a room 20 x 25 x 15 metres in dimensions.

9. Which is cheaper, milk at 7 cents per litre or 8 cents per quart?

10. If gasoline costs 35 cents per gallon in Canada, what should the price (at the same rate) be in the United States?

11. The polar diameter of the earth is 12,712.91 kilometres, the equatorial diameter 12,756.38 kilometres. Give these dimensions in miles.

12. A runner goes 100 yards in $10\frac{2}{5}$ seconds; how long should he take to go 100 metres at the same speed?

13. Express, correct to a hundredth of a millimetre, the difference between 12 inches and 30 centimetres.

11. Standards of Mass. By the *mass* of a body is meant the *quantity of matter* in it. Matter may change its form, but it can never be destroyed. A lump of matter may be transported to any place in the universe, but its mass will remain the same.

There are two units of mass in ordinary use, namely, the *pound* and the *kilogram.*

The standard *pound avoirdupois* is a certain piece of platinum preserved in the Standards Office in London, England. Its form is illustrated in Fig. 5. The *grain* is $\frac{1}{7000}$ of the pound, and the *ounce* is $\frac{1}{16}$ of the pound or 437.5 grains.

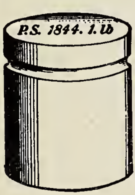

Fig. 5.—Imperial Standard Pound. Avoirdupois. Made of platinum. Height, 1.35 inches; diameter 1.15 inches. "P.S." stands for *parliamentary standard.*

The *kilogram* is the mass of a certain lump of platinum carefully preserved in Paris, and called the "Kilogramme des Archives." It was constructed by Borda (who also made the original platinum metre), and was intended to represent the mass of 1000 cubic centimetres (1 litre) of water when at its maximum density (at 4° C.).

Although the objection which had been raised against the platinum metre (namely, difficulty in reproducing it), did not hold in the case of the platinum kilogram; still the platinum-iridium alloy is harder and more durable than pure platinum, and so it was decided to make new standards out of this alloy. These are all as nearly as possible equal to the original platinum kilogram, and indeed, as they do not differ amongst themselves by more than about one part in one hundred million, they may be considered identical.

One of these was adopted as the new International kilogram, and is preserved along with the International metre at Sèvres. The others are distributed, as required, to various nations, and are their national kilograms.

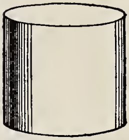

Fig. 6.—Standard kilogram, made of an alloy of platinum and iridium. Height and diameter each 1.5 inches.

These standards are plain cylinders, almost exactly $1\frac{1}{2}$ inches in diameter, and of the same height. (Figs. 6 and 7.)

The kilogram is divided decimally:—

10 milligrams (mg.)	= 1 centigram (cg.)
10 centigrams	= 1 decigram (dg.)
10 decigrams	= 1 gram (gm.)
1000 grams	= 1 kilogram (kg.)

The decagram (10 gm.) and the hectogram (100 gm.) are seldom used.

The relation of the pound to the kilogram is officially stated by the British government as follows:—

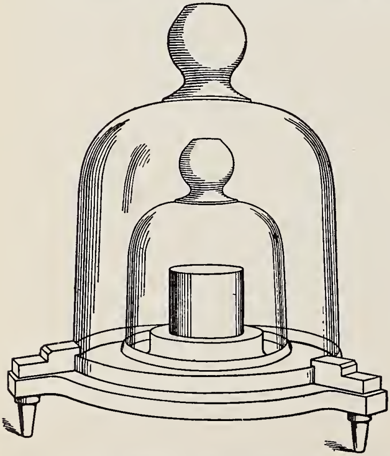

Fig. 7.—United States National Kilogram. Kept under two glass bell-jars at Washington.

1 kilogram (kg.) = 2.2046223 pounds avoir.

1 gram (gm.) = 15.4323564 grains.

1 pound avoir. = 0.45359243 kg.

1 ounce avoir. = 28.349527 grams.

Approximately 1 kg. = $2\frac{1}{5}$ lbs.; 1 oz. = $28\frac{1}{3}$ gm.

In transforming from kilograms to pounds, or the reverse, it will not be necessary to use so many decimal places as are given here. The equivalent values may be taken from the table opposite page 1.

12. Measurement of Mass. In Fig. 8 is shown a balance. The pans A and B are suspended from the ends of the beam CD, which can turn easily about a "knife-edge" at E. This is usually a sharp steel edge resting on a steel or an agate plate. The bearings at C and D, shown more fully in Fig. 8a, are nearly frictionless, so that the beam turns very freely. A long pointer P extends downwards from the middle of the beam, and its lower end moves over a scale O. When the pans are balanced and the beam is level, the pointer is opposite zero on the scale.

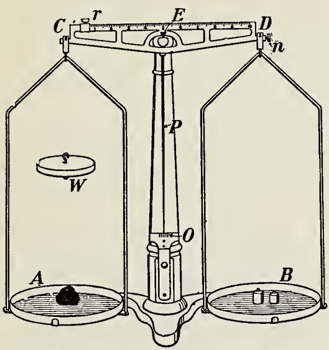

FIG. 8.—A simple and convenient balance. When in equilibrium the pointer P stands at zero on the scale O. The nut n is for adjusting the balance and the small weights, fractions of a gram, are obtained by sliding the rider r along the beam which is graduated. The weight W, if substituted for the pan A, will balance the pan B.

Suppose a lump of matter is placed on pan A. At once it descends and equilibrium is destroyed. It goes downward because the earth attracts the matter. Now put another lump on pan B. If it remains up, we say the mass on A is heavier than that on B; if the pans come to the same level and the pointer stands at zero, the two masses are equal.

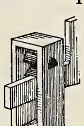

FIG. 8a. — Showing end of arm of balance.

It is the attraction of the earth upon the masses placed upon the pans which produces the motion of the balance. The attraction of the earth upon a mass is called its *weight*, and so in the balance it is the weights of the bodies which are compared. But, as is explained in Chapter V, the weight of a body is directly proportional to its mass, and so the balance allows us to compare masses.

13. Sets of Weights. We have agreed that the lump of platinum-iridium known as the International Kilogram shall be our standard of mass. (§ 11)

In order to duplicate it we simply place it on one pan of the balance, and by careful filing we make another piece of matter which, when placed on the other pan, will just balance it.

Again, with patience and care, two masses can be constructed which will be equal to each other, and which, taken together, will be equal to the original kilogram. Each will be 500 grams.

Continuing, we can produce masses of other denominations, and we may end by having a set consisting of

1,000,
500, 200, 100, 100
50, 20, 10, 10
5, 2, 2, 1 grams

and even smaller weights (Fig. 9).

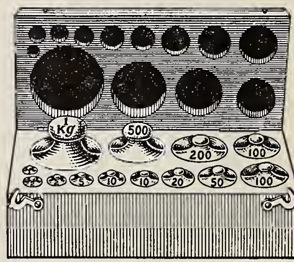

FIG. 9.—Box of Weights.

If now a mass is placed on pan A of the balance, by proper combination of these weights we can balance it and thus at once determine its mass.

The balance and the weights used by merchants throughout the country are periodically inspected by a government officer.

14. Rules for the Use of the Balance.

1. Keep the balance dry and free from dust.

2. See that the balance is properly adjusted, so that it will, when unloaded, either rest in equilibrium with the pointer at the zero mark on the scale, or will swing equally on either side of zero.

3. Place the body whose mass is to be ascertained in the left-hand scale-pan, and place the weights in the right-hand scale-pan. Until some experience in judging the mass of a body has been obtained, try all the weights in order, commencing with the largest and omitting none. When

any weight causes the right-hand pan to descend remove it. Never select weights at random.

In the balance shown in the figure any addition under 10 grams is obtained by sliding the rider *r* along the beam. It gives $\frac{1}{10}$ gram directly, and $\frac{1}{10}$ of this may be obtained by estimation.

Before beginning, the balance should be tested. Push the rider *r* over to its zero mark, and then if the pans do not balance (as indicated by the pointer *P*), turn the nut *n* until they do.

4. To determine the equilibrium do not wait until the balance comes to rest. When it swings equally on either side of zero, the mass in one pan equals that in the other.

5. Place the largest weight in the centre of the pan, and the others in the order of their denominations.

6. Keep the pans supported when weights are to be added or taken off.

7. Small weights should not be handled with the fingers. Use forceps.

8. Weigh in appropriate vessels substances liable to injure the pans. For counterpoise use shot and paper.

9. Never use the balance in a current of air.

15. Density. Let us take equal volumes of lead, aluminium, wood, brass, cork. These may conveniently be cylinders about $\frac{1}{2}$ inch in diameter and $1\frac{1}{2}$ or 2 inches in length.

By simply holding them in the hand we recognize at once that these bodies have different weights and, therefore, different masses. With the balance and our set of weights we can accurately determine the masses.

We describe the difference between these bodies by saying that they are of different densities, and we define density thus:—

The DENSITY *of a substance is the mass of unit volume of that substance.*

In the English system the foot and the pound are the units of length and mass, respectively, and the density is the number of pounds in 1 cu. ft.

In the C.G.S. system the units are the centimetre and the gram, and the density is the number of grams in 1 c.c. Since 1 litre (or 1000 c.c.) of water has the mass of 1 kilogram (or 1000 gm.), the density of water is 1 gm. per c.c. In the

English system, however, the density of water is 62.4 pounds per cu. ft.

The densities of some common substances are given in the following table:—

TABLE OF DENSITIES

(In grams per cubic centimetre)

Water	1.000	Aluminium (cast)	2.56	Paper (average)	0.9
Sea-water	1.025	Butter	0.86	Platinum	21.45
Alcohol (ethyl)	0.791	Chalk	2.4	Silver (wrought)	10.56
" (methyl)	0.810	Copper (wrought)	8.90	Tungsten	19.12
Chloroform	1.480	Diamond	3.5	Zinc (cast)	7.10
Hydrochloric Acid	1.16	Glass (ordinary)	2.6	Cork (average)	0.24
Sulphuric Acid	1.84	Gold (wrought)	19.34	Birch wood (average)	0.64
Coal Oil	0.878	Ice	0.90	Cedar (average)	0.53
Gasoline, about	0.7	Iridium	22.10	Maple (average)	0.68
Olive Oil	0.918	Iron (gray)	7.08	Oak (average)	0.75
Mercury	13.60	Lead (cast)	11.34	White Pine	0.42

Consider a piece of aluminium of volume 150 c.c. Then, since its density is 2.56 grams per c.c., its mass must be $150 \times 2.56 = 384$ grams.

Hence we have the relation,

$$\text{Mass} = \text{Volume} \times \text{Density}.$$

16. Relation between Density and Specific Gravity. By definition, the specific gravity of a body is the ratio of its weight to the weight of an equal volume of water,

$$\text{or specific gravity} = \frac{\text{weight of body}}{\text{wt. of equal vol. of water}}.$$

It is evidently a simple number, not depending on the volume of the body or on any system of units.

Methods for determining experimentally the specific gravity of a body are given in Chap. XII.

The specific gravity of water is, of course, 1; and when we say the specific gravity of gold is 19.34, we mean that gold is 19.34 times as heavy as water.

Next, let us consider a sample of cast iron, 50 c.c. in volume. By the balance it is found to weigh 361 grams, and its density is, therefore, $361 \div 50 = 7.22$ grams per c.c.

But weight of 50 c.c. of water = 50 grams, and the specific gravity of the iron is, therefore, 361 ÷ 50 = 7.22.

Thus we see that in the C.G.S. system of units the density of a substance and its specific gravity are expressed by the same number.

PROBLEMS

1. Find the mass of 140 c.c. of silver if its density is 10.5 gm. per c.c.

2. The specific gravity of sulphuric acid is 1.85. How many c.c. must one take to weigh 100 gm.?

3. A rolled aluminium cylinder is 20 cm. long, 35 mm. in diameter, and its density is 2.7 gm. per c.c. Find the weight of the cylinder.

4. The density of platinum is 21.5, of iridium is 22.4 gm. per c.c. Find the density of an alloy containing 9 parts of platinum to 1 part of iridium. Find the volume of 1 kg. of the alloy.

5. Given that 1 kg. = 2.205 lb.; and 1 in. = 2.54 cm.; find the density of water in the English system, that is, the number of pounds per cu. ft.

6. A piece of granite weighs 83.7 gm. On dropping it into the water in a graduated vessel, the water rises from 130 c.c. to 161 c.c. (Fig. 10). Find the density of the granite. What is its specific gravity? Find the density in the English system.

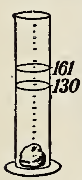

Fig. 10

7. A tank 50 cm. long, 20 cm. wide and 15 cm. deep is filled with alcohol of density 0.8 gm. per c.c. Find the weight of the alcohol.

8. A rectangular block of wood 5 x 10 x 20 cm. in dimensions weighs 770 grams. Find the density. What is the specific gravity of the wood? Find its density in the English system.

9. A thread of mercury in a fine cylindrical tube is 28 cm. long and weighs 11.9 grams. Find the internal diameter of the tube.

10. Write out the following photographic formulas, changing the weights to the metric system:—

Developer	
Water	10 oz.
Metol	7 gr.
Hydroquinone	30 "
Sulphite of Soda (dessicated)	110 "
Carbonate of Soda (dessicated)	200 "
Ten per cent. solution Bromide of Potassium	40 drops

Fixing Bath	
Water	64 oz.
Hyposulphite of Soda	16 "
When above is dissolved add the following solution:—	
Water	5 oz.
Sulphite of Soda (dessicated)	½ "
Acetic Acid	3 "
Powdered Alum	1 "

11. State some of the advantages of the metric system of weights and measures.

17. Unit of Time. If we reckon from the time when the sun is on our meridian (noon), until it is on the meridian again, the interval is a *solar day.* But the solar days thus determined are not all exactly equal to one another. The reason for this is explained in works on astronomy. In order to get an invariable interval we take the average of the solar days and call the day thus obtained a *mean solar day.* Dividing this into 86,400 equal parts we call each a *mean solar second.* This is the quantity which is "ticked off" by our watches and clocks. It is used universally by scientific men as the fundamental unit of time.

REFERENCES FOR FURTHER INFORMATION

Articles on "Metric System" and "Weights and Measures" in the *Encyclopedia Britannica.*

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PART II—MECHANICS OF SOLIDS

CHAPTER II

Velocity, Acceleration

18. Rest and Motion. When we look about, we naturally divide the things we see into two classes—those at rest and those in motion. The houses, the fences and the trees are at rest, while the motor-cars and wagons on the road, the railway-train in the distance and the birds flying over the field we declare to be in motion.

But a little thought will show that this classification of bodies is not so simple as it appears to be at first sight, that indeed a body considered from one point of view may be at rest, while from another it may be in motion. For instance, a passenger on a railway-train is at rest with respect to a fellow-traveller, but with respect to a third person on the ground outside they are both in motion. It is impossible to think of a body at rest which, when looked at in another way, would not be considered to be in motion. Thus motion is quite as natural a state as rest.

19. Velocity or Speed. A body is in motion when it is changing its position relative to another which we take to be at rest, and the change of position we call its *displacement.* Now along with the displacement which a body undergoes we generally consider the time required to produce it. Indeed, that is frequently of the utmost importance. In the case of a serious accident it is essential that the injured ones should not simply be given a "displacement" from the scene of the accident to the hospital, but that it be done in as short a time as possible—in other words, that the conveyance travel with great *speed* or *velocity*. (No distinction will be made between these two terms.)

VELOCITY *is the rate of change of position*, or, in other words, *the time-rate of displacement*.

If a train goes from Toronto to Montreal, a distance of 330 miles, in 10 hours, the *average velocity* = 330 ÷ 10 = 33 miles per hour. Sometimes the speed is greater and sometimes less than this, but this is the *average*.

$$\text{Average velocity} = \frac{\text{Space}}{\text{Time}}$$

On a stretch of level track the train may travel with approximately uniform velocity, but in motions met with in nature there are few absolutely uniform velocities.

PROBLEMS

1. An ambulance goes 8 miles in 20 minutes; find the average speed in miles per hour.

2. A train leaves Winnipeg at 10.40 p.m. and reaches Regina next morning at 9.40 as shown by the same time-piece. The distance is 357 miles. Find the average speed.

3. A train leaves Montreal at 9.45 p.m. Monday and reaches Vancouver on Saturday at 9.10 a.m., Pacific time, which is 3 hours slow of Montreal, or Eastern, time. The average speed, including stops, was $26\frac{1}{4}$ miles per hour. Find the distance.

4. Find the equivalent, in feet per second, of a speed of 60 miles per hour.

5. An eagle flies at the rate of 30 metres per second; find the speed in kilometres per hour.

6. A sledge party in the Arctic regions travels northward, for ten successive days, 10, 12, 9, 16, 4, 15, 8, 16, 13, 7 miles, respectively. Find the average velocity.

7. If at the same time the ice is drifting southward at the rate of 10 yards per minute, find the average velocity northward.

8. The speed of a street-car averages 22 ft. per sec. How far will it go in 3 hours?

9. A body is moving uniformly at the rate of a cm. in b sec. How far will it go in c hours?

10. The armature of a dynamo is 3 metres in diameter, and it revolves 150 times per minute. Find the speed of a point on its circumference.

11. Light travels at the rate of 186,000 miles per second, and it takes 8.67 years to come from Sirius, the Dog Star, to us. Find the distance of Sirius.

20. Acceleration. On coasting down an icy hill a hand-sleigh continually increases its velocity until the bottom is reached, and then as the sleigh runs up the opposite hill its velocity gradually decreases until it becomes zero. If a stone is thrown along a sheet of ice, its velocity continually diminishes until it comes to rest. These are two cases in which the velocity of a body is *not uniform*, but is changing, and we say the motion is *accelerated*. When the velocity is increased, the acceleration is *positive;* when it is decreased, the acceleration is *negative*. The latter is sometimes called a *retardation*.

Let us consider a marble rolling down a smooth board, one end of which is raised higher than the other. It begins with zero velocity, and let us suppose that at the end of the 1st, 2nd, 3rd, etc., seconds its velocity is 20, 40, 60, 80, etc., cm. per sec. The increase is 20 cm. per sec. during every second. We can represent these velocities by a diagram (Fig. 11). The horizontal line represents time. The point 1 indicates the end of the the 1st second, 2 that of the 2nd second, and so on; while the vertical lines are proportional to the velocities at the ends of the successive seconds. In this case the change in velocity is the same during each second, and we say the *acceleration is uniform.*

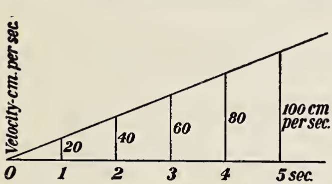


Fig. 11.—Illustrating uniform acceleration.

Next consider the stone thrown along the ice. Let it be started with a velocity of 15 ft. per sec., and during each second let its velocity be diminished 1 ft. per sec. Then the velocities at the end of each second are represented in the diagram (Fig. 12), and the stone comes to rest in 15 sec. In this case the change in the velocity during each second is the same, namely, 1 ft. per sec., and the acceleration is uniform.

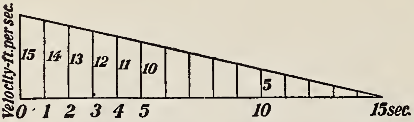


FIG. 12.—Illustrating uniform acceleration (retardation).

ACCELERATION *is rate of change of velocity.*

PROBLEMS

1. A train leaving a station has a uniform acceleration of 2 ft. per sec. per sec. What will be its velocity at the end of the 10th sec.? At the end of 15 sec.?

2. If the acceleration of a street-car is 2 ft. per sec. per sec., how many sec. will it take to acquire a speed of 30 miles per hr.?

3. An automobile going at the rate of 40 miles per hr. is brought to rest in 10 sec. Find the acceleration (retardation) if the decrease in velocity is uniform.

4. A bicyclist coasts down a hill with an acceleration of 0.5 m. per sec. per sec., and reaches the bottom in 10 sec. Find his velocity at the foot of the hill. If he then goes up the opposite hill and is brought to rest in 6 sec., what is the acceleration (retardation)?

5. A railway train changes its velocity uniformly in 2 min. from 20 km. an hr. to 30 km. an hr. Find the acceleration in cm. per sec. per sec.

6. A stone sliding on the ice at the rate of 200 yd. per min. is gradually brought to rest in 2 min. Find the acceleration in ft. and sec.

7. Change an acceleration of 981 cm. per sec. per sec. into ft. per sec. per sec. (See Table, opposite page 1.)

21. Velocity Acquired and Space Traversed. If a body starts from rest and moves with a uniform acceleration of 5 cm. per sec. per sec., it is clear that at the end of 1 sec. its velocity will be 5 cm. per sec.; at the end of 2 sec., 10 cm. per sec.; and at the end of 20 sec., 20×5 or 100 cm. per sec. Suppose now the acceleration to be a cm. per sec. per sec., and that in t sec. the velocity acquired is v cm. per sec.; then

$$v = at \text{ cm. per sec.}$$

Consider, again, the body mentioned in the previous paragraph. At the beginning of the 1st sec. the velocity is 0, and at the end of that sec. it is 5 cm. per sec. If the increase in velocity has been uniform (as we take it to be) the average velocity during the first sec. $= \frac{1}{2}(0+5) = 2\frac{1}{2}$ cm. per sec., and the space passed over $= 2\frac{1}{2}$ cm.

At the end of the 2nd sec. the velocity is 10 cm. per sec., and hence the average velocity during the first two sec. $= \frac{1}{2}(0+10) = 5$ cm. per sec., and the space traversed $= 5 \times 2 = 10$ cm.

At the end of the 20th sec. the vel. is 100 cm. per sec., and the average velocity during the first 20 sec. $= \frac{1}{2}(0+100) = 50$ cm. per sec., and the space traversed $= 50 \times 20 = 1000$ cm.

In general we see that, if a is the acceleration, the velocity at the end of t sec. is at cm. per sec., and the average velocity during these t sec. $= \frac{1}{2}(0 + at) = \frac{1}{2} at$ cm. per sec. If then s is the space traversed

$$s = \tfrac{1}{2} at \times t = \tfrac{1}{2} at^2 \text{ cm.}$$

Also, since $v = at$, $t = v/a$ and $s = \frac{1}{2} a\, v^2/a^2$ or $v^2 = 2as$.

PROBLEMS

1. Find the distance traversed by the train in problem 1, page 18, in 10 sec. and in 15 sec.

2. Find the distance travelled by the street-car in problem 2, page 18, in acquiring the speed of 30 miles per hour.

3. How far does the automobile in problem 3, page 18, go before it is brought to rest?

4. How far does the railway train in problem 5, page 18, go while increasing its speed from 20 to 30 km. per hour?

5. How far does the stone in problem 6, page 18, go before it is brought to rest?

6. From the formula $v = at$, $t = v/a$; substitute this value for t in the formula $s = \frac{1}{2} at^2$, and obtain the relation $v^2 = 2as$.

7. The acceleration of a car is 6 ft. per sec. per sec. Find the velocity acquired in travelling 100 yards.

8. An automobile driver travelling at the rate of 20 miles an hour sees an excavation in the road 50 ft. ahead. He applies the brake and stops the car just at the excavation. Find the acceleration (retardation).

9. An avalanche slides down the mountain side with a uniformly increasing velocity of 6 ft. per sec. per sec. Find its velocity when it has descended a quarter of a mile.

10. A body is moving with uniform acceleration a cm. per sec. per sec. Show that the space passed over in the 1st sec. is $\frac{1}{2}a$ cm.; in the 2nd, $\frac{3}{2}a$ cm.; in the 3rd, $\frac{5}{2}a$ cm.; in the 4th, $\frac{7}{2}a$ cm.; etc. What space will be traversed in the nth second?

22. Motion under Gravity. The most familiar illustration of motion with uniform acceleration is a body falling freely. Suppose a stone to be dropped from a height. At once it acquires a velocity downwards, which continually increases as it falls; and in a second or two it will be moving so fast that the eye can hardly follow it. In order to test experimentally the laws of motion we must devise some means of reducing the acceleration. The following is a simple and effective method of doing this.*

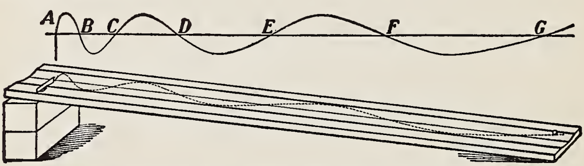


FIG. 13.—Apparatus to illustrate motion with uniform acceleration.

In a board 5 or 6 ft. long make a circular groove 4 in. wide and having a radius of 4 in. (Fig. 13). Paint the surface black and make it very smooth. Along the middle of the groove scratch or paint a straight line; and near one end of the board fasten a strip of brass accurately at right angles to the length of the groove and extending just to the middle of it.

Lay the board flat on the floor, and place a sphere (a steel ball 1 in. to $1\frac{1}{2}$ in. in diameter), at one side of the groove and let it go. It will run back and forth across the hollow, performing oscillations in approximately equal times. By counting a large number of these and taking the average, we can obtain the time of a single one.

*Devised by Prof. A. W. Duff, of the Polytechnic Institute, Worcester, Mass.

Next let one end of the board be raised and over the groove dust (through 4 or 5 thicknesses of muslin) lycopodium powder. Put the ball alongside the brass strip at one side of the groove and let it go. It oscillates across the groove and at the same time rolls down it, and the brass strip insures that it starts downwards without any initial velocity. By blowing the lycopodium powder away, a distinct curve is shown like that in the upper part of Fig. 13.

It is evident that while the ball rolls down a distance AB, it rolls from the centre line out to the side of the groove and back again; while it rolls from B to C, it rolls from the centre line to the other side of the groove and back again. These times are equal; let each be n sec. (about $\frac{1}{3}$ sec). In the same way CD, DE, EF and FG are each traversed in the same interval.

Now, $s=\frac{1}{2}at^2$, where s is the space, a is the acceleration and t is the time (§ 21).

$$
\begin{aligned}
\text{Hence } AB &= \tfrac{1}{2}\,an^2,\\
AC &= \tfrac{1}{2}\,a\,(2n)^2 = 4 \times \tfrac{1}{2}\,an^2 = 4 \times AB,\\
AD &= \tfrac{1}{2}\,a\,(3n)^2 = 9 \times \tfrac{1}{2}\,an^2 = 9 \times AB,\\
AE &= \tfrac{1}{2}\,a\,(4n)^2 = 16 \times \tfrac{1}{2}\,an^2 = 16 \times AB, \text{ etc.,}
\end{aligned}
$$

i.e., the spaces AB, AC, AD, AE, etc., are proportional to 1, 4, 9, 16, etc.; or the distance is proportional to the square of the time.

By laying a metre scale along the middle of the groove, these results can be tested experimentally.

The following are sample measurements obtained with 1 inch and $1\frac{1}{4}$ inch balls rolling down a board 6 feet long. In the third, fifth and seventh columns are shown the ratios of AB, AC, AD, AE, AF, and AG to AB.

	1 inch ball End raised 20 cm.		$1\frac{1}{4}$ inch ball End raised 22 cm.		$1\frac{1}{4}$ inch ball End raised $22\frac{1}{2}$ cm.	
	cm.	Ratio	cm.	Ratio	cm.	Ratio
AB........	4.55	1.0	4.40	1.0	4.45	1.0
AC........	18.80	4.1	18.35	4.2	18.65	4.2
AD........	40.40	8.9	39.50	9.0	40.25	9.0
AE........	70.28	15.4	70.90	16.1	72.95	16.4
AF........	111.90	24.6	108.45	24.6	111.00	24.9
AG........	161.30	35.4	157.10	35.7	161.00	36.2

These ratios are very close to the theoretical values 1, 4, 9, 16, 25, etc., the discrepancies being due to unavoidable imperfections in the board, small inaccuracies in measurement, etc.

23. To Measure the Acceleration of Gravity. The acceleration given to a falling body by the attraction of the earth is usually denoted by the letter g. If we gradually increase the height from which a body is allowed to fall until at last it just reaches the ground in 1 second, we find the distance is about 16 feet. Now the measure of the acceleration is twice that of the space fallen through in the first second, and hence $g = 32$, approximately.

The most accurate method of measuring the value of g is by means of the pendulum. In this way it is found that, using feet and seconds, $g = 32.2$; and using centimetres and seconds, $g = 981$.

These values vary slightly with the position on the earth's surface. At the equator $g = 978.10$; at the pole, 983.11; at Toronto, 980.6.

24. All Bodies falling freely have the same Acceleration. Galileo asserted that all bodies, if unimpeded, fall at the same rate. Now, common observation shows that a stone or a piece of iron, for instance, falls much faster than a piece of paper or a feather. This is explained by the fact that the paper or the feather is more impeded by the resistance of the air.

From the top of the Leaning Tower of Pisa (see § 60), Galileo allowed balls made of various materials to fall, and he showed that they fell in practically the same time. Sixty years later, when the air-pump had been invented, the statement regarding the resistance of the air was verified in the following way. A coin and a feather were placed in a tube (Fig. 14) four or five feet long and the air was exhausted. Then, on inverting the tube, it was found that the two fell to the other end together. The more completely the air is removed from the tube, the closer together do they fall.

FIG. 14—Tube to show that a coin and a feather fall in a vacuum with the same acceleration.

25. Bodies Falling Freely. Suppose a ball is thrown vertically upwards. Its velocity gradually decreases until at last it comes to rest. Then as it falls, it regains the velocities which it had on its upward path, and it ends with the same speed that it started up with. The time to come down is the same as to go up; and the speed at any point of its path is the same coming down as it was going up.

Example.—Let the ball be thrown upwards with a velocity of 19.6 m., or 1960 cm., per sec. At the end of 1 sec. its velocity will be reduced 980 cm. per sec.; at the end of the 2nd sec. it will be 1960 cm. per sec. less, that is it will be reduced to zero. Hence the ball will rise 2 sec. It will now fall for 2 sec., at the end of the 1st sec. having a velocity of 980 cm. per sec., and at the end of the 2nd sec. 1960 cm. per sec.

Going up the ball loses 980 cm. per sec. of its velocity every second, and coming down it regains that amount.

If the velocity of the ball upwards had been 1000 cm. per sec. it would have risen for $1000 \div 980 = 1.02$ sec., and then it would fall for 1.02 sec.

In this case the average velocity upwards $= \frac{1}{2}(0+1000) = 500$ cm. per sec., and the space traversed $= 500 \times 1.02\ldots = 510$ cm. (approx.).

In this example the resistance of the air is neglected.

PROBLEMS

Unless otherwise stated, take as the measure of the acceleration of gravity, with centimetres and seconds, 980; with feet and seconds, 32.

1. (*a*) A body moves 1, 3, 5, 7 ft. during the 1st, 2nd, 3rd, 4th seconds, respectively. Find the average speed.

(*b*) Express a speed of 36 kilometres per hour in cm. per second.

2. A body falls freely for 6 seconds. Find the velocity at the end of that time, and the space passed over.

3. The velocity of a body at a certain instant is 40 cm. per sec., and its acceleration is 5 cm. per sec. per sec. What will be its velocity half-a-minute later?

4. What initial speed upwards must be given to a body that it may rise for 4 seconds?

5. The Eiffel Tower is 300 metres high, and the tower of the City Hall, Toronto, is 305 ft. high. How long will a body take to fall from the top of each tower to the earth?

6. On the moon the acceleration of gravity is approximately one-sixth that on the earth. If on the moon a body were thrown vertically upwards with a velocity of 90 feet per second, how high would it rise, and how long would it take to return to its point of projection?

7. An automobile is moving at the rate of 30 miles an hour when the chauffeur sees an obstacle 66 feet ahead. He puts on his brakes and the car stops just as it reaches the obstacle. How long did it take to come to a stop? What was his acceleration (retardation)?

8. A body moving with uniform acceleration has a velocity of 10 feet per second. A minute later its velocity is 40 feet per second. What is the acceleration?

9. A body is projected vertically upward with a velocity of 39.2 metres per second. Find (1) how long it will continue to rise; (2) how high it will rise.

10. A stone is dropped down a deep mine, and one second later another stone is dropped from the same point. How far apart will the two stones be after the first one has been falling 5 seconds?

11. A stone is projected vertically upwards and returns to the ground in 10 sec. Find (1) the velocity of projection, (2) the height to which it rises, (3) its height above the ground after 1, 2, 3, 4, 5 sec., respectively.

12. A train is moving at the rate of 60 miles an hour. On rounding a curve the engineer sees another train $\frac{1}{4}$ mile away on the track at rest. By putting on all brakes a retardation of 3 feet per second per second is given the train. Will it stop in time to avoid a collision?

13. While trying to pass from one airplane to another when up in the air, an airman slipped and fell. The fall to earth occupied $17\frac{1}{2}$ sec.; from what height did he fall?

26. Motion in a Circle. Let a body M (Fig. 15) be made to revolve uniformly in a circle with centre O and radius r. A familiar illustration of this motion is seen when a stone at the end of a string is whirled about.

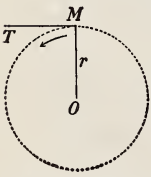


FIG. 15—Motion in a circle.

In this case the length of the line MO does not alter, and yet M has a velocity with respect to O. This arises from the continual change in the direction of the line MO. Every time the body describes a circle, its direction changes through 360°.

If the string were cut and M were thus allowed to continue with the velocity it possessed, it would move off in the tangent to the circle MT. This effect is well illustrated by the drops of water flying off from the wheels of a bicycle, or the sparks from a rapidly rotating emery wheel.

We see, then, that *one point has a velocity with respect to another when the line joining them changes in magnitude or direction.*

In the above case there is a change of velocity (being a continual change from motion in one tangent to motion in another), and hence there is an acceleration; and as the change in the velocity is uniform, the acceleration is constant. The acceleration is always directed toward the centre of the circle.

27. Translation and Rotation. If a body move so that all points have the same speed and in the same direction, we say that it has a motion of *translation* (Fig. 16). Examples: the car of an elevator, or the piston of an engine.

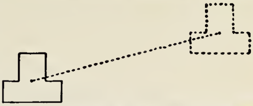


FIG. 16.—Showing motion of translation.

If, however, a body move so that all points of it move in circles having as centre a point called the centre of mass, or centre of gravity,* the motion is a *pure rotation* (Fig. 17). Example: a wheel on a shaft, such as the wheel of a sewing-machine or a fly-wheel.

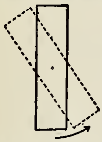


FIG. 17—Showing motion of rotation.

Usually, however, both motions are present, that is, the body has both translation and rotation. Examples: the motions of the planets, of a carriage wheel, of a body thrown up in the air.

If a body is rotating about an axis through a point O in it, it is evident that those points which are near O, such as P, Q (Fig. 18), have smaller speeds than have those points such as R, S, which are farther away.

But they all describe circles about O in the same time, and hence their *angular velocities* are all equal.

Again, consider the motions of A and B with respect to each other. To a person at A the point B will revolve about him in the same time as the body rotates about O. Also, a person at B will see A revolve about him in the same time.

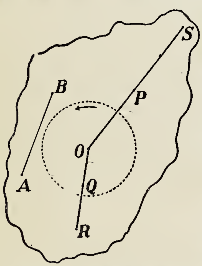


FIG. 18.—In a rotating body all points have the same angular velocity.

For instance, suppose the body to rotate once in a second. All lines in the body will change their directions in the same manner, turning through 360 degrees, and returning to their former positions at the end of a second.

28. Composition of Velocities. Suppose a passenger to be travelling on a railway train which is moving on a straight track at the rate of 15 miles per hour, or 22 feet per second. While sitting quietly in his seat, he has a motion of translation, in the direction of the track, of 22 feet per second.

Next let the passenger rise and move directly across the car, going a distance of 6 feet in 2 seconds. His velocity across will be 3 feet per second.

*Explained in Chapter VII.

In Fig. 19, A is the position of the passenger at first. If the train were at rest, in 2 seconds he would move from A to

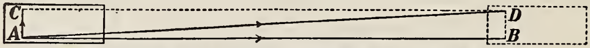


Fig. 19.—Motion of a passenger walking across a moving railway car.

C, 6 feet; while, if he sat still, the train in its motion would carry him from A to B in 2 seconds, a distance of 44 feet. It is evident, then, that if the train move forward and the passenger move across at the same time, at the end of 2 seconds he will be at D, *i.e.*, 44 feet forward and 6 feet across.

Moreover, at the end of 1 second, he will be 22 feet forward and 3 feet across, that is, half-way from A to D. The motions which he has will carry him along the line AD in 2 seconds.

29. Law of Composition. Another example will perhaps make clearer this principle of compounding velocities.

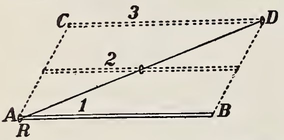


Fig. 20.—Showing how to add together two motions of a ring on a rod.

Let a ring R (Fig. 20) slide with uniform velocity along a smooth rod AB, moving from A to B in 1 second. At the same time let the rod be moved in the direction AC with a uniform velocity, reaching the position CD in a second. The ring will be at D at the end of a second.

At the end of half a second from the beginning the ring will be half-way along the rod, and the rod will be in position (2) half-way between AB and CD. It is evident that between the two motions the ring will move uniformly along the line AD, travelling this distance in 1 second.

From these illustrations we can at once deduce the law of composition of velocities.

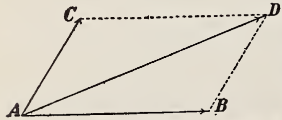


Fig. 21—The parallelogram of velocities

Let a particle possess two velocities simultaneously, one represented in direction and magnitude by the line AB, the other by AC. (Fig. 21.)

Complete the parallelogram $ABDC$. Then the diagonal AD will represent in magnitude and direction the resultant velocity.

NUMERICAL EXAMPLES

1. Suppose a vessel to steam directly east at a velocity of 12 miles per hour, while a north wind drifts it southward at a velocity of 5 miles an hour. Find the resultant velocity.

Draw a line AB, 12 cm. long, to represent the first component velocity; AC, 5 cm. long, to represent the second. (Fig. 22.)

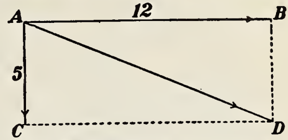


FIG. 22.—Illustrating the motion of a vessel.

Completing the parallelogram, which in this case is a rectangle, AD will represent the resultant velocity.

Here we have $\overline{AD^2} = \overline{AB^2} + \overline{BD^2} = 12^2 + 5^2 = 169 = 13^2$.

Hence $AD = 13$, *i.e.*, the resultant velocity is 13 miles per hour in the direction represented by AD.

2. A ship moves east at the rate of $7\frac{1}{2}$ miles per hour, and a passenger walks on the deck at the rate of 3 feet per second. Find his velocity relative to the earth in the following three cases: (1) when he walks toward the bow, (2) toward the stern, (3) across the deck.

3. A ship sails east at the rate of 10 miles per hour, and a north-west wind drives it south-east at the rate of 3 miles per hour. Find the resultant velocity.

To calculate the resultant accurately requires a simple application of trigonometry, but the question can be solved approximately by drawing a careful diagram. Draw a line in the easterly direction 10 inches long, and lay off from this, by means of a protractor, a line in the south-east direction, 3 inches long. Complete the parallelogram and measure carefully the length of the diagonal. (12.30 miles per hour.)

4. Find the resultant of two velocities, 20 cm. per second and 50 cm. per second, (*a*) at an angle of 60°, (*b*) at an angle of 30°. (Carefully draw diagrams, and measure the diagonals.)

5. A particle has three velocities given to it, namely, 3 feet per second in the north direction, 4 feet per second in the east direction, and 5 feet per second in the south-east direction. Find the resultant. (Carefully draw a diagram.)

CHAPTER III

INERTIA, MOMENTUM, FORCE

30. Mass, Inertia. The *mass* of a body has been defined (§ 11) as the quantity of matter in it. Just what *matter* is, no one can say. We all understand it in a general way, but we cannot explain it in terms simpler than itself. We must obtain our knowledge regarding it by experience.

When we see a young man kick a football high into the air, we know that there is not much *matter* in it. If it were filled with water or sand, so rapid a motion could not be given to it so easily, nor would it be stopped or caught so easily on coming down. A cannon-ball of the same size as the football and moving with the same speed, would simply plough through all the players on an athletic field before it would be brought to rest.

To a person accustomed to handling a utensil made of iron or enamelled ware, one made of aluminium seems singularly easy to move. If a thin rubber ball is thrown at you with great speed, you catch it with ease; if it is a base-ball, it requires a much greater effort to stop it; while if it is an iron ball, you had better let it alone.

All our experience teaches us that it requires an effort to put in motion matter which is at rest or to bring to rest matter which is in motion; or, in other words, *all matter has inertia.*

Further, the greatness of the effort which must be exerted to put a body in motion or to bring it to rest is proportional to its mass; or, *the inertia of a body is proportional to its mass.*

31. Newton's Laws of Motion: the First Law. In the preceding section we have used the word "effort" a number of times, when speaking of putting a body in motion or of bringing it to rest. In physics the word which is used in this manner is *force*. In 1687 Sir Isaac Newton published his "Principia,"* in which he gave his three famous *Laws of Motion*. The First Law is simply a statement of the conclusion which we have just arrived at, but expressed in a form which has never since been improved upon. It is as follows:—

Every body continues in its state of rest, or of uniform motion in a straight line, unless it be compelled by external force to change that state.

Sir Isaac Newton (1642-1727) at the age of 83. Demonstrated the law of gravitation. The greatest of mathematical physicists.

This is often referred to as the *Law of Inertia*.

32. Illustrations of the First Law. A ball lying at rest on the grass will not move itself. If, however, it is rolled on the grass, it 'slows up' and comes to rest. If we roll it on a smooth pavement, the motion persists longer, and if on smooth ice, longer still. It is seen that as we remove the external force (of friction), and leave the body more and more to itself the motion continues longer, and we are led to believe that if there were no friction, it would continue uniformly in a straight line.

An ordinary wheel, if set rotating, soon comes to rest. But a well-adjusted bicycle wheel, if put in motion, will continue to move for a long time. Here the external force—the friction at the axle—is made very small, and the motion persists for a long time.

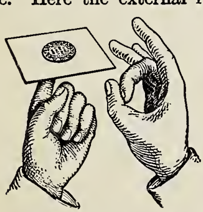


FIG. 23—Illustrating Inertia.

When a locomotive, running at a high rate of speed, leaves the rails and is rapidly brought to a standstill, the cars behind do not immediately stop, but continue ploughing ahead, and usually do great damage before coming to rest.

If one wishes to jump over a ditch, he takes a run, leaps up into the air, and his body, persisting in its motion, reaches the other side.

*The full title of the book is "Principia Mathematica Naturalis Philosophiæ," *i.e.*, "The Mathematical Principles of Natural Philosophy."

In an earthquake the buildings tend to remain at rest while the earth shakes under them, and they are broken and crumble down.

Lay a card on the end of a finger and place a small coin on the card. (Fig. 23). On 'flipping' the card suddenly with the finger it is driven out, while the inertia of the coin causes it to remain behind on the finger.

The hydraulic ram is an interesting device for utilizing the inertia of a moving column of water. It consists of a reservoir *A* fed by a natural stream, and from this a pipe *B* of considerable length leads the water to

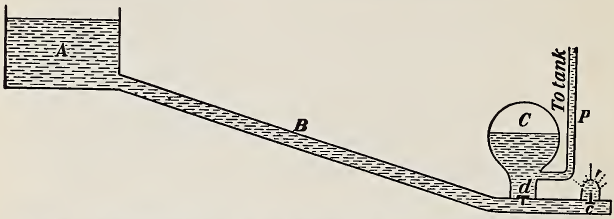


Fig. 24.—The hydraulic ram. Water is raised from A to a considerable height.

a lower level where it rushes against and closes a valve *c*. The inertia of the column carries it onward, and, pushing upward the valve *d*, some of the water enters the chamber *C* and thence goes into the pipe *P*, which runs up to a tank in the attic of a house or in some other elevated position. Immediately after coming to rest the water rebounds, and the valve *c* drops. This allows some water to escape, and the column starts moving in the pipe *B* again, and the operation is repeated. The pipe *B* should be comparatively long and straight. The greater part of the water escapes at *c*, but a fall of (say) 4 feet can raise the remaining portion to a height of perhaps 30 feet.

33. Momentum. Now from our experience we know that, in estimating the greatness of the force required to put a body in motion, we must take into account not only the mass of the body but also the velocity which is given to it. It requires a much greater force to impart a great velocity to a body than to give it a small one; and to stop a rapidly moving body is much harder than to stop one moving slowly. We feel that there is something which depends on both mass and velocity, and which we can think of as *quantity of motion*. This is known in physics as *momentum.* It is proportional to both the mass and the velocity of the body, thus

$$\text{Momentum} = \text{mass} \times \text{velocity} = mv,$$

where m is the mass of the body and v its velocity of translation.

PROBLEMS

1. Compare the momentum of a car weighing 50,000 kg. and moving with a velocity of 30 km. an hour with that of a cannon ball weighing 20,000 grams and moving with a velocity of 50,000 cm. per second.

2. A man weighing 150 pounds and running with a velocity of 6 feet per second collides with a boy of 80 pounds moving with a velocity of 9 feet per second. Compare the momenta.

3. Compare the momentum of a $\frac{1}{2}$-ounce bullet fired with a velocity of 2800 ft. per sec. with that of a 2-lb. weight which has fallen freely for 2 sec.

34. How to Measure Force. Newton's Second Law. If there is a change in the condition of a body (*i.e.*, if it does not remain at rest or in uniform motion in a straight line), then there is a change in its momentum, that is, in the *quantity of motion* it possesses. Any such change is due to some external influence which is called FORCE, and as a result of our experience we recognize that the amount of the change in a given length of time is proportional to the impressed force. Further, we recognize that the total change of momentum is also proportional to the length of time during which the force acts. We know that a motorman, wishing to stop a heavily-loaded car, must apply the brakes for a longer time than in the case where the car is empty. Again, a force must act in a definite direction, and the change in momentum must be in that direction.

Newton summed up these results of our experience in his Second Law of Motion, which is as follows:—

Change of momentum, in a given time, is proportional to the impressed force and takes place in the direction in which the force acts.

The word force is used, in ordinary conversation, in an almost endless number of meanings, but in physics the meaning is definite. If there is a change of momentum, force is acting.

Sometimes, however, a body is not free to move. In this case force would *tend* to produce a change in the momentum. We can include such cases by framing our definition thus:

FORCE *is that which tends to change momentum.*

It is to be observed that there is no suggestion as to the cause or source of force. Whatever the nature of the external influence on the body may be, we simply look at the effect; if there has been a change of momentum, then it is due to force.

It is evident, also, that the total effect of a force depends upon the time it acts. Thus, suppose a certain force to act upon a body of mass m for 1 second, and let the velocity generated be v, *i.e.*, the momentum produced is mv. If the force continues for another second, it will generate additional velocity v, or $2v$ in all, and the momentum produced will be $2mv$; and so on.

Let us state this result in symbols.

Let F represent the force, and t sec. be the time during which it acts.

At the end of t sec. the force will have generated a certain momentum, which we may write mv.

Then Force $\times$ time = momentum produced, or $Ft = mv$.

$$\text{Hence } F = m\frac{v}{t} = ma, \textit{ i.e. } \text{Force} = \text{mass} \times \text{acceleration.}$$

35. Units of Force. We can fix our ideas regarding force by considering the attraction of the earth on a mass. A bit about 1 cm. long off the larger end of a chalk crayon contains about 1 gram-mass. On picking up a gram-mass we are conscious that there is a pull downwards tending to give it a velocity, that is, to change its momentum. This pull we call a *gram-force;* it is the attraction of the earth on a gram-mass on its surface. In the same way the attraction of the earth on a pound-mass on its surface is a *pound-force.*

Next, allow the gram-mass to fall freely for 1 sec. At the end of the second it has a velocity of 980 cm. (32.2 ft.) per sec. Hence when a gram-force acts on a gram-mass for 1 sec. the momentum generated

$$= \text{mass} \times \text{velocity} = 1 \times 980 = 980 \text{ C.G.S. units.}$$

If the mass is 15 grams, the force acting on it is 15 grams-force, and the momentum generated in 1 sec.

$$= 15 \times 980 = 14{,}700 \text{ units; and so on.}$$

Now, as a body is raised above the surface of the earth, the force of attraction pulling it downwards becomes smaller, as will be explained in Chapter V. Imagine a gram-mass to be carried farther and farther away until the attraction on it is only 1-980th of what it is at the surface of the earth, that is, the force up there is 1-980 gram-force.

Then, if from this far-distant place the gram-mass is set free and allowed to move toward the earth, the velocity it will acquire in 1 sec. will be 1-980th that acquired at the earth's surface, that is, 1 cm. per sec., and the momentum generated will be 1 (gram-mass) × 1(cm. per sec.) = 1 unit of momentum.

36. The Dyne. A name has been given to that force which, when it has acted on a gram-mass for 1 sec., will have given it a velocity of 1 cm. per sec.; it is called a *dyne.*

It will be noticed that there is a distinction in nature between a gram-mass and a gram-force; and when we use the word *gram,* we must have clearly in mind whether it is a portion of matter or a force.

A gram-mass is the same wherever it be taken—to the north pole, to the moon or to a distant star, it is just so much matter; but a gram-force is not constant all over the earth's surface, as the earth's attraction on a body on its surface is different at different places. At the equator it is slightly smaller than at the poles; but a *dyne* is constant everywhere in the universe, and hence it is called an *absolute unit* of force.

A DYNE *is that force which acting on* 1 *gram-mass for* 1 *sec. will generate a velocity of* 1 *cm. per sec.*; or in other words, *a dyne is that force which acting on* 1 *gram-mass will give it an acceleration of* 1 *cm. per sec. per sec.*

PROBLEMS

1. A force of 1 dyne acts on a mass of 1 gram. Find the velocity produced in 1 sec., 2 sec., 10 sec. What is the acceleration?

2. If the mass were 10 grams, what would be the velocity produced in 1 sec., 2 sec., 10 sec.? What would be the acceleration?

3. If the force were 10 dynes and the mass 1 gram, what would be the velocity in 1 sec., 2 sec., 10 sec., and the acceleration?

4. A force of 2000 dynes acts on a mass of 400 grams. Find the velocity at the end of 1 sec., 2 sec., 10 sec.; also the acceleration, and the momentum at the end of 10 sec.

5. A force of 10 dynes acts on a body for 1 min., and produces a velocity of 120 cm. per sec. Find the mass, and the acceleration.

37. Independence of Forces. It is to be observed that each force produces its own effect, measured by change of momentum, quite independently of any others which may be acting on the body.

Suppose now a person to be at the top of a tower 64 feet high. If he drops a stone, it will fall vertically downward and will reach the ground in 2 seconds. Next, let it be thrown outward in a horizontal direction. Will it reach the ground as quickly?

By the *Second Law* the force which gives to the stone an *outward* velocity will act quite independently of the force of gravity which gives the *downward* velocity. A horizontal velocity can have no effect on a vertical one, either to increase or to diminish it. Hence the body should reach the ground in 2 seconds, just the same as if simply dropped.

This result can be experimentally tested in the following way:

A and *B* (Fig. 25) are two upright supports through which a rod *R* can slide. *S* is a spring so arranged that when *R* is pulled back and let go it flies to the right. *D* is a metal sphere through which a hole is bored to allow it to slip over the end of *R*. *C* is another sphere, at the same height above the floor as *D*.

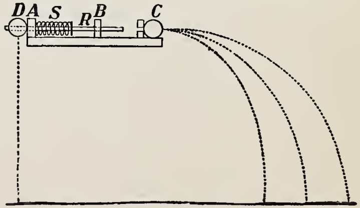


Fig. 25.—The ball *C*, following a curved path reaches the floor at the same time as *D* which falls vertically.

The rod R is just so long that when it strikes C, the sphere D is set free. Thus C is projected horizontally outwards, while D drops directly down.

By pulling R back to different distances, different velocities can be given to C, and thus different paths described, as shown in the figure.

It will be found that no matter which of the curved paths C takes it will reach the floor at the same time as D.

PROBLEMS

1. From a window 16 ft. above the ground a ball is thrown in a horizontal direction with a velocity of 50 ft. per second. Where will it strike the ground?

2. A rifle is discharged in a horizontal direction over a lake from the top of a cliff 19.6 m. above the water, and the ball strikes the water 2500 m. from shore. Find the velocity of the bullet outwards, supposing it to be uniform over the entire range.

3. In problem 2 find the velocity downwards at the moment the ball reaches the water; then draw a diagram to represent the horizontal and vertical velocities, and calculate the resultant of the two.

38. Newton's Third Law of Motion. The Third Law relates to actions between bodies.

Let us tie a string to each end of a spring balance and then have two persons, A and B, pull on the strings in opposite directions until the balance indicates (say) 15 pounds. Then it is evident that A pulls B with a force of 15 pounds, and B pulls A with an equal force and in the opposite direction; or, the *action* of A on B is equal and opposite to the *reaction* of B on A. Next, let B tie his string to a post. Then, as before, the post pulls A with the same force that A pulls it, or the action and the reaction are equal and opposite.

If one presses the table with the hand, there is an equal upward pressure exerted by the table on the hand.

A weight is suspended by a cord; the downward pull exerted on the support by the weight is equal to the upward pull exerted by the support to which the cord is fastened.

In all the above cases the action between the two bodies is either a pressure or a tension, and this force produces no motion, since neither body is free to move.

Next, consider what happens when a person jumps from a boat to the shore. The force exerted causes him to go forward and, at the same time, the boat to go backward, and the total effect of the force is the same in the two directions, that is, the momentum of the person forward is equal to the momentum of the boat backward.

On considering the above and numerous other examples, we recognize the truth of Newton's Third Law, which states:—

To every action there is always an equal and opposite reaction.

The following experiment illustrates the third law:—

A and *B* (Fig. 26) are two exactly similar ivory or steel balls, suspended side by side. *A* is drawn aside to *C*, and then allowed to fall and strike *B*. At once *A* comes to rest, and *B* moves off with a velocity equal to that which *A* had.

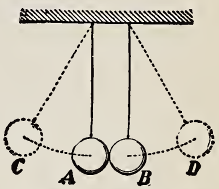


Fig. 26—The action of *A* on *B* is equal to the reaction of *B* on *A*.

Here the *action* is seen in the forward momentum of *B*, the *reaction* in the equal momentum in the opposite direction which just brings *A* to rest. Of course, if we call the latter the *action*, the former is the *reaction*.

Suppose now *A* and *B* to be sticky putty balls, so that when they collide they stick together; they will both move forward with one-half the velocity which *A* had on striking. *A* loses in momentum just what *B* gains.

PROBLEMS

1. A 200-pound man dives from the stern of a motor-boat which weighs 1200 pounds, with a speed of 6 feet per second. With what speed does the boat begin to move in the opposite direction?

2. A hollow iron sphere is filled with gunpowder and exploded. It bursts into two parts; one part, being one-quarter of the whole, flies in one direction with a velocity of 75 metres per second. What is the velocity of the other part?

3. Suspend an iron ball (Fig. 27) about 3 inches in diameter with ordinary thread. By pulling slowly and steadily on the cord below the sphere, the cord above breaks, but a quick jerk will break it below the ball. Apply the first law to explain this.

Fig. 27—An iron ball suspended by a thread.

4. A rifle weighs 8 lbs. and a bullet weighing 1 oz. leaves it with a velocity of 1500 ft. per sec. Find the velocity with which the rifle recoils.

5. An apple falls to the earth. Does the earth move to meet the apple? Can you detect it? Why?

6. Sometimes, in putting a handle in an axe or a hammer, it is accomplished by striking on the end of the handle. Explain how the law of inertia applies here.

7. A man weighing 150 lbs. jumps from a row-boat weighing 100 lbs. If his velocity forward was 10 ft. per sec., what was the velocity of the boat backwards?

8. When you stamp your feet on the pavement, why does the snow come off?

9. A bag of sand of mass 10 lbs. hangs from the end of a long cord, and a bullet of mass 1 oz. is fired into it. The bag starts moving with a velocity of 20 ft. per sec. What was the velocity of the bullet on striking the bag?

10. A motor-boat weighing 3000 lbs. and a row-boat weighing, with contents, 500 lbs. float at rest 20 ft. apart. A rope is thrown from one to the other, and by pulling on it they are brought together. How far will each move?

CHAPTER IV

Moment of a Force; Composition of Parallel Forces; Equilibrium of Forces

39. Moment of a Force. If you have to turn a nut which is rusted tight, you can do it most effectively by using a long wrench and pushing (or pulling) on the end in a direction at right angles to its length.

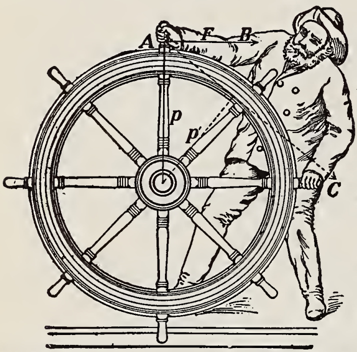


Fig. 28—The *moment of a force* depends on the force applied and its distance from the axis of rotation.

In stormy weather, in order to keep the ship on her course, the wheelsman grasps the wheel at the rim (*i.e.*, as far as possible from the axis), and exerts a force at right angles to the line joining the axis to the point where he takes hold. (Fig. 28.)

From our experience we know that the turning effect upon the wheel is proportional to the force exerted and also to the distance from the axis of the point where the force is applied.

Let F = the force applied,

p = the perpendicular distance from the axis to the line AB of the applied force.

Then the product Fp measures the tendency of the wheel to turn, or the tendency to produce angular momentum. This product is the *moment of the force*, which is defined as follows:

The MOMENT OF A FORCE *is the tendency of that force to produce rotation of a body.*

If the direction of the force F is not perpendicular to the line joining its point of application to the axis, the moment is not so great, since part of the force is spent uselessly in pressing the wheel against its axis. In Fig. 28, if AC is the new direction of the force, then p', the new perpendicular, is shorter than p, and hence the product Fp' is smaller than Fp.

40. Experiment on Law of Moments. We can test the law of moments experimentally in the following way:—

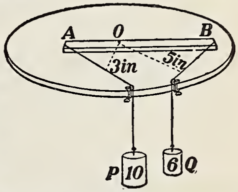


Fig. 29—Apparatus for testing the law of moments.

AB is a rod which can move freely about a pin driven in a board at O, and two cords attached to the ends A and B pass over pulleys at the edge of the board. Adjust these until the perpendicular distances from O upon the strings are 3 inches and 5 inches. Then if the weight P = 10 oz., the weight Q, to balance the other, must = 6 oz.

Here moment of force P is $10 \times 3 = 30$,
and " " " Q is $6 \times 5 = 30$.

For equilibrium of the two moments, the products of the forces by the perpendicular distances must be the same, and they must tend to produce rotations in opposite directions.

Principle of Moments: *When a body acted on by several forces in one plane is in equilibrium, the sum of the moments of the forces tending to turn the body in one way about any point in that plane is equal to the sum of the moments about the same point of the forces tending to turn the body in the other direction.*

41. Forces on a Crooked Rod. For a body shaped as in Fig. 30, with forces P and Q acting at the ends A and B, the moment of P about O is Pp, that of Q is Qq; but it is to be observed that they turn the rod in opposite directions. If we call the first positive, the other will be negative, and the entire tendency of the rod to rotate will be

$$Pp - Qq.$$

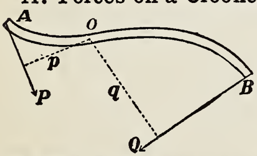


Fig. 30—Balancing forces on a rod which is not straight.

If $Pp - Qq = 0$, the rod will be in equilibrium.

42. Composition of Parallel Forces. The behaviour of parallel forces acting on a rigid body may be investigated experimentally in the following way:—

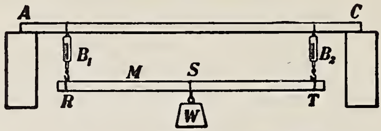


Fig. 31—Resultant of Parallel Forces.

M is a metre stick (Fig. 31) with a weight *W* suspended at its centre of gravity, and two spring balances B_1, B_2, held up by the rod *AC*, support the stick and the weight. Be careful to have the balances hanging vertically.

Take the readings of the balances B_1, B_2; let them be *P* and *Q*, respectively. Also, measure the distances *RS*, *ST*.

Then we shall find that if the weight of the stick and *W* together is *U*,

$$P + Q = U \text{ and } P \times RS = Q \times ST.$$

We conclude, then, that *the resultant of two parallel forces acting in the same direction is equal to the sum of the forces, and its point of application is situated so that its distances from the lines of action of the forces are inversely as the magnitudes of the forces.*

Again, if we take moments about *R* we should have

$$U \times RS = Q \times RT.$$

By shifting the position of *R* and *T*, various readings of the balances will be obtained.

PROBLEMS

1. A rod is 4 feet long (Fig. 32), and one end rests on a rigid support. At distances 12 inches and 18 inches from that end weights of 20 lbs. and 30 lbs., respectively, are hung. What force must be exerted at the other end in order to support these two weights? (Neglect the weight of the rod.)

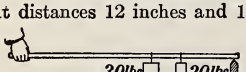


Fig. 32—What force is required to lift the weights?

2. An angler hooks a fish. Will the fish appear to pull harder if the rod is a long or a short one?

3. A stiff rod 12 feet long, projects horizontally from a vertical wall.

A weight of 20 lbs. hung on the end will break the rod. How far along the pole may a boy weighing 80 lbs. go before the pole breaks?

4. A bicycle rider presses vertically downward on the pedals. Compare the moment of the force exerted when the crank is in three positions (*a*) horizontal, (*b*) 30° below the horizontal, (*c*) 45° below.

5. Two men of the same height carry on their shoulders a pole 6 ft. long, and a mass of 121 lbs. is slung on it 30 inches from one of the men. How many pounds does each support?

6. A man carries two baskets, one on each end of a stick 30 inches long, and to balance them he grasps it 12 inches from one basket. If the total weight is 25 lbs. find the weight of each basket.

43. Unlike Parallel Forces.—Couple. Let P, P be two equal parallel forces acting on a body in opposite directions (Fig. 33). The entire effect will be to give the body a motion of rotation without a motion of translation.

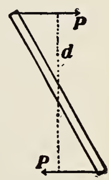


Fig. 33—Two equal opposite parallel forces produce only rotation.

Such a pair of forces is called a *couple*, and the moment of the couple is measured by the product of the force into the perpendicular distance between them. Thus if d is this distance, the magnitude of the couple is Pd. This measures the rotating power.

Next, suppose there are two unlike parallel forces P, Q, and that Q is greater than P. Then the forces P and Q are equivalent to a couple tending to cause a rotation in the direction in which the hands of a clock turn, and to a force tending to produce a motion of translation in the direction of Q, that is, to the left hand (Fig. 34).

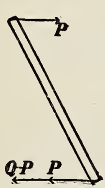


Fig. 34—Two opposite parallel but unequal forces produce both rotation and translation.

This can be seen in the following way: Divide Q into two forces P and $Q - P$. The portion P, along with P acting at the other end of the rod, forms a couple, while the force Q W P will give a motion of translation to the body in its direction.

44. Experimental Verification of the Parallelogram Law.

In § 29 it was shown that if two velocities are given to a body, it will have a resultant velocity whose magnitude and direction can be determined by the parallelogram law. If two forces act on a body, they are equivalent to a single one which can be determined in the same manner. This can be shown experimentally in the following way:—

In Fig. 35, S, S' are two spring balances hung on pins in the bar AB, which may conveniently be above the blackboard. Three strings of

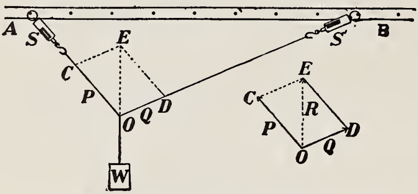


Fig. 35—How to test the law of parallelogram of forces.

unequal length are knotted together at O, and the ends of two of them are fastened to the hooks of the balances. A weight, W ounces, attached to the third string makes it hang vertically downward.

Thus three forces, namely, the tensions of the strings, pull on the knot O. The magnitude of the forces acting along the strings, OS, OS', which we shall denote by P, Q, will be given by the readings on the balances, in ounces, let us suppose. The magnitude of the force acting along OW is, of course, W ounces.

The three forces, P, Q, W, act upon the knot O, and as it does not move, these forces must be in equilibrium. The force W may be looked upon as balancing the other forces P, Q; and hence the resultant of P, Q must be equal in magnitude to W but opposite in sense.

Draw now on the blackboard, immediately behind the apparatus or in some other convenient place, lines parallel to the strings OS, OS', and make OC, OD as many units long as there are ounces shown on S, S', respectively.

On completing the parallelogram $OCED$, it will be found that the diagonal OE is vertical, and that it is as many units long as there are ounces in W.

From this experiment we see that *if two forces are represented in magnitude and direction by two sides of a parallelogram, then their resultant will be represented, in magnitude and direction, by the diagonal between the two sides.*

PROBLEMS

1. Find the resultant of 15 pounds and 36 pounds, acting at right angles to each other.

2. A weight is supported by two strings which make an angle of 90° with each other. The tension of one string is 9 pounds, that of the other 12 pounds; what is the weight?

3. Two ropes are attached to a stone, and one man exerts a force of 100 pounds, while another exerts a force of 60 pounds at an angle of 30° with the other. Draw a good diagram to scale and from it find the resultant, also the angle between its direction and that of the smaller force.

4. The resultant of two forces at right angles to each other is 80 pounds, and one force is $\frac{3}{4}$ as great as the other. Find the magnitude of each.

5. A team of horses are pulling a freight-car and exert a force of 800 pounds in a direction making an angle of 20° with the track. By means of a figure drawn to scale, find how great a force pulls the car along the track. What does the other part of the force do?

CHAPTER V

Gravitation

45. The Law of Gravitation. One of our earliest observations is that a body, when not supported, falls towards the earth, and this action we say is due to *the attraction of the earth.*

Now the earth is one of a family of planets which revolve about the sun, while certain other smaller bodies, called moons, or satellites, revolve about the planets. After many years of observation and study astronomers were able to show that each planet follows a path which has the form of an ellipse, the sun being at one focus; and that the satellites of a planet also revolve in ellipses with the planet in one focus. Certain other simple laws regarding the rate at which the bodies travel in their orbits were also discovered.

This was early in the 17th century, and various scientific people were in the habit of discussing why these motions were so and were wondering just what was the underlying cause. Then Newton examined the question, and he was able to show that, if we suppose the sun to attract the planets, and the planets to attract their satellites, according to a certain simple law, the motions of these bodies would of necessity be precisely what had been observed. The hypothesis was so simple, and it explained the motions so completely that it was at once accepted as true. This hypothesis is known as the Newtonian Law of Gravitation.

Having shown conclusively that the heavenly bodies, with their great masses, attract each other according to his law, Newton was led to the belief that all bodies, no matter what their mass or how they are distributed, attract each other in the same way. This is known as the *Law of Universal Gravitation.*

46. The Newtonian Law. Let m, m' be the masses of two particles of matter and r the distance between them. Then Newton's Law of Universal Gravitation states that the attraction between m and m' is proportional directly to the product of their masses, m and m', and inversely to the square of r, the distance between them.

In algebraical language, the Force is proportional to $\frac{m\,m'}{r^2}$, or $F = k\,\frac{m\,m'}{r^2}$, where k is a numerical constant.

If m, m' are small spheres, each containing 1 gram-mass* and r, the distance between their centres, is 1 cm., then $F = \frac{1}{15000000}$ dyne (and 1 dyne $= \frac{1}{980}$ gram-force). This is an extremely small quantity, and the attractions between ordinary masses are very small.

For example, a lead sphere 1 metre in diameter contains 5,937,600 grams or nearly 6.6 tons, yet if two such spheres be placed with just 1 cm. between their surfaces, the attraction between them will be only 227 dynes or .22 gram-force.

It is to be noted that though the Newtonian Law states *how* two bodies act toward each other it does not state *why.* The *reason why* the attraction takes place is one of the mysteries of nature.

47. The Attraction exerted by a Sphere. Some persons in years gone by used to argue that the earth could not be a

*Lead spheres 5.5 mm. in diameter (the size of a large pea) contain 1 gram-mass.

sphere, since the people on the other side from us would fall off. But if we assume that the earth attracts toward its centre all objects on its surface, then all difficulty disappears. No matter where we are on the surface of the earth, when we stand upright our feet are toward the earth's centre (Fig. 36).

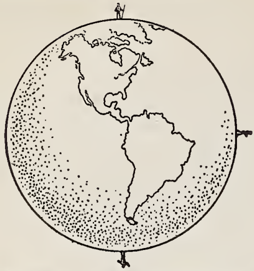


FIG. 36.—A person is attracted to the centre of the earth.

It can be shown by mathematical calculation that, assuming the truth of Newton's Law of Universal Gravitation, a homogeneous sphere attracts a body outside of it as though all its matter were collected at its centre.

48. The Weight of a Body. Consider a mass m at A on the earth's surface (Fig. 37). The attraction of the earth on the mass is the *weight* of the mass. The mass also attracts the earth with an equal force, since action and reaction are equal.

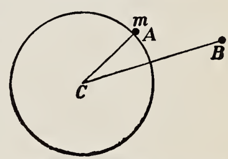


FIG. 37.—Attraction of the earth on a mass on its surface and also twice as far away from the centre.

If m is a pound-mass, the attraction of the earth on it is a *pound-force;* if it is a gram-mass, the attraction is a *gram-force.* We see then that if the whole mass of the earth were condensed into a particle at C and a pound-mass were placed 4000 miles from it, the attraction between the two would be 1 pound-force.

Next, suppose the pound-mass to be placed at B, 8000 miles from C. Then the force is not $\frac{1}{2}$ but $\frac{1}{2^2}$ or $\frac{1}{4}$ of its former value; that is, the *weight* of a pound-mass 4000 miles above the earth's surface would be $\frac{1}{4}$ of a pound-force.

If it were 2000 miles from the earth's surface or 6000 miles from its centre, this distance is $\frac{6000}{4000}$ or $\frac{3}{2}$ of its former distance, and the force of attraction

$$= \frac{1}{(\frac{3}{2})^2} = \frac{4}{9} \text{ of 1 pound-force.}$$

QUESTIONS AND PROBLEMS

1. If the earth's mass were doubled without any change in its dimensions, how would the weight of a pound-mass vary?

Could one use ordinary balances and the same weights as we use now?

2. How far from the earth's centre must a gram-mass be if the earth's attraction for it is $\frac{1}{980}$ of a gram-force? What is a dyne?

3. Find the weight of a body of mass 100 kilograms at 6000, 8000, 10,000 miles from the earth's centre.

4. The diameter of the planet Mars is 4230 miles, and its density is $\frac{7}{10}$ that of the earth. Find the weight of a pound-mass on the surface of Mars.

5. At either pole the value of g is actually $\frac{1}{191}$ greater than its value at the equator; but if the earth were at rest, other things remaining unchanged, the former value would be only about $\frac{1}{568}$ greater than the latter. Explain each of these facts.

6. A spring-balance would have to be used to compare the weight of a body on the sun or the moon with that on the earth. Explain why.

7. If two platinum spheres, each containing 11,000 kg., were placed with their centres 1 metre apart, there would be 6.87 mm. between their surfaces. Show that the attraction between the spheres would be only 806$\frac{2}{3}$ dynes, or 0.82 gram-force.

(Taking the density of platinum to be 21.45 and the value of the ratio of the circumference to the diameter to be 3.1416, it can be shown that a platinum sphere whose diameter is 99.313 cm. weighs 11,000,136 grams or about 12$\frac{1}{8}$ tons.)

CHAPTER VI

WORK AND ENERGY

49. Definition of Work. When one draws water from a cistern by means of a bucket on the end of a rope; or when bricks are hoisted during the erection of a building; or when land is ploughed; or when a blacksmith files a piece of iron; or when a carpenter planes a board; it is recognized that *work* is done.

We recognize, too, that the amount of work done depends on two factors:—

(1) The magnitude of the force required to lift the bucket or the bricks, or to draw the plough, to push the file, or to drive the plane.

(2) The distance through which the water or bricks are lifted or the plough, file or plane is moved.

In every instance it will be observed that a force acts on a body and causes it to move. In the cases of the water and the bricks the forces exerted are sufficient to lift them, *i.e.*, to overcome the attraction of the earth upon them; in the other cases sufficient force is exerted to cause the plough or the file or the plane to move.

In physics the term WORK denotes *the quantity obtained when we multiply the force by the distance in the direction of the force through which it acts.*

In order to do work, force must be exerted on a body, and the body must move in the direction in which the force acts.

50. Units of Work. By choosing various units of force and of length we obtain different units of work.

If we take as unit of force a pound-force and as unit of length a foot, the unit of work will be a foot-pound.

If 2000 pounds mass is raised through 40 feet, the work done is $2000 \times 40 = 80{,}000$ foot-pounds.

In the same way, a kilogram-metre is the work done in raising a kilogram through a metre.

If we take a centimetre as unit of length and a dyne as unit of force, the unit of work is a dyne-centimetre. To this has been given a special name, *erg*.

Now 1 gram-force = g dynes; (§§ 35, 36)

Hence 1 gram-centimetre of work = g ergs.

To raise 20 grams through 30 cm. the work required is $20 \times 30 = 600$ gram-centimetres $= 600\ g$ ergs $= 600 \times 980$ or 588,000 ergs.

51. How to Calculate Work. A bag of flour, 98 pounds, has to be carried from the foot to the top of a cliff, which has a vertical face and is 100 feet high.

There are three paths from the base to the summit of the cliff. The first is by way of a vertical ladder fastened to the face of the cliff. The second is a zig-zag path, 300 feet long, and the third is also a zig-zag route, 700 feet long.

Here a person might strap to his back the mass to be carried, and climb vertically up the ladder, or take either of the other two routes. The distances passed through are 100 feet, 300 feet, 700 feet, respectively, but the result is the same in the end—the mass is raised through 100 feet.

The force required to lift the mass is 98 pounds-force, and it acts in the vertical direction. The distance *in this direction* through which the body is moved is 100 feet, and, therefore, the

$$\text{Work} = 98 \times 100 = 9800 \text{ foot-pounds.}$$

Along the zig-zag paths the effort required to carry the mass is not so great, but the length of path is greater, and the total work is the same in the end.

PROBLEMS

1. Find the work done in exerting a force of 1000 dynes through a space of 1 metre.

2. A block of stone rests on a horizontal pavement. A spring balance, inserted in a rope attached to it, shows that to drag the stone requires a force of 90 pounds. If it is dragged through 20 feet, what is the work done?

3. The weight of a pile-driver, of 2500 pounds mass, was raised through 20 feet. How much work was required?

4. A coil-spring, naturally 30 centimetres long, is compressed until it is 10 centimetres long, the average force exerted being 20,000 dynes. Find the work done. Find its value in kilogram-metres ($g=980$).

5. Two men are cutting logs with a cross-cut saw. To move the saw requires a force of 50 pounds, and 50 strokes are made per minute, the length of each being 2 feet. Find the amount of work done by each man in one hour

6. To push his cart a banana man must exert a force of 50 pounds How much work does he do in travelling 2 miles?

52. Definition of Energy. A log, known as a pile, the lower end of which is pointed, stands upright, and it is desired to push it into the earth. To do so requires a great force, and, therefore, the performance of great work.

The method of doing it is familiar to all. A heavy block of iron is raised to a considerable height and allowed to fall upon the top of the log, which is thus pushed downwards. Successive blows drive the pile further and further into the earth until is is down far enough.

Here work is done in thrusting the pile into its place, and this work is supplied by the pile-driver weight. It is evident, then, that a heavy body raised to a height is able to do work.

Ability to do work is called ENERGY.

The iron block in its elevated position has energy. As it descends, it gives up this high position and acquires velocity. Just before striking the pile it has a great velocity, and this velocity is used up in pushing the pile into the earth. It is clear, then, that a body in motion possesses energy.

We see, thus, that there are two kinds of energy:

(1) Energy of position, or *potential* energy.
(2) Energy of motion, or *kinetic* energy.

53. Transformations of Energy. Energy may appear in different forms, but if closely analysed, it will be found that it is always either energy of position, *i.e.*, potential energy, or energy of motion, *i.e.*, kinetic energy.

The various effects due to heat, light, sound, and electricity are manifestations of energy, and one of the greatest achievements of modern science was the demonstration of the Principle of the Conservation of Energy. According to this doctrine, *the sum total of the energy in the universe remains the same.* It may change from one form to another, but none of it is ever destroyed.

A pendulum illustrates well the transformation of energy. At the highest point of its swing the energy is entirely potential, and as it falls it gradually gives up this, until at its lowest position the energy is entirely kinetic.

54. The Measure of Kinetic Energy. Suppose a mass m grams to be lifted through a height h centimetres. (Fig. 38.)

The force required is m grams-force or mg dynes, and hence the work done is mgh ergs.

Suppose now the mass is allowed to fall. Upon reaching the level A it will have fallen through a space h, and it will have a velocity v such that

$$v^2 = 2gh. \qquad (\S 21)$$

The potential energy possessed by the body when at B is mgh ergs, and as this energy of position is changed into energy of motion, its kinetic energy on reaching A must also be mgh ergs.

But $$gh = \tfrac{1}{2}v^2$$
and so the kinetic energy $= \frac{1}{2}\, mv^2$ ergs.

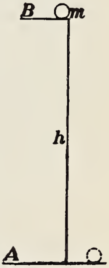


Fig. 38.—The potential energy at height B is equal to the kinetic energy on reaching the level A.

Hence a mass m grams moving with a velocity v cm. per second has kinetic energy $\frac{1}{2}mv^2$ ergs.

Now 1 gram-cm. of energy $= g$ ergs, $(g = 980)$. Hence

$$\tfrac{1}{2}mv^2 \text{ ergs} = \tfrac{1}{2}\frac{mv^2}{g} \text{ gram-cm. of energy.}$$

If the mass $= m$ lb. and its velocity $= v$ ft. per sec., its kinetic energy $= \frac{1}{2}mv^2/g$ ft.-pds., where $g = 32$.

55. Matter, Energy, Force. There are two fundamental propositions in science:—*Matter cannot be destroyed; energy cannot be destroyed.* The former lies at the basis of analytical chemistry; the latter at the basis of physics. It is to be observed, also, that matter is the vehicle or receptacle of energy.

Force, on the other hand, is of an entirely different nature. On pulling a string a tension is exerted in it, which disappears when we let it go. Energy is bought and sold; force cannot be.

56. Power. The *power,* or *activity,* of an agent is its rate of doing work.

A *horse-power* (*h.p.*) is that rate of doing work which would accomplish 33,000 foot-pounds of work per minute, or 550 foot-pounds per second.

In the centimetre-gram-second system the unit of power would naturally be 1 erg per second.

But this is an extremely small quantity, and instead of it we use 1 *watt,* which is defined thus:

$$1 \text{ watt} = 10{,}000{,}000 \text{ ergs per second.}$$

It is found that

$$746 \text{ watts} = 1 \text{ h.p.};$$

and if 1 *kilowatt* = 1000 watts, then

$$\frac{746}{1000} \text{ kw.} = 1 \text{ h.p., or } \tfrac{3}{4} \text{ kw.} = 1 \text{ h.p. (approx.)}$$

To do work requires time, and energy is measured in *kilowatt-hours* or *horse-power hours*. Thus a factory may pay 2 cents per k.w.h. or $20 per horse-power per year for the energy supplied it.

PROBLEMS

1. Why does the engineer of a heavy train try to reach the foot of a grade with a high speed?

2. If a banana man exerts a force of 50 pd. in pushing his cart and travels 2 mi. in 1 hr., at what rate does he work?

3. A man weighing 150 lbs. runs up 25 steps, each 7 in. high, in 5 sec. At what rate does he work?

4. A horse draws a carriage along a level road at constant speed. What kind of energy does it possess? If it draws the carriage up a hill, what kind of energy does it possess?

5. A steam engine is rated at 120 h.p. and drives a dynamo. If 90% of the engine's power is transformed into electric energy, find the power of the dynamo in kw.

6. A man weighing 150 lb. puts a 90-lb. bag of potatoes on his back and climbs a ladder to a height of 30 ft. in 20 sec. What is the total amount of work he does and the rate at which he does it?

7. What is the h.p. of an engine which raises 400 gal. of water per min. from a depth of 165 ft.?

8. A rifle bullet weighs 15 gm. and leaves the gun at a speed of 700 metres per sec. Find its energy.

9. A rifle bullet with a speed of 1000 ft. per sec. can penetrate 2 ft. of wood. Through how much wood should it go if its speed were 2000 ft. per sec.?

10. Find the h.p. of an engine which can raise 15 tons of coal (2000 lb. each) in 1 hr. from a pit 500 feet deep.

CHAPTER VII

Centre of Gravity

57. Definition of Centre of Gravity. Each particle of a body is acted on by the force of gravitation. The line of action of each little force is toward the centre of the earth, and hence, strictly speaking, they are not absolutely parallel. But the angles between them are so very small that we usually speak of the weights of the various particles as a set of parallel forces.

These forces have a single resultant, as can be seen in the following way:

Consider two forces F_1, F_2, acting at A, B, respectively (Fig. 39). These will have a resultant acting somewhere in the line AB which joins the points of application.

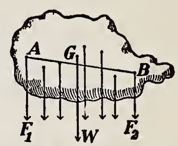


Fig. 39—The weight of a body acts at its centre of gravity.

Next, take this resultant and another force; they will have a single resultant. Continuing in this way, we at last come to the resultant of all, acting at some definite point.

The sum of all these forces is the weight of the body, and *the point G where the weight acts, is called the* CENTRE OF GRAVITY *of the body.* If the body be supported at this point, it will rest in equilibrium in any position in which it is placed.

58. To find the Centre of Gravity Experimentally. Suspend the body by a cord attached to any point A of it.

Then the weight acting downwards at G and the tension of the string acting upwards at A will rotate the body until the point G comes directly beneath A, and the line GW coincides with the direction of the supporting cord (Fig. 40). Thus if the body is suspended at A, and allowed to come to rest, the direction of the supporting cord will pass through the centre of gravity.

Next, let the body be supported at B. The direction of the supporting cord will again pass through the centre of gravity. That point is, therefore, where the two lines meet.

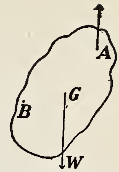


FIG. 40.—How to find the centre of gravity of a body of any form.

In the case of a flat body, such as a sheet of metal or a thin board, let it be supported at A (Fig. 41a) by a pin or in some other convenient way. Have a cord attached to A with a small weight on the end of it.

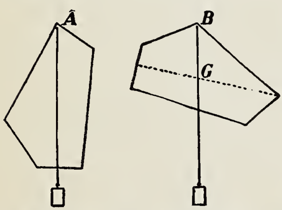


FIG. 41a FIG. 41b
How to find the centre of gravity of a flat body.

Chalk the cord and then snap it on the plate: it will make a white line across it.

Next, support the body from B (Fig. 41b) and obtain another chalk line. At G, the point of intersection of these two lines, is the centre of gravity.

59. Centre of Gravity of some Bodies of Simple Form. The centre of gravity of some bodies of simple form can often be deduced from geometrical considerations.

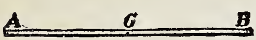


FIG. 42.—Centre of gravity of a uniform rod.

(1) For a straight uniform bar AB (Fig. 42), the centre of gravity is midway between the ends.

(2) For a parallelogram, it is at the intersection of the diagonals; and for a triangle, where the three median lines intersect. (Fig. 43.)

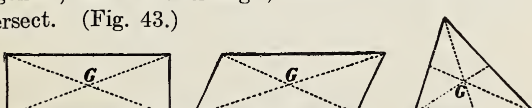


FIG. 43—Centre of gravity of a parallelogram and a triangle.

(3) For a cube or a sphere, it is at the centre of figure.

60. Condition for Equilibrium. For a body to rest in equilibrium on a plane, the line of action of the weight must fall within the

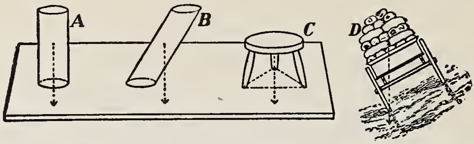


FIG. 44.—*A* and *C* are in stable equilibrium; *B* is not, it will topple over; *D* is in the critical position.

supporting base, which is the space within a cord drawn about the points of support. (See Fig. 44.)

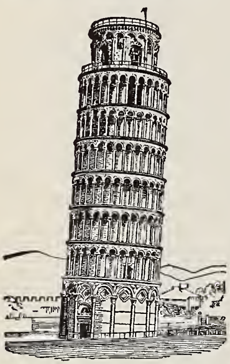


FIG. 45.—The Leaning Tower of Pisa. It overhangs its base more than 13 feet, but it is stable. (Drawn from a photograph.)

The famous Leaning Tower of Pisa (Fig. 45) is an interesting case of stability of equilibrium. It is circular in plan, 51 feet in diameter, and 172 feet high, and has eight stages, including the belfry. Its construction was begun in 1174. It was founded on wooden piles driven in boggy ground, and when it had been carried up 35 feet, it began to settle to one side. The tower overhangs the base upwards of 13 feet, but the centre of gravity is so low down that a vertical through it falls within the base, and hence the equilibrium is stable.

61. The Three States of Equilibrium. The centre of gravity of a body will always descend to as low a position as possible, or the potential energy of a body tends to become a minimum.

Consider a body in equilibrium, and suppose that by a slight motion this equilibrium is disturbed. Then if the body tends to return to its former position, its equilibrium is said to be *stable.* In this case the slight motion raises the centre of gravity, and on letting it go the body tends to return to its original position.

If, however, a slight disturbance lowers the centre of gravity, the body will not return to its original position, but will take up a new position in which the centre of gravity is lower than before. In this case the equilibrium is said to be *unstable.*

Sometimes a body rests equally well in any position in which it may be placed, in which case the equilibrium is said to be *neutral.*

An egg standing on end is in unstable equilibrium; if resting on its side the equilibrium is stable as regards motion in a circular section. A uniform sphere rests anywhere it is placed on a level surface; its equilibrium is neutral. (Fig. 46.)

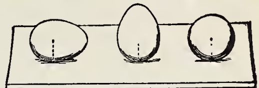


Fig. 46.—Stable, unstable and neutral equilibrium.

A round pencil lying on its side is in neutral equilibrium; balanced on its end, it is unstable. A cube, or a brick, lying on a face, is stable.

The amount of stability possessed by a body resting on a horizontal plane varies in different cases. It increases with the distance through which the centre of gravity has to be raised in order to make the body tip over. Thus, a brick lying on its largest face is more stable than when lying on its smallest.

QUESTIONS AND PROBLEMS

1. Illustrate the three states of equilibrium by a cone lying on a horizontal table.

2. Why is ballast used in a vessel? Where should it be put?

3. Why should a passenger in a canoe sit on the bottom? Why not stand up?

4. Why is a load of hay easier to upset than a load of grain?

5. Why is a pyramid a very stable structure?

6. A bowl when right-side-up is not easily balanced on the end of a finger, but if up-side-down it is. Why?

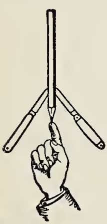


Fig. 47—Why is the pencil in equilibrium?

7. A pencil will not stand on its point, but if two pen-knives are fastened to it (Fig. 47) it rests on one's finger. Explain why.

8. A uniform rod 24 in. long and weighing 5 lb. has a 1-lb. weight on one end. Find the centre of gravity of the whole.

9. A uniform iron bar weighs 4 lb. per ft. of its length. A weight of 5 lb. is hung from one end, and the rod balances about a point 2 ft. from that end. Find the length of the bar.

10. By means of a uniform rod balanced on his shoulder a man carries a pail of water weighing 12 kg. on one end and a pail of milk weighing 8 kg. on the other. The rod is 1 metre long. If its weight be negligible, at what point should it rest on the shoulder? If it weighs 1 kg., where would it be supported?

CHAPTER VIII

Friction

62. Friction Stops Motion. A stone thrown along the ice will, if "left to itself," come to rest. A railway-train on a level track, or an ocean steamboat will, if the steam is shut off, in time come to rest. Here much energy of motion disappears, and no gain of energy of position takes its place. In the same way all the machinery of a factory when the "power" is turned off soon comes to rest.

In all these cases the energy simply seems to disappear and be *wasted*. As we shall see later, it is transformed into energy of another form, namely, heat; but it is done in such a way that we cannot utilize it.

The stopping of the motion in every instance given is due to *friction*. When one body slides or rolls over another, there is always friction, which acts as a force in opposition to the motion.

It may be observed, however, that if there were not friction between the rails and the wheels of the locomotive, the latter could not start to move.

63. Every Surface is Rough. The smoothest surface, when examined with a powerful microscope, is seen to have numerous little projections and cavities on it (Fig. 48). Hence, when two surfaces are pressed together, there is a kind of interlocking of these irregularities which resists the motion of one over the other.

Fig. 48.—Roughness of a surface as seen under a microscope.

64. Laws of Sliding Friction. Friction depends upon the nature of the substances and the roughness of the surfaces in contact. By means of the apparatus shown in Fig. 49, the laws of sliding friction can be investigated.

Let the entire weight on the pan be F; then the tension of the cord, which is the force tending to move the block M, is equal to F.

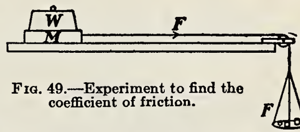


Fig. 49.—Experiment to find the coefficient of friction.

Now let F be increased until the block M moves uniformly over the surface. The friction developed just balances the force F. If F were greater than the friction, it would give an acceleration to M.

Suppose that the weight on the block is doubled. In order to give a uniform motion to M we shall have to add double the weight to the pan.

Thus the ratio F/W is constant; it is called the *coefficient of friction* between the block M and the surface.

The friction does not depend upon the area of the surfaces in contact; and it is greater at the instant of starting than during motion.

For dry pine, smooth surfaces, the coefficient is about 0.25 *i.e.*, a 40-pound block would require a 10-pound force to drag it over a horizontal pine surface.

For iron on iron, smooth but not oiled, the coefficient is about 0.2; if oiled, about 0.07. This shows the use of oil as a lubricant.

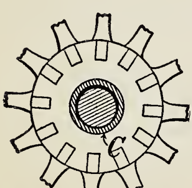


Fig. 50.—Section through a carriage hub, showing an ordinary bearing.

65. Rolling Friction. When a wheel or a sphere rolls on a plane surface, the resistance to the motion produced at the point of contact is said to be due to *rolling* friction. This, however, is very different from the friction just discussed, as there is no sliding. It is also very much smaller in magnitude.

In ordinary wheels, however, sliding friction is not avoided. In the case of the hub of a carriage (Fig. 50) there is sliding friction at the point C.

In ball-bearings (Fig. 51), which are much used in bicycles, automobiles and other high-class bearings, the sliding friction is almost completely replaced by rolling friction, and hence this kind of bearing has great advantages over the other.

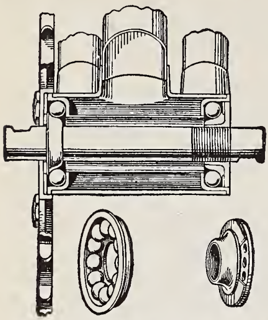


Fig. 51.—Section of the crank of a bicycle. The cup which holds the balls and the cone on which they run are shown separately below. Here the balls touch the cup in two points and the cone in one; it is a "three-point" bearing.

QUESTIONS AND PROBLEMS

1. Explain the utility of friction in: (*a*) Locomotive wheels on a railway track. (*b*) Leather belts for transmitting power. (*c*) Brakes to stop a moving car.

2. Why is sand put on the track in starting or stopping a street car or a railway train? Why are chains put on automobiles?

3. The current of a river is less rapid near its banks than in mid-stream. Can you explain this?

4. What horizontal force is required to drag a trunk weighing 150 pounds across a floor, if the coefficient of friction between trunk and floor is 0.3?

5. A brick, 2 x 4 x 8 inches in size, is slid over ice. Will the distance it moves depend on what face it rests upon?

6. A horse pulls a stone-boat weighing 50 lbs. with a barrel of water weighing 200 lbs. on it for a distance of half a mile. If the coefficient of friction is 0.6, find the work done.

7. When a force of 5 pounds is applied horizontally to a body weighing 75 lbs. on a horizontal surface, it just moves. Find the coefficient of friction.

8. A stone weighing 400 grams is thrown along the ice with a velocity of 5 metres per sec., and it comes to rest after sliding 60 metres.

(*a*) What is the initial kinetic energy, in gram-cm.?
(*b*) If F grams-force is the friction, what is the amount of work done in bringing it to rest?
(*c*) Putting (*a*) equal to (*b*), find F.
(*d*) From F and the weight of the stone find the coefficient of friction.

CHAPTER IX

MACHINES

66. Object of a Machine. A machine is a device by which energy is transferred from one place to another, or is transformed from one kind to another.

The six simple machines, usually known as the *mechanical powers*, are, the lever, the pulley, the wheel and axle, the inclined plane, the wedge, and the screw. All other machines, no matter how complicated, are but combinations of these.

Since energy cannot be created or destroyed, but simply changed from one form to another, it is evident that, neglecting friction, the amount of work put into a machine is equal to the amount which it will deliver.

67. The Lever: First Class. The lever is a rigid rod movable about a fixed axis called the fulcrum. Levers are of three classes.

First Class. In Fig. 52 AB is a rigid rod which can turn

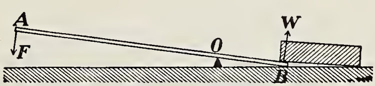


FIG. 52.—Lever of the first class.

about O, the fulcrum. By applying a force F at A a force W is exerted at B against a heavy body, which it is desired to raise.

AO, BO are called the *arms* of the lever.

Then by the principle of moments, the moment of the force F about O is equal to the moment of the force W about O, that is,

$$F \times AO = W \times BO, \text{ or } W/F = AO/BO,$$

or $$\frac{\text{Force obtained}}{\text{Force applied}} = \text{Inverse ratio of lengths of arms.}$$

This is called the *Law of the Lever*, and the ratio W/F is called the mechanical advantage.

For example, let $AO = 36$ inches, $BO = 4$ inches, then $W/F = 9$.

There are many examples of levers of the first class. Among them are, the common balance, a pump-handle, a pair of scissors (Fig. 53), a claw-hammer (Fig. 53a).

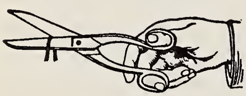


FIG. 53.—Shears, lever of the first class.

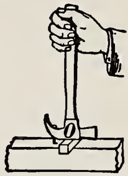


FIG. 53a.—Claw-hammer, used as a lever of the first class.

The law of the lever can be obtained by applying the principle of energy.

Suppose the end A (Fig. 54) to move through a distance a and the end

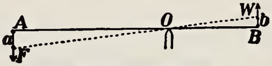


FIG. 54.—Theory of lever.

B through a distance b. It is evident that $a/b = AO/BO$

Now the work done by the force F, acting through a distance a is $F \times a$, while the work done by W acting through a distance b is $W \times b$.

Neglecting all considerations of friction or of the weight of the lever, the work done by the applied force F must be equal to the work accomplished by the force W. Hence $Fa = Wb$, and the mechanical advantage $W/F = a/b = AO/BO$, which is the law of the lever.

68. The Lever: Second Class. In levers of the second class the weight to be lifted is placed between the point where the force is applied and the fulcrum.

As before, the force F is applied at A (Fig. 55), but the force produced is exerted at B, between A and the fulcrum O.

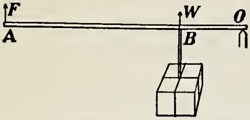


Fig. 55.—Lever of the second class.

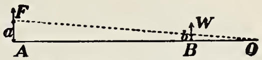


Fig. 56.—Theory of the lever of the second class.

Here we have, by the principle of moments,

$$F \times AO = W \times BO, \text{ and } W/F = AO/BO,$$

and hence the mechanical advantage in levers of this class is always greater than 1.

Or, by applying the principle of energy (Fig. 56), work done by F is Fa, by W is Wb.

Hence $Fa = Wb$, or $W/F = a/b = AO/BO$, the law of the lever.

Examples of levers of the second class:— nut-crackers (Fig. 57), trimming board (Fig. 58), safety-valve (Fig. 59), wheel-barrow, oar of a row-boat.

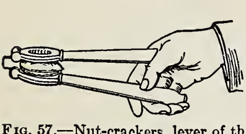


Fig. 57.—Nut-crackers, lever of the second class.

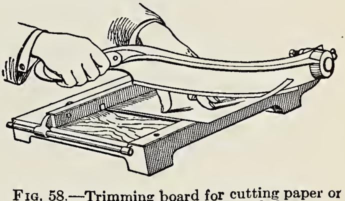


Fig. 58.—Trimming board for cutting paper or cardboard; lever of the second class.

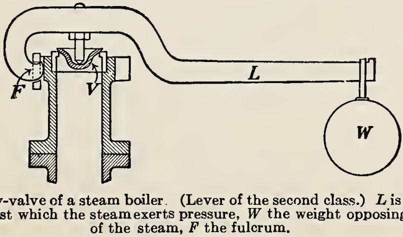


Fig. 59.—A safety-valve of a steam boiler. (Lever of the second class.) L is the lever arm, V the valve against which the steam exerts pressure, W the weight opposing the pressure of the steam, F the fulcrum.

69. The Lever: Third Class. In this case the force F is applied between the fulcrum and the weight to be lifted. (Fig. 60.)

Fig. 60.—A lever of the third class.

As before, we have

$$F \times AO = W \times BO,$$

or $W/F = AO/BO$, the law of the lever.

Notice that the weight lifted is always less than the force applied, or the mechanical advantage is less than 1.

Examples of levers of this class:— sugar-tongs (Fig. 61), the human forearm (Fig. 62); treadle of a lathe or a sewing-machine.

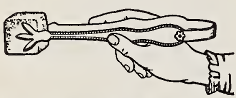


Fig. 61.—Sugar-tongs, lever of the third class.

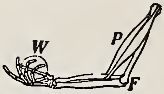


Fig. 62.—Human forearm, lever of the third class. One end of the biceps muscle is attached at the shoulder, the other is attached to the radial bone near the elbow, and exerts a force to raise the weight in the hand.

PROBLEMS

1. Explain the action of the steelyards (Fig. 63). To which class of levers does it belong? If the distance from B to O is $1\frac{1}{2}$ inches, and the sliding weight P when at a distance 6 inches from O balances a mass of 5 lb. on the hook, what must be the weight of P?

If the mass on the hook is too great to be balanced by P, what additional attachment would be required in order to weigh it?

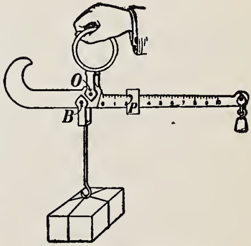


Fig. 63.—The steelyards.

2. A cubical block of granite, whose edge is 3 feet in length and which weighs 4500 lbs., is raised by thrusting one end of a crowbar 40 inches long under it to the distance of 4 inches, and then lifting on the other end. What force must be exerted?

3. A hand-barrow (Fig. 64), with the mass loaded on it, weighs 210 pounds. The centre of gravity of the barrow and load is 4 feet from the front handles and 3 feet from the back ones. Find the amount each man carries.

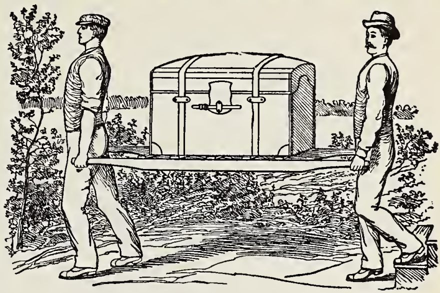


Fig. 64.—The hand-barrow.

4. To draw a nail from a piece of wood requires a pull of 200 pounds. A claw-hammer is used, the nail being $1\frac{1}{2}$ inches from the fulcrum O (Fig. 53a) and the hand being 8 inches from O. Find what force the hand must exert to draw the nail.

70. The Pulley. The pulley is used sometimes to change the direction in which a force acts, sometimes to gain mechanical advantage, and sometimes for both purposes. We shall neglect the weight and the friction of the pulley and the rope.

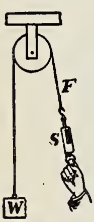


Fig. 65.—A fixed pulley simply changes the direction of force.

A single fixed pulley, such as is shown in Fig. 65, can change the direction of a force, but cannot give a mechanical advantage greater than 1. F, the force applied, is equal to the weight lifted, W.

By this arrangement a lift is changed into a pull in any convenient direction. It is often used in raising materials during the construction of a building.

By inserting a spring balance, S, in the rope, between the hand and the pulley, one can show that the force F is equal to the weight W.

Suppose the hand to move through a distance a, then the weight rises through the same distance.

$$\text{Hence } F \times a = W \times a$$
$$\text{or } F = W,$$

as tested by the spring balance.

71. A Single Movable Pulley. Here the weight W (Fig. 66) is supported by the two portions, B and C, of the rope, and hence each portion supports half of it.

Thus the force F is equal to $\frac{1}{2}$ W, and the mechanical advantage is 2.

This result can also be obtained from the principle of energy.

Let a be the distance through which W rises. Then each portion, B and C, of the rope will be shortened a distance a, and so F will move through a distance $2a$.

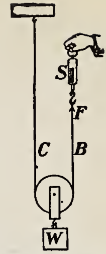


Fig. 66.—With a movable pulley the force exerted is only half as great as the weight lifted.

Then, since $F \times 2a = W \times a$

$W/F = 2$, the mechanical advantage.

For convenience a fixed pulley also is generally used as in Fig. 67.

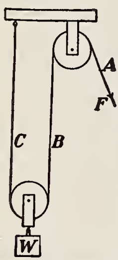


Fig. 67.—With a fixed and a movable pulley the force is changed in direction and reduced one-half.

Here when the weight rises 1 inch, B and C each shorten 1 inch and hence A lengthens 2 inches. That is, F moves through twice as far as W, and $W/F = 2$, as before.

72. Other Systems of Pulleys. Various combinations of pulleys may be used. Two are shown in Figs. 68, 69, the latter one being very commonly seen.

Here there are six portions of the rope supporting W, and hence the tension in each portion is $\frac{1}{6}$ W.

Hence $F = \frac{1}{6}$ W,

or a force equal to $\frac{1}{6}$ W will hold up W. This entirely neglects friction, which in such

Fig. 68.—Combination of 6 pulleys; 6 times the force lifted.


Fig. 69.—A familiar combination for multiplying the force 6 times.

a system is often considerable, and it therefore follows that to prevent W from descending, less than $\frac{1}{6}W$ will be required. On the other hand, to actually lift W the force F must be greater than $\frac{1}{6}W$. In every case friction acts to prevent motion.

Let us apply the principle of energy to this case. If W rises 1 foot, each portion of the rope supporting it must shorten 1 foot, and the force F will move 6 feet.

Then, work done on $W = W \times 1$ foot-pounds
" " by $F = F \times 6$ "

These are equal, and hence $W = 6F$ or $W/F = 6$, the mechanical advantage.

PROBLEMS

1. A clock may be driven in two ways. First, the weight may be attached to the end of the cord; or secondly, it may be attached to a pulley, movable as in Fig. 66, one end of the cord being fastened to the framework, and the other being wound about the barrel of the driving wheel. Compare the weights required, and also the length of time the clock will run in the two cases.

2. Find the mechanical advantage of the system shown in Fig. 70. This arrangement is called the Spanish Burton.

3. What fraction of his weight must the man shown in Fig. 71 exert in order to raise himself?

FIG. 70.—The Spanish Burton.

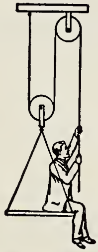


FIG. 71.—An easy method to raise one's self.


FIG. 72.—Find the pressure of the feet on the floor?

4. A man weighing 140 pounds pulls up a weight of 80 pounds by means of a fixed pulley, under which he stands (Fig. 72). Find his pressure on the floor.

73. The Wheel and Axle. This machine is shown in Figs. 73, 74. It is evident that in one complete rotation the weight F will descend a distance equal to the circumference of the wheel, while the weight W will rise a distance equal to the circumference of the axle.

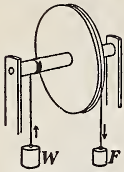


Fig. 73.—The wheel and axle.

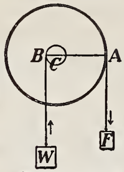


Fig. 74.—Diagram to explain the wheel and axle

Hence $F \times$ circumference of wheel $=$ $W \times$ circumference of axle. Let the radii be R and r respectively; the circumferences will be $2\pi R$ and $2\pi r$, and therefore

$$F \times 2\pi R = W \times 2\pi r, \text{ or } FR = Wr,$$

and $W/F = R/r$, the mechanical advantage.

This result can also be seen from Fig. 74. The wheel and axle turn about the centre C. Now W acts at B, a distance r from C, and F acts at A, a distance R from C.

Then, from the principle of the lever

$$F \times R = W \times r, \text{ as before.}$$

74. Examples of Wheel and Axle. The windlass (Fig. 75) is a common example, but, in place of a wheel, handles are used. Forces are applied at the handles, and the bucket is lifted by the rope, which is wound about the axle.

Fig. 75.—Windlass used in drawing water from a well.

If F = applied force, and W = weight lifted,

$$\frac{W}{F} = \frac{\text{length of crank}}{\text{radius of axle}}$$

The capstan, used on board ships for raising the anchor, is another example (Fig. 76).

The sailors apply the force by pushing against bars thrust into holes near the top of the capstan. Usually the rope is too long to be all coiled up on the barrel, so it is passed about it several times, and the end A is held by a man who keeps that portion taut. The friction is sufficient to prevent the rope from slipping. Sometimes the end B is fastened to a post or a ring on the dock, and, by turning the capstan, this portion is shortened and the ship is drawn into the dock.

FIG. 76.—Raising the ship's anchor by a capstan.

75. Differential Wheel and Axle. This machine is shown in Fig. 77. It will be seen that the rope winds off one axle and on the other. Hence in one rotation of the crank the rope is lengthened (or shortened) by an amount equal to the difference in the circumferences of the two axles; but since the rope passes round a movable pulley, the weight to be lifted, attached to this pulley, will rise only one-half the difference in the circumferences.

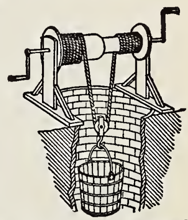


FIG. 77. — Differential wheel and axle.

It will be seen that by making the two drums which form the axles nearly equal in size, we can make the difference in their circumferences as small as we please, and the mechanical advantage will be as great as we desire.

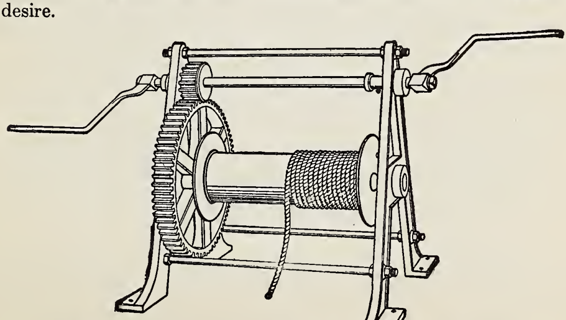


FIG. 78.—Windlass, with gearing, such as is used with a pile-driver.

PROBLEMS

1. A man weighing 160 pounds is drawn up out of a well by means of a windlass, the drum of which is 8 inches in diameter and the crank 24 inches long. Find the force required to be applied to the handle.

2. Calculate the mechanical advantage of the windlass shown in Fig. 78. The length of the crank is 16 inches, the small wheel has 12 teeth and the large one 120, and the diameter of the drum about which the rope is wound is 6 inches.

If a force of 60 pounds be applied to each crank, how great a weight can be raised? (Neglect friction.)

76. The Inclined Plane. Let a heavy mass, such as a barrel or a box, be rolled or dragged up an inclined plane AC (Fig. 79) whose length is l and height h, by means of a force F, parallel to the plane. The work done is $F \times l$.

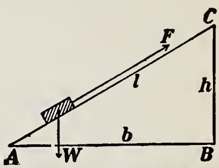


FIG. 79—Theory of the inclined plane.

Again the weight is raised through a height h and so, neglecting friction, the work done $= W \times h$.

Hence $Fl = Wh$,

and $W/F = l/h$,

that is, the mechanical advantage is the ratio of the length to the height of the plane.

The inclined plane, in the form of a plank or a skid, is used in loading goods on a wagon or a railway car.

Taking friction into account, the mechanical advantage is not so great, and, to reduce the friction as much as possible, the body may be rolled up the plane.

77. The Screw. The screw consists of a grooved cylinder which turns within a hollow cylinder or nut which it just fits. The distance from one thread to the next is called the *pitch*.

The law of the screw is easily obtained. Let l be the length of the handle by which the screw is turned (Fig. 80) and F the force exerted on it. In one rotation of the screw the end of the handle describes the circumference of a circle with radius l, that is, it moves through a distance $2\pi l$, and the work done is therefore $= F \times 2\pi l$.

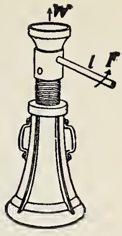


Fig. 80—The jackscrew.

Let W be the force exerted upwards as the screw rises, and d be the pitch, that is, the distance from one thread of the screw to the next. In one rotation the work done $= W \times d$.

$$\text{Hence } W \times d = F \times 2\pi l,$$
$$\text{or } W/F = 2\pi l/d,$$

or the mechanical advantage is equal to the ratio of the circumference of the circle traced out by the end of the handle to the pitch of the screw.

In actual practice, however, the advantage is much less than this on account of friction.

The screw is really an application of the inclined plane. If a triangular piece of paper, as in Fig. 81, is wrapped about a cylinder (a lead pencil, for instance), the hypotenuse of the triangle will trace out a spiral like the thread of a screw.

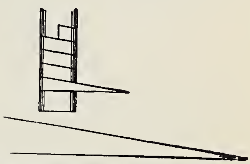


Fig. 81.—Diagram to show that the screw is an application of the inclined plane.

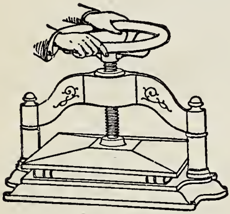


Fig. 82.—The letter press.

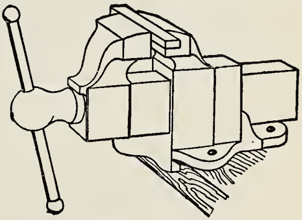


Fig. 83.—The mechanic's vice.

Examples of the screw are seen in the letter press (Fig 82), and the vice (Fig 83).

ILLUSTRATIVE PROBLEMS

1. Why should shears for cutting metal have short blades and long handles?

2. In the driving mechanism of a self-binder, shown in Fig. 84, the driving-wheel A has a diameter of 3 feet, the sprocket-wheels B and C have 40 teeth and 10 teeth, respectively. The large gear-wheel D has 37 teeth, the small one E has 12 teeth, and the crank G is 3 in. long. Neglecting friction, what pull on the driving-wheel will be required to exert a force of 10 pounds on the crank G?

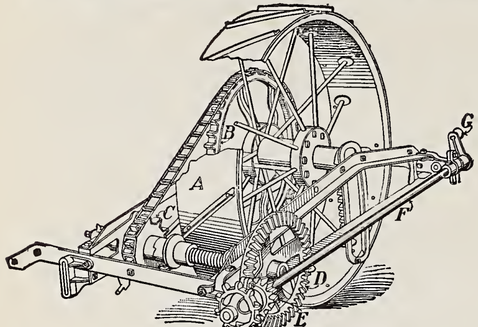


Fig. 84.—The driving part of a self-binder. The driving-wheel A is drawn forward by the horses. On its axis is the sprocket-wheel B, and this, by means of the chain drives the sprocket-wheel C. The latter drives the cog-wheel D which, again, drives the cog-wheel E, and this causes the shaft F with the crank G on its end to rotate.

3. Explain the action of the levers in the scale shown in Fig 85.

If $HF^1 = 12$ ft., $F^1D^1 = 4$ in., $MN = 36$ in., $KM = 3$ in., what weight on N would balance 2000 pounds of a load (wagon and contents?) In the scale $E^1F^1 = E^2F^2$, and $F^1D^1 = F^2D^2$, so the load is simply divided equally between the two levers.

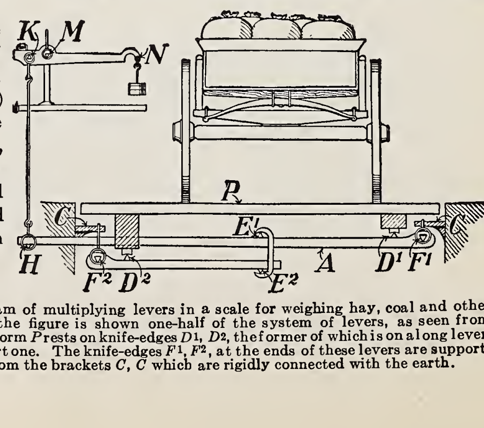


Fig. 85.—Diagram of multiplying levers in a scale for weighing hay, coal and other heavy loads. In the figure is shown one-half of the system of levers, as seen from one end. The platform P rests on knife-edges D^1, D^2, the former of which is on a long lever, the latter on a short one. The knife-edges F^1, F^2, at the ends of these levers are supported by suspension from the brackets C, C which are rigidly connected with the earth.

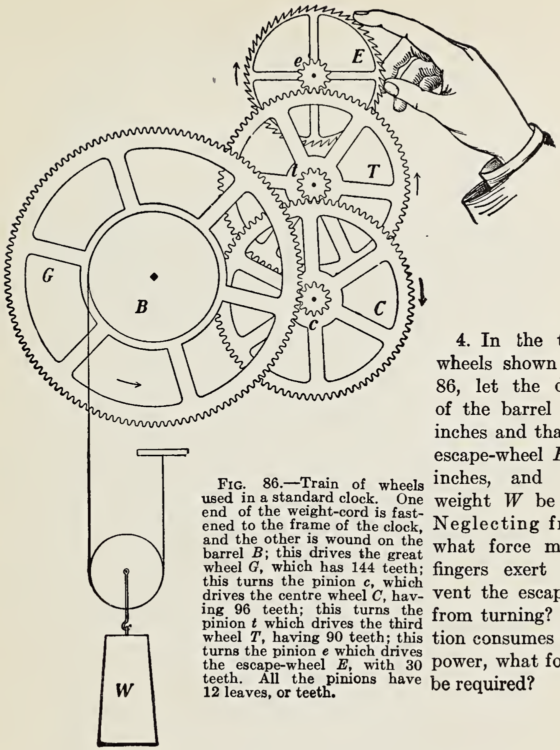


FIG. 86.—Train of wheels used in a standard clock. One end of the weight-cord is fastened to the frame of the clock, and the other is wound on the barrel B; this drives the great wheel G, which has 144 teeth; this turns the pinion c, which drives the centre wheel C, having 96 teeth; this turns the pinion t which drives the third wheel T, having 90 teeth; this turns the pinion e which drives the escape-wheel E, with 30 teeth. All the pinions have 12 leaves, or teeth.

4. In the train of wheels shown in Fig. 86, let the diameter of the barrel B be 2 inches and that of the escape-wheel E be $1\frac{3}{4}$ inches, and let the weight W be 10 lbs. Neglecting friction, what force must the fingers exert to prevent the escape-wheel from turning? If friction consumes half the power, what force will be required?

PART III—MECHANICS OF FLUIDS

CHAPTER X

Pressure of Liquids

78. Transmission of Pressure by Fluids. One of the most characteristic properties of matter is its power to transmit force. The harness connects the horse with its load; the piston and connecting rods convey the pressure of the steam to the driving wheels of the locomotive. Solids transmit pressure only in the line of action of the force. Fluids act differently. If a globe and cylinder of the form shown in Fig. 87 is filled with water and a force exerted on the water by means of a piston, it will be seen that the pressure *is transmitted*, not simply in the direction in which the force is applied, but *in all directions;* because jets of water are thrown with velocities which are apparently equal from all the apertures. If the conditions are modified by connecting with the globe U-shaped tubes partially filled with mercury, as shown in Fig. 88, it will be found that when the piston is inserted, the change in level of the mercury, caused by the transmitted pressure, is the same in each tube. This would show that the pressure applied to the piston is transmitted *equally* in all directions by the water.

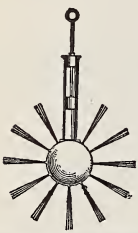


Fig. 87.—Pressure applied to the piston transmitted in all directions by the liquid within the globe.

Fig. 88.—Transmission shown to be equal in all directions by pressure gauges.

Next, let us use the apparatus shown in Fig. 89. The cylinder *C*, about 5 in. in diameter, is provided with a tightly-fitting piston *L*. On this a heavy weight (50 pounds) is placed. One end of a piece of heavy rubber tubing is attached to *C* while the other end, by means of an ordinary bicycle tire valve, is joined to the bicycle pump *P*. On working the pump the weight is raised with very little effort. Careful experiments with similar apparatus show that, neglecting friction, if the area of *L* is 50 times that of the piston of the pump, only one pound force need be applied to the pump to raise the 50-pound weight.

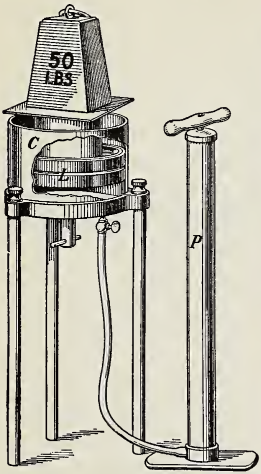


Fig. 89.—Transmission of pressure by a gas (air).

These facts may be expressed concisely as follows:—*Pressure exerted anywhere on the mass of fluid filling a closed vessel is transmitted undiminished in all directions, and acts with the same force on all equal surfaces in a direction at right angles to them.* The principal was first enunciated by Pascal, and is generally known as Pascal's Law or Principle.*

79. Practical Applications of Pascal's Principle. Pascal himself pointed out how it was possible, by the application of this principle, to multiply force for practical purposes. By experimenting with pistons inserted into a closed vessel filled with water, he showed that the pressures exerted on the pistons when made to balance were in the ratio of their

*It appears in Pascal's *Traité de l'équilibre des liqueurs*, written in 1653, but first published in 1663, one year after the author's death.

areas. Thus if the area of piston A (Fig. 90) is one square centimetre, and that of B ten times as great, one unit of force applied to A will transmit ten units to B. It is evident that this principle has almost unlimited application. Pascal remarks "Hence it follows that a vessel full of water is a new principle of Mechanics and a new machine for multiplying forces any degree we choose." Since Pascal's time the "new machine" has taken a great variety of forms, and has been used for a great variety of purposes.

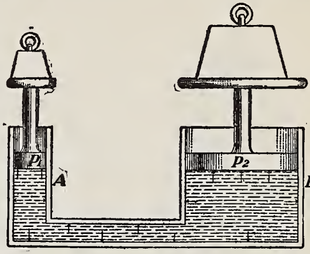


FIG. 90.—Force multiplied by transmission of pressure.

80. Hydraulic Press. One of the most common forms is that known as Bramah's hydraulic press, which is ordinarily used whenever great force is to be exerted through short distances, as in pressing goods into bales, extracting oils from seeds, making dies, testing the strength of materials, etc. Its construction is shown in Fig. 91. A and B are two cylinders connected with each other and with a water cistern by pipes closed by valves V_1 and V_2. In these cylinders work pistons P_1 and P_2 through water-tight collars, P_1 being moved by a lever. The bodies to be pressed are held between plates C and D. When P_1 is raised by the lever, water flows up from the cistern through the valve V_1 and fills the cylinder A.

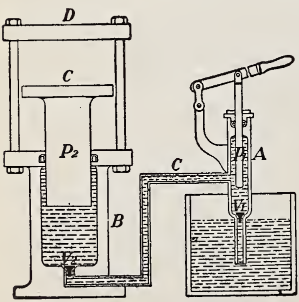


FIG. 91.—Bramah's hydraulic press.

On the down-stroke the valve V_1 is closed, and the water is forced through the valve V_2 into the cylinder B, thus exerting a force on the piston P_2, which will be as many times that applied to P_1 as the area of the cross-section of P_2 is that of the cross-section of P_1. It is evident that by decreasing the size of P_1, and increasing that of P_2, an immense force may be developed by the machine. While this is true, it is to be noted that the upward movement of P_2 will be very slow, because the action of the machine must conform to the law enunciated in §66, that is,

> the force acting on $P_1 \times$ the distance through which it moves = the force acting on $P_2 \times$ the distance through which it moves.

81. The Hydraulic Elevator. Another important application of the multiplication of force through the principle of equal transmission of pressure by fluids is the hydraulic elevator, used as a means of conveyance from floor to floor in buildings. In its simplest form it consists of a cage A, supported on a piston P, which works in a long cylindrical tube C. (Fig. 92.) The tube is connected with the water mains and the sewers by a three-way valve D which is actuated by a cord E passing through the cage. When the cord is pulled up by the operator, the valve takes the position shown at D, and the cage is forced up by the pressure on P of the water which rushes into C from the mains. When the cord is pulled down, the valve takes the position shown at F (below), and the cage descends by its own weight forcing the water out of C into the sewers.

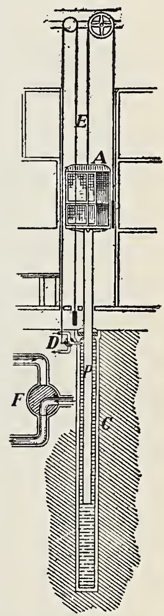


Fig. 92. — Hydraulic elevator.

When a higher lift, or increased speed is required, the cage is connected with the piston by a system of pulleys which multiplies, in the movement of the cage, the distance travelled by the piston.

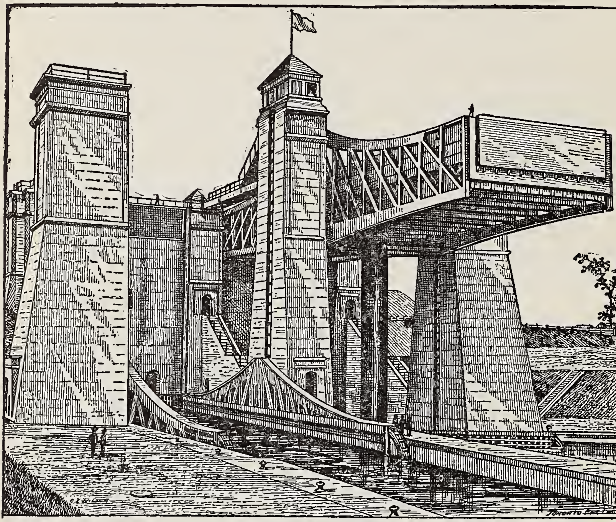


Fig. 93.—Hydraulic lift-lock at Peterborough, Ont., capable of lifting a 140-foot steamer 65 feet.

82. Canal Lift-lock. The hydraulic lift-lock, designed to take the place of ordinary locks where a great difference of level is found in short distances, is another application of the principle of equal transmission. Fig. 93 gives a general view of the Peterborough Lift-lock, the largest of its kind in the world, and Fig. 94 is a simple diagrammatic section showing its principle of operation. The lift-lock consists of two immense hydraulic elevators, supporting on their pistons P_1 and P_2 tanks A and B in which float the vessels to be raised or lowered. The presses are connected by a pipe containing a valve R which can be operated by the lockmaster in his cabin at the top of the central tower. To perform the lockage, the vessel is towed into one tank and the gates at the end leading from the canal are closed. The upper tank is then made to descend by being loaded with a few inches more of water than the lower. If now the valve is opened, the additional weight in the upper tank forces the water from its press into the other, and it gradually descends while the other tank is raised. The action, it will be observed, is automatic, but machinery is provided for forcing water into the presses to make up pressure lost through leakage.

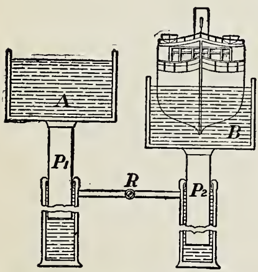


FIG. 94.—Principle of the lift-lock.

83. Pressure due to Weight. Our common experiences in the handling of liquids give us evidence of force within their mass. When, for example, we pierce a hole in a water-pipe or in the side or the bottom of a vessel filled with water, we find that the water rushes out with an intensity which we know, in a general way, to depend on the height of the water above the opening. Again, if we hold a cork at the bottom of a vessel containing water, and let it go, it is forced up to the surface of the water, where it remains, its weight being supported by the pressure of the liquid on its under surface.

84. Relation between Pressure and Depth. Since the lower layers of the liquid support the upper layers, it is to be expected that this force within the mass, due to the action of gravity, will increase with the depth. To investigate this perform the following experiment:–

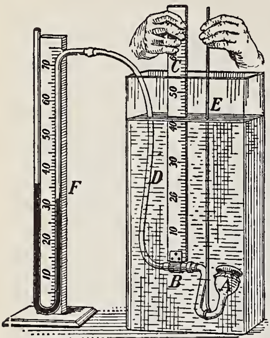


Fig. 95.—Apparatus to show that pressure is proportional to depth and is the same in all directions.

Prepare a pressure gauge of the form shown in Fig. 95 by stretching a rubber membrane over a thistle-tube A, which is connected by means of a rubber tube with a **U**-shaped glass tube F, partially filled with coloured water. The action of the gauge is shown by pressing on the membrane. The pressure is transmitted to the surface of the water in F by the air in the tube and is measured by the difference in level of the water in the branches of the **U**-tube.

Now place A in a jar of water (which should be at the temperature of the room), and gradually push it downward (Fig. 95). The changes in the level of the water in the branches of the **U**-tube indicate an increase in pressure with the increase in depth.

Careful experiments show that *this pressure increases from the surface downward in direct proportion to the depth.*

Now, by means of the wire E, turn the thistle-tube A in different directions, the centre of the membrane being kept all the time at the same depth, and observe the levels in the **U**-tube. They remain steady. Evidently the upward, downward and lateral pressures are equal at the same depth.

We find therefore that *the pressure is equal in all directions at the same depth.*

85. Magnitude of Pressure due to Weight. The downward pressure of a liquid, say water, on the bottom of a vessel with vertical sides is obviously the weight of the liquid. But, if the sides of the vessel are not vertical, the magnitude of the force is not so apparent. Let us investigate this question experimentally, using the apparatus shown in Fig. 96.

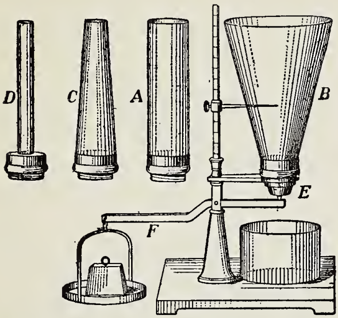


FIG. 96.—Pressure on the bottoms of vessels of different shapes and capacities.

A, *B*, *C*, and *D* are tubes of different shapes but made to fit into a common base. *E* is a movable bottom held in position by a lever and weight. Attach the cylindrical tube to the base, and support the bottom *E* in position. Now place any suitable weight in the scale-pan and pour water into the tube until the pressure detaches the bottom. If the experiment be repeated, using in succession the tubes *A*, *B*, *C*, and *D*, and marking with the pointer the height of the water when the bottom is detached, it will be found that the height is the same for all tubes, so long as the weight in the scale-pan remains unchanged.

The pressure on the bottom of a vessel filled with a given liquid is, therefore, dependent only on the depth. It is independent of the form of the vessel and of the amount of liquid which it contains.

86. Surface of a Liquid in Connecting Tubes. If a liquid is poured into a series of connecting tubes (Fig. 97), it will rise to the same horizontal plane in all the tubes. The reason is apparent. Consider, for example, the tubes *A* and *B*. Let *a* and *b* be two points in the same horizontal plane. The liquid is at rest only on the condition that the pressure

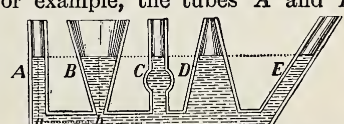


FIG. 97.—Surface of a liquid in connecting tubes in the same horizontal plane.

at *a* in the direction *ab* is equal to the pressure at *b* in the direction *ba;* but since the pressure at either of these points varies as its depth only, and is independent of the shape of the vessel, or of the quantity of the liquid in the tubes, the height of the liquid in *A* above *a* must be the same as the height in *B* above *b*.

This principle, that "water seeks its own level," is in a variety of ways, of practical importance. Possibly the common method of supplying cities with water furnishes the most striking example. Fig. 98 shows the main features of a modern system. While there are various means by

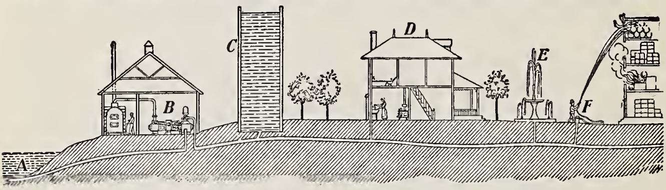


Fig. 98.—Water supply system. *A*, source of water supply; *B*, pumping station; *C*, standpipe; *D*, house supplied with water; *E*, fountain; *F*, hydrant for fire hose.

which the water is collected and forced into a reservoir or standpipe, the distribution in all cases depends on the principle that, no matter how many branches of the service pipes there may be, or whether they are high up or low down in the streets or buildings, the water in them tends to rise to the level of the water in the original source of supply connected with the pipes.

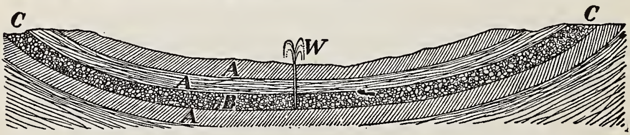


Fig. 99.—Artesian basin. *A*, impermeable strata. *B*, permeable stratum, *C*, *C*, point where permeable stratum reaches the surface. *W*, artesian well.

87. Artesian Wells. The rise of water in artesian wells is also due to the tendency of a liquid to find its own level. These wells are bored at the bottom of cup-shaped basins (Fig. 99), which are frequently many miles in width. The upper strata are impermeable, but lower down is found a stratum of loose sand, gravel, or broken stone containing water which has run into it at the points where the permeable stratum reaches the surface. When the upper strata are pierced, the water tends to rise with a force whose magnitude depends on the height of the water exerting the pressure.

PROBLEMS

1. A closed vessel is filled with liquid, and two circular pistons, whose diameters are respectively 2 cm. and 5 cm. inserted. If the pressure on the smaller piston is 50 grams, find the pressure on the larger piston when they balance each other.

2. The diameter of the large piston of a hydraulic press is 100 cm. and that of the smaller piston 5 cm. What force will be exerted by the press when a force of 2 kilograms is applied to the small piston?

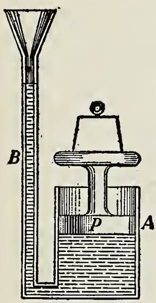


Fig. 100.

3. The diameter of the piston of a hydraulic elevator is 14 inches. Neglecting friction, what load, including the weight of the cage, can be lifted when the pressure of the water in the mains is 75 pounds per sq. inch?

4. What is the pressure in grams per sq. cm. at a depth of 100 metres in water?

Solution.—Consider 1 sq. cm. of horizontal surface at a depth of 100 metres, or 10,000 cm. The force upon it is the weight of the 10,000 c.c. of water above it, and the pressure is 10,000 grams per sq. cm.; also the pressure is the same in all directions (§ 84).

5. The area of the cross-section of the piston P (Fig. 100) is 120 sq. cm. What weight must be placed on it to maintain equilibrium when the water in the pipe B stands at a height of 3 metres above the height of the water in A?

6. The water pressure at a faucet in a house supplied with water by pipes connected with a distant reservoir is 80 pounds per sq. inch when the water in the system is at rest. What is the vertical height of the surface of the water in the reservoir above the faucet? (1 lb. water = 27.73 c. in.; see Table opposite page 1.)

7. The area of the top of a cork in the neck of a bottle is $\frac{1}{2}$ sq. in., and to push it into the bottle a force of 20 pounds is required. At what depth in a lake will the pressure exerted by the water push the cork in?

8. The Pacific Ocean near the island of Guam is 5268 fathoms deep. Find the pressure on the ocean floor there. (1 fathom = 6 feet, and 1 c. ft. of sea-water = 64 lbs.)

9. Taking the surface area of a boy's body to be 12 sq. ft., find the total force on his body if he has dived to a depth of 8 ft. below the surface of a swimming pool. (Take 1 c. ft. water = $62\frac{1}{2}$ lb.)

10. When the water pressure in the town's pipes is 75 pounds per sq. in., how high above the pipes is the water in the standpipe? (Fig. 98.)

CHAPTER XI

Buoyancy of Fluids

88. Nature of Buoyancy. When a body is immersed in a liquid, every point of its surface is subjected to a pressure which is perpendicular to the surface at that point, and which varies as the depth of that point below the surface of the liquid. When these pressures are resolved into horizontal and vertical components, the horizontal components balance each other; and since the pressure on the lower part of the body is greater than that on the upper part, the resultant of all the forces acting upon the body must be vertical and act upward. This force is termed the *resultant vertical pressure* or *buoyancy of the fluid.*

Consider, for example, the resultant pressure on a solid in the form of a cube, whose edge is 3 cm., immersed in water with its upper face horizontal at a depth of, say, 2 cm. below the surface. (Fig. 101)

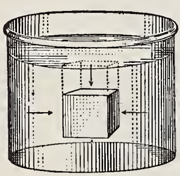


Fig. 101.—Buoyant force of a liquid on a solid.

Evidently the pressures on the vertical sides balance, and the vertical force due to the water will be equal to the difference between the pressures on the bottom and the top.

Now the pressure on the top is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 2 cm. The volume is 18 c.c. and the weight is 18 grams.

The pressure on the bottom (upwards) is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 5 cm. The volume is 45 c.c. and the weight is 45 grams.

Resultant pressure = 45 − 18 = 27 grams (upwards.)

But the volume of the cube is 27 c.c. and the weight of the water displaced = 27 grams.

89. To Determine Experimentally the Buoyant Force which a Liquid Exerts on an Immersed Body. Take a metal cylinder *A*, (Fig. 102) closed at both ends, which fits exactly into a hollow socket *B*. Hook the cylinder to the bottom of the socket, suspend them from one end of the beam of a balance, and add weights to the other end to bring the balance to equilibrium. Next, surround *A* with water, as shown in the figure. The buoyancy of the water on *A* destroys the equilibrium. Now carefully pour water in the socket *B*. It will be found that when *B* is just filled, equilibrium will be restored.

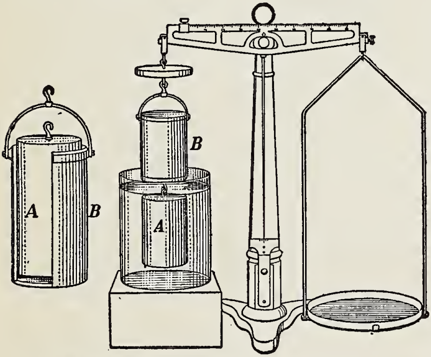


Fig. 102.—Determination of buoyant force.

Hence the buoyant force of the water upon the cylinder is equal to the weight of the water displaced.

The apparatus shown in Fig. 102 is designed especially to demonstrate the law of buoyancy, but we can easily get along without it. We need a balance, but the cylinder-and-socket arrangement can be dispensed with. The method described in the following paragraph is quite satisfactory.

By means of a fine thread suspend a heavy body, such as a stone or a piece of iron, from one end of the balance and find its weight by placing weights on the other end. Let its weight be 158 grams. Then surround the body with water as in Fig. 103 and weigh again. Let the weight now be 137 grams. The buoyant force of the water is thus 158 − 137 = 21 grams. Next, lower the body into an overflow can (Fig. 104) and catch the overflow in a beaker or other vessel whose weight has been carefully determined. Weigh again and by subtraction find the weight of the water which has been displaced by the body. It will be found to be 21 grams.

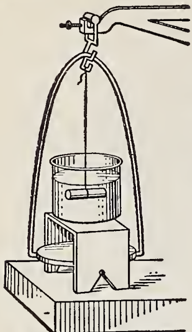


Fig. 103.—Finding the apparent loss in weight when a body is immersed in a liquid.

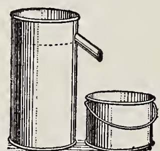


Fig. 104.—Overflow can.

If an overflow can is not available, lower the body into the water in a graduate and note the rise in the water. It will be found to be 21 c.c., the weight of which is 21 grams.

We therefore conclude:—

The buoyant force exerted by a fluid upon a body immersed in it is equal to the weight of the fluid displaced by the body; or, in slightly different words,

A body when weighed in a fluid loses in apparent weight an amount equal to the weight of the fluid which it displaces.

This is known as the Principle of Archimedes.

Archimedes had been asked by Hiero to determine whether a crown which had been made for him was of pure gold or alloyed with silver. It is said that the action of the water when in a bath suggested to him the principle of buoyancy as the key to the solution of the problem. The story is that he leaped from his bath, and rushed through the streets of Syracuse, crying "Eureka! Eureka!" (I have found it, I have found it.)

90. Principle of Flotation. It is evident that if the weight of a body immersed in a liquid is greater than the weight of the liquid displaced by it, that is, greater than the buoyant force, the body will sink; but if the buoyant force is greater, it will continue to rise until it reaches the surface. Here it will come to rest when a portion of it has risen above the surface and the weight of the liquid displaced by the immersed portion equals the weight of the body. For example, consider again the cube referred to in Fig. 101. If its weight is less than 27 grams, for definiteness say 18 grams, it will float in water. In this case the downward pressure on the top has disappeared and the weight of the cube alone is supported by the pressure on the bottom, which equals the weight of a column of water 9 sq. cm. in section and 2 cm. deep.

PROBLEMS

(Take 1 c. ft. water = 62.5 lb.)

1. A cubic foot of marble which weighs 160 pounds is immersed in water. Find (1) the buoyant force of the water on it, (2) the weight of the marble in water.

2. Twelve cubic inches of a metal weigh 5 pounds in air. What is the weight when immersed in water?

3. If 3,500 c.c. of a substance weigh 6 kg., what is the weight when immersed in water?

4. A piece of aluminium whose volume is 6.8 c.c. weighs 18.5 grams. Find the weight when immersed in a liquid twice as heavy as water.

5. If a body has a weight of 150 grams and a volume of 20 c.c., what portion of its weight will it lose if immersed in water?

6. One cubic decimetre of wood floats with $\frac{3}{5}$ of its volume immersed in water. What is the weight of the cube?

7. A cubic centimetre of cork weighs 250 mg. What part of its volume will be immersed if it is allowed to float in water?

8. The cross-section of a boat at the water-line is 150 sq. ft. What additional load will sink it 2 inches?

9. A scow with vertical sides is 25 feet long and 12 feet wide, and it sinks $2\frac{1}{2}$ inches when a team of horses walks on it. Find the weight of the team.

10. What is the least force which must be applied to a c. ft. of elm, which weighs 35 lbs. per c. ft., that it may be wholly immersed in water?

11. A piece of wood whose mass is 100 grams floats in water with $\frac{3}{4}$ of its volume immersed. What is its volume?

12. A piece of wood weighing 100 pounds floats in water with $\frac{2}{3}$ of its volume above the surface. Find its volume.

13. Why will an iron ship float on water, while a piece of the iron of which it is made sinks?

14. A vessel of water is on one scale-pan of a balance and counterpoised. Will the equilibrium be disturbed if a person dips his fingers into the water without touching the sides of the vessel? Explain.

15. A piece of coal is placed in one scale-pan of a balance and iron weights are placed in the other scale-pan to balance it. How would the equilibrium be affected if the balance, coal and weights were now placed under water? Why?

16. What is the least force which must be applied to a cubic foot of wood whose mass is 40 lbs. that it may be wholly immersed in water?

17. Referring to Fig. 94, answer the following question: If the depth of the water in the press A is the same as that in the press B which contains the vessel, which press will be the heavier?

18. A fish can rise or sink in water as it wishes. How does it produce this effect? If dead it floats; why?

19. Will there be any change in the depth to which a ship sinks as it comes from the Atlantic Ocean into Lake Ontario? Explain. If the ship weighs 1000 tons and the cross-section at the water-line is 12,000 sq. ft. find the change. (s.g. of sea water 1.025.)

20. A piece of wood floats with m/n of its volume out of the water. Find its weight per c.c and per c. ft.

21. The Royal Society of London was instituted during the reign of Charles II. It is said that at one of its meetings the king proposed the following question:—Suppose two vessels are placed on the pans of a balance and water is poured in until they exactly balance. If now a fish is put into one, why is the balance not destroyed? What is your answer to this question?

CHAPTER XII

Determination of Density

91. Finding the Density of a Solid Heavier than Water. To determine the density of a body we must find its mass and its volume. The mass is found directly by weighing; the volume usually by an application of Archimedes' Principle.

Examples. 1. Let a body weigh 20 grams in air and 16 when immersed in water. Then its mass = 20 grams. Also, since it loses 4 grams when weighed in water, that is the weight of the water displaced by the body. But 4 grams of water = 4 c.c. Hence the volume of the body = 4 c.c. and its density = 20 ÷ 4 = 5 grams per c.c.

2. Let m grams = weight of a body in air, and m_1 grams = its weight in water. Then $m - m_1$ grams = loss of weight in water. Hence $m - m_1$ c.c. = volume of the body, and

$$\text{Density (in gm. per c.c.)} = \frac{m}{m - m_1} = \frac{\text{mass (in grams)}}{\text{loss of wt. in water (in grams)}}.$$

The number thus obtained also expresses the specific gravity of the body (§16).

92. Density of a Solid Lighter than Water. Suppose the body is a block of wood. There are several methods which may be used, the following being two of the simplest:—

First Method. Weigh the wood by the balance. Then by means of a pin press the wood down into the water in an overflow vessel (Fig. 104) until it is entirely submerged. Catch the water and weigh it. This is the weight of the water displaced by the wood, which, divided into the weight of the wood in air, gives the density.

Second Method. Instead of using the balance place a graduate under the spout of the overflow vessel. Carefully lay the wood on the water in the vessel and observe the overflow into the graduate. Let it be x c.c.; its weight is x grams. Now press the wood down until entirely submerged, catching the water as before. Let it be y c.c., which weighs y grams.

Then the density (*in grams per c.c.*) = $x \div y$.

93. Density of a Liquid by the Specific Gravity Bottle. As in the case of solids, the problem is to determine the volume and the mass of the liquid.

FIG. 105.—Specific gravity bottle.

In Fig. 105 is shown a convenient bottle to use. It is provided with a closely-fitting stopper perforated with a fine bore through which any excess of liquid escapes.

First, weigh the bottle empty; then fill it with water and weigh it; then fill with the liquid and weigh again.

Let weight of bottle empty = m grams, weight of bottle filled with water $= m_1$ grams, weight of bottle filled with liquid $= m_2$ grams.

Then since $m_1 - m$ grams = weight of water filling the bottle, its volume $= m_1 - m$ c.c. Also $m_2 - m$ grams = weight of liquid filling the bottle. Hence $m_1 - m$ c.c. of the liquid $= m_2 - m$ grams and

$$\text{Density of the liquid} = \frac{m_2 - m}{m_1 - m} \text{ grams per c.c.}$$

Sometimes the volume of the bottle is marked on it (usually 50 c.c. or 100 c.c. at 15° C.), and then the weight of the bottle filled with water need not be found.

94. Density of a Liquid by Archimedes' Principle.

Archimedes' Principle may also be applied to determine the densities of liquids.

Take a glass sinker whose mass is, say, m grams, and weigh it first in the liquid whose density is to be determined, and then in water. If m_1 grams denotes the weight of the sinker in the liquid and m_2 grams its weight in water,

$m - m_1$ grams = mass of liquid displaced by sinker.

$m - m_2$ grams = mass of water displaced by sinker.

Hence volume of the sinker = $m - m_2$ c.c., and

$$\text{Density of liquid (in grams per c.c.)} = \frac{m - m_1}{m - m_2}.$$

95. Density of a Liquid by the Hydrometer. The hydrometer is an instrument designed to indicate directly the density of the liquid by the depth at which it floats in it. The principle underlying the action of this instrument may be illustrated as follows:—

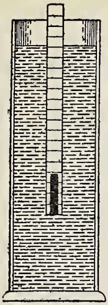


Fig. 106. — Illustration of the principle of the hydrometer.

Take a rectangular rod of wood 1 sq. cm. in section and 20 cm. long, and bore a hole in one end. After inserting sufficient shot to cause the rod to float upright in water (Fig. 106) plug up the hole and dip the rod in hot paraffin to render it impervious to water. Mark off on one of the long faces a centimetre scale. Now place the rod in water, and suppose it to sink to a depth of 16 cm. when floating. Then the weight of the rod = weight of water displaced = 16 grams.

Again, suppose it to sink to a depth of 12 cm. in a liquid whose density is to be determined.

Then, since the weight of liquid displaced equals weight of the rod,

$$12 \text{ c.c. of the liquid} = 16 \text{ grams,}$$

$$\text{And density of the liquid} = \tfrac{16}{12} \text{ gram per c.c.}$$

$$\text{Or, density of the liquid} = \frac{\text{vol. of water displaced by a floating body}}{\text{vol. of the liquid displaced by the same body}}.$$

A hydrometer for commercial purposes is usually constructed in the form shown in Fig. 107. The weight and volume are so adjusted that the instrument sinks to the division mark at the lower end of the stem in the densest liquid to be investigated and to the division mark in the upper end in the least dense liquid. The scale on the stem indicates directly the densities of liquids between these limits. The float A is usually made much larger than the stem to give sensitiveness to the instrument.

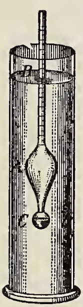


Fig. 107.—The hydrometer

As the range of an instrument of this class is necessarily limited, special instruments are constructed for use with different liquids. For example, one instrument is used for the densities of milks, another for alcohols, another for the acid in a storage battery and so on. That for testing a storage battery is illustrated in Fig. 108. The lower end is thrust into the battery and, by pressing the rubber bulb and letting it go, enough acid is drawn into the tube to float the hydrometer. The depth to which it sinks shows the general condition of the liquid in the battery.

Fig. 108.–Storage battery hydrometer.

PROBLEMS

(For table of densities see § 15)

1. A body whose mass is 60 grams is dropped into a graduated tube containing 150 c.c. of water. If the body sinks to the bottom and the water rises to the 200 c.c. mark, what is the density of the body?

2. A piece of metal whose mass is 120 grams weighs 100 grams in water and 104 grams in alcohol. Find the volume and density of the metal, and the density of the alcohol.

3. A bottle empty weighed 21.10 grams; when filled with water, 71.22 grams; when filled with alcohol, 61.73 grams. Find the density of the alcohol.

4. The specific gravity of pure milk is 1.086. What is the density of a mixture containing 500 c.c. of pure milk and 100 c.c. of water?

5. If a body when floating in water displaces 12 c.c., what is the density of a liquid in which when floating it displaces 18 c.c.?

6. A cylinder of wood 8 inches long floats vertically in water with 5 inches submerged. (*a*) What is the specific gravity of the wood? (*b*) What is the specific gravity of the liquid in which it will float with 6 inches submerged? (*c*) To what depth will it sink in alcohol whose specific gravity is 0.8?

7. A hydrometer floats with $\frac{2}{3}$ of its volume submerged when floating in water, and $\frac{3}{4}$ of its volume submerged when floating in another liquid. What is the density of the other liquid?

8. A uniform wooden rod 5 cm. square and 30 cm. long is loaded so that it floats upright in water with 20 cm. below the surface. If the rod were placed in alcohol (specific gravity 0.8) what length of the rod would be below the surface?

9. A body whose mass is 12 grams has a sinker attached to it, and the two together displace when submerged 60 c.c. of water. The sinker alone displaces 12 c.c. What is the density of the body?

10. A body whose mass is 6 grams has a sinker attached to it, and the two together weigh 16 grams in water. The sinker alone weighs 24 grams in water. What is the density of the body?

11. A mass of lead is suspected of being hollow. It weighs 2486 grams in air and 2246 grams in water. What is the volume of the cavity? (s.g. of lead, 11.3.)

REFERENCES FOR FURTHER IMFORMATION

Encyclopedia Britannica, Arts. "Density" and "Hydrometer."
Stewart & Gee, Elementary Practical Physics, Vol. I, Ch. 5.

CHAPTER XIII

Pressure in Gases

96. Has Air Weight? This question puzzled investigators from the time of Plato and Aristotle down to the seventeenth century, when it was answered by Galileo and Guericke.

Galileo (1564-1642) convinced himself that air had weight, by proving that a glass globe filled with air under high pressure weighed more than the same globe when filled with air under ordinary conditions. Guericke (1602-1686), the inventor of the air-pump, showed that a copper globe weighed more when filled with air than when exhausted.

The experiments of Galileo and Guericke may be repeated with a glass flask (Fig. 109) fitted with a stop-cock. If the flask is weighed when filled with air under ordinary pressure, then weighed when the air has been compressed into it with a bicycle pump, and again when the air has been exhausted from it with an air-pump, it is found that the first weight is less than the second but greater than the third.

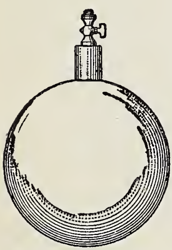


Fig. 109.—Globe for weighing air.

If a flask like that in Fig. 109 is not available, an ordinary electric light bulb may be used. First weigh it on a good balance. Then scratch the sharp projection on it with a file and strike it smartly with a piece of wood. It will break off and the air will rush in. Now weigh again, including the broken-off piece. The weight will be greater than before.

Since the volume of a mass of air varies with changes in temperature and pressure, the weight of a certain volume will be constant only at a fixed temperature and pressure. Exact quantitative experiments have shown that the mass of a litre of air at 0° C. and under normal pressure of the air at sea-level (760 mm. of mercury) is 1.293 grams.

97. Pressure of Air. It is evident that, since air has weight, it must, like liquids, exert pressure upon all bodies with which it is in contact. Just as the bed of the ocean sustains enormous pressure from the weight of the water resting on it, so the surface of the earth, the bottom of the aerial ocean in which we live, is subject to a pressure due to the weight of the air supported by it. This pressure will, of course, vary with the depth. Thus the pressure of the atmosphere at Victoria, B.C., on the sea-level is greater than at points on the mountains to the east.

The pressure of the air may be shown by many simple experiments. The following are three examples:—

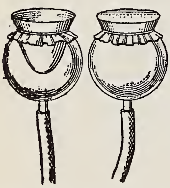


Fig. 110.—Rubber membrane forced inwards by pressure of the air.

1. Tie a piece of thin sheet rubber over the mouth of a thistle-tube (Fig. 110) and exhaust the air from the bulb by suction or by connecting it with the air-pump. As the air is exhausted, the rubber is pushed inward by the pressure of the outside air.

2. Boil water vigorously in a tin can, and when the steam is coming off freely, push into it a good rubber stopper and remove the heat. Throw cold water on it, or simply let it stand, and watch the result.

3. Thrust one end of a straw or a tube into water and withdraw the air from it by suction; the water is forced up into the tube. This phenomenon was known for ages, but it did not receive an explanation until the facts of the weight and pressure of the atmosphere were established. It was explained on the principle that Nature had a horror for empty space.

The attention of Galileo was called to this problem of the *horror vacui** in 1640 by his patron, the Grand Duke of Tuscany, who had found that water could not be lifted more than 32 feet by a suction pump. Galileo inferred that "resistance to vacuum" as a force had its limitations and could be measured; but although he had, as we have seen, proved that air has weight, he did not see the connection between the facts. After

*Horror of a vacuum.

his death the problem was solved by his pupil, Torricelli, who showed definitely that the resistance to a vacuum was the result of the pressure of the atmosphere due to its weight.

98. The Torricellian Experiment. Torricelli concluded that, since a water column rises to a height of 32 feet, and since mercury is about 14 times as heavy as water, the corresponding mercury column should be $\frac{1}{14}$ as long as the water column. To confirm his inference an experiment similar to the following was performed under his direction by Vincenzo Viviani, one of his pupils.

Take a glass tube about one metre long (Fig. 111), closed at one end, and fill it with mercury. Stopping the open end with the finger, invert it and place it in a vertical position, with the open end under the surface of the mercury in another vessel. Remove the finger. The mercury will fall a short distance in the tube, and after oscillating will come to rest with the surface of the mercury in the tube between 28 and 30 inches above the surface of the mercury in the outer vessel.

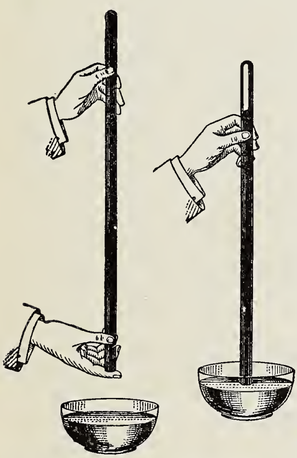


Fig. 111.—Mercury column sustained by the pressure of the air.

Torricelli concluded rightly that the column of mercury was sustained by the pressure of the air on the surface of the mercury in the outer vessel. This conclusion was confirmed by Pascal, who showed that the length of the mercury column varied with the altitude. He asked his brother-in-law, Périer, who resided in the south of France, to test it on the Puy de Dôme, a near-by mountain over 1,000 yards high. Périer filled a tube about 4 ft. long with mercury, inverted it in a vessel containing mercury and carried it to the summit. The mercury column fell more than 3 in. This result pleased them all.

QUESTIONS AND PROBLEMS

1. Fill a tumbler and hold it inverted in a dish of water as shown in Fig. 112. Why does the water not run out of the tumbler into the dish?

2. Fill a bottle with water and place a sheet of writing paper over its

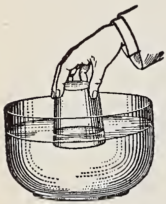


Fig. 112.

Fig. 113.

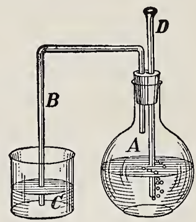


Fig. 114.

mouth. Now, holding the paper in position with the palm of the hand, invert the bottle. (Fig. 113.) Why does the water remain in the bottle when the hand is removed from the paper?

Fig. 115.

3. Boil water in a flask A arranged as in Fig. 114, conducting the steam through the tube B into cold water C. Remove the heat. Observe and explain what happens. Next remove the tube D, plugging the hole in the cork through which it passes. Repeat the experiment and explain what happens.

4. Take a bent glass tube of the form shown in Fig. 115. The upper end of it is closed, the lower open. Fill the tube with water. Why does the water not run out when it is held in a vertical position?

5. Explain the action of a fountain-pen filler.

6. Why must an opening be made in the upper part of a vessel filled with a liquid to secure a proper flow at a faucet inserted at the bottom? Can the water be emptied from a flexible rubber bag if the bag has a single small opening in it?

7. Fill a narrow-necked bottle with water and hold it mouth downward. Explain the action of the water.

8. On the tin top upon a pot of jam is sometimes seen the instruction:—"To open, puncture and push up at edge." Why puncture it?

99. The Barometer. Torricelli pointed out that the object of his experiment was "not simply to produce a vacuum, but to make an instrument which shows the mutations of the air, now heavier and dense, now lighter and thin."* The modern mercury barometer designed for this purpose is the same in principle as that constructed by Torricelli. With this instrument the pressure of the atmosphere is measured by the pressure exerted by the column of mercury which balances it, and changes in pressure are indicated by corresponding changes in the height of the mercury column.

FIG. 116. — The cistern barometer.

Two forms of the instrument are in common use.

100. The Cistern Barometer. This is simply a convenient arrangement of the original Torricellian experiment. The bowl, or cistern, and the tube are permanently mounted on a board, and a scale, engraved on the metal case protecting the glass tube, shows the height of the mercury in the tube above the surface of the mercury in the cistern.

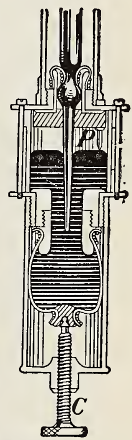


FIG. 117.—Section of the cistern.

A well-known form of this instrument is shown in Figs. 116, 117. The cistern has a flexible leather bottom which can be moved up or down by turning the screw *C*. Before taking the reading, the screw is turned until the surface of the mercury just touches the tip of the pointer *P*, which is the zero of the scale on the case. The height of the mercury is then read directly from the scale on the case. A vernier† is usually employed to determine the reading with exactness.

*Extract from letter written by Torricelli, in 1644, to M. A. Ricci, in Rome, first published in 1663.

†An explanation of the vernier is given in the *Laboratory Manual* designed to accompany this book.

101. The Siphon Barometer. This barometer consists of a tube of the proper length closed at one end and bent into **U**-shape at the other. (Fig. 118.) When filled and placed upright, the mercury in the longer branch is supported by the pressure of the air on the surface of the mercury in the shorter. A scale is attached to each branch. The upper scale gives the height of the mercury in the closed branch above a fixed point, and the lower scale the distance of the mercury in the open branch below the same fixed point. The sum of the two readings is the height of the barometer column.

Fig. 118.—Siphon barometer.

102. Aneroid Barometer. As its name implies,* this is a barometer constructed without liquid. (Fig. 119.) In this form the air presses against the flexible corrugated cover of a circular, air-tight, metal

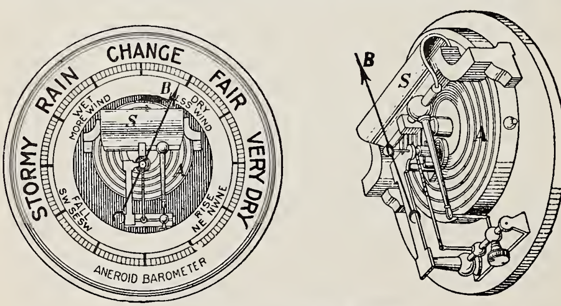


Fig. 119.—Aneroid barometer.

box *A*, from which the air is partially exhausted. The cover, which is usually supported by a spring *S*, responds to the pressure of the atmosphere, being forced in when the pressure is increased, and springing out when it is

*Greek *a* = not, *neros* = wet.

decreased. The movement of the cover is very small, but it is multiplied and transmitted to an index hand B by a system of delicate levers and a chain or by gears. The circular scale is graduated by comparison with a mercury barometer.

The aneroid is not so accurate as the mercury barometer, but, on account of its portability and its sensitiveness, is coming into very common use. It is especially useful in exploring and mountain-climbing as by its readings one can compute his height above sea-level. However, it should be compared as often as possible with a good mercury barometer in order to check its readings.

103. Practical Value of the Barometer; Atmospheric Pressure. By the barometer we can determine the pressure of the atmosphere at any point. For example, to measure the pressure per sq. cm. of the air at a point where the mercury barometer stands at 76 cm., we have but to find the weight of the column of mercury balanced by the atmospheric pressure at this point; that is, we have to find the weight of a column of mercury 1 sq. cm. in section and 76 cm. high. The volume of the column is 76 c.c., and, taking the density of mercury as 13.6 grams per c.c., this weight will be

$$76 \times 13.6 = 1033.6 \text{ grams.}$$

In general terms, if a is the area pressed, and h the height of a barometer using a liquid whose density is d,

$$\begin{aligned} ah &= \text{volume of liquid in barometric column,} \\ ahd &= \text{weight of liquid in barometric column,} \\ &= \text{pressure of atmosphere on area } a, \\ \text{and} \qquad hd &= \text{pressure of atmosphere on unit area.} \end{aligned}$$

104. Variations in Atmospheric Pressure. By continually observing the height of the barometer at any place we learn that the atmospheric pressure is constantly changing. Sometimes a decided change takes place within an hour.

Again, by comparing the simultaneous readings of barometers distributed over a large stretch of country we find that the pressure is different at different places.

QUESTIONS AND PROBLEMS

1. If the mercury barometer stands at 76 cm., how high will one filled with alcohol stand? (s.g. of alcohol, 0.8; of mercury, 13.6.)

2. During a storm the mercury in a barometer fell from 30 to 28.5 inches. If the area of the surface of a person is 12 sq. ft., what was the change in the force of the atmosphere upon him?

3. If the density of the air, like that of water, were uniform throughout its volume, how high would the atmosphere extend, assuming the height of the water barometer to be 34 ft. and the density of air to be 0.081 lb. per c. ft.?

4. Find the pressure of the atmosphere upon 1 sq. metre when the barometer stands at 75 cm.

5. If a barometer reads 30 in. at sea-level and 20 in. at the summit of a mountain would it read 25 in. half-way up? Explain.

105. Construction of the Weather Map. The Meteorological Service has stations in all parts of the country at which observers regularly record at stated hours of each day the prevailing meteorological conditions. Twice each day these simultaneous observations are sent by telegraph to the head office at Toronto. These reports include:—The barometer reading, the temperature, the direction and velocity of the wind, and the rainfall, if any. The information thus received is entered upon a map, such as that shown in Fig. 120. Places having equal barometric pressures are joined by lines called *isobars,** the successive lines showing difference of pressure due to $\frac{1}{10}$ inch of mercury. The circles show the state of the sky, and the arrows indicate the direction of the wind.

The weather map shows the distribution of atmospheric pressure at 8 a.m. on January 16th, 1923. The low area over the Gulf of St. Lawrence can be traced from Southern California and Arizona, where the first indications of disturbance were shown on the 12th. At 8 a.m., 13th, the centre of the low was in Colorado; at 8 a.m., 14th, over Lake Michigan; at 8 a.m., 15th, over Eastern Ontario and at 8 a.m., 16th, over the Gulf as indicated by the map. The spiral in-blowing of winds is well shown by this map. It will be observed that at this time there was an area of high pressure to the northward of the Great Lakes, the centre probably in Northern Manitoba. This high pressure, which had originated in the sub-arctic regions, moved southeastward during the 16th and 17th as the low area passed eastward to the Atlantic, and extremely low temperature prevailed in Ontario early on the 17th.

On account of the difference in pressure there is a motion of the air inwards towards the centre of the "low," and outwards from the centre

*Greek, *isos* = equal, *baros* = weight.

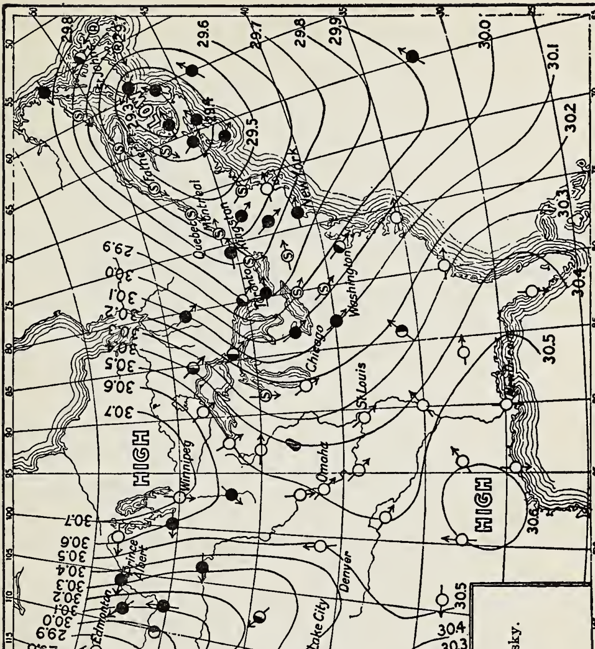


FIG. 120.—Weather Map issued at Toronto for 8 a.m., January 16, 1923. The curved lines, called *isobars*, join places where the height of the barometer is the same. The arrows show the direction of the wind.

FORECAST. *Lower Lake Region, Georgian Bay, Ottawa and Upper St. Lawrence Valleys:* Fresh north-westerly to northerly winds; snowflurries in some localities but mostly fair; becoming much colder. *Lake Superior:* Fine and very cold to-day and for first part of Wednesday; then moderating. *Manitoba and Saskatchewan:* A few local snowflurries but mostly fair; rising temperature. *Alberta:* Generally fair to-day and on Wednesday and for the most part mild.

of the "high." But these motions are not directly towards or away from the centre. An examination of the arrows on the map will show that there is a motion about the centre. In the case of the "low" this motion is *contrary to* the direction of motion of the hands of a clock, while in the case of the "high" the motion is *with* the hands of the clock. Through a combination of the motions the air moves spirally inwards to the centre of low pressure and spirally outwards from the centre of high pressure. The system of winds about a centre of low pressure is called a *cyclone* that about a centre of high pressure, an *anti-cyclone*. The disturbance in the cyclone is usually much greater than in the anti-cyclone.

At the centre of low pressure there is an ascending current of air, which rises until it reaches a great height, when it flows over into the surrounding regions. In the case of the area of high pressure there is a flow of air from the upper levels of the surrounding atmosphere into the centre of high pressure, thus raising the barometer.

It will be observed, also, that while the air in an area of low or high pressure may be only three or four miles high, these areas are hundreds of miles across.

Now it has been found that within the tropics, in the trade-wind zones, the drift of the atmosphere is towards the west and south, and disturbances are infrequent; but in higher latitudes the general drift is eastward, and disturbances are of frequent occurrence, especially during the colder months. Thus in Canada and the United States the areas of high and low pressure move eastward; the latter, however, travel faster than the former.

106. Elementary Principles of Forecasting. In using the weather map the chief aim is to foresee the movement of the areas of high and low pressure, and to predict their positions at some future time, say 36 hours hence. It is also essential to judge rightly what changes will occur in the energy of the areas shown on the map, as these changes will intensify or otherwise modify the atmospheric conditions.

As the cyclone moves eastward, the first indication of its approach will be the shifting of the wind to the eastward. The direction in which the wind will veer depends on whether the storm centre passes to the northward or the southward; and the strength of the wind will depend on the closeness of the isobars. If they are close together, the wind will be strong. If the centre passes nearly over a place, the wind will chop round to the westward very suddenly; while if the centre is at a considerable distance, the change will be more gradual.

The precipitation (rain or snow) in connection with a cyclonic area is largely dependent on the energy of the disturbance, and on the temperature and moisture of the air towards which the centre is advancing. It must, of course, be remembered that rain cannot fall unless there is moisture, and moisture will not be precipitated unless the volume of the air containing it is cooled below the dew-point (§ 276). This cooling is caused by the expansion of the air as it ascends.

Occasionally, we have a rain with a northerly wind succeeding the passage of a centre of low pressure. In this case the colder and heavier air flows in under the warmer air, lifting it to a height sufficient to cause the condensation of its moisture.

The duration of precipitation, and of winds of any particular direction, depends on the rate of movement of the storms and of the areas of high pressure. Temperature changes in any given region can be arrived at only by an accurate estimation of the distance and direction from which the air which passes over has been transferred by wind movement.

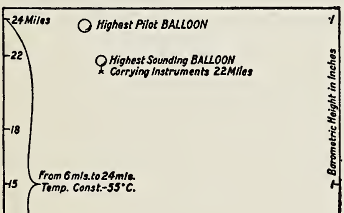


FIG. 121.—Atmospheric conditions at different heights.

Abnormally warm weather results from the incoming of warm air from more southern latitudes; and cold waves do not develop in lower middle latitudes (such as Ontario), but are the result of the rapid flow southward of air which has been cooled in high latitudes.

107. Determination of Elevation. Since the pressure of the air decreases gradually with increase in height above the sea-level, it is evident that the barometer may be utilized to determine changes in elevation. If the density of the air were uniform, its pressure, like that of liquids, would vary directly as the depth. But on account of the compressibility of air, its density is not uniform. The lower layers, which sustain the greater weight, are denser than those above them. For this reason the law giving the relation between the barometric pressure and altitude is somewhat complex. For small elevations it falls at an approximately uniform rate of one inch for every 900 feet of elevation. Fig. 121 shows roughly the conditions of atmospheric pressure at heights up to 24 miles.

108. Height of the Atmosphere. We have no means of determining accurately the height of the atmosphere. Twilight effects indicate a height of about fifty miles; above this the air ceases to reflect light. But it is known that air must extend far beyond this limit. Meteors, which consist of small masses of matter made incandescent by the heat produced by friction with the atmosphere, have been observed at heights of over 100 miles. Again, the phenomenon known as the aurora, or the northern lights, is produced in the atmosphere, and some displays have been shown to be 300 miles high.

109. Compressibility and Expansibility of Air. We have already referred to the well-known fact that air is compressible. There are many experiments which show that the volume of air is decreased by pressure. Try any of the following:—

Fig. 122.—Air compressed within a closed tube by pushing on piston.

Fig. 123.—Air compressed within a closed tube by weight of mercury in the long branch.

1. Compress a hollow rubber ball by the hand. Its volume is easily reduced.

2. Thrust a tightly-fitting piston into a tube closed at one end (Fig. 122). The air can be compressed into a small fraction of the space originally occupied by it.

3. Pour mercury into a **U**-tube closed at one end (Fig. 123). It will be found that the higher the column of the mercury in the open branch, that is, the greater the pressure due to the weight of the mercury, the less the volume of the air shut up in the closed branch becomes.

On the other hand, gases manifest, under all conditions, a tendency to expand. Whenever the pressure to which a given mass of air is subjected is lessened, its volume increases. This can also be demonstrated by many experiments, for example:—

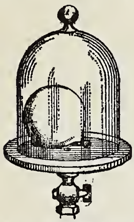


Fig. 124.—Expansion of air when pressure is removed.

1. The compressed rubber ball recovers its original volume and shape when the pressure of the hand is removed.

2. Having compressed the air in the tube (Fig. 122), let the piston go. It shoots outwards.

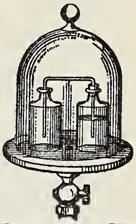


Fig. 125. — Water forced out of the closed bottle by the expansion of the air above it.

3. Place a toy balloon, partially filled with air, under the receiver of an air-pump (Fig. 124) and exhaust the air from the receiver. The balloon swells out and, if its walls are not strong, it bursts.

4. Place a bottle partly filled with water, closed with a perforated cork and connected by a bent tube with an uncorked bottle, as shown in Fig. 125, under the receiver of an air-pump and exhaust the air from the receiver. The water will be forced into the open bottle by the pressure of the air shut up in the corked bottle.

This tendency of the air to expand explains why frail hollow vessels are not crushed by the pressure of the air on their outer walls. The pressure of the air within counterbalances the pressure of the air without.

QUESTIONS AND PROBLEMS

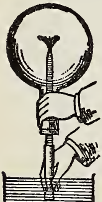


Fig. 126.


Fig. 127.

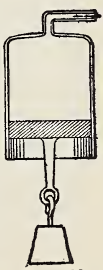


Fig. 128.

1. Arrange apparatus as shown in Fig. 126. By suction remove a portion of the air from the flask, and, keeping the rubber tube closed by pressure, place the open end in a dish of water. Now open the tube. Explain the action of the water.

2. Guericke took a pair of hemispherical cups (Fig. 127) about 1.2 ft. in diameter, so constructed that they formed a hollow air-tight sphere when their lips were placed in contact, and, at a test at Regensburg before the Emperor Ferdinand III

and the Reichstag in 1654, showed that it required sixteen horses (four pairs on each hemisphere), to pull the hemispheres apart when the air was exhausted by his air-pump. Account for this.

3. If an air-tight piston is inserted into a cylindrical vessel and the air exhausted through the tube (Fig. 128), a heavy weight may be lifted as the piston rises. Explain this action. The apparatus shown in Fig. 89 may be used for this experiment.

110. Relation between the Volume and the Pressure of Air—Boyle's Law. The exact relation between the volume of a given mass of gas and the pressure upon it was first determined by Robert Boyle in 1662. While trying to show that the Torricellian phenomenon (§ 98) was due to "the spring of the air" he devised an experiment which demonstrated that the volume of a given quantity of air varies inversely as the pressure to which it is subjected. He took a **U**-tube of the form shown in Fig. 129, and by pouring in enough mercury to fill the bent portion, inclosed a definite portion of air in the closed shorter arm. By manipulating the tube he adjusted the mercury so as to stand at the same height in each arm. Under these conditions the imprisoned air was at the pressure of the outside atmosphere, which at the time of the experiment would support a column of mercury about 29 inches high. He then poured mercury into the open arm until the air in the closed arm was compressed into

Fig. 129.—Boyle's apparatus

Robert Boyle (1627-1691). Published his Law in 1662. One of the earliest of English scientists basing their investigations upon experiment. Born at Lismore Castle, Ireland.

one-half its volume. "We observed," he says, "not without delight and satisfaction, that the quicksilver in that longer part of the tube was 29 inches higher than the other." Clearly the pressure upon the inclosed air was that produced by 29 inches of mercury and the atmosphere on its free surface. Consequently, the pressure sustained by the inclosed air was doubled when the volume was reduced to one-half. Continuing his experiment, he showed, on using a great variety of volumes and their corresponding pressures, that if the volume is reduced to $\frac{1}{3}$, the pressure is 3 times as great, if the volume is $\frac{1}{4}$, the pressure is 4 times as great, and so on. His conclusion may be stated in general terms thus:—

Let V_1, V_2, V_3, etc., represent the volumes of the inclosed air, and P_1, P_2, P_3, etc., represent corresponding pressures;
Then $V_1 P_1 = V_2 P_2 = V_3 P_3 = K$, a constant quantity.

That is, *if the temperature is kept constant, the volume of a given mass of air varies inversely as the pressure to which it is subjected.*

This relation is generally known as Boyle's Law.*

PROBLEMS

1. A tank whose capacity is 2 c. ft. has gas forced into it until the pressure is 250 pounds to the sq. inch. What volume would the gas occupy at a pressure of 75 pounds to the sq. inch?

2. A gas-holder contains 22.4 litres of gas when the barometer stands at 760 mm. What will be the volume of the gas when the barometer stands at 745 mm.?

3. Gas is forced into a cylinder 36 in. long and 14 in. in diameter until the pressure is 200 pounds per square in. If allowed to escape into an empty flexible bag much too large for it to fill completely what volume will it occupy? (Barom. 30 in.; for density of mercury see page 12).

4. Twenty-five c. ft. of gas, measured at a pressure of 29 in. of mercury, is compressed into a vessel whose capacity is $1\frac{1}{2}$ c. ft. What is the pressure of the gas?

5. A mass of air whose volume is 150 c.c. when the barometer stands at 750 mm. has a volume of 200 c.c. when carried up to a certain height in a balloon. What was the reading of the barometer at that height?

*In France it is called Mariotte's Law, having been independently discovered by Mariotte in 1676.

6. A piston is inserted into a cylindrical vessel 12 in. long, and forced down within 2 in. of the bottom. What is the pressure of the inclosed air if the barometer stands at 29 in.?

7. The density of the air in a gas-bag is 0.001293 grams per c.c. when the barometer stands at 760 mm.; find its density when the barometric height is 740 mm.?

8. An open vessel contains 100 grams of air with the barometer at 745 mm. What mass of air does it contain with the barometer at 755 mm.?

9. Oxygen gas, used for the 'lime-light', is stored in steel tanks. The volume of a tank is 6 c. ft., and the pressure of the gas at first was 15 atmospheres. After some had been used the pressure was 5 atmospheres. If the gas is sold at 6 cents a c. ft., measured at atmospheric pressure, what should be charged for the amount consumed?

10. In one form of sounding apparatus a slender glass tube closed at one end is lowered, open end down, to the bottom of the ocean, and an ingenious arrangement allows one to see to what height the water has risen in the tube. Suppose that the tube is 45 cm. long and the water. rises to within 1.5 cm. of the closed end.

(*a*) What pressure (in atmospheres) has the inclosed air been subjected to?

(*b*) Taking the barometric height to be 76 cm.; the sp. gr. of mercury to be 13.6 and that of sea-water to be 1.026, find the depth of the water.

111. Buoyancy of Gases. If we consider the cause of buoyancy, we must recognize that Archimedes' Principle applies to gases as well as to liquids. If a hollow metal or glass globe *A* (Fig. 130), suspended from one end of a short balance beam and counterpoised by a small weight *B* at the other end, is placed under the receiver of an air-pump and the air exhausted from the receiver, the globe is seen to sink. It is evident, therefore, that it was supported to a certain extent by the buoyancy of the air.

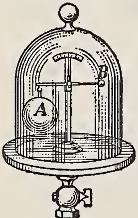


Fig. 130.—Arrangement to show the buoyancy of air.

A gas, like a liquid, exerts on any body immersed in it a buoyant force which is equal to the weight of the gas displaced by the body. If a body is lighter than the weight of the air equal in volume to itself, it will rise in the air, just as a cork, let free at the bottom of a pail of water, rises to the surface.

112. Balloons. The use of air-ships or balloons is made possible by the buoyancy of the air. A balloon is a large, light, gas-tight bag filled with some gas lighter than air, usually hydrogen or illuminating gas. It has been proposed to use helium, which will not take fire, and sufficient quantities may be available in the near future. Indeed a balloon, though not of the largest size, has been filled with helium. In Fig. 131 is illustrated the great British air-ship R-34, which during the summer of 1919 made the journey from Great Britain to the United States and back. By means of propellers it can be driven in any desired direction.

A balloon will continue to rise so long as its weight is less than the weight of the air which it displaces, and when there is a balance between

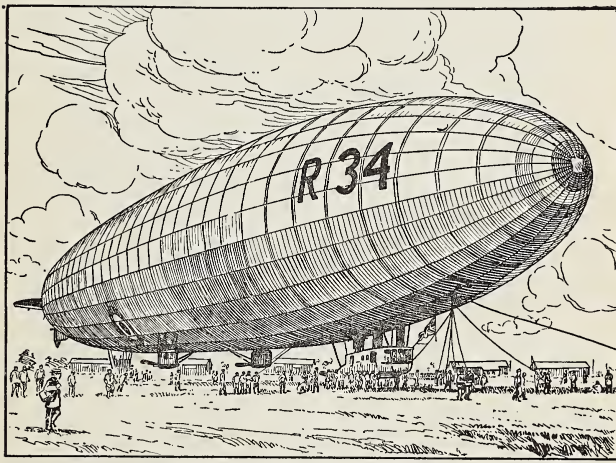


FIG. 131.—The British air-ship R-34, the first to cross the ocean. It left East Fortune, Scotland, July 2, and reached Long Island, N.Y., July 6, 1919. Time of flight, 108 hours. The return was made in 75 hours. Length, 672 ft.; diameter, 79 ft.; volume 2,000,000 cu. ft ; crew and passengers, 30.

the two forces, it simply floats at a constant height. In the case of an ordinary balloon the aeronaut maintains his position by adjusting the weight of the balloon to the buoyancy of the air. When he desires to ascend, he throws out ballast. To descend he allows gas to escape and thus decreases the buoyancy. The power air-ships can be made to rise or sink or turn to the right or left by means of suitable rudders, and so do not need to carry ballast.

QUESTIONS AND PROBLEMS

1. Why should the gas-bag be subject to an increased strain from the pressure of the gas within as the balloon ascends?

2. Aeronauts report that balloons have greater buoyancy during the day when the sun is shining upon them than at night when it is cold. Account for this fact.

3. If the volume of a balloon remains constant, where should its buoyancy be the greater, near the earth's surface or in the upper strata of the air? Give reasons for your answer.

4. The volume of a balloon is 2500 cu. m., and the weight of the gas-bag and car is 100 kg.; find its lifting power when filled with hydrogen gas, the density of which is 0.0000895 grams per c.c. while that of air is 0.001293 grams per c.c.

5. The density of helium is twice that of hydrogen. Find the lifting power of the balloon if filled with helium.

6. If the balloon were filled with illuminating gas, which is 8 times as dense as hydrogen, would it rise? Find the lifting power.

7. If the lower end of a balloon were left open would the gas begin at once to escape? Why? Explain what would happen after some time. Explain what would happen if the upper end were opened.

8. A balloon had a capacity of 80,000 c. ft. The gas-bag, net about it, and the basket together weighed 985 pounds. How great a load could it carry when filled with hydrogen? (1 c. ft. of air = 0.08 pound; of hydrogen = 0.0056 pound.)

9. The ordinary balloons used during the siege of Paris in 1870 had a capacity of about 70,000 c. ft., and the weight of the balloon and car was about 1000 pounds. Find the lifting power when filled with coal gas whose density is 0.4 that of air.

10. On rising 900 feet from sea-level the barometric height falls one inch. Will it continue to fall this amount for every 900 feet above this? Explain.

CHAPTER XIV

Applications of the Laws of Fluids

113. Air-pump. Fig. 132 shows the construction of one of the most common forms of pumps used for exhausting air

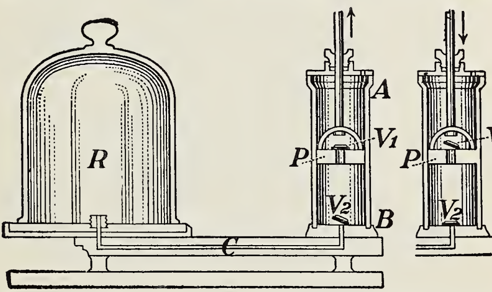


Fig. 132.—Common form of air-pump. *AB*, cylindrical barrel of pump; *R*, receiver from which air is to be exhausted; *C*, pipe connecting barrel with receiver; P, piston of pump; V_1 and V_2, valves opening upwards.

from a vessel. When the piston P is raised, the valve V_1 is closed by its own weight and the pressure of the air above it. The expansive force of the air in the receiver lifts the valve V_2, and a portion of the air flows into the lower part of the barrel. When the piston descends, the valve V_2 is closed, and the air in the barrel passes up through the valve V_1. Thus at each double stroke, a fraction of the air is removed from the receiver. The process of exhaustion will cease when the expansive force of the air in the receiver is no longer sufficient to lift the valve V_2, or when the pressure of the air below the piston fails to lift the valve V_1. It is evident, therefore, that a partial vacuum only can be obtained with a pump of this kind. To secure more complete exhaustion, pumps in which the valves are opened and closed automatically by the motion of the piston are frequently used, but even with these all the air cannot be removed from the receiver. Theoretically, a perfect vacuum cannot be obtained in this way, because at each stroke the air in the receiver is reduced only by *a fraction of itself.*

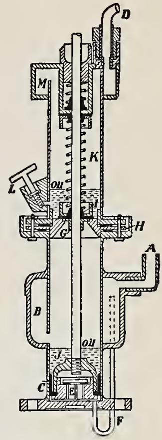


Fig. 134—Vertical section of a cylinder of an oil air-pump.

114. The Geryk*, or Oil Air-pump. This pump is much more efficient than that just described. Its action is as follows:—

The piston *J* (Fig. 134), made air-tight by the leather washer *C* and by being covered with oil, moves up-and-down in the barrel. The tube *A*, opening into the chamber *B* surrounding the barrel, is connected to the vessel from which the air is to be removed. On rising, the piston pushes before it the air in the barrel, and on reaching the top it pushes up *G* about $\frac{1}{4}$ inch, thus allowing the imprisoned air to escape through the oil into the upper part of the cylinder, from which it passes out by the tube *D*.

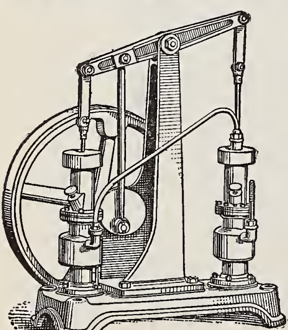


Fig. 133.—An oil air-pump with two cylinders.

When the piston descends, the spring *K*, acting upon the packing *I*, closes the upper part of the cylinder, and the piston on reaching the bottom drives whatever oil or air is beneath out through the tube *F*, or allows it to go up through the valve *E*, into the space above the piston.

Oil is introduced into the cylinder at *L*. When the pump has two cylinders, they are connected as shown in Fig. 133. With one cylinder the pressure of the air can be reduced to $\frac{1}{4}$ mm. of mercury, while with two a reduction to $\frac{1}{500}$ mm. can be obtained.

*Named after Guericke, the inventor of the air-pump.

115. Rotary Air-pump. This new type of pump (Fig. 135) will exhaust air to a pressure of .001 mm. It is of light weight, and is reliable and quiet in operation. It is also much more rapid in its action than the ordinary air-pumps.

Within the outer case of the pump is a fixed hollow cylinder provided with an inlet tube E and an outlet F, which is fitted with

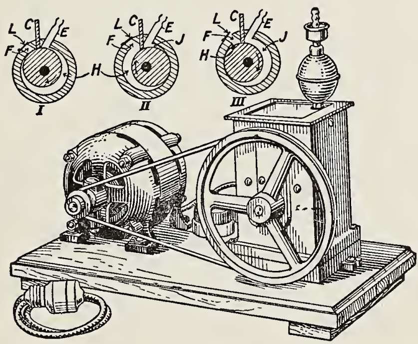


Fig. 135.—A rotary air-pump. In I, II, III is shown a vertical section of the cylinder at different stages of a rotation.

a valve L. Inside this cylinder is a second cylinder mounted eccentrically on an axle which is driven by a large pulley. As the inner cylinder rotates, it is always in contact with a portion of the outer cylinder. A metal plate C works up and down through a slot cut in the outer cylinder, always resting on the rotating cylinder. The pump case is filled with oil so that only the inlet tube E projects.

In position I the space H is in communication with the inlet tube E, which is connected to the vessel from which the air is being removed. As the cylinder rotates in the direction of the arrow to position II, the air in H is cut off from that in the vessel and is compressed, while air from the vessel expands into the space J. In position III most of the air in H has been driven out through the valve L while the space J is nearing its maximum size. As the rotation continues, position I is reached again and the cycle repeats.

116. Mercury Air-pump. When a very high vacuum is required, use is generally made of some form of mercury air-pump. There are several forms. The principle of that devised by Sprengel may be understood by reference to Fig. 136. Here the vessel *R*, from which the air is to be removed, is fused on the pump. As the mercury which is poured into the reservoir *A* falls in a broken stream through the nozzle *N* into the tube *B*, it carries air with it because each pellet of mercury acts as an air-tight piston and pushes a small portion of air before it. The density of the air in *C* and *R* is thus gradually decreased. The mercury which overflows into *D* is poured back into *A*. A vacuum as high as 0.000,007 mm. has been obtained with a mercury pump. It requires a good pump of the valved type to give an exhaustion of 1 mm.

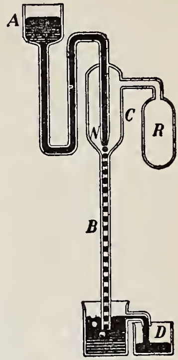


Fig. 136.—Sprengel air-pump. *A*, reservoir into which mercury is poured. *B*, glass tube of small bore, about one metre long; *R*, vessel from which air is to be drawn.

117. Bunsen Jet Pump. Bunsen devised a simple and convenient form of pump, which is much used in laboratories where a moderate exhaustion is required, as for hastening the process of filtration. In this pump (Fig. 137) water under a pressure of more than one atmosphere is forced into a jet through a tube nozzle *N*. The air is carried along by the water and is thus withdrawn from any vessel connected with the offset tube *A*.

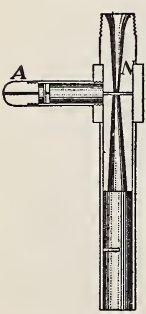


Fig. 137.—Bunsen jet pump.

118. The Hydraulic Air Compressor. An application of the principle involved in the instruments just described is to be seen in the great air compressor at Ragged Chutes, on the Montreal River, eight miles south-west from Cobalt, the centre of the great mining region in Northern Ontario.

A cement dam 660 feet long across the river raises the level of the water. By a large tube A (Fig. 138) the water is led into two vertical pipes P (only one shown in the figure), 16 feet in diameter into each of which is fitted a framework holding 66 intake pipes a, a, 14 inches in diameter. The water-line is about 10 or 12 inches above the top of the nest of intake pipes. In descending the water forms a vortex in the mouth of each pipe through which air is drawn down into the shaft below. Thus air and water are mixed together. At b the pipe is reduced to 9 feet and near the bottom, at c, is enlarged to $11\frac{1}{2}$ feet in diameter.

The water drops 350 feet, falling on a steel-covered cone B, from which it rushes into a

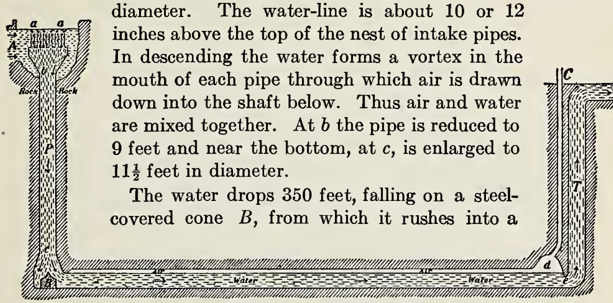


FIG. 138.—Taylor air compressor at Ragged Chutes on Montreal River (section).

horizontal tunnel over 1000 feet long, the farther end d of which is 42 feet high. In this large channel the water loses much of its speed and the air is rapidly set free, collecting in the upper part of the tunnel. At e the tunnel narrows and the water races past and enters the tail-shaft T, 300 feet high, from which it flows into the river again.

The air entrapped in the tunnel is under a pressure due to about 300 feet of water, or about 125 pounds per square inch. From d a 24-inch steel pipe leads to the surface of the earth, and from here the compressed air is piped off to the mines.

Other air compressors on the same principle are to be found at Magog, Quebec; at Ainsworth, B.C.; at the lift-lock at Peterborough (see § 82); and at the Victoria Mines in Michigan; but the one near Cobalt is the largest in existence.

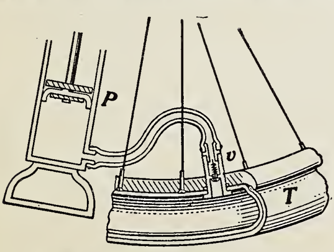


FIG. 139.—Air compressor for bicycle tires.

119. Air Compressors. The simplest compression pump is that used for inflating rubber tires. Its construction is shown in Fig. 139. When the piston P is pushed down, the air in the cylinder is forced through the valve *v* into the inner tube of the tire *T*, the valve immediately closing to check the air from going back. On lifting the piston a partial vacuum is produced in the cylinder, and the air from outside enters, going past the soft cup-shaped leather forming a part of the piston. When the piston is moving downwards, this leather is pressed against the inside of the cylinder, thus preventing the air from escaping. Each downward stroke forces more air into the tire until at last it becomes sufficiently hard. In a bicycle tire the pressure seldom exceeds 45 pounds per sq. in., while in automobile tires the pressures run from 50 to 95 pounds.

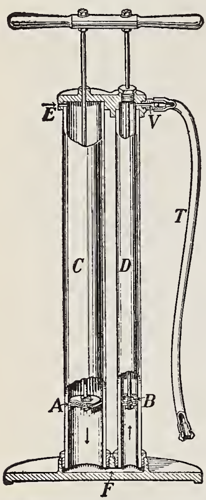


Fig. 140.—A continuous action compressor for automobile tires.

In Fig. 140 is shown a compressor used for automobile tires. There are cup-shaped washers on the pistons *A* and *B*, the edge being turned down in the former and up in the latter. When the handle is pushed down, the piston *A* drives the air from cylinder *C* through the passage *F* into cylinder *D*. Here it escapes past the washer on *B* and some of it is forced through the valve *V* into the tube *T* which is attached to the tire. The air enters at *E* to fill the cylinder *C*. On lifting the handle the air in *C* moves past the piston *A* and the air in *D* is forced through *V*. Thus on both the up and the down stroke air is forced into the tire.

Another style of compressor is illustrated in Fig. 141. A motor *A* drives the wheel *B* on the opposite end of whose axis is the crank disc *C* to which the connecting rod *D* is attached. This causes the piston *E* to move up and down in the cylinder *F*. As the piston rises, air is drawn in through the intake *I* and goes past the valve *G* into the cylinder. As it descends, this air is forced through the valve *H* into the tank *K*. The air is rapidly driven into *K* and the pressure, which is measured by the gauge *L*, quickly rises. If a vessel is connected with *I*, air will be removed from it, but this pump is not suited for producing a high vacuum.

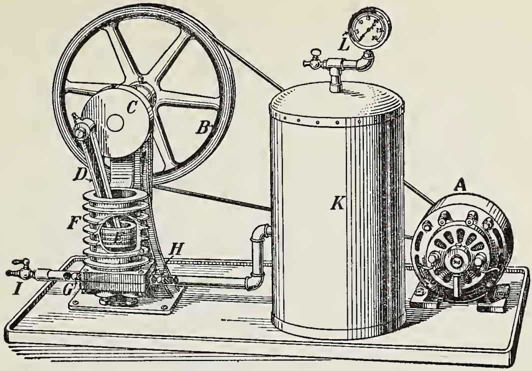


Fig. 141.—A convenient form of compressor.

120. Pressure Gauge. Its construction is shown in Fig. 142. It depends on the fact that if air or water is forced into a bent tube the tube tends to straighten out. The gauge is attached to a tank or a pipe by the nipple *G*. Within the case is a bent metal tube *AA* (the middle portion not being visible in the diagram). When air or water is forced into it under pressure, it tries to straighten out. Now the lower end is rigidly fixed, but the upper end is free to move, and the higher the pressure in the tube the greater is the motion of this end. By means of a metal strip *B* this end is joined to the short arm of the lever *C* which turns about the pin *D*. On the other end of *C* are teeth which mesh with the small pinion *E*, and the hand *F* is on the end of the axis of the pinion. Hence as the free end *A* moves, its motion is multiplied and transmitted to the hand which moves around the dial. The spring *H* takes up any loose motion in the mechanism.

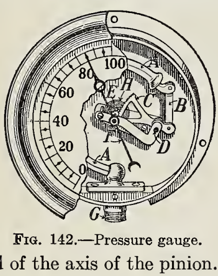


Fig. 142.—Pressure gauge.

121. Uses of Compressed Air. The air-brakes and diving apparatus are described in the next two sections. Another useful application is the pneumatic drill, used chiefly for boring holes in rock for blasting. In it the steel drill is attached to a piston which is made to move back and forth in a cylinder by allowing compressed air to act alternately on its two faces. The pneumatic hammer, which is similar in principle, is used for riveting and in general foundry work. Steam could be used, but the pipes conveying it would be hot and water would be formed from it. By means of a blast of sand, projected by a jet of air, castings and also discoloured stone and brick walls are cleaned. Figures on glass are engraved in the same way. Tubes for transmitting letters or telegrams, or for carrying cash in our large retail stores, are operated by compressed air. Many other applications cannot be mentioned here.

122. Air-brakes. Compressed air is used to set the brakes on railway cars. Fig. 143 shows the principal working parts of the Westinghouse air-brakes in common use in this country. A steam-driven air compressor pump *A* and a tank *B* for compressed air are attached to the locomotive. The equipment on each car consists of (*a*) a cylinder *C* in which works a piston *P* directly connected, by a piston-rod *D* and a system of levers

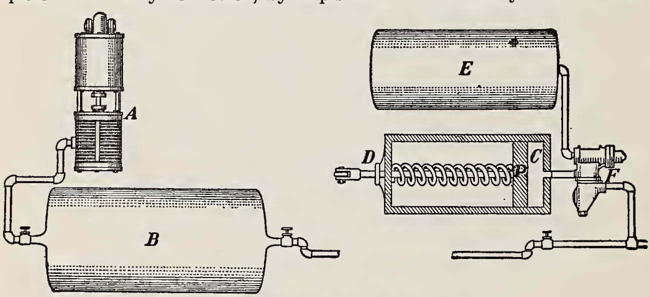


Fig. 143.—Air-brakes in use on railway trains.

with the brake-shoe, (*b*) a secondary tank *E*, and (*c*) a system of connecting pipes and a special valve *F* which automatically connects *B* with *D* when the air from *B* is admitted to the pipes, but which connects *E* with the cylinder *C* when the pressure of the air is removed.

When the train is running, pressure is maintained in the pipes, and the brakes are free, but when the pressure is decreased either by the engineer or the accidental breaking of a connection, the inrush of air from *E* to *C* forces the piston P forward, and the brakes are set. To take off the brakes, the air is again turned into the pipes when B is connected with E and the air in C is allowed to escape, while the piston P is forced into its original position by a spring.

123. Diving Bells and Diving Suits. Compressed air is also used as a reserve supply for individuals cut off from the atmosphere, as in the case of men engaged in submarine work. The diving bells and pneumatic caissons used in laying the foundations of bridges, piers, etc., are simply vessels of various shapes and sizes, open at the bottom, from which the water is kept out and workmen within supplied with air by compressed air forced in through pipes from above. (Fig. 144.) The air fills the tank completely, thus excluding the water, and escapes at the lower edges.

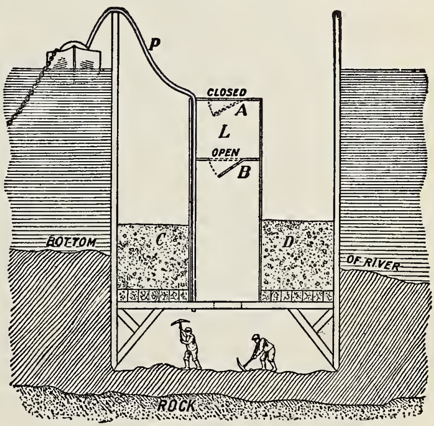


Fig. 144.—Section of a Pneumatic Caisson. The sides of the caisson are extended upward and are strongly braced to keep back the water. Masonry or concrete, C, D, placed on top of the caisson, press it down upon the bottom, while compressed air, forced through a pipe P, drives the water out of the working chamber. To leave the caisson the workman climbs up and passes through the open door B into the air lock L. The door B is then closed and the air is allowed to escape from L until it is at atmospheric pressure. Then door A is opened. In order to enter, this process is reversed. Material is hoisted out in the same way or is sucked out by a mud pump. As the earth is removed, the caisson sinks until the rock is reached. The entire caisson is then filled with solid concrete, and a permanent foundation for a dock or bridge is thus obtained.

The modern diver is incased in an air-tight weighted suit. (Fig. 145.) He is supplied with air from above through pipes or from a compressed-air reservoir attached to his suit. The air escapes through a valve into the water. Manifestly the pressure of the air used by a diver or a workman in a caisson must balance the pressure of the outside air, and the pressure of the water at his depth. The deeper he descends, therefore, the greater the pressure to which he is subjected. The ordinary limit of safety is about 80 feet· but in recent years this depth has been greatly exceeded. In 1915 a submarine sank


Fig. 145.—Diver's suit.

in Honolulu harbour, and in recovering it a diver made five descents to 306 feet. In October 1916 a descent was made in Grand Traverse Bay, Michigan, in a new diving armour to a depth of 361 feet; and about three years later a diver went down 360 feet to the bottom of the ocean near Boston, Mass.

124. Water Pumps. From very early times pumps were employed for raising water from reservoirs, or for forcing it through tubes. It is certain that the suction pump was in use in the time of Aristotle (born 384 B.C.). The force-pump was probably the invention of Ctesibius, a mechanician who flourished in Alexandria in the second century B.C. To Ctesibius is also attributed the ancient fire-engine, which consisted of two connected force-pumps, spraying alternately.

125. Suction or Lift-pump. The construction of the common suction-pump is shown in Fig. 146. During the first strokes the suction-pump acts as an air-pump, withdrawing the air from the suction pipe *BC*. As the air below the piston is removed, its pressure is lessened, and the pressure of the air on the surface of the water outside forces the water up the suction pipe and through the valve V_1 into the barrel. On the down-stroke the water held in the barrel by the valve V_1 passes up through the valve V_2, and on the next upstroke it is lifted up and discharged through the spout *G*, while more water is forced up through the valve V_1 into the barrel by the external pressure of the atmosphere. It is evident that

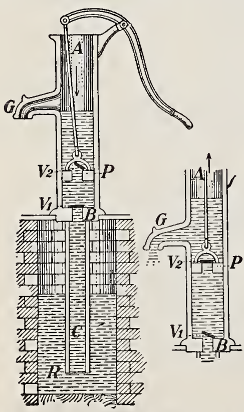


Fig. 146.—Suction-pump. *AB*, cylindrical barrel; *BC*, suction-pipe; *P*, piston; V_1 and V_2, valves opening upwards; *R*, reservoir from which water is to be lifted.

the maximum height to which water, under perfect conditions, is raised by the pressure of the atmosphere cannot be greater than the height of the water column which the air will support. Taking the relative density of mercury as 13.6 and the height of the mercury barometer as 30 inches, this height would be $\frac{30}{12} \times 13.6 = 34$ feet. But an ordinary suction-pump will not work satisfactorily if the piston is more than 25 feet above the surface of the water in the well.

126. Force-pump. When it is necessary to raise water to a considerable height, or to drive it with force through a nozzle, as for extinguishing fire, a force-pump is used. Fig. 147 shows the most common form of its construction. On the up-stroke a partial vacuum is formed in the barrel, and the air in the suction tube expands and passes up through the valve V_1. As the plunger is pushed down, the air is forced out through the valve V_2. The pump, therefore, during the first strokes acts as an air-pump. As in the suction pump, the water is forced up into the suction-pipe by the pressure of the air on the surface of the water in the reservoir. When it enters the barrel, it is forced by the plunger at each down-stroke through the valve V_2 into the discharge pipe. The flow will obviously be intermittent, as the outflow takes place only when the plunger is descending. To produce a continuous stream, and to lessen the shock on the pipe, an air chamber F is often inserted in the discharge pipe. When the water

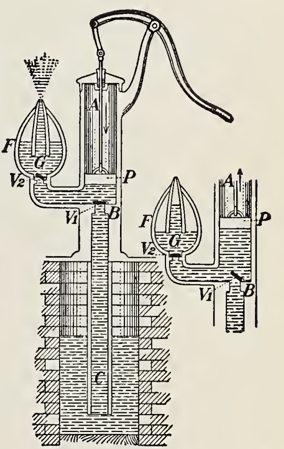


FIG. 147.—Force-pump. AB, cylindrical barrel; BC, suction-pipe; P, piston; F, air chamber; V_1, valve in suction-pipe; V_2, valve in outlet pipe; G, discharge pipe.

enters this chamber, it rises above the outlet *G* which is somewhat smaller than the inlet, and compresses the air in the chamber. As the plunger is ascending, the pressure of the inclosed air forces the water out of the chamber in a continuous stream.

127. Double-action Force-pump. In Fig. 148 is shown the construction of the double-action force-pump. When the piston is moved forward in the direction of the arrow, water is drawn into the back of the cylinder through the valve V_1, while the water in front of the piston is forced out through the valve V_3. On the backward stroke water is drawn in through the valve V_2 and is forced out through the valve V_4. Pumps of this type are used as fire engines, or for any purposes for which a large continuous stream of water is required. They are usually worked by steam or other motive power.

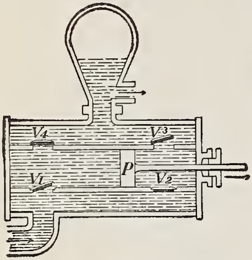


FIG. 148.—Double-action force-pump. *P*, piston; V_1, V_2, inlet valves; V_3, V_4, outlet valves.

QUESTIONS AND PROBLEMS

1. The capacity of the receiver of an air-pump is twice that of the barrel; what fractional part of the original air will be left in the receiver after (*a*) the first stroke, (*b*) the third stroke?

2. The capacity of the barrel of an air pump is one-fourth that of the receiver; compare the density of the air in the receiver after the first stroke with the density at first.

3. The capacity of the receiver of an air compressor is ten times that of the barrel; compare the density of the air in the receiver after the fifth stroke with its density at first.

4. How high can alcohol be raised by a lift-pump when the mercury barometer stands at 760 mm. if the relative densities of alcohol and mercury are 0.8 and 13.6 respectively?

5. Connect a glass model pump with a flask, as shown in Fig. 149. Fill the flask (*a*) full, (*b*) partially full of water, and endeavour to pump the water. Account for the result in each case.

6. Draw a diagram to show how water can be raised from a well one hundred feet deep.


FIG. 149.

128. Siphon. If a bent tube is filled with water, placed in a vessel of water and the ends unstopped, the water will flow freely from the tube, so long as there is a difference in level in the water in the two vessels. A bent tube of this kind, used to transfer a liquid from one vessel to another at a lower level, is called a siphon.

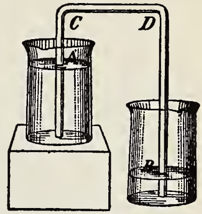


Fig. 150.—The siphon.

To understand the cause of the flow consider Fig. 150.

The pressure at A tending to move the water in the siphon in the direction AC

= the atmospheric pressure − the pressure due to the weight of the water in AC;

and the pressure at B tending to move the water in the siphon in the direction BD

= the atmospheric pressure − the pressure due to the weight of the water in BD.

But since the atmospheric pressure is the same in both cases, and the pressure due to the weight of the water in AC is less than that due to the weight of the water in BD, the force tending to move the water in the direction AC is greater than the force tending to move it in the direction BD; consequently, a flow takes place in the direction $ACDB$. This will continue until the vessel from which the water flows is empty, or until the water is at the same level in both vessels.

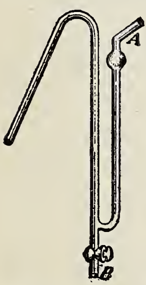


Fig. 151. — The aspirating siphon.

129. The Aspirating Siphon. When the liquid to be transferred is dangerous to handle, as in the case of some acids, an aspirating siphon is used. This consists of an ordinary siphon to which is attached an offset tube and stop-cock, as shown in Fig. 151, to facilitate the process of filling. The end B is closed by the stop-cock and the liquid is drawn into the siphon by suction at the moufb-piece A. The stop-cock is then opened, and the flow begins.

QUESTIONS AND PROBLEMS

1. Upon what does the limit of the height to which a liquid can be raised in a siphon depend? If the siphon were carried to the top of a mountain, how would its action be affected?

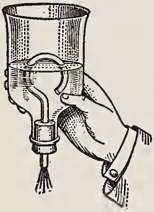


Fig. 152.

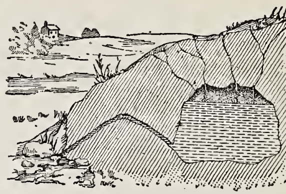


Fig. 153.—An intermittent spring.

2. Over what height can (*a*) mercury, (*b*) water, be made to flow in a siphon?

3. How high can sulphuric acid be raised in a siphon when the mercury barometer stands at 29 in., taking the specific gravities of sulphuric acid and mercury as 1.8 and 13.6 respectively?

4. Upon what does the rapidity of flow in the siphon depend?

5. Arrange apparatus as shown in Fig. 152. Let water from a tap run *slowly* into the bottle. What takes place? Explain.

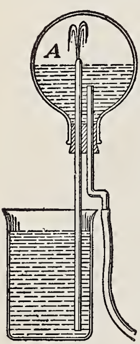


Fig. 154.

6. Natural reservoirs are sometimes found in the earth, from which the water can run by natural siphons faster than it flows into them from above (Fig. 153). Explain why the discharge through the siphon is intermittent.

7. Arrange apparatus as shown in Fig. 154. Fill the flask *A* partly full of water, insert the cork, and then invert, placing the short tube in water. Explain the cause of the phenomenon observed.

8. You are given a glass **U**-tube, a rubber tube and a vessel containing water and are asked to measure the pressure of illuminating gas at a gas-burner. How would you do it? What effect will the size of the tube have on the measurement?

9. Find the pressure upon a diver's suit at a depth of 360 feet in fresh water; also in salt water of specific gravity 1.025.

10. A person blows into a rubber tube attached to a **U**-tube containing water and is able to cause a difference in level of the water in the two tubes of 60 cm. Calculate the pressure per sq. cm. which he can exert. Find its value also in pounds per sq. in

130. Water in Motion. From early times men have used water-wheels to transform the energy of falling and running water into useful work. Many forms have been invented, the most modern and most efficient being the *Impulse* or *Pelton Wheel* and the *Reaction Turbine.*

131. The Impulse Wheel. The small water-motor (Fig. 155) used for driving washing-machines and other household appliances is an example of the impulse wheel. The water, under considerable pressure, comes to the motor by the pipe A and issues from the small nozzle with high velocity. The impact of the water on the cup-shaped buckets of the wheel causes it to rotate with great speed. Having done its work the water leaves the motor by the pipe C.

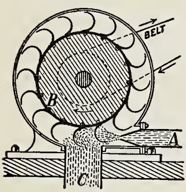


Fig. 155.—The Pelton water-wheel.

Pelton wheels are generally used where the fall of the water is very great, say above 1000 feet. They have been constructed with diameters as great as 10 feet, and sometimes the buckets are arranged in pairs about the periphery of the wheel so that two jets of water side by side may add their effects. An impulse wheel using a single jet has been made to develop 15,000 horse-power.

132. The Reaction Turbine. This type of water-wheel is now being almost universally installed in large power plants where only a moderate head of water is available. Some of the finest examples are to be found in the neighbourhood of Niagara Falls, among the largest being those of the Hydro-Electric Power Commission of Ontario.

Fig. 156 shows the general arrangement of the Commission's power plant at Queenston. Water from the Niagara River several miles above the Falls is conducted by a canal 13 miles long to the top of the cliff at Queenston where it is delivered through a steel penstock A to the 60,000 horse power turbine B which is directly connected by a vertical shaft 30 inches in diameter, to the 45,000 kilowatt generator C immediately above it. The electricity is generated at a pressure of 12,000 volts and is "stepped up" to a pressure of 110,000 volts by the transformer D from which leads off the transmission line E.

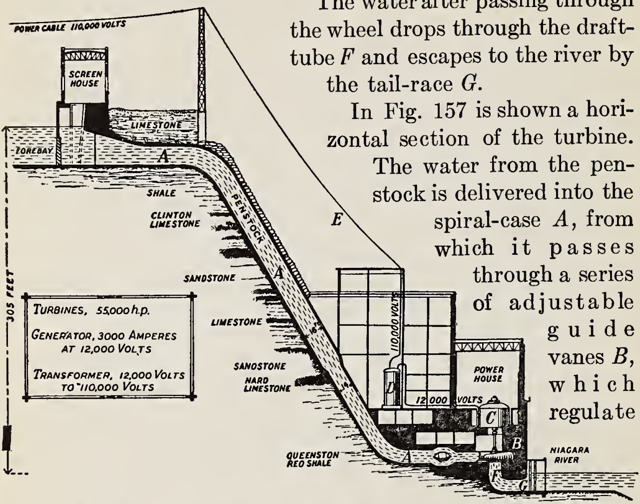


FIG. 156.—Arrangement of Hydro-Electric power plant at Queenston.

The water after passing through the wheel drops through the draft-tube F and escapes to the river by the tail-race G.

In Fig. 157 is shown a horizontal section of the turbine. The water from the penstock is delivered into the spiral-case A, from which it passes through a series of adjustable guide vanes B, which regulate the inward flow of the water and also direct it against the blades of the "runner" C in a direction best adapted to produce rotation. D is the shaft of the runner. The water moves through the runner inwards and downwards and the blades are curved to take advantage of both motions. On leaving the runner the water passes out through the draft-tube into the tail-race.

Figure 158 shows the guide vanes and the mechanism by which they are controlled. Fig. 159 gives a good idea of the enormous size of the runner and also shows how the blades are curved. The runner is of cast steel. Its outside diameter is 10 ft. 5 in., its weight is 42,000 lbs. and it rotates 187½ times per minute. The power house at Queens-

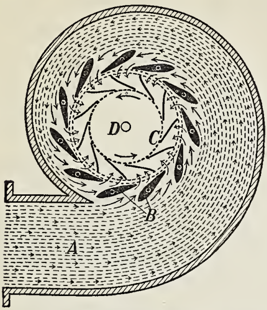


Fig. 157.—Horizontal section through the turbine showing spiral-case *A*, guide vanes *B* and runner *C*.

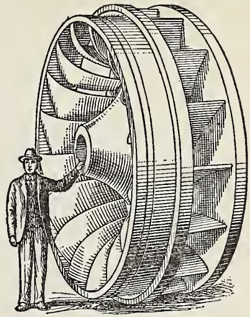


Fig. 159.—The runner or rotating part of the turbine. Note its great size and the curvature of the blades.

ton will ultimately contain 10 turbines similar to the one just described, developing a total of nearly 600,000 horse

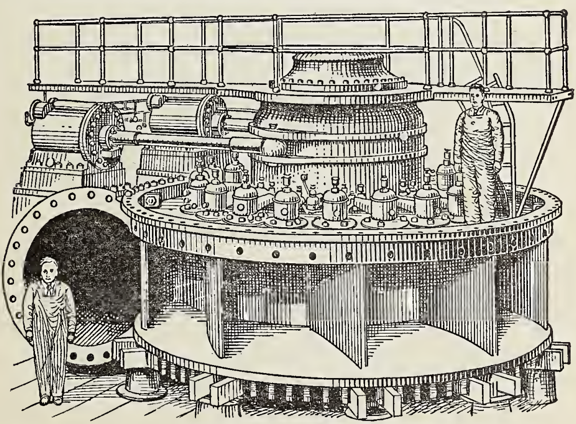


Fig. 158.—Part of the spiral case is removed showing the guide vanes. The mechanism for controlling them is seen above.

power. A brief description of the generators is given in §506.

PART IV—SOME PROPERTIES OF MATTER

CHAPTER XV

The Molecular Theory of Matter

133. Molecules and Atoms. We are all familiar with matter in its three ordinary forms—solid, liquid, gaseous—and a multitude of observations have led to the universal belief that it is composed of minute separate particles. These particles are called *molecules*. The molecules of some elements and of compound substances can be still further divided into *atoms*, but in this way the nature of the substance is altered—in other words, this is not a physical subdivision but a chemical change. Thus, the oxygen molecule has two atoms, and the water molecule consists of two atoms of hydrogen and one of oxygen.

134. Evidence suggesting Molecules. Water will soak into wood, or into beans, peas or other such seeds. On mixing 50 c.c. of water with 50 c.c. of alcohol the resulting volume is not 100 c.c., but only about 97 c.c. When copper and tin are mixed in the ratio of 2 of copper to 1 of tin, which gives an alloy used for making mirrors of reflecting telescopes, there is a shrinkage in volume of 7 or 8 per cent.

Again, various gases may be inclosed in the same space, and gases may be contained in liquids. Fish live by the oxygen which is dissolved in the water. Household ammonia is simply ammonia gas dissolved in water; and in making soda water carbonic acid gas is forced into the water under great pressure.

A simple explanation of these phenomena is that all bodies are made up of molecules with spaces between, into which the molecules of other bodies may enter. As we shall see, the molecules and the spaces between are much too small to be observed with our most powerful microscopes. The magnifying power would have to be increased at least a thousand times, but even if this magnification were obtained, it is probable that the molecules could not be seen, since there are good grounds for believing that they are constantly moving so rapidly that the eye could not follow them.

That there are pores or channels between the molecules was neatly proved by Bacon,* who filled a leaden shell with water, closed it, and then hammered it, hoping to compress the water within. But the water came through, appearing on the outside like perspiration. Afterwards the scientists of Florence tried the experiment with a silver shell, and also with the same shell thickly gilded over, but in both cases the water escaped in the same way. Many other illustrations of porosity could be given.†

135. Diffusion of Gases. The intermingling of molecules is best illustrated in the behaviour of gases. In order to investigate this question the French chemist, Berthollet, used apparatus like that illustrated in Fig. 160. It consisted of two glass globes provided with stopcocks, which could be securely screwed together. The upper one was filled with hydrogen and the lower with carbonic acid gas, which is 22 times as dense. They were then screwed together, placed in the cellar of the Paris Observatory and the stopcocks opened. After some time the contents of the two globes were tested and found to be identical,—the gases had become uniformly mixed.

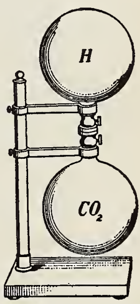


Fig. 160. — Two glass globes, one filled with hydrogen, the other with carbonic acid gas. The two gases mix until the contents of the two globes are identical.

When the passage connecting the two vessels is small, hours may be required for perfect mixing; but when it is large, a few minutes will suffice.

*Francis Bacon, 1561-1626. †See Tait's "Properties of Matter," §§ 98-100.

A simpler experiment on diffusion is the following:—

Fill one wide-mouthed jar with hydrogen and a similar one with oxygen, which is 16 times as heavy, covering the vessels with glass plates. Then put them together as shown in Fig. 161 and withdraw the glass plates. After allowing them to stand for some minutes separate them and apply a match. At once there will be a similar explosion from each, showing that the two gases have become mixed.*

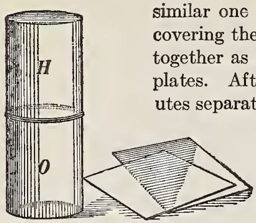


FIG. 161.—Hydrogen in one vessel quickly mixes with oxygen in the other.

It is through diffusion that the proportions of nitrogen and oxygen in the earth's atmosphere are the same at all elevations. Though oxygen is the heavier constituent, there is no excess of it at low levels.

136. Diffusion of Liquids and Solids. Liquids diffuse into each other, though not nearly so rapidly as do gases.

Carefully pour coloured alcohol (sp. gr. 0.8) on the top of clear water in a tumbler (or introduce water under the alcohol); the mixing of the two will be seen to commence at once and will proceed quite rapidly.

FIG. 162.—Copper sulphate solution in a bottle, placed in a vessel of water. In time the blue solution spreads all through the water.

Let a wide-mouthed bottle *a* (Fig. 162) be filled with a solution of copper sulphate and then placed in a larger vessel containing clear water. The solution is denser than the water, but in time the colour will be distributed uniformly throughout the liquid.

Diffusion takes place also in some metals though very slowly at ordinary temperatures. Roberts-Austen found that the diffusion of gold through lead, tin and bismuth at 550° C. was very marked; and that even at ordinary temperatures there was an appreciable diffusion of gold through solid lead. In his experiments discs of the different metals were kept in close contact for several weeks.

137. Motions of the Molecules; the Kinetic Theory. An explanation of such results as these is the hypothesis that all bodies are made up of molecules which have considerable freedom of motion, especially so in the case of gases.

*In performing this experiment wrap a cloth about each jar for safety.

The laws followed by gases, which are much simpler than those of solids and liquids, are satisfactorily accounted for by these molecular motions.

The distinguishing feature of a gas is its power of indefinite expansibility. No matter what the size of the vessel is into which a certain mass of gas is put, it will at once spread out and occupy the entire space. The particles of a gas are practically independent of their neighbours, moving freely about in the inclosure containing the gas.

A gas exerts pressure against the walls of the vessel containing it. This can be well illustrated as follows. Place a toy balloon or a half-inflated football rubber under the receiver of an air-pump and work the pump. (Fig. 163). As the air about the bag is continually removed, the bag expands; and when the air is admitted again into the receiver, the bag resumes its original volume.

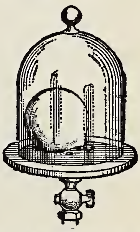


Fig. 163.—When the air is removed from the receiver, the toy balloon expands.

We may consider the bag as the seat of two contending factions—the troops of molecules within endeavouring to keep back the invading hosts of molecules without. Incessantly they rush back and forth, continually striking against the surface of the bag. As the enemies are withdrawn by the action of the pump, the defenders within gain the advantage and, pushing forward, enlarge their boundary, which at last, however, becomes so great that it is again held in check by the outsiders.

Or, we may compare the motion of the molecules of a gas to the motions of a number of bees in a closed vessel. They continually rush from side to side, frequently colliding with one another. The never-ceasing striking of the molecules of the gas against a body gives rise to the pressure exerted by the gas. This view of a gas is known as the Kinetic Theory.

138. Explanation of Boyle's Law. According to Boyle's Law (§ 113), when a gas is compressed to half its volume, the pressure which it exerts against the walls of the vessel containing it is doubled. This is just what we should expect. When the gas is made to occupy a space half as large, the particles in that space will be twice as numerous, the blows against its sides will be twice as numerous as before, and, consequently, the pressure will be doubled.

139. Effect of a Rise in Temperature. If we place the rubber bag used in § 137 in an oven, it expands, showing that the pressure of the gas is increased by the application of heat. Evidently, when a gas is heated, its molecules are made to move with greater speed, and this produces a greater pressure and causes the gas to expand.

140. Molecular Velocities. On account of numerous collisions the molecules will not all have the same velocity, but if we know the pressure which a gas exerts and also its density, it is possible to calculate the *mean* velocity of the molecules. The mean velocity,* at atmospheric pressure and freezing temperature, for four well-known gases follows:—

Hydrogen..........	1843 m. or 6046 ft.	per second.
Nitrogen...........	493 m. or 1618 ft.	"
Oxygen............	462 m. or 1517 ft.	"
Carbon Dioxide....	393 m. or 1291 ft.	"

It will be seen that the hydrogen molecules move fastest of all, being about four times as rapid as the molecules of nitrogen and oxygen, the chief constituents of the atmosphere. This is because it is much lighter. Each gas, by means of the bombardment of its molecules, is able to produce a pressure as great as that of any other gas, and hence, as hydrogen is much lighter, its molecular velocity must be much higher.

141. Passage of Hydrogen through a Porous Wall. As the velocities of the hydrogen molecules are so great, they strike much more frequently against the walls of the vessel which contains them than do the molecules of other gases. Hence, it is harder to confine hydrogen in a vessel than another gas, and it diffuses more rapidly. This is well illustrated in the following experiment:

*Strictly speaking, it is the square root of the mean square velocity which is given here.

An unglazed earthenware cup, A, (such as is used in galvanic batteries) is closed with a rubber or other cork impervious to air, and a glass tube connects this with a bottle nearly full of water (Fig. 164). A small glass tube B, drawn to a point, also passes through the cork of the bottle and reaches nearly to the bottom of the bottle.

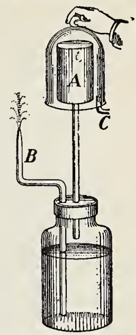


FIG. 164. — Experiment showing rapid passage of hydrogen through a porous wall.

Now hold over the porous cup a bell-jar full of dry hydrogen, or pass illuminating gas by the tube C into the bell-jar. Very soon a jet of water will spurt from the tube B, sometimes with considerable force. After this action has ceased, remove the bell-jar, and bubbles will be seen entering the water through the lower end of the tube B.

At first the space within the porous cup and in the bottle above the water is filled with air, and, when the hydrogen is placed about the porous cup, its molecules pass in through the walls of the cup much faster than the air molecules come out. In this way the pressure within the cup is increased, and this, when transmitted to the surface of the water, forces it out in a jet. When the jar is removed, the hydrogen rapidly escapes through the porous walls, and the air rushing in is seen to bubble up through the water

142. Molecular Motions in Liquids. In liquids the motions of the molecules are not so unrestrained as in a gas, but one can hardly doubt that the motions exist, however. Indeed, some direct evidence of these motions has been obtained. Brown, an English botanist, in 1827, with the assistance of a microscope, observed that minute particles like pores of plants, when introduced into a fluid, were always in a state of agitation, dancing to and fro in all directions

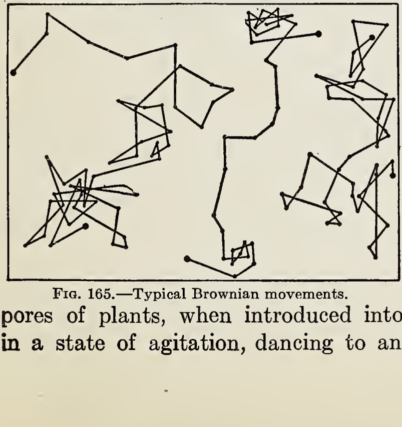


FIG. 165.—Typical Brownian movements.

with considerable speed. The smaller the particle the greater was its velocity, and the motions were apparently due to these particles being struck by molecules of the liquid. These Brownian movements are easy to see. Rosin dissolved in alcohol, mixed with water and then placed on glass under a powerful microscope; also soot, or gum mastic, or mercuric sulphide, in water will show them (Fig. 165). Recently a method has been devised for demonstrating the presence of particles too small to be seen with a microscope, and by means of it the particles obtained on making an emulsion of gamboge in water (which are too small to be observed by a microscope) have been shown to have these same Brownian motions. It is natural to infer that these motions are caused by the movement of the molecules of the liquid.

The spaces between the molecules are much smaller than in a gas, and so the collisions are much more frequent. Moreover, the molecules exert an attractive force on one another, the force of cohesion, but they glide about from point to point throughout the entire mass of the liquid. Usually when a molecule comes to the surface, its neighbours hold it back and prevent it from leaving the liquid. The molecules, however, have not all the same velocity, and occasionally, when a quick-moving one reaches the surface, the force of attraction is not sufficient to restrain it and it escapes into the air. We say the liquid *evaporates*.

When a liquid is heated, the molecules are made to move more rapidly and the collisions are more frequent. The result is that the liquid expands and the evaporation is more rapid.

In the case of oils the molecules appear to have great difficulty in escaping at the surface, and so there is little evaporation.

143. Osmosis. Over the opening of a thistle-tube let us tie a sheet of moistened parchment or other animal membrane (such as a piece of bladder). Then having filled the funnel and a portion of the tube with a strong solution of copper sulphate, let us support it, as in Fig. 166, in a vessel of water, so that the water outside is at the same level as the solution within the tube.

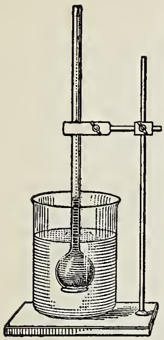


FIG. 166.—Osmosis.

In a few minutes the solution will be seen to have risen in the tube. The water will appear blue, showing that some of the solution has come out; but evidently more water has entered the tube. The rise in level continues (perhaps for two or three hours) until the hydrostatic pressure due to the difference of levels stops it.

This mode of diffusion through membranes is called *osmosis*, and the difference of level thus obtained is called *osmotic pressure.*

Substances such as common salt and others which usually form in crystals are called *crystalloids.* These diffuse through membranes quite rapidly. Starch, gelatine, albumen and gummy substances generally, which are usually amorphous in structure, are called *colloids.* These diffuse very slowly.

Osmosis plays an important part in the processes of nature.

144. Molecular Motions in Solids. As has been stated in § 136, evidences of the diffusion of the molecules of one solid into another have been observed, but the effect is very slight.

If a lump of sugar is dropped into a cup of tea, it soon dissolves, and in time its molecules spread to every part of the liquid, giving sweetness to it. In this instance the molecules of water enter into the lump of sugar and loosen the bonds which hold the molecules of sugar together. The molecules thus set free spread throughout the liquid.

Drop a minute piece of potassium permanganate into a quart jar full of water and shake the jar for a moment. The solid disappears and the water soon becomes of a rich red colour, showing that the molecules of the solid have spread to every part.

Again, ice gradually disappears even when below the freezing-point. Camphor and iodine, when gently heated, readily pass into vapour without melting. Indeed, if a piece of camphor is cut so as to have sharp corners, the wasting away at ordinary temperatures will be seen by the rounding of the corners in a very few days. This change from solid to vapour is called *sublimation.*

The motions of the molecules of a solid are much less free than those of a liquid. They vibrate back and forth about their mean positions, but as a rule are kept well to their places by their neighbours. When heated, the molecules are more vigorously agitated and the body expands, and if the heating is intense enough, it becomes liquid.

Since when a solid changes to a liquid its volume is not greatly changed, we conclude that in the two states of matter the molecules are about equally close together. But in gases they are much farther apart. A cubic centimetre of water when turned into steam occupies about 1600 c.c.

145. Viscosity. Tilt a vessel containing water; it soon comes to its new level. With ether or alcohol the new level is reached even more quickly, but with molasses much more slowly.

Although the molecules of a liquid or of a gas move with great freedom amongst their fellows, some resistance is encountered when one layer of the fluid slides over another. It is a sort of internal friction and is known as *viscosity.* Ether and alcohol have very little viscosity; they flow very freely and are called mobile liquids. On the other hand, tar, honey and molasses are very viscous.

Stir the water in a basin vigorously and then leave it to itself. It soon comes to rest, showing that water has viscosity. The viscosity of gases is smaller than that of liquids, that of air being about $\frac{1}{80}$ that of water.

146. Distinction between Solids and Liquids. We readily agree that water is a liquid and that glass is a solid, but it is not easy to frame a definition which will discriminate between the two kinds of bodies. A liquid offers no *permanent* resistance to forces tending to change its shape. It will yield to even the smallest force if continuously applied, but the rate of yielding varies greatly with different fluids, and it is this temporary resistance which constitutes viscosity.

Drive two pairs of nails in a wall in a warm place, and on one pair lay a stick of sealing-wax or a paraffin candle, on the other a tallow candle or a strip of tallow (Fig. 167). After some days (perhaps weeks), the tallow will still be straight and unyielding while the wax will be bent.

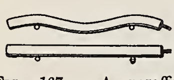


Fig. 167. — A paraffin candle bends but a tallow one keeps straight.

Lord Kelvin describes an experiment which he made many years ago. On the surface of the water in a tall jar he placed several corks, on these he laid a large cake of shoemakers' wax about two inches thick, and on top of this again were put some lead bullets. Six months later the corks had risen and the bullets had sunk half through the cake, while at the end of the year the corks were floating in the water at the top and the bullets were at the bottom of the vessel.

These experiments show that at ordinary temperatures wax is a liquid, though a very viscous one, while tallow is a true solid.

147. Solids, Liquids, Gases. The distinguishing properties of the three states of matter are the following:—

Solids have definite volume and definite shape.
Liquids have definite volume but no definite shape.
Gases have neither definite volume nor definite shape.

148. Cohesion and Adhesion. When we attempt to separate a solid into pieces, we experience difficulty in doing so. The molecules cling together, refusing to separate unless compelled by a considerable effort. This attraction between the molecules of a body is called *cohesion*, and the molecules must be very close together before this force comes into play. The fragments of a porcelain vessel may fit together so well that the eye cannot detect any cracks, but the vessel falls to pieces at the touch of a finger.

Some substances can be made to weld together much more easily than others. Clean surfaces of metallic lead, when pressed together, cohere so that it requires considerable force to pull them apart; and powdered graphite (the substance used in 'lead'-pencils), when submitted to very great pressure, becomes once more a solid mass.

Cohesion is the natural attraction of the molecules of a body for one another. If the particles of one body cling to those of another body, there is said to be *adhesion* between them. The forces in the two cases are of the same nature, and there is really no good reason for making a distinction between them. Indeed, it may be added that regarding the real nature of these forces we are entirely ignorant.

The force of cohesion is also present in liquids, but it is much weaker than in solids. If a clean glass rod is dipped in water and then withdrawn, a film of water will be seen clinging to it; but if dipped in mercury, no mercury adheres. This shows that the adhesion between glass and water is greater than the cohesion between the molecules of water, but the reverse holds in the case of mercury and glass.

149. The Size of Molecules. The problem of determining the size of molecules of matter is one of great interest, but also one of extreme difficulty. It has been attacked in various ways, and the results obtained by processes entirely different from one another agree satisfactorily, which is good evidence that they are somewhere near the truth.

It is astonishing what small quantities of some substances can be detected by sight or smell or taste. One ten-millionth of a grain, or 3.5×10^{-9} gram, of magenta dye in solution can be detected by the eye; and 1×10^{-12} gram of mercaptan, a very strong-smelling substance, can be recognized. Each of these small quantities contains many atoms or molecules.

Much of our recent knowledge regarding atoms has been obtained through radio-activity, a wonderful property which belongs to certain substances (see § 572). The number of molecules in 1 c.c. of gas at ordinary temperature and pressure is about 2.77×10^{19}; and knowing the weight of 1 c.c. of any gas we can calculate the weight of 1 molecule of it. Thus 1 c.c. of hydrogen weighs 0.00009, or 9×10^{-5} gram, and hence 1 molecule of hydrogen weighs 3.2×10^{-24} gram.

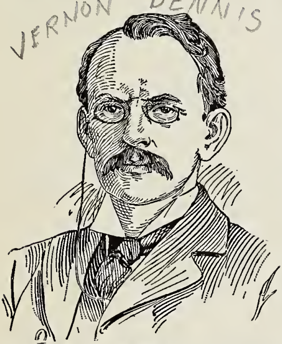
Let us consider a cubic decimetre of lead, which weighs 11.37 kg., and suppose that by three cuts at right angles to each other we divide it into eight similar cubes, each of half the linear dimensions and one-eighth the weight of the original cube. Each will have 5 cm. edges and will weigh 1.42 kg. Repeat this process again and again. After the ninth operation each cube would be just weighable on a delicate balance. The last operation possible before breaking up the lead atom would be the 28th. A single atom of lead would be contained in a cube having an edge of 3.0×10^{-8} cm. and it would weigh 3.44×10^{-22} gram.

These numbers are inconceivably small and the number of atoms in 1 c.c. is inconceivably great. Some illustrations of their magnitudes are given at the end of this chapter.

Lord Kelvin many years ago calculated in several ways the size of molecules, and he gives the following illustration: "Imagine a rain-drop, or a globe of glass as large as a pea, to be magnified up to the size of the earth, each constituent molecule being magnified in the same proportion. The magnified structure would be more coarse-grained than a heap of small shot, but probably less coarse-grained than a heap of cricket balls."

150. Nature of the Atom; the Electron Theory. So far molecules have been treated as simple bits of matter, like grains of wheat in a bushel measure, though reasons have been given for believing that they are in motion. The view ordinarily held has been that a body is built up from molecules each of which contains one or more atoms. In recent years, however, we have learned much regarding the structure of the atoms themselves. They are made up of elementary pieces of matter, which have been named *protons* and *electrons*. The protons are charged with positive electricity and the electrons with negative electricity. The nucleus of the atom contains all the protons and about half of the electrons of the atom, while the remaining electrons revolve around the nucleus much as the planets revolve about the sun. The mass of the electron is about $\frac{1}{1840}$th that of a hydrogen atom, and its dimensions are about $\frac{1}{100000}$th those of the atom given in the preceding section, while the protons are about $\frac{1}{2000}$th the size of the electrons.

VERNON DENNIS



SIR JOSEPH THOMSON. Born in Manchester, 1856. Cavendish Professor of Experimental Physics at Cambridge University, England, for many years.

PROBLEMS

1. Suppose a cubic decimetre of lead to be sliced into square plates 10 cm. square, one atom thick (3×10^{-8} cm.), and the plates to be spread out just touching each other. What area would they cover?

2. An ordinary evacuated electric-light bulb has a volume of 114 c.c. Suppose a hole to be made in it and molecules of air to rush in at the rate

of one million per second. How long will it take to fill the bulb so that the pressure within is equal to that without?

3. In a tumbler of water are approximately 10^{25} molecules, and all the water on the earth's surface contains about 5×10^{21} tumblerfuls. Suppose all the molecules in a tumbler of water to be labelled and the water then thrown into the ocean. If, after sufficient time has elapsed for the labelled molecules to be completely diffused through all the water, a tumbler of water is drawn from a tap, how many of the original molecules would there be in it?

REFERENCES FOR FURTHER INFORMATION

Watson, *A Text-book of Physics*, Chapter 15.
Burton, *The Physical Properties of Colloidal Solutions.*
White, *A Handbook of Physics*, Ch. 29 (Diffusion); 27 (Viscosity).
Loring, *Atomic Theories.*
Comstock and Troland, *The Nature of Matter and Electricity.*
Tait, *Properties of Matter.*
Edser, *General Physics*, Chapters 9, 10, 16.

CHAPTER XVI

Phenomena of Surface Tension and Capillarity

151. Forces at the Surface of a Liquid. On slowly forcing water out of a medicine dropper we see it gradually gather at the end (Fig. 168), becoming more and more globular, until at last it breaks off and falls a sphere. When mercury falls on the floor, it breaks up into a thousand shining globules. Why do not these flatten out? If melted lead be poured through a sieve at the top of a tower, it forms into drops, which harden on the way down and finally appear as solid spheres of shot.

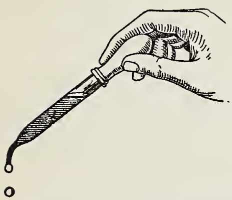


Fig. 168.—A drop of water assumes the globular form.

A beautiful way to study these phenomena was devised by the Belgian physicist, Plateau.* By mixing water and alcohol (about 60 water to 40 alcohol), it is possible to obtain a mixture of the same density as olive oil. By means of a pipette now introduce olive oil into the mixture (Fig. 169). At once it assumes a globular form. In this case it is freed from the distorting action of gravity and rests anywhere it is put.

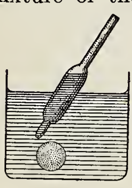


Fig. 169.—A sphere of olive oil in a mixture of water and alcohol.

When the end of a stick of sealing-wax or of a rod of glass is heated in a flame, it assumes a rounded form.

These actions are due to cohesion. A little consideration would lead us to expect the molecules at the

*Born 1801, died 1883. From 1829 his eyesight gradually deteriorated, and it failed entirely in 1843.

surface to act in a manner somewhat different from those in the interior of a liquid. Let a be a molecule well within the liquid (Fig. 170). The molecule is attracted on all sides by the molecules very close to it, within its sphere of action (which is extremely small, see § 148), and as the attraction is in all directions it will remain at rest. Next consider a molecule b which is just on the surface. In this case there will be no attraction on b from above, but the neighbouring molecules within the liquid will pull it downwards. Thus there are forces pulling the surface molecules into the liquid, bringing them all as close together as possible, so that the area of the surface will be as small as possible. It is for this reason that the water forms in spherical drops, since, for a given volume, the sphere has the smallest surface.

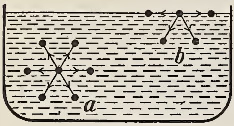


Fig. 170.—Behaviour of molecules within the liquid and at its surface.

The surface of a liquid behaves precisely as though a rubber membrane were stretched over it, and the phenomena exhibited are said to be due to *surface tension.*

152. Surface Tension in Soap Films. The surface tension of water is beautifully shown by soap bubbles and films. In these there is very little matter, and the force of gravity does not interfere with our experimenting. It is to be observed, too, that in the bubbles and films there is an outside and an inside surface, each under tension.

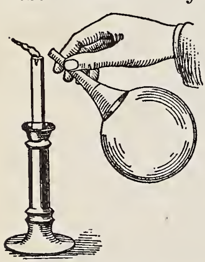


Fig. 171.—Soap-bubble blowing out a candle.

In an inflated toy balloon the rubber is under tension. This is shown by pricking with a pin or untying the mouthpiece. At once the air is forced out and the balloon becomes flat. A similar effect is obtained with a soap bubble. Let it be blown on a funnel, and the small end be held to a candle flame (Fig. 171). The outrushing air at once blows out the flame, which shows that the bubble behaves like an elastic bag.

There is a difference, however, between the balloon and the bubble. The former will shrink only to a certain size; the latter first shrinks to a film across the mouth of the funnel and then runs up the funnel handle, ever trying to reach a smaller area.

Again, take a ring of wire about two inches in diameter with a handle on it (Fig. 172). To two points on the ring tie a fine thread with a loop in it. Dip the ring in a soap solution and obtain a film across it with the loop resting on the film. Now, with the end of a wire or with the point of a pencil puncture the film within the loop. Immediately the film which is left assumes as small a surface as it can, and the loop becomes a perfect circle, since by so doing the area of the film becomes as small as possible.

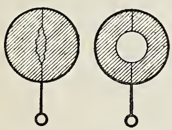


Fig. 172.—A loop of thread on a soap film.

153. Contact of Liquid and Solid. The surface of a liquid resting freely under gravity is horizontal, but where the liquid is in contact with a solid, the surface is usually curved. Water in contact with clean glass curves upward, mercury curves downward. Sometimes when the glass is dirty the curvature is absent.

Fig. 173.—Water in a glass vessel curves up, mercury curves down.

These are called *capillary* phenomena, for a reason which will soon appear. The angle of contact A (Fig. 173) between the surfaces of the liquid and the solid is called the capillary angle. For *perfectly* pure water and clean glass the angle is zero, but with slight contamination, even such as is caused by exposure to air, the angle may become 25° or more. For pure mercury and clean glass the angle is about 148°, but slight contamination reduces this to 140° or less. For turpentine it is 17°, and for petroleum 26°.

154. Level of Liquids in Capillary Tubes. In § 105 it was stated that in any number of communicating vessels a liquid stands at the same level. The following experiment gives an apparent exception to this law.

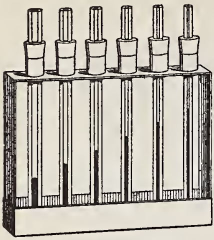


Fig. 174.—Showing the elevation of water in capillary tubes.

Let a series of capillary* tubes, whose internal diameters range from say 2 mm. to the finest obtainable, be held in a vessel containing water (Fig. 174). It will be found that in each of them the level is above that of the water in the vessel, and that the finer the tube the higher is the level. With alcohol the liquid is also elevated, (though not so much), but with mercury the liquid is depressed. The behaviour of mercury can conveniently be shown in a **U**-tube as in Fig. 175.

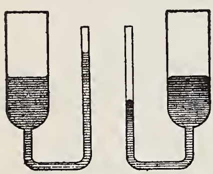


Fig. 175.—Contrasting the behaviour of water (left) and mercury (right).

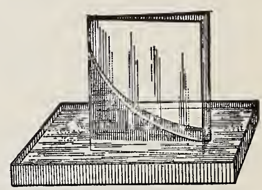


Fig. 176.—Water rises between the two plates of glass which touch along one edge.

Another convenient method of showing capillary action is illustrated in Fig. 176. Take two square pieces of window glass, and place them face to face, with an ordinary match or other small object to keep them a small distance apart along one edge while they meet together along the opposite edge. They may be held in this position by an elastic band. Then stand the plates in a dish of coloured water. The water at once creeps up between the plates, standing highest where the plates meet.

When a glass rod is withdrawn from water, some water clings to it, and the liquid is said to wet the glass. If dipped in mercury, no mercury adheres to the glass. Mercury does not wet glass.

*Latin, *capillus*, a hair.

The following are the chief laws of capillary action:—

(1) *If a liquid wets a tube, it rises in it; if not, it falls in it.*

(2) *The rise or depression is inversely proportional to the diameter of the tube.*

155. Explanation of Capillary Action. Capillary phenomena depend upon the relation between the cohesion of the liquid and the adhesion between the liquid and the tube.

In all cases the surface of a liquid at rest is perpendicular to the direction of the resultant force which acts on it. Usually the surface is horizontal, being perpendicular to the plumb-line, which indicates the direction of the force of gravity. In the case of contact between a solid and a liquid, the forces of adhesion and cohesion must be taken into account, since the force of gravity acting on a particle of matter is negligible in comparison with the attraction of neighbouring particles upon it.

Consider the forces on a small particle of the liquid at O. (Fig. 177.) The force of adhesion of the solid will be represented in direction and magnitude by the line A, that of the cohesion of the rest of the liquid by the line C. Compounding A and C by the parallelogram law (§ 44) the resultant force is R. The surface is always perpendicular to this resultant. When C greatly exceeds A, the liquid is depressed; if A greatly exceeds C, it is elevated.

Fig. 177.—Diagrams to explain capillary action.

In the case of capillary tubes the column of liquid which is above the general level of the liquid is held up by the adhesion of the glass tube for it. The total force exerted varies directly as the length of the line of contact of the liquid and the tube, which is the inner circumference of the tube; while the quantity of liquid in the elevated (or depressed) column is proportional to the *area* of the inner cross-section of the tube. If the diameter of the tube is doubled, the lifting force is doubled, and so the quantity of liquid lifted is doubled; but as the area is now *four* times as great the height of the column lifted is one-half as great.

Hence the elevation (or depression) varies inversely as the diameter of the tube.

156. Interesting Illustrations of Surface Tension and Capillarity.* It is not easy to pour water from a tumbler into a bottle without spilling it, but by holding a glass rod as in Fig. 185, the water runs down into the bottle and none is lost. The glass rod may be inclined, but the elastic skin still holds the water to the rod.

FIG. 178. — How to utilize surface tension in pouring a liquid.

Water may be led from the end of an eaves-trough into a barrel by means of a pole almost as well as by a metal tube.

When a brush is dry, the hairs spread out as in Fig. 179*a*, but on wetting it they cling together (Fig. 179*c*). This is due to the surface film which contracts and draws the hairs together. That it is not due simply to being wet is seen from Fig. 179*b*, which shows the brush in the water but with the hairs spread out.

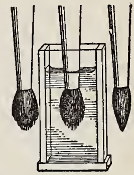


FIG. 179.—Surface tension holds the hairs of the brush together.

A wire sieve is wet by water, but if it is covered with paraffin wax, the water will not cling to it. Make a dish out of copper gauze having about twenty wires to the inch; let its diameter be about six inches and height one inch. Bind it with wire to strengthen it. Dip it in melted paraffin wax, and while still hot knock it on the table so as to shake the wax out of the holes. An ordinary pin, will still pass through the holes, and there will be over 10,000 of them. On the bottom of the dish lay a small piece of paper and pour water on it. Fully half a tumblerful of water can be poured into the vessel and yet it will not leak. The water has a skin over it, which will suffer considerable stretching before it breaks. Give the vessel a jolt, the skin breaks and the water at once runs out. A vessel constructed as described will also float on the surface of water.

Capillary action is seen in the rising of water in a cloth, or in a lump of sugar when touching the water; in the rising of oil in a lamp-wick and in the absorption of ink by blotting-paper.

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*Many beautiful experiments are described in "Soap Bubbles and the Forces which Mould Them," by C. V. Boys.

157. Small Bodies Resting on the Surface of Water.

FIG. 180.—Needle on the surface of water kept up by surface tension.

By careful manipulation a needle may be laid on the surface of still water (Fig. 180). In doing this a wire bent as in Fig. 181 may be used. The surface is made concave by laying the needle on it, and in the endeavour to contract and smooth out the hollow, sufficient force is exerted to support the needle, though its density is $7\frac{1}{2}$ times that of water. When once the water has wet the needle the water rises against the metal, and now the tendency of the surface to flatten out will draw the needle downwards.

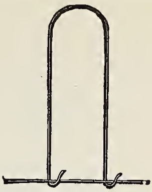

FIG. 181.—Stirrup for placing a needle on the surface of water.

If the needle is magnetized, it will act when floating like a compass-needle, showing the north and south direction.

Some insects run over the surface of water, frequently very rapidly (Fig. 182). These are held up in the same way as the needle, namely, by the skin on the surface, to rupture which requires some force.

FIG. 182.—Insect supported by the surface tension of the water.

REFERENCES FOR FURTHER INFORMATION

Boys, *Soap Bubbles and the Forces which Mould Them.*
Encyclopedia Britannica, Art. "Capillarity."
White, *A Handbook of Physics*, Chapter 28.
Chapter 23 of "*Mechanics for the Upper School*" treats Surface Tension.

PART V—SOUND

CHAPTER XVII

PRODUCTION, PROPAGATION, VELOCITY OF SOUND

158. Sound arises from a Body in Motion. The sensation of sound arises from various kinds of sources, but if we take the trouble to trace the sound to its origin, we always find that it comes from a material body in motion. There are numerous ways to demonstrate this fact.

Pluck a violin or a guitar string and watch it closely. It has a hazy outline which becomes perfectly definite when the sound dies away. Double a bit of paper and hang it on the string and then vibrate it. The paper rider is at once thrown off. Place a finger on a sounding bell. You feel the movement, and when the sound ceases the movement does also. Next, touch the surface of water with the prong of a sounding tuning-fork. The water is formed into ripples or splashes up in spray. Or hang by a fine thread a light ball or a hollow bead against the sounding bell or tuning-fork. It will be thrown off vigorously.

All our experience leads us to conclude that in every case *sound arises from matter in rapid vibration.*

159. Transverse Vibrations. Let us experiment with a simple pendulum such as that in Fig. 183.

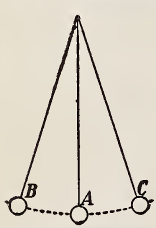

FIG. 183.—A simple pendulum.


FIG. 184.—Transverse vibrations of a rod.

Pull the bob aside to B and let it go. It will swing backward and forward through A, its position when at rest. The motion from B to C and back again to B is a *complete vibration;* and the distance AB or AC is the *amplitude* of the vibration. The length of time it takes the bob to make a complete vibration is its *period*, and the number of complete vibrations made per second is its *frequency* or vibration number.

Next, clamp a thin metal strip at one end in a vice (Fig. 184), pull the free end aside and let it go. It vibrates back and forth like the pendulum, and if the frequency is high enough, it gives forth a sound.

A violin string plucked at the centre vibrates in a similar manner.

Such vibrations as these, in which the motion is *across* the length of the vibrating body are called *transverse vibrations.*

160. Longitudinal Vibrations. The apparatus shown in Fig. 185 can be used to demonstrate the existence of a different kind of vibration.

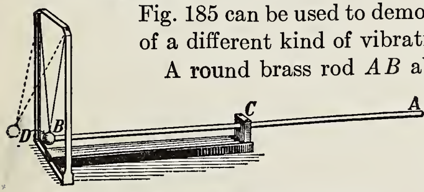

Fig. 185.—Illustrating longitudinal vibrations.

A round brass rod AB about 1 cm. in diameter and 100 cm. long is clamped at its middle point C, and an ivory ball D just touches the end B. Stroke the portion AC of the rod with a cloth on which is a little powered rosin. A high musical note is produced, and at the same time the ball rebounds from the end B vigorously, showing that the rod is in vibration in the direction of its length. In this case the ends A and B vibrate outwards from C and then back simultaneously while the centre remains perfectly steady.


Fig. 186.—Illustrating torsional vibrations.

These vibrations, in which the to-and-fro motion is in the direction of the length of the body, are said to be *longitudinal vibrations.*

161. Torsional Vibrations. By means of the apparatus shown in Fig. 186 a third kind of vibration can be illustrated.

A metal cylinder with a pointer attached to it, is suspended by a wire over a graduated circle. Twist the cylinder about its axis and let it go. The wire untwists and twists and the pointer moves back and forth around the disc performing *torsional vibrations.*

Next, grasp a piece of glass tubing about 2 cm. in diameter and 100 cm. long, at its middle with the left hand, and with a wet cloth in the right hand impart a twisting motion to the rod near the middle. The rod will twist and untwist like the wire in Fig. 186 and emit a high musical note.

Such vibrations are called *torsional vibrations.*

162. Conveyance of Sound to the Ear. In order that a sound may be perceived by our ears it is evident that some sort of medium must fill the space between the source and the ear. Usually air is this medium, but other substances can convey the sound quite as well.

By holding the ear against one end of a wooden rod, even a light scratch with a pin at the far end will be heard distinctly. One can detect the rumbling of a distant railway train by laying the ear upon the steel rail. The Indians on the western plains could, by putting the ear to the ground, detect the tramping of cavalry too far off to be seen. If two stones be struck together under water, the sound perceived by an ear under water is louder than if the experiment had been performed in the air.

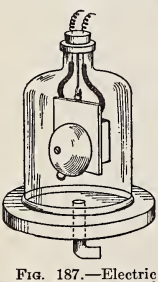

Fig. 187.—Electric bell in a jar connected to an air-pump. On exhausting the air from the jar the sound becomes weaker.

Thus we see that solids, liquids and gases all transmit sound. Further, we can show that some one of these is necessary.

Under the receiver of an air-pump, place an electric bell, supporting it as shown in Fig. 187. At first, on closing the circuit, the sound is heard easily, but if the receiver is now exhausted by a good air-pump, it becomes feebler, continually becoming weaker as the exhaustion proceeds.

If now the air, or any other gas, or any vapour, is admitted to the receiver, the sound at once gets louder.

In performing this experiment it is likely that the sound will not entirely disappear, as there will always be some air in the receiver, and, in addition, a slight motion will be transmitted to the pump by the suspension; but we are justified in believing that a vibrating body in a perfect vacuum will not excite the sensation of sound.

In this respect sound differs from light and heat which come to us from the sun and the stars, passing freely through the perfect vacuum of space.

163. Velocity of Sound in Air. It is a common observation that sound requires an appreciable time to travel from one place to another. If we watch a carpenter working at a distance, we distinctly see his hammer fall before we hear the sound of the blow. Also, steam may be seen coming from the whistle of a locomotive or a steamboat several seconds before the sound is heard, and we continue to hear the sound for the same length of time after the steam is shut off.

Some of the best experiments for determining the velocity of sound in air were made in 1822 by a commission appointed by the French Academy. The experiments were made between Montlhéry and Villejuif, two places a little south of Paris and 18.6 kilometres (or 11.6 miles) apart.

Each station was in charge of three eminent scientists and provided with similar cannon and chronometers. It was found that the interval between the moment of seeing the flash and the arrival of the sound was, on the average, 54.6 seconds. This gives a velocity of 340.9 m. or 1118.15 ft. per second. Now the temperature was 15.9° C., and as the velocity increases about 60 cm. per second for a rise of 1°C., this velocity would be 331.4 m. per second at 0° C. Other experimenters have obtained slightly different results.

VELOCITY OF SOUND IN AIR

Temperature	Velocity, Per Second
0° *C*. = 32° *F*.	332 m. = 1089 ft.
15° *C*. = 61° *F*.	341 m. = 1119 ft.
20° *C*. = 68° *F*.	344 m. = 1129 ft.
−45.6° *C*. = −50° *F*.	305.6 m. = 1002 ft.

Recent experiments made to determine the velocity of the sound produced by the discharge of cannon have shown that the velocity at a distance of 100 ft. from a 10-inch gun is about 1240 ft. per sec., or 22 per cent. above normal; at 200 ft. from the gun the velocity is only about 5 per cent. above normal; and for all distances above 500 ft. from the gun the velocity of the explosive sound from even the largest sized gun is practically normal.

The velocity at −50° F. was determined by Greely during his explorations in the Arctic regions, 1882-3.

164. Sound a Wave Motion. We have seen that for the hearing of a sound two things are necessary:—

(1) A vibrating body.

(2) A material medium extending from the vibrating body to the ear.

Just how is the disturbance which is produced by the vibrating body transmitted to the ear? Nothing material passes from one to the other. Close investigaton has led to the belief that the motion set up in the surrounding medium by the vibrating body spreads out from it in the form of waves, which, when they fall upon the drum of the ear, set it in vibration. This vibration affects the auditory nerve and we hear the sound.

We shall now proceed to learn something about wave-motion.

165. Characteristic of Wave-Motion. It is very interesting to stand on the shore of a large body of water and watch the waves, raised by a stiff breeze, as they travel majestically along. Steadily they move onward, until at last, crested with foam, they roll in upon the beach, breaking at our feet. The great ridges of water appear to be moving bodily forward towards us, but a little observation and consideration will convince us that such is not the case.

By watching a log, a sea-fowl or any other definite object floating on the surface, we see that, as the waves pass along, it simply moves up and down, not coming appreciably nearer to us.

We see, then, that the *motion* of the water is handed on but not the water itself. In the case of a flowing stream the water itself moves and, perhaps, turns our water-wheels. Equally certain it is, however, that energy (that is, ability to do work), is transmitted by waves. A small boat, though at the distance of several miles from the course of a great steamer, will, sometime after the latter has passed, experience a violent motion, produced by the "swells" of the large vessel. The water has not moved from one to the other, but it is, nevertheless, the medium by which considerable energy has been transmitted.

A peculiar characteristic of wave-motion is that, while the particles of water, or other medium, never move far from their ordinary positions of equilibrium, yet energy is transmitted from one place to another by means of the motion. In water waves, however, the particles do not simply move up and down. In deep water they move in circles in vertical planes, but as the water becomes shallow these circles are flattened into ellipses with their long axes horizontal.

166. Definition of Wave-length. A continuous series of waves, such as one can produce by moving a paddle back and forth in the water, or by lifting up and down a block floating on the water, is called a *wave-train.* The number of waves in such a train is indefinite; there may be few or many.

If now we look along such a train, we can select portions of it which are in exactly the same stage of movement, that is, which are moving in the same way at the same time. The distance between two successive similar points, such as A and B or C and D (Fig. 188), is called a wave-length. It is usual to measure from one crest to the next one, but any other similar points may be chosen.

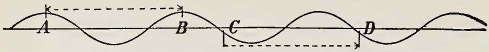

Fig. 188.—The distance AB or CD is a wave-length.

Particles which are at the same stage of the movement at the same time are said to be *in the same phase;* and so we can define a wave-length as *the shortest distance between any two particles whose motions are in the same phase.*

167. Refraction of Water Waves. It has often been observed that, when waves approach a shallow beach, the crests are usually approximately parallel to the shore line. In Fig. 189, A, B, C, etc., represent the successive positions of a wave approaching the shore. The dotted lines indicate the depth of water. It is seen that the end of the wave nearest the shore reaches shallow water first, and at once travels more slowly. This continues until at last the wave is almost parallel to the shore line.

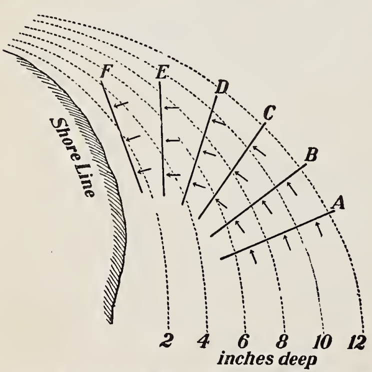

Fig. 189.—Diagram illustrating how a wave changes its direction of motion as it gets into shallower water, and is refracted.

This changing of the direction of the motion of the waves through a change in their velocity is called *refraction.*

168. Reflection of Waves. If, however, a train of water waves strike a precipitous shore or a long pier, they do not stop there, but start off again in a definite direction. This is illustrated in Fig. 190. The waves advance along *AB*, strike the pier and are reflected in the direction *BC*, the lines *AB*, *BC* making equal angles with *BD* the perpendicular to the pier. In sound and light we meet with many illustrations of reflection and refraction.

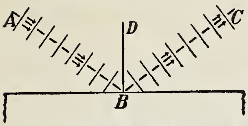

FIG. 190.—Water waves striking a long pier are reflected.

169. Transverse Waves in a Cord. Let one end of a light chain or rubber tube, 8 feet or more in length, be fastened to the ceiling or the wall of a room. Then, by shaking from side to side the free end, transverse waves will be formed and will pass freely along the tube. A rope or a length of garden hose lying on the floor may be used, but the results will not be so satisfactory.

We shall examine this motion more closely. Let us start with the tube straight as shown in (*a*), Fig. 191. The end *A* is quickly drawn aside through the space *AB*. The end particles drag the adjacent ones after them; these drag the next ones, and so on; and when the end ones have been pulled to *B*, the tube then has the form shown in (*b*).

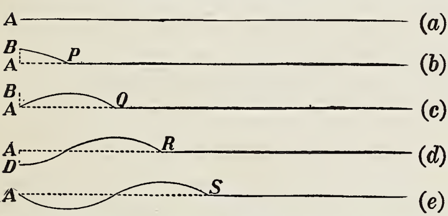

FIG. 191.—Diagram to show how a wave is formed and travels along a cord.

Instead of keeping the end at *B*, however, let it be quickly brought back to *A*, that is, the motion is from *A* to *B* and *B* to *A* without waiting at *B*. Now the particles between *B* and *P* have been given an upward movement, and their inertia will carry them further, each pulling its next neighbour after it, until when the end is brought back to A, the tube will have the form AQ, shown in (c).

Suppose, next, that the motion does not stop at A, but that it continues on to D. On arriving there the tube will have the form (d). Immediately let the end be brought back to A, thus completing the 'round trip.' The tube will now have the form shown in (e).

Notice (1) that the end has made a complete vibration, (2) that one wave has been formed, and (3) that the motion has travelled from A to S, which is a wave-length.

170. Relation between Wave-length, Velocity and Frequency. The time in which the end A executes a complete vibration is called its *period*, and the number of periods in a second is called its *frequency*, or *vibration number*. (§ 159)

We have just seen that during one period the wave-motion travels one wave-length.

Let the frequency be n per second; then the period T will be $1/n$ second.

If l = wave-length,
and v = velocity of transmission of the wave-motion;
then $l = vT$,
or $v = nl$.

This is a very important relation.

The *amplitude* of a vibration is the range on one side or the other of the middle point of the course. Thus AB or AD (Fig. 191) is the amplitude of the motion of the particle A.

171. Nodes and Loops. Next, let us keep the end of the rubber tube in continual vibration. A train of waves will steadily pass along the tube, and being reflected at the other end, a train will steadily return along it. These two trains will meet, each one moving as though it alone existed.

As the tube is under the action of the two sets of waves, the direct and reflected trains, it is easy to see that, while a direct wave may push downward any point on the tube a reflected one may lift it up, and the net result may be that the point will not move at all. The two waves in such a case are said to *interfere.*

That is just what does happen. By properly timing the vibrations of the end of the tube, the direct and reflected trains interfere, and certain points will be continually at rest.

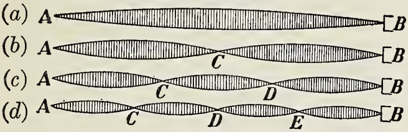

Fig. 192.—Standing waves in a cord. At A, C, D, E, B are *nodes*; midway between are *loops*.

If the end A (Fig. 192) is vibrated slowly, the tube will assume the form (a). On doubling the frequency, of vibration, it will take the form (b). By increasing the frequency other forms, such as shown in (c) and (d) may be obtained. In these cases the points A, B, C, D, E, are continually at rest and are called *nodes* The portion between two nodes is called a *ventral segment,* and the middle point of it we shall call a *loop.* The distance between two successive nodes is half a wave-length.

Such waves are called *stationary* or *standing* waves. As we have seen, they are caused by continual interference between the direct and the reflected waves.

172. Standing Waves. The most satisfactory method of producing the vibrations in a cord is to use a large tuning-fork, so arranged that the cord (which should be of silk, light and flexible) may be attached to one prong. In the absence of this the arrangement shown in Fig. 193

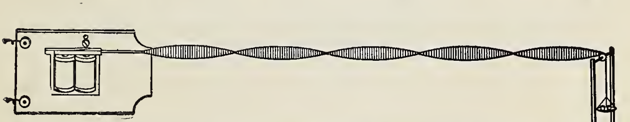

Fig. 193.—A cord is attached to the armature of an electric bell, and to the other end which passes over a pulley are added weights. By adjusting the length and the tension standing waves are produced.

may be used. The gong and the hammer of a large electric bell are removed. One end of the cord is attached to the armature, and the other passes over a pulley and has a pan to hold weights attached to it. In this way the length of the tension of the cord can be varied and the resulting standing waves studied.

The following law has been found to hold:—*The number of loops is inversely proportional to the square root of the tension.*

Instead of having the string pass over a pulley, it might be allowed to hang vertically with the weight tied on the end, the electric vibrator then being turned so that the armature is vertical. This arrangement, however, is not quite so satisfactory.

173. Longitudinal Waves in a Spiral Spring. Let us consider a long spiral spring (Fig. 194). It should be 2 or 3 m. long, and the diameter of the coils may be from 3 to 8 cm. To make such a spring one may take a hank of piano wire, slip it over a wire stretched between two supports and separate the coils, as shown in the figure. Now insert the hand between two coils at A and by a quick push compress the coils together.

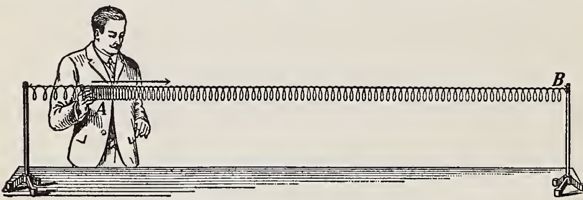

Fig. 194.—A convenient form of spiral spring.

In this way the turns of wire in front of the hand are crowded together, and the turns behind, for about the same distance, are pulled wider apart. The crowded part of the spiral may be called a *condensation*, the stretched part a *rarefaction*.

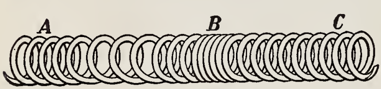

Fig. 195.—A wave consists of a condensation B, and a rarefaction A.

Now watch closely and you will see the condensation, followed by the rarefaction, run with great speed along the spiral, and on reaching the end it will give a sharp thrust against the support B. Here it will be reflected and will return to the other end, from which it may be reflected and again return to B.

If a light object be tied to the wire at any place, it will be seen, as the wave passes, to receive a sharp jerking motion forward and backward in the direction of the length of the spiral.

On a closer examination we find that by applying force with the hand to the spiral we produce a crowding together of the turns of wire in the section B, (Fig. 195) and a separation at A. Instantly the elastic force of the wire causes B to expand, crowding together the turns of wire in front of it (in the section C), and thus causing the condensation to be transmitted forward. But the coils in B do not stop when they have recovered their original position. Like a pendulum they swing beyond the position of rest, thus producing a rarefaction at B where immediately before there was a condensation. Thus the pulse of condensation as it moves forward will be followed by one of rarefaction.

174. Nature of a Sound-Wave. Let a flat strip of metal be clamped in a vice or be otherwise held in a rigid support. Draw it aside, and let go. As it moves forward it condenses the air before it, and on its return the air is rarefied. With

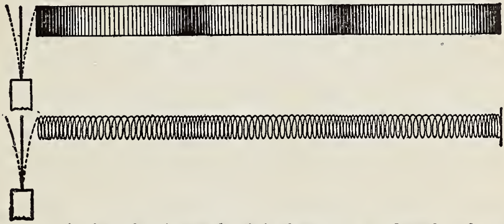

Fig. 196.—As the strip vibrates the air is alternately condensed and rarefied. A similar action takes place in the coil-spring.

each complete vibration a wave of condensation and rarefaction is produced, and during that time the sound will have travelled one wave-length, l. If the strip vibrates n times a second the space traversed in one second will be

$$nl = v, \text{ the velocity of sound.}$$

The sound, however, does not go in just one direction as shown in Fig. 196, but it spreads out in all directions, as illustrated in Fig. 197, where spherical waves move out from the sounding bell as their centre. It is evident that the vibrations in sound-waves are longitudinal.

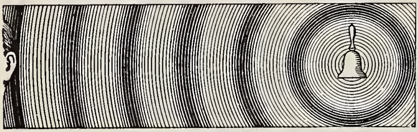

FIG. 197.—Illustrating the transmission of sound in spherical waves.

175. An Air-Wave Encircling the Earth. A wonderful example of the spread of an air-wave occurred in 1883. Krakatoa is a small island between Java and Sumatra, in the East Indies, long known as the seat of an active volcano. Following a series of less violent explosions, a tremendous eruption occurred at 10 a.m. of August 27. The effects were stupendous. Great portions of the land, above the sea and beneath it, were displaced, thus causing an immense sea-wave which destroyed 36,000 human lives, at the same time producing a great air-wave, which at once began to traverse the earth's atmosphere. It spread out circularly, gradually enlarging until it became a great circle to the earth, and then it contracted until it came together at the antipodes of Krakatoa, a point in the northern part of South America. It did not stop there, however, but, enlarging again, it retraced its course back to its source. Again it started out, went to the antipodes and returned. A third time this course was taken, and indeed it continued until the energy of the wave was spent.

The course taken by the wave was traced by means of self-registering barometers located at various observing stations throughout the world. As the wave passed over a station, there was a rise and then a fall in the barometer, and this was recorded by photographic means. In many places (Toronto included) there were four records of the wave as it moved from Krakatoa to the antipodes, and three of its return. In Fig. 198 is shown the rise in the barometer at Toronto caused by the second outward trip of the wave and the second return. The time required to go to the antipodes and return to Krakatoa was approximately 36 hours.

The sound of the explosion was actually heard, four hours after it happened, by human ears at Rodriguez, at a distance of over 2,900 miles

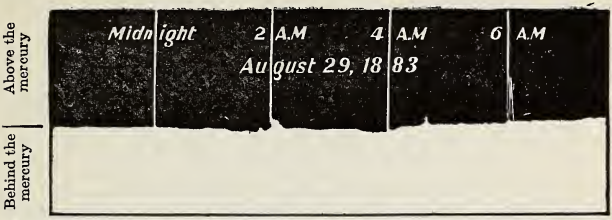

FIG. 198.—A portion of the photographic record of the height of the barometer at Toronto for August 29, 1883. To obtain the record, light is projected through the barometer tube above the mercury against sensitized paper which is on a drum behind the barometer. Every two hours the light is cut off and a white line is produced on the record. Shortly after 2 a.m., August 29, there was a rise, and at about 4.40 there was another. The former was due to the passage over Toronto of the wave on the second journey from Krakatoa to the antipodes; the latter was due to the second return from the antipodes to Toronto. (From the records of the Meteorological Service, Toronto.)

to the south-west. At the funeral of Queen Victoria, on February 1, 1901, the discharges of cannon were heard 140 miles away.

176. Velocity of Sound in Solids by Kundt's Tube. Having determined the velocity of sound in air, we can determine it in other gases and in solids by a method devised by Kundt in 1865.

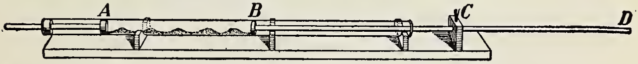

FIG. 199.—The little heaps of powder in the tube are produced by vibrations of the disc *B*.

BD (Fig. 199) is a brass rod about 80 or 100 cm. long and 8 or 10 mm. in diameter, securely clamped at the middle. To the end *B* is attached a disc of cork or other light substance, which fits loosely into a glass tube about 30 or 35 mm. in diameter. *A* is a rod on the end of which is a disc, which slides snugly in the tube, thus allowing the distance between *A* and *B* to be varied. Dried precipitated silica, or simply powder made by filing a baked cork, is scattered along the lower side of the tube.

Now with a dry cloth or piece of chamois skin, on which is a little powdered rosin, stroke the outer half of the rod. With a little practice one can make the rod emit a high musical note. At the same time the powder in the tube is agitated, and by careful adjustment of *A*, the powder will at last gather into little heaps at regular intervals.

We must now carefully measure the length of the rod and also the distance between the heaps of powder, taking the average of several experiments.

By stroking the half CD of the rod we make it alternately lengthen and shorten, and the half BC elongates and shortens in precisely the same way. Thus the mid-point of the rod remains at rest, while all other portions of the rod vibrate longitudinally, the ends having the greatest amplitude.

It is evident that the middle of the rod is a node and the ends loops (§ 171), and hence if we had a very long rod and each part of it of length BD were vibrating in the same way, we should have standing waves in the brass rod, and BD would be one-half the wave-length.

Again, as the piston at B moves forward it compresses the air in front of it and as it retreats it rarefies the air. These air-waves travel along the tube and are reflected at A and return. The two sets of waves thus meet and interfere, producing stationary waves as explained in § 171. The powder gathers at the nodes, and hence the distance between the nodes is one-half the wave-length in air of the note emitted by the brass rod.

Example.—In an experiment with a brass rod 5 ft. long the nodes were found to be $5\frac{3}{4}$ in. apart. The temperature of the room was 19° C. Find the frequency of the rod and the velocity of sound in brass.

Velocity of sound in air at 19° C. = 1127 ft. per sec.

Node to node in air = $5\frac{3}{4}$ in., and hence wave-length in air = $11\frac{1}{2}$ in.

Now $v = nl$, and substituting the values for v and l,

$1127 \times 12 = n \times 11\frac{1}{2}$, from which $n = 1176$ vibrations per sec.

Also, loop to loop in brass = 5 ft. and wave-length in brass = 10 ft., and substituting in $v = nl$, we have $v = 1176 \times 10 = 11{,}760$ ft. per sec.

By using rods of different metals we can find the velocity in each of them.

177. Velocity in Different Gases. The same apparatus can be used for different gases. To do so it is arranged as shown in Fig. 200.

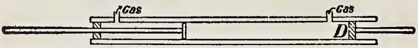

Fig. 200.—Kundt's method of finding the velocity of sound in different gases.

For this purpose a glass rod is preferable. It vibrates more easily by using a damp woollen cloth. It is waxed into the cork through which it passes. The piston D must be reasonably tight.

As before, measure the distance between adjacent heaps when the tube is filled with air. Let it be 5 in. Now fill it with carbon dioxide and let the distance be 4 in.

Then $v = nl$, and taking the velocity in air as 1120 ft. per sec.

$$1120 \times 12 = n \times 10, \text{ and } n = 1344 \text{ vibrations per sec.}$$

For carbon dioxide $v = nl$ and $v = \dfrac{1344 \times 8}{12} = 896$ ft. per sec.

Velocity of Sound in Solids, Liquids and Gases

Substance	Temperature	Velocity		Substance	Temperature	Velocity	
	°C.	m. per sec.	ft. per sec.		°C.	m. per sec.	ft. per sec.
Aluminium	....	5104	16740	Water......	9	1435	4708
Brass.....	....	3500	11480	Carbon dioxide.......	0	261.6	858
Copper...	20	3560	11670	Illuminating gas......	0	490.4	1609
Copper...	100	3290	10800	Oxygen.....	0	317.2	1041
Iron......	20	5130	16820				
Maple....	....	4110	13470				

QUESTIONS AND PROBLEMS

1. Calculate the velocity of sound in air at 5°, 10°, 40° C. (See § 163.)

2. An air-wave travelled about the earth (diameter 8000 miles) in 36 hours. Find the velocity in feet per second.

3. A thunder-clap is heard 5 seconds after the lightning flash was seen. How far away was the electrical discharge? (Temperature, 15° C.)

4. The velocity of a bullet is 1200 feet per second, and it is heard to strike the target 6 seconds after the shot was fired. Find the distance of the target. (Temperature, 20° C.)

5. At Carisbrook Castle, in the Isle of Wight, is a well 210 feet deep and 12 feet wide, the interior being lined with smooth masonry. A pin dropped into it can easily be heard to strike the water. Explain.

Find the interval between the moment of dropping the pin and that of hearing the sound. (Temperature, 15° C., $g=32$.)

6. In sound-ranging the explosion from an enemy gun is registered by a series of microphones arranged at intervals near the front-line trenches. If the sound-wave reaches three microphones A, B and C, so that B registers 1.5 sec. after A and C 2 sec. after A, how much farther is the gun from B and C than it is from A, the velocity of sound being 1120 ft. per sec. (To find the position of the gun draw circles with these distances as radii about B and C, and draw a third circle to pass through A and touch the circles. The gun is at the centre of this circle (Fig. 201).

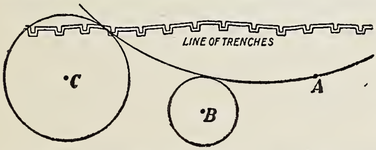

Fig. 201.—Locating an enemy gun by sound-ranging.

7. In 1826 two boats were moored on Lake Geneva, Switzerland, one on each side of the lake, 44,250 feet apart. One was supplied with a bell B (Fig. 202a), placed under water, so arranged that at the moment it was struck a torch m lighted some gunpowder in the pot P. The sound was heard at the other boat by an observer with a watch in his hand and his ear to an ear-trumpet, the bell of which was in the water. The sound was heard 9.4 seconds after the flash was seen. Calculate the velocity of sound in water.

FIG. 202a.—Apparatus for producing the sound, in Lake Geneva.


FIG. 202b.—Listening to the sound from the other side of the Lake.

8. In a Kundt's tube a brass rod is 1 m. long, and five of the intervals between the dust-heaps equal 49.5 cm. Find the velocity of sound in brass. (Temperature, 20° C.)

9. When a Kundt's tube is filled with hydrogen, the dust-heaps are 3.8 times as far apart as with air. Find the velocity of sound in hydrogen (Temperature, 20° C.)

10. The sound of a gun was heard 15 seconds after the flash was seen. If the temperature was 10°C., how far was the gun from the observer?

REFERENCES FOR FURTHER INFORMATION

Bragg, *The World of Sound*, Lectures 3, 6.
Catchpool, *Sound*, Chaps. 3, 9.
Poynting and Thomson, *Sound*, Chap. 2.

CHAPTER XVIII

Intensity, Pitch, Musical Scales

178. Musical Sounds and Noises. The slam of a door, the fall of a hammer, the crack of a rifle, the rattling of a carriage over a rough pavement,—all such disconnected, disagreeable sounds we call *noises*; while a note, such as that yielded by a plucked guitar string or by a flute, we at once recognize as musical.

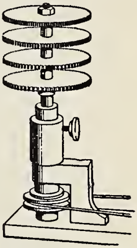

Fig. 203.—Toothed wheels on a rotating machine. On holding a card against the teeth a musical sound is heard.

A musical note is a continuous, uniform and pleasing sound; while a noise is a shock, or an irregular succession of shocks, received by the ear.

Against the teeth on a rotating disc (Fig. 203) hold a card. When the speed is slow, we hear each separate tap as a noise, but as it is increased these taps at last blend into a clear musical note.

The same result, with a rather more pleasing effect is obtained by sending a current of air through holes regularly spaced on a circle near the circumference of a rotating disc (Fig. 204). The little puffs through the holes blend into a pleasing note.

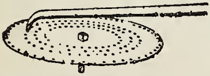

Fig. 204.—Air is blown through the holes in the rotating plate.

It is possible for a number of musical notes to be so jumbled together that the periodic nature is entirely lost, and then the result is a noise. If the holes in the disc (Fig. 204) are irregularly spaced we get a noise, not a musical note.

A musical tone is due to rapid periodic motion of a sounding body; a noise is due to non-periodic motion.

179. Distinguishing Features of Sounds. Experience has taught us to recognize three features by which musical sounds can be distinguished from one another, namely,

(1) *Intensity or Loudness,* (2) *Pitch,* (3) *Quality.*

A violinist tunes a string on his instrument to a key on a piano. The notes are then of the same *pitch* no matter what the relative *intensities* may be; but they differ in an essential something called *quality,* which enables a person, even in the dark, to recognize that the one note comes from a violin and the other from a piano.

Quality will be considered more fully at a later stage.

180. Intensity of Sound. The intensity of sound depends on three things:—

(1) *The Energy of the Vibrating Body.* The amount of energy radiated per second is proportional to the square of the amplitude of the vibrating body.

(2) *The Density of the Medium in which it is produced.* It is found that workmen in a tunnel in which the air is under pressure, though conversing naturally, appear to one another to speak in unusually loud tones, while balloonists and mountain climbers have difficulty in making themselves heard when at great heights. The denser the medium, the louder is the sound.

(3) *The Distance of the Ear from the Sounding Body.* Suppose the sound to be radiating from O (Fig. 205) as centre, and let it travel a distance OA in one second. The energy will be distributed amongst the air particles on the sphere whose centre is O and radius OA.

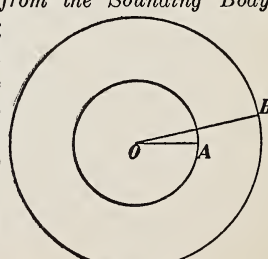

Fig. 205.—Diagram to show that the intensity of sound diminishes with the distance from the source.

In two seconds it will reach a distance OB, which is twice OA, and the energy which was on the smaller sphere will now be spread over the surface of the larger one. But this surface is *four* times that of the smaller, since the surface of a sphere is proportional to the square of its radius. Hence the intensity at B can be only one-fourth that at A, and we have the law that *the intensity of a sound varies inversely as the square of the distance from the source.*

181. Transmission by Tubes. If, however, the sound is confined to a tube, especially a straight and smooth one, it may be transmitted great distances with little loss in intensity. Being prevented from expanding, the loss of the energy of the sound-waves is caused chiefly by friction of the air against the sides of the tube. If two rooms in a house are joined by a tube, words spoken in a low tone at one end will be heard at the other.

182. Reflection of Sound. Every one has heard an echo. A sharp sound made before a large isolated building or a steep cliff, at a distance of 56 feet or more, is returned as an echo. The sound-waves strike the flat surface and are reflected back to the ear.

When there are several reflecting surfaces at different distances from the source of sound, a succession of echoes is heard. This phenomenon is often met with in mountainous regions.

In Europe there are many places celebrated for the number and beauty of their echoes. An echo in Woodstock Park (Oxfordshire, England) repeats 17 syllables by day and 20 by night. Tyndall says: "The sound of the Alpine horn, echoed from the rocks of the Wetterhorn or the Jungfrau [in Switzerland] is in the first instance heard roughly. But by successive reflections the notes are rendered more soft and flute-like, the gradual diminution of intensity giving the impression that the source of sound is retreating farther and farther into the solitude of ice and snow."

The laws of reflection of sound are the same as those of light (see § 328). Let a watch be hung at the focus of a large concave mirror (Fig. 206.) The waves strike the mirror and are returned, as shown in the figure, being brought to a focus again by a second mirror. On holding at this focus a funnel from which a rubber tube leads to the ear, the sound may be heard, even though the mirrors are a considerable distance apart.

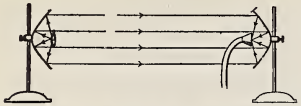

FIG. 206.—A watch is held in the focus of one concave reflector, and the ticking is heard at the focus of the other. (The foci can be located by means of rays of light.)

The Mormon Tabernacle at Salt Lake City, Utah, is an immense auditorium, elliptic in shape, 250 feet long, 150 feet wide and 80 feet high, with seating accommodation for 8000 people. A pin dropped on a wooden railing near one end, or a whisper there is heard 200 feet away at the other end with remarkable distinctness.

The bare walls of a hall are good reflectors of sound, though usually the dimensions are not great enough to give a distinct echo, but the numerous reflected sound-waves produce a *reverberation,* which appears to make the words of the speaker run into each other, and thus prevents them being distinctly heard. By means of cushions, carpets and curtains, which absorb the sound which falls upon them instead of reflecting it, this reverberation can be largely overcome. The presence of an audience has the same effect. Hence, a speaker is heard much better in a well-filled auditorium than in an empty one.

The captain of a ship can sometimes determine his distance from a shore by blowing a sharp blast on the whistle and counting the seconds until the echo is heard.

183. The Submarine Bell; Sounding the Ocean by Sound-waves. A valuable application of the fact that water is a good conductor of sound is made in a method recently introduced for warning ships from dangerous places. Light-houses and fog-horns have long been used, but the condition of the atmosphere often renders these of no avail. Submarine signals, however can be depended upon in all kinds of weather.

The submarine bell, which sends out the signals (Fig. 207a), is hung from a tripod resting at the bottom of the water or is suspended from a lightship or a buoy. The striking mechanism is actuated by compressed air or electricity supplied from the shore or the lightship.

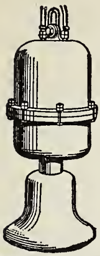

FIG. 207a.—Submarine bell, worked by compressed air supplied from the shore. The mechanism for moving the hammer of the bell is contained in the upper chamber.

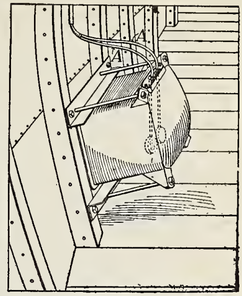

FIG. 207b.—The sound from the bell is received by two tanks placed in the forepeak of the ship, one on each side. The tank is filled with salt water, and the ship's outer skin forms one of its sides. In the water are two microphones, which are connected by wires A, A to two telephone receivers up in the pilot-house.

The receiving apparatus is carried by the ship. Two iron tanks are located in the bow of the vessel, one on each side (Fig. 207b). These tanks are filled with salt water, and the ship's outer skin forms one side of the tank. Suspended in each tank are two microphones (§ 515), which are connected to two telephone receivers up in the pilot-house. The officer on placing these to his ears can hear sounds from a bell even when more than 15 miles away; and by listening alternately to the sounds from the two tanks he can accurately locate the direction of the bell from him. Signal stations are to be found on the shores of various countries, several being located in the lower St. Lawrence and about the maritime provinces of Canada. Apparatus very similar to this was used during the war to detect the presence of a submarine.

Very recently an efficient method of determining the depth of the ocean by means of sound-waves has been devised, the operation being

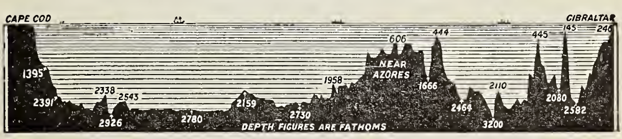

FIG. 208.—Contour of ocean floor between Cape Cod and Gibraltar.

very similar to that just described. The sound-producer, and also the sound-receivers, are carried on the ship. A sound is produced, and the exact instant when it starts out is observed. It travels to the floor of the ocean, is reflected there and returns, and the precise instant when it gets back is also observed. In the interval between these two instants the sound has travelled twice the depth of the ocean, and, knowing the velocity in sea-water, the depth can be calculated. In Fig. 208 is shown the contour of the ocean floor as determined by a U.S. Navy ship on a trip from Cape Cod to Gibraltar, the passage being made at 17 miles per hour.

184. Determination of Pitch. The number of vibrations corresponding to any given pitch may be determined by various devices. One is the toothed wheel shown in Fig. 203. Suppose we wish to find the number of vibrations of a tuning-fork. The speed of rotation is increased until the sound given by the wheel is the same as that by the fork. Then the speed is kept constant for a certain time—say half a minute—and the number of turns of the crank in this time is counted and the rotations of the wheel deduced. Then on multiplying this number by the number of teeth on the wheel, we can at once deduce the number of vibrations per second. The perforated disc may be used in the same way.

A more satisfactory instrument is that shown in Fig. 209 and known as a siren. It was invented by Cagniard de la Tour in 1819.

A perforated metal disc B rotates on a vertical axis, just above a cylindrical air-chamber C. The upper end of the chamber and also the disc are perforated at equal intervals along a circle which has as centre the axis of rotation. The upper and lower holes correspond in number, position and size, but they are drilled obliquely, those in the disc sloping in a direction opposite to those in the end of the chamber. The tube D is connected with a bellows or other blower.

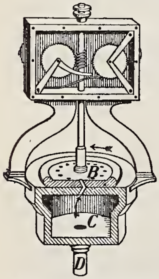

Fig. 209.—The siren. Air enters the chamber C by way of the pipe D, and on escaping causes the disc B to rotate.

When the air is forced into the chamber and passes up through the holes, the disc is made to rotate by the air-current striking against the sides of the holes in the disc, and the more powerful the air-current the more rapid is the rotation.

Vibrations in the air are set up by the puffs of air escaping above the disc as the holes come opposite each other; and by controlling the air supply, we can cause the disc to rotate at any speed, and thus obtain a sound of any desired pitch.

Having obtained this sound, a mechanical counter, in the upper part of the instrument, is thrown in gear and, keeping the speed constant for any time, this will record the number of rotations. The number of vibrations is obtained at once by multiplying the number of rotations by the number of holes in the disc and dividing by the number of seconds in the interval.

A method depending on the principle of resonance is described in § 201.

185. Limits of Audibility of Sounds. Not all vibrations, even though perfectly periodic, can be recognized as sounds, the power of detecting these varying widely in different persons. For ordinary ears the lowest frequency which causes the sensation of a musical tone is about 30 per second, the highest is between 10,000 and 20,000 per second.

In music the limits are from about 40 to 4000 vibrations per second, the piano having approximately this range. The range of the human voice lies between 60 and 1300 vibrations per second, or more than $4\frac{1}{2}$ octaves; a singer ordinarily has about two octaves.

186. Musical Combinations or Chords. A musical note is pleasing in itself, but certain combinations of notes are especially agreeable to the ear. These have been recognized amongst all nations from the earliest times, having been developed purely from the aesthetic or artistic side. The older musicians knew nothing about sound-waves and vibration numbers; they only knew what pleased the heart and expressed its emotions.

But on measuring the frequencies of the notes of the pleasing combinations, we find that the ratios between them are peculiarly simple, and indeed that the more pleasing any combination is, the simpler are the ratios between the frequencies of the notes.

187. The Octave. Pitch depends only on the number of vibrations per second; but as we compare notes of different pitch with one another—for instance the notes on a piano—

we are struck with the fact that when we have gone a certain distance upwards or downwards, the notes appear to repeat

FIG. 210.—Central part of a piano key-board. The notes marked C_2, C_1, C, C', C'', go up by octaves.

themselves. Of course the pitch is different, but there is a wonderful similarity between the notes.

On investigation a remarkable relation between the frequencies of the notes is revealed. Thus one note appears to be the repetition of another when their frequencies are as 2 to 1, and one is said to be an *octave* above the other. The combination of a note and its octave is the most pleasing of all.

Between the note and its octave custom has introduced six notes, the eight notes thus obtained usually being designated in music thus:—

$$C \quad D \quad E \quad F \quad G \quad A \quad B \quad C'$$

As we pass from C to C' by these interpolated notes we do so by steps which are universally recognized as the most pleasing to the ear. This series of notes is called the *natural* or *major diatonic* scale.

188. Major Diatonic Scale. By actual experiment it has been found that, whatever the absolute pitch may be—whether high up in the treble, or low down in the bass—the ratios between the frequencies of the different notes are constant.

Suppose the note C has a frequency 256. The entire scale is as follows:—

	C	D	E	F	G	A	B	C'
	256	288	320	$341\frac{1}{3}$	384	$426\frac{2}{3}$	480	512,
the ratios being	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2.

These ratios hold, whatever the absolute frequency may be. By international agreement the frequency of middle C of the piano is taken as 261, that of the A string of a violin being 435 vibrations per second. The numbers for the scale are then,

261 293.6 326.2 348 391.5 435 489.4 522.

189. Musical Chords. Two or more notes sounded simultaneously constitute a *chord*. If the effect is agreeable it is called *concord;* if disagreeable, *discord.*

The most perfect chord is C, C', the ratio between the frequencies being $\frac{2}{1}$. The next is C, G, the ratio being $\frac{3}{2}$. Note that in these ratios we use only the small numbers 1, 2, 3.

When the notes C, E, G are sounded together, the effect is extremely pleasing. This combination is called the Major Triad, and when C' is added to it we get the Major Chord. The frequencies of the triad have the ratios:

$$C : E : G = 4 : 5 : 6.$$

A close examination of the Major Scale shows that it is made up of repetitions of this triad. Thus C, E, G; F, A, C' and

$$C \quad D \quad E \quad F \quad G \quad A \quad B \quad C' \quad D'$$

G, B, D' are all major triads.

190. The Scale of Equal Temperament. In musical composition C is not always used as the first or key-note of the scale, but any note may be chosen for that purpose. But with each new key-note several new notes must be introduced, and at last the total number of notes becomes so great that it would be impracticable to construct an instrument with fixed notes, such as the piano or the organ, to play in all the keys.

The difficulty is overcome by *tempering* the scale, that is, by slightly altering the intervals. In the *scale of equal temperament* the octave contains 13 notes represented by the eight white keys and the five black keys of the piano, and the ratios between the frequencies of adjacent notes are all equal. Taking the key-note to have a frequency of 256 the tempered scale is as follows:—

C	*D*	*E*	*F*	*G*	*A*	*B*	*C′*
256	287.3	322.5	341.7	383.6	430.5	483.2	512.

191. The Harmonic Scale. When a note is sounded on certain musical instruments, a practised ear can usually detect, in addition to the fundamental or principal tone, tones of other frequencies. These are much less intense than the principal tone. If the frequency of a tone is represented by 1, those tones with frequencies corresponding to 2, 3, 4, 5 . . . . are said to be harmonics of the tone 1, which is called their fundamental. The entire series is known as the *Harmonic Scale.*

In the piano these harmonics are prominent. In the tuning-fork, when properly vibrated, the harmonics almost instantly disappear, leaving a pure tone.

QUESTIONS AND PROBLEMS

1. From what experience would you conclude that all sounds, no matter what the pitch may be, travel at the same rate?

2. Why does the presence of an audience improve the acoustic properties of a hall?

3. Explain the action of the ear-trumpet and the megaphone or speaking-trumpet.

4. A man standing before a precipice shouts, and 3 sec. afterwards he hears the echo. How far away is the precipice? (Temperature, 15° C.)

5. The wave-length of a sound, at temperature 15° C., is 5 inches. Find its frequency.

6. A toothed wheel has 65 teeth and revolves 240 times per minute. Find the frequency of the note produced when a card is held against the teeth. Calculate also the wave-length of the sound, taking the velocity as 1120 feet per second.

7. Find the vibration numbers of all the *C*'s on the piano, taking middle *C* as 261.

8. A wheel with 30 teeth is revolved so that when a card is held against the teeth a sound one octave above middle C, which has 261 vibrations per second, is heard. How many revolutions per minute does the wheel make?

9. Why does the sound of a circular saw fall in pitch as the saw goes farther into the wood?

10. If the vibration number of C is 300, find those for F and A .

11. If the frequency of A were 452, what would be that of C?

12. Find the wave-length of D^{IV} (*i.e.*, four octaves above D) in air at 0° C., taking the frequency of C as 261.

13. Which note has 3 times the number of vibrations of C? Which has 5 times?

14. The strings of a guitar are tuned to E_1, A_1, D, G, B, E'. If the D string has 293.6 vibrations per second, find the vibration-frequencies of the others.

15. The counter of a siren (§ ~~148~~ 184) registered 360 revolutions in 30 seconds. If the number of holes in the disc was 48, find the frequency of the note emitted by the instrument.

REFERENCES FOR FURTHER INFORMATION

Bragg, *The World of Sound*, Lecture 2.
Catchpool, *Text-book of Sound*, Chaps. 5, 8.
Poynting and Thomson, *Sound*, Chap. 3.

CHAPTER XIX

Vibrations of Strings, Rods, Plates and Air Columns

192. The Sonometer. The vibrations of strings are best studied by means of the sonometer, a convenient form of which is shown in Fig. 211. The strings are fastened to steel

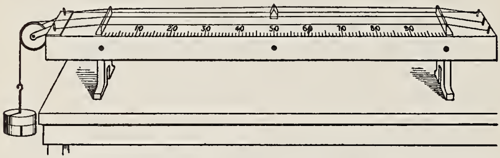

Fig. 211.—A sonometer, consisting of stretched strings over a thin wooden box. By means of a bridge we can use any part of a string.

pins near the ends of the instrument, and then pass over fixed bridges near them. The tension of a string can be altered by turning the pins with a key, or we may pass the string over a pulley and attach weights to its end. A movable bridge allows any portion of a string to be used. The vibrations are produced by a bow, by plucking or by striking with a suitable hammer.

The thin wooden box which forms the body of the instrument strengthens the sound. If the ends of a string are fastened to massive supports, stone pillars for instance, it emits only a faint sound. Its surface is small and it can put in motion only a small mass of air. When stretched over the light box, however, the string communicates its motion to the bridges on which it rests, and these set up vibrations in the wooden box. The latter has a considerable surface and impresses its motion upon a large mass of air. In this way the volume of the sound is multiplied many times.

The motions which the bridges and the box undergo are said to be *forced* vibrations, while those of the string are called *free* vibrations.

193. Laws of Transverse Vibrations of a String. First, take away the movable bridge and pluck the string. It vibrates as a whole and gives out its *fundamental* note. Then place the bridge under the middle point of the string, and pluck again, thus setting one-half of the string in vibration. The note is now an octave above the former note.* We thus obtain twice the number of vibrations by taking half the length of the string.

If, further, we take lengths which are $\frac{8}{9}$, $\frac{4}{5}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{15}$ of the full length of the string, we secure six notes which, with the fundamental and its octave, comprise the major scale. Now from § 188 we see that the relative frequencies of the notes of the scale are proportional to the reciprocals of these fractions, and hence we deduce the following important

LAW OF LENGTHS—*The number of vibrations per second of a string is inversely proportional to its length.*

Now let us see what is the effect of making the string thicker. Let us use a string of the same material but of twice the diameter. We find that the number of vibrations obtained is one-half as great. If the diameter is made three times as great, the number of vibrations is reduced to one-third; and so on. In this way we obtain the

LAW OF DIAMETERS.—*The number of vibrations per second is inversely proportional to the diameter of the string.*

On testing strings of different materials we reach the

LAW OF DENSITIES.—*The number of vibrations per second is inversely proportional to the square root of the density.*

Example:—If we take wires of steel (s.g. 7.86) and of platinum (s.g. 21.50) of the same diameter, length and under the same tension, the number of vibrations of the steel wire will be $\sqrt{\frac{21 \cdot 50}{7 \cdot 86}} = 1.65$ times that of the other.

Next, let the tension of one of the strings be so altered that it emits the same note as does that one with the weight on the end of it. Then let us keep adding to the weight until the string gives a note which is one octave higher, that is, the

*By running up the successive notes of the scale the ear will recognize the octave when the string is just half the entire length.

note now obtained is in unison with that obtained from the other string when the movable bridge is put under its middle point.

It will be found that the new weight is four times the old one. Thus we see that, in order to obtain twice the number of vibrations, we have to multiply the tension 4 times. In order to obtain 3 times the number of vibrations, we must multiply the tension 9 times; and so on. In this way we obtain the fourth important law, namely, the

LAW OF TENSIONS.—*The number of vibrations per second is proportional to the square root of the stretching weight.*

194. Nodes and Loops in a Vibrating String. The production of nodes and loops in a vibrating string can be beautifully exhibited on the sonometer (Fig. 212).

Place five little paper riders on the wire at distances $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{2}$, $\frac{5}{8}$ of the wire's length from one end. Then while a tip of the finger or a feather is gently held against the string at a distance $\frac{1}{4}$ of the length from the other end, carefully vibrate the string with a bow. The string will break up into nodes and loops, as shown in the figure, the little riders keeping their places at the nodes but being thrown off at the loops. The note emitted will be 2 octaves above the fundamental, with a frequency 4 times that of the latter.

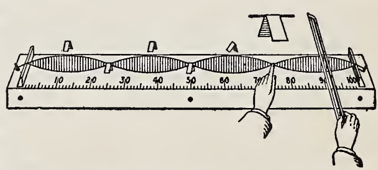

FIG. 212.—Obtaining nodes and loops in a vibrating string. The paper riders stay on at the nodes, but are thrown off at the loops.

In the same way, though somewhat more easily, the string can be made to break up into 2 or 3 segments. To obtain 2 segments, touch the string at the middle point; for 3 segments, touch it $\frac{1}{3}$ of the string's length from the end. In both cases, of course, the paper riders must be properly placed.

195. Simultaneous Production of Tones. When a string vibrates, as a whole, as shown in Fig. 213*a*, it emits its fundamental tone. To emit its first harmonic or overtone it should assume the form shown in (*b*). In the same way the forms assumed when giving the higher overtones can easily be drawn.

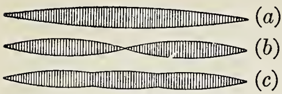

FIG. 213.—How a string vibrates when giving (*a*) its fundamental, (*b*) its first harmonic, (*c*) both of these together.

Now it is practically impossible to vibrate the string as a whole without, at the same time, having it divide and vibrate in segments. Thus with the fundamental tone of the string will be mingled its various harmonics.

The relative strengths of these harmonics will depend on the manner in which the string is put in vibration,—whether by a bow, by plucking or by striking it at some definite point. The sound usually described as "metallic" is due to the prominence of higher harmonics.

In Fig. 213*c* is shown the actual shape of the string obtained by combining (*a*) and (*b*), that is, by adding the first harmonic to the fundamental.

196. Vibrations of Rods. The vibration of a rod clamped at its middle and stroked longitudinally has been described in § 176, in connection with Kundt's tube.

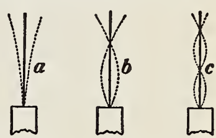

FIG. 214.—Vibrations of a rod clamped at one end.

But a rod may vibrate transversely also. Let it be clamped at one end, and the other end be drawn aside and let go. Ordinarily it will vibrate as in Fig. 214 *a*, in which case it produces its fundamental tone. But it may vibrate as illustrated in *b* and *c*, emitting its overtones.

The vibrations are due to the elasticity of the rod. The investigation of these transverse vibrations is somewhat complicated and difficult, but the following simple law has been found to be true:—

LAW OF TRANSVERSE VIBRATIONS OF RODS—*The number of vibrations per second varies inversely as the square of the length of the rod and directly as its thickness.*

Example.—A dinner gong composed of four tubes tuned to give a musical chord illustrates well the law of transverse vibrations of rods and can easily be made. Brass tubing about $\frac{3}{4}$ inch in diameter is suitable. Suppose we wish the tubes to give the notes C, E, G, C' of the diatonic scale (§ 189), the frequencies of which are proportional to 4, 5, 6, 8, respectively. The thickness (diameter) of the tubes is the same and so need not be taken into account. Let the length of the longest tube (C) be 18 inches and that of the next (E) be x inches. Then the above law says that the frequency of C is to the frequency of E inversely as the square of 18 is to the square of x, or, in symbols, $\frac{4}{5} = \left(\frac{x}{18}\right)^2$, from which $x = 16.1$ inches. Similarly the lengths of the G and C' tubes are found to be 14.7 and 12.7 inches, respectively.

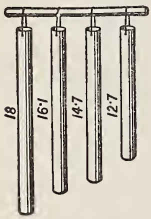

FIG. 215.—A dinner gong.

Make the tubes of these lengths and suspend them by cords through holes near one end, as in Fig. 215. Strike with a wooden or rubber hammer and they will give the major chord.

In place of tubes metal bars may be used.

197. Tuning-Fork. A tuning-fork may be considered as a rod which is bent and held at its middle point. When it vibrates, the two prongs alternately approach and recede, while the stem has a slight motion up and down. Why this is so may be seen from Fig. 216. In 1, N, N represent the nodes when the straight bar is made to vibrate. As the bar is bent more and more, the nodes approach the centre, and when the fork is obtained (as in 2), the nodes are so close together that the motion of the stem is very small. That it exists, however, can be readily shown.

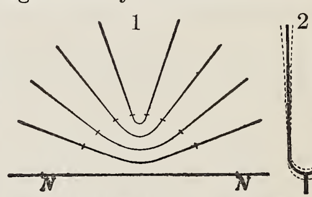

FIG. 216.—How a tuning-fork vibrates.

If a fork, after being set in vibration, is held in the hand, it will continue in motion for a long time. It gives up its energy slowly, and so the sound is feeble. But if the stem is pressed against the table, the sound is much louder. Here the stem produces forced vibrations in the table, and a large mass of air is thus put in motion. In this case the energy of the fork is used up rapidly and the sound soon dies away.

Tuning-forks are of great importance in the study of sound. When set in motion by gentle bowing, the overtones, if present at all, die away very rapidly.

With a rise in temperature the elasticity of the steel is diminished and the pitch is slightly lowered.

198. Vibrations of Plates. The plates used in the study of sound are generally made of brass or glass, and are ordinarily square or circular in shape. The plate is held by a suitable clamp at its centre, and is made to vibrate by a violin bow drawn across the edge.

Let us scatter some sand over a square plate, and while a finger-tip touches it at the middle of one side, draw the bow across the edge near one corner. At once a clear note is given, and the sand takes up the figure shown in Fig. 217*a*. If the corner is damped with the finger-tip and the bow is applied at the middle of a side, the form shown in *b* is assumed, and the note is higher than the former. By damping with two finger-tips the form *c* is obtained and a much higher note is produced.

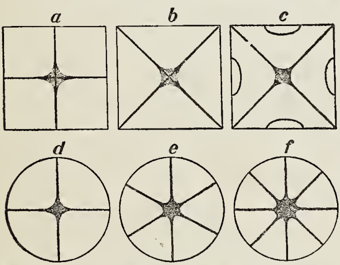

FIG. 217.—Sand-figures showing nodal lines in vibrating plates.

The sand is tossed away from certain parts of the surface and collects along the *nodal* lines, that is, those portions which are at rest.

Some of the forms assumed by the sand when a circular plate is vibrated are shown in *d*, *e*, *f*. The sand-figures always reveal the character of the vibration, and the more complicated the figure, the higher pitched the note.

199. Vibrations of Air Columns; Resonance. Let us hold a tube about 2 inches in diameter and 18 inches long with its lower end in a vessel containing water (Fig. 218); and over the open end hold a vibrating tuning-fork. Suppose the fork to make 256 vibrations per second.

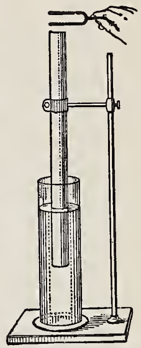

Fig. 218.—Air column in resonance with a tuning-fork.

By moving the tube up and down we find that, when it is at a certain depth, the sound we hear is greatly intensified. This is due to the air column above the water in the tube. It must have a definite length for each fork. On measuring it for this one we find it is approximately 13 inches. With higher pitched forks it is smaller than this, being always inversely proportional to the frequency of the fork.

Fig. 219.—Diagram to explain resonance in a closed tube.

The air column is put in vibration by the fork, its period of *free* vibration being the same as that of the fork. The air column is said to be in *resonance* with the fork.

200. Explanation of the Resonance of the Air Column. The tuning-fork prong vibrates between the limits *a* and *b* (Fig. 219.) As it moves forward from *a* to *b*, it produces a condensation which runs down the tube and is reflected from the bottom. When the fork retreats from *b* to *a*, a rarefaction is produced, which also travels down the tube and is reflected.

Now for resonance the tube must have such a length that in the time that the prong moves from *a* to *b* the condensation travels down the tube, is reflected, and arrives back at *b* ready to start up, along with the fork, and produce the rarefaction. Thus the vibrations of the fork and of the air column are perfectly synchronous; and as the fork continues to vibrate, the motion of the air in the tube accumulates and spreads abroad in the room, producing the marked increase of sound.

201. Determination of the Velocity of Sound by Resonance. From the explanation given of the resonance of the air column in a tube, it is seen that the sound-waves travel from A to B and back again while the fork is making half of a vibration. During a complete vibration of the fork the waves will travel *four* times the length of the air column; but we know that while the fork is making one vibration the sound-waves travel a wave-length. Thus the length of the air column is one-fourth of a wave-length of the sound emitted by the fork.*

If we know the frequency of the fork, we can, by measuring the length of the resonance column, at once deduce the velocity of sound. Also, if we know the length of the resonance column and the velocity of sound, we can deduce the pitch.

For example, using the values just obtained,

$$\begin{aligned} \text{Frequency } n &= 256 \text{ per second,} \\ \text{Wave-length } l &= 4 \times 13 = 52 \text{ inches;} \\ \text{Then } v &= nl = 256 \times 52 \\ &= 1109 \text{ feet per second.} \end{aligned}$$

202. Forms of Resonators. A resonator is a hollow vessel tuned to respond to a certain definite pitch. Two forms are shown in Fig. 220. In each case there is a larger opening, to be placed near the source of the sound, while the smaller opening is either placed in the ear, or a rubber tube leads from it to the ear. The volume is carefully adjusted so as to be in resonance with a tuning-fork (or other body) vibrating a definite number of times per second.

FIG. 220.—Two forms of resonators. The one on the right can be adjusted for different tones.

These resonators are used to analyse a compound note.

*More accurately the quarter wave-length of the sound is equal to the distance from the surface of the water to the top of the tube + 0.8 of the radius of the tube.

We can at once test whether there is present a tone corresponding to that of the resonator, by simply holding the instrument near the sounding body; if the air in the resonator responds, that tone is present, if it does not respond, the tone is absent.

The spherical form was used largely by the great German scientist Helmholtz; the other, which can be adjusted to several tones, was introduced by Koenig. They are usually made of glass or brass, but quite serviceable ones can be made in cylindrical shape out of heavy paper. (See also § 214.)

Tuning-forks which are used in acoustics are generally mounted on a light box of definite size (see Fig. 229). This is so constructed that the air within it is in *resonance* with the fork. If a fork is held with its stem resting on the table, the table is *forced* to vibrate in *consonance* with the fork.

203. Resonance of an Open Tube. Let us take two tubes, about two inches in diameter, one of them slipping closely over the other. Each may be 15 or 18 inches long (Fig. 221).

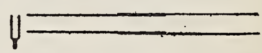

Fig. 221.—The length of an open tube when in resonance with a tuning-fork is one-half the wave-length of the sound.

Now vibrate the fork whose frequency is 256 per second and hold it over the end of the tube, varying the length at the same time.

At a definite length the air within the tube vigorously responds, and there is a marked increase in the sound. On measuring the length of the tube we find it is 26 inches, just twice the length of the tube when one end is closed.

But we found that the closed tube was one-fourth the wave-length of the sound to which it responded; hence an open tube is one-half the wave-length of the sound given by it.

The relation between the notes emitted by an open and a closed pipe of the same length can easily be illustrated by blowing across the end of a tube (say $\frac{1}{2}$ inch in diameter and 2 inches long), and observing the note produced when the tube is open and when a finger is held over one end of it. The former note is an octave higher than the latter.

204. Mode of Vibration in an Open Tube. When a rod is clamped at the middle and one half is stroked, as in § 176, we find that both halves lengthen and shorten. In this case there is a node at the middle, which is always at rest, and a loop at each end.

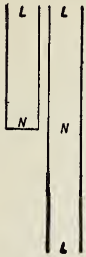

FIG. 222.—Explaining how an open pipe vibrates.

The air in an open tube vibrates quite similarly; indeed, it behaves like two closed tubes placed end to end. (Fig. 222.)

The layer of air across the middle of the open tube remains at rest while those on each side of it crowd up to it and then separate from it again. The layers at either end swing back and forth, without appreciably approaching those next to them.

FIG. 223. — Section of a wooden organ pipe.


FIG. 224.—A metallic organ pipe.

There is the greatest change of density at the middle of the tube, or the bottom of the closed tube, —*i.e.*, at the node,—while the air particles execute the greatest swing back and forth (without change in the density of the air) at the open ends. There is a loop at each end.

Exercise.—Obtain four glass or tin tubes having lengths 30, 24, 20, 15 cm. and blow across the end of each in succession,—first, with both ends open; second, with one end closed. Describe what you hear.

205. Organ Pipes. The most familiar application of the vibrations of air columns is in organ pipes. They are made either of wood or of metal. If of wood, pine, cedar or mahogany is used; if of metal, tin (with some lead in it) or zinc.

In Fig. 223 is shown a section of a rectangular wooden pipe; in Fig. 224 is a metallic cylindrical pipe. Sometimes the pipes are conical in shape.

Air is blown through the tube T into the chamber C, and escaping from this by a narrow slit it strikes against a thin lip D. In doing so a periodic motion of the air at the lip is produced, and this sets in motion the air in the pipe, which then gives out its proper note.

Organ pipes are of two kinds,—open and closed. In some open pipes reeds are used (§ 217). From the discussion in § 203 it will be clear that the note yielded by an open pipe is an octave higher than that given by a closed pipe of the same length.

206. Overtones (or Harmonics) in an Organ Pipe. The vibrations of the open and closed pipes which have been described in §§ 199, 203, are the simplest which the air column can make, and they give rise to the lowest or fundamental notes of the pipes. In order to obtain the fundamental the pipe must be blown gently. If the strength of the air current is gradually increased, other tones, namely, the overtones of the pipe, will also be heard.

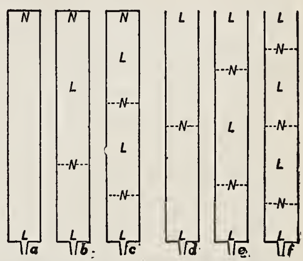

Fig. 225.—Showing the nodes and loops in open and closed organ pipes with different strengths of air currents.

In a, b, c, Fig. 225, are represented the divisions of the air column in a stopped pipe corresponding to different strengths of the air current. In a we have the fundamental vibration; here the column is undivided. The only node present is at the closed end, and there is a loop at the lip end. In b is shown the condition of the air column corresponding to the first overtone of the pipe. There is a node at the closed end, and another at a distance $\frac{1}{3}$ of the length of the pipe from the lip end. Thus the distance from a node to a loop is $\frac{1}{3}$ that in *a*, and the wave-length of the note is $\frac{1}{3}$ that of the fundamental. Its frequency is 3 times that of the fundamental.

In *c* there are three nodes and three loops, in the places indicated. From a node to a loop the distance is $\frac{1}{5}$ of the length of the pipe, and hence the wave-length of the sound is $\frac{1}{5}$ and its frequency 5 times that of the fundamental. The succeeding harmonics produced would have frequencies 7, 9, ... times that of the fundamental. Thus we see that in a closed pipe only the harmonics having frequencies an odd number of times that of the fundamental are present.

Next consider the open pipe (*d*, *e*, *f*, Fig. 225). For the fundamental the air column divides as shown in *d*, with a node at the middle and a loop at each end. With stronger blowing there is a loop at the middle as well as at each end and nodes half-way between as in *e*. In this case the wave-length is $\frac{1}{2}$ and the frequency twice that of the fundamental.

In *f* is shown the next mode of division of the air column. It will be seen that the wave-length is $\frac{1}{3}$ and the frequency 3 times that of the fundamental. By using still stronger currents of air we get harmonics with frequencies 4, 5, 6, ... times that of the fundamental. Thus in an open pipe all the harmonics (or overtones) can be produced.

QUESTIONS AND PROBLEMS

1. Why is it advisable to strike a piano string near the end rather than at the middle?

2. As water is poured into a deep bottle the sound rises in pitch. Explain why.

3. A stopped pipe is 4 feet long and an open one 12 feet long. Compare the pitch and the quality of the two pipes.

4. What would be the effect on an organ pipe if it were filled with carbonic acid gas? What with hydrogen?

5. Find the length of a stopped pipe whose fundamental has a frequency of 522. (Temperature, 20° C.)

6. A glass tube, 80 cm. long, held at its centre and vibrated with a wet cloth gives out a note whose frequency is 2540. Calculate the velocity of sound in glass.

7. If the tension of a string emitting the note A is 25 pounds, find that required to produce C'.

8. What effect will a rise in temperature have on the notes of a pipe organ?

9. One wire is twice as long as another (of the same material and diameter), and its tension is twice as great. Compare the frequencies.

10. Find the length of an air column in resonance with E. (Temperature, 20° C.; $C = 261$.)

11. A string 1 metre long, stretched by a weight of 5 pounds, gives a tone designated C. What must be the weight to give G? How much must the string be shortened to give G?

12. An open pipe 6 ft. long produces a note of frequency 256. What must be the length of a closed pipe which produces a note of frequency 512?

13. Two strings are of the same thickness and made of the same material. One is 10 inches long and has a tension of 9 pounds; the other is 12 inches long and has a tension of 16 pounds. Compare the frequencies.

14. Find the lengths of the three shortest closed pipes and of the three shortest open pipes which would respond to a tuning-fork making 256 vibrations per second. (Temperature 20° C.)

15. A steel wire 50 cm. long and under a tension of 6 pounds emits a note of 261 vibrations per second. What must be the tension of the wire if its length be increased to 100 cm. and it yields a note one octave higher?

16. Name some musical instruments in which the transverse vibrations of rods are utilized.

REFERENCES FOR FURTHER INFORMATION

Tyndall, *Sound*, Chapters 3, 4, 5.
Catchpool, *Text-Book of Sound*, Chapters 11, 12.
Poynting and Thomson, *Sound*, Chapters 6, 7, 8.
Wood, *Physical Basis of Music*.
Miller, *The Science of Musical Sounds*.

omit

CHAPTER XX

Quality, Vibrating Flames, Beats

207. Quality of Sound. It is a familiar and remarkable fact that though sounds having the same pitch and intensity may be produced on the piano, the organ, the cornet, or with the human voice, the source of the sound in each case can be easily recognized. That peculiarity of sound which allows us to make this distinction is called *quality*.

The cause of this was not explained until, in quite recent times, Helmholtz showed that it depends on the co-existence with the fundamental of secondary vibrations which alter the *forms* of the sound waves. These secondary vibrations are the overtones or harmonics, and their number and prominence determine the peculiar characteristics of a note.

In general, those notes in which the fundamental is relatively strong and the overtones few and feeble are said to be of a 'mellow' character; but when the overtones are numerous, the note is harsher and has a so-called metallic sound. If a musical string is struck with a hard body, the high harmonics come out prominently.

When a violin string is bowed, the first seven overtones are present, and give to the sound its piercing character. In the case of the piano the 1st, 2nd and 3rd overtones are fairly strong while the 4th, 5th and 6th are more feeble.

208. Vibrating Flames. The cause of *quality* was investigated by Helmholtz by means of spherical resonators (Fig. 220). But a very beautiful and simple way of investigating the complex nature of sound waves is by means of the manometric, or *pressure-measuring*, flame devised by Koenig.

A convenient form of the apparatus is shown in Fig. 226. A small chamber is divided into two compartments by a thin membrane* *m*. Gas enters one compartment, as shown in the figure, and is lighted on leaving by a fine tip. The other compartment is connected by means of a rubber tube with a funnel-shaped mouth-piece.

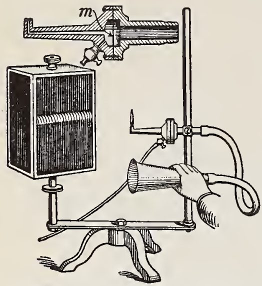

Fig. 226.—The manometric flame and mirror. A section of the gas chamber is shown separately above. On speaking into the funnel the flame dances rapidly up and down, and this motion is observed in the square mirror which is rotated by hand.

The sound waves enter the funnel, and their condensations and rarefactions produce variations in the density of the air beside the membrane. This makes the membrane vibrate back and forth, and the gas flame dances up and down. But these motions are so rapid that the eye cannot follow them, and in order to separate them they are viewed by reflection in a rotating mirror.

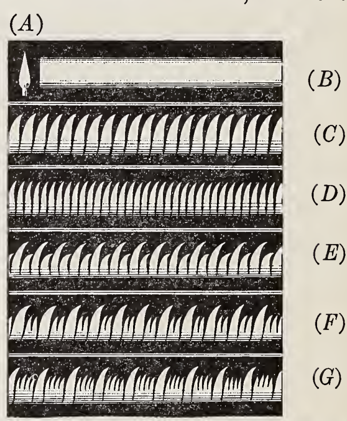

Fig. 227.—Flame pictures seen in the rotating mirror. *A*, when mirror is at rest; *B*, when flame is at rest and mirror rotating; *C*, when a tuning-fork is held before the mouthpiece; *D*, same as *C* but an octave higher; *E*, when *C* and *D* are combined; *F*, obtained with vowel *e* at pitch *C′*; *G*, with vowel ō at the same pitch.

The appearance of various images of the flame is given in Fig. 227. When the mirror is at rest, the image is seen as at *A*. If now the mirror is rotated while the flame is still, the image is a band of light, *B*. If one sings into the conical mouthpiece the sound of *oo* as in *tool*, or if one holds before it a vibrating mounted

*This may be very thin mica or rubber or gold-beater's skin.

tuning-fork the gas-jet's motion appears in the mirror like *C*. If the note is sung an octave higher, there will be twice as many little tongues in the same space, *D*. When these two tones are sung together, images as in *E* are given. On singing the vowel *e* at the pitch *C′* we obtain images as at *F*; and *G* is obtained on singing ō at the same pitch.

From the figures it will be seen that the last three notes are complex sounds. These dancing images have been successfully photographed on a moving film by Nichols and Merritt.

A simple form of the above apparatus can be constructed by anyone (Fig. 228). Hollow out a piece of wood or cork (2 inches in diameter), *A*, and across the opening stretch the membrane, *M*, keeping it in place by screwing or pinning a ring *B* against it. Gas enters by the tube *C* and leaves by the tube *D*. No mouthpiece is necessary, but a funnel, shaped as shown in the dotted line, will increase the effect. In place of the rotating mirror a piece of mirror 6 by 8 inches square, held in the hand almost vertical and given a gentle oscillatory motion will give good results.

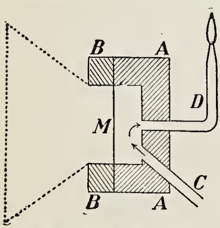

Fig. 228.—A simple form of manometric flame capsule. *AA* is a cork hollowed out, *M* is the thin membrane.

209. Sympathetic Vibrations. Place two tuning-forks which have the same vibration numbers, with the open ends of their resonance boxes facing each other and a short distance apart (Fig. 229). Now vibrate one of them vigorously by means of a bow or by striking with a soft mallet (a rubber stopper on a handle), and, after it has been sounding for a few seconds, bring it to rest by placing the hand upon it. The sound will still be heard, but on examination it will be found to proceed from the other fork.

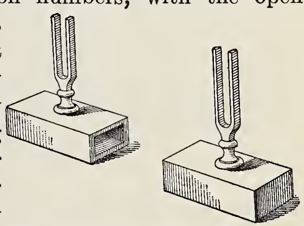

Fig. 229.—Two tuning-forks arranged to show sympathetic vibrations. When one is vibrated the other responds.

This illustrates the phenomenon of sympathetic vibrations. The first fork sets up vibrations in the resonance box on which it is mounted, and these produce vibrations in the inclosed air column. The waves proceed from it, and, on reaching the resonance box of the second, fork, put its air column in vibration. The vibrations are communicated to the box and then to the fork, which, having considerable mass, continues its motion for some time.

A single wave from the first fork would have little effect, but when a long series comes in regular succession, each helps on what the one next before it has started. Thus the effect accumulates until the second fork is given considerable motion, its sound being heard over a large room.

For the success of this experiment the vibration numbers of the two forks must be accurately equal.

210. Illustrations of Sympathetic Vibrations. The pendulum of a clock has a natural period of vibration, depending on its length, and if started, it continues swinging for a while, but at last comes to rest. Now the works of the clock are so constructed that a little push is given to the pendulum at each swing, and these pushes, being properly timed, are sufficient to keep up the motion.

Again, it is impossible by a single pull on the rope to ring a large bell, but if one times the pulls to the natural period of the bell's motion, its amplitude continually increases until it rings properly.

When a body of soldiers is crossing a suspension bridge, they are usually made to break step, for fear that the steady tramp of the men might start a vibration agreeing with the free period of the bridge, this vibration by continual additions, might reach dangerous proportions.

211. Beats. We shall experiment further with the two unison forks (Fig. 229). Stick a piece of wax* on each prong of one fork; we cannot get sympathetic vibrations now, but

*The soft modelling wax sold as "plasticine" is very convenient.

on vibrating the two forks at the same time we hear a peculiar wavy or throbbing sound, caused by alternate rising and sinking in loudness. Each recurrence of maximum loudness is called a *beat.*

We at once recognize that this effect is due to the interaction of the waves from the two forks, resulting in an alternate increase and decrease in the loudness of the sound.

Each fork produces condensations and rarefactions in the air, and since in a condensation the air particles have a *forward* motion, while in the rarefaction the motion is *backward,* it is evident that if a condensation from one fork reaches the ear at the same time as a rarefaction from the other, they will oppose their effects, and the ear-drum will have little motion—the sound will be faint. If, however, a condensation from each or a rarefaction from each arrives at the same time, the action on the ear-drum will be increased and the sound will be louder.

For simplicity, let us suppose that we have two rods vibrating transversely, one of them having a frequency of 4 and

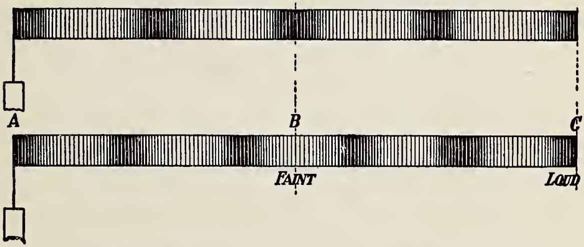

Fig. 230.—Illustrating the production of beats. Two condensations or two rarefactions coming together strengthen each other. A condensation and a rarefaction destroy each other's effect.

the other of 5 vibrations per second. If they start vibrating in the same direction at the same time, the state of affairs after one second will be as shown in Fig. 230, in which lines close together represent a condensation and lines far apart a rarefaction.

The distance AC, traversed by the sound in one second, will be 1120 ft., and it is equal to 4 wave-lengths of the first rod and 5 wave-lengths of the second rod.

Suppose one person stationed at B and another at C. From the diagram it is evident that the person at C is receiving a condensation from each rod and, consequently, hears a loud sound. At the same time the person at B is receiving a condensation from the first rod and a rarefaction from the second and hence hears a faint sound.

But the sound pulses keep travelling towards the right, and the condition which exists at B now will be at C one half second later, and after still another half second the two pulses of condensation just starting from A will arrive at C. Thus there will be *one* beat per second.

From a similar diagram drawn for rods vibrating 4 and 6 times per second it will be seen that in this case there would be *two* beats per second.

We arrive, then, at the simple law that *the number of beats per second due to two simple tones is equal to the difference of their respective vibration numbers.*

To produce beats the forks should not differ greatly in pitch.

212. Tuning by Means of Beats. Suppose we wish to tune two strings to unison. Even the most unmusical person can do it. Simply vary the tension, or the length, of one of them until, as they approach unison, the beats are fewer per second. If one beat per second is heard, there is a difference of only one vibration per second in their frequencies. Let us alter a little more until the beats are entirely gone. The strings are then in unison.

In the same way other sounding bodies, for instance two organ pipes, or a pipe and a tuning-fork, may be brought to unison.

213. Interference of Sound waves. The production of beats is but one of the many phenomena due to the interference of sound waves. Let us consider two other examples.

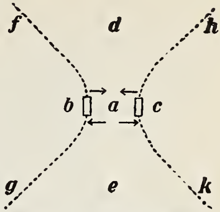

FIG. 231.—Interference with a tuning-fork.

In Fig. 231 are shown the extremities of the two prongs of a tuning-fork. They vibrate in such a way that they move alternately towards and away from each other. Thus while they produce a condensation in the space *a* between them, they produce a rarefaction at *b* and *c* on the opposite sides. In this way each prong starts out two sets of waves, which are in opposite phases. These waves travel out in all directions, and it is evident that we can find points such that when the two sets of waves arrive at them they will be in opposite phases and so will counteract each other's effects. Such points are located on two curved surfaces, of which *fg*, *hk* are horizontal sections.

This can be demonstrated by holding a vibrating fork near the ear and then rotating it slowly. When the ear is in the positions *b*, *c*, *d*, *e*, the sound is heard clearly; while if it is on either of the curved surfaces *fg*, *hk* no sound is heard.

214. Interference with Resonators. Another interesting experiment can be performed with two wide-mouthed (pickle) bottles. Vibrate a tuning-fork (256 vibrations) over the mouth of one of the bottles, and slip a microscope slide over the mouth until the air in the bottle responds vigorously. Fasten with wax the glass in the position when the bottle resounds most loudly. The bottle is then a resonator tuned to the fork.

Tune the other bottle in the same way, and then arrange them, with their mouths close together, as shown in Fig. 232. Make the fork vibrate, and then, holding it horizontally, bring it down so that the space between the prongs is opposite the mouth of the upright bottle. As it is brought into place you will observe that the sound first increases, and then suddenly fades away or disappears entirely.

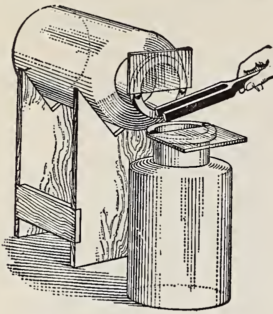

FIG. 232.—Interference with two resonators.

The reason for this is easily understood. The air in one bottle is put in vibration by the air from between the prongs, while that in the other is put in vibration by the air on the other side of the prongs; and these, as we have seen, are in opposite phases. Hence they interfere and produce silence.

If a card is slipped over the mouth of one of the bottles, that bottle's vibrations are shut off and the other sings out loudly.

215. Doppler's Principle. Suppose a body at A to be emitting a note of n vibrations per second. Waves will be excited in the surrounding air, and an observer at B will receive n waves each second. He will recognize a sound of a certain pitch.

Next, suppose that the observer approaches the sounding body; he will now receive more than n waves in a second. In addition to the n waves which he would receive if he were stationary, he will meet each second a certain number of waves, since he is nearer the sounding body at the end of a second than he was at its beginning. He will receive those waves which at the commencement of the second occupied the space he has moved. As he will now receive more than n waves per second, the pitch of the sound will appear to be higher than when there was no motion.

If the observer moves away, the number of waves received will be smaller and the pitch will be lowered.

If the observer remains at rest while the sounding body approaches or recedes, similar results will be obtained; and if we can determine the change in pitch, we can calculate the speed of the motion. This phenomenon is known as the Doppler effect, and the explanation given is known as Doppler's Principle.

The Doppler effect can be observed when a whistling locomotive is approaching or receding at a rapid rate. An automobile sounding its horn is a still better illustration as its motion makes less noise. When the machine is approaching, the sound is distinctly higher in pitch than when it is travelling away.

QUESTIONS AND PROBLEMS

1. What are the fourth and fifth overtones to *C*?

2. If one presses the loud pedal of a piano and then sings into the piano a sound will sometimes be heard after he has stopped singing. Why?

3. A tuning-fork on a resonance box is moved towards a wall, and a 'wavy' sound is heard. Explain the production of this.

4. Hold down two adjacent bass keys of a piano. Count the beats per second and deduce the difference of the vibration numbers.

5. Two persons are walking together. One takes 90 steps and the other 94 steps a minute. How many times a minute will the right feet of the two persons strike the ground together?

6. If a circular plate is made to vibrate in four sectors as in *d*, Fig. 217, and if a cone-shaped funnel is connected with the ear by a rubber tube, and the other ear is stopped with soft wax, no sound is heard when the centre of the mouth of the cone is placed over the centre of the plate; but if it is moved outward along the middle of a vibrating sector, a sound is heard. Explain these results. (For a plate 6 inches in diameter the mouth of the funnel should be $2\frac{1}{2}$ inches in diameter. Try the experiment.)

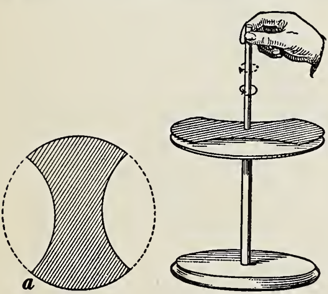

Fig. 233

7. A circular plate is made to vibrate in four sectors, and one ear (the other being covered over) is held directly over the centre of the plate at the distance of one foot or more. Little sound is heard. Why? From a circle of cardboard of the same size as the plate cut a pattern like that in *a*, Fig. 233, and having tacked this on the end of a light rod hold it over the vibrating plate and spin it round slowly. Describe the effect and account for it.

REFERENCES FOR FURTHER INFORMATION

Mayer, *Sound*, Chapters 16 to 18.
Catchpool, *Sound*, Chapter 4, 8.
Miller, *The Science of Musical Sounds*.

CHAPTER XXI

Musical Instruments; The Phonograph

216. Stringed Instruments. In the piano there is a separate string, or a set of strings, for each note. The strings are of steel wire of different diameters, and for the bass notes they are overwound with other wire, being in this way made more massive without losing their flexibility. When a key is depressed, a combination of levers causes a soft hammer to strike the string at a point about $\frac{1}{7}$ of the length of the string from the end. If the instrument gets out of tune, it is repaired by re-adjusting the tensions of the strings.

The guitar has six strings of different diameters, the three lower-pitched ones being of silk over-wound with fine wire. The strings are tuned to E_1, A_1, D, G, B, E', where D is the note next above middle C and has 293.6 vibrations per second. There are little strips across the finger-board called 'frets,' and if one presses the strings down by the fingers against them, the strings are shortened and give out the other notes (Fig. 234).

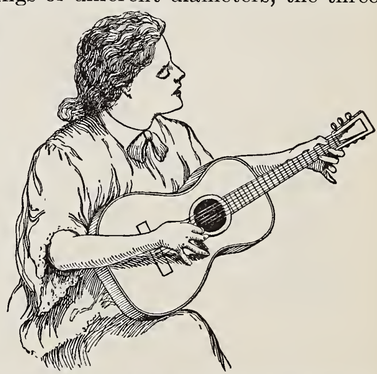

Fig. 234.—The guitar. With the left hand the strings are shortened by pressing them against the 'frets,' while the note is obtained by plucking with the right hand.

In the familiar ukelele there are four strings, tuned to G, C, E, A, with frequencies proportional to 18, 12, 15, 20. There are twelve frets, and the other notes are obtained by pressing the strings against them.

There are four strings on the violin, tuned to G_1, D, A, E', where D is next above middle C of the piano. The frequencies are proportional to 18, 27, 40, 60. The other notes are obtained by shortening the strings by means of the fingers, but as there are no frets to guide the performer, he must judge the correct positions of the fingers himself.

Exercise.—The A string of the violin has a frequency of 435 vibrations per second. Compute that of each of the others.

217. The Organ. The action of organ pipes has been explained in §§ 205, 206. In large organs they vary in length from 2 or 3 inches to about 20 feet, and some of them are conical in shape.

The flageolet or 'tin whistle' acts like an organ pipe. The part used in producing a note is that from the lip to the nearest open hole, and by closing the holes with the fingers the player lengthens the part used and obtains lower notes.

In the ordinary organ, the mouth-organ, the accordion and some other instruments the vibrating body is a reed, such as is shown in Fig. 235. The tongue A vibrates in and out of an opening which it accurately fits, the motion being kept up by the current of air which is directed through the opening.

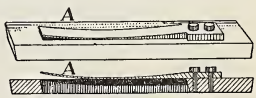

FIG. 235.—An organ reed. The tongue A moves in and out of the opening. This is called a *free* reed.

In some organ pipes there are reeds, but the note produced is due chiefly to the air column in the pipe, the reed simply serving to set it in vibration.

218. The Automobile Horn. In the common automobile horn there is a small electric motor, the armature A (Fig. 236) of which carries at one end a toothed wheel B. When the button is pressed, the armature rotates very rapidly, and the teeth of the wheel strike on a little knob C

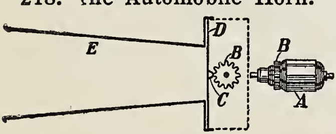

FIG. 236.—Explaining the action of an automobile horn.

at the middle of a diaphragm D. The vibrations of this diaphragm produce the sound, and the conical tube E projects it outward. The pitch of the sound at first is low, but the armature quickly gets up speed and the pitch is raised.

219. Vibrations Produced by Player's Lips. The bugle, cornet and other such instruments consist essentially of an open conical tube, the larger end terminating in a bell while at the smaller end is a cup with a rounded edge against which the tense lips of the player are steadily pressed. The lips thus constitute a reed, and by their vibrations waves are set up in the air within the tube. In this way the fundamental and the various harmonics of the air column in the tube are produced, and all but the extreme bass sounds are used in the scale.

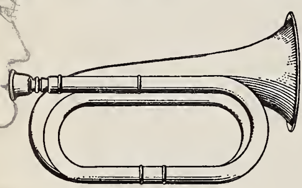

Fig. 237.—The Bugle.

The bugle is illustrated in Fig. 237. The length of tube is fixed, and the notes producible are the fundamental and about 5 overtones. Its compass is rather narrow.

In the cornet, by means of three valves a, b, c (Fig. 238), the air column may be divided into different lengths, and a series of overtones is obtained with each length.

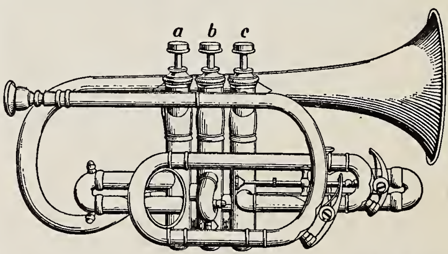

Fig. 238.—By the valves a, b, c, the air column is divided into different lengths.

220. The Phonograph. This instrument, now so familiar, was invented by Edison in 1877. Its construction is extremely simple, and one is astonished that such wonderful results can be obtained so easily.

At first the records were on tin-foil wrapped about a cylinder; after that a cylinder of wax was used; and now the

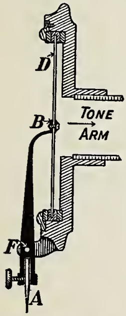

FIG. 240.—Section of reproducer for lateral or side-to-side records.

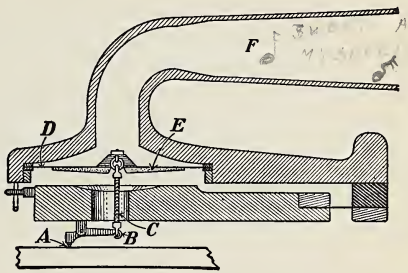

FIG. 239.—Section of the Edison reproducer, for records of the 'hill-and-dale' type.

records are on hard thin discs 10 to 12 inches in diameter. Upon the disc there is a long spiral groove, beginning at the edge and ending near the centre. In one type of record this groove varies in depth, that is, it is alternately deeper and shallower or it is a succession of "hills and dales." In the other type the groove is of uniform depth but its course is from side to side.

For the former kind the sound-box or reproducer is constructed as in Fig. 239. A small diamond with a sharp point is mounted at the end A of the short arm of a lever, while the end B of the long arm is attached by a flexible cord C to the diaphragm D, which is across the mouth of a diverging cone F, called the tone arm. The diaphragm is of vegetable fibre and is re-enforced by a layer of cork E. As the diamond point runs along the groove it moves up and down, and this motion, somewhat magnified, is transmitted to the diaphragm. The motions of the diaphragm set up waves in the air, which pass along the tone-arm, then into the horn and then into the surrounding space.

For the other style of record the reproducer is such as shown in Fig. 240. Here a needle A is in the short arm of a lever, the end B of the long arm being connected to the diaphragm D. As the end of the needle runs along the groove it moves from side to side, this motion sets up motions in the diaphragm and the sound thus produced passes along the tone arm and out as before.

The production of the records demands considerable skill and practice. Suppose we wish to make a record of a vocal solo. The singer stands in one room before a large horn which leads through the wall into another room. Across the small end of the horn is a diaphragm with a needle attached to it, much as in a reproducer. The needle rests upon a round disc of comparatively soft wax upon a turn-table. The sound waves from the voice travel up the horn, and when they strike the diaphragm they cause it to vibrate back and forth. These vibrations are handed on by a lever to the needle which, as the disc revolves, ploughs a furrow in the surface of the wax. All the little variations in the sound are recorded in the minute irregularities of the furrow. This wax disc, which is about three-quarters of an inch thick and half an inch greater in diameter than the final record is to be, is the *original* or *master record*. It could be put on a machine and used as a record in the ordinary way, but it would soon be injured, and so exact copies of it are made in a harder substance.

The method is as follows. First the wax disc is covered with very finely-powdered plumbago to make it an electrical conductor, and it is then hung in a copper sulphate bath such as is used in electrotyping. Here by means of an electric current (see § 458) copper is deposited upon it for about 24 hours, at the end of which time there is a thick layer of copper which fills every little hollow in the wax.

This copper sheet is now stripped from the wax with the greatest care and is then backed with easily melting metal in order to strengthen it. This is the *master matrix*. It is now immersed in a silver-plating bath and a thin layer of silver is deposited on the face, and then it is put in the copper bath again and a heavy layer is deposited upon the silver. It is then taken out and the layer of copper is stripped from the master matrix, the silver film allowing this to be done. This is now the *mother record.* It is an exact duplicate in copper of the original wax record.

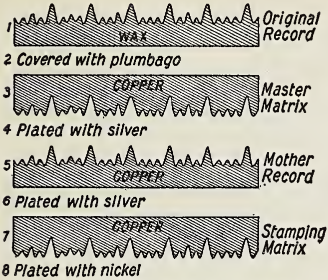

FIG. 241.—Illustrating the production of a phonogragh record.

This mother record is next covered with silver and after that with a thick layer of copper. This layer is then stripped off and backed with metal and is then known as the *stamping matrix*. Before it is used it is covered with a thin layer of nickel, which protects its surface. As many as 50 or even 100 "stampers" are made from the mother record. The master matrix is preserved with great care as all the other parts of the process depend upon it.

The composition of the material from which the records are produced differs with the different makers. In it is much shellac, rotten stone and lampblack. The ingredients are thoroughly mixed until the substance has the consistency of stiff dough. It is then moulded into discs of the required size, and while warm the hot stamping matrix is pressed against it in a hydraulic press. Then the edge is trimmed off on a lathe and the record is tested on an ordinary phonograph. The various steps in the manufacture are indicated in Fig. 241.

PART VI—HEAT

CHAPTER XXII

Nature and Source of Heat

221. Nature of Heat. It is a matter of every day experience that when motion is checked by friction or collision, heat is developed in the bodies concerned. Thus if a button is rubbed vigorously on a board or even on a piece of cloth, it may be made too hot to be handled. A drill used in boring steel quickly becomes heated. A leaden bullet shot against an iron target may be melted by the impact. The aborigines obtained fire by rubbing two dry sticks together.

For centuries it was the general belief that the heating of a body was due to the entrance into it of a subtle weightless fluid called *caloric*. The first serious attack upon this theory was made by Count Rumford* in 1798. In some experiments a bar of steel was pushed with great force into a hole in a bronze cylinder which was made to rotate. He found that as long as the cylinder was rotated heat continued to be produced, and he concluded that, as there was no limit to the amount of heat produced, it could not be a form of matter but must be a kind of motion. In the years just after this Sir Humphry Davy and others made somewhat similar experiments, but not until about the middle of the century was the old theory finally overthrown. This was due to the experiments of Joule, of Manchester, who showed that the quantity of heat produced was proportional to the mechanical work done, twice as much heat requiring twice the amount of work, and so on, from which it was clear that heat must be a form of energy.

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*Benjamin Thompson was born at Woburn (near Boston, Mass.) in 1753. In 1775 he went to England and in 1783 to Austria. He was created Count Rumford by the Elector of Bavaria. While engaged in boring cannon at Munich he made his experiments on heat. He died in France in 1814.

222. Motion of the Molecules. What becomes of the energy of the body when its motion is stopped by friction or collision? It is changed into motion of the molecules of the body, and the more vigorous the motion of the molecules the more heat the body receives. Consider what happens when a body, a piece of lead for instance, is heated. The molecules within it vibrate back and forth, striking their neighbours harder and harder as the temperature rises. This shaking about of the molecules becomes so vigorous that the bonds between them are weakened, the lead softens and then melts. After this the molecules move freely about in the liquid, and as the heat is still further applied they fly off and the liquid evaporates or turns into vapour. We are familiar with ice turning into water and then into vapour, but many other substances behave in the same way.

223. Heat from Mechanical Action. Illustrations of the production of heat by friction and percussion have already been given. It is also developed by compression. If a piece of dry tinder* is placed in a tube closed at one end, and a closely-fitting piston is pushed quickly into the tube (Fig. 242), the tinder may be lighted by the heat developed by the compression of the air. The cylinders of air compressors (automobile and bicycle pumps for instance) become heated by the repeated compression of the air drawn into them.

Conversely, if a compressed gas is allowed to expand, its temperature falls. The steam which has done work by its expansion in driving forward the piston of a steam engine escapes from the cylinder at a lower temperature than that at which it entered it.

Fig. 242.—A fire syringe.

*A pellet of cotton soaked in ether or carbon disulphide may be used instead.

224. Sources of Heat. The chief sources of heat are:

1. Mechanical action (friction, percussion, compression).
2. Chemical action (combustion).
3. Electric action (as in heaters and lamps).
4. The sun.

In every case when heat is produced, it is simply transformed from some other form of energy.

225. Heat from Chemical Action. The potential energy of chemical separation is one of our most common sources of heat. Combustible bodies, such as coal and wood, possess energy of this kind. When raised to the ignition point, they unite chemically with the oxygen of the air, and their union is accompanied by the development of heat. So far this has been the chief source of artificial heat used for cooking our food and warming our dwellings.

226. Heat from an Electric Current. An electric current possesses energy, and this, when the current is made to pass through a conductor which offers resistance to it, is transformed into heat. For example, if the terminals of a battery consisting of three or four galvanic cells joined in series are connected with a short piece of fine platinum or iron wire, it will be heated to a white heat. In electric lamps the carbon rods or the tungsten wires are heated to incandescence by an electric current. Electric heaters and electric cookers are but coils of resistance wire heated by an electric current.

227. Heat from Radiant Energy. The sun is our greatest supply of heat, but its heat, defined as the energy of molecular motion, does not come unchanged from sun to earth, as one might suppose. The atmospheres of the sun and the earth extend only a few hundred miles from these bodies, while they are 93,000,000 miles apart. It is certain that in most of the space between them there is no matter, composed of molecules, as we understand it. The direct transference of molecular motion from the sun to the earth is, therefore, impossible. To account for the transmission of energy, the physicist assumes the existence of a medium called the *ether*, which he conceives to pervade all space, penetrating between the molecules as well as reaching from star to star.

The vibrating molecules of a hot body set up disturbances in the ether, and these are transmitted in all directions by a species of wave-motion, somewhat as sound waves spread out in the air from a vibrating body. When these ether waves fall upon matter, they tend to accelerate the motion of its molecules and so to heat it. Thus the heat of the sun is first changed into *radiant energy*, or the energy of ether vibration, and the ether waves which fall upon the earth are transformed into heat. The subject is further discussed in § 310.

REFERENCES FOR FURTHER INFORMATION

Stewart and Satterly, *Text-book of Heat*, Chapter 16.
Edser, *Heat for Advanced Students*, Chapter 12.
Preston, *Theory of Heat*, Chapter 1.

CHAPTER XXIII

Expansion Through Heat

228. Expansion of Solids by Heat. There are numerous examples of the expansion of bodies through heat, and many simple experiments have been devised to illustrate it.

1. Take a brass ball (Fig. 243) which can just pass through a ring when cold, and then heat it. It will be found to be too large to go through.

2. In the apparatus shown in Fig. 244, a metal rod is fixed at one end while the other presses against the short arm of a bent lever. If the rod is heated its elongation will be shown by the movement of the end of the long arm over a scale.

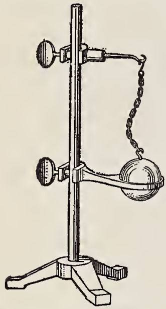

Fig. 243.—Expansion of ball by heat.

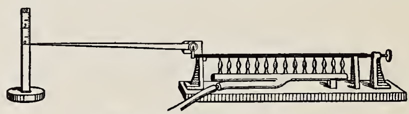

Fig. 244.—Expansion of rod by heat.

3. Construct a compound bar by riveting together strips of copper and iron (Fig. 245). When it is heated uniformly it bends into a curved form with the copper on the convex side, because the copper expands more than the iron. If placed in a cold bath, it curves in the opposite direction.

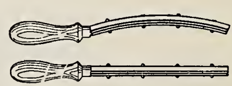

Fig. 245. — Bending of compound bar by unequal expansion of its parts.

These experiments illustrate a very general law. Solids (with very few exceptions) expand when heated and contract when cooled, but different solids have different rates of expansion.

229. Expansion of Liquids and Gases by Heat. Liquids also expand when heated. The amount of expansion varies with the liquid, but, on the whole, it is much greater than that of solids. Let us inclose liquid within a flask and connected tube, as shown in Fig. 246, and heat the flask. The liquid is seen to rise in the tube.

Fig. 246. — Expansion of liquids by heat.

The same apparatus may be used to illustrate the expansion of gases. When the flask and tube are filled with air only, insert the open end of the tube into water (Fig. 247), and heat the flask. A portion of the air is seen to bubble out through the water. If the flask is cooled, water is forced by the pressure of the outer air into the tube to take up the space left by the air as it contracts.

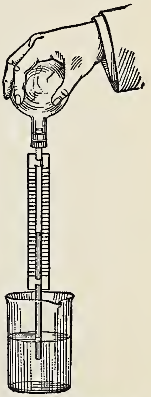

Fig. 247.—Expansion of a gas by heat.

Unlike solids and liquids, all gases have, at the ordinary pressure of the air, approximately the same rates of expansion.

230. Applications of Expansion—Compensated Pendulums. A clock is regulated by a pendulum whose rate of vibration depends on its length. The longer the pendulum, the slower the beat; and the shorter, the faster. Changes in temperature will, therefore, cause irregularities in the running of the clock, unless some provision is made for keeping the pendulum constant in length through varying changes in temperature. Several forms of compensation are in use. The Graham pendulum (Fig. 248) is provided with a bob consisting of a jar of mercury. Expansion in the rod lowers the centre of gravity of the bob, while expansion in the mercury raises it. The quantity of mercury is so adjusted as to keep the centre of gravity* always at the same level.

Fig. 248.—Graham pendulum, invented in 1722.

In the Harrison, or gridiron, pendulum (Fig. 249) the bob hangs from a framework of brass and steel rods, so connected that an increase in length of the steel rods (dark in the figure), tends to lower the bob, while an increase in the length of the brass ones tends to raise it. The lengths of the two sets are adjusted to keep the resultant length of the pendulum constant.

Fig. 249.—Harrison pendulum, invented in 1726

In the best modern clocks the pendulum consists of a rod of *invar*, an alloy of nickel and steel, the expansion of which is small, and a bob of brass, lead or other metal (Fig. 250.)

Fig. 250. — A modern pendulum, with *invar* rod and a metal bob.

231. Chronometer Balance-wheel. A watch is regulated by a balance-wheel, controlled by a hairspring (Fig. 251). An increase in temperature tends to increase the diameter of the

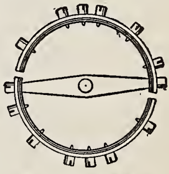

Fig. 251.—Balance-wheel of watch.

*Strictly speaking, it is a point called the *centre of oscillation*, (which nearly coincides with the centre of gravity) whose distance from the point of suspension should be kept constant.

wheel and to decrease the elasticity of the spring. Both effects would cause the watch to lose time. To counteract the retarding effects, the rim of the balance-wheel in chronometers and high-grade watches is constructed of two metals and mounted in sections, as shown in Fig. 251. The outer metal is the more expansible, and the effect of its expansion is to turn the free ends of the rim inwards, and thus to lessen the effective diameter of the wheel.

232. Thermostats. The curling up of a compound bar when heated is applied in the construction of thermostats, which are used mainly for controlling the temperature of buildings heated by hot-air furnaces or boilers. The thermostat is so constructed that, when the temperature rises to a certain degree, it sets free a current of electricity or a supply of compressed air, which closes the dampers or the steam valves; and when the temperature falls, the heat is turned on again. In Fig. 252 is shown an electric thermostat. The essential part is the compound bar *b*. When bent by the heat, it closes an electric circuit at *a*. In the pneumatic thermostat the bending of the bar shuts off the escape of compressed air and causes it to open a valve which allows a large supply of compressed air to have access to the regulators in the furnace-room.

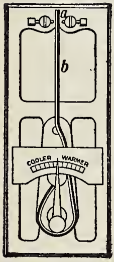

Fig. 252.—An electric thermostat.

QUESTIONS

1. A glass stopper stuck in the neck of a bottle may be loosened by subjecting the neck to friction by a string. Explain.

2. Boiler plates are put together with red-hot rivets. What is the reason for this?

3. Why does a blacksmith heat a wagon-tire before adjusting it to the wheel?

4. Why are the rails of a railroad track laid with the ends not quite touching?

5. Why does change in the temperature of a room affect the tone of a piano?

6. Glass vessels are liable to break when hot water is poured into them. Give the reason.

7. In repairing a cannon a hole was drilled into it, and a plug was made just too large to go into the hole. The plug was then held in liquid air for some time, and then it went into the hole and could never be withdrawn. Explain.

8. In constructing large guns the barrel or inner tube is surrounded by several layers, each being heated and then slipped over the layer within it. Why? When the barrel is worn out by firing, the whole gun is heated, a jet of water is sent into the barrel and the inner tube is then driven out. Explain why this is possible. (See *Encyclopedia Britannica*, Article "Ordnance.")

9. How would you provide for expansion in steam pipes?

10. A steel tape and a linen tape were equal in length when compared in a warm room, but different results were obtained when they were used to measure the length of a field on a cold day. Explain.

REFERENCES FOR FURTHER INFORMATION

Stewart and Satterly, *Text-book of Heat*, Chapters 4-6.
Edser, *Heat for Advanced Students*, Chapters 3-5.

CHAPTER XXIV

Temperature

233. Nature of Temperature. In our everyday experiences, we constantly refer to bodies as hot or cold, and use the word temperature in speaking of their conditions of hotness or coldness. Now, when is a body hot, and when cold, and what is meant by temperature?

When the blacksmith throws the red-hot iron into a tub of cold water to cool it, the iron evidently loses heat, while the water gains it. When two bodies like the iron and water are in such a condition that one grows warmer and the other colder when they are brought in contact, they are said to be at different temperatures. The body which gains heat is said to be at a lower temperature than the one which loses it. If neither grows warmer when the bodies are brought together, they are said to be at the same temperature. Temperature, therefore, may be defined as *the condition of a body considered with reference to its power of receiving heat from, or communicating heat to, another body.*

234. Temperature and Quantity of Heat. A pint of water taken from a tank is at the same temperature as a gallon taken from the same source. They will also be at the same temperature when both are brought to the boiling-point, but if they are heated by the same gas flame, it will take much longer to bring the gallon up to the boiling-point than to raise the pint to the same temperature. The change in temperature is the same in each, but the *quantity of heat* absorbed is different. A large radiator, filled with hot water may, in cooling, supply sufficient heat to warm up a room, but a small pitcher of water loses its heat with no apparent effect on its surroundings. The quantity of heat possessed by a body evidently depends on its mass as well as its temperature.

235. Determination of Temperature. Up to the time of Galileo, no instrumental means of determining temperature had been devised. Differences in the temperature of bodies were estimated by comparing the sensations resulting from contact with them. But simple experiments will show that our temperature sense cannot be relied upon to determine temperature with any degree of accuracy. Take three vessels, one containing water as hot as can be borne by the hand, one containing ice-cold water, and one with water at the temperature of the room. Hold a finger of one hand in the cold water and a finger of the other in the hot water for one or two minutes, and immediately insert both fingers in the third vessel. To one finger the water will appear to be hot, and to the other, cold. The experiment shows that our estimation of temperature depends, to a certain extent, on the temperature of the sensitive part of the body engaged in making the determination. Our ordinary experiences confirm this conclusion. If we pass from a cold room into one moderately heated, it appears warm, while the same room appears cold when we enter it from one that is overheated.

Again, our estimation of the temperature of a body depends on the nature of the body as well as upon its temperature. It is a well-known fact that on a very cold day a piece of iron exposed to frost feels much colder that a piece of wood, although both may be at the same temperature.

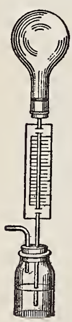

Fig. 253.—Galileo's air thermometer.

236. Galileo's Thermometer. So far as known, Galileo was the first to construct a thermometer. He conceived that since changes in the temperature of a body are accompanied by changes in its volume, these latter changes might be made to measure, indirectly, temperature. He selected air as the body to be employed as a thermometric substance.

His thermometer consisted simply of a glass bulb with a long, slender, glass stem made to dip into water, as shown in Fig. 253. Warming the bulb, caused a few bubbles of air to escape out of the stem, and when it cooled the water rose part way up the stem. Any increase in temperature was then shown by a fall of the water in the tube, and a decrease by a rise. Such a thermometer is imperfect, as the height of the column of liquid is affected by changes in the pressure of the outside air, as well as by changes in the temperature of the air within the bulb. According to Viviani, one of Galileo's pupils, 1593 was the date of the invention of the instrument.

237. Improvements on the Thermometer. About forty years later, Jean Rey, a French physician, improved the instrument by using water instead of air as the expansible substance. The bulb and a part of the stem were filled with water. Further improvements were made by the Florentine academicians, who made use of alcohol instead of water, sealed the tube, and attached a graduated scale. The first mercury thermometer was constructed by the astronomer Ismaël Boulliau, in 1659.

238. Construction of a Mercury Thermometer. Alcohol is still used to measure very low temperatures, but mercury is found in most thermometers in common use. This liquid has been selected for a variety of reasons. Among others the following may be noted:—(1) It can be used to measure a fairly wide range of temperatures, because it freezes at a low temperature and boils at a comparatively high temperature. (2) At any definite temperature it has a constant volume. (3) Slight changes in temperature are readily noted, as it expands rapidly with a rise in temperature. (4) It does not wet the tube in which it is inclosed.

To construct the thermometer a piece of thick-walled glass tubing with a uniform capillary bore is chosen, and a bulb is blown at one end. Bulb and tube are then filled with mercury. This is done by heating the bulb to expel part of the air, and then dipping the open end of the tube into mercury. As the bulb cools, mercury is forced into it by the pressure of the outside air. The liquid within the bulb is boiled to expel the remaining air, and the end of the tube is again immersed in mercury. On cooling, the vapour condenses and bulb and tube are completely filled with mercury. The tube is then sealed off.

239. Determination of the Fixed Points. Since we can describe a particular temperature only by stating how much it is above or below some temperature assumed as a standard, it is necessary to fix upon standards of temperature and also units of difference of temperature. This is most conveniently done by selecting two fixed points for a thermometric scale. The standards in almost universal use are the "freezing-point" and the "boiling-point" of water.

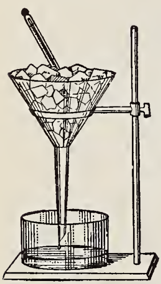

Fig. 254.—Determination of the freezing-point.

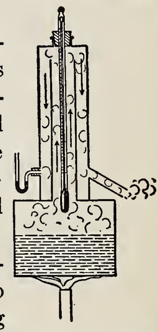

Fig. 255.—Determination of boiling-point.

To determine the freezing-point, the thermometer is surrounded with moist pulverized ice (Fig. 254), and the point at which the mercury stands when it becomes stationary is marked on the stem.

The boiling-point is determined by exposing the bulb and stem to steam rising from pure water boiling under a pressure of 76 cm. of mercury (Fig. 255). As before, the height of the mercury is marked on the stem.

240. The Graduation of the Thermometer. Having marked the freezing-point and the boiling-point, the next step is to graduate the thermometer. Two scales are in common use—the Centigrade scale and the Fahrenheit scale.

The Centigrade scale, first proposed by Celsius, a Swedish scientist, in 1740, and subsequently modified by his colleague Märten Strömer, is now universally employed in scientific work. The space between the freezing-point and the boiling-point is divided into one hundred equal divisions, or degrees, and the zero of the scale is placed at the freezing-point, the graduations being extended both above and below the zero mark.*

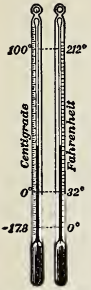

FIG. 256.—Thermometer scales.

The Fahrenheit scale is in common use among English-speaking people for household purposes. It was proposed by Gabriel Daniel Fahrenheit (1686-1736), a German instrument maker. The space between the freezing-point and the boiling-point is divided into one hundred and eighty equal divisions, each called a degree, and the zero is placed thirty-two divisions below the freezing-point. The freezing-point, therefore, reads 32° and the boiling-point 212° (Fig. 256). This zero was chosen, it is said, because Fahrenheit believed this temperature, obtained from a mixture of melting ice and ammonium chloride or sea-salt, to be the lowest attainable.

241. Comparison of Thermometer Scales. If the temperature of the room at the present moment is 68°, the temperature is 68 − 32, or 36 degrees above the freezing-point; but since 180 Fahrenheit degrees = 100 Centigrade degrees, or 9 Fahrenheit degrees = 5 Centigrade degrees, the temperature of the room is $\frac{5}{9}$ of 36, or 20 Centigrade degrees above the freezing-point; that is, the Centigrade thermometer will read 20°.

The relation between corresponding readings on the two thermometers may be obtained in the following way. Let a certain temperature be represented by $F°$ on the Fahrenheit

*Celsius at first marked the boiling-point zero and the freezing-point 100. It is said that the great botanist Linnaeus prompted Celsius and Strömer to invert the scale.

and $C°$ on the Centigrade scale. Then this temperature is $F - 32$ Fahrenheit degrees above the freezing-point, and it is also C Centigrade degrees above the freezing-point. Hence

($F - 32$) Fahr. degrees correspond to C Cent. degrees.
But 9 Fahr. degrees correspond to 5 Cent. degrees,
Therefore $\frac{5}{9}(F - 32) = C$.

242. Maximum and Minimum Thermometers. A maximum thermometer is one which records the highest temperature reached during a certain time. One form is shown in Fig. 257. It is a mercury thermometer with a constriction fixed in the tube just above the bulb (c, Fig. 257). As the temperature rises, the mercury expands and goes past the constriction; but when it contracts, the thread breaks at the constriction, that portion below it contracting into the bulb, while the mercury in the tube remains in the position it had when the temperature was highest. By gently tapping or shaking the thermometer, the mercury can be forced past the constriction, ready for use again.

Fig. 257.—A maximum thermometer (as used in the Meteorological Service).

The clinical thermometer, with which the physician takes the temperature of the body, is constructed in this way.

In another kind of maximum thermometer a small piece of iron is inserted in the stem above the mercury (Fig. 258), and is pushed forward as the mercury expands. When the mercury contracts, the iron is left behind, and thus indicates the highest point reached by the mercury.


Fig. 258.—The iron index is pushed forward by the mercury.

In the minimum thermometer, which registers the lowest temperature reached, alcohol is used. Within the alcohol a small glass index is placed (Fig. 259). As the alcohol contracts, on account of its surface tension (§ 151), it drags the index back, but when it expands, it flows past the index, which is thus left stationary and shows the lowest temperature reached. Tilting the thermometer causes the index to slip down to the surface of the alcohol column, ready for use again.

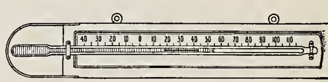
Fig. 259.—A minimum thermometer (as used in the Meteorological Service). It is hung in a horizontal position.

PROBLEMS

1. To how many Fahrenheit degrees are the following Centigrade degrees equivalent: 5, 18, 27, 65?

2. To how many Centigrade degrees are the following Fahrenheit degrees equivalent: 20, 27, 36, 95?

3. How many Fahrenheit degrees above freezing-point is 65° C.?

4. How many Centigrade degrees above freezing-point is 60° F.?

5. Convert the following readings on the Fahrenheit scale to Centigrade readings: 0°, 10°, 32°, 45°, 100°, −25°, and −40°.

6. Convert the following readings on the Centigrade scale to Fahrenheit readings: 10°, 20°, 32°, 75°, −20°, −40°, and −273°.

7. Find in Centigrade degrees the difference between 30° C. and 16° F.

8. In the Réaumur scale, (which is used for household purposes in some countries of Europe), the freezing-point is marked 0° and the boiling-point 80°. Express

   (*a*) 12° C., −10° C., 5° F., 36° F. in the Réaumur scale.

   (*b*) 16° R., 25° R., −6° R. in both the Centigrade and the Fahrenheit scale.

9. A thermometer was graduated to read 0° at the boiling-point, and 150° at the freezing-point. Find the reading on the Centigrade scale corresponding to 95° on this thermometer.

10. When the bulb of a thermometer is placed in hot water, the mercury drops perceptibly at first and then rises. Explain.

REFERENCES FOR FURTHER INFORMATION

Preston, *Theory of Heat*, Chapter 2.
Edser, *Heat for Advanced Students* Chapter 1, 2.

CHAPTER XXV

Relation Between Volume and Temperature

243. Coefficient of Expansion of Solids. It is frequently necessary to calculate with accuracy the changes in dimensions which bodies undergo through changes in temperature, as for example, when allowances are to be made for expansion and contraction in materials used for structural purposes. The important factor to be used in these calculations is the *rate* at which the substances composing the bodies expand.

The rate of expansion in length of solids is expressed as the coefficient of linear expansion, and the rate of expansion in volume, as the coefficient of cubical expansion.

The coefficient of linear expansion may be defined as the *increase in length experienced by a rod of unit length when its temperature is raised one degree.*

244. Coefficient of Linear Expansion. Let us use the apparatus shown in Fig. 260. *T* is a horizontal brass jacket tube covered with non-conducting material. Two smaller tubes, *H*, *K*, are soldered into this tube. Steam from the boiler *B* enters at *H* and passes out at *K*. A third tube at the middle, closed by a cork, has a thermometer *G* fitted into it. The rod to be experimented on is placed

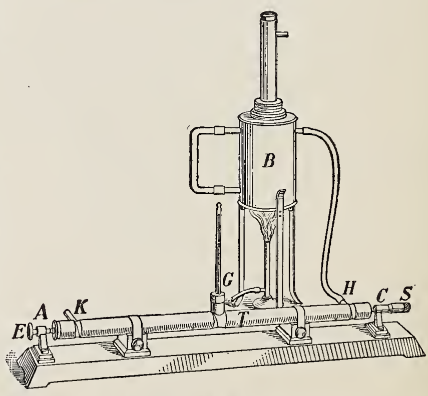

Fig. 260.—Apparatus for finding the coefficient of linear expansion of a metal rod.

within T, which is closed by conical metal caps which keep the rod central in T. This tube is carried on a rigid base, being held securely in place by clips. There are two uprights A and C, one at each end, firmly fastened to the base. One of these carries an adjusting screw E, and the other a micrometer S, which should read to 0.01 mm.

First measure the length of the rod by means of a metre stick. Then place it in the jacket T, and, having one end firm against the screw E, turn the micrometer screw S until it makes gentle contact with the other end. Take the reading of the micrometer and also that of the thermometer, which gives the temperature of the rod. Then turn the screw back two or three rotations to allow for the expansion of the rod.

Now connect the boiler and allow the steam to pass freely through for some time, until the rod has had time to be heated through and the thermometer is steady. Catch the condensed steam in a vessel placed under K, not on the base of the apparatus. Then turn the micrometer screw until it again makes gentle contact with the end of the rod. Record the temperature and read the micrometer. The micrometer readings must be taken with great care since the expansion is a very small quantity and any error made in measuring it will make a great difference in the final result.

In an actual experiment the following results were obtained:

Length of brass rod at 20° C. = 50 cm.

Increase of length when temperature of rod was raised to 100° C. = .075 cm

50 cm. brass heated through 80 Cent. deg. lengthened .075 cm.

1 cm. brass heated through 1 Cent. deg. lengthened .00001875 cm.

In general, the coefficient of linear expansion $= \dfrac{l_2 - l_1}{l_1 (t_2 - t_1)}$, where l_1 is the length at temperature t_1, and l_2 the length at temperature t_2.

The following table gives the coefficients of linear expansion of some common substances. The coefficient of cubical expansion of a solid is usually determined by a calculation from the linear coefficient.

Coefficients of Linear Expansion for 1° C.

Substance	Coefficient	Substance	Coefficient
Aluminium	0.00002313	Nickel	0.00001279
Brass	0.00001900	Platinum	0.00000899
Copper	0.00001678	Silver	0.00001921
Glass	0.00000899	Steel	0.00001322
Gold	0.00001443	Tin	0.00002234
Iron (soft)	0.00001210	Zinc	0.00002918

An alloy of nickel and steel (36 per cent. of nickel) known as "invar," has a coefficient of expansion only one-tenth that of platinum.

245. Coefficient of Expansion of Liquids. Like solids, different liquids expand at different rates. Many liquids also are very irregular in their expansion, having different coefficients at different temperatures.

Fig. 261.—Determination of the coefficient of expansion of a liquid.

The coefficient of expansion of a liquid may be determined with a fair degree of accuracy by a modification of the experiment described in § 229. The liquid is inclosed in a bulb and graduated capillary tube, shown in Fig. 261. The bulb is heated in a bath, and the position of the surface of the liquid in the tube corresponding to various temperatures is noted. Now, if the volume of the bulb in terms of the divisions of the stem is known, the expansion can be calculated. To be accurate, corrections should be made for changes in the capacities of the bulb and tube through changes in temperature.

246. Peculiar Expansion of Water; its Maximum Density. If the bulb and tube shown in Fig. 261 is filled with water at the temperature of the room—say 20° C.—and the bulb placed in a cooling bath, the water will regularly contract in volume until its temperature falls to 4° C., and then it will expand until it comes to the freezing-point. Conversely, if water at 0° C. is heated, it will contract in volume until it reaches 4° C., and then it will expand.* Hence, a given mass of water has minimum volume and maximum density when it is at 4° C.

An experiment devised by Hope shows in a simple manner that the maximum density of water is at 4° C. A metal reservoir is fitted about the middle of a tall jar, and two thermometers are inserted, one at the top and the other at the bottom, as shown in Fig. 262. The jar is filled with water at the temperature of the room, and a freezing-mixture of finely chopped ice and salt is placed in the reservoir. (The reservoir requires constant filling as the experiment proceeds.) The upper thermometer remains stationary and the lower one continues to fall until it indicates a temperature of 4° C. The lower one now remains stationary and the upper one begins to fall and continues to do so until it reaches the freezing-point.

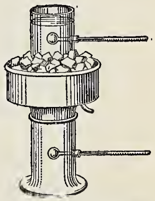
Fig. 262.—Hope's apparatus.

The experiment shows that as the water about the centre of the jar is cooled it becomes denser and continues to descend until all the water in the lower part of the jar has reached the maximum density. When the water in the middle of the jar is cooled further, it becomes lighter and ascends.

The experiment illustrates the behaviour of large bodies of water in cooling as winter approaches. As the surface layers cool, they become denser and sink, while the warmer water below rises to the top. This process continues until the whole

*In an actual experiment the contraction of the glass must be allowed for.

mass of water reaches a uniform temperature of 4° C. The colder and lighter water then remains on the surface, where the ice forms, and this protects the water below.

PROBLEMS

1. A steel piano wire is 4 feet long at a temperature of 16° C. What is its length at 20° C.?

2. A brass scale is exactly one metre long at 0° C. What is its length at 18° C.?

3. A pane of glass is 12 inches long and 10 inches wide at a temperature of 5° C. What is the area of its surface at 15° C.?

4. The bars in a gridiron pendulum are made of iron and copper. If the iron bars are 80 cm. long, what should be the length of the copper bars?

5. Show that the coefficient of area expansion of a metal is approximately twice and the coefficient of volume expansion approximately three times the coefficient of linear expansion.

6. From the table in § 244, what kind of wire would you choose as being most suitable to seal through glass for electrical purposes? Give reasons.

7. Explain where the ice would form and what would happen if water continued to contract down to 0° C., (1) if solidification produced the same expansion as it does now; (2) if contraction accompanied freezing.

8. A specific gravity bottle weighing 15.0 g. was filled with glycerine at 0° C. and the whole then weighed 65.0 g. It was allowed to stand in water at 60° C. and some of the glycerine escaped by overflowing. On weighing again, it weighed 63.5 g. Find the coefficient of expansion of glycerine. (Neglect the expansion of the bottle.)

247. Coefficient of Expansion of Gases—Charles' Law. It has been shown by the experiments of Charles, Gay-Lussac, Regnault,* and other investigators, that under constant pressure all gases expand equally for equal increases in temperature. In other words, all gases have approximately the same coefficient of expansion. Further, it was shown by Charles, that under constant pressure the volume of a given mass of gas increases by a constant fraction of its volume at

*Charles (1746-1823), Gay-Lussac (1778-1850), Regnault (1810-1878) were all French scientists.

0° C. for each increase of 1° C. in its temperature. Charles roughly determined this ratio, which was afterwards more accurately measured by Gay-Lussac, whose researches were published in 1802.

The general statement of the principle is usually known as CHARLES' LAW, but sometimes as *Gay-Lussac's Law.* It is given in the following statement:—*The volume of a given mass of any gas at constant pressure increases for each rise of* 1° *C. by a constant fraction (about* $\frac{1}{273}$*) of its volume at* 0° *C.*

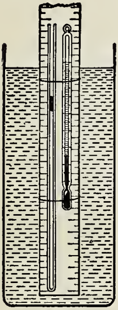

FIG. 263. — Apparatus for measuring the expansion of a gas.

An approximate verification of this law may be made in the following way:—A piece of capillary tubing of uniform bore (Fig. 263), about 60 cm. in length, is sealed at one end. Dry air is shut up in this tube by a thread of mercury, the air occupying about one-half the length of the tube at the temperature of the room. If the tube is kept in a vertical position, the inclosed air will be under constant pressure, provided no mercury escapes from the tube.

Tie the tube to a metre stick and hold it in a glass jar filled with melting ice, and when the mercury has come to rest, take the reading at the end of the air column. Note also the position of the lower end of the air column. Now replace the ice with hot water, stirring it so that the temperature is the same throughout. When the air has ceased to expand, read the end of the air column again.

Let the length of the air column at 0° C. be 20 cm. and at 80° C. let it be 25.8 cm. Then since the tube is of uniform bore, the volumes of the air at the different temperatures will be proportional to the lengths of the column. Let the area of the cross-section of the tube be x sq. cm.

Then $20x$ c.c. air when heated through 80 Cent. deg. expands $5.8x$ c.c.

and 1 c.c. air when heated through 1 Cent. deg. expands $\frac{5.8x}{20x \times 80} = \frac{1}{276}$ c.c. (approx.)

In this experiment no correction has been made for the expansion of the glass. It requires very careful work with much more complicated apparatus to obtain the correct result $\frac{1}{273}$*.

It has also been shown that if the volume remains constant, the pressure of a given mass of gas increases by the same constant fraction (about $\frac{1}{273}$) of its pressure at 0° C. for each rise in temperature of 1° C. That is to say, the volume-coefficient, and the pressure-coefficient of a gas are numerically equal. Practically, this is but a statement, in other terms, of the fact that, in obeying Charles' Law, gases also obey Boyle's Law.

248. Absolute Temperature. Since the pressure of a given mass of air increases by $\frac{1}{273}$ of its pressure at 0° C. for each increase of one degree in temperature, at 1° C. the pressure will be $\frac{274}{273}$ of that at 0° C. At 2° C. the pressure exerted by the gas will be $\frac{275}{273}$; at 100° C. the pressure will be $\frac{373}{273}$ of that at 0° C.; and so on.

Again, at $-1°$ C. the pressure will be diminished $\frac{1}{273}$, that is, the gas will exert a pressure $\frac{272}{273}$ of that at 0° C.; at $-2°$ C. the pressure will be $\frac{271}{273}$; at $-20°$ C. it will be $\frac{253}{273}$; and so on.

If we could continue lowering the temperature and reducing the pressure in this same way, then at $-273°$ C. the pressure would be nothing. But before reaching such a low temperature the gas would change to a liquid, and our method of measuring temperature by the pressure of the gas would then fail.

LORD KELVIN (SIR WILLIAM THOMSON) (1824-1907). Made important investigations in almost every branch of physics. Famous as electrician of Atlantic cables.

*For an account of Regnault's method of determining this coefficient see Edser's *Heat for Advanced Students*, page 106.

However, calculations based on the kinetic theory of gases (§137) lead to the conclusion that at $-273°$ C. the rectilinear motions of the molecules would cease; which would mean that the substance was completely deprived of heat and at the lowest possible temperature. This point is hence called the *absolute zero*, and temperature reckoned from it is called *absolute temperature*. Thus a Centigrade reading can be converted into an Absolute reading by adding 273 to it.

The lowest temperature actually reached is -272.1° C. or 0.9°A.

The method of measuring temperature on an absolute scale was proposed by Lord Kelvin in 1848.

249. Further Statement of Charles' Law. Let V_0, V_1, V_2, etc., represent the volumes of a given mass of gas, under constant pressure, at temperatures, respectively, 0°, 1°, 2°, etc., C., that is, 273°, 274°, 275°, etc., Absolute, then according to Charles' Law

$$V_0 : V_1 : V_2 : \text{etc.,} = \tfrac{273}{273} V_0 : \tfrac{274}{273} V_0 : \tfrac{275}{273} V_0 : \text{etc.}$$
$$= 273 : 274 : 275 : \text{etc.}$$

Stating this result in words, *the volume of a given mass of gas at a constant pressure varies directly as the absolute temperature.*

This manner of stating the law is often convenient for purposes of calculation.

PROBLEMS

1. If the absolute temperature of a given mass of gas is doubled while the pressure is kept constant, what change takes place in (*a*) its volume, (*b*) its mass, (*c*) its density?

2. The pressure of a given mass of gas was doubled while its volume remained constant. What change must have taken place in (*a*) its absolute temperature, (*b*) its density?

3. The pressure remaining constant, what volume will a given mass of gas occupy at 75° C. if its volume at 0° C. is 22.4 litres?

4. If the volume of a given mass of gas is 120 c.c. at 17° C., what will be its volume at $-13°$ C.?

5. A gauge indicates that the pressure of the oxygen gas in a steel gas tank is 150 pounds per square inch when the temperature is 20° C. Supposing the capacity of the tank to remain constant, find the pressure of the gas at a temperature of 30° C.

6. An empty bottle, open to the air, is corked when the temperature of the room is 18° C. and the barometer indicates a pressure of 15 pounds per square inch. Neglecting the expansion of the bottle, find the pressure of the air within it after it has been standing for some time in a water bath whose temperature is 67° C.

7. An uncorked flask contains 1.3 grams of air at a temperature of −13° C. What mass of air does it contain at a temperature of 27° C. if the pressure remains constant?

8. The volume of a given mass of gas is one litre at a temperature of 5° C. The pressure remaining constant, at what temperature will its volume be (*a*) 1100 c.c., (*b*) 900 c.c.?

9. At what temperature will the pressure of the air in a bicycle tire be 33 pounds to the square inch, if its pressure at 0° C. is 30 pounds per square inch? (Assume no change in volume.)

10. A certain mass of hydrogen gas occupies a volume of 380 c.c. at a temperature of 12° C. and 80 cm. pressure. What volume will it occupy at a temperature of −10° C. and a pressure of 76 cm.?

(1) Change in volume for change in temperature.
Since the volume varies directly as the absolute temperature and the temperature is reduced from 12° C. to −10° C. the volume will be reduced to become $\frac{273-10}{273+12}$ or $\frac{263}{285}$ of the original volume.

(2) Change in volume for change in pressure.
Since the volume varies inversely as the pressure, and the pressure is reduced from 80 cm. to 76 cm., the volume will be increased to become $\frac{80}{76}$ of the original volume.

Hence, taking into account the changes for both temperature and pressure, the volume required will be,

$$380 \times \frac{263}{285} \times \frac{80}{76} = 369.12 \text{ c.c.}$$

11. A mass of oxygen gas occupies a volume of 120 litres at a temperature of 20° C. when the barometer stands at 74 cm. What volume will it occupy at standard temperature and pressure? (0 °C. and 76 cm. pressure.)

12. The volume of a certain mass of gas is 500 c.c. at a temperature of 27° C. and a pressure of 400 grams per sq. cm. What is its volume at a temperature of 17° C. and a pressure of 600 grams per sq. cm.?

13. A mass of gas occupies 300 c.c. under a pressure of 760 mm. of mercury when at 27° C. What will the pressure become if the volume is kept constant while the temperature is raised to 327° C.?

14. A volume of gas is heated from 27° C. to 927° C., while at the same time the external pressure is raised from 15 to 60 lbs. per sq. in. How is the volume affected?

15. If one gram of a gas at 50° C. and under a pressure of 76 cm. of mercury has a volume of 800 c.c., what will be its volume at 84° C. when the pressure on it is 64 cm. of mercury?

16. The pressure upon a quantity of gas in a cylinder with a moveable piston is changed from 3 to 5 atmospheres while at the same time the temperature is changed from 20° C. to 320° C. What is the ratio of the new to the old volume?

17. The weight of a litre of air at standard temperature and pressure is 1.29 grams. Find the weight of 800 c.c. of air at 37° C. and 70 cm. pressure.

18. The density of hydrogen gas at standard temperature and pressure is 0.0000896 grams per c.c. Find its density at 15° C. and 68 cm. pressure.

19. Air is forced into a vessel whose volume is 1000 c.c. until it contains 10 gm., the temperature being 17° C. Find the pressure of the air in the vessel if the mass of one litre of air at 0° C. and 760 mm. is 1.29 gm.

20. A toy balloon is filled with carbon dioxide whose specific gravity is 1.529. To what temperature must the balloon be subjected to make it rise in air whose temperature is 20°C.? (Neglect the weight and the tension of the balloon).

CHAPTER XXVI

Measurement of Heat

250. Unit of Heat. As already pointed out (§ 234), the *temperature* of a body is to be distinguished from the *quantity of heat* which it contains. The thermometer is used to determine the temperature of a body, but its reading does not give the quantity of heat possessed by it. A gram of water in one vessel may have a higher temperature than a kilogram in another, but the latter will contain a greater quantity of heat. Again, a pound of water and a pound of mercury may be at the same temperature, but we have reasons for believing that the water contains more heat.

In order to measure heat we must choose a suitable unit, and, by common consent, the amount of heat required to raise by one degree the temperature of a unit mass has been selected as the most convenient one. The unit, will, of course, have different magnitudes, varying with the units of mass and temperature difference chosen. In connection with the metric system the unit called the CALORIE has been adopted for scientific purposes. It is *the amount of heat required to raise a mass of one gram of water one Centigrade degree in temperature.*

For example,

to raise 1 gram of water through 1° C. requires 1 calorie,

to raise 4 grams of water through 5° C. requires 20 calories,

and to raise m grams of water from t_1° to t_2° C. requires $m\ (t_2 - t_1)$ calories.

In engineering practice, the British Thermal Unit (designated B. T. U.) is in common use in English-speaking countries. It is the quantity of heat required to raise one pound of water one Fahrenheit degree in temperature.

PROBLEMS

1. How many calories of heat must enter a mass of 65 grams of water to change its temperature from 10° C. to 35° C.?

2. How many calories of heat are given out by the cooling of 120 grams of water from 85° C. to 60° C.?

3. If 1400 calories of heat enter a mass of 175 grams of water, what will be its final temperature, supposing the original to be 15° C.?

4. A hot-water coil containing 100 kilograms of water gives off 1,000,000 calories of heat. Neglecting the heat lost by the iron, find the fall in the temperature of the water.

5. On mixing 65 grams of water at 75° C. with 85 grams at 60° C., what will be the temperature of the mixture?

6. Find the resulting temperature when 100 grams of water at 20° C., 200 grams at 35° C. and 300 grams at 60° C. are mixed together.

7. On mixing 300 grams of water at 80° C. with 500 grams at an unknown temperature the resulting temperature was 36° C. What was the unknown temperature?

8. What will be the resulting temperature on mixing 400 grams of water at 10° C., 100 at 20° C., 300 at 60° C. and 200 at 80° C.?

9. One pound of a certain sample of coal contains 12,000 B.T.U. If the heat from it is applied to 120 pounds of water at 60° F., to what temperature will it be raised, assuming that one-half of the heat from the coal is lost?

10. If coal of heat value 14,000 B.T.U. per pound can be bought at $10 per ton, what price per cord should be paid for wood of heat value 8,000 B.T.U. per pound, in order that the cost for heating should be the same in the two cases. (Assume that 1 cord of wood weighs 3,500 pounds.)

251. Specific Heat. Let us take three precisely similar beakers and pour 200 grams of water into the first, 200 grams of coal-oil (petroleum) in the second and 200 grams of mercury in the third. The volumes of these substances will be 200 c.c., 228 c.c. and 14.7 c.c., respectively. (See Table, p. 12.)

Place the first beaker on the sand in a sand-bath which has been over a small Bunsen flame for some time and has become heated through (Fig. 264). Let it stay there for 30 sec. and observe the rise in temperature. Let it be 2°.

Next, place the second beaker on for the same length of time, (be careful as the oil is very inflammable). The rise in temperature will be 4°, or twice as great. Finally, put the third beaker on for the same length of time. The rise in temperature will actually be 60°, or thirty times as great.

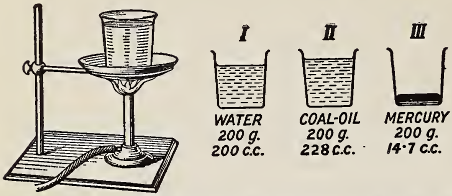

FIG. 264.—The same amount of heat is applied to the same mass of water, coal-oil and mercury.

Now the same amount of heat has been applied to each substance, and so it is clear that in order to heat a given mass of coal-oil through a certain number of degrees one need use only $\frac{1}{2}$ the amount of heat required to heat the same mass of water through the same rise in temperature, while to heat the same mass of mercury through the same rise in temperature, only $\frac{1}{30}$ as much heat is required as in the case of water. We see then that

To raise	1	gm. of	water	through	1	Cent. deg.	requires	1	calorie,
"	"	1	"	"	coal-oil	"	1	"	"	"	$\frac{1}{2}$	"
"	"	1	"	"	mercury	"	1	"	"	"	$\frac{1}{30}$	"

Taking water as the standard substance, we say that the *specific heat* of water is 1, that of coal-oil is $\frac{1}{2}$, and that of mercury is $\frac{1}{30}$.

In more general terms we can define the SPECIFIC HEAT of a substance to be *the number of heat units required to raise the temperature of a unit mass of the substance one degree.*

Hence, the quantity of heat required to warm a mass of m grams of a substance from a temperature of $t_1°$ to a temperature of $t_2° = m(t_2 - t_1)\,s$, when s is the specific heat of the substance.

PROBLEMS

1. How much heat is required to raise the temperature of 350 grams of coal-oil from 20° C. to 90° C.?

2. Calculate the amount of heat required to raise 500 c.c. of coal-oil from 15° C. to 45° C. (See Table, p. 12.)

3. Find how much heat is required to raise 500 grams of mercury from −15° C. to 175° C.?

4. How much heat is required to raise 100 c.c. of mercury from 20° C. to 200° C.?

5. If the specific heat of iron is 0.113, calculate the amount of heat in B.T.U. to raise the temperature of an iron radiator weighing 500 pounds and the 200 pounds of water which it contains from 10° C. to 85° C.

252. Capacity for Heat. Of all known substances except hydrogen, water has the greatest capacity for heat, which fact is of great importance in the distribution of heat on the surface of the earth. For example, land areas surrounded by large bodies of water are not so subject to extremes of temperature. In summer the water absorbs the heat, and, as it warms very slowly, it remains cooler than the land. In winter, on the other hand, the water gradually gives up its store of heat to the land, thus preserving an equable temperature.

The THERMAL CAPACITY of a body is defined to be *the number of heat units required to raise its temperature one degree.*

253. Determination of Specific Heat by the Method of Mixture. The method depends on the principle that *the amount of heat lost by a hot body when placed in cold water is equal to the amount of heat gained by the water.* Let us apply the method to find the specific heat of lead. Take a definite mass of lead in small pieces, say 220 grams, and heat it in steam from boiling water to a temperature of 100° C. (Fig. 265). Now place 100 grams of water at the temperature of the room, say 20° C., in a beaker and surround it it with wool or batting to keep the heat from escaping. Pour the lead into the water and, after stirring it, take the temperature. Let it be 25° C. Then the heat gained by the water

$$= 100\,(25° - 20°)\text{ cal.} = 500\text{ cal.}$$

If now no heat has escaped, 220 grams of lead must, in falling from 100° to 25°, have lost 500 cal. of heat. Or 1 gram of lead in falling 1° loses $500 \div (220 \times 75) = 0.030$ calories.

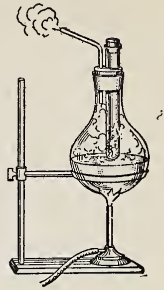

FIG. 265.—Determination of specific heat of a solid.

A general formula may be obtained as follows:—

Let m = the mass of the substance, and t = its temperature;

m_1 = the mass of the water, and t_1 = its temperature.

Let t_2 = resulting temperature after mixing.

Then, heat gained by water $= m_1\,(t_2 - t_1)$,
and heat lost by the substance $= m\,(t - t_2)\,s$, where s is its specific heat;

Therefore $m\,(t - t_2)\,s = m_1\,(t_2 - t_1)$, or $s = \dfrac{m_1\,(t_2 - t_1)}{m\,(t - t_2)}$.

254. Specific Heat of Liquids.—As a rule, the specific heat of liquids cannot be determined with any degree of accuracy by mixing the liquids with water, because many liquids do not mix readily with water, while others evolve heat when added to it; but the method of mixtures may be used for this purpose by taking a suitable body of known specific heat and dropping it into the liquid whose specific heat is to be determined. For example, a mass of 80 grams of a solid whose specific heat is 0.21 is heated to 90° C. and dropped into 100 grams of the liquid whose specific heat is to be investigated, when at a temperature of 10° C., and the resulting temperature is found to be 30° C. Then if x is the specific heat of the liquid, the heat gained by it is $100(30 - 10)x = 2000x$ calories; and the heat lost by the solid is $80(90 - 30) \times 0.21 = 1008$ calories.

Therefore $2000x = 1008$, or $x = 0.504$, the specific heat of the liquid.

Exercise.—Obtain a general formula for finding the specific heat of a liquid by this method.

Glass vessels are not the best to use in experiments involving the method of mixtures. They are easily broken, and, besides, they are slow in taking the temperature of the water which they hold. The best vessels for our purpose are of polished metal (nickel-plated brass is very good), and it is well to have the vessel containing the water within a larger one which shields it from outside radiation. Also one should be careful not to communicate heat by the hand. The vessel (or vessels) in which the experiment is made is termed a *calorimeter*. A good form of calorimeter is shown in Fig. 266. Here a polished vessel A rests on a wooden block within a larger vessel B, and the temperature can be taken by a thermometer inserted through a hole in the top.

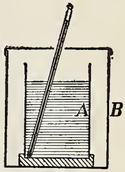

Fig. 266.—A calorimeter.

In accurate work allowance should be made for the heat gained or lost by the calorimeter.

Average Specific Heats of Some Common Substances

Aluminium.....0.214	Ice (−10° C.)..0.50	Paraffin (wax)..0.694
Brass..........0.090	Iron...........0.113	Petroleum......0.511
Copper.........0.094	Lead...........0.031	Platinum.......0.032
Glass (crown)..0.16	Marble.........0.216	Silver..........0.056
Gold...........0.032	Mercury.......0.033	Zinc............0.093

PROBLEMS

(For specific heats see table just above)

1. What is the thermal capacity of a glass beaker whose mass is 35 grams?

2. Which has the greater thermal capacity, 68 grams of mercury or 2 grams of water?

3. It requires 360 calories of heat to raise the temperature of a body 10 degrees. What is its thermal capacity?

4. The thermal capacity of 56 grams of copper is 5.264 calories. What is the specific heat of copper?

5. It requires 902.2 calories of heat to warm 130 grams of paraffin from 0° C. to 10° C. What is the specific heat of paraffin?

6. How much heat will a body whose thermal capacity is 320 calories lose in cooling from 40° to 10° C.?

7. What is the quantity of heat required to raise 120 grams of aluminium from 15° to 52° C.?

8. How many calories of heat are given off by an iron radiator whose mass is 25 kg., in cooling from 100° to 20° C.

9. A lead bullet whose mass is 12 grams had a temperature of 25° C. before it struck an iron target, and a temperature of 100° C. after impact. How many calories of heat were added to the bullet?

10. Into 120 grams of water at a temperature of 0° C. 150 grams of mercury at 80° C. are poured. What is the resulting temperature?

11. If 95 grams of a metal are heated to 100° C. and then placed in 114 grams of water at 7° C., the resulting temperature is 15° C. Find the specific heat of the metal? What metal is it?

12. A piece of iron, whose mass is 88.5 grams and temperature 90° C., is placed in 70 grams of water at 10° C. If the resulting temperature is 20° C., find the specific heat of iron.

13. A mass of zinc, weighing 5 kg. and having a temperature of 80° C., was placed in a liquid, and the resulting temperature was found to be 15° C. How much heat did the zinc impart to the liquid?

14. Find the resulting temperature on placing 75 grams of a substance having a specific heat of 0.8 and heated to 95° C. in 130 grams of a liquid at 10° C. whose specific heat is 0.6.

15. On mixing 1 kg. of a substance having a specific heat of 0.85, at a temperature of 12° C., with 500 grams of a second substance at a temperature of 120° C., the resulting temperature is 45° C. What is the specific heat of the second substance?

16. Find the resulting temperature when 20 grams of iron at 100° C. are immersed in 80 grams of water at 10° C., contained in a copper vessel whose mass is 20 grams.

17. On dropping 100 grams of iron at 200° C. into 600 grams of oil, the temperature of the oil is raised from 15° C. to 20° C. Find the specific heat of the oil.

18. The "water equivalent" of a body is the mass (in grams) of water which has the same thermal capacity as the body. Find the water equivalents of the following:—

(*a*) A copper calorimeter weighing 120 grams.
(*b*) A glass stirring-rod weighing 20 grams.
(*c*) A thermometer containing 3 grams of mercury and 10 grams of glass.

CHAPTER XXVII

Change of State

255. Fusion. Let us take some pulverized ice at a temperature below the freezing-point and apply heat to it. It gradually rises in temperature until it reaches 0° C., when it begins to melt. If the ice and the water formed from it are kept well stirred, no sensible change in temperature takes place until all the ice is melted. On the further application of heat the temperature begins again to rise.

The change from the solid to the liquid state by means of heat is called *fusion* or *melting*, and the temperature at which fusion takes place is called the *melting-point.*

The behaviour of water is typical of crystalline substances in general. Fusion takes place at a temperature which is constant for the same substance if the pressure remains unchanged. Amorphous bodies, on the other hand, have no sharply defined melting-points. When heated, they soften and pass through various stages of plasticity into more or less viscous liquids, the process being accompanied by a continuous rise in temperature. Paraffin wax, glass and wrought-iron are typical examples. By a suitable control of the temperature glass can be bent, drawn out, moulded or blown into various forms, and iron can be forged, rolled or welded.

256. Solidification. The temperature at which a substance solidifies, the pressure remaining constant, is the same as that at which it melts. For example, if water is gradually cooled, while it is kept agitated, it begins to take the solid form at 0° C., and it continues to give up heat without falling in temperature until the process of solidification is complete.

But it is interesting to note that a liquid which under ordinary conditions solidifies at a definite point may, if slowly and carefully cooled, be lowered several degrees below its normal temperature of solidification. The phenomenon is illustrated in the following experiment. Pour some pure water, boiled to free it from air bubbles, into a test-tube. Close the tube with a perforated stopper, through which a thermometer is inserted into the water. Place the tube in a freezing mixture of ice and salt. If the water is kept quiet, it may be lowered to a temperature of $-5°$ C. without freezing it, but the condition is unstable. If the water is agitated or crystals of ice are dropped in, it suddenly turns into ice and the temperature rises to 0° C.

Melting-points of Some Substances

Substance	M. P.	Substance	M. P.	Substance	M. P.
Aluminium.	659°C.	Iron (pure) . .	1530° C.	Silver	960° C.
Cobalt	1480	Lead	327	Sulphur . . .	115
Copper	1083	Mercury	-39	Tin	232
Gold	1063	Nickel	1452	Tungsten . .	3400
Iridium	2350	Platinum	1755	Zinc	419

257. Change of Volume in Fusion. Most substances suffer an increase in volume in passing from the solid to the liquid state, but some which are crystalline in structure, such as ice, bismuth, and antimony are exceptions to the rule.

The expansive force of ice in freezing is well known to all who live in cold climates. The earth is upheaved and rocks are disintegrated, while vessels and pipes which contain water are burst by the action of the frost.

Only from metals which expand on solidification can perfectly shaped castings be obtained. The reasons are obvious. Antimony is added to lead and tin to form type-metal because the alloy thus formed expands on solidifying and conforms completely and sharply with the outlines of the mould.

258. The Influence of Pressure on Melting-point. *If a substance expands on melting, its melting-point will be raised by pressure, while if it contracts, its melting-point will be lowered.* We should expect this. Since extra pressure applied to a body which takes a larger volume on melting would tend to prevent it from expanding, it would be reasonable to suppose that a higher temperature would be necessary to bring about the change; on the other hand, if the body contracts on melting, increased pressure would tend to assist the process of change, and a lower temperature should suffice.

An interesting experiment shows the effect of pressure on the melting-point of ice. Take a block of ice and rest it on two supports, and encircle it with a fine wire from which hangs a heavy weight (Fig. 267). In a few hours the wire will cut its way through the ice, but the block will still be intact. Under the pressure of the wire the ice melts, but the water thus formed is below the normal freezing-point. Hence it flows above the wire and freezes again, as the pressure there is normal. The process of melting and freezing again under these conditions is called *regelation.*

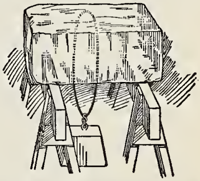

FIG. 267.—Regelation of Ice.

259. Heat of Fusion. We have seen (in § 255) that during the process of melting a crystalline body like ice, no change in temperature takes place, although heat is being continuously applied to it. In earlier times, when heat was considered to be a kind of substance, it appeared that the heat applied became hidden in the body, and it was called *latent heat.*

According to modern ideas, there is simply a transformation of energy. When a body in fusing ceases to rise in temperature, although heat is still being applied, the heat-energy is no longer occupied in increasing the average kinetic energy and to some extent the potential energy of its molecules, but is doing work in overcoming the cohesive forces which bind these molecules together in the body as a solid.

A definite quantity of heat, varying with the substance, is required to melt a definite mass of a solid. *The amount of heat required to melt one gram of a substance without a change of temperature is called its* HEAT OF FUSION. For example, the heat of fusion of ice is 80 calories, which means that 80 calories of heat are required to melt one gram of ice.

260. Determination of the Heat of Fusion of Ice. The *method of mixture* (§ 253) may be used to determine the heat of fusion of ice. For example, if 100 grams of dry snow or finely broken ice are dropped into 500 grams of water at 40° C., and the mixture is rapidly stirred until all the ice is melted, it will be found that the resulting temperature is about 20° C.

Then the amount of heat lost by 500 grams of water in cooling from 40° C. to 20° C. = 500 (40 − 20) = 10,000 calories.

This heat melts the ice and then raises the temperature of the resulting water from 0° to 20° C. But to raise the resulting 100 grams of water from 0° to 20° C. requires $100 \times 20 = 2000$ calories.

Hence the heat required to melt the 100 grams of ice = 10,000 − 2000 = 8000 calories, and the heat required to melt 1 gram of ice = 80 calories.

A general formula is obtained as follows:—

Let m = the mass of water (in grams), t_1 = its initial temperature,
t_2 = its final temperature, m_1 = the mass of the ice (in grams),
x = the heat of fusion.

Then heat lost by water in falling from t_1 to $t_2 = m\,(t_1 - t_2)$ cal.

Heat required to melt m_1 grams of ice $= m_1\,x$ cal.

Heat required to raise m_1 grams of water from 0° to $t_2 = m_1\,t_2$ cal.

But the heat lost by the water is used in melting the ice and raising the temperature of the resulting water from 0° to t_2.

$$\text{Hence, } m\,(t_1 - t_2) = m_1\,x + m_1\,t_2,$$

$$\text{and } x = \frac{m\,(t_1 - t_2) - m_1\,t_2}{m_1}.$$

HEAT OF FUSION.
M.P., Melting Point; H. of F., Heat of Fusion.

Substance	M.P.	H. of F.	Substance	M.P.	H. of F.
Iron, gray cast.	1200° C.	23	Paraffin Wax...	52° C.	35
Ice..........	0	80	Silver.........	960	21
Lead..........	327	5.4	Sulphur........	115	9.4
Mercury......	−39	2.8	Zinc...........	419	28

261. Heat given out on Solidification. All the heat required to melt a certain mass of a substance without change in temperature is given out again in the process of solidification. Thus, every gram of water, in freezing, sets free 80 calories of heat. The formation of ice tends to prevent extremes of temperature in our lake regions. Heat is given out in the process of freezing during the winter, and absorbed in melting the ice in spring and early summer.

262. Heat Absorbed in Solution; Freezing Mixtures. In cases of ordinary solution, as in dissolving sugar or salt in water, heat is absorbed. If a handful of salt is dropped into a beaker of water at the temperature of the room, and the mixture is stirred with a thermometer, the mercury will be seen to drop several degrees.

The result is much more marked if ice and salt are mixed together. Both become liquid and absorb heat in doing so. This is the principle applied in preparing freezing mixtures. The ordinary freezing mixture of ice and salt can be made to give a temperature of about −22°C. From what is the heat absorbed?

QUESTIONS AND PROBLEMS

1. Two pieces of cast iron can be joined together by the oxy-acetylene flame. How does this process differ from the blacksmith's welding?

2. Water is sometimes placed in cellars to keep vegetables from freezing. Explain the action.

3. Why is a quantity of ice at 0° C. more effective as a cooling agent than an equal mass of water at the same temperature?

4. If two pieces of ice are pressed together under the surface of warm water, they will be found to be frozen together on removing them from the water. Account for this.

5. If we pour just enough cold water on a mixture of ammonic chloride and ammonic nitrate to dissolve them, and stir the mixture with a small test-tube, into the bottom of which has been poured a little cold water, the water in the tube will be frozen. Explain.

6. Explain how the presence of ice keeps the contents of a refrigerator cool.

7. A piece of solid metal floats on the surface of the liquid metal. Does the metal expand or contract on solidifying? Explain. Melt some paraffin wax and observe if the solid floats or sinks in the liquid. Does it expand or contract on solidifying?

8. A mixture of crushed ice and water is poured into a vessel containing a thermometer. What will be the effect on the reading of the thermometer when (*a*) more water is poured in, (*b*) more ice is put in, (*c*) salt is stirred in.? Give reasons in each case.

9. What quantity of heat is required to melt 35 grams of ice at 0° C.?

10. How much heat is given off by the freezing of 15 kg. of water?

11. Find the resulting temperature when 40 grams of ice are dropped into 180 grams of water at 90° C.

12. How much ice must be placed in a pail containing 10 kg. of drinking water at 20° C. to reduce the temperature to 10° C.?

13. What mass of water at 80° C. will just melt 80 grams of ice?

14. How much heat is required to change 23 grams of ice at − 10° C. to water at 10° C.? (Specific heat of ice = 0.5.)

15. What mass of water at 75° C. will convert 120 grams of ice into water at 10° C.?

16. What mass of ice must be dissolved in a litre of water at 4° C. to reduce the temperature to 3° C.?

17. What is the specific heat of brass if a mass of 80 grams at a temperature of 100° C. just melts 9 grams of ice?

18. Fifty grams of ice are placed in 520 grams of water at 19.8° C. and the temperature of the whole becomes 11.1° C. Find the heat of fusion of ice.

19. Find the number of B.T.U.'s required to melt 1 pound of ice at the melting-point.

20. How many B.T.U.'s are needed to change 100 pounds of ice at 20° F. to water at 182° F.?

263. Vaporization. Transition from a liquid to a vapour is a familiar phenomenon. Water in a shallow dish exposed to a dry atmosphere gradually changes to a vapour and disappears in the air. If heat is applied and the water is made to boil, the change takes place more rapidly. The process of converting a liquid into a vapour is called *vaporization.* The quiet vaporization taking place at all temperatures at the surface of a liquid is known as *evaporation.*

In *ebullition,* or *boiling,* the production of vapour takes place throughout the mass, and the process is accompanied by an agitation of the liquid, due to the formation of bubbles of vapour within the liquid and their movement upward to the surface.

264. Rate of Evaporation. The rate of evaporation depends on many conditions:—

(1) *The nature of the liquid.* A little ether placed on the palm of the hand disappears almost at once and distinctly cools the skin. Water would keep the hand wet for a considerable time. Some dense oils can scarcely be said to evaporate at all. Liquids which evaporate readily are said to be *volatile.*

(2) *The temperature of both the liquid and the air.* Wet clothes and wet roads dry more rapidly on a warm day than on a cold one, if the atmosphere is equally dry on the two days.

(3) *The dryness of the air.* If there is much vapour already in the air, the evaporation is slower.

(4) *The extent of free surface of the liquid.* With a large surface the evaporation is greater than with a small one.

(5) *The renewal of the air over the liquid.* If a breeze is blowing, evaporation is more rapid. Wet articles dry very rapidly on a windy day.

The molecular explanation of evaporation is given in § 142.

265. Saturated Vapour. When a vapour has its maximum density for any given temperature, it is said to be *saturated,* and the corresponding maximum pressure is called the *saturation pressure.* Whenever a saturated vapour is either cooled or compressed, condensation takes place. For a vapour to be saturated, some of the liquid must be present.

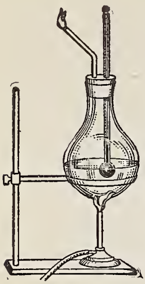

Fig. 268.—Determination of the boiling-point of a liquid.

266. Ebullition — Boiling - point. When heat is applied to water (Fig. 268), it gradually rises in temperature until vapour is disengaged in bubbles from the mass of the liquid; but no further rise in temperature takes place, however rapidly the process of boiling is maintained.

The temperature at which a liquid boils, or gives off bubbles of its own vapour, is called its *boiling-point.*

267. Effect of Pressure on the Boiling-point. The boiling-point varies with the pressure. If the pressure of the escaping steam is increased by leading the outlet-pipe to the bottom

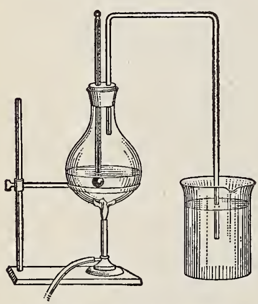

Fig. 269.—Boiling-point of a liquid raised by means of pressure.

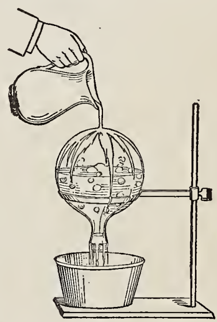

Fig .270.—Boiling-point of a liquid lowered by decrease of pressure.

of a vessel of water as shown in Fig. 269, the temperature of the boiling water is increased. On the other hand, a decrease in pressure is accompanied by a lowering of the temperature. This is shown by a familiar but striking experiment.

Half fill a flask with water and boil for a minute or two in order that the escaping steam may carry out the air. While the water is boiling, remove the flame, and at the same instant close the flask with a stopper. Invert the flask and support it on a retort stand (Fig. 270), and pour cold water over the flask. The temperature of the water in the flask is below 100° C., but it boils vigorously. The action is explained as follows. The chilling of the flask condenses the vapour within and thus reduces the pressure on the surface of the water. The water, relieved of this pressure, boils at a lower temperature. If we discontinue the cooling and allow the vapour to accumulate and the pressure to increase, the boiling ceases. The process may be repeated several times. In fact, if care is taken in expelling the air at the beginning, the water may be made to boil even when the temperature is reduced to that of the room.

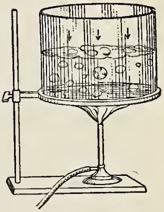

Fig. 271.—Balance between external pressure of the air and the pressure exerted by the vapour within bubble.

The reason why the boiling-point depends upon the pressure is readily found. Bubbles of vapour begin to form in the liquid only when the pressure exerted by the vapour within the bubble balances the pressure on the surface of the liquid (Fig. 271). But the pressure of a vapour in contact with its liquid in an inclosed space varies with the temperature. Hence, a liquid will be upon the point of boiling when its temperature has risen sufficiently high for the pressure of the saturated vapour of the liquid to be equal to the pressure sustained by the surface of the liquid. Therefore, when the pressure on the surface is high, the boiling-point must be high, and *vice versa.* The accompanying diagram (Fig. 272) shows graphically the relation between the pressure and the boiling-point of water ranging from 1 to 25 atmospheres.

It is to be noted that the steam bubbles begin in the small air or gas bubbles present in the water, and when these are removed by prolonged boiling, the liquid boils very irregularly

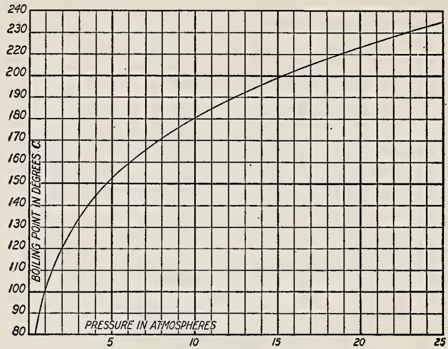

Fig. 272.—Curve showing the relation between the pressure and the boiling-point of water.

(bumps). Geyser phenomena occur because of great hydrostatic pressure due to the water.

268. Relation between Boiling-point and Altitude. Since the boiling-point is dependent on atmospheric pressure, a liquid in an open vessel will boil at a lower temperature as the elevation above the sea-level increases. This decrease is roughly 1° C. for an increase in elevation of 239 metres (= 961 feet). The boiling-point of water at the summit of Mont Blanc (15,781 feet) is about 85° C., and at Quito (9520 feet), the highest city in the world, it is 90° C.

In such high altitudes the boiling-point of water is below the temperature required for cooking many kinds of food, and artificial means of raising the temperature have to be resorted to, such as cooking in brine instead of pure water, or using closed vessels with safety devices to prevent explosions. Sometimes longer boiling is all that is required.*

In the case of liquids liable to burn, evaporation may be

*Eggs can be boiled in an open vessel on Pike's Peak, 14,108 feet high.

produced in "vacuum pans" in which boiling takes place under reduced pressure (and therefore lowered temperature). This arrangement is used, for example, in condensing milk and sugar syrups.

BOILING POINTS OF SOME SUBSTANCES (At Standard Pressure)

Substance	B.P.	Substance	B.P.	Substance	B.P.
Alcohol (Ethyl)	78° C.	Ether........	35° C.	Liquid Air....	–191°C.
Alcohol (Methyl)...	66	Hydrochloric Acid.......	110	Mercury......	357
Benzine......	80	Nitric Acid...	86	Sulphur......	444
Chloroform...	61	Sulphuric Acid	338	Turpentine (Oil).......	159

269. Heat of Vaporization. Whenever a given mass of a liquid changes into a vapour, a definite amount of heat is absorbed. Thus in the process of vaporization a certain amount of energy ceases to exist as heat, and (in a manner similar to fusion) becomes potential energy in the vapour. In accordance with the law of Conservation of Energy, all heat which thus disappears is recovered when the vapour condenses.

The amount of heat required to change one gram of any liquid into vapour without changing the temperature is called the HEAT OF VAPORIZATION, or sometimes the *latent heat of vaporization.* For example, the heat of vaporization of water is 536 calories, by which we mean that, when water is boiling under the standard atmospheric pressure (76 cm. of mercury), 536 calories of heat are required to vaporize one gram without change of temperature.

270. Determination of Heat of Vaporization. The heat of vaporization of water may be determined as follows:—By means of apparatus arranged as shown in Fig. 273, pass steam for a few minutes into a quantity of water in a vessel C. For this vessel a metal calorimeter is preferable to a beaker (see § 254), and the steam need not be delivered under the surface. Take the weight and the temperature of the water before and after the steam is conveyed into it, and find the increase in mass and temperature due to the condensation of the steam.

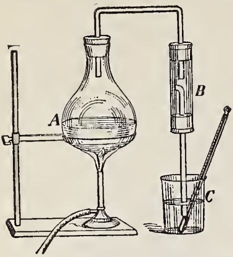

FIG. 273.—Determination of heat of vaporization of water. A, flask to contain water; B, trap to catch water condensed in the tube; C, vessel with known mass of water.

Suppose the mass of water in C at first to be 120 grams and the increase in mass due to condensation to be 5 grams; and suppose the initial and final temperatures of the water to be 10° C. and 35° C. respectively.

We can make our calculation as follows:—

Heat gained by the original 120 grams of water = 120 (35 − 10) = 3000 cal.

This heat comes from two sources,

(*a*) The heat received from the condensation of 5 grams of steam at 100° C. to water at 100° C.

(*b*) The heat received from the fall in temperature of 5 grams of water from 100° C. to 35° C. = 5 (100 − 35) = 325 cal.

Hence, the heat set free by the condensation of 5 grams of steam = 3000 − 325 = 2675 cal.

And the heat set free in the condensation of 1 gram of steam = 2675 ÷ 5 = 535 cal.

HEAT OF VAPORIZATION.

B.P., Boiling-point; H. of V., Heat of Vaporization.

Substance	B.P.	H. of V.	Substance	B.P.	H. of V.
Acetic Acid.....	118° C.	85	Chloroform ...	61° C.	58
Alcohol (Ethyl).	78	205	Ether.........	35	90
Alcohol (Methyl)	66	267	Liquid Air....	−191	55
Benzine.........	80	93	Turpentine(oil)	159	74
Carbon Disulphide.........	46	84	Water........	100	536

271. Distillation. Distillation is a process of vaporization and condensation maintained usually for the purpose of freeing a liquid from dissolved solids, or for separating the constituents of a mixture of liquids. Fig. 274 shows a simple form of distillation apparatus. The liquid to be distilled is evaporated in the flask A, and the product of the condensation of the vapour is collected in the receiver B. The pipe connecting A and B is kept cold by cold water made to circulate in the jacket which surrounds it. The separation of liquids by

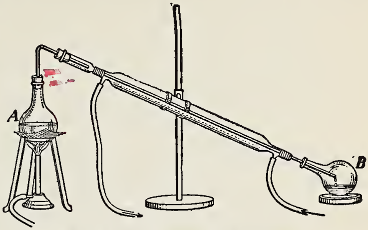

FIG. 274.—Distillation apparatus.

distillation depends on the principle that different liquids have different boiling-points, and consequently are vaporized and can be collected in a regular order. For example, when crude petroleum is heated in a still, the dissolved gaseous hydrocarbons are driven off first; then follow the lighter oils, naphtha, gasoline and benzine; in turn come the kerosene or burning oils; and later the heavier gas and fuel oils, etc. To obtain a quantity of any one constituent of a mixture in a relatively pure state, it is necessary to resort to *fractional distillation.* The fraction of the distillate which is known to contain most of the liquid desired is redistilled, and a fraction of the distillate again taken for further distillation, and so on.

QUESTIONS AND PROBLEMS

1. The singing of a tea-kettle just before boiling is said to be due to the collapse of the first bubbles formed in their upward motion through the water. Explain the cause of the collapse of these bubbles.

2. When water is boiling in a deep vessel, the bubbles of vapour are observed to increase in size as they approach the surface of the water. Give a reason for this.

3. Why does not a mass of liquid air in an open vessel immediately change into gas when brought into a room at the ordinary temperature?

4. Why is it necessary to take into account the pressure of the air in fixing the boiling-point of a thermometer?

5. The steam in a radiator and the water which is condensed from it may both be at the same temperature. Explain how the room has been warmed.

6. A vertical cylinder is one-third filled with water, and immediately above the water is fitted an air-tight piston. The piston is then drawn out nearly to the end of the cylinder. If no heat passes in or out through the piston, or the walls of the cylinder, how does the temperature of the water change? Explain.

7. Why is a pan of water often put into an oven where custards are being baked? Explain.

8. What limits the elevation of the temperature inside a loaf of bread or a piece of meat that is being baked in an oven?

9. How much heat will be required to vaporize 37 grams of water?

10. How many calories of heat are set free in the condensation of 340 grams of steam at 100° C. into water at 100° C.?

11. How much heat is required to raise 45 grams of water from 15° C. to the boiling-point and convert it into steam?

12. How much heat is given up in the change of 365 grams of steam at 100° to water at 4° C.?

13. What is the resulting temperature when 45 grams of steam at 100° C. are passed into 600 grams of ice-cold water?

14. How many grams of steam at 100° C. will be required to raise the temperature of 300 grams of water from 20° C. to 40° C.?

15. How many grams of steam at 100° C. will just melt 25 grams of ice at 0° C.?

16. How much heat is necessary to change 30 grams of ice at −15° C. to steam at 100° C.?

17. An iron radiator whose mass is 55 kg. and temperature 100° is shut off when it contains 100 grams of steam at a temperature of 100° C. How much heat is imparted to the room by the condensation of the steam and the cooling of the water and the radiator to a temperature of 40° C.?

18. If 34.7 grams of steam at 100° C. are conveyed into 500 grams of water at 20° C., the resulting temperature is 60° C. Find the heat of vaporization of water.

19. How many grams of steam at 100° C. must be passed into 200 gm. of ice-cold water to raise it to the boiling-point? What will happen if more steam than this is passed in?

20. The pool in a gymnasium is 20 metres long, 8 metres wide and 2 metres deep. How much steam at 212° F. must be passed into it to raise the temperature of the water from 50° F. to 68° F.?

272. Cold by Evaporation. In order to change a liquid into vapour, heat is always required. Water placed over a flame is turned into vapour, the heat required being supplied by the flame. If a little ether is poured on the palm of the hand, it vaporizes at once. Here the heat to produce vaporization is supplied by the hand, which therefore feels cold. For a similar reason wet garments are cold, especially if drying rapidly on a windy day.

But it is sometimes possible to produce vaporization without supplying heat from an outside source. In this case the heat comes from the liquid itself, which must, therefore, fall in temperature. Indeed, it is possible, by producing evaporation, to lower the temperature of water so much that the water will actually freeze. This is well shown in Leslie's experiment. A small quantity of cold water in a watch glass is inclosed in the receiver of an air-pump over or near a dish of strong sulphuric acid (Fig. 275). The air is then exhausted from the receiver. When the pressure is reduced sufficiently, the water begins to boil, and as the vapour is removed from the receiver, partly by being carried off with the air by the pump, and partly by absorption into the sulphuric acid, the process continues until the water is frozen.

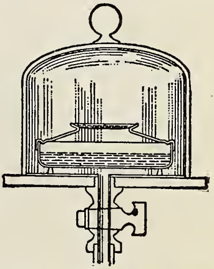

FIG. 275.—Leslie's experiment; freezing water by its own evaporation.

Similar results are shown in a more striking manner by the freezing of carbon dioxide by evaporation from the liquid form. If the liquefied gas (contained in a strong steel cylinder) is allowed to escape into a bag attached to the outlet pipe of the cylinder (Fig. 276), it will be frozen into snowy crystals by the intense cold produced in the rapid evaporation of the liquid.

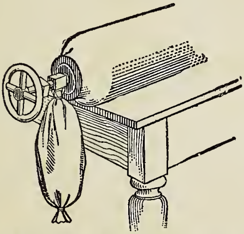

FIG. 276.—Freezing of carbon dioxide by evaporation from the liquid form.

273. Practical Applications of Cooling by Vaporization. Vaporization is our chief source of "artificial cold." The applications are numerous and varied. Fever patients are sponged with volatile liquids to reduce temperature. Ether sprays are used for freezing material for microscopic sections. Evaporation is also utilized in making artificial ice, in cooling cold-storage buildings, and in freezing shifting quicksands for engineering purposes. The liquid most commonly used for the latter purposes is ammonia liquefied by pressure. This is suitable because the gas liquefies at ordinary temperature under relatively moderate pressure (about 10 atmospheres), and it absorbs a great amount of heat in evaporation. Fig. 277 shows the essential parts of an ice-making machine. The ammonia gas is forced by the pump into the condenser coils and liquefied there by pressure, the heat given out in condensation being carried off by the water circulating on the outside of the coils. The liquid ammonia escapes slowly through the regulating valve into the low-pressure coils, where it evaporates, producing intense cold. In consequence, the brine which surrounds the coils is cooled below the freezing-point of water, and the water to be frozen, placed in cans submerged in it, is converted into ice.

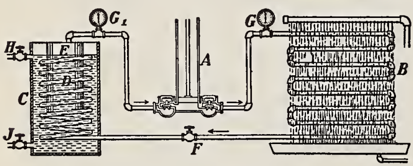

Fig. 277.—Ice-making machine. G, high-pressure gauge; G_1, low-pressure gauge; A, pump for exhausting low-pressure coils and condensing gas; B, condenser coils cooled by running water from a pipe placed above them; F, regulating valve; D, low-pressure coils; C, tank containing brine; E, can containing water to be frozen.

It will be observed that the process is continuous. The pump which forces the ammonia into the condenser coils receives its supply of gas from the low-pressure coils. The same ammonia is thus used over and over again.

In some cold-storage plants, the brine cooled as described above, is made by a force-pump to circulate in coils distributed at suitable centres throughout the building (Fig. 278.) The temperature of the air in the cold-storage rooms is thus reduced in hot weather by cold coils, very much as it may be raised in winter

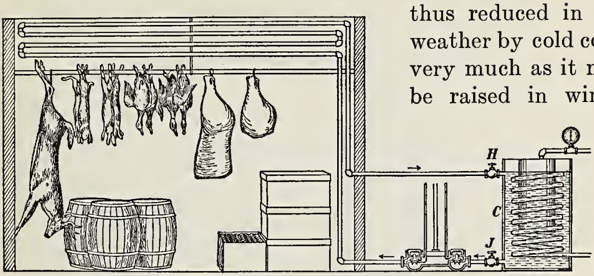

Fig. 278.—Cold-storage plant.

by similar coils containing hot water or steam.

274. Condensation—Critical Temperature. A vapour in a condition of saturation is condensed if its temperature is lowered or its pressure is increased. At this point an interesting question arises. Can an unsaturated vapour at any given condition of temperature be reduced to a liquid by increase of pressure alone? The question has been answered experimentally. It has been found that for every vapour there is a temperature above which pressure alone, however great, cannot produce condensation. This temperature is known as the *critical temperature*, and the pressure necessary to produce condensation at this temperature is called the *critical pressure*. For example, Andrews,* to whom we owe an exhaustive study of the subject, found that to reduce carbon dioxide to a liquid the temperature must be lowered to at least 30.92° C., and that above that temperature no amount of pressure would convert it into liquid form.

The critical temperature of water, alcohol, ammonia, and carbon dioxide are above the average temperature of the air, while those of the gases oxygen, hydrogen and air are much below it. The critical temperature of water is 365° C. and of air −140° C.

Below the critical temperature a further lowering of the temperature lessens the pressure necessary to condensation. For example, a pressure of 73 atmospheres is necessary to condense carbon dioxide at the critical temperature, but 60 atmospheres is sufficient at a temperature of 21.5°

*Thomas Andrews (1813-1885) Professor of Chemistry, Queen's College, Belfast, 1845-1879.

and 40 atmospheres at a temperature of 13.1°. Again, a pressure of 200 atmospheres is necessary to condense steam at the critical temperature (365° C.); at 100° C. it condenses under a pressure of one atmosphere.

275. Liquefaction of Air. The apparatus generally used to condense air into a liquid depends on the fact that when a gas is compressed its temperature rises, and when it expands, thus doing work, its temperature falls. In Fig.279 is shown the essential parts of a liquid-air machine. To one side of the pump P is joined one end of a coil of pipe A which is within J, a jacket through which cold water is always running, entering at K and leaving at H.

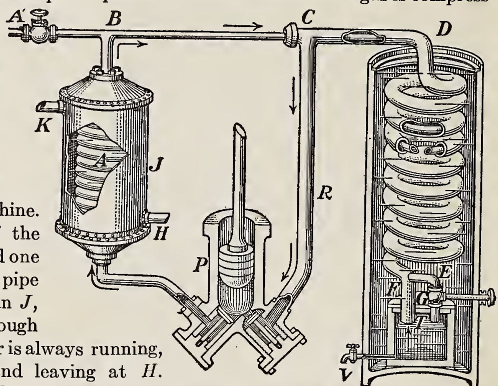

Fig. 279.—Essential parts of a liquid-air machine.

The long coil to the right is double. A small pipe runs within a larger one. In the figure the second and third turns (from the top) of the larger pipe are shown cut away, exposing the smaller pipe within. The smaller pipe enters the larger one at C, passes on to D and down to F. Here it emerges and goes over to E where its end may be closed by a valve G.

The action is as follows: The pump P draws in air from the large pipe R, and forces it, at a pressure of about 200 atmospheres, through the coil A, where it is cooled to the temperature of the water. The air passes on to B and then to C, and going through the inner coil it descends to F and then to E. Through the slightly opened valve G it expands into the vessel T being thereby cooled. From here it enters the end of the larger pipe of the coil, and ascends, it reaches D and then C, whence it goes down to enter the pump again.

Now the air on expanding into T was cooled, and hence as it ascends through the outer coil, it cools the air in the inner coil. As this process is continued, the air in the inner coil gets colder and colder until at last it becomes liquid and collects in T at a temperature of about −182° C. From this it is drawn off by the tap V.

To make 1 cubic inch of liquid about ½ cubic foot of air is required, and when the liquefaction has begun, fresh air must be supplied. It is introduced at A' from an auxiliary compressor at a pressure of about 200 atmospheres.

QUESTIONS AND PROBLEMS

1. Why does sprinkling the floor have a cooling effect on the air of the room?

2. Under what conditions will "fanning" cool the face?

3. In eastern countries and at high elevations water is poured into porous earthenware jars and placed in a draught of air to cool. Explain the cause of cooling.

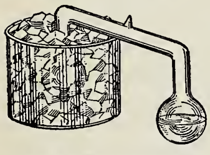

FIG. 280.—Cryophorus.

4. A tube having a bulb at each end has one of its bulbs filled with water, the remaining space containing nothing but water-vapour. The empty bulb is surrounded by a freezing mixture (Fig. 280), and after a time it is found that the water in the other bulb is frozen. Explain. (Such a tube is called a *cryophorus,* which means *frost-carrier.*)

5. A little alcohol sprinkled on the bulb of a thermometer causes an immediate lowering of the temperature registered by the thermometer, even though the thermometer shows no change in temperature when the bulb is immersed in the alcohol. Explain.

6. A beaker is placed on a board (say the lid of a chalk box), and a small amount of water placed on the board under the beaker. Ether is poured into the beaker and allowed to fill the lower part of it. Now if air is forced into the ether by means of an atomizer bulb and a glass tube and made to bubble up through it freely for a time, the beaker will be frozen to the board. Try the experiment and explain the result.

7. The boiling-point of a solution of common salt is higher than that of pure water; the freezing-point of the solution is lower than that of pure water. Use the molecular theory to explain these facts.

8. A thermometer placed in the steam above a boiling solution will indicate the boiling-point of water and not that of the solution. Explain.

9. Is there a perfect vacuum in the space above the mercury in a barometer? How would the presence of a little water in the mercury affect the height of the barometer? Under what conditions of temperature would the effect be most noticeable?

10. "Anti-freeze," used in the radiators of automobiles in cold weather, is made up mainly of methyl alcohol and water. A mixture of 40% alcohol to 60% water freezes at $-20°$ F. What will happen to the freezing-point of the mixture as evaporation proceeds, and what precautions should be observed in replacing liquid lost through evaporation?

CHAPTER XXVIII

Moisture in the Atmosphere

276. Condensation of Water-vapour of the Air—Dew-point. Evaporation is constantly taking place from water at the surface of the earth, and consequently the atmosphere always contains more or less water-vapour. This vapour will be on the point of condensation when its pressure approaches the saturation pressure. Now, since this pressure varies with the temperature, the nearness to saturation at any given time will depend on the temperature as well as upon the amount of vapour present per unit volume. Accordingly, the amount of vapour which a given space will contain rises rapidly with the temperature. Thus a given space will hold more than three times as much vapour at 30° C. as at 10° C.

If the amount of vapour in a given space remains constant and the temperature is lowered gradually, a temperature will at length be reached *at which condensation will begin to take place.* This temperature is called the DEW-POINT.

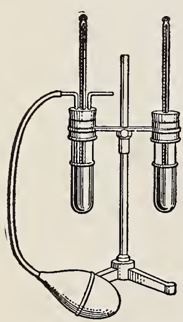
Fig. 281.—Determination of the dew-point.

The dew-point may be determined experimentally by Regnault's hygrometer (or moisture measurer) illustrated in Fig. 281. In it are two glass tubes the lower portions of which are covered with polished metal. A thermometer is inserted through the cork in each. Pour ether into the vessel fitted with the atomizer bulb and force air through it. This agitation of the ether makes it evaporate rapidly, and thus the temperature of the vessel is lowered. Note the temperature at which the polished surface surrounding the ether becomes dimmed with dew. Cease forcing the air and again note the temperature at which the moisture disappears. The mean of the two temperatures is taken as the dew-point.

The second vessel enables the observer, by comparison, to determine more readily the exact moment when condensation begins. The thermometer in this vessel gives the temperature of the air in the room at the time of the experiment.

277. Relative Humidity. The term humidity, or RELATIVE HUMIDITY, is used to denote *the ratio of the mass of water-vapour present in the air, to the mass required for saturation at the same temperature.* The air is said to be very dry when the ratio is low, and damp when it is high. These terms, it should be observed, have reference, not to the absolute amount of vapour present, but to the relative degree of saturation at the given temperature. At the present moment the air outside may be raw and damp, but after having been forced by a fan over a series of steam-heated coils, it appears in the laboratory comparatively dry. It is not to be inferred that the air has lost any of its vapour; but rather that in being heated it has acquired the capacity of taking up more.

The relative humidity is usually expressed as a percentage of the maximum amount of vapour possible at the temperature. For example, when the air contains but one-half of the amount of water-vapour necessary for saturation, its humidity is 50 per cent.

278. Chemical Hygrometer. This instrument (Fig. 282) provides a direct method for determining the mass of water-vapour present in a given volume of air.

A is a large bottle of at least 10 litres capacity, which is nearly full of water at the beginning of the experiment. By opening the tap *B* air can be drawn through the **U**-tubes *C*, *D*, *E*, in which is placed calcium chloride or some other substance which absorbs water-vapour readily. The bottle *F* contains strong sulphuric acid, which prevents any water-vapour from passing from *A* to the drying tubes *C*, *D*, *E*. These tubes are weighed before the experiment and again after a measured volume of water has escaped by the tap *B*. The average temperature of the air during the experiment is obtained by reading the thermometers *G* and *H*.

Let the volume of the water be 10 litres and the increase in the weight of the tubes be 0.05 gm. Then 1 litre of air under the given conditions contains 0.005 gm. of water-vapour.

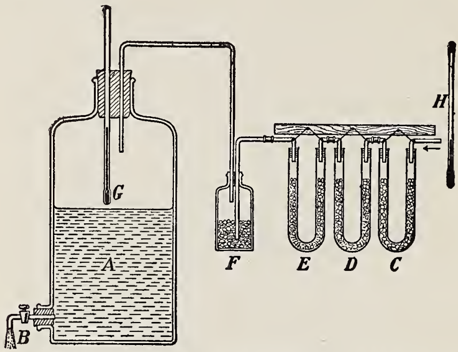

Fig. 282.—A chemical hygrometer for actually weighing the water-vapour present in a given volume of air.

We must now find the weight of water-vapour which would saturate this volume of air under the same conditions of temperature. This can be obtained by making the air first pass through tubes containing sponge or wool soaked in water before it comes to the drying tubes. Let the amount of vapour present in 1 litre of saturated air be 0.020 gm. Then the relative humidity = $\frac{0.005}{0.020} \times 100 = 25\%$.

The following table gives the mass of water-vapour present in 1 cubic metre (= 1000 litres) of saturated air at different temperatures.

Temp. Centigrade.	−10°	−5°	0°	5°	10°	15°	20°	25°	30°
Mass in grams....	2.1	3.5	4.9	6.8	9.4	12.8	17.2	22.9	30.1

Exercise.—Using this table, plot a curve (similar to that in Fig. 272) in which horizontal distances represent temperatures and vertical distances represent masses of water-vapour; and from the curve estimate the mass of water-vapour present in 1 cu. metre of saturated air at $-2\frac{1}{2}°$, $7\frac{1}{2}°$ and $27\frac{1}{2}°$ C.

279. Relative Humidity from the Dew-point. Having once obtained the table just given, the relative humidity is easily determined by a calculation from the dew-point. Take a particular example. Suppose the dew-point to be 10° C. while the temperature of the air in the room is 20° C. From the table we learn that air saturated with water-vapour at 10° C. contains 9.4 gm. per cubic metre, and at 20° C. it contains 17.2 gm. per cubic metre. Then since 1 cubic metre actually contains at 20° C. just the amount of vapour necessary for saturation at 10° C. the relative humidity = $\frac{9.4}{17.2}$ or 54.6%.

280. The Wet-and-Dry-Bulb Hygrometer. This instrument consists of two similar thermometers mounted on the same stand (Fig. 283). The bulb of one of the thermometers is covered with muslin kept moist by a wick immersed in a vessel of water. Evaporation from the wet bulb lowers its temperature, and since the ratio of evaporation varies with the dryness of the atmosphere, it is evident that the differences in the readings of the thermometers may be used as an indirect means of estimating the relative humidity of the atmosphere. The percentages are given in tables prepared by comparison with results determined from dew-point calculations.

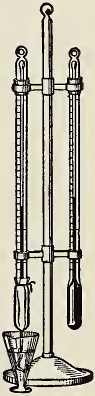

Fig. 283.—Wet-and-dry bulb hygrometer.

Example.—Suppose the dry thermometer reads 71 and the wet 62. Find 71 on the left-hand side (of the table below) and take the line running to the right from it. Then find 62 at the top and take the column running down from it. The number where these two meet is 60, and that is the relative humidity.

TABLE GIVING RELATIVE HUMIDITY

Reading of Dry Thermometer (Fahr.)	Reading of Wet Thermometer (Fahr.)																	
	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53
80	61	57	54	51	47	44	41	38										
79	63	60	57	54	50	47	44	41	37									
78	67	64	60	57	53	50	46	43	40	37								
77	71	67	63	60	56	52	49	46	42	39	36							
76	74	70	67	63	59	55	52	48	45	42	38	35						
75	78	74	70	66	63	59	55	51	48	44	40	38	34					
74	82	78	74	70	66	62	58	54	51	47	43	40	37	34				
73	86	82	78	73	69	65	61	58	54	50	46	43	40	36	33			
72	91	86	82	78	73	69	65	61	57	53	49	46	42	39	35	32		
71	95	90	86	82	77	73	69	64	60	56	53	49	45	41	38	34	31	
70		95	90	86	81	77	72	68	64	60	56	52	48	44	40	37	33	30
69			95	90	86	81	77	72	68	64	59	55	51	47	44	40	36	32
68				95	90	85	81	76	72	67	63	59	55	51	47	43	39	35
67					95	90	85	80	76	71	67	62	58	54	50	46	42	38
66						95	90	85	80	76	71	66	62	58	53	49	45	41
65							95	90	85	80	75	70	66	62	57	53	48	44
64								95	90	85	79	75	70	66	61	56	52	48
63									95	90	84	79	74	70	65	60	56	51
62										94	89	84	79	74	69	64	60	55
61											94	89	84	79	74	68	64	59
60												94	89	84	78	73	68	63

281. Relation of Humidity to Health. Humidity has an important relation to health and comfort. When the relative humidity is high, a hot day becomes oppressive because the dampness of the atmosphere interferes with free evaporation from the body. On the other hand, when the air becomes too dry, the amount of this evaporation is too great. This condition very frequently prevails in winter in houses artificially heated. Under normal conditions the relative humidity should be from 50 to 60 per cent.

282. Fog and Clouds. If the air is chilled below the temperature for saturation, vapour condenses about dust particles suspended in the air. If this condensation takes place in the strata of air immediately above the surface of the earth, we have a *fog;* if in a higher region, a *cloud.* The cooling necessary for fog formation is due to the chilling effects of cold masses at the surface of the earth; in the upper region, a cloud is formed when a stratum of warm moist air has its temperature lowered by its own expansion under reduced pressure. It would appear from recent investigations that under all conditions dust particles are necessary as nuclei for the formation of cloud globules.

The effect of dust particles in producing fog and clouds can be shown in a striking manner as follows:—

A large bottle *A* is connected to an air-pump and to a smaller bottle *B* (Fig. 284). A little water is placed in *B* to ensure that the air within it is saturated with water-vapour. The pinch-cocks *C* and *E* are closed while the air is exhausted from *A*. *D* is then closed and *C* opened. The sudden expansion of the air in *B* cools it sufficiently to produce a fog. If now a little smoke from burning paper is introduced into *B* through *E* and the experiment is repeated, the fog is many times as dense.

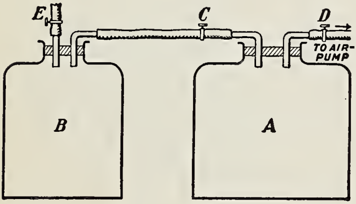

FIG. 284.—To illustrate the production of fog.

283. Dew and Frost. On a warm summer day drops of water collect on the surface of a pitcher containing ice-water, because the air in immediate contact with it is chilled below the dew-point. This action is typical of what goes on on a large scale in the deposition of *dew*. After sunset, especially when the sky is clear, small bodies at the earth's surface, such as stones, blades of grass, leaves, cobwebs, and the like, cool more rapidly than the surrounding air. If their temperature falls below the temperature of saturation, dew is deposited on them from the condensation of the vapour in the films of air which envelop them. If the dew-point is below the freezing-point, the moisture is deposited as frost.

284. Rain, Snow and Hail. The cloud globules gravitate slowly towards the earth. If they meet with conditions favourable to vaporization, they change to vapour again, but if with conditions favourable to condensation, they increase in size, unite, and fall as rain.

When the condensation in the upper air takes place at a temperature below the freezing-point, the moisture crystallizes in *snow-flakes*. At low temperatures, also, vapour becomes transformed into ice pellets and descends as *hail*. The hailstones usually contain a core of closely packed snow crystals. The exact conditions under which they are formed are not yet fully understood but probably they arise from the action of violent air currents which carry the condensed vapour up and down through alternate regions of snow and rain.

QUESTIONS AND PROBLEMS

1. As exhaustion of air proceeds, a cloud is frequently seen in the receiver of an air-pump. Explain.

2. Why can one "see his breath" on a cold day?

3. Dew does not usually form on a pitcher of ice-water standing in a room on a cold winter day. Explain.

4. On some days a locomotive leaves a long white cloud behind it and on other days only a short one. What causes the difference? Also, why does the cloud form and why does it disappear?

5. A saucer containing water is left to evaporate on a window-sill. Explain what atmospheric conditions will favour or retard the disappearance of the water.

6. On drawing 30 litres of atmospheric air through a series of drying tubes, the weight of the drying tubes increases by 0.288 gm. The temperature is 20° C. What is the relative humidity and what the dew-point?

7. Find from the table in § 278 the relative humidity corresponding to the following conditions:—

(*a*) Temp. of atmosphere 10° C.; dew-point 5° C.
(*b*) Temp. of atmosphere 15° C.; dew-point $7\frac{1}{2}$° C.
(*c*) Temp. of atmosphere 25° C.; dew-point 15° C.

8. Why does a morning fog frequently disappear with increased strength of the sun's rays?

CHAPTER XXIX

Heat and Mechanical Motion

285. Mechanical Equivalent of Heat. We have referred (§ 221) to the fact that during the first half of the nineteenth century the kinetic theory of heat, advocated by Count Rumford and Sir Humphry Davy, gradually superseded the old materialistic conception. The modern theory was regarded as established when Joule, about the middle of the century, demonstrated that for every unit of mechanical energy which disappears in the transformation of mechanical motion into heat a definite and constant quantity of heat is developed. The value of the heat unit expressed in units of mechanical energy is called the *mechanical equivalent* of heat.

James Prescott Joule (1818-1889). Lived near Manchester all his life. Experimented on the mechanical equivalent of heat for forty years.

286. Determination of the Mechanical Equivalent of Heat. The essential features of Joule's apparatus for determining the mechanical equivalent of heat are illustrated in Fig. 285. A paddle-wheel was made to revolve in a vessel of water by a falling weight connected with it by pulleys and cords. Joule measured the heat produced by the motion of the paddle and the corresponding amount of work done by the descending weight. He calculated that one B.T.U. of heat was equivalent to 772 foot-pounds of

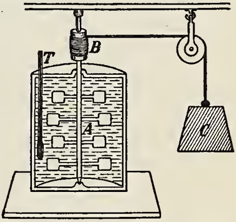

Fig. 285.—Principle of Joule's apparatus for determining the mechanical equivalent of heat.

mechanical energy. Later investigations by Rowland and others placed the constant at 778 foot-pounds for one B.T.U. of heat, which is equivalent to 4.187 joules (41,870,000 ergs), or 427 gram-metres of work for one calorie of heat.

287. Steam-engine. Mechanical motion arrested by friction or percussion becomes transformed into heat energy. On the other hand, heat is one of our chief sources of mechanical motion. In fact, it is commonly said that modern industrial development had its beginning in the invention of the steam-engine. The development of the engine as a working machine is due to James Watt, a Scottish instrument-maker, who constructed the first engine in 1768.

The essential working part of the ordinary type of steam-engine is a cylinder in which a piston is made to move backwards and forwards by the pressure of steam applied alternately to its two faces. To understand how this to-and-fro motion is kept up, study Figs. 286, 287, 288. The steam from the boiler is conveyed by a pipe into the steam-chest. From the steam-chest the steam is admitted to the cylinder by openings called ports, *A* and *B*, at the ends of the cylinder (Figs. 286, 287). The exhaust steam escapes from the cylinder by the same ports. The admission of the steam to the cylinder, and its escape after it has performed its work, is controlled by the operation of a valve. This valve is so adjusted that when the port *A* is connected with the steam-chest, *B* is connected with an exhaust pipe *P*, leading to the open air or to a condenser; and when *B* is connected with the steam-chest, *A* is connected with the exhaust pipe. Fig. 286 shows the steam entering at *A* and escaping at *B*. The piston, therefore, is being forced to the right. Meanwhile, the valve is being pushed in the opposite direction, and at the end of the stroke allows the steam to enter the cylinder at *B* and escape at *A* (Fig. 287). The piston is now being forced to the left. At the end of this stroke the steam again enters at *A* and escapes at *B* (Fig. 286). A to-and-fro motion of the piston is thus kept up. This motion is transformed into a rotary motion in the shaft by a crank

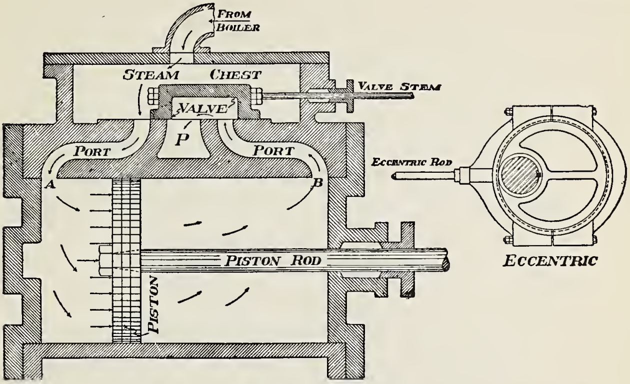

FIG. 286.—Showing the interior of the cylinder of a steam-engine. Steam entering port A, piston moving to the right.

mechanism. Fig. 288 shows how this is effected in a very common type of engine.

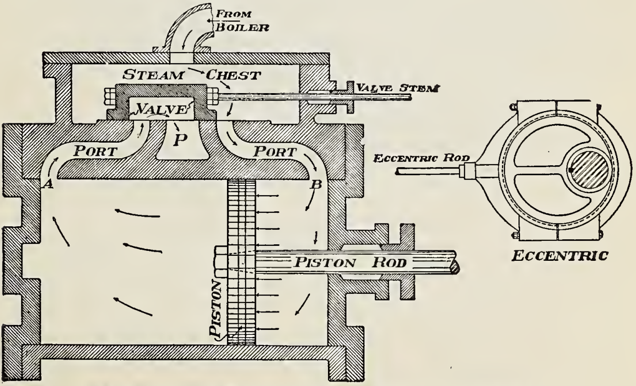

FIG. 287.—Steam entering at port B and piston moving to left.

The motion of the piston is transmitted through the piston-rod, the cross-head with its guides, and the connecting-rod to the crank pin, which is made to move in a circle about the centre of the shaft. The backward and forward motion of the piston is thus transformed by the connecting-rod into a rotary motion in the shaft and the fly-wheel. The fly-wheel serves to give steadiness to the motion and to carry the engine over the "dead centres" at the ends of the strokes.

The valve is given its backward and forward motion by an eccentric keyed rigidly to the shaft of the engine (Fig. 288).

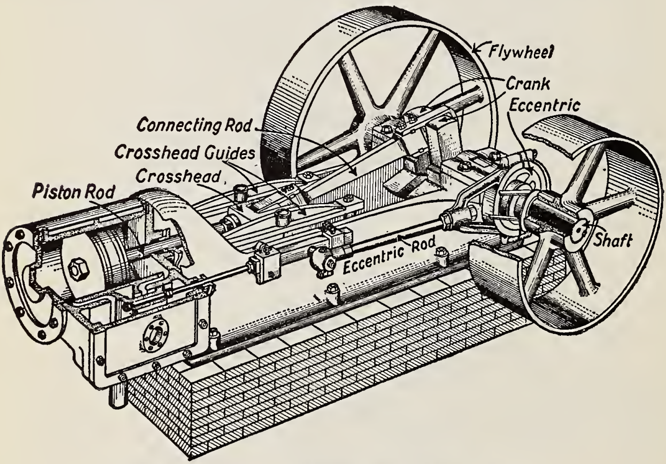

Fig. 288.—Showing the different parts of a steam-engine.

This eccentric consists of a circular disc of iron having its centre at a distance from the centre of the shaft (Figs. 286, 287). When rotating, the centre of the eccentric will, accordingly, move in a circle about the centre of the shaft; consequently, the eccentric is made to do the duty of a crank. The motion of the eccentric is communicated to the valve stem by the eccentric-rod and the collar, or strap, within which it rotates. Compare the positions of the eccentric and the valves in Figs. 286 and 287.

Exercise.—Obtain a working model of a steam-engine and note the relative positions of the piston, the connecting-rod, the crank, the eccentric, the eccentric-rod, and the valve as the fly-wheel is slowly rotated by hand.

288. High and Low Pressure Engines. In the common "high pressure" engine, the steam escapes from the cylinder directly into the air. In the low pressure, or condensing engine, the exhaust is led into a chamber (Fig. 289), where it is condensed by jets of cold water. The water is removed by an "air-pump."

Fig. 289. — Condenser of "low pressure" steam-engine.

Since a more or less perfect vacuum is maintained in the condensing chamber of a low pressure engine, it will work under a given load at a lower steam pressure than the high pressure engine, because its piston does not encounter the opposing force of the atmospheric pressure.

289. The Compound Engine. When the pressure maintained in a boiler is high, the steam escapes from the cylinder of an engine with energy capable of further work. The purpose of the compound engine is to utilize this energy latent in exhaust steam. In this type, two, three or even four cylinders with pistons connected with a common shaft are so arranged that the steam which passes out of the first cylinder enters the next, which is of wider diameter, and so on, until it finally escapes into a condensing chamber connected with the last cylinder.

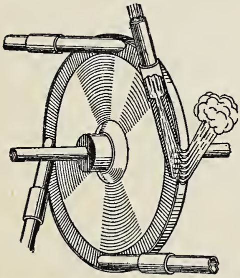

Fig. 290.—Action of steam on the blades of the drum in a turbine engine.

290. Turbine Engines. These are now being generally used in fast ocean-going steamships and many large steam power-plant installations. In this form of engine a drum attached to the main shaft is made to revolve by the impact of steam directed by nozzles against blades attached to its outer surfaces as shown in principle in Fig. 290. In actual construction there are numerous nozzles, moving blades and stationary blades, arranged with the utmost precision on the surface of the drum on the rotating shaft.

291. Gas-engines. The remarkable development of automobiles, tractors, and airplanes during the last few years,

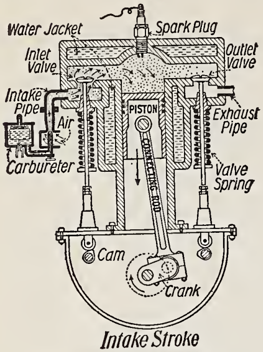

Fig. 291.—Intake Stroke.

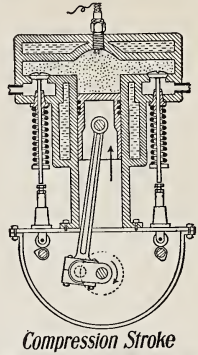

Fig. 292.—Compression Stroke.

has been made possible by improvements in gas-engines. They are used for a multitude of purposes,—in launches and as auxiliary power in sailing craft; also for pumping water, generating current for electric light, and furnishing power for other purposes, more particularly on farms and in suburban districts where the other common sources of power are not available.

The gas-engine is described as an 'internal combustion' engine, because the fuel used is burned in the cylinder of the engine itself, and the piston is driven forward by the expansion of the heated gases produced in the combustion. The fuel most commonly used is an explosive mixture of gasoline vapour and air, but in the Diesel engine, which is used in submarines and in other vessels, heavier oils are utilized.

Gas-engines are of two types, the *four-stroke-cycle*, or simply *four-cycle*, engine and the *two-stroke-cycle*, or *two-cycle*, engine.

The four-cycle engine is the type most commonly used. In this form of engine, the piston receives an impulse at the end of every fourth single stroke. To understand the action of the engine in each of the four strokes, consider Figs. 291-294. The shaft is turning in the direction of the arrows. In the

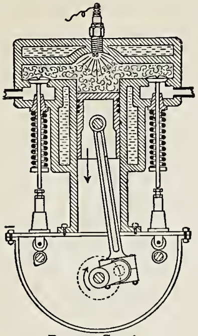

Fig. 293.—Power Stroke.

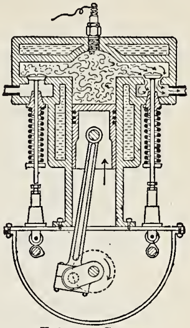

Fig. 294.—Exhaust Stroke.

intake stroke (Fig. 291), the piston is moving downwards, and a charge of the combustible gas from the carburetor is drawn into the cylinder through the inlet valve, which has been raised for the purpose by the pressure of the cam on the rod which lifts the valve. The inlet valve is closed by the valve-spring before the piston reaches the lowest point of its motion. As the piston moves upwards in the next, or compression, stroke (Fig. 292), the charge, which was drawn into the cylinder during the intake stroke, is compressed into about one-third of its volume. At a properly timed instant the compressed charge is ignited by an electric spark at the point of the spark-plug, electrically connected with a magneto or induction coil and battery and the piston is forced downwards by the expansive force of the inclosed gases. This is the third, or power, stroke (Fig. 293). Before the piston begins its upward movement, the outlet valve is opened by the action of the cam, and as the piston is forced upwards during the last, or exhaust, stroke, the burnt gases escape from the cylinder (Fig. 294). Before the end of this stroke, the outlet valve is closed by the spring, and the engine is again on the point of taking in a new charge of fuel. In Fig. 295 is a simplified diagram showing the four strokes of the cycle.

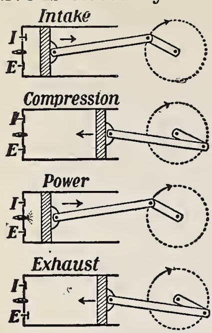

Fig. 295.—Diagram showing the four strokes.

The momentum given the balance-wheel at each explosion serves to maintain the motion until the piston receives the next impulse. To cause the pressure to be more continuous in high-speed engines, two or more cylinders have frequently their pistons connected to a common shaft (Fig. 296).

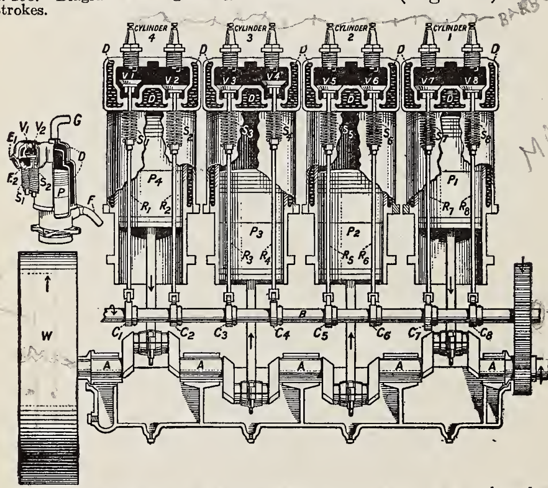

Fig. 296.—The working parts of a modern four-cylinder automobile, or launch engine A, main shaft: W, balance-wheel connected to main shaft; P_1, P_2. P_3, P_4 pistons; V_1, V_3, V_5, V_7, inlet valves; V_2, V_4, V_6, V_8, exhaust valves; R_1, R_2, etc., valve stems; S_1, S_2, etc., springs by which valves are closed; B, cam-shaft for operating valves, run by gears from main shaft; C_1, C_2, etc., cams for lifting valves; D, space to contain circulating water for cooling cylinder. The small diagram in the upper left-hand corner shows the connection between the valve-chamber and cylinder. E_1, inlet port; E_2, exhaust port; F, pipe by which cooling water enters; G, outlet for water. Two spark plugs are shown inserted at the top of each cylinder. One is connected with a battery system of ignition, the other with a magneto or dynamo. The electrical connections are so made that either may be used at will.

Figs. 291-294 illustrate the T-head form of valve arrangement, in which the inlet and outlet valves are on opposite sides of the cylinder. In Fig. 296 the cylinders are of the L-head type. Here the inlet and outlet valves are on the same side of the cylinder. In still another type, the valves are in the top of the cylinder head.

The cams which lift the valves are rigidly attached to a cam-shaft, which is run by gears from the main shaft of the engine (Fig. 296).

Exercise.—Study Fig. 296 and answer the following questions, giving in each case reasons for your answer.

1. In which stroke (intake, compression, power, or exhaust) is each of the cylinders represented?
2. In what order do the cylinders fire?
3. How does the rate of rotation of the cam-shaft compare with that of the main shaft of the engine?

292. Two-cycle Engine. The *two-cycle* engine differs from the four-cycle, in that the piston receives an impulse at the end of every second single stroke. This is accomplished as follows: Consider the piston in the position shown in Fig. 297a. During the first part of the first single stroke, the burnt gases of the previous explosion escape by the port D, and a charge stored in the crank chamber enters by the port B. In the second part of the first single stroke, the inlet and exhaust ports are covered by the piston and the charge is compressed in the cylinder, while a new charge is drawn into the crank chamber from the fuel tank through the port A (Fig. 297b). The charge in the cylinder is ignited,

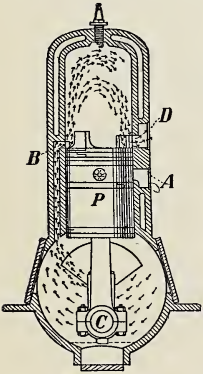

Fig. 297a.

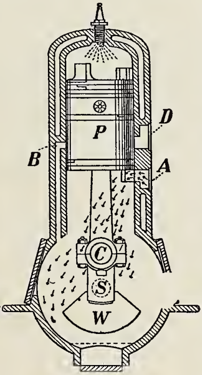

Fig. 297b.

Working parts of a two-stroke gas engine. P, piston: S, main shaft; C, crank pin; A, inlet port to crank chamber; B, inlet port to cylinder; D, exhaust port; W, counterpoise weight.

and the piston is forced forward in the second half-stroke, giving an impulse to the fly-wheel and compressing the new charge in the crank chamber. The action then goes on as before.

293. Efficiency of Heat Engines. All heat engines are wasteful of energy. The best types of compound condensing steam-engines transform only about 16 per cent. of the heat of combustion into useful work, while the ordinary high-pressure steam-engine in everyday use utilizes not more than 5 per cent. of the energy latent in the fuel. The remainder of the heat energy is lost in the ash, in radiation from the boiler, pipes and engine, or is dissipated into the atmosphere by the smoke and exhaust steam.

The best steam turbines equal in efficiency the most economical forms of reciprocating engines. The efficiency of the gas-engine is much higher than that of the steam-engine. Under good working conditions it will transform as high as 25 per cent. of heat energy into mechanical energy.

QUESTIONS AND PROBLEMS

1. The water in the boiler of a steam-engine cannot be kept boiling at 100° C. Why?

2. How does the temperature of the gaseous products of combustion in the cylinder of a gas-engine at the moment of ignition compare with the temperature at the moment of exhaust? Explain.

3. Which is in the greater need of a heavy balance-wheel, (*a*) a one-cylinder four-cycle engine or a four-cylinder four-cycle engine? (*b*) a one-cylinder four-cycle engine or a one-cylinder two-cycle engine? Why?

4. The average pressure on the piston of a steam-engine is 60 lbs. per sq. inch. If the area of the piston is 50 sq. in. and the length of the stroke 10 in., find (*a*) the work done in one stroke by the piston; (*b*) how much heat, measured by B.T.U., was lost by the steam in moving the piston.

5. The coal used in the furnace of a steam pumping-engine furnishes on an average 7000 calories of heat per gram. How many litres of water can be raised to a height of 20 metres by the consumption of 500 kg. of coal, if the efficiency of the engine is 5 per cent.?

6. Supposing that all the energy of onward motion possessed by a bullet, whose mass is 20 grams and velocity 1000 metres per sec., is transformed into heat when it strikes the target, find in calories the amount of heat developed.

7. A train whose mass is 1000 tons is stopped by the friction of brakes. If the train was moving at a rate of 30 miles per hour when the brakes were applied, how much heat was developed?

8. How much coal per hour is used in the furnaces of a steamer when the screw exerts a pushing force of 1000 kg. and drives the vessel at a rate of 20 km. per hour, if the efficiency of the engine is 10 per cent. and the coal used gives on the average 6000 calories of heat per gram.?

9. A locomotive whose efficiency is 7 per cent. is developing on the average 400 horse-power. Find its fuel consumption per hour if the coal furnishes 14,000 B.T.U.'s of heat per pound.

10. The following table gives a summary of the distribution of the heat from each pound of coal used in a condensing steam-engine plant:—

Lost in ash	100 B.T.U.
Lost in radiation from boiler	200
Carried off in smoke	2000
Lost in transmission	80
Lost in auxiliaries (feed-pump, etc.)	220
Lost in leakage and radiation from engine	200
Converted to work	2300
Rejected to condenser	8500
Total	13,600

Calculate the efficiency of the plant.

CHAPTER XXX

Transference of Heat

294. Conduction of Heat. The handle of a silver spoon becomes warmed when the bowl is allowed to stand in a cup of hot liquid; the uncovered end of a glass stirrer, under similar conditions, remains practically unchanged in temperature. Heat creeps along an iron poker when one end is thrust into the fire; while a wooden rod conveys no heat to the hand.

The transference of heat from hotter to colder parts of the same body, or from a hot body to a colder one in contact with it, is called *conduction*, when the transmission takes place, as in these instances, without any perceptible motion of the parts of the bodies concerned.

295. Conducting Powers of Solids. The above examples show clearly that solids differ widely in their power to conduct heat. The tendencies manifest in silver and iron are typical of the metals; as compared with non-metals, they are good conductors. Organic fibres, such as wool, silk, wood, and the like, are poor conductors.

The metals, however, differ widely among themselves in conductivity. This may be shown roughly as follows:—Twist two or more similar wires of different metals—say copper, iron, German silver—together at the ends and mount them as shown in Fig. 298. By means of drops of wax attach shot or bicycle balls or small nails at equal intervals along the wires. Heat the twisted ends. The progress of the heat along the wires will be indicated by the melting of the wax and the dropping of the balls. When the line of separation between the melted and unmelted drops of wax ceases to move along the wire, it will be found that the copper has melted wax at the greatest

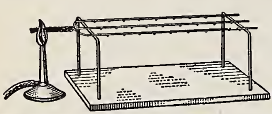

Fig. 298.—Difference in conductivity of metals.

distance from the source of heat, the iron comes next in order, and the German silver last. If the wax is distributed uniformly and the wires heated equally at their ends, the conductivities of the wires are approximately proportional to the squares of these distances.

The following table gives the relative conductivities of some of the more commonly used metals referred to copper as 100.

RELATIVE CONDUCTIVITIES OF METALS

Copper........	100	Iron (wrought)	16	Platinum.....	8
Aluminium...	52	Lead.........	9	Silver........	110
Brass.........	28	Magnesium...	41	Tin..........	17
Gold.........	77	Mercury......	1.6	Zinc..........	29

296. Conduction in Liquids. If we except mercury and molten metals, liquids are poor conductors of heat. Take water for example. We may boil the upper layers of water held in a test-tube over a lamp (Fig. 299) without perceptibly heating the water at the bottom of the tube.

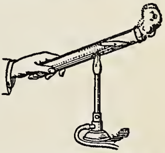

FIG. 299.—Water is a poor conductor of heat.

The poor conductivity of water is also strikingly shown in the following experiment.

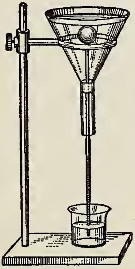

FIG. 300.—Illustration of the non-conductivity of water.

Pass the stem of a Galileo air-thermometer (§ 236) through a perforated cork inserted into a funnel as shown in Fig. 300. Then cover the bulb of the thermometer to a depth of about ½ cm. with water. Now pour a spoonful of ether on the surface of the water and set fire to it. The index of the thermometer shows that little, if any, heat is transmitted by the water to the bulb from the flame at the surface.

297. Conduction in Gases. Gases are extremely poor conductors of heat. The conductivity of air is estimated to be only about $\frac{1}{20000}$ of that of copper. Many substances, such as wool, fur, down, etc., owe their poor conductivity to the fact that they are porous and contain in their interstices air in a finely divided state. If these substances are compressed, they become better conductors.

Light, freshly fallen snow incloses within it large quantities of air, and, consequently, forms a warm blanket for the earth, protecting the roots of plants from intense frost.

Heat is conducted with the greatest difficulty through a vacuum. For holding liquid air Dewar introduced glass flasks with hollow walls from which the air has been removed. The inner surfaces of the walls are silvered to prevent radiation (§ 311). The familiar "thermos" bottle is constructed in this way. When contained in such a vessel, a hot substance will remain hot and a cold one cold for a long time.

298. Practical Significance of Conduction in Bodies. The usefulness of a substance is frequently determined by its relation to heat conduction. The materials used to convey heat, such as those from which furnaces, steam boilers, utensils for cooking, etc., are constructed, must, of course, be good conductors.

On the other hand, substances used to insulate heat, to shut it in or keep it out, should be non-conductors. A house with double walls is warm in winter and cool in summer. Wool and fur are utilized for winter clothing because they refuse to transmit the heat of the body.

In this connection the action of metallic gauze in conducting heat should be noted. Depress upon the flame of a Bunsen burner a piece of fine wire gauze. The flame spreads out under the gauze but does not pass through it (*B*, Fig. 301) Again, turn off the gas and hold the gauze about half an inch above the burner and apply above the gauze a lighted match (*A*, Fig. 301). The gas burns above the gauze. The explanation is that the metal of the gauze conducts away the heat so rapidly that the gas on the side of the gauze opposite the flame is never raised to a temperature sufficiently high to light it. This principle is applied in the construction of the Davy safety lamp for miners. A jacket of wire gauze incloses the lamp, and prevents the heat of the flame from igniting the combustible gas on the outside. (Fig. 302.)

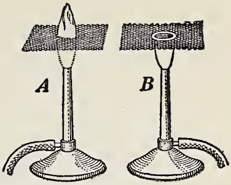

Fig. 301.—Action of metallic gauze on a gas-flame.

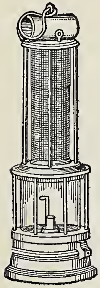
Fig. 302.—Davy safety lamp.

299. Conductivity and Sensitiveness to Temperature. We have already referred to the fact that our sensations do not give us reliable reports of the relative temperatures of bodies.

This is in part due to the disturbing effects of conduction. To take an example, iron and wood exposed to frost in winter or to the heat of the sun in summer have, under the same conditions, the same temperature; but on touching them the iron appears to be colder than the wood when the temperature is low, and hotter when it is high. These phenomena are due to the fact that the intensity of the sensation depends on the rate at which heat is transferred to or from the hand. When the temperature of the iron is low, heat from the hand is distributed rapidly throughout its mass; when hot, the heat current flows in the opposite direction.

The wood, when cold, takes from the hand only sufficient heat to warm the film in immediate contact with it; when hot, it parts with heat from this film only. In consequence, it never feels markedly cold or hot.

QUESTIONS

1. If a cylinder, half brass and half wood, be wrapped with a sheet of paper held and in the flame (Fig. 303), the paper in contact with the wood will soon be scorched, but that in contact with the brass will not be injured. Explain.

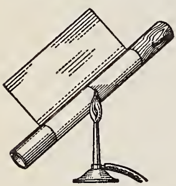

Fig. 303.

2. Why are utensils used for cooking frequently supplied with wooden handles?

3. Ice stored in ice-houses is usually packed in saw-dust. Why use saw-dust?

4. Why, in making ice-cream, is the freezing mixture placed in a wooden vessel and the cream in a metal one?

5. Water may be boiled in an ordinary paper oyster-pail over an open flame without burning the paper. Explain.

6. The so-called fireless cooker consists of a wooden box lined with felt or other non-conductor. The food is heated to a high temperature and shut up in the box. Why is the cooking process continued under these conditions?

7. Two similar cylindrical rods, one of copper and the other of lead, are covered with wax, and an end of each is inserted through a cork in the side of a vessel containing boiling water. At first the melting of the wax advances more rapidly along the lead rod, but after a while the melting on the copper overtakes that on the lead, and in the end it is 3 times as far from the hot water. Account for these phenomena. Compare the conductivities of copper and lead.

8. Which would be the more dangerous to touch in a heated oven, the pan which contains a loaf of bread or the loaf itself? Why?

9. The skin is cooled much more rapidly when placed in cold water than when placed in air at the same temperature. Why?

10. Why will a moistened finger freeze instantly to a metal door latch on a very cold day, but not to the door itself?

300. Convection Currents. The water in the test-tube (§ 296) remains cold at the bottom when heated at the top. If the heat is applied at the bottom, the mass of water is quickly warmed. The explanation is that in the latter case the heat is distributed by currents set up within the fluid.

The presence of these currents is readily seen if a few crystals of potassium permanganate are dropped into a beaker of water and the tip of a gas-flame allowed to come in contact with the bottom either at one side as in Fig. 304 or at the centre as in Fig. 305.

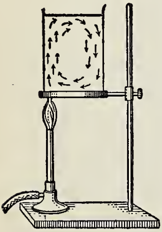

FIG. 304.—Convection currents in water heated by gas-flame placed at one side of bottom.

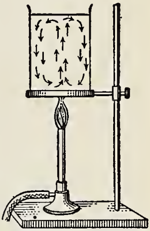

FIG. 305.—Convection currents in water heated by gas-flame placed at centre of bottom.

Such currents are called *convection currents.* They are formed whenever inequalities of temperature are maintained in the parts of a fluid. To refer to the example just cited, the portion of the water in proximity to the gas-flame is heated and its density is reduced by expansion. The body of hot water is, therefore, buoyed up and forced to the top by the colder and heavier portions which seek the bottom.

301. Transference of Heat by Convection. The transference of heat by convection currents is to be distinguished from conduction. In conduction, the energy is passed from molecule to molecule throughout the conductor; in convection, certain portions of a fluid become heated and change position within the mass, distributing their acquired heat in their progress. The water, heated at the bottom of the beaker, rises to the top carrying its heat with it.

302. Convection Currents in Gases. Gases are very sensitive to convection currents. A heated body always causes disturbances in the air about it. The rising smoke shows the direction of the air currents above a fire. Hold a hot iron—say a flat-iron—in a cloud of floating dust or smoke particles (Fig. 306). The air is seen to rise from the top of the iron, and to flow in from all sides at the bottom.

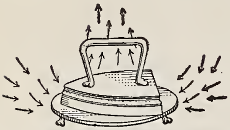

Fig. 306.—Convection currents in air about a heated flat-iron.

Make a box fitted with a glass front and chimneys as shown in Fig. 307. Place a lighted candle under one of the chimneys, and replace the front. Light some touch paper* and hold it over the other chimney. The air is observed to pass down one chimney and up the other.

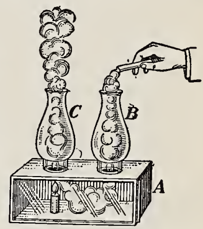

Fig. 307.—Convection currents in heated air.

303. Winds. While air currents are modified by various forces and agencies, they are, as we have seen (§ 105), all traceable to the pressure differences which result from inequalities in the temperature and other conditions of the atmosphere.

The effects of temperature differences are but manifestations, on a large scale, of convection currents, like those in the air about the heated iron. For various causes the earth's surface is unequally heated by the sun. The air over the heated areas expands, and, becoming relatively lighter, is forced upward by the buoyant pressure of the colder and heavier air of the surrounding regions.

Trade winds furnish an example. These permanent air-currents are primarily due to the unequal heating of the atmosphere in the polar and the equatorial latitudes.

We have an example also, on a much smaller scale, in land and sea breezes. On account of its higher specific heat, water

*Made by dipping blotting-paper in a solution of potassium nitrate and drying it.

warms and cools much more slowly than land. For this reason the sea is frequently cooler by day and warmer by night than the surrounding land. Hence, if there are no disturbing forces, an off-sea breeze is likely to blow over the land during the day and an off-land breeze to blow out to sea at night (Fig. 308). Since the causes producing the changes in pressure are but local, it is obvious that these atmospheric disturbances can extend but a short distance from the shore, usually not more than 10 or 15 miles.

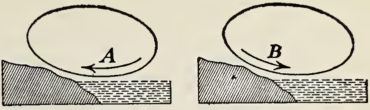

FIG. 308.—Illustration of land and sea breezes. *A*, direction of movement in sea breeze. *B*, direction of movement in land breeze.

Question.—Why do we, when turning on the draught of a stove or a furnace, close the top and open the bottom?

304. Application of Convection Currents—Cooking—Hot-water Supply. The distribution of heat for ordinary cooking operations, such as boiling, steaming, and oven-roasting and baking obviously involves convection currents.

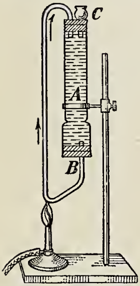

FIG. 309.—Illustration of the principle of heating water by convection currents.

When running water is available, kitchens are usually supplied with equipment for maintaining a supply of hot water for culinary purposes. The common method of heating the water by a coil in the fire-box of a stove or furnace is illustrated in the following experiment. Use a lamp chimney as a reservoir and fit up the connecting tubes as shown in Fig. 309.

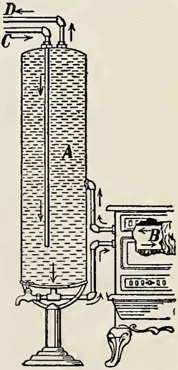

FIG. 309a.—Connections in a kitchen water heater. *A* is the hot-water tank and *B* is the water-front of the stove. The arrows show the direction in which the water moves.

Drop a crystal or two of potassium permanganate to the bottom of the reservoir to show the direction of the water currents. Fill the reservoir and tubes through the funnel *C* and heat the tube *B* with a lamp. A current will be observed to flow in the direction of the arrow. The hot water rises to the top of the reservoir, and the cold water at the bottom moves forward to be heated.

Fig. 309a shows the actual connections in a kitchen outfit. The cold water supply-pipe *C* is connected with a tank in the attic or with the water-works service pipes. The hot water is drawn off through the pipe *D*.

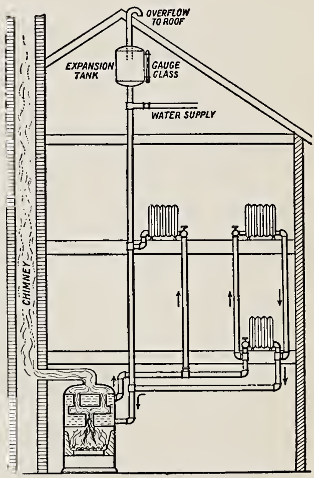

Fig. 311.—Hot-water heating system.

305. Hot-water Heating. Hot-water systems of heating dwelling houses also depend on convection currents for the distribution of heat.

The principle may be illustrated by a modification of the last experiment. Connect an open reservoir *B* with a flask, as shown in Fig. 310. Taking care not to entrap air-bubbles, fill the flask, tubes, and part of the reservoir with water. To show the direction of the currents, colour the water in the reservoir with potassium permanganate. Heat the flask. The coloured water in the reservoir almost immediately begins to

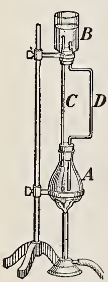

Fig. 310. — Illustration of the principle of heating buildings by hot water.

move downwards through the tube D to the bottom of the flask, and the colourless water in C appears at the top of the reservoir.

In a hot-water heating system (Fig. 311) a boiler takes the place of the flask. The hot water passes through radiators in the various rooms of the house and then returns to the furnace. An expansion tank is also connected with the system. Observe that, as in the flask, the hot water rises from the top of the heater and returns at the bottom.

306. Steam Heating. Steam also is employed for heating buildings. It is generated in a boiler and distributed by its own pressure through a system of pipes and radiators. The water of condensation either returns by gravitation or is pumped into the boiler.

307. Heating by Hot-air Furnaces. Hot-air systems of heating are in very common use. In most cases the circulation of air depends on convection currents. The development of such currents by hot-air furnaces depends on the principle that if a jacket is placed around a heated body and openings are made in its top and its bottom, a current of air will enter at the bottom and escape at a higher temperature at the top. For example, a lamp shade of the form shown in Fig. 312 forms such a jacket about a hot lamp chimney. When the air around the lamp is charged with smoke, a current of air is seen to pass in at the base of the shade and out at the top.

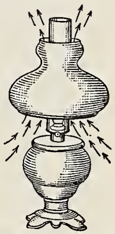

FIG. 312.—Air currents produced by placing a jacket around a heated body.

A hot-air furnace consists of simply a stove with a galvanized-iron or brick jacket (A Fig. 313) about it. Pipes connected with the top of the jacket convey the hot air to the rooms to be heated. The cold air is led into the base of the jacket by pipes connected with the outside air or with the floors of the room above.

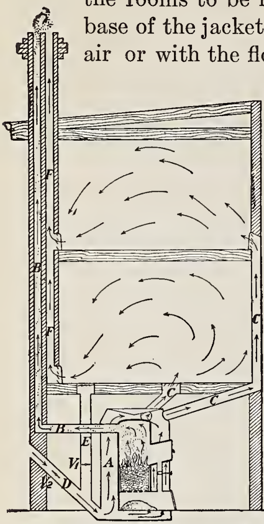

FIG. 313.—Hot-air heating and ventilating system. *A*, stove-jacket; *B*, smoke flue; *C*, warm-air pipes; *D*, cold-air pipe from outside; *E*, cold-air pipe from room; *F*, vent flue; V_1, valve in pipe *E*; V_2, valve in pipe from outside.

308. Ventilation. Most of the methods adopted for securing a supply of fresh air for living rooms depend on the development of convection currents.

When a lighted candle is placed at the bottom of a wide-mouthed jar, fitted with two tubes, as shown in *B* (Fig. 314), it burns for a time but goes out as the air becomes deprived of oxygen and vitiated by the products of combustion. If one of the tubes is pushed to the bottom (*A*, Fig. 314), the candle will continue to burn brightly, because a continuous supply of fresh air comes in by one tube and the foul gas escapes by the other.

The experiment is typical of the means usually adopted to secure ventilation in dwelling houses. A current is made to flow between supply pipes and vents by heating the air at one or more points in its circuit.

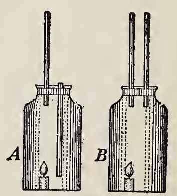

FIG. 314.—Illustration of principle of ventilation. The tubes should be at least ½ inch in diameter.

A warm-air furnace system of heating provides naturally for ventilation, if the air to be warmed is drawn from the outside and, after being used, is allowed to escape (Fig. 313). To support the circulation the vent flue is usually heated. The figure shows the vent flue placed alongside the smoke flue from which it receives heat to create a draught.

The supply pipes and vent flues are, as a rule, fitted with valves V_1, V_2, to control the air currents. When the inside supply pipe is closed and the others opened, a current of fresh air passes into and out of the house; when it is opened and the outside supply pipe and vent flue closed, the circulation is wholly within the house and the rooms are heated but not ventilated.

With a hot-water or steam-heating plant ventilation must be effected indirectly. Sometimes a supply pipe is led in at the base of each radiator and fresh air drawn in by the upward current produced by the heated coils. More frequently coils are provided for warming the air before it enters the rooms. The coils are jacketed and the method for maintaining the current differs from the furnace system only in that the air is warmed by steam coils instead of by a stove. To secure a continuous circulation in large buildings under varying atmospheric conditions, the natural convection currents are often re-inforced and controlled by a power-driven fan placed in the circuit.

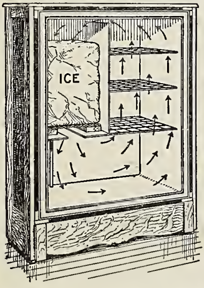

Fig. 315.—A household refrigerator with the front removed to show its construction.

309. Refrigeration. Convection currents in air may be utilized to keep an area cool as well as hot. This principle is applied in the ordinary household refrigerator (Fig. 315). Ice is placed in a chamber at the top, and provision is made for ventilation in such a manner that cool currents of air circulate over the perishable materials to be kept cool.

QUESTIONS

1. Why is ice kept in the upper part of the refrigerator?

2. Why should hot water or steam radiators be installed near outside walls or windows on the cold side of a room?

3. The apparatus shown in Fig. 315a is fitted up and filled with water until it rises a little above the end of the inner tube *C*. Describe the circulation in the water when a lamp flame is gently applied at the lower end of *B*. Fit up apparatus and try the experiment.

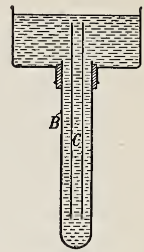

Fig. 315a.

4. Is the draft through the fire of a furnace pushed through or drawn through? Explain.

5. Why is it difficult to make accurate measurements of the conduction of heat in liquids?

6. What transformations of energy take place in the process of keeping materials cold in a refrigerator?

7. Why is a hollow wall a better non-conductor of heat than a solid wall? What effect would filling the space with saw-dust or shavings have?

8. Why is it often difficult to heat rooms on the windward side of a house by using a hot-air furnace?

9. What provision is made in a hot-air furnace for humidifying the air? Why is such provision necessary?

10. Describe the heating and ventilating systems used in your school.

310. Transference of Heat by Radiation. There is a third mode by which heat may be transferred, namely by *radiation.* It is by radiation that the sun warms the earth. By getting in the shadow we shield ourselves from this direct effect; and the face may be protected from the heat of a fire by holding a book or paper between. A hot body emits radiation *in all directions* and *in straight lines.* This is quite different from convection and conduction. Transmission by convection always takes place in one direction, namely, by upward currents; and conduction is not restricted to straight lines, for a bent wire conducts as well as a straight one.

311. Absorption and Radiation. When radiant energy falls upon a body, more or less of this energy is *absorbed,* and the temperature of the body rises. Some bodies have higher absorbing powers than others. A surface coated with lampblack or platinum-black absorbs practically all the radiation which falls upon it, and may be taken as a perfect absorber. On the other hand, a polished metal surface has a low absorbing power. Much of the energy which falls upon it is reflected from the surface instead of being absorbed by it.

This can easily be tested experimentally. Take two pieces of bright tin-plate about 4 inches square, and coat a face of one with lampblack. Then stand them parallel to each other and about 5 inches apart. They may conveniently be supported in saw-cuts in a board, and the blackened face should be turned towards the other plate. Attach with wax a bullet to the centre of the outer face of each plate. Now place midway between the plates a hot metal ball. Soon the bullet on the blackened plate will drop off while the other remains unaffected. If the blackened plate be touched with the finger, it will be found unpleasantly hot, while the other one will show a comparatively small rise in temperature.

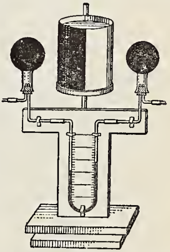

FIG. 316.—The blackened half of the vessel radiates more than the polished half.

On the other hand, a blackened surface is a good radiator while a polished surface is a bad one. To show this experimentally use an apparatus like that illustrated in Fig. 316. It consists of two blackened bulbs connected to a **U**-tube in which is coloured water. Now place between the bulbs a well-polished vessel, one half of which is blackened, and fill it with hot water. On observing the change in the level of the coloured water it will be seen that the blackened surface is radiating much more heat than the polished half.

The Crookes radiometer (Fig. 317) exhibits the effect of absorption and radiation in an interesting manner. In its simplest form it consists of a bulb from which almost all of the air has been removed. Inside the bulb is a set of four thin aluminium vanes, blackened on one side and polished on the other. The vanes are carried on light aluminium spokes radiating from a central hub, which is pivoted on the point of a needle. When the radiometer is held in sunlight or close to any body emitting radiant energy, the black faces of the vanes absorb the radiation better than the bright faces. The air in contact with the blackened faces consequently becomes hotter than that in contact with the polished faces, and differences of pressure are set up which cause the vanes to revolve.

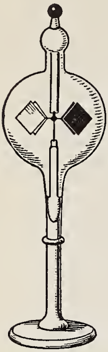

FIG. 317.—The Crookes Radiometer.

QUESTIONS

1. Explain why a sheet of zinc protects woodwork from a stove better than a sheet of asbestos. Would bright tin-plate be better still?

2. A kettle to be heated by being hung before a fireplace should have one side blackened and the other polished. Why?

3. A sign consisted of gold-leaf letters on a board painted black. It was found, after a fire on the opposite side of the street, that the wood between the letters was charred while that under them was uninjured. Explain this phenomenon.

4. Why is a frost more likely with a clear sky than with a cloudy one?

5. Why is there a greater deposition of dew on grass than upon bare ground?

6. In the Sahara the cold at night and the heat by day are equally painful to bear. Explain why.

7. Covering a plant with paper often prevents it being frozen. Why?

8. How is (*a*) conduction, (*b*) convection, (*c*) radiation reduced to a minimum in the "Thermos" bottle?

9. Describe the main differences between conduction, convection and radiation.

10. Why are the upper regions of the atmosphere very cold in spite of the fact that the radiant energy from the sun has to pass through these regions in order to reach the earth?

PART VII—LIGHT

CHAPTER XXXI

Light, its Nature and its Motion in Straight Lines

312. Light Radiation. When in the study of physics we speak of *light*, we mean the external agency which, if allowed to act upon the eye, produces the sensation of "seeing" or of "brightness." Most objects do not give out light of themselves. If you were to take them into a perfectly darkened room, you would not see them. But when light from some outside source, such as the sun, falls on them, they reflect it to our eyes and we see them. Some objects, however, such as a glowing coal or an oil lamp, can be seen without the aid of any external source of light. Such are said to be self-luminous.

Some bodies, such as glass, mica, water, etc., allow light to pass freely through them and are said to be transparent. Opaque substances entirely obstruct the passage of light; while translucent bodies, such as ground glass, oiled paper, etc., scatter the light which falls upon them, but a portion is allowed to pass through.

For the transmission of sound, the air or some other material medium is necessary (§ 162), but such is not the case with light. Exhausting the air from a glass vessel does not hinder the passage of light through it, but rather facilitates it. Again, we receive light from the sun, the stars and other heavenly bodies, and as there is no matter out in those great celestial spaces, the light must come to us through a perfect vacuum. Indeed, it travels millions of millions of miles without giving up any appreciable portion of its energy to the space it comes through.

We do not understand the process by which we obtain the sensation, but it is quite certain that to produce it work must be done. We see then that the source of light—the sun, a candle, an electric light—radiates energy, which, upon reaching the eye, changes its form and produces the sensation of brightness.

313. Light Travels in Straight Lines. It is a common observation that light travels in straight lines. We assume the truth of this in many everyday operations. The carpenter could not judge that an edge was straight nor could the marksman point his rifle properly were he not sure that three objects are precisely in a straight line when the light is *just* prevented from passing from the first to the third by the object between.

When light is admitted into a darkened room—a knot-hole in a barn, for instance—we can often trace the straight *course* of the light by the dust particles in the air. The path along which the light travels is called a *ray*. The rays themselves cannot be seen, but when they fall upon the particles of matter, these are illuminated and reflect light to the eye.

314. The Pin-hole Camera. An interesting application of the fact that light moves in straight lines is in the pin-hole camera. Let MN (Fig. 318) be a box having no ends. In front of it place a candle or other bright object, AB, and over the front end stretch tin-foil. In this prick a hole C with a pin, and over the back of the box stretch a thin sheet of paper.

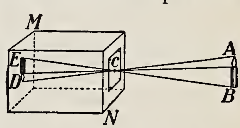

Fig. 318.—Pin-hole camera. C is a small hole in the front, and an inverted image of the candle AB is seen on the back of the box.

The light from the various portions of AB will pass through the hole C and will form on the paper an image DE, of the candle. This can be seen best by throwing over the head and the box a dark cloth. (Why?) The image is inverted, since the light travels in straight lines, and the rays cross at C.

If now we remove the paper, and for it substitute a sensitive photographic plate, a 'negative' may be obtained just as with an ordinary camera; indeed the perspective of the scene photographed will be truer than with most cameras. The chief objection to the use of the pin-hole camera is that with it the exposure required, compared to that with the ordinary camera, is very long; but excellent pictures of out-of-door scenes can be taken with a camera which almost any one can make.

It is evident that to secure a sharp, clear image, the hole C must be small. Suppose that it is made twice as large. Then we may consider each half of this hole as forming an image, and as these images will not exactly coincide, indistinctness will result. On the other hand, the hole must not be too small. As it is reduced in size, other phenomena, known as diffraction effects, are obtained. These effects show that, in all strictness, the light does not travel precisely in straight lines after all.

315. Theory of Shadows. Since the rays of light are straight, the space behind an opaque object will be screened from the light and will be *in the shadow*. If the source of the light is small, the shadows will be sharply defined, but if it is of some size, the edges will be indistinct.

FIG. 319.—If the source be small, the shadow will be sharp. A is the source, B the object, CD the shadow.

Let A (Fig. 319) be a small source—an arc lamp, for instance—and let B be an opaque ball. It will cast on the screen CD a circular shadow with sharply defined edges. But if the source is a body of considerable size, such as the sphere S (Fig. 320), then it is evident that the only portion of space which receives no light at all is the cone behind the opaque sphere E. This is called the *umbra*, or simply the *shadow*, while the portion beyond it which receives a part of the light from S is the *penumbra*. Suppose M is a body revolving about

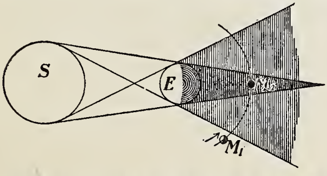

FIG. 320.—S is a large bright source, and E an opaque object. The dark portion is the *shadow*, the lighter portion the *penumbra*.

E in the direction indicated. In the first position it is just entering the penumbra; in the second position it is entirely within the shadow.

If *S* represents the sun, *E* the earth, and *M* the moon, the figure will illustrate an eclipse of the moon. For an eclipse of the sun, the moon must come between the earth and the

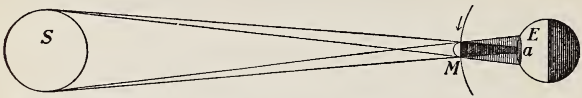

FIG. 321.—Showing how an eclipse of the sun is produced. A person at *a* cannot see the sun.

sun, as shown in Fig. 321. Only a small portion of the earth is in the shadow, and in order to see the sun totally eclipsed an observer must be at *a* on the narrow "track of totality."

316. How is Light Transmitted? Let us now consider how light may travel from one place to another. Scientists have been able to suggest only two methods by which energy can be transmitted from one place to another. A rifle bullet or a cannon ball has great energy, which it gives up on striking its aim; here the energy is transferred by the forward bodily motion of a material body. Also, as explained in § 165, energy can be handed on without transference of matter, namely by wave-motion.

The first method, which is commonly called the Emission, or Corpuscular, Theory, was strongly upheld by Sir Isaac Newton and by others following him. He believed that luminous bodies send out small particles, which produce the sensation of light when they strike the eye. This theory was commonly accepted for over a century and was finally discarded because there are some experimental results contrary to it, and others which it cannot explain. If, then, we must discard it, we necessarily turn to the second method, which has been called the Wave Theory. It is now universally accepted by scientific men.

317. Ether. But we cannot have waves without having a medium for them to travel in, and as the light-bearing medium is not ordinary matter, we are led to assume the existence of another medium which we call the *ether*. *Light is simply a motion in the ether.*

This ether must fill the great interstellar spaces of the universe; it must also pervade the space between the molecules and the atoms of matter, since light passes freely through the various forms of matter—solids, liquids and gases. We cannot detect it by any of our ordinary senses, we cannot see, feel, hear, taste, smell or weigh it, but as we cannot conceive of any other explanation of many phenomena, we are driven to believe in its existence.

318. Associated Radiations. It may be well to state here that the radiations which affect the eye never travel alone. Indeed those very radiations can also produce a heating effect and can excite chemical action—in the photographic plate, for instance. But associated with the light radiation are others which do not affect the eye at all, but which assist healthy growth and destroy obnoxious germs, give us warmth necessary for life, produce chemical effects as revealed in the colours of nature, or give us communication by wireless telegraphy.

These and many other effects are due to undulations of the ether, the chief difference among them being in the lengths of the waves. The light waves are all extremely short. The waves which produce the sensation of red are about $\frac{1}{30000}$ inch in length, while those which produce the blue sensation are only half as long.

We can *see* the waves moving on the surface of water or along a cord; we can *feel* the air, and with some effort, perhaps, can comprehend its motions; but to form a notion of how the ether is constructed and how it vibrates is a matter of excessive difficulty and indeed largely of pure conjecture. A very useful picture to have in one's mind is to think of the eye as joined to a source of light by cords of ether, and to consider the source as setting up in these cords transverse vibrations, which travel to the eye and give the luminous sensation.

319. Waves and Rays. Light thus is a form of energy, and is transferred from place to place by means of *waves*. What then is the relation of waves to rays?

Let the light spread out in all directions from a source A (Fig. 322). The waves will be concentric spheres S_1, S_2, S_3 . . . ., but the light will pass along the radii R_1, R_2, R_3 . . . ., of these spheres. The rays thus are the paths along which the waves travel, and it is seen that the ray is perpendicular to the wave-surface.

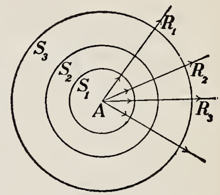

Fig. 322.—The waves are spheres with A as centre; the rays are radii of these spheres.

If we consider a number of rays moving out from A (Fig. 323), we have what is known as a divergent pencil a, and the waves are concentric spheres continually growing larger. If the rays are coming together to a point, we have a convergent pencil b, and the waves are concentric spheres continually growing smaller. If now the rays are parallel, as in c, we have a
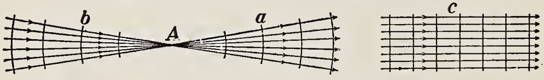
Fig. 323.—A convergent pencil, b; a divergent pencil, a; a parallel beam, c.

parallel beam, and the waves are plane surfaces, perpendicular to the rays. Such rays are obtained if the source is at a very great distance, so great that a portion of the sphere described with the source as centre might be considered a plane.

Question.—What becomes of the waves of a convergent pencil (b, Fig. 323) after they come to a point?

320. Velocity of Light. The speed with which light travels was revealed in 1675 by some observations made at the Paris Observatory by Roemer, a young Danish astronomer. In Fig. 324 let S be the sun, E_O, E', E'', E_C, the earth in various positions in its orbit; J_1, J_2 the planet Jupiter in two positions; and M one of its satellites or moons. It may be remarked that Jupiter has four large satellites and at least five smaller ones. The earth revolves about the sun in one year; Jupiter, which is $5\frac{1}{2}$ times as far from the sun, revolves in 12 years; while the satellite under consideration revolves about Jupiter in the short period of $42\frac{1}{2}$ hours. It so happens, also, that the orbits of the earth, Jupiter and the satellite are very nearly in the same plane. Jupiter, being an opaque body, casts a shadow behind it, and during every revolution of the satellite it plunges into the shadow and is eclipsed. In the position $SE_O J_1$, in which the planet and the sun are on opposite sides of the earth, Jupiter is said to be in *opposition* to the sun, while in the position $E_C SJ_2$, in which the planet and the sun appear to be in the same direction as seen from the earth, Jupiter is said to be in *conjunction* with the sun.

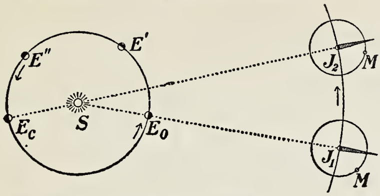

Fig. 324.—Illustrating the eclipse of Jupiter's satellite. S is the sun, E the earth, J Jupiter, and M its satellite. When the satellite passes into the shadow cast by J, it cannot be seen from E.

Suppose we observe successive eclipses near the time of opposition. We thus obtain the interval between them, and from this we can compute and tabulate the times for future eclipses. Now Roemer observed that the tabulated and the observed times did not agree, but that, as the earth moved to E', E'' and E_C, continually getting farther from the planet, the observed time lagged more and more behind the tabulated time, until when the earth was at E_C and Jupiter at J_2 (conjunction), the difference between the tabulated and the observed time of an eclipse had grown to 16 min. 40 sec., or 1000 sec.* However, by the time the earth had moved round to opposition again the observed and the tabulated times coincided.

Roemer accounted for this peculiar behaviour by noting that at conjunction the light travels the distance $E_C J_2$, which is greater than the distance $E_C J_1$, travelled at opposition, by the diameter of the earth's orbit, and hence the observed time should be later than the tabulated time by the time required by light to travel this extra distance. If we take the diameter of the orbit to be 186,000,000 miles, the velocity of light comes out to be 186,000,000 ÷ 1000 = 186,000 miles per second.

*This is the modern value; Roemer thought it was 22 m.

This is extraordinarily great, and yet in recent times physicists have been able to measure the time taken by light to travel short distances on the surface of the earth, and thus to determine its velocity. A more accurate value is 299,860 kilometres or 186,330 miles per second.

QUESTIONS AND PROBLEMS

1. A photograph is made by means of a pin-hole camera, which is 8 inches long, of a house 100 feet away and 30 feet high. Find the height of the image?

2. A pin-hole camera box is 8 inches long, and the image of a tree 200 feet away is $2\frac{1}{2}$ inches high. Find the height of the tree.

3. Why does the image in a pin-hole camera become fainter as it becomes larger (*i.e.*, by using a longer box, or pulling the screen back)?

4. Using an ordinary oil-lamp, obtain the shadow cast upon the wall by a vertical rod, (*a*) with the flame turned edgewise, (*b*) with the flat face turned to the rod. What difference do you observe? Account for it.

5. Why is the shadow obtained with a naked arc lamp sharp and well-defined? What difference will there be when a ground-glass globe is placed around the arc?

6. On holding a hair in sunlight close to a white screen the shadow of the hair is seen on the screen, but if the hair is a few inches away, scarcely any trace of the shadow can be observed. Explain this.

7. Taking the velocity of light to be 186,000 miles per second and the circumference of the earth to be 25,000 miles, how many times about the earth could light travel in one second?

8. The distance from the earth to the sun is 93,000,000 miles. Find the time required to travel this distance (*a*) by a railway train going 60 miles an hour; (*b*) by sound travelling with a speed of 1120 ft. per sec.; (*c*) by light whose speed is 186,000 miles per sec.

9. Light requires 8.6 years to come from Sirius, the brightest of the fixed stars, and 44 years to come from the Pole Star. Find the distance (in miles) of these two stars from us. (1 year = $365\frac{1}{4}$ days).

10. The sun apparently revolves about the earth 360° in 24 hours. How many degrees will it move during the time required for its light to come to us? Is the sun, then, precisely at that place in the sky where it appears to be?

REFERENCES FOR FURTHER INFORMATION

S. P. Thompson, *Light Visible and Invisible*, Lecture 1.
Edser, *Light for Students*, Chapters 1, 11, 13.
Preston, *Theory of Light*, Chapters 1, 2, 3, 19.
Duff, *Text-book of Physics*, Pages 547-564.

CHAPTER XXXII

Photometry

321. Intensity of Illumination. If you are reading a book by artificial light and cannot see the print properly, you naturally move closer to the source of light and thus increase the *intensity of illumination* of the page. You can, of course, produce the same effect by increasing the *power* of the source, for instance, by turning on another lamp or lighting additional candles.

It is evident, then, that intensity of illumination depends on two things, namely,

(1) the distance from the source,

(2) the power of the source.

322. Decrease of Intensity with Distance from the Source. Consider a small square of cardboard BC held one foot from a small source of light A (Fig. 325), and one foot behind this place a white screen DE, a sheet of white paper, for instance. The shadow cast by BC on DE is a square, each side of which is twice that of BC, and hence its area is *four* times that of BC. Next, hold the screen at FG, one foot further away, or three feet from A. The shadow of BC will now have its linear dimensions three times those of BC and its area nine times that of BC; and so on. The area of the shadow varies as the square of the distance from the source A.

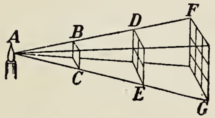

Fig. 325.—Area of DE is 4 times, and area of FG is 9 times that of BC.

Suppose now we mark off on the white screen an area 1 inch square and hold it at BC. The light which falls upon this area will produce a certain intensity of illumination. Next, hold the screen at DE. The same amount of light which fell upon 1 square inch at BC will now be spread over an area of 4 square inches, and hence the intensity of illumination will be $\frac{1}{4}$ as great as before. Again let it be held at FG. The same amount of light will now be spread over 9 square inches, and the intensity of illumination will be $\frac{1}{9}$ that at the first position; and so on.

Thus we obtain the following important law:—

The intensity of illumination varies inversely as the square of the distance from the source of light.

This law is so important that it will be well to illustrate it further.

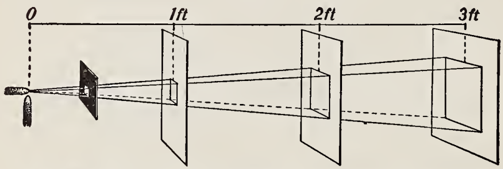

FIG. 326.—Experimental demonstration of the law of inverse squares.

Remove the condenser from a projecting lantern (preferably one with an arc lamp) and over the opening put a plate with a small square hole in it, say 1 inch to the edge (Fig. 326). Place a screen successively at distances 1 ft., 2 ft., 3 ft., etc., *from the light source*, measure the side of the square projected on the screen at each of these distances, and calculate the areas of each of these squares. It will be found that they are approximately in the proportion of 1, 4, 9, etc., that is 1^2, 2^2, 3^2, etc. But the quantity of light falling on each of these areas is the same in each case.

Thus we see that

The intensity of illumination at 2 ft. $= 1/2^2$ that at 1 ft.

The intensity of illumination at 3 ft. = $1/3^2$ that at 1 ft.
The intensity of illumination at 4 ft. = $1/4^2$ that at 1 ft.

and so on.

This is the fundamental law upon which all methods of comparing the powers of different sources of light are based.

It should be carefully observed that for this law to hold, the source of light must be small and must radiate freely in all directions. The headlight of a locomotive, for instance, projects the light mostly in one direction, and the decrease in intensity of illumination by it will not vary according to the above law.

323. Joly's Diffusion Photometer. To compare two sources of light we require some convenient method of determining equality of illumination, and various instruments, known as *photometers*, have been devised for this purpose. That suggested by Joly is one of the simplest and most satisfactory.

Two pieces of paraffin wax, each about 1 inch square and $\frac{1}{4}$ inch thick are cut from the same block of paraffin, carefully made of the same thickness and then put together with tin-foil between them (Fig. 327). Suppose we wish to compare the illuminating powers of two lamps, L_1 and L_2, for instance, an electric lamp and a candle.

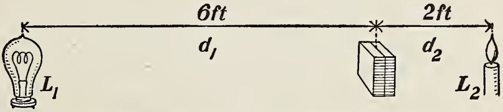

FIG. 327.—Comparing two lamps by the Joly photometer.

Move the photometer back and forth between the two lamps until the two blocks of paraffin are equally illuminated. Let its distance from L_1 be 6 ft. and from L_2 be 2 ft., that is, the distance of the photometer from the large lamp L_1 is 3 times that from the small lamp L_2.

If the two lamps were equal in power, the intensity of illumination of the left block would be $\frac{1}{9}$ that of the right block (Law of Inverse Squares). But it is equal to that of the right block.

Hence power of large lamp = 9 × power of small lamp;

$$\text{or in general, } L_1 = \left(\frac{d_1}{d_2}\right)^2 \times L_2,$$

where L_1 = power of 1st lamp,
L_2 = power of 2nd lamp,
d_1 = distance of 1st lamp from photometer,
d_2 = distance of 2nd from the photometer.

324. Verification of the Law of Inverse Squares. To do this let us use the Joly photometer (Fig. 328). Place 1 candle at one end of a board and 4 candles at the other. Now move the photometer until the line between the parafin blocks disappears, and measure its distance from the 1 candle and from the 4 candles. The latter will be twice the former. Next, replace the 4 candles by 9 and adjust as before. The distance from the 9 candles will be 3 times that from 1.

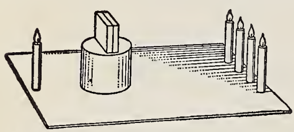

Fig. 328.—If the blocks are equally illuminated, the 4 candles are twice as far from the photometer as the single candle.

Thus if the distance is doubled, the illumination is reduced to $\frac{1}{4}$, since it requires 4 times as many candles to produce equality. In the same way, if the distance were n times as great, we should require n^2 candles to produce an illumination equal to that given by the single candle.

325. The Grease-spot or Bunsen Photometer. The essential part of this photometer is a piece of unglazed paper with a grease-spot on it. Such a spot is more translucent than the ungreased paper, so that if the paper be held before a lamp, the grease-spot appears brighter than the other portion, while if held behind the lamp, it appears darker.*

Now move the grease-spot screen between the two light-sources L_1, L_2 (Fig. 329) to be compared until it is equally bright all over its surface. Then it is evident that what illumination the screen loses by the light from L_1 passing through, is precisely compensated by the light from L_2 transmitted through it. Thus the intensity of illumination due to each lamp is the same. Hence, if d_1, d_2 are the distances from the screen, of L_1, L_2, respectively,

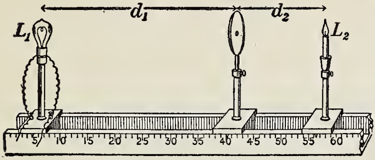

FIG. 329.—The Bunsen grease-spot photometer.

$$\frac{L_1}{L_2} = \left(\frac{d_1}{d_2}\right)^2,$$

as before.

326. The Shadow, or Rumford, Photometer. The method introduced by Rumford is to stand an opaque rod R (Fig. 330) vertically before a screen AB, and allow shadows from the two lamps to be cast on the screen.

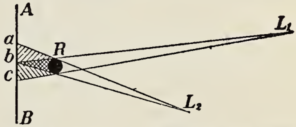

FIG. 330.—Rumford's shadow photometer. The lights L_1, L_2 are adjusted until the shadows cast by a rod R on the screen are equally dark.

If the screen is of ground-glass, it should be viewed from the side away from the lamps; if of opaque white paper (white blotting paper is best), the observer should be on the same side as the lamps.

It is evident that the portion ab is illuminated only by the lamp L_1, and the portion bc only by the lamp L_2.

*Drop a little melted paraffin on the paper (thin drawing paper answers well), and after it has become hard, remove the excess of paraffin with a knife. Place the paper between two pieces of blotting-paper and run a moderately hot iron over it, thus making a grease-spot on the paper about 1 inch in diameter. Cut from the paper a circular disc 4 or 5 inches in diameter with the spot at its centre, and mount it on a suitable support.

Now move the lamps until the portions *ab*, *bc* are equally bright (or equally dark), and then measure the distance of L_1, from *b* and of L_2 from *b*. Let these distances be d_1, d_2, respectively.

Then, as before, $\frac{L_1}{L_2} = \left(\frac{d_1}{d_2}\right)^2$.

327. Standards of Light. By the photometer we can accurately compare the strengths of two sources of light, but to state definitely the illuminating power of any lamp we should express it in terms of some fixed standard unit. We have definite standard units for measuring length, mass, time, heat, and most other quantities met with in physics; but no perfectly satisfactory standard of light has yet been devised.

The one most commonly used is the *candle*. The British standard candle is made of spermaceti, weighs 6 to the pound avoirdupois, and burns 120 grains per hour. The strength varies, however, with the state of the atmosphere and with the details of the manufacture of the wick. Yet, notwithstanding this inconstancy, it is usual to express the illuminating power of a source in terms of the standard candle.

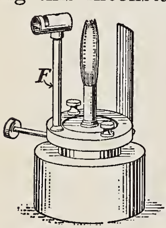

FIG. 331.—The Hefner standard lamp. The attachment F is for accurately adjusting the height of the flame.

A standard much used in scientific work is the Hefner lamp (Fig. 331). This is a small metal spirit-lamp with a cylindrical bowl 7 cm. in diameter and 4 cm. high. The wick-holder is a German-silver tube 8 mm. in interior diameter, 0.15 mm. thick, and 25 mm. high. The wick is carefully made to just fit the tube, and the height of the flame is adjusted to be 4 cm. The liquid burned is pure amyl acetate. The lamp is very constant, and its power is 98 per cent. of the British candle.

QUESTIONS AND PROBLEMS

1. Distinguish between *illuminating power* and *intensity of illumination.*

2. A candle is placed at a distance of 2 ft. from a screen, and then removed to a distance of 3 ft. Compare the intensities of illumination of the screen in the two cases.

3. A candle is placed at a distance of 10 inches from a screen and a 10 c.p. (*i.e.* candle-power) lamp is placed on the other side of the screen at a distance of 10 ft. from it. Compare the intensities of illumination on the two sides of the screen.

4. Two equal sources of light are placed on opposite sides of a sheet of paper, one 12 inches and the other 20 inches from it. Compare the intensity of illumination of the two sides of the paper.

5. When using a Rumford photometer (Fig. 330), the distance L_1b was found to be 50 inches and L_2b was 20 inches. Compare the illuminating powers of L_1 and L_2.

6. In a Rumford photometer it is found that the shadows are of equal depth when one of the lights is at a distance of 110 cm. from the screen and the other at a distance of 200 cm. from it. Compare the illuminating powers of the lights.

7. You wish to compare a candle and an oil-lamp flame by means of a shadow photometer. If the lamp is 12 times as powerful as the candle, and the latter is 15 inches from the screen, where will the lamp be when it just balances the candle?

8. In measuring the c.p. of an electric lamp with a Bunsen photometer, the distance from the standard candle to the disc is 25 cm. while the distance from the lamp to it is 100 cm. What is the c.p. of the lamp?

9. Two standard candles near together are placed on one side of the screen of a Bunsen photometer and 20 inches from it. How far must a 16 c.p. electric lamp be placed from the other side to cause the grease-spot to disappear?

10. A lamp and a candle are placed 2 m. apart, and a paraffin block is in adjustment between them when 42 cm. from the candle. Find the c.p. of the lamp.

11. For comfort in reading, the illumination of the printed page should be not less than 1 candle-foot (*i.e.*, 1 candle at a distance of 1 foot). How far might one read from a 16 c.p. lamp and still have sufficient illumination?

12. A candle and a gas-flame which is four times as strong are placed 6 feet apart. There are two positions on the line joining these two sources where a screen may be placed so that it may be equally illuminated by each source. Find these positions.

REFERENCES FOR FURTHER INFORMATION

Jones, *Lessons in Heat and Light*, Chapter 2.
Edser, *Light for Students*, Chapter 1.

CHAPTER XXXIII

Reflection of Light: Plane Mirrors

328. The Laws of Reflection. Let a lighted candle, placed in front of a sheet of thin plate glass, stand on a paper (or other) scale arranged perpendicular to the surface of the glass (Fig. 332). We see an image of the candle on the other side. Now move a second candle behind the glass until it coincides in position with the image.

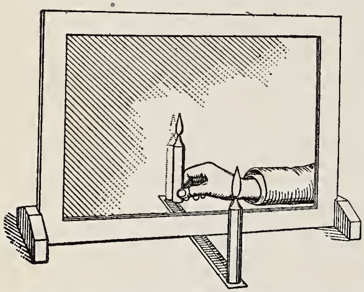

Fig. 332.—A lighted candle stands in front of a sheet of plate glass (not a mirror). Its image is seen by the experimenter, who, with a second lighted candle in his hand, is reaching round behind and trying to place it so as to coincide in position with the image of the first candle.

This can be done very accurately by employing the "method of parallax." Hold two fingers in a vertical position about six inches apart and in line with the eye and move the eye from side to side. The nearer finger appears to move to the right when the eye is moved to the left and to the left when the eye is moved to the right. Now bring the two fingers nearer together. The closer they are, the less is the relative movement. Similarly, when the image and the second candle are coincident, there will be no motion of one relative to the other when the head is moved from side to side.

It will be found that the two candles are both on the paper scale and at equal distances from the glass plate. We can state the First Law of Reflection, then, in this way:

If an object be placed before a plane mirror, its image is as far behind the mirror as the object is in front of it, and the line joining object and image is perpendicular to the mirror.

Thus light goes from the candle, strikes the mirror, from which it is reflected, and reaches the eye as though it came from a point as far behind the mirror as the candle is in front of it. Of course the image is not real, that is, the light does not actually go to it and come from it—it only appears to do so. But the deception is sometimes perfect and we take the image for a real object. This illusion is easily produced if the mirror is a good one and its edges are hidden*

This law of reflection can be stated in another way. Let MN (Fig. 333) be a section of a plane mirror. Light proceeds from A, strikes the mirror and is reflected, a portion being received by the eye E. To this eye the light *appears* to come from B, where $AM = MB$ and AB is perpendicular to MN.

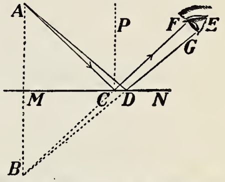

FIG. 333.—AC is an incident ray, CF the reflected ray, and CP the normal to the surface MN. Then angle of incidence ACP is equal to angle of reflection FCP.

Consider the ray AC, which, on reflection, goes in the direction CF.

In the triangles AMC, BMC, we have $AM = MB$, MC is common, and angle AMC = angle BMC, each being a right angle.

Hence the triangles are equal in every respect, and so the angle ACM = angle BCM.

But angle BCM = angle FCN, and hence the angles ACM and FCN are equal to each other.

From C let now CP be drawn perpendicular to MN. It is called the *normal* to the surface at C. At once we see angle ACP = angle FCP.

Now AC is defined to be the incident ray, CF the reflected ray, ACP the *angle of incidence* and FCP the *angle of reflection.*

Hence we can state the FIRST LAW OF REFLECTION thus:—

The angle of incidence is equal to the angle of reflection.

This statement of the law, which is precisely equivalent to the other, is sometimes more convenient to use.

A SECOND LAW should be added, namely—

The incident ray, the reflected ray and the normal to the surface are all in one plane.

*Wordsworth in "Yarrow Unvisited" refers to a case of perfect reflection:
"The swan on still St. Mary's Lake
Floats double, swan and shadow."

329. The Optical Disc. This apparatus is very useful for demonstrating the fundamental laws of optics. On a stand *A* (Fig. 334) is mounted a circular disc *B* about 13 inches in diameter, which is graduated in degrees in both directions from a zero line. It is also provided with thumb-screws, by which mirrors, lenses and prisms can be attached to it near the centre. The curved metal shield *C* is pierced by a number of narrow horizontal slits through which "rays" of light can be projected. The disc and the shield can be rotated independently about the same horizontal axis through the centre of the disc and at right angles to it.

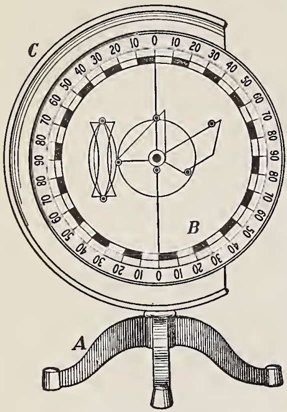

Fig. 334.—The optical disc, for demonstrating the laws of optics.

To verify the first law of reflection mount the plane mirror *D* (Fig. 335*a*) at the centre of the disc, with the face of the mirror at right angles to the zero line. Allow sunlight, or a parallel beam from a lantern, to enter by one of the slits, the others being kept covered. By moving the disc one can make the ray strike the mirror at any angle from 0° to 90°, and since it lights up the disc in its passage to and from the mirror, the angles of incidence and reflection can be read directly from the circular scale.

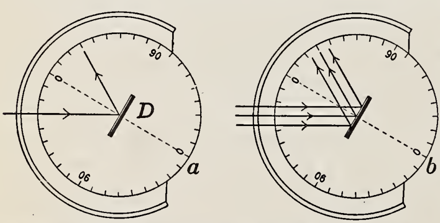

Fig. 335.—Demonstrating that the angle of incidence is equal to the angle of reflection.

If all the slits are opened the action of the mirror on a "beam" of light can be demonstrated (Fig. 335*b*).

The optical disc will be referred to later when curved mirrors, lenses and prisms are dealt with.

330. Law of Reflection in Accordance with the Wave Theory. The law of reflection, which we obtained experimentally, is just what we should expect if light is a wave motion. Let a stone be thrown into still water. Waves, in the form of concentric circles, spread out from the place A (Fig. 336) where it entered the water. If a plank lies on the surface near by, the waves will strike it and be reflected from it, moving off as circular waves whose centres B are as far behind the reflecting edge as A is in front of it. In the figure the dotted circles are the reflected waves. AM is an incident and MR the reflected "ray."

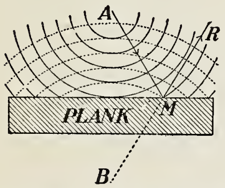

Fig. 336.—Waves on still water reflected from a plank lying on its surface.

331. Regular and Irregular Reflection. Mirrors are usually made of polished metal or of sheet glass with a coating of silver on the back surface. When light falls on a mirror, it is reflected in a definite direction and the reflection is said to be *regular*. Reflection is also regular from the still surfaces of water, mercury and other liquids.

Now an unpolished surface, such as paper, although it may appear to the eye or the hand as quite smooth, will exhibit decided inequalities when examined under a microscope. The surface will appear somewhat as in Fig. 337, and consequently the normals at the various parts of the surface will not be parallel to one another, as they are in a well-polished surface. Hence the rays when reflected will take various directions and will be scattered.

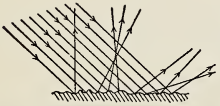
Fig. 337.—Scattering of light from a rough surface.

It is by means of this scattered light that objects are made visible to us. When sunlight is reflected by a mirror into your eyes, you do not see the mirror but the image of the sun formed by the mirror. Again, if a beam of sunlight in a dark room falls on a plate of polished silver, practically the entire beam is diverted in one definite direction, and no light is given to surrounding bodies. But if it falls on a piece of chalk, the light is diffused in all directions, and the chalk can be seen. It is sometimes difficult to see the smooth surface of a pond surrounded by trees and overhung with clouds, as the eye considers only the reflected images of these objects; but a faint breath of wind, slightly rippling the surface, reveals the water.

332. How the Eye Receives the Light. An object AB (Fig. 338) is placed before a plane mirror MM, and the eye of the observer is at E. Then the image $A'B'$ is easily drawn. The light which reaches the eye from A will appear to come from A', which is the image of A and which is as far behind MM as A is before it.

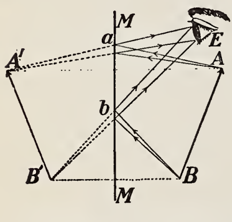

Fig. 338.—How an eye sees the image of an object before a plane mirror.

It is, therefore, by the pencil AaE that the point A is seen. In the same way the point B is seen by the small pencil BbE, and similarly for all other points of the object.

It will be observed that when the eye is placed where it is represented in the figure, the only portion of the mirror which is used is the small space between a and b.

Exercise.—Draw a figure of a person standing before a vertical mirror and show that for him to see himself from head to foot the mirror need be only half his height. Draw a figure for the person at several distances from the mirror. At what place is the used portion of the mirror the least?

333. Lateral Inversion. The image in a plane mirror is not the exact counterpart of the object producing it. The right hand of the object becomes the left hand of the image. If a printed page is held before the mirror, the letters are erect but the sides are interchanged. This effect is known as *lateral inversion.* If a word is written on a sheet of paper and at once pressed on a sheet of clean blotting-paper, the writing on the blotting-paper is inverted; but if it is held before a mirror, it is reinverted and becomes legible. The effect is illustrated in Fig. 339, showing the image in a plane mirror of the word STAR. It may be remarked, therefore, that on looking in a mirror we do not 'see ourselves as others see us.'

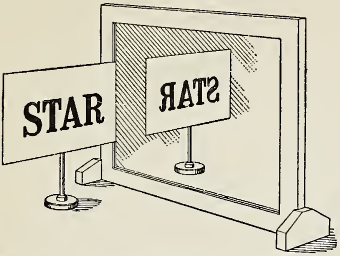

FIG 339.— Illustrating "lateral inversion" by a plane mirror.

334. Reflections from Parallel Mirrors. Let us stand two mirrors on a table, parallel to each other, and set a lighted candle between them. An eye looking over the top of one mirror at the other will see a long vista of images stretching away behind the mirror. These are produced by successive reflections.

In Fig. 340, I and II are the mirrors and O the candle.

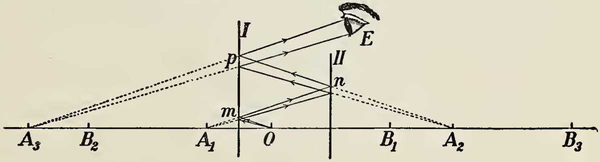

FIG. 340.—Showing many images produced by two mirrors I, II, parallel to each other.

A_1 is the image of O in I, A_2 the image of A_1 in II, A_3 that of A_2 in I, and so on. Also B_1 is the image of O in II, B_2 that of B_1 in I, B_3 that of B_2 in II, and so on. The path of the light which produces in the eye the third image A_3 is also shown. It is reflected three times, namely, at m, n and p, and from the figure it will be seen that the actual path $OmnpE$, which the light travels, is equal to the distance A_3E from the image to the eye.

335. Images in Inclined Mirrors. Let the mirrors M_1, M_2 (Fig. 341) stand at right angles to each other and O be a candle between. There will be three images, A being the first image in M_1, B the first image in M_2, while C is the image of A in M_2 or of B in M_1, these two coinciding.

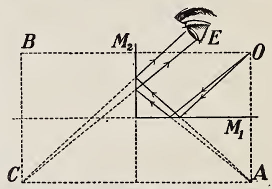

Fig. 341.—Images produced by two mirrors placed at right angles.

336. The Kaleidoscope. If the mirrors are inclined at 60°, the images will be formed at the places shown in Fig. 342. They are all located on the circumference of a circle having the intersection of the mirrors as its centre, and an inspection of the figure will show how to draw them.

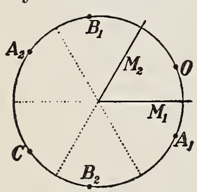

Fig. 342.—Images produced by mirrors inclined at an angle of 60°.

The kaleidoscope is a toy consisting of a tube having in it three mirrors forming an equilateral triangle, with bits of coloured glass between. The multiple images produce some very pleasing hexagonal figures. It was invented in 1816 by Sir David Brewster and created a great sensation.

QUESTIONS AND PROBLEMS

1. Why is a room lighter when its walls are white than when covered with dark paper?

2. A mirror is tilted forward until 30° from the vertical, and a candle is placed in front of it. Show by a diagram the position of the image, and also draw the rays by which an observer sees the top and the bottom of the candle.

3. The sun is 30° above the horizon and you see its image in still water. Draw a diagram to show how the light reaches your eye, and find the angles of incidence and reflection.

4. Two mirrors are inclined at 45° and a candle is placed between them. By means of a figure show the position of the images. How many are there? How many when the angle is 60° and 90°? (See Figs. 341, 342.)

5. The following rule for finding the number of images seen in two inclined mirrors has been given: "Divide the angle between the mirrors into 360 and subtract 1." Test this in the case of angles 45°, 60° and 90° (Problem 4). Draw a figure and test it for 72°. Does it hold for 40°?

6. A ray of light is reflected successively from two mirrors placed at right angles to each other. Draw a figure to show its path, and show that after the second reflection it is parallel to its original direction.

7. Two mirrors are inclined at an angle of 60°. A ray of light travelling parallel to the first mirror strikes the second, from which it is reflected and, falling on the first, is reflected from it. Show that it is now moving parallel to the second mirror.

8. In using the heliograph (Fig. 343) for signalling, the plane mirror *A* is adjusted so that the light of the sun is reflected to the distant station. Dots and dashes are then made by operating the key *B*, which tips the mirror through a small angle and so throws the light alternately on and off the receiving station. If the sun is 40° above the horizon, at what angle with the horizon will the mirror have to be set in order that the reflected ray may be horizontal? If the mirror is tilted through 2° by working the key, through how many degrees will the reflected ray be deflected?

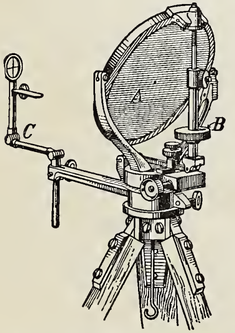

Fig. 343.—The heliograph used in the army.

9. On a moonlight night when the lake is covered with ripples, instead of an image of the moon being seen in the water, a long band of light is observed on its surface extending towards the moon. Explain this phenomenon by the aid of a diagram.

10. Show by a diagram how it is possible for a lady, by using two mirrors, to see an image of the back of her head.

REFERENCES FOR FURTHER INFORMATION

Stewart and Satterly, *Text-book of Light*, Chapter 4.
Glazebrook, *Light*, Chapter 3.
Lewis Wright, *Light*, Chapter 2. (Good experiments)

CHAPTER XXXIV

Reflection From Curved Mirrors

337. The Curved Mirrors used in Optics; Definitions. The curved mirrors used in optics are generally segments of spheres. If the reflection is from the outer surface of the sphere, the mirror is said to be *convex*; if from the inner surface, *concave*.

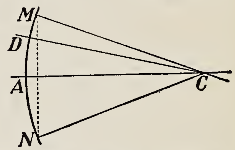

Fig. 344.—A section of a spherical mirror.

In Fig. 344 MAN represents a section of a spherical mirror. C, the centre of the sphere from which the mirror is cut, is the *centre of curvature*, and CM, CA or CN is a *radius of curvature*; MN is the *linear*, and MCN the *angular*, *aperture*; A, the middle point of the face of the mirror is the *vertex*; CA is the *principal axis*, and CD, any other straight line through C, is a *secondary axis*. Note also that all radii of a circle are at right angles to the circle, and hence CM, CD, CA and CN are normals to the mirror at the points M, D, A, N, respectively.

338. Action of Concave and Convex Mirrors. The laws of reflection hold for curved as well as for plane mirrors. The behaviour of rays of light when they fall on such mirrors can be well shown by means of the optical disc.

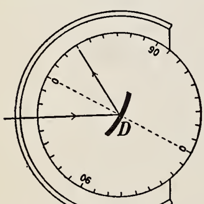

Fig. 345.—Finding the law of reflection at a concave mirror.

Fit the concave mirror D (Fig. 345) to the disc so that the zero line of the disc coincides with the principal axis of the mirror and the vertex of the mirror is at the centre of the disc. Cause a ray, or a very narrow beam, of light to fall on the mirror at the vertex.

Now, the principal axis is the normal to the mirror at the vertex and, consequently, the angles of incidence and reflection can be read directly from the scale on the disc. Repeat with different angles of incidence and observe the angle of reflection in each case. It will be found that in every case the angle of reflection is equal to the angle of incidence.

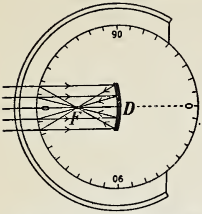

Fig. 346.—Parallel rays are converged to a point called the principal focus.

Next, let a number of rays fall on the concave mirror parallel to the principal axis (Fig. 346). After reflection from the mirror they converge approximately to a point F, which is called the *principal focus* of the mirror.

Similar experiments can be tried with a convex mirror. Attach the mirror to the disc so that its convex surface is at the centre of the disc and its principal axis is along the zero line of the disc (Fig. 347). Admit a ray and, as in Fig. 345, test with different angles of incidence. Again it will be found that the angle of reflection is always equal to the angle of incidence.

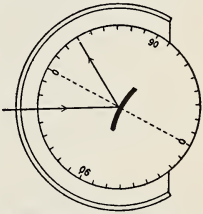

Fig. 347.—Reflection of a ray from a convex mirror.

Then allow a number of parallel rays to fall on the mirror as in Fig. 348. After reflection they diverge as though they come from the point F behind. This point is the principal focus of the convex mirror.

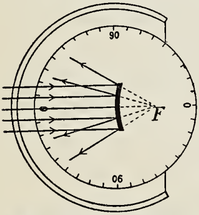

Fig. 348.—Reflection of parallel rays from a convex mirror.

339. How to Draw the Reflected Ray. Suppose QR (Fig. 349) to be a ray of light incident at R upon a concave mirror. We

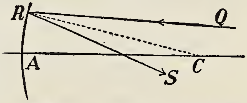

Fig. 349.—Reflection from a concave mirror.

wish to draw the reflected ray. Join R to C the centre of curvature. Then RC is the normal to the mirror at R. By making CRS (the angle of reflection) equal to CRQ (the angle of incidence) we obtain RS, the reflected ray.

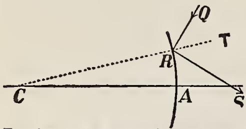

FIG. 350.—Reflection from a convex mirror.

In Fig. 350 the ray QR is incident at R upon a convex mirror. As before join R to C, the centre of curvature, and produce CR to T. Then TRQ is the angle of incidence; and making TRS equal to it we have RS, the reflected ray.

340. Principal Focus. In Fig. 351 let QR be a ray parallel to the principal axis; then, making the angle CRS = angle CRQ, we have the reflected ray RS. But since QR is parallel to AC, angle CRQ = angle RCF. Hence angle FRC = angle FCR, and the sides FR, FC are equal.

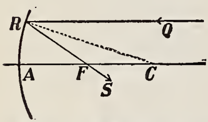

FIG. 351.—The ray QR, parallel to the principal axis AC, on reflection passes through the principal focus F.

Now if R is not far from the vertex A, FR and FA are nearly equal, and hence AF is approximately equal to FC, that is, the reflected ray cuts the principal axis at a point approximately midway between A and C.

It is evident, then, that a beam of rays parallel to the principal axis, striking the mirror near the vertex, will be converged by the concave mirror to a point F, midway between A and C. This point is called the *principal focus*, and AF is the *focal length* of the mirror. Denoting AF by f and AC by r, we have $f = r/2$.

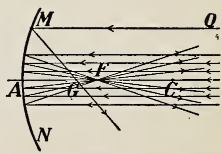

FIG. 352.—A beam of rays parallel to the principal axis passes, on reflection, through F, the principal focus.

In the case shown in Fig. 352 the rays actually pass through F, which is therefore called a *real* focus.

Rays which strike the mirror at some distance from A do not pass precisely through F. For instance, the ray QM after reflection cuts the axis at G; this *wandering* from F is called *aberration,* which amounts to FG for this ray.

For a convex mirror the same method is followed. In Fig. 353 a beam parallel to the principal axis is incident near the vertex. The reflected rays diverge in such a way that if produced backwards they pass through F, the principal focus. In this case the rays do not actually pass through F, but only appear to come from it. For this reason F is called a *virtual* focus. In the figure is also shown a ray QM, which strikes the mirror at some distance from the vertex. Upon reflection this appears to come from G, and FG is the aberration.

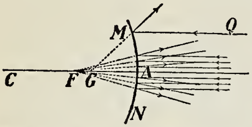

Fig. 353.—Showing reflection of a parallel beam from a convex mirror.

341. Explanation by the Wave Theory. The behaviour of curved mirrors can be easily accounted for by means of the wave theory. In Fig. 354, $a\ b$, $a_1\ b_1$, $a_2\ b_2$, ... represent plane waves moving forward to the concave mirror. The waves reach the outer portions of the mirror first and are turned back, in this way being changed into spherical waves which contract, pass through F and then expand again.

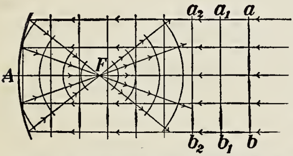

Fig. 354.—Showing how plane waves by reflection at a concave mirror are changed to spherical waves.

This action of a concave mirror is well illustrated in Fig. 355 from an instantaneous photograph of ripples on the surface of mercury. The plane waves were produced by a piece of glass fastened to one prong of a tuning-fork. They move forward, as shown by the arrows, and meet a concave reflector, by which they are changed into circular waves converging to the principal focus. They pass through this and then expand again.

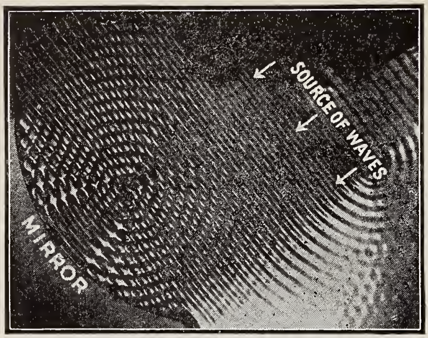

FIG. 355.—Photograph of waves on the surface of mercury.

Exercises.—(1) Draw a diagram like the photograph reproduced in Fig. 355, showing the paths of several "rays." What ultimately becomes of the waves on the surface of a liquid? (2) Draw the plane waves and the rays for a convex mirror, corresponding to Fig. 354.

342. Conjugate Foci. We have seen that light rays moving parallel to the principal axis are brought to a focus, real or virtual, by a spherical mirror, but a focus can be obtained as well with light not in parallel rays. For instance, let the light diverge from P (Fig. 356); after reflection from the concave mirror it converges to P'.

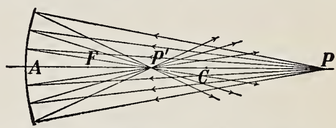

FIG. 356.—Conjugate foci in a concave mirror. P and P' are conjugate.

Now, it is evident that if the light originated at P', it would be converged by the mirror to P. Each point is the image of the other, and they are called *conjugate foci.*

In the case shown in Fig. 356, both foci are real, since the rays which come from one actually pass through the other. It is possible, however, for one of them to be virtual. Such a case is shown in Fig. 357. Here P' is conjugate to P, but is virtual. It will be noticed that P is between the mirror and F.

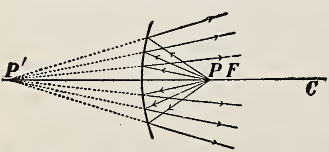

FIG. 357.—Conjugate foci in a concave reflector, one being virtual.

In these circumstances the conjugate focus is virtual, in all others it is real.

Exercise.—Draw the *waves* in these and in other cases of conjugate foci, taking P at various positions on the axis.

343. Experiments with Mirrors. *I. Concave.* (1) Into a darkened room take a concave mirror, and at the other end of the room place a lighted candle facing the mirror. The position of the image can be found by catching it on a small screen. It will be very near the principal focus, and will be real, inverted and very small. This enables us to determine the focal length of the mirror. If the sun is shining, hold the mirror in the sunlight and receive the image on a piece of paper. If sunlight is not available, find the position of the image of a distant object. Even a window at the opposite side of the room will give good results.

(2) Now carry the candle towards the mirror. The image moves out from the mirror and increases in size, but it remains real, inverted and smaller than the candle, until when the candle reaches the centre of curvature, the image is there also and is of the same size. In this way we can determine the centre of curvature and, consequently, the focal length of the mirror.

(3) Next, bring the candle nearer the mirror; the image moves farther and farther away, and is real, inverted and enlarged. When the candle reaches a certain place near the principal focus, the image will be seen on the opposite wall, inverted, and much enlarged; but when the candle is at the focus, the light is reflected from the mirror in parallel rays—the image is at infinity.

(4) When the candle is at a point between the principal focus and the vertex, the reflected rays diverge from a virtual focus behind the mirror (see Fig. 357). No real image is formed, one cannot receive it on a screen, but on looking into the mirror one sees a virtual, erect and magnified image. Its position can be determined by the method of parallax

explained in § 328. Use a hat-pin stuck in a rubber cork behind the mirror and seen above it as a 'finder.' Another hat-pin can be used as an object instead of the candle.

These results can be arranged in a table:—

IMAGES WITH A CONCAVE MIRROR

Position of Object	Position of Image	Real or Virtual	Size	Erect or Inverted
Beyond c.c.	Betw. p.f. and c.c.	Real	Smaller	Inverted
At c.c.	At c.c.	Real	Same size	Inverted
Betw. c.c. and p.f.	Beyond c.c.	Real	Larger	Inverted
At p.f.	Rays parallel			
Inside p.f.	Behind mirror	Virtual	Larger	Erect

II. Convex. If the candle is held before a convex mirror, the image is always virtual, erect and smaller than the candle. Here again the method of parallax must be used if we wish to locate the image experimentally. The image of a distant object (a chimney answers well) is practically at the focus. This method of finding the focal length requires patience but it gives very satisfactory results.

A simple example of a convex mirror is the outer surface of the bowl of a silver spoon.

344. To Draw the Image of an Object. *I. Concave Mirror.* Suppose PQ to be a small bright object placed before a concave mirror (Fig. 358). Now *all* the rays from Q after reflection pass through its image, and it is clear that we can locate the position of this image if we can draw any two rays which pass through it.

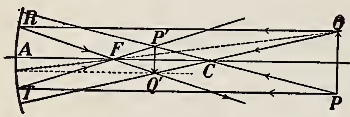

FIG. 358.—How to locate the image produced by a concave mirror.

Draw a ray QR, parallel to the principal axis; this will, upon reflection, pass through F. Also, the ray QC will strike the mirror at right angles, and, when reflected, will return upon itself. The two reflected rays intersect at Q', which is therefore the image of Q. Draw the rays PT and PC; after reflection they meet in P' which is the image of P. It is evident then that $P'Q'$ is the image of PQ.

It is evident that the ray QF will, after reflection, return parallel to the axis AC, and will, of course, also pass through Q'.

By drawing any two of the three rays QR, QC, QF we can always find Q', the image of Q. It should be observed, however, that all the rays from Q, not just those drawn, will after reflection pass through Q'. Similarly with those from P and other points on PQ.

It will be very useful to draw the image of an object in several positions. In Fig. 359 the object PQ is between A and F. By drawing QR, parallel to the axis, and QT, which passes through the centre of curvature, we obtain Q', the image of Q; and by drawing similar rays from P we obtain P', and hence $P'Q'$ the image of PQ. It is virtual and behind the mirror.

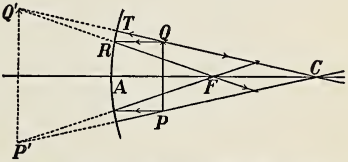

Fig. 359.—How to draw the image when the object is between the principal focus and the vertex.

Exercise.—Draw the image when the object is between F and C.

II. Convex Mirror. In Fig. 360 the mirror is convex, and the image $P'Q'$ is virtual, erect, behind the mirror and smaller than PQ. It is always so in a convex mirror.

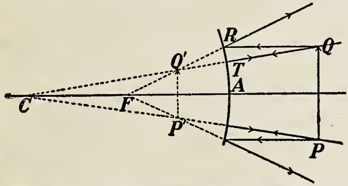

Fig. 360.— How to draw the image produced by a convex mirror.

The distances of the object and the image from the mirror are connected by a simple formula, $1/p + 1/p' = 2/r$, where p is the distance from the mirror to the object, p' the distance from the mirror to the image and r the radius of curvature.

*The proof of this formula will be found in an appendix to the *Laboratory Manual* designed to accompany this work.

345. Relative Sizes of Image and Object. Let PQ be an object and $P'Q'$ its image in a concave mirror (Fig. 361). The ray QA which strikes the mirror at the vertex, is reflected along AQ', and the angle QAP = angle $Q'AP'$. Also, angle APQ = angle $AP'Q'$, each being a right angle, and hence the two triangles APQ, $AP'Q'$ are similar to each other.

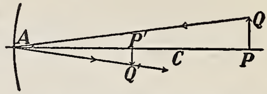

FIG. 361.—The size of the object PQ is to that of the image $P'Q'$ as their distances from the mirror.

The ratio of the length of the image to that of the object is called the *magnification*. Hence we have,

$$\text{Magnification} = \frac{P'Q'}{PQ} = \frac{AP'}{AP} = \frac{\text{distance of image from mirror}}{\text{distance of object from mirror}} = \frac{p'}{p}.$$

In the case illustrated in the figure the magnification is less than one.

346. The Rays by which an Eye sees the Image. In §344 a graphical method is given for locating the image of an object, but the actual rays by which an eye sees the image are usually not at all those shown in the figures.

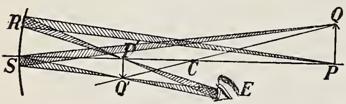

FIG. 362.—How the rays pass from the object to the eye. (Real image in concave mirror.)

In Figs. 362, 363, 364 are shown actual rays from points P and Q which reach the eye. In each figure the image is supposed to have been obtained by the graphical method. The image is real and inverted in Fig. 362, virtual and erect in the other two cases.

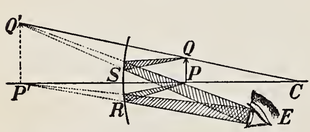

FIG. 363.—How the rays go from the object to the eye. (Virtual image in concave mirror.)

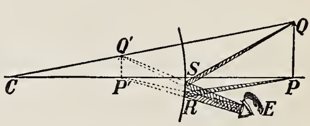

FIG. 364.—How the eye sees an object in a convex mirror. (Image always virtual.)

Now in each instance the light enters the eye as though it came from $P'Q'$. Join Q' to the outer edge of the pupil of the eye, forming thus a small cone with vertex at Q'. This cone meets the mirror at S, and it is clear that the light starts from

Q, meets the mirror at S, is reflected there and then passes through Q' (really or virtually), and reaches the eye. In the figures are shown also rays starting out from P, the other end of the object. They meet the mirror at R, where they are reflected and then received by the eye. In the same way we can draw the rays which emanate from any point in the object.

It will be seen that for the eye in the position E, shown in the figures, the only part of the mirror which is used is that space from R to S. The rays which fall on other parts of the mirror pass above or below or to one side of the eye.

347. Parabolic Mirrors. In the case of a spherical mirror, only those rays parallel to the axis which are incident near the vertex pass accurately through the principal focus; if the angular aperture is large the outer rays after reflection pass through points some distance from the focus (see Fig. 352). Conversely, if a source of light is placed at the principal focus, the rays after reflection will not all be accurately parallel to the axis, but the outer ones (Fig. 365) will converge inward, and later on after meeting will, of course, spread out. Hence at a great distance the light will be scattered and weakened.

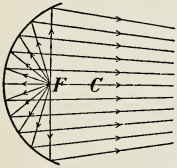

FIG. 365.—If a source of light is placed at the principal focus of a hemispherical mirror the outer rays converge and afterwards diverge again.

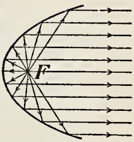

FIG. 366.—How a parabolic reflector sends out parallel rays.

Now a parabolic mirror overcomes this spreading of the rays. In Fig. 366 is shown a parabola. All rays which emanate from the focus, after reflection are parallel to the axis, no matter how great the aperture is. Parabolic mirrors are used in searchlights and in locomotive headlights. If a powerful source is used, a beam can be sent out to great distances with little loss of intensity. The latest form of signalling lamp used in the British Army is shown in Fig. 367. The filament C of the bulb B is placed at the principal focus of the concave parabolic mirror A which has a diameter of $3\frac{3}{4}$ inches. Signals sent by this lamp are readable in daylight with the naked eye at about 2 miles, and the total dispersion of the rays at that distance is only 160 yards. The bulb B is approximately 8 c.p. (4 watts).

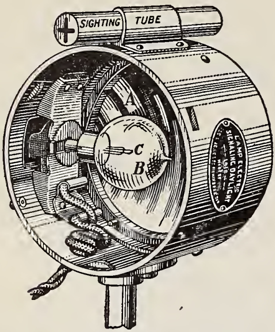

FIG. 367.—British army signalling lamp.

QUESTIONS AND PROBLEMS

1. Distinguish between a real and a virtual image.

2. Prove that the focal length of a convex spherical mirror is equal to half its radius of curvature.

3. Show by diagrams that the image of a candle placed before a convex mirror can never be inverted.

4. Find the focus conjugate to each of the following points: (*a*) the centre of curvature; (*b*) a point on the axis at an infinite distance; (*c*) the vertex; (*d*) the principal focus.

5. An object 3 cm. high is placed 30 cm. from a concave mirror of radius 20 cm. Find the position and size of the image.

First, draw a diagram carefully to scale as in Fig. 359, and measure the distance of the image from the mirror and also its height.

Next make use of the formula $1/p + 1/p' = 2/r$.

In this case $p = +30$, $r = +20$; from which we find $p' = +15$, that is the image is 15 cm. in front of the mirror.

Also magnification $= p'/p = 15/30 = 1/2$, that is the image is one-half the size of the object, or 1.5 cm. high.

The figure shows that the image is inverted.

6. If the object is 8 cm. from the mirror, find the position and size of the image.

Draw a diagram as in the previous problem.

Then using the formula, we have $p = +8$, $r = +20$ and p' comes out to be -40, that is the image is 40 cm. *behind* the mirror.

As to size, magnification $= p'/p = 40/8 = 5$, that is the image is 5 times as high as the object.

7. If the mirror in problems 5 and 6 had been convex, where would the images be and how high would they be?

REFERENCES FOR FURTHER INFORMATION.

Stewart and Satterly, *Text-book of Light*, Chapter 5.
Jones, *Lessons in Heat and Light*, Chapter 5 of *Light*.
Edser, *Light for Students*, Chapter 2.
Glazebrook, *Light*, Chapter 5.

CHAPTER XXXV

Refraction

348. Meaning of Refraction. Let us try the experiment illustrated in Fig. 368. A suitable size for the tank is 15 x 15 x $2\frac{1}{2}$ inches. The front and ends are of glass, the rest of metal. White paper with a circular disc cut out of it is pasted on the front. The edge should be graduated, and vertical and horizontal diameters marked on the glass. Pour water into the tank until up to the horizontal diameter. Now project a beam of light from a lantern or the sun and reflect it down to meet the surface of the water at the centre of the circle. If the beam is too thick, it may be made to pass through a narrow slit in a metal strip laid on top of the tank.

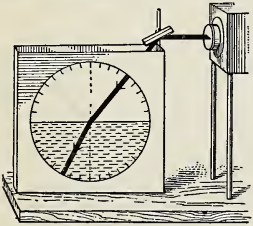

Fig. 368.—Arrangement to show refraction of light as it passes from air to water.

Observe now that while a part of the light is reflected from the surface of the water a large portion enters the water but abruptly changes its direction. If the light falls perpendicularly upon the surface, there will be no change of direction; but in all other cases there will be. This change of direction when light passes from one medium to another is called *refraction.*

Next, let us use the optical disc. Fasten the semi-circular glass plate to the disc as shown in Fig. 369. Project a 'ray' of light against the flat face of the plate. Some of the light will be reflected but a large portion will enter the glass and travel in a new direction. This is the refracted ray.

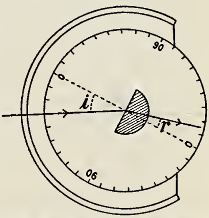

Fig. 369.—Refraction with the optical disc.

The angle between the incident ray

and the normal (marked i) is the angle of incidence, and that between the refracted ray and the normal (marked r) is the angle of refraction.

Turn the disc so that the angle i may take the values 20°, 40°, 60°, and observe the corresponding values of r. They will be approximately 13°, 25°, 35°, respectively. As we shall see later, there is a definite relation between the corresponding values of i and r.

349. Experiments Illustrating Refraction. Place a coin PQ on the bottom of an opaque vessel (Fig. 370), and then move back until the coin is just hidden from the eye E by the side of the vessel. Let water be now poured into the vessel. The coin becomes visible again, appearing to be in the position $P'Q'$. The bottom of the vessel seems to have risen and the water looks shallower than it really is.

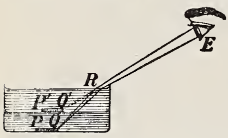

Fig. 370.—The bottom of the vessel appears raised up by refraction.

The reason for this is readily understood from the figure. Rays proceed from Q to R, and on leaving the water are bent away from the normal, ultimately entering the eye as though they came from Q'. Similarly rays from P will be refracted at the surface and will enter the eye as though they came from P'.


Fig. 371.—The stick appears broken at the surface of the water.

Another familiar illustration of refraction is the appearance of a stick—an oar, for example—when held obliquely in the water (Fig. 371). A pencil of light coming from any point on the stick, upon emergence from the water is refracted downwards and enters the eye as though it came from a point nearer the surface of the water. Thus the part of the stick immersed in the water appears lifted up.

350. Explanation of Refraction by Means of Waves. First, let us consider what might naturally happen when a battalion of soldiers passes from smooth ground to rough ploughed land. It is evident that the rate of marching over the rough land should be less than over the smooth. Let the rates be 3 and 4 miles an hour, respectively.

In the figure (Fig. 372) are shown the ranks of soldiers moving forward in the direction indicated by the arrows. The rank AB is just reaching the boundary between the smooth and the rough land, and the pace of the men at the end A is at once reduced. A short time later this rank reaches the position $a\ b$, part being on the rough and the rest still on the smooth ground. Next, it reaches the position $c\ d$, and then the whole rank reaches the position CD, entirely on the rough land. If now it proceeds in a direction at right angles to the rank, as shown by the arrows, it will move off in a direction quite different from that on the smooth ground. The succeeding ranks follow in the same manner, and the new direction of motion is AC.

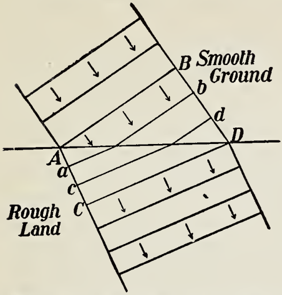

FIG. 372.—Illustrating how a change in direction of motion may be due to change in speed.

Now, it is clear that the space BD of smooth ground is marched over in the same time as the space AC of rough land, and as the rates are 4 miles and 3 miles an hour, respectively, we have

$$\frac{BD}{AC} = \frac{4}{3}.$$

We have used ranks of soldiers in the illustration but waves behave very similarly. In Fig. 373 is reproduced a photograph of waves on the surface of water. These waves were produced by attaching a piece of thin

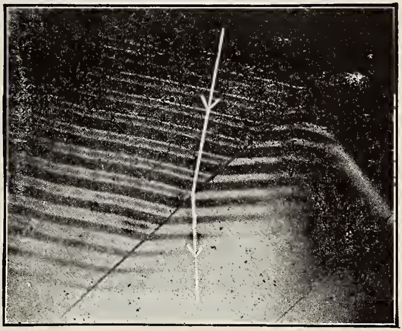

FIG. 373.—Plane waves on passing into shallower water are refracted as shown by the arrow. (Photograph by J. H. Vincent.)

glass to one prong of a tuning-fork and then vibrating it just touching the surface. The waves move forward in the direction shown by the arrow, but on reaching the shallower water over a piece of glass lying on the bottom of the vessel, their speed is diminished and the wave-fronts swerve around, thus abruptly changing the direction of propagation. (See § 167.)

351. The Laws of Refraction. We have found in § 348 that if the angles of incidence are 20°, 40°, 60°, the corresponding angles of refraction for the glass used (crown-glass) are 13°, 25°, 35°. Consider rays of light falling at O (Fig. 374) upon the surface of glass and draw AOB the normal to the surface at O. Make angles $COA = 20°$, $DOA = 40°$, $EOA = 60°$; also $cOB = 13°$, $dOB = 25°$ and $eOB = 35°$. Then Oc, Od, Oe are the refracted rays corresponding to CO, DO, EO. From C, D, E, c, d, e drop perpendiculars upon AB and measure the lengths of them. It will

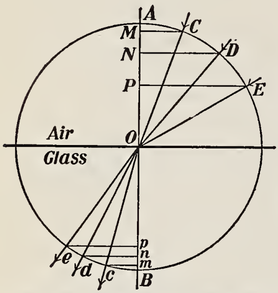

Fig. 374.—Angles of incidence and refraction.

be found that the ratios $\frac{CM}{cm}, \frac{DN}{dn}, \frac{EP}{ep}$ all have the same value, each being $= \frac{3}{2}$. This is called the *index of refraction* from air to glass. If we had used air and water (as in Fig. 368), the value of the index of refraction would have been $\frac{4}{3}$.

Next, consider the angle between the two lines AB, AC (Fig. 375). From C, any point in AC, draw CB perpendicular to AB. Then the ratio $\frac{CB}{CA}$ is defined to be the *sine* of the angle CAB.

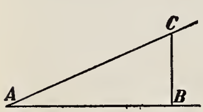

Fig. 375.—CB/AC is the sine of CAB

Coming back now to Fig. 374 we see

that $\frac{CM}{OC}$ = sine of angle COA and $\frac{cm}{Oc}$ = sine of angle cOB.

$$\text{Hence } \frac{\text{sine } COA}{\text{sine } cOB} = \frac{CM}{OC} \div \frac{cm}{Oc} = \frac{CM}{cm} = \text{index of refraction,}$$

$$\text{or } \frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}} = \text{index of refraction.}$$

We find, therefore, that *the sine of the angle of incidence divided by the sine of the angle of refraction is a constant quantity,* which is called the *index of refraction.*
This is the First Law of Refraction.

The Second Law of Refraction is:—*The incident ray, the refracted ray and the normal to the surface are in the same plane.*

Table of Indices of Refraction. The following table gives the values of the indices of refraction from air into various substances.

It must be noted, however, that the indices are not the same for lights of all colours, those for blue light being somewhat greater than those for red. The values given here are for yellow light, such as is obtained on burning sodium in a Bunsen or spirit flame.

Indices of Refraction

Crown-glass	1.51 to 1.56	Water	1.33
Flint-glass	1.61 to 1.79	Alcohol	1.36
Diamond	2.42 to 2.47	Olive-oil	1.48
Canada Balsam	1.53	Turpentine	1.47

It will be noticed that although alcohol, olive-oil and turpentine are lighter (that is, have smaller specific gravity) than water, they refract light more powerfully. They are said to be *optically denser* than water.

352. How to Draw the Refracted Ray. Suppose CO (Fig. 376) is a ray of light incident on the surface of water, at the point O. Draw the normal AO, and with O as centre describe a circle. Draw CM perpendicular to AO. Make OT three-quarters of CM, and from T draw a perpendicular to the surface, cutting the circle in c. Then Oc will be the refracted ray. If the lower medium were glass having an index of $\frac{3}{2}$, OT would be made two-thirds of CM.

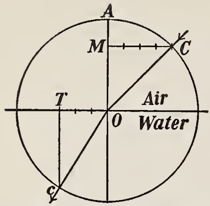

FIG. 376.—Method of drawing the refracted ray.

353. Relation between Velocity of Light and Index of Refraction. In Fig. 377 is shown the refraction of light-waves from air into water. The arrows show the direction in which the waves move before and after refraction. AB represents a wave just as it arrives at the surface of the water and CD shows it just when it is within the water. The angles i and r, of incidence and refraction, are marked and the values of the other angles are evident from the figure.

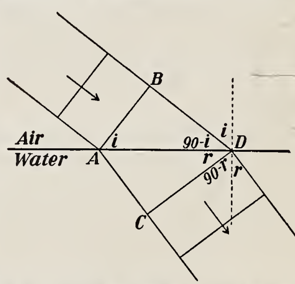

FIG. 377.—Showing passage of plane waves from air to water.

$$\text{Now} \quad \frac{\text{velocity in air}}{\text{velocity in water}} = \frac{BD}{AC} \text{ (as in § 350).}$$

$$\text{But} \quad \frac{BD}{AD} = \text{sine } BAD = \text{sine } i; \text{ and } \frac{AC}{AD} = \text{sine } CDA = \text{sine } r.$$

$$\text{Hence} \quad \frac{\text{sine } i}{\text{sine } r} = \frac{BD}{AD} \div \frac{AC}{AD} = \frac{BD}{AC} = \frac{\text{velocity in air}}{\text{velocity in water}} = \text{index of refraction.}$$

As the index for water is 4/3, light travels in air 4/3 as fast as in water

354. Refraction Through a Plate. A plate is a portion of a medium bounded by two parallel planes. In Fig. 378, $PQRS$ shows the course of a ray of light through a plate of glass. It is refracted on entering the plate and again on emerging from it. Since the normals at Q and R are parallel, the angles made with these by QR are equal. Each of them is marked r. Then, since the angles of incidence and refraction depend on the velocities of light in the two media, and if we send the light along SR, it will pass through by the course RQP, it is evident that the angle between SR and the normal at R is equal to that between PQ and the normal at Q. Each of these is marked i.

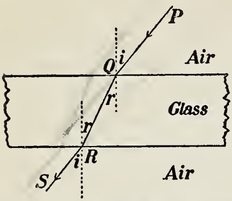

Fig. 378.—Showing the course of a ray of light through a glass plate.

It is clear, then, that the incident ray PQ is parallel to the emergent ray RS, and, therefore, that the direction of the ray is not changed by passing through the plate, though it is laterally displaced by an amount depending on the thickness of the plate. This can be easily illustrated by means of the optical disc.

355. Vision Through a Plate. Let P be an object placed behind a glass plate and seen by an eye E (Fig. 379). The pencil of light will be refracted as shown in the figure, RE, TF being parallel to PQ, PS, respectively. The object appears to be at P', nearer to the eye than P is.

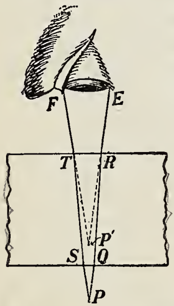

Fig. 379. — Showing why, when viewed through a glass plate, an object appears nearer.

This effect is well illustrated by laying a thick plate of glass over a printed page. It makes the print seem nearer the eye, and the plate appears thinner than it really is.

Exercise.— Draw the waves as they pass from P to the eye.

356. Total Reflection. Up to the present we have dealt mainly with the refraction of light from a medium such as air into one which is optically denser, such as water or glass. When we consider the light passing in the reverse direction, we come upon a peculiar phenomenon.

Let light spread out from the point P, under water (Fig. 380). The ray Pm, which falls perpendicularly upon the surface, emerges as mA, in the same line. The rays on each side are refracted as shown in the figure, but the ray PB upon refraction just skims along the surface. What becomes of a ray such as PC? It cannot emerge into the air, and so it is reflected back into the water. Moreover, since none of the light escapes into the air, it is *totally reflected.*

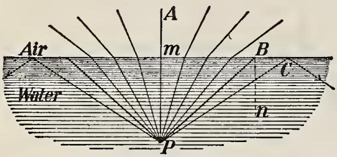

FIG. 380.—PB is the critical ray, and PBn (which is equal to BPm) is the critical angle for water and air.

It is evident that all rays beyond PB are totally reflected. Now the angle of incidence of the ray PB is PBn, which is equal to BPm. Hence if the angle of incidence of any ray is greater than PBn, it will suffer *total reflection.* This angle is called the CRITICAL ANGLE which may be defined thus:—

If a ray is travelling in any medium in such a direction that the emergent ray just grazes the surface of the medium, then the angle which the ray travelling in the (*denser*) *medium makes with the normal is called the critical angle.*

The values of the critical angles for some substances are approximately as follows:—

Water.......48½°	Crown-glass....40½°	Carbon Bisulphide..38°
Alcohol 47½	Flint-glass.....36½	Diamond..........24½

Total reflection can be illustrated by means of the optical disc.

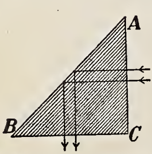

FIG. 381.—A total-reflection prism.

357. Total-reflection prisms. Let ABC (Fig. 381) be a glass prism with well-polished faces, the angles A and B each being 45°, and C therefore 90°. If light enters as shown in the figure, the angle of incidence on the face AB is 45°, which is greater than the critical angle. It will, therefore, be totally reflected and pass out as indicated. Another form of total-reflecting prism is shown in Fig. 382, in which the angle B is 135°, A and C each $67\frac{1}{2}$°. The course of the light is shown. Such arrangements are the most perfect reflectors known and are frequently used in optical instruments, for example, in the binocular (§ 392) and the prismatic compass (§ 413).

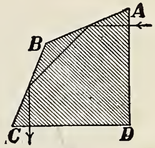

FIG. 382. — Another form of total-reflection prism.

This principle is also used in one form of the so-called 'Luxfer' prisms, two patterns of which are shown in Fig. 383. They are firmly fastened in iron frames which are let into the pavement. The sky-light enters from above, is reflected at the hypothenusal faces, and effectively illuminates the dark basement rooms.

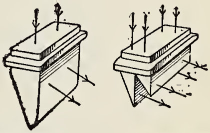
FIG. 383.—'Luxfer' prisms, useful in lighting basements.

358. Colladon's Fountain of Fire. Another beautiful illustration of total reflection is seen in the experiment known as Colladon's Fountain. A reservoir A (Fig. 384), about one foot in diameter and three feet high, is filled with water. Near the bottom is an opening B, about $\frac{1}{2}$ inch in diameter, from which the water spurts. A parallel beam of light from a lantern enters from behind, and by a lens C is converged to the opening from which the water escapes. The light enters the falling water, and, being incident at angles greater than the critical angle, it is totally reflected from side to side. The light imprisoned within the jet gives the water the appearance of liquid fire. Coloured glasses may be inserted at D, and beautify the effect.

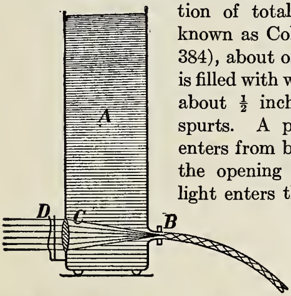

FIG. 384.—The 'fountain of fire.' The falling water seems to be on fire.

359. Refraction Through Prisms. A prism, as used in optics, is a wedge-shaped portion of a refracting substance contained between two plane faces. The angle between the faces is called the *refracting angle,* and the line in which the faces meet is the *edge of the prism.*

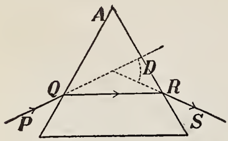

FIG. 385.—The path of light through a prism.

In Fig. 385 is shown a section of a prism, the refracting angle A of which is 60°, and $PQRS$ is a ray of light passing through it. The angle D between the original direction PQ and the final direction RS is the *angle of deviation.* The deviation is always *away from the edge of the prism.* When the light passes through the prism symmetrically, as in Fig. 385, in which the angle of emergence is equal to the angle of incidence, the deviation D is the smallest it can be, and the prism is said to be in the *position of minimum deviation.*

QUESTIONS AND PROBLEMS

1. If the index of refraction from air to diamond is 2.47, what is the index from diamond to air?

2. The index of refraction from air to water is 4/3, and from air to crown-glass is 3/2. If the velocity of light in air is 186,000 miles per second, find the velocity in water and in crown-glass; also the index of refraction from water to crown-glass.

3. Explain the wavy appearance seen above hot bricks or rocks.

4. A lighted candle is held in the beam of a projecting lantern. Explain the smoky appearance seen on the screen above the shadow of the candle.

5. In spearing fish one must strike lower than the apparent place of the fish. Draw a figure to explain why.

6. A strip of glass is laid over a line on a paper, (Fig. 386). When observed obliquely, the line appears broken. Explain why this is so.

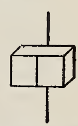
FIG. 386.—Why does the line appear broken?

7. A thick plate of glass is interposed obliquely between a lighted candle and the observer's eye. Will the apparent position of the candle be altered by the glass? Explain by means of a diagram.

8. The illumination of a room by daylight depends to a great extent on the amount of sky-light which can enter. Show why a plate of prism glass, having a section as shown in Fig. 387 placed in the upper portion of a window in a store on a narrow street is more effective in illuminating the store than ordinary plate-glass.

FIG. 387.—The plane face is on the outside.

9. How would you arrange a piece of prism glass (Fig. 387) to prevent light from an automobile headlight from being thrown upward into the eyes of the driver of another car?

10. Light passes from air into water with angles of incidence 30°, 45°, 60°. By means of carefully drawn figures show the refracted rays, and by means of a protractor measure the angles of refraction. (Index of refraction, 4/3.)

11. Make similar drawings and measurements for air into crown-glass (Index, 3/2).

12. Light passes through a 60° prism parallel to the base (as in Fig. 385). Find the angle of refraction at Q. Then draw the incident ray and also measure the angle of incidence (Index, 3/2).

13. If a thick rectangular piece of glass is placed on a printed page, the print can be read when the eye is directly above it, but if the page is viewed more and more obliquely, a point is reached when the print suddenly becomes invisible. Explain.

14. When an empty test-tube is thrust into water and placed in an inclined position, the immersed part, when viewed from above, appears as if filled with mercury. If the tube is now filled with water, the brilliant reflection disappears. Explain this phenomenon.

15. If you hold a glass of water with a spoon in it above the level of the eye and look upward at the under surface of the water, you are unable to see the part of the spoon above the water, and the surface of the water appears burnished like silver. Explain.

16. The critical angle of a substance is 41°. By means of a drawing determine the index of refraction.

17. Describe what a fish under water can see. (Take into account total reflection.)

REFERENCES FOR FURTHER INFORMATION

Wright, *Light*, Chapter 3. (Good experiments.)
Stewart and Satterly, *Text-Book of Light*, Chapter 6.
Edser, *Light for Students*, Chapter 3.
Glazebrook, *Light*, Chapter 4.

CHAPTER XXXVI

Lenses

360. Lenses. A lens is a portion of a transparent refracting medium bounded either by two curved surfaces or by one plane and one curved surface.

Almost without exception the medium used is glass, and the curved surfaces are portions of spheres.

Lenses may be divided into two classes:—(*a*) Convex or converging lenses, which are thicker at the centre than at the edge.

(*b*) Concave or diverging lenses, which are thinner at the centre than at the edge.

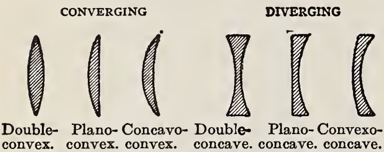

Fig. 388.—Lenses of different types.

In Fig. 388 are shown sections of different types of lenses. The concavo-convex lens is sometimes called a converging meniscus, and the convexo-concave a diverging meniscus. A meniscus is a crescent-shaped body.

361. Principal Axis. The *principal axis* is the straight line joining the centres of the spherical surfaces bounding the lens; or, if one surface is plane, it is the straight line drawn through the centre of the sphere and perpendicular to the plane surface.

362. Action of a Lens. The behaviour of the rays of light when they pass through a lens can be exhibited clearly by means of the optical disc. First, project the light through a double-convex lens, as in Fig. 389. It will be seen that the rays are bent towards the principal axis and converged to a point and then spread out. Next, use a double-concave lens, as in Fig. 390. In this case the rays diverge from the principal axis.

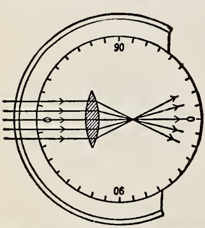

Fig. 389.—Action of a converging lens.

Similarly, we can show that all lenses which are thicker at the centre converge the rays of light while those which are thinner at the centre diverge the rays.

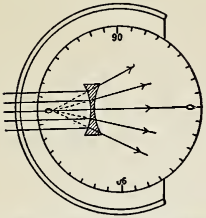

Fig. 390—Action of a diverging lens.

Consider the paths of the rays still further. Let rays parallel to the principal axis fall upon a convex lens (Fig. 391). That ray which passes along the principal axis meets the surfaces at right angles, and hence passes through without suffering any deviation. But all other rays are bent from their original paths, the deviation being greater as we approach the edge. The result is, the rays are converged approximately to a point F on the principal axis.

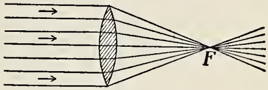

Fig. 391.—Parallel rays converged to the principal focus F.

This point is called the *principal focus*, and in the case shown, since the rays actually pass through the point, it is a *real* focus.

A parallel beam, after passing through a concave lens (Fig. 392) is spread out in such a way that the rays *appear* to come from F, which is the *principal focus* and which, in this case, is evidently *virtual.*

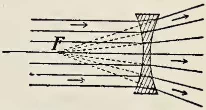

Fig. 392.—In a diverging lens the principal focus F is virtual.

The *focal length* is the distance from the principal focus to the lens, or, more accurately, to the centre of the lens.

363. Conjugate Foci. (*a*) **Converging Lens.** If the light is moving parallel to the principal axis and falls upon a convex lens, it is converged to the principal focus (Fig. 391). Next, let it emanate from a point P, on the principal axis (Fig. 393). The lens now con-

Fig. 393.—P and P' are conjugate foci.

verges it to the point P', also on the principal axis and farther from the lens than F.

Again, let us consider the direction of the light as reversed, that is, let it start from P' and pass through the lens. It is evident that it will now converge to P. Hence P and P' are two points such that light coming from one is converged by the lens to the other. Such pairs of points are called *conjugate foci*, as in the case of curved mirrors.

As P is taken nearer the lens, its conjugate focus P' moves farther from it. If P is at F, the principal focus, the rays leave the lens parallel to the principal axis (Fig. 394), and when P is closer to the lens than F (Fig. 395), the lens con-

Fig. 394.—Light emanating from the principal focus comes from the lens in parallel rays.

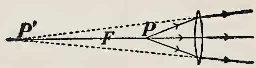

Fig. 395.—Here P', the focus conjugate to P is virtual.

verges the rays somewhat and they move off apparently from P' which in this case is a virtual focus.

(*b*) **Diverging Lens.** In the case of a diverging lens, if the incident light is parallel to the principal axis, it leaves the lens diverging from the principal focus F (Fig. 392). Let the

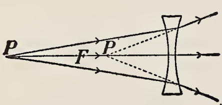

Fig. 396.–Conjugate foci in diverging lens.

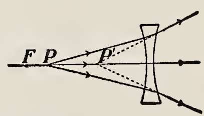

Fig. 397.—Conjugate foci in diverging lens.

light start from the point P (Figs. 396, 397). The light is made still more divergent by the lens, and, on emergence from it, appears to move off from P', which is conjugate to P and is virtual.

364. Explanation by Means of Waves. The theory that light consists of waves easily accounts for the action of lenses. Let us suppose that waves of light travelling through the air pass through a glass lens.

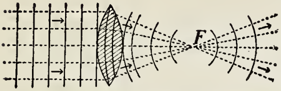

FIG. 398.—Plane waves made spherical by a converging lens.

In Fig. 398 plane waves (parallel rays) fall on the lens. Now their velocity in glass is only $\frac{2}{3}$ that in air, and that part of the waves which passes through the central part of the convex lens will be delayed behind that which traverses the lens near its edge, and the result is, the waves are concave on emerging from the lens. They continue moving onward, continually contracting, until they pass through F, the principal focus, and then they enlarge.

In Fig. 399 spherical waves spread out from P. On traversing the central portions they are held back by the thicker

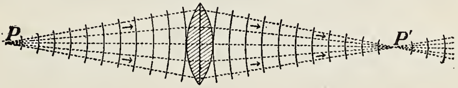

FIG. 399.—Waves expanding from P are changed by the lens into contracting spherical waves.

part of the lens, and on emerging they are concave, but they do not converge as rapidly as in the first case.

In Fig. 400 is shown the effect of a concave lens. The outer portions of the lens, being thicker than the central, retard the waves most, with the result that the convexity of the waves is increased, so that they move off having P' as their centre.

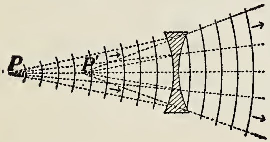

FIG. 400.—Waves going out from P are made more curved by the lens, and appear to have P' as their centre.

These results are further illustrated in a striking and beautiful manner by using an *air* lens in an '*atmosphere*' *of water*. Such a lens can be constructed without difficulty by cementing two 'watch-glasses' into a turned wooden or ebonite rim. In Fig. 401 is shown a double-concave lens immersed in water contained in a tank with plate-glass sides.

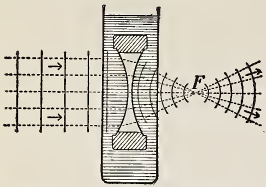

FIG. 401.—A concave air-lens in an atmosphere of water converges the light.

Plane waves from a lantern pass into the water, and on entering the lens the outer portions, since they travel in the air, rush forward ahead of the central part, thus rendering the waves concave and converging to a focus F. Thus a concave air-lens in water is converging; in a similar way it can be shown that a convex air-lens in a water atmosphere is diverging.

365. Experimental Illustrations; Determination of Focal Length. The relative positions of object and image can be easily exhibited experimentally, in a way similar to that used in the case of curved mirrors (§ 343).

I. Converging Lens. (1) First place a convex lens on the table, and place a candle as far from it as possible (for instance at the far end of the room). Then move a sheet of paper back and forth behind the lens until the small bright image is found. Examine it closely and you will see that it is inverted. The distance of the paper from the lens is its focal length (very nearly).

Hold the lens in sunlight and move the paper until the very bright image of the sun is seen. The focal length can easily be measured. Hold the lens in the sunlight for some time. The great heat of the mage will probably burn the paper. Such a lens is called a *burning-glass*.

(2) Now bring the candle slowly up toward the lens, at the same time moving the screen so as to keep the image on it. We find that the image gradually moves away from the lens, continually increasing in size as it does so.

At a certain place the image is of the same size as the object, but inverted. By measurement we find that each is twice the focal length of the lens from the lens.

(3) Bring the candle still nearer to the lens. The image retreats and is larger than the object, and when the candle is at the principal focus the image is at an infinite distance,—the rays leave the lens parallel to the principal axis.

(4) Finally, hold the candle between the principal focus and the lens; no real image is formed (Fig. 395), but on looking through the lens one sees a virtual enlarged image of the object. Its position can be found by the method of parallax.

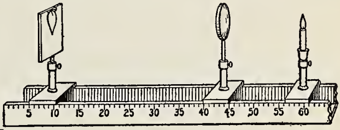

FIG. 402.—An optical bench, for studying object and image.

These results can be arranged in a table, as follows (f = focal length):—

IMAGES WITH CONVERGING LENS

Position of Object	Position of Image	Real or Virtual	Size	Erect or Inverted
Far from lens.	Beyond but near F.	Real.	Smaller	Inverted.
$2f$ from lens.	$2f$ from lens.	Real.	Same size.	Inverted.
Between $2f$ and f from lens.	Beyond $2f$ from lens.	Real.	Larger.	Inverted.
At F.	Rays parallel.			
Between F and lens.	On same side as object.	Virtual.	Larger.	Erect.

II. Diverging Lens (5) On using a concave lens we cannot obtain a real image of the object. If we view the candle through a concave lens, we always see an erect image smaller than the candle, apparently between the lens and the candle. It is always virtual (see Figs. 396, 397). The position of the image can be found by means of the method of parallax. Stand a hat-pin or knitting-needle behind the lens so that it can be seen above the lens. Move it until no relative motion between it and the image can be detected when the head is moved from side to side. If we use a distant object, the focal length can be found at once. (Another method is given in § 367.)

For making measurements of the distances of object and image from the lens, the most convenient arrangement is an optical bench, one form of which is shown in Fig. 402.

366. Focal Length and Power of a Lens. The *focal length* of a lens has been defined in § 362 as the distance from the principal focus to the centre of the lens.

The more strongly converging or diverging a lens is, the shorter is its focal length and the greater is its *power*. Hence if f is the focal length and P the power of a lens, we have

$$P = \frac{1}{f}$$

If a lens has a focal length of 1 metre, its power is said to be 1 dioptre; if the focal length is $\frac{1}{2}$ metre, the power is 2 dioptres; and so on. Conversely, let us suppose a lens to have a power of 2.5 dioptres, we must have

$$\text{Focal length, } f = \frac{1}{2.5}\text{ m.} = 40 \text{ cm.}$$

In prescribing spectacles the oculist usually states in dioptres the powers of the lenses required.

367. Combinations of Lenses. Since the action of a convex lens is opposite to that of a concave lens, as one converges while the other diverges the light, if we call one *positive*, we should call the other *negative*. Let us take the convex lens to be positive,

Consider two converging lenses of focal lengths f_1, f_2 and powers P_1, P_2.

$$\text{Then } P_1 = \frac{1}{f_1}, P_2 = \frac{1}{f_2}.$$

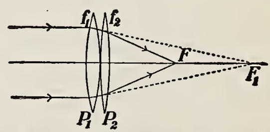

FIG. 403.—Combination of two converging lenses.

Let us put them close together (Fig. 403). Then the convergency produced by the first is increased by the second, and the power of the combination is $P_1 + P_2$. The focal length of the combination will be $\dfrac{1}{P_1 + P_2}$.

Next, consider the combined action of a convex and a concave lens (Fig. 404). Let the numerical values of the powers be P_1, P_2. Then, since the concave lens diverges, while the convex converges, the power of the combination is $P_1 - P_2$, and the focal length of the combination is

$$\frac{1}{P_1 - P_2}.$$

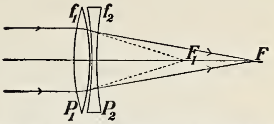

FIG. 404.—Combination of a converging and a diverging lens.

From this result we can deduce a method for finding the focal length of a concave lens.

First, find the focal length of a convex lens; let it be f_1. Then place the concave lens beside the convex one and find the focal length of the combination; let it be f. Then if f_2 is the focal length of the concave lens, we have

$$\frac{1}{f_1} = P_1, \text{ the power of the convex lens,}$$

$$\frac{1}{f_2} = P_2, \text{ the power of the concave lens (numerically),}$$

$$\frac{1}{f} = P, \text{ the power of the combination.}$$

$$\text{Now } P = P_1 - P_2,$$

$$\text{that is, } \frac{1}{f} = \frac{1}{f_1} - \frac{1}{f_2} \text{ and therefore } \frac{1}{f_2} = \frac{1}{f_1} - \frac{1}{f}.$$

In order to use this method the convex lens should be considerably more powerful than the concave one.

For example, let the focal length of the convex lens be 20 cm., and that of the combination be 60 cm.

Then $1/f_2 = 1/20 - 1/60 = 1/30$ and $f_2 = 30$ cm.

368. How to Locate the Image. Let PQ (Fig. 405) be an object placed before a convex lens A. The position of the image can be very easily located in the following way.

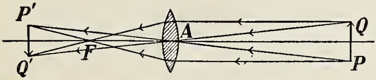

FIG. 405.—Showing how to locate the image of PQ.

From Q draw a ray parallel to the principal axis; on emerging from the lens it will pass through F, the principal focus.

Again, the ray QA which passes through the centre of the lens is not changed in direction. Let it meet the former ray in Q'. Then Q' will be the point on the image corresponding to Q on the object. From P draw a ray parallel to the principal axis; after passing through the lens it will go through F. Also draw the ray PA. It will meet the former ray in P', which will be the image of P.

The position of Q' can always be located by drawing two rays whose paths are known.

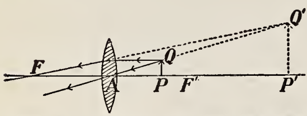

FIG. 406.—How to draw the image when the object is between the lens and the principal focus.

In a camera the lens is converging as in Fig. 405, and the image $P'Q'$ is produced on the film.

In Fig. 406 is shown the case in which the object is between the lens and the principal focus. The rays drawn parallel to the axis and through the centre of the lens do not meet after passing through the lens, but on producing them backwards they intersect at Q'. $Q'P'$ is the image of QP. It is virtual, erect and larger than the object. The image in a simple microscope is produced thus.

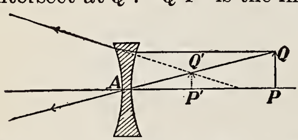

FIG. 407.—How to draw the image in a concave lens.

For a concave lens we have the construction shown in Fig. 407. The image is virtual, erect and smaller than the object.

Relation between Distances of Object and Image. There is a simple formula connecting the distances of object and image from the lens.

Consider a concave lens and let the light be coming from right to left (Fig. 407). Let $AP = p$, $AP' = p'$, and $AF = f$. (Note that in this case P, P' and F are all to the right of the lens.) Then it can be shown that $1/p' - 1/p = 1/f$.*

This formula holds for all positions of object and image and also for a convex lens, if we denote all lengths measured to the right of A "+" and those measured to the left "−".

*A proof of this formula will be found in an appendix to the *Laboratory Manual* designed to accompany this work.

369. Magnification. On examining Figs. 405-7, it will be seen that the triangles QAP, $Q'AP'$ are similar, and as before (§ 345), calling the ratio of the length of the image to that of the object the *magnification*, we have

$$\text{Magnification} = \frac{\text{P}'\text{Q}'}{\text{P Q}} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}.$$

370. Vision Through a Lens. In § 368 is explained a method of finding the position of an image produced by a lens, but it should be remembered that this is simply a geometrical construction and that the rays shown there are usually not those by which the eye sees the image. Let us draw the rays which actually enter the eye.

In Fig. 408 $P'Q'$ is the (real) image of PQ, and E is the eye. From Q' draw rays to fill the pupil of the eye. Then produce these backwards to meet the lens and finally join them to Q. Thus we obtain the pencil by which Q is seen.

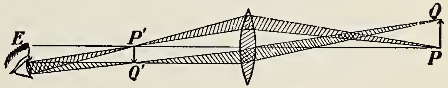

FIG. 408.—Showing the rays by which the eye sees the image of an object.

In the same way we trace the light from P to the eye.

In Fig. 409 $P'Q'$ is virtual, but the construction is the same as before. The student should draw other cases. The method is similar to that explained for curved mirrors (§ 346).

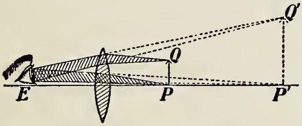

FIG. 409.—The rays reach the eye by the paths shown.

Exercise.—Draw the rays by which the eye sees an object viewed through a concave lens.

QUESTIONS AND PROBLEMS

1. What is meant by the *focal length* and the *power* of a lens? What is the relation between these quantities?

2. If a convex lens of focal length 10 cm. is placed close to a concave lens of focal length 40 cm., what will be the focal length of the combination?

3. An oculist prescribed spectacles with powers of +2.00 dioptres for the right and +3.00 dioptres for the left eye. Find the focal lengths of these lenses.

4. In another case the prescription was −4.50 for the right and −2.50 for the left eye. Find the focal lengths. (The minus sign means that the lenses were negative or diverging—for short-sightedness.)

5. The focal length of a telescope is 6 ft. The lens is composed of two separate lenses, one converging, the other diverging. The focal length of the former is 30 in.; find that of the latter.

6. In a compound lens of focal length 12 cm. there are three components placed close together, namely, two convex lenses of focal lengths 6 and 8 cm., respectively, and a concave lens between them. What is the focal length of the concave lens?

7. A lens A gives a sharp image of a distant object on a paper 10 cm. behind the lens. A second lens B is placed immediately in front of A and the image is now found 30 cm. behind A. Show in the case of each lens whether it is converging or diverging, and give its focal length.

8. Draw diagrams to show what is meant by *conjugate foci* in the case of a convex lens and also a concave lens.

9. By means of a drawing show the action of a double-convex air lens surrounded by water upon the waves of light as they pass through it.

10. A candle is placed 30 cm. from a double-concave lens of focal length 20 cm. By a drawing show the position and nature of the image; also calculate its distance from the lens and its size.

For the drawing see Fig. 407. For the calculations use the formula $1/p' - 1/p = 1/f$ (§ 368). In this case $p = +30, f = +20$, and p' comes out + 12, that is, the image is 12 cm. from the lens (on the right; see § 368).

The magnification $= p'/p = 12/30 = 2/5$, that is, the image is 2/5th the size of the candle.

11. An object is placed successively at distances 20, 40, 60 cm. from a convex lens of focal length 30 cm. Draw three diagrams to show the position, nature and size of the image in each case. Also calculate its distance from the lens and its size in each case.

For the drawings see Figs. 405, 406. For the calculations use the same formulas as in the preceding problem, being careful about the signs of the quantities.

(1) $p = +20, f = -30$; hence $p' = +60$, that is, the image is 60 cm. to the right of the lens. Magnification $= 60/20 = 3$.

(2) $p = +40, f = -30$; hence $p' = -120$, that is, the image is 120 cm. to the left of the lens. Mag. $= 120/40 = 3$.

(3) $p = +60, f = -30$; hence $p' = -60$, that is, the image is 60 cm. to the left of the lens. Mag. = 60/60 = 1.

12. A candle is placed 20 cm. from (*a*) a convex lens of focal length 10 cm., (*b*) a concave lens of focal length 30 cm. Draw a diagram for each case showing how and where the image is produced. Also calculate its position and magnification.

13. A gas flame is at a distance of 6 ft. from a wall. Where must a converging lens of 1 ft. focal length be placed in order to give a distinct image of the flame on the wall? What is the magnification?

14. A picture on a lantern slide is 2 inches wide and you wish to have an image of it 4 ft. wide on a screen 20 feet from the lantern. What will be the focal length of the lens projecting it?

Here p' and the magnitude are given, from which p can be found. Then use the formula, taking care of the signs.

15. An object 1 in. high is placed 1 ft. from a converging lens of 10 in. focal length; find the position, nature and size of the image.

16. The image of an object 20 cm. from a converging lens is magnified 4 times; what is the focal length of the lens?

17. A candle is 2 metres from a screen. Show that there are two positions between the candle and the screen where a convex lens may be placed to throw an image on the screen. If one of these positions is 40 cm. from the candle, what is the focal length of the lens? What is the magnification in the two positions of the lens?

18. Sometimes air-bubbles gather under ice and with further cold weather a new layer of ice forms under the bubbles. If a piece of ice is cut out and a vertical section made, it is sometimes found to be like Fig. 410. In the lower surface of the second layer, under the bubble, a hollow like an inverted saucer is found. Thoreau, in *Walden, or Life in the Woods*, says, "the bubbles themselves within the ice operate as burning-glasses to melt the ice beneath." Is this possible? (See § 364.)

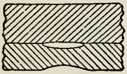

Fig. 410.—Bubble of air between layers of ice.

19. If you wanted to get a powerful burning-glass, what would you ask for (*a*) as to diameter, (*b*) as to focal length?

20. A projection lantern throws upon a screen an image of a $3\frac{1}{4} \times 4$ in. lantern slide placed 10 in. from the projecting lens. If the screen is 30 ft. from the lens, what is the size of the image.

REFERENCES FOR FURTHER INFORMATION

Stewart and Satterly, *Text-Book of Light*, Chapter 7.
Glazebrook, *Light*, Chapter 6.
Edser, *Light for Students*, Chapter 3.

CHAPTER XXXVII

Dispersion, Colour, the Spectrum, Spectrum Analysis

371. Newton's Experiment. About 1668 Newton made a famous experiment (Fig. 411). He admitted sunlight through a hole in a window-shutter, and placed a glass prism in the path of the beam. On the opposite wall, $18\frac{1}{2}$ feet from the prism, he observed an oblong image, which had parallel sides and semi-circular ends, $2\frac{1}{8}$ inches wide and $10\frac{1}{4}$ inches long. That end of the image farthest from the original direction of the light was violet, the other end red.

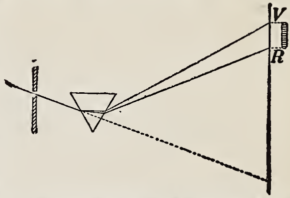

Fig. 411.—Light enters through a hole in the window-shutter, passes through a prism and is received on the opposite wall.

This image Newton called the *spectrum*. He thought he could recognize *seven* distinct colours, which he named in order:—red, orange, yellow, green, blue, indigo, violet.

It should be noted, however, that there are not seven separate coloured bands with definitely marked dividing lines between them. The adjoining colours blend into each other, and it is impossible to say where one ends and the next begins. Very often indigo is omitted from the list of colours, as not being distinct from blue and violet.

From Newton's experiment we conclude:—

(1) That white light is not simple but composite, and includes constituents of many colours.

(2) That these colours may be separated by passing the light through a prism.

(3) That lights which differ in colour differ also in degrees of refrangibility, violet being refracted most and red least.

The separation, or *spreading out*, of the constituents of a beam of light is called *dispersion*.

It will now be understood why, in § 351, when giving the indices of refraction for various substances, it was necessary to specify to what colour the values referred.

372. A Pure Spectrum. It is often inconvenient to use sunlight for this experiment, and we may substitute for it the light from a projecting lantern.

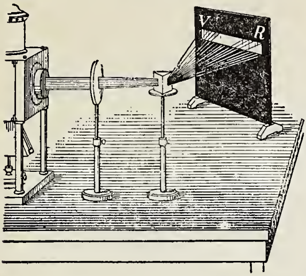

FIG. 412.—Showing how to produce a pure spectrum.

A suitable arrangement is illustrated in Fig. 412. The light emerges from a narrow vertical slit in the nozzle of the lantern, and then passes through a converging lens, so placed that an image of the slit is produced as far away as is the screen on which we wish to have the spectrum. Then a prism is placed in the path, and the spectrum appears on the screen.

The spectrum thus produced is *purer* than that obtained by Newton's simple method. Imagine the round hole used by Newton to be divided up into narrow strips parallel to the edge of the prism. Each strip will produce a spectrum of its own, but the successive spectra overlap, and hence the colour produced at any place is a mixture of adjacent spectral colours. Thus, to obtain a pure spectrum, that is, one in which the colours are not mixtures of several colours, we require a narrow slit as our source. In addition, the lens must be used to focus the image of the slit on the screen, and the prism should be placed in the position of minimum deviation (§ 359).

373. Colour of Natural Objects; Absorption. Let us produce the spectrum on the screen by means of the projecting lamp; we obtain all the colours as in *a*, Fig. 413. Next, place over the slit a red glass; the beam now transmitted consists mainly of red light, a little orange perhaps being present (*b*, Fig. 413). The glass does

FIG. 413.—A red glass transmits only red and some orange.

not owe its colour to the introduction of anything into the spectrum which did not previously exist there, but simply because it absorbs or suppresses all but the red and a little orange. We obtain similar results with green, yellow, or other colours. It is to be noted, however, that scarcely any of the transmitted colours are pure. Several colours will usually be found present, the predominating one giving its colour to the glass.

Next, hold a bit of red paper or ribbon in different portions of the spectrum. In the red it appears of its natural colour, but in every other portion it looks black. This tells us that a *red* object appears red because it absorbs the light of all other colours, reflecting or scattering only the red. In order to produce this absorption and scattering, however, the light must penetrate some distance into the object; it is not a simple surface effect. Similarly with green, or blue, or violet ribbons; but, as in the case of the coloured glass, the colours will usually be far from pure. Thus a blue ribbon will ordinarily reflect some of the violet and the green, though it will probably appear quite black in the red light.

Let us consider for a moment what happens when sunlight falls on various natural objects. The rose and the poppy appear red because they reflect mainly red light, absorbing the more refrangible colours of the spectrum. Leaves and grass appear green because they contain a green colouring matter (chlorophyll), which is able largely to absorb the red, blue and violet, the sum of the remainder being a somewhat yellowish green. A lily appears white because it reflects all the component colours of white light. When illuminated by red light it appears red; by blue, blue.

A striking way to exhibit this absorption effect is by using a strong sodium flame in a well-darkened room. This light is of a pure yellow, and bodies of all other colours appear black. The flesh tints are entirely absent from the face and the hands, which, on this account, present a ghastly appearance.

We see, then, that the colour which a body exhibits depends not only on the nature of the body itself, but also upon the nature of the light by which it is seen.

At sunrise and sunset the sun and the bright clouds near it take on gorgeous red and golden tints. These are due chiefly to absorption, not to refraction. At such times the sun's rays, in order to reach us, have to traverse a greater thickness of the earth's atmosphere than they do when the sun is overhead and the shorter light-waves, which form the blue end of the spectrum, are more absorbed than the red and yellow, which tints therefore predominate.

In § 175 reference was made to the stupendous volcanic eruption at Krakatoa in 1883. For many weeks after this the atmosphere was filled with dust, and sunsets of extraordinary magnificence were observed over the world. Somewhat similar absorption effects are produced in the neighbourhood of great forest fires, the ashes from which are conveyed by winds over considerable areas.

374. Recomposition of White Light. We have considered the decomposition of white light into its constituents; let us now explain several ways of performing the reverse operation of recombining the various spectrum colours in order to obtain white light.

(1) If two similar prisms be placed as shown in Fig. 414, the second prism simply reverses the action of the first and restores white light. The two prisms, indeed, act like a thick plate (§ 354).

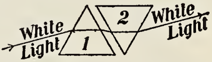

FIG. 414.—The second prism counteracts the first.

(2) By means of a large convex lens, preferably a cylindrical one (a tall beaker filled with water answers well), the light dispersed by the prism may be converged and united again. The image, when properly focussed, will be white.

(3) Next, we may allow the dispersed light to fall upon several small plane mirrors, and these, if adjusted properly, will reflect the various colours to one place on the screen, which then appears white (Fig. 415).

(4) In place of the several small mirrors we may advantageously use a single strip of thin plate-glass mirror, say

2 feet long by 4 inches wide. First, hold this in the path of the dispersed light so as to reflect it upon the opposite wall of the room. Then, by taking hold of the two ends of the strip, gently bend it until it becomes concave enough to converge the various coloured rays to a spot on the screen.

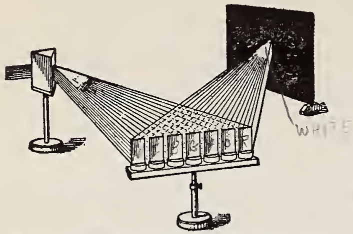

Fig. 415.—The light after passing through the prism falls on several small mirrors which reflect it to one place on the screen.

(5) In all the foregoing cases the coloured lights are mixed together outside the eye. Each colour gives rise to a colour-sensation, and a method will now be explained whereby the various colour-sensations are combined within the eye. The most convenient method is by means of Newton's disc, which consists of a circular disc of cardboard on which are pasted sectors of coloured paper, the tints and the sizes of the sectors being chosen so as to correspond as nearly as possible to the coloured bands of the spectrum.

Now put the disc on a whirling machine (Fig. 416) and set it in rapid rotation. It appears white, or whitish-gray. This is explained as follows:—

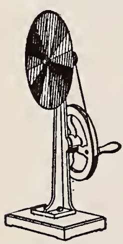

Fig. 416.—Newton's disc on a rotating machine.

Luminous impressions on the retina do not vanish instantly when the source which excites the sensation is removed. The average duration of the impression is $\frac{1}{10}$ second, but it varies with different people and with the intensity of the impression. If one looks closely at an incandescent electric lamp for some time, and then closes his eyes, the impression will stay for some time, perhaps for a minute. With an intense light it will last longer still.

If a live coal on the end of a stick is whirled about, it appears as a luminous circle; and the bright streak in the sky produced by a "shooting star" or by a rising rocket is due to this persistence of luminous impressions. In the same way, we cannot detect the individual spokes of a rapidly rotating wheel, but if illuminated by an electric spark, we see them distinctly. The duration of the spark is so short that the wheel does not move appreciably while it is illuminated.

In the familiar "moving pictures" the intervals between the successive pictures are about $\frac{1}{40}$ second, and the continuity of the motion is perfect.

If then the disc is rotated with sufficient rapidity, the impression produced by one colour does not vanish before those produced by other colours are received on the same portion of retina. In this way the impressions from all colours are present on the retina at the same time, and they make the disc appear of a uniform whitish-gray. This gray is a mixture of white and black, no *colour* being present, and the stronger the light falling on the disc the more nearly does it approach pure white.

375. Complementary Colours. Let us cut out of black cardboard a disc of the shape shown in Fig. 417, and fasten it on the axis of the whirling machine over the Newton's disc so that it just hides the red sectors. Rotate it; the colours which are exposed produce a bluish-green. It is evident, then, that this colour and red when added together will give white. *Any two colours which by their union produce white light are called complementary.* From the way it was produced we know that this blue-green is not a pure colour, but the eye cannot distinguish it from a blue-green of the same tint chosen from a pure spectrum. By covering over other colours of the Newton's disc we can obtain other complementary pairs. A few of these pairs are given in the following table:—

Fig. 417.—Disc to put over Newton's disc to cut out any desired colour.

Complementary Colours

Red	Orange	Yellow	Green-yellow	Green
Bluish-green	Green-blue	Blue	Violet	Purple

In Fig. 418 these are arranged about a circle. Note that the complement of green is purple, which is not a simple spectral colour but a compound of red and violet.

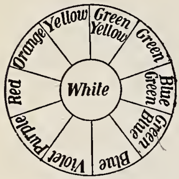

Fig. 418.—The radially opposite colours are complementary.

376. Mixture of Pigments. On rotating a disc with yellow and blue sectors,* as indicated in Fig. 419, we obtain white. On the other hand, if we mix together yellow and blue pigments, we get a green pigment. Wherein is the difference? It arises from the fact that the mixing of coloured lights is a true *addition* of the separate effects, while in mixing pigments there is a *subtraction* or *absorption* of the constituents of the light which falls on them.

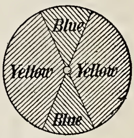

Fig. 419.—A complementary colour disc.

Ordinarily, blue paint absorbs the red and the yellow from the incident light, reflecting the blue and some of the adjacent colours, namely green and violet. Yellow paint absorbs all but the yellow and some orange and green. Hence, when yellow and blue paints are mixed, the only coloured constituent of the incident light which is not absorbed is green, and so the resulting effect is green.

Thus, mixing pigments and mixing colours are processes entirely unlike in nature, and we should not be surprised if the results produced are quite dissimilar. Indeed, the result obtained on mixing two pigments does not even suggest what will happen when two coloured lights of the same name are added together.

*We might use a single sector of blue and one of yellow, but the speed of rotation would then have to be doubled.

377. Achromatic Lenses. The focal length of a lens depends on the index of refraction of the material from which the lens is made; and as the index varies with the wave-length, or the colour, the focal length is not the same for all colours.

As the violet rays are refracted more than the red, the focal length for violet is shorter than for red. Thus, in Fig. 420, the violet rays come to a focus at V while the red converge to R, the foci for the other colours lying between V and R. A screen held at A will show a circular patch of light edged with red, while if at B, it will show a patch edged with violet. This inability to converge all the constituents of a beam of white light to a single point is a serious defect in single lenses, and is known as *chromatic aberration*.

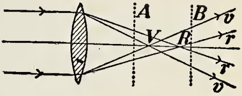

Fig. 420.—The foci for violet and red rays are quite separated.

Thus there is no single point to which all the light converges, and in determining the principal focus it is usual to find the focus for the yellow rays, which are the brightest.

Dollond, a London optician, discovered in 1757 a method of overcoming chromatic aberration. The arrangement is shown in Fig. 421. Flint-glass is more dispersive than crown-glass, and a crown-glass converging lens is combined with a flint-glass diverging lens of less power. The crown-glass lens would converge the red rays to R and the violet to V, while the flint-glass then diverges both of them so that they come together at F. Such compound lenses are said to be *achromatic*. They are used in all telescopes and microscopes.

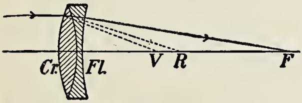

Fig. 421.—An achromatic combination of lenses.

378. The Rainbow. In the rainbow we have a solar spectrum on a grand scale. It is produced through the refraction and dispersion of sunlight by raindrops. In order to see it the observer must look towards falling rain, with the sun behind him and not more than 42° above the horizon. Frequently two bows are visible, the *primary* bow and the *secondary bow*. The former is violet on the inside and red on the outside; while in the secondary bow, which is larger and fainter, the order of the colours is reversed. Both bows are arcs of circles having a common centre (*C*, Fig. 422), which is on the line which passes through the sun and the eye of the observer.

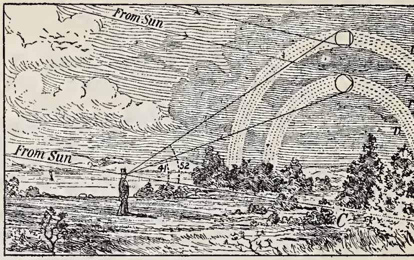

Fig. 422.—Illustrating how the rainbow is produced.

A line drawn from the eye to the primary bow makes an angle of about 41° with this line, while a line to the secondary bow makes an angle of about 52° with it.

In Fig. 422 is shown the relative positions of the sun, the raindrops and the observer. In Fig. 423 is shown the manner in which the sun's rays pass through the drops to form the primary bow. A ray of white light from the sun enters the drop at *A* and is broken up into its different colours, the violet rays being most refracted. The rays are all

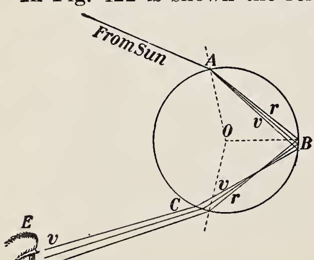

Fig. 423.—Showing how the light passes to form the primary bow.

reflected internally (though not *totally* reflected) at B; and at C they are refracted out into the air, forming a diverging pencil.

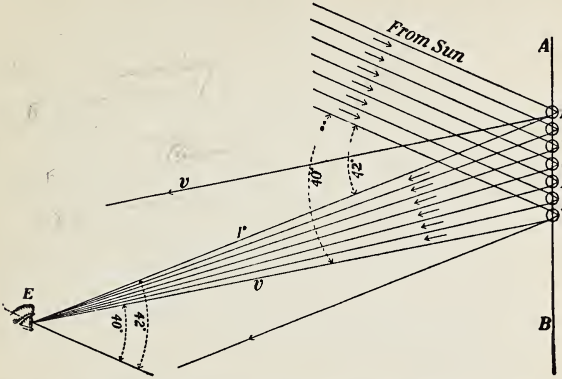

FIG. 424.—How the observer sees the different colours of the rainbow. Red comes from one drop, green from another, and so on.

Fig. 424 shows how the observer sees the different colours. Consider a series of drops arranged in the vertical line AB. The red rays from the highest drop come in greatest strength when it is in the position shown, that is, when a line from

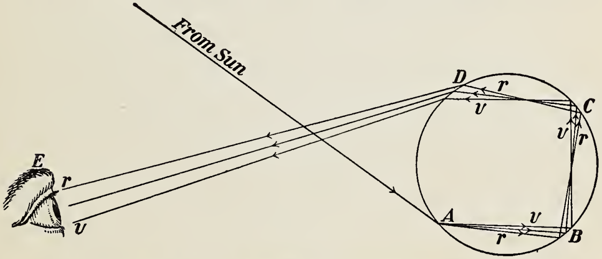

FIG. 425.—Showing how the light passes to form the secondary (outer) bow.

the eye to the drop makes an angle of 42° with a line from the sun to the drop (or to the eye). The violet rays from this particular drop pass over the observer's head, as shown in the figure. The violet rays get to the eye from the drop in the lowest position, that is, when a line from the eye to it makes an angle of 40° with the line from the sun. The red rays from this drop pass below the observer's eye. The other drops between these two supply the intermediate colours. Thus the red rays are highest in the sky, or on the outside of the bow. In order to produce the secondary bow the light enters the drop at A (Fig. 425) is reflected at B and C, and is refracted out at D.

379. The Spectroscope. This is an instrument especially designed to examine the spectra of various sources. A simple

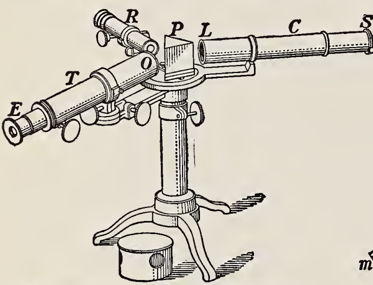

Fig. 426.—A single-prism spectroscope.

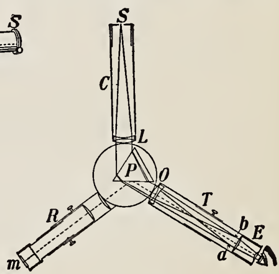

Fig. 427.—A horizontal section of a single-prism spectroscope.

form is illustrated in Fig. 426 and a sectional plan is given in Fig. 427. The tube C, known as the collimator, has a slit S at one end and a lens L at the other. The slit is at the focus of the lens, so that the light emerging from the tube is a parallel beam. It then passes through the prism P, and is received in a telescope T, the lens O of which focusses the spectrum in the plane ab. It is then viewed by the eye-piece E.

The light to be examined is placed before S. Usually a third tube R, is added. This has a small transparent scale m at one end and a lens at the other. A lamp is placed before the scale, and the light passes through the tube, is reflected from a face of the prism and then enters the telescope, an image of the scale being produced at *ab*, above the spectrum. By means of this scale any peculiarities of the spectrum of the light which is under examination can be localized or identified.

380. Direct-vision Spectroscope. If one uses three prisms, one of flint and two of crown-glass (Fig. 428), it is possible to get rid of the deviation of the middle rays of the spectrum while still dispersing the colours. Such a combination is used in pocket spectroscopes. The slit S admits the light and a convex lens converts the light into a parallel beam, which, after traversing the prisms, is seen by the eye at E. One tube can be slid over the other in order to focus the slit for the eye.

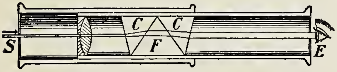

Fig. 428.—A direct-vision spectroscope.

381. Kinds of Spectra. By means of our spectroscope let us investigate the nature of the spectra given by various sources of light.

First, take an electric light. It gives a continuous coloured band, extending from red to violet without a break. A gas-flame, or an oil lamp gives a precisely similar spectrum. This is called a *continuous* spectrum.

Next, place a colourless Bunsen or alcohol flame before the slit, and in it burn some salt of sodium, chloride or carbonate of sodium, for instance. The flame is now bright yellow, and the spectrum shown in the spectroscope is a single bright yellow line.* Strontium nitrate produces a crimson flame, and the spectrum consists of several red and orange lines and a blue one. The salts of barium, potassium and other metals give similar results, each, however, with its own particular arrangement of lines. Such are a *discontinuous* or *bright-line* spectra.

*There are really *two* narrow lines very close together. which can be seen even with some pocket spectroscopes.

Again, place an electric lamp before the slit of the instrument, and then between it and the slit place a glass vessel containing a dilute solution of permanganate of potash. The spectrum is now continuous except that it is crossed by fine dark bands in the green. Using a dilute solution of human blood, we get a continuous spectrum except for well-marked dark bands in the yellow and the green. These are *absorption* spectra.

382. Spectrum Analysis. Now each element, when in the form of a vapour, has its own peculiar spectrum, the arrangement of the bright lines in no two spectra being exactly alike. Hence, by means of its spectrum the presence of a substance can be recognized. If several elements are present, their spectra will all be shown and the elements can be thus recognized. This method of detecting the presence of an element is known as *spectrum analysis*. It is an extremely sensitive method of analysis. Thus the presence of $\frac{1}{2000}$ mg. of barium, of $\frac{1}{600000}$ mg. of lithium, or of $\frac{1}{14000000}$ mg. of sodium is sufficient to show the lines characteristic of these elements.

383. The Solar Spectrum. On turning the spectroscope towards the sun, or reflecting sunlight into it, we find that the spectrum of sunlight consists of a bright band crossed by many dark lines. Fraunhofer* studied these and named the chief lines *A*, *a*, *B*, *C*, *D*, *E*, *b*, *F*, *G*, *H*, (Fig. 429) but they are all known as *Fraunhofer's lines*. They always have the same position in the spectrum.

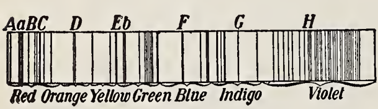

Fig. 429.—Showing some of the 'dark lines' in the spectrum of sunlight.

Photography has revealed at least 20,000 of these lines. For years attempts were made to find out the meaning of them, and in 1859 the mystery was solved by Kirchhoff.

*Fraunhofer (1787-1826) constructed his own prism and telescope, and engraved his own spectrum maps when he published his investigations in 1815.

In the orange-yellow of the solar spectrum is a prominent dark line—or rather a pair of lines very close together—named *D* by Fraunhofer. Now sodium vapour shows two fine bright yellow lines in exactly the same place in the spectrum. Surely there is some connection between sodium and the sun! The following experiment will suggest what it is.

First, place before the slit of the spectroscope an intense source of light, such as the arc light (Fig. 430). This gives a continuous spectrum with no dark lines at all. Now, while observing this, introduce between the arc light and the slit a Bunsen flame full of sodium vapour. This addition of yellow light we should naturally expect to make the yellow portion of our spectrum more intense, but that is not at all what happens. On the contrary, we see two *dark lines* in precisely the position where the bright sodium lines are produced. If one thrusts a screen between the arc light and the sodium flame the two bright lines due to sodium are seen.

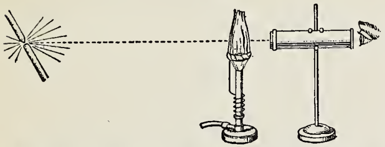

Fig. 430.—Light from the arc lamp passes through sodium vapour in the Bunsen flame and is then examined by the spectroscope. A dark band in the yellow is seen. The sodium vapour comes from the asbestos wick which has been soaked in a strong solution of common salt and is then held in the Bunsen flame.

Now, the inner portion of the sun corresponds to the arc light. It is intensely hot and would undoubtedly produce a continuous spectrum. The Bunsen flame corresponds to the atmosphere of the sun, in which, it would appear, there are vapours of sodium and other substances. These absorb some of the light from within and produce the dark lines.

It has been shown that sodium, iron, calcium, hydrogen, silver, titanium and about 30 more elements with which we are acquainted certainly exist in the sun's atmosphere. Others will probably be recognized in coming years. In 1868, observations showed the presence in the sun of a gas which was given the name *helium,* which means solar substance, and in 1895 the chemist, Ramsay, discovered it on the earth. By means of the spectroscope the astronomer has also been able to show that the stars of space are composed of substances which we have on the earth. He has also measured the speeds with which they move and has shown that our whole solar system is moving through space almost towards the bright star Vega at the rate of 12 miles per second.

QUESTIONS AND PROBLEMS

1. A ribbon purchased in daylight appeared blue, but when seen by gas-light it looked greenish. Explain this.

2. Why is it hard to distinguish between navy blue and black by candle-light.

3. One piece of glass appears dark red and another dark green. On holding them together you cannot see through them at all. Why is this?

4. Light enters a room through a red glass; what colour will a blue dress appear in it?

5. Sunlight comes through a vertical crack in the wall of a darkened room and an observer looks at the crack through a prism with its edge vertical. Draw a horizontal section through the prism and crack showing the path of the rays from the crack to the eye of the observer, and also where the crack appears to him. Mark the colours of the image.

6. Explain from the physical point of view why a rose is red, a lily is white and charcoal is black.

7. Where would you look for a rainbow in the evening? At what time can one see the longest bow? Under what circumstances could one see the bow as a complete circle?

8. What is meant by *chromatic aberration?* Explain how a combination of two lenses can get rid of it.

9. An achromatic lens is composed of a converging lens of focal length 10 cm, and a diverging lens of focal length 15 cm. What is the focal length of the combination? (§ 367.)

10. On observing the spectrum of sodium vapour in a spectroscope two fine lines are seen close together. What will be the effect of widening the slit?

REFERENCES FOR FURTHER INFORMATION

Stewart and Satterly, *Text-Book of Light*, Chapter 8.
Glazebrook, *Light*, Chapter 9.
White, *A Handbook of Physics*, Chapter 44.
Edser, *Light for Students*, Chapter 4.
Newall, *The Spectroscope and its Work* (Elementary and good).

CHAPTER XXXVIII

Optical Instruments

384. The Photographic Camera. The pin-hole camera was described in § 314. This would be quite satisfactory for taking photographs except for the fact that, as the pin-hole is very small, little light can get through it and so the time of exposure is long. This serious defect is overcome by making the hole larger and putting in it a converging lens. The greater the aperture of this lens, provided the focal length is not increased, the shorter is the exposure required.

In Fig. 431 is illustrated an ordinary camera. In the tube *A* is the lens, and at the other end of the apparatus is a frame *C* containing a piece of ground glass. By means of the bellows *B* this is moved back and forth until the scene to be photographed is sharply focussed on the ground glass. Then a holder containing a sensitive plate or film is inserted in the place of the frame *C*, the sensitized surface taking exactly the position previously occupied by the ground surface of the glass.

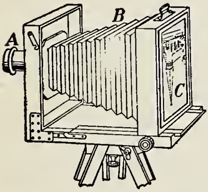

Fig. 431.—A photographic camera.

The exposure is then made, that is, light is admitted through the lens to the sensitive plate, after which, in a dark room, the plate is removed from its holder, developed and fixed.

Only in the cheapest cameras is a single convex lens used, a combination of two lenses being ordinarily found. If we wish to secure a picture which is perfectly focussed all over the plate, and to have a very short exposure, we must use one of the modern objectives, which have been brought to a high degree of efficiency. A section of one of these is shown in Fig. 432, in which it will be seen there are four

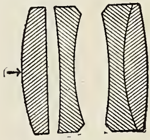
Fig. 432.—Section of a Zeiss "Tessar" photographic objective. The light enters in the direction shown by the arrow.

separate lenses combined. Others contain even more lenses, and as these are made from special kinds of glass and have surfaces with specially computed curvatures, they are expensive. Great effort and marvellous ingenuity have been expended in producing the extremely compact and efficient cameras now so familiar to us.

385. The Eye. The eye behaves much like a camera and is the most wonderful of all optical instruments. It is almost spherical in shape (Fig. 433.) The horny outer covering, the "white of the eye," is called the *sclerotic* coat. The front portion of this protrudes like a watch face and is called the *cornea,* Within the sclerotic is the *choroid* coat, and within this, again is, the *retina.*

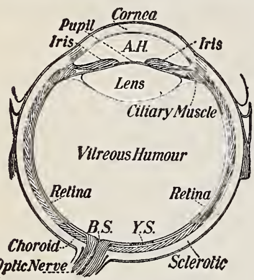

Fig. 433.—Horizontal section of a right eye. *A.H.*, aqueous humour; *Y.S.*, yellow spot; *B.S.*, blind spot.

The portion of the choroid coat visible through the cornea is called the *iris.* This forms an opaque circular diaphragm, which is variously coloured in different eyes. The aperture in it is called the *pupil,* and the size of the pupil alters involuntarily to suit the amount of light which enters the eye. When the light is feeble the pupil is large. On passing from darkness into a brilliantly lighted room the eye is at first dazzled, but the pupil soon contracts and keeps out the excessive supply of light.

Behind the pupil is the double-convex *crystalline lens.* By means of the muscles attached to the edge of the lens, the curvature of its faces, and hence its converging power, can be changed at will. The portion of the eye between the lens and the cornea is filled with a watery fluid called the *aqueous humour,* while between the lens and the retina is a transparent jelly-like substance called the *vitreous humour.* The *retina* is a semi-transparent net-work of nerve-fibres, formed by the spreading out of the termination of the optic nerve. It corresponds to the film in the camera.

386. Defects of the Eyes. A person possessing normal vision can see distinctly objects at all distances varying from 8 or 10 inches up to infinity. Light from all such is converged upon the retina (Fig. 434).

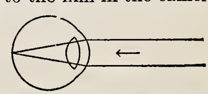

Fig. 434.—Normal eye receiving light from a distant object.

But, as we well know, there are many eyes with defects, the chief of which are *short-sightedness*, *long-sightedness*, and *astigmatism*.

A short-sighted eye cannot see objects at any considerable distance from the eye. The image of an object near at hand is produced on the retina, but the eye cannot accommodate itself for one farther off. In such a case the image is formed in front of the retina, (*a*, Fig. 435) and to the observer it appears blurred. In a short-sighted eye the lens is too strongly convergent, and in order to remedy this we must use spectacles producing the opposite effect, that is, having diverging lenses (*b*, Fig. 435).

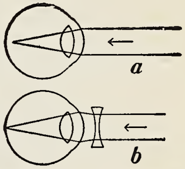

Fig. 435.—A short-sighted eye and its correction.

A long-sighted eye, in its passive condition, brings parallel rays of light to a focus behind the retina (*a*, Fig. 436). Such an eye can accommodate itself for distant objects, bringing the image forward to the retina; but for near objects its power of accommodation is not sufficient. In this case the crystalline lens is not converging enough, and in order to assist it spectacles with converging lenses should be used (*b*, Fig. 436). As a person grows older there is usually a loss of the power of accommodation, and the eye becomes long-sighted, requiring the use of converging spectacles.

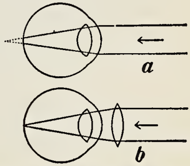

Fig. 436.—A long-sighted eye and its correction.

The defect known as astigmatism is due to a lack of symmetry in the surfaces of the cornea and the lens, but principally in the former. Ordinarily these are spherical, but sometimes the curvature is greater in one plane than in others. If a diagram, as shown in Fig. 437, be drawn about one foot in diameter and viewed from a distance of about 15 feet, an astigmatic eye will see some of the radii distinctly, while those at right angles will be blurred. In most cases the vertical section of the cornea of an astigmatic eye is more curved than a horizontal section. The proper spectacles to use are those in which one surface of the lens is a part of a cylinder instead of a sphere.

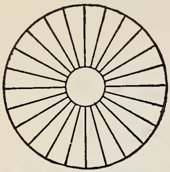

Fig. 437.—Diagram for testing for astigmatism.

387. The Projection Lantern. In Fig. 438 is shown a projection lantern and in Fig. 439 is a vertical section of it.

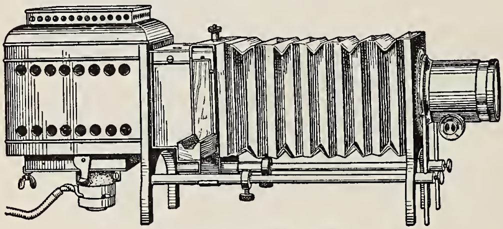

Fig. 438.—A projection lantern.

Its two essential parts are the source of light *A* and the porjection lens, or set of lenses, *D*.

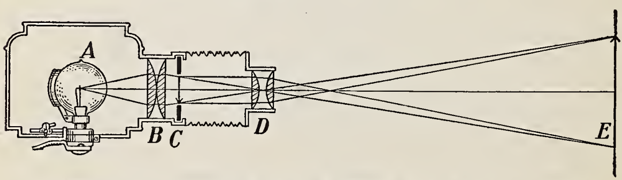

Fig. 439.—Vertical section of a projection lantern, showing how the image is produced on the screen.

The source should be as intense as possible. (Why?) In the figure it is an incandescent electric lamp, but an arc lamp, an acetylene jet or a strong oil lamp may be used. The light diverging from the source is directed by means of the so-called *condensing lenses B* upon the object *C* which we wish to exhibit on the screen *E*. This object is usually a photograph on glass, and is known as a lantern slide.

In a tube is *D*, the projecting lens. By moving this nearer the slide or farther from it a real and much enlarged image of the picture on the slide is produced on the screen. The slide and the screen are conjugate foci (§§363,368). As the image on the screen is erect, and since the projecting lens inverts the image, it is evident that the slide *C* must be placed in its carrier with the picture on it upside down.

Question.— Where must the slide be placed with reference to the focus of the projecting lens? (See § 365).

388. The Simple Microscope or Magnifying Glass. In order to see an object well, that is, to recognize details of it, we bring it near to the eye, but, as we know, when it gets within a certain distance the image is blurred. By placing a single convex lens before the eye (which is equivalent to making the eye short-sighted) we are enabled to bring the object quite close to the eye and still have the image of it on the retina distinct.

How this is done is shown in Fig. 440. (See also Figs. 406 and 409.) The object *PQ* is placed within the principal focus *F*. The image *pq* is virtual, erect and enlarged (see Fig. 406). The lens is moved back and forth until the image is focussed, in which case the image is at the least distance of distinct vision from the eye, about 10 inches. The magnification is greatest when the eye is close to the lens.

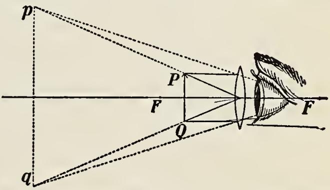

Fig. 440.—Illustrating the action of the simple microscope.

The rule for finding the magnifying power of a lens is:—*Divide the least distance of distinct vision* (10 *inches or* 25 *cm.*) *by the focal length of the lens.* The proof of this is too difficult to be given here.

For example, if the focal length is $\frac{1}{2}$-inch the magnifying power $= 10 \div \frac{1}{2} = 20$.

389. The Compound Microscope. For higher magnifications we must use a combination of convex lenses known

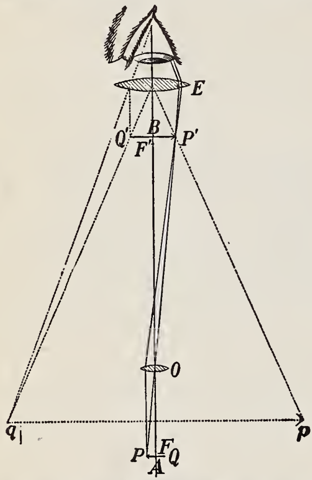

Fig. 441.—Diagram illustrating the compound microscope.

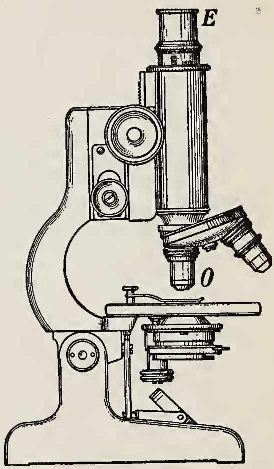

Fig. 442.—A compound microscope.

as a compound microscope. In its simplest form it consists of two lenses, the objective and the eyepiece, the action of which is illustrated in Fig. 441.

The object PQ is placed at A, before the objective O and just beyond its principal focus. Thus a real enlarged inverted image $P'Q'$ is produced at B, and the eyepiece E is so placed that $P'Q'$ is just within its focal length. The eyepiece E then acts as a simple microscope magnifying $P'Q'$, It forms an enlarged virtual image *pq* at the distance of distinct vision from the eye. This distance is approximately the length of the microscope tube. A modern compound microscope is shown in Fig. 442.

390. The Astronomical Telescope. The arrangement of the lenses in the ordinary astronomical telescope is the same in principle as in the compound microscope. In the case of the latter, however, the object to be observed is near at hand and we can place it near the objective. In these circumstances a lens of short focal length is best to use.

But the objects viewed by the telescope are far away, and we must use an objective with as great a focal length as possible. The reason for this will be evident from Fig. 443. Let AC be a ray from the upper part of the object looked at, passing through the centre C of the objective O.

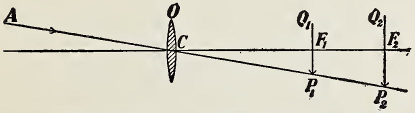

Fig. 443.—Showing why the objective of an astronomical telescope should have a long focus.

Now, the image of an object at a great distance is formed at the principal focus. If, then, F_1 is the principal focus, $P_1 Q_1$ is the image; and if F_2 is the principal focus, $P_2 Q_2$ is the image. It is clear that $P_2 Q_2$ is greater than $P_1 Q_1$, and indeed that the size varies directly as the focal length. Hence, the greater the focal length of the objective the larger will be the image produced by it.

Further, since the celestial bodies (except the sun) are very faint, the diameter of the objective should be large, in order to collect as much light from the body as possible.

A diagram illustrating the action of the telescope it given in Fig. 444. The objective forms the image at its principal focus B, that is, $OB = F$, its focal length. This is further magnified by the eyepiece E, which forms the image at *pq*.

B is just within the principal focus of the eyepiece, and so *OE*, the distance between objective and eyepiece, is approximately equal to $F + f$, the sum of their focal lengths.

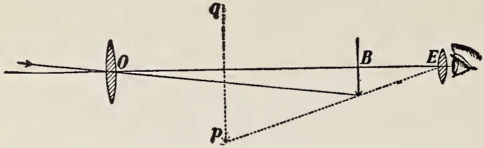

FIG. 444.—The principle of the astronomical telescope.

The magnification produced by the telescope is equal to F/f, though we cannot here deduce this formula.*

In the great telescope of the Lick Observatory the diameter of the objective is 36 inches and its focal length is 57 feet. If an eyepiece of focal length $\frac{1}{2}$-inch is used, the magnification is 1368. The diameter of the Yerkes telescope (belonging to the University of Chicago) is 40 inches and its focal length is 62 feet. A longitudinal section of a small astronomical telescope is given in Fig. 445.

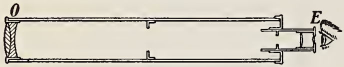

FIG. 445.—Section of a small astronomical telescope.

391. The Reflecting Telescope. In another type of astronomical telescope a concave mirror is used instead of a lens. (In Fig. 358 it is shown how a concave mirror produces the image of an object.) The mirror is usually made by hollowing out a glass disc and then depositing on it, by a chemical process, a layer of silver. When this is polished, it forms the finest of reflecting surfaces. In Fig. 446 is shown one method by which the image produced by the mirror is observed. The light rays from the distant object pass up the tube and upon striking the mirror

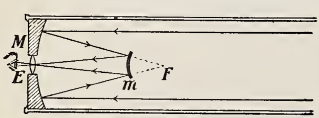

FIG. 446.—Illustrating the reflecting telescope.

*See Watson's *Physics*, p. 495, (1920 ed.); or Glazebrook's *Light*, p. 161.

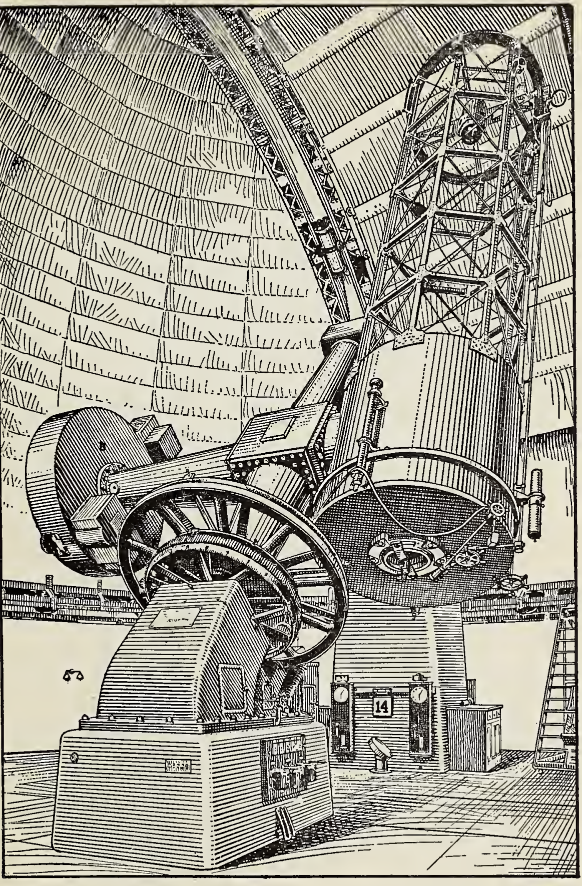

Fig. 447.—The great reflecting telescope at the Dominion Government Observatory at Victoria, B.C. At the right of the picture is seen the huge tube, which is 30 ft. long and $7\frac{1}{2}$ ft. in diameter. The upper portion (23 ft. in length) is of open steel framework. The great mirror is at the bottom of the inclosed portion. It is an immense disc of glass, actually 73 in. in diameter, 12 in. thick at the edge and 11.1 in. at the centre, where a hole 10 in. in diameter is cut out of it. It weighs 4,340 pounds. The small second mirror is seen at the top of the tube and the eyepiece projects at the centre of the bottom. The telescope is used chiefly to study the nature of the stars and the velocities with which they are moving.

M are converged towards its principal focus *F*. Before arriving there, however, they meet a convex reflector *m*, which turns them back and forms an image just in front of mirror *M*. The light, of course, does not stop there, but, proceeding onward, passes through a hole in the centre of the mirror and enters the eyepiece *E*.

In place of the convex mirror *m* one might use a concave or a plane mirror.

Some very large reflecting telescopes have been constructed. One of the largest and best is at Victoria, B.C., (Fig. 447). The mirror in it is 6 feet in diameter, and the entire moving part of the telescope weighs 90,000 pounds.

392. The Prism Binocular. Its construction is illustrated in Figs. 448 and 449. The former shows the appearance of the instrument, while the latter shows the optical arrangement. The lenses are precisely the same as in an astronomical telescope, but the compactness is obtained by using two reflection prisms. The light traverses the length of the instrument three times, which reduces the necessary length, while the reflections from the faces of the prisms erect the image. The field of view is from 7 to 10 times as great as with ordinary field-glasses of the same power.

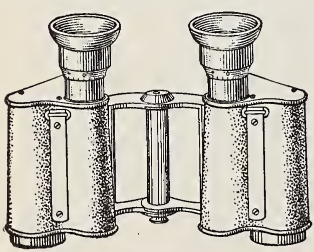

Fig. 448.—The prism binocular.

Fig. 449.—Showing the path of the light.

The use of prisms in this manner was devised by Porro about 70 years ago, but on account of difficulties in their manufacture they did not come into use until quite recently.

PART VIII—ELECTRICITY AND MAGNETISM

CHAPTER XXXIX

Magnetism

393. Natural Magnets. In various countries there is found an ore of iron which possesses the remarkable power of attracting small bits of iron. Specimens of this ore are known as *natural magnets.* This name is derived from Magnesia, a town of Lydia, Asia Minor, in the vicinity of which the ore is supposed to have been abundant. Its modern name is magnetite. It is composed of iron and oxygen, the chemical formula for it being Fe_3O_4.

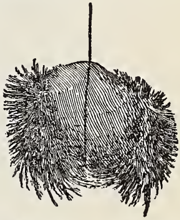

Fig. 450.—Iron filings clinging to a natural magnet.

If dipped in iron filings, many will cling to it, and if it is suspended by an untwisted fibre, it will come to rest in a definite position, thus indicating a certain direction. On account of this it is known also as a *lodestone,* (*i.e.*, leading-stone) (Fig. 450.)

394. Artificial Magnets. If a piece of steel is stroked over a natural magnet, it becomes itself a magnet. There are, however, other and more convenient methods of magnetizing pieces of steel which will be explained later (see § 479), and as steel magnets are much more powerful and more convenient to handle than natural ones, they are always used in experimental work.

Permanent steel magnets are usually of the bar, the horse-

Fig. 451.—Bar-magnets.

Fig. 452.—A horse-shoe magnet.

shoe or the compass-needle shape, as illustrated in Figs. 451-3.

395. Poles of a Magnet. Iron filings when scattered over a bar-magnet are seen to adhere to it in tufts near the ends, none, or scarcely any, being found at the middle (Fig. 454). The strength of the magnet seems to be concentrated in certain places near the ends; these places are called the *poles* of the magnet, and

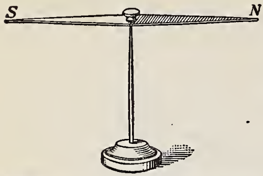

FIG. 453.—A compass-needle magnet.

FIG. 454.—The filings cling mostly at the poles.

a straight line joining them is called the *axis* of the magnet. If the magnet is suspended so that it can turn freely in a horizontal plane (Fig. 453), this axis will assume a definite north-and-south direction, in what is known as the *magnetic meridian*, which is usually not far from the geographical meridian. That end of the magnet which points north is called the *north-seeking* pole, or simply the *N*-pole, the other the *south-seeking* pole, or *S*-pole.

396. Magnetic Attraction and Repulsion. Let us bring the *S*-pole of a bar-magnet near to the *N*-pole of a compass-needle (Fig. 455). There is an attraction between them. Next present the same pole to the *S*-pole of the needle; it is repelled. Now reverse the ends of the magnet; we find that its *N*-pole attracts the *S*-pole of the needle but repels the *N*-pole.

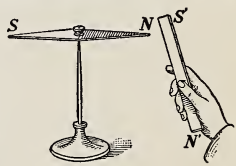

FIG. 455.—The *S*-pole of one magnet attracts the *N*-pole of another.

We thus obtain the law:—*Like magnetic poles repel, unlike attract each other.*

This experiment can be repeated with very simple means. Magnetize two sewing-needles by rubbing them, always in the same direction, against one pole of a magnet. Then thrust them into corks floating on the surface of water and push one over near the other; the attractions and repulsions will be beautifully shown.

It is to be observed that unmagnetized iron or steel will be attracted by *both* ends of a magnet. It is only when both bodies are magnetized that we can obtain repulsion.

397. Magnetic Substances. A magnetic substance is one which is attracted by a magnet. Iron and steel are the only substances which exhibit magnetic effects in a marked manner. Nickel and cobalt are also magnetic, but in a much smaller degree. In recent years Heusler, a German physicist, discovered a remarkable series of alloys possessing magnetic properties. They are composed of manganese (about 25 per cent.), aluminium (from 3 to 15 per cent.) and copper. These substances taken singly are non-magnetic, but when melted together are able easily to affect the magnetic needle.

On the other hand, bismuth, antimony and some other substances are actually repelled by a magnet. These are said to be *diamagnetic* substances, but their action on a magnet is very weak. For all practical purposes iron and steel may be considered to be the only magnetic substances.

398. Induced Magnetism. Hold a piece of iron rod, or a nail,* near one pole of a strong magnet; it becomes itself a magnet, as is seen by its power to attract iron filings or small tacks placed near its lower end (Fig. 456). Allow the nail to touch the pole of the magnet; it will be held there. A second nail may be suspended from the lower end of this one, a third from the second, and so on. (Fig. 457.) Now remove the magnet; the chain of nails falls to pieces.

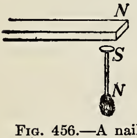

Fig. 456.—A nail if held near a magnet becomes itself a magnet by induction.

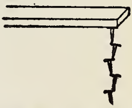
Fig. 457.—A chain of magnets by induction.

*Ordinary steel nails are not very satisfactory. Use clout nails or short pieces of stove pipe wire.

We thus see that a piece of iron becomes a *temporary* magnet when it is brought near one pole of a permanent steel magnet. Its polarity can be tested in the following way:–

Suspend a bit of soft-iron (a narrow strip of tinned-iron is very suitable), and place the *N*-pole of a bar-magnet near it (Fig. 458). Then bring the *N*-pole of a second bar-magnet near the end *n* of the strip, farthest from the first magnet. It is repelled, showing that it is an *N*-pole. Next, bring the *S*-pole of the second magnet slowly towards the end *s* of the strip. Repulsion is again observed.

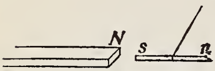

Fig. 458.—Polarity of induced magnetism.

This shows, as we should expect from the law of magnetic attraction and repulsion (§ 396), that the induced pole is opposite in kind to that of the permanent magnet adjacent to it.

399. Retentive Power. The bits of iron in Figs. 456–8, retain their magnetism only when they are near the magnet; when it is removed, their polarity disappears.

If hard-steel is used instead of soft-iron, the steel also becomes magnetized, but not so strongly as the iron. However, if the magnet is removed the steel will still retain some of its magnetism. It has become a *permanent* magnet.

Thus steel offers great resistance both to being made a magnet and to losing its magnetism. It is said to have great *retentive power*.

On the other hand, soft-iron has small retentive power. When placed near a magnet it becomes a stronger magnet than a piece of steel would, but it parts with its magnetism quite as easily as it gets it.

400. Field of Force about a Magnet. The space about a magnet, in any part of which the force from the magnet can be detected, is called its *magnetic field*. The field can be explored by means of a small compass-needle, as follows:—

Place a bar-magnet on a sheet of paper and slowly move a small compass-needle about it. The action of the two poles of the magnet on the poles of the needle will cause the latter to set itself at various points along lines which indicate the direction of the force from the magnet. These curves run from one pole to the other. In Fig. 459 is shown the direction of the needle at several points, as well as a line of force extending from one pole to the other.

FIG. 459.—Position assumed by a needle near a bar-magnet.

Another simple and effective way to map the field is by means of iron filings. Place a sheet of paper over the magnet, and sift from a muslin bag iron filings evenly and thinly over it. Tap the paper gently. Each little bit of iron becomes a magnet by induction, and tapping the paper assists them to arrange themselves along the magnetic lines of force. Fig. 460 exhibits the field about a bar-magnet, while Fig. 461 shows it about similar poles of two bar-magnets placed near together.

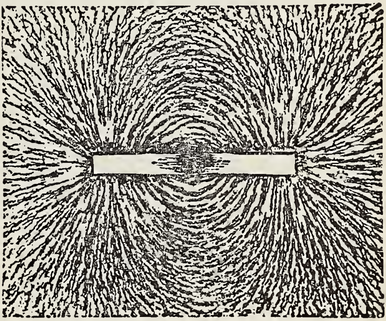

FIG. 460.—Field of force of a bar-magnet.

The magnetic force, as we have seen, is greatest in the neighbourhood of the poles, and here the curves shown by the filings are closest together. Thus the direction of the curves indicates the direction of the lines of force, and their closeness together at any point indicates the strength of the magnetic force there.

There are several ways of making these filings figures permanent. Some photographic process gives the best results, but one may use paper which has been dipped in melted paraffin. If the paper is heated, the filings sink into the wax and are held firmly in it when it cools.

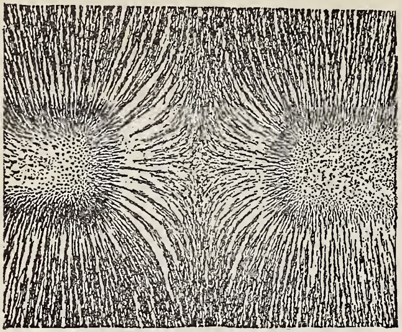

Fig. 461.—Field of force of two like poles.

401. Properties of Lines of Force. The lines of force belonging to a magnet are considered to begin at the N-pole, pass through the surrounding space, enter at the S-pole and then continue through the magnet to the N-pole again (Fig. 462). Thus each line of force is a closed curve. It is evident, also, that if we could detach an N-pole from a magnet and place it on any line of force, at A for instance, it would move along that line of force until it would come to the S-pole.

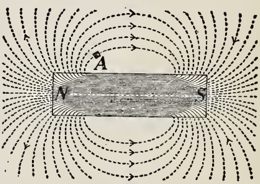

Fig. 462.—The lines of force run from the N-pole through the surrounding medium to the S-pole, and then through the magnet back to the starting point.

To illustrate this motion arrange apparatus as in Fig. 463. A bar-magnet A is placed on the bottom of the glass vessel B, which is filled with water to a depth of about 8 inches. A magnet D (a magnetized knitting-needle will serve) is

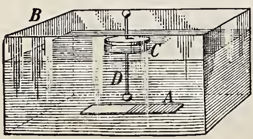

Fig. 463.—Illustrating motion along a line of force.

pushed through a cork C, which supports it in the water with its N-pole near the magnet A. When the floating magnet is placed with its lower end near the N-pole of A, it will move in a curved path until it stands over the S-pole of A. The upper pole of D is much farther from the stationary magnet than the lower pole and, consequently, the movement is practically the same as that of a detached or *free* N-pole.

Great use is made of the conception of lines of force in computations in magnetism and electricity, for example, in designing dynamos. This method of dealing with the subject was introduced by Faraday about 1830.

MICHAEL FARADAY (1791-1867). Born and lived in London. The greatest of experimental scientists. His discoveries form the basis of all our applications of electricity.

402. Magnetic Shielding. Most substances when placed in a magnetic field make no appreciable change in the lines of force, but there is one pronounced exception to this, namely, iron.

If a bar of iron is placed near the N-pole of a bar-magnet, the resulting field of force will be as shown in Fig. 464. The lines of force tend to crowd together in the iron as if they experienced less resistance to passing through it than through air. Very few lines pass through the iron into the region AB, practically all being deflected so that they follow the iron as far as possible before passing out into the air

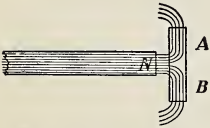

FIG. 464.—Lines of force pass through iron more easily than through air.

Place a bar-magnet with one pole about 10 cm. from a large compass-needle (Fig. 465). Pull aside the needle and let it go. It will continue vibrating for some time. Count the number of vibrations per minute. Then push the magnet up until it is 6 cm. from the needle, and again time the vibrations. They will be found to be much faster. Next, put the magnet 3 cm. from the needle; the vibrations will be still more rapid. Thus, the stronger the force of the magnet on the needle, the faster are the vibrations.

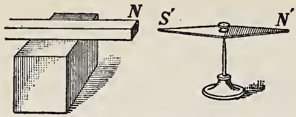

Fig. 465.—Arrangement for testing magnetic shielding.

Now, while the magnet is 3 cm. from the needle, place between them a board, a sheet of glass or of brass, and determine the period of the needle. No change will be observed. Next, insert a plate of iron. The vibrations will be much slower, thus showing that the iron has shielded the needle from the force of the magnet.

The lines of force on entering the iron are deflected as shown in Fig. 466, so that very few pass through to the compass-needle. The brass sheet produces no such effect (Fig. 467) and the field of force is unaltered by its presence. A space surrounded by a thick shell of iron is effectually protected from external magnetic force.

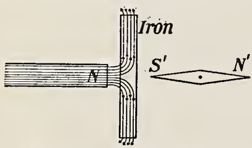

Fig. 466.—The lines of force crowd into the iron plate.

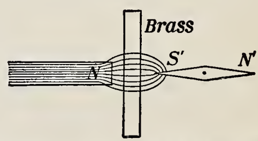

Fig. 467.—The lines of force pass freely through the brass plate.

403. Magnetic Permeability. The lines of force pass more easily through iron than through air. Thus iron has greater *permeability* than air, and the softer the iron is the greater is its permeability. Hence, when a piece of iron is placed in a magnetic field, many of the lines of force are drawn together and pass through the iron. This explains why soft-iron becomes a stronger magnet by induction than does hard-steel.

404. Each Molecule a Magnet. Let us magnetize a knitting needle or a piece of clock-spring (Fig. 468); it exhibits a pole at each end, but no magnetic effects at the centre. Now score it with a file and break it at the middle. Each part is a magnet. If we break these portions in two, each fragment is again a magnet. Continuing this, we find that each free end always gives us a magnetic pole. If all the parts are closely joined again, the adjacent poles neutralize each other and we have only the poles at the ends, as before.

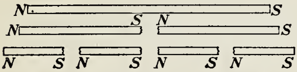

Fig. 468.—Each portion of a magnet is a magnet.

If a magnet is ground to powder, each fragment still acts as a little magnet and shows polarity.

Again, if a small tube filled with iron filings is stroked from end to end with a magnet, it will be found to possess polarity, which, however, will disappear if the filings are shaken up.

All these facts lead us to believe that each molecule is a little magnet. In an unmagnetized iron bar they are arranged in an irregular, haphazard fashion (Fig. 469), and so there is no combined action. When the iron is magnetized the molecules turn in a definite direction. Striking the rod while it is being magnetized assists the molecules to take up their new positions. On

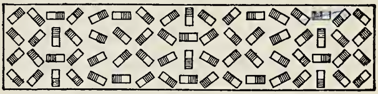

Fig. 469.—Haphazard arrangement of molecules of iron ordinarily.

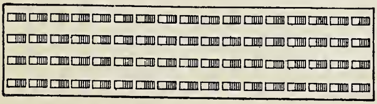

Fig. 470.—Arrangement of molecules of iron when magnetized to saturation.

the other hand, rough usage destroys a magnet. When the magnet is made as strong as it can be, the molecules are *all* arranged in regular order, as illustrated in Fig. 470. In this condition the magnet is said to be *saturated.*

The molecules of soft-iron can be brought into alignment more easily than can those of steel, but the latter retain their positions much more tenaciously.

405. Magnetizing a Steel Bar. The molecular theory of magnetism explains what happens when a bar of steel is magnetized by rubbing it with another magnet.

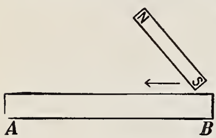

Fig. 471.—Magnetizing the steel bar *AB*.

If the steel bar *AB* (Fig. 471) is stroked in the direction of the arrow with the *S*-pole of a magnet *NS*, the molecular magnets in the bar will act like little compass-needles and will tend to turn so that their *N*-poles point towards *S* as it passes over them. Consequently, at the end of one stroke some of the molecules will have turned so that their *N*-poles point towards *A* and their *S*-poles point towards *B*. Each succeeding stroke, if made in the direction of the arrow, turns more and more molecules out of their haphazard positions and increases the strength of the new magnet. No magnetic effects are exhibited at the centre of the bar because the *N*-pole of each molecule has the *S*-pole of another molecule close to it, neutralizing its action. At the end *A*, however, we have an un-neutralized layer of *N*-poles and at the end *B* a similar layer of *S*-poles, with the result that the magnetism appears to be concentrated in these regions.

It is evident, then, that in magnetizing a bar the stroking should be done in only one direction, unless the magnet is reversed at each stroke; also, if an *S*-pole is used for stroking, the end of the bar which it touches last becomes an *N*-pole and the reverse if an *N*-pole is used for stroking.

406. Effect of Heat on Magnetization. A magnet loses its magnetism when raised to a bright red heat, and when iron is heated sufficiently, it ceases to be attracted by a magnet. This can be neatly illustrated in the following way. Heat a cast-iron ball, to a white heat if possible, and suspend it at a little distance from a magnet. At first, it is not attracted at all, but on cooling to a bright red it will be suddenly drawn in to the magnet.

The Heusler alloys, mentioned in § 397, behave peculiarly in respect to temperature. Above a certain temperature they are entirely non-magnetic. The temperature depends upon the proportions of aluminium and manganese present.

407. The Earth a Magnet. The fact that the compass-needle assumes a definite position suggests that the earth or some other celestial body exerts a magnetic action. William Gilbert,* in his great work entitled *De Magnete*, which was published in 1600, demonstrated that our earth itself is a great magnet.

In order to illustrate his views Gilbert had some lodestones cut to the shape of spheres; and he found that small magnets turned towards the poles of these models just as compass-needles behave on the earth.

The magnetic poles of the earth, however, do not coincide with the geographical poles. The north magnetic pole was found by Sir James Ross† on June 1, 1831, on the west side of Boothia Felix, in N. Lat. 70° 5′, W. Long. 96° 46′. In 1904-5 Roald Amundsen, a Norwegian, explored all about the pole. Its present position is about N. Lat. 70°, W. Long. 97°, not far from its earlier position.

The south magnetic pole was only recently attained. On January 16, 1909, three members of the expedition led by Sir Ernest Shackleton discovered it in S. Lat. 72° 25′, E. Long. 155° 16′. In both cases the magnetic pole is over 1100 miles from the geographical pole, and a straight line joining the two magnetic poles passes about 750 miles from the centre of the earth.

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*Gilbert (1540-1603) was physician to Queen Elizabeth, and was England's first great experimental scientist.

†The cost of the arctic expedition, which was made by John Ross and his nephew James, was defrayed by a wealthy Englishman named Felix Booth.

408. Magnetic Declination. We are in the habit of saying that the needle points north and south, but it has long been known that this is only approximately so. Indeed, knowing that the magnetic poles are far from the geographical poles, we should not expect the needle (except in particular places) to point to the true north. In addition, deposits of iron ore and other causes produce local variations in the needle. The angle which the axis of the needle makes with the true north-and-south line is called the *magnetic declination.*

409. Lines of Equal Declination or Isogonic Lines. Lines upon the earth's surface through places having the same declination are called *isogonic* lines; that one along which the declination is zero is called the *agonic** line. Along this line the needle points exactly north and south.

In the following table are given the values of the declination at several places in Canada and also at London, Eng., on January 1, 1910, and January 1, 1923.

MAGNETIC DECLINATION

Place	Jan. 1, 1910	Jan. 1, 1923
Halifax	21° 29′ W.	22° 15′ W.
Montreal	15° 19′ W.	16° 21′ W.
Toronto	5° 31′ W.	6° 28′ W.
Winnipeg	13° 57′ E.	13° 11′ E.
Edmonton	27° 24′ E.	27° 2′ E.
Victoria	24° 34′ E.	24° 47′ E.
Fort Norman	42° 6′ E.	41° 46′ E.
London, England	15° 40′ W.	13° 45′ W.

It will be seen that the declination at any point is subject to a slow change. At London in 1580 the declination was

*Greek, *isos* = equal, *gonia* = angle; *a* = not, *gonia* = angle.

11° 17′ E. This slowly decreased, until in 1657 it was 0° 0′. After this it became west and increased until in 1816 it was 24° 30′; since then it has steadily decreased.

In Fig. 472 is a map showing the isogonic lines for the United States and Canada for January 1, 1923.

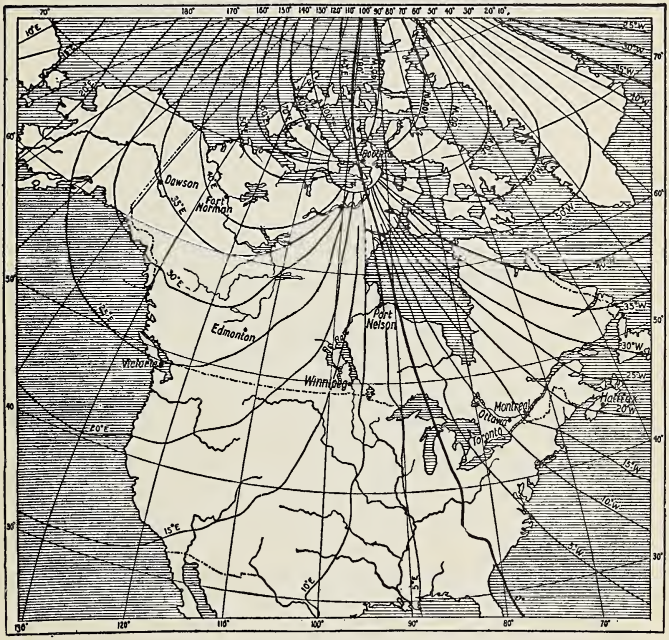


FIG. 472.—Isogonic Lines for Canada and the United States (January 1, 1923).

The data for regions north of latitude 55° are very meagre and discordant; the regions west of Hudson Bay where recent determinations have been made show considerable local disturbance; the lines north of latitude 70° are drawn largely from positions calculated theoretically, but modified where recent observations have been made. The above map was originally drawn for this work by the Department of Research in Terrestial Magnetism of the Carnegie Institution of Washington. It has been corrected to 1923 by W. E. W. Jackson, magnetician of the Meteorological Service of Canada.

410. Magnetic Inclination or Dip. Fig. 473 shows an instrument in which the magnetized needle can move in a vertical plane. The needle before being magnetized is so adjusted that it will rest in any position in which it is placed, but when magnetized, the *N*-pole (in the northern hemisphere) dips down, making a considerable angle with the horizon. If the magnetization of the needle is reversed, the other end dips down. Such an instrument is called a *dipping needle.* When in use the axis of rotation should point east and west (*i.e.*, at right angles to the *magnetic* meridian), and the needle should move with the least possible friction.

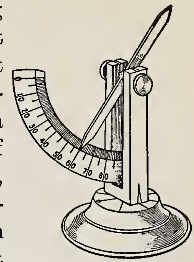


FIG. 473.—A simple dipping needle.

The angle which the needle makes with the horizon is called the *inclination* or *dip.* At the *magnetic* equator the dip is zero (or the needle is horizontal), but north or south of that line the dip increases, until at the magnetic poles it is 90°. Indeed, the location of the poles was determined by the dipping needle. In 1923 the value of the dip is:—

At Toronto, 74° 44′; Washington, 70° 59′; Mexico City, 46° 34′.

411. The Earth's Magnetic Field. As the earth is a great magnet, it must have a magnetic field about it, and a piece of iron in that field should become a magnet by induction. If an iron rod (*e.g.*, a poker, or the rod of a retort stand) is held nearly vertical, with the lower end inclined towards the north, it will be approximately parallel to the lines of force and it will become magnetized. If struck smartly when in this position, its magnetism will be strengthened. (Why?) Its magnetism can be tested with a compass-needle. Carefully move the lower end towards the *S*-pole; it is attracted. Move it near the *N*-pole; it is repelled. This shows the rod to be a magnet.

Now, when a magnet is produced by induction, its polarity is opposite to that of the inducing magnet. Hence we see that what we call the north magnetic pole of the earth is opposite in kind to the N-pole of a compass-needle.

Iron posts in buildings and the iron in a ship when it is being built become magnetized by the earth's field.

412. Mariner's Compass. In the modern ship's compass several magnetized needles are placed side by side, such a compound needle having been found more reliable than a single one. The card, divided into 32 "points of the compass," is attached to the needle, the whole being delicately poised on a sharp iridium point fixed in a bowl which is supported on gimbal rings in order that the card may remain horizontal in spite of the rolling of the vessel. (Fig. 474.)

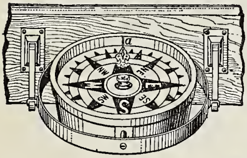


Fig. 474. — Mariner's Compass, mounted so as to remain horizontal always.

In some compasses the card is immersed in a liquid. This reduces the weight on the point and brings the needle to rest more quickly.

In order to steer in any particular direction, say north-west, the ship is turned until N.W. on the card is opposite a fixed vertical line *a* on the inside of the bowl. When the compass is installed, this "lubber's point" must be adjusted so that a line joining it to the iridium point is parallel to the keel of the ship.

In laying out a course on which to steer one must make due allowance for magnetic declination. If the true bearing* of a port is ten degrees (N. 10° E.) and the declination 6° W., the ship must be laid on a course sixteen degrees east of north, as indicated by the compass, in order that it may reach the port.

When a compass is being mounted in an iron ship masses of soft iron and also permanent magnets are arranged about it in such a way as to counteract the magnetic influence of the ship itself.

*The *true bearing* of a line is the angle the line makes with the true north line. The *magnetic bearing* of a line is the angle the line makes with the magnetic north line. In each case the angle is measured in a clockwise direction from the north line.

413. Prismatic Compass. The prismatic compass is used extensively in civil and in military reconnaisance surveying. The military form is illustrated in Fig. 475. It consists of a shallow brass box about 2 inches in diameter in which is pivoted the needle carrying a dial graduated in degrees from 0 to 360. The cover is hinged and has in it a glass window A, across the middle of which is drawn a fine black line which is vertical when a reading is to be taken. Directly opposite this line is a total reflection prism P enclosed in a hinged metal case provided with an eye-hole and slit.

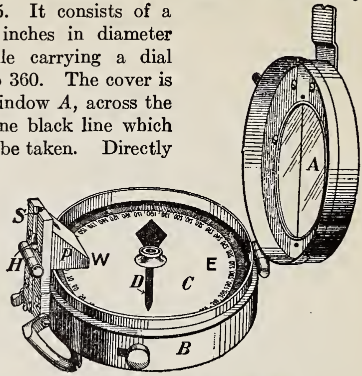


Fig. 475.—A prismatic compass.

To take the magnetic bearing of a distant object the compass is held horizontally, while the slit above the prism, the black line on the window and the object are brought into line. The observer then reads through the prism the angle which the needle makes with the line of sight. For example, a point which has a magnetic bearing of 270° is due west (magnetically) of the place from which the bearing is taken.

QUESTIONS AND PROBLEMS

1. What evidence have we that it is impossible to obtain a free magnetic pole?

2. A test-tube is filled with iron filings and the N-pole of a bar magnet is drawn from the open to the closed end several times. What change in the filings will be produced? What will be the effect of presenting the closed end of the tube to the S-pole of a compass-needle?

3. How could you magnetize a piece of steel so as to make it have an N-pole at *each* end?

4. You suspect that an iron rod is magnetized. How would you test this by means of a compass-needle?

5. Six magnetized sewing needles are thrust through six pieces of cork and are made to float near together in water with their N-poles upward. What will be the effect of holding (1) the S-pole, (2) the N-pole of a magnet above them? Try the experiment.

6. Arrange three similar bar magnets so that there will be the least possible magnetic effect on a neighbouring compass-needle.

7. Two similar bar magnets are set on end a few inches apart and a small magnetic needle is carried around the upper poles in a figure-of-eight course. How will it point in the various positions occupied (1) when the upper poles are like poles, (2) when they are unlike poles? Draw two diagrams.

8. Two precisely similar bar magnets are placed so as to form a +. Draw the field of force. Verify by performing the experiment with iron filings.

9. In dynamos and some other electrical machines it is necessary to have some parts which can be easily magnetized and which will lose their magnetism quickly when the magnetizing force is removed. What substance should be used?

10. The N-pole of a bar magnet is brought close to a point on the circumference of an iron ring. Make a sketch of the field of force about the magnet and the ring. Describe how a compass would act if placed inside the ring. Try the experiment.

11. Why is a permanent magnet injured when it is dropped or hammered?

12. If a piece of iron is heated hot enough, it loses any magnetism it may have, and if cooled when in a magnetic field, it becomes magnetized again. From our accepted theories of molecules and their motions explain these effects.

13. How would you proceed to find, experimentally, the magnetic declination in the locality of your school? Would it be better to perform the experiment in a room or in the open? Give reasons for your answer.

14. Give two methods by which the true north-and-south line can be determined at any place.

15. The true bearing of Oswego from Toronto as obtained from a map is 94°. If the average declination on Lake Ontario is 6° W., on what compass bearing would a ship have to sail in proceeding (*a*) from Toronto to Oswego, (*b*) from Oswego to Toronto?

16. The declination on Lake Ontario is changing at the rate of 3′ per annum. A ship leaves the Welland Canal for Kingston at 10 p.m., sailing by compass and allowing for a declination which was correct ten years ago. How far will the ship be off her course after a run of 100 miles? (One minute of angle is approximately equivalent to 1 inch per 100 yds.)

17. In France the end of the compass-needle pointing north is called the *Southern* pole and the end pointing south the *Northern* pole. Account for this.

18. In making a survey a church C and a water-tower D are found to have true bearings of 45° and 60° from a point A (Fig. 476). From a point B, five miles due north of A, both the church and the tower have true bearings of 90°. Calculate the distance from A to each point. If the variation at A is 10° E., what did the compass with which the bearings were taken read?

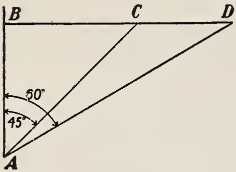


Fig. 476.

19. A base line AB is 10 miles long and lies in a true north and south direction, A being at the southern end. From A the bearing of a point C, as measured by a prismatic compass, is 20°; from B the magnetic bearing of C is 110°. The declination in each case is 10° E. Draw a careful diagram and find the lengths of AC and BC.

20. Criticise, from a scientific standpoint, the following lines in an old song:—

True as the needle to the pole
Or as the dial to the sun.
—*Barton Booth, 1681-1733.*

REFERENCES FOR FURTHER INFORMATION

Poyser and Brooks, *Magnetism and Electricity*, Chapters 1 to 4.
Glazebrook, *Electricity and Magnetism*, Chapters 7 to 12.
Hadley, *Magnetism and Electricity for Students*. Part I.

CHAPTER XL

STATIC ELECTRICITY

414. Electrical Attraction. If a stick of sealing-wax or a rod of ebonite (hard rubber) be rubbed with flannel or with cat's fur and then held near small bits of paper, pith or other light bodies, the latter will spring towards the wax or the ebonite. A glass rod when rubbed with silk acts in the same way.*

As early as 600 B.C. it was known that amber possessed this wonderful attractive power on being rubbed. The Greek name for amber is *ēlectron*, and when Gilbert (see § 407) found that many other substances behaved in the same way he called them all *electrics*. The bodies which have acquired this attractive power are said to be *electrified* or to be *charged with electricity*. In later times it has been shown that *any* two different bodies when rubbed together become electrified.

A good way to observe the force of attraction is to use a small ball of elder pith or of cork, hung by a silk thread (Fig. 477). When the rubbed glass is held near it the ball is drawn towards it.

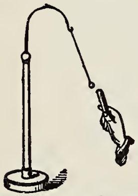


FIG. 477.—A pith ball on the end of a silk thread drawn towards the electrified rod.

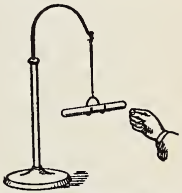


FIG. 478.—The electrified rod moves towards the hand.

It can also be shown that the electrified body is itself attracted by one that has not been electrified. Let us rub a glass rod and hang it in a wire stirrup supported by a silk thread (Fig. 478). If the hand (or other body) be held out towards the suspended body the latter will turn about and approach the hand. A rod of sealing-wax or of ebonite when rubbed acts similarly.

*In these experiments the substances should be thoroughly dry. They succeed better in winter since there is much less moisture in the air then.

415. Electrical Repulsion. Suppose, however, we allow the pith ball (Fig. 477) to touch the electrified glass rod. It clings to it for a moment and then flies off. If the end of the rod is brought near to it, the ball continually moves away from it. There is repulsion between the two. Next, rub an ebonite rod with flannel and hold it to the pith ball. It is attracted. Thus the glass now repels the pith ball, but the ebonite attracts it.

Also, two pith balls, both of which have touched either rubbed glass or rubbed ebonite and are therefore similarly electrified, will be found to repel each other; while a pith ball which has touched an electrified glass rod will attract one which has touched an electrified ebonite rod.

Again, hold a rubbed glass rod near the suspended rubbed glass rod (Fig. 478); they repel each other. Two ebonite rods behave similarly. If, however, we hold a rubbed ebonite rod near the glass rod there is attraction between them.

416. Two Kinds of Electrification. It is evident from these experiments that there are two kinds of electrification or of electrical charge, and it is customary to call that produced on rubbing glass with silk, *positive;* that produced on rubbing ebonite or sealing-wax with flannel, *negative.* The pith ball on touching the glass was repelled and must, therefore, have become charged positively. For a similar reason the ball which touched the ebonite must have acquired a negative charge.

The foregoing and numberless other experiments allow us to formulate the following:—

LAW OF ELECTRICAL ATTRACTION AND REPULSION.—*Electrical charges of like kind repel each other, those of unlike kind attract each other.*

417. Conductors and Non-conductors. We may rest a piece of electrified ebonite on another piece of ebonite or on dry glass, or sulphur or paraffin, and it will retain its electrification for some time; but if it is passed through a flame, or is gently rubbed over with a damp cloth, or simply with the hand, it loses its electrification at once. The ebonite, the glass, the sulphur and the paraffin are said to be *non-conductors* of electricity; while the damp cloth and the hand are said to be *conductors* of electricity, the electric charge escaping freely by way of them.

If we hold a piece of brass tube in the hand and rub it with fur or flannel or silk, it will show no signs of electrification; but if we fasten it to an ebonite handle and flick it with dry cat's fur, it will be negatively electrified. Approach it to a suspended rubbed ebonite rod (Fig. 478) and it will repel it. In the first case the brass was electrified, but the electrical charge immediately escaped to earth by way of the experimenter's body. In the second case the escape was prevented by the ebonite handle, and the metal remained electrified. It is to be noted, too, that a non-conductor exhibits electrification only where it is rubbed, while in a metal the charge is spread all over its surface.

Those substances which lead off an electrical charge quickly are called *conductors*, while those which prevent the charge from escaping are called *non-conductors* or *insulators*. If a conductor is held on a non-conducting support, it is said to be *insulated*. Thus, telegraph and telephone wires are held on glass insulators; and a man who is attending electric street lamps often stands on a stool with glass feet, and handles the lamps with rubber gloves.

GOOD CONDUCTORS:—metals.

FAIR CONDUCTORS:—the human body, solutions of acids and salts in water, carbon.

POOR CONDUCTORS:—dry paper, cotton, wood.

BAD CONDUCTORS, OR GOOD INSULATORS:—glass, porcelain, sealing-wax, mica, dry silk, shellac, rubber, rosin, and oils generally.

418. Nature of Electricity; the Electron Theory. It has already been stated (in § 150) that an atom of matter consists of a positively charged nucleus and a number of electrons. When a body is in its normal or unelectrified state, the sum of the negative charges on the electrons just balances the positive charges on the nuclei. It is considered that electrons are able to wander about from atom to atom in a conductor while the positively charged nuclei remain in position. Moreover, electrons can pass from one solid body to another.

We are now able to apply the idea of electrons to "explain" the phenomena which we have observed. When the ebonite and cat's fur were rubbed together, the ebonite became negatively charged, which means that it obtained an extra supply of electrons from some outside source. Did these electrons come from the cat's fur? If so, the cat's fur should be lacking the number of electrons which the ebonite has gained, and should exhibit a positive charge. This is found to be actually the case. On the other hand, the glass rod lost electrons to the silk and consequently showed a positive charge. *A negatively charged body is one which has a surplus of electrons; a positively charged body has a deficit of electrons.*

Electrons pass readily from one part of a conductor to another, while in insulators this action takes place with great difficulty.

419. The Gold-leaf Electroscope. The object of the electroscope is to detect an electric charge and to determine whether it is positive or negative. A metal rod with a knob or disc at the top (Fig. 479) extends through a well-insulated cork into a flask. From its lower end two leaves of gold or of aluminium leaf hang by their own weight. The rod may pass through a glass tube, well coated with shellac, which is inserted through the cork. The flask should be also varnished with shellac, as this improves the insulation greatly. If a charge, either positive or

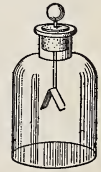


Fig. 479.—The gold-leaf electroscope.

negative, is given to the electroscope, the two leaves, being charged with electricity of the same kind, repel each other and separate.

Another form of electroscope is shown in Fig. 480. The protecting case is of wood with front and back of glass. The sides of the case are lined with tin-foil, to which a binding-post is connected. By this the case may be joined to earth and thus be kept constantly at zero potential (see § 432). The rod supporting the leaves passes through a block of unpolished ebonite or other good insulator, and the small disc on top may be removed if desired.

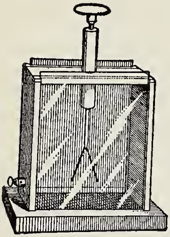


FIG. 480.—Another form of electroscope.

The electroscope may be charged by touching a charged body to the knob, or by connecting it to the knob by a conducting wire. But sometimes it is more convenient to use a *proof-plane* (Fig. 481), which is simply a small metal disc on an insulating handle. This is touched to the charged body and then to the knob of the electroscope.

FIG. 481.—A proof-plane.

420. Electrification by Contact. If the body which touches the electroscope is negatively charged, some of its surplus electrons will pass to the knob and then down to the leaves. On the other hand, if a positively charged body touches the knob, some of the electrons in the knob-leaf system pass to the positively charged body in an attempt to re-establish the balance between electrons and nuclei.

On being charged by contact, then, the electroscope acquires the same kind of charge as is on the charging body.

Another method of charging the electroscope is described in § 425.

421. Electrification by Induction. Let us slowly bring a rubbed ebonite rod towards the knob of the electroscope. The leaves are seen to separate even though the rod be a foot or more away. Some of the electrons in the knob have been repelled to the leaves by the negative charge on the ebonite. This gives the leaves an excess of negative electricity and they repel one another. This experiment shows that the mere presence of an electrified body is sufficient to produce electrification in neighbouring conductors. The charge is said to be produced by *electrostatic influence* or *induction.* As soon as the charged body is removed the leaves collapse again because the electrons which have been driven away from their nuclei are attracted back into their former position as soon as the disturbing influence disappears.

This experiment also impresses the fact that an electrified body exerts an action on bodies in the space about it. This space is called its *electrical field of force.* It can be shown, too, that the magnitude of the force exerted depends on the material filling the space. For instance, if the electrified body is immersed in petroleum the force it exerts on another body is only about one half that in air. Indeed, it is believed that the force exhibited is due to actions in the surrounding medium, which is known as the *dielectric.*

422. Testing the Charge on a Body. Suppose we have an electrified body and wish to test the nature of its charge. First, place a known charge on the electroscope; then bring up the unknown charge very slowly and watch for the first movement of the leaves. If the charge to be tested is of the same kind as that on the electroscope, the leaves will diverge still farther; if of the opposite kind, the leaves will come together. The reason is obvious.

If the unknown charge is great enough and is opposite in kind to that on the electroscope, it is possible to reverse the charge on the leaves by bringing it close to the knob. For this reason it is important to watch for the *first* movement of the leaves.

423. Nature of Induced Electrification. Let A and B (Fig. 482) be two metallic bodies placed near together on well-insulated supports.* Charge A positively by rubbing over it a glass rod rubbed with silk.

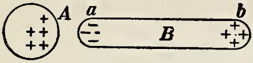


FIG. 482.—Explaining induced electrification.

First, touch A with a proof plane and carry it to the electroscope. The leaves will show a separation. Repeat and get a greater separation. Next, touch the proof plane to a, that end of B nearest A, and carry it to the electroscope. The leaves come closer together, showing that the charge on the end a is *negative*, that is, of the opposite kind to that on A.

Next, touch the proof-plane to the end b, which is farthest from A, and convey the charge to the electroscope. It makes the leaves diverge further, showing that the charge is of the same kind as that on A.

We find, therefore, that the two ends have charges of opposite signs, the charge on the end of B nearest to A being of the opposite sign to that on A. It is to be observed, also, that the electrification on B does not in any way diminish the charge on A.

424. Induced Charges are Equal. Place two insulated conductors A and B in contact and hold a positively charged rod near (Fig. 483). The conductor A will be charged negatively and B positively. While the rod is in position separate the conductors, and then remove the rod. The body A is now charged with negative and B with positive electricity.

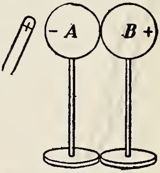


FIG. 483.—Two metal bodies on insulating stands. The charges on A and B are equal.

Bring them together carefully. A spark will be heard to pass between them and they will be entirely discharged. The two charges have neutralized each other, which shows that they must have been equal.

*The bodies may be of wood covered with tin-foil, and may rest on blocks of paraffin.

425. Charging by Induction. Let a positively charged rod be brought near an insulated conductor (Fig. 484). A considerable number of electrons will rush to the end *a* in order to get as close as possible to the positive charge on the rod. In this way the end *b* will be left positively charged. Suppose now the conductor is touched with the finger or is joined to earth* by a wire (see § 432). More electrons will be attracted from the earth by the positive charge on the rod. Now remove the finger and then the rod. The conductor will be charged negatively because it still has the excess electrons which came from the earth.

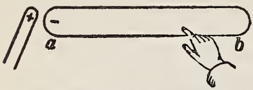


Fig. 484.—How to charge by induction.

Similarly, if the rod is charged negatively, electrons will be repelled through the finger to the ground, leaving the body positively charged.

In this way it is easy to give a charge of any desired kind and magnitude to an electroscope. Suppose we wish to give a positive charge. Rub an ebonite rod with cat's fur and bring it towards the knob of the electroscope. The knob will be charged positively and the leaves negatively by induction. Now touch the electroscope rod with the finger; the negative charge will escape. Then remove the finger and after that the ebonite rod. The positive charge will remain on the electroscope, producing a separation of the leaves.

426. The Electrophorus. By means of this instrument, which was invented by Volta, in 1775, we can electrify a conductor without using up the instrument's original charge.

It consists of a cake *A* of ebonite or of resinous wax resting on a metal plate, and a metal cover† *B*, of rather smaller diameter, provided with an insulating handle. (Fig. 485.)

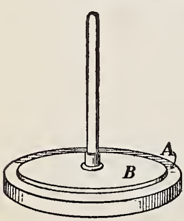


Fig. 485.—The electrophorus.

First, the cake is rubbed with cat's fur, and thus it obtains a negative charge. Then the cover is put on and touched with the finger.

*Connect to a gas or water-pipe. Connection may be made to any part of the conductor.
†This may be a wooden disc covered with tin-foil.

If it is lifted up by the handle, it will be found to be positively charged, and if it is presented to the knuckle a spark, sometimes half-an-inch long, is obtained, and the cover is discharged. The gas may be lighted with this spark; and if the cover be presented to the knob of an electroscope, the latter will be charged. The process may be repeated any number of times without renewing the charge on the cake.

Question.—Every time the cover is discharged energy disappears; where did it come from?

The action is explained thus:—When the cover is placed on the cake, which is a non-conductor, it rests upon it on a few points only and so does not remove its charge. But the negative charge on A induces on the lower face of B a positive charge, repelling to the upper face a negative charge, which escapes when the finger touches it; therefore, when the cover is lifted it has a positive charge.

427. Charges Produced by Friction Equal and Opposite. We are now in a position to demonstrate the truth of the statement made in § 418 that the cat's fur becomes positively charged when used to rub ebonite. Place a metal can, large enough to hold the cat's fur, on the plate of an electroscope (Fig. 486). Rub the ebonite and the cat's fur together and place both in the can. No motion of the leaves takes place. Remove the ebonite without touching the can with the hand. The leaves diverge, showing that the fur is charged. Replace the ebonite again and the leaves fall. The charges on the cat's fur and ebonite must, therefore, be equal in amount and opposite in kind.

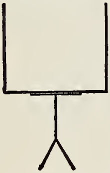


Fig. 486.—Can on electroscope.

428. Charges Reside on the Outer Surface. Place a tall metal vessel on a good insulator (Fig. 487), and electrify it by either an ebonite rod, or an electrophorus, or an electrical machine (§ 438). Disconnect from the machine. Lower a metal ball, suspended by a silk thread, into the vessel and let it touch the inner surface. Then apply the ball to the

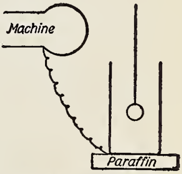


Fig. 487.—A tall metal vessel joined to an electrical machine. Removing the wire disconnects it.

electroscope; it shows no charge. Next, touch the ball to the outside of the vessel and test with the electroscope. It now shows a charge. Finally, charge the ball by the machine, then lower it into the metal vessel and touch the inner surface with it. Then test the ball with the electroscope. It will be found that its charge is entirely gone; it was given to the metal vessel, on the outer surface of which it now is.

In Fig. 488 are shown a metal sphere on an insulating stand, and two hemispheres with insulating handles which just fit over it. First, charge the sphere as strongly as possible. Then, taking hold of the insulating handles, fit the hemispheres over it, and then remove them. If now the sphere is tested with the electroscope, no trace of electricity will be found on it.

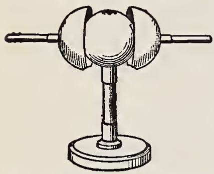


Fig. 488.—Apparatus to show that the charge resides only on the surface of a conductor.

429. Distribution of the Charge; the Action of Points. Though the electric charge resides only on the outer surface of a conductor, it is not always equally dense all over it. The distribution depends on the shape of the conductor, and experiment shows that the charge is greater at sharp edges.

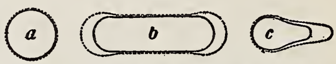


Fig. 489.—Showing the distribution of an electric charge on conductors of different shapes.

The force with which a charge tends to escape from a conductor increases with the density of the charge, and it is for this reason that a pointed conductor soon loses its charge. If a pointed wire is placed on a conductor attached to an electrical machine, the electrified air particles streaming from it may blow aside a candle flame (Fig. 490); or an "electric whirl" (Fig. 491),

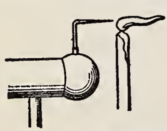


Fig. 490.—"Electric wind" from a pointed conductor blowing aside a candle flame.

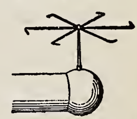


Fig. 491. — The "electric whirl" rotates by the reaction from the electric wind.

nicely balanced on a sharp point, when placed on an electrical machine is made to rotate by the reaction as the air-particles are pushed away from the points. It rotates like a lawn-sprinkler.

430. Lightning-Rods. In a thunder-storm the clouds become charged with electricity and by induction a charge of the opposite sign appears on the surface of the earth just beneath. Suppose that the cloud (Fig. 492) has a positive charge. The points of the lightning rods will acquire a very dense negative charge and consequently the air particles in contact with the points will become negatively charged and will be repelled from the points and attracted by the cloud. Every negatively charged air particle which reaches the cloud neutralizes some of its charge. In most cases the charge on the cloud will be neutralized in this way without a flash of lightning occurring.

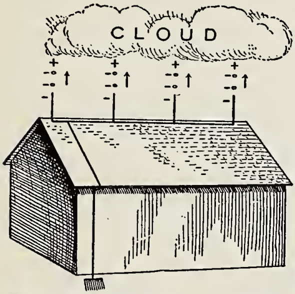


FIG. 492.—Negatively charged air particles are passing from the rods to the cloud.

If, however, the force of attraction between the charges on the cloud and the rods is great enough, there may be a very sudden rush of electrons across the intervening space, with the result that great heat is developed and the building is "struck by lightning." The wires running from the rods to the ground are, however, good conductors, and do not become sufficiently heated to set fire to the building. On the other hand, if the building is not equipped with rods, the sudden rush of electrons through the relatively poor conductors of which it is constructed generates usually sufficient heat to set the building on fire.

It is evident, then, that the lower end of a lightning-rod should be buried deep enough to be in moist earth always, since dry earth is a poor conductor.

A simple experiment to illustrate the action of lightning-rods can be performed as follows:—

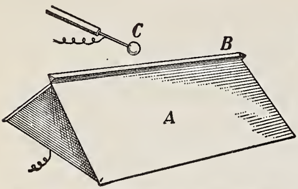


FIG. 493.—Arrangement to illustrate the use of lightning-rods.

A is a tin model of a barn with a small trough *B* placed on top, partly filled with sulphuric ether (Fig. 493). *A* is connected to one terminal of an electrical machine, while a "cloud," consisting of the brass ball *C*, mounted on an ebonite handle and connected to the other terminal of the electrical machine, is brought over the barn. A "lightning flash" occurs and the ether is ignited. Next place a row of pins along the ridge, to represent lightning rods; no flash now occurs. The action of the points neutralizes the charge on the ball as fast as it is given to it by the machine.

431. Electrical Potential. Imagine a charge of 50 electrons to be placed on each of the metal spheres *A* and *B* (Fig. 494) which are mounted on insulating supports. These electrons will spread themselves uniformly over the surface of the spheres, because each electron is being repelled by the other 49. In the case of the large sphere this force of repulsion is not so great as in the case of the small sphere because the electrons are farther apart. Consequently, the tendency for an electron to leave the sphere *B* is greater than in the case of sphere *A*; or, considering it in another way, more work would have to be done to place an additional electron upon *B* than to place it on *A*. This difference in state between *A* and *B*, when equal charges are placed upon them, is called a difference in *potential* (or sometimes, *pressure*).

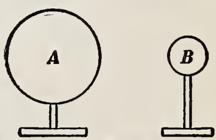


FIG. 494.—Two metal spheres on insulating supports.

If *A* and *B* are joined by a conductor, it is evident that the force tending to drive electrons in the direction *AB* is less than it is in the direction BA, and as a result there will be a flow of electrons from B to A until the potential of B becomes equal to that of A.

Question.—In what direction would electrons flow if A and B had equal positive charges on them?

Let us consider analogies in other branches of science.

Water will flow from the tank A to the tank B (Fig. 495) through the pipe C connecting them if the water is at a higher level in A than in B; or, what amounts to the same thing, if the hydrostatic pressure at a is greater than that at b. The tank B may already have more water in it, but the flow does not depend on that. It is regulated by the difference between the pressures at the two ends of the pipe and it will continue until these pressures become equal.

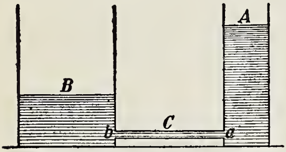


FIG. 495.—Water flows from the higher level in A to the lower level in B.

Or, consider what happens when two gas-bags filled with compressed air are joined by a tube in which is a stop-cock. If the pressure of the air is the same in each, there will be no flow from one to the other on opening the stop-cock. If there is a difference, there will be a flow from the bag at high pressure to that at low.

Again, when two bodies at different temperatures are brought together, there is a flow of heat from the one at the higher temperature to that at the lower temperature.

We see, then, that the term potential in electricity corresponds to pressure in hydrostatics and to temperature in the study of heat. If two bodies which are at different potentials are joined by a conductor, there will be a flow, or a *current*, of electricity through the conductor until the potentials are equalized.

Potential difference is usually measured in *volts*, a definition of which is given in the next chapter (§ 446).

432. Zero of Potential. In stating levels or heights we usually refer them to the level of the sea. The ocean is so large that all the rain which it receives does not appreciably alter its level. In a somewhat similar way, the earth is so large that all the electrical charges which we can give it do not appreciably alter its electrical level or potential, and so we take the earth to be our zero of potential.

Lake Superior is 602 feet above the level of the sea, and the Dead Sea, in Palestine, is 1,300 feet below it. There is a continual flow from Lake Superior to the ocean; and if a tube joined the two, there would be a flow from the ocean to the Dead Sea.

Consider the four tanks in Fig. 496. The levels of *A* and *B* are above, and those of *C* and *D* below, that of the earth. A flow would take place from *A* or *B* to the earth, or from the earth to *C* or *D*, or from any one tank to another at lower level.

Bodies which are charged positively are arbitrarily considered to be at a potential higher than that of the earth, and those charged negatively to be at a potential below that of the earth. The larger the positive charge on a body is the *higher* is its potential; the greater the negative charge the *lower* is its potential. If equal positive charges are placed on the spheres in Fig. 494, the potential of *B* will be higher than that of *A*; when the charges are negative, the potential of *B* will be lower than that of *A*. In both cases, however, the potential of *B* is *numerically greater* than that of *A*.

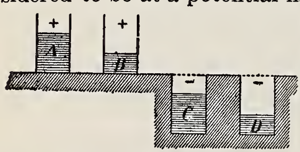


FIG. 496.—Water flows from the higher level in *A* to the lower level in *B*.

433. The Electrostatic Voltmeter. The simple electroscope can be used to indicate electrical potential since the degree of divergence of the leaves will vary with the potential of whatever body is connected to the knob. A similar instrument, calibrated to read directly in volts, is shown in Fig. 497. A light aluminium pointer *A* is pivoted at *B* on the metal support *C* which also carries the scale *D*. This support passes through an ebonite insulator *E* and has a binding-post *F* on its upper end. The case of the voltmeter is of metal and is provided with a binding-post *G*, by which it can be joined to earth to bring it to zero potential. The body whose potential (relative to the earth) we wish to measure is joined to *F*. The support *C* and the pointer *A* acquire the same kind of charge and the pointer is repelled from the support. Such voltmeters can be constructed to measure either very great or very small potentials.

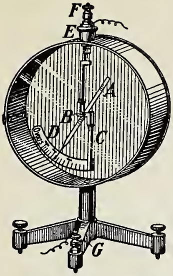


Fig. 497.—Electrostatic voltmeter.

434. Electrical Capacity. On pouring the same quantity of water into different vessels we observe that it rises to different levels; and that vessel into which we must pour the most water in order to raise its level by any amount, say 1 cm., is said to have the greatest capacity. If a vessel has a small cross-section, like a narrow tube, it will not take much water to make a great change in its level; and its capacity is small.

There is something analogous in the science of electricity. It requires different amounts of electricity to raise the potentials of different conductors by one unit, and so we say there is a difference in the *electrical capacities* of conductors.

In Fig. 494 the capacity of the large sphere is greater than that of the small one.

435. Electrical Condenser. In Fig 498 *A* and *B* are two metal plates on insulating bases. They may be of tin-plate about 10 or 12 inches square, bent at the bottom and resting on paraffin blocks, *C*, *C*, with metal blocks, *D*, *D*, to keep them in place.

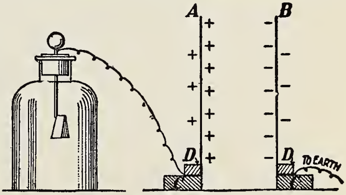


Fig. 498.—Showing the distribution of an electric charge on conductors of different shapes

First let B be at some distance from A, and charge A. The greater the charge, the higher rises the potential and the wider diverge the gold-leaves. Continue charging until the leaves are far apart.

Then, with the plate B joined to earth (simply keeping a finger on it will do), push it up towards A. As the plates get nearer together, the leaves begin to fall, showing that the potential of A has fallen through the presence of B. If now we increase the charge on A by means of a proof plane, we shall find that several times the original amount of electricity must be added to A in order to obtain the original separation of the leaves, that is, to raise it to the original potential.

The two plates and the air between them constitute a *condenser.*

The explanation of the action of the condenser is as follows. Let the charge on A be positive. When the plate B is brought up, the charge on A attracts electrons from the earth to the plate B. The movement of these charges to B is accompanied by a movement of electrons from the plate A to the leaves, and as a consequence the leaves fall.

436. The Capacity of a Condenser. The closer the plates of the condenser (Fig. 498) are brought together, the greater is the fall of the leaves. In other words, *the capacity of a condenser increases as the distance between the plates decreases.*

If we make the plate B smaller it will be found that the fall of the leaves is not so pronounced. *The capacity increases with the size of the plates.*

Next, push the plates A and B near together, and charge the plate A until the separation of the leaves is quite decided. Now insert between A and B a sheet of thick plate glass, sliding it along B, being careful not to touch A, and observe the effect on the electroscope. The leaves come closer together, showing that the potential has fallen and the capacity has increased. Ebonite or paraffin may be used as a dielectric instead of the glass, but the effect will not be so pronounced.

The capacity of a condenser using glass as a dielectric averages somewhere about 6 times that of the same condenser when air is the dielectric. This number is called the *dielectric constant* for glass. *The greater the dielectric constant, the greater is the capacity.*

Values of Some Dielectric Constants

Air	1.0	Paraffined paper	2.1 to 2.5
Glass	5.4 to 9.9	Ebonite	2.7 to 2.9
Mica	5.6 to 6.6	Sulphur	4.0 to 4.2

Capacity is measured in *farads* or *microfarads*. These units are defined in § 547.

437. Leyden Jar. This is one form of condenser. It consists of a wide-mouthed bottle (Fig. 499), the sides and the bottom of which, both within and without, are coated with tin-foil to within a short distance from the neck. The glass above the tin-foil is varnished to maintain the insulation. Through a wooden stopper passes a brass rod, the upper end of which carries a knob, the lower a chain which touches the inner coating of the jar. The two coatings form the two plates of the condenser, the glass being the dielectric.

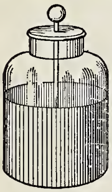
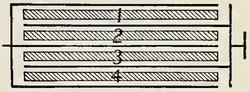


Fig. 499.—A Leyden jar.

To charge the jar the outer coating is connected to earth (or held in the hand), and the knob is joined to an electrical machine. To discharge it, connection is made between the inner and the outer coatings by discharging tongs (Fig. 500). Usually the discharge is accompanied by a brilliant spark and a loud report. (It is wise not to pass the discharge through the body.)

Fig. 500. — Discharging tongs. The handles are of glass or of ebonite.

Condensers used in electrical experiments are



Fgi. 501.—A plate condenser.

often made of a number of sheets of tin-foil separated from one another by sheets of paraffined paper or mica. Alternate sheets of the tin-foil are connected together, as shown in Fig. 501. By this means a large surface area, and consequently large capacity, can be obtained in a small volume. The condenser shown can be considered the equivalent of four two-plate condensers.

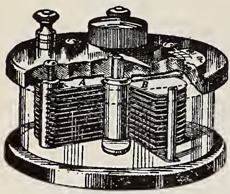


Fig. 502.—A variable air condenser.

Condensers are used extensively in line and wireless telegraphy and telephony. A variable air condenser of the type used in wireless communication is shown in Fig. 502. As the moving plates B are rotated out from between the fixed plates A, the effective area of the plates becomes less and the capacity decreases.

438. Toepler-Holtz Influence Machine. The ordinary electrical machines are simply convenient contrivances for utilizing the principle of influence illustrated in the electrophorus (§ 426).

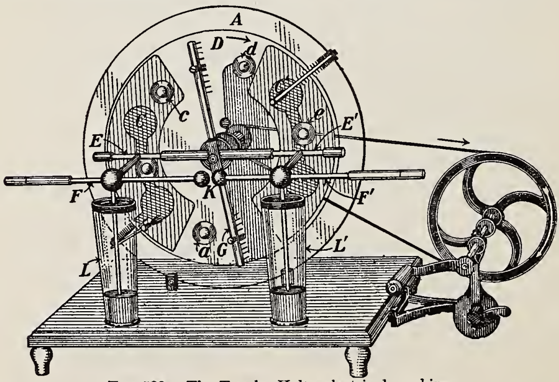


Fig. 503.—The Toepler-Holtz electrical machine.

In Fig. 503 is shown a Toepler-Holtz machine. In it are two parallel glass plates, A and D, the former being fixed while the latter can be rotated in front of it. Upon the back of A are two pieces of tin-foil C, C', called *armatures*, which are covered by paper sectors. Upon the front of the moving plate are 6 or 8 tin-foil discs, a, b, c, . . ., called carriers, each having a brass button attached at its centre. Two brass rods, one at the lower end of C, the other at the upper end of C', are bent around from the back to the front of the plates. One end is connected to an armature, while the other bears a metal brush which rubs on the brass buttons as they pass by. Upon the rods E, E' are metal combs with sharp teeth, which point toward the carriers but do not touch them. These rods are connected to the discharging knobs K, between which the sparks pass when the machine is in operation. The insulated neutralizing rod G has a brush at each end. These touch opposite pairs of carriers as they pass. Two Leyden jars L, L' are usually added and when they are charged powerful sparks are produced.

439. The Action of the Machine. The action of the machine can be explained by means of the diagram (Fig. 504) in which, for clearness, the two discs are represented as though they were two glass cylinders, one within the other, the outer one being fixed and the inner one rotating in the direction of the arrow.

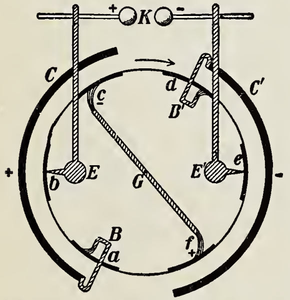


Fig. 504.—Diagram of the Toepler-Holtz machine.

Suppose a, one of the carriers, has a small positive charge. On passing the brush B a portion of its charge will be given to the armature C. The carriers c and f are (momentarily) connected by the rod G, and the charge on C will induce a negative charge on c at one end of G and a positive charge on f at the other end of G. As the carriers c and f move forward the former will give up some of its negative charge to C' and the latter some of its positive charge to C. In this way the charges on the armatures rapidly increase and these by induction increase the charge on the carriers.

Now, the carriers still retain a portion of their charges after passing the brushes B, B' and this will be collected from them by the metal combs E, E'. These are connected to the knobs K between which, when they are charged to a sufficiently high potential, a spark will pass.

Usually the machine is self-starting. If it refuses to start, a piece of rubbed ebonite or sealing-wax held opposite c or f will give the charge necessary to start it. After the machine has "picked up" the two armatures assist each other through the neutralizing rod G.

QUESTIONS AND PROBLEMS

1. If two insulated bells (Fig. 505) are joined to the coatings of a charged Leyden jar while a small brass ball is suspended between them by a silk thread, the ball will swing back and forth, causing the bells to ring. Explain the action. What will happen to the charge on the jar?

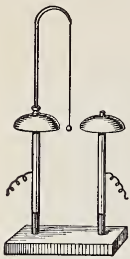


Fig. 505.—Electric chimes.

2. A piece of brass mounted on an ebonite handle is rubbed on a piece of cloth. How would you test the charge on the brass by using (*a*) a pith-ball electroscope, (*b*) a gold-leaf electroscope? Why use the ebonite handle?

3. How would you use an electroscope to test whether a moistened silk thread is as good an insulator as the same thread when dry?

4. Describe three experiments that could be performed to show that magnetism is essentially different from electricity.

5. What experiments would you perform to show that there are two kinds of electricity?

6. Why are the glass or porcelain insulators used on telephone and electric light lines made in the shape of an umbrella (Fig. 506)?

7. Telephone cables frequently contain hundreds of wires insulated from one another by dry paper, the whole being enclosed in a lead sheath. Of what use is the sheath? A small hole through the lead soon interferes seriously with the transmission of speech. Explain.

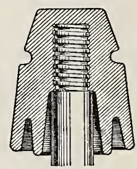


Fig. 506.—An insulator.

8. A lightly insulated field telephone cable which was buried under a road worked well for a day or two and after that the signals became very faint. The section across the road was dug up and carried over the road on poles and the line worked properly again. Explain why the fault occurred.

9. Use the electron theory to explain why charges reside on the outside of a charged conductor.

10. Explain the action of the lightning-rods (Fig. 492) when the cloud is charged negatively.

REFERENCES FOR FURTHER INFORMATION.

Ferry, *General Physics*, Chapter 21.
Smith, *Elements of Applied Physics*, Chapter 23.
Thomson, *Outline of Science*, Chapter 8.

CHAPTER XLI

The Electric Current

440. The Electric Current. As explained in Art. 431, when two bodies at different potentials are joined by a conductor, there is a passage of electricity from one to the other. For example, if the terminals of an influence machine are connected by a wire, electrons will flow along the wire from the negative to the positive terminal. We speak of an electric charge in motion as a *current of electricity.*

According to the electron theory, then, we should speak of the current as flowing from the negative to the positive terminal. Unfortunately, before the electron theory was generally accepted, *it was agreed to consider the current as flowing from the positive to the negative terminal,* and, since many of the rules governing electrical action are based on this assumption, we shall still follow the conventional method.

This at first sight may appear confusing, but the difficulty clears up when we realize that a flow of positive charges in one direction would produce the same final result as a flow of electrons (negative charges) in the opposite direction. Whichever way we consider it, the flow would tend to neutralize the charges on the terminals and to bring them to the same potential.

441. An Electric Current known by the Effects it will Produce. The electric current makes the conductor and the region surrounding it acquire new properties, one of the most striking of these being a change in magnetic conditions. This is illustrated in the following experiment.

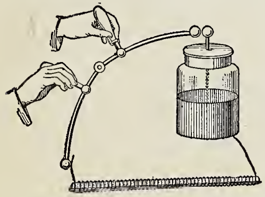


Fig. 507.—The presence of an electric current shown by its power to magnetize steel.

Insert an unmagnetized knitting needle into a small glass tube and wind copper wire in a coil about it. Connect one end of the wire with the outer coating of a powerfully charged Leyden jar, and the other with a discharger, as shown in Fig. 507. Discharge the jar through the wire. Now test the knitting needle with a magnetic compass; it will be found to be magnetized. Evidently the coil of wire in carrying the charge had the power to magnetize the steel.

442. The Voltaic Cell. A current of electricity may be generated in other ways than by the flow of charges produced by friction or induction. If a plate of zinc and one of copper are dipped into dilute sulphuric acid and connected by a conductor, as shown in Fig. 508, a current of electricity will flow through the conductor. The presence of the current can be shown by making the coil used in the experiment in the last section the conductor. When the connections are made, the knitting needle is magnetized.

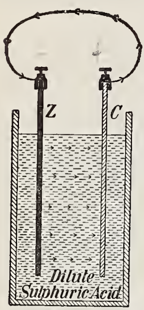


Fig. 508. — Simple voltaic cell.

The current in the wire connecting the two coatings of the Leyden jar lasts but an instant, because the coatings almost at once assume the same potential, but the current in the conductor joining the zinc and copper plates can be shown to be continuous.

This arrangement of zinc and copper plates immersed in dilute sulphuric acid for the purpose of producing an electric current is one of the simplest forms of what is known as the *Galvanic* or *Voltaic Cell.*

The cell is named from the men most immediately concerned in its development. Galvani* discovered by accident that the discharge of an electric machine connected with a skinned frog produced convulsions in the legs; and on further research he found that the same effect could be produced without the electric machine, by simply touching one end of a branched fork of copper and silver wires to the muscles in the frog's leg,

*Aloisio Galvani (1737-1798), a Physician and Professor of Anatomy in the University of Bologna.

and the other end to the lumbar nerves (Fig. 509). He attributed the results to "animal magnetism."

Allesandro Volta (1745-1827). Professor of Physics at the University of Pavia, Italy. Invented the voltaic cell.

Volta, a fellow-countryman, conceived that the electric current had its origin, not in the frog's legs, but in the contact of the metals. In his investigations he found that when discs

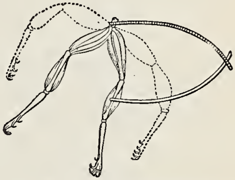


Fig. 509.—Galvani's experiment.

of copper and zinc were separated by a disc of cloth moistened with common salt brine, and joined externally by a conductor as in Fig. 509a, a current passed through the circuit. Later, he substituted zinc and copper strips for the discs and immersed them in dilute sulphuric acid.

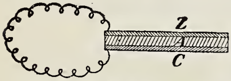


Fig. 509a.—*Z*, zinc; *C*, copper; *A*, cloth moistened with brine.

443. Plates of a Voltaic Cell Electrically Charged. Since an electric current flows through a conductor joining the plates of a voltaic cell, we should infer that the plates are electrically charged when disconnected. This can be shown to be the case by means of a *condensing electroscope*, which consists of an ordinary gold-leaf electroscope combined with a suitable condenser.

A convenient arrangement is illustrated in Fig. 510. It is unsatisfactory to work with a single voltaic cell. Three or four should be joined "in series" as shown at *B*. These cells may be small glass tubes or bottles containing dilute sulphuric acid, with strips of copper and zinc soldered together and dipping in them. The condenser consists of two perfectly flat brass plates. The lower one M is supported on an ebonite stem, and

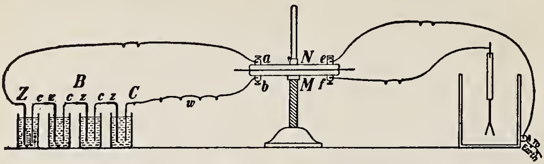


FIG. 510.—The condensing electroscope.

the upper one N is furnished with a handle. A sheet of paraffined paper or dry writing-paper or very thin mica is placed between the plates. The binding-posts e and f are joined to the electroscope, which is the same as shown in Fig. 480, with the disc removed. Its binding-post is joined to a gas-pipe or other good earth-connection. The end zinc plate Z of the battery is joined to a and the end copper plate C is joined loosely by the wire w to the binding-post b.

When the connections have been made as described, there is a charge on the gold leaves, but it is so slight that they do not diverge appreciably.

Now, by means of a glass or an ebonite rod remove the wire w from the binding-post b, and then lift off the upper plate N of the condenser. The leaves now diverge.

This action may be explained thus:—When the connections are as shown in the figure, Z and N are both joined to earth and hence are at zero potential. The lower plate M is charged positively by the battery. This charge attracts a negative charge to the lower face of N. Thus there is a considerable charge of electricity in the condenser though the potential is not high.

But when w is removed and the plate N lifted, the plate M is no longer a part of a condenser, but simply an isolated plate of much smaller capacity. The potential is, therefore, increased and the leaves diverge.

The copper plates are thus shown to have a positive charge while the zinc plates have a negative charge.

444. An Electric Circuit—Explanation of Terms. A complete circuit is necessary for a steady flow of electricity. This circuit comprises the entire path traversed by the current, including the external conductor, the plates, and the fluid. The current is regarded as flowing from the copper, or positively charged plate, to the zinc, or negatively charged plate, through the external conductor, and from the zinc to the copper plate through the fluid (Fig. 508).

When the plates are joined by a conductor, or a series of conductors, without a break, the cell is said to be *on closed circuit*; when the circuit is interrupted at any point, the cell is *on open circuit*. By joining together a number of cells we may obtain a more powerful flow of electricity, and such a combination is called a *battery*. An electric circuit is illustrated in Fig. 511. In it are two voltaic cells, two lamps L_1, L_2 and a key K. When the key is pressed, the circuit is closed; when up, open. The two cells and also the two lamps in this case are said to be joined *in series*, and the current passes through one after the other. In the arrangement shown in (Fig. 512) the cells and also the lamps are joined *in parallel.*

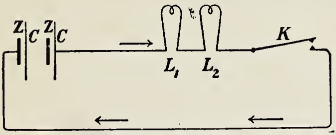


Fig. 511.—An electric circuit, with cells and lamps in series.

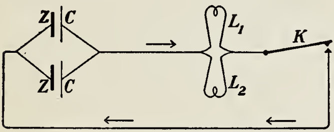


Fig. 512.—In this circuit the cells and also the lamps are arranged in parallel.

445. Electromotive Force. The term Electromotive Force is applied to *that which tends to produce a transfer of electricity*. Consider, for example, a voltaic cell on open circuit. Its electromotive force is its power of producing electric pressure, and this is obviously equal to the potential difference between the plates.

This conception can be illustrated by the analogy to two tanks of water maintained at different levels (Fig. 495). Just as the difference in level gives rise to a hydrostatic pressure which would cause a transfer of water if the tanks were connected by a pipe, so a difference of potential in the plates of the cell is regarded as producing an electrical pressure, or electromotive force, which would cause a transfer of electricity if the plates were connected by a conductor.

446. Unit of Electromotive Force. The difference of potential between two bodies is measured by the work done in transferring a certain quantity of electricity from one to the other. The practical unit of potential difference, and hence of electromotive force (E.M.F.), is known as the *volt*, and this may be taken as approximately the E.M.F. of a zinc-copper cell.

447. Oersted's Experiment. We have referred to the fact that an electric current has the power of producing magnetic effects (§ 441). This important principle was discovered by Oersted* in 1819. In the course of some experiments made with the purpose of discovering an identity between electricity and magnetism, he chanced to bring the wire joining the plates of a voltaic battery over a magnetic needle, and was astonished to see the needle turn round and set itself almost at right angles to the wire. When he reversed the direction of the current the needle turned in the opposite direction (Fig. 513). If the battery is held over the wire, the needle is deflected, thus showing that the current flows through the battery too.

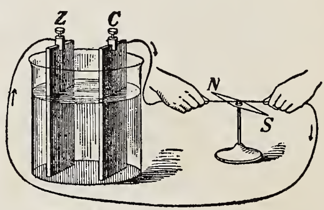


FIG. 513.—Oersted's experiment.

By holding the wire first over and then under the needle we find that, no matter in which direction the current is flowing, the needle always sets itself in accordance with the following rule:—

Imagine yourself swimming in the wire WITH *the current and* FACING *the needle; then the N-pole of the needle will be deflected toward your* LEFT *hand.*

---

*Hans Christian Oersted (1777-1851), Professor in the University of Copenhagen.

448. Electromotive Force, Current Strength, and Resistance. The water analogy may assist us still further.

The strength of the water current, that is, the quantity of water which will flow past a point in the pipe in one second, obviously depends upon the pressure resulting from differences of level, and upon the resistance which the pipe offers to the flow of water. Similarly, the *strength of the current* which passes through the conductor joining the plates of a cell *depends upon the electromotive force of the cell and the* RESISTANCE *of the circuit.*

The strength of the current may be increased, either by increasing the electromotive force, or by reducing the resistance of the circuit. The exact relation between these quantities will be discussed at a later stage (§ 525).

449. Detection of an Electric Current. Oersted's experiment furnishes a ready means of detecting an electric current. A feeble current, flowing in a single wire over a magnetic needle, produces but a very slight deflection; but if the wire is wound into a coil, and the current made to pass several times in the same direction, either over or under the needle, or, better still, if it passes in one direction over it and in the opposite direction under it, the effect will be magnified (Fig. 514). Such an arrangement is called a *Galvanoscope.* It may be used not only to detect the presence and the direction of currents, but also to compare roughly their strengths, by noting the relative deflections produced.

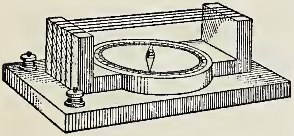


FIG. 514.—Simple galvanoscope. The wire passes several times around the frame, and its ends are joined to the binding-posts.

Exercise.—Use the 'swimmer rule' to show why a current flowing through a loop of wire should deflect a compass-needle placed between the upper and lower part of the loop more than the same current flowing through a single wire. In what circumstances would the loop produce less effect than the single wire?

CHAPTER XLII

Chemical Effects of the Electrical Current

450. Electrolysis. So far, in speaking of conductors, we have had reference mainly to metals or other solids; but an electric current may be made to flow through liquids as well. We have had a hint of this in discussing the direction of the current in the voltaic cell (§ 444).

An electric current flows through a metallic liquid like mercury in exactly the same way as through a metal in the solid state. The conductors in such cases are unaffected chemically by the currents. But certain other liquids, known as *electrolytes*, are decomposed by the currents which they carry. As an illustration, take the action of an electric current on hydrochloric acid.

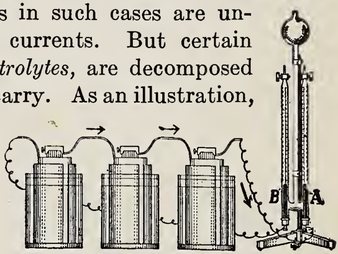


Fig. 515.—Electrolysis of hydrochloric acid. The electrodes A and B are carbon rods fitted in rubber stoppers.

Fill the tubes in the apparatus shown at the right of Fig. 515 with the acid and connect the plates of a voltaic battery consisting of three or four cells to the carbon rods A and B in such a way that the current flows in the direction of the arrows. Gases will be liberated at A and B and will fill the upper parts of the tubes. On being tested, that liberated at A will be found to be chlorine,* and that at B, hydrogen. This process of decomposition by the electric current is called *electrolysis* (*i.e., electric analysis*).

*As chlorine is very soluble, only a small portion of the gas liberated will collect at the top of the tube. If common salt is added to the hydrochloric acid, its solubility will be diminished and more gas will be collected.

The conductors by which the current enters and leaves the electrolyte are called *electrodes.* The one by which the current enters is known as the *anode,* and that by which it leaves as the *cathode*; for example, in the experiment described above, A is the anode and B the cathode.

The most common electrolytes are solutions of inorganic salts and acids in water.

451. Explanation of Electrolysis. According to the commonly accepted theory the molecules of an electrolytic salt or acid, when in solution, become more or less completely *dissociated.* The respective parts into which the molecules divide are known as *ions.*† When, for example, common salt is dissolved in water, a percentage of the molecules (NaCl) break up to form sodium (Na) and chlorine (Cl) ions. Similarly, if sulphuric acid is diluted with water, some of its molecules (H_2SO_4) dissociate into hydrogen (H) ions and sulphion (SO_4) ions.

A definite charge of electricity is associated with each ion, and when it loses this charge it ceases to be an ion. The hydrogen and sodium atoms, and the atoms of metals in general, as ions, bear positive charges, while chlorine atoms and sulphion are types, respectively, of the elements and radicals which bear negative charges.

In Fig. 516 an attempt has been made to represent the condition of a molecule of common salt before and after it is dissociated. In *a* the sodium and chlorine atoms are closely united; in *b* dissociation has taken place and the molecule has been broken up into a positively charged sodium ion and a negatively charged chlorine ion.

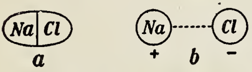


FIG. 516.—Illustrating dissociation of a molecule of common salt.

†Greek *ion* = something that moves.

The ionization theory furnishes a simple explanation of the typical results described in the preceding section. When connected with the terminals of the battery, the electrodes immersed in the hydrochloric acid become charged, the anode positively, and the cathode negatively. As a consequence, the positively charged hydrogen ions are attracted to the cathode, and the negatively charged chlorine ions to the anode. This 'migration' of positively charged ions in one direction, and negatively charged ions in the other (Fig. 517) constitutes the current in the electrolyte.

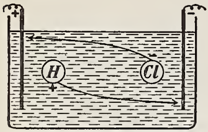


FIG. 517.—Showing migration of H and Cl ions to cathode and anode.

The ions give up their charges to the electrodes, and combine to form molecules. The gases are, therefore, liberated at these centres.

The positive charges which the hydrogen ions bring to the cathode tend to diminish the negative charge of the cathode, while the negative charges of the chlorine tend to diminish the positive charge of the anode; but a constant difference of potential between the electrodes is kept up by the current maintained in the external conductor by the battery.

452. Secondary Reactions in Electrolysis. If the hydrochloric acid is replaced by a solution of common salt (NaCl), chlorine, as before, appears at the anode, but the sodium atoms, which have parted with their charges to the cathode, instead of combining to form molecules, displace hydrogen atoms from molecules of water in order to form sodium hydroxide. Hence, hydrogen, and not sodium, is liberated at the cathode. The presence of the hydroxide in solution can be shown by adding sufficient red litmus to colour the solution. As soon as the current begins to pass, the liquid about the cathode is turned blue. The bleaching of the litmus about the anode indicates the presence of chlorine.

The foregoing experiment is typical of a large number of cases of electrolytic decomposition where secondary reactions, depending on the chemical relations of elements involved, take place. The electrolysis of water, possibly, furnishes another example.

453. Electrolysis of Water. Insert platinum electrodes into the bottom of a vessel of the form shown in Fig. 518. Partially fill the vessel with water acidulated with a few drops of sulphuric acid. Fill two test tubes with acidulated water and invert them over the electrodes. Connect the electrodes with a battery of three or four voltaic cells. Gases will be seen to bubble up from the electrodes, displacing the water in the test tubes.

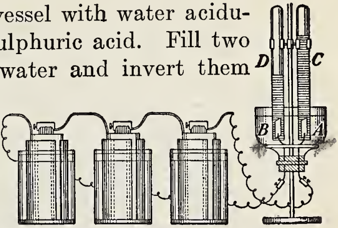


FIG. 518.—Electrolysis of water.

On testing each gas with a lighted splinter, we find that collected at the anode is oxygen, and that at the cathode, hydrogen. It will also be observed that the volume of the hydrogen collected is twice that of the oxygen.

In this case, as in the electrolysis of hydrochloric acid, there is a migration of hydrogen ions in the direction of the current, and the gas is given off at the cathode; but the liberation of the oxygen is probably due to secondary reactions. The sulphion (SO_4) ions move to the anode, where they part with their charges and combine with the hydrogen of the water, thus forming again sulphuric acid, and liberating oxygen. The quantity of the acid, therefore, remains unchanged and water only is decomposed.

454. Laws of Electrolysis. Faraday discovered that the masses of the substances liberated in the process of electrolysis are dependent only on the strength of the current and the time during which it flows, and that when the same current is made to flow through several electrolytes the masses of the various substances liberated from these electrolytes are proportional to their chemical equivalents.

His LAWS OF ELECTROLYSIS were formulated in 1833. They may be summed up in the following statements:—

1. *The mass of an ion liberated at an electrode is proportional to the strength of the current and to the time the current flows.*

2. *The masses of the elements separated from the electrolyte*

by the same electric current are proportional to their chemical equivalents.

455. Measurement of Current Strength by Electrolysis. The quantitative relation between the strength of an electric current and the mass of a substance liberated by it from an electrolyte in a given time furnishes a means of measuring the strength of the current, because a unit current may be taken as the current which liberates a certain mass of a selected element in a unit of time.

The practical unit of current strength universally accepted is the AMPERE and is defined as *the current which deposits silver at the rate of* 0.001118 *grams per second* (see § 525).

The same current deposits copper at the rate of 0.000328 grams, and hydrogen at the rate of 0.000010384 grams per second.

Other elements may be used in defining the unit, but in practice when the strength of a current is to be estimated by electrolysis, it is usually determined by ascertaining the amount of silver, copper, or hydrogen which is deposited in a specified time. If W_1, W_2, W_3 is the mass in grams of silver, copper and hydrogen, respectively, deposited in t seconds, and C the strength of the current in amperes, then

$$C = \frac{W_1}{t \times 0.001118} = \frac{W_2}{t \times 0.000328} = \frac{W_3}{t \times 0.000010384}.$$

456. Voltameters. An electrolytic cell used for the purpose of measuring the strength of an electric current is called a *voltameter*.

1. *Silver Voltameter.* The cell consists of a platinum bowl, partially filled with a solution of silver nitrate in which is suspended a silver disc (Fig. 519). When the voltameter is placed in the circuit, the platinum bowl is made the cathode and the silver disc the anode.

When the current has been passed through the solution for the specified time the silver disc is removed, the solution poured off, and the bowl washed, dried and weighed. The increase in weight gives the mass deposited and the current strength in amperes is easily calculated.

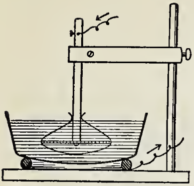


FIG. 519.—A standard silver voltameter. Cathode, a platinum bowl not less than 6 cm. in diameter and 4 cm. deep. It rests on a metal ring to ensure good connection. Anode, a disc of pure silver supported by a silver rod riveted through its centre. Electrolyte, 15 parts by weight of silver nitrate to 85 parts of water. Filter paper is wrapped about the anode to prevent loose particles of silver from falling on the cathode.

Examples.—(1) Let the original weight of the bowl be 100.0000 gm. and its weight after the current has flowed for 10 minutes be 101.3416 gm.

$$\text{Then } C = \frac{1.3416}{600 \times 0.001118} = 2 \text{ amperes.}$$

(2) Given that the original weight was 125.638 gm., the final weight 128.992 and the current 3 amperes; how long did it flow?

2. *Copper Voltameter.* In the copper voltameter two copper electrodes are immersed in a solution of copper sulphate (100 gm. of copper sulphate to 500 c.c. of water). A common form of the instrument is illustrated in Fig. 520, but two plates of sheet copper provided with binding-posts and placed in a tumbler work very well. The cathode is cleaned with sand-paper or emery-cloth and carefully weighed. It is then placed in position and the current to be measured is passed through the solution for 15 or 20 minutes. The plate is then removed, washed and dried without rubbing, and finally weighed again.

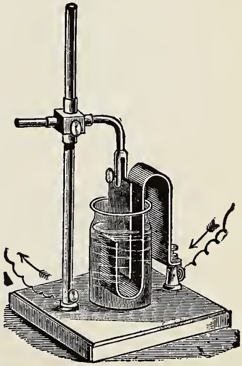


FIG. 520.—A copper voltameter.

Example.—Suppose it is desired to find the current which passes through an automobile headlight lamp when connected to a 6-volt storage battery.

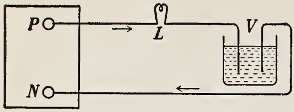


FIG. 521.—Finding the current passing through a lamp.

Arrange apparatus as in Fig. 521. P and N are the terminals of the battery, L is the lamp and V the voltameter.

Let the initial weight of the cathode be 50.342 gm. and the weight after the current has flowed for 15 min. be 51.080 gm.

$$\text{Then } C = \frac{.738}{900 \times 0.000328} = 2.5 \text{ amperes.}$$

3. *Hydrogen Voltameter.* This is simply the apparatus used for the decomposition of water (§ 453). For the purpose of measuring the current, the hydrogen alone is collected in a graduated tube. The current to be measured is passed through the acidulated water until the liquid in the tube stands at the same level as the liquid in the vessel. The time during which the current was passing is then noted. The temperature of the gas and the barometric pressure are also taken. The volume of the hydrogen liberated is read from the graduated tube, reduced to standard pressure and temperature, and the mass corresponding to this volume calculated.

The process of measuring the strength of a current by a voltameter is slow. We shall discuss in the next chapter a more convenient method by the use of ammeters. Voltameters are now used mainly in standardizing these instruments.

457. Electroplating. Advantage is taken of the deposition of a metal from a salt by electolysis in order to cover one metal with a layer of another, the process being known as *electroplating*.

Consider, as an example, the process of silver-plating. The objects to be plated are immersed in a bath containing a solution of silver salt, usually the cyanide (AgCN). A plate of silver is also immersed in the bath (Fig. 522). A current from a battery or dynamo is then passed through the bath, from the silver plate (the anode), to the objects (the cathode). The positively charged silver ions are urged to the objects, and on giving up their charges, are deposited as a metallic film upon them. Meanwhile the negatively charged cyanogen (CN) ions migrate towards the silver plate, from which they attract into solution additional silver ions. Thus the metal is transferred from the plate to the objects, while the strength of the solution remains constant.

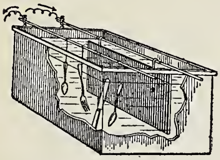


Fig. 522.—Bath and electrical connection for electroplating.

The process of plating with other metals is similar to silver-plating. The electrolyte must always be a solution of the salt of the metal to be decomposed; the anode is a plate of that metal, and the cathode the object to be plated.

For copper-plating, the bath is usually a solution of copper sulphate; for gold- and silver-plating, a solution of the cyanides; and for nickel-plating, a solution of the double sulphate of nickel and ammonium.

458. Electrotyping. Books are now usually printed from *electrotype* plates instead of from type, as the type would soon wear away. An impression of the type is made in a wax mould, the face of which is then covered with powdered plumbago to provide a conducting surface upon which the metal can be deposited. The mould is then flowed with a solution of copper sulphate, and iron filings are sprinkled over it. The iron displaces copper from the sulphate, and the plumbago surface is thus covered with a thin film of copper. The iron filings are washed off, and the mould immersed in a bath of nearly concentrated copper sulphate solution, slightly acidulated with sulphuric acid. The copper surface is then connected with the negative terminal of a battery or dynamo, and a copper plate, which is connected with the positive terminal, is immersed in the bath.

When the layer of copper has become sufficiently thick it is removed from the bath, backed with melted type-metal and mounted on a wooden block. The face is an exact reproduction of the type or the engraving.

459. Electrolytic Reduction of Ores; Electrolysis Applied to Manufactures. Electrolytic processes are now extensively used for reducing certain metals from their ores. A soluble, or fusible salt is formed by the action of chemical reagents, and the metal is deposited from it by electrolysis. For example, aluminium is reduced in large quantities from a fused mixture of electrolytes. Sodium is prepared in a similar manner.

The metallurgist also resorts to electrolysis in separating metals from their impurities. Copper, for example, is refined in this way. The unrefined copper is made the anode in a bath of copper sulphate, and the pure copper is deposited at the cathode, while the impurities fall to the bottom of the bath.

Currents of electricity are also employed in the preparation of many chemical products for commercial purposes. Caustic soda and bleaching liquors are manufactured on a large scale by electrolytic means.

460. Electrolytic Rectifiers. The current supplied for electric lighting is usually alternating, that is, it reverses in direction several times per second. Obviously such a current could not be used for electroplating. It is possible, however, to utilize electrolytic action to get rid of the current flowing in one direction and thus change the alternating into a unidirectional current suitable for electro-deposition or for charging a storage battery (§472). This process is called *rectifying* the current.

A simple rectifier can be constructed very easily by placing a lead and an aluminium plate in a jar containing borax solution (R, Fig. 523).

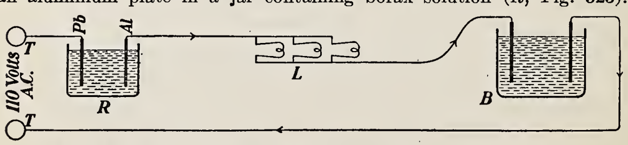


Fig. 523.—Method for using an alternating current to do electroplating or to charge a storage battery.

The rectifier is connected in series with a suitable resistance L and the electroplating bath, or storage battery, B, and the circuit is connected to the alternating current terminals T, T as shown in the figure. When the current flows in such a direction as to make the lead plate of the rectifier the anode, the oxygen which is liberated at that plate unites with the lead to form lead peroxide (PbO_2), which is a conductor and allows the current to keep flowing, but when the current reverses, a non-conducting film of aluminium hydroxide is instantly formed on the aluminium plate and further flow of the current in this direction is effectively blocked. In other words, current can flow through the rectifier from lead to aluminium but not from aluminium to lead.

On account of the resistance of the borax solution considerable heat is developed, but if there is about a litre of the solution a current of half an ampere can be used for 30-minute periods without overheating the solution. For a current of this strength the resistance can be made from three or four 32 c.p. carbon lamps connected in parallel.

QUESTIONS AND PROBLEMS

1. What weight of (*a*) copper, (*b*) silver will a current of 2 amperes deposit in 1 hour?

2. What weight of (*a*) hydrogen, (*b*) oxygen will be liberated in a water voltameter by a current of 5 amperes flowing for 2 hours?

3. At standard temperature and pressure what volume would the hydrogen and oxygen in the preceding question occupy? (1 c.c. hydrogen at N.T.P. weighs .0000895 gram.)

4. What current will deposit 1.1808 grams of copper in 10 min.?

5. A constant current is passed through a silver voltameter for a period of 20 minutes and it is found that 6.708 grams of silver have been deposited. What is the strength of the current?

6. To test an ammeter it was connected in series with a silver voltameter through which a current of 3 amperes as read by the ammeter was allowed to pass for 50 minutes. The increase in the weight of the cathode was 9.8943 grams. Find the error in the ammeter reading.

7. A copper voltameter was connected in series with a storage battery and a wireless valve for 15 minutes. If the deposit weighed 0.5904 grams, what current passed through the valve.

8. How long will it take a current of one ampere to deposit one gram of copper from a solution of copper sulphate?

9. What weight of chlorine would be set free in the experiment described in § 450, by a current of 2 amperes flowing for 100 minutes? (35.5 grams of chlorine unite with 1 gram of hydrogen to form 36.5 grams of hydrogen chloride.)

CHAPTER XLIII

Voltaic Cells; Storage Cells

461. The Essential Parts of a Voltaic Cell. We have found that when a plate of zinc and a plate of copper are immersed in dilute sulphuric acid and connected by a conducting wire, a current of electricity flows through the wire because a difference in potential is maintained between the plates. But voltaic cells may be formed with other pairs of plates and with other electrolytes as exciting fluids. The essential parts of an ordinary voltaic cell are two different conducting plates immersed in an electrolyte which acts chemically on one of them.

462. Chemical Action of a Voltaic Cell. When plates of copper and pure zinc are placed in dilute sulphuric acid to form a voltaic cell, the zinc begins to dissolve in the acid, but the action is soon checked by a coating of hydrogen which gathers on its surface. If the upper ends of the plates are connected by a conducting wire, or are touched together, the zinc continues to dissolve in the acid, forming zinc sulphate, and hydrogen is liberated at the copper plate.

These chemical changes take place within the cells in accordance with the same principles as the changes in the electrolytic cell as described in §§ 450, 451, the main difference being that in the electrolytic cell the source of the current is without the cell, while in the voltaic cell the current originates within the cell itself.

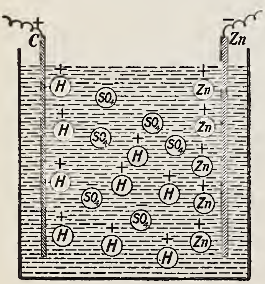


Fig. 524.—Diagram illustrating the theory of the action of a voltaic cell.

When the plates are connected, the zinc ions go into solution, carrying positive charges, and leave the zinc negatively charged, while the hydrogen ions migrate towards the copper plate and give up their charges to it. (Fig. 524.)

463. Source of Energy in the Cell. One can say with certainty that in the changes described in the preceding section chemical energy is transformed into the energy of the electric current, but the action of the forces involved in maintaining the potential difference in the plates in this transformation has not been satisfactorily explained. We may best think of the cell as a kind of furnace in which zinc is burned up chemically in order to obtain electric energy.

When the circuit is open, just enough action takes place to keep the plates at a certain difference of electrical level or potential, but when the plates are connected by a conductor the current flows and the zinc is continually consumed.

464. The Electromotive Force of a Voltaic Cell. The E.M.F. of a cell containing a given electrolyte depends on the nature of the plates. Thus the E.M.F. of a zinc-carbon cell is about one and a half times as great as that of the zinc-copper cell, when dilute sulphuric acid is the electrolyte.

When the materials used are constant, the E.M.F. is independent of the size and the shape of the plates or their distance apart.

Theoretically, a comparatively large number of substances might be selected as plates to construct a voltaic cell, but, as we shall see, some combinations are much better than others.

Let us study this question experimentally. Take plates of zinc, copper, tin and carbon and form cells by placing them, two at a time, in dilute sulphuric acid.

First use the zinc and copper plates and connect the cell so formed to a galvanoscope or a voltmeter, noting the amount and the direction of the deflection. Then substitute in succession the tin and the carbon plate for the copper plate, and from the direction of the deflection determine which is positive and which is negative in each case. Record also the amount of the deflection in each case.

Next, test copper-tin, copper-carbon and tin-carbon cells. The results will be as follows:—

Zinc −, Copper +	Copper +, Tin −
Zinc −, Tin +	Copper −, Carbon +
Zinc −, Carbon +	Tin −, Carbon +

Also, it will be found that the zinc-carbon pair gives the greatest deflection.

We learn from our tests that zinc is negative, no matter which of the other three metals is used with it; that tin is positive when used with zinc but negative with copper and carbon; that copper is positive to zinc and tin but negative to carbon; while carbon is positive to each of the others.

We can write these four metals in a series, thus:—

Zinc, Tin, Copper, Carbon,

in which any metal becomes the positive plate of a cell when used with any metal appearing before it in the series, but it becomes the negative plate if used with a metal appearing after it in the series. Moreover, the potential difference between the metals in any pair depends upon their distance apart in the series. Such a series is known as an *electromotive*, or *potential*, series.

A more comprehensive series obtained by a more extended investigation is the following:—

Magnesium, Zinc, Lead, Tin, Iron, Copper, Silver, Gold, Platinum, Carbon.

465. Local Action in Voltaic Cells. We have noted that a plate of pure zinc when used in a voltaic cell continues to dissolve only when it is connected with the copper plate. Commercial zinc will dissolve in the acid even when unconnected with another plate. The fact that the impure zinc wastes away in open circuit is possibly explained on the theory that the impurities, principally iron and carbon, take the place of the copper plate, and as a consequence currents are set up between the zinc and the impurities in electrical contact with it. Such currents are known as *local currents*, and the action is known as *local action*. This local action is wasteful. It may, to a great extent, be prevented by *amalgamating* the zinc. This is done by washing the plate in dilute sulphuric acid, and then rubbing mercury over its surface. The mercury dissolves the zinc, and forms a clean uniform layer of zinc amalgam about the plate. The zinc now dissolves only when the circuit is closed. As the zinc of the amalgam goes into the solution, the mercury takes up more of the zinc from within and the impurities float out into the liquid. Thus a homogeneous surface remains always exposed to the acid.

466. Polarization of a Cell. If the plates of a zinc-copper cell are connected with a galvanoscope, the current developed by the cell will be seen gradually to grow weaker. It will also be observed that the weakening in the current is accompanied by the collection of bubbles of hydrogen on the copper plate. To show that there is a connection between the change in the surface of the plate and the weakening in the current, brush away the bubbles and the current will be found to grow stronger. A cell is said to be *polarized* when the current becomes feeble from a deposition of a film of hydrogen on the positive plate.

The adhesion of the hydrogen to the positive plate weakens the current in two ways. First, it decreases the potential difference between the plates; because the potential difference between zinc and hydrogen is much less than between zinc and copper or carbon. Second, it increases the resistance which the current encounters within the cell, because it diminishes the surface of the plate in contact with the fluid.

Polarization may be reduced by surrounding the positive plate by a chemical agent which will combine with the hydrogen and prevent its appearance on the plate.

467. Varieties of Voltaic Cells. Voltaic cells differ from one another mainly in the remedies adopted to prevent polarization. Several of the forms commonly described have now only historic interest. Of the cells at present used for commercial purposes, the Leclanché, the Daniell and the Dry are among the most important.

468. Leclanché Cell. The construction of the cell is shown in Fig. 525. It consists of a zinc rod immersed in a solution of ammonic chloride in an outer vessel, and a carbon plate surrounded by a mixture of small pieces of carbon and powdered manganese dioxide in an inner porous cup. The zinc dissolves in the ammonic chloride solution, and the hydrogen which appears at the carbon plate is oxidized by the manganese dioxide.

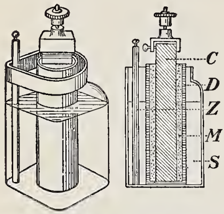


Fig. 525.—Leclanché cell. *C*, carbon; *D*, porous cup; *Z*, zinc; *M*, carbon and powdered manganese dioxide; *S*, solution of ammonic chloride.

As the reduction of the manganese dioxide goes on very slowly, the cell soon becomes polarized, but it recovers itself when allowed to stand for a few minutes. If used intermittently for a minute or two at a time, the cell does not require renewing for months. For this reason it is especially adapted for use on electric bell, telephone and other open circuits. Its E.M.F. is about 1.5 volts.

469. The Dry Cell. The so-called *dry* cell is a modified form of the Leclanché cell. The carbon plate *C* (Fig. 526) is closely surrounded by a thick paste *A*, composed chiefly of powdered carbon, manganese dioxide and ammonic chloride. This is all contained in a cylindrical zinc vessel *Z*, which acts as the negative pole of the cell. Within the zinc container is a lining of cardboard, which acts the same as the porous pot in the Leclanché cell. Melted pitch *P*, is poured on top in order to prevent evaporation, *i.e.*, to prevent the cell from becoming really *dry*.

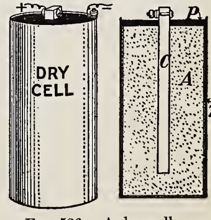


Fig. 526.—A dry cell.

470. Daniell Cell. The Daniell cell consists of a copper plate immersed in a concentrated solution of copper sulphate contained in an outer vessel and a zinc plate immersed in a zinc sulphate solution in an inner porous cup (Fig. 527).

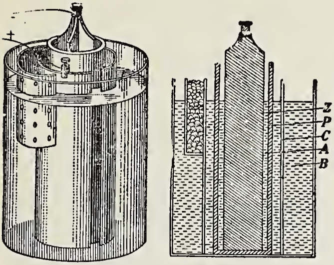


FIG. 527.—Daniell cell. *Z*, zinc; *P*, porous cup; *C*, copper; *A*, solution of zinc sulphate; *B*, solution of copper sulphate.

In a form of the Daniell cell known as the Gravity cell the porous cup is dispensed with and the solutions are separated by gravity (Fig. 528). The zinc plate, which is usually of the form shown in the figure, is supported near the top of the vessel and the copper plate is placed at the bottom. The copper sulphate, being denser than the zinc sulphate, sinks to the bottom, while the zinc sulphate floats above about the zinc plate. The copper sulphate solution is kept concentrated by placing crystals of the salt in a basket in the outer vessel (Fig. 527), or at the bottom about the copper plate (Fig. 528).

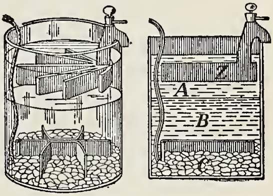


FIG. 528.—Gravity cell. *Z*, zinc plate; *A*, zinc sulphate solution; *B*, copper sulphate solution; *C*, crystals of copper sulphate.

When the plates are connected by a conductor, zinc goes into solution and copper, not hydrogen, is deposited on the copper plate. Polarization is, therefore, avoided, and the difference in potential is constantly maintained.

Accordingly, the Daniell cell is capable of giving a continuous current for an indefinite period if the materials are renewed at regular intervals; but the strength of the current is never very great because the internal resistance is high.

These cells are adapted for closed circuit work, when a comparatively weak current will suffice. The cell is damaged if left on open circuit because copper is deposited on the zinc plate and in the pores of the porous cup. The gravity type has been extensively used on telegraph lines, but in the larger installations the dynamo and the storage battery plants have now taken their place.

The E.M.F. of the cell is about 1.07 volts.

471. Storage Cells or Accumulators. If lead electrodes are substituted for the platinum in the experiment of § 453, and the battery current is made to pass through dilute sulphuric acid (1 of acid to 10 of water) for a few minutes, hydrogen will be liberated as before at the cathode, and the other lead plate (the anode) will be observed to turn a dark brown, while less oxygen is set free at its surface. The experiment may be readily performed by connecting the battery B, the electrolytic cell A, and the galvanoscope G, as shown in Fig. 529, mercury cups or keys being provided for opening and closing the circuits at C and D. When the circuit is closed at C but open at D, the electrolysis proceeds, and the galvanoscope indicates the direction of the current.

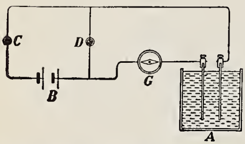


Fig. 529.—An experiment to illustrate the action of a storage cell.

If now the battery is cut out by opening C and the circuit is closed at D, a current will be found to pass through the galvanoscope in a direction opposite to the original current. The electrolytic cell now acts like an ordinary voltaic cell and can be used to ring an electric bell or for any other purpose for which a voltaic cell is used.

This experiment illustrates the principle of action of all *storage* cells or *accumulators*.

When the current is passed through the dilute acid from one plate to the other, the oxygen freed at the anode unites with the lead, forming an oxide of lead. The composition of the anode is thus made to differ from the cathode, and in consequence there arises a difference in potential between them which causes a current to flow in the opposite direction when the plates are joined by a conductor.

This current will continue to flow until the plates again become alike in composition, and hence in potential.

472. The Commercial Lead Storage Cell. Instead of using plates of solid lead, perforated plates or "grids" are usually employed (Fig. 530). When the cell is purchased in a fully charged condition, the holes in the positive plate are filled with lead peroxide (PbO_2) and those in the negative plate with spongy lead (Pb). Both of these active materials are porous and consequently present a very large surface to the action of the electrolyte. This gives the cell a greater ampere-hour capacity.

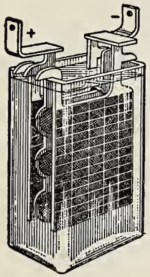


Fig. 530.—A storage cell, with one positive and two negative plates. Two positive and three negative, three positive and four negative, or even more plates may be used.

During the process of discharge both the plates are converted into lead sulphate, and a part of the sulphuric acid disappears, thus lowering the density of the electrolyte. When the cell is being charged (Fig. 531), the sulphion ions move to the anode and combine with lead sulphate and water to form lead peroxide and sulphuric acid, while the hydrogen ions on arriving at the cathode react upon the lead sulphate forming spongy lead and sulphuric acid; consequently the density of the electrolyte rises again. On account of this a battery hydrometer (§ 95) can be used to determine the general condition of a cell. The specific gravity of the solution in an

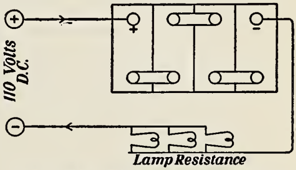


Fig. 531.—Charging a 4-cell (8-volt) storage battery from 110-volt D.C. mains.

automobile battery ranges from about 1.28 when fully charged to about 1.15 when completely discharged.

The E.M.F. of a fully charged cell is about 2.2 volts. The cell should be re-charged when the terminal voltage has dropped to about 1.8 volts at the normal rate of discharge.

The capacity of a storage battery is measured in *ampere-hours.* For example, an 80 ampere-hour battery should give a current of 1 ampere for 80 hours, or 2 amperes for 40 hours, when freshly charged.

Good lead cells have an efficiency of about 75%, that is, they give out on discharge about three-quarters of the electrical energy used in charging them.

Storage batteries are extensively used in connection with central power stations for emergency purposes. They are also used for automobile lighting and ignition, for wireless work, for driving submarines when submerged, in fire alarm and signal stations and for many other purposes.

473. Primary and Secondary Cells. The voltaic cells described in the early part of this chapter are known as *Primary Cells.* A current is available from such cells as soon as the unlike plates are immersed in the elecrolyte. A battery of voltaic cells will, of course, cease to give a current when the zinc plates are consumed, and these must be renewed from time to time.

In the storage cell the plates are alike in the beginning, and the differences in composition and potential are produced by an electrolytic process. Accordingly, such cells are known as *Secondary Cells.* As they can be discharged and recharged daily for a number of years, they furnish a cheap and reliable means of providing electric currents where portable batteries are the most convenient source of supply.

QUESTIONS AND PROBLEMS

1. What transformations of energy take place (1) in charging a storage cell, (2) in discharging it? Is anything "stored up" in the cell? If so, what?

2. Why is it possible to get a much stronger current from a storage cell than from a Daniell cell?

3. When a storage battery is discharged, the density of the electrolyte falls. Could a storage cell be re-charged by adding sulphuric acid?

4. If the capacity of a storage battery is 60 ampere-hours, how long would it give a current of 3 amperes? If its efficiency is 75%, how long would a current of 3 amperes have to be passed through it to re-charge it?

5. A storage battery is used to light 20 incandescent lamps, each requiring 0.4 amperes. If the capacity of the battery is 120 ampere-hours, how long should it light the lamps when fully charged?

6. If the electro-chemical equivalent of zinc is .000339, how much zinc must be used up in a voltaic cell to produce a current of 2 amperes for 15 min.? (Assume that no zinc is lost through local action.)

7. How many ampere-hours should be developed by the consumption in a voltaic cell of 3.051 gm. of zinc?

8. A battery of two cells is connected in series with a copper voltameter in which 3.5424 gm. of copper are deposited in 1 hr. How much zinc is dissolved in the battery in the same time?

CHAPTER XLIV

Magnetic Relations of the Current

474. Discovery of Electromagnetic Phenomena. The discovery by Oersted of the effect of an electric current on the magnetic needle (§ 447) gave a decided impetus to the study of electromagnetic phenomena. The investigations of Arago, Ampère, Davy, Faraday and others during the next ten years led to the discovery of practically all the principles that have had important applications in modern electrical development.

475. Magnetic Field Due to an Electric Current. In 1820, a year after Oersted's great discovery, Arago proved that a wire carrying a strong current had the power to lift iron filings, and hence concluded that such a wire must be regarded as a magnet. Two years later Davy showed that the apparent attraction was due to the fact that the particles of iron became magnets under the influence of the current, and that on account of the mutual attractions of the opposite poles they formed chains about the wire.

The action of the current on the filings may be shown by passing a thick wire vertically up through a hole in a card, and sprinkling iron filings from a muslin bag on the card. If the card is gently tapped while a strong current is passing through the wire, the filings arrange themselves in concentric rings about it. (Fig. 532).

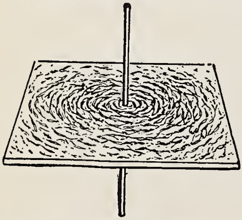


Fig. 532.—The presence of a magnetic field about a wire carrying an electric current shown by action on iron filings.

If a small jeweler's compass is placed on the card, and moved from point to point about the wire, it is found that in every position the needle tends to set itself with its axis

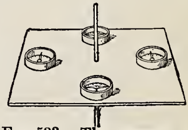


Fig. 533.—The presence of a magnetic field about a wire carrying an electric current shown by the action on a compass-needle.

tangent to a circle having the wire at its centre (Fig. 533). If one reverses the direction of the current, the direction in which the needle points is also reversed.

These experiments show that *a wire through which an electric current is flowing is surrounded by a magnetic field, the lines of force of which form circles around it.* Thus the wire throughout its entire length is surrounded by a "sort of enveloping magnetic whirl."

The direction in which a pole of the magnetic needle tends to turn depends on the direction of the current in the wire. Several rules for remembering the relation between the direction of the current and the behaviour of the needle have been given, two of them being as follows:—

1. *Imagine yourself swimming in the wire* WITH *the current and* FACING *the needle; then the N-pole of the needle will be deflected towards your* LEFT *hand.* (See § 447).

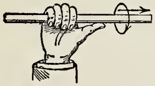


FIG. 534.—Direction of lines of force about a conductor.

2. *Suppose the right hand to grasp the wire carrying the current* (Fig. 534) *so that the thumb points in the direction of its flow; then the N-pole will be urged in the direction in which the fingers point.*

476. Magnetic Field about a Circular Conductor. Since the lines of force encircle a conductor, it would appear that a wire in the form of a circular loop carrying a current (Fig. 535) should act as a disc of steel magnetized so as to have one face an *N*-pole, the other an *S*-pole. That such is the case can be demonstrated by a simple experiment. Take a piece of copper wire and bend it into the

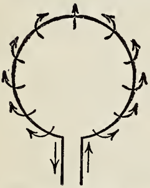


FIG. 535.—Lines of force about a circular loop.

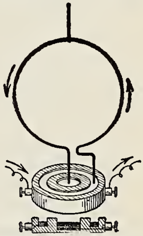


FIG. 536.—Experiment to show that a circular loop carrying a current behaves as a disc magnet.

form shown in Fig. 536, making the circle about 20 cm. in diameter. Suspend the wire by a long thread, and allow its ends to dip into mercury held in receptacles made in a wooden block of the form shown in the figure. (The inner receptacle should be about 2 cm. in diameter and the outer one 2 cm. wide, with a space of 1 cm. of wood between them.) Pass a current through the circular conductor by connecting the terminals of a battery with the mercury in the receptacles. For convenience in making connections, the receptacles should be connected by iron wires with binding-posts screwed into the block.

Now, if a bar-magnet is brought near the face of the loop, the latter will be attracted or repelled by its poles, and will behave every way as if it were a flat magnetic disc with poles at its faces. Indeed, if the current is strong, and the ends of the wire move freely in the mercury, the loop will set itself with its faces north and south under the influence of the earth's magnetic field.

In taking this position it obeys the general law, that a magnet when placed in the field of force of another magnet always tends to set itself in such a position that the line joining its poles will be parallel to the lines of force of the field in which it is placed.

To fulfil this condition the plane of the coil must become perpendicular to the direction of the lines of force of the field. *A coil carrying a current,* therefore, *always tends to set itself in the position in which the maximum number of lines of force will pass through it.*

477. Magnetic Conditions of a Helix. Ampère showed that the magnetic power of a wire carrying a current could be intensified by winding it into the form of a spiral. The magnetic properties of such a coil can be demonstrated by simple experiments.

Make a helix, or coil of wire, about three inches long, by winding insulated copper wire (No. 16 or 18) about a lead-pencil. Connect the ends of the wire with the poles of a voltaic cell, and with a magnetic needle explore the region surrounding it.

Next, make a helix somewhat larger in diameter, say about three-quarters of an inch, and place it in a rectangular opening made in a sheet of cardboard (Fig. 537). This can be done by cutting out two sides and an end of a rectangle of the proper size and then passing the free end of the strip lengthwise through the helix, and replacing the strip in position. Sprinkle iron filings from a muslin bag on the cardboard around the helix and within it. Attach the ends of the wire to the poles of a battery and gently tap the cardboard.

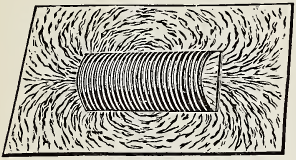


FIG. 537.—A helix carrying a current behaves like a bar-magnet.

In these experiments the helix through which the current is passing behaves exactly like a magnet, having *N* and *S* poles and a neutral equatorial region. The field, as shown by the action of the needle and the iron filings, resembles that of a bar-magnet. (Compare § 400.)

478. Polarity of the Helix and Direction of the Current. There is a fixed relation between the poles of the coil and the direction of the current passing through the wire. *If one looks at the south pole of the helix, the electric current passes through the coils in a clock-wise direction* (Fig. 538); or, we can give a "right-hand" rule similar to that in § 475, as follows:—*If the helix is grasped in the right hand,* as shown in Fig. 539, *with the fingers pointing in the direction in which the current is moving in the coils, the thumb will point to the N-pole.*

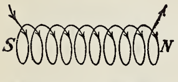


FIG. 538.—Relation of polarity of helix to the direction of the current—clock rule.

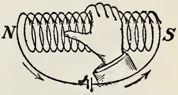


FIG. 539.—Relation of polarity of helix to direction of current—right-hand rule.

479. Electromagnet. Arago and Ampère magnetized steel needles by placing them within a coil of wire carrying a current. Sturgeon, in 1825, was the first to show that if a core of soft-iron is introduced into such a coil (Fig. 540), the magnetic effect is increased, and that the core loses its magnetism when the circuit is opened. The combination of the helix of insulated wire and a soft-iron core is called an *electromagnet.*

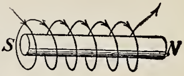


FIG. 540.—The essential parts of an electromagnet.

480. Why an Electromagnet is More Powerful than a Helix without a Core. When the helix is used without a core, the greater number of the lines of force pass in circles around the individual turns of wire, comparatively few running through the helix from end to end and back again outside the coil; but when the iron core is inserted, the greater number of lines of force pass in this latter way, because the permeability of iron is very much greater than that of air. Whenever a turn of wire is near the core, the lines of force, instead of passing in closed curves around the wire, change their shape and pass from end to end of the core. The effect of the core, therefore, is to increase the number of lines of force which are concentrated at the different poles, and consequently to increase the power of the magnet.

The strength of the magnet may be still further increased by bringing the poles close together so that the lines of force may pass within iron throughout their whole course. This is done either by bending the core into horse-shoe form, as shown in Fig. 541, or by joining two magnets by a 'yoke', as shown in Fig. 542. The lines of force thus pass from one pole to the other through the iron body held against them.

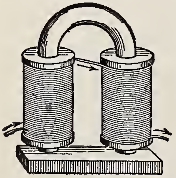


FIG. 541.—An electromagnet—horse-shoe form.

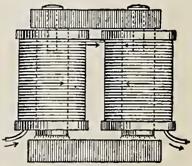


FIG. 542.—An electromagnet—yoke form.

481 Strength of an Electromagnet. The strength of an electromagnet depends equally on the strength of the current and on the number of turns of wire which encircle the core. This law is generally expressed by saying that the strength is proportional to the *ampere-turns* which surround the core, meaning that the strength varies as the product of the number of turns of wire about the core and the strength of the current measured in amperes.

ANDRÉ MARIE AMPERE (1775-1836). Born at Lyons, France. Discovered the action of one current upon another.

This law is true only when the iron core is not near to being magnetized to saturation. It should also be observed that when an electromagnet is used with a battery, or other source of current where the ends of the wire are kept at a constant difference of potential, an increase in the number of turns of the wire may not necessarily add to the strength of the magnet, because the loss in magnetizing force through loss in current caused by increased resistance may more than counterbalance the gain through the increased number of turns of wire.

QUESTIONS

1. In what circuit should a "long coil" electromagnet (one with a great number of turns of fine wire) be used—one in which the remaining resistance is great or small as compared with the resistance of the magnet? (See § 486.)

2. Why is a 2000-ohm wireless receiver usually more efficient than a 1000-ohm receiver?

482. Practical Applications of the Magnetic Effects of the Current. The discovery of the principles of electromagnetism was soon followed by practical applications. Schweigger modified Oersted's experiment by bending the wires into coils about the magnetic needle, and applied this device to detecting electric currents and comparing their strengths.

Ampère, in 1821, suggested the possibility of transmitting signals by electromagnetic action. Joseph Henry used an electromagnet at Albany, in 1831, for producing audible signals. In 1837 Morse devised the system by which dots and dashes, representing letters of the alphabet, were made on a strip of moving paper by the action of an electromagnet. About the same period, also, the possibility of producing rotary motion by the action of electromagnets was demonstrated by the experiments of Henry, Jacobi, Davenport, and others. At the present time electromagnets are used for a great variety of practical purposes, and range in size from minute ones actuated by currents measured in milli-amperes to the huge lifting magnets used in iron foundries to raise tons of iron at a time. The following sections contain descriptions of some of the more common applications.

483. The Electric Telegraph. The electric telegraph in its simplest form is an electromagnet operated at a distance by a battery and connecting wires. The circuit is opened and closed by a *key*. The electromagnet, which gives the signals, is called a *sounder*. When the current in the circuit is not sufficiently strong, on account of the resistance of the line, to work a sounder, a more sensitive electromagnet, called a *relay*, is introduced, which closes a local circuit containing a battery directly connected with the sounder.

484. The Telegraph Key. The key is an instrument for closing and breaking the circuit. Fig. 543 shows its construction. Two platinum contact points P are connected with the binding posts A and B, the lower one being connected by the bolt C, which is insulated from the frame, and the upper one being mounted on the lever L, which is connected with the binding-post B by means of the frame. The key is placed in the circuit by connecting the ends of the wire to the binding-posts.

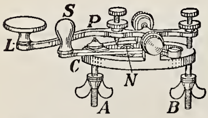


Fig. 543.—Telegraph key.

When the lever is pressed down, the platinum points are brought into contact and the circuit is completed. When the lever is not depressed, a spring N keeps the points apart. A switch S is used to connect the binding-posts, and close the circuit when the instrument is not in use.

485. The Telegraph Sounder. Fig. 544 shows the construction of the sounder. It consists of an electromagnet E, above the poles of which is a soft-iron armature A, mounted on a pivoted beam B. The beam is raised and the armature held by a spring S, above the poles of the magnet at a distance regulated by the screws C and D. The ends of the wire of the magnet are connected with the binding-posts.

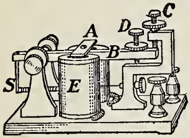


Fig. 544.—Telegraph sounder.

486. The Telegraph Relay. The relay is an instrument for closing automatically a local circuit in an office when the current in the main circuit, on account of the great resistance of the line, is too weak to work the sounder. It is a key worked by an electromagnet instead of by hand. Fig. 545 shows its construction. It consists of a "long coil" electromagnet R, in front of the poles of which is a pivoted lever L carrying a soft-iron armature, which is held a little distance from the poles by the spring S. Platinum contact points P are connected with the lever L and the screw C. The ends of the wire of the electromagnet are connected with the binding-posts B, B, and the lever L and the screw C are electrically connected with the binding-posts B_1, B_1.

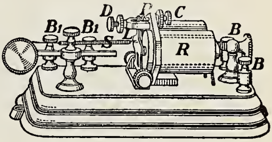


Fig. 545.—Telegraph relay.

Whenever the magnet R is magnetized, the armature is drawn toward the poles and the contact points P are brought together and the local circuit completed.

487. Connection of Instruments in a Telegraph System. Fig. 546 shows a telegraph line passing through three offices

A, *B*, and *C*, and indicates how the connections are made in each office.

When the line is not in use, the switch on each key *K* is closed and the current in the main circuit flows from the positive pole of the main battery at *A*, across the switches of the

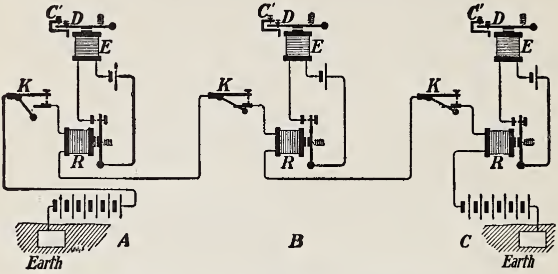


FIG. 546.—Connection of instruments in a telegraph circuit.

keys, and through the electromagnets of the relays, to the negative pole of the main battery at *C*, and thence through the battery to the ground, which forms the return circuit, to the negative pole of the main battery at *A*. The magnets *R*, *R*, *R*, are magnetized, the local circuits completed by the relays, and the current from each local battery flows through the magnet *E* of the sounder.

When the line is being used by an operator in any office *A*, the switch of his key is opened. The circuit is thus broken and the armature of the relay and of the sounder in each of the offices is released.

When the operator depresses the key and completes the main circuit, the armature of the relay in each office is drawn in, and the local circuit is completed. The screw *D* of each sounder is then drawn down against the frame, producing a 'click.' When he breaks the circuit at the key, the local circuit is again opened and the beam of each sounder is drawn up by the spring against the screw *C'*, producing another 'click' of different sound. If the circuit is completed and broken quickly by the operator, the two 'clicks' are very close together, and a "dot" is formed; but if an interval intervenes between the 'clicks', the effect is called a "dash." Different combinations of dots and dashes form different letters. The transmitting operator at A is thus able to make himself understood by the receiving operator at B or C.

488. The Electric Bell. Electric bells are of various kinds.

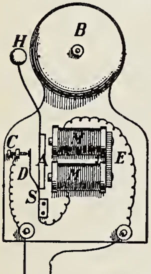
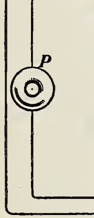


Fig. 547 shows the construction of one of the most common forms. It consists of an electromagnet M,M,E, in front of the poles of which is supported an armature A by a spring S. At the end of the armature is attached a hammer H, placed in such a position that it will strike a bell B when the armature is drawn to the poles of the magnet. A current breaker, consisting of a platinum-tipped spring D, attached to the armature, is placed in the circuit as shown in the figure.



When the circuit is completed by a push-button P, the current from the battery F passes from the screw C to the spring D and then through the

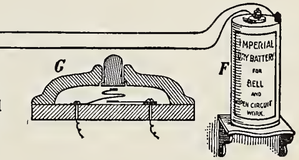


Fig. 547.—Electric bell and its connections. At G is shown a section of the push-button. The figure shows the bell when the button is not pressed. The current may pass in either direction through the electromagnets.

electromagnet to the battery. The armature is drawn in and the bell struck by the hammer; but by the movement of the armature the spring D is separated from the screw C, and the circuit is broken at this point. The magnet then releases the armature and the spring S pulls it back into its original position, thus completing the circuit again. The action goes on as before and a continuous ringing is thus kept up.

489. Galvanometers. Since the magnetic effect of the current varies as its strength, the strength of different currents may be compared by comparing their magnetic actions. Instruments for this purpose are called *Galvanometers.* There are two main types of the instrument.

In the first type, the strength of the current is measured by the deflection of a magnetic needle within a fixed coil, made to carry the current to be measured; in the second, the strength is measured by the deflection of a movable coil suspended between the poles of a permanent magnet.

The Galvanoscope described in § 449 is of the first type.

490. The Tangent Galvanometer. A more useful instrument of the first type is the *tangent galvanometer.* It consists of a short magnetic needle, not exceeding one inch in length, suspended, or poised at the centre of a large open ring or circular coil of copper wire not less than ten inches in diameter. A light pointer is usually attached to the needle, and its deflection is read on a circular scale placed under the pointer (Fig. 548)

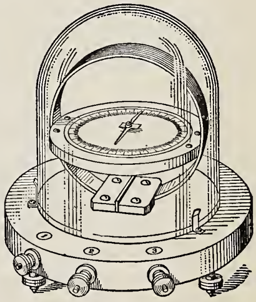


FIG. 548.—Tangent galvanometer.

This is called a 'tangent' galvanometer, because when the coil is placed parallel with the earth's magnetic meridian and a current passed through it, the *intensity of the current will vary as the 'tangent' of the angle of deflection of the needle.*

Thus, if the current corresponding to any angle of deflection, say one ampere, is known, the current corresponding to any other angle of deflection can be determined by referring to a table for the tangent of the angle, and making the necessary calculations.

491. The D'Arsonval Galvanometer. Galvanometers of the second type are generally known as *D'Arsonval galvanometers.* In this form the permanent magnet remains stationary, and a suspended coil rotates through the action of the current in the field of the permanent magnet. Fig. 549 shows the essential parts of the instrument, and in Fig. 550 a complete instrument is seen. N and S are the poles of a permanent magnet of the horse-shoe type. Between them is an elongated coil, suspended by the wires which lead the current to and from the coil. The deflection of the coil is indicated either by a light pointer and a scale (Fig. 550) or by a mirror D attached to the upper part of the coil to reflect a beam of light, which serves as a pointer to indicate the extent of the rotation. (Fig. 558).

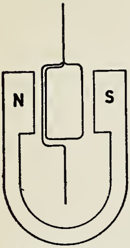


Fig. 549.—Moving coil of galvanometer.

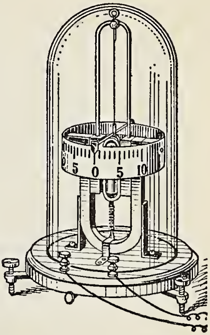


Fig. 550.—A convenient galvanometer

The coil is brought to the zero by the torsion of the suspension wires. When the current is passed through it, the coil tends to turn in such a position as to include as many as possible of the lines of force of the field of the permanent magnet, and the deflection is approximately proportional to the strength of the current. Instruments of this type may be made exceedingly sensitive.

492. Ammeters and Voltmeters. 1. *Ammeters.* A galvanometer with a scale graduated to read amperes is called an *ammeter*. The instrument must have a low resistance in order that it may be placed in the circuit without sensibly changing the strength of the current to be measured. (Fig. 551.) On account of this low resistance an ammeter should *never* be connected directly to a storage battery or even to a dry cell in good condition. A suitable resistance must always be in series with it and the cell. Indeed, the function of the ammeter is to measure the current which the cell sends through this resistance, and if the resistance is not in the circuit, or if it is not great enough, the current which passes through the ammeter may be large enough to ruin it. In all doubtful cases the ammeter should be protected by a piece of fuse wire, as shown in the figure.

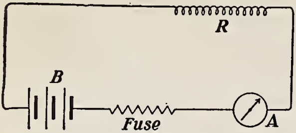


FIG. 551.—Method of inserting an ammeter A to measure the current sent by battery B through resistance R. R may be any electrical instrument or appliance.

The best portable ammeters used for commercial purposes are of the movable coil type and may be considered as modifications of the D'Arsonval galvanometer. Fig. 552 shows an instrument of this class. The coil C, having a stationary soft-iron cylinder within it, is pivoted on jewel bearings between the poles N and S of a permanent magnet of great constancy. It is brought to the zero position by a coil spring *sp*.

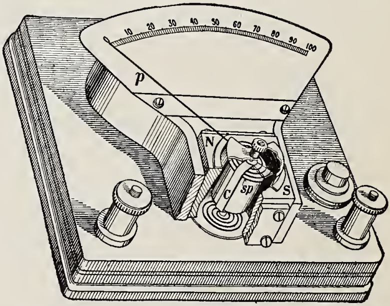


FIG. 552.—Ammeter or voltmeter. C, movable coil; *sp*, one of the springs; N, S, poles of the permanent magnet; p, pointer.

When the current is passed through the instrument, the coil, to which a pointer p is attached, reacts against the spring and turns about within the field of the magnet.

Each instrument is calibrated by comparison with a standard instrument placed *in series* with it. The method of calibrating a standard ammeter by using a voltameter has been mentioned in § 456.

2. *Voltmeters.* If the galvanometer is to be used to measure potential differences between points in a circuit, it should have high resistance, in order that the current in the main circuit may not be altered when the instrument is connected to the two points (Fig. 553). When the scale is graduated to read directly in volts, the instrument is a *voltmeter*.

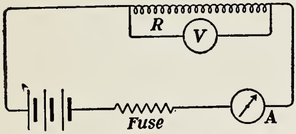


FIG. 553.—The voltmeter V is being used to measure the p. d. between the terminals of the resistance R. The ammeter A registers the current flowing through R.

Since the function of a voltmeter is to measure potential difference, and since it has a high resistance, it *can* be connected directly to a storage battery or other source of E.M.F., provided that the E.M.F. does not exceed the range of the instrument. Voltmeters are calibrated by being placed *in parallel* with a standard voltmeter.

493. Difference between Ammeter and Voltmeter. The essential difference in construction between the two classes

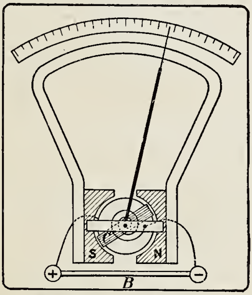


FIG. 554.—The Ammeter.

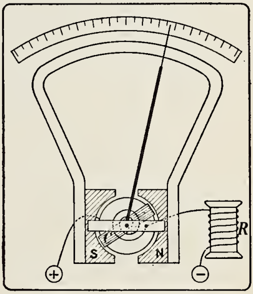


FIG. 555.—The Voltmeter.

of instrument is shown in Figs. 554 and 555. In the ammeter only a small fraction of the current passes through the coil C, the main current being carried by the shunt B, which has a very low resistance. In the voltmeter the high resistance is obtained by inserting the resistance coil R in series with the moving coil. In both instruments, then, the current actually passing through the moving coil is small.

QUESTIONS

1. Imagine an electric current to be flowing in the direction indicated in each of the loops of wire represented in Fig. 556. Which pole of the coil is at a and which at b in each case?

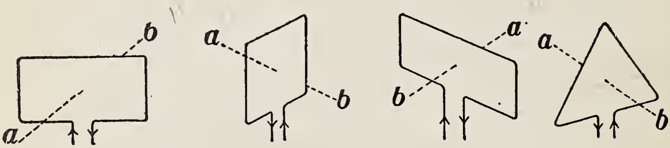


FIG. 556.—Which side of the loop is N and which S?

2. If n and s represent the poles at the faces of the coils when a current is passed through each loop of wire represented in Fig. 557, in which direction, a or b, is the current flowing in each case?

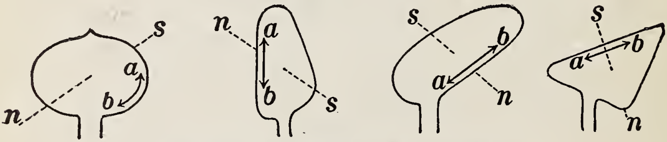


FIG. 557.—In what direction does the current flow?

3. If you were given a voltaic cell, wire with an insulating covering, and a bar of soft-iron, one end of which was marked, state exactly what arrangements you would make in order to magnetize the iron so that the marked end might be an N-pole. Give diagram.

4. A current is flowing through a rigid copper rod. How would you place a small piece of iron wire with respect to it so that the iron may be magnetized in the direction of its length? Assuming the direction of the current, state which end of the wire will be an N-pole.

5. A telegraph wire runs north and south along the magnetic meridian. A magnetic needle free to turn in all directions is placed over the wire. How will this needle act when a current is sent through the wire from south to north? Supposing the wire to run east and west, how would you detect the direction of the current with a magnetic needle?

6. An insulated wire is wound round a wooden cylinder from one end *A* to the other end *B*. How would you wind it back from *B* to *A* (1) so as to increase, (2) so as to diminish, the magnetic effects which it produces when a current is passed through it? Illustrate your answer by a diagram drawn on the assumption that you are looking at the end *B*.

7. A small coil is suspended between the poles of a powerful horse-shoe magnet, and a current is made to flow through it. How will the coil behave (1) when its axis is in the line joining the poles of the magnet? (as in Fig. 560). (2) when it points at right angles to that line?

8. If it were true that the earth's magnetism is due to currents traversing the earth's surface, show what would be their general direction.

9. An elastic spiral wire hangs so that its lower end just dips into a vessel of mercury. When the top of the spiral is connected with one terminal of a battery, and the mercury with the other, it vibrates, alternately breaking and closing the circuit at the point of contact of the end of the wire and the mercury. Explain this action.

10. Would the ammeters and voltmeters described in § 492 respond to alternating currents? Give reasons. Could alternating current instruments be constructed by substituting coils for the permanent magnets? Make a diagram.

REFERENCE FOR FURTHER INFORMATION

Lectures on Electrical Apparatus and Experiments, Weston Electrical Instrument Co., Newark N. J.

CHAPTER XLV

Induced Currents

494. Faraday's Experiments. Much of the life of the great investigator Faraday was occupied in endeavours to trace relations between the various "forces of nature"—gravitation, chemical affinity, heat, light, electricity and magnetism. Seeing that magnetic effects could be produced by an electric current, he felt sure than an electric current could be obtained by means of a magnet. During seven years (1824-31), he devoted considerable time to securing experimental proof of this, and at last, in August, 1831, was successful.

495. Production of Induced Currents. Faraday's original experiments are simple and can be performed by anyone without difficulty. Let us take a coil of many turns of fine insulated wire wound on a hollow spool of the form shown in Fig. 558 and connect the ends of the wire to a sensitive galvanometer. Thrust the pole of a bar-magnet into it, and then withdraw it; slip the coil over one pole of a horse-shoe magnet, and then remove it. In both cases the galvanometer indicates a current, in one direction when the pole passes within the coil, and in the opposite direction when it is withdrawn, but in each case *the current lasts only while the magnet and coil are in motion relative to each other.*

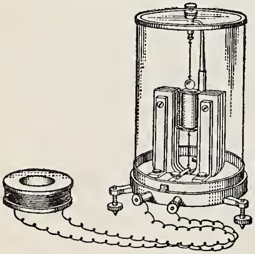


Fig. 558.—Apparatus for showing that when a magnet is thrust into or withdrawn from a closed coil a current is induced in the coil.

Faraday's discoveries may be summed up in the following statement:—*Whenever, from any cause, the number of magnetic* *lines of force passing through a closed circuit is changed, an electric current is produced in that circuit.*

Such a current is known as an INDUCED CURRENT.

496. Direction of the Induced Currents; Lenz's Law. To investigate the direction in which the induced current flows, first connect a dry cell through a suitable resistance to the galvanometer and note which way the pointer swings when the current enters the instrument by a given terminal. Having done this it will be possible, by observing the direction of the deflection, to say by which terminal the current enters the galvanometer.

Next, disconnect the cell and, having connected the coil again as in Fig. 558, insert the N-pole of a bar magnet into the coil. From the deflection of the galvanometer and the way the coil is wound, trace out the direction in which the induced current must be flowing around the upper face of the coil. It will be found that the direction is anti-clockwise, and that therefore this face of the coil is made an N-pole by the induced current.

On withdrawing the N-pole, the induced current is found to flow in the opposite direction, and hence the upper face of the coil is now made an S-pole by the induced current.

Now since like poles repel, the insertion of the N-pole must have been *opposed* by the N-pole produced by the induced current; also, since unlike poles attract, the withdrawal of the N-pole was *opposed* by the S-pole produced by the induced current. Similar results will be obtained by inserting and withdrawing the S-pole of the bar magnet.

Hence, in all cases of electromagnetic induction, *the direction of the induced current is always such that it produces a magnetic field which opposes the motion or change which induces the current.* This is known as LENZ'S LAW.

497. Primary and Secondary Currents. If an electromagnet (Fig. 559) is used in place of the bar magnet in the preceding experiments, similar results will be obtained.

Again, if an electromagnet is allowed to rest within the coil and the battery circuit is quickly broken or closed by a

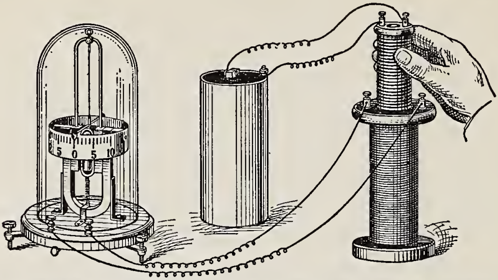


Fig. 559.—Apparatus for showing that an electromagnet acts like a permanent magnet in inducing currents in a coil.

key, an induced current will be produced in the coil each time the circuit is broken or closed. This is what we should expect, since breaking the circuit is equivalent to withdrawing the electro magnet, while making the circuit is the same as inserting the electromagnet.

The coil connected with the battery is called the *primary coil*, and the current which flows through it is called the *primary current*; the coil connected with the galvanometer is called the *secondary coil*, and the momentary currents made to flow in it, *secondary currents.*

498. Relative Directions of Primary and Secondary Currents. If the last experiment of the preceding section is repeated and care taken to trace the directions of the currents in the primary and secondary coils, it will be found that *whenever a decrease in the number of lines of force which pass through the secondary coil takes place, the secondary current flows in the same direction as the primary current, but that whenever an increase in the number of lines of force takes place, the secondary current is opposite in direction to the primary current.*

Exercise.—Show by diagrams that the relations just stated between the directions of the primary and secondary currents can be deduced directly from Lenz's Law.

499. Electromotive Force of Induced Currents. If, in any of the preceding experiments on the production of induced currents by the movement of magnets or coils, attention is given to the relation of the rapidity of motion of the magnet or coil to the effect on the galvanometer, it will be observed that the more speedy the motion, the wider is the swing of the pointer of the galvanometer. Careful experiments have shown that *the magnitude of the electromotive force induced in any circuit at a given instant, is proportional to the rate at which the number of magnetic lines of force passing through the circuit is being changed.*

It should be observed that even when the circuit is not closed a change in the number of magnetic lines of force passing through it will produce a surge of electricity through the conductors which will result in a potential difference being established between the terminals. Many of the circuits used in wireless telegraphy are of this type.

It should be noted, also, that the *motion* of a conductor within a magnetic field does not necessarily develop an E.M.F. in it. It does so only when such motion is across the direction of the lines of force, and it is obvious that it does not do so when the conductor is moved in the direction parallel with the lines of force of the field. This may be shown by connecting the coil used in the previous experiments with the galvanometer and moving it in various directions about the poles of a horse-shoe magnet. The needle is undisturbed when the coil is moved to and fro between the poles, in the position shown in Fig. 560; but if the coil is moved up and down, or is placed between the poles and turned

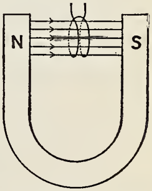


Fig. 560.—If the coil is moved to right or left no current is induced in it.

about a horizontal or a vertical axis, or is moved in any other way which causes *the number of lines of force passing through it to change,* a current is generated.

500. Self-Induction. If an electromagnet containing many turns of wire is connected with a battery and the circuit is closed and opened by touching the two ends of the connecting wires together and then separating them, a spark will be observed at the ends of the wires when they are separated, and if the hands are in contact with the bare wires, a shock will possibly be felt.

The effects observed are due to what is known as *self-induction.*

We have seen that, in the case of two distinct coils of wire near each other, when a current is started or stopped in one, a current is induced in the other. This is due to the fact that the number of magnetic lines of force passing through the second coil is thereby altered. But we can have this inductive effect with a single coil. When the current is broken the magnetic field accompanying it begins to collapse and the withdrawal of this field of force from the coil sets up a self-induced current which, in accordance with Lenz's Law, opposes the withdrawal of the field. In other words, the current tends to keep on flowing, like a car which continues to run for some time after the power which drives it is shut off.

On completing the circuit, a magnetic field is introduced into the coil and the introduction of this field produces an induced current in the coil which opposes the oncoming current. This action is similar to the inertia of a car when it is started; it takes some time for the car to acquire its maximum speed. Indeed, self-induction may be considered as electromagnetic inertia.

The direct induced current in the primary wire itself, which tends to strengthen the current when the circuit is broken, is called the *extra current.*

This self-induced current is of high E.M.F., and therefore jumps across the air space as the wires are separated, thus producing the spark.

The self-induction or, as it is frequently called, the *inductance*, of electrical circuits is of great importance in wireless telegraphy. It is applied also in "choke" coils used in connection with lightning arresters.

Inductance is measured in *henries**. A circuit has an inductance of one henry when a current changing at the rate of 1 ampere per second induces an E.M.F. of 1 volt in the circuit.

501. The Induction Coil.† In the induction coil currents of very high electromotive force are produced by the inductive action of an interrupted current. (Fig. 561.)

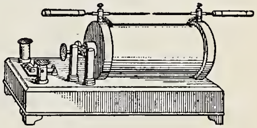


Fig. 561.—The induction coil.

The essential parts of the instrument are shown in Fig. 562. The primary coil consists of a few turns of stout insulated wire wound about a soft-iron core. The secondary coil, consisting of a great number of turns of very fine insulated wire, surrounds the primary coil. Its terminals are attached to binding-posts placed above the coil. The current-breaker is usually of the type illustrated in the electric bell (§ 488), but other forms of interrupters are often employed. The condenser C C' is made up of alternate layers of tinfoil and paraffined paper or mica, connected with the spring A and the screw B of the current-breaker in such a way that one of these is joined to the even sheets of the foil and the other to the odd ones. The core is a bundle of soft-iron wires insulated from one another by shellac. Such a core can be magnetized and demagnetized more easily than one of solid iron.

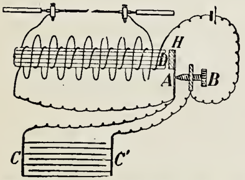


Fig. 562.—The essential parts and electrical connections in the induction coil.

*Named in honour of Joseph Henry (1799-1878), a noted scientific pioneer of the United States (See §§482, 548.)

†The induction coil was greatly improved by Ruhmkorff (1803-1877), a famous manufacturer of scientific apparatus in Paris, and is often called the Ruhmkorff coil.

502. Explanation of the Action of the Coil. When the primary circuit is completed, the battery current passes through the coil and magnetizes the core. This draws in the hammer *H*, and the circuit is broken between the spring *A* and the screw *B*. The hammer then flies back, the circuit is again completed and the action is repeated. An interrupted current is thus sent through the primary coil, which induces currents of high electromotive force in the secondary.

Self-induced currents in the primary circuit interfere with the action of the coil. When the primary circuit is completed, the current due to self-induction opposes the rise of the primary current and thus diminishes the inductive effect. Similarly, the extra-current induced in the primary coil when the circuit is broken passes across the break in the form of a spark and prolongs the time of fall of the primary current, again lessening the inductive action. The condenser is introduced to prevent this latter injurious effect. When the circuit is broken the extra-current flows into the condenser and charges it, but as the two coatings are joined between *A* and *B* through the primary coil and the battery, the condenser is immediately discharged, giving rise to a current in the opposite direction which flows back through the primary coil and *instantaneously* demagnetizes the soft-iron core. Thus, a very high E.M.F. is induced in the secondary coil when the primary circuit is broken and the potential difference between the terminals of the secondary coil can be made sufficiently great to cause a spark to pass between them, the length of the spark depending on the capacity of the coil. Coils giving sparks from 18 to 24 inches are frequently manufactured.

The smaller coils are used extensively for physiological purposes and for gas engine ignition (see § 291), and the larger for exciting vacuum tubes and for wireless telegraphy (see § 556).

Exercise.—Examine the ignition system of an automobile or a motor-boat. Observe how the current from the storage (or other) battery is made and broken in the primary circuit; and how the high-tension secondary current is distributed to the spark-plugs.

QUESTIONS

1. You have a metal hoop. By means of a diagram describe some arrangement by which, without touching the hoop, you can make electric currents pass around it, first one way, and then the other.

2. A coil about one foot in diameter, made of 400 or 500 turns of fine insulated wire, is connected with a sensitive galvanometer. When it is held with its plane facing north and south, and then turned over quickly, the needle of the galvanometer is disturbed. Give the reason for this.

3. A bar of perfectly soft iron is thrust into the interior of a coil of wire whose terminals are connected with a galvanometer. An induced current is observed. Could the coil and bar be placed in such a position that the above action might nearly or entirely disappear? Explain fully.

4. Around the outside of a deep cylindrical jar are coiled two separate pieces of fine silk-covered wire, each consisting of many turns. The ends of one coil are joined to a battery, those of the other to a sensitive galvanometer. When an iron bar is thrust into the jar, a momentary current is observed in the galvanometer coils, and when it is drawn out, another momentary current (but in an opposite direction) is observed. Account for these results.

5. A small battery was joined in circuit with a coil of fine wire and a galvanometer, in which the current was found to produce a steady but small deflection. An unmagnetized iron bar was now plunged into the hollow of the coil and then withdrawn. The galvanometer needle was observed to recede momentarily from its first position, then to return and to swing beyond it with a wider arc than before, and finally to settle down to its original deflection. Explain these actions, and state what was the source of the energy that moved the needle.

6. The poles of a voltaic battery are connected with two mercury cups. These cups are connected successively by:—(1) A long straight wire. (2) The same wire arranged in a close spiral, the wire being covered with some insulating material. (3) The same wire coiled around a soft-iron core. Describe and discuss what happens in each case when the circuit is broken.

REFERENCES FOR FURTHER INFORMATION

Hadley, *Magnetism and Electricity for Students*, Chapter 22.
Hutchinson, *Magnetism and Electricity*, Chapter 17.

CHAPTER XLVI

Applications of Induced Currents

503. The Principle of the Dynamo. In its simplest form, a dynamo is a coil of wire rotated about an axis in a magnetic field. The principle may be illustrated by connecting to the galvanometer the coil used in the experiments on current induction and rotating it about a vertical axis between the poles of a horse-shoe magnet. Continuous rotation in one direction is prevented by the twisting of the connecting wires about each other. In a working dynamo this difficulty is overcome by joining the ends of the wires to rings, from which

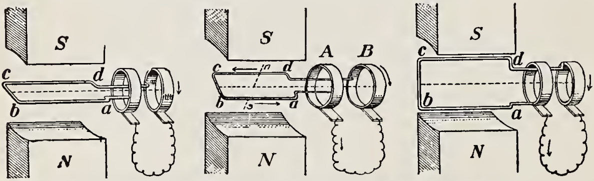


Fig. 563.—Showing what takes place in the first quarter of a rotation.

the current is taken by brushes bearing upon them.

Now as the coil rotates about its axis from the first position shown in Fig. 563, the number of the lines of force passing

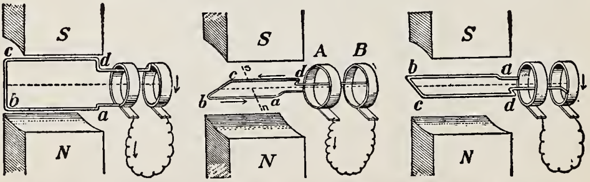


Fig. 564.—Production of a current during the second quarter of a rotation.

through the coil will be decreasing during the first quarter turn (Fig. 563), increasing during the second quarter turn, (Fig. 564), decreasing during the third quarter turn (Fig. 565),

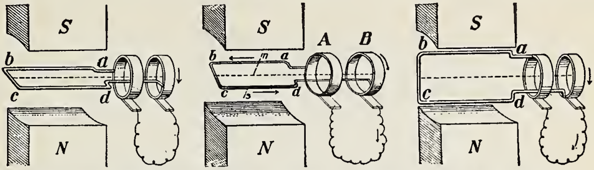


Fig. 565.—Production of a current during the third quarter of a rotation.

and again increasing during the fourth quarter turn (Fig. 566).

The number of lines of force which cut the coil is accordingly always increasing or decreasing as the coil is revolved between

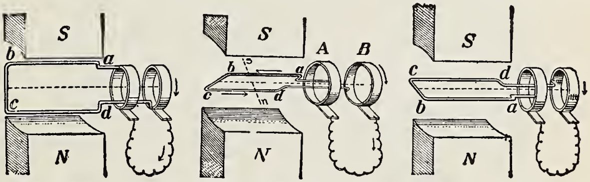


Fig. 566.—Production of a current during the fourth quarter of a rotation.

the poles of the magnet. The revolution of the coil is, therefore, accompanied by the development of induced currents within the circuit.

In what direction do these induced currents flow? In obedience to Lenz's Law (§ 496) the magnetic fields produced by the induced currents as the coil revolves upon its axis must be such as to oppose the rotation of the coil. But we have learned that a loop of wire carrying a current behaves very much like a magnetized steel disc with its opposite faces as N and S poles (§ 476). Then if N and S represent the poles of the permanent fields, and n and s the poles of the fields produced by the induced currents set up by the revolution of the coil, the n and s poles must be distributed in each position in the rotation in such a way that the motion is opposed by the attractions of the N and s poles and of the S and n poles, and by the repulsions of the N and n poles and of the S and s poles. To meet these conditions, in each of the quarter turns of the revolution, the poles must be distributed as shown for the first quarter turn by the dotted lines in Fig. 563; for the second quarter turn in Fig. 564; for the third quarter turn in Fig. 565; and for the fourth quarter turn in Fig. 566.

But in order to produce such fields the induced currents must flow in the directions shown by the arrows in the figures. (§ 478). That is to say, during the first half turn (Figs. 563, 564) the currents must flow in the direction $d\ c\ b\ a$ and during the next half turn (Figs. 565, 566) in the direction $a\ b\ c\ d$.

504. The Armature of the Dynamo. We have, for simplicity, considered in the preceding section the case of the revolution of a single coil within the magnetic field. In ordinary practice a number of coils are connected to the same collecting rings or plates. These coils are wound about a soft-iron core, which serves to hold them in place and to increase the number of lines of force passing through the space inclosed by them. The coils and core with the attached connections constitute the *armature* of the dynamo.

The armatures vary in type with the form of the core and the winding of the coils. A single coil wound in a groove about a soft-iron cylinder (Fig. 567) forms a *shuttle* armature; when a number of coils are similarly wound about the same iron cylinder the armature is said to be of the *drum* type.

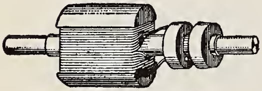


Fig. 567.—Shuttle armature.

In Fig. 568 is shown a drum armature with numerous coils, each containing several turns of wire fitted into grooves cut in the iron core parallel with the shaft. To prevent the generation of "eddy currents," within the iron itself, which are wasteful of energy and overheat the machine, the armature core is built up of thin soft-iron discs insulated from one another.

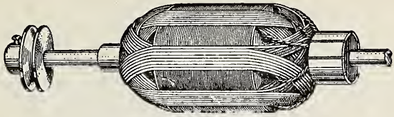


Fig. 568.—Ordinary drum armature.

505. Field Magnets. In small generators, used to develop high tension (or high potential) currents, permanent magnets are sometimes used to produce the fields. The machine is then called a *magneto*. Magnetos are used in rural and portable telephones for calling the distant station, and in gas-engines for ignition. A telephone magneto is shown diagrammatically in Fig. 569 while its general appearance is represented in Fig. 570. In all ordinary dynamos the field is furnished by electromagnets known technically as *field-magnets*. These magnets are either bipolar (Fig. 575) or multipolar (Fig. 571). In the multipolar type two or more pairs of poles are arranged in a ring about a circular yoke.

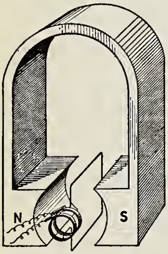


Fig. 569.—Principle of the telephone magneto.

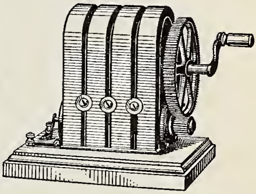


Fig. 570.—Appearance of the telephone magneto. The small gear-wheel is on the end of the armature shaft.

506. The Alternating Current Dynamo. When an alternating current is used for electric lighting or power transmission, the complete alternations* or *cycles* range from 25 to

*By a complete alternation or cycle is meant a motion of the current forwards and backwards.

60 per second. Now such a current cannot be generated in a bipolar field except by unduly increasing the rapidity of the revolutions of the armature, because the current changes direction but twice each revolution. The requisite number of cycles is secured by increasing the number of pole-pieces in the field-magnets. In the alternators in common use, the armature coils A, A . . . (Fig. 571) revolve in a multipolar field. They are wound alternately in opposite directions and connected in series, and the two free ends of the wire are brought to two collecting rings, C and D, as shown in the figure.

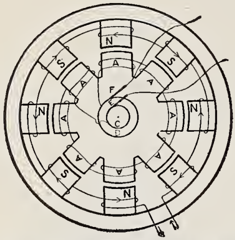


FIG. 571.—Essential parts and electrical connections in the alternating current dynamo.

To study the action, suppose the ring of armature coils to be opposite to the ring of field coils, and to be revolving in either direction. Since the armature coils leaving positions opposite N-poles in the field have currents induced in them opposite in direction to those in the coils leaving S-poles, and since these coils are wound alternately to the right and the left, it is evident that the induced current in each coil will be in such a direction as to produce a continuous current in the whole series, which will flow from one collecting ring to the other. It is evident, also, that the direction of this current will be reversed the instant the field and armature coils again face each other.

Since the number of single alternations of this current for each revolution of the armature equals the number of poles in the field-magnet, the number of single alternations per second is equal to the number of poles in the field-magnet multiplied by the number of revolutions made by the armature per second. The number of complete alternations or cycles will be one-half this number.

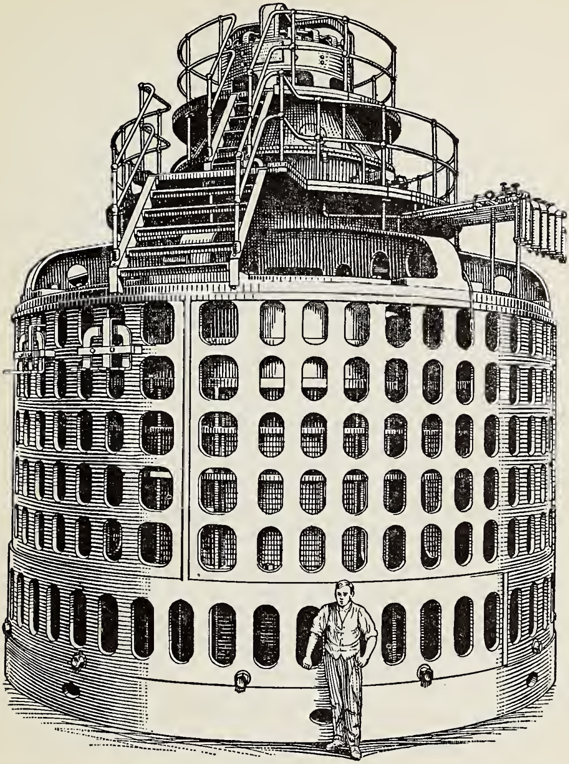


Fig. 572.—One of the great generators of the Hydro-Electric Power Commission of Ontario at Queenston (see § 132). The complete generator is $24\frac{1}{2}$ ft. in diameter, $30\frac{1}{2}$ ft. in height and weighs 1,380,000 pounds. In these machines the field-magnets rotate while the armature remains stationary. The rotor, or moving part, weighs 615,000 pounds and rotates $187\frac{1}{2}$ times per minute. The output is 45,000 kw. The current is generated at 12,000 volts, 25 cycles per second. The generators were built by the Canadian Westinghouse Co. at Hamilton and the Canadian General Electric Co. at Peterborough, Ont.

507. Production of a Direct Current—The Commutator. When an electric current flows continuously in one direction it is said to be a *direct* current. The current in an armature coil changes direction, as we have seen, at regular periods. To produce a direct current with a dynamo it is necessary to provide a device for commuting the alternating into a direct current. This is done by means of a *commutator*. It consists of a collecting ring made of segments called *commutator bars,* insulated from one another. The terminals of the coils are connected in order with the successive bars of the ring. Take, for example, the case of a single coil revolved in a bipolar field, as considered in § 503. The commutator

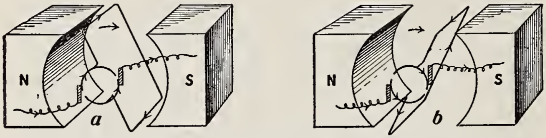


FIG. 573.—Explaining the action of the commutator. While the armature coil moves from its position in *a* to its position in *b* the direction of the current is reversed, but the direction in the external circuit is unchanged.

consists of two semi-circular bars, (Fig. 573), and the brushes are so placed that they rest upon the insulating material between the bars at the instant the current is changing direction in the coil. Then since the commutator bars change position every time the current changes direction in the coil, the current always flows in the same direction from brush to brush in the external circuit.

508. Direct Current Dynamo. The essential parts of a direct current dynamo with shuttle armature and bipolar field are shown in Fig. 574. In this particular dynamo the field-magnets are in series with the armature and the external circuit, and the same current flows through all parts of the circuit. When the armature is rotated in the clockwise direction the induced current leaves the armature by the brush A and passes around the field-magnets in such a direction as to make the right-hand pole-piece an S-pole and the left-hand one an N-pole. It then passes through the external circuit, (consisting, in this case, of two lamps,) and then back to the armature by the brush B.

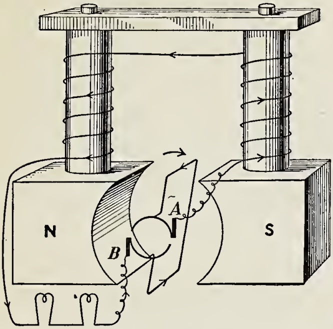


FIG. 574.—Principle of construction of a direct current dynamo.

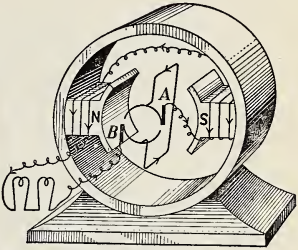


FIG. 575.—A common form of direct current dynamo.

The connections in the common circular-shaped dynamo are shown in Fig. 575.

509. Drum Armature. The action in a drum armature is similar. In order to simplify the diagram only one turn in each coil of a four-coil armature and one pole of the field-magnet have been drawn. (Fig. 576).

Consider first the winding on the armature without reference to the commutator. Starting at the point x and proceeding by the shortest path to 1, we find, on continuing along the coils, that coils 1, 2, 3 and 4 are in series in the order named and that they form a closed circuit. From the points of junction of 1 and 2 a connection is made to the commutator bar a, from the junction of 2 and 3 to bar b, from the junction of 3 and 4 to bar c, and from the junction of 4 and 1 to the bar d.

By Lenz's Law the currents induced in the coils by the rotations of the armature flow in directions which will produce a magnetic field opposing the rotation. If the armature is rotating in a clockwise direction, as shown by the arrow, this will be the case when an *s*-pole is formed in the region *A* and

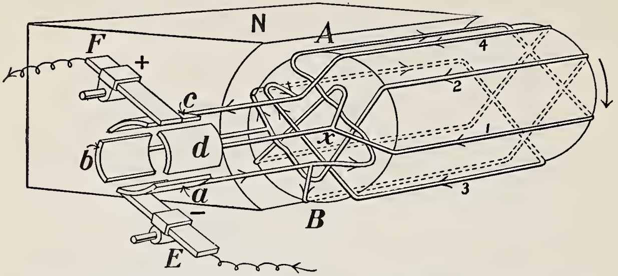


Fig. 576.—Diagram of a 4-coil drum armature, showing the connections of the coils with the commutator bars and the directions of the currents in the different coils.

an *n*-pole in the region *B*, because the rotation will then be opposed by the attractions of the *N*- and *s*-poles and of the *S*- and *n*-poles, and by the repulsions of the *N*- and *n*-poles and of the *S*- and *s*-poles. To form an *s*-pole at *A* and an *n*-pole at *B*, the induced currents in the coils must flow in the directions indicated by the arrows.

Considering now the junction of the coils 3 and 4, we see that the current is flowing towards that point in both coils, and consequently the commutator bar *c* and the brush *F* become positively charged. On the other hand, at the junction of coils 1 and 2 the flow is from the junction towards both coils, with the result that the bar *a* and the brush *E* become negatively charged. Hence a continuous current will flow

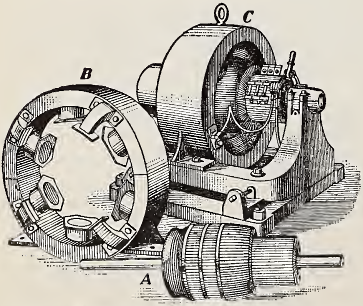


Fig. 577.—Modern direct-current dynamo. *A*, drum armature; *B*, multipolar field; *C*, dynamo complete.

from F to E through any conductor which joins these brushes.

It is obvious that in a multipolar field there must be as many pairs of collecting brushes as there are pairs of poles. Fig. 577 shows a modern direct-current dynamo with drum armature and multipolar field.

510. Excitation of the Fields in a Dynamo. In the alternating-current dynamo the electromagnets which form the fields are sometimes excited by a small direct-current dynamo belted to the shaft of the machine. In the great generator shown in Fig. 572 the exciter has a capacity of 150 kw., the voltage being 250. It is mounted above, with its armature on the upper end of the main shaft. In the direct-current dynamo the fields are magnetized by a current taken from the dynamo itself. When the *full* current generated in the armature (Fig. 578) passes through the field-magnets, which are wound with coarse wire, the dynamo is said to be *series-wound.* A dynamo of this class is used when a constant current is required, as in arc lighting. When the fields are energized by a small fraction of the current, which passes directly from one brush through many turns of fine wire in the field coils, to the other brush, while the main current does work in the external circuit (Fig. 579), the dynamo is *shunt-wound.* This type is used where the output of current required is continually changing, but where the potential difference between the brushes must be kept constant, as in incandescent lighting, power distributing, etc. The regulation is accomplished by suitable resistance placed in the shunt circuit to vary the amount of the exciting current.

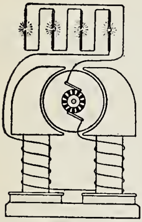


Fig. 578.—Series-wound dynamo.

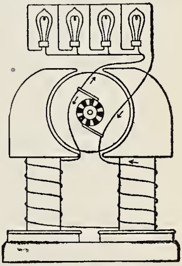


Fig. 579.—Shunt-wound dynamo.

The regulation is more nearly automatic

in the *compound-wound* dynamo. In this form the fields contain both series and shunt coils.

The field-magnets, of course, lose their strength when the current ceases to flow, but the cores contain sufficient residual magnetism to cause the machine to develop sufficient current to "pick up" on the start.

511. The Electric Motor. The purpose of the electric motor is to transform the energy of the electric current into mechanical motion. Its construction is similar to that of the dynamo. In fact, any direct-current dynamo may be used as a motor. As a simple example consider, a shunt-wound bipolar machine with shuttle armature (Fig. 580) connected with an external power circuit — in this case a number of cells.

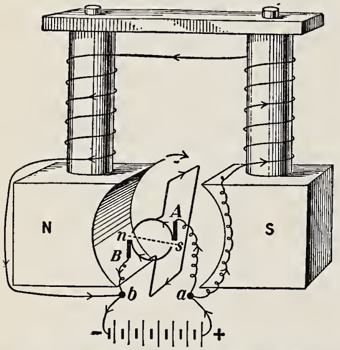


Fig. 580.—Essential parts and electrical connections in the direct-current electric motor.

The current supplied to the motor divides at a; part flows through the field-magnet coils, and part enters the armature coils by the brush A. The current through the armature coil passes out by the brush B and is joined at b by the part of the main current which flows through the field-magnet coils.

Both the field-magnet and the armature cores are thus magnetized, and the poles are formed according to the law stated in § 478. The poles of the field-magnet are as indicated in the figure. The armature will have an S-pole at s and an N-pole at n. The mutual attractions and repulsions between the poles of the armature and of the field-magnet cause the armature to revolve.

Exercise.—Trace the current through the armature and field-magnets when the negative terminal of the source of power is joined to *a* and the positive to *b*. It will be found that the armature will rotate in the same direction as before, and it should therefore rotate when a source of alternating current is applied to *a* and *b*.

Motors which will operate on both alternating and direct currents are sometimes called "universal" motors. To avoid "eddy" currents in the iron cores of the armature and field-magnets when alternating current is used, the cores should be laminated.

The larger alternating current motors are usually of the "induction" type and will not operate on direct current. An explanation of their action is beyond the scope of this book.

Question.—How could the motor be reversed?

512. Counter-Electromotive Force in the Motor. As the armature of the motor is revolved it will, as in the dynamo, develop an E.M.F. opposite to that of the current causing the motion. The higher the velocity of the armature, the greater is this counter-E.M.F. The electric motor is, therefore, self regulating for different loads. When the load is light, the speed becomes high and the increase in the counter-E.M.F. reduces the amount of current passing through the motor; on the other hand, when the load is heavy the velocity is decreased and the counter-E.M.F. is lessened, allowing a greater current for increased work.

When the motor starts from rest there is, at the beginning, no counter-E.M.F., and the current must be admitted to the armature coils gradually through a rheostat (a set of resistance coils), to prevent the overheating of the wires and the burning of the insulation.

QUESTIONS AND PROBLEMS

1. Upon what is the potential difference between the brushes of a dynamo dependent?

2. Why is more power needed to drive a dynamo delivering 20 amperes current than when delivering 10 amperes?

3. How should a dynamo be wound to produce (1) currents of high E.M.F.; (2) a current for electroplating?

4. A dynamo is running at constant speed; what effect will be produced on the strength of the field-magnets by decreasing the resistance in the external circuit (*a*) when the dynamo is series-wound; (*b*) when it is shunt-wound?

5. An alternating current dynamo has 16 poles, and its armature makes 300 revolutions per min.; find the number of cycles per sec. How many poles must the great generator in Fig. 572 have?

6. Why would an armature made of coils wound on a wooden core not be as effective as one with an iron core?

7. What would be the effect of moving the brushes of a dynamo backward and forward around the ring of commutator plates? Explain.

8. A direct-current motor is being used to drive a circular saw. An ammeter is in series with the motor. What changes will be noticed in the ammeter reading as the saw enters and leaves a piece of wood? Explain.

9. The motorman on a street-car moves his handle around slowly, admitting the current to the motors gradually. Why not move the handle completely around at once?

10. The swinging coil of a D'Arsonval galvanometer can be brought to rest quickly by short-circuiting the terminals. Explain.

513. The Transformer. If two independent coils are wound about the same iron core, as in Fig. 581, it is obvious that an alternating current in one coil A will produce an alternating current in the other B, if it is closed, because the core becomes magnetized in one direction, then demagnetized, and magnetized in the opposite direction at each change in the direction of the current in the primary circuit. Lines of force are thus made to pass through the secondary coil in alternate directions.

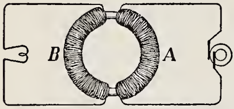


Fig. 581.— Current induced in one circuit by an alternating current in another circuit.

This is the principle of the transformer, a device for changing an alternating current of one electromotive force to that of another.

When the change is from low E.M.F. to high, the transformer is called a *step-up* transformer, and when from high to low, a *step-down* transformer. There are many forms of this instrument but the essential parts are all the same—two coils and a laminated soft-iron core, so placed that as many as possible of the lines of force produced by the current in one coil will pass through the space inclosed by the other.

Transformers used for commercial purposes are of two general types: first, transformers of the core type in which

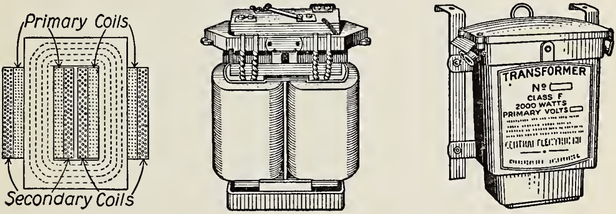


FIG. 582.—Core-type transformer, showing a vertical section, a general view and the transformer in its case on a pole.

the primary and secondary coils are wound about two parallel sides of a rectangular core, as shown in Fig. 582; and second, of the shell type, in which the coils are wound about a core shaped in the form of CD. The inner coils (Fig. 583) are the primary, and the outer the secondary. In the first case the cores are within the coils, and in the second case, the core is built around the coils.

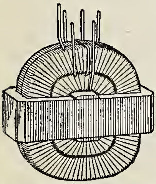


FIG. 583.—The shell-type transformer.

The electromotive force of the current generated in the secondary coil is to that of the primary current nearly in the ratio of the number of turns of wire in the secondary coil to the number in the primary.

For example, if the primary of a transformer has 200 turns and the secondary has 1000 turns, the voltage at the terminals of the secondary coil will be nearly 550 volts when an E.M.F.

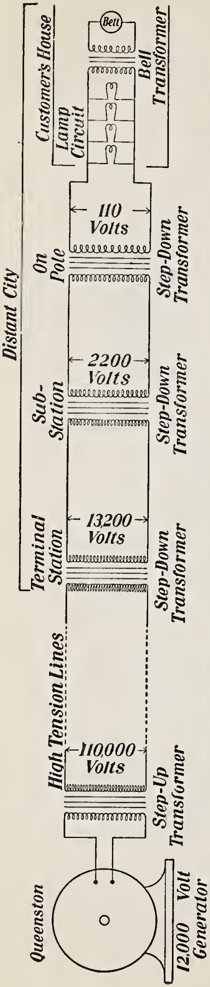


Fig. 584.—Diagram showing the different transformations which take place between Queenston and a house using Hydro-Electric Power.

of 110 volts is applied to the primary. It must be noted, however, that if a current of 2 amperes is taken from the secondary, 10 amperes (at least) must flow in the primary. The power in watts furnished by the secondary, which is obtained by multiplying the current by the voltage (2 × 550) can never be greater than that being used in the primary (10 × 110). With the numbers given, the transformer would be 100% efficient. In actual practice the efficiency of the best transformers is about 97%.

514. Uses of the Alternating Current. On account of the facility with which the E.M.F. of an alternating current may be changed by a transformer, alternating currents are now usually employed whenever it is found necessary or convenient to change the tension of a current. The most common illustrations are to be found in the case of the long distance transmission of electricity, where the currents generated by the dynamos are transformed into currents of very high E.M.F. for transmission, and again into currents of lower tension for use at the centres of distribution; and in the case of incandescent lighting, where it is advisable to have currents of fairly high tension in the street wires but, for the sake of safety and economy, currents of low E.M.F. in the lamps and house connections.

By stepping-up the voltage, the current-strength in the transmission wires is very much decreased (§ 513) and consequently the loss of energy through the development of heat in the conductor is lessened.* Also, because of the smaller current smaller conductors can be used and the cost of construction lowered. In the Hydro-Electric system which supplies many centres of Ontario with electric energy, the current when first generated at Niagara Falls is at a potential of 12,000 volts. It is then transformed to 110,000 volts and transmitted over well-insulated lines. On arriving at its destination it is transformed down again for use in lighting, power and heating.

In Fig. 584 is shown the different transformations which take place between Queenston and a house using Hydro-Electric power.

515. Telephone. The telephone, as invented by Alexander Graham Bell, employs the principle of induced currents for reproducing sound waves.

The transmitter and receiver first used were alike. Each consisted of an iron diaphragm A, supported in front of one end of a permanent bar-magnet B, about which was wound a coil of fine insulated wire, as shown in Fig. 585.

Fig. 585.—The essential parts and electrical connections in the original Bell telephone.

The terminals of the transmitter and receiver coils are connected by the line wires. The sound waves falling upon the diaphragm of the transmitter cause it to vibrate, and

*The heating is proportional to the square of the current (§§ 516, 546).

these vibrations produce fluctuations in the number of lines of force passing through the coil, which cause induced currents to surge to and fro in the circuit. The currents alternately strengthen and weaken the magnet of the receiver and thus set up vibrations in the diaphragm similar to those in the diaphragm of the transmitter.

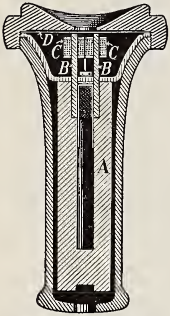


Fig. 586.—Telephone receiver.
A, permanent horse-shoe magnet; *B*, soft-iron pole-pieces; *C*, coils of fine wire; *D*, iron diaphragm.

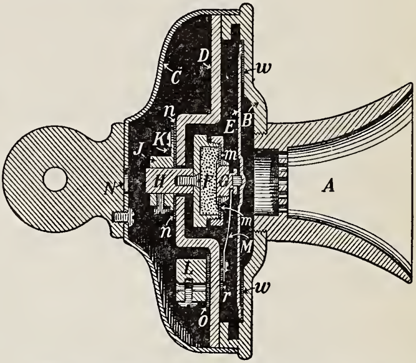


Fig. 587.—The microphone transmitter used in the the Bell system, about half natural size. *A*, mouthpiece; *E*, iron diaphragm; *F*, carbon granules of the microphone; *K* and *L*, terminals to which wires are joined. They pass out through *N*.

The Bell receiver is very sensitive and is still used on all telephone systems, but a magnet of the horse-shoe type is now usually employed instead of the bar magnet used in the original form.

The transmitter was not found satisfactory, especially on long distance lines, and has been replaced by one of the microphone type. In Fig. 587 is shown a section of a modern transmitter. It is enclosed in a metal case of which *B* is the front and *C* the back. Into *B* the mouthpiece *A* is screwed. Behind this is the soft-iron diaphragm *E*, which is separated from *B* by the insulating washer *w*, *w*. The essential part of the transmitter is the carbon microphone.

In a shallow round brass box granules of carbon *F*, in appearance resembling coal dust, are loosely packed. The front of the box is a thin disc of mica *m*, *m*, which carries a brass button *G*. In the front of this button is a non-conducting plug which rests against the centre of the diaphragm *E*. The brass box, or capsule, is fastened by a screw to a short brass rod *H*, and this is carried by a bent metal strip *D*.

The electric circuit is as follows. Let a current enter by a wire attached to *L*, which is insulated from *D* by mica washers *o*, *r*. It passes along the spring *M* to *G*, through the carbon granules to *H*, which is insulated from *D* by a mica washer *n*, *n*. Thence it leaves by a wire joined to *J* by the screw *K*. The two wires are led out through the hole *N*.

The connections of the instruments in the complete circuit are shown in Fig. 588. The transmitter acts on the principle

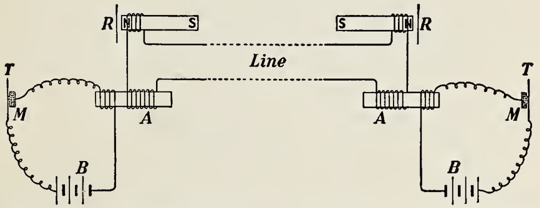


Fig. 588.—Diagram of telephone circuit. *T*, transmitter; *M*, carbon microphone; *B*, battery; *A*, step-up transformer; *R*, receiver.

that the conductivity of the granular carbon varies with the varying pressure exerted upon it, as the diaphragm vibrates under the action of the sound waves. The current passing from the battery through the primary coil of a transformer *A* will, therefore, be fluctuating in character and will induce a current of varying strength and varying direction, but of higher electromotive force, in the secondary coil which is connected in the main line with the receiver. This current will cause corresponding variations in the magnetic state of the electromagnet of the receiver and thus set up vibrations in its diaphragm, which will reproduce the sound waves that caused the diaphragm of the transmitter to vibrate.

In all telephones provision must be made for attracting the attention of the distant station. In rural and portable telephones this is done by turning the handle of a magneto (§505), which rings a bell or operates a magnetic indicator at the central switchboard. In city systems, when the receiver is lifted from its support, the support flies up and completes a circuit which lights a small lamp at 'Central.' Central establishes communication between two stations by connecting their lines which terminate in a panel before which the operator sits.

REFERENCES FOR FURTHER INFORMATION

Smith, *Elements of Applied Physics*, Chapters 29, 30, 31.
Ferry, *General Physics*, Chapter 20.
Jackson and Black, *Elementary Electricity and Magnetism*.

CHAPTER XLVII

Heating and Lighting Effects of the Electric Current

516. Heat Developed by an Electric Current. In discussing the sources of heat (§ 226) we referred to the fact that, whenever an electric current meets with resistance in a conductor, heat results. Now, as no body is a perfect conductor of electricity, a certain amount of the energy of the electric current is always transformed into the energy of molecular motion. Joule, who investigated this subject, found, by comparing the results of numerous experiments, that in a given time *the number of heat units developed in a conductor varies as its resistance and as the square of the strength of the current.*

517. Practical Applications. Resistance wires heated by an electric current are used for various purposes, such as performing surgical operations, igniting explosives, cooking, heating, etc. In electric toasters and flat-irons the resistance wire is nichrome, an alloy of nickel and chromium. This can be kept at a red heat for weeks without injury, whereas an iron wire would soon deteriorate.

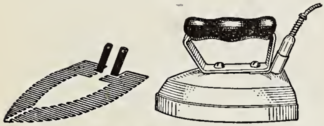


Fig. 589.—View of an electric iron and of its heating element.

Fig. 589 shows the arrangement of the resistance in an electric iron. A current of 5 or 6 amperes passes through the ordinary toaster or flat-iron used on a 110-volt circuit, which means that the power required to operate the appliance is approximately half a kilowatt.

518. Safety Fuses. When too great a current passes through any electrical machine or appliance, excessive heat is developed and the machine may be ruined through the insulation being burned or the conductors being melted. To

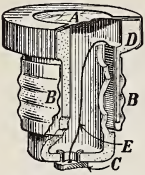


FIG. 590. — Ordinary screw plug fuse. *A*, mica; *B*, screw contact; *C*, tip contact; *D*, porcelain insulator; *E*, fuse wire.

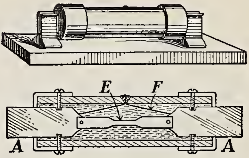


FIG. 591.—A cartridge fuse. The complete fuse inserted in the circuit is shown above; a longitudinal section below. *A*, *A*, contact strips of copper; *E*, fuse wire; *F*, a wire which chars the casing when the fuse melts.

prevent such injury, *fuses*, usually made from an alloy of lead which melts at a comparatively low temperature, are inserted in the circuit. If by accident too heavy a current is used, the fuse wire melts and opens the circuit. Two types of fuse in common use are shown in Fig. 590, 591.

519. Electric Furnace. In Fig. 592 is shown one kind of electric furnace. Carbon rods *C*, *C* pass through the asbestos walls of a chamber about 4 in. long, 2½ in. wide and 1½ in. high. Between them is a small crucible, and the space about is packed with granular carbon (arc lamp carbon rods broken into pieces about as large as coarse granulated sugar). The furnace is joined to an electric-lighting circuit through a rheostat. The resistance of the granulated carbon is considerable, and sufficient heat can be generated to melt pieces of copper in the crucible. This is a *resistance* furnace. Carborundum is produced from coke, sand, salt and sawdust in large furnaces of this type.

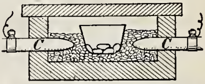


FIG. 592.—Electric resistance furnace.

In the *arc* furnace the heat is produced at a break in the circuit, as illustrated in the arc lamp (§ 523).

520. Electric Welding. Rods of metal are welded by pressing them together with sufficient force while a strong current of electricity is passed through them. Heat is developed at the point of junction, at which place the resistance is greatest, and the metals are softened and become welded together. Sometimes the rails of street-car lines are welded together and in doing so currents of as many as 28,000 amperes are used, these being supplied by special step-down transformers which yield a great current at low voltage.

521. Incandescent Lamp. The construction of the incandescent lamp in common use is shown in Fig. 593. A slender tungsten filament is attached to leading-in wires and inclosed in a pear-shaped globe, from which the air is then exhausted. The leading-in wires are made from an alloy of copper, nickel and iron which has the same coefficient of expansion as lead glass and can be fused into it. When a sufficiently strong current is passed through the filament, it is heated to incandescence and yields a bright steady light.

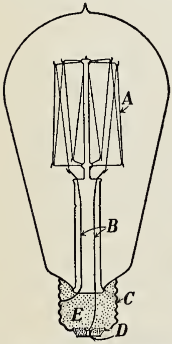


FIG. 593.—An incandescent lamp. *A*, tungsten filament; *B*, leading-in wires; *C*, screw base to which one wire is soldered the other being soldered to the tip *D*; *E*, cement.

The filament does not burn, for lack of oxygen to unite with it, but is very slowly vaporized when in use, depositing a dark metallic coating on the inner surface of the glass.

In another type of lamp the bulb is first exhausted of air and then filled with nitrogen. The presence of this gas retards vaporization and the tungsten filament may be kept at a higher temperature than in the vacuum lamp. The 'nitrogen' lamp gives for the same current a much higher candle-power than the ordinary tungsten lamp, because the increase in candle-power is proportionately greater than the increase in current necessary to produce the extra heat in the filaments. In addition, the convection currents set up in the gas by the heated filaments tend to deposit the metallic coating formed by vaporization at the top of the lamp and to leave the glass below undimmed.

In the earlier types of incandescent lamps the filament was of carbon made by carbonizing a thread of bamboo or cotton fibre at a very high temperature.

Incandescent lamps are now usually rated in watts. For example, a 55-watt, 110-volt, lamp requires ½ ampere to light it properly. The efficiency of ordinary tungsten lamps is about 1.6 watts per candle-power, whereas the older type of carbon filament lamp required about 3.4 watts per candle-power.

522. Grouping of Lamps. All the incandescent lamps to be used in the same circuit are so constructed as to give their proper candle-power when the same potential-difference is maintained between their terminals. This is generally from 100 to 110 volts. The lamps are connected *in multiple*, or *parallel*, that is, the current from the leading-in wires divides, and a part flows through each lamp, as shown in Fig. 594. The dynamo is regulated to maintain a constant potential difference between the leading-in wires.

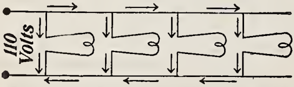


FIG. 594.—Lamps connected in multiple.

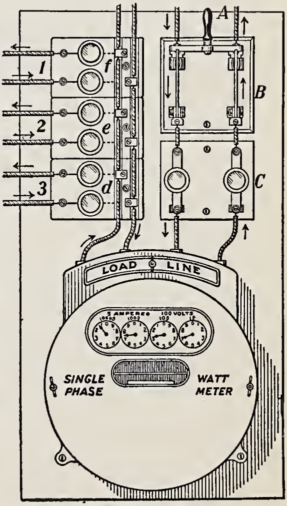


FIG. 595.—How the current enters a house.

In Fig. 595 is shown how the current is brought into a house from the street wires. It enters at A, passes through the main switch B, through the porcelain fuse-block C and then into the meter which registers the number of watt-hours of electrical energy used. It then passes through the fuse-blocks d, e, f (three in this case, though there may be any number) from which the different circuits pass through fuses to various parts of the house. Circuit 1 may be used to light the upper floors, circuit 2 the lower floors, while circuit 3 may be used for ranges, toasters, motors, etc.

523. The Arc Light. If two carbon rods, connected by conductors to the poles of a sufficiently powerful battery or dynamo, are touched together and then separated a short distance, the current continues to flow across the gap, developing intense heat and raising the terminals to incandescence, thus producing a powerful light, generally known as the *arc light.*

When the carbon points are separated by air only, the potential difference between them, when connected with the poles of an ordinary arc-light dynamo, is not sufficient to cause a spark to pass, even when they are very close together; but if they are in contact and then separated while the current is passing through them, the "extra-current" spark produced on separation (§ 500) volatilizes a small quantity of the carbon between the points and a conducting medium, consisting of carbon vapor and heated air, is thus produced, through which the current continues to flow.

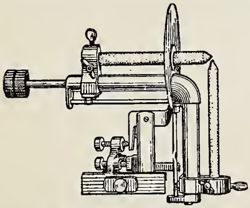


Fig. 596.—Hand-feed arc lamp.

Since this medium has a high resistance, intense heat is developed and the carbon points become vividly incandescent and burn away slowly in the air. When a direct current is used, the point of the positive carbon becomes hollowed out in the form of a crater, and the negative one becomes pointed, as shown in Fig. 596. The greater part of the light is radiated from the carbon points, the positive one being the brighter.

524. Hot-wire Ammeter. The extension which a wire experiences, when heated by a current passing through it, is utilized in the hot-wire ammeter (Fig. 597) to measure the strength of the current. The current to be measured enters by either of the binding-posts *A*, *A*, and traverses the rigid bar and the fine wire *C*, *C* before leaving by the other binding-post. To the wire *C* at the point *a* a wire *D*, *D* is attached and at the point *b* in this wire a thread *E* is attached. This thread passes around the drum *e* and the other end is fastened to the end of the spring *F* which keeps the thread and the two wires taut.

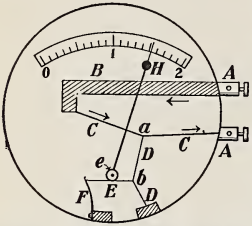


FIG. 597.—The principle of the hot-wire ammeter.

The current passing through *C*, *C* raises its temperature and causes it to lengthen. A small elongation of *C*, *C* causes a considerable movement of the end of the hand *H* which is attached to the drum.

The instrument can be calibrated by being placed in series with a voltameter or a standard ammeter. It has the disadvantage of requiring frequent adjustment, but on the other hand it can be used for measuring alternating as well as direct currents, while an ammeter of the type described in § 492 will not respond to an alternating current.

REFERENCES FOR FURTHER INFORMATION

Smith, *Elements of Applied Physics*, Chapter 26.
Tower, Smith, Turton and Cope, *Physics*, Chapter 12.
Jackson and Black, *Elementary Electricity and Magnetism*.
Barber, *General Science*.

CHAPTER XLVIII

Electrical Measurements

525. Ohm's Law. We have learned that the strength of a current, or the quantity of electricity which flows past a point in a circuit in one second, depends on the E.M.F. producing the current and the resistance of the circuit. The exact relation which exists between these quantities was first enunciated by G. S. Ohm in 1826.

Georg Simon Ohm (1789-1854). Born at Erlangen; died at Munich. Discoverer of Ohm's Law.

He found that the strength of the current was the same at *all points* in the same circuit and that the current passing through any conductor was directly proportional to the potential difference between its ends.* He called the constant quantity, obtained by dividing the potential difference by the current, the "resistance" of the conductor.

We have then:—

$$\text{Resistance of conductor} = \frac{\text{P.D. between ends of conductor}}{\text{Current flowing through conductor}}.$$

$$\text{Resistance of a circuit} = \frac{\text{Total E.M.F. in circuit}}{\text{Current flowing through circuit}}.$$

From a practical point of view this is one of the most important generalizations in electrical science. It is known as Ohm's Law.

*This is true when no change occurs in the physical conditions of the conductors. (§534).

526. Practical Electrical Units. It is evident that if units of any two of the three quantities involved in the relation stated in Ohm's Law are adopted and defined, the unit of the third quantity is also determined. This was the procedure followed at the International Congress on Electrical Units which met in London in 1908.

The following definitions of units were adopted:—

THE UNIT OF RESISTANCE. *The International Ohm is the resistance offered to an unvarying current by a column of mercury at the temperature of melting ice,* 14.4521 *grams in mass, of a constant cross-sectional area, and of a length of* 106.3 *cm.*

The area of the cross-section of such a column is 1 square millimetre.

UNIT OF CURRENT STRENGTH. *The International Ampere is the unvarying electric current which, when passed through a solution of silver nitrate under certain stated conditions, deposits silver at the rate of* 0.001118 *grams per second.*

UNIT OF ELECTROMOTIVE FORCE. *The International Volt is the electrical pressure which, when steadily applied to a conductor whose resistance is one International Ohm, will produce a current of one International Ampere.*

Then, if C is the measure of a current in amperes; R, the resistance of the circuit in ohms; and E, the electromotive force in volts; Ohm's Law may be expressed as follows:—

$$R = \frac{E}{C}, \text{ or } E = CR, \text{ or } C = \frac{E}{R}.$$

527. Demonstration of Ohm's Law. One method is illustrated in Fig. 598. A is a battery connected in series with a variable resistance B, an unknown resistance C (a metre of German silver wire of about ½ mm.

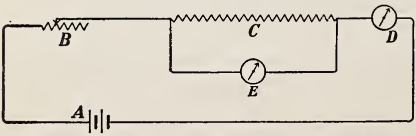


FIG. 598.—Method of demonstration of Ohm's Law.

diameter serves well) and an ammeter D. A voltmeter E is joined to the ends of C.

By reading the ammeter we find out the current flowing through *every part* of the circuit $ABCD$, while the voltmeter shows the potential difference between the ends of the wire, which is the E.M.F. required to drive that current through the wire. Divide the voltmeter reading by the ammeter reading and record the result.

Next, alter the sliding contact in B. This will change the current flowing through C and the reading on the voltmeter will alter at the same time. As before, divide the voltmeter reading by the ammeter reading and record the result.

Continue this for several more positions of B. Then if the temperature of the wire remains constant, the results will all be equal and the magnitude of the result expresses the resistance of C in ohms.

PROBLEMS

1. The electromotive force of a battery is 10 volts, the resistance of the cells 10 ohms, and the resistance of the external circuit 20 ohms. What is the current?

2. The difference in potential between a trolley wire and the rail is 500 volts. What current will flow through a conductor which joins them if the total resistance is 1000 ohms?

3. The potential difference between the terminals of an incandescent lamp is 104 volts when one-half an ampere of current is passing through the filament. What is the resistance?

4. A dynamo, the E.M.F. of which is 4 volts, is used for the purpose of copper-plating. If the resistance of the dynamo is $\frac{1}{100}$ of an ohm, what is the resistance of the bath and its connections when a current of 20 amperes is passing through it?

5. What must be the E.M.F. of a battery in order to ring an electric bell which requires a current of $\frac{1}{10}$ ampere, if the resistance of the bell and connection is 200 ohms, and the resistance of the battery 20 ohms?

6. What must be the E.M.F. of a battery required to send a current of $\frac{1}{100}$ of an ampere through a telegraph line 100 miles long if the resistance of the wires is 10 ohms to the mile, the resistance of the instruments

300 ohms, and of the battery 50 ohms, and if the return current through the earth meets with no appreciable resistance?

7. The potential difference between the carbons of an arc lamp is 50 volts and the resistance of the arc 2 ohms. If the arc exerts an opposing E.M.F. of its own of 30 volts, what is the current passing through the carbons?

8. A dynamo, of which the E.M.F. is 3 volts, is used to decompose water. What is the total resistance in the circuit when a current of one-half an ampere passes through it, if the counter electromotive force of polarization of the electrodes is 1.5 volts?

528. Fall of Potential in a Circuit. If a battery or dynamo is generating a current in a circuit, it is evident that the E.M.F. required to maintain this current in the whole circuit is greater than that required to overcome the resistance of only a part of the circuit. For example, if the total resistance is 100 ohms, and the E.M.F. is 1000 volts, the current in the circuit is 10 amperes. Here an E.M.F. of 1000 volts is required to maintain a current of 10 amperes against a total resistance of 100 ohms; manifestly to maintain this current in the part of the circuit of which the resistance is, say, 50 ohms, an E.M.F. of but 500 volts will be required. This is usually expressed by saying that there is a fall in potential of 500 volts in the part of the circuit whose resistance is 50 ohms.

In general, if there is a closed circuit through which a current is flowing, the fall in potential in any portion of the circuit is proportional to the resistance of that portion of the circuit.

529. Experimental Verification of Fall of Potential. The arrangement shown in Fig. 599 is suitable. *AB* is a metre of bare German silver wire of uniform cross-section stretched alongside a metre-stick and connected in series with a battery *C* and an ammeter *D*. One terminal of the voltmeter *E* is

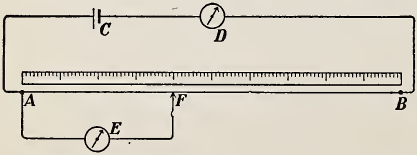


Fig. 599.—Illustration of fall of potential in a circuit.

joined to A while the other may be touched to different points.

As F is moved along the wire AB, it will be found that the reading of the ammeter does not change but that the potential difference shown by the voltmeter between A and F is directly proportional to the distance AF. (It follows from this, also, that the resistance of a conductor of uniform cross-section is directly proportional to its length.)

PROBLEMS

1. Two resistances AB and BC are connected in series and a 100 volt storage battery whose resistance is negligible has its terminals joined to A and C. If the resistance of AB is 9.6 ohms and that of BC is 2.4 ohms, what current will flow through the circuit and what will be the potential difference between A and B and between B and C?

2. The poles of a battery are connected by a wire 8 metres long, having a resistance of one-half ohm per metre. If the E M.F. of the battery is 7 volts and the internal resistance 10 ohms, find the current in the wire and the p.d. between two points on the wire which are 2 metres apart.

3. The potential difference between the brushes of a dynamo supplying current to an incandescent lamp is 104 volts. If the resistance in the wires on the street leading from the dynamo to the house is 2 ohms, that of the wires in the house 2 ohms, and that of the lamp 204 ohms, what is the fall in potential in (1) the wires on the street, (2) the wires in the house, and what is the potential difference between the terminals of the lamp?

4. A dynamo is used to light an incandescent lamp which requires a current of 0.6 ampere and a potential difference between its terminals of 110 volts. If the wires connecting the dynamo with the lamp have a resistance of 5 ohms, find the potential difference which must be maintained between the terminals of the dynamo to light the lamp properly?

5. A cell has an internal resistance of 0.3 ohm, and its E.M.F. on open circuit is 1.8 volts. If the poles are connected by a conductor whose resistance is 1.2 ohms, what is the current produced, and what would be the reading of a voltmeter connected to the terminals of the cell while the current is flowing?

6. If the E.M.F. of a cell is 1.75 volts, and its resistance 3 ohms, find the internal drop in potential when the circuit is closed by a wire whose resistance is (*a*) 4 ohms, (*b*) 32 ohms.

530. Resistance Boxes. The standard resistance was defined in § 526. It is obvious that for the purpose of comparing resistances it would be inconvenient to use mercury columns in ordinary experiments. In practical work, resistance coils are used for this purpose. Lengths of wire of known resistance are wound on bobbins and connected in sets in *resistance boxes*. Fig. 600 shows the common method of joining the coils. A current in passing from A to B meets with practically no resistance from the heavy metallic bar when all the plugs are inserted. To introduce a given resistance, the plug short-circuiting the proper coil is removed and the current is made to traverse the resistance wire. For convenience in calculation the coils are usually grouped very much as weights are arranged in boxes. For example, a set of coils of 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500 ohms may be combined to give any resistance from 1 to 1000 ohms. Fig. 601 shows an ordinary form of resistance box.

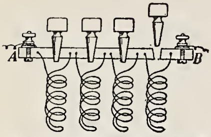


FIG. 600.—Connections in a resistance box.

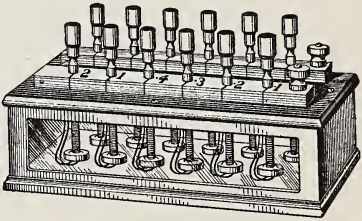


FIG. 601.—A resistance box with glass sides allowing the coils within to be seen.

531. Determination of Resistance; Method of Substitution. If the current strength and electromotive force of a current are known or can be determined with an ammeter and a voltmeter, the resistance in the circuit can be calculated from Ohm's Law $R = E/C$, in the manner described in § 527.

To determine an unknown resistance, when these factors are not known, the conductor is placed in a circuit with a cell of constant E.M.F. and a sensitive galvanometer. The deflection of the needle of the galvanometer is noted, and the unknown resistance is then replaced by a resistance box. The coils are adjusted so as to bring the needle to its former position. The resistance thus placed in the circuit is evidently the resistance of the conductor.

This method, which is usually known as the *method of substitution,* was employed by Ohm in his first experiments. Obviously, variations in the E.M.F. of the cell used will introduce errors in the determination.

532. The Wheatstone Bridge. Wheatstone, who was a contemporary of Ohm and had followed his experiments,

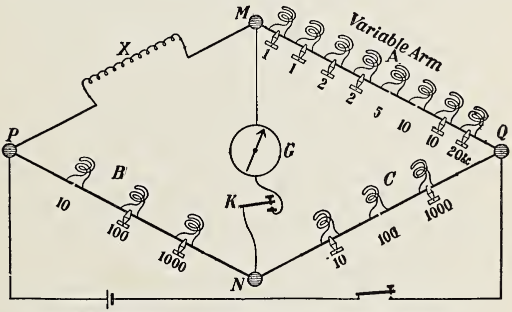


FIG. 602.—Electrical connections in the Wheatstone Bridge.

invented what is known as the "Wheatstone Bridge," an arrangement of coils which makes the determination independent of changes in the E.M.F. of the cell. The coils are arranged in three sets A, B, and C, with connections for a battery, a galvanometer and the resistance to be measured, as shown in Fig. 602.

They are mounted in a box and the changes in the resistance are made in the usual way, by inserting or withdrawing conducting plugs, as shown in Fig. 600.

The branches B and C usually have three coils each, the resistances of which are respectively 10, 100 and 1000 ohms, and the branch A has a combination of coils, which will give any number of units of resistance from, say, 1 to 11,110 ohms. The conductor, whose resistance X is to be measured is inserted in the fourth branch of the bridge (Fig. 602), and the resistances A, B and C adjusted until the galvanometer connecting M and N stands at zero when the keys are closed.

Then the current from the battery is flowing from P, partly through X and A, and partly through B and C to Q, and since no current flows from M to N, the potential of M must be the same as that of N; therefore the fall in potential from P to M in the circuit PMQ is the same as the fall from P to N in the circuit PNQ; but the fall in potential in a part of a circuit is proportional to the resistance of that portion of the circuit.

$$\text{Hence,} \qquad \frac{X}{A} = \frac{B}{C} \text{ or } X = \frac{B}{C} \times A.$$

The resistances A, B and C are read from the instrument, and the value of X is calculated from the formula.

Example.—In an experiment B was 10, C 1000 and A 3378 ohms. Then X = 33.78 ohms.

533. The Slide-wire Bridge. A simple form of bridge is shown in Fig. 603. Here a piece of German silver wire a metre long and about 0.5 mm. in diameter is made to serve the purpose of both the resistances B and C of Fig. 602. The unknown resistance X, the variable resistance box A, the galvanometer G, key K, and battery are connected as before. One terminal of the galvanometer is joined to M, a binding-post on a metal bar to which X and A are both joined and the other to a sliding contact N, by moving which the ratio B/C can be varied at will.

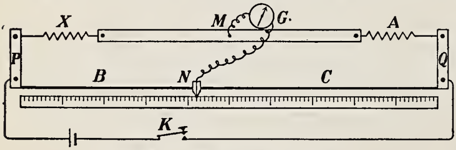


Fig. 603.—The slide-wire bridge.

The slide-wire contact is first placed at the centre of the wire and an approximate balance is secured by altering A. The contact is then moved until a perfect balance is obtained.

534. Laws of Resistance. The resistances of conductors under varying conditions have been determined by various investigators with great care. The general results are given in the following laws:—

1. The resistance of a conductor varies directly as its length.

2. The resistance of a conductor varies inversely as the area of its cross-section.

3. The resistance of a conductor of given length and cross-section depends upon the material of which it is made.

Hence, if l denotes the length of a conductor, A the area of its cross-section and R its resistance,

$$R = k\,\frac{l}{A},$$

where k is a constant depending on the material of the conductor and the units of measurement used. The constant k is known as the *Specific Resistance* of the material. For scientific purposes the specific resistance is usually expressed as the resistance in microhms, or millionths of an ohm, of a cube of this material, whose edge is one centimetre in length, when a current is made to flow parallel to one of its edges.

In engineering practice resistance is usually calculated in another way. Designating $\frac{1}{1000}$ inch as 1 *mil*, the standard of resistance is the resistance of a wire 1 mil in diameter and 1 foot in length. This is named a *mil-foot.* The resistance per mil-ft. of copper is 10.4, of aluminium 17.0 ohms, etc.

The following table gives the specific resistances in microhms and the resistance per mil-foot in ohms of some well-known substances, at 20° C. unless otherwise stated.

Specific Resistance and Resistance per Mil-foot

Substance	Sp.R.	M.-ft.	Substance	Sp.R.	M.-ft.
Aluminium wire....	2.83	17.0	Chromium.......	2.6	15.6
Carbon (filam't) 0° C.	3500	21,054	Nickel...........	7.8	46.9
" " 1500° C.	1500	9,023	Nichrome wire...	100	602
Copper wire........	1.72	10.4	Platinum wire....	10	60.2
German Silver......	21	126	Silver wire.......	1.63	9.8
Iron wire..........	10	60	Tungsten, 20° C..	5.5	33.1
Steel rails..........	11.9	71.6	" 1727° C..	59	357
Mercury...........	95.8	576	" 3227° C..	118	710

Observe that the specific resistance of chromium is 2.6 and that of nickel is 7.8, while nichrome, an alloy of these two substances, has a specific resistance of 100.

535. Resistance and Temperature. If we connect a piece of fine iron or platinum wire in a circuit with a voltaic cell and a galvanometer and note the deflection of the needle, we shall find on heating the wire with a lamp that the galvanometer indicates a weakening of the current. The rise in the temperature of this wire must, therefore, have been accompanied by an increase in its resistance. This action is typical of metals in general.

The resistance of nearly all pure metals increases about 0.4 per cent. for each rise in temperature of 1° C. The resistance of carbon, on the other hand, diminishes when heated. The filament of an incandescent carbon lamp, for instance, has when hot only about one-half the resistance which it has when cold. The resistance of an electrolyte also decreases with a rise in temperature.

It is often necessary to know the strength of current which can be carried safely by wires of different diameters. In the following table is given the carrying capacity of insulated copper wires of the ordinary sizes.

Carrying Capacity of Covered Copper Wires

Size of Wires			Insulation	
B. & S. Gauge	Diam. in Mils	Diam. in mm.	Rubber	Weather-proof
18	40.3	1.02	3 amp.	5 amp.
16	50.8	1.29	6 "	8 "
14	64.1	1.63	12 "	16 "
12	80.8	2.05	17 "	23 "
10	101.9	2.59	24 "	32 "
8	129.5	3.26	33 "	46 "

QUESTIONS AND PROBLEMS

1. What is the resistance of a column of mercury 2 metres long and 0.6 of a square millimetre in cross-section at 0° C?

2. Copper wire $\frac{1}{12}$ inch in diameter has a resistance of 8 ohms per mile. What is the resistance of a mile of copper wire the diameter of which is $\frac{1}{36}$ inch?

3. A mile of telegraph wire 2 mm. in diameter offers a resistance of 13 ohms. What is the resistance of 440 yards of wire 0.8 mm. in diameter made of the same material?

4. What length of copper wire, having a diameter of 3 mm., has the same resistance as 10 metres of copper wire having a diameter of 2 mm.?

5. On measuring the resistance of a piece of No. 30 B.W.G. (covered) copper wire 18.12 yards long I found it to have a resistance of 3.02 ohms. Another coil of the same wire had a resistance of 22.65 ohms. What length of wire was there in the coil?

6. Two wires of the same length and material are found to have resistances of 4 and 9 ohms, respectively. If the diameter of the first is 1 mm., what is the diameter of the second?

7. What must be the thickness of copper wire, which, taking equal lengths, gives the same resistance as iron wire 6.5 mm. in diameter, the specific resistance of iron being six times that of copper?

8. Find the length of an iron wire $\frac{1}{20}$ inch in diameter which will have the same resistance as a copper wire $\frac{1}{60}$ inch in diameter and 720 yards long, the conductivity of copper being six times that of iron.

9. A wire made of platinoid is found to have a resistance of 0.203 ohm per metre. The cross-section of the wire is 0.016 sq. cm. Express the specific resistance of platinoid in microhms.

10. Taking the specific resistance of copper as 1.58, calculate (1) the resistance of a kilometre of copper wire whose diameter is 1 mm., (2) the resistance of a copper conductor 1 sq. cm. in area of cross-section, and long enough to reach from Niagara to New York, reckoning this distance as 480 kilometres.

536. Resistance in a Divided Circuit; Shunts. When a current is divided and made to flow from a conductor A to another B through two parallel circuits (Fig. 604), it is often necessary to determine the resistance of a single wire, which will be equivalent to the two in parallel, and to find the fraction of the total current which flows through each wire.

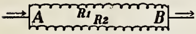


Fig. 604.—Divided circuit.

Let E denote the difference in potential between A and B and R_1 and R_2 the resistances of the wires.

Then the current through the first wire $= E/R_1$ (Ohm's Law), and the current through the second wire $= E/R_2$.

Total current through the two wires $= E/R_1 + E/R_2$.

But the total current $= E/R$, where R is the resistance of a single wire equivalent to the two.

Therefore $\dfrac{E}{R} = \dfrac{E}{R_1} + \dfrac{E}{R_2}$, that is, $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

Again, the fraction of the total current in the first wire

$$= \frac{E/R_1}{E/R_1 + E/R_2} = \frac{R_2}{R_1 + R_2}.$$

Similarly, the fraction of the total current in the second wire

$$= \frac{R_1}{R_1 + R_2}.$$

When it is undesirable to send the whole current to be measured through a galvanometer or other current-measuring instrument, a definite fractional part of the current is diverted by making the instrument one of two parallel conductors in the circuit, as shown in Fig. 605.

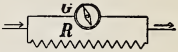


FIG. 605.— Galvanometer and shunt.

The conductor R in parallel with the galvanometer G is called a *shunt.*

If G is the resistance of the galvanometer, R the resistance of the shunt, and C the total current, the amount of current through the galvanometer

$$= \frac{R}{G + R} \times C$$

For the sake of facility in calculation, it is usual to make R $\frac{1}{9}$, $\frac{1}{99}$, or $\frac{1}{999}$ of G, when, by the above formula, the current through the galvanometer will be $\frac{1}{10}$, $\frac{1}{100}$, or $\frac{1}{1000}$, respectively, of the total current to be measured.

PROBLEMS

1. The poles of a voltaic battery are connected by two wires in parallel. If the resistance of one is 10 ohms and that of the other 20 ohms, find (1) the resistance of a single wire equivalent to the two in parallel; (2) the proportion of the total current passing through each wire.

2. Find the total resistance when the following resistances are joined in series:—$3\frac{1}{2}$ ohms, $2\frac{1}{3}$ ohms, $2\frac{1}{4}$ ohms. What would be the joint resistance if the resistances were joined in parallel?

3. What must be the resistance of a wire joined in parallel with a wire whose resistance is 12 ohms, if their joint resistance is 3 ohms?

4. The joint resistance of ten similar incandescent lamps connected in multiple is 10 ohms. What is the resistance of a single lamp?

5. Four incandescent lamps are joined in parallel on a 100-volt circuit. If the resistances of the lamps are respectively 100 ohms, 200 ohms, 300 ohms and 400 ohms, find (1) the total current passing through the group of lamps; (2) the proportion of the total current passing through the first lamp; (3) the resistance of a single lamp which would take the same current as the group.

6. A galvanometer whose resistance is 1000 ohms is used with a shunt. If $\frac{1}{11}$ of the total current passes through the galvanometer, what is the resistance of the shunt?

7. If the shunt of a galvanometer has a resistance of $1/n$ of the galvanometer, what fraction of the total current passes through the galvanometer?

8. The internal resistance of a Daniell cell is 1 ohm; its terminals are connected (*a*) by a wire whose resistance is 4 ohms, (*b*) by two wires in parallel one of the wires having a resistance of 4 ohms, the resistance of the other wire being 1 ohm. Compare the currents through the cell in the two cases.

537. Grouping of Cells or Dynamos. Electrical generators may be connected in various ways to give a current in the same circuit.

They are connected *in series* or *tandem* when the negative terminal of one is connected with the positive terminal of the next (Fig. 606), and *in multiple*, or *parallel*, when all the positive terminals are connected to one conductor and all the negatives to another (Fig. 607). Sometimes combinations of these methods of arrangement are employed as shown in Figs. 608, 609.

FIG. 606.—Cells connected in series.

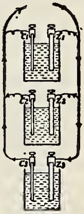


FIG. 607.—Cells connected in multiple.

538. Current Given by Series Arrangement. If n cells are arranged in series, and r is the internal resistance of each cell, it is evident that the resistance of the group $= nr$, because the current has to pass through a liquid conductor n times as long as that between the plates of a single cell.

If the potential-difference between the plates of a single cell (Fig. 606) is e, the potential-difference between Z_1 and C_1 is e; but when C_1 and Z_2 are connected by a short thick conductor there is practically no fall in potential between them, therefore the potential-difference between Z_1 and Z_2 is e. Again, the potential-difference between Z_2 and C_2 is e and therefore the potential difference between Z_1 and C_2 is $2e$. Similarly, for 3, 4, etc., cells the potential-differences are respectively $3e$, $4e$, etc. Hence, the E.M.F. of n cells in series is ne.

Suppose now that the group of cells is in circuit with an external resistance R_1.

By Ohm's Law, $C = E/R$, where E is the E.M.F. of the group and R the total resistance of the circuit; but $E = ne$, and $R = nr + R_1$;

$$\text{Hence } C = \frac{ne}{nr + R_1}.$$

539. Current Given by Multiple Arrangement. If n cells are arranged in multiple, and r is the internal resistance of a single cell, the internal resistance of the group $= r/n$, because the current in passing through the liquid from one set of plates to the other has n paths open to it, and therefore the sectional area of the column of liquid traversed is n times that of one cell, hence the resistance is only $1/n$ of that of one cell (§ 534). When all the positive plates are connected they are at the same potential; for a similar reason all the negative plates are at the same potential, hence the E.M.F. of n cells in multiple is the same as that of one cell.

This method of grouping is equivalent to transforming a number of single cells into one large cell, the Z plates being united to form one large Z plate, and the C plates to form one large C plate. It must be remembered that the potential-difference between the plates of a cell is independent of the size of the plates. (§ 464.)

If E is the E.M.F., R the total resistance of the circuit, and R_1 the external resistance,

$$C = \frac{E}{R} = \frac{e}{r/n + R_1}$$

540. Best Arrangement of Cells. It is manifest that when the external resistance is very great as compared with the internal resistance, in order to overcome the resistance the electromotive force must be increased, even at the expense of increasing the internal resistance, and the series arrangement of cells is the best. When the external resistance is very low as compared with the internal resistance, the object of the grouping is to lower as far

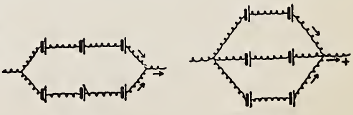


FIG. 608 FIG. 609
Cells connected in multiple-series.

as possible the internal resistance, and the multiple arrangement is the best. Between these extremes of high and low external resistance some form of multiple-series grouping (such as Figs. 608, 609) gives the strongest current.

It can be shown that for a given external resistance the maximum current from a given number of cells is obtained when the cells are so connected that the internal resistance of the battery is as nearly as possible equal to the external resistance.

PROBLEMS

1. If the E.M.F. of a Grove cell is 1.8 volts and its internal resistance is 0.3 ohm, calculate the strength of current when 50 Grove cells in series are connected to a wire whose resistance is 15 ohms.

2. If 6 cells, each with $\frac{1}{2}$ ohm internal resistance, and 1.1 volts E.M.F., are connected (*a*) all in series, (*b*) all in parallel, (*c*) in two parallel sets of three cells each (Fig. 608); calculate the current sent in each case through a wire of resistance 0.8 ohm.

3. Ten voltaic cells, each of internal resistance 2 ohms and E.M.F. 1.5 volts, are connected (*a*) in a single series, (*b*) in two series of five each, the like ends of the two series being joined together. If the terminals are in each case connected by a wire whose resistance is 10 ohms, find the strength of the current in the wire in each case.

4. The current from a battery of 4 similar cells is sent through a tangent galvanometer, the resistance of which, together with the attached wires, is exactly equal to that of a single cell. Show that the galvanometer deflection will be the same whether the cells are arranged all in multiple or all in series.

5. Calculate the number of cells in series required to produce a current of 50 milliamperes (1 milliampere = $\frac{1}{1000}$ ampere), through a line 114 miles long, whose resistance is $12\frac{1}{2}$ ohms per mile, the cells of the battery having each an internal resistance of 1.5 ohms, and an E.M.F. of 1.5 volts.

6. You have a battery of 6 Daniell cells, each of 6 ohms internal resistance, and wish to send the strongest possible current through an external resistance of 15 ohms. By means of diagrams show various ways of arranging the cells and calculate the strength of current in each case, assuming that the E.M.F. of a Daniell cell is 1.07 volts.

7. A circuit is formed of 6 similar cells in series and a wire of 10 ohms resistance. The E.M.F. of each cell is 1 volt and its resistance 5 ohms. Find the strength of the current. What would it be if the cells were joined in multiple?

541. Quantity of Electricity. Let us refer again to the flow of water through a pipe (§ 432). The *current strength* is the *rate of flow.* It depends upon the difference of pressure at the ends of the pipe, and the resistance of the pipe. But we often wish to know the *quantity* of water passing in a given time. Obviously we have the relation,

$$\text{Quantity} = \text{rate of flow} \times \text{time of flow.}$$

We might measure rate of flow in gallons-per-second, and quantity in gallons.

In electrical measurements there is something similar. We may think of the quantity of electricity passing a cross-section of a circuit in a given time, and as before we have the relation,

$$\text{Quantity of electricity} = \text{current strength} \times \text{the time.}$$

If we measure current strength in amperes, and time in seconds, the quantity will be given in *coulombs;* and we have the the definition:—A COULOMB *is the amount of electricity which passes a point in a circuit in one second when the strength of the current is one ampere.*

The ampere corresponds to gallons-per-second, the coulomb to gallons.

If the strength of a current is C amperes and the quantity flowing past a point in the circuit in t seconds is Q coulombs, then $Q = Ct$.

As already stated in §472, practical electricians frequently employ the *ampere-hour* as the unit quantity, as, for example, in estimating the capacity of a storage cell. A battery has a capacity of 100 ampere-hours, when it will furnish a current of one ampere for 100 hours, or 2 amperes for 50 hours, etc.

Question.—What is the capacity in coulombs of a 100 ampere-hour storage battery?

542. Work Done in an Electric Circuit. The water analogy will assist us again in getting a clearer grasp of the principle by which the energy expended in an electric circuit may be expressed.

Just as the work done by a stream depends on the quantity of water and the distance through which it falls, so the work done in any portion of an electric circuit depends on the quantity of electricity which passes through it and the difference in potential between its terminals. One *joule*, or 10^7 ergs, of work is done when one coulomb of electricity falls through one volt.

Hence, if Q is the quantity of electricity passing through a wire AB

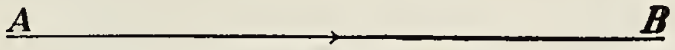


FIG. 610 Portion of an electric circuit.

(Fig. 610) and V denotes the fall in potential from A to B, the work done by the current $= QV = CVt$.

543. Rate at Which Work is Done in an Electric Circuit. The power, or rate at which work is done in an electric circuit, is estimated in *joules per sec.*, that is, *in watts* (§ 56).

Thus if a current of C amperes flows through a circuit in which there is a drop of potential of V volts, energy is being delivered at the rate of VC watts.

Hence *Power* (*watts*) = *P.D.* (*volts*) × *Current* (*amperes*).

Since one horse-power = 746 watts (§ 56).

$$\text{Power (in h.p.)} = \frac{\text{Pot. diff. (in volts)} \times \text{current (in amperes)}}{746}.$$

544. Work Done in an Electric Lamp. The efficiency of an electric lamp is usually determined in watts per candle power.

Thus if a 16-candle power incandescent lamp requires a current of ½ ampere in a 110-volt circuit, its efficiency is (110 × ½) ÷ 16 or 3.4 watts per candle power.

For commercial purposes, the energy consumed by a lamp in a given time is usually measured in *watt-hours* or *kilowatt-hours* (1 k.w. = 1000 watts). For example, if a customer has a lamp of the above description burning for 100 hours per month, he pays monthly for 55 × 100, or 5500 watt-hours, or 5.5 k.w.h., of energy.

545. The Watt-hour Meter. The object of the watt-hour meter is to show the amount of electrical energy used each month by the customer of an electric power company. The method of connecting the meter to the wires leading in from the street and to the different circuits in the building has been given in Fig. 595. In Fig. 611 is shown the outside appearance of one type of meter while a diagram of the essential working parts is given in Fig. 612.

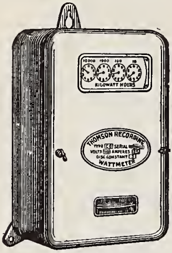


Fig. 611.—A Thomson watt-hour meter.

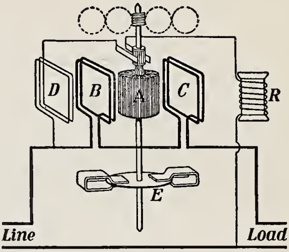


Fig. 612.—Essential parts of a Thomson watt-hour meter.

It will be observed that the meter is really a small motor without any iron in its working parts. The armature A, which is geared to the recording dials, is connected directly across the line wires, like a voltmeter, and, consequently, the current flowing through it will be directly proportional to the difference of potential between the wires. The field-coils B and C are connected, like an ammeter, in series with the "load," which will consist of lamps, heaters, etc., and therefore the field strength of the motor varies directly as the current passing through the lamps or other appliances. As a result, the turning effect on the armature is proportional to the product of the current supplied and the voltage at which it is supplied, that is, to the number of watts of power used.

The small auxiliary field-coil D is used for adjusting the meter; this is done by changing its position with reference to the armature. The aluminium disc E rotates between the poles of two permanent horseshoe magnets and the currents which are induced in it produce a braking action (Lenz's Law), which prevents the motor from running too fast and also causes it to stop as soon as the current is shut off.

It is evident that the number of turns of the armature in a given time is proportional to the power in watts and to the time, and so by a suitable arrangement of gears the dials can be made to register watt-hours or kilowatt-hours. The reading in Fig. 611 is 1727 k.w.h.

546. Relation Between Heat Energy and the Energy of the Electric Current. The mechanical equivalent of heat is 4.2 joules per calorie, that is one joule = 0.24 calorie. Hence if an electric current of C amperes is flowing in a circuit in which there is a fall in potential of V volts, and all the energy of the current is transformed into heat, $V \times C \times 0.24$ calories will be developed every second.

More frequently, however, the *quantity* of heat produced by a current is expressed in terms of the current and the resistance. By Ohm's Law, $V = C \times R$; therefore the heat developed in a circuit, whose resistance is R ohms by a current of C amperes is $C^2R \times 0.24$ calories per second, or in t seconds the heat produced $= CR^2t \times 0.24$ calories. This accords with results determined experimentally by Joule (§ 516).

PROBLEMS

1. A current of 10 amperes flows through an arc light circuit. What quantity of electricity will pass across the arc of one of the lamps in a night of 10 hours?

2. The difference in potential between a trolley wire and the rail which carries the return circuit is 500 volts, and the motor of a car takes an average current of 25 amperes. Find the average power of the motor in k.w.

3. Find the power necessary to run an electric light installation taking 125 amperes at 110 volts (*a*) in k.w., (*b*) in h.p.

4. The potential difference between the wires entering a house is 104 volts, and an average current of 8 amperes flows through them for 4 hours per day. How many watt-hours of energy must the householder pay for in a month of 30 days? Find the cost at 8 cents per kilowatt-hour.

5. Find the cost per hour of operating an electric toaster which takes 6 amperes current at 110 volts pressure, if the rate is 5 cents per k.w.h.

6. How many 40-watt tungsten lamps can be lighted by the power used in operating an electric motor which uses 2 amperes current at 220 volts?

7. Find the smallest h.p. which a gas-engine can have which is used to drive a dynamo supplying current for twenty-five 60-watt 110-volt lamps in a farm lighting system. Would it make any difference if the lamps required 55 volts pressure?

8. In a motor-generator set the motor takes 6 amperes at 550 volts, when the generator is supplying 27 amperes at 110 volts. Find the efficiency of the set.

9. A 16-candle power tungsten lamp, when used in a 25-volt circuit takes one ampere of current. Find its efficiency.

10. The resistance of the filament of an incandescent lamp is 200 ohms and it carries a current of 6 amperes. Find the amount of heat (in calories) developed in this filament per minute.

547. Unit of Capacity. We have already discussed the factors on which the capacity of a condenser depends. This capacity is measured in *farads*, or more commonly in *microfarads* (millionths of a farad).

A condenser has a capacity of one farad when a coulomb of electricity charges the plates to a potential difference of 1 *volt;*

$$\text{or Capacity (farads)} = \frac{\text{Charge (coulombs)}}{\text{P. D. (volts)}}$$

The capacity in microfarads of a simple two-plate condenser is given by the formula,

$$C = \frac{KA}{4\pi d \times 900{,}000},$$

where K is the dielectric constant of the material separating the plates, A is the area in sq. cm. of one of the plates, and d is the distance in cm. between the plates. (The derivation of this formula is too difficult for this text-book).

548. Unit of Inductance. The unit of inductance, the *henry*, has already been defined in §500. The inductance in henries of a solenoid is given by the formula,

$$L = \frac{4\pi^2 N^2 r^2 \mu}{10^9 l},$$

where L = inductance in henries, N = number of turns, r = average radius of coil in cm., μ = permeability of core, l = length of core in cm.

In tuning coils used in 'wireless' the core is air and $\mu = 1$. The permeability of iron varies between wide limits. For soft iron it may be as high as 170 and for transformer steel sheets as high as 5,000.

The henry is a large unit and the *millihenry* (1/1000th of a henry) is frequently used.

The inductance of an ordinary 50-turn honeycomb coil used in wireless telegraphy is about 0.15 millihenry, while that of a 100-turn coil is 0.6 millihenry, the inductance being proportional to the square of the number of turns when the other factors remain constant.

PROBLEMS

1. Find the capacity of a simple two-plate condenser in which the plates are each 2×3 cm., and are separated by a piece of mica $\frac{1}{10}$ mm. thick.

2. The 23-plate variable condenser in Fig. 502 is equivalent to 22 simple two-plate condensers having plates of the same size. Find its capacity if the diameter of the semi-circular plates is 7 cm., and the distance between successive plates is 1 mm.

3. Find the inductance of a 100-turn coil which is 20 cm. long and has an average radius of 3 cm., (*a*) when the core is air, (*b*) when the core is iron ($\mu = 150$).

4. Find the inductance of an air-core solenoid 50 cm. long, 4 cm. in diameter, containing 2000 turns.

REFERENCES FOR FURTHER INFORMATION

Smith, *Elements of Applied Physics*, Chapter 25.
Ferry, *General Physics*, Chapter 19.
Jackson and Black, *Elementary Electricity and Magnetism.*
Weston Electrical Instrument Co., *Monographs.*

CHAPTER XLIX

Other Forms of Radiant Energy

549. Beyond the Visible Spectrum. We have seen that when white light is passed through a prism it is separated into its different components, and on a screen placed in its path (Fig. 412) we observe a *spectrum*, with its colours ranging from violet at one end to red at the other. The wave-length of the extreme red is 0.000,8 mm. or about $\frac{1}{30000}$ inch; that of the extreme violet is 0.000,4 mm. or about $\frac{1}{60000}$ inch. If we considered these waves as we do sound waves we would say that the visible radiation corresponds to *one octave.*

The question arises, are there radiations beyond those which give rise to the red and the violet sensations?

550. Waves beyond the Violet. In order to investigate this question let us receive the spectrum upon a photographic plate. Upon developing it, we find that while it has been scarcely affected by the red and the yellow light, the blue and the violet have produced strong action, and further, that decided action has been produced *beyond the violet.* By suitable means photographic action has been traced to wave-lengths of 0.000,02 mm., that is, to more than three 'octaves' above the violet.

Quite recently it has been shown that the X-rays (see § 568) are of the same nature but having wave-lengths much shorter. They range from 1 to 120 hundred-millionths of a millimetre. Also, the Gamma rays (§573) are similar in nature, but with wave-lengths even smaller.

551. Beyond the Red. If we wish to explore beyond the red, we must use a sensitive detector of heat. Let us obtain the spectrum of the sun, and then through it, going from blue to red, pass an air thermoscope (Figs. 253, 316), the bulb of which has been coated with lamp-black. The thermoscope will show a heating effect which increases as we go towards the red, but the heating does not cease there. Beyond the red the effect is still pronounced. By means of special instruments, heat waves 0.3 mm. long have been detected and measured. Such waves are about ten 'octaves' below the longest red waves.

Bodies at ordinary temperatures emit heat waves, and as the temperature is raised they give out, in addition, those waves which affect the eye.

Still other waves are produced by electrical means and are utilized in radio telegraphy and telephony, which are briefly discussed in the following sections. Their wave-lengths run from 2 mm. to 20 km.

552. Radiant Energy. These waves of various lengths are all forms of radiant energy. While passing from one place to another they travel with the speed of light, and it is only when they fall upon some form of matter that their energy is transformed into those physical effects which we recognize as heat, light, and in other ways.

Following is a summary of radiation wave-lengths in cm:—

Electric or Hertzian waves, longest	2,000,000.0
Electric or Hertzian waves, shortest	0.2
Infra-red or heat waves, longest	0.03
Visible (red) longest	0.000 08
Visible (violet) shortest	0.000 04
Ultra-violet, photographic	0.000 002
X-rays, longest waves	0.000 000 12
X-rays, shortest waves	0.000 000 001
Gamma rays, longest waves	0.000 000 013
Gamma rays, shortest waves	0.000 000 000 7

553. Phenomenon of the Electric Spark. Let *A* and *B* (Fig. 613) be two knobs attached to an induction coil or an influence machine. On putting the apparatus in operation the potential of one knob rises until a spark passes between the knobs. Ordinarily one thinks simply that a quantity of electricity has jumped from one knob to the other in order to annul the difference of potential between A and B. But there is more in the phenomenon than that. As a matter of fact there is a rush across from A to B, then one back from B to A, then another from A to B, and so on, until the energy of the charge is dissipated. Thus, instead of a single spark there is a series of sparks between A and B. This has been demonstrated by photographing their images in a rapidly rotating mirror.

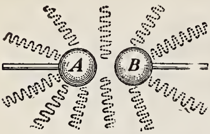


Fig. 613.—Diagram illustrating how the electric waves spread out from a spark gap.

Suppose we have a U-tube (Fig. 614) with a membrane stretched across it at D, and that the membrane is so thin that it breaks when water has been poured into the arm E to a certain height C. The water will not come to rest at once but will oscillate up and down until the motion is stopped by the friction of the tube.

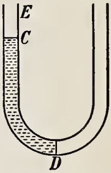


Fig. 614.—Illustrating an electric spark.

The membrane has its electrical analogy in the air-gap between the knobs A and B which breaks down when the electrical pressure across the gap becomes great enough. After this has happened the electricity surges back and forth until brought to rest by the resistance of the gap and by the loss of the energy radiated away.

Now we know that electricity in motion is always accompanied by a magnetic field, and that a reversal of current includes also a reversal of the magnetic field. A slowly alternating current passing through a wire placed over a compass needle would cause the needle to deflect first one way and then the other. A rapidly alternating current would produce a similar effect if it were not for the inertia of the needle.

In the same way the electrical surgings from knob to knob are accompanied by magnetic surgings in the surrounding space and consequently a conductor placed in this changing magnetic field should have an alternating current induced in it. In other words, when the discharge occurs, electromagnetic disturbances or waves are sent out in all directions into the surrounding space. Each discharge produces a train of waves.

554. Sympathetic Electrical Oscillations. When a tuning-fork is vibrated, air-waves spread out in all directions, and if a unison fork is placed not too far away (Fig. 229), the incident waves will excite easily observed vibrations in it (see § 209). It is possible to exhibit electrical resonance quite analogous to that obtained with the unison tuning-forks. Let us take two precisely similar Leyden jars *A* and *B* (Fig. 615), and let a wire run from the outer coating of *A* and end in a knob *c′* near to the knob *c* which is attached to the inner coating. Join these knobs to an influence machine or an induction coil.

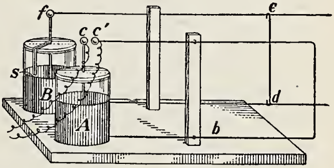


Fig. 615.—Arrangement to show electrical resonance.

Also let the inner and outer coats of *B* be connected by a wire loop *Bdef*, the portion *de* being so arranged that, by sliding it along the other wires, the area inclosed by the wire *Bdef* may be made approximately equal to that of the fixed loop on the other jar. From the inner coating of *B* a strip of tin-foil is brought down to *s*, within about 1 mm. of the outer coating.

Now cause sparks to pass between the knobs *c*, *c′*. Then if the two wire loops are nearly equal in area there will be a little spark at *s* whenever a spark passes at *c*, *c′*. If the wire *de* is slid back or forth the equality of the areas will be destroyed and the sparks will cease at *s*.

The experiment can be varied by connecting a small flash-light bulb across the gap at s or by inserting it in the circuit *Bdef*. The bulb will light when the circuits are in resonance.

When the spark passes at c, c' electricity surges back and forth between the outer and inner coatings of A, and electromagnetic waves are sent out which set up electrical oscillations in the similar circuit attached to the other jar. The *natural period* of the two circuits must be equal (or nearly so) for the sympathetic oscillations to be set up.

This natural period depends on two things, the *capacity of the circuit* and the *inductance of the circuit.* A coil of wire has more inductance than the same length of wire straightened out, and a coil of many turns has more inductance than one of a few turns.

One circuit is put in resonance with, or *tuned to,* another by altering the capacity or inductance or both.

The period in seconds is given by the formula $T = 2\pi\sqrt{LC}$, in which L is the inductance in henries and C the capacity in farads (§§ 547, 548).

In such an arrangement as here described the number of oscillations is ordinarily several millions per second.

555. Electric or Electromagnetic Waves. As early as 1864 Maxwell*, by mathematical reasoning based on experimental results obtained by Faraday, showed that electric waves in the ether must exist; but they were first detected experimentally by Hertz,† a young German physicist. Hertz showed that they travel through space with the speed of light, that they can be reflected and refracted, and that they also possess other properties similar to those possessed by light-waves.

The wave-length for any oscillatory circuit is given by the formula $l = 1885\sqrt{LC}$, where l is the wave-length in metres, L the inductance in microhenries and C the capacity in microfarads.

A 50-turn "honey-comb" coil has an inductance of about 150 microhenries. An oscillatory circuit made up of one of these coils and a 0.001

*James Clerk Maxwell, a very distinguished physicist. Born in Scotland 1831, died 1879.
†Heinrich Hertz died on January 1, 1894, in his 37th year.

microfarad variable condenser should respond to a maximum wave-length given by

$$l = 1885\sqrt{150 \times 0.001} = 730 \text{ metres (nearly).}$$

By varying the setting of the condenser the wave-length can be reduced to about 240 metres.

Also, since $v = nl$ (§ 170), and v = 186,000 miles or 300,000,000 metres per sec., for the circuit just considered n = 400,000 (approx.) oscillations or cycles per second.

556. Radio or Wireless Telegraphy. By means of electric waves it is possible to send signals from one place to another without connecting wires. A simple arrangement for doing this is shown in Fig. 616. I is an induction coil, the secondary of which is connected to the spark-gap S, the condenser C and the helix L_1 to form an oscillatory circuit similar to that used in § 554. The *aerial* wire A_1 is connected to one end of the helix L_2 and the ground wire G to the other.

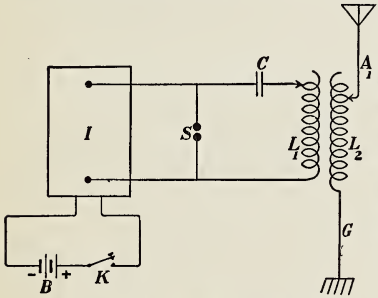


FIG. 616.—Radio telegraph transmitter.

B is a battery and K a key in series with the primary of the coil. When one presses K, each vibration of the armature produces a high-tension current in the secondary of the induction coil, which charges C until the pressure becomes so great that a discharge occurs across the gap S. When this happens, high-frequency currents surge through the oscillatory circuit and induce similar high-frequency currents in the aerial-ground circuit, with the result that electromagnetic waves are sent out in all directions. Thus each vibration of the armature produces a wave-train.

The *open* aerial-ground oscillatory circuit produces waves of larger amplitude and is therefore a much better radiator than the closed oscillatory circuit which is used to excite it.

557. Receiving Apparatus. A simple arrangement is shown in Fig. 617. Here L_3 is a tuning coil to one end of which the aerial wire A_2 is joined, while the other end is connected to the earth. In circuit with the coil are the detector D and a pair of sensitive telephone receivers T. The electromagnetic waves travel from A (Fig. 616) with the speed of light, and on reaching A_2 they excite high-frequency alternating currents in it. The detector and receivers transform this electrical energy into audible sound.

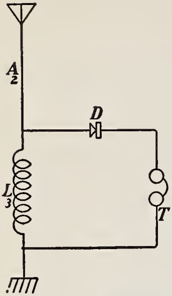


Fig. 617.—Receiving apparatus.

It should be noted that this action is essentially the same as when an alternating current in one coil induces an alternating current in another coil close to it. In radio work, however, the alternations are very rapid, the action extends over great distances, the circuits are tuned to one another and special apparatus must be used to detect the very weak currents induced in the receiving aerial.

558. Crystal Detectors. Telephone receivers can be made to respond to very weak currents and at first sight we might think that signals could be received merely by inserting the receivers in the aerial-ground receiving circuit. This does not work, however, on account of the great self-induction of the electromagnets in the receivers, which offers an enormous reactance to such rapidly alternating currents as are set up in the receiving aerial. Moreover, if the diaphragms could vibrate at so great a frequency, we would not be able to hear the note as the pitch would be too high (§ 185).

Now, certain crystals, such as galena and silicon, possess the peculiar property of rectifying an alternating current, that is, they let the current pass through them in only one direction; and this property can be utilized to produce audio-frequency vibrations of the receiver diaphragms.

The crystal is usually mounted as shown in Fig. 618. A is a brass cup containing the crystal and connected to the binding-post *B*. The adjustable metal point *C* makes contact with the crystal and is joined to the other binding-post *D*.

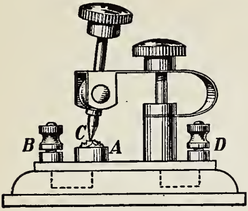


FIG. 618.—A crystal detector.

This detector is placed in circuit with the tuning coil and receivers as already described. When the alternating currents surge through the aerial-ground circuit, differences of potential are established between the ends of L_3, with the result that uni-directional pulses of current flow through the crystal and receivers. The diaphragms of the receivers do not respond to each individual impulse but to the sum total of the impulses in a wave-train. The note heard in the receivers has, therefore, the same frequency as that of the vibrating armature of the induction coil at the transmitting station.

559. Thermionic Valve. The most recent and most widely used detector, however, is the *Thermionic Valve*, which was invented by Fleming and improved by De Forest. Many forms of the valve are now obtainable but all embody the same essential features. The valve used by the British Signal Service during the World War is shown in Fig. 619. It takes the form of an ordinary incandescent light bulb in which the *filament F* is lighted by a 6-volt storage battery. Surrounding this filament is the *grid G*, consisting of a spiral of nickel wire. Around this again is a nickel cylinder *P*, called the *plate* or anode. The plate and grid are supported on wires sealed through the bulb and four terminal pins project through the base, two being used for lighting the filament, one for making connection to the grid

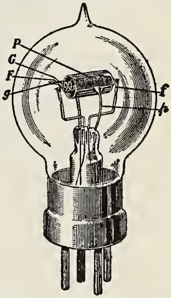


FIG. 619.—A thermionic valve. *P*, plate; *G*, grid; *F*, filament; *p*, *g*, *f* are wires supporting plate, grid, and filament, respectively.

and one to the plate. The action of the valve depends on the fact that an incandescent metal emits electrons freely.

To demonstrate this action connect apparatus as shown in Fig. 620*. A is a 6-volt storage battery connected to the filament F through the variable resistance R. A $22\frac{1}{2}$-volt battery called the "B" battery has its positive pole connected to the plate P, while the negative pole is joined through the galvanometer C to the filament. When the filament is cold no current passes through C, but as soon as it is made incandescent the galvanometer shows a deflection because the electrons emitted by the hot filament are attracted by the positive charge on the plate. This action is equivalent to a flow of positive electricity from the positive pole of B to P, across the intervening space to F and through C back to the negative pole of the battery.

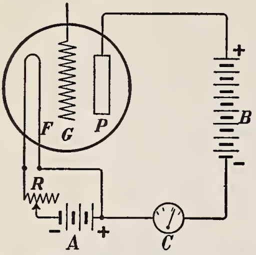


Fig. 620.—Testing the flow of current through a valve.

560. Valve Reception. A simple receiving circuit using the valve as detector is shown in Fig. 621. L_1 and L_2 are variable inductances and C_1, C_2 variable condensers used in tuning the primary (aerial-ground) and secondary circuits to the incoming waves. T is a pair of sensitive telephone receivers, replacing the galvanometer shown in Fig. 620. The connections are easily followed.

When the filament current is turned on, a steady stream of electrons will pass from F to P, and through the receivers, producing a steady deflection of the diaphragms. As soon, however, as the primary and secondary circuits are tuned to the incoming waves, feeble alternating currents will surge back and forth in the secondary circuit composed of L_2, C_2, G, F and connecting wires. As a result G will become charged

*For clearness the elements in the valve are shown diagrammatically.

alternately positively and negatively with respect to *F*. When it is charged positively it will assist *P* in pulling electrons from the filament, but when it is charged negatively

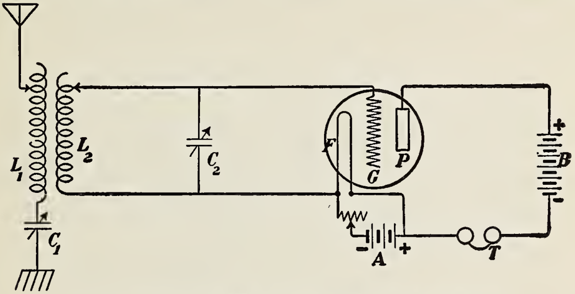


FIG. 621.—A simple receiving circuit using a valve as detector.

it will push back some of the electrons which are trying to reach *P*. The result is a *large increase* in the plate current when the grid is charged positively and a *smaller decrease* in the plate current when it is charged negatively. Each train of waves will therefore produce an *average* plate current greater than the normal current passing through the receivers when no waves are being received, and consequently will produce one vibration of the diaphragms.

Now each time the armature of the induction coil at the transmitting station vibrates, a discharge occurs and a train of waves is sent out. The frequency of the note heard in the receivers will therefore be the frequency of the armature of the induction coil.

The great advantage of the valve over the crystal detector lies in the fact that the current which actuates the receivers comes from the "B" battery and is many times stronger than the weak alternating current which sets it in operation. The effect can be likened to a boy closing a switch which sets a large machine in motion. In the case of the crystal detector the only current available to operate the receivers is the feeble incoming current itself.

561. Regenerative Circuit. If a third coil L_3, having suitable inductance, is inserted between the postive pole of the "B" battery and the plate, as in Fig. 622, and is brought close to L_2, it will act inductively on L_2 and can be made to assist and augment the current in the grid circuit. By this means greater variations will be produced in the grid

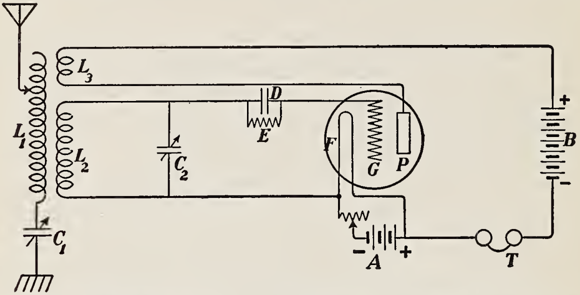


FIG. 622.—Regenerative circuit. L_1 is the primary, L_2 the secondary and L_3 the tickler coil.

potential, resulting in still greater variations in the plate current and greater motion in the receiver diaphragms.

Such a coil is called a *reactance* or "tickler" coil and the arrangement is called a regenerative circuit. With most valves the addition of the grid condenser D, shunted by the high resistance "grid leak" E, tends to increase the efficiency of the circuit.

562. Amplifiers. If we replace the telephone receivers in Fig. 622

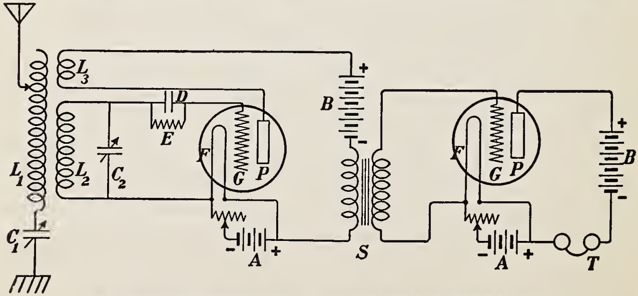


FIG. 623.—Regenerative circuit with one stage of amplification.

by the primary of a telephone transformer and join the terminals of the secondary coil to the grid and filament of a second valve in the plate circuit of which another high-tension battery and the receivers are placed, we obtain still greater surges through the receivers and a much greater volume of sound (Fig. 623). For long distance reception several of these "stages of amplification" are used.

Amplifiers of this type were used extensively during the World War to overhear enemy telephone communications.

563. Valve Transmission. The regenerative circuit described in § 561 can be used as a generator of electromagnetic waves, because when properly arranged the slightest initial alteration in the potential of the grid will cause a variation in the plate current. This will react through the tickler coil on the secondary inductance and create further changes in the grid potential which will react again on the plate current. Under such circumstances sustained electrical oscillations are produced in the secondary circuit which induce similar oscillations in the aerial circuit, with the result that *continuous* waves are sent out into space. To send a code message it is only necessary to work a key inserted in the aerial circuit.* In this case the telephones are useless and should be omitted. For effective radiation special valves are used and the E.M.F. of the "B" battery is very much increased.

In receiving these continuous waves the receiving set is made to oscillate at a frequency slightly different from that of the incoming waves. These two sets of oscillations interfere and produce a beat note whose frequency is the difference between that of the incoming wave-train and that of the free oscillations of the receiver. This is somewhat similar to the experiment with the two tuning-forks described in § 211.

Continuous-wave telegraphy has practically replaced spark telegraphy on account of the long ranges secured with small power and also because an operator can tune out undesired signals much more readily. This permits many stations to work on nearly the same wave-length without interference.

564. Wireless Telephony. In wireless telephony a valve is used to generate continuous electrical oscillations and these oscillations are *modulated* by the action of a microphone such as is used in line telegraphy.

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*The key can also be inserted in the grid or plate circuits.

A simple transmitting circuit which can be used for either continuous-wave telegraphy or wireless telephony is shown in Fig. 624. L_1 and L_2 are inductances of about 36 and 80 turns wound on forms 4 and 3 inches in diameter. C_1 is a ¼ m.f. condenser and C_2 a variable 0.001 m.f. condenser. P and S are the primary and secondary coils of a telephone transformer. M is the microphone operated by the same battery which is used to light the filament. The high-tension generator, which should have an E.M.F. of about 350 volts when a 5-watt valve is used, is connected to E and F. A is a hot-wire ammeter placed in the aerial circuit.

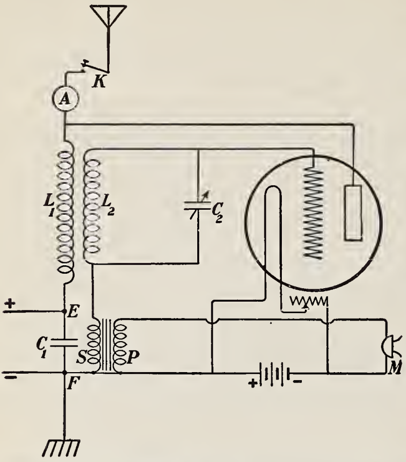

FIG. 624.—Transmitting circuit for continuous-wave telegraphy or wireless telephony.

In telephoning, the key K is kept closed and the circuits are tuned until the ammeter registers a satisfactory current. The sound to be transmitted is then produced in front of the microphone.

By this means the potential of the grid is made to vary in accordance with the sound waves falling on the microphone and, consequently, the outgoing continuous wave-train has impressed on it electromagnetic variations corresponding to the sound waves.

When a simple crystal set is used for reception the main or *carrier* wave is inaudible, but the receivers respond to the superimposed wave-train. With a valve receiving set it is possible to hear the carrier wave, but a slight adjustment in tuning will render it inaudible, while the sound produced by the superimposed wave is retained.

565. The Tungar Rectifier. This piece of apparatus (Figs. 625, 626) is a practical application of the thermionic valve which is very useful in rectifying alternating currents for charging storage batteries, for electroplating and for other purposes where direct current is necessary.

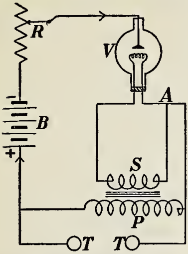

Fig. 625.—Diagram of Tungar Rectifier.

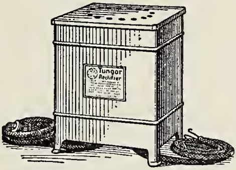

Fig. 626.—Outer view of the Tungar Rectifier.

The valve V is filled with an inert gas at a low pressure. The alternating current terminals are connected to the primary P of a small step-down transformer, to the secondary S of which the filament of the valve is attached. One terminal of the alternating source is also attached to the filament at A while the other terminal is joined to the positive pole of the storage battery B which is to be charged. The negative pole of B is connected through the variable resistance R to the plate.

When the current is turned on, the filament, which is heated to incandescence by the current induced in S, emits electrons which are attracted to the plate during the half-cycle when it is charged positively by the alternating current. Under these conditions current flows through the battery in the direction indicated by the arrow. As soon as the current reverses, however, the negatively charged electrons cannot escape from the positively charged filament and the valve becomes non-conducting until the current again reverses. Consequently, a pulsating, unidirectional current passes through the storage battery.

Rectifiers of this type can deliver as high as 12 amperes at 75 volts E.M.F. and require less care than the electrolytic rectifier described in § 460.

566. Passage of Electricity through Gases. In investigating this subject the gas is usually contained in a glass tube (Fig. 627) into the ends of which platinum wires are sealed.

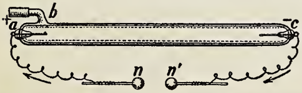

Fig. 627.—Arrangement to study the passage of electricity through a gas.

The terminals n, n' of an induction coil are joined to a and c, the electrodes of the tube. Let the electricity enter the tube at *a* and leave at *c*; these are, then, the *anode* and the *cathode*, respectively. Sometimes the electrodes have aluminium discs upon them. By connecting a side tube *b* to a good air-pump the air can be exhausted from the tube.

At first, when the air in the tube is under ordinary atmospheric pressure, the discharge passes between *n* and *n′*, but as the pressure is reduced, it begins to pass between *a* and *c*; and as the exhaustion is continued some very beautiful effects are produced.

If, however, the exhaustion is pushed still further, until the pressure within the tube is about one millionth of an atmosphere, phenomena of a different class are produced. As Sir William Crookes was the first to study these phenomena in great detail, these very highly exhausted tubes are known as Crookes Tubes.

From the cathode something is shot off which travels through the tube in straight lines and with great speed. This has been shown to consist of very small particles charged with negative electricity, and the streams of these particles are known as *cathode rays*.

567. Nature of Cathode Rays. That cathode rays really consist of streams of rapidly moving electrons can be demonstrated by the apparatus shown in Fig. 628. The Crookes tube *AB* has placed in it a longitudinal screen *CD*, coated with zinc sulphide. The end at *C* is bent at right angles to the screen and has a narrow transverse slit cut in it. The middle line of the screen makes a small angle with the axis of the tube.

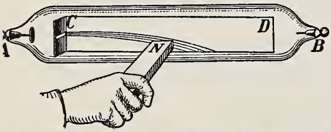

Fig. 628.—Cathode rays are deflected by a magnet.

When the electrodes are joined to an induction coil the cathode rays pass through the slit and fall on the inclined screen, producing fluorescence in a straight line (see § 569). If now the N-pole of a magnet is brought close to the side of the tube, the streak of light is deflected in the same way that a wire bearing a current would be deflected. We therefore conclude that a current is passing through the tube in the form of a stream of electrons.

568 Röntgen Rays. In 1895, Röntgen, a German physicist, while experimenting with Crookes tubes, discovered a new kind of radiation which he called X-rays, but which is more often known as Röntgen rays.

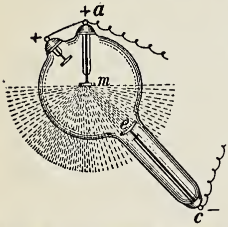

FIG. 629.—A Röntgen ray tube.

In Fig. 629 is shown a tube suitable for producing the Röntgen rays. The electrodes *a* and *c* are joined to a large induction coil. From the concave surface of the cathode *e* the cathode rays are projected, and when they strike the platinum plate *m* (or any other solid body) they give rise to the Röntgen rays, which spread out as shown in the figure, easily passing through the walls of the tube.

569. Photographs with Röntgen Rays. The Röntgen rays can affect a photographic plate just as light does. They can also pass through substances quite opaque to light, such as wood, cardboard, leather, flesh, but they do not so easily penetrate denser substances such as lead, iron and brass. If the hand be held close to a photographic plate and then exposed to the Röntgen rays, the rays easily pass through the flesh but are considerably hindered by the bones. Consequently when the plate is developed, that part which was behind the flesh is much more blackened than that behind the bones. When a print is made from the 'negative' we obtain a picture like that in Fig. 630.


FIG. 630.—From an X-ray photograph of the human hand.

In place of a photographic plate we may use a paper screen coated with crystals of barium-platino-cyanide. When the rays fall upon this, it shines with a peculiar yellow-green shimmering light. It is said to *fluoresce*. The shadow of an opaque body is clearly seen by this light.

570. Other Properties of the Röntgen Rays. If the hand or any other portion of the body is continually exposed to the Röntgen rays, serious injury may result.

Another striking characteristic of the rays is their ability to discharge an electrified body. If the air is thoroughly dry, a well-insulated electroscope (§ 419) will hold its charge for many hours; but if it is placed in the path of the Röntgen rays, the charge at once leaks away. For this to take place, the air surrounding the gold leaves must become a conductor of electricity.

The Röntgen rays are now known to be very short waves, similar in nature to light waves (§ 552).

571. Conduction of Electricity through Air. It is believed that electricity is conducted through a gas much as it is through a liquid. The latter was explained in § 451.

Let C and D (Fig. 631) be two parallel metal plates placed a few centimetres apart, and let C be joined to one pole of a

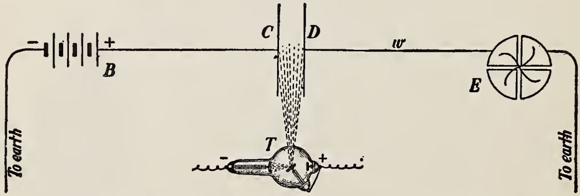

FIG. 631.—An arrangement to exhibit the conduction of electricity by air. The X-rays *ionize* the air.

battery, the other pole being joined to earth. E is an electrometer. This is a delicate instrument which measures the electrical charge given to it. First, suppose the tube T not to be in action; the needle of the electrometer will be at rest. Then let the tube be started, and let the Röntgen rays pass into the air between the plates C and D. At once the electrometer begins to receive a charge, showing that electricity has passed across from C to D and thence by the wire w to the electrometer.

When the X-rays pass through a gas, they cause the molecules of the gas to be broken up into positively and negatively charged carriers of electricity called *ions*. This process is called *ionization*. When a molecule is ionized, it is broken up into two ions, the electrical charges of which are equal in magnitude but of opposite sign. The positive ions are repelled from C to D and he negative ions are attracted by the plate C. In this way the electricity is transferred from C to D.

572. Radio-activity. In 1896, a French physicist named Becquerel discovered that the element uranium and its various compounds emitted a radiation which could affect a photographic plate; and soon afterwards it was shown that, like the X-rays, it could ionize the air. A little later it was discovered that thorium and its compounds acted in the same way. Thorium is the chief constituent of Welsbach mantles. All such bodies are said to be *radio-active*.

In searching for other radio-active bodies, Madame Curie observed that pitchblende, a mineral containing uranium, was more radio-active than pure uranium. After a very laborious chemical research, she succeeded in separating from several tons of pitchblende a few milligrams of a substance which was more than a million times as radio-active as uranium. To this substance the name of *radium* was given.

In experimenting, pure radium is not used, but radium bromide. Other radio-active substances have been discovered, *polonium* and *actinium* being the names given to two of the most powerful.

It is easy to illustrate radio-activity. Lay some crystals of a salt of uranium or thorium (uranium nitrate or thorium nitrate, for instance), upon a photographic plate securely wrapped in black paper and allow them to remain there for some hours. When the plate is developed, it will be found to be fogged. Or if the substance be held near a charged electroscope, the charge will at once leak away.

573. Different Kinds of Rays. Rutherford has shown that there are three types of rays emitted by radio-active bodies. These he named the α (alpha), the β (beta) and the γ (gamma) rays. The α rays are powerful ionizers of a gas, and it is now believed that the α particles are positively-charged atoms of helium. It takes very little to stop them. A sheet of aluminium $\frac{1}{20}$ mm. thick completely cuts them off. The β rays are much more active photographically than the α rays, but not so powerful in ionizing a gas. They consist of negatively charged particles and behave much like cathode rays. The γ rays can pass through great thicknesses of solid matter, and they are now believed to be similar in nature to the Röntgen rays.

For many years it was the dream of the old alchemists to change a base metal such as lead into a noble one such as gold, and they laboured in secret in their laboratories with this object in view. With the growth of physical science their task came to be looked upon as purely visionary, and yet in recent years something of the sort has actually been accomplished.

The study of radioactivity has led to the belief that the atoms of all substances are built up from the same elementary units—protons and electrons—and we might naturally expect that the atoms of one substance could be changed into atoms of another.

It has been found that the atom of a radio-active substance spontaneously disintegrates and gives rise to a new atom distinct in physical and chemical properties from its parent. Hydrogen can be transmuted into helium, and when this takes place a certain amount of energy is set free. It has been computed that, if we could transmute the hydrogen contained in one pint of water, the energy so liberated would be sufficient to propel the *Mauretania* across the Atlantic and back, at full speed.

It is now the dream of the physicist to discover how to utilize for the service of man the great stores of energy which are locked up in the atoms. With the poet Tennyson he can say:—

> "To those still-working energies
> I spy nor term nor bound."

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REVIEW PROBLEMS

Measurement

1. A mercury barometer read 75.28 cm. at the same time that an aneroid barometer close to it registered 30.24 in. Assuming that the former was correct, find the error in the latter in inches.

2. Amiens is 112 km. from Paris. Express this in miles.

3. In the Metric system land is measured in hectares, a hectare being a square hectometre. Find the number of acres in a hectare.

4. French field guns have a bore of 75 mm.; British eighteen-pounders measure 3.3 in. Express the difference in cm.

5. The wave-length of sodium light averages .0005893 mm. How many wave-lengths are there in a metre?

6. How many kilograms are there in a ton?

7. Find the specific gravity of an alloy consisting of 4 parts by volume of copper to 3 parts of zinc.

8. Calculate the weight of a litre of (a) ethyl alcohol, (b) mercury, (c) chloroform.

9. An oak beam measures 20 x 2 x 0.5 ft. Calculate its approximate weight.

10. Find the weight of the water in a swimming tank 75 x 25 x 8 ft. in dimensions (a) in tons (b) in kilograms.

Mechanics of Solids

1. With what velocity must a body be projected vertically upward in order that it may rise 3600 ft.?

2. A body is projected horizontally with a velocity of 30 ft. per sec. from the top of a cliff 144 ft. above the level of a lake. How far will the body travel horizontally before striking the lake?

3. A weightless rod 12 ft. long has masses of 20 and 60 lb. attached, one at each end. Find the position of the centre of gravity.

4. Two tugs pull on a vessel with forces of 1000 and 2400 pd. and their cables are at right angles to one another. Find the resultant pull on the vessel.

5. What force acting on a mass of 10 gm. for 5 sec. will produce a velocity of 100 cm. per sec.

6. A body starts from rest and moves 1000 cm. in 20 sec. under the action of a 50-dyne force. Find the mass of the body.

7. How many gallons of water can a 2 h.p. engine raise per hour from a well 20 ft. deep.

8. An automobile and load weigh 3000 lb. At what rate is the engine working when the car climbs to a vertical height of 440 ft. in 5 min.

9. The cylinder of a steam engine has an internal diameter of 5.6 inches and the average pressure of the steam during the stroke is 80 pounds per sq. in. If the length of the stroke is 10 in., find the work done per stroke in foot-pounds.

10. The load on a wheelbarrow is 270 lb. and can be considered as acting at a point $2\frac{1}{2}$ ft. from the axle of the wheel. What upward force must a man exert on the handles, which are $4\frac{1}{2}$ ft. from the axle, when pushing the barrow?

Mechanics of Fluids

1. What pressure will be produced at a tap when the vertical height of the water in the reservoir is 100 ft. above the level of the tap?

2. If the diameters of the cylinders of a hydraulic press are in the ratio 1 : 8, what will be the mechanical advantage of the press?

3. A body whose specific gravity is 8.3 weighs 124.5 gm. What will it weigh when immersed in water? What is its volume?

4. A balloon contains 2000 cu. m. of a gas whose density is .000091 gm. per c.c. If the density of air is .001293 gm. per c.c., find the total weight which the balloon will lift.

5. When the barometer is 30 in. high find the air pressure in grams per sq. cm. and pounds per sq. in. (1 in. = 2.54 cm.; 1 kg. = 2.2 lb.; s.g. of mercury = 13.6.)

6. A litre of air at 5°C. and under a pressure of 76 cm. mercury weighs 1.289 gm. What volume of air at the same temperature will weigh 1.289 gm. when under a pressure of 95 cm. mercury?

7. A certain mass of gas occupies 200 cu. ft. when the pressure is 20 pounds per sq. in. Under what pressure will the same mass of gas occupy 150 cu. ft.?

8. If the level of the water in a suction pump is 24 ft. above that of the water in the well and the diameter of the piston is 4.2 in., what force must be exerted on the piston rod to raise the piston?

9. Find the potential energy of a cubic foot of water just entering the penstock (Fig. 156) at a height of 305 ft. above that of the water in the tail-race.

10. What is the pressure against a vertical lock-gate which is 20 ft. wide when the water in the lock is 18 ft. deep? If a ship weighing 1500 tons moves into the lock and the depth of the water remains the same, what will be the pressure on the gate?

Velocity of Sound

1. In an experiment with Kundt's tube the tube was filled with oxygen and the nodes were found to be 4 in. apart. If the frequency of the rod was 1500 vibrations per sec., calculate the velocity of sound in oxygen.

2. A tuning fork has a frequency of 256 vibrations per sec., and the velocity of sound is 1120 ft. per sec. Find the wave-length of the sound produced.

3. Find the wave-length in air at 20°C. produced by a fork vibrating 400 times per sec.

4. The sound of the explosion of an enemy gun is heard 8 seconds after the flash is seen. How far away is the gun. (Temp. 15°C.)

5. How far off is a storm when there is an interval of 10 sec. between a flash of lightning and the thunder, the temperature being 18°C.?

Vibrating Strings

1. A string 80 cm. long vibrates 320 times per sec. when the tension is 40 kg. What will be the frequency when the tension is 160 kg.? What length of string under the initial tension would produce the same frequency?

2. A wire 96 cm. long makes 128 vibrations per sec. under a certain tension. What must the length become if the frequency is to become 512 per sec., the tension remaining constant?

3. An aluminium wire and another wire of the same length and diameter are stretched on a sonometer, and are in unison. If the density of the second wire is four times that of the aluminium wire, and if the tension of the aluminium wire is 20 pounds, what must be the tension of the other wire.

4. Compare the frequencies of two steel wires, the first 100 cm. long, 1 mm. in diameter and under a tension of 15 pounds.; the second 80 cm. long, ½ mm. in diameter and under a tension of 60 pounds.

5. A stretched string gives middle *C* when plucked. A bridge is placed at the middle of the wire while at the same time the tension is made four times what it was. What note will be produced by either half of the wire?

Resonance; Organ Pipes

1. A tuning-fork makes 512 complete vibrations per sec., and is in resonance with a closed tube 16.5 cm. long. Find the velocity of sound in air.

2. When the velocity of sound is 1120 ft. per sec., how long is a closed organ pipe whose first overtone has a frequency of 288 vibrations per sec.?

3. Find the length of an open organ pipe whose fundamental is in unison with a fork making 320 vibrations per second. (Temp. 20°C.)

4. An open organ pipe is 60 cm. long. What is the wave-length of its fundamental? If the velocity of sound is 342 metres per sec., find the frequency of the note.

5. An open organ pipe is 80 cm. long. What must be the length of a closed pipe which gives the octave above?

Expansion by Heat

1. A brass rod at 15°C is 62.00 cm. in length, and when raised to 55°C it is 62.05 cm. in length. Find the coefficient of linear expansion.

2. A horizontal glass tube of uniform bore is closed at one end; the other is open to the air. A drop of mercury which slides easily along the tube serves to confine a part of the air in the tube. If this drop is 20 cm. from the closed end of the tube when the temperature is 10°C., how far will it move if the temperature be raised to 60°C?

3. A mass of gas at 12°C. and under 760 mm. pressure occupies 150 c.c. If the temperature is raised to 50°C. and at the same time the pressure falls to 750 mm., calculate the new volume occupied by the gas.

4. A glass bottle has a capacity of 100 c.c. and is filled with mercury at 0°C. Neglecting the expansion of the bottle, what weight of mercury will escape from the bottle if the temperature is raised to 100°C.?*

*For mercury, coef. of vol. expansion=0.000182; density at 100°C.=13.35 gm. per c.c.

5. A certain mass of gas occupies 208 c.c. at 10°C. At what temperature will it occupy 226 c.c., if the pressure remains constant?

Specific Heat

1. Four beakers each contain 200 gm. of water, the temperatures being respectively 5°, 10°, 15°, 20°C. The water is all poured into an aluminium vessel at 15°C., weighing 50 gm. (specific heat 0.20). Find the resulting temperature. Find also the water equivalent of the aluminium vessel.

2. Find the temperature of equilibrium when 300 gm. of lead at a temperature of 98°C. is dropped into 500 gm. of water at 16°C., contained in a copper calorimeter weighing 100 gm.

3. The specific heat of sulphur is 0.2. If 1000 gm. of it at 80°C. are dropped into 200 gm. of water at 20°C. what will the resulting temperature be?

4. Find the temperature of the resulting mixture when 30 gm. of water at 40°C. are mixed with 50 gm. of alcohol at 60°C. (specific heat of alcohol = 0.60.)

5. 120 gm. of a liquid at 100°C. are poured into 300 gm. of water in a cup whose water equivalent is 8 gm. The water and cup rise from 13 to 27.5°C. What is the specific heat of the liquid?

Latent Heat

1. An iron ball whose mass is 200 gm. is heated to 100°C. and dropped into a cavity in a block of ice which is just at the melting point. If 28.25 gm. of water result from the melting of the ice find the specific heat of iron.

2. A piece of ice weighing 120 gm., at 0°C., is dropped into 200 gm. of water at 60°C. Find the temperature of the mixture when all the ice is melted.

3. A piece of ice is placed in water contained in an aluminium calorimeter. Calculate the heat of fusion from the following data:—

Mass of calorimeter, 25 gm.; specific heat of aluminium, 0.20 gm.; mass of water, 200. gm.; initial temp. of water, 20°C; mass of ice 22 gm.; final temp. of water, 10.3°C.

4. Find the temperature resulting from the mixture of 125 gm. of ice at 0°C. with 400 gm. of water at 40°C. contained in a copper vessel weighing 100 gm.

5. If 20 gm. of ice at 0°C. are dropped into 120 gm. of water at 60°C., find the temperature after all the ice has melted.

6. What weight of steam at 100°C. must be passed into 2 kg. of water at 12°C. in order that the resulting temperature may be 80°C.?

7. How many calories of heat are needed to change 1250 gm. of ice at −20°C. to steam at 100°C.?

8. How many B.T.U.'s are required to melt 100 lbs. of ice and to change the resulting water to steam at 212°F.?

9. The condensation of 50 gm. of alcohol vapour without change of temperature produces sufficient heat to raise the temperature of 666 gm. of water 15 C. deg.; find the amount of heat necessary to vaporize 1 gm. of alcohol.

10. Into 2 kg. of water at 20°C., 100 gm. of steam at 100°C. are passed. Find the final temperature of the mixture.

The Nature of Light

1. In a partial eclipse of the sun the images of the sun seen through the leaves of a tree are crescent-shaped. Explain why.

2. A man whose height is 6 feet observes that his shadow is 10 feet long at the same time that the shadow cast by a tower is 250 feet long. How high is the tower?

3. How could you use the shadow of a vertical rod to determine the true North-and-South line?

4. A pin-hole camera is 10 inches long and has a 4 x 5 inch plate at the end opposite the hole. How far from a building 80 feet high by 100 feet long must the front of the camera be held in order that the image may just cover the plate?

5. A square hole whose area is 1 square inch is cut in a piece of cardboard which is held at a distance of 2 feet from a very small bright source of light in a darkened room. A screen is placed 6 feet from the source so as to intercept the light which passes through the hole. Determine the area of the patch of light on the screen.

6. If the sun's rays make an angle of 45° with the horizon, how long will be the shadow cast by a tree 80 feet high?

7. Try to find the length of the shadow cast by a rod placed in the sunshine. Can you find it accurately? Explain.

8. In finding the velocity of sound in air by the gun method, why is no correction made for the time it takes the light to come from the gun to the observer?

9. If an aviator could fly away from the earth at a rate equal to the velocity of light and observe a clock left behind on the earth, how would the hands appear to move?

10. If the velocity of light is 186,000 miles per sec., and that of sound 1120 ft. per sec., how long would it take each to travel a distance equal to the circumference of the earth (25,000 m.)?

Photometry

1. State the law of variation of intensity of illumination with distance and describe how it is used in determining the candlepower of a lamp.

2. Compare the illumination produced by a lamp of 10 c.p. at 200 cm. distance with that produced by a 25 c.p. lamp at 100 cm. distance.

3. A 16 c.p. lamp is placed one metre from a 25 c.p. lamp. Where must a screen be placed between them in order that its sides shall be equally illuminated.

4. An arc lamp 100 ft. from a screen illuminates it as much as a 16 c.p. lamp 8 feet away. What is the c.p. of the arc?

5. A body 50 ft. from a luminous point is moved 20 ft. nearer the point. Compare the intensity of illumination received by the body in the two positions.

Plane Mirrors

1. A piece of white blotting-paper reflects most of the light which falls on it. Why can it not be used as a mirror?

2. The sun is 60° above the horizon. At what angle with the horizon will the mirror in the heliograph (Fig. 343) have to be set in order that the reflected ray may be horizontal?

3. If the mirror in the heliograph is tilted through $1\frac{1}{2}°$ by working the key, through how many degrees will the reflected ray be deflected?

4. A ray strikes a plane mirror obliquely and the mirror is turned so that the new reflected ray is at right angles to the former reflected ray. Find the angle through which the mirror has been turned.

5. How must a ruler be held in front of a plane mirror in order that its image and itself may form two sides of an equilateral triangle?

Curved Mirrors

1. Distinguish clearly between a real and a virtual image.

2. Describe two methods for finding the focal length of a concave mirror.

3. State the Laws of Reflection from curved mirrors.

4. A concave spherical mirror has a radius of curvature of 30 cm. An object is placed 75 cm. from the mirror. Where will the image be? Is it real or virtual, erect or inverted?

5. The filament of an electric lamp is 8 cm. long. The lamp is lighted and placed so that the filament is 50 cm. from a concave mirror. The image is caught on a screen which is 20 cm. from the mirror. How high is the image?

6. An object 4 cm. high is held 15 cm. in front of a convex mirror of radius 60 cm. Find the position, nature and size of the image.

7. An object 3 cm. high is placed 20 cm. from a concave mirror whose radius of curvature is 24 cm. Find the position, size and nature of the image.

8. A candle is held before a convex mirror of radius 12 inches (*a*) 8 inches, (*b*) 16 inches, from the mirror. Find the position and size of the image in each case.

9. An electric lamp stands on a table and is 8 feet from a wall. By means of a mirror you produce an image of the lamp on the wall which is 3 times as large as the lamp. What kind of mirror is it? What is its radius of curvature? Is the image erect or inverted?

10. Would an image produced by a concave mirror show any colour effect? Give one advantage of a reflecting telescope (§391) over a refracting instrument (§390).

Refraction—Lenses

1. Find the velocity of light in water and in Canada balsam, if the velocity in air is 186,000 miles per sec. (See table § 351.)

2. Find the angle of refraction when light passes from air to glass, the angle of incidence being 25° and the index of refraction being 1.5.

3. A projection lantern throws upon a screen an image of a picture 2 x 3 in. in size on a lantern slide placed 10 inches from the projecting lens. If the screen is 30 ft. from the lens, find the size of the image.

4. A lamp is 120 cm. from a screen. Where must a lens be placed in order to throw upon the screen an image of the lamp five times as long as the lamp itself? What must be the focal length of the lens?

5. Calculate the power and focal length of a compound lens composed of a converging lens having a focal length of 10 cm. and a diverging lens whose focal length is 25 cm.

6. The focal length of a converging lens is 15 cm. An object 2 cm. long is placed 10 cm. from the lens. (*a*) Is the image real or vertical? (*b*) How far is the image from the lens? (*c*) What is the size of the image?

7. If the index of refraction from air to rock salt is 1.544, find the critical angle for rock salt. (Make a careful diagram and measure the angle.)

8. Light passes through a 60° prism made of glass having an index 1.52. By means of a diagram calculate the angle of deviation when the angle of incidence is 49½°.

9. A camera having a simple lens is focussed upon a man 6 feet tall, standing 12 ft. from the lens. If the film is 6 in. from the lens, compute the size of the image. Find also the focal length of the lens.

10. The focal length of a telescope is 6 ft. The lens is composed of two lenses, one converging and the other diverging. If the focal length of the former is 30 in., find that of the latter.

Magnetism

1. A rod of soft-iron is held vertically and struck with a hammer. Describe how you would test whether the rod is magnetized. Which end should become an S-pole? Why strike it with the hammer?

2. State the Molecular Theory of Magnetism and give three facts which support the theory.

3. A circular disc of wrought iron is placed between the opposite poles of two magnets, whose axes are in the same straight line with a diameter of the disc. Make a diagram of the field of force.

4. A bar-magnet is fixed in a vertical position with its N-pole downwards, and several nails are suspended from it. A similar bar-magnet of equal strength is slid along it with the S-pole downwards. What will happen to the nails? Explain the action.

5. What effect is produced (*a*) on the weight, (*b*) on the position of the centre of gravity of a bar of iron, by magnetizing it?

6. What would be the magnetic bearing from a tower of (*a*) a tree which is due north of the tower, (*b*) a wind-mill which is due north-east of the tower, the declination being 10°E?

7. A battery moves into a new position by night and opens fire on a bridge at a distance of 7000 yards. The guns are laid by prismatic compass, the true bearing of the bridge being obtained from a map by using a protractor. If no allowance is made for declination, which is 10°W., how far from the bridge will the shells fall? (One degree equals 1.7 yds. per 100 yds. of range.)

8. Will a bar-magnet, floated in water on a piece of wood, drift towards the north? Give reasons for your answer.

9. Compasses used in surveying are usually provided with a small adjustable weight, which is used to balance the needle. Why is it necessary to make the weight adjustable? If the weight is on the S-end of the needle, how would you move it to adjust a compass, which was last used in New Orleans, for work in Ottawa?

10. Compasses become more sluggish in their movement as they approach the north magnetic pole. Account for this.

Static Electricity

1. Use the Electron Theory to explain why a pith-ball is first attracted to and then repelled from a charged ebonite rod.

2. How can an electroscope be shielded from the action of an electrical machine working near it?

3. A charge is placed on a gold-leaf electroscope. Describe what will happen to the leaves when (*a*) an insulated plate, (*b*) an earth-connected plate, is brought near the knob of the electroscope.

4. Why is repulsion between an unknown body and an electrified pith-ball a better indication that the body is electrified than is attraction?

5. Is it possible for a body on which there is a small charge of electricity to be at a higher potential than a body on which there is a much greater charge? Explain clearly what is meant by *potential* and by *capacity.*

6. An electrical line of force marks the path along which a free unit positive charge would move, when placed near a charged body. Draw the field of force for a charged two-plate condenser.

7. What is meant by electrostatic induction? If a heavy negative charge is slowly brought up to a positively charged gold-leaf electroscope, what will be the effect on the leaves? Give reasons.

8. What will be the effect of connecting a condenser across the spark-gap of an electrical machine, (*a*) on the brightness of the spark, (*b*) on the frequency with which the spark occurs?

9. A Leyden jar is held in the hand by the outer coating and the knob is touched to a terminal of an electrical machine. The jar is then placed on a table. Explain why you receive a shock on touching the knob, and why no shock is felt if the jar is placed on a dry cake of rosin.

10. Compare the maximum capacity of the twenty-three plate variable condenser (Fig. 502) with that of a three plate "vernier" condenser, the condensers being identical except for the number of plates. (The twenty-three plate condenser has twelve fixed and eleven moving plates; the three plate condenser has two fixed and one moving.)

Electrolysis

1. In calibrating an ammeter a certain current was found to deposit 0.7 gm. of silver in 40 min. What was the strength of the current?

2. An ammeter indicated 5 amperes while a current which deposited 6.2 gm. of copper in 1 hour was flowing through it. Find the error in the ammeter reading.

3. How much hydrogen and how much oxygen will be set free in a water voltameter by a current of 8 amperes flowing for 15 min.?

4. How long must a current of 80 amperes flow to refine one ton of copper?

5. The same current is passed through three electrolytic cells, the first containing acidulated water, the second a solution of copper sulphate, and the third a solution of silver nitrate. What weight of hydrogen and what weight of oxygen will be liberated in the first cell; and what weight of copper will be deposited at the cathode of the second cell; when 11.18 grams of silver are deposited on the cathode of the third cell?

Ohm's Law and Resistance

1. A flash-lamp has a resistance of 12 ohms and is lighted by a battery of 3 cells in series, each having an E.M.F. of 1.5 volts and an internal resistance of 2 ohms. The output of the battery is ½ ampere-hour. How many flashes, each five seconds long will the lamp give before a new battery is required?

2. Six cells are joined abreast in pairs and the three pairs in series. Each cell has an E.M.F. of 1.7 volts and an internal resistance of 3 ohms. Calculate the current which flows when the terminals of the battery are joined by a wire 40 ft. long, having a resistance of 4 ohms. Find also the p.d. between two points on the wire 12½ ft. apart.

3. Two wires are joined in parallel and have a joint resistance of 3 ohms. If one wire has a resistance of 12 ohms, find the resistance of the other.

4. A cell having an internal resistance of 0.2 ohm is connected to an external circuit having a resistance of 2 ohms and including an ammeter. The difference in potential between the terminals of the cells is found to be 1.25 volts. What current will the ammeter indicate? What is the E.M.F. of the cell?

5. What is the resistance of an electric toaster which allows 5 amperes to flow through it when connected to a 110-volt circuit? What current will flow through the heater when an electric lamp whose resistance is 110 ohms is connected (*a*) in series (*b*) in parallel with it?

6. Calculate the unknown resistance for each of the following measurements with a Wheatstone Bridge (§ 532).

(1) A = 19, B = 10, C = 100 ohms;
(2) A = 187, B = 1, C = 1000 ohms;
(3) A = 234, B = 1000, C = 10 ohms;
(4) A = 3, B = 1, C = 1000 ohms.

7. Find the unknown resistance for each of the following measurements with a Slide-wire Bridge (§ 533).

(1) A = 23 ohms, PN = 40, NQ = 60 cm.;
(2) A = 30 ohms, PN = 20, NQ = 80 cm.;
(3) A = 78 ohms, PN = 48, NQ = 52 cm.;
(4) A = 125 ohms, PN = 63, NQ = 37 cm.

8. Find the resistance at 20°C. of 6 miles of copper wire 50 mils. in diameter (§ 534).

9. What length of No. 14 copper wire has a resistance of 10 ohms? (§§ 534, 535).

10. What current will pass through 300 yds. of No. 18 copper wire when connected to a storage battery whose E.M.F. is 2.2 volts and whose internal resistance is negligible? (§§ 534, 535).

Electrical Energy and Power

1. An electric range takes 10 amperes at 220 volts pressure. Find the cost of using it for 2 hours at 5c. per k.w.h.

2. On May 1st a meter read 8764 k.w.h, and on June 1st, 8876 k.w.h. Make out the bill for May at 7c. per k.w.h.

3. A watt-hour meter registered 2 k.w.h. in 2 hours when the E.M.F. was 110 volts and the current flowing through the "load" was 10 amperes. Find the error in the meter reading.

4. An electric motor which actually developed 2 horse-power required 16.5 amperes at an E.M.F. of 110 volts. Find the efficiency of the motor.

5. An electric fan requires 0.25 ampere at an E.M.F. of 110 volts. How much will it cost to run it for 5 days, 10 hours per day, in a town where electric energy costs 8c. per k.w.h?

6. A 32 c.p., 120-volt lamp requires 1.25 watts per candle. Find the current which passes through the lamp and the cost of using it for 5 hours at 5c. per k.w.h.

7. What is the resistance of a 40-watt, 110-volt lamp. How many such lamps can be operated by a 5 k.w. dynamo?

8. In a transformer the input is 5 amperes at 2200 volts when the output is 98 amperes at 110 volts. Find the efficiency of the transformer.

9. A heating coil having a resistance of 50 ohms is connected to a 100-volt circuit and is placed in 1000 gm. of water at 0°C. Find the temperature of the water 10 min. later.

10. A 60-watt, 120-volt lamp is immersed in 500 gm. of water at 10°C., and the current is turned on for 30 min. Find the new temperature of the water.

ANSWERS TO NUMERICAL PROBLEMS

Part I—Introduction

Page 6. **1.** 2,500,000 mm. **2.** 299,804.97 km. **3.** 3,900,000 sq. cm. **4.** 29.921 in. **5.** 1000 l; 1,000,000 c.c. **6.** 183.49 m. **7.** 65.4 cents. **8.** 9697.5 kg. **9.** The former. **10.** 29.16 cents. **11.** 7899.80 mi., 7926-.81 mi. **12.** 11.38 sec. **13.** 4.79 mm.

Page 13. **1.** 1.47 kg. **2.** 54.05 c.c. **3.** 519.75 gm. **4.** 21.59 gm. per c.c.; 46.318 c.c. **5.** 62.44 lb. **6.** 2.7 gm. per c.c.; 2.7; 168.48 lb. per cu. ft. **7.** 12 kg. **8.** 0.77 gm. per c.c.; 0.77; 48.048 lb. per cu. ft. **9.** 1.99 mm. **10.** 283.5, 0.5, 1.9, 7.1, 13.0 gm.; 1814.4, 453.6, 141.7, 14.2, 85.0, 28.3 gm.

Part II—Mechanics of Solids

Page 16. **1.** 24 mi. per hr. **2.** $32\frac{5}{11}$ mi. per hr. **3.** 2898.44 mi. **4.** 88 ft. per sec. **5.** 108 km. per hr. **6.** 11 mi. per day. **7.** $2\frac{9}{11}$ mi. per day. **8.** 45 mi. **9.** 3600 ac/b cm. **10.** 1413.72 m. per min. **11.** $50{,}856 \times 10^9$ mi. (1 yr. = 365 d.)

Page 18. **1.** 20 ft. per sec.; 30 ft. per sec. **2.** 22 sec. **3.** $-5\frac{13}{15}$ ft. per sec. per sec. **4.** 5 m. per sec.; $-\frac{5}{6}$ m. per sec. per sec. **5.** $2\frac{17}{54}$ cm. per sec. per sec. **6.** $-\frac{1}{12}$ ft. per sec. per sec. **7.** 32.185 ft. per sec. per sec.

Page 19. **1.** 100 ft.; 225 ft. **2.** 484 ft. **3.** $293\frac{1}{3}$ ft. **4.** $833\frac{1}{3}$ m. **5.** 600 ft. **7.** 60 ft. per sec. **8.** $-8\frac{136}{225}$ ft. per sec. per sec. **9.** 125.86 ft. per sec. **10.** $(2n—1)\ a/2$ cm.

Page 23. **1.** (a) 4 ft. per sec.; (b) 1000 cm. per sec. **2.** 192 ft. per sec.; 576 ft. **3.** 190 cm. per sec. **4.** 128 ft. per sec. **5.** 7.82 sec.; 4.37 sec. (approx.) **6.** $759\frac{3}{8}$ ft.; $33\frac{3}{4}$ sec. **7.** 3 sec.; $14\frac{2}{3}$ ft. per sec. per sec. **8.** $\frac{1}{2}$ ft. per sec. per sec. **9.** 4 sec.; 78.4 m. **10.** 144 ft. or 44.1 m. **11.** (1) 160 ft. per sec. or 49 m. per sec.; (2) 400 ft. or 122.5 m.; (3) 144, 256, 336, 384, 400 ft. or 44.1, 78.4, 102.9, 117.6, 122.5 m. **12.** Yes; $29\frac{1}{3}$ ft. to spare. **13.** 4900 ft.

Page 27. **2.** 14, 8, 11.4 ft. per sec. **4.** 62.45 cm. per sec.; 68.06 cm. per sec. **5.** 7.55 ft. per sec.

Page 31. **1.** 125 : 3. **2.** 5 : 4. **3.** 175 : 256.

Page 34. **1.** 1, 2, 10 cm. per sec.; 1 cm. per sec. per sec. **2.** 0.1, 0.2, 1 cm. per sec. ; 0.1 cm. per sec. per sec. **3.** 10, 20, 100 cm. per sec.; 10 cm. per sec. per sec. **4.** 5, 10, 50 cm. per sec.; 5 cm. per sec. per sec.; 20,000 units. **5.** 5 gm.; 2 cm. per sec.per sec. **6.** 200 dynes.

Page 35. **1.** 50 ft. from house. **2.** 1250 m. per sec. **3.** 19.6 m. per sec.; 1250.15 m. per sec.

Page 36. **1.** 1 ft. per sec. **2.** 25 m. per sec. **4.** 11.72 ft. per sec. **7.** 15 ft. per sec. **9.** 3220 ft. per sec. **10.** $2\frac{6}{7}$, $17\frac{1}{7}$ ft.

Page 40. **1.** $16\frac{1}{4}$ lb. **3.** 3 ft. **4.** $2 : \sqrt{3} : \sqrt{2}$. **5.** $70\frac{7}{12}$ lb.; $50\frac{5}{12}$ lb. **6.** 15 lb., 10 lb.

Page 43. **1.** 39 lb. **2.** 15 lb. **3.** 154.9 lb; 19° (nearly). **4.** 48 lb., 64 lb. **5.** 751.7 lb.

Page 47. **1.** Doubled. **2.** 125,219.6 mi. **3.** $44\frac{4}{9}$, 25, 16 kg. **4.** 0.37 lb.

Page 50. **1.** 100,000 ergs. **2.** 1800 ft.-pd. **3.** 50,000 ft.-pd. **4.** $\frac{1}{245}$ kg.-m. **5.** 150,000 ft.-pd. **6.** 528,000 ft.-pd.

Page 53. **2.** $\frac{4}{15}$ h.p. **3.** $\frac{35}{44}$ h.p. **5.** 80.568 k.w. **6.** 7200 ft.-pd.; $\frac{36}{55}$ h.p. **7.** 20 h.p. **8.** 3675×10^7 ergs. **9.** 8 ft. **10.** 7.58 h.p.

Page 57. **8.** 10 in. from 1-lb. weight. **9.** 5 ft. **10.** 40 cm. from 12 kg. pail; $40\frac{10}{21}$ cm. from 12 kg. pail.

Page 60. **4.** 45 lb. **6.** 396,000 ft. – pd. **7.** $\frac{1}{15}$. **8.** (a) $51,020\frac{20}{49}$ gm.-cm. (b) 6000 f gm.-cm. (c) $8\frac{74}{147}$ gm. (d) $\frac{25}{1176}$.

Page 64. **1.** $1\frac{1}{4}$ lb. **2.** 225 lb. **3.** 90 lb., 120 lb. **4.** $37\frac{1}{2}$ lb.

Page 67. **1.** Twice as great; twice as long. **2.** 4. **3.** $\frac{1}{3}$. **4.** 60 lb.

Page 70. **1.** $26\frac{2}{3}$ lb. **2.** $53\frac{1}{3}$; 6400 lb.

Page 72. **2.** $20\frac{5}{9}$ lb. **3.** $4\frac{17}{27}$ lb. **4.** $\frac{1}{126}$ lb.; $\frac{1}{252}$ lb.

Part III—Mechanics of Fluids

Page 83. **1.** $312\frac{1}{2}$ gm. **2.** 800 kg. **3.** 11,550 lb. **5.** 36 kg. **6.** 184.87 ft. (nearly). **7.** 92.43 ft. **8.** 14048 pd. per sq. in. **9.** 3 tons. **10.** 173.31 ft.

Page 87. **1.** 62.5 pd.; 97.5 lb.; **2.** 4.566 lb. **3.** 2.5 kg. **4.** 4.9 gm. **5.** $\frac{2}{15}$. **6.** 600 gm. **7.** $\frac{1}{4}$. **8.** 1562.5 lb. **9.** $3906\frac{1}{4}$ lb. **10.** 27.5 lb. **11.** $133\frac{1}{3}$ c.c. **12.** 4.8 c. ft. **16.** 22.5 lb. **19.** 0.78 in. **20.** $(n-m)/n$ gm. per c.c. 125 $(n-m)/2n$ lb. per c. ft.

Page 92. **1.** 1.2 gm. per c.c. **2.** 20 c.c.; 6 gm. per c.c.; 0.8 gm. per c.c. **3.** 0.81 gm. per c.c. **4.** 1.071 gm. per c.c. **5.** $\frac{2}{3}$ gm. per c.c. **6.** (a) $\frac{5}{8}$; (b) $\frac{5}{6}$ (c) $6\frac{1}{4}$ in. **7.** $\frac{8}{9}$ gm. per c.c. **8.** 25 cm. **9.** $\frac{1}{4}$ gm. per c.c. **10.** $\frac{3}{7}$ gm. per c.c. **11.** 20.c.c.

Page 100. **1.** 12.92 m. **2.** 1272.96 lb. **3.** 26192.6 ft. **4.** 10200 kg.

Page 107. **1.** $6\frac{2}{3}$ cu. ft. **2.** 22.85 l. **3.** 75,314.7 cu. in. **4.** $483\frac{1}{3}$ in. of mercury. **5.** $562\frac{1}{2}$ mm. **6.** 174 in. of mercury. **7.** 0.0001259 gm. per c.c. **8.** 101.34 gm. **9.** $3.60. **10.** (a) 30 atmospheres. (b) 292.148 m.

Page 110. **4.** 2908.75 kg. **5.** 2685 kg. **6.** 1342.5 kg. **8.** 4967 lb. **9.** 2360 lb.

Page 122. **1.** $\frac{2}{3}$; $\frac{8}{27}$. **2.** $\frac{4}{5}$. **3.** $1\frac{1}{2}$. **4.** 12.92 m.

Page 124. **2.** (a) Height of mercury barometer; (b) 13.6 times height of mercury. **3.** $219\frac{1}{9}$ in. **9.** 22464 pd. per sq. ft.; 23025.6 pd. per sq. ft. **10.** 60 gm. per sq. cm.; 0.853 pd. per sq. in.

Part IV—Properties of Matter

Page 139. **1.** $3\frac{1}{3}$ sq. km. **2.** 100,000,000 yrs. (approx.) **3.** 2000.

Part V—Sound

Page 163. **1.** 335, 338, 356 m. per sec. **2.** 1024.3 ft. per sec. **3.** 5595 ft. **4.** 3490.2 ft. **5.** 3.81 sec. (nearly). **6.** 1680, 2240 ft. **7.** 4707.4 ft. per sec. **8.** 3474.7 m. per sec. **9.** 4290.2 ft. per sec. **10.** 5545 yd.

Page 174. **4.** 1678.5 ft. **5.** 2685.6. **6.** 260; $4\frac{4}{13}$ ft. **7.** $32\frac{5}{8}$, $65\frac{1}{4}$, $130\frac{1}{2}$, 261, 522, 1044, 2088, 4176; **8.** 1044. **10.** 400, 500. **11.** 271.2. **12.** 7.07 cm. **13.** G′, E″. **14.** 163.1, 217.5, 293.6, 391.5, 489.4, 652.5. **15.** 576.

Page 187. **3.** 3 : 2. **5.** 6.49 in. **6.** 4,064 m. per sec. **7.** 36 lb. **9.** $\sqrt{2}:1$. **10.** 10.38 in., if closed; 20.76 in., if open. **11.** $11\frac{1}{4}$ lb.; by $33\frac{1}{3}$ cm. **12.** 1.5 ft. **13.** 9 : 10. **14.** 13.23, 39.69, 66.15 in.; 26.46, 52.95, 79.38 in. **15.** 96 lb.

Page 197. **1.** E″, G″. **5.** 2 times.

Part VI—Heat

Page 219. **1.** 9, 32.4, 48.6, 117. **2.** $11\frac{1}{9}$, 15, 20, $52\frac{7}{9}$. **3.** 117. **4.** $15\frac{5}{9}$. **5.** $-17\frac{7}{9}°$, $-12\frac{2}{9}°$, 0°, $7\frac{2}{9}°$, $37\frac{7}{9}°$, $-31\frac{2}{3}°$, −40°. **6.** 50°, 68°, 89.6°, 167°, −4°, −40°, −459.4°. **7.** $38\frac{8}{9}$. **8.** (a) 9.6°, −8°, −12°, $1\frac{7}{9}°$; (b) 20°C., 68°F.; $31\frac{1}{4}$°C., $88\frac{1}{4}$°F.; $-7\frac{1}{2}$°C., $18\frac{1}{2}$°F. **9.** $36\frac{2}{3}°$.

Page 224. **1.** 4.0002 ft. **2.** 1.000342 m. **3.** 120.0216 sq. in. **4.** 57.69 cm. **5.** $(1 + x)^2 = 1 + 2x + x^2$; x is very small, and consequently x^2 can be neglected. 8. 0.000515.

Page 227. **1.** (a) Double; (b) Same; (c) One-half. **2.** (a) Double; (b) Same. **3.** 28.55 l. **4.** 107.59 c.c. **5.** 155.1 pd. per sq. in. **6.** 17.53 pd. per sq. in. **7.** 1.127 gm. **8.** 32.8°C.; −22.8°C. **9.** 27.3°C. **11.** 108.87 l. **12.** $322\frac{2}{9}$ c.c. **13.** 1520 mm. mercury. **14.** No change. **15.** 1050 c.c. **16.** 1779 : 1465. **17.** 0.837 gm **18.** 0.0000760 gm. per c.c. **19.** 6258.34 mm. **20.** 174.997°C.

Page 231. **1.** 1625 cal. **2.** 3,000 cal. **3.** 23°C. **4.** 10 C. deg. **5.** 66.5.° C. **6.** 45°C. **7.** 9.6°C. **8.** 40°C. **9.** 110°F. **10.** $10.

Page 233. **1.** 12,250 cal. **2.** 6585 cal. **3.** 3166⅔ cal. **4.** 8160 cal. **5.** 34,627.5 B.T.U.

Page 235. **1.** 5.6 cal. **2.** Mercury, 2.244 cal.; Water, 2 cal. **3.** 36 cal. **4.** 0.094. **5.** 0.694. **6.** 9,600 cal. **7.** 950.16 cal. **8.** 226,000 cal. **9.** 27.9 cal. **10.** 3.17°C. **11.** 0.113; iron. **12.** 0.113. **13.** 30,225 cal. **14.** 47.0°C. **15.** 0.748. **16.** 12.42°C. **17.** 0.678. **18.** (a) 11.28 gm. (b) 3.2 gm. (c) 1.699 gm.

Page 242. **9.** 2800 cal. **10.** 1,200,000 cal. **11.** $59\frac{1}{11}$°C. **12.** $1{,}111\frac{1}{9}$ gm. **13.** 80 gm. **14.** 2,185 cal. **15.** 166.15 gm. **16.** 12.05 gm. **17.** 0.09. **18.** 79.38. **19.** 144 B.T.U. **20.** 30,000 B.T.U.

Page 250. **9.** 19,832 cal. **10.** 182,240 cal. **11.** 27,945 cal. **12.** 230,680 cal. **13.** 44.37°C. **14.** 10.07 gm. **15.** 3.14 gm. **16.** 21,705 cal. **17.** 432,500 cal. **18.** 536.4 cal. per gm. **19.** 37.31 gm. **20.** $\frac{4}{77} \times 10^5$ kg.

Page 262. **6.** 55.8%; 10.3°C. **7.** (a) 72.3%. (b) 61.7%. (c) 55.9%.

Page 272. **4.** 2500 ft.-pd.; 3.21 B.T.U. **5.** 3,736,250 l. **6.** 2.39×10^3 cal. (approx.). **7.** 77,763.5 B.T.U. **8.** 78.06 kg. **9.** 1038.8 lb. **10.** 16.9%.

PART VII—LIGHT

Page 296. **1.** 2.4 in. **2.** 62½ ft. **7.** 7.44 times. **8.** (a) 176.86 yrs. (b) 13.89 yrs. (c) 8⅓ min. **9.** $5{,}047{,}953{,}696 \times 10^4$ mi.; $2{,}582{,}673{,}984 \times 10^5$ mi. **10.** $2\frac{1}{12}$°.

Page 302. **2.** 9:4. **3.** 72:5. **4.** 25:9. **5.** 25:4. **6.** 121:400. **7.** 51.96 in. from screen. **8.** 16 c.p. **9.** 56.56 in. **10.** 14.15 c.p. **11.** 4 ft. **12.** 2 ft. from candle towards gas-flame and 6 ft. from candle in opposite direction.

Page 311. **3.** 60°. **4.** 7; 5; 3. **8.** 110°; 4°.

Page 322. **7.** 7½ cm. behind mirror, ¾ cm. high; $4\frac{4}{9}$ cm. behind mirror, 1⅔ cm. high.

Page 332. **1.** 0.405. **2.** 139,500 mi. per sec.; 124,000 mi. per sec.; $\frac{9}{8}$. **10.** 22°, 32°, 40½°. **11.** 19½°, 28°, 35°. **12.** 30°, 48½°. **16.** 1.52.

Page 343. **2.** 13⅓ cm. **3.** 50 cm.; 33⅓ cm. **4.** $22\frac{2}{9}$ cm.; 40 cm. **5.** $4\frac{2}{7}$ ft. **6.** 4.8 cm. **7.** Converging, 10 cm.; diverging, 15 cm. **12.** (a) 20 cm. to left of lens, candle being on right; mag. = 1. (b) 12 cm. to right of lens; mag. = $\frac{3}{5}$. **13.** 1.268 ft. or 4.732 ft. from flame; mag. = 3.73 or 0.27. **14.** 9.6 in. **15.** 60 in. to left of lens; 5 in. high. **16.** If real image, 16 cm.; if virtual, 26⅔ cm. **17.** 32 cm.; mag. = 4 and ¼. **20.** 9¾ ft. × 12 ft.

Page 360. **9.** 30 cm.

Part VIII—Electricity and Magnetism

Page 386. **15.** 100°; 280°. **16.** $\frac{5}{6}$ mi. **18.** 7.07 mi., 10 mi.; 35°, 50°. **19.** 8.66 mi.; 5 mi.

Page 425. **1.** 2.3616 gm.; 8.0496 gm. **2.** 0.373824 gm.; 2.990592 gm. **3.** 4.1768 l.; 2.0884 l. **4.** 6 amperes. **5.** 5 amperes. **6.** 0.05 amperes. **7.** 2 amperes. **8.** 50 min. 49 sec. (approx.). **9.** 4.423584 gm.

Page 435. **4.** 20 hr.; $26\frac{2}{3}$ hr. **5.** 15 hr. **6.** 0.6102 gm. **7.** $2\frac{1}{2}$ ampere-hours. **8.** 7.3224 gm.

Page 472. **5.** 40 cycles; 16 poles.

Page 487. **1.** $\frac{1}{3}$ amp. **2.** $\frac{1}{2}$ amp. **3.** 208 ohms. **4.** 0.19 ohm. **5.** 22 volts. **6.** 13.5 volts. **7.** 10 amp. **8.** 3 ohms.

Page 489. **1.** $8\frac{1}{3}$ amp.; 80 volts; 20 volts. **2.** $\frac{1}{2}$ amp.; $\frac{1}{2}$ volt. **3.** 1 volt; 1 volt; 102 volts. **4.** 113 volts. **5.** 1.2 amp.; 1.44 volts. **6.** 0.75 volt; 0.15 volt.

Page 494. **1.** 3.193 ohms. **2.** 72 ohms. **3.** 20.31 ohms. **4.** 22.5 m. **5.** 135.9 yd. **6.** $\frac{2}{3}$ mm. **7.** 2.653 mm. **8.** 1080 yd. **9.** 32.48. **10.** 20.1 ohms; 75.84 ohms.

Page 496. **1.** $6\frac{2}{3}$ ohms; $\frac{2}{3}$, $\frac{1}{3}$. **2.** $8\frac{1}{12}$ ohms; $\frac{63}{73}$ohm. **3.** 4 ohms. **4.** 100 ohms. **5.** $2\frac{1}{12}$ amp.; $\frac{12}{25}$; 48 ohms. **6.** 100 ohms. **7.** $1/(n + 1)$. **8.** 9 : 25.

Page 499. **1.** 3 amp. **2.** $1\frac{14}{19}$ amp.; $1\frac{13}{53}$ amp.; $2\frac{4}{31}$ amp. **3.** $\frac{1}{2}$ amp. in each case. **5.** 50 cells. **6.** All in series, 0.126 amp.; all in parallel, 0.067 amp.; two banks in parallel, each containing three cells in series, 0.134 amp.; three banks in parallel, each containing two cells in series, 0.113 amp. **7.** 0.15 amp.; 0.092 amp.

Page 503. **1.** 360,000 coulombs. **2.** 12.5 k.w. **3.** 13.75 k.w.; 18.43 h.p. **4.** 99,840 watt-hours.; $7.99. **5.** 3.3 cents. **6.** 11 lamps. **7.** $2\frac{8}{746}$ h.p. **8.** 90%. **9.** 1.57 watts per c.p. **10.** 103,680 cal.

Page 505. **1.** 0.00032 m.f. ($K = 6$). **2.** 0.000374 micro-farads. **3.** 0.178 milli-henries; 25.565 milli-henries. **4.** 12.64 milli-henries.

Review Problems

Measurement. **1.** 0.602 in. **2.** 69.5968 mi. **3.** 2.47 acres. **4.** 0.882 cm. **5.** 1,696,928.5. **6.** 907.18 kg. **7.** 8.13 **8.**(a) 0.791 kg., (b) 13.6 kg., (c)1.48 kg. **9.** 424.75 kg. or 934.46 lb. **10.** 468 tons; 424,560.24 kg.

Mechanics of Solids. **1.** 480 ft. per sec. **2.** 90 ft. **3.** 9 ft. from 20 lb. mass. **4.** 2600 pd. **5.** 200 dynes. **6.** 10 gm. **7.** 19,800 gal. **8.** 8 h.p. **9.** $1642\frac{2}{3}$ ft.-pd. **10.** 150 pd.

Mechanics of Fluids. **1.** $43\frac{1}{3}$ pd. per sq. in. **2.** 64. **3.** 109.5 gm.; 15 c.c. **4.** 2404 kg. **5.** 1036.32 gm. per sq. cm.; 14.71 pd. per sq. in. **6.** 800 c.c. **7.** $26\frac{2}{3}$ pd. per sq. in. **8.** 144.144 pd. **9.** 19032 ft.-pd. **10.** 202,176 pd.; same.

Velocity of Sound. **1.** 1000 ft. per sec. **2.** 52.5 in. **3.** 33.87 in. **4.** 2984 yd. **5.** 3750 yd.

Vibrating Strings. **1.** 640; 40 cm. **2.** 24 cm. **3.** 80 pd. **4.** 1 : 5. **5.** C″.

Resonance; Organ Pipes. **1.** 337.92 m. per sec. **2.** 35 in. **3.** 21.17 in. **4.** 120 cm.; 285. **5.** 20 cm.

Expansion by Heat. **1.** 0.0000202. **2.** 3.53 cm. **3.** 172.27 c.c. **4.** 24.30 gm. **5.** 34.49° C.

Specific Heat. **1.** 12.53° C.; 10 gm. **2.** 17.47° C. **3.** 50° C. **4.** 50° C. **5.** 0.513.

Latent Heat. **1.** 0.113. **2.** $7\frac{1}{2}$° C. **3.** 80.09. **4.** 11.93° C. **5.** 40° C. **6.** 244.60 gm. **7.** 907,500 cal. **8.** 128,880 B.T.U. **9.** 199.8 cal. **10.** $49\frac{1}{3}$° C.

Nature of Light. **2.** 150 ft. **4.** 200 ft. **5.** 9 sq. in. **6.** 80 ft. **10.** $\frac{25}{186}$ sec.; 32.74 hr.

Photometry. **2.** 1 : 10. **3.** $44\frac{4}{9}$ cm. from 16 c.p. lamp. **4.** 2500 c.p. **5.** 9 : 25.

Plane Mirrors. **2.** 120°. **3.** 3°. **4.** 45°. **5.** Making an angle of 30° with mirror.

Curved Mirrors. **4.** 18.75 cm. to right of mirror. **5.** 3.2 cm. **6.** 10 cm. to left of mirror; $2\frac{2}{3}$ cm. **7.** 30 cm. to right of mirror; 4.5 cm. **8.** (a) $3\frac{3}{7}$ in. to left of mirror; $\frac{3}{7}$ height of candle; (b) $4\frac{4}{11}$ in. to left of mirror, $\frac{3}{11}$ height of candle. **9.** 6 ft.

Refraction. **1.** 139,849.6 mi. per sec.; 121,568.6 mi. per sec. **2.** $16\frac{1}{2}$°. **3.** 6 ft. x 9 ft. **4.** 20 cm. from lamp; $16\frac{2}{3}$ cm. **5.** 6 dioptres; $16\frac{2}{3}$ cm. **6.** 30 cm.; 6 cm. **7.** $40\frac{1}{2}$°. **8.** 39°. **9.** 3 in.; 5.76 in. **10.** $51\frac{3}{7}$ in.

Magnetism. **6.** 350°; 35°. **7.** 1190 yd.

Static Electricity. **10.** 11 : 1.

Electrolysis. **1.** 0.261 amp. **2.** 0.251 amp. **3.** 0.0747648 gm.; 0.5981184 gm. **4.** 9603.4 hr. **5.** 0.10384 gm.; 0.83072 gm.; 3.28 gm.

Ohm's Law and Resistance. **1.** 1440. **2.** 0.6 amp.; 0.75 volt. **3.** 4 ohms. **4.** 0.625 amp.; 1.375 volts. **5.** 22 ohms.; $\frac{5}{6}$ amp.; 5 amp. **6.** (1) 1.9 ohms; (2) 0.187 ohm; (3) 23,400 ohms; (4) 0.003 ohm. **7.** (1) $15\frac{1}{3}$ ohms; (2) 7.5 ohms; (3) 72 ohms; (4) 212.84 ohms. **8.** 131.7888 ohms. **9.** 3950.78 ft. **10.** 0.382 amp.

Electrical Energy and Power. **1.** 22 cents. **2.** $7.84. **3.** 0.2 k.w.h. **4.** 82.2%. **5.** 11 cents. **6.** $\frac{1}{3}$ amp.; 1 cent. **7.** 302.5 ohms; 125. **8.** 98%. **9.** 28.8° C. **10.** 61.84° C.

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