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Technical Drawing Series

Mechanical Drawing

ANTHONY

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PREFACE

THE extended use of the "Technical Drawing Series" has necessitated a very complete revision of the "Elements of Mechanical Drawing," resulting in some radical changes in the book. In adapting it to so wide a range as would include its use in the Evening Drawing School and the Technical College, it has been found necessary to separate the instruction from the problems so that it may be useful to instructors who desire a book of reference for use in connection with problems and notes of their own.

The folding plates have been abandoned, and the illustrations are now printed with the text, great care having been used to make the references to the cuts either on the same or opposite page. Many illustrations have been added, and all have been redrawn.

The problems and their lay-out are printed at the end of the book, with numerous references to the text. The number and variety have been increased, and include many of a practical character suitable for elementary courses, all of which have received the test of the classroom. The student should be required to master the principles before attempting to solve the problems, receiving such instruction as his special case may demand. By this means individual instruction may be given to large classes.

The method recommended for finishing drawings by leaving the construction lines in pencil, neither inking nor erasing them, has been found efficient for the following reasons: — It enables the instructor to follow the methods and reasoning of the student; it teaches neatness and care in the execution by preventing the free use of an eraser on the completed drawing; it is a great saving of time.

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The Third Angle Method of projection is used exclusively, in accordance with the best modern practice. Some college instructors have objected to this because it is not commonly adopted in treatises on Descriptive Geometry; but the author has used it instead of the First Angle Method for the teaching of this subject during the past four years, and it has caused no difficulty or confusion on the part of the students.

After a student has acquired the necessary skill in penmanship, the greatest stress should be laid on the subject of Projection. He should be taught to regard Graphics as a language study, the grammatical construction of which is developed in the Theory of Projection. No copying should be permitted, save in learning to use the instruments, and the subject should be taught as an art of expression rather than one of pictorial representation. Although most people recognize drawing as a medium for conveying thought, few appreciate the importance of teaching it as a language. But such it is in the fullest sense, possessing a well-defined grammatical construction, rich in varied forms of expression, forcible yet simple, and truly universal.

The author desires to express his thanks for the many suggestions and kindly criticism made by those who have found this book useful in the classroom, and who have helped to make it what it is.

GARDNER C. ANTHONY.

Tufts College,
July, 1904.

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CHAPTER I

INSTRUMENTS AND THEIR USE

1. The Outfit. It is of the first importance that the student be provided with good instruments and drawing material. The cost need not be great if they are selected by one experienced in their use. The following list comprises the smallest equipment consistent with good work, and a description of these supplies may be found in the succeeding articles.

2. Instruments and Materials. Drawing Board, 16″ × 20″, page 1. T Square, 24″, page 2. 45° Triangle, 6″, page 3. 30° × 60° Triangle, 10″, page 3. Scale, 12″, page 8. 4H Siberian Pencil, page 6. 6H Siberian Lead, page 6. Pencil Sharpener, page 6. Curves, page 10. A set of Drawing Instruments consisting of the following pieces: $5\frac{1}{2}''$ Compasses, page 12; 5″ Dividers, page 14; 3″ Bow-pencil, page 16; 3″ Bow-pen, page 16; 3″ Bow-spacers, page 16, and a 5″ Ruling Pen, page 17. Pencil Erasing Rubber, page 19. Ink Erasing Rubber, page 19. Inks, page 20. Paper, page 20. Pens, Penholder, Penwiper, and Tacks, page 20.

3. Drawing Board. A pine board $\frac{7}{8}''$ thick and measuring about 16″ × 20″ will suffice for the solution of all the problems in this book. As the upper face should be maintained a true plane, it is desirable to have two cleats on the back of the board. One of the short edges is chosen as the working edge, and made perfectly true. It should be tested from time to time, as any unevenness in this will impair the accuracy of a drawing. The working edge should be placed at the left side, and is the only one which need be used in this elementary work. It is customary for draftsmen to use the edge next the body when drawing long lines parallel to the working edge; but should this be done it will necessitate the making of this edge true, and having the angle between the two edges exactly 90°. The upper and right-hand edges must never be used.

4. T Square. This consists of a blade securely fastened to a head by means of a clamp or screws. As it is necessary that the upper edge of the blade and the inner edge of the head be maintained true, glue should never be used in the joint. These edges, together with the working edge of the board, should be examined frequently, as the accuracy of the drawing depends primarily on them. The working edge of the blade and head should be at right angles.

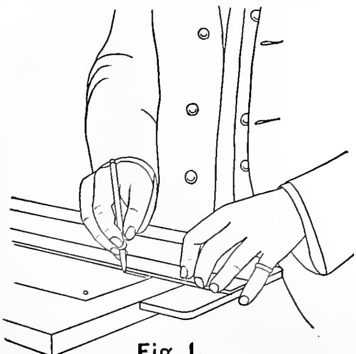


Fig. 1.

5. Use of T Square. The head should be held firmly against the left-hand edge of the board with the hand in the position shown in Fig. 1. By sliding the T square along this edge, parallel lines may be drawn. Previous to drawing a line at the extreme right, slide the hand along the blade with sufficient pressure to maintain it in position when drawing the line. Never move the T square by the blade or draw a line against the lower edge of the blade. Do not use the head in contact with the upper or right-hand edges of the board. The lower edge of the board may be used if it is known to be at 90° with the working edge.

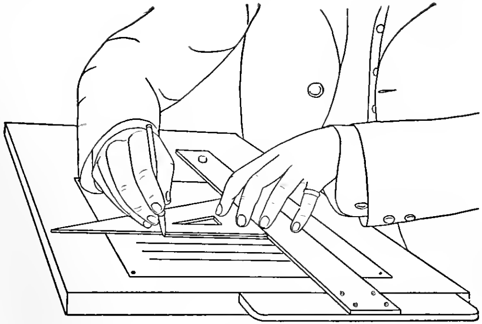


Fig. 2.

6. Triangles. The two triangles ordinarily used by draftsmen are a 45° and a 60°. The former has two equal angles and a right angle. The latter has angles of 30°, 60° and 90°. Celluloid is the best material for their construction, although wood and rubber are used also. These triangles, in combination with the T square, may be used to solve a variety of problems, and facility in rapid drawing is dependent on the skill acquired in using them.

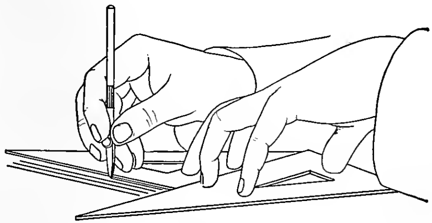


Fig. 3.

7. Use of Triangles. The position in which triangles are used for drawing lines perpendicular to the T square is shown in Fig. 2. As the drawing board should be placed so as to permit the light to come from the upper left-hand corner, this position of the T square and triangle will avoid the shadow of the blade or of the triangle being cast on the line to be drawn. Some frequent constructions involving the use of triangles are as follows:—

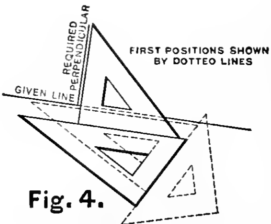


Fig. 4.

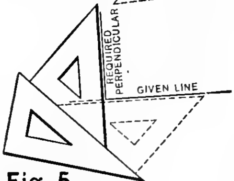


Fig. 5.

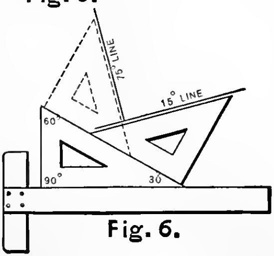


Fig. 6.

PARALLELS. Lines perpendicular to the T square blade may be drawn by sliding the triangle in contact with the blade, as in Fig. 2. When drawn at other angles, or without the aid of a T square, slide one triangle on the long edge of another, as indicated in Fig. 3, using care to hold the second triangle firmly. The first triangle is then slid in contact with the second by means of the first and second fingers.

PERPENDICULARS. If the lines be perpendicular to the T square blade, use the triangle as in Fig. 2; but when the given line does not coincide with a position of the T square, use one of the positions indicated by Figs. 4 and 5. In Fig. 4, the 60° triangle is placed parallel to, but not touching, the given line. It is then slid on the 45° triangle to a second position parallel to the first and about $\frac{1}{4}''$ from it. The 45° triangle is then moved to the second position and the required line drawn. In Fig. 5, the 45° triangle is set parallel to the given line with a short side in contact with the 60° triangle. By turning a triangle on its right angle, as in Figs. 5 and 6, a perpendicular may be drawn.

ANGLES. The method for drawing lines at angles of 45°, 30°, and 60° with the T square blade is apparent from the figures. Angles of 15° and 75° may be obtained by using the 45° and 60° triangles with the T square, as in Figs. 6 and 7. Do not draw lines within $\frac{1}{4}''$ of the corners of the triangle, and never construct angles by drawing lines along adjacent sides of the triangle.

8. To Test the Angles of Triangles. The right angle may be tested as follows: Place the triangle on the T square with the vertical edge at the right, as in Fig. 8; draw a fine line, AB, in contact with this edge, then reverse the triangle so that both edge and line may be free from shadow, and move the edge toward the line. If they coincide, the angle is 90°. If they do not coincide, and the vertex of the angle formed by line and edge is at the top, as shown by A, the angle is greater than 90° by half the angle BAC. If the vertex of the angle is below, the angle is less than 90° by half the amount indicated.

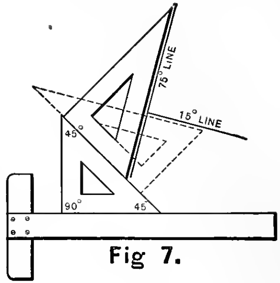


Fig 7.

Test of 45° Angle. If the 90° angle is known to be correct, place the 45° triangle on the T square, as indicated by the dotted position, Fig. 9, and draw a line to coincide with the long edge. Reverse the triangle so as to bring the second acute angle into the position of the first, and if the edge coincides with the line drawn, the acute angles are equal and therefore 45°. If the line and edge intersect at the bottom, the angle of the triangle at this point of intersection is less than 45° by half the amount indicated by AED. If they intersect at the top, the angle of the triangle at this point is less than 45° by half the amount indicated.

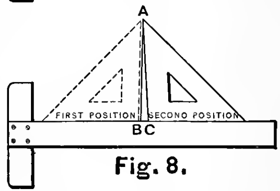


Fig. 8.

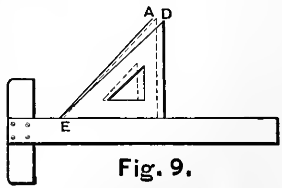


Fig. 9.

Test of 30° and 60° Angles. If the 90° angle is known to be correct, draw a line to coincide with the T square blade, and from any point on this line construct an angle of 60°, as in Art. 46, page 38. Test the angle by sliding the short edge on the T square until the hypothenuse coincides with or intersects the line measuring the angle. If the 90° and 60° angles are found to be correct, the third angle must be 30°, since the sum of the angles of any triangle is equal to 180°.

9. Pencils. The grade of lead commonly used by draftsmen is 6H; but this is extremely hard, and unless used with great care will indent the paper so that the line cannot be erased. A 4H lead requires more care and frequent sharpening, but the student will thus acquire a lightness of touch which is of value. The double-end holder with movable leads has some advantages over the lead pencil; its length remains constant; a shorter lead may be used, and leads of different grade may be used in either end.

10. To Sharpen the Pencil. Remove the wood from both ends by means of a sharp knife, exposing about $\frac{3}{8}''$ of lead. Fig. 10. One end should then be sharpened to a conical point, and the other to a chisel or wedge-shaped end. This last operation should be done with a file or pencil sharpener, but never with a knife. An excellent sharpener may be made by mounting strips of No. 0 sandpaper, $4''$ long and $\frac{5}{8}''$ wide, on a thin piece of wood so that it may be held in the fingers without soiling them. Care must be used to prevent the fine lead dust on the sharpener from falling on the paper or board, as it will work into the surface and be difficult to remove. The chisel end should be

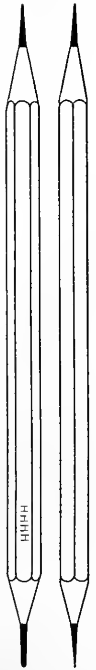


Fig. 10.

Fig. 11.

wedge-shaped, as indicated in Fig. 10, and the length of the edge should be reduced to about one-half the diameter of the lead. This may be done by first making the end slightly conical. After finishing the edge on the file or sharpener it is well to rub it on a piece of paper, rolling the pencil slightly to remove the sharp corners. Fig. 11 represents the double-end holder with leads properly sharpened. If this type is used, a 4H lead may be used for the chisel end, and a 6H for the conical end.

11. Pencilling. Good pencilling is a prerequisite to good inking. A drawing poorly pencilled is seldom well inked. Of first importance is the sharpening of the pencil, and as it wears away rapidly it must be sharpened frequently. The chisel end is used for ruling right lines, and the conical point for free-hand sketching, lettering and for marking dimensions.

The pencil should be held vertical, or nearly so, the arm free from the body, and the flat edge of the chisel end lightly touching the straight-edge. Do not attempt to draw with the pencil point in the angle made by the paper and the edge of the blade or straight-edge. Draw from left to right, or from bottom to top of board. In general, lines are drawn from the body, the draftsman facing the board when drawing horizontal * lines, and having his right side to the board when drawing long vertical lines. For the drawing of other lines the position of the draftsman should be such as to enable him to draw at ease, having a free-arm movement, even though it be necessary to draw from the opposite side of the board. The lines should be very fine although perfectly clear.

Learn to economize in the drawing of lines by omitting such portions as may be unnecessary. Invisible lines, which should be dotted when inking, may frequently be pencilled in full.

* It is customary to speak of lines drawn parallel to the T square as horizontal, and those drawn perpendicular to this edge as vertical.

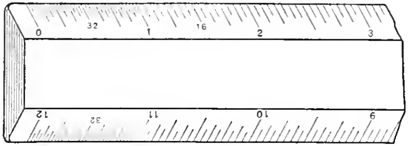


Fig. 12.

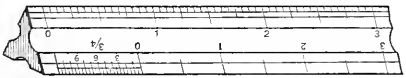


Fig. 13.

12. Scales. The best type for general drawing is that shown by Fig. 12. It should be made of boxwood, preferably with a white celluloid edge, and being graduated as in the illustration may be used for scales of Full, Half, Quarter, and Eighth size.

It should be used for dimensioning only, and not as a ruler or straight-edge. The measurements are to be taken directly from the scale by laying it on the drawing, and not by transferring the distances from the scale to the drawing by means of the dividers.

13. Use of Scales. In laying off dimensions from the scale, place it to coincide with the line to be measured, and indicate the distances by means of the conical point of the pencil. Place the point exactly opposite the required division of the scale, and having lightly marked the paper, examine the division on the scale to ascertain if the distance be correct. A steel point is also used for laying off dimensions, but in this case care must be observed in puncturing the paper, as the hole should be scarcely visible. In making successive measurements on a line, do it by addition instead of moving the scale. If it is required to make divisions of $\frac{5}{8}''$, place the 0 of the scale to coincide with the first point, and without moving it lay off $\frac{5}{8}''$, $1\frac{1}{4}''$, $1\frac{7}{8}''$, etc.

Since it is not convenient to make all representations of full size, it is customary to employ reduced sizes, or scales. Those most frequently used, together with the method of indicating them on the drawing, are as follows: —

Scale	Statement on the Drawing
Full Size.	Scale: Full Size.
One-half Size.	Scale: Half Size, or 6″ = 1 Ft.
One-fourth Size.	Scale: Quarter Size, or 3″ = 1 Ft.
One-eighth Size.	Scale: $1\frac{1}{2}$″ = 1 Ft.
One-sixteenth Size.	Scale: $\frac{3}{4}$″ = 1 Ft.
One-twelfth Size.	Scale: 1″ = 1 Ft.
One-twenty-fourth Size.	Scale: $\frac{1}{2}$″ = 1 Ft.
One-forty-eighth Size.	Scale: $\frac{1}{4}$″ = 1 Ft.

The first five of these scales may be read from a scale graduated as in Fig. 12, but the last three require graduations of forty-eighths of an inch. A form of scale commonly used, and having all of the above graduations, is represented by Fig 13.

In using the graduations of Fig. 12 for other than full or half size, the following method should be employed: In laying off quarter size dimensions, regard each quarter inch division on the scale as one inch, and subdivide the quarter inches for the fractional divisions of an inch. Thus, if it is required to measure $17\frac{3}{4}$″ at this scale, lay off 17 quarter inches, and to this add three-quarters of the next quarter of an inch. This will be found much more simple than the mental operation of obtaining the quarter part of $17\frac{3}{4}$″, and with a little practice this scale may be read as rapidly as that illustrated by Fig. 13, and without the confusion arising from the combination of a variety of divisions on one scale. In a similar manner measurements may be made at one-eighth and one-sixteenth scale.

Scales for enlarged sizes are also employed, but they are not in general use.

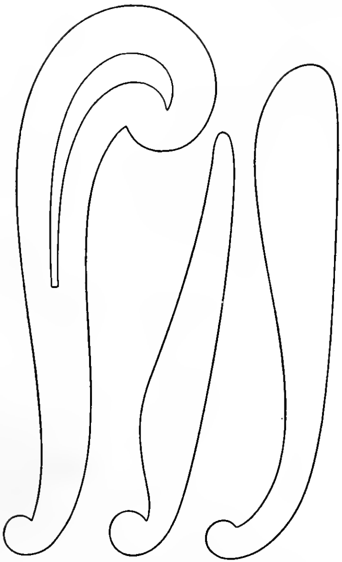


Fig. 14. **Fig. 15.** **Fig. 16.**

14. Curves. For inking lines which are neither right lines nor circular arcs, it is necessary to use irregular curves. These are made in a variety of forms, but the type having the curves of long radii, similar to those illustrated in Figs. 14, 15, 16, are the most serviceable. They are commonly constructed of rubber or celluloid. The former is poorly adapted to this use, as it is difficult to make pencil marks on its surface. Celluloid presents the same difficulty if its surface is not roughened. As the manufacturers do not furnish the curves with this surface, the draftsman may obtain the desired effect by sandpapering them. White holly is a suitable material also, and one from which the curves may easily be made.

15. Use of Curves. Many curved lines can be inked by means of the compasses, but when the radius is too great, a curve should be employed. The most important point to be observed in using the curve, is to avoid inking the curved line to the full extent of the apparent matching of the curve. In Fig. 17, the dotted position of the curve would admit of inking the line from C to E; but it will be observed that the curve matches the line from B to F. In continuing the inking to the right, the curve should be moved into the position shown by the full lines, in which it coincides with DE, a portion of the line already inked, and would enable the line to be continued to the point K.

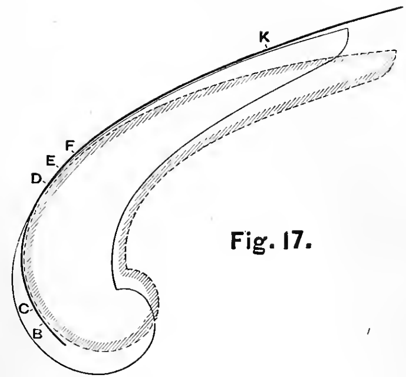


Fig. 17.

If the curved line to be inked is symmetrical with respect to an axis, as in the case of the ellipse, Fig. 18, proceed as follows: First ink the part of the curve at the extremity of the axis, WAV and XBY, by means of the compasses, using great care in selecting the radius. Next obtain, if possible, a curve to coincide with arc CW and for a short distance on either side of points C and W, so as to insure a perfect copy of the curved line. The line should now be drawn from the point W to the point C, but never should it pass to the right of the point C with the curve in this position. Now marking upon the curve a point which coincides with the point C, and reproducing this upon the opposite side, reverse the curve in order to draw lines to the right of C. Similarly ink the lower half of the ellipse. If a curve cannot be obtained to coincide with the entire arc CW, draw as much of the line as possible from the point C, and having drawn the four corresponding parts of the ellipse, select a second curve to join the curved lines with the circular arc.

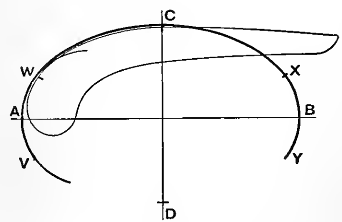


Fig. 18.

A spiral is best inked by means of the bow-pen or the compasses. Fig. 19 indicates the operations that would be required to ink the curve ACEF. Beginning at the point A, by trial determine the maximum length of an arc that can be drawn with one setting of the instrument, and ink that part of the curve. Suppose this to be AB with center at 1. Similarly obtain a second radius, but with the center on the continuation of the line B1, as at 2. This will insure a continuous curve in that AB and BC will have a common tangent at B. In like manner obtain centers 3, 4, 5, inking the curve as the centers are found. The lines B2, C3, etc. are not to be pencilled, but the direction determined by the eye.

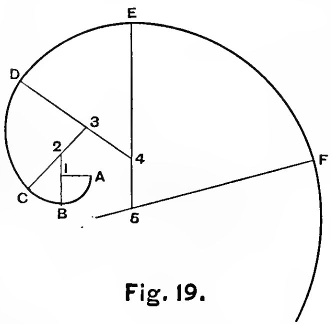


Fig. 19.

16. Compasses. Fig. 20 illustrates a compass set of approved type. There are three removable parts; the pencil-point, pen-point, and lengthening-bar. There is a joint in each leg, and the removable leg is provided with a clamp screw. The shank of the removable parts should fit the socket closely, and require but a slight effort to remove. It should not drop from the socket, even though not clamped by the screw. All joints should work smoothly. The pen is similar in design to the ruling-pen. Art. 22, page 17. The lengthening-bar is used to extend the pen or pencil-leg for the drawing of large circles.

17. The Use of Compasses. When first adjusting the compasses for use, place the pen-point in the instrument, pushing it firmly against the shoulder, and securely clamping it. Adjust the needle-point so that its point coincides with the point of the pen. Once adjusted, the needle-point should not be changed. Frequently the needle-point is made with a conical point at one end and a fine shouldered point at the other. The former should never be used, as it makes too large a hole in the paper. The alignment of the instrument may be tested by bringing the pen and pencil-point together with the legs straight, and also when bent at the joints. The points should coincide in both instances.

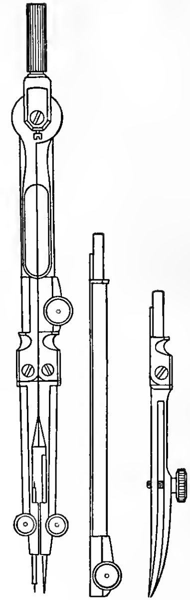


Fig. 20.

To prepare the compasses for pencilling, place a 6H lead in the pencil-point, sharpening it to a chisel end as directed for the pencil, Art. 10, page 6, save that the length of the edge should not be more than $\frac{3}{64}''$. Next remove the pen-point from the compasses and replace it with the pencil-point. Adjust the length of the lead to coincide with the needle-point, using great care to adjust the lead so that the direction of its edge shall be a tangent to the circle drawn, otherwise the width of the line will be greater than that of the chisel edge.

The compasses are held between the thumb, first and second fingers, and rotated from left to right, clockwise. The hand must never be allowed to rest on the instrument, as the needle-point would be forced into the paper, and the hole for the center made objectionably large. In placing the needle-point on a special center, the compasses may be steadied by lightly touching the point with a finger of the left hand. The point should be pressed lightly into the paper an amount sufficient to prevent slipping, but the hole should be scarcely visible. If it is necessary to designate it more clearly, sketch a pencilled line about it, but never put the point of the pencil into the hole to blacken

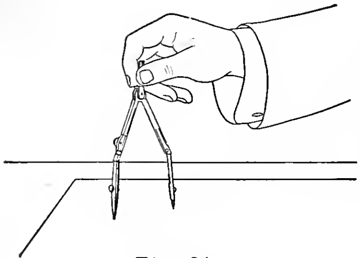


Fig. 21.

it. The needle-point must be kept vertical, and likewise the pen and pencil points. In drawing a circle, stop the line as soon as the circumference is completed, otherwise the line may be widened. If several arcs are to be drawn from one center, it is better to remove the needle-point from the center when changing the radius, as the hole may become badly worn if much pressure is exerted against it. The directions for cleaning and filling the pen are the same as for the ruling-pen, Art. 22, page 17.

When using the lengthening-bar, steady the needle-point with the left hand while moving the marking-point with the right. It is necessary to use care in doing this so as not to change the radius.

Figs. 21 and 22 illustrate the method of holding the compasses, and the position of the fingers before and after the drawing of a circle.

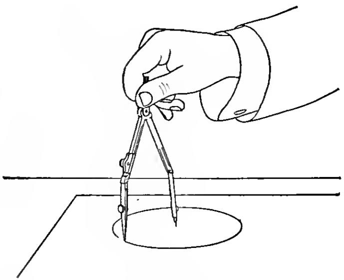


Fig. 22.

18. Dividers. This instrument is similar in design to the compasses, but the legs are fixed, and without joints. It is used to transfer measurements from one part of a drawing to another, but must not be employed for transferring from a scale to the paper. It is also used to divide a line into any number of equal parts when the divisions cannot be obtained directly from the scale. To do this proceed as follows: Suppose that it is desired to divide the line AB, Fig. 23, into five equal parts. Open the dividers to a distance equal to A1, about one-fifth of the required space, and, holding them at the joint by the thumb, first and second fingers, place one point at A, the extremity of the line to be divided, the other point being at 1. By rotating the instrument in opposite directions, as though describing a series of semicircles, lay off divisions A 1, 1 2, 2 3, 3 4, 4 5. Point 5 being beyond the extremity of the line, the divisions are too great, and should be diminished by an amount equal to one-fifth of B 5. Make a second, or third, trial if necessary, so that the last division shall fall on B. If the required number of divisions be even, bisect the given line, then bisect these divisions, and so continue as long as the remaining divisions are even.

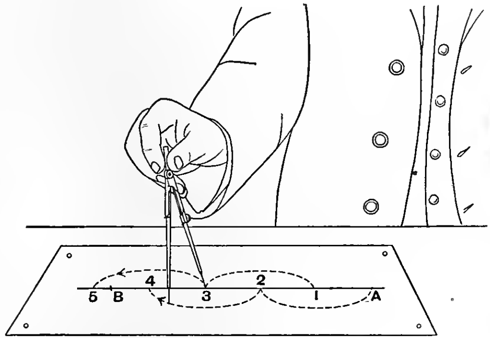


Fig. 23.

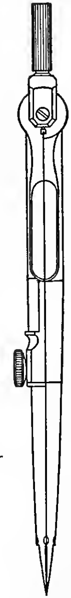


Fig. 24.

Fig. 24 illustrates a pair of hair-spring dividers. The thumbscrew is used to obtain a more delicate adjustment. The ends of the legs should be conical rather than triangular, and must be kept sharp. The puncture made by the divider points should be extremely small, but sufficiently clear to be readily seen.

19. Bow-pencil. Fig. 25. This is used for the drawing of circles from the smallest size to about $\frac{3}{4}''$ radius, if it be a $3''$ instrument. Adjust the lead as described for the compasses, but the chisel edge must be reduced nearly to a point. It should be possible to draw circles having a clean, sharp line with a radius of $\frac{1}{32}''$ or less. The adjustment for radius may be rapidly made, and with the minimum of wear on the screw, by pressing the points together, the fingers being placed below the adjusting screw.

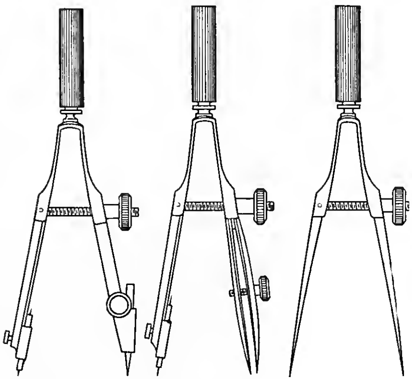


Fig. 25. Fig. 26. Fig. 27.

20. Bow-pen. Fig. 26. The range of this instrument, and its adjustment, is like that of the bow-pencil. In cleaning and filling the pen the same care must be used as described for the ruling-pen, Art. 22. An excellent test for the setting of the needle-point is as follows: Having adjusted it so that it projects slightly below the point of the pen, draw circles with the minimum radius of the instrument. If there is a tendency to lift the point from the paper, it is too short; but if it is difficult to draw the line without forcing the point into the paper, it is too long.

21. Bow-spacers. Fig. 27. These are used, like the dividers, for the spacing of distances, but they have the advantage of being fixed in position so that there is no liability of a change in the space by the handling of the instrument. In spacing distances, the instrument is rotated alternately right and left, as in the case of the dividers, and the forefinger should rest on the top of the spacers to steady them.

22. The Ruling-pen. Fig. 28 is an enlarged view of the instrument which is used for the inking of all lines other than circular arcs. The nibs must be of equal length, rounded alike, and equally sharp. The pen is filled by inserting the ink between the nibs with a common writing pen, or the quill that is furnished with many of the bottled inks. The nibs should be opened slightly to admit the ink freely. This method of filling should not necessitate wiping the outside of the nibs, but great care must be used to insure their being perfectly clean. Do not overload the pen, $\frac{1}{4}''$ of ink being sufficient for the ordinary pen. Having filled the pen, nearly close the nibs and try the width of the line on a piece of paper, opening or closing the nibs by means of the screw to vary the width of the line. The pen should be held by means of the thumb, first and second fingers, the thumb-screw being held from the body so as to be readily adjusted by means of thumb and second finger. Its position should be nearly vertical, being inclined slightly to the right. See Fig. 1, page 2. The pen should touch the straight-edge lightly, the point being about $\frac{1}{32}''$ from the edge. If both nibs touch the paper, the line should be of uniform width throughout. Varying the pressure of the nibs on the blade will cause an unevenness in the line as :

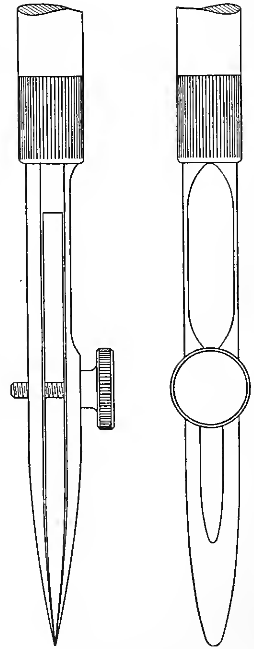


Fig. 28.

If both nibs do not touch the paper, because of their being of unequal length, or because the pen is inclined from or toward the body, the line will appear as follows :
If the distance between the nibs be unchanged, the width of the line should remain constant; but as the ink dries very rapidly, causing a slight deposit between the nibs, the width of the line will be reduced unless care is used to clean the nibs frequently, for which purpose a penwiper should always be at hand. If the ink fails to flow freely, touch the point lightly to the penwiper or the drawing board, and it will dislodge the small deposit at the point which prevents the flow. If this fails, clean the pen thoroughly, and refill. Never lay the pen aside without cleaning.

23. To sharpen the Ruling-pen. All first-class pens are properly sharpened when new, but with the cheaper qualities this is not commonly the case. The most frequent defect is in an insufficient rounding of the point. An excellent test for the ruling-pen is as follows: Having filled the pen, adjust it to draw a fine line, and without the aid of a straight-edge draw a number of lines with the pen inclined more and more from the body until the inner nib is raised sufficiently high to prevent the drawing of a line. Repeat this operation with the pen inclined toward the body. If the angles at which the last line is drawn in each case are equal, the nibs are of equal length. Next draw lines with the pen inclined to the right and to the left about 45°. If the pen moves freely, and draws clean, sharp lines, the point is sufficiently rounded. If the pen is too sharp, the cutting of the paper can easily be detected by the touch.

To sharpen the pen, the draftsman should be provided with a fine Arkansas oilstone and a magnifying glass, and proceed as follows: Close the nibs, and hold the pen on the stone as for the drawing of lines, inclining it to the right and left, so as to bring the nibs to an even length, and round the points properly. Next clean the pen, open the nibs, and grind the outside faces so that the rounded portions shall be equally sharp. In doing this, the pen should be held at an angle of about 15° with the oilstone, and rolled slightly from side to side. Examine the nibs frequently with a magnifying glass in order to note the disappearance of the bright, polished points, which indicates the degree of dullness, using care to avoid a feather or wire edge, which would indicate the shortening of the nib, and necessitate a repetition of the first operation. It is seldom necessary to touch the inner face of the nibs, but should a burr be raised on the inside, it may be removed by clamping a piece of hard paper lightly between the nibs, and moving the pen to and fro on the paper. Finally, clean and test the pen as directed, using it in connection with the straight-edge for the last test.

24. Erasers and Erasing. Pencilled lines are best erased by means of a velvet rubber. It must be used lightly, and as little as possible. The particles of rubber caused by the erasure should be removed by means of a cloth or brush, or possibly by the hand, if it be clean and dry. Do not clean the drawing until it is inked, and then with care. A sponge rubber is frequently used for this purpose. Avoid rubbing an inked line whenever possible, as the softest rubber will tend to destroy its clearness.

Inked lines must be removed with a hard rubber, which is known as an ink eraser, but never by scratching the surface with a knife. As India inks dry rapidly, and do not penetrate the hard surface of drawing papers, the object in erasing is to remove the ink without injury to the paper. By using care and time, any amount of ink may be removed from a drawing. Make sure that the ink is very dry before attempting to erase it. Use the hard rubber with a light pressure, and confine the erasing as closely to the line as possible. A card or erasing shield may be used to protect other lines and surfaces, but this is seldom necessary.

25. Inks. The bottled inks, preferably Higgins' Waterproof, are recommended in place of the Indian or Chinese stick inks, which require considerable time to prepare, and necessitate fresh mixing for each exercise. Red ink is used frequently for center lines, and blue ink for dimension lines and steel sections. Care is necessary in the use of colored inks, as they flow more freely than the India inks.

26. Paper. The paper should be of good quality, reasonably smooth, and of sufficiently close texture to admit of considerable erasing. Keuffel and Esser's Normal paper is a good grade for mechanical drawings. The size recommended for the problems of this book is $11'' \times 15''$, or one quarter of an imperial sheet, which measures $22'' \times 30''$.

27. Miscellaneous Material. One-ounce tacks of copper or iron should be used for holding the paper to the board. They are better than thumb tacks, in that they may be forced into the board so that the heads are almost flush with the surface of the paper, enabling the T square to slide freely over them.

A 170 or 303 Gillott pen with penholder will serve for such figuring and lettering as will be required.

A piece of thin chamois skin makes the best penwiper, and should always be at hand for cleaning the pens and instruments.

CHAPTER II

GENERAL INSTRUCTION

28. Preparation of the Paper. Having placed the drawing board with the long edge next the body and the working edge to the left, see that the surface is free from dust and the T square and triangles carefully wiped. Next place the long edge of the paper parallel with the long edge of the board, the paper being within about 3″ of the lower and left-hand edges of the board. The T square may be used to insure its being square with the board. Use four tacks, one at each corner, forcing them into the board so that they may be flush with the surface of the paper. The sheet should lie perfectly flat on the board, and whenever the atmospheric conditions are such as to cause the paper to swell and present an uneven surface, remove three of the tacks and again fasten the paper as before, squaring the sheet by the most important line that has been drawn. Instruction for the stretching of paper by glueing to the board is given in Art. 37, page 32.

The space within the margin line will measure 10″ × 14″, and is to be laid off in the following manner: The paper being tacked in place, obtain the center of the sheet by placing the T square blade so that it will coincide with the opposite diagonal corners of the paper, and drawing short, fine lines intersecting at C, Fig. 29. To the right and left of this point lay off 7″; above and below it lay off 5″. Next place the head of the T square firmly against the left-hand edge of the board and draw the upper and lower margin lines through the points first laid off. The right and left margin lines should be drawn by means of the 60° triangle used in connection with the T square, as shown in Fig. 29. If the triangle be too short to draw the entire line at one stroke, move the T square into such a position as will enable the remainder of the line to be drawn.

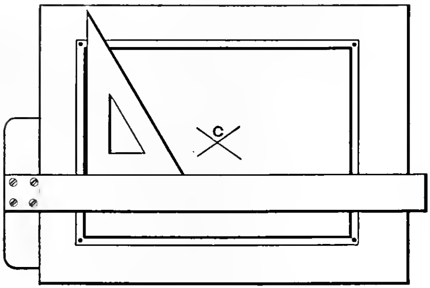


Fig. 29.

29. Character of Lines. All pencilled lines should be as fine as is consistent with clearness, and the full line used as much as possible. If the drawing is to be inked, full pencilled lines may be used in the place of dotted or broken lines, wherever confusion will not be caused.

The fine line is used for all visible lines of an object which may not be shaded.

The shade line is used to assist in the reading of a drawing by suggesting the relation of the surfaces. It is used for visible lines only.

The dotted line is used to designate the invisible lines of an object, and is never shaded. It requires more care than all other lines. The dots or dashes should be about $\frac{1}{16}''$ long and the space about $\frac{1}{32}''$. These lengths must not be increased, however long the line may be. Much of the beauty of a drawing is dependent upon the evenness of this class of line.

Center lines are frequently indicated by full red lines a little finer than the fine black lines. If drawn in black, the broken line or dot and dash line is employed. In this case the dash may be about $\frac{3}{8}''$, and the dot about $\frac{1}{16}''$ long. Extend the line beyond the surface on which it is drawn.

Dimension lines are indicated in red or blue by a very fine line similar to the center line; or in black by a series of dashes about $\frac{3}{8}''$ long. But in either case the witness or arrow point must be black and pointed, as in the figure.

Witness lines are used to indicate the extent of the surface measured when the dimension line falls outside of the surface. They are made of the same character as the dimension lines.

Border lines seldom require to be heavier than the shade lines.

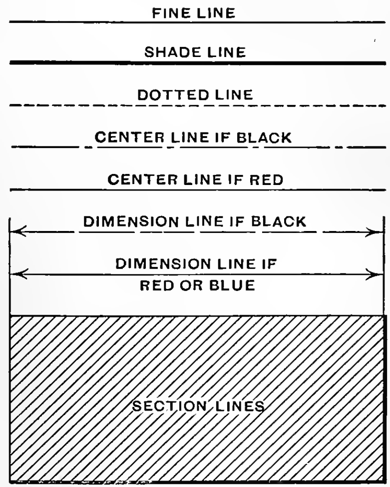


Fig. 30.

30. Shade Lines. There is but one reason that can justify the use of the shade line, and that is, added clearness to the representation by indicating the relation of the surfaces to one another. By some draftsmen the shade line is never used, while by others it is always used. Both are in error, since no law can be established concerning it, there being times when it is a mistake not to use it and others when it is equally wrong to use it.

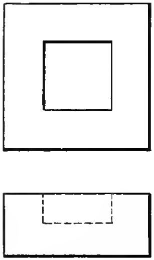


Fig. 31.

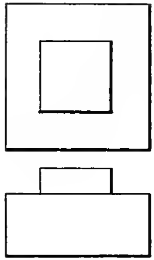


Fig. 32.

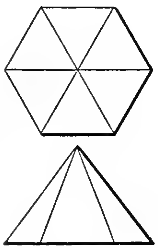


Fig. 33.

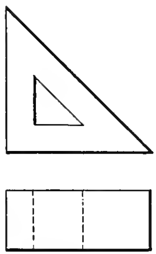


Fig. 34.

The following conventional method is much used in practice, and is recommended because of its simplicity. Shade the right-hand and lower edges of all surfaces. Do not shade the line of intersection between visible surfaces. The shade line must not encroach upon the surface which it bounds, otherwise the accuracy of the drawing would be impaired. Cylindrical surfaces may be outlined by fine lines only. Shade the lower right-hand quadrant of outside circles, and the upper left-hand quadrant of inside circles. One width of shade line is used for all right lines. Never shade pencilled lines or dotted lines.

In general the shade line is intended to represent the surface in shadow, and the light is supposed to come from the upper left-hand corner at an angle of 45°; but if there are lines of the drawing parallel with the ray of light, the angle of the ray may be considered as greater or less than 45°. Thus, in Fig. 34, it is better to shade one of the 45° lines rather than to draw both fine. Fig. 31 represents a square block with a square hole in it. This is indicated clearly by the top view alone, although it does not show the depth. Fig. 32 is a similar object, but in this case it is surmounted by a square block of smaller size. The shade lines indicate this also, but fail, of course, to give the height. Fig. 33 represents two views of a hexagonal pyramid properly shaded. The lines radiating from the center of the top view, and the two inside slant lines of the front view, are division lines between visible surfaces, and therefore not shaded.

In the shading of circles and circular arcs it is necessary to avoid the sudden transition from the shade to the fine line, and this is accomplished in the following manner: Having inked the circle with a fine line, remove the point of the compasses from the center, using care not to change the radius, and place it below and to the right of the first center, a distance equal to the desired width of the shade line. If it is an outside circle, draw the second arc on the outside, as in Fig. 35; and if it is an inside circle, it should be drawn as in Fig. 36. If the width of the shade line is such that the two eccentric arcs are not in contact throughout, the intervening space may be filled by slightly springing the instrument. Since the circular arcs are the first to be inked, care should be used to adopt a width of line that will be correct for the shaded right line.

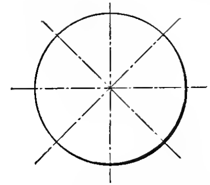


Fig. 35.

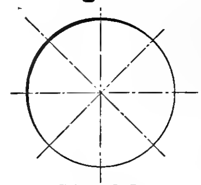


Fig. 36.

The shading of Fig. 37 differs from the two preceding in having a uniform width of shade line between points A and B, for the outside circle, and between C and D, for the inside circle. The transition from shade to fine line is made within an arc of about 45° beyond the points indicated. This method is not so simple as the preceding, and the improved appearance will rarely justify its use.

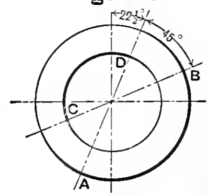


Fig. 37.

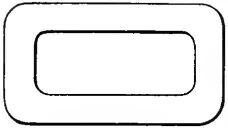


Fig. 38.

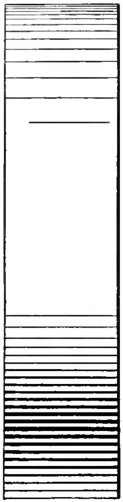


Fig. 39.

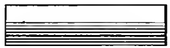


Fig. 40.

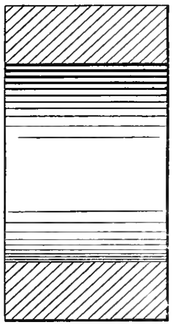


Fig. 41.

Fig. 38 represents a rectangular piece with a depression in the center, all of the corners being rounded or filleted. Small arcs, such as these, and all circles having radii less than $\frac{1}{2}''$, should be inked by means of the bow-pen, which may be used in the manner described. It is better, however, to acquire the skill necessary to spring the bow-pen, so that the shading may be done without removing the needle-point from the center. This is accomplished without changing the position of the thumb and first finger, which are used to handle the instrument. By a slight pressure of the first finger, sufficient to deflect the needle-point leg, the radius is slightly reduced when shading an inside circle, and increased when shading an outside circle. By this method the work may be much more rapidly executed.

31. Line Shading. When it is necessary to suggest the character of a surface without a second view, line shading may be used. Figs. 39, 40, and 41 illustrate good methods for the shading of cylindrical surfaces. The lines may be equally spaced, although the appearance is somewhat improved by increasing the space as one approaches the center line, and decreasing the space on the lower half, while increasing the width of the line. In shading a concave surface, as in Fig. 41, the operation is reversed. Cylindrical parts of small diameter may be shaded on the lower half only, as in Fig. 40. Conical, spherical, and other classes of surfaces may be represented by this method, but the rendering of them requires considerable skill, and it is beyond the scope of this treatise to consider the various methods.

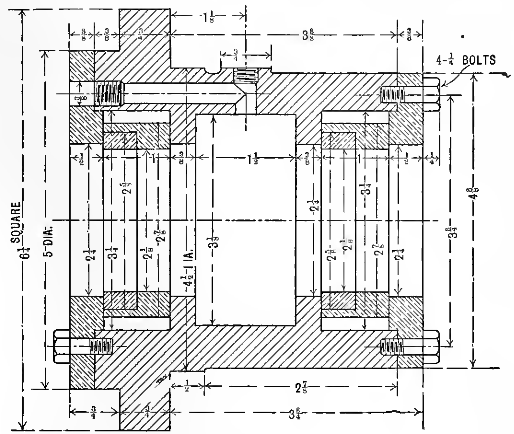


Fig. 42.

32. Sections. It is frequently necessary to make the representation of an object as it would appear if cut by a plane, and with the portion nearer the observer removed, as in Fig. 42. This cut section is indicated by a series of parallel lines, usually drawn at an angle of 45°. The width of the lines is the same as the fine line, and the distance between the lines is determined by the area and intricacy of the section. These lines are not to be drawn in pencil, but only in ink. An instrument known as a section liner is designed to insure accuracy in the spacing, but the beginner should learn to judge this by the eye. In doing so, care must be used to avoid making too small a space, and it is desirable to try the spacing on a separate sheet before sectioning the drawing. A greater degree of uniformity may be obtained by glancing back after drawing every six or eight lines.

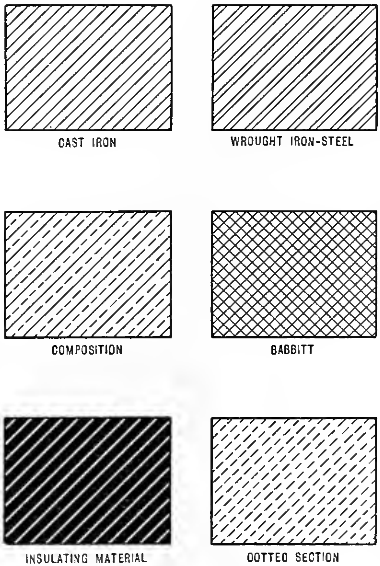


Fig. 43.

The different surfaces in the plane of the section are indicated by changing the direction of the lines, as illustrated by Fig. 42, which is the cross section of a piston rod packing. Seven distinct surfaces are clearly shown by changing the direction and angle of the section lines.

Differences in the material are also indicated by the character of the lines; but there is no general agreement as to notation. Fig. 43 illustrates six good types of sectioning, together with the names of the materials which the author has chosen to indicate by them. Whenever figures or notes occur in a section, the section lines should not be drawn across them.

33. General Instruction for Inking. Never begin the inking of a drawing until the pencilling is completed. See that the sheet is free from dust. Always ink the circles and circular arcs first, beginning with the small arcs. If the drawing is to have shade lines, shade [illegible] arc as drawn. To omit the shading of circles unt[illegible] have first been inked in fine lines will necessitate almost double the time otherwise required. It should be observed that the width of both fine and heavy lines is determined by the shading of the first circular arc. Next, ink all the full and dotted ruled lines. Begin on the upper side of the sheet and ink all the fine horizontal lines, omitting those lines which are to be shaded. Next, ink the vertical lines, beginning with those at the left. This method insures sufficient time for the drying of the ink. Do not dwell too long at the end of a line, especially if it be a heavy one, as the pressure of the ink in the pen will tend to widen the line. If a series of lines radiate from a point, allow sufficient time for the drying of each line, otherwise a blot may be made. Finally, ink all lines at other angles and those curved lines requiring the use of curves. The same order is to be followed in the inking of the shade lines, evenness being secured by ruling them at one time. Do not shade dotted lines. Ink center and dimension lines, and put on the figures and notes. Draw the section lines and put on the title.

34. Tracing. When it is desired to reproduce a drawing, transparent cloth or paper is placed above the original and the lines of the drawing traced on the new surface, as though one were inking a pencilled drawing. Tracing cloth is usually furnished with one surface glazed and the other dull. Either side may be used, but as it is difficult to erase from the dull side it is better to ink on the glazed surface. Pencilling must be done on the dull surface. Tracing cloth is used frequently in place of paper for original drawings, the pencilled paper drawing being traced on the cloth in ink. Copies of this tracing may be made by the blue print process.

As the cloth absorbs moisture quite rapidly, it shrinks and swells under varying atmospheric conditions. Because of this, large drawings which require considerable time to complete, should be inked in sections, as the cloth will require frequent adjustment in order that its surface may be smooth and in contact with the paper drawing. Only the best quality of cloth should be used, as the cheaper kinds are improperly sized and absorb ink, causing blots. If the ink fails to run freely on the glazed side, dust on the surface a little finely powdered pumice stone or chalk, rubbing it lightly across the surface with a piece of chamois skin or cloth. Use care to remove it completely from the surface.

Inked lines may be removed from the glazed side by means of a sharp knife and a hard rubber, or by dusting a little finely powdered pumice stone on the lines to be erased, and briskly rubbing them with the end of the finger or a piece of medium rubber. As the pumice becomes discolored replace it with fresh powder. In erasing lines from the dull side use the hard rubber. Pencilled lines may be removed by the ordinary pencil eraser, or by means of a cloth moistened with benzine.

35. Lettering. The subject of lettering is of such importance to the mechanical draftsman that he should adopt some clear type for general use, and acquire proficiency in the free-hand rendering of it. While at times it may be necessary to make use of instruments and mechanical aids for the construction of letters and figures, usually they may be written free-hand.

The accompanying alphabets are such as may be recommended to students, and will be acceptable in the regular practice of drafting. Their study will afford an excellent free-hand exercise, as well as skill in the figuring and lettering of drawings. Both types should be written without the aid of instruments. The first is known as the vertical Gothic, and the second as the slant Gothic. Large and small capitals may be used in the place of capitals and lower case, as illustrated. The small capitals and lower case may be made about two-thirds the height of the initial letters. This type is written quite easily by means of a hard wood stick, preferably orange or boxwood, sharpened to a point like a pencil, the size of the point being varied according to the desired width of the line. In doing this great care should be taken that the ink be black and slightly thick. The student is referred to the treatise on "Lettering," of the "Technical Drawing Series," for the further consideration of this subject.

ABCDEFGHIJKLMNOPQ
RSTUVWXYZ
1234567890 & $3\frac{5}{8}$
abcdefghijklmnopqrstuvwxyz

ABCDEFGHIJKLMNOPQ
RSTUVWXYZ
1234567890 & $3\frac{5}{8}$
abcdefghijklmnopqrstuvwxyz

36. Title. In the lay-out of the problems of this book no provision has been made for the title save in the case of applied work, such as machine drawings. It has been the custom of the author to have the name of the student printed in the right-hand lower corner just outside the margin line, and the plate number similarly placed in the upper right-hand corner. The capitals should be $\frac{3}{16}''$ high and the small letters $\frac{1}{8}''$.

In applied work the title should always be placed in the lower right-hand corner of the sheet, inside the margin line. It should designate, first, the name of the mechanism; second, the name of the special detail; third, the scale; fourth, the date, which is always that of the finishing of the drawing. The draftsman's name or initials should be printed in small type in the extreme right-hand corner within the margin.

37. Tinting. The surface of a drawing is colored or tinted for the purpose of making clear the divisions, as in map drawing; suggesting the character of the materials represented; or to indicate the character of the surface, whether plane or curved; and possibly its relation to other surfaces by the casting of shadows.

THE PAPER should be of proper quality, such as Whatman's cold pressed, and must be stretched by wetting the surface and glueing it to the board in the following manner: Having laid the paper on a flat surface, fold over about one-half inch of each edge. Thoroughly wet all of the surface save the folded edges, using a soft sponge for this purpose, but do not rub the surface. Next apply mucilage, strong paste, or a light glue, to the underside of the folded portion and press this to the board with a slight outward pressure so as to bring the surface of the paper close to the board. As the glue should "set" before the paper begins to dry and shrink, it is necessary to have the paper very wet, but no puddles must be allowed to remain on the surface after the edge is glued. The paper must be allowed to dry gradually in a horizontal position, as otherwise the water would tend to moisten the lower edge and prevent the drying of the glue. If the paper should dry too rapidly, not allowing sufficient time for the glue to "set," the surface may be moistened again.

The Color employed in making the wash or tinted surface may be a water color or ground India ink, but none of the prepared liquid inks are suitable for the purpose. The color should be very light, and when the desired shade is to be dark it should be obtained by applying several washes, allowing sufficient time for each to dry. The color must be as free from sediment as possible, but since some deposit is liable to take place, the brush should be dipped in the clear portion only, and not allowed to touch the bottom of the saucer.

The Brush should be of good size, depending somewhat on the surface to be covered, and of such quality that when filled with the color or water, it will have a good point.

Two classes of tinting are employed, the flat tint of uniform shade, and the graded tint for the representation of inclined or curved surfaces.

The Flat Tint. Remove all pencilled lines which are not to be a part of the finished drawing, and do all the necessary cleaning of the surface, using the greatest care not to roughen the paper. Inking should be done after the tinting, but if for any reason it is necessary to ink the drawing first, a waterproof ink must be employed.

In putting on the color, slightly incline the board to permit of the downward flow of the liquid, and, beginning at the upper portion of the drawing, pass lightly from left to right, using care just to touch the outline with the color, but not to overrun, and making a somewhat narrow horizontal band of color. Advance the color by successive bands, the brush just touching the lower edge of the pool of water made by the preceding wash. This lower edge should never be allowed to dry, as it would cause a streak to be made in the tinted surface. Having reached the lower edge, use less water in the brush so as to enable a better contact to be made with the outline. Finally dry the brush by squeezing it or touching it to a piece of blotting paper. It may then be used to absorb the small puddle of color at the bottom edge or corner.

Avoid touching the tinted surface until it is dry, at which time any corrections that are necessary may be made by stippling. This consists in using a comparatively dry brush and cross-hatching the surface to be corrected.

If the surface to be covered is large, it is desirable to apply a wash of clear water before applying the color. This dampened surface will prevent the quick drying of the color and insure a more even tint. When necessary to remove the tint from a surface, use a sponge with plenty of clean water, and by repeated wettings absorb the color, but do not rub the surface of the paper.

The Graded Tint may be applied by several methods, the simplest being to divide the surface into narrow bands and apply successive washes, each covering an additional band. If the tint is sufficiently light, and the bands narrow, the division line between the bands will not be very noticeable, but this may be lessened by the softening of the edges with a comparatively dry brush and clean water.

Another method, which requires considerable dexterity, is to put on a narrow band of the darkest tint that may be required, and, instead of removing the surplus water from the edge, touch the brush to some clean water and with this lighter tint continue the wash over the second band. Continue in this manner until the entire surface is covered.

There are many modifications of this method, all of which require considerable skill and are not to be recommended to the student at this stage of his progress.

CHAPTER III

GEOMETRICAL PROBLEMS

38. WHILE the majority of students are familiar with many of the propositions included in this chapter, the study of the methods best adapted to the draftsman is of great importance. It is not intended that these problems shall serve as copies, or that the examples relating to them, and given on page 122, shall be used for all students; but as reference data, and for the purpose of illustrating the draftsman's methods, they are believed to be an essential part of a text-book on Technical Drawing.

In most cases two methods are given, the first being the ordinary geometrical solution requiring the use of a straight-edge and compasses; and the second, the more direct method employed by draftsmen, involving the use of a T square and triangles as well as compasses and dividers. In the geometrical figures the given and required lines are shown in full heavy lines, and the construction in full fine lines.

It is intended that the student shall construct the propositions by the draftsman's method and then employ the method of the geometrician as a test. If the problems are performed with great accuracy, the technical skill acquired in the handling of the instruments will be correspondingly great. It is not well to ink geometrical problems, as the precision of the pencilling will be impaired.

39. To bisect a right line, AB, or the arc of a circle, ACB. Fig. 44. With centers A and B, and any radius greater than one-half of AB, describe arcs 1 and 2. Through the points of intersection of these arcs draw a line. Its intersection with the given line AB, and the arc of the circle ACB, will determine the required points.

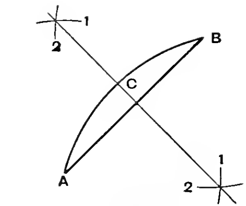


Fig. 44.

DRAFTSMAN'S METHOD. Obtain the division with the dividers as explained in Art. 18, page 14.

40. To divide a line, AB, into any number of equal parts. Fig. 45. Let the required number of divisions be five. Draw AC at any angle with AB, and lay off five equal spaces of any length. Connect the last point, 5, with B, and draw parallels through the other points intersecting AB in points 1′, 2′, 3′ and 4′ which determine the required divisions of AB. Art. 7, page 4.

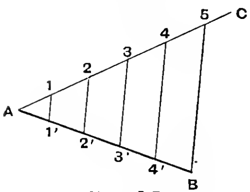


Fig. 45.

41. To draw a perpendicular to a line AB.

CASE 1. Fig. 46. When the given point C is on the line, and at or near the middle of the line.

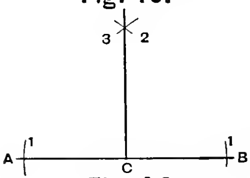


Fig. 46.

From C, with any radius, draw arcs 1 and 1, and from the point of intersection of these arcs with AB, with any radius greater than arc 1, draw arcs 2 and 3. The line drawn through the point of intersection of these arcs and the given point, C, will be the required line.

CASE 2. Fig. 47. When the point is on the line, and at or near the extremity of the line.

First Method. Let AB be the given line, and A the given point. From A, with any radius, describe arc 1. With center C, and same radius, describe arc 2. Through C, and the intersection of arcs 1 and 2, draw CE, and with same radius as before, from intersection of arcs 1 and 2, describe arc 3. A line drawn through A, and the point of intersection of arc 3 and line CE, will be the required perpendicular.

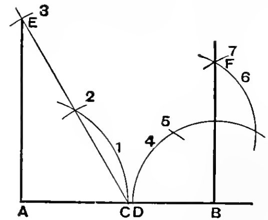


Fig. 47.

Second Method. From B, with any radius, describe arc 4. From point D, with same radius, describe arc 5. From the intersection of arcs 4 and 5, describe arc 6. From the intersection of arcs 4 and 6, describe arc 7. The line drawn through this last point of intersection, F, and the given point B, will be the required perpendicular.

Case 3. Fig. 48. When the point is outside of, and opposite, or nearly opposite, the middle of the line.

From C, with any radius, describe arcs 1 and 1. From the point of intersection of these arcs with AB, with same radius, describe arcs 2 and 3. A line drawn through this point of intersection and the given point C, will be the required line.

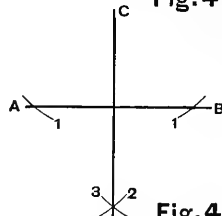


Fig. 48.

Case 4. Fig. 49. When the point is outside of, and at, or near, the extremity of the line.

From C draw any line CD. Find E, the center of CD, by dividers, or by Art. 39. On CD as a diameter, describe a semicircle. Through the given point C, and the intersection of semicircle with AB, draw CF, which will be the required perpendicular.

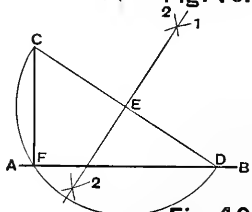


Fig. 49.

Draftsman's Method. See Art. 7, page 3.

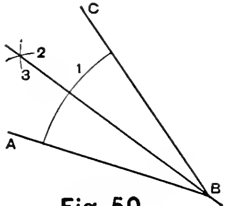


Fig. 50.

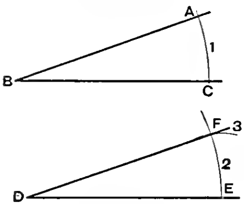


Fig. 51.

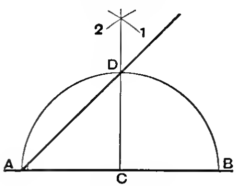


Fig. 52.

42. Angles of 15°, 30°, 45°, 60° and 75°, in either quadrant, may be constructed by means of the 60° and 45° triangles used in combination with the T square, as described in Art. 7, page 4.

43. To bisect an angle, ABC. Fig. 50. From B, with as large a radius as possible, describe arc 1. From its points of intersection with AB and CD, describe arcs 2 and 3. The line drawn through their intersection and B will bisect the given angle.

44. To construct an angle, FDE, equal to a given angle ABC. Fig. 51. Draw DE. From B and D, with equal radii, describe arcs 1 and 2. From E, with radius equal to chord AC, describe arc 3. Through D, and point of intersection of arcs 2 and 3, draw DF making the required angle.

45. To construct an angle of 45° with AB at point A. Fig. 52. Through the given point, A, describe a semicircle on AB; draw a perpendicular through the center C. A line drawn through the point A and intersection of the perpendicular with the semicircle will make an angle of 45° with AB.

46. To construct angles of 60°, 30° and 15° with AB. Fig. 53. From the given point A as a center, with any radius, describe arc 2. From B, with the same radius, describe arc 3. A line drawn through A and this point of intersection will make an angle of 60 with AB.

Through the given point A, describe a semicircle on AB. With same radius, from C describe arc 1. The line drawn through A and this point of intersection will make an angle of 30° with AB.

Having constructed an angle of 30°, as described, bisect the same, and FAB will be the required angle of 15°. In this case it would not be necessary to draw the 30° line.

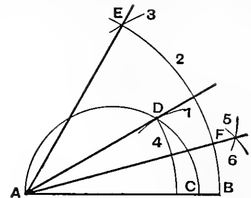


Fig. 53.

47. To construct an equilateral triangle having given the side AB. Fig. 54. Since the sides are equal, the angles will be equal, and therefore, 60°, the sum of the angles of any triangle being equal to 180°. With centers A and B and radius AB, describe arcs 1 and 2. From the point of intersection, C, draw AC and BC.

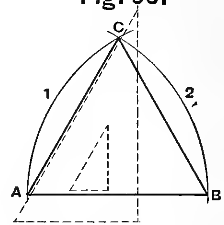


Fig. 54.

Draftsman's Method. AB being drawn with the T square, through A and B, with 60° triangle, draw AC and BC.

48. To construct an isosceles triangle. Fig. 55. Having given the base DF and the equal sides DE and EF, from centers D and F, draw arcs 1 and 2 with radius equal to the given sides. From the point of intersection, E, draw DE and EF.

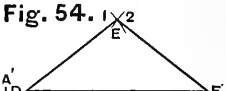


Fig. 55.

If the angle be given, construct FDE and EFD equal to the given angle, and draw DE and EF.

49. To construct a scalene triangle. Fig. 56. Having given the sides A′B′, A′C′ and B′C′. Draw AB equal to A′B′. With centers A and B, and radii equal to given sides, draw arcs 2 and 1. Draw AC and CB.

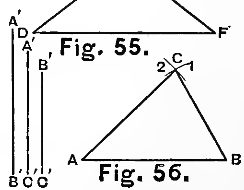


Fig. 56.

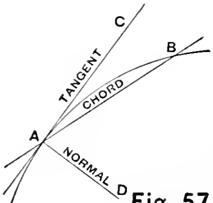


Fig. 57.

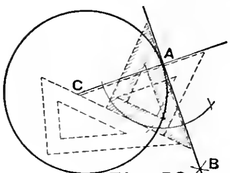


Fig. 58.

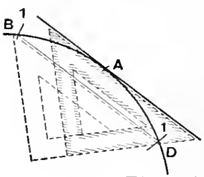


Fig. 59.

50. Tangent, Secant and Normal. Fig. 57. If a line AB cuts a curve at two points, it is called a secant. Conceive the line as revolving about the point A until the second point of intersection with the curve shall coincide with the first; the line will then be in the position AC, and called a tangent. AD is a line perpendicular to the curve at the point of tangency, and called a normal.

51. To draw a tangent, AB, to a circle.

CASE 1. When the given point A is on the circle. Fig. 58. Draw the radius AC, and erect a perpendicular, AB, at A.

DRAFTSMAN'S METHOD. Place the triangle to coincide with center C and given point A, as though to draw AC. By means of a second triangle used as a base, turn the first triangle into the second position and draw AB perpendicular to AC.

CASE 2. When the point is on the circle and the center not accessible. Fig. 59. From the given point, A, with any radius, describe arcs 1 and 1. Place the edge of the triangle to coincide with points B and D. Draw a parallel line through A.

CASE 3. When the given point, B, is without the circle. Fig. 60. Two tangents may be drawn. On BC as a diameter, describe arc 1; its intersection with the circle at A and D will be the points of tangency. The angle BAC, inscribed in the semicircle, will be 90°.

Draftsman's Method. From the given point, B, draw BA touching the circle. Through the center, C, draw a perpendicular to AB. A will be the point of tangency. In like manner obtain BD.

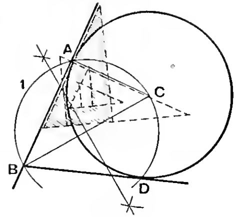


Fig. 60.

52. To lay off an arc equal to a given tangent. Fig. 61. Let AB be the given tangent and AD the arc. From the point B step off equal spaces with the dividers or bow-spacers until a point of the dividers is at or near the point of tangency A. Reverse the motion of the dividers, stepping off an equal number of spaces on the curve.

When several arcs are tangent at the same point and it is desired to lay off the length of their common tangent on each, the following approximation may be used provided the greatest arc does not exceed 60°. On AB lay off AC equal to one-quarter of AB, and from C as a center describe the arc DBEF; the arcs AD, AE and AF will closely approximate the given tangent AB.

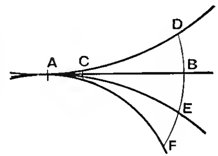


Fig. 61.

53. On a tangent to lay off a length equal to a given arc. Fig. 62. If AD be the given arc, draw the chord of this arc and continue it to C, AC being equal to one-half of AD. From C as a center, describe the arc DB intersecting the tangent at B. AB will be the required length or the rectification of the arc. This approximation should not be used for arcs greater than 60°.

The method of spacing the distance by the dividers may be employed in this case as in the previous one, Art. 52.

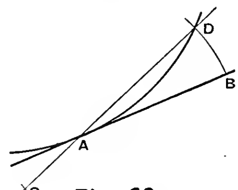


Fig. 62.

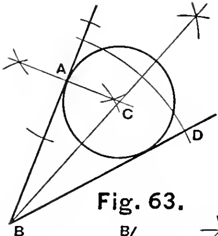


Fig. 63.

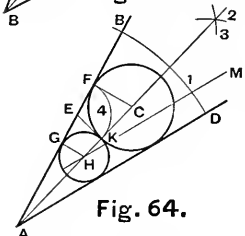


Fig. 64.

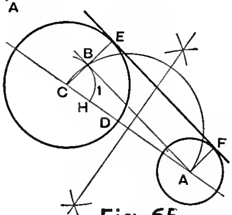


Fig. 65.

54. To draw a circle through a given point, A, and tangent to given lines, AB and BD. Fig. 63. Since the circle is to be tangent to AB and BD, its center must lie upon the bisector of the angle DBA; and because it is to be tangent to AB at the point A, its center must lie on the perpendicular to AB at A. Bisect the angle DBA, and through the point A draw AC perpendicular to AB. C will be the center of the circle, and AC its radius. The draftsman's method may be used for obtaining the perpendicular AC.

55. To draw any number of circles tangent to each other and to two given lines, AB and AD. Fig. 64. Bisect angle DAB, and with any radius, HK, draw a circle tangent to AB and AD. From K draw KE perpendicular to AK, and with radius EK describe arc 4. Through F draw FC perpendicular to AB. C will be the center and FC the radius of the second circle. Repeat the process.

If the radius of the first circle be given, draw a parallel to AD distant from it equal to the given radius. The intersection of this line, HM, with this bisector of the angle ABD will be the required center.

56. To draw a tangent to two given circles. Fig. 65. Join A and C, the centers of the given circles. From D lay off DH equal to AF. With center C and radius CH draw arc 1. From A draw a tangent to this arc. Art. 51, Case 3, page 40. Through B, the point of tangency, draw CE, and through A draw AF parallel to CE. E and F will be the points of tangency, and EF the tangent.

57. To draw a circle of a given radius, R, tangent to two given circles having centers B and C. Fig. 66. From centers B and C draw indefinitely, in any direction, lines BF and CE. Lay off HE and KF equal to the given radius R, and through F and E, from centers B and C, describe arcs 1 and 2 intersecting at A, the required center. Since these arcs intersect in a second center, there will be two solutions to this problem.

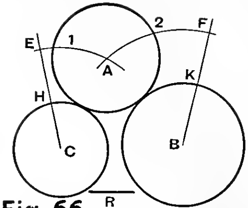


Fig. 66.

58. Through three given points, A, B and D, not in the same straight line, to draw a circle. Fig. 67. Bisect the imaginary chords AB and BD. The point of intersection, C, of the bisecting lines, will be the required center.

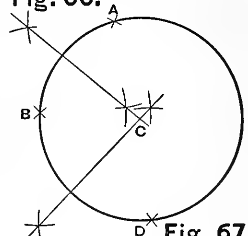


Fig. 67.

59. To circumscribe a circle about a given triangle, ABC. Fig. 68. Bisect two of the sides, as AC and BC. The point of intersection of these lines will be the center of the required circle. Draw a circle through A, B and C. This problem is identical with the preceding.

If the hypothenuse AC should pass though the center D, the angle ABC would be a right angle. See also Art. 41, case 4, page 37.

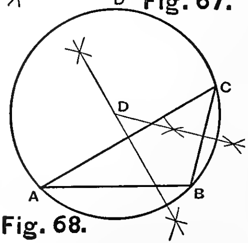


Fig. 68.

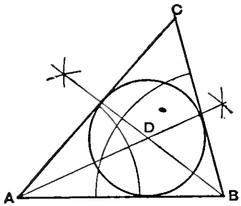


Fig. 69.

60. To inscribe a circle within a given triangle, ABC. Fig. 69. Bisect two of the angles, as CAB and ABC. The point of intersection of these lines, D, will be the center of the required circle. From this point draw a circle tangent to AC, CB and AB.

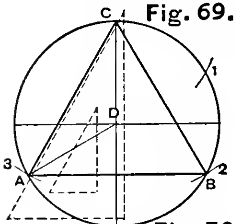


Fig. 70.

61. To inscribe an equilateral triangle within a circle. Fig. 70. From the point C draw arc 1 with a radius equal to that of the circle. From its intersection with the circle, and with the same radius, draw arc 2. From center C, and with chord CB as a radius, describe arc 3, and connect points A, B and C, which will give the required triangle.

Draftsman's Method. From point C, on the vertical diameter CD, draw CA and CB with the 60° triangle. With the T square draw AB to complete the triangle.

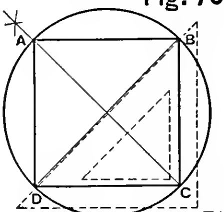


Fig. 71.

62. To inscribe a square within a circle. Fig. 71. Draw any diameter BD. Draw a second diameter, AC, perpendicular to it. Connect points A, B, C and D to complete the square.

Draftsman's Method. With the 45° triangle draw perpendicular diameters AC and BD, and connect points A, B, C and D.

63. To inscribe a pentagon within a circle. Fig. 72. Draw any diameter, GF, and a radius AK perpendicular to it. Bisect KF, and, with H as a center, and a radius AH, describe arc 3. With center A and radius AL describe arc 4. AB is the side of a pentagon. Obtain the remaining points by describing arcs 5, 6 and 7 with same radius. Connect points A, B, C, D and E to obtain the required pentagon.

DRAFTSMAN'S METHOD. Estimate an arc equal to one-fifth of the circumference, and with the dividers step off this length, dividing the circle into five parts and correcting the arc as directed for the division of a line. See Art. 18, page 14. Connect the points.

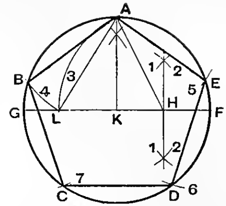


Fig. 72.

64. To inscribe a hexagon within a circle. Fig. 73. Draw any diameter FC. With centers F and C and radius equal to that of the circle draw arcs 1 and 2. Connect the points of intersection A, B, C, D, E and F to obtain the required hexagon. Observe that the angles at the center, as BKC, are 60°.

DRAFTSMAN'S METHOD. Draw a horizontal diameter FC. With 60° triangle draw diameter EB. Draw AB and ED, and with triangle draw BC, FE, AF and CD.

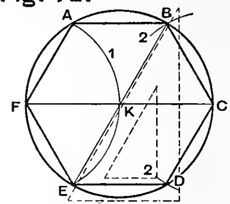


Fig. 73.

65. To circumscribe a hexagon about a circle. Fig. 74. Draw any diameter, AD. With H as center and radius equal to that of the circle, describe arc 1. Bisect the arc HLN, and through L draw AB parallel to HN. With center K and radius AK describe circle ACE. In this inscribe a hexagon by Art. 64.

DRAFTSMAN'S METHOD. With 60° triangle draw diameters AD and EB, and with same triangle draw sides AB and ED, EF and BC, AF and CD, each tangent to the given circle.

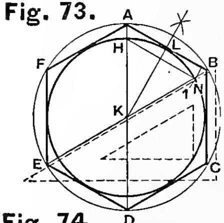


Fig. 74.

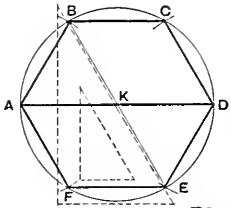


Fig. 75.

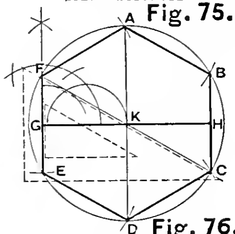


Fig. 76.

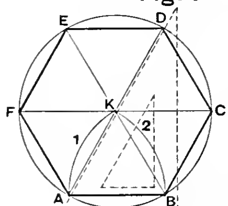


Fig. 77.

66. To draw a hexagon having given a long diameter, AD. Fig. 75. Bisect AD. With K as center, describe circle ACE, and in this inscribe a hexagon by Art. 64.

DRAFTSMAN'S METHOD. With dividers find center K. Through A and D with 60° triangle draw AB and DE. With same triangle draw BE, and through D and A draw CD and AF. Draw BC and FE to complete the hexagon. To obtain B without finding K draw a line through D at an angle of 30° with AD intersecting a 60° line through A.

67. To draw a hexagon having given a short diameter, GH. Fig. 76. Bisect GH. From G draw a perpendicular GF. From K draw FKC at 30° with GK. Through F and with center K draw circle FBD. Inscribe a hexagon by Art. 64.

DRAFTSMAN'S METHOD. With dividers find center K. With 60° triangle draw FC, and through G, K and H draw perpendiculars FE, AD and BC. Draw the sides FA and DC, AB and DE.

68. To draw a hexagon having given a side, AB. Fig. 77. With centers A and B, and radius AB, describe arcs 1 and 2. From their intersection, K, with same radius describe circle AEC. Inscribe a hexagon by Art. 64.

DRAFTSMAN'S METHOD. Through A and B with 60° triangle draw AD and BE, and through their intersection, K, draw FC. Draw FA, BC, FE, DC and ED.

69. To inscribe an octagon within a given circle, ACEG. Fig. 78. Draw any diameter GC. At center, and perpendicular to GC, draw AE. Bisect AKG and AKC. Connect the points of intersection with the circle.

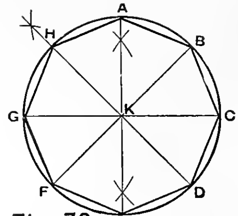


Fig. 78.

DRAFTSMAN'S METHOD. With 45° triangle and T square draw diameters AE, GC, FB and HD, and connect their extremities.

70. To circumscribe an octagon about a circle, ABCD. Fig. 79. Draw the perpendicular diameters AC and BD. With centers A, B, C and D, and radius AK, describe arcs 1, 2, 3, 4. By connecting these points of intersection a circumscribed square will be obtained. With the centers R, S, V, T, and radius RK, describe arcs 5, 6, 7, 8 to obtain the points G, H, L, N, O, P, E, F, which being connected will complete the circumscribed octagon.

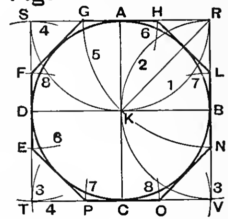


Fig. 79.

DRAFTSMAN'S METHOD. With 45° triangle and T square draw tangents FE and LN, GH and PO. At 45° draw tangents FG, ON, HL and EP, completing the octagon.

71. On a given side, AB, to construct a regular polygon having any number of sides. Fig. 80. With AB as a radius describe the semicircle D2B and divide it into as many parts as the polygon has sides; in this case five. Beginning with the second division from the left draw radial lines, A2, A3, A4. A2 will be one side of the polygon. Bisect sides AB and A2 to obtain center of circumscribing circle. The intersection of this circle with the radial lines A2, A3, and A4, will determine the vertices of the polygon.

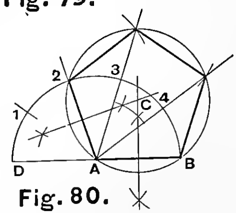


Fig. 80.

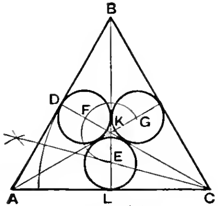


Fig. 81.

72. Within an equilateral triangle, ABC, to draw three equal circles tangent to each other and one side of the triangle. Fig. 81. Bisect the angles A, B and C. Bisect the angle DCA. E is the center of one of the required circles. With center K and radius KE describe arc EFG. F and G will be the remaining centers. From these centers with radius EL describe the required circles.

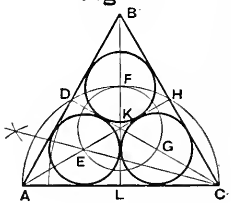


Fig. 82.

73. Within an equilateral triangle, ABC, to draw three equal circles tangent to each other and two sides of the triangle. Fig. 82. Bisect the angles A, B and C. Bisect angle DCA. E will be the center of one of the circles. With K as center and radius KE, describe arc EFG to obtain the remaining centers. Draw circles tangent to the sides AB, BC and AC.*

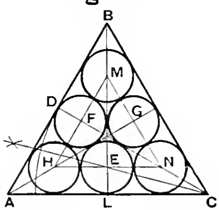


Fig. 83.

74. Within an equilateral triangle, ABC, to draw six equal circles tangent to each other and the sides of the triangle. Fig. 83. Bisect the angles and obtain E as in Art. 72. Through E draw HN parallel to AC. Draw HM parallel to AB, and MN parallel to BC. With E, H, F, M, G and N as centers, and with radius EL, describe the required circles.

75. Within a given circle, ACE, to draw three equal circles tangent to each other and the given circle. Fig. 84. Divide the circle into six

* Instead of bisecting the angle DCA, to obtain one of the centers, describe a semicircumference, AFC, on one of the sides, as AC, thus obtaining the center F.

equal parts by diameters AD, BE, CF. Produce AD indefinitely, and from E draw the tangent EG. Bisect KGE. With K as center, and radius HK, describe arc HLM, and with radius HE, from centers H, L and M, describe the required circles.

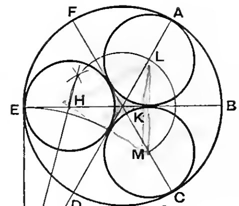


Fig. 84.

76. Within a given circle to draw any number of equal circles tangent to each other and the given circle. Fig. 85. Divide the circle by diameters into twice as many equal parts as circles required; in this case eight. Suppose the center of one of these circles to lie on AK; then the circle must be tangent to both FK and KB. Draw tangent at A intersecting KB. Since the required circle must be tangent to the given circle at A, it will also be tangent to AB, and as it must lie in the angles FKB and ABK, its center must be at D, the intersection of their bisectors. With center K draw circle through D; its intersection with EK, CK, etc., will determine the required centers. From these centers describe the required circles with radius AD.

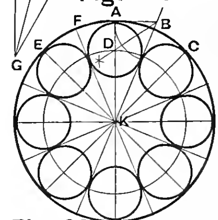


Fig. 85.

77. About a given circle to circumscribe any number of equal circles tangent to each other and the given circle. Fig. 86. Divide the circle by diameters into twice as many equal parts as circles required; in this case six. From A, the extremity of any diameter, draw tangent AB. Produce KB making BC equal to AB. At C, perpendicular to BC, draw CD intersecting AK produced, at D. This will be the center and AD the radius of one of the required circles. With center K and radius DK obtain other centers.

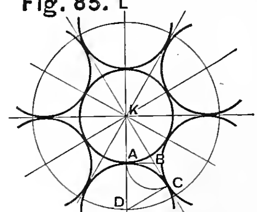


Fig. 86.

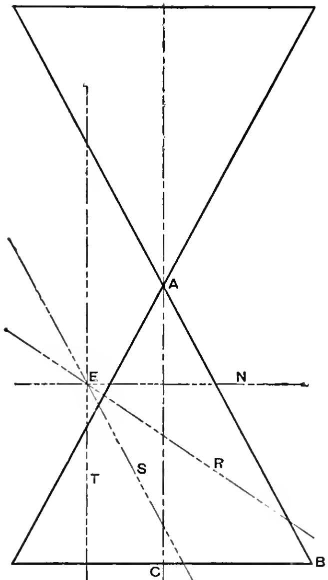


Fig. 87.

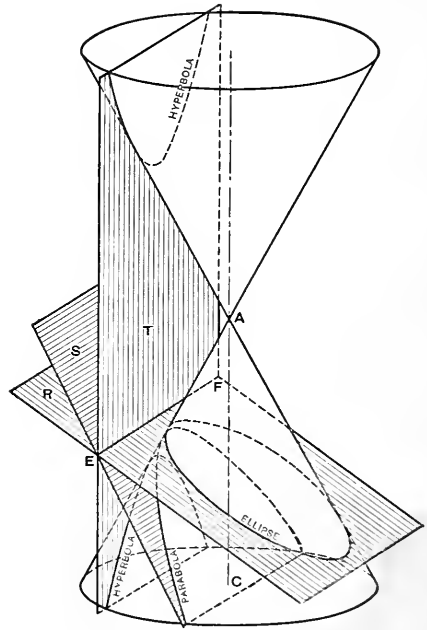


Fig. 88.

CHAPTER IV

CONIC SECTIONS

78. The Cone and Cutting Planes. Figs. 87 and 88 are illustrations of a right cone with a circular base cut by planes making several angles with the axis. It is a complete cone in that it extends as much above the vertex A as below it, the two parts being known as the upper and lower nappe. It is called a right cone because the axis is perpendicular to the base. The curves of intersection between the planes and the surfaces of the cone are known as conic sections. They are four in number: the CIRCLE, ELLIPSE, PARABOLA and HYPERBOLA. An edge view of the planes is illustrated by Fig. 87, which shows the relation they bear to the axis AC and an element AB.

The circle is obtained by a cutting plane, N, perpendicular to the axis. The ellipse is obtained by a cutting plane, R, oblique to the axis and making a greater angle with the axis than the elements do. The parabola is obtained by a cutting plane, S, making the same angle with the axis as the elements do. The hyperbola is obtained by a cutting plane, T, making a smaller angle with the axis than the elements do. All planes cutting hyperbolic curves will cut both nappes of the cone.

In Figs. 87 and 88, if we conceive the plane T as revolving about the line EF as an axis, it will cut the cone in hyperbolas from T to S. At S, parallel to AB, the curve will be a parabola. From S to N it will cut ellipses, and at N, a circle. The latter is not shown in Fig. 88.

These curves may be obtained in two ways: First, by determining the curve of intersection between the planes and the cone, as in Fig. 88; second, by known data and a knowledge of the characteristics of the curve. Only the latter is considered in this chapter.

79. The Ellipse is a curve generated by a point moving in a plane so that the sum of the distances from this point to two fixed points shall be constant. If, in Fig. 89, we conceive EKF to be a cord fastened at its extremities, E and F, and held taut by a pencil-point at K, it may be seen that as motion is given to the point it will be constrained to move in a fixed path dependent on the length of the cord. When the pencil-point is at B, one segment of the cord will equal BE and the other BF, their sum being the same as KE plus KF, and also equal to AB. The fixed points E and F are called the FOCI. They lie on the longest line that can be drawn terminating in the curve of the ellipse. The line is known as the MAJOR AXIS, and the perpendicular to it at its middle point, also terminating in the ellipse, is the MINOR AXIS. Their intersection is called the center of the ellipse, and lines drawn through this point and terminating in the ellipse are known as diameters. When two such diameters are so related that a tangent to the ellipse at the extremity of one is parallel to the second, they are called CONJUGATE DIAMETERS. KL and MN are two such diameters.

In order to construct an ellipse it is generally necessary that either of the following be given: The major and minor axes; either axis and the foci; two conjugate diameters; a chord and axis.

80. Ellipse. First Method. Fig. 89. By definition it may be seen that a series of points must be so chosen that the sum of the distances from either of them to the foci must equal the major axis. Thus, HE + HF must equal CE + CF, or KF + KE, each being equal to AB.

If the major axis and the foci be given to draw the curve, points may be determined as follows: From E, with any radius greater than AE and less than EB, describe an arc. From F, with a radius equal to the difference between the major axis and the first radius, describe a second arc cutting the first. The points of intersection of these arcs will be points, the sum of whose distances from the foci will equal the major axis, and therefore points of an ellipse. Similarly find as many points as may be necessary to enable the curve to be drawn free-hand. Lightly pencil a line through these points. For inking see Art. 15, page 11.

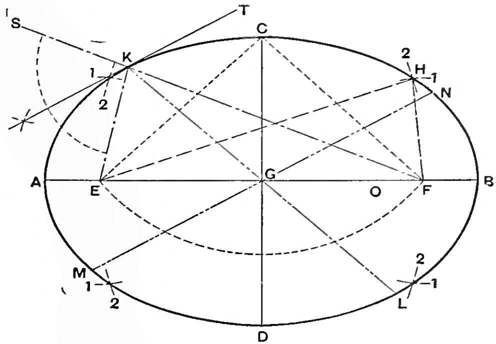


Fig. 89. ELLIPSE FIRST METHOD

Having given the major and minor axes, we can find the foci by describing, from C as a center, an arc with a radius equal to one-half the major axis. The points of intersection with the major axis will be the foci; and this must be so since the sum of these distances is equal to the major axis; and the point C being midway between A and B the two lines CE and CF must be equal. Again, if the major axis and foci are given, with a radius equal to one-half this axis describe arcs from the foci cutting the perpendicular drawn at the middle point of the major axis and thus obtain the minor axis. Having the two axes proceed as before.

A tangent to an ellipse may be drawn at any point, K, by producing FK and bisecting the angle SKE; the bisecting line, KT, will be the required tangent.

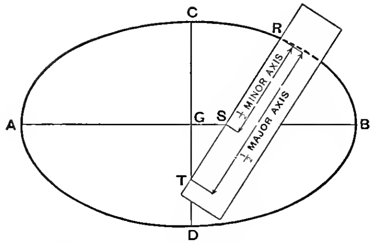


Fig. 90. ELLIPSE SECOND METHOD

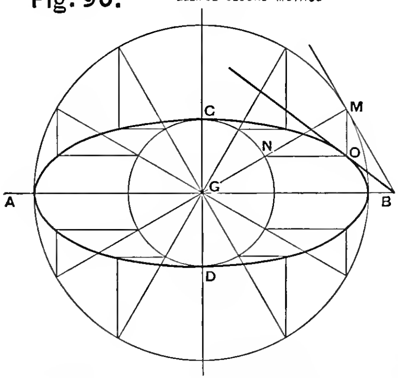


Fig. 91. ELLIPSE THIRD METHOD

81. Ellipse. Second Method. Fig. 90. Let AB and CD be the major and minor axes of an ellipse. Lay off on a piece of paper having a clean-cut edge the distance RT equal to one-half the major axis, and RS equal to one-half the minor axis. If point T be placed upon the minor axis and point S upon the major axis, and the paper constrained to move always under these conditions, the point R will describe an ellipse. Points may be laid off on the drawing to correspond with the different positions of R, and through these the required ellipse will be drawn. This is an excellent method, as construction lines are not required. It is known as the method by trammels, since an instrument called the elliptographic trammel is constructed on this principle.

82. Ellipse. Third Method. Fig. 91. Having the major axis AB and the minor axis CD, describe circles on these as diameters. Draw any radial line, as MG. From its intersection with the outer circle draw MO perpendicular to the major axis, and from its intersection with the inner circle draw NO perpendicular to the minor axis. The intersection of these lines at O will be a point in the ellipse. Similarly obtain other points.

A tangent at the point O may be obtained by drawing a tangent to the outer circle at M and from its intersection with the major axis at B, drawing the required tangent through O.

83. Ellipse. Fourth Method. Figs. 92, 93, 94. This is a very general method and may be used when we have given either the major and minor axes, one of the axes and a chord of the ellipse, or any two conjugate diameters.

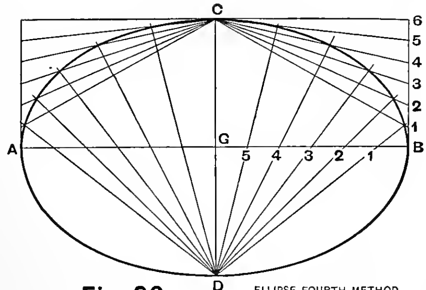


Fig. 92. ELLIPSE FOURTH METHOD CASE 1.

CASE 1. Fig. 92. Having given the major and minor axes. From the extremity of the major axis, draw B6 parallel and equal to the minor axis, and divide it into any number of equal parts; in this case six. Divide BG into the same number of equal parts. Through points 1, 2, 3, etc., on B6, draw lines to extremity C of the minor axis. From D, the other extremity of the minor axis, draw lines through points 1, 2, 3, etc., on BG, intersecting the above lines in points which will lie in the required ellipse. Construct the remainder of the ellipse in the same manner.

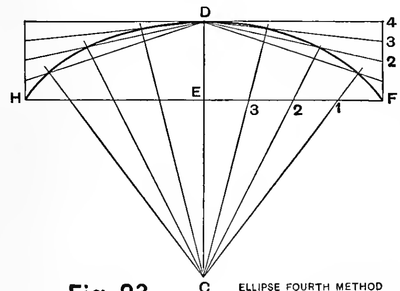


Fig. 93. ELLIPSE FOURTH METHOD CASE 2.

CASE 2. Fig. 93. Having given an axis CD and chord FH. From F draw F4 parallel to CD; divide it into any number of equal parts; in this case four. Divide the half chord FE

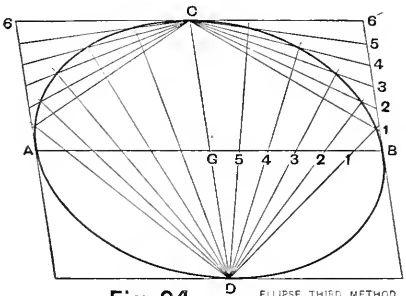


Fig. 94.

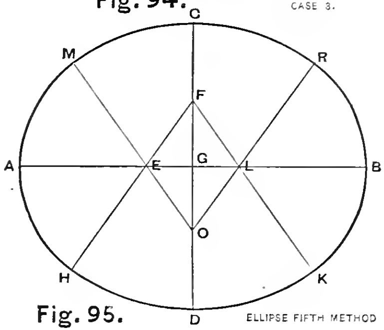


Fig. 95.

into the same number of equal parts; through these points and extremities of given axis draw intersecting lines as before, thereby obtaining the elliptical arc FD. Construct opposite side in the same manner.

CASE 3. Having given the conjugate diameters AB and CD, Fig. 94. From A and B draw lines A6 and B6 parallel to the diameter CD and equal to CG. Divide these into any number of equal parts, and, having divided BG and AG into the same number of equal parts, draw lines from these points to the extremities of diameter CD. The intersection of these lines with the former will determine points in the ellipse. In like manner describe the opposite side.

84. Ellipse. Fifth Method. Fig. 95. To describe an approximate ellipse, the major and minor axes being given. For many purposes in drawing it is sufficiently accurate to describe the ellipse by means of four circular arcs of two different radii. The following is one of several methods: On the minor axis lay off GF and GO equal to the difference between the major and minor axes. On the major axis lay off GE and GL equal to three-quarters of GF. Connect points F, E, O, L, and produce the lines. With center E and radius AE describe arc HAM. With center F and radius FD describe arc KDH. In like manner describe MCR and RBK from centers L and O. Do not use this method when the major axis is more than twice the minor.

85. The Parabola is a curve generated by a point moving in a plane so that its distance from a fixed point shall be constantly equal to its distance from a given right line. Point F, Fig. 96, is the Focus, CD is the given right line called the Directrix, and AB, a perpendicular to CD through F, is the Axis. V, the intersection of the axis with the curve, is the Vertex, and by the definition of a parabola it must be equidistant from the focus and directrix.

86. Parabola. First Method. Fig. 96. Having given the focus F and the directrix CD. Bisect FA to find the vertex V. Through any point on the axis, as L, draw MN parallel to the directrix and with radius LA describe arc 1 from focus F as center, intersecting line MN at points M and N. These are points in the parabola. Similarly obtain other points and draw the required curve. A tangent to the curve may be drawn at any point M by drawing MO parallel to the axis and bisecting the angle OMF. MT is the required tangent at the point M.

Since the angle T'MR is equal to the angle TMF, it follows that MR would be the direction of a ray of light emitted from the focus F and reflected from the parabola at M. The locomotive head light is constructed on this principle.

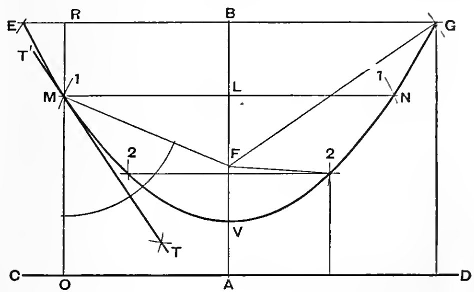


Fig. 96. PARABOLA FIRST METHOD

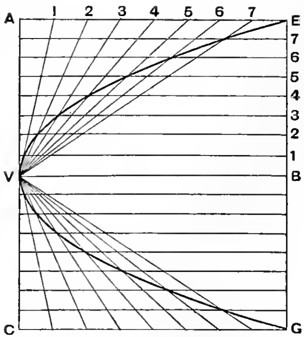


Fig. 97. PARABOLA SECOND METHOD

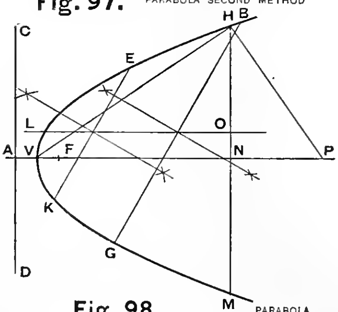


Fig. 98.

87. Parabola. Second Method. Fig. 97. Having given the abscissa VB, and the double ordinate GE. Draw AE and CG parallel and equal to VB. Divide AE and BE into the same number of equal parts. From the divisions on BE draw parallels to the axis and from the divisions on AE draw lines converging to the vertex V. The intersection of these lines, 1 and 1, 2 and 2, etc. will determine points in the required curve. In like manner obtain the opposite side.

88. Given a parabola to find its axis, focus and directrix. Draw any line BG, Fig. 98, cutting the parabola in two points. From any point E draw EK parallel to BG. Bisect BG and EK, and through these points draw LO. From any point H draw HM perpendicular to LO. The perpendicular to this line at its middle point, AN, will be the required axis.

To find the focus draw HV through the vertex, and perpendicular to it draw HP. Lay off VF equal to one-fourth of NP. F is the focus. Having laid off AV equal to VF, draw the directrix CD perpendicular to the axis.

89. The Hyperbola is a curve generated by a point moving in a plane, so that the *difference* of the distances from this point to two fixed points shall be constant. It will be observed that this definition differs from that of the ellipse by using the word *difference* instead of *sum*. The two fixed points F and F′, Fig. 99, are the FOCI, and the line VV′ is the constant distance called the TRANSVERSE AXIS. From the definition it will be seen that VV′ is equal to TF minus TF′, or NF minus NF′, etc. There will be two branches to the curve.

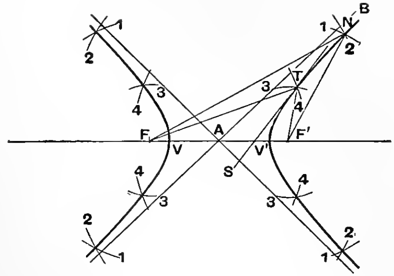


Fig. 99. HYPERBOLA: FIRST METHOD

90. Hyperbola. First Method. Fig. 99. Having given the transverse axis VV′ and the foci FF′ to describe an hyperbola. With any radius greater than F′V, from centers F and F′ describe arcs 2; from the same centers, with radii decreased by VV′, describe arcs 1 intersecting arcs 2. These points of intersection will be points in the required curve. A tangent, ST, may be drawn to any point, T, by bisecting the angle FTF′.

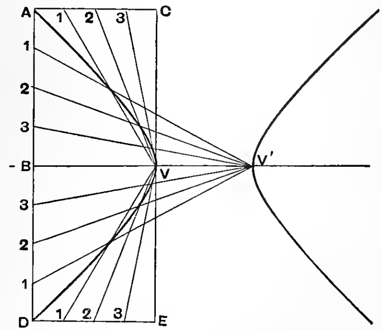


Fig. 100. HYPERBOLA: SECOND METHOD

91. Hyperbola. Second Method. Fig. 100. Having given the axis VV′, a double ordinate, AD, and BV, its distance from the vertex. Draw AC and DE parallel and equal to BV. Divide AB and AC into the same number of equal parts, and from points on AC draw lines converging to the vertex V. From points on AB draw lines converging to the vertex V′. The intersection of these lines 1 and 1, 2 and 2, etc. will be points in the required hyperbola.

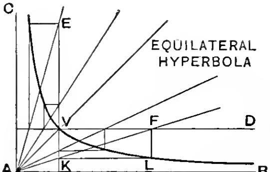


Fig. 101.

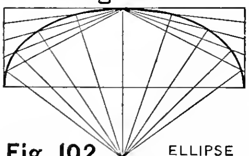


Fig. 102.

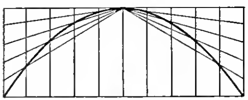


Fig. 103.

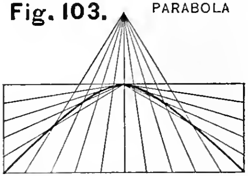


Fig. 104.

92. The Equilateral Hyperbola. Fig. 101. If the hyperbola is derived from a cone, the elements of which make angles of 45° with the axis, the curve is called an equilateral hyperbola. A knowledge of the construction of this curve is important, as it is the best approximation to the expansion curve of steam and is much used in connection with indicator diagrams.

In Fig. 101, A is the vertex and AB, AC the elements of a cone from which the curve is derived. V is the vertex of the hyperbola and AV is one-half the transverse axis. Draw VD and VE parallel to AB and AC respectively. Draw any line AF radiating from A and intersecting VD and VE or these lines produced. From the points of intersection F and K draw perpendiculars FL and KL. The intersection of these perpendiculars, L, will be a point in the hyperbola. It will be observed that the curve on the right of V is approaching AB, and on the left it is approaching AC; but it will meet and be tangent to these lines only at infinity. Lines having this relation to an hyperbola are called the ASYMPTOTES of the curve. The asymptotes of the hyperbola illustrated in Fig. 99 are AB and AC.

93. Figs. 102, 103, 104, illustrate the relation between the ellipse, parabola, and hyperbola, when constructed by the method which is common to all. In each case the height of the curve is the same and they have equal chords.

CHAPTER V

ORTHOGRAPHIC PROJECTION

94. THE previous chapters have treated of the penmanship and notation, or the alphabet and vocabulary of graphic language. The present chapter will discuss the construction of this language, and properly may be called the grammar of Graphics. These principles with their adaptation to practice by the introduction of various idiomatic construction, constitute a universal language known as Technical Drawing or Graphics.

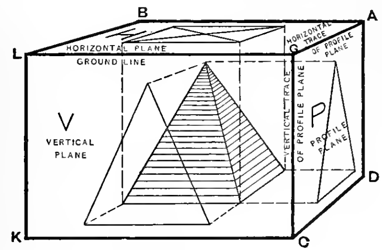


Fig. 105.

The system of representation usually employed is ORTHOGRAPHIC PROJECTION, or PROJECTION, as it is termed commonly. It is the art of delineating an object on two or more planes suitably chosen, and generally at right angles to each other, so as to represent exactly the form and dimensions of its lines and surfaces and their relation to each other.

95. Projection. Suppose that it is required to make the projections of a pyramid having a rectangular base, Fig. 105. Conceive the object as surrounded by transparent planes called PLANES OF PROJECTION or COÖRDINATE PLANES and shown in perspective by GLKC, GLBA and GCDA. We may have three representations of the object; one by looking through the front or vertical plane, one through the top or horizontal plane, and one through the side or profile plane of projection. These representations will be the correct projections of the object, if we imagine the eye as being directly opposite all of its points at one time, so that the rays of light from the points to the eye be perpendicular to the planes of projection; or we may conceive the eye as at an infinite distance from the plane, in which case all the rays will be parallel. Therefore, *if from each point of an object perpendiculars be drawn to the planes of projection, the intersection of these perpendiculars with the planes will be the required projections of the points, and the lines joining these points will be the projections of the lines and surfaces of the object.*

Thus point 1, Fig. 106, is projected on to the vertical plane GLKC at the point 1^V, on to the horizontal plane GLBA at 1^H and on to the profile plane GADC at 1^P. In like manner the points 2, 3, 4 and 5 are projected on to the three planes. The small letters ^{V}, ^{H} and ^{P}, above and to the right of the numbers, indicate the planes upon which the points lie, and when these letters are not affixed it signifies that the point or line itself is meant and not its projection. Since points 1 and 3 are projected on to the vertical plane by the same perpendicular, it follows that they will have a common point for their projection, this being designated as the projection of two points by the figures 1^V3^V, the first figure indicating the point nearer the plane of projection. Observe similar cases on the profile plane.

In order to represent these planes of projection upon a plane surface, as a sheet of drawing paper, it is necessary to revolve two of them into the plane of the other one, as in Fig. 106. The horizontal plane is revolved about GL into the position GLB′A″, and the profile plane is revolved about GC into the position GCD′A′. By this means we have obtained on a plane surface three representations of the object as they would appear on planes at right angles to each other. A good conception of the relation of the planes may be obtained by cutting a piece of paper in the form shown by B′A″GA′D′KB′ and then folding it on the lines GL and GC.

Observe that the top view is always above the front view, and the side view to the right or left of the front view according as it may be a view of the right or left side of the object.

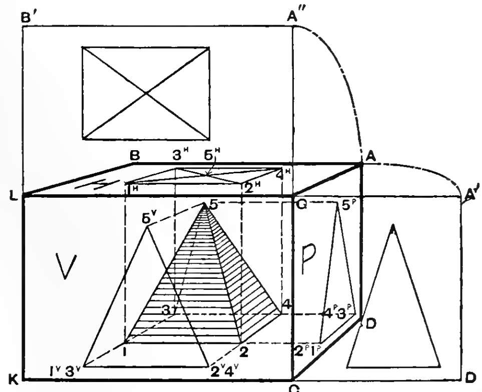


Fig. 106.

That representation which appears on the top plane of projection is called the TOP VIEW, HORIZONTAL PROJECTION or PLAN. That on the front plane is called the FRONT VIEW, VERTICAL PROJECTION or FRONT ELEVATION. That on the side plane is called the SIDE VIEW, PROFILE PROJECTION or SIDE ELEVATION. The view takes its name from the plane on which the representation is made and not from the face of the object represented. The first of these names is most consistent with general practice. The second forms are used in treatises on Descriptive Geometry. These terms are employed also in speaking of the coördinate planes, which for brevity are known as H, V and P. The third forms are used in architectural work.

GL, the line of intersection between the H and V planes, is called the GROUND LINE. GA, the intersection between the profile plane and H, is the horizontal trace of the profile plane and is indicated as the H tr. of P. GC, the intersection between the profile plane and V, is the vertical trace of the profile plane and is indicated as the V tr. of P.

From the foregoing the following laws are established: *The front and top views of any point lie in the same vertical line: The front and side views of any point lie in the same horizontal line: The top and side views of any point lie equally distant from the ground line and the Vtr. of P.*

96. To determine the projections of an object. Suppose it is required to obtain three views of a rectangular pyramid, as shown in Fig. 107, having given its dimensions. First draw the ground line and traces of P. The view included within the angle LGC will be the front view; that within the angle A″GL the top view; and that within the angle A′GC the side view. It is not necessary to limit these planes by drawing boundary lines, as in Fig. 106, since the planes are supposed to be indefinite in extent. In general, draw that view first about which most is known. In this problem the views will be drawn in the following order: front, side, and top view. At some convenient distance below GL and to the left of GC draw the line 1^V2^V of the length required to represent the base of the pyramid. At its middle point erect a perpendicular equal to the required height and connect points 1^V, 5^V and 2^V. This will complete the front view. Since the front and side views of any point lie in the same horizontal, it follows that the side view of the vertex, point 5, must lie in the horizontal line drawn through the point 5^V and at some convenient distance to the right of GC, as 5^P. Points of the base will be similarly projected, and, since the pyramid is symmetrical, the points 2^P, 1^P, and 4^P, 3^P, will be equally distant from the vertical center line drawn through 5^P, the line 2^P4^P being made of the length required to represent the depth of the pyramid.

To project points from front and side views to the top view it is necessary to observe first, that the front and top views of any point lie in the same vertical line; second, that the top and side views of any point lie equally distant from GL and V tr. of P. Therefore to project any point, as 5, into the top view, draw a perpendicular to GL through 5^V and the point will lie in this line; but its position may not be chosen as before, since two projections of a point being given the third is fixed, and the distance of this point from the front plane of projection is determined from the side view. Draw a perpendicular through the point 5^P until it intersects H tr. of P at E, and remembering that the lines GA″ and GA′ are one in space, revolve the point E by describing the arc EE′ from the center at G. From E′ draw the horizontal projecting line $E'5^H$, which will determine the top view of the point by its intersection with 5^V5^H. In like manner the points of the base may be found, and, since the vertex is connected with the four corners of the base, we shall have the lines 5^H1^H, 5^H2^H, 5^H3^H and 5^H4^H, to represent these edges.

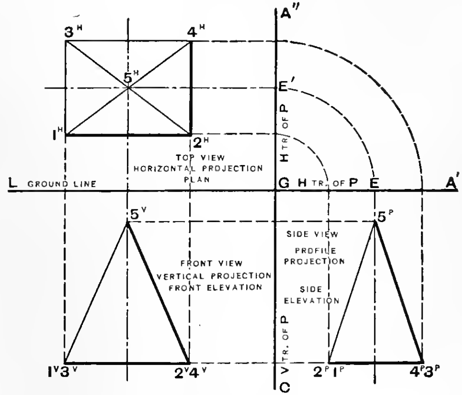


Fig. 107.

Observe the following: *The projection of a point is always a point. The projection of a line is either a point or a line. The projection of a surface is either a line or a surface.* To illustrate: The point 1 of the pyramid is projected as a point on each of the planes as shown at 1^H, 1^V, 1^P. The projection of the line 1 2 is a line on the front and top planes and a point on the side plane. The surface 1 5 2 is projected on the front and top planes at $1^V5^V2^V$ and $1^H5^H2^H$, but on the side plane by the line 1^P5^P.

Observe that the ground line is the horizontal projection of the vertical coördinate plane, and the vertical projection of the horizontal coördinate plane. In like manner the traces of P are seen to be the projections of one or the other of the coördinate planes.

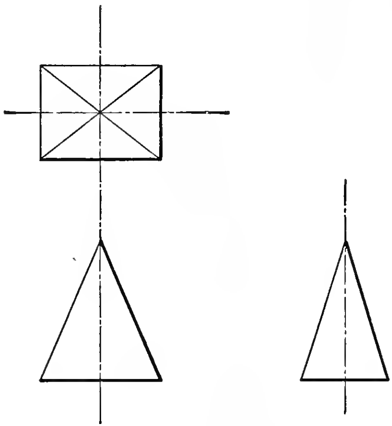


Fig. 108.

In actual practice the projection lines are seldom drawn from one view to another, only such portion being pencilled as may be necessary to locate the required points. Also, the ground line and profile traces do not appear, as it is customary to make the center lines serve this purpose wherever it may be possible. Fig. 108 is a projection of the same pyramid as was illustrated in Fig. 107 with the above-mentioned lines omitted. In projecting from the top to the side view, or *vice versa*, the dividers would be used to transfer the distances of the points from the ground line and traces of P which are now represented by the center lines of the figures.

It is better, however, for the student to make use of the ground line and projection lines until he has become thoroughly familiar with the relation of the object to the coördinate planes. The fact that these three representations of the object are made on planes perpendicular to each other must never be absent from his mind.

97. Objects oblique to the planes of projection. In studying the representation of an object which is inclined to one or more of the planes of projection, it is desirable to observe the changes which take place in its appearance when it is revolved in each of three ways: first, about an axis perpendicular to H, which is equivalent to turning it around on its base; second, by revolving it about an axis perpendicular to V, which would be inclining it to the right or left; third, by revolving it about an axis perpendicular to P, that is, revolving it forward or backward.

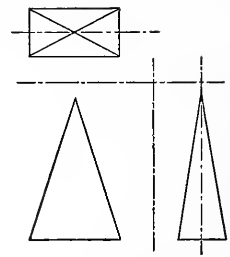


Fig. 109.

Fig. 109 represents three views of a rectangular pyramid, and it is required to represent the pyramid after it shall have been revolved about each of the three axes described above.

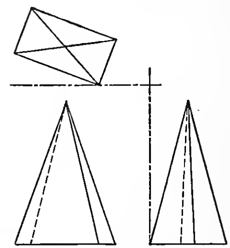


Fig. 110.

Fig. 110 is the projection of the object after having been revolved through 30° about an axis perpendicular to H. In this case the top view or horizontal projection will be the same as in Fig. 109, only the relation of the lines to the ground line being changed as the base moves in a horizontal plane. Both the front and side views are changed, but inasmuch as all the points of the object revolve about the required axis in planes perpendicular to the axis, the height of the points will remain unaltered. Therefore, having copied the top view as described, the front and side views may be projected from it as shown in Fig. 110, the height of all points of the object being the same as shown in Fig. 109.

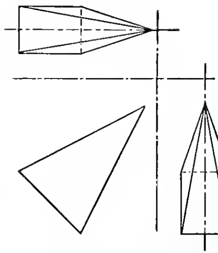


Fig. 111.

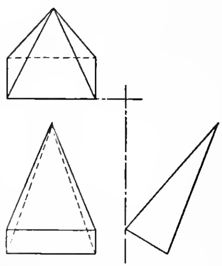


Fig. 112.

Fig. 111 is the projection of the pyramid after having been revolved from the position shown in Fig. 109 through 45° about an axis perpendicular to V. This will make no change in the front view or V projection, and it may be copied from the front view of Fig. 109,* drawing it at the required angle. The top and side views are obtained by projecting the points from the front view, observing that as all the points revolve about the axis in planes perpendicular to the axis, their distance from V will remain the same as in Fig. 109. These dimensions may be taken from GL or from the parallel plane passing through the center.

Fig. 112 represents the projection of the pyramid after having been revolved from the position shown in Fig. 109 through 30° about an axis perpendicular to P. In this case the side view will remain unchanged from that of Fig. 109, and the projections of the front and top views are obtained by noting that all points in revolving about the required axis do not change their distances from P, as they do not move to the right or the left.

From the foregoing it will be seen that there is always one view and one set of dimensions that will remain unchanged by a revolution about any axis. The following statement comprises all that has been said concerning the revolution of an object.

The unchanged view lies on that plane of projection which is perpendicular to the axis of revolution, and the unchanged dimensions are parallel to the axis of revolution.

* Fig. 113 is the same as Fig. 109 on the previous page.

In the preceding case all the revolutions were made from one position of the object; but by making successive revolutions it is possible to represent the object in any conceivable position. Having given the projections of the pyramid, as in Fig. 113, the following may be determined: —

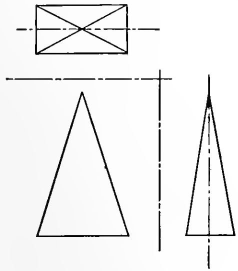


Fig. 113.

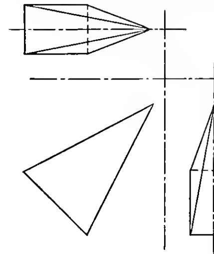


Fig. 114.

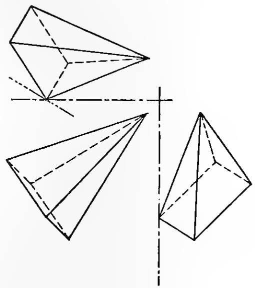


Fig. 115.

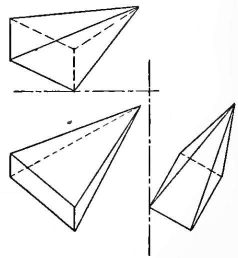


Fig. 116.

Fig. 114 represents the pyramid as revolved from the position of Fig. 113, 45° to the right about an axis perpendicular to V. The V projection is unchanged, and all the dimensions parallel to the given axis remain the same.

Fig. 116 represents the pyramid as revolved from the position of Fig. 114 backward through 30° about an axis perpendicular to P. The P projection is unchanged, and all the dimensions parallel to the given axis remain the same.

Fig. 115 represents the pyramid as revolved from the position of Fig. 116 through 30° about an axis perpendicular to H. The H projection is unchanged and all the dimensions parallel to the given axis remain the same.

98. Auxiliary planes of projection. Instead of revolving the object to make any required angle with one of the planes of projection, as was done in the preceding article, we may revolve the plane on which the projection is required. If the object be projected on to this new position of the plane, the result will be the same as by the previous method. Frequently, too, it is desirable to prevent the foreshortening of a surface by projecting it on to a plane to which it is parallel, and in such cases it is usually more simple to revolve the plane than the object.

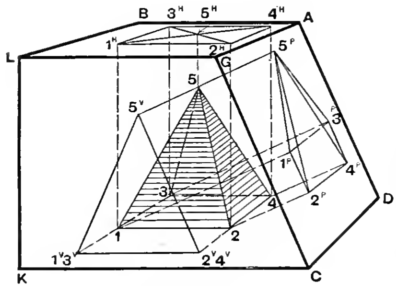


Fig. 117.

Fig. 117 is a perspective representation of a pyramid, three planes of projection, and the projection of the pyramid on these planes. This differs from Fig. 105, page 61, in that the side plane is revolved parallel to the right-hand slant face of the pyramid. The size and shape of the pyramid are unchanged, and the same notation is used. The traces of the planes of projection GL and GA remain the same, but GC is changed by reason of the revolution of the plane.

Fig. 118 illustrates these planes when revolved to coincide with V, and the projections of the object as they would appear on these planes. It will be observed that the method of projecting is identical with that already explained. Having drawn the front and top views, the side view is obtained by drawing perpendiculars from the V projection of the points to the trace GC, and determining the position of these points on the perpendiculars by projecting from the top view as indicated. Thus the side view of point 2 will be at the same distance from GC that the top view is from GL. The projecting lines may be used, or the measurements may be made with the dividers. In practice it is common to omit the traces, the center lines of the figure being substituted for them as was suggested on page 66. In the illustration the auxiliary plane is drawn parallel to the face 2 5 4, and therefore that face will not be foreshortened in this view. The lines will be seen in their true length, and the triangle in its true shape and area.

This method is used in problems relating to the development of surfaces and wherever a view of a special face is required without foreshortening. In such cases it is customary to project only those lines which lie in a plane parallel to the auxiliary plane.

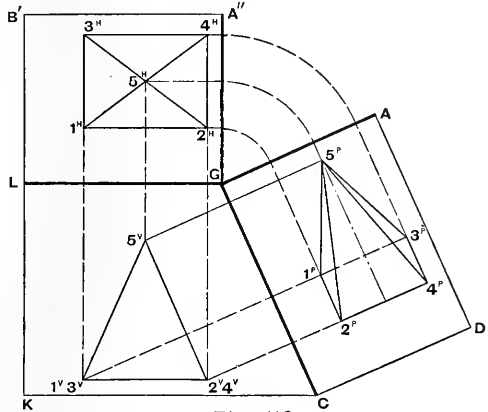


Fig. 118.

99. Views omitted by the use of auxiliary planes. It is frequently possible to lessen the labor involved in the representation of an object oblique to the coördinate planes by obtaining a view of some special face of the object having dimensions necessary to the making of the other views. In such a case an auxiliary plane parallel to the important face would be used.

Suppose that it is required to draw an octagonal prism having its edges parallel to the vertical coördinate plane, and the base making an angle of 30° with the horizontal plane. One of the long diameters of the base is parallel to the vertical plane.

Fig. 119 illustrates two views of the prism in the required position. These might be obtained by drawing the prism as resting on its base with its edges and a long diameter of the bases parallel to V, and having obtained the front and top views it would then be revolved about an axis perpendicular to V until the bases made the required angle with H; but by the following method the first of these operations will not be required. As the front view is unchanged by a revolution about an axis perpendicular to V, the upper and lower bases M^VN^V and A^VE^V may be drawn, if the diameter of the circumscribing circle of the base be given and the height of the prism be known. To determine the projection of the other edges the points $B^VC^VD^V$ must be obtained. Suppose the base to be projected on to an auxiliary plane parallel with itself, and this plane revolved about GC to coincide with V. The representation on this plane will be the same as the top view of the prism before having been revolved into the position shown, one half of which is projected by $A^VB'C'D'E^V$. By counter revolution B' will be projected at B^V, C' at C^V, and D' at D^V. The edges being drawn perpendicular to the bases will complete the view. To draw the top view it will be necessary to obtain the distances B^HL^H and C^HK^H. These can be measured from the view on the auxiliary plane, as they are double the distances $B'B^V$ and $C'C^V$. Having located the corners, complete the lower base and draw the upper base parallel to it, the foreshortened edge AM being projected from the front view.

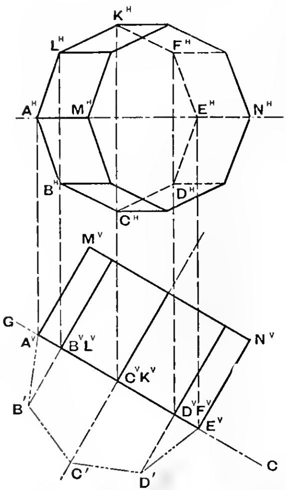


Fig. 119.

100. The projections of a circle oblique to the coördinate planes. It is required to find the front and top views of a circle lying in a plane perpendicular to V and making an angle of 30° with H; also to determine its projections after having been revolved from this position 30° about an axis perpendicular to H. The two views of the first position, Fig. 120, are to be obtained in a manner similar to that described for the base of the prism in the preceding article. Draw the diameter A^VB^V at the required angle of 30° and this will be the front view of the circle. We may conceive the circle as projected on to an auxiliary plane parallel to itself, or suppose the circle to be revolved about the diameter A^VB^V until it is parallel to V, as at $A^VE'C'B^V$. The top view of any point, as E^H, will lie in the vertical drawn from E^V and at a distance from the diameter A^HB^H equal to $E'E^V$. Obtain other points in the same manner, observing that points on the opposite side of the diameter A^HB^H may be laid off at the same time.

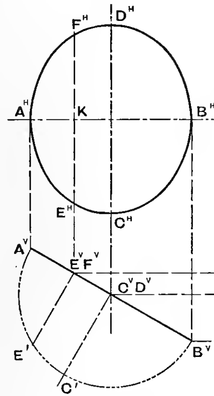


Fig. 120.

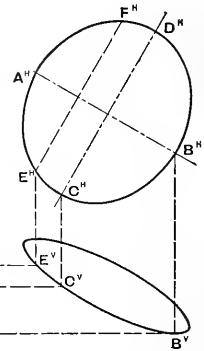


Fig. 121.

Fig. 121 illustrates two projections of this surface after having been revolved through 30° about an axis perpendicular to H. The top view being unchanged and the heights of the points remaining the same, the front view is readily obtained as indicated. It is very important that the points be obtained consecutively, and not by drawing the vertical and horizontal projecting lines and finally attempting to locate the required points at the intersection of these lines.

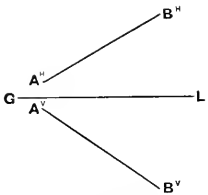


Fig. 122.

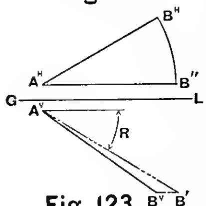


Fig. 123.

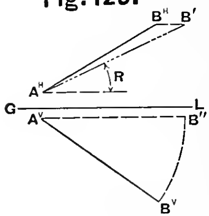


Fig. 124.

101. The projection and true length of lines. First Method. Let A^HB^H and A^VB^V, Fig. 122, represent two views of a line inclined to the coördinate planes H and V. It is required to find its length and the angles which it makes with the H and V planes of projection.

Since the line is inclined to both planes, its projections on them will be foreshortened views of the line. In order that the line shall be parallel to one of the planes, and thus seen in its true length on that plane, it will be necessary to revolve it about an axis perpendicular to one of the planes according to the principles already established for the revolution of an object. Art. 97, page 67. It may be revolved about an axis perpendicular to H until it becomes parallel to V, or about an axis perpendicular to V until parallel to H. Fig. 123 represents the line revolved parallel to V, the revolved position being shown by A^VB'. The horizontal projection of the line, which in the revolution about a vertical axis we know to be unchanged, is first drawn parallel with the ground line, in which position the line will be parallel to V. The height being unchanged the vertical projection may next be obtained, and the representation will be that of a line parallel to V and therefore seen in its true length on that plane. This view also shows the correct angle, R, which the line makes with the horizontal plane, since its position relative to that plane is unchanged by the revolution.

Similarly the line may be revolved about an axis perpendicular to V until it is parallel to H, as in Fig. 124, when its true length may be measured on the top view. This view also gives the angle, R, which the line makes with V.

Second Method. If the quadrilateral formed by a line, the projecting lines of its extremities, and its projection, be revolved about the latter until the surface coincides with the plane of projection, the revolved position of the line itself will exactly represent the length of the line, and the angle which it makes with the plane of projection will be shown on the plane into which it has been revolved.

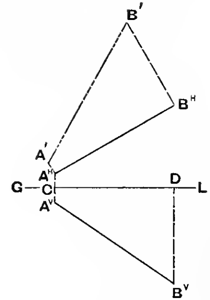


Fig. 125.

In Fig. 125, let A^HB^H be the horizontal projection of a line, and A^VC, B^VD, the vertical projection of the projection lines of its extremities. Since these last-named lines are seen in their true length in this view, we have three sides of a quadrilateral, and the fourth will be the true length of the line. To obtain this result draw A^HA' and B^HB' equal to A^VC and B^VD, respectively, and perpendicular to A^HB^H. The points A', B' will be the extremities of the revolved position of the required line, and the angle which $A'B'$ makes with A^HB^H is the angle which the given line makes with H. The plane in which this quadrilateral lies is known as the horizontal projecting plane of the line, and A^HB^H is called its horizontal trace, it being the line in which the projecting plane intersects H.

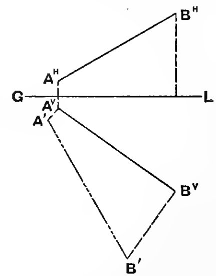


Fig. 126.

Similarly, in Fig. 126, the vertical projecting plane of the line has been used and the revolution made into V about its vertical trace A^VB^V, which is the vertical projection of the given line. The true length of the line is $A'B'$, and the angle between $A'B'$ and A^VB^V is the angle which the given line makes with V.

102. Projection of rectangular surface by auxiliary plane. An application of the foregoing principles is illustrated by Fig. 127. This represents a rectangular surface in four positions, its long sides making angles of 0°, 30°, 60° and 90° with a vertical plane, and the short sides parallel to V, and making angles of 55° with a horizontal plane. The surface might first have been drawn with its long sides horizontal, and making the required angle with the vertical plane, after which it would have been revolved about an axis perpendicular to V until the short sides made the required angle with the horizontal plane. This would have necessitated the drawing of two views, which may be avoided by the use of the following method.

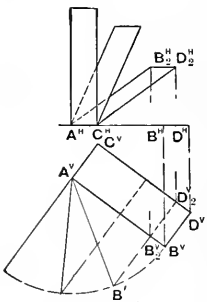


Fig. 127.

Suppose the required surface to be drawn in the plane of V as shown by $A^VB^VD^VC^V$, the short side making the angle of 55° with H. Next conceive the surface as revolved about one of the short sides as an axis until the required angles with V are obtained. A^VC^V is the vertical projection of the side which is used as an axis. Draw A^VB' equal to A^VB^V, and making an angle of 30° with it. This will represent the position of one side of the rectangle when revolved about the vertical trace of its vertical projecting plane, precisely as was done in Fig. 126. By counter revolution about this trace the line A^VB' is brought into the required position, as shown by $A^VB_2^V$. The horizontal projection of this line may now be obtained, since one of its extremities is in V and the other is determined by $B'B_2^V$ and will be laid off at B_2^H. As the short and long sides are parallel each to each, their projections will be parallel and the projections of the surface may be completed. The other positions are similarly determined.

103. Rules governing the relation of lines and surfaces to the H and V coördinate planes. If a line is perpendicular to either coördinate plane, its projection on that plane will be a point, and its other projection will be perpendicular to the ground line.

If a line is parallel to one coördinate plane and oblique to the other, its projection on the first plane will be parallel to the line itself and equal to the true length of the line. Its other projection will be parallel to the ground line and shorter than the actual line.

If a line is parallel to both coördinate planes, both of its projections will be parallel to the ground line, and their length equal to that of the actual line.

Parallel lines will be parallel in projection.

If a surface is parallel to either coördinate plane, its projection on that plane will be the true shape of the surface, and its projection on the other plane will be a line parallel to the ground line.

CHAPTER VI

ISOMETRIC AND OBLIQUE PROJECTION

104. It is frequently necessary to produce a pictorial effect in mechanical drawings while preserving the relative proportions of parts so that they may be drawn to a scale and measurements taken from them. Two methods are in general use for accomplishing this end. The first is known as Axonometric Projection, of which isometric projection is a special case; the second is Oblique Projection. Both methods employ but one plane of projection.

105. Axonometric Projection. By this system the object is so related to the plane of projection that the foreshortened length of three mutually perpendicular axes bear known relations to each other; and, by the use of special scales, lines parallel to these axes may be measured. Figs. 128 and 129 illustrate two cases of axonometric projection, the object represented being a cube. In Fig. 128 all of the edges are equally foreshortened. In Fig. 129 the axes or edges, CG and CD, are foreshortened a like amount, while CB is foreshortened twice that of the other two. The angles of these axes being known and the reduced scale due to the foreshortening being given, the making of the projection is very simple.

106. Oblique Projection. Fig. 130 is an oblique projection of the same cube. This differs from the preceding in that one face of the object is parallel to the plane of projection, and the projection is made by oblique lines instead of perpendiculars as in orthographic projection. See Art. 117, page 86.

107. Isometric Projection. Fig. 128. This is a special case of Axonometric Projection, in which the mutually perpendicular axes make equal angles with the plane of projection, and are therefore equally foreshortened.

To obtain this representation by orthographic projection, conceive the cube as resting on one face with its vertical faces making angles of 45° with V. Next suppose the cube to be revolved toward V about an axis parallel to the ground line until the diagonal of the cube drawn through C is horizontal. Then its projection will be isometric, all of the edges being of equal length and making angles of 90° or 30° with a horizontal.

108. The Isometric Axes. In Fig. 128 the lines CD, CB and CG are called isometric axes, and lines parallel to them are known as isometric lines. Planes including isometric lines are known as isometric planes. It is evident that only isometric lines may be measured, since they alone are equally foreshortened. Thus the isometric of the diagonals of the squares, AC and DB, are of unequal length, although in the original cube we know them to be equal. Likewise it is not possible to measure directly the angle between lines on an isometric drawing.

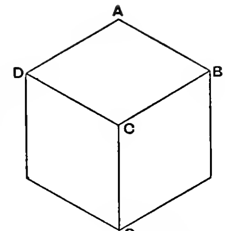


Fig. 128.

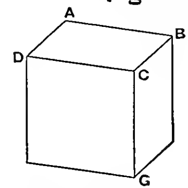


Fig. 129.

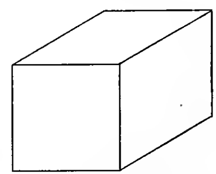


Fig. 130.

109. The Isometric Scale. If a scale should be used for making isometric projections it would be .814 of full size and constructed as follows: Fig. 133. Lay off a full size scale on a 45° line and drop verticals from these divisions on to a 30° line. The latter will be the isometric scale and may be used for measuring all lines parallel to the mutually perpendicular axes. The inconvenience due to the use of such a scale has led to the adoption of the full scale for all isometric representations; and although this results in an enlarged representation of the object, it is seldom noticeable. The term isometric drawing is usually employed to denote this enlarged representation.

110. To make the isometric drawing of a cube. Fig. 135. From the point C draw lines CB and CD at angles of 30° and equal to the required length of the edges. Draw the vertical CG of same length. As each edge of the cube is parallel to one or the other of these isometric lines, the drawing may be completed as shown. It is customary to omit the representation of invisible lines to avoid confusion. In shading an isometric drawing it is customary to draw shade lines for the division between light and dark surfaces, the direction of the light being that of the diagonal DF.

111. Non-isometric Lines. Fig. 131 is the drawing of a pentagon and Fig. 132 is an isometric drawing of it. As but one of the lines of the pentagon can be isometric, the construction is as follows: DE being chosen as an isometric line or axis, a second axis, YY, is drawn isometrically perpendicular to it. Points C and F lie on this axis, and their position is readily determined, since measurements may be made on this line. As line AB, drawn through C, is an isometric line, points A and B may be located at their proper distances to the right and left of the axis YY.

112. To make the isometric drawing of a circle. Fig. 135. Suppose the circle to be inscribed in the square D′C′G′H′. As the isometric drawing of every circle is an ellipse, it is only necessary to obtain the major and minor axes to enable the curve to be described by the method of trammels, Art. 81, page 54. These axes lie on the diagonals DG and HC, and their extremities may be determined as follows: The diagonals being non-isometric lines, the distance DK cannot be measured, and the point K must be determined by measurements parallel to the isometric axes, as PK and OK, which equal P′K′ and O′K′. Through K, an extremity of the major axis, draw KN parallel to DC; its intersection with the second diagonal at N will determine one extremity of the minor axis. Obtain M and N, and draw the ellipse by the method of trammels.

A much used approximate method is as follows: Bisect the edges of the upper face, Fig. 135, and connect these points S and T, R and V, with the vertices C and A. From their intersection, Y and Z, describe arcs RS and TV, and from centers C and A describe arcs ST and VR.

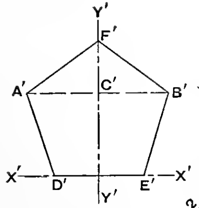


Fig. 131.

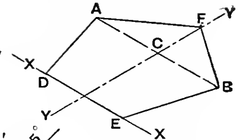


Fig. 132.

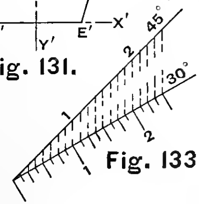


Fig. 133.

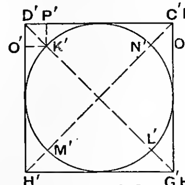


Fig. 134.

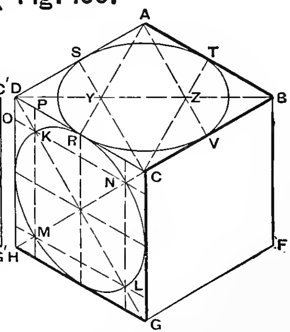


Fig. 135.

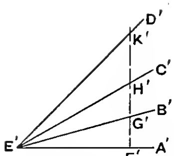


Fig. 136.

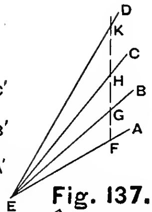


Fig. 137.

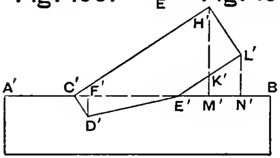


Fig. 138.

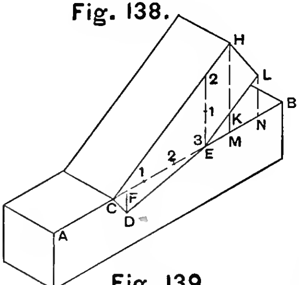


Fig. 139.

113. The measurement of angles lying in isometric planes. Suppose it is required to make the isometric drawing of the lines B′E′, C′E′ and D′E′, Fig. 136, making angles with A′E′ of 15°, 30° and 45°, respectively. Since the isometric angles cannot be measured by means of the included arcs, from any point on E′A′ draw a perpendicular, F′K′, and thus obtain two lines which will be parallel to the isometric axes and on which the angles may be measured. Having drawn EA, Fig. 137, at an angle of 30°, lay off EF equal to E′F′ and draw the vertical FK. On this lay off FG, FH and FK equal to F′G′, F′H′ and F′K′, and draw the required lines EB, EC and ED. Although the angles B′E′A′, C′E′B′ and D′E′C′ are equal, they will not be so in isometric.

114. To make an isometric drawing of an oblique timber framed into a horizontal timber. Fig. 138 illustrates a side view of the timbers, and Fig. 139 the isometric drawing. Having made the isometric of the lower piece, the cut for the oblique timber should be shown. The edges of this cut being non-isometric, they must be obtained by locating the points C, D and E, as in Fig. 138. Suppose the required pitch of the oblique timber to be two-thirds, that is, two vertical units for every three horizontal units. From C, Fig. 139, lay off on AB any three units and erect a perpendicular equal to two of the units. The point found will determine the pitch of the oblique timber. Although H′L′ is perpendicular to H′C′, Fig. 138, it is not possible to make this measurement directly on the isometric drawing, as it is a non-isometric line; but if a perpendicular, H′K′, be drawn from H′, this distance may be laid off from H, Fig. 139, thereby determining a point, K, on the lower side of the timber through which a parallel to HC may be drawn. Finally, the point L may be obtained by laying off MN equal to M′N′ and erecting a perpendicular to intersect the lower edge of the timber.

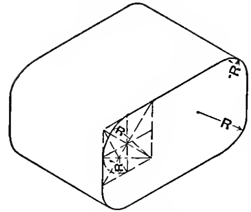


Fig. 140.

115. Suggestions for special cases. In general, make the representation of rounded surfaces, fillets and small circles by the approximate method, Art. 112, page 81, but do not sketch them free-hand. In Fig. 140, the determination of the radii R and R′ will enable the seven visible rounded corners to be easily drawn.

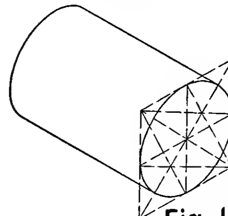


Fig. 141.

In the representation of a column or shaft, as in Fig. 141, the approximate method is sufficiently accurate, since it is not possible to measure the diameter directly, and the circumscribed square of the approximate ellipse is the same as that of the true ellipse.

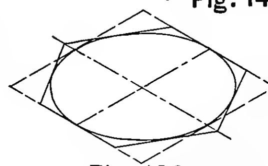


Fig. 142.

If the isometric drawing of a circle inscribed in a hexagon is desired, it will be necessary to draw the true ellipse, as in Fig. 142, otherwise the ellipse would not be tangent to the sides of the hexagon.

In representing incomplete work, such as the studs, sill and joists in Fig. 143, it is necessary that the studs be drawn of equal lengths in order that they may appear to lie in the same vertical plane. It is desirable, also, that all cut sections lie in isometric planes.

In laying out irregular curves, draw a series of ordinates perpendicular to an edge which is to be parallel to an isometric axis, as in Fig. 144. This illustrates the true section of a molding and its isometric projection.

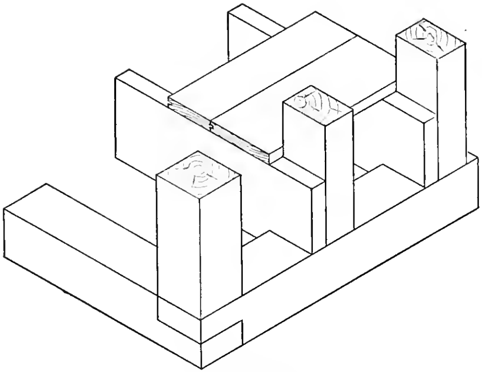


Fig. 143.

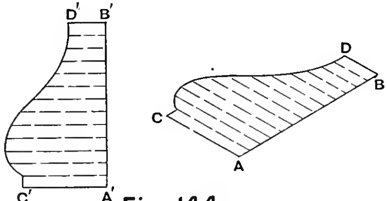


Fig. 144.

116. A useful case of axonometric projection is illustrated by Fig. 145, in which the effect produced is more nearly that of perspective. While this representation involves a little more labor than the isometric drawing, it avoids much of the distortion which characterizes the latter. The system is particularly well adapted to illustrating groups of buildings where perspective cannot be used to advantage. It is also suitable for Patent-Office drawings.

The theory upon which the representation is made is as follows: A cube is revolved into such a position that two of the axes or edges, as CG and CB, are equally inclined to the plane of projection, while the third axis, CD, is foreshortened one-half that of CG and CB. The angles which the edges make with a horizontal line are 7.2°, 41.4° and 90°. This involves the use of two special triangles, one of which must have a right angle to enable the vertical lines to be drawn. Two scales are used; a full scale for dimensions parallel to CB and CG; and a half scale for those parallel to CD. As in isometric, all dimensions must be made parallel to one of the three axes. The ellipse on the face BCGF may be described by obtaining the major and minor axes, as in the case of the isometric ellipse, and with centers on these axes describing arcs tangent to the edges. The axes of the ellipse on face DCGH do not coincide with the diagonals; but a very close approximation to them may be obtained by drawing lines KL and MN respectively perpendicular and parallel to CB. In unimportant work the extremities of these axes may be estimated by the eye; but if accuracy is required, lines KL and MN must be drawn on a square of the given size, and their intersection with the circle found as was done with the diagonals in isometric projection. A comparison of these methods is shown in the accompanying illustrations. Fig. 146 illustrates a chamfered bolt-head drawn by this method, and Fig. 147 represents the same by isometric projection.

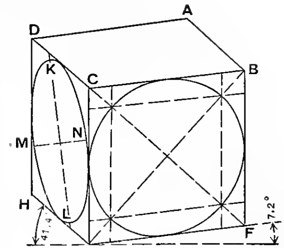


Fig. 145.

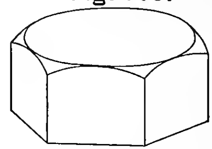


Fig. 146.

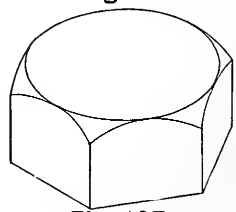


Fig. 147.

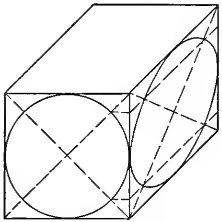


Fig. 148.

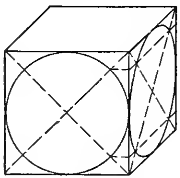


Fig. 149.

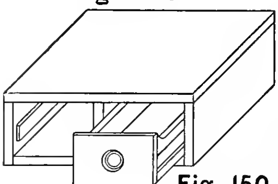


Fig. 150.

117. Oblique or Cabinet Projection. If one face of the object be parallel to the coördinate plane and the projecting lines oblique to the plane, the effect produced will be that illustrated by Fig. 148, which is the oblique projection of a cube. The face parallel to the plane is unchanged, but lines perpendicular to the plane are projected as oblique lines, the angle of which may vary. If the projecting lines make an angle of 45° with the coördinate plane, there will be no foreshortening of lines of the object which are perpendicular to the coördinate plane, as shown by the illustration, and all lines parallel to these may be measured as in isometric. In Fig. 148, the lines make angles of 45°, but any other angle might have been employed. As in isometric projection, care must be used to make all measurements parallel to the three axes, save when working on the face parallel to the coördinate plane. The oblique projection of a circle on the front face will be a circle, but on the side or top face it will be an ellipse. The axes are obtained from the front face as shown by the dotted lines, and other points similarly determined.

If the angle of the projecting lines be greater than 45°, the lines perpendicular to the front face will be foreshortened, producing an effect analogous to perspective. Fig. 149 illustrates a cube drawn by this method, in which the oblique lines are foreshortened one-half. This necessitates the use of two scales, the

45° lines being drawn half size. Circles may be obtained as in the preceding case, but it should be observed that the diagonals of the face do not coincide with the axes of the ellipse. This form of projection is sometimes distinguished from that first described by calling it CABINET PROJECTION. One advantage which oblique projection and its modifications have over isometric projection is in the representation of one face without foreshortening. It is well adapted to the representation of furniture and cabinet work.

Fig. 150 illustrates the application of these principles in the representation of a card cabinet.

CHAPTER VII

THE DEVELOPMENT OF SURFACES

118. To develop a surface. It is frequently required to illustrate the surfaces of an object in such a manner that a pattern being made from it and properly folded or rolled would exactly reproduce the object. In order to do this, an outline of each surface must be obtained as it would appear on a plane of projection parallel to it, so that there will be no foreshortening of the surface. Fig. 151 is the projection of a triangular pyramid, and it is required to produce the pattern which if properly folded would make a pyramid like the one in the drawing. This operation is called the development of the surface. Since the three slant surfaces of the pyramid are triangles, it is possible to obtain their true area by finding the length of their sides. A line is seen in its true length on a plane when it is parallel to that plane. Art. 101, page 74. Thus, the line DC may be measured on the front view because the line of the pyramid which it represents is parallel to that plane, and we know that it is parallel to the plane because the top view of the line is parallel to the ground line. Neither of the lines DA or DB may be measured from the drawing, but, since the base of the pyramid is symmetrical with respect to the axis, we know these lines or edges to be of equal length with DC. The only undetermined line of the surface BDA is AB, which may be measured on the top view, as it is parallel to H. Fig. 152 shows these lines in their proper lengths and relation to each other. Since the surfaces BDA, BDC and CDA are equal, the edges of the first having been obtained, the others may be copied from it. The bottom surface alone remains to be drawn, and this being parallel to H is already seen in its true size and shape, and may therefore be copied directly from the top view.

Having the length of one of the edges, we might have used it for a radius to describe the arc BACB, and, by spacing off the bottom edges, BA, AC and CB, have attained the same result in an easier and more practical manner.

The surface BDA may be obtained in another manner as follows: Since the true size of a surface is always to be found on a plane to which it is parallel, we have only to draw a plane, GC, parallel to this surface and project on to it, in order to obtain the required surface. Art. 98, page 70. This new plane is perpendicular to the front plane, but not to the other planes. GC is its V trace, and, if the plane be revolved to coincide with V, the true shape of the surface BDA will be obtained, as at B′D′A′.

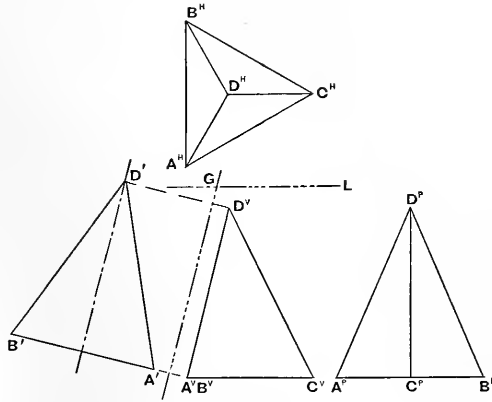


Fig. 151.

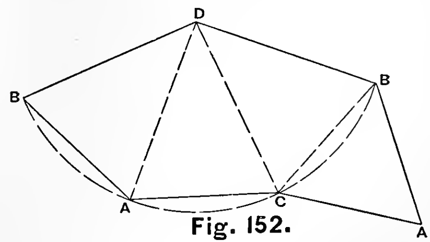


Fig. 152.

119. To develop a pyramid when cut by a plane. Fig. 153 illustrates a rectangular pyramid which it is required to develop after removing such portion of the top as lies above the plane FGHK. The operations are as follows: First, obtain the three views of the object before having been cut by the plane: Second, determine the projections of the cut surface FGHK, thus showing the pyramid as it would appear with the top portion removed: Third, obtain the true shape of the cut surface: Fourth, obtain the development of the entire pyramid, disregarding the cutting plane: Finally, determine that portion of the developed surface not removed by the cutting plane, to which must be added the section cut by the plane. As the first three operations are sufficiently well indicated by the drawing, we will consider the development only. None of the inclined edges being parallel to either of the planes of projection, it is necessary to revolve one of these edges until it shall become parallel to a plane, when it will be possible to measure it on that plane. Let AE be revolved parallel to V. Since this is a revolution about an axis perpendicular to H, the top view of the line will be changed in position, but not in length, and will be shown by $A'E^{H}$. The front view will then be $A''E^{V}$, the true length of the line. Since all the inclined edges are of the same length, with radius equal to $A''E^{V}$, describe an arc on which the chords AB, BC, CD and DA may be drawn, as shown in Fig. 154, their lengths being obtained from the top view. The development of the base is obtained directly from the top view.

Finally, having obtained the development of the entire pyramid, it is required to find the length of the edges when cut off, and the section made by the cutting plane. Since we have found it possible to obtain the true length of the inclined edges, we may in like manner find that portion of them included between the base and cutting plane. $A''E^{V}$ may be considered as the revolved position of any one of these lines, and, since the heights remain unchanged by revolving about an axis perpendicular to H, the true length of AK and DH will be A″N, and the length of BF and CG will be A″O. — It is generally better to lay off the distances from the apex instead of the base. — The cut surface F′G′H′K′ having been found by projecting it on to a plane to which it is parallel, it may now be copied as a part of the developed surface. In like manner the base should be drawn in connection with the development so that there may be a less number of edges to unite when the pattern is folded to form the required object.

The edges KF, FG, GH and HK of the developed slant faces should be equal to the edges of the surface cut by the plane, and projected on the auxiliary plane at K′F′, F′G′, G′H′ and H′K′. As these lines have been obtained independently, the comparison of their lengths will serve as an excellent test of the accuracy of the drawing.

The division lines between the faces of the prism as shown in the development are represented by dashes, because no line really exists until the surface has been folded.

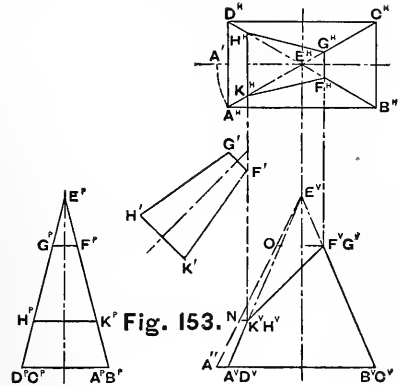


Fig. 153.

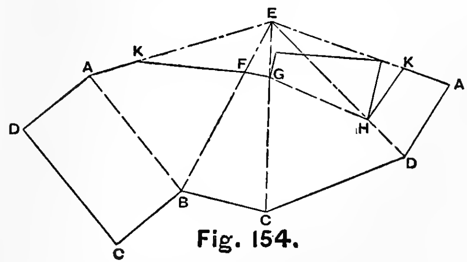


Fig. 154.

120. The development of surfaces of revolution. A surface may be conceived to be generated by the motion of a line. Such a line is called the GENERATRIX, and the different positions of the generatrix are called ELEMENTS. The line which may direct or govern the motion of the generatrix is called the DIRECTRIX.

Suppose a right line to have a motion about another right line known as the AXIS, from which it is always equidistant and to which it is parallel; then will the successive positions of this generating line constitute the surface of a cylinder. If one end of the generatrix were fixed to the axis and the other free to describe a circular path, the surface generated would be a cone. These surfaces are called SURFACES OF REVOLUTION, and may be regarded as consisting of an infinite number of lines or elements which in the first case are parallel to, and in the second case intersect, the axis. This conception of the cylinder and cone is necessary to the study of the development and intersection of surfaces.

121. The development of a cylinder. Three views of a cylinder are represented by Fig. 155, and from these it is required to develop the cylinder. Assume a number of elements, and for convenience they should be equidistant. These may be employed to obtain other views, sections and development in precisely the same manner as though they were the edges of a prism, save that, instead of connecting their extremities by right lines, a curve must be drawn through them. It will be observed that the revolved section is an ellipse, the major axis of which is equal to A^VC^V, and the minor axis equal to the diameter of the cylinder. From these data the curve might be drawn, but it is better to use this method as a test for the ellipse after having obtained the curve by means of the elements.

To develop the cylinder, Fig. 156, obtain the length of the base DD by determining the circumference of a circle having a diameter equal to that of the cylinder, and, having laid it off, divide it into as many parts as there are elements. The perpendiculars to the base drawn through these points will be the required elements, and their lengths may be obtained directly from the front view, since they are parallel to the vertical plane. A free-hand curve should be carefully pencilled through these points and afterwards neatly inked by the aid of compasses and curves.

It is unnecessary to add the base and cut section to the development, as was done in the case of pyramids, since there will be but one point of contact between these surfaces.

If the base of the cylinder had been an ellipse instead of a circle, it would have been necessary to obtain the development by spacing the length by the dividers or spacers. In such cases do not use a unit greater than 10° or 15°, and obtain the entire length of the base before locating the elements.

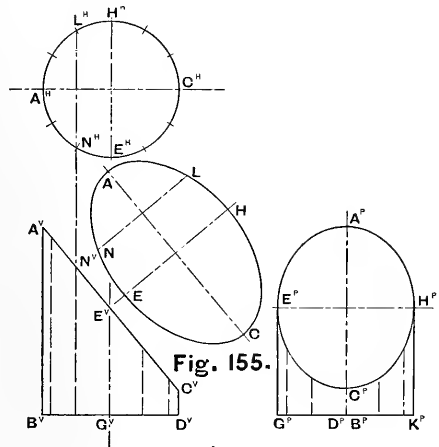


Fig. 155.

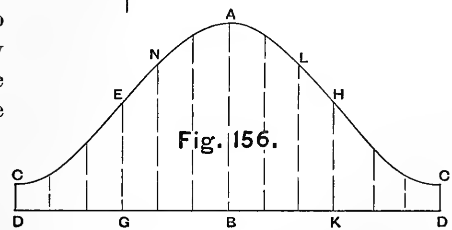


Fig. 156.

122. To develop a cone. Fig. 157 illustrates a cone cut by vertical and oblique planes. Draw the requisite number of elements by dividing the circle of the base in the top view and projecting these points into the front view, as at B^V, E^V, G^V, H^V, etc. These are then connected with the vertex A^V. It is now possible to obtain the top view of the section made by the cutting plane XY in the same manner as though the cone were a pyramid having these elements for its edges. A more accurate method is as follows: Through the front view of any point N^V, lying on the surface of the cone and also in the cutting plane, draw a horizontal line which will represent the front view of a circle of the cone, and be shown on the top view by the fine dotted circle drawn through N^H. Since the assumed point must lie on this circle as well as on its vertical projecting line, it must lie at their intersection N^H. Of course it is necessary to draw only short arcs to intersect the projecting lines. In like manner any number of points may be obtained and through them the curve described. If the elements of the cone have already been drawn, as would be necessary if it is to be developed, the points assumed had better be at the intersection of the cutting plane and the elements. This curve being an ellipse may be obtained by finding the major and minor axes and on these constructing the curve; but this method should be used only as a test of the ellipse until the problem is thoroughly understood.

The side view illustrates the true section made by the vertical cutting plane, since it is parallel to the profile plane of projection. The top view of this section is a straight line, but it is just as necessary to find the points by projection as in the case of the ellipse, since their distances on either side of the axis must be known in order to obtain the side view. Thus the top view of point O must lie in the vertical projecting line drawn through O^V, and also in a circle the diameter of which is equal to VM, therefore at O^H. From this the side view of the point may be obtained, its distance from the axis being equal to $Q^H O^H$.

Proceed with the development as in the case of the pyramid, observing that because all the elements are of equal length the development of the base will be a circular arc of length equal to the circumference of base, and radius equal to the length of an element. Divide the arc into as many parts as there are elements, and proceed to draw the elements, after which the true length of the cut portion may be found as follows: Having drawn any element, as AG, Fig. 158, it is required to find the points P and R. Since the line AG upon which they lie is not seen in its true length in any of the views, it must be revolved parallel to one of the planes. $A^V B^V$ will represent its position when revolved parallel to the front plane. The point P^V will be seen at P′, and R^V at R′, and the lengths $A^V P'$ and $P'R'$ may be laid off on the line AG of the developed surface. In a similar manner other points are found, AB serving as the revolved position for all of the elements, since they are of equal length.

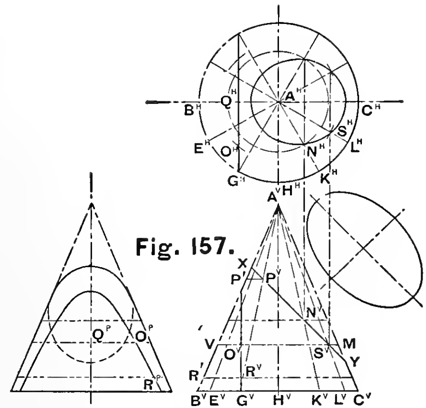


Fig. 157.

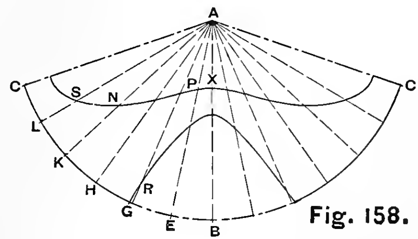


Fig. 158.

CHAPTER VIII

THE INTERSECTION OF SURFACES

123. The intersection of cylinders. Three views of two intersecting cylinders are shown in Fig. 159. It is required to determine the curve of their intersection and to develop the cylinders. By assuming elements of one cylinder and finding their intersection with the second cylinder, points in the desired curve may be obtained. Assume an element of the small cylinder, the side view of which is A^PB^P, and determine the H and V projections. The H projection, A^HB^H, is seen to pierce the large cylinder at the point B^H, which is the H projection of an element of the large cylinder. The V projection of these intersecting elements will be C^VB^V and A^VB^V. Their point of intersection will be common to both cylinders and, therefore, a point in the required curve of intersection. In like manner obtain a sufficient number of points to determine the curve. For convenience in the development of the small cylinder, it is desirable to have its elements equidistant.

In all problems relating to the development of surfaces it is important to determine those points of the curve, known as limiting points, at which the direction of curvature changes or points of tangency occur. These define the character of the curve and enable it to be drawn by the finding of a less number of points. 1^V, 2^V, 3^V, 4^V, 5^V and 6^V are limiting points of this curve.

In developing the small cylinder, Fig. 160, open it on the element L4, which will make it symmetrical with respect to the center. The method of developing does not differ from that of the cylinder in Art. 121, page 92, and the length of the elements may be taken from the front or top views.

The development of the large cylinder, Fig. 161, will be a rectangle pierced by a hole which is symmetrical with respect to a horizontal center line only. Having obtained the development of the cylinder by opening it on GH, the element DE will be drawn in the center of the surface, and from this the other elements may be determined. On DE lay off the points 3 and 5 equidistant from the center line and equal to that portion of the element cut by the small cylinder, as shown on the front view. The distance between any of these elements, as DE and CN, may be found by measuring the circular arc $D^H2^HC^H$, which should then be laid off to the left of ED, and through this point CN may be drawn. Having determined the position of an element, immediately lay off the amount cut out by the second cylinder as seen on the front view.

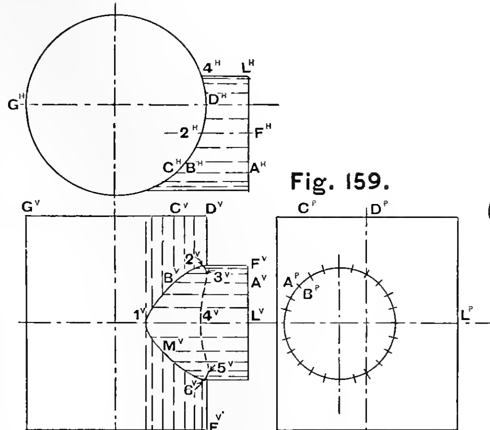


Fig. 159.

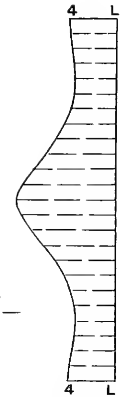


Fig. 160.

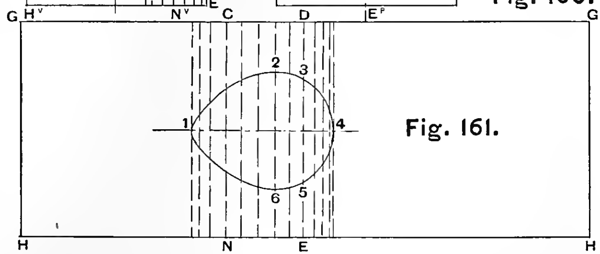


Fig. 161.

124. The use of auxiliary planes. In determining the curve of intersection between two surfaces it is customary to use a system of planes which shall cut either circles or straight lines from these surfaces. The intersection of these lines will be points of the curve. Let it be required to find the intersection of cylinders 1 and 2, Fig. 162. If we imagine them cut by a plane, VW, parallel to the axes, the appearance will be as in Fig. 164. Two elements will have been cut from each cylinder, and, since they lie in the same plane, their points of intersection will be common to both cylinders, and therefore in the required curve. Again, if we should employ a plane tangent to cylinder 2, it would cut that cylinder in a single element, and the other in two elements, as in Fig. 163. Their intersection at points 7 and 8 would be the limiting points of the curve and ordinarily the first to be determined.

To obtain the elements cut by these planes proceed as follows: Revolve the bases of the cylinders, as in Arts. 99 and 100, pages 72 and 73, and assume a cutting plane VW, shown on the top view. On the revolved bases lay off LM and NO equal to the distance of the cutting plane from the plane of the axes. Through the points M and O draw parallels to the bases, and these will indicate the amount cut from each cylinder. Next revolve the points B, D, G and K, back into the bases and through them draw the elements. Their intersection at 3, 4, 5 and 6 will be the four points determined by the plane VW. In like manner the points 7 and 8 may be found by using a plane tangent to the small cylinder. The points in the top view are obtained by projecting them from the front view on to the plane in which they lie. Thus the point 7 was found by means of the cutting plane XY, and its top view, 7^H, must lie on the line XY which is the horizontal trace of the plane. In the case illustrated by Fig. 162, it would be well to use three cutting planes between VW and XY. This will determine a sufficient number of points to sketch the curve accurately.

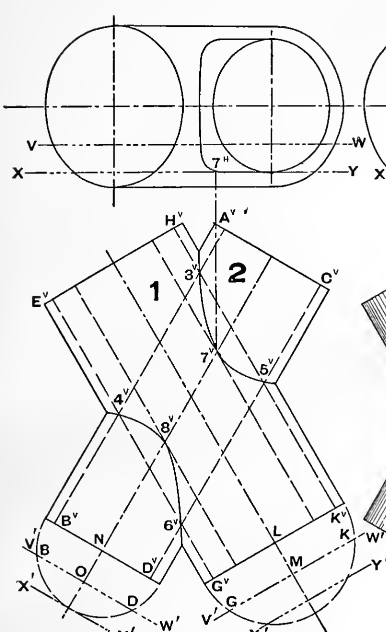


Fig. 162.

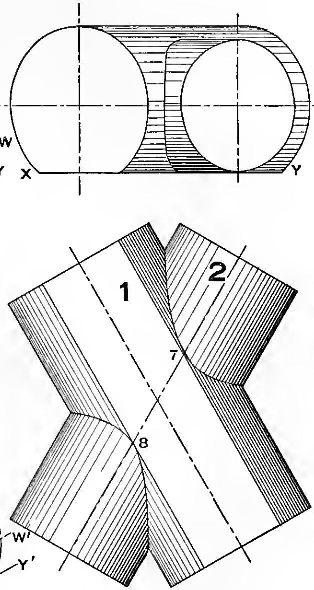


Fig. 163.

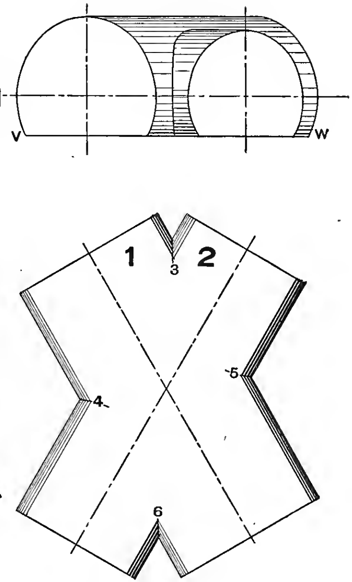


Fig. 164.

L of C

125. The intersection of an oblique and a vertical cylinder. Either of the two preceding methods may be employed in the solution of this problem. The side view not being available for spacing the elements, the following method may be pursued: Draw any element of the small cylinder, as AB, and let XY be a circle of the cylinder made by a cutting plane perpendicular to the axis. If this circle be revolved about XY parallel to the vertical plane, the point C, which is a point of both the element and the circle, will be revolved to C′, and CC′ will be the distance between the center line and the element in the top view, the latter piercing an element of the large cylinder at A^H, A^V. Since there is a second element, ED, of the small cylinder lying in the same vertical plane as AB, it will intersect the same element of the large cylinder at the point D^V. Thus find any number of points. It will be convenient to have the elements of the small cylinder spaced equally, and this may be done by spacing them on the revolved position of the circle XY, and through these points drawing the elements. In developing the small cylinder it will be found necessary to assume some line of the surface which on being developed will be a straight line, for in this problem the development of the ends of the cylinder will be curves. Such a line will lie in a plane perpendicular to the axis, and the section of the cylinder made by this plane is called a Right Section. The line XY fulfills this condition and may be used as a base line for

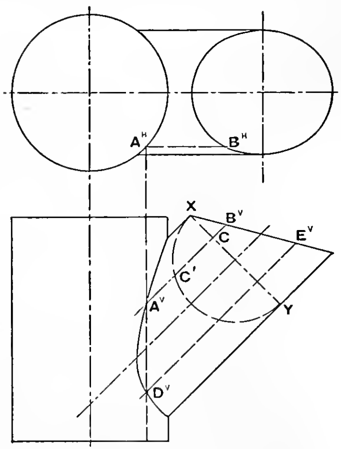


Fig. 165.

measuring the length of the elements in the development, and on its revolved position, XC′Y, the distances between the elements may be measured.

126. The intersection of prisms. In the intersection of two prisms, or two pyramids, or a prism and a pyramid, the line of penetration will be a broken line instead of a curve. As auxiliary cutting planes cannot be used to advantage, it is necessary to find the points of intersection of each edge of the one with a face of the other. To avoid confusion the points should be obtained consecutively.

Fig. 166 illustrates two intersecting hexagonal prisms. Having drawn the H and V projections, determine the points in the following manner: The edge 1 will intersect the upper face of the vertical prism at the point K. K^H is obtained directly from the V projection, K^V. Next obtain the point of intersection of the upper edge of face AB with face 1 2. The plane of the upper face of the vertical prism will cut the face 1 2 in the line KS, and, since the upper edge of face AB lies in this plane, its intersection with KS, at L, will be the desired point. The next point of intersection, M, of the edge 2 with face AB is apparent from the top view, M^H being the H projection of this point. The point N, which is the intersection of edge 3 with face AB, is similarly obtained. The next point to be determined is O, the intersection of edge A with the face 3 4. This completes one-half the intersection.

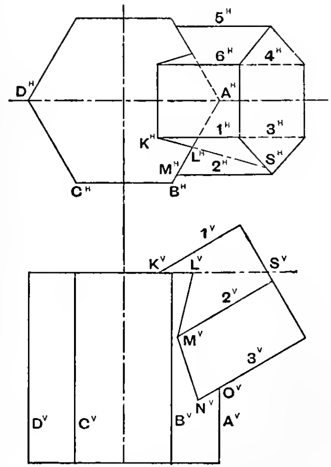


Fig. 166.

CHAPTER IX

SPIRALS, HELICES, SCREW-THREADS AND BOLT-HEADS

127. The Spiral is a curve generated by a point moving in a plane about a center from which its distance is continually increasing.

Imagine a right line, AB, Fig. 167, free to revolve in the plane of the paper about one of its extremities, A, as an axis. Also, conceive a point free to move on this line. Three classes of lines may now be derived in the following manner: If the line be stationary and the point moves, a straight line will be generated; If the point be stationary and the line revolves, a circle will be generated; If both point and line move, a spiral will be generated. By varying the relative motion of point and line the character of the curve will be changed and the various classes of spirals described.

128. The Equable Spiral. Fig. 167. If the motion of the line and the point be uniform the equable spiral will be generated. This is also called the SPIRAL OF ARCHIMEDES. The line AB is called the RADIUS VECTOR, and the radial distance traversed by the point during one revolution of the radius vector is called the PITCH. Twelve successive positions of the radius vector are shown by AC, AD, AE, etc. The distance of the point from the center being increased by one-twelfth of the pitch, AC, for each one-twelfth of a revolution of the radius vector. In practice, determine at least twenty-four points, and lightly sketch the curve freehand. This is the curve used in the cam for uniform motion. Instruction for the inking of spirals is given in Art. 15, page 12.

129. The Equiangular or Logarithmic Spiral. The construction of this curve is based on the principle that any radius vector, as AD, Fig. 168, which bisects the angle between two other radii, as AC and AB, is a mean proportional between them; that is, $\overline{AD}^2 = AC \times AB$. This spiral is called equiangular because the angle between any radius vector and the tangent to the curve at its extremity is constant.

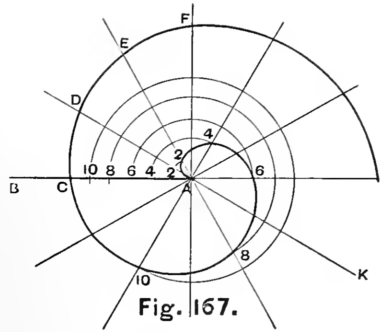


Fig. 167.

If B and C are points in the spiral and the ratio of AC to AB be given, the intermediate point D may be obtained by describing a semicircle on BC as a diameter and erecting a perpendicular at A. Its intersection with the semicircle at D will determine the required point, and AD will be a mean proportional between AC and AB. Other points on the curve lying on the diameters BC and DF may be obtained by intersecting these diameters with the perpendiculars CL, LM, MN, etc. Again, having points C and D, bisect DAC and determine a mean proportional between AC and AD. This may be done by laying off AF equal to AC and determining the mean proportional AK as before. Then lay off AE equal to AK. Other points on the diameters ER and ST may be obtained by perpendiculars as indicated.

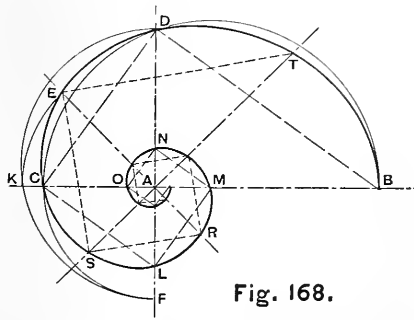


Fig. 168.

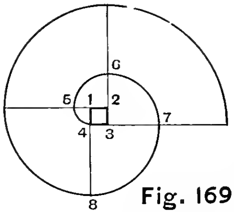


Fig. 169.

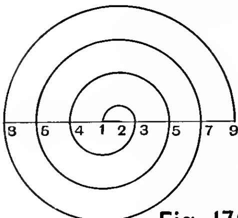


Fig. 170.

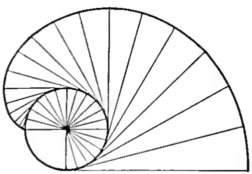


Fig. 171.

130. Involutes. This class of spirals may be generated by unwinding a perfectly flexible but inextensible cord from a polygon of any number of sides, the names of the involutes being derived from the polygons which determine their form. The curves consist of circular arcs having the vertices of the polygons for centers, and radii increasing by an amount equal to the length of the sides. Fig. 169 is the involute of a square, drawn by describing the quadrant 4–5 from center 1 with radius 1–4; then the quadrant 5–6 from center 2 with radius 5–2, and so on until the desired length of curve shall have been drawn. If two opposite sides of the square become infinitely small, the result will be a right line, the involute of which is shown in Fig. 170. Again, if the number of sides of the polygon become infinite, we shall obtain the involute of a circle, as in Fig. 171. The accuracy of this curve will depend on the number of points in the circle; in this case there are twenty-four.

131. The Helix. If the line on which the generating point is supposed to move be made to revolve about an axis with which it makes an angle of less than 90°, thus generating a cylinder or cone, a class of curves known as helices will be described. If the line AB, Fig. 172, be parallel to the axis about which it revolves, and the generating point moves on this line, three classes of lines may be described, as in the case of spirals. A circle will be generated when the point is stationary and the line revolves; a right line will be generated when the line is stationary and the point moves on the line; and since the circle and right line do not lie in the same plane, the result will be a helix when these motions take place simultaneously. The distance traversed by the generating point on the line AB during one revolution is called the PITCH. The motion of line and point being in general uniform, the curve is described as follows: Having assumed any desired number of equidistant positions of the generating element AB, as 1, 2, 3, 4, etc., the pitch should be divided into the same number of parts, as shown by the horizontal lines drawn through 1′, 2′, 3′, 4′, etc. When the element shall have moved through one of its divisions to the position 1, which in this case is one-twelfth of a revolution, the generating point will have moved on the generating line through one-twelfth of the pitch, and the point K will be determined. Also, when the element has made one-quarter of a revolution, the point will have traversed one-quarter of the pitch and be at C. As the rate of curvature is most rapid at the points of tangency, A, D and B, it is desirable to obtain a greater number of points by subdivision, as shown in the figure. In Fig. 172 the helix is right-handed, and single, but if a double helix is required, draw a parallel curve beginning at the point 6′, opposite D.

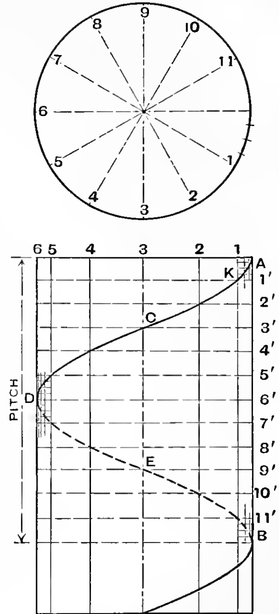


Fig. 172.

A conical helix is generated by the motion of a point on an element of a cone. The pitch is measured parallel to the axis, as in the preceding case.

132. Screw-Threads. In order to make the drawing of a screw-thread, it is necessary to know the diameter, pitch and section of the thread.

If the thread to be drawn has a V section, as in Fig. 174, and the diameter and pitch are given, begin by drawing the section of the screw as shown in dotted lines. Next, construct a templet as follows: Draw on a rather light piece of cardboard two helices of the same pitch as the required thread, one being of a diameter equal to the outside of the thread and the other equal to the diameter at the root of the thread. These may be drawn separately, or together, as in Fig. 173, and are to be carefully cut out so as to be used as a pattern in pencilling the curves. Having drawn the helices, mark the points of tangency B, G, and A, F, as well as the centers L and K, in the manner indicated. Also repeat these marks on the opposite side after cutting out. This will enable the templet to be used either side up, and be readily set to the drawing. Observe that the curve is not to be cut off abruptly at its termination, but continued a little beyond, so that in tracing the outline the pencil-point may not injure the extreme point of the templet curve. This may now be used for the drawing of all the helices on this screw, as from A to F, B to G, C to H, etc., Fig. 174.

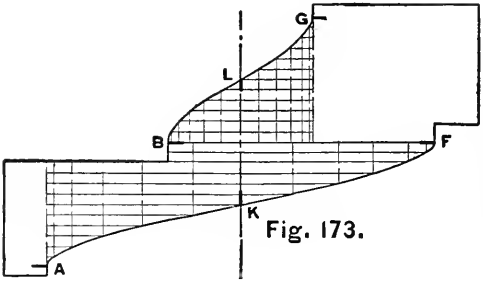


Fig. 173.

If the pitch is small in proportion to the diameter, the drawing of the screw may now be considered finished; but the contour line does not coincide with that of the section of the thread, and in order to illustrate correctly the projection of a V thread we must consider the character of the surface and apply a correction to the drawing. The surface which is being drawn is a helicoid, and is generated by the motion of a line, AB, which is made to revolve about the axis of the screw and at the same time move in the direction of the axis. This will generate the upper half of the surface of the screw. Every point of the line will describe a helix, the diameters differing, but the pitch remaining constant. The helices generated by the extremities of the line have already been drawn and the curves 1 2 3 and 4 5 6, described by two other points, 1 and 4, are shown by the fine dotted lines. The curved line M5 2N, drawn tangent to these helices, will be the visible outline of the surface instead of the dotted line which is concealed. As the labor of describing these helices would be great, it is customary to draw the outlines of the screw as follows: Having described the helices AF, BG, etc., reverse the templet and draw a small portion of the continuation of the helix on the opposite side of the screw, as at BP and CO. Then, draw the line MN tangent to the two helices, and in the other direction the tangent OP, a part of which is invisible.

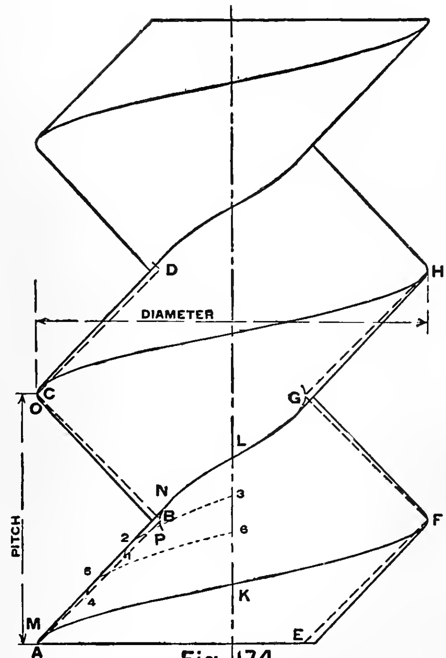


Fig. 174.

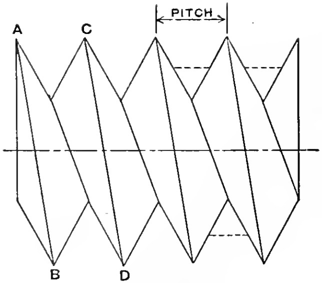


Fig. 175.

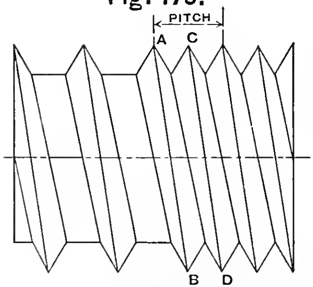
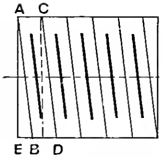
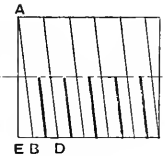


Fig. 176.

133. Conventional V Threads. In order to facilitate the drawing of screw-threads it is customary to omit the drawing of the helix, substituting therefor a right line, as in Fig. 175. In most cases, however, even this would involve too much labor as well as complication on the drawing, and the V's are likewise omitted, the representation shown in Fig. 177 being adopted. When this is done, no care is taken to make the screw of the required pitch, and the spaces between the fine lines which represent the outer helices are estimated by the eye, as in section lining. But it is imperative for the proper representation of a single thread, that the point C be over the middle of the space BD. Having drawn the line AB, make the space AC double that of EB and draw parallels. Afterwards, draw the heavy lines to represent the root of the thread. As it is difficult to make these of equal length without the aid of special lines, the method illustrated in Fig. 178 is frequently used.

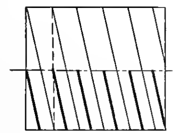
134. The Double Thread. As the use and character of the double thread are generally misunderstood, Figs. 175 and 176 have been drawn to explain this problem more clearly. Fig. 175 illustrates a screw, the diameter and pitch of which are supposed to have been given; but, as the pitch is excessive for a screw of this diameter, the diameter at the root of the thread is small, and the screw proportionally weak. The only way to strengthen the thread at this point without changing the angle of the V's is by partly filling the V's at the root, as shown by the dotted line in Fig. 175 and the left-hand portion of the complete screw in Fig. 176, thus increasing the diameter at the root. While overcoming one weakness we have introduced a second by lessening the section of the thread, so that with a nut of a given length the tendency of the thread to be stripped from the body is doubled. This last difficulty may be overcome by supposing an intermediate thread wound between the present threads, as shown by the right-hand portion of Fig. 176, which is a representation of a double thread having the same diameter and pitch as the single thread of Fig. 175, but of increased strength. It must be noted that the threads indicated by AB and CD, of Fig. 176, are entirely independent of each other and that the point C of one is diametrically opposite a point B in the parallel thread. This must be carefully observed in the practical representation of a double thread, as shown in Fig. 179. Fig. 180 represents a left-hand single thread.





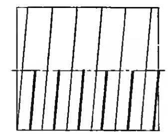
RIGHT HAND SINGLE THREADS

Fig. 177. Fig. 178.



DOUBLE THREAD

Fig. 179.



LEFT HAND THREAD

Fig. 180.

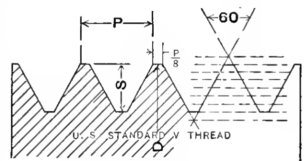


Fig. 181.

$P = 0.24\sqrt{D + 0.625} - 0.175.$

$S = 0.65\ P.$

135. U.S. Standard V Threads. The form of thread commonly used is that of the U.S. Standard, also known as the Franklin Institute Standard and illustrated by Fig. 181.

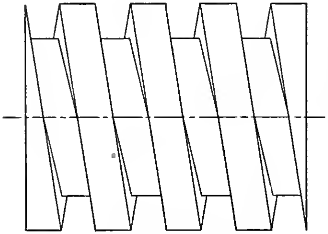


Fig. 182.

Although the pitch in a single-threaded screw is the distance between consecutive threads, the term is often applied to the number of threads per inch. Thus a screw having eight threads to an inch is frequently spoken of as 8 pitch. This is obviously wrong, but leads to no confusion, since the pitch and the number of threads per inch are reciprocals of each other. The flattening of the thread, as indicated in Fig. 181, is for the purpose of preventing injury by the bruising of the otherwise sharp V.

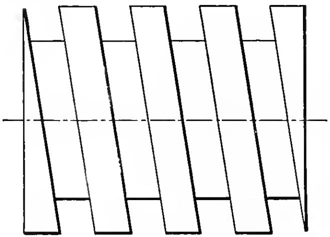


Fig. 183.

136. Square Threads. Fig. 184 represents a thread of square section. The size of this square is equal to one-half the pitch in a single thread, and one-quarter of the pitch in a double thread. The construction is similar to that described for the V thread. The outline of the section is drawn first, and a templet is prepared for the inner and outer helices. The threads are drawn as in Fig. 184 for a single right-hand thread, and as in Fig. 185 for a nut in section. Figs. 186 and 187 illustrate a double square thread and nut.

The conventional representation of the square thread is similar to that of the V thread, the right line being substituted for the helix. Fig. 182 is a correct drawing of the conventional square thread, and Fig. 183 a modification more commonly used.

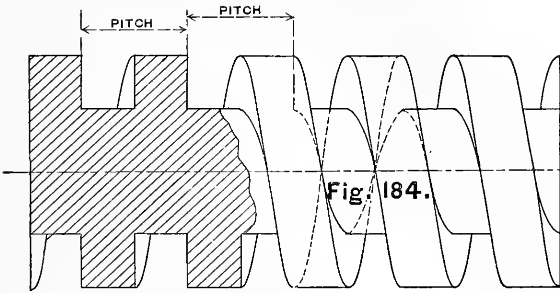


Fig. 184.

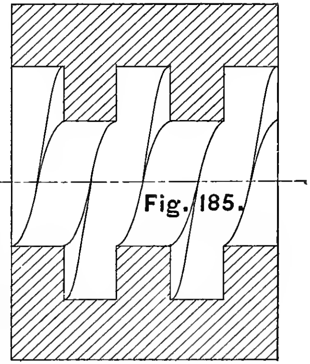


Fig. 185.

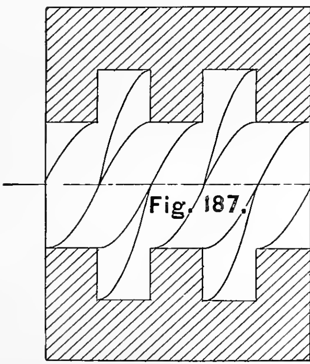


Fig. 187.

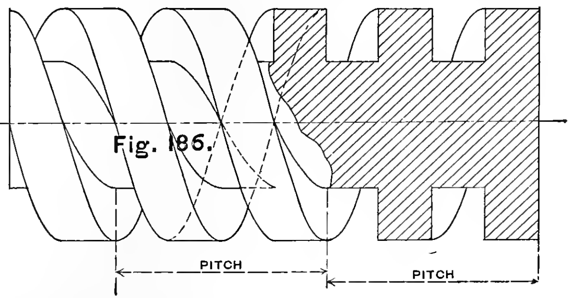
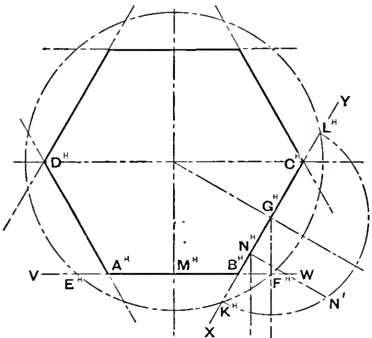


Fig. 186.

137. Sphere and Cutting Planes. If a sphere be cut by six planes equidistant from and parallel to a vertical axis, a representation will be obtained similar to that of a hexagonal bolt-head or nut. Since all planes intersect a sphere in circles, these intersections will appear as circles, ellipses or right lines. Such a representation is shown by Fig. 188, VW and XY being two of the cutting planes. The plane VW intersects the sphere in a circle the diameter of which is E^HF^H, and shown on the front plane by $E^VA^VB^VF^V$. The plane XY intersects the sphere in a circle of equal diameter, being at the same distance from the axis; but, as this plane is inclined to the V plane of projection, the circle will appear as an ellipse having a major axis, G^VH^V, equal to the diameter of the circle, and the minor axis, K^VL^V, equal to the foreshortened diameter projected from the top view. The planes VW and XY intersect in the line B^VO^V, thus cutting off a portion of the circle and ellipse. Similarly the remaining curve of intersection, D^VA^V,



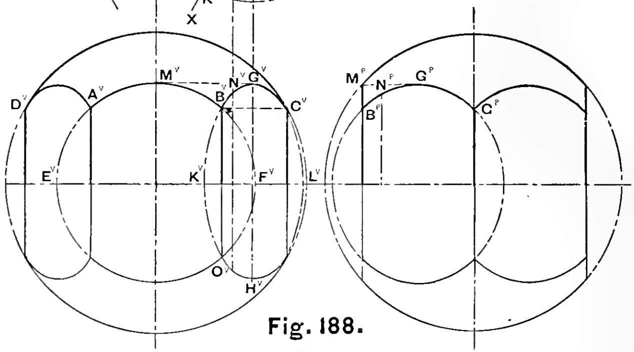


Fig. 188.

may be found and the side view likewise determined. Points in the curve may also be obtained by revolving the plane XY parallel to H, thus obtaining the height of any point, as N, which may then be projected into the other views.

Since the upper half of the two vertical projections of Fig. 188 may be regarded as a true representation of a hexagonal bolt-head, save as to proportion, it is important to consider the salient points of these projections.

First: Three faces of the head are seen when it is shown "across corners," as in the front view, and one of these is double the width of the other two.

Second: The circular arc $A^V M^V B^V$ is concentric with the circle of the sphere, and the points A^V and B^V are determined from the height of C^V, the intersection of the two planes or faces of the head and the great circle of the sphere.

Third: The major axes of the ellipses and the diameters of the circles made by the cutting planes are equal, hence points G^V and M^V are at the same height.

Fourth: Two equal faces are seen when the head is shown "across flats," as in the side view.

Fifth: The points M^P and G^P are of equal height, as in the front view, and B^P must be obtained by projection from the front view or in the same manner as C^V.

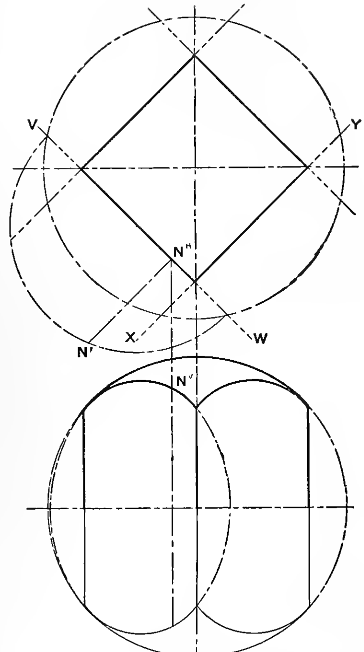


Fig. 189.

Fig. 189 is a sphere cut by four planes and similar to a square-headed bolt or nut.

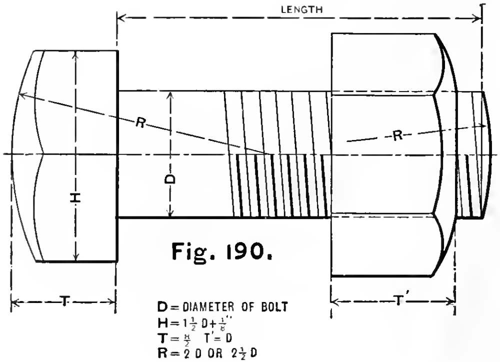


Fig. 190.

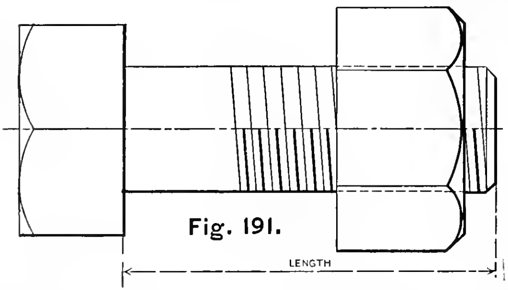


Fig. 191.

138. U.S. Standard Hexagonal Bolt-head and Nut. Two types of head and nut are illustrated, the rounded or spherical, Fig. 190, and the chamfered or conical, Fig. 191. Three dimensions are fixed by this standard: First, the distance across flats or short diameter, commonly indicated by H, and equal to one and one-half times the diameter of the bolt plus one-eighth of an inch: Second, the thickness of the head, which is equal to one-half the short diameter, or $\frac{H}{2}$: Third, the thickness of the nut, which is equal to the diameter of the bolt.

Suppose it is required to draw a rounded head "across corners," as shown in Fig. 192. Having drawn the center line, underside of head and diameter of bolt, as indicated by lines 1, 2, 3 and 4, figure the short diameter of the hexagon or distance "across flats" according to the proportion given. Lay off EF equal to one-half this amount and draw the perpendicular FG and the 30° line EG, then will the triangle EFG represent one-twelfth of the top view of the head, and EG will be equal to one-half the long diameter required. Lay off this distance on either side of E, thus determining lines 8 and 9. Draw 10 and 11, remembering that these lines equally divide the spaces between 1 and 8, and 1 and 9, which spaces also equal twice FG. Next determine the thickness of head, and with a radius * equal to twice the diameter of the bolt, describe arc 12, which determines points D, C, A and B. From the same center describe arc 14. Arcs 15 and 16 should be drawn as circular arcs, their height being determined from 14.

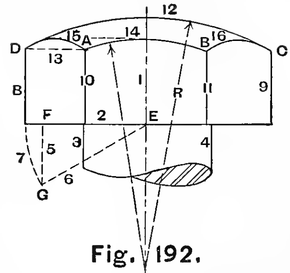


Fig. 192.

If it is required to represent a bolt-head "across flats," as in Fig. 193, proceed as before, determining the short diameter and drawing 5 and 6. Next lay off the thickness of head and describe arc 7; this will determine E and the height of arcs 12 and 13. Although these arcs are elliptical in theory, they should always be described as circular. To determine the point A, find the long diameter and obtain line 10; its intersection with 7 will be at D, equal in height to A, B and C. Finally draw arcs 12 and 13.

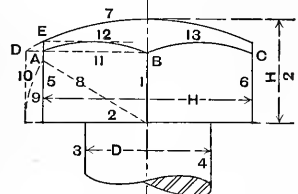


Fig. 193.

Fig. 194 illustrates a rounded nut "across corners," the order for the drawing of the lines being indicated by the figures. The thickness of a nut is equal to the diameter of the bolt. As the nut is pierced by a hole, the top will appear flat and the arc 13 must be drawn from K, but with radius equal to 2D.

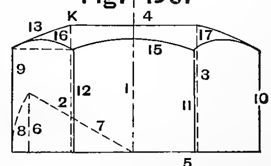


Fig. 194.

* There is no standard for this radius, but 2D is recommended as being a convenient radius for draftsmen.

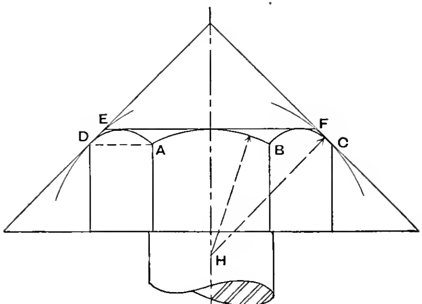


Fig. 195.

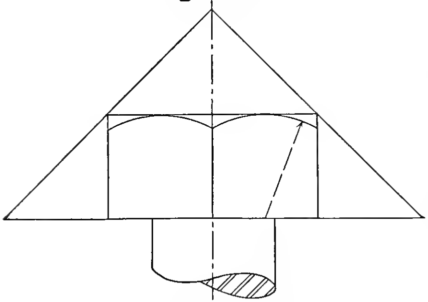


Fig. 196.

139. Chamfered Head and Nut. If we substitute a cone for the sphere in Fig. 188, page 112, cutting it by the six vertical planes parallel to the axis, and then pass a seventh plane perpendicular to the axis and tangent to the curves of intersection, a representation will be obtained similar to that of Figs. 195 and 196. This is called a chamfered head, and the curves of intersection are hyperbolas; but, since these curves approximate circular arcs, the following concise method may be employed. If it is required to draw a chamfered head or nut "across corners," construct a hexagonal prism of dimensions required for a standard bolt. From the center line, with a radius equal to diameter of bolt, describe the arc AB tangent to top of head, and from the same center draw arcs DE and CF, points D and C being determined by A and B. Finally draw arcs DA and BC tangent to EF. The method for drawing the chamfered head "across flats" is apparent from Fig. 196. A bolt with chamfered head and nut is illustrated by Fig. 191.

For the further consideration of this subject the student is referred to the chapter on Bolts and Screws in "Machine Drawing" of this series.

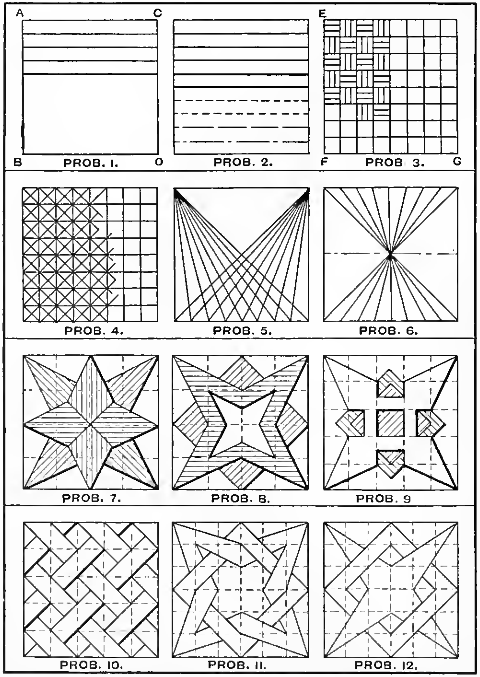
CHAPTER X

PROBLEMS

140. General instructions for performing the problems. Size of paper $11'' \times 15''$. Art. 26, page 20. Space within the margin lines $10'' \times 14.''$ Art. 28, page 21. The space required for performing the problems is given whenever the graphic statement is not made. All dimensions are from the margin lines, and in the graphic statements these are shown.

Read the articles noted before beginning to solve a problem. Note every detail of the graphic statement and observe the method required for the solution of the problem. It is not intended that the order of the problems shall necessarily be followed or that all of the problems shall be performed, but that a judicious selection be made to meet the varying needs of students. Additional problems of a more practical character may be found among the miscellaneous problems of Art. 150, page 149.

141. The use of instruments. Examples for practice. The only benefit to be derived from the making of these drawings will be in gaining a knowledge of the use of instruments; but if the directions are not observed and the prescribed methods carefully followed, much of the value of this study will be lost. These problems occupy a space of $4''$ square. Six may be drawn on each plate as indicated by the figures. Read Chapters I and II, together with such additional articles as may be prescribed in connection with the problems.



Prob. 1. By means of a triangle and T square draw AB and CD, and divide them into quarter-inches by the scale. Art. 13, page 8. Through these points draw parallel lines by the aid of a straight-edge or triangle, but do not use the T square. Their parallelism may be tested by sliding one triangle on another.

Prob. 2. Having drawn the square, divide the left-hand edge into 10 equal divisions by dividers or by Art. 40, and draw horizontal lines with the T square through these points. In inking these lines observe the variety and grade indicated by Fig. 30, Art. 29, page 22.

Prob. 3. Divide EF and FG into half-inches and divide the surface into squares. In drawing the intermediate lines, judge the spacing by the eye and draw the upper horizontal lines in the first, third, fifth and seventh squares; then the second horizontal lines in like manner, and so proceed until all the horizontal lines are drawn. Similarly draw the vertical lines. See that the lines begin and end exactly on the required points or lines.

PROB. 4. Use the T square and 45° triangle to divide the square as indicated, the size of the small squares being $\frac{1}{2}''$. This is an excellent test of precision in measurement and lining.

PROB. 5. The lower edge of this square is to be divided into half-inches and converging lines drawn to the upper corners. Care must be observed in the inking of this figure to allow each line to dry before inking the next one.

PROB. 6. This is a practice similar to that of Prob. 5.

PROB. 7. Divide the square as indicated by the dotted lines, drawing these as full lines in pencil, but do not ink them. These divisions will facilitate the drawing of the figures by making the use of a scale unnecessary. The intermediate or section lines must not be drawn with a pencil, but are to be drawn only in ink, the spacing being done by the eye. Art. 32, page 27. Ink the fine lines first, then the shade lines, and finally the section lines. Art. 30, page 23.

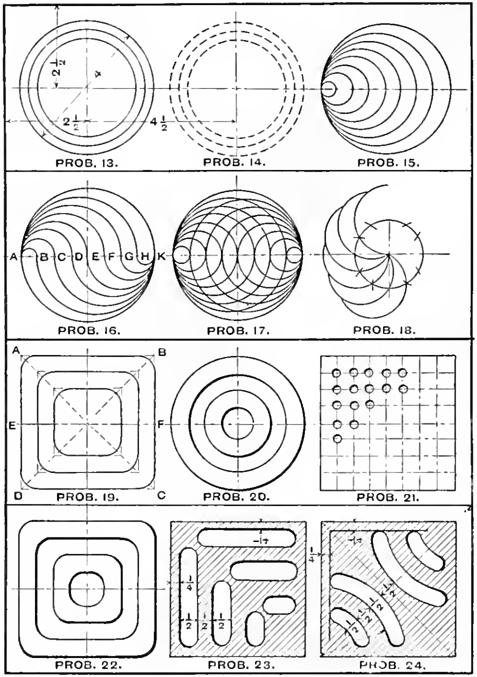
PROB. 8. Observe the instructions for Prob. 7.

PROB. 9. As the divisions of the square are not easily obtained by the scale, use the dividers as in Art. 18, page 14. Observe the instructions for Prob. 7.

PROB. 10. The lines of this figure are to be drawn by means of the 45° triangle on the T square. In pencilling, it will be found to be more convenient to draw one set of lines continuous, as the invisible portions of the bands are not readily determined at first. This figure should be shaded.

PROB. 11. The sides of this square are to be divided into six parts, which may be done by the use of dividers. Art. 18, page 14. This figure may be shaded and section lined as in Prob. 7, observing the directions given for that problem.

PROB. 12. Observe the directions for Prob. 11.



Prob. 13. Divide the horizontal diameter into quarter-inches and draw eight concentric circles through these points. Arts. 16 and 17, page 12. Use great care to avoid enlarging the center. Ink with fine lines.

Prob. 14. Draw eight concentric circles in full pencilled lines. Ink by a fine dotted line. Allow sufficient time for the drying of the ink in each circle before drawing the next.

Prob. 15. Divide the horizontal lines into half-inches and draw a series of circles through these divisions tangent to the outer circle. In inking, begin with the smallest circles and allow time for the ink on each to dry before inking the next circle. Use the bow instruments for small circles. Arts. 19 and 20, page 16.

Prob. 16. Divide the horizontal lines into half-inches and draw semicircles in the following order: AB, HK; AC, GK; AD, FK; etc. See that the circular arcs make continuous lines and are tangent to the outer circle. Ink the lines in the same order.

PROB. 17. Similar to Prob. 16, save that circles are required instead of semicircles.

PROB. 18. Draw a circle of 2″ diameter and divide it into twelve equal parts. From these divisions describe arcs, each passing through the center and terminating in the preceding arc.

PROB. 19. Make a 4″ square and divide it as follows: With a 45° triangle draw lines AC and BD through the center and construct three squares. Set the bow-pencil to a radius of $\frac{3}{8}$″ and describe a circular arc in the corner of each square just touching, but not intersecting, the sides of the square. Ink the circular arcs first.

PROB. 20. Divide a 4″ diameter into half-inches and describe circles as indicated. Do not shade the lines in pencilling. In inking this figure, suppose it to represent a series of rings of $\frac{1}{2}$″ width and shade by the method of Art. 30, page 25.

PROB. 21. This is a practice in the shading of small circular arcs with the bow-pen. Subdivide a 4″ square as indicated and describe circles of $\frac{3}{8}$″ diameter, shading them as in Art. 30, page 26.

PROB. 22. Draw the figure prescribed for Prob. 19 and, in inking, shade as in Fig. 38, page 26, supposing the figures to represent a series of hollow squares.

PROB. 23. This is a practice in shading and section lining. Construct according to dimensions and carefully observe the directions for shading. Art. 30, page 23. The section lining must be done only in ink.

PROB. 24. This is a practice in the joining of shaded arcs, the smaller of which should be drawn with the bow-pen, and the larger with the compasses. The section lining must be done only in ink.

142. Geometrical problems. A 4″ square is sufficient space for each problem. These squares may be arranged as illustrated on page 118, allowing six to each plate. All of the work is to be performed with a 4H lead pencil. The greatest possible precision must be used. Construction lines are to be made very fine; given and required lines being made stronger by pencilling a second time. When two methods are given, the problem should be constructed by the draftsman's method and tested by the geometrical method. It is desirable thoroughly to master the propositions relating to the drawing of perpendiculars and then perform the problems involving these principles. Next, study those relating to angles and similarly perform the dependent problems. Thus continue the subject according to the divisions indicated. Place the number of the problem in the right hand lower corner of the square containing it.

Do not ink the problems, as it impairs the accuracy of the work. If practice in inking is desirable, the figures of the preceding article are better adapted to this exercise.

1. Bisect a line 3″ long. Art. 39, page 36.

2. Bisect an arc of $2\frac{1}{2}$″ radius and $2\frac{3}{8}$″ chord. Art. 39, page 36.

3. Draw a line $2\frac{7}{8}$″ long and erect a perpendicular $1\frac{3}{8}$″ from one end. Do not use a T square. Art. 41, page 36.

4. Draw a line $2\frac{5}{16}$″ long and erect perpendiculars at the extremities. Use both methods. Art. 41, page 36.

5. From a point nearly over the center of a line $3\frac{1}{4}$″ long draw a perpendicular to the line. Art. 41, page 37.

6. From a point nearly over the extremity of a line $2\frac{7}{16}$″ long draw a perpendicular to the line. Do not use a T square. Art. 41, page 37.

7. Draw two intersecting lines making any angle. Construct a similar angle and bisect it. Arts. 43, 44, page 38.

8. From one extremity of a line $3\frac{1}{2}''$ long draw a second line making an angle of 45° with the first. Similarly construct an angle of 30° at the other end. Arts. 42, 45, 46, page 38.

9. From one extremity of a line $3\frac{1}{2}''$ long draw a line making an angle of $22\frac{1}{2}°$ with it. Similarly construct an angle of 15° at the other end. Arts. 42, 43, 45, 46, page 38.

10. From one extremity of a line $3\frac{1}{2}''$ long draw a second line making an angle of 60° with the first. Similarly construct an angle of 75° at the other end. Arts. 42, 45, 46, page 38.

11. Describe a circle $3\frac{1}{2}''$ in diameter and divide it into angles of 15° by means of triangles and a T square. Art. 42, page 38.

12. Construct an equilateral triangle having a base of $2\frac{15}{16}''$. Art. 47, page 39.

13. Construct an isosceles triangle having a base of $2\frac{1}{2}''$ and the equal sides $3\frac{3}{8}''$. Art. 48, page 39.

14. Construct an isosceles triangle having a base of $1\frac{1}{2}''$ and the equal angles 75°. Art. 48, page 39.

15. Construct an isosceles triangle having a base of $3\frac{5}{8}''$, the angle at the vertex being 150°. Art. 48, page 39.

16. Construct a scalene triangle having sides of $2\frac{1}{4}''$, $2\frac{3}{8}''$ and $3\frac{13}{16}''$. Art. 49, page 39.

17. Construct a right-angle triangle having a base of $2\frac{3}{4}''$ and one angle of 30°.

18. From a point on a circle $2\frac{7}{8}''$ in diameter draw a tangent. Arts. 50, 51, page 40.

19. Draw a tangent to the middle point of the arc of a circle of $2\frac{1}{4}''$ radius having a chord of $2\frac{1}{2}''$. Do not use the center of circle. Art. 51, case 2, page 40.

20. Draw a tangent to a circle $2\frac{3}{8}''$ in diameter from a point $2\frac{1}{4}''$ from center of the circle. Art. 51, case 3, page 40.

21. With a $3\frac{1}{2}''$ radius describe an arc of 45°, and from one of its extremities draw a tangent equal to the length of the arc. Art 53, page 41. From the point of tangency lay off $2\frac{1}{2}''$ on the tangent and obtain an arc of equal length. Art. 52, page 41.

22. Draw two lines making an angle of 45° with each other, and a circle tangent to these lines at a point $1\frac{3}{4}''$ from vertex of the angle. Art. 54, page 42.

23. Draw two lines making an angle of 30° with each other, and two circles tangent to each other and these lines. The diameter of the smaller circle is $\frac{3}{4}''$. Art. 55, page 42.

24. Draw a tangent to two circles having diameters of $1\frac{3}{4}''$ and $\frac{7}{8}''$, and their centers $2''$ apart. Art. 56, page 42.

25. Draw a circle having a diameter of $1\frac{1}{2}''$ tangent to two circles having diameters of $1\frac{3}{4}''$ and $1\frac{1}{8}''$ with centers $1\frac{3}{4}''$ apart. Art. 57, page 43.

26. Prescribe three points and draw a circle through them. Art. 58, page 43.

27. Circumscribe a circle about a scalene triangle having sides $1\frac{1}{2}''$, $2\frac{1}{8}''$ and $2\frac{3}{4}''$. Art. 59, page 43.

28. Inscribe a circle within an isosceles triangle having a base of $3\frac{1}{8}''$ and equal sides of $3\frac{5}{8}''$. Art. 60, page 44.

29. Draw a right-angle triangle having a side $2\frac{1}{2}''$ long and one of the oblique angles 30°. Circumscribe a circle about this triangle.

30. Within a circle $3\frac{1}{2}''$ in diameter inscribe an equilateral triangle. Art. 61, page 44.

31. Within a circle $3\frac{1}{2}''$ in diameter inscribe a square. Art. 62, page 44.

32. Within a circle $3\frac{1}{2}''$ in diameter inscribe a pentagon. Art. 63, page 44.

33. Within a circle $3\frac{1}{2}''$ in diameter inscribe a hexagon. Art. 64, page 45.

34. About a circle $2\frac{7}{8}''$ in diameter circumscribe a hexagon. Art. 65, page 45.

35. Draw a hexagon having its long diameter $3\frac{1}{2}''$. Art. 66, page 46.

36. Draw a hexagon having its short diameter $3''$. Art. 67, page 46.

37. Draw a hexagon having one side $1\frac{5}{8}''$. Art. 68, page 46.

38. Draw a hexagon having one side $1\frac{1}{2}''$ long and at an angle of 45° with the horizontal.

39. Draw a hexagon having its short diameter $2\frac{3}{4}''$ and one side horizontal. Art. 67, page 46.

40. Within a circle $3\frac{1}{2}''$ in diameter inscribe an octagon. Art. 69, page 47.

41. Circumscribe an octagon about a circle $3''$ in diameter. Art. 70, page 47.

42. Within an equilateral triangle having sides $3\frac{3}{4}''$ draw 3 equal circles touching each other and one side of the triangle. Art. 72, page 48.

43. Within an equilateral triangle having sides $3\frac{3}{4}''$ draw 3 equal circles touching each other and two sides of the triangle. Art. 73, page 48.

44. Within an equilateral triangle having sides $3\frac{3}{4}''$ draw 6 equal circles which shall be tangent to each other and the sides of the triangle. Art. 74, page 48.

45. Within a circle $3\frac{3}{4}''$ in diameter draw 3 equal circles tangent to each other and the given circle. Art. 75, page 48.

46. Within a circle $3\frac{3}{4}''$ in diameter inscribe 5 equal circles tangent to each other and the given circle. Art. 76, page 49.

47. About a circle $1\frac{1}{4}''$ in diameter circumscribe 5 equal circles tangent to each other and the given circle. Art. 77, page 49.

143. Conic Section Problems. These problems require a space of $5'' \times 7''$, or four problems to each plate. Use very fine full lines for all construction, but do not ink them. The required curves should be inked with care, observing the instructions in Art. 15, page 10.

Study Arts. 78 to 84 inclusive, before performing the following problems on the ellipse.

1. Draw an ellipse having the minor axis $4''$ and the distance between the foci $4\frac{1}{2}''$. Use the First Method. Art. 80, page 52. Draw the conjugate diameters, one of which makes an angle of 15° with the major axis. Art. 79, page 52.

2. Draw one-half of an ellipse having the major axis $6\frac{1}{2}''$ and the distance between the foci $4\frac{1}{2}''$. Use the Fourth Method. Art. 83, page 55.

3. Draw an ellipse having the major axis $6\frac{1}{2}''$ and the minor axis $3''$. Use the Second Method. Art. 81, page 54. Draw a tangent to the curve at a point $3''$ from the center. Page 53.

4. Draw an ellipse having the major axis $5\frac{1}{4}''$ and its minor axis $4\frac{1}{2}''$. Use the Fifth Method. Art. 84, page 56. Test one-quarter of the curve by the Fourth Method. Art. 83, page 55.

5. Draw an ellipse having the major axis $5\frac{1}{2}''$ and the distance between foci $3\frac{1}{2}''$. Use the First Method. Art. 80, page 52. Draw two conjugate diameters, one of which makes an angle of 75° with the major axis. Art. 79, page 52.

6. Draw an ellipse having the minor axis $3\frac{1}{2}''$ and the distance between the foci $5''$. Use the Second Method. Art. 81, page 54. Draw a tangent to the curve at a point $2\frac{3}{4}''$ distant from the minor axis. Page 53.

7. Draw one-half of an ellipse having its major axis $6\frac{1}{2}''$ and its minor axis $4\frac{1}{2}''$. Use the Fourth Method. Art. 83, page 55.

8. Draw an ellipse having the major axis $4\frac{7}{8}''$ and the minor axis $2\frac{3}{4}''$. Use the Third Method. Art. 82, page 54. Test four points by the First Method. Art. 80, page 52.

9. Draw an ellipse having the major axis $6''$, and the minor axis $4''$. Use the First Method. Art. 80, page 52. Draw a tangent to the curve at a point $2\frac{3}{8}''$ from one extremity of the minor axis. Page 53.

10. Draw an ellipse having its minor axis $4''$, and the distance between the foci $3''$. Use the Second Method. Art. 81, page 54. Draw two conjugate diameters, one of which makes an angle of 30° with the major axis. Art. 79, page 52.

11. Draw a segment of an ellipse having an axis of $4''$ and a $5\frac{1}{2}''$ chord which intersects the axis at $1\frac{1}{4}''$ from its extremity. Use the Fourth Method. Art. 83, Case 2, page 55.

12. Draw one-half of an ellipse having a major axis of $4\frac{7}{8}''$ and distance between the foci $2\frac{7}{8}''$. Use the Third Method. Art. 82, page 54. Draw the second half of the ellipse by the Fifth Method. Art. 84, page 56. Test the ellipse by the Second Method. Art. 81, page 54.

Read Arts. 85 to 88 inclusive, before performing the following problems on the parabola. Draw the axes horizontal. Ink the curves, but not the construction lines.

13. Draw a parabola having the focus $\frac{3}{4}''$ from the directrix. Draw a tangent to any point of the curve. Art. 86, page 57.

14. Draw a parabola having given the abscissa $6''$ and double ordinate $4\frac{1}{2}''$. Art. 87, page 58.

15. Draw a parabola having the focus $1''$ from the directrix. Draw a tangent to the curve at a point $1\frac{3}{4}''$ from the focus. Art. 86, page 57.

16. Draw a parabola having given the abscissa $5\frac{1}{2}''$ and double ordinate $4''$. Art. 87, page 58.

Read Arts. 89 to 93 inclusive before performing the following problems on the hyperbola. Draw the transverse axes horizontal.

17. Draw an hyperbola having its transverse axis $1\frac{1}{2}''$ and distance between the foci $2\frac{1}{8}''$. Draw a tangent to the curve at a point $1\frac{5}{8}''$ from the vertex. Art. 90, page 59.

18. Draw an hyperbola having its transverse axis $1\frac{1}{2}''$, a double ordinate $4\frac{3}{4}''$ and its distance from the vertex $1\frac{7}{8}''$. Art. 91, page 59.

19. Draw an hyperbola having its transverse axis $1\frac{3}{4}''$ and distance between the foci $2\frac{1}{4}''$. Draw a tangent to the curve from a point $1\frac{7}{8}''$ from the vertex. Art. 90, page 59.

20. Draw an hyperbola having the transverse axis $1\frac{3}{4}''$, a double ordinate $4\frac{1}{2}''$ and its distance from the vertex $2\frac{1}{4}''$. Art. 91, page 59.

144. Orthographic Projection Problems. Most of the problems are designed to occupy a space of $5'' \times 7''$, in which case there will be four problems to each plate and they should be separated by a fine inked line. All construction lines and lines of the object should be drawn very fine in pencil and *no line that has been useful in the construction of the drawing should be erased.* Invisible lines of the object are better dotted in pencil to avoid mistakes in inking. Draw no dimension lines. Study Arts. 94, 95 and 96.

Begin the drawing with that view concerning which the most information is given. It is not necessary to complete one view before beginning a second, and frequently it is desirable to proceed with the three views at one time. Carefully estimate the space to be occupied by each view when a graphic statement of the location is not given.

Only lines of the object are to be inked, the visible lines being in full and the invisible in dotted lines. Shade lines are to be used only in those problems indicated, and never shown in pencil. Observe the instructions for inking. Art. 33, page 28.

PROB. 1. Locate the ground line and traces of profile plane. Draw top and front views, and from these obtain the side view. Draw projection lines in pencil only. It is an excellent practice to number the extremity of each line of the completed projection as in Fig. 107, page 65, and thus acquire familiarity with the different views of each surface, line and point.

PROB. 2. The front and side views are given to obtain the top view. See that all the lines of the object are shown in this view.

PROB. 3. Note the difference between the top views of this figure and the preceding.

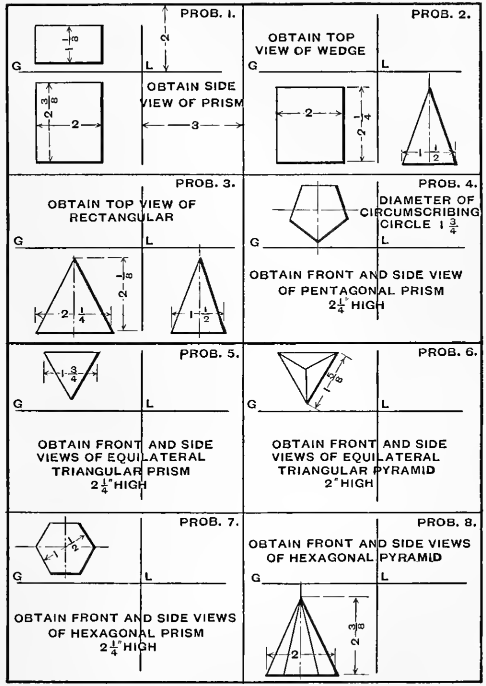
PROB. 4. Remember to represent the invisible lines.

PROB. 5. Draw the top view without the aid of compasses, it being an equilateral triangle.

PROB. 6. This problem differs from the preceding in being a pyramid instead of a prism.

PROB. 7. Do not use the compasses for the construction of the hexagon.

PROB. 8. Although the front view alone is given, it is better to draw the top view first.



PROBLEMS 9 to 16 are similar to the preceding and may be substituted for them when it is desired to omit a graphic statement. Draw three views. The space required is $5'' \times 7''$, which allows four problems to each plate.

PROB. 9. Draw a rectangular prism $2\frac{1}{8}''$ long. The bases measure $\frac{7}{8}''$ by $1\frac{7}{8}''$, and are parallel to the profile plane. The prism is resting on one of its narrow faces.

PROB. 10. Draw a wedge, the front view of which is an isosceles triangle having a base of $2\frac{1}{2}''$ and a height of $2\frac{3}{8}''$. Length of wedge $1\frac{1}{2}''$.

PROB. 11. Draw a square pyramid resting on its base with two edges of the base making angles of 15° with V. The base is $1\frac{1}{2}''$ square, and the height of the pyramid is $2\frac{3}{16}''$.

PROB. 12. Draw a pentagonal prism resting on a lateral face which is parallel to H, and the bases perpendicular to V. The bases are inscribed in a circle $1\frac{7}{8}''$ in diameter, and the sides are $2\frac{1}{8}''$ long.

PROB. 13. Draw a triangular prism resting on a face parallel to H, and the bases parallel to V. The bases are equilateral triangles having sides of $1\frac{7}{8}''$. The sides of the prism are $2\frac{1}{8}''$ long.

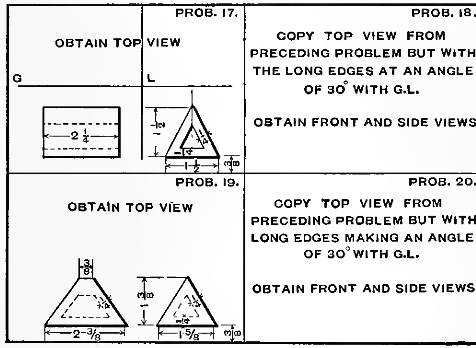
PROB. 14. Draw a triangular pyramid. The base is an equilateral triangle having sides of $1\frac{3}{4}''$ and parallel to H. One edge of the base makes an angle of 15° with V. The pyramid is $2\frac{1}{8}''$ high.

PROB. 15. Draw a hexagonal prism resting on a face parallel to H, and the bases parallel to V. The faces are $\frac{3}{4}'' \times 2\frac{1}{8}''$.

PROB. 16. Draw a hexagonal pyramid $2\frac{1}{4}''$ high. Its base is parallel to H, with two of its edges perpendicular to V. The edges of the base are $1\frac{1}{8}''$.

PROB. 17. Draw a hollow triangular prism, the faces of which are $\frac{1}{4}''$ thick. Draw the side view first.

PROB. 18. It is required to represent three views of the preceding object when turned around on its base. This will change the angle at which the top view is drawn, but does not alter the relation of the lines to each other; therefore, the top view may be copied from Prob. 17. In drawing the front and side views observe that all points of the object retain their former height, which may be obtained from the front and side views of Prob. 17. Use care to represent the invisible lines of the object.

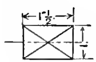


PROB. 19. The object to be represented is similar to the preceding, save that the ends are beveled and the triangular space does not pass entirely through the prism. Omit the drawing of GL, and the traces of P. Art. 96, page 66. The drawing of the projection lines may be omitted also. Use care to project all of the invisible lines of the object.

PROB. 20. It is required to represent three views of the preceding object when turned around on its base. The construction is similar to that of Prob. 18. In this case the center lines will no longer serve for the GL and traces of P. Use care to project all of the invisible lines of the object.

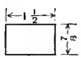
PROBLEMS 21 TO 32 DRAW THREE VIEWS FIG. 1.	DRAW THREE VIEWS OF THE OBJECT WHEN REVOLVED FROM THE POSITION OF FIG. 1, 30° TO THE LEFT ABOUT AN AXIS PERPENDICULAR TO V FIG. 2	DRAW THREE VIEWS OF THE OBJECT WHEN REVOLVED FROM THE POSITION OF FIG. 2, 25° ABOUT AN AXIS PERPENDICULAR TO H FIG. 3.
DRAW THREE VIEWS OF THE OBJECT WHEN REVOLVED FROM THE POSITION OF FIG. 1, 20° FORWARD ABOUT AN AXIS PERPENDICULAR TO P FIG. 4.	DRAW THREE VIEWS OF THE OBJECT WHEN REVOLVED FROM THE POSITION OF FIG. 2, 15° FORWARD ABOUT AN AXIS PERPENDICULAR TO P FIG. 5.	DRAW THREE VIEWS OF THE OBJECT WHEN REVOLVED FROM THE POSITION OF FIG. 5, 35° ABOUT AN AXIS PERPENDICULAR TO H FIG. 6.

145. Objects oblique to the coördinate planes. A careful study of Art. 97, page 67, must be made previous to the solution of Problems 21 to 32 inclusive. The plate will be divided into six rectangles as in the accompanying figure, the division lines being drawn in pencil only. Six positions of three views each will be required in each case. Omit the drawing of projection and shade lines. If difficulty is found with the problems, number the points as in Fig. 107, page 65, but use care to retain the same number for each point throughout the problem.



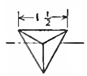
TOP VIEW OF FIG. 1.

Prob. 21. In the required positions draw a rectangular pyramid having an altitude of $1\frac{7}{8}''$.



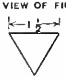
TOP VIEW OF FIG. 1.

Prob. 22. In the required positions draw a rectangular prism having an altitude of $1\frac{3}{4}''$.



TOP VIEW OF FIG. 1.

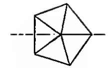
Prob. 23. In the required positions draw a triangular pyramid having an altitude of $1\frac{7}{8}''$. The base is an equilateral triangle.



TOP VIEW OF FIG. 1.

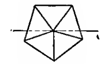
Prob. 24. In the required positions draw a triangular prism having an altitude of $1\frac{3}{4}''$. The bases are equilateral triangles.

PROB. 25. In the required positions draw a pentagonal pyramid having an altitude of $1\frac{3}{4}''$. Diameter of circumscribing circle of base $1\frac{3}{4}''$.



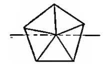
TOP VIEW OF FIG. 1.

PROB. 26. In the required positions draw a pentagonal pyramid having an altitude of $1\frac{3}{4}''$. Diameter of circumscribing circle of base $1\frac{3}{4}''$.



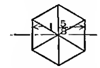
TOP VIEW OF FIG. 1.

PROB. 27. In the required positions draw a pentagonal pyramid having an altitude of $1\frac{3}{4}''$. Diameter of circumscribing circle of base $1\frac{3}{4}''$.



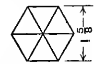
TOP VIEW OF FIG. 1

PROB. 28. In the required positions draw a hexagonal pyramid having an altitude of $1\frac{7}{8}''$.



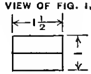
TOP VIEW OF FIG. 1.

PROB. 29. In the required positions draw a hexagonal pyramid having an altitude of $1\frac{7}{8}''$.



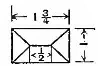
TOP VIEW OF FIG. 1.

PROB. 30. In the required positions draw a wedge having an altitude of $1\frac{7}{8}''$.



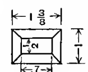
TOP VIEW OF FIG. 1.

PROB. 31. In the required positions draw a wedge having an altitude of $1\frac{1}{2}''$.

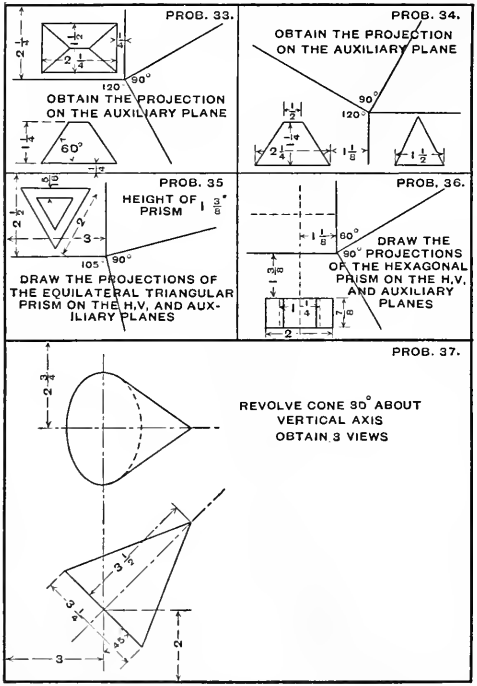


TOP VIEW OF FIG. 1.

PROB. 32. In the required positions draw the frustum of a rectangular pyramid having an altitude of $1\frac{1}{2}''$.



TOP VIEW OF FIG. 1.



PROBLEMS 33 to 37 require a knowledge of the use of auxiliary planes. Art. 98, page 70.

PROB. 33. The auxiliary plane being parallel to the plane of the right-hand end of the object, the projection on that plane will be a true representation of that surface. Other lines and surfaces will be foreshortened.

PROB. 34. In this problem the auxiliary plane makes an angle of 30° with H, and the projection on this plane is to be made in place of the top view.

PROB. 35. Observe that the prism has a triangular hole extending through it.

PROB. 36. This is similar to the preceding, save that the prism is hexagonal instead of triangular. A circular hole, $1\frac{1}{4}''$ diameter, extends from base to base.

PROB. 37. Study Arts. 99 and 100, pages 72 and 73, before solving this problem. Having obtained the base of the cone in the manner directed, locate the vertex of the cone. The tangents to the base drawn from the vertex will be the contour lines of the cone.

146. Special problems in projection. These problems require a space of $5'' \times 7''$. Three views are required in each case, and all invisible as well as visible lines should be shown on each view. Leave all construction lines in pencil. All polygons are regular polygons.

PROB. 38. Draw the frustum of an octagonal pyramid having its base parallel to H and two of its edges making an angle of 30° with V. The diameter of the circumscribing circle of its lower base is $1\frac{3}{4}''$, and of the upper base, $1\frac{5}{8}''$. The altitude is $1\frac{7}{8}''$.

PROB. 39. Revolve the pyramid of Prob. 38, 30° to right about an axis perpendicular to V.

PROB. 40. Draw a pentagonal prism resting on one of its faces and having its lateral edges at an angle of $22\frac{1}{2}°$ with V. Diameter of circumscribing circle of base $1\frac{1}{2}''$. Length of prism $2\frac{1}{2}''$.

PROB. 41. Draw an equilateral triangular prism resting on one of its faces, and its lateral edges making an angle of 15° with V. The edges of the base are $1\frac{1}{2}''$, and the length of the prism is $2\frac{1}{4}''$. There is a triangular hole extending through the bases and making the thickness of the sides $\frac{1}{4}''$.

PROB. 42. Draw a cylinder with its axis parallel to V and at an angle of 60° with H. The diameter of the base is $1\frac{3}{4}''$, and the length of cylinder, $2\frac{1}{8}''$. Obtain the ellipses by the method of trammels.

PROB. 43. Draw an equilateral triangular pyramid having an altitude of $2\frac{1}{4}''$, and the edges of the base $1\frac{7}{8}''$. The base makes an angle of 30° with H and one of its edges is perpendicular to V.

PROB. 44. Revolve the pyramid of Prob. 43, 45° forward about an axis perpendicular to P.

PROB. 45. Draw a box having the following outside dimensions. Length 2″, width $1\frac{3}{4}$″. depth, including cover, 1″. Thickness of material $\frac{1}{4}$″. The long edges of the box are parallel to H and make an angle of 30° with V. The cover is hinged on long edge and opened 30°.

PROB. 46. Draw a pyramid formed of four equilateral triangles having $2\frac{1}{8}$″ sides. The base is parallel to H and one of its edges makes an angle of 30° with V.

PROB. 47. Draw a rectangular surface, $1\frac{1}{4}$″ × $2\frac{5}{8}$″, in the following positions: The short edges parallel to H and making an angle of 75° with V; the long edges making angles of 15°, 30° and 45° with H. Art. 102, page 76.

PROB. 48. Revolve the surface from the positions required in Prob. 47, 15° forward.

PROB. 49. Draw an isosceles triangle in three positions as follows: The base lying on V and inclined at an angle of 30° with H. The altitude making angles of 90°, 30° and 15° with V. The base of the triangle is $1\frac{7}{8}$″, and the altitude $2\frac{1}{8}$″. Art. 102, page 76.

PROB. 50. Draw the same triangle revolved from the positions in Prob. 49, 30° in either direction about a vertical axis.

PROB. 51. Draw an isosceles triangle in the following positions: The base parallel to H and making an angle of 60° with V. The altitude making angles of 45° and 60° with H. The base of triangle is 2″, and the altitude $2\frac{1}{4}$″. Art. 102, page 76.

PROB. 52. Revolve the same triangle from the positions in Prob. 51, 30° backward.

PROB. 53. Draw an octagonal surface inclined at an angle of 60° with H, two of its edges being parallel to H and making angles of 15° with V. The diameter of circumscribing circle is $2\frac{1}{2}$″. Art. 102, page 76.

PROB. 54. Draw a hexagonal surface inclined at an angle of 45° with V, two of its edges being parallel to V and making angles of 30° with H. The long diameter of the hexagon is $2\frac{1}{2}$″. Art. 102, page 76.

Prob. 55. Draw the projections of a line located as follows: The left-hand extremity of the line is $\frac{1}{2}''$ behind V and $\frac{1}{4}''$ below H. The right-hand extremity is $1\frac{1}{2}''$ behind V, and $1\frac{3}{4}''$ below H. The H projection of the line makes an angle of 30° with V. Find its length by revolving it parallel to H, V and P. Art. 101, page 74.

Prob. 56. Draw the projections of a line of which the left-hand extremity is 1″ behind V and $1\frac{3}{8}''$ below H. The right-hand extremity is $\frac{3}{8}''$ behind V and $\frac{3}{4}''$ below H. The H projection makes an angle of 15° with V. Find the length of the line by revolving it into the planes of projection by the second method. Art. 101, page 75.

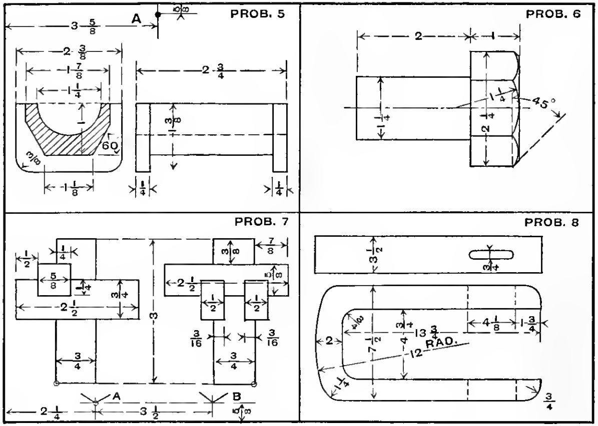
147. Isometric projection problems. Study Chapter VI, page 78. The problems require a space of 5″ × 7″. Omit the invisible lines in inking.

Prob. 1. Make the isometric drawing of a 2″ cube. Art. 110, page 80. Inscribe circles on the upper and right-hand faces, the former by the exact method and the latter by the approximate method. Art. 112, page 81. From the left-hand lower corner of the left-hand face draw lines making angles of 30°, 45° and 75° with the lower edge. Art. 113, page 82.

Prob. 2. Make the isometric drawing of the frustum of a pyramid, the lower base being 2″, and the upper base $1\frac{1}{2}''$ square. Height $1\frac{1}{4}''$. Inscribe a circle on the upper base using the approximate method. Locate the front lower corner in the center and 1″ from lower margin.

Prob. 3. Make the isometric drawing of a pentagonal plinth surmounted by a cylinder. The sides of the pentagon are 2″ and the height of the plinth is $\frac{5}{8}''$. Art. 71, page 47. The cylinder is 2″ in diameter and 1″ high. Art. 111, page 80.

Prob. 4. Make the isometric drawing of a box with cover opened through an angle of 120°. The outside dimensions are: length $2\frac{1}{4}''$, width $1\frac{1}{2}''$, depth $\frac{7}{8}''$. Thickness of material is $\frac{1}{4}''$. Locate the front lower corner in the middle of the space and $\frac{1}{4}''$ from lower margin.



PROB. 5. Make the isometric drawing of the bearing illustrated, locating the upper corner at point A.

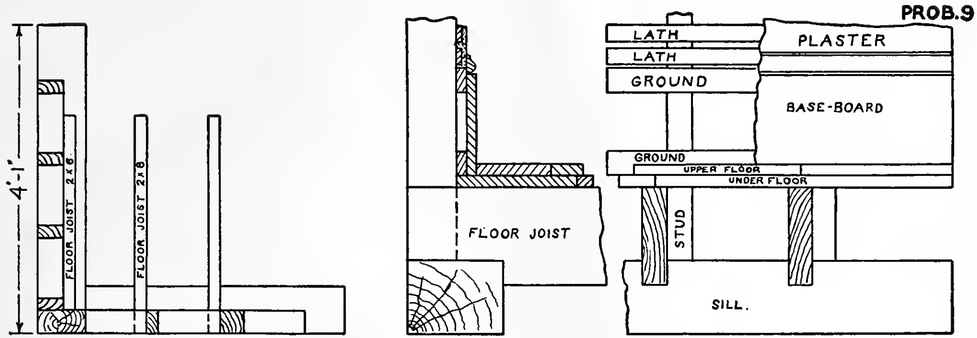
PROB. 6. Make the isometric drawing of a hexagonal bolt. The center line may be parallel with either of the isometric axes. Art. 115, p. 83.

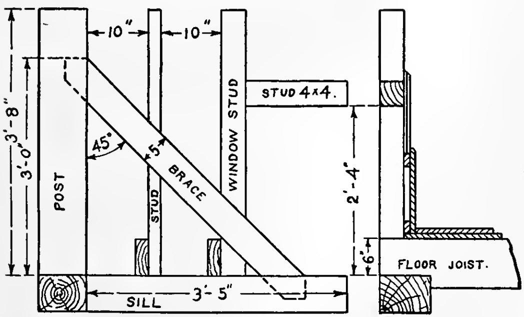
PROB. 7. Make the isometric drawing of the pieces illustrated, and a second isometric drawing of the upright block showing the cuts necessary for making the required fits. The lowest portion of the upright block will be located at A in the first case and B in the second.

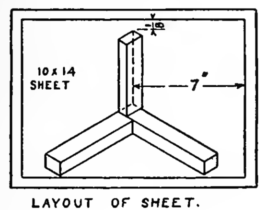
PROB. 8. Make the isometric drawing of the connecting-rod strap, the scale to be $3'' = 1$ ft. In drawing the curves observe the directions of Art. 115, page 83.

PROB. 9. Make the isometric drawing of the framing details illustrated. The dimensions of the materials are given below. The space required is $10'' \times 14''$. Draw to a scale of $1\frac{1}{2}'' = 1$ ft.

Sills,	$6'' \times 8''$.	Studs,	$2'' \times 4''$, $12''$ on centers.	Grounds,	$\frac{3}{4}'' \times 2''$.
Post,	$4'' \times 8''$.	Floor joists,	$2'' \times 8''$, $12''$ on centers.	Laths,	$\frac{3}{8}'' \times 1\frac{1}{2}'' \times 48''$, $\frac{3}{8}''$ space.
Brace,	$4'' \times 5''$.	Under flooring,	$\frac{7}{8}'' \times 10''$.	Plaster,	$\frac{3}{8}''$ thick.
Window studs,	$4'' \times 4''$.	Upper flooring,	$\frac{7}{8}'' \times 6''$.	Baseboard,	$\frac{7}{8}'' \times 10''$, including cap.

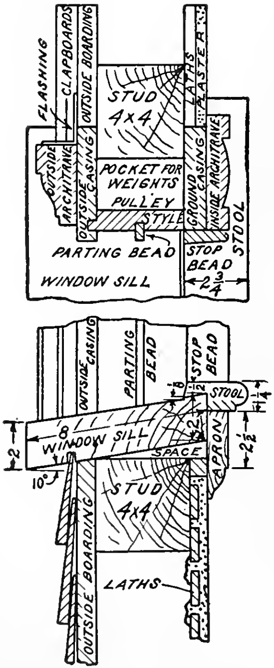


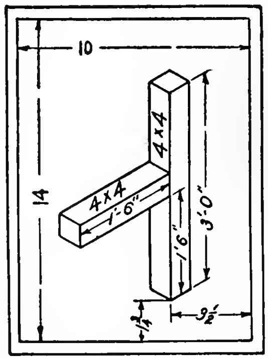




DETAILS FOR ISOMETRIC DRAWING OF FRAMING PLANS

Prob. 10. Make the isometric drawing of a window from the detailed sketches. The dimensions of the materials are given below. Show all that is given in the sections. Locate the drawing as in the lay-out sketch, observing that the short edge of the plate is horizontal. Art. 115, page 83.





LAYOUT OF SHEET

Dimensions of Materials

Studs,	$4'' \times 4''$.
Apron,	$\frac{7}{8}'' \times 3\frac{3}{4}''$.
Ground casing,	$\frac{3}{4}'' \times 4\frac{1}{2}''$.
Inside architrave,	$\frac{3}{4}'' \times 5''$.
Outside architrave,	$1\frac{3}{4}'' \times 3\frac{1}{2}''$.
Outside casing,	$\frac{7}{8}'' \times 5''$.
Outside boarding,	$\frac{7}{8}''$.
Pocket,	$2\frac{1}{4}'' \times 4''$.
Pulley style,	$\frac{7}{8}''$ thick.
Parting bead,	$\frac{3}{8}'' \times \frac{3}{4}''$.
Stop bead,	$\frac{1}{2}'' \times 1\frac{1}{2}''$.
Laths,	$\frac{3}{8}'' \times 1\frac{1}{2}'' \times 48''$, spaced $\frac{3}{8}''$.
Plaster,	$\frac{3}{8}''$ thick.

Clapboards, $\frac{1}{2}''$ at thick edge, $\times 5\frac{1}{2}'' \times 48''$, laid $3\frac{1}{2}''$ to the weather.

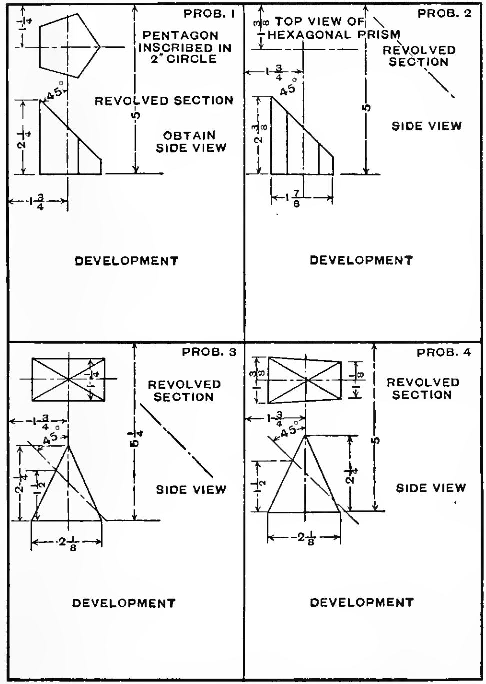
148. Development problems. Study Chapter VII, page 88. Three views and the complete development will be required in each case. In general, begin to develop the surface on the shortest edge. It will assist the student to a better understanding of this subject if the developed surface be copied on stiff paper and afterwards cut and folded. Observe the order prescribed for performing these problems. Art. 119, page 90.

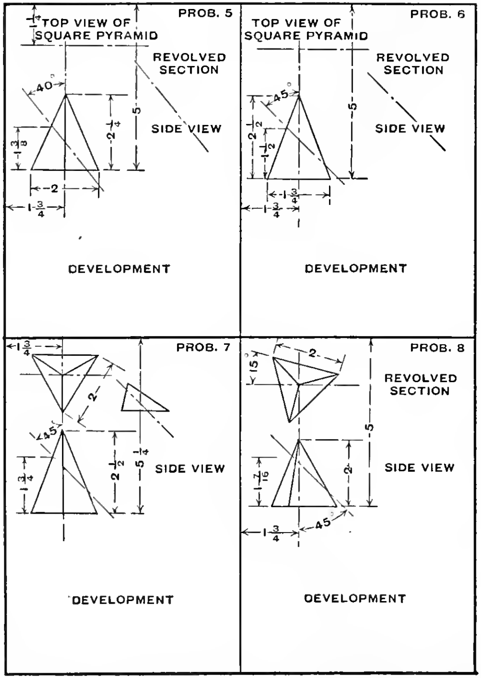
PROB. 1. This is a prism having a pentagonal base. The vertical edges being parallel to V are seen in their true length on that plane. The length of the edges of the lower base can be obtained from the top view, and the true shape of the upper base will be found by projecting it on to an auxiliary plane, as in Art. 98, page 70.

PROB. 2. Solve as for Prob. 1.

PROB. 3. In this and the following problems do not ink that portion of the surface lying above the cutting plane. Art. 119, page 90.

PROB. 4. Observe that the slant edges of this pyramid are not of equal length.





PROB. 5. This is a square prism, the diagonal of the base being 2″. Observe that the base is cut by the plane, and one of the inclined edges will not appear on the completed development. In developing the surface open it on this line.

PROB. 6. This differs from the preceding in that the base is not cut by the plane, and the position of the pyramid with respect to V is changed.

PROB. 7. The surface cut by the plane will not be symmetrical with respect to the center line as in the preceding problems. Care must be used to take no dimensions from the top view that may not be represented by lines parallel to H, the vertical projection of which will be parallel to GL.

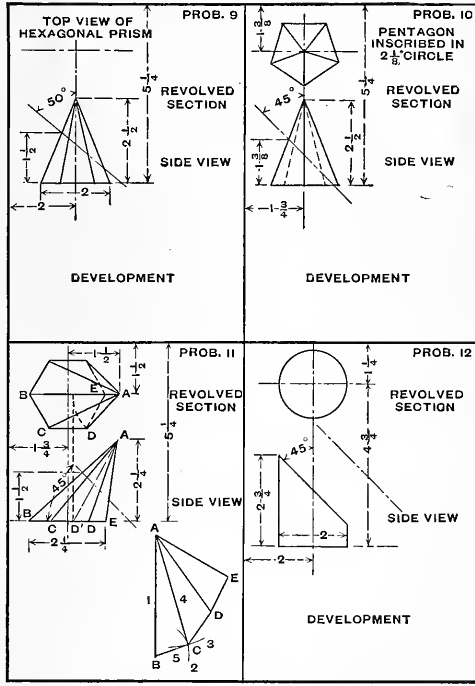
PROB. 8. This is similar to Prob. 7, only the position of the pyramid with respect to V being changed.

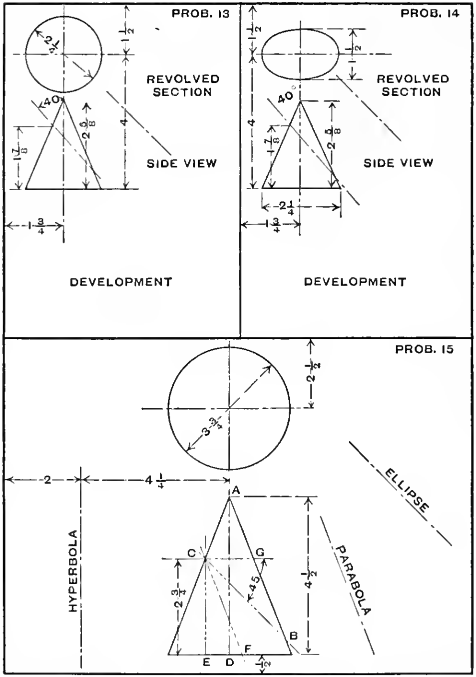
PROB. 9. The cutting plane makes an angle of 40° with H, and the auxiliary plane must be at the same angle, the projecting lines being drawn perpendicular to it.

PROB. 10. This is similar to Prob. 5, the base being cut by the cutting plane. In developing the surface open it on the uncut edge.

PROB. 11. The slant edges of this pyramid being of unequal length must be obtained separately. One of these edges, AD, is shown in its revolved position at AD′. One-half of the development is also shown, and the method and order for drawing the lines indicated by the numbers. Thus, the line AB is drawn first, and then arcs 3 and 2 described from its extremities, A and B, with radii equal to the true lengths of AC and BC: this determines the point C. In like manner the remaining points and lines are found.

PROB. 12. The development of a cylinder is required. Arts. 120 and 121, page 92. Employ twenty-four elements in obtaining the curve.





PROB. 13. Use twenty-four elements for obtaining the development. Art. 122, page 94.

PROB. 14. This being an elliptical cone, the elements will be of unequal length.

PROB. 15. The cone is cut by four planes, CG, CB, CF, CE. Determine the top view of each and their projection on planes to which they are parallel. Test the latter as follows: Determine the axes and foci of the ellipse, and test eight points by the first method, Art. 80, page 52. Test by trammels also. Test the parabola by the second method, Art. 87, page 58. The cutting plane, CF, is parallel to an element of the cone. Test the hyperbola by the second method, Art. 91, page 59. The vertex of the cone bisects the transverse axis.

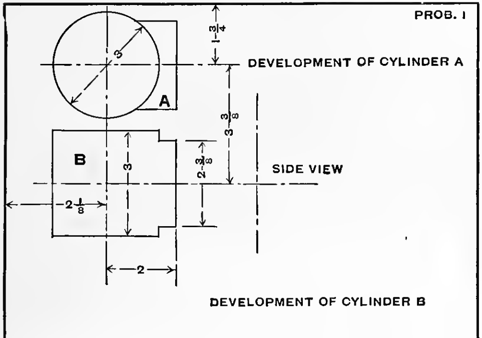
149. Intersection problems. Chap. VIII.

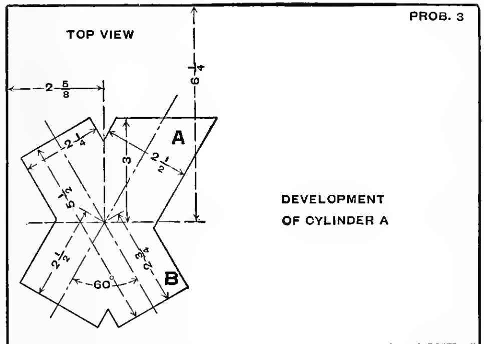
PROB. 1. Assume twenty-four equidistant elements on the small cylinder.

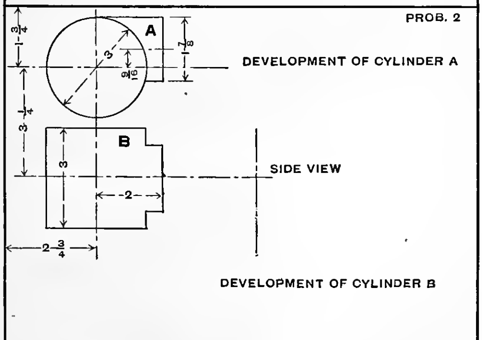
PROB. 2. The cylinders are tangent. Use care in obtaining the limiting points.

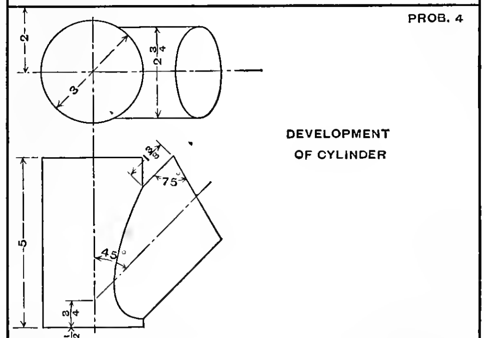
PROB. 3. Use auxiliary planes. Page 98.

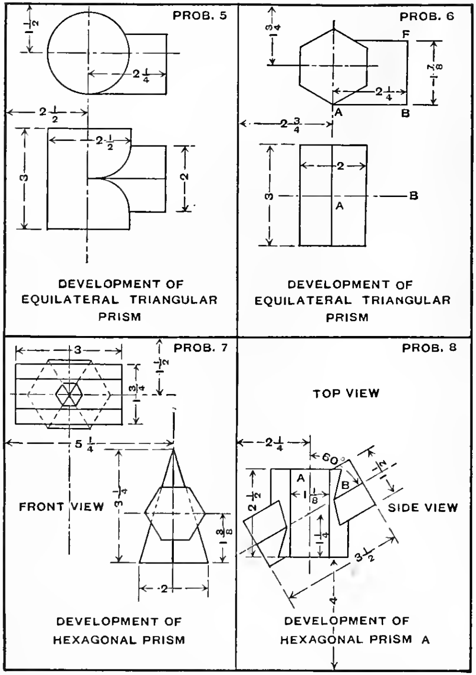
PROB. 4. Solve without auxiliary planes. Art. 125, p. 100. Develop oblique cylinder.


PROB. 1
DEVELOPMENT OF CYLINDER A
A
B
SIDE VIEW
DEVELOPMENT OF CYLINDER B
1-3/4
3-3/8
2-3/8
3
3
2-1/8
2


PROB. 3
TOP VIEW
DEVELOPMENT
OF CYLINDER A
A
B
6-1/4
2-5/8
2-1/4
2-1/2
3
5-1/2
2-1/2
2-3/4
60°


PROB. 2
DEVELOPMENT OF CYLINDER A
A
B
SIDE VIEW
DEVELOPMENT OF CYLINDER B
1-3/4
1-7/8
9/16
3
3-1/4
3
2
2-3/4


PROB. 4
DEVELOPMENT
OF CYLINDER
2
3
2-3/4
1-3/8
75°
45°
5
3/4
1/2



Prob. 5. It is required to find the intersection of a cylinder and prism without using a side view. Use cutting planes parallel with V. The base of the prism will have to be revolved in order to complete the top view and enable the intersection of the cutting planes and prisms to be determined.

Prob. 6. To determine the intersection of a hexagonal and a triangular prism. This is similar to Prob. 5, save that a prism is substituted for the cylinder. In all cases of intersection between prism and prism, it is only necessary to find the point of intersection of each edge of both prisms with a face of the other prism. Art. 126, page 101.

Prob. 7. The lines of intersection on the top view are to be completed, and the front view with its lines of intersection are required. Develop the prism only.

Prob. 8. Determine the intersection of the oblique and vertical hexagonal prisms and develop the latter. Art. 126, page 101.

150. Spirals, screw-threads and bolt-head problems. Study Chapter IX. Page 102. Problems 1 to 9 inclusive require a space of $4\frac{1}{2}''$. A graphic statement is made for the problems on screw-threads and bolt-heads.

Prob. 1. Draw an equable spiral with a pitch of $1\frac{1}{2}''$. Describe two revolutions of the radius vector. Art. 128, page 102. Sketch the curve, and ink as directed in Art. 15, page 12.

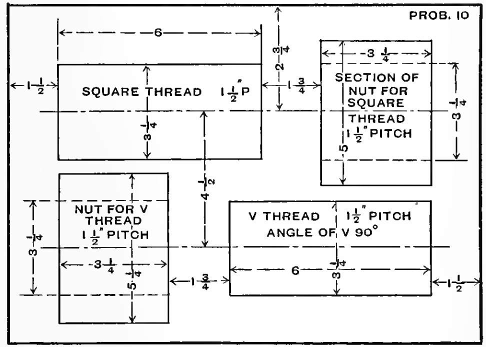
Prob. 2. Draw the involute of a $\frac{3}{8}''$ square. Art. 130, page 104.

Prob. 3. Draw the involute of an equilateral triangle having $\frac{3}{8}''$ sides. Art. 130, page 104.

Prob. 4. Draw the involute of a right line $\frac{1}{4}''$ long. Art. 130, page 104.

Prob. 5. Draw the involute of a hexagon having sides of $\frac{1}{4}''$. Art. 130, page 104.

Prob. 6. Draw the involute of a circle having a diameter of $1''$. Art. 130, page 104.

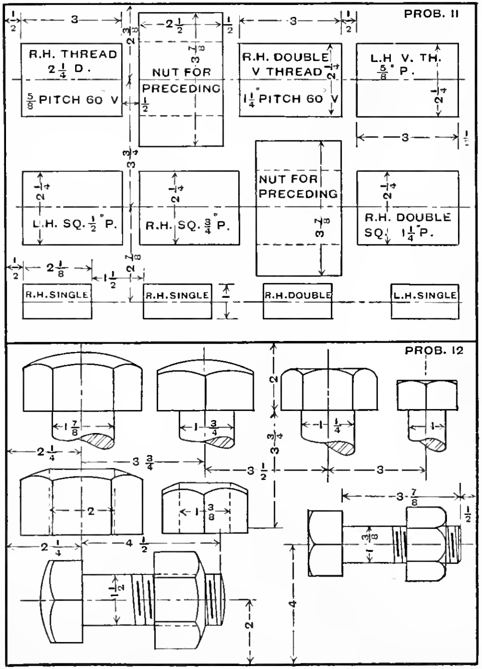


Prob. 7. Draw a right-hand helix of $1\frac{1}{2}''$ pitch, $2''$ diameter and $2\frac{1}{2}''$ length. Art. 131, page 104.

Prob. 8. Draw a left-hand double helix of $1\frac{1}{2}''$ pitch, diameter $1\frac{3}{4}''$, length $2\frac{1}{4}''$.

Prob. 9. Draw a right-hand conical helix. Pitch $1\frac{1}{4}''$, diameter of cone $2''$, height $1\frac{7}{8}''$.

Prob. 10. The diameter and pitch being the same in both cases, but two templets are required, one for the outer and one for the inner helix. Art. 132, page 106, and Art. 136, page 110.



PROB. 11. The first four examples are of conventional V threads, Art. 133, page 108. The second four are conventional square threads. Art. 136, page 110. The last four are to be drawn by the methods illustrated by Figs. 177 to 180, page 109. Make the pitches about the same as those in the illustrations, estimating the spaces by the eye. Distinguish clearly between the single and double threads.

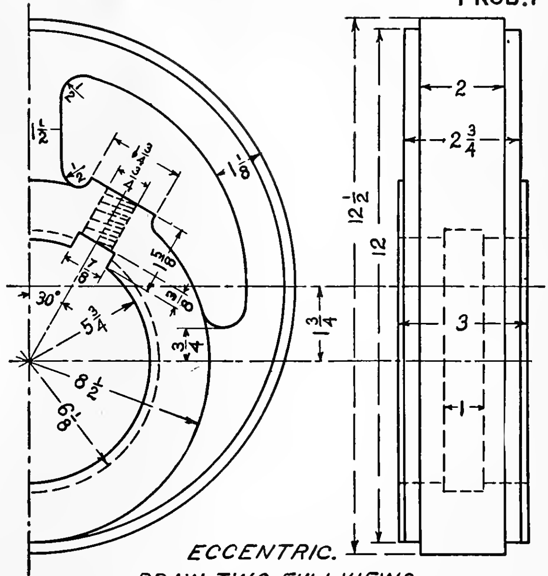
PROB. 12. Study Arts. 137, 138 and 139 before attempting this problem. The proportion and character of the heads and nuts should be so well understood that reference to the text will be unnecessary. The diameters are given, and the sketch shows the character of the bolt, whether rounded or chamfered. Observe every detail and see that the dimensions are standard. Draw the rounded heads and nuts before the chamfered type.

For the further consideration of this subject, the student is referred to the chapter on Bolts and Screws, in "Machine Drawing" of this series.

151. Miscellaneous problems. These examples of a practical character are designed for pencilling and inking practices which may be used early in the course. Problems 1 and 2 may be substituted for some of the "examples for practice," page 117 and following. They will afford an excellent practice in lettering and figuring. Problems 3 and 4 likewise require very little knowledge of projection and serve as an excellent practice in penmanship. Full instruction is given with each sketch and the proper scale is specified for a 10″ × 14″ plate. The shading of the drawing is left to the discretion of the instructor. The drawings should be correctly figured, and the title in proper form.

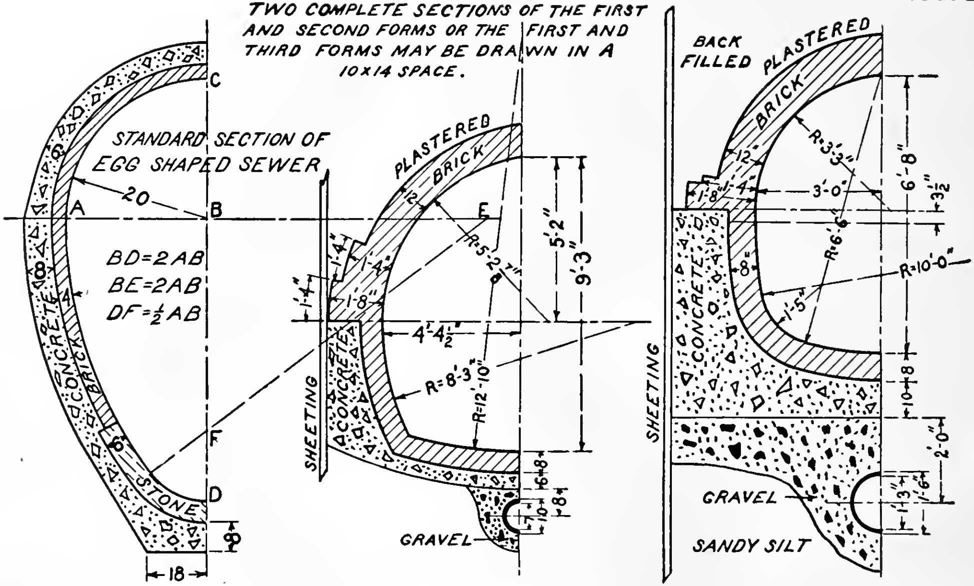
The author does not recommend the making of machine drawings, save as above, until the student has acquired a thorough working knowledge of projection, and has been taught something of the notation and idiomatic use of applied graphics.

PROB. I



ECCENTRIC.
DRAW TWO FULL VIEWS.
SHADE AND DIMENSION LINES REQUIRED.

PROB. 2

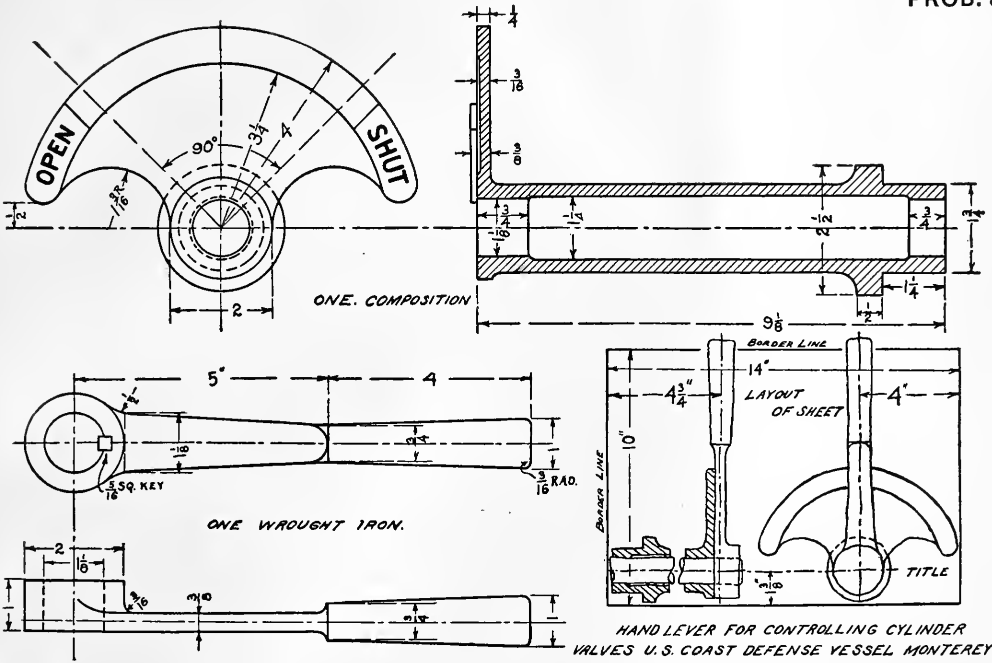


DRAW COMPLETE SECTION
SCALE 1"=1 FT.

DRAW COMPLETE SECTION
SCALE 3/4"=1 FT.

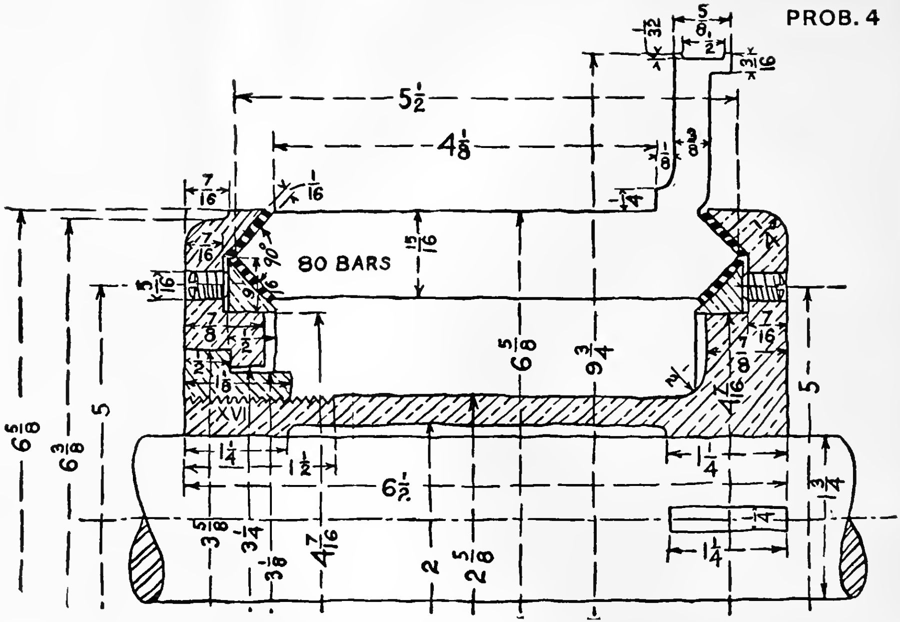
DRAW COMPLETE SECTION
SCALE 3/4"=1 FT.

PROB. 3



HAND LEVER FOR CONTROLLING CYLINDER VALVES U.S. COAST DEFENSE VESSEL MONTEREY

PROB. 4



PROB. 4. A sketch is given for the Commutator of a 10 K.W.D.C. Dynamo, and it is required to make a full-size sectional drawing of the complete commutator.

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