CONCRETE ENGINEERS' HANDBOOK

DATA FOR
THE DESIGN AND CONSTRUCTION OF PLAIN AND REINFORCED CONCRETE STRUCTURES

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FIRST EDITION
SECOND IMPRESSION

McGRAW-HILL BOOK COMPANY, INC.
239 WEST 39TH STREET. NEW YORK

LONDON: HILL PUBLISHING CO., LTD.
6 & 8 BOUVERIE ST., E. C.
1918

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First printing, May, 1918
Second printing, November, 1918

THE MAPLE PRESS YORK PA

PREFACE

This handbook has been prepared to make available in concise form the best of present day knowledge concerning concrete and reinforced concrete and to present complete data and details, as well as numerous tables and diagrams, for the design and construction of the principal types of concrete structures. Although intended as a working manual for the engineer, the first few sections of the book may be read with profit by any one engaged in concrete work. In these sections an effort has been made to present the latest authoritative knowledge in regard to the making and placing of concrete in such form that it may be applied in the field to the betterment of construction.

In preparing this book the authors have been ably assisted by Mr. S. C. Hollister, Research Engineer of the Corrugated Bar Company. Special credit is due Mr. Hollister for his application of the slope-deflection method to the development of formulas for rigid frame structures; for material relating to flexure of annular sections and to restraint of standpipe sides by connection with the base, this material being published here through the courtesy of the Corrugated Bar Company.

The authors also are greatly indebted to Messrs. Harvey Whipple, Adelbert P. Mills, Walter S. Edge, A. G. Hillberg, and Leslie H. Allen for the important chapters which they have prepared. These men are specialists; and their contributions will prove of great value to the engineering profession.

The chapter on dams, written by Mr. A. G. Hillberg, has been made brief, but a sufficiently extensive presentation is given to enable the reader to decide intelligently what type of dam to adopt. It is desired to call attention to the paragraphs dealing with siphonic spillways as there is practically no text-book information on that subject.

In writing this book the authors have drawn from the three volumes of Hool's "Reinforced Concrete Construction" only where the preparation of new material would have been substantial duplication.

The authors are under obligation to Messrs. Wm. J. Fuller and Frank C. Thiessen for many helpful suggestions and for assistance in making calculations for some of the tables and diagrams. Acknowledgments are also due to Mr. C. M. Chapman and others for suggestions and criticisms of manuscript; and to a large number of engineers who have supplied data and details, and have generously given their views in regard to both theory and practice.

The authors are indebted to Mr. Clifford E. Ives for his excellent work in preparing all drawings made expressly for this handbook.

G. A. H.
N. C. J.

April, 1918.

STANDARD NOTATION
USED THROUGHOUT THIS VOLUME
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CONCRETE ENGINEERS' HANDBOOK

SECTION I

MATERIALS

CEMENT

1. Classification, Composition, and Uses of the Principal Cementing Materials.—Cementing materials used in structural work may be divided into two main classes—non-hydraulic and hydraulic. Non-hydraulic cements, as the name implies, will not set and harden under water; while hydraulic cements will harden in either water or air. Following is a list of the structural cements of commercial importance:

Non-hydraulic:
- Gypsum plasters
- Common lime

Hydraulic:
- Hydraulic lime
  - (*Grappier cement*, a by-product)
- Puzzolan cement
- Natural cement
- Portland cement
  - (Adulterated or modified Portland cement)

1a. Gypsum Plasters.—Gypsum plasters are made by partial or complete dehydration of relatively pure or impure natural gypsum. The setting of these plasters is a recrystallization from a solution formed by admixture of the partially or totally dehydrated material with water, reforming the original substance. [Pure gypsum is a hydrous crystalline calcium sulphate ($CaSO_4 + 2H_2O$); and in its raw uncalcined state is used as an adulterant to retard the setting of Portland and natural cements. *Plaster of Paris* ($CaSO_4 + \frac{1}{2}H_2O$) is also used for the same purpose and is a refined plaster made from pure gypsum by dehydration.]

Gypsum plasters, of one variety or another, are used principally on interior walls and floors. They are also used in the form of molded hollow blocks and tiles for fireproof interior partition walls, and as one variety of "stucco" for the architectural adornment of buildings.

1b. Common Lime.—Common lime is made by burning limestone ($CaCO_3$) at a temperature of about 900°C. until its carbon dioxide (CO_2) is driven off as gas. The residue is common lime (CaO), known commercially as "quicklime." On addition of water this product slakes with evolution of heat and much increase of volume, forming a paste of lime hydrate, or calcium hydroxide ($Ca[OH]_2$) known as "lime putty" or, on dilution with water, as "cream of lime."

Even in the purest limestone to be found in nature some impurities are present. Generally a part of the lime (CaO) is found replaced by a certain percentage of magnesia (MgO), and clay is also present to some extent. [Clay is composed chiefly of silica (SiO_2) and alumina (Al_2O_3), and usually contains some iron oxide (Fe_2O_3).] In the manufacture of quicklime, magnesia acts in much the same manner and may be considered the equivalent of lime, which makes it possible to use limestone which is high in magnesia. Quicklimes are divided into four main types according to the relative content of calcium oxide (CaO) and magnesium oxide (MgO). These are:

1. High-calcium; quicklime containing 90% or over of calcium oxide.
2. Calcium; quicklime containing not less than 85% and not more than 90% of calcium oxide.
3. Magnesian; quicklime containing between 10 and 25% of magnesium oxide.
4. Dolomitic; quicklime containing over 25% of magnesium oxide.

Following is an average analysis of ten high-calcium quicklimes and two dolomitic quicklimes:

	SiO_2 %	Al_2O_3 %	Fe_2O_3 %	CaO %	MgO %
High-calcium..	0.81	0.22	0.23	94.98	1.39
Dolomitic.....	0.87	0.32	0.29	60.13	36.12

[Analysis also shows carbon dioxide (CO_2) and water (H_2O) to be present in small amounts.]

Practically all lime used in construction is made into mortar by adding sand to the paste of lime hydrate, as sand is not only cheaper than lime but diminishes the great shrinkage which accompanies the setting and hardening of lime putty. This hardening is due mainly to crystallization, but in addition some of the water in the hydroxide is gradually replaced by carbon dioxide from the atmosphere, causing a small part of the hydroxide to revert to the original calcium carbonate ($CaCO_3$).

Although common lime is used chiefly in combination with sand as a mortar in laying ordinary brick and stone masonry, it is also used extensively as an interior wall plaster and for gaging hydraulic cement mortars, either to make them easier to work or to reduce their permeability.

Hydrated lime is quicklime slaked at the place of manufacture. Its market form is that of a dry powder, and as such it can be mixed with sand more easily than can lime paste made by slaking ordinary quicklime on the work.

1c. Hydraulic Lime.—Hydraulic lime is made by burning argillaceous or silicious limestone at a temperature not less than 1000°C. When showered with water the product slakes completely or partially without sensibly increasing in volume, and possesses hydraulic properties due to the combination of calcium with silica contained in the limestone as an impurity, forming calcium silicate. It is the universal practice to slake the lime at the place of manufacture on account of the better results obtained.

Grappier cement is a by-product in the manufacture of hydraulic lime, produced by grinding the lumps of underburned and overburned material which do not slake. As might be inferred, grappier cement possesses properties similar to those of hydraulic lime.

Hydraulic lime is not manufactured in the United States on account of the abundance of raw materials suitable for the manufacture of Portland cement, with which hydraulic lime cannot compete as a structural material. A number of hydraulic limes and grappier cements are marketed as "non-staining cements"—that is, they do not stain masonry For this reason a considerable amount of this cementing material is annually imported from Europe for purposes of architectural decoration.

1d. Puzzolan or Slag Cement.—Puzzolan cement is made by incorporating hydrated lime with a silicious material, such as granulated blast-furnace slag, of suitable fineness and chemical composition. In Europe a natural puzzolanic material, such as volcanic ash.

is used at some plants in place of the blast-furnace slag. Silica, when finely enough divided, is soluble in water and chemically active. For this reason the materials are finely pulverized and intimately mixed by grinding, but are not calcined, the formation of calcium silicate taking place slowly and at ordinary temperatures.

Although this type of cement possesses hydraulic properties, it should not be confused with *slag Portland cement* (sometimes called *steel Portland cement*) which is produced by calcining finely divided slag and lime in a kiln and pulverizing the resulting clinker. Analysis of the small number of puzzolan or slag cements manufactured in this country shows approximately the following range in composition:

SiO_2 %	$Al_2O_3 + Fe_2O_3 + FeO$ %	CaO %	MgO %	S %	$CO_2 + H_2O$ %
27.2 to 31.0	11.1 to 14.2	50.3 to 51.8	1.4 to 3.4	0.15 to 1.42	2.6 to 5.3

They are normally slower in setting than Portland cements and on this account are usually treated with materials which will hasten the set—such as burned clay, high-alumina slags, caustic soda, sodium chloride, or potash. Puzzolan cements made from slag may be distinguished by their light lilac color, absence of grit, and low specific gravity (2.60 to 2.85). They also are high in sulphides, which render them liable to disintegration in air, nor are they suited for use in sea water, where there is always an excess of sulphates.

Puzzolan cement is not as strong or reliable as either natural or Portland cement and should be used only in unimportant structures or in unexposed work, such as foundations, where weight and bulk are more important than strength.

1e. Natural Cement.—Natural cement, as its name implies, is made from rock as it occurs in nature. This rock is an argillaceous (clayey) limestone, or other suitable natural rock, and it is burned at a temperature of from 900° to 1300°C., the clinker being then finely pulverized. The product does not slake, but possesses strong hydraulic properties, calcium silicate being formed and acquiring strength and rigidity through crystallization.

Unfortunately, the composition and characteristics of natural cement are subject to considerable variation. This is to be expected, since the composition of the rock from which it is made not only varies in different localities but is further subject to variation to some extent at least, even in the same deposit. Portland cement, on the contrary, is an artificial mixture, subject to control. This fact, together with its slow setting, is mainly responsible for the decrease in the use of natural cement and the adoption of Portland cement in all important structures.

In spite of these disadvantages, however, it is a significant fact that natural cements do not show disintegration with passage of time, while Portland cements frequently are most erratic in behavior. On comparing analyses of typical natural and Portland cements, it is at once noticed that natural cements have a higher percentage of silica, about the same percentage of alumina and a lower percentage of lime than have Portland cements. Excess lime, so generally prevalent in hydrated Portland cements, and not necessarily resultant on "free lime," is frequently the cause of much trouble. There is a distinct field of usefulness for natural cement which is largely overlooked by engineers at the present time. In many cases, perplexing problems could be effectively solved by its employment.

The following summary shows the range in composition of an average analysis of six well-known American natural cements:

SiO_2 %	Al_2O_3 %	Fe_2O_3 %	CaO %	MgO %
22.3 to 29.0	5.2 to 8.8	1.4 to 3.2	31.0 to 57.6	1.4 to 21.5

[Analysis also shows varying small amounts of alkalies (K_2O and Na_2O), anhydrous sulphuric acid or sulphur trioxide (SO_3), carbon dioxide (CO_2), and water (H_2O). Magnesia (MgO) is usually regarded as equivalent to lime in its action. The specific gravity of natural cements range from 2.7 to 3.1, with an average of 2.85.]

Natural cement is adapted to many uses, but its relatively low strength and slow hardening limit its field to structures where high stresses will not be imposed for several months after placing the concrete, as in large or massive structures where weight and mass are more essential than early strength—that is, in such structures as dams, abutments, foundations, and many underground structures. Mortar made with natural cement (either alone or mixed with lime mortar) is excellent for laying ordinary brick and stone masonry.

1f. Portland Cement.—Portland cement is made by finely pulverizing the clinker produced by burning a definite artificial mixture of silicious (containing silica), argillaceous (containing alumina), and calcareous (containing lime) materials to a point somewhat beyond where they begin to fuse or melt. The product is one that does not slake and possesses strong hydraulic properties. The essential components of Portland cement—namely: silica, alumina, and lime—are obtained from many different sources, but the proportions used of the raw materials are always such that the chemical composition of the different Portland cements is constant within narrow limits. The percentages of the principal components range about as follows:

SiO_2 %	Al_2O_3 %	Fe_2O_3 %	CaO %	MgO %
19 to 25	5 to 9	2 to 4	60 to 64	1.0 to 2.5

[Small amounts of alkalies (K_2O and Na_2O) and sulphur trioxide (SO_3) are also present. Magnesia (MgO) is considered by some as an impurity, while other investigators claim it is equivalent to lime (CaO) in its action. Alumina (Al_2O_3) and iron oxide (Fe_2O_3) do not act entirely alike but are usually considered to have the same functions.] The specific gravity of Portland cements range from 3.1 to 3.20, with an average of 3.15.

Portland cement is by far the most important cementing material used in modern engineering construction. It is adapted for use in concrete and mortar for all types of structures where strength is of special importance, or in structures exposed to wear or to the elements. It should invariably be employed in reinforced-concrete construction because of its high early strength and generally uniform quality.

A number of special cements employing Portland cement as a base are made by grinding in adulterating materials after calcination. These adulterants include clay, slaked lime, sand, slag, natural cement, limestones, and natural puzzolanic material or tufa. The action of these materials is essentially to promote combination between lime from the cement and silica from the adulterant, with formation of silicate of lime. In some cases these silicious adulterants improve the quality of concrete made from such cements, but this result cannot be expected from all forms of adulteration.

Sand and puzzolanic material have perhaps been used the most extensively and successfully of any of the adulterants, producing products known as *sand cement* and *tufa cement* respectively. These cements have been used principally on large work where freight rates are high and long wagon hauls combine to make the cost of undiluted Portland cement excessive. Cement specifications in common use are of a character to exclude any grinding in of materials after calcination, presumably on the ground that specifications permitting any adulteration would be subject to abuse so that the results obtained would be uncertain.

2. Portland and Natural Cements Compared.—The distinguishing properties of natural and Portland cements and the chief differences in manufacture may be summarized as follows:

	Natural cement	Portland cement
Raw material	Natural rock	Artificial mixture
Type of kiln used	Mostly the vertical stationary type	Slanting cylindrical revolving type
Calcination temperature	Low, but variable	Relatively high
Chemical composition	Variable, not under control	Controllable within narrow limits
Color	Yellow to brown	Bluish or steel gray
Specific gravity	2.7 to 3.1	3.1 to 3.2
Rate of setting	Relatively rapid	Relatively slow
Strength	Low, especially at early age	Relatively high
Degree of grinding	Usually rather coarse	Relatively fine
Soundness	Will *not* usually stand steam test	Required to stand steam test

In structures where either natural or Portland cement may be used, and where economy is the governing consideration, the choice of cement should be based on a comparison of the costs per cubic yard of the required mortar or concrete mixtures. Decisions usually are desired either between a 1:2 natural cement mortar and a 1:3 Portland cement mortar, or between a 1:2:4 natural cement concrete and a 1:4:8 Portland cement concrete.

3. Constitution of Portland Cement.[1]—The latest optical and microscopical examinations of Portland-cement clinker, and of all the substances which were formally considered as likely to be formed in manufacture, show Portland cement to be made up largely of the three compounds $3CaO{\cdot}SiO_2$, $2CaO{\cdot}SiO_2$, and $3CaO{\cdot}Al_2O_3$. The tri-calcium silicate appears the best cementing compound and it is probable that the higher its percentage, the better the cement. The small amounts present of Fe_2O_3, MgO, alkalies, etc. have but little effect on the three major compounds but their presence aids materially in manufacture by promoting the combination of CaO with Al_2O_3 and SiO_2.

A perfectly burned cement clinker consists of about 36% of tri-calcium silicate, $3CaO{\cdot}SiO_2$; 33% of di-calcium silicate, $2CaO{\cdot}SiO_2$; 21% of tri-calcium aluminate, $3CaO{\cdot}Al_2O_3$; and 10% of minor constituents. The principal cementing compound, tri-calcium silicate, $3CaO{\cdot}SiO_2$, is the last constituent to form completely in Portland-cement manufacture; and this compound is formed by the combination of CaO with $2CaO{\cdot}SiO_2$. When cement clinker is not perfectly burned there is evidently less $3CaO{\cdot}SiO_2$ formed and more $2CaO{\cdot}SiO_2$. There is also a certain percentage of free lime (CaO) present, the amount depending upon the degree of burning.

4. Setting and Hardening of Portland Cement.[2]—The setting and hardening of Portland cement is caused principally by hydration in the order named of the three major constituents—$3CaO{\cdot}Al_2O_3$, $3CaO{\cdot}SiO_2$, and $2CaO{\cdot}SiO_2$. When water is added to Portland cement, these constituents form first amorphous and later both crystalline and amorphous hydrated materials which act much as does ordinary glue, except that since they are of mineral origin and largely insoluble, hardening progresses even under water.

Of these hydration products, the compound tri-calcium aluminate ($3CaO{\cdot}Al_2O_3$) when mixed with water sets and hardens very quickly; tri-calcium silicate ($3CaO{\cdot}SiO_2$) sets and hardens somewhat less rapidly; and di-calcium silicate ($2CaO{\cdot}SiO_2$) reacts slowly. Hardening occurs only after the lapse of a long period of time. The *initial set* of cement is due undoubtedly to the hydration of $3CaO{\cdot}Al_2O_3$; the early hardness and cohesive strength is due to this hydration

[1] From paper before Am. Conc. Inst., Feb., 1916, by G. A. Rankin, Geophysical Laboratory, Carnegie Inst. of Wash.

[2] See Klein and Phillips: *Tech. Paper*, 43, U. S. Bureau of Standards.
Bates and Klein: *Tech. Paper*, 78, U. S. Bureau of Standards.

and to that of the $3CaO\cdot SiO_2$; while the gradual increase in strength is due to the further hydration of these two compounds together with the hydration of the $2CaO\cdot SiO_2$.

The compound $3CaO\cdot SiO_2$ appears to be the best cementing constituent of this group, as it is the only one of the three which when mixed with water will set and harden within a reasonable time to form a mass which is comparable in hardness and strength to Portland cement. Although $3CaO\cdot Al_2O_3$ sets and hardens rapidly, it is rather soluble in water and is not particularly durable or strong. The compound $2CaO\cdot SiO_2$, however, requires too long a time to harden to be in itself a valuable cementing material.

5. Manufacture of Portland Cement.

5a. Raw Materials.—Silica, alumina, and lime—the essential components of Portland cement—occur as ingredients in a large number of natural materials of widely varying character. In none of these, however, do the three components occur in the exact proportions required in Portland-cement manufacture so that an artificial mixture of several materials has to be resorted to. The following combinations of raw materials are used in different cement plants in this country:

1. Cement rock and limestone.
2. Marl and clay (or shale).
3. Limestone and clay (or shale).
4. Blast-furnace slag and limestone.
5. Chalk and clay.

Cement rock is an argillaceous limestone containing about 68 or 72% of lime carbonate, 18 to 27% of clayey matter, and not over 5% of magnesium carbonate. It is a dark slatey limestone, rather soft in texture, and is almost ideal for cement making due to the fact that it is easy to quarry and grind, and is usually so well balanced in composition that but a small amount of comparatively pure limestone needs to be added. This rock is found in many parts of the country but so far has been used in the manufacture of Portland cement only in the Lehigh district of eastern Pennsylvania and western New Jersey, a district producing nearly one-third the entire output of the United States.

Limestone suitable for cement manufacture is composed principally of calcium carbonate together with more or less impurities. The following shows the approximate range of composition of such limestones:

$CaCO_3$ %	SiO_2 %	$Al_2O_3 + Fe_2O_3$ %	$MgCO_3$ %
88.0 to 98.0	0.3 to 8.0	0.2 to 2.1	0.2 to 4.2

Sulphur as SO_3 and various alkalies may also be present in small percentages.

Marl is almost pure calcium carbonate. It is a soft, wet earth found in the basins of dried-up lakes and in swamp regions, deposited either by chemical agencies or through the physico-chemical agencies of certain forms of vegetable and animal life.

Clays and *shales* are of the same general composition, differing only in degree of solidification. Clays result from the decay of shales, and like their parent rock, are composed chiefly of silica (SiO_2) and alumina (Al_2O_3), and usually iron oxide (Fe_2O_3). The proportion of silica in clay suitable for cement manufacture should not be less than 55 to 65%; and the combined amount of alumina and iron oxide should be between one-third and one-half the amount of silica. A clay with these proportions of principal constituents is highly siliceous, producing a cement clinker which is comparatively easy to grind. Clay containing a greater portion of alumina produces a hard clinker and a quick-setting cement which is more severely attacked by sea water.

Blast-furnace slag is a compound formed from impurities in the iron ore and the limestone used as a flux in the blast furnace. Following is a typical analysis of slag used in the manufacture of cements:

SiO_2 %	$Al_2O_3 + Fe_2O_3$ %	CaO %	MgO %
33.10	12.60	49.98	2.45

Slag, when allowed to cool quickly, becomes a hard glassy mass, very tough and durable. In order to use slag economically in cement manufacture the molten slag is run into large cisterns, and there converted into a granular substance by directing against it innumerable little streams of air and water. This disintegrated slag has the appearance of coarse brown sugar; and in this form it is used as a raw material for Portland cement.

Chalk is a soft earthy variety of calcium carbonate, formed from the remains of minute organisms. It also sometimes contains small amounts of silica, alumina, and magnesia. Its use as a material for cement manufacture is limited.

5b. Proportioning the Raw Materials.—Two rules are in use for proportioning the raw materials used in the manufacture of Portland cement. Newberry's rule is as follows:

$$\text{Max. lime} = 2.8\ (\%SiO_2) + 1.1\ (\%Al_2O_3)$$

Eckel's rule (called the "cementation index"), which is really a modification of the above rule, takes the magnesia and iron oxide into account:

$$\frac{2.8\ (\%SiO_2) + 1.1\ (\%Al_2O_3) + 0.7\ (\%Fe_2O_3)}{\%CaO + 1.4\ (\%MgO)} = 1$$

A value of the cementation index below 1 means an excess of lime or magnesia in the cement which will cause expansion, or unsoundness. In practice it is customary to reduce by about 10% the proportion of lime found by the above rule to avoid any chance of obtaining an unsound cement. Although the above rules are not based on the most recent investigations of the constitution of Portland cement there is no immediate prospect of any change being made in practice in the methods of proportioning because the present rules are known by experience to produce excellent results.

5c. Grinding and Mixing.—The admixture and grinding of the raw materials before calcination is accomplished by either a wet or a dry process. In the wet process, principally for plants using marl, the raw materials are ground and fed into rotary kilns in the form of a slurry containing sufficient water to make it of a fluid consistency. In the dry process raw materials are ground and mixed in the dry state. The larger portion of Portland cement manufactured in the United States at the present time is made by plants using the dry process.

5d. Burning the Cement Mixture.—Rotary kilns are used in almost all American Portland-cement plants. These kilns are slightly inclined to the horizontal and revolve at about the rate of one revolution per minute. The ground raw materials are fed in at the upper end and are carried forward and tumbled over and over by the slant and revolution of the kiln. As the materials advance they are reduced by the hot gases from the burning fuel which is fed in at the lower end. The clinkers formed vary in size from ¼ in. up to about 1½ in. in diameter. It takes about an hour for a particle of raw material to traverse the entire distance from the feed to the outlet.

5e. Treatment of the Clinker.—After cement clinker is cooled it is crushed and passed through preliminary grinding mills. Then gypsum is added and the clinker ground to a fine powder. If the clinker was used without the addition of gypsum, it would take an almost immediate set. Approximately 2 lb. of gypsum ($CaSO_4$) (or *plaster of Paris*) is used to every 100 lb. of clinker.

6. Manufacture of Natural Cement.

6a. **Raw Material.**—The raw material used in the manufacture of natural cement is a natural argillaceous limestone containing from 13 to 35% of clayey material. About 15% of the clayey material is silica, the balance being alumina and iron oxide. The kind of limestone generally used contains a considerable proportion of magnesium carbonate in place of calcium carbonate. When the rock varies greatly in composition, materials from different strata are mixed together to give as uniform a product as possible. The wide range allowed in the composition of natural cement, however, does not warrant great refinement in the analysis of the rock.

6b. **Process of Manufacture.**—Natural cement is usually manufactured in vertical kilns lined with firebrick. These kilns are of the mixed-feed type, the rock (not crushed) and fuel being charged in alternate layers. The temperature required in burning is considerably below that required in Portland-cement manufacture because temperatures higher than 1300°C. would fuse the material to a slag having no hydraulic properties. The temperature employed, however, is sufficient to cause the formation of silica compounds with the lime and magnesia. The clinker is taken out at the bottom of the kiln as it is burned, and then it is crushed and ground. Grinding of the clinker is not usually carried as far as that of Portland although some of the newer mills use grinding machinery similar to that in Portland-cement plants.

7. Testing of Cement.—For standard methods of cement testing, see *Appendix A*.

7a. **Sampling.**—Tests should be conducted only on representative samples. For method of sampling, see *Appendix A*, page 834.

7b. **Uniformity in Cement Testing.**—In order to obtain results in cement testing which will be of the greatest value, definite and uniform methods should be used. Results depend not only on the quality of the cement but also on the temperature and percentage of water used in mixing, the method of mixing and molding test specimens, the temperature and humidity of the air, the character of the sand used, and the type of apparatus employed.

7c. **The Personal Factor.**—The personal factor has considerable effect on results obtained in cement testing and, on this account, only experienced, well-qualified men should be employed in making tests. Results by untrained or careless operators are really worse than nothing and may be positively misleading. The comparative results, however, by any one experienced observer are generally consistent and are of value. It is usually advisable to have the testing done at some well-established and properly equipped cement-testing laboratory.

7d. **Kinds of Tests.**—The following cement tests made regularly are recommended for construction work of importance and also in all cases where the cement to be used does not work satisfactorily:

Fineness.
Time of setting.
Tensile strength of standard mortar. (Compressive strength of standard mortar the best criterion.)
Soundness.

On unimportant construction it is generally safe to use a well-known brand of Portland cement without testing, or to make simply the test for soundness.

7e. **Fineness.**—Fine grinding has a great influence on the properties of cement. It increases the ability of the cement to react readily with water and enables the cement particles to coat the sand grains more thoroughly. In other words, the finer the cement, all other conditions being the same, the stronger will be the mortar produced with a given sand.

The fineness of cement is measured by determining the percentage by weight which will be retained on a standard 200-mesh sieve.[1] Standard specifications[2] require that the residue shall not exceed 22%. Most mills are now equipped to grind cement to such a fineness that even less than 10% is retained.

It has long been generally recognized that the coarser particles in cement are practically

[1] For description of standard sieve, see *Appendix A*, p. 836.
[2] For standard specifications, see *Appendix A*, p. 833.

inert and that at least the earlier cementing value is due chiefly to the grains that will pass the No. 200 sieve. Because of this fact the present standard method of testing for fineness is unsatisfactory, as no attempt is made therein to determine the further fineness of the greater and more valuable part of the cement. To remedy this defect an air-analyzer[1] has recently been perfected at the Bureau of Standards which makes it possible to further divide a cement which passes a 200-mesh sieve into four definite sizes. This apparatus had been standardized and will undoubtedly come into extensive use.

Increased fineness has the effect of making a cement quicker in setting and hardening, the high-alumina cements being the most affected. Fine grinding also affords additional opportunity for seasoning and thus indirectly improves the soundness of cement.

7f. Normal Consistency.—Tests for time of setting, strength, and soundness are greatly influenced by the quantity of water used in mixing. In order to have all results comparable with one another, a determination is made in each case of the quantity of water necessary to be added to a given weight of cement to give a standard or normal consistency.

A simple method of finding normal consistency is to mix a quantity of cement paste and make up from the paste a ball about 2 in. in diameter. The ball is then dropped upon the testing table from a height of 2 ft. The paste is of normal consistency when the ball does not crack and does not flatten more than one-half of its original diameter. The finer the cement, the more water is required for normal consistency. For this test the room and the mixing water should be kept at standard temperature.

Another method of finding normal consistency which is more commonly used and gives more concordant results is by the use of the Vicat needle apparatus (see Fig. 2, *Appendix A*, page 837). The manner of making this test is explained in *Appendix A*, page 838.

7g. Time of Setting.—The time of setting of a cement may vary within wide limits and is no certain criterion of quality, but it is important in that it indicates whether or not the cement can be used advantageously in ordinary construction. A cement may set so quickly that it is worthless for use as a building material (since handling cement after it commences to set weakens it and causes it to disintegrate), or it may set so slowly that it will greatly delay the progress of the work.

Age of cement has a great effect upon the setting time, and tests should preferably be made after delivery of the cement on the work. Most cements absorb moisture from the air and lose some of their hydraulic property on storage. It also occasionally happens that the gypsum added in manufacture loses its effectiveness in a short time, and in consequence the cement becomes quick setting. The cause of this loss of effectiveness of the gypsum is due usually to the composition of the cement and may be remedied by increasing the lime content.

Aside from the consideration of age, the conditions which accelerate setting are: finely ground and lightly burned material; dry atmosphere; small amount of water used in gaging; and high temperature of both water and air. Since the time of set is influenced by so many factors, tests should always be made with extreme care under standardized conditions.

There are two distinct stages in setting: (1) the initial set; and (2) the hard or final set. The best cements should be slow in taking the initial set but after that should harden rapidly. Portland cement should acquire the initial set in not less than 45 min. when the Vicat needle is used (see *Appendix A*, page 837), and hard set in not more than 10 hr.[2] The time of initial set is controlled largely by the amount of sulphate (gypsum or *plaster of Paris*) which is added in making the cement.

A cement has taken its initial set when it will not thoroughly reunite along the surfaces of a break. It has taken its final set when it begins to have appreciable strength and hardness.

There are two methods in common use for finding the time of setting. The method usually preferred is by the use of the Vicat needle apparatus explained in *Appendix A*, page 838. The

[1] Copies of *Tech. Paper* 48, the publication upon this subject, may be obtained, free of charge, upon application to the Bureau of Standards, Washington, D. C.

[2] See standard specifications in *Appendix A*, p. 833.

other method is by the use of the standardized Gillmore needles described in *Appendix A*, page 841.

7h. Tensile Strength.—The testing of cement in tension is to obtain some measure of the strength of the material in actual construction. In other words, tests of tensile strength are made primarily to determine whether the cement will be likely to have a continued and uniform hardening in the work, and whether it will have such strength when placed in mortar or concrete that it can be depended upon to withstand the strain placed upon it.

The small shapes made for testing are called *briquettes* (see details of standard test piece in *Appendix A*, page 842) and have a minimum cross-sectional area of 1 sq. in.—that is, at the place where they will break when tested. Standard mortar used in testing is composed of 1 part cement to 3 parts of standard sand[1] from Ottawa, Ill.

It is customary to store the briquettes, immediately after making, in a damp atmosphere for 24 hr. They are then immersed in water until they are tested. This is done to secure uniformity of setting, and to prevent the drying out too quickly of the cement, thereby preventing shrinkage cracks which greatly reduce the strength.

Specifications for tensile strength of cement usually stipulate that the material must pass a minimum strength requirement at 7 and 28 days. This is required in order to determine the gain in strength between different dates of testing so that some idea may be obtained of the ultimate strength which the cement will attain. A first-class cement, when tested, should give the values for tensile strength stated in the standard specifications (see *Appendix A*, page 833).

7i. Relation between Tensile and Compressive Strength.—Since cements are rarely depended upon to withstand tensile stresses, the test for tensile strength has undoubtedly become standard on account of the popular belief that there exists a more or less definite and constant relation between the tensile and compressive strengths. It can be shown, however, that the ratio of compressive to tensile strength of cement mixtures is by no means constant at all ages and varies greatly with different cements and with different mixtures. Thus the tensile strength cannot usually be regarded as any more than a very approximate indication of the probable compressive strength of the same cement.

7j. Compressive Strength.—Compressive strength of cement mortar is undoubtedly a better criterion by which to judge the suitability of a cement for use in construction. The American Society for Testing Materials has tentative specifications and methods of tests for compressive strength of Portland-cement mortar[2] which, when adopted as standard by the Society, will be inserted in and made a part of the American Specifications and Methods of Tests for Portland Cement. A foreign standard specification is as follows:

"Slowly setting Portland cement shall show a compressive strength of at least 120 kg. per sq. cm. (1710 lb. per sq. in.) when tested with 3 parts by weight of standard sand, after 7 days' hardening, 1 day in moist air and 6 days under water; after further hardening of 21 days in the air at room temperature (15° to 20°C.) the compressive strength shall be at least 250 kg. per sq. cm. (3570 lb. per sq. in.). In cases of controversies, only the test after 28 days is decisive.

"Portland cement which is intended for use under water shall show a compressive strength of at least 200 kg. per sq. cm. (2850 lb. per sq. in.) after 28 days' hardening, 1 day in moist air and 27 days in water."

7k. Soundness.—A cement to be of value must be perfectly sound; that is, it must remain constant in volume and not swell, disintegrate, or crumble. Excess of either lime, magnesia, or sulphates may cause unsoundness. The usual method of testing is to form a small pat of neat cement about 3 in. in diameter, ½ in. thick at the center, and tapering to a thin edge. This pat should remain 24 hr. in moist air and 5 hr. in an atmosphere of steam at a temperature between 98 and 100°C. upon a suitable support 1 in. above boiling water. To pass the soundness test satisfactorily, the pat should remain firm and hard, and show no signs

[1] See *Appendix A*, p. 841.
[2] See *Proc.* of the Society, vol. xvi (1916), part I (pp. 590–593).

of cracking, distortion, checking or disintegration. The steam test is what is called an *accelerated* test and is for the purpose of developing in a short time (5 hr.) those qualities which tend to destroy the strength and durability of a cement.[1]

7l. Specific Gravity.—A test for finding the specific gravity of Portland cement was originally considered to be of value in detecting adulteration and underburning, but is no longer thought to be of much importance in view of the fact that other tests lead to more definite conclusions. One trouble has been that specific gravity is not alone lowered by the above causes. Seasoning of either cement or cement clinker, for instance, although known to be desirable and in some cases absolutely necessary, lowers the specific gravity materially. On the other hand, many underburned cements show a specific gravity much higher than that set by standard specifications. These considerations, together with the fact that the principal adulterants have a specific gravity very near that of Portland cement, make it difficult in the specific gravity test to obtain results from which accurate conclusions can be drawn. The test in any case is without value unless every precaution is taken to have accurate results, as otherwise only very large amounts of adulterated material could be discovered. When the specific gravity of a cement falls below 3.10, standard specifications[2] allow a second test to be made upon an ignited sample—the idea being that ignition will lower the specific gravity of adulterated cement. This second test, however, is usually of little value as the ignition loss of most adulterants is low and as the specific gravity of an ignited sample of cement is invariably higher than that of the original sample.

7m. Chemical Analysis.[3]—If the tests of a cement for time of setting, strength, and soundness seem to indicate adulteration, resort may be had to chemical analysis. Such analysis is not usually made in routine commercial testing. Chemical analysis not only serves as a valuable means of detecting adulteration but shows the amounts of magnesia (MgO) and sulphuric anhydride (SO_3) contained in the cement. Specifications usually limit the amount of MgO to about 5% and SO_3 to about 2% because of fear that more of these materials may make the cement unsound.

8. Specifications for Cement.—Standard specifications are given in *Appendix A*.

9. Containers for Cement.—Cement may be obtained in cloth or paper bags, in bulk, and in barrels.

Cloth bags are the containers most generally used since manufacturers will refund the extra charge for the bags when returned in good condition. The consumer, however, must prepay the freight when returning the empty bags to the mill. The cloth bag will stand transportation, and its size and shape make it convenient to handle. If properly cared for, it may be used over and over again. Paper bags are more delicate and have no return value. Wooden barrels are advisable when the work is in a damp location, as in marine construction. Bulk cement requires special preparations for handling and storage.

10. Storing of Cement.—Cement either in containers or in bulk should be stored within a tight, weather-proof building, at least 8 in. away from the ground and an equal distance from any wall, so that free circulation of air may be obtained. In case the floor of a storage building is laid directly above the ground, it would be well to give the cement an additional 8-in. elevation by means of a false floor, so as to insure ventilation underneath. The cement should further be stored in such a manner as to permit easy access for proper inspection and identification or removal of each shipment. When cement is not mill-tested, a proper period before cement is needed should be allowed by the contractor for inspection and tests, this period being determined by the provisions of the specifications governing his contract.

Where cement in bags is stored in high piles for long periods, there is often a slight tendency in the lower layers to harden, caused by the pressure above; this is known as *warehouse set*.

[1] The Lackawanna Railroad Co. requires that Portland cement used in its structures shall remain sound after being subjected to boiling under a 20-atmosphere pressure. This is called the *Autoclave Test*.

[2] See *Appendix A*, p. 833.

[3] For method to be followed in making a chemical analysis, see *Appendix A*, p. 834.

Cement so [illegible] is in every way fit for service and can be reconditioned by letting each sack drop on a hard surface before using the cement contained.

11. Seasoning of Cement.—A moderate amount of seasoning in weather-tight sheds often improves the quality of the cement. Fresh cement contains small amounts of free or loosely combined lime which does not slake freely and causes expansion after the mass has set, endangering the structure in which it is used. During the time of seasoning such free lime is changed first to hydrate and then to carbonate of lime which does not swell on wetting. Usually cement is seasoned at the mills before shipping, but, with the best mills, the stock house may run so low in periods of rush that a chance will be taken on fresh material. Well-seasoned cement, therefore, may be lumpy, but the lumps are easily broken up. If, however, the cement has been subjected to excessive dampness, or has been wet, lumps will be formed which are hard and difficult to crush. A distinction should be made, so that the latter will not be used without sifting and rejection of hardened portions.

12. Use of Bulk Cement.—Within the past few years considerable cement has been shipped in bulk to cement-product factories and to construction jobs adjacent to railroad tracks. Economy has, in these instances, resulted from the saving in labor, and from the elimination of package losses and expense. There seems to be no difficulty in shipping bulk cement in tight box cars.

13. Weight of Cement.—A barrel of Portland cement weighs 376 lb., not including the barrel, and a bag of Portland cement weighs 94 lb.; in other words there are 4 bags to a barrel.

A barrel of natural cement varies in weight according to the locality in which it is manufactured. A barrel of Western cement usually weighs 265 lb. and a barrel of Eastern cement 300 lb. A bag of natural cement is usually one-third of a barrel.

A barrel of puzzolan cement is usually assumed to contain 330 lb. net, and there are 4 bags to the barrel.

A cement barrel weighs about 20 lb. on an average.

AGGREGATES

14. Definitions.—"Aggregates" is a general classifying term applied to those inert (*i.e.*, chemically inactive) materials, both fine and coarse, which, when bound together by cement, form the substance known as concrete. Fine aggregates are materials such as natural sand or rock screenings. Coarse, or large aggregates, or ballast are materials such as natural gravel, crushed rock, or by-product materials such as cinders or crushed blast-furnace slag.

15. General Requirements.—Aggregates, fine and coarse, compose approximately 90% or more of the substance of concrete. From this it follows that the properties of aggregates must correspond and be at least equal to the properties desired in the concrete.

The usual service requirements are that aggregate shall be dense, hard, durable, structurally strong and, for aggregates in concretes exposed to water action, insoluble. Further, since concrete is formed by bonding of aggregates with cement, they must permit by their physical characteristics (such as roughness) the adhesion of cement; and always all particles must be clean, so that a surface coat of one kind or another may not prevent physical contact with cement, or destroy its properties through chemical action.

16. Classification of Aggregates.—The usual classification of aggregates is into two divisions, based upon size.

Coarse aggregates are all particles of gravel, crushed stone, or other materials above ¼ in. diameter.

Fine aggregates are all particles below ¼ in. in size. Particles of such size are further divided by defining "sand" as all mineral particles from 2 mm. (1/12 in.) to 0.5 mm. in diameter; "silt," all particles from 0.5 mm. to 0.005 mm. in diameter; "clay," all particles having a diameter less than 0.005 mm.; and "loam" as a mixture of any of the above finer varieties with organic matter—*i.e.*, of vegetable or animal origin. It is particularly such organic matter rather than size of particle which renders loam unfit for concrete work, as through some chemical action not yet fully understood, possibly through formation of an organic acid, it injures or inhibits the proper action of cement.

17. Qualities of Fine Aggregates.—General.—When it is remembered that the finer natural materials are derived from rocks by disintegration and by "weathering," or breaking down through frost action, water and wind erosion, or kindred agencies, the differences in quality so often found in sand deposits, with possibly the presence of foreign materials, are not surprising. Further, sands necessarily partake of the qualities of the rock from which they are derived. Silicious quartz sands are best for concrete work, but crushed sands from any durable rock will answer, if natural sand of proper quality cannot be obtained.

18. Qualities of Coarse Aggregates.—General.—For coarse aggregates, any crushed rock of durable character, or any clean, hard, natural gravel not subject to ready disintegration may properly be used. In general, the better the stone or gravel, the better the resulting concrete. For this reason, granite, trap, or hard limestone are preferred for large aggregates, but any rock will serve which is sound, which has adequate strength and does not contain objectionable mineral inclusions liable to decompose, such as iron pyrites, FeS_2, which may form sulphuric acid by oxidation (see Fig. 1).

Fig. 1.—Iron pyrites (FeS_2) in concrete aggregate oxidizing and destroying adjacent matrix. (Magnified 40 diams.)

Since the properties of any concrete are so closely related to the properties of its components, it is essential to an understanding of the value of any stone as an aggregate that something be known of the origin, nature, and properties of the varieties in common use.

19. Materials Suitable for Coarse Aggregates.—Roughly, rocks suitable for use as aggregate fall into three groups. These are: (1) Granite and other igneous rocks; (2) sandstones and other sedimentary rocks; (3) limestones and related rocks. A fourth division comprises slates and shales, but as these weather rapidly with formation of clay, they are unsuited for use in concrete.

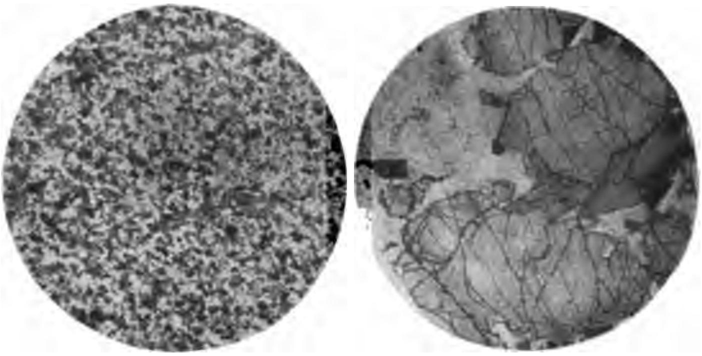
The physical character of a rock depends upon two things—its mineral constituents and its structure. If the mineral constituents are themselves durable, but massed together in a manner structurally weak, rapid weathering, with formation of sand through liberation of mineral grains, is to be expected. Such a rock would make a poor concrete. On the other hand, a dense structure with like mineral constituents would make an excellent aggregate. A dense structure and weak mineral constituents are sometimes associated, but Nature has generally cared for such rocks by bringing about their decomposition, so that they exist only as sand.

20. Igneous Rocks.—Igneous rock is a general term descriptive of all rocks formed from molten matter which has consolidated either into mineral, or glass, or both. Among such rocks are granite and trap rock. Many classifications of a more-or-less satisfactory nature have been devised; but all sorts of gradations exist between the various types, rendering their descriptive identification difficult.

20a. Granite.—Granite is well-known by its characteristic appearance (see Fig. 2). In structure, it is a blend of quartz (crystallized silica dioxide), orthoclase, and mica, though this latter may be replaced by hornblende. It is exceedingly dense, hard, and durable, consisting entirely of minerals with no glass or uncrystallized material between its constituent grains.

Granites possess the strength and durability desirable in an aggregate, but they are of low toughness. In addition, if used in concretes exposed to more than ordinary heat, as in chimneys, there is a decided tendency to disintegrate, due to unequal mineral grain expansions. Granites are not often used as aggregate, their ornamental value precluding less profitable use.

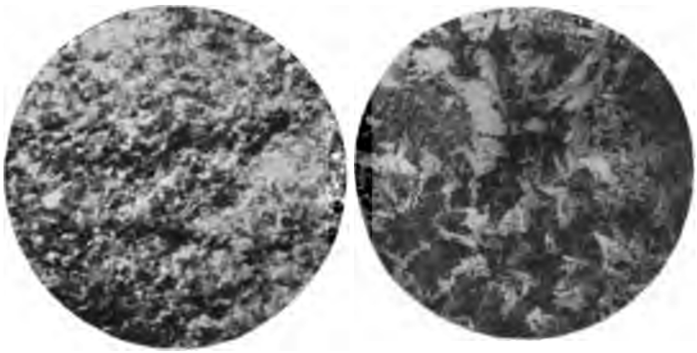
20b. Trap Rock or Diabase.—Trap rock (Fig. 3) and fine-grained basic and volcanic rocks are generally hard, of high abrasive value adhering well to cement. These rocks



Surface appearance. (Natural size.) Internal structure. (Magnified 60 diams.)
Fig. 2.—Granite.

have a closely interlaced mineral structure and generally good resistance to stress. Care should be taken not to choose a trap rock having a considerable percentage of iron present in low oxide form, as this may absorb oxygen, forming a higher oxide, with expansion and probably rupture.

In general, trap rock (and rock of similar character, in which class are included many of the "green-stones") makes a very excellent aggregate although in some respects its excellence



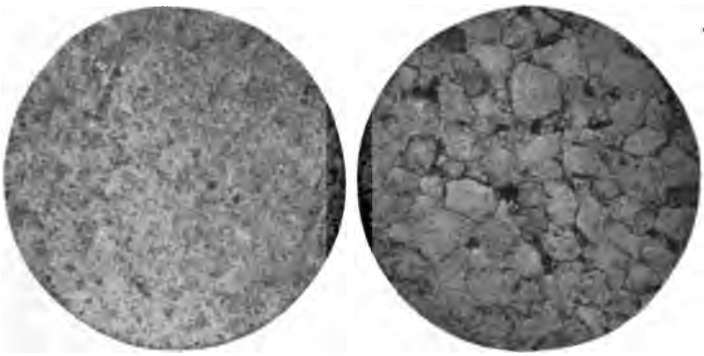
Surface appearance. (Natural size.) Internal structure. (Magnified 60 diams.)
Fig. 3.—Trap rock.

has been exaggerated. It has, however, a very high compressive strength and, as this quality is very desirable in concretes, its use has become widespread. It is not always procurable without excessive cost but, where price is not prohibitive, its use is advantageous.

21. Sedimentary Rocks.—To the sedimentary series of rocks belong all those solidified deposits which have accumulated at the bottom of bodies of water. Originally, these materials were derived from the land surface and transported to the sea or lakes, either by mechanical carriage, or by solution in water. Many of the minerals contained in sedimentary rocks were derived directly from the decay of igneous or volcanic rocks, although additional chemical changes supplementing this more-or-less complete decomposition of the original minerals may have resulted in the formation of new minerals found in the sedimentary series. With passage of time and the action of various chemical and mechanical agencies, these sedimentary deposits solidified into the stratified rocks of one kind or another found throughout the entire surface of the earth.

21a. Sandstone.—One of the most important of the sedimentary rocks is sandstone (Fig. 4). In structure, it is natural concrete, composed of finely divided mineral particles, cemented together in more-or-less close relation by iron or alumina or by calcium compounds. The character of any sandstone depends, therefore, on the mineral character of its component grains; on the size and shape of these grains; on their arrangement within the rock; and on the nature of the material cementing them together.

Quartz particles form by far the greatest percentage and the most desirable constituent of sandstone. Feldspar is also frequently present, and occasionally, hornblende, chlorite, garnet, magnetite, and calcite. In the best sandstones, the grains are arranged uniformly through the



Surface appearance. (Natural size.) Internal structure. (Magnified 60 diams.)

FIG. 4.—Sandstone.

mass, although frequently coarser and finer particles are arranged in layers, giving a stratified appearance to the stone.

So far as its use in concrete is concerned, the most important feature of a sandstone is the nature of the cementing material combining its constituent grains. *Argillaceous sandstones* in which the cementing material is lime (usually lime carbonate) may be crushed with comparative ease, but they disintegrate rapidly on exposure to weathering agents, such as water or air. Such stone may be readily identified by its effervescence when treated with a drop of hydrochloric acid. *Sandstones cemented by oxide of iron* are generally red in color, the shade being a rough indication of the amount of iron present. Many of these sandstones disintegrate very rapidly on exposure to the weather, forming the so-called "rotten stones" so often found in gravel.

Sandstones cemented solely by clay should never be used in concrete, as the simple penetration of moisture is sufficient to disintegrate them, rendering them practically valueless as aggregate. A good accelerated test is to boil ¼-in. fragments of the stone in water. Rapid disintegration indicates a weak stone, with a tendency to weather rapidly, and unsuited for use as aggregate.

21b. Limestone.—Limestone is carbonate of lime deposited on the floors of bodies of water and subsequently hardened into rock. This precipitation of lime may have been effected from the water, or through the agency of animal or vegetable life. That is to say, some limestones are chemical precipitates, while others are formed from the shells and other hard parts of animals, as well as from hardened tissues of certain plants. Such plant and animal forms fossilized are often seen in limestone fragments (see Fig. 10, page 19).

Compact limestone (Fig. 5) varies in texture from coarse to exceedingly fine. (It is practically impossible to obtain a good photograph of the surface of limestone on account of its dark color and uniform texture.) It is only occasionally pure carbonate of lime, usually containing greater or less percentages of magnesia. Either magnesian limestone, or pure calcic limestone is very well suited for use as a concrete aggregate. Any considerable percentage of clay in limestone, however, is very undesirable, as it softens the rock and renders it very liable to disintegration. Limestone is found in many colors. White, gray, yellow, blue, and green are those of most frequent occurrence.

In general, limestone makes a very good coarse aggregate for concrete. When crushed to the finer sizes, it has a flaky fracture which renders it somewhat unsuitable for use as sand unless it is rerolled. Natural limestone sands are of infrequent occurrence, as limestone is soluble to as high a percentage as 90%, so that the usual weathering processes result in solution, rather than fragmentary disintegration.

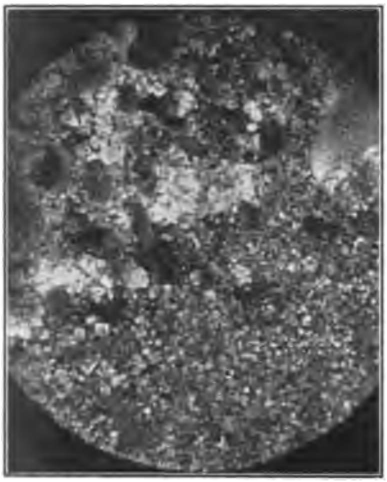


FIG. 5.— Internal structure of limestone. (Magnified 20 diams.)

22. Metamorphic Rocks.—Rocks of either igneous or sedimentary origin have often been subjected to such severe treatment in the long course of geologic history, that their ordinary character is much altered. Crushing of the earth's crust, the weight of overlying material, and contact with hot molten rock from the interior are among the causes contributing to the change. Such rocks are classed as "metamorphic."

There are many metamorphic rocks, the whole group constituting a very high percentage of the surface of the earth's crust. Some of them are of value as aggregates in concrete; while others, notably the slates and shales, have a weak stratified structure and weather so rapidly that their value in concrete is almost nothing.

The above classification gives a general indication of those rocks which when crushed are of value as concrete aggregates. The first caution to be observed in selecting them is to be sure that they do not contain objectionable impurities in their substance; and the second important caution is to be sure they are clean.

23. Gravel.—Gravel of good quality (Fig. 6) makes excellent concrete (see *Tech. Paper* 58, Bureau of Standards, Washington, D. C.). Gravel is nothing more nor less than natural rock, broken away from parent ledges and worn round by the rolling of streams. Its natural properties, therefore, are identical with the rock of which it once formed a part. Provided it has not decayed through being in relatively small masses, the properties natural to this parent rock are to be expected of a gravel. The surface of gravel is usually very rough; and from considerations of character of surface presented for adhesion of cement, it should produce as good as, and even better, concrete than crushed stone. Certainly, there is no reason against its use, provided it is clean and of good mineral quality.

It is desired to emphasize in this connection that not the least important of all the qualities of stone or gravel is its cleanness. The percentage of concretes in which cement and aggregates have little or no adhesion due to a coating of dirt (a coating of "matter out-of-place") is surprisingly large; and the careless acquiescence of engineers in the use of such materials is resulting in a general inferiority of concrete structures. "Dirt" in such cases may be visible (as when the coating is clay, or of tenacious dust due to crushing) or it may be quite invisible (such as a coating of colloidal, transparent, organic matter) requiring chemical procedure for its detection. A coating of any character is not to be disregarded, when first-quality concretes are desired. At best it is a detriment and oftentimes proves a serious defect, greatly weakening the concrete.

24. Blast-furnace Slag.—Slag from blast furnaces, crushed to proper size, has much to recommend it for mass construction. Slag is a hard though very porous material, of high compressive strength; and in certain localities is relatively cheap as compared to stone of good qualities. Offering a rough, pitted surface for the adhesion of cement, it produces a very strong concrete, but care should be taken that its sulphur content is low, else passage of time may bring about disintegration of the concrete. Some steel companies exercise great care in the preparation of slag for aggregate, weathering it in thin layers for 2 or 3 years before marketing, but the advisability of its use in concretes exposed to dampness and especially in thin sections is yet in controversy (see *Proc.* Am. Soc. Test Mat., 1913).

Fig. 6.—Enlarged surface of piece of gravel, showing roughness. (Magnified 5 diams.)

25. Cinders.—Furnace cinders as an aggregate are used only in inferior grades of mass concrete, or for fireproofing. Cinders have low structural strength, high porosity, and oftentimes as an added objection, high sulphur content. In more than one instance, sulphuric acid resulting from sulphur decomposition in cinder concrete floors has eaten away conduits and piping, and has even attacked reinforcing and structural steel. Cinder concrete is of value chiefly because of its cheapness and low specific gravity; but discrimination is required in its use.

26. Materials Suitable for Fine Aggregates.—All fine aggregates are essentially rock fragments, crushed to varying degrees of fineness, either by the natural processes of weathering, disintegration, or glacial action, or by man with his machines. Sand deposits are masses of weathered rock minerals, transported, collected, and sorted by the age-long action of streams (see Fig. 7).

Fig. 7.—Concretionary sandstone weathering to form sand.[1]

From the earliest ages the formation of sand, silt, and clay has been going on through the breaking down of rocks. The changes involved in these processes are part physical and part chemical. All changes produced at or near the surface by atmospheric agents, which result in more or less complete disintegration and decomposition, are classed under the general term of "weathering." The action of physical agents alone, which results in the rock breaking down into smaller particles without destroying its identity, is termed "disintegration" (see Fig. 8). On the other hand, the action of chemical agents destroys the identity of many of the minerals by the formation of new compounds, and this latter process is known as "decomposition." Silt and clay generally result from decomposition; and, as such chemical change has altered the character of the material (usually to its detriment so far as concrete purposes are concerned), that is one reason, but not the only reason, against permitting their presence in concrete sand.

Since coarse sands are of a size to retain and partake of the nature and properties of the parent rock, the structure of at least larger particles should be identical with the structures of

[1] From "Engineering Geology," by Ries and Watson.

such rocks; and their strength and fitness for use in concrete may be judged with more or less accuracy from a consideration of the structure and strength of those rocks known to be suitable for concrete work. There are few rocks which do not contain silica in greater or lesser quantities.

Because of its hardness and resistance to chemical agents, quartz or silica is, therefore, the commonest mineral in sand. Other minerals such as feldspar, mica, etc., though originally present, because of their lesser resistance, have been more readily decomposed by the action of the elements; and by reason of their complete disintegration with resultant fine state of subdivision, have been removed by wind and water. Quartz crystals, therefore, remain as the most evident survivors of the parent rock and their survival is evidence of their desirable qualities for concrete.

Fig. 8.—Shale deposit lying between hard sandstone ledges, disintegrating with formation of clay.

26a. Special Characteristics of Sand.—A simple and illustrative example of a rock from which quartz sand may be derived is sandstone. This stone is built up almost wholly of quartz grains, cemented together by iron oxide, calcium carbonate, or clay, and on the nature of the cementing material depends the strength and hardness of the stone. The structure of a hard sandstone is shown in Fig. 9. In this stone the cementing material is iron oxide.

Under certain conditions of use a fragment of sandstone, such as would be represented by a large sand grain, might be unfit as a material for concrete. Such a condition would be represented by subjecting concrete containing such sand to extreme of heat, as in a fire. Under like conditions it would be expected that a large fragment of the same stone would break up, or "spall," and it is actually found that some sand grains repeat in miniature the behavior of the larger pieces of stone. Such a condition of heat is, of course, unusual and extreme, and would not prejudice the use of such sand for most purposes.

If the cementing material of a sandstone sand grain be calcium carbonate, it may be dissolved by natural water, since such waters contain an appreciable percentage of carbon dioxide, or carbonic-acid gas. Such a sand, therefore, would be unsuited for use in a water-storage reservoir, or in drainage tile, or in aqueducts of any kind, or in other constructions which are designed to be impermeable to water. Fortunately, this calcium-carbonate cement in sand is easy of detection by adding a drop of muriatic acid and noting effervescence, or the lack of it.

Fig. 9.—Hard sandstone. Quartz grains cemented by iron oxide. (Natural size.)

Clay cement in a sandstone is quite undesirable. It is frequently the case that a sand thought to be quite perfect for use in concrete, by reason of its whiteness and good grading in size, is in reality quite dangerous. Clay is not a strong cement, and a sand of which the particles are built up with this cementing material is readily crushed. This is especially true in concrete, for the clay readily absorbs water and becomes a soft paste, leaving the component sand grains loose and without contact with the Portland cement save at the outer surface of the outside particles. This, of course, weakens the concrete seriously if all the sand is of this same general character.

Not all sands, however, are composed of mineral grains, as are those previously mentioned. It is not infrequently the case that the rock from which they came has been formed by the fossilizing, or partial fossilizing, of minute prehistoric shells. A section of limestone, built up in this way, is shown in Fig. 10. Other rocks contain like fossil materials in combination with quartz grains and cementing material, as in fossiliferous sandstone. In many sands derived from such rocks the structure of the shell is so perfectly preserved that the fossils retain their hollow structure and, further, are easily decomposed by agents which either reach them before their incorporation in the concrete, or afterward, by dissolving out the softer portions. The use of such a sand, therefore, is not advisable in concrete which is intended to be impervious to water, or to possess a high strength.

26b. Crushed Stone and Screenings.—Crushed stone screenings, when free from clay, usually make excellent sand. These screenings ordinarily give a stronger mortar than natural sand but are likely to contain an undue amount of dust, especially when obtained from soft stone, and should be screened and washed to get rid of the finest particles before being used in mortar or concrete.

Crushed limestone makes a concrete of excellent early strength, provided the crushings are rerolled, as limestone breaks with a flat, scaly fracture, giving particles that are structurally weak and that are very hard to compact in the manner necessary to give an impervious concrete. For work exposed to water this point is of great importance. Furthermore, if there is porosity in such concretes, the high solubility of the limestone fragments in water is a further disadvantage. In such cases, percolation proceeds at an increasing rate with passage of time, due to bodily removal of the fine aggregate by solution, leaving a honeycomb structure behind.

Fig. 10.—Fossil-bearing limestone. (Magnified 20 diams.)

26c. Sea Sand.—Sea sand is usually well suited for use as fine aggregate for concrete, so far as structure, mineral composition, and cleanness are concerned. It is, however, usually of such fineness that its use is inadvisable if undiluted by coarser particles. Saline deposits on the grains, when derived from pure sea water, should not be of a nature detrimental to concrete. It is unwise to take such sands close to tide limits, as the newer sands close to water, teem with minute organic life.

26d. Standard Sand.—The standard sand used in tests of mortars is a natural sand obtained at Ottawa, Ill., passing a screen having 20 meshes and retained on a screen having 30 meshes per lin. in., prepared and furnished by the Ottawa Silica Co. at a cost of 2 cts. per lb., f.o.b. cars, Ottawa, Ill. The grains of this sand are rounded and readily compacted, the percentage of voids being about 37%.

It is to be noted that standard sand gives about the lowest value attainable with sand in combination with cement, because of its uniform size of grain. Yet the present acceptance tests for a commercial sand provide only that it shall in like combination with a like cement, attain not less than 75% of the strength of the lowest value obtainable. This standard is decidedly low and permits the use of almost any sand, even one of poor quality.

27. Requirements of Fine Aggregate as to Shape and Size of Particles.—It is exceedingly difficult in choosing a fine aggregate for concrete work to balance all considerations. Time is a factor of utmost importance in all construction operations. Therefore, where delays would be entailed by the selection of one sand, the qualities of which are superior to those of another sand that is more readily obtained, it is more than probable that considerations of superior quality will have little weight. It is unfortunately true that regardless of all that has become known in regard to the importance of sands, their quality will be generally disregarded in favor of cheapness or convenience until engineers and owners demand and insist upon concretes of proper quality and refuse payment for those not coming up to standard. When inferior materials at a less price are as readily marketable when incorporated in concrete as first-grade materials, the contractor, as vendor, is not to be censured if he realizes every opportunity afforded him to realize the largest possible profit. The buyer and his agents receive only what they demand.

Usual specifications for concrete sands permit of little discrimination on the part of the supervising engineer, however conscientious he may be. Provision that the sand shall be "clean, sharp and coarse" means nothing, as no standards are defined as comparisons and the determination is left solely to the judgment of individuals oftentimes quite incompetent and unskilled.

Sharpness as a quality requirement for sand is archaic. It has little or no definite meaning; and rarely are two individuals agreed as to how sharpness should be determined. To some it defines the sound given off when sand is rubbed in the hand. To others, it is measured by abrasive quality, determined in the same way. To others, it indicates a certain angularity judged solely by the eye. If it were but remembered that all natural sands are water-borne and water-worn, with inevitable rounding of grains, the fallacy of "sharpness," whatever its interpretation, as a standard of quality in natural sands, would be evident.

Cleanness in sands is most important, for reasons before given. Not all dirt coatings on sand are detectable, short of laboratory procedures; and unfortunately, much sand is judged as to cleanliness by rubbing in a hand that itself is usually none too clean, the fitness of the sand being judged by the deposit it leaves behind. Judging a sand in this way without supplemental tests betokens ignorance, or carelessness, or both. Cleanness is an imperative necessity, but it should be judged by adequate tests, not by such haphazard methods as the foregoing.

Coarseness in sands, as opposed to excessive fineness, is a desirable quality, but coarseness alone, without finer materials and especially when judged without standards, is no criterion of fitness for use in concrete. As before pointed out, coarse sands have less surface area than have fine sands, requiring less cement and being more readily coated. Such a requirement, if properly judged, is therefore advantageous. As an insurance against excessive clay or loam, the requirement of coarseness may also be of benefit.

What really is needed is: (1) a general and thorough understanding by engineers and contractors alike as to the fundamental relations existing between the various materials forming concrete (see chapter on "Proportioning" in Sect. 2); (2) an appreciation of the importance of sands in the production of good concrete; (3) their selection on a basis of quality; and (4) to make the foregoing of value, a rigid insistence upon conformity to standard by tests of each shipment made to the job with ruthless rejections of inferior materials. A single test on a sample which may or may not be representative of the bulk of material is of no value whatever, unless it is supplemented by comparative tests on the materials actually delivered.

28. The Selection of Sand.—The only logical procedure in the selection of a sand for concrete is:

1. Determine its granulometric analysis by screening.
2. Determine its cleanness by washing, or by chemical tests.
3. Determine its actual strength value in concrete by test.
4. Check all shipments for cleanness and uniformity of grading.

29. Requirements of Coarse Aggregate as to Shape and Size of Particles.—Since stone is one of the strongest, if not the strongest constituent of concrete, the greater the percentage of stone (*i.e.*, the nearer concrete actually approaches natural stone in strength and density) the stronger is the concrete. It follows, then, other things being equal, that the larger the stone, the stronger will be the concrete, since each piece of stone has greater mass density than would its components unless compacted and united by Nature's unapproachable processes.

There are, however, certain limitations as to size of stone imposed by certain classes of work. In reinforced work, the plastic concrete must fit itself closely around the reinforcing metal, so that 1 to 1½ in. is the greatest diameter of particle that experience demonstrates is advisable to use.

Concrete of this character obviously requires more cement than would concrete using larger stone, since the stone surface to be coated is greater. In mass work, on the other hand, crushed stone of 2½ to 3 in. diameter may be advantageously employed, with less cement. For these reasons, if for no others, richer mixtures are specified in reinforced work and leaner mixtures in mass work. It should be borne in mind, however, that size of stone is not alone the determining factor in this regard, but that grading of stone and size and grading of sand is of even more importance as influencing the quantity of cement required, with a corresponding effect on the quality of concrete.

Plums are large stone, 5 in. or more in least diameter, thrown into plastic mass concrete, largely with the object of using them as cheap space-fillers. Their use is also to be commended for the reasons before given, provided that the plums themselves are of the proper quality of stone and that they are not of such size as to cut through the concrete section. A safe rule to follow in the use of plums is that they shall be of a maximum size such that not less than 6 in. of concrete shall intervene between them and the forms at any point. In using very large plums, this thickness of intervening concrete should be materially increased.

The shape of particle of large aggregates is of relatively little importance. Cleanliness, grading, and character of rock have far greater influence on the concrete than has angularity or roundness of particle.

30. Impurities in Aggregates.—In order that cement may adhere to sand grains and to particles of coarse aggregate, each grain or particle must bear no coating such as would prevent either proper chemical action between cement and water, or a proper bond between cement and aggregates.

This requirement applies to both fine and coarse aggregates, but is of relatively more importance with respect to the former, as sand in its natural state and as used in ordinary concrete construction is more likely to contain dirt in sufficient amount and of such kind to cause appreciable injury than is coarse aggregate.

Clay and silt are impurities of most frequent occurrence in sand and gravel. These materials are the result of decomposition of natural rock of various kinds and it is almost inevitable that they should be associated with sand.

Each of these impurities causes injury to mortar or concrete not only when it exists as a coating on the sand or gravel particles, but is equally undesirable when it occurs in such amounts, or so unequally distributed, that its extremely fine grains "ball up" and stick together when wetted, so as to remain in lumps in the finished mortar or concrete. If, however, the particles of these impurities are distributed so that they do not bind together on the addition of water; and if they are not contaminated by organic matter, experiments have shown that with sand that is not too fine, no serious harm results in lean mortars and concretes from their presence to the extent of from 10 to 15%. In fact, either clay or silt are often found beneficial as they increase the density by filling some of the voids, thus increasing the strength and watertightness besides making the mortar or concrete work smoothly. In rich mortars and concretes the density and consequently the strength is lowered by even slight additions of clay or silt as the cement furnishes all the fine material that is required.

A coating of organic matter on sand grains, such as loam, appears not only to prevent the cement from adhering but also to affect it chemically. In some cases a quantity of organic matter so small that it cannot be detected by the eye and is only slightly disclosed by chemical tests has prevented the mortar or concrete from reaching any appreciable strength.[1] Tannic acid, colloidal sewage, manure, sugar, tobacco juice are instances of organic contamination destructive to concrete.

Mica in sand or stone is objectionable because of its low mechanical strength and its laminated scaly structure. Even a small amount of this impurity in sand may seriously reduce the strength of a mortar or concrete. Mica is especially injurious in sands for concrete surface work as the scaly flakes cause the surface to dust and peel.

Furthermore, it is to be noted that water does not wet the surface of mica. Necessarily, this precludes attachment of cement, so that not only is the material weak of itself, but it lies in the mass without attachment, inviting disintegration.

Mica schist is totally unfit for use as large aggregate, both because of the foregoing reasons and also because of its rapid decomposition on exposure to air.

[1] A new process for the detection of such coatings is being developed under the auspices of Committee C-9 of the A.S.T.M., at Lewis Institute, Chicago, Ill. by Prof. Abrams and Dr. Harder. See footnote on p. 29.

Mica above 1% in concrete sands is very objectionable.

Iron pyrites or *fool's gold*—a bright, yellow substance with metallic luster—is chemically iron sulphide (Fe_2S). This is a very common impurity in stone; and its undesirability lies in its ready oxidation in the presence of water with formation of sulphuric acid (H_2SO_4), which latter readily attacks the cement of concrete with disastrous consequences (see Fig. 1, page 13).

In large fragments of stone, as in coarse aggregate, the presence of a small amount of this substance is not objectionable, but when fine aggregates are made from this same rock, the pyrites previously isolated are exposed, with resultant increase in quantity and rate of oxidation and with corresponding formation of acid.

Some sand deposits also contain unoxidized iron sulphide, though such deposits are an exception.

Finely powdered dust present in crushed stone screenings causes approximately the same effect upon the strength of mortar or concrete as does the presence of silt or clay in like quantities. It is essential for the best work that this dust be removed by screening and washing, in the same manner that silt is removed from sand and gravel, though it may later be used advantageously in known quantities by recombination.

31. Size and Gradation of Aggregate Particles.—The weakest and most changeable element in any cement (plus water)-sand-stone combination is the cement matrix in which the aggregates are embedded. The strongest and least changeable elements are the sand and stone. It follows, therefore, that strong, enduring concrete should contain as large a percentage as possible of aggregates consistent with proper embedment and cohesion, together with requisite plasticity to permit ready placing in forms. This conclusion also is true from an economic point of view, since sand and stone are much cheaper, bulk for bulk, than cement.

Further, tests of concrete show that, with the same percentage of cement to a unit volume of concrete, that mixture which gives the smallest volume and has, therefore, the greatest density,[1] usually produces the strongest and most impermeable concrete. This rule, it should be said, does not strictly apply to water-tightness, as permeability is influenced by size of voids as well as density.

31*a*. Grading of Mixtures.—Other things being equal, the best aggregates, fine and coarse, for use in concrete are those which are so graded in sizes of particles that the percentage of voids, or hollow spaces, in the resulting concrete is reduced to a minimum. The same law applies also to mortar mixtures, so that if concrete is considered as a quantity of relatively large stone set in a pudding, or bedding of mortar, the best sand as to size is one which, if mixed with the given cement in the required proportions to standard consistency, will produce the smallest volume of mortar; and the best concrete will be one in which the particles of stone are so graded as to permit in a given volume a maximum quantity of stone being bedded in such a mortar.[2]

31*b*. Grading, Density, and Strength.—It has been found that the *densest* mixture occurs with particles of *graded* sizes; and also that the *least density* occurs when the grains are *all* of the *same* size. Coarse sands, or fine sands alone are thus inferior to graded sands for concrete, but of the two extremes the coarse sand is preferable because its particles are more readily coated with cement particles. Further, a coarse sand has a less total grain surface in a unit volume than a fine sand, even when the sands considered contain the same proportion of solid matter and voids. Less total grain surface means less cement and less water required to coat the grains. Furthermore, these interactions are cumulative, for the additional amount of cement and water required in the case of fine sand reduces the density of the resulting mortar and likewise its strength as well as increasing its cost.

31*c*. Money Value of Grading.—One reason is here evident, both from an engineering as well as from a purely dollars-and-cents standpoint, that care and attention be

[1] The density of a mortar or concrete as here referred to is the ratio of the volume of the solid particles to the total volume.

[2] For the use of 6 to 8-in. aggregate, see *Eng. Rec.*, May 1, 1915.

given to the grading of sands for concrete. If it were possible to compute the total saving in the annual concrete production of the United States, both direct (by lessened quantity of cement required) or indirect (by increased durability and usefulness and prevention of disintegration) that would result from the use of proper sands, the amount would be almost beyond imagination. It is probable that from this neglect alone not less than 20% of the total annual expenditure for concrete is unnecessary waste. It has been proven in England that on a strict 1:2:4 basis, using different aggregates, the cement required for a quantity of concrete varied from 100 to 130 bags—a difference of 30% in direct cement cost, without counting the variation in quality of the several concretes, with like variation in their durability and value. At the present time, a comfortable ignorance is general among engineers and contractors alike on these important matters, but in the not distant future, an awakened intelligence on the part of all will demand reform.

32. Mechanical Analysis of Aggregates.—The value of an aggregate, sand or stone, with reference to its size may be determined by means of a sieve analysis. This analysis consists of sifting the material as supplied through several different sieves, and then plotting upon a diagram the percentage by weight which is passed (or retained) by each sieve—abscissæ (horizontal) representing size of grain and ordinates (vertical) representing percentage of any size passing each sieve.[1] Such a sieve analysis may appear of little use as regards the making and placing of 100,000 yd. of concrete, but experiment has developed definite laws establishing the relation of percentages and sizes of particles to maximum density and strength of concrete so that such a sieve analysis may be directly translated into terms of commercial and engineering economy.

A typical analysis of three natural sands—a fine, a medium, and a coarse sand is given in Fig. 11.[2] Uniform grading is indicated by an approach to a straight line and the variation from the grading found to give best results in practice is observed without difficulty.

A mechanical or sieve analysis is also useful in studying the size of the particles of the coarse aggregate.[3] Fig. 12 illustrates the analysis of a bank gravel and a crushed stone as it came from the crusher, without screening.

The following mechanical test for sand has been used in the laboratory of the Board of Water Supply of the City of New York:

Samples of the material proposed for use in mortar or concrete shall be prepared for testing by passing them through a No. 4 sieve. Of the material passing this sieve not more than 95% shall pass a No. 8 sieve, not more than 40% a No. 50 sieve, and not more than 15% a No. 100 sieve.

[1] It is greatly to be regretted that there is no standard in the United States in matters of this kind. Such variations make for confusion and waste. A proposed standard, which has much to recommend it, varies successive screen openings by a constant ratio of $\sqrt{2}$. The following sizes of sieves are desirable for analysing sand, although a very useful analysis may be made with fewer sizes:

Commercial No.	5	8	10	16	20	30	40	60	100	200
Approximate size of hole in inches.	0.165	0.096	0.073	0.042	0.034	0.020	0.015	0.009	0.0055	0.0026

Sieves are given commercial numbers, which agree approximately with the number of meshes to the linear inch. The actual size of hole, however, varies with the gage of wire used by different manufacturers and every set of sieves should be separately calibrated. The screen with ¼-in. openings is generally used for separating out large material from sand. The No. 4 sieve with four meshes per linear inch is practically its equivalent.

[2] A new portable instrument for making mechanical analysis of sands quickly in the field is now manufactured by Koleach & Co., of New York (see *Eng. Rec.*, June 26, 1915: also Fig. 2, Sect. 2, p. 69).

[3] For coarse aggregate analysis, the following sizes of sheet brass sieves with round holes are desirable; 3 in., 2½ in., 2 in., 1½ in., 1¼ in., 1 in., ¾ in., ½ in. and ¼ in. As is also the case with a like analysis of sand, a straight line on a mechanical-analysis diagram indicates a uniform grading.

Material in which the percentage passing any one sieve or two sieves exceeds the specified percentage may be used, provided there is a different percentage passing the other sieves or sieve under the limiting percentage equal to at least twice the excess.

The sieves for testing shall be defined as follows:

No. 8 mesh holes, 0.0955 in. wide, No. 23 wire.
No. 50 mesh holes, 0.110 in. wide, No. 35 wire.
No. 100 mesh holes, 0.0055 in. wide, No. 40 wire.

In the standard specifications for concrete pavement adopted by the American Concrete Institute, the fine aggregate is required to pass, when dry, a screen having ¼-in. openings. Not more than 20% is allowed to pass a sieve having 50 meshes per linear inch, and not more than 5% is allowed to pass a sieve having 100 meshes per linear inch. The coarse aggregate is specified as such as will pass a 1½-in. round opening and will be retained on a screen having ¼-in. openings. It is also required that natural mixed aggregate shall not be used as it comes from deposits, but shall be screened and used as specified.

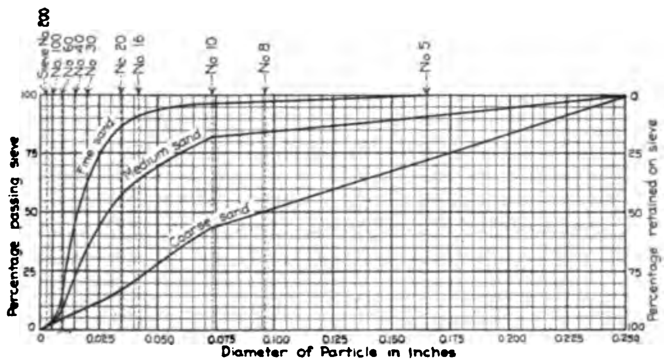


FIG. 11.—Typical mechanical analyses of fine, medium, and coarse sands.

The Joint Committee recommends that not more than 30% of sand by weight should pass a sieve having 50 meshes per linear inch.

According to heretofore accepted theory, sand for mortar or concrete should have its grains uniformly graded. Recent tests,[1] however, by Prof. McNeilly at Vanderbilt University seem to indicate that there should be a jump in the grading from the sieve No. 40 size particles to the sieve No. 10 size particles. The following conclusions were derived from these tests:

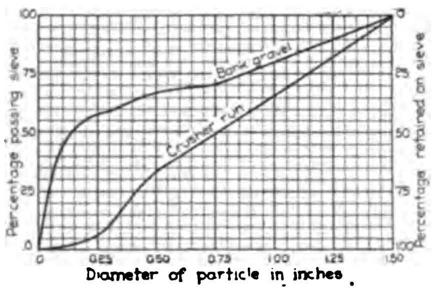


FIG. 12.—Typical mechanical analyses of bank gravel and crushed stone.

The best sizing of grains in a commercially-sieved aggregate is about as follows: 53% to be caught between the No. 4 and No. 10 sieves; 47% fines to be passed by the No. 40 sieve (this includes the cement).

There is reason to believe that the coarse aggregate in concrete could be beneficially sized on a basis similar to the above; that is to say, there should be a jump in size from coarse to quite fine instead of the usually accepted graded material.

For the method of proportioning by mechanical analysis, as developed by Wm. B. Fuller and by the U. S. Bureau of Standards, see page 68.

[1] See *Eng. Rec.*, Nov. 27, 1915. (Conclusions derived from these tests have not as yet received general acceptance.)

33. Tests for Specific Gravity of Aggregates.—The specific gravity of a substance is the ratio of the weight of an absolutely solid unit volume of the substance to the weight of a unit volume of water. This ratio for aggregates may be determined as follows:

1. By pouring a given weight of sand into a given volume of water and finding the increase in volume of the liquid. (Enough material should be used to give sufficient accuracy.)

2. By suspending pieces of coarse aggregate by a thread from chemical scales and noting weight in air and weight when hanging in water. (The difference in weight is the weight of the water which the aggregate displaces.)

Finding specific gravity of particles of sand and stone is preliminary to one method of determining the percentage of voids. The specific gravity of sand is practically a constant, with a value of 2.65. The specific gravity of gravel is also quite uniform, the average being 2.66. Average values for stone varies with the kind and the locality, ranging from 2.4 (sandstone) to 2.9 (trap). Cinders have an average specific gravity of 1.5.

Before determining specific gravity, sand or fine stone should be dried in an oven at a temperature as high as 212°F. until there is no further loss of weight. If the stone is of a porous nature, it should be moistened sufficiently to fill its pores, and then the moisture on the surface should be removed by means of blotting paper. Such a procedure, of course, does not determine absolute specific gravity but gives a result that should be used in determining the percentage of voids for proportioning concrete mixtures.

34. Voids in Aggregates.

34a. Percentage of Voids.—The percentage of voids in dry sand ranges from 28% for a coarse, well-graded natural sand to 40 to 45% for a very uniform natural or screened sand. The range for coarse aggregates is from 25 to 55%.

The percentage of voids in aggregates may be determined by two methods:

1. By determining the specific gravity of the solid particles and then weighing a given volume of the aggregate and computing therefrom the percentage of voids. (Pores in porous stone should be filled with water. See Art. 33.)

Let S = specific gravity, W = weight per cubic foot of the dry aggregate, and P = percentage of voids. Then, since water weighs 62.5 lb. per cu. ft.,

$$P = 100\left(1 - \frac{W}{62.5S}\right)$$

2. By finding the amount of water required to fill the voids in a given volume of aggregate.

Let V = volume of water required to fill the voids and T = total volume, or given volume of the aggregate. Then

$$P = \frac{V}{T}$$

With sand or fine broken stone the percentage of voids by this method should be obtained by dropping the aggregate into a vessel containing water. If K = volume displaced by the aggregate and T = given volume of the aggregate, then

$$P = \frac{T - K}{T}$$

Pouring water into fine aggregate does not give reliable results because it is physically impossible to drive out all the air.

The percentage of voids is considerably affected by the degree of compactness of the aggregate. Moderate shaking of coarse aggregate, for example, will reduce the volume of voids by as much as 10%. Loose measurement is usually considered preferable for the coarse aggregate since sand and cement separate the stones to a considerable extent in the concrete as placed. Voids in sand are usually determined with reference to the dry material well shaken.

34b. General Laws.—1. A mass of spheres of any uniform size if carefully piled in the most compact manner would have 26% voids. If the same mass of spheres were poured into a receptacle and the spheres allowed to arrange themselves, it has been found by experiment that 44% would be the smallest percentage of voids which could be obtained under the best conditions.

2. A material having particles all of a uniform size and shape contains practically the same percentage of voids as a material having particles of a corresponding similar shape but of a different uniform size.

3. In any material the largest percentage of voids occurs with particles all of the same size and the smallest percentage occurs with particles of such different sizes that the voids of each size are filled with the largest particles which will enter them. Thus, an aggregate consisting of a mixture of stones and sand has a less percentage of voids than sand alone.

4. Materials with round particles contain less voids than materials with angular particles screened to the same size

34c. Effect of Moisture on Voids in Sand and Screenings.—The percentage of voids in sand is greatly affected by moisture. The reason for this lies in the fact that when water surrounds a particle of sand it occupies space and separates this particle from grains adjacent to it. Since fine sand has a larger number of grains per unit volume, it is more affected than is coarse sand. If either loose or tamped sand is mixed with a small percentage of water and kept either loose or thoroughly tamped, it will be found to increase considerably in volume and weigh less per cubic foot. A maximum volume will be obtained with the addition of from 5 to 8% of water by weight. Greater percentages will give a less increase of volume until finally when the sand is thoroughly saturated it will have a volume slightly less than the original. A sand that has, say, 35% voids, may contain from 27 to 44% of voids depending upon the degree of compactness and the percentage of water. Natural sand as it ordinarily comes from the bank contains from 2 to 4% of moisture by weight.

34d. Percentage of Voids Determined by Weight.—The specific gravity of gravel particles and of sand grains is usually nearly constant, varying between 2.6 and 2.7.

Percentages of Voids in Sand and Gravel Corresponding to Different Weights per Cubic Foot

(Based on an Average Specific Gravity of 2.65)

Percentages of moisture by weight	Weight per cubic foot of sand or gravel										
	75	80	85	90	95	100	105	110	115	120	125
0	54.7	51.7	48.7	45.7	42.7	39.6	36.6	33.6	30.6	27.5	24.5
1	55.2	52.2	49.2	46.2	43.2	40.2	37.3	34.2	31.2	28.2	25.3
2	55.6	52.7	49.8	46.7	43.8	40.8	37.9	34.8	31.9	28.9	26.0
3	56.1	53.1	50.3	47.3	44.4	41.4	38.5	35.5	32.6	29.7	26.7
4	56.5	53.6	50.8	47.8	45.0	42.0	38.1	36.2	33.3	30.4	27.5
5	57.0	54.1	51.3	48.4	45.5	42.6	39.8	36.9	34.0	31.2	28.3
6	57.5	54.6	51.8	48.9	46.0	43.2	40.4	37.5	34.7	31.7	29.0
7	57.9	55.1	52.3	49.5	46.6	43.8	41.0	38.2	35.4	32.6	29.8
8	58.3	55.5	52.8	50.0	47.2	44.5	41.6	38.9	36.1	33.3	30.6
9	58.8	56.0	53.3	50.6	47.8	45.0	42.3	39.5	36.8	34.0	31.3
10	59.2	56.5	53.9	51.1	48.4	45.6	42.9	40.2	37.5	34.7	32.1

On account of this fact the percentage of voids in sand and gravel may be considered to vary inversely as the weight per cubic foot of dry material. Knowing the weight per cubic foot and assuming a specific gravity of 2.65, the percentage of voids in dry sand and gravel may be readily found as explained under Method (1) in Art. 34*a*. The percentage of voids in moist sand or gravel may be determined in the same manner as for the dry aggregate except that the weight per cubic foot of the moist material should be considered as decreased by the weight of moisture which the sand or gravel contains. The foregoing table gives percentages of voids for sands and gravels of different weights per cubic foot and with different percentages of moisture by weight. The table may be used for any aggregate with a specific gravity of approximately 2.65.

35. Tests of Aggregates.[1]—Tests of an aggregate for use in mortar or concrete may be divided into two general classes:

1. Tests to determine the general suitability of the aggregate.
2. Tests to determine those characteristics of the aggregate which have an influence on its general suitability.

Tests of the first class comprise those for determining the quality of the mortar or concrete that can be made from the given aggregate. These tests may be called *Tests for Acceptance.* Tests of the second class include those which may be made to determine the cause of any failure of an aggregate to pass the tests of the first class. These tests of the second class, which may be called *Tests for Quality*, are useful not only to discover the cause of failure of an aggregate to pass the tests for acceptance, but may be employed to determine the methods of improving a given aggregate and of comparing different aggregates as to special characteristics.

No standards for acceptance tests of concrete aggregates have been established by the American Society for Testing Materials, although the need of such standards is now fully realized and will soon be satisfied. The most advanced practice in this direction is probably that represented by the procedure of the Materials Testing Division of the New York Public Service Commission whose methods have been given wide publicity. The standards quoted in abridged form below are derived from this source (*Eng. Rec.*, Jan. 8, 1916 and *Eng. News*, Feb. 4, 1915).

Fine Aggregates.—Complete tests of a fine aggregate comprise:

1. Determination of % retained on No. 4 square-hole sieve.
2. Mechanical analysis of portion passing No. 4 square-hole sieve.
3. Determination of silt by washing on No. 100 sieve.
4. Determination of silt by decantation.
5. Compressive tests of 2-in. cubes.
6. Microscopical examination.
7. Weight per cubic foot.
8. Voids.
9. Specific gravity.
10. Reaction to litmus.
11. Quantitative test for organic matter as indicated by loss on ignition.[2]
12. Density in mortar.
13. Determination of insoluble silica.

It is seldom necessary to make more than the first six of the above tests and frequently only one or two of them are necessary.

1. The entire sample is screened on a No. 4 square-hole sieve and the % retained is computed upon the basis of the original weight of the sample. No correction is made for moisture contained.

(A No. 4 sieve has clear openings of 0.20 in., while the ¼-in. sieve recommended by the Joint Committee of the National Engineering Societies has 0.25-in. clear openings made by drilling round holes in a plate. The difference in results obtained with the two sieves is negligible and either is satisfactory.)

[1] See also the following articles on sand testing:

CLOYD M. CHAPMAN and NATHAN C. JOHNSON: "The Economic Side of Sand Testing," *Eng. Rec.*, June 12, 19 and 26, 1915.

CLOYD M. CHAPMAN: "The Testing of Sand for Use in Concrete," *Eng. News*, Feb. 5, 1914, and Mar. 12, 1914.

RALPH E. GOODWIN: "Standard Practice Instructions for Concrete Testing," *Eng. News*, Feb. 4 and 11, 1915.

[2] A new colorimetric test is being developed. See footnote on p. 29.

2. The mechanical analysis of the portion of the sample which has passed the No. 4 sieve is made by use of sieves Nos. 8, 16, 30, 50, and 100. About 150 grams of this material is separated by the method of quartering, and 110 grams of this is weighed and dried beneath a gas burner. One hundred grams of the dry material is placed on the No. 8 sieve, the other sieves being nested below the No. 8 in the order of increasing fineness, and the entire nest of sieves is mechanically agitated. With the agitator used, the amount of sieving is standardised by always turning the hand crank 200 revolutions. At the end of the operation the material retained on each sieve, and that which has passed the No. 100 sieve, is weighed. The % of the whole sample which passes each sieve is now computed, and the mechanical analysis curve is plotted (see Art. 32).

3. Silt by washing on the No. 100 sieve is determined for a sample obtained by quartering which weighs about 220 grams in its natural moist condition. The sample is dried slowly at temperatures not greatly above ordinary room temperature to avoid baking any clay or similar matter. Two hundred grams of the dry sample is now weighed on the No. 100 sieve, soaked in water for a few moments to soften any lumps, washed under a gentle stream of water, dried under a gas burner, and reweighed. The % of silt is the loss in weight multiplied by 100 and divided by 200.

(The washing test obtains the true silt value because it removes clay which adheres to the grains as a coating which is not separated by sieving in a dry state.)

4. Determination of silt by decantation is a test for field use only. About 20 c.c. quartered from a carefully selected sample is placed in a 100-c.c. graduated glass cylinder with about 30 c.c. of lukewarm water. The mixture is stirred with a wire for 30 sec., allowed to settle for 30 sec., and the water decanted into a second 100-c.c. cylinder. The sand left in the first cylinder is stirred up with a fresh portion of water and the process repeated. This is done 4 times. After 1 hr. the volume of silt in cylinder No. 2 and the volume of clean sand in cylinder No. 1 is noted and recorded. The % of silt is 100 multiplied by the number of cubic centimeters of silt in cylinder No. 2 and divided by the sum of the volumes of silt in cylinder No. 2 and clean sand in cylinder No. 1.

(The method of decantation is better than the method of allowing the silt to settle on top of the sand in one cylinder but is not as satisfactory as the method of washing.)

5. Compressive tests of 2-in. cubes are made by the methods recommended by the Committee on Uniform Tests of Cement, of the American Society of Civil Engineers (*Trans.* Am. Soc. C. E., vol. 75, p. 665). Fine aggregate must not be dried, but the natural moisture is determined on a separate sample, and is counted as a part of the water used for mixing, not as a part of the weight of sand. Proportions are 1 : 3 by weight. The consistency employed is 60% more water than that required to make standard Ottawa sand mortar of "normal consistency." Ottawa sand specimens of the same consistency are made with each set of specimens from commercial sand. Tests are made at ages of 3, 7, and 28 days. Specimens whose weights vary more than 3% from the average are rejected. Cubes are stored in water up to time of crushing. A spherical bearing block and two thicknesses of blotting paper above and below are used in testing. Since the wet consistency used lowers the strength at early periods, the results are permitted to fall below those for Ottawa sand specimens of standard consistency by the following amounts or less: at 3 days—10%; at 7 days—5%; at 28 days—1%.

(The wet consistency is used because it more nearly represents working conditions and because some sands fail in wet consistencies although satisfactory in "normal consistency." No tensile tests are required because "it is thought that compressive tests more nearly represent the conditions of the work and that modern practice is tending toward compressive tests.")

6. Microscopical examination is made for the purpose of detecting the presence of a crust or film of organic matter on the grains which cannot be detected by other means.

7. Weight per cubic foot is determined by using an 8 by 16-in. cylindrical concrete mold and a second cylinder of smaller diameter but high enough to have a cubic capacity slightly greater than that of the 8 by 16-in. cylinder. The smaller cylinder is placed within the larger one and filled with fine aggregate. It is now drawn out allowing the aggregate to flow out at the bottom into the larger cylinder. The material is now struck off level with the top of the measure and weighed. After weighing, the material is dried and again placed in the larger cylinder by use of the smaller cylinder as before. The weight of the material dry is then determined, the volume being found by measuring down from the top of the cylinder.

8. Voids are determined by using the sample whose apparent volume and dry weight have been determined in test No. 6. It only remains to determine the absolute volume of solid matter. This can be computed from the weight, assuming the specific gravity 2.65, or it can be found by placing the entire sample in a receptacle containing water and weighing the whole, emptying out the aggregate, and reweighing the receptacle filled with water to the previous level. The difference in weighings minus the weight of dry aggregate is the weight of a volume of water equal to the absolute volume of the aggregate. This divided by the weight of water is the absolute volume of aggregate. A hook gage clamped to the water receptacle is used to determine water levels accurately. Finally, the % of voids in aggregate

$$= \frac{100\,(\text{Apparent volume} - \text{Absolute volume})}{\text{Apparent volume}}$$

9. Specific gravity is determined by slowly and carefully introducing the aggregate into a specific gravity flask or a graduated glass cylinder containing water and noting the volume of water displaced by a known weight of material. The specific gravity of the aggregate

$$= \frac{\text{Weight of aggregate in grams}}{\text{Displaced volume in cubic centimeters}}$$

10. The reaction to litmus demonstrates the presence of injurious alkalies.

11. Organic matter as indicated by the ignition loss is determined as follows:[1]

Two 25-gram samples are weighed from a 75-gram sample containing natural moisture which has been previously selected by the method of quartering. These are placed in beakers "A" and "B" and each is covered with 60 c.c. of water at 100°F. The sand and water in beaker "A" is stirred briskly with a glass rod, allowed 30 sec. to settle, and a part of the water decanted onto a previously dried and weighed filter paper. The remaining portion is again stirred, allowed to settle and a further quantity of water decanted. The process is repeated until all the water has been filtered. An additional 30 c.c. of water at 100°F. is added to the sand in the beaker and the process is repeated again. The filtrate is allowed to drain, and the washed sand is dried under a gas burner. The same procedure is followed with beaker "B," using a second filter paper. The filter papers with filtrate on them are now dried in an oven at 100°C. to constant weight (a temperature of 100°C. must not be exceeded) and weighed. The excess in weight over the dry weight of the filter papers is the weight of silt. The filter papers are now folded carefully and ignited thoroughly in a platinum crucible. The residue is weighed, and the dry washed sand is also weighed. Finally, % loss on ignition

$$= \frac{100\ (\text{Weight of silt} - \text{Weight of crucible ash})}{\text{Weight of silt} + \text{Weight of dry washed sand}}$$

(This test is seldom necessary because more direct results are obtained by tests of 2-in. cubes.)

12. Density is determined by weighing the relative amounts of cement, sand, and water, according to the proportions used, mixing, placing mix in a graduated cylinder and noting the final volume of the set mortar. The net weight of this mortar is also determined and compared with the sum of the individual weights of cement, sand, and water in the mix. The weight of material left adhering to mixing slab and tools is thus ascertained. This loss is apportioned between the cement, sand, and water according to the relative weights of each as originally combined, and the corrected amount by weight of each constituent in the set mortar is thus computed. The corrected weights of cement and aggregate in the set mortar are now multiplied by their respective specific gravities to obtain absolute volumes and the sum of these absolute volumes divided by the total volume of set mortar is the density, or solidity ratio.

13. The determination of insoluble silica is seldom called for but when required calls for the services of an experienced analytical chemist.

Coarse Aggregates.—Complete tests of a coarse aggregate comprise:

1. Mechanical analysis.
2. Cleanliness.
3. Weight per cubic foot.
4. Voids.
5. Specific gravity.
6. Crushing strength of stone.

It is usually necessary to make only the first two of the above tests.

1. Mechanical analysis of coarse aggregate is made by mechanically agitating a nest of six sieves, the clear openings in the wire meshes of which are 2 in., 1½ in., 1 in., ¾ in., ½ in., and ¼ in. respectively. A 25-lb. sample obtained by quartering a larger sample is used. The material retained on each sieve after 100 bumps of the rocker apparatus is weighed, and the percentage passing each sieve is computed.

2. Cleanliness is judged by inspection only.

3. Weight per cubic foot is determined by pouring the material slowly into an 8 by 16-in. cylinder mold from a height of 2 ft. above the bottom, striking off the top and weighing. If the material is very wet it is previously dried sufficiently to remove surplus water, but not enough to dry out the pores.

4. Voids are determined by the method used for fine aggregate.

5. Specific gravity is determined by weighing a number of representative particles of the stone after thorough drying to remove all moisture in the pores, allowing the material to cool, filling the pores by boiling in water and

[1] A colorimetric test for organic impurities in sands is being developed under the auspices of committee C-9 of the A.S.T.M. Sodium hydroxide (NaOH) is added to a sample of sand at ordinary temperature and the depth of color resulting has been found to furnish a measure of the effect of the impurities on the strength of mortars made from such sands. See *Circular* No. 1 of Structural Materials Research Laboratory, Lewis Institute, Chicago.

The method for field tests is described in the circular as follows:

"Fill a 12-oz. graduated prescription bottle to the 4½-oz. mark with the sand to be tested. Add a 3% solution of sodium hydroxide until the volume of the sand and solution, after shaking, amounts to 7 oz. Shake thoroughly and let stand over night. Observe the color of the clear supernatant liquid.

"In approximate field tests it is not necessary to make comparison with color standards. If the clear supernatant liquid is colorless, or has a light yellow color, the sand may be considered satisfactory in so far as organic impurities are concerned. On the other hand, if a dark-colored solution, ranging from dark reds to black is obtained the sand should be rejected or used only after it has been subjected to the usual mortar strength tests.

"Field tests made in this way are not expected to give quantitative results, but will be found useful in:

1. Prospecting for sand supplies.
2. Checking the quality of sand received on the job.
3. Preliminary examination of sands in the laboratory.

"An approximate volumetric determination of the silt in sand can be made by measuring or estimating the thickness of the layer of fine material which settles on top of the sand. The % of silt by volume has been found to vary from 1 to 2 times the % by weight."

allowing the material to stand in the water until cool, removing the surface water with a towel, weighing, placing particles in a 300-c.c. graduate, pouring in a sufficient measured volume of water to cover the stone, noting the combined volume of water and stone, computing the volume of stone alone, and computing the specific gravity by dividing the original dry weight of the stone in grams by the displaced volume in cubic centimeters. Data are also afforded by this test for determining the % of absorption of the stone based on dry weight.

6. Crushing strength of the stone has never been used as an acceptance test of aggregate.

36. Notes on the Selection and Testing of Aggregates.—*Sand and stone for important or special work should be tested in some well-equipped laboratory and, of course, the tests should be made before the aggregates are purchased or the concrete mixed. The tests should be upon representative samples, and materials should be checked for uniformity as delivered. In sampling natural sands and gravels the greatest care must be used.*

Sand should also be regularly tested as construction work progresses. Often a sand that is found entirely suitable at the start will be found entirely different in later deliveries. Proportions of materials should change whenever the mechanical analysis shows a decided change in the gradation of the sand grains.

Mechanical analyses of sands, or volumetric tests of mortars or concretes made from the given sands, sometimes show that a stronger mortar or concrete may be obtained by mixing two sizes of sands from different portions of the same bank, making the single requirement that a definite percentage is to be retained on a certain sieve. Mechanical analyses and volumetric tests are also useful in studying two or more sands to determine the one most suited for the given work. Frequently a properly proportioned mixture of sand and crushed stone screenings will produce a better sand for mortar or concrete than either one used separately.

For the best results, sand for concrete requires more fine material than sand for mortar.

For maximum water-tightness a mortar or concrete may require a slightly larger proportion of fine grains in the sand than for maximum density or strength. Gravel tends to produce a more watertight concrete than broken stone.

A high unit weight of material and a correspondingly low percentage of voids are indications of coarseness and good grading of particles. However, the impossibility of establishing uniformity of weight and measurement due to different percentages of moisture and different methods of handling make these results merely general guides that seldom can be taken as positive indications of true relative values. This is especially true of the fine aggregates in which percentages of voids increase and weights decrease with the addition of moisture up to 5 to 8%.

Aggregates that contain harmful impurities may sometimes be made satisfactory for concrete work by washing.

Some sands which contain impurities have been found to prevent hardening with one brand of cement and to give satisfactory results with another brand.

A chemical analysis of aggregates is desirable in many cases, and a microscopical examination will often prove of value.

Cement adheres more readily to sand grains with rough, unpolished surfaces.

Usually an artificial sand or crushed stone will safely contain a greater percentage of fine material than natural sand.

37. Specifications for Aggregates.—The specifications adopted by the Public Service Commission for quality of concrete aggregates used in New York City subway construction are noted below (*Eng. News*, Feb. 11, 1916; *Eng. Rec.*, Jan. 8, 1916).

Fine aggregates shall conform to the following requirements:

Mechanical Grading.—

Size of opening square holes (inches)	Commercial number of sieve	Limit of fineness (% passing). Not more may pass	Limit of coarseness (% passing). Not less must pass
0.200	4	100	95
0.100	8	95	85
0.042	16	75	40
0.021	30	50	20
0.011	50	30	2
0.006	100	6	

Silt.—Not over 6% by dry weight shall pass a No. 100 sieve when screened dry. Not over 10%, dry weight, shall pass a No. 100 sieve when washed on the sieve with a stream of water.

Both of the above tests shall be made, and neither limit shall be exceeded.

The following test is for field use only:

Not over 10% by volume shall be silt when the test is made by decanting from test tubes (method described in Art. 35).

Strength in Mortar.—Fine aggregate shall be of such quality that mortar composed of 1 part Portland cement and 3 parts fine aggregate by weight will show a tensile and compressive strength at least equal to the strength of 1 : 3 mortar of the same consistency made with the same cement and standard Ottawa sand. Fine aggregate shall not be dried before being made into mortar, but shall contain natural moisture.

Organic Matter.[1]—Loss on ignition shall not exceed 0.1% of the total dry sand by weight, nor 10% of the silt obtained by decantation.

Coarse aggregate for concrete shall conform to the following requirements:

Mechanical Grading—

Size of opening square holes (inches)	Limit of fineness (% passing). Not more may pass	Limit of coarseness (% passing). Not less must pass
2	...	100
1½	100	95
1	80	40
¾	60	25
½	40	10
¼	5	..

Cleanliness.—All broken stone aggregate must be so free from dust that samples caught as the material falls from the conveyor belt at the plant will be within the limit of fineness. All gravel must be thoroughly washed at the plant.

WATER

38. General Requirements.—The water used in mixing mortar or concrete should be free from oil, acids, alkalies, or vegetable matter, and should be of a quality fit for drinking purposes. The presence of oils is easily detected by the well-known iridescent surface film. Vegetable matter can sometimes be detected by observing floating particles, or by turbidity. Chemical determinations are better and more certain.

39. Examination of Water.—Tests of water for acidity or alkalinity may be made by means of litmus paper, procured at any chemist's. If blue litmus remains blue on immersing in the water, then the property is either neutral or alkaline; if the color changes to red, then the property is acidic. If there is a dangerous amount of acid present, the change in color will be very rapid. Likewise, if red litmus changes very quickly to blue, the water will be found to contain a dangerous amount of strong alkali. If the change of color is slow and faint in either test, the indication may be disregarded. A solution of phenol-phthalein is a delicate test for alkalinity.

Whenever a water does not appear satisfactory, its effect upon the strength and setting qualities of a cement should be determined by direct test on mixtures.

40. Functions of Water.—The functions of water in concrete are:

1. Water reacts with cement to form a binding material which unites otherwise non-cohering sand and stone.

2. Water operates to flux both dissolved and undissolved cementing substances over the surfaces of sand grains and stone particles (or pieces of gravel), rendering possible extensive and close adhesion by carrying these substances into the minute and multitudinous surface irregularities of the particles, where they are absorbed as water is later absorbed or evaporated.

3. Water acts as a lubricant between sand particles and stone particles so that placement of harsh and irregular materials in molds and forms is rendered easy.

[1] For colorimetric test see footnote on p. 29.

4. Water itself occupies space in the mass.

41. Influence of Quantity of Water on Strength of Concrete.—The first function of water cited in the preceding article is basic and essential to the manufacture of concrete. If there is insufficient water, obviously its reaction with cement will prematurely cease; and if there is too much water, it is equally obvious that the cementing products may be too dilute to develop proper strength since cement depends for its early strength and for a considerable part of its later strength upon the hardening of amorphous or glue-like substances. Undue dilution of these substances is readily possible, but it is accompanied by impairment of strength, just as glue may be a valuable adhesive when of proper consistency, while the same glue, if too dilute, may be useless for like purposes although later evaporation may gradually restore its cementitious

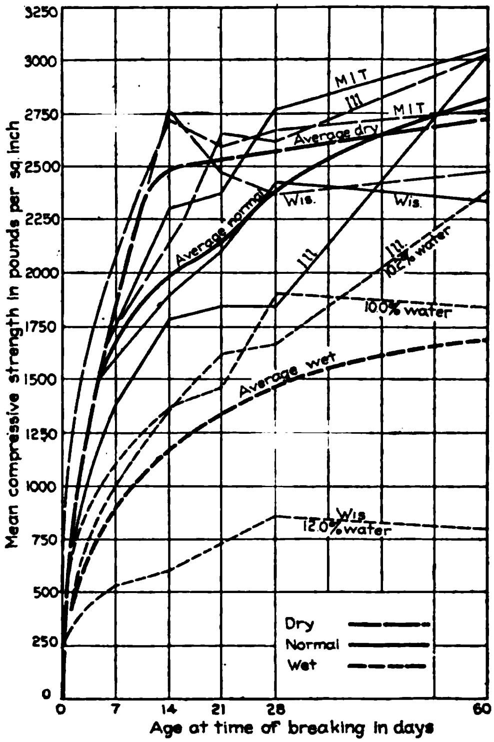


FIG. 13.—Time-strength curves for concretes mixed with different percentages of water.

value. Cement depends further for its strength upon interlacing crystals; and crystallization takes place only from saturated or supersaturated solutions. If, therefore, excess water is added, such strength as these crystals may confer is further impaired. The influence of water in greater or less quantities on the strength of concrete is shown in Fig. 13.[1]

42. Influence of Quantity of Water on Fluxing of Cement.—The second function of water cited above—its action as a carrier (or flux) of cementing substances—is obvious. In bringing sand, stone and cementing substances into intimate contact (Fig. 14), it acts physically in a manner analogous to its earlier chemical rôle.

Inevitably, however, this desirable function of water is closely dependent upon its fourth

[1] See L. N. EDWARDS: *Proc.* Am. Soc. Test. Mat., 1917.
D. A. ABRAMS: *Concrete* (C. M. Edition), July, 1917.

function—viz., its occupancy of space. It is readily seen that if the minute irregularities in the surface of stone are first filled with water, and, because of initial excess of water, the cementing solution is then too weak, a strong, intimate attachment of cement will be inhibited, both because the irregularities are already filled and also because of excessive dilution.

Furthermore, water, particularly when charged with gelatinous aluminates from cement, has ability to occlude a very high percentage of air. This air, as minute bubbles, firmly attaches itself to the sand and stone.[1] It also remains between particles to such an extent as oftentimes to completely isolate a large percentage of the materials. Given excess water, therefore, and a proportionate amount of occluded air, detriment to concrete is sure to arise from the primary fault in an increasing ratio (see Fig. 15). This explains to a large degree the lower strengths with prolonged mixing in present-type machines found by some investigators.

Fig. 14.—Cement particles fluxed over surfaces of sand grains. (Magnified 20 diams.)

43. Influence of Quantity of Water on Lubrication of Concrete Mixture.—The function of water as a lubricant of concrete is very important, but its importance can be overestimated, particularly when balanced against the detrimental effects which may and often do result. It is not necessary to add great quantities of water to concrete to make it easy-flowing if the concrete is sufficiently mixed. The more concrete is mixed, the smoother working it becomes and the less water is superficially evident. Cement is continually hydrating in the mixing action; and in process of hydration, large quantities of hydrated lime are formed. This has a very pronounced effect in lubricating the mass, and furthermore keeps it coherent. Excess water, on the other hand, promotes separation of the constituent materials, offsetting the good effects of hydration and rendering the concrete extremely harsh in working and difficult to handle. More mixing, therefore, or more efficient mixing through improved mixing devices, should be relied upon for easy placing, rather than excess water.

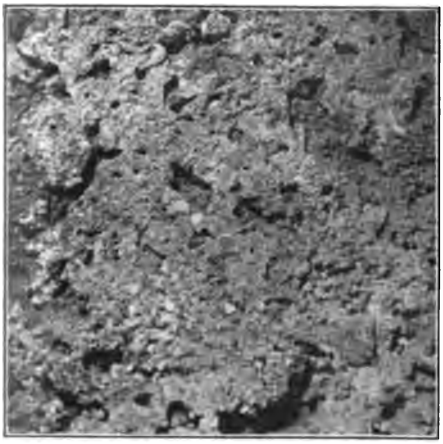


Fig. 15.—Water and air voids in concrete. (Natural size.)

44. Influence of Quantity of Water on Space Occupied in Resulting Concrete.—The fourth function of water in concrete—that of occupying space in the mass—is so important and so varied in its manifestations that a large treatise would be too small for adequate presentation. A few leading considerations may, however, serve to stimulate individual thought in this regard.

There are few substances so incompressible as water. Beyond question, although water is mobile, a given quantity occupies definite space. Concrete in forms is essentially in a confined space. In this form space are cement, sand, stone, *and water*, each occupying its proportionate share of the total volume of concrete. So long as the form remains tight, these substances must all remain substantially in place. If the form leaks, which is contrary to practice, more or less water, with a greater or less quantity of cement in solution, may escape. This escape may be before, *or after* the mass has taken either initial or final set. In this latter case, a hollow space,

[1] See Ostwald: "Colloidal Chemistry," p. 70-118.

or void of greater or less volume remains in the concrete where once was water in greater or less quantity or a solution too dilute to solidify (see Fig. 16). But leakage need not necessarily occur in this way. After forms are removed, any uncombined water or dilute solution will be free to escape, either by gravity, or by capillary suction aided by evaporation, leaving behind as a void the space it required in the mass. Such action is evidenced in hundreds of concretes examined.

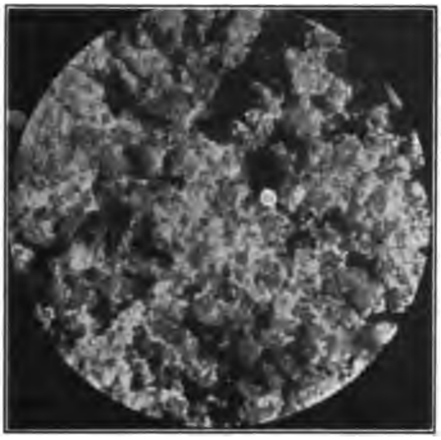


Fig. 16.—Water voids in concrete. (Magnified 18 diams.)

45. Harmful Effects of Voids Caused by Excess Water.—From the above it is evident that the more uncombined water, the more voids in the set concrete. Conversely, the more voids, the less the closeness of compacting of sand and stone; and the less this compacting, the less the density, strength, durability and value of the hardened mass. Furthermore, the proportions of the concrete are seriously unbalanced (see Arts. 2 and 16, Sect. 2).

The space loss referred to is only a small part of the ultimate damage. Physical stress due to loading may be the least intense of the stresses to which concrete is subjected. Physical, or chemical and physico-chemical stresses set up in the mass after hardening, through disruptive freezing, or through percolating water alone or carrying chemically active agents, are of far greater intensity. Each pore, or void, is a potential aid to such destructive agents; and enlarged by initial attack, soon become an active aid and abettor. First loss, therefore, may be of minor importance. Induced weaknesses, augmenting primary deficiencies, must be reckoned with to an increasing degree.

46. Excess Water the Cause of Day's Work Planes.—Perhaps the commonest evidence, to be found on every hand, as to the effects of excess water in concrete are "day's work planes." In the early life of a structure such as a buttress wall, these planes are hidden either by a smooth mortar surface at contact with forms, or by a later-applied wash or coat of cement plaster. But as months pass and the structure is subjected to water action in greater or less degree, from one source or another, these planes are made more and more evident by seepage along them. When such seepage is in quantity, it may be detected as a film of water, or, with rapid evaporation, by crystalline deposits. When seepage is less, it may be evidenced by a patch of efflorescence, but in each case the underlying cause of water passage is "laitance," which is largely caused by the use of excess water.

47. Excess Water the Cause of Large Laitance Deposits.—"Laitance," or "day's work planes," may be of small bulk, relative to the total mass of concrete, yet in some instances laitance is found to an exaggerated and oftentimes to a dangerous extent. Whenever forms are filled by dumping concretes continuously in one spot, with dependence upon hoeing-down, or natural flow for distribution of heavier materials into lower parts of forms, it is inevitable that water and the finer materials suspensible in water, including much of the cement, should separate from the heavier materials; and that they should form, when solidified above the concrete a deposit or stratum of greater or less thickness and extent, which will be entirely composed of muck or "laitance." This material is chalky and of low strength. It is very absorbent; and when saturated is of little better value than so much wet, sandy clay.

Instances of the formation of "laitance" in quantities and in situations where it is dangerous are found in columns poured in two or more sections. Although not approved by building codes, contractors, for their own convenience or to save on forms, will sometimes pour half a column, allowing it to set before continuing to the top. Inevitably, lighter materials rise in the form.

Necessarily the joint thus formed in the middle of the column is of inferior material; and of a material which cannot bond with material subsequently poured. If this procedure is again followed, the lighter part of this latter also rises so that a stratified column, with another "laitance" section at the column head will result. If after removal of forms this material should become wet from any cause, crushing and sliding is to be expected with possibly collapse of the column and its supported load. Columns should not only be poured in one section, but they furthermore should be poured of concrete of such consistency that "laitance" will not accumulate; and it would also be a desirable precaution to overflow the form to remove such accumulations as may rise. It is better to waste a portion of material at the top, in order to be sure that there may be no "laitance" at the column head, rather than to have any question as to strength or security.[1]

48. Excess Water and Waterproof Concrete.—It is difficult to find a truly waterproof field concrete, largely because excess water is so generally used in mixing. The majority of structures are of such size that they cannot be poured continuously. This necessarily means stoppage of work for greater or less intervals. Stoppage of work with wet concretes always means a layer of "laitance;" and this inevitably prevents succeeding layers from bonding, entailing a chain of consequences. A radical change in such field procedures is demanded, if these difficulties are to be overcome.

49. Excess Water Causes Unsatisfactory Concrete Floor Surfaces.[2]—It is difficult to insure that concrete floors shall be dustless. The functions of a concrete floor are to bear loads as well as to withstand abrasion and impact, these latter being the severest service to which it is subjected. It is unfortunate that the top of a concrete floor is the surface on which dependence is placed, as in possibly nine cases out of ten, this surface coat, both by virtue of its initial consistency and also because of water later brought to the surface through troweling, is partly or wholly composed of "laitance."

To remedy these defects, floor hardeners of one kind or another, are added to the concrete in mixing. Few of these substances should have any real place in the concrete-floor industry. Most are inferior to quartz sand in hardness and strength, but because of the prevalence of unsatisfactory concrete floors and because of the human tendency to escape consequences by purchasing immunities, such alleged remedies find ready sale. If, instead of buying integral floor hardeners, less water were put into concrete floors and a good quality of graded sand with Portland cement used in well-mixed and properly placed concrete, there would be less need of tonics.

50. Excess Water Prevents Bonding New Concrete to Old.—One result that can be guaranteed is the failure of effective bonding between new concrete and old. Various expedients from time to time have been claimed to bring about effective results in this regard, but little has as yet been unquestionably accomplished. Washing the surface of old concrete with hydrochloric acid is ineffective and wrong except so far as it may clean off surface dirt and carbonated deposits. Picking the surface rarely goes deep enough or covers enough surface. The inherent difficulty underlying all attempts at bonding, is the identical trouble that causes day's work planes, or that makes the wearing surface of concrete floors unsatisfactory, *i.e.*, the existence of a light, chalky, insecure material, substituted at the critical plane for a substance which should be durable and secure.

51. Excess Water and Concreting in Cold Weather.—Concreting in cold weather is always attended by some risk, even when forms remain in place until milder weather. Heating of aggregates is seldom adequate, and the heat transmitted through wooden forms after pouring is small in quantity. It should be remembered that at 40°F. the reaction between water and cement and the production of cementing strength is only one-fourth as rapid as at 50°F. and less than one-ninth as rapid as at 70°F. Dilution by excess water of such feeble solutions increases the danger, as is evidenced by frequent winter failures. Furthermore, at 39°F.

[1] See Gillmore: "On Limes, Hydraulic Cements and Mortars," 1872; p. 242.

[2] See Sect. 4, "Concrete Floors and Floor Surfaces, Sidewalks, and Roadways."

some subtle change occurs in water which decreases its chemical ability even before actual freezing;[1] and at this latter point occurs expansion of 8% by volume, with exertion of some 300 tons disruptive pressure per square inch of surface.

The greater the quantity of water in a cold-weather concrete, therefore, the greater the liability to dilution, little strength, frost disruptions, and failures. The potency of excess water in these respects is just beginning to receive due recognition.

52. Suggested Procedures to Guard Against Use of Excess Water.—Excess water in concrete should be rigidly guarded against. To insure the use of less water, specifications must embody provisions giving the engineer authority for its regulation. To this end, the following partial specification is suggested:

1. Concrete shall be an intimate mixture of sand, stone (or gravel), cement *and water* of the several kinds and qualities herein specified and in proportions as specified, subject to modification by the Engineer.

2. The proportions and quantities of all materials, *including water*, shall be as directed by the engineer and shall be subject at all times to such change as his tests or judgment may dictate as advisable.

3. *All* materials shall be accurately measured in measures of approved type and known capacity.

Cement shall be measured by the standard sack or, if in bulk, by weight, 94 lb. being taken as an equivalent of one sack. Loose measurement of cement is prohibited.

Sand and stone shall be measured in struck measures of a capacity and type approved by the engineer. Measurement in wheelbarrows of a type which do not admit of a struck measurement will not be permitted.

Water shall be measured at each mixer in containers adapted to ready adjustment and to accurate delivery of variable quantities. Supplementing the delivery of such measuring containers by additions of water, because of slowness of discharge or for any other reason, will not be permitted.

4. Concrete of a plastic consistency shall be required in all parts of the work, unless permission be given by the engineer for the use of drier and stiffer mixtures. Sloppy and overwet concretes are strictly prohibited. *The quantity of water, therefore, will be subject to regulation at all times by the engineer according to the requirements of the aggregates in use at that time.* The rejection and removal of overwet concretes either before or after placing in forms may, at the engineer's discretion, be required of the contractor without compensation.

REINFORCEMENT

53. Types of Reinforcement.—The reinforcing steel in reinforced-concrete construction is mostly in the form of rods, or bars, of round or square cross-section. These vary in size from ¼ to ⅜ in. for light floor slabs, up to 1¼ to 1½ in. as a maximum size for heavy beams and columns. Both plain and deformed bars are used. With plain bars the adhesion between steel and concrete is depended upon to furnish the necessary bond strength. With deformed bars the usual adhesion is supplemented by a mechanical bond, the amount of this bond in any given case depending upon the shape of the bar. The adhesion of concrete to flat bars is less than for round or square bars, but the flat deformed bar possesses advantages over other forms when used as hooping for tanks, pipes, and sewers where the reduced thickness of the bar allows the concrete section to have a greater effective depth for the same total thickness of concrete.

Wire fabric and expanded metal in various forms are used to a considerable extent in slabs, pipes, and conduits. These types of reinforcement are easy to place and are especially well adapted to resist temperature cracks and to prevent cracking of the concrete from impact or shock.

[1] See O. D. Van Engelen: *Century Magazine*, April, 1917.

A number of combinations of forms are employed to a greater or less extent. These combinations are known as *systems*.

54. Surface of Reinforcement.—A rough surface on steel has a higher bond value for use in concrete than a smooth surface, consequently a thin film of rust on reinforcement should not cause its rejection. In fact in the case of cold-drawn wire which presents a very smooth surface, a slight coating of rust is a decided advantage. Loose or scaly rust, however, should never be allowed. Reinforcement in this state of corrosion may be used if first cleansed with a stiff wire brush or given a bath of hydrochloric acid solution (consisting of 3 parts acid to 1 part water) and then washed in clean running water. Oiling and painting of reinforcing steel should not be permitted as its bonding value is greatly reduced thereby.

55. Quality of Steel.—Authorities differ as to the quality of steel to be used for reinforcement. Mild steel is the ordinary structural steel occurring in all structural shapes. High steel or steel of hard grade has a greater percentage of carbon than mild steel and is also known as high-carbon or high elastic-limit steel.

Brittleness is to be feared in high steel, although this quality is not so dangerous when the metal is used in heavy reinforced-concrete members—for example, in heavy beams or slabs—as the concrete to a large extent absorbs the shocks and protects the steel. All high steel should be carefully inspected and tested in order to prevent any brittle or cracked material from getting into the finished work. Steel of high elastic limit is seldom employed where plain bars are used.

Fig. 17.—Cold-twisted square bar.

Cold twisting increases the elastic limit and ultimate strength of mild-steel bars. The increase, however, is not definite, varying greatly with slight variations in the grade of the rolled steel. A square twisted bar is shown in Fig. 17.

56. Working Stresses.—The generally accepted working stress for mild steel is 16,000 lb. per sq. in. and 18,000 to 20,000 lb. per sq. in. for high steel and cold-twisted steel. A stress not greater than 16,000 lb. per sq. in. is recommended by the Joint Committee for all grades of steel.

57. Coefficient of Expansion.—The coefficient of expansion of steel is approximately 0.0000065 degree Fahrenheit.

58. Modulus of Elasticity.—The modulus of elasticity of all grades and kinds of steel is about the same and is usually taken as 30,000,000 lb. per sq. in. in both tension and compression.

59. Steel Specifications.—The following specifications are those of the Association of American Steel Manufacturers for concrete reinforcement bars rolled from billets, adopted March 22, 1910 (revised 1912 and 1914):

Manufacturers' Standard Specifications for Concrete Reinforcement Bars Rolled from Billets

1. *Manufacture.*—Steel may be made by either the open-hearth or Bessemer process. Bars shall be rolled from standard new billets.

2. *Chemical and Physical Properties.*—The chemical and physical properties shall conform to the limits as shown in the table on the following page.

3. *Chemical Determinations.*—In order to determine if the material conforms to the chemical limitations prescribed in paragraph 2 herein, analysis shall be made by the manufacturer from a test ingot taken at the time of the pouring of each melt or blow of steel, and a correct copy of such analysis shall be furnished to the engineer or his inspector.

4. *Yield Point.*—For the purposes of these specifications, the yield point shall be determined by careful observation of the drop of the beam of the testing machine, or by other equally accurate method.

5. *Form of Specimens.*—(*a*) Tensile and bending test specimens may be cut from the bars as rolled, but tensile and bending test specimens of deformed bars may be planed or turned for a length of at least 9 in. if deemed necessary by the manufacturer in order to obtain uniform cross-section.

(*b*) Tensile and bending test specimens of cold-twisted bars shall be cut from the bars after twisting, and shall be tested in full size without further treatment, unless otherwise specified as in (*c*), in which case the conditions thereon stipulated shall govern.

(*c*) If it is desired that the testing and acceptance for cold-twisted bars be made upon the hot-rolled bars before being twisted, the hot-rolled bars shall meet the requirements of the structural-steel grade for plain bars shown in this specification.

Properties considered	Structural-steel grade		Intermediate grade		Hard grade		Cold-twisted bars
	Plain bars	Deformed bars	Plain bars	Deformed bars	Plain bars	Deformed bars	
Phosphorus maximum: Bessemer....	0.10	0.10	0.10	0.10	0.10	0.10	0.10
Open-hearth..............	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Ultimate tensile strength, lb. per sq. in.	55/70,000	55/70,000	70/85,000	70/85,000	80,000 min.	80,000 min.	Recorded only
Yield point, minimum lb. per sq. in..	33,000	33,000	40,000	40,000	50,000	50,000	55,000
Elongation, % in 8-in. minimum....	1,400,000 / Tens. str.	1,250,000 / Tens. str.	1,300,000 / Tens. str.	1,125,000 / Tens. str.	1,200,000 / Tens. str.	1,000,000 / Tens. str.	5%
Cold bend without fracture: Bars under ¾-in. diameter or thickness.	180 deg. $d = 1t$	180 deg. $d = 2t$	180 deg. $d = 2t$	180 deg. $d = 3t$	180 deg. $d = 3t$	180 deg. $d = 4t$	180 deg. $d = 2t$
Bars ¾-in. diameter or thickness and over.	180 deg. $d = 1t$	180 deg. $d = 2t$	90 deg. $d = 2t$	90 deg. $d = 3t$	90 deg. $d = 3t$	90 deg. $d = 4t$	180 deg. $d = 3t$
The intermediate and hard grades will be used only when specified.							

6. *Number of Tests.*—(*a*) At least one tensile and one bending test shall be made from each melt of open-hearth steel rolled, and from each blow or lot of 10 tons of Bessemer steel rolled. In case bars differing ⅜ in. and more in diameter or thickness are rolled from one melt or blow, a test shall be made from the thickest and thinnest material rolled. Should either of these test specimens develop flaws, or should the tensile test specimen break outside of the middle third of its gaged strength, it may be discarded and another test specimen substituted therefor. In case a tensile test specimen does not meet the specifications, an additional test may be made.

(*b*) The bending test may be made by pressure or by light blows.

7. *Modifications in Elongation for Thin and Thick Material.*—For bars less than 7/16 in. and more than ¾ in. nominal diameter or thickness, the following modifications shall be made in the requirements for elongation:

(*a*) For each increase of ⅛ in. in diameter or thickness above ¾ in. a deduction of 1 shall be made from the specified percentage of elongation.

(*b*) For each decrease of 1/16 in. in diameter or thickness below 7/16 in. a deduction of 1 shall be made from the specified percentage of elongation.

(*c*) The above modifications in elongation shall not apply to cold-twisted bars.

8. *Number of Twists.*—Cold-twisted bars shall be twisted cold with one complete twist in a length equal to not more than 12 times the thickness of the bar.

9. *Finish.*—Material must be free from injurious seams, flaws or cracks, and have a workmanlike finish.

10. *Variation in Weight.*—Bars for reinforcement are subject to rejection if the actual weight of any lot varies more than 5% over or under the theoretical weight of that lot.

The following specifications are those of the American Society for Testing Materials for concrete reinforcement bars rolled from billets:

Standard Specifications for Billet-steel Concrete Reinforcement Bars

(American Society for Testing Materials)

1. (*a*) These specifications cover three classes of billet-steel concrete reinforcement bars, namely: plain, deformed and cold-twisted.

(*b*) Plain and deformed bars are of three grades, namely: structural-steel, intermediate and hard.

2. (*a*) The structural-steel grade shall be used unless otherwise specified.

(*b*) If desired, cold-twisted bars may be purchased on the basis of tests of the hot-rolled bars before twisting, in which case such tests shall govern and shall conform to the requirements specified for plain bars of structural-steel grade.

Manufacture.—3. (*a*) The steel may be made by the Bessemer or open-hearth process.

(*b*) The bars shall be rolled from new billets. No rerolled material will be accepted.

4. Cold twisted bars shall be twisted cold with one complete twist in a length not over 12 times the thickness of the bar.

Chemical Properties and Tests.—5. The steel shall conform to the following requirements as to chemical composition:

Phosphorus, Bessemer .... not over 0.10%
Open-hearth .... not over 0.05%

6. An analysis of each melt of steel shall be made by the manufacturer to determine the percentages of carbon, manganese, phosphorus and sulphur. This analysis shall be made from a test ingot taken during the pouring of the melt. The chemical composition thus determined shall be reported to the purchaser or his representative, and shall conform to the requirements specified in Sect. 5.

7. Analyses may be made by the purchaser from finished bars representing each melt of open-hearth steel, and each melt, or lot of 10 tons, of Bessemer steel. The phosphorus content thus determined shall not exceed that specified in Sect. 5 by more than 25%.

Physical Properties and Tests.—8. (*a*) The bars shall conform to the following requirements as to tensile properties:

Properties considered	Plain bars			Deformed bars			Cold-twisted bars
	Structural-steel grade	Intermediate grade	Hard grade	Structural-steel grade	Intermediate grade	Hard grade	
Tensile strength, lb. per sq. in.	55,000 to 70,000	70,000 to 85,000	80,000 min.	55,000 to 70,000	70,000 to 85,000	80,000 min.	Recorded only
Yield point, min., lb. per sq. in.	33,000	40,000	50,000	33,000	40,000	50,000	55,000
Elongation in 8 in. min. %[1]	1,400,000 / Tens. str.	1,300,000 / Tens. str.	1,200,000 / Tens. str.	1,250,000 / Tens. str.	1,125,000 / Tens. str.	1,000,000 / Tens. str.	5

[1] See Sect. 9.

(*b*) The yield point shall be determined by the drop of the beam of the testing machine.

9. (*a*) For plain and deformed bars over ¾ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sect. 8(*a*) shall be made for each increase of ⅛ in. in thickness or diameter above ¾ in.

(*b*) For plain and deformed bars under 7/16 in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sect. 8(*a*) shall be made for each decrease of 1/16 in. in thickness or diameter below 7/16 in.

10. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion, as follows:

Thickness or diameter of bar	Plain bars			Deformed bars			Cold-twisted bars
	Structural-steel grade	Intermediate grade	Hard grade	Structural-steel grade	Intermediate grade	Hard grade	
Under ¾ in.	180 deg. $d = t$	180 deg. $d = 2t$	180 deg. $d = 3t$	180 deg. $d = t$	180 deg. $d = 3t$	180 deg. $d = 4t$	180 deg. $d = 2t$
¾ in. or over	180 deg. $d = t$	90 deg. $d = 2t$	90 deg. $d = 3t$	180 deg. $d = 2t$	90 deg. $d = 3t$	90 deg. $d = 4t$	180 deg. $d = 3t$

d = diameter of pin about which the specimen is bent.
t = thickness or diameter of specimen.

11. (*a*) Tension and bend test specimens for plain and deformed bars shall be taken from the finished bars, and shall be of the full thickness or diameter of bars as rolled; except that the specimens for deformed bars may be machined for a length of at least 9 in., if deemed necessary by the manufacturer to obtain uniform cross-section.

(*b*) Tension and bend test specimens for cold-twisted bars shall be taken from the finished bars, without further treatment; except as specified in Sect. 2(*b*).

12. (*a*) One tension and one bend test shall be made from each melt of open-hearth steel, and from each melt, or lot of 10 tons, of Bessemer steel; except that if material from one melt differs ⅜ in. or more in thickness or diameter, one tension and one bend test shall be made from both the thickest and the thinnest material rolled.

(*b*) If any test specimen shows defective machining or develops flaws, it may be discarded and another specimen substituted.

(*c*) If the percentage of elongation of any tension test specimen is less than that specified in Sect. 8(*a*) and any part of the fracture is outside the middle third of the gage length, as indicated by scribe scratches marked on the specimen before testing, a retest shall be allowed.

Permissible Variations in Weight.—13. The weight of any lot of bars shall not vary more than 5% from the theoretical weight of that lot.

Finish.—14. The finished bars shall be free from injurious defects and shall have a workmanlike finish.

Inspection and Rejection.—15. The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the bars ordered. The manufacturer shall afford the inspector, free of cost, all reasonable facilities to satisfy him that the bars are being furnished in accordance with these specifications. All tests (except check analyses) and inspection shall be made at the place of manufacture prior to shipment, unless otherwise specified, and shall be so conducted as not to interfere unnecessarily with the operation of the works.

16. (*a*) Unless otherwise specified, any rejection based on tests made in accordance with Sect. 7 shall be reported within 5 working days from the receipt of the samples.

(*b*) Bars which show injurious defects subsequent to their acceptance at the manufacturer's works will be rejected, and the manufacturer shall be notified.

17. Samples tested in accordance with Sect. 7, which represent rejected bars, shall be preserved for 2 weeks from the date of the test report. In case of dissatisfaction with the results of the tests, the manufacturer may make claim for a rehearing within that time.

Reinforcing bars rolled from old rails are being used to a considerable extent in reinforced-concrete work and seem to be giving satisfaction, especially for unimportant work such as footings, retaining walls, and possibly in slabs where the failure of one rod could not wreck the structure. The specifications for rail-steel concrete reinforcement bars adopted by the Association of American Steel Manufacturers April 20, 1912 (revised April 21, 1914) are as follows:

Manufacturers' Standard Specifications for Rail-steel Concrete Reinforcement Bars

1. *Manufacture.*—All steel shall be rolled from standard section Tee rails.

2. *Physical Properties.*—The physical properties shall conform to the following limits:

Properties considered	Rail-steel grade	
	Plain bars	Deformed and hot-twisted bars
Ultimate tensile strength, minimum, lb. per sq. in.	80,000	80,000
Yield point, minimum, lb. per sq. in....	50,000	50,000
Elongation, % in 8-in. minimum......	1,200,000 / Tens. str.	1,000,000 / Tens. str.
Cold bend without fracture: Bars under ¾ in. diameter or thickness.	180 deg. $d = 3t$	180 deg. $d = 4t$
Bars ¾ in. diameter or thickness and over.	90 deg. $d = 3t$	90 deg. $d = 4t$

3. *Yield Point.*—For the purposes of these specifications, the yield point shall be determined by careful observation of the drop of the beam of the testing machine, or by other equally accurate method.

4. *Form of Specimens.*—(*a*) Tensile and bending test specimens may be cut from the bars as rolled, but tensile and bending test specimens of deformed bars may be planed or turned for a length of at least 9 in. if deemed necessary by the manufacturer in order to obtain uniform cross-section.

(*b*) Tensile and bending test specimens of hot-twisted bars shall be cut from the bars after twisting, and shall be tested in full size without further treatment, unless otherwise specified.

5. *Number of Tests.*—(*a*) One tensile and one bending test shall be made from each lot of 10 tons or less of each size of bar rolled from rails varying not more than 10 lb. per yd. in nominal weight. Should a test specimen develop flaws, or should the tensile test specimen break outside of the middle third of its gaged length, it may be discarded and another test specimen substituted therefore. In case a tensile specimen does not meet the specifications, an additional test may be made.

(*b*) The bending test may be made by pressure or by light blows.

6. *Modifications in Elongation for Thin and Thick Material.*—For bars less than 7/16 in. and more than ¾ in. nominal diameter or thickness, the following modifications shall be made in the requirements for elongation:

(*a*) For each increase of ⅛ in. in diameter or thickness above ¾ in., a deduction of 1 shall be made from the specified percentage of elongation.

(*b*) For each decrease of 1/16 in. in diameter or thickness below 7/16 in., a deduction of 1 shall be made from the specified percentage of elongation.

7. *Number of Twists.*—Hot-twisted bars of rail carbon steel shall be twisted with one complete twist in a length equal to not more than 12 times the thickness of the bar.

8. *Finish.*—Material must be free from injurious seams, flaws or cracks, and have a workmanlike finish.

9. *Variation in Weight.*—Bars for reinforcement are subject to rejection if the actual weight of any lot varies more than 5% over or under the theoretical weight of that lot.

Rerolled bar specifications have also been adopted by the American Society for Testing Materials after an extended series of tests. The specifications follow:

Standard Specifications for Rail-steel Concrete Reinforcement Bars

(American Society for Testing Materials)

1. The specifications cover three classes of rail-steel concrete reinforcement bars, namely: plain, deformed, and hot-twisted.

Manufacture.—2. The bars shall be rolled from standard section Tee rails.

3. Hot-twisted bars shall have one complete twist in a length not over 12 times the thickness of the bar.

Physical Properties and Tests.—4. (*a*) The bars shall conform to the following minimum requirements as to tensile properties:

Properties considered	Plain bars	Deformed and hot-twisted bars
Tensile strength, lb. per sq. in.	80,000	80,000
Yield point, lb. per sq. in.	50,000	50,000
Elongation in 8 in. %[1]	$\frac{1,200,000}{\text{Tens. str.}}$	$\frac{1,000,000}{\text{Tens. str.}}$

[1] See Sect. 5.

(*b*) The yield point shall be determined by the drop of the beam of the testing machine.

5. (*a*) For bars over ¾ in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sect. 4(*a*) shall be made for each increase of ⅛ in. in thickness or diameter above ¾ in.

(*b*) For bars under 7/16 in. in thickness or diameter, a deduction of 1 from the percentages of elongation specified in Sect. 4(*a*) shall be made for each decrease of 1/16 in. in thickness or diameter below 7/16 in.

6. The test specimen shall bend cold around a pin without cracking on the outside of the bent portion, as follows:

Thickness or diameter of bar	Plain bars	Deformed and hot-twisted bars
Under ¾ in.	180 deg. $d = 3t$	180 deg. $d = 4t$
¾ in. or over	90 deg. $d = 3t$	90 deg. $d = 4t$

d = diameter of pin about which the specimen is bent.
t = thickness or diameter of the specimen.

7. (*a*) Tension and bend test specimens for plain and deformed bars shall be taken from the finished bars, and shall be of the full thickness or diameter of bars as rolled; except that the specimens for deformed bars may be machined for a length of at least 9 in., if deemed necessary by the manufacturer to obtain uniform cross-section.

(*b*) Tension and bend test specimens for hot-twisted bars shall be taken from the finished bars, without further treatment.

8. (*a*) One tension and one bend test shall be made from each lot of 10 tons or less of each size of bar rolled from rails varying not more than 10 lb. per yd. in nominal weight.

(*b*) If any test specimen shows defective machining or develops flaws, it may be discarded and another specimen substituted.

(*c*) If the percentage of elongation of any tension test specimen is less than that specified in Sect. 4(*a*) and any part of the fracture is outside the middle third of the gage length, as indicated by scribe scratches marked on the specimen before testing, a retest shall be allowed.

Permissible Variations in Weight.—9. The weight of any lot of bars shall not vary more than 5% from the theoretical weight of that lot.

Finish.—10. The finished bars shall be free from injurious defects and shall have a workmanlike finish.

Inspection and Rejection.—11. The inspector representing the purchaser shall have free entry, at all times while work on the contract of the purchaser is being performed, to all parts of the manufacturer's works which concern the manufacture of the bars ordered. The manufacturer shall afford the inspector, free of cost, all reasonable facilities to satisfy him that the bars are being furnished in accordance with these specifications. All tests and inspection shall be made at the place of manufacture prior to shipment, unless otherwise specified, and shall be so conducted as not to interfere unnecessarily with the operation of the works.

12. Bars which show injurious defects subsequent to their acceptance at the manufacturer's works will be rejected, and the manufacturer shall be notified.

60. Factors Affecting Cost of Reinforcing Bars.—In order to insure minimum cost and prompt delivery of steel reinforcing bars, the steel schedule for a reinforced-concrete structure should call for bars of as few different sizes and lengths as possible. Bars of odd 16th sizes are seldom to be found in stock (except the 5/16-in. size which is frequently used for slab reinforcement) and shipments from the mill on such sizes are likely to be very slow. Designers should always bear in mind this fact and arrange to use either round or square bars in 1/8-in. sizes. Wherever possible, steel lengths that do not vary greatly on the schedule should all be made equal since an order calling for only a few different lengths will be put through the mill much faster than one calling for many different lengths.

The following size extras for bars less than 3/4-in. are standard with all mills and are the same for either round or square bars:

Size Extras for Rounds and Squares in Cents per 100 Lb.

3/4-in. and larger	Base
5/8 to 11/16-in.	5 cts. extra
1/2 to 9/16-in.	10 cts. extra
7/16-in.	20 cts. extra
3/8-in.	25 cts. extra
5/16-in.	35 cts. extra
1/4-in.	50 cts. extra

It should be noticed that a higher size extra must be paid for an odd 16th size below 3/4-in. than for the next larger 1/8-in. size. This fact alone offsets any advantage in saving steel by always calling for the nearest theoretical size whether odd or even.

Where the character of the work requires small bars a saving in cost is obtained by using round bars owing to the difference in size extras between rounds and squares of equivalent area.

Lengths less than 5 ft. should be avoided, if possible, as they are subject to the following cutting extras, whether sheared or hot-sawed:

Lengths over 24 in. and less than 60 in.	5 cts. per 100 lb.
Lengths 12 in. to 24 in. inclusive	10 cts. per 100 lb.
Lengths under 12 in.	15 cts. per 100 lb.

All orders calling for less than 2000 lb. of the same size and shape are subject to the following extras:

Quantities less than 2000 lb. but not less than 1000 lb.	15 cts. per 100 lb.
Quantities less than 1000 lb.	35 cts. per 100 lb.

61. Deformed Bars.—The following deformed bars are in common use:

61a. Diamond Bar (Fig. 18).—Furnished by Concrete-Steel Engineering Co., New York City. The standard sizes are as follows:

Diamond Bars

Size in inches	1/4	3/8	7/16	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
Area in square inches....	0.0625	0.1406	0.19	0.25	0.39	0.56	0.76	1.00	1.26	1.56
Weight per foot in pounds.	0.213	0.478	0.65	0.85	1.33	1.91	2.60	3.40	4.30	5.31

Fig. 18.—Diamond bar.

It should be noted that the weights and areas of Diamond bars are equal to those of plain square bars of like denominations.

61b. Corrugated Bars (Fig. 19).—Furnished by Corrugated Bar Co., Buffalo, N. Y. The standard sizes are as follows:

CORRUGATED ROUNDS

Size in inches	3/8	1/2	9/16	5/8	3/4	7/8	1	1 1/8	1 1/4
Net area in square inches......	0.11	0.19	0.25	0.30	0.44	0.60	0.78	0.99	1.22
Weight per foot in pounds.....	0.38	0.66	0.86	1.05	1.52	2.06	2.69	3.41	4.21

CORRUGATED SQUARES

Size in inches	1/4	3/8	1/2	5/8	3/4	7/8	1	1 1/8	1 1/4
Net area in square inches.......	0.06	0.14	0.25	0.39	0.56	0.76	1.00	1.26	1.55
Weight per foot in pounds......	0.22	0.49	0.86	1.35	1.94	2.64	3.43	4.34	5.35

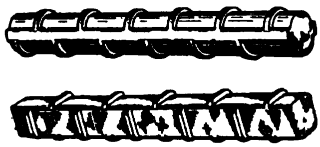


FIG. 19.—Corrugated bars.

61c. Havermeyer Bars (Fig. 20).—Furnished by Concrete Steel Co., New York City. The following table gives the weights and areas of the standard Havermeyer bars:

HAVERMEYER BARS

Size in inches	Squares		Rounds		Flats		
	Area in square inches	Weight per foot in pounds	Area in square inches	Weight per foot in pounds	Size in inches	Area in square inches	Weight per foot in pounds
1/4	0.0625	0.212	0.0491	0.167	1×1/4	0.2500	0.850
5/16	0·9770	0.332	......	.....	1×3/8	0.3750	1.280
3/8	0.1406	0.478	0.1104	0.375	1 1/4×3/8	0.4690	1.590
1/2	0.2500	0.850	0.1963	0.667	1 1/2×5/16	0.4688	1.590
5/8	0.3906	1.328	0.3068	1.043	1 1/2×3/8	0.5625	1.913
3/4	0.5625	1.913	0.4418	1.502	1 1/2×1/2	0.7500	2.550
7/8	0.7656	2.603	0.6013	2.044	1 3/4×3/8	0.6563	2.230
1	1.0000	3.400	0.7854	2.670	1 3/4×7/16	0.7656	2.600
1 1/8	1.2656	4.303	0.9940	3.379	1 3/4×1/2	0.8750	2.980
1 1/4	1.5625	5.312	1.2272	4.173			

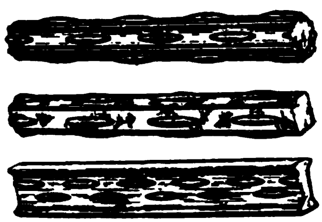


FIG. 20.—Havermeyer bars.

Special sizes of 1⅜-in. and 1½-in. square Havermeyer bars can be rolled by special arrangement, but are not carried in stock. A size extra of 10 cts. applies against 1 by ¼-in. and 1½ by 5/16-in. flats; all other sizes tabulated take the base price.

61d. Rib Bar (Fig. 21).—Furnished by Trussed Concrete Steel Co., of Youngstown, Ohio and Detroit, Mich. The following sizes are standard:

RIB BAR

Size in inches	Area in square inches	Weight per linear foot in pounds	Size in inches	Area in square inch5s	Weight per linear foot in pounds
⅜	0.1406	0.48	⅞	0.7656	2.65
½	0.2500	0.86	1	1.0000	3.46
⅝	0.3906	1.35	1⅛	1.2656	4.38
¾	0.5625	1.95			

FIG. 21.—Rib bar.

60e. Inland Bar (Fig. 22).—Furnished by Inland Steel Co., Chicago.

Sizes ⅜ in. to ⅝ in. inclusive with single row of stars on each side.

Sizes ¾ in. to 1¼ in. inclusive with double row of stars on each side.

Lengths may be obtained up to 85 ft. Supplied in both open-hearth steel and rail carbon steel.

Standard sizes are as follows:

INLAND BAR

Size in inches	⅜	½	⅝	¾	⅞	1	1⅛	1¼
Area in square inches....	0.140	0.250	0.390	0.562	0.765	1.000	1.265	1.562
Weight per foot in pounds.	0.485	0.862	1.341	1.932	2.630	3.434	4.349	5.365

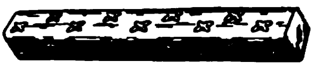


FIG. 22.—Inland bar.

Rail carbon steel bars not rolled larger than 1 in.

61f. American Bars (Fig. 23).—Furnished by American System of Reinforcing, Chicago. The following sizes are standard:

American Bars

Size in inches	Squares		Rounds	
	Net area in square inches	Weight per foot in pounds	Net area in square inches	Weight per foot in pounds
3/8	0.141	0.48	0.110	0.38
1/2	0.250	0.85	0.196	0.68
5/8	0.391	1.33	0.307	1.05
3/4	0.563	1.92	0.442	1.51
7/8	0.766	2.61	0.602	2.05
1	1.000	3.40	0.786	2.68
1 1/8	1.270	4.31	0.994	3.38
1 1/4	1.560	5.32	1.230	4.19

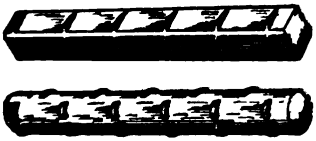


Fig. 23.—American bars.

62. Wire Fabric.—This material is used to a considerable extent for floors, roofs, walls, vaults, pavement, etc., and has been found to possess many valuable qualities. Wire fabric is made of steel wires crossing generally at right angles and secured at the intersections. The heavier wires run lengthwise and are called carrying wires; the lighter ones cross these and are called distributing or tie wires. One distinct advantage in the use of fabric is that it preserves uniform spacing of the steel.

The steel wire gage adopted as standard for all steel wire upon recommendation of the United States Bureau of Standards is given in the following table:

Steel Wire Gage

Diameter, inches	Steel wire gage[1]	Diameter, inches	Area, square inches	Pounds per foot	Pounds per mile	Feet per pound
1/2		0.5000	0.19635	0.6668	3,521.0	1.500
	7/0	0.4900	0.18857	0.6404	3,381.0	1.562
15/32		0.46875	0.17257	0.5861	3,094.0	1.706
	6/0	0.4615	0.16728	0.5681	2,999.0	1.760
7/16		0.4375	0.15033	0.5105	2,696.0	1.959
	5/0	0.4305	0.14556	0.4948	2,610.0	2.022
13/32		0.40625	0.12962	0.4402	2,324.0	2.272
	4/0	0.3938	0.12180	0.4136	2,184.0	[illegible]
3/8		0.3750	0.11045	0.3751	1,980.0	[illegible]
	3/0	0.3625	0.10321	0.3505	1,851.0	[illegible]
11/32		0.34375	0.092806	0.3152	1,664.0	[illegible]
	2/0	0.3310	0.086049	0.2922	1,543.0	[illegible]

[1] Formerly called the "American Steel & Wire Co's. Gage."

STEEL WIRE GAGE.—(*Continued.*)

Diameter, inches	Steel wire gage	Diameter, inches	Area, square inches	Pounds per foot	Pounds per mile	Feet per pound
5/16		0.3125	0.076699	0.2605	1,375.0	3.839
	0	0.3065	0.073782	0.2506	1,323.0	3.991
	1	0.2830	0.062902	0.2136	1,128.0	4.681
9/32		0.28125	0.062126	0.2110	1,114.0	4.74
	2	0.2625	0.054119	0.1838	970.4	5.441
1/4		0.2500	0.049087	0.1667	880.2	5.999
	3	0.2437	0.046645	0.1584	836.4	6.313
	4	0.2253	0.039867	0.1354	714.8	7.386
7/32		0.21875	0.037583	0.1276	673.9	7.835
	5	0.2070	0.033654	0.1143	603.4	8.750
	6	0.1920	0.028953	0.09832	519.2	10.17
3/16		0.1875	0.027612	0.09377	495.1	10.66
	7	0.1770	0.024606	0.08356	441.2	11.97
	8	0.1620	0.020612	0.07000	369.6	14.29
5/32		0.15625	0.019175	0.06512	343.8	15.36
	9	0.1483	0.017273	0.05866	309.7	17.05
1/8	10	0.1350	0.014314	0.04861	256.7	20.57
		0.125	0.012272	0.04168	220.0	24.00
	11	0.1205	0.011404	0.03873	204.5	25.82
	12	0.1055	0.0087417	0.02969	156.7	33.69
3/32		0.09375	0.0069029	0.02344	123.8	42.66
	13	0.0915	0.0065755	0.02233	117.9	44.78
	14	0.0800	0.0050266	0.01707	90.13	58.58
	15	0.0720	0.0040715	0.01383	73.01	72.32
	16	0.0625	0.0030680	0.01042	55.01	95.98
	17	0.0540	0.0022902	0.007778	41.07	128.60

The manner of securing the intersections of wire fabric has given rise to a number of different types, several of the principal ones of which are given below.

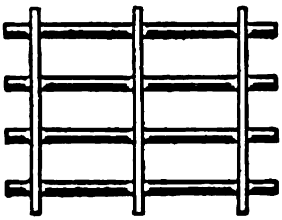


FIG. 24.—Welded wire fabric.

62a. Welded Wire Fabric.—Welded wire fabric, Fig. 24, manufactured by the Clinton Wire Cloth Co., is a galvanized wire mesh made up of a series of parallel longitudinal wires, spaced a certain distance apart and held at intervals by means of transverse wires, arranged at right angles to the longitudinal ones, and welded to them at the points of intersection by a patented electrical process. Longitudinal wires can be spaced on centers of 2 or more in., in steps of 1/2 in. Transverse wires can be spaced on centers of 1 to 18 in. inclusive, in steps of 1 in. and on centers of 10 to 18 in. inclusive, in steps of 2 in. The following table shows the sizes and areas of the wire used. Rolls kept in stock vary in length between 150 and 200 ft. and between 56 and 100 in. in width. The wire will develop an average ultimate strength of 70,000 to 80,000 lb. per sq. in.

WELDED WIRE FABRIC

Gage of longitudinal wires	Diameter of longitudinal wires (inches)	Area of one longitudinal wire (square inches)	Gage of transverse wires	Spacing of transverse wires (inches)	Area per foot of width in longitudinal wires only — Spacing of longitudinal wires: 2 in.	3 in.	4 in.	5 in.	6 in.
0000	0.394	0.122	3	16	0.735	0.490	0.367	0.294	0.245
000	0.363	0.103	4	16	0.619	0.413	0.310	0.248	0.206
00	0.331	0.086	4	16	0.516	0.344	0.258	0.207	0.172
0	0.307	0.074	6	16	0.443	0.295	0.221	0.177	0.148
1	0.283	0.063	6	16	0.377	0.252	0.189	0.151	0.126
2	0.263	0.054	8	16	0.325	0.217	0.162	0.130	0.108
3	0.244	0.047	8	16	0.280	0.187	0.140	0.112	0.093
4	0.225	0.040	9	16	0.239	0.160	0.120	0.096	0.080
5	0.207	0.034	9	16	0.202	0.135	0.101	0.081	0.067
6	0.192	0.029	10	16	0.174	0.116	0.087	0.069	0.058
7	0.177	0.025	10	16	0.148	0.098	0.074	0.059	0.049
8	0.162	0.021	10	12	0.124	0.082	0.062	0.049	0.041
9	0.148	0.017	11	12	0.104	0.069	0.052	0.041	0.035
10	0.135	0.014	12	12	0.086	0.057	0.043	0.034	0.029

62b. Triangle-mesh Wire Fabric.—Triangle-mesh steel-wire fabric, manufactured by the American Steel & Wire Co., is made with both single and stranded longitudinal or tension members. That with the single wire longitudinal is made with one wire varying in size from a No. 12 gage up to and including a ½-in. diameter, and that with the stranded longi-

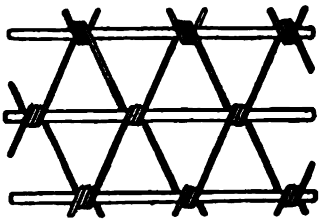


FIG. 25.—Triangle-mesh wire fabric.

tudinal is composed of two or three wires varying from No. 12 gage up to and including No. 4 wires stranded or twisted together with a long lay. These longitudinals either solid or stranded are invariably spaced 4-in. centers, the sizes being varied in order to obtain the desired cross-sectional area of steel per foot of width (see Fig. 25).

The transverse or diagonal cross wires are so woven between the longitudinals that triangles are formed by their arrangement. These diagonal cross wires are woven either 2 or 4 in. apart, as is desired. Triangle-mesh wire reinforcement is made in lengths of 150, 200, and 300 ft. and in widths from 18 to 58 in. (4-in. steps). The table following shows the number and gage of wires and the areas per foot width when the longitudinals and cross wires are spaced 4 in. on centers.

TRIANGLE-MESH WIRE FABRIC

Style number	Number of wires, each long.	Gage of wire, each long.	Gage of cross wires	Sectional area, long. square inches	Sectional area, cross wires, square inches	Cross-sectional area per foot width	Approximate weight per 100 sq. ft.
4[1]	1	6	14	0.087	0.025	0.102	43
5[1]	1	8	14	0.062	0.025	0.077	34
6[1]	1	10	14	0.043	0.025	0.058	27
7[1]	1	12	14	0.026	0.025	0.041	21
23[1]	1	¼	12½	0.147	0.038	0.170	72
24	1	4	12½	0.119	0.038	0.142	62
25	1	5	12½	0.101	0.038	0.124	55
26[1]	1	6	12½	0.087	0.038	0.110	50
27[1]	1	8	12½	0.062	0.038	0.085	41
28[1]	1	10	12½	0.043	0.038	0.066	34
29[1]	1	12	12½	0.026	0.038	0.049	28
31[1]	2	4	12½	0.238	0.038	0.261	106
32[1]	2	5	12½	0.202	0.038	0.225	92
33	2	6	12½	0.174	0.038	0.196	82
34	2	8	12½	0.124	0.038	0.146	63
35	2	10	12½	0.086	0.038	0.109	50
36	2	12	12½	0.052	0.038	0.075	37
38[1]	3	4	12½	0.358	0.038	0.380	151
39	3	5	12½	0.303	0.038	0.325	130
40[1]	3	6	12½	0.260	0.038	0.283	114
41	3	8	12½	0.185	0.038	0.208	87
42[1]	3	10	12½	0.129	0.038	0.151	66
43	3	12	12½	0.078	0.038	0.101	47

Elastic limit of regular stock is from 50,000 to 60,000 lb. per sq. in. Ultimate strength is 85,000 lb. per sq. in. or over. Higher elastic limits and breaking strengths are furnished when required. Material may be obtained either plain or galvanized. Unless otherwise specified, shipments are made of material not galvanized.

62c. Unit Wire Fabric.—A rectangular-mesh staple-locked fabric (Fig. 26) is furnished by the American System of Reinforcing. The wire used is of high tensile steel and

[1] Styles usually carried in stock.

is secured at the intersections by No. 14 wire. Standard sizes are shown in the following table. The fabric is galvanized and comes in standard widths of 3. 4, and 5 ft., 200 lin. ft. in a roll.

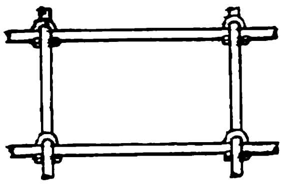


Fig. 26.—Unit wire fabric

Unit Wire Fabric

Gage of longitudinal wires	Gage of cross wires	Distance center to center in inches		Sectional area in sq. in., foot width
		Longitudinal wires	Cross wires	
11	11	6	6	0.023
10	10	6	6	0.028
9	11	6	6	0.035
9	11	4	12	0.05
9	11	3	12	0.07
8	11	4	12	0.062
7	11	4	12	0.074
6	11	4	12	0.087
5	11	4	12	0.10
4	11	4	12	0.12
3	11	4	12	0.14

62d. Lock-woven Steel Fabric.—Lock-woven steel fabric (Fig. 27) is also known as Page Special Process fabric. It is manufactured by the Page Woven Wire Fence Co., of

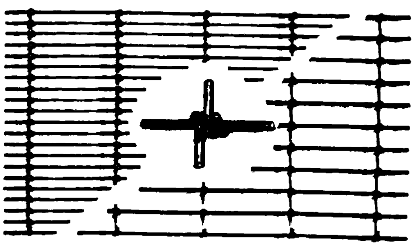


Fig. 27.—Lock-woven steel fabric.

Monessen, Pa. and is controlled by W. W. Wight & Co. of New York City. This fabric is usually made 54 in. wide with special widths from 18 to 54 in. The longitudinal wires are made by a special process which gives them an ultimate tensile strength of 180,000 lb. per sq. in. with an elastic limit of about 70% of the ultimate. The material is galvanized and is furnished in rolls of 150, 300, 450 and 600 ft. in length. The table on page 50 gives the characteristics of the different styles.

4

LOCK-WOVEN STEEL FABRIC

Style	Gage		Spacing in inches		Sectional area in sq. in. per foot width	Ultimate strength in pounds per foot width	Weight per 100 sq. ft.
	Long.	Trans.	Long.	Trans.			
14P	14	14	3	12	0.0201	3,621	11.04
13P	13	14	3	12	0.0265	4,790	12.91
12P	12	14	3	12	0.0350	6,300	15.85
11P	11	14	3	12	0.0452	8,140	17.47
9P	9	14	3	12	0.0680	12,390	28.62
8P	8	14	3	12	0.0824	14,280	34.82
7P	7	14	3	12	0.0984	17,720	39.48
14D	14	14	1½	12	0.0402	7,242	22.08
13D	13	14	1½	12	0.0532	9,580	25.82
12D	12	14	1½	12	0.0700	12,600	31.70
11½D	11½	14	1½	12	0.0795	14,313	33.25
11D	11	14	1½	12	0.0904	16,290	34.94
9½D	9½	14	1½	12	0.12498	22,450	53.43
9D	9	14	1½	12	0.1376	24,780	57.20
8D	8	14	1½	12	0.1648	29,640	69.64
7D	7	14	1½	12	0.1968	35,440	78.96

62e. Wisco Reinforcing Mesh.—Wisco mesh is manufactured by the Witherow Steel Co., Pittsburgh, Pa. It is made from the best grade of open-hearth steel and has a high tensile strength. All longitudinals are spaced 3 in. c. to c. and cross wires 12 in. c. to c. Standard rolls are 150 and 300 ft. in length. Width of rolls are furnished in any multiple of 3 in. from 18 to 54 in. Properties of the Wisco mesh are given in the following table:

WISCO MESH

Style	Sectional area per foot width	Weight per square foot	Style	Sectional area per foot width	Weight per square foot	Style	Sectional area per foot width	Weight per square foot
14	0.020	0.110	9½	0.062	0.277	6	0.116	0.465
12	0.035	0.158	9	0.069	0.286	29	0.138	0.556
11	0.046	0.175	8	0.083	0.341	27	0.197	0.775
10	0.058	0.223	7	0.098	0.395	26	0.230	1.036

63. Expanded Metal.—Expanded metal (Fig. 28) is one of the oldest forms of sheet reinforcement. It is formed by slitting a sheet of soft steel and then expanding the metal in a direction normal to the axis of the sheet. The principal advantages claimed for this type of reinforcement are the following: (1) An increased ultimate strength and high elastic limit for low-carbon steel when the diamond-shaped meshes are formed by cold drawing the metal; (2) a mechanical bond with the surrounding concrete; (3) great efficiency in the carrying of concentrated loads due to the ob-

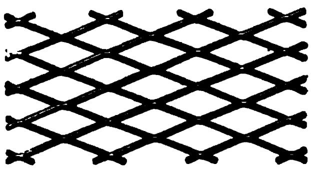


FIG. 28.—Expanded metal.

liquity of the strands; (4) an increased ductility because of the fact that the diamonds or quadrilaterals tend to close under severe loading; (5) a greater slab strength as the effect of closing up of the diamonds is to introduce a compression into the concrete at the lower part of the slab. Expanded metal and other sheet metal is made according to the U. S. Standard gage which differs but slightly from the Steel Wire gage given on page 45.

63a. Steelcrete.—Manufactured by the Consolidated Expanded Metal Cos., Rankin, Pa. The designation of the material gives the width of the diamond, the gage of the plate and the cross-section per foot of width. Size 3-9-15 means that it is a 3-in. diamond, made out of No. 9 plate, having a sectional area per foot of width of 0.15 sq. in. All standard meshes have a diamond 3 by 8 in. The standard sizes and gages are given in the following table:

"Steelcrete" Expanded Metal

Designation of mesh	Size of mesh			Wt. per sq. ft. in pounds	No. of sheets in a bundle	Size of standard sheets	No. of sq. ft. in a bundle	Wt. per bundle in lb.
	Width of diamond in inches	Length of diamond in inches	Section in sq. in. per ft. of width					
3-13-075	3	8	0.075	0.27	10	6'0" × 8'0"	480	129.6
						6'0" × 12'0"	720	194.4
3-13-10	3	8	0.10	0.37	7	6'9" × 8'0"	378	139.9
						6'9" × 12'0"	567	209.8
3-13-125	3	8	0.125	0.46	7	5'3" × 8'0"	294	135.2
						5'3" × 12'0"	441	202.9
3-9-15	3	8	0.15	0.55	5	7'0" × 8'0"	280	154.0
						7'0" × 12'0"	420	231.0
3-9-20	3	8	0.20	0.73	5	5'3" × 8'0"	210	153.3
						5'3" × 12'0"	315	230.0
3-9-25	3	8	0.25	0.92	5	4'0" × 8'0"	160	147.2
						4'0" × 12'0"	240	220.8
3-9-30	3	8	0.30	1.10	2	7'0" × 8'0"	112	123.2
						7'0" × 12'0"	168	184.8
3-9-35	3	8	0.35	1.28	2	6'0" × 8'0"	96	122.9
						6'0" × 12'0"	144	184.3
3-6-40	3	8	0.40	1.46	2	7'0" × 8'0"	112	163.5
						7'0" × 12'0"	168	245.3
3-6-45	3	8	0.45	1.65	2	6'3" × 8'0"	100	165.0
						6'3" × 12'0"	150	247.5
3-6-50	3	8	0.50	1.83	2	5'9" × 8'0"	92	168.4
						5'9" × 12'0"	138	252.5
3-6-55	3	8	0.55	2.01	2	5'3" × 8'0"	84	168.8
						5'3" × 12'0"	126	253.3
3-6-60	3	8	0.60	2.19	2	4'9" × 8'0"	76	166.4
						4'9" × 12'0"	114	249.7

The Consolidated Expanded Metal Cos. also make to order a 6-in. mesh, the size of the diamond being 6 by 16 in. The gage of plate used is No. 4, or nearly ¼ in. thick. Any cross-sectional area desired up to and including 0.4 sq. in. can be obtained. The width of the sheets depend on the sectional area. These companies also make a 4-in. mesh from No. 16 plate which is unexpanded. Any length can be obtained up to 16 ft. The cross-sectional area per foot of width is 0.093 sq. in. Special meshes can be obtained having diamonds of ¾ in., 1½ in., and 2 in.

63*b*. Kahn Mesh.—Manufactured by the Trussed Concrete Steel Co., of Youngstown, Ohio, and Detroit, Mich. The standard sizes and gages are the same as for "Steelcrete." The Kahn Mesh may also be obtained with larger diamonds for reinforcing concrete pavements. The sizes of the Kahn Road Mesh follow:

KAHN ROAD MESH

Size No.	Decimal designation	Size of mesh		Sectional area in square inches
		Width of diamond in inches	Length of diamond in inches	
15	6-13-042	6	12	0.042
20	6-13-053	6	12	0.053
22	6-13-058	6	12	0.058
25	6-13-066	6	12	0.066
28	6-13-074	6	12	0.074
30	6-9-079	6	12	0.079
32	6-9-085	6	12	0.085

No. of sheets in bundle, 10. Standard width of sheets, 5 ft. Standard lengths of sheets, 8 ft., 10 ft., 12 ft., or any equal divisions of these lengths.

63*c*. Corr-X-Metal.—Furnished by the Corrugated Bar Co., Buffalo, N. Y. The weights, sectional areas and standard sizes of sheets are given in the following table:

CORR-X-METAL

Style	Size of mesh, short way (inches)	Nominal thickness of metal (gage)	Approximate weight per square foot, (pounds)	Net sec. area per foot of width (square inches)
F	3	10	0.51	0.150
G	3	10	0.6	0.176
H	3	10	0.9	0.265
J	3	10	1.2	0.353
K	3	16	0.278	0.082
L	2¼	16	0.4	0.118
M	2¼	12	0.56	0.164
R	1½	12	0.66	0.194
S	¾	13	0.84	0.246

STANDARD SIZE SHEETS

Style	Long way of diamond	Short way of diamond
F	6′, 8′, 9′ and 10′ 8″	3′, 4′, 5′ and 6′
G	6′, 8′, 9′ and 10′ 8″	3′, 4′, 5′ and 6′
H	6′, 8′, 9′ and 10′ 8″	4′ and 5′ 4″
J	6′, 8′, 9′ and 10′ 8″	3′, 4′ and 6′
K	6′ and 8′ and 10′ 8″	3′, 4′, 5′ and 6′
L	6′ and 8′ and 10′ 6″	3′, 4′, 5′ and 6′
M	6′ and 8′ and 10′ 6″	4′ and 5′ 4″
R	6′	3′, 4′, 5′ and 6′
S	6′	3′, 4′, 5′ and 6′

63d. Econo.—Furnished by the North Western Expanded Metal Co., Chicago, Ill. Standard sizes and weights are as follows:

ECONO EXPANDED METAL

No.	Weight per square foot, pounds	Mesh and gage	Widths, feet	Lengths, feet
06-3	0.20	3″—16 ga.	3, 4, 6	8 and 12
10-3	0.34	3″—12 ga.	3, 4, 6	8 and 12
15-3	0.51	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
16-3	0.55	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
20-3	0.68	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
25-3	0.85	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
30-3	1.02	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
35-3	1.19	3″—10 ga.	3, 4, 6	8′, 10′ 6″ and 12′
40-3	1.36	3″— 7 ga.	3′ 6″, 7′ 0″	8 and 12
10-2¼	0.34	2¼″—16 ga.	3, 4, 6	8 and 12
15-2¼	0.51	2¼″—12 ga.	3, 4, 6	8 and 12
20-2¼	0.68	2¼″—10 ga.	3, 4, 6	8 and 12
40-2¼	1.36	2¼″— 7 ga.	3′ 6″, 7′ 0″	8 and 12
10-1½	0.34	1½″—18 ga.	3, 4, 6	8 only
	0.34	1½″—16 ga.	3, 4, 6	8 and 12
20-1½	0.68	1½″—12 ga.	3, 4, 6	8 and 12
15-¾	0.51	¾″—16 ga.	3, 4, 6	8 and 12
25-¾	0.85	¾″—12 ga.	3, 4, 6	8 and 12
20-½	0.68	½″—18 ga.	3, 4, 7	8 only
24-½	0.82	½″—16 ga.	2, 4	8 only

The first two figures in the first column give the area of steel and the last figure gives the short dimensions of mesh. Thus No. 30-3 has an area of 0.30 sq. in. per 12 in. of width and has a mesh 3 in. wide.

63e. GF Expanded Metal.—Manufactured by the General Fireproofing Co., Youngstown, Ohio. Standard sizes are given in the table on page 54.

GF Expanded Metal

Style	Approx. weight per sq. ft. in pounds	Deliveries	Standard size sheets: Lengths, Long way of diamond	Standard size sheets: Widths, Short way of diamond
3–10–176	0.60	Carried in stock in standard sheets	6′, 8′, 9′, 10′–8″	3′, 4′, 5′, 6′
3–10–265	0.90		6′, 8′, 9′, 10′–8″	4′, 5′–4″
3–10–353	1.20		6′, 8′, 9′, 10′–8″	3′, 4′, 6′
3–12–150	0.51		6′, 8′, 9′, 10′–8″	3′, 4′, 6′
1½–12–194	0.66		6′, 8′	3′, 4′, 6′
¾–12–246	0.84		6′, 8′	3′, 4′, 6′
3–10–324	1.10	Five days to two weeks dependent on size order and unfilled business on books	6′, 8′, 9′, 10′–8″	4′–4″
3–10–25	0.85		6′, 8′, 9′, 10′–8″	5′–8″
3–10–20	0.68		6′, 8′, 9′, 10′–8″	5′–6″
3–10–162	0.55		6′, 8′, 9′, 10′–8″	3′, 4′–6″
3–10–15	0.51		6′, 8′, 9′, 10′–8″	3′, 6′
3–12–125	0.425		6′, 8′, 9′, 10′–8″	4′–4″, 6′–6″
3–12–10	0.34		6′, 8′, 9′, 10′–8″	4′, 6′–4″
3–16–082	0.278		6′, 8′, 10′–8″	3′, 4′, 5′, 6′
3–16–059	0.20		6′, 8′, 10′–8″	3′, 4′, 6′
2¼–12–164	0.56		6′, 8′, 10′–6″	4′, 5′
2¼–16–118	0.40		6′, 8′, 10′–6″	3′, 4′, 6′
2¼–16–10	0.34		6′, 8′, 10′–6″	4′, 5′
2–12–161	0.547		6′, 8′	4′, 5′
2–16–103	0.351		6′, 8′	4′, 5′
1½–12–181	0.61		6′, 8′	4′–3″
1½–16–105	0.36		6′, 8′	4′, 5′
1½–18–088	0.308		6′, 8′	3′, 6′
1–12–234	0.796		6′, 8′	4′–8″
1–16–175	0.597		6′, 8′	3′, 4′, 6′
1–18–125	0.425		6′, 8′	4′–4″
¾–16–154	0.525		6′, 8′	4′–4″
¾–18–147	0.50		6′, 8′	3′–8″
½–18–220	0.75		6′, 8′	4′
3–7–609	2.00	Mill shipment only	6′, 8′, 9′, 10′–8′	5′
3–6–550	1.87		6′, 8′, 9′, 10′–8″	3′, 4′, 6′
3–6–500	1.70		6′, 8′, 9′, 10′–8″	4′–4″
3–6–450	1.53		6′, 8′, 9′, 10′–8″	4′–8″
3–7–400	1.36		6′, 8′, 9′, 10′–8″	5′

Note.—Interpret styles as follows: For example 3–10–176. 3 equals short dimension of diamond in inches; 10 equals approximate gage; 176 equals 0.176 sq. in. sectional area per foot of width.

64. Rib Metal.—Rib metal is manufactured by the Trussed Concrete Steel Co., and consists of nine longitudinal ribs rigidly connected by light cross members. It is made from a sheet of metal flat on one side and corrugated on the other. Strips of the metal adjacent to the ribs are stamped out, and the sheet is drawn out into square meshes (Fig. 29). The standard sheets are manufactured with meshes of from 2 to 8 in. and in all lengths up to 18 ft. The properties of rib metal are given in the table which follows:

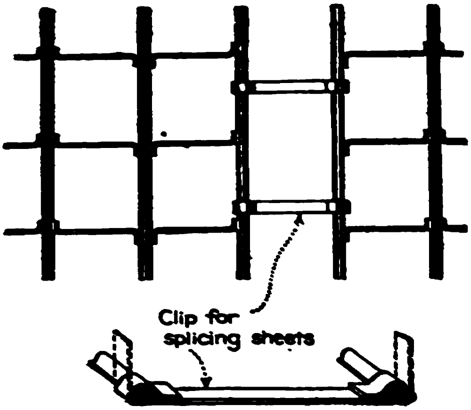


Fig. 29.—Rib metal.

Rib Metal

Size No.	Width of standard sheet, inches	Sq. ft. per linear foot of standard sheet	Area per ft. width, sq. in.	Ult. tensile strength per foot of width	Safe tensile strength per foot of width, pounds
2	16	1.33	0.54	38,880	9,720
3	24	2.00	0.36	25,920	6,480
4	32	2.67	0.27	19,440	4,860
5	40	3.33	0.216	15,552	3,888
6	48	4.00	0.18	12,960	3,240
7	56	4.67	0.154	11,088	2,772
8	64	5.33	0.135	9,720	2,430

Area of one rib = 0.09 sq. in.
Ultimate tensile strength = 6480 lb.
Safe tensile strength = 1620 lb.

65. Self-centering Fabrics.—Permanent centering fabrics (used mostly for reinforcement in concrete floor slabs resting on steel beams) are stiffened by rigid, deep ribs which do away with the use of slab forms. The mesh is made small enough to prevent ordinary concrete from passing through. The centering fabric is laid over the supports, the concrete is poured on top and the under side plastered. A simple brace along the middle of the slab span is sometimes required to give sufficient strength to the ribs until the concrete has set. The permanent centering fabrics may be obtained either in flat or segmental form.

A serious disadvantage in this type of construction is the difficulty of providing efficient fire-protection on the under side of the fabric. Bond with the concrete is also likely to be insufficient.

65a. Hy-Rib.—Hy-Rib (Fig. 30) is a steel sheathing, stiffened by deep ribs formed from a single sheet of steel. It is controlled by the Trussed Concrete Steel Co. of Youngstown, Ohio, and Detroit, Mich.

Fig. 30.—Hy-rib.

HY-RIB

Type of Hy-Rib	Formerly called	Height of ribs (inches)	Spacing of ribs (inches)	Width of sheets (inches)	Gage Nos. U. S. Standard
1½-in. Hy-Rib	Deep-Rib	1½	7	14	22, 24, 26
1 5/16-in. Hy-Rib	7-Rib	1 5/16	4	24	22, 24, 26, 28
1 3/16-in. Hy-Rib	3-Rib	1 3/16	8	16	24, 26, 28
⅜-in. Hy-Rib	6-Rib	⅜	4	20	24, 26, 28

Standard lengths, 6, 8, and 12 ft.

Other lengths are cut from standard lengths without charge except for waste.

1½-in. and 1 5/16-in. Hy-Rib are shipped in bundles of eight sheets; 1 3/16-in. and ⅜-in. Hy-Rib in bundles of sixteen sheets.

65b. Corr-Mesh.—Corr-Mesh (Fig. 31) is furnished by the Corrugated Bar Co., Buffalo, N. Y. It is a stiff-ribbed expanded metal, the ribs being spaced 3 5/32 in. c. to c. The

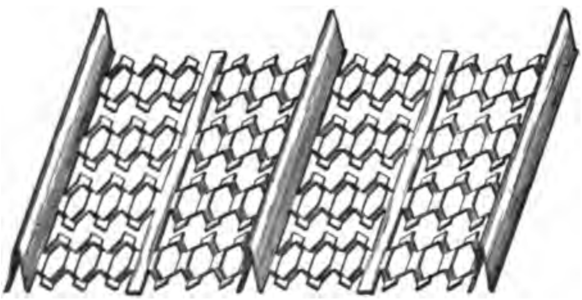


Fig. 31.—Corr-mesh.

height of the ribs is ¾ in. and the width of the sheets is 12 ⅝ in. c. to c. of outside ribs. The standard gages are No. 24, No. 26, and No. 28, although other gages can be obtained if required. The standard lengths are 6, 8, 10 and 12 ft. The sheets are furnished either flat or in various types of curves. All metal is shipped painted unless specifically ordered otherwise.

65c. Self-Sentering.—Self-Sentering (Fig. 32) is manufactured by the General Fireproofing Co., Youngstown, Ohio. It is made up of a series of heavy, cold-drawn ribs, 1 3/16 in. high, always spaced 3⅝ in. c. to c., connected by a form of expanded metal—all cut and drawn from one sheet of steel. Size of sheets—29 in. wide by lengths of 4, 5, 6, 7, 8, 9, 10, 11 and 12 ft. Longer lengths up to 14 ft. furnished on special order. Self-Sentering is made of No. 24, 26 and 28-gage metal.

65d. Chanelath.—Chanelath (Fig. 33), furnished by the North Western Ex-'nd Metal Co., Chicago, Ill., is a type of expanded metal composed of a series of heavy cold-formed steel T-ribs connected together by a mesh known as "Kno-Burn" metal lath. The T-ribs are 7/8 in. high and spaced 4 in. c. to c. The flange of the T is 1/2 in. wide. Chanelath is manufactured and carried in stock ready for immediate shipment in the following sizes of sheets: Lengths—3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 ft.; widths—4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 and 48 in.

65e. Ribplex.—Ribplex manufactured by the Berger Mfg. Co., Canton, Ohio, is an expanded metal with ribs 4.8 in. on centers and 3/4 in. high. Standard sheets are 24 in.

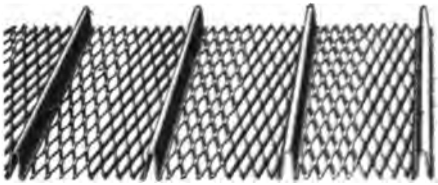


Fig. 32.—Self-sentering.

wide and are carried in stock in 4, 5, 6, 7, 8, 9, 10, 11 and 12-ft. lengths. Sheets are made in 28, 26 and 24 gages.

65f. Dovetailed Corrugated Sheets.—Sheets of thin steel corrugated so as to form dovetailed grooves are manufactured by the Brown Hoisting Machinery Co., Cleveland, Ohio, and by the Berger Mfg. Co., Canton, Ohio. The first-mentioned company manufacture a plate known as Ferroinclave and the latter company furnish two types of plates known as

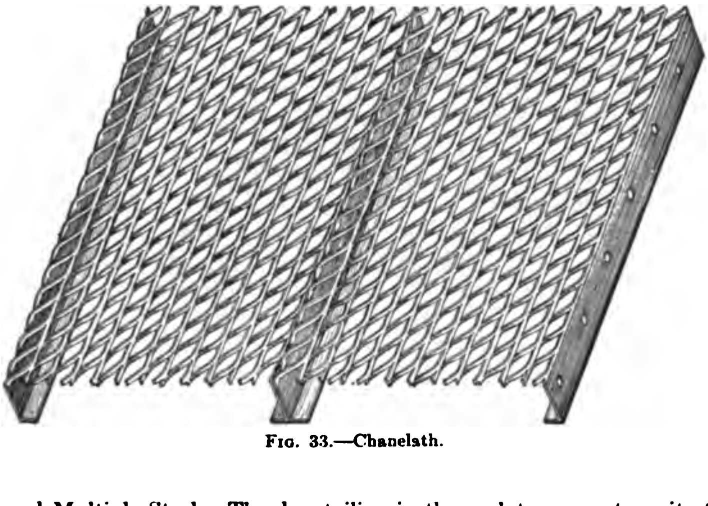


Fig. 33.—Chanelath.

Ferro-Lithic and Multiple Steel. The dovetailing in these plates serve to unite the plates to the concrete.

66. Reinforcing Systems for Beams, Girders and Columns.

66a. Kahn System.—The Kahn trussed bar (Fig. 34), named for its inventor, is rolled with flanges, which are bent up to resist the shear in the beam. For continuous beams, inverted bars are placed over the supports in the upper part of the beam, extending over the region of tension. Properties of Kahn trussed bars are shown in the following table:

KAHN TRUSSED BARS

Size in inches $a \times b$	Weight in pounds per foot	Area	Length of diagonals in inches	
			Standard	Special
Square Section Bars				
$\frac{1}{2} \times 1\frac{1}{2}$	1.4	0.41	12	6, 8, 18
$\frac{3}{4} \times 2\frac{3}{16}$	2.7	0.79	12, 24, 36	8, 18, 30
New Section Bars				
$1\frac{1}{2} \times 2\frac{1}{4}$	4.8	1.41	12, 24, 36	8, 18, 30
$1\frac{3}{4} \times 2\frac{3}{4}$	6.8	2.00	36	24, 30, 48
$2 \times 3\frac{1}{2}$	10.2	3.00	36	24, 30, 48

NOTE.—8, 12, 18, 24, 30, 36, and 48-in. diagonals are sheared alternately. Six-in. diagonals only are sheared opposite.

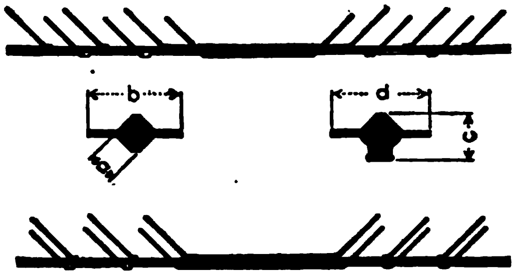


FIG. 34.—Kahn trussed bars.

What might be called the Kahn system is illustrated in Fig. 35. The collapsible column hooping is shown more in detail in Fig. 36. The hooping is shipped in the form of flat, circular coils of exact diameter and accurately spaced by means of special spacing bars. These coils spring automatically into a complete hooped column on cutting the small fastening wires. Rib bars (see Art. 61*d*) are ordinarily used as vertical reinforcement in conjunction with the hooping.

The collapsible column hooping is shipped complete with two spacing bars. Sizes of wire for hooping: $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$, and $\frac{1}{2}$-in. diameter. Diameter of coils: 9 to 30 in. Pitch: $1\frac{1}{2}$ to 12 in. Hooping, where desired, can also be obtained in bundles, coiled to the correct diameter, and with separate spacing bars, ready for assembling in the field.

66*b*. Cummings System.—The Cummings system is shown in Fig. 37. U-shaped stirrups are used on the girder frame shown. They are shipped flat with the longitudinal reinforcement, but are bent up to an inclined position on the work. The rods are held together by means of a patented chair. In the Cummings hooped column, each hoop is securely attached to the upright rods. The hoops are made of flat steel, bent to a circle, with the ends riveted or welded together in such a manner that the ends of the hoops protrude at right angles to keep them the proper distance from the mold. Reinforcement of the Cummings system is manufactured and sold by the Electric Welding Co., Pittsburg, Pa.

66*c*. Unit System.—Figs. 38 and 39 show the unit system of reinforcing con-
·· [1] by the American System of Reinforcing, Chicago, Ill. The girder frames are not stock

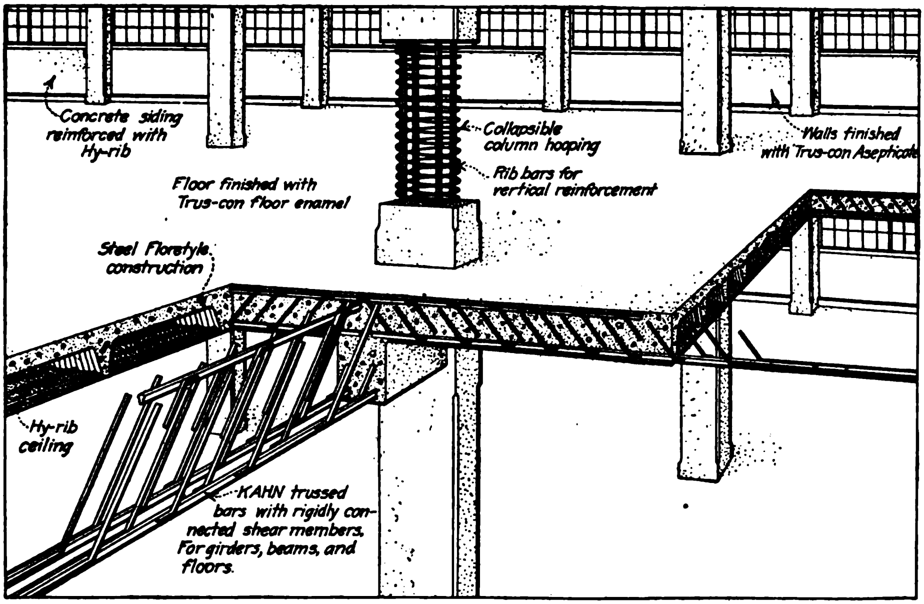


FIG. 35.—Kahn system.

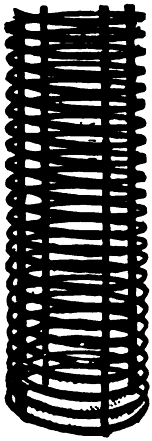


FIG. 36.—Kahn collapsible column hooping.

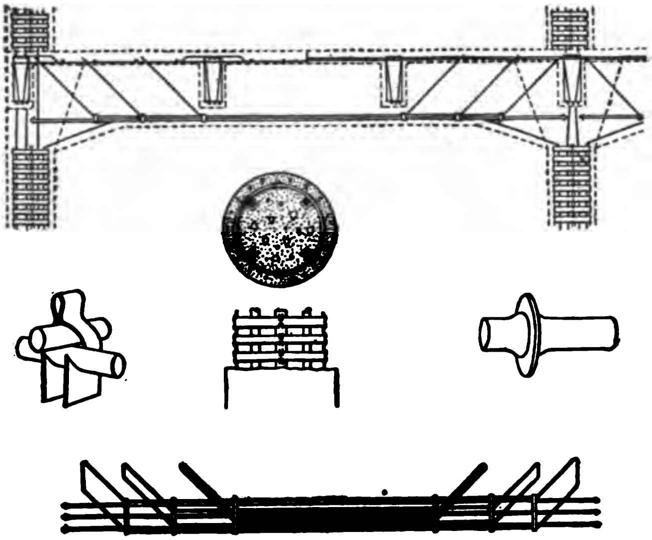


FIG. 37.—Cummings system.

frames but are built to meet the engineer's or architect's plans. Unit girder frames are provided with overlapping rods for continuous beams to reinforce against negative moment.

66d. Corr System.—Corr-bar girder frames (Fig. 40) and shop fabricated spirals (Fig. 41) are furnished by the Corrugated Bar Co., Buffalo, N. Y. As with the unit system, the girder frames are built to meet the engineer's or architect's plans. In the spiral reinforcement the spacing bars consist of two or—in large columns—four spacers made of T-section bars notched to receive the spiral. The spirals are made of cold-drawn wire and are furnished in any length, in diameters of 10 to 36 in., pitch 1 to 4 in., and of the following sizes of wire:

Gage	Dia. of wire (inch)	Wt. of wire (lb. per foot)	Practical equivalent (inch)	Gage	Dia. of wire (inch)	Wt. of wire (lb. per foot)	Practical equivalent (inch)
7/0	0.4900	0.6404	½ round	0	0.3065	0.2506	5/16 round
5/0	0.4305	0.4943	7/16 round	3	0.2437	0.0466	¼ round
3/0	0.3625	0.3505	⅜ round				

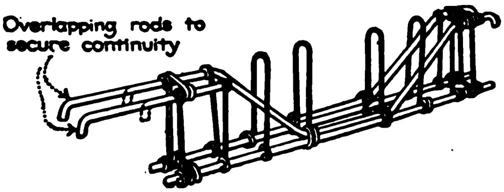


FIG. 38.—"Unit" frames.

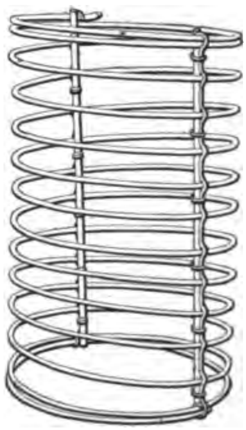


FIG. 39.—"Unit" spirals.

66e. Hennebique System.—One of the pioneers in concrete construction in Europe is Mr. Hennebique, in France, and the system which still bears his name is shown in Fig. 42.

66f. Pin-connected System.—Reinforcement in the pin-connected system consists of bars made into a truss and ready for placing in the forms (see Fig. 43).

66g. Luten Truss.—The Luten truss is shown in Fig. 44. The bars are rigidly locked together to form the truss by a clamp, with a wedge that is self-locking when driven home. The truss is especially adapted to highway culverts and bridges and is put out by the National Concrete Co.

66h. Xpantruss System.—The truss by this name is shown in Fig. 45, and is applicable chiefly to beams, girders, and heavy slabs. This system is patented by The Consolidated Expanded Metal Cos.

66i. Shop Fabricated Reinforcement System.—In this system (Fig. 46) manufactured by the Lackawanna Steel Co., Lackawanna, N. Y., the standard bar is a troughed

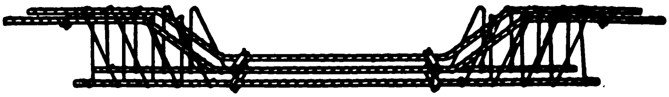


Fig. 40.—Corr-bar girder frame.

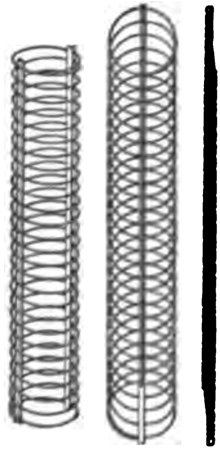


Fig. 41.—Corrugated Bar Co.'s spirals.

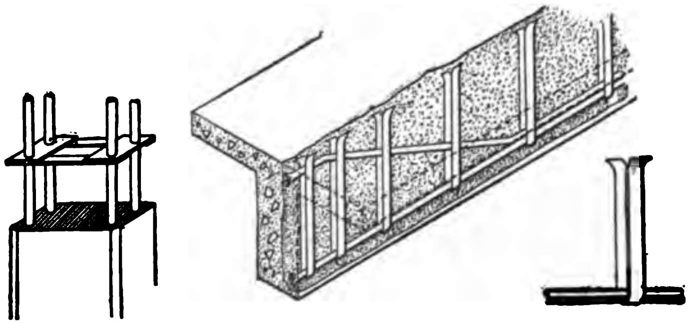


Fig. 42.—Hennebique system.

Fig. 43.—Pin-connected system.

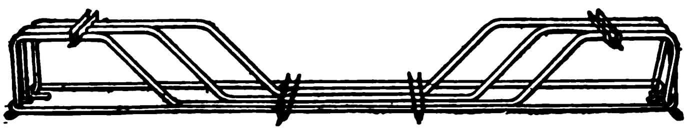


Fig. 44.—Luten truss.

section and the auxiliary reinforcing members, such as diagonal tension members, tie rods for columns, walls, etc., are flat bars (1/4 by 3/16 in.) with knobs on each edge. Fabrication is

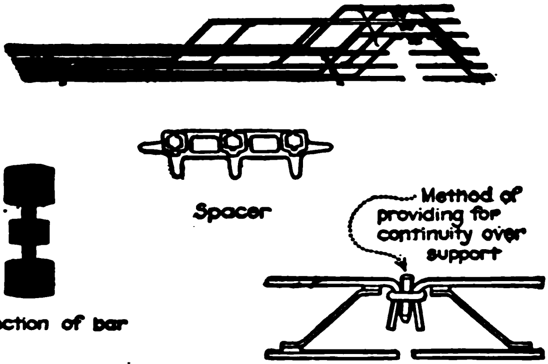


FIG. 45.—Xpantruss system.

effected by placing a portion of the auxiliary flat, properly bent, within the trough and with a bulldozer or other pressure machine squeezing the wings of the main bar and also gripping the knobs of the flat. The upper or troughed part of the main bar is a constant. Increased area is

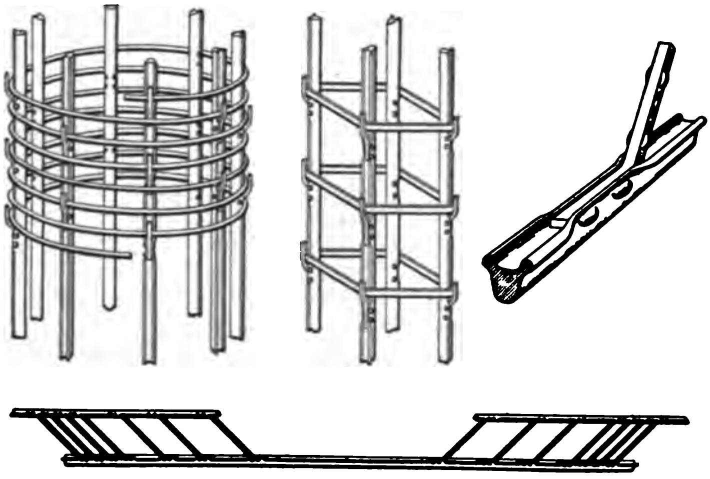


FIG. 46.—Shop fabricated reinforcement system.

developed by making the section deeper as required. Tests have shown that the rivet grip, as it is called, is greater than the strength of the auxiliary member.

SECTION 2

GENERAL METHODS OF CONSTRUCTION

PROPORTIONING CONCRETE

1. Properties of Concrete Dependent upon Properties and Relative Proportions of Constituent Materials.—Despite prevalence of careless measurement of materials and arbitrary specifying of proportions for concrete, it does not follow, as many infer, that such procedures are right, either in practice or in theory, or that they should be continued. On the contrary, such procedures are wasteful and inefficient and to meet their effects, the allowable stresses on concrete have been fixed at a standard so low that actual failure or disintegration can result only from flagrant abuses of these lax and undesirable methods. All consideration of possible perfection aside, it is known beyond question that proper proportioning of selected materials is a prerequisite to success, for concrete is wholly dependent for its properties upon the properties and proportions of its constituent materials, both severally and in combination.

2. Theory of Proportioning.—Considering the composite nature of concrete as revealed by a fractured section (Fig. 1), it will be seen at a glance that this substance is a pudding of large stone particles set in a mass, or "matrix," of other substances, which matrix is a mortar of cement and sand. If this matrix be magnified, it will be seen to be similar to the gross section, with sand grains as large particles held in a matrix of more-or-less hydrated cement. Theoretically, the large stone particles and also the sand grains should lie as closely together as possible, so that if all fragments (sand included) had come from crushing a solid cube of stone. their reassembly would approach the cube's original volume, density, and stress resistance. The aim in proportioning, therefore, is efficient recombination.

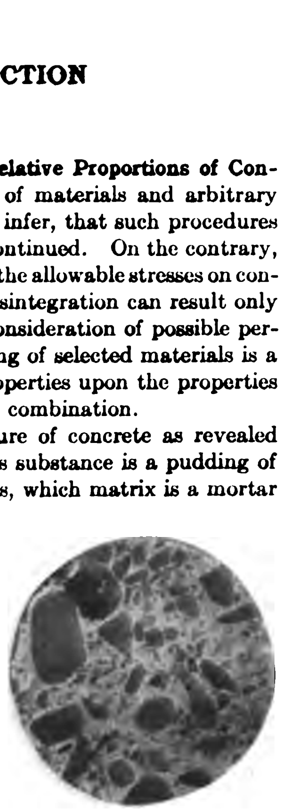


FIG. 1.—The pudding-stone structure of concrete.

It is impossible in practice to obtain the close rearrangement and interlocking of the particles that is, on all counts, desirable. There must and do remain between them, as is visually evident, inter-particle spaces or voids. To fill spaces or voids between large particles, finer materials of like nature and origin are chosen; and on theoretical grounds, it should be possible by measuring these inter-particle spaces—as, for instance, by pouring measured quantities of water into a given container filled with stone particles until the container can hold no more—to add a quantity of sand to the stone of a volume equivalent to the added volume of water, so as to render the measure almost solidly full of sand and stone particles, or rather, of stone particles since sand is itself disintegrated rock.

It is to be expected, however, as is evidenced by visual magnification, that between the sand grains must lie other and smaller inter-particle spaces in great number; and on the theoretical basis of proportioning, these should be filled by cement, so that the entire measure might be solidly full of a composite which would closely approximate natural stone in texture, density, and strength. But this ideal is not attained, partly because in this hypothesis the water necessary for reaction with cement is not allotted space, and partly because no allowance is made for the necessary surface coating of sand and stone by cement and water, with consequent dispersion of particles. The water is assumed either to lie in inter-particle spaces not filled by cement or in inter-particle spaces in the cement itself, or else to be negligible in quantity and volume.

The foregoing are the assumptions on which the void theory of proportioning is based. There is hardly one of these assumptions that does not rely on false premises, so that the whole void theory must be and is found at variance with practice, particularly when subjected to comparison with results obtained by rough field procedure. Yet the idea behind the theory is right, for it suggests by inference that density of natural stone is the ideal to be striven for and that it may be obtained: (1) by using a maximum quantity of natural stone in fragments large enough to possess unimpaired its inherent properties; and (2) by filling in the inter-fragment spaces with maximum quantities of like mineral materials. The error lies not so much in the standard thus chosen as in neglect to give proper consideration to the individual and combinative properties of the several substances included in concrete, not the least important of which is water, both as a space occupier as well as in its chemical and physical actions with cement and with inert aggregate.[1] It is always to be remembered that concrete is not concrete without water; that its proportioning is equally important with the proportioning of cement, stone, or sand; and that it affects by its quantity the proportions of the other ingredients that may be placed in any given volume.

3. The Strength Elements of Concrete.—Any concrete will have as an upper limit of stress resistance the properties of the most resistant of the materials entering into it. These materials are usually stone and sand, with some strength preference in favor of sand, as most sand particles are individual crystal units without cementing substance between them, while natural stone is an aggregation of similar particles, built up in one way or another. Since natural stone is formed under the most advantageous conditions as to arrangement of particles, pressure, etc., and since in its composite nature, it is closely analagous to concrete, it may be taken as the ultimate ideal in artificial stone made by the admixture of sand, stone, and water, with Portland cement. Furthermore, in natural stone as in concrete, the weakest element is the cementing substance which lies between the inert grains, for in each material this is alterable by various agents and is at the same time of less inherent strength than the mineral particles which it unites.

4. Proportioning for High-strength Concretes.—From the above, it follows that the greater the proportion of mineral grains in any stone, brought about through compacting and the exclusion of all but a film of cementing material (as in a very compact sandstone), the higher will be its strength. In the same way, in artificial concretes, the greater the proportion of natural stone (or of analogous mineral matter) and the less the quantity of cement, consistent with proper coating of the inert materials, the greater will be the resulting strength, for Portland cement in combination with water is the weakest element of this cement-sand-stone combination. In this connection it should be further remembered, that while ground Portland-cement clinker (commercial powdered cement) is extremely hard and of great inherent strength, the same substance in combination with water results in an entirely different product of different chemical nature and composition, having relatively low strength. It is, therefore, actually true that down to a certain limit, the less cement there is in any concrete the more enduring and, at the same time, the cheaper will be that concrete. It becomes, then, of vital importance not only to so select inert materials that they shall by their properties be able to endow the concrete with high strength, but also to so choose their proportions that they shall be a quantitative maximum in the mixture, the cement functioning in minimum quantities, as an adhesive surface coverer and as a void filler.

5. Weakness Due to Poor Proportioning.—By reason of the remarkable properties of Portland cement, carelessness in proportioning concrete has become tacitly accepted, if not permitted and sanctioned practice. Fairly good results have been obtained in spite of gross carelessness; and this has brought about a belief in the minds of perhaps a majority of constructors, that practically any materials in any proportions in combination with any cement

[1] See chapter on "Water" in Sect. 1.

will give the desired results. It is also unfortunately true, that in first results and appearance, Portland-cement concrete, even when of inferior materials and in improper proportions, appears equal to that properly made; but the rapidly increasing number of defective constructions which are being brought to light with the passage of time is bringing about an awakening in regard to the causes of such disintegrations. Such apparently easy successes have given rise to the so-called "arbitrary" proportions in widespread present use, but in probably a majority of cases, not the least important of the causes which result in ultimate failure is the use of these same arbitrary, "practical" proportions, which actually are not practical in one case out of ten, so far as results achieved in making a good product are concerned.

6. Unit of Proportioning.—In specifying concrete or mortar mixtures a unit quantity of cement is taken as a base. The cubic foot is the usual unit of quantity. On large work where overhead bins and measuring-hoppers are provided, the cubic yard may be chosen. In general the assumption is made that 1 sack of cement weighs 94 lb.; that it is ¼ bbl.; and that it occupies 1 cu. ft.[1]

7. Arbitrary Proportions.—Despite their inadvisability and incorrectness, arbitrary proportions cannot be ignored. With the cubic foot of cement as a unit, these proportions are commonly described as 1 : 2 : 4 or 1 : 3 : 6 or 1 : 4 : 8 or some other easily remembered ratio, expressed in terms of volumes of sand and stone respectively, to the unit volume of cement.

Evil as is such arbitrary choosing of proportions, when such proportions are rendered still further indefinite by inaccurate measurement of materials, the composition of the resulting concrete is indefinite and uncertain to an extent that gives a new respect for the abilities of Portland cement. It has been shown repeatedly on test, and confirmed by examination of the resulting concrete, that measurement of materials in the usual manner by wheelbarrows or shovelfuls, may bring about variations of from 100 to 200% in actual proportion of material delivered to the concrete batch. This is particularly true with regard to sand. Since sand occupies at least one-third of the volume of any concrete, and as the density and stress resistance of the mixture is so largely dependent on the quantity of sand present, the variations introduced through change of grading attendant on supplies from different localities, or through change in moisture content, will be seen to be very serious. It is all too true that in commercial work, a supposedly 1 : 2 : 4 concrete is quite as likely to be 1 : 4 : 8 or 1 : 5 : 9 or even worse, or on the other hand it may be 1 : 1 : 2, or other combinations in indefinite number, with a corresponding increase of unreliability and cement cost.

It should always be remembered that each of the inert materials of concrete has surfaces and voids peculiar to itself, and the combination of any two is peculiar to them alone. A change in either material must, therefore, result in new relations, with change in the burden imposed on the cement, which (in combination with water) must function both as an adhesive, as a surface cover, and as a void filler. Regardless of the apparent sanction generally given to the use of arbitrary proportions in making concrete and to the apparent expediting of work by slap-dash measurement, this practice should be prohibited on all work above that of the most ordinary grade.

H. C. Johnson in "What is a 1 : 2 : 4 Concrete?"[2] states that with maintenance of strict proportions with different materials, the cement demanded will range from 100 to 130 bags—a variation of 4.5% in a definite quantity of concrete. The table on page 66 summarizes the results of his experiments.

8. Proportioning by Void Determinations.—For reasons given in Art. 2 proportioning materials by void determinations is obsolescent practice, not to be more countenanced than that of arbitrary proportions, which it resembles. The determination of voids may give a rough indication as to the inter-particle spaces existing in any fine or coarse aggregate, but it does not

[1] A number of authorities have approved the adoption of 3.8 cu. ft. of cement to the barrel. This value is more nearly exact and gives 100 lb. of cement to the cubic foot or 0.95 cu. ft. per sack. One standard sack, however, may be and usually is considered as 1 cu. ft.

[2] *Concrete and Constructional Engineering* (London), Feb., 1915.

No. of mix.	Large aggregate and % voids	Small aggregate and % voids	Loose volumes: Large aggregate	Loose volumes: Small	Loose volumes: Cement	Finished volumes	Weight per cu. ft. wet, lb.	Cement in finished volume of concrete: By ratio	Cement in finished volume of concrete: By %	
1	Gravel as from river (33 %)	None	6	......	1	6.00	118	1 to 8.6	11.60	
2	Gravel, 3/4"-1/8" (38 %)	Sand from gravel, 1/8"-1/16" (40 %).	4	2	1	5.60	140	1 to 8.0	12.50	
3	Gravel, 1/2"-3/8" (42 1/2 %)	Sand from gravel, 1/8"-1/16" (40 %).	4	2	1	5.25	143	1 to 7.5	13.30	Gravel concretes (excepting Nos. 1 and 8). Average % cement = 13.15; average % voids = 38 1/2; average weight per cu. ft. = 143 1/2 lb.
7	Gravel, 3/4"-1/2" (43 %)	Sand from gravel, 1/8"-1/50" (14 %).	4	2	1	5.10	144	1 to 7.3	13.70	
8	Gravel, 3/4"-1/8" less 1/2"-3/8" material (35 1/2 %).	Sand from gravel, 1/8"-1/16" (40 %).	4	2 parts 2-1 mortar		4.96	142	1 to 9.35	10.70	
11	Washed gravel, 3/4"-1/8" (34 3/4 %)	Sand from gravel carefully washed, 1/8"-1/16" (42 %).	4	2	1	5.26	144	1 to 7.51	13.30	
12	Washed gravel, 3/4"-1/8" (34 3/4 %)	Special sand quite clean, 1/8"-1/16" (35 %).	4	2	1	5.38	146 1/2	1 to 7.7	12.96	
4	Crushed limestone size as No. 2 (45 1/2 %).	Sand as for No. 2	4	2	1	4.92	143	1 to 7.0	14.25	
5	Crushed limestone size as No. 3 (48 3/4 %).	Sand as for No. 2	4	2	1	4.90	144	1 to 7.0	14.25	
6	Crushed limestone size as No. 7 (48 %).	Sand as for No. 7	4	2	1	4.80	149	1 to 6.85	14.60	Stone concretes (excepting Nos. 9 and 10). Average % cement = 14.32; average % voids = 46 1/2; average weight per cu. ft. = 146 1/2 lb.
9	Crushed limestone size as No. 8 (45 %).	Sand as for No. 8	4	2 parts 2-1 mortar		4.40	145	1 to 8.3	12.00	
13	Washed crushed limestone size as No. 11 (45 %).	Sand as for No. 11	4	2	1	4.83	148 1/2	1 to 6.9	14.48	
14	Washed crushed limestone size as No. 11 (45 %).	Sand as for No. 12	4	2	1	5.00	149 1/2	1 to 7.14	14.00	
10	Crushed limestone, 3/4"-1/8" (46 %).	Fine sand, 1/20"-1/16" (48 %)	4	2	1	4.33	144	1 to 6.2	16.10	

afford a basis on which to proportion the materials. If a cubic foot of broken stone contains 40% of voids, the void basis of proportioning is based on the assumption that $\frac{4}{10}$ cu. ft. of sand should be added in order to fill these spaces, *i.e.*, to bring the mass up to approximate solidity; and that if this quantity of sand in turn contains 35% of voids, that $\frac{35}{100} \times \frac{4}{10}$ cu. ft. of cement should be added (plus a certain arbitrary percentage for coating sand and stone surfaces) to give to the mass actual solidity.

The inaccuracy of this method is partially due to the variation in voids in sand under different conditions, this variation being sometimes sufficient to make a difference of 30% in the amount of cement required. The following table shows variation in voids due to difference in moisture alone. The figures in this table are for sand in a loose condition and the differences would be still greater if the dry sand had been shaken and tamped. It is evident that the method of proportioning by voids is valueless unless the sand is in the same state of compactness when mixed in concrete as it is when the void test is made.

Physical Characteristics of Concrete Aggregates[1]

(For aggregates in loose condition)

	Voids (aggregate containing natural moisture)	Voids (aggregate dry, %	Weight (lb. per cu. ft., aggregate containing natural moisture)	Weight (lb. per cu. ft., aggregate dry)	% moisture in moist aggregate	Specific gravity of stone
Average of 4 good concrete sands.	53	43	82	95	5	2.65
Long Island washed gravel graded ⅜ in. to 1¾ in.	..	36	..	106	..	2.65
Commercial ¾-in. limestone.	..	44	..	97	..	2.80
Commercial 1½-in. limestone.	..	46	..	95	..	2.80
Trap rock, graded ⅜ in. to 1¾ in.	..	44	..	103	..	2.95

In practice, therefore, the proportions obtained by void determinations do not hold. As soon as water is added to sand, stone, and cement, very different physical relations are effective from those that previously existed. The particles of cement and the finer particles of sand are necessarily dispersed by the water which coats and lies between them; the larger particles of sand are dispersed by this thin mortar of cement, fine sand, and water; and the finer and larger stones in turn are dispersed in a similar manner by a like combination of the finer materials. As pointed out in Art. 2, this fact is made very evident by the examination of the fractured surface of any concrete. No matter whether the surface examined is in the gross, showing large particles of stone, or whether it is magnified to make visible the very fine particles of sand and cement, the same dispersion will be found to obtain, offering visual evidence, confirmed by test, as to the necessary inaccuracy of void determinations as a basis of proportioning.

This may be made evident by a simple illustration. Assume a vessel containing a given number of small spheres of varying sizes which may be considered as so many sand and cement grains. It is evident that if these spheres fill the measure, a certain quantity of water may be added without disturbing their inter-relations. That is to say, it is assumed that these spheres will remain in surface contact one with another even after the addition of water. If, however, the same spheres be put into a larger vessel and an additional quantity of water added, filling

[1] R. E. Goodwin in *Concrete*, Nov., 1915.

this vessel, then, if the mixture is uniform (which is the condition assumed to exist in mortar and in concrete), these spheres must be dispersed and be out of surface contact one with another. This represents in exaggerated illustration, but not in exaggerated degree the conditions as they exist in commercial concretes, dispersion therein being progressive from the finest particles to the largest, each successive grade assisting in the dispersion of the size next larger, with resultant increased demand for cement and corresponding weakening of the mass.

Rather than proportion strictly on the basis of voids, therefore, a better way is first to grade the aggregates, both coarse and fine, by sieve analyses. In this way, the voids are taken cognizance of, though in a different way. A combination of materials may then be made such as to give a mixture containing these materials in greatest quantities. The proportions of aggregates in this mixture having been determined, the amount of cement required will then depend very largely upon the strength needed or the degree of imperviousness required of the concrete. It can be approximately estimated by determining the percentage of voids in the mass, but on account of the errors introduced through the establishment of new conditions by the introduction of water, this latter assumption is not to be recommended unless checked on actual mixtures.

9. Proportioning by Mechanical Analysis.—Although the foregoing recognizes the existence of inter-particle spaces or voids, it properly should be termed "proportioning by mechanical analysis." Such proportioning is recombination after analyses are made by passing representative samples of the inert materials through successive sizes of standard-mesh screens; noting the quantity passing and the quantity retained on each screen; and plotting these as a curve, with sizes of screen openings as abscissæ and percentages of material passing as ordinates (see Art. 32, Sect. 1). By this procedure a more or less regular curve will be obtained for the sand and for the stone; and its variation from a predetermined curve, such as that of William F. Fuller,[1] or that advocated by the Bureau of Standards at Washington,[2] may be determined. The deficiencies of one material, therefore, can be balanced against the advantages of another; and by proper combination of the two, as determined from this curve, a determination may be had as to the proper proportions of the several materials. A few trials will give a very close approximation; and if the qualities of the several materials are maintained to sample throughout the work, these proportions may be safely followed. It is probable, however, that the character of the materials will change more or less throughout the job, so that it is usually necessary to continuously check the several shipments of materials as they go into the work; and if they vary seriously from the established standard, to alter the proportions of the concrete in accordance with variations noted by repetitions of the processes before noted.

10. Proportioning by Maximum Density Tests.—A direct test which reproduces actual conditions is always preferable to an indirect test based on assumptions subject to variation. The result desired in proportioning concrete is a mixture of maximum density, and the most direct means to this end is the testing of trial mixtures. These are best made with concrete, or the coarse aggregate may be omitted and the mortar alone used. The latter method, however, is not always representative, as in this case the voids in the coarse aggregate must be determined and the concrete so proportioned that the mortar will fill the voids in the gravel or the stone, with a certain arbitrary excess, thus introducing an element of error. As a factor of safety, the amount of mortar should exceed the voids in the gravel or stone about 10%. Nevertheless, this method has some justification as the percentage of voids in coarse aggregate is less variable than in sand, and also because an error in determining them has less effect on the quantity of cement used.

In proportioning by trial mixtures, definite quantities of the materials in proportions first determined by mechanical analysis are mixed with a requisite quantity of water and are put in a metal cylinder about 1 ft. long by about 1½ in. in diameter, and tamped. The volume they occupy is then determined by measuring from the top of the cylinder. When this has been

[1] See "Concrete, Plain and Reinforced," by TAYLOR and THOMPSON.

[2] See *Bull.* 58, Bureau of Standards, Washington, D. C.

determined the mixture is removed from the cylinder, the latter cleaned and a new mixture made and tried out in the same way, with a slight variation of the proportions of sand, stone and cement, but with the quantity of water constant. Very soon a mixture which will give the least volume for any given quantity of materials will be found and this mixture will give the densest, most impervious and strongest concrete with those materials.

This is known as *a mixture of maximum density*, but it is apt to be inconvenient for use in ordinary concrete work, inasmuch as it contains so large a proportion of stone that it is extremely harsh and difficult to work. To make it freer-working, more sand is generally added; and, although something of strength and density is sacrificed by so doing, the advantages of easy working and increased compactness in forms probably compensates for disadvantages arising directly from any impropriety of proportions.

Proportioning by maximum density is very readily applied in the field, all that is necessary being an iron pail and a pair of scales. Proportions can be determined for the concrete, without the use of a laboratory apparatus or any unusual equipment, by weighing out the materials, having due care that the sand is reasonably dry so that too great volumetric errors may not be introduced, and then mixing them in the pail until a mixture of maximum density is obtained. All tests are useless, however, unless the determined proportions apply to every batch mixed and placed in forms.

11. Checking Materials on the Job.[1]—When the materials used on the job are from the same sources as those tested and from which tests the proportions to be used were determined, it is a simple matter to check up their qualities. Sand and stone from the same source do not vary much in quality, except in so far as quality is influenced by size of particles. Having once established by test the suitability of sand and stone for any grade of concrete and having determined the proper proportions in which to use them to attain a certain desired result, it is only necessary thereafter to see that the size, grading, and proportions of these materials are reasonably constant to insure uniform quality of concrete. Such a check on size and grading should be had on each and every shipment of material and is easily obtained with a small set of sieves, or in the case of sand, which is by far the more important material, by means of a self-contained sand tester (see Fig. 2).

FIG. 2.—Universal Sand Tester—portable instrument for making mechanical analyses of sands.[2]

The regular and systematic testing of the size of the aggregates gives data which will permit the engineer to tell without further tests, whether the aggregates will produce a better or poorer concrete than that produced by the original or standard sample. This fact is based on the well-established principle that, other things being equal, the aggregate whose granulometric-analysis curve most nearly approaches the line of maximum density will produce the best concrete. This makes it possible to determine with reasonable certainty which of two sands of the same kind and from the same source, but differing only in fineness, will make the better concrete.

To illustrate: Concrete is to be placed in a certain locality. There are to be heavy machinery foundations and thick building foundation walls and footings below grade, with reinforced superstructure. The engineer in charge secures samples of the available concrete aggregates, both fine and coarse, and sends them to the laboratory for test. The tests show that although the best available sand has a strength in 1 : 3 mortar only 70% of that of standard Ottawa sand, yet mixed in the proportions of 1 : 1¾ : 3¾ with the cement and coarse aggregates to be used, the resulting concrete has a compressive strength of 2600 lb. per sq. in. at 28 days. Other proportions give higher and lower strengths, depending on their richness, but as the

[1] CHAPMAN and JOHNSON: *Eng. Rec.*, June 12, 19, 26, 1915.
[2] Kolesch & Co., Fulton St., New York.

design of the structure requires concrete having an ultimate strength of 2500 lb. per sq. in. the 1 : 1¾ : 3¾ proportion is used. For the foundations and footings, the designs being based on an ultimate strength of 1500 lb. per sq. in. in the concrete, the proportion of 1 : 2¼ : 4½, which gave in the test a compressive strength of 1550 lb. per sq. in., is chosen. Under the present standard method of specifying sand, this particular sand could not have been used in concrete.

Among the tests advantageously made in the field on a sand are granulometric-analysis charts made with the sand tester (Fig. 2). This sand tester has five screens having 6, 10, 20, 35, and 65 meshes per in. Each screen in succession has openings one-half the width of the openings in the preceding screen. The charts are averaged and a special guide chart is prepared for the use of the inspector on the job. In making up this guide chart a permissible variation of about 2.5% each way from the mean of the tests made on the sample, is allowed. A copy of this chart is sent to the job and a copy kept in the office files.

As the sand arrives on the job the inspector, or some one designated by the superintendent, makes tests with the sand tester and compares the resulting chart with the guide chart. If the results show greater variation than is permissible—particularly if they show the sand to be finer than shown on the guide chart—then the matter is taken up with the one who supplies the sand.

By this method the quality of the aggregates is recognized and provided for in the selection of proportions for the concrete, and enough cement is used to produce the desired quality. In this way the uncertainty which is attendant upon separately testing each of the three materials, and predicting therefrom the quality of the concrete resulting from their combination, is eliminated. The time required for testing the combination is no greater than that required for testing any one of the materials.[1]

12. Proportions and the Measurement of Materials.—Proportioning always involves measurement of materials. Even with the most exact determinations of proportions, if measurement of materials in the field is inexact and variable, concrete so made will necessarily be a substance of extremely uncertain value. Furthermore, so long as there is prevalent a tendency to use excess water, even the most exact measurement of stone, sand, and cement may be nullified. Those who seek the best results must use the utmost care not only in the initial determination of the proper proportions, but also in the measurement of the quantities of each material employed, and in checking the qualities of the materials that come on the work. There will be a proportionate improvement in the general quality of concrete as attention is more generally paid to these matters.

13. Proportioning Bank-run Gravel.—It is often questioned whether or not a natural mixture of sand and gravel as taken from the bank is suitable for concrete work. Inherently there should be no objection to this material, provided it is not contaminated by impurities, but the proportions of the several grades of sand and gravel in any bank are extremely uncertain and variable. Taking bank-run gravel and mixing it with cement in the proportions of 1 part of cement to 6 of gravel is not in any sense equivalent to 1 part of cement, 2 of sand and 4 of screened gravel, or to 1 of cement, 1½ of sand or 4½ of screened gravel, or any other equivalent summation.

Gravel of itself, if of proper quality, makes a most excellent concrete, equal to that produced by the use of crushed stone. Sand in bank-run gravel is often of excellent qualities equal in every way to that taken from large deposits of exclusively fine material. However, if bank-run gravel is to be used, the relative proportions of sand and gravel must first be determined by a series of tests on representative samples in sufficient number so that the average of the bank may be determined with fair accuracy. These materials may then be combined with cement, preferably by proportioning for maximum density. After proportions are determined in this way, it may be found possible to use a bank-run gravel as it comes. On the other hand, it may

[1] See C. M. Chapman: "Specifications for Concrete Aggregates." *Procs.* Am. Soc. Test. Mat., 1916.

be found necessary in some cases to screen out the finer materials from the coarse and recombine them in proper proportions, the defining limits between gravel, sand, and other grades of materials being as stated in Art. 16, Sect. 1.

This method of screening and recombination is always cumbersome and except on experimental or very large scales, impossible of putting into effective practice. In other cases, after existing proportions of the several grades have been determined as above, a measured quantity of sand, or of gravel, or of broken stone of a size or sizes lacking in the bank may be added to the pit-run gravel, the quantities to be added having been determined by test. In this way, the deficiencies of pit-run can be overcome by addition of other substances readily at hand, or of certain of the screened-out portions of the bank itself. Only in these ways, however, can certainty as to proportions be secured; and it must further be borne in mind that frequent tests should be made during progress of the work to insure uniformity.

Furthermore, great care should be exercised to make certain that silt in detrimental quantities, or loam, are not present in bank-run gravel. Sand pits are less likely to contain injurious quantities of silty materials than are gravel pits, by reason of the latter being the bottom of an old stream bed or a like deposit, with all materials held therein, just as they chanced to be when the waters receded. Furthermore, natural disintegration at the surface, with organic additions, affects the quality of the material. Stripping away top layers is too often omitted.

14. Proportioning Crusher-run Stone.—The statements made with respect to pit-run gravel apply in lesser degree to crusher-run stone. Different stones crush in different ways with consequent variation in the character and quantity of the fine material incident to the process. The softer stones give a larger yield of fine materials than the harder ones; and often much of the very fine material is of a character unsuited to use in concrete. This applies especially to limestone, inasmuch as limestone has a flaky fracture in its fine particles, making the particles very friable and rendering the adhesion of cement difficult, so that concrete made with such material is pervious and of low strength.

Furthermore, where there are excessive fines in crushed rock materials, some of these fines are merely impalpable dust. It is almost impossible for cement to properly coat particles of this size, as the dust particles then approach in fineness the cement particles. With this very fine material in large proportions, a considerable source of weakness is thus introduced into concrete, inasmuch as these materials cannot be covered by cement.

The use of crusher-run materials of undetermined size and grading, therefore, introduces an element of uncertainty in the making of concrete, which should not be permitted. The course to pursue is similar to that indicated for the use of bank-run gravel—*i.e.*, adequate samples of the crushed materials should be taken; the proportions and size of fine and coarse materials determined by mechanical analyses; the mixture of maximum density obtained; and the proportions noted of each of the several materials required to produce this mixture. Then, either by screening or by diluting the crusher run with screened materials or with extraneous materials, concrete of proper quality can be more nearly assured.

15. Proportioning Blast-furnace Slag and Cinders.—It is difficult to proportion for maximum density when blast-furnace slag or cinders are used as aggregate. This difficulty arises largely because of the porosity of these two materials, cinders being especially absorptive of water. A rough approximation as to proportions can be had with blast-furnace slag but with cinders it is probably better not to attempt to secure accurate proportioning, inasmuch as the use of cinder concretes is so restricted and their strength and impermeability are so low, as to render any increase obtainable by refinement of methods of secondary importance. Further reference as to the quality of these two materials will also be found in the chapter on "Aggregates" in Sect. 1.

16. Proportioning Water.—Last but not least is the question of proportioning water in concrete. This is often given so little thought as to make it considered of either minor or no importance but it can be authoritatively stated that the strength of any concrete mixture is as dependent upon the proportion of water contained as it is upon the proportions of any or all of the other materials.

Unfortunately, little is definitely known at the present time as to the proper proportions of water. It is known, however, that the quantity depends both upon the demands of the cement and also upon the character of aggregate employed, upon the surfaces to be covered, and the voids to be filled. Research has been recently directed to these lines with highly important results.

17. Success in Proportioning.—For success in proportioning, not only must the original test determinations be right, and the specifications provide proper authority for their enforcement, but these powers must be exercised and a rigid compliance compelled. Otherwise there is no use in tests, and specifications are empty words.

MIXING, TRANSPORTING, AND PLACING CONCRETE

18. Mixing Concrete.—Although with careful superintendence hand-mixing will give good results, machine-mixed concrete is usually of more uniform quality than that mixed by hand, and is less expensive—except, of course, where the quantity of concrete is so small as to prohibit the expense of purchasing or renting a mixer. The engineer should preferably reserve the right to permit hand-mixing if practically unavoidable, but this method of mixing should be resorted to only when machinery is unobtainable or where it is necessary to start work on a large job before the machinery has arrived.

Some contractors mix the materials dry until a uniform color is secured and then add the water. Others put the material and the water into the mixer at once. Either way can produce good results, except in hand-mixing, where the mixing of the cement and the sand in the dry state is the general and better practice.

The strength of concrete is very largely dependent upon the thoroughness of mixing, and much care is needed in this part of the work. No matter how suitable for the purpose the materials and proportions of the same may be, insufficient mixing will result in inferior concrete. Time of mixing is treated in Art. 23*b*, Sect. 3, and in Art. 12, Sect. 5.

The greatest care should also be exercised to make sure that the specified amounts of the materials go into each batch of concrete. For measuring concrete aggregates, it is not good practice to use the common form of contractor's wheelbarrow because the loads vary considerably with the variation in the heaping of the barrow. Special barrows constructed with sides nearly vertical can be obtained which will give the required amount when level full. The proper measuring of materials is discussed in Art. 23*e*, Sect. 3.

19. Amount of Water to be Used in Mixing Concrete.—Sufficient water should be used in mixing to obtain a concrete of sufficiently mushy consistency to be readily puddled. In reinforced work the amount of water should be such as to make the mixed concrete into a flowing paste that will flow readily around the reinforcing steel and require only light tamping or puddling to bring the mass to a homogeneous condition. A slight excess of water is preferable to not enough, but there should not be any appreciable quantity of free water present. Concrete is mixed with an excess of water if pools are immediately formed on top of the concrete when deposited in the forms. Although the quantity of water needed in different batches will vary occasionally because of the condition of the materials, the amount to use can be regulated best by measurement. A tank with a float fastened to an indicator on the outside is easily constructed in connection with a concrete mixer. The effect of consistency on strength of concrete is discussed in Art. 9, Sect. 5. For harmful effects from the use of excess water, see chapter on "Water" in Sect. 1.

The general types of mixers are described in Art. 22, Sect. 3.

20. Transporting Concrete.—The transportation of concrete is not only an engineering problem, often of first magnitude, but as a physical operation it is of prime importance in its effect on the qualities of material in the manufacture of which transportation and transportation equipment are an incident. Briefly, the transportation system must be such: (1) that the time interval elapsed between reception of concrete and its delivery to forms will not cause it to dry, or to take initial set; (2) that the system shall be tight, so that more fluid portions may not be lost in transit; (3) that the mode of transit shall not promote separation of ingredients; (4) that the delivery shall be approximately continuous, so that mixtures of varying composition may not be caused by stoppage and settling; (5) that it shall be efficient, rapid and economical. In this summary of principles, the order of importance is such as to emphasize quality of product delivered, as well as cheapness.

The varied and various appliances for the delivery of mixed concrete to forms are discussed and illustrated in Sect. 3 on "Construction Plant." To each individual need must be applied such means as careful analysis and study indicate, so correlated and systematized that the ends desired will best be served. In the proper selection of transportation plant, personal experience and judgment enter as factors of such importance that, on large operations in particular, profit or loss may depend wholly on them. In default of these, a safe rule is to study methods and equipment used on operations of like character, either by first-hand inspection or in printed reports; to supplement information so obtained by advice from those who have had direct experience; and to so adjust and modify the ways and means indicated by the foregoing as to suit them to the needs of a particular situation.

21. Depositing Concrete in Forms.—Responsibility for the character and quality of concrete does not end with its arrival at the forms. Depositing, or placement in forms is also an operation of prime importance and its conduct is governed by elementary principles which are similar to those that govern the transporting of concrete. These principles, directed toward securing quality with economy, are: (1) that the concrete shall be continuously and evenly placed in forms; (2) that it shall not be deposited continuously in one spot, with lateral flow and (in wet concretes) gravity separation of lighter, more fluid portions from those that are heavier; (3) that it shall not be deposited in forms in a manner tending to promote dissociation or segregation of the component materials;[1] (4) that so far as possible, forms shall be continuously filled without stoppage, to prevent laitance, or stoppage planes; (5) that before new concrete is deposited on concrete which has set, special precautions shall be taken to secure union between the two; (6) that concrete shall be so deposited as to minimize the entraining of air; (7) that concrete shall be joggled in the forms or that forms shall be tapped on the outside after filling, sufficiently to expel a considerable portion of entrained air; (8) that puddling and tamping shall be done sufficient to bring about close filling of forms, and close contact with reinforcement; (9) that larger aggregate shall be spaded away from forms at the concrete rises, permitting a dense mortar coat and smooth finish at the exterior surface of the casting; (10) that uncombined concrete shall not be deposited through water; (11) that concrete remixed or retempered after initial set shall not be deposited in forms; and (12) that no concrete shall be deposited in cold or very hot weather unless special and adequate precautions are taken.

22. Continuous and Even Depositing in Forms.—The temptation is very great to localize delivery of concrete at one point, with reliance on gravity or hoeing for distribution to other parts of a form. By such indulgence, one set-up of spout or barrow runways only is needed; and by adding excess water, the form gets filled with less labor than where movement of the spout, or movable or multiplication of runways are required. There are, of course, forms of such section and dimensions that localized delivery is both permissible and advisable, but where the form is long and high, localized delivery brings about a stratification or banding that not only mars the appearance of the wall, but also provides fault planes along which seepage and disintegration may proceed.

Furthermore, bearing in mind the harsh nature and heavy weight of concrete, the difficulty of manual spreading in forms of concrete dumped on one spot creates a tendency to the use of freer-flowing mixtures, and the ease with which a certain degree of flow may be brought about

[1] See N. C. Johnson: *Eng. Rec.*, Dec. 4, 11 and 18, 1915.

by the addition of water gives rise to the use of water in excess quantities in an attempt to further accelerate the placing operations. (The consequences of such additions are treated at length in chapter on "Water" in Sect. 1.) With such sloppy, or overwet concretes, but little imagination is needed to conceive what actually happens in the forms. The more fluid portions flowing off from the delivery mound carry with them much of the cement, together with the lighter portions of sand, and fill the lower unoccupied parts of the form, there to solidify

Fig. 3.—"Soup" in forms.

Fig. 4.—Loose stone section—the top of the depositing heap.

in a chalky mass of "laitance" in which is embedded much cement needed by the stripped aggregate left higher up. If materials are subsequently dumped into this "soup" before it sets, segregations result by reason of the several materials settling through this fluid in the order of their gravity.

Cause and effect are shown in Figs. 3 and 4. The cause in Fig. 3 is a knee-deep puddle of these light materials. The effect, in Fig. 4, is a reservoir wall section almost devoid of cement and sand—the top of the heap—while adjacent to it is a lower section that can be chopped like chalk.[1]

Fig. 5.—Laitance scum at top of foundation slab.

Knowing the procedures to be avoided, substitute procedures suited to individual needs may be evolved. Localized delivery brings a chain of evil consequences. Distributed delivery avoids these, at an expense only slightly greater. The gain in quality, endurance and value is worth the difference in first cost.

23. Continuous Depositing to Avoid Stoppage Planes.—Even in concretes mixed only to a plastic consistency, there is tendency for a scum of light, chalky material ("laitance") to rise. The greater the quantity of water, the thicker this deposit, which also is aggravated by silty sand or dusty stone. Such a deposit at the top of a foundation block is shown in Fig. 5. The scrolls were traced by a lath, the depth of deposit being about 3/4 in. and the thickness of block about 4 ft. Concrete was subsequently deposited directly on this layer, as it is in thousands of other instances daily, but in all of them, this will remain as a plane of weakness, ready to yield when stress of proper character is imposed. Visual evidence of such yielding is furnished by seepage of water and disintegrations starting at like planes in concretes on every hand.

[1] See N. C. Johnson: *Eng. Rec.*, Dec. 30, 1916.
D. A. Abrams: *Concrete*, April, 1917. *Proc.* Am. W. Wks. Assoc., 1916.
Carl Gaylor: *Proc.* Am. Soc. C. E., April, 1917, p. 669.

24. Bonding Set and New Concrete.—The foregoing is closely related to the problem of bonding new and old concrete or, more properly, set concrete and concrete subsequently cast upon it. Recognizing that at least on the majority of concretes, a top film or deposit of laitance exists; that this deposit is loose in texture and non-coherent; and that a portion of it is hydrolized cement, it is not to be expected that concrete subsequently placed in contact with it shall adhere. It is known and recognized that a dust film on stone or gravel will prevent adhesion of cement and it must no less be expected that a like film on solid concrete, often multiplied many times in thickness, will have like effect. Other and more complicated conditions also affect the procurement of bond, but those above given are of themselves sufficient to account for the failure of many attempts (see Art. 50, Sect. 1).

A first essential, therefore, in procuring bond is to remove this separating laitance film, whether the set concrete is hours old, or years old. It is best to remove at least ½ in. and possibly it may be necessary to remove several inches before clean, sound concrete and aggregates are exposed. This surface should then be well washed and preferably soaked with clean water, all loose material being removed. A wash of rich neat grout well scrubbed in with clean brushes will provide a good bedment; and before this has set or dried, the new concrete should be deposited, a first thin layer being tamped into place, followed by the full deposition. The foregoing gives better guaranty of success than methods usually followed, but it should be borne in mind that drying out of the fresh concrete surface, or drying or setting of the cement wash previous to applying and ramming the first layer of concrete, or failure to deposit the remainder of the concrete before this latter has taken set, will each be sufficient to cause failure to bond, as each can and will duplicate in greater or less degree the separating film which first caused trouble.

Bonding fluids and compounds are marketed under various trade names, but these cannot be successful unless conditions suitable for bond, as outlined above, are first established. Hydrochloric acid is advocated by some as a wash preparatory to bonding, but the amount that would be required if unassisted by picks or chisels in removing the usual laitance coat to a sufficient depth makes its use prohibitive, both in cost and in time and labor required. Its use, even when considerable effort is made to wash it away after use, is not to be recommended, as concrete by its porosity, is capable of absorbing harmful quantities.

25. Removal of Entrained Air.—The customary mixing and depositing processes entrain quantities of air. Even when the volumetric air content of a concrete appears low, a considerable portion of the aggregate may actually be isolated by air, with little or no attachment to cement.[1] It is doubtful if the weaknesses produced in concrete by the occlusion of air are appreciated. The evils of existing practices in this particular are to be deplored. In particular, spouting unconfined from a height, or dumping from barrows in like manner probably do maximum damage in this particular. The present type of mixers work further evil in this regard. But since many present fixed practices and equipment entail the occlusion of air, with no likelihood of an immediate change, the removal of as much as possible is logical progress.

To this end, vibrating rammers applied to the plastic concrete, or air hammers rapping the outside of forms, or even sledge or maul blows[2] have been used with good effect. In concrete-products plants, vibrated molds have been used to obtain superior density; and in road work, vibration by motor, applied to mats on the fresh-laid concrete are said to produce superior wearing qualities.[3] Certainly if the introduction of an objectionable impurity in a structural material cannot be prevented, but its removal can be later effected, it is the part of constructive engineering to overcome the undesirable effects while seeking to remove the cause.

26. Spading, Puddling and Tamping.—Forms should be closely filled, and, so far as possible, close contacting of form surfaces with smooth, plastic material should be brought about. Since large aggregates tend to bridge over, or jam, leaving unsightly surface pockets, they should,

[1] See N. C. Johnson: *Eng. Rec.*, Jan. 23, 1915.
C. B. McCullough: *Concrete*, April, 1917.

[2] See H. S. Carpenter; *Eng. Rec.*, March 31, 1917.

[3] The Vibrolithic Pavement of R. S. Stubbs Co., Austin, Tex.

as the form is filled, be spaded back from form surfaces so that a dense, smooth mortar may lie at exposed surfaces. Furthermore, since it is essential for structural strength and for preservation of steel that the embedment of reinforcement be adequate with close contacting of mortar, puddling of concrete should be progressively carried on as forms are filled.

The temptation to use excessively wet concretes to lessen labor in the two foregoing operations is prevalent. For intricate reinforcement, a free-flowing concrete must be used, but it is better to obtain the requisite flow by sufficient mixing, by the use of finer ballast and by puddling than by indulging in excess water, which so generally defeats the intent of its use.

27. Depositing Concrete Through Water.—Care should be taken in depositing concrete under water that it is not deposited through water, unless confined.

Underwater concretes are usually deposited by means of a tremie—a tube of about 1 ft. diameter at the top, slightly flaring at the bottom and at the start of a length sufficient to reach to the bottom. As deposition proceeds, the delivery end may be raised, but not out of the soft deposited concrete, else water will enter, causing washing of concrete subsequently deposited. The tremie must be kept full of concrete at all times; and deposition is assisted by moving the bottom of the pipe slowly about, permitting gradual discharge. If the charge is lost, and the tremie becomes filled with water, it is wise to add extra cement to the next charge, in order to compensate for that which will be lost through washing away. Necessarily, a tremie is heavy, so that scow, derrick or other handling arrangements must be provided. Care also must be exercised in order that waves from passing boats may not lift the tremie as well as the scow, causing loss of charge.

Underwater buckets, which are substantially boxes with bottom-dumping doors, have been used in some underwater concreting, but their use is more costly than that of tremies and possibly less satisfactory. Tilting buckets are not suited to underwater work, inasmuch as their dumping subjects the concrete to washing.

Depositing in cloth bags of greater or less size to hold together the mixed concrete in passing through the water, has been successfully accomplished.[1] Paper bags are less successful than are those of jute or burlap. The adhesion of successive bags is dependent upon transfusion between and saturation of the bags with dissolved cementitious products, but in view of the great mass in which the concrete is used in such operations, and its gravity functioning, lack of strength at joining planes is of little moment.

28. Remixed and Retempered Concrete.—It is erring on the side of safety to reject all concrete or mortar which has taken pronounced set, whether initial or final, or which requires the addition of water and reworking to have requisite plasticity. The exact actions which take place during initial set are not precisely known, but it is probable that in this process is begun an interlacing crystallization which is later augmented by other crystallizations and depositions of colloidal (amorphous or non-crystalline) material in the processes of final set and the subsequent hardening. But whatever the exact process, it is known that retempering and reworking of Portland-cement mixtures after initial set is decidedly disadvantageous at best, resulting in a loose, unresistant product of inferior strength and coherence. This practice, therefore, is to be avoided; and the operations of transporting and placing should never be of such duration as to permit initial set, even in hot weather.

29. Concreting in Hot Weather and in Cold Weather.—The basis of all concrete is the union of inert materials by substances produced through chemical reaction between Portland cement and water. Any acceleration or retardation of this chemical process affects the quantity and quality of binder resultant from this reaction; and any such alteration affects critically the quality, strength, and endurance of concrete formed by admixture of this binding product with sand and stone..

Temperature is known to control the rate of all chemical reactions. In general, heat

[1] *Proc.* Am. Soc. C. E., vol. 39, p. 126; and vol. 47, p. 101.

accelerates and cold retards chemical union.[1] Furthermore, solution of some products, such as gypsum ($CaSO_4$) contained in Portland cement, is active at relatively low temperatures and inactive at higher temperatures, while solution of other products takes place in reverse order. Relative evaporation speeds at different temperatures are also to be considered, with correlative effect on the strength of concrete produced at any given time. It is reasonable to expect, therefore, as is borne out in fact, that hot (weather) concretes are quick-setting and of early strength and that cold (weather) concretes are slow-setting and of low strength; and on forgetfulness of these obvious but inescapable facts rests responsibility for many a failure.

Particularly is this true of cold-weather concreting. At 40°F. concrete requires four times as long a period to attain a given strength as the same concrete at 50°F.; and at 40°F. about nine times as long as at 70°F. Below 40°F. the ratio still further increases. Many so-called "mysterious" failures, in which the concrete is obviously not frozen, are to be explained by delayed set and hardening, due to low temperatures alone. Below 40°F. the set is so delayed down to and including 32°F. where rupture by ice formation occurs (requiring a later extra period at elevated temperatures to induce reconsolidation in addition to that normally required for setting at the average temperature prevailing) that computation must be made for each instance to insure safety.

Using Portland cement of normal hardening rate, the following periods before removal of forms in summer weather are suggested as representative of correct practice:

For concrete in mass work	24 to 48 hr.
For concrete in thin sections	48 to 60 hr.
For concrete columns	48 to 60 hr.
For concrete in beams and girders	12 to 21 days
For concrete in long span slabs	14 to 21 days

The period required in cold weather will be more or less protracted according to the average temperature prevailing[2] both prior to and during the setting period, inasmuch as temperatures prior to mixing and placing will hold for the aggregates, even though in many cases attempts at preheating have been made.

29a. Preheating Aggregates and Water.—Preheating sand, stone, and water previous to admixture is an operation difficult adequately to perform. Each cubic yard of materials will require approximately 1000 B.t.u. per degree rise in temperature. With an indeterminate factor of heat transference, the fuel required on a day's operations may be computed or, better still, such computation may be neglected and fuel added until the temperature of the materials has been sufficiently raised. It is erring on the side of safety to have this temperature judged by an unsensitive, calloused hand, rather than by a thermometer. Water may more easily be made too hot, inducing flash set when mixed with cement.

29b. Means for Heating Aggregates.—An old smokestack section, buried in sand or stone, and fired with wood, is perhaps the best construction-job means of heating aggregates. Steam jets are the least efficient. Water may be heated by either immersed steam coils, or by steam jets, or by externally applied heat. A gasoline torch playing directly into the mixer drum is sold as a concrete heater.

29c. Enclosure and Heating of Forms.—In cold-weather building operations in particular, enclosure by canvas is desirable. Salamanders, or other heating units are kept burning within to keep the temperatures somewhat elevated. It must be borne in mind, however, that at best the temperature of the enclosure is low; and that heat transference through wooden forms to the concrete is slow. Such precautions, therefore, do not admit of dispensing with preheating of aggregates and water, or of leaving forms in place for a requisite time.

[1] The speed of chemical reactions is approximately as the sixth power of the absolute temperature

[2] See A. B. McDaniel: *Proc. Am. Con. Inst.*, 1915; also Art. 16, Sect. 5.

29d. Protection Against Frost.—The employment of manure in contact with concrete is seriously objectionable. The heat of manure is derived from the decomposition of its organic portions and in this process, compounds destructive of concrete are formed. Clean straw, clean sawdust, or canvas will assist in protection against frost, but in addition, artificial heat must be employed for temperatures below 35°F. if assurance of safety is desired.

29e. Freezing of Concrete.—If frozen before initial set, concrete will reconsolidate on later elevation of temperature with seemingly no impairment of strength. This holds particularly for sections where there is sufficient hydrostatic head to recompact the mass as the expansively disrupting ice is thawed, chemical reactions, in the interval, having been suspended. It is better, however, to prevent freezing than to take chances.

29f. Use of Anti-freezing Mixtures.—Common salt (NaCl), or calcium chloride, ($CaCl_2$), is the basis of most anti-freezing mixtures. Glycerine and alcohol also have been tried, but both tend to lower the strength and there is also question as to the propriety of using glycerine, because of possible organic decomposition and injury to the concrete. Calcium chloride or salt added to water will lower its freezing point, and in proportions of $CaCl_2$ not to exceed 2% of the weight of cement or proportions of salt from 2 to 10% of the weight of water, have been recommended and used, but the ill effects of salt so far outweighs its benefits—as for instance, by promoting corrosion of steel—that it is better omitted. No anti-freezing compound is better than salt; and none is equal to adequate heating of materials with proper maintenance of temperature during the setting period.

29g. Protection Against Heat.—Aside from slab and thin wall construction, protection of concrete against heat is rarely needed. For such purposes, protecting coverings of straw, sawdust, sand, or canvas[1] are usually sufficient. Evaporation must be guarded against, as must also working after initial set, as in floating floor or sidewalk surfaces. Hot-weather evils, however, are less troublesome than are those incident to cold-weather concreting and are provided against with corresponding ease.

FIELD TESTS OF CONCRETE

30. Object of Field Tests.—The primary object of making field tests of concrete is either to obtain information as to the strength of field concretes or assurance as to the strength and integrity of a commercial structure.

31. Limitations Inherent in Field Tests.—Necessarily, field tests of concrete must be made on specimens of such section—usually 6-in. cubes, or better, 6 by 12-in. or 8 by 16-in. cylinders—that the maximum strength to be anticipated shall not exceed the capacity of testing apparatus available. This limits the size of specimen to a considerable degree, which affects the relationship between strength of test specimen and strength of a like section in the structure according as aggregates of greater or less size are used.

Necessarily, also, the strength of test specimens has dependence upon the degree of compacting and care of molding employed in their manufacture. In actual structures, quite dissimilar internal conditions exist, with static head playing a more or less important part in consolidation, this static head varying continually in each portion of the structure. It is therefore difficult, if not impossible, to duplicate in test specimens pressure conditions obtaining in a structure.

It is assumed, furthermore, that the materials incorporated in a small test specimen are representative of, and in like quantities to, those making up concrete in the structure. It is a regretable fact that a concrete mix is rarely of uniform composition in its several parts; and that so great is this variation found to be that relatively small portions of any mix may or may not represent in their constitution and properties when hardened a fair average of the concretes in the structure.

Temperature conditions further increase discrepancies between test specimens and struc-

[1] See Sect. 4 on "Concrete Floors and Floor Surfaces, Sidewalks, and Pavements."

tural concretes. When concrete is in considerable mass, temperature rise due to chemical reactions between cement and water are largely retained,[1] and atmospheric variations exercise less effect on the proper increase of strength. In small specimens, on the contrary, moicture and temperature conditions are subject to abrupt change with consequent variation of properties in the hardened concrete.[2] The mode of applying stress is another factor tending to dissimilarity and to misleading conclusions.

32. Comparative Tests on Field-molded and Structural Specimens.—Two notable series of investigations are on record with respect to the value of field tests of concrete. Those of the Public Service Commission of New York[3] give comparative values between field-molded specimens and specimens cut from the actual structure. Those of Kansas City, Mo.,[4] were tests on field-molded specimens alone, without comparative tests on specimens cut from the structure.

33. Value of Tests on Field-molded Test Specimens.—The indications of the tests above mentioned are not favorable so far as consistency between laboratory and field is concerned, and this is to be expected, as the practice of sampling concrete from a mixer; molding such samples more or less inexpertly into small specimens; curing them under conditions dissimilar to those structurally existing; and applying a breaking stress, must, obviously, give results of doubtful value. Then again, by the time these test specimens are matured and broken, tons upon tons of concrete have been piled on or around that portion of the structure of which they might have been part, so that the removal of this concrete would be next to impossible, even though test results should be adverse and indicate a low strength. The best that might be done would be to so vary mixtures or procedures in subsequent parts of the work as to produce more nearly the values desired.

34. Transverse Tests on Beam Specimens.—A variation in form of specimen and method of testing introduced in the Welland Canal tests is of interest, though subject to all limitations above set forth. In these tests[5] the test specimen is a beam of rectangular section 4¾ by 3½ in. and 3 ft. long, tested transversely. Such tests give little if any indication as to the ability of a concrete to withstand applications of stress other than exactly similar to those applied in the test.

35. Core Drill Test Specimens from Actual Structures.—In certain instances core borings to secure test specimens have been made in completed structures. Such cores are more representative of mass conditions than other specimens. In taking borings at the Municipal Filter Plant, Cleveland, Ohio, both 6-in. and 4-in. cores were taken with a diamond drill bit, the cores being subjected to examination and tests of various kinds. Somewhat similar work but with a shot drill was done at the Ashokan Reservoir of the New York City water supply system in 1915.

Either a shot or a diamond bit may be used in core borings. The shot bit is slower, cuts reinforcing steel more readily, but gives a rough core. The diamond bit cuts rapidly, gives a smooth, even core, but the diamond loss may be a serious item of expense, particularly where tie-wires or reinforcing steel is encountered. Either bit is almost helpless where segregated pockets of loose material are encountered.

36. Suggested Methods for Making and Testing Field Specimens of Concrete.—The following methods for making and testing field specimens of concrete are taken by permission from the report of Committee C-9 of the Am. Soc. Test. Mat., June, 1917.

The following methods are presented not as final recommendations but as an outline of what in the opinion of the committee represents the best practice at the present time. The necessity for greater attention to testing concrete in construction, and for the adoption of a proper method for sampling the concrete to represent the product of the various field operations, is recognized by engineers and contractors, especially in view of the tend-

[1] Paul and Mathew: *Trans.* Am. Soc. C. E., 1915, pp. 1225–1267.
[2] A. B. McDaniel: *Bull.* 47. Univ. of Ill., 1915.
Withey: Wisc. Engr., Feb., 1915.
[3] *Eng. Rec.*, Sept. 4, 1915.
[4] *Eng. News*, Sept. 10, 1914.
[5] *Eng. Rec.*, July 24, 1915, p. 112.

ency in many quarters to use a wet, sloppy concrete which may give a final strength much lower than that upon which the design is based.

The tests are designed to provide an indication of the quality of the concrete which is placed in the structure and character of workmanship in mixing. By providing damp sand storage for the test specimens, the variable weather conditions are purposely disregarded although these sometimes greatly affect the final strength of the concrete. In comparing the results, the temperature and weather conditions must be taken into account.

Size and Shape of Specimen.—The test specimen should be of cylindrical form, with length twice the diameter. When the coarse aggregate does not exceed 1¼ in. in diameter, a 6 by 12-in. cylinder may be used, although an 8 by 16-in. cylinder gives more concordant results. For larger-size aggregate a mold whose diameter is not less than 4 times the diameter of the largest size aggregate should be used.

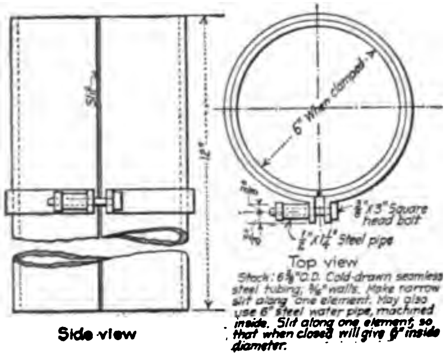


FIG. 6.

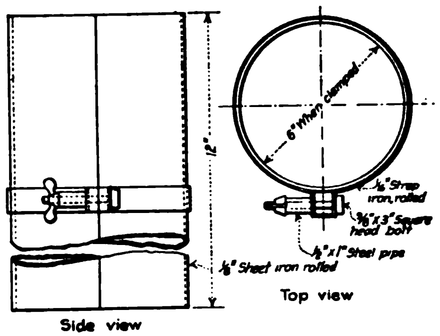


FIG. 7.

Molds and Apparatus.—Figs. 6 to 8 show types of molds which should be used in the field. Figs. 6 and 7 show types which are designed for repeated use; while the mold shown in Fig. 7 is destroyed in removing test piece, or else has to be resoldered. While this latter mold is not adapted to continuous use and therefore is more expensive in actual cost, it is quite convenient to use where but few tests are to be made and particularly advantageous where specimens are to be shipped at early stages as the specimens can be left in the molds for shipment. Various modifications of each of these forms will suggest themselves, the only requirements being that they hold their shape during the molding of the specimen and that the ends remain perpendicular to the side.

When bottomless forms are used, it will be necessary to provide a plane surface on which to mold the specimens. Individual plates of glass ¼ to ⅜ in. thick, or metal plates with plain surfaces about 2 in. larger than the diameter of the mold, may be used, placing one under each test piece. A piece of wax paper should be provided to place under each test specimen to prevent the concrete from adhering to the plate, or the plate may be oiled.

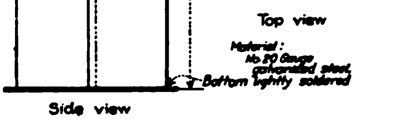


FIG. 8.

A central place should be selected for molding the specimens, and a sand pile provided so that they may be kept in damp sand as described below, to prevent undue evaporation and obtain uniformity of storage conditions.

Sampling the Concrete.—Concrete for the test specimens should be taken immediately after it has been placed in the forms. Each sample should be taken from one place. A sufficient number of samples—each large enough to make one test specimen—should be taken at different points so that the specimens made from them will give a fair average of the work. The location from which each sample is taken should be clearly noted for future reference.

In securing samples, the concrete is taken up from the mass by a shovel or similar implement and placed in a large pail or in some other receptacle for transporting to the place where the specimens are molded. Care should be taken to see that each specimen represents the total mixture of the concrete at that place.

Molding the Specimen.—The pails containing the samples of concrete should be taken to the place selected for making the test pieces as quickly as possible. To offset segregation of materials during transportation, each sample should then be dumped out of the pail into a non-absorbent water-tight receptacle, and without further mixing immediately placed in the mold. Different samples should not be mixed together, but each sample should make one specimen.

For working the concrete around the edges of the sides of the mold, a 3-in. round steel rod, 2 ft. long, should be used.

Ramming should be avoided, but care should be taken to remove air pockets. The freshly made specimen should be struck off and troweled level with the top of the form. The specimen should preferably be capped in the field while it is in the mold so as to be ready for the testing machine. After the concrete has stiffened appreciably and before the molds are removed, neat cement or a rather stiff 1 : 2 mortar may be used to fill the molds level full. A piece of plate glass or machined metal plate should then be worked around on the top of the mortar until it rests on the form. This plate should be oiled or a piece of wax paper be placed between it and the concrete. If the forms are carefully made, this will give top and bottom surfaces perpendicular to the sides of the specimens. To prevent the specimen from drying out, it should be covered or otherwise protected. If desired, the mold itself may be buried in sand while the specimen is being molded.

At the end of 48 hr. the specimens should be removed from the mold and buried in damp sand. In case the molds shown in Fig. 8 are used, specimens may be buried in damp sand without the removal of the forms, thus permitting shipment of the specimens in the molds. Test specimens made in the mold shown in Fig. 8 may be removed by opening the soldered joint with a sharp tool.

Testing.—Ten days prior to the date of test, specimens should be well packed in damp sand or wet shavings and shipped to the testing laboratory, where they should be stored either in a moist room or in damp sand until the date of the test. It is assumed that ordinarily a 28-day test will be made, although tests at 7 and 14 days will give some indications of the results to be expected at 28 days. In case 7-day tests are made, the test pieces should remain at the job as long as possible to harden, and should be shipped so as to arrive at the laboratory in time to make the test on the required date.

The foregoing recommendations of the Am. Soc. Test. Mat. are subject to revision, it being recognized that they are, by the nature and cost of equipment specified and require-

Fig. 8*A*.—Paraffined paper carton mold.

Fig. 8*B*.—Slitting the mold.

Fig. 8*C*.—The released specimen with end pads.

ments for curing conditions, more nearly allied to laboratory procedures than to testing in the field. To overcome these limitations, the author has, with uniform success, used stock cartons of paraffined paper, such as those shown in Fig. 8*A* for field molds. As will be seen, they are simply stout cartons with caps and they may be had in quantities at prices ranging from 1½ cts. for 3½ by 7-in., to 6 cts. for 6 by 12-in. sizes.

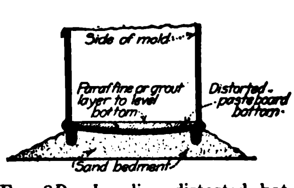


Fig. 8*D*.—Leveling distorted bottoms of waxed ends.

Molding and puddling are accomplished in the usual manner, the mold retaining its shape, and when full, capped, with identifying data written directly on the cap. When the specimen has matured, the mold is slit down the side with a sharp knife, as in Fig. 8*B*, and the shell removed. This leaves a perfect specimen, as in Fig. 8*C*, whereon, it will be noted, the top and bottom cardboards remain as cushions for the testing machine heads.

To insure even bottoms, it is well to set the empty cartons on loose sand during molding and until set. For such bottoms as are sprung, or out of true, a little melted paraffin, or of cement grout (if time to set is permitted) poured in with the mold vertical, will ensure a bedment so even as to make plaster preparation unnecessary (see Fig. 8*D*).

The advantages of this mold for field work are:

1. It is readily and cheaply procurable anywhere.

2. Temperature changes excepted, curing conditions for all specimens are always uniform and alike, without bedding in sand or immersing in water, as the waxed carton retains all water.

3. Shipment of specimens from job (where molded) to laboratory (for crushing) may be made in any manner convenient, curing proceeding uniformly throughout this period.

4. There are no molds to clean or to re-ship.

37. Pre-use Tests of Materials.—It is to be observed that recognition is accorded in the above recommendations to the questionable commercial values of field tests of concrete. The art is at present in a transition state.

Some field tests, however, are of value. One of these is as follows: It seems to hold true that the strength of concrete is directly dependent upon the size-grading of its aggregate; that this is particularly true as respects the fine aggregate; and that of a selection of sands, concrete will be strongest when made with that sand whose sum of percentage passing a given series of screens is lowest.

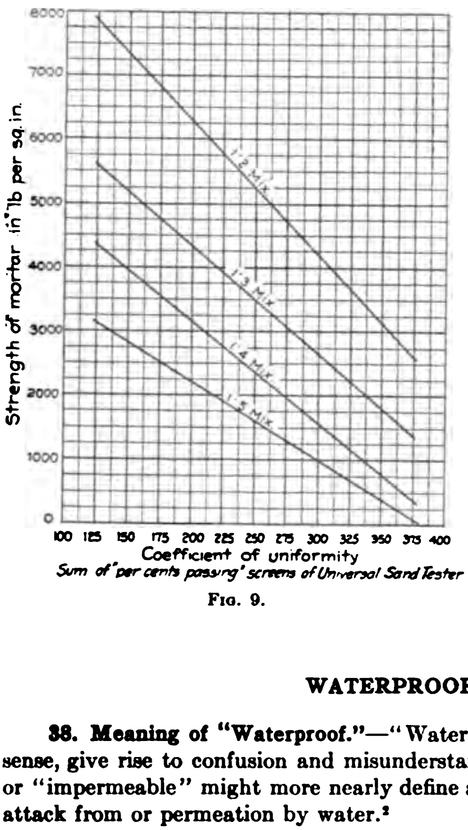


Fig. 9.

In Fig. 9 is shown a series of curves prepared by C. M. Chapman illustrative of this point. Percentages passing in these curves are taken from record cards of the Universal Sand Tester and illustrate the field practice of Westinghouse Church Kerr & Co. in the selection of sands. It is obvious from this chart, that if the percentages passing is known for any sand, the strength of a mortar made from it in given proportions at any given age (in this case, 28 days) may be read directly. Although it has been held that the strength of a mortar is not a measure of the strength of a like mortar in concrete, the relationship is not entirely misleading. On the contrary, it now seems probable that pre-use tests of materials; the establishment and maintenance of correct proportions; and refinement of processes of mixing and placing will afford the greatest developments in the concrete art.[1]

WATERPROOFING CONCRETE

38. Meaning of "Waterproof."—"Waterproof" as applied to concrete may, in its literal sense, give rise to confusion and misunderstanding. "Water-resistant" to a specified degree, or "impermeable" might more nearly define and delimit the abilities of concrete to withstand attack from or permeation by water.[2]

39. Resistance of Concretes to Water Action.—Few concretes are free from one manifestation or another of water action. Except for minor surface attack, such action follows water penetration, which latter may result from an actual hydraulic head, as in a dam, sewer, aqueduct, or reservoir; or from a negative head induced by evaporation from an exposed surface, as in a retaining wall, subaqueous tunnel, or sidewalk; or it may be caused by surface wetting and mass absorption, as in a concrete building, or in stucco. Chemical attack by sol-

[1] See R. E. Goodwin: *Concrete*, Nov., 1915.

[2] See M. O. Withey: "Permeability Test of Gravel Concrete." *Proc.* Western Soc. Engr's., 1914.

vents excepted (but inclusive of secondary frost action) the effects of penetrant water on concretes are generally alike, though differing in degree. Water penetration is directly or indirectly the cause of the majority of disintegrations in concrete and the degree to which water penetration is permitted by the texture of any concrete is a direct measure of its strength and endurance.

40. Resistance of Concretes to Water Penetration.—Penetration of water into concrete is readiest by an actual physical passageway or passageways. Obviously, a given quantity in a given time may enter by one large passageway, or by a multiplication of minute passageways. Securing resistance to penetration is, therefore, to be accomplished by reduction of such passageways to a minimum, both as to size and number, or by sealing them off. It follows, therefore, that concretes of given materials are water-tight and water-resistant, as well as strong and enduring, in proportion to their absolute densities. Conversely, concretes are weak, permeable, and of low endurance in proportion to their porosities.[1]

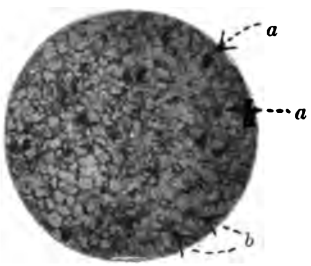


FIG. 10.—Medina sandstone showing pores which render the stone absorbent. (Magnified 20 diams.)

41. Degree of Impermeability Attainable.—Absolute freedom from water penetration is probably impossible of attainment in the commercial manufacture of concrete. Certainly, the average results of present practice warrant that belief. An improvement in present-day work is not only an imperative necessity, but, fortunately, practicable as well.

Illustrative of the difficulty of obtaining absolute impermeability in artificial concretes, the structure of sandstone (Fig. 10) is worthy of study. This has before been cited[2] as an ideal concrete in structure, in that it has in combination silica (sand) particles closely compacted, with a minimum of cementitious material between them. Yet sandstone of this grade is known to be absorptive of water and to weather (disintegrate) rapidly. Arrows (*a*) and (*b*) indicate the minute passageways *in the cementing material between the silica particles* through which water entrance is secured and at which disintegrations center. And in further likeness to artificial concretes, the more cementing material in any sandstone, the higher its porosity and the lower its strength and endurance.

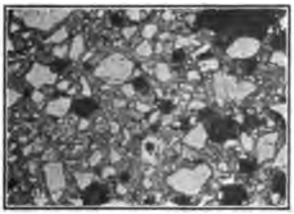


FIG. 11.—Pinholes passageways in commercial concrete. (Magnified 10 diams.)

FIG. 12.

42. Porosity of Commercial Concretes.—Necessarily, because of limitations imposed in artificial concretes by inadequate compacting and consolidating processes, a density equal even to sandstone cannot be obtained. Dispersion of aggregates in concrete, both coarse and fine, with corresponding increase of cementing material between and around them, has been before

[1] See chapter on "Proportioning Concrete," Sect. 2, and on "Properties of Plain Concrete," Sect. 5.
[2] See chapter on "Aggregates" in Sect. 1.

noted.[1] Isolation of aggregates by water[2] and occlusion of air by mixing and placing methods in current vogue[3] have been pointed out at various times. The value of proper proportioning as an aid to water-tightness has been the subject of frequent papers, discussions, and writings. Field methods, however, provocative of undesirable conditions, have remained unchanged. This argues either an apathy not creditable to the engineering profession and inimical to concrete, or else a confession of inability to remedy recognized evils. A present lack of adequate presentation of the problem of impermeable concrete may be one reason for this state of the art.

Search in commercial concretes for passageways capable of conveying water need not be protracted to meet with reward. Such passageways vary in size from "pinholes," indicated by the surface shown in Fig. 11, to those of finger size in Fig. 12. "A wall you could throw a cat through" is verbatim repetition of a field characterization which is not infrequently applicable. It is often objected that "pinhole" passageways are not continuous, but no proof of such assertion is offered; and while the converse is equally difficult of proof, the porosity of pinholed concretes under test and the presence of like pinholes throughout any and every section of such concretes gives warrant for belief that they are, by their multitude, of great importance when their combined water-conveying abilities are considered.

43. Excess Water as a Cause of Porosity.—Aside from segregated pockets of stone (which also are caused by excess water), water voids are, however, quantitatively more important than are airholes as passageways for penetrant or percolating water. Water voids are relatively massive, approaching segregations even when not so classified; and inasmuch as the water once lying in them has either flowed away, or been evaporated, continuity of passageways for subsequent water flow is strongly indicated, if not proven by the fact of this loss. An example of such water voids and flow passages in a commercial concrete, intended to be of superior grade, is shown in Fig. 13. This is typical of innumerable passageways of like character found generally in overwet concretes.

FIG. 13.—Water voids passageway in concrete. (Magnified 3 diams.)

44. Shrinkage Cracks.—Shrinkage cracks in concrete are of a general type and so universal as to be viewed by the majority either with eyes unseeing, or regarded with the contempt that comes from familiarity. Their importance both as a condition and as an indicator of internal processes is, however, of the greatest importance.

Shrinkage cracks are of a general type, irregular in line and radiating from a common center, usually a pore of greater or less size. This is to be expected, inasmuch as such a pore is at least a possible point of egress for water from the mass immediately surrounding. Further, flow is freest from such an open center, so that under evaporation or other processes it soon becomes a point of dryness; and inasmuch as it is already a point of weakness, relief planes radiate from it gradually as drying proceeds inward, until shrinkage stresses are balanced by internal resistance.

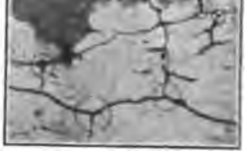


FIG. 14.

44a. Types of Shrinkage Cracks.—Perhaps the commonest type of shrinkage crack is three-branched. This is to be seen on every hand in concretes and stuccos, both in the gross (Fig. 14) and in microscopic sizes (Fig. 15). Where numerous pore centers exist, complicated systems build up by the junction of a multitude of like radiating cracks from "crazing", with oftentimes, deeper and more serious disruptions, as in Fig. 16. In both Fig. 14 and Fig. 16 the centers have been outlined in circles. Like cracks in

[1] See chapter on "Proportioning Concrete" in Sect. 2, and on "Water" in Sect. 1.
[2] N. C. JOHNSON: *Eng. Rec.*, Jan. 23, 1915; Dec. 30, 1916.
[3] N. C. JOHNSON: *Eng. Rec.*, Dec. 4, 1915.

drying earth are everywhere to be observed, the three-branched crack permitting spherical contraction and relief with minimum disturbance.

44b. Shrinkage Cracks and Porosity.—Necessarily, such shrinkage cracks in concrete are open passageways for water. This is attested on every hand by the crystalline filling of dissolved salts left behind in such shrinkage cracks. Fig. 17 is typical of such conditions, which exist where the fluid supply is, or becomes, somewhat limited, so that supersaturation and crystallization may be brought about. Moreover, as is evidenced by such crystalline fillings, these cracks once conveyed fluid through the concrete. Cracks of like formation are equally potent to convey other fluid; and, if the supply is ample, to remove soluble portions of the concrete, with mechanical dislodgment and removal of inert particles released by such solution. The original passageway is thus speedily enlarged, possibly to harmful proportions and certainly to increased water-carrying capacity. Evidence as to such quantitative removal of material is given on sheltered surfaces of concrete, such as inspection galleries

Fig. 15.—Microscopic shrinkage crack. (Magnified 150 diams.)

Fig. 16.

Fig. 17.

of concrete dams, where deposits removed from the concrete and aggregating many tons are not infrequently piled on floors and cling to walls.

44c. Prevention of Shrinkage Cracks.—Shrinkage cracks like the foregoing are difficult of prevention. They may be minimized in number and severity by: (1) use of graded materials; (2) avoiding the use of excess water; (3) adequate mixing; (4) careful placing to avoid segregation; and (5) curing (annealing) under proper conditions of moisture, so that shrinkage stresses will be developed only at a rate commensurate with the slow increase of strength in concrete. This, in connection with slow drying and hardening of colloids, largely explains the high strength of concretes cured under water, and conversely, explains the easier disintegration of concretes in which too rapid drying occurs.

But though the foregoing five principles, if made effective in practice under skilled direction, would result in better concretes as to water-tightness, with correlative strength and endurance, their field observance is so limited as to be negligible. Adequate mixing will not be had, so long as engineers countenance and tacitly, if not openly, approve inadequate mixing in the interest of quantity output. Graded sand and stone will not be used unless cheaper than other available materials, so long as the same price per yard in forms is paid for one as for the other. Excess water will be used until insistence is had for the use of lesser quantities. Careful, uniform placing needs standardization and enforcement by engineers of such standards. Moist curing, or annealing of large sections is often a physical impossibility, but often it can be done if required. The practice of curing commercial concrete is now extending; a fact which offers much encouragement. Though many problems yet remain unsolved, our present knowledge is sufficient for great improvement, once engineering sentiment for right practice is brought to the point of putting them into effect. The cause of pervious concretes lies not so much in lack of knowledge as to how to make concretes that are impervious, as in neglect to put into effect the knowledge at hand.

45. Pervious Concretes and Laitance.—Laitance—the porous, chalky material which rises during deposition to greater or less extent at the surface of concretes—is a chief foe of water-tightness. The deposit is particularly deep with excess water, or too fine or dusty aggregates, or both. Concrete subsequently placed on this laitance fails utterly to bond; and seepage readily takes place along this construction or "day's work" joint, often followed by later disintegration. Laitance an inch or more thick, scrolled with a lath, on top of a newly poured foundation block 4 ft. in depth, is shown in Fig. 5, page 74. In sections of greater height, as in reservoir walls, especially where overwet concretes are indulged, the deposit will be of greater depth, usually extending through the body of the wall to form a horizontal joint, open to percolating water.

Occasionally such laitance deposits are localized, forming pockets; and inasmuch as such pockets are formed from the finer material assumedly lying distributed in inter-particle spaces throughout the concrete mass, their isolation implies and often proves the existence of segregated and open pockets of ballast at other points. This is easily to be understood when continuous deposition in one part of overwet concretes is observed in field work with runoff of lighter materials as the mound grows.

In all concrete work subject to water action in any degree, laitance planes may be substantially avoided by filling forms without permitting set of one portion before the portion next above is deposited. Running off "soupy" portions from the top of forms while the mass is fluid is a palliative measure that may be used, or removal of laitance after setting may be attempted. No measures yet devised are wholly adequate, in that none basically remove the cause of complaint.

46. Effect of Temperature and Atmospheric Effects on Water-tightness.—Temperature effects in finished structures, and those due to atmospheric changes, may result in opening passageways capable of conveying water. The coefficient of expansion of concrete is approximately the same as that of steel (see Art. 32, Sect. 5). Adopting a linear unit, the movement at any point may be found by multiplying the distance expressed in terms of this unit of this point from a fixed point by the degrees change of temperature experienced or anticipated. In massive masonry, the interior experiences little thermal change. Actual dimensional changes in such structures are probably compensated for by internal flow.[1]

Variations in moisture content, even after prolonged set, affect the volume of concrete from 0.05 to 0.08% in the usual atmospheric range. This may be outwardly evidenced by cracks, but is more generally taken up in internal stresses in the concrete and reinforcement. Such changes are to be anticipated from our knowledge as to the colloid content of concretes and the absorptive and dessicative properties of such materials.

If consideration of anticipated temperature and moisture changes indicates that a given structure will be liable to cracking by stresses thus induced, expansion joints must be provided. Their use is preferable to their omission in most cases, but they must be most carefully formed. Copper or lead flashing, or asphalt or elaterite mastic and fabrics have been found efficacious when properly applied, but all precautions must be observed as to obtaining density in the surrounding concrete. With mastic compounds, dryness of the adjacent concrete must be secured in order to obtain proper bond, else leakage at the joint will occur.

47. Integral Waterproofing Compounds.—The foregoing paragraphs have given an insight into the causes of porous or leaking concretes. Where penetration and flow of water occur, it has been pointed out that an actual, physical passage or passages exist, that these passageways may be: (1) pinholes, or pores, resulting from occluded air; (2) water voids, or spaces left by excess water; (3) shrinkage cracks radiating from a pore, or hole of greater or less size, with joining of a myriad of like cracks into complicated systems of cracks in concretes that dry too rapidly; (4) segregation due largely to excess water, with open pockets of stone; (5) laitance, in pockets or strata, due largely to excess water; and (6) temperature or other cracks, due to atmospheric changes.

[1] F. R. McMillan: *Bull.* University of Minnesota.

It is to be expected that the customary violations of the natural laws governing concrete which bring such passageways into existence should cause wide demand for something purchasable which would afford relief from consequences. If any agent exists or is to be found, which, when added to concrete made with lack of care, is capable either of preventing or of closing the passageways through which water penetration or transmission is brought about, without detriment to the strength or other properties of the concrete, it may be ranked as a noteworthy discovery. There is some question, however, as to whether any substance, particularly in economical percentages, will accomplish this end to any save a minor degree. Consideration of the open void volumes and segregated areas in many concrete structures reflect the magnitude of the task. It is probable that proper practices and materials and they alone are adequate as well as unsurpassed in securing water-tight and enduring concretes.

47*a*. Integral Waterproofing Classification.—Integral waterproofings now on the market may be grouped under four heads:

(*a*) Special materials added to the mixing water.

(*b*) Special materials added dry to the cement at the job.

(*c*) Cement to which has been added the special materials during manufacture.

(*d*) Special materials and cement applied as a plaster, this being intended to so bond with the concrete surface as to become integral with it.

The special materials employed in the foregoing are substantially as follows:

(*a*) Various forms of metallic salts, such as chloride of lime; oil emulsions; lime soaps, suspended in water; and like compounds. The actions of oil emulsions is to form soaps in combination with the lime of cement; that of soap solutions as lubricants and formers of insoluble fillers by reaction with cement. Lime chloride has a catalytic action difficult properly to define, but tending to hasten set rather than either to lubricate, or to form pore-filling compounds.

(*b*) Dry powders of floury consistency, formed of metallic stearates, such as lime soap, often with alum and hydrated lime. Their properties are claimed to be void-filling and lubricating.

(*c*) Like substances, or glycerides of limes, mixed with cement during manufacture.

(*d*) The same as (*c*), used as a surface plaster.

47*b*. Value of Integral Waterproofing Compounds.—There is no general authoritative conclusion yet determined as to the value of integral compounds. Field testimony differs, probably according as the methods and materials of one use have, through inherent excellence or weakness, proven either adequate or inadequate to produce impervious concrete. The most extensive work that has thus far been done is published in *Tech. Paper* 3, by R. J. Wig and R. H. Bates of the U. S. Bureau of Standards. A majority of present commercial waterproofers were tested in the course of the work therein detailed, but a subsequent series requested by several manufacturers of tested compounds, with concretes to be made under commercial conditions, has not yet matured.

The conclusion of the foregoing tests is that no additive, proprietary, or open, will of itself overcome initial, serious deficiencies of material, or admit of defective practices; and no additive so far known is superior in results to an excess of cement and the use of graded sand of proper quality with a little water as circumstances permit.

47*c*. Rendering Defective Structures Impervious.—It is often necessary to render an existing structure as nearly waterproof as possible. The end to be attained is, of course, the closing of all water passageways. The proper method to use is dependent upon the size, character, and origin of the pores or passageways in the concrete. If the pores are very small, some inert filler such as clay or silt may be sufficient; or a soap and alum mixture, such as that employed in the Sylvester process, may be applied. If the pores are of slightly larger size, paraffine or a paraffine-carrying oil, or bitumen, or an asphaltic oil may be successfully used. Paraffine may be applied either hot or cold. If applied cold, it is dissolved in a volatile carrier in saturated solution. Applied to the surface of the concrete, it penetrates to a greater or less depth according to the dryness and porosity of the concrete. Within a short time the volatile carrier is evaporated, leaving the paraffine in the holes. Paraffine may also be applied in a molten condition and, to render successful its use, the concrete must first be rendered sufficiently warm by artificial heat so that the melted paraffine may be thoroughly rubbed in. Hot paraffine treatment is one of the most durable of waterproofing methods for work exposed to weather, but it requires considerable experience to secure a successful result.

Bitumens of one grade or another are applied either in solution or hot, as in the paraffine surface treatment. They also may be incorporated in paints which are applied to the surface.

Any bitumens employed must possess a high degree of elasticity and durability and must have considerable bonding ability with the concrete. To this end all concrete surfaces to which bitumens are applied should be thoroughly dried and preferably should be warm at the time of applications. Material should be well rubbed into corners and recesses; and the waterproofing film should be continuous throughout.

In applying bituminous paints and solutions it is a prerequisite to success that the coating shall be applied on that side of the concrete against which the water pressure is exerted. If this is done, the materials will be carried into the water passageways, but if this is neglected the materials will be forced out so that their application is waste.

48. Waterproofing by Cement Grouting.—Neat cement grout has often been tried as a waterproofing coating, applied either as a surface plaster or as a surface wash. It has also been used as a crack filler, but inasmuch as it is virtually impossible to make a coating or filling of this kind adhere to set concrete, its use is rarely, if ever, attended with success. In difficult situations attempts have been made to use cement grout under pressure as a waterproofer, but the instances on record where this has been successfully done do not indicate generally satisfactory results. It seems to be requisite that any waterproofing mixture shall be more or less plastic and viscous and that it shall be so applied as to deform and closely fill passageways in the concrete under pressure of the water.

49. Membranous Waterproofings.—Membranous waterproofing is an elastic, continuous sheet or membrane completely covering or surrounding a structure to be waterproofed (see Fig.

FIG. 18.

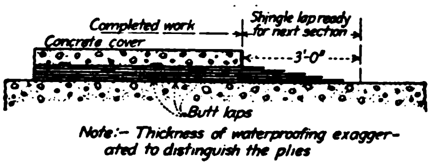


FIG. 19.

18). This membrane is laid in several overlapping layers (Fig. 19), impregnated and fastened down with some bituminous compound. The membranous system of waterproofing is adapted principally to the waterproofing of structures in course of erection, such as subways, tunnels, building foundations, retaining walls, arches, reservoirs, etc.

The bituminous materials employed as sealing compounds in the membrane method of waterproofing are: (*a*) coal-tar pitch applied hot; (*b*) asphalts applied hot; (*c*) asphalt mastic applied hot; (*d*) especially prepared asphaltic compounds sold under various trade names.

The membranes to be used with the above sealing compounds are: (*a*) tarred felt; (*b*) asphalted felt; (*c*) burlap; (*d*) burlap saturated with asphalt or tar; (*e*) combinations of canvas and felt, or canvas and burlap, or felt and burlap.

49a. Application of Membranous Waterproofing.—Success of membranous waterproofing depends largely upon the care with which the materials are applied. It is necessary first to prepare the concrete surface. It must neither be too rough, nor too wet, nor covered with dirt or foreign substance; and it must not possess a glaze due to richness of cement surface. It is, therefore, necessary: (1) that all dirt and foreign matter shall be removed before waterproofing is applied; (2) that when it is applied the concrete shall be rendered dry, either by drainage and evaporation or by the application of artificial heat; (3) that the concrete shall be thoroughly set (as is indicated in the requirement for dryness); (4) that any glazed surfaces shall be picked or rubbed down in order that the materials may adhere; (5) that form ties or other projections that might puncture the waterproofing shall be removed; and (6) that any metal surfaces encountered shall be dry, clean, and free from rust or dirt.

49b. Continuity of Membrane.—Lack of continuity may be fatal to the success of any waterproofing membrane. The waterproofing sheet must, therefore, be applied continuously over the whole surface to be treated, footings and foundations included. All joints in the membrane must be broken at least 4 in. on cross joints and 12 in. on longitudinal; and at least 12 in. of lap must be left at corners to form good junctions with adjoining sections. Where it is necessary to stop work, a lap of at least 12 in. shall be provided for joining on new work. Each layer of bituminous or other material must completely cover the surface on which it is spread, without cracks or blow-holes; and the fabric must be rolled out smoothly and pressed over the cementing material so as to insure its sticking thoroughly and evenly over the entire surface.

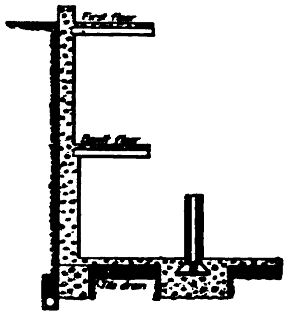


Fig. 20.

49c. Protection of Waterproofing.—After the waterproofing has been put in place it must be properly protected from injury. Such injury may occur when backfilling with earth; when depositing concrete against the waterproofing (see Fig. 20); when laying brick or rubble, or from careless piling of materials on the completed waterproofing work. Injury from workmen's shoes is not infrequent. It should be remembered that a completed membranous waterproofing is usually soft and liable to injury and the chances of so doing should be reduced to a minimum. A single point of entry for water, particularly if inaccessible when the work is completed, may render ineffective all precautions against leakage.

The following table gives the numbers of ply of waterproofing required with various heads of water:

Number of Ply of Waterproofing Required for Varying Heads of Water[1]

Head of water	Material			
	Coal tar and felt	Commercial asphalt and felt	Special felts and compounds	Asphalt mastic, thickness in inches
0	2	2	1	¼
1	3	3	2	⅝
2	4	4	3	⅝
6	5	5	4	⅝
8	6	6	5	¾
10	7	7	6	¾
15	8	8	7	¾
20	9	9	8	¾

[1] From "Modern Method of Waterproofing," M. H. Lewis.

50. Rules for Making Concrete Impervious.—(*a*) To make concrete that shall be impervious, the rules basically governing the making of dense concrete apply with special force. These are:

1. *Use proper materials, i.e.*, clean and preferably graded sand; hard, durable stone; cement that conforms to standard; and clean water.

2. *Use proper proportions of proper materials, i.e.*, avoid arbitrary proportions; use careful measurement for each batch; test each shipment of sand for uniformity of grading; if variation is found, properly compensate by variation of proportions.

3. *Properly and adequately mix the materials, i.e.*, not only stir together the several ingredients, but prolong the operation sufficiently to secure the needful consistency and distribution, particularly of the cement.

4. *Use a minimum of water* that will permit adequate filling of forms and contact with reinforcement.

5. *Place carefully in forms* to avoid segregation or unequal distribution.

6. *Expel as much as possible of occluded air* by puddling, or vibrating, or jarring of forms as filling proceeds.

7. *Fill forms continuously to top, preferably overflowing,* to avoid stoppage, or laitance planes.

8. *Properly protect concrete against rapid evaporation and against unusual heat or cold during the setting, hardening, and curing periods* to avoid shrinkage, frost, or other crackings and disruptions.

9. *Remove visible segregations as soon as discovered,* replacing with good concrete, well rammed into place. Do not rely on surface plastering of defects. Such attempts are unworkman-like and ineffective.

10. *Construct expansion or contraction joints with extreme care.* Do not rely for water-tightness on any supposed bond between abutting concrete sections.

(*b*) *Integral waterproofings* cannot be relied upon to avert the consequences of improper manufacture.

(*c*) *Membranous waterproofings* are of service in closing water passageways in existing defective structures when, and only when, carefully applied.

FINISHING CONCRETE SURFACES

51. Character of Surface Finish Desired.—The character of finish desirable to produce on concrete surfaces is determined by the ends to be served. The majority of requirements for special finishes are architectural, varying according to the character of the structure and with the location of the surface.

52. Removing Form Marks.—For all concrete work exposed to view, forms should be exceedingly well constructed, producing plane surfaces and straight, sharp lines and true angles in the finished concrete. In work of this character, extra care is usually taken to obtain even-textured, dense surfaces by using a mixture of proper consistency and by careful spading and puddling. Such surfaces necessarily reproduce all defects of the mold, so that after-treatment is necessary to remove the form marks, as well as to relieve the "dead" color due to excess of cement at the surface, with oftentimes efflorescence, or other whitish deposits, indirectly occasioned thereby.

FIG. 21.—Rotary concrete surfacer being used. Note contrast between finished and unfinished surface.

It is elementary optics that blemishes are least visible on a non-uniform light-diffracting "matt" or stippled surface. Form marks, therefore, are concealed by producing such a surface. This may be done by tooling, by rubbing, by brushing, or by sand-blasting, and such treatments have a further advantage of modifying the dead color above referred to by exposing a multitude of sand grains to light, so that by reflection from their facets, the gray-green of cement is relieved and brightened.

52a. Tooling.—Tooling concrete surfaces is more or less costly, depending upon the length of time the concrete has set and upon its hardness. Bush-hammering, crandalling, or axing may be done by hand, or a pneumatic or electric tool may be employed at considerable

advantage. One type of surfacing tool permitting of tooling or of grinding and the method of its use is shown in Fig. 21.[1] Its capacity is rated at 60 to 70 sq. ft. of surface per hour on concrete from 1 to 21 days old. Fig. 22 shows a hand bush-hammered surface of colored aggregates, and Fig. 23 a picked surface.

Only small-sized aggregate should be used in facing material to be tooled, as it is hard to dress and to obtain uniform results on surfaces where large angular stones are encountered. The concrete should be thoroughly hardened before work is commenced, especially if sharp clean surfaces are desired. The concrete should preferably be about 2 months old, although if it is allowed to stand too long the labor involved will be unnecessarily great.

A variety of surface effects may be obtained by tooling, as the effect produced in any given case depends upon the kind of tool used. Some variation in the appearance of the finished surface may also be obtained by the manner in which the tool is handled. By striking a perpendicular blow no lines or marks are left in the surface, whereas with a glancing blow, tooth marks are left which can be made parallel to each other or at various angles. Tooling cannot ordinarily

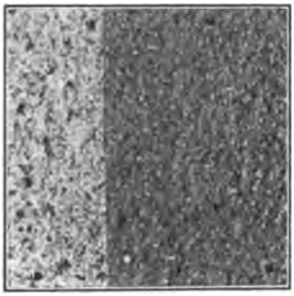


Fig. 22.

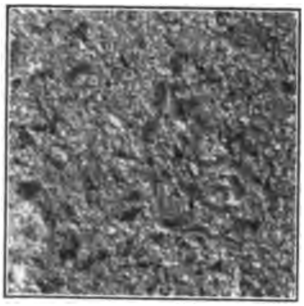


Fig. 23.

Fig. 22.—Surface finish obtained by hand bush-hammering. The contrast of shades is produced by using different colored aggregates. The concrete in the dark portion was made with red sandstone, while the light portion was made with trap rock.
Fig. 23.—Picked surface.

be performed satisfactorily on gravel concrete, as the pebbles will be dislodged before being chipped.

52b. Rubbing.—If a rubbed surface finish is desired, the coarse aggregate should be well spaded back from the face of the work and the forms should be removed before the concrete has set hard, preferably in a day or two after the concrete is poured. It is necessary with green concrete to use care in removing forms to avoid spalling, as it is very difficult, if not impossible, to adequately repair such spalls by patching with mortar. It is necessary, also, to remove form wires or other projections before rubbing and to point with mortar any pockets or open places in the surface. The process of rubbing consists in grinding down the surface of the concrete sufficiently to remove all impressions of the timber or other irregularities, using a brick of carborundum, emery, concrete, or soft natural stone. In connection with the rubbing (which is accomplished with a circular motion), a thin grout composed of cement and sand should be applied to the surface, well rubbed in, and the work afterward washed down with clean water. The grout is used simply to fill surface imperfections and care must be taken not to allow it to remain as a film on the surface.

This method of treatment produces a comparatively smooth surface of uniform color much superior to that obtained by the all too prevalent method of painting with a grout, which almost invariably crazes, cracks, and peels off. Rubbing is a very acceptable, cheap way of finishing concrete surfaces.

[1] The Berg Electric Rotary Surfacer, Elevator Supplies Co., N. Y.

4. *Use a minimum of water* that will permit adequate filling of forms and contact with reinforcement.

5. *Place carefully in forms* to avoid segregation or unequal distribution.

6. *Expel as much as possible of occluded air* by puddling, or vibrating, or jarring of forms as filling proceeds.

7. *Fill forms continuously to top, preferably overflowing,* to avoid stoppage, or laitance planes.

8. *Properly protect concrete against rapid evaporation and against unusual heat or cold during the setting, hardening, and curing periods* to avoid shrinkage, frost, or other crackings and disruptions.

9. *Remove visible segregations as soon as discovered,* replacing with good concrete, well rammed into place. Do not rely on surface plastering of defects. Such attempts are unworkman-like and ineffective.

10. *Construct expansion or contraction joints with extreme care.* Do not rely for water-tightness on any supposed bond between abutting concrete sections.

(*b*) *Integral waterproofings* cannot be relied upon to avert the consequences of improper manufacture.

(*c*) *Membranous waterproofings* are of service in closing water passageways in existing defective structures when, and only when, carefully applied.

FINISHING CONCRETE SURFACES

51. Character of Surface Finish Desired.—The character of finish desirable to produce on concrete surfaces is determined by the ends to be served. The majority of requirements for special finishes are architectural, varying according to the character of the structure and with the location of the surface.

52. Removing Form Marks.—For all concrete work exposed to view, forms should be exceedingly well constructed, producing plane surfaces and straight, sharp lines and true angles in the finished concrete. In work of this character, extra care is usually taken to obtain even-textured, dense surfaces by using a mixture of proper consistency and by careful spading and puddling. Such surfaces necessarily reproduce all defects of the mold, so that after-treatment is necessary to remove the form marks, as well as to relieve the "dead" color due to excess of cement at the surface, with oftentimes efflorescence, or other whitish deposits, indirectly occasioned thereby.

FIG. 21.—Rotary concrete surfacer being used. Note contrast between finished and unfinished surface.

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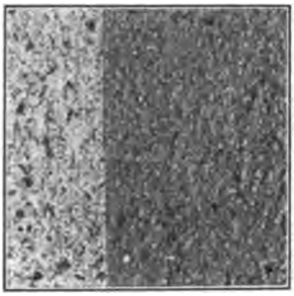
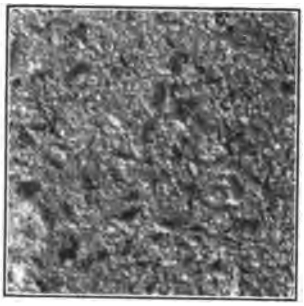


Fig. 22.



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[1] The Berg Electric Rotary Surfacer, Elevator Supplies Co., N. Y.

joints, also, should be made tight enough to prevent any material leakage of the liquid mass, as such leakage will mar the appearance of the finished work.

Forms should have sufficient strength to properly support the loads which they are called upon to carry. Horizontal members, such as floor sheathing and supporting joists, should be able to support the weight of the concrete and the construction load. Vertical members, such as wall sheathing and supporting studs, should be designed to resist the hydrostatic pressure of wet concrete which is about 145 lb. per sq. ft. for each vertical foot of height.

60. Economical Considerations.—The cost of forms constitutes a large item of expense in the building of reinforced-concrete structures and this cost varies to a considerable extent with the kind of form construction adopted. Formwork, of course, should in every case leave the finished concrete true to line and surface, but even with this accomplished a great deal can be done in so designing forms and in so planning the detailed methods of their construction that erection and removal will be greatly facilitated without undue waste of lumber. As a general rule, the most important consideration is that of ease in form removal as great economy may be effected by using form units over and over again with a minimum of repairs.

Simplicity and symmetry in formwork should always be given consideration. In buildings, uniform story heights should be selected whenever possible in order to prevent continual remaking of column forms, and frequent changes in column sizes should be avoided (at least in the case of light-floor construction), not only on account of the column forms themselves, but on account of the beam and girder forms or slab forms which frame into them. Also, where it is feasible, beam sizes should be so chosen that local standard widths of lumber may be employed without splitting; and, at the same time, the sizes and spacing of the beams should be made so uniform that the contractor may use his forms repeatedly, thus greatly reducing the expense for lumber and eliminating the cost of making new forms. In many cases it will be found that a slight excess of concrete will save many times its cost in carpenter work and lumber.

61. Lumber for Forms.—The kind of lumber to use for forms depends upon the character of the work and the available supply in the local yards. Spruce seems to be the best all-round material. It can readily be obtained in almost any locality and is undoubtedly an excellent lumber to use for joists, studs, and posts. For sheathing, however, white pine is better than spruce by reason of its smoothness and its resistance to warping, but this wood is generally too expensive (except for cornice and ornamental work), and spruce makes a good substitute. If white pine is to be used for sheathing, the fact should not be overlooked that this kind of lumber has little durability on account of its extreme softness and would not give good results if the forms were to be used many times.

Aside from spruce or white pine, Norway pine and southern pine are generally the most available and give satisfaction. Hemlock is not usually desirable, especially for that part of form work which comes into contact with the concrete, but it is sometimes used for ledgers, studs, and posts. This wood is too coarse-grained to be suitable for sheathing and is liable to curl when exposed to the weather or to wet concrete.

It is safe to say that lumber which is only partially seasoned should be the kind employed in form construction. Kiln-dried has a tendency to swell when soaked by the concrete, and this swelling causes bulging and distortion of the forms. Green lumber, on the other hand, dries out and shrinks if allowed to stand too long before the concrete is placed; fortunately, though, this tendency of the green lumber to check and warp may be prevented to some extent by keeping the boards thoroughly saturated with water. When using natural, well-seasoned lumber care should be taken not to drive the work up too close, since forms should always be left in a position to experience some slight swelling without any undesirable results.

Sheathing lumber should be dressed at least on one side and both edges, even for non-exposed surfaces, as the removal and cleaning of the forms are greatly facilitated thereby. In face work, where a smooth and true surface is quite important, the lumber employed should be dressed on all four sides. Lumber which is dressed in this manner is easy to work up and place, and this fact alone usually more than offsets the cost of dressing.

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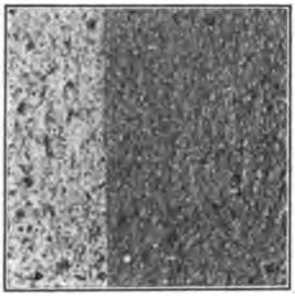
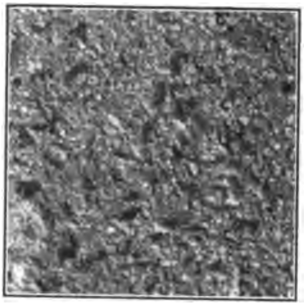


Fig. 22.



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using them. The following rules are recommended by the Illinois Department of Factory Investigation:[1]

TIME REQUIRED BEFORE REMOVING FORMS

	Above 60°F.	50° to 60°F.	40° to 50°F.	Less than 40°F.
Columns.............	Within 3 days	5 days	Not less than 10 days	Not until tests have been made indicating that the concrete is set.
Side forms for girders and beams.	Within 4 days	6 days	Not less than 10 days	
Bottom forms of slabs (6 ft. or less span).	Within 4 days[2]	8 days	Not less than 14 days	
Bottom forms of beams and girders (less than 14-ft. span).	Within 14 days[2]	18 days	Not less than 14 days	

Form work in buildings should be so designed that the column forms may be removed without in any way disturbing the supports of the beams and girders. This practice bares the concrete of the column to the hardening action of the air and permits a defect to be detected and remedied before any load is brought to bear upon the column. The beam and girder sides are next taken down, but this is not done until the slabs are strong enough to stand up without support. Sometimes, however, as a matter of safety, the slab supports (namely, the sheathing and joists) are replaced with temporary uprights bearing against a plank on the underside of the slab. The beam and girder bottoms and the posts supporting them are left in place longer than any of the other formwork, and should not be taken down until there is absolutely no doubt as to the strength of the members supported. Walls are usually built separately from the other parts of the building, and a wall form may be removed whenever the concrete in the wall is hard enough to bear its own weight.

63. Number of Sets of Forms in Building Work.—The number of sets of forms which are necessary in building work depends upon the weather, the variation in the shape of the members from floor to floor, and the floor area. Under reasonably good summer conditions 1½ sets of forms (forms for 1½ stories) will serve the purpose and allow the work to progress at the usual rate of a story in a week or 10 days. If the floor framing, however, is particularly complicated, one set of forms may be the more economical if the speed of construction is not the leading consideration. In such a case, of course, extra lumber must be used for beam and girder bottoms and for supports which must be left in place. Where the floor area is large and the construction is fairly uniform throughout, even less than one set of forms with additional beam bottoms and posts may be sufficient.

64. Examples of Form Design.

64a. Column Forms.—Fig. 25 shows a typical form for a rectangular column. Two sides of the column are held together, as shown, by bolts, and the two opposite sides by hardwood wedges between the bolt and the form. The sheathing or lagging boards run the entire length of the column and are made up into panel units by means of the cleats, which serve also as clamps. A somewhat similar column form is shown in Fig. 38, page 102.

A common type of rectangular column form where wooden wedges are used to do all the clamping is illustrated in Fig. 26. The tightening is made possible by the use of stop blocks on each clamping piece. When a reduction is made in the size of column, these blocks must either be ripped off or additional blocks nailed on. The boards comprising each side of the column form are usually battened or cleated together so as to form a panel unit. This practice allows the clamps to be put on separately and thus permits stop blocks on the clamps to be easily

[1] See also rules given in "Concrete, Plain and Reinforced," by TAYLOR and THOMPSON (1916 edition).
[2] Add 1 day for each additional foot of span.

changed when the column form is remade. It is a common practice to make up the sides of column forms with narrow strips of sheathing in order to facilitate the reduction in size of columns from floor to floor. A method of reduction sometimes employed by the Aberthaw Construction

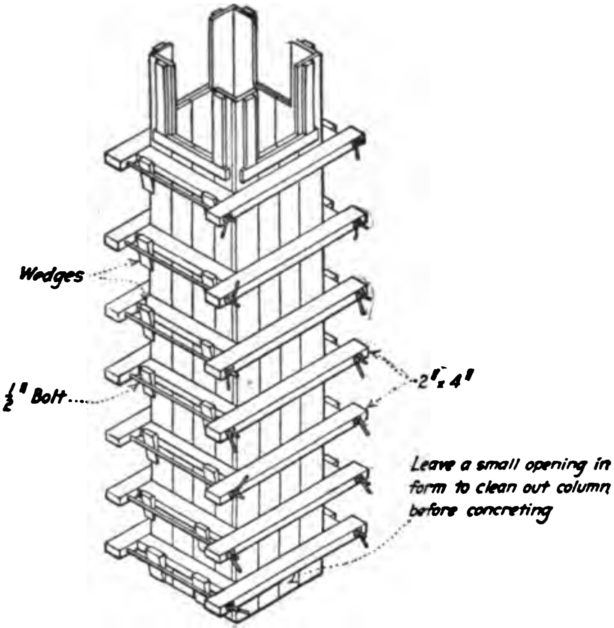


FIG. 25.

Co. is shown in Fig. 27. The column form represented here is also illustrated in Plate III, page 114. The use of narrow sheathing strips applies especially to interior column forms since exterior or wall columns are usually of the same width from basement to roof and change but little in thickness.

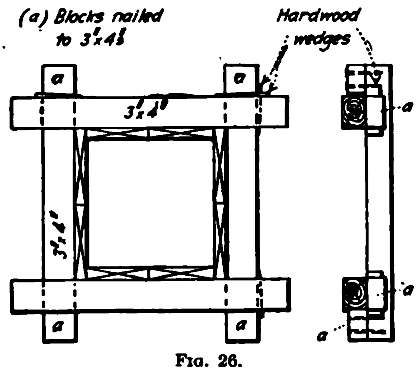


FIG. 26.

Fig. 28 and Plate II, page 113, show a method of bracing column forms by means of 4 by 4-in. "whalers" or posts. The use of Henderson clips should be noted. Sometimes the bolting is done directly to the whalers.

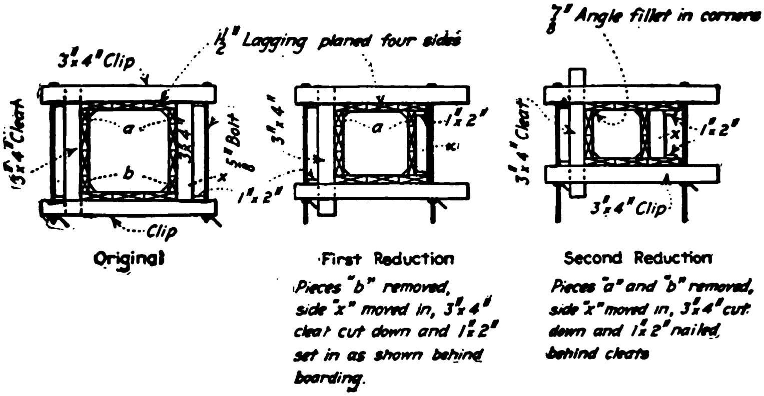


Fig. 27.

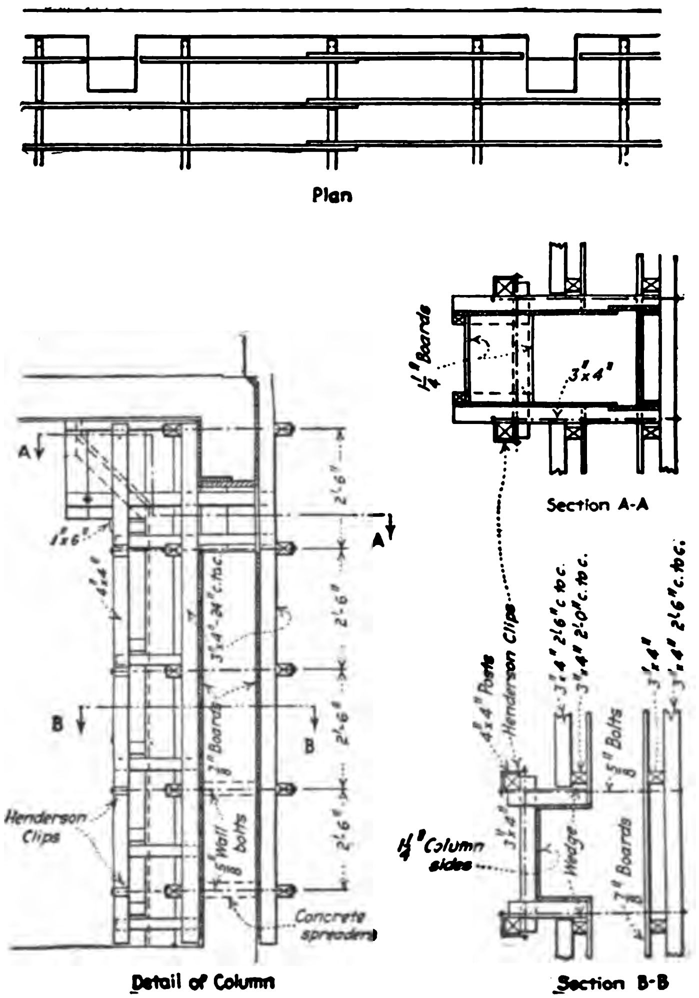


Fig. 28.—Wall column forms employed in Morrill building, Portland, Me.

Other types of rectangular column forms in common use are shown in Figs. 29, 30, and 31. The cleats which form part of the clamps are nailed to the sides when making. The clamp shown in Fig. 31 is patented and known as the "New England Column Clamp," controlled by the N. E. Column Clamp Co., Boston, Mass. It consists of angle irons punched at frequent intervals with bolts for holding the form together wherever needed. Angles and bolts may be bought in the open market. The New England Column Clamp Co. sells only the rights to use.

Fig. 32 illustrates still another type of rectangular column form. The use of horizontal sheathing should be noted. The sheathing is nailed to the vertical pieces before erection.

A patented steel clamp, known as the "Gemco Column Clamp" and manufactured by the Gemco Mfg. Co., Milwaukee, Wis., is shown in Fig. 33. These clamps are described by the manufacturers as follows:

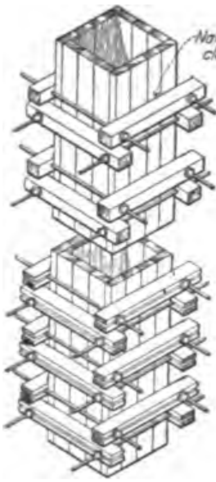


Fig. 29.

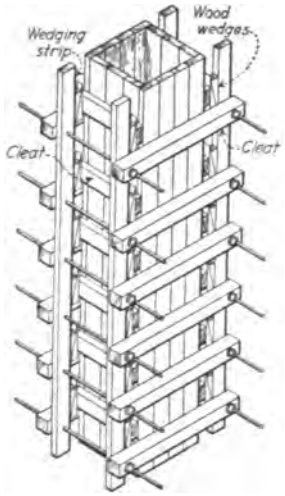


Fig. 30.

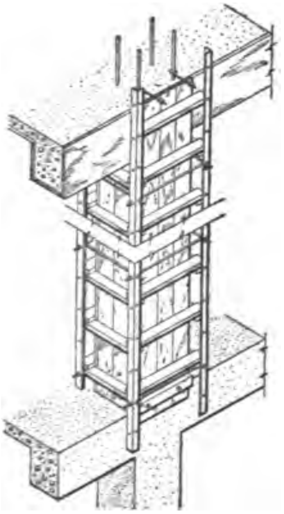


Fig. 31.—New England column clamp.

A Gemco Square-column Clamp is composed of four, straight, interchangeable, steel bars, each 2 in. wide by $\frac{5}{16}$ in. thick. One end of each bar is provided with a hardened toothed locking dog pivoted between two plates which are firmly riveted to the opposite sides of the bar. These plates fully protect the locking dog from damage, but sufficient space is allowed to permit the bars to slide freely when the locking dog is not engaged.

The clamp is set by pressure on the tightening lever which slides over the free end of the bars, as shown in Fig. 33. When the bars are drawn to position, the locking dog is set by pressure of the fingers or light tap of a hammer and it positively locks the clamp. Tightening lever is detachable and can be used on any Gemco Square-column Clamp.

A Gemco Clamp can be assembled, tightened and set in 1 min. It automatically squares a column and can be tightened from all sides so as to positively close all cracks, thus retaining all the valuable part of the mixture which would otherwise flow out with the water. Gemco Clamps will square a rectangular pier or column in the same manner.

These clamps can be removed in the same time which it takes to adjust them. The tightening lever is used to partially relieve the strain when the locking dog can be disengaged by the use of a claw hammer and a nail or punch inserted in the hole at the corner of the dog. After two diagonally opposite corners are loosened the entire clamp is free. By working two men on opposite sides of the column and at diagonal corners of the clamps, the work of setting or wrecking can be completed in a very short time.

When knocked down these straight, interchangeable bars of steel are easily transferred from floor to floor or building to building.

To avoid unnecessary expense by using large clamps on small columns the Gemco Square-column Clamps are manufactured in three stock sizes. Special sizes made only on specific order.

Stock No.	Column size, inches	Weight, pounds
C-10................	24	29½
C-11................	30	35
C-12................	36	40½
C-13, Tightening lever	..	4¾

Another patented type of all-steel column clamp which requires no wood framing is shown in Fig. 34—manufactured by the K. & W. Clamp Co., Minneapolis, Minn.

All square or rectangular-column forms should have bevel strips in the corners. Sharp corners in concrete work should always be avoided, not only because it is difficult to tamp concrete so as to obtain perfect corners, but because the edges are likely to be chipped off when the forms are removed or while the concrete is still green. The bevel strips are usually nailed to two opposite panel units when the forms are being made up.

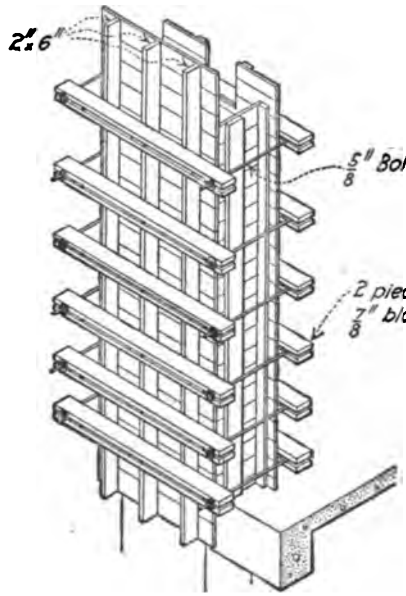


FIG. 32.

Octagonal and round-column forms with wooden clamps are not susceptible of ready arrangement into units, and are clumsy and quite expensive to make. The problem seems to be solved in present-day practice by the use of either iron bands or chains. Fig. 35 shows a form clamp manufactured by the Sterling Wheelbarrow Co., Milwaukee, Wis., which can be used on any type of column form. The band-iron placed around the form (view *a*) is passed through the clamping head, and drawn tight. The manufacturers describe the operation as follows:

The operator grasps the band with one hand and ratchets the lever several times. This ratcheting grips and releases the band, drawing it perfectly tight no matter how kinked the band was before being put around the form. When he sees that the band is tight enough, the operator presses the lever down close to the form, thus crimping the band in such a way that slippage is impossible (view *b*), and locks the clamp by inserting the lever into one of the openings in the locking device (view *c*). When knocking down the form, it is necessary simply to release the lever from the locking device, return the clamping head to position, and pull the band out.

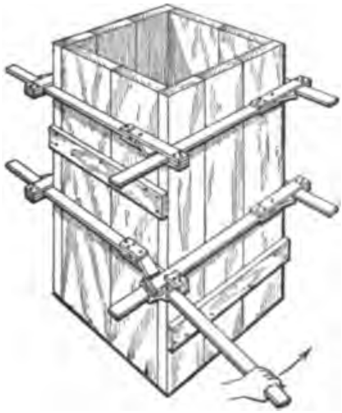


Fig. 33.—Gemco square column clamp.

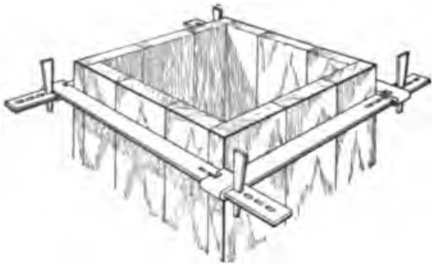
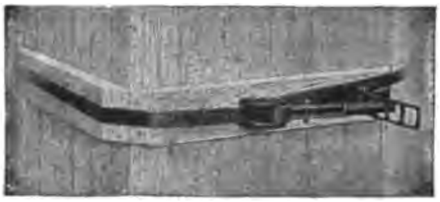


Fig. 34.—K. & W. clamp.

a

b



c

Fig. 35.—Sterling form clamp.

A Gemco Clamp for round columns (Fig. 36) is described by the manufacturers in the following manner:

A Gemco Round-column Clamp is made of a strip of 1¼-in., 16-gage band iron, one end of which carries a malleable-iron casting which holds the hardened tooth locking dog. When in use the free end of the band iron is slipped under the locking dog and the clamp can be quickly and easily drawn to and set at any desired degree of tension with a tightening lever. The lever is detachable and can be used on any size of Gemco Round-column Clamp.

A clamp may be struck or wrecked in less time than required for setting, and in this operation the detachable lever is used to relieve the strain.

Gemco Round Clamps are made in two stock sizes—for 24-in. and 36-in. columns. Special sizes made to order.

Stock No.	Column size, inches	Weight, pounds
C-20	24	3
C-21	36	3¾
C-22 Tightening Lever	..	2¾

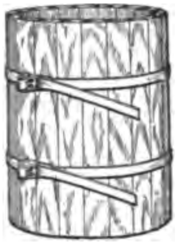


Fig. 36.—Gemco round-column clamp.

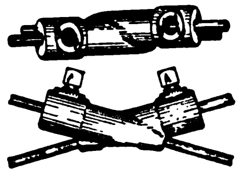


Fig. 37.—Universal round-column clamp.

Another patented clamp for circular columns is shown in Fig. 37, manufactured by the Universal Form Clamp Co., Chicago, Ill. These clamps are made in one size only, *i.e.*, size

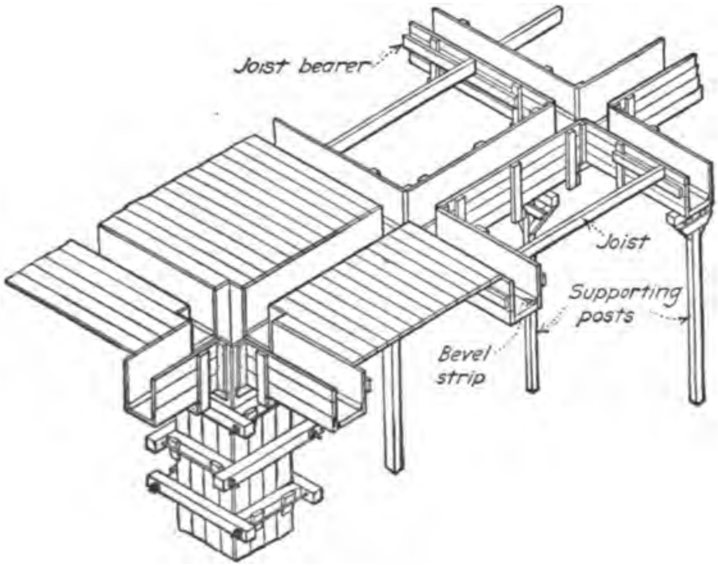


Fig. 38.

C-1 for ¼-in. rods, 5 in. long overall. A tightening wrench shown in Fig. 60, page 117, is used with this clamp.

Wire and rod clamps described under the heading "Wall and Pier Forms" (page 111) are used to some extent on forms for columns.

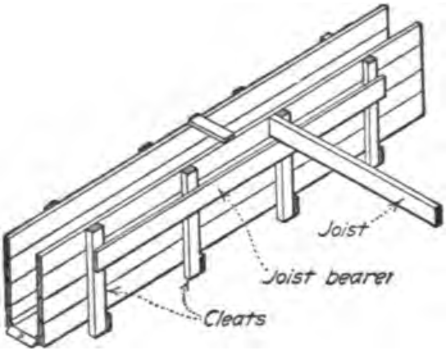


Fig. 39.

64b. Beam and Girder Forms.—Fig. 38 is an isometric view of a typical floor. Joists support the slab sheathing and are carried in turn by ledgers or joist bearers (see also Plates I and IV, pages 112 and 115 respectively). The method of clamping the beam and girder forms is shown more in detaii in Figs. 39 and 40. Separate blocks are sometimes used instead of a joist bearer. Plate II, page 113, shows a method of supporting slab forms by notching out the beam cleats to receive the joists.

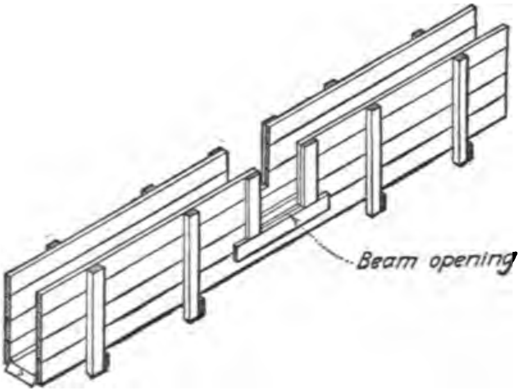


Fig. 40.

A number of different methods are employed to clamp beam and girder forms and prevent them from spreading due to the pressure of wet concrete. The method referred to above is perhaps the most common, but bolts either above or below the beam bottom are sometimes employed. Clamping by means of *ribbands* is well illustrated in Plates I to III inclusive, pages 112 to 114.

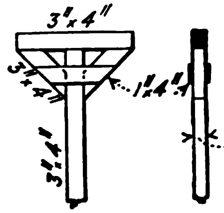


Fig. 41.

Typical girder-form construction is shown in Fig. 40. Sometimes the girder sides are not erected complt te, the portion between the beams being erected in separate sections, as shown in Plate I, page 112. The lower part of the girder sides are made of thick plank in order to provide a suitable support for the beams.

A common method of bracing the outer side of a wall beam is shown in Plate I. Another method is by means of a cantilever brace. Methods shown in Fig. 28 and Plate II, page 113, should also be noted. Fig. 41 shows a form of ordinary post or shore in common use. Haunch design may be seen in Plates I and IV, pages 112 and 115 respectively.

64c. Slab Forms.—Two types of slab forms need to be considered for ordinary construction where beams and girders are employed: (1) the panel type (the only type of form construction so far treated); and (2) slab forms made up into box shape, comprising the joists and the sides of the beams and girders.

In the usual type of construction, designated above as type 1, the beam and girder forms are either erected complete and the posts set in place after the erection, or the beam and girder bottoms are spiked in place to the post caps, and the sides are erected as separate units. After this much is accomplished by either method, the joists are then put in position and the panel

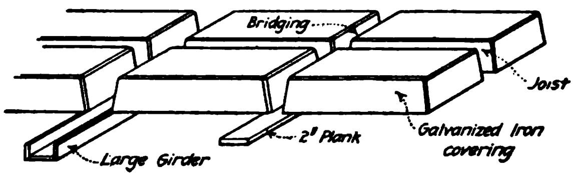


Fig. 42.

units are placed on top of them without nailing. Notching out of the slab panels is usually necessary at the columns. When the forms are to be used but once, the slab sheathing is not made up in advance into panels, but is lightly nailed to the joists after they are in place.

Fig. 42 shows one method of making up slab forms into box shape—the form construction being that used at the University of Wisconsin. Planks extend in two directions supporting the cells. The steel and continuous stirrups are set for demonstration only.

A collapsible core box is used by at least one prominent builder (Fig. 43). A plank resting on cleats on the sides of the cores forms the bottom of the beam mold. The main girders are molded in similar spaces between the ends of the cores in one panel and those in the next panel. The molds are made in two equal parts with a hinged joint through the longitudinal center line of the upper surface, and are held open by transverse struts between the lower edges. When the forms are to be removed, the transverse struts are knocked out and the boxes are collapsed. The use of these molds presupposes a standardized layout.

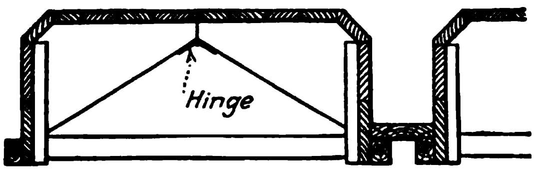


Fig. 43.

Forms for flat-slab floors are much simpler than for the ordinary beam-and-girder construction—with the exception, of course, of the forms for the flaring column heads. Typical designs of forms for flat-slab floors are shown in Fig. 44 and Plate V, page 116. Corrugated iron has been used to a considerable extent for sheathing where the ceiling surface does not need to be absolutely smooth.

A new and novel system of form work (Fig. 45) has been devised by Jesse E. Hodges, of Cincinnati, which is applicable to both beam-and-girder and to flat-slab constructions. Edward O. Keator & Co. of Cincinnati, Ohio, are sole agents for this system.

The slab forms consist of very light metal sheathing held on a matting (usually wooden strips 2 in. by 2½ in. by 4 ft. 6 in.) which is supported by stringers placed about 4 ft. on centers.

The matting is formed by connecting the small wooden strips by a light metal chain near each end so as to allow an opening of about $2\frac{1}{2}$ in. between the strips. The mats are made of short lengths so that they can be rolled into bundles and not be too heavy for one or two men to carry. Usually one stringer is used halfway between beams, and ledgers on the beam sides hold the ends of the mat. The stringers do not have to be changed in length for different jobs as they can be used in long lengths and lapped.

Fig. 44.

The adjustable shores (Fig. 46) are made from 8-ft. lumber, with a 4 by 4-in. post and two 2 by 4-in. side pieces, and are adjustable from 8 to 14 ft. This range is sufficient for ordinary work but longer shores can, of course, be built for special work. In erecting one of these shores, the clamps are loosened and the 4 by 4 drawn out to length designated by a measured pole in the usual manner. Clamps are then tightened and the shores raised. The feature of this

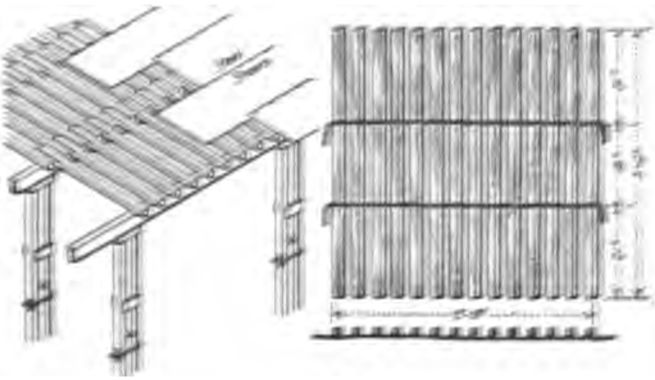


Fig. 45.—Hodges system of formwork.

type of shore is a positive clamp which sets itself when the cam is in driving position and the shore is under load. This shore also has a detachable head which may be left on, in setting shores under a beam, or removed in using the shores for flat-slab floors. With the head removed the form stringers rest in a fork made by the two upper pieces of the shore, which makes it unnecessary for a man to climb up and nail cleats on each side of the joist after the shore is in place

Wedges are unnecessary with the Hodges shore as leveling the centering for the floor may be accomplished with a kind of racket-jack arrangement shown in Fig. 47. To adjust the length of shore a steel bar which is notched to form a rack is attached to one face of the 4 by 4. A fork-

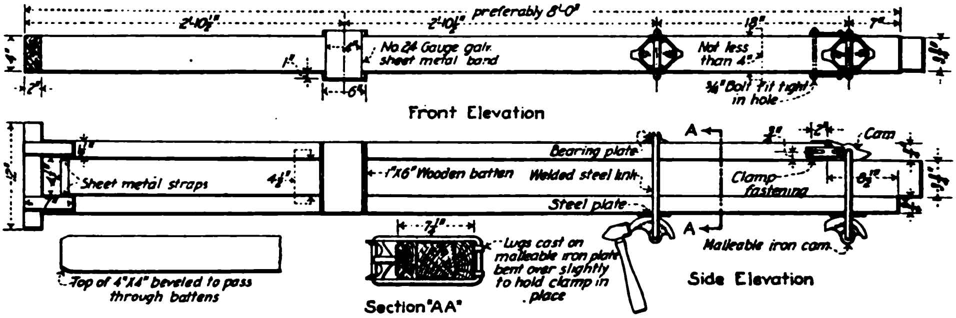


FIG. 46.—Hodges adjustable shore.

ended lifting lever is then fitted over the timber, with its fulcrum bolt resting in one of the slots of the bar and the ends of the fork engaging the lower ends of the 2 by 4-in. sticks. With the

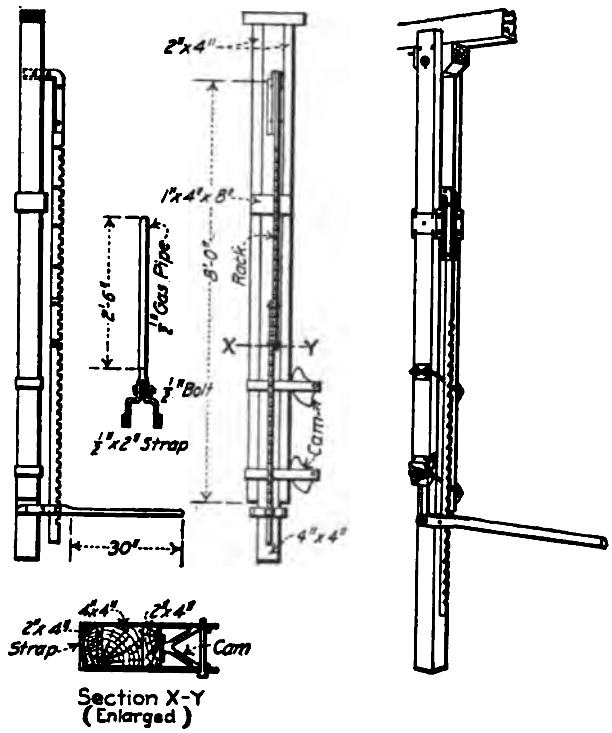


FIG. 47.

cams loosened, the upper portion of the shore is raised by bearing down on the lever handle, and when in proper position is locked by the cams.

Another type of adjustable shore is shown in Fig. 48, manufactured by the H. W. Roos

Co., Cincinnati, Ohio, and known as the "Roos Self-lock Adjustable Shore." It is made up for average use with a 6-ft. piece of standard pipe and two pieces of 2 by 4's dressed side and edge, 8 ft. long. Only the necessary castings are sold by the company above mentioned. The operation of the shore consists in taking the extension member by the two legs and lifting it to the

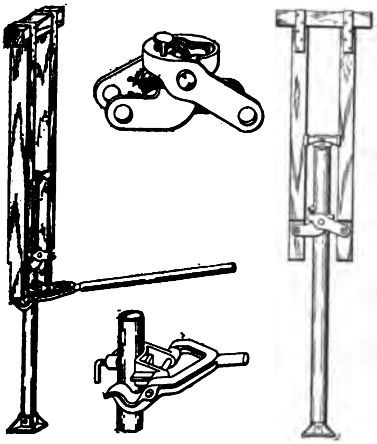


Fig. 48.—Roos self-lock adjustable shore.

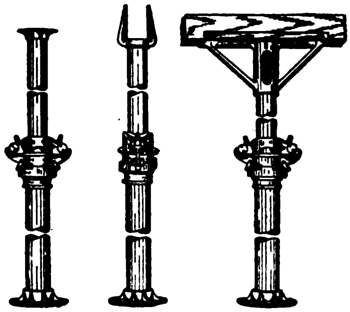


Fig. 49.—Gemco adjustable steel shoring.

desired height. The slightest pressure on the two legs of the extension member toward the pipe causes the yokes to grip and the shore is ready for the load. The greater the load the greater the grip. For finer adjustment the shore can be raised or lowered with the jacking device which can be attached to any point under the two legs of the shore by turning the handled screw

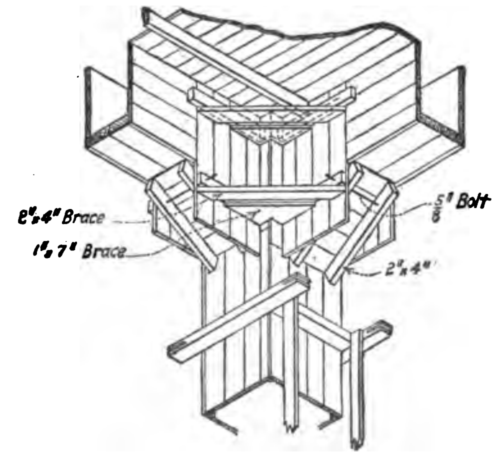


Fig. 50.

and the upper member can be set at its final height with the adjusting lever. A turn of the thumb-screw lock sets the yokes and prevents any possible release through outside jarring. The jacking device is then removed for adjusting the next shore.

"Gemco Adjustable Steel Shoring" is shown in Fig. 49. The shores are adjustable in height from 9 ft. to 12 ft. 6 in. For adjusting the height or raising the load, a detachable lever is used. These shores are manufactured by the Gemco Mfg. Co., Milwaukee, Wis.

There are many other types of wooden floor forms but those described above illustrate the main features in floor-form design. For steel floor forms see page 135.

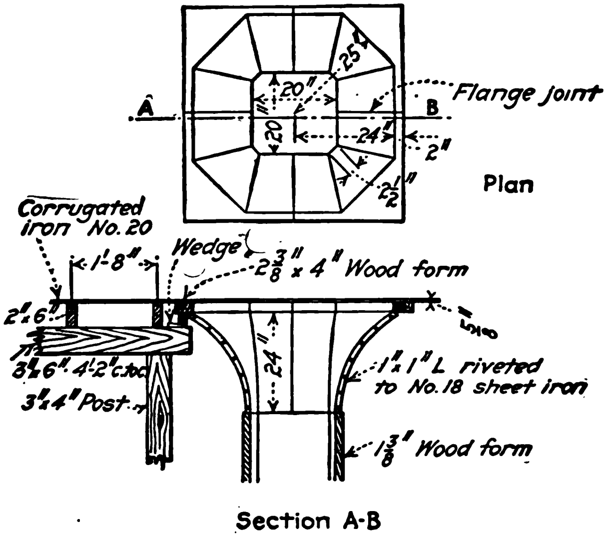


FIG. 51.

64d. Column Heads.—The tops of column forms are sometimes made separate from the column forms proper. Generally this is done either to avoid remaking the column tops to fit varying sizes of beams and girders, or to facilitate the construction of special form

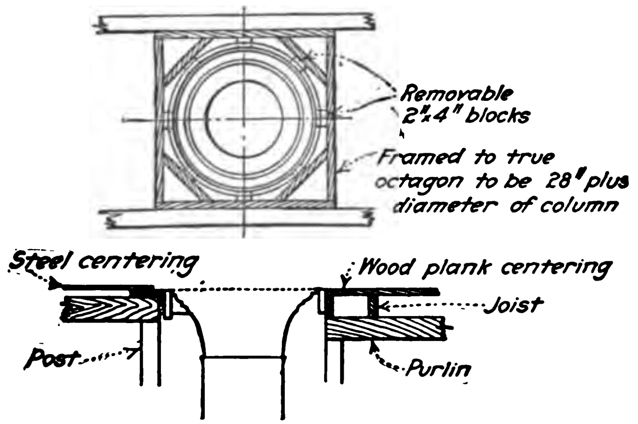


FIG. 52.

heads. Fig. 50 shows an assembly of column heads for regular interior columns of the Larkin Co's. building, Buffalo, N. Y., designed by the Aberthaw Construction Co., the contractors. These heads are also shown, but not in detail, in Plate IV, page 115.

The problem of the column head in flat-slab construction has been met practically from

Fig. 53.—Steel column form used in Reid-Murdock Co.'s building, Chicago, Ill.

Fig. 54.—Steel column head to fit octagonal column.

the first by the use of metal in some form. The metal mold used by the Aberthaw Construction Co. in the Massachusetts Cotton Mills at Lowell, Mass., is shown in Fig. 51. Note that the head is very simple, a square where it leaves the column, with flaring beveled corners, which finally form an octagon at the ceiling level.

Fig. 52 shows a type of column-head form used in constructing the Larkin warehouse at Chicago, Ill. It is one of the standard types of flaring column caps manufactured by the Des-

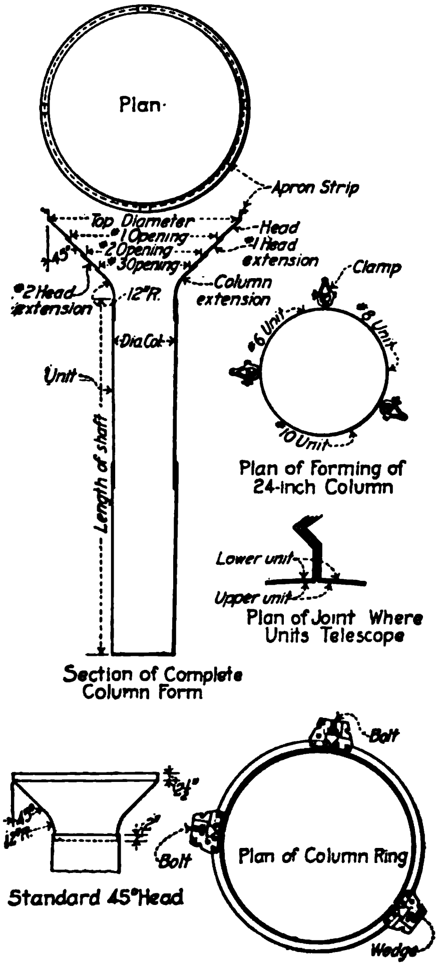


Fig. 55.—Adjustable steel column heads, Hydraulic Pressed Steel Co.

lauriers Column Mold Co., St. Paul, Minn. The columns themselves were molded by steel forms and these forms were placed after the erection of the upper floor falsework. The column-cap section was suspended from the floor forms and the remaining column sections bolted to it.

Figs. 53 and 54 show adjustable column heads as manufactured by the Blaw Steel Construction Co.; conical and octagonal heads only are illustrated here, but this company also makes rectangular head molds and molds which spread from a rectangular section at the column to an octagonal shape at the floor slab. The conical head is in two parts; the top portion remains the same for all columns while the bottom is fitted to columns of various diameters. The octagonal mold is also adjustable at the lower end and may be used in connection with either octagonal and circular steel column forms or with wooden octagonal forms. The Hydraulic Pressed Steel Co., Cleveland, Ohio, manufacture round steel column forms with both standard and adjustable column heads (see Figs. 55 and 56).

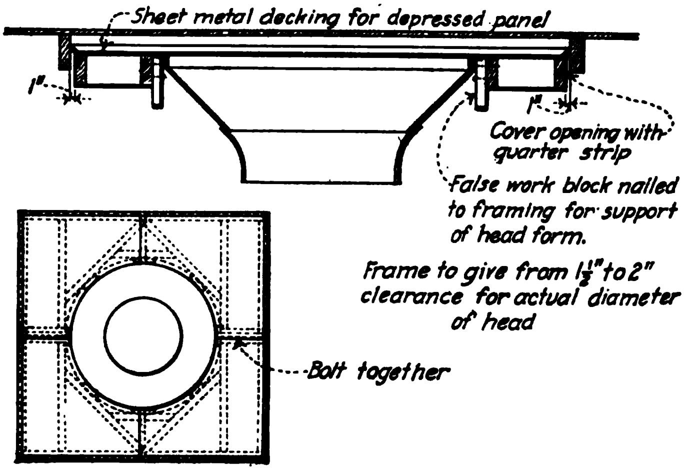


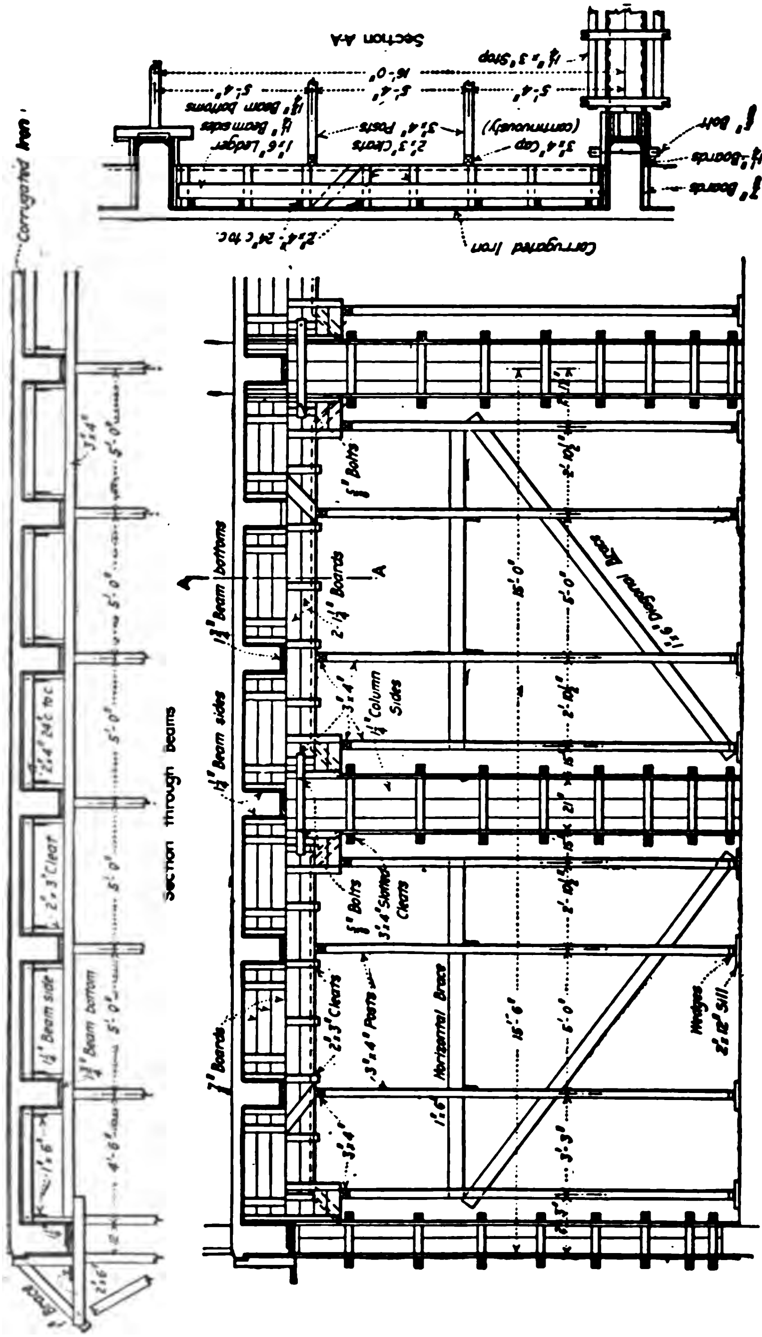
FIG. 56.—Method of forming depressed panels.

64e. Wall and Pier Forms.—Forms for bridge piers and for walls of appreciable height are usually constructed of either 1-in. or 2-in. plank nailed to studs and held by horizontal waling pieces, with tie bolts extending across the pier or wall between opposite wales. The wales, which often consist of two planks fastened together but separated by spacing blocks, are set edgewise against the form studs and the tie bolts are carried through the openings which occur in the waling pieces. Wire is sometimes used for bracing and is tightened either by wedges or by twisting. The wire pulls against spreaders which are inserted between forms and which are removed as the concrete level rises.

Where bolts are employed in pier and wall construction, a number of different methods are used for withdrawing the bolts. One method is to cover each bolt with old pipe cut somewhat shorter than the inside dimensions of the forms, and to place a wood washer at each end of the pipe. When the forms are taken down, the bolts are easily drawn out of the pipes, the wood washers are then cut out of the face of the concrete, and the holes pointed up. Another method is to make the bolts in three pieces, with the middle piece occupying the same position between the forms as the pipe above described. This middle section is connected with the end pieces by means of ordinary unions. When the concrete has set sufficiently, one turn releases the end sections and the holes left in the work are plugged with mortar. Still another method is to use a patented casting with set screw, also a tightening wrench and rod puller. The tightening wrench is a device for the purpose of exerting a pressure against the form to draw it in line or to desired dimensions. The rod puller is for removing or pulling the tie rods from the concrete after the concrete has set sufficiently to stand alone.

The second method described is shown in Figs. 57, 58 and 59. (In Fig. 59 wire is used instead of the middle piece of bolt.) The bar couplings shown in Fig. 57 are manufactured by the Marion Malleable Iron Works, Marion, Ind., and are stocked in seven sizes threaded to take bolts $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, 1, $1\frac{1}{8}$, and $1\frac{1}{4}$ in. "Universal Cone Nuts," shown in Fig. 58, are manu-

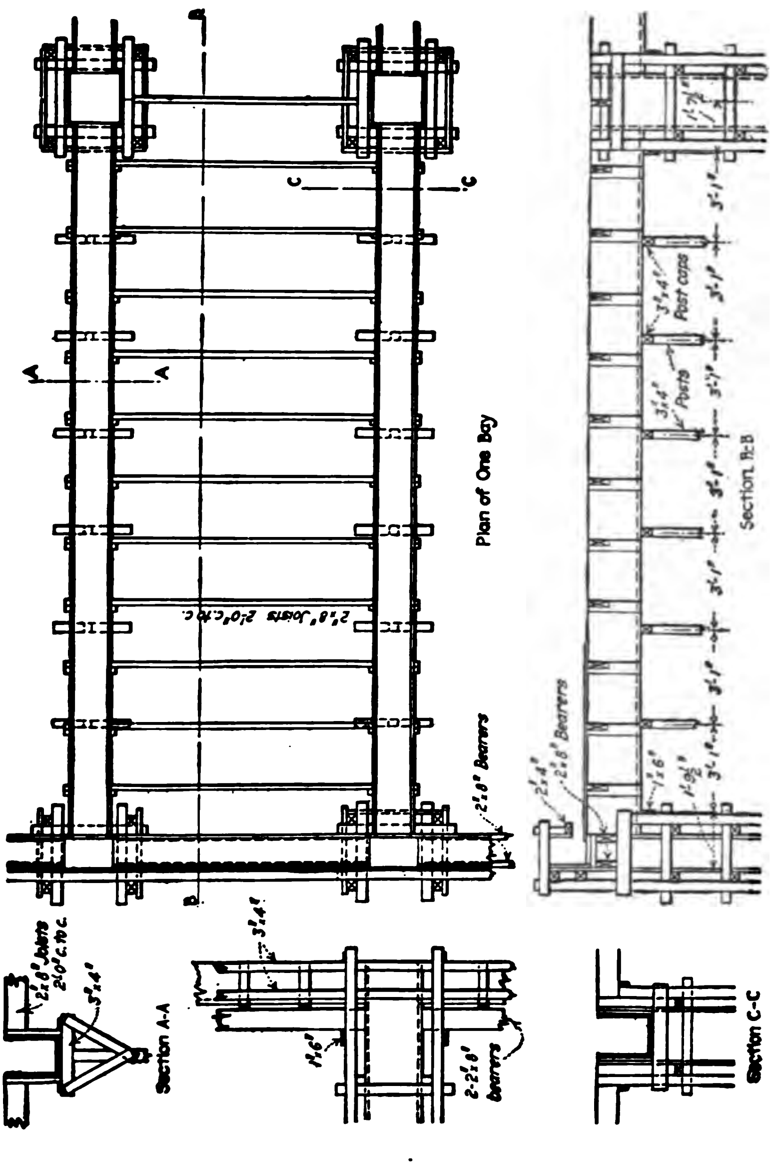
PLATE I.



Section parallel to Girder

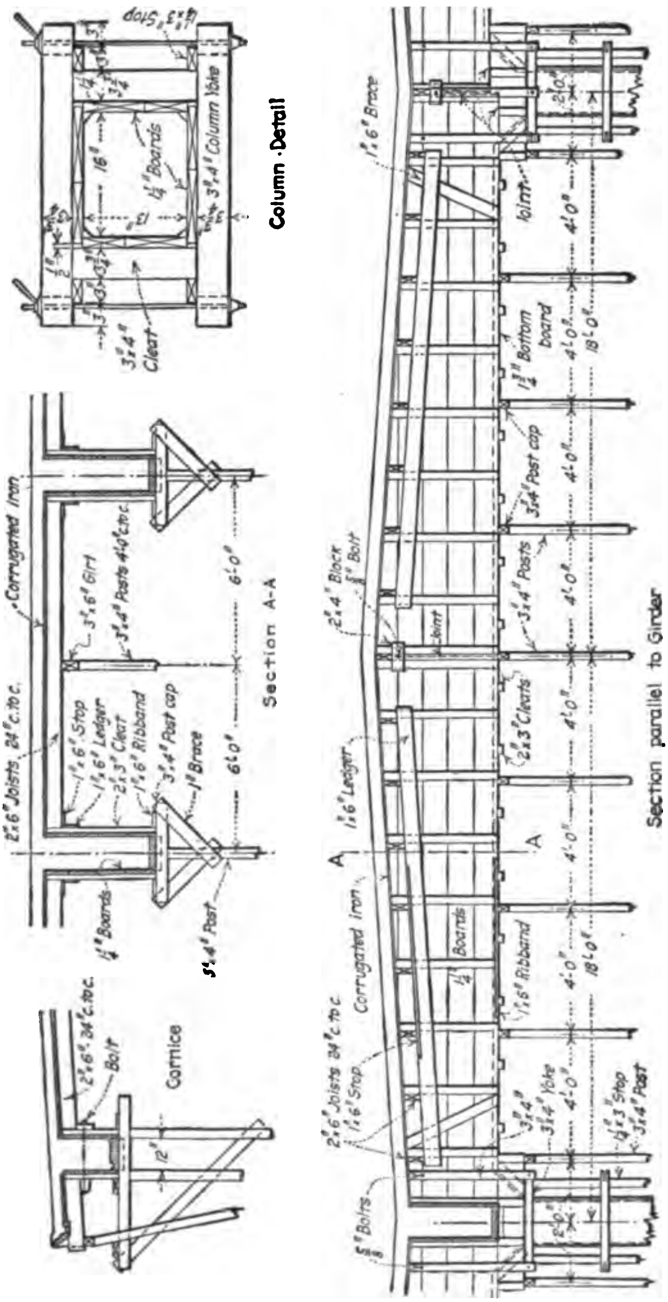
Assembly of forms in Hard Fiber Plant, Tonawanda N. Y.

PLATE II.



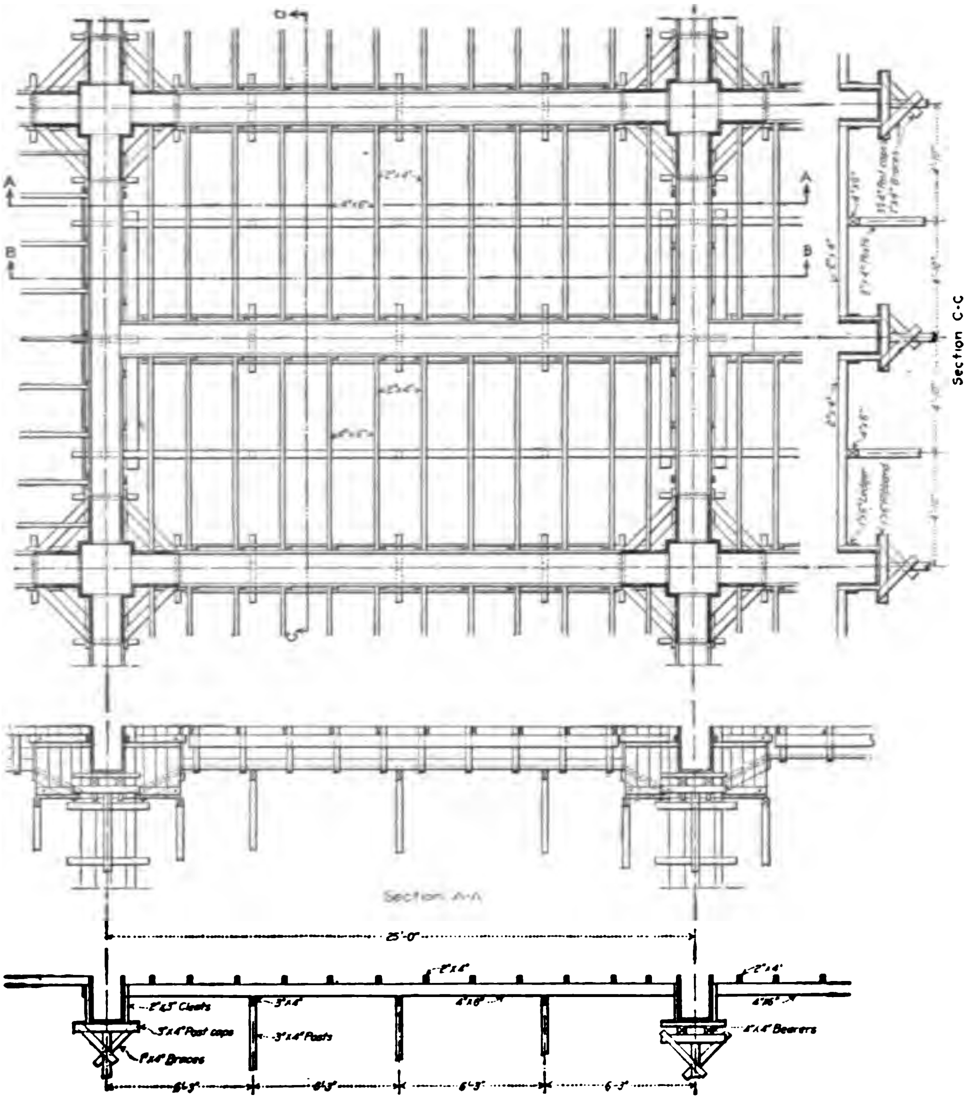
Assembly of beam and column forms for Lamson Consolidated Store Service Co.'s building, Lowell, Mass.

PLATE III.



Assembly of forms for 36-ft. spans in Hard Fiber Plant, Tonawanda, N. Y.

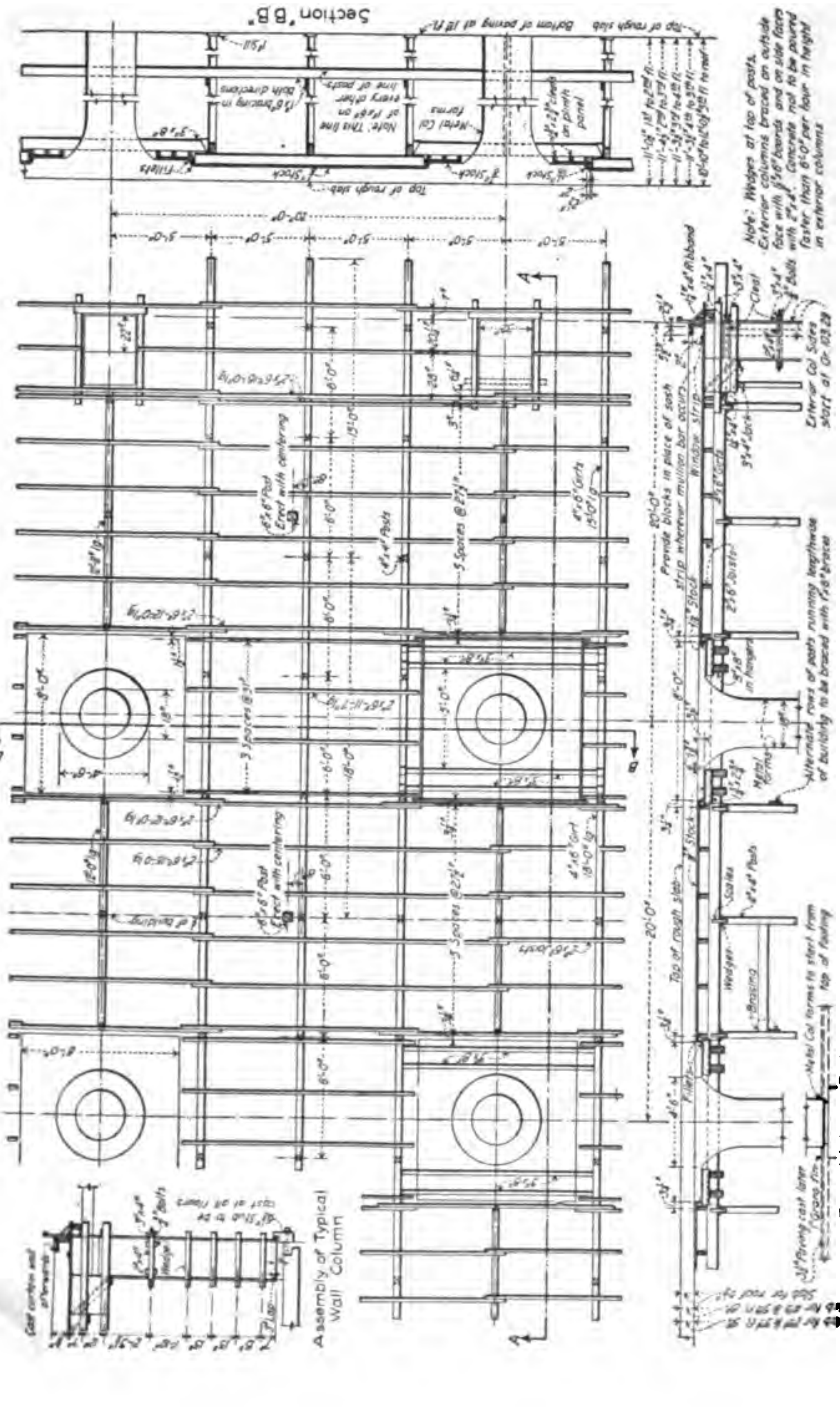
PLATE IV.



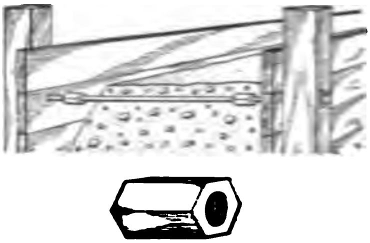
Section B-B

Assembly of forms, Larkin Co.'s building, Buffalo, N. Y.

PLATE V.



Assembly of typical forms in Oakville Co.'s building, Waterbury, Conn.



Coupling used for form work

Fig. 57.

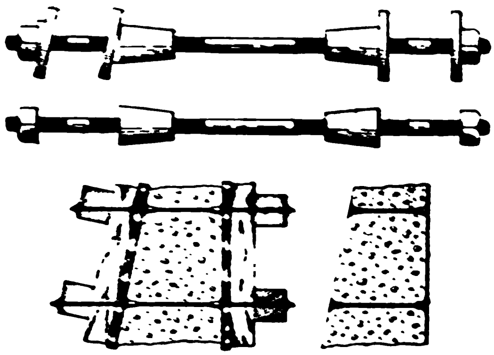


Fig. 58.

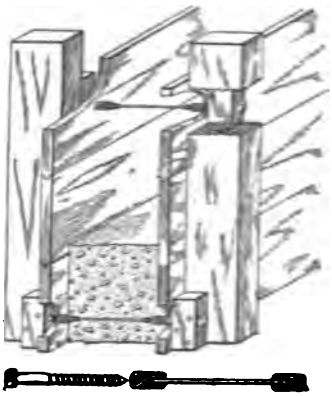


Fig. 59.

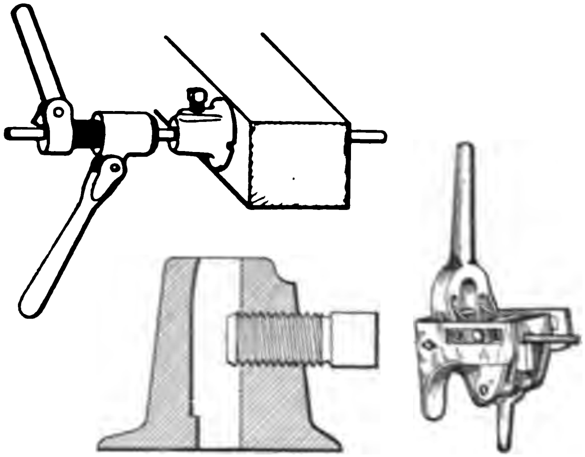
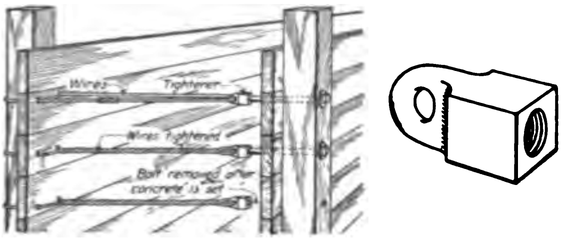
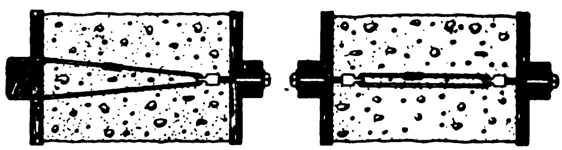


Fig. 60.—Universal rod clamp, rod tightener, and rod puller. Rod puller at the right.

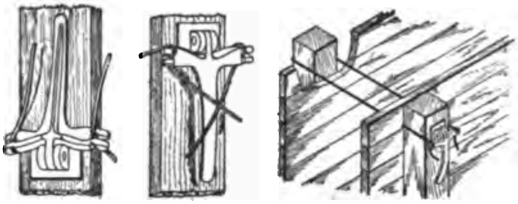


Use of wire form tightener



One face of wall to be finished *Both faces of wall to be finished*

Fig. 61.



(a) (b) (c)

FIG. 62.

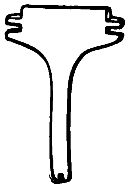


FIG. 63.

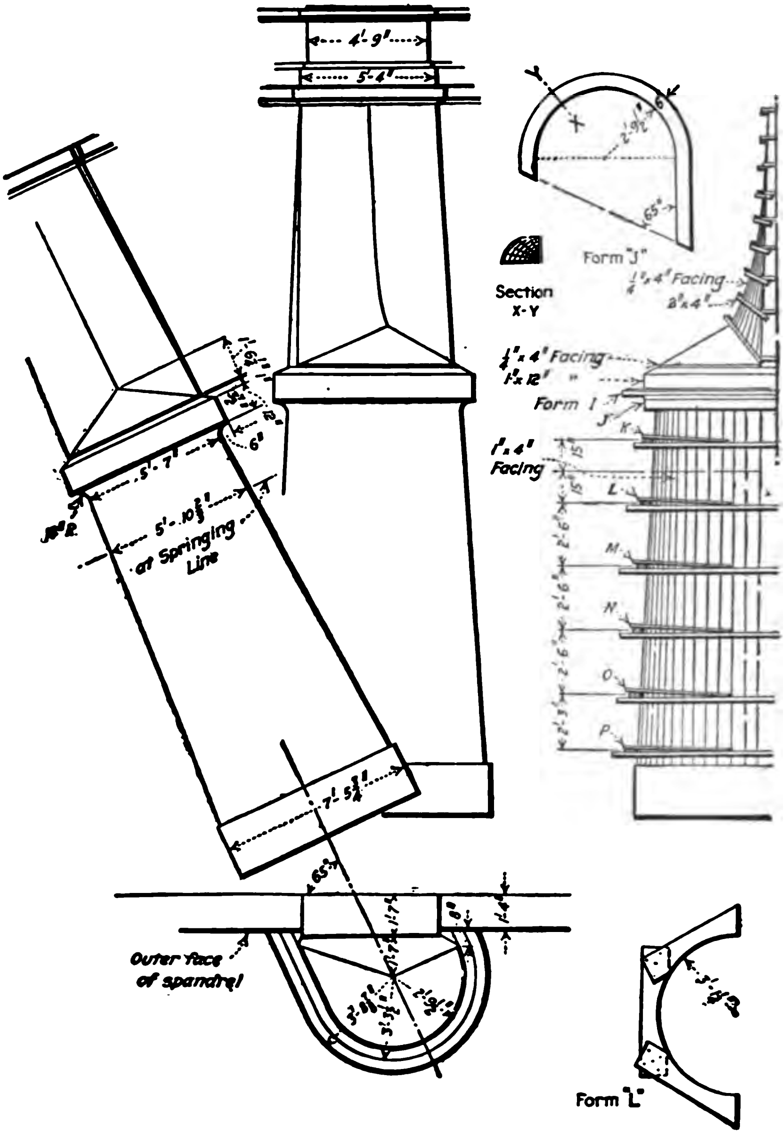


FIG. 64.—Typical pier forms used in bridges of Luten design.

factured by the Universal Form Clamp Co., Chicago, Ill. Cut shows washers used with cones and countersunk into forms, but this is not necessary. Universal rod clamps (Fig. 60) can be used on the outside of forms in place of nuts and washers. The device shown in Fig. 59, known as "Tyscru," is being marketed by the Unit-Wall Construction Co., New York City.

A patented casting, rod tightener, and rod puller such as used in the third method above described are shown in Fig. 60. These devices are manufactured by the Universal Form Clamp Co., Chicago, Ill. The rod clamps are made in six sizes, namely: No. 1 for ¼-in. and $\frac{5}{16}$-in. rods; No. 2 for ⅜-in. rods; No. 3 for ½-in. rods; No. 4 and No. 4 extra heavy for ⅝-in. rods; and No. 5 for ¾-in. rods. The No. 4 is recommended for column clamping and No. 4 extra heavy for large retaining walls.

Patented wire form clamps are illustrated in Figs. 61, 62 and 63. The wire form tightener, shown in Fig. 61, is manufactured by the Marion Malleable Iron Works, Marion, Ind., and is made in three sizes, threaded to take ½, ¾, and 1-in. bolts. The tightener is simply attached to a wire slipped through from one side of the forms and a bolt is screwed into it from the face

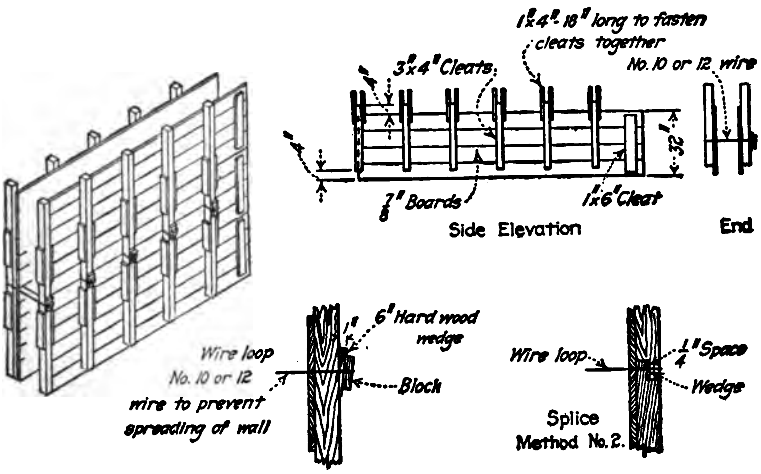


Fig. 65.

side. By turning this bolt any degree of tension may be obtained in the wire. The "Universal Wire Clamp" is illustrated in Fig. 62. The clamp is placed against wales or studding and wires bent as shown in (a). The clamp should be placed in a position so as to allow the wires to enter the slots at nearly right angles. For locking all that is necessary is to pull handle down so that it takes a position as shown in (b) and (c). The "American Wire Clamp" is shown in Fig. 63 and is similar in operation to the clamp just mentioned.

Where especially good work is desired, forms are lined with galvanized iron. For high piers or walls, the forms are constructed in large panels. After the concrete has been constructed to a proper height and the last course has set several days, the panels are disconnected and hoisted to a higher position and then reassembled for concreting, and so on.

When the ends of bridge piers are rounding, special forms are necessary. In the construction of the Atherton Ave. bridge over the Pennsylvania R. R. tracks in Pittsburg, the forms for the curved ends of the piers were built of 1-in. by 2-in. strips nailed to horizontal segmental wales. These wales were nailed to the wales of the side forms. The rounding forms were kept in place by wiring to dowels set in the foundation concrete. Before starting the erection of the formwork, a flexible panel was made by nailing galvanized-iron sheets to the 1-in. by 2-in. strips. This panel was bent against the wales, which acted like hoops. Curved pier forms used in bridges of Luten design is shown in Fig. 64.

Fig. 65 illustrates a simple method employed by the Aberthaw Construction Co. for building a wall of considerable height by means of movable forms. A simple method is also shown in Fig. 66.

Wall forms of small height may be braced by battered posts outside. Fig. 67 shows forms for a concrete foundation wall for small buildings where no cellar is necessary. Such forms may

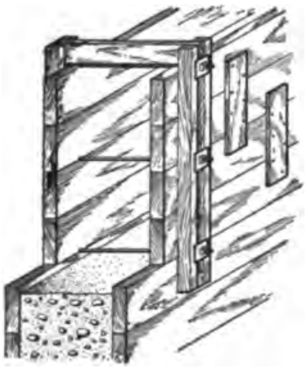


Fig. 66.

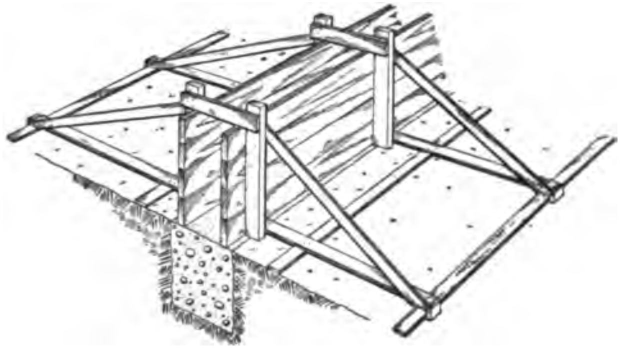


Fig. 67.

either be constructed in sections or built in place. It should be noted that the forms are suspended over the trench and are not allowed to rest upon the new concrete. Figs. 68 and 69 show other methods of form construction for low walls. No outside form is needed in the lower part of such walls as shown in Fig. 68 when the earth is extremely hard and firm.

Fig. 70 shows a common type of form design for curtain walls below windows. Curtain walls for complete wall panels are of similar construction.

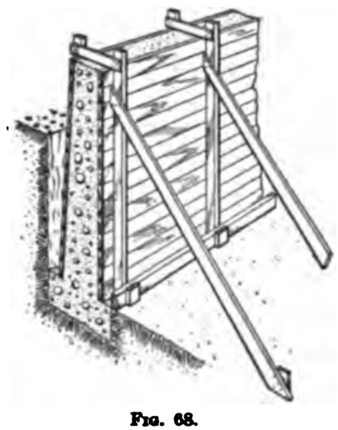


Fig. 68.

Fig. 69.

65. Design of Forms.—Although form design in most cases has been left entirely to the discretion of the superintendent or foreman on the job, it has been found by a number of engineering contractors that it pays on large or important work to have the forms designed in the drafting room, provided such designing work is done under the direction of a man of practical ability. This plan is not only more economical, but also eliminates the possibility of an error being made in the proper size and spacing of form members.

Many details of form design depend almost entirely upon judgment and experience; but the size and spacing of supports for sheathing in slab, column, and wall forms and the spacing of posts for slabs, beams, and girders are capable of being determined by scientific calculation and such practice results in the use of a minimum quantity of lumber consistent with the deflec-

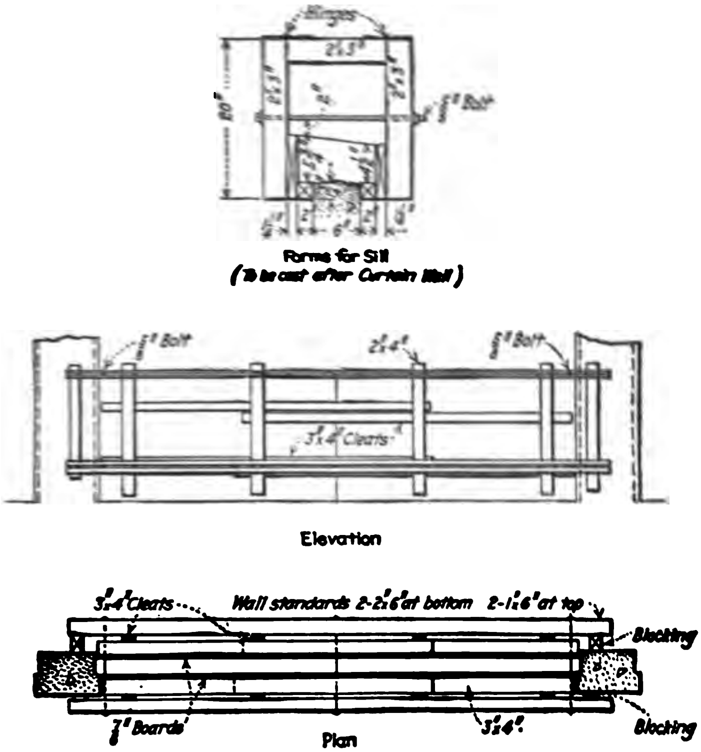


Fig. 70.—Forms for curtain wall and sill, American Hide and Leather Co.'s building, Lowell, Mass.

tion allowed. In no case should the spacing of the supports be greater than a safe span for the sheathing. Deflection should always be considered in order to give sufficient stiffness and thus prevent partial rupture of the concrete.

65a. Values to Use in Design.—In calculating floor forms, we must add to the weight of the concrete itself, a live load which may be assumed as liable to come upon the concrete while it is setting. This live load is usually taken at 75 lb. per sq. ft. except in cases where the floor is made a storage for cement or sand.

The pressure of concrete against forms depends principally upon the rate of filling and the temperature. The accompanying table[1] is from tests made by Major Francis R. Shunk, Corps of Engineers, U. S. A.

Rate of filling (vertical feet per hour)	Temperature				
	80°	70°	60°	50°	40°
2	530	560	600	680	790
3	690	720	810	920	1,080
4	820	870	980	1,130	1,340
5	930	990	1,120	1,310	1,570
6	1,020	1,090	1,250	1,480	1,780
7	1,090	1,170	1,350	1,620	1,970
8	1,130	1,240	1,440	1,740	

Tests on the pressure of wet concrete

[1] From *Eng. Rec.*, Jan. 15, 1910.

poured rapidly have been made under average conditions by the Aberthaw Construction Co., Boston, Mass. The following table[1] gives the results observed in the pouring of two columns of small cross-sectional area. It should be noticed that the wet concrete exerted a hydrostatic pressure equivalent to that of a liquid weighing from 140 to 150 lb. per cu. ft. or very nearly that of a liquid having the same weight as the concrete.

Description of concrete		Head (ft.)	Pressure (lb. per sq. ft.)	Hydraulic equivalent (lb. per cu. ft.)
Time of pouring, 9 min	No. 1 column	3.08	460	149
Mixture, 1:1½:3		6.08	900	148
Stone, 1-in. run-of-crusher		9.08	1,330	146
		12.08	1,710	142
Wt. of mixture, 152 lb. per cu. ft.		15.08	2,110	140
Time of pouring, 14 min	No. 2 column	2.75	407	148
		5.75	840	146
Mixture, 1:1:1		8.75	1,280	146
Stone, 1-in. run-of-crusher		11.75	1,700	145
		14.75	2,080	141
Wt. of mixture, 149 lb. per cu. ft.		17.75	2,450	138

From the results of the tests just mentioned it would seem that 145 lb. per sq. ft. per foot of height is a rational value to use for the lateral pressure of concrete in the design of forms. This same value was also arrived at from laboratory and field tests made under the supervision of Prof. A. B. McDaniel and N. B. Garver at the University of Illinois in 1913, 1914, and 1915.

Opinions differ as to the proper coefficient to use in the moment formula when making computations for the strength of floor sheathing and joists. It is probable that the formula $M = \frac{wl^2}{10}$ may safely be used in all cases except for single-span joists where the concrete is conveyed to place by small dump cars on a portable track. With dump cars the concentrations on a single-span joist may be so heavy that the negative bending moments of the dead load will be relatively very small as compared to the positive moments and the formula $M = \frac{wl^2}{8}$ should undoubtedly be employed. When the bending moment is figured as $M = \frac{wl^2}{10}$, the deflection should be determined by the deflection formula $D = \frac{3}{384} \cdot \frac{w''(l'')^4}{EI}$ (see "*Notation,*" page 124). This formula is the ordinary one for calculating deflection except that the coefficient is taken as a mean between $\frac{1}{384}$ for a beam with fixed ends and $\frac{5}{384}$ for a beam with ends simply supported. For lumber commonly used in form construction E may be assumed at 1,200,000 lb. per sq. in.

The fiber stresses allowed in form design may well be higher than those it would be desirable to use in more permanent construction. The following values are usually employed:

Maximum fiber stress in spruce or equal:	
for timbers	1,200 lb. per sq. in.
for column yokes	1,800 lb. per sq. in.
Horizontal shear for spruce or equal	200 lb. per sq. in.
Crushing perpendicular to grain in spruce or equal	400 lb. per sq. in.

[1] From *Concrete-Cement Age*, Oct., 1913.

The crushing of form lumber perpendicular to the grain should not be overlooked. Although a 3 by 4-in. or 4 by 4-in. post may be braced at least every 6 ft. in height and have apparently an allowable compressive strength of about 800 lb. per sq. in., such a post should not be permitted to carry more than one-half such a load due to crushing of the lumber perpendicular to the grain in the crosspiece or girt over the post. Especially is this true where a granolithic finish is to be cast with the slab, as settlement would ruin the slab surface. Large hardwood wedges should be used to prevent any settlement due to crushing under the post.

The maximum deflection which may be allowed a form timber is not definitely known. Some designers allow very small deflections for joists and use full live and dead load in designing. Others reason that any serious deflection will be caused only by the dead weight of the wet concrete and for that reason allow comparatively large deflections. Deflections allowed in practice vary from an arbitrary maximum of $\frac{1}{8}$ in. for all timbers to maximum deflections of $\frac{1}{360}$ of the span.

65b. Drafting-room Methods.—The following article is taken by permission from a paper presented at the Twelfth Annual Convention of the American Concrete Institute by R. A. Sherwin, Resident Engineer, Aberthaw Construction Co.:

The first study and drawings to be made form a general assembly. Usually several different combinations of timbers and methods of assembling the panels are sketched up and compared as to cost. Other things being equal, of course, the cheapest design is adopted. A record of costs of the various units which go to make up the complete centering scheme is therefore necessary.

Points to be remembered in this study are:

1. Joists and girts should be in as few lengths as possible to save time in sorting on the job.
2. Use stock sizes and lengths of lumber.
3. Keep number of panels and pieces to a minimum.
4. Provide easy stripping.
5. Allow clearance enough for slight inaccuracies in making up and erecting, swelling of panels, etc.
6. Panels should be a whole number of boards in width, if possible, for ease in making up.
7. Units to be as big as can be handled and joists used as panel cleats where possible.
8. Provide for re-use of panels.
9. Beams to be handled as trough units when the job is regular and units can be re-used.
10. Consider use of floor domes or inverted boxes when beams are close together. In this way beam sides and slab are erected, stripped and moved as a unit. When either of the last two systems is used the beam sides should be given a slope to prevent hard stripping.
11. Provide for reshoring if necessary.
12. Have bracing above the men's heads.
13. When four beam haunches occur at a column consider making haunches as a unit similar to a column head in flat-slab construction.
14. Consideration of steel forms.

A general assembly shows how the various parts are to be put together, supported, tied, and braced to resist the concrete pressure and the weights coming upon the members. This drawing in its final shape can be made on tracing cloth with soft pencil. The dimension arrows, lettering and figures should be inked so that clear prints can be made to avoid errors in the field. These assemblies should be filed permanently for reference on future work.

When it is necessary to strip the floor centering for re-use before the concrete has been in place long enough to gain its full strength, provisions for proper support of the green slab must be made in the design of the floor forms. The safest and cheapest method of doing this, in the writer's opinion, is to place boards between the floor panels and wedge posts up to a bearing under these boards before the centering posts are knocked out. In this way the slab is never left unsupported, as is the case when posts are replaced after all the centering is down. These boards should be placed according to a plan and shall be located so as to shorten the spans of the main reinforcing bands.

After the general scheme for the forms has been decided upon the detail panel drawings are prepared. By panel, in this paper, is meant several boards cleated together into a unit to be used as a form for some part of a concrete member. Every different-sized panel is first sketched roughly on standard 6 by 9-in. sketching pads. These pads have holes punched at the top, so that as sketches of the different kinds of panels are made they can be bunched together and brass rivets put through the holes. These sketches are transferred in more detail onto thin tracing paper. The standard sheet is 25 by 33 in., divided by a 1-in. space into two halves of five spaces each. These details are used at the job mill for making up the panels. The blue-prints sent to the mill man are cut up into 5 by 6-in. units, with one detail on each, and are given to the carpenters at the making-up benches. On these detail sheets for each panel there is given the panel mark (which is stenciled onto the finished panel), the number wanted, and the floor on which they are to be used. The sizes of the stock, dimensions and alterations, when panels are to be re-used, should be clearly marked.

A system of symbols as follows is easily learned by the workmen and should be kept standard:

B, beam side; *BB*, beam bottoms; *F*, floor slab panels; *P*, plinth forms; *H*, haunch forms; *C*, column sides; *W*, wall panels.

Thus *B*18 means beam side number 18, and its location is shown on the key plan.

These detail sheets can be drawn up entirely with soft pencil on paper, as they are not valuable after the forms are once made. Rubber stamps for a great deal of the lettering will save time.

Where time allows and plans are complete it is advantageous to get out one kind of panel at a time. In any case the first panels to be sent to the making-up mill should be the typical floor panels. These can be used economically in building foundation walls, etc., and in this way the job can get an extra use out of them.

It is impossible to get a satisfactory cost unit for doing this form drawing. A price per square foot or per detail varies widely according to the number of details required and the number of square feet that can be made from one detail in different buildings. A big building where a large duplication of the different kinds of units is possible will cost much less per square foot for this work than a small building which is badly cut up. The average cost per sheet has been found to be about \$4.50 complete. This includes time spent on studies, assemblies, key plans, scheduling, and checking. The number of sheets required on buildings of different sizes of the same type is fairly uniform according to the size. The number varies from 30 sheets on a small building up to 100 or more on a large building with irregularities. By reducing the cost of the sheets to drafting-room hours, by dividing the probable total cost by the average rate per hour, a fairly close estimate of the number of men required to turn out a job in a specified time can be made. This is useful on rush work.

The key plan mentioned above is really a diagram of the floor plan upon which are shown the locations of the various form panels. This plan is the only one which the workmen consult in erecting the formwork. It must, therefore, be complete and clear.

This plan can best be made by tracing the floor plan on cloth from a blue-print, indicating the beams and holes in the floor and the columns and walls in the story below. The lines should be heavily inked and the figures large enough and spaced so that the tracing can be reduced and the photo reduced to convenient size for the foremen to handle on the job. A big blue-print is inconvenient and fades in the strong sunlight. As a rule, to avoid misunderstandings, a key plan should be prepared for each floor in the building.

The key plan may be supplemented by a letter to the job in which should be noted the assumptions made in preparing the details. These notes might include such information as clearances allowed, grades at which forms are to start, scheme for re-use, etc.

66. Tables and Diagrams for Designing Forms.

66*a*. Notation.—The following notation is used in the tables and diagrams:

b = breadth of member in inches.
d = depth of member in inches.
l = span of member in feet.
l'' = span of member in inches.
w = uniform load per linear foot.
w'' = uniform load per linear inch.
w' = total load on floor in pounds per square foot = dead weight of slab per square foot plus 75 lb. per sq. ft. live load.
h = head in feet.
D = deflection in inches.
E = modulus of elasticity in pounds per square inch.
I = moment of inertia in inches^4.
f = maximum fiber stress in pounds per square inch
M = bending moment in foot-pounds.
M_r = resisting moment in inch-pounds.
s = spacing in inches.
n = number of spaces.
n' = number of spaces between column yokes.
k = largest dimension of column in inches.
P = concentrated load in pounds.
V = total maximum vertical shear in pounds.
v = maximum unit horizontal shear (pounds per square inch) $= \frac{3}{2}\left(\frac{V}{bd}\right)$.

66*b*. Fiber Stresses Allowed.—The fiber stresses allowed are those given in Art. 65*a*.

66c. Formulas Used.—The spacing of joists was determined in Table I by the formulas:

For flexure

$$s = 2000\frac{bd^2}{w'l^2}$$

For deflection

$$D = 0.0225\frac{l^2}{d}$$

When $D = \frac{1}{8}$ in. $\quad l = \frac{\sqrt{800d}}{12}$

When $D = \frac{1}{4}$ in. $\quad l = \frac{\sqrt{1600d}}{12}$

When $D = \frac{l''}{360}$ $\quad l = \frac{40}{27}d.$

For horizontal shear

$$s = 3200\frac{bd}{w'l}$$

(Depths for joists are taken $\frac{1}{4}$ in. less than nominal sizes.)

In Table II, Part A:

For flexure

$$s = 2000\frac{bd^2}{w'l^2}$$

For $D = \frac{1}{8}$ in.

$$s = 11,100\frac{bd^3}{w'l^4}$$

For horizontal shear

$$s = 3200\frac{bd}{w'l}$$

In Table II, Part B:

For flexure

$$s = 1600\frac{bd^2}{w'l^2}$$

For $D = \frac{l''}{360}$

$$s = 1780\frac{bd^3}{w'l^3}$$

For horizontal shear

$$s = 3200\frac{bd}{w'l}$$

Table III:

Deflection of $\frac{1}{8}$ in. governs.

$$l'' = \sqrt[4]{\frac{1,590,000(145)d^3}{w'}}$$

In Table IV the concentrated loads from the joists were considered on a simple span in calculating the bending moment. The worst case was assumed—that is, when one joist comes at mid-span. Since a girt is usually continuous for at least two spans and the full live load never reaches it, the moment of resistance of the timbers was multiplied by 1.2. Nominal sizes of the girts were used in the computations.

For flexure

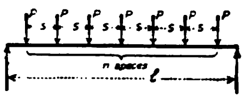
$$l = \frac{240bd^2 + \frac{s}{8}P(n^2 + 2n)}{3P(n + 1)}$$

Horizontal shear is to be considered separately. From the above considerations it would seem that an allowable horizontal shear of (200)(1.2) = 240 lb. per sq. in. may safely be used.

Diagram I:

For flexure

$$s = \sqrt{\frac{336{,}000d^2}{145h}}$$

For deflection

$$s = \sqrt[4]{\frac{1{,}590{,}000d^3}{h}}$$

Diagram II:

For flexure

$$s = 2380\frac{bd^2}{h(l'')^2}$$

For deflection ($D = \frac{1}{8}$ in.)

$$l'' = \sqrt{\frac{95{,}400d}{238}}$$

When $d = 2$ in. $l'' = 28.3$ in. When $d = 4$ in. $l'' = 40.0$ in. When $d = 6$ in. $l'' = 49.0$ in.

These results are based on a value of $k = l''$ and the values given may at least be increased to 30, 42, and 50 respectively.

For shear ($v = 200$ lb. per sq. in.)

$$l'' = 9d - (l'' - k)$$

Table V:

$$n' = \frac{(hl'')^2}{19{,}060}$$

$$h = \frac{1}{l''}\sqrt{19{,}060n'}$$

Formulas to use with table:

$$\text{Diameter of bolt} = \sqrt{\frac{0.095bd^2}{2l'' - k}}$$

$$\text{Net area of washer} = \frac{3bd^2}{2l'' - k}$$

Illustrative Problems.—1. Required the proper spacing of 2 by 8-in. joists having a span of 7.5 ft. to support forms for a 5-in. slab in beam-and-girder construction, assuming 1-in. sheathing.

Table I shows that 2 by 8-in. joists spaced 31 in. on centers will give sufficient strength if the bending moment is assumed equal to $\frac{wl^2}{10}$. For this spacing the table indicates that the deflection is somewhat over $\frac{1}{8}$ in. but less than $\frac{1}{4}$ in. and much less than $\frac{1}{360}$ span. Accurately, $D = 0.0225\frac{(7.5)^2}{7.75} = 0.16$ in. From Table III we find that the spacing cannot be greater than 30 in. without the deflection of the sheathing exceeding $\frac{1}{8}$ in., which is not advisable.

For $M = \frac{wl^2}{8}$, the spacing would be (0.8) (31) = 25 in. from Table I and the deflection $D = 0.030\frac{(7.5)^2}{7.75} = 0.22$ in. From Table II, the spacing would be $22 + \frac{1}{3}(8) = 24.7$ in.

For $M = \frac{wl^2}{10}$ and the deflection limited to $\frac{1}{8}$ in., Table II shows the proper spacing to be $21 + \frac{1}{3}(8) = 23.7$ in.

2. Assume joists in the preceding problem to be supported midway between beams. Determine their economical size and proper spacing, assuming $M = \frac{wl^2}{10}$ and deflection limited to $\frac{1}{8}$ in.

It will be sufficiently accurate to assume the span as 4 ft. From either Table I or II, we find that 2 by 4-in. joists may be employed spaced 25 in. on centers.

3. Determine size and span length of girt in the preceding problem to support the joists midway between beams.

Load coming from each joist is (3.75) ($\frac{25}{12}$) (137.5) = 1075, or say 1000 lb., accurately enough. From Table IV we find that a 3 by 4-in. girt with posts spaced 3.8 ft. c. to c. could be used, or a 3 by 6-in. girt with posts 5.6 ft. c. to c., or a 4 by 6-in. girt with posts 6.6 ft. c. to c.

Horizontal shear must be considered separately considering a joist to occur close to one support. Assuming a 4 by 6-in. girt with posts 6.6 ft. on centers, $V = 2260$ lb. and $v = \frac{3}{2}\frac{2260}{(4)(6)} = 142$ lb. per sq. in., which is less than the allowable value.

TABLE I. SPACING OF JOISTS

Based on $M = \frac{wl^2}{10}$ Deflection unlimited

For supported ends $\left(M = \frac{wl^2}{8}\right)$, deduct 20 per cent. from values given—except for the few values determined by shear which should be multiplied by $\frac{d}{2l}$.

Deflection for any spacing not governed by shear:

For $M = \frac{wl^2}{10}$, $D = 0.0225\frac{l^2}{d}$ For $M = \frac{wl^2}{8}$, $D = 0.030\frac{l^2}{d}$

Distance Center to Center of Joists in Inches

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
3 in.	2X4	31	24	20	16	14*	12*	10*						
	2X6			47	39	32	28	24	21	19	16	14*	13*	12*
	2X8						50	43	38	33	29	26	23	21
	2X10									52	47	41	37	32
	3X4	47	37	30	25	21*	18*	15*	13*	11*	10*			
	3X6					49	41	36	31	27	24	21*	19*	17*
	3X8								57	50	44	39	35	32
	3X10												56	50
	4X6						55	48	41	36	33	30	26	23
	4X8											52	47	42
	4X10													
4 in.	2X4	28	22	18	15	12*	10*							
	2X6		52	42	35	29	25	21	19	18	14	13*	11*	10*
	2X8					35	45	39	34	30	26	23	21	19
	2X10								54	47	42	37	33	30
	3X4	42	33	27	22	18*	16*	13*	12*	10*				
	3X6				52	44	37	32	28	25	22	19*	17*	16*
	3X8								51	45	40	35	32	29
	3X10											56	50	45
	4X6						50	43	37	33	30	27	23	21
	4X8										53	47	42	38
	4X10													
5 in.	2X4	25	20	16	13	11*								
	2X6		47	38	32	26	23	19	17	15	13	12*	10*	
	2X8					48	41	35	31	27	24	21	19	17
	2X10							56	49	43	38	34	30	27
	3X4	38	30	24	20	17*	14*	12*	11*					
	3X6			57	47	40	34	29	25	22	20	17*	16*	14*
	3X8							53	46	41	36	32	29	26
	3X10											51	45	41
	4X6					53	45	39	34	30	27	24	21	19
	4X8									52	48	43	38	35
	4X10													53
6 in.	2X4	23	18	15	12	10*								
	2X6	55	43	35	29	24	21	18	15	14	12	11*	10*	
	2X8				53	44	38	32	28	25	22	20	17	16
	2X10							57	45	39	35	31	28	25
	3X4	35	27	22	18	15*	13*	11*	10*					
	3X6			53	43	36	31	27	23	20	18	16*	14*	13*
	3X8						56	49	42	37	33	29	26	24
	3X10										52	47	42	38
	4X6					49	41	36	31	27	25	22	19	17
	4X8								56	50	44	39	35	32
	4X10												56	50
7 in.	2X4	21	17	14	11									
	2X6	51	40	32	27	22	19	16	14	12	11	10*		
	2X8				49	41	35	30	26	23	20	18	16	15
	2X10						55	47	41	36	32	29	26	23
	3X4	32	25	20	17	14*	12*	10*						
	3X6			48	40	34	29	25	21	19	17	15*	13*	12*
	3X8						52	45	39	34	30	27	24	22
	3X10									55	48	43	39	35
	4X6				53	45	38	33	29	25	23	21	18	16
	4X8								52	46	41	36	32	29
	4X10												51	46

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
8 in	2X4	20	16	13	10									
	2X6	47	37	30	25	21	18	15	13	12	10			
	2X8			55	45	38	32	26	24	21	19	17	15	13
	2X10						51	44	38	34	30	27	24	21
	3X4	30	23	19	16	13*	11*	10*						
	3X6		55	45	37	31	27	23	20	17	15	14*	12*	11*
	3X8						48	42	36	32	28	25	23	20
	3X10									51	45	40	36	32
	4X6				50	42	35	31	27	23	21	19	16	15
	4X8							56	48	42	38	34	30	27
	4X10											53	48	43
9 in	2X4	18	15	12	10									
	2X6	44	35	28	23	19	16	14	12	11				
	2X8			51	42	35	30	26	23	20	17	16	14	13
	2X10					55◎	46	41	36	31	28	25	22	20
	3X4	28	22	18	15	12*	10*							
	3X6		52	42	35	29	25	21	18	16	14	13*	11*	10*
	3X8					53	45	39	34	30	26	23	21	19
	3X10								54	47	42	37	33	30
	4X6			56	46	39	33	28	25	22	20	18	15	14
	4X8							52	45	40	35	31	28	25
	4X10										56	50	45	40
10 in	2X4	17	14	11										
	2X6	41	32	26	22	18	15	13	11	10				
	2X8			48	40	33	28	24	21	18	16	15	13	12
	2X10					52◎	45	39	34	29	26	23	21	19
	3X4	26	21	17	14	11*	10*							
	3X6		48	39	32	27	23	20	17	15	13	12*	11*	10*
	3X8					50	42	36	32	28	25	22	20	18
	3X10								50	44	39	35	31	28
	4X6			53	43	36	31	27	23	20	19	17	14	13
	4X8							49	42	37	33	29	26	24
	4X10										52	47	42	38
11 in	2X4	16	13	10										
	2X6	39	30	25	20	17	14	12	11					
	2X8			45	37	31	27	23	20	17	15	14	12	11
	2X10					49◎	42	36	32	28	24	22	20	18
	3X4	25	19	16	13	11*								
	3X6		46	37	31	26	22	19	16	14	13	11*	10*	
	3X8					47	40	34	30	26	23	21	18	17
	3X10							55	48	42	37	33	29	27
	4X6			49	41	34	29	25	22	19	18	16	13	12
	4X8						53	46	40	33	31	28	23	22
	4X10									56	49	44	39	35
12 in	2X4	15	12	10										
	2X6	36	29	23	19	16	14	12	10					
	2X8		49◎	42	35	29	25	22	19	16	15	13	12	10
	2X10				50◎	46◎	40	34	30	26	23	20	18	17
	3X4	23	18	15	12	10*								
	3X6	55	43	35	29	24	21	18	15	13	12	11*		
	3X8				53	44	38	32	28	25	22	19	17	16
	3X10							52	43	39	35	31	28	25
	4X6			47	39	32	28	24	21	18	17	15	13	11
	4X8						50	43	38	33	29	28	24	21
	4X10									52	46	41	37	33

To left of solid zigzag lines, deflection is less than $\frac{1}{2}$ in, using formula $\frac{5}{384}\cdot\frac{w(l)^4}{EI}$.
To right of dotted zigzag lines deflection is greater than $\frac{1}{2}$ in, using formula $\frac{5}{384}\cdot\frac{w(l)^4}{EI}$.

* indicates deflection is greater than $\frac{1}{360}$ of the span.
◎ indicates spacing is determined by horizontal shear.

TABLE II.—SPACING OF JOISTS

Deflection limited

Distance center to center of joists in inches

Part A.—Based on $M = \frac{wl^2}{10}$ with deflection limited to ½ in.

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
		Span of joists in feet												
3 in.	2X4	31	24	16	11									
	2X6			47	39	29	21	15	12					
	2X8						50	38	29	22	17	14	11	
	2X10								57	44	35	28	22	18
	3X4	47	37	25	17	12								
	3X6					43	31	23	18	14	11			
	3X8							57	43	33	26	21	17	14
	3X10										52	41	34	27
	4X6					58	42	31	23	18	14	11		
	4X8								58	45	35	28	22	18
	4X10											55	45	36
6 in.	2X4	23	18	12										
	2X6	55	43	33	29	21	15	11						
	2X8				53	44	38	28	21	17	13	10		
	2X10							51	43	33	26	21	17	13
	3X4	35	27	18	12									
	3X6			53	43	32	23	17	13	10				
	3X8						56	43	32	25	20	15	12	10
	3X10									50	39	31	25	20
	4X6					43	31	23	18	13	11			
	4X8							57	43	33	26	21	17	14
	4X10										52	41	34	27
9 in.	2X4	18	15	10										
	2X6	44	35	28	23	17	12							
	2X8			51	42	35	30	23	17	13	10			
	2X10					55	48	41	34	27	21	16	13	11
	3X4	28	22	15	10									
	3X6		52	42	35	26	19	14	10					
	3X8					53	45	34	26	20	16	12	10	
	3X10								52	40	31	25	20	16
	4X6			56	46	34	25	18	14	11				
	4X8							46	35	27	21	16	13	11
	4X10									53	42	33	27	22
12 in.	2X4	15	12											
	2X6	36	29	23	19	14	10							
	2X8		49	42	35	29	25	19	14	11				
	2X10				50	46	40	34	29	22	17	14	11	
	3X4	23	18	12										
	3X6	53	43	35	29	21	15	11						
	3X8				53	44	38	28	22	17	13	10		
	3X10							52	43	33	26	21	17	13
	4X6			47	39	29	21	15	12					
	4X8						50	38	29	22	17	14	11	
	4X10								57	44	35	28	22	18

Strength governs to left of zigzag lines.

Part B.—Based on $M = \frac{wl^2}{8}$ with deflection limited to $\frac{1}{360}$ of the span.

Slab thickness	Size of joist (inches)	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
		Span of joists in feet												
3 in	2X4	25	18	13	10									
	2X6	58	46	37	31	26	22	17	14	11				
	2X8				56	47	40	35	30	26	23	20	17	14
	2X10							55	48	42	37	33	30	27
	3X4	37	27	20	15	11								
	3X6			56	46	39	32	26	21	17	14	12	10	
	3X8							52	45	40	35	30	25	22
	3X10										56	50	44	40
	4X6					52	43	35	28	23	19	16	14	12
	4X8									53	47	40	34	29
	4X10													54
6 in	2X4	18	13	10										
	2X6	44	34	28	23	19	16	13	10					
	2X8			51	42	35	30	26	22	20	17	15	13	11
	2X10						48	41	36	31	28	25	22	20
	3X4	28	20	15	11									
	3X6		52	42	35	29	24	19	16	13	11			
	3X8					53	45	39	34	30	26	22	19	16
	3X10								54	47	42	37	33	30
	4X6			56	46	39	32	26	21	17	14	12	10	
	4X8							52	45	40	35	30	25	22
	4X10										56	50	45	40
9 in.	2X4	15	11											
	2X6	35	27	22	18	15	13	10						
	2X8		50	41	34	28	24	21	18	16	14	12	10	
	2X10				53	45	38	33	29	25	22	20	18	16
	3X4	22	16	12										
	3X6	52	41	33	28	23	19	15	13	10				
	3X8				50	42	36	31	27	24	21	18	15	13
	3X10						57	49	43	38	33	30	27	24
	4X6		55	45	37	31	26	21	17	14	11	10		
	4X8					57	48	41	36	32	28	24	20	17
	4X10								57	50	45	40	36	32
12 in.	2X4	12												
	2X6	29	23	18	15	13	11							
	2X8	53	42	34	28	23	20	17	15	13	12	10		
	2X10			54	44	37	32	27	24	21	18	16	15	13
	3X4	18	13	10										
	3X6	44	34	28	23	19	16	13	10					
	3X8			51	42	35	30	26	22	20	17	15	13	11
	3X10						48	41	36	31	28	25	22	20
	4X6	58	46	37	31	26	22	17	14	11				
	4X8				56	47	40	34	30	26	23	20	17	14
	4X10							55	48	42	37	33	30	28

Deflection governs to right of zigzag lines.

A 3 by 4-in. post could sustain (3) (4) (400) = 4800 lb. without injuring the fibers of the girt. It would only be required to support 3450 lb., consequently this size of post is suitable.

4. Assuming the cross-section of beam below slab as 14 by 18 in. (23 in. total depth) determine the safe span for the beam bottom to be made of 2-in. plank.

Live plus dead load on beam bottom is $75 + {}^{23}\!/_{12}(150) = 3625$ lb. per sq. ft. Diagram I shows the maximum span to be 43 in.

5. Find the proper spacing of 3 by 4-in. posts to support the forms for 8 by 16-in. beams (cross-section given below slab) spaced 6 ft. on centers with a 4-in. floor slab. Assume that no girt is placed at midspan of joists.

Total load on beam per linear foot is $(125)(6) + \frac{(8)(16)(150)}{144} = 883$ lb. not considering the weight of the forms, which may be neglected. Safe bearing of post on fibers of cap = (3) (4) (400) = 4800 lb. Safe spacing of posts $= \frac{4800}{883} = 5.4$ ft.

6. Determine the size and spacing of joists, girts, and posts to support an 11-in. flat slab floor.

Assuming 1-in. sheathing we find from Table III that the joists cannot be placed more than 27 in. on centers. Table I shows that for 2 by 8-in. joists the spacing of girts may be made 6.5 ft. The load coming from each joist is $(6.5)({}^{27}\!/_{12})(212.5) = 3110$ lb., or accurately enough 3000 lb. From Table IV we find that for 4 by 6-in. girts the posts may be placed 3.8 ft. on centers. Horizontal shear on girts must be considered separately. $v = {}^{3}\!/_{2}\frac{4380}{(4)(6)} = 273$ lb. which is somewhat greater than the allowable value and a 6-ft. spacing of the girts is necessary.

TABLE III.—SAFE SPAN FOR FLOOR SHEATHING

(inches)

Based on $M = \frac{wl^2}{10}$ with deflection limited to ⅛ in.

Slab thickness	Weight in pounds per square foot (live plus dead)	1" Stock	1¼" Stock	1½" Stock	2" Stock	Slab thickness	Weight in pounds per square foot (live plus dead)	1" Stock	1¼" Stock	1½" Stock	2" Stock
3 in.	112.5	32	29	46	57	8 in.	175.0	29	35	41	51
4 in.	125.0	31	38	45	56	9 in.	187.5	28	35	41	50
5 in	137.5	30	37	44	54	10 in.	200.0	28	34	40	50
6 in.	150.0	30	37	43	53	11 in.	212.5	27	33	39	49
7 in.	162.5	29	36	42	52	12 in.	225.0	27	33	39	48

7. What spacing of vertical studs is required for a wall form with 1½-in. sheathing and a height of 12 ft. Diagram I shows the spacing to be 18 in.

8. Assuming a 24 by 24-in. column with $l'' = 37$ in., determine the spacing of the column yokes.

For 2 by 4-in. yokes placed on edge, the value of l'' to be used in Table V should be

$$(37)\ (1.23)\sqrt{\frac{(2)\ (37)\ (24) - (24)^2}{37^2}} = 42.5 \text{ in., say 42 in.}$$

For 4 by 4-in. yokes, the value of l'' to be used should be $(37)(0.87)(0.935) = 30$ in.

For shear the actual value of l'' must not be taken less than $9d - (l'' - k) = (9)(4) - (37 - 24) = 23$ in. for either the 2 by 4-in. or the 4 by 4-in. yokes. Evidently the shear in the yokes will be less than the allowable.

Other things being equal, Diagram II shows that 1-in. sheathing would be more economical than 1½-in. when 2 by 4-in. yokes are used. The table shows that the same number of yokes would be used in the two cases.

The spacing center to center for the 2 by 4-in. yokes for a 10-ft. column with 1-in. sheathing would be as follows in inches starting at the top: 30–20–14–11–10–9–8–7.

Where the strength of yokes governs their spacing, the bolts must have a diameter of $\sqrt{\frac{0.095bd^2}{2l'' - k}}$ using actual values of k and l''. Thus for the 2 by 4-in. yokes, diameter of the bolts must be at least $\sqrt{\frac{(0.095)\ (2)\ (16)}{50}} = 0.25$ in. The net area of washer should be $\frac{3bd^2}{2l'' - k} = \frac{(3)\ (2)\ (16)}{50} = 1.92$ sq. in.

9. Assuming a 12 by 12-in. column with $l'' = 22$ in., determine the spacing of 2 by 4-in. yokes placed on edge.

The limiting value of l'' for shear on yoke is $(9)(4) - (10) = 26$ in. Thus actual values of l'' and k must be considered as 26 in. and $(26 - 10) = 16$ in. respectively, which gives a value of l'' to be used in Table V of

$$(26)\ (1.23)\sqrt{\frac{(2)\ (26)\ (16) - (16)^2}{(26)^2}}\sqrt{\frac{12}{16}} = 25.6 \text{ in., say 26 in.}$$

Table IV.—Span of Girts

Based on $M_r = \frac{1}{6} fbd^2$ (1.2)

Distance center to center of posts in feet*

Spacing of joists	Size of girts (inches)	500	1000	1500	2000	2500	3000	3500	4000	4500	5000	6000	7000	8000
12 in.	3X4	3.9	2.6	2.2										
	3X6	5.9	4.1	3.2	2.8	2.5	2.3	2.1						
	3X8	7.8	5.4	4.4	3.9	3.3	3.0	2.8	2.6	2.4	2.3	2.2		
	3X10	9.8	6.8	5.6	4.8	4.3	4.0	3.6	3.3	3.1	2.9	2.6	2.4	2.3
	4X4	4.4	3.0	2.4	2.2									
	4X6	6.7	4.7	3.9	3.2	2.8	2.6	2.4	2.3	2.2				
	4X8	9.0	6.3	5.1	4.4	4.0	3.6	3.3	3.0	2.8	2.7	2.4	2.3	2.2
	4X10	11.2	8.0	6.4	5.6	4.9	4.5	4.2	4.0	3.7	3.4	3.1	2.8	2.6
	6X6	8.3	5.8	4.7	4.1	3.6	3.2	2.9	2.7	2.6	2.5	2.3	2.1	
	6X8	11.0	7.8	6.3	5.4	4.8	4.4	4.1	3.9	3.6	3.3	3.0	2.8	2.6
	6X10		9.7	8.0	6.8	6.1	5.6	5.1	4.8	4.5	4.3	4.0	3.6	3.3
18 in.	3X4	4.6	3.3	2.5										
	3X6	7.1	4.9	3.9	3.4	3.1	2.9	2.4						
	3X8	9.5	6.6	5.4	4.5	4.0	3.7	3.4	3.3	3.1	3.0	2.5		
	3X10	12.0	7.2	6.8	6.0	5.2	4.6	4.3	4.0	3.8	3.6	3.3	3.1	3.0
	4X4	5.4	3.7	3.1	2.5									
	4X6	8.2	5.8	4.5	3.9	3.5	3.3	3.1	2.8	2.5				
	4X8	11.0	7.7	6.3	5.4	4.7	4.3	3.9	3.7	3.5	3.3	3.1	2.9	2.5
	4X10		9.7	7.8	6.8	6.1	5.5	5.0	4.6	4.3	4.1	3.8	3.5	3.3
	6X6	10.0	7.0	5.8	4.8	4.3	3.9	3.6	3.4	3.2	3.1	2.9	2.4	
	6X8		9.5	7.7	6.6	6.0	5.4	4.9	4.5	4.9	4.0	3.7	3.4	3.2
	6X10		12.0	9.7	8.4	7.4	6.8	6.3	6.0	5.5	5.2	4.6	4.3	4.0
21 in.	3X4	4.9	3.6	2.5										
	3X6	7.6	5.2	4.2	3.8	3.4	2.9	2.4						
	3X8	10.3	7.2	5.7	4.9	4.3	4.0	3.8	3.6	3.4	3.0	2.5		
	3X10	12.8	9.0	7.4	6.3	5.5	5.0	4.6	4.3	4.1	3.9	3.6	3.4	3.0
	4X4	5.7	4.0	3.4	2.5									
	4X6	8.8	6.1	4.9	4.2	3.8	3.6	3.3	2.8	2.5				
	4X8	11.8	8.3	6.9	5.7	5.0	4.6	4.3	4.0	3.8	3.7	3.4	2.9	2.5
	4X10		10.5	8.4	7.4	6.6	5.9	5.3	5.0	4.7	4.4	4.1	3.8	3.6
	6X6	10.9	7.6	6.7	5.2	4.6	4.2	3.9	3.7	3.6	3.4	2.9	2.4	
	6X8		10.3	8.3	7.3	6.4	5.7	5.2	4.9	4.6	4.3	4.0	3.8	3.6
	6X10		12.8	10.5	9.0	8.0	7.4	6.9	6.3	5.9	5.5	5.0	4.6	4.3
24 in.	3X4	5.2	3.8	2.5										
	3X6	8.3	5.5	4.6	4.1	3.4	2.9	2.4						
	3X8	10.9	7.7	6.0	5.2	4.7	4.3	4.1	3.8	3.4	3.0	2.5		
	3X10		9.6	8.0	6.6	5.8	5.3	4.9	4.6	4.4	4.2	4.0	3.4	3.0
	4X4	6.1	4.4	3.4	2.5									
	4X6	9.4	6.5	5.2	4.5	4.2	3.8	3.3	2.8	2.5				
	4X8	12.7	8.9	7.2	6.1	5.4	4.9	4.6	4.3	4.2	4.0	3.4	2.9	2.5
	4X10		11.2	9.0	8.0	6.9	6.2	5.7	5.3	5.0	4.8	4.4	4.2	4.0
	6X6	11.7	8.2	6.5	5.5	4.9	4.5	4.3	4.1	3.8	3.4	2.9	2.4	
	6X8		10.9	8.9	7.8	6.7	6.1	5.6	5.2	4.9	4.7	4.9	4.1	3.8
	6X10			11.2	9.6	8.6	8.0	7.2	6.6	6.2	5.8	5.3	4.9	4.6
27 in.	3X4	5.6	3.8	2.5										
	3X6	8.8	5.9	4.9	4.3	3.4	2.9	2.4						
	3X8	11.5	8.1	6.4	5.5	5.0	4.7	4.4	3.8	3.4	3.0	2.5		
	3X10		10.2	8.3	7.0	6.2	5.6	5.3	5.0	4.7	4.6	4.0	3.4	3.0
	4X4	6.4	4.7	3.4	2.5									
	4X6	10.0	6.8	5.5	4.9	4.5	3.8	3.3	2.8	2.5				
	4X8		9.5	7.5	6.4	5.7	5.3	4.9	4.7	4.5	4.1	3.4	2.9	2.5
	4X10		11.8	9.6	8.3	7.2	6.5	6.0	5.6	5.3	5.1	4.7	4.5	4.0
	6X6	12.3	8.7	6.8	5.8	5.3	4.9	4.6	4.3	3.8	3.4	2.9	2.4	
	6X8		11.5	9.5	8.1	7.1	6.4	5.9	5.5	5.2	5.0	4.7	4.4	3.8
	6X10			11.8	10.2	9.2	8.3	7.5	7.0	6.5	6.2	5.6	5.3	5.0
30 in.	3X4	5.9	3.8	2.5										
	3X6	9.1	6.2	5.2	4.3	3.4	2.9	2.4						
	3X8	12.1	8.4	6.7	5.9	5.4	5.0	4.4	3.8	3.4	3.0	2.5		
	3X10		10.8	8.6	7.3	6.5	6.0	5.6	5.3	5.1	4.8	4.0	3.4	3.0
	4X4	6.7	5.0	3.4	2.5									
	4X6	10.6	7.1	5.9	5.2	4.6	3.8	3.3	2.8	2.5				
	4X8		10.1	7.9	6.7	6.0	5.6	5.3	5.0	4.5	4.1	3.4	2.9	2.5
	4X10		12.4	10.2	8.6	7.6	6.9	6.3	6.0	5.7	5.4	5.1	4.5	4.0
	6X6	12.9	9.1	7.1	6.2	5.6	5.2	4.9	4.3	3.8	3.4	2.9	2.4	
	6X8		12.1	10.1	8.4	7.4	6.7	6.2	5.9	5.6	5.3	5.0	4.4	3.8
	6X10			12.4	10.8	9.7	8.6	7.9	7.3	6.9	6.5	6.0	5.6	5.3
36 in.	3X4	6.6	3.8	2.5										
	3X6	9.8	6.9	5.7	4.3	3.4	2.9	2.4						
	3X8		9.1	7.4	6.5	6.0	5.1	4.4	3.8	3.4	3.0	2.5		
	3X10		12.0	9.3	8.0	7.2	6.6	6.3	6.0	5.3	4.8	4.0	3.4	3.0
	4X4	7.4	5.1	3.4	2.5									
	4X6	11.7	7.8	6.2	5.7	4.6	3.8	3.3	2.8	2.5				
	4X8		10.8	8.5	7.4	6.7	6.3	5.8	5.1	4.5	4.1	3.4	2.9	2.5
	4X10			11.1	9.3	8.2	7.5	7.0	6.6	6.3	6.1	5.3	4.5	4.0
	6X6		9.7	7.8	6.9	6.3	5.7	4.9	4.3	3.8	3.4	2.9	2.4	
	6X8			10.8	9.1	8.1	7.4	6.9	6.3	6.2	6.0	5.1	4.4	3.8
	6X10				12.0	10.4	9.3	8.5	8.0	7.5	7.2	6.6	6.3	6.0
48 in.	3X4	7.7	3.8	2.5										
	3X6	11.1	8.2	5.7	4.3	3.4	2.9	2.4						
	3X8		10.4	8.7	7.6	6.1	5.1	4.4	3.8	3.4	3.0	2.5		
	3X10		12.2	10.6	9.3	8.5	8.0	6.8	6.0	5.3	4.8	4.0	3.4	3.0
	4X4	8.7	5.1	3.4	2.5									
	4X6		9.2	7.7	5.7	4.6	3.8	3.3	2.8	2.5				
	4X8		12.1	9.9	8.7	8.0	6.8	5.8	5.1	4.5	4.1	3.4	2.9	2.5
	4X10			12.4	10.6	9.6	8.9	8.3	8.0	7.1	6.4	5.3	4.5	4.0
	6X6		11.1	9.1	8.2	6.9	5.7	4.9	4.3	3.8	3.4	2.9	2.4	
	6X8			12.1	10.4	9.4	8.7	8.2	7.6	6.8	6.1	5.1	4.4	3.8
	6X10					11.7	10.6	9.9	9.3	8.9	8.5	8.0	6.8	6.0

(Load coming from each joist in pounds.)

* Horizontal shear must be considered separately using formula $v = \frac{3}{2} \cdot \frac{V}{bd}$

67. Systematizing Formwork on Buildings.[1]

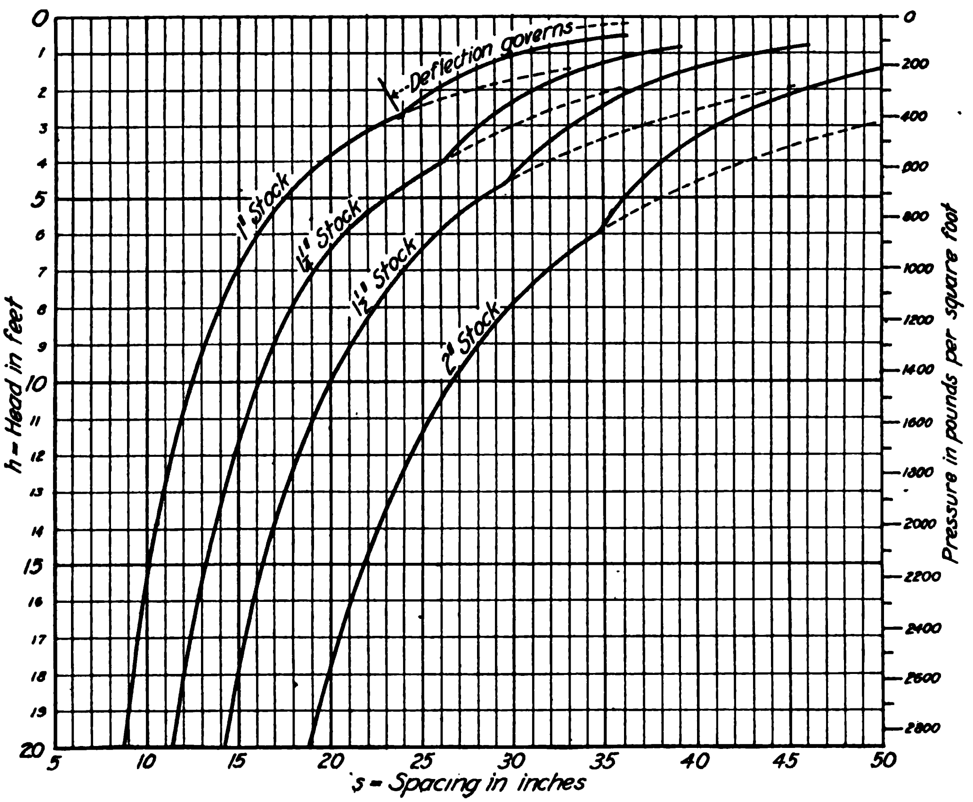
67a. Sawmill and Yard.—The sawmill should be located after a study of the job site in a position where there is plenty of space for piling the stock and finished panels. The sawmill shed and equipment should be standard so that it can be erected and put in operation as soon as possible in order to reduce hand-sawing to the minimum. A sawmill at its best is dangerous. For this reason every precaution should be taken to protect the workmen by efficient saw, machine, and belt guards. The mill should be near the building so as to reduce the cost of moving panels. This moving cost is an important item in obtaining low erection costs and will amount to a

DIAGRAM I

SPACING OF VERTICAL AND HORIZONTAL STUDS (OR COLUMN YOKES) AT ANY GIVEN DEPTH BELOW SURFACE OF CONCRETE

(Based on strength and deflection of sheathing)

$M = \frac{wl^2}{10}$ Deflection limited to ⅛ in.



considerable item when the mill is poorly located, either from lack of planning or from lack of space for the plant around the building.

The mill yard should be so arranged that the stock will go in one direction from the lumber piles to the saws, to the making-up benches and to the finished panel piles, which should be nearest the building.

The lumber when received is checked as to quality and quantity. The stock is then sorted and piled according to size and length, each pile having its size plainly marked.

It is then an easy matter for the millman to make a lumber ledger of all the stock in his yard. Thus at all times it is possible to know the stock available, for as the lumber is used deductions can be made and the running total kept up to date.

67b. Shop Procedure.—The millman divides the panel details into boards to make the required width of the panel. The number of boards of each kind needed to make the number of panels wanted are ordered moved to the saw to be cut to proper length and thence to the make-up bench, where cleats of the proper size and

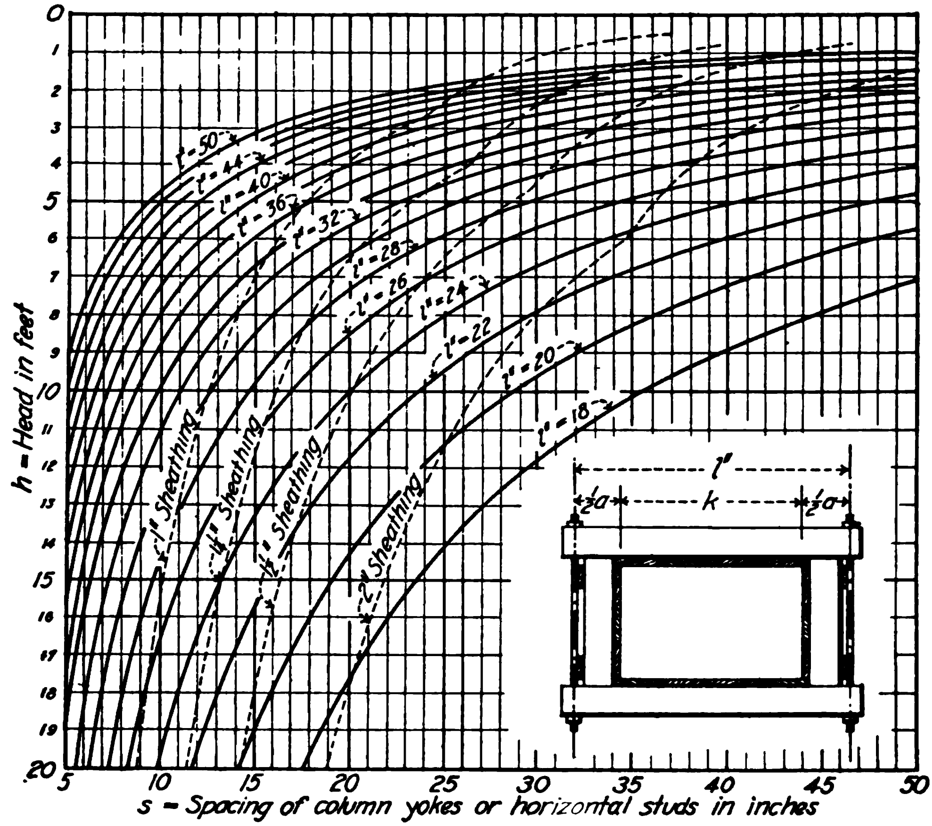
[1] By R. A. SHERWIN, Resident Engineer, Aberthaw Construction Co. From paper presented at the Twelfth Annual Convention, American Concrete Institute.

Diagram II

Spacing of Column Yokes or Horizontal Studs at Any Given Depth Below Surface of Concrete

(Based on strength and deflection of the yokes or studs)

Based on $M = \frac{wl^2}{8}$ Deflection less than 1/8 in.



Directions for Using Diagram II and Table V

For 2×4-in. yokes or studs (flat) multiply actual l'' by 1.73 before using diagram or table.
For 2×4-in. yokes or studs (on edge) multiply actual l'' by 1.23 before using diagram or table.
For 3×4-in. yokes or studs (on edge) multiply actual l'' by 1.00 before using diagram or table.
For 4×4-in. yokes or studs (on edge) multiply actual l'' by 0.87 before using diagram or table.
For 3×6-in. yokes or studs (on edge) multiply actual l'' by 0.67 before using diagram or table.
For 4×6-in. yokes or studs (on edge) multiply actual l'' by 0.58 before using diagram or table.
For 6×6-in. yokes or studs (on edge) multiply actual l'' by 0.47 before using diagram or table.

For $b \times d$-in yokes or studs (on edge) multiply actual l'' by $\sqrt{\frac{48}{bd^2}}$ before using diagram or table.

For columns the value of l'' to be used in diagram or table should be the value of l'' as found above multiplied by $\sqrt{\frac{2l''k - k^2}{(l'')^2}}$, in which expression the actual values of l'' and k are to be substituted.

In determining spacings from the diagram or table for $\left\{\begin{matrix} 3 \times 4\text{-in. and } 4 \times 4\text{-in.} \\ 3 \times 6\text{-in., } 4 \times 6\text{-in., and } 6 \times 6\text{-in.} \end{matrix}\right\}$ yokes, actual values of l'' greater than $\left\{\begin{matrix} 42.0 \\ 50.0 \end{matrix}\right\}$ in. will give a deflection of yokes greater than 1/8 in., and actual values of l'' greater than $\left\{\begin{matrix} 60.0 \\ 72.0 \end{matrix}\right\}$ in., will give a deflection greater than 1/4 in.

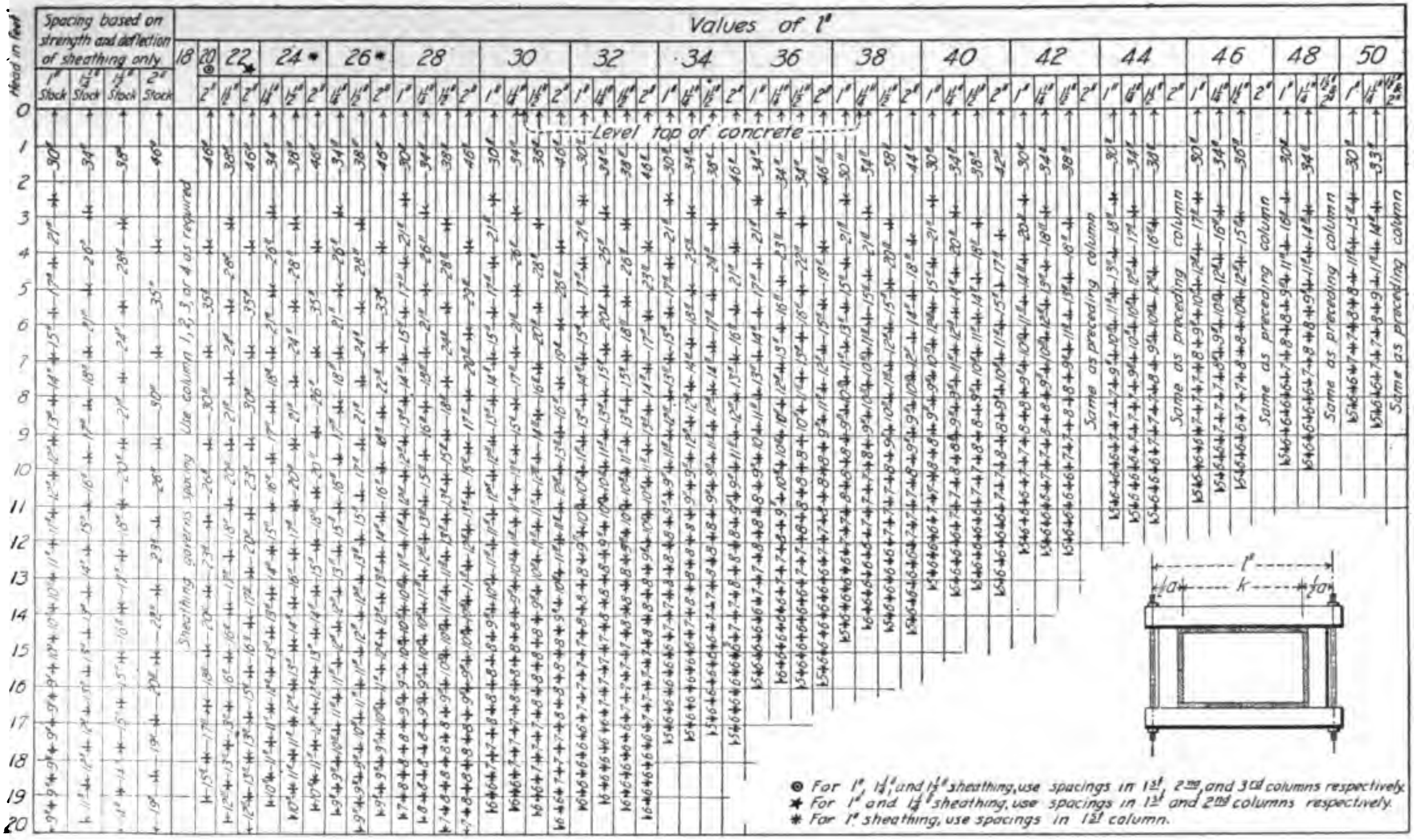
In determining spacings from the diagram or table, *actual* values of l'' must not be considered as less than determined by the formula

$$l'' = 9d\text{-}(l'' - k)$$

otherwise horizontal shear will be greater than 200 lb. per sq. in. The corresponding *actual* value of k (which will be called k') should be determined by subtracting the value of a (see sketch) from the value of l'' found by the above formula. The value of l'' to use in diagram or table should then be found as explained above and finally multiplied

TABLE V.—SPACING OF COLUMN YOKES OR HORIZONTAL STUDS

Based on $M = \frac{wl^2}{8}$ Deflection less than 1/8 in.



length are already waiting, having been previously ordered through the mill. Another order to the bench carpenters, which is clipped to the blue-print sketch of the panel, enables them to cleat together the boards into the finished panel. The panel is then taken away by a laborer, stenciled with its location mark, oiled and piled until ready for use in the building.

All this is done by orders written on standard order forms of three kinds, one for moving the stock, another for sawing, and the last for making up. Duplicates of the orders are placed on the millman's progress board so that he knows at all times how the work is progressing, for when an order is completed and returned to him its duplicate is taken down. Each kind of order is of different color so as to be easily identified.

There are several points in connection with making up panels which may be considered here.

As much assembling as possible should be done at the bench. Rangers for wall beams can be attached and ledgers to support joists can be nailed to the interior beam side cleats at the proper depth. When inserts need to be placed in the sides of beams, the holes for the bolts which hold them can be 'ocated on the details and the holes bored at the mill. The groove strip for steel sash and also the corner fillet can be put on the column sides and beam bottoms. Bevel key strips for walls should also be nailed lightly to the panels when necessary.

Heavy cleats for big wall columns should be cut and bored, but not attached to the panels. The panel can be cleated with 1¼-in. boards, and is thus made much lighter and easier to handle in erecting. Cleanouts at the bottom, on two opposite sides of all columns, should be made at the mill. Reduction strips should be nailed at the edge of all panels which reduce in size when reused.

All small pieces liabl· to get lost or used for other purposes can be dipped in red paint so that their small size will be respected by the carpenters when looking for loose boards.

Much waste in making up ⅞-in. panels can be prevented if the floor is laid out to make the majority of the panels a whole number of boards wide. Roofers come 5⅜ to 5½ in. wide, and it is an easy matter to plan the panels to come, say, seven or eight boards wide, which means no waste in ripping one board to make the width. The lengths should be planned as near stock lengths as possible Panels made of spliced boards are expensive to make and are easily broken.

New or Used Material?—It is not economical in labor to make up panels from lumber which has already been in contact with concrete several times. If, however, panels are in good condition, they can be cleaned and repaired at a considerable saving, both in labor and material.

67c. Cleat Spacing.—The cleat spacing for the various kinds of panels should be kept uniform so that the strips on the benches, for spacing the cleats, w ll not need to be moved for every set of panels.

For beam sides 2 by 3-in. cleats, flat, can be used on panels up to 30 in. deep; 2 by 4-in., flat, up to 42 in.; and 3 by 4-in., on edge, above 42 in. deep. Nails should be specified as follows:

No.	Size	Width of board, inches
3...........	10d	6¾ and 7¾
2...........	10d	2¾ to 5¾
1...........	10d	Less than 2¾
	12d	In 3 × 4 cleats

8d coated and clinch ends in ⅞-in. boards.

Wire nails should be used for formwork because of ease in driving and drawing. The holding power is also sufficient. Double-headed nails should be used in securing all boards which have to be loosened before stripping.

67d. Planning of Field Work.—An important feature of the field work is a planning department, whose business it is to plan the work in advance so that at all times the several gangs will have definite tasks to do and be supplied with sufficient material with which to do the work.

The moving boss has an important position in this field work. He is responsible for getting the proper panels to their correct location in the building and for having all other stock called for on the assembly plan on the job ahead of the erection carpenters. When the forms are stripped he must move the panels, which are to be remade before their next use, to the remaking benches, and thence to their next location. It is expensive to have high-priced carpenters waiting for stock or using stuff not suited for the job they have to do.

The erection of the centering and the assembling of the panels, after the latter have been moved to their proper location in the building, are done by carpenters. It is economical to employ good carpenters and to keep them employed constantly. A gang of men that understand formwork on concrete buildings, the methods used, and the grade of work desired, is a valuable asset for any firm specialising in reinforced-concrete work.

A competent foreman should be in charge of all carpenter labor. He should be consulted by the planning department so that "team work" will prevail.

Quality in concrete building work is usually more desirable than low costs. Good lines and surfaces will be remembered after the cost is forgotten. These results can be obtained only by the careful supervision of the erection of forms.

67e. Stripping of Forms.—When forms are to be reused, as is usually the case, stripping should be done as carefully as possible. An intelligent foreman in charge of a stripping gang is a good investment. Any man can wreck forms cheaply, but a man who can strip the forms and leave them in good condition for reuse is in the end the cheapest stripper. He saves the time of the high-priced carpenters in remaking and fitting broken panels

The question of when to strip is one for the building designer to decide, and his instructions should be carefully followed. It is cheap insurance to make enough forms so that no chances need be taken of weakening the structure by stripping while the concrete is still green.

The panels which need to be altered before reuse should be remade from the blue-prints showing the necessary changes. A bench for this purpose can be set up in the story where the forms are stripped. It is sometimes advisable in a high building to set up power saws in one of the upper stories for cutting off panels and getting out stock for remaking and repairing.

A large gang of carpenters working overtime is sometimes necessary in order to deliver a building on time. This is expensive, however. The workmen cannot give back in efficient labor the value of their high wages. Ten hours is about all the average man can work without falling off greatly in efficiency. Night work on forms should be avoided if possible.

When speed is not the essence of the contract, there is time to plan the erection work more carefully. Smaller gangs can be used and their tasks and the material for them routed by slips similar to those used in making up panels.

An important item in the field work is the erection of economical safe stages. The stages should be carefully designed as to the worst possible load to come upon them, and the design should be strictly followed in the field. A man thoroughly familiar with lumber should be made inspector as to the quality of the stock used for stages. A good stage makes the workmen more efficient and is a good insurance investment.

68. Steel Forms.—Steel forms have always been used more or less for sewers, curbs, and sidewalks, but now the application of steel forms to floor and column construction is advancing

Fig. 71.—Blaw light wall forms on foundation wall for plant of Hubbard & Co., Pittsburgh, Pa.

at an exceedingly rapid rate. Steel forms are almost universally employed for circular columns and for flaring column heads. Wall forms are also much used and have given satisfaction, especially in residences and other structures of the foundation-wall type.

Several types of floor forms are on the market and are used to some extent. The same is true of forms for rectangular and octagonal columns.

The adjustment in height of steel forms for circular columns is obtained by telescoping the ends. Such forms are usually made up of a series of panels of thin galvanized steel held rigidly in place, like staves in a barrel, by means of stiff steel bands. The panels are somewhat flexible and are sprung in or out depending upon the size of the column.

FIG. 72.—Steel floretyles.

Fig. 53, page 109, and Plate V, page 116, show steel forms for circular columns, in place, and ready for the pouring of the concrete. The construction around the column heads should be noted. The form illustrated in Fig. 53 is manufactured by the Blaw Steel Construction Co. and is so designed that all variations are taken care of with a single set of forms. Vertical ad-

FIG. 73.—Steel floretyles with end caps.

justment is obtained by telescoping one form section inside of the section below it, an adjustment of 18 in. being permitted between each two sections. Diameter adjustment is provided for by the use of form panels of various standard widths. The edges of the panel sheets are bent back to form flanges, and these flanges are slotted. The panels are kept to any desired

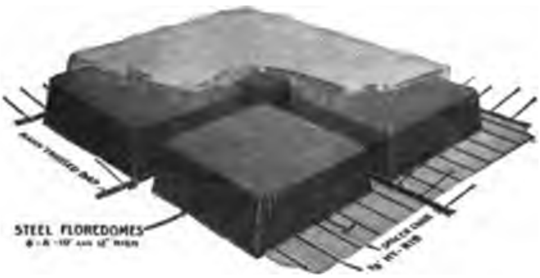


FIG. 74.—Steel floredomes.

curvature by segmental steel bands which slip through the slots in the panel flanges, and are drawn tight by means of keys or wedges. The bands are not adjustable, and a complete set must be provided for each diameter of column to be built. The form panels are made in three standard lengths, 6 ft., 4½ ft., and 3 ft. The steel column forms are not used to support any part of the floor, as is usually the case in wood-form construction, so that the floor forms may be erected complete before the column molds are set up.

The "Hydraulic" column forms shown in Fig. 55, page 110 (manufactured by the Hydraulic Pressed Steel Co., Cleveland, Ohio) consist of galvanized-iron units held together at the

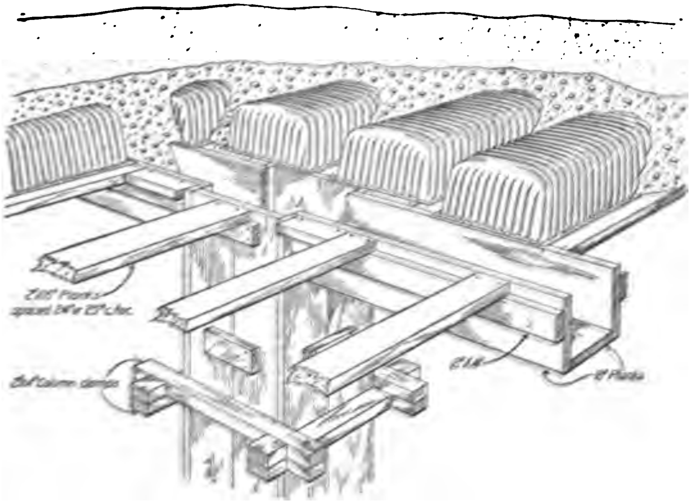


Fig. 75.—G. F. steel-tile.

joints with quick-acting clamps and shaped with steel ring. Any height of column or size of head may be obtained.

Fig. 71 shows Blaw light wall forms for foundation-wall work where wire ties are used. The standard wall-form panel is 2 ft. square and provided with holes in the four flanging angles for the passage of fasteners. Slots are also provided in the plate to permit of the insertion of the wire ties. The horizontal and vertical liners are used to keep the form straight and to connect panels together so that they may be shifted in larger units than single panels. This type of form is not limited to two-course working, but may be used in pouring any desired height of wall at one operation.

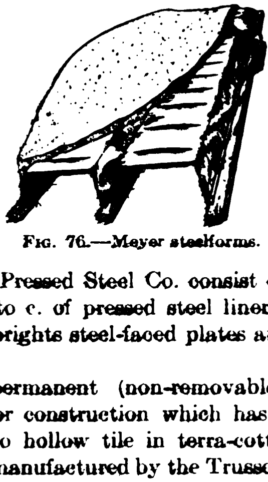


Fig. 76.—Meyer steelforms.

Wall and foundation forms manufactured by the Hydraulic Pressed Steel Co. consist of uprights which are aligned and accurately spaced 3 ft. 3 in. c. to c. of pressed steel liners. Between these uprights steel-faced plates are clamped.

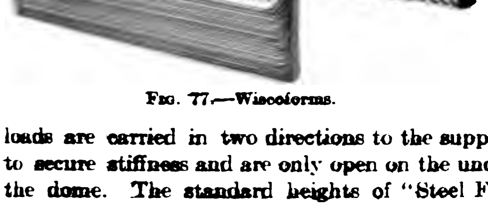


Fig. 77.—Wiscoforms.

A type of permanent (non-removable) steel form for floor construction which has a similar function to hollow tile in terra-cotta hollow tile floors, manufactured by the Trussed Concrete Steel Co., is shown in Figs. 72 and 73. This type of floor form is also shown in Fig. 35, page 59. "Steel Floredomes" shown in Fig. 74 are manufactured by the same company for two-way construction in which the loads are carried in two directions to the supports. The metal domes are deeply corrugated to secure stiffness and are only open on the underside so that the joists extend on all sides of the dome. The standard heights of "Steel Floretyles:" 6, 8, 10, 12, and 14 ir mate width at base: 20½ in., exclusive of 1-in. flanges along bottom edges, which add 2 in. to this dimension. Standard lengths (nominal) of all sizes: 4 ft. and 3 ft.—actual lengths are 4 ft. 1 in. and 3 ft. 1 in. to provide for end lap. "Steel Floredomes" may be obtained in depths of 6, 8, 10, and 12 in. and 21 by 21 in. base. Permanent steel floor forms similar to "Steel Floredomes" are furnished by the General Fireproofing Co., Youngstown, Ohio (known as G.F. Steel-Tile, Fig. 75). Steel floor forms which are removable and may be re-used in successive floors of a building are furnished by the Concrete Engineering Co., Omaha, Neb. (known as Meyer Steelforms, Fig. 76); and by the Witherow Steel Co., Pittsburg, Pa. (known as Wisco-forms, Fig. 77).

69. Construction Notes.—Nails should be used sparingly in the construction of forms except in those sections which are to be used over and over again without change. Unnecessary nailing not only adds to the labor of wrecking but is liable to render the lumber unfit for continued use. Where nails must be used in the connection of form sections, the heads should be left protruding so that they may be drawn without injury to the lumber. A special form of double-headed nail is now on the market and gives satisfaction.

The location of column forms from floor to floor of a building should be determined by means of the transit, and special care should be given to the erection of these forms in order to make sure that they are set true to line and level. Column forms should be held in position by diagonal braces in two directions nailed to adjustable or slotted blocks which are bolted to the concrete slab—the bolts being placed when the floor is poured. Sometimes small pieces of plank are employed instead of blocks, and these are nailed directly to the floor slab within 2 or 3 days after pouring and while the concrete is still green.

Trouble in the erection of floor forms may usually be traced to inaccuracies in form measurements. If the column forms are of the proper widths and if the beam and girder forms are cut to exact lengths, no trouble of great consequence can arise. A variation of more than ¼ in. from the sizes shown on the drawings should not be permitted.

One method of erecting forms for rectangular columns is to nail three of the sides together lightly before raising them to place, and then to set the remaining side afterward. This enables the column reinforcement to be put in place before the form is set. Another method in common use is to assemble the column form complete before raising, in which case the form must be raised above the projecting reinforcement (belonging to the footing or column below) and then lowered.

All forms for concrete require a coating of some lubricant to prevent the concrete from adhering to the wood and making a rough, unpleasing appearance. Crude oil or petroline is used to a considerable extent and preserves the forms against damage by alternate wetting and drying. The forms should preferably be oiled before they are set in place.

Oil should not be used on forms against surfaces which are to be plastered, as oil prevents the adhesion of the plaster. Wetting with water in such cases will be sufficient.

Beam and girder forms should be raised slightly higher at the center than at the ends in order to prevent sagging. If this is done, deflection and compression of the supports will finally leave the beams and girders in a level position. A deflection equal to ⅜ in. in every 10 ft. of length is usually provided for.

The sides of beam and girder forms should project over the edges of the bottom plank. By so doing it becomes possible to leave the beam and girder bottoms in place after the sides have been dropped.

Slab forms of the ordinary panel type should be made in sections (usually four to a floor panel) in order to prevent binding and permit easy removal. A splice of ¾ in. is usually allowed between adjacent sections, and this space is covered with a strip of sheet metal, thus giving some leeway in fitting the sheathing panels into place without unnecessary cutting. Sometimes it is a good plan to provide for a loose board between two panels near the center of the span so that temporary uprights may be used to support the floor slab when the forms are stripped.

All posts or shores should rest on large hardwood wedges, driven in pairs to an even bearing. Hard driving of wedges should not be permitted as it is sure to injure the concrete which is setting under them. In some instances wedges have been placed at the top of the posts instead of at the bottom, as is the usual custom. The disadvantage of this method lies in the difficulty of driving wedges while standing on a temporary support but, on the other hand, by top-wedging, the shores or posts may be permanently braced before the formwork is leveled.

Concrete, when poured under horizontal or inclined forms (such as in footings), will exert an upward pressure, and such forms should be securely anchored.

When posts are placed on plank sills, great care should be exercised to avoid settlement because of the likelihood of hollows coming under the sills due to unevenness of concrete floors or to thawing out of frozen ground.

Deflection should be carefully guarded against at a window head as a slight deflection at this point will cause considerable trouble and expense in setting the window sash.

Forms which are to be used again should be cleaned as soon as they are taken down.

In removing forms the green concrete must not be disturbed by prying against it.

BENDING AND PLACING REINFORCEMENT

70. Checking, Assorting, and Storing Steel.—Steel should be checked, assorted, and stored as soon as it is delivered at the site. It should be blocked up several inches from the ground and should be stored in such a manner that those rods needed first may be easily reached.

71. Bending of Reinforcement.—Such a simple matter as bending of rods for concrete reinforcement might seem to be almost too unimportant a subject to be worthy of very much attention. However, when it comes to the proposition of making thousands of bends per day, factors of time and expense in this work are most important. Of course, the structural design should be such as to require the steel bends to be as few in number and kind as possible, but much can be done to lessen expense by the manner of making these bends. In addition to economical considerations, care should be taken to see that the bends are made true to line and plane, and that the steel is not injured during the operation.

71*a*. Types of Bends.—In general, there are five different types of bends to be made with reinforcing rods which may be indicated as follows: (1) bending of heavy beam and girder rods; (2) bending of the vertical reinforcing rods of columns at or near floor level where columns change size; (3) bending of stirrups and column hoops; (4) bending of slab reinforcement, and (5) the coiling of rods or wire to form spiral column reinforcement. In making each type of bend, the work should always be so arranged that all rods of the same size and shape are bent at the same time. This avoids remeasuring and resetting of templets.

71*b*. Hand Devices.—A simple method of bending heavy rods is shown in Fig. 78. Either steel bars or steel plugs 5 or 6 in. long are placed in holes in the bending table, these holes being bored at the points where the bends are to be made. A piece of plank is nailed to the table, as shown, in order to hold the rod in place while being bent. The rod is then placed in the position *GH* and bent around the plugs *C* and *D*, and then around the plugs *B* and *E* until the ends *AB* and *EF* are parallel to *GH*.

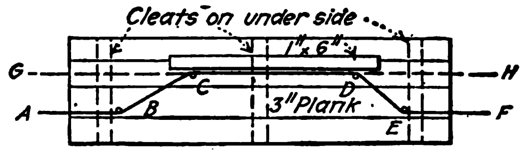


Fig. 78.

The manner in which rods are to be bent is generally left to the discretion of the steel foreman. The general practice is to bend beam and girder reinforcement by using a heavy pipe slipped on the rod and then making the bends as described, using either steel plugs or angles bolted to the table.

In building two large reinforced-concrete buildings for the General Electric Co. at Schenectady, N. Y., the Stone & Webster Engineering Corporation, of Boston, accomplished the bending of the heavy rods by means of two $\frac{3}{8}$-in. steel plates mounted one on top of the other on a bending table, and, in these, holes were drilled 3-in. on centers in both directions. Steel pegs were dropped in the holes the proper distance and angle apart, and the rods were then bent by hand using a 2-in. steam pipe.

R. C. Hardman, in *Engineering Record*, describes as follows a small home-made bar bender for bending cold bars up to $1\frac{1}{4}$-in. diameter, where not greater than 90-degree bends are required:

The apparatus consists essentially of a cast-iron plate containing two lugs between which the bar is placed, a steel lever fastened to the plate by means of a steel pin about which it acts, and a set of fillers, as shown in Fig. 79.

The cast-iron plate can be cast of coarse metal in any foundry, the top of the plate with which the lever comes into contact being machined. The bolt holes, certain ones of which must be countersunk to allow free action of the lever, may be either cored or drilled. The space between the two lugs should be slightly larger than the largest size bar to be bent. The filler is a device made of strap iron by any blacksmith, which fits around the lug opposite the lever, to insure a tight fit for bars smaller than the maximum. A set of these to accommodate the various commercial sizes of steel bars can be made at small cost. To insure a good fit the edges of the lug around which the "filler" is placed should be machined. The lever is made of 1 by 2-in. flat steel forged to shape, with the face engaging the bar slightly upset on the upper side. Its length is about 4 ft.

The operation of the apparatus can be readily seen in Fig. 79, in which the lever, bar to be bent and a filler are shown dotted. The apparatus is fastened to a bench by means of bolts, and countersunk so that the top of the plate is flush with the top of the bench.

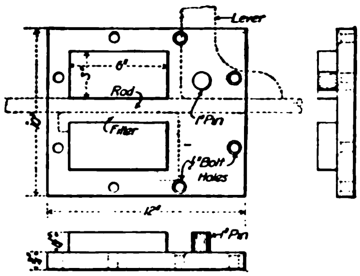


Fig. 79.—Small home-made bar bender.

Two men can make cold bends under 1 in. in diameter. On larger sizes three men are required unless recourse is had to a pipe extension to the lever.

The cost of the apparatus should not exceed $12 to $15, and it will readily pay for itself on a small job which will not admit of a more versatile bender.

Several hand and power devices are on the market for the bending of steel reinforcement and for the making of spiral coils. These all have their merits and have given satisfaction.

Fig. 80 shows the Universal bar bender which may be fastened to any bench or plank. It is a light, portable device weighing about 60 lb. and capable of bending all ordinary sizes of reinforcing bars to any angle desired, without any adjustment being necessary. The top half

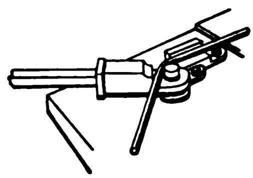


Fig. 80.—Universal bar bender.

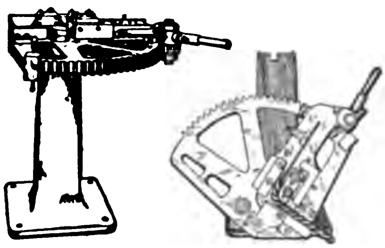


Fig. 81.—Wallace bar bender.

of the bender can be removed and used to bend bars after they are in place. The bar rolls around the pin in bending, thus distributing the strain along the bar and reducing the chances of fracture at the bend.

The bender is equipped with a 5-ft. crowbar for a handle, which may be removed and used for other purposes. To bend large bars easily, the handle should be lengthened by using an iron pipe over the crowbar.

A bar bender designed for heavy work and manufactured by the Wallace Supplies Mfg. Co., Chicago, Ill., is shown in Fig. 81. This machine has an auxiliary ratchet lever which operates a pinion against a series of teeth in the frame at a large ratio, thus developing great power. The

ratchet panel may be thrown out of engagement and machine operated with the regular lever for light work.

A bender manufactured by the Waterloo Construction Co., Waterloo, Iowa, is shown in Fig. 82. This bender bends reinforcing bars up to and including 1¼ in. The machine is furnished with a detachable handle 7 ft. long for convenience in handling.

71c. Power-operated Benders. Fig. 83 shows a power-operated truck-mounted bar bender designed to bend any size of reinforcing rod that is likely to be used in building operations.

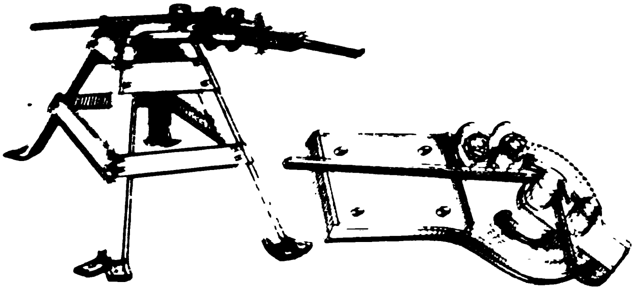


Fig. 82.—Waterloo bar bender.

tions. Any size of bar from [illegible] in. round, square or deformed, can be bent to any angle desired, or spirals formed from [illegible]-in. diameter to any required size. Weight, complete, ready for shipment, 2700 lb. The machine is manufactured by Kardong Bros., Minneapolis, Minn.

71d. Care to be Exercised in Bending. Bending reinforcement should be done in such a manner that the bars will not break or crack at the bends; that is, the bends should not be too sharp and the bending force should be applied gradually and not with a jerk. Reinforcing bars should also be bent cold except for the unusual sizes of [illegible] in. and upward, when heat

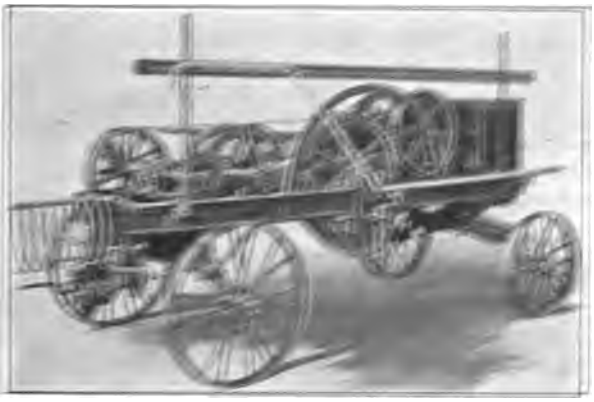


Fig. 83.—Power-operated bar bender.

may be required. [illegible]

Structural grade bars will [illegible] with less danger of injury than either the hard grade [illegible] reinforcement will usually require heating of the larger sizes before bending can be safely attempted. Square cold-twisted bars frequently give trouble in bending, due to the fact that they are apt to twist out of their proper plane.

71e. Bending of Slab Reinforcement.—Slab reinforcement is usually bent after it is in place on the floor. A tool for this purpose is shown in Fig. 84. Sometimes, however, slab rods are bent before being placed. This latter method seems preferable since the bends can be more accurately made. To keep the cost of such bending as low as possible, the bending machine should be constructed so that the bends may be made with great rapidity. Such a machine is shown in Fig. 85 and was employed on the General Electric Co.'s buildings referred to above.

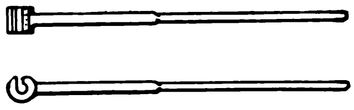


Fig. 84.

This machine consists of two units which can be placed any distance apart and securely bolted down to a table. Each part consists of a steel plate on which is mounted two smaller plates having angle stops attached. Between the stops is located a casting, which is pivoted so

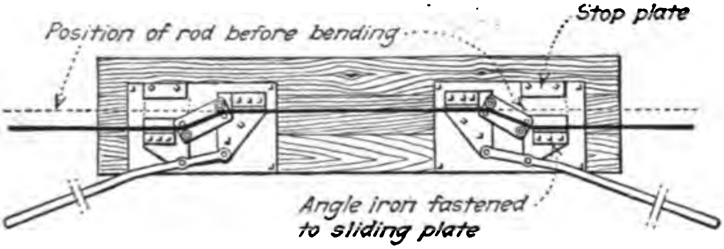


Fig. 85.

that it can readily take any desired position between the angle stops. With the steel rod in position, this casting can be turned by means of a long lever attached so as to move one of the mounted plates. On the upper side of the casting are four lugs, and two of these lugs are constructed so that adjustable collars can be slipped over them, thus making a tight fit for the rods which are to be bent. The amount and the angle of the offset can be regulated by changing the distance through which the lever is turned. This machine offsets a rod parallel to itself and with one pull of the lever two bends can be made.

72. Placing of Reinforcement.—Steel should be thoroughly cleaned before being placed in the forms in order to obtain a positive adhesion of the concrete to the steel. A slight film of red rust is not objectionable, but no rod should be set in place on which rust scales have formed (see Art. 54, Sect. 1).

All reinforcing metal should be securely fastened in correct positions by wiring or otherwise before the placing of the concrete is begun. Particular attention should be given to loose-bar reinforcement—that it is accurately and properly supported in position and that it is not disturbed until the concrete is poured.

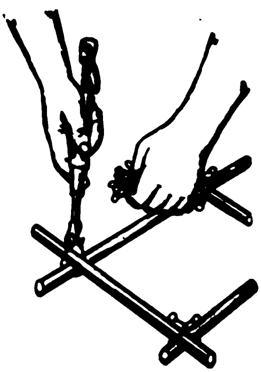


Fig. 86.—Wiring of reinforcement using Curry Tyer.

The advantages to be derived from placing beam-and-girder reinforcement in frames has been considered in Art. 11, Sect. 11. With steel in frames the erector has simply to line and level them in the forms, place braces where necessary, and make end connections with abutting frames. Column reinforcement should be made up into frames, the same as for beams and girders.

Slab and wall rods should be tied with wire at their intersections to prevent them from slipping or getting out of place. The usual method has been to use a pair of pliers and to cut the wire into convenient lengths. A device known as the Curry Tyer has recently been placed on the market for wire tying which has made a great record for itself in a very short time. Fig. 86 shows this to be a simple and practical device. The ties used for the binding are uniform in length and the mechanical action of the tying tool gives the wire a uniform number of twists. The tie is looped around the rods and the ends are placed on the hooks of the tying tool, then a quick upward jerk of the wooden handle whirls the teeth and draws the wire up tightly.

Another method of tying slab and wall rods together at their intersections is shown in Fig. 87. These *Bar-tys* are manufactured by the Concrete Steel Co., in the three types illustrated.

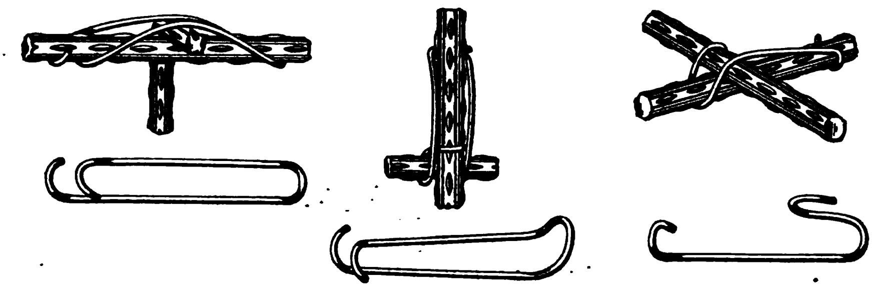
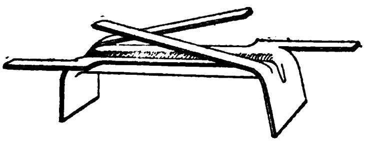


Fig. 87.—Bar-tys.

The tys are quickly put in place and when once snapped on the bar will resist a tremendous pressure.

73. Devices for Supporting Reinforcing Bars.—There are many excellent devices on the market for supporting reinforcing bars at the proper distance from the forms. Some of these devices not only support the rods but give them the proper spacing and lock them in position.

Beam spacers sold by the Universal Form Clamp Co., of Chicago, Ill., are shown in Fig. 88. They are made in three sizes—namely: 4 in., 5 in., and 6 in.—and space the bars accurately both from the sides of form and center to center of bars. *Easy chairs* sold by the same company are adaptable to either a one-way or a two-way system of slab reinforcement. Fig. 89 shows the chair before being placed in position. The upright tying finger enables the chair to be readily seized and put in position. Fig. 90 shows the



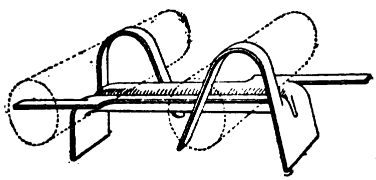


Fig. 88.—Beam spacers.

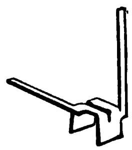


Fig. 89.—Easy chair.

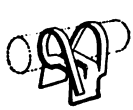


Fig. 90.

chair applied to a one-way system of reinforcement. After the chair has been placed in position the flexible tying fingers are bent over the bar from opposite sides by hand, firmly securing the chair to the bar. This chair can readily be applied to a two-way system of reinforcement. *Easy chairs* are made in three sizes so as to provide for reinforcing bars varying in diameter from $\frac{3}{8}$ to 1 in.

A device for supporting slab rods, known as the *Securo locking spacer*, is a light bar passing beneath the rods and having depending lugs which rest on the bottom of the form and so keep the rods at the desired height. At the location of each rod the bar has two flexible clips which

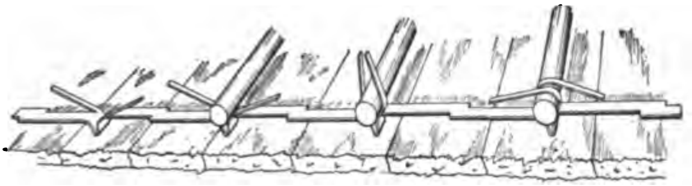


Fig. 91.—Securo slab bar spacer.

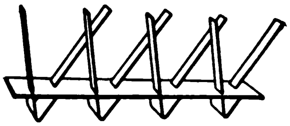


Fig. 92.—Securo beam spacer.

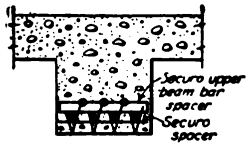


Fig. 93.

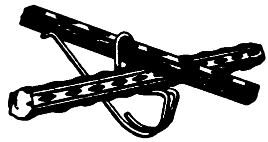


Fig. 94.—Ty-chair.

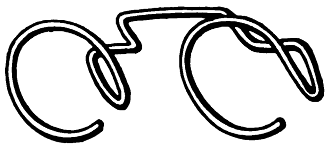


Fig. 95.—Easel-chair.

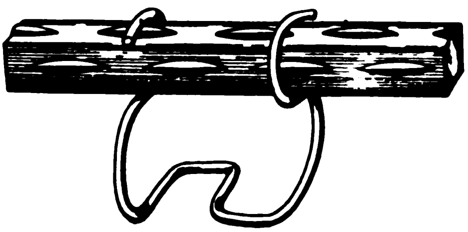


Fig. 96.—Bar-chair.

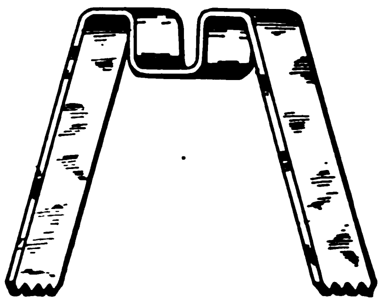


Fig. 97.—Hy-chair.

are bent around it, and these serve to insure the use of the proper number of rods. Three strips of *Securo slab bar spacers* are used per panel for ordinary spans. Two additional upper spacers are used where slab bars run in two directions. Upper spacers are also used in flat-slab construction. The device is shown in Fig. 91 and is made by the Metal Building Materials Co. of Chicago, Ill.

Securo supporting and locking spacer for beam bars is shown in Fig. 92. Three spacers are used per beam for ordinary spans. Upper beam spacers are employed when beam rods

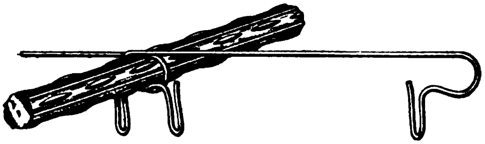


Fig. 98.—Chair spacer.

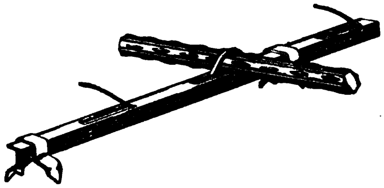


Fig. 99.—Continuous slab spacer.

are to be placed in two or more layers (Fig. 93). *Securo beam spacers* are furnished for any widths of beams and for any number of bars in lower and upper layers.

Supporting and spacing devices manufactured by the Concrete Steel Co. are shown in Figs. 94 to 100 inclusive. *Ty-chairs,* shown in Fig. 94, are made of spring steel wire for tying any combination of reinforcing bars. These chairs are made in the following standard sizes:

No. 2	No. 3	No. 4
Combinations of cross bars, inches	Combinations of cross bars, inches	Combinations of cross bars, inches
1/4 and 3/8	1/2 and 1/2	3/4 and 3/4
1/4 and 1/2	1/2 and 5/8	3/4 and 7/8
1/4 and 5/8	1/2 and 3/4	3/4 and 1
3/8 and 3/8	5/8 and 5/8	3/4 and 1 1/8
3/8 and 1/2	5/8 and 3/4	7/8 and 7/8
		7/8 and 1

The *Easel-chairs* shown in Fig. 95 are designed particularly for terra-cotta or steel tile and joist construction. Single chairs supporting one bar are made 2 in. wide, and double chairs supporting two bars are 4 in. wide and will fit any size bar. The standard Easel-chairs space the underside of the bar 1 in. from the form.

Bar-chairs (Fig. 96) are used for supporting single bars. They are easily sprung into position, locking on the bar with a strong tension grip. Bar-chairs are made for each size and shape of reinforcing bars. Standard distance from underside of bar to forms is 1 in.

Hy-chairs are illustrated in Fig. 97. They are made from 2 by 1/8-in. flat steel with any required height, and are used for supporting single reinforcing bars of any type or size. They are particularly useful on flat-slab buildings for rigidly holding the column bars.

Chair spacers and *continuous slab spacers* are shown in Figs. 98 and 99 respectively. The "chair spacers" are made of spring steel wire. The "slab spacers" are made from 1/2-in. cold-rolled angle and the prongs are so flexible that they can be readily bent around the bars by hand. In both types of spacers standard distance from underside of bar to form is 1 in.

Adjustable beam saddles (Fig. 100) are made from sheet steel and used in beams and girders for accurately spacing the bars and holding them the required distance from the forms. Standard distance from underside of bar to forms is 1½ in. and 2 in.

The *chair lock* shown in Fig. 101 is manufactured by the Electric Welding Co., Pittsburg, Pa. Stock sizes are as follows: ⅜-in. round cross rod by ⅜, ½, ⅝, ¾, and ⅞-in. round main rods. Any size, however, can be furnished to fit any shape of rod. *Chair pinchers* are furnished in two sizes for fastening the locks to cross rods.

Staple chairs shown in Fig. 102 are made from extra stiff sheet steel, cut and bent to develop two pairs of pointed prongs projecting in opposite chairs. The chairs are driven into the formwork as far as is desired and in the exact position the steel is to occupy. The bars are placed and the upper prongs bent down over the bar with a quick blow of the hammer. The driven-in points do not seem to make form removal difficult.

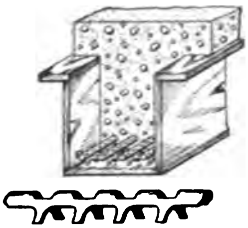


Fig. 100.—Adjustable beam saddle.

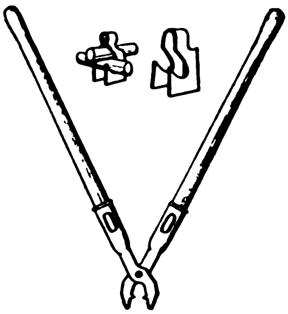


Fig. 101.—Chair lock

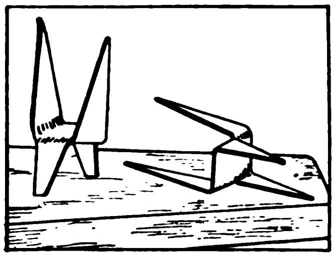


Fig. 102.—Staple chairs.

THE MANUFACTURE AND USE OF CONCRETE STONE, BLOCK AND BRICK

By Harvey Whipple[1]

74. Development of the Industry.—Although the development of the concrete products industry, embracing, first of all, the manufacture of building units, began earlier than reinforced concrete building construction, it has lagged behind it. This has been due, in general, to a lack of trained management. And the trained management has been lacking, perhaps, because of the apparent simplicity of operations involved in the manufacture of concrete building units. In the early days, the use of cheap machines of questionable value was common and many of them were sold on such a basis as to tempt the more slovenly artisans. Men who had made failures of other work sought easy money by making block beside a gravel bank and following the inadequate directions which came with the machine.

Attracted by the seeming lack of any complexity in the essential operations of block manufacture, by the cheapness of the machines with which the work was to be done, and by the seeming lack of any necessity for any auxiliary equipment, it is not to be wondered at that there were many enterprises in concrete products manufacture of woodshed and backyard magnitude. The result was that many cheap buildings put up with this new product soon developed the weaknesses which brought down upon an entire industry the condemnation which was earned by its incompetent putterers.

Many of these early products were not even sound structurally. Many of them were extremely porous. The appearance of a concrete block wall after a rain and the slowness with which the moisture disappeared in subsequent sunshine, gave rise to the early belief that concrete as a building material makes for dampness and is unfit for dwellings. The fact that the early block had hollow spaces giving a cored wall equal to 25 to 50% of the cross-sectional

[1] Managing Editor *Concrete*. Author "Concrete Stone Manufacture."

area, led to the extravagant claims not only that no moisture could get through the wall from the outside, but that such a wall was sufficiently insulated so that there could be no possible danger of condensation on the interior wall surfaces. The blockmakers had the architect against them. The commonest type of block was an imitation of pitch face stone, which the architect objected to, not so much because it was an imitation, as because it was a very poor imitation of the real thing. Architects also objected to the early units because of their proportions in height and length, commonly 8 in. high by 16 in. long.

The conditions which are gradually driving the incompetent out of the industry, and which are closing down those plants which are inadequately equipped, poorly managed, and under-capitalized, are bringing into the industry men who were first unattracted by what seemed to be a "small fry" business. The possibilities of concrete stone manufacture have been developed very remarkably in the last 10 years, and much more rapidly in the last 4 or 5 years, so that many of the leading architects are now specifying manufactured stone on an equal basis with natural stone, and in some cases, in preference to natural stone in important building enterprises.

There are very few who doubt the value of well-made concrete building units and it remains only for intelligent manufacturers to develop their business along lines entirely different from those which characterized the industry's early efforts.

A most important thing for them to appreciate at the outset is the difference between concrete units which are suitable for exposed walls and for the trim of first-class buildings, and those other units which demand no architectural consideration and which are used to replace common brick in foundation walls and such walls as are to be faced with some other material.

75. Two Main Lines of Work.—There are in general two types of concrete building units. Their manufacture involves two distinct lines of work. One is in the production of standard units in quantity; units which are structurally sound but have no special claim for use where any architectural purpose is to be served. This is a simple bulk proposition, where the constant flow of materials and the quantity of manufactured output are largely the determining factors in the success of the enterprise. The other line of work is in the manufacture of trim stone or standard units which are specially faced or otherwise surface treated to make them suitable for exposed walls or trim in competition with natural stone, face brick, terra-cotta, and other well-known building materials similarly used.

The success of one enterprise or the other depends quite as much upon the natural supply of other building materials in the community as upon the enterprise and ability of the particular manufacturer. A successful enterprise in the manufacture of trim stone and ornamental work necessitates the employment of modellers, pattern makers, mold makers, workers in glue, plaster, and wood; it involves the use of selected aggregates, more skilled workmen in the molding department, and frequently the employment of stone cutters in the finishing department.

76. Methods of Manufacture.—In the production of concrete building units there are the wet process and the dry process. There is no well-understood definition which sharply distinguishes between a dry mixture and a wet mixture and the consistency of the concrete used varies all along the line from that mixture which will just stick together when squeezed in the hand, to the other extreme, a mixture which is of a soupy consistency.

There are three general classifications in manufacturing methods, each calling for the use of different equipment. These three are the so-called dry-tamp method of manufacture; the so-called pressure method; and the wet-cast method.

76a. Dry-tamp Method.—The dry-tamp method is the one most commonly employed. It is the method whose products have in the main given concrete block its early bad reputation; it is a method whose uses are quite satisfactory when in intelligent hands and where there are methods of curing the products which contain so low a percentage of gaging water; and it is the method whose abuses have resulted in many of the bad products that have brought unmerited criticism in some cases of the entire output of the concrete products industry.

The dry-tamp product may be made in an ordinary wooden box, but as commonly known it is made in the mold boxes of simple machines, so familiar everywhere (see Fig. 103). The mixture should have just as much water in it as will permit the quick removal of the product from the mold in which it is made. The abuse of the method is in using too little water.

76b. Pressure Method.—Pressure machines are not so common in the field as they were at one time although their use is undoubtedly increasing at present. There are in use, however, machines applying pressure hydraulically, and others in which the pressure is exerted mechanically by means of toggles operated by hand (see Fig. 104). It has been urged by some that the application of pressure which is exerted evenly over one entire face of a product does not result in so dense a unit as is possible through tamping, the contention being that this even pressure, allowing less free displacement of individual particles than by tamping with a small-headed hammer, induces an arching action, particularly when crushed stone is used, this arching action among the particles of stone resulting in voids. There appears, however, to be little practical evidence of the correctness of this belief.

Fig. 103.—Hand tamp block machine.

76c. Wet-cast Method.—Wet-cast concrete products are not easily defined nor are the factors which enter into them easily outlined. There is more variation in the equipment used and in the methods pursued in wet-cast work than in the other two methods. The quantity of water varies a great deal in wet-cast work, as between the consistency used in sand molds and that used in metal molds, for instance. With sand molds, it is possible to use a very high percentage of water in the mix, providing the mixture is constantly agitated before being deposited in the molds, because the excess moisture is, in large measure, taken up by the sand of the mold. The use of a very wet mixture in metal gang molds (Fig. 105), or in a fairly tight mold of any kind, would result in a poor product, due to the fact that much of the moisture could not readily escape and would be almost sure to result in a porous product. The tendency to be overcome is the use of two much water.

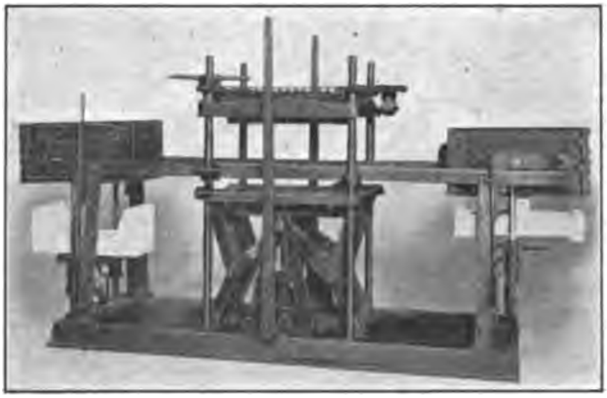


Fig. 104.—Pressure block machine.

77. Consistency.—The consideration of the various processes of manufacture has involved some thought of consistency which, in a measure, defines those processes. It is now fairly well established in the concrete field that the ideal consistency is in that mixture which will barely retain its shape when the forms are removed immediately after the concrete has been deposited and pressed into shape. This is just a little wetter than can readily be used in dry-tamp machines. It is just a little wetter than is ordinarily used in pressure machines and it is a great deal drier than the mixture which is ordinarily used in the wet-cast concrete. Since, however, various other conditions which contribute to the production of good concrete are more susceptible of control in the case of factory work than in field work, the concrete products manufacturer has a somewhat wider latitude in the matter of consistency.

Concrete products manufacturers using dry-tamp equipment have constant difficulty with employees in trying to get them to use a mixture of the wettest consistency which it is possible to use in the mold boxes of their machines. The drier the mix, granting that it is just wet enough to stick together under tamping, the easier it is to remove the product from the mold without damage. The quantity of water which gives the mixture an ideal consistency resulting in water marks on the outside of the product when it is removed from the mold will, it is generally contended in the field, cause the product to stick to the face plates, resulting in damaged products and in retarding the work. This sticking is undoubtedly due to a combination of the water and fineness of the facing material in causing a suction on the face plate which mars the fresh product. A wetter mixture with a slightly coarser facing material is not so liable to cause damage in removal from the mold.

Fig. 105.—Metal gang molds mounted on car.

The results of dry mixtures are not nearly so bad as might be expected. When the products are removed from the molding room promptly and put into a steam curing room where a warm atmosphere saturated with moisture does not permit the evaporation of any of the moisture which has entered into the block in the first place, very good products can be obtained.

Mixtures so wet that they would give very unsatisfactory results under ordinary conditions give high-class products when poured into sand molds. A recent tendency among sand-cast stone manufacturers is toward less water and toward longer mixing, the additional mixing serving in the place of so great an excess of water in obtaining a smoothly flowing mixture. High crushing strengths and a high degree of density are obtained. Numerous architects show a preference for the manufactured product over the natural product because of less tendency to discolor through the absorption of moisture and dirt. Where a mixture is poured, however, into a rigid and non-absorptive mold, as in the use of steel gang molds, in block and brick manufacture, it is important that the moisture content be kept down just as low as is possible consistent with a ready flow of the material into the forms and around the cores.

78. Commercial Molds.—Commercial molding equipment is, for the most part, very simple, the essential requirement being a mold box from which the product can be readily removed when shaped. An idea which has been almost inseparable from concrete block from the inception of the molding machinery for its production, is that it shall provide a partially hollow wall. This is very clearly shown in Fig. 106. The various sketches show how the designers of different machines have varied the provisions for air space either in the unit itself or in the wall as the block are laid up. The block shown at *A*, *B*, *C*, *D* and *I* are in one class, being complete units in each case providing the entire wall thickness. At *E* are two separate thin-wall slabs which, when laid in the wall, are held by metal ties. The unit shown at *H* is for light residence or other light wall construction, providing the plain outer wall surface on what is here shown as the upper side of the sketch. The projecting lugs provide a base for attaching furring strips for lath and plaster. Blocks *F* and *G* each consists of two separate parts held by metal ties, cast in the blocks. The broad U-shape block *J* is designed for an interlocking arrangement as laid in the wall, the straight faces of this block forming both interior and exterior wall surfaces, giving a complete and continuous air space in the wall. The block shown at *K* is for a similar construction, the lug on the block giving the desired bond between the two sides of the wall. Still another type of block has three rows of vertical ducts, and two horizontal. The center row is filled with slush concrete as laid up and provides for reinforcing rods.

The hollow space serves to economize in material, to make a lighter, more easily-handled building unit, to provide as laid up in the wall, either a series of vertical air ducts, as in the block shown at *A*, *B*, *C*, *D* and *I*, or a more nearly continuous air space throughout the wall, as in the block shown at *E*, *F*, *G*, *H*, *J* and *K*. In the varied manner of providing air space in the wall lies the chief difference between many of the machines on the market.

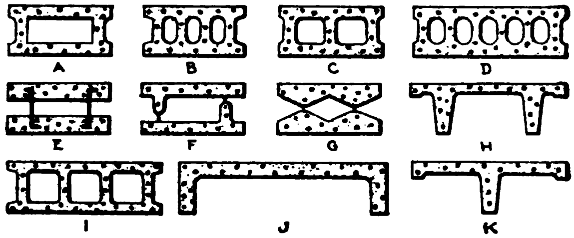


FIG. 106.—Horizontal cross-sections of representative types of concrete block.

79. Operation of Machines.—In the operation of the machines, considering more particularly the mold boxes, the general type shown in Fig. 103 is made either to make block face down, face up, or with the face at the side. If block are to be made with a special face design, as for instance, the lamentable example of the rock-face block which is poorly conceived as an imitation of pitch-face stone, then the face plate, which is to give this design, is usually at the bottom. If the face of the block, however, is to be faced with a separate mixture of material for a special texture or color, then the machine will be of either the face-down type or face-up type. If block are to be produced solely for structural purposes, without face design, or without a facing material used on the face side, the so-called stripper machines have given very satisfactory results. In these machines, the block is produced upright in the mold just as it is used in the wall and the cores of the machine are introduced and removed by upward and downward movements. The sides of the block are thus always perpendicular to the bed of the machine, hence the block is stripped out of the mold with a troweling action. Other types of machines tip the block over before it is removed on the pallet.

In some machines the cores are removed with a downward motion after the block has been tipped over. The common way, however, is to withdraw the cores while the block is still face down, leaving the hollow spaces lying in horizontal position. After the cores are removed a block is turned over on the pallet. All this is done by the movement of two levers, one to remove cores and one to tip over the mold box, or by automatic mechanism which is set in motion by one lever movement.

One of the pressure machines has a sort of track of equal length on each side of the pressure head (see Fig. 104). Mold boxes travel on this track, one box at each end. The box is filled clear to the top, if it is to be a plain block, or if it is to be a faced block the backing material is struck off at a depth of ¼ in. below the top and the facing material is put on and struck off. The box is then rolled on the track to the center of the machine under the pressure head. When the levers are released, the box is rolled back to its first position. A pallet is placed on top and clamped in position. The box is then turned over and lowered so that the pallet rests upon a stand placed to receive it. The block is thus released face downward on the pallet.

79a. Tamping.—In the use of machines in which block are compacted by means of tamping, this tamping is done in three ways: (1) by hand solely; (2) by hand-operated pneumatic tampers; or (3) by means of machine tampers suspended above the mold box. Hand-tamping is most common, the common type of tamper (Fig. 107) being a double-ended instrument with a center bar for a handle, and with one narrow spade-like head and one broad flat head, the narrow head being used beween cores and the broad head over the full block area before and after the cores are introduced. In hand work, everything depends upon the operator and most manufacturers maintain that the operator relaxes his efforts to a considerable extent toward the end of the day. In spite of this, a man who is accustomed to this work gives excellent results with hand-tamping.

Machines for tamping have to be built to withstand severe jarring. They are supported either by a framework from the floor of the factory or suspended from a framework above. Such tampers have several feet at the ends of plungers so arranged as to fit the type of machine in use. These plungers strike the concrete with equal force over the entire open area of the mold. The plungers are so adjusted as to be responsive to the depth of the material in the mold box, so that the length of stroke varies as the mold box is filled.

Fig. 107.—Hand tamp.

80. Gang Molds for Wet-cast Products.—A type of equipment for making wet-cast products that is coming into more general use consists of gang molds. One kind is mounted upon cars (Fig. 105), which are run on tracks to receive the concrete at the mixer, and from there to curing tunnels. Another kind is located in such a way throughout a stretch of floor as to require the mixed concrete to be brought to the molds. This process thus involves either the use of a very large number of individual molds in which the products must remain arranged over a very large casting area for from 12 to 48 hr., or else it involves the use of molds set up in such a way that they may be conveyed to a curing place when they are filled. When the products have become hard, the molds are simply taken down by a removal of the core pieces, sides and division plates, and are oiled and set up again, either on platform or car.

81. Materials.

81a. Cement—Storage and Conveying.—As a rule concrete products manufacturers are satisfied to use any standard brand of Portland cement, which can usually be depended upon to conform to the specifications of the American Society for Testing Materials. There is, however, a disposition among some manufacturers, particularly those making a high class of trim stone and more particularly also where a rather wet mixture is used, to select their brand of cement with some care. Some of these manufacturers believe that some brands have a tendency to cause crazing, which is one of the bugbears of the concrete stone manufacturer. No manufacturer has been found, however, who can explain just exactly the reason for his preference for one brand over another, except so far as his experience has seemed to show that the use of one brand resulted in less crazing than another. There is a belief among some manufacturers that a cement which has been aged much longer than is ordinarily demanded is desirable in concrete stone manufacture and it is said that this older cement is less likely to give hair-checking or crazing. In other respects, the quality of cement required in concrete products manufacture scarcely differs from that in general concrete work.

In a small plant the cement is ordinarily stored on a platform as near as possible to the level of the hopper which feeds the mixer. This may be, in some plants, at the second floor level, chutes being used for cement as well as aggregates, or it may be at a level between the first and second floor, determined by the level of the mixer itself. If stored in bags on the second floor level, an elevator of some kind is provided unless the plant is so small as not to warrant the use of equipment of this kind. It is possible, sometimes, to have a railway siding on a trestle and to use gravity conveyors with ball-bearing rollers to carry pallets bringing bags of cement. With an arrangement of this kind, the cement is brought into the storage space direct from the car with very little handling. In another plant the track may be slightly above the level of the floor on which the mixer stands; or, even with it on the same level, it is possible to build a platform at the level of the car floor and to pile bags of cement in such a way that they may be emptied into a cart filled with gravel from a bin along the side and dumped into a mixer which stands just under the platform.

Bulk cement has not been used extensively in products manufacture but has been very successfully used by a few. The cement may be scraped from the door of the car into a chute feeding into the bottom of an elevator boot, the elevator lifting the cement into a bin in the top of the plant from which it falls by gravity into a measuring box above the mixer hopper. In another plant the cement has been loaded from the car to wheelbarrows, handled over a runway to the hopper of a mixer. Where the output of the plant is large enough and it has been possible to hold a car to use up its entire contents, this handling of the cement has been very economical. When this has not been possible, the cement has been dumped from the wheelbarrows into a bin close to the mixer from which it is shoveled into the mixer hopper.

81*b*. Aggregates—Kind and Quality.—In the main, the aggregates used in the general field of concreting, are suitable, except as to size, for concrete products manufacture. In general, fine materials are used throughout the concrete products field. Better, cheaper products can of course be made when larger aggregate can be used, the maximum size equal to one-half the smallest dimension of the product.

Aside from quality as to cleanness, hardness, and so on, the concrete products manufacturer has been most concerned with the consistency of the mixture with which the size and quality of the aggregate have a great deal to do. A very dry mixture on one hand and a very wet mixture on the other—both of them more common in the field of products manufacture than any intermediate mixture—have both been an influence in favor of rather fine materials. Most manufacturers using dry-tamp equipment appear to be convinced that coarse materials cannot be successfully used in a mixture containing little water because of a tendency of coarse materials to fall out of the product on removal from the molds and to cause a high percentage of breakage.

Crushed limestone, especially when there is a rather high percentage of fine material, undoubtedly permits the use of more water than does sand. It is still a question whether or not the excess of moisture used in a mixture of such material, becomes available for the hydration of the cement in the curing period which follows the molding.

The prevalent belief is that a high percentage of fine materials should not be used, the usual specifications being that a percentage no higher than 5 or 10% passing a 100-mesh screen shall be used in the fine aggregate.

The most desirable qualities in concrete building units are, of course, strength and the quality of resisting the attacks of the elements, to the end that they will not disintegrate the concrete nor spoil its beauty through absorption of discolorative agents.

Bank-run and crusher-run materials should not be used. It is important that a concrete products manufacturer's output be of even quality. To this end he should maintain a constant supply of an aggregate of uniformly high quality, grading in size from fine to coarse. Any amount of tamping or pressing, or care in puddling and pouring, or in placing the concrete, is entirely unavailing if these operations have not been preceded by scrupulous care in the choice, grading, and mixture of the materials, as essential to securing density in the product. It is strongly recommended in connection with this chapter that reference be had to the chapter on "Aggregates" in Sect. 1 and the chapter on "Proportioning" in Sect. 2, as these chapters treat in detail of the selection and grading of the materials in order to obtain the best results. No manufacturer should determine his proportions arbitrarily but should first examine or have examined samples of the material which he proposes to use and which he has reason to believe will come to him in unvarying quality. To make sure that this quality is unvarying, frequent tests should be made to determine the grading which should precede any decision as to the proportions in the mixture.

82. Mixing.—No one in the concrete field has a wider latitude in selecting mixing equipment than the concrete products manufacturer. His plant is stationary, the requirements of the plant are more or less regular; and he has no problems of getting about, here and there, under varying conditions.

82a. Mixers—General Type.—The commonest types of mixers in concrete products plants are those with the simple cylindrical drums in which a comparatively dry mixture of concrete ordinarily is turned out, and the continuous mixer, for a long time much despised in the general field. In many concrete products factories where a wet mixture is being turned out in large quantities, types of mixers are used similar to those found in large construction work in the field. In addition to these types, the small cylindrical mixer for handling facing materials is common in almost every concrete products factory. There are frequently two or three of these so that they may be used for facing mixtures of various kinds without change. In connection with the mixers employed there is one thing which is perhaps unique in the products field and that is the adaptation of some of the best-known makes of continuous mixers to the specific needs of the factory in which they are used. Local mechanical ability in each case has been able to set up these machines so as to give very satisfactory results. Not only has the flow of dry materials been fixed under close control but the water is added in definite quantity to give a continuous flow of a like mixture. It is almost invariable that the use of a continuous mixer in a concrete products factory requires the erection of bins and hoppers over those with which the machine is equipped.

82b. Mixing Dry and Mixing Wet.—It is coming to be more general practice to mix the concrete materials dry in one mixer and to add the water in a second mixer. In a plant where stone of a very high quality is manufactured, materials are stored on the second floor. They are shoveled into a car in definite proportions with the cement on top. This car is shoved along the track in front of the bins from which the materials are obtained and is dumped into a cylindrical drum mixer where the materials are mixed dry. They are then dropped through a chute to a continuous mixer on the first floor where the water is added, the quantity of water being very carefully gaged to give a like consistency all the time.

Long and thorough mixing is particularly important in concrete products manufacture where a homogeneous mixture of like color throughout is particularly desirable. Mixing the materials dry and then adding water in another mixer is almost sure to make for greater thoroughness in a combination of the materials.

82c. Agitation Subsequent to Mixing in Wet-cast Work.—Where materials are mixed wet for casting in sand molds, it is highly important that the agitation of the mixture be continued so as to prevent segregation of the materials. Even in mixtures where the size of stone is little more than ¼ in., it is impossible in the wet mixture which is used, to prevent this stone settling to the bottom of the receptacle as it is poured out from the mixer. For this reason it is common practice to provide some means of keeping this mix agitated up to the time it is placed in the molds. In the largest plants this is done in an auxiliary mixer travelling on an overhead crane, driven by an electric motor—really a mixer in itself. It takes the materials from the mixer proper, keeps them constantly agitated until the mix is deposited through a pipe 3 or 4 in. in diameter to molds in a sand bed. In smaller plants where such equipment seems unwarranted, it is common to use a large wooden cask either swung from a travelling hoist or mounted on a truck moving about on tracks through the casting area. A workman simply turns a crank operating paddles to keep the mixture agitated while it is being run off through a spigot into the molds.

82d. Mixing Facing Materials.—There are small cylindrical drum mixers specially provided for mixing facing materials in small batches. Thorough mixing of the facing mixture is highly desirable so that there may be, in the case of special aggregates or the use of color in any form, a thorough distribution of the material in order that it will not be spotted or in any way uneven either in color or in texture. In the coarse-textured concrete stone, which is becoming more popular, the mixture used is comparatively lean in cement and thorough mixing is, therefore, necessary to be sure of cementing the particles in place. These products are afterward brushed and it is important that the mixing be thorough so as to be sure of embedding the stone particles.

83. Placing.—In a small concrete products plant where perhaps but two hand-operated block machines are used, and where only two men may be employed at this work, it is not uncommon to operate the mixer for a short period; pile up a batch of concrete in front of the two machines and shovel it from the floor direct in the machines, each workman serving himself at this labor. This, however, is not the way of the modern concrete products plant which does away as much as possible with wasteful hand methods, and by increasing the capacity and output of the plant and increasing the quantity of machinery used, lowers the cost of the product and in most cases improves its quality.

83a. Buckets and Hoppers.—In the majority of concrete products factories, the mixed concrete is conveyed from the mixer by a bucket travelling by a trolley system to serve the trim stone department and a row of block machines. From this travelling bucket the concrete is deposited in 4 to 5-cu. ft. batches in hoppers feeding to sloping tables just behind the block machines (Figs. 108 and 109), the operators of the machines scraping the mixed concrete into the mold boxes with very little lost effort.

Fig. 108.—Factory layout. (Block department in background.)

R = Curing rooms.
S = Second floor, or rather an intermediate floor above curing rooms where a battery of mixers (5) are located.
E = Elevator for boxes of mixed facing material.
B = Bucket (a part of an electric monorail system) for delivering mixed concrete to machines and at bankers.
H = Hoppers to receive mixed concrete.
C = Block car.

In a large factory, where there are several block machines, and a large-dimension stone department, there is a battery of mixers at an intermediate level between the first floor and the second floor, so arranged that the raw materials come in by conveyor belting at the level of the hoppers over the various continuous mixers. The mixed concrete is fed into buckets which are hooked to a monorail system operated electrically so that any workman anywhere in the large molding room can have a box of mixed concrete delivered to him suited to his special work, by means of tracks and switches controlled from the starting point. The empty buckets are then returned to be refilled. This is an elaborate system and an expensive installation. Few existing factories probably have an output to warrant it. In a factory making but one type of units a conveyor belt brings the mixed material from mixer to machines.

The market offers machinery which couples the block machine with elevating equipment and delivers concrete direct from the mixer to a hopper in the top of the machine and drops it as required into the mold box under the tampers. Such equipment is usually coupled with machine tampers.

83b. Wheelbarrows.—It is far more common in factories which put most of their effort on dimension stone, to find that concrete is handled chiefly in wheelbarrows. One of the largest wet-cast stone factories in the East, and one of the largest factories making dry-tamp dimension stone in the Middle West, make use of wheelbarrows in transporting the mixed concrete to the molds. In the case of the wet concrete, four pails are carried in a barrow. The situation is quite different, however, in various localities in respect to labor, and the situation is also affected by the fact that, in a factory making dimension stone which is to sell in competition with natural stone, the actual labor in molding, tamping, and finishing is much more per cubic foot of concrete than is the ordinary standard product made in machines. The bulk of the concrete handled is, therefore, of less consequence than where standard units are the chief products.

83c. Pallets.—The pallets used in standard block machines and brick machines are commonly of two kinds—wood and iron. The pallets stand repeated changes from wet to dry and are subject to severe wear. If iron pallets are neglected, they become coated with rust and concrete so as to be useless. It is recommended that to keep iron pallets in proper condition they should be kept coated with paraffine oil, or that they be dipped in a mixture of

FIG. 109.—Block machines with hoppers above are shown in the background and block cars in the foreground. Note the two floor levels. The cars run on tracks into pits so that four decks can be loaded without too high a reach. The empty "decks" from the car at the left are placed on the car at the right as it is piled.

kerosene and axle grease, when the concrete can easily be wiped off at the end of each day's work. The same treatment is recommended for the metal parts of the block machine. Another manufacturer suggests that iron pallets be dipped in a solution of 1 part lard oil and 1 part kerosene, to keep the pallets clean and to prevent rusting. With wood pallets, the difficulties are from swelling, splintering, warping and so on. It is necessary to use wood which is as little subject to warping as possible, and have the pallets well treated in order to overcome warp so far as it can be done. It is pointed out that pallets should not be made of one solid piece but of strips not more than 4 in. wide, with slightly open joints to allow for some expansion. Some manufacturers recommend dipping wood pallets into hot linseed oil. Difficulty from slight swelling of the pallets is not important except in machines where pallets must fit very accurately as is not the case with most tamp machines. There are machines, however, where the pallets must fit with great nicety and in such cases considerable difficulty has been experienced in getting a pallet which will resist the severe treatment. One manufacturer suffering such conditions finally adopted a combination wood and metal pallet.

Most blocks are delivered on a pallet right side up, face perpendicular as they will in a wall. The condition of the pallet in this case is not so important. Where, however, the block is turned over face down and delivered on a pallet in that position, it is very important, particularly for face block, that the pallet be very true, and metal pallets are recommended for such work where a true, smooth face is required. For rough-textured block, this is, of course, not necessary.

83d. Bankers.—Practically all dimension stone is made in special molds, these molds resting on heavy plank pallets, supported in turn by bankers which may be of reinforced concrete to minimize vibration. Such bankers are used in factories where dimension stone is made by the dry-tamp process. For work of this kind, the placing of the concrete is much slower than with wet-cast work, as the facing materials have to be built up vertically 2 or 3 in. at a time on such faces of the stone as are to be exposed, besides placing on the bottom, and the backing must be tamped in as the facing is brought up. Thus for the convenience of the workmen the mold is placed on bankers at a height which is convenient for his work.

84. Curing.—In from 20 min. to 1 hr. after water has been added and the mixing of concrete completed, this mixture must be placed and it must not only be so handled subsequently as not to disturb the hardening process but it must be kept in a condition which will aid that process. Conditions for curing must be such that the product will not be rapidly dried, yet as temperature influences the hardening—heat quickening it, cold retarding it, and freezing interrupting the hardening process for the period in which the low temperature continues—it is important that these things be considered in caring for products when they have been molded or cast.

The problems of curing do not present themselves as so serious a matter to the manufacturer of wet-cast stone as to the manufacturer of dry-tamp products. In the wet-cast work where there is already an excess of moisture, sufficient heat is practically the only essential, with conditions which will prevent too-rapid drying. In tamp products, and for the most part in pressed products where the moisture entering the mixture is only just about (or even a little less than) that actually required for thorough hydration, it is very important that none of this moisture be permitted to escape before complete hydration.

In curing wet-cast stone, block, and brick made in gang molds on cars, one method is to move these cars on tracks to curing tunnels which are heated by steam. The tunnels are made of concrete and just high enough to admit loaded cars. They are heated to give a rapid hardening of the concrete. The cars are usually removed 24 hr. after they are placed in the tunnels, the molds are taken down, and the products are carefully piled under sheds. The molds are oiled, set up again on the cars and returned to the mixer to be refilled. In other wet-cast work, where gang molds on cars are not employed, or where other molds are used with a wet mixture on a large casting floor, it is common to leave these molds in place until the product is sufficiently hard to be handled. The molds prevent a rapid escape of moisture in the early stages of hardening, and particularly is this true in sand cast work, where the sand is always damp from having absorbed the excess moisture of the mixture.

84a. Natural Curing.—The recommendations in the old "Standard Practice" of the American Concrete Institute with respect to natural curing are usually regarded as sound. They are as follows:

> *Natural Curing.*—The concrete products shall be protected from the sun and strong currents of air for a period of at least 7 days. Throughout this period they shall be sprinkled at such intervals as is necessary to prevent drying, and maintained at a temperature of not less than 50°F. Such other precautions shall be taken as to enable the hardening to take place under the most favorable conditions. Products must not be removed from the yard until they are 21 days old.

Where products are cured in this way, it is necessary that racks or cars be used so that block on the pallets may be piled up in tiers. As standard practice requires that products be sprinkled for 7 days, it is obvious that there must be curing shed space for 7 days' output and a very large yard storage space in order to keep products until 21 days old. Building regulations in numerous cities require that products be at least 30 days old when cured in this way, without regard to the crushing strength.

Inasmuch as this method of curing would require the use of a very large number of cars, on which to tier up the products, it is common, when natural curing is used, to apply the rack system of storage. It will be obvious that this can be used only in rather small plants. The racks are usually built of 2 by 4's, each rack 16 to 20 ft. long. One rack can thus readily accommodate 26 blocks 8 by 8 by 16 in. These racks can be piled about four high. Four rows of racks will thus accommodate about 400 blocks.

With natural curing, the products are either moved on cars and placed upon racks, or are carried on pallets direct from the machine to the racks. This latter method can only be used in the smallest plants because the labor of carrying the block the distance required by racking in a large plant would make the cost excessive.

Sometimes products are made which are too large or too heavy or of too awkward shape for removal to curing sheds. Until hardening has progressed to a considerable extent precaution should be taken to see that these products are kept under suitable conditions to attain strength. When left in the molding room, the products should be covered with wet cloths and the cloths kept wet. This applies particularly to tamp products where the molds are removed a day or so after manufacture. Sprinkling may be done systematically and thoroughly with a nozzle, which gives a fine, well-diffused spray. The nearer the spray approximates a floating mist the more thoroughly it will do the work, reaching all the surface of products stored in tiers on racks or cars.

Where the products are removed to sheds and cured in the natural way, it is obvious that considerable labor will be required to use the hose on the products and it would be difficult to use the hose in any effectual way. It is common, therefore, to install permanent sprinklers in the curing sheds.

In an Eastern factory turning out high-class products from wood and plaster molds, the time for the products to remain in the molds is from 5 hr. for small units up to 24 to 48 hr. for the larger and more complicated pieces. Until the molds are removed it is not necessary to apply additional water to prevent the escape of the moisture contained in the product because there is considerable water used in the mix. As soon as the molds are removed, sprinkling begins, using a fine spray. This factory, which is of old construction, has wooden columns about 10 ft. apart in each direction. Pipes have been placed so that there is a water outlet at every column. A man is kept busy all day sprinkling the products and another man continues the work at night. Great care is taken to keep the products moist until they dry out evenly.

In some ornamental work, manufacturers frequently make use of total immersion of products to cure them. It is common in such cases to cover the product with wet cloths just as soon as it has been removed from the mold and allow it to remain in this way until it has attained sufficient strength to be handled and placed in the tank. Other manufacturers do not use the immersion system but simply use the wet cloths.

84b. Steam Curing.—General practice in steam curing makes use of a wet steam and a low pressure to create a dense warm fog with all the moisture which can be introduced at a temperature in the curing rooms between 100° and 130°F. While common practice in the field does not warrant the use of high-pressure steam and while investigations with high-pressure steam in curing have not gone far enough to suggest its adoption on a commercial basis, there has been some investigation tending to show that steam under pressure up to 80 lb. can be used with success. This steam has to be employed in steam-tight compartments. The ordinary curing rooms of the concrete products plant are not steam-tight. They are wide enough to accommodate the cars which usually run on 24-in. track and high enough to permit four decks of standard block to be piled on the cars, leaving some room for the accommodation of special products to which it may be necessary to give steam-curing treatment. The curing tunnels of the plant are usually about 60 to 90 ft. long, built of concrete block, with arched or "A-shaped" ceiling—preferably a ceiling made by applying Portlant-cement plaster to a ribbed reinforcing mesh. The ceiling is built so as to carry the condensed moisture of the room to the sides and away from the fresh products which dripping would damage. Curing rooms are sometimes built wide enough to accommodate two or even three tracks, so laid out that in the case of very wide products to be admitted to a curing room, only the middle track can be used, allowing plenty of room for the projection of the products at the sides. At one end the curing rooms usually open into the molding department as convenient as possible to the machines supplying the greatest number of products to be cured, and the other end frequently opens into a passageway connecting with the yard, or to the yard direct.

The construction of the doors has given concrete products manufacturers considerable difficulty from time to time, due to the fact that metal doors rust, unless kept in perfect condition, wooden doors swell with the steam on one side while they remain dry and of their original size on the other side, and canvas curtains are very short-lived. These curtains roll up and are fastened at the sides, usually, with carriage buttons. In using them, allowance has to be made for shrinkage. Galvanized sheet metal for doors lightly framed with wood, or small angle irons, have given satisfaction.

The following recommendations of the American Concrete Institute with respect to steam curing were made some time ago but are still considered as representing good practice.

The products shall be removed from the molds as soon as conditions will permit and shall be placed in a steam-curing chamber containing an atmosphere of steam saturated with moisture for a period of at least 48 hr. The curing chamber shall be maintained at a temperature between 100° and 130°F. The products shall then be removed and stored for at least 8 days. (This does not apply to high-pressure steam curing.)

From an excellent discussion of the proper use of steam in curing concrete products, the following by W. M. Kinney in *Concrete* is quoted:

The principal object in curing concrete products with steam is to accelerate the hardening by means of heat without endangering the concrete through loss of moisture by evaporation. Saturated steam will provide not only heat but sufficient moisture to insure against injury from drying.

In the early history of concrete products manufacture it was customary to use exhaust steam for heating the curing chamber, but as a sufficient quantity was not always to be procured, the natural resort was to use steam direct from the boiler. The records show that in many cases large quantities of good concrete came to grief due to its drying action, which was not at that time explained.

Especially difficult is the maintenance of a sufficient quantity of steam in a boiler at low pressure to heat sufficiently any number of curing rooms. Coupled with this difficulty is the danger of the pressure rising considerably above that necessary for proper curing. With this in view we have been recommending steam under pressure, that is, around 30 to 45 lb., provided it be admitted through water.

The most satisfactory way of admitting steam in this manner is through a perforated pipe embedded in a trough of water running through the center or along the sides of the curing chamber. The floor should be so sloped that any water or condensation on the products or on the walls of the curing chamber will be returned to the trough for re-evaporation. In this manner the trough is automatically kept full of water and we have yet to record a case of trouble when this method of curing was employed. All of the heat which the steam contains on being emitted is taken up immediately by the water and the result shown by evaporation. The temperature of the water is at the boiling point due to the fact that steam is continually being forced through it and what heat is taken up by the water is used in evaporating the water. This water evaporated at 212°F. is just as useful in warming the room as is the steam at the same temperature.

To explain the drying action of steam under pressure when admitted to the curing chamber, let us assume that we are taking steam from a boiler operating at 30-lb. gage pressure. The temperature of the steam in the boiler is 252° atmospheric pressure. It is, of course, understood that steam under atmospheric pressure and having a temperature of 252° is in an abnormal condition which we technically call superheated. This steam is in a similar condition to that which would be obtained if water vapor at 212° were heated up to 252° away from contact with water. Naturally the first thing that steam in this condition does is to avail itself of the first opportunity to reach normal conditions and the most ready way is to absorb water from anything in its vicinity which is so possessed The result is a drying out of the concrete products which happen to be stored in the vicinity.

Another method of introducing steam into curing rooms is described by A. E. Cline as follows:

The simplest way is to have a main pipe over the top of the curing-room doors, then from this lead a separate 1½-in. pipe to each room. Run this along one of the walls close to the floor but with slant enough to drain to the farther end. Join each length with a T, having the third hole of the T reduced to ¼ in. and turn this at right angles to the wall. The 1½-in. pipe can be reduced to 1 in. at half the length of the room, and this on a supposition that the curing room is 80 ft. long. If using steam for power, have this main pipe connected with the exhaust during the day and with the boiler at night. If the boiler pressure at night is greater than 10 lb. it is best to have a reducing valve in the main line as high pressure is not wanted.

85. Special Molds.—Although the commercial market affords a wide range of machines for molding various concrete units and a large number of special molds—not only for sills, lintels, and so on, used in building, but for various ornamental objects and special architectural pieces —the progressive concrete products manufacturer no longer considers that his plant is fully equipped until he has a department for making molds to meet the demands of architects in turning out dimension stone according to special designs as may be required.

85*a*. Wood Molds.—The material most commonly used in making these molds for dimension stone is wood and a large plant will have an extensive wood-working department with power saws, planers, etc., for cutting down the labor cost. By far the greater part of dimension-stone work in most factories will be made in wood molds—preferably white pine. This will include sills, lintels, belt courses, cornices and so on. In general, for a plain piece of work the mold consists of side planks resting on a pallet with end pieces fitting inside (see Fig. 110).

All pieces have to be carefully finished, all small holes or cracks filled, ordinarily with plaster, the entire work shellacked and then oiled before use. By clamping the side pieces firmly, the end pieces are held in place on the outside against cleats. When the facing mixture has been placed on the bottom of the mold and up the front side and part way on the ends, as the case requires, the backing follows. When it is tamped all the way up, the work is struck off at the top and a plank very carefully bedded on top by means of a layer of bedding sand. This plank is then clamped in place, the clamps passing over the bottom pallet and the entire work is turned over so that when the mold pieces are released the stone is right side up, having been tamped in place in an up-side-down position. It is not necessary to use the bedding sand on very small products which will bed readily on a smooth plank surface.

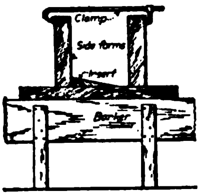


Fig. 110.—Cross-section of simple wood mold.

In wet-cast work in which the molds cannot be removed immediately, it is common to use a large casting floor smoothly finished with concrete. This is shellacked and oiled as an ordinary mold surface would be treated and on it are set up side rails for plain work, with the necessary insert pieces and dividing partitions to produce the plain units in necessary lengths. Sometimes for greater convenience the bottom of these molds is provided in a bench or table with a concrete slab top.

Where long side rails are necessary channel irons of proper widths can be used to advantage. Properly cleaned and oiled they will give long service and the principal thing to recommend them is the fact that they do not warp as wood does frequently unless the grain is very heavily filled and the surface shellacked and oiled, nor do they spring out of line from the weight of the concrete. This is something that has to be carefully considered in long work, where even a slight wind due to the springing of the form frequently prevents the use of the stone on nice fitting work.

The market affords a great many standard molds made of metal for various ornamental pieces and standard architectural units. Manufacturers who are catering to discriminating architects will not depend upon standard units, however, but will be prepared to meet the designs of architects.

85*b*. Plaster Molds.—Plaster molds are very extensively used in concrete stone manufacture. In fact, plaster is used not only in making molds but in making models, and not only in plaster molds themselves but in making molded inserts for wood and metal molds. Fig. 111 shows how, by means of a template of thin metal, stiffened by a wood frame operated on a smooth oiled surface, it is possible to make moldings for various purposes in mold manufacture. The template is guided by a straight-edge and moves on the table, pushing aside the soft plaster, except in the desired section described by the template. Similar work requiring curved outline is handled by mounting a template at the end of a pivoted arm at such a length as to describe the required arc. The template then has a circular movement.

Plaster is readily used in making models from architects' drawings. When this is done the plaster is cast in a large block from which the model can be carved with knives and suitable chisels after the details have been outlined with a pencil on the various faces of the block. The plaster model is then shellacked and oiled and the mold made over this model, the mold also being made of plaster.

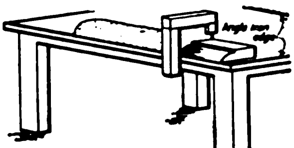


Fig. 111.—Making plaster molding.

Flat panels in moderate relief without undercut may be cast in draw molds. The model should be framed with wood strips or clay "fences" to control the plaster. It should then be given two coats of shellac (orange) and when dry, should be greased, using a mixture of 1 part stearine and 2 parts kerosene (combined hot). If a clay model is being reproduced, grease with lard oil. Sift plaster into a pan half full of water, until plaster lies about an inch below the surface of the water. Stir thoroughly. When it has thickened to a creamy consistency, apply to model, going over the entire surface thinly at first and jarring the model which is best supported by a rigid frame. The jarring eliminates bubbles and pinholes. Then pour in the plaster, reinforcing as may be necessary with strips of burlap, excelsior or wood frame for heavy work. Jars, urns, capitols and similar objects must have piece molds, the model surface being divided by clay fences against which the plaster is applied. When a section of plaster mold is hard, the fence is removed and notches are cut in the plaster edge to key the adjoining mold section (see Fig. 112). The edges are then shellacked and greased for ready separation of the mold parts. When the parts are set up for casting the concrete, the mold previously shellacked and greased is held by rope or by chain and turnbuckle, or by clamps, as the size, weight, or shape of the work may necessitate (see Fig. 113).

Fig. 112.—Plaster molds. The dark modeled part is the plaster model. The parts *A* and *B* are the first two pieces of a plaster mold.

85c. Glue Molds.—The use of gelatin or glue molds is necessary in all work where there is intricate undercut in the model to be reproduced, it not being possible to remove a rigid mold over these undercuts unless a plaster mold, for instance, is made in many pieces to join at the undercut and thus pull away. When a glue mold is to be made, the model is greased and covered, first with paper and then with modeling clay to the thickness necessary for the thickness of the glue mold. This clay covering is then greased and plaster is applied over it, to form a shell with a hole or several holes at the top with air vents at various heights. This is illustrated in Fig. 114, the lion head being the model to be reproduced. The space immediately around it is first filled with clay and over this is the plaster shell with a funnel at the top. When the plaster mold is hard it is removed and the clay and paper

FIG. 113.—Plaster mold with plank pallets clamped on top and bottom being turned over on banker after tamping full, before release of mold parts from the fresh product.

cleaned from the model. The plaster shell and model are shellacked and oiled again and the shell is then fitted in place over the model and the space between the model and the shell is filled with the glue, the vent holes being stopped with clay as the shell is filled. For ready handling, the plaster shell is usually divided in a number of pieces, together with the glue mold, the division being made with a knife. Glue should be of the best quality and should be melted in a double receptacle with very little water used in the vessel with the glue. Over-heating takes the "life" or elasticity out of the glue.

FIG. 114.—Making a glue mold.

85*d*. Combination Molds.—The manufacturer who has a well-developed model and mold department will have workers in wood, in clay, and in plaster, and men also familiar with the making and use of glue or gelatin molds. As the work progresses, a resourcefulness in meeting special requirements will lead the manufacturer to make combinations of various mold-making materials as, for instance, plaster inserts in wood molds, and small glue molds in connection with plaster molds to take care of small areas of undercut in the model to be reproduced.

85*e*. Waste Molds.—Ornamental pieces, especially when there is to be considerable duplication and rapid work is necessary, are sometimes made in so-called "waste" molds of plaster. If the model is intricate, a glue mold is made first and in the glue mold a glue model is cast. From the glue model as many duplicate molds are made of plaster as there are pieces to be cast. When the concrete has become hard within the plaster mold, the plaster is cut away and the concrete surface cleaned. Panels, without undercut, are reproduced in a similar way without the necessity for first making a glue model.

86. Sand Molds and Casting in Sand.—Sand-molding of concrete has not been in extensive use except by a few manufacturers particularly in the East, until recently. Patents covering important features of sand-molding are just about to expire.

For ordinary work the sand, or the mixture of sand and stone dust with a little loam or other ingredients to make the sand particles adhere, as in iron foundries, is used in large beds on a big casting floor. The sand is packed around models; the models are so made that they can be withdrawn at the top and the molds are filled with a very thoroughly mixed concrete at a flowing consistency in which there are usually no particles larger than ½ in. and most of the aggregates much smaller than this. Less water is used than the consistency might suggest. The ideal mix has no free water. The mixture is constantly agitated after leaving the mixer proper so as to prevent segregation of materials. Mixing frequently is continued for from 10 to 15 min. which greatly facilitates the smoothness of the flow. Ordinarily, the concrete is deposited through a spout and some means adopted (as for instance holding a small board near the bottom of the mold) to prevent injury to the sand mold by the heavy stream of concrete. To preserve the edges at the back of this stone, which is usually the surface upward, and to permit subsequent troweling of this upward surface, it is common to use wood strips placed in the sand to give a more stable edge against which to work. It is customary to fill the sand molds throughout one floor area; allow an hour for the very wet mixture to stiffen and settle and then trowel the upper surfaces, filling in a little additional concrete on the backs of the stones where it has settled.

When balusters, capitols, and similar pieces are to be made, having no large flat surface to which the model parts will "draw" on the upper side of the sand bed, it is necessary to use the flask method. A box for each of two or more portions of a pattern supports the sand for a section of the flask. The boxes are assembled to complete a mold. The sand is sometimes mixed with a very small percentage of plaster to give stiffness to the mold.

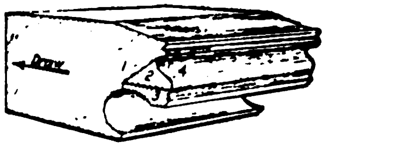


FIG. 115.—Splitting pattern for making sand mold.

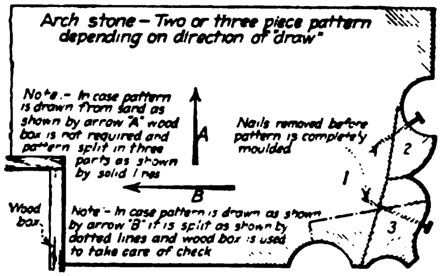


FIG. 116.—Two ways of splitting a pattern.

A great deal of the skill in successful stone manufacture with sand molds is in making the models, around which the sand must be packed. Take for instance a stone like that illustrated in Fig. 115. The molded surface at the right is to be down and the model drawn from the sand in the direction indicated by the arrow. It is necessary, therefore, to make a four-piece pattern as indicated by the numbers 1, 2, 3, and 4. The parts are held lightly when first made by nails as shown in Fig. 116, which shows two ways of splitting a pattern somewhat similar to that in Fig. 115. The nails are loosened when the pattern is packed into the sand. Note the small pattern part at 4 in Fig. 115. It is the custom among many manufacturers to eliminate the undercut in such a small part in making a pattern, it being cheaper in such a case to let the stone cutter put in the undercut after the concrete is hard. If a great number of these pieces were to be made, it would, in most cases, be cheaper to make the pattern complete to avoid so much stonecutting.

There is a tendency in practically all sand-cast concrete, for some of the sand in the surrounding bed to be held by the cement and so leave lumps and uneven places on the casting. Surface treatment of sand-cast concrete stone is, therefore, necessary, in most cases.

87. Surfaces.

87a. Face Design in Standard Units.—Design of units is a matter best left to the judgment of architects who are specialists in such matters. Some early manufacturers of block machinery and of concrete block were led astray by the ease with which the face of a block could be cast to imitate anything. Face plates were supplied to imitate pitch-face stone, cobble stone, bush-hammered stone, tooled stone and with all sorts of panels and borders. Not satisfied with this, manufacturers of block and special stone, still drunk with the plastic possibilities of so tractable a material, impressed it with the designs of rubber matting and ed-steel ceilings. These errors the industry is outgrowing. The most persistent crime against good taste is a rock-face block which is bad chiefly because it does not imitate successfully one of the least desirable types of natural stone. This much should be of record in this connection. If block makers will now devote, as many of them are doing, as much thought to making concrete look like itself as their predecessors have to making it look like the least desirable natural stone, a great future seems probable for the manufacturers of the material.

In making special units, the manufacturer will do well to follow the designs which architects work out for him. In making standard units, he will more frequently have the architects on his side if he eliminates face designs and makes a plain unit whose title to beauty is in the honesty of its appearance, in the tones and colors of its exposed aggregates, or in the light and shade of a rugged texture.

87b. Facing Materials.—Most concrete block and special-dimension stone is not of the same mix throughout. When such stone is made of a dry-tamp mixture, it is a simple matter to face it, on the surfaces which are to be exposed, using a special aggregate which will give the desired qualities in color and texture in the finished product. The facing mixture can be backed up with plain concrete. When stone is cast in sand molds—in fact, in the manufacture of most wet-cast stone—the conditions in the work necessitate that the concrete shall be the same all the way through. In such work, therefore, facing mixtures are not used. There are some exceptions to this rule in the use of commercial equipment for the manufacture of concrete block in gang molds. One exception is in the coating of face plates with a thin film of glue on which, after the glue has become sufficiently thick and sticky, the facing aggregate, with no cement, is sprinkled so as to form a complete layer over the face plate. This plate, placed in the bottom of a mold is filled with a wet mixture of concrete. The water loosens the glue and the facing aggregate bonds with the wet backing. Another method of facing a wet-cast material is in gang molds in which the product is made face up. The molds are filled with a slushy mixture, not quite to the surface, then the facing mixture is spread on after the backing concrete has partially hardened. This is troweled into place on the surface In general, with regard to facing mixtures, it is common to make them of a 1:2 proportion of cement and a special aggregate, or 1:2½ or 1:3, depending not only upon the grading of the mixture but upon the effect desired in smoothness or roughness in the product. Materials commonly used include special sand, in white, buff, and so on, crushed marbles in the more choice work, crushed granite, trap rock, even crushed cobble stones (which are within the reach of almost anybody in a glacial country), and crushed limestone. Micaspar crystals, so-called, are used by many manufacturers in producing a gray granite effect in either dark or light shades. Other facing materials are on the market containing mica which gives a sparkle and life to the finished product. It has been very common for most manufacturers to use fine sand for facing material, particularly in tamp work, and to make a special effort to get a very smooth, fine face on their products. Some architects are encouraging work in another direction by the interest which they have shown in products having a rough texture. This is secured by using coarser aggregates not so well graded—that is, not so much fine material to fill up all the spaces—and with just as little cement as can be depended upon to bind the aggregates thoroughly in the face. It is common in such work to use 1:3 mixtures and to use facing aggregates as large as ½ in. The results which can be produced in this way are limitless and their effectiveness depends largely upon the taste and judgment of the manufacturer. The use of yellow marble or black marble (crushed trap rock is frequently used in place of black marble), red granites, and so forth, lead to possibilities in colors in concrete stone which only need experiment to prove.

If a smooth block or polished finish is desired, the product should be made on as smooth a surface as possible so that there will be few projections to work down. Where a well-graded aggregate of a polishable material is used—as, for instance, granite or marble—and this material is well distributed over the surface area with few spaces between, it is possible to polish a concrete product just as the granite or marble itself is polished. A recent development in the manufacture of concrete stone lies in the direction of obtaining a smooth surface by means of a sheet of very heavy paraffined paper of glossy finish, which is placed in the bottom of the mold of whatever kind is used, the facing material being placed in on top of this and the backing tamped behind it. This gives a very smooth surface which requires only a minimum of rubbing to give an excellent finish.

Aside from the special methods and devices in obtaining various products which have already been described, facing mixtures are used chiefly with tamped products. In such work, the facing mixture is generally a relatively dry mixture, placed on the bottom or face-side of the mold box, and the backing tamped in behind it. When it is necessary to fill up the facing material on the side of a mold of any kind, this can be done by piling it up 2 or 3 in. at a time and filling in the backing behind it, or it may be done by the use of a thin piece of metal of a size equal to the side of the mold which is to be faced. With the use of this dividing plate the facing mixture is placed on one side and the backing on the other, the plate being gradually raised as the two are tamped together to form a bond. This facing mixture, while relatively dry, must neither be too dry nor too wet. It must be just sticky enough to hold its position when pressed into shape.

Whenever a facing mixture is used, it is desirable to finish the work in such a way that the special aggregates of whatever nature are used, are exposed to lend color and to give better texture to the work by the removal of the cement which covers the surfaces, leaving the cement in the matrix to bind the aggregates in their position. Various methods for finishing the stone, not only that which is faced with a special mixture, but also that which is of like character throughout. will be considered.

87c. Colors.—Great care should be exercised in the selection of colors. They cannot be used in rich mixtures without destroying a great deal of the binding value of the cement with which they are mixed, and the strongest colors cannot be obtained unless rich mixtures are used. For both these reasons there has been some tendency to discourage the use of mineral colors in cement mixtures.

J. H. Jackson, authority on colors, writes in *Concrete* as follows:

Mineral colors of the highest degree of purity are the only ones to use in coloring cement. The permanency of shade or color obtained depends upon the elimination by the color manufacturer of anything in the color that the cement itself will destroy. Few contractors realize that the more intense and brilliant are the colors, the more quickly they fade, and, in more instances than one, help to disintegrate the concrete, for none of the mineral colors useful in cement work, is found naturally brilliant and the addition of chemically prepared colors or the treatment of native colors chemically to give them intensity is a positive detriment under all conditions. All true cement colors will withstand the acid treatment (1 part acid, 5 parts to 6 parts water), scrubbing and troweling, but in polishing after setting and trowel polishing before setting, special care should be taken when yellow, green and similar colors are used for the metallic polishing is likely to darken the color. Never give a smooth finish to outside concrete when color is used.

The standard proportions for colors generally used are 6 to 6½ lb. of color to every 100 lb. of cement. The amount of color can be increased slightly if a deeper shade is desired, but you should not use more than 10 lb. of color to every 100 lb. of cement, for an excess of color reduces the binding power of the cement.

A table of color quantities and the results by L. C. Sabin from his "Cement and Concrete" is as follows:

Colored Mortars

Colors given to Portland-cement Mortars Containing 2 Parts River Sand to 1 Cement

Dry material used	Weight of dry coloring matter to 100 lb. cement			
	½ lb.	1 lb.	2 lb.	4 lb.
Lamp black	Light slate	Light gray	Blue gray	Dark blue slate
Prussian blue	Light green slate	Light blue slate	Blue slate	Bright blue slate
Yellow ochre	Light green			Light buff
Ultramarine blue		Light blue slate	Blue slate	Bright blue slate
Burnt umber	Light pinkish slate	Pinkish slate	Dull lavender-pink	Chocolate
Venetian red	Slate, pink tinge	Bright pinkish slate	Light dull pink	Dull pink
Chatt. iron ore	Light pinkish slate	Dull pink	Light terra-cotta	Light brick red
Red iron ore	Pinkish slate	Dull pink	Terra-cotta	Light brick red

Coloring by absorption is described in the *Concrete* by Adolph Schilling:

To color in reds and in browns use sulphate of iron in a solution of 1 lb. of sulphate to 1 gal. of water; for greens 1 lb. sulphate of copper to 3 gal. water. The older the concrete the longer the bath must continue. It must be borne in mind that the coloring stops to a very great extent the hardening of the concrete. Therefore, it is necessary to permit the concrete to attain such strength as is necessary in the ornamental work before immersing it. Immersion may continue for a few minutes or for several days, depending upon the age of the concrete and upon the depth of color desired. The effects may be varied by using an aggregate which is not highly absorptive so that this stands out while the matrix surrounding it is colored. Effects can be heightened in elaborate designs by "picking out" certain parts, coloring them with a brush with cement stains.

Rough-textured work is sometimes beautified by having stain stippled on or brushed over the high points. The manufacturer, of course, will always have to use his judgment in the application of these colorings. The great danger is in overdoing.

Aside from the use of white cement to obtain light colors in concrete, it is possible to make an ordinary gray cement surface lighter by rubbing with a concrete brick made of fine materials. The work should be kept wet while being rubbed.

A method of obtaining white surfaces is described by John Oursler, in *Concrete*, as follows:

A wash made of 1 lb. of concentrated lye, 4 lb. of alum and 5 gal. of water, with enough cement added to make the wash of a good consistency for spreading with a brush, has been used to give a white surface.

87d. Spraying.—One method for removing the surface film of concrete block and special stone where there is a special facing mixture, is the spraying method, described in *Concrete* by the early user of the method, E. J. Thompson:

Immediately upon removing the block from the machine, place it where there will be a good light on the face and spray it, using a fine vapor spray, such as is used in spraying fruit trees. The outlet holes in such sprays are about the size of an ordinary pin, and, for the best results, should be used in connection with a water pressure of 40 lb. or more. This spray nozzle attached to a length of ½-in. hose is all the equipment needed. The spraying is a simple operation. It can be done, after a little practice, by any intelligent laborer. The effect of the spray, which lasts only momentarily, is to wash off the surface film of cement and expose the aggregate (see Fig. 117). The spraying must not be continued until the surface begins to run or furrow, but just a little practice teaches the operator when to stop.

Some manufacturers prefer spraying their products when they are lying face up; others prefer to have them with the face perpendicular. The perpendicular method is the commonest, but the face-up method has a tendency to leave all the cement on the surface, to wash it into the pores. One manufacturer who advocates this method urges that it not only exposes the aggregate but makes the face of the product more dense. When this work is done on standard concrete block, it is done very rapidly and adds very little to the cost of the block.

87e. Brushing.—Brushing the surfaces of concrete stone in standard and special units is particularly desirable where a rough-textured effect is desired (see Figs. 117 and 118). It is commonly used where a graded aggregate, with larger particles than is ordinarily used in facing mixtures, is employed. The brushing is done, ordinarily, while the product is comparatively green, using a fiber brush with stiff bristles and using considerable water while the brushing is in progress. Care must be taken that brushing is not started too soon so that the face of the products will be damaged, and on the other hand, the work must be done before the product is too hard, or the work becomes expensive. The manufacturer will find that the time for brushing depends a great deal upon the conditions of curing. For surfaces which have been allowed to become partially hardened, a brush about 4 in. wide made by clamping together a number of sheets of wire cloth has been found more effective than the wire brushes which are ordinarily sold for this purpose. Care must be taken in brushing not to injure the edges of the products. Sometimes it is desirable for the operator to use a small frame or at least a straight-edge to protect the edges of the product while brushing.

Fig. 119 shows three views of turning stand for handling green concrete block to be brushed. This stand is used for handling products which are delivered on pallets face down. A block on pallet is placed on stand as in the first position. The shelf is turned over, so that the block rests on its side as in the second position. The shelf which first supported the block is then turned down leaving the face exposed for brushing or other treatment.

Where steam-curing is employed, it is common to run the block into the steam rooms for a few hours and out again, when the brushing is done, and the products are returned to the curing rooms.

87f. Rubbing.—Rubbing, as a finish in concrete stone, is commonly used where a limestone finish is desired. In wet-cast work when air bubbles and slight imperfections occur on the surfaces of concrete stone, it is the usual practice to pour on the surface a creamy grout

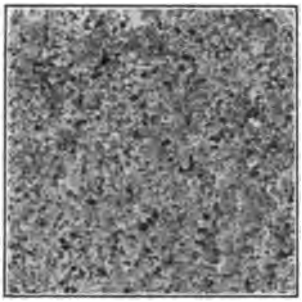


FIG. 117.—Surface obtained with relatively fine materials, either by spraying, by light brushing or by acid washing.

FIG. 118.—Brushed surface. Fine- and coarse-grained surfaces in contrast (crushed limestone).

of cement and water. This is rubbed in first with a brush or swabbed on with a cloth, and a few minutes later rubbed in with a small wood block. When hard, the stone is rubbed down with abrasives. The rubbing removes the effect of the painted surface. It is common to use cement brick made with very fine material in rubbing concrete work. Ordinary commercial abrasives are also employed.

For finishing marble concrete which may be cast in large blocks and cut up by gang saws into slabs, or cast in thin sections in pressure machines as for floor tiles, the methods are like those in factories where natural marble is handled. The pieces are first smoothed down on a revolving rubbing bed, sand and water being fed to the grinding surface. Then power-operated carborundum discs are employed, or a second rubbing bed with a finer grit, for a dull gloss surface, or when a polish is desired, the work going under hand or power-operated mops, employing oxalic acid and putty powder in the process.

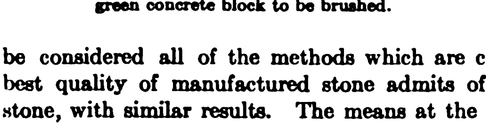


FIG. 119.—Three views of turning stand for handling green concrete block to be brushed.

87g. Tooling.—In considering the tooling of concrete stone in various surface treatments now put on it, there might be considered all of the methods which are common to the finishing of natural stone. The best quality of manufactured stone admits of the same treatments as are given to natural stone, with similar results. The means at the disposal of the manufacturer of high-class concrete stone include the ordinary hand work with rasps and chisels, bush hammers, crandalls, planers, polishers, and so on. To make possible the use of tools of this kind in obtaining satisfactory finish, it is necessary that the concrete have a uniform texture, with no aggregates of unusual hardness; that is, the materials shall be of a like character throughout. The aggregates used in such work, ordinarily pass a ½-in. screen. They include crushed limestone, marble, granite, and trap rock. While faced products are frequently tooled, it is more common to find such finishing methods applied to products which are of like character throughout.

The use of pneumatic tools in bush-hammering and crandalling concrete surfaces is becoming more common and resulting in great economy over the use of hand tools. A finish in parallel grooves, very common on natural stone, is put on concrete stone using power equipment which revolves a gang of thin carborundum wheels mounted together, so that considerable surface is covered at one time. Methods of cutting which are common in the natural stone field and which are used, though are not so common in the concrete stone industry, include the cutting up of large slabs of stone by means of saws which make deep grooves, so that a slab is easily split up on the job just before the stone is laid, thus preserving the edges and keeping them clean and sharp.

For fine ornamental details and for finishing undercuts which are not easily included in patterns and molds, expert stone carvers are employed in some of the best plants making a high quality of concrete stone.

87h. Mosaics.—The subject of the surface treatment of concrete would not be complete without the mention of the possibilities which lie in the use of mosaics in large and small ornamental surfaces. These may be glued into a mold on strips of paper where the concrete surface is to be flush with the mosaic itself; or the inlay, consisting of tile or bits of marble or other stone, may be set in places which have been provided in the products when cast. After the product is complete and before the curing period is entirely ended, the grooves or spaces left where inlays are to be inserted are thoroughly wet and grouted in to hold the inlays in place.

87i. Efflorescence.—Efflorescence is ordinarily considered in connection with surfaces because it is a surface disfiguration. The cause, however, goes far back of the surface treatment and lies in the fact that the concrete mixture is not so dense as should be obtained. Efflorescence is, in reality, a disfiguration which is common to brick, to concrete and to natural stone. It occurs, as a direct result of porosity in the material on which it appears. An impervious substance is not subject to efflorescence. When a substance is porous, water which is absorbed dissolves certain salts found in the material; as for instance in concrete block and brick, the salt which is dissolved is principally lime carbonate. A concrete product which soaks water like a sponge after a rain, subsequently dries out generally from exposure to the sun. In drying out, any salt solution is brought to the surface and left there when the water evaporates. The best remedy is in prevention by maintaining a dense mixture. Efflorescence can usually be removed with a solution of muriatic acid and water although this is not always successful.

87j. Air Bubbles.—Air bubbles, like efflorescence, are a part of the subject of density. They are formed when there is insufficient care in placing the concrete. A wet mixture should be spaded against the forms so that these little air pockets will not form. They are also prevented by tapping the molds, or by vibrating platforms, actuated sometimes by machinery. The object of such equipment, however, is not so much to avoid the pinholes, which become surface blemishes, as to produce a dense concrete by consolidating the mass.

The removal of air bubbles is frequently accomplished by tooling the surface when the concrete is removed, and sometimes it is done by filling the surface a cement paste being rubbed in, as is described in connection with rubbed surfaces.

87k. Crazing.—Crazing, or the formation of hair-cracks in the surface of concrete stone, is more frequently encountered in wet mixtures than in dry mixtures; is more common in rich mixtures than in lean mixtures; and it is said by some manufacturers that the tendency to surface crazing diminishes greatly when a well-aged cement is used. The writer has never known but one manufacturer using a wet mixture of concrete who claimed entire freedom from crazing in his products. This manufacturer uses a graded mixture of trap rock.

The reason for the hair-checking is in the greater tendency of the surface of concrete stone to undergo temperature changes, leaving the interior of the stone comparatively unaffected.

These surface cracks do not cause any structural harm, as they extend scarcely more than $\frac{1}{32}$ in. into the stone. They are entirely removed in ordinary tooling treatments to which much of the wet-cast stone (in which crazing is most common) is subjected. While these cracks are believed by some manufacturers to be characteristic of the very rich surface skin of cement which forms on spaded mixtures of concrete or on very rich mixtures of fine material, other manufacturers find that even in tooling this thin mortar skin on the surface, the crazing is not entirely done away with as it may craze again later on. Its only disadvantage is in the fact that it provides minute lodging spaces for dirt. It does not seem possible to say just how this crazing can be avoided in concrete stone, because it is maintained by many manufacturers that two pieces of stone made in the same way, from the same batch and cured in the same way, will not show the same results, one piece being covered in a short time with surface cracks and the other piece entirely free from them. A manufacturer of concrete monuments says that he obviates surface crazing by keeping his products buried in damp sand for 30 days after manufacture. In manipulating the drier mixtures of concrete, manufacturers frequently insist that workmen shall not trowel the surfaces of products after they are molded, this troweling having a tendency to bring the fine cement particles to the surface and result in crazing.

88. Specifications of the American Concrete Institute.—Newly adopted (1917) specifications and building regulations of the American Concrete Institute for manufacture and use of concrete architectural stone, building block and brick provide as follows:

1. Concrete architectural stone and building blocks for solid or hollow walls and concrete brick made in accordance with the following specifications and meeting the requirements thereof may be used in building construction.

2. *Tests.*—Concrete architectural stone, building blocks for hollow and solid walls and concrete brick must be subjected to (*a*) compression and (*b*) absorption tests. All tests must be made in a testing laboratory of recognized standing.

3. *Ultimate Compressive Strength.*—(*a*) In the case of solid stone, blocks, and brick, the ultimate compressive strength at 28 days must average fifteen hundred (1500) lb. per sq. in. of gross cross-sectional area of the stone as used in the wall and must not fall below one thousand (1000) lb. per sq. in. in any case.

(*b*) The ultimate compressive strength of hollow and two-piece building blocks at 28 days must average one thousand (1000) lb. per sq. in. of gross cross-sectional area of the block as used in the wall, and must not fall below seven hundred (700) lb. per sq. in. in any test.

4. *Gross Cross-sectional Areas.*—(*a*) Solid concrete stone, blocks and brick. The cross-sectional area shall be considered as the minimum area in compression.

(*b*) Hollow building blocks. In the case of hollow building blocks, the gross cross-sectional area shall be considered as the product of the length by the width of the block. No allowance shall be made for the air space of the block.

(*c*) Two-piece building blocks. In the case of two-piece building blocks, if only one block is tested at a time, the gross cross-sectional area shall be regarded as the product of the length of the block by one-half of the width of the wall for which the block is intended. If two blocks are tested together, then the gross cross-sectional area shall be regarded as the product of the length of the block by the full width of the wall for which the block is intended.

5. *Absorption.*—The absorption at 28 days (being the weight of the water absorbed divided by the weight of the dry sample) must not exceed ten (10) % when tested as hereinafter specified.

6. *Samples.*—At least six samples must be provided for the purpose of testing. Such samples must represent the ordinary commercial product. In cases where the material is made and used in special shapes and forms too large for testing in the ordinary machine, smaller specimens shall be used as may be directed. Whenever possible, the tests shall be made on full-sized samples.

7 *Compression Tests.*—Compression tests shall be made as follows: The sample to be tested must be carefully measured and then bedded in *plaster of Paris* or other cementitious material in order to secure uniform bearing in the testing machine. It shall then be loaded to failure. The compressive strength in pounds per square inch of gross cross-sectional area shall be regarded as the quotient obtained by dividing the total applied load in pounds by the gross cross-sectional area, which area shall be expressed in square inches computed according to Art. 4.

When such tests must be made on cut sections of blocks, the pieces of the block must first be carefully measured. The samples shall then be bedded to secure uniform bearing, and loaded to failure. In this case, however, the compressive strength in pounds per square inch of net area must be obtained and the net area shall be regarded as the minimum bearing area in compression. The average of the compressive strength of the two portions of blocks shall be regarded as the compressive strength of the samples submitted. This net compressive strength shall then be reduced to compressive strength in pounds per square inch of gross cross-sectional area as follows:

The net area of a full-sized block shall be carefully calculated and the total compressive strength of the block will be obtained by multiplying this area by the net compressive strength obtained above. This total gross compressive strength shall be divided by the gross cross-sectional area as figured by Art. 4 to obtain the compressive strength in pounds per square inch of gross cross-sectional area.

When testing other than rectangular blocks, great care must be taken to apply the load at the center of gravity of the specimen.

8. *Absorption Tests*.—The samples shall be first thoroughly dried to a constant weight at a temperature not to exceed two hundred and twelve (212) degrees Fahrenheit, and the weight recorded. After drying, the sample shall be immersed in clean water for a period of 48 hr. The sample shall then be removed; the surface water wiped off, and the sample reweighed. The percentage of absorption shall be regarded as the weight of the water absorbed divided by the weight of the dry sample multiplied by one hundred (100).

9. *Limit of Loading*.—(*a*) Hollow walls of concrete building blocks. The load on any hollow walls of concrete blocks, including the superimposed weight of the wall, shall not exceed one hundred and sixty-seven (167) lb. per sq. in. of gross area. If the floor loads are carried on girders or joists resting on cement pilasters filled in place with slush concrete mixed in proportion of one (1) part cement, not to exceed two (2) parts of sand and four (4) parts of gravel or crushed stone, said pilasters may be loaded not to exceed three hundred (300) lb. per sq. in. of gross cross-sectional area.

(*b*) Solid walls of concrete blocks. Solid walls built of architectural stone, blocks or brick and laid in Portland-cement mortar or hollow block walls filled with concrete shall not be loaded to exceed three hundred (300) lb. per sq. in. of gross cross-sectional area.

10. *Girders and Joists*.—Wherever girders or joists rest upon walls in such a manner as to cause concentrated loads of over four thousand (4000) lb. the blocks supporting the girders or joists must be made solid for at least eight (8) in. from the inside face of the wall, except where a suitable bearing plate is provided to distribute the load over a sufficient area to reduce the stress so it will conform to the requirements of Art. 9.

When the combined live and dead floor loads exceed sixty (60) lb. per sq. ft., the floor joists shall rest on a steel plate not less than three-eighths (3/8) in. thick and of a width 1/2 to 1 in. less than the wall thickness. In lieu of said steel plate the joists may rest on a solid block which may be three (3) or four (4) in. less in wall thickness than the building wall, except in instances where the wall is eight (8) in. thick, in which cases the solid blocks shall be the same thickness as the building wall.

11. *Thickness of Walls*.—(*a*) Thickness of bearing walls shall be such as will conform to the limit of loading given in Art. 9. In no instance shall bearing walls be less than eight (8) in. thick. Hollow walls eight (8) in thick shall not be over sixteen (16) ft. high for one story, or more than a total of twenty-four (24) ft. for two stories.

(*b*) Walls of residences and buildings commonly known as apartment buildings not exceeding four stories in height, in which the dead floor load does not exceed sixty (60) lb. or the live load over sixty (60) lb. per sq. ft., shall have a minimum thickness in inches as shown in Table I.

TABLE I

No. of stories	Basement, inches	First story, inches	Second story, inches	Third story, inches	Fourth story, inches
1	8	8	..	..	..
2	10	8	8	..	..
3	12	12	10	8	..
4	16	12	10	10	8

12. *Variation in Thickness of Walls*.—(*a*) Wherever walls are decreased in thickness the top course of the thicker wall shall afford a solid bearing for the webs or walls of the course of the concrete block above.

13. *Bonding and Bearing Walls*.—Where the face wall is constructed of both hollow concrete blocks and brick, the facing shall be bonded into the backing, either with headers projecting four (4) in. into the brickwork, every fourth course being a header course, or with approved ties, no brick backing to be less than eight (8) in. thick. Where the walls are made entirely of concrete blocks, but where said blocks have not the same width as the wall, every fifth course shall overlap the course below by not less than four (4) in. unless the wall system alternates the cross bond through the wall in each course.

14. *Curtain Walls*.—For curtain walls the limit of loading shall be the same as given in Art. 9. In no instance shall curtain walls be less than eight (8) in. in thickness.

15. *Party Walls*.—Walls of hollow concrete blocks used in the construction of party walls shall be filled in place with concrete in the proportion and manner described in Art. 9.

16. *Partition Walls*.—Hollow partition walls of concrete blocks may be of the same thickness as required in hollow tile, terra-cotta or plaster blocks for like purposes.

SECTION 3

CONSTRUCTION PLANT

PREPARATION OF CONCRETE AGGREGATES

1. Preparation of Crushed-stone Aggregate.—The preparation of crushed-stone aggregate has grown to be an industry of such size that marked refinements in methods have been introduced in recent years in the better plants. The general scheme, however, is (1) the breaking down of the ledge, by one means or another, into pieces which can be readily handled and fed into crushing machinery; (2) the breaking of these larger pieces in crushers of one type or another; and (3) the separation of the crushed material into various sizes.

1a. Preparation of Site for Quarrying.—When a ledge is located for quarrying, it is necessary to strip off the overburden in order to expose clean rock and to prevent the workings from being filled with dirt and débris. Quarrying is carried on in more-or-less distinct levels one above another, the overburden being stripped back a distance from the top and the ledge quarried down a convenient depth, gradually working backward into the face. A lower terrace will then be started, working backward for a distance, until the vertical face of the first portion is encountered. Successive levels may be quarried in this way, the top ledge being successively stripped back to greater distances and the lower ledges being worked again in the same sequence. It is usually sought to quarry inward from the side face of a ledge whose top is a considerable distance above the side of the crushing plant, in order that the rock may be brought within reach by gravity.

1b. Quarrying.—The general method of quarrying is to bring rock down by charges of explosive set off in holes drilled in the rock ledge. These holes are put down to the depth to which the rock is to be split. The requisite amount of powder is charged into the hole, covered by sand, and fired by means of a fuse or by electricity. In larger operations charges in a line of drilled holes are fired simultaneously by electricity. Gunpowder is the explosive mostly used, although nitroglycerine and dynamite are often preferred both because of the larger quantity of rock which can be brought down per batch and also because of the shattering effect of these quick-acting explosives. Various refinements in minor details are of great importance and have a distinct bearing on the effect of the explosive charge. Even such an apparently small matter as the form of the bottom of the drill hole has a very marked effect. When bored with a hand drill, the hole is triangular at the bottom and the blast in such a hole will break rock in three directions. Explosives in a squared bottom hole have a more distinctly lateral effect. An expert rock man will shoot approximately that portion of the rock which he desires to bring down.

1c. Drills.—*The jumper* is a drill similar to that used for drilling holes for plug and feather work in dimension-stone quarries, except that it is larger and longer. It is usually held by one man, who rotates it between the alternate blows from hammers in the hands of two other men. *Churn drills* are long heavy drills measuring from 6 to 8 ft. in length. They are raised by a workman, let fall, caught on the rebound, raised and rotated a little and then dropped again, thus cutting a hole without being driven by hammer. They are more economical than jumpers as they cut faster and make larger holes. *Machine rock drills* bore much more rapidly than hand drills and are more economical in most operations for preparing rock for concrete aggregate, where the work is of sufficient magnitude to justify the preliminary outlay. They drill in any direction and can often be used in boring holes so located that they could not be bored by hand. They are worked either by steam or by compressed air, and may be either percussion or rotary. The action of a percussion drill is the same as that of a churn drill already described, a piston moved by steam or compressed air being attached to the drill in such manner as to make a stroke at every complete movement of the piston, an automatic device rotating the drill slightly at each stroke. *Rotary drills* may be either shot drills or diamond drills and they are more often used for prospecting than for drilling holes for explosives, inasmuch as in their use a core is obtained which is of value mainly as indicating the strata penetrated.

1d. **Stone Crushers.**—Crushers are of two general kinds: jaw crushers and gyratory crushers. The former type is better adapted to small or portable plants, while the latter is used in larger operations. A convenient size of jaw crusher for a portable plant is about 10 by 16 in. This will crush from 50 to 100 cu. yd. per day, depending upon the size of the stone to be crushed.

Both types of crushers have means for regulating openings so that by using a proper opening together with a proper crushing plate, almost any size of crusher product can be obtained, the size being limited by a small opening at the crusher-plate end of the machine. The output of any crusher will depend to a large extent upon the plant arrangement. Necessarily also, the more finely a stone is crushed the more work must be done upon it and the less the output. In deciding upon the type of crusher to be installed at any plant, it is best to get comparative estimates, costs, and tables of weight and output from manufacturers of various types of apparatus, balancing the advantages of one against those of another and finding the machine best adapted to the purpose in mind. Machinery of this kind is constantly being improved and changed in type, so that accurate data representative of the latest practice is difficult to give.

1e. **Screening and Grading of Crushed Stone.**—As stone comes from the crusher, it is carried by some elevating means, usually a bucket elevator, to revolving screens fixed over bins. Elevating and screening plants can be furnished in either portable units (in which case they are so arranged that they can be readily dismantled for transportation) or in fixed units with the machinery more massive. The usual type of screen is a rotary screen inclined on its longitudinal axis, screens of various-size holes disposed successively throughout its length forming the screen barrel. The stone as received from the elevating buckets is fed into the fine screen end. Through the openings in this screen the very fine materials, dust, etc., are taken off; and as the stone progresses down the screen barrel, the several sizes fall into bins arranged below them, from which they are drawn off into conveyances as required. The storage bins vary in size from those having a capacity of 13 tons to those having a capacity of 50 tons. In some of the modern types of bins, provisions are made so that a bin may be raised to a height sufficient to permit wagons being driven under gate spouts.

1f. **Washing Crushed Stone.**—Crushed stone is often covered with a tenacious film of dust of which it is very hard to get rid. Although seldom if ever done, it would be advantageous to wash stone after crushing and screening, inasmuch as this dust is of such size that it is impossible to coat it with cement, and so tenacious that it prevents the cement from being in contact with the aggregate.

1g. **Crushed Limestone.**—Limestone crushes with a flaky fracture and a considerable amount of dust. If the finer screenings are to be used, it is well to roll them between rollers, inasmuch as this flaky fracture renders them extremely friable and unsuited to the production of concretes impervious to water or of high strength.

2. Screening of Sand and Gravel.—Screening of sand and gravel may be done by hand or by machinery. Hand-screening is adapted to small jobs and light work. Power-screening is adapted to handling larger quantities of material. Screening of gravel or sand containing large amounts of coarse material can be done more cheaply by mechanical than by hand means, using either revolving screens or fixed screens placed upon an incline. The type of screening equipment is largely determined by location and natural topography, and the availability of power. Revolving screens are most effective and economical for large quantities, the material being conveyed by bucket elevators to the screens and then falling into bins provided with [illegible] convenient for unloading.

Large and elaborate plants for screening of sand and gravel are being installed in increasing numbers in situations where dredging of these materials from river beds is practicable. Many of these plants are now turning out aggregate of exceedingly high quality, the screening and grading operation being incidental to the elevation of this material. Necessarily also these materials are washed while being screened and graded.

3. Washing of Sand and Gravel.—Gravel is not infrequently coated with a tenacious film of material which, if not removed, may greatly reduce the strength of the concrete. Sand also is not infrequently contaminated with clay, loam, or slit coatings of organic matter. Such coatings are responsible for a great deal of difficulty and many defects in concrete; and their removal, while not easy even by washing, is decidedly essential.

Various means have been proposed for washing sand and gravel. Attempts have been made to wash them in piles with a hose but this is always difficult and usually impossible to carry out properly, inasmuch as the materials washed from the upper layers are carried down by the stream of water to the lower portions, where they are rarely dislodged. Another scheme is shoveling sand into one end of a V-trough, washing it down with a hose and endeavoring to carry off the finer materials in the runoff water. For large quantities of material a combination of a trough of this kind with an ejector has been successfully used. A concrete mixer has also proven adaptable to this kind of work, water being turned in and the mixer allowed to overflow while the drum is in rotation. Necessarily all of these processes are somewhat expensive and add to the cost of the aggregate, but where tests indicate that the quality of concrete will be seriously affected by uncleanness of sand or stone, it should be undertaken without hesitation even at the increased cost.

HANDLING AND STORAGE OF MATERIALS

4. General Considerations.—The handling of materials, and their storage and disposal is of great economic importance in concrete work. Hundreds of tons are removed, loaded, transported, unloaded, piled and elevated oftentimes to considerable heights before the making of concrete has begun; and all this material as concrete must be rehandled one or more times before it is delivered to forms. The engineering problems prior to placement are, therefore, highly important; and their adequacy not only determines rate of progress, but their efficiency may decide whether there is to be a quick turnover or a slow one, with corresponding profit or possibly loss.

It is axiomatic that gravity should be employed in such work whenever possible by utilizing natural advantages of site to the fullest extent. It should also be borne in mind that there is approximately twice the quantity of stone to be handled as sand; and twice the quantity of sand as cement. In planning a job, careful routing and proportionate disposal of these materials should therefore receive early and adequate attention.

It is also trite and almost unnecessary to say that sufficient storage room should be provided for supplies of materials, ample to insure continuous prosecution of work when desired. Shipments may be held up through any number of unexpected and unforseen happenings, so that unless there is reserve supply, a shutdown must result. Specifications often stipulate that a certain reserve quantity of material shall be maintained on the job, but even where this salutary provision is omitted, contractor and owner's engineer alike, for their individual and mutual protection, should have regard for this very important feature.

It would seem that remarks as to providing proper care for materials on delivery to the work were equally unnecessary, yet disregard of these important matters is seen every day. Cement *always* reacts with water, whether such water comes from the mixer measuring tank, or whether it comes from rain, or from condensed steam, or from dew, or from water absorbed from the ground. Cement, therefore, should always be stored in weather-tight houses, having floors raised at least 6 in. above the ground; and any cement directly at the work should be kept off the ground and carefully covered with an impervious covering, whether or not the atmosphere seems damp, or rain actually falling, or the weather threatening.

5. Storage and Care of Stone.—One of the essential qualities of large aggregate for concrete is cleanness. Stone, therefore, should not be dumped indiscriminately, so that when it is rehandled, dirt and rubbish are carried into the concrete. A thick layer of sand, or preferably a platform of planks should be placed on the ground before stone is deposited; and not only will this precaution be found to keep the material clean, but it furthermore will often pay for its own cost, both in the quantity of stone saved by this means in rehandling and also in assurance as to freedom of the concrete from deleterious foreign matter.

6. Shoveling Materials Directly from Cars to Ground.—Where a siding extends to the construction site, sand and stone may be shoveled directly from the cars to the ground, which should have been previously smoothed. That portion of the ground designed for the stone should first be spread with a layer of sand at least 1 in. thick, this layer serving to keep the stone clean and also working economy in subsequent shoveling. In order that materials may be piled high and the track be kept clear, a bulkhead may be built of a double row of 2 by 12-in. plank with 1 by 3-in. cross-ties, having stops as indicated in Fig. 1. This method requires no fitting or cutting of the plank.

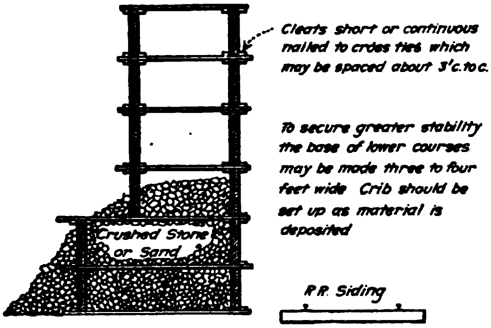


Fig. 1.

7. Storage and Care of Sand.—Sand should not be dumped directly on the ground, but for like reasons, should receive care similar to that described for stone. Although sand is a common material—"common as dirt"—all dirt is not sand and dirt rarely, if ever, makes good concrete. It is only by attention to such seemingly small matters that a job can be well organized, made profitable, and the best results secured.

8. Conveyance Economics.—The type of vehicle in which sand and stone are conveyed exercises a large influence on the economy of a job. As an excellent employment of gravity to cut labor costs, bottom-dumping conveyances, whether horse-drawn or motor-driven, or railroad cars, are conspicuous for economy. In certain situations, of course, gravity cannot be employed. In these the use of a locomotive crane with grab bucket is increasingly prevalent; and the type of car or barge used to convey materials within reach of the bucket will have a pronounced effect on the amount of material the bucket can handle in a given time.

As an example of comparative conveyance economies, consider the following with respect to two types of railroad cars:

Two quarries are located on different railroads. Railroad "A" is prepared to supply only hopper-bottom cars; railroad "B" can furnish only flat-bottom cars. Assume gravity dump to be impossible, and that the plan of operation involves unloading the stone by shoveling. Assume also that one quarry, on railroad No. "A" quotes \$1.30 per cu. yd.; and the other, on railroad No. "B" quotes \$1.32 per cu. yd. It will be found cheaper in the end to order material at \$1.32 per cu. yd. in flat-bottom cars for the reason that more than the difference in first cost can be saved by lessened labor in unloading.

The reason for this is self-evident. A good man, under efficient superintendence, can unload 2 cu. yd. per hr. from a flat-bottom car, as against 1½ cu. yd. per hr. from a hopper-bottom car—a saving of at least 3 cts. per cu. yd.—and this proportional saving increases as less efficient labor is employed. These figures represent average results. A good man working under a yardage or carload system will perhaps average 50% more than the above, but the ratio will not change between the two types of cars. The often-neglected matter of shoveling, therefore, becomes a matter of moment.

9. Unloading Economies.—Low-side cars are more economical for unloading than high-side, except where a wagon loader (Figs. 2 and 3) is used. Where such a loader is employed,

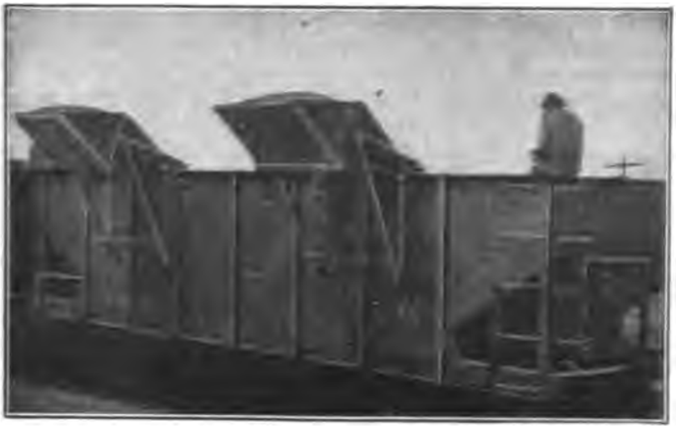


Fig. 2.

the car side to which it is attached should either be of sufficient height to give wagon clearance, or stakes must be provided to raise the hopper to the proper level. The use of a hopper expedites unloading the car, relieving possible demurrage charges and cutting down team and wagon hours. One, two, or three hoppers may be attached to a single car and one or more men put in the car for each hopper, depending upon the number of teams available. Hoppers should be filled, ready to dump when the teams range alongside the car; and the more quickly materials are discharged into wagons, possibly even without stopping, the more economical will be the process.

Fig. 3.

10. Proper Size and Type of Shovel.—Although refinements in efficiency can be carried to extremes, a potent factor in securing a proper output of work in so simple an operation as shoveling is a proper size and type of shovel. A good worker will always look for a good shovel, which of itself is eloquent testimony as to the importance of this tool. Frederick W. Taylor found that a shovel adapted to a load of 21 lb. gave the best results. Such a shovel corresponds to the standard No. 4 size shovel. The No. 3 shovel is somewhat smaller. The size of shovel should always be chosen with reference to the weight of the materials to be handled.

The better the quality of material in a shovel, the longer will it last. A shovel is a conveying tool and, at the same time, it is a cutting tool no less than a chisel or a drill. No contractor would consider using an inferior steel for such cutting instruments, yet in buying shovels many consider first cost only, having little regard to performance or endurance. It is elementary economics to balance the value of a shovel costing $9 a dozen which lasts about 2 months, against the value of a shovel costing $5 a dozen, which lasts 1 month or even less, particularly when its performance, in addition to its absolute existence, is taken into account.

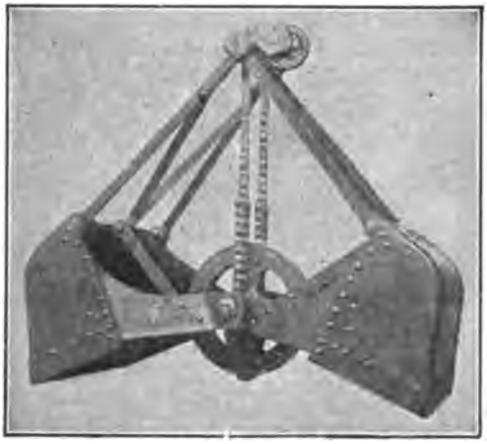


Fig. 4.

11. Clam-shell Buckets.—When gravity dumping is precluded, a most efficient unloading device is the clam-shell bucket (Fig. 4), hung from derrick or locomotive crane. With this combination a skilled engineman can unload a large quantity of material in a day, but care should be taken that the bucket chosen is one suited to the work. Some buckets tend to ride

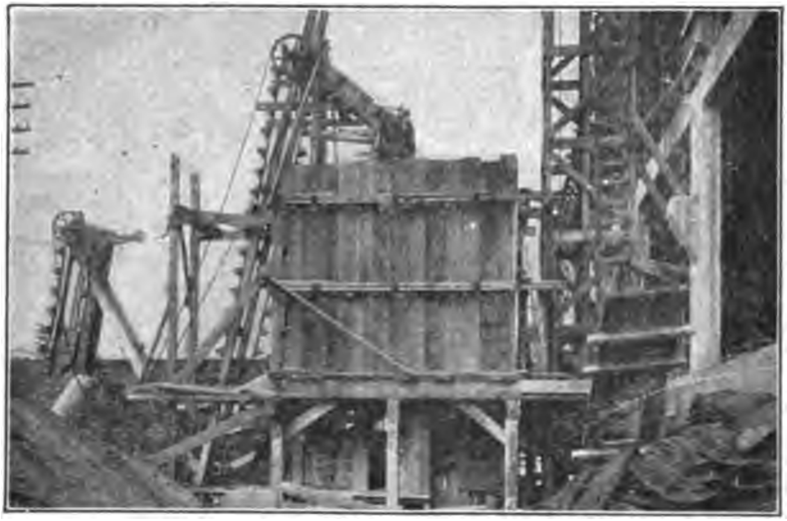


Fig. 5.

the material when it is at all resistant, while others are so designed that they fill to the capacity at each bite, with proportionate efficiency and economy. The cost of equipment of this type is necessarily large, so that proportionate care should be taken in choosing the type of bucket. No matter how excellent the derrick, or crane, or engine, or operative, a poor bucket will negative them all and run up handling costs at an alarming rate. An orange-peel bucket is a digging rather than an unloading tool and is not well adapted to the usual rehandling of materials.

12. Bucket Unloaders and Conveyors.—There are an almost endless variety of elevating bucket unloaders each suited to some particular need. When the quantity of materials to be handled and stored is considerable, a bucket elevator installation (Fig. 5) may prove very economical, the more particularly as it makes possible the use of gravity discharge from conveyance to elevating buckets as well as gravity discharge from storage bins through measuring hoppers to the mixer. The installation and power equipment required for such elevating mechanisms is not necessarily expensive or extensive. Some lighter types are shown in Figs. 6 and 7, and from these, the plant may range up to the heavier types, capable of handling very

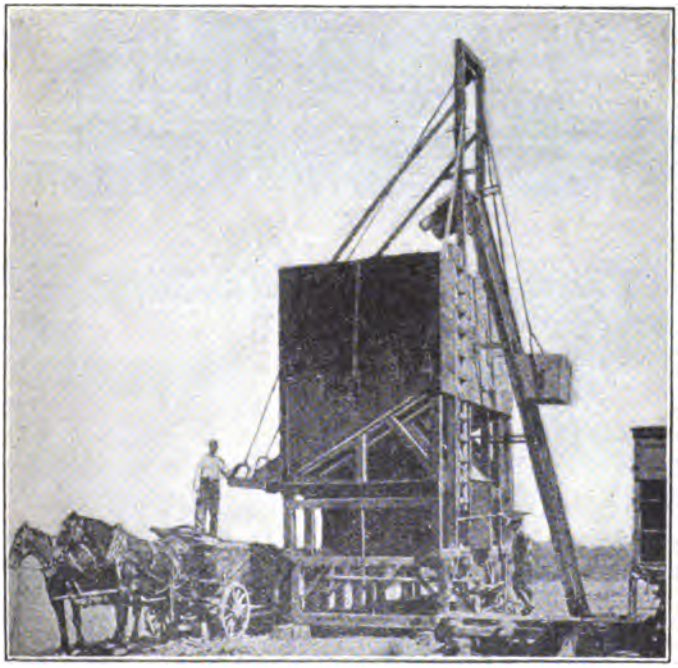


Fig. 6.

large quantities. In each individual layout, the needs should be studied and the economy or lack of it determined. No hard and fast recommendation can be made that will apply to all situations.

13. Belt Conveyors.—In certain situations an endless belt, grooved to V-form by pulleys over which it runs, furnishes a rapid and excellent means of handling raw concrete materials. It cannot, however, raise the materials so nearly vertically as can the bucket conveyor, its elevating slope being limited to about 1 vertical foot for every 2½ horizontal feet. The applications of conveying belts are without number, but, as in the case of the bucket conveyor, no blanket recommendation can be made. The manufacturers of conveying machinery are in position to advise with respect to types, applicability, and relative economies of various types of conveyors for any given set of conditions, and should be consulted for individual needs.

14. Storage and Handling of Sack Cement.—Cement, because of ease in handling, is usually ordered in cloth or paper sacks, each holding 94 lb. Cloth sacks are less liable to rupture than are paper sacks, but their cost is greater. When sack cement is received, a chain of laborers is formed between cars and storage house, each man shouldering one sack. Economies in such procedure may be introduced by insuring ready entrance and exit both to cars and to storage

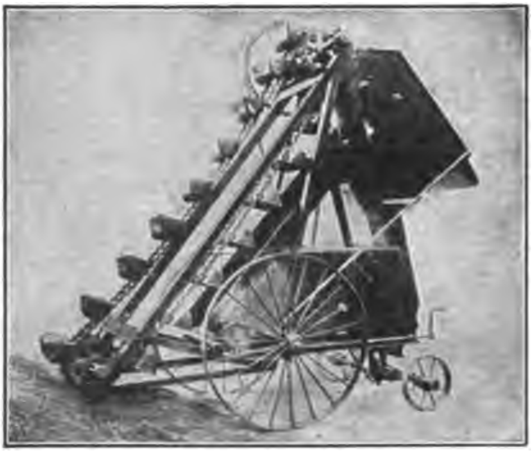


Fig. 7.

houses, with adequately wide gang planks to cars, so that confusion and interference may be prevented. The speed at which men will work is then a question of the personality and driving force of the superintendent or foreman.

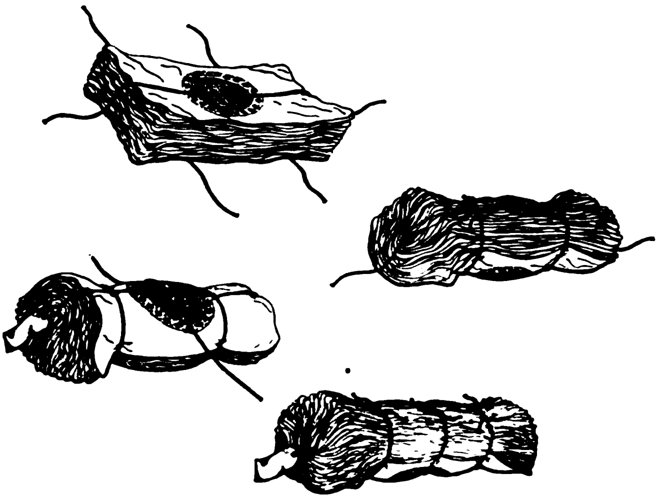


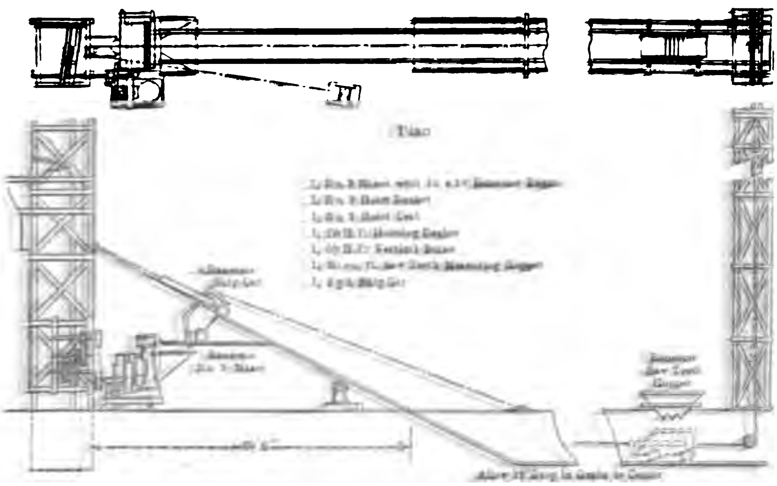
Fig. 8.—Proper method of bundling cement sacks.

Upper left: A bundle of 50 cement sacks laid out flat with 2 ropes 40 in. long, under the pile, and with a longer rope of about 8 ft., resting on top.
Upper right: The first operation in bundling is to bring two of the ropes over the pile, as shown, tying tightly.
Lower left: After the short ropes have been tied, the bundle is turned over, and the long rope brought around and crosses in the middle of the bundle, engaging first the shorter ropes.
Lower right: Bundle of 50 cement sacks tied and tagged ready for shipment.

15. Bundling and Storage of Empty Cement Sacks.—Inasmuch as each cement sack has a return value of 10 cts., it is important that none should be lost, damaged, or destroyed. Furthermore, the cleaning, bundling, and shipping of these sacks becomes an operation of importance on large jobs. Much cement is lost when bags are insufficiently shaken, and this retained cement further adds to the weight and bulk of sacks bundled for return. The proper method of bundling sacks for return shipment is shown in Fig. 8. In showing a bundle of 50 sacks, it has been with the purpose of emphasizing the greater convenience in handling with equal advantages as a counting unit of the 50-sack bundle, rather than the 100-sack bundle.

16. Storage and Handling of Water.—In most instances water in pipes is within reach of a concrete job. Where this is not so and water must be pumped, reservoirs of adequate capacity properly protected against contamination should be supplied. From such reservoirs, water can be distributed in pipes to be used as needed. The storage of water is so simple and so easily carried out that the needs of each individual situation are of greater determining importance than any particular type of pumping or distributing apparatus.

17. A Typical Installation.—Fig. 9 illustrates storage arrangements and handling facilities as installed in connection with a large building for the Singer Manufacturing Co. at Elizabethport, N. J. The available storage space in this instance was a narrow strip 20 ft. wide between



Elevation

Fig. 9

the railroad track and the building, of which the work in question formed an extension. A trench was made, extending the length of this available space, and this was sheathed and converted into a tunnel by a covering of 2-in. plank. In the bottom of the trench was laid a suitable track for the operation of a skip car. On the ledger pieces were mounted two saw-toothed measuring hoppers, fitted with wheels.

Materials were unloaded from the cars and piled over the trench, sand and stone in alternate piles. The saw-toothed measuring hoppers were kept continually against the toe of the sand and stone piles to facilitate charging by breaking down of the face of the piles. Four men in this way handled the materials for about 30 cu. yd. of concrete per hr. Materials were automatically dropped from the measuring hoppers into the skip car as it passed below the hoppers on its way to the mixer. The A-frame shown in Fig. 10 has been used to good advantage as a substitute for the trench. With this arrangement one man only was needed to charge the measuring hopper.

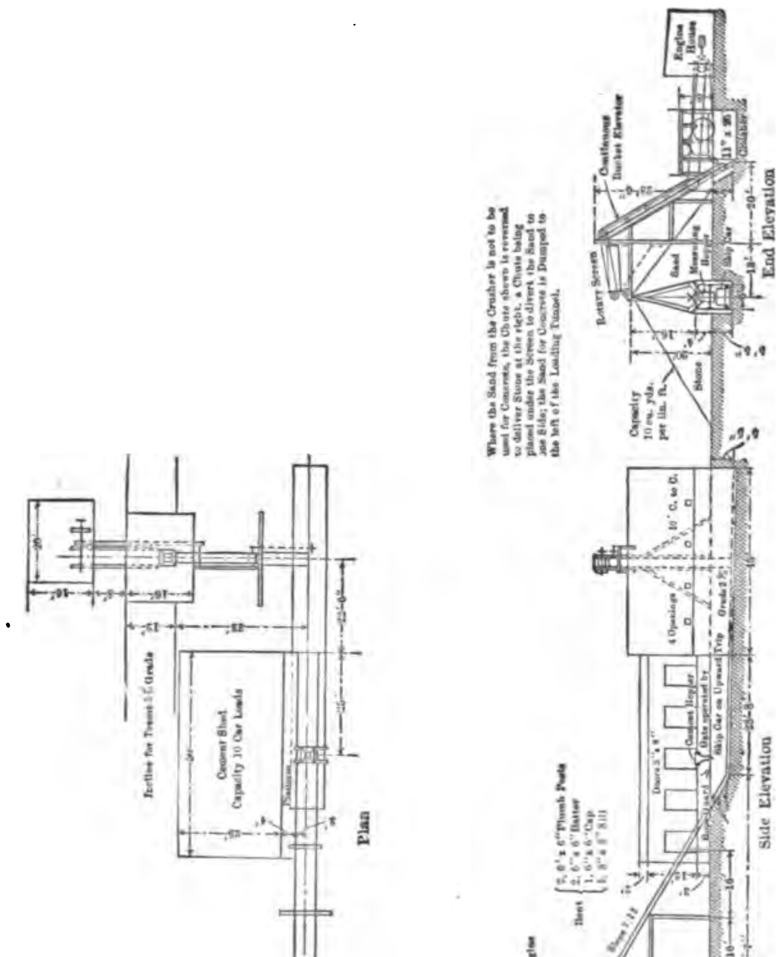

Plan
Cement Shed
Capacity 10 Car Loads
Side Elevation
End Elevation
Rotary Screen
Continuous Bucket Elevator
Engine House
25 H.P. Hoisting Engine
12 Sheave Wheel
Capacity 10 cu. yds. per lin. ft.
Stone
Sand
Cement Hopper
4 Openings
10' C. to C.
Where the Sand from the Crusher is not to be used for Concrete, the Chute shown is reversed to deliver Stone at the right, a Chute being placed under the Screen to divert the Sand to one Side; the Sand for Concrete is Dumped to the left of the Loading Tunnel.

FIG. 10.

CONCRETING PLANT

18. Plant Economics.—Necessarily much of the plant for the handling and storage of materials must be considered as a part of the concreting plant proper. The economies of the whole plant therefore depend upon the individual and collective economies of its elements. The main factors affecting these are *first cost, cost of installation, cost of operation, cost of maintenance, cost of removal, salvage, and interest on the investment.*

18a. First Cost.—First cost of plant includes many items in addition to prices for machinery. Bonus for quick delivery, where equipment is required in a hurry; express charges; tracing charges; and a hundred other items all swell the quoted figures. And it is not always best to consider first cost too closely. A plant that is cheapest in first cost may not be the most economical; and labor losses due to delay speedily offset differences in price between good and poor equipment. Furthermore, low costs of operation and maintenance with higher salvage returns still further reconcile any disparity.

18b. Cost of Installation.—Cost of installation varies with the character of the plant, cost of labor, location of the work and a variety of factors which must be separately considered for each situation.

18c. Cost of Operation.—Cost of operation depends both upon plant arrangement and upon organization. The concreting plant should be of a type, size, capacity, and arrangement to permit continuous operation during working hours, assuming an organization so coördinated as to make this possible and desirable; and the character and arrangement of plant will depend to a large extent upon local conditions, such as contour of the ground, class of construction, manner in which materials are delivered to the site, total yardage to be placed, time limit, bonus, penalty and other financial considerations which permit the use of equipment more or less expensive and elaborate. In addition, attention must be given to the time of the year during which the work is to be done, the normal temperature at that season for the particular locality, and the amount of land available for plant and material, since storage is always a factor in operation.

18d. Cost of Maintenance.—Cost of maintenance includes upkeep of machines, repairs, oil, etc., this being greater or less according to the mechanical excellence of the plant and to its disposition and treatment.

18e. Cost of Removal.—Cost of removal includes clearing the site of the plant and its appurtenances after completion of the work. This cost will vary, according as more or less of the plant is sold or junked, with proportionate lessening of care and labor required in loading on cars, and of transportation.

18f. Salvage.—The salvage value of machinery is always problematical. It usually is worth what can be obtained for it. Certainly, depreciation on contractor's machinery is very large and most estimates of salvage value should be liberally discounted.

19. Balancing the Plant.—The general layout of the work will probably be the determining factor in the choice of means adopted for carrying out each portion of the work. The total yardage of concrete will also have a pronounced effect, possibly suggesting two or more separate installations of medium size, or a single installation of greater size, or a number of smaller mixers placed on different parts of the work. Various factors must be balanced one against the other and various layouts planned, with a following through from delivery of raw materials to delivery of concrete in the forms, with juggling of one scheme with another until the most advantageous result, consistent with allowable cost, is secured. Careful planning of plant before starting the job is well repaid in results, and a well-balanced plant is far more profitable than one poorly balanced. Installation of a mixer of double the capacity of the charging facilities, or of a fraction of the capacity of the handling facilities for the mixed materials is sheer waste.

20. Typical Plants.—Some typical examples of plants which have proven successful in service, are given in the following paragraphs:

Fig. 11 shows an arrangement on the work of Cramp & Co., Philadelphia. It will be noted that materials are delivered in bottom-dump wagons upon the incline, and pass by way of bucket elevator to the bins above the mixer. Once in the bins, it is a gravity process through measuring hopper to mixer. Messrs. Cramp & Co. report 289 cu. yd. with this plant in 8½ hr.,

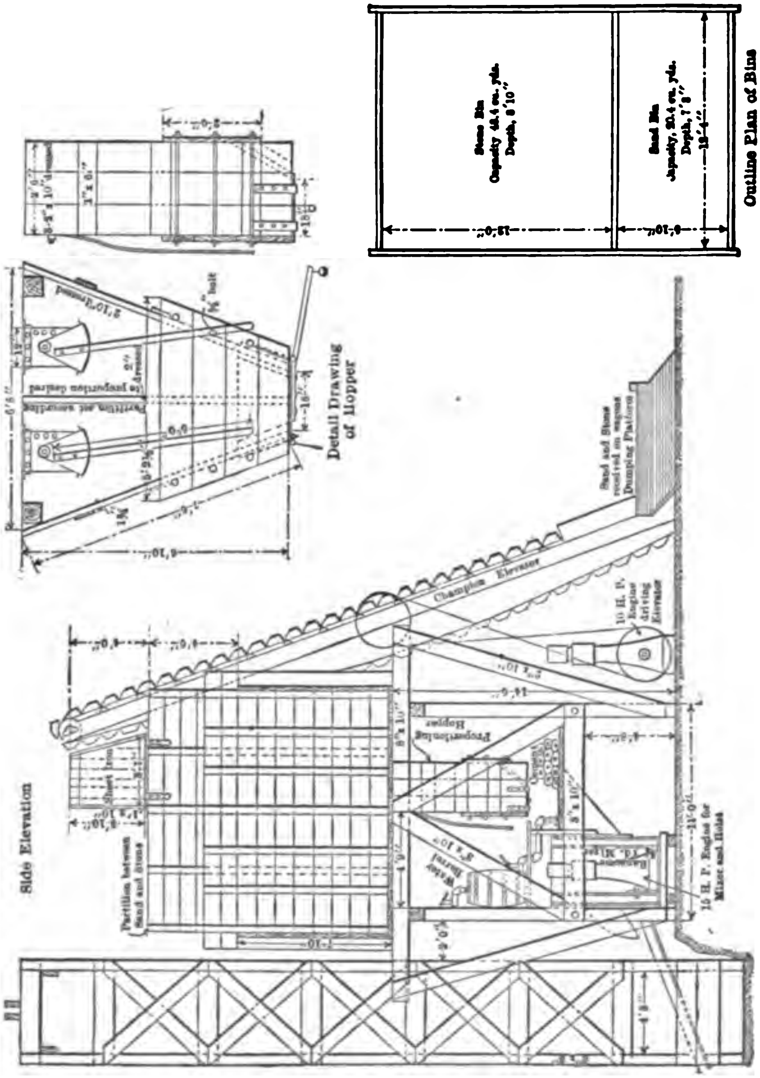


FIG. 11.

with a crew of eight men, and covering all handling from wagons and cement storage to delivery bin on hoist tower. The mixer is ½-yd. capacity.

Fig. 12 illustrates an arrangement adopted by the Turner Construction Co. on the Bush Stores, South Brooklyn. Materials were delivered to the work in standard cars, and unloaded by shovel into special hoppers as indicated. These hoppers were readily portable, and each hopper had a capacity for approximately 1 cu. yd. Materials were drawn off as required into cars with a capacity of 6 cu. ft., and wheeled to the mixer which was set at a lower level so that the fixed hopper was on a level with the ground. The bumping post *A* facilitated discharge of carts into the hopper. Carts were wheeled up against the bumper when a slight lift on the handles did the trick.

Fig. 13 illustrates the plant used in the erection of the buildings for Foster Armstrong Co. at Despatch, N. Y. The work on these buildings was carried on through the winter months and the bins indicated provided the readiest means for heating the materials to the desired temperature of 90 to 100°F. An auxiliary measuring tank took care of the salt solution used in the concrete mixture. Steam coils also served to warm the water used in mixing the concrete. When mixed, the concrete was placed immediately; in no case more than 10 min. elapsed. When the concrete had been placed, it was protected against the action of frost by a solid wood covering, blocked up at least 6 in. above the surface of the floor in a manner to permit free

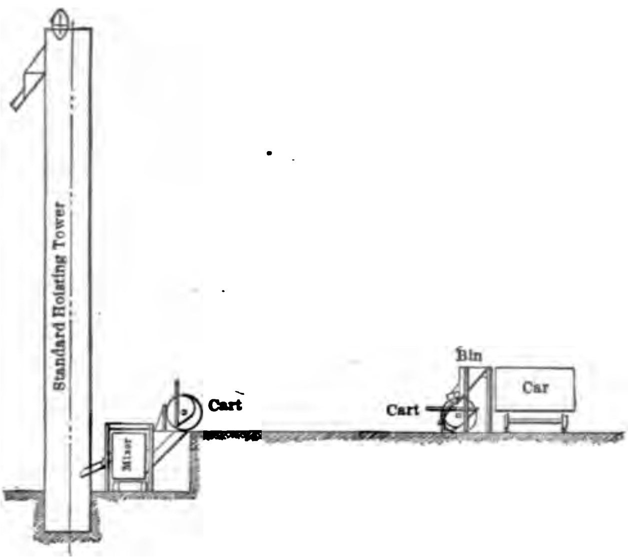


Fig. 12.

circulation of air beneath the covering. Heat was introduced beneath the floor (or in the case of ground floors, beneath the board covering) by means of steam coils and salamanders, provision being made for the escape of sufficient steam beneath the covering to prevent premature drying out of the concrete. Salamanders were sprinkled freely with water, thus producing the necessary amount of moisture, and small openings were left in the floor slab to permit the warm air to circulate over the upper surface of the floor. The sides of the floor were protected by canvas curtains which extended downward; to the floor next below.

There were placed beneath the floor and beneath the panels on top of the floor, at intervals of 10 ft., self-registering thermometers, which in no case showed lower than 32°. This temperature was maintained until the test cubes which had been allowed to set on the floor and beneath the top covering showed the strength used as a basis for the design.

The extra plant involved in carrying on winter work involves considerable outlay, and work in freezing weather should not be undertaken without a thorough understanding of all that is involved.

Fig. 14 indicates a more or less elaborate plant, designed for large work, or for cramped quarters. The plant consists of a suitable bucket elevator, designed to handle the entire aggregate. This elevator discharges the materials into trough screens as indicated. While passing through these screens, the coarse materials are washed by a flow of water applied at the head, and the sand is still more thoroughly washed by passing through the water boot and inclined worm. The washed materials are discharged from the first flight of trough screens upon inclined troughs leading to a fixed measuring hopper at the mixer. The lower end of the

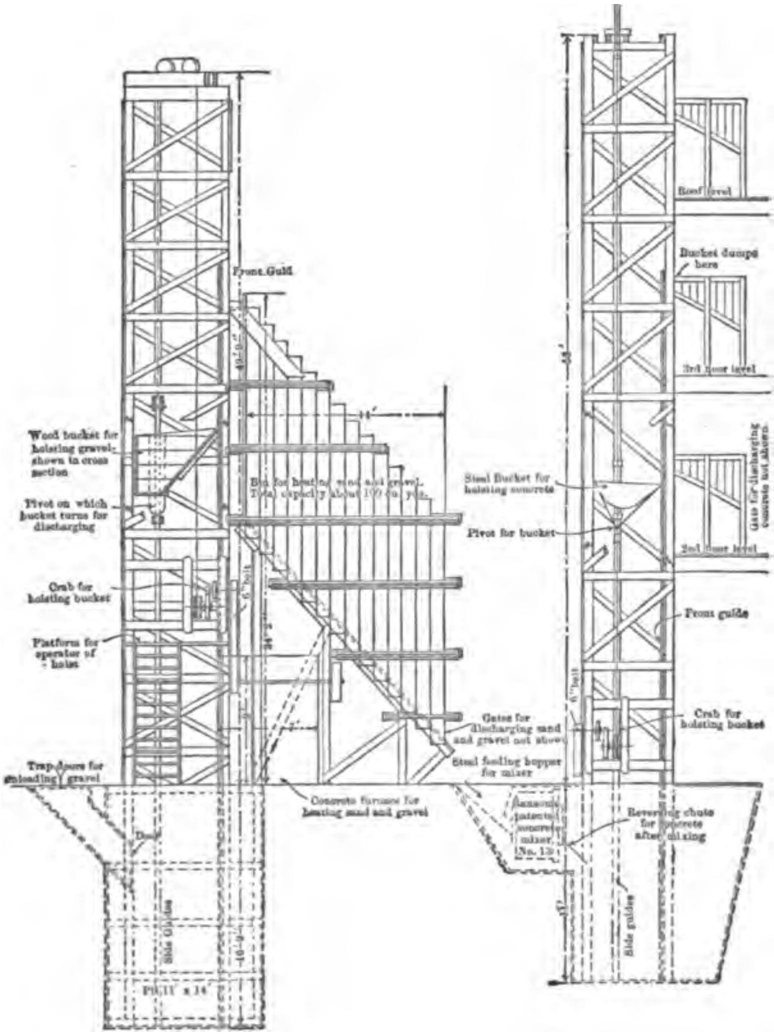


FIG. 13.

troughs is fitted with a gate to control the flow. Such excess material as cannot be cared for by the measuring troughs falls to the ground in piles as indicated. Two belt conveyors operating in tunnels beneath the piles permit ready draft against these reserve piles, delivering materials to the original elevator, and thence to the measuring troughs. With such a plant it is a simple matter to handle upward of 50 cu. yd. per hr. with four men.

Figs. 15 and 16 illustrate two set-ups of practically the same plant, and illustrate forcibly the importance of proper arrangement. With the arrangement shown in Fig. 16, 17 men handled 72 cu. yd. in $4\frac{1}{2}$ hr. With the arrangement shown in Fig. 15, 15 men handled 87 cu.

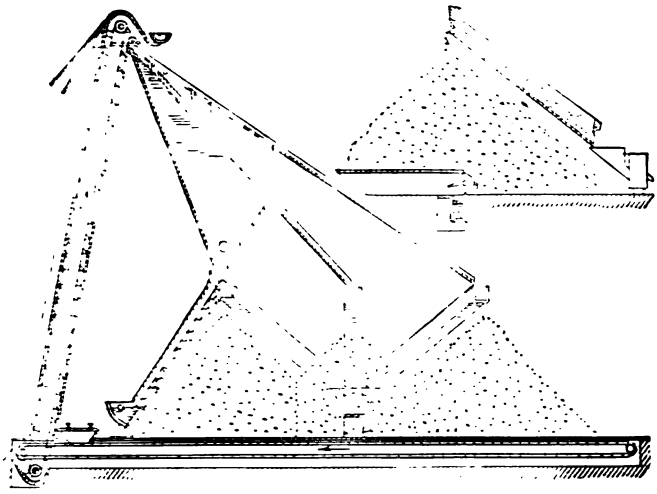


Fig. 14.

yd. in $4\frac{1}{2}$ hr., a saving of approximately 13 cts. per cu. yd. In Fig. 15 the runway and platform are too small, with the result that the men interfere with each other, and cannot work to

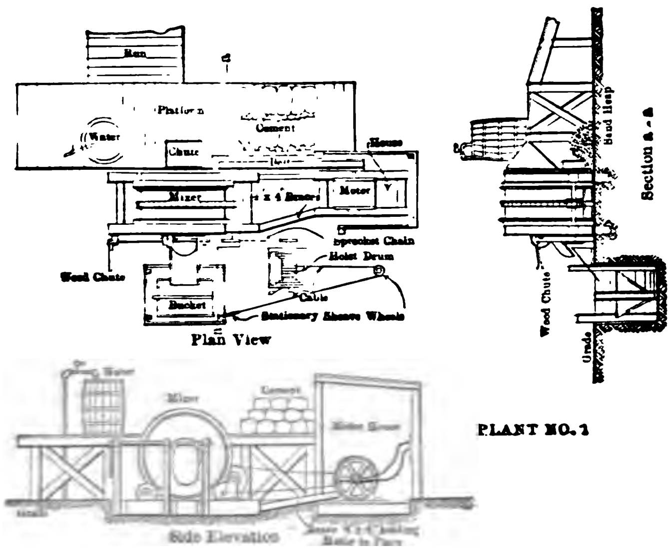


Fig. 15.

advantage. Furthermore, the use of the feed chute involves assembly of the batch in the mixer drum. Water was fed to the machine a bucketful at a time, requiring an extra man for this purpose. In Fig. 16 a charging hopper is substituted for the feed chute, and both platform and runway are increased in size. Water is fed to the mixer through a pipe, and all operating levers are controlled by one man. Provision is also made to take care of any material working down beneath the mixer, with the result that wear and tear on journals, etc., is reduced.

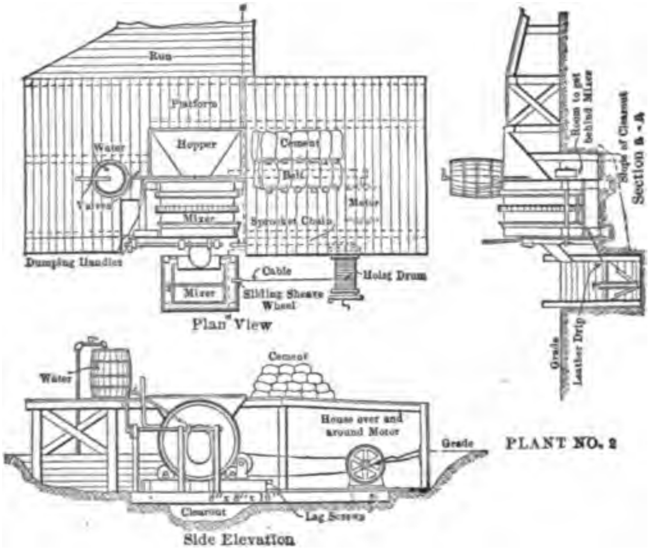


Fig. 16.

The difference in cost of the two arrangements amounted to $59. The illustrations are taken from actual experience, and indicate results secured under different superintendents.

21. Machine vs. Hand-mixing.—Except in relatively small quantities, hand-mixing of concrete is not to be economically considered. Furthermore, hand-mixing is inferior to ma-

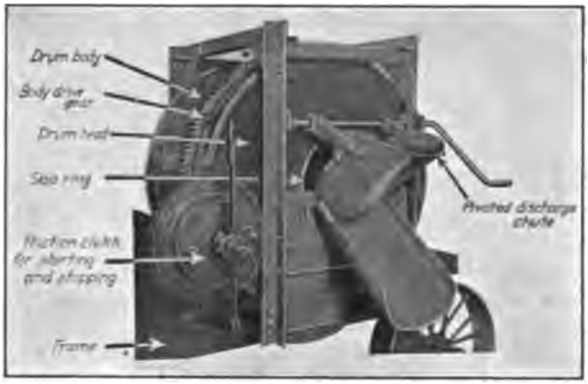


Fig. 17.—Drum mixer.

chine-mixing, with no comparison in quantity output. The province of a mixing machine is essentially the thorough incorporation of materials—one of the fundamentals in the production of sound, enduring concrete. Mixing, therefore, should be accorded the respect due its impor- and the best possible means chosen for its accomplishment.

22. Types of Mixers.—The general types of mixers which have endured and are on the market at the present time may be classified as *drum mixers, trough mixers, gravity mixers, and pneumatic mixers.*

22a. Drum Mixers.—Drum mixers (Fig. 17) are essentially of a type, differing mainly in excellence of mechanical construction and arrangement. The action of all of them

Fig. 18.—Low charging drum mixer.

is about the same so far as mixing is concerned, the operation being accomplished by agitation, lifting, and pouring of the several materials by blades and scoops attached to the inside of the mixer drum. With the exception of tilting mixers, discharge of the materials from the drum is accomplished by inserting a trough into one side of the drum, in such position as to catch the concrete as it is poured from the mixing buckets. Minor differences in charging mechanisms and arrangements are to be noted in different makes, but the action of all is essentially

Fig. 19.—Small pot mixer.

the action of a churn, in which capacity they would function if filled with cream, instead of with stone, sand, cement, and water.

Of the low-charging mixers, the mixer shown in Fig. 18 is typical. Small pot mixers such as shown in Fig. 19 are excellent for small work.

22b. Trough Mixers.—Trough mixers are paddle mixers of one type or another.

They may be batch mixers of the shoveling type (Fig. 20), or continuous mixers (Fig. 21), in which a sectional screw rotates in an open trough. Continuous mixers have not met with general favor as have batch mixers since many engineers object to these mixers on the grounds of uncertainty of mixing operation.

22c. Gravity Mixers.—Gravity mixers are essentially a series of large funnels or pans suspended one above another with bottom gates which can be opened successively, permitting materials to flow from one into the other with incidental mixing to a greater or

Fig. 20.—Batch mixer of the shoveling type.

less extent. Gravity mixers are often urged in preference to power-driven mixers on grounds of cheapness in operation and low first cost, permitting their being scrapped when worn; but many engineers do not advocate their use because of the inherent uncertainty of their mixing operation and oftentimes the requirement of deterimental quantities of water to prevent the mass sticking in the pans.

22d. Pneumatic Mixers.—Pneumatic mixers have been developed by various inventors. At the present time there are two main types on the market. In some of these

Fig. 21.—Continuous mixer.

machines premixing is had before delivery, either mechanically or by the agitation of air pressure, while in others the charge is introduced into a chamber, dependence for mixing being placed on what may happen in transit through pipes under the delivering air pressure. Pneumatic mixers have their own particular field—that of placing concrete in forms where access is particularly difficult—but because of the large compressor plant which must be installed for each mixer, and for other reasons which are valid and of importance in many classes of work, use is relatively restricted.

23. Machine Mixing.

23a. Time of Mixer Operations.—Considering the concreting plant proper as an installation for mixing together raw materials to form concrete, the plant cycle can be considered as complete in three operations, viz., charging, mixing, and discharging.

In charging and discharging the mixer, a time limit is imposed both by the physical laws governing the flow of materials from one container to another, and also (in the case of power-loading, or side-loading hoppers in particular) by the physical limitations of operatives and of the mechanism itself. As plant refinements are given consideration (particularly with regard to the gravity loading of measuring or charging hoppers from overhead bins) this loading time is diminished; but when a side-loading hopper, or a measuring hopper is charged by wheelbarrows, the time is lengthened more-or-less according to the perfection of the runway arrangements and the speed at which the men work.

In the following table is given the result of timing of different types of mixers on different classes of work. These studies were made both with a seconds clock, motion picture camera, and with a stop-watch, and are the summary of a large number of observations. From this it will be seen that the loading periods vary greatly; that the unloading periods have an equally-great variation; that the mixing periods are usually dependent upon the time taken in loading and unloading; and that successive operations overlap, the endeavor of the mixer man being to get out his material on as near a batch-a-minute schedule as is possible. In a number of instances it will be noted from this table, such a procedure gives a negative mixing time.

Summary of Timing Data on Concrete Mixers

Time given in minutes and seconds.

Run no.	Kind of mixer	Loading means	Loading	Unloading	Actual mixing	Actual total	Loading and unloading, total	Time of mixing batch, minimum schedule
1	Lakewood, 1 yd...	Batch hopper.	0:51	0:59	0:11	2:02	1:50	—0:50
2	Koehring, 1 yd....	Batch hopper.	0:36	0:34	0:42	1:51	1:10	—0:10
3	Smith, ¾ yd......	Batch hopper.	0:15	0:19	0:25	1:01	0:34	+0:26
4	Foote, ¼ yd......	Side loader...	0:16	0:17	0:20	0:54	0:33	+0:27
5	Foote, ½ yd......	Side loader...	0:23	0:27	0:25	1:15	0:50	+0:10
6	Chain Belt, ½ yd..	Side loader...	0:07	0:35	0:28	1:10	0:42	+0:18
7	Koehring, ½ yd...	Side loader...	0:12	0:32	0:21	1:05	0:44	+0:16
8	Lakewood, 1 yd...	Side loader...	0:11	1:02	1:11	2:25	1:12	—0:12
9	Ransome, 1 yd....	Batch hopper.	0:35	0:40	1:40	2:57	1:15	—0:15
10	Ransome, ¼ yd...	Side loader...	0:08	0:12	0:54	1:10	0:20	+0:40
11	Chain Belt, ¼ yd.	Side loader...	0:13	0:27	0:11	0:51	0:40	+0:20
12	Ransome, ¼ yd...	Side loader...	0:18	0:38	0:46	1:32	0:56	+0:40
13	Koehring.........	Side loader...	0:17	0:29	0:20	1:06	0:46	+0:14
Average..............................			0:21	0:33	0:28	1:29	0:53	+0:70
Average, omitting 8 and 9............			0:19	0:29	0:17	1:17	0:49	+0:11

Note.—Mixer 8 had very poor blading. Mixer 9 was fed by derrick bucket. Long mixing due to inability to get raw material and to dispose of mixed concrete.

23b. Time of Mixing.—Insufficient time is given to the mixing operation itself in most commerical work. Too long a period may possibly be indulged, but it usually is not; and no fear need be entertained of injuring the concrete by a mixing interval up to and including 30 min. The mixing operation proper comprehends not only admixture of materials, but also reaction between cement and water with distribution of the products of this reaction over the surfaces of sand and stone. The time required for such thorough incorporation, to a certain extent, for the hastening of the reaction between cement and water, depends the adequacy of the blading and cleanness of the mixer. Oftentimes mixers are put on with the drum (Fig. 22) half-choked with concrete or full of holes, or the blading so worn they cannot handle the materials. Necessarily such mixers will not produce the same as a clean mixer, properly bladed and having a tight drum. Also, mixers are not all eq efficient.

So many factors enter into the making of good concrete, that a hard and fast rule appli to all cases cannot be made, but in general it may be said 30 sec. or even 1 min. of mixing adequate. It is far better, when it is desired to do a thoroughly first-class job, to er more mixers even at a higher first cost for equipment and work them on a longer schedule, it is to attempt with one mixer to get out concrete on a rapid-fire schedule. The latter m often brings a chain of unfortunate consequences, for not only is the concrete inadequ mixed and the cement insufficiently used, but also excess water is nearly always added in to make the mass free-working and to diminish the labor of mixing.

23c. Drum Speeds.—Extended experimentation has established star drum speeds for various sizes of mixers. Engines and motors as supplied with them a regulated as to maintain these speeds practically constant. Necessarily, as the art of con making advances, changes will result, but the present rotational speeds of standard m seem suited to the requirements of average practice. Obviously, a slower drum speed v

FIG. 22.—Mixer drum.

FIG. 23.—Charging hopper mounted on mixer f

result in less thorough incorporation of materials and a greater speed might cause the m to stick to the drum through centrifugal action.[1] It is best, therefore, to adhere to ratings prescribed by manufacturers unless such speeds are patently inefficient. In as any concrete mixer that will perform its operations better and more quickly than petitors is sure to have correspondingly greater sales, it is safe to assume that mixe facturers have adopted for their product the maximum speed consistent with proper op It is not well, therefore, for the user to attempt economies by changing speed of the mix

23d. Loading the Mixer.—There are many time economies that may be in loading the charge of materials into the mixer. Various types of loading mechan been designed to meet different conditions of service and the time cycle of each is A study of each type will show its adaptability to particular needs.

Charging Hoppers.—Where a charging hopper mounted on the mixer frame can as in Fig. 23, the limitation to charging time is dependent upon the design of this hop the slope of its sides and upon the size of opening from hopper to drum. Inasmu type of charging device is usually loaded by gravity from superposed measuring ho

[1] For studies of mixer actions see N. C. JOHNSON: *Eng. Rec.*, Dec. 4, 1915.

considerations must be taken into account in their design; and always there must be promptitude in releasing of gates, etc. In some very large operations such as the Elephant Butte Dam, pneumatic opening devices have been installed with an interlocking system, so that a sequence of operations is carried out with almost perfect regularity and great efficiency.

Power Loaders.—Side loaders or power loaders are often attached to mixers in order to give the advantages of low loading, as well as those of relatively high discharge of mixed materials. The general type of mechanism employed is shown in Fig. 24. The type of loading hopper or skip varies with different manufacturers, some hoppers having a raised back, requiring a slight incline for wheelbarrows that must be dumped into the hopper, while others permit running wheelbarrows directly on to the hopper back itself. Through a friction clutch, the power loader is elevated by the same motive power which drives the mixer drum. Inasmuch as it is required to hoist such loading skips to a considerable height before materials will run

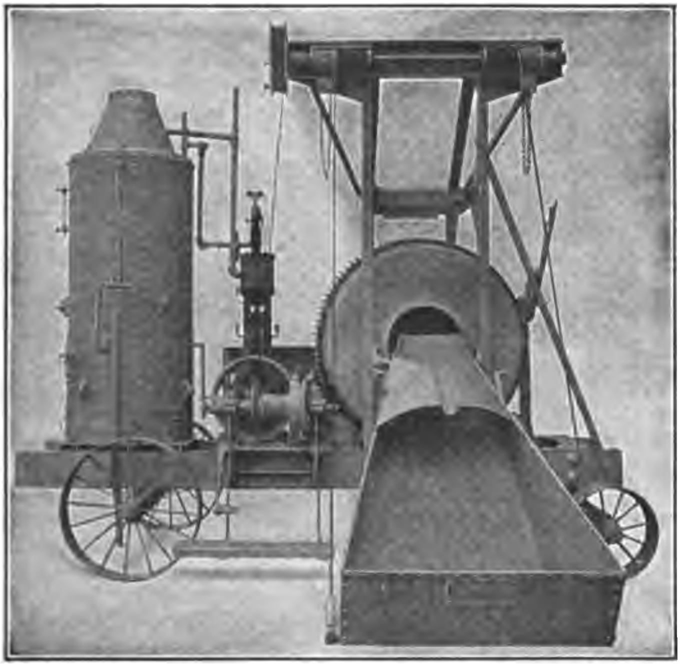


FIG. 24.

from them into the mixer drum, it is essential that sufficient power be provided to hoist this skip rapidly, as otherwise an undue amount of time will be consumed in this elevating operation. The mixers of different manufacture vary widely as to speed of hoisting; and it will generally be found that the more expensive mixers have a better and more rapid hoisting mechanism, in addition to their other economies, than have the cheaper types of mixing machines.

Low-charging Mixers.—Low-charging mixers (see Fig. 18), particularly in smaller units, have of recent years been meeting with favor. In such mixers the opening at the charging end is relatively larger than in other types of drum mixers and blading about this opening on the interior of the drum is so disposed as to draw the materials within the drum from a relatively small hopper of low height into which they are charged by wheelbarrows. With such mixers an inclined runway platform of 2½ to 3 ft. in height is required. Their advantages, therefore, consist in a simplification of charging and the absence of hoisting mechanisms rather than in any particular efficiency of mixing operation. Furthermore, these machines are relatively low in price and a number of small units, gasoline or electric motor-driven, are often very advantageous when distributed about the work. From a standpoint of thorough mixing and flexibility of operation, there is much to recommend this practice, inasmuch as the needs of one part of the work can be supplied without reference to other parts or causing an overdraft on any one machine with consequent speeding up of operations as is the case when all parts of the work are demanding concrete at the same time from a single, centralized plant. Without disparaging in any way the importance of the time-cost element in concrete mixing operations, it is yet to be regretted that considerations of quality and ultimate satisfaction of the customer, rather than first cost, do not more often govern both the selection of the plant and its operation.

23e. Measuring Materials.—It is often taken for granted that measurement of materials for a concrete batch is of little or no importance and that it can be accomplished in almost any way. It is probable that the average mix varies at least 50% in its proportions from those desired, and for this reason alone it is not to be wondered that much concrete found on every hand is so variable in quality.

Materials should be measured either in bottomless boxes placed on wheelbarrows, or like devices, or else in a barrow pan permitting of struck measurement. A measuring barrow of known capacity permitting struck measurement is shown in Fig. 25. At the same time these

FIG. 25.

barrows are adapted by reason of their balance, to the conveyance of considerable quantities of material at one time. Measuring hoppers of known capacity, if carefully filled, can be made to function quite accurately; but where they are not struck, or where there is pronounced variation in the moisture content of the sand, the quantities of materials obtained per batch will be found surprisingly variable.

It is difficult to convince the average contractor that economies can result from careful measurement. It may seem a useless task to confine field men to struck measure of sand and stone, but if the comparative quantities of cement required for accurately proportioned and inaccurately proportioned mixes were taken into account, the cost balance would usually be found in favor of the careful proportioning and measurement. And in addition, there should be considered and there will be considered with increasing force with passage of time, the ultimate performance and endurance of the concrete produced. The time is not long distant when owners will demand of contractors guarantees as to the quality of the product which they are to receive and only by careful proportioning and measurement and placing of the concrete can a reasonable basis for such guarantees be established.

23f. Discharge of the Mixer.—A further economy of time can often be had by giving attention to the proper and rapid discharge of materials from the mixer. Many mixers have inadequate and insufficient blading due to having become worn with passage of time. Many also are partially choked by concrete hardened inside the drum. Both insufficient blading and choked mixer drums mean a relatively slow discharge of materials from the drum which cuts into the essential mixing operation.

24. Transporting and Placing of Concrete.—Providing means for transporting mixed concrete and for placing it properly in forms is an art in itself. These operations both in first cost and in ultimate effect rank equal in importance with the operations of conveying, proportioning, and of mixing raw materials. In mixed concrete, not only are the raw materials to be handled and oftentimes conveyed to considerable distances, but in addition this must be done at low unit cost and in such a manner and so expeditiously as to protect the mixed mass from injury.

The means usually adopted for the conveyance and placing of concrete are some sort of bucket or cableway, or else open spouts or chutes through which the concrete flows by gravity, or else in barrows, carts, or cars. The particular means adopted in any case, will depend upon the size of the operation, upon the physical conditions attendant and upon the financial limitations to plant imposed by commercial considerations.

24a. Barrows.—As affecting perhaps the great bulk of concrete used today, it will be proper to first consider the use of barrows or carts. This method involves less original

Fig. 26.

plant outlay than the others before enumerated. In many instances, the cost of installation of an elaborate plant would cover not only the cost of the barrows themselves, but a great part of the entire cost of distribution of the concrete by other means.

The ordinary wheelbarrow (Fig. 26) having a flat pan is not well adapted to the distribution of concrete. In such a barrow a man can handle about 1½ to 2 cu. ft. of mixed concrete. This load he can wheel about 25 ft. every 3 min., the objection to the pan wheelbarrow being that the man's working rate is necessarily cut down by the care which is required to keep the materials from slopping over the sides. Furthermore by the design of the barrow a large proportion of the weight of the load is on the man's arms, rather than on the wheel. Deep pan barrows have been designed to overcome this difficulty, but have not wholly accomplished the desired end.

24b. Concrete Carts.—Two-wheel concrete carts (Fig. 27) are better adapted to this work than wheelbarrows, both because they can carry a larger load and also because this load is balanced on the wheels themselves with little or no strain on the man. The usual two-wheel concrete car is of 6-cu. ft. capacity in which about 4½ cu. ft. of mixed concrete can be carried by one man.

In this comparison there are, however, certain cost offsets to be made. Wheelbarrows require less scaffolding than do the heavier and wider carts, so that the cost of this runway must be carefully estimated. When runways must be elevated, the showing becomes more favorable for carts, as bents or supports for wheelbarrows must be practically of the same size and strength as those for carts. Turnouts and gangways must in both cases be of ample width so that there

Fig. 27.

may not be congestion in the passing of full and empty carts going to and returning from the forms.

24c. Buckets.—There is a great variety in types of buckets adapted to the distribution of concrete. Some of these buckets are straight-side skips, as in Fig. 28, adapted

Fig. 28.—Tilting bucket.

Fig. 29.—Round self-tilting bucket.

to dump by overturning. Others are bottom-dumping buckets operated by a man at the form; and these bottom-dumping buckets may be of various patterns, adapted to some particular use. An example of this sort of bucket is shown in Fig. 31, in which the bottom is so constructed as to form a long narrow opening, actuated through a powerful lever mechanism. A great variety of these devices is on the market and the needs of each particular situation must be studied and met by as specialized a product for that use, as financial considerations will permit.

24d. Cableways and Buckets.—Cableways usually require large initial outlay but on large operations they may be found very economical. Usually they consist of a strong

Fig. 30.—Bottom-dump bucket for large forms.

Fig. 31.—Bottom-dump bucket for narrow forms.

messenger cable or cables (carried between either fixed or movable towers) with actuating cables to hoist and carry the buckets to any desired spot on the work. Cableway buckets may be of various types, but that shown in Fig. 33 has proven its worth in many constructions. The bucket shown in this illustration is a 2-yd. bucket. Its deep upper body and steep sloping bottom provide capacity and free flow, while a gate controlled by levers regulates the discharge of concrete.

Fig. 32.—Tower-hoist tilting bucket.

Fig. 33.—Bottom-dump cableway bucket.

24e. Spouts or Chutes.—The handling of concrete through spouts or chutes is a development of the last 8 years. This system at the present time is in more extensive use than any of the foregoing methods of distribution, with the possible exception of distribution in carts. The economic features of spouting are undeniably attractive. To raise concrete vertically in a tower by means of a skip bucket and engine located at the central mixer plant, then distributing by gravity through channels which can be arranged in convenient sections to cover any area with a radius from 10 to 300 ft. from the base of the tower, appeals strongly both to engineering and to business sense. Further, the ease of handling by gravity is usually greater and the time cost per cubic yard for placing is usually less than in transferring the same quantity of material in hand-barrows, in cableway buckets, or in cars. Yet in spite of its many good points, the convenience of spouting has brought about many abuses.

For instance, it is obvious that in order to flow readily through chutes, concrete must be smooth and plastic, whereas the materials of which concrete is composed, with the exception of water, are all exceedingly sharp and gritty. It is not to be wondered then that lubrication and ease of flow secured by increased wetness, has encouraged the use of excess water, especially where for reasons of cost, it is desired to erect only a relatively low tower causing the angle of the spout to be comparatively flat. Furthermore, many spouting equipments have been installed with ease of distribution alone in view, the first cost of plant and rapid deterioration not being taken into account, so that saving has later been sought by cutting corners to make up for the initial mistake.

In all spouting installations, care must be taken to have the chutes at a workable inclination. Furthermore, it is important to maintain a uniform pitch throughout the entire line, in order that the flow may be thorough and uninterrupted and not subject to slackening at one part and accelerated flow in another. The pitch also must be greater when the material is to be carried to a considerable distance than when it is to be carried only a short distance, for as the distance increases, the friction of the concrete in a chute tends to overcome its initial momentum. Whereas, therefore, a wet concrete will flow 50 ft. with the pitch of 1 in 6 it becomes necessary to increase this pitch to 1 in 4 for a distributing distance of 100 ft., while a distance of 300 or 400 ft. will require a pitch of 1 in 3. The slopes as above described are based upon chute rigidly supported having uniform pitch throughout; and it would be even better to increase this pitch in order that concretes of a drier consistency may be used.

Various methods have been proposed for increasing the ease of flow of concrete in chutes. Hydrated lime in one proportion or another has probably proven the most effective, but there is no standard procedure in this regard, nor is the exact quantity of hydrated lime required for any given concrete prescribable without experimental knowledge of the aggregates separately and in combination. Hydrated lime added to concrete has some undesirable features, but even aside from these it is an expensive diluent of inferior strength, and inasmuch as practically the same effect through the same agency may be realized by a longer mixing of materials, the wisdom of its use is not yet beyond question.

The unfortunate tendency, as before pointed out, is to add more water to spouting concretes to make them flow freely. This, however, defeats its own end, inasmuch as segregation takes place very readily from wet mixtures, so that there is initially a rapid rush of semifluid materials down the chute, with afterward a slow dribbling of the heavier and harsher materials, oftentimes requiring men in the rigging with hoes to keep the unwatered sand and stone from stopping. With sloppy mixtures, therefore, not only is the quality of concrete impaired but also the cost of delivery and placing is very largely increased. On the other hand, thoroughly mixed concrete without excessive water may be successfully delivered through spouts disposed at proper pitch without segregation or the loss in value attendant upon the use of excessively wet mixtures.

24f. Sections Used in Spouting.—It is desirable that concrete spouting be arranged in a series of units which may be assembled in various combinations. Continuous-line spouting should be changeable to swivel-head, or swivel-head to continuous-line, as the conditions of the work require, it being necessary, of course, to have in stock a supply of the necessary units. This interchangeability is of great value in service, for spouts wear at the head and foot of each unit of length. By reversing a trough section, end for end, when showing heavy wear at one ·t, a new, unworn surface may be put at point of greatest wear.

A standard trough section, Fig. 34, is made of No. 14 gage steel, forming a trough 8½ in. deep by 10 in. wide on top. The bottom is curved to practically a semicircle of 4-in. radius, the upper part of the sides being straight and tangent to the curve. Each section is punched with standard spacing, arranged for connecting all of the various attachments.

FIG. 34.

The hopper head, Fig. 35, attached at one end for receiving the concrete from the bin, or from an upper trough section, forms one point of support of the next trough section. At the other end is the splash hood, Fig. 36. By fastening the hopper head to the trough section at one end, and the splash hood at the other, we have the complete trough section, Fig. 37.

FIG. 35.

These 24 by 24-in. hopper heads, as well as the splash hoods, can be bolted to either end of any standard trough section.

Standard trough sections are joined for continuous-line spouting by bolting together their angle-iron yokes or flanges and bolting on the compression plate part. Thus, several sections are joined together, with a hopper head at one end of the entire group, and a splash hood at the other end.

Fig. 38 shows the swivel-hook used in making the flexible joint between successive trough sections for swivel-head spouting and shows one of these joints, in which the upper line of spouting is supported by a fall and tackle attached to the bail on the splash hood; while the lower line is supported by the swivel-hook, connecting the lower hopper head with the splash hood of the upper line. The swivel-hook is kept clear of the path of the concrete.

In some cases it is desirable to have a flexible joint in continuous-line spouting. In this case the two sections are put together in a different manner, Fig. 39, where both the hopper

Fig. 36.

head and the splash hood are dispensed with. The hanger plate is here used in conjunction with a special yoke, after one of the angle-iron yokes has been removed. This allows a slight movement sideways, without requiring the attachments for the swivel-head operation.

Various types of spouting have been tried, ranging from round pipe to rectangular troughs. Best results have been secured from the use of 5-in. pipes, or 10-in. open troughs, the latter

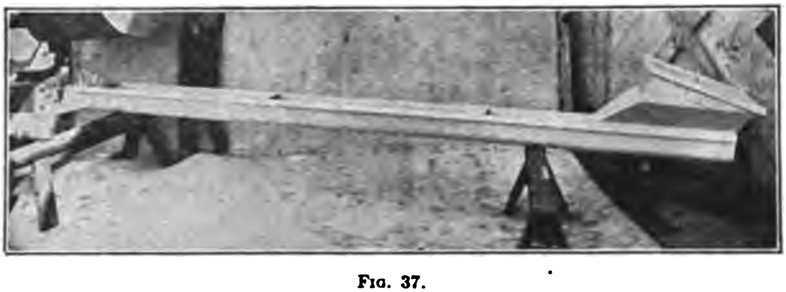


Fig. 37.

having the preference for flat slopes, and the former where there is necessity for varying pitch, with a likelihood of steeper pitch than named above.

With open spouting the use of remixing hoppers (Fig. 40), in connection with flexible spouting (Fig. 41), accomplishes satisfactorily the necessary changes in pitch.

The greatest items of expense in spouting plants are first cost, installation, and maintenance. Maintenance charges are particularly heavy. The ordinary stock spouting which is made of No. 14 gage metal will seldom handle more than 2000 cu. yd. without renewal. This is due to the abrasive action of the material, especially as affecting the rivets which join the various sections.

A recent development is a spout made up of two or more longitudinal sections of the shape indicated in Fig. 42. The various sections are interchangeable, and there are no bolts or rivets

Fig. 38.

extending through the spout, all joint bolts or rivets passing through the flanges, and the various longitudinal joints made secure by fish plates. This type of spouting has the further

Fig. 39.

advantage of making possible renewal of worn sections severally, instead of renewing the length of spout as a whole. This type furthermore ensures a spout which is stiff in all directions, a point of considerable importance.

24g. Hoists.—Whether the distribution is by spouts, by carts, or by barrows, it has become general practice on all work extending above ground to hoist the concrete. For this purpose a tower is practically indispensable.

FIG. 40.

It will ordinarily be found advisable to install the hoist at the beginning of operations, since by so doing the mixer may readily be set so that the operation of charging may be facilitated, principally by cutting out inclines, with resultant saving in labor.

FIG. 41.

Towers are constructed of steel or wood. The hoist bucket should be constructed on the simplest lines without catches or trips. A substantial bail made of two 3-in. Z-bars back to back, is arranged to operate between two 5¾-in. wooden guides, and is fitted at the lower end with journals in which rests the bucket trunnion. In setting up the tower and bucket, it is advisable in all cases to set the bucket so that it is balanced, and to this end the front guide should be so set as to be almost in contact with the nose of the bucket when the latter is pushed back to a point where the load will tend slightly to press the stops on the sides of the bucket backward against the bail. Friction of the nose against the guides is, by this

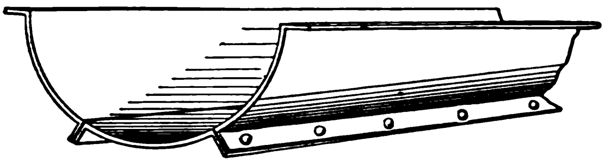


Fig. 42.

means, cut down. By removing the front guide at any point in the height of the tower, and placing a block on the back of the latter, the bucket is canted forward so that it will drop its contents out through the opening made by the removal of the front guide. The bucket automatically rights itself, and is pulled back into position by the weight of the bail when the operator releases the brake.

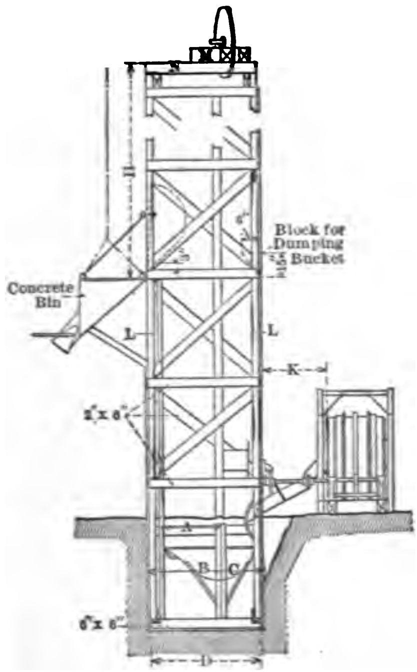


Fig. 43.

A typical hoist is shown in Fig. 43, operating in connection with a mixer, the power being taken from an extension of the mixer shaft. The power equipment of the latter should be of sufficient capacity to operate both mixer and hoist at the same time. A variation of this plant showing a direct-connected hoist is shown in Fig. 44, but for ordinary conditions the first-described arrangement is preferable.

At any desired height a bin or hopper is set, into which the material is discharged by the

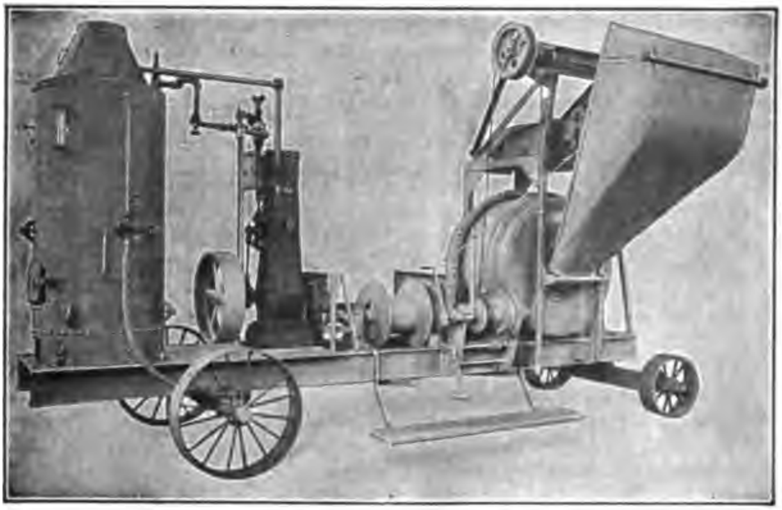


Fig. 44.

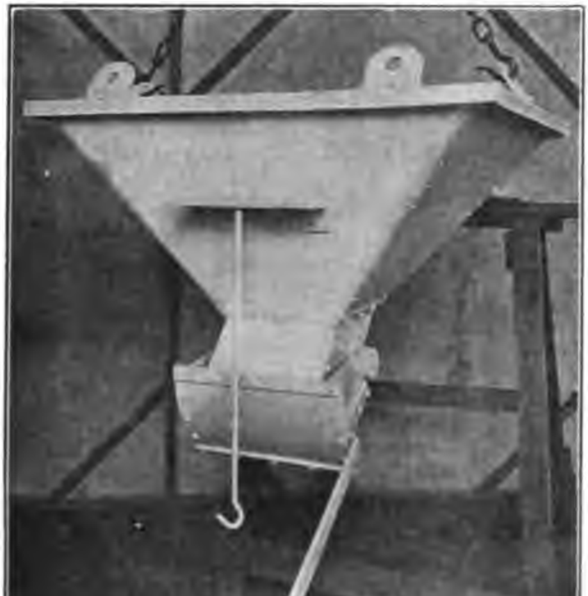


Fig. 45.

hoist bucket. From this point distribution may be effected by wheelbarrow, cart, car, or spout, either separately or in combination. A concrete bin such as shown in Fig. 45 forms the upper end of a spouting system, the gate of the hopper under manual control regulating the flow.

25. Spouting Plants.—Spouting plants may be classed as *boom plants*, *guy-line plants*, and *tower plants*.

25a. Boom Plants.—In boom plants, the first and second sections of spouting are mounted on a bracket attached to the hoisting tower, the free end being moved by tag lines to the position desired. This rig offers advantages of flexibility and freedom of movement not often obtained in placing concrete. Oftentimes open-throated booms (through which the first section of spouting is carried) are used, these having the advantage of lending lateral stability to the spout itself as well as of economizing space.

25b. Guy-line Plants.—In guy-line plants, the spout is suspended by blocks and falls from guy lines, or special cables suspended between towers, or other supports especially set up for the purpose. The advantage of this type of spouting plant lies in its ready adaptability. It is limited, however, in lateral movement unless its deficiencies are supplemented by take-offs at various points with small boom plants or supplementary guy-line plants.

25c. Tower Plants.—Tower plants are of like general feature, but the spouting line is supported at ends of successive sections by movable towers or tripods. A plant of this kind is less flexible than a boom plant, but is more flexible than a guy-line plant, inasmuch as the various supports in the line may be moved successively, rendering possible the covering of a very wide area from a single hoisting tower. A guy-line plant, on the contrary, requires under like circumstances that the whole line be dismantled and set up again in the new location.

25d. Combinations of Spouting Systems.—Combinations of the above systems are used advantageously in one way and another in order to surmount special obstacles. Among such combinations may be mentioned a rehoisting tower which permits covering a wider area. In such a plant the concrete is distributed from mixer and first tower through chutes to a hopper at the base of the second tower, when it is again elevated and distributed throughout the work. A careful study is required in order to make spouting plants thoroughly effective; and this study should always be made before the job is started to make sure that the proper radius of delivery and best arrangement is secured.

25e. Regulating Flow of Concrete in Spouting Plants.—It is quite essential for the proper operation of the spouting plant that concrete should be uniformly and continuously carried down the chutes. To this end a receiving hopper is placed at the head of the elevating tower, with a man in control of its gate. Upon this man then depends to a large extent the success of the operation. If he permits a proper amount of material to flow into the chutes, they can usually be relied upon to carry it freely providing they are disposed at proper inclination. If he sees the line becoming choked, upon his slackening or shutting off the delivery depends either a speedy clearing of the line with relatively continuous operation, or shutting down for an indefinite period.

No matter what type of mixing equipment, or what system of distribution is adopted, there should be kept in constatnt view the object to be attained, namely, the economical production and placement, not merely of materials which will fill form spaces with possible acceptance, but rather of materials which in forms will solidify and endure under stress, whatever the nature of such stress may be. Each yard of poor concrete carelessly placed gives concrete a black eye. Each yard of good concrete properly placed is testimony as to the abilities of this inherently wonderful material.

SECTION 4

CONCRETE FLOORS AND FLOOR SURFACES, SIDEWALKS, AND ROADWAYS

CONCRETE FLOORS AND FLOOR SURFACES

1. The Concrete-floor Problem.—Concrete floors in modern commercial buildings are of peculiar importance. Not only is the floor slab an integral part of the structure, making possible its usefulness by supporting applied loads—such as machinery, or stored goods— but such floors must further perform unusual service in withstanding severe concentrated stresses—as, for instance, those due to passing trucks heavily loaded—and particularly must they withstand at their top surfaces not only the crushing above referred to, but, in addition, a constant and severe abrasion through the impact of shoes, or the movement of loads, with oftentimes attack from chemicals used in manufacturing processes.

No part, therefore, either of aggregates or of the cement matrix in which they are embedded, may give way without "dusting," or progressive destruction, of the floor to greater or less degree, causing not only annoyance and inconvenience, but possibly more serious consequences by reason of the released particles being carried into machinery and manufactured products. These particles are abrasive, gritty, and may be chemically injurious. The problem of satisfactory concrete-floor surfaces becomes, therefore, essentially the problem of producing a concrete (1) of a strength requisite to resist compression and shear due to floor loads; and (2) of sufficient top-surface resistance to withstand the mechanical attack of normal service. Chemical attack must be provided for by special supplementary treatment with a resisting paint or varnish.

FIG. 1.—Rough fracture through porous floor slab. (Magnified 2 diams.)

The requisite first named is met with relative ease. Density and strength through use of proper aggregates and good cement, thoroughly mixed, without excess water, and carefully placed are the procedures to be followed (see chapters on "Aggregates" and "Water" in Sect. 1 and on "Mixing, Transporting, and Placing Concrete" in Sect. 2). Porous floor slabs, such as that shown in Fig. 1 do not make either for strength or for any of the qualities desired in good concrete.

The requisite second named—that of producing a resisting top surface is more difficult to meet with full satisfaction. If the surface of concrete floors (or of concrete roads or sidewalks) could be natural stone of proper quality molded with the same ease as is concrete and made integral both as a monolith and by tying with steel to the rest of the structure, there would be little cause for complaint on the score of dusting, wear, or structual functioning. Yet the aggregate employed in concrete is natural stone in fragments. Since reinforcing steel does not affect wearing qualities at the surface, the difficulty must, therefore, lie either in the choice of the natural materials, in the proportion of these materials exposed as resistants to abrasion, or else in the quality or quantity of the cementing material holding them vise-like against the abrading forces.

The ideal in artificial stone floors is the terazzo floor in which an even surface (95% or more of which is natural stone with 5% or less of cementing material) is presented to wear. Grinding concrete floors is necessarily expensive, but removing a surface layer by such means produces a superior result that is found to justify the cost. The significance of these facts, together with what is known of the general top-surface character of concretes, leads to the conclusion, which is fully borne out, that in the extreme top layers of a concrete floor is to be found much of whatever difficulty is experienced.

2. "Dusting" of Concrete Floors.—Research has shown that "dusting" floors are concretes which are at least locally poor, such local weakness (most evident in the top coat) being shown typically in Fig. 2, wherein the sand grains are seen to be uncertainly held in a loose and easily abraded matrix of what appears to be of the nature of efflorescence. All dusting-floor surfaces, however, are not identical with the one shown, nor are the causes of dusting necessarily the same. Nevertheless, the process of progress of progressive destruction or "dusting" is much the same in most cases and is largely due to a loose condition of the cement binder which permits the resistant sand grains to fall out, exposing fresh surfaces of soft, hydrated cement to attrition, with repetition of the process until remedies are applied, or until resistant strata at irregular depths below the surface are reached. These actions are augmented at times by chemical action, either from moisture or from other atmospheric or fluid agencies.

Fig. 2.—Dusting surface of concrete floor. (Magnified 5 diams.)

3. Making Good Concrete Floors and Floor Surfaces.—A good concrete floor is essentially a good concrete. The principles of making good concrete—the right materials and the right proportions of same, including water; thorough mixing; careful placing; careful curing—epitomize the making of good floors.

To these axiomatic and self-evident general principles should be added the following:

1. Whenever possible, run top coat and base together. If this is not possible or advisable, remove top surface of base to a depth of at least ½ in. Then, before placing top coat, roughen the base surface well and wash clean, using hose or brushes. This procedure is necessary to procure a proper bond between top coat and base.

2. Use coarse, rather than too fine material in the top coat.[1]

4. Special Surface Finishes.

4a. Surface Grinding.—"Granolithic" is a term applied alike to concrete floors having cement and sand finish, and to those having a surface layer of crushed granite, or other hard, enduring rock bonded with cement. As above noted, a desirable finish to such floors, a finish that gives a pleasing appearance and removes much of any tendency there may be to dusting or surface disintegration of any kind, may be produced by surface grinding when the concrete is from 4 to 7 days old by means of a machine similar to that employed in grinding terazzo floors. Such grinding removes any laitance or loose material from the surface, produces a smooth though not polished surface and, by selection of aggregates before laying with special reference to color, gives an unusually pleasing effect.

4b. Integral Pigments.—Pigments of one coloration or another, chemically inert toward concrete, can be had of a number of dealers. Inasmuch as surface color only is desired, it is advantageous to apply them only in a relatively-thin mortar layer at the top surface. This layer should be truly integral with the layers below, else it will scale off. It should further be borne in mind that the coloring value in concrete of any integral pigment will be affected strongly by the color of Portland cement, which is itself a pigment, white when hydrated, gray-green when unhydrated. According, therefore, to the color added, the effect of pigment in concrete will be more or less intense according to its percentage presence as related to the percentage of cement in the mixture and to the degree of hydration of the latter. The color of aggregates may also affect the result. Care should, therefore, be exercised in attempting to secure a given intensity of final color, not to use pigments in such quantity as to be detrimental to the concrete. Small trial batches will aid in securing the effect desired.

[1] See L. C. Wason: *Trans.* A.S.M.E., 1914, p. 400.

4c. Finish Produced by Removal of Water From Surface.—A process of finishing floors said to give excellent results is the abstraction of excess surface water through absorption by dry cement laid on webbing over the soft floor.[1] This method should be advantageous in many instances, particularly where excess water is used. The process is proprietary.

4d. Integral Hardeners and Surface Compounds.—A number of compounds are on the market designed to be incorporated with the surface to make it resistant. One of these is carbide of silicon (carborundum) under one name or another. This material unquestionably has great abrasive resistance and if properly held in place by cement should produce a surface capable of withstanding severe traffic. There is, however, no reason to expect better conditions of manufacture attending its use than would obtain where it is omitted; and as good quartz sand or crushed durable rock properly bedded in cement is capable of supplying most needs; and inasmuch as the sparkling, glistening effect incident to the use of carborundum is often objectionable, the advantages to be expected from it should be carefully looked into before it is employed.

Common iron, powdered, is extensively marketed as a surface hardener for concrete floors. A variety of claims, many of them conflicting, are advanced by its advocates. Some assert that the iron oxidizes (rusts) with expansive filling of pores and prevention of further moisture penetration. Soil-ammoniac may even be added to promote this rusting. Others claim no rusting with the virtue residing in the superior hardness of such iron as remains at the surface.

It is unquestionably a fact that the average iron in contact with moist air will rust. This produces a characteristic red stain of rust in the concrete, but it is doubtful if as an incident to mixing or placing, this iron rust can be so directed, distributed, and placed as to constitute a reliable pore filler; and it is further more likely to be an attractor of moisture than a preventive of moisture penetration, since rust ($Fe_2O_3 \cdot X\ H_2O$) is deliquescent. Further, iron is so inferior in hardness to common quartz sand as to make the ratio of comparison about 230 (for quartz) to 18 (for iron),[2] so that with equally satisfactory embedment in cement, iron should prove inferior to ordinary sand. In addition, factory grease is often not removed from the iron, so that attachment of cement is hindered, if not inhibited.

There is no top coat superior in all-around qualities to good quartz sand of proper size and grading, nor is there any additive at present known qualified *per se* to overcome initial deficiencies resulting from faulty manufacture or inferior materials.

5. Causes of Common Defects in Concrete Floors.—A statement of defects commonly found in concrete floors and the causes which give rise to them is conversely an aid to the production of floors of proper endurance. Avoidance of wrong practices is the surest guaranty of success. Such a listing of the causes of defects, therefore, follows:

(*a*) *Poor Cement.*—This cause is infrequent. It is true that defective Portland cements are occasionally manufactured and that they are marketed, but misuse of cement is more frequent than deficiencies in the cement itself. Other causes should be sought and eliminated before blame is attached to the cement.

(*b*) *Poor Quality of Sand.*—This cause is relatively frequent. Sands, as before noted, are derived from the breakdown of natural rocks; and in most sand deposits the grains have existed for millions of years, so that their inherent quality and endurance is vouched for, but decomposing cementing materials, such as clay (uniting very small mineral particles to form the larger grains), or organic matter, or dirt, or other impurities, may render the best sand unfit for use in concrete. *Be sure of the quality of sand before laying the floor* (see Art. 30, Sect. 1).

(*c*) *Poor Grading of Sand.*—This cause is relatively frequent. It needs no demonstration to prove that the greatest quantity of sand in a given volume will be obtained when the particles are so graded in size that smaller grains will lie between the spaces of larger grains, progressively down the scale of sizes. Likewise, the least quantity of sand in a given volume will be had when all the grains are of one uniform size. This truth is sometimes stated by saying

[1] See P. M. Bruner: *Proc.* Am. Con. Inst., 1915.

[2] Scleroscope scale.

that "minimum voids" of "maximum density" is obtained with graded sizes of grains; and that "maximum voids" or "minimum density" is obtained when the grains are all of one size. Sand for concrete floors and particularly for the top coat should, therefore, be graded in size in such manner as to give "maximum density" with maximum quantity of enduring silicious material of proper grain size at the surface. *Be sure of the grading of sand before laying the floor.*

(d) *Dirty Water.*—The occurrence of defects due to the use of dirty water is relatively infrequent. Dirty water is not merely water that carries fine silty matter in suspension, but more particularly contaminated water, carrying organic matter, such as stable or barnyard drainage. Organic matter of this kind is very injurious to concrete and may even cause failure to set, or total disintegration. *Use only water of unquestionable cleanness.*

(e) *Too Much Water.*—The use of too much water is of general occurrence. With excess water in a mix, the fine particles of stone, and the fine particles of sand, and the finest particles of floor cement separate and rise, forming a thick scum at the top of the slab. This is where the slab should be most enduring, but if too much water is used, a material having about the resistance and character of chalk is substituted for the enduring materials desired. *Avoid excess water. Use thorough mixing to obtain the plasticity desired.*

(f) *Wrong Proportions of Materials.*—Arbitrary proportions in concrete making have little except careless convenience to recommend them. Through lack of understanding and because of the supposed difficulty of proper proportioning, they have remained in practice. It should be borne in mind that each sand and each gravel has properties peculiar to itself; and that the proportions in which they should be used in combination with cement and water apply to them only, so that such proportions cannot be taken as a criterion for the use of other sands or gravels. *Proportion the materials so as to obtain maximum density* (see chapter on "Aggregates" in Sect. 1 and on "Proportioning Concrete" in Sect. 2).

(g) *Insufficient Mixing.*—One of the reasons that excess water is so commonly used in concrete is that it renders mixing easy. The desire for ease and rapidity of working tends to carry the speeding-up process beyond allowable limits so that insufficient mixing is more often indulged in than is realized. *Whether the mixture is sloppy, plastic, or dry, mix it not less than* 1 *min. in a batch machine, or an equivalent amount if other form of mixing is employed.*

(h) *Too Much Tamping.*—Concrete may be tamped too much. With a medium wet concrete or concrete of plastic consistency, tamping to a certain degree is desirable to compact the mass. But tamping to more than the required degree brings unfortunate results. It should be recognized that sand and stone and cement in a fluid or semifluid concrete are non-coherent and that by the agitation of tamping, heavier materials sink and lighter materials rise. This causes separation or "segregation," of necessity putting fine, chalky materials, not adapted to resist abrasion, at the wearing surface. *Don't flood the surface by tamping.*

(i) *Too Little Tamping.*—It is frequently the case that a concrete floor is deposited in such haste and with so little care that it does not compact; and particularly, it is not sufficiently joggled to remove air from the mass. In all mixing operations, air is stirred into the plastic mass, much as it might be into a stiff batter; and if such air is not removed in placing, a honeycombed structure will result. Further, the tendency of entrapped air is to concentrate at or near the upper, or wearing surface, so that when the cement has set, it will be molded as bubbles in the mass. Needless to say, such bubbles are holes in the concrete; and holes offer very poor resistance both to stress and to abrasion. *Tamp enough to compact the concrete, but not enough to flood it.*

(j) *Too Much Troweling.*—After a concrete floor form is filled and screeded, the surface is rough and irregular. When the slab has taken its initial set, the finisher rubs or floats the surface to an even finish with a steel or wooden trowel. This process brings considerable water to the surface, acting in a manner analogous to the tamping operation before referred to, so that the very fine particles of cement and sand will rise to the surface. *Don't flood the surface by too much troweling.*

(k) *Use of Cement as a Surface Dryer.*—It is not infrequently the case, particularly when very wet concrete is used and it is desired to hasten finishing operations, that dry cement is sprinkled over the surface of the partially set mass, and worked smooth with a trowel or float. In this case, the liquid it is sought to absorb is a strong solution brought up from the body of the slab; and to this solution the new cement further contributes like substances. This results in a deposit of fine silicious material from the sand and hydrolized cement a little below the finish with a skin of nearly pure cement directly at the surface. Necessarily, therefore, the body of the concrete slab and this thin veneer of neat cement at the surface are separated by a loose, non-adhering laitance film, so that scaling in patches of greater or less size soon results and will continue indefinitely until loose portions are entirely removed. *Use less water in mixing and avoid adding cement as a dryer either neat or mixed with sand.*

(l) *Use of Retempered Concrete.*—The setting of Portland cement takes place in two stages: (1) a gradual stiffening, known as initial set; and (2) an attainment of rigidity, with later slow hardening with passage of time. If the concrete for a floor has been mixed for some time, it may have taken its initial set. Adding more water and reworking renders the mass again plastic, so that it can be deposited much as might freshly mixed concrete. A certain valuable property, however, has become lost by this retempering, or rewetting, inasmuch as among other actions interlacing crystallization had already begun, and by rewetting and remixing, these crystals have been damaged and their reticulation destroyed. Such a floor, therefore, will be of inferior strength and density and possibly crumbly. *Avoid using concrete which has taken its initial set.*

(m) *Loosening of Top Coat from Base.*—Where floors are deposited in two layers and particularly where the under floor as deposited is overwet, the upper layer (top coat) will be separated from the base by a scum of laitance deposited at the top surface of the under portion previous to setting. If the top coat in such floors is of good quality and sufficiently thick, it may stand ordinary shocks and wear, but if it is thin, or is subjected to sufficiently intense stress, it will become loosened and at best be unpleasant in use, giving a hollow sound when struck, or walked over, or in extreme cases, will become shattered and crumble into pieces of greater or less size. *Cast top coat and base at one operation wherever possible;* and in other cases, remove surface of base to a depth of ½ in. or more and wash thoroughly, bonding to top coat with layer of rich cement grout. Do not permit this latter to dry out or set before top coat is applied.

(n) *Placing Concrete in Freezing Weather without Protection.*—Contact between Portland cement and water results in a chemical reaction. As is well known, the speed of a chemical reaction is a function of the temperature. Below 50°F. this reaction is very slow, and so long as this temperature persists, there is comparatively slight formation of the binding or cementing substance, though it may be formed later, after the temperature has risen. Furthermore, when water freezes it expands with a force of approximately 300 tons per sq. in. with a volumetric increase of 8%, so that in frozen concrete there is a general disruption and dispersion of components, possibly during the setting process, and to such an extent that in concrete floors, due to lack of hydrostatic head, they rarely, if ever, become again fully consolidated, regardless of how fully the cement may later react with water. Concrete floors which have been frozen, therefore, are weak and scaly. *Use heated aggregates, heated water, and proper protection when concreting in freezing weather.* Avoid the use of salt. Its advantages are not commensurate with its disadvantages.

6. Remedial Measures.—Obviously, the best remedial measures, so far as the generality of concrete floors is concerned, are the avoidance of improper procedures and the unceasingly careful institution of proper ones. The foregoing list of the causes of defects is, therefore, a list of remedial measures, so far as floors yet to be laid are concerned. Where, however, recognized defects exist in floors in place, it is a matter of serious moment to effect their repair.

6a. Retopping.—Where head room and goods or machinery in place will permit, removal of existing top down to sound concrete, with thorough chipping, roughening and cleansing of the exposed surface and the careful laying of a new top of proper quality is sor times the most advisable and in the end, the cheapest procedure. Details of bonding top c to base have been previously given.

6b. Chemical Hardeners.—Sodium or magnesium fluosilicate is marketed unc various trade names as a liquid hardener. A pronounced change is to be noted in the appe ance of concrete so treated with a greater or less hardening of the surface and correspondi resistance to abrasion, according to the initial condition of the concrete. The use of fluosi cates, originally marketed as "fluates" has been known for many years, with a comparative recent revival through aggressive selling campaigns since the extension of uses for concre buildings, with corresponding increase in the number of defective floors.

6c. Use of Oils.—Linseed oil, both boiled and raw, with and without additior and adulterants has been tried as a binder for dusting concrete floors, but has not proven ade quate for all uses. Its value may be gaged by the resistive and retentive values that an oxidize film of a like oil might be expected to possess when subjected to the same traffic or other condi tions which gave rise to the original complaint.

China-wood oil, sometimes called Chinese wood-oil, or Tung oil, either alone or in com bination with linseed or other oils or resins has proven somewhat more effective than plair linseed. It is today in extensive use as a basis for various concrete floor, wall, and water-resisting paints and in proper combinations is very effective. But excellent as are the properties of this oil it is not necessarily an effective remedy for all dusting floors. The conditions attendant on each individual case should be minutely studied and the chances of success estimated, rather than attempting the haphazard application of any palliative, proprietary or public, on the chance that the result desired will be secured.

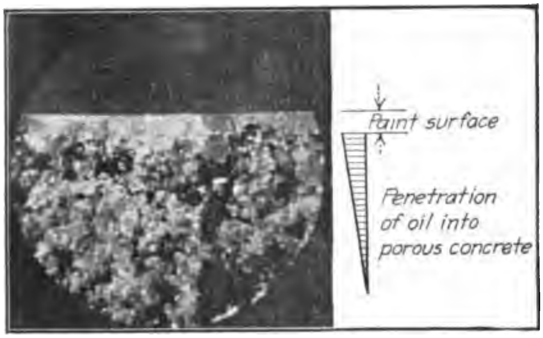


Fig. 3.—Paint film on concrete floor. (Magnified 20 diams.)

6d. Floor Coatings and Paints.—It is oftentimes desirable for reasons of sanitation or of appearance, on good as well as on dusting floors, to use a surface coat of paint. A number of excellent floor paints are on the market which can be had in a variety of colors, the surface obtained being hard and sufficiently resistant to abrasion for most purposes and capable of withstanding the action of water almost perfectly. The manner in which such a floor paint penetrates into the body of the concrete, leaving a hard resistant film at the top, is shown in the micrograph of Fig. 3.

CONCRETE SIDEWALKS

7. Structural Functions.—Concrete floors and concrete sidewalks have similarity of functioning as abrasion and impact-resisting surfaces and dissimilarity of functioning in that sidewalks have solid bedding and low concentrated load and are rarely called upon to act as beams. There is further dissimilarity in that the perfection of surface of a concrete floor is not required of the concrete sidewalk, since the latter is in the open where slight surface disintegrations do no harm and are not noticeable.

8. Essential Qualities.—The same general conditions governing the manufacture of concrete floors apply to the manufacture of concrete sidewalks since their requisites are held in common. A concrete sidewalk is also subject to the same character of disintegrations as is a concrete

...aggravated by year-round exposure to weathering and like influences. Like precautions ...should be observed in its manufacture.

9. The Making of Concrete Sidewalks.—The disintegrations of sidewalks seen on every ... point strongly to the need of either more careful procedures than those usually indulged ... to improved procedures, but in view of the many successful concrete sidewalks made ...careful following of accepted procedures, indications are that more care is a particularly in... demand.

9a. Porous Subbase.—One thing that must be always guarded against in a con... pavement is the action of frost. For this reason, adequate drainage of subsoil must ...vided. The porous, or draining foundation for outdoor construction should, there... be from 6 to 12 in. thick, dependent upon the climate in which it is situated and upon ...character of soil. On an extremely porous or sandy soil without frost, the foundation ...be omitted entirely, but it is erring on the side of safety to use a porous subbase in all ...

Such a subbase may be cinders, coarse sand, or preferably broken stone or gravel. This ...terial should be thoroughly rammed to present a firm and unyielding stratum and, at inter... drains of coarser sand or of gravel, or even open-tile drains may be advantageously in... to carry off water which may collect. Cinders or sand should be thoroughly wetted and ...pacted by ramming.

9b. Concrete Base.—On top of this draining foundation is placed a slab of coarse ...crete rammed into place, of a quality at least equivalent to that obtained with arbitrary ...portions of 1 : 3 : 6. This base is usually 3 to 3½ in. in thickness and it is separated into ...ks by form boards so that unequal settlement through freezing, rising through the penetra... of tree roots or other vegetation, or buckling through temperature changes may not injure the walk. Such divisions are quite essential and should not be omitted.

9c. Top or Wearing Surface.—Cast upon the concrete base and preferably made ...egral with it is the top coat, the upper surface of which is the wearing surface. As in concrete ...oors, the more silica or like rock material which can be exposed to wear in this surface, consist... with proper gripping by cement, the more enduring will be this surface. The principles ...ated for producing a surface of like character on concrete floors apply with equal or with even greater force to concrete sidewalks.

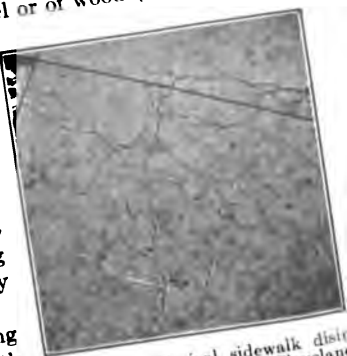


Fig. 4.—Typical sidewalk disintegration. (Public Square, Cleveland, Ohio.)

9d. Surface Finishing.—After screeding it is customary to finish the top surface of pavements either by floating with a flat trowel, either of steel or of wood (with or without later brooming to roughen the surface) or to groove or checker the floated surface, both with the object of securing rough... and of improving its appearance. All working of the surface has an effect similar to that of floating, and as pointed out in Art. 5 under "Floors" it may cause an excess of water at the wearing surface.

As in making concrete floors, the use of dry cement, or dry cement and fine sand sprinkled over an excessively wet surface, is a common reliance for speeding-up the finishing process. Destruction of top coat is often to be traced directly to this practice.

9e. Surface Protection and Curing.—Laying cement sidewalks in freezing weather is always attended with risk. Even with heated aggregates and heated water (cement is rarely, if ever heated) it is difficult if not impossible to maintain the temperature of the mass at a point sufficiently high and for a time sufficiently long to insure reaction between the water and cement so as to prevent disruption either before final set, or so near that time that reconsolidation at elevated temperatures will later take place. It should further be borne in mind, that lowered temperature greatly prolongs the period of plasticity, leaving water uncombined and free to form disrupting ice.

Even with heated aggregates, therefore, and later surface protection, great care must be exercised to be sure that minute local disruptions (Fig. 4) due to frost have not made the concrete, particularly at the top, ready for further disintegrations, either through attrition or by other actions. The best and safest plan is not to lay concrete sidewalks in freezing weather.

9f. Protecting Sidewalks in Hot Weather.—In view of what has been said in Art. 5 under "Floors" with regard to the effects of floating and troweling in bringing water to the surface and remembering further the requirements of cement in regard to quantities of water necessary for slow hydration and chemical interactions, it is readily to be understood that rapid evaporation in hot, dry weather will cause hair cracks, through a too rapid removal of water, leaving behind and directly at the surface such of its products as may be in solution, with further weakening of the concrete through deprivation of the remaining cement of its necessary reacting water.

To guard against such happenings, as soon as possible after finishing, the pavement should be covered with a moist protecting canvas supported above the surface by a frame, or by a layer of moist sand, either canvas or sand being kept moist for several days. Only by observing precautions such as these can success be obtained.

9g. Special Surface Finishes.—The same special finishes such as carborundum, or iron, suggested for concrete floors (Art. 4) are advocated for concrete pavements by their sponsors. The same remarks apply equally to both uses, as does a restatement of the fact that no additive so far brought out is capable *per se* of overcoming defects in manufacture and that none is superior in all-around qualities to good clean quartz of proper grading.

10. Vault-light Pavements.—Basement areas under sidewalks in cities are a valuable asset but it is necessary that they be cheaply lighted. This desirable end is accomplished through the use of a combination of round or square lenses of thick glass set in cement mortar, formed into pavement blocks of requisite size, reinforced with steel rods and supported on steel or concrete girders. A template is used in spacing the lenses, the mortar being troweled in place as in usual cement mortar work. From the standpoint of considerations previously presented, this type of sidewalk should and does prove very enduring, by reason of the large area of resistant glass exposed to abrasion and the lessened area of cement mortar, but care should be taken to have the cement portion as carefully proportioned, mixed, and laid, as it would be where the entire wearing surface is of cement mortar. Joints between adjacent blocks and adjoining constructions are sealed with a waterproofing compound. Shattered vault light sidewalks are frequently seen, this shattering being due to the wheels of heavily laden hand trucks, or to improper working or use of the cement mortar, with later disintegration through frost or other actions, with loosening of lenses and permitting of continued spalling of their edges until the light-transmitting properties of the glass are impaired, or the lens broken completely. These difficulties, however, do not reflect upon the value of this construction properly executed.

11. Concrete Curbing.—Concrete in curb and gutter construction has met with much favor. It proves generally excellent provided it is properly underdrained. Integral curb and gutter blocks are made in lengths having definite cross-joints but no longitudinal joints, as such, by the action of frost or vegetation, would invite separation by frost or by the action of penetrant vegetation.

12. Summary.—Concrete sidewalks, no less than other constructions, are structures of concrete; and to be enduring, they must be good concretes. Furthermore, they must be particularly protected against natural disintegrating forces, chief among which is frost. This requires good subdrainage as a prime requisite; and of almost equal importance, a dense structure resistant to the penetration of water. To secure this, rigid observance of the principles of mixing good concrete must be insisted upon. The man with "20 years' experience" may only be one who persists in the least advisable procedures.

CONCRETE ROADWAYS

13. Structural Functions.—Concrete sidewalks and concrete roadway pavements are similar in that they are impact and abrasion-resisting surfaces bedded on a continuous subbase; and they are dissimilar in the perfection of the surface required and in the degree of impact and abrasion which they must sustain.

14. Essential Qualities.—Concrete roads, no less than concrete floors, are, first of all, concretes, so that their character is governed by the laws basically controlling the making of good concrete. Particularly must provision be made against disintegration through weathering, through the heavy impact of shod hoofs and tires of loaded vehicles, and through the raveling action of fast motor traffic. The best means for this purpose at present known are: (1) proper subdrainage; and (2) the securing of high-density concrete having adequate cementation of adequate quantities of rock products in the wearing surface, through care in selection and proper proportioning of materials; through adequate mixing, careful placing, and proper curing.

15. One-course and Two-course Pavements.—The tendency at the present time is toward the use of one-course, rather than two-course roadway pavements. A certain roughness of surface is very desirable to prevent slipping; and in a pavement with the concrete properly proportioned and mixed there is a requisite amount of rock material in the surface to withstand abrasion, with the added advantage that the slab is monolithic, without separation planes, as is so often the case where one course is laid upon another which has already set. The general method of finish is, however, similar screeding, floating, and troweling being done in the usual manner.

16. The Making of Concrete Roadways.

16a. Porous Subbase.—Inasmuch as a concrete roadway pavement should not be called upon to sustain beam stresses, a continuous unyielding bed must be provided, else cracking and unequal settlements will occur. Such a solid bedment of the concrete slab requires special provision against upward heaving by frost action, which necessitates either a porous subbase of a depth sufficient (together with side drains) to clear the ground of water below frost line, or else one having a sufficiently yielding nature to permit local ground eruptions without extension of the disturbance to the concrete slab above it, though this latter is almost impossible to secure.

It is regretable that so little attention is paid to this important feature of subdrainage. The majority of concrete roadways are placed directly on rolled ground, often without pretense of drainage and in some cases, with even a pronounced dish toward the center. To counteract the effect of settlement or frost action encouraged by this initial fault, reinforcement is embedded in the slab, $\frac{1}{10}$ of 1% of steel being the Joint Conference recommendation, but twice to three times that amount being actually necessary to secure a modicum of assurance. Concrete roadways should be capable of enduring and rendering splendid service without reinforcement. Actually, even the reinforcement too often proves inadequate to the unnecessary task imposed on it.

16b. Proportioning and Selecting of Materials.—Arbitrary proportions are generally used in pavement work, regardless of possible benefits obtainable by better grading. For one-course work, average specifications call for 1:2:3 concrete; and for two-course work, 1:2½:5 concrete in the base with a topping of 1:2 mortar, although a concrete, using fine stone instead of sand requiring even less cement, would be preferable.

The principles of proportioning and selection of materials for concrete roadways as laid down by the Aggregate Committee of the 1914 National Conference are as follows:

1. **For fine aggregate,** use only sand or other fine aggregate free from very fine particles, and which has been actually tested by mechanical analysis, and for the tensile strength of standard mortar.
2. Use coarse-grained sands or hard stone screenings with dust removed.

3. Use sand or other fine aggregate that is absolutely clean.

4. For coarse aggregate, use hard stone, such as granite, trap, gravel, or hard limestone.

5. If bank gravel or crushed stone is used, always remove the sand or screenings and remix in the proper proportions.

If local conditions prevent following any of these rules, adopt some other material than concrete for your pavement.

More detailed requirements for fine aggregate are:

The size of the fine aggregate shall be such that the grains will pass when dry a screen having ¼-in. openings. In the field a ⅜-in. mesh or in some cases a ½-in. mesh screen may be used for this separation.

Not more than 10% of the grains below the ¼-in. size shall pass a sieve having 50 meshes to the linear inch, and not more than 2% shall pass a screen having 100 meshes to the linear inch. This is an exceptionally coarse sand, but coarse sand is a necessity for a durable pavement.

16c. Joints.—Expansion joints must be provided to prevent cracking due to temperature changes. These should be provided at linear intervals of from 30 to 50 ft. depending upon the climate of the region in which the pavement is situated. Joints should also be placed at changes in grade, and longitudinally between curbs. The usual joint is from ¼ to ½ in. wide and the National Conference recommends a preformed plastic filler. A variety of special compounds for this purpose are on the market and also metal-and-plastic inserts intended to reduce the surface exposure of joints to a minimum.

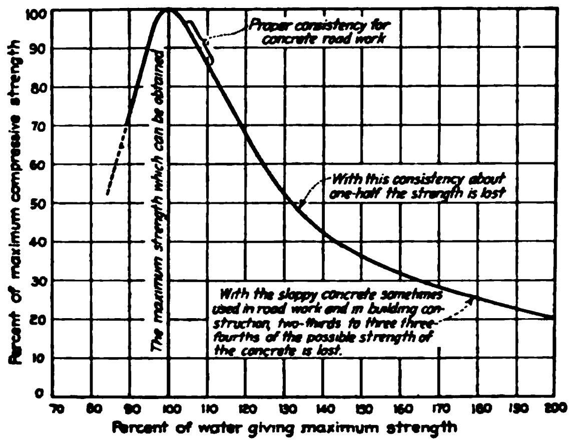


Fig. 5.—Proper consistency of concrete for road work.[1]

16d. Curing.—Difficulties that have arisen in the use of concrete pavements have forced recognition of some usually neglected though well-known principles of concrete making. Among these is protection of the pavement for a period of time against drying in warm weather and frost in cold weather. For the purpose first named, earth dams permitting flooding the pavement with water have been used with success; also protecting canvases, or layers of sand, or earth, or sawdust have been used each being kept moist. In the winter time protection against frost has been obtained by the use of hay or straw and sometimes manure. The use of the latter, however, is dangerous, inasmuch as manure not only discolors the concrete, but may bring about disintegration through penetration and decomposition of organic acids.

16e. Consistency.—Observation indicates a general tendency to mix pavement concretes too wet (see Fig. 5). The proper consistency is plastic, permitting compacting and molding, with surface finishing, but without runoff, or separation of ingredients. Overwet concretes in roadway pavements cannot be expected to have better qualities than the same character of mix possesses in other types of structures.

[1] D. A. Abrams: Bulletin Portland Cement Assoc.

SECTION 5

PROPERTIES OF CEMENT MORTAR AND PLAIN CONCRETE

By Adelbert P. Mills[1]

STRENGTH OF CEMENT MORTAR AND PLAIN CONCRETE

1. Strength in General.—The strength of mortar and concrete made with a given cement is dependent primarily upon: (1) the inherent strength of its aggregates, particularly the large aggregates; (2) the proportion of cement per unit of volume of mortar or concrete; (3) the degree of compactness or density of the mortar or concrete; and (4) the time afforded subsequent to final deposition in molds or forms.

(1). With a given aggregate (sand, or sand and broken stone or gravel) the mortar or concrete strength will, other things being equal, increase with increased proportion of cement in the mixture.

(2). With a given proportion of cement in the mixture the mortar or concrete strength will, other things being equal, increase with increased density of the mixture.

(3). Strength normally increases with age for an indefinite period.

The first of these rules does not apply in comparing mortars or concretes which have been made with different aggregates, with different cements, with different proportions of water used in gaging, or with different methods of mixing or placing the mixture, nor in comparing mortars or concretes which have cured under different conditions.

The density of the mixture secured will depend primarily upon the gradation in sizes of the aggregate, but may also be affected by the manner and extent of manipulation of the material in mixing and placing, and by varying the proportion of water used in gaging.

The second rule fails as a basis of comparison when different cements are used, when the aggregates possess a different mineral character or are to a differing degree contaminated with mineral or organic impurities, and may not apply when methods or conditions of making, placing, and curing of the mixture vary.

The third rule is independent of most other factors except that certain circumstances such as the proportion of water used in gaging and the conditions of curing may affect the rate of increase of strength with age. An apparent falling off or retrogression in tensile strength after from 3 to 6 months is commonly noted in tests of neat cement, and less commonly in tests of mortars. A less-marked retrogression in compressive strength is also frequently observed in tests of neat cement, and a slight retrogression in compressive strength of mortars and concretes is not infrequently observed after long periods.

A number of empirical formulas have been derived by various experimentors which attempt to express a definite mathematical relation between strength of mortar and concrete and the absolute volume of cement and sand, or cement, sand, and coarse aggregate, in the mixture. Owing to the fact that the effect of many factors, some of which are mentioned above, cannot be taken into account by such formulas, their application is restricted to certain classes of laboratory investigations which do not involve many of the variables encountered in the use of similar materials on construction work.

2. Laboratory Tests, Their Use and Significance.—The value of cement mortars and concretes as structural materials depends primarily upon their mechanical strength and dura-

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bility when hardened. The conditions encountered in practical use, however, are necessarily so variable as to exclude the possibility of the establishment of standards based directly upon practical experience.

The only established American standards for mortars are those for tensile strength (see *Appendix* A). These standards merely fix values of tensile strength, determined under laboratory conditions, which experience has shown may be expected of cements and sands found satisfactory in practical construction work, in order that inferiority in any particular mortar materials may be detected by deviation from such standards. In other words, knowing that good concrete or mortar sands will in laboratory tests show a tensile strength not inferior to that of standard sand mortars, it is assumed that any sand which exhibits like tensile properties in the laboratory will not fail to satisfactorily meet the conditions of structural use.

The conditions encountered by a mortar in a structure are not duplicated in the laboratory, but the laboratory method is so standardized that the external conditions of the test may be duplicated elsewhere or at a different time.

Two factors operate to lessen the importance of laboratory tests of mortars as an indication of suitability of the material for construction uses: (1) the closeness of the relation between the results of tensile tests and the qualities which a mortar in a structure will be called upon to show may properly be considered open to question, and (2) laboratory tests of this class of material cannot be made with any great degree of precision, and the results may be very much in error if the work is not performed under proper conditions by a skilled operator.

Mortars are never used structurally in such a way that they will be depended upon to carry tensile stress, while they are commonly used to carry compressive stress. From this circumstance it may be argued that a compressive test in the laboratory will afford a more direct indication of the structural qualities of the material than does the tensile test. The compressive test is less easily made than the tensile test, however, and calls for more elaborate, more expensive, and less portable equipment. It is not easy to establish the relationship between laboratory test results and structural qualities of mortars, but the tensile test has been made so much more generally than the compressive test, that the average man has no experience by which he may judge the value of the latter, while he has observed that materials which pass the tensile test infrequently prove unsatisfactory in a structure. Whether the compressive test will fail any less frequently as an indication of unsuitability remains to be conclusively shown, but the somewhat inadequate data available seem to point toward this conclusion. It is undoubtedly a fact that certain natural impurities in sands, as well as certain classes of material sometimes intentionally added to mortars for special purposes such as decreasing permeability or altering the appearance, reveal their injurious character to a much more marked extent in compressive tests than they do in tensile tests.

The results of laboratory tests of mortars are affected by a number of factors, not all of which are readily subject to control. It is never possible to determine precisely the relative qualities of mortar materials tested in different laboratories or by different operators. The atmospheric condition of the laboratory, with respect to both temperature and humidity, is one important factor which ought not to be subject to variation, but is so nevertheless. The most important consideration, however, is the fact that very slight variations in the detail methods of manipulation of the materials in making test specimens affect the test results to so great an extent that a very close check between the results of two different operators is impossible.

An experienced operator may be able to check his own results within say 5%, but a second equally-experienced operator who can also check his own results thus closely, may not be able to check the first man within less than 15 or 20%. Each man has developed invariable methods of manipulation, but the methods of the two men will never be identical. This fact need not invalidate comparative tests, however, and the results of tests of standard mortar and mortar made by the same experienced operator with a commercial sand substituted for the standard sand should be truly comparable.

Bearing in mind the considerations above discussed, it may be concluded: (1) that laboratory acceptance tests of mortar must not be considered to show the absolute strength which may be developed in a structure, but merely to indicate the approximate relative value of the proposed materials and materials whose suitability has been proved; and (2) that laboratory tests must never be entrusted to anyone who has not had a large experience in making this particular kind of test with the advantage of a fully equipped laboratory. The testing of concrete materials is not the job for a novice, and the average field laboratory is not a fit place to do the work.

3. Neat, Mortar, and Concrete Strength Compared.—A comparison of the strength of cement, cement mortar, and concrete can only be made when the many variable factors which influence the results of tests have been eliminated so far as is practically possible. This means that only tests made with identical materials under the same auspices are truly comparable.

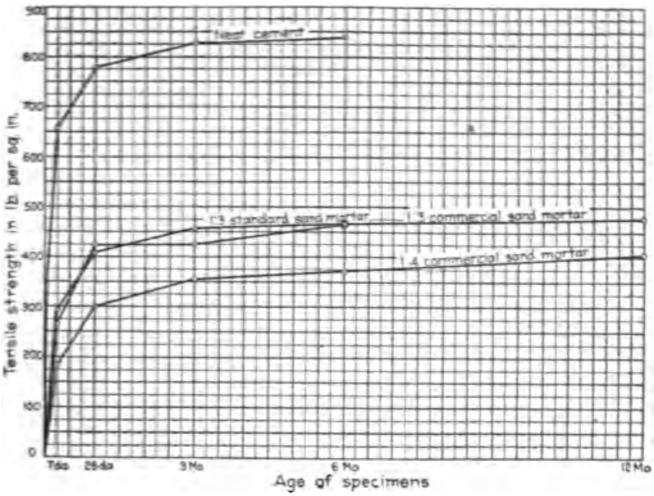

FIG. 1.—Relation between tensile strength of neat cement and cement mortars (9 brands of cement).

The strengths of the cement, mortar, and concrete mixtures shown graphically by the diagrams of Figs. 1 and 2 are based upon one series of tests made by the Technologic Branch of the U. S. Geological Survey at the Structural Materials Testing Laboratories formerly located at St. Louis, Mo. The complete report of these tests is contained in *Bulletin* 344 of the U. S. Geological Survey and *Technologic Paper* 2 of the U. S. Bureau of Standards.

The diagrams for neat cement and 1:3 standard sand mortar are averages of three tests of each mixture for each of nine separate brands of cement. For the commercial sand mortars and all of the concrete mixtures, a blend of these nine cements was used. The diagrams average the results of three tests of each mortar and 18 to 21 tests of each concrete. (The irregularities shown by the diagrams for the commercial sand mortars would doubtless not appear if they represented the average of a larger number of tests.) The same commercial sand was used in the mortars and all of the concretes. It is described as Merrimac River sand and is composed of flint grains having comparatively smooth surfaces. It has a fairly well-graded composition, its void content is not particularly high, but it is finer than is desirable. The four coarse aggregates are typical well-graded aggregates of the classes indicated. It should not be concluded from these tests that granite or gravel aggregate can be depended upon to excel limestone in all cases. Very slight differences in two apparently similar materials of the same class will often make a great difference in their value as concrete aggregate.

It is not intended that the diagrams of Figs. 1 and 2 shall form a basis for definite conclusions concerning the relative strengths of various cement mixtures, but only to show in a general way the results of tests of typical materials in various mixtures. The value of any given material can be determined only by tests of its qualities regardless of the qualities other materials of the same type may show. The more important of the many factors which affect mortar and concrete strength are considered below.

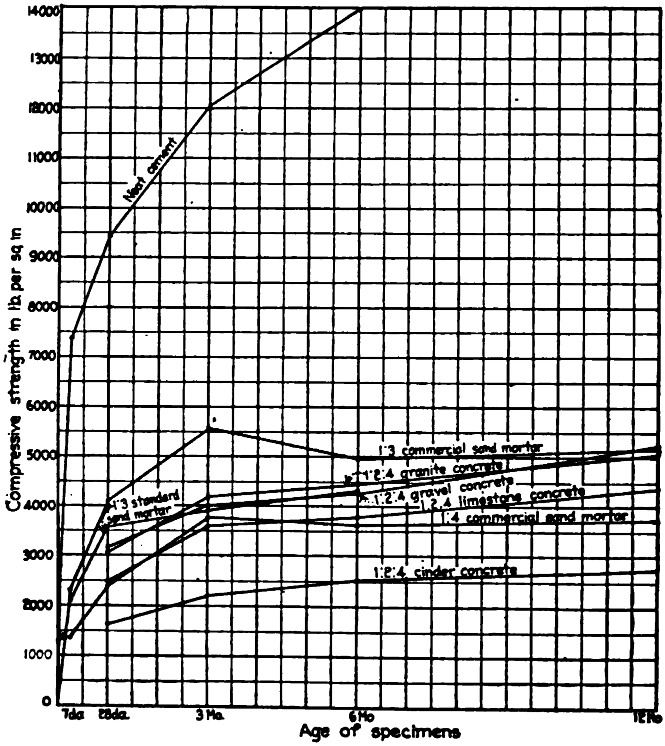

FIG. 2.—Relation between compressive strength of neat cement, cement mortars, and concretes (9 brands of cement).

4. Aggregates of Mortar and Concrete.—Testing of the cement used in all important concrete structures has been common practice for many years, but the importance of the quality of the aggregate, in its relation to the quality of mortar and concrete of which it forms a constituent, has not yet come to be adequately appreciated by the majority of engineers, architects, and contractors. While cement of good quality is essential to the making of good concrete, its manufacture has today been standardized to such an extent by exacting specifications that in the average case it is actually safer to assume, without tests, that the cement is satisfactory than to assume that the aggregate materials most readily available may properly be used without careful experimental determination of their quality.

The principal requisites for concrete aggregates are structural strength and durability, a proper gradation of sizes of particles, and cleanliness or freedom from deleterious matter.

The unsuitability of a weak, soft, or porous material is quite obvious, but a well-graded limestone aggregate may make better concrete than a harder granite aggregate whose void content is high, and a coating of matter partly or wholly of organic origin upon the particles of the best-graded granite aggregate obtainable may cause the concrete in which it is used to have exceptionally poor qualities.

The physical testing of aggregates is, unfortunately, not controlled by any generally-accepted specifications, but methods are at the present time undergoing standardization by the technical committees of the most interested National engineering societies. Some of the largest engineering organizations, as well as State, Federal, and municipal public service commissions, employing large quantities of concrete, make a regular practice of subjecting all aggregate materials used to a systematic physical examination. The variability of aggregate materials available in different localities, and even of an aggregate from a single source of limited extent, magnifies the importance of tests not only in choosing the most suitable aggregates from those available for construction work in any given locality, but also in certifying the quality of all shipments of that aggregate to the job. This entails a considerable expense for the testing of materials which possess a very low intrinsic value. It is an expense which may be justified, however, by the direct benefit gained by a thorough knowledge of how available materials may best be utilized. An unsatisfactory material may be greatly improved by washing, perhaps, or by screening and readmixture of the different grades in different proportions, or the judicious mixture of two available materials may be found to yield a material greatly superior to either one alone.

It is very desirable, also, that the proportions of the mixture of cement and fine and coarse aggregate be not rigidly specified, but only that the physical properties of the resulting concrete shall be up to a fixed standard of strength or, in some cases, density or impermeability. This will often mean that when the local materials are inferior, a concrete of the required quality may be attained either by using the local materials in a rather rich mixture, or by importing better aggregate from a distance, using a leaner mixture. Thus the relative costs of the additional cement used with the local materials, or the freight charges on imported aggregate will be the factor which determines the choice.

For tests, specifications, and properties of aggregates, see chapter on "Aggregates" in Sect. 1.

5. Effect of Mineral Character of Aggregates.—The structural strength and durability of concrete aggregates is dependent upon the mineral character of the rock from which it is derived. In the case of coarse aggregate of artificially crushed stone, the original qualities of the rock have obviously not been altered. When the rock has been broken down into gravel or sand through the operation of natural agencies, the structural qualities of the individual particles of the material will still be identical with those of the parent rock except for possible changes effected by chemical agencies.

The principal classes of rocks from which concrete aggregates are derived are granites, trap-rocks, limestones, and sandstones. *Granite* is an igneous rock of variable structure and texture, whose principal mineral constituents are quartz and feldspar with varying amounts of mica, hornblende, etc. The structural qualities of granites vary greatly but granites as a class rank among the hardest, strongest and most durable stones. The term *trap-rock* is commonly used to include basalt, diabase, and a number of other igneous rocks possessing similar chemical and physical properties. The principal mineral constituents of most of these rocks are pyroxene and feldspar. They are commonly rather fine-grained, hard, tough, and durable. *Limestones* contain carbonate of lime, calcite, or carbonate of lime together with the double carbonate of lime and magnesia, dolomite, as the essential constituent. Sand and clay are common impurities and some varieties contain large amounts of shells and other fossils. Limestones vary greatly in structure, strength, hardness, and durability. Some of the limestones are superior in structural qualities to some of the granites, but the average limestone is inferior to the average granite or the average trap-rock as concrete aggregate. *Sandstones* consist of grains of varying sizes, chiefly quartz, bound together by silica or iron oxide or, less frequently, by lime carbonate or clay. The structural qualities, strength, hardness, and durability of sandstones vary greatly according to the texture, and the class of the binder. A silicious binder excels any other in all respects; an iron oxide binder is usually, though not always, superior to one of lime carbonate; and sandstones having a clay binder are in all respects least valuable as concrete aggregate.

Fig. 3, which is based upon tests made at the U. S. Bureau of Standards (*Tech. Paper* 58), shows the great variability in strength exhibited by concretes made with different classes of coarse aggregate and with different aggregates of the same general class. The proportions were 1 : 2 : 4 in all cases, and the same cement, a blend of nine standard brands of Portland cement, was used throughout. Two river sands of similar character and somewhat similar granular analysis were used in making the specimens of each coarse aggregate. It will be noted that, except in the case of the granite concretes which included only four different aggregates, the range in strength of concretes with different aggregates of the same class is often more than 100% of the average strength.

All sands are derived from rocks which have been broken down or disintegrated through the operation of purely-physical agencies, without change of mineral identity, or which, in addition to disintegration, have been more or less decomposed by chemical

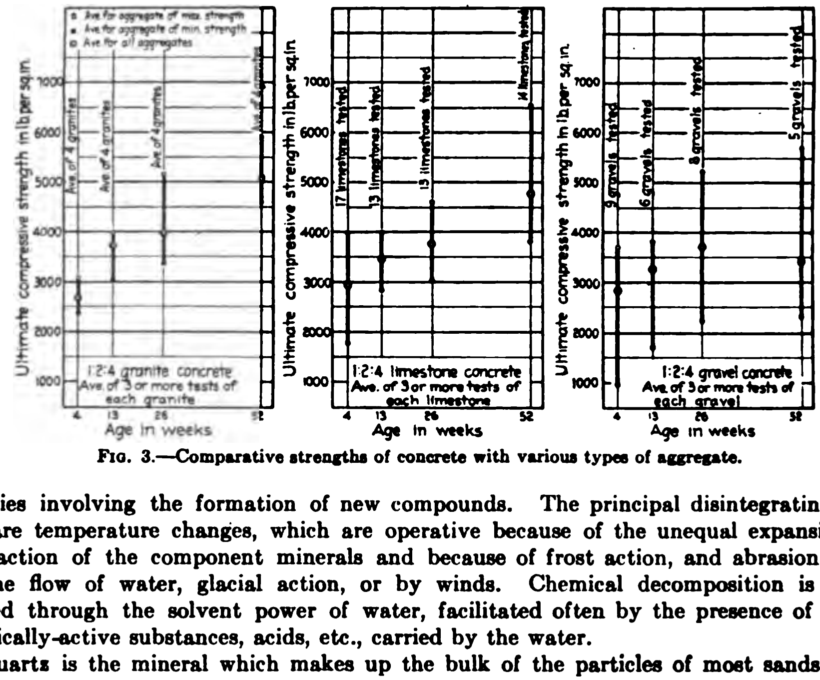

Fig. 3.—Comparative strengths of concrete with various types of aggregate.

agencies involving the formation of new compounds. The principal disintegrating agencies are temperature changes, which are operative because of the unequal expansion and contraction of the component minerals and because of frost action, and abrasion caused by the flow of water, glacial action, or by winds. Chemical decomposition is accomplished through the solvent power of water, facilitated often by the presence of various chemically-active substances, acids, etc., carried by the water.

Quartz is the mineral which makes up the bulk of the particles of most sands. This is due to the fact that only the harder constituents of rocks survive as sand, and quartz is not only a very common constituent of rocks, but is also very hard and resists chemical decomposition. The fact that quartz is the principal constituent of a sand does not insure its suitability for concrete, however. Comparatively small amounts of certain minerals like mica or even feldspar or hornblende, or very small amounts of organic impurities will render a quartz sand altogether unfit for use. Sandstone is a common source of quartz sand, and the quality of the sand will depend upon the character of the binder of the original rock, since the individual particles of sand are made up of still smaller particles of quartz bound together by silica, iron oxide, lime carbonate, or clay according to the class of the rock. Sands are seldom derived from trap-rock or granite directly, though sand beds may often be largely made up of the constituent minerals of these rocks. Pyroxene and hornblende are complex silicates possessing a degree of hardness, strength, and durability slightly inferior to that of quartz. Hornblende particularly has inferior weathering qualities. Feldspars are essentially silicates of alumina with potash, soda, or lime. They are considerably less strong and durable than quartz. Mica is a very objectionable constituent of sands for concrete. It is soft, has low strength, particularly in shear, and its laminated structure promotes the percolation of water. Its surface is also of such a character that the bond secured by cement is very poor. Limestones do not serve as a source of concrete sands although calcite and dolomite may occur in sands derived from sandstones having a calcium carbonate binder. Limestone screenings or crusher dust are also sometimes used as fine aggregate though the concrete made therewith is usually inferior in strength to that made with an average sand, and it is also apt to be, or will in time become, more permeable, owing to the solubility of calcite and dolomite.

Sand deposits being rarely of a residual character, but usually deposited by stream or glacial action, and being also of such a character that the percolation of surface waters through the beds is very easy, the material is often contaminated by matter, much of it of organic origin, carried in suspension by water. Thus the coating of the grains by such substances as tannic acid is frequently encountered. The effect of such impurities is extremely detrimental and the difficulty with which their presence may be detected increases the importance of careful tests of concrete sands.

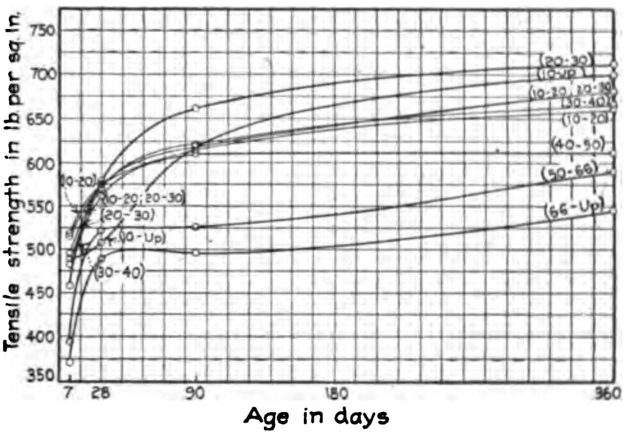

FIG. 4a.—Effect of size of sand upon tensile strength 1:1 mortar. (Tests of R. P. Davis.)

6. Effect of Shape and Size of Aggregates.—Specifications frequently call for a sharp sand, *i.e.*, one composed of rough angular particles, in spite of the fact that in many localities river or beach sands having somewhat rounded particles are the only ones obtainable, and have been used with perfect satisfaction. The shape of the particles is chiefly important in so far as it affects the void percentage of the material, and rough, irregular particles do not compact better than rounded particles. On the contrary, rounded particles which afford no opportunity for bridging will compact into the densest mass. In some instances the surface facets of angular particles may afford a better adherence for cement than the rounded surfaces of water worn particles, but this factor is less influential than is the density of the mass, and mortars and concretes made with aggregates composed of worn, rounded particles are not inferior in strength to those made with "sharp" aggregates, and very often excel the latter in strength. The same considerations apply in the case of crushed stone *vs.* gravel as coarse aggregate. Concrete of excellent quality or very inferior quality may be made with either class of material, the mineral character of the particles and the gradation of sizes being much more influential factors than the shape of the particles of aggregate. The requirement of sharpness is based upon an erroneous idea of the positive advantage gained; it often works hardship, or injury, or is unenforcible, and should be omitted from specifications.

The prevailing size of the particles of aggregates is more important than the shape, but far less important than the gradation of sizes because of the direct relation of the latter to density and strength. A comparatively coarse sand is always preferable to fine sand. It has a smaller surface to be coated with cement per unit of volume and therefore requires less cement to produce a mortar of a given strength. It is less difficult to fill its interstices with cement than in the case of fine sand, and a denser mass is usually secured with the same proportion of cement. A composition of coarse sand and finer material which will serve to fill the voids in the coarse material will usually excel either a very coarse or a very fine sand alone and will lead to economy of cement. This is particularly true when permeability is important, a mortar made with a combination of coarse and fine sand, or one in which the sizes of particles are well graded from coarse to fine, being less permeable than one made with either exclusively coarse or exclusively fine material.

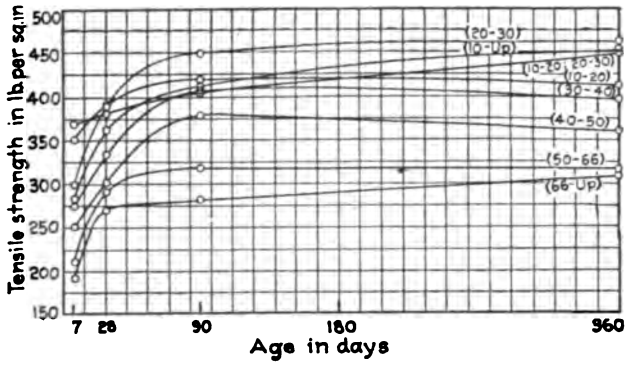

FIG. 4b.—Effect of size of sand upon tensile strength 1:2 mortar. (Tests of R. P. Davis.)

The relation between size of sand and tensile strength of mortar is shown by Figs. 4a, 4b, and 4c for 1 : 1, 1 : 2, and 1 : 3 mortars, respectively. These diagrams are based upon tests made in the laboratories of the College of Civil Engineering, Cornell University, by R. P. Davis ("Materials of Construction," by A. P. Mills, pp. 152–153). The various sands used were prepared artificially by separating a natural beach sand of nearly pure quartz into eight sizes or combinations of sizes. It is shown that the sand which passes the 20-mesh sieve and is retained on the 30-mesh sieve produces the strongest mortar for all mixtures and at all except the early ages. The sand of all sizes finer than that passing the 10-mesh sieve ranks second for all mixtures, the blend of equal amounts of 10–20 and 30–40 sand ranks third, 10–20 sand ranks fourth, and all finer sizes of sand rank lower in the order of their fineness.

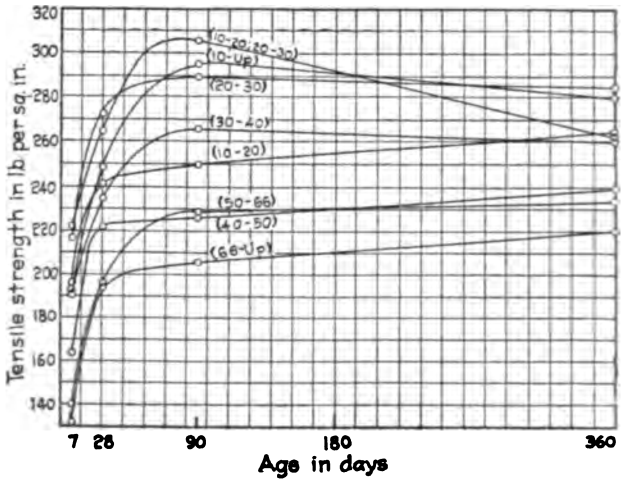

FIG. 4c.—Effect of size of sand upon tensile strength 1:3 mortar. (Tests of R. P. Davis.)

7. Relation Between Density and Strength.—The term *density* is employed referring to mortars and concretes, meaning the ratio of the sum of the absolute volumes or absolutely solid substance of the individual constituents contained in a measured unit volume of mortar or concrete to the measured unit volume of the materials combined in the form of mortar or concrete, water being neglected as an individual constituent. In other words the *density* is the solidity ratio, the ratio of the volume of solids to the volume of the mass of mortar or concrete.

Many experimental studies have shown that the strength of mortars and concretes is directly proportional to the density of the mixture. The density of the mixture is dependent partly upon the thoroughness of mixing, the amount of water used in gaging, etc., but is primarily dependent upon the gradation of sizes of the aggregates. Aggregates in which the relative amount of particles of different sizes is such that the particles of one size just suffice

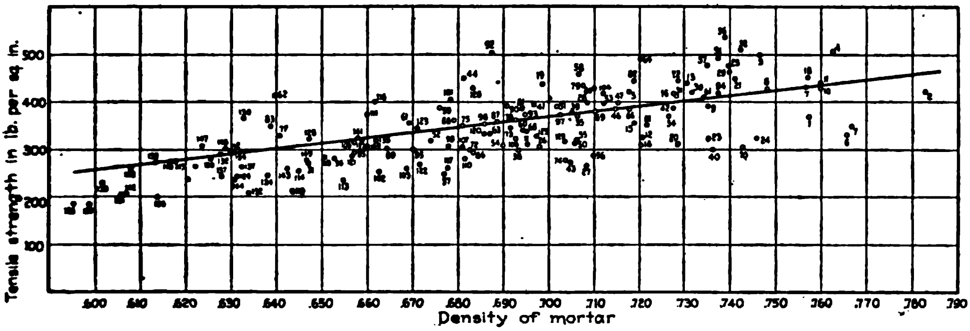

FIG. 5a.—Relation of "density" or solidity ratio of 1:3 mortar to tensile strength. Age, 13 weeks.

to fill the voids of the next larger size will have a minimum void space, and will therefore require a minimum proportion of cement to secure a product of given strength. The ideal concrete of maximum density would be made with aggregates of this character, and, in addition, would be so proportioned that the mortar just suffices to coat the particles and fill the voids of the coarse aggregate, and the cement just suffices to coat the particles of sand and fill its voids. This ideal can of course only be approximated in practice because the larger particles of each

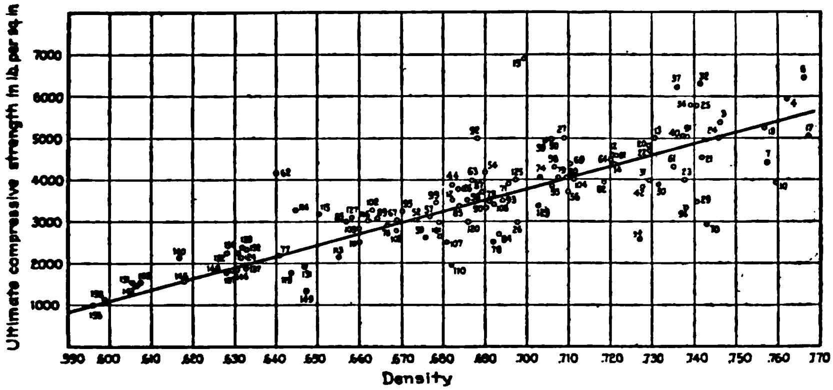

FIG. 5b.—Relation of "density" or solidity ratio to compressive strength of 1:3 mortars. Age, 13 weeks.

aggregate will not closely approach each other. Owing to the wedging action of the smaller particles the larger stones and grains of sand are forced apart so that the density of the mixture is certain to be less than that theoretically possible. A slight excess of mortar over that theoretically required is usually beneficial to density and strength.

The relation of density of 1 : 3 mortars to tensile strength is shown by Fig. 5a which comprises tests of 157 different natural sands made by the U. S. Bureau of Standards (*Tech. Paper*

58). The numbers on the diagram indicate the order of fineness of the sands, No. 1 being the coarsest, and No. 157 the finest. The corresponding relation of density and compressive strength for the same mortars is shown by Fig. 5b. Note that Figs. 5a and 5b show that the density of mortars is in general inversely proportional to the fineness of the sand used, the finer the sand, the lower the density.

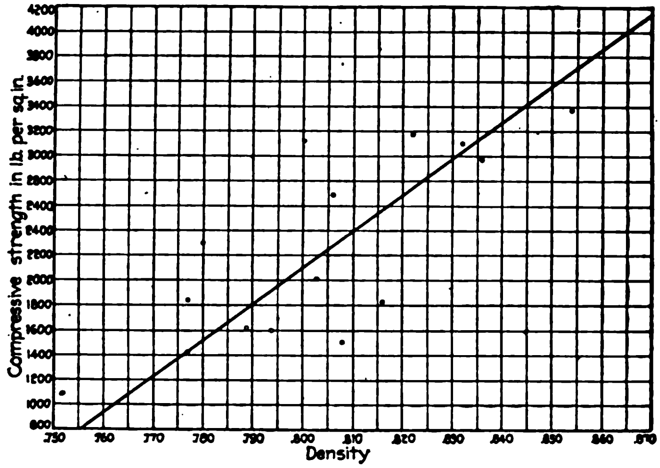

FIG. 6.—Relation between density and compressive strength of concrete. (Mix 1 : 2 : 4. Age, 4 weeks.)

The relation of density to compressive strength of 1 : 2 : 4 concretes made with various aggregates is shown by Fig. 6. The diagram is based upon tests made by the U. S. Bureau of Standards (*Tech. Paper* 58, Tables 23–26). The data are not extensive enough to definitely fix the relation sought because of the inevitable wide variation in test results due to variable

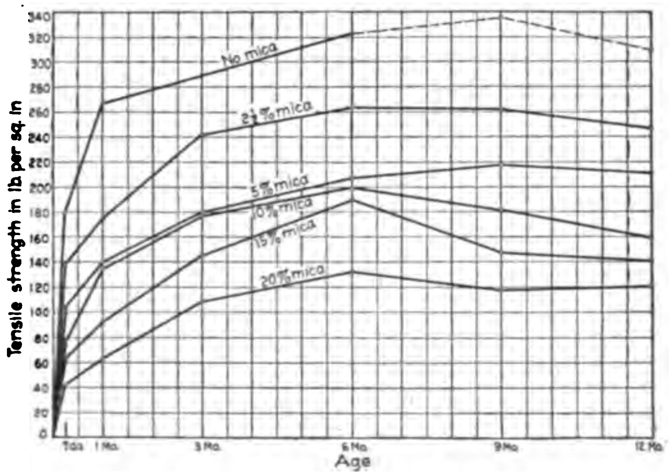

FIG. 7.—Effect of mica upon tensile strength of 1 : 3 standard sand mortar.

factors other than density. That the relation is a direct one, approximately that shown by the straight line upon the diagram, is, however, a justifiable conclusion.

8. Effect of Mica, Clay, and Loam in Aggregates.—The occurrence of mica, clay, and ...m in aggregates has been explained in connection with the consideration of mineral composition in Art. 5. The very detrimental effect of mica upon the strength of 1 : 3 standard sand mortar is shown by Fig. 7. The diagrams are based upon tests made by W. N. Willis (*Eng. News*, vol. 54, p. 145). The loss in strength amounts to 15 to 25% with 2½% of mica in the sand, 25 to 45% with 5% of mica, and becomes still more marked as the proportion of mica is increased. Mr. Willis also observed that increasing the proportion of mica increased the voids in the sand from 37% with no mica, to 67% with 20% of mica. The weight was at the same time lowered 20%, and the amount of water required in gaging the mortar was 3 times that required in gaging mortars free from mica.

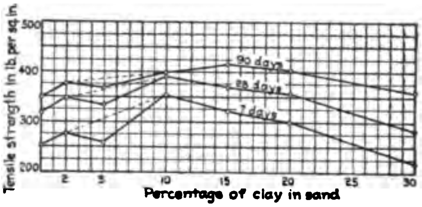

FIG. 8.—Effect of clay upon strength of 1 : 3 commercial sand mortar. (All mixtures artificially made in the laboratory.)

The effect of clay in sands is dependent upon its state of subdivision and the uniformity with which it is distributed through the sand. In most laboratory tests the addition of clay in moderate amounts has been found to be beneficial. Typical results of laboratory tests are exhibited by Fig. 8 which is derived from tests made by F. L. Roman (*Eng. & Cont.*, vol. 43, p. 403). With the materials here used the maximum increase in strength due to clay additions was observed to be about 20%, and was secured with additions of 10 to 15% of clay. Similar tests made by L. T. B. Southwick and G. A. Wellman (*Eng. Rec.*, vol. 63, p. 332) show that maximum strengths of 1 : 1½, 1 : 3, 1 : 4½, and 1 : 6 mortar mixtures are secured with 3%, 10%, 15%, and 20% of clay, respectively. These laboratory results do not prove that similar percentages of clay will be beneficial or harmless in natural clayey sands. The manner of distribution and degree of fineness of the clay in concrete sands will be the determining factors, and the amount permissible will not usually approach the above limits. Lumps of clay do not become broken up in concrete mixing, and should be carefully excluded from aggregates.

Loam, in the usual acceptance of the term, is earth which is made up of vegetable mold together with clay or sand or both. It is extremely injurious to mortars and concretes because of its content of organic matter. Fig. 9, derived from the series of tests of F. L. Roman above referred to, shows the effect of loam and organic matter in sand upon strength. From these tests it appears that 5% of this loam (about 1.5% organic matter in the sand) reduces the strength of the mortar about 20%, and other amounts are nearly proportionally detrimental. Organic matter naturally occurring in sands is frequently found detrimental to an even greater extent than is indicated by these tests, wherein the organic loam in the shape of fine powder was mixed with the sand. Organic matter not infrequently covers sand particles with a film which is not easily perceptible, but which tremendously retards the normal rate of hardening and gaining strength. An investigation made by Sanford E. Thompson (*Trans.* Am. Soc. C. E., vol. 65) led to the conclusion that organic matter constituting over 10% of the silt and at the same time over 0.1% of the sand is distinctly injurious.

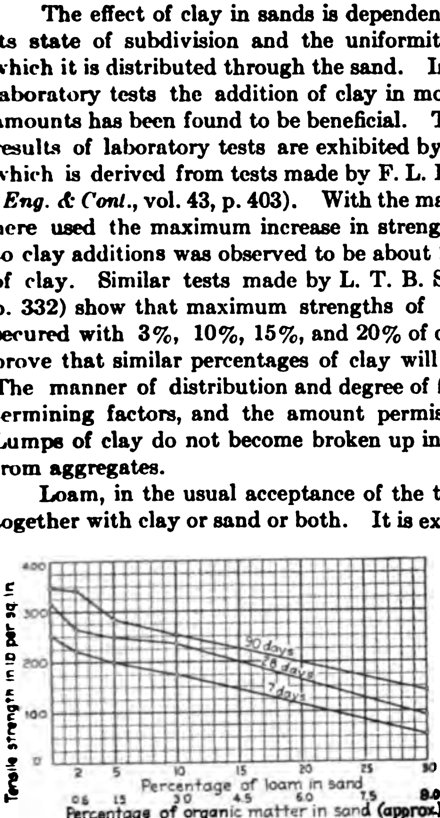

FIG. 9.—Effect of organic loam upon strength of 1 : 3 mortar.

9. Effect of Consistency.—The important relation of consistency to the strength of mortar and concrete is shown by Figs. 10 and 11. The amount of water is expressed as a percentage of the total dry weight of aggregates and includes any moisture carried by the sand or coarse aggregate in its natural condition. These diagrams are based upon tests made in the laboratories of the Sheffield Scientific School under the direction of Prof. Barney (*Eng. and Cont.*, vol. 42, p. 244).

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These tests show that a quite definite percentage of water is required to produce a mortar or concrete of maximum strength with given materials. For the particular materials used in this case the maximum 1 : 2 mortar strength was attained with about 15.6% of water, and the maximum strength of 1 : 2 : 4 concrete with about 8.4% of water. For other materials or other mixes these consistencies for maximum strength will not remain the same, but for any

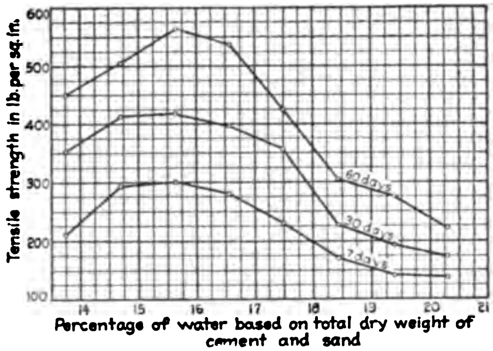


FIG. 10.—Effect of consistency upon strength of 1 : 2 mortar.

mixture of given materials there is a critical consistency which will be productive of higher strength than any drier or wetter consistency. This fact is particularly important in view of the common practice of using extremely wet concrete mixes in order to be able to deliver the concrete on the work cheaply and expeditiously by the use of chuting devices between mixer and forms. The fact should be understood that such wet mixtures may be used only with a

FIG. 11.—Effect of consistency upon strength of 1 : 2 : 4 concrete.

considerable sacrifice in strength of the concrete placed. On the other hand, these tests indicate that nothing is gained by making a concrete so dry that it must be rammed or tamped in place instead of being puddled. Concretes containing 8 to 9% of water are of a sufficiently mushy consistency to be readily puddled, but from 12 to 15% of water, or even more, is commonly used to produce the fluid consistency desirable for chute or spout delivery.

For harmful effects from the use of excess water and for suggested procedures, see chapter on "Water" in Sect. 1.

10. Compressive and Tensile Strengths Compared.—The compressive and tensile strengths of the same mortar mixture may be contrasted by a comparison of the diagrams of Figs. 1 and 2, pages 217 and 218. A direct comparison for 1 : 3 standard sand mortar is afforded by Fig. 12 which is based upon tests of seven brands of cement made in the Structural Materials Laboratory formerly maintained at St. Louis, Mo., by the U. S. Geological Survey (U. S. Geol. Surv.

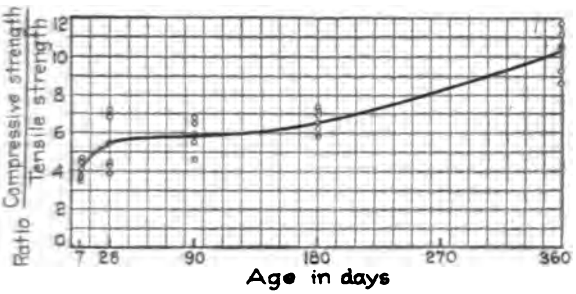


FIG. 12.—Ratio of compressive to tensile strength of 1 : 3 standard Portland cement mortar. (Each result is average of 30 tests of one brand. Curve is average of 7 brands.)

Bull. 331). The specimens used were standard tension briquettes and 2-in. compression cubes. It is shown by the diagram that the average value of the ratio of compressive strength to tensile strength is far from being constant as the age increases because of the relatively more rapid rate of gain in tensile strength during the first few weeks and the very slight gain or actual retrogression which characterizes the tensile strength after the first 6 months. The average

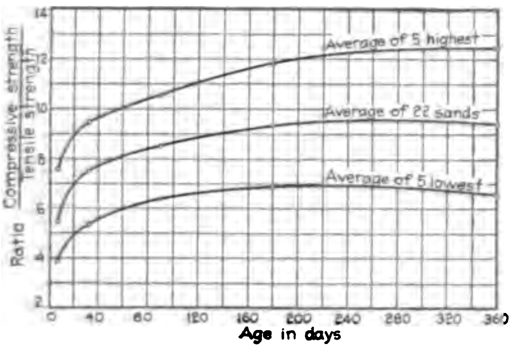


FIG. 13.—Ratio of compressive to tensile strength of 1 : 3 mortar using blend of 7 Portland cements and 22 commercial sands.

value of the ratio for all cements is about 6 between the 1-month and the 6-month periods, but the individual brands of cement show variations of from 15 to 40% from this average.

A similar series of tests, made under the same auspices, with a blend of the above 7 brands of cement and 22 commercial sands in 1 : 3 mortars has been made the basis of the diagrams of Fig. 13. Because of the very wide variations shown by the 22 different mortars, only the average for all the mortars, the average for the 5 mortars showing the highest value of the ratio and the average for the 5 mortars showing the lowest value of the ratio are plotted. The same variation in the ratio of compressive to tensile strength with age shown by standard sand mortars is exhibited by the commercial sand mortars.

An inspection of Figs. 12 and 13 will show that the tensile strength of mortars is not more than a very approximate indication of the probable compressive strength of similar mortars with the same cement, and even then the age, the mixture, and the sand used are sources of variation which must be taken account of.

The tensile strength of concrete is a property of little importance because, being low in comparison with the compressive strength, concrete is practically never designed to carry tensile stress. When concrete is used in situations involving tensile stress it is more economical

TENSILE AND COMPRESSIVE STRENGTHS OF CONCRETE

Character of coarse aggregate and mix	Age, months (approx.)		Compressive strength (lb. per sq. in.)	Tensile strength (lb. per sq. in.)	Ratio $\frac{\text{tensile strength}}{\text{compressive strength}}$
	Tensile tests	Compression tests			
Limestone 1:2:4.....	6	1	2,206	278	
			2,708	308	
			2,500	253	
				306	
				264	
				257	
	Average....	....	2,505	278	11.1%
Sandstone 1:2:4....	6	1	1,069	149	
			1,375	142	
			1,417	133	
			1,722	178	
			2,000	158	
			2,139	128	
				153	
				150	
				161	
	Average....	....	1,620	150	9.3%
Sandstone 1:2½:5...	6	2	1,028	121	
			1,639	114	
			972	106	
			889	158	
			1,042	114	
			2,083	97	
			1,472	179	
			1,889	129	
			1,639	139	
	Average....	....	1,406	129	9.1%

to use steel reinforcement than to use the very large sections which would be required if the concrete were depended upon to carry tension.

An indication of the comparative strength of concrete in tension and compression is afforded by the table shown on page 228. These data were derived in tests made by the writer in the laboratories of the College of Civil Engineering, Cornell University. The concrete was mixed and the specimens molded in the field.

Note that the values of the ratio of tensile to compressive strength in this table would have been somewhat lower had the specimens been tested in compression at the same age they were in tension.

11. Strength of Plain Concrete Columns.—The strength of plain concrete columns, as determined by tests of laboratory specimens whose dimensions are comparable with those of columns used in structures, is usually not less than 75 nor more than 90% of the strength of cubes of the same concrete, the column length not exceeding 10 to 12 diameters.

A number of series of tests of plain concrete columns are tabulated below and on page 230. The strength of cubes of similar concrete is indicated where data from comparable tests are available.

STRENGTH OF PLAIN CONCRETE COLUMNS

Watertown Arsenal Tests[1]

Mixture and character of coarse aggregate	Age, months (approx.)	Compressive strength (lb. per sq. in.)	Cross-section, inches (approx.)	Length, feet (approx.)
1:1 Mortar	6	5,011+	12.5 × 12.5	8
1:2 Mortar	6	3,652	12.5 × 12.5	8
1:2 Mortar	6	2,488	12.5 × 12.5	8
1:3 Mortar	6	2,062	12.5 × 12.5	8
1:3 Mortar	6	2,692	12.5 × 12.5	8
1:4 Mortar	6	1,564	12.5 × 12.5	8
1:4 Mortar	6	1,471	12.5 × 12.5	8
1:5 Mortar	6	1,038	12.5 × 12.5	8
1:5 Mortar	6	1,082	12.5 × 12.5	8
1:1:2 (Pebbles)	5	1,525	12.5 × 12.5	8
1:1:2 (Pebbles)	8	1,720	12.5 × 12.5	8
1:1:2 (Trap rock)	5	3,900	12.5 × 12.5	8
1:2:3 (Pebbles)	8	1,769	12.5 × 12.5	8
1:2:4 (Pebbles)	3½	1,710	12.5 × 12.5	8
1:2:4 (Pebbles)	5	1,506	12.5 × 12.5	8
1:2:4 (Trap rock)	5	1,750	12.5 × 12.5	8
1:2:4 (Trap rock)	6	1,990	12.5 × 12.5	8
1:2:5 (Pebbles)	3	1,100	12.5 × 12.5	8
1:3:6 (Pebbles)	5	700	12.5 × 12.5	8
1:3:6 (Pebbles)	8	462	12.5 × 12.5	8
1:3:6 (Trap rock)	4	1,350	12.5 × 12.5	8
1:2:4 (Cinders)	5½	871	12.5 × 12.5	8
1:3:6 (Cinders)	5	1,060	12.5 × 12.5	8

[1] "Tests of Metals," 1904, 1905.

UNIVERSITY OF ILLINOIS TESTS[1]

Ratio col. strength / cube strength	Mixture (coarse aggregate, crushed limestone)	Compressive strength of columns (lb. per sq. in.)	Age columns, months (approx.)	Compressive strength of cubes (lb. per sq. in.)	Age cubes, months (approx.)	Cross-section, inches (approx.)	Length, feet (approx.)
	1:1½:3	2,120	2	......[2]	......	12 in. cyl.	10
95.3	1:1½:3	2,480	2	2,600	2	12 in. cyl.	10
69.9	1:2:3½	1,710	2	2,443	2	12 × 12	12
	1:2:3½	2,004	2	......	......	9 × 9	12
	1:2:3½	1,610	2	......	......	12 × 12	12
	1:2:3½	1,709	2	......	......	12 × 12	12
60.5	1:2:3½	1,189	2	1,962	2	12 × 12	6
	1:2:3½	1,079	2	......	......	9 × 9	6
	1:2:3½	2,650	12	......	......	12 × 12	12
	1:2:3½	2,770	16	......	......	12 × 12	12
	1:2:4	1,165	2	......	......	12 in cyl.	10
	1:2:4	2,000	2	......	......	12 in. cyl.	10
	1:2:4	2,210	2	......	......	12 in. cyl.	10
78.1	1:2:4	1,590	2	2,035	2	12 in. cyl.	10
	1:2:4	1,945	2	......	......	12 in. cyl.	10
78.3	1:2:4	1,460	2	1,865	2	12 in. cyl.	10
	1:2:4	1,810	2	......	......	12 in. cyl.	10
80.4	1:2:4	1,925	6	2,390	6	12 in. cyl.	10
99.7	1:2:4	1,845	6	1,850	6	12 in. cyl.	10
99.7	1:2:4	1,770	6	1,775	6	12 in. cyl.	10
99.8	1:2:4	2,680	6	2,685	6	12 in. cyl.	10
85.3	1:2:4	2,160	6	2,530	6	12 in. cyl.	10
74.6	1:2:4	1,770	6	2,370	6	12 in. cyl.	10
	1:3:6	955	2	......	......	12 in. cyl.	10
	1:3:6	1,110	2	......	......	12 in. cyl.	10
	1:4:8	575	2	......	......	12 in. cyl.	10
	1:4:8	575	2	......	......	12 in. cyl.	10
UNIVERSITY OF WISCONSIN TESTS[3]							
84.0	1:2:4	2,040	2	2,427	2	12 × 12	10
88.0	1:2:4	2,110	2	2,395	2	12 × 12	10
91.7	1:2:4	2,055	2	2,240	2	12 × 12	10
88.1	1:2:4	2,080	2	2,360	2	12 × 12	10

[1] *Bulls.* 10 and 20 of the Univ. of Ill. Eng. Exper. Station.

[2] Data from tests of cubes made at ages which do not correspond even approximately to the age of the column made from the same concrete have been omitted.

[3] *Bull.* 300.

12. Effect of Method of Mixing.—The method of mixing mortars and concretes may vary with respect to: (1) amount of water used; (2) duration of mixing operation; and (3) detail method of manipulation. The effect of variation in the amount of water used is considered in Art. 9. The effect of the duration of the mixing operation is shown by Fig. 14 which is based upon tests by Prof. Scofield of Purdue University (*Eng. and Cont.*, Jan. 17, 1915). All of the concrete was mixed in a Chicago Cube Mixer of 2½-cu. ft. capacity, run at the rate of 26 revolutions per min. These concretes are all much stronger than the average commercial concrete but this fact does not affect the significance of the test results. It appears that with the particular materials used a very decided advantage is gained by operating this mixer much longer than the usual period of mixing. The actual time of mixing most advantageous to the quality of concrete produced under given conditions will probably vary greatly with different materials and with different mixers. It is very probable, however, that the average concrete used in every day practice would be considerably improved in quality, if it were mixed for a longer period. This is certainly true of concretes which are turned out at a rate of a batch per minute as is sometimes the case. A distinct advantage is gained by mixing beyond the point which produces a batch of even color. The mass becomes more viscous; there is less danger of separation of fine and coarse material; for a given water content it appears to possess a wetter consistency and flows better in transporting by chute and in depositing in the forms; and it forms a concrete of greater density, less permeability, and greater strength.

It cannot be said that machine-mixing will invariably produce better concrete than hand-mixing, but for all except the smallest work it is less expensive and is, therefore, generally preferred. Hand-mixing is more apt to be severely slighted than machine-mixing because of the heavy labor involved and the comparatively long time required.

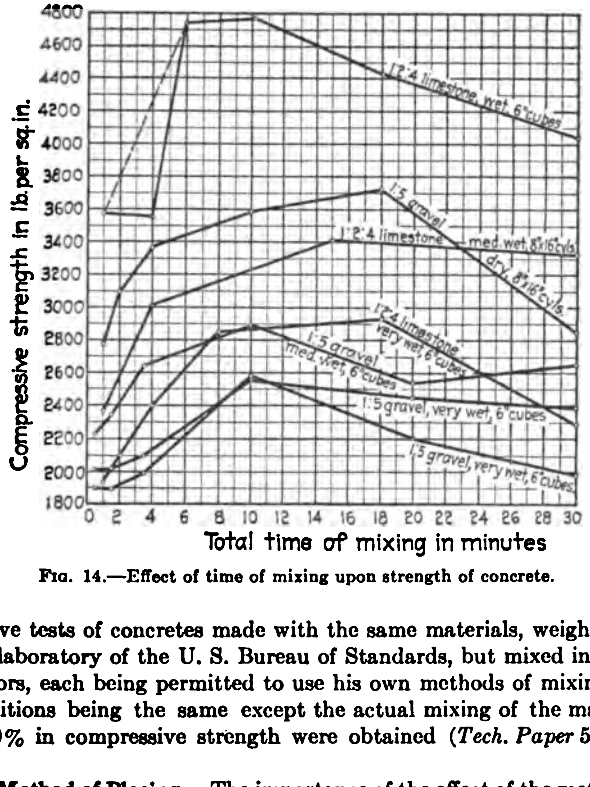


FIG. 14.—Effect of time of mixing upon strength of concrete.

In comparative tests of concretes made with the same materials, weighed and molded by employees of the laboratory of the U. S. Bureau of Standards, but mixed in the field by three different contractors, each being permitted to use his own methods of mixing, both hand and machine, all conditions being the same except the actual mixing of the materials, variations of as much as 70% in compressive strength were obtained (*Tech. Paper* 58, U. S. Bureau of Standards).

13. Effect of Method of Placing.—The importance of the effect of the methods of manipulating and molding laboratory specimens of mortar upon the qualities of the specimens shown by tests has been discussed in Art. 2. The same factors are operative in the case of molding laboratory specimens of concrete, and their disturbing effect becomes even more pronounced when the work is done in the field. Experimental data are lacking which might show the extent of the effect of variations in molding methods, but an indication of the importance of this factor is afforded by any series of tests of mortar or concrete specimens made from the same batch of material and stored and tested in an identical manner. A number of such apparently identical specimens may perhaps vary in strength less than 5% if made by an experienced operator, but a second equally-experienced operator using the same materials and the same general methods will often obtain results which are not within 20% of those of the first man. deviation must be attributed primarily to slight differences in detail methods of molding specimens.

When special methods of handling the materials are considered, only very scanty compa tive data are available. The following tests reported by R. E. Goodwin of the Mater Testing Division of the New York Public Service Commission indicate that concrete placed mass may possess greater strength than when it is cast in molds of the size ordinarily u (*Eng. News*, Feb. 18, 1915). Several pieces of concrete were cut from existing subway str tures at places designated before the concrete was placed in the forms. While the concr was being poured in the forms, samples from the same batches were cast in molds. A porti of the specimens thus molded were stored in moist sand on the work, while others were sto in the laboratory moist room. In addition similar specimens were made from the same ma rials brought from the work and mixed and molded in the laboratory. The pieces of concre taken from the work were cut from various portions of 12-in. walls at the age of 2 mont and after being rough-dressed were polished smooth to exact dimensions. All tests were ma at the age of 90 days; the concrete was a 1:2:4 mixture; and all specimens were 6 by 6 by 12 i

Compressive Strength of Field and Test-sample Concrete

Concrete made on the work (All in one line are from the same batch)				Concrete made in the laboratory (Samples of same materials from the work usec		
Specimens cut from 12-in. wall	Specimens poured in molds and stored on the work	Specimens poured in molds and stored in moist room	Consistency of batch	Specimens made in laboratory and stored on the work	Specimens made in laboratory and stored in moist room	Consistency of batch
(4) 3,095	(4) 2,880	(4) 2,870	wet	(4) 2,135	(4) 2,080	wet
(4) 2,410	....	(4) 1,720	very wet	(4) 2,225	(4) 1,930	wet
(4) 2,415	(2) 2,585	(3) 2,060	wet	(4) 1,980	(4) 1,705	wet
(4) 2,760	(4) 2,020	(4) 1,775	very wet	(4) 1,900	(4) 1,910	wet
Average (16) 2,670	Average (10) 2,480	Average (15) 2,110	....	Average (16) 2,060	Average (16) 1,910	

(Figures in parentheses indicate number of specimens averaged for each result.)

The quality of concrete deposited under water is usually considered to be decidedly inferior to that of concrete placed under normal conditions where water is not encountered. This is doubtless true if the material is permitted to fall freely through the water, or if the circumstances are such that the formation of laitance is facilitated. That first quality concrete can be made in subaqueous construction was shown, however, during the construction of the Detroit River Tunnel. In this case concrete of 1:3:6 mix was deposited at a depth of 60 to 80 ft. below the water surface through 12-in. tremies. Test cores cut from the tremie-deposited concrete by a 6-in. shot drill showed a compressive strength of from 2740 to 4000 lb. per sq. in. 1 year after deposition. Other specimens in the shape of roughly-cut 6-in. cubes of 1:2:4 tremie-deposited concrete developed a strength of from 1800 to 3040 lb. per sq. in., and it was believed that their strength was impaired by the operation of cutting (*Trans.* Am. Soc. C. E., vol. 74, p. 338). The matter of the pressure under which this concrete was deposited probably has some bearing upon the quality, for in this case the hydrostatic pressure at the bottom of a tremie tube was about 30 lb. per sq. in.

14. Effect of Regaging.—The Joint Committee on Concrete and Reinforced Concrete recommends that "the remixing of mortar or concrete that has partly set should not be permitted" (Proc. Am. Soc. C. E.. Dec., 1916, p. 1673), and most engineers specify that mortar or concrete shall be used within 1 hr. or even ½ hr. after it is gaged. It is undoubtedly generally advisable on construction work to adhere to the practice of not permitting regaging and it is particularly important that concrete which has stood undisturbed for some time be not permitted to get into the form in its non-plastic condition, but the harmful effect of regaging is often less pronounced than is commonly believed, and exceptions to the general rule may under certain circumstances properly be made.

The data on this page show that the strength of Portland-cement mortars is not injuriously affected by allowing them to stand for periods of from 1 to 3 hr., and then regaging and molding. In fact the delayed treatment appears slightly beneficial owing probably to the increased amount of working given the material. This effect is one that has been shown with singular unanimity by a considerable number of experimenters using all classes of cement. One fact brought out particularly by the tests of Mr. Sabin is that the material regaged should not merely be remixed, but should have sufficient water added so that the original consistency will be restored after regaging.

Effect of Regaging upon Strength of Mortars—Age 4 Months

Office of Public Roads Tests[1]

Mix	Mortar made up into briquettes immediately after mixing	Mortar broken up after initial set and made into briquettes. Water added to restore normal consistency	Mortar broken up after final set and made into briquettes. Water added to restore normal consistency
		Tensile strength in lb. per sq. in.	
Neat	657	653	540
1:1	628	678	563
1:2	504	554	499
1:3	407	326	353
Initial set	1 hr. 42 min.		
Final set	7 hr. 15 min.		

Effect of Regaging upon Tensile Strength of 1:2 Mortar—Age 1 Year

Tests of L. C. Sabin[2]

Molded immediately	Stood 1 hr., regaged and molded	Stood 3 hr., regaged each hour and then molded	Stood 3 hr., regaged each hour with water added to restore original consistency and then molded	Stood 5 hr., regaged each hour and then molded	Stood 5 hr., regaged each hour with water added to restore original consistency and then molded	Stood 5 hr., regaged and then molded	Stood 5 hr. regaged and then molded
No water added in regaging							
579	565	569	....	570	....	568	
Water added to restore original consistency							
554	579	...	627	....	624	....	560

Natural cements and quick-setting Portland cements appear to be less capable of showing an undiminished strength after regaging than do normal or slow-setting Portlands.

The most pronounced effect of regaging of mortars and concretes is in the direction of retarding the set and delaying the hardening, thus reducing the strength at early periods. Candlot ("Ciments et Chaux Hydrauliques," 1898, p. 358) found that mortars regaged after attaining their final set all required 8 to 10 hr. to set regardless of the rapidity or slowness with which the mortar originally set. This effect of regaging alone will often be a sufficient cause

[1] U. S. Department of Agriculture, *Bull.* 235.
[2] "Cement and Concrete," p. 253.

for prohibiting regaged mortar or concrete on construction work requiring an early assumption of load.

Candlot also found that regaging had a very detrimental effect upon adhesive strength of mortars to stone, the loss being often 50%. Sabin ("Cement and Concrete," p. 290) also found that regaging was detrimental to adhesive strength of mortar to stone, the effect being more pronounced with rich mortars. Earnest McCullough (*Eng. News*, Jan. 11, 1906) found that regaged mortars showed a loss in power to adhere to old mortar or concrete, but found that the addition of 10 to 12% of lime to the regaged mortar produced a material whose adhesive strength considerably excelled that of mortar placed when freshly mixed.

15. Effect of Curing Conditions.—The principal variations in curing conditions which affect the process of hardening and gaining strength of mortars and concretes are: (1) variations of moisture conditions, and (2) variations of temperature.

The effect of different conditions of exposure to moisture is shown by the following data based upon tests made at the U. S. Bureau of Standards (*Tech. Paper* 58). The test specimens were 8 by 16-in. cylinders.

EFFECT OF VARIATION OF MOISTURE CONDITION IN CURING PERIOD

Tests of Bureau of Standards[1]

Mix, class of concrete, and curing conditions	Compressive strength (lb. per sq. in.)				
	1 week	4 weeks	13 weeks	26 weeks	52 weeks
1:6 gravel (quaking): In damp closet entire period.	...	1,898	1,968	2,172	2,400
1:6 gravel (quaking): 4 weeks in damp closet, then removed.	...	1,648	1,825	2,063	2,220
1:2:4 trap rock (quaking): Immersed immediately after molding.	...	2,851	3,570+	4,094+	3,956
24 hr. in damp closet, then immersed............	...	3,978+	3,978+	4,100	4,247+
8 weeks in damp closet, then immersed...........	...	.....	3,190	3,457	3,389
1:2:4 gravel (mushy): Sprinkled daily for 1 week, then stored indoors in dry room.	481	1,104	1,469		
4 weeks in damp room, then placed in open, exposed to weather.	...	1,834	2,500		
1:2:4 gravel (quaking): In damp closet entire period	...	2,612			
Open air, exposed to weather entire period......	...	2,085			

These tests indicate: (1) that concrete specimens cured in the moist air of the damp closet until tested become somewhat stronger than ones immersed in water after a longer or shorter period in the damp closet, or ones immersed immediately after molding. (This may be due in part at least to the fact that the immersed specimens were tested while still holding much more water than the damp closet specimens.) (2) Specimens cured in the damp closet are considerably stronger than ones cured in the comparatively dry air of the laboratory, even though

[1] *Tech. Paper* 58.

the latter were sprinkled daily for the first week. (3) Specimens gained strength in the open air exposed to the weather to a considerably greater extent than did others cured indoors in a comparatively dry room, but not to as great an extent as ones stored in the damp closet.

The relation between the mean temperature encountered during the curing period and the strength of 1:2:4 concrete is shown by the diagram of Fig. 15. This diagram was constructed by A. B. McDaniel as a summary of the results of tests made at the Engineering Experiment Station of the University of Illinois (*Bull.* 81). The concrete used was a 1:2:4 mixture by weight (1:2.2:3.6 by volume), the coarse aggregate being crushed limestone. A portion of the specimens were 6 by 6-in. cylinders, some were 6-in. cubes and the balance were 8 by 16-in. cylinders. All values were reduced to an equivalent value for 8 by 16-in. cylinders, however. Ten sets of specimens were made, and each set was subjected to a different mean temperature throughout the period of curing. The mean temperatures employed were 26.5°, 27.1°, 30°, 34.7°, 35.5°, 48.5°, 68°, 71.8°, 72.8°, and 95.6° respectively. The marked effect of low temperatures in at least delaying, if not permanently preventing, the hardening process is excellently shown by the diagram of Fig. 15. The diagram represents the results obtained with only one mixture of one class of materials, but the relative effect of various temperatures on other concretes may be expected to at least approximate the relation found for this concrete, and the diagram should furnish suggestive information useful in estimating the strength of concrete cured at abnormal temperatures.

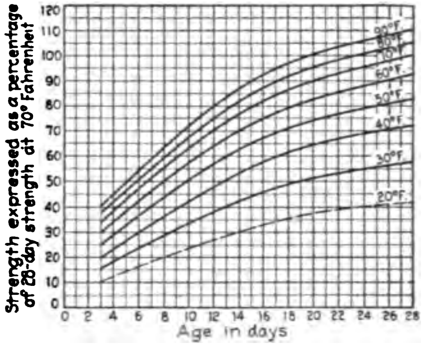


FIG. 15.—Effect of temperature of curing upon compressive strength of 1 : 2 : 4 concrete. (The temperatures given are the mean temperatures encountered during the period of curing.)

EFFECT OF CURING 1 : 4 MORTAR UNDER STEAM PRESSURE

(8 × 16-in. cylinders)

Tests of Bureau of Standards[1]

Gage pressure (lb. per sq. in.)	Temperature (Fahr.)	Duration of exposure to steam (hours)	Compressive strength			
			2 days	7 days	14 days	28 days
Not steamed.	...	...	613	1,296	1,528	1,727
0	212	48	1,267			
2	218	24	1,808	1,792	1,805	
10	239	24	1,786	1,555	1,701	1,902
20	258	24	2,139	2,284	2,740	
40	286	24	3,292	3,381	3,984	
80	323	24	3,964	3,966	4,433	
80	323	24	4,487	4,187	4,840	

Certain classes of concrete products such as tiles and blocks are subjected to an accelerated hardening treatment by the use of steam. The effect of such treatment upon strength is shown by data given on this page from tests of the U. S. Bureau of Standards (*Tech. Paper* 58).

The results show that up to 80 lb. per sq. in. gage pressure, steam has an accelerating action upon the hardening of cement mortar, and that the compressive strength increases with the pressure as well as with the time of exposure to steam. A compressive strength considerably (in some cases over 100%) in excess of that obtained normally after aging for 6 weeks may be obtained in 2 days by using steam under pressure for curing. Furthermore, the steam permanently accelerates the hardening of the

[1] *Tech. Paper* 58.

concrete which subsequently increases in compressive strength with age upon exposure to the atmosphere.

It was noted that the steam-cured concrete was more uniform in appearance and lighter in color than normally-aged mortar from the same materials. These tests were made upon Portland-cement mortars, but the same conclusions were found to apply to concretes. The mortar or concrete should obtain an initial set before exposure to the steam treatment.

16. Effect of Freezing.—The effect of low temperatures in delaying or permanently preventing the hardening of mortar and concrete has been shown by Fig. 15. In the event of the temperature being close to the freezing point of water from 4 to 8 times as long a period is required to obtain a final set as is required at normal room temperatures. If water in mortar or concrete freezes before the cement has set, it is not available for the chemical action of setting and hardening and hence the concrete or mortar will not set at all until the ice melts. These facts must be borne in mind when removing forms from concrete placed during cold weather. If the temperature hovers above the freezing point for some time after concrete is deposited, there is a possibility of the water drying out before the greatly delayed setting has taken place. If, however, the concrete has begun to set before the temperature drops considerably below the freezing point, the expansion of the water in solidifying produces an expansive force in excess of the cohesive strength of the green concrete. This action results in a destruction of the bond and crumbling of the concrete when the ice melts. If the temperature does not fall more than a very few degrees below freezing, the result may simply be the further delaying of the set without appreciable injury.

Two factors operate to lessen the injurious effect of freezing upon mortar and concrete: (1) concrete is a rather poor heat conductor, the outer portion therefore serving as an insulation for the bulk of the material and preventing an injurious lowering of the temperature in the interior of the mass; and (2) the chemical action of setting and hardening of cement being an endothermic reaction, the heat evolved serves to raise the temperature of the material and so offsets to a degree the loss of heat by radiation and conduction. Experimental data secured during the construction of the Arrow Rock Dam and the Kensico Dam (see Art. 42) indicate that mass concrete shows a rise in internal temperature of from 20 to 40°F. above the initial temperature within a period of from 15 to 30 days.

Serious injury is often suffered by concrete which encounters temperatures considerably below the freezing point within the first few hours after placing, but this injury is usually confined to the outermost portion of the work and seldom penetrates more than an inch or two of depth. A very frequent form of injury is a scaling off of a very thin crust of rich material which has been flushed to the surface in finishing the work.

Numerous experimental studies of the effect of frost action on mortars have been made and have led to somewhat conflicting conclusions, but practically all of these have involved the use of such small specimens, briquettes, 2-in. cubes, etc., that the condition of exposure is comparable only to that of the outermost surface layer of concrete.

Authorities are quite in accord in prescribing that "concrete should not be mixed or deposited at a freezing temperature, unless special precautions are taken to avoid the use of materials containing frost or covered with ice crystals, and to provide means to prevent the concrete from freezing after being placed in position and until it has thoroughly hardened."

17. Effect of Salts.—Common salt (NaCl) is frequently used as an ingredient of the mixes of concrete or mortar which must be placed in cold weather. Its primary effect is the lowering of the temperature at which water will freeze. Approximately 1% of salt in the mixing water lowers the freezing point 1°F. Calcium chloride ($CaCl_2$) is also used to a lesser extent to serve the same purpose.

The effect of common salt and calcium chloride upon the strength of a 1:2:4 limestone concrete is shown by the diagrams of Fig. 16 which are derived from tests made by H. E. Pulver and S. E. Johnson of the University of Wisconsin (*The Wisconsin Engineer*, October, 1913). The specimens were 4-in. cubes, and the tests were made in duplicate, one series of specimens being cured indoors at normal temperatures (60°–75°F.), the other cured out of doors or in a refrigerator at temperatures below 32°F. The amounts of the salts used are expressed as percentages by weight of the mixing water.

The tests show that common salt used alone is quite injurious to the strength of concrete cured at normal temperatures, the loss being roughly proportional to the amount of salt used. With concrete cured at temperatures below freezing, however, it facilitates the hardening process. The tests show an increase of strength for the addition of NaCl up to 12%, after which there is a decrease. Common salt retards the setting of concrete to a considerable degree.

Calcium chloride used alone is beneficial to the strength of concrete, whether cured at normal temperatures or below freezing, up to about 4% $CaCl_2$, at which point the maximum

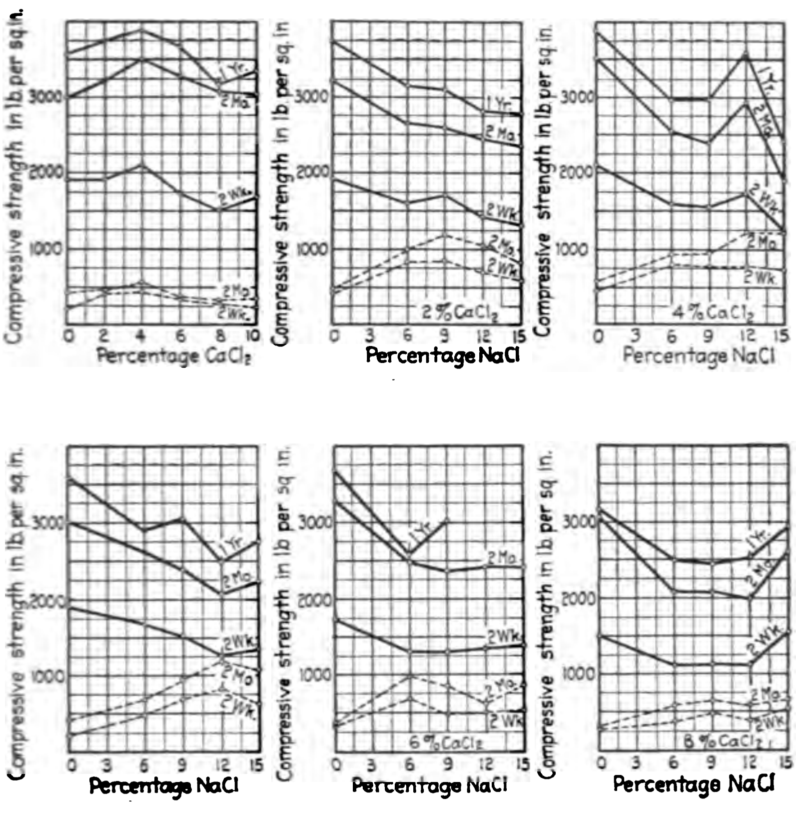


FIG. 16.—Effect of salts upon strength of 1 : 2 : 4 concrete.
—— specimens cured at normal temperatures (60 to 75° F.).
- - - - - specimens cured at low temperatures (below 32° F.).

strength is obtained. This maximum strength, however, in the case of the cold-cured specimens is only about one-half the maximum strength of the cold-cured specimens having 12% of common salt. Calcium chloride accelerates the setting of concrete.

With concretes cured at low temperatures the best effect was obtained with a mixture of 2% of $CaCl_2$ and 9% of NaCl. This mixture gave about the same strength as the cold-cured concrete having 12% of NaCl alone, and was not as detrimental to the strength of normally cured concrete as the NaCl alone seemed to be.

18. Effect of Hydrated Lime and Waterproofing Compounds.—The addition of hydrated lime in small percentages has not a very marked effect upon the strength of laboratory specimens of mortar and concrete. Fig. 17, which is based upon tests made by Prof. Harry Gardner of the University of Kansas (*Eng. Rec.*, vol. 64, p. 3C9), shows the effect of varying percentages by weight of hydrated lime upon the tensile strength of 1 :3 standard sand mortars. The replacement of the cement by hydrated lime appears to be slightly beneficial to strength up to about 15%, except in the case of tests made at the ages of 3 and 7 days wherein the effect was generally detrimental in proportion to the amount of lime used. Other tests, notably those of H. S. Spackman (*Concrete-Cement Age*, vol. 4, p. 112 and *Eng. Rec.*, vol. 69, p. 25) have shown that small amounts of hydrated lime sometimes appear to affect the strength favorably, sometimes unfavorably. The most pronounced effect of hydrated lime added to mortars and concretes is its producing a more plastic, better-working material. The fat, viscous mortar produced spreads better under the trowel, and in the case of concrete the presence of a small amount of lime hydrate tends toward the production of a mixture of greater uniformity by prevention of the separation of fine and coarse materials. This fact may constitute a distinct advantage in the case of the average construction job which would not be noted under the ideal conditions of mixing and molding in the laboratory.

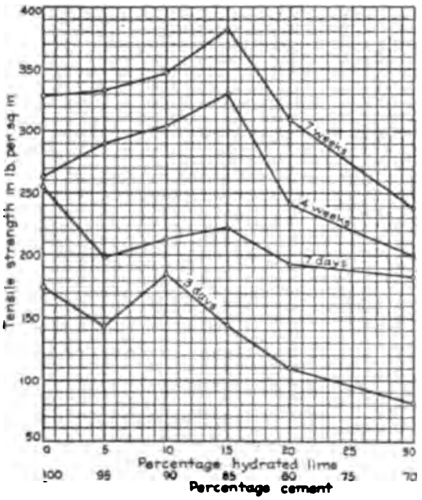


FIG. 17.—Effect of hydrated lime upon tensile strength of 1 : 3 standard mortar.

The effect of a large number of commercial waterproofing compounds upon the tensile and compressive strengths of mortars in 1 : 4, 1 : 6, and 1 : 8 mixtures has been investigated by the U. S. Bureau of Standards (*Tech. Paper* 3). Figs. 18*a* and 18*b* show the results of tests of the effect of 15. such compounds upon the compressive strength of 1 : 4 mortar. These results are typical of the results obtained with all mortar mixtures, and the results obtained in tensile tests do not differ greatly from those obtained in compression. The proprietary compounds are, of course, not identified by name, but they are classified as follows: No. 27 is dolomitic hydrated lime; No. 28 is a solid chemically active filler designed to form an insoluble lime resinate void filler; Nos. 29 to 36 inclusive are water-repelling solid substances consisting essentially of stearic acid with soda and potash or lime, designed to form an insoluble lime soap; No. 37 is cement containing a water-repelling substance; Nos. 38 to 40 inclusive are chemically active liquid fillers designed to fill the voids with either tar, insoluble lime silicate, etc.

19. Effect of Sea Water Used in Gaging.—The use of sea water to gage cement mortars and concretes is almost invariably forbidden by specifications for work done in localities where the use of sea water might be convenient. It has not been conclusively shown, however, that the use of sea water instead of fresh water has a particularly harmful effect. Messrs. Taylor and Thompson ("Concrete, Plain and Reinforced," 1916, p. 166) found by a very limited number of tests of 1 :2 :4 concrete cubes that there was no appreciable difference in strength of specimens gaged with sea water and other specimens gaged with fresh water.

Results obtained in comparative tensile tests of mortar briquettes made by Cloyd M. Chapman are shown by Figs. 19,*a*, 19,*b* and 19,*c* (*Eng. News*, vol. 63, p. 291). These tests were made upon three sets of specimens: series (A) specimens were gaged with fresh water and cured in fresh water; series (B) specimens were gaged with fresh water and cured in sea water; and series (C) specimens were gaged with sea water and cured in sea water. The specimens were tensile briquettes and the sea-water curing was done in tanks in the laboratory using a frequently changed bath of sea water. These tests indicate that the use of sea water for gaging

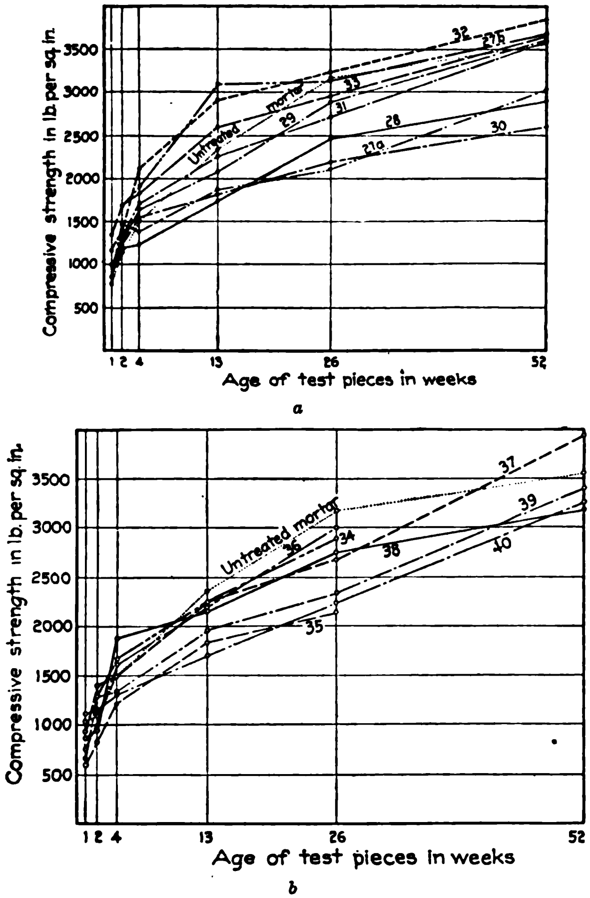


FIG. 18.—Compressive strength of waterproofed mortars. (One part Portland cement to 4 parts No. 1 sand.)

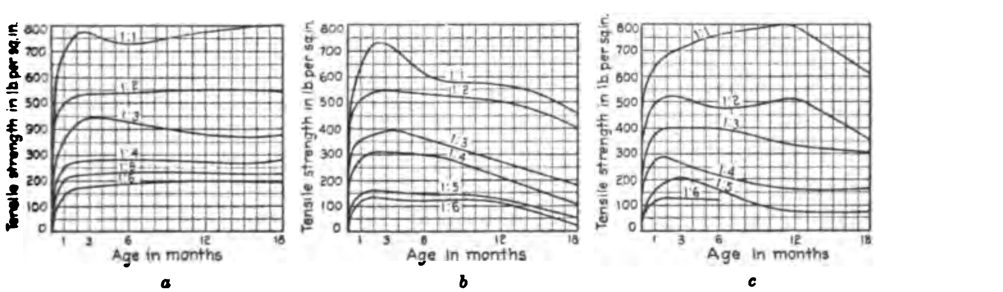


FIG. 19.—Effect upon tensile strength of gaging cement mortars with sea water. *a*, Specimens gaged with fresh water—cured in fresh water: *b*, specimens gaged with fresh water—cured in sea water; *c*, specimens gaged with sea water—cured in sea water.

is not particularly detrimental to the tensile strength of mortars except in very lean mixtures. The condition of curing of these specimens was probably not as severe a test as actual immersion in moving sea water would have been. On the other hand, the section of the specimens was so small that any superficial or surface effect of the sea water would appear to have an injurious effect much greater than that suffered by concrete of large bulk exposed in sea water.

20. Effect of Oils Used in Gaging.—The use of certain classes of mineral residual oils in gaging mortars and concretes with the object of dampproofing them or reducing their permea-

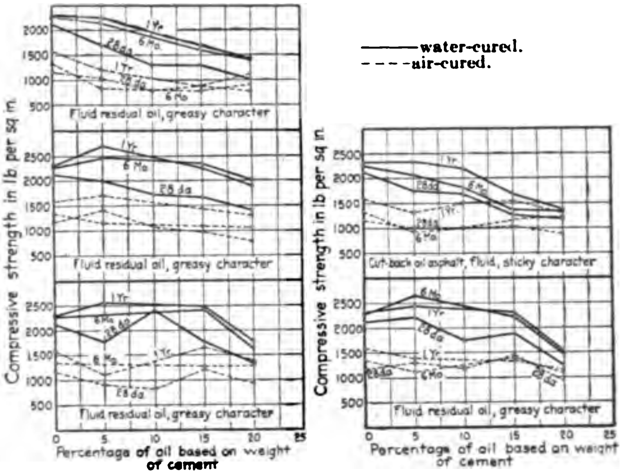


FIG. 20.—Effect of mineral residual oils upon compressive strength of 1 : 3 mortar. (Tests of Logan Waller Page.)

bility lends importance to the consideration of the effect of such oils upon the strength of the mortars or concretes in which they are used. Figs. 20 and 21 present the results of tests made by Logan Waller Page, Director of the Office of Public Roads (U. S. Dept. of Agriculture, *Bull.* 230). It appears from these tests that the use of mineral oils up to from 5 to 10% of

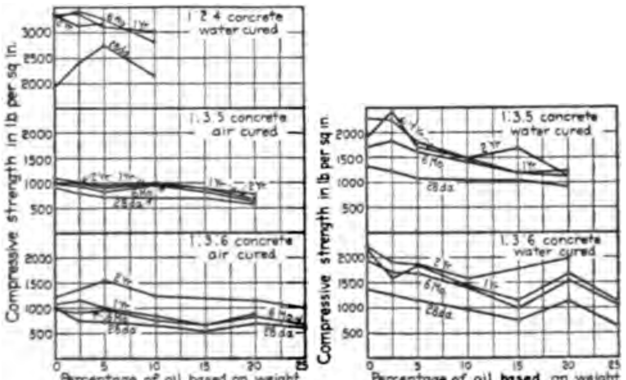


FIG. 21.—Effect of mineral residual oils upon compressive strength of concrete. (Tests of Logan Waller Page.)

the cement lowers the compressive strength to a moderate degree only but that larger amounts may be very injurious.

Other tests made by Arthur Taylor and Thomas Sanborn (*Trans.* Am. Soc. C. E., vol. 76, p. 1094) using Western asphaltic oils showed a more marked falling off in strength of mortars than was observed in Page's tests. At 28 and 50 days the compressive strengths of 3-in. cubes of 1 : 3 mortar made by the incorporation of Western oils were as follows:

EFFECT OF ASPHALTIC OILS UPON STRENGTH OF 1 : 3 MORTAR

Compressive Strength

Character and percentage of oil used		28 days		50 days	
		(lb. per sq. in.)	Relative value	(lb. per sq. in.)	Relative value
No oil		3,950	100.0	4,400	100.0
Boiler fuel	5	2,435	61.6	3,620	82.4
Boiler fuel	10	1,780	45.1	2,460	56.0
Boiler fuel	15	1,460	37.0	2,000	45.5
Boiler fuel	25	712	18.2	1,000	22.8
Richmond fuel	10	1,640	41.5		
Road oil No. 6	10	1,235	31.2		
Liquid asphalt	10	1,080	27.4		

Many oils used for various commercial purposes contain animal oils, vegetable oils or admixtures of these. Such oils have been found to be capable of not only weakening cement mortars and concretes, but actually to disintegrate concrete in some cases, the effect being most pronounced in the early stages of setting and hardening (see tests of James C. Hain, *Eng. News*, March 16, 1905).

21. Effect of Laitance.—Laitance is a whitish substance which is washed out of concrete and subsequently deposited as a scum when there is an excess of water used in mixing (see chapter on "Water" in Sect. 1), or when concrete is deposited in water, or when water collects in pools on the surface of freshly laid concrete. The laitance consists of the finest flocculent matter in the cement together with some silt and clay from the aggregates. Its occurrence is explained by the formation of amorphous hydrates in the early stages of the setting of cement. The composition of laitance is practically identical with that of cement, but it hardens only very slowly and never acquires much strength. As a consequence, if not removed by water and brushes or by a steam jet, it forms a distinct plane of weakness between successive layers of concrete. The washing out of a portion of the finest part of the cement means the loss by the concrete of just so much of its most valuable constituent, because it is the impalpably fine portion of the cement which is most active in binding together the inert particles of the aggregate. This same material alone does not develop great cohesiveness, however. A familiar example of this fact is afforded by the relative behavior of very finely ground cements and cements of only average fineness in neat and mortar mixtures. The cement of average fineness will greatly excel in neat strength, but the cement which is ground so finely that the proportion of impalpably fine material is very large will form mortars of greatly superior strength.

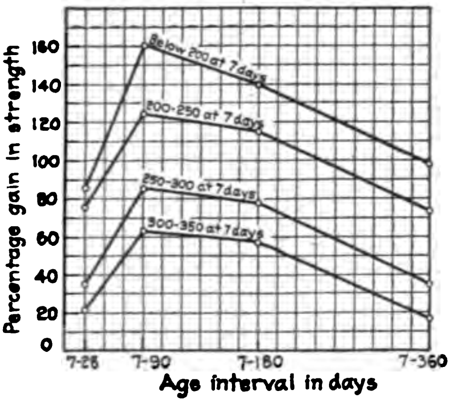


FIG. 22.—Rate of increase in tensile strength of standard 1 : 3 mortar. (Each result the average of 9 to 70 tests.)

22. Rate of Increase in Mortar Strength—Retrogression.—The relation between the early test strength and the subsequent gain in strength is shown by Figs. 22 and 23 which are based upon the tests of the former Structural Materials Laboratory of the U. S. Geological Survey in St. Louis (U. S. Geol. Surv. *Bull.* 331). The rate of gain in both tensile and compressive strength for these 1 : 3 mortars (made with seven typical Portland cements) is shown to be approximately inversely proportional at all ages to the strength at 7 days, those mortars which show the lowest tensile or compressive strength at 7 days maintaining the best rate of gain in strength at all ages.

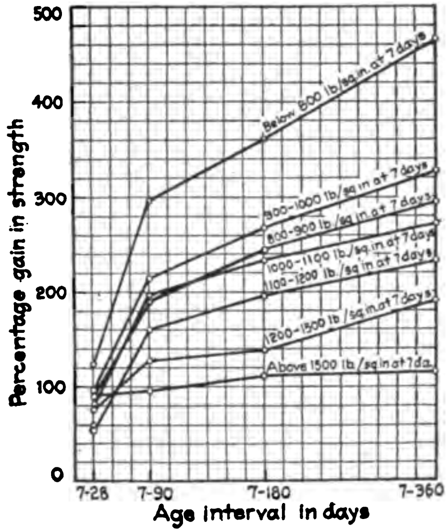


FIG. 23.—Rate of increase in compressive strength of standard 1 : 3 mortar. (Each result the average of 20 to 70 tests.)

The following table indicates the relation between the early strength of mortar and subsequent retrogression in strength as determined by the Structural Materials Laboratory Tests above quoted:

RETROGRESSION IN STRENGTH OF 1:3 STANDARD MORTARS

Tests of Structural Materials Laboratory

Strength at 7 days (lb. per sq. in.)	% showing retrogression between ages of			
	7 and 28 days	28 and 90 days	90 and 180 days	180 and 360 days
		Tension		
Below 200	0	0	86	86
200- 250	0	0	62	71
250- 300	0	0	48	100
300- 350	0	0	57	100
		Compression		
Below 800	0	0	0	20
800- 900	0	0	0	14
900-1,000	0	0	8	25
1,000-1,100	0	0	0	12
1,100-1,200	0	0	0	20
1,200-1,500	0	0	22	0
Above 1,500	0	40	40	20

23. Transverse Strength.—The transverse strength of granular brittle materials like mortars and concretes is best expressed by the *Modulus of Rupture.* The modulus of rupture is the apparent stress in the extreme fiber of a transverse test specimen under the load which produces rupture. For specimens of rectangular section of breadth b and height h, loaded centrally on a span l, the breaking load being W, the modulus of rupture is computed by the formula

$$\text{Modulus of rupture} = \frac{3Wl}{2bh^2}$$

The extreme fiber stress thus computed is not the actual fiber stress because the formula involves the inaccurate assumption that the material deforms elastically for all stresses up to rupture. The comparative relations between results are not affected by this inaccuracy of the formula, however, when the tests compared are made upon specimens of similar material, because the computed values of the modulus of rupture are very nearly proportional to the actual stresses.

Since the extreme fiber stresses on the tension side and on the compression side of a beam of homogeneous material are equal, and the tensile strength of mortar or concrete is only a small fraction of the compressive strength, the transverse strength of mortar or concrete is almost wholly dependent upon the tensile strength. The modulus of rupture found in transverse tests will invariably be considerably in excess of the tensile strength, however, because the computed stress in the extreme fiber considerably exceeds the actual stress.

The data on this page constitute a summary of a portion of an extensive series of tests of transverse strength of mortars and concretes made by Wm. B. Fuller. ("Concrete, Plain and Reinforced" by Taylor and Thompson, p. 334.) The tests were made upon specimens 6 by 6 in. in section, supported on spans of 30 and 60 in. One brand of cement and the same sand and crushed trap-rock aggregate were used throughout the series of tests. The specimens were broken at the age of 1 month.

24. Shearing Strength.—The shearing strength of mortars and concretes possesses great significance because compressive failure of short compressive specimens or structural members is usually failure by shearing on a diagonal plane, and because shearing stresses are important considerations in all cases of concrete beams. It is very difficult, however, to make experimental determinations of pure shearing strength because most methods and devices which may be used to make shearing tests involve either a cutting action, bearing pressures, or beam stresses. The data on the shearing strength of mortars given on page 244 are derived from tests made by Feret. The specimens used were prisms, 2 by 2 cm. in section, subjected to single shear, the conditions being such that beam stresses probably affected the results considerably. Specimens were tested after 5 months curing.

Transverse Strength of Mortars and Concretes

Tests of William B. Fuller

Proportions by weight, cement: sand: stone	Proportions by volume, cement: sand: stone	Modulus of rupture (lb. per sq. in.)		
		Maximum	Minimum	Average of 6
1:0:0	1:0:0	968	856	906
1:1:0	1:1.17:0	866	628	734
1:2:0	1:2.34:0	640	592	616
1:3:0	1:3.51:0	432	392	418
1:4:0	1:4.68:0	294	262	279
1:5:0	1:5.85:0	180	170	173
1:6:0	1:7.02:0	94	92	93
1:1:2	1:1.17: 2.06	798	646	710
1:1:3	1:1.17: 3.09	732	573	655
1:2:4	1:2.34: 4.12	480	399	439
1:2:5	1:2.34: 5.17	413	349	380
1:3:5	1:3.51: 5.17	308	262	285
1:3:6	1:3.51: 6.21	246	213	226
1:4:8	1:4.68: 8.25	158	156	157
1:6:10	1:7.02:10.34	91	87	89

Shearing Strength of Cement Mortars

Tests of R. Feret[1]

Character of sand	Approximate proportions by weight		Ultimate strength (lb. per sq. in.)			Ratio of shear to compression
	Cement	Sand	Shear	Tension	Compression	
Very coarse granite sand	1	18.6	170	69	240	0.71
	1	9.9	570	146	870	0.66
	1	6.9	1,070	212	1,540	0.70
	1	5.2	1,440	258	2,350	0.61
	1	4.1	2,000	314	3,320	0.60
	1	3.2	2,560	367	4,170	0.61
	1	2.5	2,790	421	5,210	0.54
	1	1.8	3,580	480	5,970	0.60
	1	1.2	3,930	537	6,670	0.59
	1	0.7	3,640	563	6,810	0.65
Medium-sized very shelly sand	1	12.9	256	81	310	0.83
	1	7.0	669	182	950	0.70
	1	5.0	1,040	240	1,510	0.69
	1	4.1	1,350	278	1,990	0.68
	1	3.1	1,810	320	2,720	0.67
	1	2.5	2,250	368	3,430	0.66
	1	2.0	2,650	415	4,380	0.61
	1	1.4	2,750	521	5,440	0.50
	1	0.9	3,580	541	6,100	0.59
	1	0.5	3,540	602	6,720	0.53
Very fine silicious sand	1	12.3	156	67	160	0.97
	1	5.8	370	126	540	0.69
	1	3.5	768	214	1,230	0.62
	1	2.4	1,410	302	1,940	0.73
	1	1.8	2,130	364	2,840	0.75
	1	1.3	2,570	436	3,710	0.69
	1	1.0	2,750	510	5,000	0.55
	1	0.7	3,070	574	5,760	0.53
	1	0.5	3,570	647	6,500	0.55
	1	0.3	4,120	691	7,110	0.58
Equal parts of coarse medium and fine ground quartzite	1	5.0	1,720	328	2,350	0.73
	1	3.0	3,100	450	4,010	0.77
	1	2.0	3,070	518	4,810	0.64
20-31-mesh ground quartzite	1	3.0	......	456	3,640	
Neat Portland cement	1	0.0	3,680	698	8,040	0.46

The data at the top of page 245, showing the results of shearing tests, are derived from tests made at the Massachusetts Institute of Technology under the direction of Prof. C. M. Spofford. The specimens were cylinders 5 in. in diameter and 15½ in. long. The ends were securely

[1] Data taken from "Concrete, Plain and Reinforced" by Taylor and Thompson, p. 136, 1909 Edition.

clamped in cylindrical bearings and the load was applied along the middle third of the length by a semi-cylindrical block. The final failure appeared to be by true shear.

The following data are taken from tests made at the University of Illinois Engineering Experiment Station under the direction of Prof. A. N. Talbot (*Bull.* 8). Two methods of testing were used. In the first a 6-in. hole was punched in a concrete plate or block; in the second, a short beam 4 by 4 in. in cross-section was securely clamped at the ends and the middle third of the length loaded. Three forms of specimens were used in the punching tests: (1) plain concrete plate; (2) recessed

SHEARING STRENGTH OF CONCRETE

Summary of Massachusetts Institute of Technology Tests[1]

Proportions	Method of storing	Shearing strength (lb. per sq. in.)			Compressive strength (lb. per sq. in.) 5 by 15½-in. cylinders	Ratio of shear to compression
		Max.	Min.	Ave.		
1:2:4	air	1,630	960	1,310	2,070	0.63
1:2:4	water	2,090	1,180	1,650	2,620	0.63
1:3:5	air	1,590	890	1,240	1,310	0.94
1:3:5	water	1,380	840	1,120	1,360	0.32
1:3:6	air	1,450	950	1,180	950	1.25
1:3:6	water	1,200	1,030	1,120	1,270	0.88

SHEARING STRENGTH OF CONCRETE

Summary of University of Illinois Tests

Proportions	Form of specimens	Method of storing	Shearing strength (lb. per sq. in.)	Compressive strength (lb. per sq. in.)		Ratio of shear to compression	
				Cube	Cylinder	Cube	Cylinder
1:3:6	Plain plate..............	air	679	1,230	.....	0.55	
1:3:6	....	water	729	1,230	.....	0.59	
1:3:6	....	damp sand	905	2,428	1,322	0.37	0.68
1:3:6	....	damp sand	968	1,721	1,160	0.56	0.83
1:2:4	Plain plate..............	damp sand	1,193	3,210	2,430	0.37	0.49
1:3:6	Recessed block..........	air	796	1,230	.....	0.65	
1:3:6	....	water	692	1,230	.....	0.56	
1:3:6	....	water	879	1,230	.....	0.71	
1:3:6	....	damp sand	1,141	2,428	1,322	0.47	0.86
1:3:6	....	damp sand	910	1,721	1,160	0.53	0.78
1:2:4	Recessed block..........	damp sand	1,257	3,210	2,430	0.39	0.52
1:3:6	Reinforced recessed block	air	1,051	1,230	.....	0.86	
1:3:6	....	damp sand	1,821	2,428	1,322	0.75	1.38
1:3:6	....	damp sand	1,555	1,721	1,160	0.90	1.39
1:2:4	Reinforced recessed block	damp sand	2,145	3,210	2,430	0.67	0.88
1:3:6	Restrained beam........	damp sand	1,313	2,428	1,322	0.54	1.00
1:3:6	....	damp sand	1,020	1,721	1,160	0.59	0.88
1:2:4	Restrained beam........	damp sand	1,418	3,210	2,430	0.44	0.58

[1] Taken from *Bull.* 8, Uni. of Ill. Eng. Exp. Sta.

Adhesion of New to Old Concrete

Transverse Strength of Joints—Tests of Hector St. George Robinson

Method employed to secure a bond	Computed tensile stress in extreme fiber (lb. per sq. in.)	Efficiency of bond, %
Solid specimens with no joint	302	
	362	
	289	
	340	
	352	
Average	329	100.0
Surface (molded against rough board) merely wetted	140	
	78	
	130	
	110	
	172	
Average	126	38.3
Surface roughened with a chisel, cleaned and wetted	194	
	170	
	205	
	142	
	165	
	234	
Average	185	56.2
Surface roughened, cleaned, and thoroughly coated with neat cement grout	325	
	272	
	280	
	248	
Average	281	85.5
Surface treated with hydrochloric acid, washed, brushed, and wetted	300	
	248	
	260	
	201	
	340	
	271	
Average	270	82.0

concrete block; (3) recessed concrete block reinforced outside of the area subjected to the direct action of the punch.

25. Adhesive Strength.—The adhesion of mortars to various building materials is a matter of much importance which has, however, been insufficiently investigated. Fig. 24 presents the results of tests made by General E. S. Wheeler (Report Chief of Engineers, U. S. A., 1895, p. 3019, and 1896 pp. 2799, 2834). Discs of the material concerned, 1 by 1 in. square and ¼ in. thick, were prepared and inserted in the center of briquette molds. The molds were subsequently filled with mortar and the specimens were tested in the usual manner in tension. Mr. Wheeler found that a consistency wetter than that which gives a maximum tensile strength is required to give a maximum adhesive strength of mortar to stone, even though the surface of the stone be saturated with moisture. Irregularities of the surface of stone or brick appeared not to affect adhesive strength, but a dirty surface, or insufficient moistening of the surface greatly reduced adhesion.

25a. Adhesion to Concrete Previously Placed.—The adhesion of concrete to old work of the same character constitutes an important problem in many classes of construction work, but few experimental determinations of the bond between new and old work have been made. The data on this page have been derived from a series of tests made by Hector St. George Robinson in 1912 (*Proc.* Institute of Civil Engineers, vol. 189, p. 310). The specimens used were prisms of 1 : 2 : 4 concrete 30 in. long and 4 by 4 in. in section. One set were solid prisms. The remainder were made by placing a stop-board in the mold 8 in. from one end and allowing the concrete molded in this end to harden for 7 days before the stop-board was removed and the balance of the mold filled with additional fresh concrete. All specimens were tested after further hardening for

28 days. Four different treatments accorded the face of the old concrete to improve the bond are enumerated. The tests were made by rigidly clamping the 8-in. portion of each beam in a fixed support and loading the cantilever beam at a point 20 in. from the support. The relative strengths of the various joints thus tested was determined by computing the extreme fiber stress on the tension side of the joint (modulus of rupture). The apparent tensile strength thus computed is much higher than the actual tensile stress, but the relative efficiencies of the various methods of securing a bond are nevertheless shown.

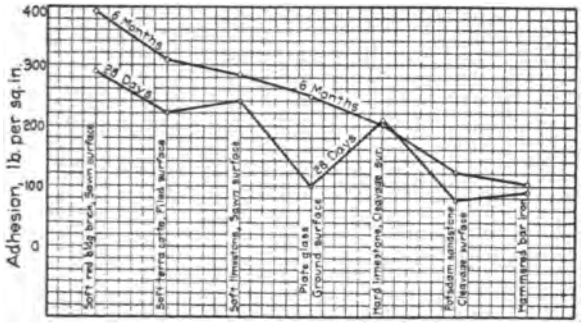


FIG. 24.—Adhesive strength of Portland cement mortar. 1 part cement; 1 part crushed quartz (Nos. 20–30). (Wheeler, Report of Chief of Eng'r's, 1895.)

25b. Adhesion or Bond to Steel.—See Art. 2, Sect. 6.

26. Strength of Natural Cement Mortar and Concrete.—The production and use of natural cement in the United States has declined so rapidly since 1899, when the amount produced reached its maximum of nearly 10,000,000 bbl. per year, and greatly exceeded the output of Portland cement, that the present importance of natural cement as a material of engineering construction is almost negligible in comparison with that of Portland cement. The reasons for the great decline in importance of natural cement are briefly: (1) the great improvement in quality and lowering of cost of Portland cement; (2) the inferiority of the average natural cement to the average Portland cement in structural qualities; (3) the great variability in quality shown by natural cements owing to the lack of close control of the manufacturing process; and (4) a general distrust of natural cement among engineers and others which is often alone responsible for its use being forbidden by specifications.

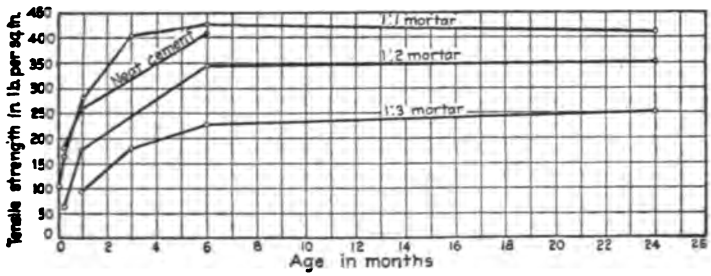


FIG. 25a.—Tensile strength of natural cement—average of 10 brands. (*Sabin.*)

Natural cement, mortars, and concretes vary greatly in strength owing to a great variability in both composition and constitution of the cement. "This variation is found not only in comparing cements from different localities, but even in comparing samples taken at different times from the output of any one locality. The only general statements that may be made concerning their strength is that natural cements rarely show more than half the tensile strength of Portland cements of the same age, and their compressive strength rarely exceeds one-third that of Portland cement in similar mixtures" ("Materials of Construction," by A. P. Mills). The diagrams of Figs. 25*a* and 25*b* average the results obtained from tensile tests of mortars of ten representative brands of natural cement made by L. C. Sabin ("Report Chief of Engineers," 1895, p. 2937), and compressive tests of eight to nine brands of natural cement made by Clifford Richardson (*Brickbuilder*, vol. 6, p. 253).

Very few data are available showing the strength of natural cement concretes. Tests made at the Watertown Arsenal in 1899 show the following strengths of 12-in. cubes of 1 : 2 : 4

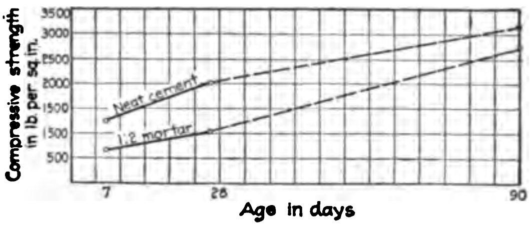


FIG. 25*b*.—Compressive strength of natural cement—average of 8 to 9 brands (1 brand only at 90 days). (*Richardson.*)

trap-rock concrete made with one brand of typical natural cement ("Tests of Metals," 1899 and 1901):

	Max. (lb. per sq. in.)	Min. (lb. per sq. in.)	Average of 5 (lb. per sq. in.)
Compressive strength at 3 months.....	460	263	332
Compressive strength at 14 months.....	914	585	715

Sabin ("Cement and Concrete," p. 314) quotes the following tests made by A. W. Dow for the Engineer Commissioner of the District of Columbia in 1897. The specimens were 12-in. cubes of 1 : 2 : 6 concrete, made with six different coarse aggregates and tested at the age of 12 months. Comparative tests of a Portland-cement concrete made with the same aggregates with the same proportions are also reported.

	Max. (lb. per sq. in.)	Min. (lb. per sq. in.)	Average (lb. per sq. in.)
Compressive strength of 1 : 2 : 6 Natural-cement concrete...............	915	763	844
Compressive strength of 1 : 2 : 6 Portland-cement concrete..............	3,060	1,850	2,670

The standard specifications of the American Society for Testing Materials (A. S. T. M. Standards, 1916) require that the minimum tensile strength of 1 : 3 natural cement mortar made with standard Ottawa sand shall be:

7 days (1 day in moist air, 6 days in water)—50 lb. per sq. in.
28 days (1 day in moist air, 27 days in water)—125 lb. per sq. in.

27. Strength of Cinder Concrete.—The compressive strength of a number of mixes of concrete made with anthracite coal cinders and six different Portland cements is shown by the following summaries of two series of tests made at the Watertown Arsenal ("Tests of Metals," 1898, 1903, and 1904). The specimens of each test series were 12-in. cubes, and the average values given are means of from two to four tests.

STRENGTH OF CINDER CONCRETE

Watertown Arsenal Tests—Tests Made in 1898

Brand of cement	Proportions of mixture	Average compressive strength (lb. per sq. in.)	
		1 month	3 months
A	1:1:3	1,466	2,001
B	1:1:3	1,032	1,393
C	1:1:3	2,329	2,834
D	1:1:3	1,602	2,414
E	1:1:3	1,438	1,890
F	1:1:3	1,379	1,788
A	1:2:3	1,098	1,634
A	1:2:4	904	1,325
A	1:2:5	769	1,084
B	1:2:5	471	685
C	1:2:5	940	1,600
D	1:2:5	696	1,223
E	1:2:5	744	880
A	1:3:6	529	788

TESTS MADE IN 1903 AND 1904 (ONE BRAND OF CEMENT)

Proportions of mixture	Compressive strength (lb. per sq. in.)								
	5 weeks			32 weeks			1 year, 15 weeks		
	Max.	Min.	Ave. of 3	Max.	Min.	Ave. of 2	Max.	Min.	Ave. of 4
1:2 :4	2,430	1,950	2,143	2,600	2,500	2,550	2,610	2,410	2,488
1:2½:5	1,400	1,570	1,457	2,020	1,980	2,000	1,950	1,480	1,700
1:3 :6	1,200	1,350	1,293	1,730	1,560	1,645	1,400	1,290	1,363

More recent tests of cinder concrete, the results of which should be indicative of the range of quality of the cinder concrete used in building construction, are summarized in the table on page 250. These tests constitute a portion of a study of "Cinder Concrete Floor Construction" by Harold Perrine and by George E. Strehan (*Trans.* Am. Soc. C. E., vol. 79, p. 523). The specimens were 8 by 16-in. cylinders made by competent men with laboratory training, but the material was taken from that going into the floors of various structures then in process of construction in New York City. The samples were taken without advance

STRENGTH OF CINDER CONCRETE

Perrine and Strehan Tests

Class of concrete	Compressive strength (lb. per sq. in.)			
	1 month	2 months	6 months	1 year
1:2:5 continuous-mixer concrete, low-grade cinders..	407	701	933	913
1:2:5 batch-mixer concrete, good cinders	818	1,254	1,744	1,465
1:2:5 batch-mixer concrete, good cinders..........	980	1,035	1,478	1,475
1:2:5 hand-mixed concrete, good cinders	507	662	754	813

notice been given the contractor, and the specimens, after being molded on the job, were tested in the Columbia University Laboratory.

STRENGTH OF CINDER CONCRETE

Structural Materials Laboratories' Tests

	Compressive strength (lb. per sq. in.)			At age of 52 weeks
	4 weeks	13 weeks	26 weeks	
Max........	1,964	2,445	2,792	2,958
Min........	1,499	1,981	2,187	2,493
Ave........	1,647	2,217	2,525	2,761

Tests of 1:2:4 cinder concrete made at the Structural Materials Testing Laboratories at St. Louis in 1909 are summarized on this page (*Tech. Paper* 2, U. S. Bureau of Standards). Tests of 21 cylinders 8 by 16 in. are averaged.

28. Working Stresses.—For working stresses recommended by the Joint Committee, see *Appendix B.*

ELASTIC PROPERTIES OF CEMENT MORTAR AND CONCRETE

29. Stress-strain Curves for Mortars and Concretes.—Typical stress-strain curves for a number of classes of 1:2:4 concrete at the age of 1 year are presented by Fig. 26. These curves average the results of tests of 21 specimens (8 by 16-in. cylinders) for each class of con-

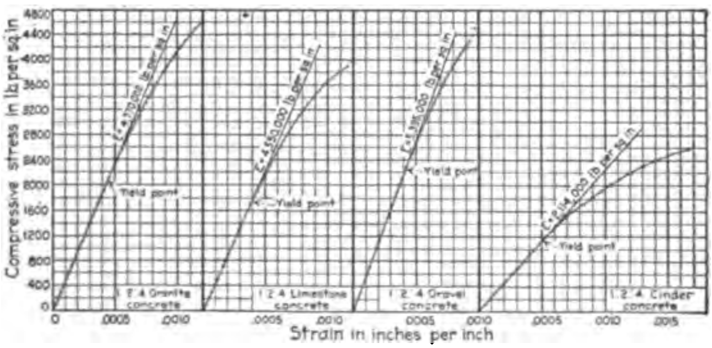


FIG. 26.—Stress-strain curves for concretes. (Curves are averages for 21 tests. Age 12 mo.)

crete, made at the Structural Materials Laboratories at St. Louis (U. S. Bureau of Standards *Tech. Paper* 2). At earlier ages tests of these same concretes resulted in stress-strain curves closely resembling these 1-year-test curves except that the slope of the curves is less, and the curvature greater, at the earlier test periods.

Stress-strain curves for mortars exhibit the same characteristics as do these curves for concretes.

30. Yield Point.—Mortars and concretes are not perfectly elastic materials for any range of loading, there being a slight decrease in the proportion of stress to strain as the stress increases, and a slight permanent set for very low stresses. Even the first portion of the stress-strain curve is, therefore, not a perfectly straight line, but for purposes of curve plotting it is sensibly so up to a certain point. This point where deviation from a practically straight-line relation between stress and strain is first perceived is designated the *yield point*. Beyond the yield point the slope of the curve decreases at an increasingly rapid rate, and the permanent set increases correspondingly.

The following table gives values of the yield point in tests of mortars of three proportions made by the Bureau of Standards (*Tech. Paper* 58).

YIELD POINT OF MORTARS

Proportions by volume	Age 4 weeks			Age 13 weeks		
	Yield point (lb. per sq. in.)	Modulus of elasticity (lb. per sq. in.)	Ultimate strength (lb. per sq. in.)	Yield point (lb. per sq. in.)	Modulus of elasticity (lb. per sq. in.)	Ultimate strength (lb. per sq. in.)
1:1	1,834	4,243,000	5,613	2,600	4,153,000	6,739
1:2	....	....	3,070	1,833	4,673,000	4,560
1:4	400	2,120,000	1,432	700	2,200,000	1,663

The yield points of various classes of 1 :2 :4 and 1 :3 :6 concretes at ages up to 1 year are shown by Fig. 27 which is based upon tests of the Bureau of Standards (*Tech. Paper* 58).

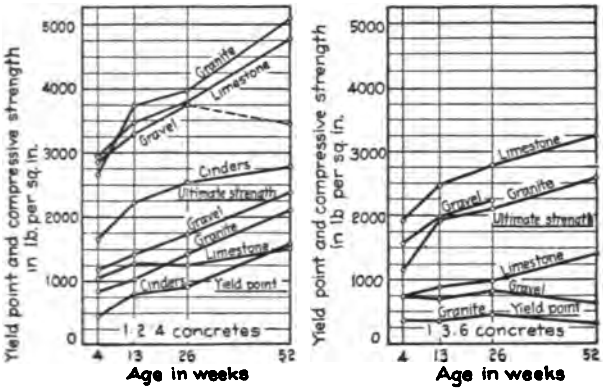


FIG. 27.—Yield point of 1 : 2 : 4 and 1 : 3 : 6 concretes.

Values of ultimate compressive strength for the same concretes are shown for purposes of comparison. These tests indicate that the yield point of the average concrete is in the neighborhood of three-tenths of the compressive strength at 1 month and about four-tenths of the compressive strength at 1 year.

31. Modulus of Elasticity.—The modulus of elasticity of an elastic material is the quotient obtained by dividing unit stress by the corresponding unit deformation or strain, the limit of elastic behavior not being exceeded. In American practice the unit of measurement is pounds per square inch. Since mortars and concretes are not perfectly elastic materials, the deformation not bearing a constant relation to the stress for any range of loading, the quotient of stress divided by strain will vary, decreasing as the stress increases. Properly speaking an inelastic material has no modulus of elasticity, but practice has sanctioned the use of the term in connection with mortars and concretes, meaning the quotient of any small stress increment by the corresponding strain increment. The value of E thus computed is, therefore, the slope of a short chord of the stress-strain curve, or if the stress increment be very small, E at any point on the stress-strain curve is represented by the tangent to that curve. Within the limits of working stresses for mortars or concretes the value of E changes only very slightly, and its initial value may, therefore, properly be used for purposes of design. This initial value of E may most conveniently be determined by noting the slope of the tangent to the first portion of the stress-strain curve.

The values of the initial modulus of elasticity for the three mortar mixtures mentioned in the last preceding chapter are given in the above table. Values of E found for the various concretes also mentioned above are given in Fig. 28. Little may be said by way of generalization concerning E for concretes. It increases with age and with the richness of the mix, but varies greatly with different classes of aggregate materials and with different aggregates of the same general class. E for cinder concrete appears to be something less than one-half the average value found with rock concretes.

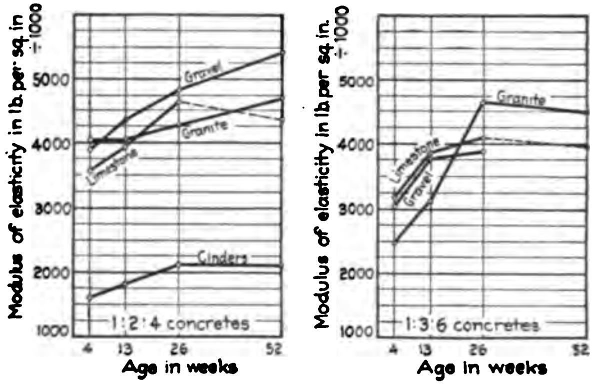


FIG. 28.—Modulus of elasticity of 1 : 2 : 4 and 1 : 3 : 6 concretes.

The design values of E recommended by the Joint Committee are given in *Appendix B.*

CONTRACTION AND EXPANSION OF CEMENT MORTAR AND CONCRETE

32. Coefficient of Expansion.—Mortars and concretes expand as the temperature is raised and contract as the temperature is lowered. The coefficients of linear expansion per degree Fahrenheit for a series of mortars and crushed stone concretes tested under the writer's direction in the laboratories of the College of Civil Engineering, Cornell University, in 1916 are listed in the table on this page. All of the specimens were molded in the shape of bricks 8 in. long, 4 in. wide, and 2 in. thick. Heating was done in a specially-constructed resistance type of electric furnace, and distortion was measured by a special extensometer actuated by fused quartz contact bars extending through the furnace walls. Measurements were made to the nearest 0.000,000,9 in. per in. of length of the specimen. The test results listed are averages of from two to ten tests of each class of material, the ages varying from 1 to 8 months. The range of temperatures employed was from about 70° to 212°F.

Mortars		Concretes	
Mixture	Coefficient of expansion per degree Fahrenheit	Mixture	Coefficient of expansion per degree Fahrenheit
Neat	0.000,007,83	1:1½:3	0.000,006,77
1:1	0.000,007,43	1:2 :4	0.000,006,60
1:2	0.000,006,00	1:2½:5	0.000,005,58
1:3	0.000,006,05	1:3 :6	0.000,005,37
1:4	0.000,005,94		
1:5	0.000,005,77		

These tests show that the coefficient of expansion of mortars and concretes increases with increase of richness of the mix, but that the range of values between a very lean concrete and neat cement is comparatively short. An average concrete will have a temperature coefficient almost exactly equal to that of the average steel. This fact is one of great importance in all cases of reinforced concrete.

33. Moisture Changes.—Mortars and concretes expand in volume if kept wet or immersed in water, and contract if exposed in air. Experiments made by Prof. A. H. White at the University of Michigan (*Proc.* Am. Soc. Test. Mat., vols. 11 and 14) indicate that this property is not confined to the early hardening period but is characteristic of mortars and concretes even after 20 years in service. Fig. 29*a* shows the variations in length observed in two 1-in. by 1-in. by 4-in. bars of 1 : 3 mortar tested by Prof. White. These bars appear to have suffered no impairment of the ability to expand immersed and contract when exposed to air even after a period of nearly 5 years. These particular specimens, when about 4 years old, show increases in length of about 0.05% in a 3 to 4-month period when placed in water after thorough drying out, and their contraction in air is scarcely less rapid. Specimens of the same mixes made with other cements showed in a number of cases more extensive volume changes than do the ones shown in Fig. 29*a*. One mortar stored in air contracted about 1.10% in the first 3 months. A specimen cut from the rich mortar top coat of a sidewalk which had been in service for 20 years expanded about 0.16% when stored in

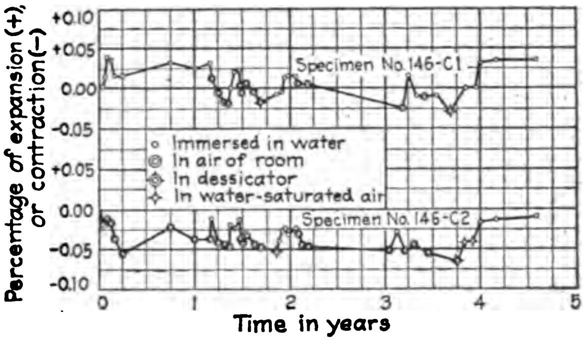


FIG. 29*a*.—Expansion and contraction of 1 : 3 mortars when alternately wet and dried.

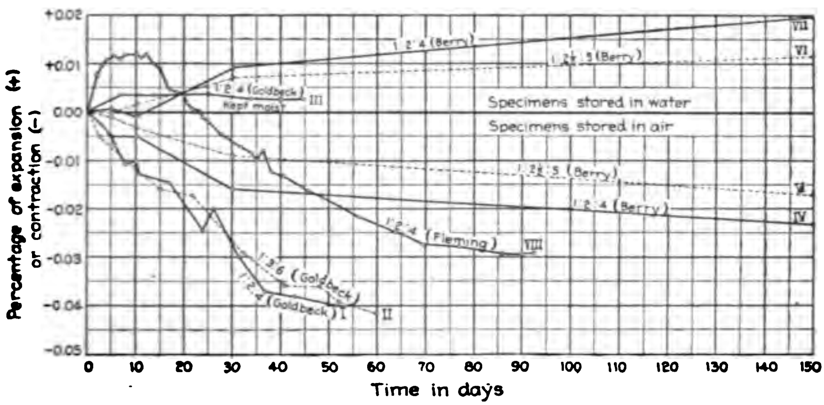


FIG. 29*b*.—Expansion and contraction of concretes. Air and water storage.

water for 2 years, and a portion from the gravel concrete base of the same walk expanded about 0.12% in the same period.

The results of a number of tests of concrete made under various auspices are shown by Fig. 29*b*. Curves I, II, and III show the changes in length of concretes tested by A. T. Goldbeck in the laboratory of the Office of Public Roads, U. S. Department of Agriculture (*Proc.* Am. Soc. Test. Mat., vol. 11). The specimens were 8 in. square and 5 ft. long. Specimens I and II were stored in air; specimen III was wrapped in burlap which was kept moist continuously. Curves IV, V, VI, and VII are derived from tests made by Prof. H. C. Berry in the laboratories of the University of Pennsylvania (*Proc.* Am. Soc. Test. Mat., vol. 11). The specimens were 9 in. square and the gaged length was 20 in. Specimens IV and V were stored in air while specimens VI and VII were stored in water. Curve VIII is the record of a test of a slab of concrete 12 in. wide, 6 in. deep, and 10 ft. long between gage points, made by Prof. B. P. Fleming of the State University of Iowa. The slab was exposed in the air of the laboratory throughout the period of observation. Its behavior is notable in one respect in that a pronounced expansion was observed for the first 10 days, amounting to about 0.012%. After about 12 days it contracted continuously for about 75 days longer, at which time it shows a net contraction of about 0.03%. Mr. Goldbeck states that some of his mixtures showed a tendency to expand in the early hardening period of a few days but the amount of expansion was very slight.

The tests of Fig. 29*b* indicate that the changes in volume of concretes when exposed in either a wet or a dry situation are rather variable with different cements and aggregates. It appears, however, that a concrete which dries out in the air may be expected to contract from 0.02 to 0.05%, and when immersed in water may expand at least half this amount. If the concrete in a structure is so restrained that it is not free to expand or contract, it is possible therefore that stresses amounting to from 400 to 1000 lb. per sq. in. in tension may occur, E being considered to be 2,000,000 lb. per sq. in. This means that the tensile strength of concrete is exceeded and the concrete will, and commonly does, crack. Difficulty caused by the expansion of concrete in a damp or wet situation is not so commonly encountered, and the stresses introduced will never cause compressive failure. They may, however, cause a buckling action in the case of continuous surfaces of large extent.

DURABILITY OF CEMENT MORTAR AND CONCRETE

34. Fire-resistance Properties.—Concrete ranks highly as a fire-resistant and fireproofing material principally because it possesses a low rate of heat conductivity and has a low coefficient of expansion practically equal to that of steel, in addition to being incombustible. Other masonry materials like some of the natural stones and terra-cotta are no less incombustible than concrete, but are inferior to the latter as a fireproofing material because they possess either greater conductivity or a higher coefficient of expansion.

Tests of the conductivity of concretes made by Prof. Ira H. Woolson (*Proc.* Am. Soc. Test. Mat., vols. 5, 6, and 7) led to the conclusions: "That all concretes"—stone gravel, and cinder—"have a very low thermal-conductivity, and herein lies their ability to resist fire. That when the surface of a mass of concrete is exposed for hours to a high heat, the temperature of the concrete 1 in. or less beneath the surface will be several hundred degrees below the outside. That a point 2 in. beneath the surface would stand an outside temperature of 1500°F. for 2 hr. with a rise of only 500° to 700°, and points with 3 in. or more of protection would scarcely be heated above the boiling point of water."

The low thermal-conductivity of concrete is partly due to its porosity, air spaces affording efficient protection against conduction, and partly due to the absorption of heat of vaporization by the water of combination in the set cement when the temperature of dehydration of the latter is reached. This dehydration probably begins at about 500°F. and is completed at about 900°F. (S. B. Newberry, *Cement*, May, 1902, p. 95). The absorption of heat by the surface material in becoming itself dehydrated retards the dehydration of the underlying material. The surface concrete which is injured by heat, but which remains in place, affords protection for the material beneath, for it becomes a poorer conductor than the original concrete. The Joint Committee on Concrete and Reinforced Concrete recommends that "metal be protected by a minimum of 2 in. of concrete on girders and columns, 1½ in. on beams, and 1 in. on floor slabs."

The experience gained in great conflagrations like the Baltimore fire, the San Francisco fire, the Edison plant fire, etc., has been that concrete exposed to intense heat for considerable periods becomes calcined to a depth of from ¼ to ¾ in. but shows no tendency to spall off except at exposed corners and edges (see reports of Captain J. S. Sewell to the Chief of Engineers, U. S. A., and of Prof. Norton to the Insurance Engineering Experiment Station, on the Baltimore fire, *Eng. News*, March 24, and June 2, 1904; the report of S. A. Reed to the National Board of Fire Underwriters on the San Francisco fire, *Eng. News*, vol. 56, p. 137; the comments of various engineers upon the Edison plant fire, *Eng. News*, vol. 73, p. 38; and the report on the Edison fire of the National Fire Protective Association, obtainable in booklet form from the New York Board of Fire Underwriters).

Prof. Norton and others have concluded that there is little difference in the action of fire on stone concrete and cinder concrete.

35. Weathering Qualities.—The principal agencies affecting the durability of concretes and mortars which are classed as weathering agencies are changes of atmospheric temperature, wind and rain, and changes of atmospheric moisture. The expansion and contraction of mortars and concretes subjected to variations of temperature and moisture conditions are responsible for practically all failures of these materials under conditions of exposure to the weather. Either temperature effects or moisture effects may be alone operative, or both effects may be combined. Temperature stresses caused by the shrinkage of continuous large surface areas are particularly apt to cause cracking. This cannot be wholly prevented, but cracks can be made less harmful by the use of steel reinforcement so placed that a multitude of small cracks, which do not open up much, replace a few large and deep cracks. In the average situation the introduction of dangerous stresses caused by a tendency to expand or contract is more apt to be due to moisture changes than to temperature changes, because the volumetric changes in the latter case are less marked. The expansion and contraction of rich mortars and concretes is considerably more extensive than that of leaner mixes when the moisture condition varies, and the same thing appears to be true to a lesser extent when the temperature varies. This circumstance is responsible for the difficulty often encountered in causing a surface coating of comparatively rich material plastered upon a leaner base material to adhere permanently if the bond between the two is at all defective. The surface material tends to expand and contract more than the underlying material not only because it is richer in cement, but also because it protects the underlying material from as extensive temperature and moisture changes as it itself experiences. The result is the introduction of excessive stresses in the surface material, the opening up of tension cracks, or buckling due to compressive stress, and the ultimate spalling off of the surface layer. The principal preventive measures which may be adopted are the use of as lean a surface coat as is practicable, the use of as thin a plaster coating as possible thus favoring the formation of many small cracks rather than a few large ones, and the adoption of all measures tending to make a strong bond between the two classes of material. An excessive amount of troweling of surfaces is to be avoided because of the flushing to the surface of a film of nearly neat cement which will tend to peel off in some cases.

36. Abrasive Resistance.—The abrasive resistance of mortars "is primarily of importance in the determination of the best mortar for use in the top coat of concrete floors, walks, and pavements. Resistance to abrasion will always be dependent not only upon the cement, as regards the tenacity with which it clings to the sand grains, which will be largely dependent upon its fineness and its lime content, but also upon the hardness of the sand used. Abrasion either wears away the cement and the sand grains, or it pulls the sand grains out of the cement matrix.

"With soft sand particles the resistance to abrasion with a given cement decreases constantly as the percentage of sand is increased. With hard sand grains the abrasive resistance increases as the proportion of sand increases, until the volume of cement becomes relatively too small to bind the sand grains together thoroughly. This limit is found to be reached when the mortar contains not more than two parts of sand to one of cement." ("Materials of Construction," by A. P. Mills.) The abrasive resistance of concretes is dependent almost wholly upon that of the mortar made by its cement and fine aggregate. The coarse aggregate is practically always forced back from the surface of concrete exposed to abrasion. If it is exposed, the same considerations of relative hardness of the stone particles and proportion of the mix applicable in the case of mortars apply to the concrete.

37. Action of Sea Water.—The behavior of concrete in sea water is a problem which has occupied much of the attention of engineers for many years. The question has often been discussed, and many attempts have been made to determine experimentally the exact action of sea water upon concrete, and the causes of that action. The amount of accurate information available is rather meager, however, and the results of experimental investigations are inconclusive and often contradictory. Many concrete structures in sea water out of the range of frost action have remained intact and uninjured for many years. Others have been seriously disintegrated, particularly between high and low tide levels. The disintegration is evidently often due in part to frost action, but chemical action is frequently indicated by the softening of the mortar, and the complete disintegration of mortar and concrete specimens by subjection to the action of sea water at normal temperatures in the laboratory has been accomplished. The exact nature of the chemical action involved cannot be definitely stated. It is commonly believed, however, that the magnesium sulphate in the sea water is the most injurious constituent, and that the magnesium chloride and calcium chloride are somewhat less active. The magnesium sulphate attacks the lime in the cement, also the alumina, forming large and rapidly growing crystals of hydrated magnesia and calcium sulpho-aluminate. Both magnesium chloride and sodium chloride attack the silicates of the cement.

The chemical action is accompanied by various physical phenomena. Sometimes the mass swells, cracks, and gradually falls apart; sometimes the mortar softens and becomes disintegrated leaving the coarse aggregate exposed and finally permitting it to fall away; and occasionally a crust forms on the surface which later cracks off.

An important symposium of European investigators' studies of the problem of concrete in sea water is afforded by the several papers of Chapter XVII of the *Proceedings* of the Sixth Congress of the International Association for Testing Materials held in New York in 1912.

The conclusions arrived at from a study of important German and Scandinavian tests have been expressed, in part, as follows (*Concrete and Constructional Engineering*, January, 1910):

1. Good Portland cements such as are now on the European market, are very resistant to the action of sea water. A marked difference in the behavior of cements of slightly different composition has not been found, except that a high proportion of aluminates tends to cause disintegration.
2. In a dense mortar, the chemical action is confined to an outer layer of small depth, further action being checked by the slowness of diffusion. A porous mortar, by admitting salt water to the interior, is apt to crack by expansion owing to chemical change.
3. The main agency in the destruction of mortar and concrete in marine embankments, harbor works, groynes, etc., is not chemical action, but the alternations of saturation, drying in the sun, freezing, etc., due to the alternate exposure and covering by the rise and fall of the tide.
4. The denser the mortar the better (1 cement : 3 sand is too poor). An admixture of fine sand with the ordinary sand increases the closeness of the mixture. A well-graded aggregate would be advantageous for the same reason.
5. The addition of finely ground silica or trass to the cement before mixing is possibly advantageous in the case of weaker mortars. It is very doubtful whether anything is gained by adding trass to the richer mortars.
6. The destructive action of the sea being mainly physical and mechanical, and not chemical, tests by mere immersion in still sea water are of very little value in determining the behavior of concrete in marine engineering works. A mixture which disintegrates under this test is certainly useless, but a mixture which passes the test may disintegrate under the more stringent conditions of practical use.
7. As long a period as is practicable should be allowed for the hardening of concrete blocks before placing in the sea.
8. The behavior of test specimens for the first 12 months is very irregular, and definite conclusions can only be drawn from the results of long-period tests.

The most notable American investigations of the subject are one made by the Bureau of Standards (*Tech. Paper* 12) and one begun in 1908 by the Aberthaw Construction Co. in coöperation with the United States Navy Department (reported in a pamphlet issued by the Aberthaw Construction Co. in 1914; also in *Eng. Rec.*, March 21, 1914). Few general conclusions may be drawn from the results of the first 5 years' observations of the Aberthaw tests. This is particularly true since European experience has shown that the first indications of injury to many concretes appear only after from 5 to 10 years' exposure, and in some cases only after more than 20 years. The most notable indications afforded by the Aberthaw tests after 5 years' exposure in a latitude involving wide variations of temperature and frequent freezing and thawing, are:

1. Concrete which is alternately immersed and exposed as the tide rises and falls is most subject to injury.

2. Lean mixtures (1 :3 :6) are very much more subject to attack than rich mixtures (1 :1 :2), and medium mixtures (1 :2 :4) are more vulnerable than rich ones.

3. Concrete mixed with a plastic or even a very wet consistency appears to withstand sea water attack better than concrete of a dry consistency.

4. The relative immunity from attack by sea water of concretes made with cements of various classes, including a low-iron cement, high- low- and average-alumina cements, an iron-ore cement, and a slag Portland cement, has not been conclusively established.

The Joint Committee makes the following recommendations for concrete placed in sea water:

To effect the best resistance to sea water, the concrete must be proportioned, mixed, and placed so as to prevent the penetration of sea water into the mass or through the joints. The aggregates should be carefully selected, graded, and proportioned with the cement so as to secure the maximum possible density; the concrete should be thoroughly mixed; the joints between old and new work should be made water-tight; and the concrete should be kept from exposure to sea water until it is thoroughly hard and impervious.

38. Action of Alkali.—The effect of alkali on concrete is a problem resembling in many respects that of the action of sea water on concrete. The problem is of especial interest in connection with concrete construction in the arid regions of the West, where soluble salts are present in the soil to an extent not usually found elsewhere.

The principal salts encountered in alkali waters usually include: magnesium sulphate, calcium sulphate and sodium sulphate, magnesium chloride, sodium chloride, and potassium chloride, together with carbonates of magnesium, sodium, and potassium. Of these the sulphates appear to be most active in causing disintegration of concrete; the chlorides also are active, while the carbonates appear to be without effect.

The attempts at an explanation of the manner of attack of these salts upon concrete have hitherto encountered the same difficulty found in the case of sea water—an unsatisfactory knowledge of the constitution of cement. From the physical point of view the action exactly resembles the action of frost except that it is more rapid. There exists, apparently, a disruptive force which quickly destroys the bond and causes disintegration. This action appears to proceed most rapidly in the parts of a structure subjected to alternate wetting with alkali water and drying in the air. In porous concrete the action proceeds much more rapidly than in dense concrete, where, indeed, it may make no progress at all.

As in the case of the injurious action of sea water on concrete, instances of failure caused by alkali waters are merely isolated ones, presenting an interesting field for study, but not constituting a very serious menace to the future of concrete construction in the arid regions of the West. The remedy in the present state of our knowledge is, as in the case of marine structures, a matter of the possible physical precautions only—the securing of the densest possible concrete, thus preventing injury by the exclusion of the salt-bearing waters.

39. Action of Acids, Oils, and Sewage.—The Joint Committee Report makes the following statements concerning the effect of acids and oils upon concrete:

Dense concrete thoroughly hardened is affected appreciably only by acids which seriously injure other materials. Substances like manure, that contain acids, may injuriously affect green concrete, but do not affect concrete that is thoroughly hardened.

Concrete is unaffected by such mineral oils as petroleum and ordinary engine oils. Oils which contain fatty acids produce injurious effects, forming compounds with the lime which may result in a disintegration of the concrete in contact with them.

The use of concrete sewer pipes has led to considerable study of the effect of sewage and sewage gases upon concrete. Sidney H. Chambers concluded from an investigation reported before the Concrete Institute (Great Britain) in 1910:

That the gases in solution in sewage and those expelled from it, arising from its decomposition, do act injuriously upon Portland-cement concrete, notwithstanding the fact that the concrete is constituted of sound and good materials, when the following conditions prevail: (1) A high degree of putrescence of the sewage; (2) a moistened surface, which held or absorbed the putrid gases; (3) the presence of a free air supply. Further, that in the absence of one or the other of the above-enumerated factors little danger from erosion need be feared.

Rudolph Hering is responsible for the following statements concerning the effect of the acids in sewage upon concrete (quoted from a report to the President of the Borough of Brooklyn in 1908 by Gustave Kaufman in *Proceedings* of the National Association of Cement Users, vol. 8, 1912, p. 725).

Portland cement used for the manufacture of concrete pipes is attacked by certain strong acids, such as sulphuric acid, which converts the carbonate into sulphate of lime, which is comparatively soft and easily eroded. Therefore cement pipe cannot be used where strong acids are known to enter the sewers.

The acid question should be viewed in a reasonable light. When the dilution of sewage is sufficient the discharge of a small amount of even strong acid will not cause objectionable effects, as evidenced by European cities where the use of concrete sewers is almost exclusive in some cities, as Paris and Vienna. In England concrete sewers are also very common.

The greasy substance which is usually found to coat the perimeter of a sewer under the water line tends to protect the cement from the action of acids to some extent.

Over 400 miles of concrete sewer pipe laid in the City of Brooklyn during a period of over 50 years are giving eminent satisfaction.

40. Electrolysis in Concrete.—Experience and laboratory tests have shown that under certain conditions concrete may be seriously damaged by electrolytic action caused by the flow of electric current between the concrete and iron or steel embedded therein. The phenomena assumes importance under certain conditions of use of reinforced concrete, also the use of concrete foundations and footings in which the bases of columns of buildings, bridges, and elevated railway structures are embedded. A very important laboratory and field study of the entire problem has been made by the U. S. Bureau of Standards, and the conclusions arrived at in this investigation constitute the authority for the statements made here (*Tech. Paper* 18, Bureau of Standards).

The electrolytic effect differs according to the direction of flow of the current. If electrically positive iron or steel is in contact with concrete the iron will become corroded provided the concrete is moist or wet and the potential gradient is high enough to heat the junction to a temperature not below about 45°C. (113°F.). The minimum potential gradient found effective in causing corrosion in moist concrete was about 60 volts per ft. Iron when corroded expands to about 2.2 times its original volume and causes mechanical pressure found in some cases to reach values as high as 4700 lb. per sq. in. This causes cracking of the concrete. The passivity of iron below 45°C. is due chiefly to the inhibiting effect of $Ca(OH)_2$ in the concrete, and on this account old concrete in which the $Ca(OH)_2$ has been largely carbonated is probably more susceptible to electrolysis than new concrete in the same moisture condition. For air-dried concrete a much higher potential gradient is required to produce the temperature at which corrosion becomes dangerous than for moist concrete. Under actual conditions, therefore, corrosion from stray currents may be expected only under special or extreme conditions. Normal wet concrete carrying current also increases its resistance a hundredfold in the course of a few weeks, owing partly to the precipitation of $CaCO_3$ which fills up the pores. This further lessens danger of trouble.

Electrolysis of the concrete in contact with negative iron is manifested in a different way. The concrete near the cathode becomes softened, beginning at the cathode surface, and extending to a depth of ¼ in. or more. This softening practically completely destroys the bond between iron or steel and concrete. While the anode effect becomes serious in normal concrete only on comparatively high voltages, decreases much more rapidly than the voltage, and almost disappears at voltages likely to be encountered in practice, the cathode effect develops at all voltages, the rate being roughly proportional to the voltage. The softening effect is due to the gradual concentration of alkalies, Na and K, near the cathode, these finally becoming strong enough to attack the cement. The cathode effect is wholly limited to the vicinity of the cathode, the strength of the mass of the concrete not being affected.

Salt or calcium chloride, even in very small amounts (a fraction of 1%), multiplies the rate of corrosion of iron at the anode many hundredfold because it increases the conductivity of wet concrete, destroys the passivity of iron at ordinary temperatures, and prevents the increase in resistance with flow of current by preventing the precipitation of $CaCO_3$. Salt should therefore not be used in structures subject to electrolytic action, and special consideration should be given to the possibility of electrolytic action in the cases of all concretes exposed to sea water or salt brine.

The danger of electrolysis of reinforced-concrete structures through the operation of stray currents has been overestimated. Certain precautions are necessary under special conditions, but there is no cause for serious alarm. Non-reinforced-concrete structures are practically immune from injury by electrolysis.

The precautions to be adopted in the special cases where electrolysis is to be feared include avoidance of grounds in direct-current circuits in buildings; providing insulating joints in pipe lines which enter buildings outside the walls; completely isolating the buildings by insulating joints in pipe lines which enter the building and also continue on beyond; providing a copper cable shunt around the building if the potential drop is large; isolating from the concrete lead-covered cables entering the building; and interconnecting all metal work in the building if practicable, but without connecting this metal work to ground plates or to pipe lines outside of the insulating joints.

41. Effect of Manure.—Manure is occasionally used to cover up fresh concrete in freezing weather, not only because it is a poor conductor of heat when rather dry, but also because its decomposition is a source of heat. Experience has shown (see *Eng. News*, vol. 49, pp. 11, 104, 126, 127, and 175; also *Journal of the New England Waterworks Association*, vol. 22, p. 242) that manure not only discolors the work, but that it also has a marked disintegrating effect if placed in contact with freshly placed concrete. The injury is especially pronounced if rain wets the manure during the early hardening period and carries the uric acid into the concrete. The use of manure as a preventive of freezing of fresh concrete may be considered permissible only if the work is covered first by a material which will be sufficiently impermeable to prevent the seepage of acid into the concrete.

Concrete which has once thoroughly hardened appears not to be susceptible to injury by contact with manure except that it is in some cases somewhat discolored.

MISCELLANEOUS PROPERTIES OF CEMENT MORTAR AND CONCRETE

42. Rise of Temperature in Setting.—The chemical combination of water with Portland cement is an endothermic reaction, the heat evolved being sufficient to materially raise the temperature of mortars and concretes during the period of setting and hardening. The total rise in temperature, the rate of increase, and the time interval before the maximum temperature is reached are all variable, depending upon the character of the cement used, the proportions of the mixture, the size of specimen or bulk of material involved (in so far as this determines the distance of the point at which temperatures are measured from any exposed surface of the material), external temperature conditions, the amount of water used in mixing, etc.

The results of observations of temperatures acquired at the center of 12-in. cubes of cements and mortars tested at the Watertown Arsenal ("Tests of Metals," 1901, p. 493), are shown by Fig. 30*a*. With these specimens the maximum temperature was attained in from 12 to 18 hr. except in the case of natural cements which reached their maximum temperature much earlier. The heating effect is less than one-half as great with 1:1 mortar as with neat cement, and is quite small with leaner mixtures.

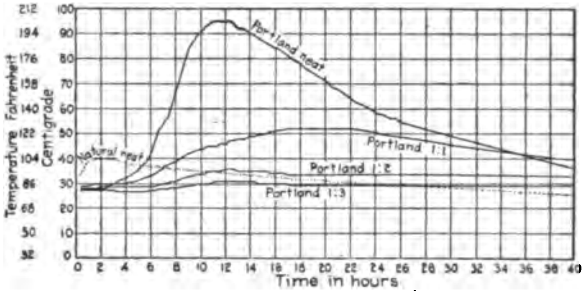


FIG. 30a.—Rise in temperature of cement and mortar in setting and hardening. (12-in. cubes.)

Radiation has a great deal to do with the temperatures observed at the centers of these comparatively small specimens, as is shown by comparison of the curves for mortars with Fig. 30b which shows the rise in temperature of a 1:2½:6 concrete, the temperature having been measured in the midst of a very large mass of concrete with the thermometer covered by a

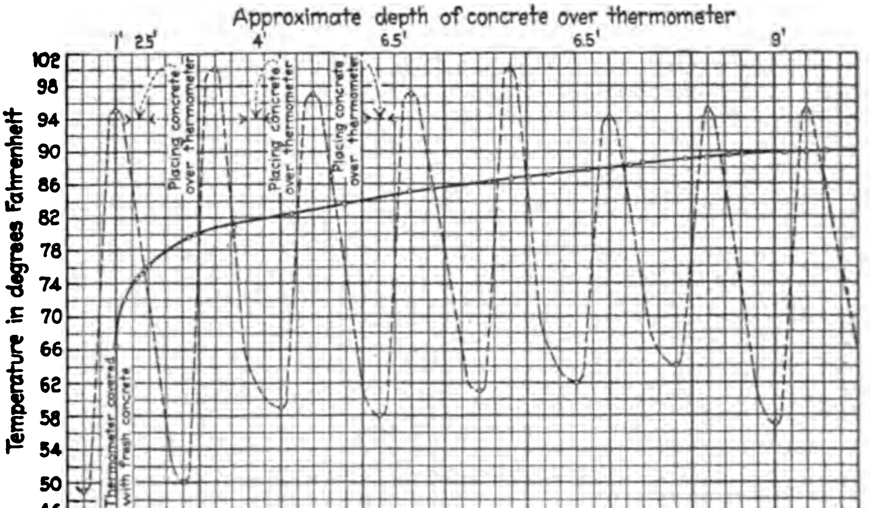


FIG. 30b.—Rise in temperature of mass concrete in setting and hardening.

Composition of concrete: 1 part sand-cement, 2½ parts sand, 6 parts gravel, 2 parts large cobbles.

Dotted lines indicate daily variation in atmospheric temperature.

depth of from 1 to 9 ft. of concrete as indicated. Fig. 30b constitutes a portion of a report upon an investigation of the temperature changes in mass concrete made during the construction of the Arrowrock Dam by the United States Reclamation Service near Boise, Idaho ("Temperature Changes in Mass Concrete" by Charles H. Paul and A. B. Mayhew, *Trans.* Am. Soc.

C. E., vol. 79, p. 1225, 1915). A summary of the results obtained in these tests is given in the following table.

TEMPERATURE CHANGES IN MASS CONCRETE

Arrowrock Dam Tests

Proportions of mix (parts by volume)				Distance to nearest face (ft.)	Depth of concrete over thermometer (ft.)	Rise in temperature in degrees Fahr.	Highest temperature in degrees Fahr.	Time reaching highest temperature, hr.
Sand cement	Sand	Gravel	Cobbles					
*1	2	4	2½	1.0	2.0	2.5	78.0	1
*1	2	4	2½	2.0	1.5	3.6	77.6	1
1	2½	5	2	10.0	35.0	16.6	91.6	32
1	2	4¾	1½	3.0	3.0	19.5	74.5	5
1	2½	6	2	19.5	3.5	20.6	64.6	15
1	2¾	4	1½	3.5	6.0	26.9	69.8	5
1	2½	6	2	76.0	15.5	27.5	94.0	31
1	2½	5½	2¾	31.0	80.0	35.7	96.2	101
1	2½	6	2	20.0	28.5	36.7	86.2	45

* Thermometer bedded in fresh concrete in shallow trench dug in concrete 11 days old.

Similar tests made during the construction of the Kensico Dam at Valhalla, N. Y., by George T. Seabury, are reported to have established that (*Proc.* Am. Soc. C. E., vol. 29, p. 1247–1253):

With the 1 : 3 : 6 concrete used, thermometers being inserted as soon as the concrete was placed and immediately covered to as great a depth as possible, the rise in temperature was uniformly about 40°F.

The maximum concrete temperature reached was often well above 100°F. in summer, in one instance, 118.5°. This maximum was usually reached in about 15 days.

The rate of increase in temperature was about 1° per hr. for 4 or 5 hr. gradually increased to 8°–10° per hr. during the period of final set of the cement, and then dropped suddenly to about ½° per hr. for a considerable time. A total rise in temperature of from 25° to 30°F. often occurred subsequent to the period of final setting.

43. Porosity.—The porosity of a mortar or concrete is expressed by the percentage of void space (space filled by air or uncombined water) in terms of the total volume. It is determined experimentally by subtracting from the total apparent volume the volume of solid matter and dividing by the total apparent volume. The total apparent volume may be determined by direct measurement or by determination of the volume of water it displaces, absorption being prevented by a waterproof coating of grease or varnish. The volume of solid matter is determined by weighing the specimen dry in air, and subsequently in water after the pores of the material have been thoroughly impregnated with water. The difference in these weights divided by the weight of a unit volume of water is the volume of solid matter. Extreme accuracy in determining porosity is not possible because of the difficulty encountered in completely fiilling all the voids in the material.

The porosity of mortars and concretes is principally dependent upon the consistency of the mixture, and the granularmetric composition of the aggregates. Plastic or wet consistencies will in general produce mortars having less void space, and therefore lower porosity, than dry consistencies, and a well-graded aggregate will form less porous mortar than one whose particles are not well graded in size. A concrete will usually be considerably less porous than a mortar because of its proportion of comparatively non-porous coarse aggregate, and a fine-sand aggregate will in general produce more porous mortar than a coarse sand because of the larger amount of water required in gaging the latter.

The porosity of mortars will usually be found within the limits of 15% and 30%, an average 1:2 mortar showing 20 to 25% porosity. Concretes show from about 12 to about 20% porosity, the lower figure applying to concretes having a relatively large proportion of coarse aggregate and the higher figure to concretes having a relatively low proportion of coarse aggregate.

44. Permeability and Absorptive Properties.—The permeability of mortar or concrete is a measure of the rate at which water under a given pressure will pass through a given thickness of the material. The absorptive properties of a mortar or concrete constitute a measure of the rate at which moisture will be absorbed when the material is exposed in damp situations or covered with water under negligibly small heads.

Permeability is an important consideration where water-tightness of walls, etc., is required and percolation of water is not admissible.

Absorptive properties of a mortar determine its value as a dampproofing coat, particularly in the event of its use as a plaster over metal lath, which must be protected to prevent corrosion. In view of the disintegrating effect of expansion and contraction of mortars used as a plaster, etc., the moisture content (which largely affects this expansion and contraction) should not be greatly variable. Thus the least absorptive mortar will be most durable, up to the limit reached when the cement content is relatively so high that the expansion and contraction is disproportionately increased.

The determining of precise information concerning each of these properties is dependent upon a standardization of methods of conducting tests. Such standard methods have not yet been adopted, and it is therefore impossible to quote data as to the absolute permeability or absorptive power of mortars.

Tests to determine the relative permeability and absorptive power of mortars were made at the Structural Materials Laboratory as St. Louis in 1909, and are reported in *Technologic Paper* 3 of the Bureau of Standards. Owing to the small number of tests made and certain unsatisfactory features of the testing method employed, only a few general conclusions will be drawn from the report of these tests. (1) Permeability decreases rapidly for all mixtures with increase in age of the specimens when tested; (2) permeability decreases considerably with the continuation of the flow; (3) permeability increases with the leanness of the mixture, the dryness of the mixture, and increased coarseness of the sand.

Absorption was found to be dependent upon the same factors: it decreased with the age of the mortar as a rule, but not as rapidly as did the permeability (especially with the leaner mixtures); it decreased but slightly with increased richness of the mixtures; and the wetter mixtures were slightly less absorptive than the dryer mixtures.

The permeability of mortars and concrete is closely related to the porosity, but the relationship is not always direct, and is by no means constant, since the continuity and size of the pores determines permeability more than does the actual percentage of voids.

The Joint Committee Report says concerning the permeability and the waterproofing of concrete:

Many expedients have been resorted to for rendering concrete impervious to water. Experience shows, however, that when concrete or mortar is proportioned to obtain the greatest practicable density and is mixed to the proper consistency, the resulting mortar or concrete is impervious under moderate pressure.

On the other hand, concrete of dry consistency is more or less pervious to water, and though compounds of various kinds have been mixed with the concrete or applied as a wash to the surface, in an effort to offset this defect, these expedients have generally been disappointing, for the reason that many of these compounds have at best but temporary value, and in time lose their power of imparting impermeability to the concrete.

In the case of subways, long retaining walls and reservoirs, provided the concrete itself is impervious, cracks may be so reduced, by horizontal and vertical reinforcement properly proportioned and located, that they will be too minute to permit leakage, or will be closed by infiltration of silt.

Asphaltic or coal-tar preparations applied either as a mastic or as a coating on felt or cloth fabric, are used for waterproofing, and should be proof against injury by liquids or gases.

For retaining and similar walls in direct contact with the earth, the application of one or two coatings of hot coal-tar pitch, following a painting with a thin wash of coal-tar dissolved in benzol, to the thoroughly dried surface of concrete is an efficient method of preventing the penetration of moisture from the earth.

45. Protection of Embedded Steel From Corrosion.—The Joint Committee Report says concerning the corrosion of metal reinforcement in concrete:

Tests and experience indicate that steel sufficiently embedded in good concrete is well protected against corrosion, no matter whether located above or below water level. It is recommended that such protection be not less than 1 in. in thickness. If the concrete is porous so as to be readily permeable by water, as when concrete is laid with a very dry consistency, the metal may corrode on account of the presence of moisture and air.

The historic tests made by Prof. C. L. Norton for the Insurance Engineering Station in Boston in 1902 led to the following conclusions the validity of which has never been disproved by either tests or experience (*Eng. News*, vol. 48, p. 333):

1. Neat Portland cement, even in thin layers, is an effective preventive of rusting.
2. Concretes, to be effective in preventing rust, must be dense and without voids or cracks. They should be mixed quite wet where applied to the metal.
3. The corrosion found in cinder concrete is mainly due to the iron oxide, or rust, in the cinder, and not to the sulphur.
4. Cinder concrete, if free from voids and well rammed when wet, is about as effective as stone concrete in protecting steel.

Further tests made by Prof. Norton in 1903 showed conclusively that steel reinforcement, corroded before being embedded in concrete, does not corrode further, provided only that it has a continuous unbroken coating of concrete. This fact is important since it is almost impossible to prevent exposure of reinforcing steel on construction work to the elements, and a large proportion of such steel is therefore corroded when placed in the work.

Experience has abundantly shown that if concrete be mixed sufficiently wet so that it will flow about the reinforcement with only a moderate amount of puddling, the thin film of rich mortar which coats the steel affords a perfect preventive of corrosion.

46. Weight of Mortar and Concrete.—The weight of mortars and concretes varies with the proportions of the mixture, the consistency used in mixing, and the character and granularmetric composition of the aggregates. William B. Fuller found the following range of weights of mortars of various proportions made with the same sand and cement:

Proportions of mixture..........	1:1	1:2	1:3	1:4	1:5	1:6	1:7
Ave. weight (lb. per cu. ft.)......	145.1	143.3	140.0	137.7	138.6	135.5	137.6

The Bureau of Standards found the following relation between weight of gravel concrete and the proportions of mixture (*Tech. Paper* 58):

Proportions of mixture.............	1:1:2	1:1½:3	1:2:4	1:2½:5	1:3:6	1:4:8
Ave. weight (lb. per cu. ft.).........	147	145	144	143	142	140

The same series of tests of the Bureau of Standards indicate the following relations to hold between weight and consistency:

Proportions by volume 1 : 2 : 4 Coarse aggregate	Weight per cubic foot		
	Watery or fluid consistency	Soft, mushy consistency	Stiff quaking consistency
Cinder.........	115.2	114.9	113.1
Granite........	147.6	147.7	148.9
Gravel.........	139.6	142.7	144.5
Limestone......	144.7	145.9	147.8

The cinder concrete appears to be slightly heavier the wetter the mixture, but all of the rock concretes are slightly heavier the drier the mixture. No universally applicable relation between weight of concrete and the class of aggregate used is shown by tests. Different gravel concretes, for instance, will show greater variations in weight than the difference in average weight of gravel concretes and granite concretes. For purposes of design the weight of any class of stone concrete may be assumed to be 150 lb. per cu. ft., while cinder concrete may be assumed to weigh 115 lb. per cu. ft.

SECTION 6

GENERAL PROPERTIES OF REINFORCED CONCRETE

1. Advantages of Combining Concrete and Steel.—Steel can be put into a form to resist a given tensile stress much more cheaply than to resist an equal amount of compressive stress. This comes from the fact that the solid bar is well adapted to take tensile stresses, while for compressive stresses the steel must be made into forms of more extended cross-section in order to provide sufficient lateral rigidity. Other facts to be noted are the lack of durability of steel in many locations and its failure to stand up under a high heat.

Concrete, on the other hand, cannot be used in tension except to a very limited extent, but its compressive strength is sufficiently high to be of structural importance. It is also a good fireproof material and has great durability. In addition, concrete is a cheaper material than steel, can readily be obtained in almost any locality, and tests and the results of observations show that it thoroughly protects embedded steel from corrosion.

From the above considerations it follows that the advantages to be gained by using concrete reinforced with steel instead of either material separately will vary with different types of structures. In structural members subjected to both tension and compression, as in all forms of beams, the proper combination of the two materials meets with the best success. Steel rods embedded in the lower side of the beam carry the tensile stresses while the compressive stresses are carried by the concrete. Here the steel is used in its cheapest form and the construction may be made strong, economical, and very durable. In compressive members of appreciable length, such as columns, a combination of the two materials is also quite advantageous, although to a varying degree depending upon whether the reinforcing steel is used in the form of small rods or as structural-steel shapes.

2. Bond Between Concrete and Steel.—Most of the tests on bond have been made by embedding a short reinforcing bar in a block of concrete and pulling it out in a testing machine. In the kind of tests referred to, the concrete is in compression and conditions do not correspond to those ordinarily encountered in beams and slabs. Pull-out tests of this character, however, have been found to be of valuable aid in determining actual values for bond. This conclusion was reached through an extended series of experiments at the University of Illinois during the period 1909–1912; a series which include both pull-out tests and beam tests, as described further on in this article.

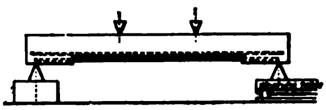


FIG. 1.

In a series of experiments made at the University of Wisconsin, test beams were arranged as shown in Fig. 1, the reinforcing bars being embedded only a short distance from each end, leaving the middle portion exposed. The stress in the rods were computed from the observed deformations. The beam was prevented from failing in the early stages of the tests, by an upper set of auxiliary rods. Failure finally occurred by the pulling out of the lower rods, as intended. Pull-out tests were also made with concrete cylinders and rods arranged as shown in Fig. 2. Cylinders designated as (*a*) were tested in the ordinary way with their upper surfaces bedded against the lower face of the pulling head. In cylinders (*b*) tension was applied to both upper and lower rods, bringing tension also in the concrete. The principal conclusions arrived at were as follows (the word *static* is used in connection with beams progressively tested to failure under loads gradually applied, and the word *repeated* occurs in connection with those beams subjected to repeated loadings):[1]

[1] *Bull.* 5, vol. 5, University of Wisconsin,

The static bond between 1 : 2 : 4 concrete and plain round steel rods increases with age at least up to 6 months. About 80% of the 6 months' bond strength is developed in 1 month.

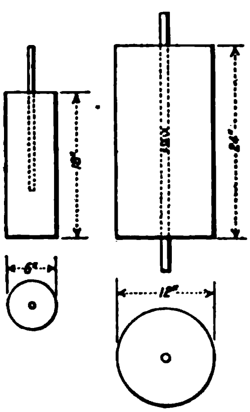
Owing to the variation in the results of individual tests and the difference between laboratory and practical working conditions, it does not seem as though the maximum static bond between concrete of the class used and plain rods less than ¾ in. diameter should be assumed greater than 250 lb. per sq. in. or for the rods or larger size 200 lb. per sq. in.

The method of making bond tests by pulling a rod from a cylinder of concrete in such a manner that the concrete around the rod is compressed gives results which are neither of quantitative nor qualitative value. The results obtained are dependent largely upon the compressive stress acting on the head of the cylinder. Cylinder tests in which the rod and concrete are both subjected to a tensile stress give results more in accord with the bond values obtained from beam tests.

The static bond between the class of concrete employed and corrugated bars is about twice as great as that which can be developed with plain round rods of about the same size. The static bond between concrete and rusted rods is very much greater than that obtained where plain round rods are used.

From the tests under repeated loadings it seems evident that 50 to 60% of the static bond between concrete and plain round rods may be repeated a large number of times without failure in bond; that 60 to 70% are the corresponding figures for corrugated bars. Under repeated loadings the bond between concrete and rusted round rods is considerably greater than that between concrete and plain round rods.

Considering the severity of the tests made there seems to be no valid reason for believing that the bond between concrete and plain round rods will be destroyed under repeated loadings, providing a proper working value is used. Such a value for concrete of the class used in these tests should not be over 50 lb. per sq. in.



(a) (b)

Fig. 2.

In tests at the University of Illinois, attention was given to obtaining accurate measurement of slip of bar through the concrete as the loading progressed, in both the ordinary pull-out tests and in tests on beams. In the beams of this series the concrete was *not* cut away from the rods as in the tests at the University of Wisconsin.

In the pull-out tests the amount of movement of the free end of the embedded bar was measured by means of an Ames gage. In the beam tests, the Ames gage was used to measure center deflections and the movement of the ends of the reinforcing bars. In many of the tests, observations were also made on the amount of slip of the reinforcing bar with respect to the adjacent concrete at several points along the length of the beam.

The concrete blocks used in the pull-out tests were usually 8 in. in diameter and 8 in. long, with the bar embedded axially. The beams tested were 8 by 12 in. in section with an effective depth of 10 in. The span length was generally 6 ft. All beams were tested with two symmetrical loads, generally at the one-third points of the span. With the exception of six tests, the longitudinal reinforcement consisted of a single bar of large diameter placed horizontally throughout the length of the beam. The principal results of these tests and the conclusions reached were as follows:[1]

Pull-out Tests

Bond between concrete and steel may be divided into two principal elements, adhesive resistance and sliding resistance. The source of adhesive resistance is not known, but its presence is a matter of universal experience with materials of the nature of mortar and concrete. Sliding resistance arises from inequalities of the surface of the bar and irregularities of its section and alignment together with the corresponding conformations in the concrete. The adhesive resistance must be overcome before sliding resistance comes into action. In other words, the two elements of bond resistance are not effective at the same time at a given point. Many evidences of the tests indicate that adhesive resistance is much the more important element of bond resistance.

Relation of Bond Stress to Slip of Bar as Load Increases.—Pull-out tests with plain bars show that a considerable bond stress is developed before a measurable slip is produced. Slip of bar begins as soon as the adhesive resistance is overcome. After the adhesive resistance is overcome, a further slip without an opportunity of rest is accompanied by a rapidly-increasing bond stress until a maximum bond resistance is reached at a definite amount of slip (see Fig. 3).

[1] *Bull.* 71, Engineering Experiment Station, University of Illinois.

The true relation of slip of bar to bond stress can best be studied by considering the action of a bar over a very short section of the embedded length. The difficulties arising from secondary stresses made it impracticable to conduct tests on bars embedded very short lengths. The desired results (Fig. 3) were obtained by varying the forms of the specimens in such a way that the effect of different combinations of dimensions could be studied.

Pull-out tests with plain bars of the same size embedded different lengths furnish data which suggest the values of bond resistance over a very short length of embedment, or indicate values of bond resistance which are independent of the length of embedment. Tests with bars of different size which were embedded a distance proportional to their diameters give the true relation when the effect of size of bar is eliminated. Two series of tests of this kind on plain round bars of ordinary mill surface gave almost identical values for bond resistance after eliminating the effect of length of embedment and size of bar, and we may consider that these values represent the stresses which were developed in turn over each unit of area of the embedded bar as it was withdrawn by a load applied by the method used in these tests. These tests showed that for concrete of the kind used (a 1 : 2 : 4 mix, stored in damp sand and tested at the age of about 60 days) the first measurable slip of bar came at a bond stress of about 260 lb. per sq. in., and that the maximum bond resistance reached an average value of 440 lb. per sq. in. If we conclude that adhesive resistance was overcome at the first measurable slip, it will be seen that the adhesive resistance was about 60% of the maximum bond resistance. This ratio did not vary much for a wide range of mixes, ages, size of bar, condition of storage, etc.

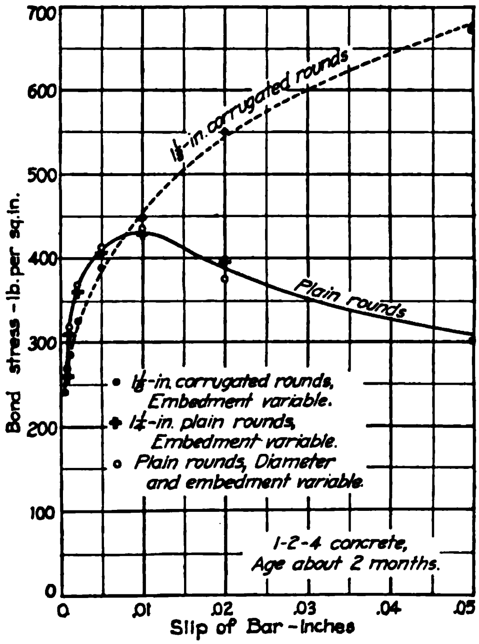


Fig. 3.—Relation of bond to slip of bar as load increases.

Sliding resistance reached its maximum value for plain bars of ordinary mill surface at a slip of about 0.01 in. The constancy in the amount of slip corresponding to the maximum bond resistance for a wide range of mixes, ages, sizes of bar, conditions of storages, etc., is a noteworthy feature of the tests. With further slip the sliding resistance decreased slowly at first, then more rapidly, until with a slip of 0.1 in. the bond resistance was about one-half its maximum value.

Bond Resistance in Terms of Compressive Strength of Concrete.—Pull-out tests with plain round bars show end slip to begin at an average bond stress equal to about one-sixth the compressive strength of 6-in. cubes from the same concrete; the maximum bond resistance is equal to about one-fourth the compressive strength of 6-in. cubes. These values were about the same for a wide range of mixes, ages, and conditions of storage. In terms of the compressive strength of 8 by 16-in. concrete cylinders these values would be about 13% for first end slip and 19% for the maximum bond resistance.

Distribution of Bond Stress Along a Bar.—The tests indicate that bond stress is not uniformly distributed along a bar embedded any considerable length and having the load applied at one end. Slip of bar begins first at the point where the bar enters the concrete, and the bond stress must be greater here than elsewhere until a sufficient slip has occurred to develop the maximum bond resistance at this point. Slip of bar begins last at the free end of the bar. After slip becomes general, there is an approximate equality of bond stress throughout the embedded length.

Variation of Bond Resistance with Size, Shape, and Condition of Surface of Bar.—The maximum bond resistance was not materially different for bars of different diameters.

Rusted bars gave bond resistances about 15% higher than similar bars with ordinary mill surface.

The tests with flat bars showed wide variations of bond resistance and were not conclusive. Square bars gave values of unit stress about 75% of those obtained with plain round bars.

T-bars gave lower unit bond resistance than plain round bars, but gave about double the bond resistance per unit of length that was found for the plain round bars of the same sectional area.

With polished bars the bond resistance is due almost entirely to adhesion between the concrete and steel. Numerous tests with polished bars embedded in 1: 2: 4 concrete and tested at 60 days indicated a maximum bond resistance of about 160 lb. per sq. in., or about 60% of the bond resistance of bars of ordinary surface at small amounts of slip.

Adhesive resistance must be destroyed, sliding resistance largely overcome, and the concrete ahead of the projections must undergo an appreciable compressive deformation before the projections on a deformed bar become effective in taking bond stress. The tests indicate that the projections do not materially assist in resisting a force tending to withdraw the bar until a slip has occurred approximating that corresponding to the maximum sliding resistance of plain bars. As slip continues a larger and larger portion of the bond stress is taken by direct bearing of the projections on the concrete ahead.

In determining the comparative merits of deformed bars, the bar which longest resists beginning of slip should be rated highest, other considerations being equal.

The concrete cylinders of the pull-out specimens with deformed bars were reinforced against bursting or splitting, because it was desired to study the load-slip relation through a wide range of values. In only a few tests was the maximum bond resistance reached at an end slip less than 0.1 in. It should be recognized that, in general, the bond stresses reported for deformed bars at end slip of 0.05 and 0.1 in. could not have been developed with bars embedded in unreinforced blocks. These high values of bond resistance must not be considered as available under the usual conditions of bond action in reinforced-concrete members. In the tests in which the blocks were not reinforced, evidence of splitting of the blocks was found at end slips of 0.02 to 0.05 in.

The normal components of the bearing stresses developed by the projections on a deformed bar may produce very destructive bursting stresses in the surrounding concrete. The bearing stress between the projections and the concrete in the tests with certain types of commercial deformed bars was computed to be from 5800 to 14,000 lb. per sq. in. at the highest bond stresses considered in these tests. The large slip and the high bearing stresses developed in the later stages of the tests show the absurdity of seriously considering the extremely high values that are usually reported to be the true bond resistance of many types of deformed bars.

Round bars with standard V-shaped threads gave much higher bond resistance at low slips than the commercial deformed bars. The average bond resistance at an end slip of 0.001 in. was 612 lb. per sq. in. The maximum bond resistance was 745 lb. per sq. in. These were the only deformed bar tests in which failure came by shearing the surrounding concrete.

The 1-in. twisted square bars gave a bond resistance per unit of surface at an end slip of 0.001 in., only 88% of that for the plain rounds. Following an end slip of about 0.01 in., these bars showed a decided decrease in bond resistance, and a slip of 5 to 10 times this amount was required to cause the bond resistance to regain its first maximum value. After this, the bond resistance gradually rose as the bar was withdrawn. Some of the bars were withdrawn 2 or 3 in. before the highest resistance was reached. The apparent bond stresses at these slips were very high; but, of course, such stresses and slips could not be developed in a structure and could not have been developed in the tests had the blocks not been reinforced against bursting. Such values are entirely meaningless under any rational interpretation of the tests.

Anchoring of Reinforcing Bars.—The tests with plain round bars anchored by means of nuts and with washers, only showed that the entire bar must slip an appreciable amount before these forms of anchorage come into action. Anchorages of the dimensions used in these tests did not become effective until the bar had slipped an amount corresponding to the maximum bond resistance of plain bars. With further movement the apparent bond resistance was high, but was accompanied by excessive bearing stresses on the concrete.

The load-slip relation for bars anchored by means of hooks and bends was not determined.[1]

Influence of Method of Curing Concrete.—Tests on specimens stored under different conditions indicate that concrete stored in damp sand may be expected to give about the same bond resistance and compressive resistance as that stored in water. Water-stored specimens gave values of maximum bond resistance higher in each instance than the air-stored specimens; the increase for water storage ranged from 10 to 45%. The difference seemed to increase with age. The presence of water not only did not injure the bond for ages up to 3 years, but it was an important factor in producing conditions which resulted in high bond resistances. However, it was found that specimens tested with the concrete in a saturated condition gave lower values for bond than those which had been allowed to dry out before testing. The bars in specimens which had been immersed in water as long as 3½ years showed no signs of rust or other deterioration.

Influence of Freezing of Concrete.—Specimens made outdoors in freezing weather, where they probably froze and thawed several times during the period of setting and hardening, were almost devoid of bond strength.

Influence of Age and Mix of Concrete.—Pull-out tests made at early ages gave surprisingly high values of bond resistance. Plain bars embedded in 1 : 2 : 4 concrete and tested at 2 days did not show end slip of bar until a bond stress of 75 lb. per sq. in. was developed. Bond resistance increases most rapidly with age during the first month. The richer mixes show a more rapid increase than the leaner ones. The tests on concrete at ages of over 1 year showed that the bond resistance of specimens stored in a damp place may be expected ultimately to reach a value as much as twice that developed at 60 days.

The load-slip relation of leaner and richer mixes was similar to that for 1: 2: 4 concrete. For a wide range of mixes the bond resistance was nearly proportional to the amount of cement used. This relation did not obtain in a mix from which the coarse aggregate had been omitted.

Effect of Continued and Repeated Load.—When the application of load was continued over a considerable period of time or when the load was released and reapplied, the usual relation of slip of bar to bond resistance was considerably modified. The few tests which were made indicate that the bond stress corresponding to beginning of slip is the highest stress which can be maintained permanently or be reapplied indefinitely without failure of bond.

[1] Other tests have shown that a semicircular hook of 4 times the diameter of the bar and well embedded in concrete may be assumed to develop the elastic limit of the steel without exceeding the bearing strength of the concrete. The curved ends should consist of bends through 180 deg. with a short length of straight rod beyond the bend. A short cross rod aids greatly in distributing the bearing stress in the concrete. Short ...re hooks upon the ends of bars are not of great value.

Effect of Concrete Setting Under Pressure.—Bond resistance of plain bars is greatly increased if the concrete is caused to set under pressure. With a pressure of 100 lb. per sq. in. on the fresh concrete for 5 days after molding, the maximum bond resistance was increased 92% over that of similar bars in concrete which had set without pressure. The greater density of the concrete and its more intimate contact with the bar seems to be responsible for the increased bond resistance. Light pressures gave an appreciable increase in bond resistance. With polished bars the effect of pressure was slight.

As might have been expected, the compressive resistance of concrete setting under pressure was increased in much the same ratio as the bond resistance. At the age of 80 days the initial modulus of elasticity in compression for concrete which set under a pressure of 100 lb. per sq. in. was about 37% higher and the compressive strength was increased by about 73% over that of concrete which had set without pressure. The density of the concrete, as determined by the unit weights, was increased about 4% by a pressure of 100 lb. per sq. in. on the fresh concrete. The increase in strength and density was relatively greater for the low than for the high pressures. A pressure continued for 1 day, or until the concrete had taken its final set and hardening had begun, seems to have produced the same effect in increasing the strength and elastic properties of the concrete as when the pressure was continued for a much longer period.

Beam Tests

The mean computed values for bond stresses in the 6-ft. beams in the series of 1911 and 1912 were as given below. All beams were of 1 : 2 : 4 concrete, tested at 2 to 8 months by loads applied at the one-third points of the span. Stresses are given in pounds per square inch.

	Number of tests	First end slip of bar	End slip of 0.001 in.	Maximum bond stress
1 and 1¼-in. plain round...	28	245	340	375
¾-in. plain round..........	3	186	242	274
⅝-in. plain round..........	3	172	235	255
1-in. plain square..........	6	190	248	278
1-in. twisted square........	3	222	289	337
1⅛-in. corrugated round....	9	251	360	488

In the beams reinforced with plain bars end slip begins at 67% of the maximum bond resistance; for the corrugated rounds this ratio is 51%, and for the twisted squares, 66%.

The bond unit resistance in beams reinforced with plain square bars, computed on the superficial area of the bar, was about 75% of that for similar beams reinforced with plain round bars of similar size.

Beams reinforced with twisted square bars gave values at small slips about 85% of those found for plain rounds. At the maximum load, the bond-unit stress with the twisted bars was 90% of that with plain round bars of similar size.

In the beams reinforced with 1⅛-in. corrugated rounds, slip of the end of the bar was observed at about the same bond stress as in the plain bars of comparable size. At an end slip of 0.001 in., the corrugated bars gave a bond resistance about 6% higher and at the maximum load, about 30% higher than the plain rounds.

The beams in which the longitudinal reinforcement consisted of three or four bars smaller than those used in most of the tests gave bond stresses which, according to the usual method of computation, were about 70% of the stresses obtained in the beams reinforced with a single bar of large size. It seems probable that the lower computed bond stresses in these tests are due to errors in the assumptions made as to the distribution of bond stress and not to actual differences of bond resistance in the bars of different size.

The tests on beams with the loads placed in different positions with respect to the span gave little variation in bond resistance during the early stages of the tests. The maximum bond resistances increased rapidly as the load approached the supports. These tests indicate that the variation in the maximum bond stresses must be due to the presence of other than normal beam action.

The bond stresses developed in the beam tests indicate that with beams of the same cross-section the bond stresses are distributed in the same way during the early stages of the test in beams varying widely in span length and loading. During the later stages of the test, the distribution of bond stress seems to depend largely upon the conditions of stress in the concrete through the region of the span where beam bond stresses are high. The distribution of bond stresses in beams of different cross-section apparently varies with the relative dimensions of the beam and the reinforcing bars.

In the reinforced-concrete beams it was found that very small amounts of slip at the ends of the bar represented critical conditions of bond stress. For beams failing in bond the load at an end slip of 0.001 in. was 89 to 94% of the maximum load found in beams reinforced with plain bars, and 79% of the maximum load for similar beams reinforced with corrugated bars. As soon as slip of bar became general, other conditions were introduced which soon caused the failure of the beam.

The bond stresses developed in a reinforced-concrete beam by a load applied as in these tests varies widely over the region in which beam bond stresses are present. High bond stresses are developed just outside the load points at comparatively low loads. The load which first developed a bond stress nearly equal to the maximum bond resistance in the region of beam bond stresses produced a stress near the support which was not more than about 15 to 40% of the maximum bond resistance. As the load is increased, the region of high bond stress is thrown nearer and nearer the support, and at the same time the bond stress over the region just outside the load point becomes steadily smaller. This indicates a piecemeal development of the maximum bond stress as the load is increased. The actual bond stresses in certain tests varied from less than one-half to more than twice the average bond resistance computed in the usual manner.

Slip of bar in a reinforced-concrete beam has a marked influence in increasing the center deflection during the later stages of loading.

The comparison of the bond stresses developed in beams and in pull-out specimens from the same materials is of interest. Such a comparison should be made for similar amounts of slip. In the pull-out tests the maximum bond resistance came at a slip of about 0.01 in. for plain bars. The mean bond resistance for the deformed bars tested was not materiallv different from that of the plain bars until a slip of about 0.01 in. was developed; with a continuation of slip the projections came into action and with much larger slip high bond stresses were developed The beam tests showed that about 79 to 94 % of the maximum bond resistance was being developed when the bar had slipped 0.001 in. at the free end; hence the bond stress developed at an end slip of 0.001 in. was used as a basis of the principal comparisons in the pull-out tests. However, it is recognized that, under certain conditions, the stresses developed at larger amounts of slip may have an important bearing on the effective bond resistance of the bar.

The pull-out tests and beam tests gave nearly identical bond stresses for similar amounts of slip in many groups of tests, but it seems that this was the result of a certain accidental combination of dimensions in the two forms of specimens and did not indicate that the computed stresses in the beams were the correct stresses. However, it is believed that a properly designed pull-out test does give the correct value of bond resistance, and gives values which probably closely represent the bond stresses which actually exist in a beam or other member as slipping is produced from point to point along the bar. The relative position of the bar during molding may be expected to influence the values of bond resistance found in the tests.

A working bond stress equal to 4 % of the compressive strength of the concrete tested in the form of 8 by 16-in. cylinders at the age of 28 days (equivalent to 80 lb. per sq. in. in concrete having a compressive strength of 2000 lb. per sq. in.) is as high a stress as should be used. This stress is equivalent to about one-third that causing first slip of bar and one-fifth of the maximum bond resistance of plain round bars as determined from pull-out tests. The use of deformed bars of proper design may be expected to guard against local deficiencies in bond resistance due to poor workmanship and their presence may properly be considered as an additional safeguard against ultimate failure by bond. However, it does not seem wise to place the working bond stress for deformed bars higher than that used for plain bars.

3. Length of Embedment of Reinforcing Bars to Provide for Bond.—Let f_s be the working tensile strength of the steel, A_s the area of bar, o the circumference of bar, d the diameter or thickness of bar, u the working unit bond strength, and x the required length of embedment (or grip) for the above values of f_s and u. Then, to develop the strength of the steel, using either round or square bars,

$$xou = A_s f_s = \frac{\pi d^2}{4} \cdot f_s$$

or

$$x = \frac{f_s d}{4u}$$

4. Ratio of the Moduli of Elasticity.—Let f_s = unit stress in steel, f_c = unit stress in concrete, E_s = modulus of elasticity of steel, and E_c = modulus of elasticity of concrete. Since the modulus of elasticity of a material is the ratio of stress to deformation, it follows that, for equal deformations, the stresses in the steel and concrete will be as their moduli of elasticity. Thus,

$$\frac{f_s}{f_c} = \frac{E_s}{E_c}$$

This ratio of the moduli is generally denoted by the letter n, or

$$f_s = nf_c$$

The equation just given shows that if the stress in either the steel or concrete of a concrete column is known, the stress in the other material can be found, and this relation is made use of in the derivation of column formulas. Fig. 26, Sect. 5, page 250, shows that the modulus of elasticity of concrete in compression is less for the greater loads, and hence the value of n is greater. Thus, it is plain that with increasing loads in concrete columns the steel receives a greater proportionate stress, the variation in the amount carried by the steel depending on the variation in the value of n. In order to take account of the fact that under increasing loads the steel receives an increasing proportion, it is desirable to use a value of n in the computations for design somewhat larger than that which is obtained by taking a value of E_c corresponding to working loads on small prisms (about 10). A value of 15 for n may well be used for the ordinary 1 : 2 : 4 mix.

In concrete beams, experiments show that the tension which remains in the concrete just below the neutral axis, and properly not allowed for in the derivation of the beam formulas, has its effect in the position of the neutral axis and the strength of the beam. It is found that a value of 15 for n is not too large for calculations of strength of beams, assuming the ordinary 1 : 2 : 4 mix, although great accuracy in this respect is not necessary. This value of 15 for n is the one most generally used, but a value of 12 is also frequently employed. The value of 15 corresponds to a value of E_c of 2,000,000 which is somewhat low as determined by compressive tests.

For the proper values of n to use for other mixtures see recommendations of the Joint Committee, *Appendix B.*

Comparatively few tests have been made on the elasticity of concrete in tension, but these seem to indicate that for small stresses, it is practically the same as in compression, although probably slightly less.

5. Behavior of Reinforced Concrete Under Tension.—Early tests indicated that the ultimate stretch of reinforced concrete in tension is as much as 10 times that of plain concrete, but such results were due to the fact that it was found extremely difficult to determine just when the concrete begins to crack. Cracks do not become noticeable, even on very close examination, until a stretching occurs corresponding to a tensile stress much beyond the ultimate tensile strength of the concrete. The steel causes a uniform elongating of the concrete so that the cracks which open up are very small and remain invisible for some time.

A method of detecting minute cracks in the tensile side of beams was accidentally discovered in 1901–02 in some experiments made at the University of Wisconsin. It was found that when beams were hardened in water and only partially dried before testing, very fine hair-cracks became noticeable at a moderate load. Before these cracks occurred, however, dark wet lines appeared across the beam, and it was observed that each of these lines was later followed by a very fine crack. These water-marks were proven to be incipient cracks by the sawing out of a strip of concrete along the outer part of the beam. Careful measurements of extension showed that these streaks, or water-marks, occurred at practically the same deformation at which the concrete ruptured when not reinforced. This same phenomenon has since been observed by many careful experimenters, and the fact is now generally established that concrete, reinforced with steel, does not elongate under tensile stress to any greater extent before cracking than plain concrete.

A reinforced-concrete beam for working loads is usually more heavily stressed on the tension side than the ultimate tensile strength of plain concrete—enough steel being usually embedded near the lower face to permit the full allowable compressive strength of the concrete to be utilized. The presence, then, of the cracks above referred to, accurring long before a reinforced-concrete beam has obtained its working load, must seriously affect the tensile strength of the concrete. The moment formulas now in most general use for the design of reinforced-concrete beams neglect entirely the tensile strength of the concrete.

Experiments have shown that concrete when well placed and mixed somewhat wet, completely protects the steel in the tensile side of a beam from corrosion, even when the unit stress in the steel somewhat exceeds the elastic limit.

6. Shrinkage and Temperature Stresses.—In reinforced-concrete structures which are free to contract and expand, the stresses occurring from temperature changes and from shrinkage in hardening are due wholly to the mutual action of the steel and concrete. Of the stresses produced from these two causes, those which result from hardening are the greater, but experiments show that even these are not sufficient to be of practical importance. In regard to the temperature stresses, they are negligible by reason of the nearly equal rates of expansion of the two materials.

On the other hand, if reinforced-concrete structures are restrained by outside forces, or if they are of such dimensions that they cannot be considered as sufficiently well bonded to act as a unit—such as long retaining walls—then the stresses resulting are much greater, and the tensile strength of the concrete will be reached (this will occur with a drop in temperature somewhere between 10 and 20°F.), thus producing cracks, called contraction cracks. To prevent plainly noticeable cracks due to shrinkage and lowering of the temperature, all exposed surfaces should be reinforced with about 0.3 of 1% of steel, based on the cross-section of the concrete. This is less than the amount required theoretically, but experience shows this amount to give very satisfactory results where the foundations are stable. If the structure is fixed in two directions, the reinforcement must be placed accordingly. The above percentage of steel should be figured for an area of cross-section of maximum thickness of about 12 in.

No amount of reinforcement can entirely prevent contraction cracks. The steel can, however, if of small diameter and placed close to the surface, force the cracks to take place at such frequent intervals that the required deformation occurs without any one crack becoming large. No cracks will open up to be plainly noticeable until the steel is stressed beyond its elastic limit. The amount of steel should be such, then, that without being stressed beyond its elastic limit, it will withstand the tensile stress resulting from the maximum fall of temperature (usually considered to be 50°) in the steel itself plus the tensile stress necessary to crack the concrete. A high elastic-limit steel is thus advantageous.

The size and spacing of the cracks will also depend upon the bond strength of the reinforcing rods. The distance between cracks in any given case will be the length required to develop a bond strength equal to the tensile strength of the concrete. Thus, bars with irregular surfaces which provide a mechanical bond with the concrete are in general more effective than smooth bars.

7. Weight of Reinforced Concrete.—Reinforcing steel in the usual proportions adds from 3 to 5 lb. to the weight of plain concrete per cubic foot. The weight of plain concrete for the various kinds of aggregate may be found on page 263. Reinforced concrete is usually assumed as 150 lb. per cu. ft. in making computations for design.

SECTION 7

BEAMS AND SLABS

RECTANGULAR BEAMS AND SLABS

1. Forces to be Resisted.—As expressed by the Joint Committee the forces to be resisted are those due to:

1. *The dead load*, which includes the weight of the structure and fixed loads and forces.

2. *The live load*, or the loads and forces which are variable. The dynamic effect of the live load will often require consideration. Allowance for the latter is preferably made by a proportionate increase in either the live load or the live-load stresses. The working stresses recommended (see *Appendix B*) are intended to apply to the equivalent static stresses thus determined.

2. Distribution of Stress in Homogeneous Beams.—The following statements and formulas are in accordance with the theory of homogeneous beams:

1. At any cross-section the internal forces, or stresses, may be resolved into normal and tangential components. The components normal to the section are stresses of tension and compression, while the tangential components add together and form a stress known as the resisting shear.

2. The shear at any cross-section is borne by the tangential stresses in that section. The moment at any section is borne by the component stresses normal to that section.

3. The neutral axis passes through the center of gravity of the cross-section.

4. The intensity of stress normal to the section increases directly with the distance from the neutral axis and is a maximum at the extreme fiber (Fig. 1). The intensity of this stress at any given point in the cross-section is given by the formula

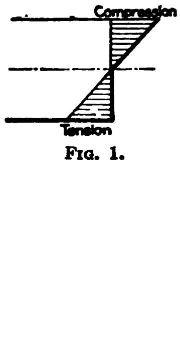


FIG. 1.

$$f = \frac{My}{I}$$

in which f = fiber stress at distance y from neutral axis.

M = external bending moment at section in inch-pounds.

y = distance in inches from neutral axis to any fiber.

I = moment of inertia of the cross-section about the neutral axis.

5. The general formula which gives the longitudinal shear per square inch (v) at any desired point in the cross-section is

$$v = \frac{VQ}{Ib'}$$

in which V = total shear at the section in pounds.

Q = statical moment about the neutral axis of that portion of the cross-section lying either above or below (depending upon whether the point in question is above or below the neutral axis) an axis drawn through the point in question parallel to the neutral axis.

I = moment of inertia of the cross-section about the neutral axis.

b' = width of beam at the given point.

In the above formula, by the term *statical moment* is meant the product of the area mentioned by the distance between its center of gravity and the neutral axis. For example, the longitudinal shearing intensity at a point c in a rectangular beam, Fig. 2, may be expressed as follows:

$$v = \frac{VA'r}{Ib}$$

For rectangular beams and all beams of uniform width, the largest value of v for any given section will occur at the neutral axis since the statical moment Q has its maximum value for a point on this axis, and b is constant.

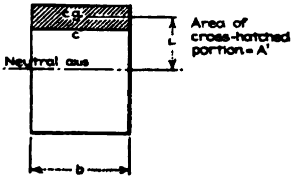


Fig. 2.

6. If a beam is of constant cross-section throughout, the maximum values of f and v will occur at the section where M and V respectively have maximum values.

7. In addition to the longitudinal or horizontal shear at any point there coexists a vertical shear and the intensity of this vertical shear is equal to the intensity of the horizontal shear.

8. The intensity of the shear at the top and bottom of a beam is zero and the intensity of shear (horizontal and vertical) along a vertical cross-section for a rectangular beam varies as the ordinates to a parabola, as shown graphically in Fig. 3. The maximum value occurs at the neutral axis and is $\frac{3}{2}$ the average intensity, or $\frac{3}{2} \cdot \frac{V}{bd}$.

9. At the neutral plane there exists a tension and compression at angles of 45 deg. to the horizontal, and the intensity of these forces is equal to that of the shear.

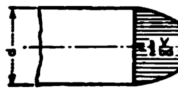


Fig. 3.

10. At the end of a simply supported beam where the shear is a maximum and the bending moment a minimum, the stresses lie practically at 45 deg. to the horizontal throughout the entire depth of beam.

11. At the section of maximum moment, the shear is zero and the stresses are horizontal.

12. If f represents the intensity of horizontal fiber stress and v the intensity of vertical or horizontal shearing stress at any point in a beam, the intensity of the inclined stress will be given by the formula

$$t = \frac{1}{2}f \pm \sqrt{\frac{1}{4}f^2 + v^2}$$

and the direction of this stress by the formula

$$\tan 2K = \frac{2v}{f}$$

where K is the angle of the stress with the horizontal.

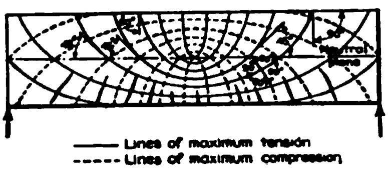


Fig. 4.

13. At any given point maximum compressive stress and maximum tensile stress make an angle of 90 deg. with each other.

14. The directions of the maximum stresses for a simply supported beam uniformly loaded are as given in Fig. 4. The general direction of the stresses in a beam with any given loading may be determined by means of the formulas for t and K given above.

15. The common theory of flexure gives the unit stress correctly at the important section of maximum moment and also for the extreme fibers in other sections, since at these points the shear is zero. Where the shear is not zero an inclined stress is the result and the flexure formula gives only the horizontal component of this stress—namely, the fiber stress.

3. Assumptions in Theory of Flexure for Homogeneous Beams.—The two main assumptions in the common theory of flexure are:

1. If, when a beam is not loaded, a plane cross-section be made, this cross-section will still be a plane after the load is put on and bending takes place (Navier's hypothesis).

2. The stress is proportional to the deformation—namely, to the amount of elongation or compression per unit of length (Hooke's Law).

From the first assumption it follows that the unit deformations of the fibers at any section of a beam are proportional to their distance from the neutral axis. By means of the second assumption the important principle is established that the unit stresses in the fibers are also proportional to the distances of the fibers from the neutral axis.

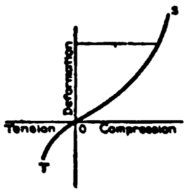


Fig. 5.

4. Plain Concrete Beams.—The first assumption in the common theory of flexure, as given in the preceding article, may be applied directly to plain concrete and also to reinforced-concrete beams. Careful measurements seem to show some deviation from a plane, but in general this assumption seems to be warranted. From this fact it follows (as stated above) that deformations of the fibers are proportional to the distances of the fibers from the neutral axis. *OS* in Fig. 5 is the stress-deformation diagram for concrete in compression with the deformations represented vertically. The curve *OT* is the stress-deformation diagram for concrete in tension. For working loads the curves *OS* and *OT* do not vary materially from straight lines and the unit stresses in the fibers at any section of a plain concrete beam may thus be assumed to vary directly as the deformations and consequently as the distances of the fibers from the neutral axis. Hence, the common flexure formula for homogeneous beams applies when the loads are working loads. For ultimate loads, however, ths formula does not strictly apply.

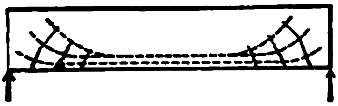


Fig. 6.

A plain concrete beam will fail by cracks opening up along the uneven lines which are shown in Fig. 4 on account of the low strength of concrete in tension. If concrete were only stronger in tension, then the plain concrete beam might be of some structural value. In order to offset this disadvantage of plain concrete, steel is used.

5. Purpose and Location of Steel Reinforcement.—Steel reinforcement should have the general directions shown in Fig. 6 in order to take the tension in the beam and prevent the cracks starting along the lines indicated. Fig. 7 is the simplest method of reinforcement and quite often used for light loads. In beams highly stressed, curved or inclined reinforcement is needed, in addition to the horizontal rods. The most common method is to use several bars for the horizontal reinforcement and then to bend up some of these at an angle of from 30 to 45 deg. as they approach the end of the beam and where they are not needed to resist bending stresses. The concrete is depended upon to take care of the compressive and pure shearing stresses, its resistance to such stresses being large.

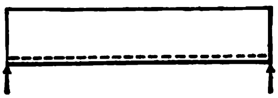


Fig. 7.

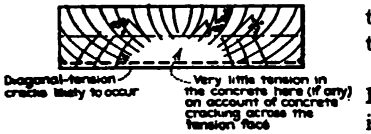


Fig. 8.

6. Tensile Stress Lines in Reinforced-concrete Beams.—Lines of maximum tension in the concrete of reinforced-concrete beams are considerably inclined immediately above the line of the steel. The inclination of these lines is greater, the greater the shear, and the less the horizontal tension. The inclination, therefore, increases toward the end of the beam. At points nearer the neutral plane, the horizontal tensile stresses become less and the inclined tension approaches the value of the shearing stress, while its inclination approaches 45 deg. Fig. 8 is an attempt to represent roughly the general direction of the inclined tensile stresses in a simply supported beam uniformly loaded and with horizontal reinforcement.

7. Flexure Formulas for Reinforced-concrete Beams.—A great many varieties of flexure formulas have been proposed from time to time to be used in the design of reinforced-concrete beams. As might be expected, many of the earlier formulas considered the concrete to carry its share of the tension which we know now cannot be done with safety. Only two classes of flexure formulas are at the present time in practical use. In each of these classes, tension in the concrete is neglected and a plane section before bending, is assumed to be a plane after bending takes place.

The formulas almost universally used and made standard by the Joint Committee relate to working stresses and safe loads, and are based on the straight-line theory of stress distribution. The other formulas referred to above relate to ultimate strength and ultimate loads and the stress-deformation curve for concrete in compression is assumed to be a full parabola. Ultimate-load formulas are used to such a limited extent that only a few pages of this handbook are devoted to their consideration—namely, Arts. 10 and 11.

8. Assumptions in Flexure Calculations.—The following assumptions are made in deriving the flexure formulas: (1) the adhesion of concrete to steel is perfect within the elastic limit of the steel; (2) no initial stresses are considered in either the concrete or the steel due to contraction or expansion; (3) the applied forces are parallel to each other and perpendicular to the neutral surface of the beam before bending; (4) sectional planes before bending remain plane surfaces after bending within the elastic limit of the steel; (5) no tension exists in the concrete; (6) modulus of elasticity of concrete is constant.

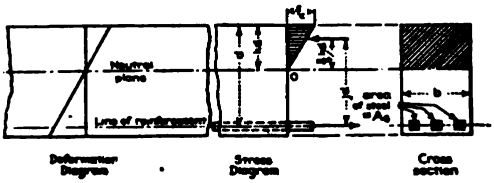


FIG. 9.

9. Flexure Formulas for Working Loads—Straight-line Theory.—The unit stress in the steel is within the elastic limit, and the unit stresses in the concrete at the given section of the beam are considered to vary as the ordinates to a straight line (see Fig. 9). Tension in the concrete is neglected. The formulas follow[1] (see *Notation, Appendix D*):

[1] The formulas may be derived as follows:

Total compressive resistance = total tensile resistance, or

$$\tfrac{1}{2}f_c kbd = A_s f_s \qquad (a)$$

From the assumption that deformations vary as the distances of the fibers from the natural axis and assuming stress proportional to deformation

$$\frac{f_c}{E_c kd} = \frac{f_s}{E_s d(1-k)}$$

which reduces to

$$f_s = f_c n\frac{1-k}{k}, \text{ or } f_c = \frac{f_s k}{n(1-k)}, \text{ or } k = \frac{1}{1+\dfrac{f_s}{nf_c}} \qquad (b)$$

The total resisting moment of the beam is the sum of the moments of the total compressive stresses and of the total tensile stresses about the neutral axis, or

$$\begin{aligned} M &= \tfrac{2}{3}kd(\tfrac{1}{2}f_c kbd) + d(1-k)A_s f_s \\ &= \tfrac{1}{3}f_c k^2bd^2 + A_s f_s d(1-k) \qquad (c) \end{aligned}$$

Eliminating k between equations (a) and (b), the following formula for steel ratio results

$$p = \frac{\tfrac{1}{2}}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c}+1\right)}$$

Introducing the value of f_s from equation (b) into equation (a), we have

$$\tfrac{1}{2}k^2bd - A_s n(1-k) = 0$$

or

$$\tfrac{1}{2}k^2b - pbn(1-k) = 0$$

$$k = \sqrt{2pn + (pn)^2} - pn = \frac{1}{1 + \frac{f_s}{nf_c}} \tag{1}$$

$$j = 1 - \tfrac{1}{3} k \tag{2}$$

$$p = \frac{A_s}{bd} = \frac{\tfrac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = \frac{f_c k}{2f_s} \tag{3}$$

$$M_c = \tfrac{1}{2} f_c kj(bd^2), \quad \text{or} \quad bd^2 = \frac{2M}{f_c kj}, \quad \text{or} \quad f_c = \frac{2M}{kjbd^2} \tag{4}$$

$$M_s = pf_s j(bd^2), \quad \text{or} \quad bd^2 = \frac{M}{pf_s j}, \quad \text{or} \quad f_s = \frac{M}{A_s jd} \tag{5}$$

$$f_c = \frac{2f_s p}{k} \quad \text{or} \quad \frac{f_s k}{n(1 - k)} \tag{6}$$

The above formulas show that for a given ratio of $\frac{f_s}{f_c}$, p and k remain the same for all sizes of beams.. The formula for M_c gives the resisting moment when the maximum allowable value of f_c is introduced as the limiting factor and the formula for M_s gives the resisting moment when the maximum allowable value of f_s is the limiting factor. The lesser of these two resisting moments, when proper working values are assigned to f_c and f_s, is the safe resisting moment of the beam in question.

Unlike steel beams, reinforced-concrete beams require a preliminary formula to be solved before the formula for resisting moment may be employed. Solving this preliminary formula locates the position of the neutral axis which is in the same position only for beams of a given percentage of steel reinforcement.

The method of procedure in flexure formulas is to determine the vertical section of the beam where the moment is a maximum and apply the formulas at that section. Either formula for p, containing the values of f_c and f_s, determines the amount of steel reinforcement which is needed to cause the beam to be of equal strength in tension and compression. The formulas for resisting moment determine the bending moment which a beam will safely withstand (for an existing structure) or the size of the beam needed to resist a given bending moment (for a proposed structure).

If a beam is over-reinforced, its resisting moment depends on M_c, and if under-reinforced on M_s.

If it is desired to find the fiber stresses in concrete and steel of a given beam, the formulas $f_s = \frac{M}{A_s jd}$ and $f_c = \frac{2M}{kjbd^2}$ $\left(\text{or } f_c = \frac{2f_s p}{k}\right)$ should be used, where M is the external bending moment in each case. For a given external M, either $bd^2 = \frac{2M}{f_c kj}$ or $bd^2 = \frac{M}{pf_s j}$ may be used to de-

---

from which

$$k = \sqrt{2pn + (pn)^2} - pn$$

Substituting the value of $A_s f_s$ from (a) into (c), we get

$$M_c = \tfrac{1}{2} f_c k(1 - \tfrac{1}{3} k)bd^2$$

or

$$M_c = \tfrac{1}{2} f_c kjbd^2$$

Substituting the value of f_c from (a) into (c), and remembering that $A_s = pbd$,

$$M_s = pf_s jbd^2$$

Equation (a) may be solved to give

$$f_c = \frac{2f_s p}{k}, \text{ or } p = \frac{f_c k}{2f_s}$$

7. Flexure Formulas for Reinforced-concrete Beams.—A great many varieties of flexure formulas have been proposed from time to time to be used in the design of reinforced-concrete beams. As might be expected, many of the earlier formulas considered the concrete to carry its share of the tension which we know now cannot be done with safety. Only two classes of flexure formulas are at the present time in practical use. In each of these classes, tension in the concrete is neglected and a plane section before bending, is assumed to be a plane after bending takes place.

The formulas almost universally used and made standard by the Joint Committee relate to working stresses and safe loads, and are based on the straight-line theory of stress distribution. The other formulas referred to above relate to ultimate strength and ultimate loads and the stress-deformation curve for concrete in compression is assumed to be a full parabola. Ultimate-load formulas are used to such a limited extent that only a few pages of this handbook are devoted to their consideration—namely, Arts. 10 and 11.

8. Assumptions in Flexure Calculations.—The following assumptions are made in deriving the flexure formulas: (1) the adhesion of concrete to steel is perfect within the elastic limit of the steel; (2) no initial stresses are considered in either the concrete or the steel due to contraction or expansion; (3) the applied forces are parallel to each other and perpendicular to the neutral surface of the beam before bending; (4) sectional planes before bending remain plane surfaces after bending within the elastic limit of the steel; (5) no tension exists in the concrete; (6) modulus of elasticity of concrete is constant.

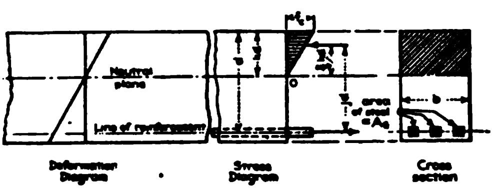


FIG. 9.

9. Flexure Formulas for Working Loads—Straight-line Theory.—The unit stress in the steel is within the elastic limit, and the unit stresses in the concrete at the given section of the beam are considered to vary as the ordinates to a straight line (see Fig. 9). Tension in the concrete is neglected. The formulas follow[1] (see *Notation, Appendix D*):

[1] The formulas may be derived as follows:
Total compressive resistance = total tensile resistance, or

$$\tfrac{1}{2}f_c kbd = A_s f_s \qquad (a$$

From the assumption that deformations vary as the distances of the fibers from the natural axis and assumi stress proportional to deformation

$$\frac{f_c}{E_c kd} = \frac{f_s}{E_s d(1-k)}$$

which reduces to

$$f_s = f_c n\frac{1-k}{k}, \text{ or } f_c = \frac{f_s k}{n(1-k)}, \text{ or } k = \frac{1}{1+\dfrac{f_s}{nf_c}}$$

The total resisting moment of the beam is the sum of the moments of the total compressive stresses an the total tensile stresses about the neutral axis, or

$$\begin{aligned} M &= \tfrac{2}{3}kd(\tfrac{1}{2}f_c kbd) + d(1-k)A_s f_s \\ &= \tfrac{1}{3}f_c k^2 bd^2 + A_s f_s d(1-k) \end{aligned}$$

Eliminating k between equations (a) and (b), the following formula for steel ratio results

$$p = \frac{\tfrac{1}{2}}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c}+1\right)}$$

Introducing the value of f_s from equation (b) into equation (a), we have

$$\tfrac{1}{2}k^2 bd - A_s n(1-k) = 0$$

or

$$\tfrac{1}{2}k^2 b - pbn(1-k) = 0$$

[illegible]

[illegible] formulas show [illegible]

[illegible] maximum [illegible]

[illegible] moments [illegible] working [illegible]

[illegible] beam in question

[illegible] steel beams [illegible] reinforced-concrete [illegible]

[illegible] formula for resisting moment [illegible]

[illegible] position of the neutral axis [illegible]

[illegible] steel reinforcement.

[illegible] procedure in [illegible] formulas is to determine [illegible]

[illegible] moment is a maximum and [illegible]

[illegible] moment determine the bending moment [illegible]

[illegible] size of the beam needed to resist [illegible]

[illegible] resisting moment [illegible]

[illegible] $V = $ [illegible]

[illegible] agram [illegible]

[illegible] principle [illegible] is equal to [illegible] ce. Thus,

[illegible] (1

[illegible] unless in each [illegible]

termine cross-section, when the p used is obtained from the formula $p = \dfrac{\frac{1}{2}}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c} + 1\right)}$ or from $p = \dfrac{f_c k}{2f_s}$, in which $k = \dfrac{1}{1 + \dfrac{f_s}{nf_c}}$.

Illustrative Problem.—What will be the resisting moment (M) for a beam whose breadth (b) is 8 in. with a distance from the center of the reinforcement to the compression surface (d) of 12 in., the area of steel section being 0.96 sq. in.? Assume $n = 15$; $f_c = 650$ lb. per sq. in.; and $f_s = 16{,}000$ lb. per sq. in.

$$p = \frac{A_s}{bd} = \frac{0.96}{(8)(12)} = 0.01$$

From (1)

$$k = \sqrt{(2)(0.01)(15) + (0.01)^2(15)^2} - (0.01)(15) = 0.418$$
$$j = 0.861$$

From (4)

$$M_c = \tfrac{1}{2}(650)(0.418)(0.861)(8)(12)^2 = 134{,}700 \text{ in.-lb.}$$

From (5)

$$M_s = (0.01)(16{,}000)(0.861)(8)(12)^2 = 158{,}700 \text{ in.-lb.}$$

M_c is the lesser of the two resisting moments and hence controls in the design.

Illustrative Problem.—Assume the beam of the preceding problem to be 14 in. deep and subjected to a bending moment of 130,000 in.-lb. Compute the maximum unit stresses in the steel and concrete.

$$p = \frac{A_s}{bd} = \frac{0.96}{(8)(14)} = 0.0086$$

From (1)

$$k = \sqrt{(2)(0.0086)(15) + (0.0086)^2(15)^2} - (0.0086)(15) = 0.395$$
$$j = 0.868$$

From (4)

$$130{,}000 = \left(\frac{f_c}{2}\right)(0.395)(0.868)(8)(14)^2$$
$$f_c = 480 \text{ lb. per sq. in.}$$

From (5)

$$130{,}000 = (0.0086)(f_s)(0.868)(8)(14)^2$$
$$f_s = 11{,}100 \text{ lb. per sq. in.}$$

Illustrative Problem.—A beam is to be designed to withstand a bending moment of 300,000 in.-lb. and to have equal strength in tension and compression. A 1 : 2 : 4 concrete will be used with $E_c = 2{,}000{,}000$ and $f_c = 600$ lb. per sq. in. The pull in the steel is to be limited to 14,000 lb. per sq. in. Its modulus of elasticity E_s is 30,000,000.

$$n = \frac{E_s}{E_c} = 15 \qquad \frac{f_s}{f_c} = \frac{70}{3}$$

From (1) and (2)

$$k = \frac{1}{1 + \dfrac{14000}{(15)(600)}} = 0.391 \text{ and } j = 0.870$$

From (3)

$$p = \frac{(600)(0.391)}{(21)(14{,}000)} = 0.0084$$

Either (4) or (5) may now be used in determining b and d since the amount of steel to be employed will cause simultaneous maximum working stresses.

From (5)

$$bd^2 = \frac{300{,}000}{(0.0084)(14{,}000)(0.870)} = 2930$$

Many different values of b and d will satisfy the last equation. If b is taken as 10 in., then

$$d^2 = \frac{2930}{10} = 293, \text{ or } d = 17\tfrac{1}{4} \text{ in.}$$

Finally

$$A_s = (0.0084)(10)(17.25) = 1.45 \text{ sq. in.}$$

If $1\frac{3}{4}$ in. is allowed between the tension surface of the concrete and the center of the steel, the entire depth of the beam should be 19 in.

10. Flexure Formulas for Ultimate Loads.—The stress-deformation curve for concrete in compression is assumed to be a full parabola. Experiments show this to be very nearly the case for ultimate loads. The amount of reinforcement is considered as sufficient to develop the full compressive strength of the concrete without stressing the steel beyond its yield point. Failure under such conditions (Fig. 10) will occur by crushing the concrete.

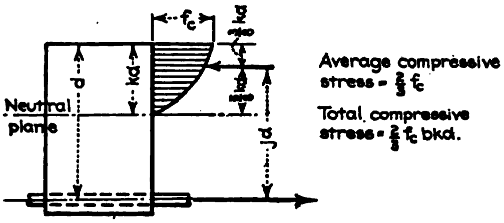


FIG. 10.

The formulas follow (see *Notation, Appendix D*):

$$k = \sqrt{3pn + \left(\frac{3}{2}pn\right)^2} - \frac{3}{2}pn = \frac{1}{1 + \dfrac{f_s}{2nf_c}} \tag{1}$$

$$j = 1 - \tfrac{3}{8}k. \tag{2}$$

$$p = \frac{A_s}{bd} = \frac{\tfrac{2}{3}}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{2nf_c} + 1\right)} = \frac{2}{3} \cdot \frac{f_c k}{f_s} \tag{3}$$

$$M_c = \tfrac{2}{3} f_c kj(bd^2), \quad \text{or} \quad bd^2 = \frac{M}{\tfrac{2}{3} f_c kj} \tag{4}$$

$$M_s = pf_sj(bd^2), \quad \text{or} \quad bd^2 = \frac{M}{pf_sj} \tag{5}$$

$$f_c = \frac{3f_sp}{2k} \tag{6}$$

In the above formulas, f_s = elastic limit of the steel, and f_c = ultimate compressive strength of concrete.

When using the above formulas, it should be remembered that the amount of steel in the beam is assumed as sufficient to cause the ultimate resisting moment to be due to the concrete. Thus, the resisting moment of the beam may be figured by using the formula for M_c. If an amount of steel is used such that the ultimate strength of the concrete and the elastic limit of the steel would be reached simultaneously, either M_c or M_s may be used to determine the ultimate resisting moment. If a less amount of steel is used than the amount just mentioned, the conditions of the assumption do not hold, and the formulas given above cannot be used. When this happens the ultimate moment may be figured by means of formulas based on a parabolic variation of compression in the concrete and applicable for *any* load up to the ultimate. The parabola for such a case is not a full one and the formulas are cumbersome to use and not at all fitted for practical use.

The formulas for ultimate loads, however, can readily be employed to design a beam for equal strength in tension and compression. The method is to first find the required amount of steel. Then either $bd^2 = \dfrac{M}{\tfrac{2}{3} f_ckj}$ or $bd^2 = \dfrac{M}{pf_sj}$ may be used to determine the size of beam necessary.

ILLUSTRATIVE PROBLEM.—A beam is to be designed to have equal strength in tension and compression and to safely withstand a bending moment of 150,000 in.-lb., the ultimate compressive strength of the concrete being taken at 2000 lb. per in. and the elastic limit of the steel at 40,000 lb. per sq. in. Assume $n = 15$.

$$\frac{f_s}{f_c} = 20$$

$$k = \frac{1}{1 + \frac{20}{30}} = 0.598$$

$$j = 1 - \tfrac{3}{8}k = 0.775$$

$$p = \frac{2}{3} \cdot \frac{0.598}{20} = 0.02$$

With a factor of safety of 4, the ultimate bending moment is 600,000 in.-lb. and

$$bd^2 = \frac{600{,}000}{(\tfrac{2}{3})(2000)(0.598)(0.775)} = 972$$

With $b = 8$ in., then

$$d^2 = \frac{972}{8} = 121.5, \text{ or } d = 11 \text{ in.}$$

Also,

$$A_s = (0.02)(8)(11) = 1.76 \text{ sq. in.}$$

11. Flexure Formulas for Working Loads and for Ultimate Loads Compared.—Formulas for ultimate loads are open to the objection that when a factor of safety is applied which will bring the stress in the concrete to about a good working stress, the stress in the steel becomes unduly low from a standpoint of economy. A factor of safety of 3 or 4, as is usually taken, leaves a high stress in the concrete with the stress in the steel far below what is usually considered a safe stress. Beams designed by the ultimate load formulas will generally be of smaller cross-sectional dimensions than when the straight-line formulas are employed; but, on the other hand, a larger amount of steel is required. Practically identical results will be obtained by the two classes of formulas if about 15% lower compressive stress is permitted in the concrete by the ultimate load formulas than by the formulas based on the straight-line theory. There seems to be no good reason why the simple formulas based on the theory of straight-line stress variation should not be used for purposes of design, safe working stresses being employed.

12. Lengths of Simply-supported Beams.—The span length for beams and slabs simply supported should be taken as the distance from center to center of supports, but need not be taken to exceed the clear span plus the depth of beam or slab.

13. Shearing Stresses.—In Fig. 11 is shown a small portion of a concrete beam, so short that no appreciable portion of the load on the beam acts directly upon it. The opposing total compressive forces are denoted by C' and C; and the tension in the steel on each face by T' and T. The tension in the concrete may be neglected. Let V be the total shear on this small portion of the beam. From conditions of equilibrium, $C' = T'$ and $C = T$. The total horizontal shearing stress upon a horizontal section immediately above the steel is $T' - T$, and if b denotes the breadth of the beam and v the unit shear (horizontal or vertical) at any point between the neutral axis and the steel, then

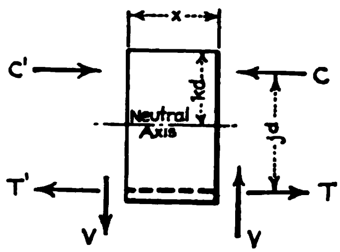


FIG. 11.

$$v = \frac{T' - T}{bx} \tag{1}$$

The various couples acting upon the element produce equilibrium; hence

$$Vx = (T' - T)jd$$

or

$$(T' - T) = \frac{Vx}{jd}$$

Substituting this value in equation (1) there results

$$v = \frac{V}{bjd} \tag{2}$$

which is the value of shear intensity at any point between the neutral axis and the steel.

The value of j for working loads varies within narrow limits and v will change but slightly if the different values of j are inserted in equation (2). The average value of j for beams in ordinary construction is $\frac{7}{8}$. Using this value, equation (2) reduces to

$$v = \frac{8}{7} \cdot \frac{V}{bd} \tag{3}$$

Shearing stress is the same at all points between the neutral axis and the steel, and above the neutral axis it follows the parabolic law. Fig. 12 represents the distribution of shearing stress on a vertical cross-section assuming no tension in the concrete.

The longitudinal tension in the concrete near the end of beam modifies the distribution of the shear, increasing the shearing stress somewhat at the neutral axis and decreasing it at the level of the reinforcement. Equation (3), however, gives results which are sufficiently accurate and are derived for beams having the horizontal bars straight throughout. When any web reinforcement is used, the distribution and the amount of the shearing stresses at the end of a simply supported beam are materially different from the foregoing. The analysis of the stresses becomes more complex and a determination of their value impracticable. Even here, however, the above formula serves a useful purpose. It is found that shear is the chief factor in the failure of a beam by diagonal tension and either formula (2) or formula (3) may be used in design if properly controlled by the results of experiments.

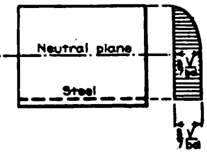


Fig. 12.

Failure by the actual shearing of the concrete in a beam is not a likely occurrence under any conditions as the shearing strength of concrete is at least one-half the crushing strength.

14. Methods of Strengthening Beams Against Failure in Diagonal Tension.—The intensity of the diagonal tensile stress at any point in a beam depends upon the shear and horizontal tension in the concrete, with shear as the chief factor. The percentage of horizontal reinforcement must also be considered, since the amount of steel employed affects the horizontal deformation and consequently the tension in the concrete. Thus beams may be strengthened against failure in diagonal tension by keeping the horizontal tension small through the use of considerable horizontal steel at points of heavy shear, by avoiding heavy shearing stresses, and by providing some type of web reinforcement. A low unit working stress in whatever type of web reinforcement is employed is also much to be preferred.

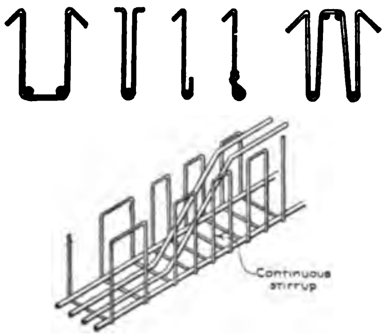


Fig. 13.

The most unfavorable part of a beam as regards diagonal tension is at points of excessive shear combined with considerable bending moment. A sufficient number of reinforcing rods should be extended horizontally to the ends of the beam to provide for bending with low unit stresses in the steel. In small beams, vertical stirrups looped about the horizontal rods may be employed throughout for web reinforcement but in large beams under heavy shearing stresses, both stirrups and bent rods should be used. The stirrups in large beams should be securely fastened to the longitudinal rods in such a way as to prevent slipping of bar past the stirrup. Inclined web members may also be used in place of vertical stirrups if securely attached to the horizontal rods. Vertical stirrups may be made in various forms, as indicated in Fig. 13.

15. Moment and Diagonal-tension Tests—General.[1]—When a beam begins to fail by yielding of the steel at, or near, the section of maximum bending moment, any further load

[1] For detailed treatment of this subject, see "Concrete, Plain and Reinforced" by Taylor and Thompson (1916 Edition).

rapidly increases the deformation, large cracks open up in the concrete on the tension side, the neutral axis rises on this account, and the ultimate failure soon occurs by the crushing of the concrete. A steel tension failure is found to occur when the amount of steel used is less than the amount determined by theoretical formulas which makes the beam of equal strength in tension and compression. This result agrees, then, with what is expected. Likewise it is found that with a larger amount of steel than is theoretically required, the yield point of steel is not reached and the beam fails directly by crushing of the concrete. Beams with no web reinforcement and with the existence of large shearing and moment stresses, fail by inclined cracks opening up in the concrete, thus substantiating to a considerable degree the theoretical deductions regarding the internal stresses in beams.

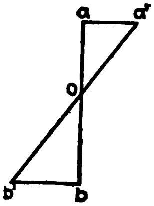


Fig. 14.

The results of breaking tests on reinforced beams with different percentages of steel reinforcement compare well with the results derived from theoretical formulas. Considering the nature of the material, the calculations by the two assumptions of stress variation are found to agree sufficiently with the experimental results to justify their use in problems of design.

Another method of testing reinforced-concrete beams is by the use of extensometers to measure distortions, so that the deformation of the steel and of the extreme fiber of the concrete may be calculated and the neutral axis determined. In making beam tests it is customary to place equal loads at points dividing the length into three equal parts. The advantage of this arrangement lies in the fact that the bending moment is practically uniform between the loads and, if measuring devices are attached, the deformations of the fibers at the top and at the bottom may be easily determined. If in Fig. 14 the deformations be aa' and bb', the neutral axis O is located by connecting a' and b' with a straight line, intersecting ab at O.

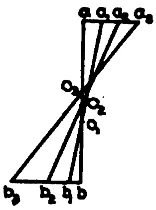


Fig. 15.

In moment calculations, the position of the neutral axis is of prime importance and once this is known, the actual strength may be determined with little uncertainty. The formula for k shows that the position of the neutral axis depends only upon the percentage of steel employed and upon the ratio $\frac{E_s}{E_c}$ or n. The value of E_c is the only value in the formula which is uncertain. It might be well to take the value as determined by the ordinary compression test for use in theoretical formulas, but closer results can be obtained from these formulas if the value of n is taken so that for average conditions the neutral axis is in as nearly as possible the same position theoretically and experimentally. The reason that E_c, as thus determined, will give better results at working loads, is due to the effect of the remaining tension in the concrete below the neutral axis—a stress which is properly not allowed for in the resisting moment.

In making the experiments above described, it was observed that the neutral axis raised as the loading increased, k being approximately $\frac{3}{8}$ at working loads. It was also noted that for the lower loads the neutral axis as determined from the theoretical formulas is more uncertain and generally lower in the beam than for the higher loads. This is undoubtedly due to the relatively large influence of the tensile strength of the concrete in such cases. This rise of the neutral axis as the load increases is shown in Fig. 15. Consider a_1b_1 to be the plane ab after a bending takes place just sufficient to bring the maximum tensile stress in the concrete to its ultimate value. When loads are applied which cause a greater bending moment, the concrete in tension becomes broken by fine cracks, and the steel takes a greater part of the tensile stress. The elongation at b now increases faster than at a, and the neutral axis rises

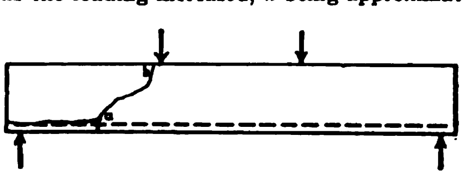


Fig. 16.

rapidly. When working loads are reached, the position of the neutral axis moves but little, and the steel takes all the tension.

Fig. 16 illustrates a typical diagonal tension failure in beams reinforced with only horizontal bars. The initial crack forms at *a* and branches toward *b*. A little later the concrete begins to fail in a horizontal tension crack just above the rods, running from *a* toward the end of the beam. This horizontal crack is brought about by the new conditions which exist after the concrete has become cracked along the diagonal line and the normal diagonal tension has thus ceased to act. Sometimes this horizontal crack does not extend to the end of the beam—the final failure occurring either by the diagonal crack extending to the top of the beam or the horizontal rods pulling out. Thus final failure often occurs from stresses which are developed after initial failure has occurred. However, the initial failure and its cause is what is of importance in design.

Tests show that it is possible to provide sufficient web reinforcement by means of stirrups and bent bars to develop the full strength of the beam whether governed by the crushing strength of the concrete or the elastic limit of the steel. It is found that part of the diagonal tension is taken by the concrete so that web reinforcement need not be designed to take all the diagonal tensile stresses.

Vertical stirrups spaced a distance apart equal to, or greater than, the depth of beam help but little in preventing diagonal cracks between successive stirrups. They may prevent final failure, however, by preventing the extension of a crack horizontally along the reinforcing rods. Stirrups are found by tests to be most effective when spaced a distance apart equal to one-third the depth of beam. To give the best results they should be securely fastened to the longitudinal bars in the tension side of the beam.

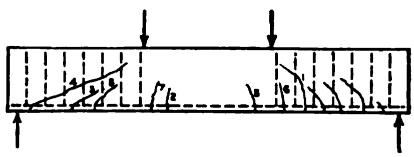


Fig. 17.

Tests in which curved and inclined rods were used, but in which no rods continued straight for the entire length of the beam, showed results very little better than for straight rods.

Vertical stirrups and bent rods combined are found by tests to give the very best results. Tests also seem to indicate that too much reliance should not be placed upon one or two bent rods. For this reason, even if one or two rods are bent up properly to take the diagonal tension, it would be good design to consider this rod, or rods, as not taking any diagonal tensile stress and to provide a thorough web reinforcement by means of stirrups.

Tests show that bent bars may be inclined at any angle between 30 and 45 deg. without the beam showing any marked difference in strength. Beams having sharp bends in the reinforcing bars are found to have less strength than beams with bars having circular bends of a radius about 12 diameters.

Fig. 17 represents the conditions which developed in the test of a beam. The cracks are numbered in the order of their appearance, final failure occurring at crack No. 4 and being due to inadequate web reinforcement. The stirrups were stressed beyond their yield point.

It appears from tests of beams in which bent rods were employed with a good anchorage at their ends, that the anchorage is quite advantageous in increasing web resistance. This form of construction is also found to be an insurance against failure at low loads through defective concrete or insufficient bond.

Hooks at the ends of the horizontal tensile bars prevent slipping of the bars in the concrete and are found to increase the strength of the beam materially.

The results of experiments show that the ultimate compressive strength of concrete in a beam is at least equal to its crushing strength as determined by tests on cubes hardened under similar conditions; also, that the yield point of the steel should be regarded as ultimate strength as far as reinforced beams are concerned. When the steel reaches its yield point, the beam deflects, and failure soon occurs by the crushing of the concrete.

16. Bond Stress.—The tension in the horizontal steel near the lower surface of a reinforced-concrete beam is a maximum near the center of beam and decreases each way toward the end. The difference in the tension between any two points is transmitted to the concrete by the bond between the steel and the concrete.

A formula for bond may be derived for beams in which the reinforcement is horizontal or straight throughout. The total shearing stress per linear inch between the steel and the concrete, considering a length of beam equal to x, is

$$\frac{T' - T}{x}$$

From Fig. 11

$$Vx = (T' - T)jd$$

or

$$\frac{T' - T}{x} = \frac{V}{jd} \qquad \text{(bond stress per linear inch)}$$

and the bond stress per square inch of the surface of the steel bars is $\frac{V}{jd}$ divided by the sum in inches of the circumference of the bars at the given vertical cross-section. If u = unit bond stress, and Σo the total circumference of all bars in a beam at the given section, then

$$u = \frac{V}{\Sigma ojd}$$

The above formula shows that theoretically the bond stress is a simple function of the shear and varies with the shear. Thus, shear diagrams may be used to represent the variation of bond stress along a beam. When using the above formula, the average value of $j = \frac{7}{8}$ may be taken.

If we consider simply supported beams, tests on rectangular and T-beams loaded at the quarter points show that when stirrups are used the beam is stiffened and the bond stress along the horizontal rods near the end of beam is somewhat reduced. A reason for this may be shown in the fact that, after the concrete begins to crack from diagonal tension, the stirrups aid in carrying part of the tensile stress which results from the bending moment then existing at the line of the diagonal crack; the stress in the horizontal rods at the end of beam is thus reduced and likewise the liability of failure through bond. A greater reduction of the bond stress has been found to exist when the web reinforcement is provided by means of bent rods and stirrups. The reduction becomes considerable when about one-half of all the rods are bent up, provided, however, that a sufficient number of rods be thus employed. Results seem to indicate that no reduction should be considered in design unless the number of rods bent be greater than two or three and that the bends be made at least at two points at each end of beam. Tests show that for conditions especially favorable, an average of 50% more bond stress may safely be allowed on the horizontal rods at the end of beam than would be considered safe by the above formula. No allowance should be made when only stirrups are employed for the web reinforcement.

It has been found in the testing of simply supported T-beams with steel straight throughout and loaded at the one-third points, that the bond stress along the horizontal steel is affected by the presence of tensile stresses in the concrete and that this bond stress is usually a maximum just outside the load points. The observed bond stress at these points was in some cases as much as 50% greater than the computed stress.

In beams reinforced for diagonal tension the bond stresses along the horizontal bars are not distributed as uniformly as in beams having the reinforcement horizontal or straight throughout. The bond stresses are found to be concentrated at and near stirrups and at and near points of bending of the longitudinal rods. Such concentration of bond stress causes local slip of the longitudinal bars unless the web reinforcement is well distributed along the beam. If a stirrup is not rigidly attached to the horizontal rods and local slip of bars occurs, the effectiveness of the stirrup is somewhat impaired.

The bond stress in continuous beams is treated in Art. 39.

The bond strength of vertical (or inclined) stirrups may be insufficient to develop the required strength of the stirrups with respect to tension. This possibility must also be investigated in the design of beams having web reinforcement in the form of bent rods. Tests show that it is safe to assume that the stress in a stirrup or bent-up bar may be transferred to the concrete above a point 0.6 the depth of beam from the upper surface. In most cases it is found that stirrups must have hooked ends.

For illustrative problem, see page 298.

17. Web Reinforcement in General.—Inclined web reinforcement may be separate members firmly connected with the horizontal reinforcement to prevent slipping, or some of the horizontal bars may be bent up near the ends of the beam where they are not needed to resist bending. The vertical reinforcement may be used separately or in combination with inclined reinforcement, depending upon the preference of the designer and upon the amount of diagonal tension to be provided for. Vertical stirrups should be looped around the horizontal bars and in important beams should also be firmly secured to these bars by wiring or otherwise. Stirrups should usually be looped or hooked at the top in order to prevent slipping due to insufficient bond (see Art. 19).

The proportioning of web reinforcement cannot be done with any degree of exactness since very little experimental work has been performed along this line. However, rough determinations of what is required may be obtained on rational grounds. The only information from tests is the value of the maximum shearing stress which measures diagonal tension failure—(1) for beams with horizontal bars only, and (2) for beams having an effective system of web reinforcement. Also, tests on beams, with and without web reinforcement, show that when reinforcement is provided for diagonal tension, the concrete may be assumed to carry its full value of the shear and the steel the remainder. It is generally conceded as safe practice in the designing of beams to use only two-thirds of the external vertical shear in making calculations of the stresses to be taken by the web reinforcement.

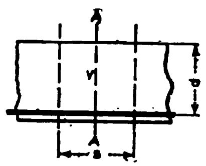


Fig. 18.

Consider now Fig. 18, in which V represents the average total shear over the portion s of the beam. Let v' represent average unit horizontal shear on any plane below the neutral axis. Then (see Art. 13)

$$v' = \frac{V}{bjd}$$

The total shear over any such horizontal plane is $v'bs$; whence

$$v'bs = \frac{Vs}{jd}$$

The function of stirrups, either vertical or inclined, is to resist by their tensile strength that portion of the above shearing stress which is not carried by the concrete.

Assume a vertical stirrup to be placed at the section A–A, and to oppose the shear over the portion of the beam. The total stress in the stirrup is A_sf_s (in a U-shaped stirrup, A_s is the sum of the areas of the two legs), and it is produced by that part of the total shear over the horizontal plane bs not taken by the concrete. Assuming the steel to take two-thirds of the total shear, then

$$A_sf_s = \frac{2}{3}\ v'bs = \frac{2}{3}\cdot\frac{Vs}{jd}$$

or

$$V = \frac{3}{2}\cdot\frac{A_sf_sjd}{s} \qquad (1)$$

Solving

$$A_s = \frac{2}{3}\cdot\frac{Vs}{f_sjd}\ \text{(vertical stirrups)} \qquad (2)$$

or

$$s = \frac{3}{2} \cdot \frac{A_s f_s jd}{V} \tag{3}$$

For inclined members and bent-up bars, the lines on a beam representing the direction in which the diagonal tensile cracks are likely to occur, are crossed more times per unit of length for a given horizontal spacing than would be the case if vertical stirrups were employed; that is, a given amount of inclined steel is much more effective in taking diagonal tension than the same amount of vertical steel. It may be assumed that the allowable stress in the inclined bars is approximately $\frac{A_s f_s}{\sin 45°}$ and the required area of steel, assuming the steel to take two-thirds of the shear, is

$$A_s = \frac{2}{3} \cdot \frac{0.7(Vs)}{f_s jd} \tag{4}$$

or

$$s = \frac{3}{2} \cdot \frac{A_s f_s jd}{0.7V} \tag{5}$$

Since tests show that bent bars may be inclined at any angle between 30 and 45 deg. without a beam showing any marked difference in strength (see Art. 15), the Joint Committee recommends that the longitudinal spacing of vertical stirrups should not exceed one-half (½) the depth of beam and that of inclined members and bent-up bars should not exceed three-fourths (¾) of the depth of beam.

18. Region Where No Web Reinforcement is Required.—There is a region near the center of most beams in which the shear does not exceed that permissible for plain concrete. In this region no shear reinforcement is required. The distance from one support to a point beyond which no stirrups are required may be found as follows for a uniformly loaded beam.

Let

l = span of beam in feet.
w = uniform load in pounds per foot.
x_1 = distance in feet from left support beyond which no stirrups are required.
v_1 = unit working shear for plain concrete.
V_1 = total working shear, producing unit shear of v_1.

From equation (2) on page 280, it is obvious that, where $v_1 = v$, no stirrups are required. At this point

$$v_1 = \frac{V_1}{bjd}$$

But

$$V_1 = \frac{wl}{2} - wx_1$$

Whence

$$x_1 = \frac{l}{2} - \frac{v_1 bjd}{w}$$

or, in terms of v at the end of beam,

$$x_1 = \frac{l}{2}\left(1 - \frac{v_1}{v}\right)$$

When v at the end of beam equals $3v_1$, then

$$x_1 = \frac{l}{3}$$

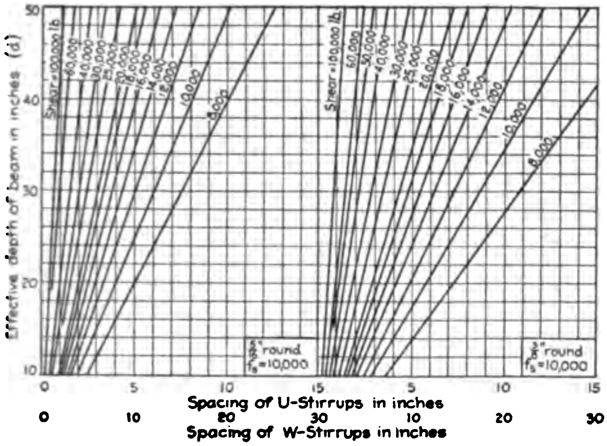
This derivation applies only to a static uniformly distributed load over the whole span.

Suppose a 10-ft. beam (b = 10 in. and d = 20 in.) is uniformly loaded with a static load of 2900 lb. per ft. and assume v_1 = 40 lb. per sq. in. according to recommendation of Joint Committee for 2000-lb. concrete. Also assume $jd = \frac{7}{8}d$. Then

$$x_1 = \frac{10}{2} - \frac{(40)(10)(17.5)}{2900} = 2.6 \text{ ft.}$$

When designing for floor systems it is often more proper to consider the uniform load to

DIAGRAM I



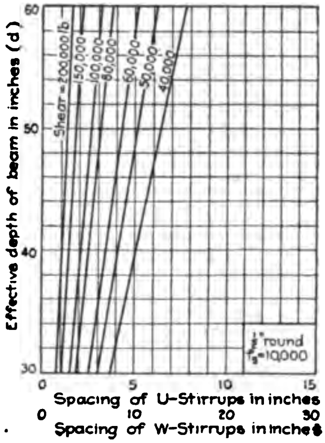
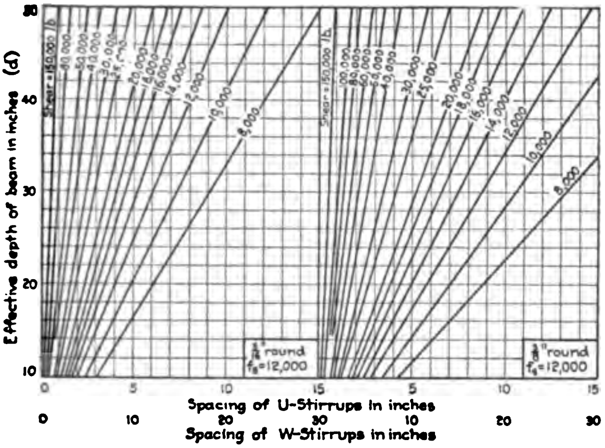


DIAGRAM II



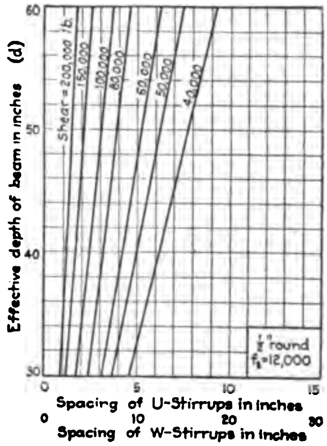
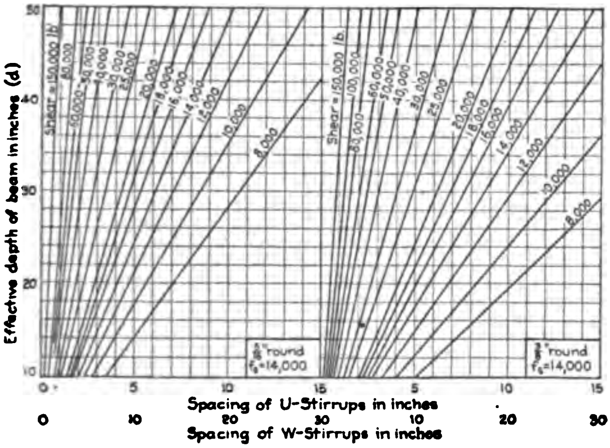


DIAGRAM III



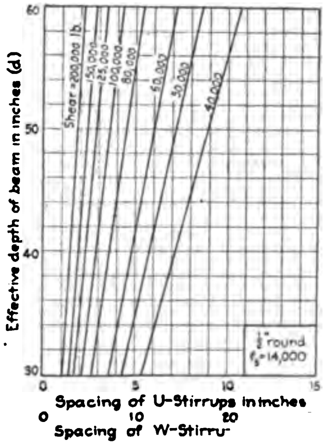


Diagram IV

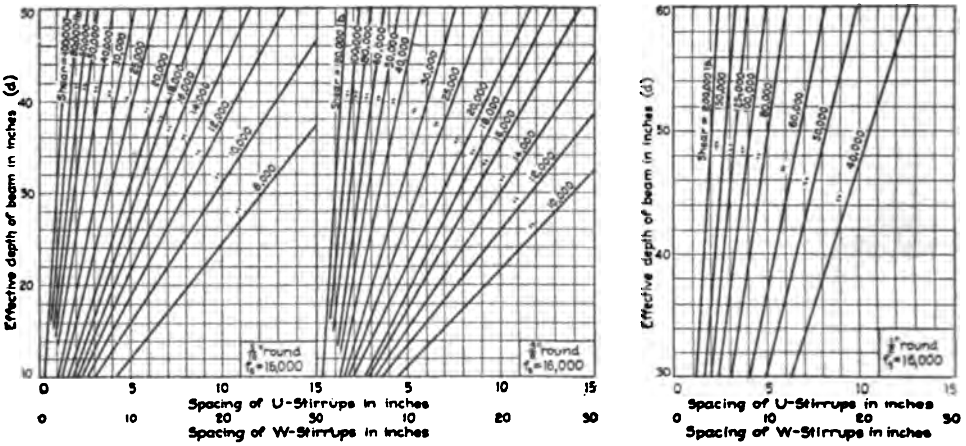


Diagram V

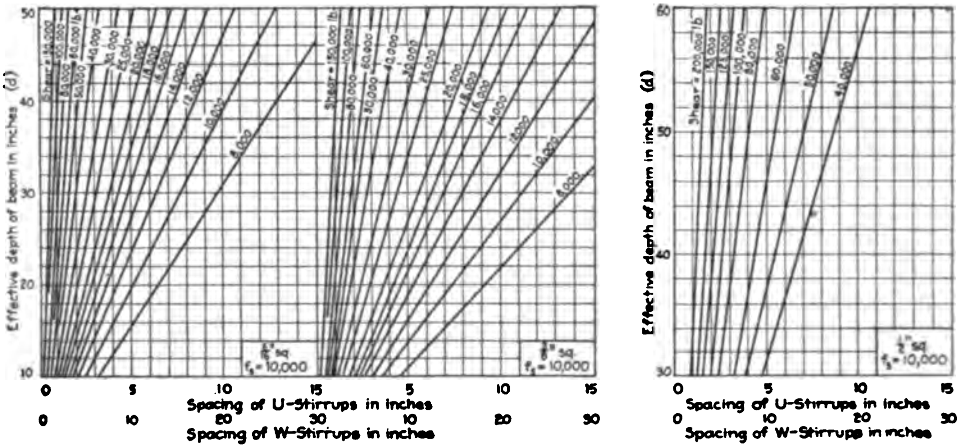


Diagram VI

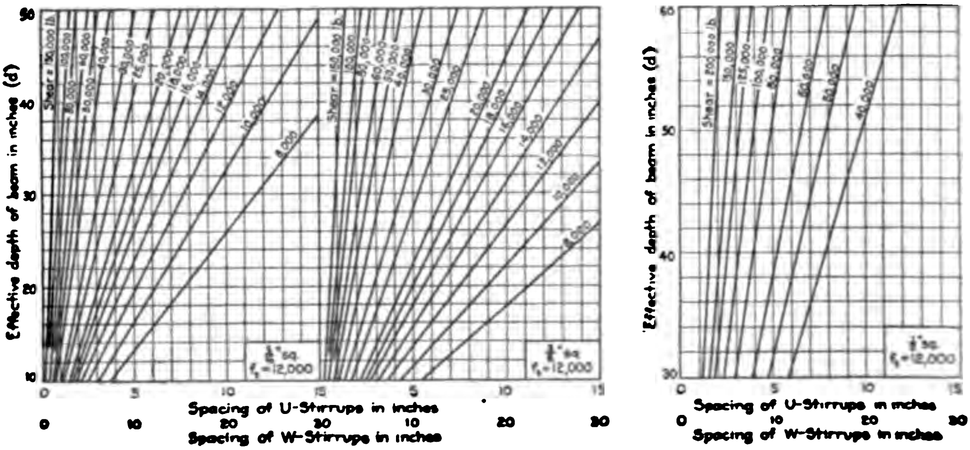


DIAGRAM VII

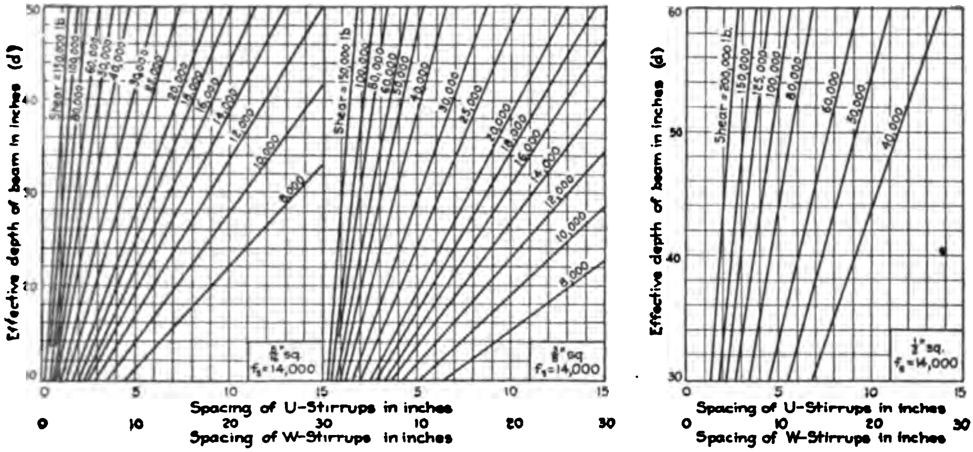
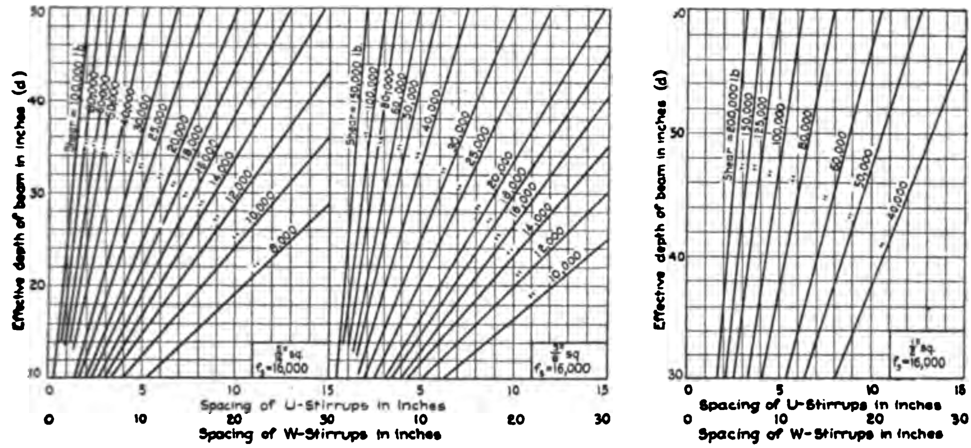


DIAGRAM VIII



be a moving load. In this case the shear at the center of the span is not zero, but is equal at maximum to $\frac{w'l}{8}$, where w' is the live load per foot. When plotting the shear diagram this value should be plotted at the center of the beam and the total end shear at the support. The variation in shear between these two points may be safely assumed to be a straight line. The shear to be carried by the concrete being known, the point in the beam beyond which no shear reinforcement is required may be quickly located.

19. Vertical Stirrups.—The required total area of cross-section of a vertical stirrup may be determined by the formula (see page 285)

$$A_s = \frac{2}{3} \cdot \frac{Vs}{f_s jd}$$

assuming the web reinforcement to carry two-thirds of the total shear. (For U-shaped stirrup, A_s is the sum of the areas of the two legs.) With a given size of stirrup, this formula may be solved to give the spacing required, or

$$s = \frac{3}{2} \cdot \frac{A_s f_s jd}{V}$$

The value of V should be taken at the section where the spacing is desired. This spacing formula may be solved directly by means of Diagrams 1 to VIII inclusive for three sizes of stirrups.[1]

If the shear diagram is drawn for any given beam, it is convenient to use the above formula in the form

$$V = \frac{3}{2} \cdot \frac{A_v f_v jd}{s}$$

and, for various even-inch spacings (s) of the stirrups, to solve for the corresponding total external shears. At the point where the ordinate to the shear diagram scales a computed total shear, there the spacing may be made the even inch used for s in the formula.

Tests have shown that little or no value is derived from stirrups spaced a distance apart equal to, or greater than, d (see page 283). A practical limit suggested by the Joint Committee is one-half the depth.

In restrained beams the first stirrup should be placed no farther than one-half the minimum spacing from the edge of support and, in beams simply supported, the first stirrup should be placed not farther than one-half the minimum spacing from the *center* of support.

The variation of shear intensity along a uniformly loaded beam is shown in Fig. 19. The area $ABCD$ represents the total stress to be taken by the stirrups at each end of beam. The ordinate AB represents two-thirds of the shear at the support per 1-in. length of beam.

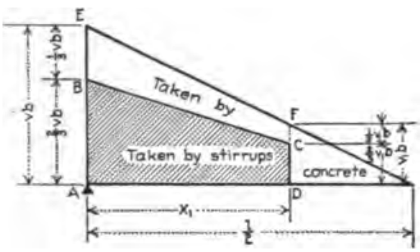


Fig. 19.

Some attention must be paid to the diameter of stirrup which it will be possible to employ in any given case. The diameter should not be so small that the stirrups will be placed too close together for convenience in construction, yet not so far apart that the limiting value $\frac{1}{2}d$ is exceeded. But, in addition to such consideration, the bond strength of the stirrup must be investigated if the stirrups are not made with hooked ends, since the danger of slipping determines the maximum diameter which may be employed.

The distribution of bond stresses developed on the surface of the stirrups is indeterminate. Evidently it must not be expected that tension will be transferred through bond to the concrete until the compression area of the beam is reached, or until a point but little below is reached. Experiments show that it is safe to assume the grip of a stirrup to be 0.6 the depth of beam. Using notation on page 270.

$$f_s A_s = 0.6\, dou$$

or

$$\frac{A_s}{o} = 0.6 \frac{u}{f_s} d$$

But, for round or square stirrups, letting i = maximum diameter of stirrup,

$$\frac{A_s}{o} = \frac{\pi \frac{i^2}{4}}{\pi i} = \frac{1}{4} i$$

Then

$$i = \left(2.4 \frac{u}{f_s}\right) d$$

If each end of the stirrup is bent into a prong or hook, then stirrups of larger diameter may be used than is indicated by the above formula. Tests show that if hooks with a semicircular

[1] Diagrams similar to those by Frank S. Bailey in *Eng. News*, Oct. 12, 1916.

bend of 4 diameters are well embedded in concrete, the stress in the bar will reach the elastic limit before slipping takes place (see page 268).

It is considered good practice to use $\frac{5}{16}$-in. stirrups with hooked ends for beams from 10 to 25 in. deep, $\frac{3}{8}$-in. stirrups for beams from 25 to 40 in. deep, and $\frac{1}{2}$-in. stirrups for beams from 40 to 60 in. deep.

ILLUSTRATIVE PROBLEM.—A simply supported beam is 9 by 16 in. in cross-section and the tension reinforcement is 2 in. above the lower face of the beam. Span of the beam is 8.5 ft. Uniform load of 1800 lb. per ft. If necessary, the web is to be reinforced against diagonal tension using vertical stirrups. Allowable $f_s = 10{,}000$; $v = 40$; $u = 80$.

$$v = \frac{V}{bjd} = \frac{7650}{(9)(\frac{7}{8})(14)} = 69 \text{ lb. per sq. in.}$$

The allowable shear is 40 lb. per sq. in., hence stirrups are necessary.

The diameter of a stirrup without any prong or hook should not exceed

$$i = 2.4\frac{80}{10{,}000}(14) = 0.27 \text{ in.}$$

If the stirrups are to be bent at the upper end, $\frac{5}{16}$-in. round bars may be considered secure against slipping.

Stirrups are unnecessary at a distance from support equal to

$$x_1 = \frac{8.5}{2}\left(1 - \frac{40}{69}\right) = 1.80 \text{ ft.}$$

The minimum spacing of stirrups (U-shape) will occur at the supports, or

$$s = \frac{3}{2}\cdot\frac{2(0.077)(10{,}000)(\frac{7}{8})(14)}{7650} = 3.70 \text{ in.}$$

The shear diagram for one-half the beam is shown in Fig. 20. For a 4-in. stirrup spacing

$$V = \frac{3}{2}\cdot\frac{2(0.077)(10{,}000)(\frac{7}{8})(14)}{4} = 7090 \text{ lb.}$$

The point where $V = 7090$ lb. is easily found by scaling to the shear diagram. For a 5-in. stirrup spacing $V = \frac{4}{5}(7090) = 5670$ lb. For a 6-in. spacing $V = \frac{4}{6}(7090) = 4730$ lb., etc. Time can be saved by using Diagram 1 in finding values of V directly.

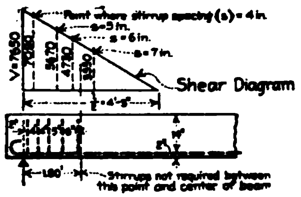


FIG. 20.

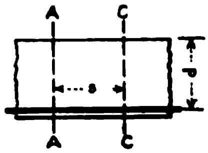


FIG. 21.

20. Method of Placing Stirrups from the Moment Diagram.—It is a well-known principle of mechanics that the difference in moment between any two points along a beam is equal to the average total shear over the distance between the points multiplied by that distance. Thus, from Fig. 21,

$$Vs = (M_A - M_C)$$

or,

$$V = \frac{(M_A - M_C)}{s} \quad . \quad . \quad . \qquad (1)$$

For loads concentrated at points along a beam this law is not strictly true, unless in each case the concentration occurs at a point midway between the transverse sections chosen; but in the case of concentrated loadings, by beams cast against girders in concrete construction, and even by loadings on slabs transmitted to the floor beams, the concentration may not be sharply defined, and there is no determinate law of shear variation over such a region. Moreover, as this discussion will show later, the distance s is relatively small where shear

Within the limits of actual conditions in reinforced-concrete construction, therefore, the above statement may be considered very approximate to the truth, for the beam loaded with concentrated loads.

Substituting the above value for V in equation (1) on page 285, we have

$$\frac{(M_A - M_C)}{s} = \frac{3}{2} \cdot \frac{A_s f_s jd}{s}$$

whence

$$M_i = (M_A - M_C) = 1.5A_s f_s jd \tag{2}$$

in which M_i is the increment of moment between the ends of the portion of the beam to be reinforced with the stirrup.

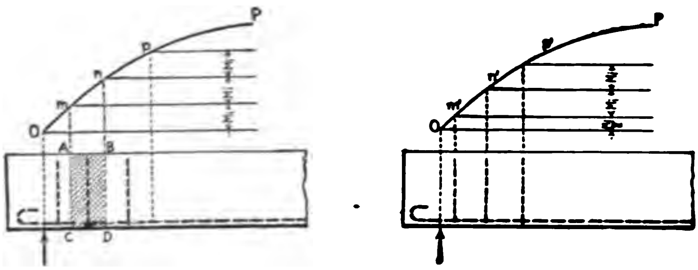
If the stirrup is inclined at an angle θ with the horizontal, then

$$M_i = \frac{1.5A_s f_s jd}{\sin \theta} \tag{3}$$

When θ = 45 deg. (or, accurately enough, any angle between 30 and 45 deg.)

$$M_i = 2.1A_s f_s jd \tag{4}$$

Consider the portion of the beam shown in Fig. 22a, loaded in such a manner as to produce the moment curve OP. It is desired to reinforce the portion shown with vertical stirrups, keeping in mind the principles just laid down. A certain stirrup has been adopted which, for this particular beam, gives the value of M_i from equation (2) equal to the vertical distance



(a) (b)

FIG. 22.

shown in Fig. 22a. The first increment intercepts the portion Om of the curve OP; the second, mn, and so on Let one of these intercepts, as mn, be projected on the beam, thereby defining the area $ABDC$ on the diagram of the beam. This area is the portion of the beam over which the adopted stirrup will exactly carry the shear. The length of the portion is seen to vary as the shear varies along the beam. Since the stirrup is required to carry the shear for this portion of the beam, it should be placed through the center of gravity of the shear area for this portion. Likewise, each other portion of the beam defined by the projection of M_i would have a stirrup through the center of gravity of its shear area.

To eliminate the feature of having to locate this center of gravity, the following method is proposed: Lay off as the first value ½ M_i (Fig. 22b). Let all other spaces be equal to M_i as before. These increments have m', n', etc., for points of intersection on the moment curve. Let these points be projected on the beam. Each projection will thus determine the position of a stirrup. This scheme gives very closely the same results as before, the stirrup being placed slightly nearer the support than when placed through the center of gravity of the shear area for the portion of the beam; this error, however, is well inside the accuracy of placing this part of the reinforcing.

Diagrams IX to XII inclusive are plotted for a ready solution of equations (2) and (4) given above. Their use is explained in an illustrative problem on page 295.

DIAGRAM X

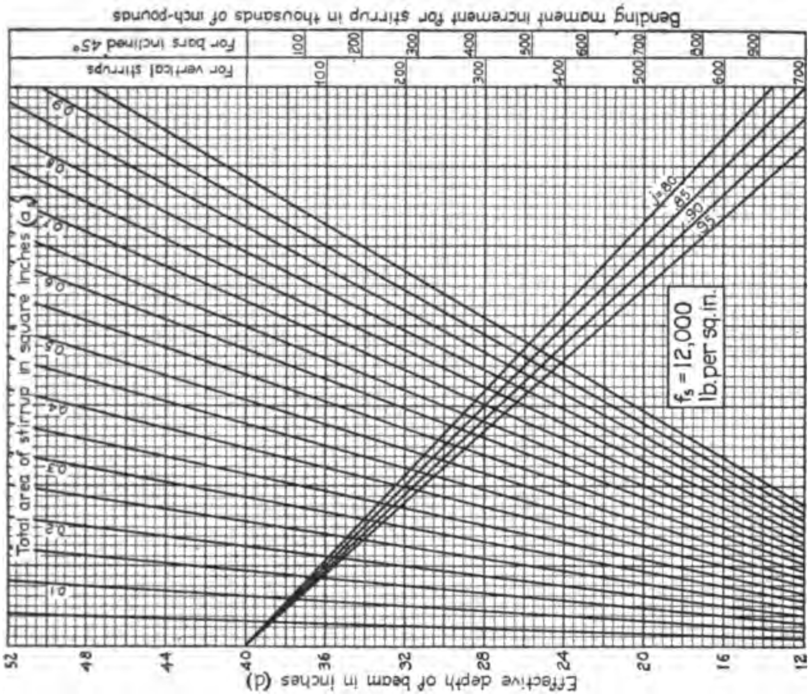


DIAGRAM IX

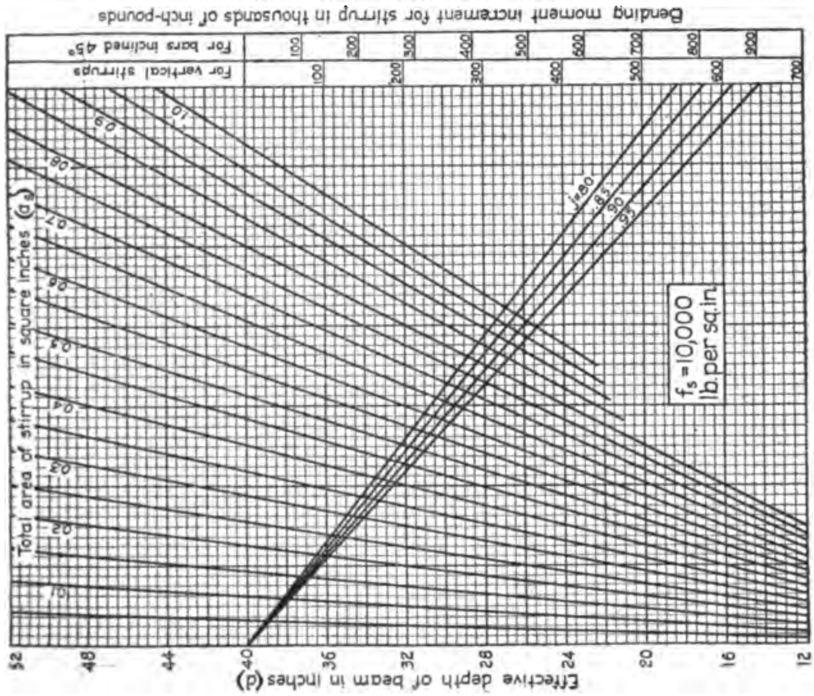


DIAGRAM XII

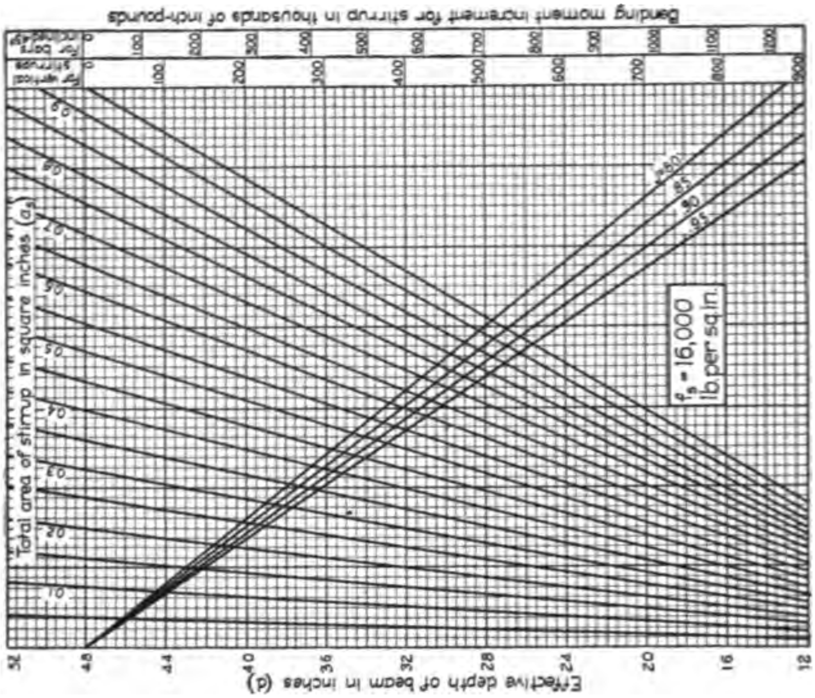
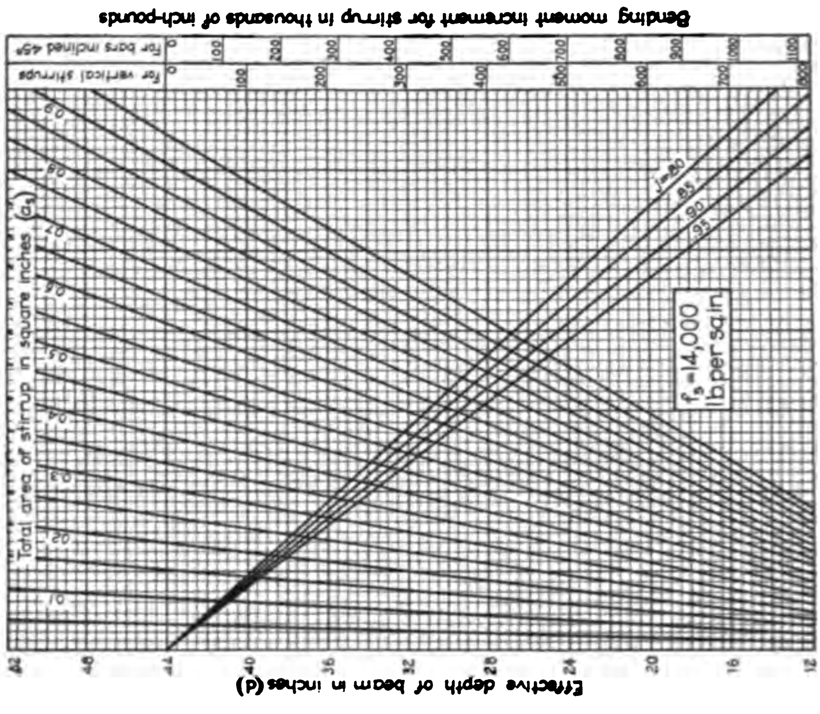


DIAGRAM XI



A simple method of constructing the parabola or moment diagram for uniform loading is as follows: Let AG, Fig. 23, represent the base of the parabola, with a middle ordinate of 4.5 at D. Divide the base into any desired number of equal parts, as for instance, six. Number these points from each end beginning with zero. Divide the middle ordinate by the product of the numbers at that point, as $\frac{4.5}{3 \times 3} = 0.5$. This constant, if multiplied by the product of the pair numbers at any point gives the ordinate at that point. For example, the ordinate at C is $2 \times 4 \times 0.5 = 4.0$. If an ordinate is desired at a point between the equal divisions, as X, the fractional part of the division may be expressed for the point from each way. At X the distance from A is 2.6 units, and from G, 3.4 units. The ordinate at X is $2.6 \times 3.4 \times 0.5 = 4.42$. If the middle ordinate does not fall at an even division, as would be the case if an odd number of units were used, the fractional values for the mid-point would be used the same as the full values in the above case.

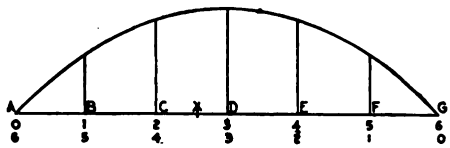


Fig. 23.

Illustrative Problem.—Determine the size and spacing of vertical stirrups for shear reinforcement in a beam loaded as shown in Fig. 24*a*.

For the static uniform load of 1000 lb. per ft., the moment at the center is

$$M = \frac{wl^2}{8} = \frac{(1000)(21)^2(12)}{8} = 662{,}000 \text{ in.-lb.}$$

For the concentrated loads

$$M = (1000)\ (7)\ (12) = 840{,}000 \text{ in.-lb.}$$

which moment obtains through the central third of the beam. The moment curves for each condition are plotted separately and the ordinates of the two combined to form the curve of total moments, shown in Fig. 24*b*.

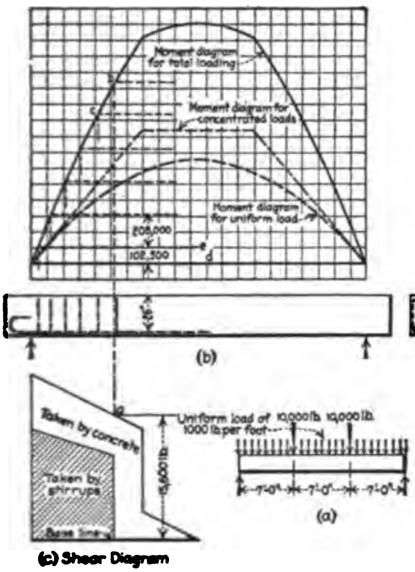


Fig. 24.

The total end shear is (1000) (21) ½ + 10,000 = 20,500 lb. Just outside the third point the shear is 20,500 − (1000) (7) = 13,500 lb. Just inside the third point it is 13,500 − 10,000 = 3500 lb. At the center the shear is zero. The shear diagram is shown in Fig. 24*c*.

In flexure the following values will be adopted: $f_s = 16{,}000$, $f_c = 650$; $n = 15$. It is found that for equal strength in tension and compression a depth (d) of 26 in. is required when $b = 20$ in. Also $j = 0.875$.

The allowable shear to be carried by the concrete alone will be taken as 30 lb. per sq. in. The concrete will then carry (30)(20)(26) = 15,600 lb. This value is plotted as point a on the shear diagram, Fig. 24*c*; and the distance from the support to point a is the distance along the beam where shear reinforcement is required. In this problem the stirrups will be hooked at the ends and an allowable stress of 10,000 lb. per sq. in. will be assumed. It will be necessary to select a size of stirrup that will not be so small that the spacing will be too small for convenient construction, nor yet large enough to cause the spacing to be greater than ½d, a limit recommended by the Joint Committee. To accomplish this, let a vertical line be projected from a, Fig. 24*c*, to the moment curve, as at b. Assume some distance along the moment curve, as bc, whose horizontal projection is approximately ½d, and note the value of its vertical projection (in this case 200,000 in.-lb.). With this value for vertical stirrups, enter Diagram IX at the right and move to the left until the value of $j = 0.875$ is reached; then vertically until an intersection is made with a horizontal line from $d = 26$ in. This point is found to give a stirrup area equal to 0.58 sq. in.

Four $\frac{7}{16}$-in. round rods give a combined area of 0.601 sq. in., and by entering Diagram IX at $d = 26$ in., thence horizontally to $A_s = 0.60$ sq. in., thence vertically to $j = 0.875$ and finally to the right, $M = 205{,}000$ in.-lb. for vertical stirrups.

Beginning as in Fig. 22b, ½(205,000) is laid off at the base of the moment curve, as de, Fig. 24b, after which equal increments of 205,000 are laid off until the point b is passed. Through the points thus fixed, horizontal lines are drawn to intersect the moment curve. From each intersection a vertical line projected to the beam locates a stirrup.

21. Bent Bars and Vertical Stirrups for Web Reinforcement.—The ends of the horizontal bars in a reinforced-concrete beam may usually be bent up to assist vertical stirrups in preventing diagonal tension failure. Although in some cases these bars may be found theoretically to take all the diagonal tensile stresses not taken by the concrete, vertical stirrups are always desirable, as shown by tests.

Plain rods bent up to provide web reinforcement often lack sufficient bond strength to render them fully effective. Where bent up at a considerable angle they should be turned again horizontally and extend some distance along the upper part of the beam, as shown in Fig. 25. In heavy construction the ends of all bars should be bent into a hook. The most convenient method of using reinforcement is to bend up two rods at a time and make all the bars inclined at the same angle with the horizontal. The bars bent should theoretically be such as to keep the center of gravity of the beam cross-section in the line drawn vertically through the center of the section. An exception occurs to the bending of two rods at a time, in the case of an odd number of horizontal rods. Here, one of the bends may consist of either one or three rods.

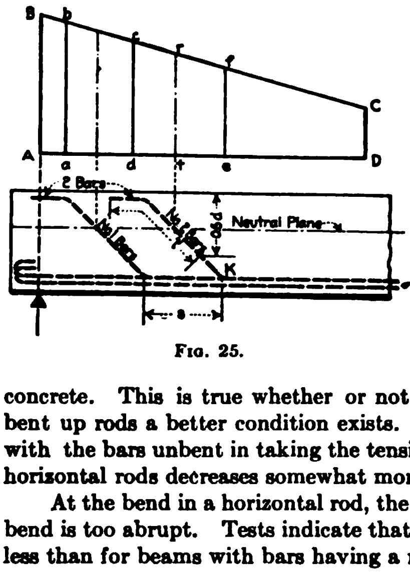


FIG. 25.

If bent rods are not required to provide for diagonal tension, then the horizontal rods may be dispensed with at the points, beyond which they are not needed to provide for tension due to bending. This method of stopping off the horizontal rods is not desirable, however, as the bond in the concrete near the middle of the beam is not as good as would be the case near the end where the moments are smaller. Also, when a bar is discontinued, the stress in those which remain is immediately increased tending still further to impair the bond between the steel and the concrete. This is true whether or not a hook is employed on the discontinued rods. With bent up rods a better condition exists. The horizontal components of the upturned bars act with the bars unbent in taking the tension due to bending, and so in general the tension in the horizontal rods decreases somewhat more gradually toward the end of beam as it should.

At the bend in a horizontal rod, the unit stress in the concrete may become excessive if the bend is too abrupt. Tests indicate that the strength of beams with bars having sharp bends is less than for beams with bars having a radius of bend equal to about 12 diameters.

The bent rods, if of the same diameter, should be so arranged that each rod will take an equal part of the diagonal tension—that is, if they can be bent in this way and still provide satisfactorily for the horizontal tension. The points where the bent rods should cross the neutral axis of the beam may be found by any of the methods given for determining the spacing of vertical stirrups, remembering that a rod bent at 45 deg. (or, as tests show, at any angle between 30 and 45 deg.)[1] is $\frac{1}{0 \cdot 7} = 1.43$ times as effective per unit area as a vertical stirrup. If the rods cannot be bent as desired, then vertical stirrups must be used to provide for some of the diagonal tension, and it would be preferable in all cases to use stirrups even in that part of the beam where the bars are bent up.

Consider uniform loading and let $ABCD$ in Fig. 25 be the diagonal tension area. Assume that four rods may be bent near the end of beam and assume also that the first two bars cannot be bent up nearer the center of beam than the point K, Fig. 25. The area $cdef$ should be made

[1] See p. 286.

equal to 1.43 times the allowable tensile stress in the two No. 2 bars; likewise area *abcd* should represent 1.43 times the allowable tensile stress in the No. 1 bars. (The vertical *rt* should theoretically pass through the center of gravity of the area and, if *dt* is made $\frac{5}{12}$ and *te* $\frac{7}{12}$ of *de*, this will be practically accomplished. The error is not serious if *dt* is made equal to *te*.) If the No. 1 bars cannot be bent up so that the areas *abcd* and *cdef* have the line *cd* in common, then a vertical stirrup, or stirrups, will be needed to take care of the space between these areas. If the areas overlap, the No. 1 bars may be bent up nearer the end of beam if no other condition governs. In any case the distance *s* should not be greater than $\frac{3}{4}d$—that is, the maximum spacing at which inclined web reinforcement can be considered effective. Stirrups will at least be needed to the left of *ab* and to the right of *ef*. The area taken care of by a vertical stirrup is equal to its tensile value.

The Joint Committee recommends that bent bars be considered as adding to diagonal tension resistance for a horizontal distance from the point of bending equal to $\frac{3}{4}d$. The Joint Committee also recommends for the case where stirrups are needed in combination with bent-up bars, that the stresses in the stirrups be determined by finding the amount of the total shear which may be allowed by reason of the bent-up bars, and substracting this shear from the total external vertical shear. Two-thirds of the remainder is recommended as the shear to be considered as carried by the stirrups.

The bond strength of inclined bars must be investigated. This strength should be provided in the upper portion of the beam. As with vertical stirrups, it is arbitrarily assumed that no stress is transmitted from the steel to the concrete below a point which is 0.6*d* below the upper surface of the beam.

Assume that the stress in an inclined bar is its working stress. This gives the maximum condition. Using the notation of page 270 and l' for length

$$l'ou = A_s f_s$$

and for round or square bars,

$$l'u = \frac{f_s i}{4}$$

or

$$l' = \frac{f_s}{4u} \text{ diameters}$$

If the allowable $f_s = 10{,}000$ lb. per sq. in. and the allowable $u = 80$ lb. per sq. in., then l' (as shown at No. 2 bars in Fig. 25) should equal 31 diameters by the above formula. If $l' =$ 16,000 lb. per sq. in., $l' = 50$ diameters.

The value of hooks on the ends of bars is discussed on page 268.

See illustrative problem on page 298.

22. Points Where Horizontal Reinforcement May be Bent.—For uniformly loaded beams the bending-moment curve is a parabola as shown in Fig. 26.

Let a = area of bars required at the center of beam.

a_2 = area of bars to be bent.

$p_2 = 100\frac{a_2}{a} =$ % of total steel that may be bent up.

M = maximum moment $\frac{wl^2}{\phi}$.

x_2 = distance from support to point where bars may be bent.

M_{x_2} = moment at distance x_2 from support.

Then

$$\frac{M}{M_{x_2}} = \frac{a}{a - a_2}$$

Substituting values of M and M_{x_2}, and solving for x_2,

$$x_2 = \frac{l}{2}\left(1 - \sqrt{\frac{8}{\phi} \cdot \frac{p_2}{100}}\right)$$

Fig. 26,[1] based on the above formula, indicates the points at which rods may be bent up for three types of beams with the maximum bending moments specified. To illustrate the use of the diagram, assume that a beam designed for $M = \frac{wl^2}{10}$ requires 3.5 sq. in. of steel at the center. To find the point where 40% of the steel may be bent up and still leave sufficient steel to carry the tension, trace horizontally from the 40% mark at the right to the curve $M = \frac{wl^2}{10}$ and then vertically to the lower margin where $0.22l$ is read.

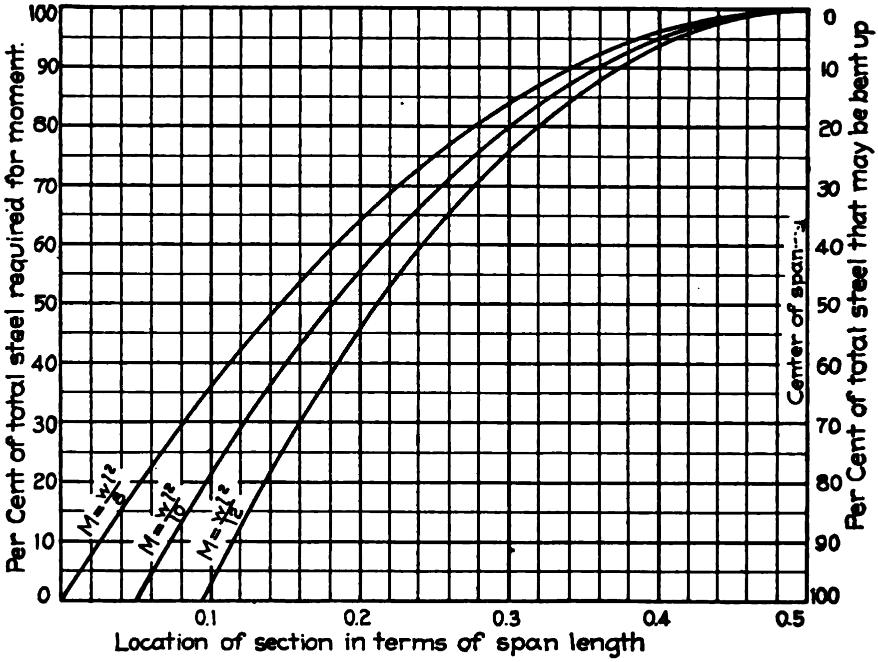


Fig. 26.

If it is desired to bend up a number of rods two or more at a time, then x_1 should be determined for each bend. After this is done, the remaining horizontal bars should be secure against slipping.

For concentrated and unsymmetrical loading, the maximum moments at various sections will need to be determined, in order to ascertain the points where the horizontal bars may be bent up. From these maximum moments obtain the required area of horizontal rods at the different points (1, 2, 3, and 4, Fig. 27). Plot a curve to scale, as shown. Thus, *ab* represents the area required at the point *a*. On the center ordinate lay off the required areas of the rods, and draw horizontals as shown. The rods may be bent up where these horizontals cut the curve but it would be better, however, to carry them a short distance beyond the theoretical points.

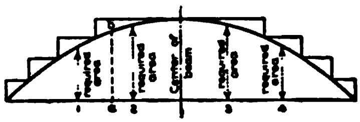


Fig. 27.

Illustrative Problem.—Design the left end of a simply supported beam to span 15 ft. and to support the loads shown in Fig. 28 with equal strength in tension and compression. Allowable $f_c = 650$; $u = 80$; $v = 40$ without web reinforcement and 120 with an effective web reinforcement.

[1] Diagram taken from an article in *Eng. News*, Aug. 19, 1916, by Karl D. Schwendener.

The reactions are readily found and are given in the sketch. The maximum moment occurs at the center load since the shear passes through the value zero at this point. We shall assume the weight of beam included in the uniform load of 1000 lb. per ft.

$$M = (25{,}500)(8)(12) - (23{,}000)(4)(12) = 1{,}344{,}000 \text{ in.-lb.}$$

It is found that a depth (d) of 28 in. is required when $b = 16$ in.

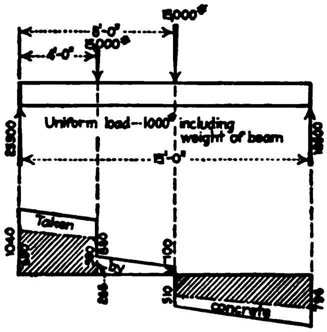


Fig. 28.

$$v = \frac{V}{bjd} = \frac{25{,}500}{(16)(\frac{7}{8})(28)} = 65 \text{ lb. per sq. in.}$$

Thus web reinforcement is needed.

$$A_s = (16)(28)(0.0077) = 3.45 \text{ sq. in.}$$

We shall select eight $\frac{3}{4}$-in. round rods = 3.53 sq. in. Bond for one rod at the left end of beam is

$$u = \frac{25{,}500}{(2.356)(\frac{7}{8})(28)} = 443 \text{ lb. per sq. in.}$$

For plain rods, the number which must extend straight to the left end of beam is, for bond conditions especially favorable (see page 284),

$$\frac{443}{(1.5)(80)} = 4$$

Thus, at this end of beam four rods may be bent up.

The concrete will be found to take care of any diagonal tension between the concentrated loads since concrete will take (40)(16) = 640 lb. per lin. in. Horizontal shear (which measures diagonal tension) at the support is

$$\frac{V}{jd} = \frac{25{,}500}{(\frac{7}{8})(28)} = 1040 \text{ lb. per lin. in.}$$

and to the left of the adjacent concentrated load, it is

$$\frac{21{,}500}{(\frac{7}{8})(28)} = 880 \text{ lb. per lin. in.}$$

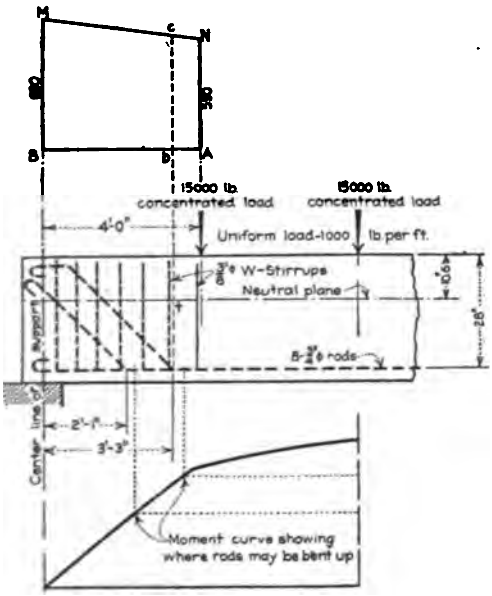


Fig. 29.

The total diagonal tension to be taken by the web reinforcement is represented by a trapezoid (Fig. 28), the parallel sides of which are ⅔ (1040) = 690 lb. and ⅔ (880) = 590 lb. and the length 4 ft. Hence, total stress in this part of beam to be taken by the concrete and by the web reinforcement is

$$\frac{690 + 590}{2} \times 4 \times 12 = 30{,}720 \text{ lb.}$$

Since four rods are to be bent, their comparative tensile value is

$$(4)(0.4418)(16{,}000)(1.43) = 40{,}400 \text{ lb.}$$

Thus, the tensile value of the rods is in excess of the stress to be provided for.

An investigation must now be made to see whether the tensile stresses in the bottom of the beam will permit the bending of the bars. Fig. 29 shows the bending-moment curve plotted to scale, and the points where the bars may be bent up are determined by the method described on page 298. It is clear that the bars cannot be bent up as desired to provide thoroughly for diagonal tension. The points where the bars are actually bent up are about 2 in. beyond the theoretical points as determined by moment.

If each bent-up bar is assumed to take diagonal tension to the left only of its point of bending, then the area $bANc$ remains unprovided for. Stirrups will be provided to take diagonal tension between the point t, where the line bc produced meets the neutral line, and the adjacent load. Only one stirrup is required, but it would seem advisable in a design of this kind to also place stirrups at the positions indicated by the dotted lines. The length of embedment of the inclined bars should be 50 diameters, or 37½ in.

23. Transverse Spacing of Reinforcement.—The shearing stress along *ab*, Fig. 30, should equal the amount of stress transmitted by bond along *bcd*. If bond and shearing strengths were equal, *ab* should equal *bcd*, and the clear space between bars should be $\frac{3.1416}{2}$ diameters = 1.57 diameters. But shearing strength here employed is controlled by the diagonal tension. For v = 90 lb. per sq. in. and u = 80, *ab* should be at least 1.40 diameters; and for v = 120 lb. per sq. in. and u = 80, *ab* should be at least 1.05 diameters. There is likely to be more or less tension in the concrete surrounding the bars, and, besides, since the concrete is not easily placed between the rods, it may have a lower strength in that vicinity. A clear spacing of 1½ to 2 diameters is advisable unless it is determined by computation that the bond stress is very much lower than the allowable. In the above discussion plain bars only have been considered. Deformed bars, if stressed to their full bond value, should be spaced farther apart than plain bars.

The Joint Committee recommends that the lateral spacing of parallel bars should not be less than 3 diameters, center to center, and that the distance from the side of the beam to the center of the nearest bar should not be less than 2 diameters. In order that concrete may be readily placed between bars and also give sufficient concrete on the sides of the beam for fire protection, it is also advisable to require that the spacing of rods be not less than 1 in. in the clear (if the maximum size of aggregate does not exceed 1 in.) and that 1½ in. in the clear be also considered the minimum distance of the rods from the sides of the beam. Thus, the least width of beam should be the greater of the two values determined from the following formulas:

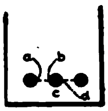


FIG. 30.

$$b = [3(n - 1) + 4]d_1$$
$$b = a_g(n - 1) + nd_1 + 3$$

in which b = least width of beam in inches.
d_1 = thickness of the rods in inches.
n = maximum number of rods which occurs in a horizontal layer.
a_g = maximum size of aggregate in inches.

For an aggregate with a maximum size of 1 in., the width of beam for all rods greater than ⅝ in. in diameter will ordinarily be governed by the first formula and for ⅝-in. rods and less, by the second formula.

Where two or more layers of rods are used, the rods should be so placed as to permit the mortar to run between them. The Joint Committee specifies a limiting clear space of 1 in. and does not recommend the use of more than two layers "unless the layers are tied together by adequate metal connections, particularly at and near points where bars are bent up or bent down." The Joint Committee also advises that "where more than one layer is used, at least all bars above the lower layer should be bent up and anchored beyond the edge of the support."

24. Depth of Concrete Below Rods.—Tests show that a 2-in. thickness of concrete is necessary to thoroughly protect embedded steel from the direct action of flames. Flat slabs are found to be affected to a less depth than projecting members such as beams and columns. The Joint Committee suggests that "the metal in girders and columns be protected by a minimum of 2 in. of concrete; that the metal in beams be protected by a minimum of 1½ in. of concrete; and that the metal in floor slabs be protected by a minimum of 1 in. of concrete."

The following depths of concrete below the center of steel may ordinarily be employed except where conditions are unusually severe.

SLABS

Depth to steel (d)	Depth below center of steel
3¼ in. and under	¾ in.
Between 3¼ in. and 4¾ in.	1 in.
4¾ in. and over	1¼ in.

BEAMS AND GIRDERS

Depth to steel (d)	Depth in the clear below steel
10 in. and under	1 in.
Between 10 in. and 20 in.	1½ in.
20 in. and over	2 in.

25. Ratio of Length to Depth of Beam for Equal Strength in Moment and Shear.—With given working stresses in concrete and steel, there is a definite ratio of length to depth of beam which will give equal strength in moment and shear. First, consider beams simply supported. For a single concentrated load at the center of span

$$\frac{l}{d} = \frac{2f_s p}{v_1}$$

in which v_1 = allowable shearing stress and f_s = working stress in steel. For a uniformly-distributed load

$$\frac{l}{d} = \frac{4f_s p}{v_1}$$

For beams loaded with equal loads at the third points,

$$\frac{l}{d} = \frac{3f_s p}{v_1}$$

Taking for example, $v_1 = 40$ lb. per sq. in., $f_s = 16{,}000$, $f_c = 650$, $n = 15$, and, using an average value of $\frac{7}{8}$ for j, we have the following ratios for $\frac{l}{d}$.

For concentrated load at center of span	$\frac{l}{d} = 6.16$
For uniformly distributed load	$\frac{l}{d} = 12.32$
For equal loads at the third points	$\frac{l}{d} = 9.24$

It should be clear that the strength of beams of greater relative length than obtained by the formulas will be determined by their moment of resistance, while that of shorter beams by their shearing resistance.

In the case of continuous beams the above formulas will apply if l is taken as the length between points of inflection. It is often convenient to know the extreme limit in design. The Joint Committee recommends 120 lb. per sq. in. for the shearing strength of concrete when adequately reinforced against diagonal tension. This is a low figure but is adopted in order to prevent any likelihood of cracks opening up in the concrete. Suppose then, it is required to know the minimum value of $\frac{l}{d}$ for a given beam, uniformly loaded. From the formula for uniform load, using the working stresses given above,

$$\frac{l}{d} = \frac{(4)(16{,}000)(0.0077)}{120} = 4.11$$

At the same time that the ratio of length to depth is being investigated for moment and shear, there are other conditions which must be considered. For instance, the ratio of length to breadth of beam should not exceed a value of about 25 if the beam is not supported laterally. The reason for this is found in the fact that the upper part of the beam is a column, and to prevent additional stress due to side bending the length should not exceed about 25 times the width. On the other hand, the best-shaped beam is one in which b lies between $\frac{1}{2}d$ and $\frac{3}{4}d$. In any given case, to satisfy all requirements and arrive at a satisfactory design, two or three trials may be required.

26. Economical Proportions of Rectangular Beams.—Without taking the cost of web reinforcement into consideration, it can be shown mathematically that the cost of a rectangular reinforced-concrete beam to resist a given bending moment and be of equal strength in tension and compression, varies inversely with the depth, directly with the square root of the breadth, and directly with the cube root of the ratio of breadth to depth.

The breadth and depth of a rectangular beam to be of equal strength in tension and compression may be found by means of the formulas (see page 277)

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad j = 1 - \tfrac{1}{3}k \qquad p = \frac{f_c k}{2f_s}$$

$$bd^2 = \frac{2M}{f_c kj} \qquad bd^2 = \frac{M}{pf_s j} \qquad A_s = pbd$$

If b or d, or the ratio $\frac{b}{d}$ is decided upon, the proportions and steel area of a beam to resist a given bending moment are definitely determined. Thus for a fixed breadth, fixed depth, or fixed ratio of breadth to depth, the cost of a beam will vary with the working stresses employed since values of k, j, and p depend wholly on values of f_c, f_s, and n.

Where the depth is fixed, it is found that, if the ratio

$$r = \frac{\text{cost of steel per unit volume}}{\text{cost of concrete per unit volume}}$$

does not exceed a value of 60 to 80 (60 a common value), no economy results from using f_s greater than 16,000 lb. per sq. in. when f_c = 600 to 700 lb. per sq. in., or from using f_s greater than 12,000 lb. per sq. in. when f_c = 400 to 500 lb. per sq. in. Somewhat higher values than these may be economically used for f_s when the breadth of beam is fixed. In both cases cost decreases as f_c increases. When the ratio $\frac{b}{d}$ is fixed, no economy results from using f_s greater than 16,000 to 18,000 when f_c = 400 to 600, or from using f_s greater than 14,000 when f_c = 700. In this case cost increases as f_c increases.

In the case where the cross-section of beam is determined by shear, the maximum depth theoretically permissible is that for which bd is just large enough to carry the shear. With a beam designed for moment alone, the cost decreases as the depth increases, but the area of the cross-section becomes less. A point must be reached when the beam will be of just the required strength in moment and shear (see Art. 25). The question which now arises is whether or not a still greater depth will result in greater economy. The quantity bd must now remain constant for the greater depths. But bd^2, on the other hand, is increased and the concrete stress (f_c) decreased. A smaller value for f_c permits the use of a smaller percentage of steel, and the cost is still further reduced. Thus it should be clear that the proportions of a beam will not be determined by shear excepting as to minimum cross-section—an increase in depth always resulting in a gain in economy. It should be noted in this connection, however, that although deep beams are economical of concrete, the wooden forms cost more than they do for shallow beams.

27. Rectangular Beams with Steel in Top and Bottom.—Compressive stresses are usually carried by concrete more economically than by steel. It is sometimes desirable, however, to place steel in the compression as well as in the tension side of the beam. When a rectangular beam is limited as to size, double reinforcement is sometimes the result, and in such cases the value of the steel reinforcement on the compressive side needs to be known. The effectiveness of steel in compression has sometimes been questioned, but the results of tests indicate that the steel does its share of the work.

The Joint Committee recommends that "the reinforcing bars for compression in beams should be straight and should be 2 diameters in the clear from the surface of the concrete. For the positive bending moment, such reinforcement should not exceed 1% of the area of the concrete."

The formulas which are used in the design of double-reinforced rectangular beams are derived by means of the same fundamental principles as for beams with single reinforcement. In deriving the following formulas the compression in the concrete is assumed to follow the linear law and the tension in it is neglected; the formulas then apply to working conditions only.

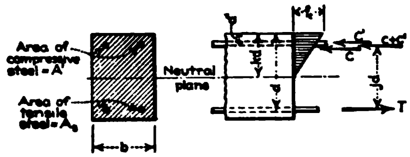


Fig. 31.

Let A' = cross-sectional area of compressive reinforcement (Fig. 31).

d' = distance from the compressive face of the beam to the center of the compressive reinforcement.

p' = ratio of cross-section of steel in compression to cross-section of beam above the tensile steel $= \frac{A'}{bd}$.

f'_s = compressive unit stress in steel.

$$k = \sqrt{2n\left(p + p'\frac{d'}{d}\right) + n^2(p + p')^2} - n(p + p') \qquad (1)$$

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (1A)$$

$$j = \frac{k^2\left(1 - \frac{1}{3}k\right) + 2p'n\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{k^2 + 2p'n\left(k - \frac{d'}{d}\right)} \qquad (2)$$

$$f_s = \frac{M}{A_s jd} = \frac{M}{pjbd^2} \qquad (3)$$

$$f_c = \frac{f_s k}{n(1 - k)} \qquad (4)$$

$$f'_s = f_s \frac{k - \frac{d'}{d}}{1 - k} \qquad (5)$$

$$f_s = \frac{f_c n(1 - k)}{k} \qquad (6)$$

$$M_s = bd^2 f_s pj \qquad (7)$$

$$d = \frac{M}{A_s f_s j} \qquad (8)$$

The cases which may be met with in practice, with the method of solution in each instance indicated, are as follows:

1. To determine fiber stresses.

   Compute p, p', and $\frac{d'}{d}$.

   Solve for k from formula (1) and j from formula (2).

   Substitute value of j in formula (3) and value of k in formulas (4) and (5).

   Solve directly for fiber stresses.

2. To determine moment of resistance.

   Compute p, p', and $\frac{d'}{d}$.

Solve for k and j.

Substitute value of k in formula (6) and find value of f_s when the maximum allowable value of f_c is substituted.

Solve formula (7) for M_s using either the value of f_s determined from formula (6) or the allowable value of f_s, whichever is the lesser.

3. To determine d for a given b and given values of A_s, $\frac{p}{p'}$, f_c, and f_s.

(Trial method. Best shown by use of diagrams. See illustrative problem, page 347.)

4. To determine p and p', or only p' (see Art. 27a).

Formulas for shear, bond, and web reinforcement are the same for double-reinforced beams as for beams with tensile steel only. When using formulas for shear and bond stress along horizontal tension rods of beams double-reinforced, an average value of $j = 0.85$ may be taken.

27a. Formulas for Determining Percentages of Steel in Double-reinforced Rectangular Beams.[1]—The formulas given below are based on the fundamental fact that for any given values of f_c and f_s, k has exactly the same value regardless of the shape or type of beam. This single value for all beams is expressed by the formula

$$k = \frac{1}{1 + \frac{f_s}{nf_c}}$$

It follows from this that if steel is added to the section without changing the extreme fiber stresses, this added tensional and compressive steel must form a balanced couple whose stresses conform to the stresses already in the section.

Let p_1 = steel ratio for the beam without compressive steel.
p_2 = steel ratio for the added tensional steel.
$p = p_1 + p_2$.
p' = steel ratio for compressive steel.
M_1 = moment of the beam without compressive steel.
M_2 = moment of the added steel couple.
$M = M_1 + M_2$.

Then

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \qquad (1)$$

$$p_1 = \frac{f_c k}{2f_s} \qquad (2)$$

$$M_1 = f_s p_1 \left(1 - \frac{k}{3}\right) bd^2 \qquad (3)$$

$$M_2 = M - M_1 \qquad (4)$$

$$p_2 = \frac{M_2}{f_s\left(1 - \frac{d'}{d}\right) bd^2} \qquad (5)$$

$$p = p_1 + p_2 \qquad (6)$$

$$p' = p_2 \frac{1 - k}{k - \frac{d'}{d}} \qquad (7)$$

(See page 348 for illustrative problem.)

28. Deflection of Rectangular Beams.—Fig. 32 gives the general form of a deflection diagram for a reinforced-concrete beam. The portion AB shows the deflection before the concrete has begun to fail in tension near the center of the beam, BC shows the deflection during

[1] From thesis by ROBERT S. BEARD submitted to graduate school of University of Kansas in partial fulfillment of the requirements for the Master's Degree.

a second or readjusting stage, and CD the deflection with the steel near the center of beam carrying practically all the tension.

28a. Maney's Method.[1]—The deflection of a reinforced-concrete beam of whatever shape may be determined by the formula

$$D = c\frac{l^2}{d}(e_c + e_s)$$

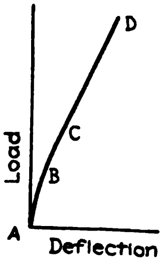


Fig. 32.

where

D = maximum deflection (if desired in inches, the units specified below should be used).

l = span (inches).

d = depth of the beam to the center of the steel (inches).

e_c = unit deformation in extreme fiber for the concrete $= \frac{f_c}{E_c}$.

e_s = unit deformation in extreme fiber for the steel $= \frac{f_s}{E_s}$.

$c = \frac{c_1}{c_2}$ in which

c_1 = the numerical coefficient in the formula for deflection of homogeneous beams, $D = c_1\frac{Wl^3}{EI}$, depending on the loading and on how the ends are supported.

c_2 = the numerical coefficient in the formula for bending moment, $M = c_2wl^2$,

for a simple beam loaded at center, $c = 1/12$ or 0.0833
uniformly loaded, $c = 5/48$ or 0.1041
loaded at the third points, $c = 23/216$ or 0.1065
for a beam with fixed ends, loaded at center, $c = 1/24$ or 0.0416
uniformly loaded, $c = 1/32$ or 0.0313
loaded at the third points, $c = 5/144$ or 0.0347

28b. Turneaure and Maurer's Method.[2]—Turneaure and Maurer recommend that 8 to 10 be used for n in the formulas which they have derived, and which are given below. They also state that the formulas presented are the result of modifying the deflection formulas for homogeneous beams in accordance with the following assumptions:

1. The representative or mean section has a depth equal to the distance from the top of the beam to the center of the steel.

2. It sustains tension as well as compression, both following the linear law.

3. The proper mean modulus of elasticity of the concrete equals the average or secant modulus up to the working compressive stress.

4. The allowance for steel in computing the moment of inertia of the mean section should be based on the amount of steel in the mid-sections, since stirrups and bent-up rods do not affect stiffness materially for working loads.

The following are the deflection formulas for rectangular reinforced concrete beams:

$$D = \frac{c_1}{E_s} \cdot \frac{Wl^3}{bd^3} \cdot \frac{n}{\alpha} \tag{1}$$

$$\alpha = 1/3[k^3 + (1 - k)^3 + 3np(1 - k)^2] \tag{2}$$

$$k = \frac{1 + 2np}{2 + 2np} \tag{3}$$

From equations (2) and (3), the value of α for any values of p and n may be computed, and then the deflection from equation (1). The notation employed in the above formulas is as follows:

[1] See paper by G. A. Maney, presented before the seventeenth annual meeting of the American Society for Testing Materials.

[2] "Principles of Reinforced Concrete Construction" 2d Edition, p. 116.

D = maximum deflection (if desired in inches, the units specified below should be used).
b = breadth of the beam (inches).
d = depth of the beam to the center of the steel (inches).
W = total load (pounds).
l = span (inches).
p = steel ratio.
E_s = modulus of elasticity of the reinforcing steel (pounds per square inch).
n = ratio of the moduli of elasticity of steel and concrete.
α = a numerical coefficient depending on p and n.
k = proportionate depth of the neutral axis.
c_1 = the numerical coefficient in the formula for deflection of homogeneous beams, $c_1 \frac{Wl^3}{EI}$, depending on the loading and support. For example,

for a cantilever loaded at the end, $c_1 = \frac{1}{3}$
for a cantilever uniformly loaded, $c_1 = \frac{1}{8}$
for a simple beam loaded at center, $c_1 = \frac{1}{48}$
for a simple beam uniformly loaded, $c_1 = \frac{5}{384}$
for a beam with fixed ends, load at the center, $c_1 = \frac{1}{192}$
for a beam with fixed ends, uniformly loaded, $c_1 = \frac{1}{384}$

The following are the deflection formulas for reinforced concrete T-beams (referred to later):

$$D = \frac{c_1}{E_s} \cdot \frac{Wl^3}{bd^3} \cdot \frac{n}{\beta}$$

$$\beta = \tfrac{1}{3}[k^3 - \left(1 - \frac{b'}{b}\right)\left(k - \frac{t}{d}\right)^3 + \frac{b'}{b}(1-k)^3 + 3pn(1-k)^2$$

$$k = \frac{np + \frac{1}{2}\left[\frac{b'}{b} - \frac{b'}{b}\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^2\right]}{np + \frac{b'}{b} - \frac{b'}{b}\left(\frac{t}{d}\right) + \frac{t}{d}}$$

in which β is a coefficient depending upon the steel ratio and n, and other symbols as before.

29. Slabs.—A reinforced-concrete slab should be figured in the same manner as a rectangular beam, the bending moment being usually computed for a width of slab equal to 1 ft. The ratio of steel in a slab is most readily found by dividing the cross-section of one bar by the area between the centers of two adjacent bars, this area being the spacing of the bars multiplied by the depth of steel below the top of slab.

Slab bars should not be placed too far apart to properly take stress directly nor yet should they be spaced so close that the concrete cannot be properly placed between them. The main tensile reinforcement should not be spaced farther apart than $2\frac{1}{2}$ times the thickness of the slab. The minimum limit should be about the same as in beams.

Shearing failures are not usually important in slabs, but in special cases of heavy loading the same care should be used as in the design of large beams.[1]

29a. Moments in Continuous Slabs.—For uniformly loaded spans, fully continuous over two or more intermediate supports, a moment of $\frac{1}{12}wl^2$ may be used both in the centers of all spans and over all supports, for both dead and live loads. For slabs continuous for two spans only, with ends restrained, the bending moment both at the center support and near the middle of span should be taken as $\frac{wl^2}{10}$. For very unequal spans or spans of unusual length, the moments should be computed more accurately.

29b. Provision for Negative Moment in Continuous Slabs.—Slabs having spans of any appreciable length should be reinforced against negative moment. This may be done by bending up a part of the rods in the spans on each side of a support and extending each

[1] See p. 344 for illustrative problems using diagrams. For flat slab floors, see chapter in Sect. 11.

set of bent rods along the top of beam into the adjoining span. The bend in the bars should be near the ¼ points in the span, and usually at an angle of 30 deg. with the horizontal. Too sharp an angle may tend to crack the slab.

When placing slab reinforcement in long spans, a top reinforcement at least to the third point will be desirable. In ordinary spans the steel should at least be lapped a sufficient distance over supports to provide adequate bond strength, and the steel should be bent up far enough from the support to provide properly for negative moment.

29c. Floor Slabs Supported Along Four Sides.—When a floor panel is square, or nearly so, the slab may advantageously be reinforced in both directions. Exact analysis of stresses in such a case is impossible, but some important facts have been brought out by approximate solutions for uniform loading. The theory applied in such an analysis depends upon the fact that the load at any point on the slab is distributed to the two systems of reinforcing bars at that point, in proportion to the stiffness of the beam elements lying in those directions.

The following recommendations are from the report of the Joint Committee:

Floor slabs having the supports extending along the four sides should be designed and reinforced as continuous over the supports. If the length of the slab exceeds 1½ times its width, the entire load should be carried by transverse reinforcement.

For uniformly distributed loads on square slabs, one-half the live and dead load may be used in the calculations of moment to be resisted in each direction. For oblong slabs, the length of which is not greater than 1½ times their width, the moment to be resisted by the transverse reinforcement may be found by using a proportion of the live and dead load equal to that given by the formula

$$r = \frac{l}{b} - 0.5$$

where l = length and b = breadth of slab. The longitudinal reinforcement should then be proportioned to carry the remainder of the load.

In placing reinforcement in such slabs account may well be taken of the fact that the bending moment is greater near the center of the slab than near the edges. For this purpose two-thirds of the previously calculated moments may be assumed as carried by the center half of the slab and one-third by the outside quarters.

29d. Cross-reinforcement in Slabs.—When the length of a floor panel is large compared to its breadth, the longitudinal reinforcement (that is, reinforcement parallel with the length) is of little value in carrying loads, but a small amount is nevertheless generally desirable in preventing shrinkage and temperature cracks and in binding the entire structure together. It is more important for wide beam spacing than when the beams are closely spaced. The amount of steel to use is usually selected somewhat arbitrarily, and ¼-in. or ⅜-in. rods spaced 18 to 24 in. apart is common practice. The top of the slab over a girder should be reinforced transversely not only for stiffening the girder, but also to provide for the negative bending moment produced with the bending of the slab at right angles to the direction of the principal slab steel.

T-BEAMS

30. T-Beams in Floor Construction.—When a slab and beam (or girder) are built at the same time and thoroughly tied together by means of stirrups, bent-up rods, and cross-slab reinforcement, a part of the slab may be considered to act with the upper part of the beam in compression. This form of beam is called a T-beam, and the extra amount of concrete in the compressive part of such a beam makes possible a considerable saving over the rectangular form. The thickness of the flange is fixed by the thickness of slab required to support its load, but the width of slab which can be taken as effective flange width must be selected somewhat arbitrarily.

31. Tests of T-beams.—T-beams are found to fail under essentially the same conditions as rectangular beams, and the same general principles apply. Tests show that the maximum load carried can be materially increased by placing cross bars in the top of slab and by adding fillets between the flange and the beam. Cross bars are found to be actually needed to insure T-beam action but fillets are not required in ordinary cases.

T-beams with projections of flange on each side of web of 10.5 times the thickness of the slab have been found to carry a load only 5% larger than beams with projections of 6.8 times the thickness of the slab. No appreciable difference was found in the latter beams between the deformations at the edge of flange and the deformations in the flange at the web.

32. Flange Width.—The Joint Committee has recommended the following rules for determining flange width:

(*a*) It shall not exceed one-fourth of the span length of the beam.

(*b*) Its overhanging width on either side of the web shall not exceed 6 times the thickness of the slab.

Beams in which the T-form is used only for the purpose of providing additional compression area of concrete should preferably have a width of flange not more than 3 times the width of the stem and a thickness of flange not less than one-third of the depth of the beam.

33. Bonding of Web and Flange.—The web and flange of a T-beam can be considered well tied together when slab reinforcement crosses the beam and when the web reinforcement extends well up into the slab. The bonding should be especially well looked after near the end of beam, and this is generally accomplished by means of the bent rods and stirrups brought up as high as possible, in addition to the slab reinforcement (as mentioned) acting at right angles to the length of the beam. Along the center of the beam the differential stresses between the beam and slab are not large, but it is better to insert vertical stirrups extending up into the slab at occasional intervals since shrinkage of the concrete is apt to part the slab from the beam if there is not some means to hold the two together mechanically. The thinner the sections, the more thorough should be the bonding.

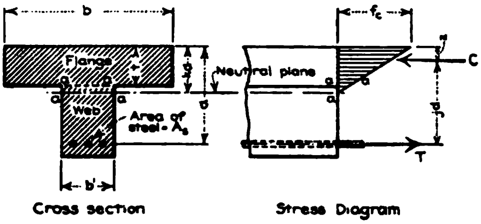


FIG. 33.

34. Flexure Formulas.—With a T-beam it is necessary to distinguish two cases; namely, (1) the neutral axis in the flange, and (2) the neutral axis in the web.

Case I. The Neutral Axis in the Flange.—All formulas for "moment calculations" of rectangular beams apply to this case. It should be remembered, however, that b of the formulas denotes flange width, not web width, and p (the steel ratio) is $\frac{A_s}{bd}$, not $\frac{A_s}{b'd}$ (Fig. 33).

Case II. The Neutral Axis in the Web.—The amount of compression in the web (*aaaa*, Fig. 33) is commonly small compared with that in the flange, and is generally neglected. The formulas to use, assuming a straight-line variation of stress and neglecting the compression in the web, are:

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \tag{1}$$

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \tag{2}$$

$$k = \frac{pn + \frac{1}{2}\left(\frac{t}{d}\right)^2}{pn + \frac{t}{d}} \tag{3}$$

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \tag{4}$$

$$jd = d - z \tag{5}$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3 \left(\frac{1}{2pn}\right)}{6 - 3\frac{t}{d}} \tag{6}$$

$$f_s = \frac{M}{A_s jd} = \frac{M}{pjbd^2} \tag{7}$$

$$f_c = \frac{f_s k}{n(1 - k)} \tag{8}$$

$$f_s = \frac{f_c n(1 - k)}{k} \tag{9}$$

$$M_s = f_s A_s jd \tag{10}$$

$$M_c = f_c\left(1 - \frac{t}{2kd}\right) bt \cdot jd \tag{11}$$

Approximate formulas can also be obtained. From the stress diagram, Fig. 33, it is clear that the arm of the resisting couple is never as small as $d - \frac{1}{2}t$, and that the average unit compressive stress is never as small as $\frac{1}{2}f_c$, except when the neutral axis is at the top of the web. Using these limiting values as approximations for the true ones,

$$M_c = \tfrac{1}{2}f_c bt(d - \tfrac{1}{2}t) \tag{a}$$

$$M_s = A_s f_s(d - \tfrac{1}{2}t), \text{ or } A_s = \frac{M}{(f_s)(d - \frac{1}{2}t)} \tag{b}$$

The errors involved in these approximations are on the side of safety.

Formulas which take into account the compression in the stem are recommended where the flange is small compared to the stem. Such formulas may be found in the report of the Joint Committee, and are as follows:

$$kd = \sqrt{\frac{2ndA_s + (b - b')t^2}{b'} + \left(\frac{nA_s + (b - b')t}{b'}\right)^2} - \frac{nA_s + (b - b')t}{b'}$$

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)b + [(kd - t)^2(t + \frac{1}{3}(kd - t))]b'}{t(2kd - t)b + (kd - t)^2b'}$$

$$jd = d - z$$

$$f_s = \frac{M}{A_s jd}$$

$$f_c = \frac{2Mkd}{[(2kd - t)bt + (kd - t)^2b']jd}$$

34a. Case II Formulas for Determining Dimensions and Steel Ratio for Given Working Stresses.[1]—The following formulas are sometimes useful in the design of T-beams when the neutral axis is in the web:

$$\frac{t}{d} = \frac{R_1 - \sqrt{R_2^2 - 12f_c R_1}}{2R_1} \tag{12}$$

in which

$$R_1 = f_c + \frac{f_s}{n}$$

$$R_2 = 3\left(\frac{M}{bt^2} + f_c + \frac{f_s}{2n}\right)$$

and

$$p = \left(\frac{t}{d}\right)\left(\frac{f_c}{f_s}\right) - \left(\frac{t}{d}\right)^2\left(\frac{R_1}{2f_s}\right) \tag{13}$$

(Formula (12) should be solved by exact methods as the slide rule does not give satisfactory results.)

If the depth of beam is fixed by the headroom available, formula (13) gives directly the proper percentage of steel for given working stresses.

[1] From thesis by Robert S. Beard submitted to graduate school of University of Kansas in partial fulfillment of the requirements for the Master's Degree.

35. Designing for Shear.—Since a T-beam will usually have ample strength in compression for any ordinary depth of beam likely to be selected, the design of the stem of the T, or the beam below the slab, is largely a question of providing sufficient concrete to take care of the shearing stresses and to give a good layout of the tension rods. The manner of providing reinforcement for shearing stresses in T-beams is similar to that described for rectangular beams. In T-beams, however, the reinforcement for shear should run well up into the slab in order to tie the beam and slab together. The shearing strength of a T-beam is about the same as that of a rectangular beam of the same depth and a width equal to the width of the stem of the T.

36. General Proportions of T-beams.—T-beams should not be made too deep in proportion to the width of stem as such forms are relatively weak at the junction of stem and flange. The width should preferably be from one-third to one-half the depth in ordinary cases. For large beams the width may be made from one-third to one-fourth the depth.

All re-entrant angles in concrete are points of weakness and such angles should, therefore, be avoided.

37. Economical Considerations.—When a floor slab forms the flange of a T-beam, it is possible to determine economical proportions for the stem.

Consider a portion of a rectangular beam one unit in length.

Let c = cost of concrete per unit volume.
r = ratio of cost of steel to cost of concrete per unit volume.
C = cost of beam per unit length.
d = depth of beam below slab.

Then

$$C = c\left[b'd' + \frac{rM}{f_s(d' + \frac{1}{2}t)}\right]$$

using the approximate formula (*b*) on page 309.

When d' is fixed by the headroom available, the cost will be a minimum when b' is made as small as possible, and its value will then be determined by the shearing stress or by the space required for the rods. The expression also shows that the cost will decrease with increased values of f_s, and that with a fixed value of $b'd'$ the cost decreases with increase in depth. If the value of b' is assumed as fixed, then there is a definite value of d which will give minimum cost. The following expression has been deduced from the preceding equation and will give the value of d for minimum cost when the value of b' is fixed:

$$d = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2}$$

From this expression the best depths for various assumed widths may readily be determined and the desirable proportions finally selected.

The following table is convenient in determining values of r:

Cost of steel, cts. per lb.	Cost of concrete, dollars per cubic yard							
	5	6	7	8	9	10	11	12
1	26	22	19	16				
1½	40	33	28	25	22			
2	53	44	38	33	29	26		
2½	66	55	47	41	37	33	30	
3	80	66	57	50	44	40	36	33
3½	92	78	66	58	51	46	42	38
4	..	88	75	66	59	53	48	44

38. Conditions Met with in Design of T-beams.—In practice the design of T-beams will take one of the following forms with method to be followed in each case indicated:

1. To find moment of resistance or fiber stresses.

   The values of k and j may be found from equations (3) and (6), or from equations (2), (4) and (5), on page 308, and then the values of the fiber stresses from equations (7) and (8), or the moment of resistance from equations (9) and (10). When the moment of resistance depends upon the concrete, equation (9) is useful in determining the value of f_s to use in equation (10). To obtain this value, the maximum allowable value of f_c should be inserted in equation (9). (If the value of k is found to be less than $\frac{t}{d}$, then the problem falls under Case I and the formulas for rectangular beams apply.)

2. To design a T-beam in which the flange forms a portion of a floor slab.
   Depth and width of stem of beam should be selected with reference to shearing strength, space for necessary rods, and other considerations. The depth having been selected, the amount of steel may be approximately determined by equation (*b*). The amount of steel being known, the value of j may be determined by equation (6). The value of k should also be found from equation (2) or (3) in order to ascertain if the beam falls under Case I or Case II. The stress in the concrete, corresponding to the allowable working stress in the steel, is then found from equation (8).
3. To find the minimum depth for a single-reinforced T-beam.
   The value of $\frac{t}{d}$, or d, may be found from equation (12) for given working stresses, and the amount of steel from equation (13). The width of stem should then be selected with reference to shearing strength, etc.
4. To design a T-beam not connected with a floor system.
   First method:
   First, select suitable proportions for the web. A flange thickness is then assumed such as to give satisfactory proportions between t and d. The value of $\frac{t}{d}$ is then known and k and j can be determined from equations (1), (4), and (5). The area of steel and the breadth of flange are then found from equations (10) and (11) respectively.
   Second method:
   For any assumed thickness and width of flange, the depth of beam may be determined by equation (12) and the percentage of steel from equation (13). The width of stem should then be selected with reference to shearing strength, etc.

(When making approximate computations for shear or bond stress along the horizontal tension rods, an average value of $j = \frac{7}{8}$ may be assumed, as for rectangular beams.)

39. Design of a Continuous T-beam at the Supports.—Negative bending moment exists at the supports of continuous beams and tensile steel must be placed in the top of beams over supports to prevent cracks opening up at these points. For the usual case of equal spans and indefinite live load, the common method of providing for this negative moment is by bending up one-half of the rods on each side and extending each set over the supports into the adjoining span. The remaining lower horizontal rods in each span are carried horizontally through the supporting columns.

In a design of continuous T-beams at the supports it should be noted that the flange is under tension, that the stress in the concrete is negligible above the neutral axis and that a rectangular section may be considered at such points. The method of design is thus similar to the design of a double-reinforced rectangular beam at the center of span with the exception that the compressive and tensile stresses about the neutral axis are inverted (see page 302).

Since a T-beam in the center of span becomes a rectangular beam over supports, the stress in the tensile steel at the support will generally be greater in ordinary designing than the corresponding stress at the center of beam; that is, this stress will be greater if half the rods are bent up on each side and lap over the support. For this reason, then, when selecting the steel at the center of span, a little more than the required amount at that point should be chosen. It should be noticed, however, that the column has some strengthening action at the support and it will not be necessary to keep too closely to the allowable stress.

T-beams with projections of flange on each side of web of 10.5 times the thickness of the slab have been found to carry a load only 5% larger than beams with projections of 6.8 times the thickness of the slab. No appreciable difference was found in the latter beams between the deformations at the edge of flange and the deformations in the flange at the web.

32. Flange Width.—The Joint Committee has recommended the following rules for determining flange width:

(a) It shall not exceed one-fourth of the span length of the beam.

(b) Its overhanging width on either side of the web shall not exceed 6 times the thickness of the slab.

Beams in which the T-form is used only for the purpose of providing additional compression area of concrete should preferably have a width of flange not more than 3 times the width of the stem and a thickness of flange not less than one-third of the depth of the beam.

33. Bonding of Web and Flange.—The web and flange of a T-beam can be considered well tied together when slab reinforcement crosses the beam and when the web reinforcement extends well up into the slab. The bonding should be especially well looked after near the end of beam, and this is generally accomplished by means of the bent rods and stirrups brought up as high as possible, in addition to the slab reinforcement (as mentioned) acting at right angles to the length of the beam. Along the center of the beam the differential stresses between the beam and slab are not large, but it is better to insert vertical stirrups extending up into the slab at occasional intervals since shrinkage of the concrete is apt to part the slab from the beam if there is not some means to hold the two together mechanically. The thinner the sections, the more thorough should be the bonding.

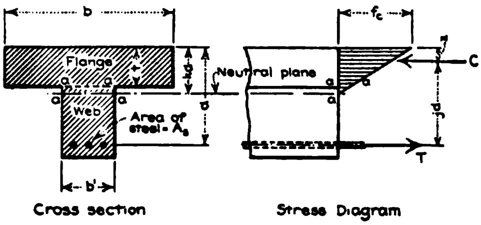


FIG. 33.

34. Flexure Formulas.—With a T-beam it is necessary to distinguish two cases; namely, (1) the neutral axis in the flange, and (2) the neutral axis in the web.

Case I. The Neutral Axis in the Flange.—All formulas for "moment calculations" of rectangular beams apply to this case. It should be remembered, however, that b of the formulas denotes flange width, not web width, and p (the steel ratio) is $\frac{A_s}{bd}$, not $\frac{A_s}{b'd}$ (Fig. 33).

Case II. The Neutral Axis in the Web.—The amount of compression in the web (*aaaa*, Fig. 33) is commonly small compared with that in the flange, and is generally neglected. The formulas to use, assuming a straight-line variation of stress and neglecting the compression in the web, are:

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \tag{1}$$

$$kd = \frac{2ndA_s + bt^2}{2nA_s + 2bt} \tag{2}$$

$$k = \frac{pn + \frac{1}{2}\left(\frac{t}{d}\right)^2}{pn + \frac{t}{d}} \tag{3}$$

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \tag{4}$$

$$jd = d - z \tag{5}$$

$$j = \frac{6 - 6\left(\frac{t}{d}\right) + 2\left(\frac{t}{d}\right)^2 + \left(\frac{t}{d}\right)^3\left(\frac{1}{2pn}\right)}{6 - 3\frac{t}{d}} \tag{6}$$

[illegible]

Approximate [illegible] that the [illegible] compressive [illegible] web. Using these [illegible]

[illegible]

The errors [illegible]

Formulas [illegible] flange is small [illegible] Committee, and are as follows:

$$kd = \sqrt{[illegible]}$$

$$z = [illegible]$$

$$jd = d - z$$

$$f_s = \frac{M}{A_s jd}$$

$$f_c = [illegible]$$

34d. Case II Formulas for [illegible] Given Working Stresses.—[illegible] beams when the neutral axis is in the [illegible]

[illegible]

and

$$p = [illegible] \qquad (13)$$

(Formula (12) should be solved by exact methods as the slide rule does not give satisfactory results.)

If the depth of beam is fixed by the headroom available, formula (13) gives directly the proper percentage of steel for given working stresses.

[1] From thesis by Robert S. Beard submitted to graduate school of University [illegible] fillment of the requirements for the Master's Degree.

A higher compressive stress may be allowed in the concrete at the supports than at the middle of span, because of the fact that the negative moment decreases very rapidly and only a short section is under maximum stress. Also, a slight excess of stress at this point does not in any way endanger the structure but merely increases somewhat the positive moment on the beam.

There are three methods of reducing the compressive stress in the concrete at the bottom of the beam over supports: (1) by increasing the amount of compressive steel in the bottom of the beam; (2) by increasing the area of compressive concrete, which may or may not require a flat haunch depending upon the width of the support; and (3) by increasing the areas of both steel and concrete.

The bond stress along the horizontal rods at the top of a continuous beam over supports may be found by the same formula as is employed for the tension rods at the end of a simply supported beam. However, if bent up rods are employed for web reinforcement and if these same rods are employed to take the tension over supports, the beam is greatly stiffened and the bond stress along the top rods is undoubtedly reduced appreciably below that given by the theoretical formula. This bond stress is affected by the amount of web reinforcement used in a somewhat similar manner to the way the bond stress is affected along the rods at the end of a simply supported beam (see page 284).

In beams considered uniformly loaded, the rods which are bent should extend beyond the center of support to about the fourth point, or in beams of very definite live load to the third point (point of zero moment varies for different loadings), to provide thoroughly for negative moment, and this length should be increased if it is not sufficient to transfer to the concrete through bond, the greatest allowable tensile stress in the rods.

If half of the rods from each span are used over the support, then half of the total number will extend to about the fourth point where the tension due to negative moment becomes zero. At this point the shear is only one-half of what it is at the end of span, if the beam is considered uniformly loaded. Since bond stress due to increment (or decrement) tension varies as shear, a sufficient number of rods are thus run out to the fourth point, and with the bent rods being added gradually to this number until all the rods are acting in this manner at the support, it is clear that this method of design is satisfactory even when the bond stress at the support is the maximum allowable.

Rods should be bent in a position to take as much diagonal tension as possible, usually at an angle between 30 and 45 deg., and the points where the rods are no longer needed at the bottom of beam to resist tension may be found as explained on page 297. It is also necessary to determine where the rods over supports may be bent down. It will be on the safe side, and sufficiently accurate, to consider the curve for negative moment as a straight line between the support and the point of zero moment at the fourth point. (For a very definite live load, zero moment should be assumed at the third point.) With this variation of the moment in mind, it is an easy matter to find where the rods may be bent down at the top of the beam. The designer must use his judgment in the matter, but this much may be said: if a bend cannot be made in a rod, as proposed, due to the controlling points for bending at the top and bottom, a greater number of rods may be employed at the center of span in order to make this bending possible, and the design governed accordingly. It is evident from the above that it will not always be possible to place the rods so as to take all the diagonal tension, in which case both stirrups and bent rods must be used.

Another point to be noted in the design of a continuous beam at the supports is the bond stress of the compressive reinforcement. It can be shown that the bond stress per square inch for the tension and compression rods will be proportional to the product of the diameters by their distances from the neutral axis. Since the compressive steel will generally be nearer the neutral axis than the tensile steel, it follows that, if the compression bars are no larger in diameter than the tension bars, the bond stress per square inch will always be less than that of the tension bars. It is sufficient to consider simply the compressive stress in the steel and provide a sufficient length from this point to the end of the bar to transmit this stress. The working strength of the steel in compression cannot be reached without exceeding the compressive strength of the concrete in which it is embedded. Consequently, in common design it will be satisfactory to provide a lap beyond the support sufficient to take care of compressive stress in the steel equal to the maximum as determined by the concrete.

40. T-beams with Steel in Top and Bottom.—The following formulas correspond to those for rectangular beams given on page 303. $\Delta = \frac{t}{d}$.

$$k = \frac{p + p'\left(\frac{d'}{d}\right) + \frac{\Delta^2}{2n}}{p + p' + \frac{\Delta}{n}} \tag{1}$$

$$j = \frac{\Delta(2k - \Delta) - \frac{\Delta^2}{3}(2k - 2\Delta) + 2p'n\left(k - \frac{d'}{d}\right)\left(1 - \frac{d'}{d}\right)}{\Delta(2k - \Delta) + 2p'n\left(k - \frac{d'}{d}\right)} \tag{2}$$

$$f_s = \frac{M}{A_s jd} = \frac{M}{pjbd^2} \tag{3}$$

$$f_c = \frac{f_s k}{1 - k} \tag{4}$$

$$f'_s = f_s \frac{k - \frac{d'}{d}}{1 - k} \tag{5}$$

$$f_s = \frac{f_c n(1 - k)}{k} \tag{6}$$

$$M_s = bd^2 f_s pj \tag{7}$$

40a. Formulas for Determining Percentages of Steel in Double-reinforced T-beams.[1] $\Delta = \frac{t}{d}$.

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \tag{1}$$

$$p_1 = \Delta\frac{f_c}{f_s}\left(1 - \frac{\Delta}{2}\right) - \frac{\Delta^2}{2n} \tag{2}$$

$$j = \frac{6 - 6\Delta + 2\Delta^2 + \Delta^3\left(\frac{1}{2p_1n}\right)}{6 - 3\Delta} \tag{3}$$

$$M_1 = f_s p_1 jbd^2 \tag{4}$$

$$M_2 = M - M_1 \tag{5}$$

$$p_2 = \frac{M_2}{f_s\left(1 - \frac{d'}{d}\right)bd^2} \tag{6}$$

$$p = p_1 + p_2 \tag{7}$$

$$p' = p_2 \frac{1 - k}{k - \frac{d'}{d}} \tag{8}$$

41. Deflection of T-beams.—Formulas are given in Art. 28.

[1] From thesis by Robert S. Beard submitted to graduate school of University of Kansas in partial fulfillment of the requirements for the Master's Degree. See p. 304 for notation, etc.

SPECIAL BEAMS

42. Wedge-shaped Beams.[1]—The analysis of wedge-shaped beams is useful chiefly in the design of counterforts and buttresses, and in the design of cantilever beams for overhanging sidewalks or roadways on deck bridges. Formulas follow for the general case shown in Fig. 34:

$$k = \sqrt{\frac{2pn \cos \beta_t}{\cos^2 \beta_c} + \frac{p^2n^2 \cos^2 \beta_t}{\cos^4 \beta_c}} - \frac{pn \cos \beta_t}{\cos^2 \beta_c} = \frac{1}{1 + \frac{f_s}{nf_c}}$$

$$j = 1 - \tfrac{1}{3} k$$

$$p = \frac{A_s}{bd} = \frac{\tfrac{1}{2}\left(\frac{\cos^2 \beta_c}{\cos \beta_t}\right)}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)} = \frac{f_c k}{2f_s}\left(\frac{\cos^2 \beta_c}{\cos \beta_t}\right)$$

$$M_c = \tfrac{1}{2} f_c kj\,(bd^2) \cos^2 \beta_c, \text{ or } bd^2 = \frac{2M}{f_c kj(\cos^2 \beta_c)}, \text{ or } f_c = \frac{2M}{kj(bd^2)(\cos^2 \beta_c)}$$

$$M_s = pf_s j\,(bd^2) \cos \beta_t, \text{ or } bd^2 = \frac{M}{pf_s j(\cos \beta_t)}, \text{ or } f_s = \frac{M}{A_s jd\,(\cos \beta_t)}$$

$$f_c = \frac{2f_s p}{k}\left(\frac{\cos \beta_t}{\cos^2 \beta_c}\right) \text{ or } \frac{f_s k}{n(1 - k)}$$

When the compression side of the beam is horizontal, $\cos \beta_c$ becomes equal to unity. Likewise, when the tension side of the beam is horizontal, $\cos \beta_t = 1$. With both top and bottom faces of the beam horizontal, the above formulas reduce to those for a rectangular beam given in Art. 9. The above formulas should not be applied to beams having β_c greater than 45 deg. on account of approximations in the theory which depart more and more from accuracy as β_c increases.

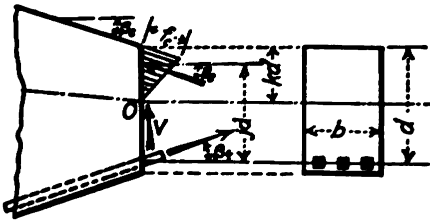


FIG. 34.

Let V_1 represent the total shear to be taken by concrete and web reinforcement, and let V represent total shear on section computed as for a rectangular beam. Then

$$V_1 = V - \frac{M}{d}(\tan \beta_c + \tan \beta_t)$$

β_c and β_t are to be taken as positive when they bear the same relation to the direction of V as shown in Fig. 34, and are to be taken as negative when they bear the reverse relation. The formulas for shear, bond, stirrup spacing, etc. in rectangular beams apply to wedge-shaped beams if V in formulas is replaced by V_1.

43. Beams of Any Complex or Irregular Section.

43a. Analytical Method.[2]—Long and cumbersome formulas for the analysis of complex sections can be avoided by a simple application of general methods of solution to the homogeneous transformed section. Such a general solution is here presented, based upon the standard notation and using the homogeneous section easily obtained by multiplying the steel areas by n, the ratio of the moduli of elasticity of steel and concrete. Thus an equivalent concrete section is obtained, to which the ordinary principles of analysis for homogeneous beams are applied. An equivalent steel section could as well be used if desired.

Adopting the usual fundamental assumptions for the analysis of reinforced-concrete

[1] See *Appendix I* of "Earth Pressure, Retaining Walls and Bins" by WILLIAM CAIN. See also vol. I of "Bridge Engineering" by J. A. L. WADDELL.

[2] Method as given by J. H. CISSEL in *Eng. Rec.*, March 24, 1917.

beams, based upon the straight-line theory, it can easily be shown that an equivalent homogeneous concrete section will be obtained by multiplying the steel area by n. The neutral axis can then be located at the centroid, or center of gravity, of the transformed section. This is done most conveniently by equating the statical moment of the equivalent area on one side to the statical moment of that on the other side.

Knowing the position of the neutral axis, the moment of inertia of the transformed section with respect to this axis may be calculated, and the well-known fundamental formula $M = \frac{fI}{y}$ used to find the resisting moments for a given section, or the fiber stresses produced by a given bending moment.

Fig. 35 gives the dimensions and notation used for the exact analysis of a reinforced-concrete T-beam, including the effect of the compression in the concrete of the stem above the neutral axis. To compute the safe resisting moment for this beam when $A_s = 4$ sq. in., $n = 15$, $f_s = 16{,}000$ and $f_c = 650$, the compressive area is divided, for convenience, into the parts shown in the figure. Equating the statical moments with respect to the neutral axis of the transformed steel and the concrete,

$$(32 \times 4)(y - 2) + \frac{12y^2}{2} = 60(24 - y)$$

Solving,

$$y = 7.31 \text{ and } d - y = 16.69$$

The moment of inertia about this axis can then be computed as follows:

$$nI_s = 15 \times 4 \times (16.69)^2 = \qquad 16{,}713$$
$$I_c = \left(44 \times 7.31^3 \times \tfrac{1}{3}\right) - \left(32 \times 3.31^3 \times \tfrac{1}{3}\right) = \qquad 5{,}342$$
$$I = I_c = nI_s = \qquad 22{,}055$$

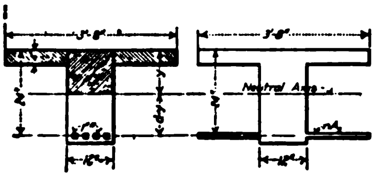


Fig. 35.

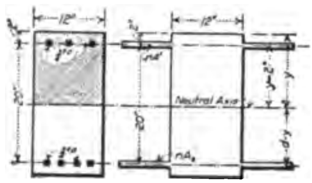


Fig. 36.

The resisting moments are therefore:

$$\frac{f_c I}{y} = M_c = \frac{650 \times 22{,}055}{7.31} = 1{,}960{,}000 \text{ in.-lb.}$$

$$\frac{f_s I}{n(d-y)} = M_s = \frac{16{,}000 \times 22{,}055}{15 \times 16.69} = 1{,}410{,}000 \text{ in.-lb.}$$

The safe resisting moment is thus 1,410,000—given by the steel.

Given the rectangular beam shown in Fig. 36 subjected to a positive bending moment of 760,000 in.-lb with $n = 15$, $A_s = 2.25$ sq. in., and $A' = 1.69$ sq. in., to compute the fiber stresses in the steel and concrete, proceed as follows:

The transformed area of compressive steel $= 15 \times 1.69 = 25.4$
The transformed area of tensile steel $= 15 \times 2.25 = 33.8$

Equating statical moments of tensile and compressive areas about the neutral axis (using transformed section):

$$\frac{12y^2}{2} + 25.4(y - 2) = 33.8(22 - y)$$

Solving,

$$y = 7.58 \text{ in. and } d - y = 14.42 \text{ in.}$$

Then the moment of inertia will be:

$$nI'_s = 15 \times 1.69 \times (5.58)^2 = 789$$
$$nI_s = 15 \times 2.25 \times (14.42)^2 = 7018$$
$$I_c = 12 \times (7.58)^3 \times \tfrac{1}{3} = 1742$$
$$I = I_c + nI_s + nI'_s = 9549$$

Therefore the fiber stresses are:

$$f_c = \frac{My}{I} = \frac{760{,}000 \times 7\ 58}{9549} = 605$$

$$f'_s = \frac{nM(y - 2)}{I} = \frac{760{,}000 \times 5\ 58 \times 15}{9549} = 6680$$

$$f_s = \frac{nM(22 - y)}{I} = \frac{760{,}000 \times 14.42 \times 15}{9549} = 17{,}300$$

43b. Graphical Method.[1]—Application of graphical methods to the location of the neutral axis and the determination of effective depth and resisting moment of reinforced-concrete beams is here proposed for cases of complex sections where the steel is distributed as in beams reinforced for compression or where several layers of rods are used. It is assumed that graphical methods are familiar to the reader, and these methods will be illustrated both for investigating a given section and for designing a section to resist a given bending moment.

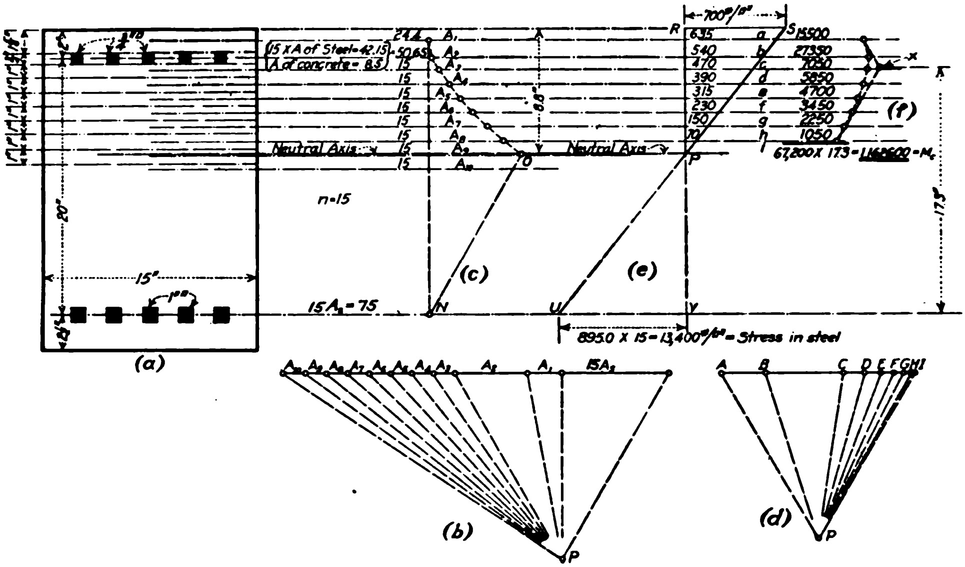


FIG. 37.

To Find the Neutral Axis.—Consider the problem of finding the neutral axis and resisting moment of a given reinforced-concrete beam, reinforced on the compression side, as in Fig. 37*a*. Applying the usual method of finding the centroid of an area, the compression side of the beam is divided into thin slices, taken small in order to avoid large errors, and each slice is represented by a force A_1, A_2, etc. In order to obtain a homogeneous section, the area of the steel is multiplied by $n = 15$. The force diagram for these "area forces" is then drawn, Fig. 37*b*, beginning with nA_s at one end, A_1 next, etc., as shown.

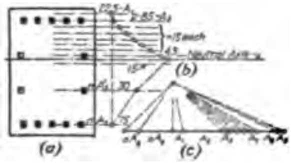


FIG. 38.

The funicular polygon for locating the resultant of these forces is then drawn, Fig. 37*c*, and the intersection *O* of the line *NO*, with the lines parallel to the rays for the compressive areas, gives the position of this resultant, or the centroid of the section, and hence locates the neutral axis. Note that none of the area below *O* is used, according to the usual assumption that tension in concrete is neglected.

To Find the Resisting Moment.—Now in Fig. 37*e* the point *P* is on the neutral axis just found, *RS* is drawn to convenient scale to represent the allowable stress in the concrete, 700 lb. per sq. in., and the straight line *PS* is drawn to represent the variation of stress across the section. By scaling the ordinates to this line at centers of the slices considered, and multiplying by their areas, the values of the resisting forces were obtained, after noting that the steel stress in tension is $UV \times 15$, or 13,400 lb., which is less than the allowable value, showing that the concrete governs. If the steel stress were found to exceed the allowable value, the line *UV* should be made $16{,}000/n$ to locate the stress line *PS*, and the steel would govern. The compressive forces are plotted in the force polygon, Fig. 37*d*, in order to locate their resultant by the polygon, Fig. 37*f* at *X*.

Knowing the magnitude of the compressive forces, by summation, to be 67,200 lb. the effective depth is scaled off as 17.3 in. and the resisting moment is then $67{,}200 \times 17.3 = 1{,}162{,}600$ in.-lb. This has been checked closely by the usual formulas.

[1] Method as given by W. S. WOLFE in *Eng. Rec.* March 24, 1917, and June 27, 1917.

Application to complex sections, as Fig. 38a, can be made as shown by Figs. 38b and 38c, locating the neutral axis.

Designing Double-reinforced Beams.—To design a double-reinforced beam for a given bending moment, first assume the position of the steel and the allowable total depth, and assume a convenient width of, say, 10 in. and

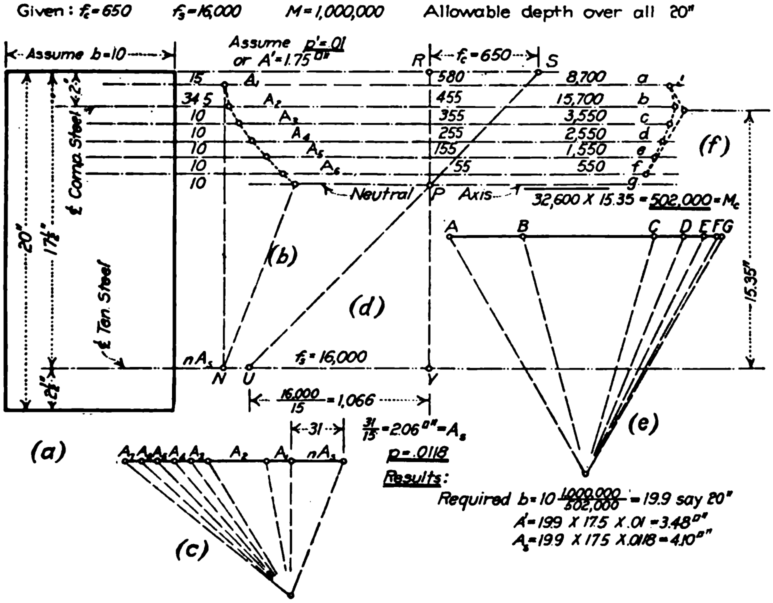


Fig. 39.

a proper value for the compression steel ratio p'. The stress line US is then drawn in Fig. 39d by making RS equal to the allowable compressive stress in the concrete (650) and UV equal to the allowable stress in the steel in tension (16,000) divided by n (15), or 1066. The intersection P then locates the neutral axis.

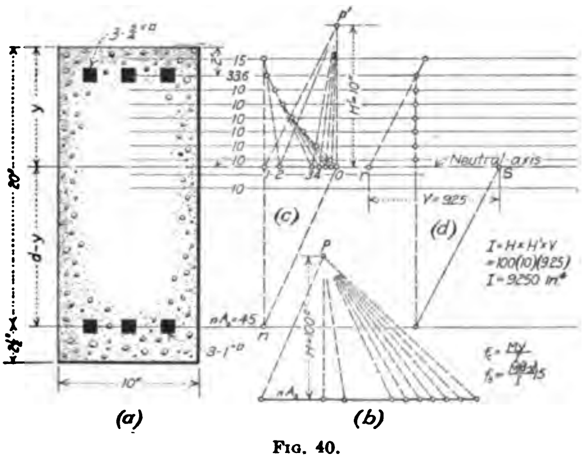


Fig. 40.

The compression side is divided into a number of slices as shown, and the areas, A_1, A_2, etc., computed, knowing the value of A_1, because the compressive steel area is known from the given ratio p'. To find the area of tension steel required, the value of nA_s is obtained from the polygon, Fig. 39b, and the force diagram Fig. 39c, using the closing line NO. nA_s is then scaled off (31) and divided by n (15), giving 2.06 sq. in. for the area desired. The steel ratio p is then easily computed. The resisting moment is obtained as before, using the diagrams, Figs. 39d, 39e, and 39f, by which a resisting moment of 502,000 in.-lb. is obtained, as shown.

Now, if the width is increased and the steel ratios are kept constant, the resisting moment will increase directly as the increase in width. Therefore, if the assumed width be multiplied by the ratio of the required moment to the moment obtained for this width, the required width is found. This computation, given on the diagram, shows that a 20-in. width is necessary. The required areas of steel in compression and tension can then be computed as given, knowing the values of the steel ratios p and p'.

Moment of Inertia of Complex Beam Sections.—Fig. 40a shows the cross-section of a double-reinforced concrete beam, the moment of inertia of which is desired. The compression side of the beam is divided into small slices, the area of the steel being multiplied by 15, and the areas of these slices are laid off in the force polygon, Fig. 40b, together with nA_s, or 15 times the area of the tension steel. For convenience the pole p is taken with the pole distance H equal to 100 sq. in. to the scale at which the areas were laid out in the force polygon. From Fig. 40b the funicular polygon Fig. 40c is drawn locating the neutral axis by the intersection O. It will be noted that all the strings in this funicular polygon are extended until they intersect the neutral axis, the axis about which I is desired, at points 1, 2, 3, etc. Now for convenience the pole p' is taken so that the pole distance H' equals 10 in. to the scale at which the section of the beam was drawn, and p' is connected with points 1, 2, 3, etc. Now, parallel to these rays the corresponding strings in the funicular polygon, Fig. 40d, are drawn, thus following Culmann's approximate method for finding the moment of inertia graphically. From this construction we get $I = H \times H' \times V = 100 \times 10 \times 9.25 = 9250$ in.^{4}, from which f_c and f_s can easily be obtained for any given bending moment by using the formulas

$$f_c = \frac{My}{I} \text{ and } f'_s = \frac{M(d-y)}{I} \quad (15)$$

SHEAR AND MOMENT IN RESTRAINED AND CONTINUOUS BEAMS

44. Span Length for Beams and Slabs.—The Joint Committee recommends the following:

The span length for beams and slabs simply supported should be taken as the distance from center to center of supports, but need not be taken to exceed the clear span plus the depth of beam or slab. For continuous or restrained beams built monolithically into supports, the span length may be taken as the clear distance between the faces of supports. Brackets should not be considered as reducing the clear span in the sense here intended, except that when brackets which make an angle of 45 deg. or more with the axis of a restrained beam are built monolithically with the beam, the span may be measured from the section where the combined depth of beam and bracket is at least one-third more than the depth of the beam. Maximum negative moments are to be considered as existing at the end of the span as here defined.

When the depth of a restrained beam is greater at its ends than at mid-span and the slope of the bottom of the beam at its ends makes an angle of not more than 15 deg. with the direction of the axis of the beam at mid-span, the span length may be measured from face to face of supports.

45. Recommendations of Joint Committee as to Positive and Negative Moments.—In computing the positive and negative moments in beams and slabs continuous over several supports, due to uniformly distributed loads, the Joint Committee recommends the following rules:

(a) For floor slabs, the bending moments at center and at support should be taken at $\frac{wl^2}{12}$ for both dead and live loads, where w represents the load per linear unit and l the span length.

(b) For beams, the bending moment at center and at support for interior spans should be taken at $\frac{wl^2}{12}$ and for end spans it should be taken at $\frac{wl^2}{10}$ for center and interior support, for both dead and live loads.

(c) In the case of beams and slabs continuous for two spans only, with their ends restrained, the bending moment both at the central support and near the middle of the span should be taken as $\frac{wl^2}{10}$.

(d) At the ends of continuous beams, the amount of negative moment which will be developed in the beam will depend on the condition of restraint or fixedness, and this will depend on the form of construction used. In the ordinary cases a moment of $\frac{wl^2}{16}$ may be taken; for small beams running into heavy columns this should be increased, but not to exceed $\frac{wl^2}{12}$.

For spans of unusual length, or for spans of materially unequal length, more exact calculations should be made. Special consideration is also required in the case of concentrated loads.

Even if the center of the span is designed for a greater bending moment than is called for by (a) or (b), the negative m ·ment at the support should not be taken as less than the values there given.

46. Theorem of Three Moments.—By means of the *theorem of three moments*, the moments at the supports of a continuous beam may be deduced. The theorem, assuming level supports, is in its general form as follows (Fig. 40A)[1]:

$$M_1 l_2 I_3 + 2M_2 (l_2 I_3 + l_3 I_2) + M_3 l_3 I_2 = -P_2 l_2^2 I_3 (k_2 - k_2^3) - P_3 l_3^2 I_2 (3k_3 - 2k_3^2 + k_3^3) \quad (1)$$

[1] Derivation of the *theorem of three moments* is as follows:

Let the origin of coördinates be B (Fig. A), with x measured positively toward the left. Considering only the deformation due to the bending moment, and neglecting the deformation due to shearing forces, the equation of the elastic curve is given by

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

in which M is the bending moment at any point x, y. If this expression is integrated once, there results an expression of $\frac{dy}{dx}$, the slope of the tangent to the elastic curve at any point x, y. Thus the slope of the tangent at P is

$$\frac{dy}{dx} = \phi = \int_B^P \frac{M\,dx}{EI}$$

and for the whole member the change in slope of the tangent becomes

$$\phi_1 = \int_A^B \frac{M\,dx}{EI} \quad (a)$$

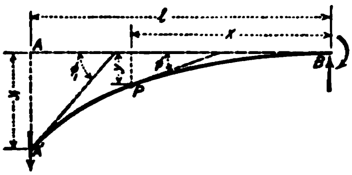


Fig. A.

In Fig. B is shown a beam resting on several supports, all on the same level. Let us consider the portion of the beam between A and B, by cutting it out close to the support by the planes m and n. The part of the beam cut out is shown in Fig. C. Each end has the same shear and moment acting upon it as it did in its original position, thus causing stresses throughout the portion AB identical with those acting before the beam was cut. The moment at any point L, to the left of the load P, is

$$M_L = M_1 + V_1 x' \quad (b)$$

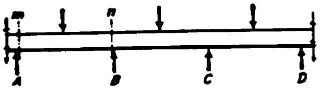


Fig. B.

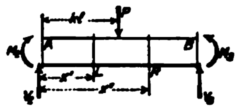


Fig. C.

The moment at any point R to the right of the load is

$$M_R = M_1 + V_1 x'' - P(x'' - kl) \quad (c)$$

By taking moments first about A, and then about B, the values of V_2 and V_1 are found to be

$$V_2 = \frac{M_2 - M_1 + Pkl}{l} \quad (d)$$

$$V_1 = \frac{M_1 - M_2 + P(l - kl)}{l} \quad (e)$$

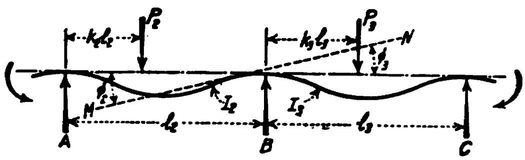


Fig. D.

It is apparent that the moment at any point in the beam may be found if the moment at the reaction can be found. We will, therefore, proceed with the solution of a general case of a continuous girder with concentrated loads.

Consider now a beam resting on several supports on the same level, and loaded as shown in Fig. D. The slope of the tangent to the deflection curve at B, considering the portion of the beam to the left of B, is ϕ_2; and that for the tangent to the deflection curve at B, considering the portion to the right of B, is ϕ_3. Since the deflection curve of a beam must necessarily be continuous, MN is a straight line, and

$$\phi_2 = -\phi_3 \quad (f)$$

This is the general equation of three moments for one concentrated load in each span. When there is more than one concentrated load in each span, the loading may be broken up into a series of cases similar to the above. For each set of loads the right-hand portion of equation (1) is solved, and the results finally combined, and placed equal to the left-hand portion of equation (1).

The theorem may be applied to a beam with uniform loads. For the loading shown in Fig. 41 the theorem has the form

$$M_2l_2I_3 + 2M_3(l_2I_3 + l_3I_2) + M_4l_3I_2 = -\frac{w_2l_2^3I_3}{4} - \frac{w_3l_3^3I_2}{4} \qquad (2)$$

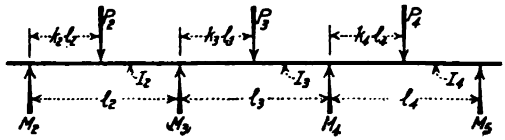


FIG. 40A.

For a constant moment of inertia and all spans equal, equation (1) reduces to

$$M_2 + 4M_3 + M_4 = -P_2l(k_2 - k_2^3) - P_3l(2k_3 - 3k_3^2 + k_3^3)$$

When $k_1 = k_2 = 0.5$ this equation reduces to

$$M_2 + 4M_3 + M_4 = -0.375P_2l - 0.875P_3l$$

When in equation (2) the moment of inertia is a constant and all spans carry the same uniform load

$$M_2l_2 + 2M_3(l_2 + l_3) + M_4l_3 = -\frac{w}{4}(l_2^3 + l_3^3)$$

and when $l_2 = l_3$,

$$M_2 + 4M_3 + M_4 = -\frac{wl^2}{2}$$

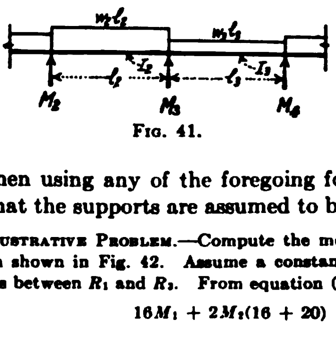


FIG. 41.

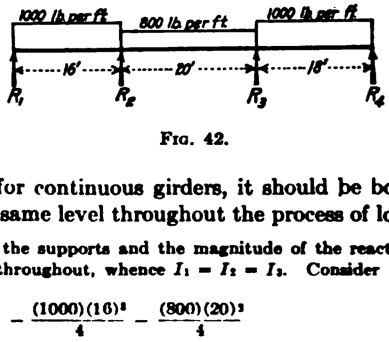


FIG. 42.

When using any of the foregoing formulas for continuous girders, it should be borne in mind that the supports are assumed to be on the same level throughout the process of loading.

ILLUSTRATIVE PROBLEM.—Compute the moments at the supports and the magnitude of the reactions for the beam shown in Fig. 42. Assume a constant section throughout, whence $I_1 = I_2 = I_3$. Consider first the two spans between R_1 and R_3. From equation (2)

$$16M_1 + 2M_2(16 + 20) + 20M_3 = -\frac{(1000)(16)^3}{4} - \frac{(800)(20)^3}{4}$$

If the values of M in equations (b) and (c) are substituted into equation (a), and the latter integrated from A to P_2, and from P_2 to B, and finally the values of V_2 and V_3 as given in equations (d) and (e) are inserted, there results for ϕ_2 the value

$$\phi_2 = \frac{M_2l_2^2 + 2M_3l_2^2 + P_2l_2^3(k_2 - k_2^3)}{6EI_2l_2}$$

In a similar manner the values of ϕ_3 may be found by considering C as the origin of x, and integrating toward the left, whence

$$\phi_3 = \frac{M_4l_3^2 + 2M_3l_3^2 + P_3l_3^3(2k_3 - 3k_3^2 + k_3^3)}{6EI_3l_3}$$

Inserting these values into equation (f) there results

$$M_2l_2I_3 + 2M_3(l_2I_3 + l_3I_2) + M_4l_3I_2 = -P_2l_2^2I_3(k_2 - k_2^3) - P_3l_3^2I_2(2k_3 - 3k_3^2 + k_3^3) \qquad (g)$$

A development precisely similar to the foregoing may be applied to a beam with uniform loads.

whence

$$4M_1 + 18M_2 + 5M_3 = -656{,}000 \qquad (a)$$

Applying equation (2) to the two spans between R_2 and R_4, and noting the change in subscripts accordingly, we obtain

$$20M_2 + 2M_3(20 + 18) + 18M_4 = -\frac{(800)(20)^3}{4} - \frac{(1000)(18)^3}{4}$$

whence

$$10M_2 + 38M_3 + 9M_4 = -1{,}529{,}000 \qquad (b)$$

Since all supports are free, $M_1 = M_4 = 0$. Noting this, and solving (a) and (b) simultaneously,

$$\begin{aligned} 90M_2 + 25M_3 &= -3{,}280{,}000 \\ 90M_2 + 34M_3 &= -3{,}761{,}000 \\ \hline 317M_3 &= -10{,}481{,}000 \\ M_3 &= -33{,}100 \text{ ft.-lb.} \end{aligned}$$

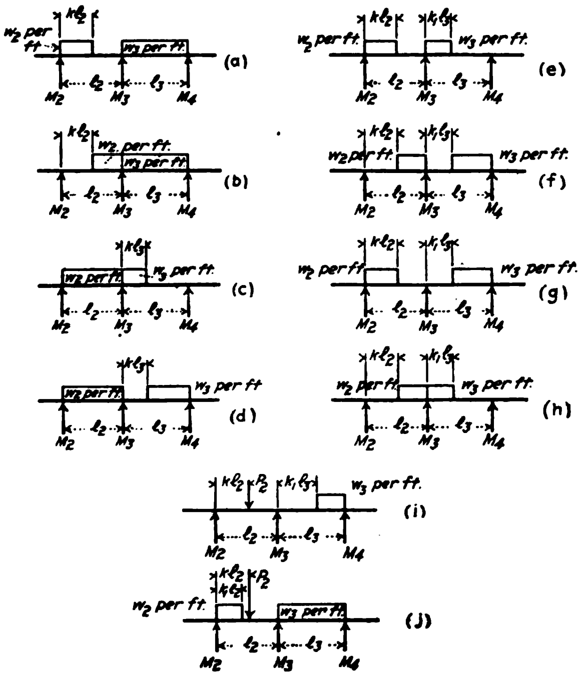


FIG. 43.

Substituting this into (b) we find

$$M_2 = -45{,}600 \text{ ft.-lb.}$$

We may now find the reactions.

$$16R_1 - \frac{(1000)(16)^2}{2} = -45{,}600$$

$$16R_1 = 128{,}000 - 45{,}600 = 82{,}400$$

$$R_1 = 5150 \text{ lb.}$$

$$(5150)(36) - (1000)(16)(28) + 20R_2 - \frac{(800)(20)^2}{2} = -33{,}100$$

$$20R_2 = -33{,}100 - 185{,}400 + 448{,}000 + 160{,}000 = 389{,}500$$

$$R_2 = 19{,}470 \text{ lb.}$$

$$18R_4 - \frac{(1000)(18)^2}{2} = -33{,}100$$

$$R_4 = 7{,}160 \text{ lb.}$$

$$(7160)(38) - (1000)(18)(29) + 20R_3 - \frac{(800)(20)^2}{2} = -45{,}600$$

$$20R_3 = -45{,}600 - 272{,}000 + 552{,}000 + 160{,}000$$

$$R_3 = 18{,}220 \text{ lb.}$$

As a check,

$$R_1 + R_2 + R_3 + R_4 = \text{total load} = 50{,}000 \text{ lb.}$$

Very often a continuous girder may have a uniform load over one or more spans. Fig. 43 will be found to give the possible cases, and the corresponding formulas follow. Should no load be on one of the spans, w for that span becomes zero, and the term containing it will drop out. Only the right-hand side of the general equation is affected by the loading.

Let $M_2l_2 + 2M_3(l_2 + l_3) + M_4l_3$ be denoted by r.

Then (a) $r = -w_2l_2^3\left(\frac{k^2}{2} - \frac{k^4}{4}\right) - \frac{1}{4}w_3l_3^3.$

(b) $r = -w_2l_2^3\left(\frac{1}{4} - \frac{k^2}{2} + \frac{k^4}{4}\right) - \frac{1}{4}w_3l_3^3.$

(c) $r = -\frac{1}{4}w_2l_2^3 - w_3l_3^3\left(k^2 - k^3 + \frac{k^4}{4}\right).$

(d) $r = -\frac{1}{4}w_2l_2^3 - w_3l_3^3\left(\frac{1}{4} - k^2 + k^3 - \frac{k^4}{4}\right).$

(e) $r = -w_2l_2^3\left(\frac{k^2}{2} - \frac{k^4}{4}\right) - w_3l_3^3\left(k_1^2 - k_1^3 + \frac{k_1^4}{4}\right).$

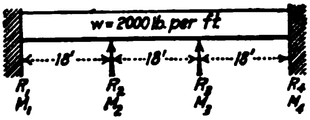


Fig. 44.

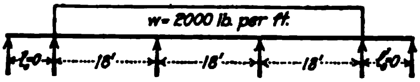


Fig. 45.

(f) $r = -w_2l_2^3\left(\frac{1}{4} - \frac{k^2}{2} + \frac{k^4}{4}\right) - w_3l_3^3\left(\frac{1}{4} - k_1^2 + k_1^3 - \frac{k_1^4}{4}\right).$

(g) $r = -w_2l_2^3\left(\frac{k^2}{2} - \frac{k^4}{4}\right) - w_3l_3^3\left(\frac{1}{4} - k_1^2 + k_1^3 - \frac{k_1^4}{4}\right).$

(h) $r = -w_2l_2^3\left(\frac{1}{4} - \frac{k^2}{2} + \frac{k^4}{4}\right) - w_3l_3^3\left(k_1^2 - k_1^3 + \frac{k_1^4}{4}\right).$

(i) $r = -\Sigma P_2l_2^2(k - k^3) - w_3l_3^3\left(\frac{1}{4} - k_1^2 + k_1^3 - \frac{k_1^4}{4}\right).$

(j) $r = -\Sigma P_2l_2^2(k - k^3) - w_2l_2^3\left(\frac{k_1^2}{2} - \frac{k_1^4}{4} - \frac{1}{4}w_3l_3^3\right).$

Spans similarly located and similarly loaded have the same representative term. Cases (i) and (j) are given to show how concentrated loads may be considered with uniform loads.

When a beam has fixed ends—that is, when the slope of the tangent to the elastic curve at the end is constant for all loading—the theorem of three moments may readily be applied. This is accomplished by supplying a span of zero length at each fixed end, and then by proceeding as before. This satisfies the requirement made in the above definition for fixed ends.

Illustrative Problem.—Determine the moments at the supports and the magnitude of the reactions for the beam shown in Fig. 44. In Fig. 45 the fixed ends have been removed and the spans l_0 and l' put in their places

The equations now become, remembering that $R_0 = R' = 0$, and $M_1 = M_2 = 0$,

$$2M_1 + M_2 = -\frac{1}{4}wl^2$$

$$M_1 + 4M_2 + M_3 = -\frac{1}{2}wl^2$$

$$M_2 + 4M_3 + M_4 = -\frac{1}{2}wl^2$$

$$M_3 + 2M_4 = -\frac{1}{4}wl^2$$

If these equations are solved simultaneously, there results

$$M_1 = M_2 = M_3 = M_4 = -\frac{1}{12}wl^2 = -54{,}000 \text{ ft.-lb.}$$

$$R_1 = R_4 = 18{,}000 \text{ lb.}$$

$$R_2 = R_3 = 36{,}000 \text{ lb.}$$

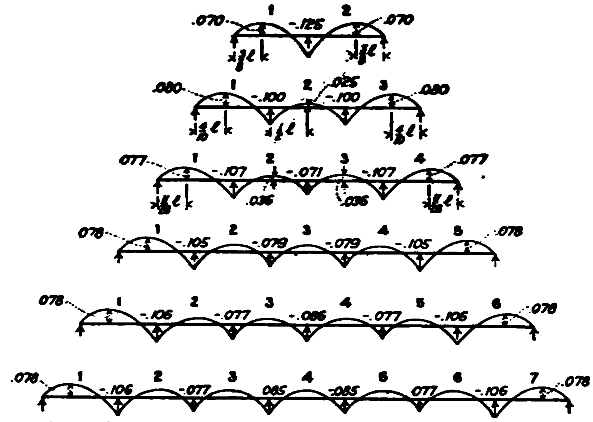


FIG. 46.—Moments in continuous beams; supported ends; uniform load on all spans; spans all equal. Coefficients of (wl^2).

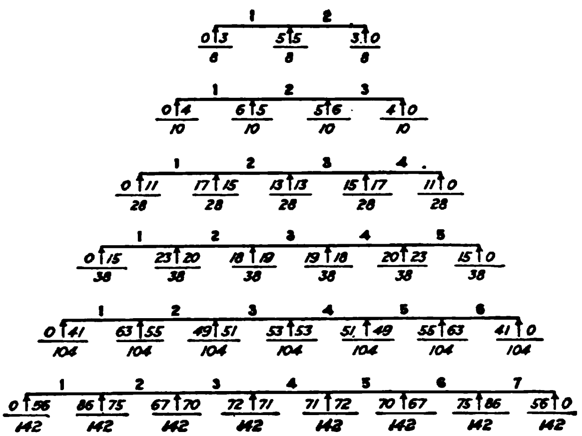


FIG. 47.—Shears in continuous beams; supported ends; uniform load on all spans; spans all equal. Coefficients of (wl).

47. Uniform Load Over All Spans.—In Fig. 46 are given the moments in continuous beams for a uniform load over all spans. Supported ends and equal spans are assumed. Positive moments are plotted above the beam. The shears on each side of the supports are given in Fig. 47. The reaction at any given support is the sum of the two shears at that support.

The maximum moments at (or near) the center of many of the interior spans are not given, as they are small and do not vary greatly from those given for the continuous beam of four spans. Maximum positive moment occurs for zero shear as in simple beams.

In continuous beams with fixed ends, and with uniform load and equal spans (t above), the maximum positive moment occurs at the center of each span and its value in every case is $\frac{wl^2}{24}$. The negative moment over each support equals $\frac{wl^2}{12}$. Shears close to the supports are all equal with a value of $\frac{1}{2}wl$.

The maximum positive moment on a beam of one span, with one end fixed and the other end free, and uniformly loaded is $\frac{9}{128}wl^2$ and the negative moment at the fixed end is $\frac{1}{8}wl^2$.

48. Fixed and Moving Concentrated Loads.—Continuous beams of one, two, and three equal spans and any span length may be figured for fixed and moving concentrated loads by means of the influence lines of Figs. 48 to 52 inclusive.

48a. Influence Lines.—As a load moves over a beam, the shear and moment at a given section will vary. If the value of moment at any point A is plotted as an ordinate at the point where the load is applied, and this process repeated for each position of the load, the result is called an *influence diagram* for the moment at point A; and the curve generated by the extremities of all ordinates is called an *influence line* for the moment at point A. Similar lines may be drawn for shear and for deflections. In structures, influence lines may also be drawn for stress intensities at a given point. The curve gets its name because of the fact that for any

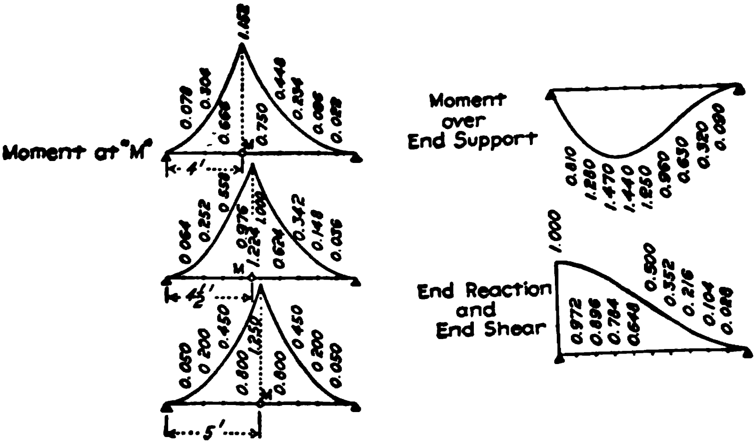


FIG. 48.—Influence lines for one span, fixed ends. (Spans, 10 ft. Load, unity.)

chosen point, it gives the influence on a certain function at that point, for varied positions of the load.

It should be noted that the influence line for moment—for a simple beam, for instance—differs from the moment diagram for that beam. The moment diagram gives the *moment at any point* for *one* position of the load; while the influence line for moment gives the moment at *one* point for *any* position of the load. For each point in the beam there may be drawn an influence line, but each influence line is descriptive of but one point. In Fig. 53 there is drawn an influence line for moment at A. The moment at A is $\frac{Pab}{l}$, and that is the value of the ordinate at A. The ordinate at B is $\frac{Pab}{l} \cdot \frac{x}{a}$ and is the moment at A when the load P is at B.

Suppose the beam to have a load of 1 lb. moving across it. The ordinate at A is then $\frac{ab}{l}$. Usually influence lines are drawn for unit loads. The ordinate at B is then the moment at A when a unit load is placed at B. If the load at B is not unity, then the moment at A will be equal to the load times the ordinate at B for the 1-lb. load.

If the beam is loaded with a uniform load, the moment at A is equal to the load per foot

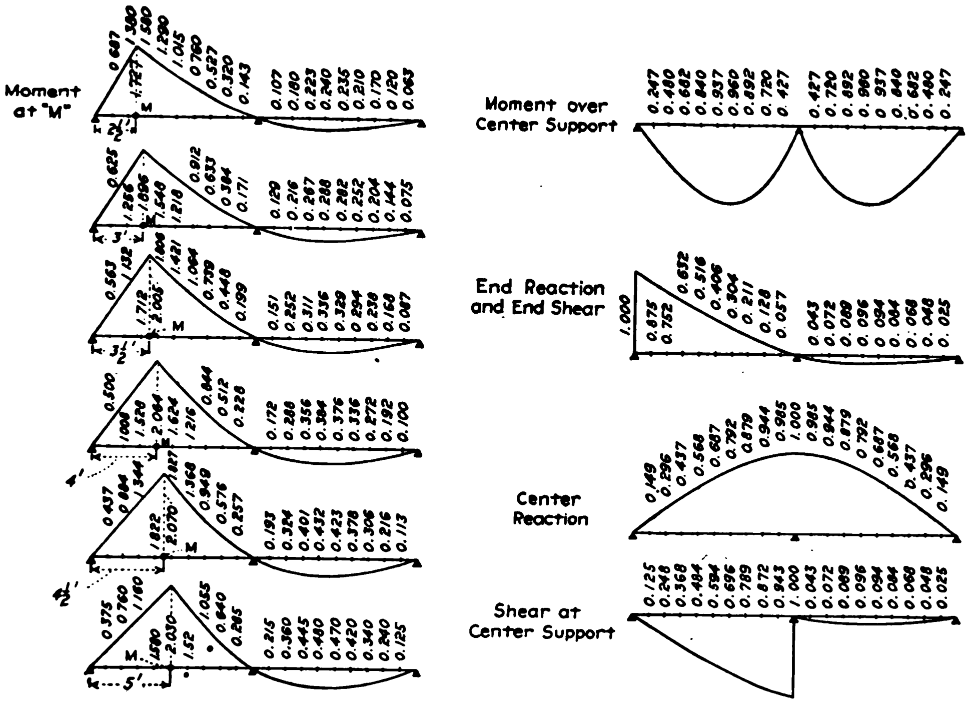


Fig. 49.—Influence lines for two equal spans, supported ends. (Spans, 10 ft. Load, unity.)

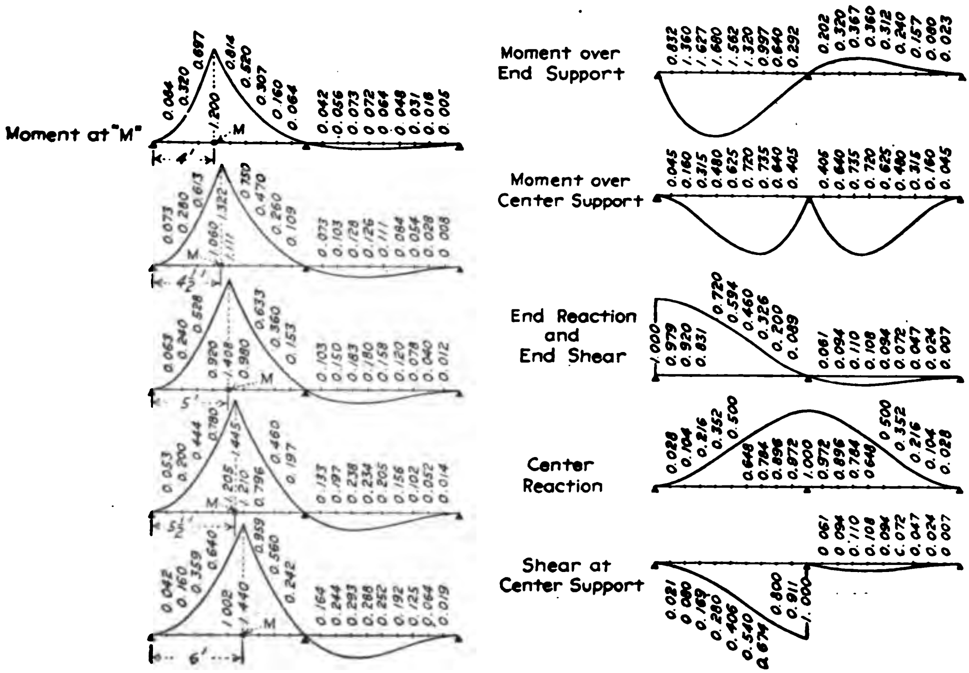


Fig. 50.—Influence lines for two equal spans, fixed ends. (Spans, 10 ft. Load, unity.)

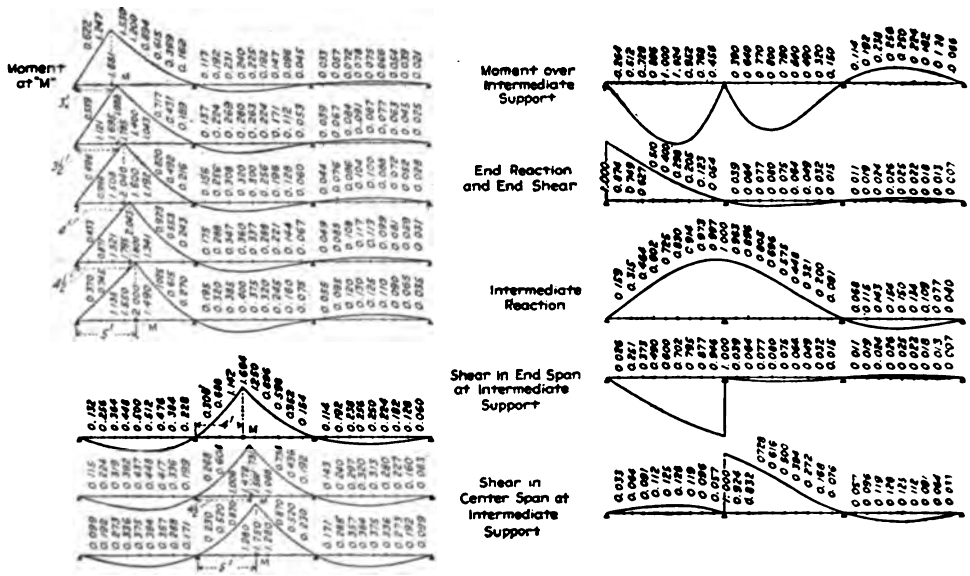


FIG. 51.—Influence lines for three equal spans, supported ends. (Spans, 10 ft. Load, unity.)

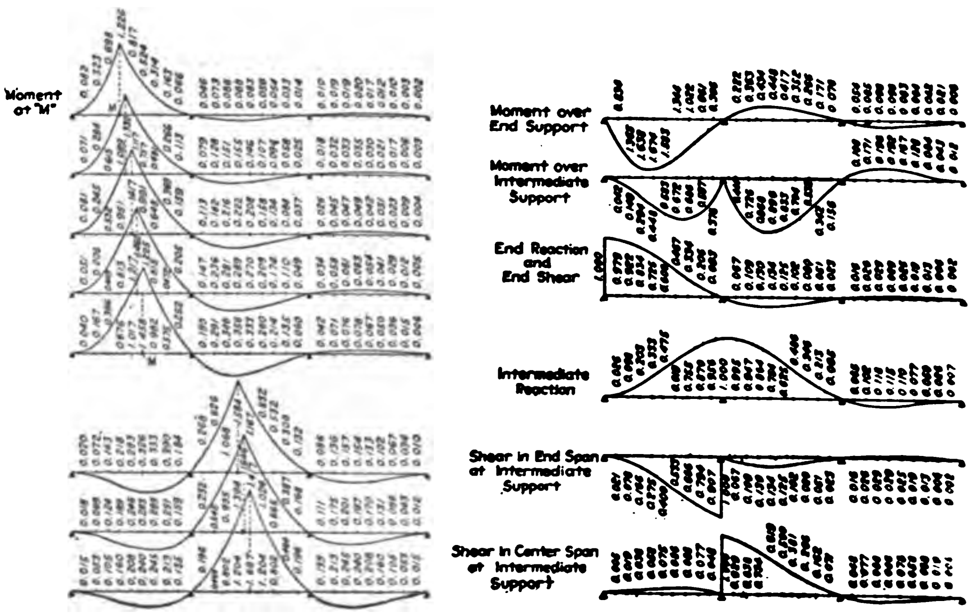


FIG. 52.—Influence lines for three equal spans, fixed ends. (Spans, 10 ft. Load, unity.)

times the area of the influence diagram for the moment at A. In Fig. 53 this is $\left(w \cdot \frac{ab}{l} \cdot l \cdot \frac{1}{2}\right)$ or $\frac{w}{2} \cdot ab$, which is readily recognized as the moment at A for a uniform load. For a partial uniform loading, the load per foot multiplied by the area of the influence diagram for the loaded portion will give the moment at A.

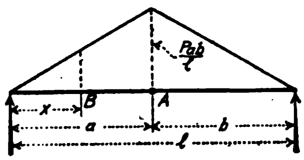


Fig. 53.

Influence lines have been constructed in Figs. 48 to 52 inclusive for continuous beams of equal spans, each of 10 units in length, with the outer ends either fixed or simply supported. If the influence lines are desired for equal spans other than 10 ft. in length, they may be constructed by regarding one unit of length as one-tenth the span. The ordinates will be the same as those plotted here. The ordinates for positive moment are plotted above the line.

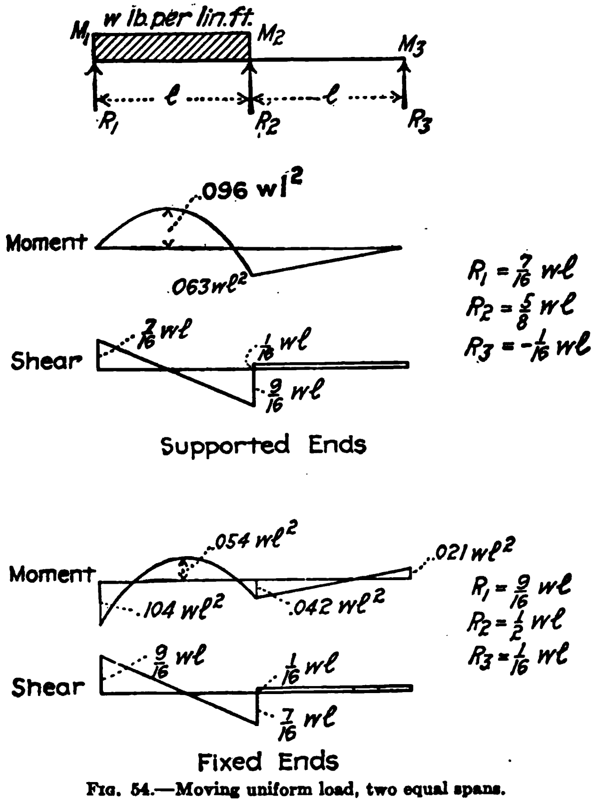


Fig. 54.—Moving uniform load, two equal spans.

For two equal spans with ends either supported or fixed, it is readily seen that the greatest positive moment at any point M will be obtained when the load covers only the span in which

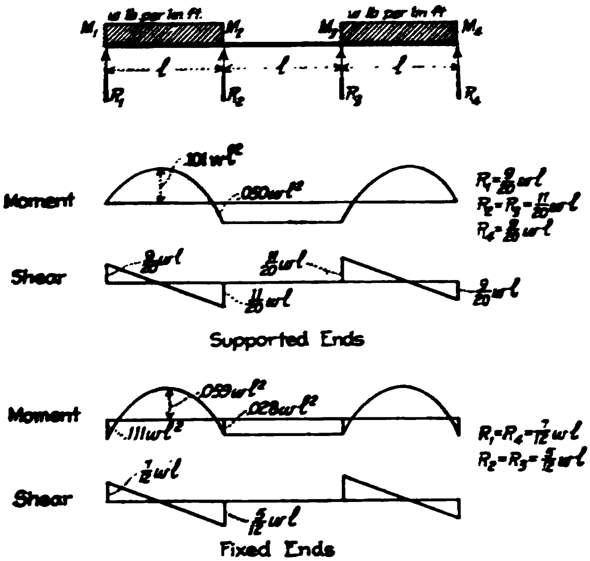


FIG. 55.—Moving uniform load, three equal spans.

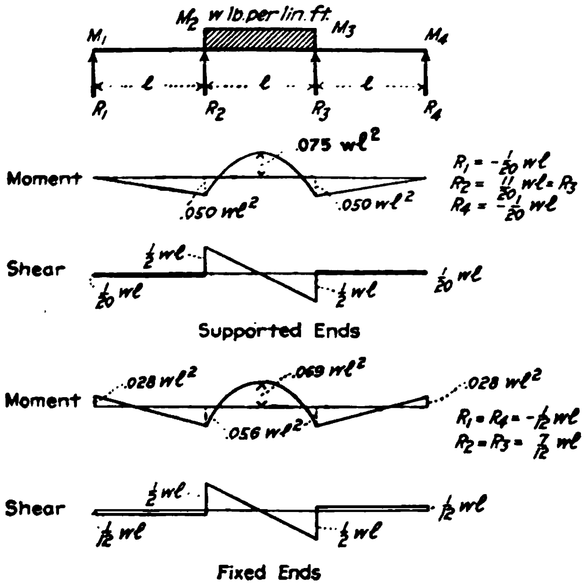


FIG. 56.—Moving uniform load, three equal spans.

M occurs, since the area for that span is positive. The greatest end reaction and end shear will also be obtained when the load is over one span. The greatest center reaction, negative moment (over center support), and shear at center support, will be obtained by fully loading both spans.

For three equal spans, the uniform live load should cover alternate spans to give the greatest positive moment in any span, and to give the greatest end reaction and end shear. For

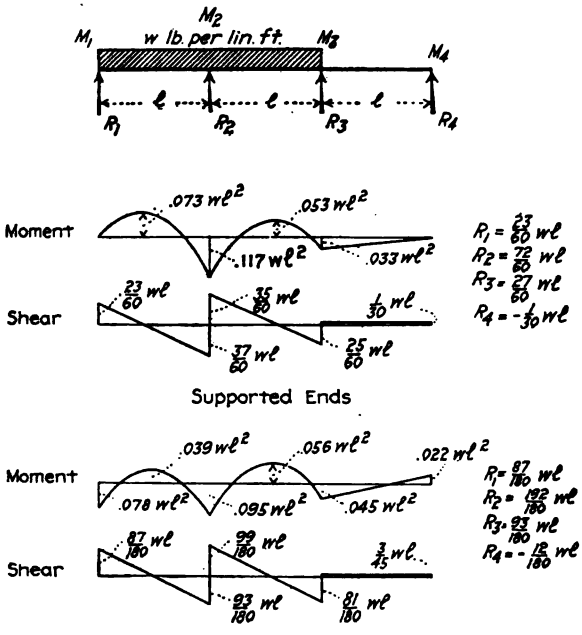


FIG. 57.—Moving uniform load, three equal spans.

intermediate reactions and negative moment over intermediate reactions, the spans adjacent to the reaction in question should be fully loaded.

49. Moving Uniform Loads.—If a uniform load is considered, influence lines indicate the spans which should be loaded in order to obtain the maximum values of the given functions.

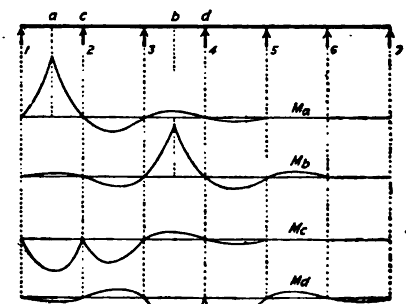


FIG. 58.

Fig. 54 represents the variation in moment and shear for a uniform load on one span of a beam of two equal spans, both fixed and supported ends. Figs. 55, 56 and 57 give values for various loadings with three equal spans.

To illustrate the effect of loads on various spans upon the bending moments, influence lines have been drawn for six equal spans (Fig. 58), for moments at the centers of span 1–2 and 3–4, and at supports 2 and 4. A maximum moment at the center of a span requires each alternate span to be loaded, and a maximum moment at the support requires the two adjacent spans to be loaded and then each alternate span. The effect of loads on remote spans is seen to be small. Influence lines are not drawn for shears since, in a large number of spans, the shears do not differ greatly from those in simple beams.

50. Maximum Moments from Uniform Loads.—The following table gives the values of maximum negative and positive moments for girders of equal spans. Each column headed "Fixed load" gives the maximum moment at the point under consideration for a uniform load covering the entire length. The columns headed "Moving load" give the maximum moment at the point under consideration when the uniform load is placed on certain spans to cause the maximum moment.

MAXIMUM MOMENTS IN CONTINUOUS BEAMS; SUPPORTED ENDS; UNIFORM FIXED AND MOVING LOADS

Coefficients of (wl^2)

No. of spans	Intermediate spans				End spans			
	Fixed load		Moving load		Fixed load		Moving load	
	At center +	At support (−)	At center +	At support (−)	At center +	At 2d support (−)	At center +	At 2d support (−)
Two	.....	.....	.....	....	0.070	0.125	0.096	0.125
Three	0.025	.....	0.075	....	0.080	0.100	0.101	0.117
Four	0.036	0.071	0.081	0.107	0.077	0.107	0.098	0.120 (0.115)[1]
Five	0.046	0.079	0.086	0.111 (0.106)[1]	0.078	0.105	0.099	0.120 (0.116)[1]
Six	0.043	0.086	0.084	0.116 (0.106)[1]	0.078	0.106	0.099	0.120 (0.116)[1]
Seven	0.044	0.085	0.084	0.114 (0.106)[1]	0.078	0.106	0.099	0.120 (0.116)[1]

[1] Where two adjacent spans only are loaded.

The fixed-load coefficients will apply to the dead load when finding the maximum coefficients due to a moving uniform load—the case ordinarily encountered in building construction. Inasmuch as the theoretical maximum moments in continuous beams of five or more spans would involve unreasonable assumptions as to position of the live loads, the values of moment coefficients in small table may be taken.

Nature of load	Intermediate spans		End spans	
	At center	At support	At center	At 2d support
Dead load	0.046	0.086	0.080	0.107
(two spans)	.....	.....	(0.070)	(0.125)
Live load	0.086	0.107	0.101	0.117
(two spans)	.....	.....	(0.096)	(0.125)

Combining the dead and live loads into a single unit for the purpose of determining general moment coefficients which will apply to all ordinary cases, we obtain the values given in table on page 331.

In continuous-beam computations, the beam is assumed as freely supported at the interior supports and the assumption is made that the supports are of no appreciable width. For beams in concrete construction, therefore, the coefficients in the third column of the table should be reduced, and could very well be taken equal to those in the second column. The live load will generally range from two to five times the dead load, but the ratio of 10:1 is given to show the slight variation in moment coefficients for ratios above 5 : 1.

Ratio of live to dead	Intermediate spans		End spans	
	At center	At support	At center	At 2d support
Three or more spans				
2:1	0.073	0.100	0.094	0.114
5:1	0.079	0.104	0.098	0.115
10:1	0.082	0.105	0.099	0.116
Two spans				
2:1	.....	.....	0.087	0.125
5:1	.....	.....	0.092	0.125
10:1	.....	.....	0.093	0.125

From a study of the table, reducing the bending-moment coefficients at the interior supports as above proposed, it is seen that the bending moment at the center and at the support for interior spans may be taken as $\frac{wl^2}{21}$ (0.083 wl^2), and for end spans it may be taken as $\frac{wl^2}{10}$ for center and adjoining support, where w includes both dead and live loads. In the case of two spans only, the bending moment at the center support may be taken as $\frac{wl^2}{8}$, and near the middle of the span as $\frac{wl^2}{10}$. Where the ends of a two-span beam are restrained, the bending moment may well be taken as $\frac{wl^2}{10}$, both at the center support and near the middle of the span.

The shear at each support of continuous beams with fixed ends may be taken as one-half the span load. If the ends are simply supported, the shear in the end spans near the second support will be approximately 0.6wl.

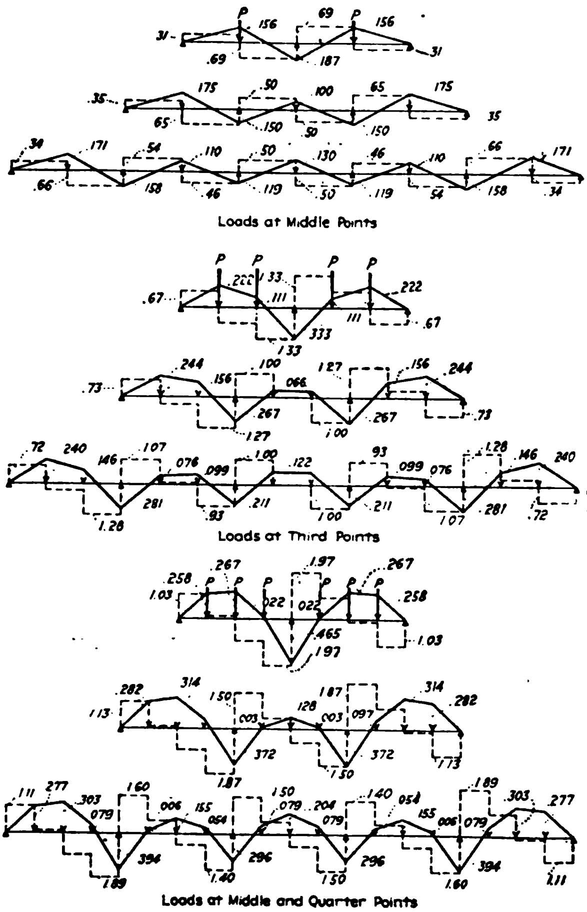


FIG. 59.—Moments and shears in continuous beams; supported ends; spans all equal; concentrated loads as shown. Coefficients of (Pl) for moment. Coefficients of (P) for shear.

51. Beam Concentrations.—Floor systems of beam-and-girder construction impose concentrated loads on the girders at the ends of the floor beams. There may be one or more floor beams built into each girder, depending upon the shape of the panels.

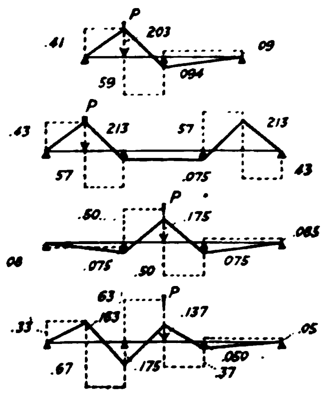


FIG. 60.—Concentrated loads as shown; loads at middle points; two and three equal spans; supported ends. Coefficients of (Pl) for moment. Coefficients of (P) for shear.

Figs. 59 and 62 inclusive give the shears and moments caused by beam concentrations on continuous girders. The girder spans are assumed equal and the ends of the girders as simply supported. In girders with fixed ends and with full loading as shown in Fig. 59, the maximum positive moment for loads at the middle points is the same as the maximum negative moment and equals $0.125Pl$ in every case. For loads at the third points, the maximum positive moment is 0.110 Pl and the maximum negative moment is exactly twice this value. For loads at the middle and quarter points, the maximum positive moment is $0.187Pl$ and the maximum negative is $0.313Pl$. The maximum shears in all cases are the same as in simple beams.

In the following table are given the maximum positive and negative moments due to beam concentrations. Ends of beams are assumed as simply supported.

It is quite evident that the variation of moment coefficients is very nearly the same as shown in the table previously given for uniform loads.

MAXIMUM MOMENTS IN CONTINUOUS GIRDERS DUE TO BEAM CONCENTRATIONS; SUPPORTED ENDS

Coefficients of (Pl)

No. of spans	Intermediate spans				End spans			
	Fixed load		Moving load		Fixed load		Moving load	
	At center +	At support (−)	At center +	At support (−)	At center +	At 2d support (−)	At center +	At 2d support (−)
Loads at middle points								
Two	......	......	......	......	0.156	0.187	0.203	0.187
Three	0.100	......	0.175	......	0.175	0.150	0.213	0.175
Five	0.130	0.119	0.191	0.156[1]	0.171	0.158	0.211	0.174[1]
Loads at third points								
Two	......	......	......	......	0.222	0.333	0.278	0.333
Three	0.066	......	0.200	......	0.244	0.267	0.289	0.311
Five	0.122	0.211	0.228	0.276[1]	0.240	0.281	0.286	0.309[1]
Loads at middle and quarter points								
Two	......	......	......	......	0.267	0.465	0.383	0.465
Three	0.128	......	0.312	......	0.314	0.372	0.406	0.438
Five	0.204	0.296	0.352	0.389[1]	0.303	0.394	0.401	0.435[1]

[1] Two adjacent spans only are loaded.

By similar reasoning to that employed in deriving moment coefficients for uniform loads, we obtain the following values:

Nature of load	Intermediate spans		End spans	
	At center	At support	At center	At 2d support
Loads at middle points				
Dead load...........	0.130	0.119	0.175	0.158
(two spans).........	.....	.....	(0.156)	(0.187)
Live load............	0.191	0.156	0.213	0.175
(two spans).........	.....	.....	(0.203)	(0.187)
Loads at third points				
Dead load...........	0.122	0.211	0.244	0.281
(two spans).........	.....	.....	(0.222)	(0.333)
Live load............	0.228	0.276	0.289	0.311
(two spans).........	.....	.....	(0.278)	(0.333)
Loads at middle and quarter points				
Dead load...........	0.204	0.296	0.314	0.394
(two spans).........	.....	.....	(0.267)	(0.465)
Live load............	0.352	0.389	0.406	0.438
(two spans).........	.....	.....	(0.383)	(0.465)

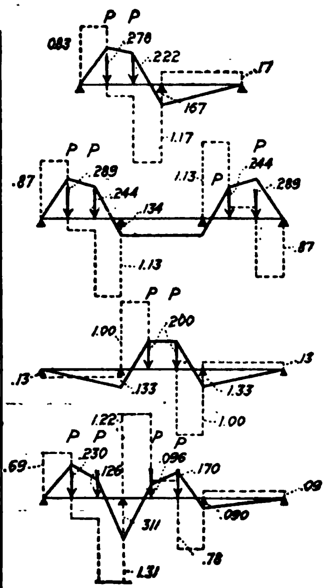


Fig. 61.—Concentrated loads as shown; loads at third points; two and three equal spans; supported ends. Coefficients of (P) for shear.

Combining the dead and live loads into a single unit for the purpose of determining general moment coefficients, we obtain the following:

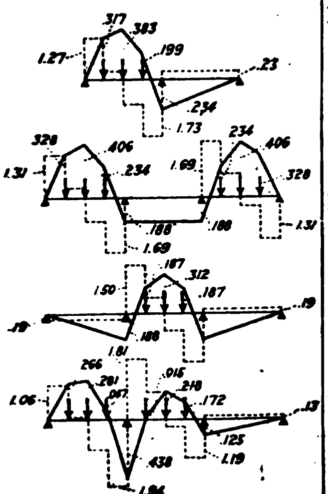


Fig. 62.—Concentrated loads as shown; loads at third points; two and three equal spans; supported ends. Coefficients of (Pl) for moment. Coefficients of (P) for shear.

Ratio of live to dead	Intermediate spans		End spans	
	At center	At support	At center	At 2d support
Loads at middle points				
2:1	0.171	0.143	0.200	0.169
5:1	0.181	0.150	0.207	0.172
(Two spans)				
2:1	.....	.....	0.187	0.187
5:1	.....	.....	0.195	0.187
Loads at third points				
2:1	0.193	0.254	0.274	0.301
5:1	0.210	0.265	0.281	0.306
(Two spans)				
2:1	.....	.....	0.259	0.333
5:1	.....	.....	0.269	0.333
Loads at middle and quarter points				
2:1	0.303	0.358	0.375	0.423
5:1	0.327	0.374	0.391	0.431
(Two spans)				
2:1	.....	.....	0.344	0.465
5:1	.....	.....	0.364	0.465

The following table shows that a floor girder carrying one or more beams and subjected to an indefinite live load may be computed with sufficient accuracy by considering it simply supported and then reducing the maximum moment so found (and an equal negative moment) by the same ratio of reduction used with uniform loading. For example, suppose the maximum moment due to given concentrated loads is K (considering the beam supported), then if $\frac{1}{12}wl^2$ is used for uniform loading instead of $\frac{1}{8}wl^2$, $\frac{8}{12}$ of K, or $\frac{2}{3}K$, may be used for the concentrated loads. The table gives moment coefficients according to this rule. These coefficients should be compared with those in the preceding table.

No. of spans	Intermediate spans	End span and adjoining support
Loads at middle points		
Three or more spans.....	0.167	0.208
Two spans..............	0.208	0.250
Loads at third points		
Three or more spans......	0.222	0.278
Two spans..............	0.278	0.333
Loads at middle and quarter points		
Three or more spans......	0.333	0.417
Two spans..............	0.417	0.500

In a beam loaded at the middle points we find that the moment at the center of intermediate spans may have a value about 8% greater than the recommended value for use in design. (This, however, is considering the beam as freely supported at the interior supports, and the supports of no appreciable width.) All other values for this loading are somewhat less than those recommended. In fact, the specified moment coefficient for the center support of a two-span beam may be reduced and made the same as for the center of span. In beams of three or more spans and for the same loading as just mentioned, the moment coefficient for the inner supports of end spans may be reduced so as to have the same value as specified for the interior spans.

The moment at interior supports in beams loaded at the third points may have a value about 19% greater than that specified, but the width and monolithic character of the supports will offset this to a considerable extent. It is preferable, however, to make some allowance for this in design although for simplicity this has not been done in this handbook. The same is true for the inner supports of the end spans although the increase in moment over that specified is about 10%.

The reader may draw his own conclusions in regard to moment coefficients in beams loaded at the middle and quarter points.

The recommendations for shear given for uniform loads will apply in the case of beam concentrations.

52. Negative Moment at the Ends of Continuous Beams.—The amount of negative moment at the ends of continuous beams depends upon the manner in which the ends are restrained. A beam cannot be entirely fixed unless the restraint is sufficient to cause the neutral surface at the ends to be horizontal. The moment coefficient must be left to the judgment of the designer, but the shear and moment diagrams shown in Figs. 54 to 57 inclusive (for beams with fixed ends) and in Figs. 70 and 72 will prove useful in this connection (see also recommendations of Joint Committee on page 318).

53. Bending Up of Bars and Provision for Negative Moment.—In Figs. 63 to 65 inclusive are given bending-moment curves which apply to continuous beams, supported ends, for uniform loads (both live and dead) on two, three, and four equal spans. The dead-load curves are the same as shown in Fig. 46 for uniform load over all spans. In plotting the live-load curves, the loadings were considered which give maximum and minimum values at each of the

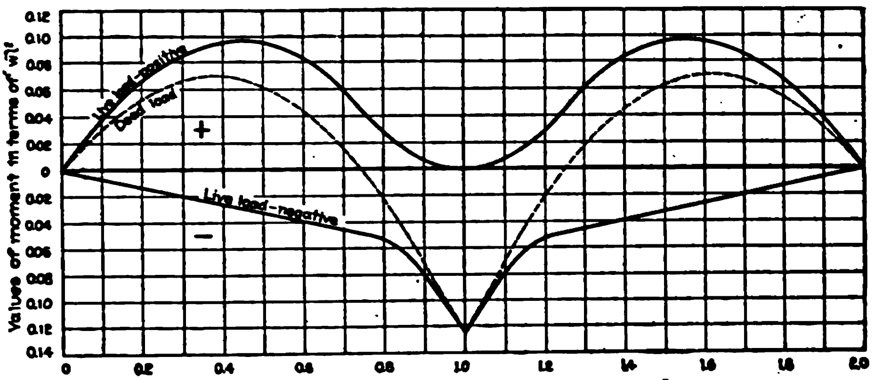


FIG. 63.—Moment curves for uniform load; two spans; supported ends.

one-tenth division sections. Thus, these curves do not represent any one condition of loading, but may be used to determine the extreme values of the live-load moment at any given point in the span.

It should be noted that some portions of the live-load curves are quite different from those shown in Figs. 46 to 57 inclusive. For example, the part of the maximum live-load curve close to the center support in the beam of two spans (Fig. 63) is quite different from a similar portion

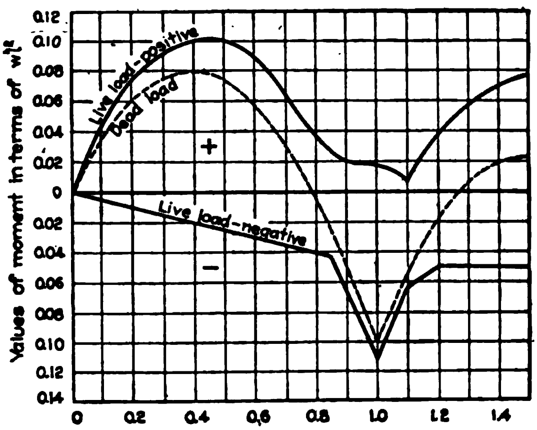


FIG. 64.—Moment curves for uniform load; three spans; left-hand half; supported ends.

in any of the curves shown in Figs. 46 and 54. This is due to the fact that for these sections maximum and minimum moments are caused by only partial loading, and not by having either one or both spans fully loaded. If influence lines were plotted for these sections, this point would be clearly brought out.

In the two-span beam shown in Fig. 66, maximum and minimum bending-moment curves are given for a 3 : 1 ratio of live to dead load. To obtain these curves from Fig. 63, points should be determined for each one-tenth of the span. The following notation will be employed:

w_d = dead load per unit of length.
w_l = live load per unit of length.
$w_t = w_d + w_l$ = total load per unit of length.

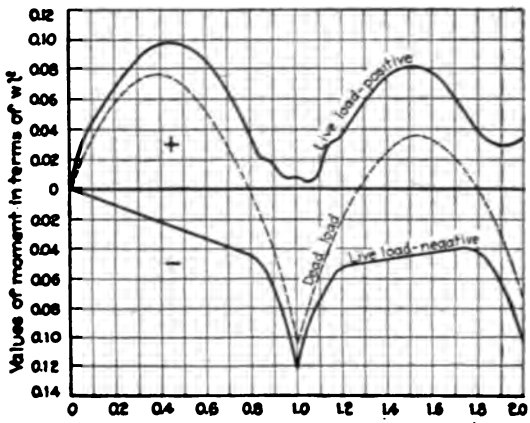


FIG. 65.—Moment curves for uniform load; four spans; left-hand half; supported ends.

Consider a point in either span of a two-span beam (Fig. 63) at a distance 0.2l from the center support. The moment will vary between

$$-0.02w_dl^2 + 0.03w_ll^2$$

and

$$-0.02w_dl^2 - 0.05w_ll^2$$

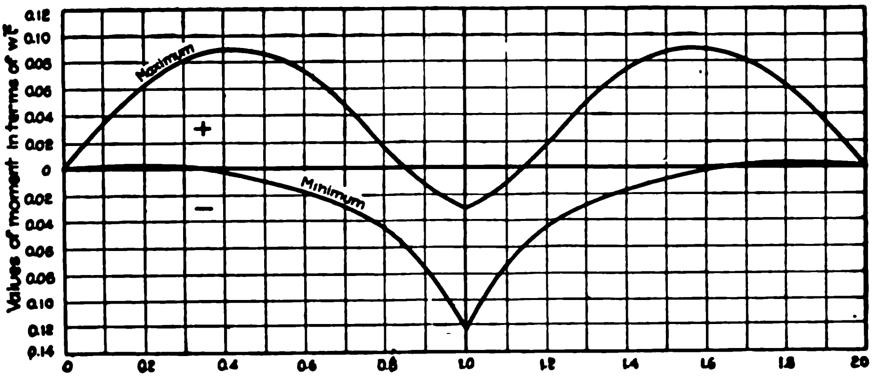


FIG. 66.—Moment curves for continuous beams of two spans; supported ends. Ratio of live load to dead load 3 : 1.

Assuming $w_l = 3w_d$, the moment varies from $+0.07w_dl^2$ to $-0.17w_dl^2$; that is (since $w_d = \frac{1}{4}w_t$) from $+0.0175w_tl^2$ to $+0.0425w_tl^2$.

Curves such as shown in Fig. 66 show what positive and negative moments should be provided for at any point in the span. They also indicate in what manner the steel may safely be bent up from the lower side of the beam. The curves in Fig. 66 show that a negative moment is likely to occur over one-half of each span of a two-span beam. Any fixing of the ends, however, will reduce this length.

Referring to Figs. 66, 67 and 68, it is clear that negative moment may, under extreme conditions, occur entirely across the beam. For ordinary cases, then, it would seem that top

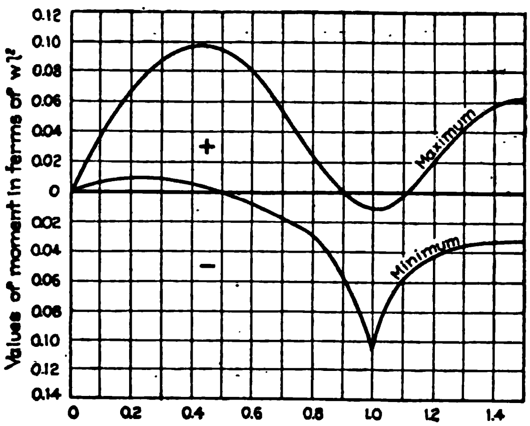


Fig. 67.—Moment curves for continuous beams of three spans; left-hand half; supported ends. Ratio of live load to dead load 3 : 1.

reinforcement should extend to at least the fourth point. In special cases, however, it may be desirable to provide for negative-tension reinforcement over the entire span. If reinforcing frames are used, the top rods employed for handling and for fastening the stirrups into a unit will aid materially in taking care of any tensile stresses which may occur in the top of the beam.

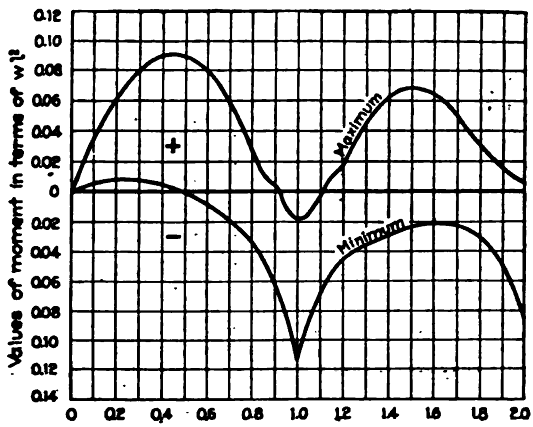


Fig. 68.—Moment curves for continuous beams of four spans; left-hand half; supported ends. Ratio of live load to dead load 3 : 1.

It is also true that in monolithic floor construction, the adjoining slab will help considerably in preventing top tensile stresses at the center of the span.

In view of the above considerations, it would seem that rods may be bent up with sufficient accuracy (for ordinary cases where uniform live load is somewhat indefinite), by applying the method of Art. 22. For special cases, curves should be drawn similar to those shown in Figs. 66 to 68 inclusive.

Maximum and minimum moment curves for concentrated loads are shown in Figs. 69 to 73 inclusive. Three-span beams only are considered and curves are given for both supported and fixed ends. From a study of these curves and in view of the considerations presented previously in this article, it would seem (assuming the usual indefinite live load) that rods in girders loaded at the third points may safely be bent up as explained in the floor-bay design of Art. 11, Sect. 11. The curves are given for a 4 : 1 ratio of live to dead load. The moment

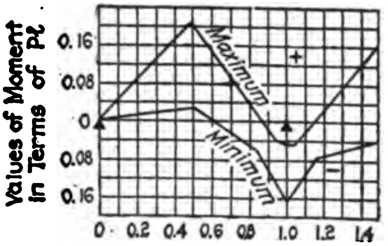


Fig. 69.—Moment curves for continuous beams of three spans; supported ends; loads at middle points. Ratio of live load to dead load 4 : 1.

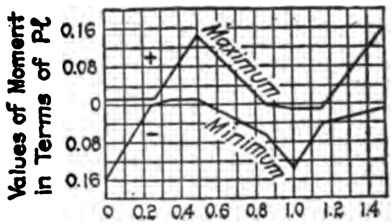


Fig. 70.—Moment curves for continuous beams of three spans; fixed ends; loads at middle points. Ratio of live load to dead load 4 : 1.

due to dead weight of girder stem has little effect in considerations regarding the bending up of rods.

The formulas given below may be of use when considering the variation of moment in beams of many equal spans for different kinds of loading. In deriving the formulas for maximum positive moment, alternate spans were considered as covered with live load. This gives the worst condition of loading for positive moment. Likewise in deriving the formulas for maximum negative moment, the worst condition of loading for negative moment was assumed

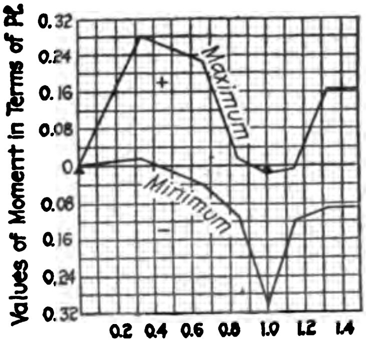


Fig. 71.—Moment curves for continuous beams of three spans; supported ends; loads at third points. Ratio of live load to dead load 4 : 1

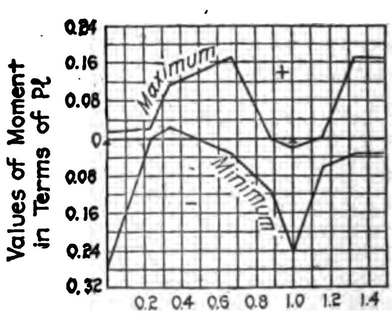


Fig. 72.—Moment curves for continuous beams of three spans; fixed ends; loads at third points. Ratio of live load to dead load 4 : 1.

—that is, two adjacent spans were assumed as fully loaded alternating with one span unloaded. For the same load on each span, the formulas reduce to simple terms, and give the moments on continuous beams with fixed ends and with any number of spans.

The following notation will be employed:

$M_{max.\ p.}$ = maximum positive moment at center of span.
$M_{min.\ p.}$ = minimum positive moment (or maximum negative moment) at center of span.
$M_{max.\ n.}$ = maximum negative moment at the support.

Uniform loads:

$$M_{max.\ p.} = \frac{l^2}{24}(2w_t - w_d)$$

$$M_{min.\ p.} = -\frac{l^2}{24}(w_t - 2w_d)$$

$$M_{max.\ n.} = -\frac{l^2}{36}(4w_t - w_d)$$

Loads at middle points:

$$M_{max.\ p.} = \frac{l}{16}(3P_t - P_d)$$

$$M_{min.\ p.} = -\frac{l}{16}(P_t - 3P_d)$$

$$M_{max.\ n.} = -\frac{l}{24}(4P_t - P_d)$$

Loads at third points:

$$M_{max.\ p.} = \frac{l}{9}(2P_t - P_d)$$

$$M_{min.\ p.} = -\frac{l}{9}(P_t - 2P_d)$$

$$M_{max.\ n.} = -\frac{2l}{27}(4P_t - P_d)$$

Triangular distribution of load:

$$M_{max.\ p.} = \frac{5l}{96}(2.2W_t - W_d)$$

$$M_{min.\ p.} = -\frac{5l}{96}(W_t - 2.2W_d)$$

$$M_{max.\ n.} = -\frac{5l}{144}(4W_t - W_d)$$

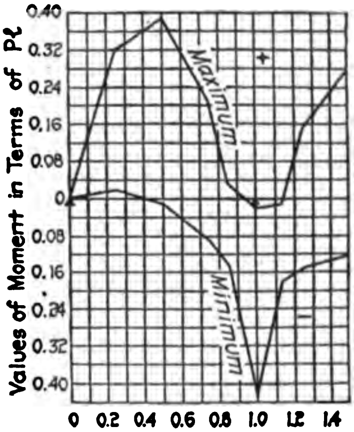


FIG. 73.—Moment curves for continuous beams of three spans; supported ends; loads at middle and quarter points. Ratio of live load to dead load 4 : 1.

54. Continuous Beams with Varying Moment of Inertia.—It is the practice of some designers to place more steel between the supports of continuous beams than the amount just sufficient to resist the bending moment specified by the Joint Committee. They consider that by doing this the stresses over the supports are reduced and the design is more economical. They maintain, also, that this method of procedure is advisable in order to provide for imperfect continuity of the beam and to take care of unknown stresses caused by unequal settlements of the supports. It is important, therefore, to determine the actual moments which occur at the center and supports in such cases.

A study of this kind, assuming uniform loads, has been made by R. E. Spaulding[1] for beams with fixed ends and for beams with one end fixed and one end free. The curves shown in Fig. 74 are the result of his investigations and apply to either rectangular or T-beams. Mr. Spaulding assumed a uniform moment of inertia for that portion of the beam over the support in which negative moments exist, and another moment of inertia for the central portion of the span—that is, between points of inflection.

The condition of fixed ends is obtained for full loading on the lower floors of a building having very heavy columns. The case of one end fixed and one end free is found in either span of a two-span beam with the same uniform load on both spans.

Referring to the curves of Fig. 74 it is clear that with both ends fixed and with the same moment of inertia throughout, the moment at the center is $0.042wl^2 = \frac{wl^2}{24}$ and at the end is

[1] *Eng. News*, Jan. 13, 1910.

$0.083wl^2 = \frac{wl^2}{12}$, as is well known. If, as is sometimes done, a small amount of steel is placed over the supports, such that $\frac{I_t}{I} = 0.20$, and the full bending moment $\frac{wl^2}{8}$ is provided for at the center, the actual moment at the center for fixed ends will be $0.069wl^2 \left(55\% \text{ of } \frac{wl^2}{8}\right)$ and at the support will be $0.056wl^2$. Such a condition stresses the steel at the support to a value 2.2 times the working stress, since provision is made at the support for a moment of only $(0.20)(\frac{1}{8}wl^2) = \frac{1}{40}wl^2 = 0.025wl^2$—that is, of course, assuming the amount of steel used to be proportional to the moments of inertia. Again, suppose the amount of steel at the support of a beam with fixed ends to be made one-half that at the center and that the center be designed for $\frac{1}{10}wl^2$. The actual moment at the center will be $0.053\ wl^2$ and the end bending moment will be $0.073wl^2$. Thus, with this distribution of the reinforcement, the steel at the support will be overstressed to an amount $\frac{0.073}{0.050} = 1.5$ times the assumed working stress. It may be seen from the curves of Fig. 74 (for beams with fixed ends) that if a beam is figured as simply supported, then about 0.6 of the amount of steel that is employed in the middle of the beam should be placed over supports in order to make as economical a design as possible under the conditions. For example, with this distribution of steel, provision is made at the support for a moment of

$$0.6\left(\frac{wl^2}{8}\right) = 0.075wl^2$$

and this is exactly the actual moment caused by such an arrangement. The assumption which is made that the moments of inertia are proportional to the amount of steel used makes practically no difference in these comparative results.

In the above discussion no compressive steel has been considered at the supports. In rectangular beams, of course, there is none needed but in T-beams the section becomes rectangular at the supports and the stress in the concrete needs attention. Referring to Table 2, page 355, for n = 15, it may be seen that for an allowable compressive stress in the concrete at the support of 750 lb. per sq. in. and 16,000 lb. in the steel, the required percentage of tensile steel is practically 1%. It is thus possible to get along without any additional strengthening of the compressive part of the beam at the support only when the beam is reinforced at the center of span with less than 1% ÷ 0.6 = 1.7% of the area of the stem (including the portion of the slab directly above it), whether or not 1.7% is less than the amount determined for the full bending moment of $\frac{wl^2}{8}$.

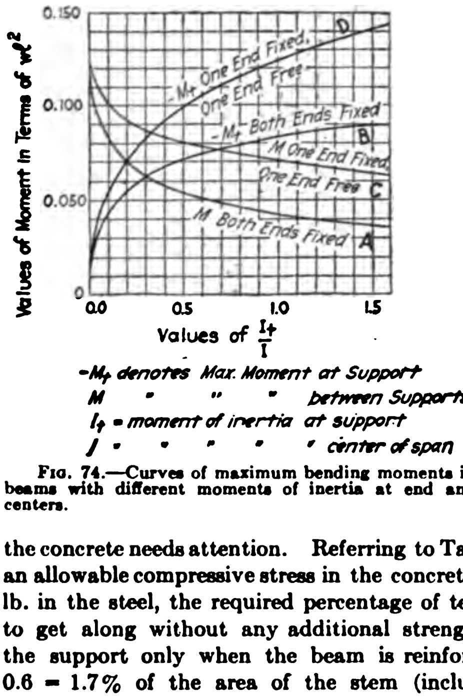


FIG. 74.—Curves of maximum bending moments in beams with different moments of inertia at end and centers.

The question naturally arises whether it is more economical to design for the full bending moment of $\frac{wl^2}{8}$ and provide steel over supports equal in amount to 0.6 of the steel at the middle of beam, or, as recommended by the Joint Committee, to design for $\frac{wl^2}{12}$ both at the center of span and over supports. Consider a given case where a continuous beam designed as simply supported requires 3 sq. in. of steel at the center of span. Of this amount $(0.6)(3) = 1.8$ sq. in. should be bent up and carried over the top of the support (disregarding amount of steel required for bond), the rest of the rods being horizontal in the bottom of the beam. The approximate volume of this steel may be taken as the area of the steel in the middle times the span in inches, or 3 sq. in. $\times\, l = 3l$ cu. in. The same beam designed according to the Joint Committee's recommendations would require 2 sq. in. of steel at the center of span and 2 sq. in. over the top at the support, and the volume of steel may be taken as the volume of the long rods, the area of which amounts to 2 sq. in., plus the volume of steel carried over from the adjoining spans and extending to say the one-fourth point of the span, or approximately (2 sq. in.)$(l) + (2)(\frac{1}{4}l) = 2\frac{1}{2}l$ cu. in. The reader should notice in the above comparison that the first design is favored to the extent that no top steel is assumed to extend beyond the center of the support. Assume this steel to extend to only the one-fifth point of the span, then the volume of steel in the first case becomes equal to $3l + (1.8)(\frac{1}{5}l) = 3.4l$ cu. in. Thus it is clearly seen that a beam designed for a moment $\frac{wl^2}{8}$ in the center of span requires approximately 25% more steel than the beam designed with $\frac{wl^2}{12}$, for both positive and negative moments.

Mr. Spaulding in his study of continuous beams (referred to above) assumed that the moment of inertia is constant between the points of inflection, and there changes abruptly to another value which is constant for the ends of the beam. This assumption neglects in all cases the value of concrete below the neutral axis because it is in tension and is liable to crack. This is true, however, only for sections subjected to the maximum bending moments. In fact, the variation of the moment of inertia throughout the beam may be represented by a curve with maximums at the points of inflection and minimums at the middle of the beam and at the supports. Sanford E. Thompson in an article published in the issue of *Engineering News* of Jan. 13, 1910, under the title "Continuity in Reinforced-concrete Beams," states that extensive studies made in his office, considering the moment of inertia to vary in this way, gave results which substantially agree with those obtained by Mr. Spaulding and prove the latter's assertion that the assumption of constant moment of inertia between points of inflection is sufficiently accurate for all practical purposes.

On page 340 it is shown that a beam reinforced for the full bending moment of $\frac{wl^2}{8}$ at the center of span will induce a bending moment (if properly designed) of approximately $0.075wl^2$ over supports. This is only about 10% less than the moment of $\frac{wl^2}{12}$ recommended by the Joint Committee.

DESIGNING TABLES AND DIAGRAMS FOR BEAMS AND SLABS

55. Illustrative Problems.—The use of designing tables and diagrams can best be explained by giving the solutions of typical designing problems. The following working stresses will be assumed throughout:

$f_c = 650$. $f_s = 16{,}000$. $n = 15$. $v = 40$ without web reinforcement and 120 when thorough web reinforcement is provided.

Bond stress will not be considered here.

. *Design a rectangular beam to span* 40 *ft. and to support a load of* 600 *lb. per ft.* (*including weight of beam*). *Beam is assumed to be simply supported.*

Solution using Tables.

From Table 2, for $n = 15$, $f_s = 16{,}000$ and $f_c = 650$

$$K = 107.4$$

$$M = \frac{wl^2}{8} = \frac{(600)(40)^2(12)}{8} = 1{,}440{,}000 \text{ in.-lb.}$$

$$bd^2 = \frac{1{,}440{,}000}{107.4} = 13{,}400.$$

Assume $d = 1.5b$. Then Table 4 shows that $b = 18$ in. and $d = 27\frac{1}{2}$ in. will be satisfactory. Area of cross-section, $bd = (18)(27.5) = 495$ sq. in.

$$A_s = (495)(0.0077) = 3.81 \text{ sq. in.}$$

We shall select four $1\frac{1}{8}$-in. round rods = 3.98 sq. in. (see Table 1 or Table 5).

$$v = \frac{V}{bjd} = \frac{12{,}000}{(18)(\frac{7}{8})(27.5)} = 28 \text{ lb. per sq. in.}$$

Web reinforcement is not theoretically needed.

The beam may be reviewed as follows:

$$p = \frac{A_s}{bd} = \frac{3.98}{495} = 0.0080$$

From Table 3, for this value of p,

$$k = 0.384 \quad j = 0.872$$

Then,

$$f_s = \frac{1{,}440{,}000}{(3.98)(0.872)(27.5)} = 15{,}100 \text{ lb. per sq. in.}$$

$$f_c = \frac{(2)(15{,}100)(0.0080)}{0.384} = 630 \text{ lb. per sq. in.}$$

Solution Using Diagrams.—In Diagram 2,[1] the intersection of the curves $f_c = 650$ and $f_s = 16{,}000$ is first found. Tracing down, p is found to be 0.0077, and tracing horizontally $K\left(\frac{M}{bd^2}\right)$ is found to be 107.3. The solution then follows as in the use of tables given above.

Diagrams 1 and 2 may also be employed to determine the safe resisting moment of a given beam and the greatest unit stresses in the steel and concrete due to a given bending moment.

To determine the safe resisting moment of a given beam, the value of p should be computed. After finding this value on the lower margin, trace vertically, stopping at the first of the two curves $f_c = 650$ and $f_s = 16{,}000$ (assuming these the allowable stresses). Now trace horizontally to either side margin and the value of K is found. Then, $M = Kbd^2$. Consider a beam of the above dimensions to have 1% of steel. Tracing vertically from this value on the lower margin of Diagram 2, the 650 curve is the first curve to be reached and at a value of $K = 117.0$. Then $M = (117)(18)(27.5)^2 = 1{,}593{,}000$ in.-lb.

To determine the greatest unit stresses in the steel and concrete of a given beam due to a given bending moment, the value of p should be computed as before. Also, K should be computed from the formula $K = \frac{M}{bd^2}$. With these values of p and K, find the intersection of the vertical and horizontal lines through these values respectively, and from the adjacent steel and concrete curves the values of f_c and f may be estimated. Consider a beam of the above dimensions and with 0.7% of steel, to be subjected to a bending moment of 1,200,000 in.-lb. or $K = \frac{1{,}200{,}000}{(18)(27.50)^2} = 88.2$. The intersection of the vertical and horizontal lines through these values respectively in Diagram 2 gives $f_c = 550$ and $f_s = 14{,}400$. This procedure is followed in reviewing beam design.

Diagrams 1 and 2 may also be employed to find minimum allowable depth of beam for a given percentage of steel and various assumed widths, also to find the amount of steel for a beam with given loading.

To find the depth of beam for a given percentage of steel and given allowable stresses, select the lower value of K determined by the intersection of the allowable stress curves with

[1] Diagram first given by ARTHUR W. FRENCH, *Trans.* Am. Soc. C. E., vol. 56, 1906, p. 362.

the vertical line representing the given steel percentage. This value of K substituted in formula

$$M = Kbd^2, \text{ or } d = \sqrt{\frac{M}{bK}}$$

gives the smallest permissible depth for various assumed widths.

To find the percentage of steel for a given beam, compute the value of K from formula $K = \frac{M}{bd^2}$. Locate this value at the left of the diagram. Trace horizontally to the right until the proper allowable stress curve is reached. Thus, if $K = 80$, the curve of $f_c = 650$ intersects the horizontal through the given value of K at a value for p of 0.003, but f_s for this percentage is seen to be over 22,000. The desired percentage is 0.0056, determined by the intersection of the curve $f_s = 16{,}000$ with the horizontal in question.

2. *A beam of 16-in. width, having its compression face inclined at an angle of* 30 *deg. and its tension face at an angle of* 20 *deg., is to be designed for a moment of* 3,000,000 *in.-lb. What depth and percentage of reinforcement are necessary?*

Diagram 2 gives $K = 107.3$ and $p = 0.0077$ (see preceding problem).

These values should be multiplied by $\cos^2\beta_c$ and $\frac{\cos^2\beta_c}{\cos\beta_t}$ respectively. The products may be obtained directly from Diagram 3.

Entering the diagram with a value of 10.73 (resetting the decimal in $K = 107.3$) on the lower margin, trace vertically to inclined line for $\beta_c = 30$ deg. and then horizontally to the left-hand margin where a value of 10.73 $\cos^2$ (30 deg.) = 8.05 is found. Pointing off properly, the proper value of K to use is 80.5. Then

$$d = \sqrt{\frac{3{,}000{,}000}{(80.5)(16)}} = 48\tfrac{1}{2} \text{ in.}$$

To find $0.0077 \frac{\cos^2\beta_c}{\cos\beta_t}$, enter the diagrams with a value of 7.7 on the lower margin, trace vertically to inclined line for $\beta_c = 30$ deg., then horizontally to inclined line for $\beta_t = 20$ deg., and then vertically upward to the upper margin where a value of 6.1 is found. The proper value of p to use is 0.0061.

Diagram 3 may be employed when either $\beta_c = 0$ or $\beta_t = 0$. The procedure would be the same as above.

3. *A beam with* $b = 16$ *in.,* $d = 45$ *in., and* $p = 0.007$ *is subjected to a moment of* 2,500,000 *in.-lb. Assuming the compression face inclined at an angle of* 20 *deg. and the tension face at* 25 *deg., what are the unit stresses* f_c *and* f_s*?*

$$K = \frac{2{,}500{,}000}{(16)(45)^2} = 77.2$$

Before using Diagram 2, the values of K and p should be multiplied by $\frac{1}{\cos^2\beta_c}$ and $\frac{\cos\beta_t}{\cos^2\beta_c}$ respectively. These products may be obtained directly from Diagram 3.

Entering the diagram with a value of 7.72 on the left-hand margin, trace horizontally to inclined line for $\beta_c = 20$ deg. and then vertically downward to the lower margin where a value of $\frac{7.72}{\cos^2(20 \text{ deg.})} = 8.75$ is found. Pointing off properly, the proper value of K to use in Diagram 2 is 87.5.

To find $7.00 \frac{\cos\beta_t}{\cos^2\beta_c}$, enter the diagram with a value of 7.00 on the upper margin, trace vertically downward to inclined line for $\beta_t = 25$ deg., then horizontally to inclined line for $\beta_c = 20$ deg., and then vertically downward to the lower margin where a value of 7.25 is found. The proper value of p to use in Diagram 2 is 0.0072.

Using $K = 87.5$ and $p = 0.0072$ in Diagram 2 gives $f_c = 540$ and $f_s = 13{,}700$.

Diagram 3 may be employed when either $\beta_c = 0$ or $\beta_t = 0$. The procedure would be the same as above.

4. *Determine the approximate weight of a rectangular beam 24 in. wide, with a clear span of 25 ft. and carrying a load of 5000 lb. per ft.*

The load per foot length per inch width will be:

$$\frac{5000}{24} = 209 \text{ lb.}$$

From Diagram 4[1] we obtain $c = 0.885$. If we take $M = \frac{wl^2}{8}$, then the weight of the beam per foot length will be:

$$w' = [0.885 + (25)(0.003)]\frac{(5000)(25)}{100} = 1200 \text{ lb. per ft.}$$

5. *What safe load per square foot (including dead weight) can be supported by a slab 6 in. deep ($d = 4\frac{3}{4}$ in.) and 10-ft. span reinforced with ½-in. round rods placed 8 in. apart? The slab is simply supported and reinforced in only one direction.*

$$p = \frac{0.1963}{(8)(4.75)} = 0.0052$$

Referring to Diagram 6, Part 2, and tracing vertically from this value of p on the lower margin to an intersection with the curve of $d = 4\frac{3}{4}$ in., and then tracing horizontally to the left-hand margin, a bending moment of 20,100 in.-lb. is found.

Select this value of the bending moment on the left-hand margin of Diagram 5 and trace horizontally to the right to an intersection with a vertical line through 10, denoting span length. The safe load, based on $M = \frac{wl^2}{10}$ can now be estimated directly by means of the curved lines and is found to be 168 lb. per sq. ft.

(168) (0.80) = 134½ lb. per sq. ft., safe load for slab simply supported.

6. *Design a slab to span 6 ft. and to carry a live load of 250 lb. per sq. ft. Slab is to be fully continuous and reinforced in only one direction.*

Assume the weight of slab at 50 lb. per sq. ft. Total load for slab is thus 300 lb. per sq. ft.

From Diagram 5 for this span length and load per square foot, a bending moment of 11,000 in.-lb. is found, based on $\frac{wl^2}{12}$. Diagram 6, Part 1, shows that a depth (d) of 3 in. will be ample—total depth $3\frac{3}{4}$ in. Also, $A_s = 0.275$ sq. in.

From Table 6, we may use ⅜-in. round rods spaced $4\frac{3}{4}$ in. on centers.

The assumed and actual dead weights are close enough, and the slab need not be redesigned. The slab should be reinforced against negative moment at the supports. The slab should also be reinforced transversely in order to prevent shrinkage and temperature cracks. Shear at ends of slab in direction of reinforcement is (300) (3) = 900 lb. per ft. of breadth. Allowable shear = (12)(3)(40) = 1440. Thus no web reinforcement is needed, as is usually the case except for excessive loading.

7. *Design a slab for a 10 by 10-ft. panel to carry a live load of 250 lb. per sq. ft. Slab is to be fully continuous and reinforced in both directions.*

The dead load will be assumed at 60 lb. per sq. ft. Total moment to be resisted in each direction according to recommendations of the Joint Committee (Art. 29c, page 307) is

$$M = \frac{1}{2}\cdot\frac{wl^2}{12} = \frac{(310)(10)^2(12)}{2(12)} = 15{,}500 \text{ in.-lb.}$$

(Diagram 5 may be used, assuming $M = \frac{wl^2}{12}$, and dividing result by 2.)

Using Diagram 6, Part 1, the slab is seen to be of very nearly equal strength in tension and compression when $d = 3\frac{1}{4}$ in. and $A_s = 0.32$ sq. in. The required spacing for center half

[1] Diagram and formulas taken from article by M. J. Lorentz in *Eng. News*, March 20, 1913.

of slab, then, is 4 in. on centers for $\frac{3}{8}$-in. round bars. The bars should be spaced the same throughout the center half of slab and then the spacing gradually increased to the edge of the slab, using one-half as many bars in the outside quarters. The slab should be reinforced against negative moment at the supports.

The depth of the slab should be made 5 in. in order to have the upper reinforcing system at the minimum distance $3\frac{1}{2}$ in. from the surface of the slab. The lower system will then be slightly stronger than necessary. The dead weight is approximately that assumed. For safety in construction, it is preferable to require the two systems of reinforcement to be fastened together at frequent intervals. Web reinforcement is not necessary.

8. *Design the center cross-section of a T-beam in a floor system; the beam is to have a span of 12 ft. and be fully continuous. Maximum shear (live plus dead) is closely equal to 12,200 lb. Maximum moment (live plus dead) = 356,300 in.-lb. Supported slab is 6-in. thick.*

The only purpose of the concrete below the neutral axis is to bind together the tension and compression flanges, and consequently its section is determined by the shearing stresses involved and space for the necessary bars. The shearing stress v should not be greater than 120. The area $b'd$ (unless the value of j should turn out to be less than $\frac{7}{8}$) should not be less than

$$\frac{12{,}200}{(\frac{7}{8})(120)} = 116 \text{ sq. in.}$$

The following formula of Art. 37, gives the most economical depths for various assumed web widths:

$$d = \frac{rM}{f_s b'} + \frac{t}{2}$$

Assuming r as 60, then

$$\text{for } b' = 9 \text{ in.} \qquad d = 15.2$$
$$\text{for } b' = 10 \text{ in.} \qquad d = 14.6, \text{ etc.}$$

Some rough calculations show that if four bars are to be used and all in one row, the breadth of stem necessary for the bars controls. A breadth b' of 10 in. and a depth d of 15 in. (total depth 17 in.) will be tried.

Diagrams 7 and 8 cannot be employed to solve for the resisting moment of a given beam but are useful in designing. Formula (11), Art. 34, may be put in the following form

$$\frac{M}{bd^2} = f_c\left(1 - \frac{t}{2kd}\right)\frac{t}{d}j$$

k and j in this equation are functions of f_c and f_s, and hence the variables are f_c, f_s, and the ratio $\frac{t}{d}$. The curves at the left in Diagrams 7 and 8 are plotted from this equation with a fixed value of f_s = 16,000 lb. per sq. in. Values of f_c may be determined for various values of $\frac{M}{bd^2}$ and $\frac{t}{d}$, or values of $\frac{M}{bd^2}$ may be determined for various values of f_c and $\frac{t}{d}$. It must not be overlooked, however, that these diagrams will apply only when the amount of steel is such that f_s = 16,000 lb. per sq. in. This amount of steel may be easily determined when the corresponding j is found from the curves at the right of the diagram. Suppose $\frac{M}{bd^2} = 80$ and $\frac{t}{d} = 0.2$, then the intersection of horizontal and vertical lines through these values respectively in Diagram 8 shows f_c to equal 600, and then tracing from this intersection horizontally to the right until the vertical line is reached indicating $f_c = 600$ (at the right-hand side of the diagram), we find j equal to 0.91. Finally, $A_s = \frac{M}{f_s jd}$, in which $j = 0.91$, $f_s = 16{,}000$, and M and d are known. Diagram 8 will now be employed in working out the problem stated at the beginning of this discussion.

The breadth of the flange is controlled by one-fourth the span,[1] or 36 in. Assuming a depth (d) of 15 in.

$$\frac{M}{bd^2} = \frac{356,300}{(36)(15)^2} = 44$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d} = \frac{6.0}{15.0} = 0.40$, we find from the diagram that this beam falls under Case I; that is, the neutral axis is in the flange.

Diagrams 1 and 2 may be used for T-beams under Case I. In the problem at hand, a horizontal line through the value 44 for K, in Diagram 2, intersects the oblique line $f_s = 16,000$ at a value of $f_c = 370$. The value of p corresponding is 0.003. Then $A_s = pbd = (0.003)(36)(15) = 1.62$ sq. in. Table 5 shows that four ¾-in. round bars will give the required steel area.

9. *The flange of a T-beam is 24 in. wide and 4 in. thick. The beam is to sustain a bending moment of 480,000 in.-lb. What depth of beam and amount of steel are necessary?*

We will try $d = 18$ in.

$$\frac{M}{bd^2} = \frac{480,000}{(24)(18)^2} = 61.6$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d} = \frac{4}{18} = 0.222$, we find from Diagram 8, $f_c = 485$ lb. per sq. in. and $j = 0.910$. Then

$$A_s = \frac{480,000}{(16,000)(0.910)(18)} = 1.84 \text{ sq. in.}$$

The stress in the concrete of 485 is permissible and the beam as designed will be considered satisfactory. (Formula (12) in Art. 34*a* may be used to find minimum depth for a given flange width without trial.)

Suppose 2.0 sq. in. of steel were inserted in a beam of the above dimensions, and suppose that the safe resisting moment is desired. Diagram 10 must be used for this case.

$$p = \frac{2.0}{(24)(18)} = 0.0046$$

$\frac{t}{d} = 0.222$ as before. Tracing vertically from this value on the lower margin of the left diagram to a value of $p = 0.0046$ and then tracing horizontally to the left margin, we find a value of $k = 0.32$. In a similar manner we find j equal to 0.91

$$M_s = f_sA_sjd = (16,000)(2.0)(0.91)(18) = 525,000 \text{ in.-lb.}$$

$$f_c = \frac{f_sk}{n(1-k)} = \frac{(16,000)(0.32)}{(15)(1-0.32)} = 502 \text{ lb.}$$

or, from Table 8,

$$f_c = (0.0314)(16,000) = 502 \text{ lb.}$$

Since f_c is less than 650, the resisting moment depends upon the steel, or $M_s = 525,000$ in.-lb.

10. *A continuous T-beam, uniformly loaded, has a bending moment at the center of each span of 356,300 in.-lb. Negative bending moment at the supports and the positive bending moment at the center of span are figured by the formula,* $M = \frac{wl^2}{12}$. *The tensile steel at the center of span consists of four ¾-in. round bars.* $b' = 10$ *in.* $d = 15$ *in. Design the supports.*

At the supports the flange of the T-beam, being in tension, is negligible and the T-beam changes into a rectangular beam with steel in top and bottom. Two of the tension bars on each side of the supports will be bent up and made to lap over the top of the supports, while the other two bars on each side will be continued straight and lapped over supports at the bottom of beam.

The ratios of steel in tension and compression are the same, and are respectively:

[1] See recommendations of the Joint Committee, Art. 32.

$$p = p' = \frac{1.77}{(10)(15)} = 0.0118$$

(1.77 in above equation taken from Table 5.)

$$\frac{d'}{d} = \frac{2}{15} = 0.133$$

From Diagram 12, knowing $p' = p$, we obtain

$$\text{For } \frac{d'}{d} = 0.10 \ldots \begin{cases} k = 0.361 \\ j = 0.888 \end{cases}$$

$$\text{For } \frac{d'}{d} = 0.15 \ldots \begin{cases} k = 0.377 \\ j = 0.866 \end{cases}$$

Thus,

$$\text{For } \frac{d'}{d} = 0.133 \ldots \begin{cases} k = 0.372 \\ j = 0.873 \end{cases}$$

(It is usually well within the precision of the actual work, and on the safe side, to use the curves for the value of $\frac{d'}{d}$ next larger than the actual value. Thus in this problem the values of k and j for $\frac{d'}{d} = 0.15$ could be used with sufficient accuracy.)

Then

$$f_s = \frac{M}{A_s jd} = \frac{356{,}300}{(1.77)(0.873)(15)} = 15{,}400 \text{ lb. per sq. in.}$$

and, using Table 9,

$$f_c = \frac{k}{n(1-k)} \cdot f_s = (0.0394)(15{,}400) = 607 \text{ lb. per sq. in.}$$

The stresses in the concrete and steel are within the allowable and no haunch or additional steel are necessary.

The moment of resistance at the supports may be found as follows:

$$f_s = \frac{f_c n(1-k)}{k} = \frac{650}{0.0394} = 16{,}500 \text{ lb. per sq. in.}$$

Thus the moment of resistance depends on the steel and

$$M_s = bd^2 f_s pj = (10)\,(15)^2\,(16{,}000)\,(0.0118)\,(0.873) = 371{,}000 \text{ in.-lb.}$$

11. *At the support of a continuous T-beam the following values are known:* $b = 12$ *in.,* $p = p'$, $A_s = 3.0$ *sq. in.,* $M = 750{,}000$ *in.-lb.,* $f_c = 750$ *and* $f_s = 16{,}000$. *Find the required depth of beam.*

Assume $$\frac{d'}{d} = 0.10$$

From formula on Diagram 12,

$$f_s = \frac{M}{A_s jd} \quad \text{or} \quad jd = \frac{M}{A_s f_s} = \frac{75{,}000}{(3)\,(16{,}000)} = 15.6$$

Assume $j = 0.87$. Then $d = 17.95$

Adopting $d = 18$ in. $$p = \frac{3.0}{(12)(18)} = 0.0139$$

Diagram 12 for $\frac{d'}{d} = 0.10$ and $p = p' = 0.0139$ shows $j = 0.886$

Assume $j = 0.886$. Then $d = 17.6$ in.

Adopting $d = 17\frac{3}{4}$ in. $p = \frac{3.0}{(12)(17.75)} = 0.0141$

Diagram 12 shows $j = 0.886$

Thus $d = 17\frac{3}{4}$ in is satisfactory provided $\frac{d'}{d}$ is approximately 0.10.

12. *In a double-reinforced rectangular beam* $b = 12$ *in.*, $d = 18$ *in.*, $\frac{d'}{d} = 0.10$, $M = 750{,}000$, $f_c = 650$ *and* $f_s = 16{,}000$. *Determine the required percentages of tensile and compressive steel.*

Following the method outlined in Art. 27*a*, we have, using Table 2,

$$k = 0.378,\ p_1 = 0.0077,\ \text{and}\ K = 107.4$$

$$M_1 = 107.4\ (12)(18)^2 = 418{,}000 \text{ in.-lb.}$$

$$M_2 = 750{,}000 - 418{,}000 = 332{,}000 \text{ in.-lb.}$$

$$p_2 = \frac{332{,}000}{16{,}000\ (0.9)(12)(18)^2} = 0.0059$$

$$p = p_1 + p_2 = 0.0136$$

$$p' = (0.0059)\left(\frac{1 - 0.378}{0.378 - 0.1}\right) = 0.0132$$

56. Leffler's Comprehensive Beam Chart.[1] *Simple Rectangular Beams.*—The procedure to design a beam simply reinforced to carry a certain bending moment M with certain allowable stresses is as follows: Divide the allowable f_s by the allowable f_c thus obtaining the value of $\frac{f_s}{f_c}$. Find this value on the chart (Diagram 13) on the curve $\frac{f_s}{f_c}$. From it move vertically to the $p' = 0$ curve. This point on the $p' = 0$ curve lies where some p curve intersects the $p' = 0$ curve. . The value of this p curve gives the p to use. The abscissa of this point on the $p' = 0$ curve gives the value of L_s to be used in $L_s = \frac{M}{bd^2f_s}$. Equation (I) given on the diagram can then be solved for bd^2 which completes the solution for moment. To b and d such values can then be assigned as shear requirements may demand. If shear requirements are so large as to govern the dimensions of the beam, the design for moment is accomplished by solving (I) for L_s, f_s being used at its allowable value. The p to use is that of the p curve which intersects the $p' = 0$ curve at the point whose abscissa is L_s.

Doubly Reinforced Rectangular Beams.—Suppose the dimensions b and d of a beam to carry a certain M, with certain allowable stresses, are arbitrarily fixed by such limitations as headroom, clearance, architectural effects, shear requirements, etc., the procedure would then be as follows: Solve equations (I) and (II) for L_s and L_c. Using L_s as an abscissa and L_c as an ordinate, plot a point on the chart. If the point falls below or on the $p' = 0$ curve, the beam can be designed as a simple beam; the value of p to use being that of the p curve which intersects the $p' = 0$ curve at the point whose abscissa is L_s. If the point falls well above the $p' = 0$ curve, the beam must be a doubly reinforced one. The values of p and p' to use are those of the p and p' curves on which the point falls. If the point falls above the $p' = 0$ curve and beyond the scope of the chart, the beam is probably impossible as a reinforced concrete beam and recourse must be had to structural steel.

If the point falls but a short distance above the $p' = 0$ curve, it is possible to design the beam as a simply reinforced one. The procedure is as follows: Proceed horizontally from the point until the $p' = 0$ curve is intersected. The value of the p curve that meets the $p' = 0$ curve at this point of intersection gives the value of p to use for designing the beam with tensile reinforcement only. The actual f_c will be the same as the allowable f_c, but the actual f_s will be less than the allowable f_s. Carefully made cost figures are the only means of determining whether this simply reinforced beam will be cheaper than a doubly reinforced one.

To find M when p, p', b, d, the allowable f_s, and the allowable f_c are given, proceed as follows: Read off the abscissa L_s of the intersection of the p and p' curves, then evaluate equa-

[1] By Ralph R. Leffler, Chicago, Ill.

tion (I) for M; read off the ordinate L_c of the intersection of the p and p' curves and evaluate equation (II) for M. Use the smaller value.

T-beams.—A T-beam can be regarded as a large rectangular beam minus two rectangular beams, or, if we combine the two small beams, as a large rectangular beam minus a smaller rectangular beam. Referring to the beam-section on the chart, $ABCD$ is the large rectangular beam and the empty rectangular spaces under the flanges of the T-beam added together form the smaller rectangular beam.

Let M, the allowable f_s, and the allowable f_c be given. First, design a simple rectangular beam that will carry the given M with the given allowable f_s and f_c. Having found bd^2, assign to b a value in accordance with some rules, such as those of the Joint Committee, and then solve for d. Ascertain the amount of tensile steel reinforcement required and make the width of the stem such that it can be properly arranged. We now have the approximate dimensions of the T-beam. At this point it is well to investigate the shear in different sections of the beam. Assuming that we have found that the shear is sufficiently well taken care of, we can proceed to finish the design for moment.

The expression for the value for k can be written as a function of $\frac{f_s}{f_c}$ (assuming $n = 15$). This means that for every value of $\frac{f_s}{f_c}$ there will be found on the same vertical line a point on the k curve (or straight line) whose ordinate gives the value of k corresponding to the value of $\frac{f_s}{f_c}$.

Having found the value of k, locate the neutral axis of the T-beam. If kd is less than, equal to, or very nearly equal to t (the thickness of the flange usually determined by the floor slab design) the neutral axis lies in the flange, at the bottom of the flange or but a short distance below the flange, in which case the design of our T-beam for moment is already complete. If kd is much greater than t, the neutral axis lies well down in the web. To secure a correct theoretical solution it then is necessary to ascertain how much M is carried by the two imaginary beams under the flanges. At this point a little thought discloses that since these imaginary beams cause a loss of resisting M, it is necessary to deepen d. Having depeened d according to our best judgment, the next thing is to find by the formula $M = L_s bd^2 f_s$ how much resisting M the deepened beam is capable of sustaining at the allowable unit stresses, the beam being considered as a simple rectangular beam, of width b, and depth d, equal to the deepened d. The p to use is that of the p curve which intersects the $p' = 0$ curve at the point whose abscissa is L_s. Again the neutral axis is located by the same process as before, and as before, if the neutral axis is in the flange, in the lower edge of the flange or very close up to the bottom of the flange the design for moment is complete. If the neutral axis comes well down in the web it is necessary to ascertain how much resisting moment is carried by the imaginary beams under the flanges. From the stress diagram, $k_1 = \frac{kd - t}{d_1}$, the symbols for the imaginary beam being distinguished by the subscript 1. The L_{s1} corresponding to this k_1 is the abscissa of the k_1 on the chart. Solve the equation $M_1 = L_{s1}b_1d_1^2f_s$ in which b_1 is the combined width of the two imaginary beams and d_1 their depth. This M_1 is the resisting moment carried by the two imaginary beams. The p_1 to use is that of the p curve which intersects the $p' = 0$ curve at the point whose abscissa is L_{s1}. If to M_1 we add the M caused by the loading and thus obtain a sum equal to the resisting M of the enlarged beam, our solution for moment is complete. If their sum is considerably different from it, d should be increased or decreased until they are closely equal, the same procedure being gone through as before. The amount of tensile steel needed for the T-beam is equal to $pbd - p_1b_1d_1$. If k_1 should be so small as not to come within the scope of the chart, M_1 can be obtained from the equation

$$M_1 = \frac{f_{c1}k_1b_1d_1}{2}\left(d_1 - \frac{k_1d_1}{3}\right) = \frac{f_{c1}k_1b_1j_1d_1^2}{2} \text{ in which } f_{c1} = \frac{f_ck_1d_1}{kd}. \quad \text{Also } A_{s1} = \frac{M_1}{j_1d_1f_s} = p_1b_1d_1$$

in which f_s is the same as in the large beam $ABCD$ for the f_s of the large beam is the same as the f_s of the smaller imaginary beam.

Note that this solution of a T-beam does not neglect the stress in the stem.

Doubly Reinforced T-beams.—After the reader has thoroughly grasped the foregoing solutions of the "Doubly Reinforced Rectangular Beams" and the "Simply Reinforced T-beam" he will find it easy to devise a method, parallel to that given for simple T-beams, for designing doubly reinforced T-beams; it being only needful to remember that the resisting M of the larger beam is that of a doubly reinforced beam and that k is a function of $\frac{f_s}{f_c}$ only, and as such is primarily independent of the amount or location of the compressive steel reinforcement. It is j that is primarily dependent on the amount and location of the compressive steel reinforcement.

57. Beard and Schuler's Comprehensive Charts.[1] *Rectangular Beams and Floor Slabs.*—The moment caused by a uniformly distributed load at any point on any beam which is fixed, partially restrained, or simply supported at the ends may be expressed in inch-pounds by the formula, $M = 12\frac{w}{\phi}l^2$, in which w is the uniform load in pounds per foot, l is the span in feet, and ϕ is the moment denominator. ϕ is 8 for the moment at the center of a simply supported beam.

The formula for the resisting moment of a simple rectangular reinforced-concrete beam is $M = Kbd^2$ in which $K = \frac{1}{2}f_ckj$ for the compression couple and f_spj for the tension couple. When the beam is supporting a uniformly distributed load the general formula may be expanded into the two forms

$$12\frac{w}{\phi}j^2 = f_spjbd^2$$

and

$$12\frac{w}{\phi}j^2 = \frac{1}{2}f_ckjbd^2$$

In the upper left-hand quadrant of Diagram 14, which is called the moment chart, the logarithmic abscissas and ordinates represent the spans in feet and moments in inch-pounds respectively. Each sloping line on the diagram represents a particular value of $\frac{w}{\phi}$. With $\frac{w}{\phi}$ a constant, the formula takes the form $M = Cj^2$ and, when expressed in logarithms, the form $\log M = \log C + 2\log j$. The moment chart is a graphical representation of this family of curves, which are parallel straight lines.

The upper right-hand quadrant, or the depth chart, is the logarithmic plat of the equation $M = Kbd^2$. In this case also the logarithmic ordinates represent the moments but the logarithmic abscissas represent values of K. Each sloping line represents a particular value of bd^2. b is taken as 12 in. in all cases and the line is designated by the corresponding value of d.

This plat then represents the general equation

$$\log M = \log C' + \log K$$

It is a series of parallel 45-deg. lines.

The lower right-hand quadrant or the stress chart is a logarithmic plat of the two families of curves:

$$K = f_spj \text{ and } K = \tfrac{1}{2}f_ckj$$

In the stress chart the logarithmic abscissas and ordinates represent values of K and p respectively, and the sloping lines represent particular values of f_c and f_s.

$A_s = pbd$. In the lower left-hand quadrant or the steel chart, the lines sloping upward to the right represent constant values of d and the abscissas and the ordinates, as numbered at the right-hand side of the diagram, represent values of A_s and p respectively.

When the cross-sectional area of a round rod is a and the rod spacing is s, $A_s = \frac{12a}{s}$. Each

[1] By Robert S. Beard and Don B. Schuler.

line sloping upward to the left in the steel chart is marked with the diameter of the rod whose area it represents. The abscissas represent the steel area and the ordinates, as marked at the left side of the diagram, represent the round-rod spacing. The spacing for square rods is $\frac{4}{\pi}$ times the spacing of round rods of the same thickness. The k and j curves are also platted in this steel chart.

Illustrative Problem.—Suppose that it is required to design a slab to carry a total live and dead load of 400 lb. per sq. ft. over a simple span of 20 ft. with limiting stresses of $f_c = 650$ and $f_s = 16{,}000$ lb.

Before entering Diagram 14 the load per square foot, 400 lb., must be divided by the moment denominator which is 8 in this case.

$$\frac{w}{\phi} = \frac{400}{8} = 50$$

In the stress chart, the intersection of the values $f_c = 650$ and $f_s = 16{,}000$ gives a value of $K = 107$. Any designer who uses a particular set of stresses constantly remembers the corresponding value of K, and omits this operation.

Now in the moment chart find the intersection of the sloping line $\frac{w}{\phi} = 50$ with the line representing a span of 20 ft., as indicated in Fig. 75. The ordinate of this intersection corresponds to a moment of 240,000 in.-lb. Follow this ordinate into the depth chart to its intersection with $K = 107$. At this intersection $d = 13.6$ in. It is decided to use a depth of 14 in. which corresponds to $K = 102$ for this moment. Follow the line $K = 102$ into the stress chart. At the point where $f_c = 650$, $f_s = 17{,}600$, at the point where $f_s = 16{,}000$, $f_c = 630$. This second set are therefore the limiting working stresses. At this point $p = 0.0073$. Follow this abscissa into the steel chart to $d = 14$ in. At this intersection $A_s = 1.23$ sq. in. Follow this A_s line to the 1-in. line. The required spacing for 1-in. round rods is 7.7 in. Use a spacing of $7\frac{1}{2}$ in. If it is required to use 1-in. square rods, the necessary spacing is $\frac{4}{\pi} \times 7.7 = 9.8$ in. Use $9\frac{3}{4}$-in. spacing.

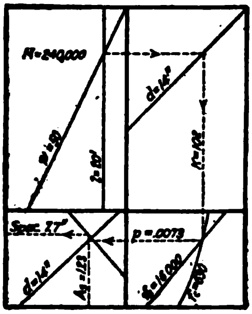


Fig. 75.

If it is desired to find the resisting moment when every other rod is turned up, this process is reversed. In the steel chart follow the rod spacing of 15 in. to the 1-in. line. $A_s = 0.626$ sq. in. Follow this A_s line to $d = 14$ in., $p = 0.00372$. Follow this p line to $f_s = 16{,}000$, $f_c = 430$, $K = 54$. Follow the $K = 54$ line to $d = 14$ in. in the depth chart, $M = 128{,}000$ in.-lb.

T-beams.—The moment caused by a uniformly distributed load at any point on any beam may be expressed in inch-pounds by the formula, $M = \frac{w}{\phi} l^2$.

The resisting moment of the steel reinforcement in a T-beam is Rbt^2 where $R = \frac{f_s pj}{\Delta^2}$ and $\Delta = \frac{t}{d}$. Then when the T-beam is supporting a uniformly distributed load

$$M = 12\frac{w}{\phi}j^2 = Rbt^2$$

In order to make Diagram 15 of more general application the moment is divided by b, the breadth of the beam in inches. The moment formula then takes the form

$$\frac{M}{b} = \frac{12w}{\phi b}j^2 = \frac{w}{\phi B}j^2 = Rt^2$$

B is used to express the width of the beam in feet, $\frac{w}{B}$ then expresses the uniformly distributed load in terms of live load per square foot of flange.

In the upper left-hand quadrant of Diagram 15 the logarithmic abscissas and ordinates represent the spans in feet and moments in inch-pounds divided by the breadth of beam in inches respectively. Each sloping line represents a particular value of $\frac{w}{\phi B}$. With $\frac{w}{\phi B}$ a constant, the formula takes the form $\frac{M}{b} = Cj^2$. The moment chart is a logarithmic graphical representation of this family of curves.

The upper right-hand quadrant or the slab chart is the logarithmic plat of the equation $\frac{M}{b} = Rt^2$. The abscissas represent values of R, and the ordinates, values of $\frac{M}{b}$. The sloping lines correspond to particular values of t.

The lower right-hand quadrant or steel chart is a logarithmic plat of the equation $R = f_s\frac{pj}{\Delta^2}$. In this steel chart the abscissas and ordinates represent values of R and $\frac{pj}{\Delta^2}$ or y respectively. The sloping lines represent particular values of f_s.

In the lower left-hand quadrant or proportional chart, the lines sloping upward to the right are the logarithmic plat of the family of curves $y = \frac{pj}{\Delta^2}$. The abscissas and ordinates of the proportional chart represent values of p and y respectively, and the sloping straight lines represent particular values of Δ.

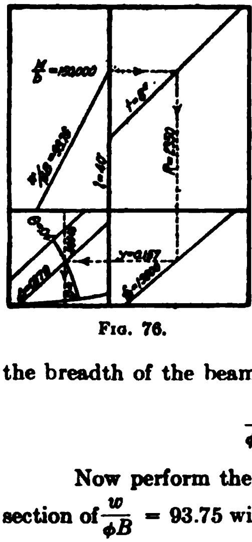


Fig. 76.

The curved lines on the proportional chart have been platted from this formula by solving for the values of p corresponding to particular values of $\frac{f_c}{f_s} = \theta$ and Δ, and then drawing the θ curves through the intersections of these values of p with the corresponding Δ lines in the proportional chart. The curve $\Delta = k$ is the line of division between the T-beam and the simple beam.

Illustrative Problem.—Design a simply supported T-beam with the following factors predetermined: $l = 40$ ft., $w = 4000$ lb. per lin. ft., $f_c = 600$, $f_s = 15{,}000$, $t = 8$ in., and $b = 64$ in.

Before entering Diagram 15, the total load per linear foot of 4000 lb. must be divided by both the moment denominator, 8, and the breadth of the beam in feet, $5\frac{1}{3}$

$$\frac{w}{\phi B} = \frac{4000}{8 \times 5\frac{1}{3}} = 93.75. \quad \theta = \frac{600}{15{,}000} = 0.04$$

Now perform the operations on the T-beam chart indicated in Fig. 76. At the intersection of $\frac{w}{\phi B} = 93.75$ with the line $l = 40$ ft., the ordinate is $\frac{M}{b} = 150{,}000$ at the intersection of this ordinate with $t = 8$ in., $R = 2350$. At the intersection of the abscissas, $R = 2350$ with $f_s = 15{,}000$, $y = 0.157$. Now follow the ordinate $y = 0.157$ to its intersection with the line $\theta = 0.04$. At this point $\Delta = 0.179$ and $p = 0.00548$.

$$d = \frac{t}{\Delta} = \frac{8}{0.179} = 44.7 \text{ in.}$$

$$A_s = pbd = 0.00548 \times 64 \times 44.7 = 15.67 \text{ sq. in.}$$

The T-beam chart is worked in the reverse direction when it is desired to know what resisting moment a given section can exert. Suppose that the bending up of four rods has reduced the steel ratio to 0.00358.

In the proportional chart follow the abscissa, $p = 0.00358$ to its intersection with the sloping line $\Delta = 0.1791$. At this point $\theta = 0.00285$ and $y = 0.103$.

$$\text{If } f_c = 600, \qquad \theta = \frac{600}{f_s} = 0.00285, \qquad f_s = 21{,}100 \text{ lb.}$$

$$\text{If } f_s = 15{,}000, \qquad \theta = \frac{f_c}{15{,}000} = 0.00285, \qquad f_c = 428 \text{ lb.}$$

This second group are the working stresses. Now follow the ordinate $y = 0.103$ to its intersection with the value $f_c = 15{,}000$. At this point $R = 1540$. Trace this abscissa, $R = 1540$, to its intersection with the line $t = 8$ in. At this point $\frac{M}{b} = 98{,}000$. Then $M = 98{,}000 \times 64 = 6{,}270{,}000$ in.-lb.

The handling of problems on both Diagrams 14 and 15 is facilitated by the use of two pointers. The last value found is held by one pointer while the other is used to pick out the value determined by the next step in the problem.

TABLE 1.—AREAS, PERIMETERS, AND WEIGHTS OF RODS

	Round rods			Square rods		
Size (inches)	Area (square inches)	Perimeter (inches)	Weight per foot (pounds)	Area (square inches)	Perimeter (inches)	Weight per foot (pounds)
1/4	0.0491	0.785	0.17	0.0625	1.00	0.21
5/16	0.0767	0.982	0.26	0.0977	1.25	0.33
3/8	0.1104	1.178	0.38	0.1406	1.50	0.48
7/16	0.1503	1.374	0.51	0.1914	1.75	0.65
1/2	0.1963	1.571	0.67	0.2500	2.00	0.85
9/16	0.2485	1.767	0.85	0.3164	2.25	1.08
5/8	0.3068	1.964	1.04	0.3906	2.50	1.33
11/16	0.3712	2.160	1.26	0.4727	2.75	1.61
3/4	0.4418	2.356	1.50	0.5625	3.00	1.91
13/16	0.5185	2.553	1.76	0.6602	3.25	2.25
7/8	0.6013	2.749	2.04	0.7656	3.50	2.60
15/16	0.6903	2.945	2.35	0.8789	3.75	2.99
1	0.7854	3.142	2.67	1.0000	4.00	3.40
1 1/8	0.9940	3.534	3.38	1.2656	4.50	4.30
1 1/4	1.2272	3.927	4.17	1.5625	5.00	5.31
1 3/8	1.4849	4.320	5.05	1.8906	5.50	6.43
1 1/2	1.7671	4.712	6.01	2.2500	6.00	7.65
1 5/8	2.0739	5.105	7.05	2.6406	6.50	9.98
1 3/4	2.4053	5.498	8.18	3.0625	7.00	10.41
1 7/8	2.7612	5.891	9.39	3.5156	7.50	11.95
2	3.1416	6.283	10.68	4.0000	8.00	13.60
2 1/4	3.9761	7.069	13.52	5.0625	9.00	17.22
2 1/2	4.9087	7.854	16.69	6.2500	10.00	21.25
2 3/4	5.9396	8.639	20.20	7.5625	11.00	25.72
3	7.0686	9.425	24.03	9.0000	12.00	30.09

TABLE 2.—USE FOR RECTANGULAR BEAMS AND SLABS

$$k = \frac{1}{1 + \frac{f_s}{nf_c}}, \quad j = 1 - \frac{k}{3}, \quad p = \frac{\frac{1}{2}}{\frac{f_s}{f_c}\left(\frac{f_s}{nf_c} + 1\right)}, \quad K = pf_sj, \text{ or } \frac{f_ckj}{2} \text{ (from Formula } M = Kbd^2\text{)}$$

Ratio of Moduli $n = 12$

f_s	f_c	k	j	p	K	f_s	f_c	k	j	p	K
12,000	500	0.332	0.889	0.0069	73.6	16,000	500	0.273	0.909	0.0043	62.0
	550	0.354	0.882	0.0081	85.7		550	0.292	0.903	0.0050	72.2
	600	0.375	0.875	0.0094	98.4		600	0.310	0.897	0.0058	83.2
	650	0.394	0.869	0.0107	111.3		650	0.328	0.891	0.0067	95.0
	700	0.412	0.863	0.0120	124.4		700	0.344	0.885	0.0075	106.2
	800	0.444	0.852	0.0148	151.3		800	0.375	0.875	0.0094	131.3
	900	0.475	0.842	0.0177	178.8		900	0.403	0.866	0.0113	156.5
14,000	500	0.300	0.900	0.0054	67.5	17,000	500	0.261	0.917	0.0038	59.2
	550	0.320	0.893	0.0063	78.6		550	0.280	0.907	0.0045	69.4
	600	0.340	0.888	0.0073	90.6		600	0.298	0.901	0.0052	79.6
	650	0.358	0.881	0.0083	102.5		650	0.314	0.895	0.0060	91.3
	700	0.375	0.875	0.0094	114.8		700	0.331	0.890	0.0068	102.9
	800	0.407	0.864	0.0116	140.4		800	0.361	0.880	0.0085	127.1
	900	0.435	0.855	0.0140	167.5		900	0.390	0.870	0.0103	152.2
15,000	500	0.286	0.905	0.0048	64.7	20,000	500	0.230	0.923	0.0029	53.1
	550	0.306	0.898	0.0056	75.4		550	0.248	0.917	0.0034	62.4
	600	0.325	0.892	0.0065	86.7		600	0.264	0.912	0.0040	72.2
	650	0.343	0.886	0.0074	98.4		650	0.280	0.907	0.0046	82.4
	700	0.360	0.880	0.0084	110.3		700	0.295	0.902	0.0052	93.3
	800	0.391	0.870	0.0105	135.7		800	0.324	0.892	0.0065	115.6
	900	0.418	0.861	0.0125	161.5		900	0.351	0.883	0.0079	139.5

Ratio of Moduli $n = 15$

f_s	f_c	k	j	p	K	f_s	f_c	k	j	p	K
12,000	500	0.384	0.872	0.0080	83.7	16,000	500	0.319	0.894	0.0050	71.3
	550	0.407	0.864	0.0093	96.4		550	0.339	0.887	0.0058	82.3
	600	0.428	0.857	0.0107	110.0		600	0.358	0.881	0.0067	94.4
	650	0.448	0.851	0.0121	123.6		650	0.378	0.874	0.0077	107.4
	700	0.467	0.844	0.0136	138.0		700	0.397	0.868	0.0087	120.6
	750	0.484	0.839	0.0151	152.0		750	0.414	0.862	0.0097	133.8
	800	0.501	0.833	0.0167	166.9		800	0.429	0.857	0.0107	146.7
14 000	500	0.348	0.884	0.0062	76.7	17,000	500	0.306	0.898	0.0045	68.5
	550	0.372	0.876	0.0073	89.5		550	0.326	0.892	0.0053	80.4
	600	0.391	0.870	0.0084	102.0		600	0.346	0.885	0.0061	91.8
	650	0.410	0.863	0.0095	114.8		650	0.365	0.878	0.0070	103.5
	700	0.428	0.857	0.0107	128.3		700	0.382	0.873	0.0079	116.1
	750	0.446	0.851	0.0120	142.3		750	0.398	0.866	0.0088	129.5
	800	0.462	0.846	0.0132	156.3		800	0.415	0.862	0.0097	141.9
15,000	500	0.334	0.889	0.0056	74.1	20,000	500	0.272	0.909	0.0034	61.8
	550	0.355	0.882	0.0065	86.1		550	0.292	0.903	0.0040	72.2
	600	0.375	0.875	0.0075	98.3		600	0.311	0.897	0.0047	83.7
	650	0.393	0.869	0.0085	111.3		650	0.328	0.891	0.0053	94.4
	700	0.411	0.863	0.0096	124.2		700	0.344	0.885	0.0060	106.2
	750	0.429	0.857	0.0107	137.9		750	0.359	0.880	0.0067	117.9
	800	0.445	0.852	0.0118	151.2		800	0.374	0.875	0.0075	130.9

TABLE 3.—VALUES OF k AND j FOR RECTANGULAR BEAMS AND SLABS

$$k = \sqrt{2pn + (pn)^2} - pn \qquad j = 1 - \tfrac{1}{3}k$$

p	$n = 12$		$n = 15$		p	$n = 12$		$n = 15$	
	k	j	k	j		k	j	k	j
0.0010	0.145	0.952	0.158	0.947	0.0090	0.370	0.877	0.402	0.866
0.0012	0.155	0.948	0.169	0.944	0.0092	0.373	0.876	0.405	0.865
0.0014	0.166	0.945	0.181	0.940	0.0094	0.376	0.875	0.407	0.864
0.0016	0.177	0.941	0.192	0.936	0.0096	0.379	0.874	0.411	0.863
0.0018	0.186	0.938	0.202	0.933	0.0098	0.381	0.873	0.414	0.862
0.0020	0.196	0.935	0.217	0.928	0.0100	0.385	0.872	0.418	0.861
0.0022	0.204	0.932	0.222	0.926	0.0102	0.387	0.871	0.420	0.860
0.0024	0.212	0.929	0.231	0.923	0.0104	0.391	0.870	0.423	0.859
0.0026	0.220	0.927	0.240	0.920	0.0106	0.394	0.869	0.426	0.858
0.0028	0.227	0.924	0.248	0.917	0.0108	0.396	0.868	0.429	0.857
0.0030	0.235	0.922	0.258	0.914	0.0110	0.398	0.867	0.432	0.856
0.0032	0.241	0.920	0.263	0.912	0.0112	0.402	0.866	0.434	0.855
0.0034	0.248	0.917	0.271	0.910	0.0114	0.404	0.865	0.437	0.854
0.0036	0.254	0.915	0.277	0.908	0.0116	0.407	0.864	0.440	0.853
0.0038	0.260	0.913	0.284	0.905	0.0118	0.410	0.863	0.443	0.852
0.0040	0.266	0.911	0.292	0.903	0.0120	0.412	0.863	0.446	0.851
0.0042	0.270	0.910	0.297	0.901	0.0122	0.415	0.862	0.448	0.851
0.0044	0.276	0.908	0.303	0.899	0.0124	0.417	0.861	0.451	0.850
0.0046	0.281	0.906	0.309	0.897	0.0126	0.419	0.860	0.454	0.849
0.0048	0.286	0.904	0.315	0.895	0.0128	0.422	0.859	0.457	0.848
0.0050	0.291	0.903	0.320	0.893	0.0130	0.424	0.859	0.459	0.847
0.0052	0.295	0.901	0.324	0.892	0.0132	0.427	0.858	0.461	0.846
0.0054	0.300	0.900	0.329	0.891	0.0134	0.429	0.857	0.464	0.845
0.0056	0.304	0.899	0.333	0.889	0.0136	0.432	0.856	0.466	0.845
0.0058	0.309	0.897	0.337	0.888	0.0138	0.434	0.855	0.468	0.844
0.0060	0.314	0.895	0.344	0.885	0.0140	0.436	0.855	0.471	0.843
0.0062	0.317	0.894	0.348	0.884	0.0142	0.437	0.854	0.473	0.843
0.0064	0.322	0.893	0.352	0.883	0.0144	0.440	0.853	0.475	0.842
0.0066	0.325	0.892	0.356	0.881	0.0146	0.442	0.853	0.477	0.841
0.0068	0.330	0.890	0.360	0.880	0.0148	0.444	0.852	0.479	0.840
0.0070	0.334	0.889	0.365	0.878	0.0150	0.446	0.861	0.481	0.840
0.0072	0.338	0.887	0.369	0.877	0.0152	0.449	0.850	0.483	0.839
0.0074	0.342	0.886	0.372	0.876	0.0154	0.451	0.850	0.485	0.838
0.0076	0.345	0.885	0.376	0.875	0.0156	0.453	0.849	0.487	0.838
0.0078	0.349	0.884	0.380	0.873	0.0158	0.455	0.848	0.489	0.837
0.0080	0.353	0.882	0.384	0.872	0.0160	0.457	0.848	0.493	0.836
0.0082	0.356	0.881	0.387	0.871	0.0170	0.467	0.845	0.502	0.833
0.0084	0.360	0.880	0.390	0.870	0.0180	0.476	0.841	0.513	0.829
0.0086	0.363	0.879	0.394	0.869	0.0190	0.485	0.838	0.522	0.826
0.0088	0.366	0.878	0.398	0.867	0.0200	0.493	0.836	0.531	0.823

TABLE 4.—VALUES OF bd^2 FOR DIFFERENT VALUES OF b AND d

Values of bd^2	$b = 12$		$b = 10$		$b = 15$		$d = 0.5b$			$d = b$		$d = 1.5b$			$d = 1.7b$			$d = 2b$		
	d	bd	d	bd	d	bd	b	d	bd	b	bd	b	d	bd	b	d	bd	b	d	bd
108	3	36	3.3	32.8	2.7	40.5	7.6	3.8	28.8	4.8	23.0	3.6	5.4	19.8	3.4	5.7	19.4	3.0	6.0	18.0
192	4	48	4.4	43.7	3.6	54.0	9.2	4.6	41.4	5.8	33.6	4.4	6.6	29.0	4.1	6.9	28.3	3.6	7.3	26.3
300	5	60	5.5	54.8	4.5	67.5	10.6	5.3	56.2	6.7	44.9	5.1	7.7	39.2	4.7	8.0	37.6	4.2	8.4	35.2
432	6	72	6.6	65.7	5.4	81.0	12.0	6.0	72.0	7.6	57.8	5.8	8.7	50.1	5.4	9.1	49.2	4.8	9.5	45.6
588	7	84	7.7	76.5	6.3	94.5	13.4	6.7	89.8	8.4	70.6	6.4	9.6	61.5	5.9	10.0	59.0	5.3	10.6	56.2
768	8	96	8.8	87.5	7.2	108.0	14.6	7.3	106.5	9.2	84.6	7.0	10.5	73.5	6.4	10.9	69.8	5.8	11.5	66.8
972	9	108	9.9	98.5	8.1	121.3	15.8	7.9	124.9	9.9	98.0	7.6	11.4	86.7	6.9	11.8	81.4	6.3	12.5	78.8
1,200	10	120	10.9	109.4	8.9	133.5	17.0	8.5	144.5	10.7	114.5	8.1	12.1	98.0	7.5	12.7	95.2	6.7	13.4	89.8
1,452	11	132	12.1	120.2	9.8	147.0	18.0	9.0	162.0	11.4	129.7	8.7	13.1	114.0	8.0	13.6	109.0	7.2	14.3	103.0
1,728	12	144	13.1	131.2	10.7	160.5	19.0	9.5	180.5	12.0	144.0	9.2	13.8	127.0	8.4	14.3	120.0	7.6	15.2	115.6
2,028	13	156	14.3	142.3	11.6	174.0	20.0	10.0	200.0	12.7	161.3	9.7	14.6	141.6	9.0	15.2	136.8	8.0	15.9	127.0
2,352	14	168	15.4	153.6	12.6	189.0	21.0	10.5	220.2	13.3	176.9	10.2	15.3	156.0	9.4	15.9	149.3	8.4	16.8	141.0
2,700	15	180	16.4	164.0	13.5	202.5	22.0	11.0	242.0	13.9	193.2	10.7	16.1	172.0	9.8	16.7	163.5	8.8	17.6	155.0
3,072	16	192	17.5	175.0	14.3	214.5	23.0	11.5	264.2	14.6	213.2	11.2	16.8	188.2	10.2	17.4	177.5	9.2	18.3	168.0
3,468	17	204	18.6	186.0	15.2	228.0	24.0	12.0	288.0	15.2	231.0	11.6	17.4	202.0	10.6	18.1	192.0	9.6	19.1	184.0
3,888	18	216	19.7	197.0	16.1	241.5	25.0	12.5	312.2	15.7	246.5	12.1	18.1	219.0	11.0	18.8	207.0	9.9	19.8	196.0
4,332	19	228	20.8	208.0	17.0	255.0	25.8	12.9	332.7	16.3	265.7	12.5	18.8	235.0	11.5	19.5	224.0	10.3	20.6	212.0
4,800	20	240	21.9	218.8	17.9	268.5	26.8	13.4	359.5	16.9	285.6	12.9	19.3	249.0	11.9	20.2	240.5	10.7	21.3	228.0
5,292	21	252	23.0	229.8	18.8	282.0	27.8	13.9	387.0	17.5	306.2	13.3	19.9	265.0	12.2	20.8	254.0	11.0	22.0	242.0
5,808	22	264	24.1	240.6	19.7	295.5	28.6	14.3	409.0	18.0	324.0	13.8	20.7	286.0	12.7	21.5	273.0	11.4	22.7	258.5
6,348	23	276	25.2	252.0	20.6	309.0	29.4	14.7	432.0	18.6	345.9	14.2	21.3	302.0	13.0	22.1	288.0	11.7	23.4	274.0
6,912	24	288	26.3	263.0	21.5	323.0	30.2	15.1	456.0	19.1	364.8	14.6	21.9	320.0	13.4	22.8	306.0	12.1	24.1	292.0
7,500	25	300	27.4	274.0	22.4	336.0	31.0	15.5	480.0	19.6	384.1	15.0	22.5	338.0	13.8	23.4	323.0	12.4	24.7	306.0
8,112	26	312	28.5	285.0	23.2	348.0	32.0	16.0	522.0	20.2	408.0	15.4	23.1	356.0	14.1	24.0	339.0	12.7	25.4	323.0
8,748	27	324	29.6	295.8	24.1	361.5	32.8	16.4	537.5	20.6	424.4	15.8	23.7	375.0	14.5	24.6	356.0	13.0	26.0	338.0
9,408	28	336	30.7	307.0	25.0	375.0	33.6	16.8	565.0	21.2	449.4	16.1	24.1	388.0	14.8	25.2	373.0	13.3	26.6	354.0
10,092	29	348	31.9	318.2	25.9	388.0	34.4	17.2	592.0	21.7	470.9	16.5	24.8	410.0	15.2	25.8	392.0	13.7	27.3	374.0
10,800	30	360	32.9	328.3	26.8	402.0	35.2	17.6	620.0	22.1	488.4	16.9	25.4	430.0	15.5	26.4	410.0	14.0	27.9	390.0
11,532	31	372	33.9	339.4	27.7	416.0	35.8	17.9	642.0	22.6	510.8	17.3	26.0	450.0	15.9	27.0	429.0	14.3	28.5	408.0
12,288	32	384	35.0	350.0	28.6	429.0	36.6	18.3	670.0	23.1	533.6	17.6	26.4	465.0	16.2	27.6	447.0	14.6	29.2	424.0
13,068	33	396	36.2	361.8	29.5	442.5	37.4	18.7	700.0	23.6	557.0	18.0	27.0	486.0	16.5	28.1	464.0	14.9	29.7	443.0
13,872	34	408	37.2	372.1	30.4	456.0	38.2	19.1	730.0	24.1	580.8	18.3	27.5	503.0	16.9	28.7	485.0	15.2	30.4	462.0
14,700	35	420	38.3	383.3	31.3	469.0	38.8	19.4	753.0	24.5	600.3	18.7	28.0	524.0	17.2	29.3	504.0	15.5	30.9	478.0
15,552	36	432	39.4	394.0	32.1	482.0	39.6	19.8	784.0	25.0	625.0	19.1	28.6	546.0	17.6	29.8	525.0	15.8	31.5	498.0
16,428	37	444	40.6	406.0	33.0	495.0	40.4	20.2	816.0	25.4	645.2	19.4	29.1	565.0	17.9	30.4	544.0	16.1	32.1	517.0
17,328	38	456	41.6	416.0	33.9	508.0	41.2	20.6	847.0	25.9	670.8	19.8	29.7	588.0	18.2	30.9	562.0	16.4	32.7	537.0
18,252	39	468	42.7	427.1	34.8	522.0	41.8	20.9	875.0	26.4	697.0	20.1	30.1	605.0	18.5	31.4	581.0	16.6	33.2	552.0
19,200	40	480	43.8	438.0	35.7	536.0	42.4	21.2	900.0	26.8	718.2	20.4	30.6	624.0	18.8	32.0	602.0	16.9	33.8	572.0

TABLE 5.—NUMBER OF RODS AND SECTIONAL AREA IN SQUARE INCHES FOR BEAM AND COLUMN REINFORCEMENT

Size of rod		2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
½ in.	round	0.39	0.58	0.78	0.98	1.18	1.37	1.57	1.77	1.96	2.16	2.36	2.55	2.75	2.94	3.10
	square	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	3.75	4.04
9/16 in.	round	0.49	0.74	0.99	1.24	1.49	1.74	1.99	2.24	2.48	2.73	2.98	3.23	3.48	3.73	3.98
	square	0.63	0.94	1.27	1.58	1.90	2.21	2.53	2.85	3.16	3.48	3.80	4.11	4.43	4.75	5.06
⅝ in.	round	0.61	0.91	1.23	1.53	1.84	2.15	2.45	2.76	3.07	3.37	3.68	3.99	4.30	4.60	4.91
	square	0.78	1.07	1.56	1.95	2.34	2.73	3.12	3.52	3.91	4.30	4.69	5.08	5.47	5.86	6.25
11/16 in.	round	0.74	1.11	1.48	1.86	2.23	2.60	2.97	3.34	3.71	4.08	4.45	4.83	5.20	5.57	5.94
	square	0.94	1.41	1.89	2.36	2.84	3.31	3.78	4.25	4.73	5.20	5.67	6.15	6.62	7.09	7.56
¾ in.	round	0.88	1.32	1.77	2.21	2.65	3.09	3.53	3.98	4.42	4.86	5.30	5.74	6.19	6.63	7.07
	square	1.12	1.68	2.25	2.81	3.38	3.94	4.50	5.06	5.62	6.19	6.75	7.31	7.88	8.44	9.00
13/16 in.	round	1.03	1.55	2.07	2.59	3.11	3.63	4.15	4.67	5.18	5.70	6.22	6.74	7.26	7.78	8.30
	square	1.32	1.98	2.64	3.30	3.96	4.62	5.28	5.94	6.60	7.26	7.92	8.58	9.24	9.90	10.56
⅞ in.	round	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01	6.61	7.22	7.82	8.42	9.02	9.62
	square	1.53	2.29	3.06	3.83	4.59	5.36	6.12	6.89	7.66	8.42	9.19	9.95	10.72	11.48	12.25
15/16 in.	round	1.38	2.07	2.76	3.45	4.14	4.83	5.52	6.21	6.90	7.59	3.28	8.97	9.66	10.35	11.04
	square	1.75	2.63	3.52	4.39	5.27	6.15	7.03	7.91	8.79	9.67	10.55	11.43	12.30	13.18	14.06
1 in.	round	1.57	2.35	3.14	3.93	4.71	5.50	6.28	7.07	7.85	8.64	9.43	10.21	11.00	11.78	12.57
	square	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	14.00	15.00	16.00
1⅛ in.	round	1.98	2.98	3.98	4.97	5.96	6.96	7.95	8.95	9.94	10.94	11.93	12.92	13.92	14.91	15.90
	square	2.53	3.79	5.06	6.33	7.59	8.86	10.12	11.39	12.66	13.92	15.19	16.45	16.72	18.98	20.25
1¼ in.	round	2.45	3.68	4.91	6.14	7.36	8.59	9.82	11.04	12.27	13.50	14.73	15.95	17.18	18.41	19.64
	square	3.12	4.68	6.25	7.81	9.37	10.94	12.50	14.06	15.62	17.19	18.75	20.31	21.87	23.44	25.00

Table 6.—Spacing of Round Rods in Slabs

Diameter (inches)	Sectional area of steel per foot of slab when spaced as follows: 2 in.	2½ in.	3 in.	3½ in.	4 in.	4½ in.	5 in.	5½ in.	6 in.	7 in.	8 in.	9 in.	10 in.	12 in.
¼	0.29	0.23	0.20	0.17	0.15	0.13	0.12							
5/16	0.46	0.36	0.31	0.26	0.23	0.20	0.18	0.17	0.15	0.13				
⅜	0.66	0.53	0.44	0.38	0.33	0.29	0.26	0.24	0.22	0.19	0.17	0.15	0.13	
7/16	0.90	0.72	0.60	0.51	0.45	0.40	0.36	0.33	0.30	0.26	0.23	0.20	0.18	0.15
½	1.18	0.94	0.78	0.67	0.59	0.52	0.47	0.43	0.39	0.34	0.29	0.26	0.24	0.20
9/16	1.49	1.19	0.99	0.85	0.75	0.66	0.60	0.54	0.50	0.43	0.37	0.33	0.30	0.25
⅝	1.84	1.47	1.23	1.05	0.92	0.82	0.74	0.67	0.61	0.53	0.46	0.41	0.37	0.31
11/16	2.23	1.78	1.48	1.27	1.11	0.99	0.89	0.81	0.74	0.64	0.56	0.49	0.45	0.37
¾	2.65	2.12	1.77	1.51	1.32	1.18	1.06	0.96	0.88	0.76	0.66	0.59	0.53	0.44
13/16	3.11	2.48	2.07	1.78	1.56	1.38	1.24	1.13	1.04	0.89	0.78	0.69	0.62	0.52
⅞	3.61	2.88	2.40	2.06	1.80	1.60	1.44	1.31	1.20	1.03	0.90	0.80	0.72	0.60
15/16	4.14	3.31	2.76	2.37	2.07	1.84	1.66	1.51	1.38	1.18	1.03	0.92	0.83	0.69
1	4.71	3.77	3.14	2.69	2.36	2.09	1.88	1.71	1.57	1.35	1.18	1.05	0.94	0.78
1⅛	....	4.77	3.98	3.41	2.98	2.65	2.39	2.17	1.99	1.70	1.49	1.33	1.19	0.99
1¼	....	.....	4.91	4.21	3.68	3.27	2.95	2.68	2.45	2.10	1.84	1.64	1.47	1.23
1⅜	....	.....	.....	5.09	4.45	3.96	3.56	3.24	2.97	2.55	2.23	1.98	1.78	1.48
1½	....	.....	.....	.....	5.30	4.71	4.24	3.86	3.53	3.03	2.65	2.36	2.12	1.77

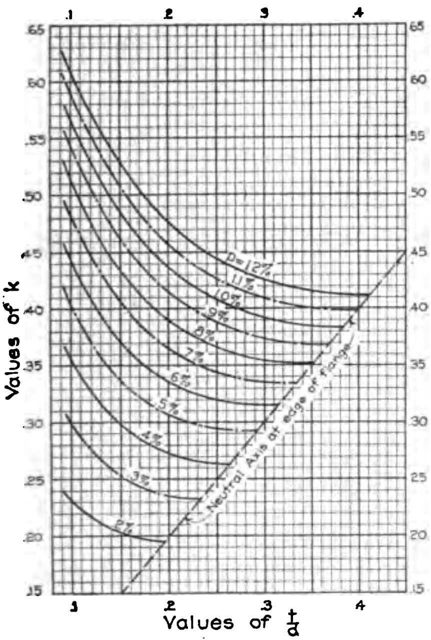
Table 7.—Spacing of Square Rods in Slabs

Dimension (inches)	Sectional area of steel per foot of slab when spaced as follows: 2 in.	2½ in.	3 in.	3½ in.	4 in.	4½ in.	5 in.	5½ in.	6 in.	7 in.	8 in.	9 in.	10 in.	12 in.
¼	0.37	0.30	0.25	0.21	0.19	0.17	0.15	0.13	0.12					
5/16	0.59	0.47	0.39	0.33	0.29	0.26	0.23	0.21	0.19	0.17	0.15	0.13		
⅜	0.84	0.67	0.56	0.48	0.42	0.37	0.34	0.31	0.28	0.24	0.21	0.19	0.17	0.14
7/16	1.15	0.92	0.77	0.66	0.57	0.51	0.46	0.42	0.38	0.33	0.29	0.25	0.23	0.19
½	1.50	1.20	1.00	0.86	0.75	0.67	0.60	0.55	0.50	0.43	0.37	0.33	0.30	0.25
9/16	1.90	1.52	1.27	1.08	0.95	0.84	0.76	0.69	0.63	0.54	0.47	0.42	0.38	0.32
⅝	2.34	1.87	1.56	1.34	1.17	1.04	0.94	0.85	0.78	0.67	0.59	0.52	0.47	0.39
11/16	2.84	2.27	1.99	1.62	1.42	1.33	1.13	1.03	0.94	0.81	0.71	0.66	0.57	0.47
¾	3.37	2.70	2.25	1.93	1.69	1.50	1.35	1.23	1.12	0.96	0.84	0.75	0.67	0.56
13/16	3.96	3.17	2.64	2.26	1.98	1.76	1.58	1.44	1.32	1.13	0.99	0.88	0.79	0.66
⅞	4.59	3.67	3.06	2.62	2.30	2.04	1.84	1.67	1.53	1.31	1.15	1.02	0.92	0.77
15/16	5.27	4.22	3.52	3.01	2.64	2.34	2.11	1.92	1.76	1.51	1.32	1.17	1.05	0.88
1	6.00	4.80	4.00	3.43	3.00	2.67	2.40	2.18	2.00	1.71	1.50	1.33	1.20	1.00
1⅛	....	6.08	5.06	4.34	3.80	3.37	3.04	2.76	2.53	2.17	1.89	1.69	1.52	1.27
1¼	...	.....	6.25	5.36	4.69	4.17	3.75	3.41	3.12	2.68	2.34	2.08	1.87	1.56
1⅜	...	.....	.....	6.48	5.67	5.04	4.54	4.12	3.78	3.24	2.84	2.52	2.27	1.89
1½	...	.....	.....	....	6.75	6.00	5.40	4.91	4.50	3.86	3.37	3.00	2.70	2.25

Diagram 9.

Use for T-beams.

Values of k and j for various percentages of steel. Based on $n = 12$.



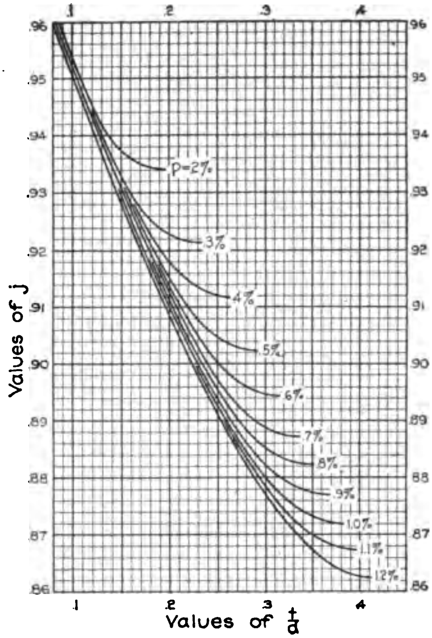
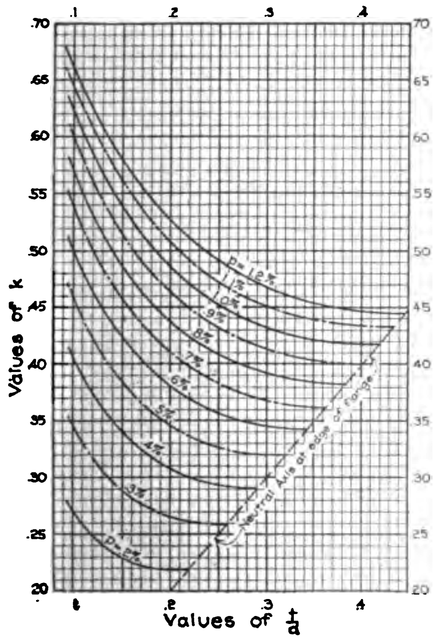


Diagram 10.

Use for T-beams.

Values of k and j for various percentages of steel. Based on $n = 15$.



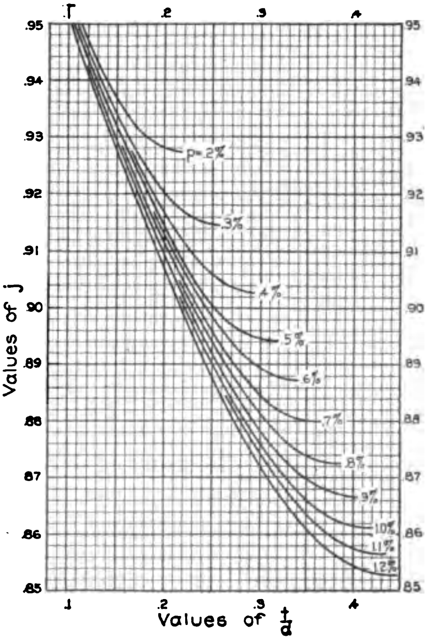


DIAGRAM 1.

USE FOR RECTANGULAR BEAMS AND SLABS.

Based on $n = 12$.

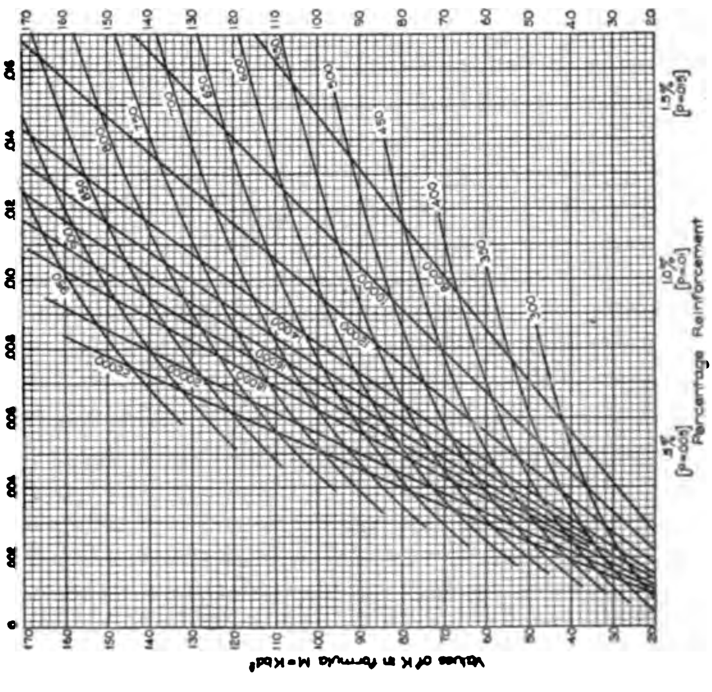


DIAGRAM 2.

USE FOR RECTANGULAR BEAMS AND SLABS.

Based on $n = 15$.

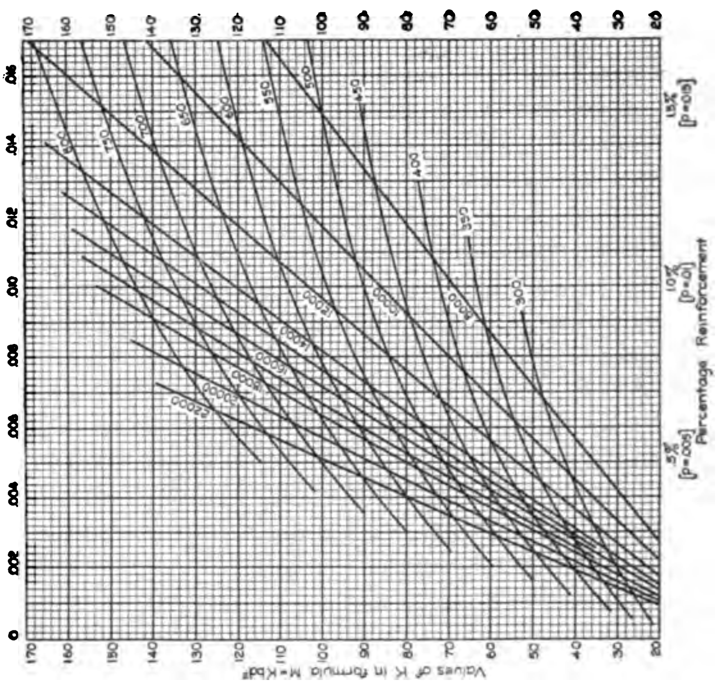


Diagram 3.—Use for Wedge-shaped Beams.

In *designing*, use Diagram 1 or 2 and then multiply the values of K and p obtained by $\cos^2\beta_c$ and $\frac{\cos^2\beta_c}{\cos\beta_t}$ respectively. (These products may be obtained directly from diagram given below.)

In *reviewing*, determine K and p in the usual manner and then multiply these values by $\frac{1}{\cos^2\beta_c}$ and $\frac{\cos\beta_t}{\cos^2\beta_c}$ respectively before using Diagram 1 or 2. (These products may be obtained directly from diagram given below.)

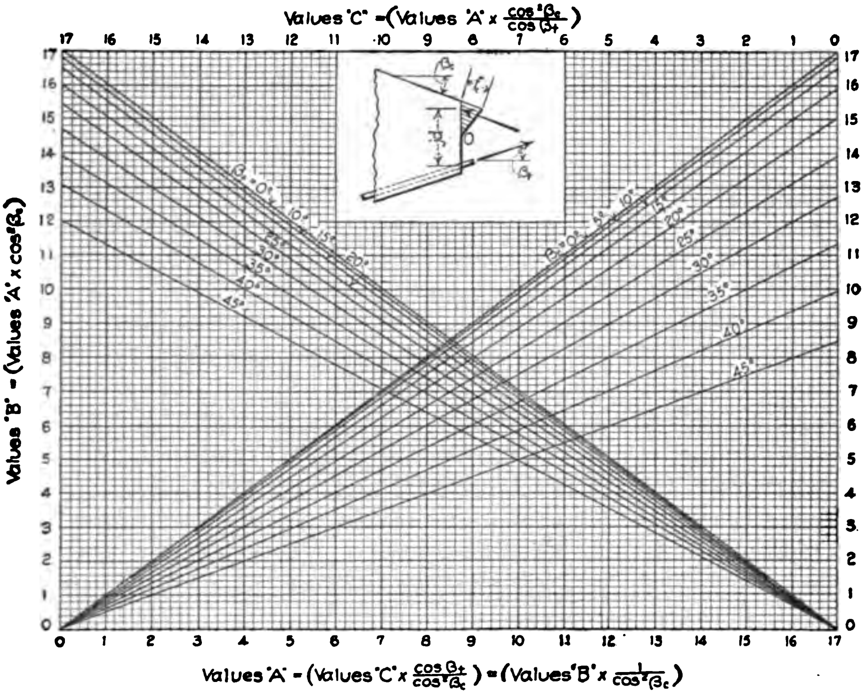


Diagram 4.—Use for Finding Approximate Weight of Rectangular Beams.

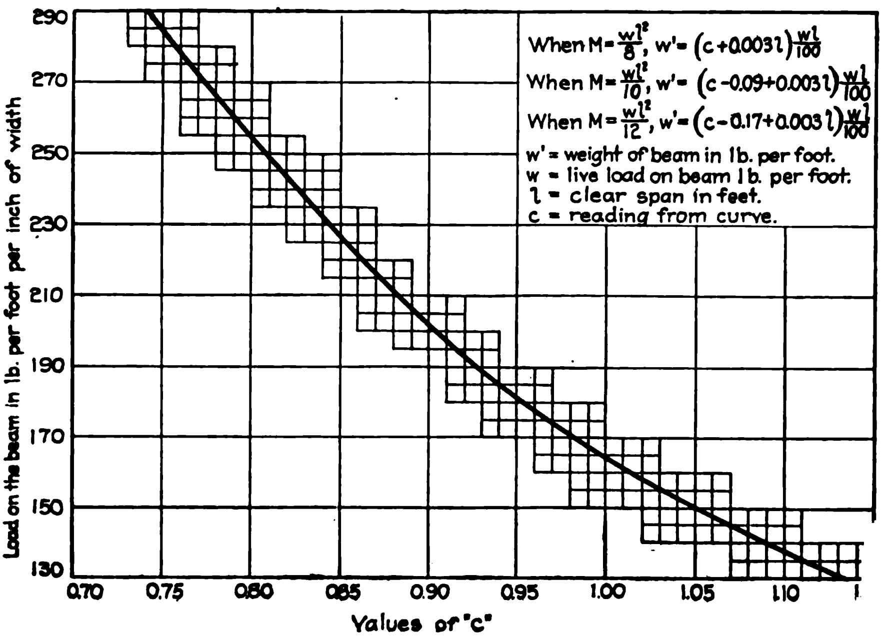


Diagram 5.

Bending Moments in Slabs for Uniformly Distributed Loads.

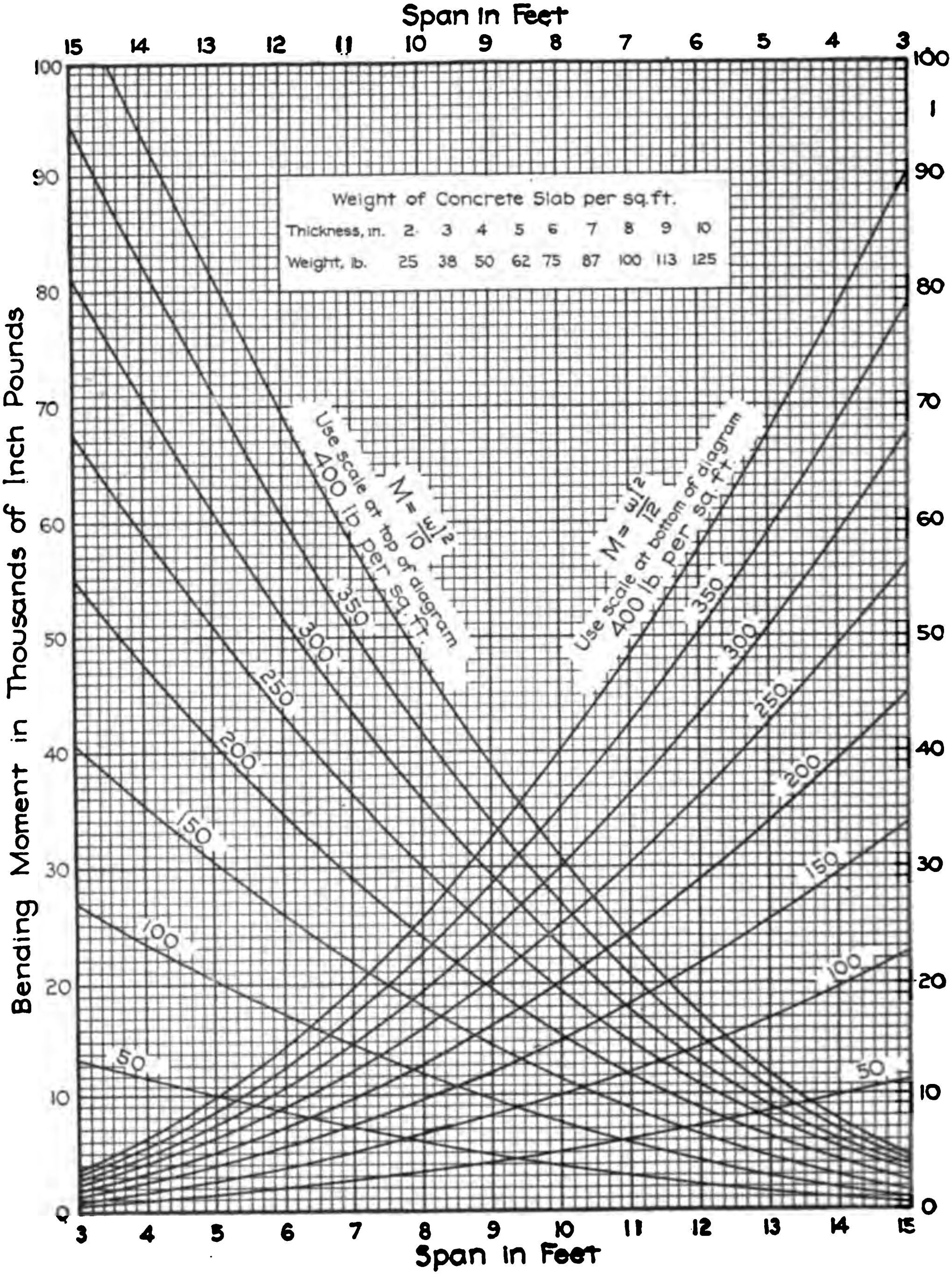


DIAGRAM 6, PART 2.
USE FOR SLABS.
Based on $f_s = 16{,}000$; $f_c = 650$; $n = 15$.

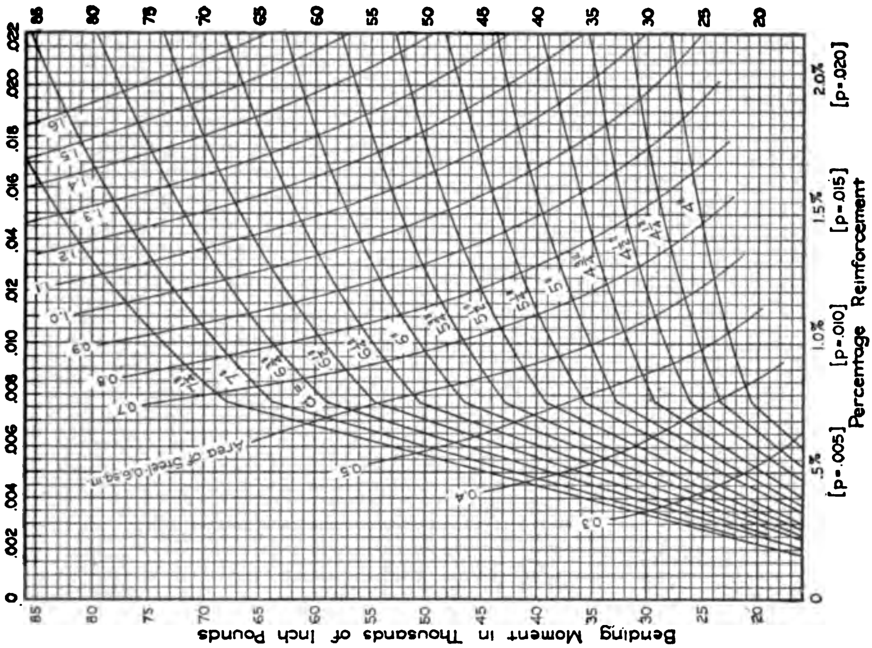


DIAGRAM 6, PART 1.
USE FOR SLABS.
Based on $f_s = 16{,}000$; $f_c = 650$; $n = 15$.

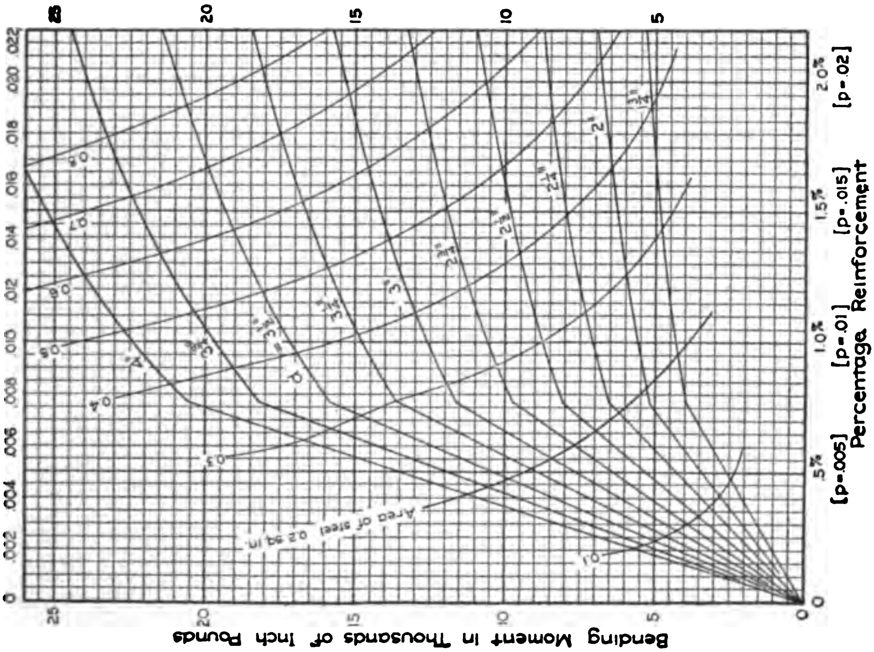


Diagram 7.

Use for T-beams.

Based on $f_s = 16{,}000$; $n = 12$.

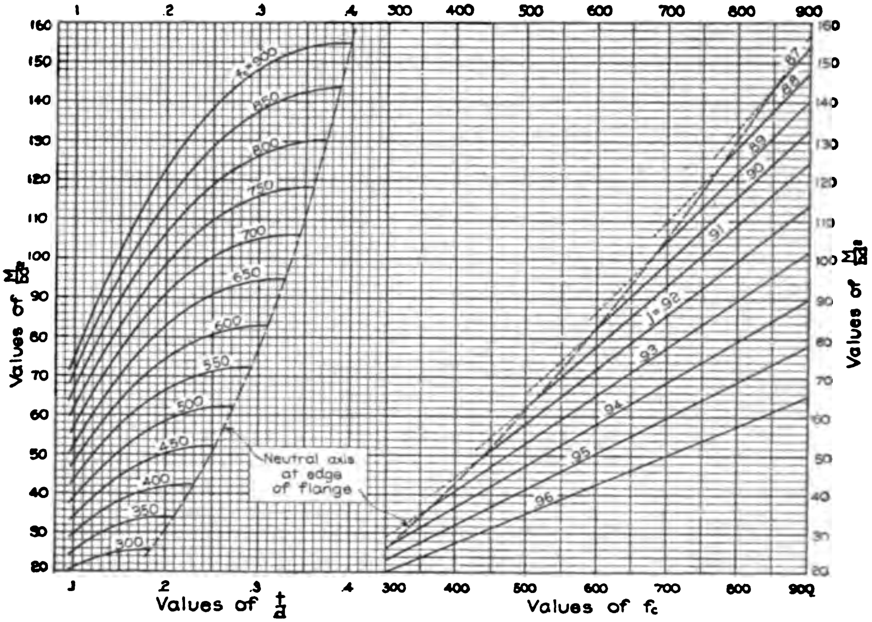


Diagram 8.

Use for T-beams.

Based on $f_s = 16{,}000$; $n = 15$.

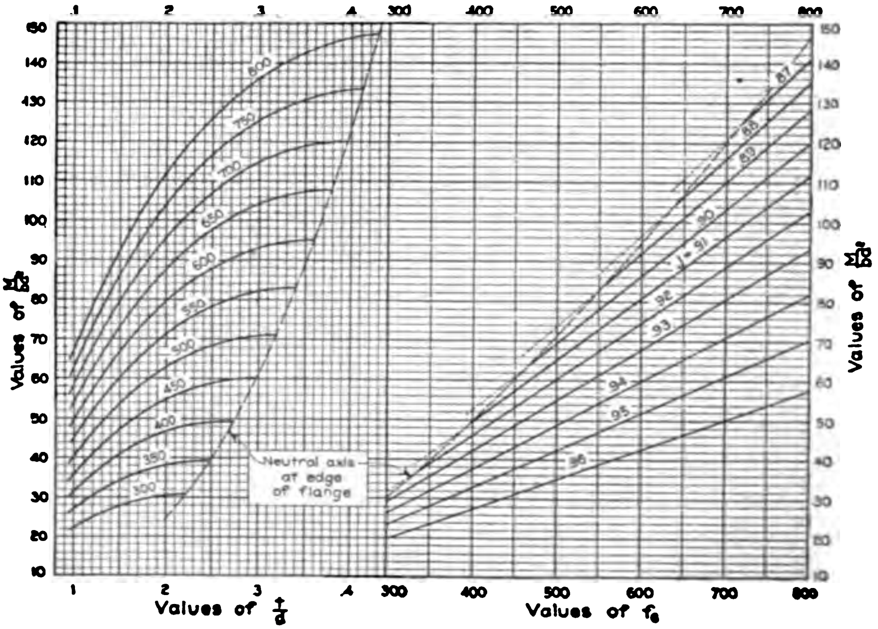


Diagram 9.

Use for T-beams.

Values of k and j for various percentages of steel. Based on $n = 12$.

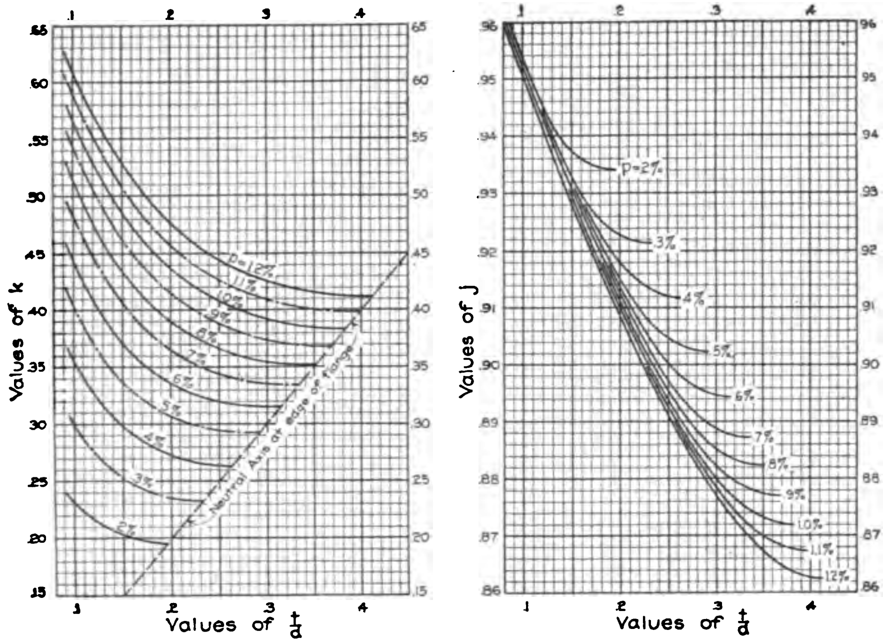


Diagram 10.

Use for T-beams.

Values of k and j for various percentages of steel. Based on $n = 15$.

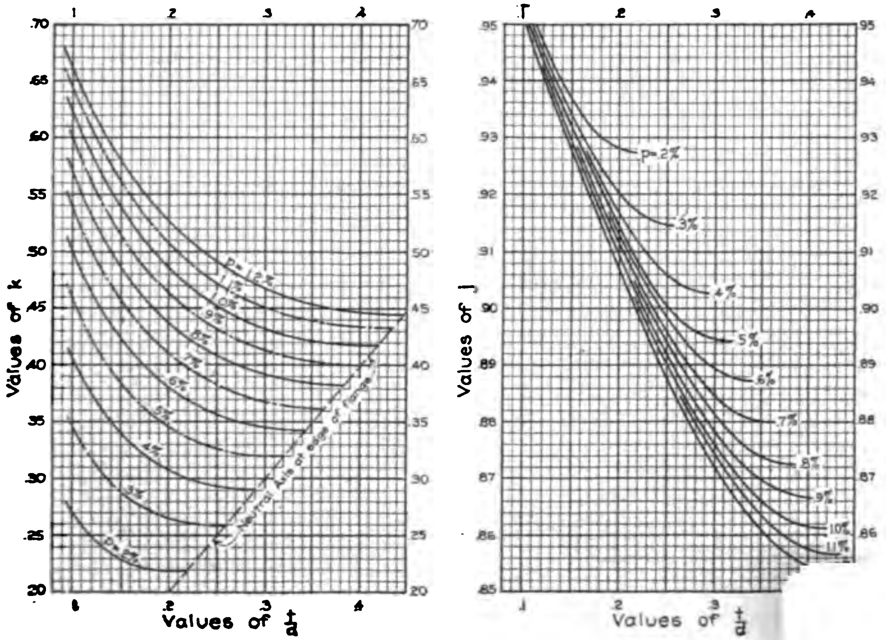
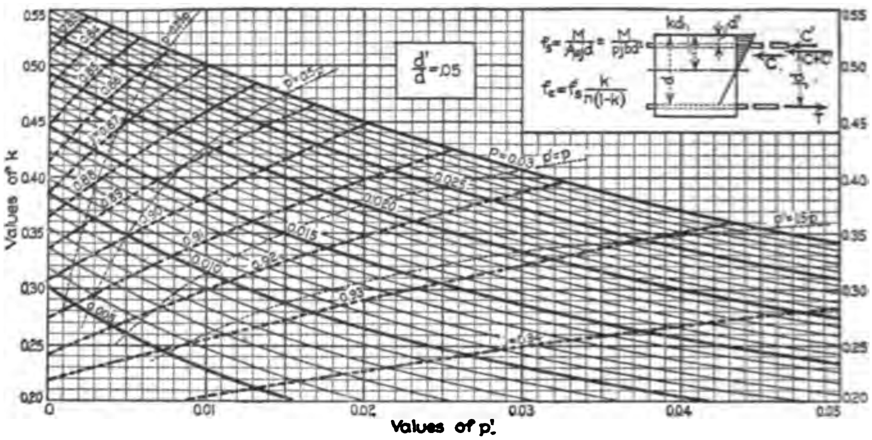
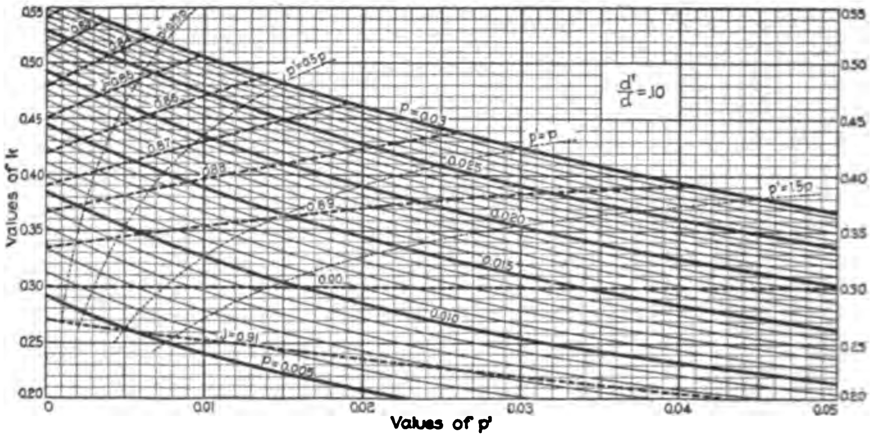


Diagram 11.

Use for Rectangular Beams With Steel in Top and Bottom.

Based on $n = 12$.





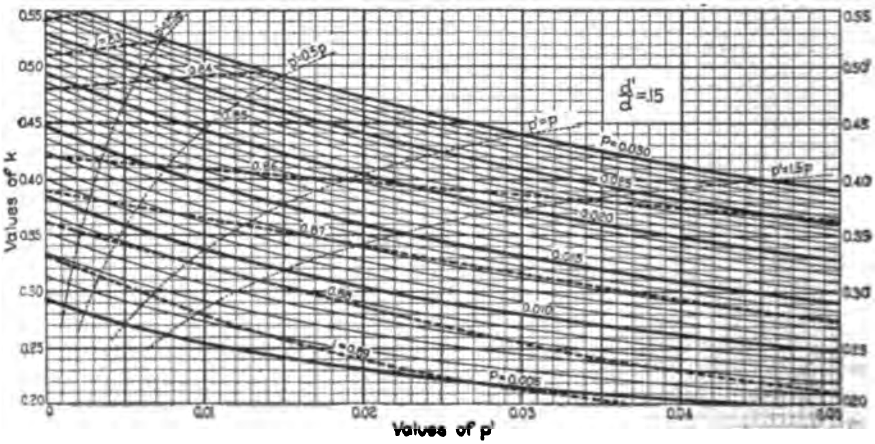
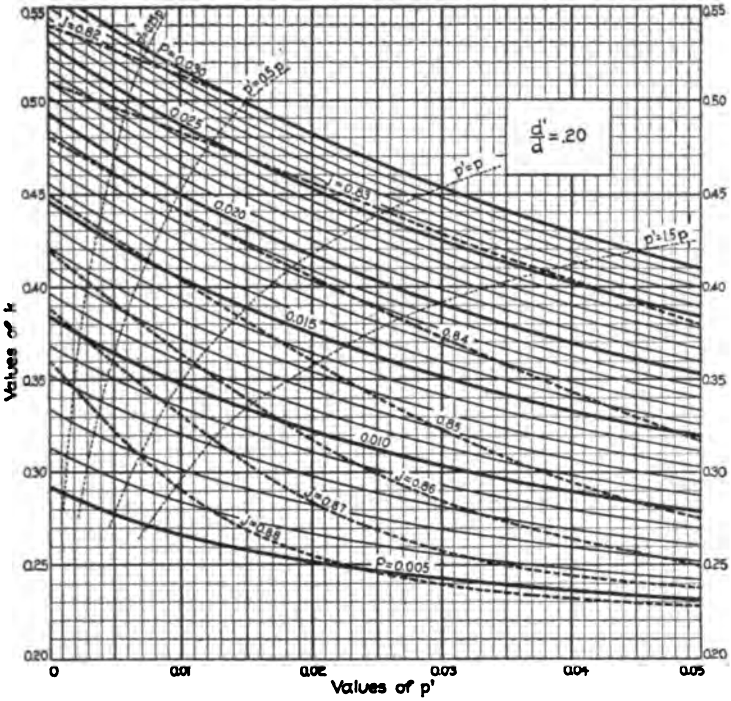


DIAGRAM 11.—(*Continued*)



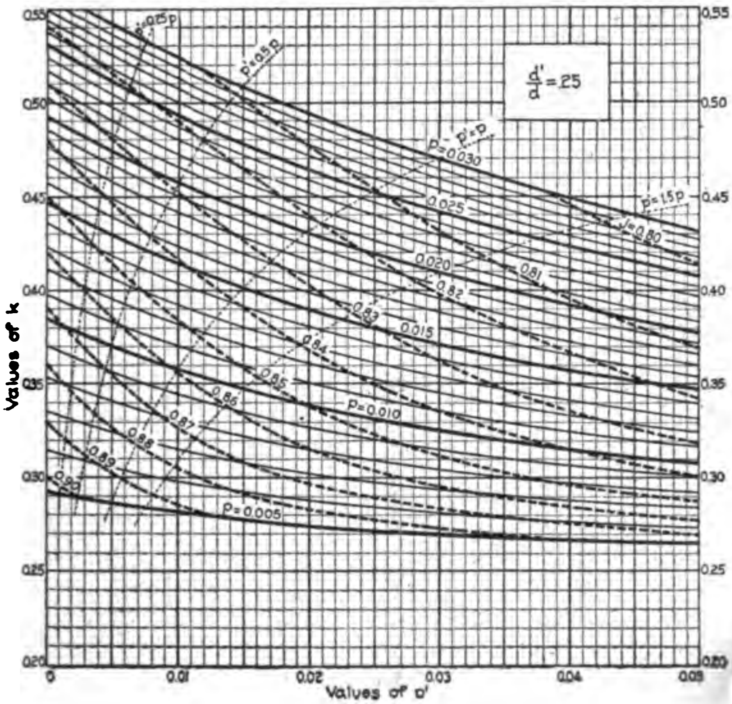


Diagram 12.

Use for Rectangular Beams With Steel in Top and Bottom.

Based on $n = 15$.

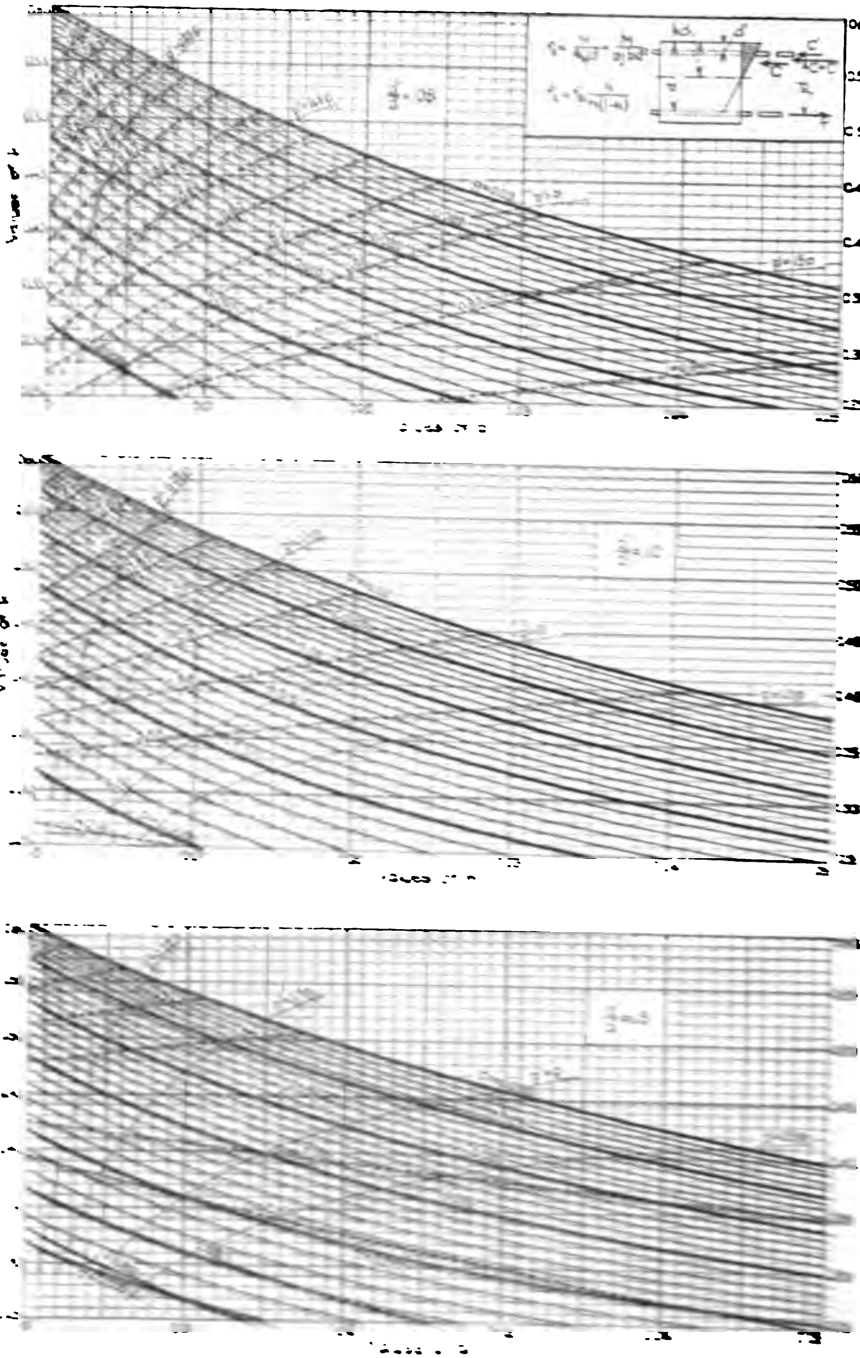
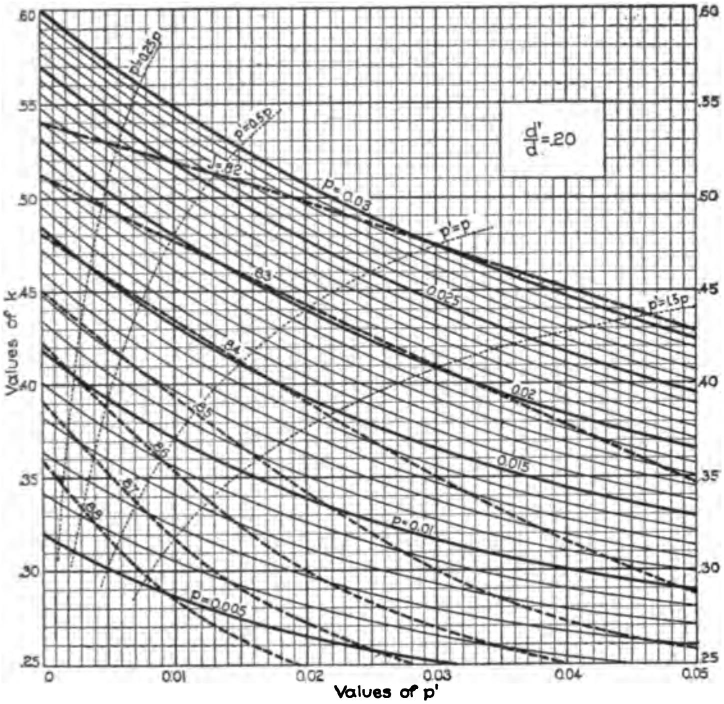


DIAGRAM 12.—(*Continued*)



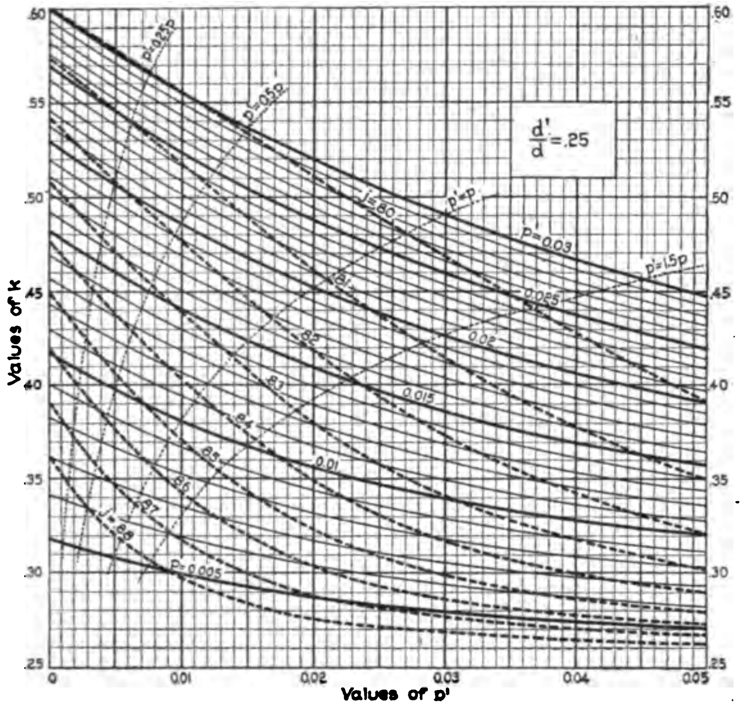
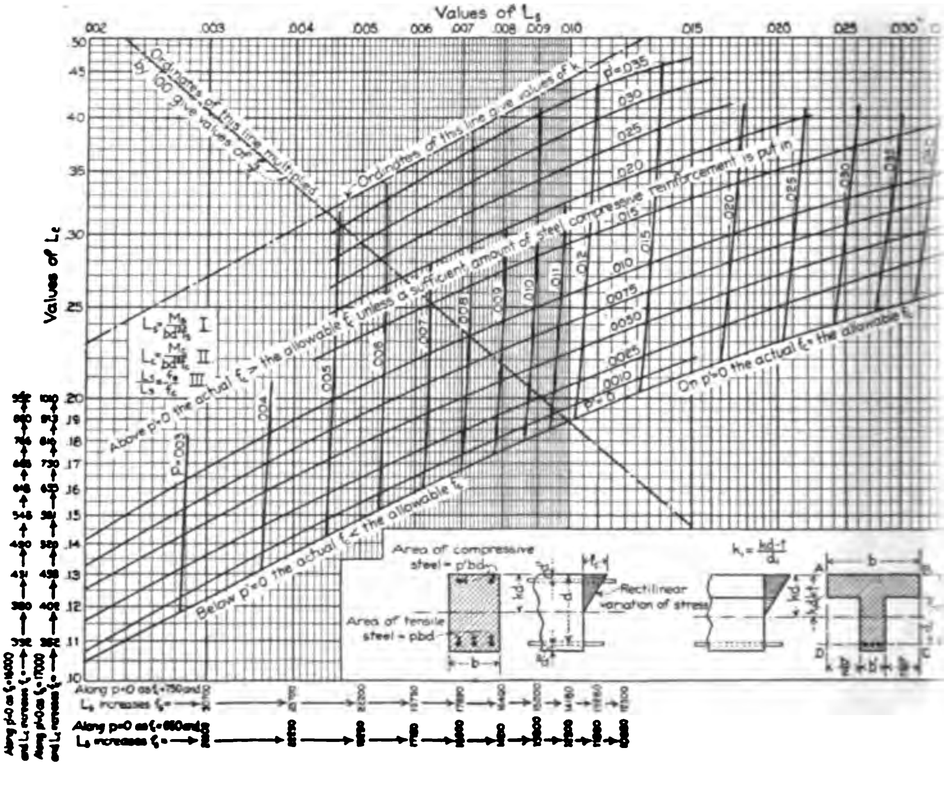


DIAGRAM 13.

LEFFLER'S COMPREHENSIVE BEAM CHART.

Curves for $\frac{d'}{d} = 0.10$ and $n = 15$.



SECTION 8

COLUMNS

1. Column Types.—Concrete columns are of four principal types:

1. Plain concrete columns or piers.
2. Columns reinforced with longitudinal rods only.
3. Columns reinforced with both hoops and longitudinal rods.
4. Columns reinforced with structural-steel shapes.

2. Plain Concrete Columns or Piers.—The Joint Committee does not consider any compression member a column unless it is reinforced and has a ratio of unsupported length to least width greater than four (see Art. 7 and *Appendix B*). Compression members in which the unsupported length to least width is four or less are referred to as piers. Piers may or may not be reinforced depending upon the stresses to which they are subjected.

Bending stresses in columns due to eccentric loads must be provided for by increasing the section until the maximum stress does not exceed the allowable. A formula for homogeneous columns or piers follows. Formulas applicable to reinforced-concrete columns or reinforced piers are given under "Bending and Direct Stress," Sect. 9.

The ordinary formula for the compressive fiber stress due to eccentric loading upon solid rectangular columns or piers of homogeneous materials (Fig. 1) is as follows:

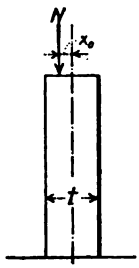


Fig. 1.

N = total load.
A = area of column.
x_0 = eccentricity.
t = breadth of column.
f_c = total unit pressure on outer fiber nearest to line of vertical pressure.

Then

$$f_c = \frac{N}{A}\left(1 + \frac{6x_0}{t}\right)$$

and the additional intensity of compressive stress due to eccentric loading is seen to be equal to $\frac{N}{A} \cdot \frac{6x_0}{t}$.

3. Columns with Longitudinal Reinforcement.—Since the modulus of elasticity of a material is the ratio of stress to deformation, it follows that for equal deformations the stresses in the steel and concrete of a concrete column will be as their moduli of elasticity. Thus

$$\frac{f_s}{f_c} = \frac{E_s}{E_c}, \text{ or } f_s = f_c n$$

Let A = total net area.
A_c = area of concrete.
A_s = area of longitudinal steel.
p = steel ratio $= \frac{A_s}{A}$.
f_s = tensile unit stress in steel.
f_c = compressive unit stress in concrete.

$n = \frac{E_s}{E_c}$.

P = total strength of a reinforced column for the stress f_c.

Then

$$P = f_cA_c + f_sA_s = f_c(A - pA) + f_cnpA$$

or

$$P = f_cA\,[1 + (n - 1)p]$$

Tests[1] on columns with vertical steel bar reinforcement indicate that the steel may be counted upon in design to take its portion of the loading as computed from the above equation.

The economy of steel reinforcement is dependent upon the working stresses permissible in the concrete and the value of n, since the stress in the steel $= f_cn$. The stresses in the steel will be relatively low except in the unusual combination of high working stresses in the concrete with a large value of n.

4. Columns with Hooped and Longitudinal Reinforcement.—Whenever a material is subjected to compression along one axis, then, as a consequence, there will be an expansion of the material along axes which are perpendicular to the one first considered. Thus, if the material of a column is held laterally, then lateral compressive stresses are developed which tend to neutralize the effect of the longitudinal compressive stresses and thus to increase the resistance against failure. This is the principle involved in the use of spiral or banded reinforcement. The addition of bands or spirals to columns having longitudinal reinforcement does not have much effect upon the deformation of such columns up to the point of failure without hooping. In fact the elastic limit and rigidity of the column appears to be decreased if anything. The effect of such hooping, however, raises slightly the ultimate strength and increases the capacity of the column to deform at loads beyond the elastic limit, so that a somewhat higher working stress may be employed on the concrete than for plain concrete columns. Tests show that about 1% of a closely spaced spiral of high-carbon steel is sufficient to prevent the longitudinal rods from bulging outward and will provide a satisfactory amount of toughness with a corresponding raising of the ultimate strength beyond the elastic limit.

The following notation will be used for hooped reinforcement.

Let d_1 = diameter of enclosed concrete to center line of hooping.

A_h = sectional area of one strand of hooping for given pitch.

s = pitch allowed.

l_h = length of hooping in 1 ft. in height of column.

p = percentage of hooping.

Then

$$(p)\left(\frac{\pi d_1^2}{4}\right)(s) = A_h\pi d_1$$

$$A_h = \frac{spd_1}{4}$$

For $p = 1\%$

$$A_h = (0.0025)(s)(d_1) \quad \text{or} \quad s = \frac{A_h}{0.0025d_1}$$

Also

$$A_hl_h = \frac{\pi d_1^2}{4}(12)(0.01)$$

$$l_h = \frac{37.7d_1}{s}$$

Results for banded and spiral reinforcement will not differ appreciably and the above formulas may be used for both bands and spirals.

5. Columns Reinforced with Structural-steel Shapes.—If a structural-steel column is designed to take all the load and then is simply fireproofed with a covering of concrete, it

[1] For tests on columns, see "Concrete, Plain and Reinforced," by TAYLOR and THOMPSON, 3d Edition, 1916.

cannot properly be called a reinforced-concrete column. To be classed under this heading the steel must be designed so that it takes a load in combination with the concrete; that is, the steel must be figured in the same way as vertical rods and the stresses determined by the formulas previously given.

Structural-steel reinforcement is sometimes in the form of a cross in the center of the column or more often angles are employed connected by riveted latticing. Tests of columns of this character generally show lower ultimate strength than similar columns reinforced with the same quantity of steel in the form of vertical rods. This is most likely due to the difficulty of properly placing the concrete around the steel, and, furthermore, to the fact that the adhesion of concrete to steel where the latter presents broad flat surfaces is not good.

To be able to count upon the concrete in columns reinforced with structural forms, the steel should be well enclosed either by the form itself or by means of bands or hooping. However, when the amount of steel becomes very large, the relative value of the concrete becomes more uncertain, and it would be good design to neglect its element of strength.

6. Working Stresses.—Working stresses recommended by the Joint Committee are given in Art. 7.

7. Recommendations of the Joint Committee.—The form given below is essentially that of the Special Committee on Concrete and Reinforced Concrete of the American Society of Civil Engineers, Proc. Am. Soc. C. E., Dec., 1916, p. 1688.

By columns are meant compression members of which the ratio of unsupported length to least width exceeds about four, and which are provided with reinforcement of one of the forms hereafter described.

It is recommended that the ratio of unsupported length of column to its least width be limited to 15.

The effective area of hooped columns or columns reinforced with structural shapes shall be taken as the area within the circle enclosing the spiral or the polygon enclosing the structural shapes.

Columns may be reinforced by longitudinal bars; by bands, hoops, or spirals, together with longitudinal bars; or by structural forms which are sufficiently rigid to have value in themselves as columns. The general effect of closely spaced hooping is to greatly increase the toughness of the column and to add to its ultimate strength, but hooping has little effect on its behavior within the limit of elasticity. It thus renders the concrete a safer and more reliable material, and should permit the use of a somewhat higher working stress. The beneficial effects of toughening are adequately provided by a moderate amount of hooping, a larger amount serving mainly to increase the ultimate strength and the deformation possible before ultimate failure.

Composite columns of structural steel and concrete, in which the steel forms a column by itself, should be designed with caution. To classify this type as a concrete column reinforced with structural steel is hardly permissible, as the steel generally will take the greater part of the load. When this type of column is used, the concrete should not be relied upon to tie the steel units together nor to transmit stresses from one unit to another. The units should be adequately tied together by tie-plates or lattice bars, which, together with other details, such as splices, etc., should be designed in conformity with standard practice for structural steel. The concrete may exert a beneficial effect in restraining the steel from lateral deflection and also in increasing the carrying capacity of the column. The proportion of load to be carried by the concrete will depend on the form of the column and the method of construction. Generally for high percentages of steel, the concrete will develop relatively low unit stresses, and caution should be used in placing dependence on the concrete.

The following recommendations are made for the relative working stresses in the concrete for the several types of columns:

(a) For concentric compression on a plain-concrete pier, the length of which does not exceed four diameters, 22.5% of the compressive strength may be allowed.

(b) Columns with longitudinal reinforcement to the extent of not less than 1% and not more than 4%, and with lateral ties of not less than ¼ in. in diameter, 12 in. apart, nor more than 16 diameters of the longitudinal bar: the unit stress recommended for (a).

(c) Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with circular hoops or spirals not less than 1% of the volume of the concrete and as hereinafter specified: a unit stress 55% higher than given for (a), provided the ratio of unsupported length of column to diameter of the hooped core is not more than 10.

The foregoing recommendations are based on the following conditions:

It is recommended that the minimum size of columns to which the working stresses may be applied be 12 in., out to out.

The hoops or bands are not to be counted on directly as adding to the strength of the column.

Longitudinal reinforcement bars should be maintained straight, and shall have sufficient lateral support to be securely held in place until the concrete has set.

Where hooping is used, the total amount of such reinforcement shall not be less than 1% of the volume of the column, enclosed. The clear spacing of such hooping shall be not greater than one-sixth the diameter of the enclosed column, and preferably not greater than one-tenth, and in no case more than 2½ in. Hooping is to be circular and the ends of bands must be united in such a way as to develop their full strength. Adequate means must be provided to hold bands or hoops in place so as to form a column, the core of which shall be straight and well-centered. The strength of hooped columns depends very much upon the ratio of length to diameter of hooped core, and the strength due to hooping decreases rapidly as this ratio increases beyond five. The working stresses recommended are for hooped columns with a length of not more than 10 diameters of the hooped core. The Committee has no recommendations to make for a formula for working stresses for columns longer than 10 diameters.

Bending stresses due to eccentric loads, such as unequal spans of beams, and to lateral forces, must be provided for by increasing the section until the maximum stress does not exceed the values above specified. Where tension is possible in the longitudinal bars of the column, adequate connection between the ends of the bars must be provided to take this tension.

8. Tables and Diagrams.—Table 1 gives the allowable load P for different sizes and shapes of columns reinforced with longitudinal bars and reinforced with both longitudinal bars and spiral reinforcement. The area of longitudinal steel is given for each size of column and percentage of reinforcement. It is assumed that the longitudinal bars in the round and octagonal columns are arranged to form a circle in the cross-section of the column with the hooping immediately outside this circle.

Table 2 gives the sectional area of hooping and length of hooping per foot of column for a maximum pitch of one-sixth (1/6) the diameter of the enclosed concrete but not to exceed 2½ in. according to the recommendations of the Joint Committee (see Art. 7). For some diameters, values are given for a pitch of about one-tenth (1/10) the diameter of enclosed concrete. This table can be used for spiral reinforcement without material error.

Table 3 gives the volume of column in cubic feet per foot of length for diameter D.

The column diagram given can be used for either designing or reviewing designs of columns.

Illustrative Problem.—What size of square column reinforced with 2% of longitudinal steel and with the required number of lateral ties will be required to support a centrally applied load of 900,000 lb.? A 3000-lb. concrete is to be used and the unsupported length of column is less than 15 diameters.

From Art. 3,

$$P = f_cA[1 + (n - 1)p] \quad \text{or} \quad A = \frac{P}{f_c[1 + (n - 1)p]}$$

From Art. 7(*b*), f_c = 22.5% of 3000 lb. = 675 lb. per sq. in. Also $n = 10$ (see *Appendix B*). Then the effective area of column

$$A = \frac{900,000}{675[1 + (10 - 1)(0.02)]} = \frac{900,000}{797} = 1130 \text{ sq. in.}$$

The value of $f_c[1 + (n - 1)]$ = 797 may be obtained directly from the column diagram.

The side of effective square area

$$d = \sqrt{1130} = 33.6 \text{ in., say } 34 \text{ in.}$$

A column 37 in. square will suffice considering 1½ in. of concrete all around as fireproofing.

The problem may be readily solved by using Table 1. For a 3000-lb. concrete and $p = 0.02$, we find that a column 36 in. square will support 868,000 lb. and a 37-in. square column will support 922,000 lb. Thus a column 37 in. square is ample.

Illustrative Problem.—What size of round column and area of longitudinal steel will be required to support a load of 1,100,000 lb.? A 3000-lb. concrete is to be used with 1% of spiral reinforcement. Unsupported length is less than 10 diameters. Take $p = 0.025$ and $n = 10$.

From Art. 7(*c*) we find that the value of f_c may be taken 55% greater than for (*a*), or (1.55)(3000)(0.225) = 1050 lb. per sq. in.

Then from Art. 3

$$P = f_cA[1 + (n - 1)p]$$

The column diagram shows $f_c[1 + (n - 1)p] = 1287$. Therefore

$$P = 1287(A) \text{ or } A = \frac{1,100,000}{1287} = 855 \text{ sq. in.}$$

$$\frac{\pi d^2}{4} = 855 \qquad d = \sqrt{\frac{(855)(4)}{\pi}} = 33 \text{ in.} \qquad D = 36 \text{ in.}$$

$$A_s = (0.025)(855) = 21.4 \text{ sq. in.}$$

From Table 1 we find directly that a 36-in. column will be of sufficient size and that A_s = 21.4 sq. in.

Table 2 shows that for a column with d = 33 in., the pitch of spiral should not exceed 2½ in., with a sectional area of spiral of 0.206 sq. in. and with a length of spiral per foot of column height of 498 in.

TABLE 1.—USE FOR ROUND, OCTAGONAL AND SQUARE COLUMNS

$$P = f_c A[1 + (n - 1)p] \qquad A_s = pA$$

D = outside diameter. For octagonal columns D = short diameter. For square columns D = side of square.

$$A = \frac{\pi d^2}{4} \text{ for round and octagonal columns.}$$

$$A = d^2 \text{ for square columns.}$$

Diameter of columns (D)	Effective diam. of columns (d)	1 : 2 : 4 (2,000-lb. concrete) n = 15			1 : 1½ : 3 (2,500-lb. concrete) n = 12			1 : 1 : 2 (3,000-lb. concrete) n = 10			A_s (sq. in.)	
		f_c = 450 lb. per sq. in.		1% spiral f_c = 700 lb. per sq. in.	f_c = 565 lb. per sq. in.		1% spiral f_c = 870 lb. per sq. in.	f_c = 675 lb. per sq. in.		1% spiral f_c = 1,050 lb. per sq. in.	Square	Round or octagonal
		Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.		
		P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)		
						p = 0.01						
10	7	25,100	19,700	30,700	30,700	24,100	37,200	36,100	28,300	44,100	0.5	0.4
11	8	32,800	25,800	40,100	40,100	31,500	48,600	47,100	37,000	57,600	0.6	0.5
12	9	41,500	32,600	50,800	50,800	39,900	61,500	59,600	46,800	72,800	0.8	0.6
13	10	51,300	40,300	62,700	62,700	49,300	75,900	73,600	57,800	89,900	1.0	0.8
14	11	62,100	48,800	75,800	75,800	59,600	91,800	89,100	70,000	108,800	1.2	1.0
15	12	73,900	58,100	90,200	90,200	71,900	109,300	106,000	83,300	129,700	1.4	1.1
16	13	86,700	68,200	105,900	105,900	83,500	128,200	124,400	97,600	152,000	1.7	1.3
17	14	100,700	79,000	122,300	122,800	96,600	148,700	144,200	113,200	176,000	2.0	1.5
18	15	115,500	90,800	141,000	141,000	110,800	171,000	166,000	130,000	203,000	2.3	1.8
19	16	131,500	103,200	161,000	161,000	126,100	194,000	188,000	148,000	230,000	2.6	2.0
20	17	148,300	116,400	181,000	181,000	142,300	219,000	213,000	167,000	260,000	2.9	2.3
21	18	166,000	130,500	203,000	203,000	159,000	246,000	239,000	188,000	291,000	3.2	2.6
22	19	185,000	145,500	226,000	226,000	178,000	274,000	266,000	209,000	325,000	3.6	2.8
23	20	205,000	161,000	251,000	251,000	197,000	303,000	295,000	231,000	360,000	4.0	3.1
24	21	226,000	178,000	276,000	276,000	217,000	335,000	325,000	255,000	397,000	4.4	3.5
25	22	249,000	195,000	303,000	303,000	235,000	367,000	356,000	280,000	435,000	4.8	3.8
26	23	272,000	213,000	332,000	332,000	260,000	401,000	389,000	306,000	476,000	5.3	4.2
27	24	295,000	232,000	361,000	361,000	283,000	437,000	424,000	333,000	518,000	5.8	4.5
28	25	321,000	252,000	392,000	392,000	308,000	474,000	460,000	361,000	562,000	6.3	4.9
29	26	347,000	272,000	424,000	424,000	333,000	513,000	498,000	381,000	608,000	6.8	5.3
30	27	374,000	294,000	457,000	457,000	359,000	553,000	537,000	422,000	656,000	7.3	5.7
31	28	403,000	316,000	491,000	491,000	386,000	595,000	577,000	453,000	705,000	7.8	6.2
32	29	432,000	339,000	527,000	527,000	414,000	638,000	619,000	486,000	757,000	8.4	6.6
33	30	462,000	362,000	564,000	564,000	443,000	633,000	663,000	520,000	808,000	9.0	7.1
34	31	493,000	387,000	602,000	602,000	473,000	729,000	708,000	555,000	864,000	9.6	7.6
35	32	526,000	413,000	642,000	642,000	504,000	777,000	755,000	592,000	922,000	10.2	8.0
36	33	559,000	438,000	683,000	683,000	536,000	826,000	802,000	630,000	980,000	10.9	8.6
37	34	593,000	466,000	725,000	725,000	569,000	877,000	851,000	668,000	1,039,000	11.6	9.1
38	35	629,000	494,000	768,000	768,000	603,000	929,000	901,000	708,000	1,100,000	12.3	9.6
39	36	665,000	523,000	812,000	812,000	638,000	983,000	953,000	750,000	1,170,000	13.0	10.2
40	37	703,000	542,000	858,000	858,000	675,000	1,040,000	1,007,000	792,000	1,230,000	13.7	10.8
41	38	742,000	582,000	905,000	905,000	711,000	1,100,000	1,060,000	835,000	1,300,000	14.4	11.3
42	39	780,000	613,000	954,000	954,000	749,000	1,150,000	1,120,000	879,000	1,370,000	15.2	12.0
						p = 0.015						
10	7	26,700	21,000	32,600	32,200	25,300	39,000	37,500	29,500	45,900	0.7	0.6
11	8	34,900	27,400	42,600	42,100	33,100	51,000	49,100	38,500	59,900	1.0	0.8
12	9	44,100	36,700	53,900	53,300	41,800	64,500	62,100	48,800	75,800	1.2	1.0
13	10	54,500	42,800	66,500	65,800	51,700	79,600	76,600	60,200	93,600	1.5	1.2
14	11	65,900	51,800	80,500	79,700	62,500	96,400	92,700	72,800	113,200	1.8	1.4
15	12	78,500	61,700	95,800	94,800	74,400	114,700	110,300	86,600	134,800	2.2	1.7
16	13	92,100	72,400	112,400	111,100	87,400	134,600	129,400	101,700	158,000	2.5	2.0
17	14	106,800	83,900	130,400	129,000	101,300	156,000	150,000	118,000	183,000	2.9	2.3
18	15	122,700	96,300	150,000	148,000	116,400	179,000	172,000	135,300	211,000	3.4	2.7
19	16	139,500	109,600	170,000	169,000	132,300	204,000	196,000	154,000	240,000	3.8	3.0
20	17	157,500	123,700	192,000	190,000	149,400	230,000	221,000	174,000	271,000	4.3	3.4
21	18	176,500	138,600	216,000	213,000	168,000	258,000	249,000	195,000	304,000	4.9	3.8
22	19	196,800	155,000	240,000	237,000	187,000	288,000	277,000	217,000	338,000	5.4	4.3
23	20	218,000	171,000	266,000	263,000	207,000	319,000	307,000	241,000	374,000	6.0	4.7
24	21	240,000	189,000	293,000	290,000	228,000	351,000	338,000	265,000	413,000	6.6	5.2
25	22	264,000	207,000	322,000	319,000	250,000	385,000	371,000	291,000	453,000	7.3	5.7
26	23	288,000	227,000	352,000	348,000	273,000	422,000	415,000	318,000	495,000	7.9	6.2
27	24	314,000	247,000	383,000	379,000	297,000	459,000	442,000	347,000	538,000	8.6	6.8
28	25	341,000	267,000	416,000	411,000	323,000	498,000	479,000	376,000	585,000	9.4	7.4
29	26	368,000	289,000	450,000	444,000	349,000	538,000	518,000	407,000	633,000	10.1	8.0
30	27	397,000	312,000	485,000	480,000	377,000	581,000	558,000	439,000	682,000	10.9	8.6
31	28	428,000	335,200	522,000	516,000	405,000	624,000	601,000	465,000	733,000	11.8	9.2
32	29	458,000	360,000	559,000	554,000	434,000	670,000	644,000	506,000	787,000	12.6	9.9
33	30	481,000	385,000	599,000	593,000	465,000	716,000	690,000	541,000	841,000	13.5	10.6
34	31	524,000	412,000	639,000	632,000	497,000	765,000	736,000	577,000	899,000	14.4	11.3
35	32	559,000	438,000	681,000	674,000	529,000	816,000	775,000	616,000	953,000	15.4	12.1
36	33	594,000	466,000	724,000	717,000	563,000	867,000	835,000	655,000	1,019,000	16.3	12.8
37	34	630,000	495,000	769,000	761,000	598,000	921,000	887,000	696,000	1,081,000	[illegible]	[illegible]
38	35	668,000	525,000	815,000	806,000	634,000	976,000	938,000	737,000	1,146,000	[illegible]	[illegible]
39	36	706,000	555,000	862,000	853,000	670,000	1,032,000	993,000	770,000	1,212,000	[illegible]	[illegible]
40	37	746,000	586,000	911,000	901,000	708,000	1,090,000	1,050,000	824,000	1,280,000	[illegible]	[illegible]
41	38	787,000	618,000	961,000	950,000	746,000	1,150,000	1,110,000	868,000	1,350,000	[illegible]	[illegible]
42	39	829,000	651,000	1,012,000	1,000,000	786,000	1,211,000	1,170,000	915,000	1,420,000	[illegible]	[illegible]

Diameter of columns (D)	Effective diam. of columns (d)	1 : 2 : 4 (2,000-lb. concrete) $n = 15$			1 : 1½ : 3 (2,500-lb. concrete) $n = 12$			1 : 1 : 2 (3,000-lb. concrete) $n = 10$			A_s (sq. in.)	
		f_c = 450 lb. per sq. in.		1 % spiral f_c = 700 lb. per sq. in.	f_c = 565 lb. per sq. in.		1 % spiral f_c = 870 lb. per sq. in.	f_c = 675 lb. per sq. in.		1 % spiral f_c = 1,050 lb. per sq. in.		
		Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or octagonal
		P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)		
						$p = 0.02$						
10	7	28,200	22,200	34,500	33,800	26,500	40,800	39,100	30,700	47,700	1.0	0.8
11	8	36,900	29,000	45,000	44,100	34,600	53,300	51,000	40,100	62,300	1.3	1.0
12	9	46,700	36,700	57,000	55,700	43,800	67,500	64,600	50,700	78,800	1.6	1.3
13	10	57,600	45,300	70,400	68,900	54,100	83,300	79,700	62,600	97,400	2.0	1.6
14	11	69,600	54,800	85,200	83,400	65,500	100,800	96,400	75,800	117,800	2.4	1.9
15	12	83,000	65,200	101,300	99,300	78,000	120,000	114,600	90,100	140,000	2.9	2.3
16	13	97,300	76,500	118,900	116,500	91,500	140,800	134,800	105,800	165,000	3.4	2.7
17	14	113,000	88,700	137,900	135,000	106,100	163,000	156,000	123,600	191,000	3.9	3.1
18	15	129,600	101,800	153,000	155,000	121,800	187,000	179,000	140,900	219,000	4.5	3.5
19	16	147,400	115,900	180,000	176,000	138,600	213,000	204,000	160,000	249,000	5.1	4.0
20	17	166,000	130,700	203,000	199,000	156,000	241,000	230,000	181,000	281,000	5.8	4.5
21	18	187,000	146,500	228,000	223,000	175,000	270,000	258,000	203,000	316,000	6.5	5.1
22	19	208,000	163,000	254,000	249,000	195,000	301,000	288,000	226,000	351,000	7.2	5.7
23	20	230,000	181,000	281,000	276,000	217,000	333,000	319,000	250,000	389,000	8.0	6.3
24	21	254,000	200,000	310,000	304,000	239,000	367,000	351,000	276,000	430,000	8.8	6.9
25	22	279,000	219,000	341,000	333,000	262,000	403,000	386,000	303,000	471,000	9.7	7.6
26	23	305,000	239,000	372,000	364,000	286,000	441,000	422,000	331,000	515,000	10.6	8.3
27	24	332,000	261,000	366,000	397,000	311,000	480,000	459,000	361,000	561,000	11.5	9.0
28	25	360,000	283,000	440,000	431,000	338,000	521,000	498,000	391,000	608,000	12.5	9.9
29	26	389,000	306,000	476,000	466,000	366,000	563,000	538,000	423,000	658,000	13.5	10.6
30	27	420,000	330,000	513,000	503,000	395,000	607,000	581,000	456,000	710,000	14.6	11.5
31	28	452,000	355,000	552,000	540,000	424,000	653,000	625,000	491,000	763,000	15.7	12.3
32	29	485,000	380,000	592,000	579,000	455,000	701,000	670,000	526,000	818,000	16.8	13.2
33	30	519,000	407,000	633,000	620,000	487,000	750,000	717,000	563,000	876,000	18.0	14.1
34	31	554,000	435,000	676,000	662,000	520,000	801,000	766,000	603,000	935,000	19.2	15.1
35	32	590,000	464,000	721,000	706,000	554,000	853,000	817,000	641,000	997,000	20.5	16.1
36	33	628,000	493,000	766,000	750,000	589,000	907,000	868,000	682,000	1,060,000	21.8	17.1
37	34	666,000	523,000	814,000	797,000	625,000	963,000	922,000	723,000	1,125,000	23.1	18.2
38	35	706,000	555,000	862,000	844,000	663,000	1,021,000	976,000	767,000	1,190,000	24.5	19.2
39	36	746,000	587,000	912,000	893,000	702,000	1,080,000	1,032,000	811,000	1,260,000	25.9	20.4
40	37	789,000	620,000	963,000	943,000	742,000	1,141,000	1,090,000	857,000	1,330,000	27.4	21.5
41	38	832,000	653,000	1,016,000	985,000	782,000	1,200,000	1,150,000	904,000	1,410,000	28.9	22.7
42	39	876,000	688,000	1,070,000	1,048,000	824,000	1,270,000	1,210,000	952,000	1,480,000	30.4	23.9
						$p = 0.025$						
10	7	29,800	23,400	36,370	35,300	27,700	42,700	40,500	31,800	49,500	1.2	1.0
11	8	38,900	30,600	47,500	46,100	362,00	55,800	52,900	41,600	64,700	1.6	1.3
12	9	49,300	38,700	60,100	58,200	45,800	70,600	67,000	52,600	81,800	2.0	1.6
13	10	60,800	47,800	74,200	72,000	56,500	87,100	82,700	64,500	101,000	2.5	2.0
14	11	73,500	57,800	89,800	87,100	68,500	105,400	100,000	78,600	122,000	3.0	2.4
15	12	87,600	68,800	106,900	103,700	81,500	125,400	119,000	93,500	145,600	3.6	2.8
16	13	102,900	80,700	125,000	121,800	95,600	147,200	140,000	109,800	171,000	4.2	3.3
17	14	119,200	93,600	145,000	141,200	111,000	171,000	162,000	127,200	198,000	4.9	3.9
18	15	136,900	107,500	167,000	162,000	127,000	196,000	186,000	146,100	227,000	5.6	4.4
19	16	156,000	122,300	190,000	184,000	145,000	223,000	212,000	166,000	259,000	6.4	5.0
20	17	176,000	138,000	215,000	208,000	163,000	252,000	239,000	188,000	292,000	7.2	5.7
21	18	197,000	155,000	240,000	233,000	183,000	282,000	268,000	211,000	328,000	8.1	6.4
22	19	219,000	173,000	268,000	260,000	204,000	314,000	298,000	235,000	365,000	9.0	7.1
23	20	243,000	191,000	297,000	288,000	226,000	348,000	331,000	260,000	404,000	10.0	7.9
24	21	268,000	211,000	327,000	317,000	249,000	384,000	365,000	286,000	445,000	11.0	8.7
25	22	295,000	231,000	359,000	348,000	274,000	422,000	400,000	314,000	489,000	12.1	9.5
26	23	322,000	253,000	393,000	381,000	299,000	461,000	438,000	344,000	534,000	13.2	10.4
27	24	351,000	275,000	428,000	415,000	325,000	502,000	476,000	374,000	582,000	14.4	11.3
28	25	380,000	299,000	464,000	450,000	353,000	544,000	517,000	406,000	631,000	15.6	12.3
29	26	418,000	323,000	502,000	486,000	382,000	589,000	559,000	439,000	683,000	16.9	13.3
30	27	443,000	348,000	541,000	525,000	413,000	635,000	603,000	474,000	736,000	18.2	14.3
31	28	477,000	374,000	582,000	565,000	443,000	683,000	649,000	509,000	792,000	19.6	15.4
32	29	512,000	459,000	594,000	606,000	476,000	733,000	696,000	546,000	850,000	21.0	16.5
33	30	548,000	430,000	668,000	648,000	508,000	784,000	744,000	535,000	908,000	22.5	17.7
34	31	585,000	459,000	713,000	692,000	544,000	837,000	795,000	624,000	970,000	24.0	18.9
35	32	628,000	489,000	760,000	738,000	579,000	892,000	848,000	665,000	1,034,000	25.6	20.1
36	33	663,000	520,000	808,000	784,000	616,000	949,000	901,000	708,000	1,100,000	27.2	21.4
37	34	703,000	553,000	858,000	833,000	654,000	1,007,000	958,000	751,000	1,170,000	28.9	22.7
38	35	745,000	585,000	909,000	882,000	693,000	1,070,000	1,012,000	796,000	1,240,000	30.6	24.1
39	36	789,000	619,000	962,000	933,000	733,000	1,130,000	1,110,000	842,000	1,310,000	32.4	25.5
40	37	833,000	654,000	1,016,000	985,000	775,000	1,190,000	1,130,000	889,000	1,380,000	34.2	26.9
41	38	878,000	690,000	1,070,000	1,040,000	817,000	1,260,000	1,190,000	938,000	1,460,000	36.0	28.4
42	39	925,000	727,000	1,130,000	1,095,000	860,000	1,320,000	1,260,000	988,000	1,540,000	38.0	29.9

Diameter of columns (D)	Effective diam. of columns (d)	1 : 2 : 4 (2,000-lb concrete) $n = 15$			1 : 1½ : 3 (2,500-lb. concrete) $n = 12$			1 : 1 : 2 (3,000-lb. concrete) $n = 10$			A_s (sq. in.)	
		$f_c = 450$ lb. per sq. in.		1% spiral $f_c = 700$ lb. per sq. in.	$f_c = 565$ lb. per sq. in.		1% spiral $f_c = 870$ lb. per sq. in.	$f_c = 675$ lb. per sq. in.		1% spiral $f_c = 1,050$ lb. per sq. in.		
		Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or octagonal
		P (lb.)	P (lb.)	P (lb).	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P(lb.)	P (lb.)		
						$p = 0.03$						
10	7	31,300	24,600	38,260	36,800	28,900	44,500	42,000	33,000	51,400	1.5	1x2
11	8	41,900	32,100	50,000	48,100	37,800	58,200	54,800	43,100	67,100	1.9	1.5
12	9	51,800	40,600	63,200	60,700	47,800	73,600	69,400	54,500	84,900	2.4	1.9
13	10	63,900	50,200	78,100	75,100	59,000	90,900	85,700	67,300	104,700	3.0	2.4
14	11	77,300	60,700	94,500	90,800	71,400	110,000	103,700	81,450	126,700	3.6	2.9
15	12	92,100	72,300	112,400	108,100	85,000	131,000	123,300	96,900	151,000	4.3	3.4
16	13	108,000	84,800	132,000	127,000	99,700	154,000	144,800	113,800	177,000	5.1	4.0
17	14	125,200	98,200	153,000	147,200	115,600	178,000	168,000	132,000	205,000	5.9	4.6
18	15	143,900	113,000	176,000	169,000	132,700	204,000	193,000	151,000	236,000	6.8	5.3
19	16	164,000	128,400	200,000	192,000	151,000	233,000	219,000	172,000	268,000	7.7	6.0
20	17	185,000	145,000	226,000	217,000	171,000	263,000	248,000	194,000	303,000	8.7	6.8
21	18	207,000	163,000	253,000	243,000	191,000	294,000	278,000	219,000	339,000	9.7	7.6
22	19	231,000	181,000	282,000	271,000	213,000	328,000	309,000	243,000	378,000	10.8	8.5
23	20	256,000	201,000	312,000	300,000	236,000	363,000	343,000	269,000	419,000	12.0	9.4
24	21	282,000	221,000	344,000	331,000	260,000	401,000	378,000	297,000	462,000	13.2	10.4
25	22	310,000	243,000	378,000	363,000	286,000	430,000	415,000	326,000	507,000	14.5	11.4
26	23	338,000	265,000	413,000	397,000	312,000	481,000	453,000	356,000	555,000	15.9	12.5
27	24	368,000	289,000	450,000	433,000	349,000	523,000	494,000	387,000	604,000	17.3	13.6
28	25	400,000	314,000	488,000	469,000	369,000	568,000	536,000	421,000	655,000	18.8	14.7
29	26	432,000	339,000	528,000	508,000	399,000	614,000	579,000	455,000	708,000	20.3	15.9
30	27	466,000	366,000	569,000	548,000	430,000	662,000	625,000	491,000	765,000	21.9	17.2
31	28	501,000	393,000	612,000	588,000	462,000	712,000	672,000	528,000	822,000	23.5	18.5
32	29	537,000	422,000	657,000	632,000	496,000	764,000	721,000	566,000	881,000	25.2	19.8
33	30	575,000	452,000	703,000	676,000	531,000	818,000	772,000	605,000	943,000	27.0	21.2
34	31	614,000	482,000	750,000	722,000	567,000	873,000	824,000	646,000	1,006,000	28.8	22.6
35	32	655,000	514,000	799,000	769,000	604,000	931,000	878,000	689,000	1,072,000	30.7	24.1
36	33	695,000	546,000	850,000	818,000	643,000	990,000	933,000	733,000	1,140,000	32.7	25.7
37	34	739,000	580,000	902,000	867,000	682,000	1,050,000	991,000	778,000	1,210,000	34.7	27.2
38	35	783,000	615,000	956,000	920,000	723,000	1,113,000	1,050,000	825,000	1,280,000	36.8	28.9
39	36	829,000	650,000	1,012,000	973,000	765,000	1,178,000	1,120,000	873,000	1,360,000	38.9	30.5
40	37	875,000	687,000	1,069,000	1,028,000	808,000	1,240,000	1,170,000	921,000	1,440,000	41.1	32.3
41	38	923,000	725,000	1,127,000	1,084,000	852,000	1,310,000	1,240,000	972,000	1,510,000	43.3	34.0
42	39	972,000	764,000	1,187,000	1,140,000	898,000	1,380,000	1,300,000	1,023,000	1,590,000	45.6	35.8
						$p = 0.035$						
10	7	32,900	25,800	40,200	38,400	30,100	46,400	43,500	34,200	53,200	1.7	1.4
11	8	42,900	33,800	52,400	50,200	39,400	60,600	56,800	44,600	69,400	2.2	1.8
12	9	54,300	42,700	66,400	63,300	49,800	76,700	72,000	56,500	87,900	2.8	2.2
13	10	67,100	52,700	81,900	78,300	61,500	94,600	88,800	69,800	108,500	3.5	2.8
14	11	81,100	63,800	99,100	94,800	75,500	114,500	107,500	84,500	131,300	4.2	3.3
15	12	96,500	75,900	118,000	112,800	88,600	136,300	127,800	100,500	156,000	5.0	4.0
16	13	113,400	89,100	138,400	132,400	104,000	160,000	150,000	117,900	183,000	5.9	4.6
17	14	131,500	103,300	161,000	153,000	120,000	186,000	174,000	136,800	213,000	6.9	5.4
18	15	151,000	118,700	184,000	176,000	138,400	213,000	200,000	155,000	244,000	7.9	6.2
19	16	172,000	135,000	210,000	201,000	158,000	242,000	227,000	179,000	279,000	9.0	7.0
20	17	194,000	152,000	237,000	226,000	178,000	274,000	257,000	202,000	313,000	10.1	7.9
21	18	217,000	171,000	265,000	254,000	199,000	307,000	289,000	226,000	352,000	11.3	8.9
22	19	242,000	190,000	296,000	283,000	222,000	342,000	320,000	252,000	391,000	12.7	9.9
23	20	268,000	211,000	328,000	313,000	246,000	379,000	355,000	279,000	434,000	14.0	11.0
24	21	296,000	232,000	361,000	345,000	271,000	417,000	392,000	308,000	478,000	15.4	12.1
25	22	325,000	255,000	396,000	379,000	298,000	458,000	430,000	338,000	525,000	17.0	13.3
26	23	355,000	279,000	433,000	414,000	325,000	501,000	470,000	369,000	560,000	18.5	14.5
27	24	386,000	303,000	472,000	451,000	354,000	545,000	512,000	402,000	625,000	20.2	15.8
28	25	419,000	329,000	512,000	490,000	384,000	592,000	555,000	435,000	678,000	21.9	17.2
29	26	453,000	356,000	554,000	529,000	416,000	640,000	600,000	471,000	733,000	23.7	18.6
30	27	489,000	384,000	597,000	571,000	448,000	690,000	648,000	509,000	791,000	25.5	20.0
31	28	526,000	413,000	642,000	614,000	482,000	742,000	697,000	547,000	850,000	27.4	21.5
32	29	564,000	443,000	689,000	659,000	517,000	796,000	746,000	586,000	912,000	29.5	23.1
33	30	604,000	464,000	737,000	705,000	553,000	852,000	800,000	628,000	976,000	31.5	24.7
34	31	645,000	506,000	787,000	752,000	591,000	910,000	854,000	670,000	1,042,000	33.7	26.4
35	32	688,000	540,000	839,000	802,000	620,000	969,000	910,000	714,000	1,110,000	35.8	28.1
36	33	731,000	574,000	892,000	853,000	670,000	1,031,000	968,000	760,000	1,181,000	38.1	29.9
37	34	776,000	610,000	947,000	905,000	711,000	1,094,000	1,027,000	816,000	1,254,000	40.4	31.8
38	35	822,000	646,000	1,003,000	959,000	754,000	1,159,000	1,088,000	855,000	1,330,000	42.8	33.7
39	36	870,000	683,000	1,062,000	1,015,000	798,000	1,230,000	1,150,000	904,000	1,410,000	45.3	35.6
40	37	919,000	722,000	1,120,000	1,072,000	842,000	1,300,000	1,220,000	955,000	1,490,000	47.8	37.6
41	38	969,000	761,000	1,180,000	1,130,000	888,000	1,370,000	1,280,000	1,007,000	1,570,000	50.6	39.7
42	39	1,021,000	802,000	1,250,000	1,190,000	936,000	1,440,000	1,350,000	1,061,000	1,650,000	53.2	41.8

Diameter of columns (D)	Effective diam. of columns (d)	1 : 2 : 4 (2,000-lb concrete) $n = 15$, $f_c = 450$ lb. per sq. in., Square, P (lb.)	1 : 2 : 4, $f_c = 450$ lb. per sq. in., Round or oct., P (lb.)	1 : 2 : 4, 1% spiral $f_c = 700$ lb. per sq. in., Round or oct., P (lb).	1 : 1½ : 3 (2,500-lb. concrete) $n = 12$, $f_c = 565$ lb. per sq. in., Square, P (lb.)	1 : 1½ : 3, $f_c = 565$ lb. per sq. in., Round or oct., P (lb.)	1 : 1½ : 3, 1% spiral $f_c = 870$ lb. per sq. in., Round or oct., P (lb.)	1 : 1 : 2 (3,000-lb. concrete) $n = 10$, $f_c = 675$ lb. per sq. in., Square, P (lb.)	1 : 1 : 2, $f_c = 675$ lb. per sq. in., Round or oct., P (lb.)	1 : 1 : 2, 1% spiral $f_c = 1,050$ lb. per sq. in., Round or oct., P (lb.)	A_s (sq. in.) Square	A_s (sq. in.) Round or octagonal
						$p = 0.04$						
10	7	34,400	27,000	42,000	39,900	31,300	48,200	45,000	35,300	55,000	2.0	1.5
11	8	44,900	35,300	54,900	52,100	41,000	63,000	58,800	46,200	71,800	2.6	2.0
12	9	56,900	44,600	69,500	65,800	51,800	79,700	74,400	58,400	90,900	3.2	2.5
13	10	70,200	55,100	85,800	81,400	64,000	98,400	91,800	72,100	112,200	4.0	3.1
14	11	84,900	66,700	103,800	98,500	77,400	119,100	111,000	87,300	135,800	4.8	3.8
15	12	101,000	79,400	123,500	117,200	92,000	141,700	132,200	103,800	162,000	5.8	4.5
16	13	118,600	93,200	144,900	137,600	108,100	166,000	155,000	121,900	190,000	6.4	5.3
17	14	137,500	108,000	168,000	160,000	125,300	193,000	180,000	141,400	220,000	7.8	6.2
18	15	158,000	124,000	193,000	183,000	143,900	221,000	207,000	162,000	252,000	9.0	7.1
19	16	180,000	141,200	220,000	208,000	164,000	252,000	235,000	185,000	287,000	10.2	8.0
20	17	203,000	159,000	248,000	235,000	185,000	284,000	265,000	208,000	324,000	11.6	9.1
21	18	227,000	179,000	278,000	264,000	207,000	319,000	297,000	234,000	364,000	13.0	10.2
22	19	253,000	199,000	310,000	294,000	231,000	355,000	331,000	261,000	405,000	14.4	11.3
23	20	281,000	221,000	343,000	326,000	256,000	394,000	367,000	289,000	448,000	16.0	12.6
24	21	310,000	243,000	378,000	349,000	282,000	434,000	405,000	318,000	495,000	17.6	13.9
25	22	330,000	267,000	415,000	394,000	309,000	476,000	445,000	349,000	543,000	19.4	15.2
26	23	371,000	292,000	454,000	430,000	338,000	521,000	486,000	382,000	593,000	21.2	16.6
27	24	404,000	317,000	494,000	468,000	368,000	567,000	529,000	415,000	646,000	23.0	18.1
28	25	439,000	345,000	536,000	509,000	399,000	615,000	574,000	451,000	701,000	25.0	19.6
29	26	474,000	373,000	580,000	550,000	432,000	665,000	620,000	487,000	758,000	27.0	21.2
30	27	512,000	402,000	625,000	593,000	466,000	717,000	670,000	526,000	818,000	29.2	22.9
31	28	551,000	432,000	672,000	638,000	501,000	772,000	720,000	565,000	880,000	31.4	24.6
32	29	591,000	464,000	721,000	685,000	538,000	828,000	772,000	606,000	943,000	33.6	26.4
33	30	632,000	496,000	772,000	733,000	575,000	886,000	826,000	648,000	1,009,000	36.0	28.3
34	31	675,000	530,000	824,000	782,000	614,000	946,000	882,000	693,000	1,080,000	38.4	30.2
35	32	719,000	665,000	878,000	834,000	655,000	1,008,000	940,000	738,000	1,150,000	41.0	32.2
36	33	764,000	601,000	934,000	886,000	698,000	1,072,000	1,000,000	785,000	1,220,000	43.6	34.2
37	34	811,000	637,000	991,000	941,000	739,000	1,140,000	1,060,000	834,000	1,300,000	46.2	36.3
38	35	860,000	675,000	1,051,000	996,000	784,000	1,210,000	1,130,000	883,000	1,370,000	49.0	38.5
39	36	910,000	715,000	1,112,000	1,054,000	828,000	1,280,000	1,190,000	935,000	1,450,000	51.8	40.7
40	37	961,000	755,000	1,170,000	1,110,000	876,000	1,350,000	1,260,000	987,000	1,540,000	54.0	43.0
41	38	1,014,000	796,000	1,240,000	1,180,000	923,000	1,420,000	1,330,000	1,0[illegible]0,000	1,620,000	57.8	45.4
42	39	1,068,000	838,000	1,300,000	1,240,000	973,000	1,500,000	1,400,000	1,097,000	1,710,000	60.8	47.8
						$p = 0.045$						
10	7	36,000	28,100	43,900	41,400	32,500	50,100	46,500	36,500	56,800	2.2	1.7
11	8	47,000	36,900	57,400	54,100	42,500	65,400	60,700	47,700	74,200	2.9	2.3
12	9	59,500	46,700	72,600	68,400	53,800	82,800	71,800	60,300	93,800	3.7	2.9
13	10	73,400	57,500	89,600	84,500	66,400	101,200	94,800	74,500	115,900	4.5	3.5
14	11	88,800	69,700	108,400	102,200	80,300	123,600	114,800	90,100	140,200	5.4	4.3
15	12	105,700	83,000	129,000	121,700	95,500	147,000	136,500	107,200	167,000	6.5	5.1
16	13	124,000	97,500	151,000	142,800	112,200	173,000	160,000	126,000	196,000	7.6	6.0
17	14	143,800	113,000	176,000	165,600	130,000	200,000	186,000	146,000	227,000	8.8	6.9
18	15	165,000	130,000	202,000	190,000	149,000	230,000	213,000	168,000	261,000	10.2	8.0
19	16	188,000	148,000	229,000	216,000	170,000	262,000	243,000	191,000	297,000	11.5	9.0
20	17	212,000	167,000	259,000	244,000	192,000	295,000	274,000	215,000	335,000	13.0	10.4
21	18	238,000	187,000	290,000	274,000	215,000	331,000	307,000	242,000	376,000	14.6	11.5
22	19	265,000	208,000	324,000	305,000	240,000	369,000	342,000	269,000	418,000	16.3	12.8
23	20	294,000	231,000	358,000	338,000	265,000	409,000	389,000	298,000	463,000	18.0	14.1
24	21	323,000	254,000	395,000	372,000	293,000	451,000	418,000	329,000	511,000	19.9	15.6
25	22	356,000	279,000	434,000	409,000	321,000	495,000	459,000	361,000	561,000	21.8	17.1
26	23	388,000	305,000	474,000	447,000	351,000	541,000	502,000	394,000	613,000	23.8	18.7
27	24	423,000	332,000	516,000	486,000	384,000	589,000	546,000	418,000	668,000	25.9	20.3
28	25	459,000	365,000	560,000	528,000	415,000	639,000	593,000	465,000	724,000	28.1	22.1
29	26	496,000	390,000	606,000	571,000	448,000	691,000	641,000	503,000	783,000	30.4	23.9
30	27	535,000	420,000	653,000	616,000	484,000	745,000	691,000	543,000	845,000	32.8	25.8
31	28	576,000	452,000	703,000	663,000	520,000	801,000	744,000	584,000	908,000	35.3	27.7
32	29	617,000	485,000	754,000	711,000	558,000	859,000	805,000	626,000	974,000	37.9	29.7
33	30	661,000	518,000	807,000	760,000	597,000	920,000	854,000	670,000	1,042,000	40.5	31.8
34	31	705,000	554,000	861,000	812,000	638,000	982,000	911,000	715,000	1,110,000	43.3	34.0
35	32	752,000	590,000	918,000	865,000	679,000	1,046,000	971,000	762,000	1,190,000	46.1	36.2
36	33	800,000	628,000	976,000	920,000	723,000	1,110,000	1,032,000	812,000	1,260,000	49.0	38.5
37	34	849,000	666,000	1,036,000	977,000	767,000	1,180,000	1,100,000	861,000	1,340,000	51.9	40.8
38	35	899,000	706,000	1,098,000	1,035,000	813,000	1,250,000	1,160,000	913,000	1,4[illegible]0,000	55.2	43.3
39	36	951,000	747,000	1,160,000	1,094,000	860,000	1,320,000	1,230,000	965,000	1,5[illegible],000	58.3	45.7
40	37	1,005,000	790,000	1,230,000	1,160,000	910,000	1,400,000	1,300,000	1,020,000	1,590,000	61.5	48.4
41	38	1,060,000	833,000	1,290,000	1,220,000	958,000	1,480,000	1,370,000	1,075,000	1,670,000	65.0	51.0
42	39	1,109,000	877,000	1,360,000	1,290,000	1,010,000	1,550,000	1,440,000	1,130,000	1,760,000	68.4	53.8

Diameter of columns (D)	Effective diam. of columns (d)	1 : 2 : 4 (2,000-lb. concrete) $n = 15$			1 : 1½ : 3 (2,500-lb. concrete) $n = 12$			1 : 1 : 2 (3,000-lb. concrete) $n = 10$			A_s (sq. in.)	
		$f_c = 450$ lb. per sq. in.		1% spiral $f_c = 700$ lb. per sq. in.	$f_c = 565$ lb. per sq. in.		1% spiral $f_c = 870$ lb. per sq. in.	$f_c = 675$ lb. per sq. in.		1% spiral $f_c = 1{,}050$ lb. per sq. in.	Square	Round or octagonal
		Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.	Square	Round or oct.	Round or oct.		
		P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)	P (lb.)		
						$p = 0.05$						
10	7	37,500	29,400	45,800	42,900	33,700	51,900	48,000	37,700	58,600	2.5	1.9
11	8	49,000	38,500	59,800	56,100	44,000	67,800	63,300	49,200	76,600	3.2	2.5
12	9	62,000	48,700	75,700	70,800	55,700	85,800	80,200	62,300	96,900	4.1	3.2
13	10	76,500	60,100	93,500	87,600	68,800	106,000	97,900	76,900	119,600	5.0	3.9
14	11	92,600	72,700	113,000	106,000	83,300	128,000	118,400	93,000	144,900	6.1	4.8
15	12	110,200	86,500	134,600	126,000	99,000	153,000	142,400	110,700	172,000	7.2	5.7
16	13	129,400	101,600	158,000	148,000	116,300	179,000	167,200	130,000	202,000	8.5	6.6
17	14	150,000	117,800	183,000	172,000	134,800	208,000	192,000	151,000	234,000	9.8	7.7
18	15	172,000	135,200	210,000	197,000	155,000	238,000	220,000	173,000	269,000	11.3	8.8
19	16	196,000	154,000	239,000	224,000	176,000	271,000	251,000	197,000	306,000	12.8	10.1
20	17	221,000	174,000	270,000	253,000	199,000	306,000	283,000	222,000	346,000	14.5	11.4
21	18	248,000	195,000	303,000	284,000	223,000	343,000	317,000	250,000	388,000	16.2	12.7
22	19	278,000	217,000	337,000	316,000	248,000	383,000	353,000	278,000	432,000	18.1	14.2
23	20	306,000	241,000	374,000	350,000	275,000	424,000	392,000	308,000	478,000	20.0	15.7
24	21	337,000	265,000	412,000	386,000	303,000	467,000	432,000	339,000	528,000	22.1	17.3
25	22	371,000	291,000	452,000	424,000	333,000	513,000	474,000	372,000	579,000	24.2	19.0
26	23	405,000	318,000	494,000	464,000	364,000	560,000	518,000	406,000	633,000	26.5	20.8
27	24	441,000	346,000	548,000	505,000	396,000	610,000	564,000	443,000	689,000	28.8	22.6
28	25	478,000	376,000	584,000	548,000	430,000	662,000	612,000	481,000	748,000	31.3	24.5
29	26	517,000	406,000	632,000	592,000	465,000	716,000	662,000	520,000	809,000	33.8	26.6
30	27	558,000	438,000	680,000	639,000	512,000	772,000	714,000	561,000	872,000	36.5	28.6
31	28	600,000	471,000	733,000	686,000	539,000	831,000	768,000	603,000	938,000	39.2	30.8
32	29	644,000	501,000	786,000	737,000	578,000	891,000	823,000	647,000	1,006,000	42.1	33.0
33	30	688,000	541,000	841,000	788,000	619,000	954,000	881,000	692,000	1,077,000	45.0	35.3
34	31	735,000	578,000	898,000	841,000	666,000	1,018,000	941,000	739,000	1,150,000	48.1	37.7
35	32	784,000	615,000	957,000	897,000	704,000	1,085,000	1,002,000	787,000	1,230,000	51.2	40.2
36	33	833,000	655,000	1,018,000	954,000	750,000	1,150,000	1,066,000	837,000	1,300,000	54.5	42.8
37	34	885,000	695,000	1,080,000	1,012,000	795,000	1,220,000	1,130,000	889,000	1,380,000	57.8	45.4
38	35	937,000	736,000	1,140,000	1,073,000	843,000	1,300,000	1,200,000	942,000	1,470,000	61.3	48.1
39	36	992,000	779,000	1,210,000	1,130,000	892,000	1,370,000	1,270,000	996,000	1,550,000	64.8	50.9
40	37	1,047,000	823,000	1,280,000	1,200,000	942,000	1,450,000	1,340,000	1,052,000	1,640,000	68.5	53.8
41	38	1,105,000	879,000	1,350,000	1,270,000	993,000	1,530,000	1,410,000	1,110,000	1,730,000	72.2	56.7
42	39	1,160,000	914,000	1,420,000	1,330,000	1,046,000	1,610,000	1,490,000	1,170,000	1,820,000	76.1	59.7

TABLE 2.—HOOPED COLUMN REINFORCEMENT

Diameter of enclosed concrete to center line of hooping (inches)	Pitch (inches)	Sectional area of hooping (square inches)	Length of hooping in 1 ft. in height (inches)	Diameter of encosed concrete to center line of hooping (inches)	Pitch (inches)	Sectional area of hooping (square inches)	Length of hooping in 1 ft. in height (inches)
8	1 1¼ max.	0.020 0.025	302 242	24	2⅜ 2½ max.	0.142 0.150	381 362
9	1 1½ max.	0.022 0.034	339 226	25	2½ max.	0.156	377
10	1 1⅝ max.	0.025 0.041	377 232	26	2½ max.	0.162	392
11	1⅛ 1¾ max.	0.031 0.048	369 237	27	2½ max.	0.169	407
12	1¼ 2 max.	0.037 0.060	362 226	28	2½ max.	0.175	422
13	1⅜ 2⅛ max.	0.045 0.069	356 230	29	2½ max.	0.181	437
14	1⅜ 2¼ max.	0.048 0.079	384 234	30	2½ max.	0.187	452
15	1½ 2½ max.	0.056 0.094	377 226	31	2½ max.	0.194	467
16	1⅝ 2½ max.	0.065 0.100	371 241	32	2½ max.	0.200	483
17	1¾ 2½ max.	0.074 0.106	366 256	33	2½ max.	0.206	498
18	1⅞ 2½ max.	0.084 0.112	362 271	34	2½ max.	0.212	513
19	1⅞ 2½ max.	0.089 0.119	382 287	35	2½ max.	0.219	528
20	2 2½ max.	0.100 0.125	377 302	36	2½ max.	0.225	543
21	2⅛ 2½ max.	0.112 0.131	373 317	37	2½ max.	0 231	558
22	2¼ 2½ max.	0.124 0.137	369 332	38	2½ max.	0 238	574
[illegible]	2¼ 2½ max.	0.130 0 144	386 347	39	2½ max.	0.244	588

TABLE 3.—VOLUME OF COLUMN IN CUBIC FEET PER FOOT OF LENGTH FOR DIAMETER *D*

D	Square	Round	Octagonal	*D*	Square	Round	Octagonal
10	0.69	0.55	0.58	27	5.06	3.98	4.19
11	0.84	0.66	0.70	28	5.44	4.28	4.51
12	1.00	0.79	0.83	29	5.84	4.62	4.84
13	1.17	0.92	0.97				
14	1.36	1.07	1.13	30	6.25	4.91	5.18
				31	6.68	5.24	5.53
15	1.56	1.23	1.29	32	7.11	5.58	5.88
16	1.78	1.40	1.47	33	7.55	5.94	6.26
17	2.01	1.58	1.66	34	8.02	6.31	6.64
18	2.25	1.77	1.87				
19	2.51	1.97	2.08	35	8.50	6.67	7.05
				36	8.98	7.05	7.46
20	2.78	2.18	2.30	37	9.49	7.46	7.88
21	3.06	2.41	2.54	38	10.00	7.87	8.30
22	3.36	2.64	2.78	39	10.60	8.29	8.75
23	3.67	2.89	3.05				
24	4.00	3.14	3.32	40	11.10	8.71	9.20
				41	11.70	9.15	9.67
25	4.34	3.41	3.59	42	12.30	9.61	10.10
26	4.70	3.68	3.89				

9. Reduction Formula for Long Columns.—Where long columns must be used, the reduction formula which follows, taken from the Los Angeles Building Ordinance, may be safely employed in the design of columns whose unsupported length (l) is between 15 and 30 times the least dimension of effective section (d). Let r represent the quantity by which the working stress for columns with $\frac{l}{d}$ less than 15 should be multiplied to give a working stress which may be used for long columns. Then

$$r = 1.6 - \tfrac{1}{25}\left(\frac{l}{d}\right)$$

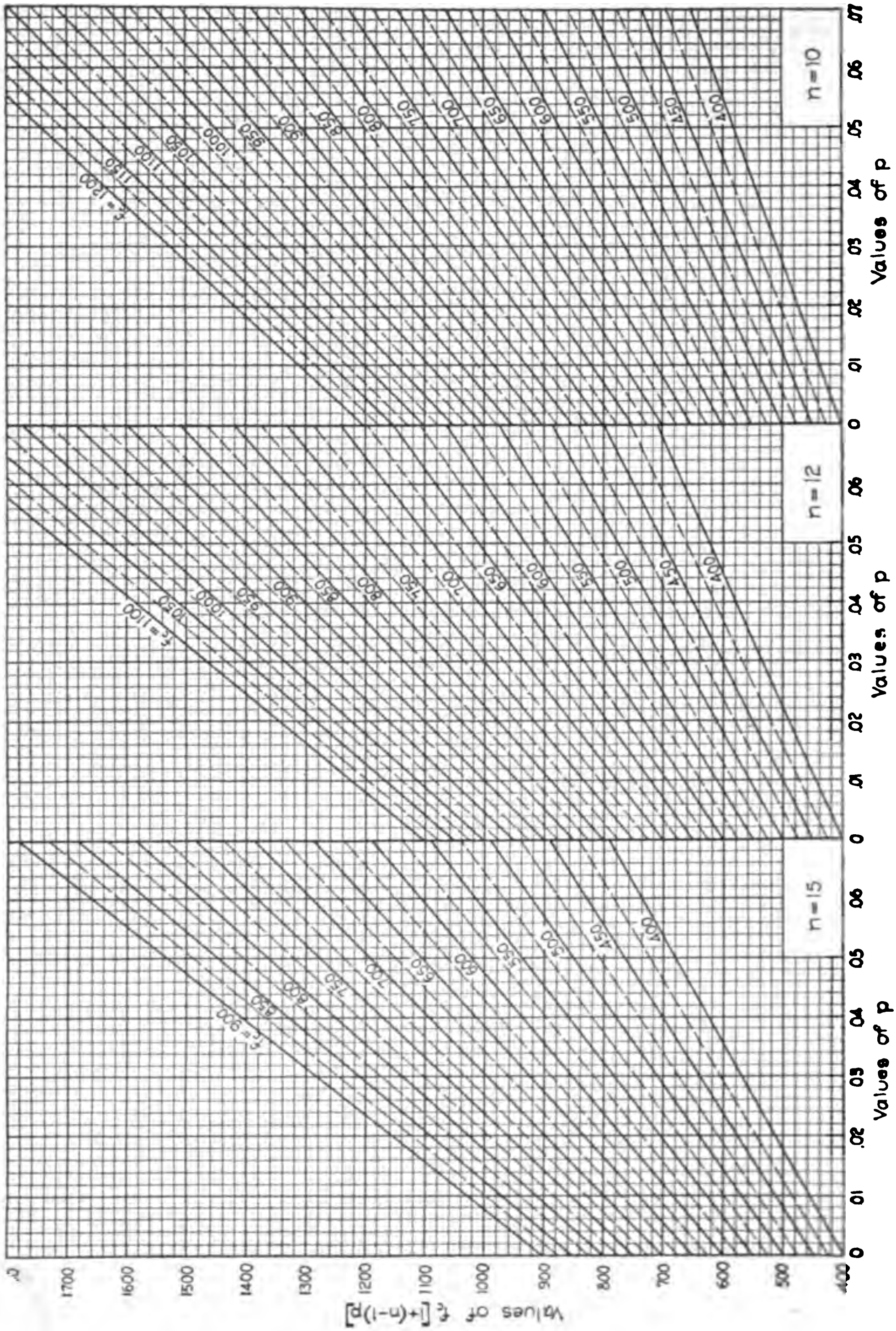
10. Columns Supporting Bracket Loads.—A column supporting a roof is frequently made to carry a traveling crane which runs on a track supported by side brackets or at one side of the column. To compute the maximum stress in such a column, it is necessary to find the maximum bending moment at whatever section it occurs, and then combine the stresses due to bending and thrust by the general method explained in Sect. 9.

The maximum bending moment occurs *at* the load, and depends upon the height at which the load is placed and the end conditions of the column. To simplify the calculations, the depth of the bracket will be considered small in comparison with the length of the column. This is not strictly correct but the error involved will be on the safe side since any increase in the depth of a bracket reduces the maximum amount.

Columns supporting bracket loads will be considered for the three conditions: (1) both ends free to turn, (2) both ends fixed, and (3) one end fixed and the other end free. Columns in practice will have conditions intermediate to these and good judgment as to flexibility of the end connections is necessary to arrive at correct results in any particular case.

Column Free to Turn at Both Ends.—The bending-moment diagram for this case is shown in Fig. 2. The bending moment Px of the eccentric load is resisted by horizontal forces at the ends of the column which form a couple, the value of which is also Px. The maximum pos-

COLUMN DIAGRAM



mum value of the bending moment is evidently Px, which would occur with the load at either end of the column. The minimum value of the maximum bending moment occurs when $a = b = \frac{L}{2}$, and equals $\frac{1}{2}Px$.

Column Fixed at Both Ends.—Analysis shows that the least value of the maximum bending moment is $\frac{1}{4}Px$ and occurs when b has the following values: $0.211L$, $0.500L$, and $0.789L$. The

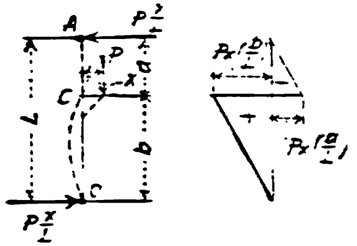


FIG. 2

moment diagrams for these cases are shown in Fig. 3. The greatest value of the maximum moment is Px and occurs when the bracket is either at the top or bottom of the column.

The following general formulas may also be obtained:

At A Fig. 3

$$M_A = Px\frac{b}{L}\left(2 - 3\frac{b}{L}\right)$$

$$R = \frac{6Pxb}{L^2}\left(1 - \frac{b}{L}\right)$$

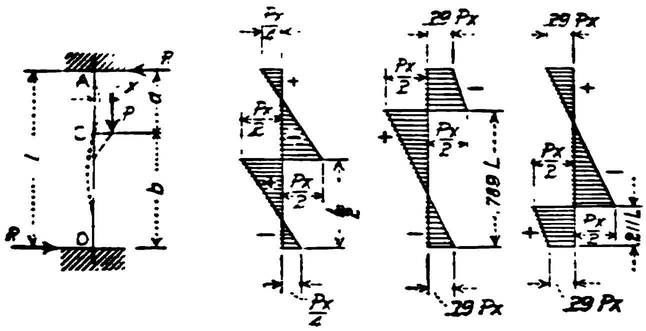


FIG. 3

Just above C,

$$M = M_A - Ra$$

Just below C,

$$M = M_A - Ra + Px$$

or

$$M = M_0 + Rb$$

At O,

$$M_0 = Px\left[1 - 4\left(\frac{b}{L}\right) + 3\left(\frac{b}{L}\right)^2\right]$$

Column Fixed at One End and Free to Turn at the Other.—The minimum value of the maximum bending moment equals $\frac{1}{2}Px$ and occurs when b is equal to $0.258L$ or $0.605L$.

ing-moment diagrams for these cases are shown in Fig. 4 which also shows the condition to make the bending moment at the base zero, and the particular case of $b = L$.

The following general formulas apply:

$$R = \frac{3Pxb}{2L^2}\left(2 - \frac{b}{L}\right)$$

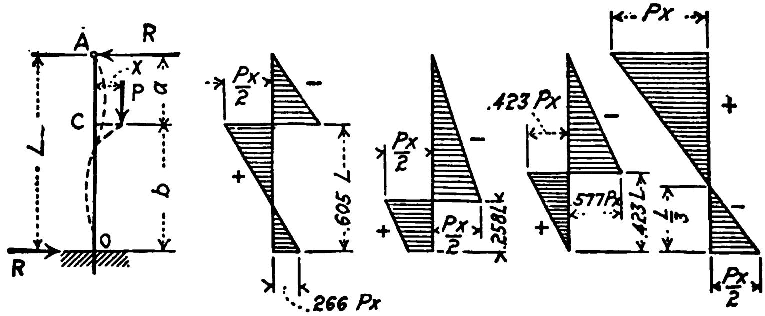


Fig. 4.

Just above C (Fig. 4),

$$M = -Ra$$

Just below C,

$$M = Px - Ra$$

At O

$$M_0 = Px - \tfrac{3}{2}Px \cdot \frac{b}{L}\left(2 - \frac{b}{L}\right)$$

SECTION 9

BENDING AND DIRECT STRESS

1. Theory in General.—If a beam is acted upon by forces which are all normal to its length, then the stresses resulting are due to simple bending, and the formulas deduced in Sect. 7 may be employed. If, however, any of the forces acting throughout the length of a beam be inclined, or if additional forces be applied at the ends, then our beam formulas for simple bending will not apply. Likewise, in columns, if the load be eccentrically applied or if lateral pressure be exerted, both bending and direct stresses will result and the ordinary column formulas given in Sect. 8 cannot be used except to give approximate results when the amount of bending is small.

The same combination of stresses occurs also in arch rings and may occur in special cases. The formulas to be derived can be employed in any type of reinforced-concrete structure provided the normal component of the resultant thrust on the given section acts with a lever arm about the center of gravity of the section. In long beams and columns, the deflection resulting from flexure should be given consideration when determining the eccentricity of the axial and inclined forces.

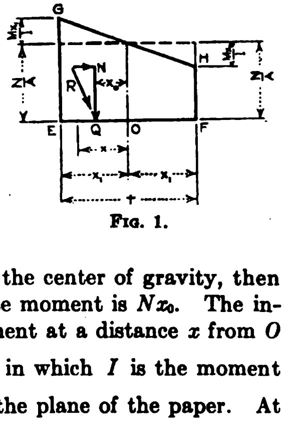


FIG. 1.

Let us first consider structures of plain concrete. The distribution of pressure on any section due to a resultant pressure acting at different points will be explained. Consider a section represented in projection by EF, Fig. 1. When the resultant R acts at the center of gravity O, the intensity of stress is uniform over the section and is equal to the vertical component of R divided by the area of section, or $\frac{N}{A}$. If R acts at any other point, as Q, and if the projection of the section is taken such that the distance x_0 represents the true lever arm of N about the center of gravity, then the force N is equivalent to an equal N at O and a couple whose moment is Nx_0. The intensity of the uniformly varying stress due to this bending moment at a distance x from O is (by the common flexure formula for homogeneous beams) $\frac{Nx_0x_1}{I}$, in which I is the moment of inertia of the section about an axis through O at right angles to the plane of the paper. At the edges E and F this intensity $= \frac{Nx_0x_1}{I}$. Regarding compressive and tensile stresses as positive and negative respectively, the intensity of stress at edge E is

$$f_c = \frac{N}{A} + \frac{Nx_0x_1}{I}$$

At edge F it is

$$f_c' = \frac{N}{A} - \frac{Nx_0x_1}{I}$$

If the stress f_c' comes out minus, the value obtained is the maximum tension as shown in Fig. 2. In plain concrete construction a greater tension than about 50 lb. per sq. in. should not be allowed.

When we come to reinforced concrete, which is composed of two materials (concrete and steel) with different values of E, then the steel area at any given cross-section may be replaced by an area of concrete equal to n times the area of the steel, placed in the plane of the steel reinforcement. This section may be called the transformed section, or section of concrete theoretically equivalent in resistance to the actual section. Under this heading rectangular sections only will be considered and Fig. 3 represents a transformed section as referred to above.

Thus, if A_c is the area of the concrete, and A_0 is the area of the steel $= A_s + A'$; then the equivalent area

$$A = A_c + nA_0 = bt + n(A_s + A')$$

If I_c is the moment of inertia of the concrete about the gravity axis, and I_s is the moment of inertia of the steel about the same axis, then

$$I = I_c + nI_s$$

and

$$\begin{matrix}(f_c)\\(f_c')\end{matrix} = \frac{N}{A_c + nA_0} \begin{matrix}(+)\\(-)\end{matrix} \frac{Nx_0x_1}{I_c + nI_s}$$

If we denote p and p' by $\frac{A_s}{bt}$ and $\frac{A'}{bt}$ respectively, then the distance from the face most highly stressed to the center of gravity of the transformed section is (by moments)

$$u = \frac{bt\frac{(t)}{2} + nA_sd + nA'd'}{A} = \frac{\frac{bt^2}{2} + nA_sd + nA'd'}{bt + n(A_s + A')} = \frac{t/2 + npd + np'd'}{1 + np + np'}$$

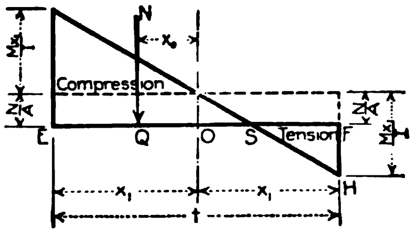


FIG. 2.

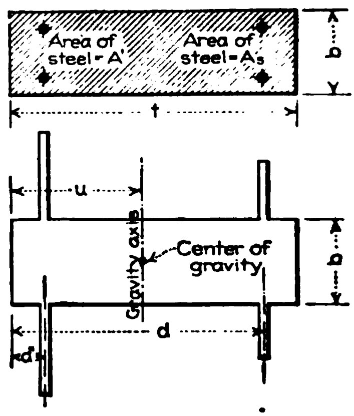


FIG. 3.

$$I_c = \tfrac{1}{3}bu^3 + \tfrac{1}{3}b(t - u)^3 = \frac{b}{3}\left[u^3 + (t - u)^3\right]$$

$$I_s = A_s(d - u)^2 + A'(u - d')^2$$

$$I = I_c + nI_s = \frac{b}{3}\left[u^3 + (t - u)^3\right] + nA_s(d - u)^2 + nA'(u - d')^2$$

If the reinforcement is symmetrical, then $u = \frac{t}{2}$ and

$$I = \tfrac{1}{12}bt^3 + 2nA_s(\tfrac{1}{2}t - d')^2 = \tfrac{1}{12}bt^3 + 2npbt\,(\tfrac{1}{2}t - d')^2$$

Since, $A = bt + n(A_s + A') = bt + nbt(p + p')$

$$\frac{(f_c)}{(f_c')} = \frac{N}{bt + nbt(p + p')} \begin{matrix}(+)\\(-)\end{matrix} \frac{Nx_0 \frac{t}{2}}{\frac{1}{12}bt^3 + 2npbt(\frac{1}{2}t - d')^2}$$

2. Analytical Determination of Stresses in Rectangular Sections.

2a. Compression Over the Whole Section—Steel Top and Bottom (Case I).—The formulas developed in preceding article apply when the stress is either compression over the entire section, or when there is compression over a portion of the section with a tension over the remainder not exceeding the allowable tensile stress in the concrete. The formulas we shall use will apply to rectangular sections with symmetrical reinforcement and are given in the following form for convenience, letting p_0 denote the quantity $p + p'$:

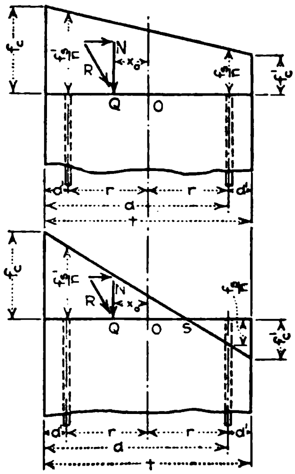


FIG. 4.

$$r = \frac{t}{2} - d'$$

$$\frac{(f_c)}{(f_c')} = \frac{N}{bt}\left[\frac{1}{1 + np_0} \begin{matrix}(+)\\(-)\end{matrix} \frac{6x_0 t}{t^2 + 12np_0 r^2}\right] \qquad \begin{matrix}(1)\\(2)\end{matrix}$$

By referring to Fig. 4 it will be clear that the stress in the steel is always less than $n \times f_c$; thus, if f_c is kept within its allowable value, the steel is sure to be safely stressed.

Equation 2 gives a means of determining the eccentricity of the resultant force, or x_0, for which there can be neither tension nor compression at the surface opposite to that near which the thrust acts. To obtain the value of x_0 which gives a zero value to f_c', equate the two terms within the brackets, and solve.

$$\frac{1}{1 + n(p + p')} = \frac{6x_0 t}{t^2 + 12np_0 r^2}$$

or

$$x_0 = \frac{t^2 + 24npr^2}{1 + n(p + p')} \cdot \frac{1}{6t} \qquad (3)$$

If n is assumed to be 15, and, if the steel is embedded in the concrete one-tenth of the total depth from each surface so that $2r = \frac{4}{5}t$, formula (3) becomes

$$\frac{x_0}{t} = \frac{1 + 28.8p_0}{6 + 90p_0} \qquad (4)$$

If the values $n = 15$ and $2r = \frac{4}{5}t$ are substituted in formula (1), this formula becomes

$$f_c = \frac{N}{bt}\left[\frac{1}{1 + 15p_0} + \frac{x_0}{t} \cdot \frac{6}{1 + 28.8p_0}\right] \qquad (5)$$

or if the expression in the brackets is denoted by K,

$$f_c = \frac{NK}{bt} \qquad (6)$$

Diagrams 1 to 6 inclusive give values of K for various values of p_0, $\frac{x_0}{t}$, and $\frac{d'}{t}$, and for both $n = 12$ and $n = 15$. The termination of the curves are determined in Diagram 5 by equation (4) and in the other diagrams by similar equations. For greater values of $\frac{x_0}{t}$, Case I does not apply; that is, there is tension in the concrete and Case II must be employed.

DIAGRAM 1

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 12$ and $A' = A_s$.

$d' = 0.05t$

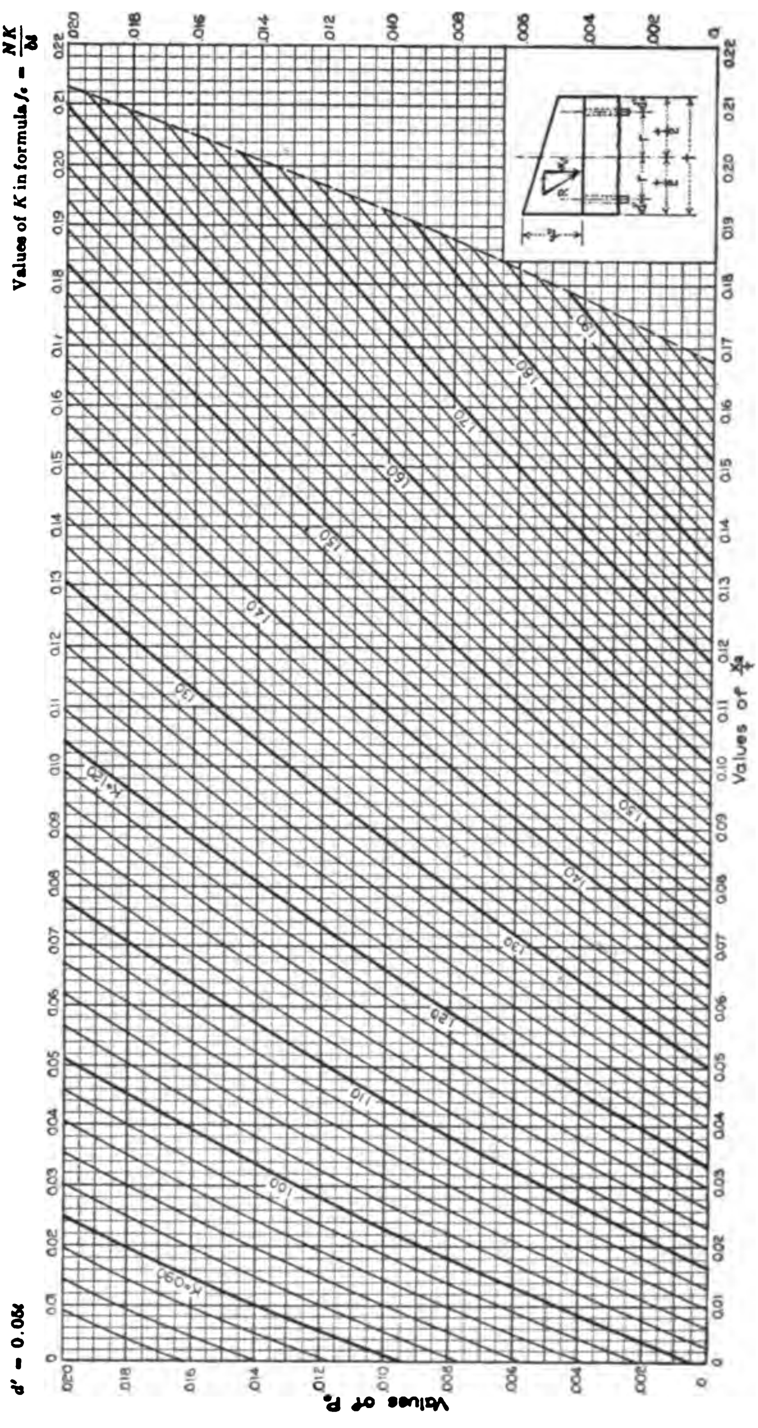


DIAGRAM 2

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 12$ and $A' = A_s$.

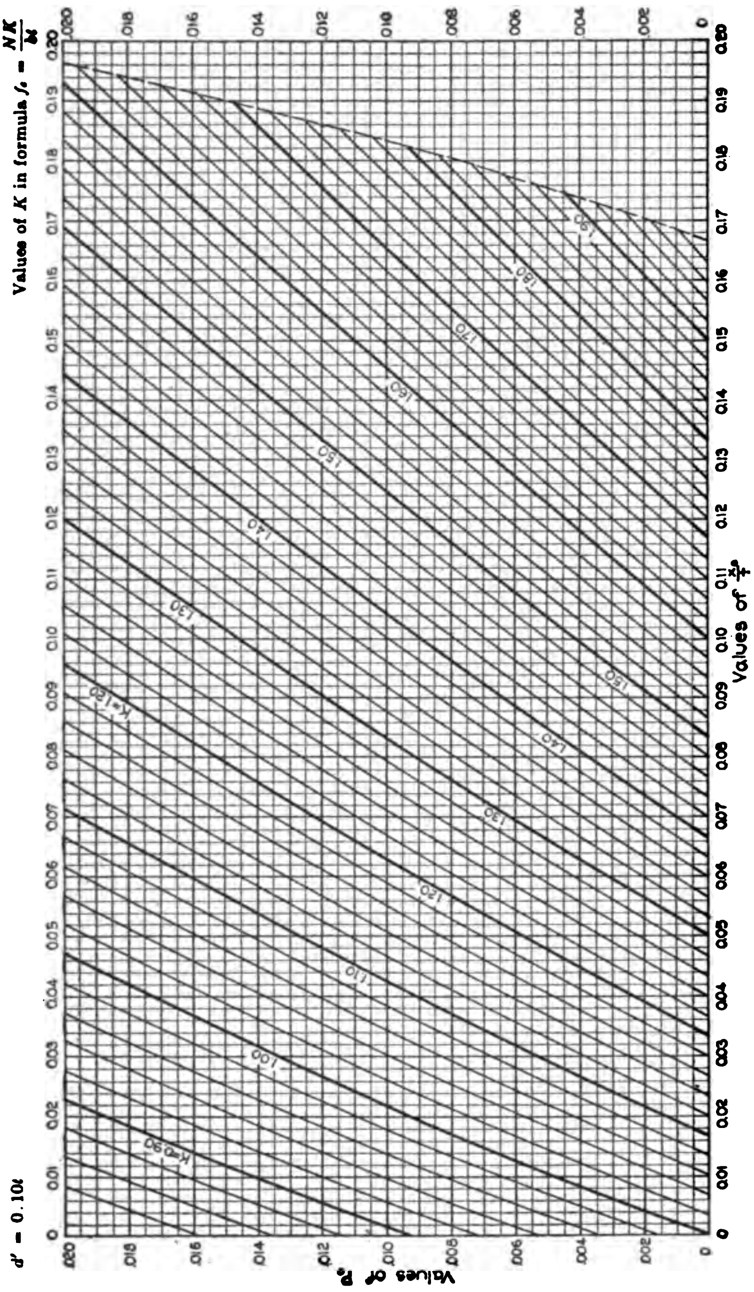


DIAGRAM 3

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 12$ and $A' = A_s$.

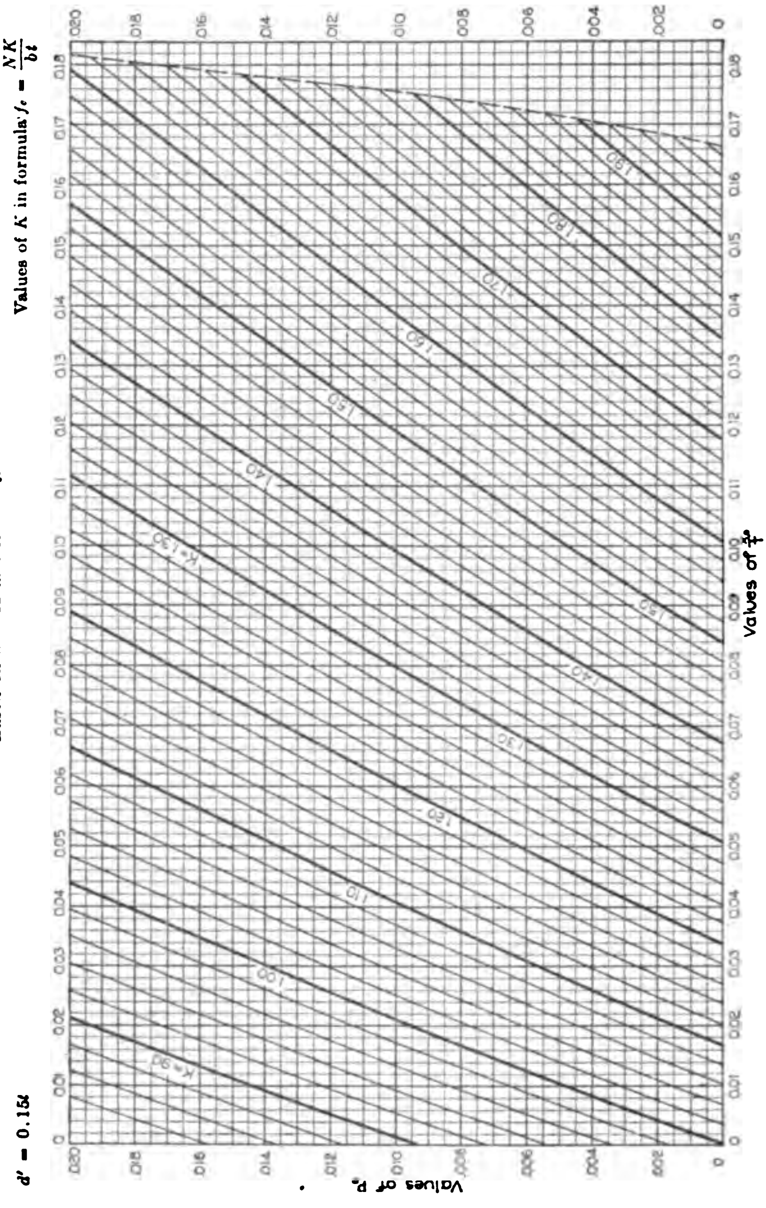


DIAGRAM 4

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 15$ and $A' = A_s$.

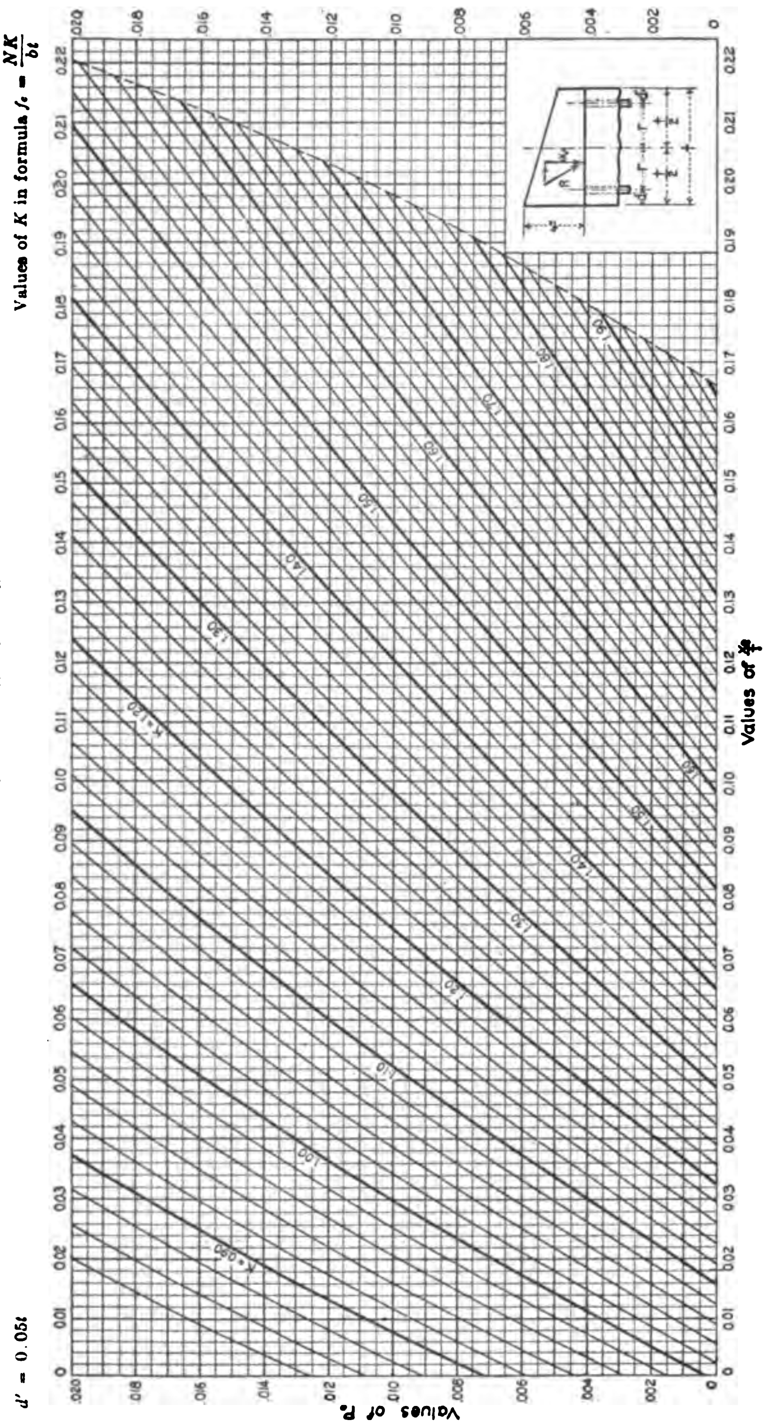


DIAGRAM 5

BENDING AND DIRECT STRESS—COMPRESSION OVER WHOLE SECTION.

Based on $n = 15$ and $A' = A_s$.

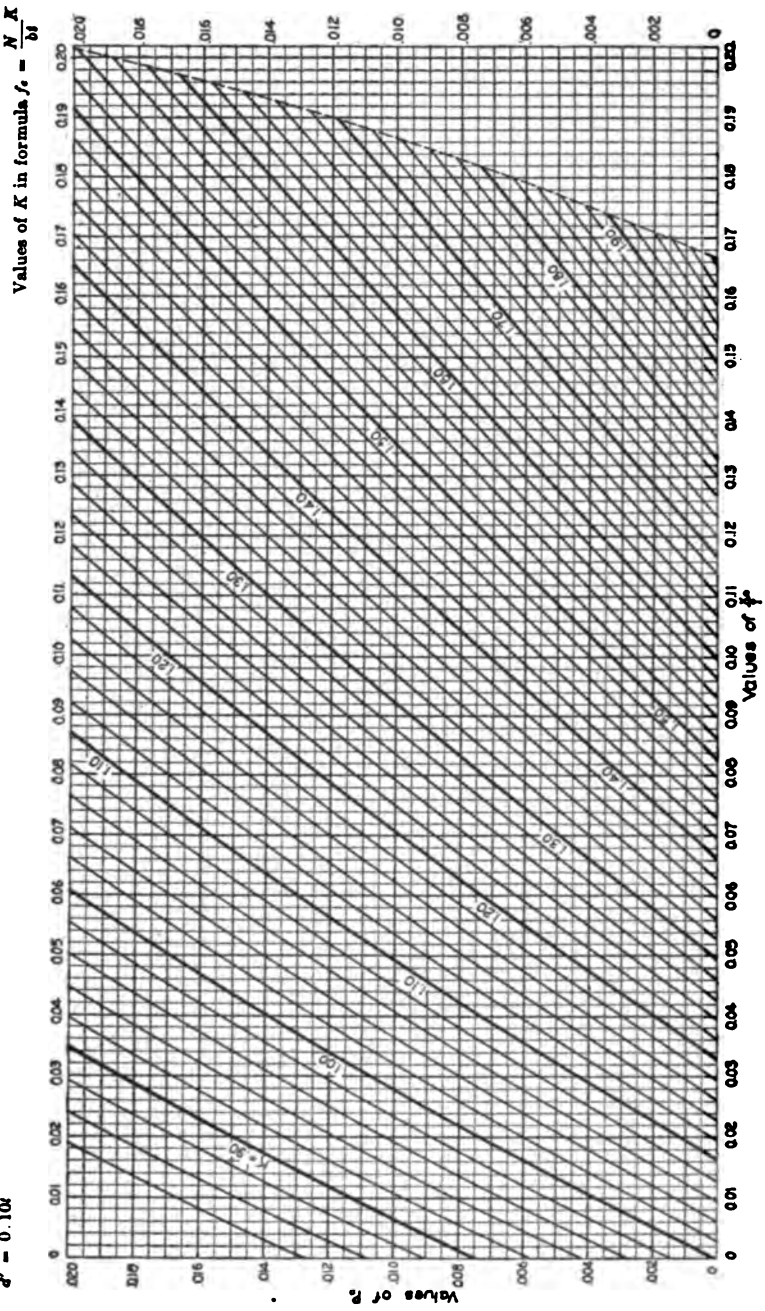
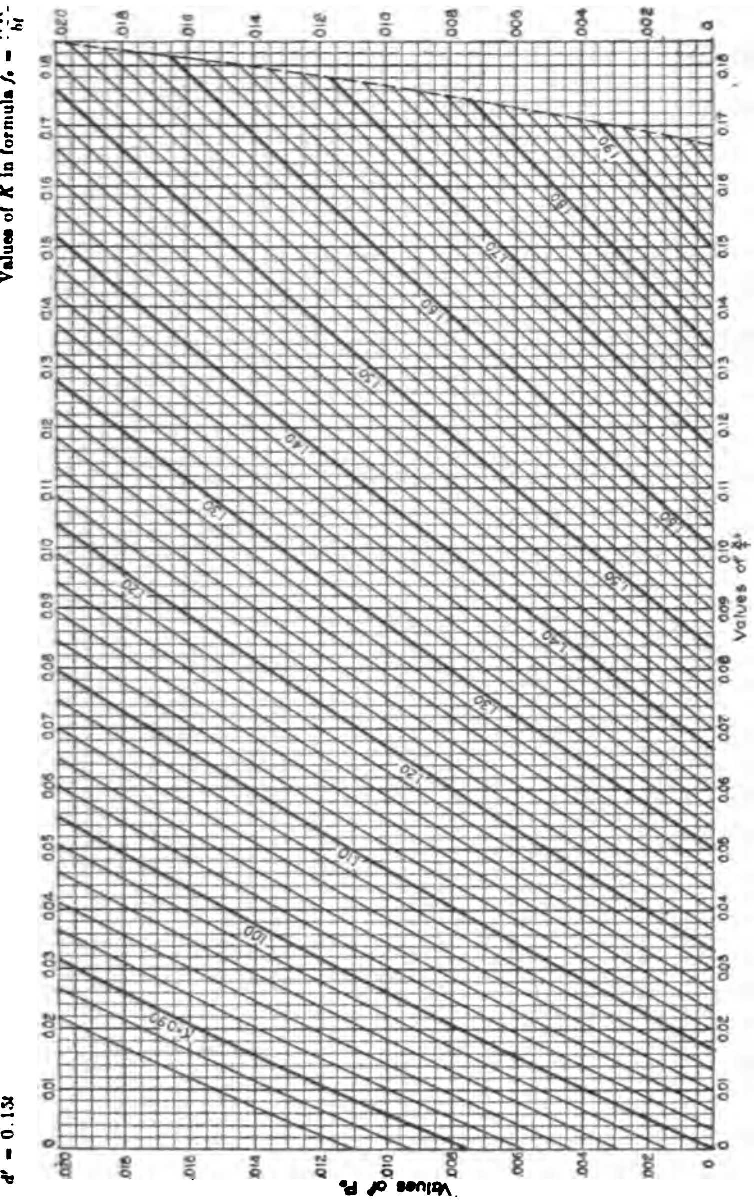


Diagram 6

Bending and Direct Stress—Compression Over Whole Section.

Based on $n = 15$ and $A' = A_s$.



2b. Tension Over Part of Section—Steel Top and Bottom (Case II).—It will be on the safe side and convenient as regards the construction of working diagrams to consider that, when any tension exists in the concrete, the steel carries all the tensile stresses. In this case there are three unit stresses to be determined: namely, maximum unit compression in concrete f_c, maximum unit compression in steel f_s', and maximum unit tension in steel f_s. The general formulas developed in Art. 1 are not applicable to this case and the following method may be used:

Referring to Fig. 5, it follows that

$$f_s' = nf_c\left(1 - \frac{d'}{kt}\right) \qquad (7)$$

and

$$f_s = nf_c\left(\frac{d}{kt} - 1\right) \qquad (8)$$

Since the resultant fiber stress equals N

$$N = \frac{f_s'p_0bt}{2} + \frac{f_cbkt}{2} - \frac{f_sp_0bt}{2}$$

Eliminating f_s' and f_s by means of equations (7) and (8)

$$N = \frac{f_cbt}{2}\cdot\frac{k^2 + 2np_0k - np_0}{k}$$

$$= \frac{f_cbt}{2}\cdot\frac{k^2 + 2nkp_0 - np_0}{k} \qquad (9)$$

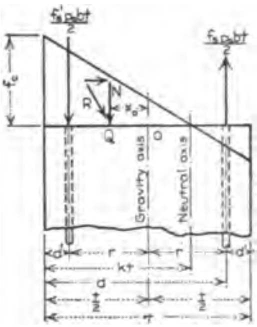


Fig. 5.

The moment of the stresses about the gravity axis, eliminating f_s' and f_s as before, is

$$M = f_cbt^2\left[\frac{np_0r^2}{kt^2} + \frac{k}{12}(3 - 2k)\right] \qquad (10)$$

or, if the quantity within the brackets is designated by L, then

$$M = f_cbt^2L, \text{ or } f_c = \frac{M}{Lbt^2} \qquad (11)$$

The position of the neutral axis must be determined before equation (11) can be used. Since $Nx_0 = M$ we may multiply equation (9) by x_0 and equate it to equation (10). Proceeding in this manner the following equation results

$$k^3 - 3\left(\tfrac{1}{2} - \frac{x_0}{t}\right)k^2 + 6np_0k\frac{x_0}{t} = 3np_0\left(\frac{x_0}{t} + 2\frac{r^2}{t^2}\right) \qquad (12)$$

Diagrams 7, 8, 9, 11, 12 and 13, based on equation (12), give values of k for various values of p_0, $\frac{x_0}{t}$, and $\frac{d'}{t}$, and for both $n = 12$ and $n = 15$. Diagrams 10 and 14 give values of L.

The method of procedure in solving problems under Case II is as follows: (1) Determine k from the proper diagram; (2) find L from Diagram 10 or 14; (3) solve equation (11) for f_c; (4) find unit stresses in the steel from formulas (7) and (8).

Illustrative Problem.—A beam is 9 in. wide and 20 in. deep. The reinforcement both above and below consists of one steel rod 1 in. in diameter embedded at a depth of 2 in. At a certain section, the normal component of the resultant force is 60,000 lb., acting at a distance of 3.4 in. from the gravity axis. Assume $n = 15$. Compute the maximum unit compressive stress in the concrete.

$$p_0 = \frac{A_s + A'}{bt} = \frac{(2)(0.7854)}{(9)(20)} = 0.0087$$

$$\frac{x_0}{t} = \frac{3.4}{20} = 0.17$$

DIAGRAM 7

BENDING AND DIRECT STRESS. TENSION OVER PART OF SECTION.

Based on $n = 12$ and $f'_s = f_s$.

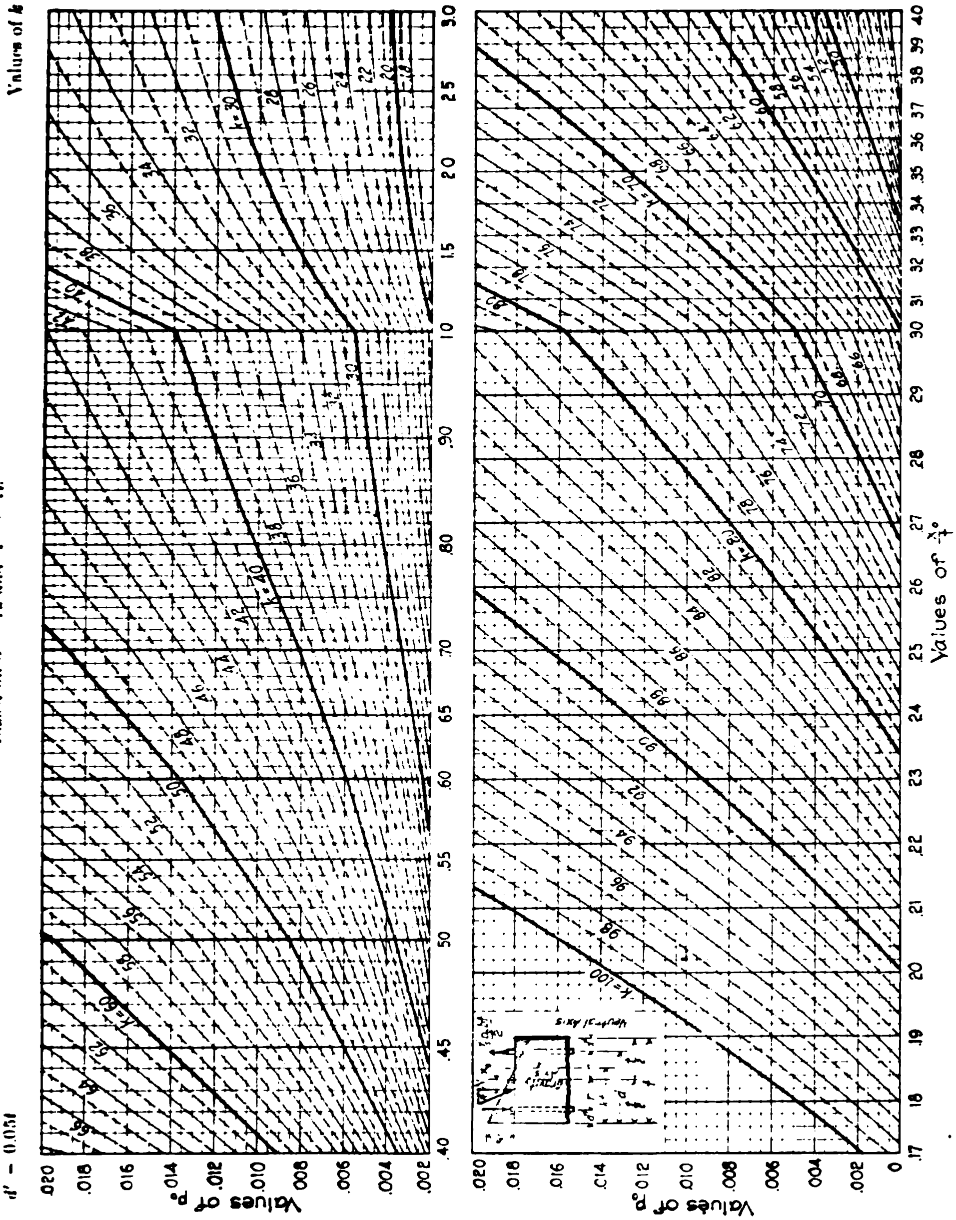


DIAGRAM 8

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 12$ and $A' = A$.

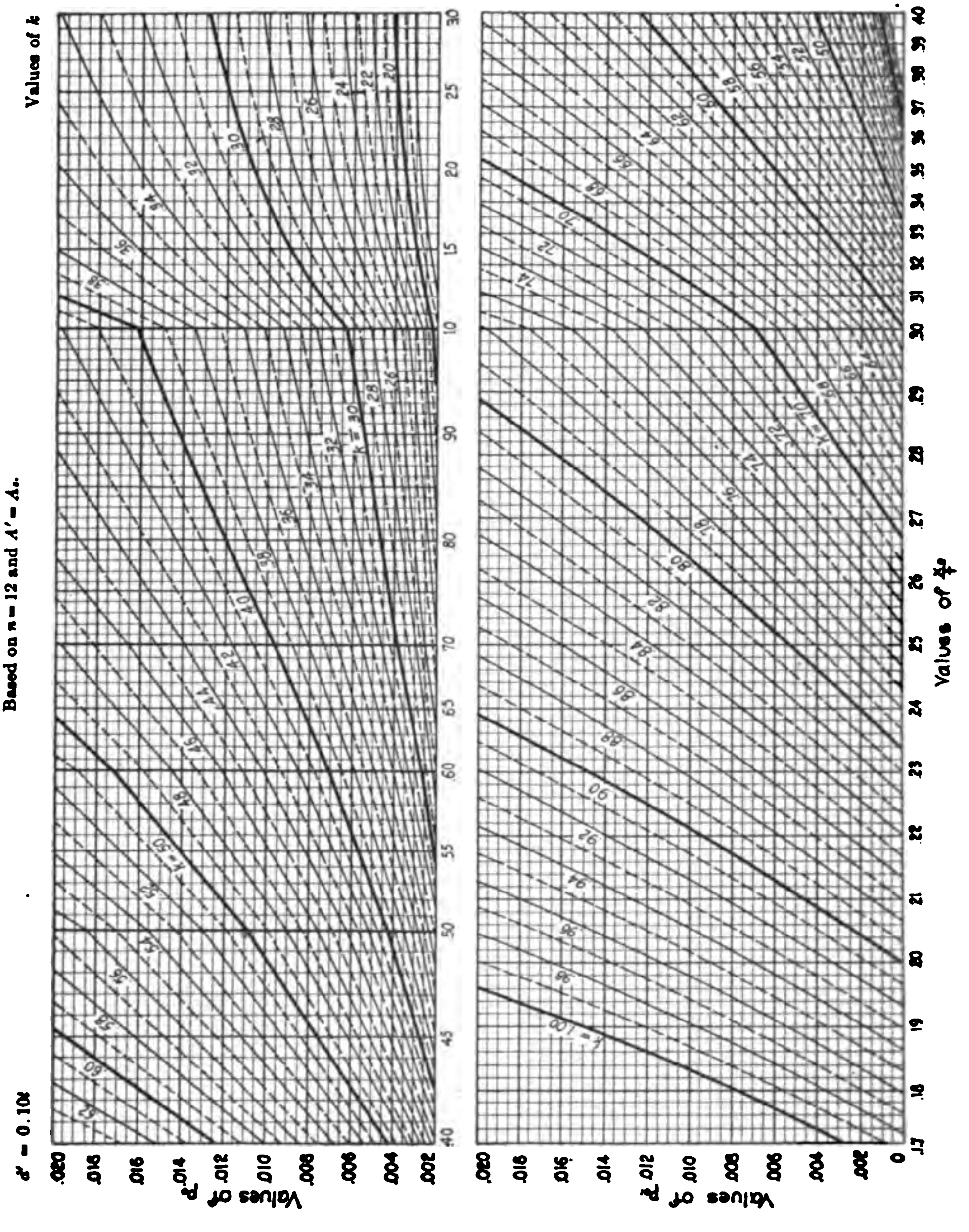


DIAGRAM 9

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 12$ and $A' = A_s$.

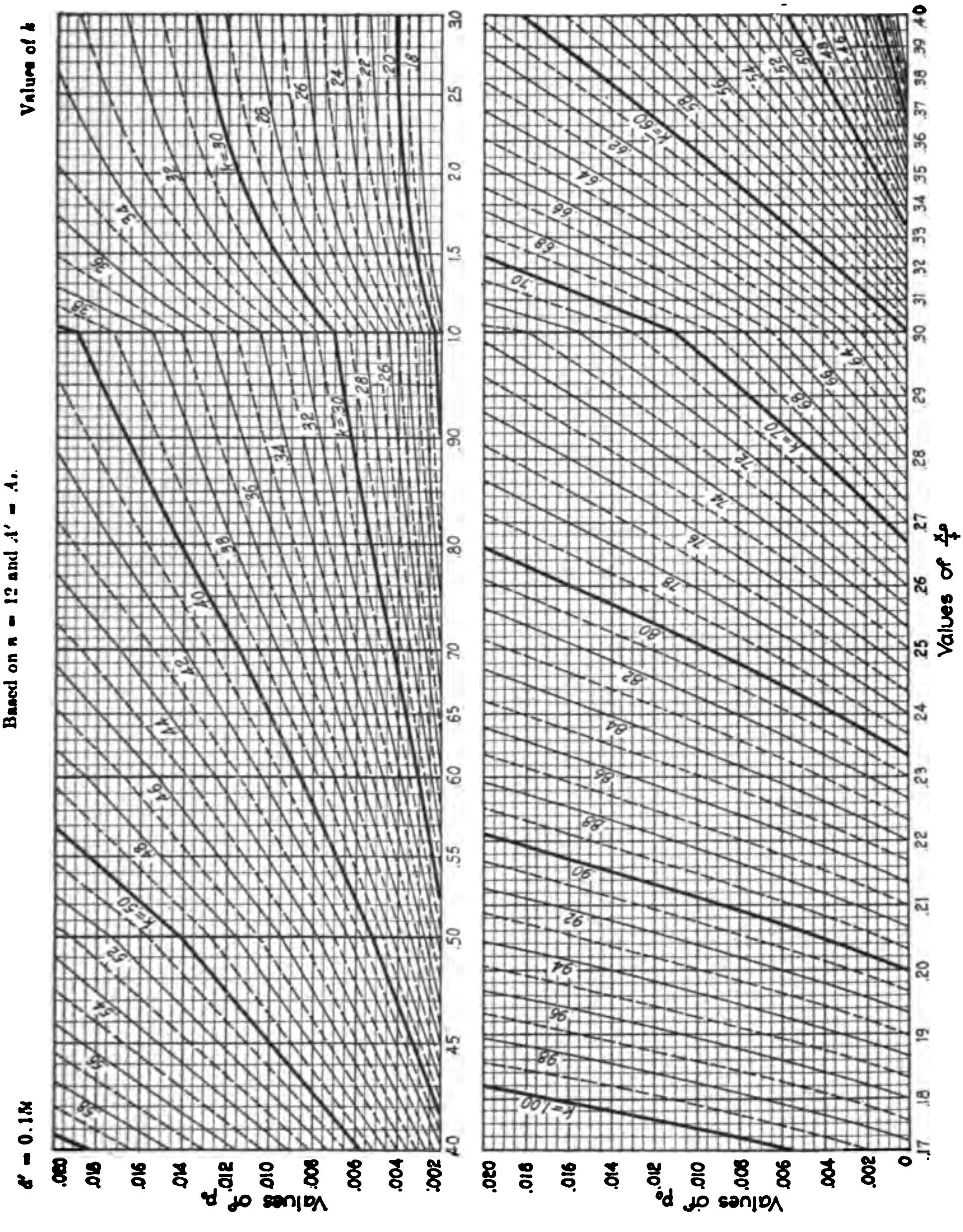
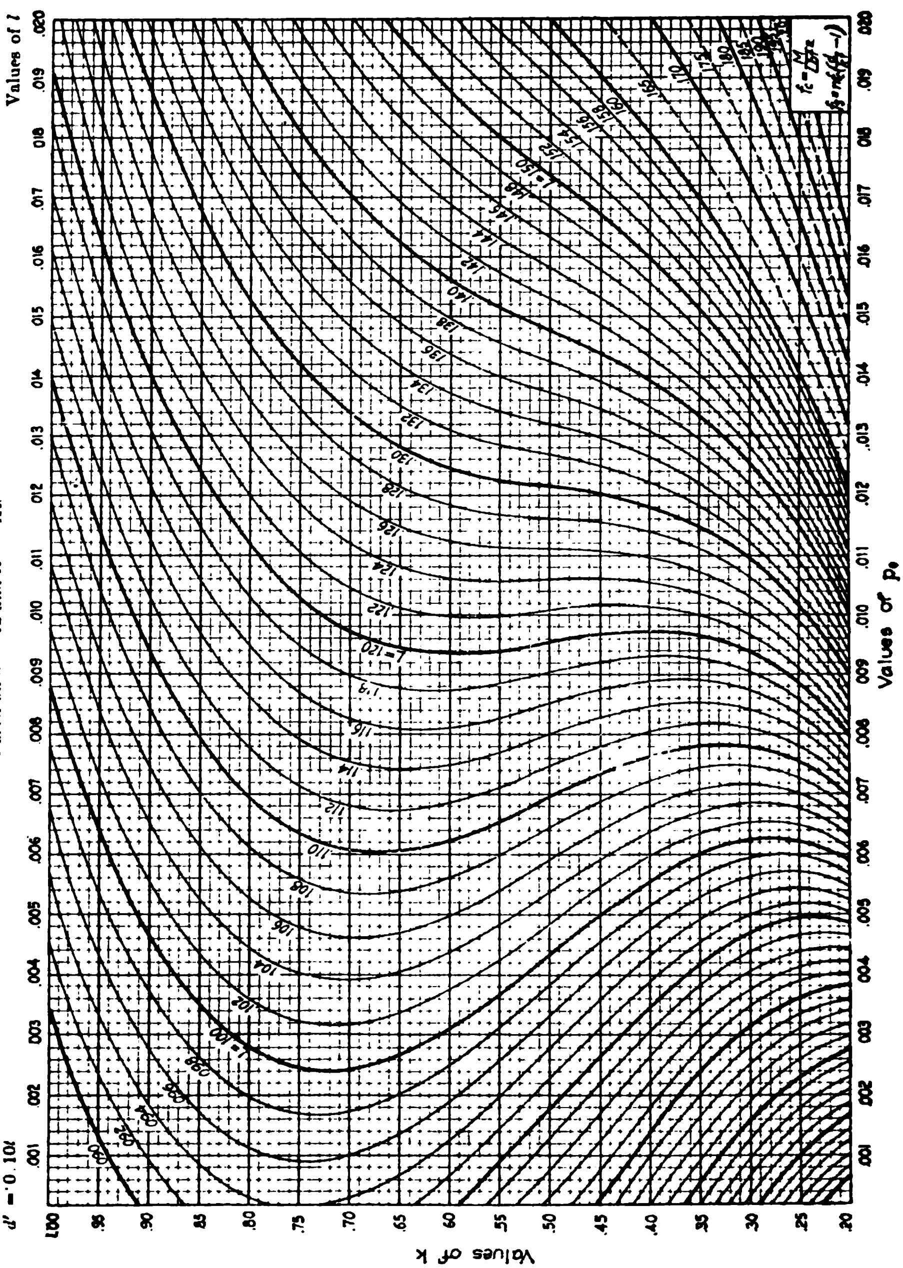


DIAGRAM 10

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 12$ and $A' = A_s$.



For $d' = 0.05t$, divide p_e by 0.700 and find value of l from above diagram
For $d' = 0.15t$, divide p_e by 1.300 and find value of l from above diagram

DIAGRAM 11

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 15$ and $A' = A_s$.

$d' = 0.05t$

Values of k

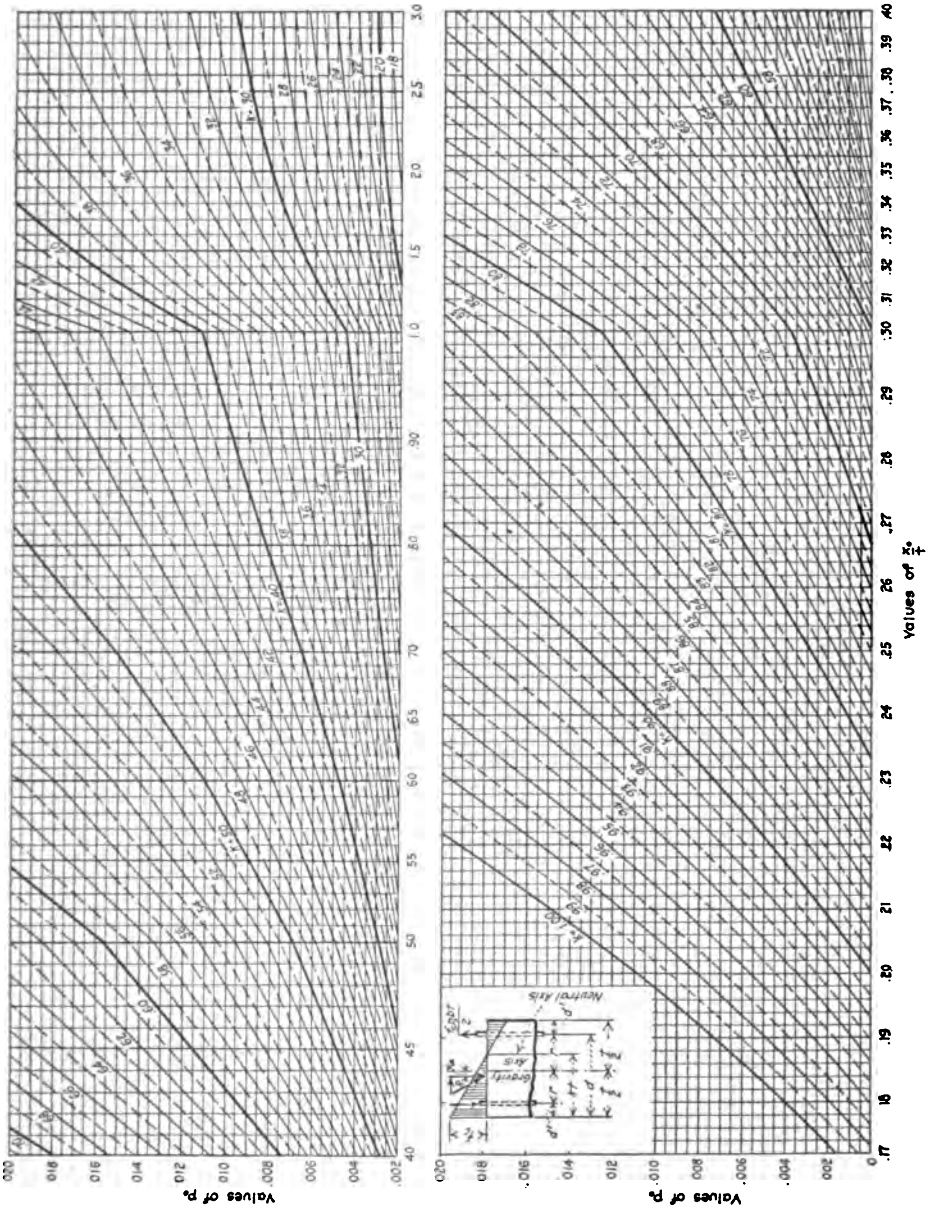


DIAGRAM 12

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 15$ and $A' = A$.

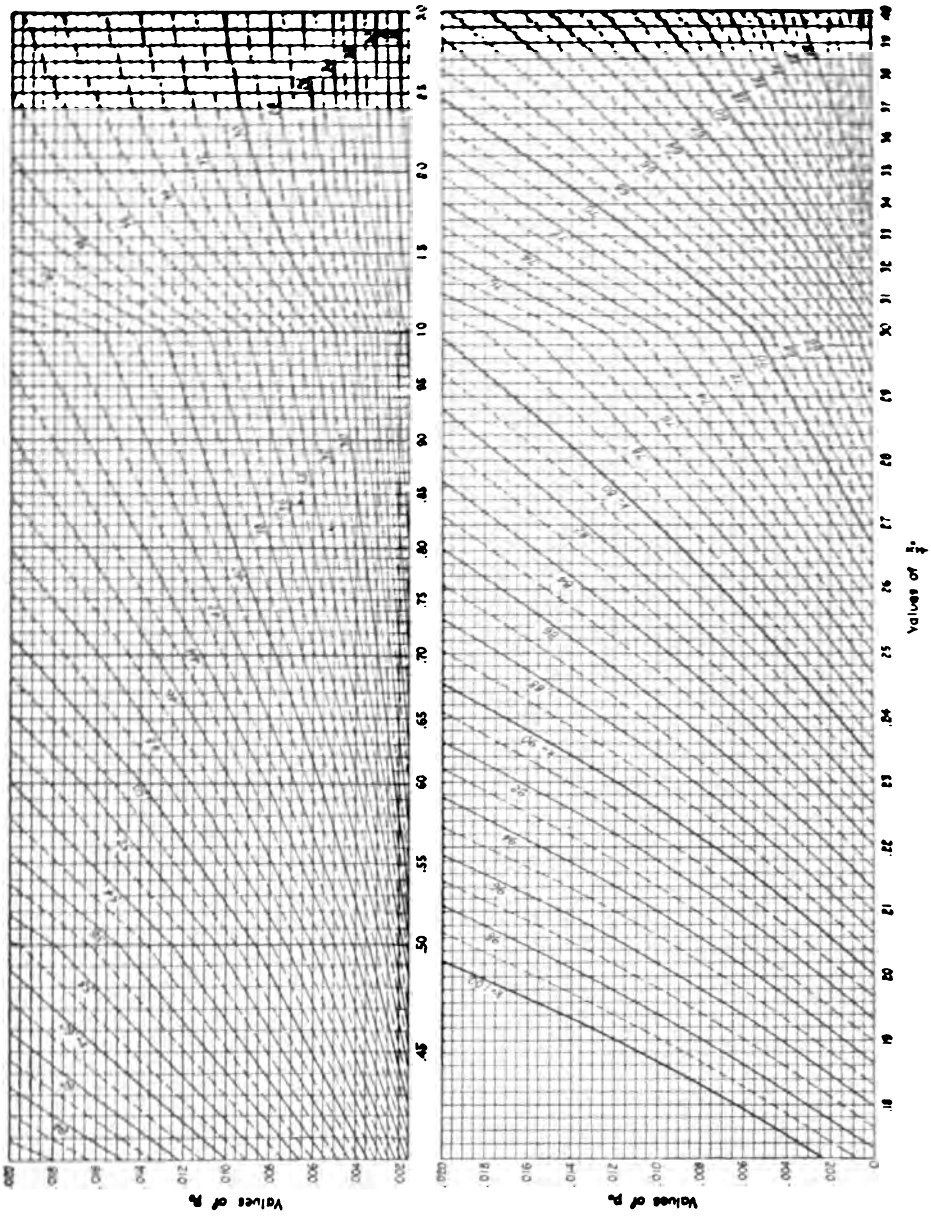


DIAGRAM 13

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION

Based on $n = 15$ and $A'_s = A_s$

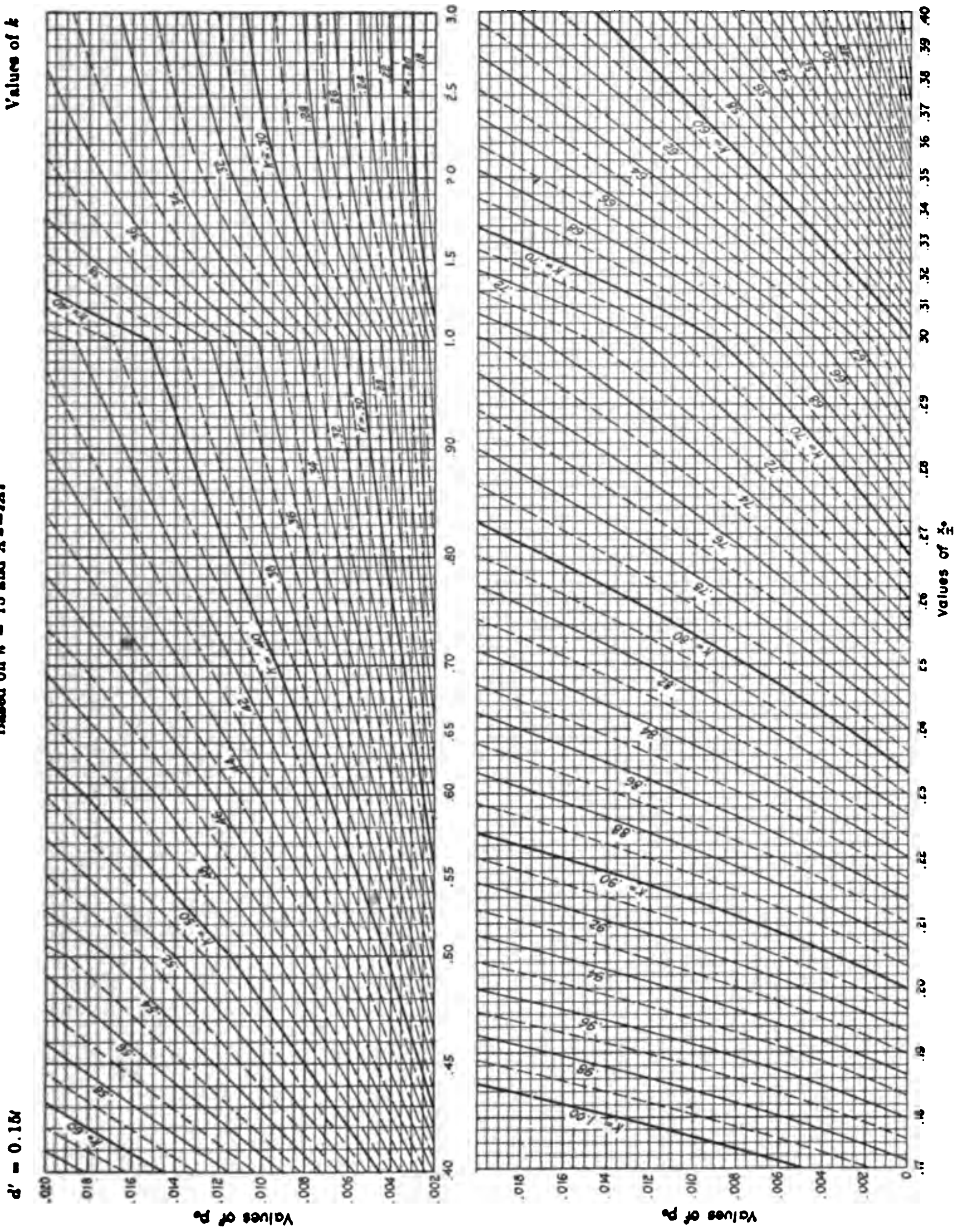
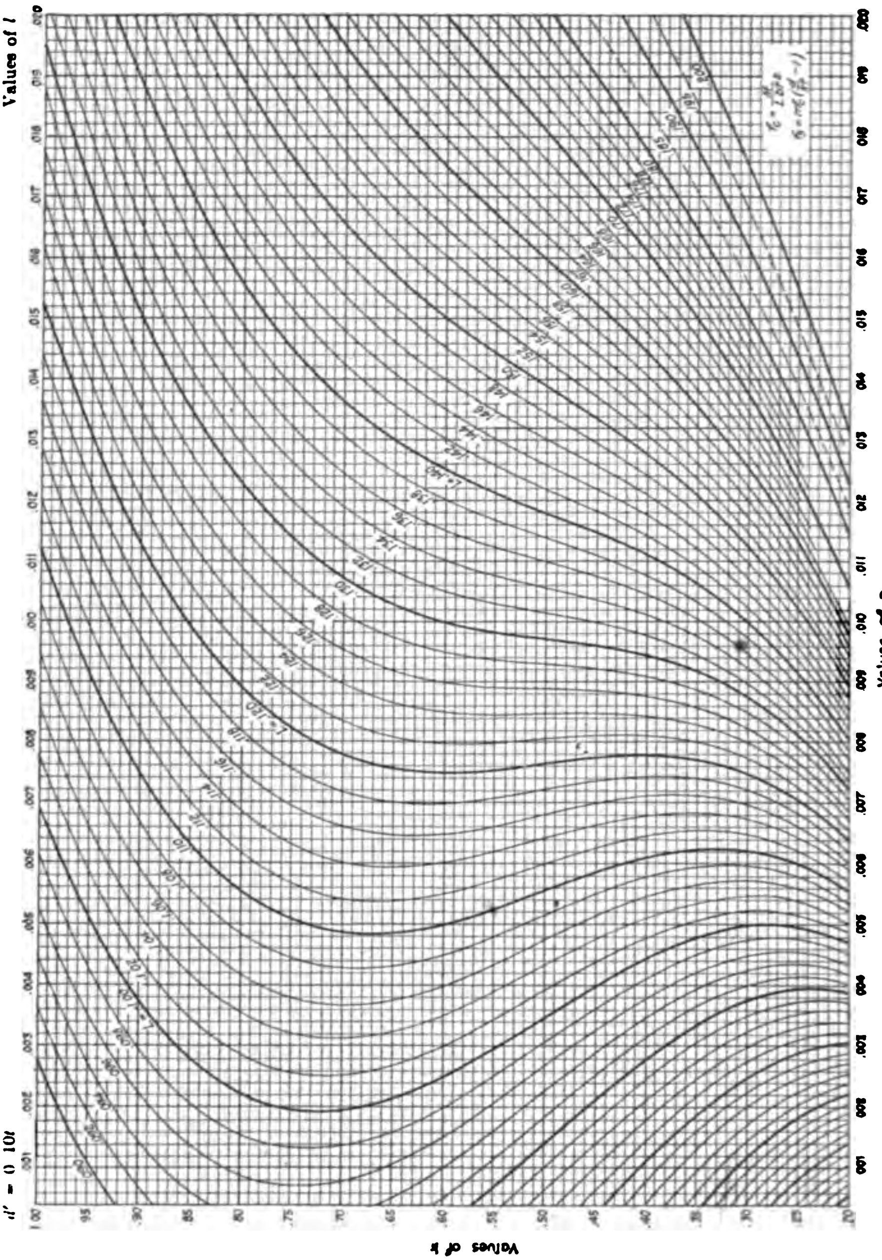


DIAGRAM 14

BENDING AND DIRECT STRESS—TENSION OVER PART OF SECTION.

Based on $n = 15$ and $A' = A_s$.



For $d' = 0.07t$, divide p_s by 0.700 and find value of L from above diagram.
For $d' = 0.14t$, divide p_s by 1.300 and find value of L from above diagram.

For these values of p_0 and $\frac{x_0}{t}$, Diagram 5 gives $K = 1.70$ and shows that the problem falls under Case I Then by formula (6)

$$f_c = \frac{NK}{bt} = \frac{(60{,}000)(1.70)}{(9)(20)} = 567 \text{ lb. per sq. in.}$$

Illustrative Problem.—Change the eccentricity of the preceding problem to 6 in. and solve.

$$\frac{x_0}{t} = \frac{6}{20} = 0.30$$

For $p_0 = 0.0087$ and $\frac{x_0}{t} = 0.30$, Diagram 5 shows that $\frac{x_0}{t}$ is too great for the problem to come under Case I. The method of procedure for Case II must then be followed.

Diagram 12 gives $k = 0.73$ for the values of p_0 and $\frac{x_0}{t}$ given above. With $k = 0.73$ and $p_0 = 0.0087$, Diagram 14 shows L to be 0.123. Solving equation (11)

$$f_c = \frac{M}{Lbt^2} = \frac{(60{,}000)(6)}{(0.123)(9)(20)^2} = 815 \text{ lb. per sq. in.}$$

Using formula (8) gives

$$f_s = nf_c\left(\frac{d}{kt} - 1\right) = (15)(815)\left(\frac{18}{0.73 \times 20} - 1\right) = 2830 \text{ lb. per sq. in.}$$

The stress f_s' may be found by formula (7) but is always less than $n \times f_c$.

Illustrative Problem.—An arch is 20 in. deep and is reinforced with three rods ¾ in. in diameter to each foot of width, both above and below. If the rods are embedded to a depth of 2 in. and the normal component of the resultant thrust on a section is 100,000 lb. for 1-ft. width of arch, with an eccentricity of 3.4 in., determine the maximum intensity of compressive stress on the concrete. Assume $n = 15$

$$p_0 = \frac{(6)(0.4418)}{(12)(20)} = 0.0110$$

$$\frac{x_0}{t} = \frac{3.4}{20} = 0.170$$

Diagram 5 gives $K = 1.63$ and the problem comes under Case I. Then by formula (6)

$$f_c = \frac{NK}{bt} = \frac{(100{,}000)(1.63)}{(12)(20)} = 679 \text{ lb. per sq. in.}$$

2c. Tension Over Part of Section—Steel in Tension Face Only (Case III).—Referring to Fig. 6 and taking moments about the center of the steel we have

$$Ne' = \tfrac{1}{2}f_ckjbd^2$$

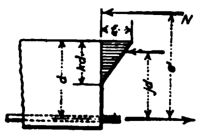


Fig. 6.

Since the algebraic sum of the compressive and tensile forces must equal N, we may write

$$N = \tfrac{1}{2}f_ckbd - f_spbd$$

We also know (see page 276) that

$$f_c = \frac{kf_s}{n(1 - k)}$$

From these three equations may be obtained the formulas:

$$k^2 - 2pn\ (1 - k) = k^2j\frac{d}{e'}$$

$$j = 1 - \tfrac{1}{3}k$$

$$p = \frac{k^2}{2n(1 - k)}\left[1 - j\frac{d}{e'}\right]$$

$$K = \frac{M_c}{bd^2} = \frac{Ne'}{bd^2} = \tfrac{1}{2}f_ckj$$

$$f_c = \frac{2Ne'}{kjbd^2}$$

$$f_s = nf_c\frac{1 - k}{k}$$

Diagrams 15 and 16[1] may be used in designing and reviewing structures subjected to bending and direct stress with steel in tension face only.

[1] Scheme of diagrams proposed by Robert S. Beard, Asst. City Engineer, Kansas City, Mo.

DIAGRAM 15

BENDING AND DIRECT STRESS—STEEL IN TENSION FACE ONLY.

TENSION OVER PART OF SECTION.

Based on $n = 12$.

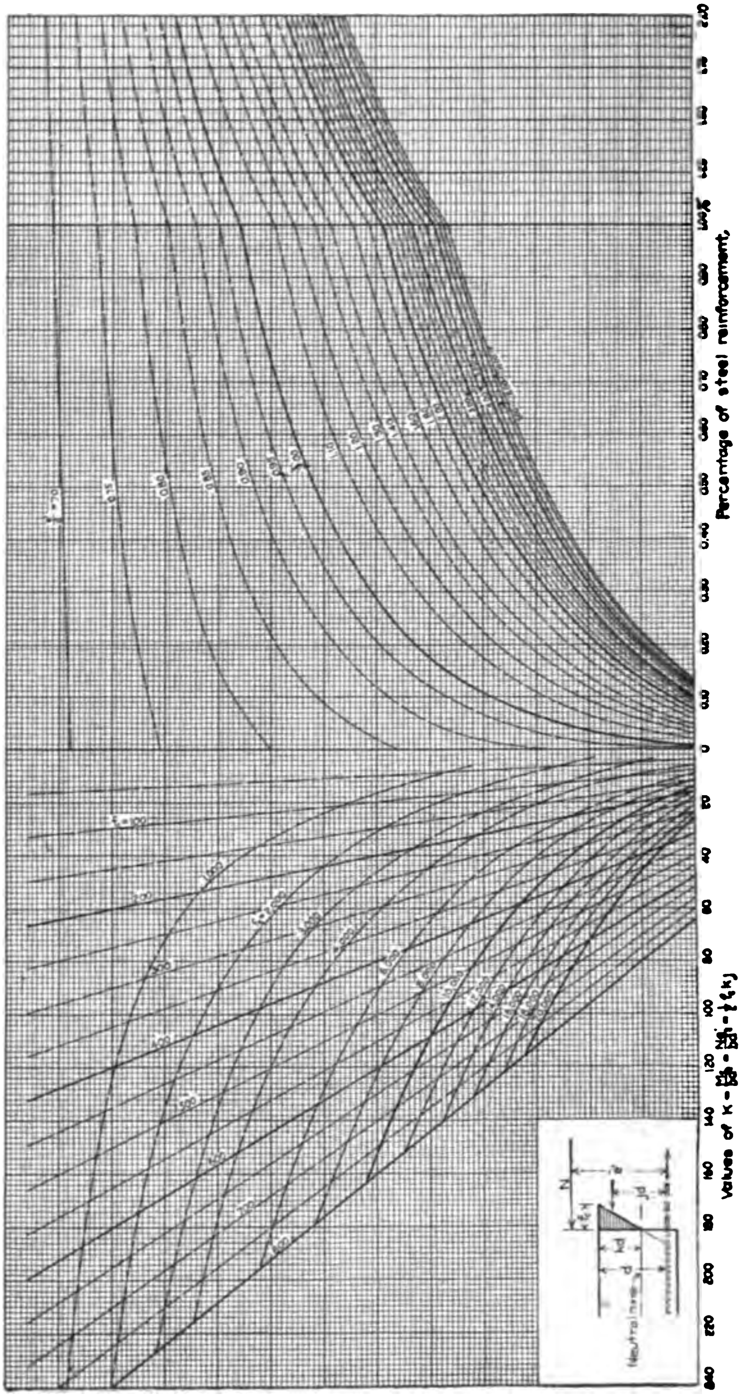
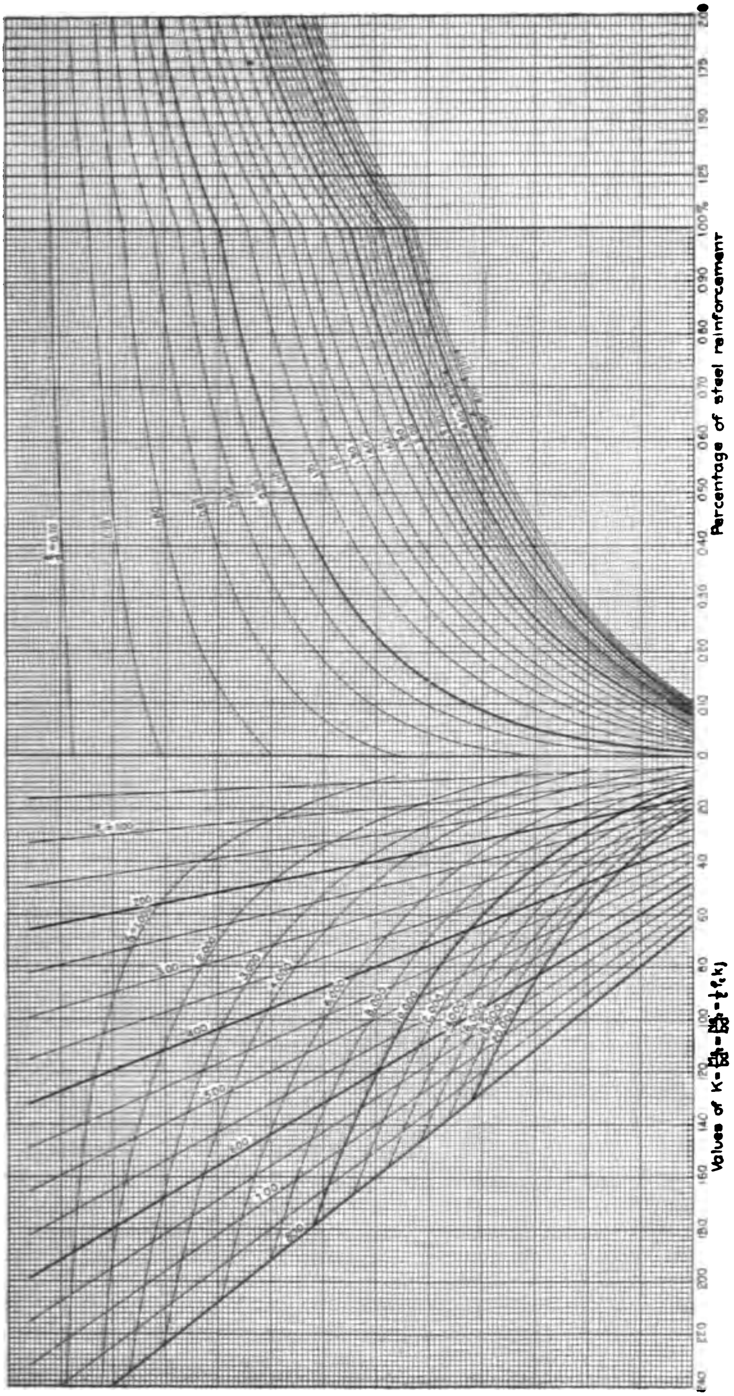


DIAGRAM 16

BENDING AND DIRECT STRESS—STEEL IN TENSION FACE ONLY.

TENSION OVER PART OF SECTION.

Based on $n = 15$.



ILLUSTRATIVE PROBLEM.—The vertical wall of a cantilever retaining wall is subjected to an earth pressure of 2400 lb. applied at a distance of 4.92 ft. above the top of footing. The weight of vertical walls is 2200 lb. Determine the unit stresses f_c and f_s, assuming $n = 15$, $p = 0.0077$ and $d = 10.5$ in.

The moment at the top of footing

$$M = (2400)(4.92)(12) = 141{,}700 \text{ in.-lb.}$$

$$K = \frac{141{,}700}{(12)(10.5)^2} = 107$$

$$\frac{e'}{d} = \frac{141{,}700}{(2200)(10.5)} = 6.1$$

Entering Diagram 16 with a value of $p = 0.0077$ on the lower right-hand margin and tracing vertically to a value of $\frac{e'}{d} = 6.1$, then horizontally to the left to a point vertically above $K = 107$, we find $f_s = 14{,}000$ and $f_c = 610$.

ILLUSTRATIVE PROBLEM.—Design the vertical wall of the retaining wall described in the preceding problem so that $f_c = 750$ and $f_s = 16{,}000$. Assume the weight of wall at 2000 lb.

For these unit stresses the left-hand part of Diagram 16 shows $K = 133.8$. Then

$$d = \sqrt{\frac{141{,}700}{(133.8)(12)}} = 9.4 \text{ in., say } 9\tfrac{1}{2} \text{ in.}$$

$$\frac{e'}{d} = \frac{141{,}700}{(2000)(9.5)} = 7.45$$

Following across the diagram horizontally to the right to a value of $\frac{e'}{d} = 7.45$ and then vertically downward to the lower right-hand margin we find $p = 0.0085$.

3. Graphical Determination of Stresses.[1]

3a. Rectangular Sections.—Fig. 7a shows the cross-section of a column, reinforced with twelve 1-in. square bars, and of dimensions as shown. This column is loaded with an eccentric load of 75,000 lb., acting 1 in. outside the edge of the column. It is desired to find the maximum unit stress in the concrete and steel, assuming that the load is symmetrically placed about the axis XX, Fig. 7a. In this solution the effect of the bending moment produced by the eccentricity of the load will first be considered, after which the effect of the direct load will be added.

The neutral axis is located by means of Figs. 7b and 7c, and the base line ef is drawn in Fig. 7d. The length eb is measured off to represent some convenient number of pounds per square inch, in this case 400, and the line bcr is drawn. From the intercepts between the lines cf and cr, the stress in the tension steel may be found. Figs. 7e and 7g locate the resultant of the compressive stresses. Figs. 7f and 7h locate the resultant of the tensile stresses, and the effective depth is found to be 14.6 in., from which 656,000 in.-lb. is found to be the resisting moment when the maximum stress in the concrete is 400 lb. per sq. in. The eccentricity of N measured from the neutral axis is approximately 8.25 in., therefore the load produces a moment of 619,000 in.-lb. Now, if the resisting moment is 656,000 in.-lb. when the maximum concrete stress is 400 lb. per sq. in., by proportion the maximum stress in the concrete produced by a bending moment of 619,000 in.-lb. will be 377 lb. per sq. in., shown to scale as ea in Fig. 7d.

The effect of the direct load will now be considered. A direct load applied at the neutral axis will have the effect of moving the base line ef to the left, thus increasing the compressive stresses and decreasing the tensile stresses. Assume the base line ef to be moved to e_1f_1, a distance of 100 lb. per sq. in. The increase of compressive stresses is added to the decrease in the tensile stresses and found to be 33,600 lb., which is laid off to scale along the line e_1g, thus locating the point g. In other words, a load of 33,600 lb. applied approximately at the neutral axis would move the base line ef to e_1f_1, increasing the compressive stresses in the concrete 100 lb. per sq. in. and decreasing the tensile stress in the tension steel $100(n) = 1500$ lb. per sq. in. The base line is now moved another 100 lb. per sq. in. to the left, and the additional increase in compressive stresses plus the decrease in the tensile stresses is added to the 33,600 lb. already

[1] Method as given by W. S. WOLFE, Instructor in Architectural Engineering, University of Illinois, in *Eng. & Cont.*, April 25, 1917, and May 23, 1917.

obtained, making 71,400 lb. which is laid off along e_2h, thus locating h. Again the base line is moved 100 lb. per sq. in. to the left, reaching the position e_3f_3 and i is found to be out a distance of 112,400 lb. Now draw the curve $eghi$ and locate k, the point where the 75,000-lb. line cuts; then draw the base line kxy, which shows ax = 587 lb. per sq. in. = the maximum stress in the concrete, and $yd(n)$ = the maximum tensile stress in tensile steel. That is, a load of 75,000 lb. applied approximately along the neutral axis will move the base line ef to the position xy, thus increasing the compressive stress in the concrete 210 lb. per sq. in. and decreasing the tensile stress in the steel $yf(n)$ = 210(15) = 3150 lb. per sq. in. Now a slight approximation has been made in assuming that a load applied at the neutral axis moves the base line directly to the left. This assumption is exactly correct to start with, but as the base line moves to the left, more area comes under compression and the position of the load must move slightly toward the ten-

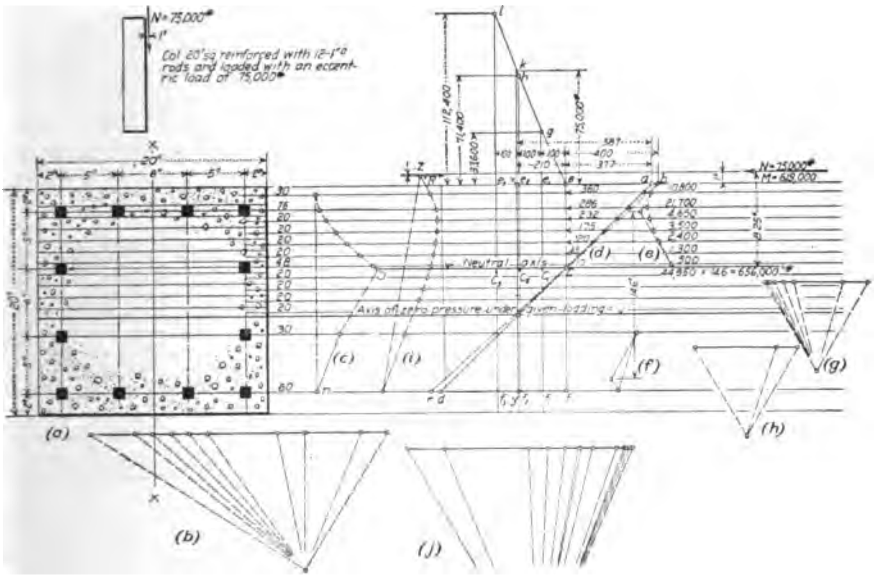


Fig. 7.

sion side of the beam in order to keep the base lines parallel. This deviation from the neutral axis, however, is only slight as long as there is very much tension in the column. For example, when the base line has moved to e_1f_1 the load should act through c_1 in place of through c; when the base line has moved to e_2f_2 the load should act through c_2; etc.

As a graphical check on the work the compressive and tensile stresses in the column when the base line is xy are found and laid out in the force polygon, Fig. 7j, from which the funicular polygon, Fig. 7i, is drawn, locating their resultant, which, of course, should have the same action line as the applied load N. The error is found to be z, which is just about as large as the error made in assuming that the eccentricity was the distance from the neutral axis to the action line of N. If it is desired to make a correction for this error, add z to the eccentricity used. This will increase M slightly, which will increase the length of ea, in this case from 377 to 383 and the maximum stress in the concrete is 583 lb. per sq. in.

3b. Hollow Circular Sections.—When the section of a reinforced-concrete chimney, or any hollow circular section such as shown in Fig. 8a is considered as having an eccentric load, we have a very difficult problem as far as an analytical solution is concerned.[1] However-

[1] See analytical method used in the design of chimneys, Art. 15, Sect. 18.

SECTION 10

MOMENTS IN RIGID BUILDING FRAMES

1. Importance of the Subject.—The reinforced-concrete building frame differs from other frames particularly in its rigidity at the junction of members, when proper provision has been made for continuity. This applies not only to columns extending from the basement to the roof, and to beams continuous for more than one span; but more particularly to the rigidity existing between the column system and the continuous beams. Many recent tests[1] upon completed structures show that the concrete building frame is a rigid one, and, if properly reinforced, may well be designed as such. Many tests also show that, whether so designed or not, the reinforced-concrete frame, due to its rigidity, causes column stresses hitherto not considered, and of a magnitude to cause more careful investigation during future design.

2. Method of Analysis.—The method of analysis which follows is that of slope-deflections developed with the aid of the principle of area moments.

From the property of the $\frac{M}{EI}$ curve for beams we have the following important laws:[2]

1. The change in angle between the tangents at any two points on the elastic curve of a member, which change is caused by the action of moments on the members at those points, is equal to the area on the $\frac{M}{EI}$ diagram of these moments, included between the two points.

2. The lateral displacement of one end of a member, when that member is acted upon by moments, is equal to the statical moment of the $\frac{M}{EI}$ diagram about the displaced end. This is the principle of area-moments (see *Mich. Technic*, 1910).

[1] *Bulls.* 64 and 84, Eng. Exp. Sta., Univ. of Ill.; *Jour. Am. Conc. Inst.*, vol. II, No. 6, 1914.

[2] For any small portion ds of a member (see Fig. A), there is a certain value of x which may be designated by dx. Let tangents be drawn to the curve at the extremities of the portion ds; then the angle between these tangents may be called $d\phi$. For every value of $d\phi$ there is a corresponding elementary portion dy intercepted on $B'B$. This distance dy is equal to $x \cdot d\phi$ (since the curvature is in reality very slight); and, similarly, y_1 is equal to

$$\int_B^P dy = \int_B^P x \cdot d\phi = \int_B^P \frac{Mxdx}{EI}$$

(see Art. 46, Sect. 7.)

Suppose that a curve be plotted such that the ordinate at any point represents the bending moment at the corresponding point in the member, divided by EI. Thus, for any point P, the ordinate to the curve is $\frac{M_P}{EI}$. The area under this $\frac{M}{EI}$ curve for a distance dx along its axis (shaded in the figure) is equal to $\frac{M}{EI} \cdot dx$; likewise, under the whole curve it is

$$\int_A^B \frac{Mdx}{EI}$$

But this is the expression for the change of slope ϕ of the tangent at A with respect to the tangent at B; hence the first law is evident.

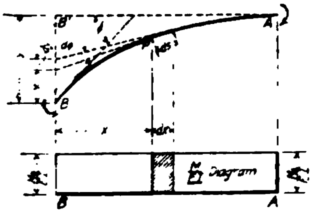


FIG. A.

The shaded area in Fig. A has a moment about the end B equal to $\frac{M}{EI}dx \cdot x$. This is seen to be the value of dy. The moment about B of the total area under the $\frac{M}{EI}$ curve is

$$\int_A^B \frac{Mxdx}{EI}$$

which is the value of y, the linear deflection of one end normal to the original position of the member. The second law then follows.

Before proceeding with the application of these laws or principles, it is necessary to establish rules concerning the signs of moments and deflections.

The moment at the end of any member will be considered *positive* if the external moment at that end acts in a *clockwise* direction. The subscript of the moment will determine the member and end in question; thus M_{AB} is the moment at the end A of the member AB.

The change in slope of the tangent to the elastic curve of a member will be considered positive when the tangent has turned in a clockwise direction.

The deflection of any point in the member will be measured normal to, and away from, the base line or line of original position. The sign of the deflection will be considered positive when measured in the same direction from the base line as are positive slopes.

$\frac{M}{EI}$ diagrams will be plotted on the tension side of the member.

In Fig. 1a there is shown a member AB acted upon by moments and shears at the ends. Let AB' be the original position when considering the member AB, and $A'B$ the original position when considering the member BA. The angles θ_A and θ_B are negative, as is also d. According to the scheme of signs, the moments at both ends are positive. However, when the $\frac{M}{EI}$ diagram is plotted with respect to the member, as indicated above, there is seen to occur a point of inflection. Fig. 1b is the $\frac{M}{EI}$ diagram for the member under the loading shown.

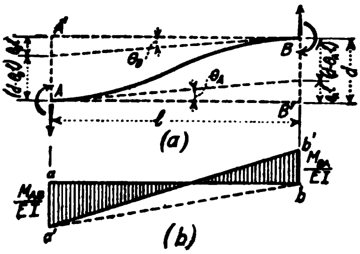


FIG. 1.

Since the end A is not fixed, the total deflection d of B is due partly to the change in slope at A and partly to the flexure of the member. Thus, the distance $d - \theta_A l$ is the deviation due to flexure, and is equal to the statical moment of the $\frac{M}{EI}$ diagram about B. The area of the $\frac{M}{EI}$ diagram is seen to be the algebraic sum of the areas $a'ab$ and $a'bb'$; whence

$$-(d - \theta_A l) = \frac{l^2}{EI}\left(\frac{M_{AB}}{3} + \frac{M_{BA}}{6}\right)$$

The change in slope of the member from B to A is $\theta_A - \theta_B$, and is equal to the area of the $\frac{M}{EI}$ diagram, or

$$\theta_A - \theta_B = \frac{l}{2EI}(M_{AB} + M_{BA})$$

Eliminating M_{BA} from the two equations above, there results

$$M_{AB} = \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3d}{l}\right)$$

Substituting K for $\frac{I}{l}$ and R for $\frac{d}{l}$,

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) \qquad (1)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) \qquad (1a)$$

This is the general equation for moment at one end of a member carrying no transverse load, in terms of the relative changes in position of the ends.[1]

When there is a transverse load upon the member an expression similar to equation (1) may be derived. Consider the member shown in Fig. 2a. The deflection of B from the tangent at A is $d - \theta_A l$ and equals the moment of the $\frac{M}{EI}$ diagram about B (see Fig. 2b).

[1] The above method for the general treatment of rigid frames first appeared in Bulletin 1, University of Minnesota, "Secondary Stresses and other Problems in Rigid Frames," by G. A. Maney, March, 1915. The method was applied to the special case of wind stresses in office buildings, Bulletin 80, University of Illinois, by W. Wilson and G. A. Maney (June, 1915). The notation here used is from the latter bulletin.

This diagram is similar to that shown in Fig. 1b, except that the area caused by the load is added to the diagram of the other moments at A and B.

$$(d - \theta_A l) = \frac{l}{EI}\left[\frac{M_{AB}l}{3} + \frac{M_{BA}l}{6} + \frac{Pab}{6l}(l + b)\right]$$

The difference in slope between the two ends is $(\theta_A - \theta_B)$, and is equal to the area of the $\frac{M}{EI}$ diagram.

$$(\theta_A - \theta_B) = \frac{l}{2EI}\left(M_{AB} + M_{BA} + \frac{Pab}{l}\right)$$

From these two equations M_{BA} may be eliminated, whence

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - \frac{Pab^2}{l^2} \qquad (2)$$

Similarly

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + \frac{Pa^2b}{l^2} \qquad (2a)$$

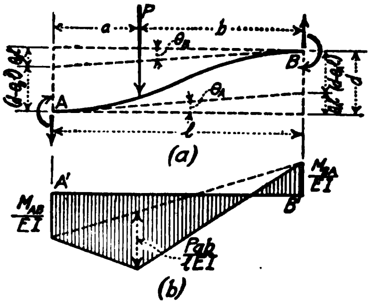


Fig. 2.

If the transverse load on the member is symmetrical, then equation (2) reduces to

$$M_{AB} = 2EK(2\theta_A + \theta_B - 3R) - \frac{F}{l} \qquad (3)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A - 3R) + \frac{F}{l} \qquad (3a)$$

in which F is the area of the moment diagram for the given symmetrical loading on a simple span.[1] For different types of symmetrical load the values of $\frac{F}{l}$ are given below:

Values of $\frac{F}{l}$, Moment Factor for Symmetrical Loads*

Loading	Moment diagram	Moment factor $\frac{F}{l}$
$\frac{l}{2}$	$\frac{Pl}{4}$	$\frac{Pl}{8}$
P, P; a, b, a; l	$\frac{P}{a}$	$\frac{Pa}{l}(l-a)$
P, P; $\frac{l}{3}$, $\frac{l}{3}$, $\frac{l}{3}$	$\frac{Pl}{3}$	$\frac{2}{9}Pl$
P, P, P; a, b, b, a; l	$P(a+\frac{b}{2})$; $\frac{3}{2}Pa$	$\frac{P}{2l}(3a^2+6ab+b^2)$
P, P, P; $\frac{l}{4}$, $\frac{l}{4}$, $\frac{l}{4}$, $\frac{l}{4}$	$\frac{Pl}{2}$; $\frac{3}{8}Pl$	$\frac{5}{16}Pl$
w lb. per ft.; l	$\frac{wl^2}{8}$	$\frac{wl^2}{12}$
a, b, a; w lb. per ft.	$\frac{wab}{2}$	$\frac{wb}{12l}[6a(b+a)+b^2]$
a, b, a; w lb per ft.	$\frac{wa^2}{2}$	$\frac{wa^2}{6l}(3l-2a)$

* Prepared from a table by F. E. Richart, Master's Thesis, 1916, Univ. of Ill.

It should be noted that in equations (2) and (3) the sign of the last term is negative. In equations (2a) and (3a) the sign of the last term is positive. A general rule results:

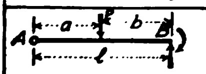
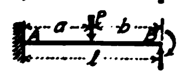
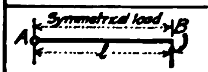
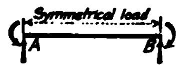
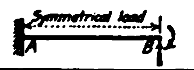
[1] The last term of equations (2) and (3) is the moment at the end of a fixed beam loaded similarly to member AB.

If the load, independent of the member, tends to rotate around the joint under consideration in a *clockwise* direction, the sign of the last term of equations (2) and (3) is *negative*. If the load, independent of the member, tends to rotate around the joint under consideration in a *counter-clockwise* direction, the sign of the last term of equations (2) and (3) is *positive*, as in equations (2*a*) and (2*b*).

By comparison of equations (1) to the succeeding ones, the last term in equations (2) and (3) are noted to be "load terms"; that is, they are the modifying factors of the equations due to the presence of the loads.

A number of special equations arise from those in (1), (2) and (3) when various end restraints are imposed upon the member. These, for convenience, have been tabulated below. The above statement concerning signs applies here also.

Special Moment Equations (R=0)

Member and loading	θ	M_{AB}	M_{BA}	Eq No.
	$2\theta_A = -\theta_B$	Zero	$3EK\theta_B$	1b
	$\theta_A = -\theta_B$	$2EK\theta_B$	$2EK\theta_B$	1c
	$\theta_A = 0$	$2EK\theta_B$	$4EK\theta_B$	1d
	$\theta_A = \theta_B$	$6EK\theta_B$	$6EK\theta_B$	1e
		Zero	$3EK\theta_B + \frac{Pab}{2l^2}(l+a)$	2b
	$\theta_A = 0$	$2EK\theta_B - \frac{Pab^2}{l^2}$	$4EK\theta_B + \frac{Pa^2b}{l^2}$	2c
		Zero	$3EK\theta_B + \frac{3}{2}\frac{F}{l}$	3b
	$\theta_A = -\theta_B$	$2EK\theta_A - \frac{F}{l}$	$2EK\theta_B + \frac{F}{l}$	3c
	$\theta_A = 0$	$2EK\theta_B - \frac{F}{l}$	$4EK\theta_B + \frac{F}{l}$	3d

3. Application of Method of Analysis to Simple Cases.—1. *Let it be desired to determine the moment at the central support of the beam shown in Fig. 3.*

Referring to equation 2*b* in the above table of special moment equations,

$$M_{BA} = 3EK_1\theta_B + \frac{Pab}{2l_1^2}(l_1 + a)$$

$$K_1 = \frac{I_1}{l_1} \qquad K_2 = \frac{I_2}{l_2}$$

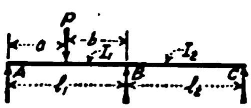


FIG. 3.

and from equation 1*b*,

$$M_{BC} = 3EK_2\theta_B$$

Since the summation of moments around the joint B must equal zero for equilibrium,

$$M_{BA} + M_{BC} = 0$$

$$(3EK_1 + 3EK_2)\theta_B + \frac{Pab}{2l_1^2}(l_1 + a) = 0$$

$$\theta_B = -\frac{1}{3EK_1 + 3EK_2}\left[\frac{Pab}{2l_1^2}(l_1 + a)\right]$$

whence

$$M_{BC} = -\frac{K_2}{K_1 + K_2}\left[\frac{Pab}{2l_1^2}(l_1 + a)\right] \text{ or } M_{BA} = \frac{K_2}{K_1 + K_2}\left[\frac{Pab}{2l_1^2}(l_1 + a)\right]$$

either of which equations gives tension at the top of the beam over the support B.

If $K_1 = K_2$, and $l_1 = l_2$,

$$M_{BC} = -M_{BA} = -\frac{Pab}{4l_1^2}(l_1 + a)$$

2. *Let it be desired to find the moment at the top of the lower column in the frame shown in Fig. 4. Let the values of K for the column be the same, and let K for the girders, also, be alike.*

From equation 3*d* in the above table,

$$M_{BA} = 4EK_1\theta_B + \frac{F}{l} \tag{a}$$

From equation 1*d*,

$$M_{BD} = M_{BE} = 4EK_2\theta_B$$

and

$$M_{BC} = 4EK_1\theta_B$$

Since the sum of the moments around the joint B must equal zero,

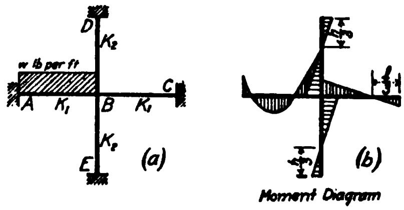


Fig. 4.

$$M_{BA} + M_{BC} + M_{BD} + M_{BE} = 0$$

$$(8EK_1 + 8EK_2)\theta_B + \frac{F}{l} = 0$$

$$\theta_B = -\frac{1}{8EK_1 + 8EK_2}\cdot\frac{F}{l} = -\frac{1}{8EK_1 + 8EK_2}\cdot\frac{wl^2}{12}$$

Substituting into the above equations there results

$$M_{BA} = \left(\frac{K_1 + 2K_2}{2K_1 + 2K_2}\right)\frac{wl^2}{12}$$

$$M_{BD} = M_{BE} = -\left(\frac{K_2}{2K_1 + 2K_2}\right)\frac{wl^2}{12}$$

$$M_{BC} = -\left(\frac{K_1}{2K_1 + 2K_2}\right)\frac{wl^2}{12}$$

If $K_2 = 0$, that is, a simple support replaces the columns at B,

$$M_{BA} = -M_{BC} = \frac{wl^2}{24}$$

which is the moment at the central support of a continuous girder with fixed ends loaded on one span with a uniform load.

4. Conception of Rigidity of Building Frames.—Suppose a point in space to be held to some infinitely rigid body by several elastic members, each of which is fixed at the end farthest from the point in question. Thus, in Fig. 5, the point A is the rigid junction of the members. The members 1 to 5 are fixed at points a, b, c, d and e, and are rigidly connected to each other at A. Let AB be a loaded member, tending to turn point A in a clockwise direction. If the members 1 to 5 are infinitely rigid or any one of them is infinitely rigid, the slope of the tangent to AB at A will remain in a fixed position throughout loading, and AB would be said to be fixed at A. Again, if the members 1 to 5 were not at all rigid, A would turn and there would be no bending restraint upon AB, through the members meeting at point A. If there is no load acting upon members 1 to 5, there are two limiting conditions of rigidity at the joint A—namely: (1) zero restraint and (2) fixity, due,

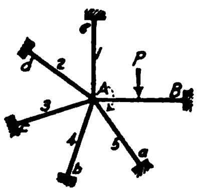


Fig. 5.

respectively, to whether the members 1 to 5 have zero stiffness, or have infinite stiffness. It is evident, since the sum of the effects of each of the members is the total effect of all of the members in restraining the rotation of point A, that the effect of all together may be thought of as the effect of but one member capable of giving the same restraint.

In a building frame of reinforced concrete, any beam is restrained a certain amount at each end, due to the rigid connection existing between it and the columns above and below, and the girder beyond. The restraint causes negative moments in the ends of the beam, and in turn the load on the beam, in causing a tendency for a rotation of A, causes flexural stresses in the restraining members.

Fig. 6 shows a uniform load over one span in a building frame, and the deformations caused. The members radiating from A and B are restrained at their outer ends to an extent varying between a hinged and a fixed condition. If loads were put on aA and Bd, the deflections in the columns at A and B would be practically removed, and AB would be practically fixed. Suppose, however, that instead of this the spans beyond a and d were loaded. This would add to the deflections shown. Further deformation could be obtained by loading alternate panels in all directions.

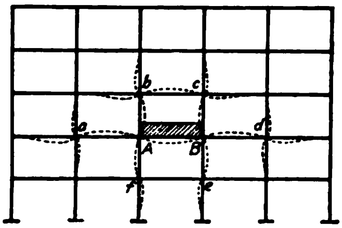


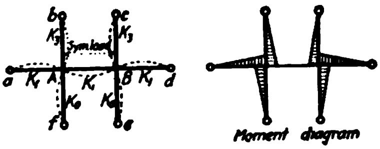
Fig. 6.

A different effect could be obtained by loading the panels above and below AB. This would develop a point of contraflexure in the column, but would give a constant moment in the unloaded girders. By loading alternate bays the maximum effect of this kind would be obtained.

It will be possible to remove the portion $abcdef$ of the frame, and impose upon the extremities of the members such moments as will imitate any condition of full or partial loading. It will be assumed in these cases that the three girders have equal K's; that the lower columns have equal K's; and that the upper columns have equal K's. This will be recognized as a common condition; and though cases may arise which are different, this case will aid the judgment of the designer.

5. Moments at Interior Columns in Beam-and-girder Construction.

5a. All Terminals Hinged (Case I).—From the fact that the frame is symmetrically loaded and is symmetrically rigid $\theta_A = -\theta_B$; hence, from equation 3c,



$$M_{AB} = 2EK_1\theta_A - \frac{F}{l}$$

From equation 1b,

$$M_{Af} = 3EK_0\theta_A$$
$$M_{Aa} = 3EK_1\theta_A$$
$$M_{Ab} = 3EK_2\theta_A$$

But for equilibrium,

$$M_{Af} + M_{Aa} + M_{Ab} + M_{AB} = 0$$

Substituting and solving,

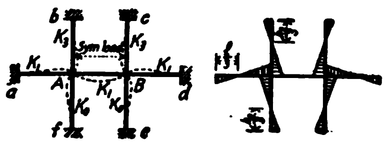
$$\theta_A = \frac{F}{l} \cdot \frac{1}{3K_0 + 5K_1 + 3K_2}$$

Substituting this value into the above moment equations,

$$M_{AB} = -\frac{F}{l}\left[\frac{3(K_0 + K_1 + K_2)}{3K_0 + 5K_1 + 3K_2}\right] \tag{Ia}$$

$$M_{Af} = \frac{F}{l}\left[\frac{3K_0}{3K_0 + 5K_1 + 3K_2}\right] \tag{Ib}$$

5b. All Terminals Fixed (Case II).—As in the preceding development,



$$M_{AB} = 2EK_1\theta_A - \frac{F}{l}$$

From equation 1*d*,

$$\begin{aligned} M_{Af} &= 4EK_0\theta_A \\ M_{Aa} &= 4EK_1\theta_A \\ M_{Ab} &= 4EK_2\theta_A \end{aligned}$$

Since

$$M_{Af} + M_{Aa} + M_{Ab} + M_{AB} = 0$$

it is found that

$$\theta_A = \frac{F}{l} \cdot \frac{1}{4K_0 + 6K_1 + 4K_2}$$

By substitution,

$$M_{AB} = -\frac{F}{l}\left[\frac{2(K_0 + K_1 + K_2)}{2K_0 + 3K_1 + 2K_2}\right] \tag{IIa}$$

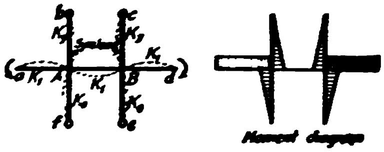
$$M_{Af} = \frac{F}{l}\left[\frac{2K_0}{2K_0 + 3K_1 + 2K_2}\right] \tag{IIb}$$

Equation (II*a*) could have been derived from (I*a*) by replacing the $3K$ of each outstanding member by $4K$, as would be clear from a study of equations 1*b* and 1*d*.

From these two cases, Diagram 1 on page 421 was prepared. Instead of plotting the values of K directly, the ratio of each K to K_1 was plotted. Thus $K_0' = \frac{K_0}{K_1}$ and $K_2' = \frac{K_2}{K_1}$. The diagram shows clearly the small difference in the negative moment at the end of the loaded girder between hinged and fixed terminals. When $K_2' = 0$ there are no upper columns, as is the case in a viaduct bent, or a one-story deck structure. When $K_2' = K_0' = 0$, the girder becomes one on simple intermediate supports.

If a structure of two stories in height were extended indefinitely in either direction and alternate spans were loaded, Cases I and II would be modified by giving the two outside girders a constant moment over their length, and having their terminals neither fixed nor hinged. Expressions for M_{AB} and M_{Af} for each case follow.

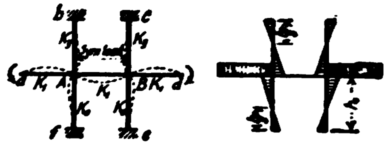
5c. Columns Hinged. Outer Girders with Constant Moment (Case III).—



$$M_{AB} = -\frac{F}{l}\left[\frac{3K_0 + 2K_1 + 3K_2}{3K_0 + 4K_1 + 3K_2}\right] \qquad \text{(IIIa)}$$

$$M_{Af} = -\frac{F}{l}\left[\frac{3K_0}{3K_0 + 4K_1 + 3K_2}\right] \qquad \text{(IIIb)}$$

5d. Columns Fixed. Outer Girders with Constant Moment (Case IV).—



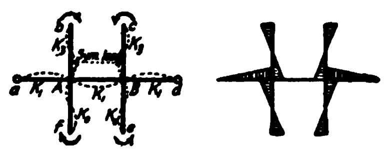
$$M_{AB} = -\frac{F}{l}\left[\frac{2K_0 + K_1 + 2K_2}{2K_0 + 2K_1 + 2K_2}\right] \qquad \text{(IVa)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{K_0}{K_0 + K_1 + K_2}\right] \qquad \text{(IVb)}$$

Equations (IIIa) and (IVa) are plotted on Diagram 2. It indicates, as before, the small difference in the value of M_{AB} between fixed and hinged columns.

A third condition which will develop high stresses in the columns, and which will give a high degree of rigidity to joints A and B, is a case of loading in which alternate bays are loaded, and all the spans of one bay for the full height of the structure are loaded. When all panel loads are the same, or when the loads are such as to give the same rotation at each joint, the point of inflection will be at the center of the column. The girders may have their extremities either hinged or fixed, or the outer girders may have constant moments over their length. Each of these conditions is treated in a following case.

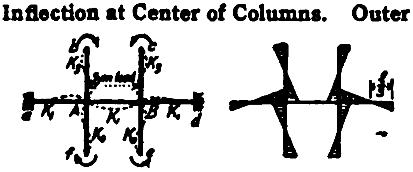
5e. Point of Inflection at Center of Columns. Outer Girders Hinged (Case V).—



$$M_{AB} = -\frac{F}{l}\left[\frac{6K_0 + 3K_1 + 6K_2}{6K_0 + 5K_1 + 6K_2}\right] \qquad \text{(Va)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{6K_0}{6K_0 + 5K_1 + 6K_2}\right] \qquad \text{(Vb)}$$

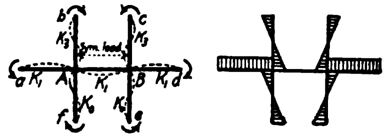
5f. Point of Inflection at Center of Columns. Outer Girders Fixed (Case VI).—



$$M_{AB} = -\frac{F}{l}\left[\frac{3K_0 + 2K_1 + 3K_3}{3K_0 + 3K_1 + 3K_3}\right] \qquad \text{(VI}a\text{)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{K_0}{K_0 + K_1 + K_3}\right] \qquad \text{(VI}b\text{)}$$

5*g*. Point of Inflection at Center of Columns. Outer Girders with Constant Moment (Case VII).—



$$M_{AB} = -\frac{F}{l}\left[\frac{3K_0 + K_1 + 3K_3}{3K_0 + 2K_1 + 3K_3}\right] \qquad \text{(VII}a\text{)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{3K_0}{3K_0 + 2K_1 + 3K_3}\right] \qquad \text{(VII}b\text{)}$$

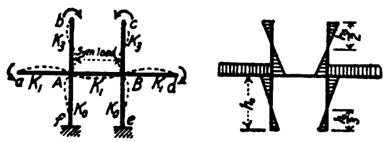
Equations (V*a*), (VI*a*) and (VII*a*) have been plotted in Diagram 3. It should be noted, according to Diagrams 2 and 3, that the effect of the constant moment along the outer girders is to cause a rotation *in addition* to that caused normally by the load; and that this additional rotation causes a decrease in negative moment in the girder AB from what it would have been had the girders been either hinged or fixed at their outer ends. This is consistent with the fundamental conception of rigidity.

Diagram 4 has been plotted to show comparatively the results of the foregoing cases. Since both K_0' and K_3' have the same coefficients in the foregoing expressions for M_{AB}, Diagram 4 is plotted between the moment and the sum of K_0' and K_3'. The resulting curves are identical with those for $K_3' = 0$ in Diagrams 1 to 3.

The variation of moment is for nearly all cases not over 10% of the load factor $\frac{F}{l}$. For values of $(K_0' + K_3') > 5$, the moment is proportional to the value of $(K_0' + K_3')$. The small difference in moments between hinged and fixed terminals is again emphasized. All cases approach a moment $M_{AB} = \frac{F}{l}$ as a maximum limit.

On the first floor there may arise a condition in which the first tier of columns may be hinged or fixed, while the second tier of columns may have a central point of inflection. Since the greatest stress will arise from fixity of the lower tier, rather than from a hinged condition, it will be investigated. The outer girders will be located as in Cases V to VII above.

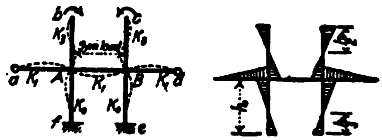
5*h*. Point of Inflection at Center of Upper Columns. Lower Columns Fixed. Outer Girders with Constant Moment (Case VIII).—



$$M_{AB} = -\frac{F}{l}\left[\frac{2K_0 + K_1 + 3K_3}{2K_0 + 2K_1 + 3K_3}\right] \qquad \text{(VIII}a\text{)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{3K_3}{2K_0 + 2K_1 + 3K_3}\right] \qquad \text{(VIII}b\text{)}$$

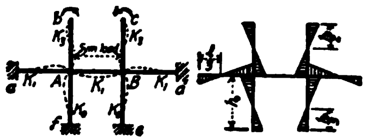
5*i*. Point of Inflection at Center of Upper Columns. Lower Columns Fixed. Outer Girders Hinged (Case IX).—



$$M_{AB} = -\frac{F}{l}\left[\frac{4K_0 + 3K_1 + 6K_2}{4K_0 + 5K_1 + 6K_2}\right] \quad \text{(IXa)}$$

$$M_{Af} = \frac{F}{l}\left[\frac{6K_2}{4K_0 + 5K_1 + 6K_2}\right] \quad \text{(IXb)}$$

5j. Point of Inflection at Center of Upper Columns. Lower Columns Fixed. Outer Girders Fixed (Case X).—



$$M_{AB} = -\frac{F}{l}\left[\frac{2K_0 + 2K_1 + 3K_2}{2K_0 + 3K_1 + 3K_2}\right] \quad \text{(Xa)}$$

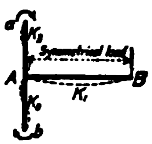
$$M_{Af} = \frac{F}{l}\left[\frac{3K_2}{2K_0 + 3K_1 + 3K_2}\right] \quad \text{(Xb)}$$

The moment M_{AB} as given by each of the above three cases is plotted in Diagram 5. There is little difference between the moments on this diagram and those on Diagram 3, for corresponding K_2'.

A review of the foregoing ten cases shows that the negative moments in the loaded girders cannot in any way exceed those for a continuous girder whose ends are similarly restrained. It is justifiable, therefore, to design the girders as continuous, so far as negative moments are concerned; and for either positive or negative moments the recommendations of the Joint Committee (see Art. 45, Sect. 7) are entirely adequate as a basis of computation.

The effect on the columns of unbalanced floor load is shown for each of the ten cases on Diagrams 6 to 9 inclusive. The greatest moment in the interior columns is found at either end of the second-tier columns when the first tier has fixed bases (Case VIII, Diagram 9). The value of this moment is shown on Diagram 10. Very rarely will it exceed 60% of $\frac{F}{l}$, which for uniform loads gives $0.05\,wl^2$.

6. Moments at Exterior Columns in Beam-and-girder Construction.—Since exterior columns must resist all the moment in the end of the joining beam, the moments will be liable to be somewhat higher than in interior columns, where the girder beyond takes its share of the moment.



It has already been shown (page 418) that when moments are applied at a and b such that points of inflection are caused at the mid-height of the columns, the columns more firmly retard the rotation of the joint A. The moment, therefore, is greater in the columns when the moments at a and b are thus applied. The condition of restraint of B may vary from

DIAGRAM 1

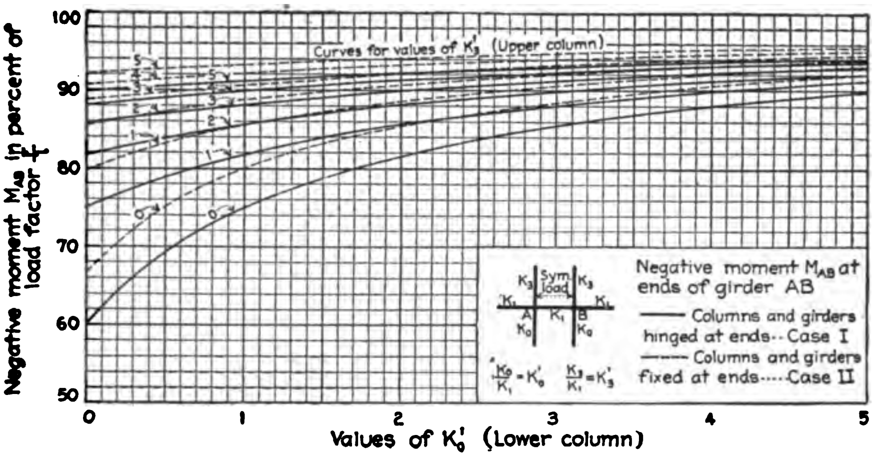


DIAGRAM 2

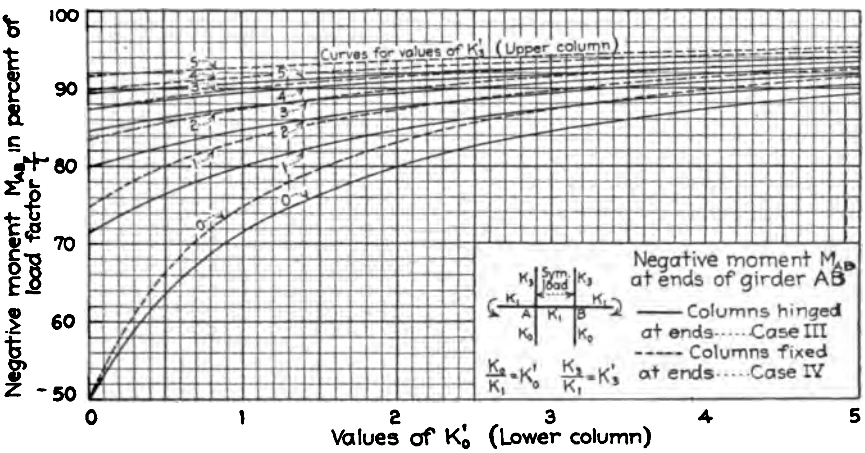


DIAGRAM 3

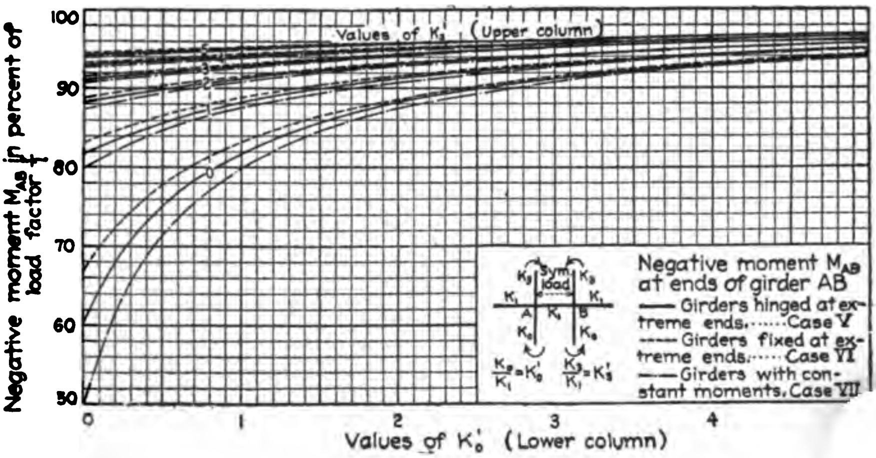


DIAGRAM 4

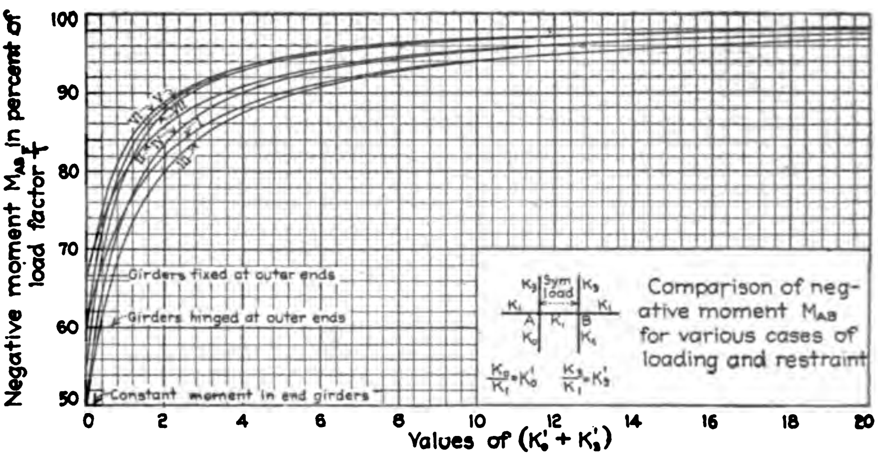


DIAGRAM 5

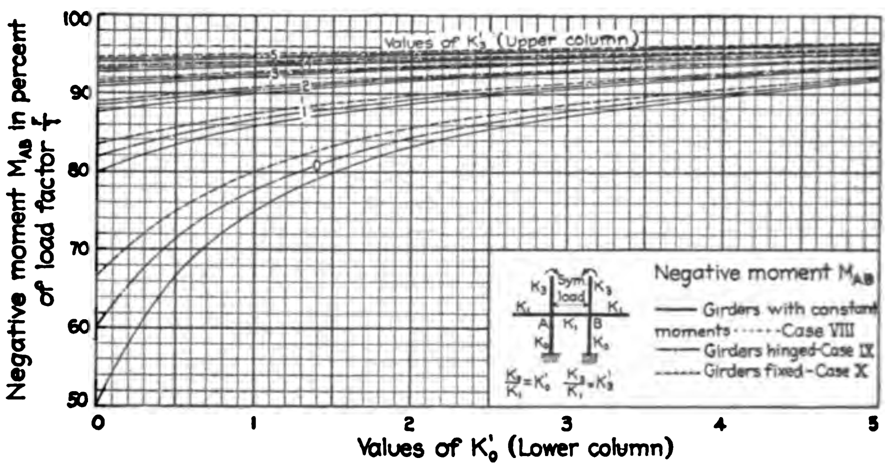


DIAGRAM 6

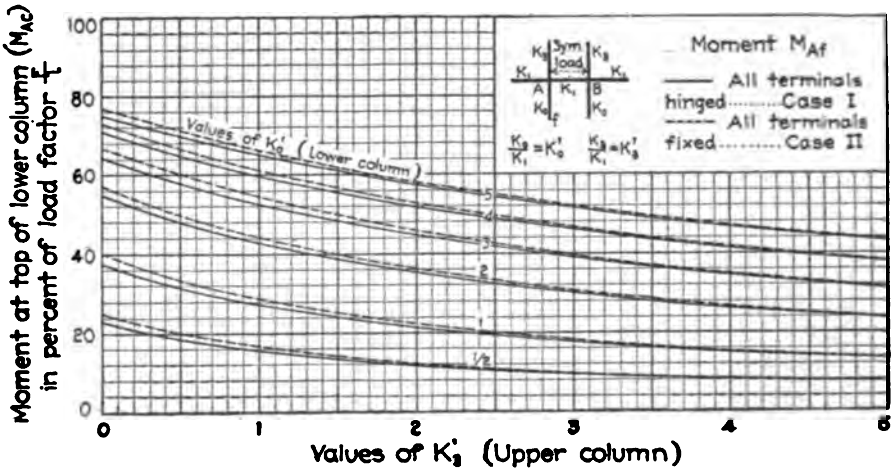


DIAGRAM 7

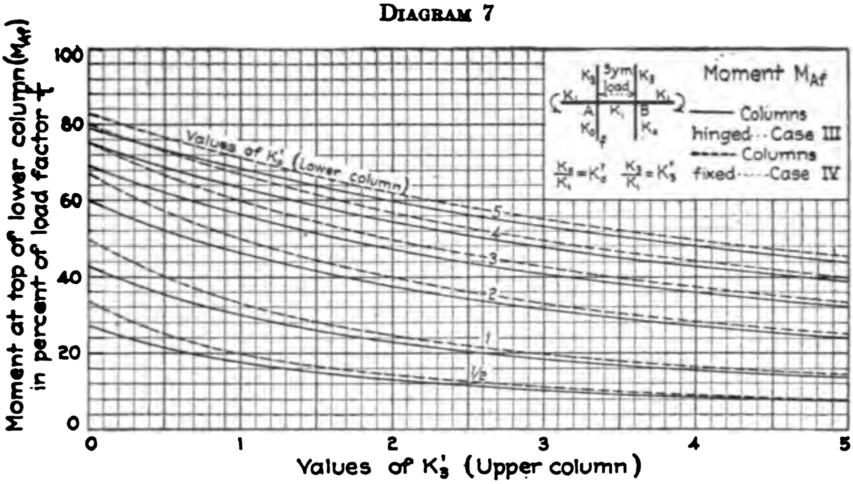


DIAGRAM 8

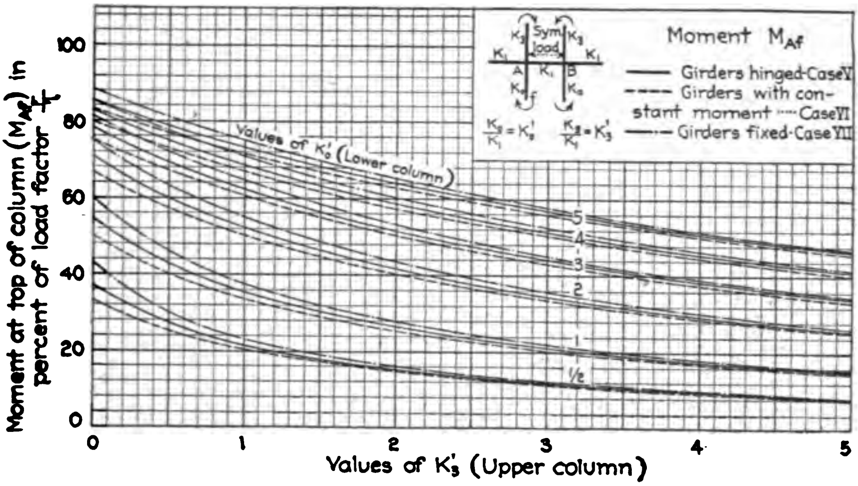


DIAGRAM 9

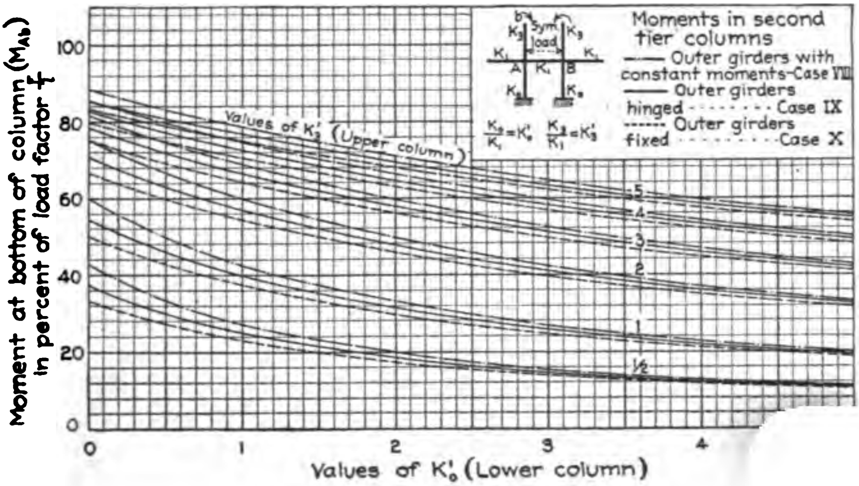


DIAGRAM 10

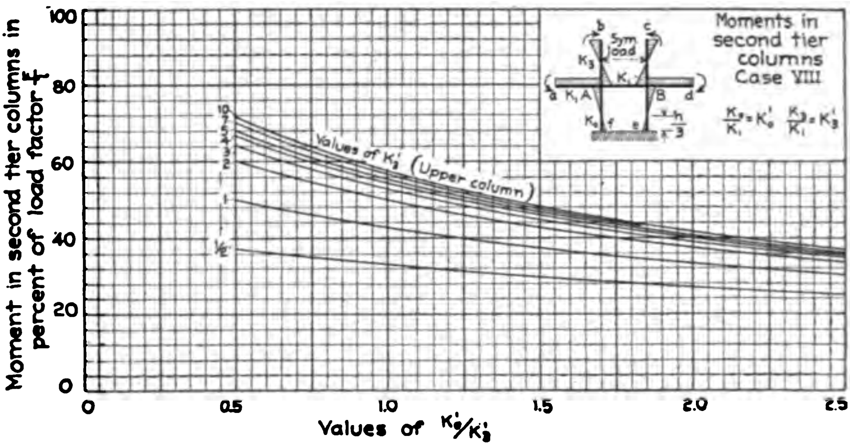
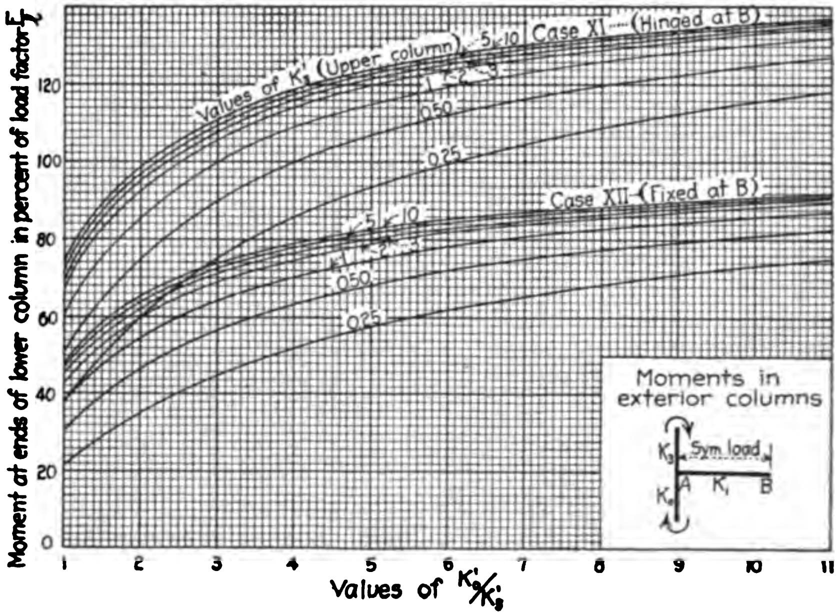


DIAGRAM 11



fixity to a hinged support. It will be assumed that A and B do not receive translation during the loading of the frame. The following moments at the head of the lower column result:

6a. Inner End of Girder Hinged (Case XI).—

$$M_{Ab} = \frac{F}{l}\left[\frac{3K_0}{2K_0 + K_1 + 2K_3}\right] \qquad \text{(XI)}$$

6b. Inner End of Girder Fixed (Case XII).—

$$M_{Ab} = \frac{F}{l}\left[\frac{3K_0}{3K_0 + 2K_1 + 3K_3}\right] \qquad \text{(XII)}$$

Diagram 11 was plotted from these two cases. Case XI gives an increase in moment of approximately 40% over Case XII, and this shows the greatly undesirable result of putting an expansion joint in the girder system only one span from the exterior columns. If the girders are continuous past the second column from the end, few cases will arise in which the moment at the head of the lower exterior column (M_{Ab}) will exceed 60% of the load factor $\left(\frac{F}{l}\right)$, which for a uniform load over AB gives 0.05 wl^2.

7. Moments in Columns in Flat-slab Construction.—In flat-slab construction, it is necessary to estimate the stiffness of the floor before it is possible to compute the moments at the heads of the column. This may be done by replacing the floor slab by an "equivalent girder." Just what portion of the floor slab may be considered as a girder resisting deformation in the columns depends upon the type of reinforcing employed.

Consider a flat-slab structure in which four-way reinforcing has been employed. If one column is submitted to a bending moment, the rigidity of the joint at its end depends upon the rigidity of the column beyond and upon the stiffness of the floor slab. The moment taken by the flat slab may be taken partially by the diagonal system of reinforcement to columns lying diagonally across the panel from the column in question, and partly by the rectangular system to columns in the same bent as the column in question. If this same structure is loaded on its floor in alternate panels, the moment caused in the slab will be carried to the columns by the two systems of reinforcement.

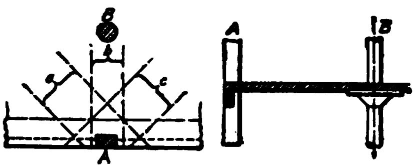


Fig. 7.

Consider now an exterior column as shown in Fig. 7. Let two adjacent spans be loaded with a uniform load. Moment is brought to the column by the three bands of reinforcement a, b, and c. As the structure is loaded symmetrically to the right and to the left of band b, the moments brought by the bands a and c will be equal. The proportion of moment carried by bands a and c to that carried by band b depends upon the rigidity of the two systems of paths.

Suppose there exist no diagonal bands a and c. The reinforcement would now follow a two-way system and reinforcing for the central portions of the panel would cross over the bands of reinforcing between columns. If in this system the load is applied symmetrically with respect to the band b, the moment of the load carried to A would reach A largely through the beam action of the bands of reinforcing extending between columns. It is here assumed that the band b has a width equal approximately to the width of the depressed head.

Whereas, in the case of the four-way reinforcement a considerable amount of moment is brought by bands a, b and c to column A, which in the two-way reinforcement would have reached the column A through the band b and through the band along the outer wall, it will be assumed for purposes of estimation that the amount of moment brought to A may be determined by assuming a beam action of the rectangular reinforcement.

It will be considered first that the stiffness between columns A and B is furnished by a strip of floor slab the width of band b. Later a discussion concerning the error involved in this assumption will be undertaken.

The moment brought to the head of the column will be estimated by considering a load over the area between A and B having the width of one panel. The frame in sectional view appears on page 420. The moment M_{AB} is given under Case XII. It will be sufficient in computing the value of I_1 for K_1 to consider only the concrete area, neglecting the cross-section of the reinforcement. In the columns, however, the reinforcement should be considered in computing the moment of inertia. Having solved for the moment by formula (XII), the necessary flexural reinforcement at the ends of the column may be computed. The moments in the various bands in the slab are found in the computations on flat slab floors (see Art. 20, Sect. 11).

Concerning the portion of the slab between columns A and B, it has been assumed above as having a width equal to the width of the band b. It should be noted that the moment of inertia I_1 of the cross-section of this strip varies directly with its width. Hence K_1 also varies directly with its width. An examination of formula (XII) indicates that considering the values K_0 and K_3 as constants, the moment is decreased by an increase in K_1, that is to say, by an increase of the width of the strip. Just how extensive this variation may be can readily be determined from Diagram 11.

8. Criteria for Maximum Combined Stresses in Columns.—Maximum stresses in columns may occur from two sources: first, from a maximum bending moment and direct stress; and, secondly, from a maximum deflection of the column together with direct stress. Maximum deflection of the column, and maximum moment in the column occurring from moment introduced into the column from the girder, do not occur simultaneously. Although it is not always true, it is usually the case that the maximum moment, together with the combined stress, will give the greatest fiber stress in the column.

It may be necessary to design certain columns for a given static load, or it may be necessary to investigate columns for special loading in a given panel or panels. For that reason the following criteria are given.

8a. Interior Columns.—Maximum moments are caused by loading the bay adjacent to the column in question for the full height of the structure, and, where possible, by loading alternate bays in both directions from that bay. The maximum moment will be found in the second-tier columns. The maximum stress in these columns will be found by combining the stress caused by this maximum moment with the axial stress produced by the load on the floors above the first floor.

Maximum deflections may be caused in the interior columns by loading spans alternate in all directions. In the case of very slender columns, the deflection may add a moment caused by the direct load to that moment causing the deflection. The stress thus produced, when combined with the dead load and such live load as is transmitted to the column in question will produce the maximum fiber stress for that case of loading.

8b. Exterior Columns.—Loading to produce the maximum stress due to moment in the exterior columns is the same as that for interior columns. Likewise the loading causing the maximum deflection in the exterior column is applied in the same manner as the loading causing the maximum deflection in the interior columns. Exterior columns, however, will receive more moment than interior columns because of the fact that the columns are not assisted by a girder beyond in restraining the moment generated. Whatever negative moment there is in the end of the girder in the exterior bay, it must be balanced by a moment in the exterior column above and below that girder. The stresses due to this moment combined with the stresses due to axial loading will cause the maximum fiber stress in the column.

A very important consideration in the exterior columns is the moment caused in the corner columns. When the floor is constructed of slabs and beams, the moment may be considered as being introduced into the column by the two girders meeting at right angles to each other, and these moments may then be combined to give a diagonal resulting moment. The column in such construction is usually square, and there will, therefore, be a change of axes for this resultant bending. When the floor is a flat slab, moments are brought into the corner column from three sources (in four-way reinforcing systems), from the lintel beams and from the diagonal band of reinforcing that crosses the corner panel. It has been found from tests on buildings that this diagonal band carries extremely high stresses in the reinforcing and that these high stresses introduce into the corner columns very large moments. An estimate of the moment in the corner column can be made by considering an equivalent beam action (see page 425) to replace this diagonal band. The negative moment at the extremity of this equivalent beam may be computed and this moment may then be combined with the resultant moment of the moments at the ends of the lintel beams. It should be here noted that the length of the equivalent girder is the length of the diagonal across the panel. If the corner column is not round, the transfer of axes is necessary before computing maximum fiber stress.

9. Wind Stresses in Building Frames.—It is often necessary to determine the stresses set up in a large building frame by wind pressure on some exposed face of the building. Exact methods[1] of the analysis of these stresses are very long and tedious, and some of them are so laborious that it is impracticable if not impossible to apply them to a building of any considerable height.

An exhaustive study of stresses due to wind pressures in buildings has been made in *Bulletin* 80 of the Engineering Experiment Station of the University of Illinois by Prof. W. M. Wilson and G. A. Maney. In this bulletin a number of approximate methods have been studied with a view to the determination of their accuracy, and a comparison of exact methods has been made to determine their applicability, after which the writers of the bulletin present an original procedure for exact analysis by slope-deflections.

Tall building frames which are not symmetrical about a vertical center line or which have a considerable variation in the sizes of adjacent members should be analyzed by some one of the exact methods, of which that of slope-deflections is probably the most usable.

For bents of a given building frame having practically equal spans and equal column sections, an approximate method based on the following assumptions has been found to give quite accurate results.[2]

1. Points of inflection in columns are located at their mid-height.
2. The point of inflection of each girder is at its mid-length.
3. The shear on each interior column is equal, and the shear on each exterior column is equal to one-half the shear on any interior column.

The wind is considered as being applied at each floor level. If W represents the total wind force applied to the structure above a given floor, then according to assumption (3), the shearing force W is distributed among the several columns just above this floor in such a manner that each interior column takes an equal amount and each exterior column takes half of the amount taken by an interior column. Thus if n is the number of panels, $\frac{W}{n}$ is the amount taken by each interior column, and $\frac{W}{2n}$ is the amount taken by each exterior column.

[1] The reader is referred to the following articles, any one of which gives an exact solution of the problem:

"Wind Stresses in the Frames of Office Buildings," by Albert Smith, *Journal Western Society of Engineers*, vol. 20, No. 4, p. 341.

"Stresses in Tall Buildings," by Cyrus A. Melick, *Bull.* 8, College of Engineering, University of Ohio.

"The Theory of Frameworks with Rectangular Panels and its Application to Buildings Which Have to Resist Wind," by Ernst F. Johnson, *Trans.* Am. Soc. C. E., vol. 55, p. 413.

"Wind Stresses in the Steel Frames of Office Buildings," *Bull.* 80, Engineering Experiment Station, University of Illinois.

[2] "Wind Stresses in Frames of Office Buildings," by Albert Smith of Purdue University, *Journal of Western Society of Engineers*, vol. 20, No. 4, p. 341.

Axial stresses occur only in the exterior columns and are equal. They may be found by dividing the moment of the total wind pressure about a given floor level by the length of the building in the direction of the wind pressure. These axial stresses are not important except in very tall, narrow, buildings. The direct stress in a column due to wind should be combined with the stress due to vertical loading.

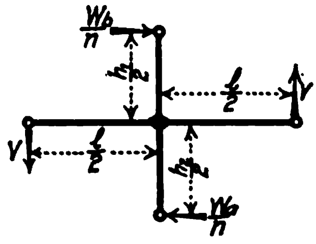


FIG. 8.

Since, according to assumptions 1 and 2 above, points of inflection occur at the center of each column and at the center of each girder, a single joint may be removed as a free body as in Fig. 8. The moments in the members about the joint may readily be found from such a figure.

The couple caused by the shears on the columns must be resisted by the shears on the girders. The girders are assumed as having the same length. Then

$$V = \frac{W_a h_1 + W_b h_2}{2ln} \qquad (1)$$

assuming a and b to refer to two adjacent stories.

The moment at the base of the upper column is equal to $\frac{W_b h_1}{2n}$; and that at the top of the lower column, $\frac{W_a h_2}{2n}$. The moment at the end of either girder due to the wind is $\frac{Vl}{2}$.

For an exterior panel (Fig. 9) the value of V is given by equation (1). The moment at the base of the upper column is equal to $\frac{W_b h_1}{4n}$; at the top of the lower column, $\frac{W_a h_2}{4n}$; and at the end of the girder, $\frac{Vl}{2}$.

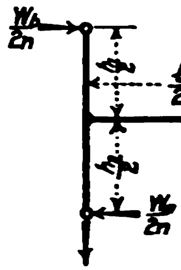
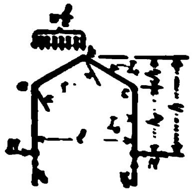


FIG. 9.

10. Roof Frames.—It would be impractical to give here a complete set of formulas for the large range of possible roof frames. It will be of use, however, to give some of the more common types of frames.

Type I.—*Case I.*

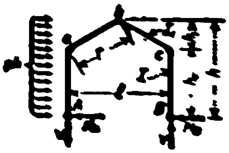


$$R = \frac{r}{h_0} \qquad S = \frac{I_1}{I_0}$$

$$V_A = \tfrac{3}{8}wl \qquad V_B = \tfrac{1}{8}wl$$

$$H = \frac{wl^2}{64}\,\frac{R(3h_0 + 5h)}{h_0^2 S + R(3h_0 h + h_1^2)}$$

Case II.

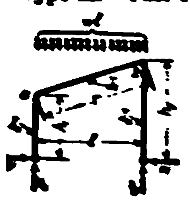


$$V_A = V_B = \frac{p}{2l}(h_0^2 + h_1(h + h_0))$$

$$H_B = \frac{p}{16}\left[\frac{5h_0^2h_1(S + 3R) + 5h_1^2R(3h_0 + h) + 5h_0^2hS + 6h_0^2R(h_0 + h)}{h_0^2S + R(3h_0h + h_1^2)}\right]$$

$$H_A = ph - H_B$$

Type II.—*Case I.*

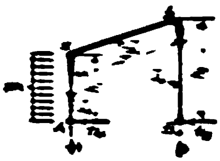


$$\frac{h_2}{h_1} = k \qquad \frac{r}{h_1} = R \qquad \frac{I_1}{I_0} = S$$

$$V_A = V_B = \frac{wl}{2}$$

$$H = \frac{wl^2}{8h_1}\left[\frac{R(1 + k)}{S(1 + k^3) + R(1 + k + k^2)}\right]$$

Case II.

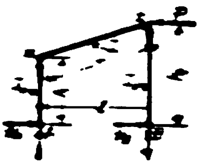


$$V_A = V_B = \frac{wh_1^2}{2l}$$

$$H_B = \frac{wh_1}{8}\left[\frac{3S - R(4 - 2k)}{S(1 - k^3) - R(1 - k - k^2)}\right]$$

$$H_A = wh - H_B$$

Case III.

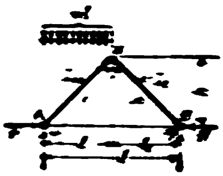


$$V = \frac{Ph_2}{l}$$

$$H_A = \frac{Pk}{2}\left[\frac{2k^2S - R(1 - 2k)}{S(1 - k^3) - R(1 - k - k^2)}\right]$$

$$H_B = P - H_A$$

Type III.—*Case I.*



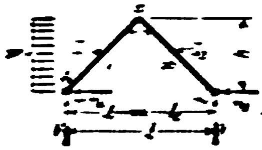
$$R = \frac{l}{s} \qquad S = \frac{I_2}{I_1}$$

$$V_A = \frac{wl}{2}(l - l_2)$$

$$V_B = \frac{wl_2^2}{2l}$$

$$H = \frac{wl^2}{8h}\cdot\frac{4l_2R - S(l - 4l_2)}{R - S}$$

Case II.

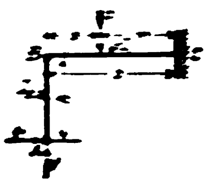


$$V_A = V_B = \frac{wh^2}{2l}$$

$$H_B = \frac{wh}{8}\cdot\frac{4l_2R - S(l - 5l_2)}{R - S}$$

11. L-frames.—The L-frame may occur either with a fixed or hinged column base; or with a fixed or hinged girder-end. In the four cases which follow, the girder-end is fixed and the column base hinged. The frame may, with its loading, be revolved through 90 deg. to suit the reverse of the cases shown. Frames of this nature which are fixed at both extremities seldom occur in practice as a separate structure, and hence are not given here.

Case I.



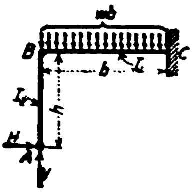
$$R = \frac{h}{l} \qquad S = \frac{I_2}{I_1}$$

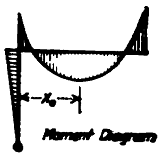
$$H_A = \frac{3Pam^2}{Rh^2(3R - 4S)}$$

$$V_A = \frac{2S(a - 2b) - 3R(l - 2a)}{3R - 4S}$$

$$M_B = -H_Ah \quad M_C = -H_Ah - V_Al - Pm \quad M_D = -H_Ah - V_Aa$$

Case II.





Moment Diagram

$$R = \frac{b}{h}$$

$$S = \frac{I_1}{I_2}$$

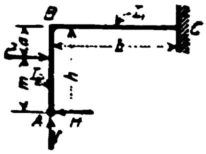
$$H_A = \frac{wh}{4} \cdot \frac{R^3}{3R + 4S}$$

$$V_A = \frac{3wb}{2} \frac{(R + S)}{(3R + 4S)}$$

$$M_B = -H_A h \qquad M_c = -H_A + V_A b - \frac{w[illegible]}{2}$$

$$X_0 = \frac{V_A}{w} \qquad M_x\text{, (max.)} = \frac{V_A^2}{2w} - H_A [illegible]$$

Case III.

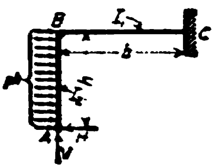


$$H_A = \frac{3Pa[2h^2S - aS(2b + m)]}{Rh^3(3R + 4S)}$$

$$V_A = \frac{2P}{h^3} \frac{[18Rh^2 + a^2S(2b + m)]}{(3R + 4S)}$$

$$M_p = H_A m \qquad M_B = H_A h - Pa \qquad M_c = M_B + V_A b$$

Case IV.



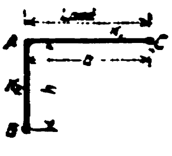
$$H_A = \frac{3ph(R + S)}{2(3R + 4S)}$$

$$V_A = \frac{3ph}{4R} \cdot \frac{S}{(3R + 4S)}$$

$$M_B = H_A h - \frac{ph^2}{2} \qquad M_c = M_B + V_A b$$

$$y_0 = \frac{H_A}{p} \qquad M_y\text{, max.} = \frac{H_A^2}{2p}$$

A very common form of the L-frame is the one which is hinged at both extremities. It is not uncommon for the beam to have some slope other than horizontal. Both cases [illegible] briefly:



$$M_{AC} = 3EK_1\theta_A - N$$

$$M_{AB} = 3EK_2\theta_A$$

$$M_{AB} = -M_{AC}$$

$$\theta_A = \frac{M_{AB}}{3EK_2}$$

$$M_{AC} = \frac{M_{AB}}{K_2} \cdot K_1 - N = -M_{AC}\frac{K_1}{K_2} - N$$

$$M_{AC}\left(\frac{K_1 + K_2}{K_2}\right) = -N$$

$$M_{AC} = -\frac{NK_2}{K_1 + K_2}$$

Suppose the "load" to be a concentrated load P, placed a distance a from C. Then $N = \frac{Pab}{2l^2}(l + a)$. If the "load" is a symmetrically placed load with respect to the member [illegible] $N = \frac{F}{l}$, the moment at the end of a fixed beam which carries that particular loading (see page 413 for values of $\frac{F}{l}$ for various loads).

When the beam AC is not horizontal, but slopes upward to the end [illegible] the solution [illegible] frame may be obtained by letting [illegible] in Type II [illegible] Rigid Frames [illegible] become [illegible]

SECTION 11

BUILDINGS

FLOORS—GENERAL DATA

1. General Types of Concrete Floors.—There are four general types of concrete floors: (1) monolithic beam and girder construction, (2) flat-slab construction, (3) unit construction and (4) steel-frame construction with concrete slabs. In the fourth type mentioned, the beams and girders are usually covered with concrete for fire protection.

2. Floor Loads.—The following extract from the Seattle Building Code illustrates good practice:

All floors shall be constructed to bear a safe live load per superficial square foot of not less than the following amounts.

	Pounds
Public buildings	100
Detention buildings, in cells or wards	60
Churches, chapels, theatres, assembly halls or court rooms with permanent seats	80
Lobbies, passageways, corridors and stairways of the same	100
Assembly halls with movable seats	100
Halls used for dancing, or roller skating	150
Lobbies, passageways, corridors and stairways of the same	100
Stables	80
Dwellings, apartment houses, flat buildings and lodging houses	50
Class rooms in schools	60
Assembly rooms in schools	80
Office buildings and hotels, ground floor	125
For floors above the ground floor	75
Store buildings for light merchandise, ground floor	125
For floors above the ground floor	100
Store buildings for heavy merchandise, such as grocery stores or hardware stores	150
Warehouses	200
Factories and workshops, when the nature of the work is general	125
Machine shops, armories, drill rooms and riding schools	250

Floors in a building to be used for the sale, storage or manufacture of heavy machinery, shall be proportioned to the load they may have to carry.

In addition to specifying minimum floor loadings for which the various types of buildings must be designed, the Rochester Building Code requires also the following:

The weight placed on the floors of any building, now or hereafter constructed, shall be safely distributed thereon. The bureau may require the owner or occupant of any building or portion thereof to redistribute the load on any floor, or to lighten such load when deemed necessary, even if not greater than the minimum in this section prescribed.

To prevent overloading in all warehouses, storehouses, factories, workshops and stores, now or hereafter constructed, where heavy materials are kept or stored, or machinery introduced, the weight that each floor will safely sustain upon each square foot thereof, or upon each varying part of such floor, shall be estimated by a competent person employed by the owner or occupant, or the bureau may make such estimate, and said estimate shall be placed permanently on a stone or metal tablet in a conspicuous place in the hallway of each story or varying parts of each story of the building to which it relates.

No person shall place or permit to be placed on the floor of any building, now or hereafter constructed, any greater load than the safe load thereof as correctly estimated and ascertained as herein provided.

The working stresses usually employed, and those recommended by the Joint Committee are intended to apply to static loads only. Proper allowance for the dynamic effect of the live load should be taken into account by adding the desired amount to the live load to produce an equivalent static load before applying the unit stresses in proportioning parts. An allowance for impact will be necessary only in special cases, as in the case of floors supporting heavy machinery. The amount to add to the live load because of impact will vary all the way from 25 to 100% depending upon the proportion of the specified live load which may be subject to motion.

The dead load of any floor may be estimated from the following approximate data—weights are per square foot of floor surface:

Wooden wearing surfaces, 4 to 6 lb.	Plaster, 5 lb.
Screeds or nailing strips, 2 lb.	Suspended ceiling, 10 lb.
Cinder-concrete filling (2 in. thick), 15 lb.	Cinders, 7 lb.
Hollow tile, see Art. 12.	

3. Economic Considerations.—The size of floor bays depends upon the loading, the uses to which the building is to be put, and the size and shape of the ground area. In a large building it is frequently worth while to make several comparative estimates with different layouts of the beams, girders and columns, so as to obtain the most economical arrangement under the given conditions.

The cost of changing forms to meet changes in size of beams and columns in different stories must be kept constantly in mind. It is more economical to vary the depths of beams from floor to floor than the widths, on account of the slab panels; and to vary square columns in one dimension rather than in two, both on account of the columns themselves and the beams which frame into them. It is not advisable to change the size of members from floor to floor for small differences in computed dimensions; nor is it advisable on any floor, if simplicity in field work is desired, to make slight changes in the sizes of beams and in the thicknesses and reinforcement of the slabs, unless such variations occur over large areas. When slabs of considerable variation in span alternate, it is better to vary the thickness and keep the reinforcement the same, than to vary the reinforcement and use the same thickness of slab. Although slight differences in dimensions are not desirable, the designs made should be considered carefully in every detail.

4. Floor Surfaces.—A concrete floor usually has a mortar or granolithic finish as wearing surface. Such a surface if not allowed to set rapidly is hard and practically impervious to water.

The usual proportions for granolithic finish are 1 part Portland cement, 1 part sand, and 1 part crushed stone which passes through a ¼-in. mesh screen. This mortar surfacing is laid over the concrete slab and troweled to a hard finish. If placed before the concrete below has set, it may be from ¾ to 1 in. thick but where the concrete of the slab becomes old before the granolithic finish is laid, the thickness should be at least 2 in.

Granolithic finish is screeded to grade with a straight-edge, smoothed with a wooden float, and finished with a steel trowel. It is often marked off into blocks, or sections, of suitable size by shallow grooves. The object in dividing the surface into panels is purely ornamental, since reinforcement against shrinkage cracks is provided in the lower portion of the concrete slab.

For a complete treatment of concrete-floor surfaces, see Sect. 4.

Some engineers prefer to lay a cinder-concrete base, not less than 2 in. thick, before placing the cement finish. The advantage of this method is that should there be any reason for removing a section of the finished floor, it can be accomplished without injury to the reinforced-concrete slab. On account of the porosity of the cinder concrete it is difficult, however, to obtain a satisfactory cement finish if the same is placed before the cinder concrete base has set.

If a wood floor is desired, 2 by 3-in. or 2 by 4-in. sleepers are usually laid on top of the rough concrete slab, and cinder concrete or stone concrete poor in cement is run between these nailing strips. For ordinary cases, ⅞ to 1¼-in. maple flooring nailed to 2 by 3-in. sleepers, 16 in. apart, is found satisfactory. The proportions of cinder concrete specified vary considerably—average proportions are 1 : 3 : 6. The sides of sleepers are usually beveled, but this does not prevent them from becoming loose if the wood shrinks. The best method of holding the sleepers in place is to drive 40d. nails in the sides of the sleepers at intervals of 3 ft. on alternate sides. These nails key with the concrete and prevent movement of the sleeper.

There has been much dissatisfaction with the above method of constructing wooden floor surfaces on account of the liability of the sleepers to decay by dry rot. To lessen decay the sleepers should be well seasoned and might advantageously be impregnated with preservative of some sort, and care should be taken to permit no moisture to reach them even after the top flooring is in place. Sufficient time should be allowed the concrete to dry out before the finished flooring is laid. Dry rot is a fungus disease of wood that thrives in the presence of moisture and absence of circulating air. It makes little difference in result whether the moisture is the sap of the green wood, or water that soaks into the wood after it has been dried.

Sleepers are sometimes laid in dry cinder fill. In such construction the chance of dry rot is probably small, since there is some opportunity for air circulation through the porous cinders. Cinders also tend to absorb moisture.

A sand-and-tar base for plank flooring on reinforced-concrete floor slabs has been employed to some extent to avoid any danger from dry rot. A layer of sand mixed with coal tar is spread about 1¼ in. thick on the floor slab and leveled while still warm and soft with a straight-edge. On this layer is then placed first a layer of 2-in. plank, then ⅞-in. rough pine board, and finally a wearing surface of ⅞ to 1¼-in. maple. The different layers of planking should be placed in different directions.

The following construction has been used in schoolhouse floors with satisfactory results: At the completion of the floor slab, a 1-in. footing of sand is placed on top of the same and brought to a true level by screeding. On top of the sand are placed 1¼-in. boards laid diagonally and nailed together at the edges. The form lumber used for centering is generally employed for this work. On top of the 1¼-in. boards is then placed building paper, which in turn is covered by the finished flooring.

So-called "nailable concretes" have been attempted but without substantial success. They are, in effect, poor concretes, made from cinders and sand with a small amount of asbestos or like substance, into which flooring nails may be driven. Their prolonged retention of water, generation of acid from wet cinders and faulty hold on nails has prevented their extensive use.

In hotels and similar establishments, linoleum or carpeting over a fairly smooth cement base seems to give satisfaction. If the floor is very wet and alkaline, saponification may take place, with destruction of the linoleum. A thoroughly good cement is insurance against such occurrences. Sodium silicate should not be used, as it liberates free sodium.

Flat tiles are sometimes employed as a floor surface in reinforced-concrete construction. The most durable and sanitary tile is the vitreous-clay tile. The *ceramic* tile is perhaps the most often used and is a vitreous-clay tile manufactured in square, hexagonal, and round shapes. The squares range in size from ½ to ¾ in. and the hexagonal shapes from ¾ to 1 in. Tiles are laid some little time after the floor slab is poured and, if set in Portland-cement mortar, should be embedded in a layer not less than 2 to 2½ in. thick in order to prevent *curling* of the tile due to shrinkage of the base. There are now on the market, however, some patented bases which seem satisfactory and which will allow a bedding thickness of only 1 in. over and above the thickness of the floor slab proper.

5. Small Floor Openings.—The methods of arranging slab reinforcement around small openings, such as for chimneys and ventilators, needs some consideration. Both a wrong and a right method are shown in Fig. 1. The openings are shown against a wall, and the floor slab is reinforced in only one direction.

The method shown of placing the reinforcing rods parallel to the opening makes a neat looking job, but it should be evident that no proper provision is necessarily made to carry the load which would ordinarily come on the wall at the openings. The right method shown is less artistic, but, as a general rule, is vastly more efficient. The cross rods serve only to reinforce the slab locally. All such designs should be carefully studied to make sure that the required strength is secured at all points. Slab rods should never be curved horizontally to dodge floor openings as a curved rod will tend to rotate.

Wrought-iron and galvanized-iron sleeves are built into the construction work for all steam, return, sprinkler, sewer, gas, and similar pipes. All floor sleeves should be flush with the ceiling line and should extend about 2 in. above the floor line. Pipe-risers should not be allowed to come up through columns as repairs and alterations are difficult, if not impossible, under such an arrangement; electric conduits, however, form an exception to this rule. Special shafts with fireproof walls are sometimes used for plumbing and vent pipes, and this practice has much to commend it since a floor to be a perfect fire cutoff should be solid from wall to wall, with stairways, elevators, and all openings enclosed in vertical fireproof walls.

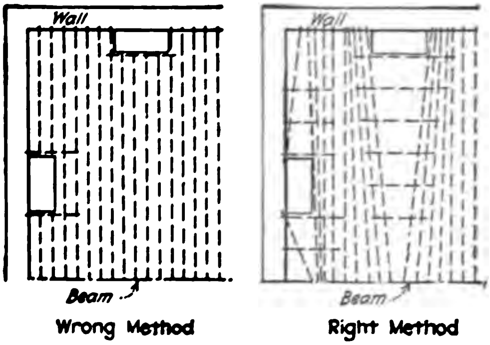


Fig. 1.—Method of placing reinforcement around small floor openings.

6. Provision for the Attachment of Shaft-hangers and Sprinkler Pipes.—There are many different methods of attaching shafting, sprinkler pipes, and machinery to the under side of a concrete floor. The method adopted should be as flexible as possible in order to accommodate future changes in the line of shafting, additions, or improved machinery. All bolts and sockets should be placed before the concrete is run, as drilling is expensive and reinforcing bars are liable to be encountered, which would cause more or less difficulty, and might lead to serious trouble if cut off.

A convenient method is to place suitable bolts at intervals, usually 4 ft. on centers, with their threaded ends projecting from the concrete. By means of these projecting bolts, timbers may be fastened wherever desired and the shaft-hangers lag-screwed in place.

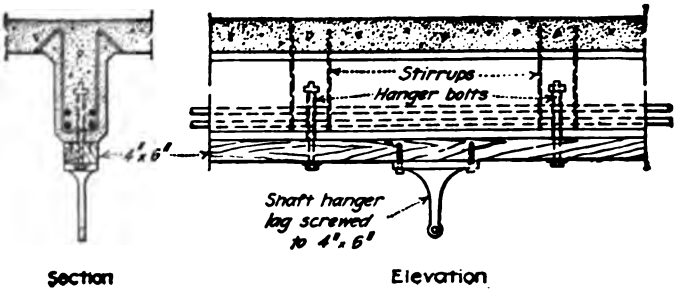


Fig. 2.

The heads of bolts projecting into concrete should be enlarged or bent so that the bolt will not tear out. If desired, a washer or plate may be employed for this purpose underneath the head of the bolt. Figs 2 and 3 illustrate the use of large washer nuts. These are temporarily held in position by a thin iron tube resting on the centering and by a bolt projecting through the hole in the bottom of the beam box. The washer nut is tightened down to the top of the tube which secures it firmly in an upright position (see detail shown in Fig. 4). When the beam boxes or centering are removed, the bolts are easily unscrewed from the nut, leaving a clear passage through the concrete to the nut above. The stirrups shown are to prevent excessive deformation in the beam. By the method shown in Fig. 3, the shaft-hangers may be securely fastened by bolts.

When projecting bolts are used, it is seldom that all are made use of, and those not used present an unsightly appearance. This is the main objection to the scheme shown in Fig. 5. Saddles and check nuts serve to hold the long bolts in position.

The Unit socket is shown in Fig. 6. This device serves also as a support and spacer for the beam rods, keeping them all properly spaced from each other and from the beam box or centering.

A form of adjustable socket for use in beams, principally, is shown in Fig. 7. These castings are made in convenient lengths and the slot in the bottom makes it possible to place hangers

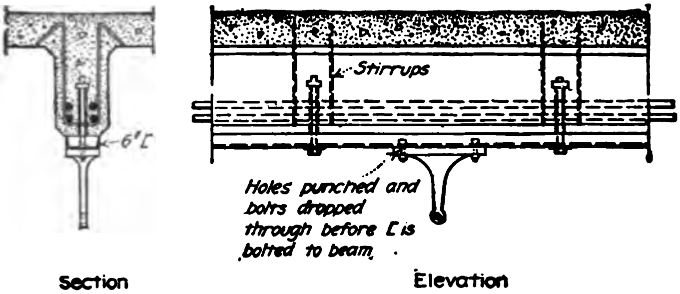


Fig. 3.

or bolts at any desired location along the length of the insert. The casting can be anchored as securely in the concrete as may be necessary by the use of stirrups passed through the open spaces.

Figs. 8 and 9 illustrate two other methods used for attaching shaft-hangers to beams. Fig. 8 is what is known as a pipe-slot hanger. Fig. 9 can be made a strong and serviceable support for motors and machinery.

The schemes shown in Figs. 10 and 11 have been used quite extensively by the Turner Construction Co. of New York City.

Fig. 12 shows a hanger socket mainly for use in slabs. The casting varies in length with the depth of the slab and is made smaller at the tapped end than at the top, so there will be no possibility of the binding of the tap-screw when screwed in. The cross-pin shown passes transversely through a cored hole in the upper end of the casting. Fig. 13 shows a somewhat similar type of socket. A bolt through a hole in the form secures it in position during concreting. Figs. 14 and 15 show two other methods used for slabs between beams.

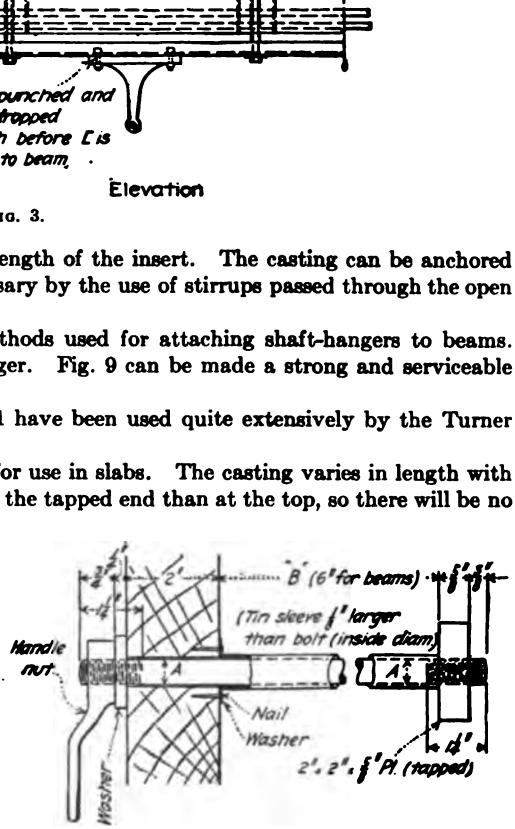


Fig. 4.—Detail of socket and bolt.

A flexible method of attachment is shown in Fig. 16—a method used in the shops of the United Shoe Machinery Co., Beverly, Mass. The anchor bolts were spaced 3 ft. on centers in all transverse floor and roof girders. A transverse line of bolts alternately spaced 1 ft. and 6 ft. centers was also built in the middle of each floor and roof panel. In the girder arrangement, the nuts on the lower ends of the anchor bolts engage cast-iron saddles which clamp against pairs of angles with wood fillers. Fig. 16 shows how the "T"-headed attaching bolts may move freely along the slot formed by the angles. In the floor panels, the bolts have simple heads or nuts upon the upper ends which bear upon flat washers. In thin slabs the heads bear upon washer plates in the upper surface of the slab (Fig. 16).

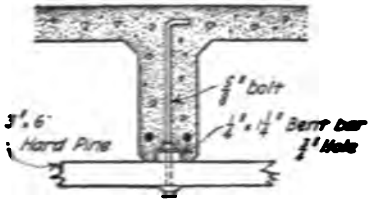


Fig. 5.

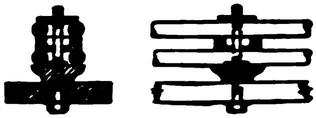


Fig. 6.

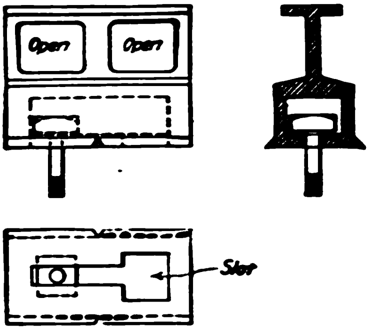


Fig. 7.

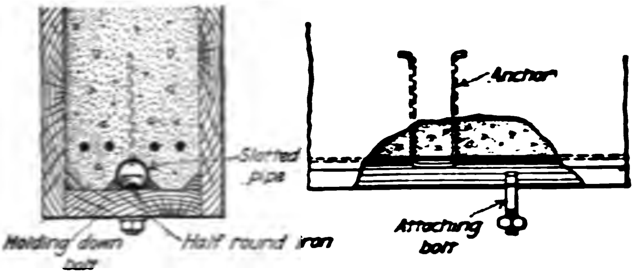


Fig. 8.

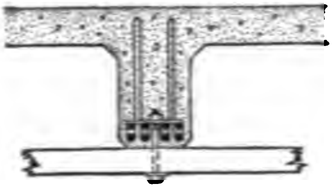


Fig. 9.

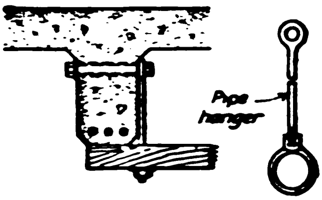


Fig. 10.

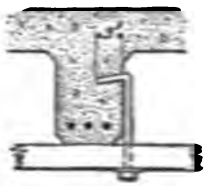


Fig. 11.

It is not good practice to depend upon reinforcing rods for direct support of shaft-hangers, as thereby too great a strain is placed locally on the rods and the concrete under them will become loosened from the continuous pull and vibration of the shafting.

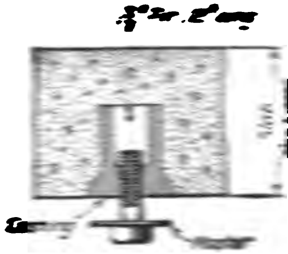


FIG. 12

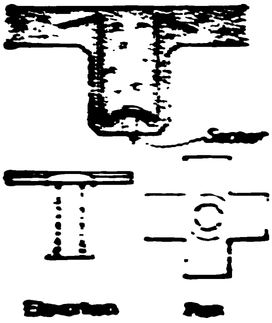


FIG. 13

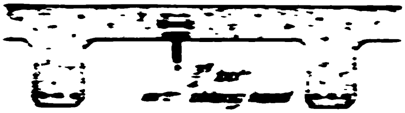


FIG. 14

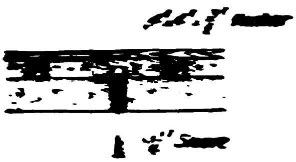


FIG. 15

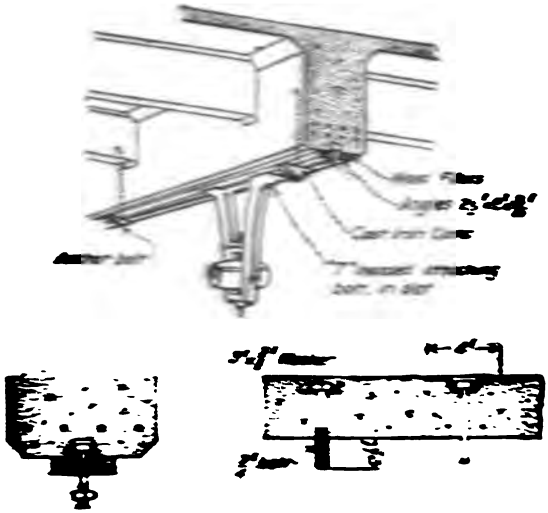


FIG. 16

7. **Bedding Machinery.**—Machines can readily be connected to the concrete by drilling small holes in the floor at the proper points and setting lag-screws or through-bolts. If through-bolts are required, the holes should be located before the floor is concreted. A permanent level base may be obtained by leveling the machine an inch or two above the foundation proper and grouting it in place. A dam of sand is first built around the machine. Then the grout,

in proportions 1 part cement to 1 or 2 parts of sand mixed to the consistency of thick cream, is poured so as to run under the casting. As the mortar sets it is rammed with a rod to prevent shrinkage.

In most cases machinery runs better and lasts longer if placed on solid foundations, but there are exceptional cases where machines seem to require more or less *give* on their bases, as in certain textile mills. In such cases a wood-finished floor can be laid or wooden timbers fastened to the concrete floor. Sheet cork and linoleum carpet have also been used for this purpose.

8. Waterproof Floors.—In case of fire, a water-tight floor in factory buildings prevents damage from water to the machinery or materials in the stories below. According to *Insurance Engineering*, the damage from water after a fire in fireproof factory buildings is much greater than that from the fire itself. A concrete floor with granolithic surface is practically impervious to water but unless a floor is made self-draining, water will get down through the floor openings. Raised sills should be provided around all openings, whether or not a cement finish is used, and scuppers of ample size should be placed in the outside walls to carry water from the floors to the outside of the building. Fig. 17 is a detail of wall scuppers used in the Liberty Silk warehouses, New York City. The waterproofing of floors may be accomplished by using an impervious finished flooring or by using an undercoating of asphalt felt laid in hot asphalt.

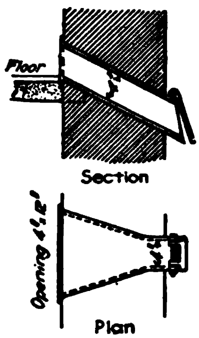


Fig. 17.

9. Tests.—Within the last few years full-sized floor panels of both the monolithic beam and girder and the flat-slab types, have been tested for the elastic deformation of the concrete and the steel under increasing loads. The tests consisted in loading, with an increasing load, a number of consecutive panels of a reinforced-concrete building, and of observing the exact contraction or expansion of certain portions of the steel and of the concrete by means of delicate instruments. In order to make such observations, the concrete was removed from the steel in given places, and drill holes were made in the steel thus uncovered. Into these holes the extensometer points were placed. Small holes were also drilled in the concrete for the same purpose. From the elastic deformations obtained for moduli of elasticity of the steel and concrete, the stresses in the members were computed and the results compared with those for which the building was designed. Much valuable data pertaining to proper methods for design have been obtained from these tests.

10. Basement Floors.—A basement floor in dry ground is usually made of 1 : 3 : 5 concrete, 3 or 4 in. thick. Usually no wearing surface is needed other than the ordinary concrete troweled to a hard finish, but, where considerable wear is expected, the usual mortar coat may be laid as on the upper floors. To prevent shrinkage cracks, the floor should be divided into blocks about 8 or 10 ft. square. This may be accomplished by laying alternate blocks, and then filling in the intermediate ones after the adjoining concrete has set. Basement floors are constructed in the same manner as concrete sidewalks (see Sect. 4).

It is not safe to depend upon the concrete itself being water-tight. If the basement is below tide-water or ground-water level, a layer of waterproofing consisting of three to six layers of waterproof felt (cemented together and to the concrete by coal-tar pitch or asphalt) should be spread on the concrete and carried up in continuous sheets on the walls to above water level. The whole surface should then be covered with another layer of concrete at least 3 or 4 in. thick.

The earth under the basement floor should be well drained, and drains of tile pipe, or of screened gravel and stone, may be placed in trenches just below the floor. Sometimes it is necessary to cover the entire area with cinders or stone; and sometimes the concrete must be made extra thick, or reinforcement added, to resist the upward pressure of the water.

MONOLITHIC BEAM-AND-GIRDER CONSTRUCTION

11. Ordinary Type of Beam-and-girder Construction.—The theory and methods of design of slabs, beams, and girders are given in Sect. 7. If a floor slab is reinforced in one direction only, the load will practically all be transmitted to the beams at right angles to the direction of the reinforcing rods. A small part, however, will be transferred directly to the girders at the sides of the panels, but this may well be neglected in the calculations for cross-beams. In fact, even with reinforcement in two directions, the load should be assumed as all transferred to the cross-beams unless the panel is nearly square.

If panels, nearly square, are reinforced in both directions, the loads carried to the cross-beams and girders will not be uniformly distributed over the length of such beams and girders, but may be assumed to vary in accordance with the ordinates of a triangle. This assumption is surely on the safe side in regard to moment, if the area of the triangle is made equal to that part of the total load on the panel which is transmitted to the beam in question—as determined by the formula of Art. 29c, Sect. 7. Assumptions of this load being either uniformly distributed, or varying as the ordinates of a parabola, give a lower resulting moment than the triangle method.

Let w be the uniform load per unit of area on the slab, and w_2 and w_1 the parts of this unit load that go to the shorter and longer beams respectively. Applying the loads in the form of a triangle having its apex at the middle of the beam, the maximum moment will be for the longer beam, and, if this beam is considered simply supported and as carrying the load from one panel only,

$$M = \tfrac{1}{12}w_1bl^2$$

and for the shorter beam

$$M = \tfrac{1}{12}w_2b^2l$$

If the slab is square, w_1 is $\frac{w}{2}$, and $M = \frac{1}{24}wl^3$. For beams made continuous, the bending moment may be multiplied by the coefficient $\frac{2}{3}$ or $\frac{3}{4}$ as the case may be. With beams or girders common to two panels, the bending moment should be multiplied by 2.

Unless the panel is nearly square, floor slabs should not be reinforced in two directions, as it is evident that no economy results from double reinforcement when the ratio of length to breadth of panel is greater than about 1.2. If the length of the slab exceeds 1.5 times its breadth, the entire load should surely be carried by the transverse reinforcement.

When the floor surface is given a granolithic finish, this finish under certain conditions may be considered to act with the slab proper in taking the stresses under loading. Usually, however, the designer has no way of finding out whether the finish will be placed at the same time as the concrete, or run a number of hours later, and this alone should call for caution on the part of the designer in figuring the finish as a part of the slab. If the superintendent on the job is known as a careful man and if there is to be very careful supervision, the floor may be sometimes figured in this way, but never without putting an underscored note on the drawing to the effect that the finish shall be run immediately after the pouring of the concrete slab. Also the surface of the floor should be blocked off only along the center line of columns and no joint should be made between column lines, as such joints would affect the needed strength of the slab.

Where care in construction is not assured, or where any appreciable wear on the floor is expected, the finish should properly not be included in the effective slab thickness. It is also advisable not to figure this way for a winter job under any conditions.

It is possible, by taking great precautions, to bond a wearing surface to a concrete slab after the concrete in the slab has set. This requires special treatment including thorough cleaning and soaking of the old concrete, providing a bond layer of neat cement mortar, and placing the surface before this neat cement has begun to harden. Poor joints occur quite frequently, however, and it is advisable not to figure the finish as a part of the slab in a case of this kind, even though considerable care is assured at the time of construction.

If it is deemed advisable in any given design to consider the finish as an integral part of the slab, the maximum fiber stress in compression may be determined by the method outlined in Fig. 18. The distance b is made equal to the maximum allowable stress on the ordinary concrete, and the distance a which represents the maximum stress on the cement finish is then computed on this ratio. The later stress, however, should not be in excess of the maximum allowable stress for cement finish.

Fig. 19 shows an arrangement for slab steel such that there is the same steel area at the top of slab over the cross-beams as at the bottom of the slab midway between these beams. All rods are bent and are identical in shape—namely, straight at one end, bent up at the center over a beam, and bent over a beam at the other end. The required arrangement is effected by

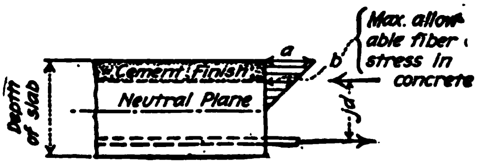


Fig. 18.

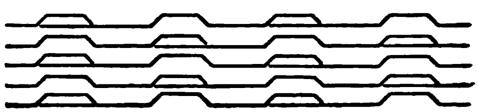


Fig. 19.

shifting alternate rods 7 ft. ahead. Some designers bend up all the rods over supports, but a better method is to continue some steel at the bottom of slab and thus make sure that no point in tension is unprovided with steel. The rods arranged as in Fig. 19 may be carried over three spans and still obtain the same amount of steel over supports as in the center of span, but the amount of steel at the bottom of slab near the supporting beams becomes very small.

Fig. 20 shows another arrangement for the slab steel. Both straight and bent-up rods are employed, each rod extending over three slab spans. The joints in the bent rods occur over supports and the steel is lapped a sufficient distance to provide adequate bond strength.

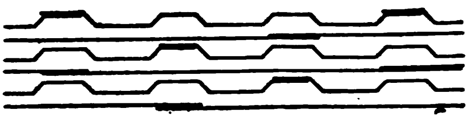


Fig. 20.

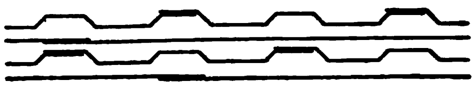


Fig. 21.

This lapping is so arranged that two-thirds as much steel occurs over the supports as in the center of span.

Fig. 21 shows an arrangement of steel which gives three-fourths the center-of-span area over supports. The arrangement is similar to that shown in Fig. 20 except that the rods extend over only two spans.

The arrangement shown in Fig. 20 requires the least steel and that shown in Fig. 19 requires the most. Fig. 19 shows undoubtedly the best design for spans over 6 or 7 ft.

For moments in beams and girders due to building acting as a rigid frame, see Sect. 10.

Illustrative Problem.—Design an interior floor bay to support a live load of 250 lb. per sq. ft. with the columns spaced 21 ft. by 21 ft. on centers and with two intermediate beams. Working stresses recommended by the Joint Committee for a 2000-lb. concrete (see *Appendix B*) are to be employed throughout. (Tables and diagrams referred to are those of Sect. 7.)

Since an interior floor bay has been assumed, the bending moment $\frac{wl^2}{12}$ may be employed for the three parts of the floor bay, namely: the slab, the beam, and the girder. Fig. 22 shows the proposed arrangement of beams and girders.

The floor surface will be given a granolithic finish, consisting of a layer of 1 : 2 mortar, 1 in. thick, spread upon the surface of the concrete slab before it has begun to set, and troweled to a hard finish. For simplicity, the weight of finish will be assumed as included in the specified live load of 250 lb. per sq. ft. Of course it should be readily understood that in practice, where a definite live load is required, the finish should be considered separately as a superimposed dead load.

Slab.—The main reinforcement will be placed in the direction AA', Fig. 19, and the span of slab will be 7 ft. Slab is to be fully continuous and its total depth will be taken to the nearest ½ in.

Diagrams 5 and 6, Sect. 7, show that a 4¼-in. (d = 3½-in.) slab with span of 7 ft. will sustain a load (live plus dead) of approximately 325 lb. per sq. ft. Corresponding weight of slab is 56 lb. per sq. ft., and the total load per square foot for the slab to carry is therefore 250 + 56 = 306 lb.

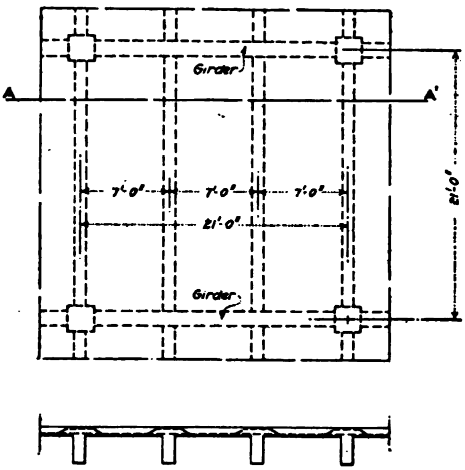
For a slab with d = 3½ in., Diagram 6, Part 1, gives A_s = 0.325 sq. in. Referring to Table 6, it is evident that ⅜-in. round rods spaced 4 in. on centers will give sufficient steel area. If desired, the span of slab may be taken as the clear distance between faces of supports (see Art. 44, Sect. 7). Four round rods ⅜ in. in diameter will be placed transversely in each 7-ft. panel to prevent shrinkage and temperature cracks, and to bind the entire structure together.

Cross-beams (*Eight-rod Design*).—The cross-beams have a span of 21 ft. The beams and slab will be poured at the same time and thoroughly tied together so that a T-beam section may be considered. The distance between beams is 7 ft., and the dead and live loads of the slab per foot length of beam is equal to 7 × 306 = 2140 lb. Assume dead load of the stem of beam at 240 lb. per lin. ft. Then the total loading per foot of length equals 2380 lb. The maximum shear

$$V = \frac{(2380)\,(21)}{2} = 25{,}000 \text{ lb.}$$

and the maximum moment

$$M = \frac{(2380)\,(21)^2\,(12)}{12} = 1{,}050{,}000 \text{ in.-lb.}$$



Cross-section A-A'.
Fig. 22.

(If desired, the span of beam may be taken as the clear distance between faces of supports. See Art. 44., Sect. 7.) The required cross-section of web as determined by shear

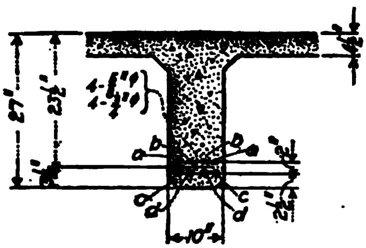
$$\frac{(25{,}000)}{105} = 238 \text{ sq. in.}$$

The following formula of Art. 37, Sect. 7, gives the most economical depths for various assumed web widths:

$$d' = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2}$$

with r in this design as 60, then

for b' = 9 in.	d = 23.1 in.
for b' = 10 in.	d = 22.0 in.
for b' = 11 in.	d = 21.2 in., etc.



Cross-section of Crossbeam
Fig. 23.

In order to provide for most of the diagonal tension by means of bent-up rods, it is proposed to use eight rods placed in two rows, four rods to a row. A value of b' = 10 in. may be satisfactory as regards rod spacing, but $b' \times d$ as given above is not great enough to provide for shear. It will be more economical to deepen the beam of 10-in. width to a depth of 23½ in. than to adopt a width of 11 in. and a depth of about 21.2 in. Fig. 23 shows the arrangement of the steel which will be tried in the bottom of the beam at the center of the span. This exact arrangement will also be tried at the top of beam over supports. It is quite likely that there will be eight rods in the girder of about 1-in. or 1⅛-in. diameter and, since the cross-beam rods should fit nicely with the girder rods over supporting columns, the two layers of rods will be placed 2 in. center to center as shown.

The weight of the stem is $\frac{(10)\,(22.5)\,(150)}{144}$ = 235 lb. per ft. and the weight assumed thus is satisfactory.

The width of the flange of the T-beam is controlled in this design (see Art. 32, Sect. 7) by 12 times the thickness of slab plus the width of stem, or 64 i Then

$$\frac{M}{bd^2} = \frac{1{,}050{,}000}{(64)(23.5)^2} = 29.7$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d} = \frac{4.5}{23.5} = 0.19$, Diagram 8 shows f_c = 300 lb. per sq. in. and j = 0.93. Then

$$A_s = \frac{1{,}050{,}000}{(16{,}000)\,(0.93)\,(23.5)} = 3.00 \text{ sq. in.}$$

Four ¾-in. round rods and four ⅝-in. round rods will be selected (Fig. 23), having a total area of 3.00 sq. in. (see Table 5). Eight 1⅛-in. round rods would do, but the ¾- and ⅝-in. rods are more likely to be found in stock. The four ¾-in. rods will be placed in the lower row and it will be sufficiently accurate to consider the center of gravity of the steel area as midway between the two rows. It is evident now that the width of beam for rod spacing was correctly assumed. (Note the rod spacing recommended by the Joint Committee in Art. 23, Sect. 7.)

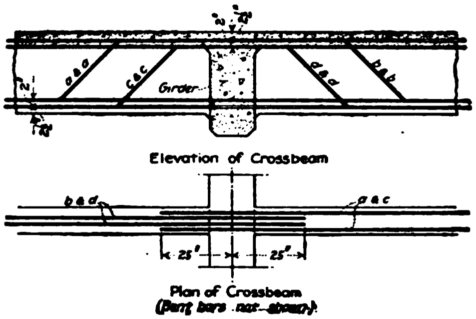


Fig. 24.

Four rods will be bent up and lap over the top of the support. The other four will be continued straight and lap over support at the bottom of beam (see Fig. 24). The bond stress along the eight rods at the top of beam near support

$$u = \frac{25{,}000}{[(4)(2.356) + (4)(1.964)](0.85)(23.5)} = 73 \text{ lb. per sq. in}$$

which is satisfactory.

(The average value of j is taken in the above equation, see Art. 27, Sect. 7.) The proposed arrangement of rods shown in Fig. 24.

The rods at the top of beam over supports will have the same effective depth (d) as the rods at the bottom of beam at the center of span (Figs. 23 and 24). Then

$$\frac{d'}{d} = \frac{3.5}{23.5} = 0.149$$

$$p = \frac{3.00}{(10)\,(23.5)} = 0.0128 = p'$$

The following values may be obtained from Diagram 12:

$$k = 0.384$$
$$j = 0.864$$

Then, using also Table 9,

$$f_s = \frac{1{,}050{,}000}{(3.00)(0.864)(23.5)} = 17{,}200 \text{ lb. per sq. in.}$$

$$f_c = (17{,}200)(0.0415) = 715 \text{ lb. per sq. in.}$$

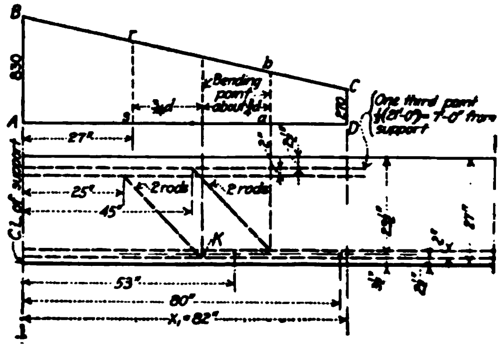


Fig. 25.

Allowable compression in concrete at the support may be 750 lb. per sq. in., and hence no haunch or extra steel is necessary (see *Appendix B*). The tensile stress in the steel is greater than the allowable but will be considered as satisfactory here simply for the purpose of presenting several comparative designs using different numbers of rods. If a little more than the required amount of steel had been selected at the center of span—say eight ¾-in. rounds—the stress in the steel over supports would not have figured out greater than the allowable (see Art. 39, Sect. 7). The diagram given on page 298 may be employed to find the points in the beam where the lower horizontal rods may be bent up. Bending up the first two rods is equivalent to bending up $\frac{(2)(0.307)}{3.00}$ = 0.205 or 20½% of the steel. After bending up the next two rods 50% of all the steel is bent. The diagram above referred to shows that those bends may be made at 0.318l (80 in.) and 0.210l (53 in.) from the center of support.

The rods at the top near support should be bent down as explained in Art. 39, Sect. 7; the first two rods at a distance not less than $\frac{(2)(0.442)}{3.00}$ of $\frac{l}{3}$ = 25 in. from the center of support (assuming zero moment at the third point): and the next two at a distance ½ of $\frac{l}{3}$ = 42 in. from the same point (see Fig. 25).

The distance from the support to the point where web reinforcement is not needed (see Art. 18, Sect. 7):

$$x_1 = \frac{21}{2} - \frac{(40)(10)(0.93)(23.5)}{2380} = 6.86 \text{ ft.} = 82 \text{ in.}$$

Also,

$$BC = \frac{2}{3} \cdot \frac{25{,}000}{(0.85)(23.5)} = 830 \text{ lb.}$$

and

$$CD = \frac{2}{3} \cdot (40)(10) = 270 \text{ lb.}$$

Fig. 25 shows the diagonal-tension trapezoid.

The points to bend rods at the top of beam control the design, as shown in Fig. 25. The horizontal distance between bent rods is about the allowable—namely, about ¾d. Since bending of longitudinal reinforcing bars at an angle across the web of beam may be considered as adding to diagonal tension resistance for a horizontal distance from the point of bending equal to ¾d (see Art. 21, Sect. 7), stirrups will be needed only for the areas $ABbc$ and $abCD$. Diagonal tension represented by the area $abCD$ will be cared for by the additional stirrups which will be inserted to secure good T-beam action. The distance from the center of support to the point where stirrups are not necessary scales 27 in. The stirrups near the end of beam will be looped about the upper rods, and hence will be in an inverted position to those in a simply supported beam.

We shall use ⅜-in. round U-shaped stirrups bent at the ends (see Art. 19, Sect. 7). The minimum spacing of stirrups will occur at the support, and this spacing is given in Diagram IV, Sect. 7, page 288, as 4.2 in. Spacing at other points along the beam may be found readily by means of this diagram. The first stirrup will be placed 2 in. from the edge of girder, assuming the girder to have a less width than the column. In order to secure good T-beam action, the web and flange will be tied together with vertical stirrups placed about 18 in. on centers and looped about the lower rods for the center half of beam.

The bars bent over the support should run to the third point of the adjoining span to provide thoroughly for negative moment, assuming a very definite live load (see Art. 39, Sect. 7). The allowable stress in the compression rods at the support is 715 × 15 = 10,725 lb. per sq. in., and the necessary length for bond of ¾-in. round rods is $\frac{(10{,}725)(0.75)}{(4)(80)} = 25$ in. (see Art. 21, Sect. 7). This length is shown in Fig. 24.

Cross-beams (*Six-rod Design*).—Four ⅞-in. round rods and two ⅝-in. round rods will give the required area of 3.00 sq. in. The arrangement shown in Fig. 26 will be adopted.

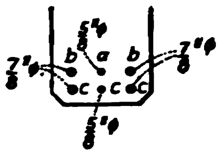


Fig. 26.

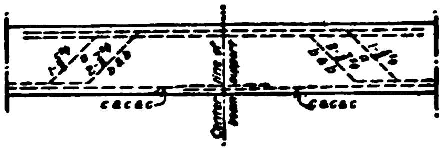


Fig. 27.

The three rods in the upper row will be bent up and lap over supports as shown in Fig. 27. The other three will lap over support at the bottom of beam. The bond stress along the six rods at the top of beam near support

$$u = \frac{25{,}000}{[(4)(2.749) + (2)(1.964)](0.85)(23.5)} = 84 \text{ lb. per sq. in.}$$

which will be considered satisfactory for the arrangement of rods shown in Fig. 27 (see Art. 16, Sect. 7).

The first ⅝-in. rod may be bent at 93 in. from center of support and the next two ⅞-in. rods may be bent at 53 in. from center of support.

The two ⅞-in. rods may be bent down at a distance not less than $\frac{(2)(0.6013)}{3.02}$ of $\frac{l}{3} = 33$ in. from the center of support, and the ⅝-in. rod at ½ of $\frac{l}{3} = 42$ in. from the same point.

The points to bend rods at the top of beam control the location of the bends of the two ⅞-in. rods. The ⅝-in. rod will be bent so as to be about ¾d from the other two rods (see Fig. 28). The spacing of stirrups, and the distance from the center of support to the point where stirrups are not necessary may be found in the same manner as for the eight-rod design.

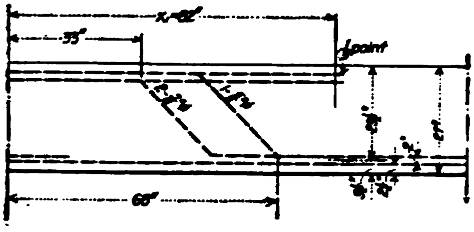


Fig. 28.

Cross-beams (*Four-rod Design*).—Four ⅞-in. square rods will give an area of 3.06 sq. in. which is only slightly more than is required. These rods will all be placed in one row as shown in Fig. 29.[1] Fig. 30 shows the complete

[1] The beam is slightly narrow according to the rod spacing recommended by the Joint Committee.

design. Stirrups are provided to take all the diagonal tension—the strengthening action of the bent rods not being considered. It should be noted that the design is not conservative with respect to the negative-tension reinforcement, since the upper rods run only to the fourth point and the curve for negative moment has not been considered in bending down the rods (see Art. 39, Sect. 7). Nevertheless a design of this kind is generally considered good. In fact, it is all that is desired under ordinary conditions in roof design because of the character and amount of the live load.

Of course, it should be realized that some latitude may generally be allowed in that part of beam design referred to above, on account of the improbability of obtaining maximum conditions, but it is a good idea to have

FIG. 29.

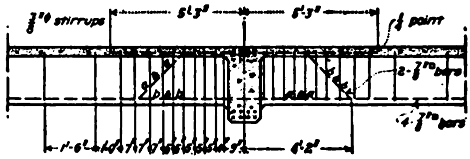


FIG. 30.

some conservative plan in mind, and to live up to that plan as nearly as circumstances will permit. At any rate, designs should never become so radical as to include rods bent up close to the support in the computations for negative reinforcement.

Fig. 31 shows a common continuous-beam design using separate straight rods over the supports. All the diagonal tensile stresses are cared for by vertical stirrups.

Girder.—The girder has a span of 21 ft. with concentrated loads at the third points. The weight of the stem will be assumed at 500 lb. per lin. ft. Reaction of concentrated loads = 2 × 25,000 = 50,000 lb.

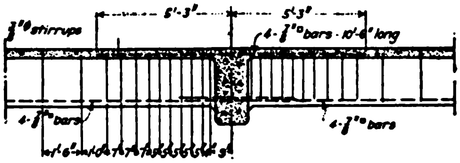


FIG. 31.

Maximum moment of concentrated loads with ends of beam simply supported would be

$$(50,000)(7)(12) = 4,200,000 \text{ in.-lb.}$$

Taking $M = \frac{wl^2}{12}$, this moment reduces to

$$\begin{aligned} \tfrac{2}{3}(4,200,000) &= 2,800,000 \text{ in.-lb.} \\ \text{Moment of dead load} &= \underline{220,500 \text{ in.-lb.}} \\ \text{Total moment} &= 3,020,500 \text{ in.-lb.} \end{aligned}$$

If desired, the span of girder may be taken as the clear distance between faces of supports (see Art. 44, Sect. 7).

The total maximum shear

$$V = 50,000 + 10.5(500) = 55,300 \text{ lb.}$$

The cross-section of web as determined by shear $= \frac{55,300}{105} = 527$ sq. in. Using the formula for economical depths, we have:

b' (inches)	d (inches)	$b' \times d$ (square inches)
12	33.1	397
14	30.8	431
15	29.8	447
16	29.0	464

It is quite probable that a breadth of 12 in. will give proper rod spacing, but the corresponding depth of $\frac{527}{12} = 44$ in. is likely to be too great when the cost of columns and walls is taken into consideration.[1] Besides, the depth would be

[1] In order to include the effect of columns and walls in the formula $d = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2}$, first determine the total horizontal area covered by the stems of the T-beams under consideration and the total horizontal sectional area of the columns and walls at the level of the beams. If the cost per cubic foot of the columns and walls is greater or less than the cost per cubic foot of the T-beam stems, increase or decrease their area in proportion to the difference in cost. Then increase the cost per cubic foot of the T-beam stems in the ratio which the total corrected area bears to the area of the T-beam stems in order to obtain the unit cost c which is used to determine the ratio r.

great in proportion to the breadth of stem and the beam would be relatively weak at the junction of stem and flange. Illumination from windows must also be considered. The following cross-section will be taken as satisfactory:

$$b' = 15 \text{ in.} \qquad d = 32\tfrac{1}{2} \text{ in.}$$

The breadth of the flange of the T-beam is controlled in this case by 12 times the thickness of slab plus the width of stem, or 69 in. Then

$$\frac{M}{bd^2} = \frac{3{,}020{,}000}{(69)(32.5)^2} = 41.5$$

For this value of $\frac{M}{bd^2}$ and for $\frac{t}{d} = \frac{4.5}{32.5} = 0.14$, Diagram 8 shows $f_c = 415$ lb. per sq. in., and $j = 0.935$. Then

$$A_s = \frac{3{,}020{,}000}{(16{,}000)(0.935)(32.5)} = 6.21 \text{ sq. in.}$$

Eight 1-in. round rods (total area 6.28 sq. in.) will be chosen. The bond stress along the eight rods at the top of beam close to the support,

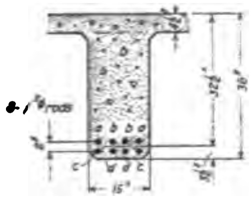
$$u = \frac{55{,}300}{(8)(3.14)(0.85)(32.5)} = 80 \text{ lb. per sq. in.}$$

which is satisfactory.

Fig. 32 shows sketch of adopted cross-section. The weight of the stem is

$$\frac{(15)(31.5)(150)}{144} = 492 \text{ lb. per lin. ft.}$$

and the value assumed is on the safe side.



Cross-section of Girder

FIG. 32.

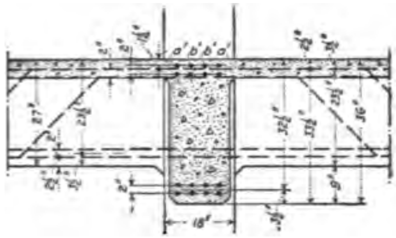


FIG. 33.

The rods at the top of girder over supporting columns will be placed as shown in Fig. 33 in order to fit in nicely with the cross-beam rods. It should be noticed that the value of d at the support is 33.5 in., or 1 in. more than at the center of span. Then

$$\frac{d'}{d} = \frac{2.5}{33.5} = 0.075$$

$$p = \frac{6.28}{(15)(33.5)} = 0.0125$$

$$p' = 0.5p \text{ (see Fig. 34)}$$

The following values are obtained from Diagram 12 and Table 9,

$$k = 0.403 \qquad j = 0.882 \qquad \frac{k}{n(1-k)} = 0.0449$$

Then

$$f_s = \frac{M}{A_s jd} = \frac{3{,}020{,}000}{(6.28)(0.882)(33.5)} = 16{,}200 \text{ lb. per sq. in.}$$

$$f_c = (16{,}200)(0.0449) = 730 \text{ lb. per sq. in.}$$

These values will be considered satisfactory.

It is proposed to have bent rods take as much of the diagonal tension as possible. The total maximum shear = 55,300 lb. The shear on the support side of the third point = 55,300 − 500(7) = 51,800 lb. On the side of the third point toward the center of span, the shear = 51,800 − 50,000 = 1800 lb., or v = 4 lb. per sq. in. Thus, web reinforcement is needed only from the support to the point where the beam intersects the girder.

Horizontal shear (measures diagonal tension) at the support

$$\frac{V}{jd} = \frac{55{,}300}{(0.85)(33.5)} = 1950 \text{ lb. per lin. in.}$$

and at the third point, it is

$$\frac{51{,}800}{(0.93)(32.5)} = 1720 \text{ lb. per lin. in.}$$

The total diagonal tension is represented by a trapezoid, the parallel sides of which are 1950 lb. and 1720 lb., and the length 7 ft. Hence total diagonal tension is

$$\frac{1950 + 1720}{2}(7.0)(12) = 154{,}100 \text{ lb.}$$

Two-thirds of this amount, or 102,700 lb., will be taken by the web reinforcement. If six rods are to be bent, their tensile value is

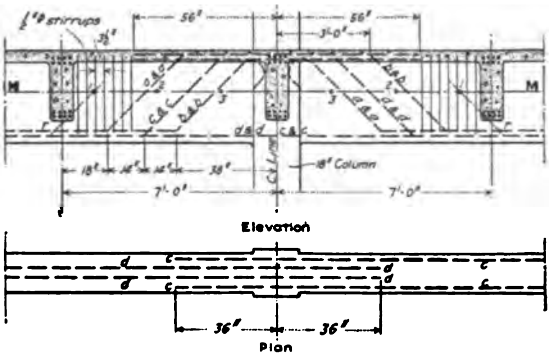
$$(6)(0.785)(16{,}000)(1.43) = 108{,}000 \text{ lb.}$$

which is in excess of the stress to be provided for.

Now shear is nearly uniform between the supports and the third point, and, as far as diagonal tension is concerned, it would be sufficiently accurate to give equal spacing to the inclined rods. Since the size of columns is not given, an 18-in. diameter of column will be considered. The spacing suggested above, then, would be taken between a point 9 in. from the center of support (that is, at the edge of column) and a point where the center of the beam intersects the girder. The plan proposed is to bend six rods, two at a time, and the points to bend for diagonal tension should be laid off on a line approximately midway between the neutral axes for positive and negative moment—as *MM*, Fig. 34. These points (1, 2, and 3) may be determined by dividing the distance mentioned above into three equal parts and locating a point at the center of each part.

An investigation must now be made to determine whether or not the tensile stresses in the beam will permit the bending of the rods as above suggested. From a study of moment curves of continuous beams loaded at the third points, for different conditions, it is found sufficiently on the safe side, as regards bending up rods, to consider the point of zero moment to occur at a distance of ¾ of l from the third point measured toward the support (see Art. 53, Sect. 7). Also when considering the bending down of rods, the same distance, or ¾ of $\frac{l}{3}$ (56 in. in this case), may be safely taken as the distance out from the support to the point of zero moment. The curve of bending moments in each case is to all practical purposes a straight line. Thus, the point where the first two rods may be bent up, using the above data, is about $\frac{(56)(2)}{8} = 14$ in. from the center of the intersection of the cross-beams. Allowing say 4 in. beyond the theoretical point for bending, this distance becomes 18 in.

As regards diagonal tension, the rod to intersect the center line at point 1 should be bent at r, as shown by the dotted line. Since rods cannot be bent at r, stirrups will be employed to take the diagonal tension between the bent rods and the cross-beam. (Stirrups will also be placed at occasional intervals throughout the girder to bind together the web and flange.) Round rods of ½-in. diameter may be used for stirrups if bent at the upper end. The tensile value of each stirrup (U-shape) is $(2)(0.196)(16{,}000) = 6270$ lb. The shear to be provided for in 1-in. length of beam is 1720 lb. and it will be necessary to space the stirrups $\frac{6270}{1720} = 3.6$ in., say 3½ in., apart as shown in Fig. 34.



(Bent bars not shown)

Fig. 34.

It should be noticed that in bending up the lower rods attention should be paid to the points where the upper rods may be bent down. In the design at hand, using 45-deg. angle bends, the rods may be bent approximately as planned and the design will be accepted. The horizontal spacing of the bent rods should not be greater than the distance between points 1 and 2, or between points 2 and 3, unless stirrups are provided where such spacing occurs.

The rods at the top of girder should extend each side of the center of support far enough to obtain their full strength in bond, which is 50 in.

The maximum stress in the compression rods at the support is $730 \times 15 = 11{,}000$ lb. per sq. in. The necessary length for bond of 1-in. rods is $\frac{(11{,}000)}{(4)(80)} = 35$ in., say 36 in. This length is shown in Fig. 34.

The top of the slab over the girder will be reinforced transversely with ⅜-in. rods spaced 12 in. c. to c., in order to provide for the negative bending moment produced with the bending of the slab next to the girder.

Designing Plates.—Plates I to IV inclusive give different complete designs for the 21 by 21-ft. floor bay in question. If desired, all the cross-beam and girder reinforcement may be made into frames, except the cross-beam reinforcement running into columns in the designs of Plates I and III. Even for these beams, however, the stirrups may be rigidly spaced if wired at the upper turns to longitudinal rods of small diameter. After the stirrup steel is suspended in the forms, the bent rods can then be easily slipped into place and wired.

Fig. 35 and Plate II show a girder design such that both girder and cross-beam reinforcement may be built into frames. In the arrangement shown, the girder frames (with the exception of the rods over supports) would be put in place first, then the cross-beam frames running into columns, next the negative tension rods of the girders, and finally the two intermediate cross-beam frames. The arrangement as regards strength is not as ideal as in the girder design of Plates I, III, and IV since the bent-up rods do not extend over supports, but in the design shown there is a surplus of diagonal-tension reinforcement which offsets this weakness. To be sure the bent rods are anchored by transverse rods placed between layers of steel, but the danger lies in the liability of these bars to slip horizontally when a force inclined to the vertical is exerted upon them. The additional amount of diagonal-tension reinforcement, however, over and above that figured will make the design a safe one. The liability to slip applies more especially to the lower transverse rods, as the upper rods are long and well anchored in the slab.

If the cross-beam steel in Plate III could have been placed over the top of the girder steel at columns, then all reinforcement shown on this plate could have been made up into frames before being placed in the forms. The reason why the cross-beam rods were not placed above the girder rods in the design in question was because the cross-beam rods would then interfere with the main reinforcement of the slab. To prevent this, the girder rods over supports would need to be lowered and this could not be accomplished in the design in question because of the compressive stress in the concrete being already a maximum at this point. It could be accomplished, however, by either forming a flat haunch, by inserting extra compressive steel, or by deepening the girder.

Placing of the reinforcement in the forms in frames insures accurate location of all steel. If a loose-rod method is used, great care is required in construction to make sure that the steel is not disturbed during the pouring of the concrete. Usually small diameter rods are needed to make the frames, in addition to the main reinforcement.

The placing of inverted stirrups near the supports has not been considered in the above discussion. These are placed after all other reinforcement is in its proper position and are slipped down over the negative-tension steel and wired to it. The continuous stirrup shown in some of the designs is convenient for this purpose. In schemes 3 and 4 the continuous stirrup is also used in place of some of the upright U-stirrups.

Spacing rods should be placed in all beams and girders between the upper and lower rows of steel. These are plain rods of the desired size and should be placed transversely not more than 5 ft. on centers. Special frame

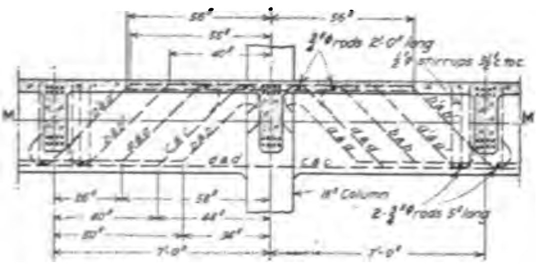


Fig. 35.

supports may be provided, if desired, or U-shaped stirrups may be employed if the length of the hook is made sufficient to permit the stirrups to rest on the slab form. If special supports are used, they should be spaced about 5 ft. on centers. In the steel schedules given, special frame supports and spacing rods are omitted for simplicity.

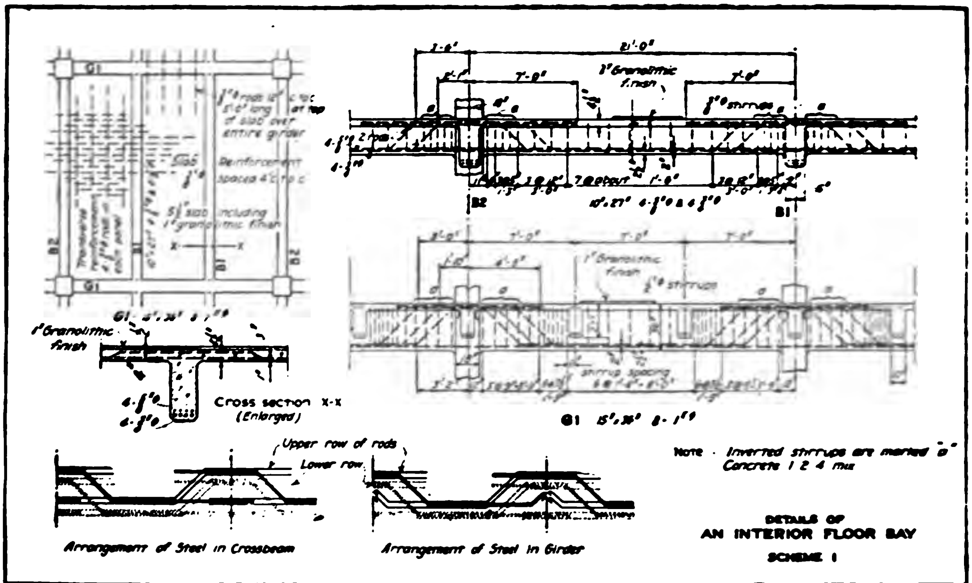
The width of stirrups is given in the bending schedules as the clear width inside of the outer strands, and the vertical height is given inside the turns. A little thought will make clear that these are the dimensions needed in bending.

12. Hollow-tile Construction.—Hollow-tile construction is used to a considerable extent for light buildings such as modern store buildings and office structures.

Fig. 36 shows a typical one-way hollow-tile slab and Fig. 37 a two-way tile construction. No cross-beams are employed in the one-way type except the small ribs of the floor slab formed between the rows of hollow tile. In the two-way type, cross-beams are placed at the columns. The tiles are placed directly upon the forms with the reinforcing rods in the spaces between them, and the concrete is filled in between the tiles and poured over the top to form the floor. The ribs form a series of comparatively light T-beams side by side with flanges usually 2 or more inches in thickness. The main beams or girders are also of T-shape. The flanges of these beams or girders are usually of the same thickness as the floor slab, but lighter tiles are sometimes used near the stem, in which case the flange becomes thinner than when the tiles are entirely omitted at this part of the floor. The function of the tiles is simply to create a void in the concrete and thus to decrease the dead weight of slab, and they do not enter into the calculations for strength of floor.

Either hard-burned or semi-porous tile may be used in reinforced-concrete floor construc-

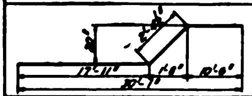
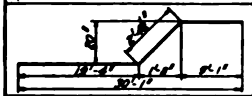
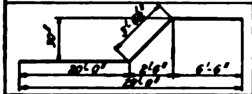
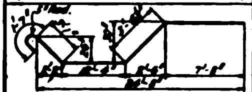
PLATE I



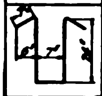
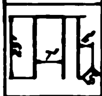
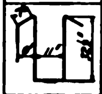
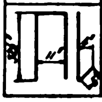
DETAILS OF AN INTERIOR FLOOR BAY SCHEME I

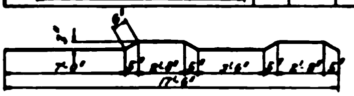
PLATE I(A)

BEAM AND GIRDER RODS

Bends	Size	Length	For beam or girder Mk.	No.	Number wanted for each beam	Total number wanted
	⅞"φ	30'-9½"	B1	2	4	12
			B2	1	4	
	⅝"φ	30'-9½"	B1	2	4	12
			B2	1	4	
	1"φ	30'-0½"	G1	1	4	4
	1"φ	28'-2½"	G1	1	4	4

STIRRUPS

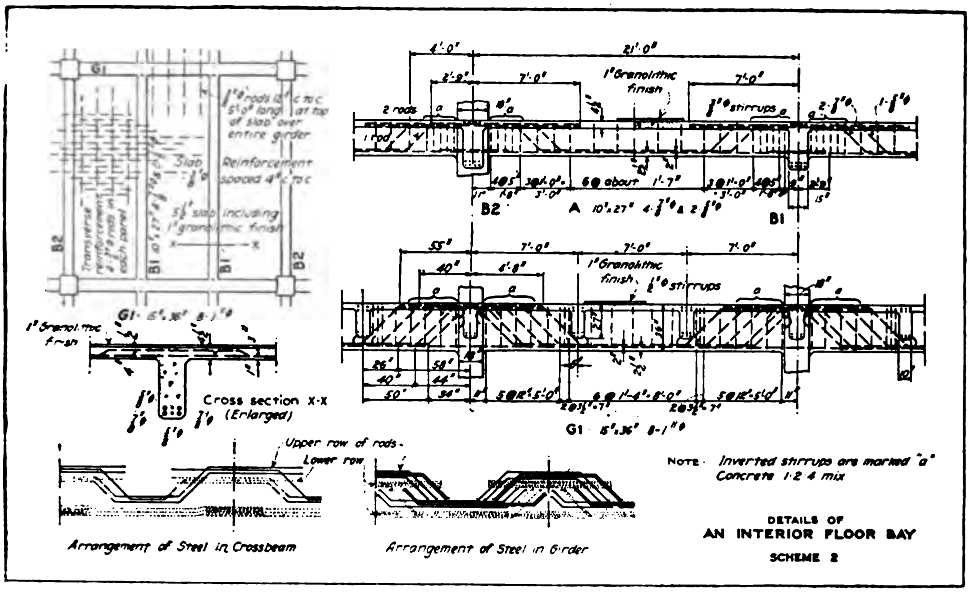
Bends	Size	Length	For beam or girder Mk.	No.	Number wanted for each beam	Total number wanted
	⅜"φ	5'-9"	B1	2	12	36
			B2	1	12	
	⅜"φ	5'-3"	B1	2	10	30
			B2	1	10	
	½"φ	7'-6"	G1	1	21	21
	½"φ	7'-2"	G1	1	8	8



⅜"φ Slab Steel Total length 17'-10"

STEEL BENDING SCHEDULE FOR AN INTERIOR FLOOR BAY SCHEME I

PLATE II



DETAILS OF
AN INTERIOR FLOOR BAY
SCHEME 2

PLATE II(A)

BEAM AND GIRDER RODS

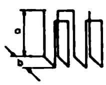
Bends	Size	Length	For beam or girder Mk	No	Number wanted for each beam	Total number wanted
11'-0" 1'-8" 8'-10" 1'-6" 11'-0"; 35'-0"	5/8"φ	36'-4"	B1	2	1	3
			B2	1	1	
9'-9" 1'-6" 12'-4" 1'-8" 9'-9"	7/8"φ	36'-4"	B1	2	2	6
			B2	1	2	
4'-8"	5/8"φ	25'-4"	B1	2	1	3
	7/8"φ	25'-4"	B2	1	2	6
5"Rod 20" 28½" 1'-8" 13'-4" 1'-4½" 16'-4½" 1'-7" 5"Rod	1"φ	22'-3"	G1	1	4	4
30½" 5'-7" 20'-8" 2'-6½" 23'-2½" 1'-7" 5"Rod	1"φ	25'-10"	G1	1	4	4
5"Rod 30½" 2'-6½" 7'-11" 2'-6½" 13'-0" 5"Rod	1"φ	18'-3"	G1	1	4	4
Straight	1"φ	9'-4"	G1	1	4	4

U-STIRRUPS

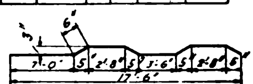
Bends	Size	Length	Beam or Girder Mk	No	Number for each beam	Total No. wanted
7" 6" 7" 24"	3/8"φ	5'-9"	B1	2	11	33
			B2	1	11	
7" 6" 11" 32½"	1/2"φ	7'-6"	G1	1	13	13

CONTINUOUS STIRRUPS

a	b	Spacers	Size	Length	Beam Mk	No	No for each beam	Total No wanted
24	7	4-4@5	3/8"φ	29'-6"	B1	2	2	4
		4@5		24'-7"	B2	1	2	2
32½	11	3@12	1/2"φ	28'-4"	G1	1	2	2

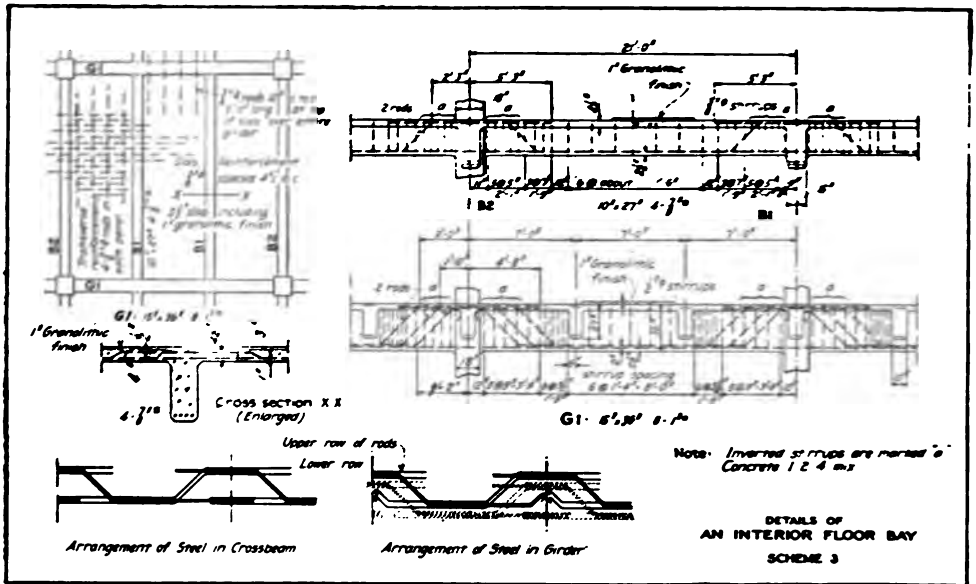


Continuous Stirrup



STEEL BENDING SCHEDULE
FOR
AN INTERIOR FLOOR BAY
SCHEME 2

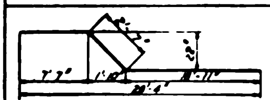
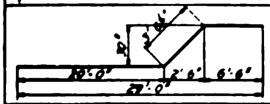
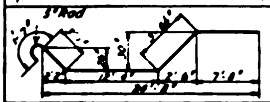
PLATE III



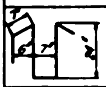
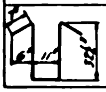
DETAILS OF
AN INTERIOR FLOOR BAY
SCHEME 3

PLATE III(A)

BEAM AND GIRDER RODS

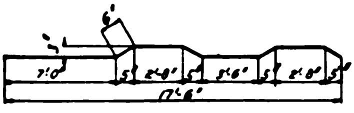
Bends	Size	Length	For beam or girder: Mk.	No.	Number wanted for each beam	Total number wanted
	⅞″□	29′-1″	B1	2	4	12
			B2	1	4	
	1″□	30′-0½″	G1	1	4	4
	1″□	28′-2½″	G1	1	4	4

U-STIRRUPS

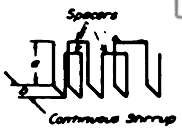
Bends	Size	Length	For beam or girder: Mk.	No.	Number wanted for each beam	Total number wanted
	⅜″⌀	5′-9″	B1	2	15	45
			B2	1	15	
	½″⌀	7′-6″	G1	1	5	5

CONTINUOUS STIRRUPS

a	b	Spacers	Size	Length	For beam or girder		Number for each beam	Total number wanted
24	7	4-4@5	⅜″⌀	29′-6″	B1	2	2	6
		4@5		24′-7″	B2	1	2	
32½	11	9-3½-3½-3½-3½-3½-3½	½″⌀	53′-2″	G1	1	2	2
		3@9″		27′-7″			2	2



⅜″⌀ Slab Steel Total length 17′-0″



STEEL BENDING SCHEDULE
FOR
AN INTERIOR FLOOR BAY
SCHEME 3

PLATE IV

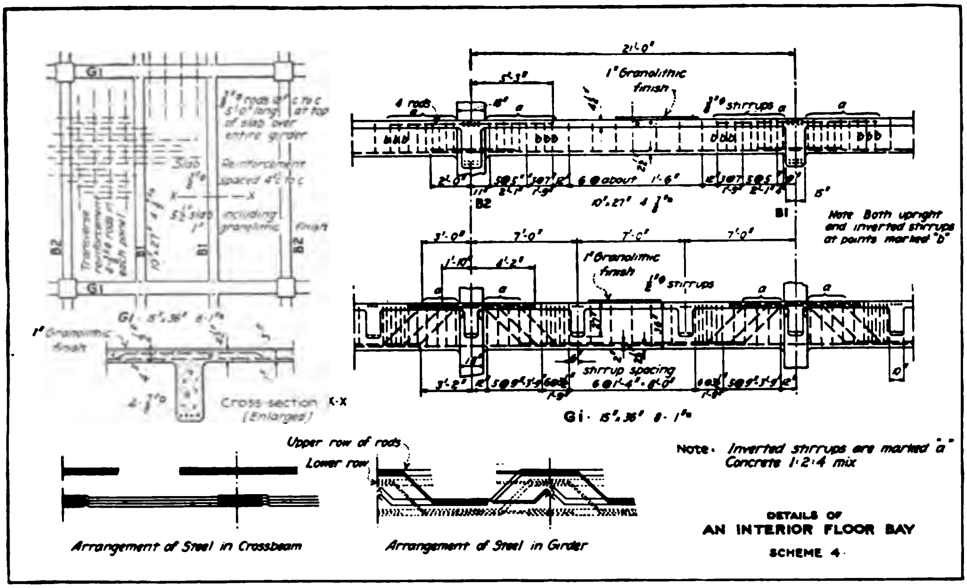


PLATE IV(A)

BEAM AND GIRDER RODS

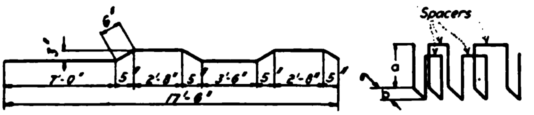
Bends	Size	Length	For beam or girder Mk	No	Number wanted for each beam	Total No wanted
Straight	7/8"□	10'-6"	B1	2	4	12
			B2	1	4	
20'-6", 4'-6", 25'-0"	7/8"□	25'-0"	B1	2	4	12
			B2	1	4	
30", 3'-6½", 20'-0", 2'-6", 6'-6", 29'-0"	1"□	30'-0½"	G1	1	4	4
5" Rod, 30", 2'-2", 12'-4", 2'-6", 7'-8", 28'-8"	1"□	28'-2½"	G1	1	4	4

U-STIRRUPS

Bends	Size	Length	For beam or girder Mk	No	Number for each beam	Total No wanted
7", 6", 7", 24"	3/8"φ	5'-9"	B1	2	15	45
			B2	1	15	
7", 6", 11", 32½"	1/2"φ	7'-6"	G1	1	5	5

CONTINUOUS STIRRUPS

a	b	Spacers	Size	Length	Beam or Girder Mk	No	No for each beam	Total No wanted
24	7	4-5@5-3@7	3/8"φ	50'-0"	B1	2	2	6
		5@5-3@7		45'-1"	B2	1	2	
32½	11	9-3½-3½-3½-3½-3½-3½	1/2"φ	53'-2"	G1	1	2	2
		3@9		27'-7"			2	2



3/8"φ Slab Steel Total length 17'-10"

Continuous Stirrup

STEEL BENDING SCHEDULE
FOR
AN INTERIOR FLOOR BAY
SCHEME 4

tion. Hard-burned tile, due to its density, has a higher crushing strength and will, therefore, undergo a greater stress without any sign of failure, but it does not seem to be as good a fire-resisting material as the semi-porous.

The following table gives average weights of the common sizes of hollow tile:

WEIGHTS OF HOLLOW TILE

4 by 12 by 12	16 lb.	8 by 12 by 12	30 lb.
5 by 12 by 12	20 lb.	9 by 12 by 12	33 lb.
6 by 12 by 12	22 lb.	10 by 12 by 12	35 lb.
7 by 12 by 12	27 lb.	12 by 12 by 12	40 lb.

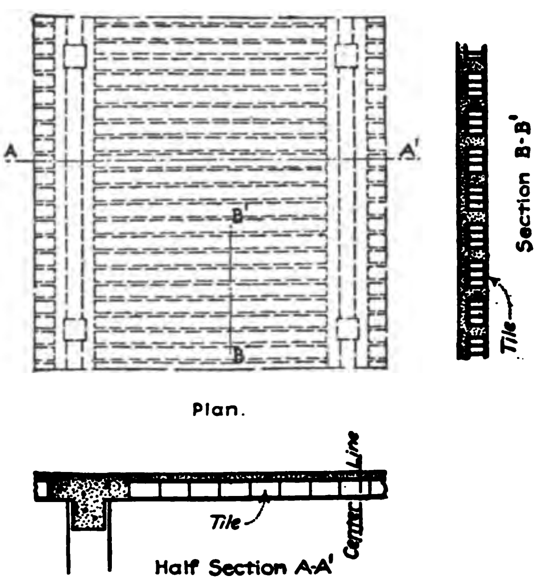


FIG. 36.

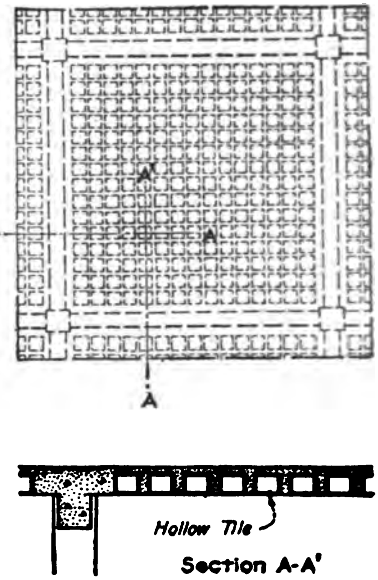


FIG. 37.

The commercial sizes of tiles are usually 12 by 12 in. in plan and vary in depth from 4 to 16 in. The depth of a tile concrete floor should be designed so as to allow for these commercial sizes. The standard sizes manufactured by the National Fire Proofing Co. are all 12 by 12 in. in plan with the following depths: 4 in., 5 in., 6 in., 7 in., 8 in., 9 in., 10 in., 12 in., 15 in. Special sizes of tile may be obtained if the order is of sufficient size to warrant the manufacturing of the same.

Tiles are likely to vary 3/8 in. from the dimensions specified so that the plans should show the full thickness of the floor and the minimum amount of concrete topping. If the tiles are small, due to shrinking in burning, the thickness of floor should be made up in concrete.

Unless the tile is thoroughly sprinkled before the floor is poured, slight depressions will occur over the ribs. This is because the hollow tile absorbs the moisture in the concrete of the top coat, causing it to set more quickly than the rib with its greater body of concrete and greater shrinkage. Sprinkling of the tile should be insisted upon, especially in hot weather.

Hollow-tile floors are generally plastered on the underside as it is only in the roughest kind of work that this is not done. The surface of the tiles should be deeply scored so that the plaster will bind firmly.

Ordinary hollow tile is open at both ends and cannot be used when the floor is reinforced in both directions. For such floors two-way tile should be procured.

Hollow-tile floors are used mostly in long-span construction, and where the loads are light and distributed. The reason for this is the small dead load, the flat ceiling, and the simplicity of the formwork. The cost of the solid type of beam and girder construction for the conditions of long span and light loads is generaly much greater than for hollow tile.

ILLUSTRATIVE PROBLEM.—Design an interior panel of a one-way hollow-tile floor to carry a live load of 100 lb. per sq. ft. with the rows of girders spaced 21 ft. on centers. The recommendations of the Joint Committe will be followed for a 2000-lb. concrete (see *Appendix B*). The ratio of the unit cost of steel in place to unit cost of concrete in place (r) will be taken at 70, with 20 cts. as the cost of concrete per cubic foot.

The finished flooring will consist of ⅞-in. maple boards nailed to 2 by 3-in. sleepers. The sleepers will be placed on the concrete slab and cinder concrete in proportions 1 : 3 : 6 filled in between them. The following weights will be taken in pounds per square foot of floor area: wooden floor, 5; sleepers or nailing strips, 2; concrete filling, 15; plaster, 5.

Ribs 4 in. wide will be assumed making a 16-in. width of flange for the small T-beams. The concrete topping will be made 2 in. If a 9-in. tile is assumed, the total load per linear foot of beam will be 274 lb., made up of the following items:

$$
\begin{aligned}
\text{Live load} &= 100 \times {}^{16}\!/\!_{12} = 133 \text{ lb.}\\
\text{Wood floor} &= 5 \times {}^{4}\!/\!_{3} = 7 \text{ lb.}\\
\text{Sleepers} &= 2 \times {}^{4}\!/\!_{3} = 3 \text{ lb.}\\
\text{Concrete filling} &= 15 \times {}^{4}\!/\!_{3} = 20 \text{ lb.}\\
\text{Concrete topping} &= 25 \times {}^{4}\!/\!_{3} = 33 \text{ lb.}\\
\text{Tile} &= 33 \text{ lb.}\\
\text{Stem} &= \frac{(4)(9)}{144} \times (150) = 38 \text{ lb.}\\
\text{Plaster} &= 5 \times {}^{4}\!/\!_{3} = 7 \text{ lb.}\\
\hline
\text{Total} &= 274 \text{ lb.}
\end{aligned}
$$

The bending moment,

$$M = \frac{wl^2}{12} = \frac{(274)(21)(21)(12)}{12} = 121{,}000 \text{ in.-lb.}$$

The economical depth of floor now needs to be determined. Using the same notation as in Art. 37 of Sect. 7 and, in addition, using the term c_t to represent the variation in cost of tile in place per 1-in. change in depth, we have

$$C = cb'd' + \frac{crM}{f_s\left(d' + \frac{t}{2}\right)} + d'c_t$$

as the total cost of the small T-beams per unit length. The following expression has been deduced from the preceding equation by the aid of the calculus, and will give the value of d for minimum cost when the value of b' is fixed:

$$d = \sqrt{\frac{c_c r M}{f_s(b'c_c + 144c_t)}} + \frac{t}{2}$$

The term c_c in this formula means the cost of concrete per cubic foot. A value of 1¼ cts. will be given to c_t. Then

$$d = \sqrt{\frac{(20)(70)(121{,}000)}{(16{,}000)(260)}} + \frac{2}{2} = 7.4 \text{ in.}$$

The effective depth d must be taken so as to provide for a commercial depth of tile. A 7½-in. effective depth will be tried with a 1½-in. fireproof covering below the center of steel. This assumption would permit of a 7-in. tile.

Proceeding with the design, however, we find that

$$\frac{M}{bd^2} = \frac{121{,}000}{(16)(7.5)^2} = 135$$

and

$$\frac{t}{d} = \frac{2}{7.5} = 0.27$$

Diagram 8, Sect. 7, shows the stress in the concrete to be above 650 lb. per sq. in., which cannot be allowed. By trial it is found that a 9½-in. effective depth is needed to bring the concrete stress to an allowable value. For this depth,

$$\frac{M}{bd^2} = \frac{121{,}000}{(16)(9.5)^2} = 83.8$$

$$\frac{t}{d} = \frac{2}{9.5} = 0.21$$

From Diagram 8, $j = 0.91$. Then

$$A_s = \frac{121{,}000}{(16{,}000)(0.91)(9.5)} = 0.88 \text{ sq. in.}$$

Two ¾-in. round rods will be employed in each rib—one will lie straight and the other will be bent up at both ends and extend along the top of beam to the quarter point of the adjoining span. This arrangement will give the same steel over supports as in the center of span.

The total shear close to support

$$V = \frac{(274)(21)}{2} = 2880 \text{ lb.}$$

The bond along the two rods at the top over supports

$$u = \frac{2880}{(2)(2.36)(0.85)(9.5)} = 76 \text{ lb. per sq. in.}$$

The distance from the support to where stirrups are unnecessary

$$x_1 = \frac{21}{2} - \frac{(40)(4)(0.91)(9.5)}{274} = 5.4 \text{ ft.} = 65 \text{ in.}$$

All the diagonal tension not taken by the concrete will be provided for by vertical stirrups; in other words, the strengthening action of the bent rod will not be considered. Round stirrups ¼-in. diameter will be employed, bent at the ends. Stirrup spacing near the support:

$$s = \frac{3}{2} \cdot \frac{(0.049)(2)(16{,}000)(0.85)(9.5)}{2880} = 6\tfrac{1}{2} \text{ in.}$$

The spacing adopted is shown in Plate V.

The moment, shear, and bond considerations given above do not take into account the strengthening action of the flange of the T-shaped girders, and allowance may be made for this when thought necessary. It is quite evident that the concrete is not overstressed in compression over supports, but the stress at the edge of girder flange should be investigated. For the width of girder flange shown in Plate V, the bending moment may be taken at 6/7 (121,000) = 104,000 in.-lb. This value is obtained by considering the point of zero moment at the third point.

$$\frac{d'}{d} = \frac{1.5}{9.5} = 0.158$$

$$p' = p = \frac{0.88}{(4)(9.5)} = 0.023$$

Diagram 12 and Table 9 give the values:

$$k = 0.440 \quad j = 0.848 \quad \frac{k}{n(1-k)} = 0.0524$$

Then

$$f_s = \frac{M}{A_s jd} = \frac{104{,}000}{(0.88)(0.848)(9.5)} = 14{,}700 \text{ lb. per sq. in.}$$

$$f_c = (14{,}700)(0.0524) = 770 \text{ lb. per sq. in.}$$

The load on the floor is (274) (12/16) = 205 lb. per sq. ft., and the girder therefore carries a load of (205)(21) = 4300 lb. per lin. ft., to which should be added the weight of the girder itself. This weight will be assumed at 360 lb, making a total load, which may be considered uniform, of 4660 lb. per lin. ft. The bending moment

$$M = \frac{(4660)(21)(21)(12)}{12} = 2{,}055{,}000 \text{ in.-lb.}$$

Assume the total depth of girder to be limited to 36 in., effective depth to 32½ in. Then $\frac{t}{d} = \frac{11}{32.5} = 0.34$. Diagram 8 shows that, for $\frac{t}{d} = 0.34$ and $f_c = 650$, $\frac{M}{bd^2} = 107$, or

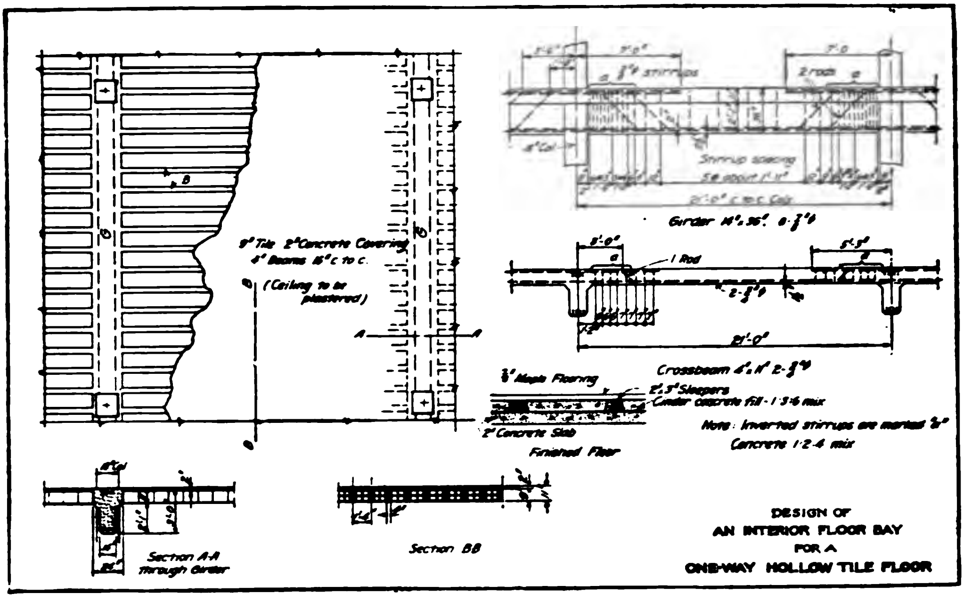
$$b = \frac{2{,}055{,}000}{(32.5)^2(107)} = 18.2 \text{ in.}$$

An arbitrary value of 24 in. will be adopted by b, which makes $\frac{M}{bd^2} = 81$. Diagram 8 shows the neutral axis to lie in the flange. Diagram 2 shows $p = 0.0057$, or

$$A_s = (0.0057)(24)(32.5) = 4.4 \text{ sq. in.}$$

Eight ⅞-in. round rods will be selected, with a total area of 4.81 sq. in. The width of stem will be made 14 in to provide properly for shear. The remaining computations for stresses in the steel and concrete at the support, and

PLATE V



DESIGN OF
AN INTERIOR FLOOR BAY
FOR A
ONE-WAY HOLLOW TILE FLOOR

PLATE V(A)

BEAM AND GIRDER RODS

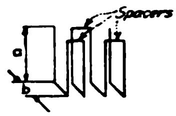
Bends	Size	Length	Beam or Girder	No wanted for each beam	Total No wanted
18'-10" · 2'-5" · 6'-8" · 29'-11"	7/8"φ	30'-11"	G	4	4
17'-0" · 2'-5" · 10'-6" · 29'-11"	7/8"φ	30'-11"	G	4	4
8'-3" · 8" · 13'-8" · 8" · 8'-3" · 31'-6"	3/4"φ	32'-1"	B	1	
Straight	3/4"φ	25'-0"	B	1	

U-STIRRUPS

Bends	Size	Length	Beam or Girder	No wanted for each beam
6" · 10" · 31"	3/8"φ	7'-8"	G	8
2" · 2 3/4" · 9"	1/4"φ	2'-4"	B	6

CONTINUOUS STIRRUPS

a	b		Size	Length	Beam or Girder	No for each beam
31"	10	6@3'-3@4'-6'	3/8"φ	69'-0"	G	2
9	2 3/4	6-6-6-7	1/4"φ	10'-6 3/4"	B	2



Continuous Stirrup

STEEL BENDING SCHEDULE
FOR
AN INTERIOR FLOOR BAY
ONE-WAY HOLLOW TILE FLOOR

for shear and bond, are similar in every way to those given under cross-beam design in two-intermediate beam construction. In Plate V an 18-in. column is assumed for convenience.

As in solid concrete floors, the proper depth for girders in hollow-tile construction depends upon the use to which the building is to be put, and the cost of all columns and walls in the building per unit increase in height. The cost of formwork should also receive attention. Of course, as regards the materials in the girder itself, cost decreases with depth. The problem resolves itself into finding the limiting depth of the girder in view of the many conditions which must be considered. Very often a perfectly flat ceiling is desired, and very wide girders must then result.

The following table has been prepared to simplify the computations in the design of the small T-beams of a hollow-tile floor. The table shows at a glance, for any given thickness of concrete topping, the minimum depth of tile needed for any given bending moment, and the corresponding steel area required. Greater depths of tile may sometimes prove more economical.

It is quite evident from a study of the table that different thicknesses of concrete topping should be considered in order to arrive at the most economical design. When the rib width does not change in the economical considerations, the economy of the various designs may be based on the cost per foot length of floor having a width, the distance center to center of ribs; but when the steel area is such that the width of rib varies, the economy of the designs should be based on the cost per square foot of floor.

Table for Hollow-tile Floors

Depth of tile (inches)	Based on $f_c = 650$: $f_s = 16{,}000$; $n = 15$ (1½ in. allowed from center of steel to bottom of slab)							
	Bending moments and steel areas for various depths of tile and concrete (Moments in thousands of inch-pounds; steel areas in square inches)							
	4-in. rib For any other width of rib multiply all values in table by distance center to center of ribs and divide by 16							
	Thickness of concrete topping (inches)							
	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
4	*27/0.49	*35/0.55	*43/0.62	*52/0.68				
5	41/0.58	52/0.68	*69/0.74	*72/0.80	*84/0.86			
6	56/0.65	71/0.78	84/0.86	*96/0.92	*110/0.98	*124/1.04		
7	71/0.70	90/0.84	107/0.96	124/1.04	*139/1.10	*155/1.17	*172/1.23	
8	86/0.73	110/0.90	131/1.03	152/1.14	171/1.23	*189/1.29	*208/1.35	*227/1.41
9	104/0.76	130/0.94	156/1.09	181/1.21	204/1.32	226/1.41	248/1.48	*268/1.53
10	117/0.78	150/0.98	181/1.14	210/1.28	233/1.40	243/1.51	290/1.59	314/1.67
12	147/0.81	190/1.03	231/1.22	269/1.38	309/1.52	340/1.65	374/1.77	407/1.87

* Neutral axis in flange

FLAT-SLAB CONSTRUCTION

By Walter S. Edge[1]

13. General Description.—The term *flat-slab construction* as here employed may be considered to include that type of building construction employing a reinforced-concrete slab in which the load upon the floor is carried directly to the columns without the agency of other elements, such as beams or girders.

As commonly constructed, a reinforced-concrete floor slab of uniform thickness (for all or the greater part of its area) is supported symmetrically upon columns provided with wide conical-shaped capitals at their junction with the under side of slab (see Fig. 38.) The slab may be uniform in thickness from the edge of one capital to another; or a portion of it, symmetrical with respect to the column, may be increased in thickness, forming a drop panel (see Fig. 39). Another form occasionally used, carries the thickened slab from column to column, thus forming in reality shallow beams between columns, and giving the effect of a paneled ceiling.

The methods of reinforcing the slab and columns differ radically in different systems, and are described in Art. 17. In general it may be said that the slab reinforcement consists of a

Fig. 38.—Velie Motor Vehicle Co.'s factory building, Moline, Ill.

large number of comparatively small-diameter rods, a considerable percentage of which radiate from the center of the various columns and are commonly located in the bottom of the slab at the center of the span, and in the top of the slab over the column head. While in some respects flat-slab construction is structurally the simplest form of concrete-steel design, it is of comparatively recent development, and has a present-day use out of all proportion to the time that has elapsed since its first introduction. Since the erection in 1903 of the first flat-slab building, their relative number has steadily increased until at present, probably 80% of new reinforced-concrete construction (in which the live load is 100 lb. per sq. ft. or more) is of this type.

14. Advantages Over the Beam-and-girder Type.—The reasons for its popularity as might be supposed are economic ones, and may be summarized as follows:

1. The ceiling, being flat or practically so, offers no obstruction to the passage of light; and, as the windows may extend to the underside of floor, good daylight illumination may be obtained when desired for the maximum width of building (see Figs. 38 and 39).

2. The failure, or partial failure, of concrete structures under fire has been first to develop at the corners of the columns, beams, and girders where spalling of the concrete is apt to take place. A flat-slab floor supported upon round concrete columns offers practically no sharp

[1] Consulting Engineer, New York City.

angles for spalling to begin, and experience shows that such construction will suffer little damage where a beam-and-girder building would be seriously injured.

3. The automatic sprinkler system is after all the final safeguard where inflamable materials are stored in the building, and a much more efficient installation can be made with the flat ceiling, than with an unfinished ceiling of the beam-and-girder type (see Fig. 39).

4. A considerable saving in story height and in total height of a multi-storied building (or an increase in clear story height with the same height of building) may be secured by the use of flat-slab construction as compared with the common form of beam-and-girder design (see Fig. 40).

5. The danger of sudden collapse from excessive overload—particularly in the case of the so-called four-way type of floor, due to the interlacing of a great number of small steel rods running in four directions—is much less than in beam-and-girder type of construction.

Fig. 39.—Stewart Warner Speedometer Corporation building, Chicago, Ill.

6. The slab formwork is much simplified and no added complication is introduced by the round columns or ornamental column head, since it is now common practice to use metal forms which are fairly well standardized and may be rented from a number of firms making a specialty of this work.

7. For average conditions, the flat-slab type of construction is more economical than the beam-and-girder type for live loads of 100 lb. per sq. ft. and over, and the economy increases with the load.

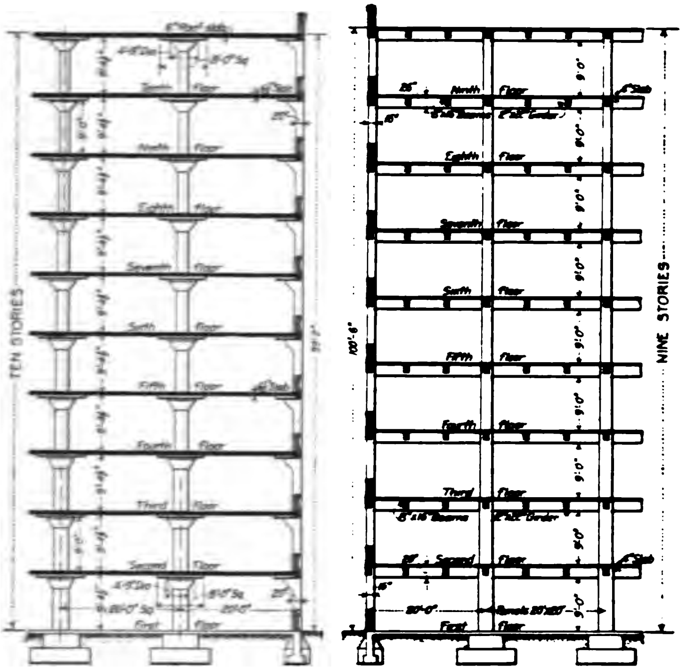
15. Classes of Buildings to Which Adapted.—The many advantages which the flat-slab system of floor construction possesses has developed an excess of enthusiasm for its use and has caused its adoption in some cases where an adherence to the beam-and-girder type would undoubtedly have been the part of wisdom. The stern realities, however, of the economic side of the case, due to the present high cost of structural-steel shapes, has brought about the adoption of flat-slab floors in many types of buildings which were heretofore almost universally constructed with structural-steel skeleton frames. Among these may be mentioned apartment houses, stores, hotels, and loft buildings for light manufacturing.

Structures to which this system is best adapted may be summarized as follows:

1. Warehouses.
2. Factories.
3. Cold-storage plants.
4. Garages and auto service stations.
5. Wharves.
6. Coal-storage bins, etc.
7. Railroad terminals.

The conditions which make for economy in the use of flat-slab floors are as follows:

1. An approximately uniform and equal spacing of supporting columns.
2. The absence of frequent large openings in the floor.
3. A superimposed live load of 100 lb. per sq. ft. or more.



FLAT SLAB CONSTRUCTION BEAM AND GIRDER CONSTRUCTION

Fig. 40.

While other considerations may make the use of flat-slab construction advisable, it will frequently be found that other types—such as steel-tile construction or combination terra-cotta block and concrete—will prove to be more economical for lighter loads or irregular spans.

In the great majority of buildings in which this system of floor construction is employed, utilitarian considerations alone prevail; nevertheless, it can, on occasion, be combined with a simple scheme of decoration to give a very pleasing effect.

16. Remarks Regarding Design.—The methods used in the design of flat-slab floors are almost entirely based on the results of extensometer measurements of the deformation of concrete and steel in actual buildings under test loads (see Art. 19). The theoretical analyses of stress which have been made have proved to be ultraconservative as compared to test results, due undoubtedly to the fact that it is extremely difficult to take all factors into consideration in any formula. No theoretical method of analysis has been developed which really meets the conditions of actual design, and it seems highly improbable that any method will be devised that will give more than a close approximation, which the present methods of empirical design undoubtedly give. The reason for this lies in the materials which go to make up the construction. Steel is a thoroughly tested material and may be secured with well-determined physical properties. The quality of concrete, however, is far more uncertain and its physical properties can only be controlled within certain limits and cannot be accurately forecast. Even if samples are taken and tested, it cannot be safely predicted that the quality of the material in the structure will agree with that in the sample to the degree of accuracy that a theoretical analysis would require. Further than this, there are shrinkage and temperature stresses set up in any structure having considerable area, and these stresses are not subject to accurate determination beforehand since the methods of construction cannot be always known in advance.

Flat-slab floors, as ordinarily designed, are figured to carry uniform loads or, at most, only moderate concentrations, such as light partitions, etc. When concentrated loads must be carried in addition to the uniform live load, it is common to introduce beams for this purpose. These may be either of customary sections or they may be of the wide and shallow type involving only a small loss in clear story height of the building. The same practice is followed with reference to openings of stairs, etc.

The calculation of beams of this class involve many considerations. Under some conditions, safe results will be secured if the beam is computed for the concentrated load alone, but usually a portion of the live and dead load of the slab should be included. Where the continuity of the slab is destroyed by an opening, the marginal beams should be figured to carry their share of the floor in addition to the concentrated loads and, of course, their own weight.

When necessary to secure drainage, flat-slab floors or roofs may be pitched and this is common practice in warehouse construction. When it is necessary to introduce steps or sudden changes of slope, however, special treatment will be required.

Probably the great majority of floors of this type are laid with a cement finish. If this is laid at the same time or very shortly after the pouring of the structural slab and before the concrete has taken its final set, it may be safely considered as a part of same and the reinforcing steel so calculated. In the absence of positive information to this effect, it is better to include the weight of the finish but not its thickness in the computations for reinforcement, for in the majority of cases the finish is laid at some later time and does not bond with the floor slab.

Exterior columns, particularly in the upper stories of buildings, require special reinforcement to resist bending. Interior columns are also subject to bending stresses but usually they are more heavily reinforced than the exterior columns and, further, the slab on the unloaded side of the column acts to a certain extent to relieve the bending in the column. While this point should be given due consideration it is seldom that the interior columns need extra steel if the recommendations of the Chicago Code or the American Concrete Institute (A. C. I.), as regards size, are followed. The interior columns under the roof are really very unlikely to be subjected to unbalanced live load and in the lower stories where it may occur, the direct load and column diameter are both greater, tending to reduce the bending effect.

A majority of the designs of flat-slab structures have been made by specialists who have made a special study of this particular form of construction. It is well that such is the case, for the subject is a specialty and, while simple enough in many cases, the production of a safe and economical design is a task that demands a skill not quickly acquired. It is manifestly impossible in a book of this character to embody and arrange sufficient information so that any one could proceed with safety with its aid to lay out any and every problem, and such is not its purpose.

Failures of flat-slab structures have occurred and other structures which have so far escaped will certainly give trouble if they ever receive their full designed load. Causes for such shameful occurrence are usually easily to be found. The first and probably most common cause is a fierce commercial competition which, in the absence of strict building code requirements, has led unscrupulous designers to place a sublime confidence in thin sections which no tests can justify. Another cause has been poor construction which, of course, is fatal to any design but probably in no class of construction is first-class concrete work more essential to success. A third contributing cause has been gross ignorance.

It is the part of wisdom and true economy, therefore, to entrust work of this class only to those who have had experience under a competent designer or, better still, to have the design made by or reviewed by a consulting engineer who has successful constructions of this class to his credit.

17. Systems.—A number of systems of designing and reinforcing flat-slab floors have been developed which, so far as the external appearance of the finished building is concerned, are very similar but differ radically in structural features and in method of design. They may be divided into four general classes with regard to the method of reinforcement which, stated in the order of the total number of buildings constructed of each class, are: (1) The four-way system; (2) the two-way system; (3) the circumferential system; and (4) the three-way system.

In the four-way system the slab may be designed either of a uniform thickness, or drop panels may be used at the column capitals. The reinforcement is placed in four direct bands running in two directions and two diagonal bands which pass diagonally across the panel from column to column. In this system the center line of each band passes over the center of the supporting columns. A portion of the rods are usually bent up over the column capital. Several systems of this type do not follow the rule in this respect, however. Additional transverse steel is placed in the top of the slab over the direct bands in many cases.

In the two-way system the reinforcement is all placed in two directions. The direct bands of reinforcement are carried from column to column and the rectangular area remaining is reinforced in both directions parallel to the direct bands by similar bars which pass across them. It is common practice in this system to carry the bars of the direct bands in the top of the slab at the column head and in the bottom of the slab at the center of span. Also the slab bars are commonly bent up where they pass over the direct bands.

The circumferential system makes use of both radial and circumferential reinforcement around the column head. The region between column heads which is commonly occupied by the direct bands is also reinforced with concentric rings which overlap with those around the column head. Finally, the slab in the center of the panel is reinforced in a similar manner.

The three-way system requires a special arrangement of columns and, while it possesses certain theoretical advantages over the four-way and two-way systems, its use so far has been comparatively limited. In this system the interior columns are placed in such a manner that the lines connecting their center lines form equilateral triangles. By this arrangement all the bands of reinforcement have equal spans and all pass over the column heads.

The majority of the systems herein described are operating under license from the Flat Slab Patents Co. of Chicago and are protected by special detail patents.

The more important systems will now be described in detail.

17*a*. Barton Spider Web System.—The Barton Spider Web system is similar to other flat-slab systems as to the arrangement of columns, column heads, and drop panels, but differs radically in the type of reinforcement used. As regards the slab, it is a four-way system and over the column head it is a two-way system. It will be seen by referring to Fig. 41 that the slab reinforcement is made up of straight rods of small diameter which run from column to column and are not continuous. The negative reinforcement over the column head consists of two systems of bent rods at right angles to each other which are placed in the top of slab and have their looped ends bent down.

Two methods of fabricating this steel are in use. In the first, that shown in Fig. 41, the column-head steel is in the form of a fabricated mattress of bars in one direction which comes to the job bent and ready to place. In this case the mat serves to space and support the floor-slab reinforcement. In the other type the column head steel consists of loose bars supported from the forms by blocks of concrete cast in the field. A zig-zag stirrup hangs from these bars and supports the ends of the direct belts of steel in the bottom of the slab. The negative moment in the slab across the line between columns is taken care of by a belt of steel in the top of the slab at right angles to and over the direct belt.

The designers of this system recognize the necessity of additional vertical reinforcement in the exterior columns to resist bending, particularly in the upper stories.

In order to govern the design, it is assumed that the loads will seek to enter the columns by the shortest possible route and will, therefore, cause a radiation of stress out from the column in all directions. Also, it is assumed that these stresses will pass through a point of inflection on account of the slab being fixed. This point of contraflexure cannot be definitely located, inasmuch as variations in the loading, such as naturally occurs on all floors, will cause the point to shift either toward or away from the columns.

To meet the above conditions the Barton Spider Web system is a four-way system in which all the major slab reinforcement radiates from the column head and also is composed of separate units of negative (column head) and positive (slab center) steel, which lap past

Fig. 41

each other so as to provide steel at top and bottom of the slab in the region of the shifting point of inflection.

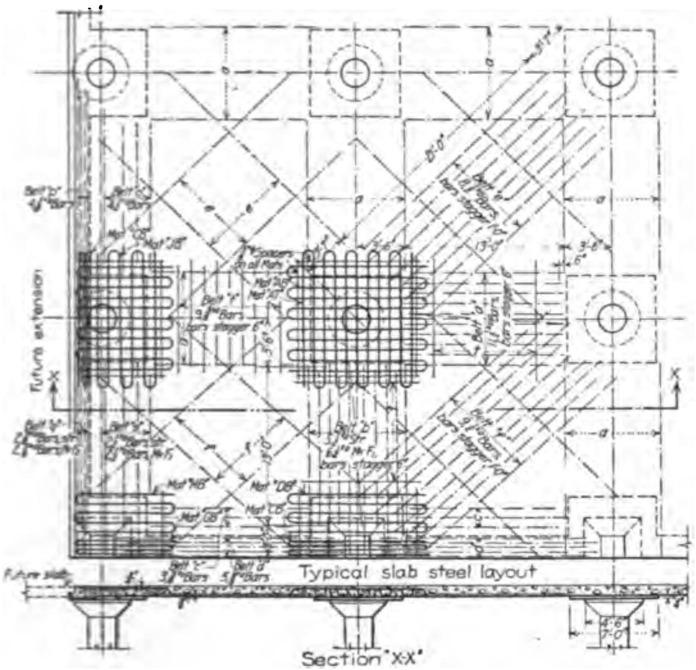
So far as the methods of arriving at bending moments in the different belts are concerned, the engineers of the Barton Spider Web system believe the American Society method as the most mathematically correct, but that their coefficients are needlessly conservative. They believe the Chicago Code to be the most satisfactory for practical use and, as every test made on these floors in that city gave a deflection of less than $^{1}/_{180}$ of the span for a test load of twice the live load plus a superimposed dead load and showed perfect elasticity, it is felt that the Chicago coefficients are amply safe. The Cleveland Code with respect to columns is favored above others by the designers of this system.

Fig. 39 is a typical interior of a building constructed according to this system.

Fig. 42 is copied from the working plans of a building designed under this system and illustrates its practical application. Much labor is saved on drawings by the method of [illegible] bands here adopted. The method of framing openings is by the use of wide, shallow beams, which method is quite commonly used by other designers.

For cuts and descriptive matter, the writer is indebted to the Barton Spider Web system.

17b. Cantilever Flat-slab Construction.—Flat-slab floors are designed by the Concrete Steel Products Co., Consulting Engineers, Chicago, under the trade name of "Canti-



Mk	No. of Belts	No. of Bars	Size	Length	Mk	Width of Belt	Remarks
a	19	11	[illegible]	12'-0"		7'-6"	
b	12	5 / 6	[illegible]	12'-6" / 15'-6"	[illegible]	7'-6"	alternate bent & str't
c	15	3	[illegible]	12'-0"		7'-6"	
d	11	5	[illegible]	16'-6"		7'-6"	
e	18	6	[illegible]	[illegible]		7'-6"	
f	28	9	[illegible]	20'-0"		7'-6"	
g	2	4	[illegible]	12'-0"		3'-6"	
T	35	9	[illegible]	9'-0"		8'-0"	top bars

Fig. 42.

Fig. 43.

lever Flat-slab Construction." In their designing, use is made of either a true flat slab or a slab stiffened by the use of drop panels around the column capitals as is done with other systems.

Earlier designs prepared by this company made use of radial rods, rings around the column heads, and column rods bent down into the slab; but as extensometer tests proved these to be inefficient, their use has been discontinued in later work.

The system of reinforcement commonly employed is the four-way system which was developed and perfected by the engineers of this company. On account of the greater stiffness and economy of materials secured, the drop-panel type of floor is always favored, although many floors have been designed and built of the true flat type.

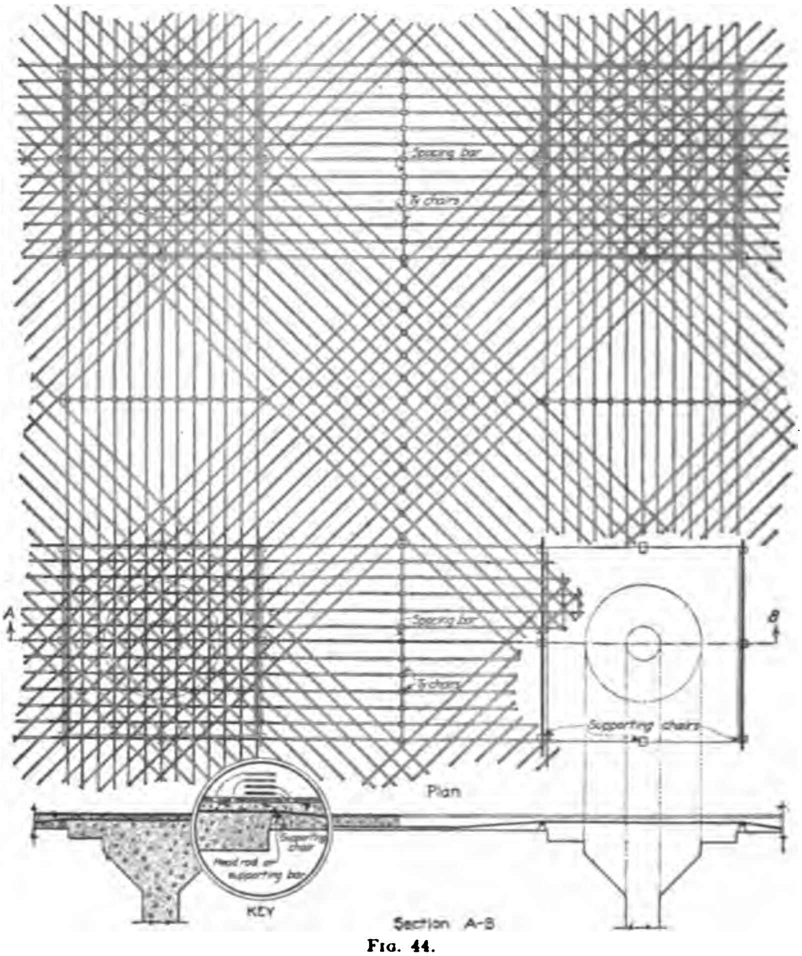


FIG. 44.

Great stress is laid on the accurate placing and anchoring of the reinforcement in place. The negative reinforcement passing over the column head is supported by head rods which in turn are carried on concrete blocks and in the center of span the accurate spacing of bars is maintained by bar spacers, tying devices, or wiring. A view of a typical floor is shown in Fig. 43.

Hundreds of buildings have been constructed according to this system and a number of these located in Chicago have been tested with very satisfactory results (see Art. 19).

For information and cut the writer is indebted to the Concrete Steel Products Co. of Chicago.

17c. Simplex System.—The Simplex system of flat-slab construction developed by the Concrete Steel Co. of New York is a four-way system with certain added refinements in the way of devices for anchoring the rods in place (see Figs. 44 and 45). It is designed on conservative lines—particularly with respect to the stresses on the concrete which are not allowed to exceed the New York City Building Code requirements. In the great majority of cases a drop head is used around the column capital. The steel reinforcement is commonly calculated on the basis of the Pittsburgh or Chicago Ruling, although it may be designed to meet the requirements of any building code. In the design of the interior and exterior columns the Chicago Ruling is used.

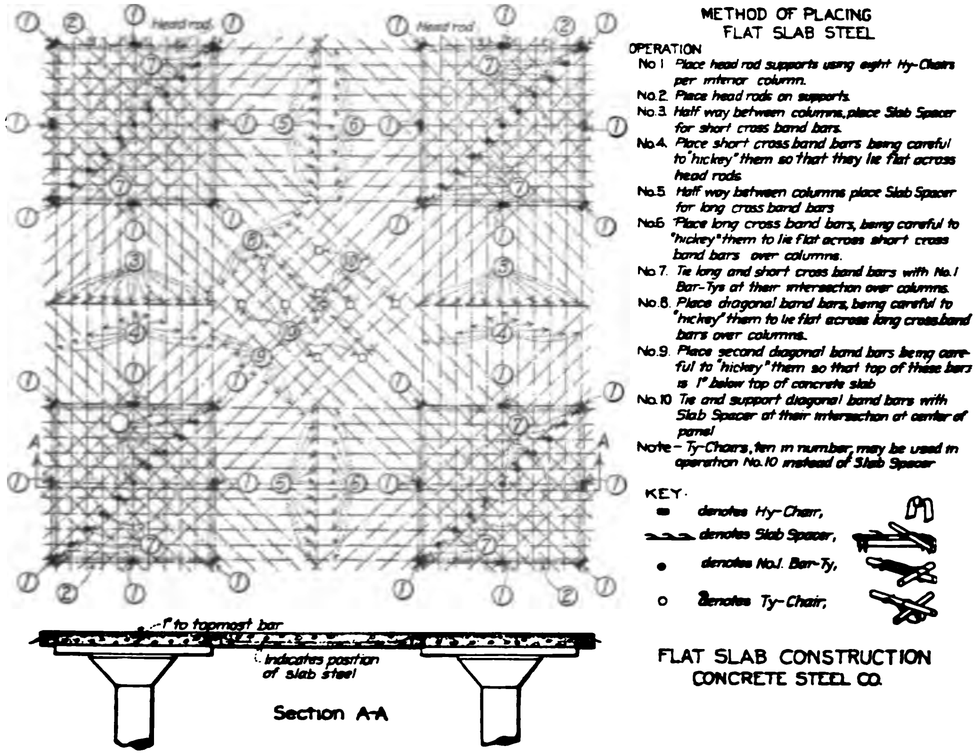


Fig 45.

The concrete sizes to use with the system for the Pittsburgh and Chicago Rulings have been computed and are given in the tables of Art. 21. The dimensions there given apply only to square interior panels. Exterior panels may require special treatment.

The method of placing the steel reinforcement is explained in detail in Fig. 45. In practice it is frequently found that the weight of the rods themselves will give them sufficient sag so that little or no bending is required.

This system has been used very extensively and with great success and, since the policy of its sponsors has always been a conservative one, it has never failed to give satisfaction.

17d. The Mushroom System.—The Mushroom system was developed by C. A. P. Turner and was in very extensive use until quite recently. Further use of the system, unless licensed by the Flat Slab Patents Co., was prevented by order of Court on account of alleged infringement of the Norcross patent (see Art. 18).

The system, as originally developed, was a four-way system with certain added features. In some cases the column bars were bent down into the slab around the column head to form a radial reinforcement. In other cases special elbow rods were inserted in the column head for the same purpose. A series of ring rods, spaced by radials forming a spider, were also used as a reinforcement at the column head and as a support to the slab steel. In some cases a flat spiral was used in the same position. The use of this system has been pushed with much business enterprise, but the severe service to which this class of buildings is almost sure to be subjected has shown beyond question, that the concrete sections used in many of the earlier

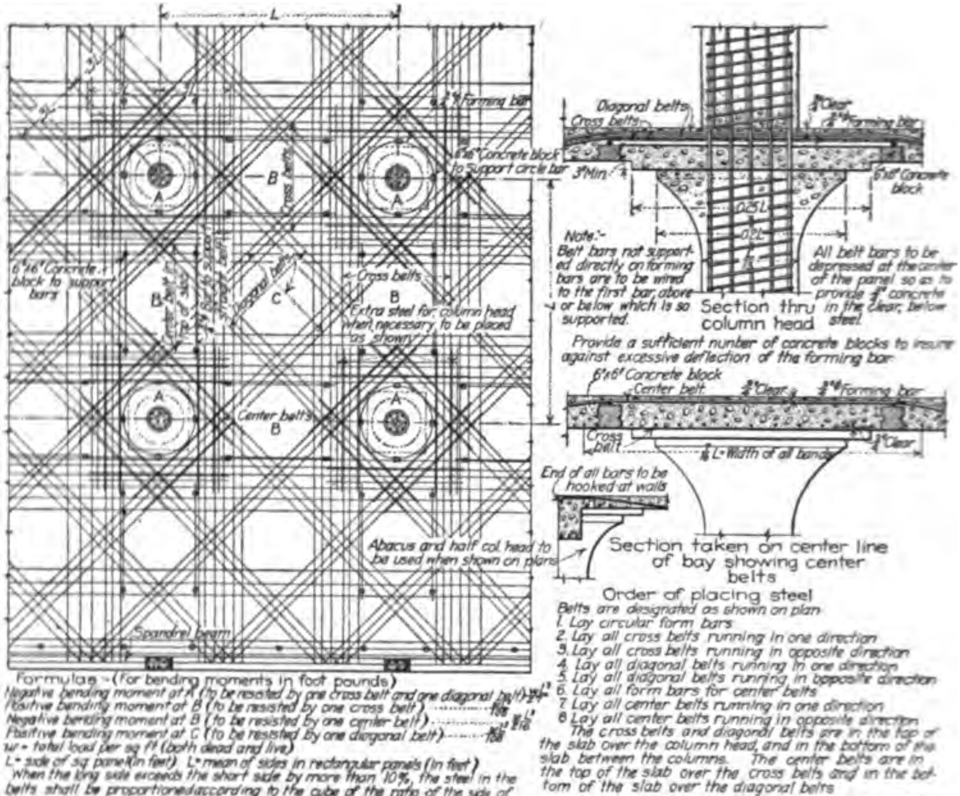


FIG. 46.

designs were entirely too light, since sagging and cracking of floors have resulted. The right requirements of the various building codes are intended to and do remedy this condition, so that now all systems must compete on a substantially uniform basis.

17e. Watson System.—**A modified type of the four-way system is that developed by Wilbur J. Watson & Co. of Cleveland, Ohio. This system differs from the standard four-way system in the introduction of center belts or bands of reinforcement which do not pass over the column heads, in addition to the usual direct and diagonal bands.**

The details of this system together with the methods of computation and methods of placing are clearly shown in Fig. 46.

17f. Akme System.—The Akme system of girderless-floor construction was developed by the Condron Co., Structural Engineers of Chicago. It is a two-way system of very simple construction and has had a wide and successful use.

When building codes are in use which have special rulings regarding flat-slab floors, the system is designed to meet their requirements but, in order to govern design where such rulings do not apply, the Condron Co. have prepared a set of instructions which are given below. These are to be used in connection with design standards (see Figs. 47 and 48).

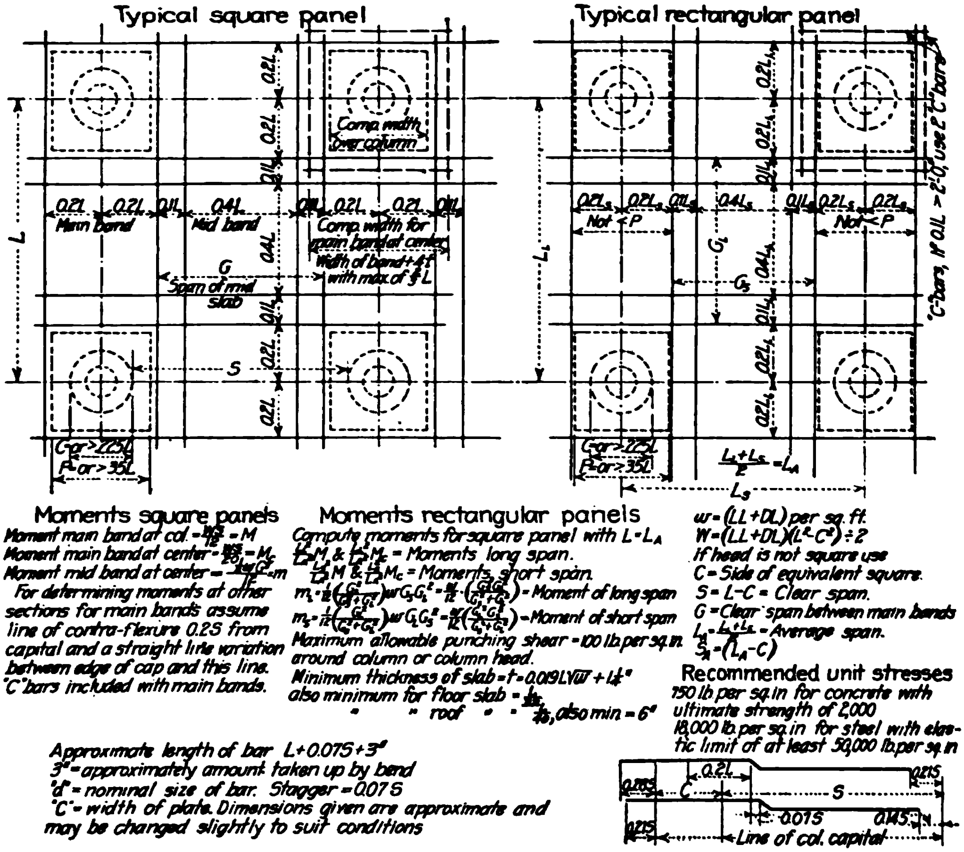


FIG. 47.—Akme system. Drop panel type. Design standards, Condron Co.

RULES FOR THE DESIGN OF GIRDERLESS FLOORS

(To Accompany Akme Design Standards)

The term girderless floors as herein used refers to flat slabs of uniform or varying thickness supported without beams or girders on columns having flaring heads.

Flat-slab Type.—In this type the slab thickness is uniform between column heads.

Drop-panel Type.—In this type the lower face of the slab is dropped so as to increase the thickness of the slab above the column head. The lateral dimensions of this portion of the slab, which is usually made square, should be not less than 0.35L.

Paneled-ceiling Type.—This type may conform in general to either of the above types with the exception that the slab is reduced in thickness in the central portion of the panel.

Columns.—The diameter or side of any interior concrete column shall be not less than one-thirteenth of the panel length or one-twelfth of the clear story height, except that for columns supporting roofs only this dimension shall be not less than one-fifteenth of the panel length. In any case the diameter or side of the column shall be not less than 12 in.

Bending in Columns.—Exterior or wall columns supporting floors or roofs shall be designed to resist, in addition to direct load, 40% of the negative bending moment for exterior floor panels or 80% for exterior roof panels.

Column Head.—The diameter of the column head, measured where it intersects the underside of the slab, should be approximately 0.235L, but may vary to suit conditions. It shall have a vertical face below the slab of 1½ in., below which the surface of the head shall have a slope of 45 deg. to the vertical face of the column shaft. If other shapes of column head are used, the surface of the same shall nowhere fall inside of the surface of the above-defined conical head. Heads may be round, octagonal, or square.

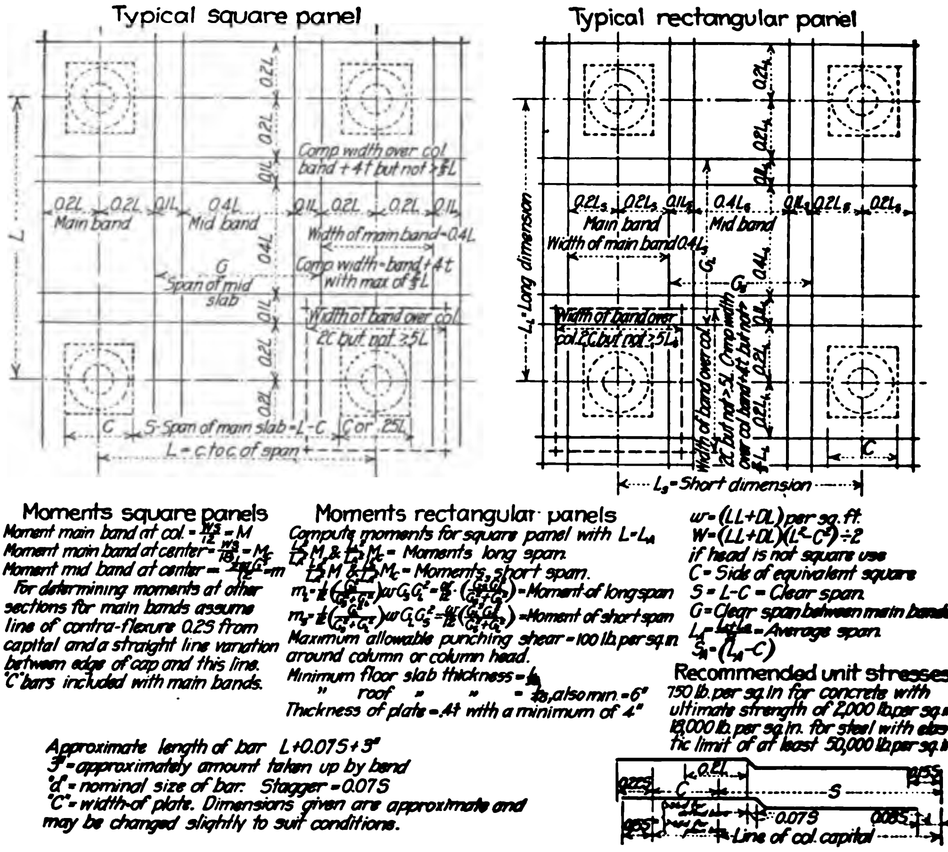


FIG. 48.—Akme system. Flat slab design standards, Condron Co.

If round or octagonal heads are used, the diameter of head to be used in the slab calculations shall be the side of an equivalent square. Where a square plate is used as part of the column head and its lateral dimension is within the 45 deg. slope of the conical head, the size of said square plate shall be used as the diameter of column head in making slab calculations, provided the thickness of said plate is equal to or greater than one-half the thickness of the slab and not less than 4 in.

Slab Thickness.—The minimum thickness of the slab (except in paneled-ceiling type) shall be not less than $\frac{L}{32}$ for floors and $\frac{L}{40}$ for roofs, nor less than given by the following formula:

$$t = 0.019L\sqrt{w} + 1\tfrac{1}{4} \text{ in.}$$

where t = total slab thickness in inches; L = panel length in feet; and w = total live and dead load in pounds per square foot.

In the paneled-ceiling type the thickness of the enclosed panel shall be not less than one thirty-second of its clear span.

Drop Panel.—The depth of drop panel where used shall be determined by using its width at the section considered as the full width to resist compression resulting from negative moment.

Panel Strips—For purposes of computation each panel of the slab is to be divided into two sets of strips called A (main slab strips) and B (mid-slab strips). Strips A extend from column to column and have a width equal to $\frac{L}{2}$, and strips B occupy the space between strips A, and likewise have a width of $\frac{L}{2}$.

Reinforcement in strips A shall be placed symmetrically about column centers for a width of approximately $0.4L$ at mid-span and approximately $0.5L$ over columns. The width for compression shall be taken as the width of the belts of reinforcement, plus 4 times the thickness of the slab, but shall not exceed $\frac{3}{4}L$. The width of main belts of reinforcement over the columns shall not exceed twice the width of the column head.

Bending-moment Coefficients, Interior Panels.—For the flat-slab type the negative bending moment taken at a cross-section of each strip A at the edge of a column head shall be $\frac{WS}{12}$. The positive bending moment taken at a cross-section of each strip A midway between column supports shall be $\frac{WS}{18}$. The positive and negative bending moments taken at a cross-section of each strip B at the middle of the panel on the center line of columns, respectively, shall be $\frac{\frac{w}{2}G^2}{12}$.

For the drop-panel type the corresponding moments at the above-mentioned section shall be $\frac{WS}{12}$, $\frac{WS}{20}$, and $\frac{\frac{w}{2}G^2}{12}$.

For paneled-ceiling type the moment coefficients shall be the same as for the flat-slab type.

For determining moments at other sections of main strips A, the line of contraflexure shall be assumed to be at a distance equal to $\frac{L}{4}$ from the center of column, with a straight-line variation moment between the edge of the head and the said line of contraflexure.

In the above W = one-half total live and dead load on the panel, exclusive of the area over the column head; S = the clear span in feet between column heads; w = total live and dead load per square foot; and G = the clear distance in feet between main belts of bars at the section midway between columns.

Bending-moment Coefficients, Exterior Panels.—For exterior panels without cantilever overhang, where wall columns with flaring heads or brackets are used, and for other spans not continuous over both supports, the positive bending moment coefficients shall be increased 20%.

When bearing walls or piers and girders are substituted for the above wall columns with flaring heads or brackets, compute the moments for the exterior panels of such construction by assuming the distance from the face of column head to inside face of wall or girder as S; and the distance between the first interior main belt and the inside face of wall or girder as G.

Oblong Panels.—For oblong panels the moments shall first be determined for an assumed square panel with sides equal to the mean of the length and breadth of the oblong panel. The moments thus found for strips A shall be multiplied by the ratio of the square of the span in question and the square of the span of the assumed square panel, and the moments thus found used in determining the steel required in strips A.

The moments for strips B shall be computed as follows: The load carried by the long and short span strips B shall be in the proportion of the ratio of the square of the short span to sum of squares of long and short spans and the ratio of the square of the long span to sum of squares of long and short spans respectively. The moments shall then be found as for square panel using the proportion of w carried by the span in question instead of $\frac{w}{2}$.

When the length of panel does not exceed the breadth by more than 5%, all computations may be made on the basis of a square with sides equal to the mean of the length and breadth. The rules given herein shall not be used for rectangular panels in which the length exceeds four-thirds of the breadth, but special consideration shall be given to such cases.

Stresses in Steel and Concrete.—The stresses shall be calculated on the basis of the straight-line formula, neglecting the tension value of the concrete. The depth of the slab for calculation of stresses shall be taken as the distance from the compressive face to the center of gravity of the belt of reinforcement in a given strip. The tensile stress in steel reinforcement should not exceed 16,000 lb. per sq. in. for structural-steel grade nor 18,000 lb. per sq. in. for cold-twisted or high-carbon deformed bars. The maximum allowable compression in the concrete shall not exceed 750 lb. per sq. in. The allowable punching shear on the perimeter of the column head shall not exceed 100 lb. per sq. in. Where governing ordinances or laws require lower allowable unit stresses, such unit stresses shall be substituted for the above.

Walls and Openings.—Where necessary, slabs shall be thickened or girders or beams shall be used under walls and around openings to carry concentrated loads.

Placing of Reinforcement.—Reinforcement shall be rigidly held in its designed position while pouring concrete. The bars in the upper portion of the slab should be rigidly supported by frames or transverse bars resting on concrete blocks of proper height. Bars in the lower portion of the slab should be raised from the forms and

held in proper position, preferably by a continuous combined spacing and raising device. The lateral spacing of bars shall not exceed 1½ times the thickness of the slab, nor more than 12 in.

Bars shall be bent to conform to the bending diagrams shown in Figs. 47 and 48 and shall be so placed in the slab that they will not be nearer than ¾ in. from the face of the concrete.

A great number of buildings have been constructed under this system and many of them have been tested with very satisfactory results. Fig. 49 shows one of these floors during the steel-placing process in which the simplicity of the arrangement is apparent. For data and cuts the writer is indebted to the Condron Co. of Chicago.

Fig. 49.

17g. Corr-plate Floors.—A type of flat-slab floor known commercially as the Corr-plate floor, developed by the Corrugated Bar Co. of Buffalo, N. Y., is having a very wide and successful use.

The general features of the slab and columns as used with this system are similar to those used with other types, and it may be designed either with or without drop heads. When drop heads are used, they are frequently made somewhat shallower than is the practice of other designers. In other respects there is little to distinguish the system from others so far as the exterior appearance goes.

It has been the practice of the engineers of this system to give due consideration to the reinforcement of the exterior columns to resist their share of the bending, the necessity of which has been demonstrated by numerous load tests of actual buildings.

The method of reinforcement used is that known as the two-way system and was developed primarily from a series of very interesting tests performed upon a small model. For information regarding this the reader is referred to Art. 19.

The practice of the designers of this system has been somewhat modified as the result of tests which have been made on actual Corr-plate floors but only as regards minor details, the original findings having been demonstrated to be substantially correct.

With regard to the design of exterior columns and lintel beams, the engineers of the Corrugated Bar Co. feel that a universal practice should be laid down which should be followed by all, such as the recommendations of the American Concrete Institute or the Special Committee of the A. S. C. E. This system may be designed to meet the requirements of any code.

Among its advantages are the use of bars of moderate length and extending over one span only (Fig. 49A). It is also common practice to use bars of a larger diameter than is customary with other systems, thus giving a saving in unit price for slab reinforcement extends in two directions at right angles to each other. A portion of the rods in the center of the slab are bent up over the center line of columns so that extra rods are not needed at this point in the top of slab.

17A. S-M-I System.—This system, more commonly known as the Smulski system, was invented and patented by Edward Smulski and is the best-known if not the only type of circumferential system. Its introduction is of very recent date but it has been quite extensively used, particularly in certain portions of the Eastern States.

The feature which differentiates this system from others that have been described is the arrangement of the reinforcement for, as regards the slab, the proportions are much the same in all systems. Use is made of circumferential and radial reinforcement in both top and bottom of the slab with only a small amount of steel passing from column to column. For the following description and theoretical discussion the writer is indebted to the inventor of the system:

Fig. 49A.

Description of the System

The reinforcement of a typical interior panel, fully illustrated in Figs. 50 and 51, consists of three types of units.

1. Unit *C* at the column head composed of rings and radial bars in the shape of hair pins, the upper prong of which resists tension while the lower prong resists compression (see Fig. 52).
2. Unit *A* between columns consisting of two trussed bars and rings.
3. Unit *B* in central portion consisting of four diagonal trussed bars and rings.

Units *T* are sometimes used as shown in "Top Reinforcement," Fig. 50.

The radial bars are provided with a semicircular hook of sufficient dimensions to transfer the stresses into the concrete by bond and bearing. The center ring which they sometimes engage keeps them in place and forms an additional factor of safety.

The trussed bars of Units *A* and *B* are bent up near the points of inflection and carried near the top and parallel to the surface of the slab to the column head, where they engage the center ring. The bent portion resists shear and binds the column-head section to the rest of the slab. The straight portion of the trussed bars in the center of the slab and at the column head resists tension due to the positive and negative bending moments respectively.

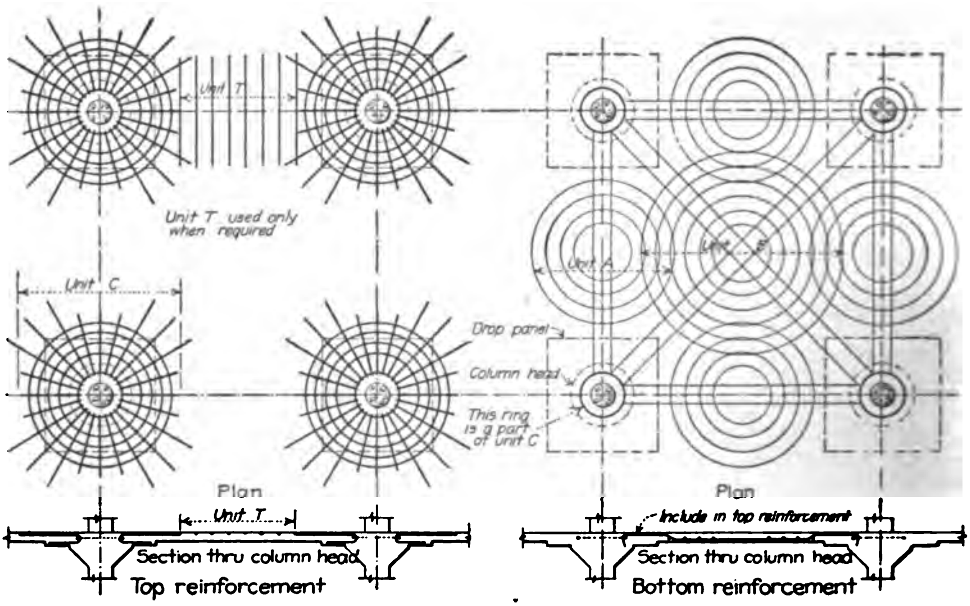

Unit T
Unit T used only when required
Unit C
Plan
Unit T
Section thru column head
Top reinforcement
Unit A
Unit B
Drop panel
Column head
This ring is a part of unit C
Plan
Include in top reinforcement
Section thru column head
Bottom reinforcement

Fig. 50.

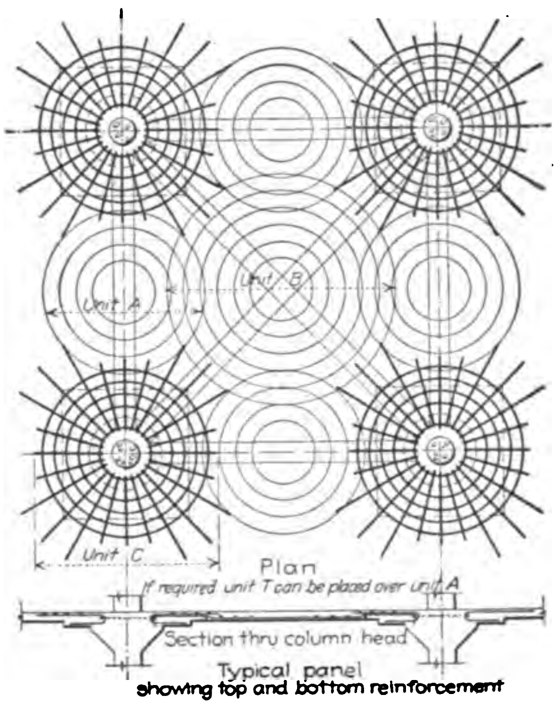

Unit A
Unit B
Unit C
Plan
If required unit T can be placed over unit A
Section thru column head
Typical panel
showing top and bottom reinforcement

Fig. 51.

The trussed bar extends into the column head a sufficient distance beyond the point of maximum stress (i.e., the edge of the column head) to develop, in combination with the hook, their full tensile strength. The ring which they engage serves to distribute the bearing stresses laterally on to a large area of concrete.

Position of Units.—Units *A* and *B* are placed near the bottom while unit *C* is near the top of the slab.

Compression Reinforcement.—By introduction of compression reinforcement in the shape of lower prongs of the radials, the slab is stiffened at the support, and the compression stresses in concrete reduced. If desired, therefore, it is possible to omit the drop panel at the column head and use an altogether flat ceiling. This is often desirable either for the sake of appearance or to simplify shafting or piping.

Secondary Reinforcement.—Sometimes to prevent cracks on the top of the slab between columns, additional secondary reinforcement consisting of short straight bars, and called Units *T*, is used. These bars are usually placed after the concrete of the slab is poured.

Fig. 52.

Theoretical Discussion

The scientific basis of the S-M-I system is evident from the following discussion of the action of a flat slab under load.

Shape of the Slab after Deflection.—After deflection a flat slab assumes a composite shape, namely, the shape of an umbrella at the column head, and the shape of a saucer in the central portion.

Lines of Equal Deflection.—The shape of a deflected slab can be seen better from Fig. 53, which shows in sections the deflection curve along the side and the diagonal of the panel, and in plan the lines of equal deflection. The lines of equal deflection, which are based on tests, were obtained by connecting the points which deflected an equal distance below their original position.

Direction of Stresses and Reinforcement.—By referring to the plan and the sections, it is evident that deformation of fibers are equal and therefore the fiber stresses act perpendicularly to the lines of equal deflection as indicated by arrows in Fig. 53. The best method of resisting these stresses, or preventing the deformation, is either by placing the bars perpendicularly to the lines of equal deflection, or by enclosing them by means of a ring, the hooping action of which is explained later. Fig. 54 shows the deflection lines in light dash lines and the reinforcement according to the S-M-I system in heavy lines. The radials and trussed bars are perpendicular to the lines of equal deflection. The rings either intersect the deflection lines at angles close to 90 deg., or they enclose the same and prevent the enclosed concrete from spreading. The combination therefore fulfills all the requirements of efficient and economical reinforcement.

Continuous Construction Separated into Simple Parts.—In continuous beams and slabs the bending moment at the points of inflection is zero. Therefore, it is possible to separate the structure at these points without chang-

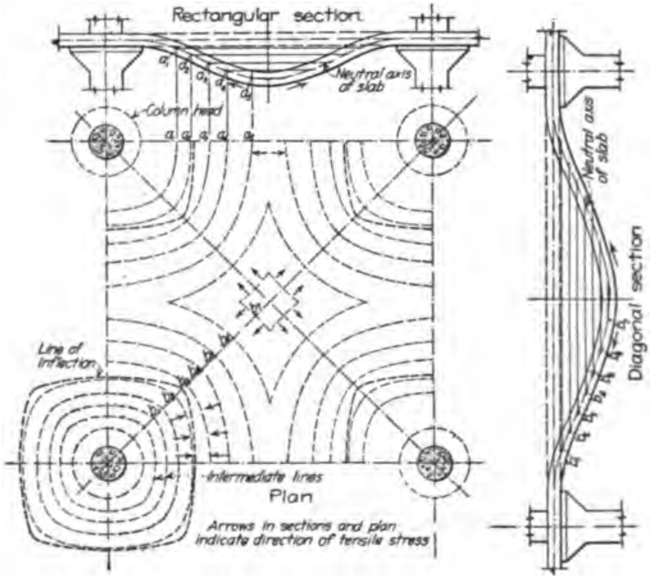


FIG. 53.

ing the stresses in the remainder of the structure. This may be accomplished by insertion of a hinge or other connection capable of transferring shear. The same is possible in flat-slab construction.

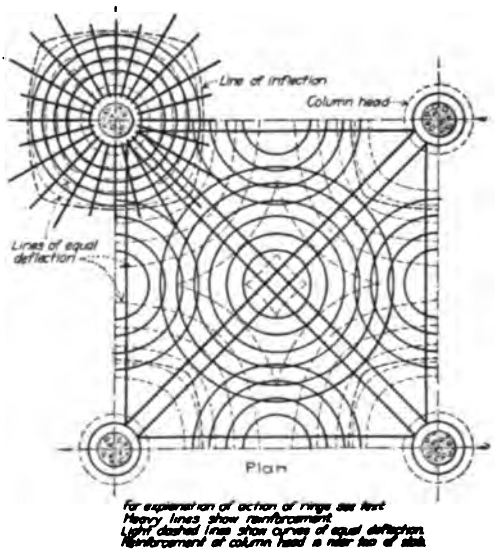


FIG. 54.

Separating Flat Slab into Simple Parts.—As explained above, a flat slab may be separated into simple parts, namely: Circular cantilevers at the column head; slabs between columns; and slabs supported at four points subjected to stresses in all directions.

In designing the reinforcement it is permissible to treat the separate parts independently and to provide in each of them a sufficient amount of steel to resist the particular bending moments to which they may be subjected.

The unit shear at the points of inflection is always low, not exceeding 40 lb. per sq. in., so that concrete is capable of taking care of the shearing stresses. Since it is not advisable to rely on concrete alone, the parts of the slab subjected to positive bending moment and reinforced by Units *A* and *B* are tied securely to the circular cantilever at the column head by the bent portions of the trussed bars and by overlapping of Units *A* and *C*.

The position of the points of inflection is variable for different positions of the live load. To provide for this and also to prevent secondary cracks due to temperature and shrinkage, the reinforcement in the various units overlaps, thereby tying the slab together and enabling it to act as a whole if such action is required by any contingencies.

Column-head Section.—At the column head the portion of the slab within the points of inflection acts like a circular cantilever loaded uniformly over its area, and also along its circumference by the loads transferred to it from the rest of the slab. This portion is subject to negative bending moments: *i.e.*, the particles in the upper part of the slab elongate while the particles in the lower part compress.

The negative bending moment at the column head is larger than the positive bending moment in the center of the slab. The amount of steel required there is consequently larger than in any other part of the slab.

The most unfavorable condition of loading for the column-head section is when all the spans surrounding the column are loaded. In such case the shape of the cantilever will be as shown in Fig. 55. Since after deflection any circle increases its radius as well as its circumference, the particles must elongate in radial as well as in circumferential direction and are therefore subjected to radial and circumferential stresses. The most effective tensile reinforcement is by means of rings and radial bars.

Compressive stresses act also in radial and circumferential direction. The compression acting radially composes the bulk of compressive stresses and may be resisted by the compressive prongs of radial bars in combination with concrete. The efficiency of steel in resisting compression is well established by tests of Prof. Withey in America and Prof. Bach in Germany. These tests are described in Taylor and Thompson's "Concrete, Plain and Reinforced, 3d Edition.

Slabs Between Columns.—The principal stresses in this part act mainly in one direction which at first is parallel to the edge of the panel and then gradually becomes inclined, as is evident from Figs. 53 and 54. In addition secondary stresses due to cross-bending and also due to shrinkage and temperature changes act across the principal stresses.

The advantages of using rings in this part to resist the various stresses are as follows: (1) They intersect the lines of equal deflection more nearly at right angles than straight bars (see Fig. 54); (2) they bind the Units *A* and *B*, thereby preventing secondary cracks; (3) the rings in the two units supplement each other; and (4) the arrangement is economical as the rings cover the whole surface without waste of material.

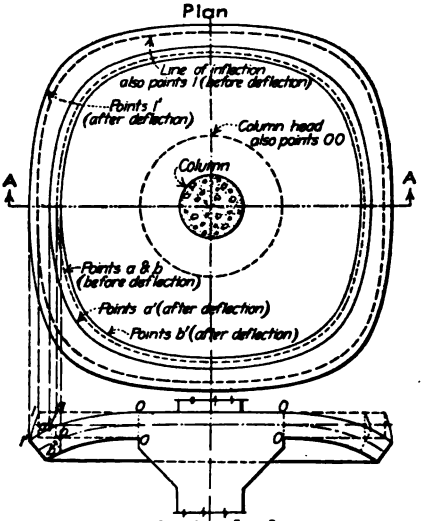


Fig. 55.

Central Part of Slab.—The central portion acts like a slab supported at four corners and loaded with uniform load. The bending moment is positive so that the top is in compression and the bottom in tension. As evident from Fig. 53, the stresses act in all directions; the reinforcement consisting of rings, therefore, is fully effective.

Action of Rings.—The following discussion demonstrates that rings resist effectively stresses acting within the ring in any direction.

In considering the action of rings it must be remembered that they are filled with solid concrete which governs their shape. The deformation and stresses in the rings are caused by the pressure of the concrete on their circumference so that the rings assume the shape of the solid concrete which they enclose.

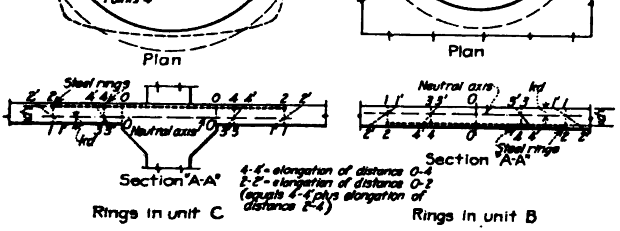


Fig. 56.

Fig. 56 shows parts of the slab at the column and also in the center of the slab, subjected to stresses in all directions. *AA* is a section in any direction.

Under load the slab compresses near one surface and expands near the other surface so that 0–3 shortens by 3–3′, while 0–4 expands by 4–4′. The same is true of a section in any direction; therefore the circle 4–4 tends to assume the shape of 4′–4′. Before assuming the new position the concrete must stretch the ring by which it is enclosed; *i.e.*, increase its radius and therefore its circumference. The concrete exerts a pressure along the circumference of the ring similar to the pressure of water in a reservoir. Since the modulus of elasticity of the steel is different from that of the enclosed concrete, the steel ring by its tensile resistance prevents partially the movement. It stretches, however, to some extent, causing tensile stresses in the steel.

Considering the second ring, it is evident that the movement of point 2 consists not only of the elongation of the distance 0-4 but also of the elongation of the distance 2-4. The outside ring therefore shares the stresses with the ring inside. Any deformation of the concrete irrespective of its direction is taken up at once by all the rings placed outside of the place of deformation. All rings are therefore effective in resisting stresses (see also Art. 19).

Forces Acting in All Directions.—Where the forces act in all directions as in the center of the slab and at the column head, the ring stretches uniformly along its circumference. After deformation the shape of the ring remains substantially circular and the stresses are uniform along its circumference.

Forces Acting Principally in One Direction.—If the forces act principally in one direction, as in unit *A*, the condition is similar to that of a solid disc of concrete with a tight-fitting steel ring around it, subject to a force in one direction. Under the pressure of the enclosed concrete the shape of the ring changes gradually into an oblong curve with the concrete following and still pressing tightly on the ring. In this case the stresses in the ring are a maximum at the sections cut by a diameter perpendicular to the direction of the stress, and decrease to zero at points 90 deg. from the point of maximum stress.

From the above it is evident that the stresses due to the principal bending moment are small in the parts of rings of Unit *A* which are near the column head, so that they can resist stresses in the diagonal direction in places where they run almost parallel to the diagonal trussed bars.

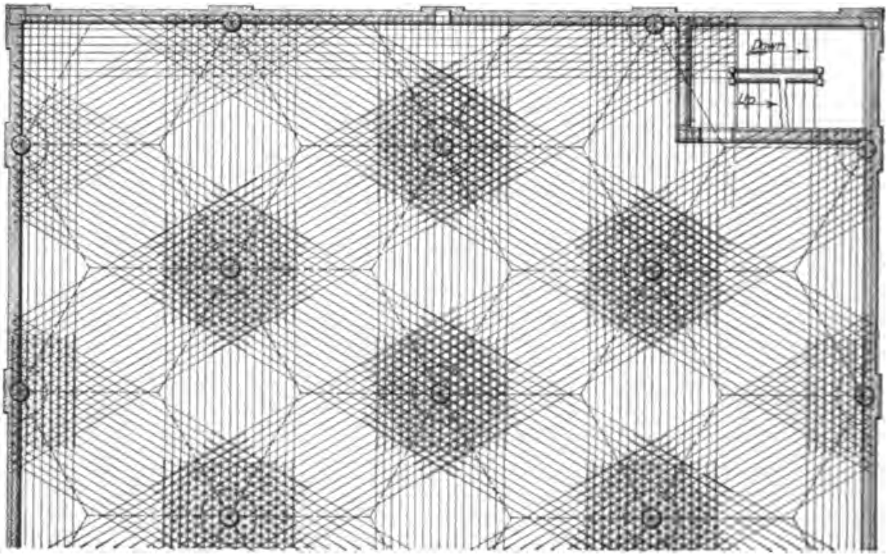


Fig. 57.

The advantages claimed for this system by its sponsors are: (1) An economy of steel over other systems; (2) freedom from obstruction over the column head, an advantage when structural-steel column cores are used; and (3) ease of pouring column head and slab due to the large diameter and comparatively small number of rods used.

The bending of the circular slab rods may be done in a shop and the large rings shipped in two parts with a liberal lap provided, or it may be bent on the job with a form of tire bender.

This system has been used in the construction of about 70 buildings to date, all of which have given satisfactory service.

17*i*. Three-way System.—The three-way system was invented and patented (Patent No. 1,064,850) by David W. Morrow, Civil and Architectural Engineer of Cleveland, Ohio, and has been successfully used in buildings for the Cleveland Railway Co. of Cleveland.

The principal novelty of this design lies in the arrangement of interior columns which are located at the apices of equilateral triangles. Under this arrangement the bands of steel reinforcement are all of equal span. Fig. 57 is a plan of a building laid out on this system, Fig. 58 shows the detail at the column head for the same design.

The advantages claimed for this system are as follows: Right-angle turns are avoided and in passing from one aisle to another it is only necessary to turn through an angle of 60 deg., thus making it easy for the running of vehicles or the free movement of overhead carriers handling long material.

The reinforcing steel over the column head is placed in three layers, thus giving it a possible slight advantage over the four-way system in effective depth at this point.

The interior column spacing being equal, the spans and consequently the amount of steel and length of rods in the bands will be equal.

It is claimed that the three-way system is particularly adapted to garages on account of the ease with which a car may be turned into one of the aisles between columns, the plan being, of course, to park the cars in the diagonal aisles.

The following data with regard to one of the buildings designed, according to this system, has been furnished by Mr. Morrow and may be useful to those who may wish to compare this type with others.

The floor, in what is known as the storeroom, is 184 ft. long and 120 ft. wide, the columns being spaced 23 ft. c. to c., giving 20 ft. wide longitudinal and diagonal aisles. It was designed for a live load of 350 lb.

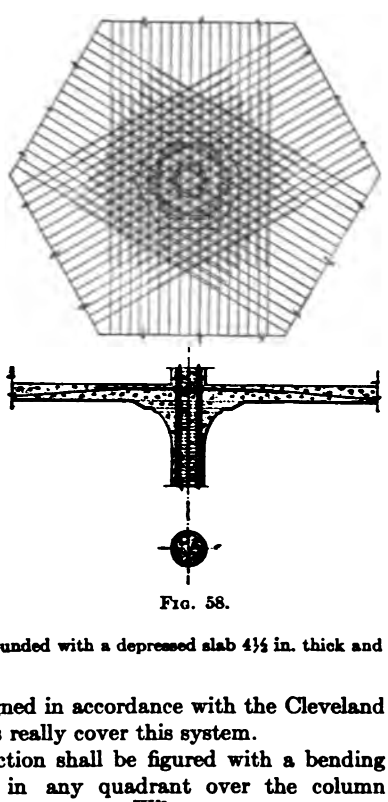


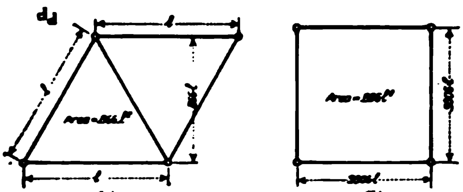
Fig. 58.

The floor slab is $10\frac{1}{2}$ in. thick reinforced with twenty-one $1\frac{1}{4}$-in. round rods per band, spaced 6 in. c. to c., all rods lapping over the column head and extending 5 ft. 6 in. beyond the center of columns, the rods being approximately 34 ft. long.

The floor slab is supported on 20-in. spirally reinforced columns with a flaring cap 5 ft. in diameter, and a hexagonal drop panel $4\frac{1}{2}$ in. thick and 8 ft. across. The outside edge of the slab rests on a 17-in. brick wall with pilasters at column points and is bounded with a depressed slab $4\frac{1}{2}$ in. thick and 4 ft. wide.

Buildings designed by Mr. Morrow have been designed in accordance with the Cleveland Building Code and, of course, none of the building codes really cover this system.

The Cleveland law requires that flat-slab construction shall be figured with a bending moment in any quadrant over the column head of not less than $\frac{Wl^2}{27}$ in foot-pounds, in which W equals total weight per square foot, dead and live load, and L equals length in feet of a side of an equivalent square in rectangular panels, and the side of a square in square panels. The length L shall be taken center to center of columns. For the three-way system, L is taken as the side of a square which has the same area as a parallelogram panel (see Figs. 59a and 59b).



(a) (b)

Fig. 59.

The bending moment of the Cleveland Code in the quadrant over the head is $\frac{WL^2}{27}$. Therefore the bending moment in the sextant $= \frac{2}{3} \times \frac{WL^2}{27}$ and substituting the value of L in terms of l, we have for the bending moment in the sextant, $M = \frac{Wl^2}{50}$ at the column cap. The bending moment at the center of span is taken to be one-half of the above, or $M = \frac{Wl^2}{100}$.

Mr. Morrow has also figured this system on what might be called a cantilever and suspended-span method. In this he assumes the line of inflection to be out three-tenths of the span from the center of column and that the minimum size of the column cap is two-tenths of the span.

The bending moment on a sextant at the edge of the head by this method is $M = \frac{Wl^2}{42}$, which is more conservative than the Cleveland Code.

By the Pittsburgh Ruling we would have $M = \frac{WL^2}{16}$ for the center of span for the entire periphery which would be equal to $\frac{WL^2}{96}$ for a single band at the center of span. The method of computing the steel required over the column head by the Pittsburgh Ruling depends upon the size of column cap, but since the steel in the center of span is usually the determining element, it is evident that this ruling checks the Cleveland Code quite closely in this case.

In all the buildings so far constructed under this system, a depressed head has been used to assist in taking care of the bending stress and shear in the concrete at the edge of the column cap. It has been the practice to lap all rods over the columns, extending them three-tenths of the span beyond the center.

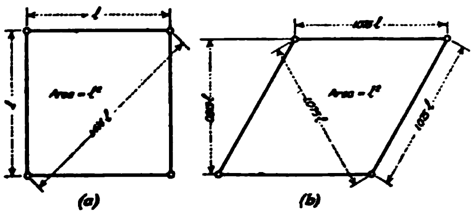


FIG. 60.

A computation made by Mr. Morrow would tend to show that the bending moment in the three-way system is less than that in a four-way system and that the diagonal bands are less efficient than the direct bands in the four-way, while the equal spans and equal loads of the three-way system are probably more efficient. The demonstration of this is as follows.

Comparing two panels of equal area [see Fig. 60(*a*) and (*b*)] the one (*a*) a square panel supported by the ordinary arrangement of columns, and the other (*b*) a parallelogram panel such as is found in the three-way system. Let (*a*) represent a panel having four-way reinforcing. Then the loads in the panel are carried to the columns by four half bands on the sides and two full bands on the diagonals, making four full bands per panel.

Let (*b*) represent a parallelogram panel having three-way reinforcing. In this case the loads are carried by four half bands around the edge, and one full diagonal band, making three full bands per panel. The average span of the bands in the square panel is

$$\frac{2 \times 1.414L + 2L}{4} = 1.207L$$

The span of the bands in the parallelogram are all of the same length being $1.075L$.

Assuming that the bending moment varies as the span, we can make a comparison of the bending moments in the two panels by comparing their average span:

$1.207L$ = average span in square panel.

$1.075L$ = average span in parallelogram panel.

$$\frac{0.132L}{1.075L} = 0.123 = 12.3\%$$

Therefore, if the above assumptions are correct, the bending moment in a square panel is 12.3% more than the bending moment in a hexagonal panel carrying the same load.

The question naturally arises as to whether or not one-half of the load of the panel goes to the diagonals.

If we lay out a square panel so that the sides of the diagonal bands intersect on the sides of the square bands, all bands being equal to $0.414L$ in width, and figure the area in the panel falling in common over side and diagonal bands, we will find them to be as follows:

Area in panel falling in common over side and diagonal bands, 48.5%.

Area in panel falling over side bands, 17.2%.

Area in panel falling only over diagonal bands, 34.3%.

This would apparently tend to show that a large portion of the load falls on the diagonals and that they, due to their long span, are not able to carry it and that it is transferred to the side bands producing an inverse moment over their center. As is well known, cracks usually appear at these points in loading tests of flat-slab floors which are not reinforced to resist this moment.

18. Patents.—O. W. Norcross, on Nov. 22, 1901, applied for a patent on a "new and useful flooring for buildings." On April 29, 1902, he was granted Patent No. 698,542 covering the following claims:

1. The combination of separate posts or supports, and a flooring consisting of a metallic network formed by strips of wire netting enclosed therein, so as to radiate from the posts or supports on which the floor rests.
2. A flooring resting on separate supports and consisting of concrete with metallic network so arranged therein that the amount of metal will be greatest at the points where the greatest tensile and shearing strains are to be supported.
3. A flooring resting on separate posts, and consisting of metallic network formed by strips of wire netting laid from post to post, to cross each other in cob-house fashion, and concrete enclosing the metallic network.
4. A flooring resting on separate posts, and consisting of metallic network formed by strips of wire netting laid from post to post, and in the diagonals of the figures outlined by the posts, and concrete enclosing the metallic network.
5. A flooring resting on separate supports, consisting of concrete having a metallic network enclosed in the bottom layer thereof, with the body portion of said concrete of lighter material than the bottom layer thereof.

On the same date, April 29, 1902, Patent No. 698,543 was also granted to Mr. Norcross, covering flooring for buildings, the claims of which are similar to No. 698,542 except that instead of columns, walls or other longitudinal supports are specified.

Norcross Patent No. 698,542 was sustained by the United States Circuit Court of Appeals for the Eighth Judicial District on Dec. 10, 1914, in the case of John L. Drum *vs.* C. A. P. Turner. Upon petition of defendant, that Court decided not to reopen the case or to permit the introduction of additional evidence offered by the defendant. On June 1, 1915, the Supreme Court of the United States refused to set aside the decision of the lower court or grant the defendant any relief whatever. On June 9, 1915, the United States District Court for the District of Minnesota, entered a decree against the defendant, C. A. P. Turner, for infringement of the Norcross patent and an injunction against further infringement.

On Aug. 3, 1915, the United States District Court for the District of Minnesota, held that Mr. Turner's "Spiral Mushroom System" was an infringement of the said Norcross patent, and imposed a fine for the violation of the injunction. On Jan. 21, 1916, the Circuit Court of Appeals for the Eighth Circuit unanimously refused to reconsider the decision of the District Court on the question of infringement, and denied Mr. Turner's petition for a writ of certiorari.

On Oct. 4, 1916, the United States District Court for the District of Minnesota in the case of C. A. P. Turner, plaintiff, and Deere Webber Building Co. and Deere Webber Co., defendants, decided "that the defendants' structure does not infringe any of claims 1, 2, 4, 6 or 8" of C. A. P. Turner Patent No. 985,119, and "that said claims and each of them are void for lack of invention in view of the prior art, as held in the case of Turner *vs.* Moore supra."

In the United States District Court in the District of New Jersey in the case of C. A. P. Turner, plaintiff, *vs.* Lauter Piano Co. and American Concrete Steel Co., defendants, the Court held that the claims of Patents, Nos. 985,119 and 1,003,384, granted to the plaintiff and which were relied upon by him in this suit are invalid.

As the matter now stands, therefore, Mr. Turner is under injunction both as to his original construction and his so-called "Spiral Mushroom System."

The leading flat-slab promoters in the United States are now licensed under the Norcross patent. This is true of two-way, three-way and four-way flat-slab advocates alike.

Besides the Norcross patent which has been declared basic, there are several other patents which have been granted covering special methods of construction and reinforcement. Some of these have been tested in the courts. It is evident, therefore, that the field for the designer of flat slabs is not a free one and, unless he can invent a method of reinforcement that is entirely new and an advance in the art, he must be licensed under one of the "Systems" so-called. The possibilities are so well covered that it is difficult to see how a really new system can be devised. Rather should we seek to advance by improving our materials or methods of construction, or both.

19. Loading Tests.—Since the first introduction of the flat-slab type of construction there have been erected in the United States many hundreds of buildings of this kind. The majority have been subjected to test loads over one or more floor panels of at least 1¾ times the designed live load, and have passed the test without cracking or serious deflection. There have been a few cases of flat-slab floors which have given trouble just as there have been cases of poor design with other systems, but it is undoubtedly true that the flat-slab type will stand more abuse in this respect than any other form of reinforced-concrete construction. The continuous bands of small steel rods running in two or more directions and all passing over the column heads form a network of remarkable strength. When a structure of this type is loaded to destruction it shows a very slow yielding, and it is safe to say that a properly designed flat slab with continuous bands of steel passing over the column heads cannot show a sudden collapse either under test or in actual service.

Besides the loading tests before referred to, which have been made to satisfy architects and building superintendents, in which at most the deflection of the floor slab only was measured, there have been numerous extensometer tests (so-called) made of complete buildings under a variety of conditions of loading.

In these tests a portion of the floor slab in a completed building has been subjected to load very much as a beam is tested in a testing laboratory, and the actual deformations of exposed portions of the reinforcing steel at various points in the slab have been measured. Also, measurements have been made of the deformations of the concrete of the slab. Measurements of the deformations of the steel and concrete in both the exterior and interior columns have also been made in many of the recent tests.

Much of this information has been held as confidential by various designers, but the results of many such tests have been published and are thus available for the use of all. Tests of this kind giving as they do actual distortion in the individual steel rods and in the concrete itself, give a very accurate indication of the stresses in the structures at the time of making the test due to the applied load. Of course, there are initial stresses due to dead load and temperature which can be computed with more or less accuracy and stresses due to the shrinkage of concrete, which are more uncertain, which must be allowed for in addition to the live-load stress.

The performing of tests of this kind entailed a great amount of physical labor in their actual execution. A high order of ability was shown in planning and organizing the operation, and an equally high degree of skill was required to secure satisfactory results in the use of laboratory instruments under such trying conditions. The cost of these tests was great, but the value of the results obtained and the advance given to the art of design has justified the cost in money and in personal sacrifice.

A list of the more important tests relating to flat-slab floors is given below:

Experiments on Models.—An interesting series of experiments upon laboratory models was performed by the research department of the Corrugated Bar Co. to obtain a scientific basis for the design of flat-slab floors. A rubber plate 0.5 in. thick was stretched over a box which had cylindrical plugs projecting from the bottom to just touch the rubber sheet, so spaced that they would, acting as miniature columns, divide the slab up into nine equal panels. Half rounds were used at the sides to act as wall columns. Load was applied by exhausting air from the box so that the rubber sheet deflected just as a flat slab is supposed to do and this load was maintained very exactly by means of a water jet aspirator and U-tube manometer. Deflection readings were taken at points 0.1 of the span apart.

Test of Flat-slab Floor of the Deere & Webber Co.'s Building, Minneapolis, Published December, 1910.—The type of floor is four-way and is typical of the standard practice of the Concrete Steel Products Co. at that time. Deformation measurements were made on concrete and steel in the slab only. Tests are described by Arthur R. Lord in papers entitled "A Test of a Flat-slab Floor in a Reinforced Concrete Building" and "A Discussion of the Basis of Design of Reinforced-Concrete Flat Slabs," presented before the Annual Convention of the National Association of Cement Users (Am. Conc. Inst.). These articles appeared in *Engineering News*, Dec. 22 and 29, 1910 and Jan. 12, 1911.

Test of Franks Building, Chicago.—Floors were constructed according to the Cantilever Flat-slab system by the Leonard Co. of Chicago. The panel size was 19 ft. 4 in. by 20 ft. 3 in. Four interior panels located on the tenth (top) floor were tested. The slab thickness was 9¼ in., and 13¼ in. at the drop panels.

Test of Powers Building, Minneapolis.—Described in a paper by F. J. Trelease read before the American Concrete Institute in March, 1912 (vol. viii, *Proceedings*).

Test of Larkin Warehouse.—Test of a four-way slab described in a paper presented by Arthur R. Lord before the American Concrete Institute in December, 1912. Test was also published in the *Engineering Record*, January, 1913 and in the *Cement Era*, January, 1913.

Test of Soo Line Terminal, Chicago.—Test of single-story structure which carries railroad tracks on the upper deck. For complete description of this building see *Engineering Record*, Aug. 16, 1913, *Engineering News*, Aug. 21, 1913, and *Railway Age Gazette*, Aug. 22, 1913. For description of test see article in *Bulletin* 84 of the Engineering Experiment Station of the University of Illinois.

Test of Schulze Baking Co.'s Building, Chicago.—Floor slab is of the four-way type with short transverse bars in top of slab on the center line of columns. Test is described in *Bulletin* 84 of the Engineering Experiment Station of the University of Illinois.

Tests of S-M-I Flat Slabs.—Test made under the supervision of Sanford E. Thompson of a slab 20 ft. square supported upon columns spaced 12 ft. on centers so that the slab projected 4 ft. on all sides in order that a condition of continuity over the columns would exist.

Test of Shredded Wheat Factory Niagara Falls, N. Y.—Type of floor is two-way, designed by the Corrugated Bar Co. and known commercially as Corr-plate Floor. Test is described in *Bulletin* 84 of the Engineering Experiment Station of the University of Illinois. It is also reported in the *Journal of the American Concrete Institute*, vol. ii, No. 6, 1914, in an article by W. A. Slater.

Tests of Circumferential Cantilevers.—Tests were made upon octagonal slabs each carried upon an 8-in. square column having an octagonal flare head 2 ft. in diameter. For description of tests see *Engineering Record*, vol. 73, page 249.

Worcester Slab Test.—The structure upon which this test was performed was constructed especially for that purpose at Worcester, Mass. The objects were to determine the effect of different steel arrangements and the effect of variation in size of the column capital. A complete description of this test will be found in *Bulletin* 84 of the Engineering Experiment Station of the University of Illinois.

Test of Schwinn Building, Chicago.—The floors are of the four-way type and were designed before the present Chicago ruling was adopted. Description of this test is given in an article by Arthur R. Lord in vol. xiii of the *Proceedings* of the American Concrete Institute, 1917.

Test of Curtis-Ledger Factory, Chicago.—Floor tested was designed according to the Barton Spider Web system. Summary of this test is given in *Bulletin* 84 of the Engineering Experiment Station of the University of Illinois.

Test of Sears-Roebuck Building, Seattle.—Floor was designed according to the Akme Two-way system. Test was performed under the supervision of the Building Department of the City of Seattle and is reported in the *Proceedings* of the PacificNorth west Society of Civil Engineers, vol. xv for January and February, 1916, in a paper presented by D. E. Hooker.

Test of Bell Street Warehouse, Seattle.—The type of floor used in this structure is a four-way system designed according to the Mushroom system by C. A. P. Turner. The first test made on this building is described in an article by D. E. Hooker in *Engineering Record*, vol. 73, page 647.

Since the slab tested was cast in freezing weather and the temperature near the freezing point for several days, it was claimed by the designers that the concrete was not thoroughly cured at the time of making the test and that, therefore, conclusions drawn from the test were unwarranted. In order to satisfy all concerned a second test was made upon four panels of the same floor slab previously tested. Description of the second test is given in *Engineering News—Record*, April 19, 1917.

Test of Building of Pierce Arrow Motor Car Co., Buffalo, N. Y.—Floor is a four-way system designed according to the Chicago Code.

Test of S-M-I Flat Slab at Purdue University, April 19, 1917.—Test was performed upon a structure erected for the purpose and did not form a part of a building built for commercial use. Test was reported by Prof. F. K. Hatt of Purdue University to Edward Smulski.

Discussion of Tests.—To those who are familiar with the accurate results which are to be obtained in laboratory testing, the results secured in field tests of buildings are apt to be a disappointment.

Many factors enter into these tests which cannot be entirely controlled and inconsistencies develop in the measured results which cannot always be explained. In the first place the structure is composed of two materials of radically differing properties. The steel possesses definite qualities, properties, and areas which can be accurately measured. Its accurate location in the structure also can be secured by the exercise of proper care. The concrete, on the other hand, as at present manufactured, does not possess qualities which can be accurately forecast either in similar structures or in different parts of the same structure and, as is well known, those which it does possess are subject to change with time, but even this change cannot be known in advance. Accurate test results are, therefore, not to be expected.

The tested slab in the majority of cases form but a small portion of the floor and the distribution of stress to the surrounding portions has a modifying effect upon the results. Also, the relative size and stiffness of the columns above and below the floor will cause differing results in the tests of floors otherwise similar. Shrinkage and temperature changes produce effects which are difficult to measure and eliminate, and additional complications are introduced by settlement of the columns, either due to yielding of the foundation or to shortening in the columns themselves under direct load.

A brief summary of the more important results of tests is given below:

1. Tests of wide beams and later of flat-slab buildings proved that a beam may be made much wider than the support, with only a moderate loss of efficiency.

2. Tests of models and of actual buildings have demonstrated the advisability of transverse reinforcement placed on the edge of the panel in the top of slab, particularly where no drop panel is used.

3. For the true flat slab, *i.e.*, where no drop heads are used, the critical section so far as the steel is concerned, is at the edge of the column capital.

4. With sections and reinforcement as specified under the Chicago ruling, surprisingly low stresses have been measured in the steel. This is particularly true of slabs having drop panels.

5. Eccentric action of the load produces a marked bending action in interior and particularly in exterior columns which should be reinforced accordingly. Columns for single-story structures, where unbalanced live load is carried on the roof, should receive attention in this regard.

6. Footings for single-story structures of this class should be designed with a liberal factor of safety as the danger of settlement is apparently greater in this class of structure.

7. Structures with relatively thick slabs show low steel stress as compared to thinner slabs designed in the same manner.

8. Stresses due to dead load can be measured if proper precautions are taken during construction.

9. Different arrangements of load have different effects upon the same panel and single-panel loading does not necessarily develop the greatest stresses.

10. In flat-slab design, deflection rather than stress in the steel controls because large deflection results in serious cracking of the slab. Floors designed under the Chicago ruling do not exceed one-eight-hundredth of the span and most of them show but slightly more than one-half of this amount.

11. Relative deformations in the concrete are of value as a comparison, but as an accurate indication of stress they have little weight due to lack of uniformity in the elastic properties of concrete.

12. Buildings designed according to the flat-slab rulings of any of our better cities, under any of the systems, and constructed with a reasonable amount of intelligent supervision, give entire satisfaction in service. Of these rulings, that of Chicago is the best and most conservative.

13. Many more tests on four-way reinforced slabs are available than of other systems and for this reason comparisons, which are largely a matter of judgment, cannot be made with certainty. Analyses of published data made by a number of engineers would indicate that given the same span, load, and concrete sections, the circumferential system will show the lowest stress per pound of steel, the four-way system next, and the two-way system last. This is not necessarily their order with regard to total economy, however. Bending and placing costs and the unit price of steel are contributing factors which modify the final result.

14. The ruling adopted by the American Concrete Institute meets certain objections to the Chicago ruling and harmonizes better in some respects with the results of later tests. It is recommended for use as the best that is now available.

15. The ruling adopted by the Special Committee of the American Society of Civil Engineers, while theoretically sound in some respects, is incorrect in others, and adopts coefficients which are unnecessarily severe and not justified by the facts. This ruling, if generally adopted, would, in the author's opinion, impose a needless burden on the industry.

The following very able analysis of this subject based on four-way tests is taken from a paper by Arthur R. Lord, Structural and Testing Engineer of Chicago, read before the Southwestern Cement Association at Kansas City, Feb. 22, 1917.

The total positive and negative moment is taken in the A. C. I. report as 0.09 $wl_1(l_2 - qc)^2$ and the division of this moment between the various sections is given in Table I. The Joint Committee report increases the total moment to $0.107wl_1(l_2 - \frac{2}{3}c)^2$ and allows a much less flexible design than the A. C. I. report. Reduced to the same basis the Chicago Ruling total moment is $0.092wl_1(l_2 - \frac{2}{3}c)^2$.

TABLE I[1]

Code	Type of flat slab	Total − and + mom., coef.[2]	Total − mom., %	Total + mom., %	Distribution limits				
					Col. head section, %	Middle section, %	Outer section, %	Inner section, %	Sum, %
1	2	3	4	5	6	7	8	9	10
Chicago	4-way	0.087	66.7	33.3	53.2	13.4	20.0	13.4	100
Ruling...	2-way	0.093	62.5	37.5	50.0	12.5	25.0	12.5	100
A. C. I.	Drop	0.090	70.0 to 60.0	30.0 to 40.0	60.0 to 50.0	10.0 to 20.0	18.0 to 28.0	12.0 to 22.0	100
report	No drop	0.090	70.0 to 50.0	30.0 to 50.0	60.0 to 40.0	10.0 to 30.0	18.0 to 38.0	12.0 to 32.0	100
J C	Drop	0.107	62.5	37.5	50.0	12.5	28.1 to 22.5	9.4 to 15.0	100
report	No drop	0.107	62.5	37.5	50.0 to 40.6	12.5 to 21.9	28.1 to 20.6	9.4 to 16.9	100

Total Moment Requirement.—It will be seen from Table I that the A. C. I. report and the Chicago Ruling adopt the same total moment coefficient. In passing it may be mentioned that this coefficient is the highest called

[1] Table taken from discussion by T. L. CONDRON at the A. C. I. convention at Chicago, Feb. 8, 1917.

[2] Coefficient in formula, Total mom. = $Kwl_1(l_2 - gc)^2$.

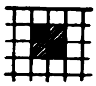
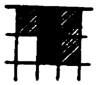
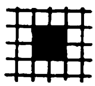
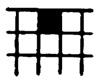
for in any building code in the United States so far as I know and the highest heretofore used in practical designing. It represents the most conservative past practice. The J. C. report increases this moment 18.5%. Let us investigate what extensometer tests of buildings erected under Chicago regulations show as to the actual stresses developed in flat slabs. Table II gives a summary of four Chicago tests, all of four-way flat slabs of the drop type—that is, with

TABLE II

Test	Stresses over column head[1]				Steel stresses			
	In steel		In concrete		At panel center[2]		Top rods	
	Test[3]	Chicago[4]	Test[5]	Chicago[4]	Test[3]	Chicago[4]	Test[3]	Chicago[4]
1	2	3	4	5	6	7	8	9
Franks	9,200	23,700	1,110	1,400	10,100	31,000		
Larkin	8,500	23,000	800	1,120	16,000	37,000		
Schulze	7,100	22,500	530	1,230	6,200	34,000	4,900	71,500
Schwinn[6]	9,300	38,000	300	1,160	18,900	39,000	9,900	70,000

a greater thickness adjacent to the support than in the slab proper. All these floors were loaded to or in excess of twice the total dead and live design load as indicated in Table III. It certainly seems to me that these tests indicate eminently safe and conservative design. The average steel stress at the support measured about 9000 lb. per sq. in. against a computed stress of about 27,000 lb. per sq. in., and the concrete stress about 700 lb. per sq. in. against a computed value of about 1250 lb. per sq. in. At the panel center the average steel stress was about 13,000 lb. per sq. in. against a computed value of 37,000 lb. per sq. in. The concrete stress at the center is always very low—less than half that at the supports. The average stress in the top rods was less than 8000 lb. per sq. in. against a computed value of over 70,000 lb. per sq. in. Taking into account the possible and reasonable effect of various loadings and of long-continued loading, it would appear that a factor of safety of four was largely exceeded.

TABLE III

Test	Panel size		Slab thickness		Test load: live dead	total	Area loaded	Twice design load in place	Floor tested
	Length	Breadth	Slab	Drop					
Franks	20′-3″	19′-4″	9.0″	13.00″	624 115	739		24 hr.	10th (top)
Larkin	24′-2″	20′-0″	9.0″	15.75″	618 120	738		24 hr.	2d
Schulze	20′-0″	17′-6″	9.0″	15.00″	722 122	844		24 hr.	1st
Schwinn	26′-0″	25′-0″	10.0″	16.00″	450 125	575		379 days	6th (top)

[1] Average of readings in fully loaded area.
[2] Average of readings on direct and diagonal bands.
[3] Observed steel stresses as reported in tests.
[4] Stresses calculated by Chicago Ruling for a design load equal to applied test load.
[5] Concrete stresses reduced to 80% of those reported for the tests and computed on the basis of the initial modulus of elasticity.
[6] Stresses given are for readings taken with full test load in place over 1 year.

Table III gives information about the Chicago tests referred to in this paper. Table IV gives further data including a calculation of the various resisting moments from the measured steel stresses. In considering these tests the stress in slab steel of direct bands at the edge of the loaded area has been increased so that the stress in rods outside the loaded area has been added to the stress in the rods under the load and the total stress figured on this basis. Also, in column 12 of Table IV, all the head rods are considered as stressed as highly as the slab rods, though they lie much nearer the neutral axis of the slab. Even on this extremely conservative basis the average total moment is only $0.025WL$, equal to $0.035wl_1(l_2 - qc)^2$, as against the value of $0.09wl_1(l_2 - qc)^2$ used in the Chicago and A. C. I. Codes. It is possible that other loadings may increase even the *sum* of the moments somewhat and that very long continued loading may have its effect. One test involved a full year under load and the increase in the total moment was about 7%, with no increase during the last 3 or 4 months. But taking all influences into account I do not see how any one can fairly ask for a more conservative moment coefficient than that adopted by the A. C. I. and Chicago Codes. In one test at Seattle of a flat-slab building built on a very deficient basis as compared with either of these codes, and in which as a result exceedingly high stresses were developed, reaching as high as 50,000 lb. per sq. in. in the steel, and in which modifying actions were reduced to a minimum, the total resisting moment on the basis of the A. C. I. Code was $0.066\ wl_1(l_2 - qc)^2$ against the code specification of $0.09wl_1(l_2 - qc)^2$.

TABLE IV

Test	Section considered	Right area of all slab rods and effective laps	Effective section, all slab rods and effective laps	d, for steel areas in columns 3 and 4	Right area of slab rods, laps and head rods	Effective section, slab rods, laps and head rods	d, for steel areas given in 6 and 7	Resisting moment of observed steel stresses			
								All slab rods and effective laps		Slab rods, laps and head rods	
								Right area	Effective section	Right area	Effective section
1	2	3 in.	4 sq. in.	5 sq. in.	6 sq. in.	7 sq. in.	8 in.	9	10	11	12
Franks...	Over column head.........	10.40	13.31	10.88	11.98	14.88	10.68	0.0129WL	0.0165WL	0.0149WL	0.0185WL
	Center of panel	6.88	8.33	7.62	.....	.....	.....	0.0068WL	0.0081WL	0.0068WL	0.0081WL
	Top rods										
	Sum.......	.....	.....	.....	.....	.....	.....	0.0197WL	0.0246WL	0.0217WL	0.0266WL
Larkin...	Over column head.........	11.33	13.70	13.87	12.53	14.89	13.75	0.0123WL	0.0149WL	0.0135WL	0.0161WL
	Center of panel	7.75	9.37	7.84	.....	.....	.....	0.0090WL	0.0109WL	0.0090WL	0.0109WL
	Top rods										
	Sum.......	.....	.....	.....	.....	.....	.....	0.0213WL	0.0258WL	0.0225WL	0.0270WL
Schulze..	Over column head.........	9.20	10.84	12.10	9.20	10.84	12.10	0.0105WL	0.0123WL	0.0105WL	0.0123WL
	Center of panel	5.95	7.18	7.75	.....	.....	.....	0.0038WL	0.0046WL	0.0038WL	0.0046WL
	Top rods......	1.10	1.10	7.88	.....	.....	.....	0.0006WL	0.0006WL	0.0006WL	0.0006WL
	Sum.......	.....	.....	.....	.....	.....	.....	0.0149WL	0.0175WL	0.0149WL	0.0175WL
Schwinn.	Over column head.........	13.74	16.42	10.40	13.74	16.42	10.40	0.0102WL	0.0122WL	0.0102WL	0.0122WL
	Center of panel	9.57	11.58	8.85	.....	.....	.....	0.0122WL	0.0148WL	0.0122WL	0.0148WL
	Top rods......	1.77	1.77	8.75	.....	.....	.....	0.0012WL	0.0012WL	0.0012WL	0.0012WL
	Sum.......	.....	.....	.....	.....	.....	.....	0.0236WL	0.0282WL	0.0236WL	0.0282WL

In view of the extensometer test record and the fact that hundreds of other buildings designed on the Chicago basis have indicated by deflections under test load that they have equally low stresses and moments, I believe the position of the Joint Committee is ultraconservative in this requirement.

Distribution of the Total Moment.—The question as to the amount of total moment to be specified in design is, as I view it, one of conservative *vs.* ultraconservative opinion. The distribution of the total moment, however, does not admit of so many opinions. The J. C. report makes a rigid division of the moment into 62.5% negative and 37.5% positive for any or all types of flat slab. The A. C. I. report leaves a portion of the moment to be assigned in accordance with the physical details of the slab and its reinforcement. I cannot see that there is any room for debate on this difference. If the J. C. division is right for a flat slab with a drop and supported by stiff columns, it is wrong for a flat slab without a drop and supported by relatively slender columns. The evidence of the tests also is positive and conclusive on this. For four panels loaded uniformly, the tests show a distribution of approxi-

mately $\frac{5}{8}$ − and $\frac{3}{8}$ +, with individual cases in which as high as 70% of the total moment has been negative (see Table V). These figures are for buildings with a much deeper drop than the J. C. permits. With other conditions of loading, certainly the center or positive moment would be greater and the ratio of negative moment to total moment reduced. And with no drop present, the change would be even greater as shown in the test of the Bell Street Warehouse at Seattle where only 40% of the total moment was negative and 60% positive—practically a reversal of the conditions where a large drop is used. This data gives ample basis for the A. C. I. recommendation that permits an extreme division of 50% − and 50% + where the drop is not used, and also permits from 60 to 70% negative where the drop is used. This division accords much more closely with the facts than the rigid J. C. distribution, which forces the designer to either use an unbalanced design or employ additional material where the total material already involved is more than ample but improperly distributed.

Table V

Test	Steel area in square inches			Observed steel stress			Total tension carried		% of whole	
	Two* diagonal bands	Long direct band	Short direct band	Diagonal band	Long direct band	Short direct band	Diagonal bands	Direct bands	Diagonal bands	Direct bands
Franks	10.00	3.54	3.14	6,950	7,350	10,100	69,000	57,000	54.8	45.2
Larkin	11.13	5.10	2.55	12,900	10,200	24,200	143,800	113,800	55.9	44.1
Schulze	8.48	3.45	2.55	4,220	8,400	7,200	36,000	47,400	43.2	56.8
Schwinn	13.89	5.10	4.12	21,900	16,000	18,600	304,000	159,000	65.6	34.4
Average									54.9	45.1

* Effective section.

The J. C. also assigns a fixed proportion of the moment—12.5% of the total moment—to the mid-section in flat slabs with a drop. The whole report shows no conception of the fact that the distribution of the moment is different in two-way and four-way flat slabs—a fact very clearly brought out in tests. In four-way flat slabs tested in Chicago as shown in Table II, columns 8 and 9, the calculated stress across the mid-section was over 70,000 lb. per sq. in. and the actual stress under 10,000 lb. per sq. in. even after a year. In the test of the Sears-Roebuck Building at Seattle, a two-way flat slab gives a computed stress at this section of 27,200 lb. per sq. in. and an actual stress of 23,500 lb. per sq. in. This would indicate that the moment across the mid-section is several times as great with the two-way arrangement of the steel as it is with the four-way arrangement. The J. C. assignment of moment to this section is too large for four-way slabs and too small for two-way slabs, and again the result is unbalanced design. The A. C. I. report permits a distribution that will fit either type.

Customary practice in four-way design makes a distribution of the steel between direct and diagonal bands varying from a 1 : 1 ratio to a 1.5 : 1 ratio and the results of tests as shown in Table V show that these ratios are about right. The average distribution of the moment was about even between direct and diagonal bands and this would result in a 1.4 : 1 ratio between direct and diagonal bands. The J. C. report specifies as a *minimum* that 60% of the positive moment must be taken by the outer section and this gives a minimum relation of 2.1 times as much cross-sectional steel area in the direct as in the diagonal band. Just why all previous practice is passed over and a novel arrangement required, I am at a loss to know after studying the available data.

Arbitrary Limitations.—Arbitrary limits, established in addition to the natural limitations of the moment and shear requirements, should be reduced to a minimum. With flat-slab construction the moment at the center is too small to establish a slab thickness such as we deem advisable from the standpoint of deflection and some arbitrary limit is desirable here, as found in all three codes cited above. At the support, however, the shear determines an adequate thickness and no other limit is needed.

An investigation of the Chicago buildings tested in the past does not appear to call for the limitation of $0.4L$ as the minimum drop dimension nor for one-half the slab thickness as the maximum drop thickness. These and other J. C. limitations handicap the designer in the economic use of his materials and do not seem to be founded on any experience in the past. The A. C. I. report contains all the limitations that seem necessary to safe design.

Column Moment.—The amount of moment in the column will depend on the type of flat slab and on the stiffness of the columns above and below the slab. For the Franks test, made on an upper floor and with 20-in. columns on a 20-ft. span, the observed moment was $0.0138wl_1(l_2 - qc)^2$. For small columns in upper stories of buildings the A. C. I. specification of $0.022wl_1(l_2 - qc)^2$ should be ample even where the drop type of flat slab is used as was the case in the Franks building. For lower stories the column moment undoubtedly largely increases but it would not be right to specify these larger moments as a minimum for all columns. This is a matter where the judgment and calculations of the engineer must decide on the proper increase but the specification of a definite *minimum* moment by the A. C. I. Committee seems to me a step in the right direction. The J. C. report and the Chicago Ruling leaves this in very indefinite shape, with the result that many designers acting in good faith will entirely overlook the design of the columns for bending moment.

20. Methods of Design and Problems.—Many attempts have been made to develop a scientific method of analysis of the stresses which will be developed in flat-slab floors as at present constructed. On account of the complex nature of the problem and the various modifying factors which enter into it no satisfactory method of design, so far as the writer is aware, has been presented.

A very comprehensive testing program of flat slabs has been carried out in which actual stresses have been measured (see Art. 19). With these results as a foundation, rules governing the design of such structures have been adopted by various city building departments and technical societies, and the adequacy of these rulings has been proven in turn by tests of buildings designed in accordance with their provisions (see *Appendix C*). A study of the provisions of the various rulings reveals at once the fact that all methods of design now in common use are to a large extent empirical.

The following general remarks regarding the design of flat-slab buildings may be of aid to those who are inexperienced in this field.

1. The live load for which the structure is to be designed must be established either by the architect or the engineer, or, if the structure comes under the jurisdiction of a building department of a city, the Building Code will be found to contain rules which will apply. Frequently, it is necessary to obtain a special ruling from the building department in a particular case.

2. The story heights, etc., having been decided upon with due consideration of the use to which the building is to be put, the next question to be settled is the column spacing. Frequently this is decided by the owner or the architect but in industrial buildings, the machinery to be housed may be the controlling factor. In garages and service stations of moderate width, a wide central span and narrow side panels may be necessary. However, the following general rules may be stated.

(*a*) In general the short span construction is the cheapest, but it is very seldom that spans less than 16 ft. are used, and probably a 20-ft. panel represents the average.

(*b*) The ideal design has square panels.

(*c*) The next best arrangement consists of identical rectangular panels.

(*d*) Oblique panels have been used, but their use is not recommended.

(*e*) The outer spans are frequently made shorter than the interior spans in order to make the bending moment in exterior panels equal to that in interior panels.

3. The method of design to be used for the slab depends on whether the structure comes under the jurisdiction of a city building department. If it does, and the department has adopted a ruling on flat-slab design, that ruling must be followed in so far as it applies. If no ruling has been adopted, then much time and trouble will be saved by visiting the building examiners and obtaining a statement as to just what they will accept. In cases where no city authorities have jurisdiction, the designer is at liberty to use any of the rulings which have been adopted by our larger cities or technical societies, but the ruling of the American Concrete Institute is recommended.

The following recommendations regarding designs of flat-slab floors are made by Arthur R. Lord after a thorough study of all the available test data and an analysis of the various rulings which have been adopted (see paper by A. R. Lord entitled, "Extensometer Test Data and Recommended Practice in the Design of Flat Slabs," read before the Southwestern Cement Association, Feb. 23, 1917). The intimate connection which Mr. Lord has had with the development of this branch of engineering and his thorough knowledge of the practical construction side entitles him to speak with authority and with his recommendations we are thoroughly in accord.

For engineers who desire to be conservative and who are designing for normal variations in the conditions of manufacture of the concrete in various localities, I would personally advise the use of the A. C. I. Committee report as the best and most thoroughly considered design basis now available. For engineers designing structures to be erected by the most improved means under positive controls, ensuring a superior grade of concrete, I would approve the use of the A. C. I. report with a moment coefficient of $0.08wl_1(l_2 - qc)^2$ instead of the value given in their report $0.09wl_1(l_2 - qc)^2$.

For four-way flat slabs with drops which is the economical type to use, I should advise the following distribution of the total moment:

Column-head section	50%
Mid-section	10%
Outer section (direct bands)	20%
Inner section (diagonal bands)	20%

Where no drop is used, I should advise taking from 6 to 10% (depending on the column stiffness) of the total amount away from the column-head section and distributing it equally over the inner and outer sections.

For two-way flat slabs with drops, I would advise the following distribution of the total moment:

Column-head section	50%
Mid-section	15%
Outer section	23%
Inner section	12%.

The writer agrees with these recommendations and knows them to be very conservative. With first-class construction and supervision, satisfactory results have been obtained with the four-way system using drop panels and concrete sections as given in the tables of Art. 21 under the Pittsburgh Ruling, and using low steel stresses (never over 16,000 lb. per sq.in.). This is somewhat less than the minimum recommended by Mr. Lord, both as regards concrete and steel, and its universal adoption is not recommended.

The Pittsburgh Ruling may be regarded as giving the minimum design which has been used with satisfactory results. With a uniformly high grade of concrete assured and properly seasoned before loading, it is the writer's opinion that the Pittsburgh Ruling would give good results. With conditions as they are, Mr. Lord's recommendations above quoted represent the best practice.

A series of designs of typical panels will now be given in order to illustrate the method of application of the various rulings.

It is not the intention to compare the relative economy of the various systems and the reader should be slow to make such comparisons for, unless all factors are considered, they are sure to be misleading. As a matter of fact there is very little difference between different designs which are made upon exactly the same basis.

20a. Computations for Akme System.—The following computations are published through the courtesy of the Condron Co. of Chicago and are based on the Chicago Ruling. For method of analysis and moment distribution see Art. 17*f*; also *Appendix C*.

Span 22 ft. 0 in. by 25 ft. 0 in. 200 lb. L.L. 9½-in. slab

Slab = 119 lb. per sq. ft.
L.L. = 200 lb. per sq. ft
Total = 319 lb. per sq. ft.

Col. head section $= \frac{WL^2}{15}$ ft.-lb. w = load per sq. ft.

Outer section $= \frac{WL^2}{30}$ ft.-lb. $W = \frac{Lw}{2}$

Inner & mid. section $= \frac{WL^2}{60}$ ft.-lb.

Head = (0.225)(23.5) = 5.29, say 5 ft. 6 in. round head.
Plate = (22)(0.35) = 7.70, say 8 ft. 0 in. short direction.
Plate = (25)(0.35) = 8.75, say 9 ft. 0 in. long direction. Use 9 ft. 0 in. square

$$\frac{22 + 25}{2} = 23.5 \text{ avg. span.}$$

$$\text{Col. head moment} = \frac{(319)(23.5)^2(12)}{(2)(15)} = 1{,}655{,}980 \text{ in.-lb.}$$

f_s = 18,000 lb. per sq. in.
f_c = 700 lb. per sq. in.

Cubes

$\overline{22}^3 = 10{,}648$
$\overline{23.5}^3 = 12{,}978$
$\overline{25}^3 = 15{,}625$

$$\frac{(1{,}655{,}980)(15{,}625)}{12{,}978} = 1{,}995{,}000 \text{ in.-lb., long span}$$

$$\frac{(1{,}655{,}980)(10{,}648)}{12{,}978} = 1{,}360{,}000 \text{ in.-lb., short span}$$

	Long span	Short span	
Col. head section	1,995,000	1,360,000	$9\frac{1}{2}$ in.
Outer section	997,500	680,000	$1\frac{1}{8}$ in.
Inner & mid. section	340,000	498,750	
			$8\frac{3}{8}$ in.

Long span $\dfrac{997{,}500}{(8\frac{3}{8})^2(141)} = 101 = K$ $\quad p = 0.63\%$ $\quad$ Assume plate.... $7\frac{1}{2}$ in.

$(0.0063)(141)(8\frac{3}{8}) = 7.45$ sq. in. $\quad 7\frac{1}{2} + 9\frac{1}{2} = 17$ in.

$17 - \frac{3}{4}\phi = 7.51$ sq. in. $\quad 1\frac{1}{8}$ in.

$d = 15\frac{7}{8}$ in., high bars

$\frac{3}{4}$ in.

$d' = 15\frac{1}{8}$ in., low bars

Short span $\dfrac{680{,}000}{(8\frac{3}{8})^2(141)} = 89 = K$ $\quad 0.42\%$

$(0.0042)(8\frac{3}{8})(141) = 4.96$ sq. in.

$12 - \frac{3}{4}\phi = 5.30$ sq. in.

Long span $\dfrac{1{,}995{,}000}{(15\frac{7}{8})^2(108)} = 74 = K$ $\quad 0.46\%$

$(0.0046)(15\frac{7}{8})(108) = 7.88$ sq. in.

$17 - \frac{3}{4}\phi + 2 - \frac{1}{2}\phi = 7.90$ sq. in.

Short span $\dfrac{1{,}360{,}000}{(15\frac{7}{8})^2(108)} = 55 = K$ $\quad 0.34\%$

$(0.0034)(15\frac{1}{8})(108) = 5.56$ sq. in.

$12 - \frac{3}{4}\phi + 2 - \frac{1}{2}\phi = 5.69$ sq. in.

Slab $\quad 9\frac{1}{2} - 1$ in. $= 8\frac{1}{2}$-in., short bands

$9\frac{1}{2} - 1\frac{1}{2}$ in. $= 8$-in., long bands

Long bands $\dfrac{340{,}000}{(8)(0.9)(18{,}000)} = 2.62$ sq. in. $\quad 14 - \frac{1}{2}\phi = 2.74$ sq. in.

$\dfrac{498{,}750}{(8\frac{1}{2})(0.9)(18{,}000)} = 3.63$ sq. in. $\quad 19 - \frac{1}{2}\phi = 3.73$ sq. in.

Make long main band..................10 ft. 0 in. Mid. band..............................10 ft. 0 in.
Make short main band..................9 ft. 6 in. Mid. band..............................8 ft. 6 in.

20b. Computations for Corr-plate Floors.—The following computations may be taken as typical of the method used in the design of Corr-plate floors based on the Standard Building Regulations of the American Concrete Institute, 1917. As will be noted, the designs are for typical interior panels only, and special conditions and exterior panels will require a different treatment. The computations were furnished the writer through the courtesy of the Corrugated Bar Co. of Buffalo, N. Y.

The moment distribution usually adopted in Corr-plate floors is given in Fig. 61. In the problems here presented the moment distribution recommended by the American Concrete Institute is used.

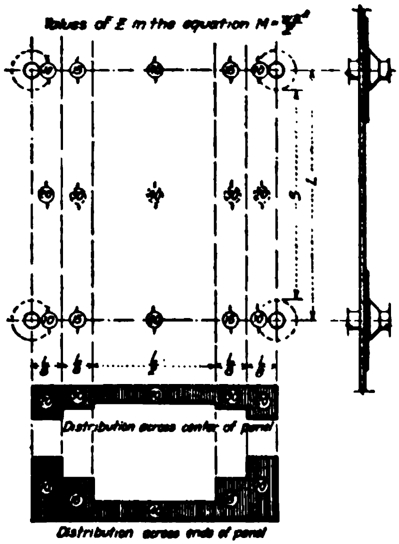


FIG. 61.

Typical Interior Panel (Fig. 62).—

Data: Panel = 20 ft. 4 in. by 20 ft. 4 in.

Live load = 175 lb. per sq. ft.

$f_s = 18{,}000 \qquad f_c = 750$

$l_1 = l_2 = 20.33$

$c = 4.5 \qquad qc = \frac{2}{3}(4.5) = 3$ ft. 0 in.

Assuming that a $7\frac{3}{4}$-in. slab will be required, we find from the thickness formula that the total slab thickness

$$t = 0.02 \times 20.33 \times \sqrt{175 + 93} + 1 = 7.65$$

Therefore the assumption made is satisfactory.

The sum of the positive and negative moments in inch-pounds or the total moment

$$= (0.09)(268)(20.33)(17.33)^2(12)$$
$$= 1{,}767{,}500 \text{ in.-lb.}$$

Corr-plate moment distribution to agree with A. C. I. Standards of 1917			
A. C. I. section	Negative moment in % of total moment	A. C. I. section	Positive moment in % of total moment
Col. head section.......	2 bands A = 31%	Outer section.....	2 bands A = 13%
	2 bands B = 21%		2 bands B = 9%
Middle section............	1 band C = 13%	Inner section........	1 band C = 13%

Negative Moment Steel in Each Direction.—

At drop panel, $jdf_s = (0.875)(9.5)(18{,}000) = 149{,}625$

At outer section, $jdf_s = (0.895)(6.5)(18{,}000) = 104{,}100$

$$2 \text{ bands } A = \frac{(1{,}767{,}500)(0.31)}{149{,}625} = 3.67 \text{ sq. in.}$$

$$2 \text{ bands } B = \frac{(1{,}767{,}500)(0.21)}{149{,}625} = 2.49 \text{ sq. in.}$$

$$1 \text{ band } C = \frac{(1{,}767{,}500)(0.13)}{104{,}100} = 2.21 \text{ sq. in.}$$

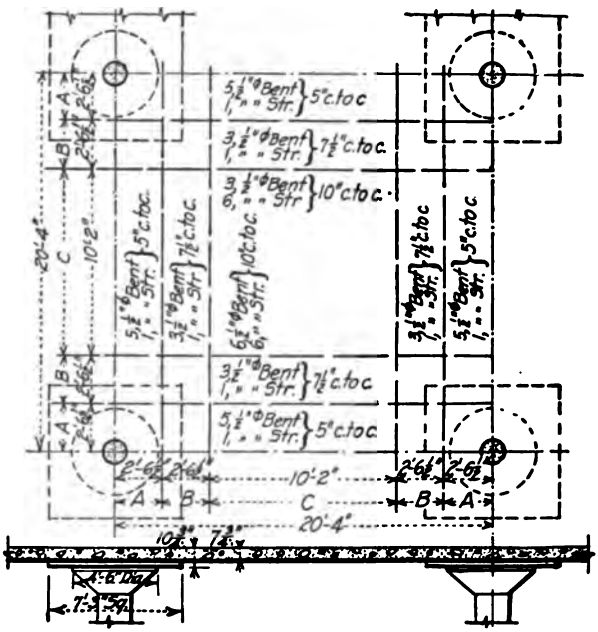


Fig. 62.

Positive Moment Steel in Each Direction.—

$$2 \text{ bands } A = \frac{(1{,}767{,}500)(0.13)}{104{,}100} = 2.21 \text{ sq. in.} \quad \text{Use 12 -- ½-in. rounds, bend 10.}$$

$$2 \text{ bands } B = \frac{(1{,}767{,}500)(0.09)}{104{,}100} = 1.53 \text{ sq. in.} \quad \text{Use 8 -- ½-in. rounds, bend 6.}$$

$$1 \text{ band } C = \frac{(1{,}767{,}500)(0.13)}{104{,}100} = 2.21 \text{ sq. in.} \quad \text{Use 12 -- ½-in. rounds, bend 6.}$$

The area of steel required for negative moment in the several bands is secured by bending up the necessary number of bars from the bottom of the slab or by bending up alternate bars and supplying the deficiency in negative reinforcement by introducing straight bars in the top of the slab.

The unit shearing stress $u = \dfrac{(0.3)(20.33)^2(268)}{(122)(0.875)(9.5)} = 33$ lb. per sq. in.

Typical Interior and Exterior Panel (Fig. 63).—

$$\begin{aligned} \textit{Data} \quad \text{Panel} &= 20 \text{ ft. } 4 \text{ in. by } 22 \text{ ft. } 6 \text{ in.} \\ \text{Live load} &= 175 \text{ lb. per sq. ft.} \end{aligned}$$

$$f_s = 18{,}000 \qquad f_c = 750$$

$$l_1 = 20.33 \text{ ft.} \qquad l_2 = 22.5 \text{ ft.}$$

$$r = 5.0 \qquad qr = \tfrac{2}{3}(5.0) = 3.33$$

$$L = 21.42$$

Assuming an 8-in. slab

$$t = 0.02 \times 21.42 \times \sqrt{175 + 96} + 1 = 8.06.$$

The assumption made is therefore satisfactory.

Design for the Long Span, $l_2 = 22.5$ *ft.—*

$$\begin{aligned} \text{Total moment} &= (0.09)(271)(20.33)(19.17)^2(12) \\ &= 2{,}182{,}000 \text{ in.-lb.} \end{aligned}$$

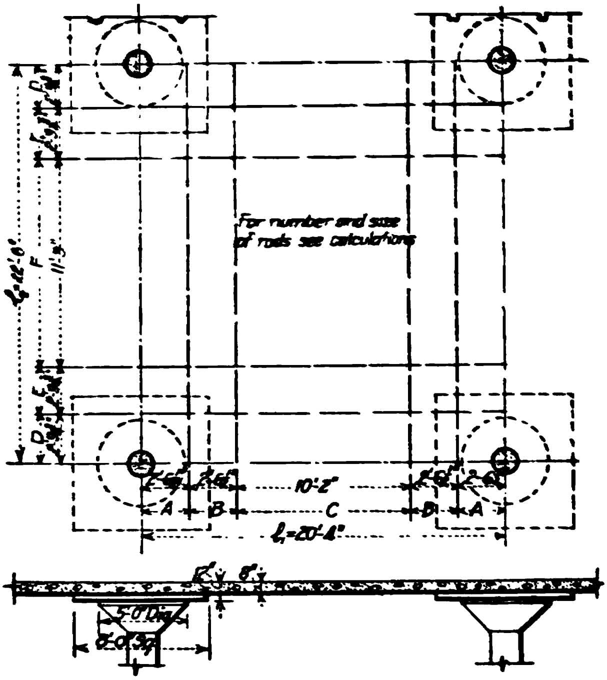


Fig. 63.

Note.—The same distribution of moment applies for the rectangular panel as that given for the square panel.

Negative Moment Steel.—

$$\begin{aligned} \text{At drop panel} \quad jdf_s &= (0.875)(10.5)(18{,}000) = 165{,}400 \\ \text{At outer section } jdf_s &= (0.895)(\ 7.0)(18{,}000) = 112{,}800 \end{aligned}$$

$$2 \text{ bands } A = \frac{(2{,}182{,}000)(0.31)}{165{,}400} = 4.08 \text{ sq. in.}$$

$$2 \text{ bands } B = \frac{(2{,}182{,}000)(0.21)}{165{,}400} = 2.77 \text{ sq. in.}$$

$$1 \text{ band } C = \frac{(2{,}182{,}000)(0.13)}{112{,}800} = 2.52 \text{ sq. in.}$$

Positive Moment Steel.—

$2 \text{ bands } A = \dfrac{(2{,}182{,}000)(0.13)}{112{,}800} = 2.52$ sq. in. Use 9 — ½-in. square, bend up 8

$2 \text{ bands } B = \dfrac{(2{,}182{,}000)(0.09)}{112{,}800} = 1.75$ sq. in. Use 8 — ½-in. square, bend up 6

$1 \text{ band } C = \dfrac{(2{,}182{,}000)(0.13)}{112{,}800} = 2.52$ sq. in. Use 10 — ½-in. square, bend up 5

Design for the Short Span, $l_1 = 20.83$ *ft.*—

$$\text{Total moment} = (0.09)(271)(22.5)(17)^2(12) = 1{,}903{,}000 \text{ in.-lb.}$$

Negative Moment Steel.—

At drop panel $jdf_s = (0.875)(11)(18{,}000) = 173{,}200$

At outer section $jdf_s = (0.895)(6.5)(18{,}000) = 104{,}700$

$$2 \text{ bands } D = \frac{(1{,}903{,}000)(0.31)}{173{,}200} = 3.41 \text{ sq. in.}$$

$$2 \text{ bands } E = \frac{(1{,}903{,}000)(0.21)}{173{,}200} = 2.31 \text{ sq. in.}$$

$$1 \text{ band } F = \frac{(1{,}903{,}000)(0.13)}{104{,}700} = 2.36 \text{ sq. in.}$$

Positive Moment Steel.—

$$2 \text{ bands } D = \frac{(1{,}903{,}000)(0.13)}{104{,}700} = 2.36 \text{ sq. in.}$$ Use 10 – ½-in. square, bend up 7

$$2 \text{ bands } E = \frac{(1{,}903{,}000)(0.09)}{104{,}700} = 1.63 \text{ sq. in.}$$ Use 6 – ½-in. square, bend up 5

$$1 \text{ band } F = \frac{(1{,}903{,}000)(0.13)}{104{,}700} = 2.36 \text{ sq. in.}$$ Use 10 – ½-in. square, bend up 5

NOTE.—For exterior panels—provided the span lengths remain unchanged—the positive moment steel, and the negative moment steel at the first row of interior columns, in the bands normal to the wall is to be increased by 20%.

20c. Computations Based on Pittsburgh Ruling.—The following design of a typical panel according to the Pittsburgh Ruling is here inserted to illustrate this method of

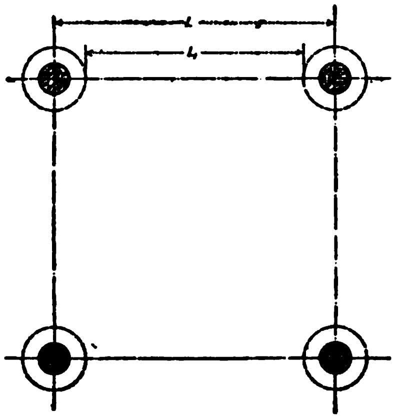


FIG. 64.

computation and it is not recommended for universal adoption. In order to give a design the equal of that recommended by the American Concrete Institute, the value of M and M' will have to be increased or a lower steel stress used. It will be observed that the concrete sections required are thinner and the amounts of steel computed are less than that required by either the Chicago Code or the American Concrete Institute Rulings.

Panel = 20 ft. 10 in. by 22 ft. 2¼ in.	Live load = 250 lb. per sq. ft.
Assume 8½-in. slab and a 4½-in. drop.	Dead load = 106 lb. per sq. ft.
Column cap = 4 ft. 9 in. diameter.	Total load = 356 lb. per sq. ft.

Using the Pittsburgh Ruling which is based on the radial distribution of stress theory, the total bending moment in the center of the suspended slab is $M = \frac{WL}{16}$, where W is the total live and dead load on the panel and L is the panel side (Fig. 64).

$f_s = 16{,}000$ lb. per sq. in.

$f_c = 750$ lb. per sq. in.

For the short direct bands,

$$M = \frac{(356)(20.83)^2(20.83)(12)}{16} = 2{,}420{,}000 \text{ in.-lb.}$$

$d = 8½$ in. $-$ 1 in. $=$ 7½ in. Area of steel = 22.54 sq. in.

118 – ½-in. round rods

This steel is distributed among eight bands and the proportion for one short direct band is $\frac{113}{8} = 14 - \frac{1}{2}$-in. round rods per band.

For the long direct bands we have

$$M = \frac{(356)(22.18)^2(22.18)(12)}{16} = 2,910,000 \text{ in.-lb.}$$

$$d = 7\frac{1}{2} \text{ in.} \quad \text{Area} = 26.90 \text{ sq.in.} = 135 - \tfrac{1}{2}\text{-in. rounds}$$

$$\frac{135}{8} = 17 - \tfrac{1}{2}\text{-in. rounds per band}$$

The diagonal bands in an oblong panel are calculated on the basis of the average panel length.

$$\text{Average panel} = \frac{(20.83)(22.18)}{2} = 21.5 \text{ ft.}$$

$$M = \frac{(356)(21.5)^2)(21.5)(12)}{16} = 2,650,000 \text{ in.-lb.}$$

$$d = 7\frac{1}{2} \text{ in.} \quad \text{Area of steel} = 24.52 \text{ sq. in.} = 123 - \tfrac{1}{2}\text{-in. round rods}$$

$$\frac{123}{8} = 16 - \tfrac{1}{2}\text{-in. round rods per band.}$$

Check of steel over column head.

Using the same ruling as before, the bending moment at the column head to be resisted by eight bands is $M = \frac{W'L'}{11}$, where W' is the total panel load exclusive of that over the column capital and L' is the span measured from edge of cap to edge of cap. In this case, as the difference of spans is small, we can use the average span for L'

$$\begin{aligned} \text{Area of slab} &= (20.83)(22.18) = 462.0 \\ \text{Area of col. cap.} &= \underline{17.7} \\ & \qquad\qquad\qquad\quad 444.3 \end{aligned}$$

$$W' = (444.3)(356) = 158,000. \qquad L' = 21.5 - 4.75 = 16.75 \text{ ft.}$$

$$M = \frac{(158.000)(16.75)(12)}{11} = 2,892,000 \text{ in.-lb.}$$

$$d = 13 \text{ in.} - 2 \text{ in.} = 11 \text{ in.} \quad \text{Area of steel} = 18.77 \text{ sq. in.} = 94 - \tfrac{1}{2}\text{-in. round rods}$$

The total band width is 0.4 of the average span, which is 8.6 ft. The 94 — ½-in. round rods are not to be distributed over the whole band width but over the width of the column cap plus twice the effective depth. This is 4 ft. 9 in. + 2 × 11 in. = 6 ft. 7 in.

The code specifies that over the column head the computed spacing must be maintained for the full band width. The steel required then in the eight bands radiating from the column capital will be $\frac{8.6}{6.58}(94) = 123 - \frac{1}{2}$-in. round rods

We actually have

2 short direct bands	@ 14	=	28
2 long direct bands	@ 17	=	34
4 diagonal bands	@ 16	=	64
			126 — ½-in. round rods

which gives a margin of safety over the required amounts.

Shear at the edge of the column cap $= \frac{158,000}{(179)(11)} = 80$ lb. per sq. in. with an allowable punching shear of 120 lb. per sq. in.

Stirrups are not necessary in this form of construction as all the slab rods are in the top of the slab over the drop head and are in the bottom at the center of the span.

$$p = \frac{18.77}{248 \times 11} = 0.0069 \text{ (allowable} = 0.0097 \text{ for } 16,000 \text{ and } 750) \text{ O. K. for } f_c$$

The percentage of steel in the center of the span is always low and compression of that point is never a controlling factor.

If standard concrete sections are used, it will seldom be necessary to check other portions of the slab design, except under special conditions. When such is not the case, the compression in the concrete should be checked at the edge of the drop head. Where a smaller depth of drop head is used or is omitted altogether, the compression around the edge of the column cap will be the controlling feature and, in order to keep to the allowable stress, compression steel may have to be inserted.

Where continuity cannot be secured, as for example at the exterior columns, the direc[illegible] and diagonal bands ending at the exterior columns should be increased 20%.

20d. Computations Based on Chicago Ruling.—The computations of a simple panel which follow are inserted to illustrate the method of applying the Chicago Ruling to a four-way system. Results by this method are very similar to those obtained by the A. C. I. Ruling. The across direct bands which are called for under this ruling are not required under the Pittsburgh Ruling and many designers who use the Chicago method do not put these rods in the top of the slab across the column center lines. While a small amount of reinforcement is required at this point, it is doubtful whether its omission would entail serious cracking along the edge of the panel in the drop-head type of construction where conservative concrete sections are used.

Panel 24 ft. 8 in. by 24 ft. 8 in.	Live load = 200 lb.
	Finish = 12 lb.
Assume a 10½-in. slab	Dead load = 138 lb. per sq. ft.
and a 6½-in. drop.	350
Drop head 8 ft. 9 in. by 8 ft. 9 in.	f_s = 16,000 lb. per sq. in.
Col. cap 5 ft. 9 in. diameter.	f_c = 750 lb. per sq. in.

The formula for the minimum thickness of slab as given by the Chicago Code is $t = 0.023\ L\sqrt{w}$ where t is the total thickness of slab in inches, L is the panel length in feet and w is the total live and dead load in pounds per square foot.

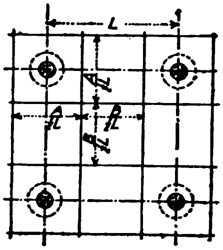


FIG. 65.

$t = 0.023 \times 24.67\sqrt{350} = 10.6$ in. so our assumption is satisfactory, as $\frac{L}{32} =$ 9.25 in., the other code requirement, is exceeded.

The column cap must be at least $0.225 \times L = 5.56$ ft. which the assumed dimension of 5 ft. 9 in. satisfies.

The width of strips A and B (Fig. 65) as used in the moment computations will be 12 ft. 4 in. W, used in the formulas, will be $(12.33)(350) = 4315$ lb.

Negative moment for strip A.

$$M = \frac{WL^2}{15} = \frac{(4315)(24.67)^2\ (12)}{15} = 2{,}103{,}000 \text{ in.-lb.}$$

Positive moment for strip A

$$M = \frac{WL^2}{40} = \frac{(4315)(24.67)^2\ (12)}{40} = 787{,}500 \text{ in.-lb.}$$

Positive moment for strip B

$$M = \frac{WL^2}{60} = \frac{(4315)(24.67)^2\ (12)}{60} = 525{,}700 \text{ in.-lb.}$$

Negative moment for strip B

$$M = \frac{WL^2}{60} = 525{,}700 \text{ in.-lb.}$$

The negative moment in strip A is resisted by one diagonal and one direct band.

$$\text{Area of steel} = \frac{2{,}103{,}000}{(0.87)(17 - 2)(16{,}000)} = 10.08 \text{ sq. in.}$$

40 — ½-in. sq. rods.

which are not necessarily divided equally between one direct and one diagonal band. For distribution see the following computation:

The positive moment in each strip A is resisted by the steel in one cross or direct band.

$$d = 10\tfrac{1}{2} \text{ in.} - 1 \text{ in.} = 9\tfrac{1}{2} \text{ in.}$$

$$\text{Area of steel} = \frac{787{,}500}{(0.91)(9.5)(16{,}000)} = 5.70 \text{ sq. in.}$$

23 — ½-in. sq. rods.

The positive moment of each strip B shall be resisted by the steel in one diagonal band.

$$\text{Area of steel} = \frac{525{,}700}{(0.91)(9.5)(16{,}000)} = 3.785 \text{ sq. in.}$$

16 — ½-in. sq. rods.

The negative moment of strip B is resisted by the across direct bands which are short rods placed in the top of the slab over and at right angles to the direct bands.

Area of steel = 3.785 sq. in.
= 16 — ½-in. sq. rods.

The proper steel distribution will then be:

24 – ½-in. sq. rods in direct bands.
16 – ½-in. sq. rods in diagonal bands.
16 – ½-in. sq. rods across direct bands.

For exterior panels the amount of reinforcement is to be increased by the use of the moment coefficients specified for this case.

If sections given in the tables are used, it will not be necessary to check the shears or concrete stresses except in special cases.

The methods of computation used for rectangular panels are clearly explained in the text so that the detailed method will not be given here. In fact the entire ruling is very clearly written and covers the vital points of design very well.

20e. Computations Based on Ruling of American Concrete Institute.—The following design of a simple panel of a four-way flat slab under the A. C. I. Ruling will serve to make clear the wording of the ruling. As in all other problems in this chapter, the ordinary straight line stress variation is assumed and, as in other cases, the critical sections so far as compression in the concrete goes are either around the column capital or at the edge of the drop panel. This method gives a very similar steel distribution and general results much like those obtained by the Chicago Ruling.

Given an interior panel 25 ft. 6 in. by 25 ft. 6 in. to be designed to carry a live load of 175 lb. per sq. ft. plus a 1-in. cement finish. Assume a slab thickness t = 10¼ in. and a drop 6¼ in. thick and 9 ft. 0 in. by 9 ft. 0 in. The column capital will be 5 ft. 9 in. diameter. (Refer to Fig. 3, *Appendix C*, page 858.)

Testing the assumed dimensions by the ruling we have as follows:

$t = 0.02L\sqrt{w} + 1$ in.
$t = 0.02 \times 25.5\sqrt{316} + 1$ in.
$t = 10.07$ in.

$\frac{L}{32} = \frac{25.5}{32} = 0.797$ ft. $= 9.57$ in.

Live load = 175 lb. per sq. ft.
Dead load = 129 lb. per sq. ft.
Finish load = 12 lb. per sq. ft.
Total load w = 316 lb. per sq. ft.
f_s = 16,000 lb. per sq. in.
f_c = 750 lb. per sq. in.

The assumption of t = 10¼ in. is thus seen to be satisfactory.

The drop where used shall not be less than $0.3L = (0.3)(25.5) = 7.65$ ft. and the assumed size of drop 9 ft. by 9 ft. need not be changed.

The sum of the positive and negative moments shall not be less than $0.09wl_1(l_2 - qc)^2$, where w is the unit live and dead load in pounds per square foot, l_1 is the span in feet center to center of columns parallel to sections at which moments are considered, l_2 is the span in feet center to center of columns perpendicular to sections at which moments are considered, c the average diameter of column capital in feet at the point where its thickness is 1½ in., and q is the distance from center line of capital to the center of gravity of the periphery of the half capital divided by $\frac{c}{2}$ (for our case $q = ⅔$).

$$\text{Total moment} = (0.09)(316)(25.5)(25.5 - ⅔ \times 5.75)^2(12) = 4{,}086{,}600 \text{ in.-lb.}$$

Under the ruling 50% of this must be resisted by the steel over the column head. (Refer to Fig. 3, page 858.)

Not less than 10% shall be resisted in the mid-section, not less than 18% shall be resisted in the outer sections, and not less than 12% of the total moment shall be resisted on the inner section.

The following distribution is adopted:

Column-head section	50%
Mid-section	10%
Outer section (direct bands)	20%
Inner section (diagonal bands)	20%
Total	100%

The moments to be taken by the different sections will be as follows:

Column-head section	2,043,300 in.-lb.
Mid-section	408,660
Outer section	817,320
Inner section	817,320
Total	4,086,600 in.-lb.

The column-head section includes two diagonal bands and one direct band.

The effective depth is 16½ in. − 2 in. = 14½ in. (assuming 2 in. to the center of gravity of the steel). With $f_s = 16{,}000$ and $f_c = 650$, we have the area for three bands in top of slab over the column head

$$A_s = \frac{2{,}043{,}300}{(14.5)(0.874)(16{,}000)} = 10.08 \text{ sq. in.}$$

which is equal to 40 − ½-in. sq. bars.

For the mid-section $d = 10\frac{1}{4} - 1\frac{1}{4} = 9$ in.

$$A_s = \frac{408{,}660}{(9)(0.91)(16{,}000)} = 3.12 \text{ sq. in.}$$

which is equal to 13 − ½-in. sq. bars in the across direct bands located in the top of the slab across the sides of the panel.

In the outer section $d = 9$ in.

$$A_s = \frac{817{,}320}{(9)(0.89)(16{,}000)} = 6.33 \text{ sq. in.}$$

which is equal to 26 − ½-in. sq. rods for each direct band.

Under this system of moment distribution, the effective area of steel for the inner section—*i.e.*, the two diagonal bands—will be the same as that for the direct band or 6.23 sq. in. The actual area then of each diagonal band must be $\frac{6.33}{(2)(0.707)} = 4.47$ sq. in., which is 18 − ½-in. sq. rods.

Assuming that all the diagonal rods are bent up over the head to form negative reinforcement, the amount of steel in a direct band which will have to be bent up will be 10.08 − 6.33 (effective area of two diagonal bands) = 3.75 sq. in., which is equal to 15 − ½-in. sq. rods. As there are 26 − ½-in. sq. rods in each direct band, 11 may be stopped at the edge of the column capital or carried through as compression reinforcement. Another allowable arrangement would be to bend up all the bars in the direct band over the column head, which would require an effective area of 10.08 − 6.33 = 3.75 sq. in. in the two diagonal bands to be bent up, which equals $\frac{3.75}{(2)(0.707)} = 2.65$ sq. in., or 11 − ½-in. sq. rods per diagonal band. These rods should be carried (0.35)(25.5) = 8.92 ft., or 8 ft. 11 in. past the center line of the column.

The balance of the rods in the diagonal band should extend (0.35)(25.5) = 8 ft. 11 in. on either side of the center of panel—that is, they must be at least 17 ft. 10 in. long in this case.

It is common practice to use rods in the direct bands which are long enough to cover two spans and when this is done the laps should be staggered. Since they are spliced over the column head, each bar should extend 40 diameters or 20 in. past the center line of splice in this case.

Still another possible arrangement of steel would be to arrange the diagonal bands as just outlined and then to only bend up half the bars in the direct bands, lapping them 8 ft. 11 in. past the center line from each side so as to make up the necessary area over the head. This, of course, only applies where the steel in the direct bands extends for one span only. Where rods covering two spans are used in the direct bands, this arrangement will have to be modified or extra short rods will have to be added over the column head.

20f. Roof Design.—In the design of flat-slab roofs it should be remembered that the deflection rather than the strength may be and usually is the controlling factor. For this reason very thin slabs should not be used. The Chicago and A. C. I. Rulings limit the thickness of the roof slab to one-fortieth of the span as a minimum. For a 20-ft. span this would give a thickness of 6 in. which has been found by experience to be about right for a roof slab of this span.

There are two methods of constructing roofs in common use. The first method is to pitch the slab, laying the roofing surface directly on the concrete. The second method is to cast the roof slab level and obtain the required pitch by means of a cinder-concrete fill (of varying depth) which is surfaced with a cement finish and upon which the roofing membrane is laid. The first method involves a comparatively small dead load and, as the roof live load is small, —generally 40 lb. per sq. ft.—the total load for which the slab will be designed is small. In the second case the dead load may be very considerable in certain parts of the slab where a deep cinder fill is necessary. This matter should obviously receive consideration in the design.

The writer has made it a practice in the first case mentioned—*i.e.*, a pitched slab roof—to never design for a lighter live load than 75 lb. per sq. ft. This means a total load of 75 lb. plus the dead load of the slab and roof finish. Some other engineers use even more than this. The reason for such a rule is that the roof slab, being subjected to the maximum temperature variation, both daily and seasonal, is very apt to crack if the percentage of steel is too small and designing with light loads will not give sufficient reinforcement to resist these stresses.

Frequently in roof design the question of the best design to accommodate sawtooth skylights will arise. An economy in the use of concrete and a more rational design will often result if a beam-and-girder design is used instead of the flat-slab type for such cases. Some designers prefer the older type for the roofs of all flat-slab buildings.

20g. Beams in Flat-slab Floors.—Two general cases of the design of beams forming a part of flat slabs arise. The first is that of beams located between the exterior columns and supporting the edge of the slab and carrying either a low wall under the windows, or, where no windows occur, a wall extending from the floor to the underside of the beam above. The stresses to be withstood are torsional, due to the deflection of the panel, and tension, due to bending under applied vertical load. For this reason a wide shallow beam as nearly square in section as possible is to be preferred. Where a drop panel is used, it is advantageous to make the beams the same depth as the drop. Lighting consideration make it advisable to keep the depth as small as possible. Parallel and adjacent to the beam there is a half band of steel in the bottom of the slab which has a smaller effective depth than the reinforcement of the wall beam. It is evident, therefore, that a part at least of this band cannot be stressed to its full value until the steel in the bottom of the beam is overstressed.

It is the practice of some designers to calculate the steel in the wall beams for the wall load and beam dead load only, and there are several cases where such a practice has given satisfactory results. With this method, however, there is every chance of having cracked beams and exterior walls. The writer believes that in addition to the load above specified, the live load on a portion of the floor should be included. It is recommended that in addition to the weight of the beams and wall, the exterior beams be calculated to carry, as a uniform load, the live load of a strip of floor adjacent to the beam, of length equal to the span, and width equal to one-sixth the span at right angles to the exterior wall of the building.

For the second-class beams located in interior panels, the same rule may be used, except that where the beam carries concentrated or stair loads these must be provided for in addition. Some designers prefer the wide, shallow type of beam with a depth equal to the drop head which has, besides the advantage of increased clearance, the added advantage of less liability of unsightly cracking under excessive deflection. As will be evident from a study of any building design, cases of this kind are bound to arise which are indeterminate. Under these conditions the application of such theoretical analysis as will apply must be modified by experience based on former designs, successful or otherwise.

Continuity should be secured in all beams whenever possible, particularly in the wall beams. Adequate temperature reinforcement in exposed concrete surfaces should always be provided. Where the wall beams do not extend below the floor slab, it is usually possible to utilize the entire wall below the window as a beam and a very stiff construction can be secured. The danger of cracking, due to torsional stresses, is greater, however, in this type of design.

20h. Columns.—The calculation of the columns supporting a flat slab is one of the most important parts of the entire design and is more frequently overlooked than any other portion of the work.

Tests have shown conclusively that both interior and exterior columns below and above the loaded floor are stressed by an unbalanced live load in the panel which they support. This point is well covered by the A. C. I. Ruling and was also covered in a way by the Chicago and Pittsburgh Rulings, but in many cases little consideration has been given to it.

There is a provision in the Chicago Ruling that no column shall be less than one-twelfth the panel length or one-twelfth the clear height in thickness. For the interior columns in all stories below the roof story, this rule is good and should be used even though it will often give a larger section than the direct load will require. An exception may be made where structural-steel cores are used. In the story below the roof somewhat smaller columns may be used as there is little possibility of unbalanced live load on the roof, but it is the writer's opinion that 12 in. diameter should be the minimum size for an interior column.

For the exterior columns the Chicago Ruling will frequently give rather thicker sectic than are necessary, particularly when wide columns are desired on account of the exterior appearance of the building. The following minimum thicknesses are recommended for exterior reinforced-concrete columns:

16-ft. span	14 in.
20-ft. span	16 in.
24-ft. span	18 in.
28-ft. span	20 in.

The method of computing the moment in and designing columns supporting flat-slab buildings as recommended by the American Concrete Institute will give conservative results and should meet with universal adoption. At present commercial competition has led to a slighting of the reinforcement to resist bending in the exterior columns in many cases, with the result that hair cracks have developed on the outside of the column at the base of the bracket. No serious consequences accompany such cracks, but if the adjacent panels were overloaded, a dangerous condition would probably develop.

As a practical minimum for vertical reinforcement in columns, the equivalent of four ¾-in. round rods has been adopted by some designers and is a safe minimum for general use.

The bending stresses in columns will be found to be more important in the upper stories than in the lower floors on account of the increase in the direct load as the footings are approached.

For moments in columns in flat-slab construction, see also Art. 7, Sect. 10. The methods of computing stresses in columns due to bending and direct stress are given in Sect. 9.

20i. Brick Exterior Wall Supports.—Several of the building codes allow the use of brick bearing walls for the exterior support of flat-slab floors. The fact that such a support cannot develop a large negative moment is offset by providing an excess of 40% of steel in the adjacent slabs as against 20% where reinforced-concrete columns are used. Some structures of this class have not given entire satisfaction and one failure is attributed to the weakness of the brick supporting walls (*Engineering News*, vol. 76, page 262). Two difficulties arise in this connection; one is to determine what thickness of wall is necessary and the other is to secure first-class brickwork. Another element which is almost certain to cause trouble is that the interior columns and brick exterior walls settle at different rates which, in a multi-story building, will introduce serious strains in the slabs. Where such a construction must be used, it is recommended that pilasters of a total thickness at least equal to one-twelfth of the span be used and that a heavy continuous beam be carried around the edge of the floor.

It is inadvisable to use corbels or brackets on the brick pilasters as they will increase the bending stress in the pilaster and may actually endanger the structure. If they are required for shear, the slab drop or beam should be increased in size to take care of it and no attempt should be made to restrain the slab at a brick wall or pilaster bearing. Usually there is little if any saving in total cost by the use of brick, and the reinforced-concrete skeleton building veneered with brick is superior structurally.

21. Tables. *Pittsburgh Ruling.*—The following tables have been computed for a standard four-way flat slab having square panels, drop heads, and reinforced according to the Pittsburgh Ruling.

The stresses used are as follows:

f_s = 16,000 lb. per sq. in.
f_c = not more than 750 lb. per sq. in. at the edge of column capital.

The shear at the edge of the drop head

$$v = \frac{V}{bjd} \text{ is not greater than 60 lb. per sq. in.}$$

For 100 lb. per sq. ft. superimposed load, the drop head is $0.35L$ square.

For 150 to 400 lb. per sq. ft. superimposed load, the drop head is 0.4L square, where L is the center to center distance between columns.

The superimposed load means the live or live and dead load above the structural slab and may or may not include the weight of the cement finish, depending upon whether it is laid at the same time as the slab or at a later date if it forms a part of the structure at all.

The computations are based on a typical interior panel and the amounts of steel given in the tables must be increased in exterior panels of the same span in accordance with the Pittsburgh Code requirements. Where concrete exterior columns are used and are so constructed and reinforced that they will withstand the bending stresses developed by the unbalanced load, the amount of steel given in the tables should be increased 20% for all the bands which end at the outside of the building or are parallel to and adjacent to the wall beam.

Bands which extend from wall to wall, such as diagonal bands at corner panels, should be increased 40% over the values given in the tables. It is the writer's opinion that the same method should be followed in treating exterior panels where brackets are not provided at the wall columns and insufficient vertical steel is used in them to properly resist the bending stresses due to the unbalanced loading, *i.e.*, the slab reinforcement should be increased 40%.

Where reinforced-concrete interior columns are used and the exterior panels are carried on brick piers or walls, the Pittsburgh Ruling recommends that the steel in the exterior panels be increased 40%.

Where, as is frequently the case, the span of the exterior panels is greater or less than that of the interior panels, due consideration must be given to these conditions, and the preceding remarks regarding exterior panels will be modified thereby.

These tables are given here principally as a check on design and as an aid to estimating, and it is not the intention that they should take the place of a carefully considered design in any special case. Particularly is this true where conditions, due to irregular panels, openings, or concentrated loading make an accurate analysis of actual stresses imperative.

Where panels are not square and where the angles between column center lines are not right angles, the tables do not apply and cannot be used without modification.

For intermediate spans and loads the proper values (for square panels) may be obtained from the tables approximately by interpolation.

FLAT-SLAB PANELS—PITTSBURGH REGULATIONS

Interior Panels—Superimposed Load 100 lb. per sq. ft.

Panel	Capital diameter	Head	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band		Steel in lb. per sq. ft.
						Direct	Diagonal	
16′ × 16′	3′-6″	5′-7″ × 5′-7″	8″	5″	0.465	11-⅜″ϕ	11-⅜″ϕ	1.460
17′ × 17′	3′-9″	6′-0″ × 6′-0″	8½″	5½″	0.492	12-⅜″ϕ	12-⅜″ϕ	1.491
18′ × 18′	4′-0″	6′-4″ × 6′-4″	9″	5¾″	0.513	14-⅜″ϕ	14-⅜″ϕ	1.510
19′ × 19′	4′-3″	6′-8″ × 6′-8″	9½″	6″	0.535	15-⅜″ϕ	15-⅜″ϕ	1.610
20′ × 20′	4′-6″	7′-0″ × 7′-0″	9¾″	6¼″	0.559	17-⅜″ϕ	17-⅜″ϕ	1.700
21′ × 21′	4′-9″	7′-4″ × 7′-4″	10¼″	6½″	0.580	15-⅜″□	15-⅜″□	1.780
22′ × 22′	5′-0″	7′-8″ × 7′-8″	10¾″	7″	0.622	17-⅜″□	17-⅜″□	1.980
23′ × 23′	5′-3″	8′-1″ × 8′-1″	11¼″	7¼″	0.644	19-⅜″□	19-⅜″□	2.140
24′ × 24′	5′-6″	8′-5″ × 8′-5″	11¾″	7½″	0.669	15-½″ϕ	15-½″ϕ	2.190
25′ × 25′	5′-9″	8′-9″ × 8′-9″	12¼″	7¾″	0.691	16-½″ϕ	16-½″ϕ	2.260
26′ × 26′	5′-9″	9′-2″ × 9′-2″	13¼″	8¼″	0.740	18-½″ϕ	18-½″ϕ	2.430
27′ × 27′	6′-0″	9′-6″ × 9′-6″	13¾″	8½″	0.763	19-½″ϕ	19-½″ϕ	2.480

Interior Panels—Superimposed Load 150 lb. per sq. ft.

Panel	Capital diameter	Head	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band		Steel in lb. per sq. ft.
						Direct	Diagonal	
16′ × 16′	3′–6″	5′–7″ × 5′–7″	9″	5¼″	0.475	11–⅜″□	11–⅜″□	1.78
17′ × 17′	3′–9″	6′–0″ × 6′–0″	9½″	5¾″	0.518	12–⅜″□	12–⅜″□	1.84
18′ × 18′	4′–0″	6′–4″ × 6′–4″	10″	6″	0.543	13–⅜″□	13–⅜″□	1.91
19′ × 19′	4′–3″	7′–0″ × 7′–0″	10½″	6¼″	0.571	15–⅜″□	15–⅜″□	2.06
20′ × 20′	4′–6″	7′–4″ × 7′–4″	11″	6½″	0.594	17–⅜″□	17–⅜″□	2.15
21′ × 21′	4′–9″	7′–8″ × 7′–8″	11½″	7″	0.635	18–⅜″□	18–⅜″□	2.21
22′ × 22′	5′–0″	8′–1″ × 8′–1″	12¼″	7¼″	0.662	15–½″φ	15–½″φ	2.44
23′ × 23′	5′–3″	8′–5″ × 8′–5″	12¾″	7¾″	0.705	16–½″φ	16–½″φ	2.50
24′ × 24′	5′–6″	8′–9″ × 8′–9″	13½″	8¼″	0.744	17–½″φ	17–½″φ	2.52
25′ × 25′	5′–9″	9′–2″ × 9′–2″	13¾″	8½″	0.770	19–½″φ	19–½″φ	2.70
26′ × 26′	5′–9″	9′–8″ × 9′–8″	14¾″	8¾″	0.800	17–½″□	17–½″□	2.91
27′ × 27′	6′–0″	10′–6″ × 10′–6″	15¼″	9″	0.830	18–½″□	18–½″□	2.98

Flat-slab Panels—Pittsburgh Regulations

Interior Panels

Superimposed load			200 lb. per sq. ft.						250 lb. per sq. ft.					
Side of panel	Capital diameter	Side of head	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band: Direct	Steel in each band: Diagonal	Steel in lb. per sq. ft.	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band: Direct	Steel in each band: Diagonal	Steel in lb. per sq. ft.
16′	3′–6″	6′–5″	10″	6″	0.555	12–⅜″□	12–⅜″□	2.00	10¾″	6½″	0.600	13–⅜″□	13–⅜″□	2.16
17′	3′–9″	6′–10″	10½″	6¼″	0.578	13–⅜″□	13–⅜″□	2.02	11¼″	7″	0.645	14–⅜″□	14–⅜″□	2.18
18′	4′–0″	7′–3″	10¾″	6¾″	0.620	15–⅜″□	15–⅜″□	2.19	12″	7½″	0.684	16–⅜″□	16–⅜″□	2.34
19′	4′–3″	7′–8″	11½″	7″	0.645	16–⅜″□	16–⅜″□	2.27	12½″	7¾″	0.710	18–⅜″□	18–⅜″□	2.92
20′	4′–6″	8′–0″	12″	7¼″	0.665	18–⅜″□	18–⅜″□	2.34	13¼″	8¼″	0.755	14–½″φ	14–½″φ	2.54
21′	4′–9″	8′–5″	12¾″	7½″	0.696	15–½″φ	15–½″φ	2.56	13¾″	8½″	0.778	16–½″φ	16–½″φ	2.74
22′	5′–0″	8′–10″	13¼″	7¾″	0.720	17–½″φ	17–½″φ	2.78	14½″	9″	0.825	17–½″φ	17–½″φ	2.78
23′	5′–3″	9′–3″	14″	8¼″	0.765	18–½″φ	18–½″φ	2.88	15″	9½″	0.867	19–½″φ	19–½″φ	2.99
24′	5′–6″	9′–8″	14½″	8¾″	0.811	16–½″□	16–½″□	3.02	15¾″	10″	0.908	16–½″□	16–½″□	3.02
25′	5′–9″	10′–0″	15¼″	9¼″	0.849	17–½″□	17–½″□	3.06	16½″	10½″	0.955	18–½″□	18–½″□	3.24
26′	5′–9″	10′–5″	16¼″	9¾″	0.901	18–½″□	18–½″□	3.08	17½″	11″	1.010	19–½″□	19–½″□	3.28
27′	6′–0″	10′–10″	16¾″	10¼″	0.946	20–½″□	20–½″□	3.30	18¼″	11½″	1.050	21–½″□	21–½″□	3.48

Flat-slab Panels—Pittsburgh Regulations

Interior Panels

Superimposed load			300 lb. per sq. ft.						350 lb. per sq. ft.					
Side of panel	Capital diameter	Side of head	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band: Direct	Steel in each band: Diagonal	Steel in lb. per sq. ft.	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band: Direct	Steel in each band: Diagonal	Steel in lb. per sq. ft.
16′	3′-6″	6′-5″	11½″	7¼″	0.665	13-⅜″□	13-⅜″□	2.16	12¼″	8″	0.725	13-⅜″□	13-⅜″□	2.16
17′	3′-9″	6′-10″	12″	7¾″	0.702	15-⅜″□	15-⅜″□	2.33	12¾″	8½″	0.765	15-⅜″□	15-⅜″□	2.33
18′	4′-0″	7′-3″	12¾″	8¼″	0.750	16-⅜″□	16-⅜″□	2.34	13½″	9″	0.810	17-⅜″□	17-⅜″□	2.49
19′	4′-3″	7′-8″	13¼″	8¾″	0.790	18-⅜″□	18-⅜″□	2.48	14¼	9½″	0.860	14-½″φ	14-½″φ	2.69
20′	4′-6″	8′-0″	14¼″	9¼″	0.840	15-½″φ	15-½″φ	2.72	15″	10″	0.900	15-½″φ	15-½″φ	2.72
21′	4′-9″	8′-5″	15″	9¾″	0.882	16-½″φ	16-½″φ	2.75	15¾″	10½″	0.945	17-½″φ	17-½″φ	2.92
22′	5′-0″	8′-10″	15½″	10¼″	0.928	18-½″φ	18-½″φ	3.03	16½″	11″	0.995	18-½″φ	18-½″φ	2.94
23′	5′-3″	9′-3″	16″	10¾″	0.970	15-½″□	15-½″□	2.98	17¼″	11¾″	1.058	16-½″□	16-½″□	3.16
24′	5′-6″	9′-8″	17″	11¼″	1.020	17-½″□	17-½″□	3.20	18″	12½″	1.115	17-½″□	17-½″□	3.20
25′	5′-9″	10′-0″	17½″	11¾″	1.055	18-½″□	18-½″□	3.26	18¾″	13¼″	1.180	18-½″□	18-½″□	3.24
26′	5′-9″	10′-5″	18¾″	12¼″	1.105	20-½″□	20-½″□	3.46	20″	14″	1.250	20-½″□	20-½″□	3.46

Flat-slab Panels—Pittsburgh Regulations

Interior Panels

Superimposed load			400 lb. per sq. ft.					
Side of panel	Capital diameter	Side of head	T Total drop	t Slab	Concrete in cu. ft. per sq. ft.	Steel in each band: Direct	Steel in each band: Diagonal	Steel in lb. per sq. ft.
16′	3′-6″	6′-5″	12¾″	8¾″	0.785	14-⅜″□	14-⅜″□	2.31
17′	3′-9″	6′-10″	13½″	9¼″	0.828	15-⅜″□	15-⅜″□	2.33
18′	4′-0″	7′-3″	14¼″	9¾″	0.878	17-⅜″□	17-⅜″□	2.49
19′	4′-3″	7′-8″	15¼″	10¼″	0.924	14-½″φ	14-½″φ	2.69
20′	4′-6″	8′-0″	16″	10¾″	0.968	16-½″φ	16-½″φ	2.90
21′	4′-9″	8′-5″	16¾″	11½″	1.025	17-½″φ	17-½″φ	2.92
32′	5′-0″	8′-10″	17½″	12¼″	1.090	15-½″□	15-½″□	3.10
23′	5′-3″	9′-3″	18¼″	13″	1.155	16-½″□	16-½″□	3.16
24′	5′-6″	9′-8″	19″	13¾″	1.220	17-½″□	17-½″□	3.20
25′	5′-9″	10′-0″	19¾″	14½″	1.280	19-½″□	19-½″□	3.44
26′	5′-9″	10′-5″	21″	15″	1.333	21-½″□	21-½″□	3.63

Chicago Ruling.—The following tables have been computed for a four-way flat slab having square panels, drop heads, and reinforced according to the Chicago Ruling. They include designs for from 100 to 400 lb. per sq. ft. live load and spans of from 16 to 27 ft. in some cases. Being based on square interior panels they will not apply without modification to other conditions and are useful mainly for estimating and as a check on actual designs. The general remarks given under the head of Pittsburgh Ruling apply in this case.

For methods of computation see Art. 20*d* and *Appendix C*.

The stresses used are: f_s = 18,000 and f_c = not over 750 lb. per sq. in.

Flat-slab Panels—Chicago Regulations
Interior Panels

Superimposed load			100 lb. per sq. ft.							150 lb. per sq. ft.						
						Steel in each band							Steel in each band			
Side of panel	Capital diameter	Side of head	T total drop	t slab	Concrete in cu. ft. per sq. ft.	Direct	Across direct	Diagonal	Steel in lb. per sq. ft.	T total drop	t slab	Concrete in cu. ft. per sq. ft.	Direct	Across direct	Diagonal	Steel in lb. per sq. ft.
16'	3'-6"	5'-7"	9½"	6"	0.536	13-⅜"φ	9-⅜"φ	10-⅜"φ	1.68	9¾"	6"	0.540	17-⅜"φ	11-⅜"φ	12-⅜"φ	2.07
17'	3'-9"	6'-0"	10¼"	6½"	0.581	15-⅜"φ	10-⅜"φ	11-⅜"φ	1.74	10¼"	6½"	0.581	19-⅜"φ	13-⅜"φ	14-⅜"φ	2.22
18'	4'-0"	6'-4"	11"	6¾"	0.605	17-⅜"φ	11-⅜"φ	12-⅜"φ	1.82	10¾"	6¾"	0.608	21-⅜"φ	15-⅜"φ	16-⅜"φ	2.35
19'	4'-3"	6'-8"	11½"	7¼"	0.648	19-⅜"φ	13-⅜"φ	14-⅜"φ	1.97	11½"	7¼"	0.648	19-⅜"□	13-⅜"□	14-⅜"□	2.48
20'	4'-6"	7'-0"	12"	7½"	0.670	17-⅜"□	11-⅜"□	12-⅜"□	1.80	12¼"	7½"	0.672	21-⅜"□	14-⅜"□	15-⅜"□	2.55
21'	4'-9"	7'-4"	12¾"	8"	0.715	19-⅜"□	12-⅜"□	13-⅜"□	2.13	13"	8"	0.715	17-½"φ	11-½"φ	12-½"φ	2.58
22'	5'-0"	7'-8"	13¼"	8¼"	0.740	21-⅜"□	14-⅜"□	15-⅜"□	2.30	13½"	8¼"	0.742	19-½"φ	12-½"φ	13-½"φ	2.83
23'	5'-3"	8'-1"	14¼"	8¾"	0.785	17-½"φ	11-½"φ	12-½"φ	2.46	14½"	8¾"	0.795	21-½"φ	13-½"φ	14-½"φ	2.94
24'	5'-6"	8'-5"	14¾	9"	0.810	19-½"φ	12-½"φ	13-½"φ	2.58	14¾"	9"	0.808	23-½"φ	15-½"φ	16-½"φ	3.15
25'	5'-9"	8'-9"	15½"	9½"	0.855	21-½"φ	13-½"φ	14-½"φ	2.69	15½"	9½"	0.850	20-½"□	13-½"□	14-½"□	3.34
26'	5'-9"	9'-2"	15¾"	9¾"	0.875	23-½"φ	15-½"φ	16-½"φ	2.93	16"	10"	0.895	22-½"□	14-½"□	15-½"□	3.45
27'	6'-0"	9'-6"	16¼"	10¼"	0.931	19-½"□	13-½"□	14-½"□	3.01	17¼"	10½"	0.946	23-½"□	15-½"□	16-½"□	3.75
			200 lb. per sq. ft.							250 lb. per sq. ft.						
16'	3'-6"	5'-7"	10"	6¼"	0.558	16-⅜"□	10-⅜"□	11-⅜"□	2.43	11"	6¾"	0.605	17-⅜"□	11-⅜"□	12-⅜"□	2.62
17'	3'-9"	6'-0"	10¼"	6½"	0.578	18-⅜"□	12-⅜"□	13-⅜"□	2.63	11½"	7¼"	0.650	19-⅜"□	13-⅜"□	14-⅜"□	2.81
18'	4'-0"	6'-4"	11¼"	7"	0.625	19-⅜"□	13-⅜"□	14-⅜"□	2.64	12½"	7¾"	0.695	21-⅜"□	14-⅜"□	15-⅜"□	2.88
19'	4'-3"	6'-8"	12"	7½"	0.675	22-⅜"□	15-⅜"□	16-⅜"□	2.86	13½"	8¼"	0.741	17-½"φ	11-½"φ	12-½"φ	3.03
20'	4'-6"	7'-0"	12¾"	8"	0.720	18-½"φ	12-½"φ	13-½"φ	3.07	14"	8¾"	0.785	19-½"φ	13-½"φ	13-½"φ	3.17
21'	4'-9"	7'-4"	13¾"	8½"	0.765	19-½"φ	13-½"φ	14-½"φ	3.10	15"	9¼"	0.826	21-½"φ	14-½"φ	15-½"φ	3.37
22'	5'-0"	7'-8"	14¾"	9"	0.809	21-½"φ	14-½"φ	15-½"φ	3.22	15¾"	9¾"	0.871	23-½"φ	15-½"φ	16-½"φ	3.47
23'	5'-3"	8'-1"	15½"	9½"	0.852	23-½"φ	15-½"φ	16-½"φ	3.29	16½"	10¼"	0.920	20-½"□	13-½"□	14-½"□	3.66
24'	5'-6"	8'-5"	16¼"	10"	0.898	20-½"□	13-½"□	14-½"□	3.49	17½"	10¾"	0.970	22-½"□	14-½"□	15-½"□	3.78
25'	5'-9"	8'-9"	17"	10½"	0.945	22-½"□	14-½"□	15-½"□	3.62	18¾"	11½"	1.035	23-½"□	15-½"□	16-½"□	3.80
26'	5'-9"	9'-2"	17½"	11"	0.986	24-½"□	16-½"□	17-½"□	3.87	19½"	12"	1.080	25-½"□	17-½"□	18-½"□	4.06
27'	6'-0"	9'-6"	18½"	11½"	1.033	26-½"□	17-½"□	18-½"□	3.97	20½"	12½"	1.120	27-½"□	18-½"□	19-½"□	4.17

Flat-slab Panels—Chicago Regulations

Interior Panels—Superimposed Load 300 lb. per sq. ft.

Panel	Capital diameter	Head	T total drop	t slab	Concrete in cu. ft. per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct	Across direct	Diagonal	
16′ × 16′	3′-6″	5′-7″ × 5′-7″	11½″	7¼″	0.648	19-⅜″□	12-⅜″□	13-⅜″□	2.88
17′ × 17′	3′-9″	6′-0″ × 6′-0″	12¾″	8″	0.715	20-⅜″□	13-⅜″□	14-⅜″□	2.90
18′ × 18′	4′-0″	6′-4″ × 6′-4″	13¾″	8½″	0.766	16-½″φ	11-½″φ	12-½″φ	3.13
19′ × 19′	4′-3″	6′-8″ × 6′-8″	14½″	9″	0.810	18-½″φ	12-½″φ	13-½″φ	3.23
20′ × 20′	4′-6″	7′-0″ × 7′-0″	15½″	9½″	0.854	20-½″φ	13-½″φ	14-½″φ	3.27
21′ × 21′	4′-9″	7′-4″ × 7′-4″	16¼″	10″	0.899	22-½″φ	15-½″φ	16-½″φ	3.57
22′ × 22′	5′-0″	7′-8″ × 7′-8″	16¾″	10½″	0.945	20-½″□	13-½″□	13-½″□	3.71
23′ × 23′	5′-3″	8′-1″ × 8′-1″	18″	11″	0.990	21-½″□	14-½″□	15-½″□	3.88
24′ × 24′	5′-6″	8′-5″ × 8′-5″	19″	11¾″	1.055	23-½″□	15-½″□	16-½″□	4.00
25′ × 25′	5′-9″	8′-9″ × 8′-9″	20″	12¼″	1.100	25-½″□	16-½″□	17-½″□	4.11
26′ × 26′	5′-9″	9′-2″ × 9′-2″	20¾″	12¾″	1.153	27-½″□	18-½″□	19-½″□	4.34

Interior Panels—Superimposed Load 350 lb. per sq. ft.

Panel	Capital diameter	Head	T total drop	t slab	Concrete in cu. ft. per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct	Across direct	Diagonal	
16′ × 16′	3′-6″	5′-9″ × 5′-9″	12½″	8″	0.718	19-⅜″□	13-⅜″□	13-⅜″□	2.91
17′ × 17′	3′-9″	6′-1″ × 6′-1″	13¾″	8½″	0.768	21-⅜″□	14-⅜″□	15-⅜″□	2.97
18′ × 18′	4′-0″	6′-6″ × 6′-6″	14½″	9″	0.810	18-½″φ	11-½″φ	12-½″φ	3.10
19′ × 19′	4′-3″	6′-11″ × 6′-11″	15½″	9½″	0.855	19-½″φ	13-½″φ	14-½″φ	3.48
20′ × 20′	4′-6″	7′-4″ × 7′-4″	16½″	10″	0.905	21-½″φ	14-½″φ	15-½″φ	3.53
21′ × 21′	4′-9″	7′-5″ × 7′-5″	17¼″	10¾″	0.965	19-½″□	12-½″□	13-½″□	3.77
22′ × 22′	5′-0″	8′-0″ × 8′-0″	18″	11¼″	1.010	21-½″□	13-½″□	14-½″□	3.92
23′ × 23′	5′-3″	8′-4″ × 8′-4″	19″	12″	1.075	22-½″□	15-½″□	15-½″□	4.00
24′ × 24′	5′-6″	8′-9″ × 8′-9″	20¼″	12½″	1.130	24-½″□	16-½″□	17-½″□	4.12
25′ × 25′	5′-9″	9′-3″ × 9′-3″	21″	13″	1.175	27-½″□	18-½″□	18-½″□	4.40
26′ × 26′	5′-9″	9′-5″ × 9′-5″	22¼″	13¾″	1.240	28-½″□	19-½″□	20-½″□	4.54

Interior Panels—Superimposed Load 400 lb. per sq. ft.

Panel	Capital diameter	Head	T total drop	t slab	Concrete in cu. ft. per sq. ft.	Steel in each band			Steel in lb. per sq. ft.
						Direct	Across direct	Diagonal	
16′ × 16′	3′-6″	6′-1″ × 6′-1″	13″	8¼″	0.746	20-⅜″□	14-⅜″□	14-⅜″□	3.09
17′ × 17′	3′-9″	6′-4″ × 6′-4″	14½″	9″	0.814	22-⅜″□	15-⅜″□	16-⅜″□	3.24
18′ × 18′	4′-0″	6′-9″ × 6′-9″	15″	9½″	0.855	25-⅜″□	17-⅜″□	18-⅜″□	3.47
19′ × 19′	4′-3″	7′-2″ × 7′-2″	16¼″	10″	0.905	21-½″φ	13-½″φ	14-½″φ	3.63
20′ × 20′	4′-6″	7′-6″ × 7′-6″	17½″	10¾″	0.978	22-½″φ	15-½″φ	16-½″φ	3.77
21′ × 21′	4′-9″	8′-0″ × 8′-0″	18¼″	11¼″	1.022	25-½″φ	16-½″φ	17-½″φ	3.91
22′ × 22′	5′-0″	8′-4″ × 8′-4″	19½″	12″	1.090	21-½″□	14-½″□	14-½″□	3.98
23′ × 23′	5′-3″	8′-8″ × 8′-8″	20″	12½″	1.130	24-½″□	15-½″□	16-½″□	4.29
24′ × 24′	5′-6″	9′-2″ × 9′-2″	21″	13¼″	1.200	26-½″□	17-½″□	18-½″□	4.52
25′ × 25′	5′-9″	9′-5″ × 9′-5″	22¼″	14″	1.270	28-½″□	18-½″□	19-½″□	4.60
26′ × 26′	5′-9″	9′-11″ × 9′-11″	23¼″	14½″	1.315	30-½″□	20-½″□	20-½″□	4.71

Corr-plate Floors.—The following table gives data regarding designs which are based on the Corr-plate method of computation for square interior panels and apply only to such cases.

Live load, lb.	f_s = 18,000 lb.			Corr. plate floor panels						f_c = 700 lb.
	Size of panel	Slab	Column head	Cap	20 bands	30 bands	40 bands	Extra bars over column	Reinforcement per sq. ft., lb.	Concrete per sq. ft., cu. ft.
40		5"			16–3/8ϕ@ 6"c/c	12–3/8ϕ@ 8"c/c	16–3/8ϕ@11½"c/c	12–3/8ϕ–8'–0"	1.61	0.463
150		6"	42"		16–½ϕ@7"c/c	12–3/8ϕ@7"c/c	20–3/8ϕ@9"c/c	12–½ϕ–8'–0"	2.19	0.544
200	16'–0"×16'–0"	6½"	mini-	6'–6"×6'–6"×2"	16–½ϕ@7"c/c	16–3/8ϕ@6"c/c	20–3/8ϕ@8"c/c	14–½ϕ–8'–0"	2.34	0.585
250		6½"	mum		16–½ϕ@6"c/c	12–½ϕ@8"c/c	16–½ϕ@11½"c/c	14–½ϕ–8'–0"	2.72	0.583
300		7"			16–5/8ϕ@8"c/c	12–½ϕ@8½"c/c	16–½ϕ@11"c/c	10–5/8ϕ–8'–0"	3.25	0.621
40		5½"	42"		16–3/8ϕ@6"c/c	16–3/8ϕ@8"c/c	16–3/8ϕ@8"c/c	14–3/8ϕ–9'–0"	1.55	0.500
150		6½"	48"		12–5/8ϕ@10"c/c	12–½ϕ@10"c/c	16–½ϕ@12"c/c	14–½ϕ–9'–0"	2.55	0.590
200	18'–0"×18'–0"	7"	mini-	7'–3"×7'–3"×2"	12–5/8ϕ@9"c/c	16–½ϕ@9"c/c	16–½ϕ@11"c/c	12–5/8ϕ–9'–0"	2.83	0.631
250		7½"	mum		12–5/8ϕ@8"c/c	16–½ϕ@8"c/c	20–½ϕ@10"c/c	12–5/8ϕ–9'–0"	3.01	0.672
300		8"			16–5/8ϕ@7"c/c	16–½ϕ@8"c/c	16–½ϕ@11½"c/c	14–5/8ϕ–9'–0"	3.18	0.711
40		6"	42"	8'–0"×8'–0"×2"	16–½ϕ@7"c/c	16–3/8ϕ@7"c/c	28–3/8ϕ@9"c/c	12–½ϕ–10'–0"	2.02	0.539
150		7"	54"	8'–0"×8'–0"×2"	12–5/8ϕ@8½"c/c	16–½ϕ@8½"c/c	24–½ϕ@11"c/c	12–5/8ϕ–10'–0"	2.88	0.638
200	20'–0"×20'–0"	7½"	mini-	8'–0"×8'–0"×2"	16–5/8ϕ@7"c/c	16–½ϕ@7½"c/c	24–½ϕ@11"c/c	14–5/8ϕ–10'–0"	3.19	0.677
250		8"	mum	8'–0"×8'–0"×2"	20–5/8ϕ@7"c/c	16–½ϕ@7½"c/c	24–½ϕ@9"c/c	14–5/8ϕ–10'–0"	3.45	0.715
300		8½"		8'–0"×8'–0"×3"	20–5/8ϕ@7"c/c	12–5/8ϕ@10"c/c	16–5/8ϕ@13"c/c	14–5/8ϕ–10'–6"	3.64	0.767
40		6½"	48"	8'–0"×8'–0"×2"	16–½ϕ@7"c/c	16–½ϕ@10"c/c	20–½ϕ@12¼"c/c	18–½ϕ–11'–0"	2.47	0.580
150		8"	60"	8'–0"×8'–0"×2"	16–5/8ϕ@8"c/c	16–½ϕ@8"c/c	28–½ϕ@10¼"c/c	10–5/8ϕ–11'–0"	3.09	0.720
200	22'–0"×22'–0"	8½"	mini-	8'–0"×8'–0"×2"	20–5/8ϕ@7"c/c	20–½ϕ@7"c/c	28–½ϕ@9"c/c	18–5/8ϕ–11'–0"	3.53	0.758
250		8½"	mum	8'–0"×8'–0"×2½"	20–5/8ϕ@6½"c/c	16–5/8ϕ@9½"c/c	20–5/8ϕ@12"c/c	18–5/8ϕ–11'–0"	3.88	0.761
300		9"		8'–0"×8'–0"×3"	24–5/8ϕ@6"c/c	16–5/8ϕ@9"c/c	20–5/8ϕ@11½"c/c	18–5/8ϕ–11'–0"	4.12	0.805
40		7"	48"	9'–6"×9'–6"×2"	20–5/8ϕ@9"c/c	16–½ϕ@9"c/c	20–½ϕ@12¼"c/c	14–5/8ϕ–12'–0"	2.73	0.623
150		8½"	63"	9'–6"×9'–6"×2"	20–5/8ϕ@7"c/c	16–5/8ϕ@10¼"c/c	20–5/8ϕ@13"c/c	22–5/8ϕ–12'–0"	3.64	0.766
200	24'–0"×24'–0"	9"	mini-	9'–6"×9'–6"×3"	20–¾ϕ@9"c/c	16–5/8ϕ@9"c/c	20–5/8ϕ@12¼"c/c	20–5/8ϕ–12'–0"	4.09	0.819
250		9½"	mum	9'–6"×9'–6"×3½"	20–¾ϕ@9"c/c	16–5/8ϕ@9"c/c	20–5/8ϕ@12¼"c/c	20–5/8ϕ–12'–0"	4.09	0.863
300		10"		9'–6"×9'–6"×4"	20–¾ϕ@8"c/c	16–5/8ϕ@8"c/c	28–5/8ϕ@11"c/c	20–5/8ϕ–12'–0"	4.52	0.908

NOTE.—These designs are for panels continuous on all four sides. They are designed to carry in addition to the live loads, a finish load of 15 lb. per sq. ft. Reinforcement per square foot includes spacers and chairs. Concrete per square foot includes caps and heads.

The stresses used are

$$f_s = 18{,}000 \text{ lb. per sq. in.}$$
$$f_c = 700 \text{ lb. per sq. in.}$$

They are based on the Corr-plate method of moment distribution and, of course, will not apply where other rulings are in force. The main value of tables of this kind is for estimating and as a check on design. The general remarks under the head of Pittsburgh Ruling apply in this case.

For methods of computation see Arts. 17*g* and 20*b*.

Akme System.—The designs given in the following tables were not computed by the Condron Co., but were worked out by one of the construction companies who have used the Akme system extensively. As given here the steel stresses govern, the compression in the concrete being always less than the allowable.

The data here given is useful for estimating and as a check on design, and applies only to square interior panels using the stresses specified. For any other conditions special treatment will be required.

The stresses used are

$$f_s = 18{,}000 \text{ lb. per sq. in.}$$
$$f_c = 750 \text{ lb. per sq. in.}$$

These tables should be used in connection with Figs. 47 and 48, pp. 467 and 468. For method of computation see Arts. 17*f* and 20*a*.

AKME SYSTEM—FLAT SLAB

			100 lb. per sq. ft.					150 lb. per sq. ft.				
Size of panel	Capital diameter	Side drop head	Drop	Slab	Reinforcing steel Band *A*	Reinforcing steel Band *B*	Concrete, cu. ft. per sq. ft.	Drop	Slab	Reinforcing steel Band *A*	Reinforcing steel Band *B*	Concrete, cu. ft. per sq. ft.
15′ × 15′	3′-6″	5′-3″	4″	6″	10-½φ	5-½φ	0.541	4″	6″	12-½φ	6-½φ	0.541
16′ × 16′	3′-9″	5′-9″	4″	6″	12-½φ	6-½φ	0.543	4″	6″	10-⅝φ	7-½φ	0.543
17′ × 17′	4′-0″	6′-0″	4″	6½″	9-⅝φ	6-½φ	0.583	4″	6½″	11-⅝φ	8-½φ	0.583
18′ × 18′	4′-3″	6′-6″	4″	7″	10-⅝φ	9-½φ	0.626	4″	7″	12-⅝φ	9-½φ	0.626
19′ × 19′	4′-6″	6′-9″	4″	7½″	12-⅝φ	8-½φ	0.667	4″	7½″	10-¾φ	10-½φ	0.667
20′ × 20′	4′-6″	7′-0″	4″	7½″	13-⅝φ	10-½φ	0.666	4″	7½″	12-¾φ	12-½φ	0.666
21′ × 21′	4′-9″	7′-6″	4″	8″	15-⅝φ	11-½φ	0.709	4″	8″	13-¾φ	9-⅝φ	0.709
22′ × 22′	5′-0″	7′-9″	4½″	8½″	11-¾φ	12-½φ	0.755	4½″	8½″	14-¾φ	10-⅝φ	0.755
23′ × 23′	5′-3″	8′-0″	5″	9″	12-¾φ	9-⅝φ	0.800	5″	9″	15-¾φ	10-⅝φ	0.800
24′ × 24′	5′-6″	8′-6″	5″	9″	14-¾φ	10-⅝φ	0.802	5″	9″	12-⅞φ	12-⅝φ	0.802
25′ × 25′	5′-9″	8′-9″	5″	9½″	15-¾φ	10-⅝φ	0.843	5″	9½″	14-⅞φ	13-⅝φ	0.843
			200 lb. per sq. ft.					250 lb. per sq. ft.				
15′ × 15′	3′-6″	5′-3″	4″	6½″	9-⅝φ	7-½φ	0.583	4″	6½″	11-⅝φ	8-½φ	0.583
16′ × 16′	3′-9″	5′-9″	4″	6½″	11-⅝φ	8-½φ	0.584	4″	7″	12-⅝φ	9-½φ	0.626
17′ × 17′	4′-0″	6′-0″	4½″	7″	12-⅝φ	9-½φ	0.630	4½″	7½″	13-⅝φ	10-½φ	0.672
18′ × 18′	4′-3″	6′-6″	4½″	7½″	10-¾φ	10-½φ	0.674	5″	8″	11-¾φ	11-½φ	0.721
19′ × 19′	4′-6″	6′-9″	4½″	8″	11-¾φ	11-½φ	0.714	5″	8½″	12-¾φ	12-½φ	0.761
20′ × 20′	4′-6″	7′-0″	4½″	8″	13-¾φ	9-⅝φ	0.713	5″	9″	13-¾φ	9-⅝φ	0.801
21′ × 21′	4′-9″	7′-6″	4½″	8½″	14-¾φ	10-⅝φ	0.756	5″	9″	15-¾φ	10-⅝φ	0.803
22′ × 22′	5′-0″	7′-9″	5″	9″	15-¾φ	10-⅝φ	0.802	5½″	9½″	16-¾φ	12-⅝φ	0.846
23′ × 23′	5′-3″	8′-0″	5½″	9½″	12-⅞φ	12-⅝φ	0.847	6″	10″	13-⅞φ	13-⅝φ	0.892
24′ × 24′	5′-6″	8′-6″	5½″	9½″	14-⅞φ	13-⅝φ	0.849	6″	10½″	15-⅞φ	14-⅝φ	0.938
25′ × 25′	5′-9″	8′-9″	6″	10″	15-⅞φ	14-⅝φ	0.895	6½″	11″	15-⅞φ	15-⅝φ	0.983

Size of panel	Capital diameter	Side drop head	300 lb. per sq. ft.					350 lb. per sq. ft.				
			Drop	Slab	Reinforcing steel		Concrete, cu. ft. per sq. ft.	Drop	Slab	Reinforcing steel		Concrete, cu. ft. per sq. ft
					Band A	Band B				Band A	Band B	
15′ × 15′	3′-6″	5′-3″	4″	7″	12-⅝φ	9-½φ	0.624	4½″	7½″	13-⅝φ	9-½φ	0.671
16′ × 16′	3′-9″	5′-9″	4½″	7½″	9-¾φ	9-½φ	0.673	5″	8″	10-¾φ	10-½φ	0.721
17′ × 17′	4′-0″	6′-0″	4½″	8″	11-¾φ	10-½φ	0.713	5″	8½″	11-¾φ	11-½φ	0.760
18′ × 18′	4′-3″	6′-6″	5″	8½″	12-¾φ	12-½φ	0.763	5″	9″	13-¾φ	12-½φ	0.840
19′ × 19′	4′-6″	6′-9″	5½″	9″	9-⅞φ	9-⅝φ	0.808	5½″	9½″	10-⅞φ	9-⅝φ	0.949
20′ × 20′	4′-6″	7′-0″	5½″	9½″	10-⅞φ	10-⅝φ	0.848	5½″	10″	12-⅞φ	11-⅝φ	0.889
21′ × 21′	4′-9″	7′-6″	5½″	9½″	12-⅞φ	11-⅝φ	0.850	6″	10″	13-⅞φ	12-⅝φ	0.897
22′ × 22′	5′-0″	7′-9″	6″	10″	13-⅞φ	12-⅝φ	0.895	6″	10½″	14-⅞φ	10-¾φ	0.937
23′ × 23′	5′-3″	8′-0″	6½″	10½″	11-⅞φ	10-¾φ	0.941	7″	11″	15-⅞φ	10-¾φ	0.982
24′ × 24′	5′-6″	8′-6″	7″	11″	15-⅞φ	11-¾φ	0.990	7½″	11½″	12-1φ	11-¾φ	1.037
25′ × 25′	5′-9″	8′-9″	7″	11½″	13-1φ	11-¾φ	1.030	7½″	12″	13-1φ	12-¾φ	1.076
			400 lb. per sq. ft.									
15′ × 15′	3′-6″	5′-3″	5″	8″	13-⅝φ	9-½φ	0.718					
16′ × 16′	3′-9″	5′-9″	5″	8½″	10-¾φ	10-½φ	0.762					
17′ × 17′	4′-0″	6′-0″	5½″	9″	12-¾φ	12-½φ	0.807					
18′ × 18′	4′-3″	6′-6″	5½″	9½″	13-¾φ	9-⅝φ	0.851					
19′ × 19′	4′-6″	6′-9″	6″	10″	11-⅞φ	9-⅝φ	0.896					
20′ × 20′	4′-6″	7′-0″	6″	10½″	12-⅞φ	11-⅝φ	0.936					
21′ × 21′	4′-9″	7′-6″	6½″	10½″	13-⅞φ	9-¾φ	0.944					
22′ × 22′	5′-0″	7′-9″	6½″	11″	15-⅞φ	10-¾φ	0.984					
23′ × 23′	5′-3″	8′-0″	7″	11½″	12-1φ	11-¾φ	1.029					
24′ × 24′	5′-6″	8′-6″	7½″	12″	13-1φ	12-¾φ	1.078					
25′ × 25′	5′-9″	8′-9″	8″	12½″	14-1φ	12-¾φ	1.123					

22. Construction Methods and Safeguards.—The care with which the actual construction of a reinforced-concrete building is carried out has more to do with its success than any other part of the operation and is certainly far more important than the design. This is just as true of the flat-slab type as of any other. The present practice, therefore, of several of the larger bar companies in furnishing design without inspection is to be regretted.

The more important features regarding construction as they appear to the writer may be summarized as follows:

1. *Column Forms.*—The present practice is to use metal forms for the interior columns, including the column capital, which are commonly leased from one of the steel form companies. Forms for the exterior columns are commonly built of wood, although in some cases metal forms are also used. Great care should be taken to see that these forms are tight, particularly around the capital, as leakage at this point always results in a poor casting which never can be properly repaired. The same is true where beams join the interior columns.

2. *Pouring Columns.*—Columns should be cast to the bottom of the column capital and allowed to set 24 hr. before casting the capital and slab. Otherwise, there is likely to be a separation at this point.

3. *Floor Forms.*—Both matched and edged lumber is used for floor forms with satisfactory results. In the latter case, however, more care must be exercised to prevent leakage which is always attended with serious consequences.

4. *Methods of Securing Reinforcing Steel in Position.*—In all reinforced-concrete construction the accurate location and securing of the reinforcing steel in its proper position until the casting of the concrete is completed is of the utmost importance. Until recently, this fact was not -rally realized and some of the older constructions, which have been lately demolished,

exhibit a remarkable displacement of both major and secondary reinforcement. Beam bars were crowded together in one side of the beam or forced up several inches above their correct position and stirrups were in almost all conceivable positions.

The necessity of the positive anchoring of the reinforcement in position during the casting period is even greater in the case of flat-slab floors than with other forms of construction, because a small amount of vertical displacement will have a much greater relative effect.

This fact has been well understood by engineers engaged in this class of construction and the result has been the development of numerous special devices for spacing rods uniformly and maintaining them at the proper distance above the forms. Cuts of some of these devices are shown in Art. 73, Sect. 2. These are carried in stock by, or can be quickly obtained from, their various makers and are all of proven merit. It is strongly recommended that devices of this type or of equal efficiency be specified and their use insisted on as experience has shown that slipshod methods have no place in this class of construction.

5. *Casting Floor Slab.*—The ideal floor is one that is cast at a single operation. It is not always possible to do this but the fewer construction joints in the floor, the better. It is usual to make these in the center of panels and it is the practice of some engineers to use extra short steel rods across these joints. A thorough cleaning of the finished surface together with the liberal use of neat cement on this surface before casting the adjoining section is advantageous. Care should be taken, by the frequent use of grade points, to see that the slab is cast to the thickness specified. Variations of an inch or more have frequently been observed in practice.

6. *Floor Finish.*—Two methods of applying the cement finish to the floor slab are in common use. The first, known as the monolithic method, consists in smoothing down the structural slab and applying a thin finish before the concrete has thoroughly set. In this case the finish is properly considered as a part of the slab in the computations for strength.

In the second method, more commonly used in cold-weather work, the finish to a thickness of $\frac{3}{4}$ in. or more is applied after the structural slab has set and cannot be considered as an integral part of it for in many cases it does not bond uniformly with it but, of course, adds to the dead load.

7. *Wall Beams.*—Where the wall beams do not extend above the slab they are cast with it, but in some cases where they extend above they are cast part with the floor and the balance afterward. When this is the case, it is advisable to provide additional stirrups to bond the two sections together.

8. *Concrete.*—Very wet concrete should not be permitted and the remarks on this subject in Sects. 1 and 2 apply here with full force. A mixture no leaner than 1 : 2 : 4 should be used in the slabs with an aggregate not coarser than 1 in. For columns the writer prefers a richer mixture and has recommended 1 : 1½ : 3 wherever possible.

9. *Steel.*—Deformed or square twisted bars are to be preferred for all flat-slab work on account of their superior bonding qualities. Where market conditions permit, hard-grade steel is recommended for slabs on account of its higher elastic limit. It has been the usual practice in four-way flat slabs to specify the exact points of lapping of the various bars. Just as good results may be secured by allowing the splices to come where they will but so arranging the bars that the splices are well staggered. The ability to use steel of random lengths frequently results in a considerable saving.

10. *Removal of Forms.*—It is not advisable to remove the support from a flat-slab floor as soon after casting as with the beam-and-girder type. Especial care must be exercised in this respect in cold-weather work. Yielding of the green concrete in the slab may be sufficient to crack the exterior columns, a thing which has happened in the past.

It is usually possible to so arrange the floor forms that they may be removed without disturbing some of the props which support the slab, and this practice should be followed. It is unwise to leave this matter to the judgment of an inexperienced contractor, who will likely as not pull out the forms and put in the props afterward. No rule can be given but it may be said that the time that the slab should be supported is a function of the mean temperature, provided it has been prevented from actually freezing, and cases are on record where 40 days was insufficient.

In winter work heating must frequently be resorted to, both of the materials before mixing and of the slab after pouring, but unless the heated air can pass over the top of the slab, but little benefit will be derived from the latter method.

UNIT CONSTRUCTION

23. Method of Construction in General.—The two main systems of "unit" construction for buildings are the *Unit-bilt* system and the Ransome Unit system. In the first system mentioned, all members are cast in forms on the ground and set in place, when hard, by a derrick. The Ransome system differs principally in that the slab is poured in place after the unit beams, girders, and columns have been erected. In the *Unit-bilt* system all the units are tied together by virtue of bars projecting into pockets or open spaces in which concrete is poured.

24. Advantages of the Unit Method.—The greatest advantage obtained from the use of separately molded members occurs when a large number of the same size of beams, columns, and girders are to be employed. The same forms can then be used over and over again. From this it follows that the unit type of construction is not likely to have universal application due to the multiplicity of shapes in complex structures. Study of designs, however, should in most cases reduce the number of shapes to a workable minimum. Restriction in sizes of discontinuous members is also likely to be a controlling factor and, under some conditions, narrow the application of this system to certain layouts governed by the capacities of the handling apparatus.

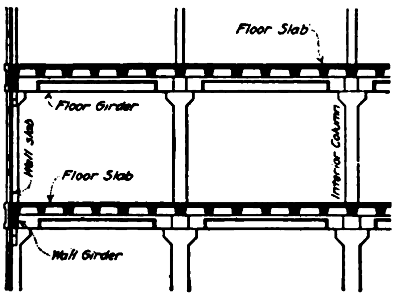


Fig. 66.—Part section of building on the *Unit-bilt* system of reinforced-concrete construction.

Where a large number of members of the same dimensions are required, *unit* construction is likely to be cheaper than monolithic construction for three main reasons: (1) the greatly reduced amount of falsework required as compared with the monolithic type, (2) the reduction in the number of men required for the work of construction, and (3) the chance to carry on the work under cover in all kinds of weather.

Shrinkage cracks do not usually occur in buildings of unit construction since all the shrinkage has taken place in the individual units before their incorporation into the structure. Every element may be inspected and approved before it is placed, or, if desired, a given percentage of the units may be tested before erection.

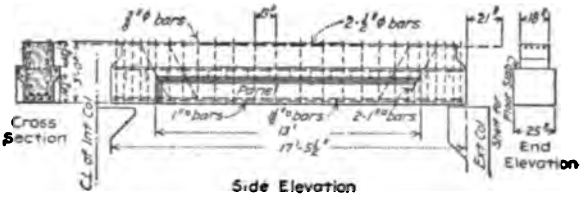


Fig. 67.—Typical girders for *Unit-bilt* system.

25. "Unit-bilt" System.—Figs. 66 to 69 inclusive, and Plate VI show the principle of construction of the *Unit-bilt* system used by the Unit Construction Co. of St. Louis. The columns are provided with brackets for the support of the girders, and the girders are set on a mortar bed at a distance back from the center of column sufficient to allow the column rods to overlap. The girder rods to take negative moment over supports project into the space over the columns—the girders being cut back so as to give the necessary length for embedment (Fig. 67 and Plate VI). The ribbed slabs rest on shelves or ledges on the sides of the girder, with the top of slab above the top of girder so as to permit the slab reinforcing rods to project into the space thus formed and provide for negative tensile stresses. The outer face of each side rib of slab has a groove, so that the two grooves of adjacent slabs will form a key for the grout filling. No attempt is made in the field to obtain a complete bond between the filled concrete and the concrete of the separately molded members, since the filled concrete is used either in direct shear or in compression, or as a means for bonding the projecting rods in the space between the units.

It would be impossible to enumerate all constructions that have been carried out by *Unit-bilt* methods. The more important applications are as follows: sea walls, caissons, docks, retaining walls, tunnel linings, culverts, pipe, sewers, fences, fence posts, telegraph posts, piles, lamp posts, warehouses, factories, elevators, cotton-handling plants, cotton mills, bridges, and viaducts, fireproof residences, school buildings, railroad stations, roundhouses and train sheds.

Railroad work offers a wonderful field for the development of the "unit" method. If it were possible to standardize such structures as engine houses, freight sheds, snowsheds, train-sheds and small stations, the construction of these by the Unit method on a factory basis at central locations would effect great economies.

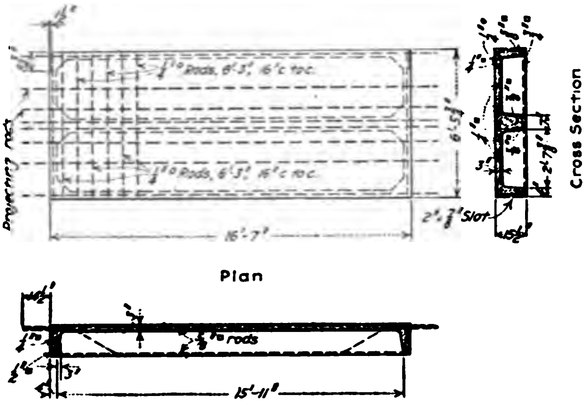


Fig. 68.—Typical floor slab for *Unit-bilt* system.

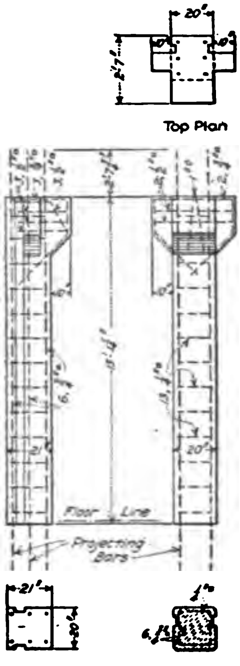


Fig. 69.—Details of column construction, *Unit-bilt* system.

26. Ransome Unit System.—The main details of the Ransome Unit system are shown in Plate VII. The usual reinforcement is placed near the sides of the columns and, in addition, a longitudinal rod is inserted in a central cored hole extending lengthwise through the column. The grouting of the column is done from the top after the setting of the girders and beams. The cored hole is enlarged and flared out at the bottom in order to insure an even bed for the column. The main column reinforcement is not continuous from story to story and the caps and bases of the columns are enlarged so that, at these points, the concrete alone will be able to transfer the weights.

The main girders are placed on top of the columns—the ends of each girder being enlarged so as to almost cover the cap of the column. The beams are made with dove-tailed ends and fit into pockets in the girders (Plate VII). In the design of this system, the vertical stirrups

PLATE VI

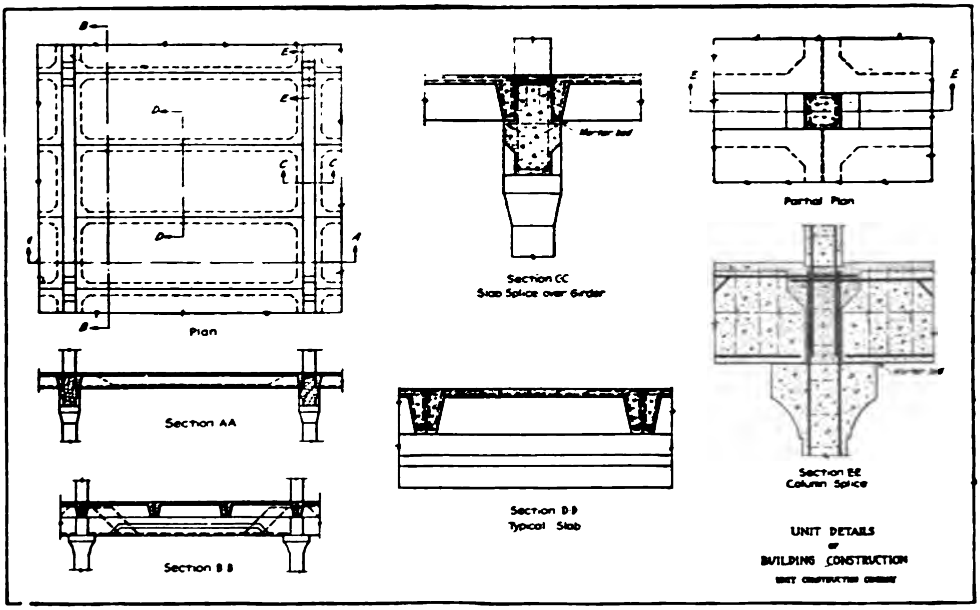
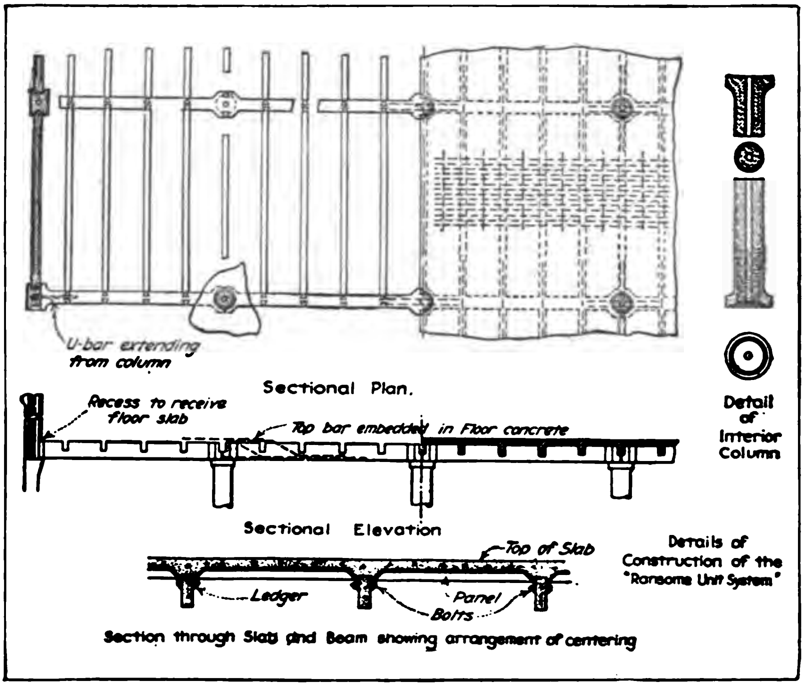


PLATE VII



Details of Construction of the "Ransome Unit System" must be so arranged as to thoroughly bind the slab and beam, and make these members act as a unit. The beams and girders are usually cast of a depth equal to the distance from the bottom to the neutral axis only (Plate VII). The slab forms are erected between the beams (which are usually spaced about 4 ft. on centers) and rest upon ledgers bolted to the sides of the beam. The beam and girders are joined to the slab with a beveled joint, giving a slight draft to the forms, thus affording considerable concrete around the connecting rods.

There is no shoring placed under the floor during construction so that the beam-and-girder units should be designed strong enough to carry their own weight plus that of forms and wet concrete slabs, in addition to an allowance for impact from buckets and so forth. The slab panels may be removed in a much shorter time than would be permitted on monolithic construction, since the previously molded beams and girders take care of the entire load up to the time when the full live load is brought on the floors.

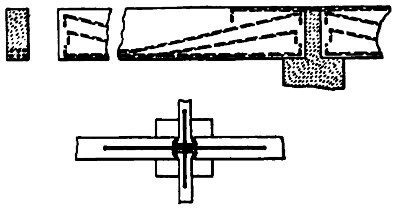


Fig. 70.

The beams in some buildings have been connected by means of tie bars as shown in Fig. 70. At about the quarter points and in the top of each beam and girder a hole was cast, extending down into the beam about 3 in. From this to the end of beam a slot was formed. With two beams end to end, a rod with a right-angle bend at each end could then be slipped in place before the slab was poured. In the more recent developments of the system (Plate VII), the ends of the reinforcing rods project above the tops of the beam and girder units and a loose rod is inserted in the slab to provide for negative moment. The beams and girders are figured as simply supported although, with proper design, the continuity might probably be taken advantage of to some extent.

STEEL-FRAME CONSTRUCTION WITH CONCRETE SLABS

27. Types of Construction.—The steel skeleton consists of columns, girders and cross-beams—the same arrangement as in the monolithic beam-and-girder construction—the beams usually being spaced about 6 ft. on centers. The many types of this form of construction differ from each other in the form of slab steel used, the position of the concrete relative to the beam, and in the use of curved or flat slabs.

28. Wrapping of I-beams.—I-beams completely enclosed in concrete should be wrapped with wire, wire mesh, or metal lath, to prevent the concrete below the bottom flange from cracking and dropping off. The material used should be of sufficient size to render efficient service even though it should, by some accident, become corroded. It should be wrapped securely to the I-beam flange, and, at the same time, sufficient space should be left to effect a concrete clinch between the wrapping material and the beam.

29. Types Illustrated.—Fig 71 shows the floor slab placed directly on the tops of the steel beams. The reinforcement of the slab may be either small rods, a wire fabric, or sheet reinforcement. The slab as shown must be calculated as a simple beam, since reinforcement is not provided for negative moment over the supports.

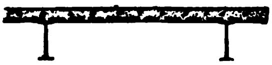


Fig. 71.

Fig. 72 is a very common form of concrete floor supported by steel girders. The form for constructing same is also shown. The haunches of the slab are carried down to the lower flange of the I-beam, the under surface of which may be covered with metal lathing and plaster for fire protection (see Figs. 73 and 74). The I-beam is sometimes entirely encased in the concrete but it is difficult to place the material under the lower flange.

Concrete floors are sometimes laid as continuous slabs with only the upper part of the I-beams in the concrete, and sometimes the slab is so located with reference to the I-beams that the metal is placed between the beams instead of running over them, as in Figs. 75 and 76. Still another type of floor consists of arches sprung between the lower flanges of the I-beams and filled to the floor level with cinders.

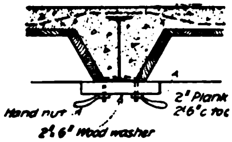


FIG. 72.

FIG. 73.

FIG. 74.

The Roebling slab floor is of many types, a common form being shown in Fig. 77. The reinforcement is flat bars, which are bent at the beams so as to connect with the flange as shown. Spacers, which supply the place of distributing rods, are fitted into slots in the bars. A 16-ft. span may be constructed with this type of floor. A flat ceiling is obtained by suspending metal lathing from beam to beam and plastering. A Roebling segmental concrete arch floor is shown in Fig. 78.

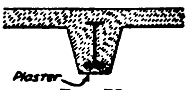


FIG. 75.

FIG. 76.

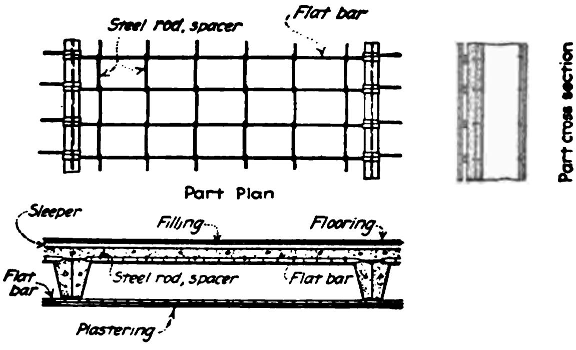


FIG. 77.—Roebling slab floor.

ROOFS

30. Structural Design.—Concrete roofs of the usual type are designed in the same manner as floors. Any likelihood of vapor condensing on the underside of roofs in buildings where steam-laden or moist air is to be found may be avoided by proper insulation and good ventilation.

When roof girders or frames are built monolithic with the columns in order to reduce sizes of members, the method and formulas given in Sect. 10 may be employed to determine the resulting moments.

31. Loading.—The roof of a reinforced-concrete building should be designed to carry the weight of roof covering and snow which may come upon it. If the roof has considerable pitch, wind pressure should also be considered. A roof load commonly assumed in temperate climates for flat roofs is 40 lb. per sq. ft., in addition to the weight of the concrete itself.

The snow load varies with the latitude and the humidity. As a maximum it is approximately 30 lb. per horizontal sq. ft. in Canada and northern Wisconsin, 20 lb. in the City of Chicago, 10 lb. in Cincinnati, and rapidly diminishes southward.

The wind load, which acts horizontally, varies with the velocity of the wind. A pressure of 30 lb. per sq. ft. of vertical surface is usually assumed. Several formulas are in existence for determining wind pressure on inclined surfaces. Duchemin's formula which follows, is preferred by many engineers as it is based upon carefully conducted experiments:

$$P = P_1 \frac{2 \sin A}{1 + \sin^2 A}$$

where P = normal pressure of wind in pounds per square foot of inclined surface.
P_1 = pressure of wind in pounds per square foot on a vertical surface.
A = angle of inclination of the roof.

The dead load of any roof may be estimated quite closely from the following data—weights are per square foot of roof surface:

Five-ply felt and gravel roof, 6 lb.
Four-ply felt and gravel roof, 5½ lb.
Three-ply ready roofing, 0.6 to 1 lb.
Slates, 3⁄16 in. thick, 7¼ lb.; ¼ in. thick, 9.6 lb. (the common thickness is 3⁄16 in. for sizes up to 10 by 20 in.).

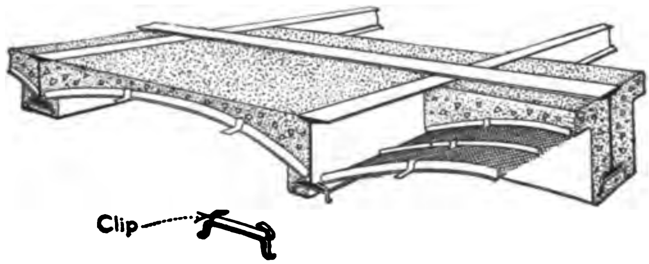


FIG. 78.—Roebling segmental concrete arch floor.

Shingle clay tiles, 11 to 14 lb.
Spanish tile, 8 lb.
Vitrified roofing tile, 1 in. thick, 9 lb. (including asphalt and five thicknesses of felt, 11½ lb.).
Slate tile, ⅞ to 1 in. thick, 13 lb. (including asphalt and felt, 15½ lb.).
Tin roofing, sheets or shingles, 1 lb.
Copper roofing, sheets, 1½ lb.; tiles 1¾ lb.
Corrugated iron, 1 to 3 lb.
Skylights with galvanized-iron frames, ¼-in. glass, 4½ lb.; 5⁄16-in., 5 lb.; ⅜-in., 6 lb.
Plaster, 5 lb.
Suspended ceilings, 10 lb.
Cinders, 45 lb. per cu. ft.
Cinder concrete, 112 lb. per cu. ft.

32. Prevention of Condensation on Concrete Roof Slabs.[1]—In storage warehouses and buildings of a similar nature, where no artificial heating is required, condensation can be almost eliminated by proper ventilation. Buildings of this type may be classed among those requiring little or no insulation for concrete roofs. Power houses, paper mills, roundhouses and similar structures with concrete roofs, however, are a class of buildings which require the best of insulation and ventilation to prevent condensation. Between these two extremes lie many manufacturing and industrial buildings which, if built of concrete, will require a more or less positive type of insulation for the roof slab.

With these facts in mind it can be seen at once that it would be folly to use the same kind of insulation for all classes of buildings and expect to obtain good results and at the same time exercise economy. In one case, a certain method of insulation may meet all the requirements, while in another, on account of different conditions, the results may be entirely unsatisfactory.

[1] By ALBERT M. WOLF, C. E., in *Concrete-cement Age*, May, 1914.

For this reason the writer will refer so far as possible to the particular kind of condensation insulation which is applicable to each type of building.

The types of insulation to be discussed are as follows:

1. Roofing felts and quilts.
2. Cinder fill (with cement finish upon which the roofing is laid).
3. Cinder-concrete fill (covered with roofing).
4. Hollow tile (with mortar top coat upon which roofing is laid).
5. Combination hollow tile and cinder fill.
6. Double concrete roof (light concrete slab above the main roof slab).
7. Suspended ceilings.

Roofing Felts and Quilts.—If the concrete roof is pitched or sloped to provide for drainage and the building is to be used for ordinary light manufacturing, warehouse or storage purposes and very little steam or moisture is present, a heavy roofing quilt or insulator placed under the tar-and-gravel or prepared waterproof roofing, will furnish sufficient insulation. Such insulation is easily applied, is of light weight and low first cost. The cost in general amounts to about 1¼ cts. per sq. ft. in place. It has a disadvantage which somewhat offsets the low cost, that of being soft and cellular and therefore easily pierced or broken down, thus destroying its insulating value and necessitating a renewal.

Cinder Fill.—One of the most common methods of insulating roofs is to place a fill of steam-boiler cinders on the roof pitched to provide for drainage and covered with a coating of cement mortar about 1 in. thick upon which the roofing is placed. This method of insulation permits the use of a level roof slab, which in itself is quite a saving. The extra cost of form-work with a roof sloped in one general direction amounts to about ½ to 1 ct. per sq. ft., while if the surface is warped this extra cost will amount to 3 or 4 cts. per sq. ft.

The cinders, which should be a porous grade of steam-boiler cinders, free from refuse or slag, should be wet down thoroughly, then placed on the roof, arranged to the proper slopes and tamped to an even surface. The minimum thickness of cinders at any place should be not less than 3 in. Before the cinders have had a chance to dry out a 1 : 3 cement mortar coat, mixed quite wet, should be placed on the cinders to a depth of about 1 in. and given a smooth float finish. After the mortar has thoroughly set the roofing may be placed.

It is essential that the cinders be wet down before hoisting to the roof, for if this is done after placing on the roof slab the excess water will stand on the slab and cause trouble by seeping through the ceiling. If the cinders are not wet down before placing, they do not tamp well and when the surface finish of cement mortar is applied, the dry cinders will take up the water in the mortar and decrease its strength and value.

With insulation of this sort it is necessary to provide for expansion of the top surface. The top portion of the fill and the mortar finish should therefore be kept 1 in. or so from all parapet walls to allow for expansion joints to be filled with some bituminous or asphalt paving pitch.

A cinder fill weighs on an average from 65 to 75 lb. per cu. ft. and it is therefore important that the downspouts be so arranged as to keep the average depth to a minimum, which will usually be about 12 in. The cost of this type of insulation for an average depth of 12 in. is about 9 or 10 cts. per sq. ft.

This insulation is solid enough to bear all the usual weights coming upon it, gives no trouble from expansion and can be readily used on concrete slabs which are designed as future floors in case of the desire for the addition of future stories because of the ease with which it can be torn up and re-used. It has been used extensively on concrete warehouses and manufacturing buildings and is a very satisfactory insulation for any type of building except power houses, paper mills and other similar buildings where much steam is present.

Cinder-concrete Insulation.—A porous cinder concrete mixed in proportions of 1 part by volume of cement to 8 parts or 10 parts of porous screened steam-boiler cinders has been used to a considerable extent as an insulator. It should be placed carefully so as not to lose the porosity, in much the same manner as a cinder fill, and finished off with a mortar coat on which to lay the roofing. Expansion joints should be provided at all walls the entire depth of the fill, since on hot days such a fill expands and exerts considerable pressure. Many parapet walls have been pushed out of place because of failure to observe this rule. A cinder-concrete fill insulation should primarily be very porous, with a rich mortar finish to seal the dead air space in the fill.

A cinder-concrete fill is not so efficient an insulator as a cinder fill, the cost is higher and the danger of expansion is greater. The excessive weight, about 100 lb. per cu. ft., is the main disadvantage, which means that the roof construction must be considerably heavier in order to carry the load. The cost of the cinder-concrete fill with an average depth of 12 in. will vary from 13 to 15 cts. per sq. ft., depending on the height of building and the cost of cinders.

Hollow-tile Insulation.—Hollow clay tile laid on a concrete roof slab and covered with a cement-mortar finish upon which the roofing is laid forms a good insulation against heat and cold and resulting condensation. The tile used are 3 or 4 in. thick, of the ordinary soft clay partition-tile variety, with scratched surfaces to furnish a key for the cement-mortar surfacing about ¾ or 1 in. thick. The tile should be laid end to end to form continuous air spaces, with tar or asphalt expansion joints at all walls.

This insulation can be used on sloping roofs only and in fact is the ideal one for such roofs, since it combines the advantages of light weight (32 to 35 lb. per sq. ft.) comparatively low cost and positive insulation. It can be constructed very rapidly and easily and can be used for almost any type of structure. The average cost of this construction will be about 10 to 12 cts. per sq. ft.

Combination Hollow Tile and Cinder Fill.—Without doubt the best insulator is the combination hollow tile and cinder fill, for it combines and augments the advantages of each method considered separately. It is constructed in the same manner as the cinder fill except that the hollow tile are first placed end to end on the roof slab, and the cinders and mortar finish placed thereon. The weight of this construction for an average depth of 12 in. amounts to from 70 to 75 lb. per sq. ft. and the cost is about 12 to 13 cts. per sq. ft.

This insulation is as nearly perfect as can be made without the use of expensive cork insulation combined with tile, etc. For power houses, paper mills, roundhouses and structures of a similar nature it is as nearly positive as can be constructed, and with proper ventilation no trouble should be experienced from condensation. The dead air space directly over the roof slab afforded by the tile, and the protecting covering of cinders (which at the same time forms the roof slopes) to keep the temperature of the air in the dead air space at the normal temperature, allow little chance of condensation except when the ventilation is insufficient. The principal objection is the weight of this type of insulation, but where a positive insulator is required the advantages cited overcome the disadvantages of weight.

Double-roof of Concrete Slabs.—A somewhat new and little-used type of construction is that of secondary concrete roof slab pitched for drainage, supported on a wooden framework resting on the concrete roof slab. The framework can be built up of 2 by 6-in. stringers set on edge on the slab and in turn supporting by means of short struts and braces, 2 by 6-in. rafters directly above at the required height and slope. The several frames thus formed are tied together with longitudinal and diagonal braces at such distances apart as can be spanned by the thin concrete slab to be placed on the stiffened metal lath fastened to the top of frames. The ribbed metal lath should be lapped at the sides and the ends, and securely fastened together. It will be found most economical to use the maximum span allowable for the heaviest metal lath obtainable. This is a No. 24-ga. stiffened metal lath which will support without other centering a 2-in. concrete slab before the same has set on a span of 6 ft. After hardening, such a slab will readily carry a live load of 25 lb. per sq. ft.

The concrete of a 1 : 2 : 4, or better still, a 1 : 1½ : 3 mixture should be mixed rather dry, for if there is an excess of water some of the cement will be carried away when the water drips through the mesh. The coarse aggregate used should be a crushed stone or gravel passing a

¾-in. mesh. The slab should be given a smooth float finish ready to receive roofing or waterproofing, which should be of the best, for if any moisture reaches the metal mesh it will soon rust and the wood supports rot out and the roof be destroyed. This construction weighs about 30 to 35 lb. per sq. ft. and will cost on an average about 16 cts. per sq. ft.

This construction gives a continuous dead air space over the roof slab and if the work is well done and the ends effectively closed it provides a very effective insulation for any type of building. It has the disadvantages of high first cost and the use of a wooden framework, which does not bring the construction within the fireproof classification.

Suspended Ceilings.—Suspended ceilings are used quite frequently to prevent heat radiation and condensation on concrete roofs. In beam-and-girder construction good insulation can be obtained by fastening a metal lath to the bottoms of the beams with wires or expansion bolts and applying a cement plaster. Where the spans are short an ordinary metal lath will suffice but for long spans (over 2 ft.) a stiffened metal lath should be used. The lath should be lapped and wired together at sides and ends so as to form a stiff surface. The type of ceiling just described will cost about 6 to 7 cts. per sq. ft.

For flat-slab roofs without beams or girders a different type of suspended ceiling is used. This construction consists mainly of a No. 26-ga. stiffened metal lath ceiling wired at every rib to ¼ by 1½-in. flats or 1¼-in. channels, 5 ft. c. to c., running at right angles to the ribs, the flats or channels being supported by No. 7 wire hangers or by ¼ by 1-in. flats spaced about 5 ft. c. to c. hung from the concrete roof slab and placed at the time of pouring the latter. If a lighter or No. 24-ga. stiffened metal lath is used, the supports should be not more than 4 ft. c. to c. Such a ceiling will cost on an average 10½ cts. per sq. ft.

These ceilings should be plastered with a 1 : 5 : 12 plaster consisting of hydrated lime, Portland cement and sand, respectively, thoroughly mixed while dry before adding water. Long cow-hair should be used in the plaster.

On account of the dead air space between roof and ceiling this type of construction gives a very positive insulation. The metal lath, however, has a tendency to rust and experience has shown that suspended ceilings will break down in hot fires. Then again the roof slab must be sloped or other provision made for drainage, which adds to the cost. Suspended ceilings have been used as insulators in nearly every class of buildings including power houses, mills and roundhouses and have in general given good service.

33. Concrete Roof Surfaces.—Although concrete roofs have been designed to be impervious without a covering of any other material, it seems to be the general opinion that it is difficult to secure absolute imperviousness in this type of roof because of the likelihood of shrinkage cracks. Some engineers, however, believe that the cracks may be kept so minute, by properly reinforcing against all the tensile stresses due to expansion and contraction, that the roof will stay waterproof. In fact, some large buildings have been erected on this theory, and have shown no signs of leakage. There is no doubt, however, that a concrete roof without a special covering of any sort is at least well adapted to all structures where absolute imperviousness is not essential, as in train sheds, reservoir roofs, and in *out-of-door* structures in general.

Shrinkage joints are sometimes introduced in concrete roof slabs. These joints are made water-tight by the introduction of a trough-shaped strip of copper or lead which will take contraction and expansion. If a sufficient number of joints of this nature are made, shrinkage cracks will undoubtedly be reduced to a minimum. To make shrinkage joints the slab must be poured separately from the beams and girders, and no bond should be allowed between the slab and the roof framing. If steel beams are used, there is no danger, but in all-concrete construction, galvanized strips, well oiled, or some equivalent should be introduced.

A maximum amount of reinforcement against shrinkage cracks will be of little avail in a concrete roof unless the concrete itself is impervious. Methods of waterproofing concrete may be roughly classified as follows: (1) Use of a rich-concrete mixture of such proportions of sand and stone as to produce maximum density; (2) use of waterproof coatings or washes;

(3) admixture of substances designed to produce impermeability. It is often advisable to combine two or more of these methods.

Methods of proportioning concrete for maximum density are given in Sect. 2. Plastering a concrete roof surface with a layer of rich mortar has been found effective when care has been taken to place the same before the concrete has taken its final set. If such care is not exercised, variation in temperature and moisture between the concrete and plaster will be almost certain to cause a separation. Troweling concrete to a dense, hard surface, in the same manner that granolithic work is troweled, makes concrete impervious and nearly equal to a surfacing of rich mortar.

The methods employed to waterproof concrete by using washes or by introducing foreign ingredients into the mixture are given in chapter on "Waterproofing Concrete" in Sect. 2. The general remarks made there apply to roofs as well as to other types of structures.

A hard wearing surface is sometimes required in roof construction—for example, when it forms the floor of a roof garder. The roof surface in such a case should have a granolithic finish the same as in floors, and a form of bitumen emulsion mixed with cement mortar may be used in making the surface impervious.

It is not good practice to use a concrete roof without covering on steep slopes, as the slabs must then either be laid quite dry (which does not favor imperviousness) or else top forms must be used to retain the concrete. The latter method is slow and expensive.

34. Separate Roof Coverings.—The felt and gravel roof is the most common and efficient form of roof covering. The covering is composed of layers of waterproof felt, cemented together and to the concrete by coal-tar pitch or asphalt. The method followed is to lay from four to eight thicknesses of felt over the roof surface, each ply being cemented to the preceding layer, and the entire surface then floated with a heavy, flowing coat of pitch or asphalt, in which (while hot) clean, dry, uniformly screened gravel or slag is embedded, sufficient in quantity to cover the surface thoroughly.

Asphalt has in the past been preferred to coal-tar pitch as a binding medium, but at the present time coal-tar products appear to be satisfactory when made to contain a large percentage of carbon, and are being used by many in preference to asphalt. Asphalt, however, seems to be much superior to coal-tar pitch in ready roofings.

The following specification for a five-ply tar and gravel or slag roof has been taken from the *Proceedings* of the American Railway Engineering Association, 1910 and is essentially the Barrett Co.'s specification.

There shall be used five (5) thicknesses of saturated felt weighing not less than fourteen (14) lb. per one hundred (100) sq. ft., single thickness; not less than two-hundred (200) lb. of pitch; and not less than four hundred (400) lb. of gravel or three hundred (300) lb. of slag from ¼ to ⅝-in. size, free from dirt, per one hundred (100) sq. ft. of completed roof.

The material shall be applied as follows: First, coat the concrete with hot pitch mopped on uniformly. Second, lay two (2) full thicknesses of tarred felt, lapping each sheet seventeen (17) in. over the preceding one, and mop with hot pitch the full width of the seventeen (17) in. lap, so that in no case shall felt touch felt. Third, coat the entire surface with hot pitch, mopped on uniformly. Fourth, lay three (3) full thicknesses of felt, lapping each sheet twenty-two (22) in. over the preceding one, mopping with hot pitch the full width of the twenty-two (22) in. lap between the piles, so that in no case shall felt touch felt. Fifth, spread over the entire surface of the roof a uniform coat of pitch, into which, while hot, imbed the gravel or slag. The gravel or slag in all cases must be dry.

The coal-tar pitch or asphalt is the life of the roof, particularly in the topcoating. The gravel or slag should be applied liberally, in order to completely bury the coat of waterproof material and protect it from injury due to walking on the roof, and from the action of the sun. In the following statements asphalt only will be mentioned, but it must be understood that the same statements will apply to coal-tar pitch.

In waterproofing, the object of using saturated felt is merely to provide a medium to hold the asphalt together, and thus allow for expansions, contractions and settlings. The greatest care and judgment must be exercised in laying felt to see that it is properly stretched, laid smoothly, and that no wrinkles appear. All of the roofing felts on the market are made of wool or flax, saturated with a preserving material that will harmonize with the asphalt. Saturated wool roofing felts are compact, and although they absorb little of the asphalt, they hold it in repeated layers to constitute the body material of the roof. The flax felt is porous and of strong texture, and absorbs the asphalt, holding it within its fibers. Unsaturated burlap or canvas can be made to hold asphalt if first run through a bath of liquid asphalt or tar.

The greater the amount of felt and asphalt used, the greater the life of the roof. However, if too much asphalt is used on top, it will run; hence the flatter the roof, the greater the life. No roof should have less than 100 lb. of asphalt per 100 sq. ft.

What is known as a felt and gravel roof should not be used in cold climates, on a surface of greater pitch than about 3 in. to the foot. In hot climates 1 in. to the foot is about the proper maximum. Where the roof is of greater pitch, the gutters may be put in with galvanized iron, copper, tin or piles of felt and asphalt, and the steeply pitched surface covered with clay tile or slate shingles with lap joints. Vitrified clay tile are made in a great variety of forms, flat, ribbed, and corrugated; but those of some interlocking pattern are best. Nailing strips (1 by 2 in.) should be inserted in the concrete under each row of tile or slate, but it is important that they should be so placed as not to affect the strength of the roof slab. Slate or tile should not be used on a surface with a pitch of less than 6 in. to the foot.

A most durable roof covering for flat surfaces (slope not exceeding ¼ in. to the foot) is vitrified roofing tile embedded in asphalt. The tile consists of flat rectangular terra-cotta tile about 1 to 1¼ in. thick, bedded in hot asphalt, on top of four to six thicknesses of felt of the kind mentioned above. The joints between the tiles should be filled with asphalt. Slate tiles also make a good wearing surface. They are usually ⅞ to 1 in. thick, by 12 by 12 in. in area.

Tin, corrugated iron, and copper roofings are sometimes placed on reinforced-concrete buildings. These roofings do not have a long life if laid directly on the concrete, but seem to give satisfaction when a wooden sheathing is used as a separator. Copper is expensive as compared with other types of roofing and is usually employed only on very costly buildings.

There are many roofings on the market which come ready to lay, made on the principle of the built-up roof. These roofings are especially valuable for use in small and isolated buildings, as they do not require the expert help which is necessary for a built-up roof. In a preparation which comes rolled, however, it is impossible to obtain a great deal of waterproof material, and there is an additional weakness in some coverings of this type due to the fact that the waterproofing sheets are nailed together with a short exposed lap. Expansion and contraction of the body material in the course of time opens holes alongside the nails and permits water to enter. In spite of adverse criticism, however, there are many ready roofings which seem to give satisfaction and are used frequently on pitched surfaces, and to some extent on flat slopes.

A number of types of metal and composition shingles are on the market. Reinforced-concrete slabs from 5 to 6 ft. long have been used for this purpose.

35. Drainage.—Felt and gravel roofs should have a pitch of at least ⅜ in. to 1 ft. in order to provide proper drainage. Flat tile, however, may be employed on surfaces with a very slight pitch—preferably not over ¼ in. per ft.

Gussets in flat roofs are generally formed by placing a well-tamped cinder filling over the concrete slab and then laying a 1 to 2-in. surface of cinder or stone concrete. This type of roof generally permits the use of the floor forms without much change and there is the advantage of having all the column heights of the top story the same. Stone-concrete surfacing seems to be the most satisfactory but a cinder mixture is sometimes specified.

If a flat ceiling is not required, or if it is the intention to employ a suspended ceiling (Fig. 79), the roof beams and roof girders may be so inclined as to give the required roof pitch. This method is advantageous when cold weather is likely to overtake the work before it can be closed in, for, with the use of cinders as a filling, there is delay in placing the final roof surface.

Where concrete walls project above the roof proper, grooves or reglets about 1 by 1½ in.

must be left in the concrete wall in which to insert the edge of the flashing (see Fig. 80). To make a reglet, a strip of wood is nailed to the forms and the strip is taken out when the forms are removed. The flashing is keyed with the reglet, as shown, and cemented up with a rich cement—sometimes rubber cement.

When metal standing flashing is specified on felt roofs, it is necessary to nail through the metal and felt into wooden strips embedded in the concrete slab in order to hold the flashing in place (see Fig. 80). Since expansion and contraction will soon loosen the nails, the nails and the flange of the flashing should be covered with a felt strip prior to coating the roof with the top coat of asphalt and gravel. A reinforcement of flax felt, mopped on solidly, appears to be as satisfactory as metal flashing in every way (see Fig. 81). All such felt flashings should be turned up at the parapet walls and curbs at least 4 in. at the highest points of the roof, and not less than 12 in. high as the outlets are approached, in order to avoid overflows should the outlets become clogged.

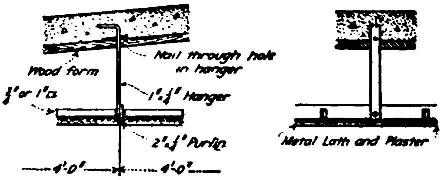


Fig. 79.—Suspended ceiling construction.

Fig. 80 shows the method employed in applying a galvanized-iron flashing for a felt and gravel roof on a large reinforced-concrete warehouse. The reglet was made to taper slightly—that is, wider at the outside—in order to permit the easy withdrawal of the wood strip that was nailed to the forms. The groove was made 10 in. above the roof line. When the concrete was being poured for the roof floor, a heavy strip of wood was laid with its upper surface flush with the top surface of the concrete, and 6 in. from, and parallel with, the parapet wall. This strip was employed to nail the edge of the flashing to the roof after the roof had been covered with felt and asphalt. The nails and the flange of the flashing were covered with a felt strip prior to coating the roof with the top coat of asphalt and gravel.

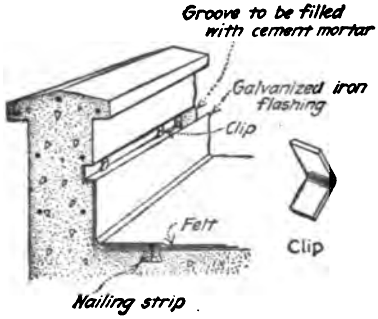


Fig. 80.

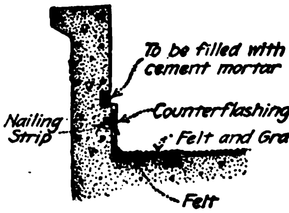


Fig. 81.

The flashing was secured in the reglets by band-iron clips, as shown. These bands (2 in. long) were bent midway to an angle of about 30 deg. After insertion they were struck with the hammer on the bend, which straightened them out and jammed them in place, thereby holding the flashing. The reglet was pointed up with a rich cement to insure a water-tight connection. Figs. 81 to 84 inclusive, show different methods of flashing which may be employed (see also Figs. 89 and 90, and Art. 36).

In flat-roof construction with parapet walls, all valleys in the roof surface should lead to drain boxes. Fig. 85 shows the drainage scheme for part of the roof of a shoe factory at Cambridge, Mass.

Drain boxes, or conductor boxes as they are usually called, have been made in many ways—lead or copper being generally used. Fig. 86 shows a double copper box for conductors used on a factory building at Roxbury, Mass. Fig. 87 is a sketch of a conductor box installed in a warehouse at Portland, Maine. Wire screens or strainers should always be specified since they prevent the downspouts from becoming clogged by leaves, etc. The downspouts may be

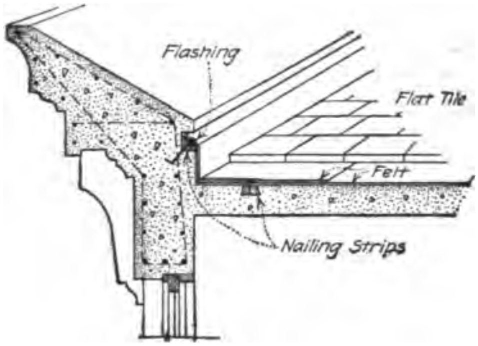


FIG. 82.

carried down inside or outside the building—depending upon preference and sewer conditions. The conductors should be cast iron when carried inside the buildings, while corrugated iron,

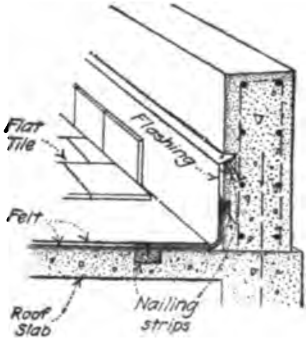


FIG. 83.

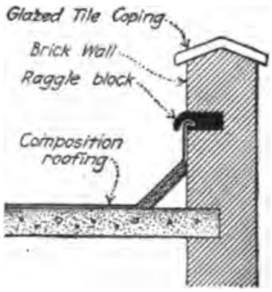


FIG. 84.

due to its ability to expand without breaking, is better on the outside of the building where freezing of water in the conductors may be expected. All vapor and soil pipes extending

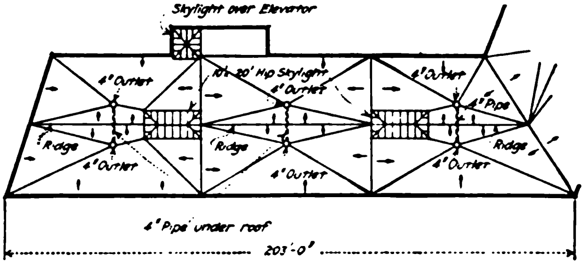


FIG. 85.

through the roof should have an expansion sleeve soldered on and counterflashed with an inverted copper cone attached to pipe. For discussion of conductor heads for roof drainage, see article by A. M. Wolf in *Engineering News*, May 11, 1916, page 901.

Fig. 88 shows the method of draining a roof surface of considerable pitch—a condition under which parapet walls could not be used. This is called hanging-gutter construction. The gutters should have a slope of about 1 in. in 16 ft.

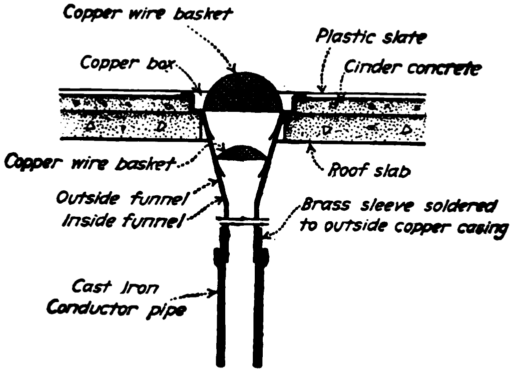


FIG. 86.

The following table will serve as a guide by which to proportion the size of gutters and downspouts:

Span of roof	Gutter	Conductor
Up to 50 ft.	6 in.	4 in. every 40 ft.
50 to 70 ft.	7 in.	5 in. every 40 ft.
70 to 100 ft.	8 in.	6 in. every 40 ft.

There is wide variation in practice in the number of square feet of roof surface to 1 sq. in. of leader opening, but the above table should give some idea of the general practice.

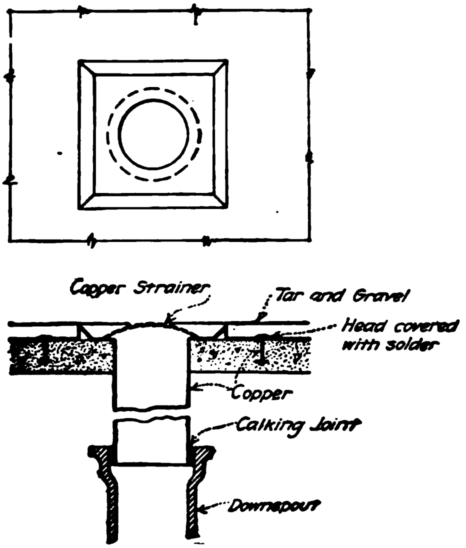


FIG. 87.

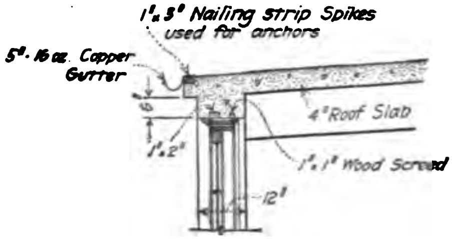


FIG. 88.

36. Parapet Walls.[1]—Parapet walls are of two general types, brick or reinforced concrete, or a combination of both, the type used depending upon the architectural treatment desired. Where a roof is enclosed by parapet walls, overflow openings in the latter should be provided at suitable points to prevent flooding of the roof in case the downspouts become stopped up. Such openings can be provided by simply leaving a hole in the parapet, or better still, the

[1] By ALBERT M. WOLF, C. E., in *Concrete*, Dec., 1916.

openings (4 to 8 in. square) can be lined with sheet metal or a pipe projecting at least 1 in. beyond the face of the wall.

Brick Parapet Walls.—Brick parapet walls have been used extensively, but unless properly built, they are very likely to give more or less trouble, and if built to avoid this, the usual economy over concrete (in localities where brick are comparatively cheap) is lost. This has given rise to the use of concrete parapets either exposed or covered with a veneer of brick or terra-cotta.

The vital part of the brick parapet wall is the inner side, and usually this is made up of common brick laid up in ordinary lime mortar. As a result, many brick parapets in a few years become a crumbling mass, owing to the freezing of the brick just above the roof flashing, after being saturated with water splashing up on them during rains, or from snow piled on the roof. Such a condition increases the cost of maintaining the roof in proper condition, for once the brick start crumbling, the flashing becomes loosened, water finds its way back of the roofing and down through the slab. This means that the parapet must be rebuilt, the flashing and probably the roofing also, must be replaced.

Since the life of a good roofing is materially shortened by the conditions cited above, first-class roofers now recommend that the inner side of the brick parapets be built of hard-burned vitrified or paving brick laid up in cement mortar, and covered with a waterproof coping. In addition to this, the coating of the parapet (on roof side) with roofing tar or pitch up to the underside of the coping will do much toward making it more permanent.

Where cinder or cinder-concrete fills are used to form the drainage slope, special attention should be paid to the anchoring of parapet walls to the concrete slab and the provision of expansion joints between walls and roof filling. When cinder fills are used, a coating of cement mortar from 1 to 1½ in. thick is placed on the top after grading the cinders to the proper slopes, to produce a firm foundation for the roofing. The mortar coat expands considerably, as does cinder-concrete filling, which is sometimes used, and expansion joints from 1 to 2 in. wide should therefore be placed at all walls extending down through the mortar topping or cinder concrete. These joints should be filled with a tar or asphalt paving pitch which will perform its function of completely filling the joint under all conditions of weather.

Lack of proper provision for expansion of roof fills has been the cause of the pushing out of line, and rendering dangerous many brick parapets. For this reason, in addition to the expansion joints, it is well to anchor the brick walls to the spandrel slabs or girders by means of stub bars bent up from the roof slab or spandrel. No rules for exact size and spacing of such anchor bars can be given, but in general ½ in. diameter bars projecting up into the wall for about 2 ft. at a spacing of from 18 in. to 2 ft. will do the work. Of course it is highly essential that the wall be built in solid around the rods with good cement mortar.

Another detail of importance is the flashing slot and strip left in the parapet to provide a means of fastening the flashing and counterflashing and making waterproof the roofing at the parapets. A very satisfactory detail for flashing of ordinary tar and gravel roofing at parapets is shown in Fig. 89. This detail, recommended as good practice by the American Railway Engineering Association, makes use of a 2 by 4-in. timber with one edge beveled, laid continuous in the parapet at the proper height in place of a stretcher course of brick. This serves as a nailing strip for a light wooden strip holding the flashing and counterflashing in place. After placing the flashing the slot is completely sealed up with cement grout or roofing cement.

Concrete Parapets.—For the proper flashing of concrete parapet walls the detail shown in Fig. 90 can be recommended. A 2 by 4-in. piece of lumber is ripped on the diagonal as shown and then placed in the forms at the desired height, the upper strip being securely nailed thereto, so as to insure its removal when forms are taken down, while the lower piece is just tacked to forms (from outside) with wires or nails driven into it as shown to anchor it to the concrete. The flashing and counterflashing are then placed in the same manner as for brick walls.

As generally designed, concrete parapets, in addition to retaining or masking the drainage slopes, carry a portion of the roof load as beams, but owing to the fact that they are generally much deeper than required to simply carry the load, other considerations besides the load must be taken into account. That is, enough reinforcing must be provided and distributed in a manner which will prevent the formation of expansion and contraction cracks resulting from the excessive changes of temperature to which side walls are subjected. Ordinarily parapets are at least 3 ft. deep overall, and usually this depth is much more than is required to resist the bending moments induced therein, especially in flat-slab construction where the portion of the flat slab adjacent to the parapet carries most of the roof panel load. This means that a very low percentage of steel will be required to resist the tensile stresses produced by bending, in fact, it may be such small amount as to be incapable of resisting the stresses set up by temperature changes. For this reason, it is always well to so detail the reinforcement of the portion of the parapet above the roof as to have not less than 25 to 30% of longitudinal reinforcement arranged somewhat as shown in Fig. 91 with plenty of vertical reinforcement in the form of

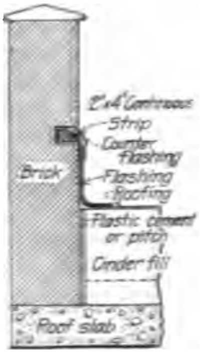


Fig. 89.

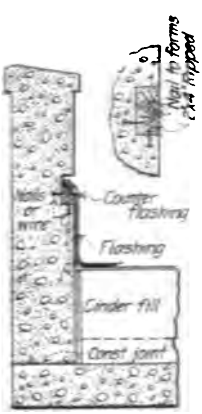


Fig. 90.

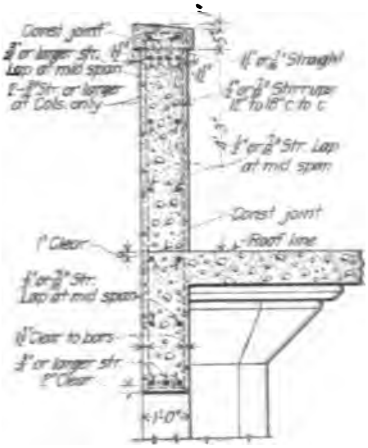


Fig. 91.

stirrups to tie together the portions of parapet poured at different times (portion below and above roof line).

Concrete parapets should always be designed and reinforced as fully or partially continuous beams, depending upon the location thereof and the condition of end supports, for unless this is done unsightly cracks are sure to appear near the supports. If the parapet walls are of such form as to require pouring in two operations (as in Fig. 91) they will, of course, not be so strong as if poured in one operation, and, therefore, if the total depth is to be considered effective a bending moment coefficient somewhat lower than used in the formula for fully continuous beams, viz., $M = \frac{wl^2}{12}$, should be used. In the writer's opinion, this should be, for beams of the type in question, $M = \frac{wl^2}{10}$ for interior spans and $M = \frac{wl^2}{9}$ for end spans at support and mid-span.

Parapets seldom require very much diagonal tension reinforcement owing to the depth of same and the relatively light loads to be carried, and the use of bent bars is therefore seldom warranted, since the stirrups used to tie the portions of wall together can be made of sufficient number to care for all diagonal tension stresses in excess of that which the concrete alone will resist. At corners extra horizontal bars should be provided, bent around the corner so as to lap with the main bars, for unless this is done, cracks are sure to develop, owing to expansive and contractive forces acting at right angles to each other. In large buildings it will be found advisable to provide expansion joints in parapets about every 200 ft. over columns, the spans adjacent to such joints being designed and reinforced same as end spans.

Where concrete parapets are covered with brick or terra-cotta which forms a part of the architectural treatment, they must be reinforced to resist the twisting action produced by the

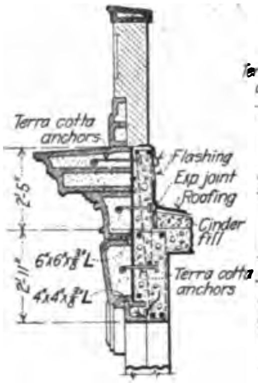


FIG. 92.

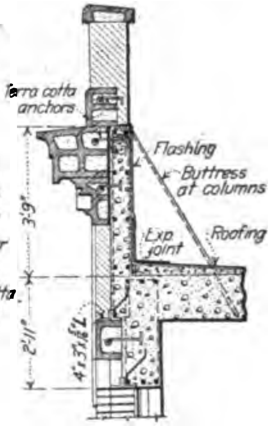


FIG. 93.

weight of the material hung from the concrete (see Figs. 92 and 93). When the concrete wall supporting the ornamental part of the parapet or cornice is not very high, as in Fig. 92, the stresses thus produced can be taken care of by placing vertical bars or stirrups so as to reinforce

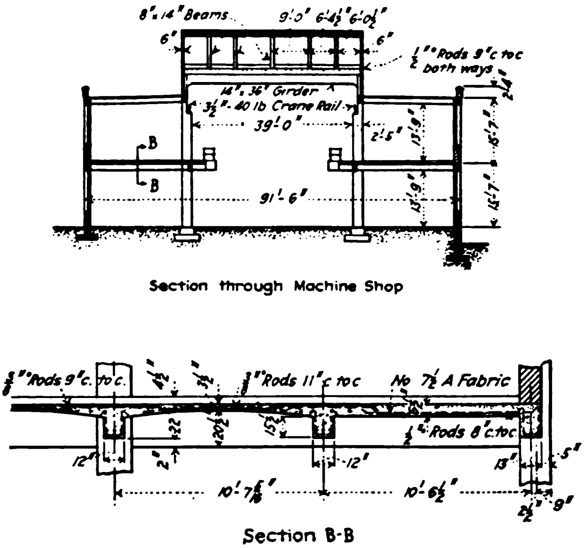


FIG. 94A.

the wall as a cantilever with sufficient longitudinal reinforcement to resist the tensile stresses due to beam action in a vertical plane. If the concrete portion of the parapet is relatively high, and the ornamental stonework or terra-cotta projects a considerable distance beyond the face of the wall, or, in other words, if the center of gravity of the entire parapet lies outside the face of the concrete wall, buttresses should be used at the columns and the wall reinforced as a counterfort wall between the buttresses. The main reinforcement in the buttresses (at the back) should be well anchored into the floor construction or into the column below and after extending up the back of the buttress be bent down in front of the longitudinal wall bars, thus giving a positive support for the continuous wall slab. Buttressed parapet walls should also be reinforced to take care of the ordinary beam action which may develop due to the weight of the parapet. The same bending-moment coefficients are recommended as for ordinary roof parapets previously described.

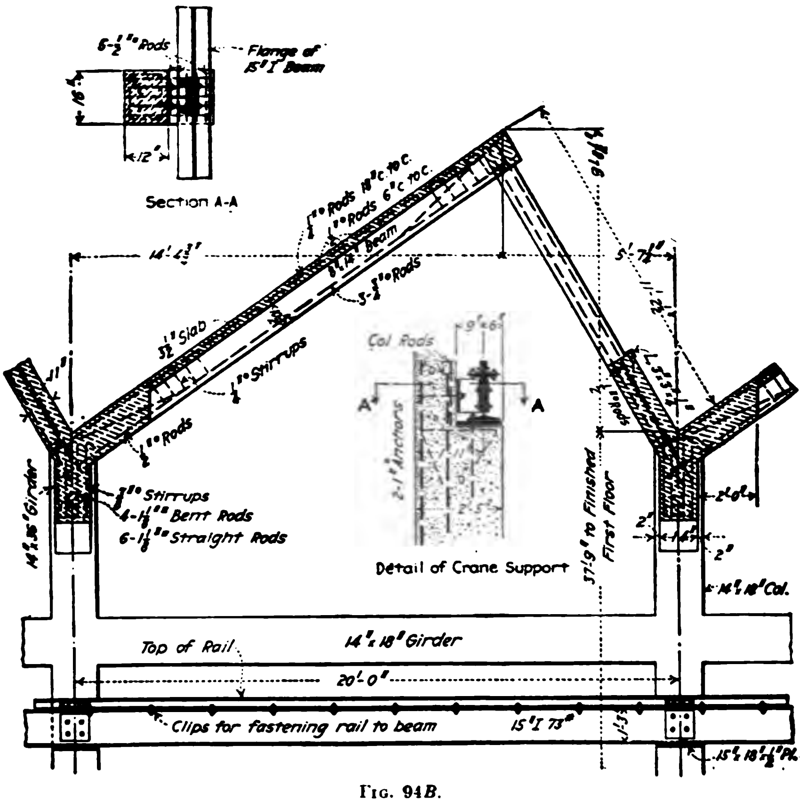


Fig. 94B.

If the covering of brick or terra-cotta is only a few inches thick, it can be supported by spandrel angles anchored to the concrete, and placed wherever offsets occur in the veneer. In addition to these angles, corrugated metal brick ties set in the concrete should be built into the brick joints to aid in holding the brick in place. Terra-cotta or stonework which projects 9 in. or more beyond the face of concrete should be anchored to the parapet by bolts passing through the webs of the blocks of terra-cotta or by bent plate anchors securely fastened into the stone courses. These anchors should be used in addition to the spandrel angle supports previously mentioned (see Figs. 92 and 93). When a well-designed concrete parapet is used, there is no danger of failure of the main support of the cornice, and if the spandrel angles and anchors are properly placed and well covered with mortar when placing the veneer, practically all danger of part of the cornice falling is removed.

37. Sawtooth Construction.—Sawtooth roofs, in general, have been found especially well adapted for machine shops and factories, where it is desirable to have a uniform daylight

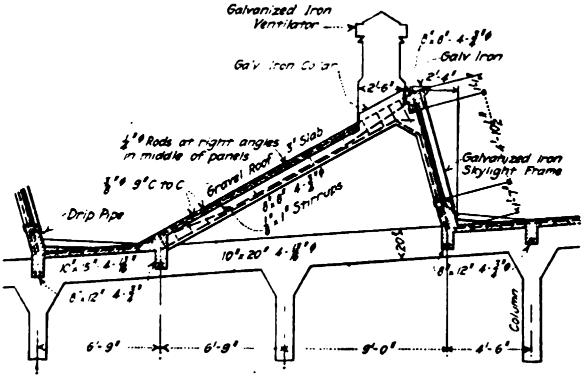


FIG. 95.—Cross-section detail of sawtooth roof.

illumination over the entire floor area. In reinforced concrete, unfortunately, this type of construction is expensive because of the irregularities of the forms, and has been employed only

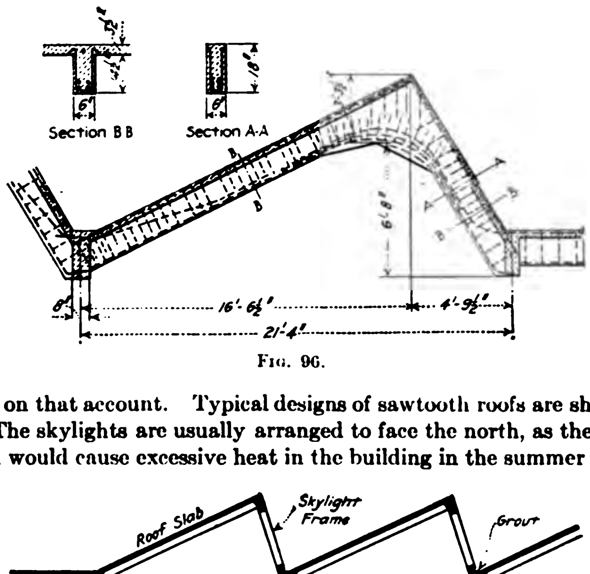


FIG. 96.

to a limited extent on that account. Typical designs of sawtooth roofs are shown in Figs. 94*A*, 94*B*, 95 and 96. The skylights are usually arranged to face the north, as the sun's rays would be undesirable and would cause excessive heat in the building in the summer time.

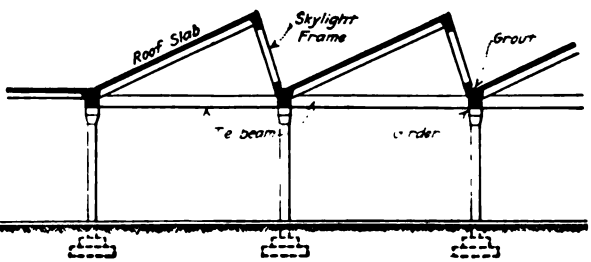
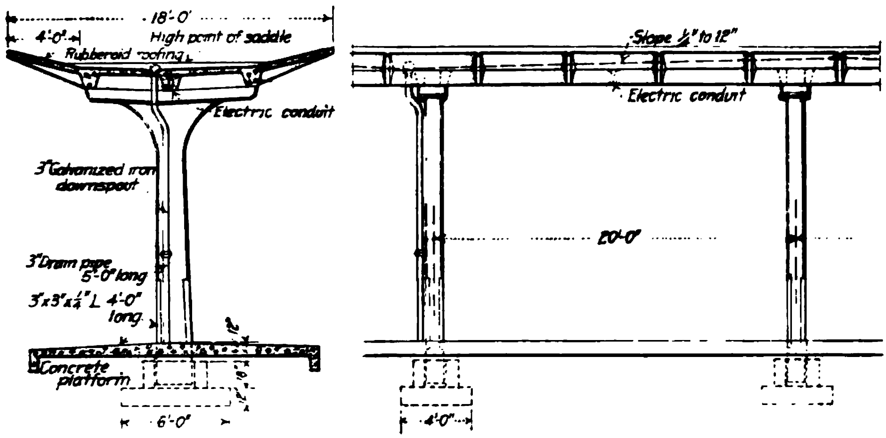


FIG. 97.—Cross-section showing typical arrangement of units in sawtooth construction, *Unit-bilt* system

Fig. 96 is a sketch of a sawtooth roof used in a cotton mill at East Boston, Mass. The girders supporting the sawtooth were made of sufficient stiffness so that no horizontal tie rods were necessary. The lines of columns for the sawtooth span are 20 ft. c. to c. and two girders are located in each bay.

A sawtooth roof construction using separately molded members has been developed by the Unit Construction Co. of St. Louis (see Art. 25). Fig. 97 is a cross-section showing the typical arrangement of units. The roof portion of the sawtooth rests at its lower end on a ledge in the girder and at the upper end on a ledge in the skylight frame. The lower end of the frame also rests on a ledge in the girder and the horizontal beams are provided in order to tie in the building, and for the possible support of shafting or other installations. The skylight frame extends from column to column and consists essentially of a large flat plate into which the window frames are cast. The connections are made in practically the same manner as the floor connections for this system of unit construction as described in Art. 25.



Transverse section Typical elevation

Fig. 98

Sawtooth skylights are usually glazed with wire glass, held in metallic frames, so that the entire construction is fire-resisting.

38. Trainshed of "Unit-bilt" Construction.—A trainshed built by the *Unit-bilt* system is shown in Fig. 98 (see Art. 25).

COLUMNS

39. Details of Design.—Columns exceeding 15 diameters in length should be avoided as but few tests have been made on columns of such slender proportions. Fortunately, however, columns longer than 15 diameters are quite rare except in roof stories where a light roof load often requires very light sections. Where long columns must be used, the reduction formula given in Art. 9, Sect. 8 may be used in their design.

A high percentage of longitudinal reinforcement—say above 4 to 6%—is undesirable on account of the uncertainty of the strength of such columns. In fact, increase in the proportion of cement is usually much to be preferred to a high steel percentage, not only because the cement gives a more definitely known increase in strength, but also because it is a cheaper column reinforcement than steel. In no case should a leaner concrete mixture than 1 : 2 : 4 be used.

All columns should be reinforced with at least four ¾-in. round rods or four ⅝-in. square bars whether or not such reinforcement is theoretically required. This should be done to guard against the possibility of eccentric loading and for safety in construction.

Provision should be made in all cases for the bending moment which will be developed by unequally loaded panels, eccentric loading, or uneven spacing of columns. Especially is this true for wall columns and corner columns. Moment in columns due to unsymmetrical floor loading may be found as explained in Sect. 10. Moments in columns due to bracket loads may be found by the formulas of Art. 10, Sect. 8. The stresses and amount of reinforcement required for a column subjected to bending as well as direct stress may be found as explained in Sect. 9.

The maximum allowable stress in a column due to bending and direct stress may be taken considerably greater than for direct stress only, since the maximum stress does not occur over the whole cross-section of the column but simply at the outside edge. The bending moment also rapidly diminishes along the column from the maximum value used in the stress computations. A unit stress 15% greater than for direct compression may be permitted provided the unit stress due to the maximum loading centrally applied is not greater than the allowable for direct compression.

In structures with inflammable contents where a special fireproofing is not provided around concrete columns, 1½ in. on all sides should be considered as protective covering and then this thickness should not be included in the calculations for strength. A less thickness than 1½ in. should be sufficient where the contents of a building are not especially inflammable. In no case, however, should the steel be nearer the surface than 1½ to 2 in. since it is desirable to prevent any tendency of the vertical rods to buckle under working loads. For square or rectangular columns the corners should be beveled or rounded, as sharp corners are more seriously affected by fire than round ones.

It is advisable in all cases to place occasional horizontal hoops around the vertical steel. Although tests do not show that they are absolutely necessary if a 1½ to 2-in. concrete covering is provided, yet it is good practice to employ them as an additional precaution against any buckling of the rods under small loads. Such hoops, also, serve to keep the rods in place during the pouring of the concrete, and should not be farther apart than 12 in. nor exceeding about 16 times the diameter of the vertical rods. For columns of moderate size, ¼-in. wire hoops are generally used 12 in. on centers.

Bands, hoops or spirals to be considered effective should have a clear spacing not greater than one-sixth (preferably one-tenth) the diameter of the enclosed column—in no case, however, more than 2½ in.—and should be equal in amount to at least 1% of the volume of the column inside the hoops (see report of the Joint Committee in Art. 7, Sect. 8). It should be noted that the Joint Committee recommends an increase in the allowable working stress on the concrete of hooped columns only when the ratio of the unsupported length of column to the diameter of the hooped core is not greater than 10.

It should always be kept in mind that hoops and spirals are tension reinforcement and subject to all the rules governing the design of such steel. If hoops are used, the ends should lap a sufficient distance to secure the requisite grip, or else the ends should be bent toward the center of the column to accomplish the same purpose. When spiral reinforcement is employed, it is important to have one continuous piece from top to bottom, or else joints should be made as just explained.

Hooping should extend from the bottom of the column to the under side of the slab above and adequate means should be provided to hold it rigidly in place so as to form a column, the core of which will be straight and well-centered. For ordinary percentages of hooping, tests show that wire of high-carbon steel gives a much larger increase in the ultimate strength of column than wire of mild steel.

The core of a hooped column should preferably be circular in cross-section, but square and rectangular hoops are sometimes employed in practice. (The Joint Committee recommends that only circular hoops be employed.) In such cases, the hoops are probably less effective. but it is not known how much. The ideal condition, theoretically, is hooped columns of circular cross-section with circular cores.

When light vertical rods are used they may be spliced by lapping a sufficient distance above the floor level (about 30 diameters) to develop the requisite bond strength. They should be bent slightly inward at the top in order to come just within the rods of the column above, thus preventing any tendency of the load coming down from the upper rods to bulge the column. Rods much over an inch in diameter should be milled square and butted together in a secure manner.

The splicing of large rods may be effected by joining them in rather closely fitting pipe sleeves. All such splices should be made just above the floor level and not more than 12 in. above the same. Since each rod should find a bearing on a rod below, the number of rods increases downward in the building and, unless the loading is eccentric, great care should be taken to have the rods symmetrically arranged in each story. A splice of this kind is good for compression in the rods but is of no use when the stress is tension. Rods in tension may be spliced by using short extra rods at the splice. These short rods should extend above and below the joint far enough to develop the requisite bond strength. Rods may extend through several stories if so desired, but it is difficult to hold them in place while concreting the lower lengths.

At the bottom of the columns, the loads from the rods should be distributed over the footing by means of a steel bearing plate. In order not to overstress the concrete in the column, the concrete immediately below the plate should be enlarged so as to bring the average pressure within the allowable. Tests show that if considerable lengths of the longitudinal reinforcing rods are bent outward into a reinforced-concrete footing, the strength of column will be about as great as when these rods are bedded on a metal plate. The results, however, also show that the use of metal plates leads to greater uniformity and strength. Base plates should be set in a thin grout of proportions about 1 : 2 in order to prevent hollow places under the plates.

In our modern city buildings where the floor space on the first floors is valuable, the use of a steel column core presents a very economical and efficient construction. To be able to count upon the concrete in such columns, the steel itself should be sufficiently rigid to act as a column and the concrete should be well enclosed either by the steel form itself or by means of bands or hooping. However, if the amount of steel is very large, the relative value of the concrete is more uncertain, and its element of strength should be neglected. The best connection between column and beam is a structural-steel seat riveted to the column, and of sufficient size to carry the entire load in bearing. These seats do not appear in the finished surface of rooms and their stiffness is considerably assisted by the concrete shell. Where structural-steel columns are used, the floor beams and girders should be figured as simply supported unless the usual continuity rods are run through the column.

Wherever concrete walls are to be built against concrete columns at some later time, weather breaks or keyways should be formed in the columns. This is accomplished by nailing small strips on the inside of the forms. Where windows are to be set into column reveals, window ties should be embedded. In order to obtain a low cost for formwork it is better to vary square columns in one dimension rather than in two, both on account of the columns themselves and the beams framing into them.

For the same reason it is advisable to keep the story heights uniform so that the forms will not have to be cut off or spliced from story to story. If the story heights vary it is better to have the high stories at the bottom as it is cheaper to cut off the forms than to splice them.

Cinder-concrete cylinders have been used to some extent as a protective covering for reinforced-concrete columns and also to take the place of column forms. In Factory No. 1 of the plant of the Bush Terminal Co., South Brooklyn, N. Y., the interior columns are cylindrical and composed of an outside shell of cinder concrete 2½ in. thick. These cinder-concrete cylinders were prepared in advance in 2-ft. lengths, in a zinc mold, with spiral hooping and expanded metal forming the inner surface. After hardening, they were set one upon another in the building and filled with concrete.

Hooped concrete columns with cast-iron cores known as the Emperger columns have been receiving some attention. It is claimed that by using the hooped reinforcement high working stresses may be used, thus reducing the size of column (see *Engineering Record*, March 3, 1917).

Hollow reinforced-concrete columns have been used in heating and ventilation systems, the air heating and fan systems being in a pent house on the roof. The heated air is forced down the hollow columns and is distributed to the different floors through openings in the columns.

40. Loading.—In addition to its own dead weight, a column should be designed to carry the live and dead loads of the roof and floors above. The full live load on floors must be used in designing columns for warehouses or buildings for heavy mercantile and manufacturing purposes. In other buildings, however, of five stories or more in height, where it seems reasonable to suppose that a full live load will never occur on all floors simultaneously, reductions may be made as follows:

The live load on the floor next below the top floor may be assumed at 95% of the allowable live load, on the next lower floor at 90%, and on each succeeding lower floor at correspondingly decreasing percentages, provided that in no case shall less than 50% of the allowable live load be assumed.[1]

41. Column Brackets.—Column brackets, such as are shown in Fig. 99, are serviceable in stiffening the building frame and in decreasing the stress in the girders. They should not, however, be considered as decreasing the girder span, but simply as so much additional protection against failure.

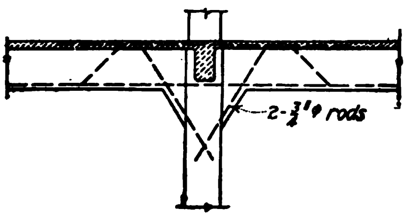


FIG. 99.

Brackets are usually required in high buildings in order to brace the frame properly against wind stresses. If, however, buildings are both high and narrow, the stresses in the members due to wind should be figured and proper provision made for the same. For method of computing such stresses, see Art. 9, Sect. 10.

A column supporting a roof may also serve as a crane post or to carry a heavy load on a bracket. For method of computing the bending moment due to such eccentric loading see Art. 10, Sect. 8

ILLUSTRATIVE PROBLEM.—In order to show in detail the method of column design, the computations will be given for the design of a typical interior column of a three-story building assuming that no bending moment is likely to be caused by unequally loaded panels or eccentric loading. The columns will be spaced 21 ft. on centers both ways and the two intermediate beam design of Plate I, page 448, will be adopted as the typical floor-bay design for all three floors.

The height from finished floor to finished floor will be made 13 ft., except the basement which will be made 11 ft. A concrete mixture of 1 : 1½ : 3 will be used, which will allow a working stress of 565 lb. per sq. in. when longitudinal steel only is employed and 870 lb. per sq. in. when effective hooping is provided. As recommended in the final report of the Joint Committee (see *Appendix B*) the concrete will be assumed to have a modulus of elasticity one-twelfth that of the steel. The size of octagonal columns will be given in terms of the short diameter.

The load coming from the roof may be found by multiplying the total shear on both girder and cross-beam by 2, and adding. Assume end reaction from roof beam at 16,100 lb. and from roof girder at 36,200 lb. Then the roof load is

$$(2)\ (36{,}200) + (2)\ (16{,}100) = 104{,}600 \text{ lb.}$$

Similarly, each floor load is

$$(2)\ (55{,}300) + (2)\ (25{,}000) = 160{,}600 \text{ lb.}$$

The building in question will be considered as for manufacturing purposes and no reduction in live load will be made. The complete design of column is given on page 531. Tables of Sect. 8 will be used in the design.

The recommendations of the Joint Committee will be followed and the amount of vertical steel is in every case less than 4% and more than 1%. The pitch of the spiral hooping is given to the nearest ⅛ in., as the mills that fabricate spirals can manufacture them to such a pitch. The weight of column has been figured for a length 2 ft. less than the story height in order to compensate somewhat for taking the weight of beams and girders as extending from center to center of supports. It is the intention to splice all rods, not lapped, by means of a tightly fitting pipe sleeve about 12 in. long.

[1] From 1916 New York City Building Code, Bureau of Manhattan.

Third-floor Columns.—Assume weight of column at 2700 lb. Then the total load is 104,600 + 2700 = 107,300 lb.

Taking the percentage of longitudinal steel at 0.025, Table 1 shows that a column whose effective diameter is 14 in. (total diameter 17 in.) is good for 111,000 lb. Table 3 shows weight of column per foot to be (1.66) (150) and the total weight (1.66) (150) (11) = 2740 lb. Table 1 shows the steel area to be 3.9 sq. in. and from Table 4 we get seven 7/8-in. round rods.

Second-floor Columns.—Assume weight at 3800 lb., then total weight is 107,300 + 160,600 + 3800 = 271,700 lb.

Table 1 shows that with 1% spiral and 3.5% longitudinal rods, a column whose effective diameter is 17 in. (total 20 in.) is satisfactory. Table 3 shows the weight to be (2.30) (150) (11) = 3800 lb. Table 1 gives the steel area of 7.9 sq. in. and Table 4 shows that fourteen 7/8-in. round rods will be sufficient. From Table 2 for pitch of 2 1/2 in., the area of spiral is 0.106 sq. in. (say 3/8-in. round) and the length of spiral per foot is 256 in. For minimum pitch of 1 3/4 in. the area of spiral is 0.088 sq. in. (say 3/8-in. round) and the length per foot is 366 in. The value of $\frac{l}{d}$ is $\frac{11 \times 12}{18} = 7\frac{1}{3}$.

First-floor Columns.—Total load is 271,700 + 160,600 + 6500 = 438,800 lb.

Table 1 shows that a column of 26 in. outside diameter and with 2% longitudinal steel will answer. Weight is (3.89) (150) (11) = 6420 lb. Steel area is 8.3 sq. in., or fourteen 7/8-in. round. Area of spiral is 0.144 sq. in. (say 7/16-in. or 3/8-in. round) and length per foot is 347 in. For minimum pitch of 2 1/4 in., area is 0.13 sq. in. (say 7/16-in. round) and the length per foot is 386 in.

Basement Columns.—Total load is 438,800 + 160,600 + 7500 = 606,900 lb.

Size of column from Table 1 is 31 in. outside diameter or 28 in. effective for $p = 1.5\%$. Steel area is 9.2 sq. in. Use fourteen 1-in. round rods. Weight of column is (5.53) (150) (9) = 7500 lb. Area of spiral is 0.175 sq. in. (say 1/2-in. round) and the length of spiral per foot is 422 in.

COLUMN SCHEDULE

Story	Kind of load	Amount of load	Shape and size of column (short diameter)	Vertical steel (rounds)	Spiral (core 3 in. less than diameter of column)
Third...	Roof.....	104,600	17 in.	7-7/8 in.	
	Column..	2,700	octagonal		
	Total...	107,300			
Second...	Floor....	160,600	20 in.	14-7/8 in. (lap	dia. = 3/8 in.
	Column..	3,800	octagonal	above 27 in.)	pitch = 2 1/2 in.
	Total....	271,700			
First	Floor ..	160,600	26 in.	14-7/8 in. (lap	dia. = 7/16 in.
	Column..	6,500	octagonal	above 27 in.)	pitch = 2 1/2 in.
	Total..	438,800			
Basement	Floor....	160,600	31 in.	14-1 in.	dia. = 1/2 in.
	Column..	7,500	octagonal		pitch = 2 1/2 in.
	Total...	606,900			

ILLUSTRATIVE PROBLEM.—It will be of interest to investigate the columns in the foregoing problem concerning stresses due to unbalanced load. Since the columns just designed represent standard practice, the following investigation will serve to point out the effect of unbalanced loading upon such a design. All diagrams referred to in this problem will be found in Sect. 10.

From a comparison of Diagrams 6 to 10, it may be seen that the largest moments in interior columns due to unbalanced loads will occur in the second tier columns.

Assuming the neutral axis of the girder in Plate I to be governed by the fact that the beam is doubly reinforced, kd is found to be 13.1 in.

$$I = I_c + nI_s$$

$$I_c = \frac{(15)(13.1)^3}{3} + \frac{(15)(22.9)^3}{3} = 71,100 \text{ in.}^4$$

$$nI_s = 47,300 \text{ in.}^4$$

Whence

$$I = 118{,}400 \text{ in.}^4 \text{ and } K_1 = \frac{118{,}400}{252} = 471 \text{ in.}^3$$

For the basement column, similarly,

$$I = 41{,}760 \text{ in.}^4 \text{ and } K_0 = 316 \text{ in.}^3$$

For the first tier column,

$$I = 20{,}300 \text{ in.}^4 \text{ and } K_2 = 131 \text{ in.}^3$$

$$K_0' = 0.675 \qquad K_2' = 0.278 \qquad K_0'/K_2' = 2.42.$$

From an examination of Diagram 10 it may be seen that, for these values of K_2' and $K_0' + K_2'$, the moment at the end of the first story columns will not exceed 20% of $\frac{F}{l}$. Therefore, for the live-load moment,

$$f = \frac{My}{I} = Wl \cdot \frac{y}{I} = \frac{(110{,}250)(21)(11.5)(0.20)}{20{,}300} = 262 \text{ lb. per sq. in.}$$

This is the extreme compressive unit stress in the concrete due to unbalanced loading. It may be noted that the panels either way from the loaded one have no live load. Thus, referring to the design of this column in the previous problem, the total load carried by the column under present conditions is

$$438{,}800 - \frac{(21)^2(250)}{2} = 383{,}800 \text{ lb.}$$

Now for a given column, in which no load is assigned to the spiral steel, the concrete unit stress due to direct load is

$$f_c = \frac{P}{A\,[1 + (n - 1)\,p]}$$

Then in the above case

$$f_c = \frac{383{,}800}{(415.5)[1 + (0.02)(11)]} = 757 \text{ lb. per sq. in.}$$

This stress, when combined with that caused by the unbalanced load, gives

$$f_c(\text{max}) = 262 + 757 = 1019 \text{ lb. per sq. in.}$$

A re-design is advised.

Illustrative Problem.—The moment in an outer column of the same size as the interior column of the preceding problems may be computed in the following manner. The dead-load moment may be computed with the girder-end fixed, and the live-load moment with the girder-end hinged. The two results may then be added together as the most probable moment in the column. For the live load (Case XI),

$$M_{Ab} = \frac{F}{l}\left[\frac{3(131)}{2(131) + 471 + 2(48.3)}\right] = 0.474\,\frac{F}{l}$$
$$= (0.474)(110{,}250)(21) = 1{,}097{,}000 \text{ in.-lb.}$$

For the dead load (Case XII),

$$M_{Ab} = \frac{F}{l}\left[\frac{3(131)}{3(131) + 2(471) + 3(48.3)}\right] = 0.265\frac{F}{l}$$
$$= (0.265)(50{,}350)(21) = 280{,}000 \text{ in.-lb.}$$

$$\text{Total moment} = 1{,}377{,}000 \text{ in.-lb.}$$

$$\text{For flexure } f = \frac{(1{,}377{,}000)(8.5)}{20{,}300} = 576 \text{ lb. per sq. in.}$$

which is the stress due to flexure only, at the top or bottom of the first-story column. The direct load on this outside column will be 438,800 + 2, plus, say, 50,000 lb. for the spandrel load above the first floor. The direct stress is

$$f_c = \frac{269{,}400}{(415.5)\,[1 + (0.2)(12)]} = 523 \text{ lb. per sq. in.}$$

This, added to the flexure stress above, gives for a total stress

$$f_c(\text{max}) = 576 + 523 = 1099 \text{ lb. per sq. in.}$$

WALLS AND PARTITIONS

42. Bearing Walls.—The two principal types of reinforced-concrete office and factory buildings differ only in the character of the outside walls. A skeleton type is the common form of construction but bearing walls are built occasionally.

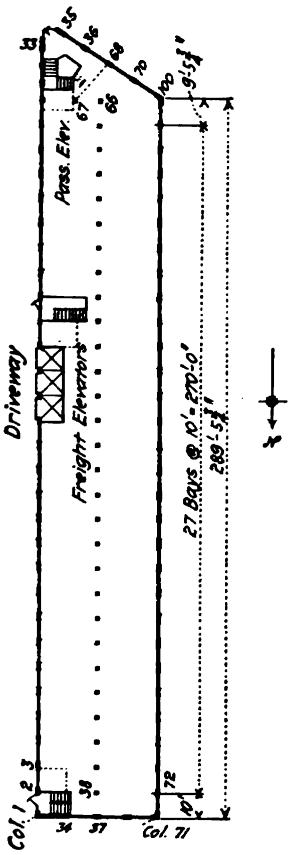
Although the bearing-wall type of construction is naturally the only type employed in concrete residences, it has not been found in much favor for manufacturing and commercial buildings in general. Of course, a building with monolithic wall construction possesses great rigidity when properly designed and reinforced, but it cannot be erected as rapidly as buildings with other wall types, and, unless great care is employed in construction, there is a likelihood of defects occurring in casting the work. There is also difficulty in obtaining a uniform color for the wall surfaces.

Concrete walls are either of single thickness, or of double thickness with an air space between. One method of obtaining an air space is by building in a central course of hollow-tile blocks. Double walls render the interior of the building less subject to changes in temperature and more completely moisture-proof, but, in the majority of office and manufacturing buildings, single walls are not objectionable and make a great saving in floor space. Occasional projections or pilasters improve the appearance and add to the strength of a single wall. Building codes usually specify that concrete bearing walls are to be made the same thickness as brick bearing walls.

Except in residences, bearing-wall footings must usually be reinforced. A cantilever projection is formed on each side of the wall and the amount of reinforcement may be determined as for a simple cantilever beam (see Art. 5, Sect. 12).

43. Curtain Walls.—In the usual type of construction, wall girders are placed at each floor and the reinforced-concrete walls (called *curtain* walls) are designed merely to fill in the panels between the columns and girders which form the skeleton frame of the building. They are not intended to carry any weight but need to be strong enough to withstand wind pressure of 30 lb. per sq. ft. Such walls may be designed as slabs carrying a uniformly distributed load and supported on all four sides. Figuring in this way, using the above value for the wind pressure, a wall in ordinary building construction will never exceed 3½ to 4 in. in thickness, which is the practical limit for ease in construction. In fact, 6-in. walls will generally be required in order to obtain sufficient imperviousness to moisture. Also, a 6-in. wall is usually cheaper than a 4-in. wall owing to the greater ease of pouring the concrete and placing the reinforcing steel in position. A 4-in. wall, however, is all that is needed for fire protection. Building codes usually specify a minimum thickness of 8 in.

In addition to the consideration of wind pressure, concrete walls are reinforced generally for the purpose of preventing cracks and guarding against accidents during or immediately after construction, except, of course, where lateral pressures occur, when the beam action must be considered. The reinforcement generally consists of ¼ to ½-in. rods placed both horizontally and vertically from 12 to 18 in. apart.



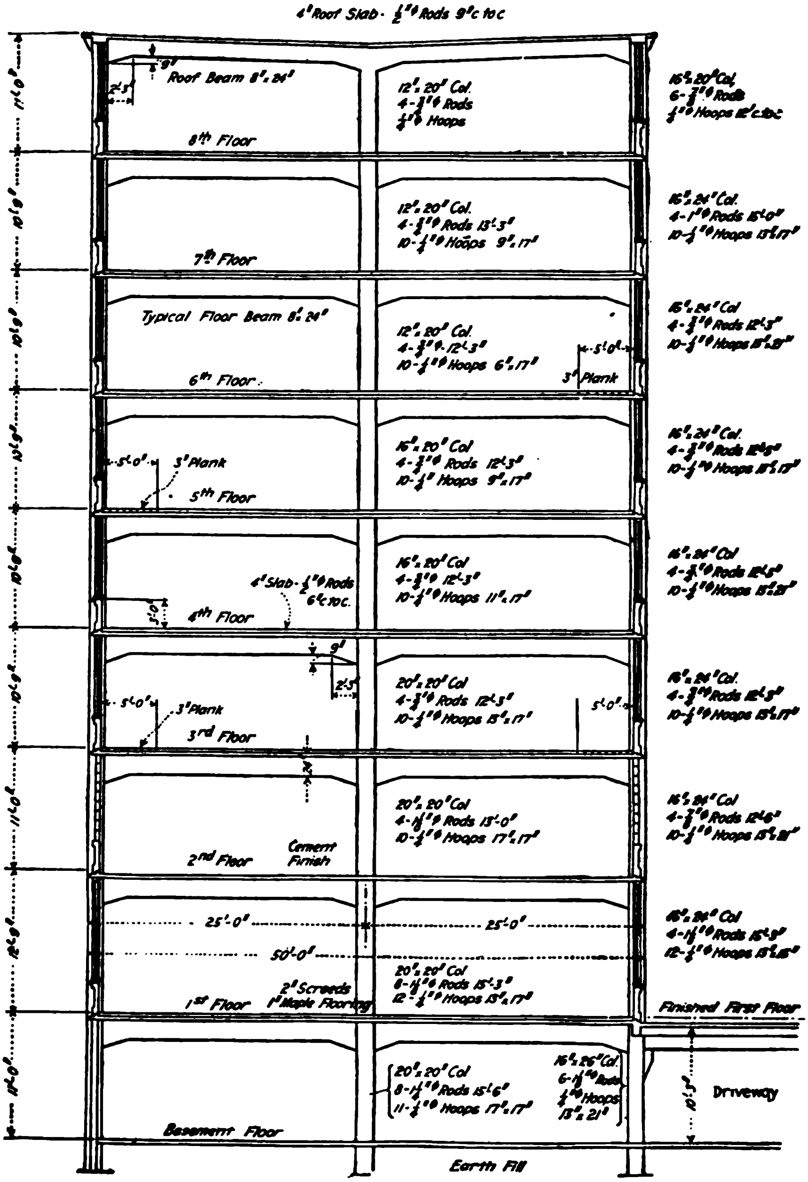
First Floor Plan

Fig. 100.—Lang building, Haverhill, Mass.

Slots in the columns are made by nailing a strip on the inside of the column forms. In this way the curtain walls are mortised into the columns and a contraction joint is formed, so that the contraction to be provided for is due only to the curtain wall itself.

When curtain walls are employed with a small percentage of window area, it is customary to use both horizontal and vertical reinforcement, the same as when the wall covers the entire panel, and to place one or more rods near the edge of the slab about all openings. When wire fabric is used for reinforcement, the edges of the openings are stiffened by bending back the fabric into a U-shape.

Where a large percentage of the outside wall of a building is used for windows, as is gener-



TYPICAL CROSS SECTION THROUGH BUILDING

FIG. 101.—Lang building, Haverhill, Mass.

ally the case in factory buildings, the wall proper consists of little more than columns with deep wall beams or wall girders forming belt courses between them. The spandrels, or walls directly beneath the window sills, may be reinforced so as to act as the upper part of the wall beams, but the usual method is to consider this portion as separate from the beam and merely reinforce with small rods (in some cases with wire fabric) so as to prevent cracks and to reinforce the construction during the setting of the concrete. If this is done, the spandrels may be put

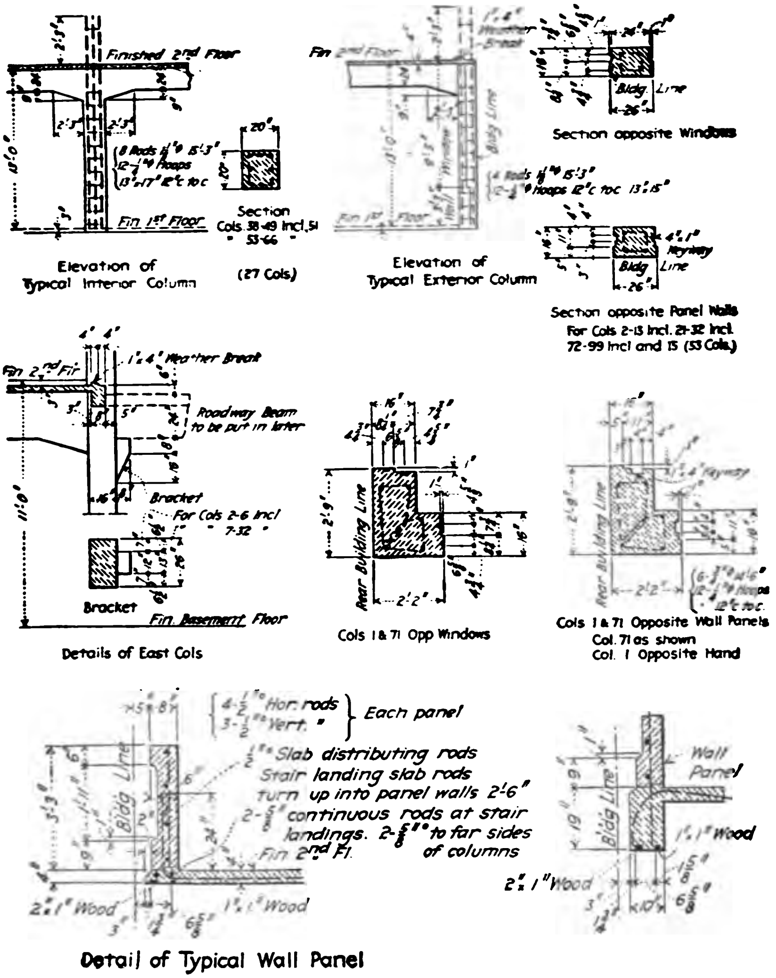


FIG. 102.—Details of Lang building, Haverhill, Mass.

in after the main structural parts have been cast, in the same manner as complete curtain walls. This saves time in the erection of the building and allows the use of more care in obtaining a neat finish on the spandrel walls.

In Fig. 102 two spandrel sections are shown, one where wall beams are employed and the other where only the floor slab and spandrels form the exterior belt course. Figs. 100 to 106 inclusive show many of the details of the Lang Building at Haverhill, Mass. Many of the details are referred to elsewhere.

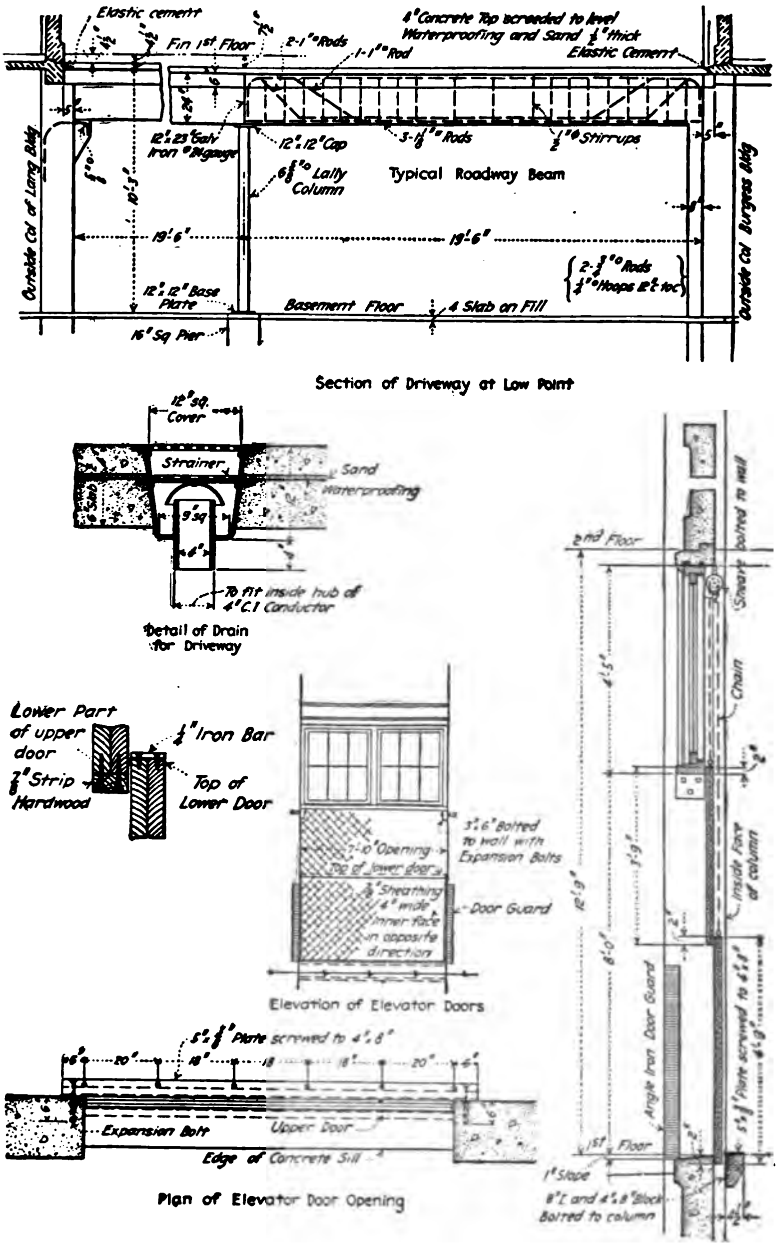


Fig. 103.—Details of Lang building, Haverhill, Mass.

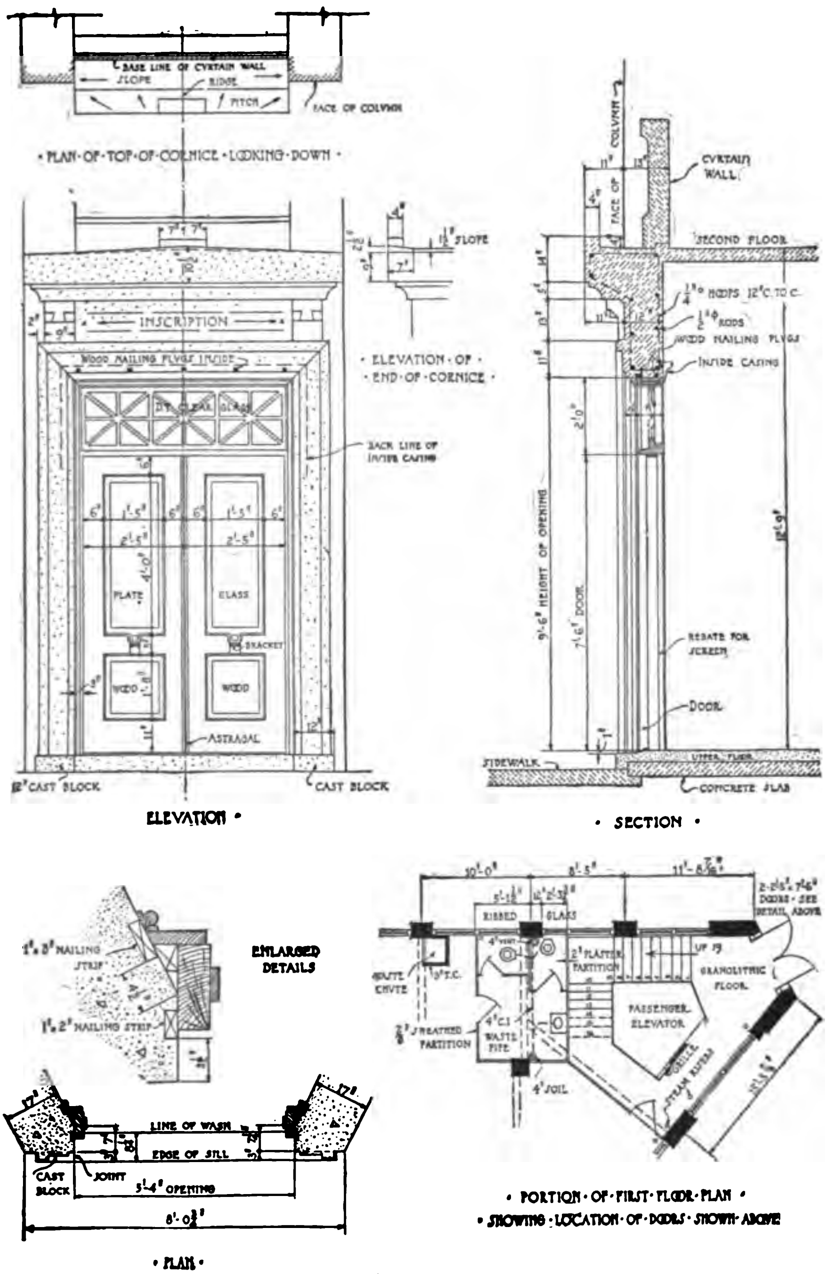


FIG. 104.—Details of Lang building, Haverhill, Mass.

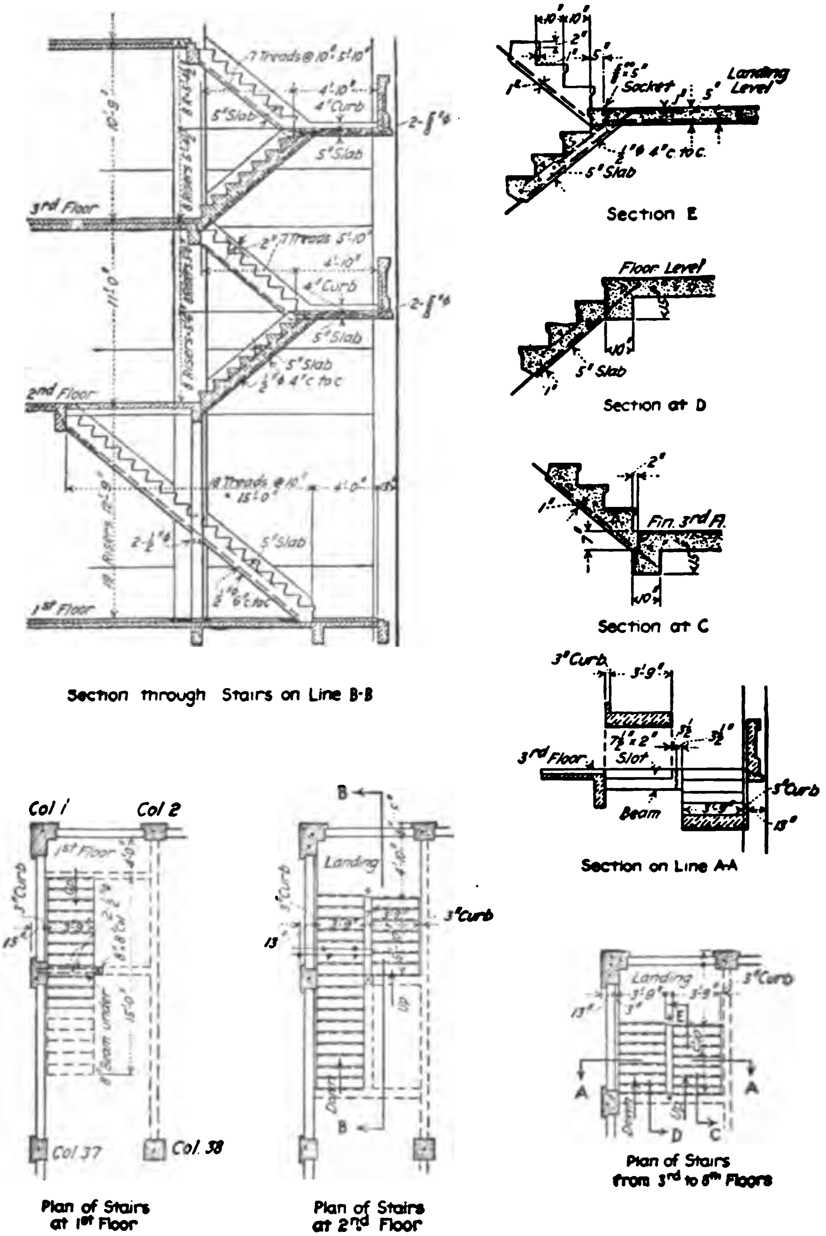


FIG. 105.—Details of Lang building, Haverhill, Mass.

Reinforced concrete is well adapted to the construction of walls where considerable strength is required, but in very thin walls, such as curtain walls, it becomes a relatively expensive building material on account of the cost of forms. Brick, concrete blocks, or terra-cotta are often used on this account, not only for spandrel walls, but to cover entire wall panels.

44. Brick and Other Veneer.—From an architectural standpoint it sometimes becomes necessary to cover the exterior columns and walls with brick, terra-cotta, marble, limestone, or other material. The facing or veneer is usually laid after the structural frame is completed.

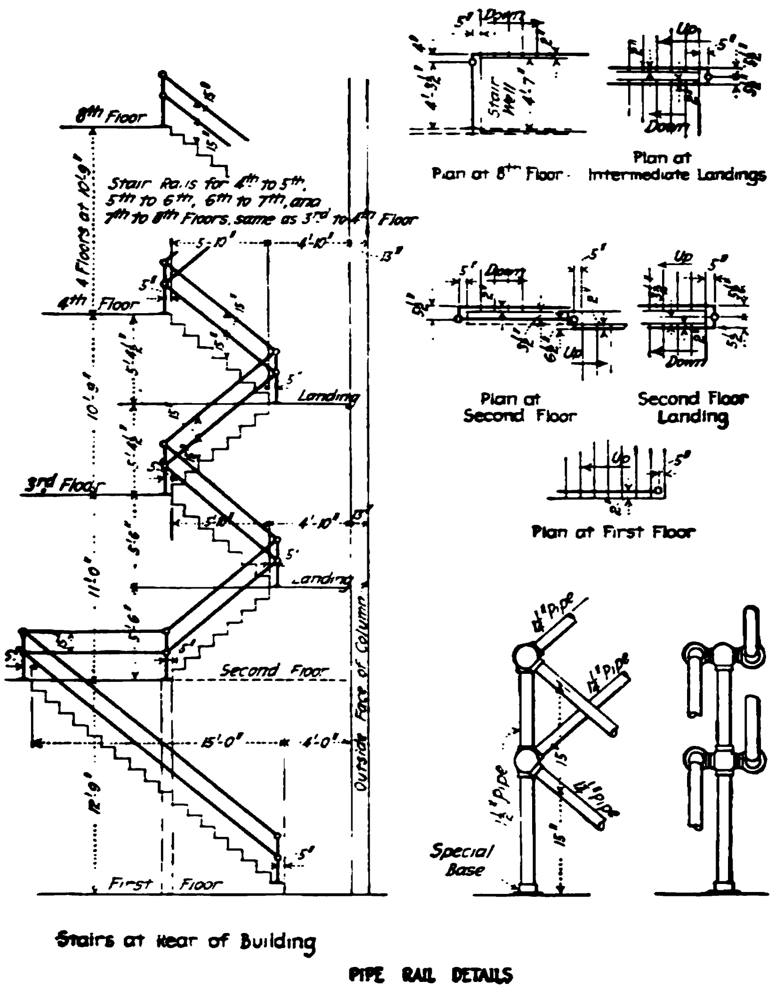


Fig. 106.—Details of Lang building, Haverhill, Mass.

A brick veneer generally consists of one thickness of brick, and this is tied into the concrete work by means of copper or galvanized-iron ties. These ties are usually about ¾ in. wide and are tacked to the form boards in such a manner that about 4 in. of the tie will be embedded in the concrete work. When the forms are removed, the portion of the tie lying flat against the forms is bent out as shown in Fig. 107. A tie should be provided for about every 4 sq. ft. of wall surface. Brick facings should be supported, at least at every floor, by a ledge formed in the concrete. Angle irons are usually employed to carry stone and brickwork over door and window openings.

Fig. 108 shows one method of supporting a stone lintel. The anchor at the top should be noted.

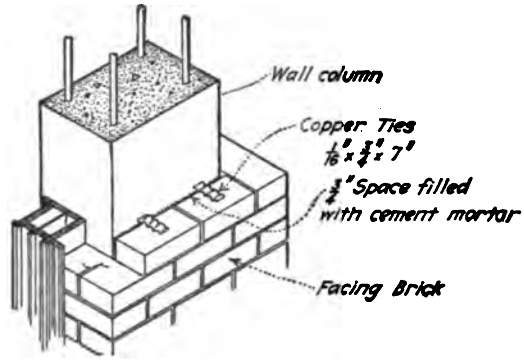


FIG. 107.

The arrangement of the ties and supports for terra-cotta varies greatly with the design. The facing is usually supported by projections in the concrete fitting into openings in the terra-cotta, forming a sort of dovetailing. Iron anchors then tie the two together. It is very important that suitable play be provided for, as terra-cotta cannot be made to exact dimensions. Stone facings are also supported by projections in the concrete, the same as terra-cotta, but the stone can be made to more nearly fit any simple shape given to the concrete ledge.

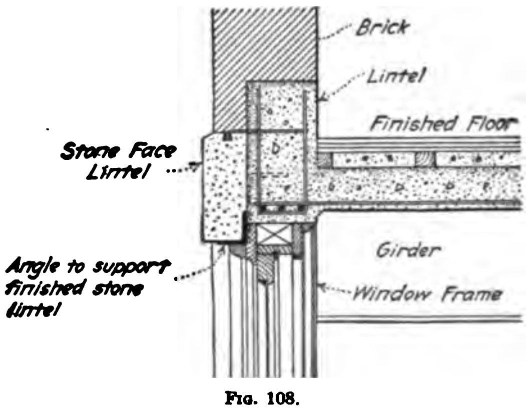


FIG. 108.

45. Window Openings.—The arrangement of floor slab and wall beam in Fig. 108 makes it possible to obtain the maximum amount of light within the building; that is, the window frames are run as close as possible to the underside of the floor slab. It should be noted that to accomplish this, the beams should either be run parallel with the walls of the building, so that none of the beams will take bearing on the lintel, or else the floor arrangement should be similar to that shown in Fig. 101 where only parallel girders are employed spaced relatively close together. If beams run parallel with the walls, the spacing of the beams may be returned at the corners by a diagonal girder as illustrated in Fig. 109. By so doing the windows may be made the same height throughout.

Reinforced-concrete buildings to be considered fire-resisting should have windows of wire glass, held in metallic frames. Wire glass is either ribbed, rough, maze, or polished plate having wire embedded in its center during the process of manufacture. The wire netting used for this purpose is similar to ordinary *chicken netting* with about a 1-in. mesh. It is embedded in the glass at a very high temperature which insures adhesion between the netting and glass, and the two materials become one and inseparable. If the glass is broken by shock or by intense heat, it remains intact. It is this property which gives wire glass its fire-retarding qualities.

Metal frames are made in a great variety of forms to meet all purposes. Sashes may be stationary, sliding, pivoted either horizontally or vertically,

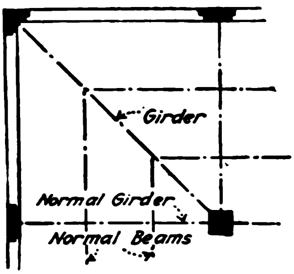


Fig. 109.

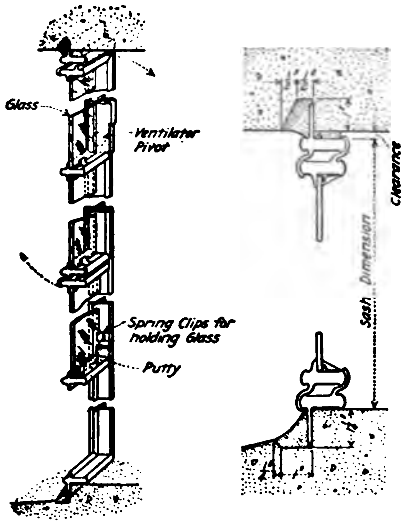


Fig. 110.

hinged, or double hung with weights like an ordinary window. Sash may also be obtained which will close automatically and lock under fire by the fusing of a link or other means to accomplish the same result. A number of sections of steel-sash windows are shown in Figs. 110, 111, and 112. These sections are included to show a few of the different methods of fastening metal frames to the walls or wall columns.

When properly constructed, a tin-covered wood shutter is a most effective window protection. There is one objection, however, to the use of shutters on window openings and that is, they must necessarily be open while the building is in use, and, when

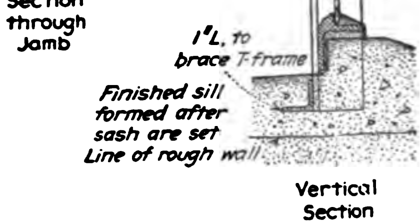


Fig. 111.

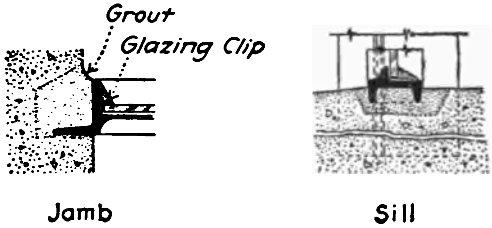


Fig. 112.

the need comes for them, they are apt to be overlooked and not closed. They, moreover, hide a fire and are unsightly for many locations. Frames and sash of wood covered with metal are sometimes used—the wood furnishing the strength and the metal the protection. Note the combination of wood, tin and steel in the window design shown in Fig. 113.

Automatic steel rolling fire shutters, placed either on the outside of window openings or in the window reveals immediately in front of the window frames and sash, form the least objectionable appearing type of shutter protection for windows, and seem to be entirely ade-

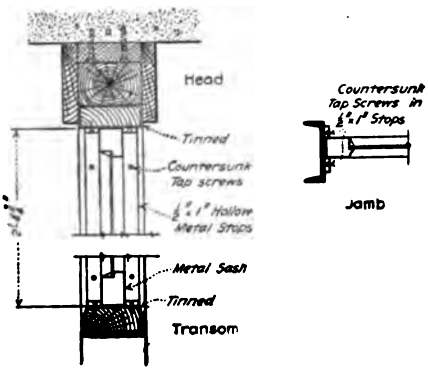


Fig. 113.

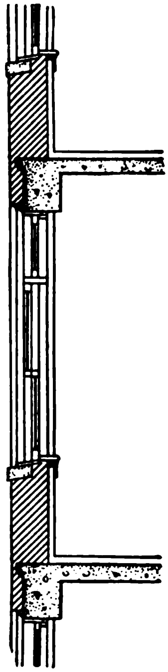


Fig. 114.

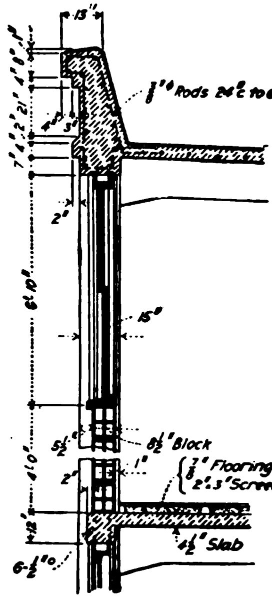


Fig. 115.

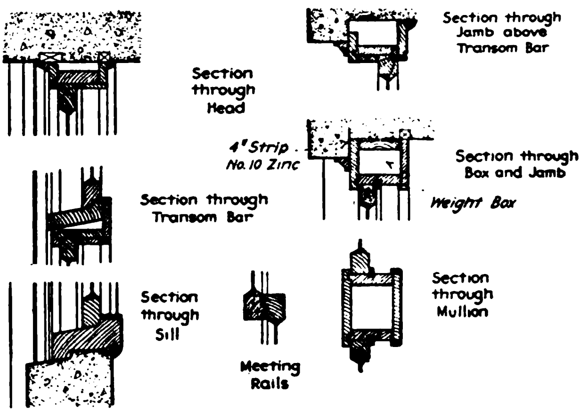
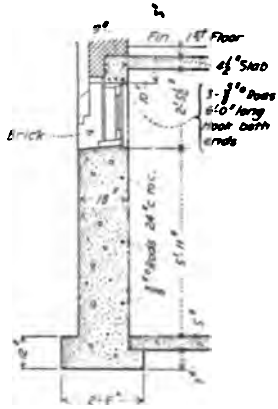


Fig. 116.



Section through Basement Window

Fig. 117.

quate for all ordinary conditions. Open sprinklers, or *water curtains* as they are frequently called, are sometimes used in connection with shutters or wire glass windows, but do not seem to be very efficient in themselves except in very moderate exposures.

Fig. 114 shows a window construction with ordinary wooden sash employed in a large hospital in the Middle West. Fig. 115 shows the window construction in a shoe factory at Cambridge, Mass. Double-hung window details for a building at Boston, Mass., are shown in Fig. 116. A detail of a basement window opening is given in Fig. 117. Fig. 118 shows a common window and wall construction.

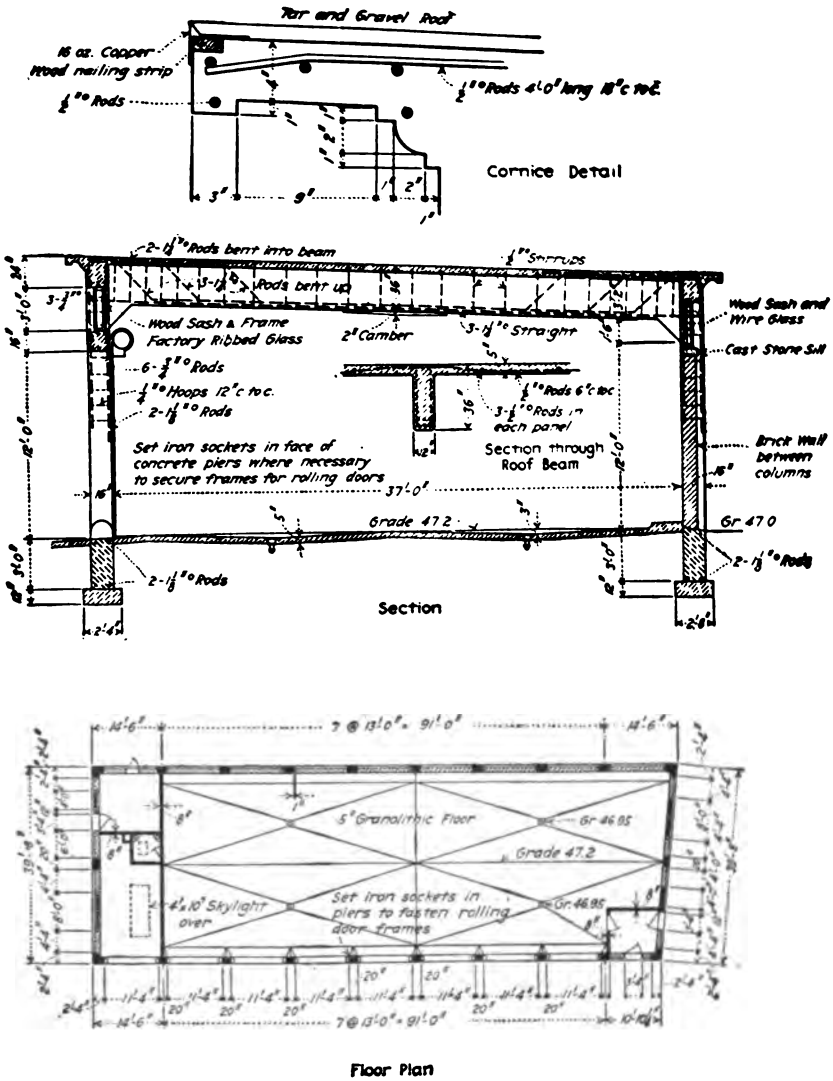


FIG. 118.—Truck garage, Pacific Mills, Lawrence, Mass.

46. Door Openings.—Doors in a reinforced-concrete building should be fire-resisting, the same as windows, and may be of either the hinged, sliding, or rolling type. If tin-covered wood doors are provided on openings in interior partition walls, they are generally hung on inclined tracks so that they will close automatically. Where it is desirable to keep them open, an automatic release operated by a fusible link is provided.

Sliding and rolling doors are generally used only for large openings such as driveway and freight-elevator entrances and are usually arranged to close automatically in case of fire. Sliding doors should have the rails on which they operate protected by metal shields to prevent obstruction. Steel rolling doors are made in horizontal, jointed, sectional strips, which wind

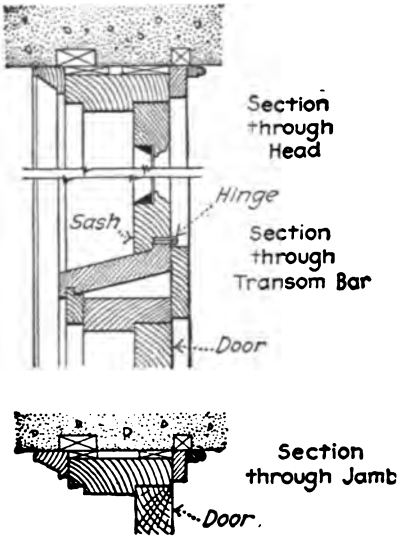


Fig. 119.

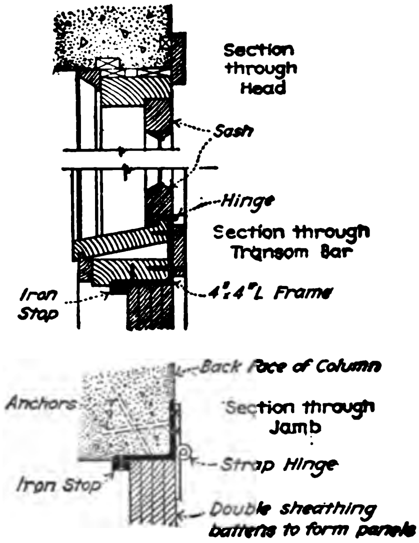


Fig. 120.

up on a roller placed in a pocket above the opening, the ends moving in metal grooves to hold them in place (see Fig. 118). There are a number of types of vertically operated doors. A common type for freight elevators is shown in Fig. 103.

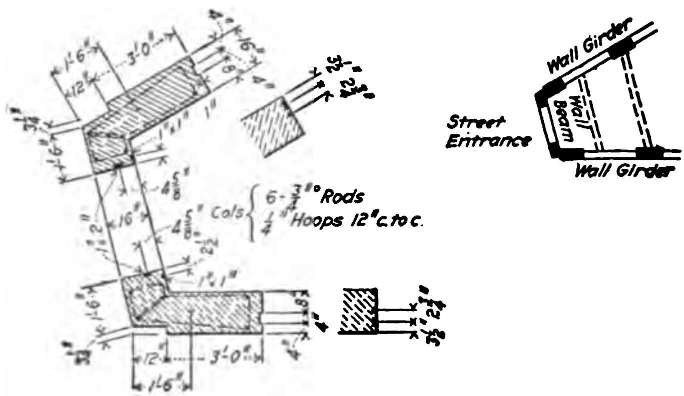


Fig. 121.

Fig. 119 shows the details of an ordinary outside wooden door. Fig. 120 gives the details of large swinging driveway doors used in a building at Boston, Mass. The arrangement around the main-street entrance doors to the same building is shown in Fig. 121. Similar door details in the Lang Building (Fig. 104) should also be noted.

47. Basement Walls.—Basement walls to sustain earth may be designed as simply supported slabs, the earth-pressure reactions being usually taken by the basement and first floors. The common construction is to employ curtain walls at least 8 in. thick between wall columns, and, in addition to reinforcing them vertically to take the earth pressure, to place rods near the bottom of the wall so as to make the wall carry itself as a beam from footing to footing. This type of wall thus requires no foundation of its own and may be built in the same way as the curtain walls in the upper stories. Sometimes it becomes necessary to reinforce horizontally for the earth pressure. This brings a lateral force on the columns, but the resulting eccentricity of column loading may be disregarded for ordinary cases unless a very light wall column is used. It is customary in the design of basement walls to assume the earth pressure as due to a fluid weighing 30 lb. per cu. ft.

Sidewalk lights are formed of circular pieces of plate glass set in reinforced-concrete slabs, and supported by steel beams. A typical vault-light construction, using concrete beams is shown in Fig. 122. The spacing of the beams should be governed by the thickness of the slab—the usual spacing being 3 to 4 ft.

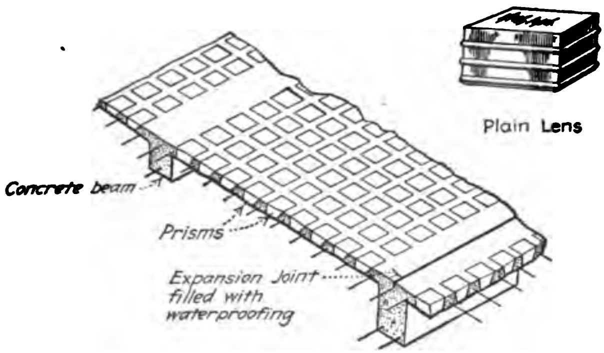


Fig. 122.

48. Partitions.—Solid concrete partition walls may be made 3 or 4 in. thick if reinforced in a similar manner to exterior curtain walls. Extra rods should be placed near the edges of all openings, and rods should project into the floor and ceiling for anchorage. It is usually convenient to pour the concrete after the floors are laid, and, where partitions are not located under beams, this may be done by leaving a slot in the floor at the proper place.

Concrete blocks have been used to some extent, but their thickness is often objectionable. In a warehouse at Nashville, Tenn., concrete blocks were employed 8 by 8 by 24 in. in size with two hollow spaces. The blocks around the elevators were 4 by 4 by 6 in. solid. Rabbets were formed in each end and in top and bottom surfaces, and these were filled with cement mortar as the blocks were laid, in order to secure as perfect a bond as possible.

A solid concrete wall 4 in. thick makes a very efficient fire-resisting partition, but is heavy, and difficult to install. For this reason metal lath and plaster, terra-cotta tile, and plaster-block partitions are generally used in preference to concrete.

Metal lath and plaster partitions are extensively employed in concrete buildings and make a fairly good (although not first-class) fire-resisting construction. Some tests of plaster partitions in actual fires have shown such constructions to be reliable under fairly severe conditions, while other tests have shown failure, due to the difficulty of obtaining sufficient bond between the plaster and the metal lath to resist successfully the combined action of fire and water.

The ordinary form of metal lath and plaster partition consists of channel-iron or other steel studding set crosswise of the partition, and connected at the top and bottom with the floor. The metal lath is then fastened to both sides of the studding and each side plastered with a mixture of lime and cement mortar. Three coats of plaster are generally applied—the first or scratch coat containing the usual quantity of hair or fiber. Openings are cased on all sides with a steel framework to which the lath is firmly fastened. This type of a partition is usually made from 4 to 6 in. thick, and made either hollow or with a central body of cinder concrete. A 2-in. solid partition, however, is sometimes used and consists of only one thickness of metal lath wired to the steel uprights.

Metal lath and plaster partitions made by the Roebling Construction Co. are shown in Figs. 123, 124 and 125. The studs are fastened top and bottom by means of knees. Where flats are used for uprights, the knees are formed by simply bending the ends of the studs. Where cement-finished floors are used, wood plugs or expansion bolts are inserted for fastening. Wood furrings, where required for nailing purposes, are attached to the studs by means of

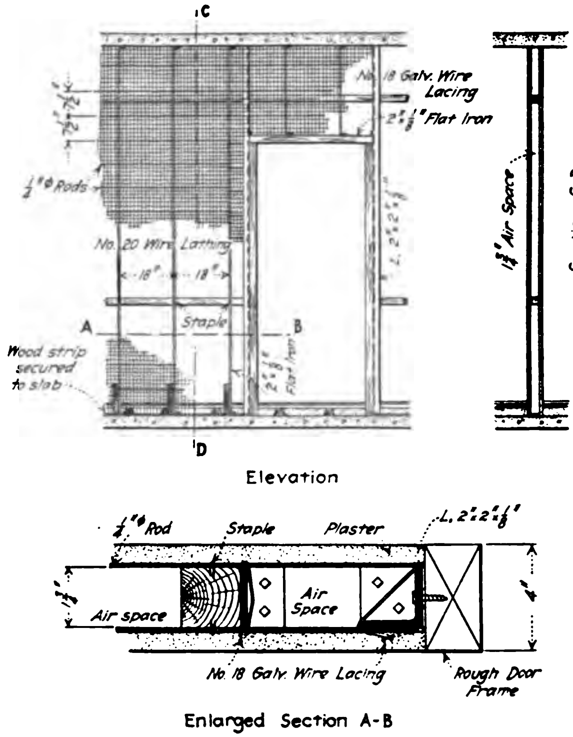
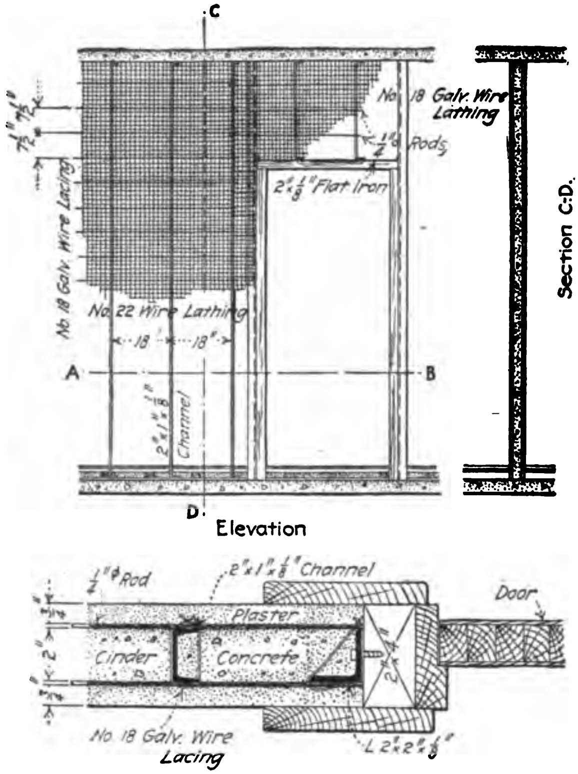


FIG. 123.

staples. Furrings are usually required to receive the baseboards, chair rail, and picture molding. They are not needed, however, where cinder concrete is filled in between the lath since cinder concrete will receive nails readily. The vertical members at door openings are made to extend the full height from floor to ceiling. Such members around openings are punched at intervals with holes to permit the fastening of wood frames or other trim. Of course, from a standpoint of fire-resistance, metal or plaster trim is preferable to wood.

Two types of plaster partitions erected by the Expanded Metal Cos. are shown in Figs. 126 and 127. A metal and plaster partition known as the *hy-rib* construction is shown in Fig. 128. The ribs of the hy-rib take the place of studs and eliminate the necessity of attaching sheets of lath to studs. The manner of attaching a hy-rib partition to floor and ceiling is shown.



Enlarged Section with Door Casing

FIG. 124.

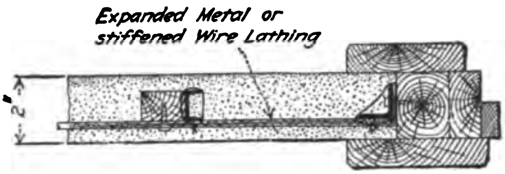


FIG. 125.

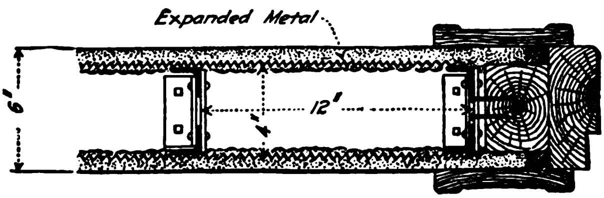


FIG. 126.

Where hollow partitions are required, two thicknesses of hy-rib are used, separated by means of rib studs.

Terra-cotta tile partitions are very satisfactory light-weight partitions if made of semi-porous or porous material. Partition blocks are made in thicknesses varying from 2 to 12 in., the 4-in. block being the most common. Plaster on both sides of a terra-cotta partition increases the thickness by 1½ in., that is, ¾ in. to a side. Two-in. and 3-in. tile partitions are not recommended for dependable efficiency in case of fire. The safe height of a terra-cotta partition may be approximated by multiplying the thickness of the block in inches by 40. This will give the safe height in inches.

Full-porous terra-cotta blocks are slightly more expensive than semi-porous, as they weigh more per square foot, and have heavier faces and webs, but they make a partition which is decidedly more dependable under fire test. At least a portion of a partition should be built of full-porous blocks in order to provide for the nailing on of wood trim.

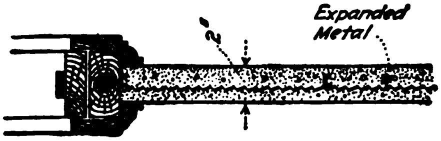


FIG. 127.

Terra-cotta partitions should be securely braced with slate at the ceiling. The blocks should be wet before being laid and also before being plastered, and should be laid in a mortar composed of 1 part lime-putty, 2 parts cement, and 2 or 3 parts sand.

Plaster blocks as used in partition construction are made principally of gypsum or plaster-of-Paris, with an admixture of wood fiber, reeds, or other suitable material. They are extremely light, easy to handle, and can readily be cut and sawed, but possesses several disadvantages in actual use. They also offer poor resistance to hose streams in case of fire.

The following table gives weights of various forms of partitions. The weights for the block partitions do not include the plastered surfaces. If a partition is to be plastered on both sides, add 10 lb. per sq. ft.


Hy-rib

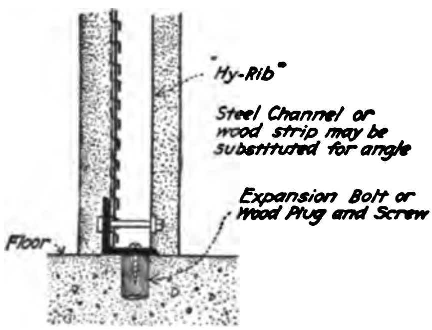


FIG. 128.

WEIGHTS OF PARTITIONS

Kind of partition	Thickness (inches)	Weight (lb. per sq. ft. of partition)
Solid plaster....	2	20
	4	32
Hollow plaster..	4	22
Terra-cotta	2	12 to 14
	3	15 to 17
	4	16 to 18
	5	18 to 20
	6	24 to 26
Plaster block....	2	7
	4	12
	8	22

Large buildings should be divided by fire walls of either concrete or brick, so that if fire occurs in any part of the structure it will be kept from spreading. A fire wall to be thoroughly effective should run from basement to roof and be provided with automatic fireproof doors. Severe exposures require fire doors on each side of such walls.

Stairs and elevator shafts should be enclosed in either concrete or brick partitions. Where considerations of appearance or light prevent the use of opaque enclosures, the fire hazard should be reduced by using wire glass in metal frames. Open grille-work around passenger elevators should be of this construction.

STAIRS

49. General Design.—The usual type of reinforced-concrete stairway consists of an inclined slab with the steps formed upon its upper surface. The design of such stairs is a simple problem, the slab being figured as freely supported and with a span equal to the horizontal distance between supports. Transverse reinforcement is used only for stiffening and to prevent shrinkage cracks. A common load used in designing stairs in commercial and manufacturing buildings is 100 lb. per horizontal sq. ft.

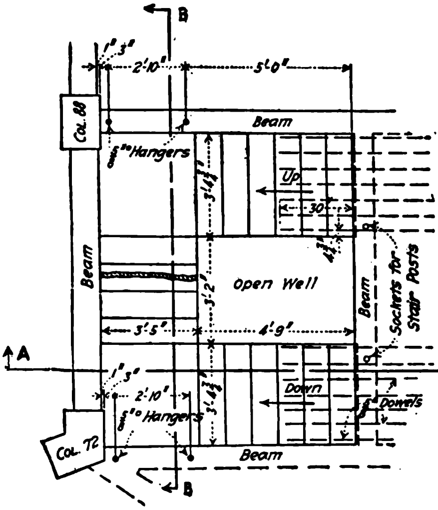
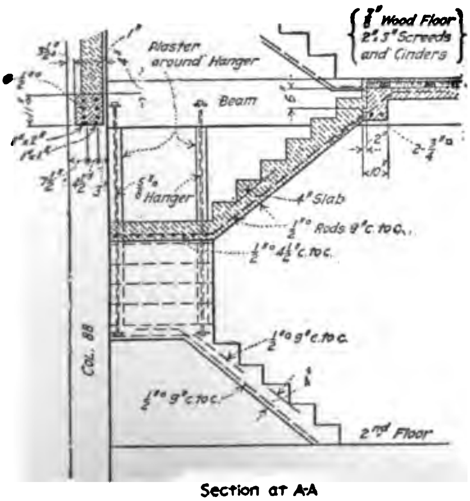


Fig. 129A.

A simple method of design is to support the ends of a stairway slab directly on floor beams or floor girders, or on some special beam or beams inserted for the purpose. This, however, cannot always be accomplished conveniently and it is common design to make the span of a stairway slab to include a platform slab as well. Stresses in slabs of this type are somewhat indeterminate on account of the angle which occurs at the edge of the platform; but many such slabs, however, have been computed as simple slabs freely supported and have given satisfaction in every case.

In stairway design, some arrangement is usually made whereby short spans may be employed, as slab construction is not suitable for long flights. When it becomes necessary to use



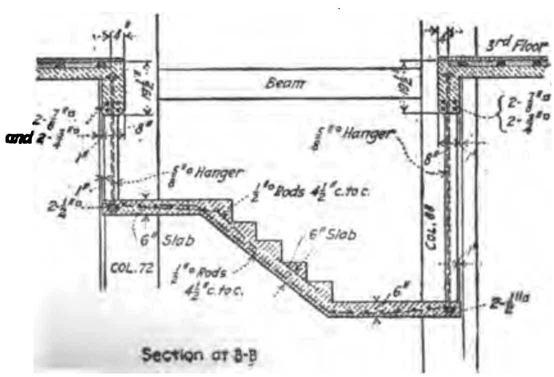


Fig. 129B.

stair spans of great length, however, without any chance of intermediate support, a side girder construction may be employed.

Stairways should be enclosed in fire-resisting partitions in order to prevent the spread of fire (see Art. 48).

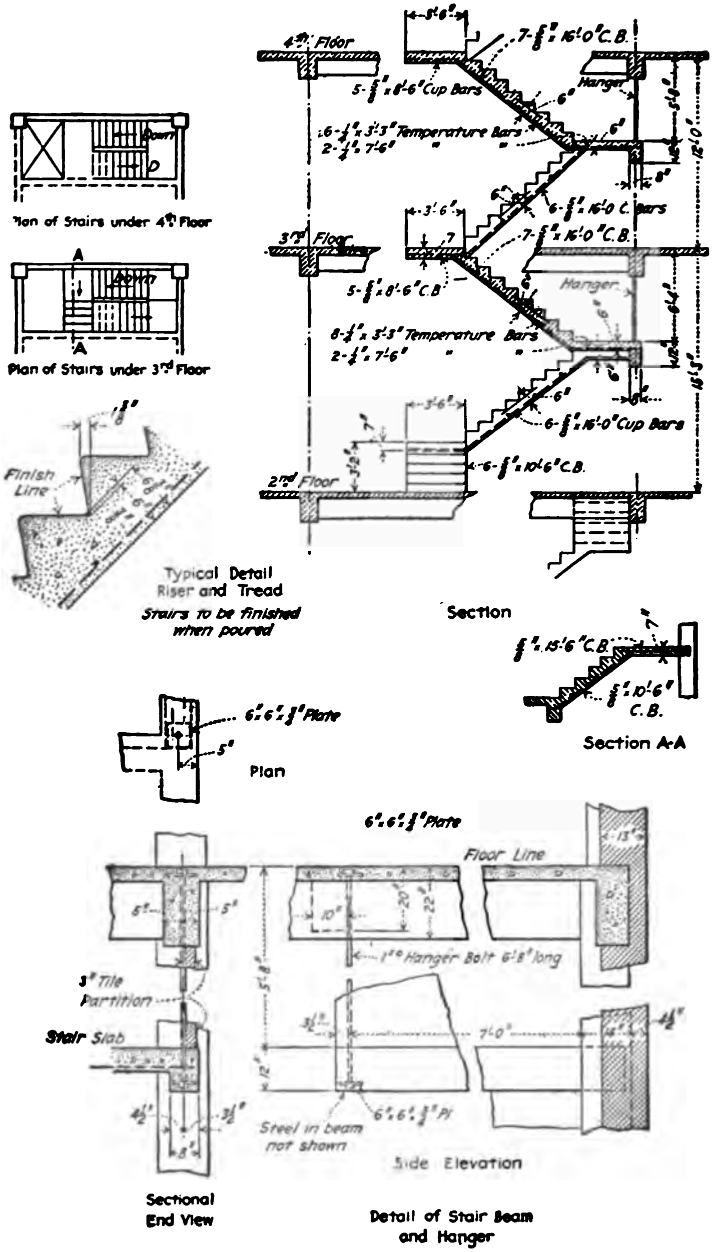


FIG. 130.

50. Methods of Supporting Stairs.—The slab method of construction is usually the cheaper and employed wherever possible. In long straight flights, a beam is often used to shorten the span. This beam may run between the regular columns which make up the structural frame, or additional short posts may be provided. Fig. 105, page 538, shows a stairway supported in this way. The design is that employed near columns 1 and 2 of the Lang Building, Fig. 100, page 533.

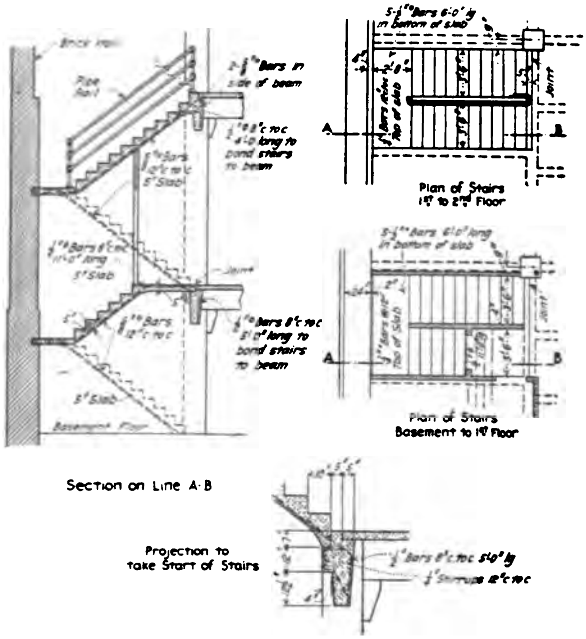


Fig. 131.

Where a stairway must be broken into two or more runs to the story, it is customary to employ either rod hangers or beams intermediate in the story height. Figs. 129*A* and 129*B* show a stairway supported by rod hangers only. In Fig. 130 both intermediate beams and rod hangers are employed.

Stairways are generally constructed at some convenient time after the structural frame is completed. Where stair-runs start from floor beams or floor girders, a plank should be nailed to the side of the beam forms to cause rabbets in the concrete, and dowels should also be provided.

The ends of a stairway slab are usually fixed to more or less extent and the dowels provided in the course of construction may well be made of such length and in such numbers as to give a sufficient reinforcement for negative moment. In short spans fixed in this way, the moment at the center of slab may be computed with safety using the formula $M = \frac{wl^2}{10}$.

Fig. 132.

A stairway supported by brick walls is shown in Fig. 131. The same arrangement would be employed with concrete bearing walls.

Steps are sometimes molded after the rough concrete stair slab is in place. The steps may then be molded separately and set on the slab, or they may be poured in place. In the former case, rods should be embedded in each step to permit of handling without injury, while in the latter case the steps are all poured at once in a similar manner to the way the pouring is done when they form an integral part of the slab.

51. Stair Details.—In factory construction stair treads are usually surfaced with a 1 or

1½-in. cement top finish. A method of constructing the steps is illustrated in Fig. 132. The form board for the riser is arranged as shown at *A* and the top of this board is beveled so that by placing an upper form board on the outside a nosing or projection in the concrete work is formed.

Another type of step with cement finish is shown in Fig. 130. This step makes a good appearance, but is a trifle more difficult to construct than the step previously mentioned.

The outer edge of concrete steps are protected in many cases by a light steel angle which runs the entire width of the stairs. When linoleum is used, this angle is raised so as to have the upper leg flush with the finished tread. Metal non-slipping treads embedded in the concrete are sometimes employed.

Fig. 129*B* shows the method of connecting concrete stairs with a wood-finished floor. Fig. 133 gives the details of wood-covered concrete stairs employed in a fireproof residence. In the best buildings, treads and risers are often covered with marble slabs having plaster soffits.

The *rise* of a stair is the height from the top of one step to the top of the next. The *run* is the horizontal distance from the face of one riser to the face of the next. The *run* is usually less than the width of tread on account of the nosing. To secure a comfortable stair, the run must bear a certain relation to the rise. One rule is that the sum of the rise and run should be equal to from 17 to 17½ in. For ordinary use, a rise of 7 to 7½ in. is about right. Stairs

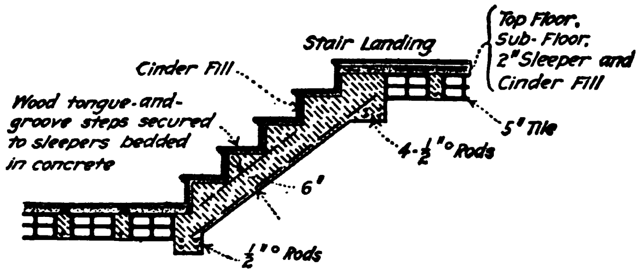


FIG. 133.

having a rise greater than 7¾ in. are steep. Properly designed stairs without nosings should have at least 12-in. treads to be comfortable and the above rule does not apply to this type of stair.

The common railing is of galvanized-iron pipe put together with malleable-iron fittings and having the stanchions rigidly secured in sockets in the concrete work. Fig. 106 gives details of a railing of this type used in the Lang Building. The hand-rail should be placed at a height of about 2 ft. 6 in. above the tread on line with the face of riser. Rails monolithic with the stairs are sometimes used.

ELEVATOR SHAFTS

52. Elevator-shaft Pits.—Elevator-shaft pits should not be less than 3 ft. below the basement floor. When oil-cushion buffers are employed, however, the pit depth should vary according to the speed of the elevator and the stroke of the buffer designed for that particular speed. With a car speed of 600 ft. per min., the pit should be about 11 ft. deep at the center where the buffer is placed.

If the first floor is made the bottom landing, a pit pan may be suspended from the first floor, and buffers, if used, may be placed on the basement floor and arranged to project up into the pit through special openings in the pan. This method is generally employed when it is advantageous to gain headroom in the basement underneath the pit pans. All footings for columns and foundations adjoining elevator shafts should be kept below the floor of the elevator-shaft pit, and the pit itself should be finished to plumb-line dimensions.

In localities where the water level in the soil is very high, or where there are underground springs, the pits should be thoroughly waterproofed. This may be done in the same manner as for basement floors (see Art. 47, Sect. 11). If the water pressure is very great, waterproof pans may be employed as shown in Fig. 134.

The sides of the pits are usually made of concrete 6 to 8 in. thick and the bottom of the pit is made a slab of the same thickness as the basement floor. The sides of the pit are supported

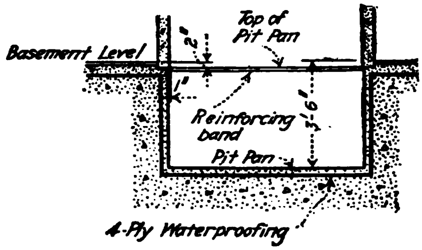


Fig. 134.

at the top and bottom, and may be figured as a simple slab acted upon by the earth pressure and by a pressure due to the live load on the basement floor. The steel used in the side walls should be run into the basement floor in order to take the reaction at the upper end of the slab.

53. Pent Houses.—The heights of pent houses over elevator shafts vary according to the number of sets of sheave beams and the size of the sheaves. If the machine is located in the

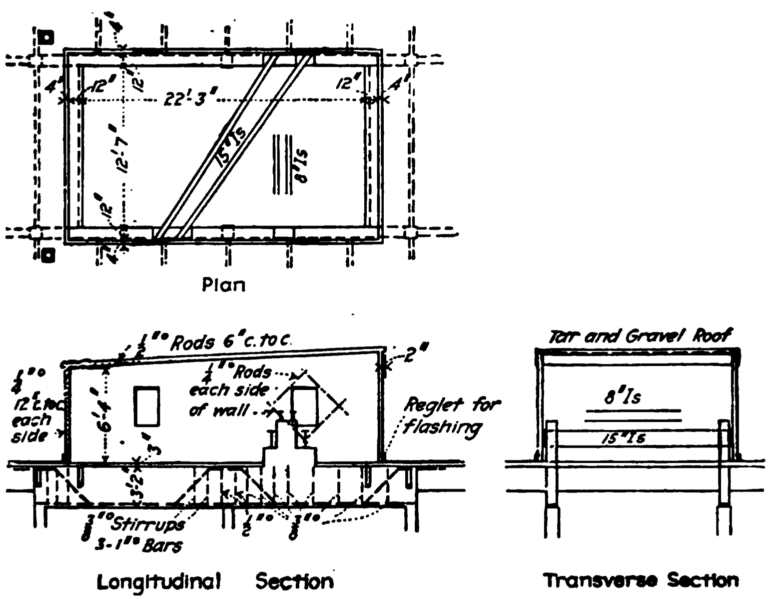


Fig. 135.—Pent-house details.

basement or on any intermediate floor, the height of the pent house will vary according to the location of the counterweights with respect to the machines. If the machine is placed over the shaft, the size and height of the pent house may be affected by the dimensions of the machine itself. Figs. 135 and 136 give details of actual pent-house constructions.

In all machine rooms, doors should be provided of sufficient size to permit any part of the machinery to be removed in case of repairs. The construction should also be such as to admit of easy access for the purpose of oiling sheave bearings, etc.

Local regulations of some localities require that a grating be installed in all elevator shafts. just below the overhead machinery. This grating should be constructed and properly supported to sustain the weight of a number of men.

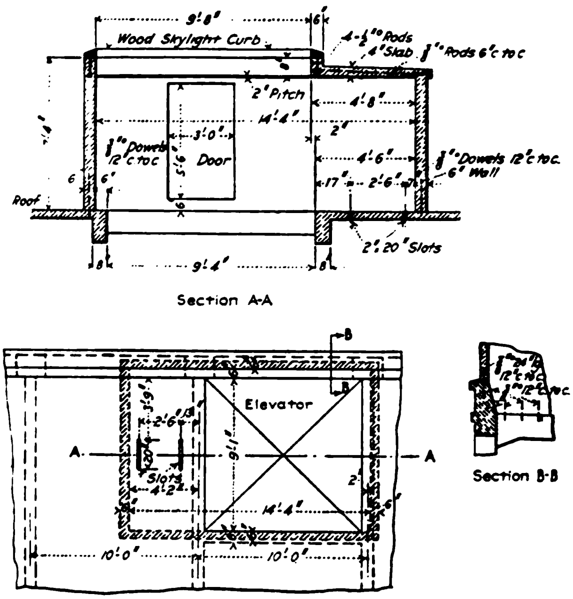


FIG. 136.—Pent-house details.

PROVISION FOR CONTRACTION AND EXPANSION

54. Methods Employed.—There is no well-established practice of providing for contraction and expansion in reinforced-concrete buildings as a whole. Most engineers construct curtain

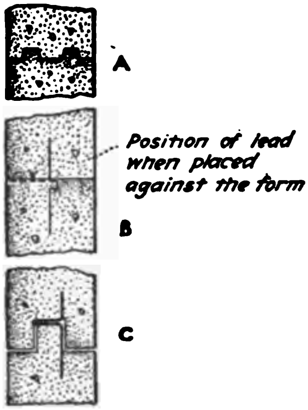


FIG. 137.

walls into keyways leaving only the wall beams and wall girders subject to contraction, and it has been found that these can usually be reinforced sufficiently to prevent cracking. Temperature reinforcement in wall beams should be placed horizontally near the outer surface and should be well distributed throughout the beam depth. This longitudinal reinforcement should be well lapped at the corners of the building. Buildings of a length of 300 ft. have been constructed in this manner with no expansion joints and have not cracked.

When used, expansion joints to be a success should completely separate the building from bottom to top. They should preferably be made by means of a double column supporting a double girder, and the joints should be waterproofed. Some engineers specify that a complete separation shall be made every 100 ft. in length of the building.

Rather than provide sufficient steel to take contraction in long walls, expansion joints are often resorted to. Fig. 137 shows different methods of making such joints. In type A

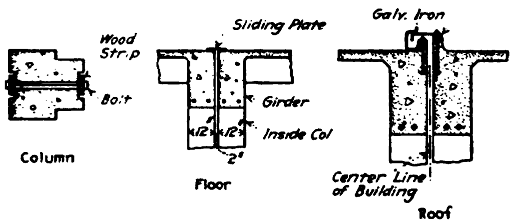


Fig. 138.

grooves are simply cast in the end of one section and coated with cold-water paint or pitch. In building the next section a tongue is formed fitting the groove. This type of joint is generally reinforced with burlap when placed in front of an earth fill. In type *B*, Fig. 137, sheet lead is used. The projecting portion of the sheet is bent up against the form while concrete is being placed in the first section. After removing the forms it is bent down parallel with the wall surface so as to extend into the abutting concrete. A U-shaped bend is formed in the lead sheet at the concrete joint. In type *C* the tongue and groove is formed as in *A*, and in addition a bent lead or copper strip is inserted.

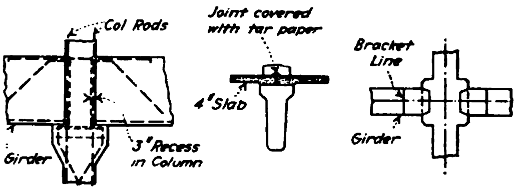


Fig. 139.

Fig. 138 shows the manner of providing expansion joints in the United Shoe Machinery Co.'s factory at Beverly, Mass.

Fig. 139 shows expansion joints in a sawmill at Waycross, Ga. An expansion joint occurs directly on every column line. This was done by pouring columns and all transverse beams on column line monolithic, and by making all intermediate beams and girders distinct members. Recesses were made to receive these. The slab was also an after consideration and was poured independently of beams and girders. The walls were poured after the columns had been stripped. The monolithic feature of concrete construction was entirely eliminated. This was done not only for contraction and expansion, but to permit the pouring of the concrete at convenient times. The roof slabs were cut on all column and ridge lines for movement, and afterward capped. No other covering than concrete was used for the roof and this was waterproofed by introducing a foreign ingredient into the concrete mixture.

SECTION 12

FOUNDATIONS

1. Bearing Capacity of Soils.—The bearing capacity of soils depends upon their composition, the degree to which they are confined, and the amount of moisture which they contain. An approximate idea of the loads which may be safely placed upon uniform strata of considerable thickness may be obtained from the table on page 583.

There are many kinds and mixtures of soils and it is not always possible to judge the safe bearing capacity of any given soil by reference to a table such as that above mentioned. When there is any doubt, tests or borings should be made. After opening the trenches, the best method is to load known areas and observe the settlement, but in interpreting the results, the fact should not be overlooked that a small area will bear a larger load per unit of area for a short time than a larger area will perpetually. Thus, the area tested should be as large as practicable and the test should continue for some time.

Ordinary soils will usually bear more weight the greater the depth, owing to the fact that they become more condensed from the superimposed load. With clay, depth is especially important as there is less liability of its being displaced laterally due to other excavations in the immediate vicinity, and also because at greater depths the amount of moisture in it is not subject to so much variation. In any soil, the bed of the foundation should be below the reach of frost.

It is safe to say that any rock in its native bed will bear the heaviest load that can be brought upon it by any masonry construction. It scarcely ever happens that rock is loaded with the full amount of weight which it is capable of sustaining. In preparing a rock bed, all loose and decayed portions should be cut away and the bed dressed to a plane surface as nearly perpendicular to the direction of the pressure as is practicable. Any fissures or seams in the rock should be filled with concrete. A sloping surface should be stepped or the foundation designed with sufficient toe to prevent sliding.

Sand when dry, or wet sand when prevented from spreading laterally, forms one of the best beds for a foundation. However, porous, sandy soils are easily removed by running water and require extreme care at the hands of the constructor.

Piles are used in such soils as are not able to bear the weight of structures without an excessive spread of the footing base. If a pile is driven so that its lower end rests upon a hard stratum, the loading is limited by the strength of the pile considered as a column. The load on an ordinary bearing pile is carried by the friction of the earth on the sides of the pile. The only formulas in anything like general use for friction piles are the following, known as the *Engineering News* formulas:

For a pile driven with a drop hammer, $P = \dfrac{2Wh}{s + 1}$

For a pile driven with a steam hammer, $P = \dfrac{2Wh}{s + 0.1}$

in which P is the safe load in pounds, W the weight of the hammer in pounds, h the fall of the hammer in feet, and s the penetration or sinking in inches under the last blow, assumed to be sensible and at an approximately uniform rate. The above formulas were deduced for wood piles, but they are the best there are for concrete piles. They are also claimed to be safe, for ordinary weights of hammer and the usual height of fall, for a pile that acts as a column.

In the driving of piles, Joseph R. Worcester of Boston, advises for piles which meet a hard resistance, a penetration of 1 in. under a 2000-lb. hammer falling 10 ft.; and for piles held by friction a penetration of 3 in. under a 2000-lb. hammer falling 15 ft. Ordinary piles of spruce and Norway pine will usually sustain 10 tons by friction and 15 tons in bearing. These piles should never be less than 6 in. in diameter at the small end and never more than 18 in. at the large end. They may be driven 2 to 3 ft. apart depending upon their length, the hardness of the soil and the size of the butts. It has been found that little or no additional bearing power is secured if the spacing is much less than 2 ft. on centers. Concrete piles are manufactured in different sizes and shapes, and their bearing capacity should be thoroughly considered for each case.

2. Pressure on the Soil.—A footing must be spread until the safe bearing capacity of the soil is not exceeded. An effort need not be made to eliminate all settlement, but rather to so plan the structure that whatever settlement does take place will be uniform. In other words, the center of gravity of the loads from the columns should coincide with the center of gravity of the upward reactions, or with the center of gravity of the base if the base rests directly on the soil.

In buildings subject to shock or constant live load, the area of the footing should be proportioned for the full live and dead loads. In other buildings, that footing should be chosen in which the live load bears the highest percentage to dead load and its area determined for the total load at the allowable bearing in the soil; the pressure of the *dead* load per unit area should then be determined and the area of all other footings should be proportioned for dead load only with this unit pressure. Of course, the preceding statement applies without change only when the soil is uniform throughout.

The pressure on the foundation of a tall building should be considerably less than that which may be allowed on the foundation of a low one-story structure. In the former case a slight inequality of bearing power, and consequent unequal settling, might imperil the safety of the entire building, while in the latter case no serious harm would result.

3. Plain Concrete Footings.—The depth of a plain concrete footing must be sufficient so that the allowable tensile strength of the concrete is not exceeded. If the area of the base of the footing is considerably greater than the required area for the top, the footing may be stepped or sloped. The top area required depends upon the unit bearing pressure allowed on the concrete (see recommendation of Joint Committee in regard to bearing pressure in *Appendix B*). The base area required is governed by the safe bearing capacity of the soil.

In finding the maximum tensile stress in the concrete, the projection should be treated as a cantilever loaded uniformly by the soil reaction. In stepped footings, the depth of the steps should be made such that the tension in the concrete nowhere exceeds the allowable.

In a series of tests on unreinforced-concrete column footings made at the University of Illinois,[1] the bending moment was computed by the method shown in Art. 6 for reinforced footings and the moduli of rupture were calculated by using a resisting moment based upon the full width of the footing; that is, by considering the fiber stress in the concrete at the bottom of the footing to be uniform over the length of a section passing through the face of the wall, instead of taking into account the variation of stress across the section. As is usually the case when plain concrete is used in flexure, the unreinforced footings showed considerable variation in results. The variations were such as not to permit a method of determining the effective width of resisting section to be established or to obtain a formula for resisting moment. Based upon the full section of the footing, the moduli of rupture obtained were considerably less (averaging about one-third less) than the moduli of rupture of control beams made with the same concrete.

4. Advantages in Using Reinforced Concrete for Foundations.—Reinforced concrete is well adapted to the construction of foundations. As compared with plain concrete, its advan-

[1] See *Bull.* 67, University of Illinois, Engineering Experiment Station.

tages for spread footings are a reduction in the amount of excavation required, a saving in material, and a reduction in the weight of the foundation itself.

5. Wall Footings.—Except in residences, bearing-wall footings must usually be reinforced. A cantilever projection is formed on each side of the wall and the amount of reinforcement may be determined as for a simple cantilever beam.

In figuring maximum bond stress, tests show that the total external shear at the wall-face section should be used in the formulas of Art. 16, Sect. 7. For diagonal tension, the shear should be computed at a distance from the wall face equal to the effective depth of the footing. It is good practice to design small wall footings so that no diagonal tension reinforcement is required. In stepped and sloping footings the depth to steel at a distance (d) from the face of wall is less than in footings of uniform depth so that diagonal tension is quite likely to control in such footings. In large important footings, where diagonal tension is a critical element, web reinforcement should be employed preferably made up in some type of unit frame for convenience in construction and to ensure the accurate placing of the steel.

6. Types of Column Footings.—Foundations for columns are of four principal types: (1) single footings; (2) combined footings, including two or more columns; (3) cantilever footings; and (4) raft foundations covering the whole foundation area.

7. Single Column Footings.—A single symmetrical slab either square or rectangular is the most common form of spread footing. The reinforcing bars are placed at the bottom of the footing and run in either two or four directions.

Depth for Punching Shear.—The punching shear on a column footing, on an area equal to the perimeter of the column times the depth to the steel should not exceed the allowable value, which for a 1 : 2 : 4 concrete is 120 lb. per sq. in. (see Report of Joint Committee, *Appendix B*). The load producing punching shear may be found by multiplying the column load by the ratio

$$\frac{\text{footing area minus column area}}{\text{footing area}}$$

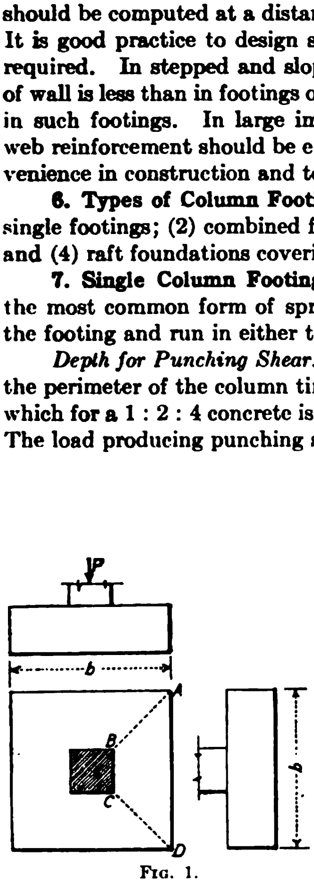


Fig. 1.

Maximum Bending Moment.—The maximum bending moment occurs at the face of the column. Referring to Fig. 1, the bending moment at BC is due to the load on the area $ABCD$. The distance out from BC to the center of gravity of the area $ABCD$—distance x, Fig. 2—may be formed for a square footing by the following formula

$$x = \frac{\frac{ac}{2} + \frac{2}{3}c^2}{a + c}$$

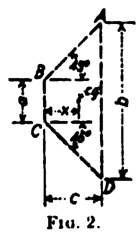


Fig. 2.

Then, for a square footing with square or round column

$$M = \frac{1}{4}\left(\frac{b^2 - a^2}{b^2}\right)Px = \frac{(b - a)^2(2b + a)}{24b^2}P$$

where P is the total column load. If we let $C_1 = \frac{(b - a)^2(2b + a)}{24b^2}$, then

$$M = C_1P$$

Diagram 1 gives values of C_1 for various values of a and b.

DIAGRAM 1

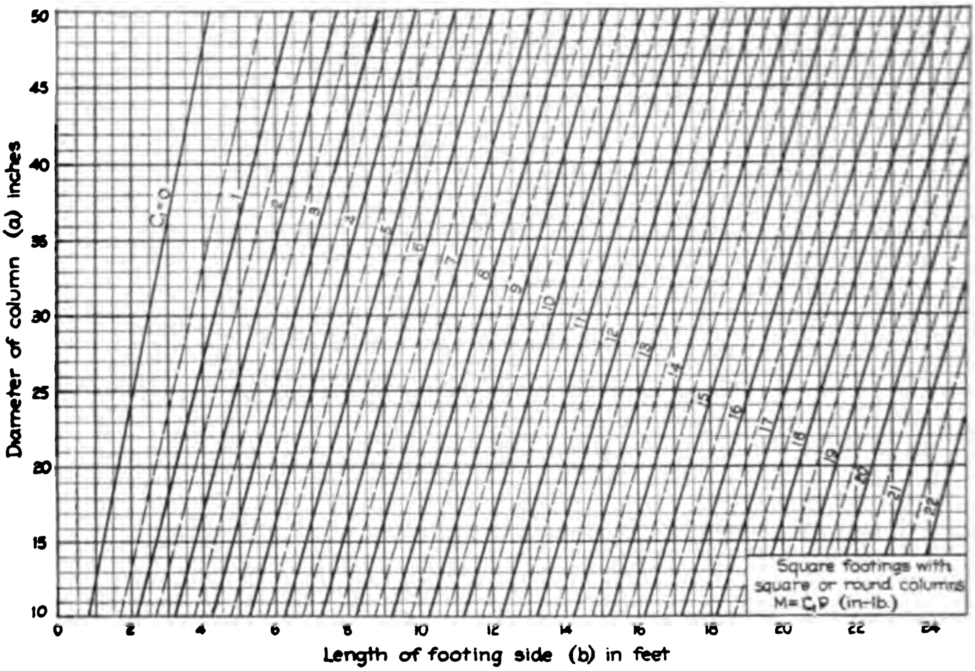
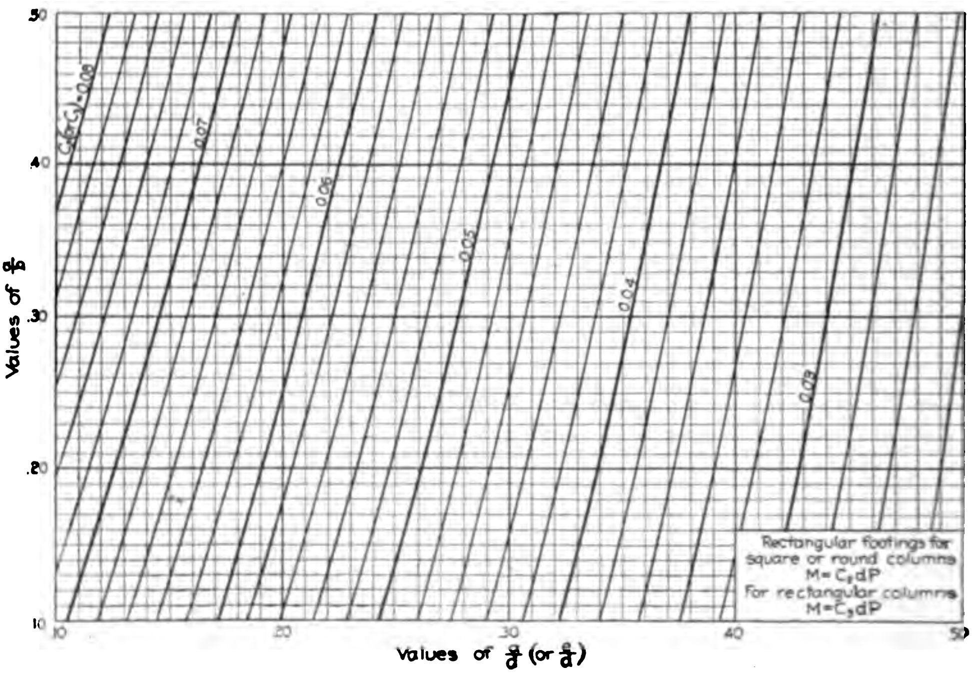


DIAGRAM 2



For a rectangular footing with a square or round column (Fig. 3), the distance

$$x = \frac{ac + \frac{2}{3}c(b - a)}{(a + b)}$$

and the moment at BC is

$$M = C_2dP$$

where

$$C_2 = \frac{1}{24}\left(2 + \frac{a}{b}\right)\left(1 - \frac{a}{d}\right)^2$$

Diagram 2 gives values of C_2 for various values of $\frac{a}{b}$ and $\frac{a}{d}$.

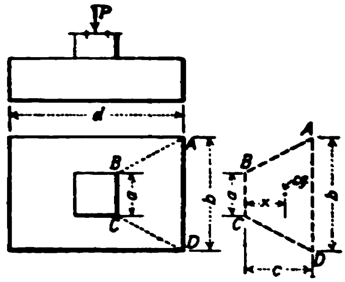


Fig. 3.

For a rectangular footing with a rectangular column (Fig. 4), the moment at BC is

$$M = C_3dP$$

where

$$C_3 = \frac{1}{24}\left(2 + \frac{a}{b}\right)\left(1 - \frac{e}{d}\right)^2$$

Diagram 2 gives values of C_3 for various values of $\frac{a}{b}$ and $\frac{e}{d}$.

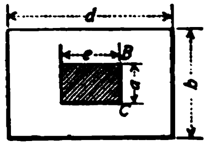


Fig. 4.

Width of Footing to Use in Flexure Computations for Two-way Reinforcement.[1]—With two-way reinforcement evenly spaced over the footing, it seems that the tensile stress is approximately the same in bars lying within a space somewhat greater than the width of the pier and that there is also considerable stress in the bars which lie near the edges of the footing. For intermediate bars stresses intermediate in amount will be developed. For footings having two-way reinforcement spaced uniformly over the footing, the method proposed for determining the maximum tensile stress in the reinforcing bars, is to use in the calculation of resisting moment at a section at the face of the pier the area of all the bars which lie within a width of footing equal to the width of pier plus twice the thickness of footing, plus half the remaining distance on each side to the edge of the footing. This method gives results in keeping with the results of tests. When the spacing through the middle of the width of the footing is closer, or even when the bars are concentrated in the middle portion, the same method may be applied without serious error. Enough reinforcement should be placed in the outer portion to prevent the concentration of tension cracks in the concrete and to provide for other distribution of stress.

No failures of concrete have been observed in tests and none would be expected with the low percentages of reinforcement used.

Bond Stresses.[1]—The method proposed for calculating maximum bond stress in column footings having two-way reinforcement evenly spaced, or spaced as noted in the preceding paragraph, is to use the ordinary bond stress formula, and to consider the circumference of all the bars which were used in the calculation of tensile stress, and to take for the external shear that amount of upward pressure or load which was used in the calculation of the bending moment at the given section.

Bond resistance is one of the most important features of strength of column footings, and probably much more important than is appreciated by the average designer. The calculations of bond stress in footings of ordinary dimensions where large reinforcing bars are used show that the bond stress may be the governing element of strength. Tests show that in multiple-way reinforcement a special phenomenon affects the problem and that lower bond resistance may be found in footings than in beams. Longitudinal cracks form under and along the reinforcing bar due to the stretch in the reinforcing bars which extend in another direction, and these cracks act to reduce the bond resistance. The development of these cracks along the reinforcing bars must be expected in service under high tensile stresses, and low working bond

[1] From *Bull.* 67, University of Illinois, Engineering Experiment Station.

stresses should be selected. An advantage will be found in placing under the bars a thickness of concrete of 2 in., or better 3 in., for footings of the size ordinarily used in buildings.

Difficulty may be found in providing the necessary bond resistance, and this points to an advantage in the use of bars of small size, even if they must be closely spaced. Generally speaking, bars of ¾-in. size or smaller will be found to serve the purpose of footings of usual dimensions. The use of large bars, because of ease in placing, leads to the construction of footings which are insecure in bond resistance. Column footings reinforced with deformed bars develop high bond resistance. Curving the bar upward and backward at the end increases the bond resistance, but this form is awkward in construction. Reinforcement formed by bending long bars in a series of horizontal loops covering the whole footing gives a footing with high bond resistance.

The use of short bars placed with their ends staggered increases the tendency to fail by bond and cannot be considered as acceptable practice in footings of ordinary proportions. In footings in which the projection is short in comparison with the depth, the objection is very great.

Diagonal Tension.[1]—As a means of measuring resistance to diagonal tension failure, the vertical shearing stress should be calculated by using the vertical sections formed upon the square (assuming square column) which lies at a distance from the face of the pier equal to the depth of the footing. This calculation gives values of the shearing stress, for footings which failed by diagonal tension, which agree fairly closely with the values which have been obtained in tests of simple beams. The formula used in this calculation is $v = \frac{V}{bjd}$, where V is the total vertical shear at this section taken to be equal to the upward pressure on the area of the footing outside of the section considered, b is the total distance around the four sides of the section, and jd is the distance from the center of reinforcing bars to the center of the compressive stresses. The working stress now frequently specified for this purpose in the design of beams, 40 lb. per sq. in., for 1 : 2 : 4 concrete, may be applied to the design of footings.

Four-way Reinforcement.[1]—Footings having reinforcement placed in the direction of the diagonals as well as parallel to the sides (four-way reinforcement) give good tests. The significance of the results is so obscured by the variety of manner of failure (bond, diagonal tension, and perhaps tension) and by variations in the quality of the concrete, that a comparison with two-way reinforcement on the basis of loads carried would not be of value. This type of distribution of reinforcement should be included in further tests. Measurements of deformation in the bars are needed to determine the division of stress among the four sets of bars.

Stepped and Sloping Footings.[1]—In stepped footings, the abrupt change in the value of the arm of the resisting moment at the point where the depth of footing changes may be expected to produce a correspondingly abrupt increase of stress in the reinforcing bars. Where the step is large in comparison with the projection, the bond stress must become abnormally large. It is evident that the distribution of bond stress is quite different from that in a footing of uniform thickness. The sloped footing also gives a distribution of stress which is different from that in a footing of uniform thickness. However, for footings of uniform thickness the bond stress is a maximum at the section at the face of the pier; in a sloped footing the bond stress at the section at the face of the pier would be less accordingly than in a footing of uniform thickness, and a moderate slope may be found to distribute the bond stress more uniformly throughout the length of the bar. This is not of advantage if the full embedment of the bar is effective in resisting any pull due to bond.

Illustrative Problem.—Design a single square footing for a round column of 24 in. diameter carrying 300,000 lb., when the safe bearing capacity of the soil is 2 tons per sq. ft. Use a 2000-lb. concrete with medium steel reinforcement. Two sets of bars are to be used placed at right angles to each other and parallel with the sides of the footing.

[1] From *Bull.* 67, University of Illinois, Engineering Experiment Station.

The required area of footing is found by dividing the load on the column plus the assumed weight of footing (36,000 lb.) by the safe bearing capacity of the soil, or $\frac{336,000}{4000} = 84$ sq. ft. A base 9 ft. 2 in. square will be selected (area 84.0 sq. ft.).

The load producing punching shear is $\frac{84.0 - 3.1}{84.0}(300,000) = 289,000$ lb. (The weight of footing need not be considered except in determining the area of base of footing as the upward reaction of the soil due to weight of footing is equal and opposite to this weight.) The minimum depth, then, for punching shear is

$$\frac{289,000}{(2)(3.14)(12)(120)} = 32 \text{ in.}$$

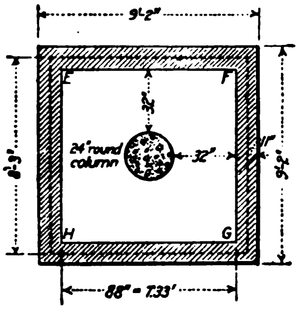


Fig. 5.

Shear, as measuring diagonal tension, is measured at a distance from face of column equal to the depth of the footing to the steel, or 32 in. in this problem. The reaction of the soil on the cross-hatched area, Fig. 5, is what causes shear on the vertical planes through *EFGH*.

$$\text{Total shear on } EFGH = \frac{84.0 - (7.33)^2}{84.0}(300,000) = 108,000 \text{ lb.}$$

$$v = \frac{V}{bjd} = \frac{180,000}{(4)(88)(0.875)(32)} = 11 \text{ lb. per sq. in.}$$

Thus the footing needs no stirrups for a depth (d) of 32 in.

Whenever the unit shear is found too great at the given distance out from the face of column, the plane where the shear is just the allowable should be determined. Then the total tension to be taken by stirrups divided by the tensile value of one stirrup gives the number of stirrups required.

In nearly all cases a slight deepening of the footing is all that is necessary to avoid using stirrups. The placing of stirrups is troublesome and footings should be designed so that web reinforcement will not be needed, if this can be done without greatly increasing the depth.

The bending moment acting on each set of rods is, by means of Diagram 1,

$$M = 6.2(300,000) = 1,860,000 \text{ in.-lb.}$$

$$A_s = \frac{M}{f_s jd} = \frac{1,860,000}{(16,000)(0.875)(32)} = 4.2 \text{ sq. in.}$$

Fourteen $\frac{5}{8}$-in. round bars will be employed in each band, of width equal to diameter of column plus twice the thickness of footing, plus half the remaining distance on each side to the edge of the footing. The effective width in this problem is 8 ft. 3 in. (see Fig. 5). We will place 16 bars throughout the entire width of footing, as shown in the complete design, Fig. 6. The bond stress

$$u = \frac{V}{\Sigma o j d} = \frac{289,000(\frac{1}{4})}{(1.96)(14)(0.875)(32)} = 94 \text{ lb. per sq. in.}$$

Since the allowable bond stress for plain bars is 80 lb. per sq. in. either deformed bars must be used or a larger number of smaller plain bars.

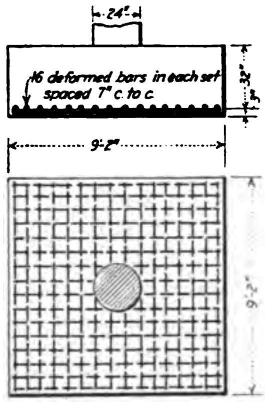


Fig. 6.

Illustrative Problem.—Design a single square sloping footing for a round building column of 30-in. diameter, carrying 610,000 lb. Consider the safe bearing capacity of the soil at 2½ tons per sq. ft. and use four-way reinforcement. $f_s = 16,000$. $f_c = 650$.

It will be assumed that satisfactory soil may be found at the required depth below the basement floor. A base plate will be provided under the column rods, and this plate will be placed on top of the finished footing. The top of the footing will be made with an area of about twice that of the column and a bearing pressure of 700 lb. per sq. in. will be permitted (see recommendations of the Joint Committee, *Appendix B*). Investigation shows that this stress is not exceeded. If desired, the base plate may be placed some distance down in the footing, but under such an arrangement the column rods for the first story must be placed while the footing is being poured, which is troublesome. Sometimes, however, this is avoided, especially where the footing is at considerable depth, by inserting only short column rods of such a length that they may be properly spliced immediately above the basement floor. If this is done, the first floor can be laid entire and the first-story columns started above it.

The top of the footing will be made square, 42 in. on a side. This provides a 6-in. ledge all around so that, if it is desired to erect the first-story columns before pouring the basement floor, the column forms will have some place on which to rest. The depth of the footing at the outer edge will be made 12 in. The dead weight of the footing will be taken at 60,000 lb.

The required area of footing is found by dividing the total load by the safe bearing capacity of the soil.

$$\frac{670{,}000}{(2.5)(2000)} = 134 \text{ sq. ft.}$$

We shall select an area 11 ft. 6 in. square.

Four-way reinforcement for single column footings is shown in Fig. 8. After deducting the area of the column base, the remainder of the footing slab is considered as eight cantilevers, four running parallel to the sides and four on the diagonals.

Depth for punching shear,

$$d = \left(\frac{132.2 - 4.9}{132.2}\right)\frac{610{,}000}{(3.14)(2.5)(12)(120)} = 52 \text{ in.}$$

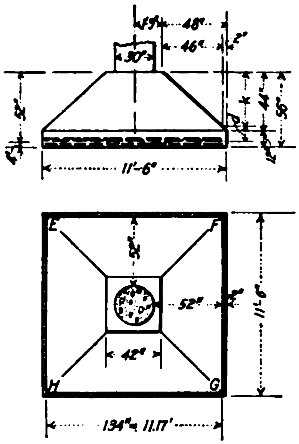
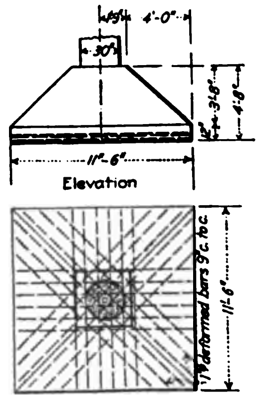


Fig. 7.



Plan

Fig. 8.

Referring to Fig. 7, the reaction of the soil on the cross-hatched area is what causes shear on the vertical planes *EFGH*

$$d' = 52 - k = 52 - \frac{(44)(46)}{48} = 9.8 \text{ in.}$$

$$\text{Total shear on } EFGH = \frac{132.2 - (11.17)^2}{132.2}(610{,}000) = 34{,}200 \text{ lb.}$$

$$v = \frac{34{,}200}{(4)(134)(0.875)(9.8)} = 7.4 \text{ lb. per sq. in.}$$

The total moment acting around the entire column perimeter is found from Diagram 1 to be

$$M = (4)(7.8)(610{,}000) = 19{,}030{,}000 \text{ in.-lb.}$$

$$A_s = \frac{19{,}032{,}000}{(16{,}000)(0.875)(52)} = 26.2 \text{ sq. in.}$$

This total amount of steel will be divided by eight in order to determine the amount of steel in each band. If desired, more bars can be placed parallel to the sides of the footing than in a diagonal direction. In this design six 1-in. round bars will compose each set.

In view of tests made on footings with two-way reinforcement it would seem that the bars need not all pass directly under the column. In fact, if the width of bands needs to be increased in order to cover the entire area of the footing, it would be conservative to say that this may be done provided that the increase in the width of each band is not great. Until more tests have been made along this line, it would seem wise (especially in footings in which the depth decreases toward the outer edge) not to be too radical in the design of such important structures as footings. Where a small increase in the width of bands does not suffice to cover the entire area of footing, a few short cross rods may be employed to span the open spaces. Increase in band width should preferably be made in the bands parallel to the sides of the footing as, in these bands, the lengths of the rods do not change.

It should be noticed that the diagonal rods have a greater length subjected to cantilever action than the rods parallel to the sides of the footing. Since it is difficult to calculate accurately the stresses in a square footing, it does not seem proper to attempt any but equal division of moment between the rods in the different directions. Some designers prefer an octagonal shape of footing so that all rods will have approximately the same length. The rods should be cut from 2 to 4 in. shorter than the total width of footing, but if desired, the rods may be cut considerably shorter and staggered so as to allow for the decrease in bending moment from the column toward the edges of the footing.

The bond stress along the horizontal tension bars must now be investigated.

$$u = \left(\frac{132.2 - 4.9}{132.2}\right)\frac{610{,}000}{(48)(3.14)(0.875)(52)} = 86 \text{ lb. per sq. in.}$$

Deformed bars will be used. The complete design is shown in Fig. 8.

Where, on account of soil conditions, a greater depth is needed below the basement floor than that required for the footing, the pier or column between the top of the footing and the basement floor should be flared in a similar manner (but inverted) to the flare in columns just below flat-slab floors, as by so doing the bending moment and shear on the footing may be decreased. This should be clear from the discussion on flat-slab floors.

8. Combined Column Footings.—It is sometimes necessary to support a column on, or very near, the edge of a property line in order not to encroach upon adjacent property. In such a case a single symmetrical footing cannot be used. The nearest interior column is usually selected and a combined footing constructed under both columns. Sometimes a combined footing will include more than two columns.

The design of combined footings consists in constructing a base of such shape that the center of gravity of the loads will coincide with the center of gravity of the upward reaction. In addition, the base must have sufficient area so that the allowable pressure on the soil will not be exceeded.

The combined footing may be either a slab of uniform thickness or an inverted T-beam. If the slab type of combined footing is used with two columns, the slab must have a trapezoidal shape when the columns are placed at opposite ends of the footing. A rectangular shape may be used where a longitudinal projection is possible beyond the heavier load of a sufficient length to cause the center of gravity of the rectangle to coincide with the center of gravity of the loads. Transverse reinforcement is needed when the width of footing is much larger than the width of columns.

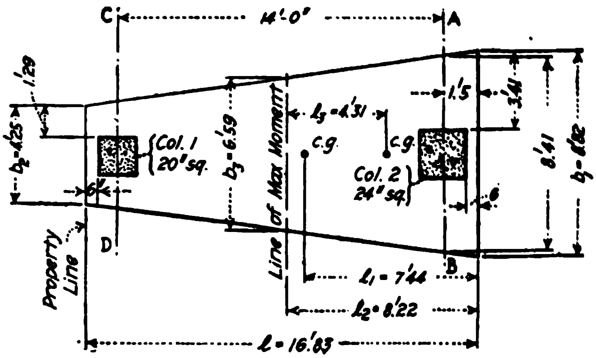


Fig. 9.

Illustrative Problem.—Design a combined footing for columns 1 and 2, Fig. 9. Column 1 is 20 in. square and sustains a load of 280,000 lb. Column 2 is 24 in. square and sustains a load of 380,000 lb. Distance between centers of columns is 14 ft. Allowable soil pressure is 6500 lb. per sq. ft. Assume a 2000-lb. concrete with medium steel reinforcement.

Load on column 1 = 280,000 lb.
Load on column 2 = 380,000 lb.

660,000 lb.

Assumed weight of footing = 55,000 lb.

Total = 715,000 lb.

$$\frac{715{,}000}{6500} = 110 \text{ sq. ft.}$$

Pressure on the soil due to column loads only is

$$\frac{660{,}000}{110} = 6000 \text{ lb. per sq. ft.}$$

The lengths of the parallel sides are unknown and two equations will be needed to solve. First equation may be obtained from the formula that the area of a trapezoid equals the average of the sum of the parallel sides multiplied by its length. The second equation may be found from the principle stated above—that the center of gravity of the trapezoid must coincide with the center of gravity of the combined column loading. This must be so in order that the pressure on the soil, and the consequent settling (if any), may be uniform—also to prevent dangerous transverse stresses in the columns.

$$\text{Area of footing} = \frac{b_1 + b_2}{2}(16.83) = 110$$

$$b_1 + b_2 = 13.07$$

$$l_1 - 1.5 = \frac{280{,}000}{660{,}000}(14) = 5.94 \qquad (1)$$

or

$$l_1 = 7.44 \text{ ft.}$$

Using the common equation for the center of gravity in a trapezoid

$$l_1 = 7.44 = \frac{16.83}{3}\cdot\frac{b_1 + 2b_2}{b_1 + b_2} \qquad (2)$$

Solving equations (1) and (2) for b_1 and b_2

$$b_1 = 8.82 \text{ ft. and } b_2 = 4.25 \text{ ft.}$$

The values of b_1 and b_2 may also be found by considering the footing 1 ft. wide and using the formulas on page 582 to find intensity of pressure at each end. Thus, if we let p_1 denote the intensity at the column 2 end and p_2 the intensity at the column 1 end, we have

$$p_1 = \frac{660{,}000}{16.83}\left[1 + \frac{6\left(\frac{16.83}{2} - 7.44\right)}{16.83}\right] = 52{,}900 \text{ lb.}$$

$$p_2 = \frac{660{,}000}{16.83}\left[1 - \frac{6\left(\frac{16.83}{2} - 7.44\right)}{16.83}\right] = 25{,}500 \text{ lb.}$$

Then

$$b_1 = \frac{52{,}900}{6000} = 8.82 \text{ ft.}$$

$$b_2 = \frac{25{,}500}{6000} = 4.25 \text{ ft.}$$

To find the necessary depth of footing and amount of steel required, we must find the section maximum moment (which is the section of zero shear) and then determine the center of gravity of the area to one side of this section as an aid in finding the value of this maximum moment.

$$\left[8.82l_2 - \frac{8.82 - 4.25}{(2)(16.83)}l_2^2\right]6000 = 380{,}000$$

$$l_2 - 0.0154l_2^2 = 7.18$$

$$l_2 = 8.22$$

And by trial,

$$b_1 - b_2 = 8.82 - 4.25 = 4.57$$

$$\frac{b_3 - b_2}{4.57} = \frac{16.83 - 8.22}{16.83}$$

$$b_3 - b_2 = 2.34$$

$$b_3 = 2.34 + 4.25 = 6.59$$

$$l_3 = \frac{l_2}{3}\cdot\frac{b_3 + 2b_1}{b_3 + b_1} = \frac{8.22}{3}\cdot\frac{6.59 + 2(8.82)}{6.59 + 8.82} = 4.31$$

We can now compute the value of the maximum moment.

$$M = 380{,}000\ (8.22 - 1.5 - 4.31) = 915{,}800 \text{ ft.-lb.}$$

or

$$M = (915{,}800)(12) = 10{,}989{,}600 \text{ in.-lb.}$$

The moment for 1 in. of width along the line of maximum moment, $M = \frac{10{,}989{,}600}{(6.59)(12)} = 139{,}000$ in.-lb. The footing must be considered as an inverted beam at this section, and the required depth and the area of the steel may be computed by the usual methods.

If the moment had been taken about a line through the center of gravity of the entire trapezoid, the result would be 133 000 in.-lb., or an error of only about 2.2%. This approximate solution is often used in practice as the error is small.

From Diagram 2, page 360, $K = 107.4$ and $p = 0.0077$ for the stresses as recommended by the Joint Committee.

$$d = \sqrt{\frac{139{,}000}{107.4}} = 36.0 \text{ in.}$$

$$A_s = (0.0077)(12)(6.59)(36) = 21.9 \text{ sq. in. for the whole width.}$$

Eighteen 1¼-in. round rods will be needed.

The depth required for punching shear at column 2 is

$$d = \frac{(388{,}000 - 6000)(4)}{(24)(4)(120)} = 41$$

and at column 1 it is

$$d = \frac{(288{,}000 - 6000)(1.67)}{(20)(4)(120)} = 28 \text{ in.}$$

In each case the depth is less than that required for moment.

The depth required for diagonal tension must now be determined. The shear at column 2 equals 388,000 lb. minus the part of this load distributed on the soil over the area *mrm*, Fig. 11, or

$$V = 388{,}000 - (8.14)(2.5)(6000) = 258{,}000 \text{ lb}$$

and

$$d = \frac{258{,}000}{(8.14)(12)(0.875)(120)} = 25 \text{ in.}$$

The shear at column 1 is

$$V = 288{,}000 - (4.88)(2.17)(6000) = 217{,}500 \text{ lb}$$

and

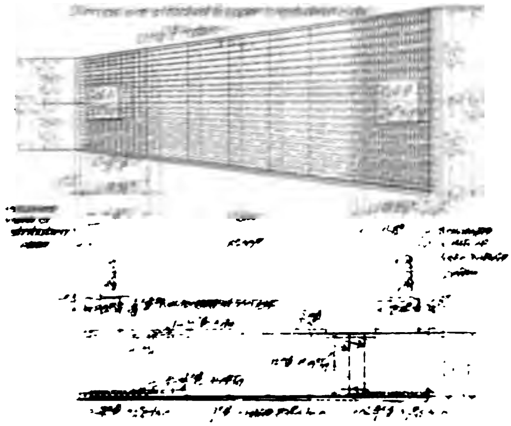
$$d = \frac{217{,}500}{(4.88)(12)(0.875)(120)} = 36 \text{ in.}$$

which is just that required for moment.

Adopting a depth (d) of 36 in. and assuming that deformed rods are used, then the number of rods required for bond will be (assuming $u = 100$)

$$u = \frac{V}{\Sigma o j d}, \text{ or number of rods} = \frac{258{,}000}{(3.93)(0.875)(36)(100)} = 21$$

Thus the bond controls the number of horizontal rods required and 21 rods will be employed (see Fig. 10).



[illegible]

Transverse reinforcement should be provided to withstand bending ... [illegible] ... column 2, ... at which distributing beams will be ... [illegible]

should ... [illegible]

... [illegible]

[illegible]

In the same manner, the distributing reinforcement for column 1 is determined.

$$M = \left(\frac{280{,}000}{2}\right)\left(\frac{4.25 - 1.67}{4.25}\right)\left(\frac{1.29}{2}\right)(12) = 658{,}000 \text{ in.-lb.}$$

$$d = \sqrt{\frac{658{,}000}{(107.4)(3.5)(12)}} = 1.33 \text{ in.}$$

Since the depth of the whole slab must be used, the necessary steel is found as above.

$$A_s = \frac{658{,}000}{(16{,}000)(0.875)(36)} = 1.3 \text{ sq. in.}$$

It sometimes happens that the required depth of the distributing beam may be larger than the depth of the whole slab. In such cases the footing may have an increased thickness under the column or else steel may be introduced at the top and bottom. Double reinforcement, however, should not be employed when excavation can readily be made.

The bond stress along the rods of the distributing beams must now be investigated. Considering the beam under column 2, the maximum shear is

$$\left(\frac{380{,}000}{2}\right)\left(\frac{3.82 - 2.0}{8.82}\right) = 147{,}000 \text{ lb.}$$

Assuming that deformed rods are used, then the number of rods required for bond will be ($u = 100$)

$$u = \frac{V}{\Sigma o j d}, \text{ or number of rods} = \frac{147{,}000}{(2.36)(0.875)(36)(100)} = 20$$

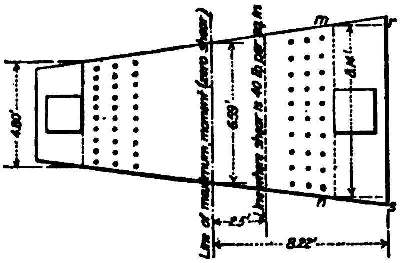


FIG. 11.

Thus the number determined for bond controls. In a similar manner, 12 rods are needed in the distributing beam under column 1.

Fig. 10 shows the spacing of the transverse reinforcement. Steel should be introduced between the end distributing beams about 24 in. c. to c. in order to make the footing more rigid and more capable of directly transmitting the loads.

Web reinforcement is not needed in the distributing beams, as each distributing beam and column has a similar load to that of a single footing, and for such footings we know that the intensity of shearing stress, as measuring diagonal tension, may be computed on a section at a distance out from the face of column equal to the depth of the footing to the steel. In a longitudinal direction, however, shear should be computed on a section close to the support, as *mn*, Fig. 11, since in this direction the tension steel is at the top of the footing and cracks may open up in the web as in simple beams. The spacing of stirrups is given in Figs. 10 and 11, determined as in a simple beam.

The loads beyond the centers of the columns in a longitudinal direction cause negative moment at the supports; that is, the load to the left of column 1 and to the right of column 2 cause tension in the bottom of the footing at the sections *AB* and *CD* respectively (Fig. 9). This tension, however, does not exceed the tensile strength of the concrete. At column 1, using the common flexure formula,

$$f = \frac{My}{I} = \frac{(8.82)(1.5)(6000)(9)(18)(12)}{(8.41)(12)(36)^3} = 33 \text{ lb. per sq. in.}$$

A few rods 4 ft. long will be placed at the bottom of footing across the sections *AB* and *CD* in order to provide for any possible defects in the bed of the foundation.

9. Cantilever Footings.—The cantilever type of construction may be employed in place of a combined footing under the usual conditions; that is, when encroachment upon adjacent property must be avoided and when, at the same time, it becomes necessary to make use of the land close to the property line. In cantilever construction, the wall-column footing and the footing of the nearest interior column are connected by a beam or strap, and this strap is extended so as to support the wall column. To save excavation, the top of the strap is usually placed at the same level as the tops of the footings.

ILLUSTRATIVE PROBLEM.—Let us assume the size of columns and column loading as shown in Fig. 12. The distance from the center of interior column to the property line will be made 12 ft. and the outer edge of the wall column will be placed on the property line. In addition to the above, we shall consider that piles are necessary and that it is not feasible to drive such piles closer than 2 ft. apart in any direction. The safe bearing capacity of each pile will be taken at 10 short tons. The working stresses as recommended by the Joint Committee for a 2000-lb. concrete and medium steel will be adopted.

Approximate figuring shows that about 12 piles will be needed under the wall footing, and 18 piles under the interior footing; that is, if the center of the wall footing is placed about as shown in Fig. 12. Knowing the number of piles required, the shape of the footings may be easily determined. The arrangement shown for the piles insures a spacing of at least 2 ft. in every direction.

On account of cantilever action, the uplift on the interior column may be due to the combined live and dead load of the wall column, but, since the live load may not always be present, the uplift from the dead load only should be considered. Disregarding the strap weight to the right of the center of wall footing, this uplift is

$$\frac{(130{,}000)(2.33) - (1200)(9)(4.5)}{9.00} = 28{,}000 \text{ lb.}$$

The strap will now be designed for the total live and dead load on the exterior column since this loading gives maximum conditions. The total load on the wall footing is

$$\frac{(175{,}000)(11.33) + (1200)(12)(6)}{9.00} = 230{,}700 \text{ lb.}$$

The uplift on the interior column may be found in a similar manner to that for dead load only, or more accurately (taking moments about the outer edge of the wall column)

$$\frac{(230{,}700)(3.00) - (175{,}000)(0.67) - (1200)(12)(6)}{12.00} = 40{,}700 \text{ lb.}$$

The loads on the strap are shown in Fig. 13. The maximum moment occurs where the shear is zero, or accurately enough,

$$\frac{(175{,}000)(6)}{230{,}700} = 4.5 \text{ ft.}$$

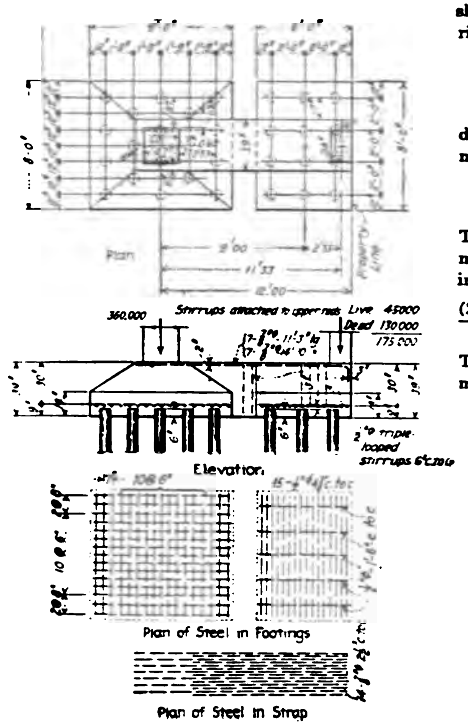


Fig. 12.

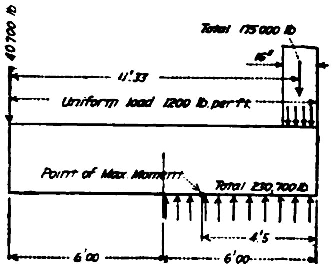


Fig. 13.

from the outside edge of wall column.

$$M = (175{,}000)(3.83) - \left(\frac{230{,}700}{6}\right)(4.25)(2.25) = 281{,}000 \text{ ft.-lb.}$$
$$= 3{,}372{,}000 \text{ in.-lb.}$$

Then, assuming $b = 39$ in.

$$d = \sqrt{\frac{3{,}372{,}000}{(107.4)(39)}} = 28\tfrac{1}{2} \text{ in.}$$

The shear in the strap at the inside edge of wall column should not exceed 120 lb. per sq. in. with an effective system of web reinforcement. This is due to the fact that tension exists in the upper portion of the strap; in other words, the inward spread of the wall-column load will not aid to any appreciable extent in reducing the tendency to fail through diagonal tension. For the dimensions determined for moment

$$v = \frac{(175{,}000) - \left(\frac{230{,}700}{6}\right)(1.33)}{(39)(0.875)(28.5)}$$
$$= \frac{123{,}900}{(39)(0.875)(28.5)} = 127 \text{ lb. per sq. in.}$$

The strap will be deepened so that $d = 30$ in. and total depth $= 32$ in. Stirrups will be needed from the inner edge of the exterior column to a point 3.5 ft. from the property line where the shear becomes 10 lb. per sq. in. Triple-looped ½-in. round stirrups will be employed and these will be spaced 6 in. on centers, or

$$s = \frac{3}{2}\,\frac{A_v f_v jd}{V} = \frac{3}{2}\,\frac{(6)(0.196)(16{,}000)(0.875)(30)}{123{,}900} = 6 \text{ in.}$$

The unit shear at the inner edge of wall footing is

$$\frac{(40{,}700) + (1200)(6)}{(39)(0.875)(30)} = 47 \text{ lb. per sq. in.}$$

Two triple-looped ½-in. round stirrups placed just to the left of this point will make the design satisfactory

The number of horizontal rods will now be determined.

$$A_s = \frac{M}{f_s jd} = \frac{3{,}372{,}000}{(16{,}000)(0.875)(30)} = 8.1 \text{ sq. in.}$$

Fourteen ⅞-in. round deformed rods will be used.

$$u = \frac{123{,}900}{(14)(2.75)(0.875)(30)} = 122 \text{ lb. per sq. in.}$$

This value will be considered satisfactory on account of bending the ends of the rods as shown.

The horizontal tension rods should have a sufficient length for straight bond to the left of the section of maximum moment. Alternate rods may stop off at 10 ft. from the property line, which is about 5 in. beyond the point where they are no longer needed in tension.

The interior footing may be designed in the same manner as explained in Art. 8, except that in this problem we have the moment from concentrated loads instead of from uniform loads.

$$M = (80{,}000)(2.37)(2) + (20{,}000)(0.62)(2) + (20{,}000)(1.87)(2) + (40{,}000)(0.87)(2)$$
$$= 548{,}400 \text{ ft.-lb., or } 6{,}581{,}000 \text{ in.-lb.}$$

The depth to steel will be made 30 in. in order to provide properly for the strap. Web reinforcement is not needed

$$A_s = \frac{M}{f_s jd} = \frac{6{,}581{,}000}{(16{,}000)(0.875)(30)} = 15.6 \text{ sq. in.}$$

The steel will be placed in two directions. Thus, for moment, 3.9 sq. in. or seven ⅞-in. round rods will be needed in each band. Let us now determine the number of ⅞-in. round deformed rods in each set that will be required for bond.

$$u = \frac{V}{\Sigma o jd}, \text{ or number of rods } = \frac{\frac{(16)(20{,}000)}{4}}{(100)(2.75)(0.875)(30)} = 11$$

This number controls.

The weight of footing, as designed, deducting the weight of strap already considered, is approximately 22,000 lb. This gives a total pressure on the piles of 360,000 + 22,000 − 28,000 = 354,000 lb. Thus the number of piles was correctly chosen. In this design there is no danger of failure by direct shear above the tops of the piles as for this kind of shear about one-half the compressive strength of the concrete may be allowed. There is also no danger from punching shear around the base of the column.

The footing under the wall column acts as a simple cantilever. The moment on each edge of strap

$$M = (60{,}000)(17) = 1{,}020{,}000 \text{ in.-lb.}$$

As in the interior footing, the depth to steel will be made 30 in.

$$A_s = \frac{1{,}020{,}000}{(16{,}000)(0.875)(30)} = 2.4 \text{ sq. in.}$$

Twelve ½-in. round rods are satisfactory for moment. For bond

$$\text{number of deformed rods} = \frac{60{,}000}{(100)(1.57)(0.875)(30)} = 15$$

A few cross rods will also be employed to better distribute the load.

The weight of wall footing deducting the weight of strap is approximately 10,000 lb., giving a total pressure on the piles of 230,700 + 10,000 = 240,700 lb., or almost exactly 10 tons per pile, as planned.

It is generally considered good practice to lay the concrete directly upon the heads of the piles. The ground is usually excavated around the piles, the depth depending upon soil conditions, and then a layer of broken stone is spread and rammed before the concrete is laid. The supporting power of the soil between the piles is thus utilized.

10. Raft Foundations.—The raft foundation in building construction may be considered as an extension of the single or combined footing until the foundation covers at least a considerable, if not the entire, area of the building. Its use is limited to sites where the allowable pressure on the soil is very small or where the building is supported by piles sustained by friction.

Raft foundations may be divided into the following three general classes:

1. Flat slabs of plain or reinforced concrete.
2. Beams or girders with a slab underneath.
3. Beams or girders with a slab on top.

A flat-slab foundation may be designed in the same manner as a flat-slab floor (see Art. 20, Sect. 11). The foundation is considered as an inverted flat slab loaded by the reaction of the ground and supported by the columns. The column base should be made large enough to prevent excessive moments and shears in the concrete.

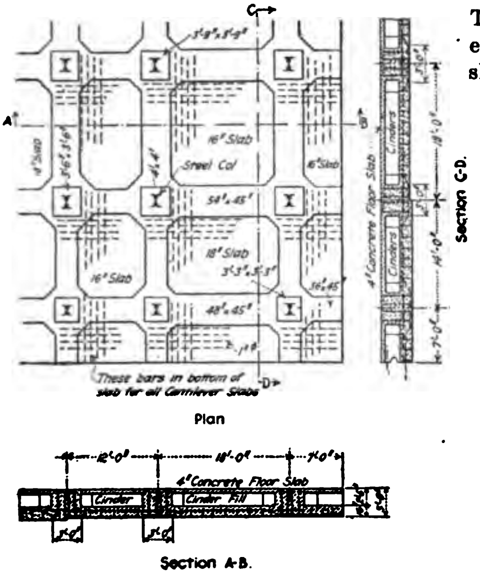


FIG. 14.

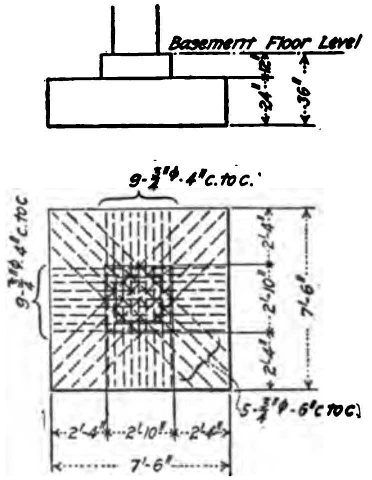


FIG. 15.

Fig. 14 shows a design of a raft foundation of the second class. The action of forces is exactly as in an inverted floor and calls for similar treatment in designing. As compared with Class 3, Class 2 permits a T-beam design, but on the other hand, requires an extra fill and separate floor surface in the basement.

11. Examples of Column Footings.—Fig. 15 shows a design of a single footing for the typical interior columns of a factory building for the Bradley Knitting Co., Delavan, Wis. The design of the top of footing should be noted.

Fig. 16 is an example of a combined footing employed in an automobile depot in Boston.

Fig. 17 is a special foundation of novel design used in the Garage Building of the Decanville Automobile Co., New York City.

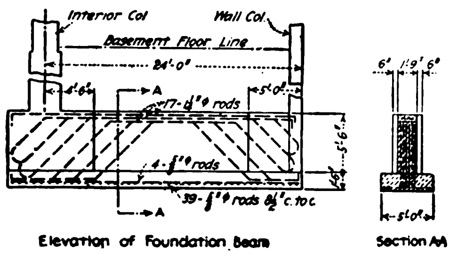


FIG. 16.

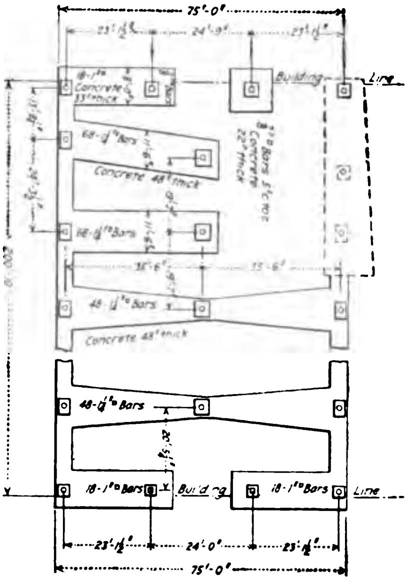


FIG. 17.

12. Concrete Piles.—Concrete piles are usually employed where wooden piles would be subject to decay or to destruction by the action of marine worms. By using one of the types of concrete piles, they may be employed under almost any circumstance where wooden piles would be suitable. Wooden piles, to insure permanency must be cut off below permanent water line, as, when subjected to an atmosphere which is alternately wet and dry, they will decay. Concrete piles are not so restricted, as they are permanent above water line as well as below it, and the cutoff line can therefore be as high as is consistent with the depth actually required in the foundations which they support. The cost of the concrete piles, themselves, is somewhat greater than that of wooden piles, but this cost is, in a great many cases, more than offset by the fact that, due to their size and shape, they are capable of sustaining greater loads than wooden piles, and by the saving in excavation, pumping, sheeting and masonry made possible by their use.

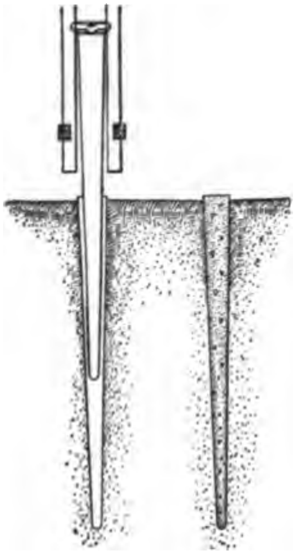


Fig. 18.—Raymond pile core collapsed and partly withdrawn. Completed Raymond pile without reinforcement.

Reinforced-concrete sheet piling has been used quite extensively for permanent bulk-heads, piers and the like, and is designed to meet the requirements of the particular situation where it is employed. This sheet piling may be designed with an interlocking feature, or without, as may best and most economically meet the particular requirement.

Concrete piles are of two general types—those molded in place and those molded before driving.

12a. Piles Molded in Place.—The Raymond, Simplex and Pedestal piles are the forms of this type which are most widely used.

Raymond Piles.—Raymond concrete piles are made by driving a tapering reinforced steel shell to refusal, by means of a collapsible steel core, withdrawing the core and thereupon filling the shell with concrete. There is thus a shell or form for every pile and the concrete within the shell may be reinforced or not, as may be desired. Fig. 18 shows a partly and an entirely completed Raymond pile. Raymond piles are of 8-in. diameter at the point and taper 0.4 in. per ft. up to 37 ft. in length.

The shell of a Raymond pile is made of sheet steel of a thickness depending upon the elasticity and crushing tendency of the earth, and is reinforced by means of steel wire spirally wound on the interior of the shell on a 3-in. pitch and held in place by grooving the shell around the wire and fastening the ends by electric welding. The weight of both shell and wire may be varied to suit conditions, but 24-gage steel in the shell and No. 3 wire are most commonly used.

A boot or point of pressed steel is placed on the end of the core or mandrel and completely encloses the bottom of the shell.

After the core is collapsed and withdrawn, the shell is inspected to insure its perfection and it is then filled with concrete.

A mechanical problem that for several years taxed the ingenuity of engineers connected with the Raymond system, was to provide a shell of sufficient strength to withstand the back pressure of the soil after the driving core had been withdrawn. This difficulty has been successfully overcome by means of the spirally reinforced steel shell, which is made on machines specially designed for the purpose. This shell is now universally used in connection with this method and is illustrated in Fig. 19.

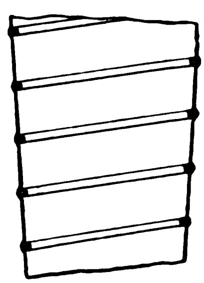


Fig. 19.

Simplex Piles.—In the Simplex pile a hollow cylindrical steel tube or form, 16 in. in diameter and 3/4 in. thick, is driven to a suitable bearing. When the required depth is reached, the form is filled with concrete to a sufficient height, and then withdrawn. The driving point, depending on soil conditions, may be either a round cast-iron point that is set in place, or a hinged cutting edge called an alligator point which opens as the tube is withdrawn. Fig. 20 shows the standard Simplex pile which fills the great majority of requirements and is adaptable in all ordinarily unreliable ground. The method shown is termed "standard," as about 75% of Simplex piling stands on the detachable cast-iron point. The alligator point is limited in its use to a certain class of soil—that is, one that stands well after penetration. As generally used, no reinforcement is required, but the system admits of placing any desired reinforcing members within the form. In very wet soil a permanent casing of slightly less diameter than the inside of the cylindrical form is used to prevent washing or displacement of the concrete. Sometimes a pile is molded on the surface and lowered through the form, which is then withdrawn, leaving the molded pile in position.

Pedestal Piles.—Pedestal piles are made in the following manner: (1) a core and steel casing are driven to the required depth, (2) the core is removed and a charge of concrete is dropped in, (3) the core is used as a rammer to compress the concrete into the surrounding soil, (4) this process is

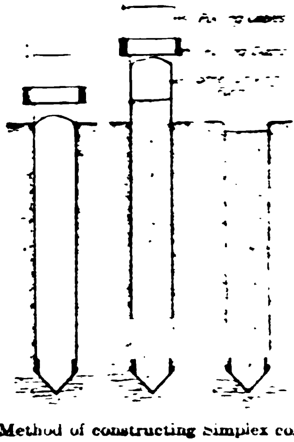


Fig. 20.—Method of constructing Simplex concrete piles.

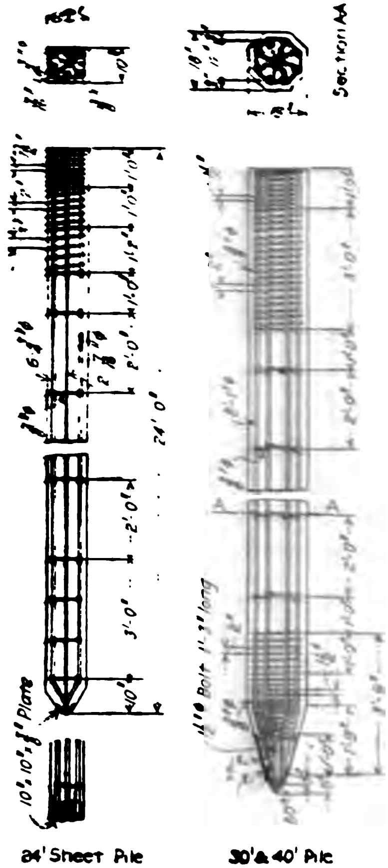


Fig. 21.—Details of piles used in ore dock construction at Cleveland.

repeated until a pedestal base about 3 ft. in diameter is formed, (5) the casing is withdrawn leaving the completed pile with an enlarged base.

Composite Wood and Concrete Piles.—In order to eliminate the great cost of placing extremely long concrete piles, a method has been devised by the Ryamond Concrete Pile Co. to install what is know as a composite pile. This consists of a Raymond pile superimposed upon a wood pile. The length of the concrete pile is determined by the depth to which it is necessary to drive the wood pile in order to insure its cutoff being below water. A dowel provided between the wood and concrete pile which is strong enough to withstand any lateral displacement. By this method long wood piles can be used, where necessary, with short concrete piles superimposed upon them, thus avoiding the necessity of excavation, sheeting and pumping in material which is apt to be extremely difficult and costly to handle.

12b. Piles Molded Before Driving.—Cast piles must be reinforced to permit handling, to withstand the shock due to driving, and to withstand the lateral strains, if conditions are such as to require it.

Precast piles are usually designed to meet the particular conditions under which they are to be used. In some cases they are provided with cast- or wrought-iron or steel driving points and may also have an iron pipe cast in the center for jetting. Usually, however, the iron or steel point is omitted, and jetting, if found necessary, is done by means of pipes, which are not fastened to the pile.

To protect the pile from shattering under the severe blows of the hammer, laminated wood blocks may be used, although some contractors provide a special driving head in which a cushion of sand, rope, or other material is placed between a driving block of wood and the concrete to prevent crushing the head of the pile. These piles can be built in any required size, and have been placed up to 90 ft. in length, though considerably shorter piles are more frequently employed. On dock and pier work premolded concrete piles are used both as supporting piles and as sheet piles to retain the ground behind them. Such piles must not only be reinforced against driving and handling stresses, but must be designed to safely support the soil load to which they will be subjected. These piles must ordinarily be cured for from 30 to 60 days after casting before they can be used, which, on rush work, is considerable detriment to their use. It is also extremely difficult to predetermine the exact length which will be required, and if it is necessary to cut off any unused portion of the pile this must be done by breaking away the concrete from the reinforcement and either bending this latter down into the superimposed caps or cutting it off with hack saws or acetylene torches.

Fig. 21 shows the details of the concrete piles employed in the construction of ore docks for the Pennsylvania Railroad Co. at Cleveland, Ohio. The piles, octagonal in shape and reinforced with eight 1-in. round rods, were cast vertically in steel forms, a cast-iron shoe being fitted into the form and becoming a part of the finished pile. A 1:2:4 mixture of both gravel and broken stone was used. The forms were removed in from 12 to 24 hr. after pouring, depending upon the weather. No pile was driven before it was at least 30 days old. Sheet piles were also used, as shown in Fig. 21.

SECTION 13

RETAINING WALLS

A retaining wall is a wall of masonry—such as stone, plain concrete, or reinforced concrete—built to sustain the lateral pressure of earth or of other material possessing more or less frictional stability. In stone masonry and plain concrete walls the section must be made heavy enough so that the weight of the structure will prevent overturning, whence the name gravity section. In reinforced-concrete walls the weight of a considerable part of the sustained material is utilized to maintain stability and, in addition, the sections may be designed to more nearly develop the full strength of the concrete.

The treatment of retaining walls here given deals (1) with the earth thrust or pressure which acts against the wall, (2) with the forces which maintain the stability of the wall under this thrust, and (3) with the design of walls to withstand these attacking and stabilizing forces.

1. Earth Pressure.—In Fig. 1 let *AB* represent the back of a retaining wall, and *AC* the surface of the ground. The earth has a tendency to break away and come down some line, as *CB*, thus producing pressure on the wall. The weight of the earth tends to cause this breaking away while the resisting forces are the friction on the face *AB* and on the plane *BC* (the latter called internal friction); the cohesion along the line *BC*; and the resistance of the wall due to its stability against overturning and sliding. The coefficients of these frictions, and of cohesion, vary with slope of the surface *AC*, the fineness of the retained material, and its moisture content. Cohesion is influenced greatly by moisture and the vibration of moving loads, and seldom obtains in a newly made untamped fill. Most earth pressure theories, therefore, treat the limiting case of an ideal granular mass possessing no cohesion.[1] This has the effect of reducing the curve of rupture *BC* to a straight line.

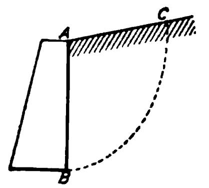


FIG. 1.

All earth pressure theories given here assume (1) that the surface of rupture *BC* is a plane, (2) that the point of resultant pressure is at one-third the height of the wall from the bottom when the surface of the material meets the top of the wall, and (3) that the resultant pressure makes some definite angle with the horizontal. These theories result in a triangular distribution of pressure against the face of the wall when the surface of the material meets the top of the wall; and since the resultant pressure must pass through the centroid of this pressure-distribution triangle, it must act at one-third the height of the wall from the bottom. Its inclination varies with different conditions and theories.

TABLE 1.—ANGLES OF REPOSE AND WEIGHTS PER CUBIC FOOT FOR VARIOUS EARTHS[2]

Material	Slope	Angle of repose, degrees	Weight in lb. per cu. ft.
Sand, dry	2.8 :1 to 1.4 :1	20 to 35	90 to 110
Sand, moist	1.75:1 to 1:1	30 to 45	100 to 110
Sand, wet	2.8 :1 to 1.2 :1	20 to 40	110 to 120
Ordinary earth, dry	2.8 :1 to 1:1	20 to 45	80 to 100
Ordinary earth, moist	2.1 :1 to 1:1	25 to 45	80 to 100
Ordinary earth, wet	2.1 :1 to 1.75:1	25 to 30	100 to 120
Gravel, round to angular	1.75:1 to 0.9 :1	30 to 48	100 to 135
Gravel, sand and clay	2.8 :1 to 1.3 :1	20 to 37	100 to 115

[1] For theory including cohesion, see CAIN'S "Earth Pressure, Walls and Bins." It is valuable for the investigation of stability of existing walls backed by earth which has been compacted in some manner.

[2] From CAIN'S "Earth Pressure, Walls and Bins," p. 9.

TABLE 2.—COEFFICIENTS OF INTERNAL FRICTION[1]

Kind of material	Tangent of angle of internal friction	Approximate corresponding		Authority
		Angle, degrees	Slope	
Coal, shingle, ballast, etc.	1.423	54	0.7 to 1	B. Baker
Bank sand	1.423	54	0.7 to 1	Goodrich
Riprap	1.097	48	0.9 to 1	Goodrich
Earth	1.097	48	0.9 to 1	B. Baker
Quicksand, 100 up	0.895	42	1.1 to 1	Goodrich
Clay	0.895	42	1.1 to 1	B. Baker
Quicksand, 50–100	0.750	37	1.3 to 1	Goodrich
Earth	0.750	37	1.3 to 1	Steel
Bank sand	0.750	37	1.3 to 1	Wilson
Sand, 50–100	0.549	29	1.8 to 1	Goodrich
Bank sand	0.549	29	1.8 to 1	Goodrich
Clay	0.474	25	2.1 to 1	Goodrich
Cinders	0.474	25	2.1 to 1	Goodrich
Gravel, ½-in.	0.474	25	2.1 to 1	Goodrich
Gravel, ¼-in.	0.350	19	2.9 to 1	Goodrich
Bank sand	0.350	19	2.9 to 1	Goodrich
Sand, 30–50	0.258	14	3.9 to 1	Goodrich
Sand, 20–30	0.179	10	5.6 to 1	Goodrich

1a. Rankine's Formula for Resultant Active Earth Pressure.—Rankine has developed the following formula[2] for the case in which (1) the total active thrust P acts upon a vertical plane, (2) acts parallel to the surface of the earth for all cases in which $\theta \geq 0$, (3) acts in a material of indefinite extent, and (4) the earth carries no load except its own weight (Fig. 2):

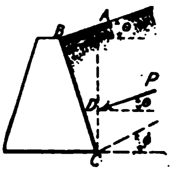


FIG. 2.

$$P = C_e \frac{wh^2}{2}$$

in which

P = total active thrust of earth against the vertical plane as described above.

w = weight per cubic foot of retained material.

h = height of vertical section considered, as AC.

$$C_e = \cos\theta \cdot \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}$$

where

θ = angle of surcharge.

ϕ = angle of internal friction.

Diagram 1 gives the values of C_e for various values of θ and ϕ. It should be noted that P is parallel to the surface AB, when θ is either positive, or zero; and that it acts at a point D, $\frac{h}{3}$ above C, or in other words, $CD = \left(\frac{AC}{3}\right)$.

[1] E. P. GOODRICH: *Trans.* Am. Soc. of C. E., vol. 53, p. 301.

[2] For development see BAKER'S "Masonry Construction," 10th Ed., p. 493.

When $\theta = \phi$, then

$$P = \frac{1}{2} wh^2 \cdot \cos \phi$$

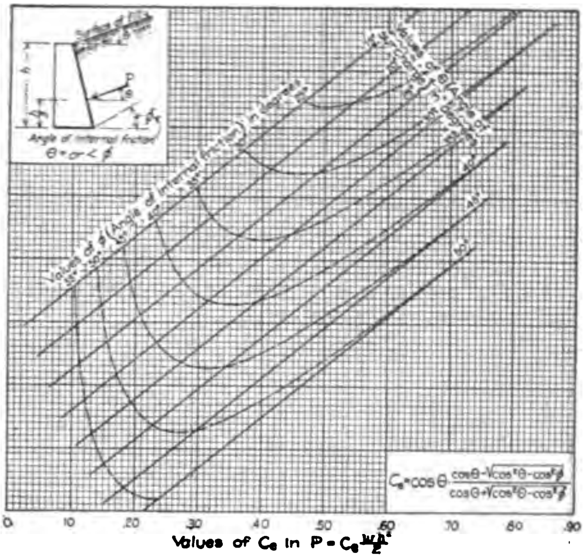
When $\theta = 0$

$$P = C_e' \frac{wh^2}{2}$$

in which

$$C_e' = \tan^2 (45° - \tfrac{1}{2}\phi)$$

DIAGRAM 1



The following table gives values of C_e' for varying values of ϕ.

ϕ	C_e'	ϕ	C_e'	ϕ	C_e'	ϕ	C_e'
20°	0.490	30°	0.3333	40°	0.2174	50°	0.1325
25°	0.406	35°	0.2710	45°	0.1718	55°	0.0994

1b. Coulomb's Wedge of Maximum Pressure.—Coulomb advanced the theory that the wedge *ACF* (Fig. 3), lying between the surface of the earth *AF* and the plane of rupture *CF* would move down against the vertical plane *AC* due to its own weight, causing the resultant pressure *P'*. The prism itself is in equilibrium through a force acting upward against *CF*, and making the angle ϕ with *CF*; and a force opposing *P'* due to the resistance of the wall *BC* and of the prism *BAC* (if *BC* is not vertical). If angle *BCA* is relatively large, there will be found a plane of rupture *SC*, in the prism *BAC*, tending to force a portion of this prism upward. This plane of rupture makes angle β with *AC*, which is dependent upon θ and ϕ. Its value is given in Table 3.

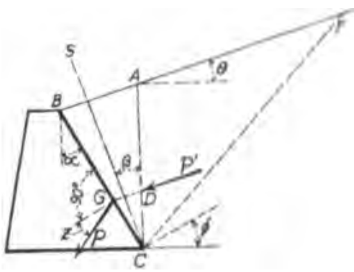


FIG. 3.

TABLE 3

θ	ϕ	β	Batter of β (inches per foot)	θ	ϕ	β	Batter of β (inches per foot)
15°	15°	0° 0′	0	20°	20°	0° 0′	0
	20	20° 25′	4½		25	15° 30′	3⅜
	25	21° 05′	4⅝		30	18° 25′	4
	30	21° 55′	4⅞		35	19° 10′	4⅛
	35	21° 30′	4¾		40	18° 55′	4⅛
	40	20° 35′	4½		45	18° 05′	3⅞
	45	19° 15′	4⅛		50	16° 45′	3⅝
	50	17° 35′	3¾		55	15° 10′	3¼
	55	15° 45′	3⅜				
				30°	30	0° 0′	0
25°	25	0° 0′	0		35	12° 05′	2½
	30	13° 35′	2⅞		40	14° 25′	3⅛
	35	16° 15′	3½		45	15° 0′	3¼
	40	16° 55′	3⅝		50	14° 40′	3⅛
	45	16° 40′	3⅝		55	13° 40′	2⅞
	50	15° 45′	3⅝				
	55	14° 25′	3⅛	40°	40	0° 0′	0
					45	9° 45′	2
35°	35	0° 0′	0		50	11° 30′	2⅜
	40	10° 50′	2¼		55	11° 40′	2½
	45	10° 55′	2¼				
	50	10° 15′	2⅛	45°	45	0° 0′	0
	55	12° 45′	2¾		50	8° 40′	1⅞
					55	10° 05′	2⅛

Let the point G on BC be so located that $CG = \frac{1}{3}BC$. This will be the point of application of the resultant earth pressure P. Its line of action as applied to the wall makes the angle Z with a normal to the wall at G.

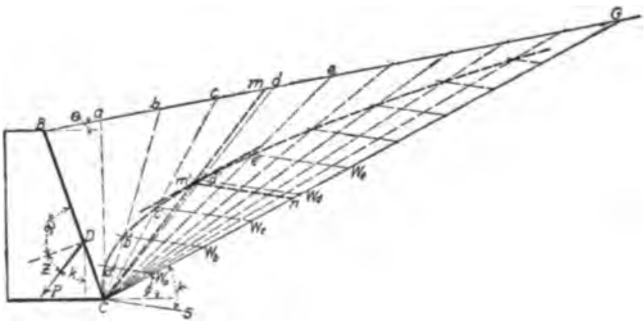


FIG. 4.

When the face BC of the wall is battered such that $\alpha < \beta$, Construction I should be followed:
When $\alpha = \beta$, follow Construction I if $Z > \phi$; otherwise follow Construction II.
When $\alpha > \beta$, follow Construction II.

Construction I.—Let the wall BC (Fig. 4) retain material whose surface slope is θ, and whose angle of internal friction is ϕ. It will be assumed that the resultant thrust P will be applied at the third point D, so that it makes the angle ϕ with a normal at D. [It may be assumed to make the angle of friction (Ƶ of the material against the wall) with the normal when that angle does not exceed ϕ.] Beginning at B, lay off on BG arbitrary distances, equal, for convenience, as Ba, ab, etc., and connect these points a, b, c, etc. with C. Compute the weight of these prisms thus formed, with length of 1 ft. normal to the drawing, and lay them off to some convenient scale, on CG, beginning at C, as W_a, W_b, etc. If the distances on BG are equal, those on CG will likewise be equal. Draw CS making an angle k with CG, where k is the angle made by P with the vertical. Draw $a'W_a$, $b'W_b$, etc., all parallel to CS, and through these points a', b', c', etc., thus obtained, draw a curve. A tangent to this curve parallel to CG is tangent at m', through which Cm may be drawn. Thus the prism BCm is that which causes the maximum pressure P for the conditions assumed; and $m'n$, drawn parallel to CS, and scaled off to the scale of the weights on CG, gives the maximum value of this thrust P.

When $\theta = 0$ the surface of fill is level, and Cm then bisects angle BCG, and may thus be drawn at once. The weight of prism BCm may be laid off on CG and $m'n$ drawn to get the corresponding value of P.

Construction II (Fig. 5).—When it has been found from Table 3 that the wall BC makes a greater angle than β with the vertical, it becomes necessary to determine the pressure P'

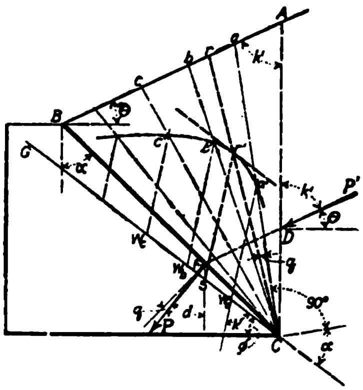


FIG. 5.

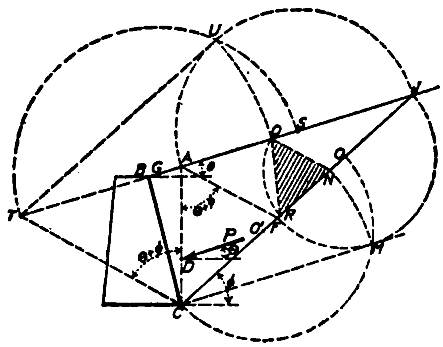


FIG. 6.

acting parallel to AB, against a vertical plane AC through C. Lay off for convenience equal distances Aa, ab, etc., and connect a, b, c, etc. with C. Compute the weights of the prisms thus formed, and plot these weights to some convenient scale on CG (which makes an angle ϕ with a horizontal through C), as CW_a, W_a, W_b, etc. Draw through these points the lines W_aa', W_bb', etc., making the angle k' (equal to $90° - \theta$) with CG. A smooth curve may then be drawn through a', b', c', etc. Point r' is located by drawing a tangent to this curve parallel to CG. The plane rC is the plane of rupture for the wedge BCr, which might be forced upward due to the pressure P' and the resistance of the wall. The resultant pressure P' acting at D in either direction on the line FD is equal to sr' scaled to that of the plotted weights on CG.

Let the inclination of BC to the vertical be α. Extend CG beyond C and lay off α counterclockwise; then lay off 90 deg. The line thus obtained will make an angle q with the plane of rupture Cr.

Now if q is less than Ƶ, where Ƶ is the angle of friction of the material on the wall, produce P' to F, on BC. Through F the resultant P on the face BC will act, such that its angle of inclination with the vertical (d) equals $\angle rCG$, or what is the same thing, such that it makes the angle q with a normal to BC. The magnitude of P is $r'S$ scaled to that of the weights on CG.

If q is greater than Ƶ, perform Construction I making P slope the angle Ƶ with the normal to BC, Fig. 4.

When θ is negative and not large, or when α is negative but not to exceed 10 deg., the constructions I and II may still be used.

Rebhann's Construction.—The value of P by the Coulomb theory may be found in the following manner: Let BC, Fig. 6, be the back face of a retaining wall with the earth surface on the line BJ. AC is the vertical plane against which P acts, at point D, $\frac{1}{3}\overline{AC}$ from C. CJ makes the angle of internal friction ϕ with the horizontal. Draw AF so that $\angle CAF = \theta + \phi$. With O as the center and FJ as the diameter, describe the arc FMJ. Draw CM tangent to this arc. (The point of tangency M may be located accurately by describing the semicircle CMO on the diameter CO.) Make $CN = CM$, and through N draw $QN \parallel AF$. Making $RN = QN$ forms the triangle QRN. The magnitude of P is

$$P = (w)(\text{Area } QRN)$$

Its direction corresponds to Rankine's thrust, parallel to AJ.

Point Q may be located by making a construction similar to that for finding N. Draw $TC \parallel AF$. Describe arc AUJ on the diameter AJ. Locate U by drawing arc TUS on diameter TS, whence TU is tangent to AUJ at U. Make $TQ = TU$.

When $\theta = \phi$, AJ and CJ do not meet; also, $\angle CAF = 2\phi$.

When $\theta = 0$, $\angle CAF = \phi$, and P is horizontal.

1c. Comparison of Coulomb and Rankine Results.—Rebhann's construction, also that of Fig. 4, gives results which check those of the Rankine formula, upon a vertical plane, when the earth extends indefinitely. The theories differ somewhat for other cases.

1d. Useful Interpretation of Results of Earth Pressure Theories.—Assume a wall BC, Fig. 7, to retain a fluid. Since the pressure of fluid is dependent upon the depth, the intensity of the pressure at any depth $y = wy$, in which w is the weight per cubic foot of the fluid in pounds. Then the pressure at CD is wh. Thus the variation in pressure intensity is as a straight line, BD. The resultant of this *triangle of pressure* is $P = \frac{1}{2}wh^2$. It acts horizontally, normal to BC and through the centroid of BCD.

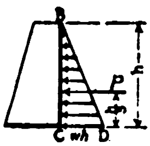


FIG. 7.

Suppose the top of the wall to be covered at a depth h', Fig. 8, with this liquid. The pressure at B is $BF = wh'$. Similarly, $CD = w(h' + h)$. The total pressure on BC is equal to the area of the *trapezoid of pressure* $BFDC$; and its resultant P acts through the centroid of this trapezoid distance y from CD. It acts normal to BC. The distance y may be found from the formula

$$y = \frac{h}{3} \cdot \frac{h + 3h'}{h + 2h'}$$

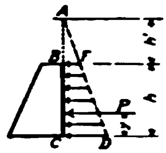


FIG. 8

Now the precise difference between earth (granular mass) and fluid pressures, is that in the case of earth there is not equal pressure in all directions at a given point. If E_v represents vertical pressure at a point in an earth mass, and E_l the lateral pressure at that point, then $E_l = C_e E_v$. Since in each of the foregoing formulas it is obvious that the intensity of pressure varies with the depth, the pressure areas caused by the earth are similar to those caused by a fluid. Thus, from Diagram 1 on page 577, if $w' = C_e w$, then P would equal the pressure caused by a *fluid* whose weight is w', and whose direction of acting upon the plane considered would be parallel to the direction of P. In like manner any of the foregoing developments may be converted into fluid-like action.

2. Live Load on Top of Fill—Equivalent Surcharge.—When a live load is applied in a direction normal to the face of the wall, as for instance upon a track or roadway at right angles to a bridge abutment, Fig. 9, the earth thrust P is larger than before. Let the weight of the applied live load (including impact) be replaced by an equal weight represented by the prism AMN, of the same material as that retained by the wall. Let the depth of this "equivalent surcharge" be h'.

• The total pressure on MC is given by the trapezoidal pressure $ACSR$, whose resultant is P. It may be obtained by determining an equivalent fluid of weight w' as described in the preceding article; or it may be determined graphically by finding the thrust on MC, then on MA, and taking their difference. In usual cases, P will be nearly equal to $\frac{w'}{2}(h' + h)^2$, the error being less than 5% on the safe side. As before,

$$y = \frac{h}{3} \cdot \frac{h + 3h}{h + 2h'}$$

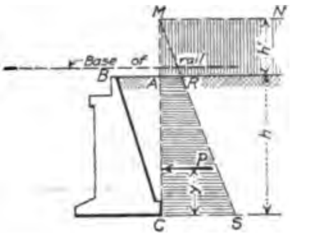


Fig. 9.

The thrust P is finally prolonged to meet the face BC, where it is combined with the weight of the prism BAC, to determine the final thrust against the wall.

Construction I, page 578 could have been employed to find directly the thrust against BC by placing the surcharge up to B.

When roadways, tracks, or other live loads are placed close to, and parallel to, the wall, the method described above will apply. If these live loads are remote from the wall, the method of procedure described in the following article is recommended. It will, of course, apply to both cases, but is a saving for the conditions cited.

3. Live Load on Top of Fill—Pressure Distribution.—Let AD, Fig. 10, represent a track parallel to the wall BC. Tests[1] show that the pressure on AD is practically all distributed between AF and DG, which make 30 deg. with the vertical. Assuming uniform distribution throughout this region, the pressure per square foot on the horizontal plane FG is computed, F being the point where AF strikes the wall. Let the intensity of pressure on ab, without the load AD, be shown by the triangle abc. Let de $(= cf)$ equal the unit pressure on FG due to AD multiplied by some factor N dependent upon the material in the fill. The resultant of abc, acting $\frac{h}{3}$ above C, and the resultant of $defc$, acting $\frac{1}{2}\overline{CF}$ above C, are then combined to get the final thrust on BC.

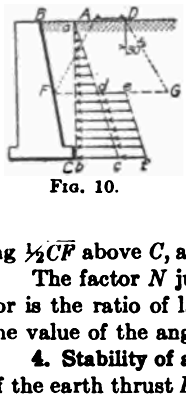


Fig. 10.

The factor N just referred to may be determined from the table on page 577. This factor is the ratio of lateral unit pressure to vertical unit pressure, and is thus dependent upon the value of the angle of internal friction. Thus $N = C_e'$ in the table referred to.

4. Stability of a Retaining Wall.—Two motions of the wall tend to result due to the action of the earth thrust P: (1) a tendency to slide forward; and (2) a tendency to tip forward about some point on the base.

The tendency for the wall to slide forward may be stated as being equal to the tangential component of the resultant force R acting upon the base, or plane of bearing. In walls having a horizontal plane of bearing it is equal to the horizontal component R_H, Fig. 11. The resistance to sliding may be developed in three ways: (1) by the friction on the plane AD, (2) by the depth of A below the surface of the ground in front of the wall,[2] and (3) by a key wall projecting downward from the plane AD. The frictional resistance of the base plane AD may be taken as the total component normal to AD multiplied by the coefficient of friction of the wall material upon the supporting soil (see Table 4). The key wall will cause compression in the soil before it, the intensity of which should not exceed seven-tenths of the maximum working unit pressure.

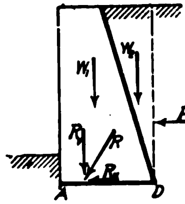


Fig. 11.

The resistance to overturning the wall is afforded by a distributed reaction of the bearing soil upward against the base of the wall. Since the bearing capacity of the soil must not be exceeded, it is necessary to study the distribution of this reaction, and to determine simple rules which may govern to secure stability.

[1] See tests by Prof. M. L. Enger, *Eng. Rec.*, Jan. 22, 1916, p. 106–8.
[2] See Art. 15, Sect. 17, page 763.

TABLE 4.—COEFFICIENTS AND ANGLES OF FRICTION BETWEEN EARTH AND OTHER MATERIALS

Materials	$f = \tan\phi$	ϕ
Masonry upon masonry	0.65	33°
Masonry upon wood, with grain	0.60	31°
Masonry upon wood, across grain	0.50	26° 40′
Masonry on dry clay	0.50	26° 40′
Masonry on wet clay	0.33	18° 20′
Masonry on sand	0.40	21° 50′
Masonry on gravel	0.60	31°

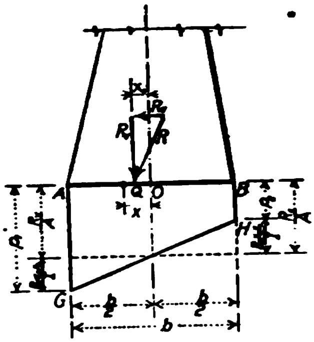


FIG. 12.

Consider the base of a 1-ft. section of wall to be represented in projection by AB, Fig. 12. When R acts at the center of gravity O, the intensity of pressure is uniform over the base plane and is equal to the vertical component of R divided by the base area, or $\frac{R_v}{A} = \frac{R_v}{b}$. If R acts at any other point, as Q, the force R_v is equivalent to an equal R_v at O and a couple whose moment is R_vx_0. At any point distant x from O the intensity of the pressure due to this moment is $\frac{R_vx_0x}{I}$, in which I is the moment of inertia of the base plane about an axis through O at right angles to AB and lying in the base plane. At the edges A and B this intensity $= \frac{R_vx_0b}{2I} = \frac{6R_vx_0}{b^2}$. The intensity of pressure at edge A is

$$p_1 = \frac{R_v}{b} + \frac{6R_vx_0}{b^2} = \frac{R_v}{b}\left(1 + \frac{6x_0}{b}\right) \qquad \text{(lb. per sq. ft.)}$$

and at edge B it is

$$p_2 = \frac{R_v}{b}\left(1 - \frac{6x_0}{t}\right) \qquad \text{(lb. per sq. ft.)}$$

Since the base plane cannot resist tension, the second term of p_2 must not exceed the first. As a limiting condition, if the two terms are just equal, $p_2 = 0$, or

$$\frac{R_v}{b} = \frac{6R_vx_0}{b^2}$$

in which x_0 is the distance to Q when the eccentricity R_v just causes this equality. Solving

$$x_0 = \frac{b}{6} \qquad \text{(ft.)}$$

The rule thus determined follows: *The resultant force acting upon the base plane must strike it back (toward B) of the forward third point of the base plane, if no tension is to be taken by that plane.*

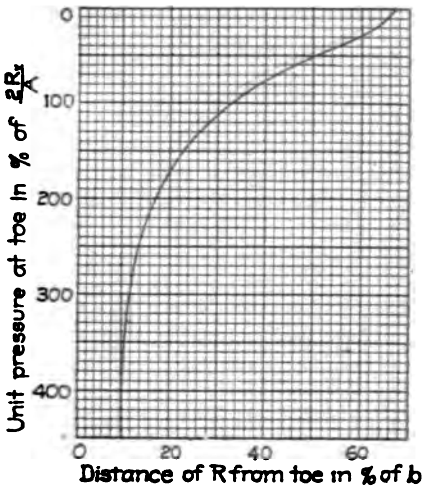
When $p_2 = 0$,

$$p_1 = \frac{2R_v}{A} \qquad \text{(lb. per sq. ft.)}$$

The relation between the point of application of the resultant R upon the base and the unit stress in the soil under the toe is shown in Diagram 2. The rapid rise in unit pressure due to a small forward movement should be noted. Since it has been advised in this discussion to limit the resultant to the forward third point, the unit pressure under the toe for that condition is taken as 100%. Its maximum value depends upon the decision of the engineers regarding the quality and supporting power of the soil. If R falls inside of the middle third, the pressure at the toe will be less than 100%. The difference between its percentage and 100% will give the unit pressure at the heel in % of $\frac{2R_v}{A}$.

Table 5 gives allowable unit pressures upon various soils. If high walls are to be built, tests should be made at the site, as described in Art. 1, Sect. 12. The tendency of the pressure on the bearing soil to heave the earth in front of the wall should also be investigated.

Diagram 2



When the soil pressure under the toe is greater than the allowable unit pressure, the base area should be increased by extending the toe forward. On a solid wall this may be done by projecting a toe from the front face, whose top surface slopes from 30 to 60 deg. with the face of the wall, and whose bottom surface is an extension of the base plane. Such a projecting toe must be designed for shear and moment, the same as the toe on a reinforced-concrete wall.

Table No. 5.—Safe Bearing Capacity of Soils in Short Tons per Square Foot

Kind of material	Minimum	Maximum
Rock, the hardest, in thick layers in native bed........................	200.0	
Rock equal to best ashlar masonry....	25.0	30
Rock equal to best brick masonry.....	15.0	20
Rock equal to poor brick masonry.....	5.0	10
Clay in thick beds, always dry........	6.0	8
Clay in thick beds, moderately dry....	4.0	6
Clay, soft..........................	1.0	2
Gravel and coarse sand, well cemented.	8.0	10
Sand, dry, compact and well cemented.	4.0	6
Sand, clean, dry.....................	2.0	4
Quicksand, alluvial soils, etc..........	0.5	1

On a reinforced-concrete wall the unit pressure under the toe may be reduced by extending the toe farther from the face of the wall. Some designers extend the back slab to obtain more weight of fill.

When a base slab is used under the body of the wall for capping piles, or for providing suitable bed, the unit pressure at the front edge of the wall at the top surface of the base slab must not exceed the allowable compressive unit stress in the concrete. Should it exceed this value, provision may be made as above described. Care should always be exercised i-keying the body of the wall against sliding or overturning on the base slab. Keyed joints and dowel rods may be used for this purpose.

4a. So-called "Factor of Safety."—In masonry walls it has been customary to define the factor of safety against overturning by the relation

$$\text{factor of safety} = \frac{\text{moment of resisting forces}}{\text{moment of overturning forces}}$$

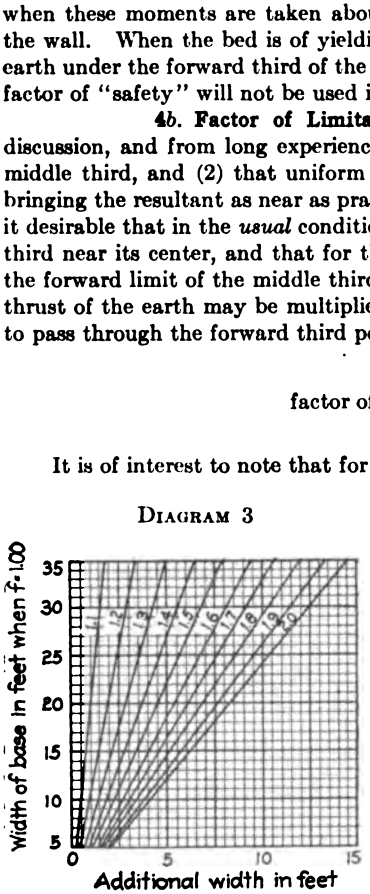
when these moments are taken about the toe of the base. This would assume a rigid bed for the wall. When the bed is of yielding material, the wall will not rock on the toe because the earth under the forward third of the base will crush, allowing the wall to settle as it tips. This factor of "safety" will not be used in this discussion.

4b. Factor of Limitation.—Two considerations resulting from the foregoing discussion, and from long experience, are (1) that the resultant of pressures stay within the middle third, and (2) that uniform settlement should, as far as possible, be provided for by bringing the resultant as near as practicable to the center of the base. These conditions make it desirable that in the *usual* condition of loading the resultant should pass through the middle third near its center, and that for the unexpected or unusual loadings it should never exceed the forward limit of the middle third. The "factor of limitation" is that factor by which the thrust of the earth may be multiplied to determine that thrust which will cause the resultant to pass through the forward third point. In other words,

$$\text{factor of limitation} = \frac{\text{limiting thrust}}{\text{actual thrust}}$$

It is of interest to note that for a wall of given height, supporting a level fill, the factor of limitation may be increased a certain percentage by increasing the width of base a definite amount, which amount is the same for plain walls and for reinforced-concrete cantilever or counterfort walls. This fact is illustrated in Diagram 3. For example, suppose a given wall with a 30-ft. base to have been designed with a factor of limitation of one. If 12.5 ft. were added to the width of base, the factor of limitation for the wall would be doubled.

DIAGRAM 3



Level fill against plain concrete walls and reinforced concrete cantilever or counterfort walls.

5. Types of Retaining Walls.—Retaining walls of concrete may be of plain concrete, or of reinforced concrete. Plain concrete walls have proportions similar to those of masonry, and their section is usually the gravity section. Reinforced-concrete walls, however, are not as massive. The various types in common use are: (1) the T- or cantilever wall; (2) the counterforted wall; (3) the buttressed wall; and (4) the cellular wall. Other special forms have been developed for various purposes.

The following discussion is intended to give preliminary proportions before the final investigation is made for stability. For the cases here given no further designing need be done; but for irregular embankments and other unusual conditions the cases cited will aid in choosing the preliminary form of wall.

6. Design of Plain Concrete Walls.

6a. Formulas and Diagrams for the Two Principal Types.—For type (*a*) shown in Fig. 13, let

A = area of cross-section.
W = weight of 1 ft. of wall = $150A$ for concrete.
t = width at top = $\frac{b}{6}$.
b = width at base.
h = height of wall.
e = angle between P and horizontal.
c = distance from outer third point Q to centroid of section.
f = factor of limitation, or the number of times P_H may be increased before the resultant passes through the third point Q.

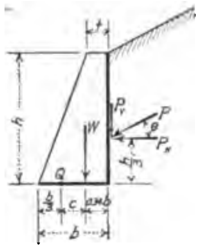


Fig. 13.

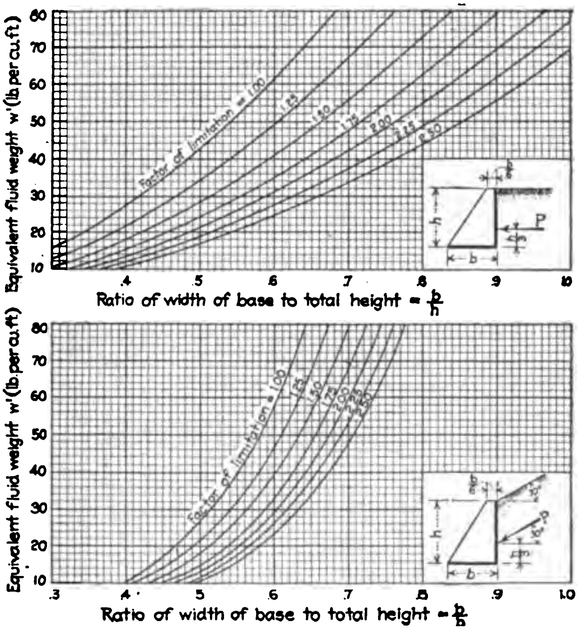
Assuming the resultant to be passing through Q, then the algebraic sum of the moments about Q equals zero, or $\Sigma M_Q = 0$,

$$Wc + \frac{2P_v b}{3} = f P_H \frac{h}{3}$$

When $t = \frac{b}{6}$, and $W = 150A$, then $c = 0.326b$, whence, since $P = \frac{w'h^2}{2}$,

$$28.5b^2 + 0.333\, fw'bh \sin e - 0.1667fw'h^2 . \cos e = 0$$

Diagram 4



Many designers prefer to disregard the frictional component P_v. It cannot well be developed upon a vertical face. Walls should have a slight batter on the back, although presumably vertical. Letting $e = 0$,

$$\frac{b}{h} = 0.0764 \sqrt{fw'}$$

When e = 30 deg.

$$28.5 \frac{b^2}{h^2} + 0.1667fw' \frac{b}{h} - 0.1442fw' = 0$$

Diagram 4 gives values of $\frac{b}{h}$ for various values of w' and f, when e = 0 and 30 deg.

Type (*b*), (*Fig.* 14).—For comparison with type (*a*), a level filling will be assumed. It may be noted that a prism of earth *BAC* will add to the stability for this type. However, the wall in general is less stable because its weight *W* lies forward from *Q*. As before, assuming $\Sigma M_Q = 0$, there results,

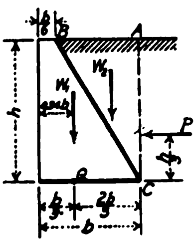


FIG. 14.

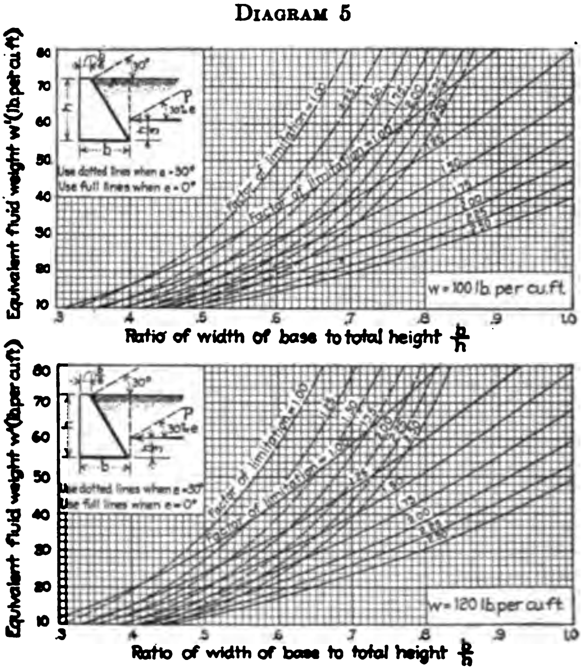
$$\frac{b}{h} = 0.10\sqrt{fw'} \quad \text{when } w = 100 \text{ lb. per cu. ft.}$$

and

$$\frac{b}{h} = 0.0915\sqrt{fw'} \quad \text{when } w = 120 \text{ lb. per cu. ft.}$$

in which w is the weight per cubic foot of the material comprising the material in prism *ABC*. Diagram 5 gives values of $\frac{b}{h}$ for these two cases when f and w' have varied values as well as with e = 30 deg. or zero.

DIAGRAM 5



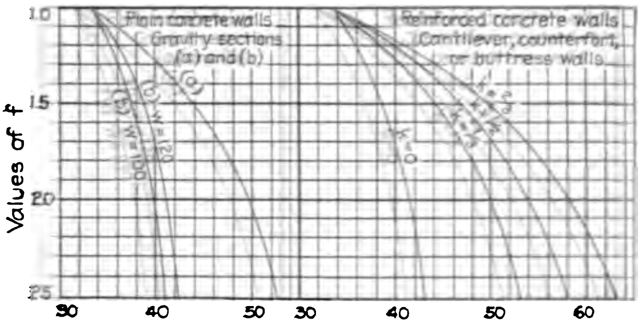
Ratio of height of earth to height of wall above ground	Ratio of thickness of base to total height of wall = $\frac{b}{h}$	Ratio of height of earth to height of wall above ground	Ratio of thickness of base to total height of wall = $\frac{b}{h}$
1.0	0.35	2.0	0.58
1.1	0.42	2.5	0.60
1.2	0.46	3.0	0.62
1.3	0.49	4.0	0.63
1.4	0.51	6.0	0.64
1.5	0.52	9.0	0.65
1.6	0.54	14.0	0.66
1.7	0.55	25.0	0.68
1.8	0.56	or more	

6b. Trautwine's Table.—Table on this page taken from Trautwine's "Civil Engineer's Pocketbook" gives values of $\frac{b}{h}$ for various heights of surcharge, as for instance, roadbeds. Values here given are empirical, and may be used for average conditions. They are the result of long practice in retaining-wall design. The earth is assumed to slope up from the top of wall till it reaches a level at the height indicated by the ratio in the first column.

Trautwine recommends that when the backing is somewhat consolidated in horizontal layers, each of these thicknesses may be reduced, but that no rule can be given for this. He also states that since sand and gravel have no cohesion, the full dimensions as above should be used with these materials, even though the backing be deposited in layers. A mixture of sand, or earth with a large proportion of pebbles, bowlders, etc., will exert a greater pressure against the wall than the materials ordinarily used for backing; and hence when such backing has to be used, the above thicknesses should be increased, say, about $\frac{1}{8}$ to $\frac{1}{6}$ part.

6c. Selection of Preliminary Section.—The above methods of proportioning the gravity section enables the selection of a preliminary wall against which the earth pressures

Diagram 6



may be determined, and the actual resultant obtained. The resulting soil pressures are then determined and compared to the allowable pressures. Analysis should be made for the lateral pressure $f \cdot P_H$ (the resultant for which is to pass through the forward third point) as well as for P_H. Likewise resistance to sliding should be great enough to prevent sliding under the action of $f \cdot P_H$.

Since the thrust of $f \cdot P_H$ will cause the resultant to pass through the forward third point, the actual thrust P_H will cause a resultant much nearer the center. Diagram 6 has been drawn to show the effect of various values of f upon the actual position of the resultant. For instance, for type (a), by designing with $f = 1.75$, the actual resultant will pass through a point $0.475b$ from the toe. If the actual thrust, causing this actual resultant, should be multiplied by 1.75, the resultant thus caused would pass through the forward third point.

Having the position of the resultant ($0.475b$ from the toe), the unit soil pressure under the toe may be found, from Diagram 2, to be 55% of $\frac{2R_v}{A}$, that is, 55% of the allowable maximum unit pressure assigned to the case when R passes through the third point.

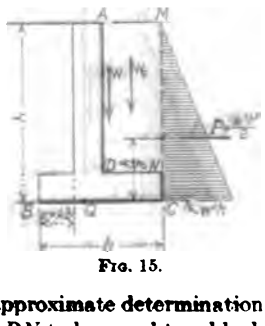


Fig. 15.

7. Design of Cantilever or T-walls of Reinforced Concrete.—A cantilever or T-wall of reinforced concrete consists of a cantilever stem AD, Fig. 15, rigidly attached to a base slab, BC. Since stability greatly depends upon the weight W_2 of the earth $ADNM$, it is likewise affected by the distance x of the face of the stem from the toe B.

7a. To Determine Approximate Base Width.—As an approximate determination (within 10%), we will assume the weight of the wall and earth above DN to be equal to a block of earth of section $h(b - x)$, of the same weight, w, of the earth in the fill.[1] Assume

[1] See article by H. M. Gibb, *Eng. News*, July 24, 1913.

resultant to pass through the forward third point, the total moment about that point must equal zero. The weights of the length x of the base will be neglected. Thus,

$$\frac{wh}{G}(b - x)(3x + b) = \tfrac{1}{2} f w' h^2 \cdot \frac{h}{3}$$

Whence

$$\frac{b}{h} = \frac{\sqrt{fw'}}{\sqrt{w(1 + 2k - 3k^2)}}$$
$$= C\sqrt{fw'}$$

in which

$$C_1 = \frac{1}{10\sqrt{1 + 2k - 3k^2}} \quad \text{for } w = 100 \text{ lb. per cu. ft.}$$

or

$$C_2 = \frac{1}{10.95\sqrt{1 + 2k - 3k^2}} \quad \text{for } w = 120 \text{ lb. per cu. ft.}$$

Values of C_1 and C_2 (to be introduced in place of C above) may be taken from Diagram 7.

DIAGRAM 7

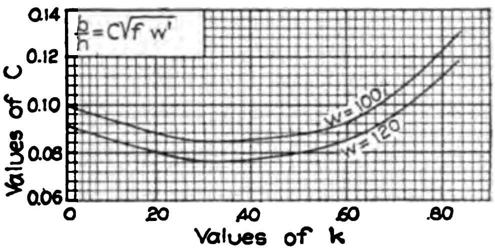
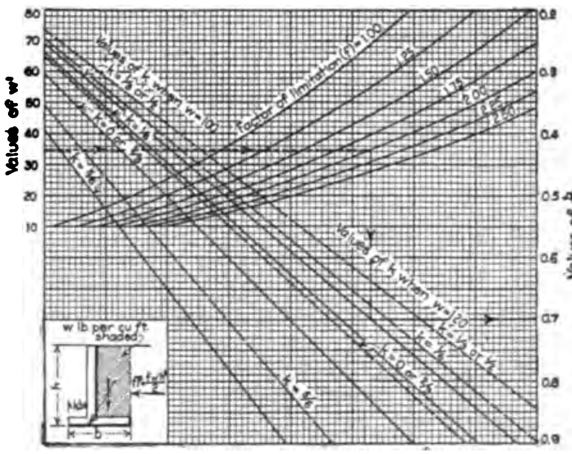


DIAGRAM 8



Proportions of cantilever, counterfort, or buttress retaining wall.

It is important to note in Diagram 7 that for any given value of w' and f, the minimum value of $\frac{b}{h}$ is that for $k = \frac{1}{3}$; and that when $k = \frac{1}{2}$ it is very nearly the same. Since it is obvious that resistance to sliding is increased by increasing the weight on the base, it is common practice where possible to place the stem so that $k = \frac{1}{3}$.

The width of base of a cantilever wall may be taken from Diagram 8, by entering with a value of w' and moving to the right to the selected value of f; thence down to the desired value of k; thence to the right margin where the proper value of $\frac{b}{h}$ may be read.

Illustrative Problem.—Desired cantilever wall, $h = 20$ ft.; $k = \frac{1}{3}$, $f = 2$. For material to be retained, $w = 100$, $\theta = 0$, $\phi = 30$ deg. Determine width of base. From the table on page 577 $C_e' = 0.3333$. From Art. 1d, $w' = C_e'w$ or $w' = 34.7$. Entering Diagram 8 with this value, thence horizontally to $f = 2.00$, thence downward to $k = \frac{1}{3}$ (for $w = 100$), thence horizontally to the right margin, $\frac{b}{h} = 0.70$. Therefore $b = 20 \times 0.7 = 14$ ft. From the right-hand side of Diagram 6 it is found that for $f = 2$ and $k = \frac{1}{3}$, the resultant will strike the base at a point $0.50 \times 14 = 7.00$ ft. from the toe. The corresponding pressure under the toe for that position is, from Diagram 2, about 50% of the working value $\frac{2R_v}{A}$.

7b. Stem.—The tendency of the earth pressures is to break the stem of the wall, similar to a cantilever beam. This moment is greatest at the junction of the wall with the base. Fig. 16 shows a cantilever wall, with the total earth pressure area divided into two parts—the pressure on the stem, and the pressure on the base slab. The thrust P_1 is the same as that for a depth of earth equal to the height of the stem above the upper side of the base slab. The bending moment per foot of wall at its junction with the base surface is equal to the moment of P_1 about that same line; or

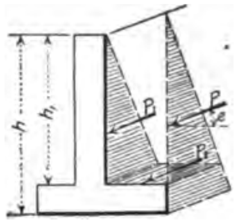


Fig. 16.

$$M = f P_1 \cos e \cdot \frac{h_1}{3} = \frac{fw'h_1^3}{6} \cdot \cos e \qquad \text{(ft.-lb.)}$$

But for balanced stresses in the reinforced concrete, $M = Kbd^2$ (see Diagram 2 of Sect. 7, page 360). Whence for a 12-in. width of stem,

$$d = 0.408h_1^{3/2}\sqrt{\frac{fw'}{K}\cdot \cos e} \qquad \text{(in.)}$$

in which d = effective depth of stem slab in inches, h_1 = total height of stem in feet, and w' = fluid equivalent in pounds per cubic foot.

When $e = 0$,

$$d = 0.408h_1^{3/2}\sqrt{\frac{fw'}{K}} \qquad \text{(in.)}$$

When $e = 30$ deg.,

$$d = 0.380h_1^{3/2}\sqrt{\frac{fw'}{K}} \qquad \text{(in.)}$$

The vertical reinforcing will be on the face next to the backing.

Diagram 9 gives the effective depth d for various values of fw' and K. The latter term includes the selection of working stresses and the percentage of steel. To this effective depth is added sufficient concrete to properly imbed the steel. The thickness of wall at the top is arbitrary, but never less than 6 in.

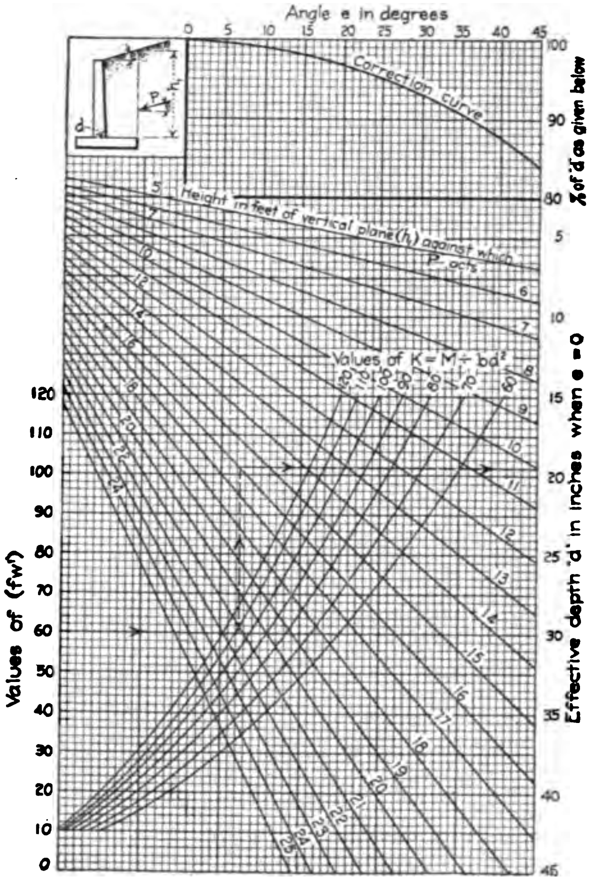
Illustrative Problem.—The stem of a wall is 16 ft. above the top of the base slab. Required its maximum thickness for $w' = 40$, $f = 1.50$, $n = 15$, $f_s = 16{,}000$, and $f_c = 650$. Slope of surcharge (hence e) is 30 deg. From Diagram 2, Sect. 7, $K = 107$, $p = 0.0077$.

Entering Diagram 9 with $fw' = (40)(1.50) = 60$; thence to $K = 107$; thence vertically to $h_1 = 16$; thence to right margin, where $d = 19.5$ in. Due to the fact that $e = 30$ deg., from the correction curve at the top it is found that d for this case should be 93.2% of 19.5, or 18.2 in. Total thickness, therefore, should be 21 in.

It may be necessary in some cases to take into account the weight of the vertical slab in finding the maximum thickness required. The resultant of the earth pressure and weight of stem may readily be found graphically and then the eccentricity of this resultant on the section at the junction with the base slab may be scaled. The required thickness and percentage of reinforcement may then be found by means of Diagram 15 or 16 of Sect. 9, pages 404 and 405.

It is not necessary to carry all the steel to the top of the stem. The moment diagram should be plotted for the stem and some of the rods cut off a sufficient distance above the point where they are not needed to secure anchorage (see Arts. 16 and 22, Sect. 7).

DIAGRAM 9

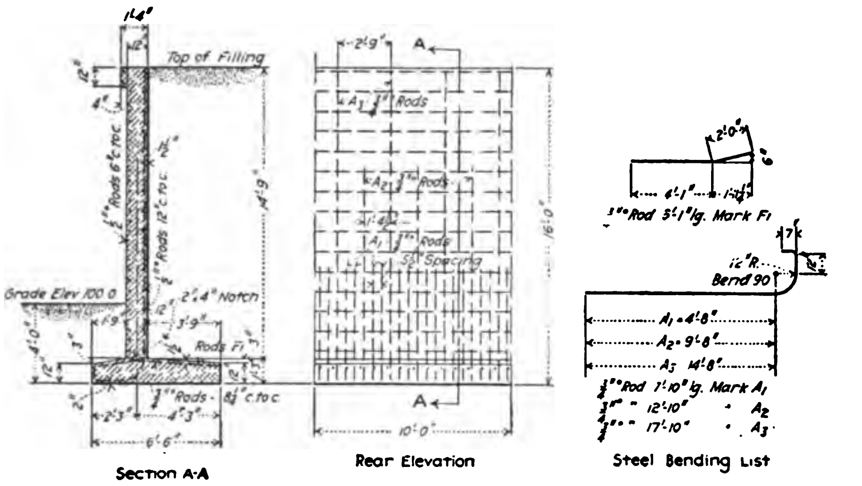


7c. Base Slab.—The greatest stress in the base slab will be developed when the resultant is at the most forward position—that is, in the present discussion at the forward third point. Under this condition the toe has a maximum upward pressure, while the heel has a maximum downward pressure combined with a minimum upward pressure.

The toe slab *MN* (Fig. 17) must be designed for the moment and shear at *N* caused by the pressure area *A*. Investigation must also be made for bond stress. Reinforcing will be along the bottom for this portion of the base. Diagram 14 (see page 596) may be used in the shear investigation. Shear usually governs toes of length $\frac{h}{3}$ or less.

The heel slab carries the upward pressure represented by the area *B*; the downward component of P_2 shown by P_2'; and the dead weight of all earth above the slab *RS*. The thrust

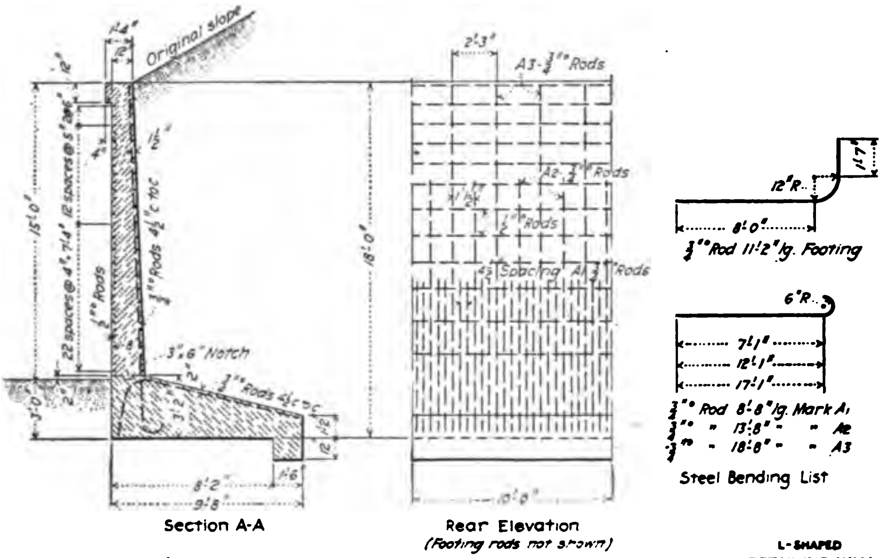
PLATE I



Concrete 1:2:4 mix 8.1 cu yd per 10 ft length of wall.
Steel: 770 lb per 10 ft. length of wall. Deformed bars.
Use No. 16 Wire (Steel Wire Gauge) for tying.
To splice longitudinal reinforcement overlap 2 ft. and wrap with wire

CANTILEVER TYPE
RETAINING WALL
SCALE DATE
DRAWN BY TRACED BY
CORRECT
APPROVED

PLATE II



Concrete 1:2½:5 mix 16.4 cu yd per 10 ft of wall
Steel: 1290 lb per 10 ft of wall Deformed bars
Use No. 16 Wire (Steel Wire Gauge) for tying
To splice longitudinal reinforcement overlap 2' and wrap with wire

L-SHAPED
RETAINING WALL.
SCALE DATE
DRAWN BY TRACED BY
CORRECT
APPROVED

P_2 has the distribution shown by area C, varying from f_1 at S to f_2 at R. Vertical components f_1' and f_2' of f_1 and f_2 respectively, designate the pressure area D. The pressure of the weight of earth above RS is not shown. Reinforcing for moment requires rods near the upper surface. Shear and bond should be provided for.

Sliding on the base may be figured from R_v and the coefficient of friction of the base material on the soil (see Table 4, page 582). If $f \cdot P$ causes more tendency to slide than resisted by friction, small projecting key walls should be cast on the under side of the base and anchored to it by dowel rods (see page 581).

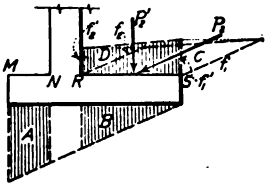


Fig. 17.

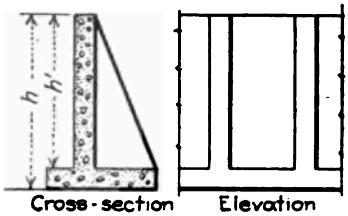
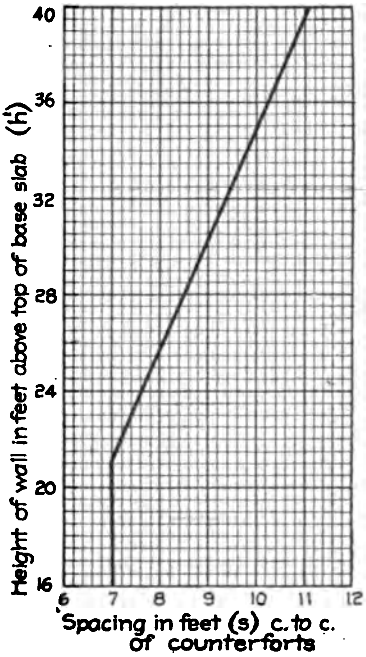


Fig. 18.

Diagram 10



7d. Expansion Joints.—Expansion joints should divide the structure into sections to relieve continuity and to relieve temperature and shrinkage stresses. These latter stresses should be cared for by the addition of about 0.3 of 1% of steel placed horizontally in the stem, to confine cracking of any extent to the joints. The joints should be lock-joints, and in general should be waterproofed to prevent unsightly stains.

Typical designs are shown in Plates I and II.

8. Design of Counterforted Walls.—For walls above about 20 ft. in height, the cantilever type of retaining wall requires a wall of great thickness to be self-supporting. A great saving is effected in the amount of material used for high walls by placing ribs, or counterforts, at intervals on the back side of the wall, tying it to the back of the base slab (see Fig. 18). The vertical wall is therefore a slab continuous over the counterforts, and loaded by horizontal loads, thus saving much material in the wall itself.

The width b of the base and height of the wall may be found as though the wall were of the cantilever type, by using Diagrams 1 and 8. Minimum material is required when the vertical wall is from $\frac{b}{2}$ to $\frac{b}{3}$ from the toe, as was shown for cantilever walls in Diagram 7.

8a. Thickness and Spacing of Counterforts.—The counterforts should have a thickness equal to one-twentieth of the height of the wall, and perferably never less than 12 in. Their spacing should be such as to give the minimum amount of material required for the wall. This spacing s, measured in feet from center to center, should be

$$s = 2.46 + 0.216h \qquad \text{(ft.)}$$

and should not be less than 7 ft. for walls under 20 ft. in height. The spacing may be obtained directly from Diagram 10.

The reinforcing of the counterfort will be discussed in an illustrative problem.

8b. Vertical or Face Wall.—The face wall, as noted above, is a slab supported at its junction with the counterforts and the base slab. Its thickness is commonly computed by considering it to be made up of horizontally loaded continuous beams, as AB, Fig. 19. At any depth h_x the load per square foot would be $h_x \cdot w'$, and the moment at the center would be $h_x w' \frac{s^2}{10}$. From this the thickness at that depth could be computed. The maximum thickness would be required in a strip at the bottom of the vertical wall, when h_x would equal h', if we neglect any restraint offered by the base slab. Then

$$M_{max.} = \frac{12fw'h's^2.}{10} = Kbd^2$$

Since $b = 12$ in. for convenience,

$$d = 0.32s\sqrt{\frac{fw'h'}{K}} \qquad \text{(in.)}$$

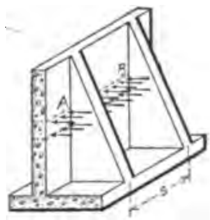
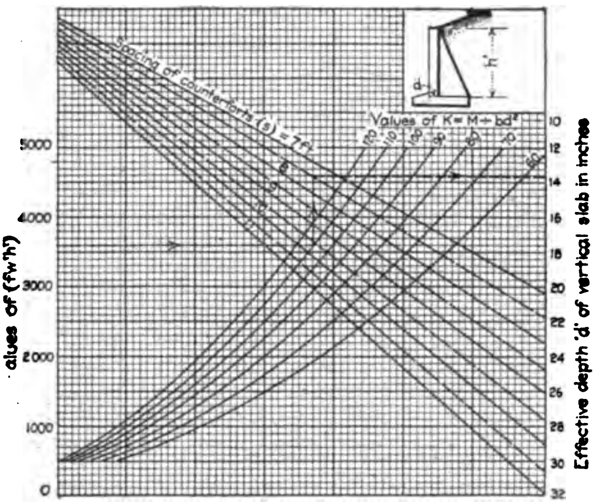


FIG. 19.

which is the effective depth of the slab at the bottom. The total thickness is found by adding protective covering for the steel. The thickness of the wall varies from this value at the bottom to a thickness not less than $\frac{1}{20}h$ at the top. The equation given above is the basis for Diagram 11.

If it is desired to design for $\frac{h_x w' s^2}{12}$, use 0.9 of the effective depth given in Diagram 11.

DIAGRAM 11



ILLUSTRATIVE PROBLEM.—Given $f = 2$, $w' = 50$, $h' = 36$ ft. To find thickness of face wall. Counterforts are spaced 7 ft. 6 in. c. to c. Use $f_s = 15{,}000$, $f_c = 650$, whence $K = 110$. Enter Diagram 11 with $(fw'h') = 3600$, thence to right to $K = 110$; thence vertically to $s = 7.5$ ft.; thence to right where d is given as 13.7 in. Total thickness, say, 15 in.

8c. Back Floor Slab.—This is the portion of the base slab to the rear of the vertical wall. It must be designed to resist the upward reaction of the soil against that portion of the base; and the weight of the earth immediately above it, together with the vertical component of the inclined earth thrust if the surcharge is sloped. Fig. 20 shows a counterforted wall with the earth sloped to the line AD. The total pressure on the plane DC is P, whose effect above the base slab is represented by the triangle of pressure Djh. The portion Ddg (with resultant P_1) is transferred to Ab, as Abc. The portion $dghj$ (resultant P_2) is transferred to bj as $bcjh$. Vertical components of bc and jh, at b and j, would determine the downward component pressure of P_2. This is the same as was given for the cantilever wall, Fig. 17.

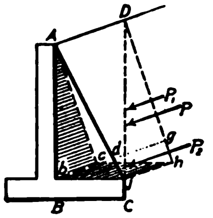


Fig. 20.

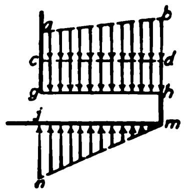


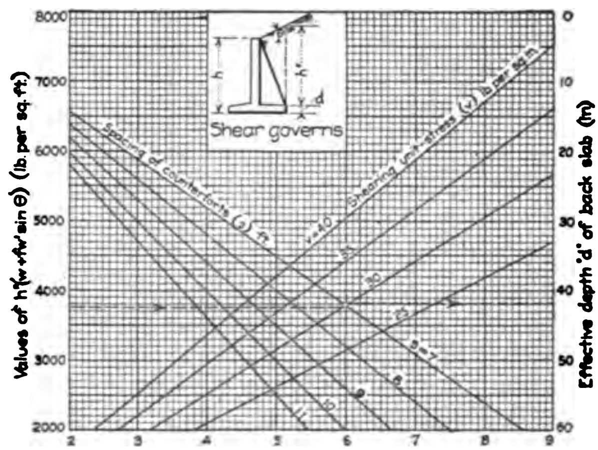
Fig. 21.

The various pressures acting are shown in Fig. 21. Trapezoid $abhg$ represents the vertical component $acdb$ of P_2, plus the pressure $cghd$ of the earth immediately above gh ($ADjb$, Fig. 20). The area jmn represents the upward pressure of the soil.

When the surcharge is level, $abdc = 0$.

The whole floor slab may now be designed for either shear at the counterforts, or moments in the slab treated as continuous over the counterforts. In either case the pressure on a strip at the back edge hm would determine the depth of the back slab. If the slab is not reinforced

Diagram 12



adequately against shear and diagonal tension, by the addition of bent-up bars or stirrups, then the allowable shearing unit stress will be low, and shear will govern the depth of the slab. In this case, the effective depth of slab in inches is given by the relation

$$d = \frac{sh''}{21v}(w + fw' \sin \theta)$$

in which h'' is the depth in feet of the earth above the slab at the heel. Diagram 12 is based on this formula.

ILLUSTRATIVE PROBLEM.—Given: Stem of wall 20 ft. high, surcharge slope, θ = 30 deg.; angle of internal friction, ϕ = 40; f = 1.75; length of base of counterfort = 7.5 ft.; spacing of counterforts, center to center = 7 ft.; weight of earth, w = 120 lb. per cu. ft. Find: depth of back floor slab when v = 30 lb. per sq in.

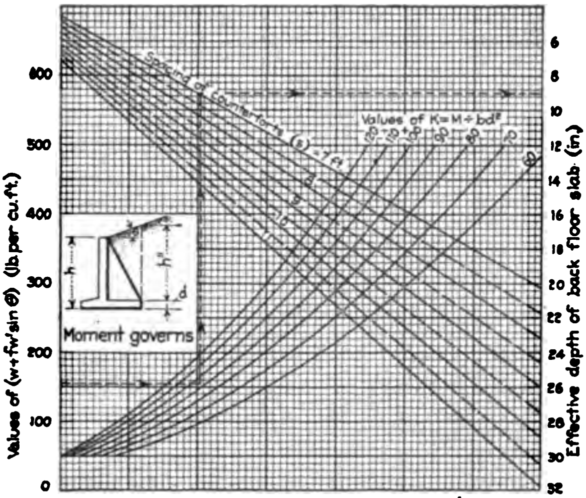
$$h'' = 20 + (7.5)(\tan 30 \text{ deg.}) = 24.4 \text{ ft.}$$

From Diagram 1, C_e = 0.316; hence $w' = C_e w$ = 38 lb. per cu. ft.

$$(w + fw' \sin \theta) = 120 + (1.75)(38)(0.5) = 153.3 \text{ lb. per cu. ft.}$$

$h''(w + fw' \sin \theta)$ = (24.4)(153.3) = 3745 lb. per sq. ft. This is the downward pressure per square foot at the top of the back edge of the slab. Entering Diagram 12 from the left margin with this value, we move toward the right to v = 30; then vertically to s = 7; then out to the right margin, where we find d = 41.9 in., say 44 in. total depth, allowing proper covering for the steel at the lower face. This is the total thickness required for shearing forces. Investigation must of course be made of this for unit stresses in moment and bond.

DIAGRAM 13



When stirrups or bent-up bars are placed in the slab to adequately resist the shearing forces, then the bending moments will govern the thickness of the slab, for walls up to at least 40 ft. high, since the allowable shearing unit stress will be high. Under these conditions, since the base slab, like the vertical wall, is continuous over supports,

$$Kbd^2 = 1.2(w + fw' \sin \theta)s^2$$

Whence

$$d = 0.32s\sqrt{\frac{(w + fw' \sin \theta)}{K}}$$

in which d is the effective depth in inches. The effective depth may be taken directly from Diagram 13, which is based upon the above formula. The use of the diagram is essentially the same as the use of Diagram 11.

If it is desired to design for a moment of $(w + fw' \sin \theta)\frac{s^2}{12}$, use 0.9 of the effective depth given in Diagram 13.

ILLUSTRATIVE PROBLEM.—Given same wall as above, but with v = 120 lb. per sq. in. In this case the thickness will be governed by the moment in the slab. Let K = 95. Entering Diagram 13 with $(w + fw' \sin \theta)$ = 153.3, proceed to the right to K = 95; then vertically to s = 7 ft.; then to the right margin where d = 9 in. Use 11 in. total thickness.

Some designers favor placing a beam along the back edge of the back floor slab, to stiffen it. The design of this beam is arbitrary when a panel of the back floor is either square, or as is usually the case, has its shortest span between counterforts.

The methods of reinforcing the back slab are given in Art. 8e with a discussion of the reinforcement of the base slab as a whole.

8d. Cantilever Toe Slab.—When the resultant pressure R strikes the base at the forward third point, the upward pressure on the base is triangular, and the pressure per square foot at the toe A, Fig. 22, is equal to twice the vertical component of R divided by the area of the base over which R acts, expressed in square feet. Thus the total shear on the section CD, neglecting the small downward weight of AD, is equal to $R_v(2 - k)k$, for a length of 1 ft. of wall (perpendicular to the paper). From Art. 13, Sect. 7, $v = \frac{V}{bjd}$, whence, assuming $j = \frac{7}{8}$,

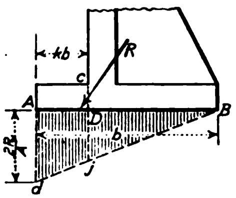


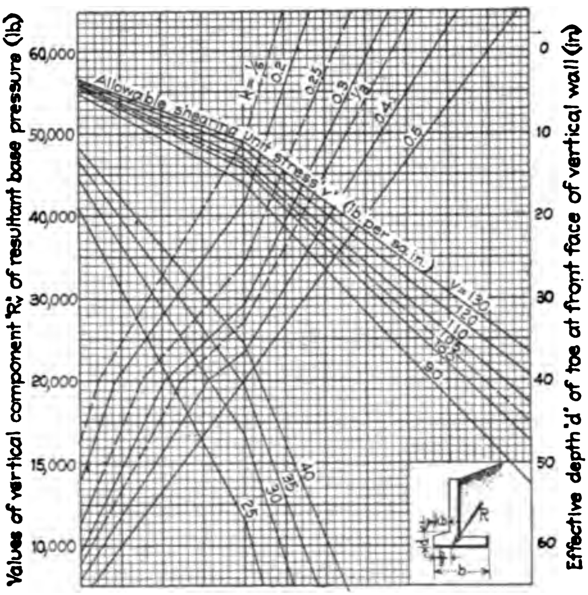
FIG. 22.

$$d = 0.095\frac{R_v(2 - k)k}{V}$$

in which d is the effective depth of the toe slab in inches, at the section CD. It may be tapered toward A. Diagram 14 has been prepared to give this depth from the above formula.

ILLUSTRATIVE PROBLEM.—Given R_v = 18,000 lb.; k = 0.3; v = 35 lb. per sq. in. Find d. Enter Diagram 14 at the left with R_v = 18,000; then on a horizontal line to k = 0.3; then vertically to v = 35; and lastly, to the right margin where d = 25 in. This must be compared to the value obtained from the moment requirement.

DIAGRAM 14



8e. Methods of Reinforcing Counterforted Wall.—In the preceding discussion only the critical sections of the main units of the wall have been dealt with. It should be noted that those same sections should be investigated for diagonal tension and for bond, as in the design of continuous slabs. Important considerations in the arrangement of the reinforcement will be discussed for each part of the wall.

Vertical Slab.—The reinforcing bars in the vertical wall should be at the outer face at the center of the span, and at the back face at the counterforts. All of the reinforcement needed at the front at the center of the span is needed at the back face at the counterfort, since the moment at the support should be taken as that at the center. It is not advisable, however, to bend all of the rods to the back face at the supports, but to have about one-fourth of them run through. This amount of steel should be supplied at the back face by short rods running about $\frac{1}{3}$ s each way from the counterfort, or further if necessary for bond.

About 0.3 of 1% of steel should be placed vertically in the wall, and continued into the base slab. This serves two purposes: it provides a support for the horizontal steel before pouring; and it resists the tendency to form large horizontal cracks, particularly near the bottom of the face wall.

Base Slab.—The toe, after having been proportioned for shearing forces, must be reinforced for bending moment at the section CD, Fig. 22. Rods should be placed near the bottom face of the toe and anchored at its outer end. Part of these rods should extend the entire width of the base, near its lower face; and part of them may well be bent up, with a long radius, to the back face of the vertical wall, extending upward from the base a distance of about $\frac{1}{2}$ s, or more. About 0.3 of 1% of steel should run parallel to the face wall, near the bottom of the toe.

The principal reinforcement at the back slab runs parallel to the face wall. The rods at the center of the span s are at the bottom of the slab, and at the supports (counterforts) near the top. The points of bending the rods should be between the quarter and third points of the span, as determined from Fig. 26, Sect. 7, page 298. Not all of the rods should be bent up at these points, but some should run straight through near the bottom face. The extra amount required at the top face at the support may be made up by rods placed near the top face and extending $\frac{1}{3}$ s either way from the counterfort, or further if necessary for moment or bond. Stirrups, if placed near the counterforts, should be placed in accordance with the shear reinforcement in continuous beams or slabs.

When the vertical wall is in front of the center of the base, longitudinal steel is likely to be required in the bottom face of the back slab between the center of the base and the vertical wall, as in this portion the upward pressures may be greater than the downward pressures. This is especially notable when the face wall is at the front edge of the base slab (see Plate IV).

The Counterfort.—The reinforcement of the counterfort must take the tension developed in the direction AC, Fig. 23; the tension across BC due to the overturning moment of the stem AB; the tension across AB due to the tendency of the face slab to move forward away from the counterforts; the shear on BC due to the earth thrust on AB; and the shear on AB due to the resultant downward pressure on BC.

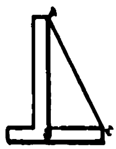


Fig. 23.

The counterfort is designed as a wedge-shaped cantilever beam fixed to the base slab. Such an assumption requires that the base slab be rigidly attached to the bottom of the counterfort over its entire length. The principal reinforcing in the counterforts is the steel along the inclined face tying the upper end of the face wall to the back edge of the base. Both ends of this steel must be thoroughly anchored. This may be done by hooking, or by hooking to cross rods, provided that in the latter case the bearing of the cross rods does not exceed the compressive working unit stress of the concrete.

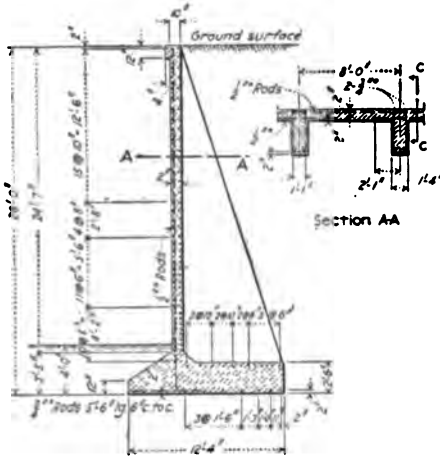
Illustrative Problem.—Given a counterfort 20 ft. high, 16 in. thick, and 8 ft. wide at the base slab; required the tensile steel necessary to prevent exceeding the unit stresses $f_c = 650$, and $f_s = 16{,}000$ lb. per sq. in. Moment on the counterfort = 7,000,000 in.-lb. Allowing 0.2 ft. for steel covering,

$$K = \frac{M}{bd^2} = \frac{7{,}000{,}000}{(16)(7.8)^2(12)^2} = 50$$

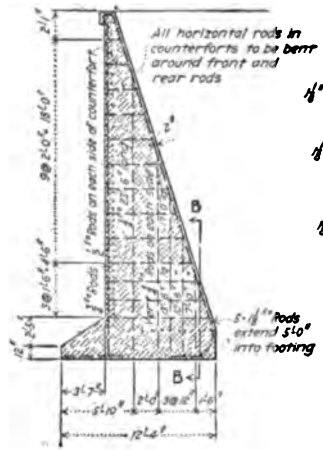
Entering Diagram 2, Sect. 7, page 360, at 50, for $f_s = 16{,}000$, $p = 0.0035$. Slope of back face = tan $^{-1}$0.4 = about 22 deg. Entering Diagram 3, page 361, at the lower margin with 3.5, trace vertically upward to $\beta_c = 0$, then horizontally to $\beta_t = 22$ deg., then vertically upward to values of C, where 3.85 is found. Pointing this off properly, the corrected value of $p = 0.00385$.

$$A_s = (16)(7.8)^2(12)^2(0.00385) = 5.4 \text{ sq. in.}$$

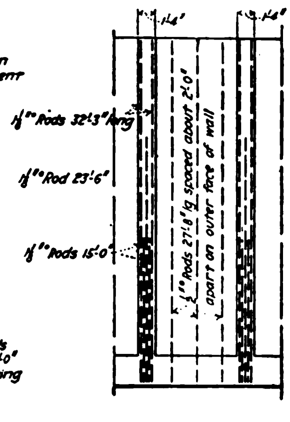
PLATE III



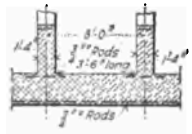
Section C-C between Counterforts



Section through Counterforts



Rear Elevation



Section B-B

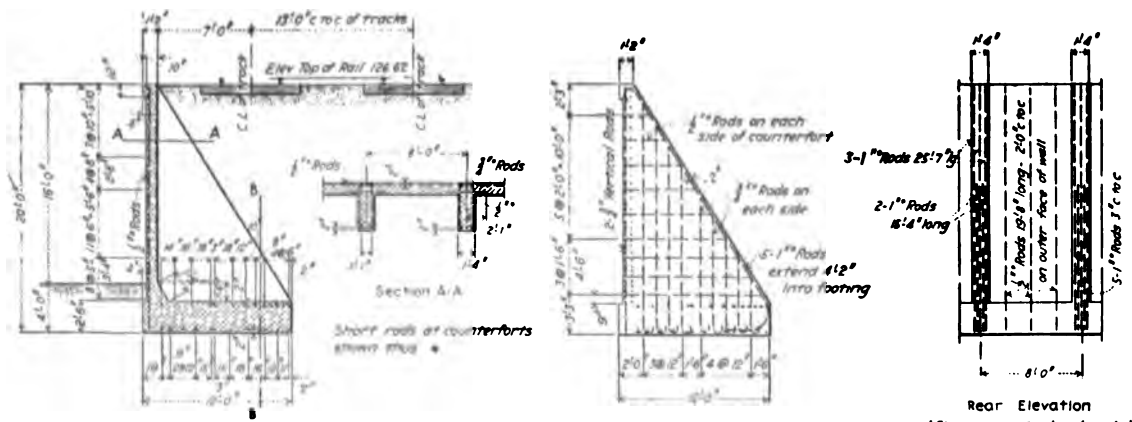
Steel for One Bay and Counterfort

Footing	7	$\frac{1}{2}$"	8'-0"	56 Lin. ft.	84 lb.
	16		5'-6"	88 "	132 "
	12		3'-6"	42 "	63 "
Wall, Horizontal	41	$\frac{5}{8}$"	8'-0"	328 "	220 "
	41		4'-2"	171 "	115 "
Vertical	2	$\frac{3}{4}$"	27'-8"	55 "	83 "
	3	$\frac{1}{2}$"	27'-8"	83 "	55 "
Counterfort	2	$1\frac{1}{8}$"	32'-3"	65 "	220 "
	1		23'-6"	24 "	81 "
	2		15'-0"	30 "	101 "
Vertical	2	$\frac{3}{4}$"	25'-6"	47 "	71 "
	2		17'-0"	34 "	51 "
	2		13'-6"	27 "	41 "
	2		10'-6"	21 "	32 "
	2		7'-0"	14 "	21 "
					1370 "

Counterfort	2	$\frac{1}{2}$"	1'-9"	4	1370
	2		2'-6"	5	
	2		3'-0"	6	
	2		3'-6"	7	
	2		4'-2"	8	
	2		5'-0"	10	141 ft. 94 lb
Horizontal rods	2		5'-6"	11	
	2		6'-0"	12	
	2		6'-9"	14	
	2		7'-3"	15	
	2		7'-9"	16	
	2		8'-3"	16	
	2		8'-6"	17	
					Total 1470 lb

Concrete 1:2½:5 mix
Deformed bars. Use No. 16 Wire (Steel Wire Gauge) for tying

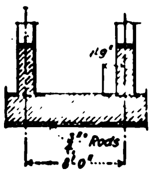
COUNTERFORT RETAINING WALL
SCALE $\frac{1}{8}$" = 1 FOOT OCT. 16, 1912
DESIGNED BY TRACED BY
CHECKED BY APPROVED



Section between Counterforts

Section through Counterforts

Rear Elevation
(Showing vertical rods only)



Section B-B

Steel for One Bay and Counterfort

Footing	10	3/4"	8'0"	80 lin ft.	120
	14		3'6"	49	74
Wall. Hor	31	1/2"	8'0"	248	165
	31		4'2"	129	87
Vertical	2	3/4"	19'8"	39	69
	3	1/2"	19'8"	59	39
Counterfort	3	1"	25'7"	77	206
	2		15'4"	31	83
Vertical	2	3/4"	19'0"	38	57
	2		17'0"	34	51
	2		15'6"	31	46
	2		14'0"	28	42
	2		11'6"	23	31
	2		10'0"	20	30
	2		8'3"	17	26
	2		7'0"	14	21
	2		5'0"	10	15
					1152

					1152
Counterfort, Hor	2	1/2"	2'6"	5	
	2		3'9"	8	
	2		5'0"	10	
	2		6'3"	12	
	2		7'6"	15	87
	2		8'6"	17	
	2		9'6"	19	
	2		10'6"	21	
	2		11'6"	23	
			Total		1240 lb

Notes: Concrete 1:2½:5 mix
Deformed bars. Use
No. 16 Wire (Steel Wire Gauge)
for tying

COUNTERFORT
RETAINING WALL
SCALE DATE
DESIGNED BY TRACED BY
CHECKED BY APPROVED

It will be found by considering various horizontal sections of the counterfort that it will not be necessary to run all of this steel to the full height of the counterfort. Wherever any steel is stopped because it is no longer needed for moment, it should be extended sufficiently to develop the full strength by bonding, or should be adequately anchored.

Horizontal steel should extend from the counterfort into the front face to keep the latter from being torn fron the counterfort. This steel should be hooked to the horizontal rods in the slab, or should be bent either way into the slab to thoroughly attach it to the vertical slab.

Similar steel should be extended and anchored into the footing slab.

If the cross-section of the base of the counterfort is not large enough to carry in shear the thrust against the wall, the thickness of the counterfort should be increased, or stirrups should be extended from the base slab into the counterfort.

Typical designs are shown in Plates III and IV.

9. Special Types of Reinforced-concrete Walls.—Fig. 24 shows the cellular type of reinforced-concrete retaining wall. This wall is formed of two longitudinal curtain walls (*A*) and (*B*), connected by transverse walls (*C*). The vertical space between the parallel longitudi-

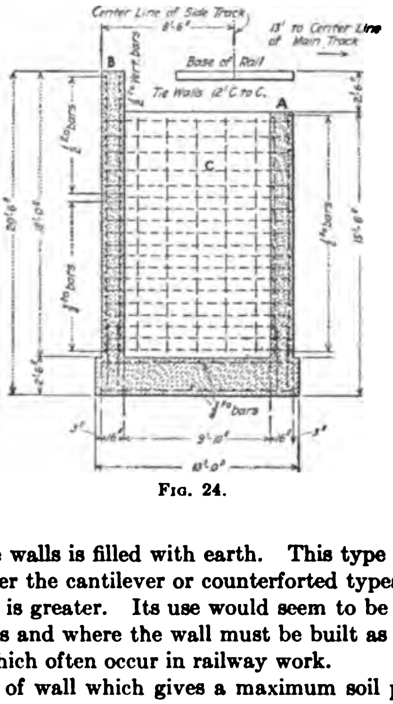


Fig. 24.

nal walls and the transverse walls is filled with earth. This type of wall gives a lower maximum soil pressure than either the cantilever or counterforted types, but under average conditions its cost per linear foot is greater. Its use would seem to be restricted to poor soil with no opportunity to drive piles and where the wall must be built as close as possible to a given property line—conditions which often occur in railway work.

Fig. 25 shows a design of wall which gives a maximum soil pressure even less than the cellular type and which costs approximately the same per linear foot of wall. This design was employed by the C. M. & St. P. Ry. in a long retaining wall at Elgin, Ill., and was chosen in preference to the design shown in Fig. 24 on account of the much lower bearing pressure on the soil. The wall consists of a footing (*Y*) which supports a longitudinal wall (*T*) and the cross walls (*U*). The cross walls at the outer end support the girder (*X*), which spans from one cross wall to another. The slabs (*V*) are supported at one end by the girder and at the other end by the longitudinal wall. The great reduction in soil pressure is brought about mainly by eliminating the weight of the earth directly above the footings, the space between the cross walls (*U*) being an open and empty space in this design. In this type of wall, and in the cellular type as well, stability against sliding should be carefully investigated.

10. Construction of Retaining Walls.

10a. Backfilling and Drainage.—Quite as much attention should be paid to the earth filling and to its drainage as to the design and construction of the retaining wall proper or to the matter of providing a suitable foundation. If the earth is deposited in layers inclined from the wall, the pressure will be small compared to that resulting if the layers are sloped toward the wall. It is quite often the case that by depositing the backfilling so as not to

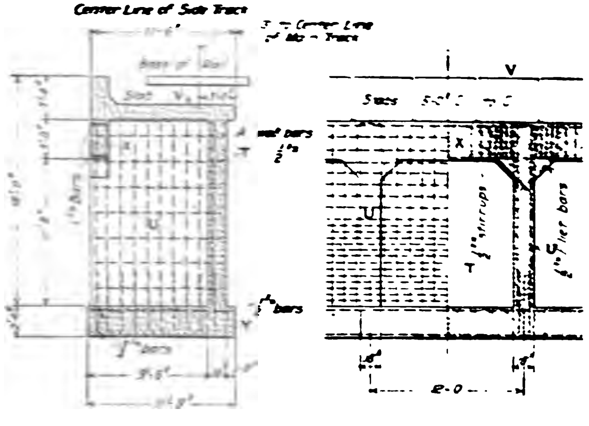
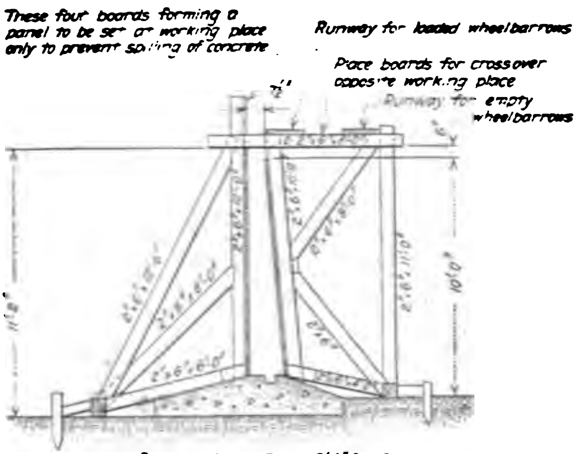


Fig. 25.



Bracing of Wall Forms 2'-6" C. to C.

Fig. 26

slide against the wall, a light wall may be made to stand where, under the same conditions, if the earth is dumped so as to slide against the wall, even a heavy wall will fail.

When placing a backfill near a steep undisturbed slope of earth or rock, care should be taken that the earth does not arch, or does not form a wedge. If the proper precautions are not taken, a heavy lateral thrust will occur near the top of wall.

Water behind a wall is a frequent cause of failure. It adds to the weight of the backing and softens the material so that the lateral thrust is increased. Also, undrained backfilling will freeze and create lateral thrust due to the consequent expansion. To drain the backing, *weepers or weep holes* should be left through the wall just above the footing. Tile, 3 or diameter is generally used and, in the North Central States, placed usually not more than 10 to 15 ft. apart. The tile should be connected with a longitudinal drain in front of the wall. If the backing is retentive of water, a vertical layer of broken stone, coarse gravel, or cinders should be placed next to the wall to act as a drain. The filling in front of the wall should also be carefully drained.

10b. Forms.—Formwork as applied to buildings is treated in detail in Sect. 2. Since the method of constructing forms and the directions for their removal are very much the same for different types of structures, very little need be said under this heading.

The lumber used for forms should have a nominal thickness of at least 1½ in. before surfacing and should be of a good quality of Douglas fir or Southern long-leaf yellow pine.

FIG. 27.

The lumber for face work should be dressed on one side and on both edges to a uniform thickness and width. The lumber for backing and other rough work may be unsurfaced and of an inferior grade of the kinds above mentioned.

Forms should be substantial and unyielding, and built so that the concrete will conform to the dimensions shown on the designer's plans, and they should also be tight so as to prevent the leakage of mortar. Forms may be either continuous or sectional, or a combination of both, depending upon the economy of the work. The concrete in any given section should be allowed to harden for 36 hr. before the forms are removed and, in freezing weather, extra care must be taken to make sure that the concrete has had sufficient time to become thoroughly set. Material once used for forms should be cleaned before being used again.

A design of a form for a reinforced-concrete cantilever wall is shown in Fig. 26.

Fig. 27 shows the method of constructing and reinforcing the counterforts of a retaining ıt Buffalo, N. Y.

SECTION 14

SLAB AND GIRDER BRIDGES

The methods of designing slabs, beams, and girders are explained at length in Sect. 8, and no attempt is made in this section to treat of these methods in detail. Many things must be considered, however, in designing bridges of this class, aside from the proportioning of simple and continuous beams, and such matters are given due consideration.

The erection of forms and other operations in slab-and-girder bridge construction are essentially the same for ordinary conditions as the corresponding operations in the construction of buildings. On this account, constructional methods are referred to only incidentally under this heading.

The loadings to use in design are, of course, the same as for the floors in arch bridges of open-spandrel construction (see Art. 6, Sect. 16). In fact, it should be noted that the framed structure which is supported by a ribbed arch is virtually a trestle form of girder bridge and the loadings and general design are identical.

Impact may properly be neglected in arch-ring analysis but becomes important in bridge-floor design. An increase of 25% is usually made in the live load or live-load stresses for highway bridges and 50% in those for railroad bridges.

From the standpoint of economy, slab bridges should in general be limited in span length to about 25 ft, and ordinary girder bridges to about 50 ft.

SLAB BRIDGES

1. Slabs Under Concentrated Loading.

1a. Illinois Tests.—From tests made on simply supported slabs at the University of Illinois, Prof. W. A. Slater has recommended that where the total width of slab is greater than twice the span, the effective width e (Fig. 1) be assumed as

$$e = \tfrac{4}{3}x + d$$

where x is the distance from the concentrated load to the nearest support and d is the width at right angles to the span over which the load is applied.

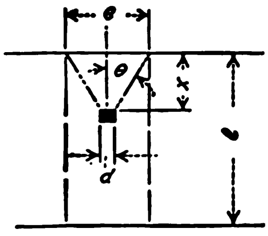


FIG. 1.

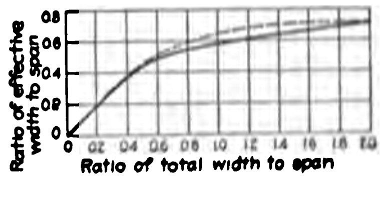


FIG. 2.

Tests showed the effective width to be but little influenced by the depth of the slab or by the percentage of longitudinal reinforcement. Prof. Slater has recommended, however, that the latter be limited to 1% "because of the possibility that in a beam with a large amount of longitudinal reinforcement and a relatively small depth, failure may be caused by transverse tension in the concrete and not by longitudinal steel stress."

The above formula refers to a total width of slab greater than twice the span. For a slab whose total width is less than this, the effective width may be found from Fig. 2 (full line) which shows the ratio of the effective width of the span as determined from the measured steel stresses in the University of Illinois tests.

1b. Ohio Tests.[1]—The object of the tests was to obtain, if possible, a sufficient knowledge of the distribution of loads through and by concrete floor slabs to enable the designer to rationally proportion the joists of a slab floor, and also the slab itself, to carry concentrated loads.

The following conclusions regarding the distribution of concentrated loads on a reinforced concrete slab, to the floor joists, seem to be warranted by these tests:

1. The percentage of reinforcement has little or no effect upon the load distribution to the joists, so long as safe loads on the slab are not exceeded.
2. The amount of load distributed by the slab to other joists than the one immediately under the load, increases with the thickness of the slab.
3. The outside joists should be designed for the same total live load as the intermediate joists.
4. The axle load of a truck may be considered as distributed uniformly over 12 ft. in width of roadway

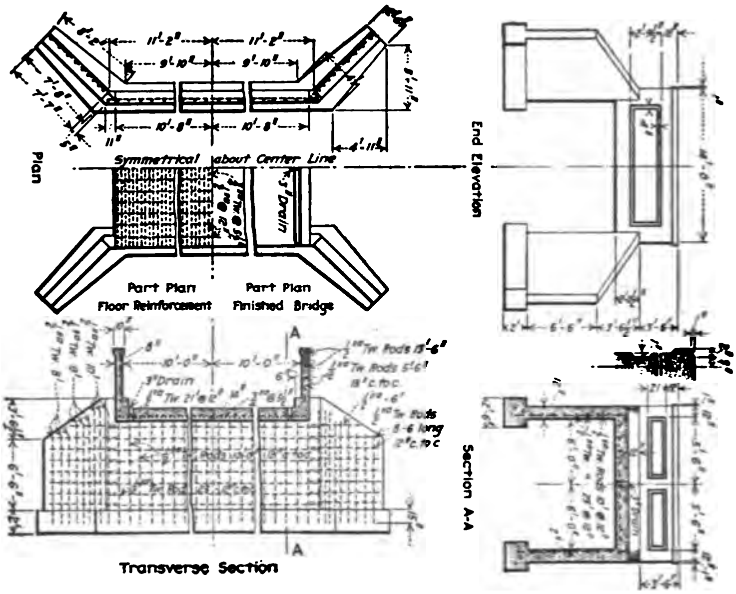


FIG. 3.—Standard design for slab bridges of 12-ft. span, Wisconsin Highway Commission.

In a slab of a certain span and indefinite width, there is some width symmetrical with the load, beyond which a single concentrated load will have no effect. The stresses in this slab will be a maximum under the load and will decrease in each direction from it.

The "effective width" of a slab is that width used in designing over which a single concentrated load may be considered as uniformly distributed on a line down the middle of the slab parallel to the supports.

The tests of slabs seem to warrant the following conclusions:

1. The "effective width" is effected very little by the percentage of transverse reinforcement (parallel to supports).
2. The "effective width" decreases somewhat as the load increases.
3. The "effective width" in percentage of the span, decreases as the span increases
4. The following formula will give a safe value of "effective width" where the total width of slab is greater than $1\frac{1}{2}l + 4$ ft.

$$e = 0.6l + 1.7 \text{ ft.}$$

where e is effective width in feet and l is the span in feet.

[1] From *Bull.* 28, State of Ohio, Highway Department

1c. Tests by Goldbeck.—The results of these tests are shown by the dotted curve in Fig. 2.

2. Slab Bridges of Single Span.—The floor of a slab bridge may be designed as any simply supported slab except that shearing stresses may require very careful attention where the live

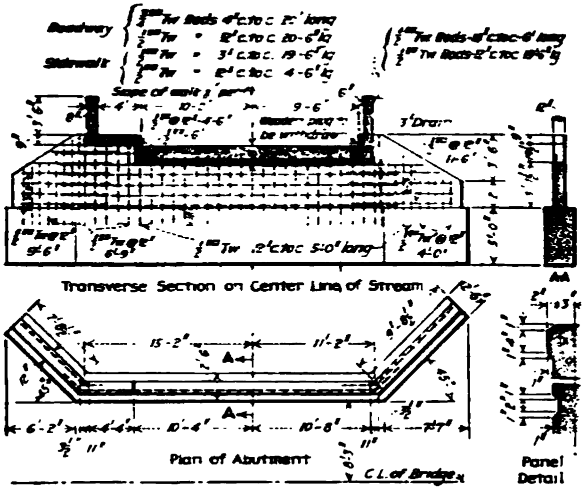


FIG. 4.—Details of Christiana bridge, Town of Cross Plains, Dane County, Wis.

load is relatively large, as in railroad structures. The whole reinforcing system may be made absolutely rigid by wiring the main reinforcing rods to the transverse spacing rods at the ends

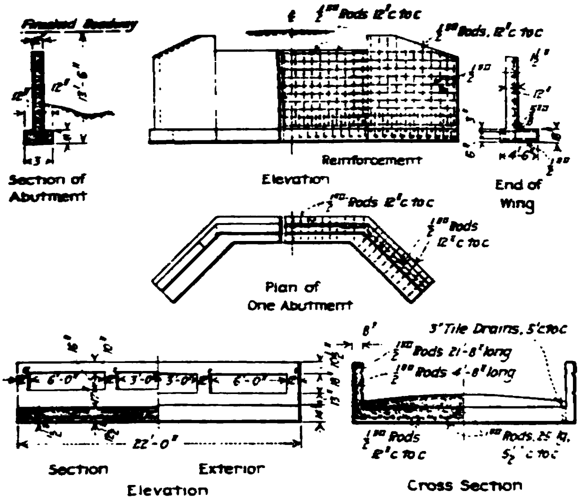


FIG. 5.—Typical slab bridge, Illinois Highway Commission. Abutment wings of the cantilever type. Main wall vertical slab supported at top by superstructure and at base by footing.

of the bent-up steel. Both the straight longitudinal rods and those bent up to provide for diagonal tension should be hooked at the ends.

Practice varies in regard to the use of expansion joints between the slab floor and the abutments. There are none provided in Figs. 3, 4 and 5, but such joints are placed at both abutments in Fig. 6. In the latter figure, vertical end expansion joints are provided at the angle points between abutment wings and slab, making it necessary to cantilever the outer portions of the slab width on account of insufficient abutment support.

In Figs. 3, 4 and 5 the main wall or vertical slab of the abutment is supported at the top by the floor and at the bottom by the footing, and may be figured for earth pressure as a simple slab with the main steel near the outer or stream face. Of course, the main wall should also be designed to act as a column to support the superstructure. Since a joint of low friction is

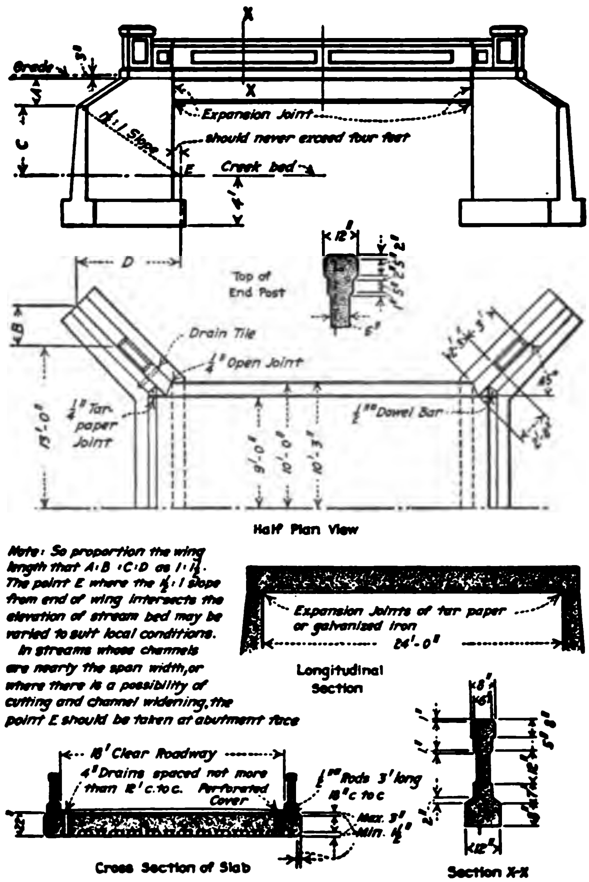


FIG. 6.—Typical slab superstructure, Iowa Highway Commission.

not provided between floor and abutments, the unit tensile stress in the steel of the superstructure is usually kept low (12,000 lb. per sq. in.) so as to provide properly for the additional tension in the steel caused by the contraction of the bridge in cold weather. The wings may be designed as self-supporting retaining walls of the cantilever type, using the methods explained in Sect. 13. Theoretically the maximum efficiency of the footing for the wing walls can be obtained by placing the wing wall at about the outer middle-third point of the base, but in many cases considerable saving in excavation may make it more desirable to shift the footing a little toward the stream bed. Counterforted walls are advisable only for abutments over 20 ft. in height.

Figs. 6 and 7 show an unusual abutment design adopted by the engineers of the Iowa Highway Commission for both slab and girder bridges. Expansion joints being provided at each end of the superstructure, both the main portion and the wings were designed as self-supporting retaining walls. The main portion, however, was not only analyzed in the ordinary manner for pressure on the base, but was also analyzed taking into account the stability due to the weight of the wings. The mean value of the maximum pressure at the toe of the foundation by the two methods being found safe, and an analysis for sliding and overturning being satisfactory, the dimensions shown were adopted. The horizontal rods designated as tension bars were inserted in order to utilize the weight above mentioned. These rods are placed in the main stem near the upper surface and extend continuously through the wings, with splices at the center of the main portion. The effect of considering the wings as a part of the

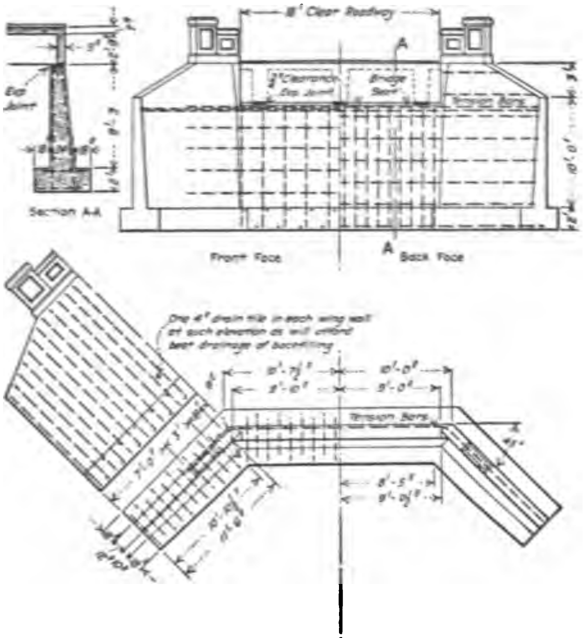


Fig. 7.—Typical substructure for slab and girder bridges, Iowa Highway Commission.

abutment body is to shift the center of gravity of the entire mass farther from the stream face and thus reduce the eccentricity of pressure on the foundation. The horizontal reinforcing rods shown near the stream face of the abutment, and which are carried about 5 ft. into the wings, were employed to counteract a tendency to the formation of vertical cracks on the outside at the corner of wing and abutment. The vertical rods in the front face serve as a framework upon which to build the horizontal rods and they also prevent any tendency toward the formation of horizontal cracks in the stream face due to the clogging of an expansion joint.

3. Slab Bridges of Multiple Spans.—Slab bridges of multiple spans will be treated under the four following headings:

Concrete pile trestles.	Trestles with framed bents.
Pier trestles.	Cantilever flat-slab construction.

3a. Concrete Pile Trestles.—Figs. 8 and 9 give the essential details of design of the pile trestles built by the Illinois Central Railroad. They can be considered typical of concrete pile trestles in general. These trestles replace similar wooden structures over swamps

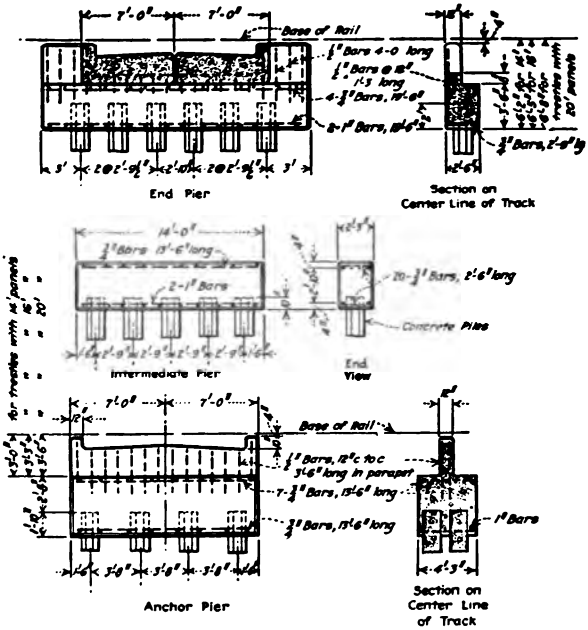


FIG. 8.—Details of substructure, standard concrete pile trestle, Illinois Central R. R.

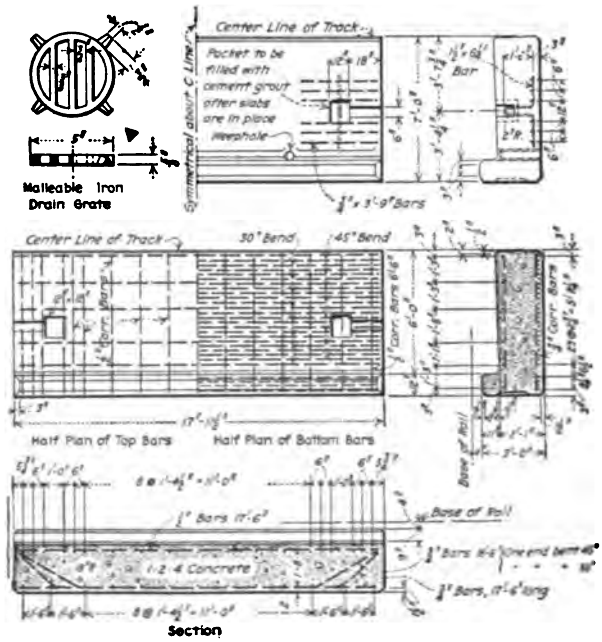


FIG. 9.—Standard slab for clear spans of 16 ft., Illinois Central R. R.

and shallow streams which may not be filled and where bridges on more permanent supports would be extremely expensive because of their great length. The construction consists of pile bents spaced generally from 16 to 20 ft. c. to c., and with a height above ground not greater than the span. The piles are capped with reinforced-concrete girders which support the floor slabs.

The piles and deck slabs are usually cast in a convenient yard, allowed to season from 60 to 90 days, and are then hauled to the bridge site. The lifting stirrups shown permit of the slabs being set in place by a wrecking crane. The ballast and track are laid directly on the slabs after the longitudinal and transverse joints (except at anchor bents) are filled with cement

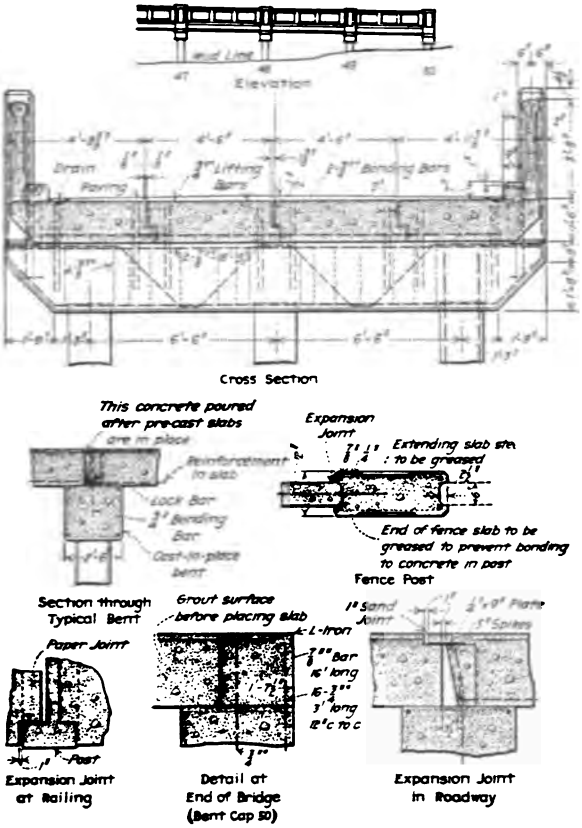


FIG. 10.—Details of pile trestle across the Miles River near Easton, Md.

mortar and after the floor surface is thoroughly waterproofed. The slabs are set on a bed of grout on the pile caps. An anchor bent is used at suitable intervals to take up longitudinal stresses due to tractive force and, by means of an expansion joint, to prevent any great accumulation of movement of the deck due to temperature changes.

A concrete pile trestle for carrying a highway is shown in Fig. 10[1]. It was found economical to cast the piles, deck slabs, and railing slabs at Baltimore, 60 miles away, and transport them to the site on scows. Expansion joints were located in the roadway slabs, curb, and railing slabs at every fifth bent.

[1] See also *Eng. News*, Feb. 5, 1914.

3b. Pier Trestles.—Thin concrete piers are preferable to pile bents when the height of bridge above the ground line is greater than about 16 ft. Fig. 11 shows a typical trestle of the solid bench-wall type built by the Illinois Central Railroad.

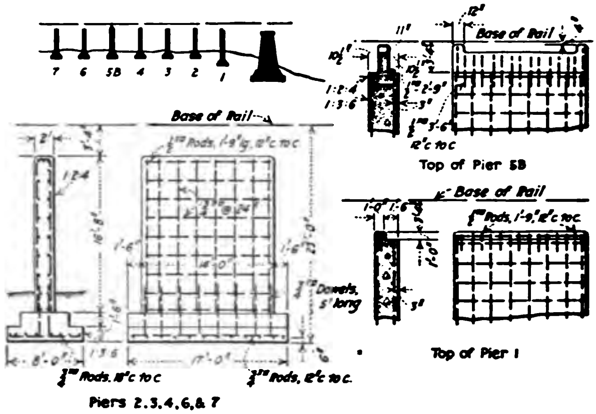


FIG. 11.—Pier details, Illinois Central R. R. trestle over Kaskaskia River near New Athens, Ill.

3c. Trestles with Framed Bents.—Slab bridges with framed bents forming subways are used on at least fifteen railroads in this country. A design which may be considered

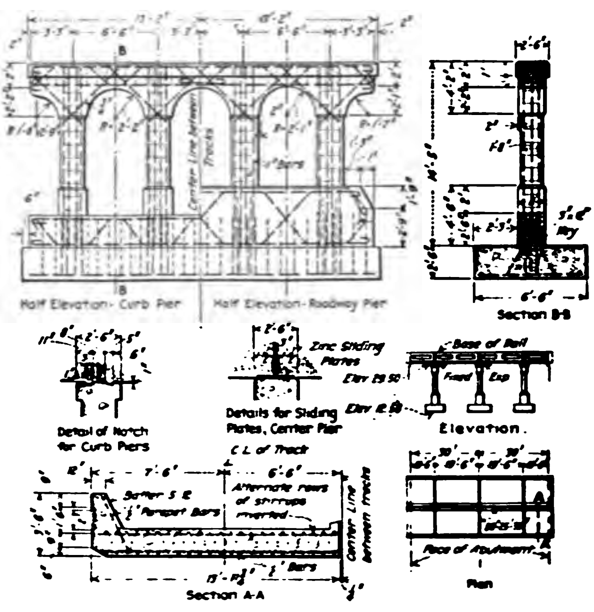


FIG. 12.—Details of Mozart Street subway, Bloomingdale Road track elevation, Chicago, Milwaukee & St. Paul Railway.

typical is shown in Fig. 12. The deck slabs may either be cast in place or cast at some central yard and placed in a similar manner to the slabs for pile or pier trestles. In Fig. 12 the design is shown for slabs to be cast in place..

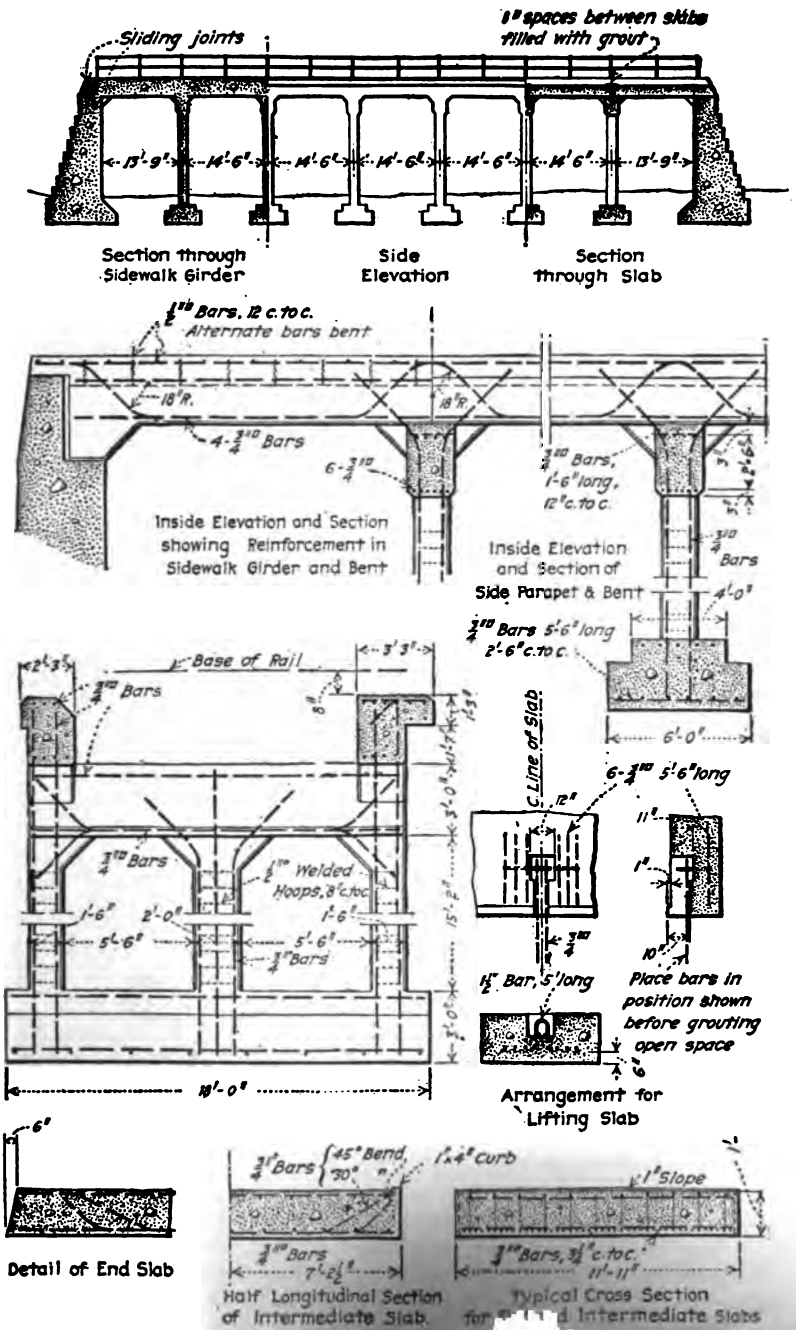


Fig. 13.—Details of trestle approach to Cumberland [illegible] Illinois Central R. R.

A framed-bent trestle with continuous side girders to resist stresses due to traction is shown in Fig. 13. The girder on one side acts simply as a tie and parapet, while the other with a cantilever projection at the side acts also as a sidewalk. The slabs were cast in a yard at some distance from the bridge site, loaded on flat cars, taken to the job, and swung into place with derricks. Expansion is allowed for at both ends by providing a sliding joint between the bridge superstructure and the abutments. The shallow 4-in. curbs at the ends of the slabs were provided to prevent seepage from getting into the 1-in. grout joint between slabs. The pier footings are reinforced longitudinally in top and bottom, and were figured as continuous T-

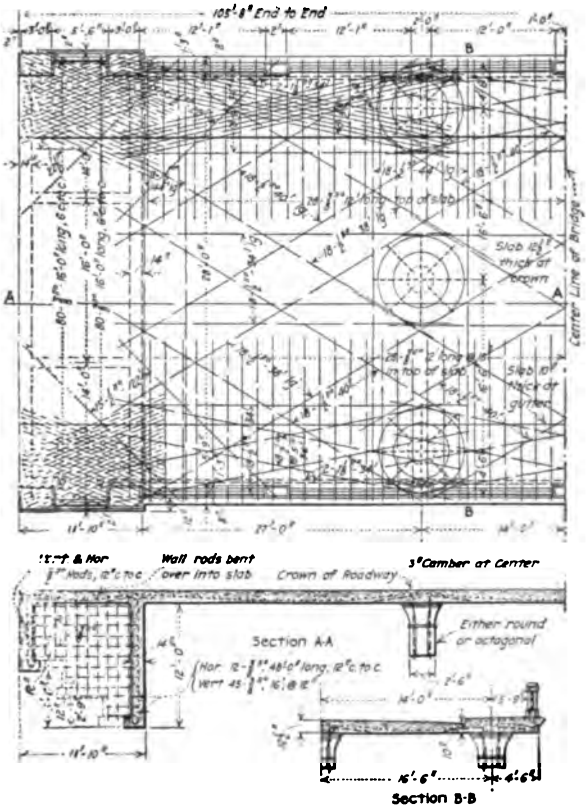


FIG. 14.—Cantilever flat-slab bridge on Mississippi River Boulevard, St. Paul, Minn.

beams uniformly loaded by the pressure on the soil. The cross girders at the top of the columns support the deck slabs previously referred to, and are made continuous.

3*d*. Cantilever Flat-slab Construction.—Fig. 14 is a flat-slab structure of the Turner Mushroom type. The methods which may be used in the design of the roadway slab are treated in Sect. 11. The hollow abutments should be noted.

Many cantilever flat-slab bridges are built in which the abutments are of the ordinary reinforced-concrete type. The abutment walls are considered as held at the top by the superstructure to which they are anchored by bending the vertical rods into the slab.

SIMPLE GIRDER BRIDGES

4. Deck Girders.—The deck-girder type of construction usually proves more economical than the through-girder type wherever sufficient headroom is available. The girders, of course, should be relatively thin and deep for the greatest economy, and a curtain wall should be provided between the girders at each end of span to retain the earth fill, thereby avoiding complicated parapet walls on the abutments.

Standard details of deck-girder bridges designed by the engineers of the Iowa Highway Commission are shown in Fig. 15. The floor slab was analyzed both as fixed and as continuous

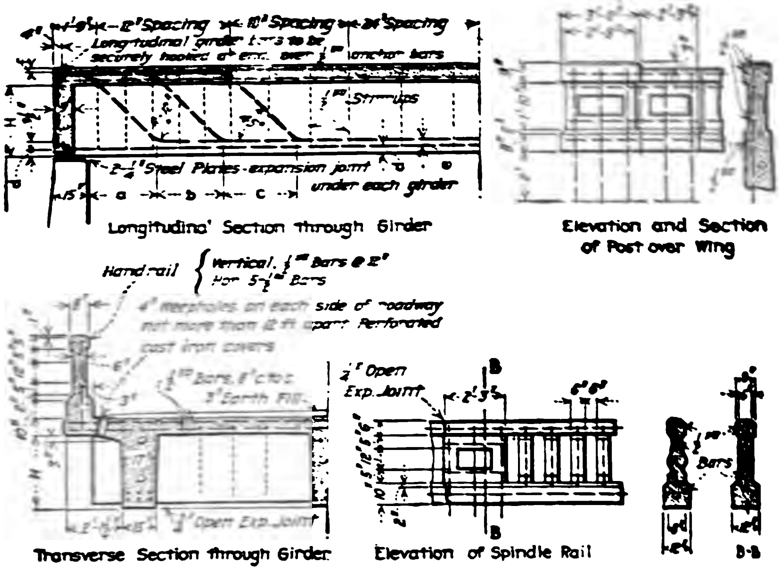


Table of Dimensions and Girder Reinforcement

Span	H	d	e	f	Stirrup Spaces 12"	Stirrup Spaces 18"	Stirrup Spaces 24"	a	b	c	Girder Bars Top Row	Girder Bars Bottom Row
24'	2'-2"	2½"	3"	2½"	10	6	2	2'-3"	2'-3"	2'-0"	3 - 1"	3 - 1"
26'	2'-4"	2½"	3"	2½"	12	6	2	2'-6"	2'-1"	2'-5"	3 - 1"	3 - 1"
28'	2'-6"	2½"	3"	2½"	12	6	3	2'-5"	2'-2"	2'-11"	3 - 1"	[illegible]
30'	2'-9"	3"	3½"	2½"	10	6	5	2'-8"	2'-6"	2'-10"	3 - 1"	[illegible]
32'	2'-10"	3"	3½"	3"	10	6	6	2'-7½"	2'-6½"	3'-1"	[illegible]	[illegible]
34'	3'-1"	3"	4"	3"	10	10	4	2'-11"	2'-10"	3'-3"	[illegible]	[illegible]
36'	3'-3"	3"	4"	3"	10	10	5	3'-0"	2'-11"	3'-10"	[illegible]	[illegible]
38'	3'-5"	3"	4"	3"	10	10	6	3'-4"	2'-10"	3'-4"	[illegible]	[illegible]
40'	3'-8"	3½"	4"	3½"	12	6	9	3'-6"	3'-2"	3'-10"	[illegible]	[illegible]

Fig. 15—Standard details of concrete deck-girder bridges, Iowa Highway Commission

and was designed to resist maximum stresses caused by either method of analysis. The method of fastening the girder steel to anchor rods should be noted. Expansion joints are provided under the girder stems by means of sliding steel plates anchored into the body of both superstructure and substructure.

Fig. 16 illustrates the type of deck-girder bridge adopted as standard by the Illinois Highway Commission. The following description of the methods employed in providing for expansion in girder bridges is given in the fourth report of the Commission:

Two methods of providing for expansion in girder bridges have been used and both have proved satisfactory In one method, the wing walls of one abutment are entirely separated from the abutment wall proper, the latter

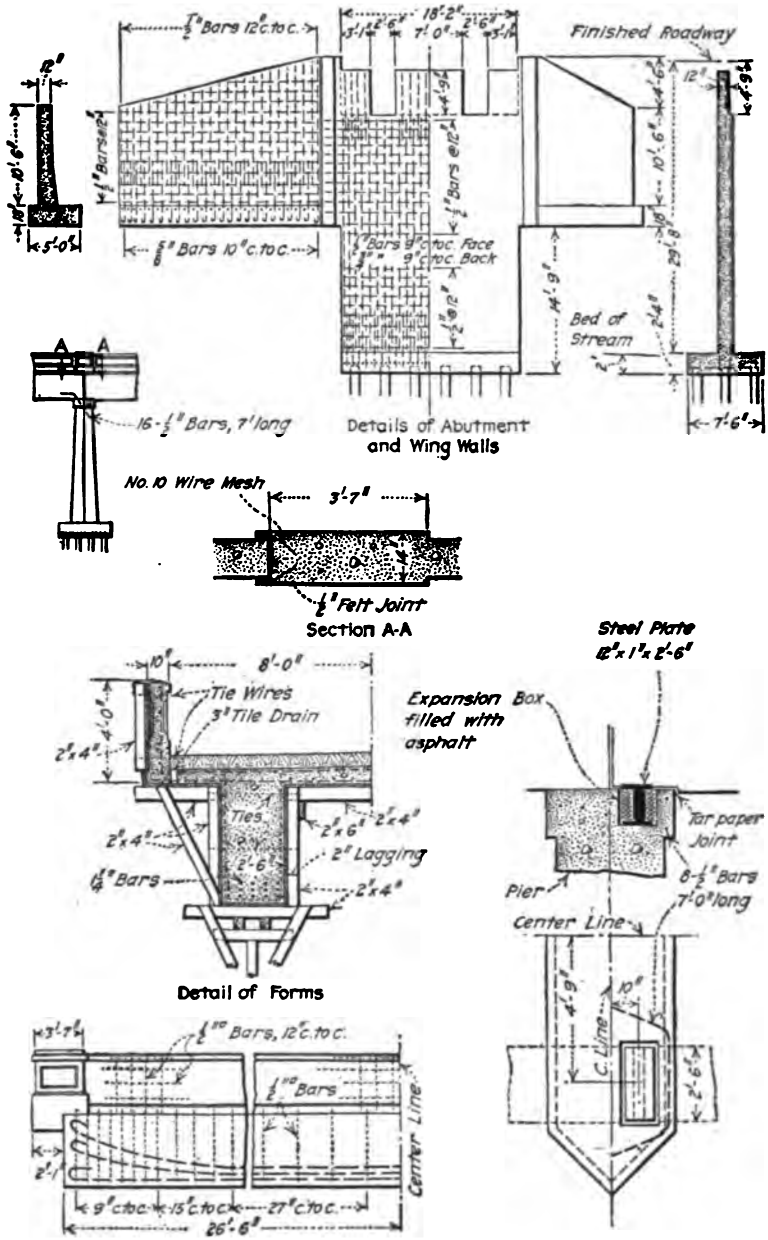


FIG. 16.—Details of Embarrass River bridge, Cumberland Co., Ill.

being free to move at the top with the expansion or contraction of the superstructure. The wing walls are designed to be self-supporting. As girder spans designed by the Commission have so far been limited to 60 ft., the amount of movement either way from the normal is small and is taken up by deflection of the main wall or a slight rocking of the wall on the footing. Earth pressure against the wall is of little importance in this connection as it but tends to reduce the tension in the girder steel during expansion and to cause the abutment wall to follow the superstructure during contraction. It does not increase the stress in the compression area of the girder as the load is applied at the bottom of the girder, tending by this eccentricity of application to reverse the dead- and live-load stresses in the girder.

This method has been found to be entirely successful, but is somewhat objectionable as a slight movement of the wings due to earth pressure and unequal settlement sometimes causes the wing walls to move forward slightly at the top, making a somewhat unsightly offset between the wing and abutment walls. This has never been more than 2 or 3 in. for the highest walls, but as it is not understood by the ordinary observer, an impression of weakness is sometimes caused.

The present method of providing for expansion is to design the abutments and wings in the ordinary way, separating the superstructure completely from one of the abutments by a thick paper joint and supporting each girder at the free end on a single cast-iron rocker of large diameter. The reaction is transmitted to the girder and abutment from the rocker through planed structural-steel plates stiffened with I-beams when necessary. The rocker surfaces in contact with the bearing plates are turned to insure perfect bearing on the plates. The diameter of the rocker is made proportional to the load imposed per linear inch, in the same manner as is commonly used in proportioning roller nests for steel bridges. The upper and lower plates are bedded in the concrete of the superstructure and abutment. The rocker is located in a pocket built in the abutment. This pocket is filled with a soft asphalt to prevent the entrance of water or dirt and to protect the metal from corrosion.

The rocker method of providing for expansion has proved very satisfactory and is but little more expensive than the other method, especially when it is considered that the wings may be tied to the main wall when rockers are used and advantage taken of the mutual support thus obtained.

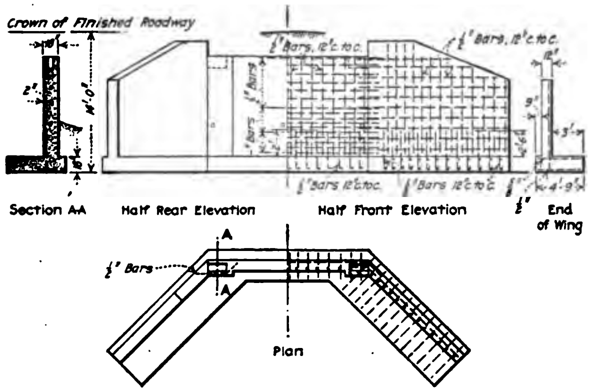


FIG. 17.—Type of abutment used for girder bridges by the Illinois Highway Commission. Wings of cantilever type. Main wall supported by wings as counterforts.

Fig. 17 shows a type of abutment adopted by the Illinois Highway Commission in cases where the girders are supported on cast-iron rockers and the wings are nearly parallel to the roadway or make an angle of more than 45 deg. with the face of the abutment. The wing walls are considered to act as counterforts and the reinforcing steel in the main walls is horizontal and placed near the stream face of the wall.

Fig. 18 gives the details of a girder bridge, the main portion of the abutments and the wings of which were designed and figured in the same manner as the slab bridges of Figs. 3, 4 and 5.

In the structure shown in Figs. 19 to 23 inclusive, cross girders and stringers were provided in addition to the longitudinal girders, this type of floor system being found to be economical for wide bridges of long span. One end of each span is anchored to the pier and the other end is allowed to expand and contract in a joint packed all around with ½ in. of tar paper and bearing on a pair of milled steel plates. Expansion joints were also made in the roadway slab and railing. Cast-iron scuppers were placed in each curb on 25-ft. centers. For maximum stresses in the cross girders, the sidewalks were assumed to be unloaded.

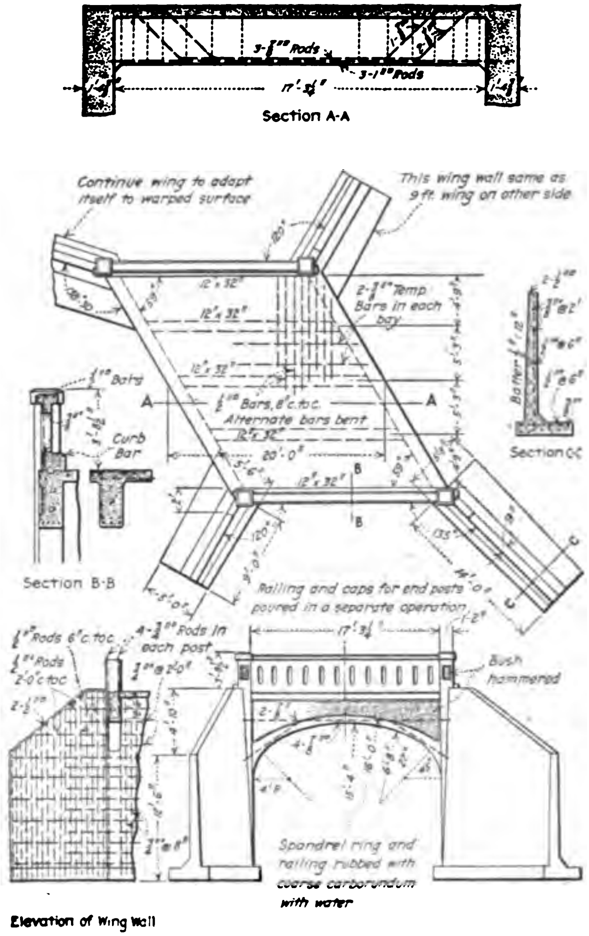


FIG. 18.—Lindsey Creek crossing near Wyeth, Oregon.

FIG. 19.—North Samuels Avenue viaduct, Fort Worth, Texas.

Reinforced-concrete deck-girder bridges for railroad traffic [illegible] such structures are [illegible] The superstructure [illegible] plain-concrete piers and abutments.

b. Through Girders.—From the standpoint of economy, [illegible] through-girder [illegible] not be built except where insufficient headroom or other [illegible] conditions [illegible] deck girder.

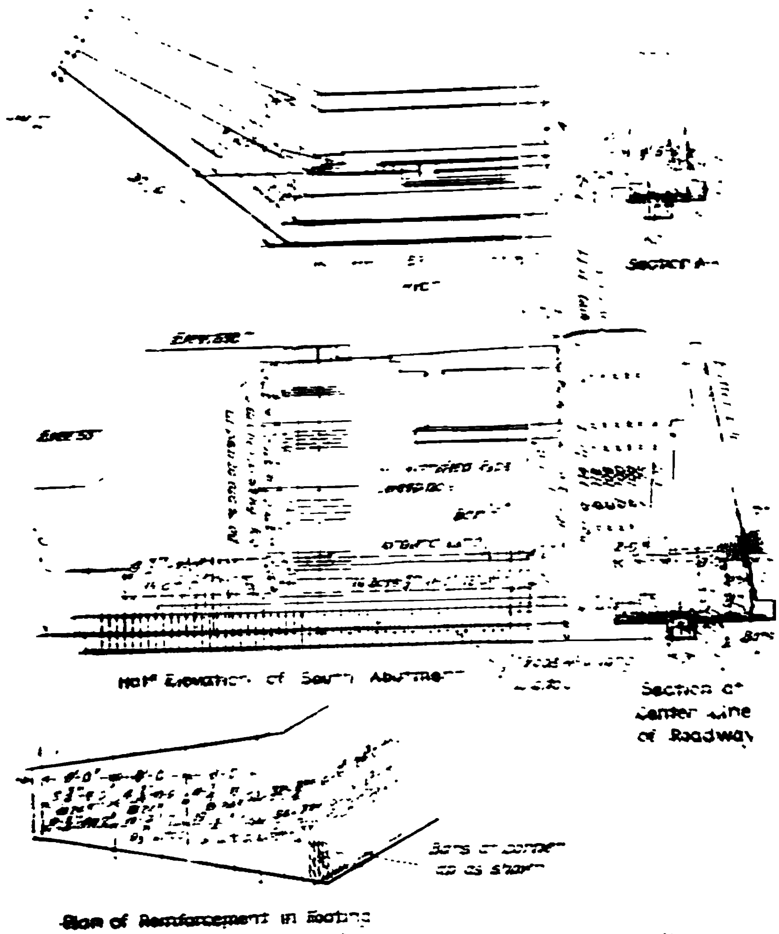


Fig. [illegible].—Details of south abutment [illegible] Fort Worth, Texas.

The standard type of through-girder bridge, adopted by the Iowa Highway Commission for locations where the deck type is impracticable, is shown in Fig. [illegible] The girders themselves were designed as simple beams and the floor as a slab partially fixed, the point of contraflexure being arbitrarily assumed at about 1 ft. from the edge of the girder. This is approximately in accordance with tests made by the Illinois Highway Commission in 190[illegible]

In the tests referred to, stress measurements were made on a through-girder span with an 18-ft. roadway loaded with crushed stone and pig iron. The full applied load was 418 tons, which gave a distributed load per square foot of floor of 1450 lb. Deformation mea[illegible]

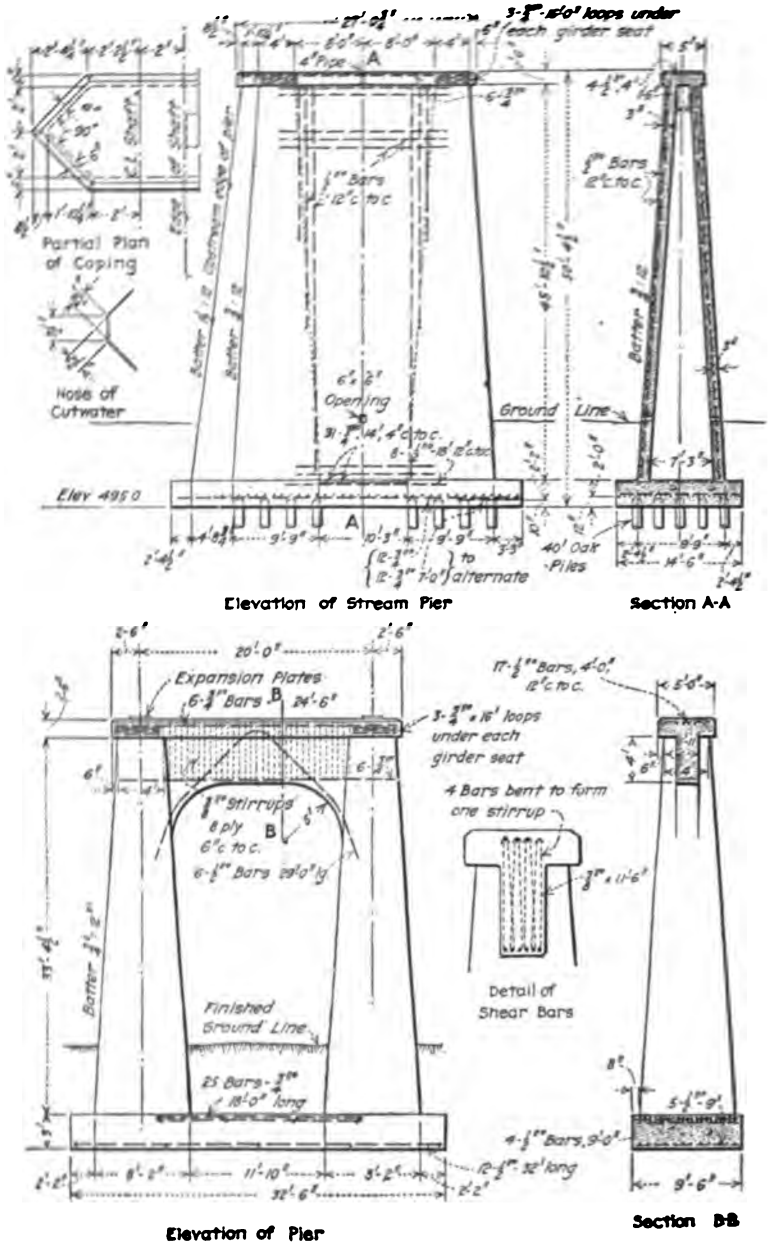


FIG. 21.—Details of typical piers, North Samuels Avenue viaduct, Fort Worth, Texas.

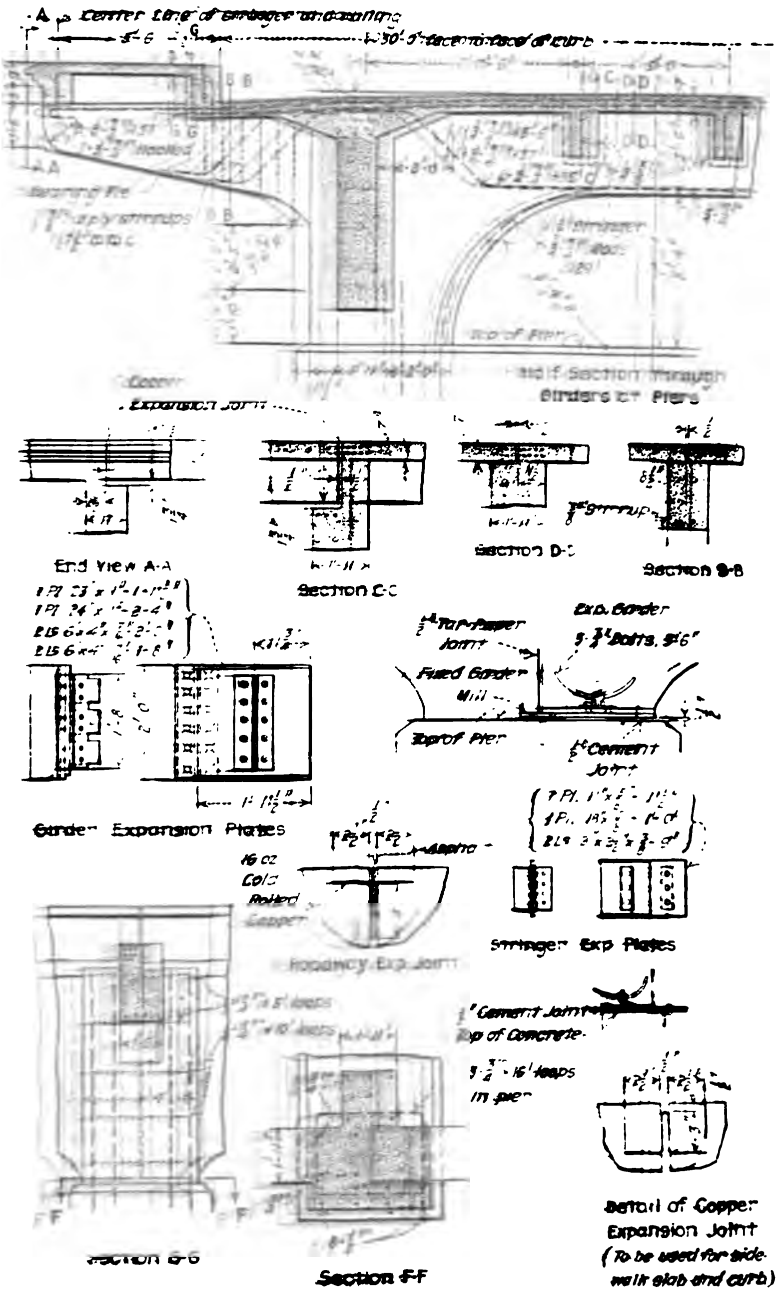


FIG. 22.—Details of floor system, North Samuels Avenue viaduct Fort Worth, Texas.

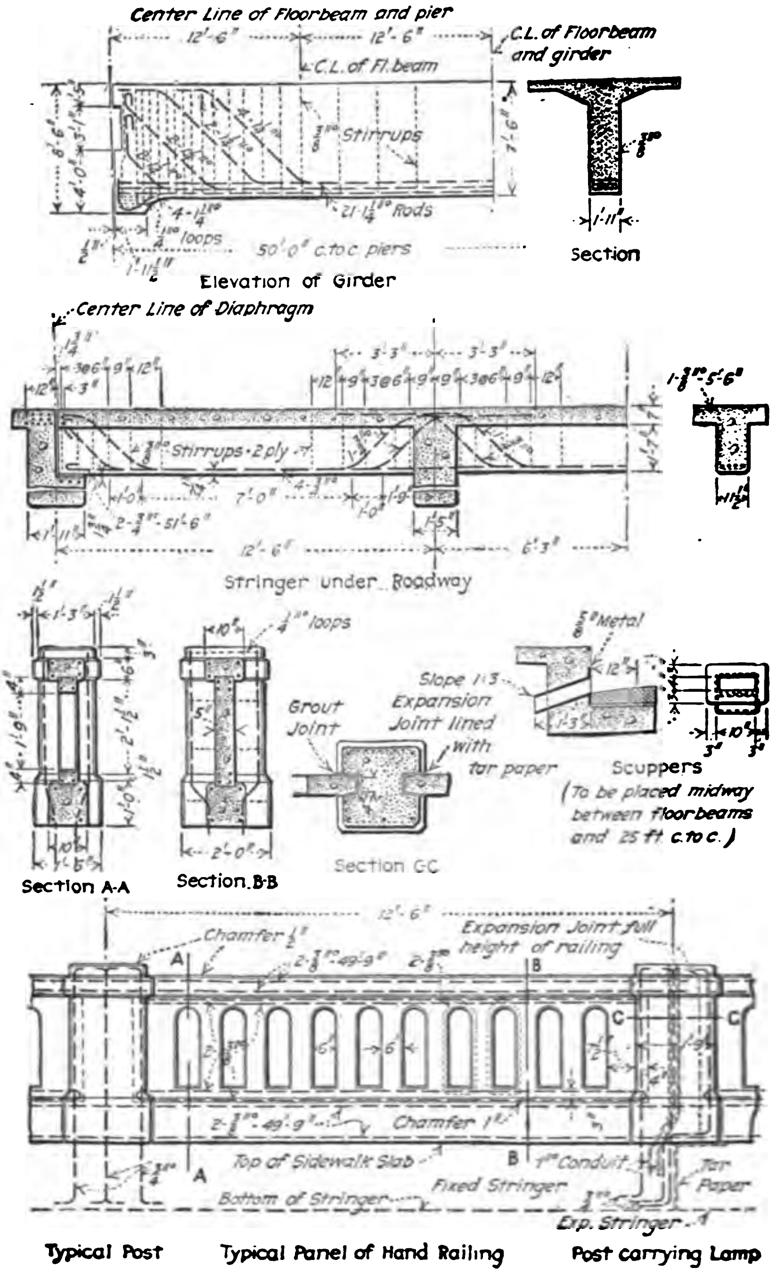


FIG. 23.—Details of floor system and railing, North Samuels Avenue viaduct, Fort Worth, Texas.

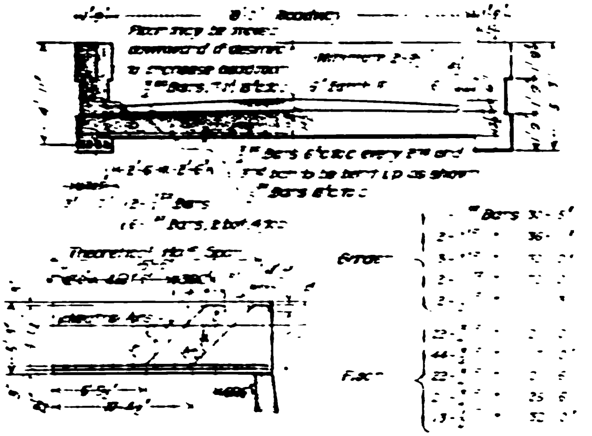


FIG. 24—Standard sections for through girder bridges of 30-ft. span. Iowa Highway Commission

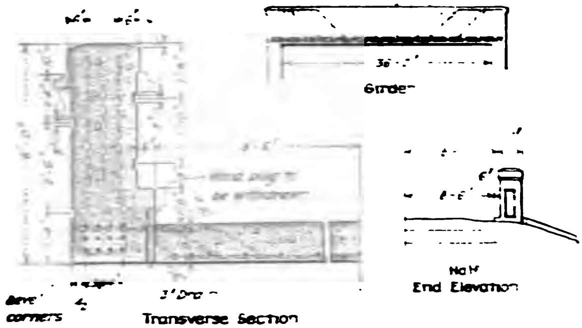


FIG. 25—Details of superstructure of Funkelien bridge, Town of Christiana, Dane Co., Wis

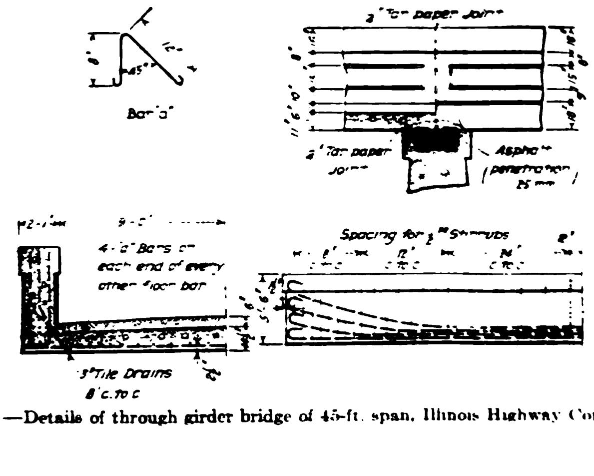


FIG. 26—Details of through girder bridge of 45-ft. span, Illinois Highway Commission

were made in the reinforcing steel of the suspended floor which indicated stress equivalent to that theoretically resulting from a sinple span of 15.5 ft. In other words, since the roadway was 18 ft. between girders, the point of contraflexure was apparently about 15 in. from each girder face.

Figs. 25, 26 and 27 illustrate other design of through-girder superstructures.

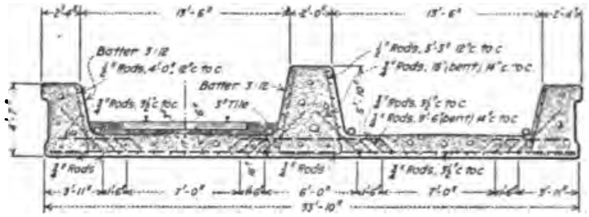


FIG. 27.—Cross-section of through girder bridge, C. B. & Q. R. R.

Fig. 28 shows a rather unusual type of through bridge on account of the fact that the girder reinforcement is in the form of a truss of sufficient strength to carry the dead and construction loads. The piers are simply columns braced between by either a vertical slab or by struts. As no falsework is necessary, this type of construction is especially adapted for highway bridges over railroads and electric lines, or at locations where the soil is very soft.

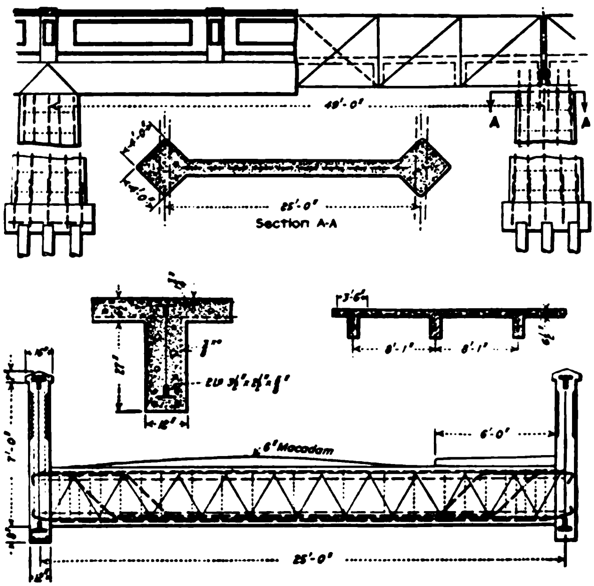


FIG. 28.—Details of bridge over Muddy Creek, Hamilton County, Ohio

CONTINUOUS-GIRDER BRIDGES

Monolithic Construction

6. Expansion Joints.—In order to prevent an accumulation of movement due to contraction and expansion, expansion joints should be provided at least every 100 ft. in length of the structure. If this is not done, severe stresses are likely to occur in the end columns.

In long bridges an expansion joint is usually provided between the superstructure and the abutments for the reason that, if an abutment with a heavy pressure of earth against it is rigidly connected with a number of continuous spans, the expansion and contraction tend to act in one direction only—that is, away from the abutment—the earth pressure back of the abutment not allowing movement in the opposite direction. Such a condition would lead to difficulties at the center of the bridge, or over the expansion piers next to the abutments, and the abutments and piers would also be severely overstressed due to the continuous movement in one direction. Each time an abutment would move slightly due to contraction, the earth against it by reason of the heavy moving loads would fill in the small space left by the contractive movement, and when expansion again took place, the abutment would be restrained by the earth so that enormous stresses might be developed.

7. Examples of Typical Bridges of the Continuous-girder Type.—A rather simple highway trestle, applicable to comparatively low crossings, is shown in Fig. 29. The longitudinal beams are continuous over three spans, an expansion joint occurring over every third pier. Intermediate piers are made monolithic with the floor by means of rods from the columns and stirrups from the cross beams. Because of the indeterminate degree of fixity, the lightness of the structure, and unknown construction factors, all members affected were designed both as fixed

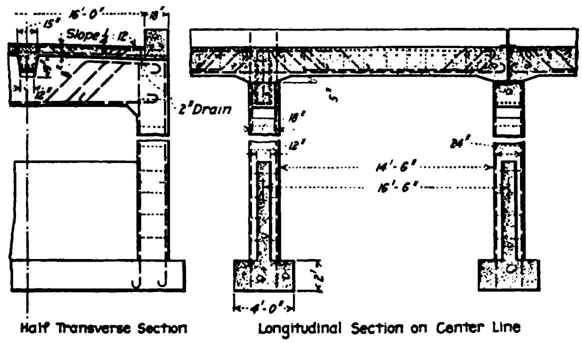


FIG. 29.—Details of standard trestle spans, Engineering Department, State of Arizona.

beams and as beams freely supported. The slabs were designed as continuous, with equal positive and negative steel throughout. The center longitudinal beam was designed as a T-beam with a 36-in. flange. The pier web, or wall between supporting columns of bents, is carried 2 ft. above high water.

A low trestle or viaduct type of construction is shown in Fig. 30. The slab-beam-and-girder spans were selected since arches, it was thought, would not appear to advantage for such low construction. The cost of the girder type was also found to be much less than for a series of arches, due principally to decrease in the dead weight of the structure and to simplicity in formwork. Expansion joints occur about every 200 ft. and make the viaduct virtually a number of independent structures, a double row of columns being provided at each joint. The arched girders capping the column bents were designed as straight rectangular beams and no account was taken of the possible arch action. The entire top surface of the roadway slab was waterproofed with a layer of burlap and two layers of felt laid in hot asphalt.

Fig. 31 gives the details of a continuous-girder bridge to span a stream which is generally dry, but which at flood times reaches over a wide area of bottom lands. This railway bridge is also within the backward area of the White Rock reservoir of the Dallas water supply so that ample provision had to be made for high-water conditions. The floor consists of a double T-beam which is monolithic with the bents and the abutments. There is no expansion joint in the structure since it was considered of sufficient strength to take all movement from end to end. Both ends of the structure are open between girder supports with short trestles con-

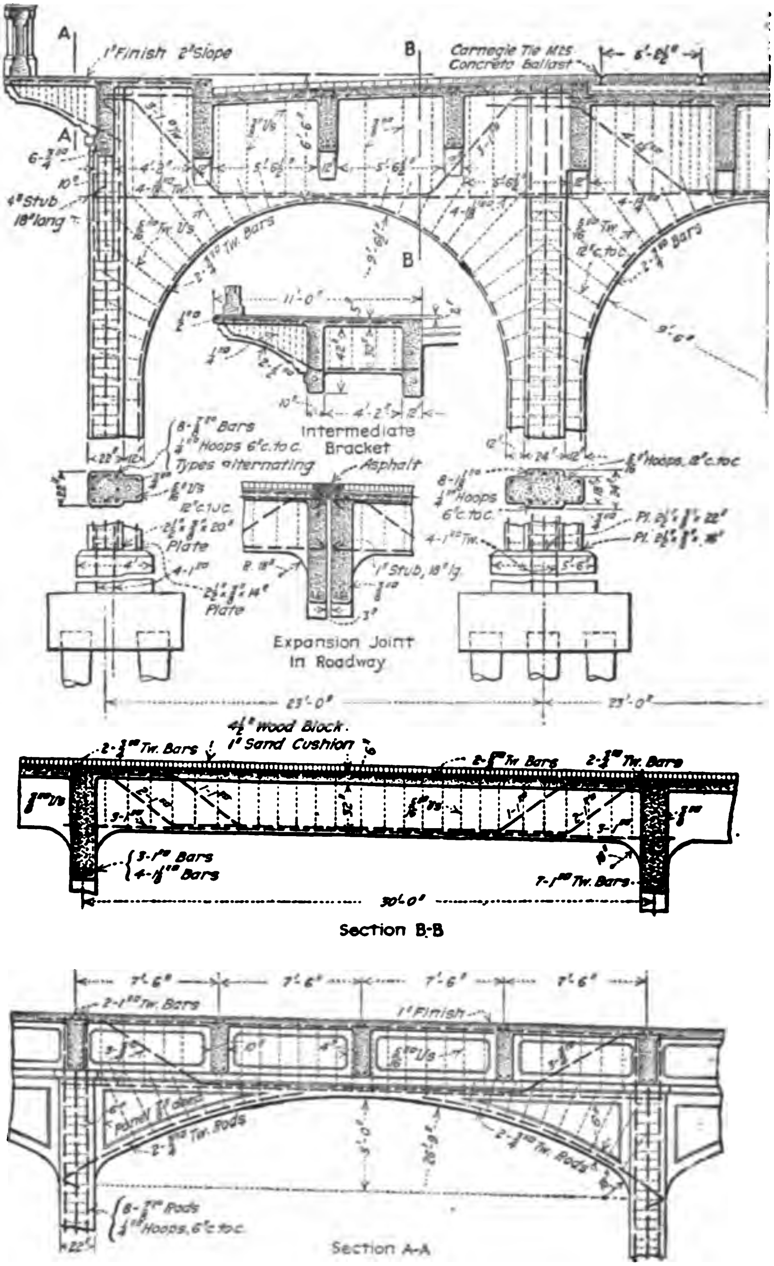


FIG. 30.—Details of Gilbert Avenue viaduct, Cincinnati, Ohio.

necting with the bridge which at some future time may be filled in if high water gives no trouble at this point. The girders were designed for Cooper's E-30 loading with an impact allowance of 100% of the live load. The ribbed abutments should be noted.

A continuous-girder bridge or trestle with no expansion joints and with abutments of the ordinary type is shown in Fig. 32. The structural-steel core in the column bents is unusual but undoubtedly adds greatly to the rigidity.

Fig. 33 gives the details of one of the typical bridges in the Chicago, Milwaukee & St. Paul track depression work at Minneapolis. The girders are continuous from end to end with expansion joints at the abutments—that is, the girders were considered continuous over the two interior supports and simply supported at the ends. The moments and shears were calculated by influence lines in accordance with the theory of continuous structures assuming constant I and unyielding supports (see Art. 48a, Sect. 7). Corrections were made for ⅛-in.

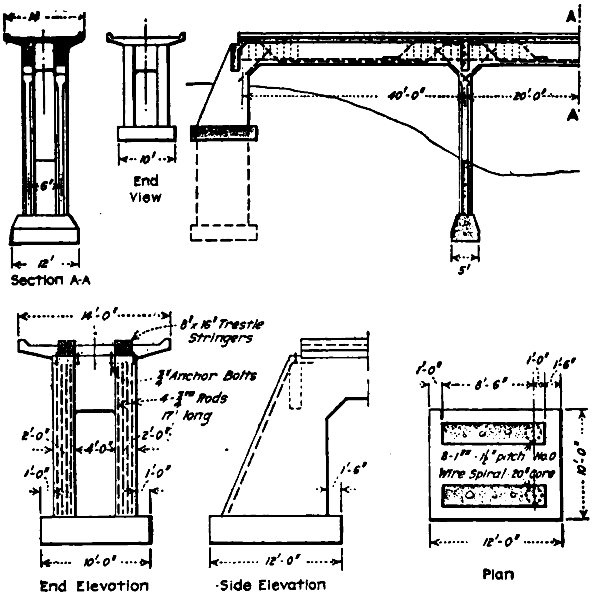


FIG. 31.—Details of railway bridge over White Rock Creek near Dallas, Texas.

settlement of supports for variable I. An unusual feature of the bridges is a curved shelf on the outer face of the outside girder which is intended to act as a smoke shield by diverting the smoke from the parapet as engines pass under the bridge.

8. Analysis of Stresses in Rigid Viaduct Structures.—A viaduct structure, composed of one or more cross frames or bents, and two or more spans of longitudinal deck girders, is in reality a rigid frame when girders and bents are rigidly attached to each other. Reinforced-concrete structures of this kind will act as rigid frames between expansion joints, and should be investigated as such. The general problem may be reduced to two problems for analysis: the stresses caused in the frame as seen in longitudinal elevation (hereafter referred to as the viaduct frame); and the stresses in the cross frame seen in a transverse section of the structure (hereafter called the cross frame, or bent). These problems will be treated in the order here given.

The viaduct frame should be designed to withstand: (1) the dead load of the frame; (2) the vertical live load, plus impact; (3) the horizontal traction forces, or braking forces, plus

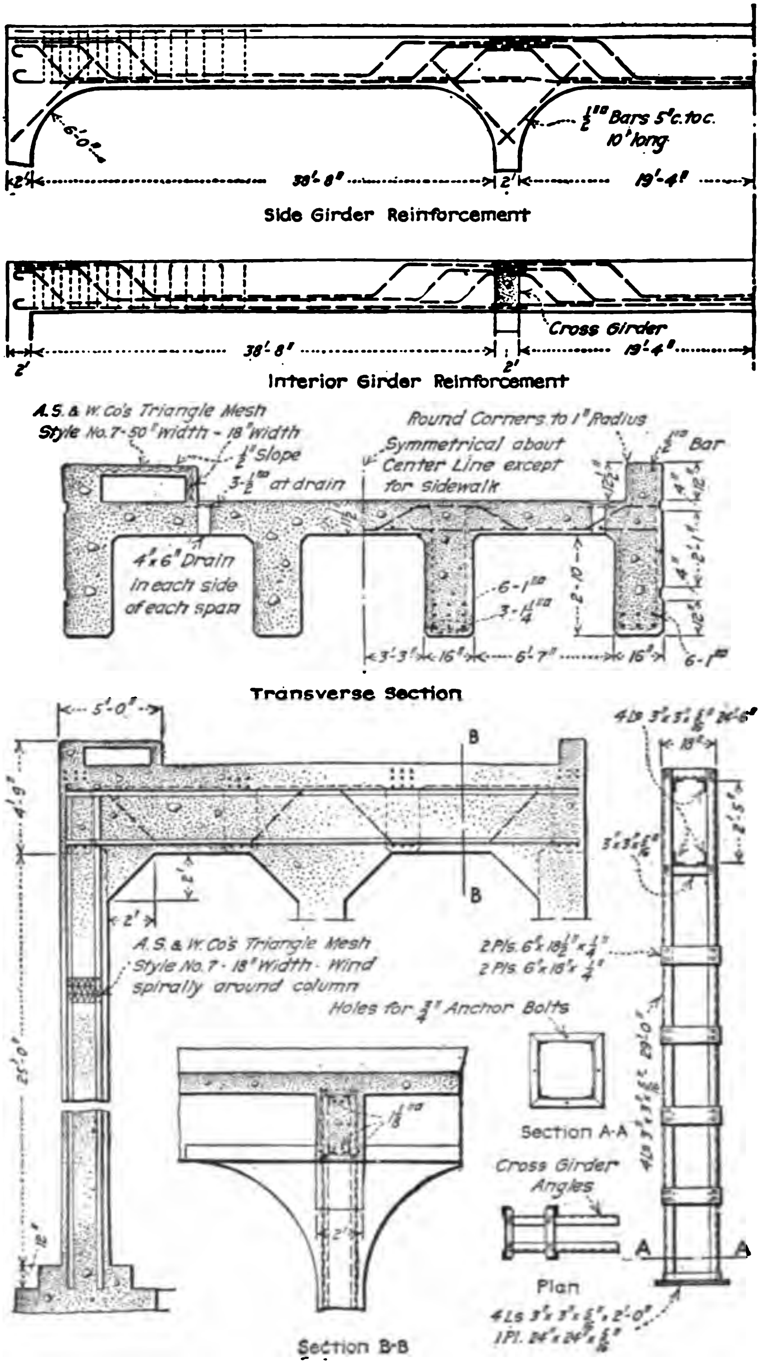


FIG. 32.—Details of Mill Creek bridge, Village of Casenovia, Richland Co., Wis.

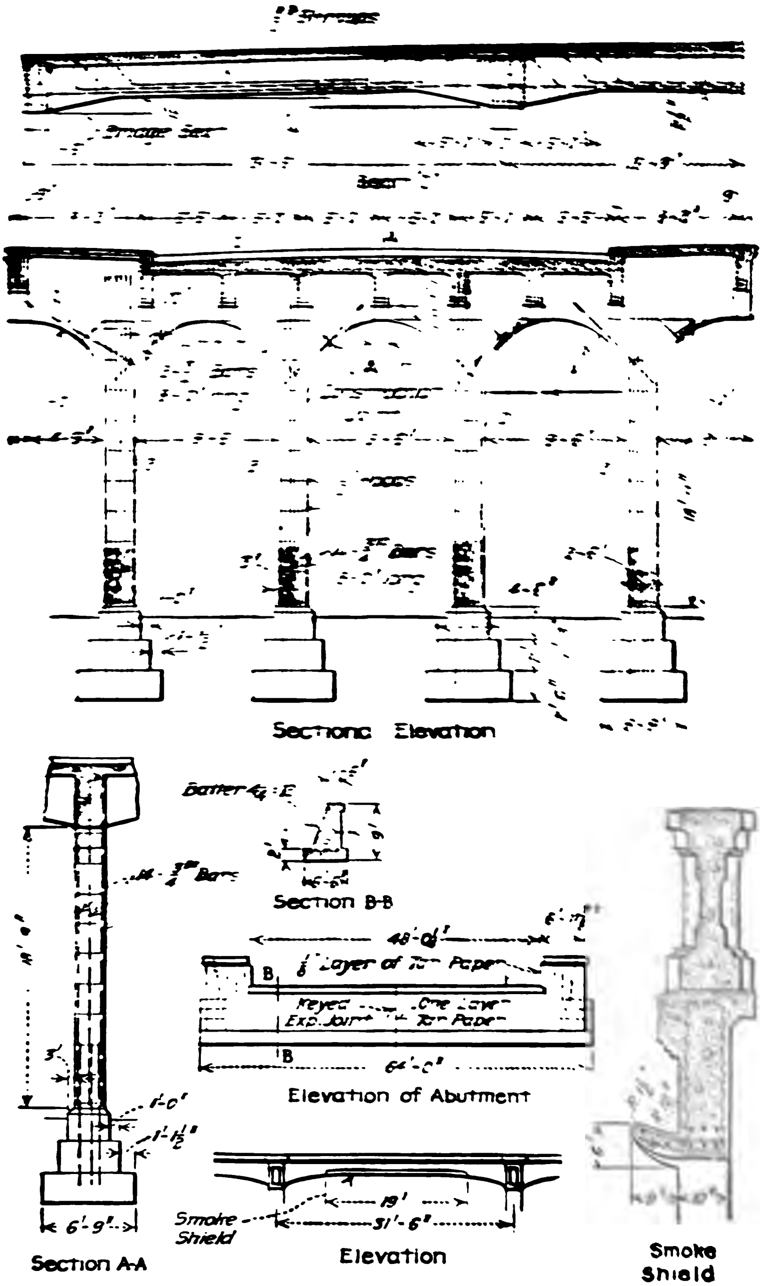


FIG. 33.—Details of Bryant Avenue bridge over tracks of the C. M. & St. P. Ry., Minneapolis, Minn.

impact; (4) stresses due to changes in temperature. Any one of these cases may develop large moments in both girders and columns. The reactions on the footings cannot be determined in an unsymmetrical frame, or in a symmetrical frame with unsymmetrical loading, without consideration of the elastic distortion of the structure. The dead load would consist of the estimated weight of the frame. The live load and its impact would be of the classes of loading given for arches (see Art. 6, Sect. 16).

8a. Viaduct Frames.—Before proceeding with the analysis of the viaduct frame, it is necessary to determine the conditions of support of the bases of the columns and of the outer ends of the girders. The greatest moments in the columns due to unbalanced loads on adjacent spans will occur when the column bases are fixed; and the greatest column stresses due to lateral, or tractive, forces will occur when the column bases may be considered as hinged. Since the tractive forces produce very large column stresses, and since there is usually great difficulty in securing a perfectly fixed column base, the discussion here given will apply first to viaduct frames whose column bases are hinged, after which certain modifications of the development will be suggested to care for the case of the perfectly fixed column base. End girders will be assumed to have frictionless bearings in the expansion pockets, thus allowing the maximum horizontal deformation of the frame to occur.

The extent to which the bent will act as a pair of columns in the viaduct frame should also be determined. In Fig. 34 is shown a two-legged bent with slightly battered columns. Girders attached at B and C will deflect in vertical planes; hence it is convenient to replace AB and CD with columns lying in vertical planes, as $A'B$ and CD', whose stiffness, or ability to restrain the points B and C, is just equal to that of the legs AB and CD. The length of these equivalent columns will be h, and their moments of inertia about a horizontal axis lying in the plane of the bent will be $I_{A'B} = I_{AB} \cos \theta$. When the bent is composed of more than two columns, each column should be replaced by a vertical one, as above; and the sum of the moments of inertia of all equivalent columns on one side of the axis of symmetry should be considered as the moment of inertia of a single resultant column $A'B$. This resultant column will be called "the column" in the following treatment of viaduct frames.

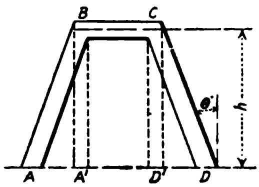


Fig. 34.

It is immaterial whether or not the whole viaduct frame is split lengthwise into two parts for analysis. The designer should arrange this matter to suit his convenience in computations.

Three cases of loading will be applied to the viaduct frame: (1) a moving vertical concentrated load; (2) a vertical symmetrical load, placed symmetrically on a single span; (3) a horizontal load acting axially on the deck girder. It will be possible from these investigations to plot influence lines for live load, and to determine the effect of dead load on the span.

The following series of solutions will begin with the general case of the four-span frame. From this case the other cases may be deduced, since each of the cases following is a special form of the general case. All solutions will be made by the method of slope-deflections (see Art. 2, Sect. 10). The nomenclature used is as follows:

M = moment. Subscripts will denote where this moment is taken—as, M_{AB} = moment at A in the member AB.

θ = change in slope of the tangent to the elastic curve at a given joint. Subscripts will denote the joint in question.

d_1 = lateral movement of the top of any first-tier column due to eccentric vertical, or lateral, loads.

d_2 = lateral movement of the top of any second-tier column.

h = column height. Subscript designates the column in question.

l = girder length. Subscript designates the girder in question.

$n = \frac{1}{h}$. Subscript designates the column in question.

$K = \frac{1}{l}$ for girder; or $\frac{1}{h}$ for column. Subscript indicates member in question.

$\frac{F}{l}$ = constant for symmetrical loading. See page 413 for values for different types of loading.

X = moment effect at left end of girder due to load P on that girder, and in general equals $\frac{Pab^2}{l^2}$, in which a is the distance from the left end of the girder to P, and $a + b = l$. Subscripts will denote the girder in question.

Y = moment effect at right end of girder due to load P on that girder, and in general equals $\frac{Pa^2b}{l^2}$. Subscripts will denote the girder in question.

8b. Four-span Viaduct Frame with Rigidly-connected Column Tie (Type I).—The general case of the unsymmetrical frame shown in Fig. 35 will be analyzed for the loads P_1 and P_2 and Q occurring separately.

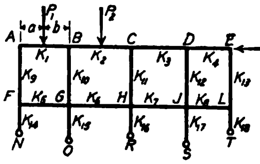


Fig. 35.

Since the structure and its loads are for the general case unsymmetrical, there will be a horizontal movement of the members AB, and of the members FL, besides rotation at all of the joints. Vertical components of this nearly horizontal motion will be neglected in this analysis.

Referring to Arts. 2 and 3, Sect. 10, the equations for the moment at the end of each may be written as follows:

$$M_{FN} = 3EK_{14}(\theta_F - n_{14}d_1)$$
$$M_{FG} = 2EK_5(2\theta_F + \theta_G)$$
$$M_{FA} = 2EK_9(2\theta_F + \theta_A - 3n_9d_2)$$
$$M_{AF} = 2EK_9(2\theta_A + \theta_F - 3n_9d_2)$$
$$M_{AB} = 2EK_1(2\theta_A + \theta_B) - X_{AB}$$
$$M_{BA} = 2EK_1(2\theta_B + \theta_A) + Y_{BA}$$
$$M_{BG} = 2EK_{10}(2\theta_B + \theta_G - 3n_{10}d_2)$$
$$M_{BC} = 2EK_2(2\theta_B + \theta_C) - X_{BC}$$
$$M_{CB} = 2EK_2(2\theta_C + \theta_B) + Y_{CB}$$
$$M_{CH} = 2EK_{11}(2\theta_C + \theta_H - 3n_{11}d_2)$$
$$M_{CD} = 2EK_3(2\theta_C + \theta_D)$$
$$M_{DC} = 2EK_3(2\theta_D + \theta_C)$$
$$M_{DJ} = 2EK_{12}(2\theta_D + \theta_J - 3n_{12}d_2)$$
$$M_{DE} = 2EK_4(2\theta_D + \theta_E)$$
$$M_{ED} = 2EK_4(2\theta_E + \theta_D)$$
$$M_{EL} = 2EK_{13}(2\theta_E + \theta_L - 3n_{13}d_2)$$
$$M_{LE} = 2EK_{13}(2\theta_L + \theta_E - 3n_{13}d_2)$$
$$M_{LT} = 3EK_{18}(\theta_L - n_{18}d_1)$$
$$M_{LJ} = 2EK_8(2\theta_L + \theta_J)$$
$$M_{JL} = 2EK_8(2\theta_J + \theta_L)$$
$$M_{JS} = 3EK_{17}(\theta_J - n_{17}d_1)$$
$$M_{JD} = 2EK_{12}(2\theta_J + \theta_D - 3n_{12}d_2)$$
$$M_{JH} = 2EK_7(2\theta_J + \theta_H)$$
$$M_{HJ} = 2EK_7(2\theta_H + \theta_J)$$
$$M_{HR} = 3EK_{16}(\theta_H - n_{16}d_1)$$
$$M_{HC} = 2EK_{11}(2\theta_H + \theta_C - 3n_{11}d_2)$$
$$M_{HG} = 2EK_6(2\theta_H + \theta_G)$$
$$M_{GH} = 2EK_6(2\theta_G + \theta_H)$$
$$M_{GO} = 3EK_{15}(\theta_G - n_{15}d_1)$$
$$M_{GB} = 2EK_{10}(2\theta_G + \theta_B - 3n_{10}d_2)$$
$$M_{GF} = 2EK_5(2\theta_G + \theta_F)$$

Since from statics, any joint in a structure is in equilibrium, the sum of the moments about that joint must equal zero. Thus,

$$M_{FN} + M_{FG} + M_{FA} = 0$$

The values of each moment (from above) may be substituted into this equation.

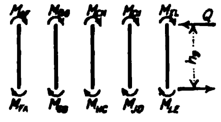


Fig. 36.

In like manner the equations above for the members concurring at any joint may be summed up and this sum set to zero. This will result in ten equations, one for each of the joints A to L, inclusive, which will involve twelve unknowns; that is, a θ for each of these joints, and in addition, d_2 and d_1. For solution it is necessary to write as many equations as there are unknowns. One of the two additional equations may be supplied from the following condition:

The sum of the moments at the top and bottom of all columns of one tier, plus the product of the shear (Q), and the height of the tier (h_9), equals zero. Thus, referring to Fig. 36,

$$M_{AF} + M_{BG} + M_{CH} + M_{DJ} + M_{EL} + M_{FA} + M_{GB} + M_{BC} + M_{JD} + M_{LE} + Qh_9 = 0$$

The second of the two equations may be written from the requirement that the sum of all horizontal components of reactions at the bases N to T, inclusive, must equal Q. Thus,

$$M_{FN}n_{14} + M_{GO}n_{15} + M_{HR}n_{16} + M_{JS}n_{17} + M_{LT}n_{18} - Q = 0$$

These two equations, and the ten equations mentioned above, have been tabulated in Table I, after having first been divided through by E. All terms having no variables have been put on the right-hand side of the equations, and have been tabulated as three cases: (1) with the load P_1 on AB; (2) with the load P_2 on BC; (3) with the horizontal load Q at E. Vertical loads on other spans may be treated as in Cases (1) and (2). This arrangement allows any case to be analyzed entirely separately. In either Case (1) or (2), the position of the load may be altered by changing the distances a and b, hence changing the values of X and Y (see page 629, Fig. 35). It was found convenient to divide the last two equations by a common constant term.

After a table similar to Table I has been prepared, the known terms may be evaluated, and the resulting simultaneous equations solved as in the illustrative problem under Type VII., Art. 8h.

TABLE 1.

	Variable Terms (Left hand side of equations)												Constant Terms (Right hand side of equations)		
Eq No.	θ_A	θ_B	θ_C	θ_D	θ_E	θ_F	θ_G	θ_H	θ_J	θ_L	d_2	d_1	Case I (Concentrated load—Span AB)	Case II (Concentrated load—Span BC)	Case III (Horizontal load on deck)
1	$2K_9$					$3K_{14}+4K_5+4K_9$	$2K_5$				$-6K_9n_9$	$-3K_{14}n_{14}$			
2	$4K_1+4K_9$	$2K_1$				$2K_9$					$-6K_9n_9$		X_{AB}/E		
3	$2K_1$	$4K_1+4K_2+4K_{10}$	$2K_2$				$2K_{10}$				$-6K_{10}n_{10}$		$-Y_{BA}/E$	X_{BC}/E	
4		$2K_2$	$4K_2+4K_3+4K_{11}$	$2K_3$				$2K_{11}$			$-6K_{11}n_{11}$			$-Y_{CB}/E$	
5			$2K_3$	$4K_3+4K_4+4K_{12}$	$2K_4$				$2K_{12}$		$-6K_{12}n_{12}$				
6				$2K_4$	$4K_4+4K_{13}$					$2K_{13}$	$-6K_{13}n_{13}$				
7					$2K_{13}$				$2K_8$	$4K_8+4K_{13}+3K_{18}$	$-6K_{13}n_{13}$	$-3K_{18}n_{18}$			
8				$2K_{12}$				$2K_7$	$4K_7+4K_8+4K_{12}+3K_{17}$	$2K_8$	$-6K_{12}n_{12}$	$-3K_{17}n_{17}$			
9			$2K_{11}$				$2K_6$	$4K_6+4K_7+4K_{11}+3K_{16}$	$2K_7$		$-6K_{11}n_{11}$	$-3K_{16}n_{16}$			
10		$2K_{10}$				$2K_5$	$4K_5+4K_6+4K_{10}+3K_{15}$	$2K_6$			$-6K_{10}n_{10}$	$-3K_{15}n_{15}$			
11						$K_{14}n_{14}$	$K_{15}n_{15}$	$K_{16}n_{16}$	$K_{17}n_{17}$	$K_{18}n_{18}$		$-(K_{14}n_{14}^2+K_{15}n_{15}^2+K_{16}n_{16}^2+K_{17}n_{17}^2+K_{18}n_{18}^2)$			$Q/3E$
12	K_9	K_{10}	K_{11}	K_{12}	K_{13}	K_9	K_{10}	K_{11}	K_{12}	K_{13}	$-3(K_9n_9+K_{10}n_{10}+K_{11}n_{11}+K_{12}n_{12}+K_{13}n_{13})$				$Qh_9/6E$

It should be noted both in the moment equations, and in Table I, that when the load is on AB, X_{BC} and Y_{CB} both equal zero.

When the load is symmetrically placed on a member, $X = Y = \frac{F}{l}$. Thus, if a load is symmetrically placed on AB, $M_{AB} = 2EK_1(2\theta_A + \theta_B) - \frac{F}{l}$. Equation (2), Table I, equals $\left(\frac{F}{lE}\right)$ for Case I. Equation (3), Table I, equals $\left(-\frac{F}{lE}\right)$ for Case I. The same scheme holds for other members. For values of $\frac{F}{l}$ for various symmetrical loadings, see page 413.

Case a (Fig. 37).—The solution for this case may be obtained from the general equations of Table I, by substituting $K_3 = K_{13} = K_{18} = 0$.

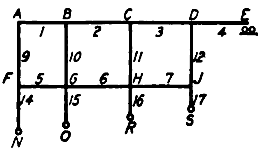


FIG. 37.

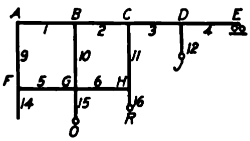


FIG. 38.

It is important to note the variation if $h_{17} = 0$, and a hinge is placed at J. Then JD and JH would be hinged at J. In this special case $K_{17} = 0$; $M_{JD} = M_{JH} = 0$; $d_1 = 0$;

$M_{DJ} = 3EK_{12}\ (\theta_D - n_{12}d_2)$; $M_{HJ} = 3EK_7\theta_H$. With these modifications a table like Table I may then be constructed.

Case b (*Fig.* 38).—In this case $K_7 = K_8 = K_{13} = K_{17} = K_{18} = 0$; $M_{JD} = 0$; $M_{DJ} = 3EK_{12}$ $(\theta_D - n_{12}d_2)$. These variations from the general moment equations, pages 629 and 630, will allow a new table, similar to Table I, to be made up.

Case c (*Fig.* 39).—The equations of Table I apply to this frame when $K_5 = K_6 = K_9 = K_{13} = K_{14} = K_{18} = 0$.

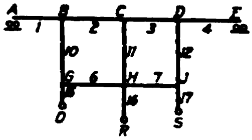


FIG. 39.

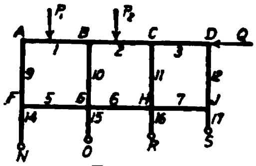


FIG. 40.

8c. Three-span Viaduct Frame with Rigidly-connected Column Tie (Type II).—Equations of moment at the ends of each member (Fig. 40) in the frame may be written as in Type I.

$M_{FN} = 3EK_{14}(\theta_F - n_{14}d_1)$
$M_{FG} = 2EK_5(2\theta_F + \theta_G)$
$M_{FA} = 2EK_9(2\theta_F + \theta_A - 3n_9d_2)$
$M_{AF} = 2EK_9(2\theta_A + \theta_F - 3n_9d_2)$
$M_{AB} = 2EK_1(2\theta_A + \theta_B) - X_{AB}$
$M_{BA} = 2EK_1(2\theta_B + \theta_A) + Y_{BA}$
$M_{BG} = 2EK_{10}(2\theta_B + \theta_G - 3n_{10}d_2)$
$M_{BC} = 2EK_2(2\theta_B + \theta_C) - X_{BC}$
$M_{CB} = 2EK_2(2\theta_C + \theta_B) + Y_{CB}$
$M_{CH} = 2EK_{11}(2\theta_C + \theta_H - 3n_{11}d_2)$
$M_{CD} = 2EK_3(2\theta_C + \theta_D)$
$M_{DC} = 2EK_3(2\theta_D + \theta_C)$
$M_{DJ} = 2EK_{12}(2\theta_D + \theta_J - 3n_{12}d_2)$
$M_{JD} = 2EK_{12}(2\theta_J + \theta_D - 3n_{12}d_2)$
$M_{JS} = 3EK_{17}(\theta_J - n_{17}d_1)$
$M_{JH} = 2EK_7(2\theta_J + \theta_H)$
$M_{HJ} = 2EK_7(2\theta_H + \theta_J)$
$M_{HR} = 3EK_{16}(\theta_H - n_{16}d_1)$
$M_{HC} = 2EK_{11}(2\theta_H + \theta_C - 3n_{11}d_2)$
$M_{HG} = 2EK_6(2\theta_H + \theta_G)$
$M_{GH} = 2EK_6(2\theta_G + \theta_H)$
$M_{GO} = 3EK_{15}(\theta_G - n_{15}d_1)$
$M_{GB} = 2EK_{10}(2\theta_G + \theta_B - 3n_{10}d_2)$
$M_{GF} = 2EK_5(2\theta_G + \theta_F)$

The general equations set up from these moment equations are the first eight of the following. The ninth and tenth are found in the same manner as the eleventh and twelfth of the preceding type (see page 630).

(1) $2K_9\theta_A + (3K_{14} + 4K_5 + 4K_9)\theta_F + 2K_5\theta_G - 6K_9n_9d_2 - 3K_{14}n_{14}d_1 = 0$

(2) $(4K_1 + 4K_9)\theta_A + 2K_1\theta_B + 2K_9\theta_F - 6K_9n_9 = \dfrac{X_{AB}}{E}$

(3) $2K_1\theta_A + (4K_1 + 4K_2 + 4K_{10})\theta_B + 2K_2\theta_C + 2K_{10}\theta_G - 6K_{10}n_{10}d_2 = -\dfrac{Y_{BA}}{E}$

(4) $2K_2\theta_B + (4K_2 + 4K_3 + 4K_{11})\theta_C + 2K_3\theta_D + 2K_{11}\theta_H - 6K_{11}n_{11}d_2 = 0$

(5) $2K_3\theta_C + (4K_3 + 4K_{12})\theta_D + 2K_{12}\theta_J - 6K_{12}n_{12}d_2 = 0$

(6) $2K_{12}\theta_D + 2K_7\theta_H + (3K_{17} + 4K_7 + 4K_{12})\theta_J - 6K_{12}n_{12}d_2 - 3K_{17}n_{17}d_1 = 0$

(7) $2K_{11}\theta_C + 2K_6\theta_G + (3K_{16} + 4K_6 + 4K_7 + 4K_{11})\theta_H + 2K_7\theta_J - 6K_{11}n_{11}d_2 - 3K_{16}n_{16}d_1 = 0$

(8) $2K_{10}\theta_B + 2K_5\theta_F + (3K_{15} + 4K_5 + 4K_6 + 4K_{10})\theta_G + 2K_6\theta_H - 6K_{10}n_{10}d_2 - 3K_{15}n_{15}d_1 = 0$

(9) $K_{14}n_{14}\theta_F + K_{15}n_{15}\theta_G + K_{16}n_{16}\theta_H + K_{17}n_{17}\theta_J - (K_{14}n_{14}^2 + K_{15}n_{15}^2 + K_{16}n_{16}^2 + K_{17}n_{17}^2)d_1 = 0$

(10) $K_9\theta_A + K_{10}\theta_B + K_{11}\theta_C + K_{12}\theta_D + K_9\theta_F + K_{10}\theta_G + K_{11}\theta_H + K_{12}\theta_J - 2d_2(K_9n_9 + K_{10}n_{10} + K_{11}n_{11} + K_{12}n_{12}) = 0$

The right-hand side of the above equations is given for a vertical load P_1 in span AB. For a vertical load P_2 on span BC, replace $(-Y_{BA}/E)$ in equation (3) with (X_{BC}/E); set equation (4) equal to $(-Y_{CB}/E)$ instead of zero, and set equation (2) to zero. For a horizontal load at D, set equation (9) equal to $(Q/3E)$; set equation (10) equal to $(-Qh_9/6E)$; and all other equations to zero. It will be noted that the modifications are in accordance with Cases I, II and III of Table I.

When any horizontal member is loaded symmetrically, then for that member, $X = Y = \frac{F}{l}$. Thus, if a load is placed symmetrically on AB, $M_{AB} = 2EK_1(2\theta_A + \theta_B) - \frac{F}{l}$; $M_{BA} = 2EK_1(2\theta_B + \theta_A) + \frac{F}{l}$; equation (2), page 631, would equal (F/lE); and equation (3), page 631, would equal $(-F/lE)$. Values of F/l for various symmetrical loadings are given in the table on page 413.

These ten equations may now be tabulated as in Table I, and may be solved by the same method as that employed in a problem under Type VII (see Art. 8h).

Case a (*Fig.* 41).—This frame may be analyzed by letting $K_{12} = K_7 = K_{17} = 0$, in the general equations of Type II, page 631.

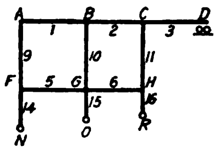


Fig. 41.

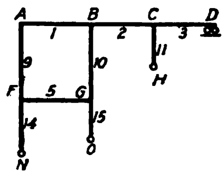


Fig. 42.

Suppose $h_{16} = 0$, and a hinge is placed at H_1 so that HC and HG are hinged. Then $K_{16} = 0 = M_{HC} = M_{HG} = 0$; $M_{GH} = 3EK_G\theta_G$; $M_{CH} = 3EK_{11}(\theta_C - n_{11}d_2)$; $d_1 = 0$. These modifications would be made in the moment equations on page 631, and a new set of general equations would be written.

Case b (Fig. 42).—For this case, $K_6 = K_7 = K_{12} = K_{16} = K_{17} = 0$; $M_{HC} = 0$; $M_{CH} = 3EK_{11}(\theta_C - n_{11}d_2)$. These values would be set into the moment equations on page 631 and the resulting modifications would be made in the ten general equations.

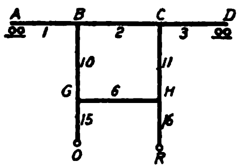


Fig. 43.

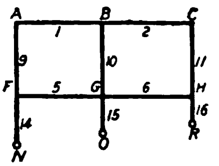


Fig. 44.

Case c (*Fig.* 43).—Equations (1) to (10), on page 631, may be used for this case by placing in them $K_5 = K_7 = K_9 = K_{12} = K_{14} = K_{17} = 0$.

8d. Two-span Viaduct Frame with Rigidly-connected Column Tie (Type III).—The method of analysis does not differ from that of the preceding cases. The moment equations are as follows (see Fig. 44):

$$M_{FN} = 3EK_{14}(\theta_F - n_{14}d_1)$$
$$M_{FG} = 2EK_5(2\theta_F + \theta_G)$$
$$M_{FA} = 2EK_9(2\theta_F + \theta_A - 3n_9d_2)$$
$$M_{AF} = 2EK_9(2\theta_A + \theta_F - 2n_9d_2)$$
$$M_{AB} = 2EK_1(2\theta_A + \theta_B) - X_{AB}$$
$$M_{BA} = 2EK_1(2\theta_B + \theta_A) + Y_{BA}$$
$$M_{BG} = 2EK_{10}(2\theta_B + \theta_G - 3n_{10}d_2)$$
$$M_{BC} = 2EK_2(2\theta_B + \theta_C)$$
$$M_{CB} = 2EK_2(2\theta_C + \theta_B)$$

$$M_{CH} = 2EK_{11}(2\theta_C + \theta_H - 3n_{11}d_2)$$
$$M_{HC} = 2EK_{11}(2\theta_H + \theta_C - 3n_{11}d_2)$$
$$M_{HR} = 3EK_{16}(\theta_H - n_{16}d_1)$$
$$M_{HG} = 2EK_6(2\theta_H + \theta_G)$$
$$M_{GH} = 2EK_6(2\theta_G + \theta_H)$$
$$M_{GB} = 2EK_{10}(2\theta_G + \theta_B - 3n_{10}d_2)$$
$$M_{GO} = 3EK_{15}(\theta_G - n_{15}d_1)$$
$$M_{GF} = 2EK_5(2\theta_G + \theta_F)$$

General equations:

(1) $(4K_9 + 4K_1)\theta_A + 2K_1\theta_B + 2K_9\theta_F - 6K_9n_9d_2 = X_{AB}/E$
(2) $2K_1\theta_A + (4K_1 + 4K_{10} + 4K_2)\theta_B + 2K_2\theta_C + 2K_{16}\theta_G - 6K_{10}n_{10}d_2 = -Y_{AB}\ E.$
(3) $2K_2\theta_B + (4K_2 + 4K_{11})\theta_C + 2K_{11}\theta_H - 6K_{11}n_{11}d_2 = 0$
(4) $2K_9\theta_A + (3K_{14} + 4K_5 + 4K_9)\theta_F + 2K_5\theta_G - 3K_{14}n_{14}d_1 - 6K_9n_9d_2 = 0$
(5) $2K_{10}\theta_B + 2K_5\theta_F + (4K_5 + 4K_{10} + 4K_6 + 3K_{15})\theta_G + 2K_6O_H - 3K_{15}n_{15}d_1 - 6K_{10}n_{10}d_2 = 0$
(6) $2K_{11}\theta_C + 2K_6\theta_G + (4K_{11} + 4K_6 + 3K_{16})\theta_H + 3K_{16}n_{16}d_1 - 6K_{11}n_{11}d_2 = 0$
(7) $K_{14}n_{14}\theta_F + K_{15}n_{15}\theta_G + K_{16}n_{16}\theta_H - (K_{14}n_{14}^2 + K_{15}n_{15}^2 + K_{16}n_{16}^2)d_1 = 0$
(8) $K_9\theta_A + K_{10}\theta_B + K_{11}\theta_C + K_9\theta_F + K_{10}\theta_G + K_{11}\theta_H - (K_9n_9 + K_{10}n_{10} + K_{11}n_{11})2d_2 = 0$

For a horizontal load at C, equation (7) would equal $\frac{Q}{3E}$; equation (8) would equal $\frac{-Qh_9}{6E}$; and all others would equal zero.

See preceding case for directions for analysis with symmetrical loading on a horizontal member.

Case a (*Fig.* 45).—Solution of this frame is accomplished by the above set of eight general equations, by letting $K_6 = K_{11} = K_{16} = 0$.

8e. One-span Viaduct Frame with Rigidly-connected Column Tie (Type IV).—The general moment equations for this case (Fig. 46) are as follows:

$M_{FN} = 3EK_{14}(\theta_F - n_{14}d_1)$
$M_{FG} = 2EK_5(2\theta_F + \theta_G)$
$M_{FA} = 2EK_9(2\theta_F + \theta_A - 3n_9d_2)$
$M_{AF} = 2EK_9(2\theta_A + \theta_F - 3n_9d_2)$
$M_{AB} = 2EK_1(2\theta_A + \theta_B) - X_{AB}$

$M_{BA} = 2EK_1(2\theta_B + \theta_A) + Y_{AB}$
$M_{BG} = 2EF_{10}(2\theta_B + \theta_G - 3n_{10}d_2)$
$M_{GB} = 2EK_{10}(2\theta_G + \theta_B - 3n_{10}d_2)$
$M_{GF} = 2EK_5(2\theta_G + \theta_F)$
$M_{GO} = 3EK_{15}(\theta_G - n_{15}d_1)$

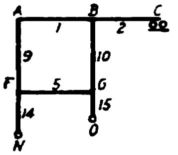


Fig. 45.

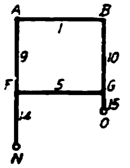


Fig. 46.

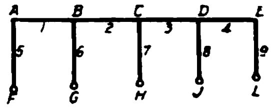


Fig. 47.

General equations:

(1) $(4K_9 + 4K_1)\theta_A + 2K_1\theta_B + 2K_9\theta_F - 6K_9n_9d_2 = X_{AB}/E$
(2) $2K_1\theta_A + (4K_1 + 4K_{10})\theta_B + 2K_{10}\theta_G - 6K_{10}n_{10}d_2 = -Y_{BA}/E$
(3) $2K_9\theta_A + (3K_{14} + 4K_5 + 4K_9)\theta_F + 2K_5\theta_G - 3K_{14}n_{14}d_1 - 6K_9n_9d_2 = 0$
(4) $2K_{10}\theta_B + 2K_5\theta_F + (4K_5 + 4K_{10} + 3K_{15})\theta_G - 3K_{15}n_{15}d_1 - 6K_{10}n_{10}d_2 = 0$
(5) $K_{14}n_{14}\theta_F + K_{15}n_{15}\theta_G - (K_{14}n_{14}^2 + K_{15}n_{15}^2)d_1 = 0$
(6) $K_9\theta_A + K_{10}\theta_B + K_9\theta_F + K_{10}\theta_G - (K_9n_9 + K_{10}n_{10})2d_2 = 0$

For a horizontal load at B equation (7) would equal $(Q/3E)$; equation (8) would equal $(-Qh_9/6E)$; and all others would equal zero.

For a load placed symmetrically on AB, replace the terms X_{AB} and Y_{AB} by F/l in above equations, without altering the present signs. Values of F/l for various symmetrical loads are given on page 413.

8f. Four-span Viaduct Frame (Type V).—The general case here given (Fig. 47) deals with a frame that is either symmetrical or unsymmetrical. A load on a single span will cause a sidewise movement of the deck. The vertical component of this movement is insignificant here, and will be neglected.

The following moment equations and general conditional equations will treat of a vertical load P_1 on AB; a vertical load P_2 on BC; and a horizontal load Q at E.

$M_{AF} = 3EK_5(\theta_A - n_5 d)$
$M_{AB} = 2EK_1(2\theta_A + \theta_B) - X_{AB}$
$M_{BA} = 2EK_1(2\theta_B + \theta_A) + Y_{BA}$
$M_{BG} = 3EK_6(\theta_B - n_6 d)$
$M_{BC} = 2EK_2(2\theta_B + \theta_C) - X_{BC}$
$M_{CB} = 2EK_2(2\theta_C + \theta_B) + Y_{CB}$
$M_{CH} = 3EK_7(\theta_C - n_7 d)$
$M_{CD} = 2EK_3(2\theta_C + \theta_D)$
$M_{DC} = 2EK_3(2\theta_D + \theta_C)$
$M_{DJ} = 3EK_8(\theta_D - n_8 d)$
$M_{DE} = 2EK_4(2\theta_D + \theta_E)$
$M_{ED} = 2EK_4(2\theta_E + \theta_D)$
$M_{EL} = 3EK_9(\theta_E - n_9 d)$

General equations (E is constant) for loads P_1 and Q:

(1) $(3K_5 + 4K_1)\theta_A + 2K_1\theta_B - 3K_5 n_5 d = X_{AB}/E$
(2) $2K_1\theta_A + (3K_6 + 4K_1 + 4K_2)\theta_B + 2K_2\theta_C - 3K_6 n_6 d = -Y_{BA}/E$
(3) $2K_2\theta_B + (3K_7 + 4K_2 + 4K_3)\theta_C + 2K_3\theta_D - 3K_7 n_7 d = 0$
(4) $2K_3\theta_C + (3K_8 + 4K_3 + 4K_4)\theta_D + 2K_4\theta_E - 3K_8 n_8 d = 0$
(5) $2K_4\theta_D + (4K_4 + 3K_9)\theta_E - 3K_9 n_9 d = 0$
(6) $K_5 n_5 \theta_A + K_6 n_6 \theta_B + K_7 n_7 \theta_C + K_8 n_8 \theta_D + K_9 n_9 \theta_E - (K_5 n_5^2 + K_6 n_6^2 + K_7 n_7^2 + K_8 n_8^2 -$
$K_9 n_9^2)d = Q/3E$

When only P_1 is acting, $Q = 0$. When only Q acts, $X_{AB} = Y_{AB} = 0$. When only P_1 acts on BC, $X_{AB} = Y_{BA} = 0$; $Q = 0$; equation (2) equals (X_{BC}/E); equation (3) equals $(-Y_{CB}/E)$. When a symmetrical load is placed on a member, X and Y for that member are replaced by $\frac{F}{l}$ of the load. Values of $\frac{F}{l}$ for various symmetrical loads may be found on page 413.

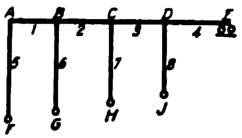


FIG. 48.

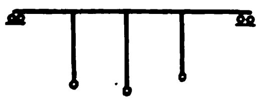


FIG. 49.

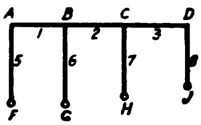


FIG. 50.

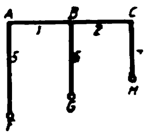


FIG. 51.

Case a (*Fig.* 48).—A special case arises when $h_9 = 0$, or joint E rests on rollers. A solution of this case may be reached by making $K_9 = 0$ in the general equations of Type V.

Case b (*Fig.* 49).—This case is very common. It may be analyzed by placing $K_5 = K_9 = 0$ in the general equations for Type V.

8g. Three-span Viaduct Frame (Type VI).—A solution of this frame (Fig. 50) may be made from the equations given on page 633, by substituting into those equations $K_4 = K_9 = 0$. It will be noted that θ_E drops out, and in accordance, equation (5) disappears. In all other respects the solution is identical.

8h. Two-span Viaduct Frame (Type VII).—General equations for this frame (Fig. 51) may be set up by placing $K_3 = K_4 = K_8 = K_9 = 0$ in the general equations for Type V. Hence, for the loads P_1 and Q:

(1) $(3K_5 + 4K_1)\theta_A + 2K_1\theta_B - 3K_5 n_5 d = X_{AB}/E$
(2) $2K_1\theta_A + (3K_6 + 4K_1 + 4K_2)\theta_B + 2K_2\theta_C - 3K_6 n_6 d = -Y_{BA}/E$
(3) $2K_2\theta_B + (3K_7 + 4K_2)\theta_C - 3K_7 n_7 d = 0$
(4) $K_5 n_5 \theta_A + K_6 n_6 \theta_B + K_7 n_7 \theta_C - (K_5 n_5^2 + K_6 n_6^2 + K_7 n_7^2)d = Q/3E$

In these equations, when P_1 acts alone, $Q = 0$. When Q acts alone, $P_1 = 0$. When a load is placed symmetrically on AB, X_{AB} and Y_{BA} are replaced by $\frac{F}{l}$ for that load (see page 413).

ILLUSTRATIVE PROBLEM.—A viaduct frame of Type VII has two spans of 17.0 ft. each, with columns of length 42.5 ft., 34.0 ft., and 29.75 ft. The moments of inertia are: $I_1 = I_2 = 80{,}000$ in.^{4}; $I_5 = 100{,}000$ in.^{4}; $I_6 = I_7 = 10{,}000$ in.^{4}. Determine horizontal reactions at F, G, and H, when a tractive force of 12,000 lb. acts along the deck (as at C). From these data the following table of constants is made up:

Member	Length (in.)	I (in.^{4})	$\frac{I}{l} = K$	$\frac{1}{l} = n$	$3Kn$	$3Kn^2$
1	204	80,000	392.50			
2	204	80,000	392.50			
5	510	100,000	196.20	0.00196	1.154	0.002263
6	408	10,000	24.50	0.00245	0.180	0.004410
7	357	10,000	28.05	0.00281	0.236	0.000664

From this table of constants the four general equations were reduced to the following equations, in which $E = 3{,}000{,}000$ lb. per sq. in. The fourth equation has been multiplied by 3, which permits values to be drawn directly from the above table. All equations were finally divided by 10.

Left-hand side of equations					Right-hand side
Equation No.	θ_A	θ_B	θ_C	d	Constant terms
1	215.86	78.50	....	−0.1154	0
2	78.50	321.35	78.50	−0.0180	0
3	....	78.50	165.42	−0.0236	0
4	0.1154	0.0180	0.0236	−0.0003368	0.0004
1′	1.0	0.364	....	−0.000534	0
2′	1.0	4.08	1.0	−0.000229	0
4′	1.0	0.156	0.2045	−0.00292	0.00347
2′ − 1′ = 5	....	3.716	1.0	0.000305	0
− 4′ = 6	....	3.924	0.7955	0.002691	−0.00347
5′	....	1.0	0.2693	0.0000822	0
6′	....	1.0	0.2027	0.000685	−0.000884
3′	....	1.0	2.106	−0.0003008	0
3′ − 5′ = 7	....	....	1.8367	−0.0003830	0
− 6′ = 8	....	....	1.9033	−0.0009858	0.000884
7′	....	....	1.0	−0.0002085	0
8′	....	....	1.0	−0.0005170	0.000464
8′ − 7′ = 9	....	....	....	−0.0003085	0.000464

Solving equation (9) gives $d = -1.503$ (inches lateral shifting).

$\theta_C = -0.0003135$ (rotation of C in radians).

$\theta_B = +0.0002078$ (rotation of B in radians).

$\theta_A = -0.000876$ (rotation of A in radians).

$M_{AF} + E = 3K_5\theta_A - 3K_5n_5d = -(588.6)(0.0008786) + (1.154)(1.503) = 1.223$

$M_{BG} + E = 3K_6\theta_B - 3K_6n_6d = (73.5)(0.0002078) + (0.18)(1.503) = 0.2863$

$M_{CH} + E = 3K_7\theta_C - 3K_7n_7d = -(84.15)(0.0003135) + (9.236)(1.503) = 0.3286$

$M_{AF} = 3{,}669{,}000$ in.-lb. $\qquad H_F = 7{,}190$ lb.

$M_{BG} = 858{,}900$ in.-lb. $\qquad H_G = 2{,}110$ lb.

$M_{CH} = 985{,}000$ in.-lb. $\qquad H_H = 2{,}760$ lb.

Total to check = 12,060 lb. (error 0.5%)

Case a (*Fig.* 52).—The solution may be made by using the general equations of Type VII, making $K_7 = 0$.

Case b (*Fig.* 53).—In this case $K_5 = K_7 = 0$. Substituting these into the general equations of Type VII, the solution is found at once.

8i. One-span Frame, Unequal Columns (Type VIII).—The solution for this frame (Fig. 54) may be obtained from the equations of Type VII, by making $K_2 = K_7 = 0$. This will eliminate equation (3), and modify the other equations.

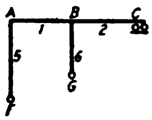


FIG. 52.

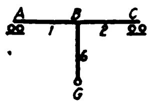


FIG. 53.

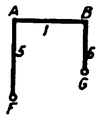


FIG. 54.

8j. Temperature Stresses.—Stresses caused by changes in temperature may become very large, and require thorough investigation. They come about from the fact that the members tend to change length, whence each causes a lateral displacement of a member at right angles to it.

The following analysis will be made for a frame of Type I, in which the lower tier of columns increase in length toward the left (see Fig. 55). We will note later that this was to cause all values of Δ to be positive; and will then give the effect upon the resulting equations when Δ is negative. The change of temperature is assumed in the development to be positive. Later, corrections will be noted for cases when the change is negative.

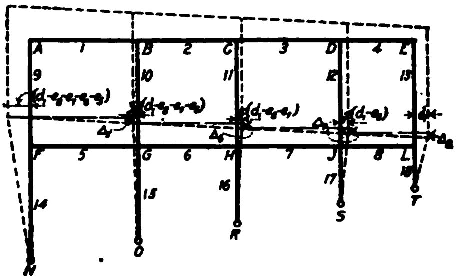


FIG. 55.

If the rise in temperature is t, then for a material having a coefficient of expansion of C_t, the increase in a length l is $+ C_t tl$. Its value is negative for a drop in temperature. This increase in length should be computed and tabulated for each member.

Suppose for the present that the column LT is held vertically during a rise in temperature. Then the horizontal movement of any joint to the left of L, will equal the elongation occurring between L and that joint. LT does not really remain vertical, however, since L will move to the right an amount d_1, dependent upon the elastic rigidity of the frame. Hence the horizontal movement of any joint to the left of L, will be equal to d_1 minus the elongation occurring between L and that joint. The shifting of the frame causes flexure in the columns, which in turn develops horizontal reactions at the bases. The sum of these horizontal reactions must equal zero.

The moment equations for the frame are as follows:

$$M_{AF} = 2EK_9(2\theta_A + \theta_F - 3n_9 d_2) \qquad M_{DE} = 2EK_4(2\theta_D + \theta_E - 3n_4\Delta_8)$$
$$M_{AB} = 2EK_1(2\theta_A + \theta_B - 3n_1\Delta_5) \qquad M_{ED} = 2EK_4(2\theta_E + \theta_D - 3n_4\Delta_8)$$
$$M_{BA} = 2EK_1(2\theta_B + \theta_A - 3n_1\Delta_5) \qquad M_{EL} = 2EK_{13}(2\theta_E + \theta_L - 3n_{13}d_2)$$
$$M_{BG} = 2EK_{10}(2\theta_B + \theta_G + 3n_{10}d_2) \qquad M_{LE} = 2EK_{13}(2\theta_L + \theta_E - 3n_{13}d_2)$$
$$M_{BC} = 2EK_2(2\theta_B + \theta_C - 3n_2\Delta_6) \qquad M_{LJ} = 2EK_8(2\theta_L + \theta_J - 3n_8\Delta_8)$$
$$M_{CB} = 2EK_2(2\theta_C + \theta_B - 3n_2\Delta_6) \qquad M_{LT} = 3EK_{18}(\theta_L - n_{18}d_1)$$
$$M_{CH} = 2EK_{11}(2\theta_C + \theta_H - 3n_{11}d_2) \qquad M_{JL} = 2EK_8(2\theta_J + \theta_L - 3n_8\Delta_8)$$
$$M_{CD} = 2EK_3(2\theta_C + \theta_D - 3n_3\Delta_7) \qquad M_{JD} = 2EK_{12}(2\theta_J + \theta_D - 3n_{12}d_2)$$
$$M_{DC} = 2EK_3(2\theta_D + \theta_C - 3n_3\Delta_7) \qquad M_{JS} = 3EK_{17}[\theta_J - n_{17}(d_1 - e_8)]$$
$$M_{DJ} = 2EK_{12}(2\theta_D + \theta_J - 3n_{12}d_2) \qquad M_{JH} = 2EK_7(2\theta_J + \theta_H - 3n_7\Delta_7)$$

$$M_{HJ} = 2EK_7(2\theta_H + \theta_J - 3n_7\Delta_7)$$
$$M_{HC} = 2EK_{11}(2\theta_H + \theta_C - 3n_{11}d_2)$$
$$M_{HR} = 3EK_{16}[\theta_H - n_{16}(d_1 - e_8 - e_7)]$$
$$M_{HG} = 2EK_6(2\theta_H + \theta_G - 3n_6\Delta_6)$$
$$M_{GH} = 2EK_6(2\theta_G + \theta_H - 3n_6\Delta_6)$$
$$M_{GB} = 2EK_{10}(2\theta_G + \theta_B - 3n_{10}d_2)$$
$$M_{GF} = 2EK_5(2\theta_G + \theta_F - 3n_5\Delta_5)$$
$$M_{FG} = 2EK_5(2\theta_F + \theta_G - 3n_5\Delta_5)$$
$$M_{FA} = 2EK_9(2\theta_F + \theta_A - 3n_9d_2)$$
$$M_{FN} = 3EK_{14}[\theta_F - n_{14}(d_1 - e_8 - e_7 - e_6 - e_5)]$$
$$M_{FN}n_{14} + M_{GO}n_{15} + M_{HR}n_{16} + M_{JS}n_{17} + M_{LT}n_{18} = 0$$
$$M_{AF} + M_{FA} + M_{BG} + M_{GB} + M_{CH} + M_{HC} + M_{DJ} + M_{JD} + M_{EL} + M_{LE} = 0$$
$$M_{GO} = 3EK_{15}[\theta_G - n_{15}(d_1 - e_8 - e_7 - e_6)]$$

General equations:

(1) $(4K_1 + 4K_9)\theta_A + 2K_1\theta_B + 2K_9\theta_F - 6K_9n_9d_2 = 6K_1n_1\Delta_5$

(2) $2K_1\theta_A + (4K_1 + 4K_2 + 4K_{10})\theta_B + 2K_2\theta_C + 2K_{10}\theta_G - 6K_{10}n_{10}d_2 = 6K_2n_2\Delta_6 + 6K_1n_1\Delta_5$

(3) $2K_2\theta_B + (4K_2 + 4K_3 + K_{11})\theta_C + 2K_3\theta_D + 2K_{11}\theta_H - 6K_{11}n_{11}d_2 = 6K_2n_2\Delta_6 + 6K_3n_3\Delta_7$

(4) $2K_3\theta_C + (4K_3 + 4K_4 + 4K_{12})\theta_D + 2K_4\theta_E + 2K_{12}\theta_J - 6K_{12}n_{12}d_2 = 6K_4n_4\Delta_8 + 6K_3n_3\Delta_7$

(5) $2K_4\theta_D + (4K_4 + 4K_{13})\theta_E + 2K_{13}\theta_L - 6K_{13}n_{13}d_2 = 6K_4n_4\Delta_8$

(6) $2K_{13}\theta_E + (3K_{18} + 4K_8 + 4K_{13})\theta_L + 2K_8\theta_J - 3K_{18}n_{18}d_1 - 6K_{13}n_{13}d_2 = 6K_8n_8\Delta_8$

(7) $2K_{12}\theta_D + (3K_{17} + 4K_7 + 4K_8 + 4K_{12})\theta_J + 2K_7\theta_H + 2K_8\theta_L - 3K_{17}n_{17}d_1 - 6K_{12}n_{12}d_2 = 6K_7n_7\Delta_7 + 6K_8n_8\Delta_8 - 3K_{17}n_{17}e_8$

(8) $2K_{11}\theta_C + (3K_{16} + 4K_6 + 4K_7 + 4K_{11})\theta_H + 2K_6\theta_G + 2K_7\theta_J - 3K_{16}n_{16}d_1 - 6K_{11}n_{11}d_2 = 6K_6n_6\Delta_6 + 6K_7n_7\Delta_7 - 3K_{16}n_{16}(e_7 + e_8)$

(9) $2K_{10}\theta_B + (3K_{15} + 4K_5 + 4K_6 + 4K_{10})\theta_G + 2K_5\theta_F + 2K_6\theta_H - 3K_{15}n_{15}d_1 - 6K_{10}n_{10}d_2 = 6K_5n_5\Delta_5 + 6K_6n_6\Delta_6 - 3K_{15}n_{15}(e_6 + e_7 + e_8)$

(10) $2K_9\theta_A + (3K_{14} + 4K_5 + 4K_9)\theta_F + 2K_5\theta_G - 3K_{14}n_{14}d_1 - 6K_9n_9d_2 = 6K_5n_5\Delta_5 - 3K_{14}n_{14}(e_5 + e_6 + e_7 + e_8)$

(11) $K_{14}n_{14}\theta_F + K_{15}n_{15}\theta_G + K_{16}n_{16}\theta_H + K_{17}n_{17}\theta_J + K_{18}n_{18}\theta_L - d(K_{14}n_{14}^2 + K_{15}n_{15}^2 + K_{16}n_{16}^2 + K_{17}n_{17}^2 + K_{18}n_{18}^2) = -\left[e_8 \sum_{14}^{17} Kn^2 + e_7 \sum_{14}^{16} Kn^2 + e_6 \sum_{14}^{15} Kn^2 + e_5(K_{14}n_{14}^2)\right]$

(12) $6K_9\theta_A + 6K_{10}\theta_B + 6K_{11}\theta_C + 6K_{12}\theta_D + 6K_{13}\theta_E + 6K_9\theta_F + 6K_{10}\theta_G + 6K_{11}\theta_H + 6K_{12}\theta_J + 6K_{13}\theta_L - 2d(K_9n_9 + K_{10}n_{10} + K_{11}n_{11} + K_{12}n_{12} + K_{13}n_{13}) = 0$

The left-hand side of the above equations is identical to that of the equations in Table I. All values on the right-hand side are known as soon as one assumes a given change in temperature and computes the corresponding changes in length. They may be put into a separate column, similar to Case I, Table I, for instance. Solution of this column may be made after some one other case has been solved. Thus, in the problem on page 635, if a column for temperature change is added to the right-hand side, its solution could be made by following the procedure noted in the column headed "Equation No." at the extreme left of the table.

In the foregoing equations, it was assumed that all values of Δ were positive. For instance, before the rise in temperature, F and G were on the same level. After the temperature change the final position of G is *below* the final position F by an amount equal to $+\Delta_5$, since to attain this sloped position the member FG would have to rotate in a *positive* direction about either F or G. Suppose, however, that GO for a given case is longer than FN. Then the final position of G would be *above* the final position of F by an amount equal to $-\Delta_5$. Hence, for such a case, all terms involving Δ_5 in equations (1) to (11), above, would become negative. The following rule may therefore be stated: Beginning at the left end of the frame after distortion from temperature change has taken place, if a normally horizontal member slopes downward to the right, Δ for that member is positive, and its value is equal to the vertical displacement of its right end. If the member slopes upward to the right, Δ for that member is negative, and its value is equal to the vertical displacement of its right end. This relation between the final positions of points holds for either a rise or fall of temperature. If all columns of the structure have the same length, then all values of Δ are zero.

The value of e in the foregoing equations would become negative for a fall of temperature.

8k. Effect of Fixed Bases.—Certain modifications made in the foregoing equations will permit them to be used for the analysis of frames of the type which they affect, but with fixed, instead of hinged bases. Take, for instance, a portion of the frame of Type I$_4$ (see page 629). There is no rotation at N (Fig. 56) when N is perfectly fixed, hence

$$M_{FN} = 2EK_{14}\,(2\theta_F - 3n_{14}d_1)$$

This would replace that for M_{FN} given on page 629. Equations precisely similar, except for subscripts, may be written for the moment in the top of all other lower columns of the frame. These values of moment would replace corresponding ones in the equations on pages 629 and 630. Having made these changes in the moment equations, the general conditional equations will be affected thereby, and must be revised accordingly.

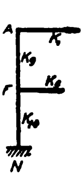


Fig. 56.

After the conditional equations have been solved, the moments at the ends of each member may then be found. The moment at the base of the fixed column FN above, is

$$M_{NF} = 2EK_{14}(\theta_A - 3n_{14}d_1)$$
$$= \tfrac{1}{2}(M_{FN} + 6EK_{14}n_{14}d_1)$$

Having found the moment at the ends of each column, the point of inflection may readily be obtained; and, supposing a hinge to be introduced at the point of inflection, the horizontal reaction at this "hinge"—or shear on the column—may be found as for a frame with hinged bases. The vertical reactions may likewise be found as for hinged bases.

The same modification to take into account fixity of the base of supporting columns may be applied to any other of the foregoing types of viaduct frame.

8l. Viaduct Bent.—The cross-frame or bent, provides the lateral stiffness for the viaduct structure, in addition to being the supporting unit. It should be designed to withstand: (1) the dead load of the entire structure; (2) the direct and flexural stresses set up in the "columns" of the viaduct frame, due to live load, as determined in the foregoing discussion; (3) lateral forces of wind, and centrifugal forces on curves; (4) lateral expansion; (5) moments due to loads on floor girder; (6) moments due to overhanging floor beams carrying walks, etc.

For bents having a batter steeper than 1 to 6, the procedure of analysis for the first five cases of loading given above is identical with that for the viaduct frame of similar dimensions. Frames with excessive batter, however, require consideration of the vertical component of lateral displacement due to lateral forces or to underbalanced loads (see assumptions, page 628). The relation is, of course, purely trigonometrical, so that the method of attack is as already outlined, save for the adding of this component of lateral movement to the lateral displacement of the ends of horizontal members.

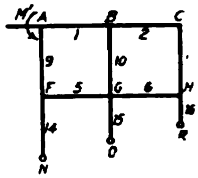


Fig. 57.

When the bent is symmetrical and carries lateral loads at the joints, there is a point of inflection of moment in each horizontal member at the point where it is intersected by the axis of symmetry. For the analysis of such a frame see "Modern Framed Structures" by Johnson, Bryan and Turneaure, Part II, Arts. 283–4.

The sixth case of loading—that of a known moment applied at a joint may be treated by a slight modification to the foregoing equations. Fig. 57 shows a bent or cross-frame of Type III, of viaduct frames, with a cantilever at A applying a known moment M'. Referring now to page 632, the equations there given for M_{AB} and M_{AF} are equally true for this case. It will be noted, however, in forming equation (1) on page 633, where no load is acting on AB, that the sum of the moments in the A-ends of members meeting at A is zero. In this case where M' is acting, equation (1) would now equal M', since the moments in the members must offset the external moment at any joint. M' as shown in the figure is negative on the right-hand side of the equation, since the moment required to resist it is clockwise.

CANTILEVER BRIDGES

A type of bridge which in appearance is a concrete arch, but which in reality is composed of balanced cantilevers, is shown in Figs. 58 to 61 inclusive. A structure cf this type can be made with longer spans than the ordinary girder and is suited to locations where the real arch would be exceedingly costly on account of unsatisfactory foundation conditions.

9. Theory of Design.—A pier and the cantilever arms on each side compose a unit, the arms being balanced for dead load and for full live load. The piers are designed for bending

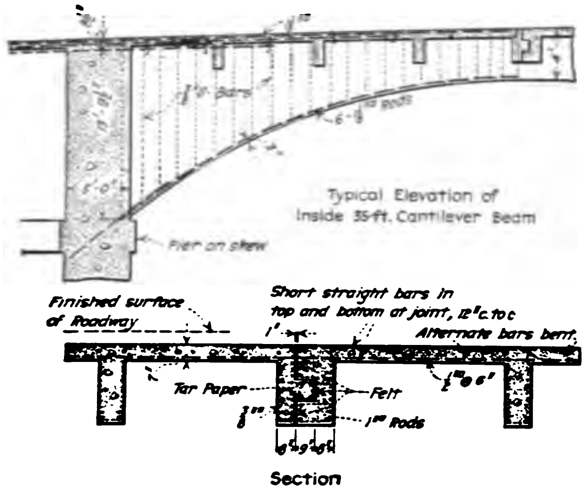


FIG. 58.—Details of Hopple Street viaduct, Cincinnati, Ohio.

due to the maximum eccentric load that can be applied, and considering the load on only one of the cantilever arms at a time. The pier footings are designed so that the pressure on the base of a pier due to this same eccentric loading will not cause an intensity greater than the unit bearing value of the soil.

10. Examples of Cantilever Bridges.—The viaduct shown in Fig. 58 consists of twenty-five skewed spans, each span comprising two curved cantilever arms supported on reinforced-

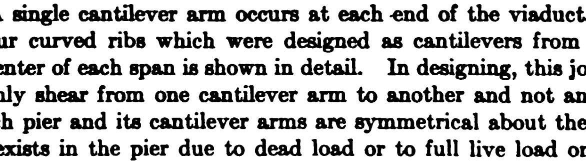


FIG. 59.—Half-elevation and detail of expansion joint of arch-shaped cantilever bridge over Rouge River, Wayne County, Mich.

concrete piers. A single cantilever arm occurs at each end of the viaduct. Each cantilever arm comprises four curved ribs which were designed as cantilevers from the skewed piers. The joint at the center of each span is shown in detail. In designing, this joint was considered as transmitting only shear from one cantilever arm to another and not any bending or arch action. Since each pier and its cantilever arms are symmetrical about the center line of the pier, no bending exists in the pier due to dead load or to full live load on both cantilevers. Piers, footings, and piling were designed to withstand the overturning effect produced by the loading of a single cantilever with the full live load.

The expansion joint at the center of the structure, shown in Fig. 59 is entirely different from that employed in the bridge just described. It should be noted, however, that the

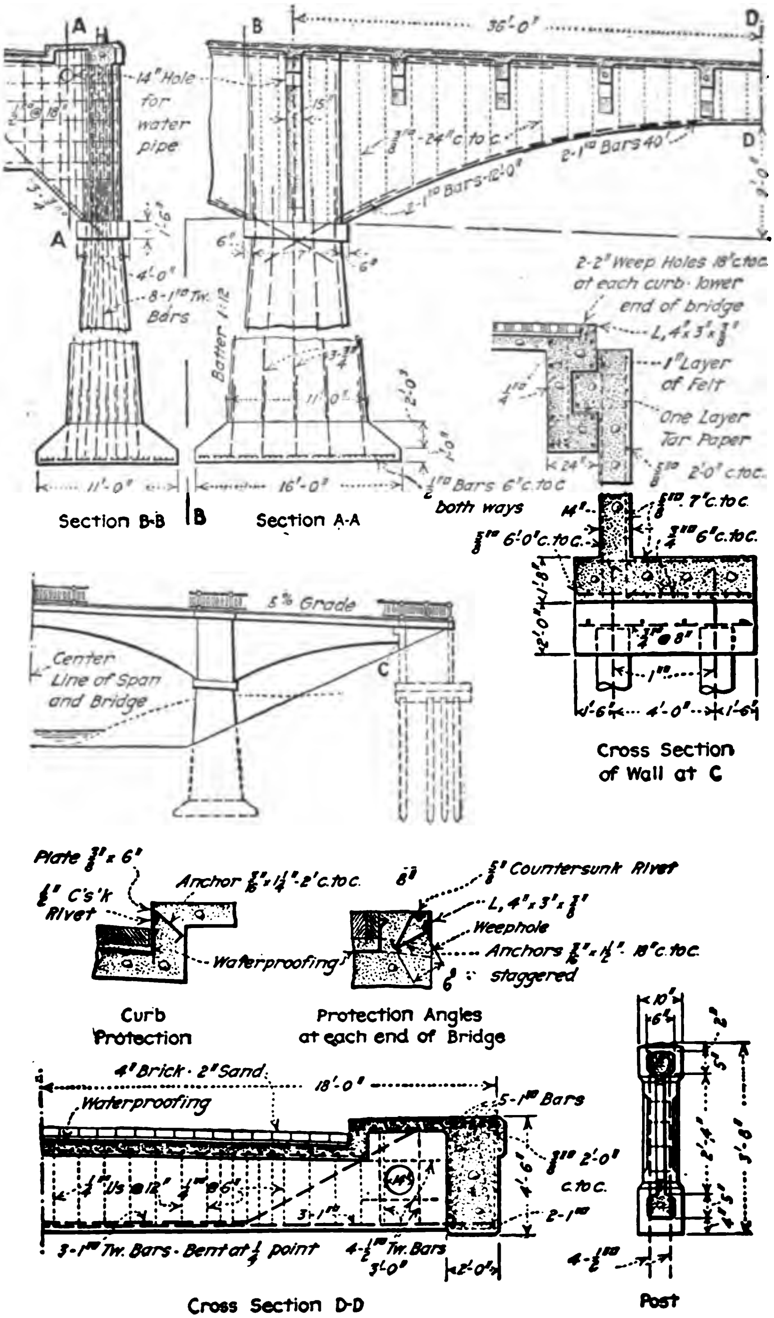


FIG. 60.—Details of Runnymede Avenue bridge over West Fork Creek, Cincinnati, Ohio.

result is accomplished—that is, that the two abutting arms are permitted to move longitudinally, but not laterally or vertically. The joint consists of two 3-ft. piers of 2¼-in. steel shafting each inserted in two 1½-ft. pieces of 3-in. gas pipe embedded in the concrete. End joints were made by placing sheets of three-ply tar paper on top of the abutments before the end cantilevers were poured, thus permitting a slight movement of the ends of the structure under changes of load and temperature. There was no apparent deflection at any of the expansion joints due to live load.

The joints at the ends of the bridge shown in Fig. 60 are of the same type as employed in the viaduct spans of Fig. 58. The absence of such a joint at the center of the middle span is, however, the principal feature. In spite of this continuity between piers, no account was taken of continuous action upon supports and the bridge was designed in the same manner as the cantilever viaduct previously referred to. In fact, it has been found that a joint at the center of span is an unnecessary refinement in cantilever-bridge design, and might be omitted.

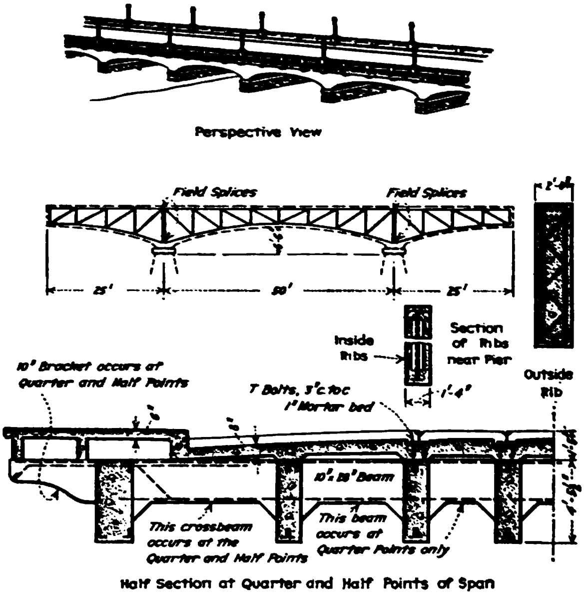


FIG. 61.—Details of Washington Street bridge, Norwalk, Conn.

The bridge was designed and constructed so as not to rest on the abutments at all, the abutments being used merely to hold back the earthfill at each end and to serve as anchorage for the end cantilevers. A structure of this type with end openings closed by earthfill has all the appearances of a real arch. In such a case abutments are not needed.

If a number of bridges similar to the bridge of Fig. 60 were placed end to end, the result would be essentially a structure such as shown in Fig. 61. It should be noted that the Norwalk bridge is continuous in sections of maximum length of 100 ft. The structural-steel work was designed to be self-supporting during erection and to carry the erection stresses of the forms and the fluid concrete in the ribs, cross-girders, and sidewalk brackets. Although a deflection of ½ in. at the free ends of the cantilevers was anticipated, a deflection of only ⅟₁₆ to ⅛ in. actually resulted due principally to the rigidity of the forms and to the fact that the concrete was continuously setting during the process of placing. The combined steel and concrete in the ribs was proportioned to carry the roadway slab, paving, and all live loads. The trusses were proportioned to carry all shear not safely taken by the concrete but were not proportioned to carry all the tension developed by the bending moment since extra horizontal rods were embedded in the concrete adjacent to the top chords of the trusses.

SECTION 15

CONCRETE FLOORS AND ABUTMENTS FOR STEEL BRIDGES[1]

1. Concrete Floors on Steel Bridges.—Within recent years it has become the general opinion among railroad engineers that a ballasted solid floor is the most satisfactory form of floor for steel bridges. Perhaps the best type of such a floor is the reinforced-concrete slab resting directly on the steel-floor members.

Figs. 1 and 2 show details of reinforced-concrete deck slabs for plate-girder spans. The slab floors are seen to rest directly upon the top flange of the steel girders. Comparison of the two designs is of value since they show a wide difference in the concrete details and in the arrangement of the reinforcement. The slabs are usually made at some convenient location and hoisted into place when sufficiently cured. Before adding the ballast, the upper surface of the slabs is thoroughly waterproofed by painting with tar paint. Drain holes are placed in such a position as to keep the drip clear of the steel members.

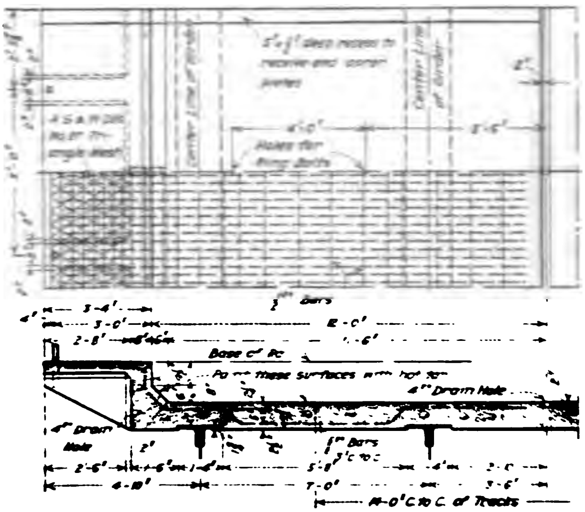


Fig. 1.—Reinforced-concrete deck for steel-girder spans on D. M. & N. Ry.

A reinforced-concrete floor for a through plate-girder bridge is shown in Fig. 3. The concrete of the floor slab is seen to extend up on the sides to form curbs, and these curbs extend entirely around the gusset plates. Steel trough floors filled with concrete are also used in through plate-girder bridges.

Steel I-beams encased in concrete and supporting a reinforced-concrete floor slab is the most common type of highway bridge with steel-floor members. On account of the ease with which forms may be constructed to hold the concrete, this bridge for short spans is sometimes

[1] For treatment of concrete piers for steel bridges see "Foundations of Bridges and Buildings," by Jacoby and Davis, or "Structural Engineers' Handbook," by Ketchum.

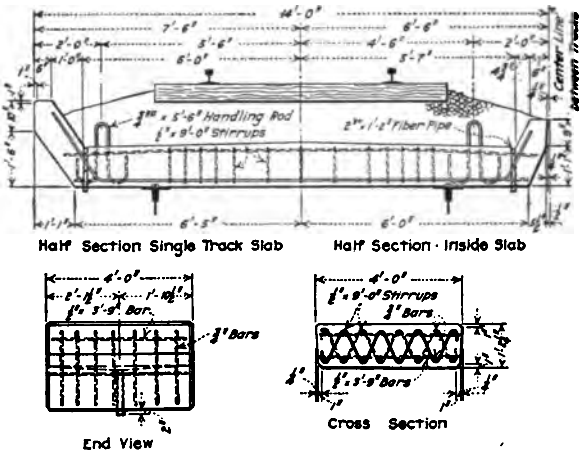


FIG. 2.—Standard reinforced-concrete slab for deck girders of C. M. & St. P. Ry.

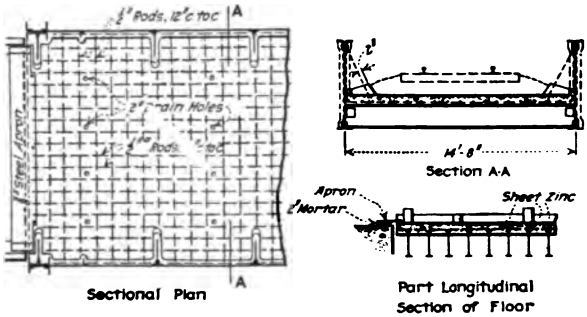


FIG. 3.—Reinforced-concrete floor for through plate-girder bridge, C. B. & Q. R. R.

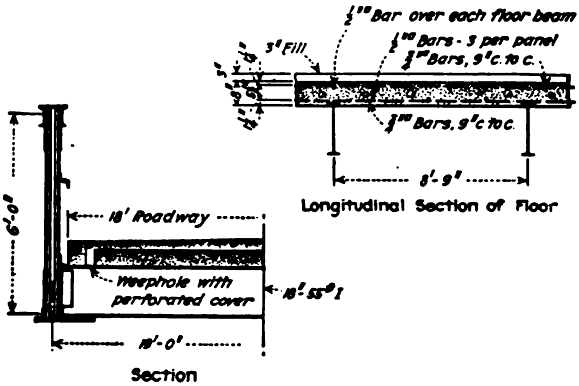


FIG. 4.—Concrete slab floor for highway steel truss span, Iowa Highway Commission.

used in preference to slab bridges of all concrete. The only disadvantage of this bridge is in point of economy.

Timber floors for highway bridges are not in great favor at the present time. Since the expense of maintaining wood floors is considerable, the engineers of a number of highway commissions design practically all-steel bridges with concrete floors covered by a wearing surface of gravel or macadam. For exceedingly light traffic on country bridges, driving is sometimes allowed directly on top of the floor slab, making an allowance of at least 1 in. in the thickness of the slab for wear and cutting transverse grooves to prevent slipping. Figs. 4 and 5 show typical designs of reinforced-concrete floors for steel-truss spans.

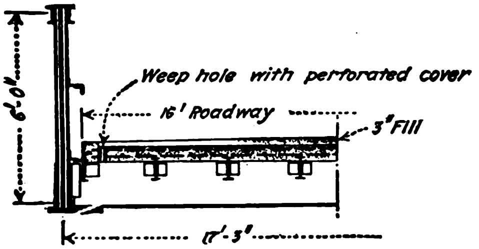


Fig. 5.—Concrete slab floor for highway steel-truss span, Iowa Highway Commission.

2. Abutments for Steel Bridges.—An abutment in its simplest form is a retaining wall terminating the approach embankment to a bridge, and provided with a bridge seat for the end of the first span to bear upon. The discussion here given is limited to those abutments of plain or reinforced concrete which receive a vertical downward bearing from the bridge. Abutments for arches are treated in Arts. 4 and 39, Sect. 16.

3. Types of Abutments.—Bridge abutments of concrete may be classified according to general form as follows:

1. Pier abutments.
2. Wing abutments.
3. Cellular abutments.
4. U-abutments.
5. T-abutments.
6. Buried pier abutments.
7. Skeleton and arched abutments.

The design and advantages of each form will be discussed separately.

4. Pier Abutments of Plain Concrete.—This is the simplest form of abutment (Fig. 6). Since many of the other forms are elaborations of this one, its stability will be studied in detail.

The thrust P of the earth against the back is found from the method of equivalent surcharge h', as described on page 581. The force F is due to frost expansion, and depends upon the depth to which the ground or ballast may freeze. It can at best only be estimated, on the basis of ice pressures.

The vertical load of the trusses or girders with their live load, cause two forces B bearing on the bridge seat. The dimensions s, d, and n depend upon the structure supported. The intensity of B depends upon the span and loading, and is taken from the design of the superstructure. The force T may either be caused by the tractive effort of the train on the bridge; by braking of the train; or by temperature changes not wholly adjusted by poorly operating expansion joints. All of these forces are determined from the design of the superstructure.

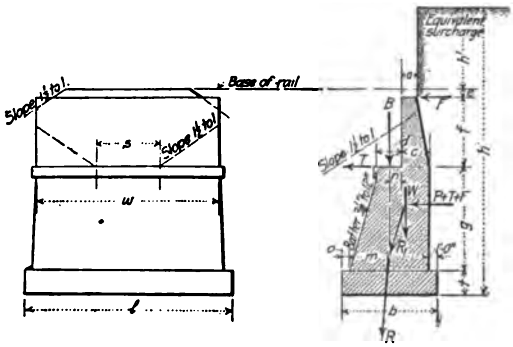


Fig. 6.—Plain concrete-pier abutment.

The best form of abutment is that which puts the resultant pressure very close to the center of the base. This requirement is particularly desirable in yielding soil, since the vibratory loads will nearly always cause settlement. Many abutments have tipped forward notice-ably because the pressure caused a non-uniform pressure on the soil, and hence a non-uniform settlement.

The back wall should have a thickness a at its top of at least 12 in., and more for railway bridges. Owing to the uncertainty of actual freezing forces, the thickness c at the bottom of the back wall may be taken as $0.4f$ to $0.45f$, the larger value for railway structures. These thicknesses have given good results in practice for back walls not reinforced.

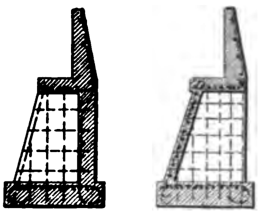
The length of the back wall should be such that when material spills around its ends at a slope of 1½ to 1, it should not strike the pedestal of the bearing shoe. The batter of the back face of the back wall should be about 2 in 12. The height f is of course set by the superstructure. The length w of the bridge seat is governed by the overall dimensions of the structure being supported. J. E. Grenier[1] specifies a minimum thickness of the coping as 18 in. for railway, and 12 in. for other bridges. He also specifies the distance d to be "at least 12 in. greater than required for the bedplates of steel superstructures."

It is a general rule that, below the coping, the plain concrete abutment shall have a thickness at any point of from 0.4 to 0.5 of the depth of the point below the base of rail. The thickness m should be such that the compressive unit stress at the forward edge is within that allowed for plain concrete. This stress is determined precisely as though the wall rested on the soil at that plane.

The base slab should have a forward projection o sufficient to keep the forward soil pressure within the allowable range. The thickness of such a projection will determine the thickness t of the slab. It is found the same as that for the toe of a reinforced retaining wall. The whole slab should preferably be reinforced.

The body of the wall should be well bonded by dowel rods to the base slab, so that it can neither rock nor slide upon it.

5. Pier Abutments of Reinforced Concrete.—Pier abutments of reinforced concrete are divided into two general groups: (1) buttressed, and (2) counterforted. Fig. 7a shows the general form of a buttressed abutment. The back wall is designed as a cantilever wall, while below the bridge seat, the back slab is designed as a continuous slab spanning horizontally the space between buttresses. The back slab must be well anchored to the base slab to prevent rocking forward on the base; and ample steel should be provided at the junction of the upper portion of the wall with the base slab to prevent failure in shear (sliding) on the plane of the top face of the base slab. Reinforcing in the body of the buttress is very light, and is necessary only to prevent cracks.



(a) (b)

FIG. 7.—Reinforced-concrete pier abutments.

Fig. 7b shows a counterforted pier abutment. The back wall, as before, is designed as a cantilever wall. The whole structure below the bridge seat is designed as a counterfort retaining wall, and the discussion of the design of that wall will apply directly here.

It is of course the best arrangement to place the counterfort or buttress immediately beneath the bridge seat, and at such other points as is necessary to provide against the thrust of the earth.

6. Wing Abutments.—If wing walls are extended out beyond the ends of the bridge seat of a pier abutment, the structure is called a wing abutment. The wings may either be on a line with the face of the abutment, or deflected backward from the face (Fig. 8). The straight wing is used where dry crossings are made, as for instance, street or railroad crossings. They usually extend to the foot of the supported embankment, and are caped about 2 ft. above the surface of the slope. When the toe of the slope at the end of the wing wall requires protection from stream flow, deflected wings are usually built. They are usually put at 30 deg. with the face wall.

The wing walls are designed for earth thrust, as in a retaining wall. The best design is

[1] GRENIER'S, "General Specifications for Bridges."

one which gives the same intensity of pressure over the base as is developed under the body of the abutment; and it is particularly desirable that the resultant pressure on the base of the wing wall cuts at relatively the same point as that of the body of the wall. Expansion joints are often placed at the junction of the wing with the body. Such joints should be lock-joints so that uneven tipping or settling will not cause unsightly offsets to develop between the wings and the body. Where no joints are provided, especial attention should be paid to the similarity of pressure on the entire foundation, as noted above.

Because of the desire not to obstruct stream flow, reinforced-concrete wing abutments are usually of the counterfort type rather than the buttress type. Such a wall is shown in Fig. 9. The body of the wall is designed like the same form of pier abutment. The counterforts of the wing walls may either be parallel to the track, or normal to the wings.

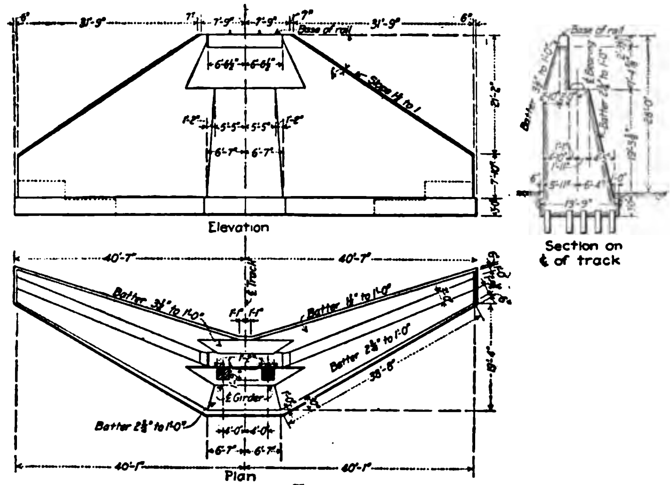


Fig. 8.—Plain concrete wing abutment.

7. Cellular Abutments.—The cellular abutment, like the cellular retaining wall, consists of a box-shaped pocket buried in the fill, to increase stability against overturning. A modified form consists of a pier abutment, with wings running normal to the face of the abutment, and a tie wall across the outstanding ends of these wings. Such an abutment is more costly than the pier form, and has not been in common use.

8. U-abutments.—A very common form of abutment, called the U-abutment because of its shape, is shown in Fig. 10. It consists of the face wall with bridge seat, and two wings at right angles to the face wall. The pocket thus formed has a floor formed by the footing slab. Such a type of abutment is very useful at the end of a moderately high fill which has a long slope in the direction of the track. This form of abutment is usually of plain concrete and is cast in one piece. Its total length is generally about 1½ times its height.

The constituent parts of the U-abutment are designed like a pier abutment or retaining walls. The outside faces are either vertical or slightly battered. The inside faces are either heavily battered or stepped, the latter being the more common. The fill is allowed to slope from the top of the outer ends of the wing walls in all directions away from the track at about 1½ to 1. The inner faces should be battered 2 in 12 for the upper 3 ft., to provide for frost expansion. The whole abutment should be thoroughly drained.

9. T-abutments.—This form of abutment is shown in Fig. 11. It is of practically the same cost as the U-abutment. The front face and stem are usually of plain concrete and the floor over the stem of reinforced concrete. The wall is secure against tipping forward.

10. Buried-pier Abutments.—Usually when extremely high abutments are required, or when the abutment extends to considerable depth for suitable foundation, the earth slope may be allowed to spill freely around the abutment. For this purpose a tall pier may be used, with short wing walls to protect the bridge seat from the earth slope. An abutment of this type is called a buried-pier abutment. It should be designed for the vertical reaction of the span,

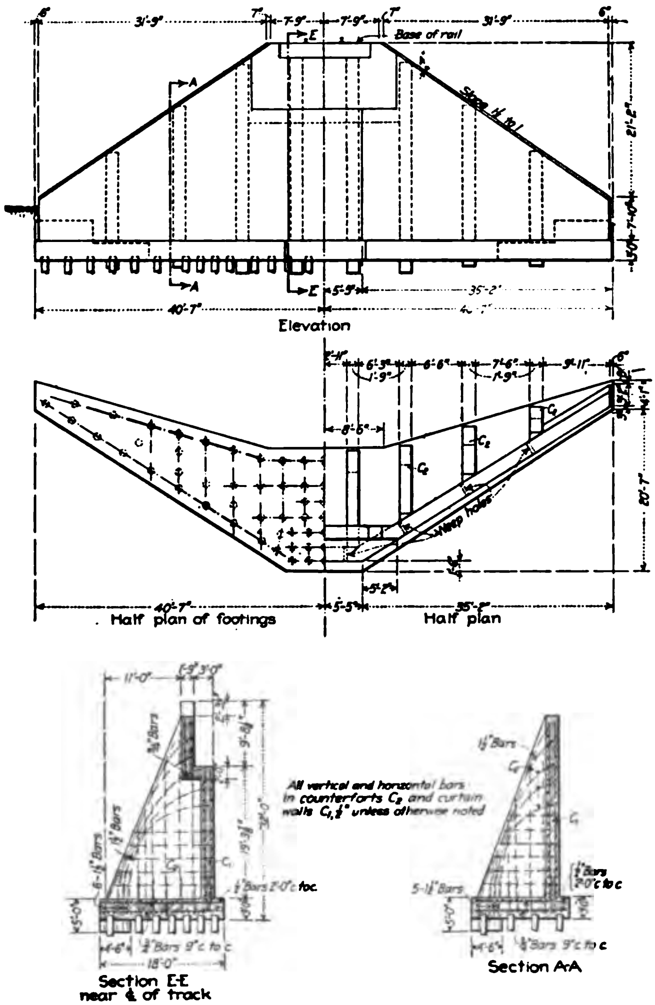


Fig. 9.—Reinforced-concrete counterfort wing abutment.

and the lateral earth and traction forces. It is especially desirable that the resultant pressure on the base should pass through its center, particularly when the subsoil is yielding. The economy of this abutment makes it more desirable than the wing abutment for high embankments.

11. Skeleton and Arched Abutments.—A large number of special forms of abutments have been developed recently, the most notable from an economic standpoint being the skeleton and arched abutments. The skeleton abutment consists of a very heavy viaduct frame of two or more spans, upon the outer end of which is placed the bridge seat. The arched abutment differs only in that it is a series of small arches on high walls. The proper length of these abutments is governed by the cost per additional foot of abutment and of superstructure.

The analysis of the skeleton abutment does not differ from that of a viaduct frame (see Art. 8, Sect. 14). The arched abutment may be analyzed as a system of arches; or more easily by an approximate solution of a rigid viaduct frame of similar proportions, having girders of a section equal to that at the crown of the arches. This is of course approximate; but limiting conditions may be assumed, remembering that if the girder section is assumed too small the

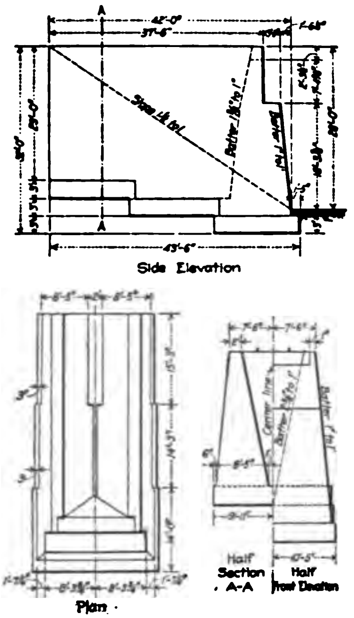


FIG. 10.—U-abutment.

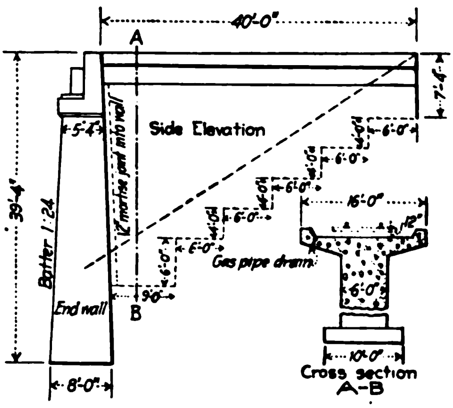


FIG. 11.—T-abutment.

stresses in the columns or bents will be too large, and *vice versa*. The deck girders may be analyzed as though continuous, and then their under sides arched, for rigidity and for appearance. Two very important considerations should be borne in mind, namely, tractive or braking forces, and temperature variation.

12. Care in Constructing Abutments.—In abutments, as in retaining walls, special care should be taken to obtain a structure that is impervious to water. Fills and embankments accumulate water, which will seek outlet through the abutment. Unless the wall is of proper density throughout, without reliance on a smooth face, disintegration along percolating planes and later partial or total destruction is sure to take place.

For valuable material on the economic selection and design of abutments see J. H. Prior, *Proc.* A. R. E. A., vol. 13 (1912), page 1085.

SECTION 16

ARCHES

GENERAL DATA

1. Definitions.—The following are some of the common technical terms applied to the various parts of an arch (see Figs. 1*A* and 1*B*).

Soffit.—The under or concave surface of an arch.

Back.—The upper or convex surface of an arch.

Skewback.—The surface upon which the end of the arch rests. This definition applies particularly to the stone or brick arch since the surface mentioned is purely imaginary in the case of the concrete arch. The term, however, is useful in concrete-arch analysis.

Springing Line.—The line in which the soffit meets pier or abutment—that is, the inner edge of the skewback.

Span.—The horizontal distance between springing lines measured parallel to the center line of roadway.

Intrados.—The line of intersection of the *soffit* with a vertical plane taken parallel to the center line of roadway.

Extrados.—The line of intersection of the *back* with a vertical plane taken parallel to the center line of roadway.

Crown.—The highest part of the arch ring.

Rise.—The height of intrados at crown above level of springing lines.

Haunch.—The portion of the arch ring about midway between the springing line and crown.

Spandrel.—The space between the back of arch and the roadway.

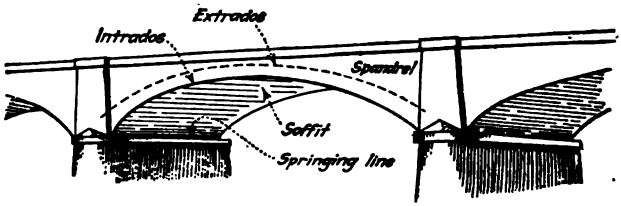


FIG. 1*A*.

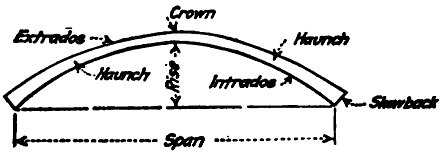


FIG. 1*B*.

Arches are divided into *right* arches and *skew* arches, depending upon the angle made by the springing lines with the center line of roadway. A *right* arch is one that makes this angle exactly 90 deg.

2. Curve of the Intrados.—The form or general outline of an arch is the first consideration in its design. According to the curve of the intrados, arches are usually divided into circular, multi-centered, elliptical, and parabolic. If the intrados is a semicircle, the arch is a semicircular arch; and, if the intrados is less than a semicircle, it is a segmental arch. A multi-centered arch is one in which the intrados is composed of several arcs of circles tangent to each other. Semicircular and semi-elliptical arches are full centered—that is, they spring from horizontal beds—while segmental and parabolic arches spring from inclined beds called skew-backs (see Fig. 1*B*). Multi-centered arches may have beds either inclined or horizontal.

The parabola may be modified for the sake of appearance by short circular curves at its ends. made tangent to the parabola and to the vertical side of the pier or abutment. Minor curves joining the arch soffit to the pier are not effective, however, and should not be considered as part of the arch rise.

2a. Three-centered Curve.—A segmental arch cannot often be used to advantage, for it seldom can be made to fit the line of pressure. The three-centered arch is perhaps the most common for solid-spandrel construction and gives a pleasing and generally an economical design. The formula for the radius of a circular segment when the chord distance (span) and mid-ordinate (rise) of the segment are known is as follows:

$$\text{Radius} = \frac{(\tfrac{1}{2}\text{ chord})^2 + (\text{mid-ordinate})^2}{2 \times \text{mid-ordinate}}$$

Following are the formulas for the radii of a three-centered curve (see Fig. 2).

$$R = \frac{x^2 + y^2}{2y}$$

$$r = \frac{1}{2} \cdot \frac{\overline{AF}^2 + \overline{FE}^2}{FE \cos\theta - AF \sin\theta}$$

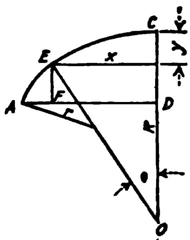


Fig. 2.

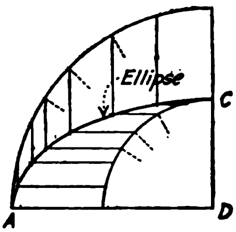


Fig. 3.

2b. Semi-ellipse.—The multi-centered curve can be made to approximate an ellipse. Entirely graphical methods of obtaining the semi-ellipse and corresponding approximate multi-centered curves are as follows:

Let AD and CD (Fig. 3) be the semi-major and semi-minor axes, respectively, of the ellipse. With D as a center, draw circular arcs with radii AD and CD. From points where a common radius intersects the two circular arcs, draw vertical and horizontal ordinates. The intersection of these ordinates gives one point on the ellipse. Other points may be found in a similar manner.

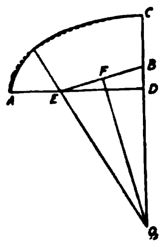


Fig. 4.

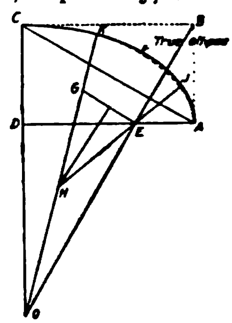


Fig. 5.

Suppose now that a three-centered intrados is required which approximates a true ellipse. The form of the true ellipse is first drawn by the method given above and is shown in Fig. 4 by the full line. The approximate form, shown dotted, is what is required. Assume any two equal distances CB and AE more than one-half of the semi-minor axis. Join BE and bisect the line BE at F. Through F draw a perpendicular to BE, intersecting the line CD at O. The two points O and E will be centers of two circular arcs which will form an approximate ellipse. By first selecting the position of the point E so that the circular arc described from E as center will conform as closely as possible with the true ellipse, satisfactory curves will easily be found.

The method of drawing an approximate ellipse using a five-centered curve will now be explained. In order to have a check on the work, it is advisable to first draw the form of the true ellipse by the method given above. Let AD and CD (Fig. 5) be the given semi-axes. Join A and C, and through B draw a perpendicular to AC, determining E and O, two of the centers. From O, with OC as radius, draw an arc CK as long as thought suitable, and join K with O. Make KG equal to AE. Join E and G. At the center of EG draw a perpendicular to EG, and note its intersection H with KO. From H, with radius HK, draw an arc to HE (extended); and from E, with EA as radius, complete the curve.

2c. Parabola.—The equation of the parabola, Fig. 6, is as follows:

$$y = \frac{x^2 b}{a^2}$$

Divide the line OR into any number of convenient equal parts, and number the points of division 1, 2, 3, etc., beginning at the point nearest O. Then to find the values of y, for the various abscissas x, the numbers 1, 2, 3, etc., should be inserted in the above equation for values of x, and the total number, which in the illustration is 6, should be inserted for the value of a.

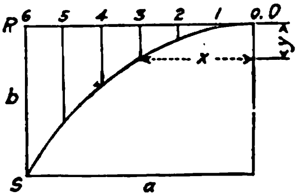


FIG. 6.

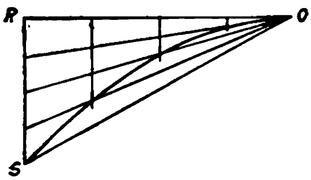


FIG. 7.

A very simple graphical method of drawing the parabola is to lay off on the vertical line RS, Fig. 7, the same number of equal divisions as are made on the horizontal axis OR, and from O draw radiating lines to the various division points on the vertical axis RS. From the various points on the horizontal line OR draw vertical lines intersecting the radiating lines from O. The points of intersection of these vertical lines with corresponding radiating lines are points on the required parabolic curve.

3. Arrangement of Spandrels.—Arch spandrels may be entirely filled with earth, or they may be left more or less open and the roadway supported on a series of transverse walls, or on a complete superstructure of columns, girders, beams, and slabs. If, as is rarely the case, a heavy or massive appearance is desired in open-spandrel construction, then side curtain walls may be used and all spandrel openings closed. In the open-spandrel type, the arch ring may be either solid or composed of two or more longitudinal ribs.

With filled spandrels, the filling material is held in place laterally by retaining walls which rest upon the arch ring. These retaining walls may be of either the gravity or the reinforced type, or they may consist of thin vertical slabs tied together by reinforced-concrete cross walls. Solid fillings increase the weight of the superstructure and make necessary thicker arch rings and larger foundations. Open-spandrel construction, on the other hand, requires a relatively larger amount of formwork. At the present cost of labor and materials in this country, the filled type of arch spandrel is preferable from the standpoint of economy for all arches of moderate rise with spans less than about 100 ft. and also for flat arches of greater span where the ratio of rise to span is not more than one-tenth. Fortunately, a proper artistic appearance is usually obtained in satisfying these economical requirements.

4. Piers and Abutments.—The springing lines, or springs of an arch, should be located as near the foundation as conditions will permit. This will often make possible a less expensive design for the abutments and, where piers are employed, will reduce the overturning effect on the piers to a minimum.

In the case of long bridges with a series of arches, what are called abutment piers should be placed at frequent intervals (usually every five or six spans) so as to act as an abutment in case of failure of one or more of the arches. This type of pier is made of sufficient thickness to resist the pressure for either arch standing and the other arch removed, and for both arches standing. The ordinary arch pier should be analyzed for one adjacent arch without live load and the other adjacent arch with live load over the whole span.

Arch bridges of four and six spans do not present a desirable appearance. For esthetic effect, an odd number of spans should be selected and the span lengths should decrease each way from the center of bridge.

The depth of arch foundations and the shape of abutments and piers is dependent upon local conditions, and in some difficult cases have to be chosen after thorough study. A certain shape of abutment or pier is first assumed; and this is then reviewed to see that the load upon the ground does not exceed the allowable and that it is well distributed. Great saving is effected in some cases by the use of hollow, or ribbed, abutments and piers.

5. Depth of Filling at Crown.—In making a preliminary design for an earth-filled arch bridge, it is necessary to know approximately the required crown thickness of the arch ring and also the amount of earth filling over the crown. This must be known in order to determine the remaining distance from the crown to the springing line—that is, the available rise for the arch. For highway bridges, a depth of filling including the pavement of from 1 to 2 ft. will be sufficient; but for railroad structures a minimum depth of from 2 to 3 ft. below the ties will be needed in order to form a cushion for the ties, to distribute the load, and to absorb the shock from passing trains.

6. Loads.—The dead weight of the arch ring itself and of the superimposed material constitute usually the principal loads on an arch ring. With open-spandrel construction, the dead loads act vertically upon the arch ring or arch rib through the transverse walls or columns, and are hence definitely known. With filled spandrels, the pressure produced on the arch ring by the earth filling is really inclined and the dead load cannot be so accurately determined.

On flat earth-filled arches, it is better to consider only vertical loads as acting on the arch ring, for the conjugate horizontal forces are small and may be neglected. On earth-filled arches with large rise, the horizontal thrusts become great, especially close to the springing lines, and it may be advisable in some cases to take these horizontal components into account. The omission of these horizontal thrusts, however, is always on the side of safety.

A common assumption for weight of earth fill where the actual value is unknown is 100 lb. per cu. ft. When sand is used, its weight should be taken at 120 lb. Pavement is usually assumed as 12 in. thick and as weighing 150 lb. per cu. ft.

The live load to be used in the investigation of an arch bridge should be the greatest that comes or is liable to come upon the roadway. Each location should be studied and the live load chosen to fit the requirements. For ordinary conditions a standard loading is commonly employed. Wind pressure is considered only on light or exceptionally high structures.

In earth-filled bridges where there is sufficient thickness of filling to distribute the concentrated loads over a considerable area of arch ring, uniform live loads are used in the arch-ring design. City highway bridges are generally designed for 50-ton electric cars and for such bridges, with spans of 200 ft. or more, a uniform load of 1200 lb. per lin. ft. is usually taken on each railway track together with a uniform load of 80 lb. per sq. ft. over the remaining area of roadway and sidewalks. For spans between 100 and 200 ft., the loads are taken proportionally. The loads specified above for city bridges may be reduced by about 20% to apply to the arch rings of light country bridges. The load on each street railway track is generally assumed to cover a width of 9 ft.

In addition to the above loads, city bridges and bridges on thoroughfares likely to be used for heavy hauling should be designed to carry 20-ton trucks, with axles about 10 ft. c. to c., 14 tons on rear axle and 6 tons on front axle; wheels about 5 ft. c. to c.

Because of the permanent character of concrete bridges it may be wise to provide a larger margin for increase of loading than is above suggested, or than is usually allowed in steel-bridge design. Fortunately, in the case of concrete-arch bridges a large increase can be provided for with only a slight increase of expense due, of course, to the controlling influence of the dead load.

Following is an extract from the report of a Committee on Reinforced Concrete Highway Bridges and Culverts, American Concrete Institute, presented at the Annual Convention at Chicago, Feb. 17, 1914:

Class "A" Bridges.—Main thoroughfares leading from large towns. In view of the extensive introduction of the heavy motor trucks and traction engines, and the probable general use of such vehicles in the future, it is recommended that bridges on main thoroughfares and other roads which are likely to be used for heavy hauling, be designed to carry 20-ton trucks, with axles about 10 ft. c. to c., 14 tons on rear axle and 6 tons on fore axle; wheels about 5 ft. c. to c. Outside of the large cities it is recommended that only one such vehicle be assumed to be on the bridge at any one time; the likelihood of more than one being on the bridge, in a position to produce maximum stresses at the same time, is so remote that this assumption is considered safe. It is advised that such very heavy loads be considered as occupying only the ordinary width of the road, about 8 ft. in width and about 35 ft. in length. Congested traffic of heavily loaded wagons or motor trucks will rarely impose a load of more than 100 lb. per sq. ft. over a considerable area. The above-mentioned 20-ton truck gives a load of about 140 lb. per sq. ft., on the area actually occupied, but it is considered extravagant to assume that a large bridge is covered with such heavy loads. One hundred pounds per square foot is thought ample to assume for the loading of spans more than 60 ft. long in designing the trusses or main girders. It is thought to be safe to reduce this assumed load in the case of longer spans, to the following amounts:

Length of span (ft.)	Assumed load (lb. per sq. ft.)
80	90
100	80
125	75
200 and over	70

with all intermediate spans in proportion.

The greatest load that is liable to be imposed on a bridge sidewalk occurs when there is some excitement in the neighborhood which attracts a large crowd, and for which the bridge affords an especially good point of view. In that case the crowd forms a compact mass against the railing, not more than 4 ft. deep, making a load seldom exceeding 100 lb. per sq. ft. over a very considerable space. The remaining portion of the sidewalk may be covered by a moving crowd which can scarcely weigh more than 40 lb. per sq. ft. It may be advisable, sometimes, to so design sidewalk slabs, that if a street car or motor truck accidentally gets upon the sidewalk, it will not go through. Such accidents are so rare, that it is thought safe to allow materials to be stressed somewhat beyond the elastic limit in such cases.

Class "B" Bridges.—Although it is impossible to determine beforehand, especially in the newer parts of the country, whether any given road is to be used for heavy traffic, it seems extravagant, at least in the cases of larger spans, to design bridges to carry much heavier loads than can be expected to come upon them. It is recommended that bridges of this class be designed to carry 15-ton trucks, with axles 10 ft. apart, 5 tons on the front and 10 tons on the rear axle. This will allow for a considerable overloading of existing motor trucks. It is further recommended that only one truck be assumed to be on the bridge at one time, in designing the floor system, that it be assumed to cover a width of 8 ft. and a length of 35 ft. and that the remainder of the bridge be covered with a load of about 90 lb. per sq. ft., for spans up to 60 ft.

For longer spans, the trusses and main girders should be designed for the following loads:

Length of span (ft.)	Assumed load (lb. per sq. ft.)
80	80
100	70
125	65
150	60
200 and over	55

with intermediate spans in proportion.

Sidewalks should be designed to carry the same loads as in the case of Class "A" bridges.

Special Bridges.—City bridges and bridges carrying traffic connected with mines, quarries, lumber regions, mills, manufactories, etc., require special consideration and should, of course, be designed to carry any load which can reasonably be expected to pass over them, bearing in mind the likelihood of heavy traction engines and motor trucks coming into extensive use in the not distant future.

Bridges Carrying Electric Cars.—Electric traction is still in its infancy and nobody is able to forecast its future development. It seems probable, however, that it will not be profitable to run cars weighing more than 50 tons each, at a speed that would be permitted on any public road. If very high speeds are desired, the traction company will doubtless be required to operate over its own right-of-way. It is recommended that bridges carrying either urban or interurban electric cars be designed to carry 50-ton cars on two trucks, spaced 30 ft. c. to c., each truck having two axles spaced 7 ft. c. to c. The Committee sees no reason for changing the customary practice of assuming that an axle load is distributed over three ties.

For railroad bridges, Cooper's Standard Loadings are generally specified, the particular loading to be used depending upon the location of the line and the future traffic that may be expected. As regards the arch ring in earth-filled arches, where the thickness of filling is sufficient to distribute the concentrated loads over a considerable area, an equivalent uniform loading per linear foot per track is generally substituted. A load of 700 lb. per sq. ft. is common for railroad traffic on spans, say, over 80 ft. in length. A uniform load of 1000 lb. per sq. ft. is frequently adopted for shorter spans. The impact of live loads is not usually considered except for the floors in all arches of open-spandrel construction. Braking or tractive stresses are important only for bridges on heavy grades.

A concentrated load should be assumed to be distributed downward through the fill on a 30-deg. slope with the vertical starting from the ends of the ties.[1] An axle load is assumed as distributed over three ties in the direction of the track.

7. Empirical Rules for Thickness of Arch Ring.—Various empirical formulas have been developed for the trial thickness of arch ring at the crown and are an aid to the judgment.

F. F. Weld[2] gives the following formula:

$$h = \sqrt{l} + \frac{l}{10} + \frac{w}{200} + \frac{w'}{400}$$

where

h = crown thickness in inches.
l = clear span in feet.
w = live load in pounds per square foot, uniformly distributed.
w' = weight of dead load above the crown of the arch in pounds per square foot.

W. J. Douglas gives the following tabulated formulas for different highway spans, the values of h being given in feet:

Under 20 ft. $h = 0.03(6 + l)$*
20 to 50 ft. $h = 0.015(30 + l)$*
50 to 150 ft. $h = 0.00010(11{,}000 + l^2)$†
Over 150 ft. $h = 0.016(75 + l)$‡

* For railroad arches, add 25%.
† For railroad arches, add 20%.
‡ For railroad arches, add 15%.

Joseph P. Schwada gives the following formula which is founded on a rational basis but has been checked with results from actual designs:

$$h = \frac{l^2}{57.6(r - h)f_cK}\left[\frac{B}{20} + r + 8h + 6F + \frac{w}{20}\right]$$

where

h = crown thickness in feet.
l = clear span in feet.
r = rise of intrados in feet.
F = depth of fill at crown in feet (not including track and ballast or pavement).
B = weight of track and ballast or pavement in pounds per square foot.
w = uniform live load in pounds per square foot.

The following paragraphs and Figs. 8 and 9 are from Mr. Schwada's article in *Eng. News*:[3]

In Fig. 8 is shown the application of the formula for crown thickness to a series of railroad arches for ratios of rise to span from ¼ to ⅛ and for conditions as noted in the figure. For convenience the coefficients of stress for certain spans are arranged in tabular form. Intermediate values can be determined and the crown thickness for any other conditions easily obtained.

[1] See tests by M. L. Enger, *Eng. Rec.*, Jan. 22, 1916, p. 106.
[2] *Eng. Rec.*, Nov. 4, 1905, p. 529.
[3] *Eng. News*, Nov. 9, 1916, p. 880.

To adapt the formula for crown thickness to highway arches a slight change must be made in the value of coefficients for short spans, because of the light loads involved and because of a desire to obtain thicknesses which are practical. The coefficients for f_c for spans up to about 60 ft. therefore differ from the coefficients for the same

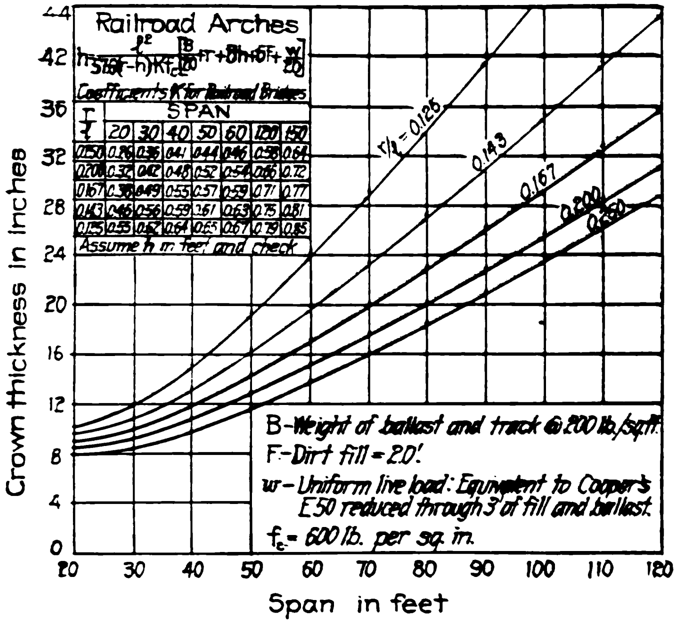


Fig. 8.

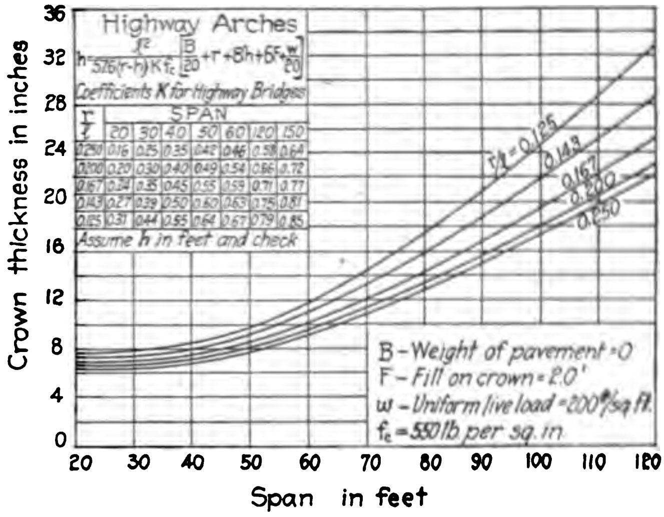


Fig. 9.

spans for railroad arches. For spans over 60 ft. the coefficients are the same. A complete table of values for highway arches is given in Fig. 9, with an application to a series of highway arches for conditions shown in the figure.[42]

To illustrate the application of the formula to a highway arch assume: span, 90 ft.; rise, 12 ft.; weight of pavement, 100 lb. per sq. ft.; fill, 1 ft.; live load, 300 lb. per sq. ft.; unit stress, f_c, 550 lb. per sq. in. Then K = 0.71. Assume depth at crown to be 23 in. = 1.92 ft. According to the expression,

$$h = \frac{90 \times 90}{57.6 \times 10.08 \times 0.71 \times 550}[5 + 12.0 + 15.36 + 6 + 15] = 1.91 \text{ ft.} = 23 \text{ in.}$$

If the resulting thickness does not check the assumed thickness, another trial must be made.

In applying the formula for crown thickness it should be understood that f_c in the expression represents an approximate maximum value of stress which one may reasonably expect to obtain if the arch is designed according to the rules for curvature and thickness followed in the design of the arches here considered. The maximum value of stress for these arches varied from 575 lb. per sq. in. to 650 lb. per sq. in., with an average value of about 625 lb. per sq. in. A value of 550 lb. per sq. in. to 600 lb. per sq. in. for f_c is suggested for use in the formula.

The empirical rules given above, should be used only for trial. The exact shape of the arch ring and the thickness at different sections must be determined by analysis.

8. Approximate Formula for Best Shape of Arch Axis.—Victor H. Cochrane has derived equations[1] giving approximately the best shape of axis for both open-spandrel and filled-spandrel arches. The equations are given in Art. 32 of this section.

9. Proper Thickness of Arch Ring in the Haunch for Given Thicknesses at Crown and Springing.—Victor H. Cochrane has analyzed a series of typical arches[1] so chosen as to be applicable to any span length, rise-ratio (ratio of rise to span), thickness at crown and springing, and manner of loading. The characteristics of these typical arches are given in Art. 33 of this section.

10. Dead Loads and Their Action Lines.—After a trial arch ring has been assumed, the dead loads may be determined.

Assume, for example, a railway earth-filled arch. The earth filling and ballast, ties, and rails should be reduced to an equivalent height of masonry, as shown in Fig. 10. If the earth filling is taken at 120 lb. per cu. ft. and the ballast, ties, and rails at 150 lb. per sq. ft. of roadway, then since the earth filling reaches 1 ft. above the extrados at the crown, the vertical distance *ab* should be laid off equal to $\frac{120}{150}$ = 0.8 ft. In a similar manner the distance *be* should be laid off equal to $\frac{150}{150}$ = 1 ft. Points *d* and *f* should be determined for the loading at *c*, and similar points should also be found for other places along the arch ring, a sufficient number being taken to fully determine the curved line *fe*. This line is called the *reduced-load contour*.

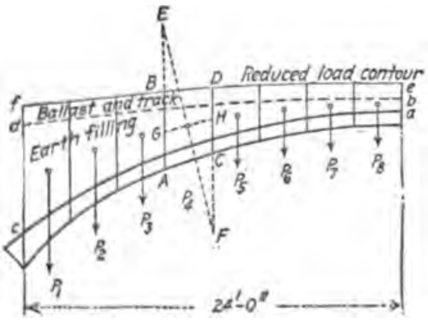


FIG. 10.

The arch with its load should now be divided by vertical lines into trapezoids, or what are nearly so. For testing the trial arch by the approximate method (presented in the following article), the horizontal distance between springing lines may be conveniently divided into divisions of equal horizontal length. Fig. 10 shows eight divisions on each side of the crown.

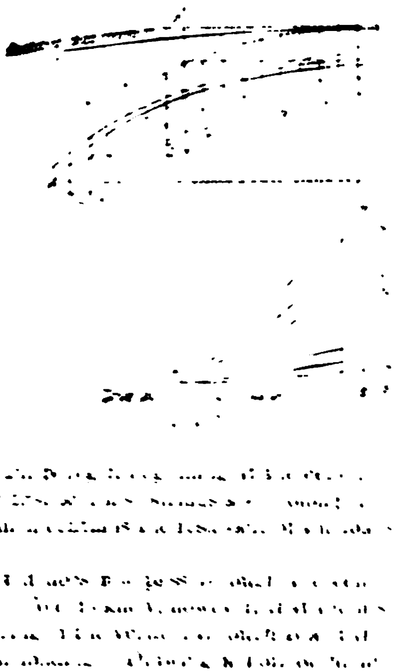
The next step is to determine the area and center of gravity of each trapezoid. With the area known, the load corresponding to each trapezoid is found by multiplying by the weight of a cubic foot of concrete, the arch considered being included between two longitudinal vertical planes 1 ft. apart. The center of gravity for each of the trapezoids may be found as follows: Extend *AB*, Fig. 10, so that *BE* = *CD*, and in the opposite direction extend *CD* so that *CF* = *AB*. The intersection of *EF* and the median *GH* is the center of gravity sought.

11. Approximate Method of Testing Trial Arch.—Since the dead load usually controls the shape of the arch ring, it is desirable to test the trial arch for this loading, employing an

[1] Proc. Engineers' Society of Western Pennsylvania, vol. 32, No. 8

[illegible]

[illegible]



[illegible]

[illegible]

In both preliminary and final analyses [illegible] = 3 [illegible] drafting work so as to have the dimensions exact [illegible] lines sharp.

12. Use of Temporary Hinges in Arch Erection.—Temporary hinges were employed by the late George S. Morison in the construction of arches; they have also been used in a number of European bridges. In arches with a rise less than one-fourth the span, these hinges materi[illegible] decrease stresses in the arch due to the elimination of all dead-load bending stresses (including arch shortening produced by dead-load compressive stresses) in addition to stresses from shrinkage or settlement. Concrete should be poured in joints to close hinges only after full dead load is on the structure and the shortening and shrinkage stresses have taken place.

13. Use of Reinforcement in Concrete Arches.—It would seem from purely theoretical considerations that but little could be gained by the use of reinforcement in a concrete arch since the direct compression usually controls to such an extent that the allowable stress in the concrete permits of but a small unit tensile stress in the steel. From a broader viewpoint, however, it is clear that the steel adds greatly to the reliability of the construction and makes possible a higher working stress in the concrete than could properly be employed in the design of plain-concrete structures. Higher working stress produces a thinner arch ring, and consequently less dead load and lighter abutments. Undoubtedly, a large saving may result from this cause in the case of long-span arches.

A considerable portion of an arch ring is subject to both positive and negative moments, and for this reason the reinforcement should be placed, for some distance at least, near both upper and lower surfaces. The general practice is to carry both rows of steel throughout the entire span thereby eliminating any possibility of failure due to an inadequate provision for tensile stresses. On account of the heavy compressive stress in arch rings, the upper and lower reinforcement should be tied together to prevent buckling.

The percentage of longitudinal steel in arch rings is to a certain extent arbitrary. An amount of steel between ½ and 1½% of the ring at the crown seems to be good practice in the ordinary full-barrel arch-ring design although the exact amount depends upon the loading and the form of arch selected, and must be finally tested by computation. Transverse rods at right angles to the longitudinal are generally used to prevent cracks in the concrete and to distribute the loads laterally. Web reinforcement is not required in ordinary construction.

14. Classification of Arch Rings.—Arches may be classified as hinged or hingeless. A hingeless arch is one having fixed ends, while a hinged arch may have a hinge at the crown, a hinge at each end, or a hinge at each end and one at the crown. Arches of one and two hinges are not used to any extent in masonry construction since the three-hinged arch offers the advantage of more definitely fixing the line of pressure throughout the ring and thus makes possible a saving of material. Hinges, however, are often an expensive detail and the three-hinged arch is by no means so common as the concrete arch having fixed ends. Friction on hinges is also an important consideration.

Three-hinged arches are treated in Arts. 43 to 47 inclusive.

ANALYSIS OF THE ARCH BY THE ELASTIC THEORY[1]

15. Deflection of Curved Beams.—Deflection formulas for curved beams (in which the radius of curvature is large as compared with the depth) are employed in the development of arch theory.

Let AB, Fig. 12, be any portion of a curved beam in its *unstrained* form and $A'B$ the same portion in its *strained* form, assuming the beam rigidly fixed at B. Let $X - X$ and $Y - Y$ be rectangular axes with origin at A, and denote the components of A' as Δx and Δy. AO, tangent to the arch axis at A, moves through the angle k. Formulas below give the following values: (1) angular change of AO, (2) component Δx of A', (3) component Δy of A'.

$$k = \sum_A^B \frac{Ms}{E_cI} \tag{1}$$

$$\Delta y = -\sum_A^B \frac{Mxs}{E_cI} \tag{2}$$

$$\Delta x = \sum_A^B \frac{Mys}{E_cI} \tag{3}$$

[1] Method of analysis as given in Turneaure and Maurer's "Principles of Reinforced Concrete Construction," 2nd Edition, pp. 335 to 344.

The smaller the elementary lengths of beam considered, the more accurately will the above formulas apply. In the derivation of the formulas the values of M and I have been regarded as constant quantities for each particular elementary length considered. Since this is not true in practice on account of each element having appreciable length, a close approximation to the actual M and I for a given element may be obtained by taking the values of the bending moment and moment of inertia at the mid-point of s. Distances x and y should also be measured to this point. As in simple beams, M is considered positive when it tends to increase the compression on the back of the arch. The minus sign is used in formula (2) because the effect of a positive value of M in any element causes an upward deflection—that is, a minus value of Δy, considering only the effect of bending in the element in question.

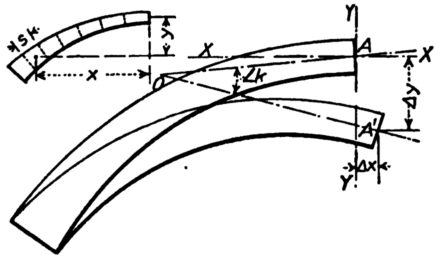


Fig. 12.

16. General Procedure in Arch Analysis.—A concrete arch with fixed ends is statically indeterminate. There are, in all, six unknown quantities—three at each support (the vertical and horizontal components of the reaction, and the bending moment; or, what is the same thing, the magnitude, direction, and point of application of the reaction)—and it is possible to determine only three unknowns by the principles of statics. The three additional equations may be found from the following conditions:

The change in span of the arch	$= \Delta x = 0$
The vertical displacement at one end relative to the other end	$= \Delta y = 0$
The angle between the tangents to the arch axis at the two ends of the arch remain unchanged, or	$\angle k = 0$

These three conditions must be true since the arch is fixed at the abutments.

Instead of actually finding the components of the reactions and the moments at the supports as outlined above, it is simpler for symmetrical arches to take the origin of coördinates at the crown and find the thrust, shear, and moment at that point. With these three unknowns determined, each half of arch may then be treated as statically determinate.[1]

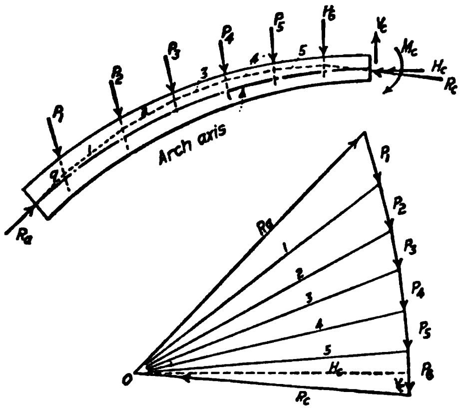


Fig. 13.

The analysis of a symmetrical arch consists in finding the thrust, shear, and bending moment at the crown and at intermediate sections in the arch ring or arch rib, and then finding the stresses resulting therefrom. (The method of finding stresses on an arch section, knowing moment and thrust, is explained in Sect. 9.) The thrust is here taken to be the normal component of the resultant force on the section, and the shear is the component at right angles to the normal. The bending moment will be considered positive when it tends to increase the compression on the back of the arch, this being the same convention as for beams.

A horizontal thrust is produced at the crown when the arch is loaded symmetrically. For non-symmetrical loading, an inclined pressure acts at the crown, but its horizontal component is called the horizontal thrust for that loading. Its vertical component is the shear at the crown.

[1] For unsymmetrical arches, see Art. 28.

Assume the arch as cut at the crown and consider each half to act as a cantilever sustaining exactly the same forces as exist in the arch itself.

The external forces holding a semi-arch in equilibrium (Fig. 13) are the loads P_1, P_2, etc., the horizontal thrust H_c, the vertical shear V_c, and the reaction at the skewback R_a.

After H_c, V_c, and M_c have been computed, the line of pressure (accurately enough represented by the equilibrium polygon) can be constructed by help of the force polygon, Fig. 13. The value of M_c definitely determines the point of application of H_c and makes the construction of the exact line of pressure possible. (For a positive value of M_c, the thrust H_c acts above the arch axis.) From this line of pressure and the accompanying force polygon may be obtained the thrusts, shears, and bending moments at intermediate points of the arch. The force polygon gives directly the thrusts and shears, while the line of pressure makes possible the determination of the bending moment at any section, the bending moment being equal to the resultant pressure at the given point multiplied by the perpendicular distance from the arch axis to the line of pressure. Usually the line of pressure is drawn to serve only as a check on the computations, and the bending moments at the various points are determined algebraically.

The line of pressure of an arch is a continuous curve, but differs very little from an equilibrium polygon for the given loads, Fig. 13. In fact this curve becomes tangent to the equilibrium polygon between the angle points. The greater the number of loads, the nearer the polygon approaches the line of pressure.

With H_c, V_c, and M_c determined, all external forces are known except the reaction at the skewback, and this is determined by the closing line of the force polygon. An equilibrium polygon may then be constructed as already mentioned, the first side being in the line of R_c produced, the second parallel to the ray 5, and so on until the last side through q gives the position of R_a.

17. Notation.—The following notation will be employed:

Let s = length of a division of the arch ring measured along the arch axis.
n_h = number of divisions in one-half the arch.
l = span of arch axis.
c_a = average unit compression in concrete of arch ring due to thrust.
t_c = coefficient of linear temperature expansion.
t_D = number of degrees rise or fall in temperature.
E_c = modulus of elasticity of concrete.

At the crown, let

H_c = horizontal thrust.
V_c = vertical shear.
R_c = resultant of H_c and V_c.
M_c = bending moment.

At any point on the arch axis, with coördinates x and y referred to the crown as origin, let

N = thrust (normal) on radial section.
S = shear on radial section.
R = resultant force on radial section, resultant of N and S.
x_0 = eccentricity of thrust on section, or distance of N from the arch axis.
t = depth of section.
I = moment of inertia of section including steel = $I_c + nI_s$.
A = area of section including steel = $A_c + nA_s$.
p_0 = steel ratio for total steel at section.
d' = embedment of steel from either upper or lower surface.
M = moment = Nx_0.
m_L = moment at any point on left half of arch axis of all external loads (P_1, P_2, etc.) between the point and the crown.
m_R = moment at any point on right half of arch axis of all external loads between the point and the crown.
m = moment at any point on either half of arch axis of all external loads (P_1, P_2, etc.) between the point and the crown.

18. Formulas for Thrust, Shear, and Moment.—When the arch ring is so divided into sections that $\frac{s}{I}$ is a constant, then the following formulas apply:

Loading:

$$H_c = \frac{n_h \Sigma(m_L + m_R)y - \Sigma(m_L + m_R)\Sigma y}{2[n\Sigma y^2 - (\Sigma y)^2]} \tag{4}$$

$$V_c = \frac{\Sigma(m_L - m_R)x}{2\Sigma x^2} \tag{5}$$

$$M_c = \frac{\Sigma(m_L + m_R) - 2H_c\Sigma y}{2n_h} \tag{6}$$

$$M = M_c + H_c y + V_c x - m_L \tag{7}$$

$$M = M_c + H_c y - V_c x - m_R \tag{8}$$

All values of m_L, m_R, x, and y should be substituted as positive. All summations refer to one-half of the arch axis. Positive value of V_c indicates that the line of pressure slopes upward toward the left; a negative value, downward toward the left. Positive value of M_c indicates that the thrust H_c acts above the arch axis. Signs preceding terms M_c and $V_c x$ in formulas (7) and (8) depend upon the results of (5) and (6).

Temperature:

$$H_c = \frac{I}{s} \cdot \frac{t_c t_D l n_h E_c}{2[n_h\Sigma y^2 - (\Sigma y)^2]} \qquad (t_D \text{ should be inserted as } + \text{ for a rise; } - \text{ for a drop.}) \tag{9}$$

$$M_c = -\frac{H_c\Sigma y}{n_h} \tag{10}$$

$$M = M_c + H_c y \tag{11}$$

The value of t_D should be inserted as plus for a rise of temperature; minus (−) for a drop. Signs preceding H_c in formulas (10) and (11) depend upon the result of formula (9). Sign preceding M_c in formula (11) depends upon the result of formula (10). Thus for fall of temperature, thrust and moment are of opposite sign from those for a rise. l = span of arch axis.

Rib shortening:

$$H_c = -\frac{I}{s} \cdot \frac{c_a l n_h}{2[n_h\Sigma y^2 - (\Sigma y)^2]} \tag{12}$$

$$M_c = -\frac{H_c\Sigma y}{n_h} \tag{13}$$

$$M = M_c + H_c y \tag{14}$$

Values of moments and thrusts for rib shortening are of same sign as for temperature fall. l = span of arch axis.

19. Division of Arch Ring for Constant $\frac{s}{I}$.—The graphical method shown in Fig. 14 is usually employed. AB is drawn to any convenient scale equal in length to one-half the arch axis. The curve EF is then drawn through points whose ordinates are the values I and whose abscissas are the corresponding distances along the arch axis from the skewback. (In order to make the drawing clear, the ordinates and corresponding abscissas which determine the curve EF are not shown.) A length AH is then assumed, a perpendicular LC erected at its center, and the lines AC and CH determined. Starting from point H, lines are drawn parallel alternately to AC and CH, as shown in Fig. 14. Only three or four trials will usually be required to divide the line AB into the desired number of divisions. The base of each triangle thus formed corresponds to s and its altitude to I. Since all the triangles are similar by construction, the term $\frac{s}{I}$ is constant throughout.

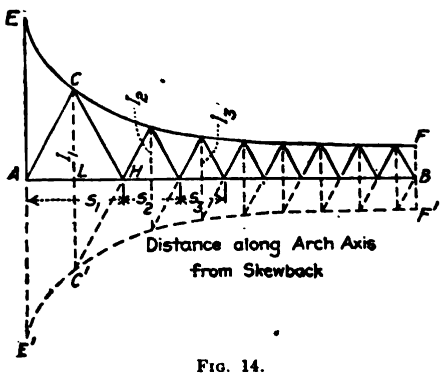


Fig. 14.

A convenient modification of the above method is to draw a second curve $E'F'$ below AB, using the same ordinates as for EF. AH is then assumed as before and the perpendicular CC' erected at its center. Starting with C', diagonals and verticals are drawn alternately making the diagonals parallel to AC. This method offers the advantage of drawing all the diagonals parallel to the same line.

20. Loadings to Use in Computations.—Arches should be analyzed for live load over three-eighths of the span, five-eighths of the span, the middle one-fourth of the span, and the end three-eighths of the span. These uniform loadings are only approximations to the true loadings which produce the maximum stresses. The exact loadings to cause maximum conditions may be found by the use of influence lines.

21. Use of Influence Lines.—It is common practice in arch design to consider the live load as extending over certain definite portions of the span and to assume that these loadings produce the maximum effects. For example, it is often assumed that by loading either the whole span or the half span the greatest possible stresses at any given section are obtained. In general this assumption is only a very rough approximation, and considerable inaccuracy may result from such a method of procedure. In fact, in the case of large and important structures, the only satisfactory way to analyze for maximum stresses is by what might be called a *unit-load* or *influence-line* method. By this method the arch is first analyzed for a load of unity at the several load points and then influence lines[1] are drawn for either moment and thrust or for fiber stress.

The position of the live load on an arch to cause maximum stress at any given section cannot be determined in advance in the common method of analysis. An investigation will show that different loadings are required for sections similarly located in arches of different proportions. The only accurate method, then, is to draw a proper number of influence lines as above described. In arches continuously loaded no definite load points exist at which to place the load of unity in influence-line analysis, but in arches of this class points may be chosen for this purpose sufficiently close together to give any desired degree of accuracy.

22. Internal Temperature Investigations.—Comparatively few experiments have been made which furnish data on the internal temperature range in concrete structures. Undoubtedly the most important are those completed under the direction of the Engineering Experiment Station at Ames, Iowa, on two highway arch bridges of the earth-filled type. These experiments[2] are described in detail in *Bulletin* 30 of the Iowa State College of Agriculture and Mechanic Arts where a summary is also given of the other tests that have been made on internal temperature variation.

The writers of the bulletin conclude that "to render an arch structurally safe, provision should be made (in the latitude where the bridge tests were conducted) for stresses induced by a temperature variation of *at least* 40°F. each way from an assumed temperature of no stress. Particular circumstances may demand that a greater variation be used for drop in temperature to prevent the appearance of cracks. This will always remain largely a matter of judgment with the designing engineer."

23. Shrinkage Stresses.—Shrinkage stresses are at present ignored in arch analysis, as the shrinkage coefficients on actual arches have not been determined.

24. Deflection at Any Point.—The deflection at any point in an arch may be found by formula (2), Art. 15, or

$$\Delta y = -\frac{s}{E_cI}\Sigma Mx$$

The arch should be assumed as cut at the point in question, and either portion of the arch may be considered. The cantilever selected should be subjected to exactly the same forces as exist in the arch itself.

If the deflection of the crown of a symmetrical arch is desired, the value of M due to loading for any section of the left cantilever may be found from formula (7), Art. 18; or, substituting this value in the above equation, we have

$$\Delta y = -\frac{s}{E_cI}(M_c\Sigma x + H_c\Sigma xy + V_c\Sigma x^2 - \Sigma m_Lx)$$

[1] See Art. 48a, Sect. 7; also Art. 34 of this section.

[2] By Messrs. C. S. Nichols and C. B. McCullough.

For temperature changes, formula (11) of Art. 18 may be substituted in place of formula (7), or

$$\Delta y = -\frac{s}{E_c I}(M_c \Sigma x + H_c \Sigma xy)$$
$$= -\frac{t_c A_D l(n_h \Sigma xy - \Sigma x \Sigma y)}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

25. Method of Procedure in Arch-ring Design.—The main steps that need to be taken in the design of an arch ring may be enumerated as follows:

1. Assume a thickness for the arch ring at the crown and at the springing, using empirical formulas, if desired, as an aid to the judgment.
2. Lay out the curve assumed for the intrados.
3. Lay out a curve for the extrados to give as nearly as possible the assumed ring thickness at the springing.
4. Draw the arch axis between the extrados and intrados.
5. Divide the arch axis into an even number of divisions such that the ratio $\frac{s}{I}$ is constant for all.
6. Compute the dead and live loads, and indicate these loads properly on the drawing.
7. Compute H_c, V_c, and M_c at the crown for the different conditions of loading.
8. Draw the force polygons for the different conditions of loading and the corresponding equilibrium polygons, or lines of pressure.
9. Determine the thrusts, shears, bending moments, and eccentric distances at the centers of the $\frac{s}{I}$ divisions of the arch ring for the different conditions of loading.
10. Compute the thrust and moment at the crown due to variation in temperature; also the moments on the various sections, and the corresponding thrusts and shears by resolving the crown thrust into tangential and radial components.
11. Where necessary, compute the thrust and moment at the crown, and the thrust, shear, and moment at various sections due to rib shortening.
12. Combine the thrusts, shears, and moments due to the different conditions of loading with the thrusts, shears, and moments due to temperature and rib shortening. (The results usually show that the shearing unit stresses are very small and need not be considered.)
13. Compute the maximum stresses—compression in the concrete and tension in the steel—due to the thrusts and moments. If the stresses are either too small or too large, the dimensions or even the shape of the arch ring must be changed and the computations repeated.

26. Uncertainty as to Fixedness of Ends of Arch.—This uncertainty can be reduced or entirely eliminated by taking the skewback for purposes of analysis at a plane where the ends of the arch are virtually fixed. Whenever the abutments are of such a form that there is no pronounced change of section at the springing lines, then the analysis should include the whole structure down to the point where the distortion due to the live load on the arch will be inappreciable. In some cases this may be the very bottom of the abutment.

27. Skew Arches.—Skew arches may be treated exactly as right arches, the span being taken parallel to the center line of roadway and not at right angles to the springing lines of the arch.

28. Unsymmetrical Arches.—Unsymmetrical arches are sometimes desirable in the end spans of a series of two or more arches in order to reduce material in abutments, and at the same time, to provide ample waterway area over streams. Also, arches of this type are often necessary under other conditions, as, for example, when a river in a deep ravine is bordered by a railway requiring maximum clearance near the abutments.

Formulas for unsymmetrical arches depend upon the location of the coördinate axes.

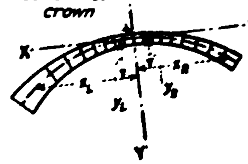


Fig. 15.

28a. Origin of Coördinates Between Divisional Lengths.—In the analysis of unsymmetrical arches, the entire arch ring should be divided into a sufficient number of $\frac{s}{I}$ divisions to obtain the desired degree of accuracy. The origin of coördinates may then be taken at the center of any one of the sections occurring between the divisional lengths, but for convenience in scaling the values of x and y, this origin should be placed at one of the sections near the crown. The X and Y axes should be drawn perpendicular and parallel respectively to this section so as to permit the thrust at the section to be determined directly without composition and resolution of forces. Fig. 15 shows how these axes should be drawn.

The flexure formulas for H_0, V_0, and M_0 for unsymmetrical arches, considering the origin of coördinates near the crown, are exceedingly complex and inconvenient for use in practice. The best plan is to use formulas as given below and to solve simultaneously for the above values after the numerical values of the coefficients are substituted. Following are the formulas to be solved in this way:

Loading:

$$H_0\Sigma y^2 + V_0(\Sigma x_L y_L - \Sigma x_R y_R) + M_0\Sigma y - \Sigma my = 0$$
$$H_0(\Sigma x_L y_L - \Sigma x_R y_R) + V_0\Sigma x^2 + M_0(\Sigma x_L - \Sigma x_R) - \Sigma m_L x_L + \Sigma m_R x_R = 0$$
$$H_0\Sigma y + V_0(\Sigma x_L - \Sigma x_R) + nM_0 - \Sigma m = 0$$
$$M_L = M_0 + H_0 y_L + V_0 x_L - m_L$$
$$M_R = M_0 + H_0 y_R - V_0 x_R - m_R$$

All values of m_L, m_R, x_L, x_R, y_L, and y_R should be substituted as positive. The subscripts L and R refer to summations to left and right of the chosen section respectively. No subscript indicates that summation is for entire arch. Positive value of M_0 indicates that the thrust H_0 acts above the arch axis. Considering the chosen section as nearly vertical, a positive value of V_0 indicates that the line of pressure slopes upward toward the left; a negative value, downward toward the left. Signs preceding terms M_0, V_0x_L, and V_0x_R in the last two formulas depend upon the signs of M_0 and V_0 resulting from the three simultaneous equations.

Temperature:

$$H_0\Sigma y^2 + V_0(\Sigma x_L y_L - \Sigma x_R y_R) + M_0\Sigma y - \frac{I}{s}\cdot t_c t_D l E_c = 0$$
$$H_0(\Sigma x_L y_L - \Sigma x_R y_R) + V_0\Sigma x^2 + M_0(\Sigma x_L - \Sigma x_R) = 0$$
$$H_0\Sigma y + V_0(\Sigma x_L - \Sigma x_R) + nM_0 = 0$$
$$M_L = M_0 + H_0 y_L + V_0 x_L$$
$$M_R = M_0 + H_0 y_R - V_0 x_R$$

The value of t_D should be inserted as plus (+) for a rise of temperature; minus (−) for a drop. Signs preceding terms M_0, H_0y_L, H_0y_R, V_0x_L, and V_0x_R in the last two formulas depend upon the signs of M_0, H_0, and V_0 resulting from the three simultaneous equations. l = span or arch axis measured parallel to X axis.

Rib shortening:

$$t_D = \frac{c_a}{E_c A_e}$$

Rib shortening causes the same effect as a lowering of the temperature. Solving for t_D gives equivalent temperature drop.

28b. Origin of Coördinates at Crown.—Assuming that by "crown section" is meant a section at the highest point of the arch, the following formulas result, where q equals $\left(\frac{s}{I}\right)_R$ divided by $\left(\frac{s}{I}\right)_L$.

Loading:

$$H_c(\Sigma y^2_L + \Sigma y^2_R q) + V_c(\Sigma x_L y_L - \Sigma x_R y_R q) + M_c(\Sigma y_L + \Sigma y_R q) - \Sigma m_L y_L - \Sigma m_R y_R q = 0$$
$$H_c(\Sigma x_L y_L - \Sigma x_R y_R q) + V_c(\Sigma x_L^2 + \Sigma x_R^2 q) + M_c(\Sigma x_L - \Sigma x_R q) - \Sigma m_L x_L + \Sigma m_R x_R q = 0$$
$$H_c(\Sigma y_L + \Sigma y_R q) + V_c(\Sigma x_L - \Sigma x_R q) + M_c(n_L + n_R q) - \Sigma m_L - \Sigma m_R q = 0$$
$$M_L = M_c + H_c y_L + V_c x_L - m_L$$
$$M_R = M_c + H_c y_R - V_c x_R - m_R$$

Temperature:

$$H_c(\Sigma y_L^2 + \Sigma y_R^2 q) + V_c(\Sigma x_L y_L - \Sigma x_R y_R q) + M_c(\Sigma y_L + \Sigma y_R q) - \frac{t_c t_D l E_c}{\left(\frac{s}{I}\right)_L} = 0$$
$$H_c(\Sigma x_L y_L - \Sigma x_R y_R q) + V_c(\Sigma x_L^2 + \Sigma x_R^2 q) + M_c(\Sigma x_L - \Sigma x_R q) = 0$$
$$H_c(\Sigma y_L + \Sigma y_R q) + V_c(\Sigma x_L - \Sigma x_R q) + M_c(n_L + n_R q) = 0$$
$$M_L = M_c + H_c y_L + V_c x_L$$
$$M_R = M_c + H_c y_R - V_c x_R$$

Rib shortening:

$$t_D = \frac{c_a}{E_c A_e}$$

All values of m_L, m_R, x_L, x_R, y_L, and y_R should be substituted as positive.

28c. Origin of Coördinates at Left Springing.—Fig. 16 shows how the coördinates x and y should be measured. The directions H_1, V_1, and M_1 are shown for values considered as positive in the formulas given below. Values of y measured below the axis $X - X$ should always be considered as negative.

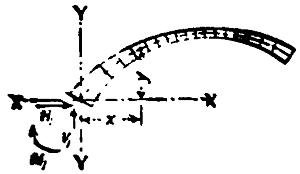


Fig. 16.

Loading.

$$H_1\Sigma y^2 - V_1\Sigma xy - M_1\Sigma y - \Sigma my = 0$$
$$H_1\Sigma xy - V_1\Sigma x^2 - M_1\Sigma x + \Sigma mx = 0$$
$$H_1\Sigma y - V_1\Sigma x - nM_1 + \Sigma m = 0$$

or

$$H_1 = \frac{ab - cd}{ae - cf} \qquad V_1 = \frac{b - H_1e}{c} \qquad M_1 = \frac{H_1\Sigma y - V_1\Sigma x + \Sigma m}{n}$$

in which

$$a = \Sigma x\Sigma y - n\Sigma xy \qquad d = \Sigma m\Sigma y - n\Sigma my$$
$$b = \Sigma mx\Sigma y - \Sigma x\Sigma my \qquad e = \Sigma x\Sigma y^2 - \Sigma xy\Sigma y$$
$$c = \Sigma x^2\Sigma y - \Sigma x\Sigma xy \qquad f = n\Sigma y^2 - (\Sigma y)^2$$

Then

$$M = M_1 + V_1x - H_1y - m$$

All values of m and x should be substituted as positive. Values of y below the $X - X$ axis should be taken as negative. The summations refer to entire arch. Positive value of M_1 indicates that the reaction acts to the left of the arch axis at the springing. Positive values of H_1 and V_1 indicate that the reaction acts upward to the right. Signs preceding terms M_1, V_1x, and H_1y in the last formula depend upon the signs of M_1, V_1, and H_1 resulting from the preceding equations.

Temperature:

$$H_1\Sigma y^2 - V_1\Sigma xy - M_1\Sigma y = \frac{I}{s}\cdot t_c t_D l E_c$$
$$H_1\Sigma xy - V_1\Sigma x^2 - M_1\Sigma x = 0$$
$$H_1\Sigma y - V_1\Sigma x - nM_1 = 0$$

or

$$H_1 = \frac{a\Sigma x - nc}{ae - cf}(k) \qquad V_1 = \frac{k\Sigma x - H_1e}{c} \qquad M_1 = \frac{H_1\Sigma y - V_1\Sigma x}{n}$$

in which

$$k = \frac{I}{s}\cdot t_c t_D l E_c$$

Then

$$M = M_1 + V_1x - H_1y$$

The value of t_D should be inserted as plus (+) for a rise of temperature; minus (−) for a drop. Signs preceding terms M_1, V_1, and H_1 in the last formula depend upon the signs of M_1, V_1, and H_1 resulting from the preceding equations. l = span of arch axis measured horizontally; that is, parallel to X axis.

Rib shortening:

$$t_D = \frac{c_a}{E_c t_c}$$

Rib shortening causes the same effect as a lowering of the temperature. Solving for t_D gives equivalent temperature drop.

29. Arch Structure of Two Spans with Elastic Pier.[1]—A structure of this type is shown in Fig. 17.

Fig. 17.

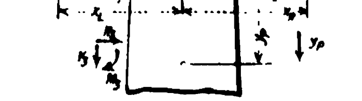


Fig. 18.

The points indicated as fixed may be the bottoms of the pier and abutments, or they may be at intermediate sections, depending upon where the designer considers the structure fixed. The horizontal section A–A where arches and pier join is the section considered. In Fig. 18 the top of the pier is shown in detail.

[1] An arch structure of more than two spans and with elastic piers may readily be analyzed by the ellipse-of-elasticity method explained in vol. III of Hool's "Reinforced Concrete Construction."

What may be called the skewbacks of the arches are shown. The weight of the material between the skewbacks and the section A–A need be considered only in finding the resultant thrusts on the pier sections. The origin of coördinates x and y for each arch is taken at the middle point C of the section A–A instead of at the center of the skewbacks.

The following notation is employed:

Let x_L, y_L = coördinates of any point on the axis of the left arch referred to the center of the section A–A as origin. Values of y_L should be considered plus when measured above the axis X–X, and as negative when measured below that axis.

x_R, y_R = coördinates of any point on the axis of the right arch referred to the center of the section A–A as origin. Values of y_R should be considered plus when measured above the axis X–X, and as negative when measured below that axis.

y_P = depth of any point on the vertical axis of the pier below the section A–A. All values positive.

m_L, m_R = moment at any point on axis of left arch and right arch respectively of all external loads between the point in question and the top of the pier.

M_L, M_R, M_P = moment at any point on axis of left arch, right arch, and pier respectively.

n_L, n_R, n_P = number of $\frac{s}{I}$ divisions in the left arch, right arch, and pier respectively.

c_L, c_R, c_P = values of $\frac{s}{I}$ for left arch, right arch, and pier respectively.

H_1, V_1 = horizontal and vertical components of the thrust from the left arch at the top of pier.

M_1 = moment at section A–A due to thrust from left arch = vertical component of thrust from left arch multiplied by the distance from the point C (the center of the section) to where this thrust produced cuts the section A–A.

H_2, V_2 = horizontal and vertical components of the thrust from the right arch at the top of pier.

M_2 = moment at section A–A due to thrust from right arch.

H_3 = resultant shear on section A–A = $H_1 - H_2$.

V_3 = resultant thrust (normal) on section A–A = $V_1 + V_2$.

M_3 = resultant moment on section A–A = $M_1 - M_2$.

The arrows in Fig. 18 indicate what are considered positive values of the quantities.

The following six equations for loading may be solved simultaneously for the values of H_1, V_1, M_1, H_2, V_2 and M_2:

Loading:

$$c_L(M_1\Sigma y_L - H_1\Sigma y_L^2 + V_1\Sigma x_L y_L - \Sigma m_L y_L) = -c_R(M_2\Sigma y_R - H_2\Sigma y_R^2 + V_2\Sigma x_R y_R - \Sigma m_R y_R)$$
$$M_1\Sigma x_L - H_1\Sigma x_L y_L + V_1\Sigma x_L^2 - \Sigma m_L x_L = 0$$
$$M_2\Sigma x_R - H_2\Sigma x_R y_R + V_2\Sigma x_R^2 - \Sigma m_R x_R = 0$$
$$c_L(n_L M_1 - H_1\Sigma y_L + V_1\Sigma x_L - \Sigma m_L) = -c_R(n_R M_2 - H_2\Sigma y_R + V_2\Sigma x_R - \Sigma m_R)$$
$$c_P[(M_1 - M_2)\Sigma y_P + (H_1 - H_2)\Sigma y_P^2] = c_L[M_1\Sigma y_L - H_1\Sigma y_L^2 + V_1\Sigma x_L y_L - \Sigma m_L y_L]$$
$$c_P[n_P(M_1 - M_2) + (H_1 - H_2)\Sigma y_P] = -c_L[n_L M_1 - H_1\Sigma y_L + V_1\Sigma x_L - \Sigma m_L]$$

Bending Moment at any point:

$$M_L = M_1 - H_1 y_L + V_1 x_L - m_L$$
$$M_R = M_2 - H_2 y_R + V_2 x_R - m_R$$
$$M_P = (M_1 - M_2) + (H_1 - H_2)y_P$$

Values of y_L and y_R should be considered plus when measured above the axis X–X, and as negative when measured below that axis. The values of H_3, V_3, and M_3 may be obtained from the following relations:

$$H_3 = H_1 - H_2 \qquad V_3 = V_1 + V_2 \qquad M_3 = M_1 - M_2$$

All values of m_L and m_R should be substituted as positive.

For arch bridges symmetrical about the center line of pier, the labor involved in solving the simultaneous equations mentioned above will be greatly reduced.

All the simultaneous equations given above pertain to the unknown forces acting at the section A–A. With these completely determined, however, the moment and thrust at any section may be found in same manner as for the single symmetrical arch. Each of the three members must be considered separately and each subjected to exactly the same force that is found to act upon it at the top of pier in the monolithic structure.

When the two arch spans are equal, either arch may be analyzed for temperature and rib shortening stresses in the same manner as for a single arch having immovable or fixed supports.

COCHRANE'S FORMULAS AND DIAGRAMS FOR USE IN THE DESIGN OF SYMMETRICAL ARCHES IN ACCORDANCE WITH THE ELASTIC THEORY[1]

30. Accuracy of Formulas and Diagrams.—The formulas and diagrams in this chapter are of sufficient accuracy to use for the final design in many cases, and in practically all instances they will afford a means of determining the form and dimensions of the arch with assurance that the final analysis will show that but slight changes, if any, are required.

31. Difficulties and Uncertainties Involved in Applying the Elastic Theory.—There are many reasons for concluding that the use of rigidly exact theoretical formulas for arch design is wholly unwarranted, and that if by making certain reasonable practical assumptions we can devise greatly shortened methods of design, we are quite justified in so doing. For, in the first place, the elastic theory as applied to hingeless arches is based on the assumption that the ends of the arch are rigidly attached to immovable abutments, so that a section of the arch at the skewback or springing is subject neither to vertical or horizontal displacement nor to rotation. This assumption is never in entire agreement with actual conditions, and is only admissible as an approximation in cases where the piers or abutments will, by reason of their size and the character of the foundations, be subject to but slight distortions or movements. There are other approximate assumptions made in deriving the working formulas, such as that the material is homogeneous, that the modulus of elasticity is constant, and that the effect of the radius of curvature may be neglected. Furthermore, it is necessary to replace the definite integrals of the three equations of condition by summations. But this is not all. Even if the thrusts and moments at any section for any given condition of loading, etc., could be found with certainty by the elastic theory the resultant stresses could not even then be exactly determined, on account of the approximate character of the flexure formulas and the uncertain tensile stresses in the concrete. Finally, it may be pointed out that the live load itself is subject to much uncertainty, both as to its amount and its distribution, and that the effect of variations in temperature, while in many cases quite severe, cannot in the present state of our knowledge concerning the matter be determined with any great degree of certainty. The effect of the shrinkage of the concrete is even more uncertain. These and other like considerations justify the use of approximate short-cut methods such as those proposed in this chapter.

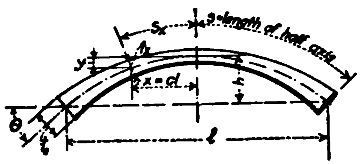


FIG. 19.

32. Best Shape of Arch Axis.—Using the notation as shown in Fig. 19; also

$$r = \text{rise ratio} = \frac{h}{l}.$$

$$u = \text{ratio of thickness at any point to thickness at the crown} = \frac{t_x}{t_0}.$$

$$u_s = \frac{t_s}{t_0}.$$

then the equations derived giving approximately the best shape of arch axis for both open-spandrel and filled-spandrel arches may be expressed as follows:

For open-spandrel arches:

$$y = \frac{8rl}{6 + 5r}(3c^2 + 10c^4r)$$

$$\tan\theta = \frac{8r}{6 + 5r}(3 + 5r)$$

[1] Taken verbatim, for the greater part, from paper on the "Design of Symmetrical Hingeless Concrete Arches" presented before The Engineers' Society of Western Pennsylvania by VICTOR H. COCHRANE. This paper was published in the Nov., 1916, issue of the Proc., vol. 32, No. 8, pp. 647–713. Diagrams on pp. 682–685 inclusive were supplied by Mr. Cochrane and take the place of formulas presented in the original.

For filled-spandrel arches:

$$y = \frac{4rl}{1 + 3r}(c^2 + 24c^5r)$$

$$\tan \theta = \frac{4r}{1 + 3r}(1 + 7.5r)$$

33. Variation in Thickness of Arch Ribs.—Mr. Cochrane made a number of complete designs and investigated a number of designs found in technical literature, in order to determine what the thicknesses of the arch should be at various points in the haunch to give the same fiber stresses as at the crown and the springing; or in other words, to determine the variation in u with respect to $v = \frac{s_x}{s}$ for equal maximum stresses throughout the arch ring. Plotting the values of v as abscissas with the values of u as ordinates, curves were obtained of which those shown in the full lines OAB and OCD in Fig. 20 are typical. In each case it was found that a certain portion of the haunch might, so far as the stresses were concerned, have been made thinner than the crown section. It was also found in each case that the thinnest section required is at about the quarter point, and that between this point and the springing the thickness should increase almost uniformly. It is easy to see why this should be the case, since the moments due to rib shortening and temperature variations are small near the quarter point and greatest at the crown or springing. A slight increase of thickness was required near the crown, as shown at X in OCD, in some arches of large rise-ratio designed for a relatively heavy live load.

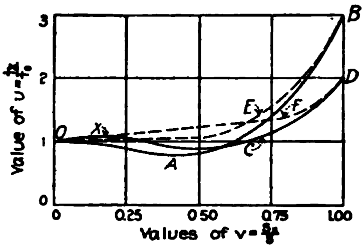


Fig. 20.

An arch of this shape would be unsightly and difficult to build, and consequently about the best that can be done is to make the arch rib of uniform thickness, or but slightly increasing thickness, from the crown to about the quarter point, and to increase the thickness almost uniformly from this point to the springing. The point at which the thickness should begin to increase rapidly is nearer the springing in arches of large rise-ratio carrying heavy live loads than in flat arches carrying light live loads. The dashed curve OFD in Fig. 20 represents an arch of the former kind and the curve OEB an arch of the latter kind, these curves representing the actual thicknesses to be used instead of the possible minimum thicknesses shown by OCD and OAB. The flatter arches generally require a greater relative thickness at the springing than do those with large rise-ratios.

These conclusions led to the selection of a number of typical curves showing the variation of u with respect to v for various values of u_s. These curves are shown in Diagram 1 and are designated as Types $A_{3.25}$, A_3, $A_{2.75}$, $A_{2.5}$, $A_{2.25}$, A_2, $A_{1.75}$ and $A_{1.5}$, the subscript in each case denoting the value of u_s for that type. These curves were chosen after a careful study of existing well-designed arches, and it is believed that the arch thicknesses required in any given case will be closely represented by some one of them; we have only to select the one best suited to the given conditions.

Since in each case the typical arch has a greater thickness in the haunches than would be required to keep the extreme fiber stresses within desired limits, we reach the important conclusion that *it is unnecessary to compute the stresses in any but the crown and springing sections.* The writer has yet to find a single well-designed arch having thicknesses corresponding closely to those represented by one of these typical curves, in which the stresses at the crown and springing are not very nearly the maximum found anywhere in the arch, provided the arch-shortening effect is considered. Perhaps the only good reason for computing the stresses anywhere except at these two sections is in order to determine whether the amount of the steel reinforcement may be reduced near the quarter point.

Many arches have been built having a much greater relative thickness in the haunch than indicated by these diagrams, but such practice is not to be recommended. Not only do such arches require more material than necessary, but on account of the thicker haunches have greater arch-shortening and temperature stresses at the crown and springing sections than do those having thicknesses corresponding to the diagrams. Thickening the haunch does not

DIAGRAM 1

Equations of Arch Axes:

For open-spandrel arches

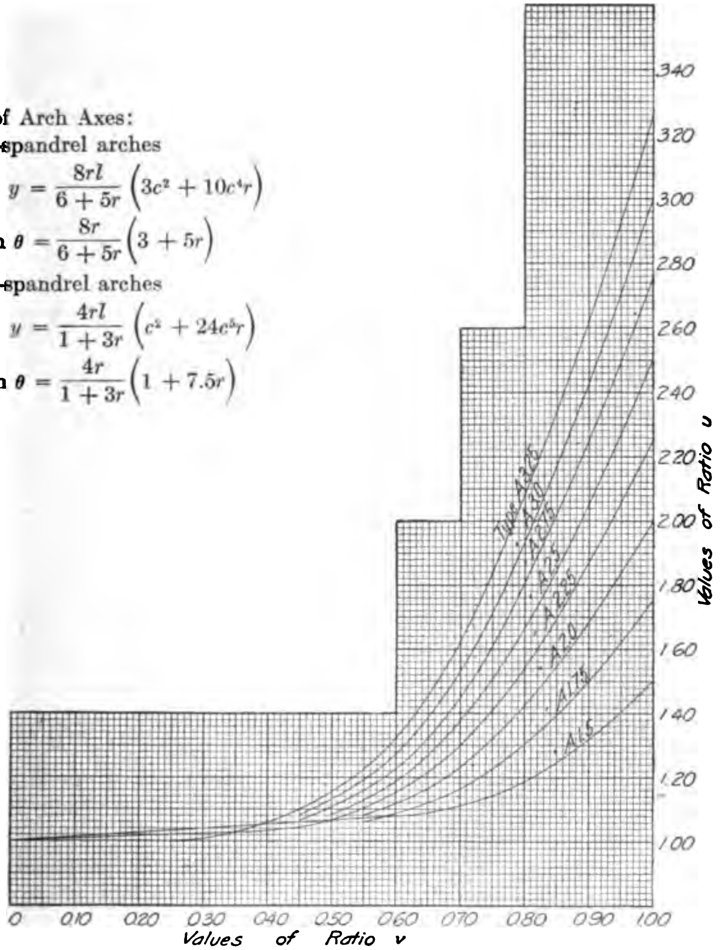
$$y = \frac{8rl}{6 + 5r}\left(3c^2 + 10c^4 r\right)$$

$$\tan \theta = \frac{8r}{6 + 5r}\left(3 + 5r\right)$$

For filled-spandrel arches

$$y = \frac{4rl}{1 + 3r}\left(c^2 + 24c^5 r\right)$$

$$\tan \theta = \frac{4r}{1 + 3r}\left(1 + 7.5r\right)$$



Assumed thickness of typical arches.

seem to improve the appearance of the arch. It is therefore clearly advisable to make the haunch as thin as practicable.

If the half axis is divided into ten equal sections the ratio of the depth of the arch at the center of each section to the depth at the crown is given in the following table:

TABLE 1.—THICKNESSES OF TYPICAL ARCHES

Value of $v = \frac{s_x}{s}$	Values of $u = \frac{t_x}{t_0}$ for type							
	$A_{1.5}$	$A_{1.75}$	A_2	$A_{2.25}$	$A_{2.5}$	$A_{2.75}$	A_3	$A_{3.25}$
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.05	1.007	1.006	1.005	1.004	1.003	1.002	1.001	1.000
0.15	1.021	1.018	1.015	1.012	1.009	1.006	1.003	1.000
0.25	1.035	1.030	1.025	1.020	1.015	1.010	1.005	1.000
0.35	1.049	1.042	1.035	1.028	1.023	1.021	1.023	1.030
0.45	1.063	1.054	1.048	1.048	1.057	1.070	1.083	1.101
0.55	1.077	1.072	1.085	1.105	1.133	1.165	1.193	1.231
0.65	1.095	1.125	1.168	1.215	1.269	1.328	1.385	1.455
0.75	1.145	1.223	1.311	1.403	1.508	1.625	1.737	1.865
0.85	1.245	1.393	1.547	1.700	1.862	2.025	2.185	2.355
0.95	1.406	1.621	1.837	2.055	2.277	2.495	2.709	2.932
1.00	1.500	1.750	2.000	2.250	2.500	2.750	3.000	3.250

We can readily obtain a formula for the areas of the vertical faces of these typical arches. This formula is

$$A = \frac{182.2 + 18.68u_e + 5.48u_e^2}{100} t_0 s$$

The value of s, the length of the half axis, may be determined by scaling from a drawing, or it may be taken by interpolation from the following table:

TABLE 2.—LENGTHS OF THE HALF ARCH AXIS s IN TERMS OF THE SPAN LENGTH

Kinds of arches	Values of s for rise-ratio r =				
	0.10	0.15	0.20	0.25	0.30
Open-spandrel arches......	$0.513l$	$0.529l$	$0.551l$	$0.577l$	$0.607l$
Filled-spandrel arches.....	$0.515l$	$0.534l$	$0.559l$		

Table 3 gives values of $\frac{\Delta s}{I}$ (referred to as $\frac{s}{I}$ in preceding chapters for the typical arches, the half axis in each case divided into 10 parts). I_0 = moment of inertia at the crown.

TABLE 3

Type	Value of $\frac{\Delta s}{I}$
$A_{1.5}$	$0.0769\ s \div I_0$
$A_{1.75}$	$0.0732\ s \div I_0$
A_2	$0.0699\ s \div I_0$
$A_{2.25}$	$0.0672\ s \div I_0$
$A_{2.5}$	$0.0647\ s \div I_0$
$A_{2.75}$	$0.0626\ s \div I_0$
A_3	$0.0606\ s \div I_0$
$A_{3.25}$	$0.0586\ s \div I_0$

The approximate value of s may be taken from Table 2 above.

The following table gives the location of the centers of divisions having a constant ratio $\frac{\Delta s}{I}$. These points will be referred to for convenience as "division centers," or "centers of divisions."

TABLE 4.—LOCATION OF CENTERS OF DIVISIONS HAVING A CONSTANT RATIO $\frac{\Delta s}{I}$ (10 divisions)

Point at center of division. No. from crown	Values of $v = \frac{s_x}{s}$ for type							
	$A_{1.5}$	$A_{1.75}$	A_2	$A_{2.25}$	$A_{2.5}$	$A_{2.75}$	A_3	$A_{3.25}$
1	0.039	0.037	0.036	0.034	0.033	0.032	0.031	0.030
2	0.119	0.112	0.107	0.102	0.098	0.095	0.092	0.089
3	0.201	0.190	0.180	0.172	0.165	0.159	0.153	0.149
4	0.286	0.270	0.255	0.243	0.232	0.222	0.214	0.208
5	0.374	0.352	0.332	0.316	0.301	0.287	0.275	0.265
6	0.464	0.436	0.411	0.389	0.370	0.353	0.338	0.325
7	0.558	0.522	0.491	0.466	0.443	0.423	0.405	0.389
8	0.657	0.614	0.578	0.549	0.524	0.502	0.481	0.463
9	0.766	0.722	0.684	0.652	0.625	0.600	0.578	0.558
10	0.912	0.890	0.872	0.856	0.842	0.829	0.817	0.806

To find the distances from the crown to the division centers, multiply the above coefficients by the length of the half axis. These coefficients apply to an axis of any shape.

Table 4 was computed on the basis of a 1 %crown reinforcement, but it may be used for plain arch ribs, since the moments and thrusts are changed but little on account of a considerable change in the location of the division centers.

34. Influence-line Diagrams.—The formulas used in figuring the influence-line values of Diagrams 2 to 13 inclusive were the same as given in Art. 18.

The use of these influence-line diagrams requires little explanation. The designer having determined by means of the diagrams referred to in Art. 35 (or otherwise) the approximate actual or relative thicknesses at crown and springing, selects the typical arch best suited to the particular case, and reads from the diagrams, by interpolation if necessary, the values of the moments and thrusts at crown and springing. Having these values he constructs influence line diagrams to any desired scale and reads off from these the moments and thrusts due to the specified concentrated loads. If no concentrated loads are specified, or if uniform loads may be substituted for the concentrated loads, the influence lines are not needed and the diagrams of Art. 35 are used instead.

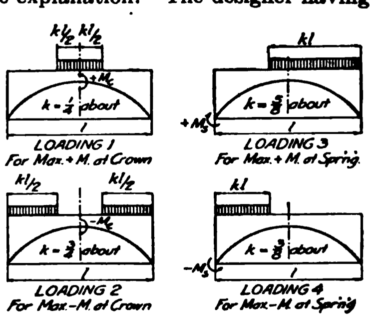
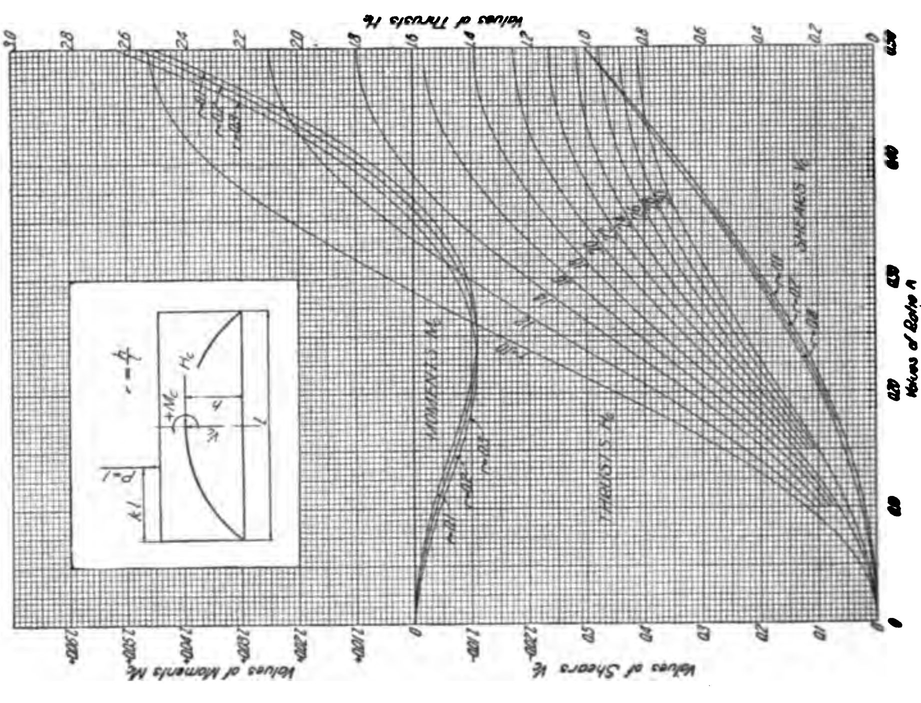


FIG. 21.—Typical loadings for maximum moments at crown and springing.

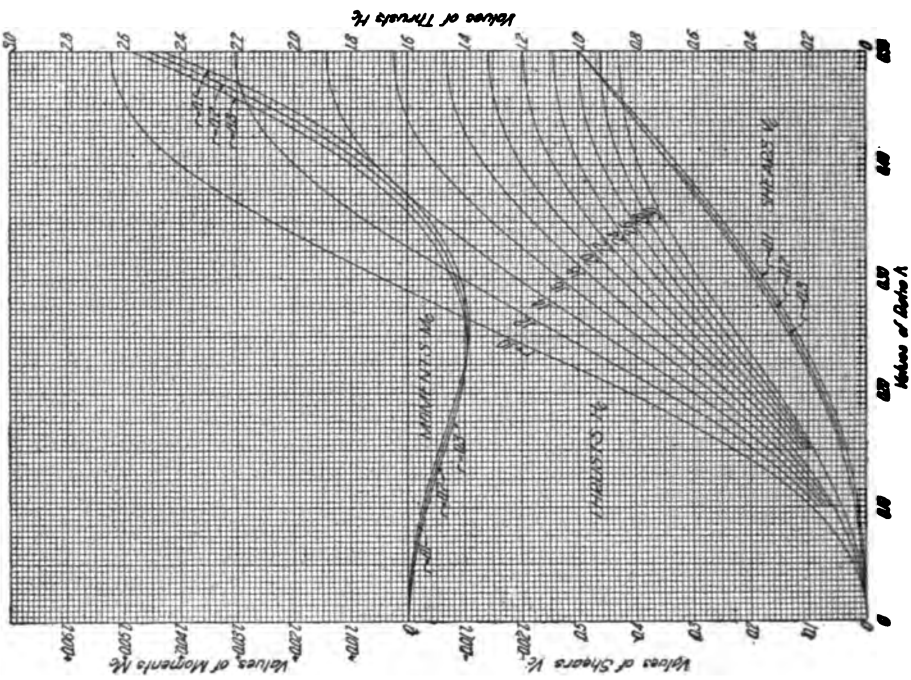
It will be noted that the maximum live-load moments at the crown and springing sections are determined by four typical arrangements of the live load designated in Fig. 21 as loadings 1, 2, 3 and 4. Loadings 1 and 2 are for the maximum positive and negative moments at the crown respectively, and when combined cover the entire span. Loadings 3 and 4 are for the maximum positive and negative moments at the springing section, respectively, and when combined likewise cover the span. In the figure, k represents the ratio of the length loaded to the span length. Loadings 1, 3 and 4 are continuous, while loading 2 is discontinuous.

DIAGRAM 2



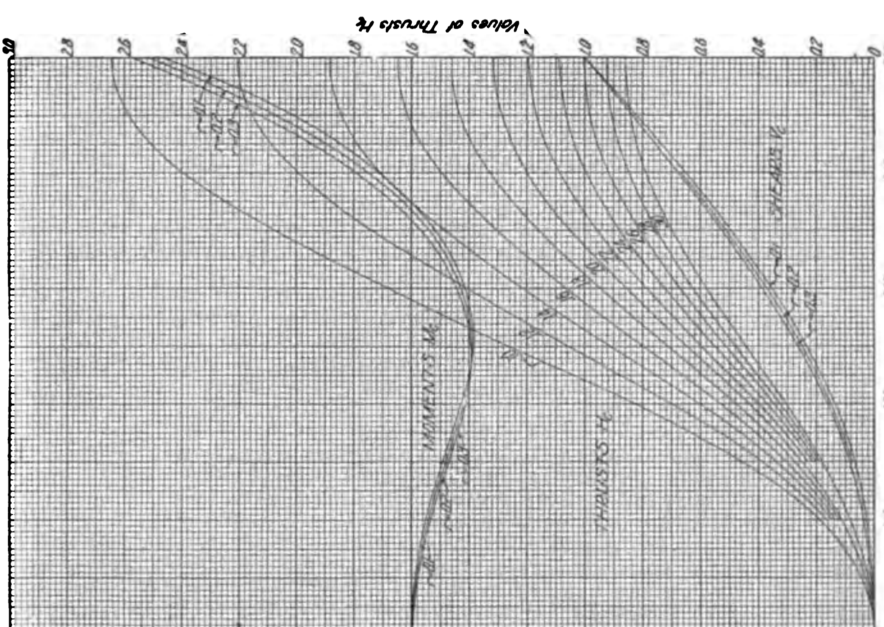
type A1.4.

DIAGRAM 3



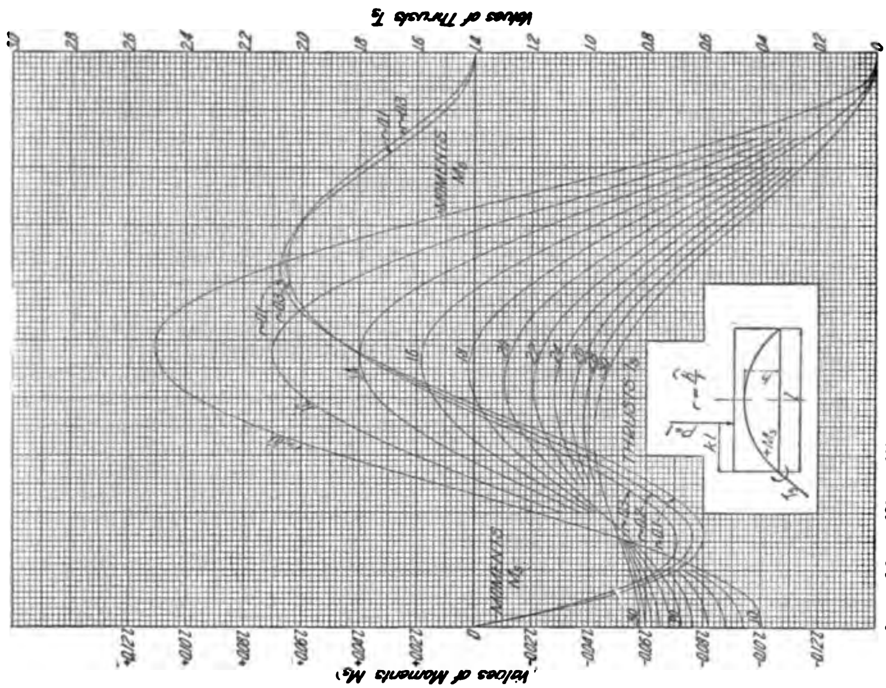
Influence lines for moments, thrusts and shears at crown; open spandrel arch, type A1.6.

DIAGRAM 4



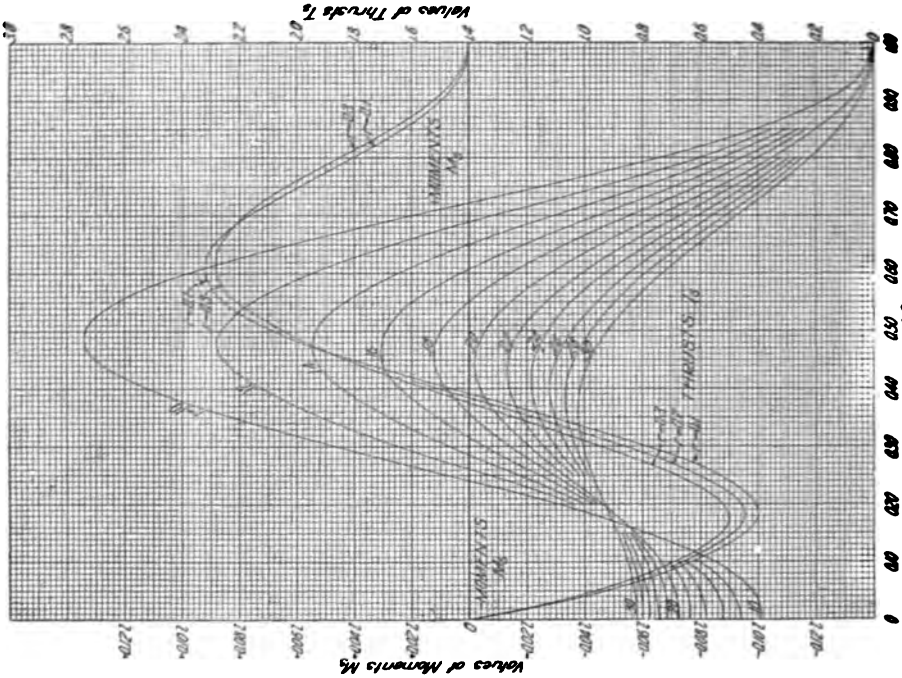
Influence lines for moments, thrusts and shears at crown: open spandrel arch, type $A_{1.6}$.

DIAGRAM 5



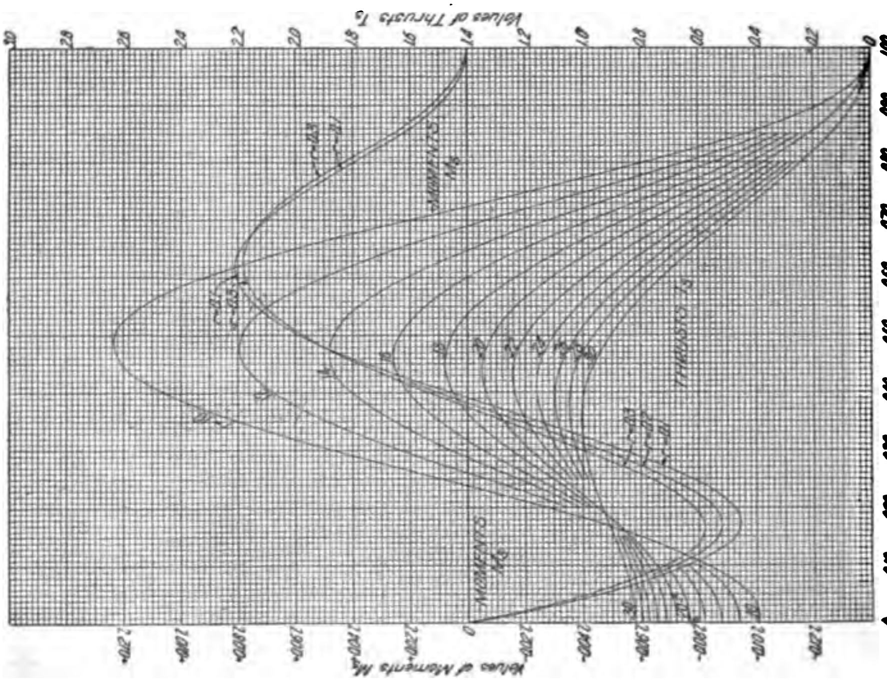
Influence lines for moments and thrusts at springing; open spandrel arch, type $A_{1.6}$.

DIAGRAM 7



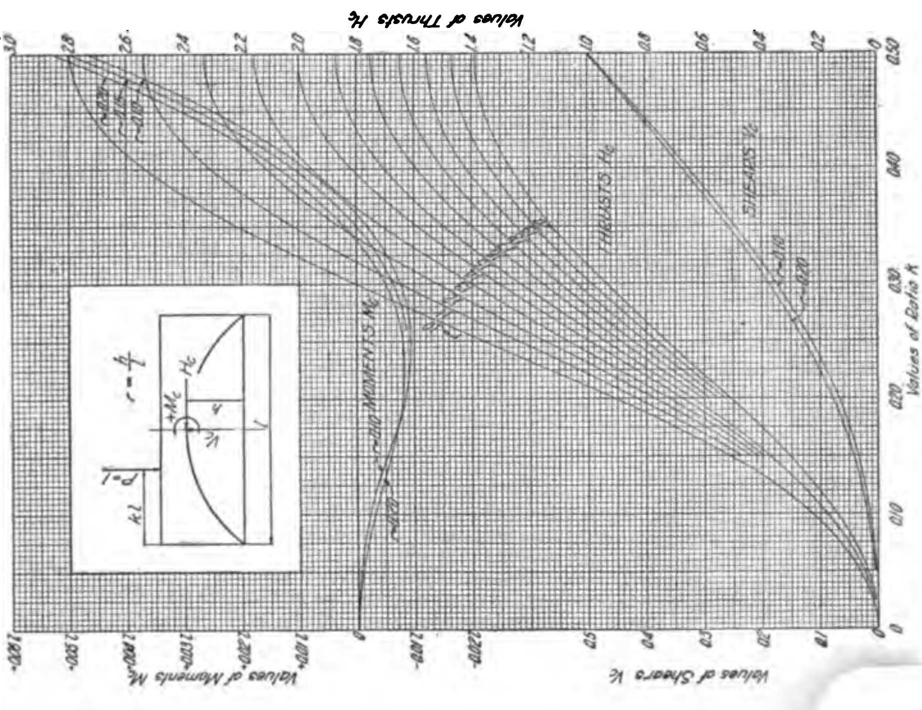
Influence lines for moments and thrusts at springing; open spandrel arch, type $A_{5.5}$.

DIAGRAM 6



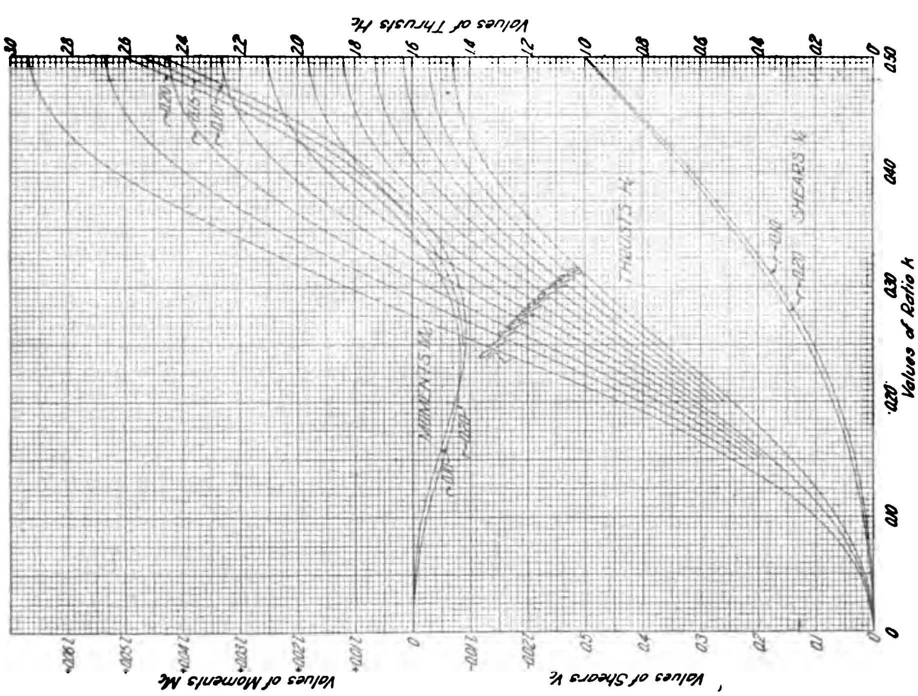
Influence lines for moments and thrusts at springing; open spandrel arch, type $A_{5.0}$.

Diagram 8



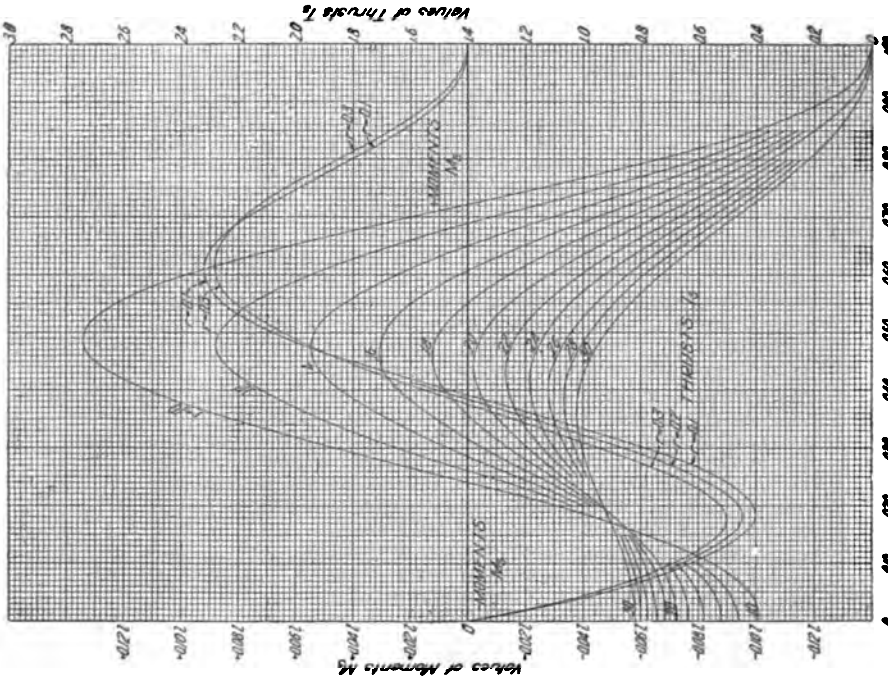
Influence lines for moments, thrusts and shears at crown; filled spandrel arch, type $A_{2.0}$.

Diagram 9



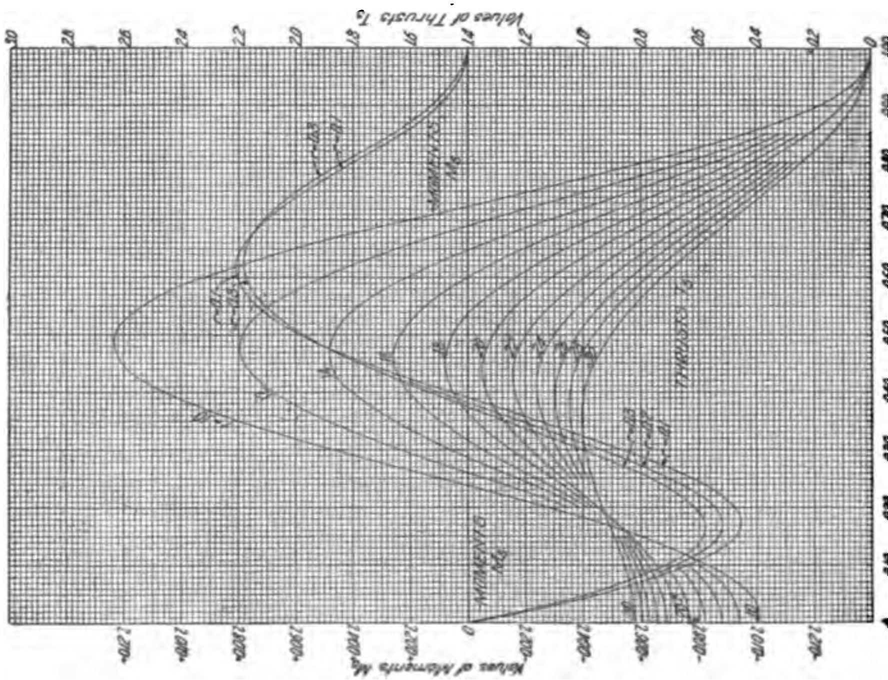
Influence lines for moments, thrusts and shears at crown; filled spandrel arch, type $A_{2.5}$.

DIAGRAM 7



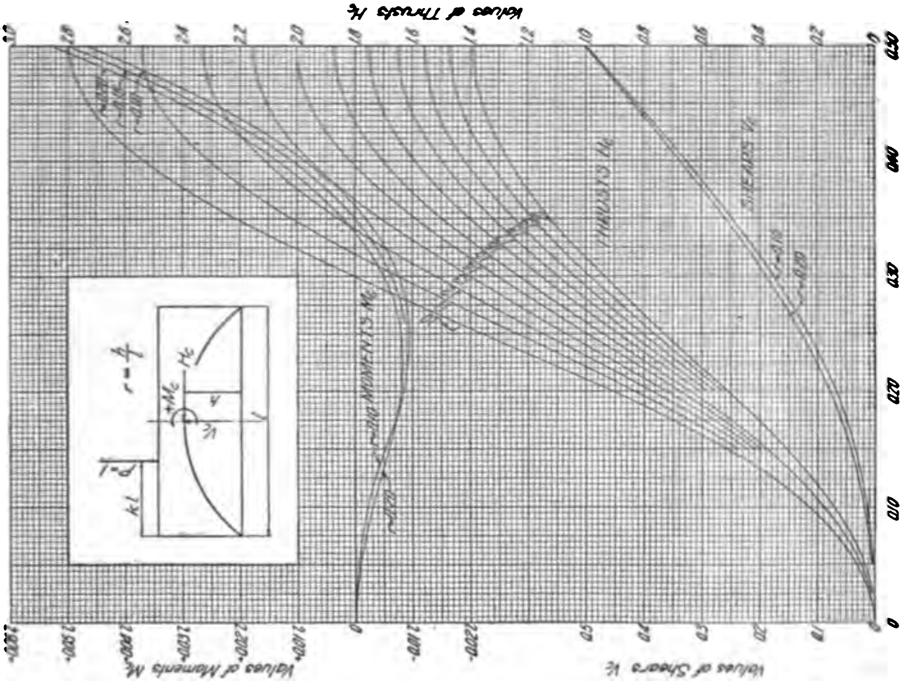
Influence lines for moments and thrusts at springing; open spandrel arch, type $A_{3.8}$.

DIAGRAM 6



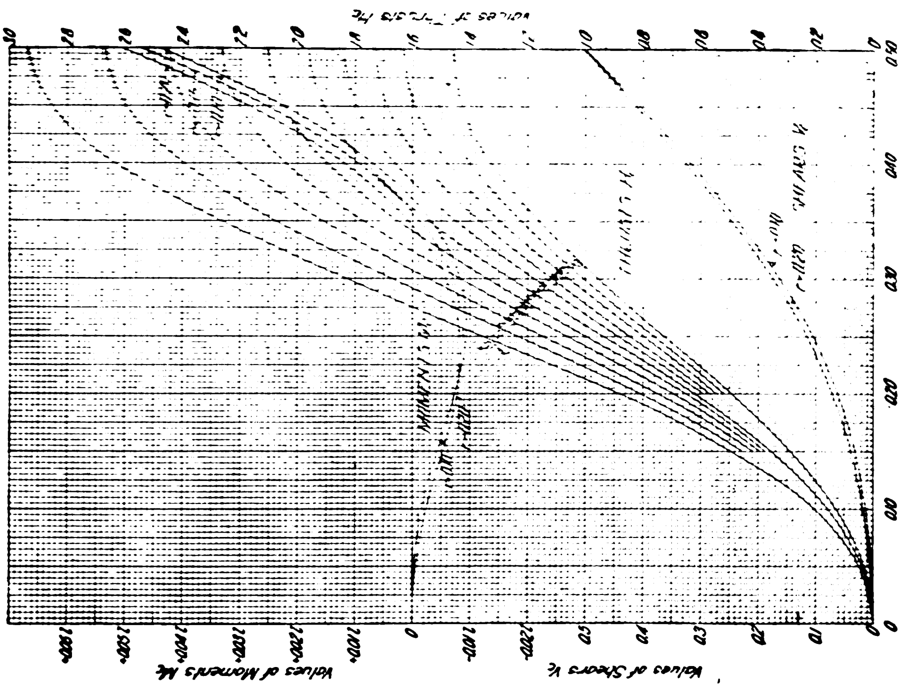
Influence lines for moments and thrusts at springing; open spandrel arch, type $A_{3.6}$.

DIAGRAM 8



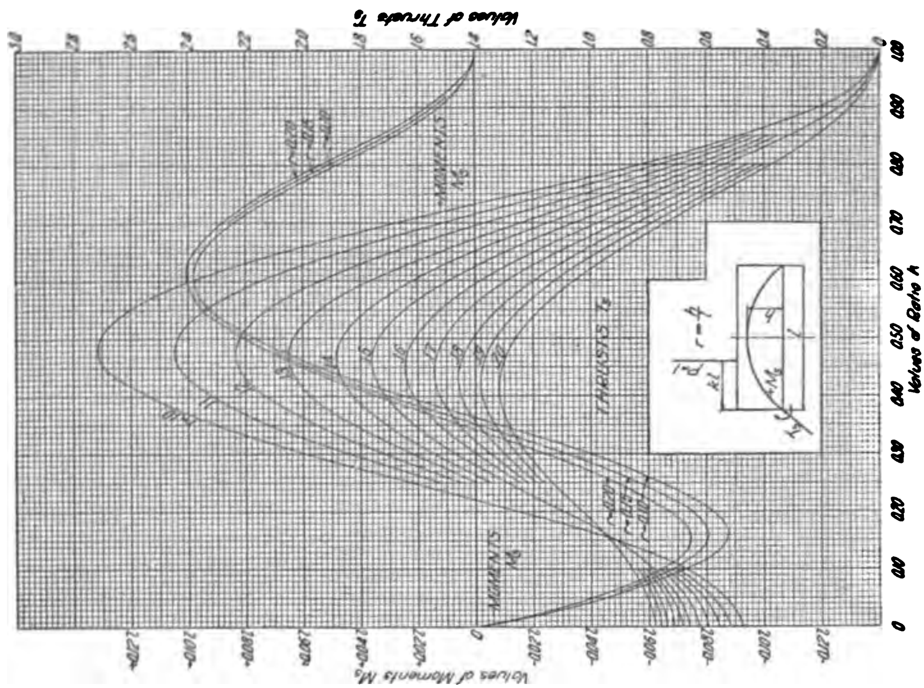
Influence lines for moments, thrusts and shears at crown; filled spandrel arch, type $A_{2.6}$.

DIAGRAM 9



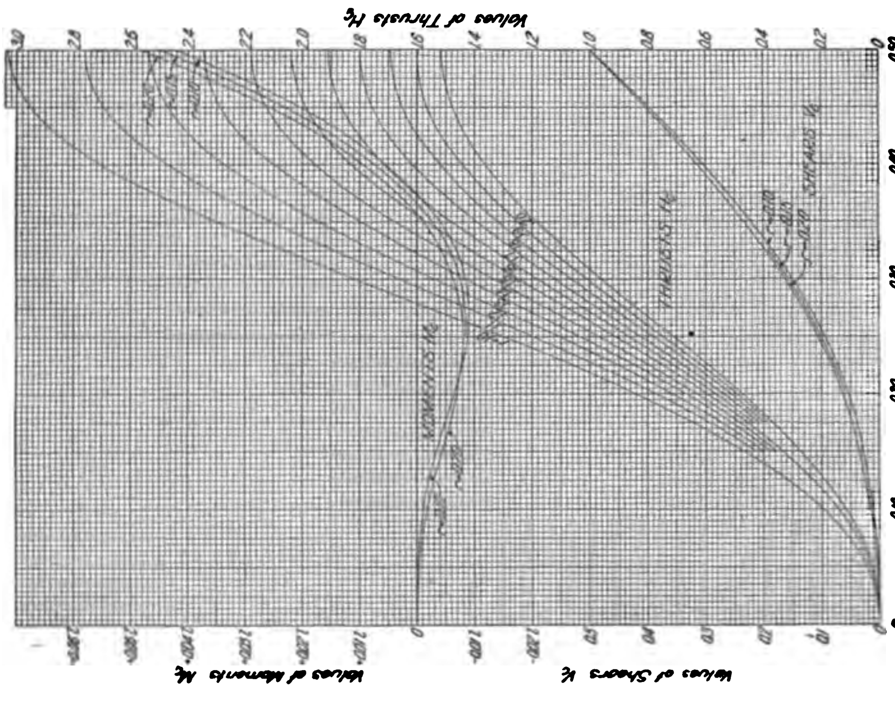
Influence lines for moments, thrusts and shears at crown; filled spandrel arch, type $A_{2.8}$.

DIAGRAM 11



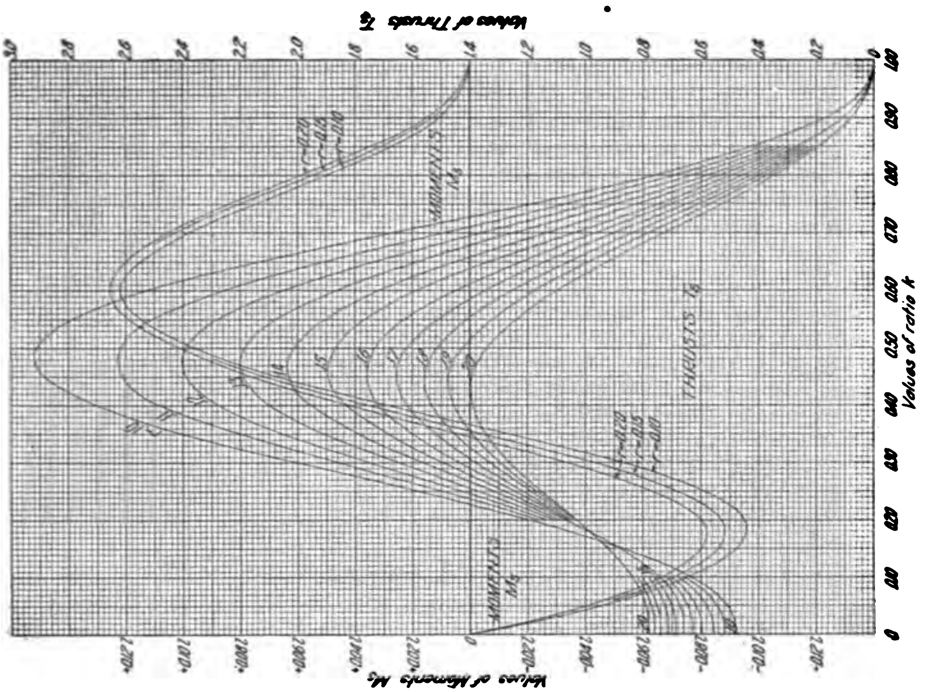
Influence lines for moments and thrusts at springing; filled spandrel arch, type A3.0.

DIAGRAM 10



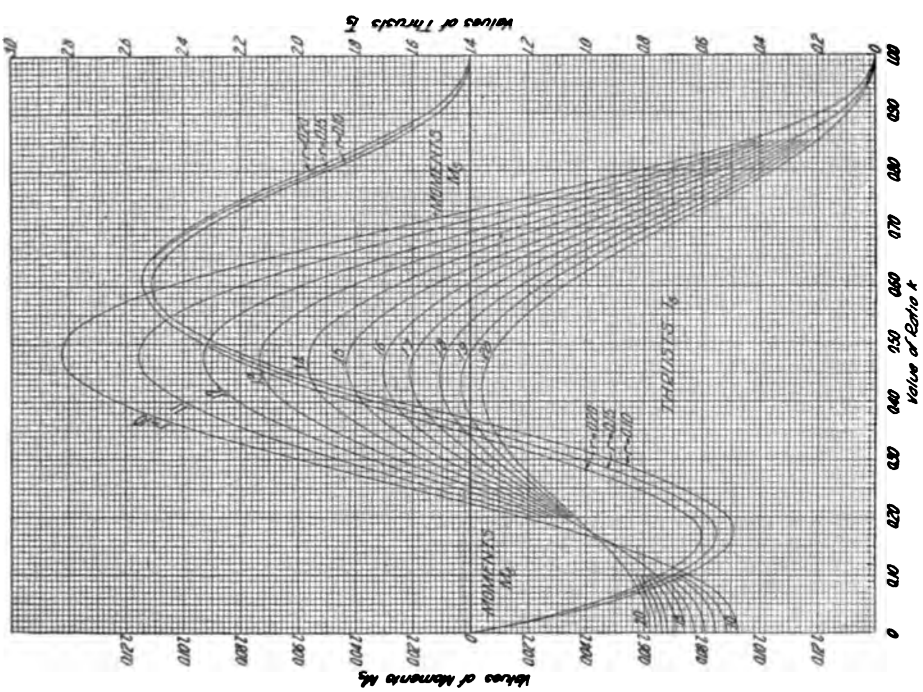
Influence lines for moments, thrusts and shears at crown; filled spandrel arch, type A3.0.

DIAGRAM 13



Influence lines for moments and thrusts at springing; filled spandrel arch, type $A_{3.0}$.

DIAGRAM 12



Influence lines for moments and thrusts at springing; filled spandrel arch, type $A_{2.5}$.

An inspection of the moment diagrams reveals an unexpectedly small variation in the value of the ratio k for each loading, the variation being greater with respect to the rise than with respect to the type of arch. It appears that using approximate average values for k, the maximum positive moment at the crown occurs when the live load covers one-fourth of the span at the middle, and the maximum negative moment occurs when the load covers the two end three-eighths. The maximum positive moment at the springing is produced by a load covering five-eighths of the span beginning at the opposite end, and the maximum negative moment occurs when the load covers the adjacent end three-eighths of the span. It may be stated further that loading 1 and loading 4 combined produce loading 3, and that one-half of loading 2 is the same as loading 4.

Another striking fact is that for any given type of arch the coefficients for moments vary only slightly with respect to the rise-ratio. Hence we may say that the moments for any typical arch of given span length are practically independent of the rise-ratio.

As the ratio of springing thickness to crown thickness increases, the arch axis remaining the same, the moments at the springing increase and the moments at the crown decrease, as might be expected. The change in the moments as the arch axis changes in form, the ratio u_s remaining the same, may be seen by comparing the diagrams for an open-spandrel arch with those for a filled-spandrel arch having the same ratio u_s. Table 5 presents a typical case. The positive moments at crown and springing increase as the arch axis departs farther from the parabolic form and becomes more nearly elliptical, while the negative moments at crown and springing decrease.

Evidently the areas enclosed by a moment influence line above and below the line of zero moments are a measure of the maximum moments due to a uniform live load on the span. The algebraic sum of the positive and negative areas is a coefficient of the moments due to live load over the entire span. It is evident, therefore, from an inspection of the diagrams that the moments for live load over the whole span are generally much smaller numerically than the maximum moments, and are always positive at crown and springing.

Another interesting fact is that for arches of the same type (that is, having the same ratio u_s), but having axes of different form, the arithmetical sum of the positive and negative moments is practically the same. For example, take an open-spandrel and a filled-spandrel arch of the same rise-ratio $r = 0.2$ and the same thickness-ratio $u_s = 2.5$ (Diagrams 4, 7, 9 and 12), and the moments are as follows:

TABLE 5.—COMPARISON OF MOMENTS FOR TYPE $A_{2,5}$ OPEN-SPANDREL AND FILLED-SPANDREL ARCHES ($r = 0.20$)

Section	Kind of moment	Open-spandrel arch	Filled-spandrel arch
Crown	Max. + moment	$+0.00474wl^2$	$+0.00637wl^2$
	Max. − moment	$-0.00378wl^2$	$-0.00302wl^2$
	Algebraic sum	$+0.00096wl^2$	$+0.00335wl^2$
	Arithmetical sum	$0.00852wl^2$	$0.00939wl^2$
Springing	Max. + moment	$+0.02990wl^2$	$+0.04110wl^2$
	Max. − moment	$-0.02330wl^2$	$-0.01750wl^2$
	Algebraic sum	$+0.00660wl^2$	$+0.02360wl^2$
	Arithmetical sum	$0.05320wl^2$	$0.05860wl^2$

Thus while the maximum positive moments are more than one-third greater for the filled spandrel arch than for the open-spandrel one, the arithmetical sums of the moments are only one-tenth greater. These two arch axes differ widely, and hence for two axes differing but little we may assume that the arithmetical sum of the maximum positive and negative moments is the same for each arch. This fact leads to an easily applied approximate method for correcting the known moments for an assumed axis to suit an actual arch having its axis differing in form from the assumed axis. This method is given in Art. 36.

The moments in any typical arch due to *unit* loads are proportional to the span length, while the thrusts are the same for all span lengths. The shears at the crown are nearly independent of the rise and are the same for all span lengths. The thrusts at the crown increase as u_s increases, and also as the arch axis approaches an elliptical form, and very approximately inversely as the rise-ratio r.

35. Diagrams for Moments, Thrusts, and Average Stresses.—The notation used in Diagrams 14 to 21 inclusive is as follows:

l = the span length of the arch axis in feet.
h = the rise of arch axis in feet.
r = the rise-ratio h/l.
y = the ordinate of the arch axis at any point the abscissa of which is cl.
u_s = ratio of the thickness at springing to the thickness at crown.
M_c = moment at crown in foot-pounds.
T_c = thrust at crown in pounds.
M_s = moment at springing in foot-pounds.
T_s = thrust at springing in pounds.
V_s = approximate dead load vertical end reaction, or one-half dead weight of span in pounds.
w_c = weight of arch at the crown, plus average weight of arch superstructure at the crown in pounds per lineal foot of span.
w = live load in pounds per lineal foot of span (not necessarily the same for all positions of the live load).
t_e = the coefficient of linear expansion due to temperature changes.
t_D = change in temperature in degrees Fahrenheit.
E = modulus of elasticity of concrete in pounds per square foot.
I_c = the moment of inertia of the arch rib at the crown in feet[4].
f_a = average direct stress throughout arch in pounds per square foot.
f_{ac} = direct stress at crown section in pounds per square foot.

From what has been said the use of these diagrams will be readily understood. Following is a summary of the steps required in designing an arch by the use of the diagrams. It is assumed that the arch superstructure has been designed, and that the dead load per lineal foot at the crown, exclusive of the arch rib, is known; also that the span and rise of the axis have been fixed.

1. Assume a crown thickness in accordance with one of the empirical formulas in use, or by comparison with a previous design, and compute the total dead load per lineal foot of span at the crown (= w_c).

2. Assume the arch type, or the thickness-ratio u_s. For open-spandrel arches this ratio will usually be from 1.5 to 2.5, and for filled-spandrel arches from 2 to 3.25.

3. Determine from the diagram the dead-load thrusts and the maximum positive and negative moments and corresponding thrusts at the crown and springing due to live load, temperature variation and arch shortening, and calculate the extreme fiber stresses due to the proper combinations of moments and thrusts. If the stresses so found are too great or too small, change the thickness at the crown or at the springing, or both, and repeat the above operation. The second trial will usually be sufficient. If the thicknesses originally assumed are not correct, the fact will usually be revealed before the first set of calculations is completed.

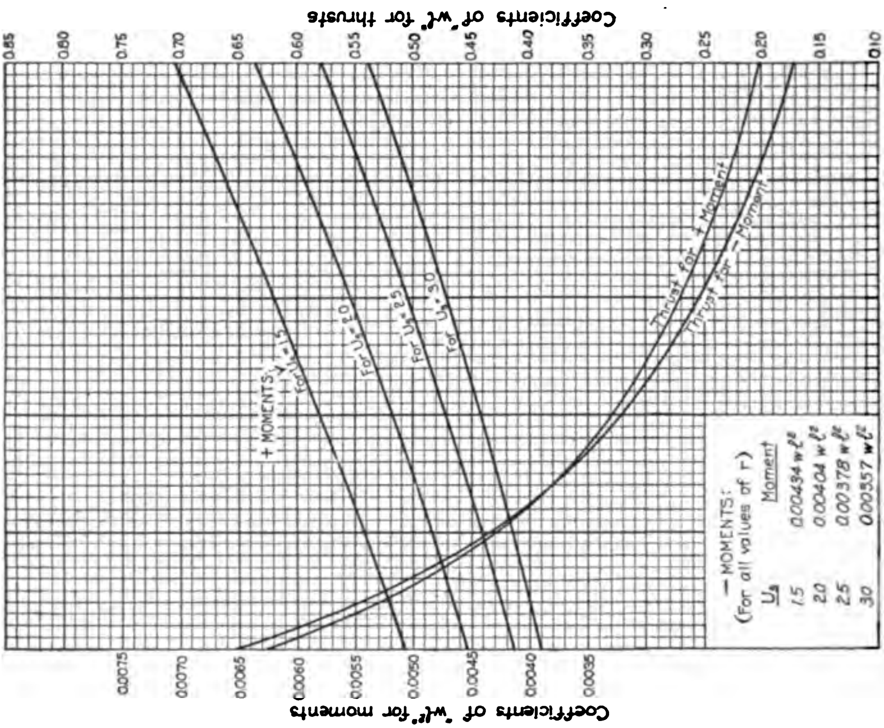
4. Lay out the assumed arch axis as per Art. 32 and divide it into 10 or more equal parts, locating the center of each division. At the points thus found lay off the half thickness, as given in Diagram 1 or in Table 1, above and below the assumed axis, and through the points thus determined pass, as nearly as practicable through all the points, segmental, three-centered, or five-centered curves. A set of railroad curves is useful for this purpose.

5. Compute the dead loads at the panel points or at suitable intervals and lay out an equilibrium polygon passing through the crown and springing, the value of the horizontal thrust being first computed as for a three-hinged arch and used as the pole distance for the force polygon (see Art. 11).

6. By trial alter the shape of the arch axis so that it will fit the dead-load equilibrium polygon as nearly as practicable, lay out the arch thicknesses again, and determine the radii of the intradosal and extradosal curves.

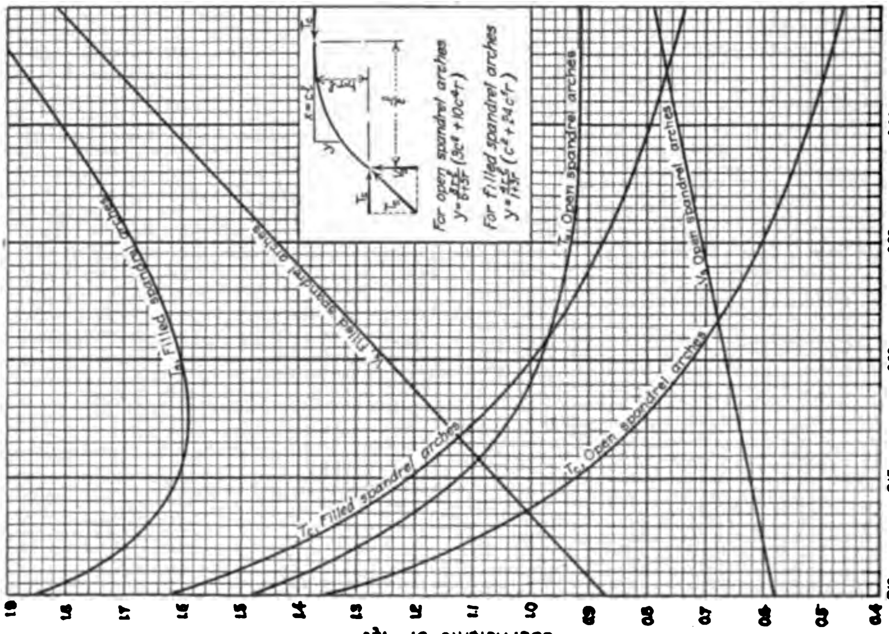
7. If the actual axis departs considerably from the assumed axis, scale the dead-load thrusts from the force polygon and correct the maximum live-load moments by the method given in Art. 36, revising the stresses and changing the thicknesses if necessary. This last step will seldom be required.

DIAGRAM 15



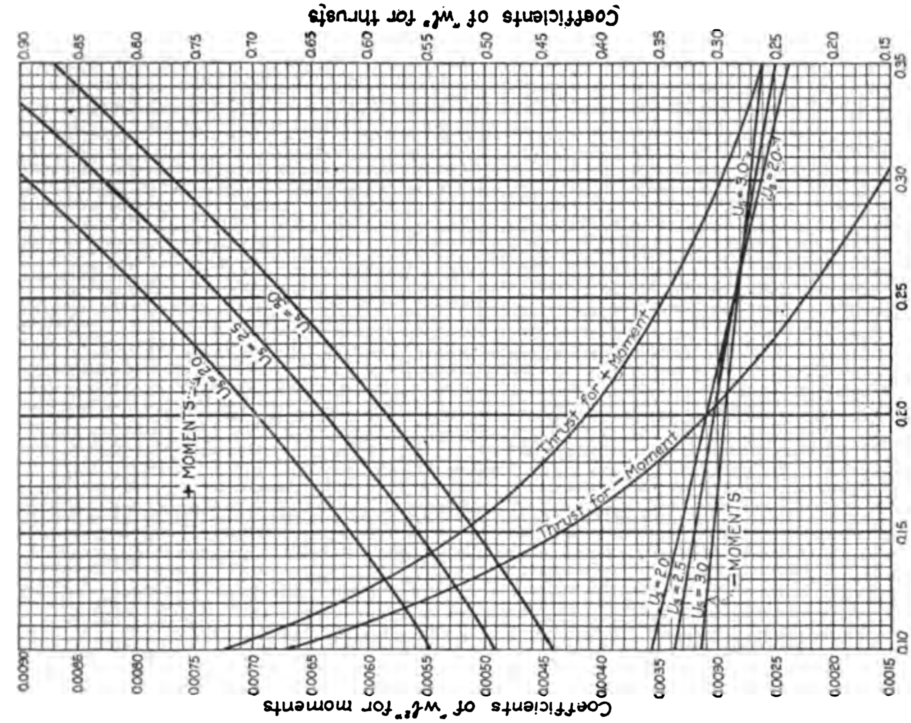
Live-load thrusts and moments at crown; open spandrel arches.

DIAGRAM 14



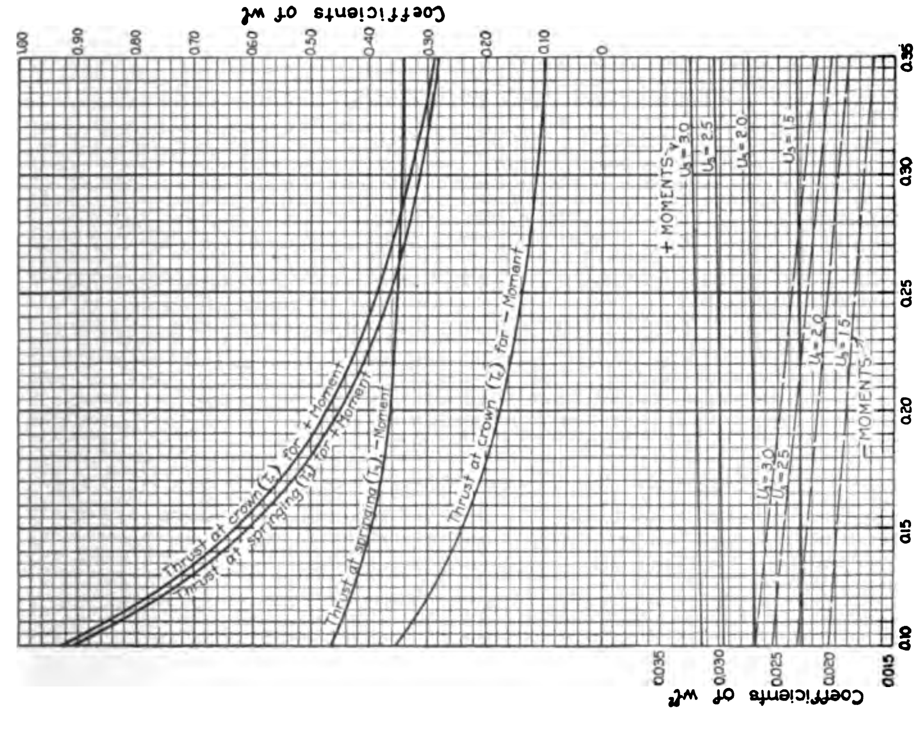
Approximate dead-load thrusts and moments.

DIAGRAM 17



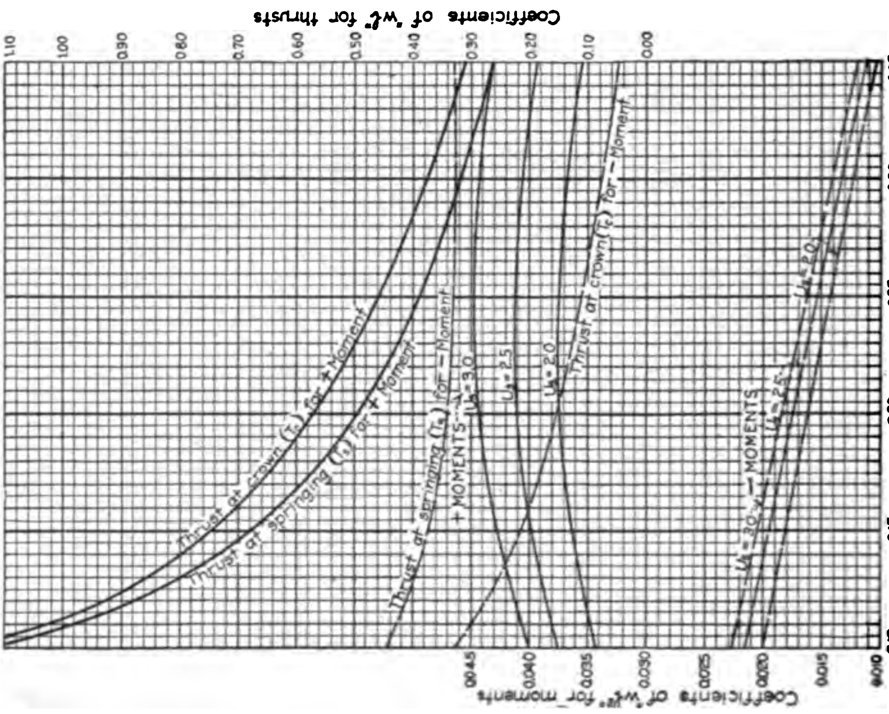
Live-load thrusts and moments at crown; filled spandrel arches.

DIAGRAM 16



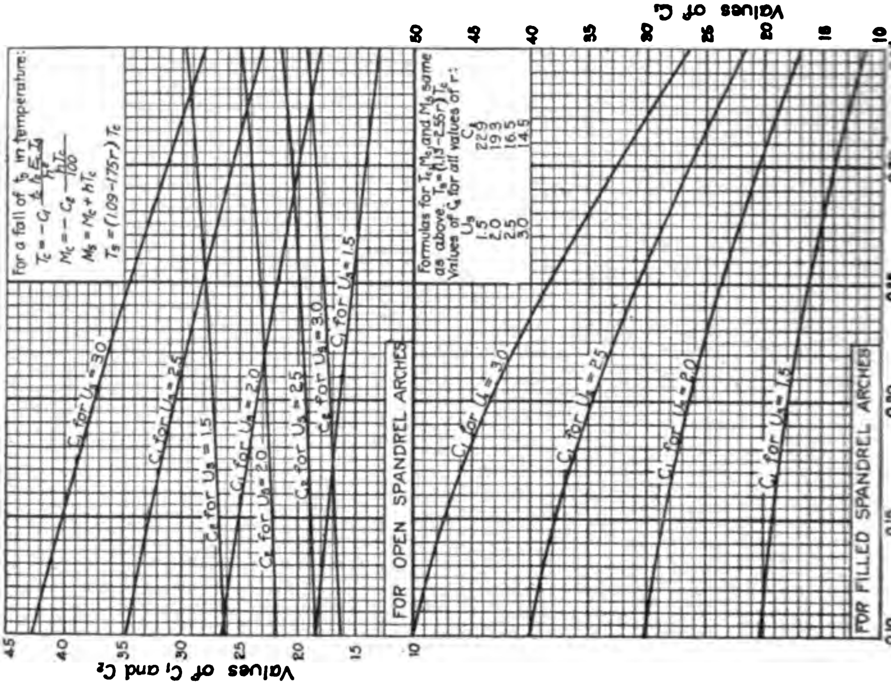
Live-load thrusts and moments at springing; open spandrel arches.

DIAGRAM 18



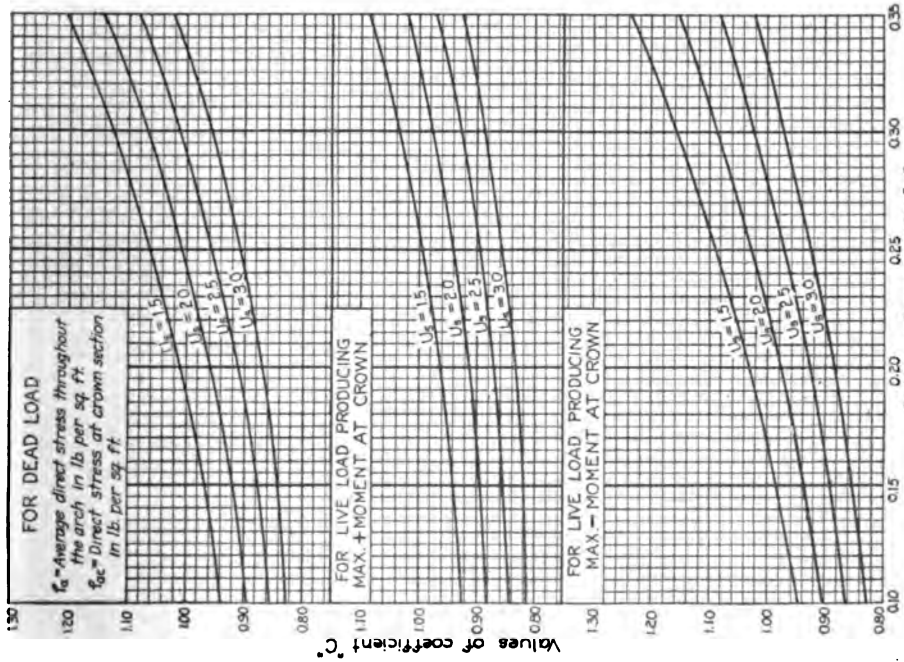
Live-load thrusts and moments at springing; filled spandrel arches.

DIAGRAM 19



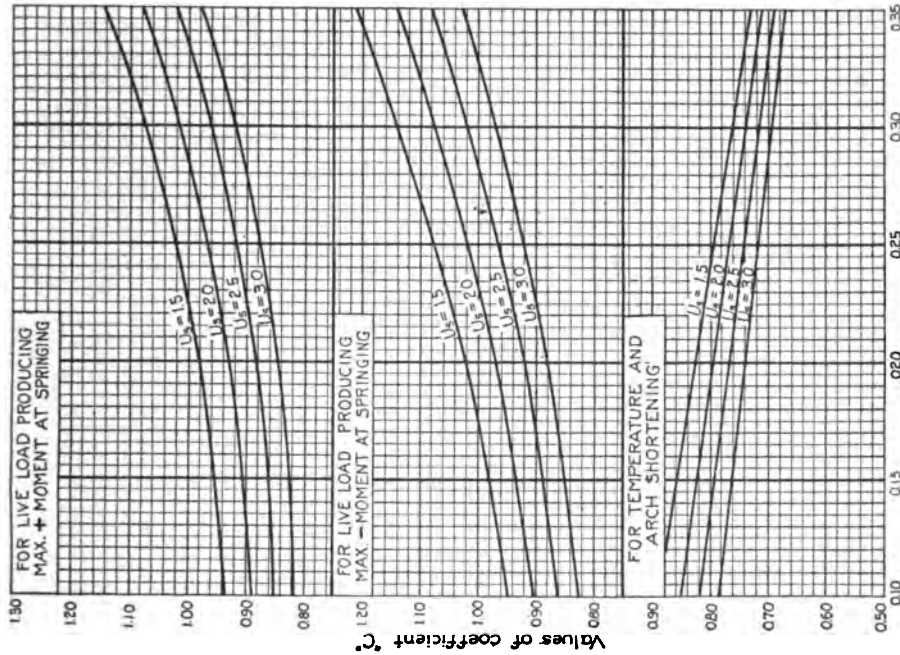
Temperature and rib shortening

DIAGRAM 20



Average stresses. Values of "C" in formula $f_a = Cf_{ac}$.

DIAGRAM 21



Average stresses. Values of "C" in formula $f_a = Cf_{ac}$.

36. Approximate Method of Correcting Maximum Moments when Actual Arch Axis Deviates from Assumed Axis.—It has been found that for any given typical arch—that is. for an arch having a constant ratio u_s—the resistance line for live load over the entire span crosses the arch axis at about the same horizontal distance from the center line of the span. regardless of the shape of the arch axis. Also it has been found that for two arch axes of the same type and which differ but little, the arithmetical sum (S_1) of the maximum positive and negative moments is the same for each arch. Making use of these facts, the formulas given below were obtained.

In Fig. 22, the line $CABS$ represents the axis of an open-spandrel arch of type $A_{2.5}$ and rise-ratio = 0.2, while $CDES$ represents the axis of a type $A_{2.5}$ filled-spandrel arch. The

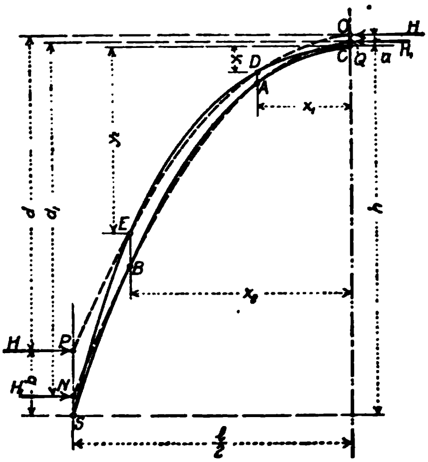


FIG. 22.—Resistance lines for an open-spandrel and a filled spandrel arch; live load over whole span.

vertical scale is 3⅓ times the horizontal. The resistance lines for full live load are $QABN$ and $ODEP$, respectively, and it will be seen that the intersection points A and D are about equidistant from the center line, as are also B and E. It appears, then, that if the full-load resistance line for one axis, as $CABS$, is known, the resistance line for another axis, as $CDES$, may be approximately determined by passing the parabola $ODEP$ through the two points D and E having the abscissas x_1 and x_2 the same as for the known intersection points A and B, and the ordinates y_1 and y_2, referred to C as origin.

Let d = the rise of the required resistance line for live load over the entire span (parabolic).
d_1 = the corresponding known term for the assumed arch.
a = the vertical intercept between the arch axis and the required resistance line at the crown.
b = the vertical intercept at the springing.
H_1 = horizontal thrust for known resistance line $QABN$.
M_p' = known maximum *positive* moment at crown or springing for the assumed axis $CABS$.
M_n' = known maximum *negative* moment at crown or springing for the assumed axis $CABS$.
M_p and M_n = corresponding terms for the actual axis $CDES$.
S_1 = arithmetical sum of the moments M_p' and M_n' = $M_p' - M_n'$.

Then

$$M_p = \frac{H_1 a \frac{d_1}{d} + S_1}{2}$$

$$M_n = \frac{H_1 a \frac{d_1}{d} - S_1}{2}$$

For the springing moments substitute b for a in these formulas.

These formulas will give close approximations to the true values of the moments even if the axis used deviates considerably from the assumed axis.

Illustrative Problem.—The design will be that of the 132-ft. arch, Kansas River bridge at Lawrence, Kan. This is a rather flat open-spandrel arch. Typical half sections of the span are shown in Fig. 23. Following are the dimensions:

$$l = 132 \qquad h = 16 \qquad \therefore r = 0.121$$
$$t_0 = 2.5 \qquad t_s = 5.63 \qquad \therefore u_s = 2.25$$

The live loads are as follows:

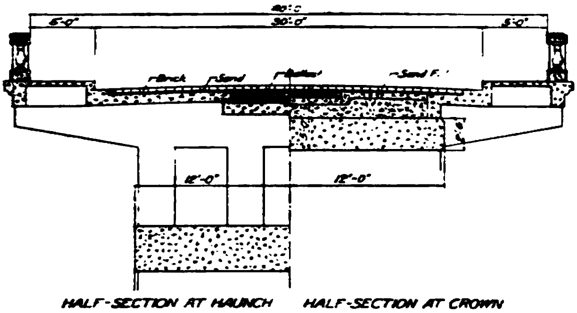


Fig. 23.

Live Loads

Member loaded	Portion of structure loaded		
	Track	Remainder of roadway (18 ft.)	Sidewalks
For roadway floor slabs, fascia-girders, cantilever beams, cross-girders, spandrel columns and approach girder spans.	Electric railway loading (assumed to occupy a transverse width of 12 ft.).	20-ton road roller or 200 lb. per sq. ft.	150 lb. per sq. ft.
For arch rings and abutments.	Electric railway loading (assumed to occupy a transverse width of 12 ft.).	60 lb. per sq. ft.	50 lb. per sq. ft.
	Or the equivalent uniform loads given below.		
For the unbalanced load on piers	3500 lb. per lin. ft. span	50 lb. per sq. ft.	40 lb per sq. ft.

The wheel loads are shown in Fig. 24.

The working stresses assumed are given on page 688.

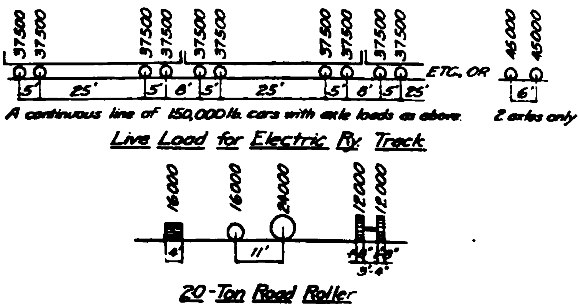


Fig. 24.

WORKING STRESSES IN POUNDS PER SQUARE INCH

Member	Kind of stress	Stress
Floor slabs	Compression in concrete	600
	Tension in steel	16,000
Arch rings	Compression on concrete, including L. L., D. L., and rib shortening	600
	Compression on concrete, including L. L., D. L., rib shortening and temperature (for a variation of $\pm 40°$)	800

$E_s = 30{,}000{,}000$ $E_c = 2{,}000{,}000$ $n = 15$ t_c = coefficient of expansion = 0.0000055.

In order to employ the formulas it is necessary to use a uniform live load in lieu of the concentrated loads shown in Fig. 24 for the electric railway track. It is difficult, if not impracticable, to work out equivalent uniform loads for arches such as are in use for simple beam and truss spans. It will usually be sufficiently accurate for practical purposes to use the average weight per lineal foot of car. Hence we assume the uniform load as follows:

Average weight of car	$= \frac{150{,}000}{43}$	= 3,490
Uniform load on roadway	$= (30 - 12) \times 60$	= 1,080
Uniform load on sidewalks	$= 10 \times 50$	= 500
Total		5,070 lb.

Then we have $wl = 5{,}070 \times 132 = 669{,}000$ lb.
and $wl^2 = 5{,}070 \times 132^2 = 88{,}340{,}000$ ft.-lb.

It will be assumed to begin with that rough preliminary calculations have been made from which it is found that the crown thickness t_0 may be assumed = 2.5, and the ratio $u_s = 2.25$. The first step is to compute the dead load per lineal foot at the crown (w_c), thus:

Weight of track, paving, and paving base on sand fill	4,000
Weight of concrete in floor slabs, fascia girders and side walls	3,170
Weight of cantilever brackets, per foot of span	1,300
Weight of sand fill	2,880
Weight of railings	800
Weight in conduit spaces	250
Weight of arch rib, 24 × 2.5 × 150	9,000
Total u_c =	21,400 lb.

Dead Load.—From Diagram 14, T_c (or H_c)
$= (1.135)(21{,}400)(132) = 3{,}210{,}000$ lb.
$V_s = (0.600)(21{,}400)(132) = 1{,}700{,}000$ lb.
$T_s = 3{,}630{,}000$ lb.

Live Load, Max. + Mom. at Crown.—From Diagram 15,
$T_c = (0.524)(669{,}000) = 351{,}000$ lb.
$M_c = (0.00445)(88{,}340{,}000) = +393{,}000$ ft.-lb.

Live Load, Max. − Mom. at Crown.—From Diagram 15,
$T_c = (0.545)(669{,}000) = 365{,}000$ lb.
$M_c = (-0.00391)(88{,}340{,}000) = -345{,}000$ ft.-lb.

Live Load, Max. + Mom. at Springing.—From Diagram 16,
$T_s = (0.746)(669{,}000) = 499{,}000$ lb.
$T_c = (0.775)(669{,}000) = 518{,}000$ lb.
$M_s = (0.0282)(88{,}340{,}000) = +2{,}490{,}000$ ft.-lb.

Live Load, Max. − Mom. at Springing.—From Diagram 16,
$T_s = (0.431)(669{,}000) = 288{,}000$ lb.
$T_c = (0.295)(669{,}000) = 197{,}000$ lb.
$M_s = (-0.024)(88{,}340{,}000) = -2{,}120{,}000$ ft.-lb.

The arch is reinforced with 24 lines of 1¼-in. square bars at top and bottom. The centers of the bars are 3 in. from the face of the arch, or 1 ft. from the axis at the crown. Hence the moment of inertia

$$I_0 = \frac{(24)(2.5)^3}{12} + \frac{(24)(2)(1.56)(1)^2(15)}{144} = 39.05$$

The equivalent area A_0 at the crown =

$$2.5 + \left[\frac{(2)(1.56)(15)}{144}\right](24) = 67.8 \text{ sq. ft.}$$

Fall of Temperature of 40°.—
$t_c t_D E = (0.0000055)(40)(288{,}000{,}000)$
$= 63{,}400$ lb. per sq. ft.

From Diagram 19,

$$T_c = \frac{(29.9)(63,400)(39.05)}{(16)(16)} = -289,000 \text{ lb.}$$

$$M_c = \frac{(20.1)(289,000)(16)}{100} = +929,000 \text{ ft.-lb.}$$

$$T_s = (1.09 - 1.75 \times 0.121)(289,000) = -254,000 \text{ lb.}$$

$$M_s = +929,000 - (16)(289,000) = -3,695,000 \text{ ft.-lb.}$$

Average Stresses.—From Diagrams 20 and 21

For dead load,

$$f_a = (0.88)\left(\frac{3,210,000}{67.8}\right) = 41,700 \text{ lb.}$$

For L. L. producing max. + moment at crown,

$$f_a = (0.87)\left(\frac{351,000}{67.8}\right) = 4500 \text{ lb.}$$

For L. L. producing max. − moment at crown,

$$f_a = (0.90)\left(\frac{365,000}{67.8}\right) = 4800 \text{ lb.}$$

For L. L. producing max. + moment at springing

$$f_a = (0.87)\left(\frac{518,000}{67.8}\right) = 6700 \text{ lb.}$$

For L. L. producing max. − moment at springing,

$$f_a = (0.89)\left(\frac{197,000}{67.8}\right) = 2600 \text{ lb.}$$

For temperature drop of 40°,

$$f_a = (0.83)\left(\frac{289,000}{67.8}\right) = -3500 \text{ lb.}$$

For each combination of loading, the arch-shortening thrusts and moments bear the same ratio to the thrusts and moments due to a fall of 40° in temperature, as does the total average stress to the stress $\omega t_0 E$ (= 63,400 lb.).

I. Summary for Maximum + Moment at Crown

(a) D. L., L. L. and Arch Shortening

	Thrust	Moment	Average stress
D. L.	+3,210,000	....	+41,700
L. L.	+ 351,000	+393,000	+ 4,500
Arch S.	− 200,000	+642,000	− 2,400
Total	+3,361,000	+1,035,000	+43,800

(b) D. L., L. L., Temperature Variation, and Arch Shortening

	Thrust	Moment	Average stress
D. L. and L. L.	+3,561,000	+393,000	+46,200
Temperature	− 289,000	+929,000	− 3,500
Arch S.	− 185,000	+593,000	− 2,200
Total	+3,087,000	+1,915,000	+40,500

II. Summary for Maximum − Moment at Crown

(a) D. L., L. L., and Arch Shortening

	Thrust	Moment	Average stress
D. L.	+3,210,000	....	+41,700
L. L.	+ 365,000	−345,000	+ 4,800
Arch S.	− 201,000	+646,000	− 2,400
Total	+3,374,000	+301,000	+44,100

It appears from the above that there is no negative moment at the crown.

III. Summary for Maximum + Moment at Springing

(a) D. L., L. L., and Arch Shortening

	Thrust	Moment	Average stress
D. L.	+3,630,000	....	+41,700
L. L.	+ 499,000	+2,490,000	+ 6,700
Arch S.	− 184,000	−2,675,000	− 2,500
Total	+3,945,000	− 185,000	+45,900

(b) D. L., L. L., Temperature Variation, and Arch Shortening

	Thrust	Moment	Average stress
D. L. and L. L.	+4,129,000	+2,490,000	+48,400
Temperature	+ 254,000	+3,695,000	+ 3,500
Arch S.	− 197,000	−2,860,000	− 2,700
Total	+4,186,000	+3,325,000	+49,200

IV. Summary for Maximum − Moment at Springing

(a) D. L., L. L., and Arch Shortening

	Thrust	Moment	Average stress
D. L.	+3,630,000	....	+41,700
L. L.	+ 288,000	−2,120,000	+ 2,600
Arch S.	− 168,000	−2,450,000	− 2,300
Total	+3,750,000	−4,570,000	+42,000

(b) D. L., L. L., Temperature Variation, and Arch Shortening

	Thrust	Moment	Average stress
D. L. and L. L.	+3,918,000	−2,120,000	+44,300
Temperature	− 254,000	−3,695,000	− 3,500
Arch S.	− 155,000	−2,257,000	− 2,100
Total	+3,509,000	−8,072,000	+38,700

The following table gives the approximate extreme fiber stresses in pounds per square inch computed from the above thrusts and moments on the assumption that the concrete takes no tension:

Case I(a)	Case I(b)	Case IV(a)	Case IV(b)
580	750	420	730

The stresses computed by this method will be found to be somewhat less than those figured by the exact method, but if some arbitrary allowance should be made for the dead-load moments the difference would be less. A good rule for figuring the arbitrary allowance for dead-load moments (in case the arch axis is made to fit the dead-load equilibrium polygon) is to assume an eccentricity of application of the dead-load thrust above or below the axis equal to one-fortieth the depth of section.

DETAILS OF ARCH BRIDGES

37. Spandrel Details in Earth-filled Bridges.—The filling material in solid-spandrel bridges is held in place laterally by retaining walls which rest upon the arch ring. These retaining walls may be of either the gravity or the reinforced type, or they may consist of thin vertical slabs tied together by reinforced-concrete cross walls. In the usual type of solid-spandrel construction, the sidewalk rests upon the earth filling, which is the type shown in Figs. 25*A* and 26. Where the counterforted type of spandrel wall is employed, sidewalks are sometimes

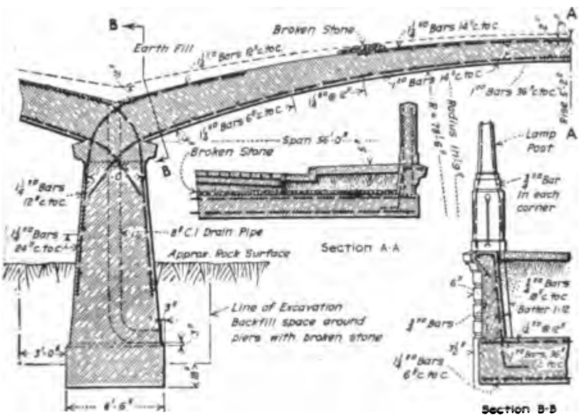


FIG. 25*A*.—Details of Pine Street bridge, Lima, Ohio.

cantilevered beyond the faces of the arch ring, as illustrated in Fig. 27. The faces of spandrel walls may be entirely plain, or panels of approximately a triangular shape may be formed, either by indenting the portion above the arch ring or by nailing beveled strips to the form-work. Brick and stone are used in some cases as a facing for arch rings and spandrel walls.

Figs. 25*A* and 25*B* show a portion of a flat-arch bridge designed for the city of Lima, Ohio. The spandrel walls are of the reinforced cantilever type.

A bridge with gravity spandrel walls is shown in Fig. 26. The brick facing for the arch ring and the cast concrete and brick belt courses should be noted. The spandrel walls rest partly on the brick facing and partly on the concrete portion of the arch, and are keyed into the concrete portion by means of a projection which fits into a 6 by 12-in. groove in the arch.

A counterforted type of spandrel wall is shown in Fig. 27. These walls are 12 in. thick and are reinforced on both faces with a double system of rods. The counterforts occur at about 9-ft. intervals, and cantilever brackets are placed at these counterforts to support the sidewalks. The following description is taken from *Engineering Record*, Feb. 22, 1913.

The entire width of the arch ring between outside faces of spandrel walls is 35 ft., and the roadway above is 39 ft. wide, thus giving an overhang of 2 ft. on each side of the bridge between the curb lines. This 2-ft. overhang constitutes the concrete gutter of the roadway and, as such, will be subject to heavy concentrated wheel loads com ing upon the cantilever section. It was, therefore, built as a heavily reinforced concrete beam. This beam is 2 ft. 9 in. wide, having a depth of 15 in. at the spandrel wall and 10 in. at the curb, and is reinforced with fourteen ¾-in. rods, with additional reinforcement at the brackets.

The bridge is designed for trolley traffic, and provision is made for the trolley poles by anchoring sections of 10-in. cast-iron water pipe in the brackets of the piers and abutments. Drainage is provided by means of 6-in. cast-iron drains in the gutters over the piers.

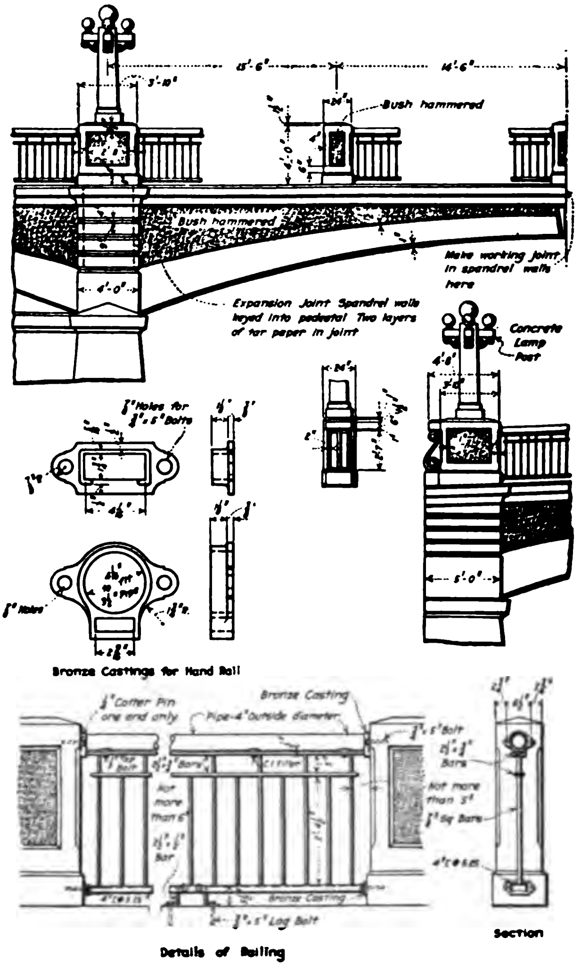


Fig. 25B.—Details of Pine Street bridge, Lima, Ohio.

Details of the arch bridge over the Olentangy River on King Avenue, Columbus, Ohio, is shown in Figs. 28A, 28B, and 28C. The type of spandrel walls without pilasters over piers should be noted. Since the space beneath each sidewalk is hollow, the inner wall was designed as a slab with the principal steel placed horizontally between cross walls. The longitudinal walls under the car track were employed to prevent the usual settlement of the track when laid on a new fill. The ties were laid directly on top of these walls and earth filling was dumped both sides of, and also between, the longitudinal walls.

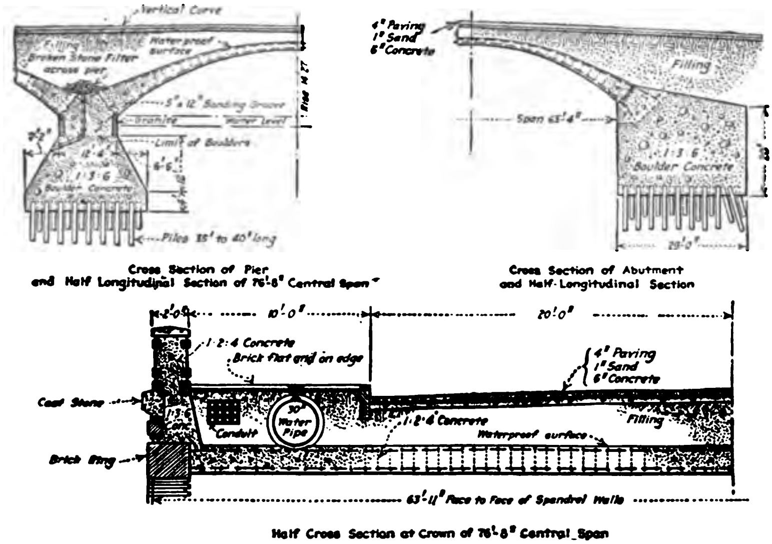


FIG. 26.—Details of Lars Anderson bridge over the Charles River, Cambridge and Boston, Mass.

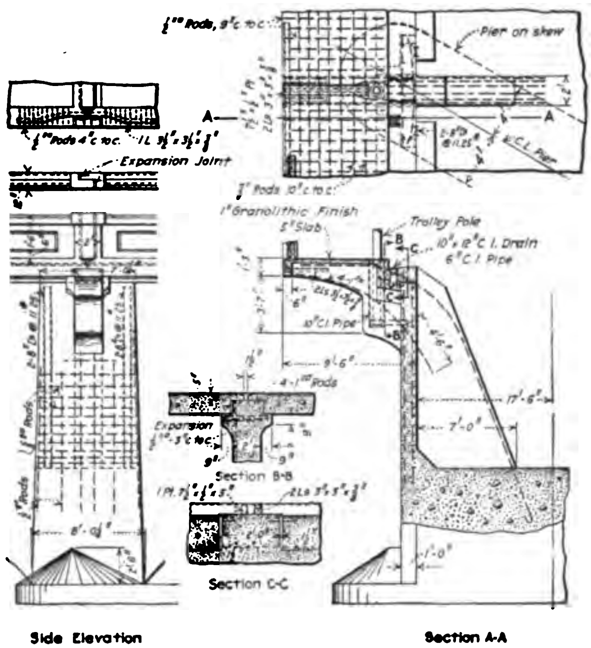


FIG. 27.—Counterforted spandrel wall, highway bridge at Ansonia, Conn.

Drains should be placed on each side of the roadway of a concrete bridge at intervals of 30 to 40 ft. when the roadway is level and about every 100 ft. when on a grade. These drains should have a diameter of not less than 3 in. The minimum area of a drain in square inches may be computed by the formula

$$a = \frac{A}{200}.$$

where A = area of the surface drained in square feet.

One type of expansion joint in a simple cantilever wall is shown in Fig. 29.

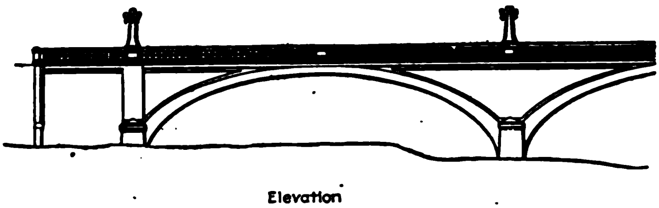


FIG. 28A.—Elevation of west end of Olentangy River bridge on King Avenue, Columbus, Ohio.

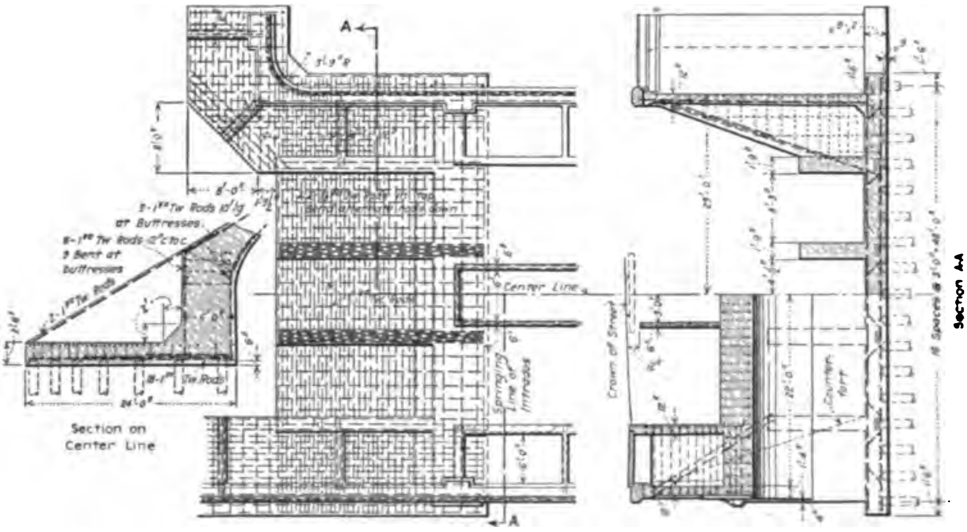


FIG. 28B.—Details of west abutment, Olentangy River bridge on King Avenue, Columbus, Ohio.

38. Spandrel Details in Open-spandrel Bridges.—The general types of open-spandrel bridges have been described in Art. 3. Figs. 30 to 35 inclusive will serve to illustrate details of some of these types.

Fig. 30 shows a full-barreled arch reinforced with typical Melan trusses made up of 3 by 3 by 5/16-in. angles and 2¼ by ¼-in. lattice bars. The floor system is carried on a series of transverse spandrel walls and the floor slab is provided with expansion joints as shown. The sidewalks leave an overhang of about 3 ft. and are supported on cantilever brackets. The

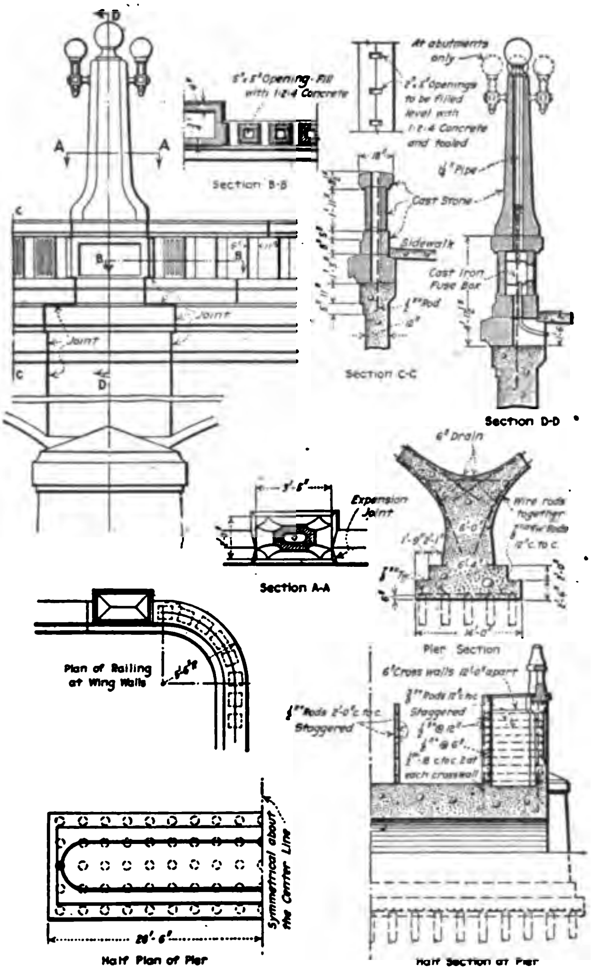


Fig. 28C.—Details of Olentangy River bridge on King Avenue, Columbus, Ohio.

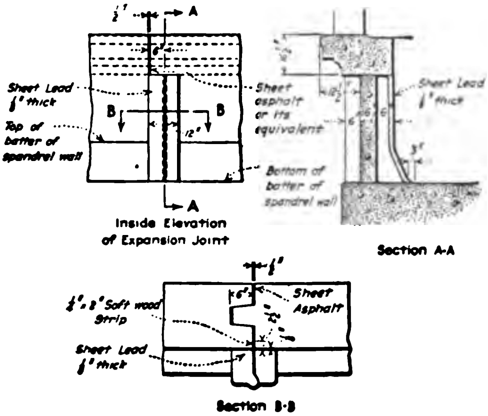


Fig. 29.—Details of expansion joint in highway arch bridge over Chattahoochee River at Columbus, Georgi

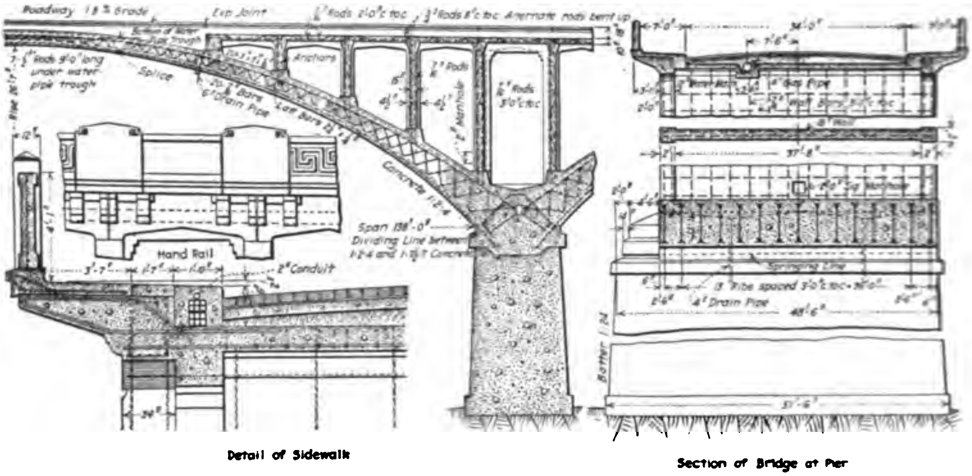


Fig. 30.—Details of Broadway bridge across the Oswego River at Fulton, N. Y.

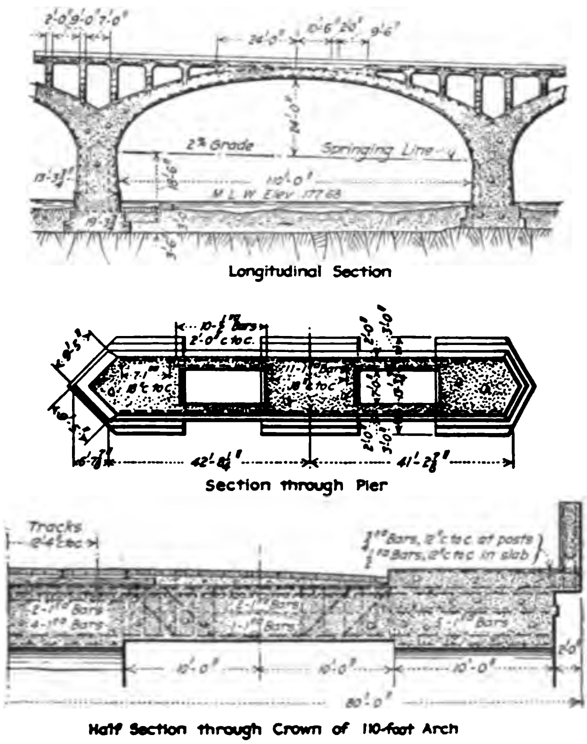


Fig. 31.—Details of Penn Street viaduct, Reading, Pa.

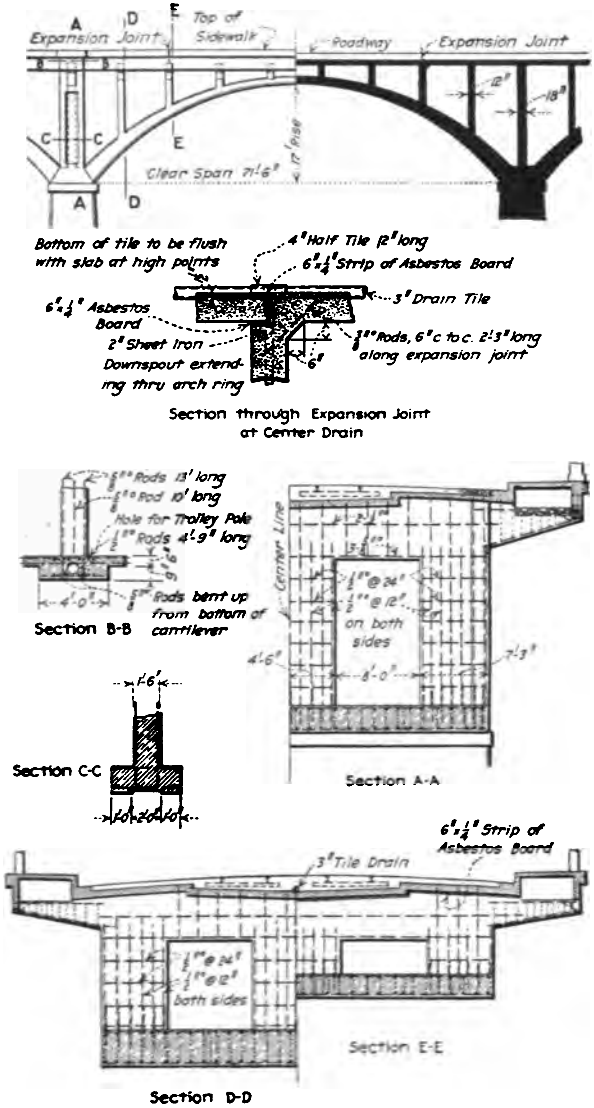


FIG. 32A.—Details of the Dallas-Oak Cliff viaduct, Dallas, Texas.

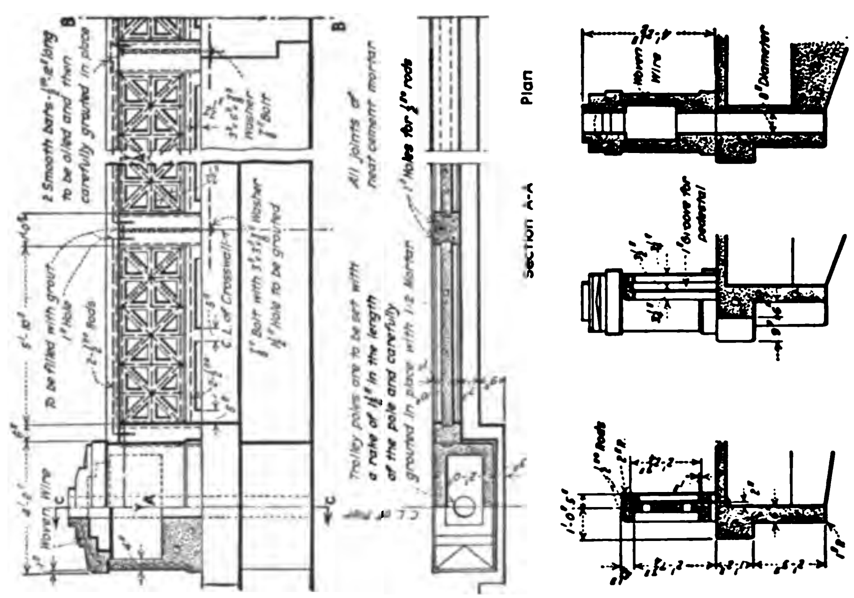


FIG. 32B.—Details of the Dallas-Oak Cliff viaduct, Dallas, Texas.

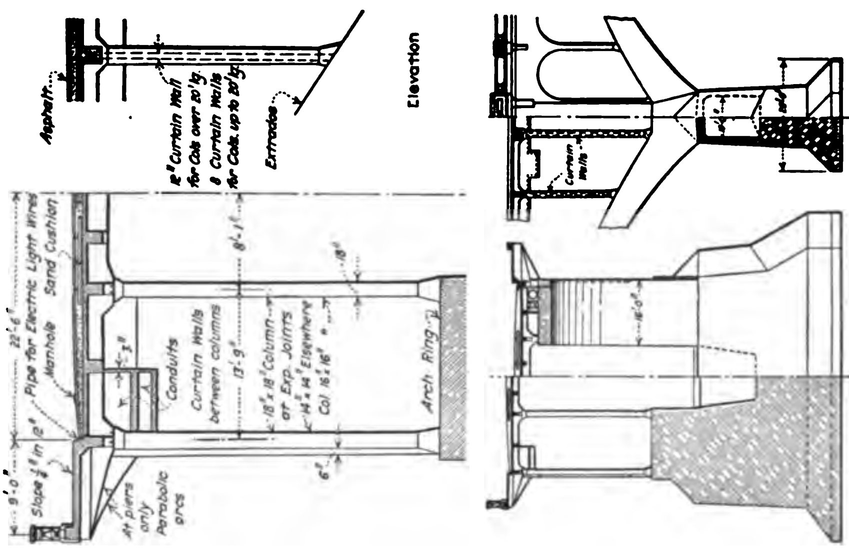


FIG. 33.—Bridge over Saskatchewan River at Saskatoon, Canada.

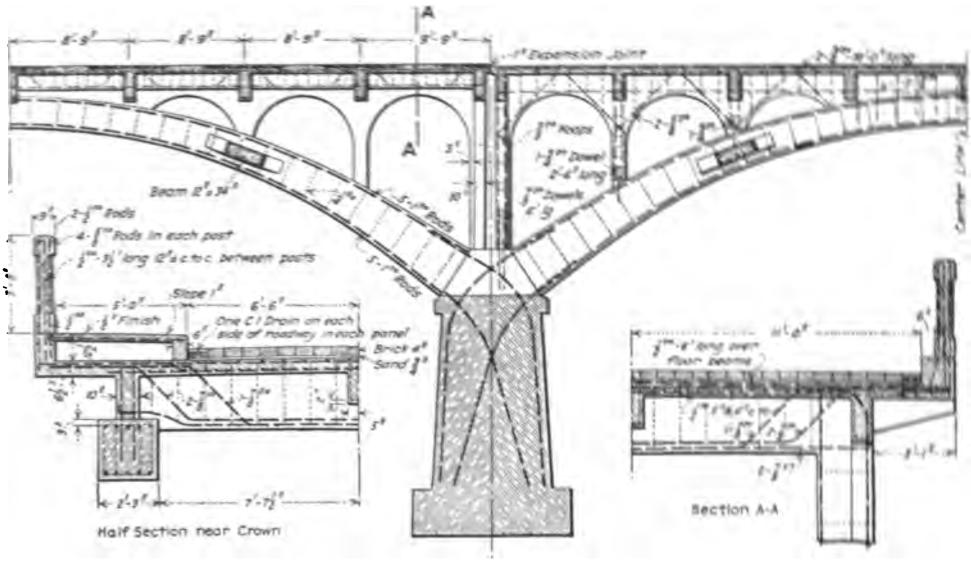


FIG. 34.—Wisconsin approach to high wagon bridge at Winona, Minn.

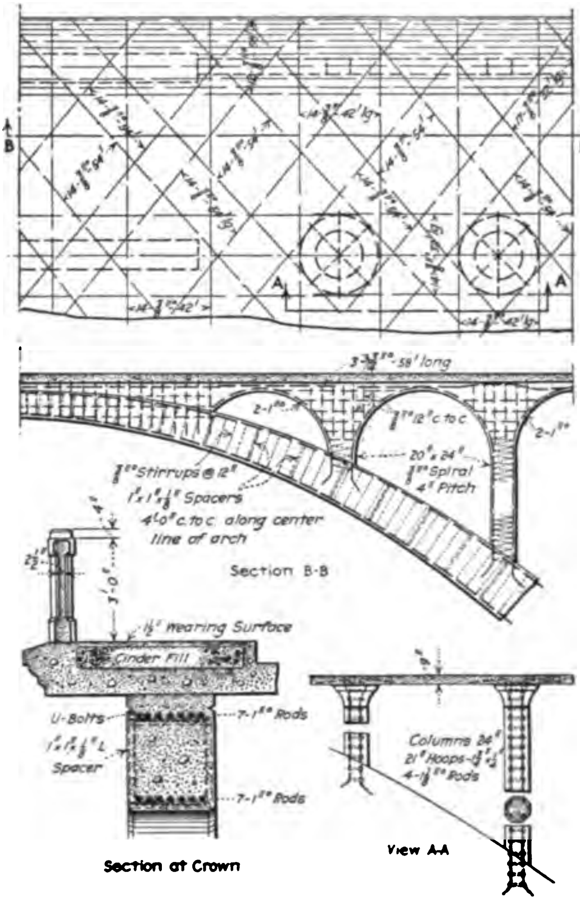


Fig. 35.—Bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

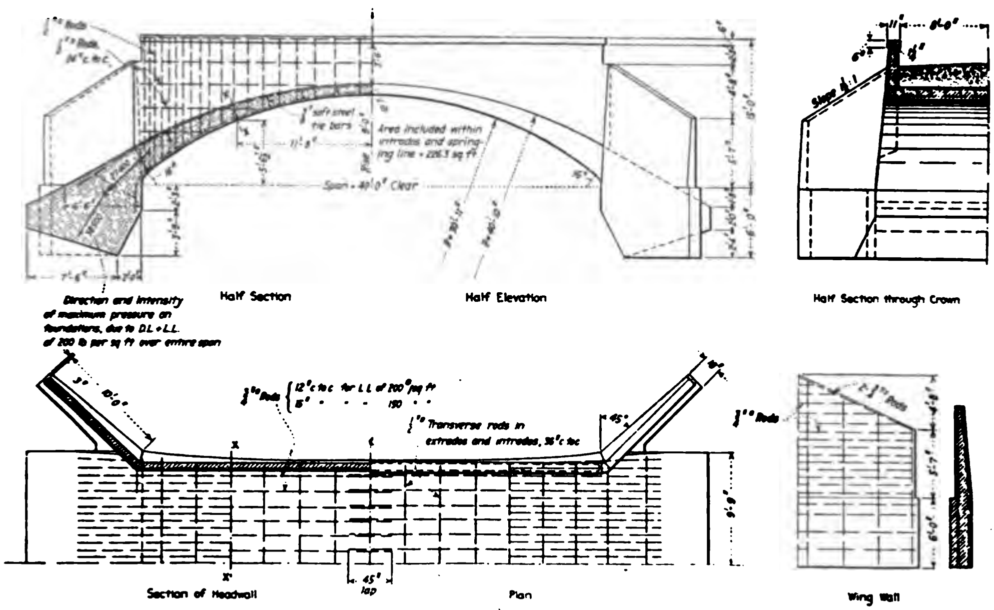


Fig. 36.—Standard 40-ft. arch, State of Missouri Highway Department.

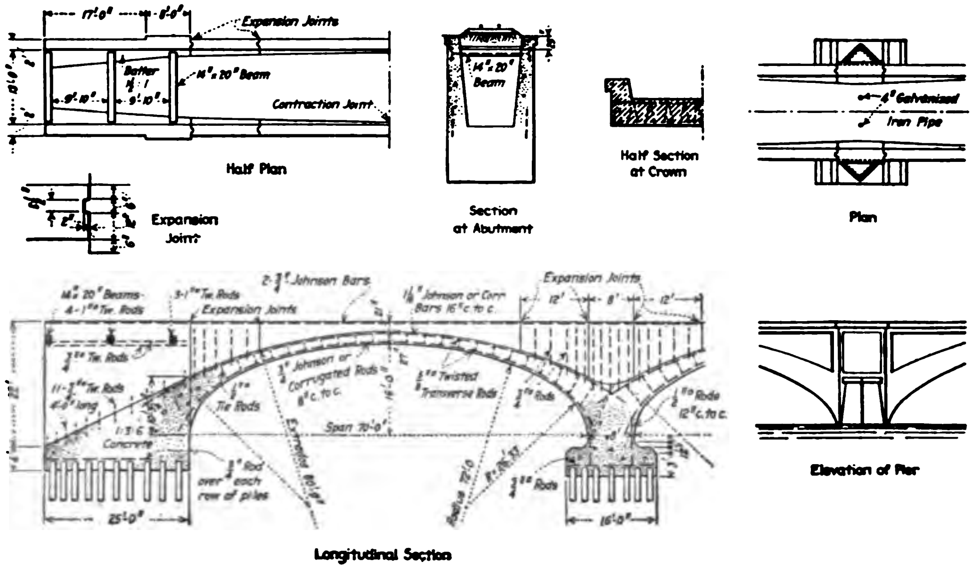


Fig. 37.—Elk Run bridge across Cedar River, W. C. F. & N. Ry.

cantilever section of the sidewalk is cast in units 5 ft. long and laid in place. Tile conduits are provided under each sidewalk for necessary wires, and a 4-in. gas pipe and 12-in. water main are laid in specially designed reinforced-concrete troughs beneath the roadway.

Transverse spandrel walls with openings to save material are shown in Figs. 32*A* and 32*B*. The method of carrying the sidewalk should be noted.

The curtain walls between columns in Fig. 33 were considered simply as a bracing system. The columns were designed to carry all loads, but doubtless the curtain walls help to distribute the total loading. In taking the loading for the arch rings, it was assumed that the loading from columns was equally distributed over 12 ft. of arch ring instead of having two loads concentrated on an arch ring 16 ft. wide.

39. Piers and Abutments.—The resistance offered by piers to the passage of water varies with the type of starling. Experiments show that the value in this respect of the different shapes of piers is in the following order: first, elliptical horizontal sections; second, rectangular body with starlings formed by two circular arcs, tangent to the sides and described on the sides of an equilateral triangle; third, rectangular body with semicircular starlings; fourth, rectangular body with triangular starlings, the angle at the nose being 90 deg.; and fifth, rectangular body without starlings.

The ordinary type of abutment in earth-filled arches is shown in Figs. 36 and 37. What might be called a buttressed abutment is shown in detail in Fig. 28*B*. Fig. 38 represents a very wide abutment in which a network of rods has been employed to make the entire abutment act as a unit. Hollow piers for ribbed-arch structures are shown in Figs. 31 and 33.

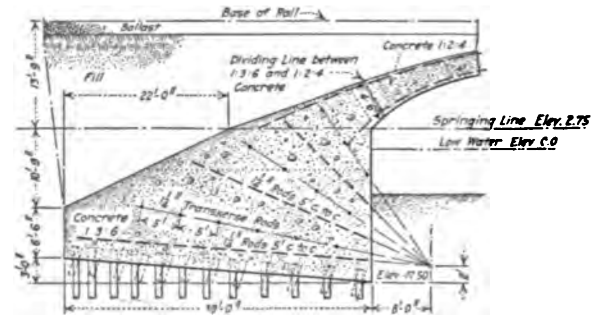


FIG. 38.—Abutment of causeway arch construction, Galveston, Texas.

40. Railing and Ornamental Details.—A spindle balustrade is the common type of railing. In such railings the spindles or balusters are usually the only members which are not cast in place. Expansion joints should be provided each side of the posts and also over the spandrel joints. Railing and ornamental details of various kinds are shown in Figs. 25*B*, 28*C*, 30 and 32*B*.

CONSTRUCTION OF ARCHES

41. Arch-ring Construction.—Arch rings with span lengths less than about 90 ft. are usually constructed in longitudinal ribs 3 or 4 ft. wide, or in fact of such a width that one entire rib can be poured in approximately 1 day's time. In narrow arches the entire arch ring is sometimes poured at one operation. This method of construction has been successfully used for much greater spans than 90 ft. but, unless special care is taken to make the centering very stiff, the construction of any one rib may deform the arch center to such an extent as practically to strike the center under the completed ribs. Of course, the ribs should be poured continuously from each abutment toward the crown so as to obtain a symmetrical loading on the falsework and thus eliminate distortion of the centering so far as possible.

For spans of 90 ft. or over it is usually preferable to construct an arch ring or arch rib by what is known as the alternate block or voussoir method. The arch is constructed in transverse blocks of such size that each block can be completed at one pouring, or with about a day's work. Obviously this method reduces shrinkage stresses in the arch ring to a minimum.

For the best results the blocks should be poured in such order as to give a uniform settlement of the centering, and also prevent the crown of the arch from rising as the lower arch loads are placed. If blocks close to the crown section are not placed before the blocks at the haunch and springing sections, the centering will rise at the crown and the placing of the crown loads will be likely to cause cracks at the middle of the haunch. Even in the construction of an arch by the longitudinal-rib method, a temporary loading of the crown is often necessary.

The order followed in the construction of the Philadelphia and Reading R. R. bridge across the Delaware River at Yardley, Pa.—an earth-filled bridge with clear span of 90 ft. 9 in.—is shown in Fig. 39, the sections being concreted in the alphabetical order shown. The section D is the keying section, and the section E a haunching section (quite unusual construction) which was added after the lower portion of the arch ring was completed. The part of the arch ring close to the springing lines was placed monolithic with the piers and abutments, making

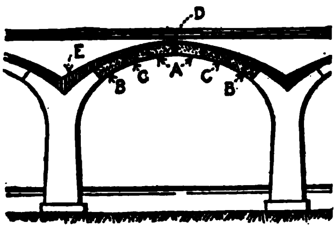


Fig. 39.

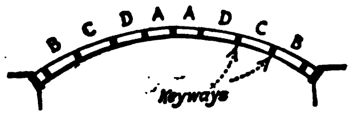


Fig. 40.

what is called an umbrella form for the piers. In large arches this umbrella type of construction is frequently adopted. The pier forms in such cases are more expensive, but this increase in expense for the piers is usually more than offset by the saving in the falsework for the arch ring.

Fig. 40 shows the method of constructing the Larimer Avenue bridge at Pittsburgh—a bridge of the open-spandrel type, with two ribs, having a clear span of approximately 300 ft. The blocks were placed in alphabetical order and later the keys between them were concreted to make the closure.

In constructing an arch rib or arch ring by the alternate block method the individual sections or block spaces are closed off at the ends by timber bulkheads. On the steepest slopes of the lagging these bulkheads adjoining keying sections are held in place by temporary struts between voussoirs. A top form is usually needed for the block sections near the piers and abutments. This top form should be laid up as the concreting progresses.

If arch reinforcement for large arches is put in place in long lengths, the settlement and deformation of the centering during the pouring of the concrete will cause buckling of the steel which will prevent the reinforcement from lying in its theoretical position. For this reason steel lengths should not exceed about 30 ft. and the splicing should occur in the keyways. An effort should be made to stagger the splices of adjacent rods and to locate the splices where the tension in the steel is a minimum.

The upper reinforcement in arch rings may be kept in position either by means of spacing boards nailed to props or by the steel being wired directly to transverse timbers supported above the surface of the finished concrete. In the first method mentioned, the props and spacing boards must be knocked out as the concrete is brought up with likelihood of disturbing the steel.

When steel ribs (either rolled sections or built-up lattice girders) are used as arch reinforcement, great care should be taken to fix the ribs in the proper position, and in this position they should be braced until the concrete is placed. The use of such ribs is known as the Melan system.

Spandrel walls for earth-filled arches are either built on top of the arch ring, or include a portion of the arch, the bottom inner edge of the spandrel retaining wall lapping a short distance over the completed arch ring.

42. Centering.—The bent type of timber falsework is the type of centering generally employed in arch construction except where a deep gorge is to be spanned or where a large clearance under the arch is necessary while the bridge is under erection. Timber arches, Howe trusses and bowstring trusses are sometimes employed when it is impossible to use the bent type of centering, but these forms are expensive to build, deform badly under loading, and have but small salvage value. Before using any of these types, consideration should be given to the use of steel centers.

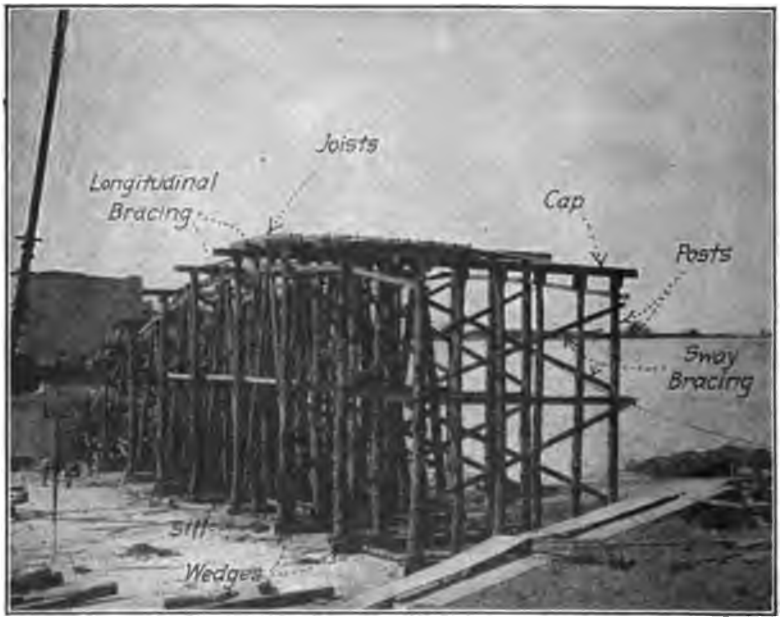


Fig. 41.—Common form of timber centering for arches of low rise.

42a. Timber Centers.—A simple and common form of timber centering for arches of low rise is shown in Fig. 41. The lagging had not been placed at the time this photograph was taken, but some of the joists for supporting the lagging were already in place. The joists, of course, extended from abutment to abutment and were supported by transverse bents of round timber resting on sills the full length of each bent. The falsework rested on the concrete floor of a spillway channel so that mud sills, piles, or specially constructed concrete footings were not necessary. Wedges were placed at the bottom of the posts so that the center might be lowered conveniently after the arch ring was completed and ready to bear its load.

The centering used in the Third Avenue bridge at Cedar Rapids, Iowa, is shown in Fig. 42. In a paper presented before the Western Society of Engineers, April 13, 1914, Barton J. Sweatt described the construction of this centering as follows:

The falsework for supporting the arches consisted of pile bents, the first bent being 6 ft. 6 in. from the face of piers and abutments, the second bent 12 ft. 6 in. from the first, and the intermediate bents were 14 ft. 6 in. centers. Oak piles were used and as a rule were driven to bed rock, the spacing was 6 ft. 0 in. for the three outside piles and

8 ft. 0 in. for the intermediate. The caps used were 12 by 12-in. yellow pine, false caps 6 by 10 in., joists 4 by 14 in., spaced 24 in. on centers and the lagging was 2 by 8 in. The proper curve for the intrados was obtained by the use of 2-in. strips cut to the proper curve and tacked to the regular joists. Oak wedges were used between the main and false caps. These wedges were placed in pairs and spaced about 4 ft. apart. Small wedges were used under the ends of the joists to bring them to the proper height.

In constructing the centering, an allowance of 1½ in. was made for camber and ½ in. for settlement after the centering was removed. The actual settlement of the crown after removing the centering was ¾ in.

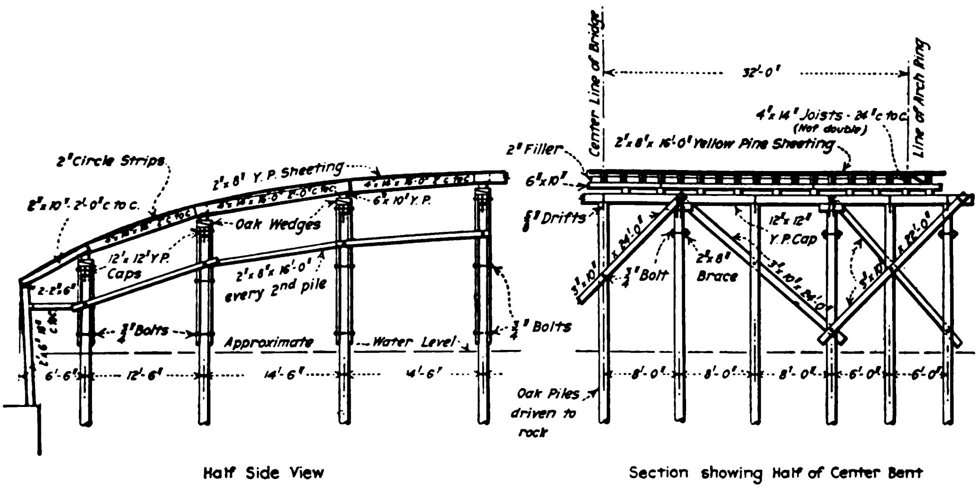


Fig. 42.—Centering for Third Avenue bridge, Cedar Rapids, Iowa.

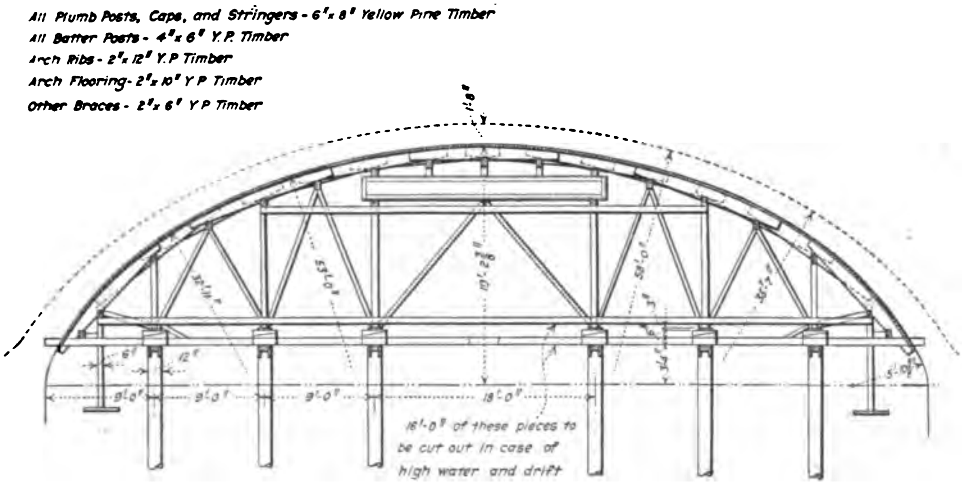


Fig. 43.—Falsework and centering for Cleveland Avenue bridge, Kansas City, Mo. Note structural steel girder carrying the center of the span to allow for flood.

An article in *Cement Age*, March, 1912, describes the construction of the centering shown in Fig. 43 in the following manner:

The centering for the arch consisted of four pile bents of four piles each, and two center pile bents of five piles each. These bents were capped, top of caps being elevation of spring line of arch, and four lines of 6 by 8-in. stringers were placed continuous from abutment to abutment—the ends, at elevation of the spring line, bearing 4 in. on the concrete abutments. At completion the ends of stringers were bored out and the holes in the abutments filled with concrete. On these four lines of stringers were placed a set of pine wedges over each pile, there being four and five sets of wedges to each bent, and 3 by 12-in. timber was laid on the wedges over each bent through the width of the arch. On these 3 by 12-in. timbers, the arch bents were erected, each having four and five 6 by 8-in vertical posts V-braced, with 6 by 8-in. caps set edgeways, top of caps 12 in. below intrados of arch. On these caps were placed 2 by 12-in. ribs, dapped to take square bearings on caps. These ribs were placed 18 in. centers across the arch and were cut from timber of sufficient dimensions to make them lap over alternate caps. The ribs were covered with 2-in. planking to form the intrados of the arch.

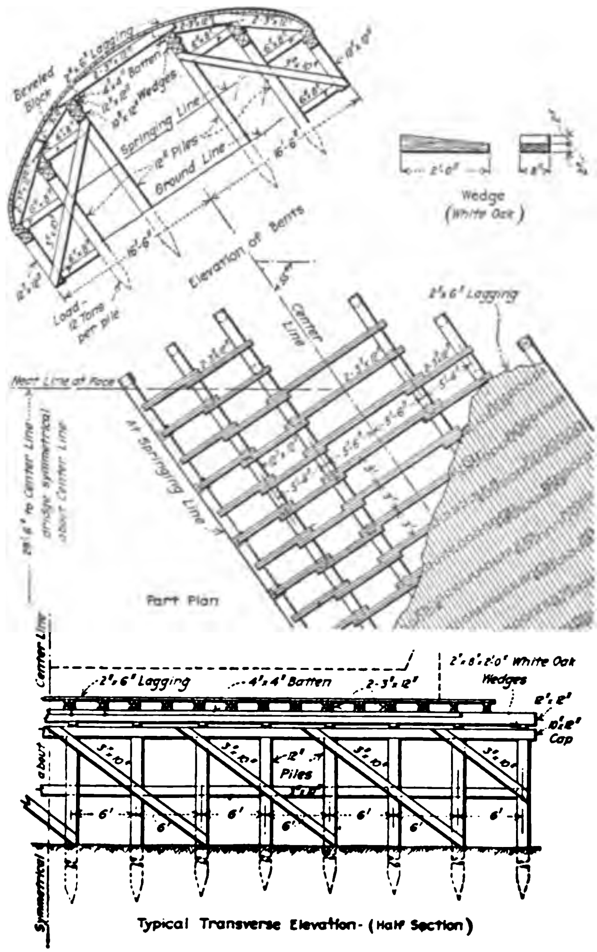


FIG. 44.—Details of arch centers for proposed renewal, C. R. R. Co. of N. J.

A very simple and accurate method of laying out the ribs was used which consisted of laying out the full-size arch intrados radii on a level place near the bridge site. The timber for the ribs was, therefore, marked by a full-size drawing. Probably the most interesting feature of this arch centering was the simple straight work giving maximum strength and maximum safety in every respect at the lowest cost. The five steel floor beams of the old bridge were utilized to make an 18-ft. clear opening of maximum height in the centering. This was economical, as it saved one pile bent and one centering bent, but its main purpose was to allow drift to pass through in case of high water during construction.

Figs. 44, 45, 46 and 47 show timber centers similar to the one just described.

A patented type of centering is shown in Fig. 48, known as the Luten arch centering. The idea in this center is to dispense with the usual wedges employed in lowering the falsework. The top part of the uprights consists of two thin members with major dimensions transverse to each other. These are arranged in the form of a T-column, and wired together at frequent intervals. Each member separately is made too light to carry its loading so that clipping the wires permits each member to buckle, which lowers the center.

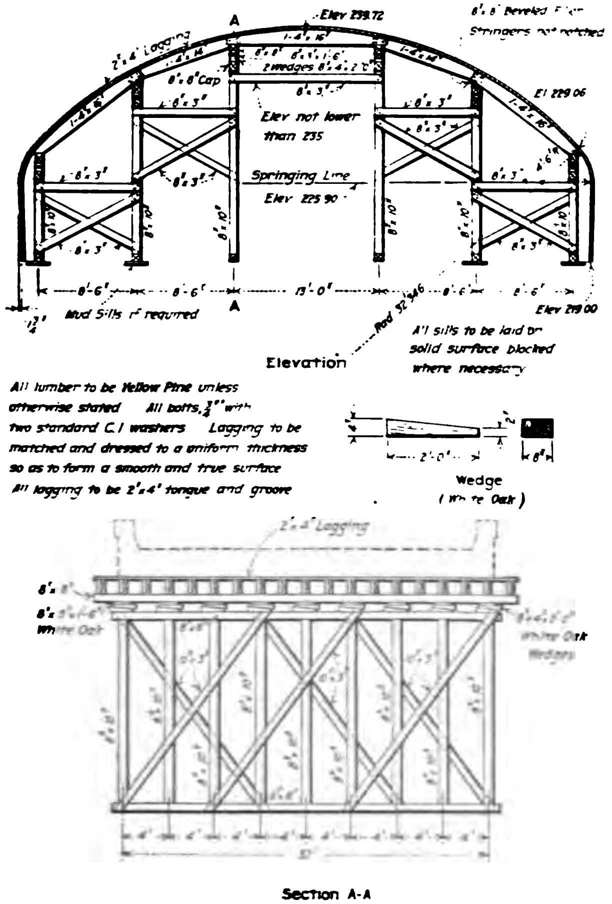
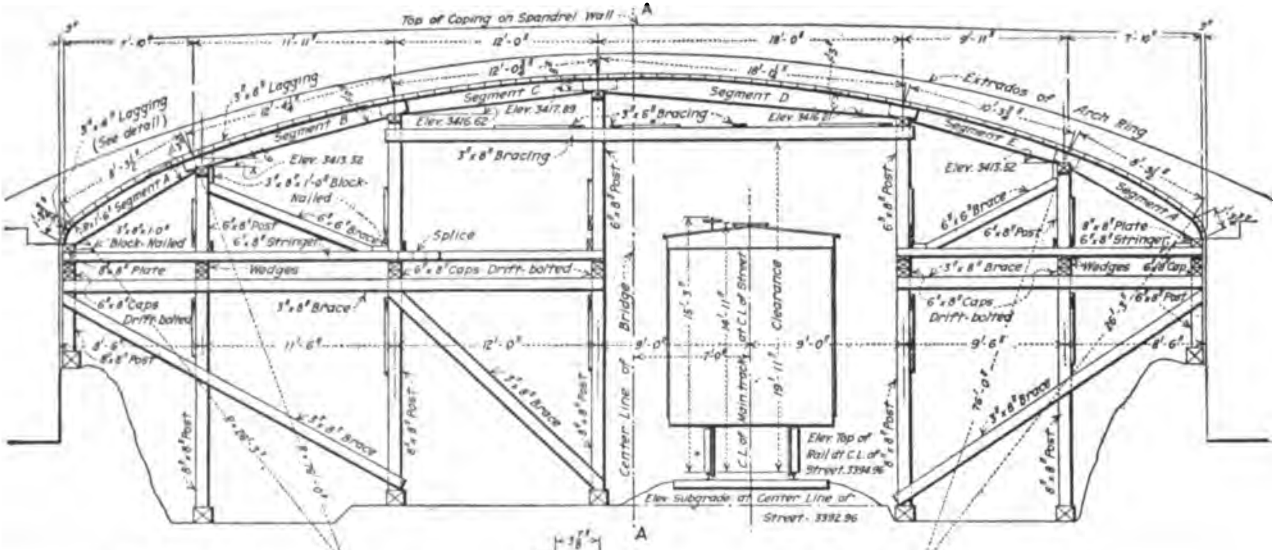


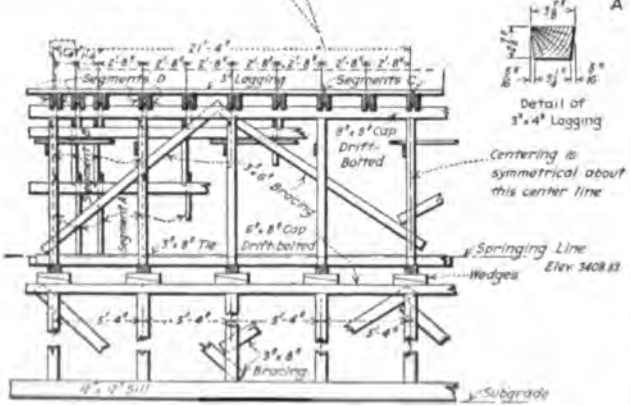
Fig. 45.—Details of arch centers for Center Street bridge, Phillipsburg, N. J.

W. W. Washburn in an article in *Concrete-cement Age*, August, 1914, writes as follows in regard to the centering shown in Fig. 49:

Since Buffalo Bayou is a navigable stream, it was necessary to leave an opening in the arch centering for the passage of tugs and boats during construction. This opening was 24 ft. vertically above average water level and 38 ft. wide. Protection piles on each side of the passageway were necessary, consequently the total span of the opening in the arch centering was 49 ft. To carry the load over this opening nine 30-in., 200-lb. I-beams were used. These I-beams will be used in the construction of other bridges.

Longitudinal bracing was arranged so as to counter, as much as possible, the "bucking up" tend centering at the crown as the arch was concreted. On account of the arch being skewed, special stud





Half Section A-A

Bill of Material

No.	Size	Ordered Length	Location
21	12"×12"	16'-0"	Sills
14	8"×8"	18'-0"	Caps
54	8"×8"	14'-0"	Posts
7	8"×8"	12'-0"	Caps
9	6"×8"	22'-0"	Stringers
23	6"×8"	18'-0"	Caps and Stringers
16	6"×8"	12'-0"	" " "
18	6"×8"	10'-0"	Posts
18	6"×8"	8'-0"	"
9	6"×6"	12'-0"	Braces
9	6"×6"	10'-0"	"
57	3"×16"	20'-0"	Segments D & A
38	3"×12"	14'-0"	Segments B & C
13	3"×12"	12'-0"	Segment E

No.	Size	Ordered Length	Location
18	3"×8"	24'-0"	Bracing
9	3"×8"	22'-0"	"
9	3"×8"	18'-0"	"
55	3"×8"	16'-0"	"
9	3"×8"	14'-0"	"
116	3"×8"	18'-0"	Lagging D & M
116	3"×8"	16'-0"	"
116	3"×8"	12'-0"	"
16	3"×6"	16'-0"	Bracing
40	3"×6"	14'-0"	Bracing
11	3"×4"	18'-0"	Lagging S-2-S & E-E (See Sketch)
11	3"×4"	16'-0"	"
11	3"×4"	12'-0"	"
140	6"×8"	2'-0"	Wedges
140	3/4"×1'-4"	Drift Bolts without heads or points	

Fig. 46.—Details of arch centers, 25th Street crossing, Great Falls, Mont., C. M. & St. P. Ry.

during the placing of braces, etc., so as to take care of all side thrusts. Struts were placed diagonally between bents at right angles. Spacing of centering piles was such that no pile received a load of over 12 tons.

Timber centering for a bridge over railroad tracks at Fall River, Mass., is shown in Fig. 50 and described in *Engineering Record*, April 26, 1913, as follows:

In building the arch centering it was necessary to provide for certain requirements that necessitated a design similar to the one shown in the accompanying drawing.

These requirements called for an overhead clearance of 18 ft. at a point 11 ft. 3 in. from the property line, a clear span of 40 ft. between inside vertical posts, and a minimum clearance of 15 ft. from the top of rail to the under

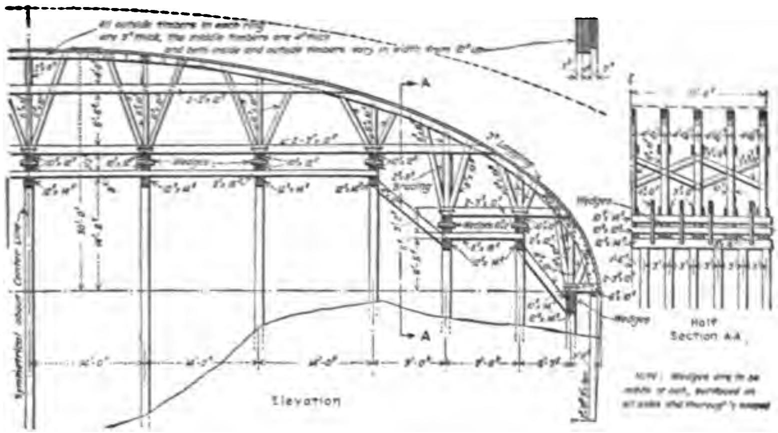


Fig. 47.—Falsework for bridge over Big Muddy River, I. C. R. R.

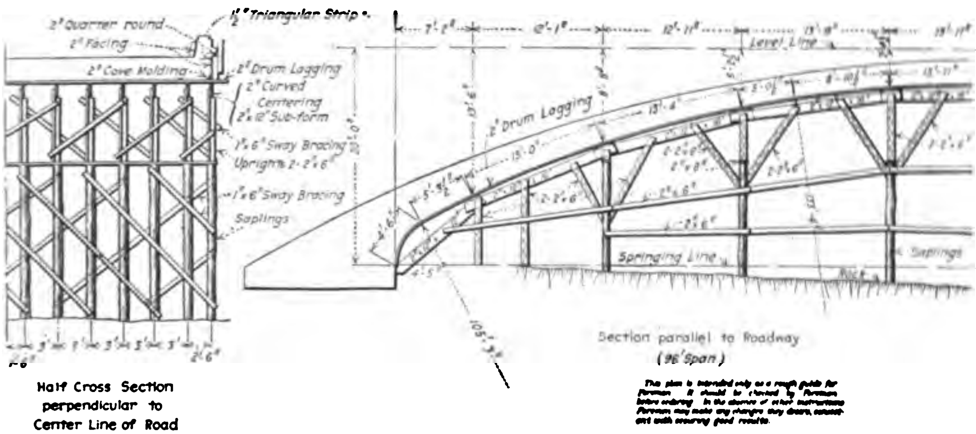


Fig. 48.—Centers of Luten Design for Cicott Street bridge over Wabash River, Logansport, Ind.

side of the truss. The work had to be conducted without interruption of train service. The uprights supporting the centering were carried on concrete mud sills and 2-in. tongue-and-groove lagging was used over the entire arch ring. The centering was designed for a deflection equal to one eight-hundredth of the span.

Figs. 51*A* and 51*B* show the type of arch centering used in constructing a three-ribbed arch on the Mississippi River Boulevard, St. Paul, Minn. Boxing for the ribs is also shown.

Posts in timber centering have sometimes been placed approximately normal to the arch soffit, but such instances are quite rare. Since specially constructed footings are necessary for inclined members, this form of center may be used economically only when rock or other suitable foundation lies near the ground surface.

Sand boxes have been used to a very limited extent in this country in place of wedges for the striking or lowering of arch centers. These boxes have given satisfaction in most instances, but great care must be taken to keep the sand dry while the arch ring is being constructed. This type of lowering device is expensive, but the extra first cost may be offset in large arches by the high cost of striking wooden wedges.

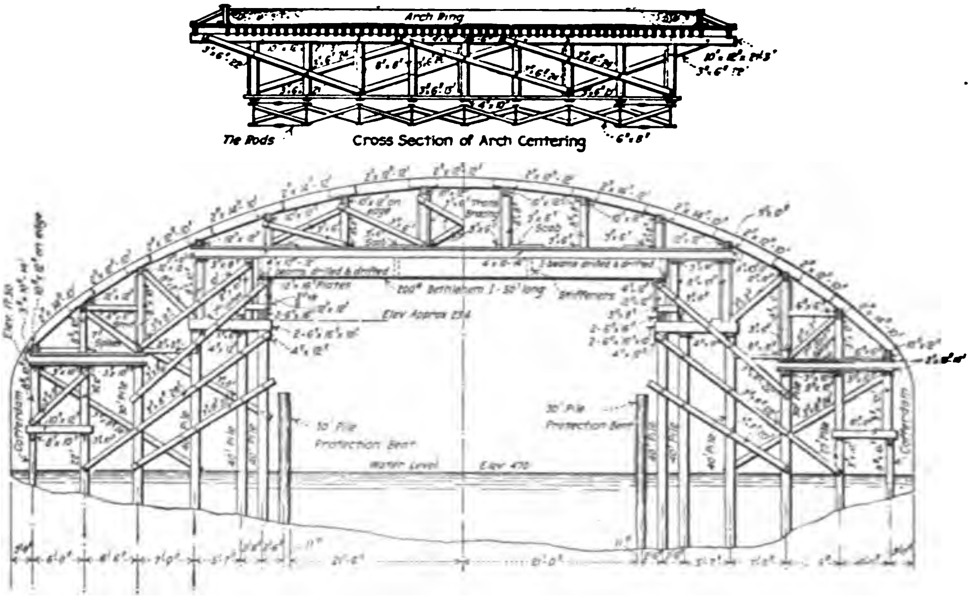


FIG. 49.—Details of arch centering and supports for 110-ft. span of San Jacinto bridge, Houston, Texas. Note size of opening for navigation.

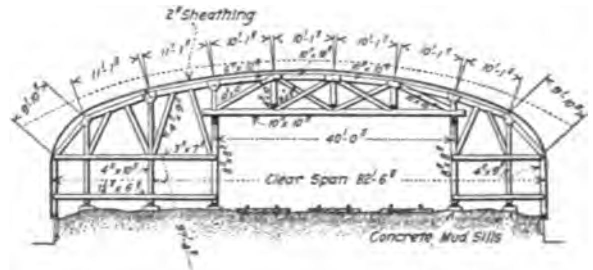


FIG. 50.—Highway bridge at South Park, Fall River, Mass., over tracks and right-of-way of the N. Y., N. H. & H. R. R.

The chief disadvantage of using sand boxes lies in the fact that the sand will compress as the weight on the centering increases. The amount of this compressibility is considerable, greatly increasing deflection unless the sand is put under an initial compression, which is seldom feasible.

A sand box used in the main arch of the Edmondson Avenue bridge, Baltimore, is shown in Fig. 52. The center was lowered by allowing the sand to run out through a 1-in. circular hole in the oak bottom of the steel-plate cylinder. This hole was closed by a wooden plug while

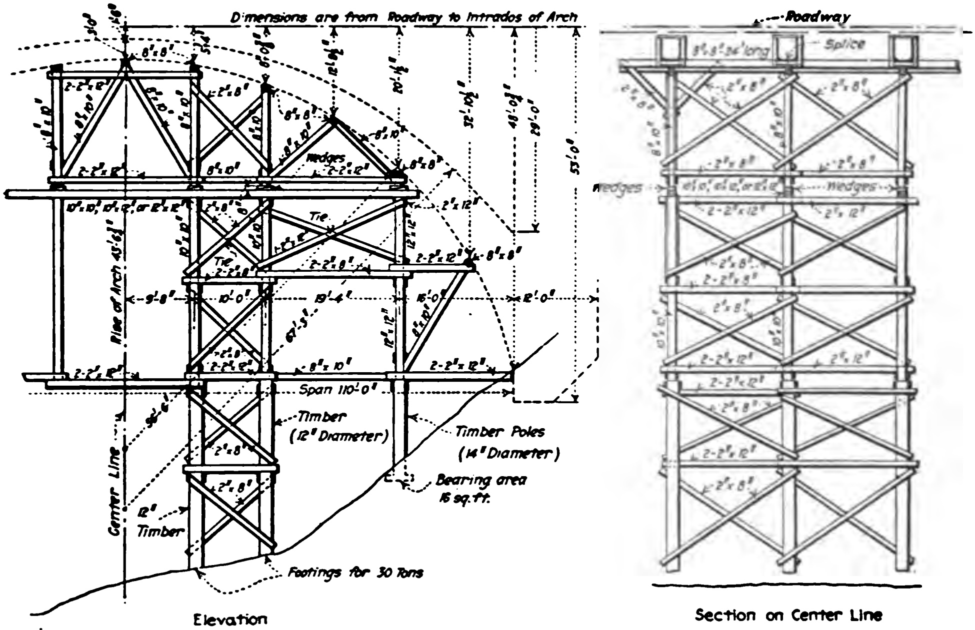


Fig. 51A.—Centering for bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

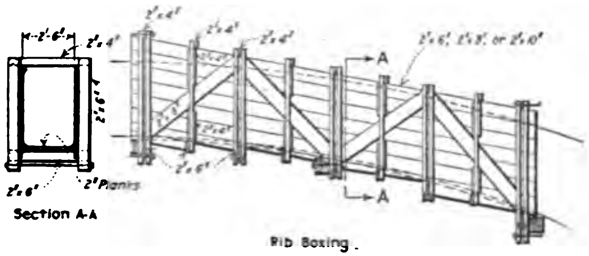


Fig. 51B.—Rib boxing for bridge over ravine on Mississippi River Boulevard, St. Paul, Minn.

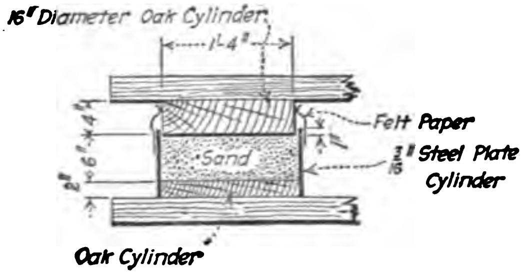


Fig. 52.—Sand box.

the centering supported its load. A disadvantage in the use of sand boxes lies in the fact that the centering cannot be raised before the arch ring is poured in order to adjust the top members to the curve of the arch intrados.

In the design of large arch centers an uncertainty exists regarding the pressures from voussoirs placed on the steepest portions of the lagging. Either of two assumptions are usually made as to the forces acting on the centering due to the weight of such voussoirs. In the common method of design, the assumption is made that the centering sustains only the radial components of the voussoir weights, the tangential components being resisted by temporary struts between voussoirs so as to be transferred to the abutments. The more accurate method is to assume that tangential pressures (in addition to the radial pressures) act on the centering which, from any voussoir, may be as great as the product of the radial component and the coefficient of friction between the voussoirs and lagging. The original tangential component is then reduced by this amount.

Since a timber center is only a temporary structure and has a high salvage value, great accuracy in the design of the separate members is not necessary. The method of design need only be such that the size of each member is well on the safe side. Then, too, rigidity is quite as important as strength, so that all things considered, close figuring is out of the question. Obviously the weight of centering may be omitted except for high arches. For the method of designing lagging, joists, and posts see Arts. 65 and 66, Sect. 2.

As a rule, only hardwood should be used for caps and sills, although long-leaf pine may be sufficiently hard in many cases. Wedges, however, should be made of hardwood without exception. It is always advisable to reduce the number of joints in side-grain compression to a minimum on account of the low bearing value of timber across the grain. Steel distributing plates are of advantage in this connection.

Care should be taken to prevent lateral displacement of vertical posts due to radial pressure from the arch ring. This may be avoided either by proper longitudinal bracing or by notching out the joists and shimming them tight against the caps.

Many practical notes on the design and erection of falsework may be found in Sect. 7 of the "American Civil Engineers' Pocket Book."

In striking arch centers, wedges should be lowered gradually beginning at the crown and working toward the springing lines. The lowering should be done symmetrically with respect to the center of the arch ring. In a series of arches, centers between abutments or abutment piers should be struck simultaneously. As a rule, centers should not be struck from arches in less than 28 days under favorable weather conditions, and it is desirable that a longer period should elapse if possible.

42b. Steel Centers.—Steel centers of the arch-trussed type should receive consideration when arches are to be built in series or where the character of the stream or crossing renders timber and pile falsework impossible or expensive. Undoubtedly the cost of a steel center is usually high, but if it can be used a number of times, as in a large series of arches, it may not prove any more costly than timber.

It is generally recognized that there are some well-defined advantages in using three-hinged arch centers. In the first place, the crown deflection using steel centers is usually much less than that obtained by employing timber falsework. Furthermore, it is possible to compute the deflection of each point of a steel center with some degree of accuracy while, in the case of a wooden center, the probable settlement at each bent is pretty much a matter of guesswork. Steel centers also have the additional advantages of allowing an obstructed opening for railroad or other traffic and of eliminating danger from flood and ice in the construction of arches over streams. The advantage of allowing the deflection to be quite accurately computed makes it possible to give the centers a preliminary camber so that when the concrete is in place and the centering withdrawn, the arch ring will assume its true position.

One disadvantage of using steel in arch centering lies in the fact that it is materially affected by temperature changes. For this reason, in constructing large arches, only the alternate block method should be employed.

The steel centering used in constructing the three-span earth-filled arch structure which carries Atherton Avenue in the City of Pittsburgh across the four tracks of the Pennsylvania Railroad is shown in Fig. 53. This centering, fabricated by the Blaw Steel Cons. Co., consisted of steel arch trusses spaced 5 ft. 5¼ in. on centers. The trusses carried timbers and lagging, and were supported on framed trestle bents placed close to the pier faces. Sufficient trusses were at first erected to concrete one-half the width of each arch ring, then the centers were shifted transversely to themselves and the concrete placed for the second half of the structure. The method used in construction and the details of the arch centering are described in *Engineering and Contracting*, Feb. 19, 1913, as follows:

A six-post bent was erected on the footing shelf of the pier, the idea being to have a bent-post under each arch rib. On the bent caps over each post was placed a block and double wedge and on these supports a 12 by 12-in. plate on which rested the ribs. Between each pair of ribs a dolly was fastened to the bent-caps.

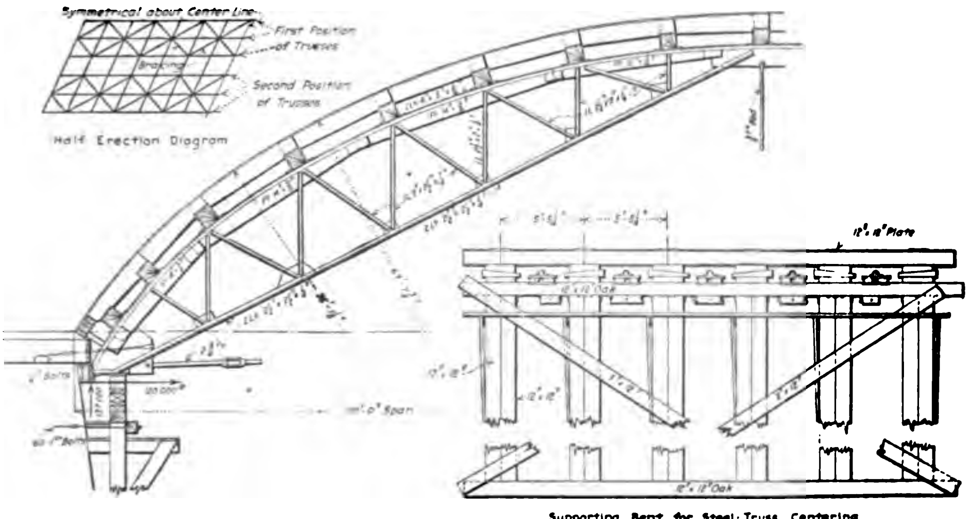


FIG. 53.—Steel centering for Atherton Avenue bridge, Pittsburgh, Pa.

The shifting of the center to construct the second half of the arch was accomplished as follows: Jacks were set up on the bent-caps alongside the dollies, and a strain taken on them until the wedges were loosened sufficiently to be easily removed. The jacks were then lowered until the weight of the centers rested on the dollies. To prevent the lagging and cross-timbers from being lifted off the ribs by sticking to the soffit of the arch ring, one end of the center was lowered ahead of the other so as to give a stripping action in freeing the lagging. When lowered into the dollies the whole center was shifted sidewise rubbing on the dollies, until it rested on the six-post bents under the second half of the arch. The jacks were then placed on the caps of the second bents and the center raised and the blocks and wedges inserted. A steamboat ratchet was used to pull the center on the dollies. Four men working 8 hr. shifted a center. Incidentally the tie rods connecting the opposite ends of the ribs were found, when planked across, to provide a most convenient bridge for the workmen engaged in shifting and adjusting the centers.

The lateral thrust on the centers due to their skewed position was taken care of by suitable lateral bracing of the ribs. In anticipation of the center rising at the crown in concreting from the haunches upward, the ribs were anchored back to the pier masonry. The joining carried by the ribs consisted of cross-timbers over which were notched stringers with curved top edges. The stringers were spaced 11½ in. apart and were lagged with ⅞-in boards. The bearings of the stringer ends against the piers were formed by wedges.

Steel centering employed in the construction of the South Eighth Street Viaduct, Allentown, Pa. (a two-ribbed arch structure of nine 120-ft. spans) is shown in Fig. 54. The *Engineering News*, April 17, 1913, describes this centering as follows:

For the nine 120-ft. arches three full sets of steel arch centers were used, using each set for three of the arches Each set of centers consisted of two independent-trussed arches of the outlines shown in Fig. 54, each arch supporting one of the twin concrete arch ribs and being itself made up of two steel arch ribs interbraced with steel struts. Across the upper chords of these steel ribs, which were curved to the curve of the concrete arch, was bolted the wooden lagging on which the concrete was deposited. The twin centering arches were held together by a timber cross-beam and diagonal steel rods.

The arch trusses were fabricated in six sections and riveted on the ground into semi-arches, which were lifted by derricks into place, to be bolted at the base to the supporting columns. At the crown it was riveted solidly in the bottom chord, but bolted through slotted holes at the upper chord, to insure the stress passing through the lower chord.

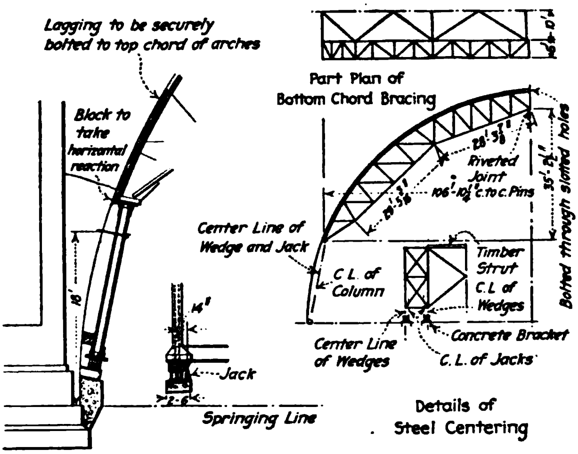


Fig. 54.—Details of steel centering for South Eighth Street viaduct, Allentown, Pa.

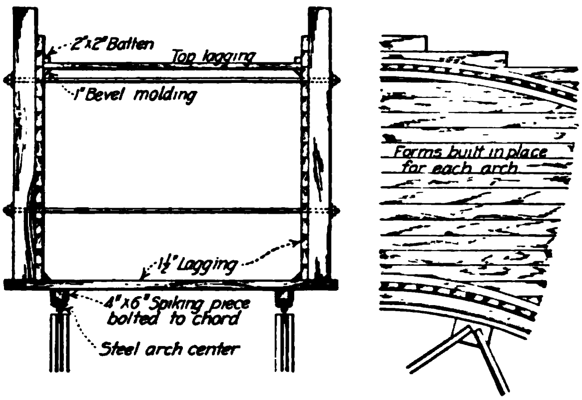


Fig. 55.—Arch rib forms on steel centers for Brooklyn-Brighton viaduct, Cleveland.

The centers were supported on inclined steel columns which footed on concrete steps purposely projected from the main section of the pier and cut off after the centers were struck. The base plates of the columns rest on cast-iron wedges which in turn rest on I-beam grillages, footing on the aforementioned concrete projection. Between the column base and the projection, 10-ton screw jacks are interposed to aid in the alignment and leveling of the centers; they are allowed to remain in place, though the load passes directly to the wedges which are used for striking centers. A U-shaped clamp, made of a 1-in. bolt (not shown in the drawing) is passed around each pair of wedges to prevent any possible lateral motion. A similar bolt is used for the same purpose higher up on the main column.

The unique feature in the steel centering used in constructing the Tunkhannock Creek Viaduct on the relocation of the Delaware, Lackawanna, and Western Railroad was an adjustable panel at the crown of the steel arch trusses.

Arch rib forms on steel centers are shown in Fig. 55. These forms were used in constructing the Brooklyn-Brighton viaduct, Cleveland.

THREE-HINGED ARCHES

43. General Discussion.—An arch with three hinges is statically determinate and consequently can be analyzed much more readily for a given loading than is possible in the case of a fixed-ended or solid arch. Furthermore, three-hinged arches do not need to be analyzed for temperature changes, the hinges allowing contraction and expansion of the ribs without causing any stress throughout the arch. Obviously this statement does not take into account the effect which results from friction on the hinges, but such effect is usually considered to be negligible. Whether or not hinge friction is likely to cause appreciable error in the analysis of three-hinged arches is still a matter, however, in regard to which there seems to be a decided difference of opinion.

Three-hinged arches are especially adapted to sites where abutments and piers must be founded on compressible soil or on piles. The hinges permit of considerable settlement without failure of the arch or without causing the huge cracks which are sure to develop in a fixed-ended structure under like conditions. Of course, a solid arch may be designed on the assumption that the abutments are yielding, but this is rarely done and such computations in any event could not take into account such settlement as might come from an unexpected source.

Hinges in arch-bridge construction are likely to be an expensive detail, especially in short-span structures. The claim is made, however, that in arches of large span, the saving in concrete as compared with the fixed-ended type much more than pays for the hinges.

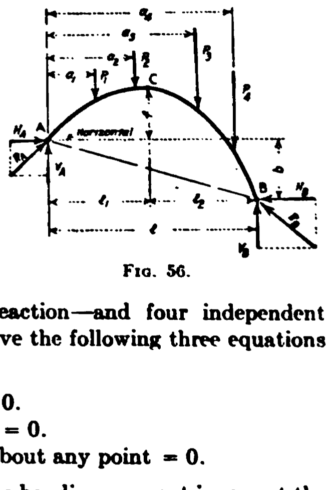


FIG. 56.

44. Methods of Analysis.—Consider first the general case of an unsymmetrical three-hinged arch subjected to a number of vertical concentrated loads. By referring to Fig. 56, it is seen that there are four unknown quantities—namely, the horizontal and vertical components of each reaction—and four independent equations are necessary to solve for these unknowns. We have the following three equations from statics:

ΣV = algebraic sum of the vertical components = 0.
ΣH = algebraic sum of the horizontal components = 0.
ΣM = algebraic sum of moments of all the forces about any point = 0.

The additional equation may be obtained from the fact that the bending moment is zero at the crown hinge. Thus we have the following four equations with respect to the arch of Fig. 56.

$$V_A + V_B - \Sigma P = 0$$
$$H_A - H_B = 0$$

Taking moments about the left hinge

$$H_B b - V_B l + \Sigma P a = 0$$

Since the moment at the crown hinge is zero

$$V_A l_1 - H_A f - \Sigma_0^{l_1} P (l_1 - a) = 0$$

These four equations may be solved simultaneously to obtain the horizontal and vertical components of the two reactions.

The calculations may be simplified by resolving each reaction into a vertical force and a force in the direction of the closing chord (Fig. 57). The four equations in this case are as follows (since $H_A = H_B$ from Fig. 56):

$$\begin{aligned} V_1 + V_2 - \Sigma P &= 0 \\ H_1 - H_2 &= 0 \\ - V_2 l + \Sigma Pa &= 0 \\ V_1 l_1 - \Sigma_0^{l_1} P\,(l_1 - a) - H_1 r &= 0 \end{aligned}$$

or

$$V_1 l_1 - \Sigma_0^{l_1} P(l_1 - a) - H_A c = 0$$

(The values of V' have not been considered in the first equation as they are equal and opposite in direction.) With the components of either reaction determined by these equations, the line of thrust may be drawn throughout the arch as described in Art. 16 for the arch with fixed ends.

It should be noted that the values of V_1 and V_2 may be obtained from the above equations (or by using $\Sigma M = 0$ at both points A and B) in the following form:

$$V_1 = \frac{1}{l}\,\Sigma P(l - a) \tag{1}$$

$$V_2 = \frac{1}{l}\,\Sigma Pa \tag{2}$$

These forces are thus identical with the reactions of a simple beam of the same span and similarly loaded.

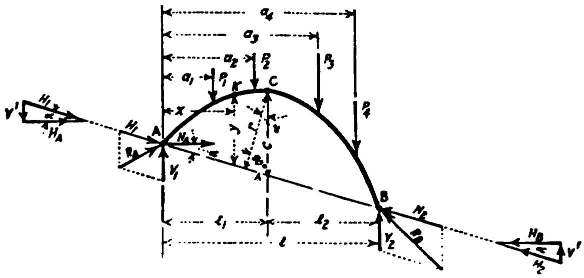


FIG. 57.

The bending moment at any point K (Fig. 57) may be expressed as follows:

$$\begin{aligned} M &= V_1 x - \Sigma_0^{x} P(x - a) - H_A y \\ &= M_K - H_A y \end{aligned} \tag{3}$$

where M_K is the bending moment at the point K of a similarly loaded beam. At the crown hinge, letting M_C denote the moment of the vertical forces about the point C, we have

$$M = M_C - H_A c = 0$$

or

$$H_A = \frac{M_c}{c} \tag{4}$$

Equations (1) to (4) inclusive are the formulas commonly employed in the analysis of three-hinged arches—supplemented, of course, with the force and equilibrium polygons as in the case of arches with fixed ends.

For symmetrical arches, H_1 and H_2 are horizontal and the line of thrust need be drawn for only one-half the arch when the loading is symmetrical about the crown hinge. In such a case of loading, the thrust at the crown hinge is horizontal and the line of thrust may be determined by trial in the manner described in Art. 11. This trial method gives exact results when applied to a three-hinged symmetrical arch on account of there being two known points (hinge points) on the line of thrust for each half of arch.

The computations for uniform live loading are extremely simple and should be made separately from those for dead load or concentrated live loads. For full loading, with the crown hinge at mid-span, formula (4) gives

$$H_A = \frac{1}{8}\cdot\frac{wl^2}{c} \tag{5}$$

where w is the uniform load per foot. The following equation, determined by substituting in formula (3) gives the bending moment at any point (coördinates x and y):

$$M = \frac{1}{2}wx(l - x) - \frac{1}{8}\cdot\frac{wl^2}{c}\cdot y \tag{6}$$

(For an arch of parabolic form, $M = 0$, and only axial stress occurs throughout the arch for full uniform loading.) With only one-half of the span loaded

$$H_A = \frac{1}{16}\cdot\frac{wl^2}{c} \tag{7}$$

or one-half that due to full loading. The bending moment at any point in the loaded half equals

$$M = \frac{1}{8}wx(3l - 4x) - \frac{1}{16}\cdot\frac{wl^2}{c}\cdot y \tag{8}$$

and in the unloaded half

$$M = \frac{1}{8}wlx - \frac{1}{16}\cdot\frac{wl^2}{c}\cdot y \tag{9}$$

(In equations (8) and (9), the value of x is measured from that end of the arch which is nearer to the point in question.)

A three-hinged arch is commonly analyzed for (1) dead and uniform live load over the entire span, (2) for dead and uniform live load over the right half of span, and (3) for dead and uniform live load over the left half of span. Full loading gives maximum stresses for the sections near the hinges, while the half-span loadings give the greatest stresses near the quarter points of the span. The usual method of design is to locate the hinges at the proper points and to draw the force lines representing the load concentrations. These loads can be determined quite accurately by making a complete design of the spandrels prior to the arch design and by approximating the weight of the arch ring—the arch ring, however, need not be drawn. The lines of thrust for the three conditions of loading stated above are then drawn as shown in Fig. 58. With the lines of thrust known, it then becomes possible to determine the correct thickness of the arch at any point and decide upon a suitable arch ring which, of course, should not differ appreciably in weight or position from the arch ring previously assumed or else a second analysis should be made.

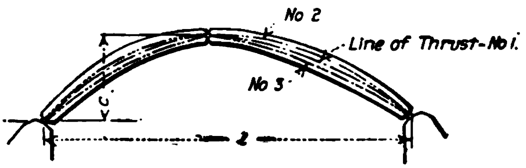


FIG. 58.

For a load of unity at the point L_2 in Fig. 59, the direction of one of the reaction lines is given by the line connecting the two hinges to one side of the load. The direction of the other reaction is then known. It is thus an easy matter to construct influence lines similar to those of Art. 34 and determine the exact maximum loadings. Method of constructing influence lines is explained in Art. 48a, Sect. 7.

45. Common Type of Hinges.—The most common form of arch hinge consists of a structural or cast-steel pin bearing on two steel castings. Hinges of this type are shown in Figs. 60 to 63 inclusive.

46. Methods of Construction.—Three distinct methods of construction have been employed in the erection of three-hinged arches: (1) casting the concrete ribs in forms on the ground and then hoisting them into place; (2) erecting structural-steel reinforcement to be employed in

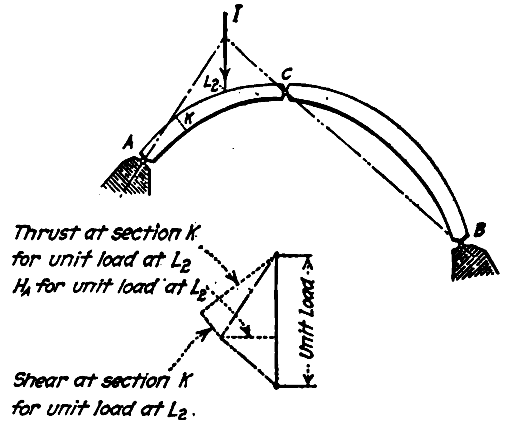


FIG. 59.

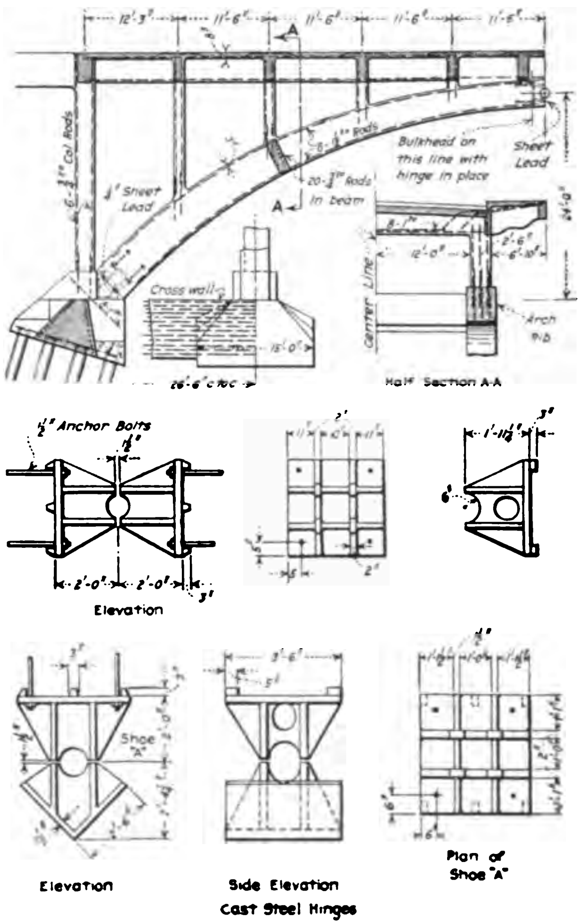


FIG. 60.—Details of Fourth Street bridge, Paducah, Ky.

the arch ribs and using this reinforcement to support the weight of the forms and plastic concrete during construction; and (3) employing the usual type of centering and casting the ribs in place. The first method is the one usually followed. Method No. 2 is of advantage when a stream to be spanned is subject to sudden freshets and a minimum of falsework is required. Method No. 3 is necessary only under unusual conditions.

The cheapest type of the three-hinged arch and also the type that is lightest and best adapted to the use of hinges is one of detached ribs supporting spandrel columns. Such a type of arch lends itself readily to the unit method of construction should this form of erection be desired, and also eliminates the necessity for waterproofing which is a serious problem in the case of a solid filled arch.

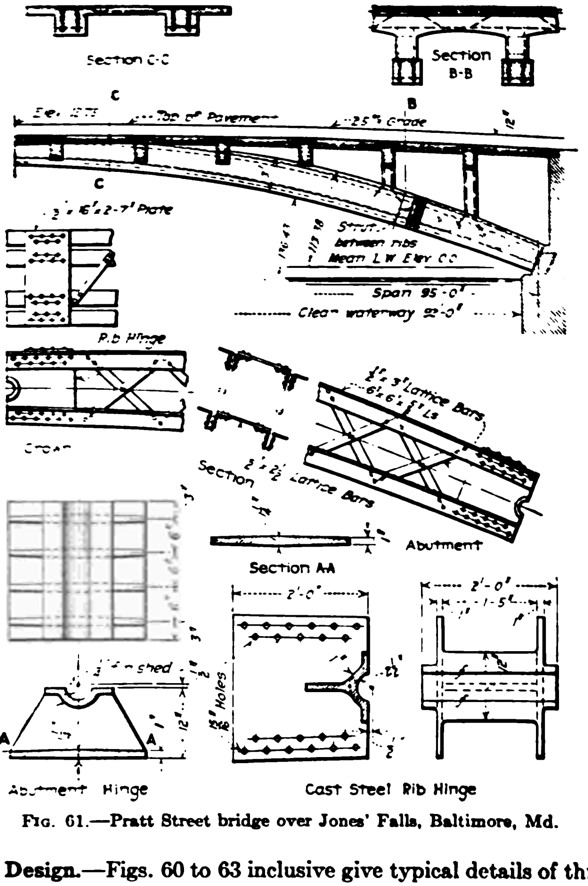


FIG. 61.—Pratt Street bridge over Jones' Falls, Baltimore, Md.

47. Details of Design.—Figs. 60 to 63 inclusive give typical details of three-hinged arches.

The arch shown in Fig. 60 is founded on Ohio River mud, Raymond concrete piles being used for the foundations. The reason for the use of the cast hinges in this case is thus apparent, as settlement of foundations was anticipated. No appreciable settlement, however, has ever taken place.

The two halves of each rib of the bridge shown in Fig. 61 were designed to be erected simultaneously, without falsework, by derricks on opposite sides of the stream, and to be self-supporting as soon as the crown-hinge connection was made. Temporary sway bracing was provided to insure lateral stability while the forms were being built and filled with concrete.

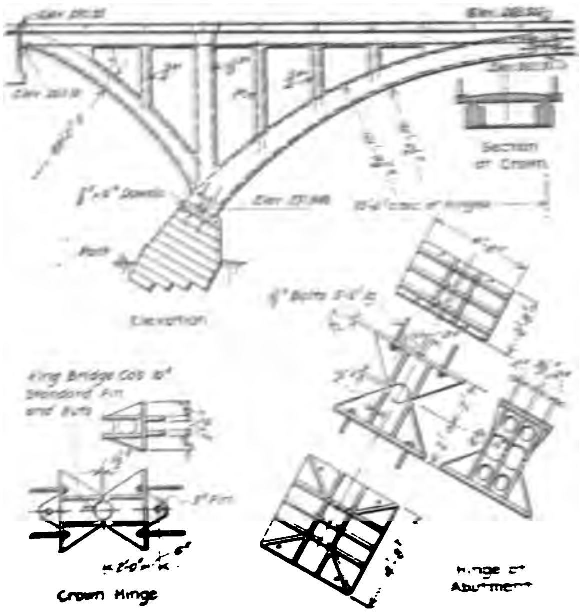


Fig. 62.—Bridge over the Vermillion River at Wakeman, Ohio.

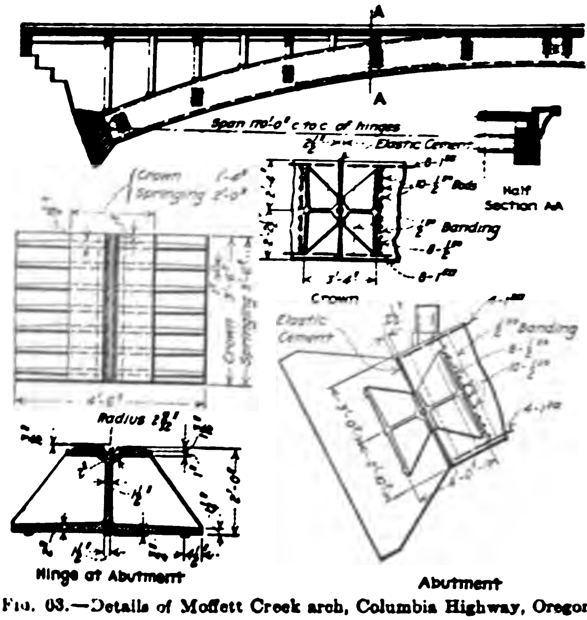


Fig. 63.—Details of Moffett Creek arch, Columbia Highway, Oregon.

The three-hinged arch construction with cantilever ends, shown in Fig. 62, is unusual, but was found to be more economical for an arch of this type and dimensions than a hingeless structure. The cantilever ends were rendered necessary on account of the fact that there were no stable foundations for abutments at the top of the fill at the ends of the bridge. The cantilever ends decreased the dead-load thrust on the center hinges about one-quarter and decreased considerably the angle which the resultant thrust on the lower hinges made with the vertical, thus decreasing the size of abutment required. The cost of the hinges necessary for the three-hinged arch was very considerably less than the cost of the additional steel reinforcing in the arch ring required to take care of the additional bending moments in the hingeless arch. No connection whatever was needed between the ends of the cantilevers and the abutments on account of the extremely small amount of vertical motion at these points. A common type of centering was used in constructing the arch ribs in place. The cast-steel hinges were entirely encased in concrete after the centers were struck, a ¼-in. plate of sheet lead having been placed at the center of each hinge to allow the necessary motion of the arch rib under live load and temperature stresses. In this way all possibility of corrosion of the steel hinges was avoided.

SECTION 17

HYDRAULIC STRUCTURES

DAMS

By A. G. Hillberg[1]

A dam is built for the purpose of holding back a certain volume of water and usually to raise the water level at a given point, so as to create a fall or head. Low dams designed to permit the water to flow over them are termed weirs, while higher structures are called spillways. Dams designed to hold back the water and not intended to assist in passing the discharge are classified as bulkheads.

Dams, as a rule, can be divided into two main groups: Fixed and movable. The former consist of masonry, earth, rock, steel, timber, or combinations of these materials and are designed as immovable; while the second group contains movable structures, such as gates, stop logs, flashboards, needles, rollers, etc., designed to be removed as required.

Often dams are combinations of these two groups as, for instance, at Keokuk, Iowa, where 11-ft. steel gates have been placed on top of a 32-ft. spillway dam of mass concrete.

1. Preliminary Studies.—Before the actual designing of a dam can be undertaken, a great deal of knowledge of its purpose and the site must be at hand. As a rule, the geological formation is the deciding factor as to what type to adopt, while very often the hydrographical and topographical conditions decide the height of the structure.

1a. Locating.—When investigating a river with the view of finding a suitable dam site, the engineer, naturally, first looks for the narrow canyons. The reason for this is the desire to get as short a structure as possible and also because the rock formation in such a narrow place is usually firm and hard. However, the writer once investigated such a narrow box-canyon where the sides consisted of good, hard, blue limestone and thought the site to be excellent, while borings afterward showed the canyon to be a volcanic split in the rock, probably 1500 ft. deep below the river bed.

When storage is required, a narrow box-canyon, while offering good dam sites, may not give the desired reservoir capacity. It is, therefore, often necessary to investigate a number of possible dam sites, as the one apparently best suited may not have any rock foundation nor give the necessary storage capacity. A site, which at first does not seem suitable, might after all be the most favorable because the formation is solid bedrock located not far below the surface and the basin upstream such as to give the desired capacity with a structure of low height.

1b. Geological Investigations.—These investigations must be very thorough and the capital spent on careful explorations will be saved many times over during the construction. The first matter of importance is to know whether or not bedrock can be reached. If so, then it is important to know whether it is stratified, decomposed, or homogeneous; whether hard or soft; or whether the formation is primeval, sedimentary, or volcanic.

Such data can be obtained only through core borings with diamond drills, and it is advisable to test the holes by means of air or water under pressure. In one case the exact locations of fissures were determined by having two gaskets on the pipe, so that air under pressure could

[1] Hydraulic Engineer, New York City.

be applied to any predetermined portion of the hole.[1] Should it be found that the formation is stratified or otherwise permeable, it is sometimes necessary to grout it.

In soft foundations, wash borings must be used and a record kept of the materials penetrated. Even here the main point is whether the formation is permeable or not. As a rule, clay or hard-pan offer good foundations, while glacial deposits are always treacherous.

Hard-pan is not always to be depended upon since, if saturated with water under high pressure, it will often disintegrate into yellow clay and gravel.[2]

As soon as the necessary data have been obtained, a subsurface map should be made. From this map it can be decided what type of dam is most suitable and if any precautions have to be taken in improving the foundations.

1c. Selecting a Suitable Type of Dam.—As a rule, an engineer will first think of a gravity-section dam, but if bedrock cannot be reached this type is out of the question. It is also usually out of the question if no suitable materials for concrete are available at the site, because hauling materials is expensive. For such conditions dams of reinforced concrete are well worth investigating. However, the type of dam depends directly on the foundation conditions, which can be divided into two groups: Rock foundations and soft foundations.

Rock Formation.—A type of dam always suitable in rock formations is the gravity-section. However, if the site is narrow, an arched dam as a rule shows a considerable saving in material. A still greater saving can sometimes be obtained by a reinforced-concrete dam.

If no concrete materials are available, an earthen dam might be the solution, but, in order to make it tight, there must be both clay and sand or gravel available, as at least 20% of clay is necessary to make it tight.

In many instances rock-fill dams have been used, but, in order to make them tight, a watertight membrane must be provided on the upstream side. This can be made of gravel, sand, fine sand and clay graded so that they will form an impervious stratum. Reinforced concrete has also been used, but as a rule it is cheaper to support such a deck on concrete buttresses rather than on a rock fill. To place a core wall of concrete near the center of a rock-fill dam is not good practice, as the upstream portion of the rock fill is then submerged.

If good timber is available and the structure to be built is not too high, a rock-fill crib dam will often be worth investigating.

Steel dams are structurally all right, but their maintenance is high.

Soft Foundations.—In soft foundations, gravity-section dams are out of the question, but sometimes engineers have put low gravity-section structures on piling. The most suitable dams for such conditions are earthen dams; reinforced-concrete structures placed on continuous foundation mattresses; rock-fill timber cribs; or framed timber dams.

1d. Height of Structure.—Several conditions influence the height of a dam. If it is to be used for storing water for irrigation, the reservoir capacity must be sufficient to regulate the natural flow so that a certain area can be irrigated; if built in connection with a hydro-electric power plant, the head created is just as important as the storage or the regulation of the river discharge; if a low diversion dam, it must be high enough to divert water into the pipe line, flume, or ditch at all stages; if built for flood protection, it must have sufficient capacity to retain all discharge in excess of the maximum carrying capacity of the river channel below the dam, etc.

Very often the interference of the backwater with structures of immovable character, such as operating mills, other power plants, buildings, railroads, etc., will limit the height of a dam.

The top of the bulkhead section of a dam should always be higher than the maximum water level, as wave action will occur on the surface of the reservoir. Such wave action is a function of the exposed length of surface and, according to Stevenson

$$H = 1.5\sqrt{l} + (2.5 - \sqrt[4]{l}) \quad \text{(ft.}$$

[1] C. W. Smith: "Construction of Dams."

[2] "Failure of Stony River Dam." *Eng. Rec.*, Jan., 1914, p. 115.

where H = height of wave in feet; and l, length of exposed water surface in miles, measured along a line perpendicular to the dam.

D. C. Henny gives the following formula:

$$H = 0.075\ (V - 8.5) \qquad \text{(ft.)}$$

where V = velocity of wind in miles per hour. For dams designed by the U. S. Reclamation Service, the following wave heights have been used:

Wind velocity, miles per hour.	Wave height, feet.
35	2.00
40	2.50
44	2.50
48	3.50
50	3.50
56	4.00
75	5.00

The top of the dam should, of course, be somewhat higher than the wave height, especially if it is an earthen dam, which might be severely eroded if water splashes over it.

1e. Hydrographic Investigations.—For whatever purpose a dam is built an intimate knowledge of the river and its drainage basin is required. The best possible data to work on are actual observations on the river itself, such as are kept and published by the U. S. Geological Survey and by the various State engineering offices.

If no such records are available at the point where the dam is to be built, records at other points, preferably one above and one below the site, can be made use of and a rating curve established. Should no records at all be available on the stream itself, the best method is to compare it with adjacent streams on which records have been kept.

In many cases it is necessary to resort to runoff calculations, basing them on the rainfall. Such computations are always very unreliable and should be made and used with the greatest caution. The best available method is that developed by Vermeule.[1]

$$E = F(15.50 + 0.16R) \qquad \text{(in.)}$$

where E = yearly losses due to evaporation; $F = (0.05T - 1.48)$, where T = mean yearly temperature in degrees Fahrenheit; and R = yearly rainfall in inches. Vermeule found by investigating the rivers in the East, especially those in New Jersey, that the monthly relation between evaporation (including absorption by crops, etc.) and rainfall was as follows:

December	$e = 0.42 + 0.10r$	June	$e = 2.50 + 0.25r$
January	$e = 0.27 + 0.10r$	July	$e = 3.00 + 0.30r$
February	$e = 0.30 + 0.10r$	August	$e = 2.62 + 0.25r$
March	$e = 0.48 + 0.10r$	September	$e = 1.63 + 0.20r$
April	$e = 0.87 + 0.10r$	October	$e = 0.88 + 0.12r$
May	$e = 1.87 + 0.20r$	November	$e = 0.66 + 0.10r$

Each of these monthly evaporations is to be multiplied by $f = (0.05t - 1.48)$, where t = average monthly temperature in degrees Fahrenheit.

Where gage records are available they should be worked up as shown in Fig. 1, giving the rating curve, the gage heights, and the duration per year of each gage height. The longer the record, the better the average will be.

It is of great importance to determine, as correctly as possible, the maximum discharge that can possibly occur. As the records given might not cover a sufficiently long period of time to embrace such a maximum discharge, a cautious designer should increase the given maximum

[1] New Jersey Geological Survey, 1894.

from 20 up to 50%, and even then have sufficient freeboard to safeguard the bulkhead portion from being overtopped.

1f. Capacity of Reservoir.—As soon as the dam has been located definitely and the reservoir surveyed, a capacity diagram should be made as shown in Fig. 2. Such a diagram

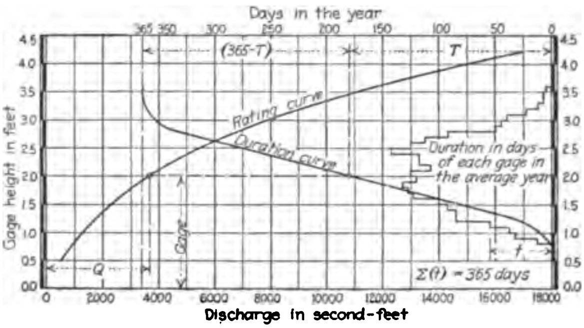
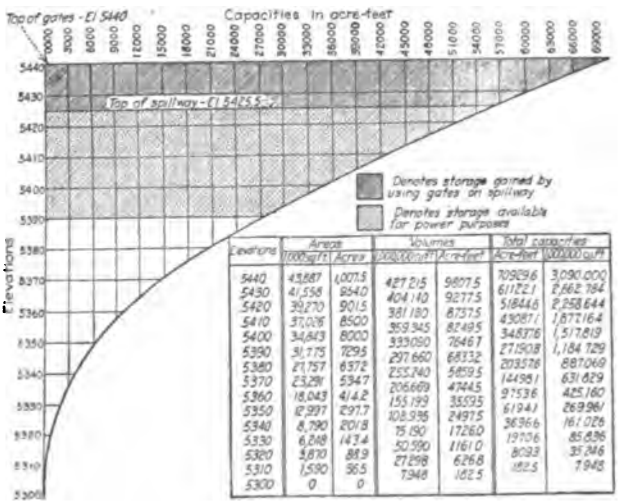


FIG. 1.

is obtained by determining the areas enclosed by each contour, calculating the mean areas between any two contours and multiplying by the distance between them. A table should also be made giving the areas and capacities. For convenience the unit measure is 1 acre-foot which equals 43,560 cu. ft. Such a diagram is approximate only, as it is based on the assumption that



Elevations	Areas, 1000 sq. ft.	Areas, Acres	Volumes, 1,000,000 cu. ft.	Volumes, Acre-feet	Total capacities, Acre-feet	Total capacities, 1,000,000 cu. ft.
5440	43,887	1,007.5	427.215	9807.5	70929.6	3,090.000
5430	41,558	954.0	404.140	9277.5	61122.1	2,662.784
5420	39,270	901.5	381.180	8737.5	51844.6	2,258.644
5410	37,026	850.0	359.345	8249.5	43087.1	1,877.164
5400	34,843	800.0	333.090	7646.7	34837.6	1,517.819
5390	31,775	729.5	297.660	6833.2	27190.8	1,184.729
5380	27,757	637.2	255.240	5859.5	20357.6	887.069
5370	23,291	534.7	206.669	4744.5	14498.1	631.829
5360	18,043	414.2	155.199	3559.5	9753.6	425.160
5350	12,997	297.7	108.935	2497.5	6194.1	269.961
5340	8,790	201.8	75.190	1726.0	3696.6	161.026
5330	6,248	143.4	50.590	1161.0	1970.6	85.836
5320	3,870	88.9	27.298	626.8	809.3	35.246
5310	1,590	36.5	7.948	182.5	182.5	7.948
5300	0	0				

FIG. 2.

the water level is horizontal; while, as a matter of fact, it is a modified parabola tangent to the horizontal at the dam, and, at the upper end, to the slope of the water surface of the river feeding into the reservoir.

The line thus formed is called the *backwater curve* and its exact shape cannot be calculated. However, several methods have been devised whereby a fairly intelligent guess can be made.

If the dam were put in a channel with vertical sides or nearly so, the shape would be a parabola and the length l of its horizontal projection

$$l = \frac{2h}{S}$$

where h is the height of the new water level above the natural slope S of the water surface at the dam before it was built (Fig. 3).

However, as the shape of a reservoir basin is such that the respective cross-sections are not uniform, the best method to apply is that of trial and error. Therefore, select first cross-sections at suitable intervals, $l_1, l_2, l_3, \ldots l_n$, and determine the area A, the wetted perimeter

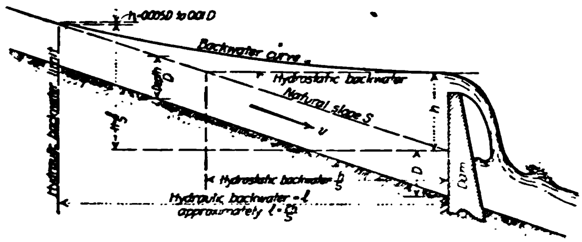


Fig. 3.

p, the hydraulic radius R and the velocity v at different elevations for each one of them. Then assume a value of n in Kutter's formula, basing it on the prevailing n at high and low water in the river. This value is variable because the frictional resistance is greater for low stages than for high, not only because of the increase in R but because of the decrease in velocity as well. In many cases, especially if the dam is high, it is advisable to make a curve showing the variation in n and extending it to cover stages as high as will obtain after the dam is built.

As the water level at the dam is known, the computation is started by guessing at the slope S_1 in the first section l_1 (Fig. 4). This will give a certain elevation at the first cross-section so that A, p, R and v can be calculated ($v = Q/A$, Q being the discharge). Calculating the

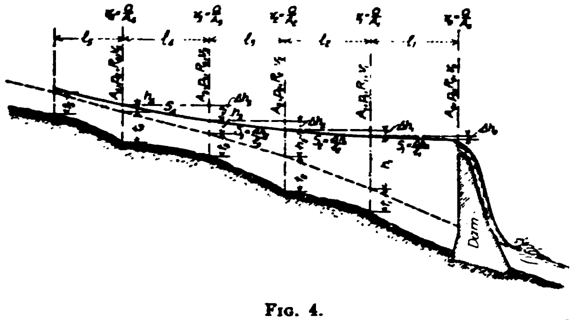


Fig. 4.

average values of A_m, p_m, R_m and v_m for the section and inserting them in the Chezy formula

$$C_1 = \frac{v}{\sqrt{RS}}$$

a value of C_1 is determined.

By doing the same in the Kutter formula

$$C_2 = \frac{41.65 + \dfrac{0.00281}{S_1} + \dfrac{1.811}{n}}{1 + \left(41.65 + \dfrac{0.00281}{S_1}\right)\dfrac{n}{\sqrt{R}}}$$

a value of C_2 is obtained. Of course, if S_1 were correct, both values would have been the same. By guessing at S_1 and correcting A_m, p_m, R_m, and v_m, values are finally obtained that will make

$$C_1 = C_2$$

This is the correct value of S_1, and it determines the height of the backwater curve at the cross-section in question.

The next section is then taken and dealt with in the same way, and so on until finally a point l_n is reached, where the slope coincides with the natural slope S of the water surface.

As all backwater computations are based on a given volume of discharge Q, it is obvious that in order to find the limits of the influence of the water backed up, at least two sets of calculations must be made, one for Q_{max} and one for Q_{min}.

The *inflow* in a reservoir is

$$I = D + E + S$$

where I = inflow; D, outflow; E, evaporation and seepage; and S, storage, which can be positive, negative or zero.

It is difficult to measure the inflow, as all of it might not be surface water, while the discharge D can be measured directly and E calculated from evaporation records obtained from pans and seepage records of similar reservoirs.

2. Design of Foundation.—When designing a dam, the substructure is just as important as the superstructure, as a structure is no safer than its weakest part. If bedrock of good quality is available, a gravity-section dam can, as a rule, be placed directly upon it, provided the upper strata are removed. This is done not only to remove disintegrated portions, but also to obtain a rugged surface with which concrete will give a good bond.

2a. Grouting.—Should the rock be disintegrated or fissured, it is often necessary to grout it. This is done by drilling into it and pressing grout into the fissures through the drill holes. The consistency of such grout must depend upon the size of the fissures, which generally are so thin that neat cement and water must be used. Should any larger fissure obtain, sand is mixed with the cement in various proportions and the grouting pressure is reduced so as not to force the cement out of the mixture.

2b. Cut-off Walls.—In fissured rock, cutoff walls of concrete are often used instead of grouting, the underlying idea being to establish a long seam offering excessive resistance to seepage. Of course, structures placed on such foundations must be calculated to resist a considerable uplift.

2c. Caissons.—If the rock is so disintegrated that it consists of boulders loosely cemented together between which a rapid percolation takes place, it might be found necessary to use pneumatic caissons, as was done at Hale's Bar, near Chattanooga, Tenn.[1]. Several dams in Europe have been placed on such foundations.

In soft foundations it has also been found expedient to use open caissons as foundation for the core wall in earthen dams. The main difficulty in using caissons, whether pneumatic or open, lies in the difficulty in making water-tight the joints between them.

2d. Pilings.—In soft foundation, piling is often used for improving the ground. However, a portion of these piles must be placed in an inclined position parallel to the resultant of all the forces acting on the structure above the plane of loading of the piles. Otherwise, there will be no resistance to the horizontal component of the hydrostatic pressure and the dam might move in a downstream direction.

2e. Sheet Piling.—In soft foundations, where it is impossible to reach bedrock by any of the means given above, sheet pilings are often used. These can be of wood, steel or reinforced concrete, but one thing applies to them all: they must be strong enough to withstand the bending moment of the forces acting on their upstream side while transmitting the load to the materials on the downstream side. As is obvious a considerable force must be

[1] "Foundations for the Hale's Bar Dam." *Eng. Rec.*, Feb., 1913, p. 178.

resisted on the downstream side at the top of such sheet piling, and it is, therefore, customary to imbed the top firmly in the masonry.

One thing applies to all details appurtenant to the improving of foundations, whether firm or soft: the improved portion must extend deeply enough so that the weight of the materials including the weight of the structure itself as a surcharge on the downstream side more than balances the forces on the upstream side of an arbitrary plane generally located at the upstream side of the dam or through the center of the cutoff wall, if such is made use of. The forces on the upstream side of such a plane are due to the full hydrostatic pressure only, if the foundation is bedrock. If it is a soft foundation, the active pressure of the saturated soil must be added to the water pressure, while on the downstream side the weight of the structure, acting as a surcharge, naturally increases the passive, or resisting, earth pressure.

3. Design of Dams of Gravity Section.—This type of dam resists the hydrostatic pressure through its great weight, which is sufficient to form, with the horizontal component of the water pressure, a resultant passing the plane between masonry and foundation at a point located within the middle third. As masonry cannot be depended upon to resist more than a small amount of tension, the design must be investigated at different elevations to insure against tensile stresses.

3a. Hydrostatic Pressure.—In the analysis it is customary to consider a strip of the structure 1 ft. wide. As the weight of 1 cu. ft. of water is w—assumed at 62.5 lb., which is somewhat too high —the intensity of pressure per square foot at any depth is

$$p = wh \qquad \text{(lb.)}$$

where h = depth in feet below surface of water.

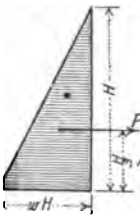


Fig. 5.

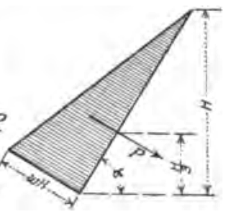


Fig. 6.

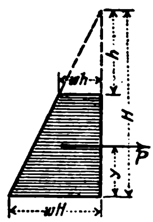


Fig. 7.

The total pressure P on a vertical plane extending from the water surface to a depth H increases proportionally from zero at the top to wH at the bottom (Fig. 5) and is expressed by

$$P = wH \times \frac{H}{2} = \frac{wH^2}{2} \qquad \text{(lb.)}$$

Should the plane be inclined an angle α with the horizontal (Fig. 6), the maximum intensity of the pressure remains the same, wH, while the length increases to $H \times \frac{1}{\sin \alpha}$, so that

$$P = wH \times \frac{H}{2 \sin \alpha} = \frac{wH^2}{2 \sin \alpha} \qquad \text{(lb.)}$$

If the plane is submerged, the pressure will be the difference of the total pressure P minus the pressure p corresponding to the upper part of the pressure triangle (Fig. 7), or

$$P_1 = P - p = \frac{wH^2}{2} - \frac{wh^2}{2} = \frac{w}{2}(H^2 - h^2) \qquad \text{(lb.)}$$

For inclined surfaces multiply by $\frac{1}{\sin \alpha}$ as shown above.

The resultant force of the hydrostatic pressure is always located through the center of gravity of the pressure diagram and normal to the plane in question. When the pressure diagram is a triangle, the resultant is located a distance $\frac{H}{3}$ above the lowest point (Figs. 5 and 6), while if it is a trapezoid (Fig. 7), the distance above the lower side is

$$y = \frac{H - h}{3} \times \frac{wH + 2wh}{wH + wh} = \frac{H - h}{3} \times \frac{H + 2h}{H + h} \qquad \text{(ft.)}$$

3b. Profiles of Dams.—The simplest form of a dam is a triangle with its upstream side vertical (Fig. 8).

The overturning moment is

$$\frac{PH}{3} = \frac{wH^2}{2} \times \frac{H}{3} = \frac{wH^3}{6} \qquad \text{(ft.-lb.)}$$

while the resisting moment is

$$W \times \frac{2X}{3} = \frac{mHX}{2} \times \frac{2X}{3} = \frac{mHX^2}{3} \qquad \text{(ft.-lb.)}$$

where m = weight of 1 cu. ft. of masonry.

Dams are designed with a factor of safety against overturning of at least two: Therefore,

$$\textit{Resisting Moment} = 2(\textit{Overturning Moment})$$

or

$$\frac{mHX^2}{3} = 2 \times \frac{wH^3}{6}$$

and the length of base will be

$$X = H\sqrt{\frac{w}{m}} \qquad \text{(ft.)}$$

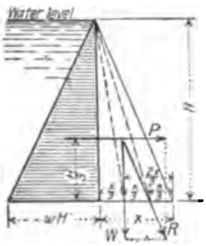


FIG. 8.

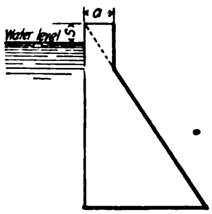


FIG. 9.

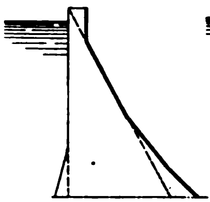


FIG. 10.

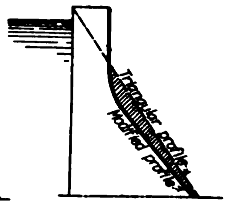


FIG. 11.

That for a factor of safety of two the resultant cuts the base at the outer edge of the middle third can be proved as follows (Fig. 8):

$$P:W = \frac{X}{3}:\frac{H}{3}, \text{ but } P = \frac{wH^2}{2} \text{ and } W = \frac{mHX}{2}$$

so that

$$X = H\sqrt{\frac{w}{m}} \qquad \text{(ft.)}$$

The triangular shape of dam as an economical type was first pointed out by Edward Wegmann.[1] He recommended that the water level for safety in the calculation be assumed as extending to the top of the dam and that the superelevation (s) of the masonry above the actual future water level and the top width (a) of the dam crest, each be made one-tenth of the height of the dam, limiting the former to a maximum of 10 ft. and the latter to a minimum of 5 ft. (Fig. 9).

He also recommended that the upstream side be kept vertical until the unit stresses for "reservoir empty" approach the permissible maximum. Then the side should be sloped, so as to keep the stresses at or somewhat below this stress.

As the limiting pressure will be reached sooner in the downstream than in the upstream face, it is obvious that a similar slope must be started at a higher elevation on that side (Fig. 10).

Wm. P. Creager[2] found that the economical top width of a dam is a function of the acting forces (hydrostatic pressure, ice pressure, uplift, etc.) and that it should vary from 10 to 17% of the height. However, as the height varies from nothing to a maximum and again to nothing along the length of a dam, Creager's method would give a variable top width in addition to irregularities in the face of the dam. An average section must, therefore, be adopted if this method is used.

[1] WEGMANN: "The Design and Construction of Dams," 1st Ed., 1888.

[2] CREAGER: "The Economical Top Width of Non-overflow Dams," *Trans., Am. Soc. C. E.*, vol 61, Nov., 1915.

In designing a dam it is advisable to start with a triangular profile and in that way obtain a tentative section. The top width is then determined from practical considerations. As such added weight at the top tends to place the resultant of the structure nearer the upstream face, it is obvious that the profile can be somewhat reduced, and its downstream face given a slope asymptotic to the face of the triangular profile (Fig 11). Thus a saving is effected as is shown shaded in the figure. It is on this saving in material Creager has based his theory.

3c. Uplift.—Should a dam be founded on pervious material, uplift will result. Its magnitude is a function of the permeability of the foundation and can vary from full hydrostatic pressure all along the base to nothing. Experiments at the Oester Dam in Germany showed that in various sections the following pressures existed:

Section I—full head at heel to one-half at toe.
Section II—full head at heel to one-fourth at toe.
Section III—three-fourths at heel to one-fourth at toe.
Section IV—full head at heel to zero at toe.

At the Neye Dam in Germany similar observations were made. This dam has its foundation 26 ft. below the surface of the rock, or about twice that at Oester. Here 57% of the full pressure was observed at the heel and 32% near the toe.

In America it is customary to assume two-thirds of the full hydrostatic pressure at the heel, diminishing in a straight line to zero at the toe (Fig. 12a). However, a more correct way would be to assume full pressure at the heel due to the water on the upstream side and full pressure at the toe due to the water on the downstream side (Fig. 12b).

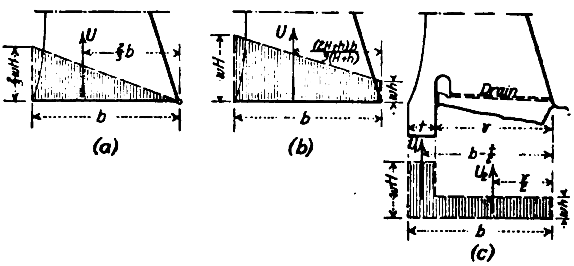


FIG. 12.

A still better way is to provide drainage immediately back of the cutoff wall and to use an uplift corresponding to the elevation of the water surface below the dam under the entire base excepting the portion in front of the drains on which full uplift is acting (Fig. 12c).

Depending upon which of the above assumptions is used, the pressures and moments referred to the toe of the section will be as follows:

	Force in pounds	Moment in foot-pounds
Fig. 12a	$U = \frac{1}{3}wHb$	$M = \frac{2}{9}wHb^2$
Fig. 12b	$U = \frac{w}{2}(H + h)b$	$M = \frac{w}{3}\left(H + \frac{h}{2}\right)b^2$
Fig. 12c	$U = U_1 + U_2 = w(Ht + hv)$	$M = w\left[Ht\left(b - \frac{t}{2}\right) + \frac{hv^2}{2}\right]$

Some engineers apply the uplift theory to every horizontal joint in a dam and use then the assumption shown in Fig. 12a for these joints. However, if care is taken and the upstream side made as water-tight as possible and, in addition, vertical drains, or weepers, are provided a few feet inside the face and connected to a drainage gallery, such uplift can be eliminated entirely.

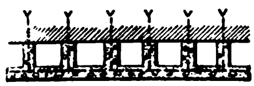


FIG. 13.

In order to get as impervious a surface as possible and at the same time to provide as ample a drainage as possible, it has been proposed to build in front of gravity section dams cellular facings of reinforced concrete. This facing is anchored to the masonry, and at the bottom the cells are piped through the base of the dam. The cells should be interconnected at certain intervals from the bottom up (Fig. 13).

3d. Wind Pressure.—No wind pressure needs to be considered, as when the reservoir is full its action can only be on the downstream side, thus decreasing the unit stresses in the masonry. When the reservoir is empty, such pressure on the upstream side will be considerably less than the future hydrostatic pressure and, therefore, need not be considered. If it acts on the downstream side, it will throw the line of pressure backward approximately

$$X = \frac{\theta\left(3\sqrt{\rho} - \sqrt{\frac{1}{\rho}}\right)}{3mH + 6\theta}$$

where θ = wind pressure in pounds per square foot; ρ, specific gravity of masonry (generally 2.5); m, unit weight of masonry per cubic foot; and H, height of structure above plane in consideration. However, the influence of wind on the stability is so small as to make it negligible.

3e. Ice Thrust.—It is a much disputed point whether ice pressure should be considered or not. Many engineers say that, as the dam is located in a narrow place, arching stresses between the banks will relieve the dam of the ice pressure. However, several failures are directly traceable to such pressure, but then, as in the case of Waldron, Ill.,[1] at Minneapolis,[2] and at Thomaston, Conn.,[3] the sheet of ice was confined and the fluctuations in water level introduced toggle joint effects in the ice.

Reputable engineers, acting conservatively, have recommended and used the following ice pressures:[4]

	Pounds per lin. ft.
St. Maurice, Que	50,000
Wachusett, Mass	47,000
Olive Bridge, N. Y.	47,000
Kensico, N. Y.	47,000
Croton Falls, N. Y.	30,000
Cross River, N. Y.	24,000
Keokuk, Iowa, on piers only	20,000

Such pressures are, as a rule, applied at the water surface, but as the sheet of ice is assumed to be about 2 ft. thick, the actual point of application is about 1 ft. below the surface. If, therefore, a joint is considered located at a depth h below the water surface and an ice pressure I is assumed acting at a depth of 1 ft. below the surface, the moment will be

$$M = I(h - 1) \qquad \text{(ft.-lb.)}$$

The influence of the ice pressure on the profile of the dam decreases with the depth, necessitating a material increase of the upper portions only, if the force is to be sustained by gravity action. However, because of this decreasing influence, it is often advisable to reinforce the upstream face, so as to enable the section to withstand the ice pressure by cantilever action.

3f. Initial Stress.—As the construction of a large dam covers several seasons, the temperatures under which masonry is laid varies considerably. As yet no rules have been laid down for taking into account in the design the effect of such stresses.

3g. Temperature Stresses.—Seasonal changes in the temperature and, to a somewhat smaller extent, daily changes will cause the masonry to expand or contract. It is, therefore, impossible to avoid cracking, especially in the upper and thinner portion. In order to limit this cracking to a minimum and to locate such cracks where they cannot have any harmful influence, it is customary to provide cracks or expansion joints at certain intervals. In order to make them water-tight the masonry is often dovetailed together, or strips of copper flashing used.

[1] *Eng. News*, Apr. 23, 1908.
[2] *Eng. News*, May 11, 1899.
[3] Flinn: "Water Works Handbook," p. 119.
[4] Smith: "Construction of Dams," p 117.

Such expansion joints should not be located closer than 20 ft. and not more than 50 ft. apart. If an abrupt change in the foundation occurs, an expansion joint should be located there as cracking cannot be prevented at such points.

3h. Stresses in Masonry and on Foundation.—After the resultant of all the forces acting on a section of a dam located above a certain plane has been determined as to magnitude and location, it is necessary to calculate the unit stresses in the joint, so as to insure against overloading. If in Fig. 14, the resultant R (consisting of the components V and H) cuts the joint a distance l from its center, it is possible, without changing the conditions, to imagine two forces equal to V and opposite in direction placed in the center of the joint. One of these forces can then be combined with the component V to form a couple Vl, and the other force V gives an equal loading of the joint. The resultant unit pressures will then be a combination of the corresponding pressures due to this central load and the couple, which for the central load V is

$$P_1 = \frac{V}{a} \qquad \text{(lb.per sq. in.)}$$

where a is the area of the joint and equal to bd, or b, if the width d is 1 ft.

For the couple the maximum unit stresses are

$$P_2 = \pm \frac{Vl}{S} \qquad \text{(lb.per sq. in.)}$$

where S = section modulus for the area of the joint referred to an axis through its center, or $\frac{b^2}{6}$ if the joint is 1 ft. wide. Therefore,

$$P = P_1 \pm P_2 = \frac{V}{a} \pm \frac{Vl}{\frac{b^2}{6}} = \frac{V}{b} \pm \frac{6Vl}{b^2} = \frac{V}{b}\left(I \pm \frac{6l}{b}\right) \qquad \text{(lb.per sq. in.)}$$

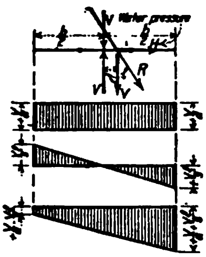


Fig. 14.

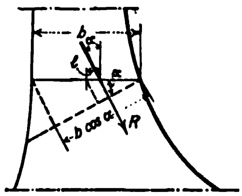


Fig. 15.

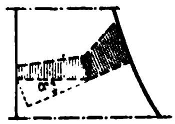


Fig. 16.

Of course, as a negative stress would mean tension and, as such stresses are not permitted, such outcome of the calculation proves that the length b of the joint is too short, or, in other words, the resultant falls outside the middle third.

Prof. Rankine argued that the pressures near the faces are tangential to them and that, therefore, in considering the pressures as above—*viz.*, normal to a given horizontal joint—the maximums are not obtained. M. Bouvier recommended that the pressure be calculated on a plane normal to the resultant (Fig. 15). If the resultant R cuts the horizontal joint at a distance l from its center and under an angle α with the vertical, the inclined joint will also make an angle α with the horizontal. Consequently, the values in the above equations will be R instead of V, $b \cos \alpha$ instead of b, and $l \cos \alpha$ instead of l, so that

$$P = \frac{R}{b \cos \alpha}\left(I \pm \frac{6\,l}{b}\right) \qquad \text{(lb. per sq. in.)}$$

In Europe it is customary to figure the stresses on joints normal to both faces and bent at an angle inside the dam (Fig. 16). The bend is located at a point where the unit stresses on the projected horizontal joints are equal.

Several dam profiles (Rankine's, etc.) show a small amount of tension in the downstream face when horizontal joints in the upper part are analyzed for the condition "reservoir empty" and using the formulas given above. So does the triangular profile, because the added portion at the top draws the pressure line upstream of the middle third. It is extremely doubtful that tensional stresses actually develop in the joint, so long as the resultant remains within the limits of the masonry. If the joints were open, tension would be out of the question and a corresponding increase in the maximum compressive stress would be the result. In view of the slight ability of concrete and especially cyclopean masonry to resist tensional stresses, it is advisable to consider the joints as being open. If then the resultant R (Fig. 16A) cuts a joint at a point located a distance c from the edge of the masonry, and $c < \frac{b}{3}$, the pressure diagram would be a triangle with the base $3c$. If, furthermore, the width d of the joint is 1 ft.

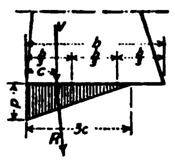


FIG. 16A.

$$V = \frac{P \times 3c}{2} \quad \text{or} \quad P = \frac{2V}{3c} \qquad \text{(lb. per sq. in.)}$$

where V is the vertical component of the resultant R.

The maximum permissible compressive unit stress should not exceed 300 lb. per sq. in., corresponding to 21.6 tons per sq. ft. If the stress is more, it will be necessary to increase the length of the joint—that is, increase the area and the section modulus, thus reducing the maximum stress.

3i. Shearing Stresses.—Each joint should also be investigated for shearing stress. As the horizontal component is P, the unit shear s in the horizontal joint would be

$$s = \frac{P}{a} \qquad \text{(lb. per sq. in.)}$$

where a is the area in square inches.

If a joint such as d (Fig. 17) is considered, it is obvious that the shearing force is composed of the hydrostatic pressure $P \cos \beta$ and a portion of the weight of the masonry $w' = W \sin \beta$, so that the total shearing force is

$$S = P \cos \beta + W \sin \beta$$

Several joints must be investigated by the trial and error method until that giving the maximum stress is found.

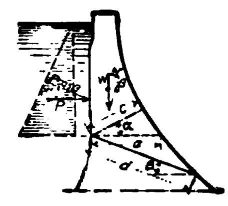


FIG. 17.

Two dams built by French engineers in Africa failed at points located one-third down from the top because of excessive diagonal shear. The permissible unit shearing stresses should not exceed 100 lb. per sq. in. and many engineers do not like to exceed 70 lb. Often the joint between rock and masonry is investigated for friction, neglecting the adhesion, but this is not necessary, as (1) such adhesion exists, and (2) the roughness of the rock surface gives a good mechanical bond with the concrete.

3j. Final Calculation.—After it has been decided what assumptions are to be made as to the forces acting on the dam, a tentative triangular section calculated, and the top width and the super elevation determined, it is necessary to investigate the design at a number of different elevations, so as to be certain that the unit stresses do not exceed the permissible adopted for the design, and that, on the other hand, the actual stresses are not so low as to make the design uneconomical. The aim is to keep the pressure lines inside the middle third in such a way that for the condition "reservoir empty" the line falls closely to the upstream limit of this middle third and for "reservoir full" as closely to the downstream limit as possible. The ideal dam would have these pressure lines coinciding with the middle third limits, but this cannot, for practical reasons, be accomplished at the top of the structure. Often the pressure line for "reservoir empty" will fall slightly outside and upstream of the middle third limit, thus causing a slight tension in the downstream face. In one case a dam was on purpose designed so as to lean upstream, or against the water pressure (Hauser Lake, Mont.).

As a rule, both sides of a dam are kept parallel until a point is reached where the resultant of the forces passes through the outer limit of the middle third (Fig. 18a) or

$$P : W = \frac{a}{6} : \frac{h}{3}$$

If now, as recommended by Wegmann, the water surface is assumed as extending to the top of the dam, $P = \frac{wd^2}{2}$ and $W = mad$, which inserted in the equation above gives

$$\frac{wd^2}{2mad} = \frac{a}{2d} \text{ or } d = a\sqrt{\frac{m}{w}} \qquad \text{(ft.)}$$

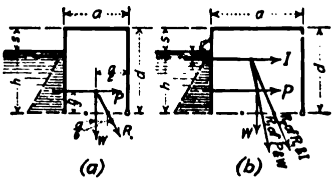


Fig. 18.

If an ice pressure I is to be taken into consideration, it is impossible for the water surface to reach the top of the dam. In this case moment equations are the more convenient to use, which, established in respect to the downstream edge (Fig. 18b) with a factor of safety of two, would read

$$2\left[I(h - 1) + \frac{wh^2}{2} \times \frac{h}{3}\right] = mad \times \frac{a}{2} = \frac{mda^2}{2}$$

In this equation it is best to consider the top width a as the variable, unless the dam is reinforced when the usual reinforced-concrete formula for cantilevers can be added to the right side of the equation.

If uplift of $\frac{2}{3}h$ diminishing to zero is assumed in the joint, the corresponding moment must be added to the left side of the equation (see Art. 3c).

Elevation	Total weight of masonry (lb)	Reservoir empty: Gravity line, From back (feet)	Reservoir empty: Gravity line, From toe (feet)	Reservoir empty: Moment about toe (ft-lb)	Reservoir empty: Intensity of pressure, At back	Reservoir empty: Intensity of pressure, At toe	Reservoir full: Water pressure (lb)	Reservoir full: Lever arm (feet)	Reservoir full: Moment about toe (ft-lb)	Reservoir full: Factor of safety	Reservoir full: Gravity line, From back (feet)	Reservoir full: Gravity line, From toe (feet)	Reservoir full: Intensity of pressure, At back	Reservoir full: Intensity of pressure, At toe	Reservoir full: Shear (lb per sq in.)
5425	28,000	5.00	5.00	140,000	+19.50	+19.50	7,032	5.0	35,160	3.98	6.25	3.75	+4.90	+34.10	4.88
5405	74,600	7.33	16.00	1,192,200	+47.20	-2.60	38,280	11.67	446,000	2.67	13.33	10.00	+12.80	+31.80	11.40
5385	158,520	11.47	25.20	3,990,000	+63.70	-3.70	94,530	18.33	1,732,730	2.30	22.39	14.27	+10.10	+49.90	17.90
5365	280,000	16.00	34.00	9,520,000	+80.80	-3.10	175,780	25.00	4,395,000	2.16	31.70	18.30	+7.60	+70.10	24.40
5345	438,660	20.48	42.85	18,785,200	+99.10	-2.90	282,030	31.67	8,931,000	2.10	40.83	22.50	+6.20	+90.00	31.00
5325	634,690	25.00	51.67	32,784,000	+117.50	-2.50	413,280	38.33	15,841,000	2.07	50.00	26.67	+5.20	+109.80	37.40
5305	868,000	29.50	60.50	52,500,000	+136.20	-2.20	569,530	45.00	25,628,850	2.05	59.25	30.75	+3.40	+130.60	44.00
5285	1,138,630	34.00	69.33	78,934,800	+154.80	-2.00	750,780	51.67	38,792,800	2.03	68.20	35.13	+3.10	+149.70	50.50
5265	1,446,700	38.50	78.17	113,094,800	+173.90	-1.90	957,030	58.33	55,823,600	2.03	77.10	39.57	+2.90	+169.10	57.00

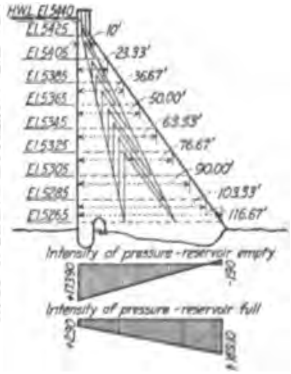


Fig. 19.

In this way each joint is analyzed keeping in mind that all the forces acting on the dam at or above the joint in question must be included.

A drawing should be prepared showing the profile of the dam, the limits of the middle thirds and the pressure lines of "reservoir empty" and "reservoir full." A table should also be provided giving in detail the various weights, moments, stresses, factors of safety, etc., as shown in Fig. 19.

Every construction joint in the dam offers chances for defective work. Laitance, dirt, etc., tend to establish open joints through which water might percolate. Such joints, of course.

do not offer the proper resistance to shearing, and joints should, therefore, never be made horizontal. They should be stepped and located at different elevations in adjacent sections. Rocks should be left projecting halfway up to enable the next course of masonry to get a good hold. Before starting this next course the surface should always be scrubbed thoroughly with steel brushes applying water and neat cement.

In Europe it is customary to place the courses in such a way that the top surfaces of them are always normal to the faces of the dam and, in many cases, the courses are broken and the outer part placed normal to the pressure line "reservoir full" (Fig. 20).

In many cases dams designed as gravity-section structures have been arched in plan (Roosevelt, Arrowrock, Tallulah Falls). The reason for this is that in such dams temperature stresses are counteracted by arch action in the structure. However, at Tallulah Falls when the dam contracts it pulls away from the rock abutments, proving the necessity of providing sections at the abutments where concrete should be placed in cold weather so as to insure a tight joint under maximum contraction. This course was followed by the U. S. Reclamation Service when building the Arrowrock Dam.

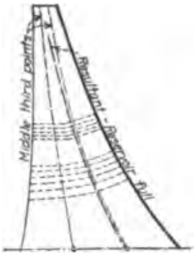


FIG. 20.

4. Design of Arched Dams.—If the dam site is narrow and good rock abutments are available, it is likely that an arched dam will prove economical. Since, however, an arch is longer than a straight line, the economy over a gravity-section dam depends upon whether the arch, with its smaller cross-section, greater length, more expensive formwork, greater accuracy in instrument work, sometimes richer mix of concrete and possible reinforcement requirements, will prove the less expensive.

Arched dams can be divided into three classes:

Buttressed Arches.—These are generally built of reinforced concrete and termed multiple-arch dams (see under "Reinforced-concrete Dams", Art. 5*f*).

Constant-radius dams, which have a constant length of radius of either the upstream side, the center, or the downstream side.

Constant-angle dams, which have a constant subtended angle, but a variable radius being a function of the cord lengths at the different elevations.

4a. Constant-radius Dams.—Dams of this type are also called "true arches" as they are built mainly on the principle that the pressure line of a uniform load acting on the periphery of a circle has a circular line polygon. Consequently, as soon as the upstream radius R_u has been determined upon, the stresses in the arch rings, assumed to be 1 ft. high, can be calculated by the formula

$$S = PR_u \qquad \text{(lb.)}$$

where P = water pressure in pounds per square foot measured to the center of the height of the arch ring or wH, where w is the weight of water per cubic foot and H the depth in feet of the center of the arch ring below the water surface.

As the pressure line coincides with the center of the circular arch, a correction in the value of S in the above formula is required.

$$S_1 : S = R_u : \frac{R_u + R_d}{2} \text{ or } S_1 = S\frac{2R_u}{R_u + R_d} \qquad \text{(lb.)}$$

where R_d is the radius of the downstream side or $R_d = R_u - T$, where T is the thickness o the arch ring in feet.

If the arch ring is h in. high and t in. wide, its area is ht sq. in. The permissible compressive unit stress is q lb. per sq. in., so that the total resistance is

$$Q = qht \qquad \text{(lb.)}$$

As now $$Q \geq S_1, \text{ obviously } S_1 \leq qht$$

or the required thickness t, if all other qualifications are given,

$$t = \frac{S_1}{qh}$$

A correction must also be made for the vertical pressures due to the weight of the dam, which will increase the stress S_1 to $S_1 + \Delta S$.

For a dam with a vertical downstream face and a battered upstream side (Fig. 21a)

$$\Delta S = \frac{\rho - 1}{3\mu}\left(1 + \frac{R_u}{R_u + R_d}\right)P$$

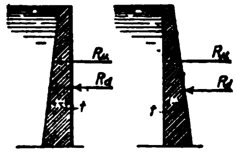


FIG. 21.

and for a dam with a battered downstream face and a vertical upstream side (Fig. 21b)

$$\Delta S = -\frac{\rho}{3\mu}\left(1 + \frac{R_u}{R_u + R_d}\right)P$$

where ρ is the specific weight of masonry or $\frac{\text{weight}}{62.5}$, and μ, the reciprocal of Poisson's ratio. For concrete $\frac{1}{\mu}$ is from 0.16 to 0.22 or as an average $\mu = 5$.[1]

An arched dam is, as a rule, reinforced in both faces, in the upstream face so as to take up tensile stresses due to cantilever action and in the downstream face so as to insure against cracks due to tension caused by vertical beam action. The action of the dam, because of its resisting moment when considered as a vertical beam 1 ft. wide, and because of the support at the bottom, which is fixed, is to reduce the loading of the intermediate arch rings and to increase it correspondingly in the upper rings.

By applying these formulas the thickness of the arch can be determined at as many elevations as desired and the cross-section obtained. However, the formula is correct only for an arch of circular form with fixed ends, while an arched dam is also fixed along its bottom. Often the lower part is so short and the cross-section so thick that horizontal beam action will result. It is, therefore, necessary to investigate the tentative section for a variety of different conditions: First, as a gravity section, until tension develops; second, as a cantilever section, until a deformation occurs sufficient to develop arching stresses; third, as a beam tending to equalize such arching stresses, decreasing them at the bottom and increasing them at the top; and fourth as an arch.[2]

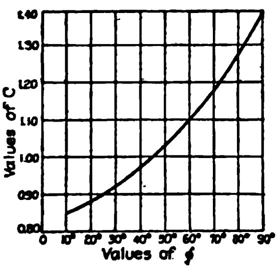


FIG. 22.

Approximate methods for finding what proportion of the load is taken by the arch and by the cantilever respectively have been discussed by Harrison and Woodard.[3] Shirreffs in his discussion of that paper develops the following formula for the crown deflection of an arch dam:

$$D_c = \frac{C + P_1 R_u^2}{Et}$$

where $$P_1 = P \times \frac{R_u}{R_m}$$

and, $$C = \frac{\frac{2 \sin \varphi}{\varphi}(1 - \cos \varphi) + \frac{1}{2}(\cos 2\varphi - 1)}{\frac{3\varphi}{\sin \varphi} + \cos \varphi - 4} + (1 - \cos \varphi)$$

[1] BELLET: "Barrages en Marconnerie."

[2] "For a full discussion of these stresses see MORRISON and BRODIE: "Masonry Dam Design."

[3] Lake Cheesman Dam and Reservoir," *Trans.* Am. Soc. C. E., vol. 53, p. 89.

and is a factor which takes the curved beam action into consideration and can be found directly from Fig. 22. E is the modulus of elasticity and t, the thickness of the arch ring.

This formula and curve, however, do not take the initial stresses in the dam structure into consideration, and, therefore, before applying it for finding arch deflections it is necessary to determine how much of this load is carried by the arch due to initial stresses, because only after deducting this part of the load does the remainder divide up between arch and cantilever action.

By initial stresses are meant stresses principally due to the weight of the structure and to the water pressure. Therefore, these stresses reach their maximum values at or near the foundation, and are zero at the crest.

A number of arched dams of very slender dimensions have been built in Australia by the English engineer Wade. The following table has been taken from Wegmann, "The Design and Construction of Dams."

Locality	Max. height, feet	Top width, feet	Base width, feet	Radius of curve, feet	Max. pressure, tons per square foot	Top above water surface, feet
Parramatta	52.0	4.8	15.0	160	15	2.0
Lithgow No. 1	35.0	3.5	10.9	100	10	3.5
Parkes	33.5	3.0	13.5	300	24	5.0
Cootamundra	46.0	3.0	13.0	250	25	1.0
Picton	28.0	7.0	13.6	120	12	10.0
Tamworth	61.0	3.0	21.5	250	20	2.0
Wellington	48.0	3.0	10.0	150	20	2.0
Mudgee	50.0	3.0	18.0	253	20	1.0
Wollongong	42.0	3.5	11.6	200	20	1.0
Katcomba	25.0	3.0	20.3	220	15	1.0
Lithgow No. 2	87.0	3.0	24.0	100	10	3.0
Medlow	65.0	3.5	9.0	60	12	3.0
Queen Charlotte Vale	32.0	3.0	8.6	90	10	2.0

Most of these dams are reinforced to take care of cantilever and temperature stresses.

The largest arched dams in the United States are located at Pathfinder and Shoshone, both in Wyoming. They are of almost identical design, with a radius of 150 ft. figured to the center of the top width. The upstream slope is 0.15 : 1 and the downstream 0.25 : 1 in both cases. The former has a top width of 14 ft., a bottom width of 94 ft. and a total height of 210 ft.; the latter a top width of 10 ft., a bottom width of 108 ft. and a total height of 328.4 ft. The lower 85 ft. do not taper but are of a uniform thickness of 108 ft. Both dams are built by the U. S. Reclamation Service.

4b. Constant-angle Dams.—If a circular arch of given chord length is investigated for economy, it will be found that the most economical shape has a subtended angle $2\varphi = 133.5$ deg. It can also be shown that for variations between 120 and 150 deg. the increase in area is negligible.[1] As for any elevation the chord C is known, the corresponding mean radius is (Fig. 23).

$$R_m = \frac{C}{2 \sin \varphi}$$

The volume in a given section is equal to the area multiplied by the length of the mean arc,

$$V = \text{area} \times R_m \times 2\varphi$$

where φ is given in terms of π.

[1] LARS JORGENSEN: "The Constant Angle Arch Dam," *Trans.* Am. Soc. C. E., vol. 78, p. 685, 1914.

Poisson's ratio for lateral strains is taken into consideration in determining the relative arch action in a dam of this type, a value of one-fifth being adopted, and the initial stresses induced axially by the weight of the dam on the foundation, together with the water load, being utilized to help support the latter. (See above under "Constant-radius Dams.") However, dams designated in accordance with this method are liable to have an overhang, necessitating a thickening of its lower part. This is, as a rule, accomplished by increasing the downstream radius R_d for these lower arch rings, while keeping the former radius R_u. The result is an arch thicker at the crown than at the haunches.

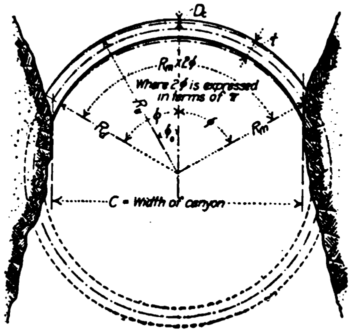


FIG. 23.

When the tentative cross-section has been determined it must be analyzed in the same way as that of a constant-radius dam.

Several dams of this type have been built in the West and in Alaska. One constructed for the Alaska-Gastineau Mining Co. near Juneau, Alaska, is 168 ft. high. Another dam, which ultimately will be 305 ft. high, has been built for the Pacific Gas & Electric Co. at Lake Spaulding, Cal. The maximum stress in this dam is 23.8 tons per sq. ft.

5. Design of Reinforced-concrete Dams.—Reinforced-concrete dams possess many theoretical advantages and they are by far the most satisfactory dam, so far as the type goes. The few failures on record of such dams have never been due to fault of the dam proper; they have always been traced to faulty foundation. Overconfidence in this type has led engineers to neglect the facts that a secure foundation, sufficiently deep cut-off walls, adequate resistance against sliding in soft ground, etc., are just as important details as the design and construction of the superstructure itself.

Because of the comparatively light weight of the structure, it is customary to support the apron or water-tight membrane on a series of triangular buttresses in such a way that its inclina-

FIG. 24.

FIG. 25.

tion is about 45 deg. (Figs. 24 and 25). Obviously, the horizontal component of the water pressure thus equals the vertical and the resultant has an inclination of approximately 45 deg. From Fig. 26 it is evident that if the buttresses are of the shape indicated, the resultant of the water pressure cuts the base in the outer edge of the middle third. The weight of the structure draws the resultant somewhat inside this point. The corresponding unit pressures on the foundation are thus greater at the downstream edge than at the upstream.

If soft foundation prevails, it is desirable that it be loaded as uniformly as possible. The buttresses are then increased in length so that the resultant will cut it near the center of the base. If the upstream slope of a bulkhead dam is 1 : 1 and the downstream batter of the buttresses 4 : 1, or, in other words, the length of the base is $\frac{5}{4}H$, where H is the height of the dam, experience has shown that the resultant of water and concrete cuts the base almost exactly in the center. n addition, the buttresses are placed on spread footings on a continuous foundation mattress, so that the load will be distributed evenly over the whole base.

A dam of this type can be bulkhead (Fig. 24) or spillway section (Fig. 25). It can be placed on any kind of foundation, and in 1909 a dam 22 ft. high was built on baled hay at Anadarko, Okla. The top strata consisted of a peculiar waxy clay or marl under which a pocket of floating quicksand was found. To control the quicksand baled hay was laid down on the surface of the sand pocket and on this was distributed a number of perforated pipes, which were again covered with more hay. The pipes led to a sump outside the dam, from which, during construction, the water was constantly pumped. On the sand and hay foundation a continuous foundation mattress was built and the dam and power house placed upon it. Although several severe floods have passed over this structure, so far no defects have developed.

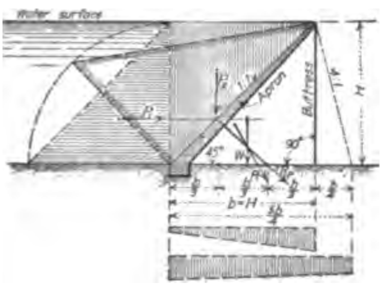


FIG. 26.

In 1908 a dam was constructed near Douglas, Wyo., 135 ft. high above the water line, of which 80 ft. in the center are founded on clay and hard-pan. The unit load under the mattress is 5.2 tons per sq. ft.

5a. Cut-off Walls.—When built on soft foundations, cutoff walls of reinforced concrete (sometimes extended by sheet pilings of steel, wood, or reinforced concrete) are built at the upstream heel of the dam. The required depth of such cutoffs can be calculated by means of the usual earth and hydrostatic pressure theories after the necessary data have been obtained through test pits or borings. However, as a rule, if such cutoffs extend below the ground level or bottom of the superstructure to a depth one-half the height of the superstructure, they are safe.

As such cutoffs must be strong enough to resist the unequal pressures acting on them, they must be reinforced and the superstructures designed for taking up a considerable reaction from them.

5b. Foundation Mattress.—In soft ground the buttresses are placed on a continuous foundation mattress, which must be reinforced in order to distribute the load from the buttresses uniformly over the foundation. Weep holes are often provided so as to prevent uplift from leakage, which possibly might find its way around the cutoff.

When the buttresses are placed on this mattress on widely spread footings, a certain amount of inverted arch action will take place, thus reducing the reinforcement materially. The concrete mixture in the mattresses is as a rule 1 : 3 : 5. The pressures on the foundation are calculated by the formulas used for figuring the stresses in a gravity-section dam (see Art. 3).

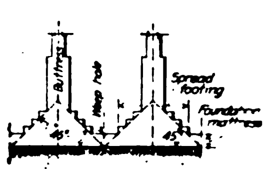


FIG. 27.

5c. Buttresses.—On top of the mattress or placed directly on the rock, if such can be reached, are the buttresses. If built on a mattress or soft rock, such buttresses are given a wide base by stepping them. Such widening should, if not reinforced, be made in such a way that lines drawn at 45 deg. and intersecting between the buttresses at the underside of the mattress will lie entirely within the steps (Fig. 27). If this is not done, the steps must be reinforced.

Because of the reinforcement, the buttresses in some dams have been designed to take tensile stresses in their upper parts (acting as cantilevers). In such cases the usual reinforced-concrete beam formulas are used. Further down, where the buttresses are longer, no tensile stresses are permitted and at the base the resultant must be well within the middle third.

The most dangerous stresses in a buttress are due to shear. As the apron or water-tight membrane placed on the upstream side carries the load to the buttresses, it is obvious that each buttress must withstand the load on one entire span. As the spacing, as a rule, is about 18 ft. these forces are considerable. So far no satisfactory method has been developed whereby the exact lines of shear can be determined, and the designer, therefore, must be satisfied by figuring the shearing stresses on horizontal joints, checking himself roughly to be certain that the areas parallel to the water pressure are large enough.

As the shear increases with the depth, the buttresses become thicker. In many cases the top portions have been made 12 in. thick, but this is somewhat too small and 16 in. should be adopted as the minimum.

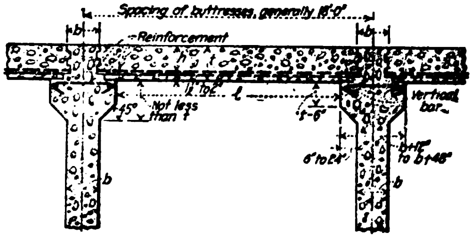


FIG. 28.

In high dams the bottom thickness has been as great as 72 in., but this, of course, is a function of the length of the buttress.

In no case should the permissible unit shear exceed 100 lb. per sq. in. and even then, some reinforcement should be used. In walls not reinforced the stress should be limited to 70 lb. per sq. in.

Sometimes when the buttresses are very long, as for instance in high overflow dams, their weight will draw the resultant downstream of the center. There is no objection to this, provided the foundation is good and able to resist the unbalanced pressure. However, if the foundation is soft and an equal loading is desired under the entire base, the portions of the buttresses next to the apron are thickened sufficiently to place the resultant in the center.

In order to form a seat for the slabs of the apron, the buttresses at their upstream edge are shaped as shown in Figs 28 and 29. The width of this seat varies from 6 to 24 in., depending upon the thickness t of the slab. At the buttress the thickness of the haunch must at least be t in order to develop the same resistance to shearing as the slab itself.

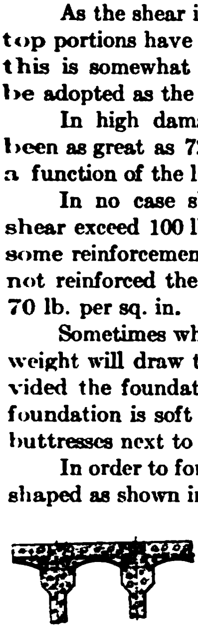


FIG. 29

The concrete mix in buttresses is generally 1 : 3 : 5, but sometimes a richer mix is used.

5d. Bracing.—As buttresses are very high as compared with their width, it is often necessary to brace one against the other. This is done by means of rectangular beams as shown in Fig. 24.

5e. Apron.—The apron consists of slabs (Ambursen type) or arches (multiple-arch type). Its sole function is to act as a water-tight membrane and to transfer the hydrostatic pressure to the buttresses. Slabs are designed as uniformly loaded and the usual formulas apply. As a rule the moment is

$$M = \frac{Wl}{8} \qquad \text{(ft.-lb.)}$$

where W is the load in pounds on a strip 1 ft. wide. As the unit load is wh lb. per sq. ft. and the length of the loaded strip l ft., $W = whl$ and

$$M = \frac{whl^2}{8} = 7.8125\ hl^2 \qquad \text{(ft.-lb.)}$$

or

$$M = 93.75\ hl^2 \qquad \text{(in.-lb.)}$$

The reason for using this formula is that the buttresses are built first and the slabs afterward and in such a way that no continuity of action can be depended upon (Fig. 28).

As these slabs are submerged, and as it is difficult to make them absolutely water-tight, some engineers believe there is danger that in time the reinforcement will corrode. In order to

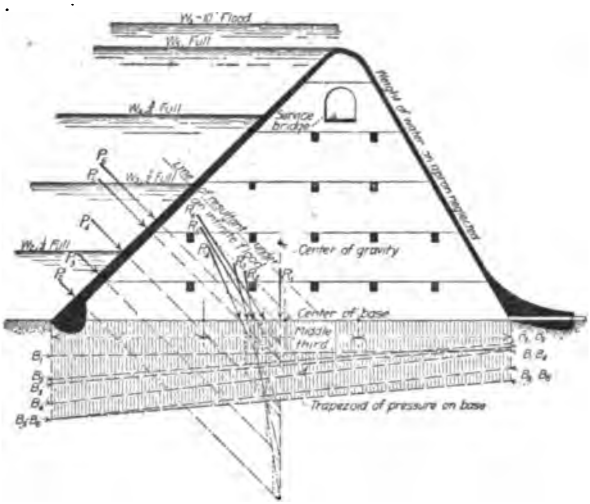


FIG. 30.—Design of Ambursen dam.

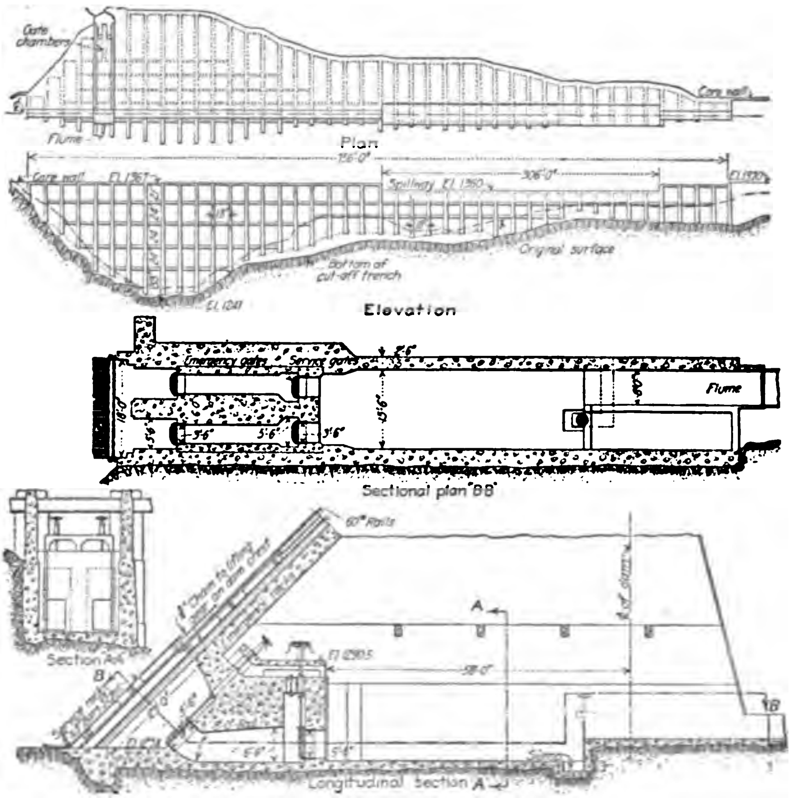


FIG. 31.—Arrangement of waste gates, emergency racks, etc.

overcome this objection, slabs have sometimes been built as semi-arches, as shown in Fig. 29, but the haunches must be designed for arch action and not like those shown in the figure. As a rule, the concrete mix for these slabs or arches is 1 : 2 : 4.

After the proper slab thickness, the buttresses, etc., have been calculated, one entire panel of the dam is analyzed for safety of design and the corresponding unit stresses determined (Fig. 30).

Some practical details of design are shown in Fig. 31. It is also customary to provide a service bridge inside the dam, as shown in Fig. 30, so that the underside of the slabs and their joints with the buttresses can be inspected without difficulty as often as desired.

The advantages of the hollow reinforced dam have led a number of designers to develop various types. Many of them have never been built like those developed by Ransome (Fig. 32) and Morton (Fig. 33). The Ransome dam consists of buttresses placed at a certain angle so that they intersect. The slabs are thus of a variable span. This is a very strong structure, but the cost of forms excessive. In the Morton dam the apron is supported on inclined columns braced by intermediate beams. The disadvantages are that these columns are liable to bend, that the load is concentrated on the column foundations, that the form cost is high and that the placing of reinforcement and concrete difficult.

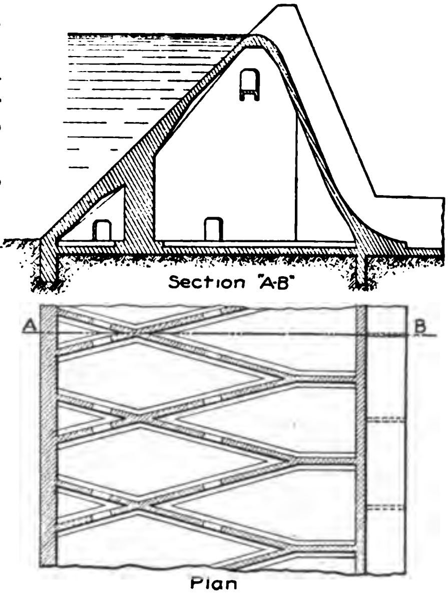


Fig. 32.—Ransome dam.

The Edge dam, used in a somewhat modified form in the reconstruction of the Austin Dam in Texas, offers several structural advantages (Fig. 34). The apron is divided into square panels reinforced both ways, which gives it great structural strength and the buttresses are thoroughly braced by intermediate walls supporting the deck. However, it is not always possible to take advantage of the structurally slender thickness of the apron, as a certain thickness against leakage is required. Furthermore, the form cost is excessive and the saving in concrete slight.

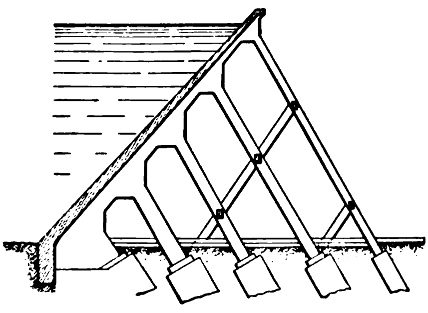


Fig. 33.—Morton dam.

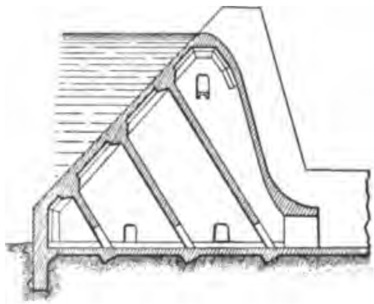


Fig. 34.—Edge dam.

5f. Multiple-arch Dams.—To overcome what he considered objectional features of the Ambursen dam—viz., the reinforced-concrete slabs and the many buttresses—Eastwood developed a type consisting of a series of circular arches of long spans. The first dam built

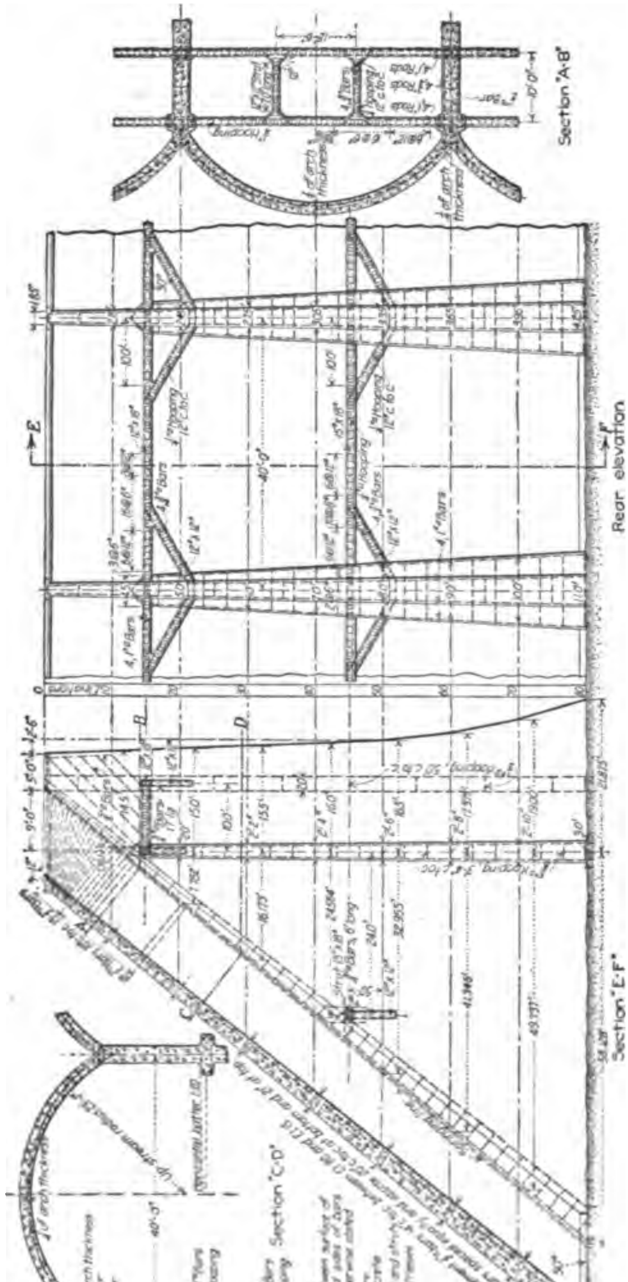


FIG. 35.—Multiple arch dam.

of this type was for the Hume-Bennett Lumber Co. in California in 1908. It consists of 12 circular arches of 50-ft. span supported on 13 buttresses.[1]

A later dam of this type was condemned by the California Railroad Commission because of insufficient bracing of the buttresses. The advantage of the slab type is, that should one series of slabs collapse, the adjacent buttresses will prevent further damage to the structure, while if a series of arches collapses the whole structure will fall because of the absence of a counterforce at the buttress.

In some recent dams designed by Jorgensen[2] the buttresses have been braced so that they can, by cantilever action, sustain the arch reaction should the arches in the adjacent panel be missing (Fig. 35). Quoting Jorgensen, the most economical spacing of the buttresses lies between 30 and 50 ft. For low dams the lower limit would be the best and for high dams the upper.

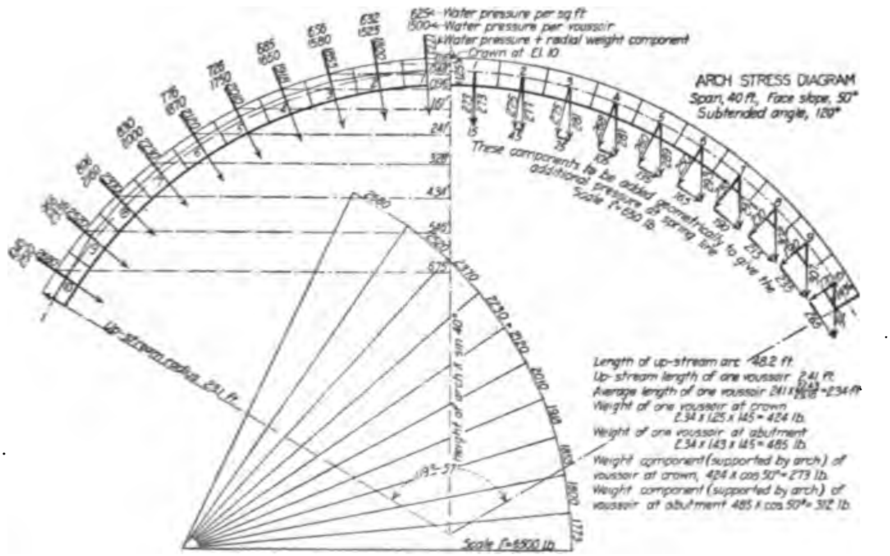


FIG. 36.

It should be noted that for arches with a large subtended angle and placed in an inclined position, the crown lies at a considerably higher elevation than the haunches and that the load varies due to the variation in hydrostatic pressure.

For designing the thickness of the arches, the methods given under "Arched Dams" (Art. 4) can be used. However, if the subtended angle is large and the inclination considerable, it is more convenient to use graphic statics, drawing one pressure line for the weight and another for the hydrostatic pressure and then combining them (Fig. 36).

6. Earthen Dams With Concrete Core Wall.—The design of earthen dams is purely a matter of experience. The ruling factor is preventing water from traversing the dam. This is effected by providing an impermeable core of clay, puddle, concrete, masonry, steel plates protected with concrete or asphalt coatings, and, especially in smaller dams, sheet pilings of lumber. It is absolutely necessary if an earthen dam is decided upon, that the necessary materials are available at or near the site, as the quantities are so voluminous that it is impossible to haul them any great distance.

[1] WEGMANN: "The Design and Construction of Dams," 1911 Ed., p. 436.
[2] LARS JORGENSEN: "Multiple-arch Dams on Rush Creek, California." *Trans.* Am. Soc. C. E., March, 1917.

An earthen dam may consist of:

1. A homogeneous bank of earth.
2. A bank of earth having a puddle core.
3. A bank of earth having a masonry core wall.
4. A bank of earth having a puddle placed on the water slope.

The first method can be used only when the required quantity of earth or gravel, containing enough clay to make it water-tight, can be obtained at a reasonable cost.

In cases where this would prove too expensive, a central core is provided of clayey earth or gravel, ordinary earth being used in the other portions of the dam. In order to prevent leakage under the dam this core should be extended down in a trench to an impervious stratum, or at least far enough down to make the path of such leakage as long as possible.

In the third plan, masonry is substituted for the puddle core. This plan is adopted when no clayey earth can be obtained at a reasonable cost. As a rule it is more expensive than plans 1 and 2, but has great advantages as regards safety.

As regards plan 4, it is open to two objections: The puddle is injured if the slope settles, which nearly always happens, and cracks will occur in the water line due to alternate wetting and drying.

In some cases concrete paving, plain or reinforced, has been used on the upstream side of an earthern dam for making the dam water-tight However, such paving is apt to slide and should be heavily anchored down. Even then, settlements in the slope will cause cracking, and leakage is bound to take place.

The theory of earth-dam design is very simple: The upstream portion and the core wall serve to reduce the leakage to such an extent that the hydraulic gradient falls inside the dam body, and the downstream portion, through its weight, will hold the saturated mass in position and prevent it from sliding in a downstream direction. It has often been noted that earthen dams have a tendency to fail through the caving of the downstream slope, which proves that the hydraulic gradient did not fall inside the dam but intersected the slope some distance above the foundation.

In order to broaden the base and to bring the downstream slope out as far as possible on high dams, it is customary to step the slope (so-called berms) at elevations 20 to 30 ft. apart, which obviously means a saving in materials. Such berms are usually paved and provided with gutters on the inside.

In order to confine the hydraulic gradient to the embankment, the Pierson Engineering Corporation, when building a series of earthen dams in Spain,[1] placed drainage dykes of broken rock near the downstream side of each dam at the toe of a somewhat pervious section, and provided outlets for any leakage collected by these dykes.

When sheet pilings are used below a core wall and the foundation is very soft, it is often necessary to increase the downstream portion of the dam, in order to create a surcharge sufficient to counterbalance any unbalanced forces on the sheet piling due to hydrostatic pressure.

Experience has shown what general dimensions are most suitable for earthen dams. They are, of course, to a certain extent a function of the materials used and the height of the structure, additional strength being given to high dams:[2]

Top width	10 to 30 ft.
Superelevation above high water	5 to 25 ft.
Upstream slope	1:2: to 1:3
Downstream slope	1:1½ to 1:2½

The upper portion of the upstream slope should be protected against erosion by waves and burrowing animals by a riprap, 15 to 20 in. thick, or concrete paving, 5 to 8 in. thick, ex-

[1] *Eng. Rec.*, Aug. 29, 1914, p. 250.

[2] WEGMANN: "The Design and Construction of Dams."

tending some distance below and 5 to 15 ft. above the water line. The top of the dam, the downstream slope and the upstream slope above the paving are generally covered with good soil and sodded.

A fairly satisfactory formula for the top thickness is given by Lyndon:[1]

$$T = 5 + 0.2\,H \qquad \text{(ft.)}$$

where H = height of dam in feet.

As the earth fill is bound to settle it is customary to increase the profile from one-fifteenth to one-twentieth of its height (Fig. 36*A*). This shrinkage is, of course, a function of the quality of the materials and the degree to which they have been rammed. As the height varies from zero to a maximum, this method will make the crest of the dam arched in the longitudinal section.

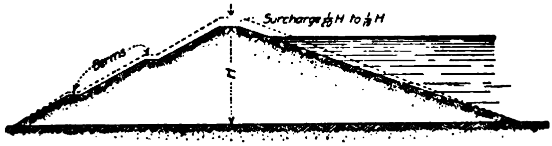


Fig. 36*A*.

Core walls are never designed to resist the whole water pressure as they serve merely as a water-tight membrane and are backed by the downstream portion of the dam. They should extend from 1 to 2 ft. above the highest water level in the reservoir and have a top width of from 2.5 to 6 ft. The sides are battered from 16:1 to 20:1, so that the thickness at the natural ground surface is from one-sixth to one-seventh of the height. Instead of battering the faces, the increase in thickness can be obtained by offsets.

Parker[2] gives a method, whereby the stability of core walls can be determined: Draw in Fig. 37 on the water side a line *ab* making with the horizontal an angle ϕ equal to the angle of

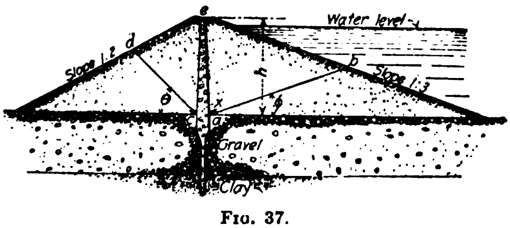


Fig. 37.

repose of the saturated earth (usually ϕ = 20 to 23 deg.). Then draw on the downstream side a line *cd*, making with the horizontal an angle Θ equal to the angle of repose of the rammed earth (generally Θ = 45 to 55 deg.).

Calculate the areas located above these lines. For ϕ = 20 deg. and a 1:3 slope of the upstream side, the area *abc* becomes very nearly $\frac{3}{4}h^2$; and with Θ = 45 deg. and a 1:2 slope of the downstream side the area *cde* is $\frac{1}{3}h^2$. The weight of rammed earth can be taken at 132 lb. per cu. ft. and that of saturated at 160. The thrust on the core wall would thus be

$$\tfrac{3}{4}h^2\,(160) - \tfrac{1}{3}h^2\,(132) = 76h^2 \qquad \text{(lb. per lin. ft.)}$$

If the thickness of the core wall is x ft., its ultimate resistance to shear, when of concrete, is about 30,000 x lb. per lin. ft. If a factor of safety f (usually f = 2) is used,

$$30{,}000\,x = 76h^2f \text{ or } x = \text{about } \frac{h^2f}{400} \qquad \text{(ft.)}$$

[1] Lamar Lyndon: "Hydro-electric Power," vol. I, p. 279.
[2] Philip A. Morley Parker: "The Control of Water," p. 318.

Unless h is great this formula will give results somewhat smaller than experience has shown as requisite to stop percolation through an ordinary mix of concrete. Especially for reinforced concrete, it is necessary to use a very dense mix or to coat the upstream surface with an impervious material.

Herschel[1] gives the following practical rules for first-class work: 4 to 5 ft. thick at bottom of trench enlarged to 8 ft. at natural surface and tapering to 4 ft. at top of core wall. However, for smaller dams these dimensions are entirely too clumsy. Herschel states also that a wall 2 ft. thick throughout is sufficient to stop percolation.

When building earthen dams the materials should be deposited in layers from 8 to 12 in. thick and each course should be well packed. This is best accomplished by steam rollers, of which the first one has grooved rollers and the other smooth ones. Especially in dams without core walls, every layer should be somewhat inclined toward the water side of the dam and arched upward, so that, later on, when the materials settle, no pockets are formed. Such pockets, if impervious, will retain water and the hydrostatic pressure will be carried out to the downstream slope, which then will cave. It has happened that dams, saturated in this way, after having been in successful operation for several years have failed in an upstream direction, because of the outer hydrostatic pressure being removed through a lowering of the water surface.

Hollow, cellular core walls of reinforced concrete extending to the top of the dam have been proposed because they offer a convenient way of providing a continuous overflow along the crest of the dam, thus preventing overtopping.

7. Passing the Discharge.—Openings must be provided in, at, or near a dam at such an elevation, that when the water in the reservoir rises above a certain level, it will escape through one or more of these openings, thus preventing the bulkhead portion from being overtopped. Such relief must be designed to discharge the maximum flood observed or anticipated at the dam site. If the reservoir has a large surface, the rise in water level to an elevation required to give the necessary head on the spillway is sometimes considerable, and, if in addition the floods are of short duration, a reduction can be made in the spillway requirements.

7a. Form of Spillway.—There are several ways of passing floods at dams. Pipes laid at or near the base of the structure, thus acting under a great hydraulic head, are sometimes used, but, as a rule, in conjunction with other types of spillways. Overflow spillways are of many designs. Sometimes they are located apart from the main dam and the water discharged into a gulley, canal, or channel conveying it back into the river a safe distance below the dam; in other cases they are located in the dam itself, part of which has been lowered for the purpose.

When spillways are located in places where attendance is possible, such as for power plants with the station at the dam, they are often outfitted with some type of movable dam, generally gates, as it provides means for keeping the water level at a higher elevation, thus increasing not only the storage but also the head. Another advantage of this system is that the spillway can be shortened materially as the discharge openings can be made very deep, without necessitating a sacrifice in the storage capacity. Consequently, such dams, as built at McCalls Ferry and for the Ozark Power Company on the White River in Missouri, are not suitable. They are designed to resist and to pass the maximum floods, 16 and 12.5 ft., respectively, and, because of the lack of movable dams or gates, the water level, after the flood has passed, soon falls to the crest of the dam. Flashboards about 5 ft. high are now used at both places, but such provisions are more or less to be considered as makeshifts.

When spillways are located in places where they cannot be under constant surveillance, they must be given ample length, so that when the water level rises a comparatively large discharge can be accommodated, thus preventing a rapid rise which might endanger the main structure. In order to obtain a great length of spillway in as short a distance as possible, they are sometimes arched or zig-zagged in plan. However, because of the interference at the corners of such broken lines the effective length is shorter than the actual total length. Sometimes,

[1] *Proc.* I. C. E., vol. 132, p. 255.

for high heads the water flows over the whole structure in such a way that the effective length is the straight line between the end points.

7b. Discharge Capacity.—The discharging capacity of a spillway depends upon its shape. The general formula used reads:

$$Q = \tfrac{2}{3}\,\mu l h\sqrt{2gh} \text{ or } Q = \tfrac{2}{3}\,\mu l\sqrt{2g}\,(h)^{3/2} \qquad \text{(sec.-ft.)}$$

where Q = discharge in second-feet; l, length of the spillway; h, head acting on it; g = 32.16; and μ an empirical coefficient depending upon the shape of the spillway, and given in Fig. 38.

If the velocity of approach v_0 is considerable, its influence on the discharge must be taken into account. This is done by transforming it into the corresponding velocity head $k = \frac{v_0^2}{2g}$ and introducing it in the equation, which thus becomes:

$$Q = \tfrac{2}{3}\,\mu l\sqrt{2g}\,[(h + k)^{3/2} - k^{3/2}] \qquad \text{(sec.-ft.)}$$

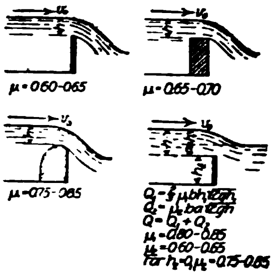


Fig. 38.

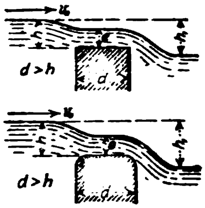


Fig. 39.

Should the crest of the dam be very wide $d > h$ (Fig. 39) with sharp corners, the discharge is:

$$Q = 0.35l\sqrt{2g}\,(h + k)^{3/2} \quad \text{or } \mu = 0.525$$

and

$$e = \tfrac{2}{3}\,(h + k)$$

and for rounded corners

$$Q = 0.40l\sqrt{2g}(h + k)^{3/2} \quad \text{or } \mu = 0.60$$

Francis found that for vertical, sharp-crested, rectangular weirs with complete contractions and free overfall

$$Q = 3.33\left[l - \frac{nh}{10}\right]h^{3/2}$$

where n = number of lateral contractions, 0, 1, or 2. His constant 3.33 corresponds to μ = 0.62 in the above given general formula. As a rule the end contractions can be neglected when a spillway is considered so that the formula becomes:

$$Q = 3.33lh^{3/2}$$

and the influence of the velocity of approach v_0 is introduced as given above.

Because of the lack of experimental data on weirs with $h < 2$ ft., engineers are using for higher heads, formulas developed for smaller heads. However, it has been shown that the Francis formula gives reasonably accurate results for heads up to 5 ft.

7c.—Profiles of Spillways.—Furthermore, spillways are seldom built in the shape of sharp-crested weirs. They are, as a rule, given an ogee shape, so as to prevent a vacuum below the falling sheet of water, the so-called nappe.

It is, therefore, customary to design the shape of the nappe for the maximum head a on the spillway using the general form as found by Bazin for sharp-crested weirs, and then to form the concrete work accordingly. It is advisable to let the concrete on the downstream side of the apex of the curve for the lower nappe encroach somewhat on the water, so as to be certain that a vacuum cannot form (Fig. 40*A*). If the abscissæ are x, the ordinates for the upper

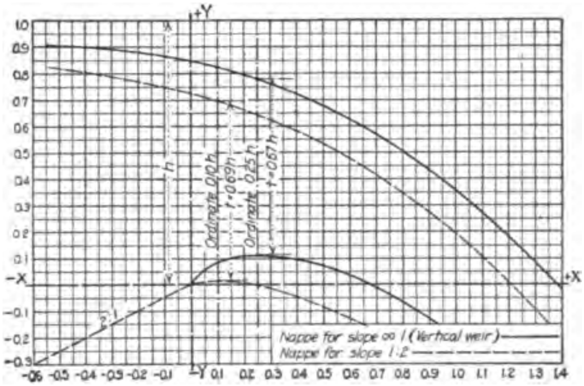


FIG. 40.

nappe y_u and for the lower y_l and the head h, Bazin found the following values for verticle sharp-crested weirs:

$\frac{x}{h}$	$\frac{y_u}{h}$	$\frac{y_l}{h}$
−3.00	0.997	.....
−1.00	0.963	.....
0.00	0.851	0.000
0.05	.....	0.059
0.10	0.826	0 085
0.15	.....	0.101
0 20	0.795	0.109
0.25	0.782	0.112
0.30	0.762	0.111
0.35	.....	0.106
0.40	0.724	0.097
0.45	.....	0.085
0.50	0.680	0.071
0 55	.....	0.054
0.60	0.627	0.035
0.65	.....	0.013
0.70	0.569	−0.009
1.40	−0.020	.. ..

Consequently, if the head is 10 ft., $\frac{x}{10} = -3.00$ or $x = -30$ ft. and $\frac{y_u}{10} = 0.997$ or $y_u = 9.97$ ft., etc. It is obvious that the lower nappe rises from origin to a height 0.112h in the distance 0.25h and that the shape is approximately an ellipse with the major axis horizontal and equal to $\frac{1}{4}h$ and the minor axis vertical and equal to $\frac{1}{8}h$.

Boussinesque found that on the ordinate 0.25h the flow was practically horizontal.[1] From this observation can be deducted the velocity in the upper film of the nappe $v_u = 0.475\sqrt{2gh}$ and that in the lower film $v_l = 0.946\sqrt{2gh}$. Assuming that the pressure increase on this ordinate is in a direct proportion to the depth, the average velocity takes place one-third from the bottom of the nappe.

For a sharp-crested weir the discharge per linear foot is

$$Q = \tfrac{2}{3}\mu h\sqrt{2gh} \text{ with } \mu = 0.62$$

On the ordinate 0.25h the thickness t of the nappe is

$$t = (0.782 - 0.112)h = 0.67h \text{ (see Bazin's table)}$$

so that the mean velocity is

$$v_m = \frac{Q}{t} = \frac{2}{3} \times \frac{0.62\sqrt{2gh}}{0.67} = 0.62\sqrt{2gh}$$

and

$$\frac{2v_m^2}{g} = 2 \times 0.62^2 \times 2h = 1.54h$$

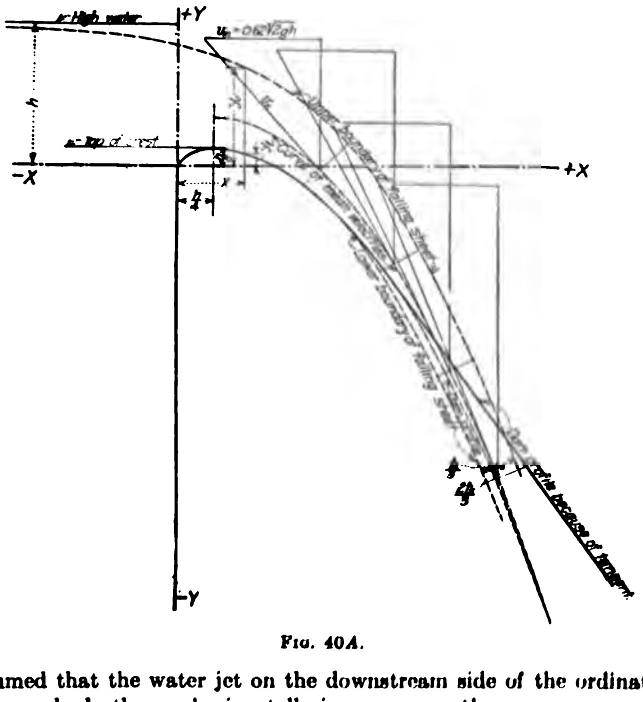


Fig. 40A.

If it is assumed that the water jet on the downstream side of the ordinate 0.25h follows the laws for a heavy body thrown horizontally in a vacuum, then

$$x = v_m t \text{ and } y = \frac{gt^2}{2}$$

or

$$y = \frac{g}{2} \times \frac{x^2}{v_m^2}$$

[1] *Comptes rendus de L'Académie des Sciences*, July-Oct., 1887.

Introducing in this parabolic expression the value found above, or

$$\frac{2v_m^2}{g} = 1.54h \text{ it becomes } y = \frac{x^2}{1.54h} \text{ and } x^2 = (1.54h)y$$

By plotting this parabola the thickness of the nappe at any point can be found by constructing graphically the tangential velocity v_x, keeping in mind that the horizontal velocity remains constant $v_m = 0.62\sqrt{2gh}$ (Fig. 40A). As the discharge Q also remains constant, the thickness t_x of the nappe is expressed by

$$t_x = \frac{Q}{v_x}$$

Of this thickness $\frac{t_x}{3}$ is plotted downward on the normal to the parabola in the point in question and $\frac{2t_x}{3}$ upward. By combining the upper and the lower points respectively, the shape of the nappe is found.

A parabola can be calculated, which approximates the shape of the underside of the nappe. As the stability of overflow dams is calculated in the same manner as that for gravity-section dams, reinforced-concrete dams, etc., it is not always possible to keep this parabolic shape of the spillway all the way to the toe. Generally a tangent is drawn to the parabola at such an angle that its intersection with the base provides the required width (Fig. 41).

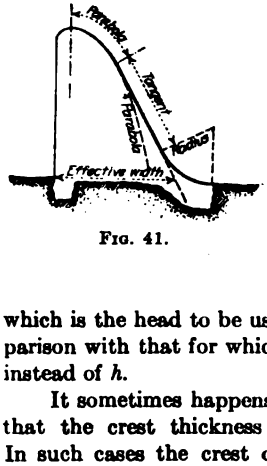


Fig. 41.

From the above it is obvious that the true head for figuring the discharge is larger than the head on the crest of the spillway. This true head can always be found if the elevation of the crest of the spillway and the surface of the water perpendicularly above it are known. If this thickness t of the nappe is known, the true head h is

$$h = \frac{t}{0.782 - 0.112} = \frac{t}{0.67} = 1.49t$$

which is the head to be used in the discharge formula. If the discharge is very small in comparison with that for which the spillway was figured, the Francis formula must be used with t instead of h.

It sometimes happens, especially when ogee curves are fitted to spillways in arched dams, that the crest thickness is insufficient to accommodate the curvature. In such cases the crest of the spillway is cantilevered over the upstream face as shown in Fig. 42.

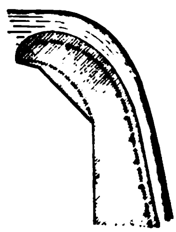


Fig. 42.—Cantilevered spillway.

Reinforced-concrete dams have decks sloping about 1 : 1, so that the velocity of approach is increased gradually. Therefore, the oblique pressure on the nappe, due to the part of the discharge coming from the body of water located below the elevation of the crest, is at a smaller angle with the horizontal, than for a vertical face of the dam. In consequence, the curvature of the underside of the nappe is not so pronounced.

Bazin observed the flow over a sharp-crested weir inclined downstream on a slope of 1 : 2 (Fig. 40). The coördinates observed are as follows:

$\frac{x}{h}$	$\frac{y_u}{h}$	$\frac{y_l}{h}$
0.00	0.730	0.000
0.10	0.700	0.011
0.20	0.666	0.005
0.30	0.630	−0.014
0.40	0.585	−0.044
0.50	0.535	−0.083
0.60	0.480	−0.130
0.70	0.418	......
0.80	0.350	......
0.90	0.276	......
1.00	0.196	......
1.10	0.109	......
1.20	0.009	......
1.30	−0.098	......

For the same head h Bazin found that the inclined weir gave nearly 13% greater discharge.

Knowing that for a slope $1 : \infty$, or the horizontal, the water flows horizontally until the crest (or in this case the end of the channel) has been reached and that then the shape of the nappe follows the laws for a heavy body thrown horizontally (see under Boussinesque) that for slopes of $1:2$ and $\infty:1$, or the vertical, the shapes are as given above, it is comparatively easy to determine approximately by interpolation the shape of the nappe for any slope. However, in order to be safe, the concrete lines should be designed so as to encroach upon the lower nappe for the maximum head on the spillway, as otherwise a vacuum will form

Horton[1] in figuring the discharge over ogee curves uses the formula

$$Q = Clh^{3/2}$$

where h is measured from the crest of the curve. A correction is introduced in C, which is expressed

$$C = [3.62 - 0.16(S - 1)]\, h^{1/20}$$

where S is the slope of the approach to the crest, or

$$S = \frac{\text{horizontal run}}{\text{vertical rise}}$$

This is obviously the best formula to use for dams with inclined water surface like reinforced-concrete structures.

If thus S is 1 vertical to 2 horizontal and $h = 4.0$ ft., $C = 3.71$.

Actual experiments have shown that C is 3.74 which proves that the formula gives somewhat conservative values. It is to be noted that the portion upstream of the ordinate $0.25h$ (see Bazin's method) must be at least 3 ft. in width for this increase in C to take place.

7d. **Overflow Dams.**—When a dam acts as an overflow and its height is comparatively large, the energy of the water when reaching its toe is sometimes considerable. The force P on a plane normal to the nappe is:

$$P = \frac{Qw}{g}v = \frac{Qw}{g}\sqrt{2gH} = Qw\sqrt{\frac{2H}{g}}$$

where v is the velocity ; Q, the quantity; w, the unit weight per cubic foot of water = 62.5; and g, gravity = 32.16.

[1] *Water Supply Paper* 200, p. 131.

If the water below the dam is of sufficient depth to act as a shock absorber, it is customary to provide the spillway at its bottom with a sweep, so that the water is discharged from it in a horizontal direction against the water below the dam (Fig. 41).

Should the water level below the dam be too low to act as a cushion, it is sometimes necessary, as was done at Gatun, Panama Canal, to place concrete blocks in the path of the water, thus breaking up its force by eddie formations.

Another method is by providing stilling pools, which often are formed by a low secondary dam placed some distance below the main dam.

Often aprons are constructed to protect the river bed from erosion. Wegmann recommends that if L is the length of such an apron in the downstream direction and H, the height of the crest of the spillway above the top of the apron

$$L = 2H$$

with the intention that the standing wave will occur in this distance. However, in many instances this is not the case and in India L is made much longer. Often

$$L = 3 \text{ to } 4H$$

and sometimes extended by a riprap $1.5H$ in length.

Rehbock[1] suggests that for the maximum head on the spillway h_{max} the length be made

$$L = 1.5H + 6h_{max} \text{ to } 2H + 8h_{max}$$

7e. Sluices.—Dams are generally provided with pipes laid through their base, so-called sluices, to enable the drawing down of the water level below the spillway crest. Often such sluices are designed with the intention of removing possible silt deposits, but their efficiency is doubtful. Should a flood occur, greater than that for which the spillway is designed, such sluices are very useful as their discharge capacity is great.

Such sluices should have their valves or gates placed in an open shaft, and stop logs should be provided at their upstream side, so that the valves can be inspected or removed for repairs. Coarse screens are sometimes placed in the conduit to prevent water-logged timbers or other large objects from entering. Bell mouths should be formed in the concrete in order to make the losses due to entry as small as possible. If h is the head measured from the center of the opening to the water surface, or the difference in elevation between the upstream and downstream water surfaces, if the downstream opening of the conduit is submerged, and if l is the length of the conduit the losses in the conduit are

$$h = \frac{v^2}{2g} + \mu\frac{v^2}{2g} + C\frac{P}{a}l \times \frac{v^2}{2g} = \frac{v^2}{2g}\left(1 + \mu + C\frac{P}{a}l\right)$$

where the first term is the losses due to the velocity of the flow; the second, due to entry; and the third, the frictional resistance in the conduit.

The entry coefficient

$$\mu = \frac{1}{k^2} - 1$$

so that μ varies from 0.06 for a pipe with a bell-mouthed entry to 0.50 for a pipe projecting into the reservoir.

The coefficient C depends upon the frictional resistance in the conduit, p is the wetted perimeter and a is the net area of the conduit. For round pipes

$$\frac{P}{a} = \frac{\pi d \times 4}{\pi d^2} = \frac{4}{d}$$

[1] "Handbuch der Igenieurwissenschaften." "Der Wasserbau," p. 37.

and for rectangular conduits t ft. high and b ft. wide

$$\frac{P}{a} = \frac{2(t + b)}{tb}$$

A good trial value of C is 0.0075.

Generally the quantity Q is given and it is desired to find the area of the conduit or the diameter d of a pipe sufficiently large to discharge this quantity under a given head; thus

$$h = \frac{v^2}{2g}\left(1 + \mu + C\frac{4l}{d}\right)$$

introducing

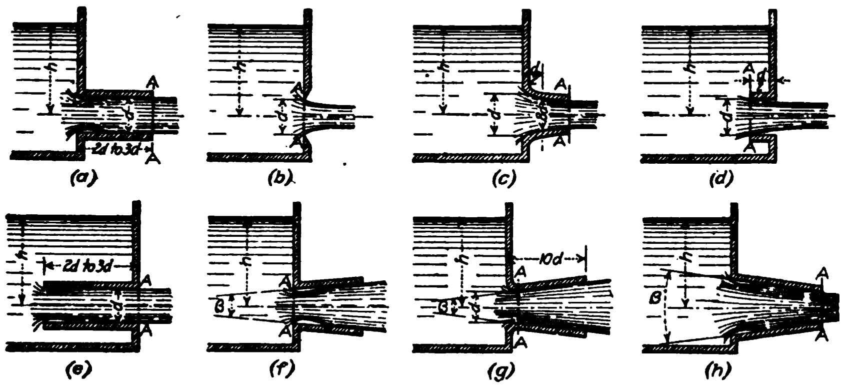
$$\mu = 0.5,\ C = 0.0075, \text{ and } v = \frac{Q}{a}, \text{ where } a = \frac{\pi d^2}{4}$$

and solving

$$d^5 = \frac{8Q^2}{h\pi^2 g}\left(1.5d + 0.03l\right)$$

In this equation the only variable is d which is easiest found by trial. A good first trial value is

$$d = 0.30\sqrt[5]{\frac{Q^2 l}{h}}$$



Area A in each case measured on section "A-A"

(a) Standard mouthpiece—$v = 0.82\sqrt{2gh}$—$Q = Av = 0.82A\sqrt{2gh}$. (e) Re-entrant tube—$v = 0.72\sqrt{2gh}$—$Q = Av = 0.72A\sqrt{2gh}$.
(b) Sharp edged orfice—$v = 0.97\sqrt{2gh}$—$Q = 0.64Av = 0.62A\sqrt{2gh}$. (f) Conical diverging tube—$Q = 0.95A\sqrt{2gh}$.
(c) Streamline contour—$v = 0.96\sqrt{2gh}$—$Q = Av = 0.96A\sqrt{2gh}$. (g) Venturi adjutage Angle $\theta = 5°$ to $8°$—$Q = 1.5A\sqrt{2gh}$
(d) Borda's mouthpiece—$v = 0.99\sqrt{2gh}$—$Q = 0.54Av = 0.53A\sqrt{2gh}$. (h) Conical converging tube—Angle $\theta = 5°$ to $10°$—$Q = 0.93A\sqrt{2gh}$.

Fig. 43.

In reinforced-concrete dams the conduit is, as a rule, very short and serves merely as a setting for the sluice gate. It is continued through the dam in the open channel formed by the buttresses. In such a case the conduit can be considered as a mouthpiece and the velocity determined directly from Fig. 43.[1]

7f. Siphonic Spillways.—In places where the discharge to be handled is comparatively small, siphonic spillways can be used to advantage.[2] As such devices generally are designed for automatic operation, it is obvious that a close regulation of the water surface will be obtained. As, furthermore, the operating head is the difference in elevation between the water surfaces above and below less the hydraulic losses in the siphon, the discharging capacity per linear foot is considerably in excess of that of an overflow spillway.

[1] Slocum: "Elements of Hydraulics," Ed. II, p. 69.
[2] Hillberg: "Spillways of the Siphonic Type," *Eng. Rec.*, May 3, 1913, p. 488.

In the design three details are of importance:

1. The upper part must be so made that as soon as the water rises above the level to be maintained, the siphon intake is sealed to the air and is kept sealed until the water level has been drawn down again to normal. The air openings must then be large enough to admit quickly sufficient air to break the siphonic action. Both of these features can be secured by having long and sharp edges on the intake to the siphon at the normal water level.

2. The lower end of the siphon must be submerged deeply enough to secure a constant seal from the beginning of the siphonic action. The upper edge of this opening must be as sharp as possible to permit an easy escape of any air carried out by the water.

3. The cross-sectional area should be as large as possible at the intake to reduce losses due to entry. Back of the intake provisions should be made for an efficient absorption of the enclosed air. This is obtained by building a channel around the opening, so that in the beginning water will flow into it from all sides forming a spray. The neck of the siphon is generally curved, so that the water pressure on any entrained pocket of air will move it downward flattening it so that the friction at the contact surface will tear off layers of air until all of it has been carried out. A gradual narrowing of the cross-sectional area up to this point is desirable because the increase in velocity head will lessen the static pressure thus creating an overpressure on the upper part of such air pockets. Below this point no enlargement of the area is permissible as the corresponding decrease in velocity will release a certain portion of the entrained air, which, especially if the siphon is curved, will collect and cause interruption of the siphonic action taking the form of pulsations. From this it is obvious that the best practice is to incline the lower leg, as released air would thus quicker reach the concrete wall and be again entrained in the water.

In a siphon the sum of all losses must equal the difference in elevation E between the water surfaces or

$$E = \frac{v^2}{2g}\left(1 + \mu + x + k + C\frac{pl}{a}\right)$$

where v = velocity; g, 32.16 or gravity; μ, coefficient for loss due to entry (probable maximum 0.50); x, coefficient for loss due to friction in upstream leg (probable maximum 0.10); C = coefficient for loss due to friction in downstream leg (varies from 0.005 to 0.009); p, its wetted perimeter; a, its area; l, its length; and k, coefficient for loss due to bends or curves (probable maximum 0.25).

This equation is applicable for all conditions up to E = 33.9 ft., which is the suction limit. Should E be greater than this, the losses must be kept equal to or smaller than 33.9 ft. by tapering the downstream leg in such a way that its smallest cross-section is at or near the lowest point. As the maximum velocity will take place at this section, it is obvious that a certain amount of head is still in the form of pressure in the sections above. This can be formulated in the following: Calculate a conduit with a hydraulic gradient so that the sum of all losses is 33.9 ft. or less. One siphon at Gibswil, Switzerland (Fig. 44) operates under an elevation difference of 52.48 ft.

Experiments made in Switzerland on the operation of siphons shows that the general formula

$$Q = av = a\mu\sqrt{2gh} \qquad \text{(sec.-ft.)}$$

can be used with a coefficient μ varying from 0.55 for smaller heads to 0.70 for higher. It is always convenient to use this formula for trial computations, but to check the final design with the more correct theoretical formula. It is to be remembered, however, that h_{max} = 33.9 ft.

If the discharge is too voluminous to be handled in one conduit, several are used and placed side by side.

The materials used for building smaller siphons are steel pipes with reinforced-concrete hoods or reinforced concrete is used throughout. Larger siphons are always built of concrete

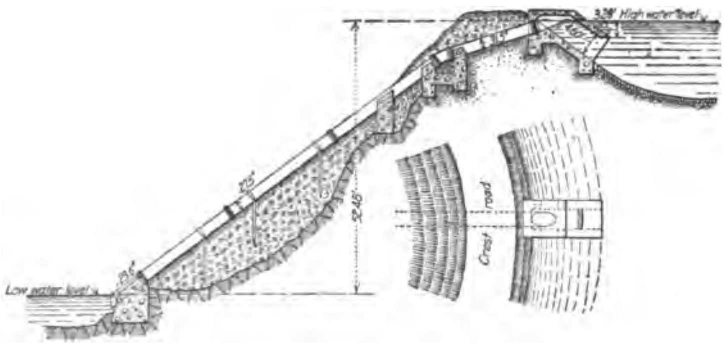


FIG. 44.—Elevation of high-head siphon at Gibswil, Switzerland.

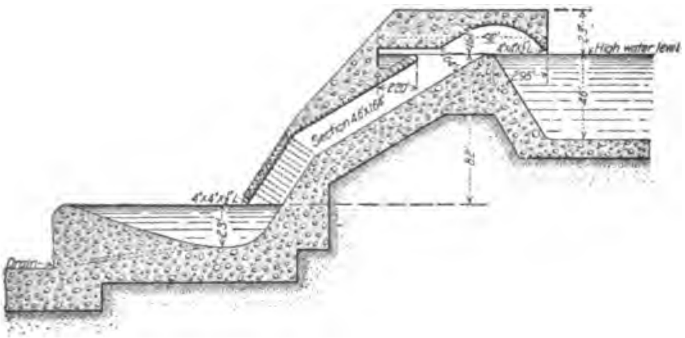


FIG. 45.—Spillway in use at Seon, Switzerland.

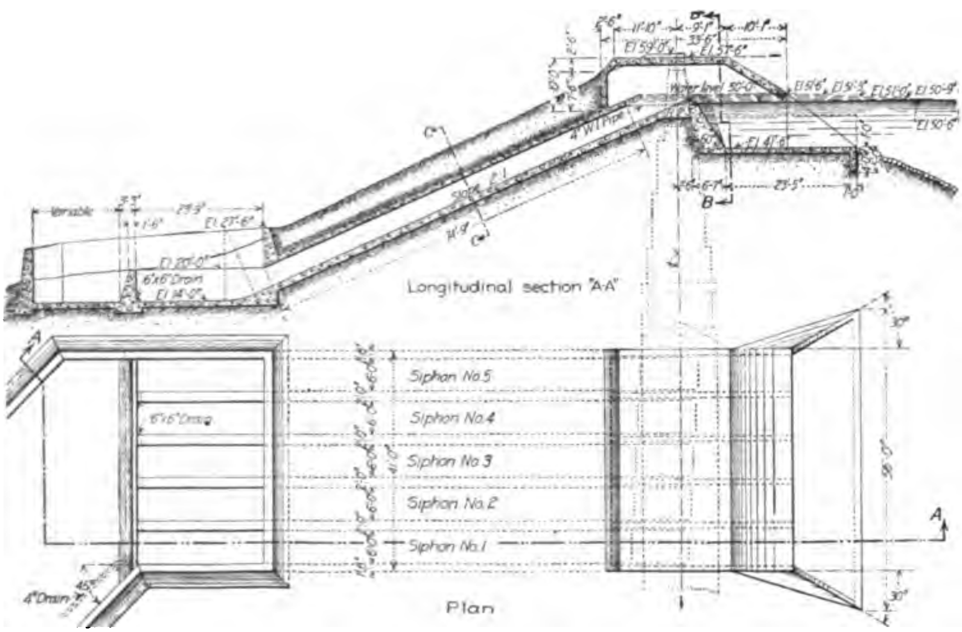


FIG. 46*A*.—General design of siphon for Dunning's dam, Scranton Gas & Water Co.

which can be either mass or reinforced. One siphon built of reinforced concrete in Switzerland is shown in Fig. 45.

Because of the suction the exterior load on the siphon walls equals the full atmospheric pressure of 14.7 lb. per sq. in. or 2120 lb. per sq. ft. On the underside this load can be reduced because of the weight of the water and the structure itself.

The biggest siphon so far proposed is designed for the Dunning's Dam of the Scranton, Pa. water supply (Figs. 46*A* and 46*B*). It consists of five conduits 4 ft. high and 6 ft. wide and

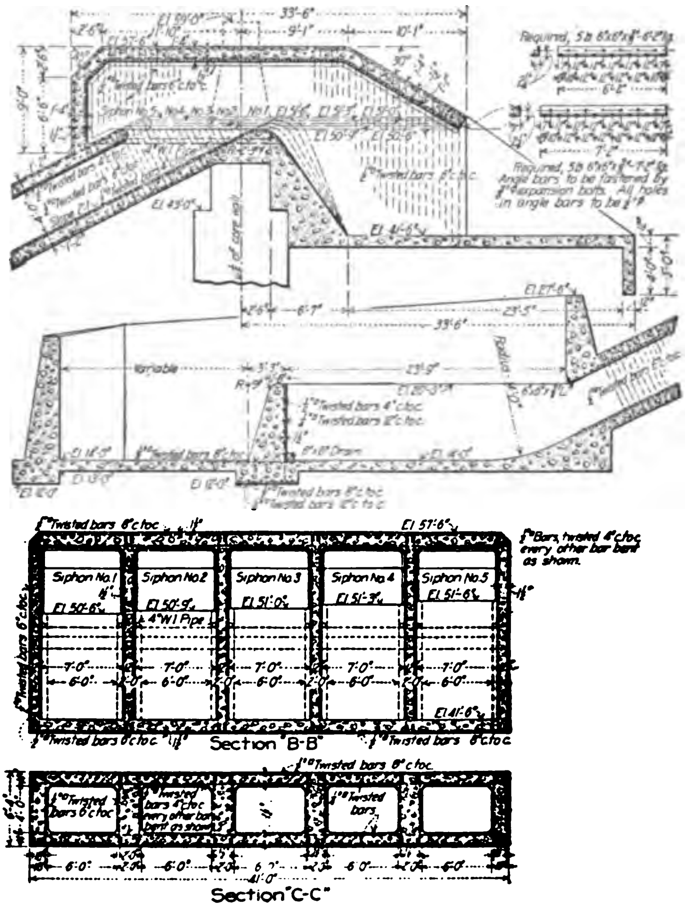


Fig. 46*B*.—Details of siphon for Dunning's dam, Scranton Gas & Water Co.

its total discharge capacity is 3750 sec.-ft. Its overall width is 38 ft. and the length 90 ft. The maximum stress in the concrete is 543 lb. and in the steel 13,750 lb. per sq. in.

8. Movable Dams.[1]—Movable dams are used in places where a wide opening is required to accommodate the flood discharge. Where such dams are used for river regulation they are generally placed straight across the channel and erected on a substructure which is nothing more than a low sill. The movable parts are so made that they can be laid down on this sill,

[1] The best textbook on movable dams is "Handbuch der Ingenieurwissenschaften, Part III" "der Wasserbau," Chapter III "Die Beweglichen Wehre," by Prof. K. E. Hilgard.

removed entirely to the shore or hoisted between piers, so as to leave an unobstructed channel of practically the same width as that before the placing of the dam.

Movable crests are often used in combination with spillways for dams in places where it is desired to maintain as nearly constant as possible the elevation of the water surface above the dam.

All types of movable dams can be divided into three groups: (1) Requiring operating machinery; (2) operating under hydrostatic pressure differences; and (3) automatically operating.

8a. Requiring Operating Machinery.—These dams can be operated manually, electrically, hydraulically, or in any other mechanical way. The operating machinery is often mounted on a traveler or a barge so that it can be moved along the dam and used at any desired point. The principal types are: Stop logs, needle dams, *A*-frame dams, curtains, flashboards, gates, wicket gates, bridge dams, taintor gates and rolling dams.

8b. Operating under Hydrostatic Pressure Differences.—These dams are generally so designed that they are operated by the opening or closing of valves in conduits connecting an inclosed chamber under the gate with the high water above and the low water below the dam. The usual types are: Bear traps, drum dams and butterfly dams.

8c. Automatically Operating.—For close regulation of water levels, automatically operating devices have come to the front during the past 10 years. The first to be built was in Connecticut as early as in 1902.[1] They were made of oak and about 3.5 ft. high and 6 ft. wide. Each gate operated on the principle, that when the water level reached its top, the resultant of the hydrostatic pressure fell above the point of support. In order to prevent the gate from opening with an accelerated speed, the shaft was provided with toothed cams at the ends, shifting the point of support upward on the gate when opening. In 1914 gates of similar design and from 5 to 15 ft. high and 18.5 ft. wide were installed at Austin, Tex.[2] They consist of steel frames with wooden covering. Their lower part is filled with concrete to place the center of gravity below the point of support. However, such gates have not been a success because of the difficulty of designing a cam or a bascule which will prevent the gate from gaining in speed when opening. Because of this the impact, when the gate reaches the open position, is, by larger gates, sufficient to destroy them. Practically the only effective way of making gates automatically operating is by providing them with counterweights, so designed that their moments increase in proportion to the increase in moment due to hydrostatic and impact pressures on the gate leaf. Among such balanced gates are those with overhead rolling counterweight, underhung counterweight, counterweights suspended at ends of levers, and those with variable counterweights.

9. Fish Ladders.—To permit fish to pass dams in search of spawning grounds and of food, the laws of many States require that fishways be provided. Such ways or ladders consist of a number of compartments arranged in steps and separated by cross-partitions or baffles. The construction varies depending upon the habits of the fish living in the stream in question, but they can be divided into two main groups: Jump ladders and swim ladders. Combinations of both are sometimes built if the prevailing conditions should so require. All fishways must be so designed that the outlet is below low water level and so located as to have an unobstructed discharge of water in order to attract the fish. The intake or upstream end should be not less than 1 ft. lower than the crest of the dam. The slope of the bottom should never be more than 1:4 and if possible be 1:10 or even less. The width should not be less than 4 ft. and often it is up to 10 ft. No compartment should be shorter than 4 ft. or in depth less than 2.5 ft. Plenty of light should be admitted or the fish will not use it. However, to protect the fish from birds and human beings, fishways should be covered by gratings so built as to facilitate inspection and cleaning. There should be no regulating gates at the intake, necessitating attendance. Fishways are built of wood, steel, masonry, or reinforced concrete.

[1] *Eng. Rec.*, Mar. 8, 1902, p. 222.

[2] LAMAR LYNDON: "Hydro-electric Power," vol. 1, p. 285.

RESERVOIRS

10. General Types.—There are two general types of reservoirs—open reservoirs and covered reservoirs. Open reservoirs vary in size from great catchment basins closed by dams or embankments, either of earth, or of masonry, or both, to smaller containers constructed largely or wholly of masonry, such as sewage treatment basins. Fig. 47 shows a 5,000,000-gal. open circular basin constructed by the city of Duluth, Minn.

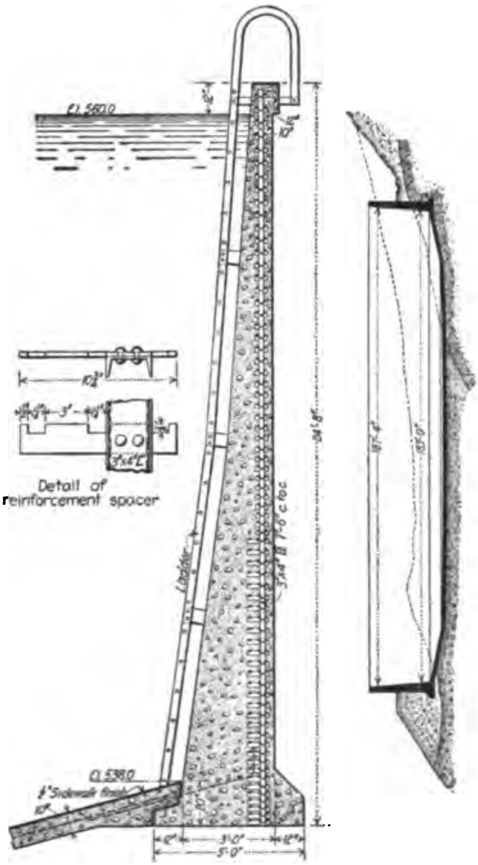


Fig. 47.—Duluth circular reservoir. Hoop reinforcement for wall was supported by notched steel plate brackets riveted to vertical channels.

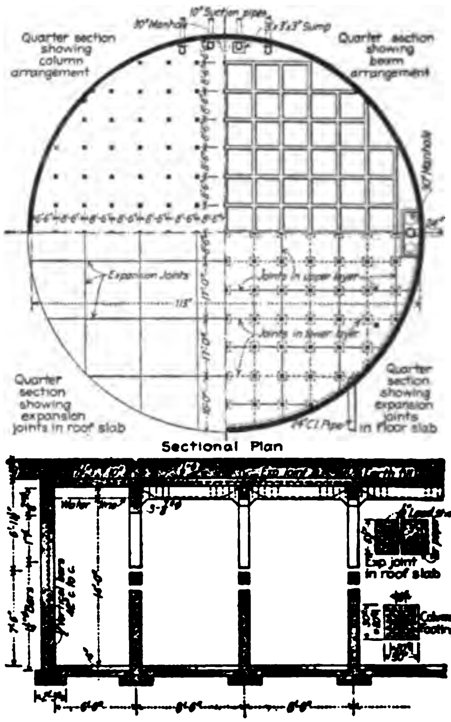


Fig. 48.—Covered equalizing and storage reservoir for Hibbing water-works.

Covered reservoirs are of relatively limited size and usually are constructed of concrete masonry in floors, roof, and walls, with earth covering on roof and against walls (see Figs. 48 and 49). Covered reservoirs prevent freezing or disagreeable warming of water, as well as pollution from outside sources and organic growths which require sunlight.[1]

11. Quality of Concrete for Reservoir Masonry.—Concrete for reservoirs has density and correlatively, impermeability as basic requisites. To this end, the selection of materials, the

[1] For discussion see Ellms: "Water Supply;" Flinn, Weston and Bogert: "Water Works Handbook;" Hasen: "American Civil Engineers Pocket Book;" and others.

proportioning, the mixing, the placing and, particularly, the quantity of water employed should be subject to rigid regulation. Percolating and entrant water is the most active disintegrating agent to which concrete is normally subject. The use of arbitrary proportions and careless methods of manipulation in such concretes is therefore not only poor engineering, but a direct courting of trouble. The number of specifications and textbooks permitting and advocating such practices is at present regrettably large, either in ignorance of, or regardless of, the definite authoritative and generally available knowledge as to their danger and incorrectness.

12. Open Basins with Embankment Walls.—With soil of proper character, open reservoirs may be constructed without masonry. It is usually preferable, however, after forming dense banks from excavated or other materials, to cast upon them and upon the bottom,

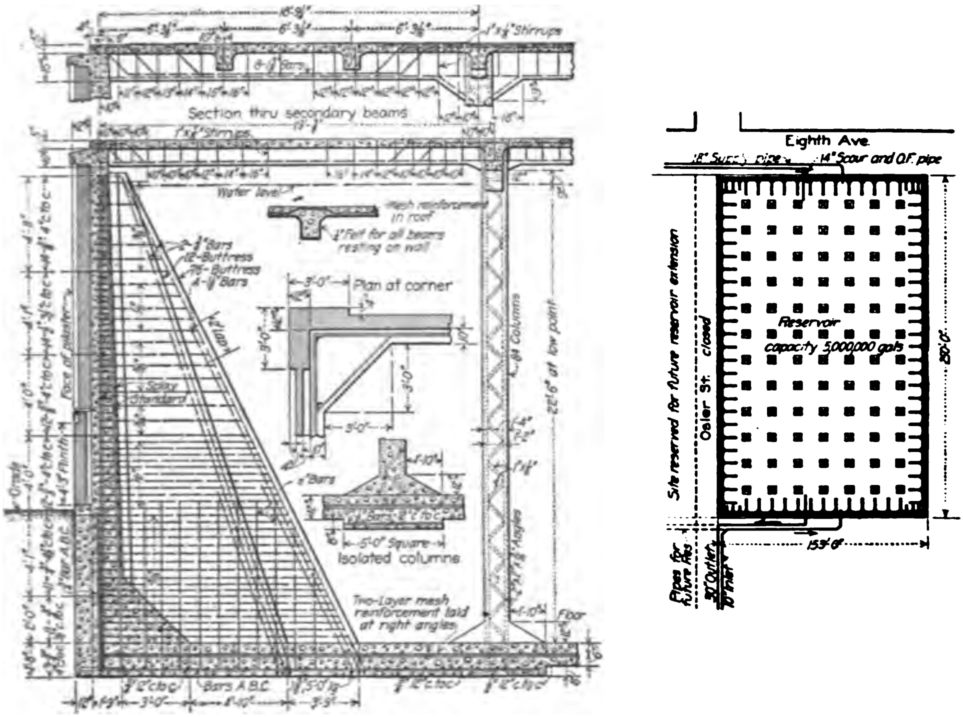


Fig. 49.—Covered water-works reservoir at Regina, Sask.

slabs of concrete, reinforced or not, according to the stability and uniformity of the foundation and embankments.

For embankment walls, concrete of rather stiff consistency is cast directly on a sand coat over a layer of puddled clay. This clay layer should be from 10 to 24 in. in thickness; and the slope about 3 : 1 and never steeper than 2 : 1. Reinforcing mesh or rods may be embedded in embankment slabs if individual circumstances of location or material indicate this procedure as advisable.[1] Concrete core walls may be used with earth embankment, with or without masonry facing. In all cases, whether or not concrete facings are used, embankments should be well settled and compacted and well worked into a thoroughly stripped and scarified subsoil.

13. Concrete Floors for Reservoirs.—Concrete floors for reservoirs are laid in one or more separate layers, either as a continuous slab or superposed slabs, or in rectangular blocks with

[1] See *Eng. News*, vol. 74, p. 267, 1915.

closely abutting joints. As before stated, the concrete should be as impervious as possible; and to prevent leakage, continuous construction of a monolithic slab is advantageous, inasmuch as the often considerable leakage at joints is thereby prevented. The strength of the concrete should be such as to support the weight of water over slight inequalities of bottom when the reservoir is full; and at least sufficient to withstand upward pressure of ground water with the reservoir empty. A thickness of from 6 to 12 in., depending upon subsoil conditions, is usually sufficient.

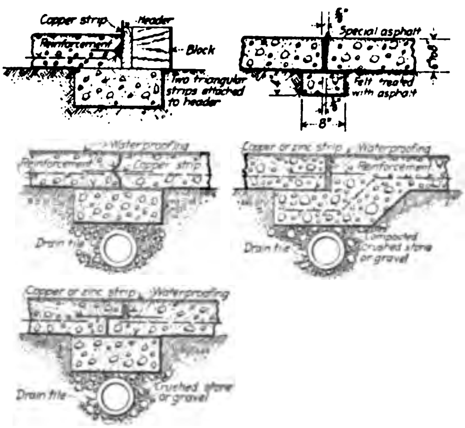


Fig. 50.

When concrete floors are divided into panels, the blocks of from 15 to 60 ft. square are connected at the edges by some sort of lock-joint. This is to provide for flexibility during settlement, and for contracton. There is usually a beam of concrete laid along and below the joint. Details of such joints are shown in Fig. 50. If a tongue-and-groove joint is used, the tongue should be V-shaped, otherwise it will be broken off.

Pipes and other fixtures passing through the bottom should be flanged and well water-proofed.

14. Groined and Flat Floors.—Inverted groins have become a familiar type of reservoir bottom in view of the advantages offered by footings adapted to distribute column pressures formed by the thicker portions, with channels for water flow and cleansing offered by the continuous inverts. This type of floor is generally used with covered reservoirs, and it may be

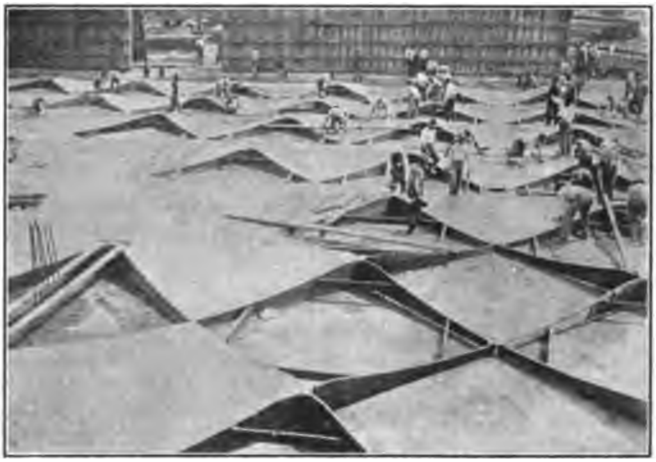


Fig. 51.—Construction view of groined reservoir floor being placed in two layers.

laid in one or two layers, as desired. In Fig. 51 is shown in process of construction such a floor, placed in two layers, the lower layer being run flat.

A modification of the groined floor is shown in Fig. 52. This type offers the additional constructional advantage of permitting continuous placing of plastic concrete. This design was developed by the John F. Casey Co. in their repairs to the Filtered Water Reservoir at the Division Pumping Station, Cleveland, Ohio.[1]

Flat floors may be used in either open or covered reservoirs. In large open reservoirs they are customary as groined slabs are more difficult of construction and not advantageous where no piers are to be supported, all necessary water channels being readily formed by sloping slabs. A notable example of large, open concrete reservoir construction of this type is the Hillview Reservoir of the Catskill Water Supply for the City of New York.[2]

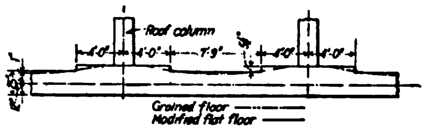


FIG. 52.—Modification of groined floor.

15. Concrete Walls for Open Reservoirs.—Concrete walls for reservoir sides are designed as retaining walls, except that reservoir walls must withstand earth thrust from outside when the reservoir is empty, and water pressure from within when the reservoir is full. The resistance of outside earth embankments against water pressure is sometimes deducted, but this procedure requires that to bear upon the earth, the wall may undergo no deflection; *i.e.*, the earth does not shrink away from the outer face of the wall but is constant in bearing. Various means are used to prevent such loss of contact between wall and bank. The wall may be battered, or better, stepped slightly. The portion of the fill next to the wall is sometimes puddled, but where this is done, provision against excessive thrust must be made in the design, or the wall must be braced.

When the embankment on one side of a wall is caused to resist pressure from the opposite side of the wall, there is a tendency to force out a prism of earth. The largest thrust which the earth will resist is called the *passive thrust.*

In Fig. 53 let AB be a given wall with an embankment whose earth surface is $Abfn$ tending to resist a pressure acting against the wall from the left. The friction between wall and earth will act downward, since the earth will tend to move upward. The reaction of the earth P will thus be directed upward making an angle above the normal to the wall of Z, but never greater than ϕ (see Art. 1b, Sect. 13). Lay off on the surface line points a, b, c, etc. arbitrarily. Draw lines from these points to B, thus bounding a series of prisms of length 1 ft. perpendicular to the drawing. Compute the weights of these prisms and to any convenient scale lay them off in order, on the line BW which makes an angle ϕ with the horizontal as w_a, w_b, w_c, etc. Draw through one of these points a line, as tw_d, which makes an angle Z to the right of a normal to BW. Then through each point on BW draw a line parallel to this direction tw_d, to an intersection with corresponding ray through B, as w_e to B_e at S_e; w_f to B_f at S_f; etc. Through the points S draw a smooth curve. (This curve will have breaks at rays connecting B with points of change of slope.) Draw a tangent v to the curve parallel to BW, and through the point of tangency draw a ray from B, as BS_gg. This line will represent the plane of rupture of the embankment if the passive thrust is exceeded. The distance w_gS_g between BW and the tangent v, measured parallel to the direction tw_d and to the same scale as that used in laying off the weights on BW, is the magnitude of the passive thrust.

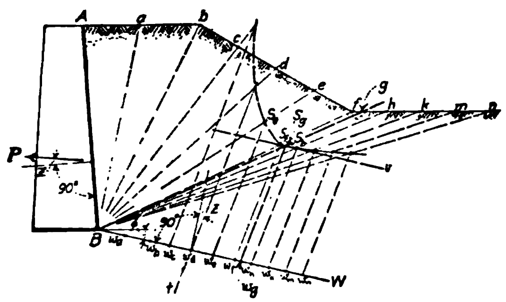


FIG. 53.

[1] See *Eng. Rec.*, Dec. 9, 1916, p. 702.

[2] See *Proc.* Am. Conc. Inst., 1915. See also TAYLOR and LYNDON on pavement of circular reservoir at Austin, Tex., *Proc.* Am. Conc. Inst., 1916, p. 143.

The point of application of P, the reaction of the earth against the wall, will be one-third of AB above B.

Proof of the construction may be at once seen by revolving the force triangle BS_gw_g to such a position that BW_g lies on AB. Bw_g is then seen to be the weight of the block of earth $AbfgB$ lying above the plane of rupture Bg; BS_g is the resultant pressure on the plane Bg and making the internal friction angle ϕ thereto; and as previously noted, w_gS_g denotes the passive thrust, at an angle (90 deg. $\pm$ Z) to the wall.

The moment of the passive thrust is added to the moment of resistance, taken about the third point toward the earth, when the reservoir is full; and the moment of the *active* thrust is taken as the attacking moment about the third point toward the water, when the reservoir is empty.

16. Partition and Outside Walls.—Walls subdividing the basin for purposes of filling or emptying are usually of concrete. They consist either of a cantilever wall or a double-counterforted wall with the stem at the center of the base. In the latter type the double-counterfort acts as a wedge-shaped beam with both faces inclined, and is reinforced to act in either direction. The stem of the cantilever wall may be designed in the same manner. The base slab is designed to take the resultant pressure from either side.

Vertical joints in concrete walls and partitions should be made at intervals not to exceed 60 ft. The tongue-and-groove type of keyway with a folded metal strip, as in slabs, makes a satisfactory joint. It should be thoroughly waterproofed.

Concrete in such walls should be well spaded next to the forms to obtain a smooth surface. The whole material should be well tamped. The thinner the wall, the more essential is proper selection and proportioning of materials, adequate mixing, and careful placing.

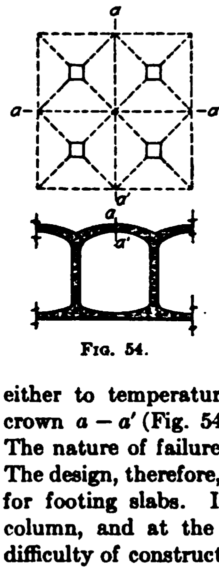


Fig. 54.

17. Provision for Ice.—All walls for reservoirs in which ice is likely to form, should be battered at the water line to heave the ice as it expands. Paved embankments usually give little trouble in this respect.

18. Covers or Roofs for Reservoirs and Basins.—The purpose of a roof may be (1) to prevent contamination or organic growths, (2) to prevent freezing, or (3) to maintain lower water temperature in warm climates. Three types of construction are used: beam-and-slab, flat-slab, and groined-(elliptical) arch construction, support being afforded by walls, by columns, or piers extending to the floor. The first two types are the same as those for floors. The last requires separate analysis.

19. Groined-arch Construction.—Experience shows that due either to temperature or to settlement, cracks are likely to form at construction joints at the crown $a - a'$ (Fig. 54); and when such cracks have formed, little or no arch action prevails. The nature of failures indicates that arch action is not sufficient to be considered in the design. The design, therefore, should be made for shear and cantilever moment, and may be made as for footing slabs. It is advisable to reinforce the "umbrella" at the upper face over the column, and at the lower face of the crown. Due to the increased cost of formwork and difficulty of construction, this type of roof is not often as economical as the usual flat-slab, or other types of floor construction.

20. Construction Details of Columns and Roof.—Footings for walls and columns are laid below the pavement or else the pavement is thickened at bearing points. To prevent undue settlement pressures on footings should closely approximate those on floors of basins. If separate footings are provided, expansion joints should be made in the pavement about the column. Roof slab should be reinforced against temperature changes and shrinkage by adding 0.4% of steel. Many reservoir roofs have been built continuous with the side walls, but if the width of structure is considerable, or if material temperature change is anticipated, expansion joints should be provided between roof and walls.

STANDPIPES AND SMALL TANKS

A standpipe is a cylindrical tank resting directly upon its foundation, and usually, though not always, having a height greater than its diameter. It may be used for the storage of fluids other than water, though the latter use is the most common.

Standpipes of concrete require particular care in construction. Not only must the concrete be of sufficient strength to withstand, without outside support, bursting from pressure of water or cracking from temperature changes, but further the concrete must be so continuous and of such strength, density, and finish that percolation at work planes or other points will not take place.

21. Analysis of Stresses in Standpipes.—Since the pressure of a fluid varies in intensity with the depth, the unit pressure at a depth y is

$$p_y = wy$$

Suppose a circumferential strip, 1 ft. wide to be cut from the shell at a depth y (Fig. 55). The total tension on any diameter is

$$T = p_y r$$

where p_y is in pounds per square foot; r is the inner radius of the tank in feet; and T the pull on one side of the tank per foot of height.

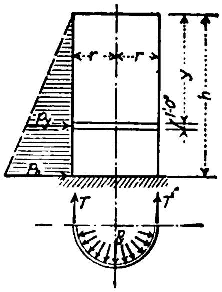


Fig. 55.

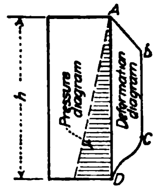


Fig. 56.

In a standpipe of concrete the tensile resistance is necessarily low, since at a given tensile unit stress the concrete will crack, permitting seepage under high heads. Tests indicate that concrete of good quality will crack minutely at an elongation of 0.00016 to 0.0006 inches per inch. The value 0.00010 corresponds to a unit stress of 200 lb. per sq. in. in the concrete, and (when bars are present) 3000 lb. per sq. in. in the steel. Stresses above these values require full tensile resistance of the steel. Where high heads are used, low stresses should be employed; but in tanks having a head of 30 ft. or less, steel stresses of 8000 to 10,000 lb. per sq. in. have proven successful, since the cracks do not open enough to cause appreciable seepage at these low heads.

European practice employs a very rich mix for use in standpipes, varying from 1 cement: 1¼ graded aggregate, to 1 cement: 2¼ graded aggregate, these values varying for tanks from 100 ft. high to 30 ft. or less in height.

22. Restraint at Base.—The deformation of a standpipe is not wholly due to the elongation of the hoops under stress. At the base the rigidity of the bottom prevents hoop expansions and introduces a restraining moment on a vertical element. If the side of the tank were of uniform thickness and a homogeneous material were employed, the deformation of the rings would vary as the depth of water. But in the design of a concrete standpipe with steel reinforcement, the hoop elongation will be limited to that elongation which corresponds to the adopted working unit stress in the steel. Thus in Fig. 56, the portion BC of the exaggerated deformation is a constant for any depth between B and C. The deformation to AB may be a straight line or not; however, it is the gradual increase of deformation up to that corresponding to the working hoop stress.

The restraint of the bottom is such that at D no hoop stress exists, but rather a fixity, similar to a vertical beam fixed at the lower end. This restraint decreases so that at some point C only hoop stress, and likewise hoop deformation, prevails. It is evident that at this point C, the tangent to the deformation curve is again vertical, thus parallel to the tangent to the curve at the bottom.

Suppose the lower portion of the shell of the tank or standpipe to be made up of vertical beams restrained at their lower end, and having a length sufficient to include the deformation CD, Fig. 56. Then (Fig. 57) the water pressure varies from its upper end with $w(h - l)$ to its lower end with wh. Let the beam be acted upon at its upper end by a moment M', sufficient to maintain the slope at that end parallel to that at the base. Then the slope at C is zero with respect to that at D. The deformation Δ at C must be equal to that caused by ring working stress only.

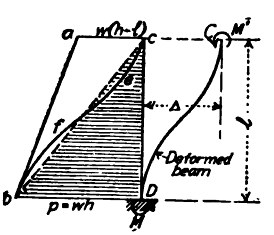


Fig. 57.

Now imagine the elastic hoops existing between the levels C and D to restrain the deformation of the beam somewhat. Let the deformation of the beam thus modified be that shown by CD, Fig. 57. The hoops stresses between the levels C and D will thus vary as abscissas to the deformation curve CD. Before we may again consider the vertical beams separately, the effect produced by these hoop stresses should be used to modify the water pressure on this beam. The deformation area DcC may be converted to a corresponding stress area, and subtracted from the pressure trapezoid $abDc$ by placing cD upon ab. The loading left to the restrained cantilever beam to be resisted by *beam action* is therefore represented by the area $bfecD$.

Since the actual form of the curve $bfec$ is still unknown, the resultant loading is likewise unknown. So far as concerns the deflection Δ of the end C a very close approximation is apparent by using the triangular loading bcD. It should be noted that this triangle exceeds the actual loading at e (curve $bfce$) but is less than that at f; but since the upper portion of the beam contributes more than half of the deflection Δ, the approximation becomes, if anything, slightly on the safe side. For the loading adopted, and for a homogenous beam,

$$M = \frac{pl^2}{8} \qquad M' = \frac{pl^2}{24} \qquad \Delta = \frac{pl^4}{80EI} \tag{1}$$

From the theory of deflections of reinforced-concrete beams[1] the deflection Δ of the beam CD becomes, instead of that just given for a homogeneous beam,

$$\Delta = \frac{1}{80E_s} \cdot \frac{62.5}{24} \frac{hl^4}{bd^3} \cdot \frac{n}{\alpha} = \frac{1}{368.8E_s} \frac{hl^4}{d^3} \cdot \frac{n}{\alpha} \tag{2}$$

in which

h = depth of water at bottom of tank, in inches.
l = length of beam element CD, in inches.
d = effective depth of beam element CD, in inches.
E_s = modulus of elasticity of steel, pounds per square inch.
n/α = numerical coefficient dependent upon p and n, in which n is recommended by Turneaure and Maurer to be 8 or 10 (see Fig. 58).
Δ = deflection of end C, in inches.

Referring again to Figs. 56 and 57, it will be noted that Δ must equal the deflection, or change in radius due to hoop tension. This may be expressed in terms of the steel working stress f_s, the radius r in inches, and E_s.

[1] See Art. 28, Sect. 7.

$$\Delta = \frac{f_s}{E_s} r \tag{3}$$

Since this must equal the beam deflection, equating (2) and (3) gives

$$l^4 = \frac{4425 f_s r d^3}{h\left(\frac{n}{\alpha}\right)}$$

whence

$$l = 8.16 \sqrt[4]{\frac{f_s r d^3}{h \frac{n}{\alpha}}} \qquad \text{(in.)} \tag{4}$$

This is the value of l which will give the desired deflection Δ. Having thus found l, the moments of its loading may be determined. Thus, with all linear dimensions in inches, M in inch-pounds is from (1) and (4),

$$M = 3.612 \sqrt{\frac{f_s r d^3 h}{\frac{n}{\alpha}}} \tag{5}$$

In using this formula it should be noted that f_s is the unit stress in the hoops. M' is given by the relation

$$M' = \frac{1}{3} M \tag{6}$$

The point of inflection is at $0.63l$ above the bottom. At this point the reinforcement may swing from the inner to the outer face. Although only a third of the steel is needed at the outer face, the remaining steel should be carried up sufficiently to provide ample bond.

The moment above the point C may be assumed to vary as a straight line from the value M' at C to zero at the top. This is not strictly true but is sufficiently close to provide a means of cutting some of the steel near the outer face if desired.

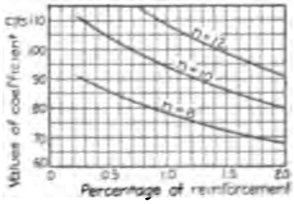


FIG. 58.

23. Shear at Base.—The shear at the base may be seen from Fig. 57 to be equal to the triangular loading on the beam. Thus, the shear per foot of circumference becomes

$$V = 0.217hl$$

where h and l are in inches.

ILLUSTRATIVE PROBLEM.—Let h = 40 ft., diameter = 20 ft., thickness of shell = 10 in. Use f_s = 8000 lb. per sq. in. for hoop stress. Assume n = 10 and p = 1.5%.

From formulas (2) and (3) in Art. 28b, Sect. 7, or Fig. 58, page 767, $\frac{n}{\alpha} = 86$. At the base

$$M = 3.612 \sqrt{\frac{8000 \times 120 \times (8.5)^3 \times 480}{86}} = 207{,}000 \text{ in.-lb.}$$

Assuming j = 0.83 (now n = 15) and noting from Diagram 2 on page 369 that when p = 1.5%, f_s = 10,500 when f_c = 650, the moment value of 1 sq. in. of steel thus stressed is, for d = 8.5 in.,

$$M = f_s jd = (10{,}500)(0.83)(8.5) = 74{,}100 \text{ in.-lb.}$$

$$\frac{207{,}000}{74{,}100} = 2.81 \text{ sq. in. steel}$$

Checking.

$$p = \frac{A_s}{bd} = \frac{2.8}{(8.5)(12)} = 0.028, \text{ nearly } 3\%$$

It will be necessary to taper the wall at the base. Solving for the effective depth required for p = 1.5% (K = 131),

$$d^2 = \frac{207{,}000}{(131)(12)} = 131.7 \qquad d = 11.5 \text{ in. or 14 in. total}$$

$$A_s = 12 \times 11.5 \times 0.015 = 2.07 \text{ sq. in. per ft. of shell.}$$

Use ⅞-in. sq. bars spaced 4½ in. on centers.

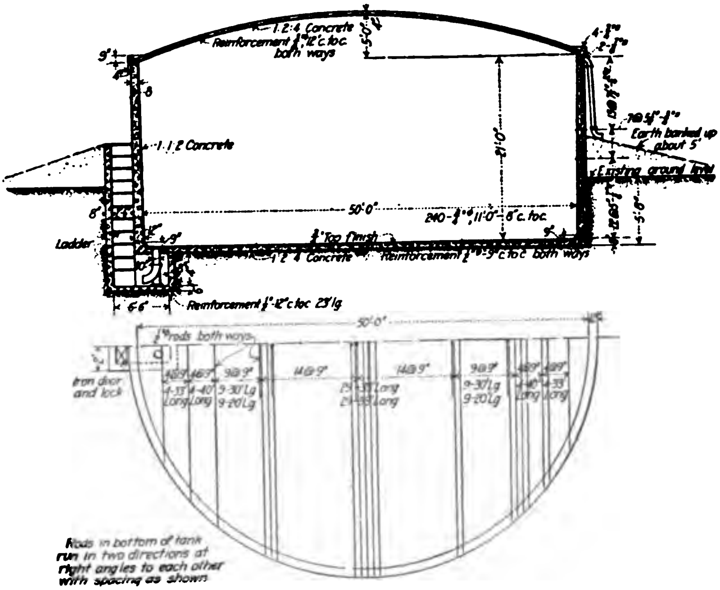


Fig. 62.—Low standpipe at Penetanguishene, Ont.

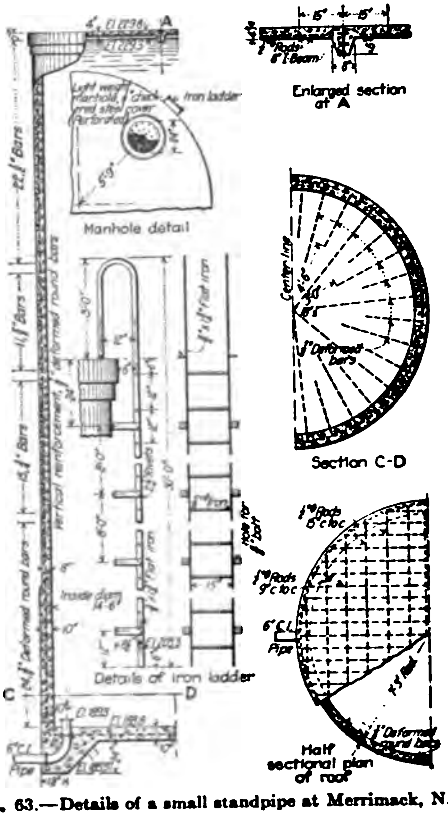


Fig. 63.—Details of a small standpipe at Merrimack, N. H

Fig. 63[1] shows the details of a small standpipe of 14½ ft. inside diameter and 40 ft. high, built at Merrimack, N. H. The standpipe has a capacity of 50,000 gal. The concrete was poured in one continuous operation which lasted 39½ hr.

26. Precautions in Construction.—The construction of standpipes of concrete has not been thus far wholly satisfactory. It is a relatively simple matter to design against water pressure alone, but true monolithic construction, without leaks at junction of base and walls or at work planes, or cracks from secondary stresses, is less easy.

In addition to the procedures for producing dense concrete before referred to and treated in detail in Sect. 2, care should be taken to so place the concrete as to secure thorough compacting in the forms and close contacting with forms and with steel. To this end, forms in relatively shallow lifts are advantageous for puddling and compacting, together with expulsion of entrained air, which can be further facilitated by tapping forms with mauls or mallets, or better yet, with air hammers.

Although shallow lifts are advantageous, provision must be made in their design for rapid addition of higher lifts, since the best construction, with avoidance of work planes, requires continuous deposition, even though day and night work is entailed, preferably without permitting any portion to take even initial set before new concrete is placed upon it. Excess water is a potent source of trouble in constructions of this character, as through its use, laitance in bands and pockets is encouraged.

Where continuous placement of concrete is impossible for one reason or another, vertical diaphragms of sheet metal may be embedded in the concrete as work joints. Sheet copper, with asphalt or asphalt mastic filler, can be made to give water-tight construction, but the sightliness of the standpipe is usually impaired by the irregular belt of friable and easily weathered material at each division.

ELEVATED TANKS

Elevated tanks may be classed in groups according to the type of floor employed: (*a*) suspended, (*b*) flat, (*c*) beam-and-slab, (*d*) dam, or (*e*) double-dome.

The suspended bottom was employed in the Middleboro tank[2] (see Fig. 64). It was designed in a manner similar to the same type of steel tank. For the analysis of such a bottom, see Ketchum's "Structural Engineers' Handbook," page 366.

Only small tanks have been constructed with flat circular floors. Such construction follows that for flat slab floors. Floors of beams and slab are also designed after similar building floors.

When the floors restrain the sides of the tank, the negative moment at the base of the wall may be provided for in a manner similar to that employed for standpipes.

Dome floors are common in Europe, and some have been built in this country. The analysis of the double-dome floor follows. Tanks with only the curved dome have been built. It is reported that trouble has been experienced due to restraint between walls and dome, and to the lateral thrust of the dome. This difficulty may be eliminated by using the same unit stress in the steel in the ring of the dome that is used in the hoops of the sides. Then the dome ring will expand under load an amount equal to that of the sides and there will be no restraint between the two. The analysis of a single dome will be that for a similar portion in the following analysis.

27. Analysis of Stresses.—The roof A (Fig. 65) is a simple dome, and may be designed as a thin spherical segment. In this case, all stresses are assumed to be tangential to the spherical surface.

Assuming a uniformly distributed load of w lb. per sq. ft. on a horizontal projection, which will include the weight of the dome since it is relatively flat, the vertical component per foot of rim is $w\frac{D}{4}$. If the tangent to the shell at a vertical diameter makes the angle ϕ' with the horizon-

[1] *Eng. News*, Dec. 23, 1915.
[2] *Eng. & Contg.*, vol. 44, p. 473, Dec. 22, 1915.

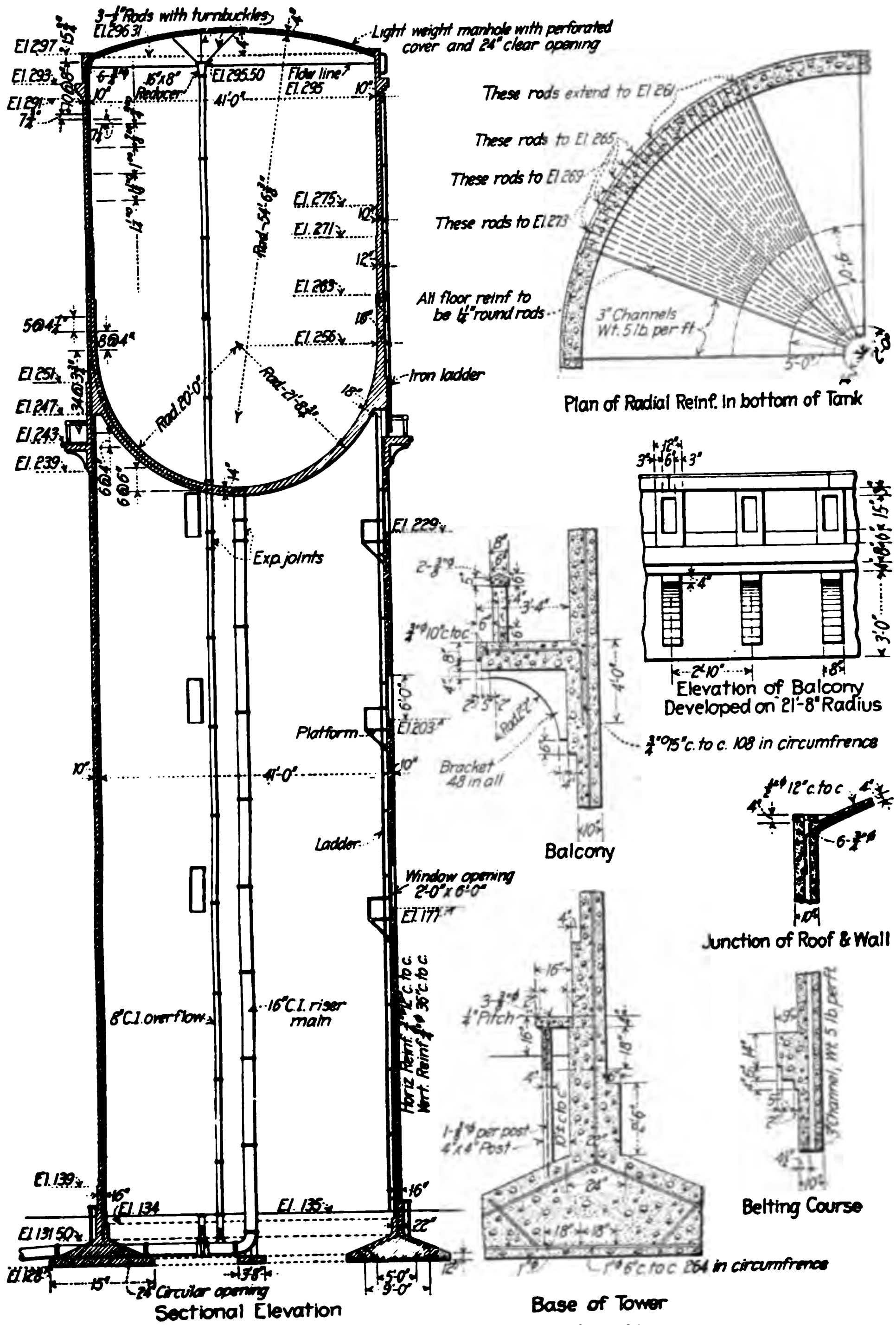


Fig. 64.—Details of tower tank at Middleboro, Mass.

tal, the outward pressure T_1 per foot of rim is $\frac{WD}{4 \tan \phi'}$ (since $V_1 + T_1 = \tan \phi'$). The thrust tangent to the dome at its rim will be the resultant of these two forces, or $\sqrt{T_1^2 + V_1^2}$. Taking a value of f_c in pounds per square inch and letting t = thickness of shell,

$$12\,t f_c = \sqrt{T_1^2 + V_1^2}$$

$$t = \frac{1}{12 f_c}\sqrt{T_1^2 + V_1^2}$$

It will be found that for diameters D less than 20 ft. the controlling factors in determining the thickness will be practicability of building the thin shell, and provision for accidental punching shear. Fairly satisfactory proportions for large diameters, remembering these limitations, will be obtainable by assuming a low concrete unit stress, say 100 lb. per sq. in. Metal fabric or lath should be used as an added provision against possible concentrations. Vertical shear at the base ring should be computed.

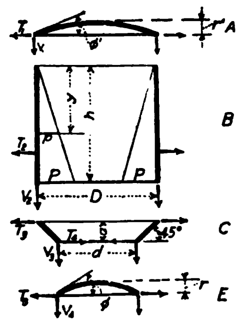


FIG. 65.

The thrust T_1 should be resisted by a ring of reinforcement at the top of the wall. The tension S in this ring will be

$$S = \frac{T_1 D}{2}$$

Tan ϕ' may be found from the relation

$$\tan\frac{\phi'}{2} = \frac{2r'}{D}$$

The steel required for the hoop will be

$$A_s = \frac{S}{f_s} = \frac{T_1 D}{2 f_s}$$

The tensile unit stress in the steel should not exceed about 8000 lb. per sq. in.

ILLUSTRATIVE PROBLEM.—D = 40 ft., r' = 4 ft. Adopted load of 150 lb. per sq. ft. on horizontal projection, including dead load. Let f_s = 5000.

$$V_1 = \frac{wD}{4} = \frac{150 \times 40}{4} = 1500 \text{ lb. per ft.}$$

$$\text{Tan}\,\frac{\phi'}{2} = \frac{1}{5}, \text{ whence } \frac{\phi'}{2} = 11°18'$$

$$\phi' = 22°36' \qquad \tan\phi' = 0.416.$$

$$T_1 = \frac{V_1}{\tan\phi'} = \frac{1500}{0.416} = 3600 \text{ lb. per ft.}$$

$$t = \frac{1}{12 f_c}\sqrt{T_1^2 + V_1^2} = \frac{1}{12 \times 100}\,(3920) = 3.3, \text{ say } 3\tfrac{1}{2} \text{ in.}$$

$$S = \frac{3600 \times 40}{2} = 72{,}000 \text{ lb.}$$

$$A_s = \frac{72{,}000}{5000} = 14.5 \text{ sq. in., say 14 1-in. sq. rods.}$$

A beam may be cast around the top of the tank in which half of the steel may be encased. The remainder may be put in the dome itself near the rim.

The shell of the tank B (Fig. 65) is designed like the sides of a standpipe. Reinforcement consists of rings, and sufficient vertical steel to support them.

The conical portion C carries a vertical load of V_2 per foot of upper rim, which is equal to the dead weight of the tank shell and the roof. In addition it carries a normal water pressure of wh lb. per sq. ft. at the upper rim, and one of $w\,(h + a)$ lb. per sq. ft. at its lower rim. The sides of the cone are assumed to slope at 45 deg. The shear per lineal foot of the bottom rim is equal to the weight of the structure above this level plus the weight of water above the conical portion, divided by the perimeter of the rim. Since the slope of the cone is 45 deg., the thrust T_4 has the same value per lineal foot as V_3. Likewise the thrust at the upper rim, T_3, is equal to the shear V_2 at that level, or the weight of structure above it. Two rings may be used, one to resist T_3, and one to resist T_4, with reinforcement in the slab to span between these rings; or, rings of steel rods may be distributed somewhat, through the conical side, to aid the resistance of the sloping rods spanning between rings. This will be modified with respect to the action of T_4 when the dome thrust T_4 is determined. Vertical shear at the base should be computed and provided for.

The dome of the floor has a vertical shear at its perimeter equal to its weight plus the weight of water directly above it. This shear is the vertical component of the thrust at the base ring acting tangent to the same surface, or at the angle ϕ. Hence,

$$T_5 = V_4 \cot \phi$$

This is the ring thrust acting at the base of the dome.

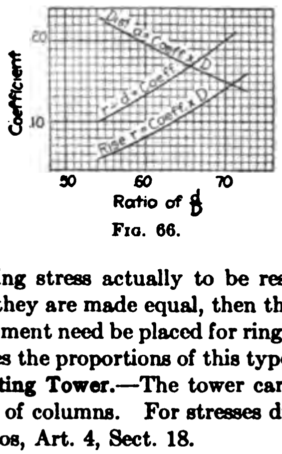


FIG. 66.

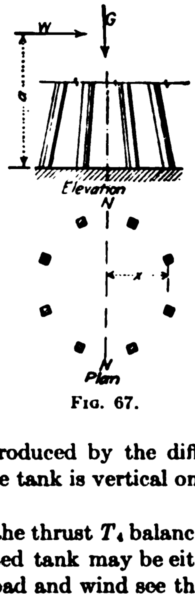


FIG. 67.

Now the ring stress actually to be resisted is that produced by the difference between T_4 and T_5. If they are made equal, then the reaction of the tank is vertical only at this point, and no reinforcement need be placed for ring tension.

Fig. 66 gives the proportions of this type of floor when the thrust T_4 balances the thrust T_5.

28. Supporting Tower.—The tower carrying an elevated tank may be either a cylindrical shell or a group of columns. For stresses due to vertical load and wind see the design of deep grain bins or silos, Art. 4, Sect. 18.

A tower formed of a group of columns is shown in Fig. 67, acted upon by a total wind pressure of W lb. at a height of a in. Let the distance from the axis of the group to the column farthest out be x in. Then the unit stress at this distance x due to W will be

$$f_x = \frac{Wax}{I_c + nI_s} + \frac{G}{cA}$$

assuming little or no tension to exist on the section under the action of W and G. The number of columns is denoted by c. This unit stress must be multiplied by the area A of the column to obtain its load. Other columns will carry a similar stress when the wind is in another direction.

Investigation of compressive stress due to wind should be made on the leeward side of the tank at its base and at other sections when shear is vital.

Bracing in reinforced-concrete towers consists usually of horizontal struts only, designed to resist moment. The analysis of stresses in a bent of the tower may be made by the use of the method of slope-deflections (see Sect. 10).

Fig. 68[1] shows an elevated tank with double-dome bottom, built at Kitchener, Ont. It forms an excellent example of this type of structure.

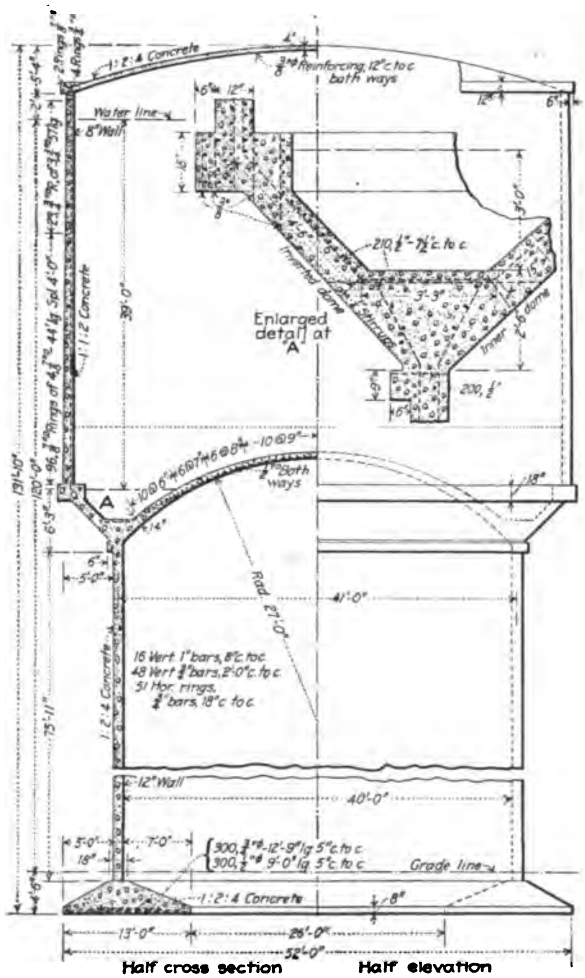


FIG. 68.—Tank for water-works at Kitchener, Ont.

CULVERTS

29. General Considerations.—The term culvert is usually applied to structures built to carry surface water or small streams through highway or railroad embankments. When the area of waterway required is comparatively small, a pipe culvert is usually the most economical. For the larger openings either the box or arch form should be employed depending upon the avail-

[1] *Eng. News*, vol. 69, p. 309, Feb. 13, 1913.

able headroom, the depth of fill, the condition of the foundation, and whether or not an artistic arch design is especially desirable.

The ordinary type of arch culvert and the box culvert without a load-supporting floor (called *open-box* culvert) are in reality small bridges, and it is sometimes a question of how large such structures may become before they should be considered strictly in the bridge class. No arbitrary division is adhered to in the following articles except that a culvert is considered a structure which can be completely and economically standardized, based on a given area of waterway and height of embankment.

30. Factors in Culvert Design.

30*a*. Culvert Efficiency.—A culvert to be efficient in the amount of water it can discharge should have its headwalls or wings arranged so as to facilitate the flow, and its bed should be considerably inclined for those cases where the channel below the culvert will permit the water to flow away freely. If any well-defined stream bed exists, the bed of the culvert should have the same inclination as that of the stream, as otherwise either the outlet or inlet end will clog depending upon whether the slope of the culvert is greater or less respectively than the slope of the stream bed.

Any projections in the culvert bed should be avoided as they will retard the water and diminish efficiency. It is also important that culverts be placed across roadways in the direction of the stream flow since, if this is not done, clogging and subsequently washouts will be likely to occur.

A culvert will discharge twice as much under a head of 4 ft. as under a head of 1 ft., but water should not be allowed to dam up in this manner unless the culvert is well constructed through a water-tight embankment.

30*b*. Waterway Required.—Assuming an efficient culvert design, the area of waterway required depends principally upon the maximum rate of rainfall, the area and shape of the watershed, the kind and condition of the soil throughout this watershed area, and the character and inclination of both drainage surface and stream bed. A number of empirical formulas have been proposed by which to calculate the required culvert opening, but obviously a problem of this kind does not admit of an exact mathematical solution and the desired size of culvert should be determined by direct observation whenever that is possible.

In a new country an empirical formula is often the only method by which the required area of waterway can be determined. Talbot's formula is the one most generally employed and is as follows:

$$A = C\sqrt[4]{a^3}$$

where A = area of waterway in square feet, a = drainage area in acres, and C is a coefficient which varies from 1 to $\frac{1}{6}$ in the following manner:

For steep and rocky ground, C varies from $\frac{2}{3}$ to 1. For rolling agricultural country subject to floods at times of melting of snow, and with the length of valley three or four items its width, C is about $\frac{1}{3}$; and if the stream is longer in proportion to the area, decrease C. In districts not affected by accumulated snow, and where the length of the valley is several times the width, $\frac{1}{5}$, $\frac{1}{6}$, or even less, may be used. C should be increased for steep side slopes, especially if the upper part of the valley has a much greater fall than the channel at the culvert.[1]

The proper area of waterway can best be determined by knowing the dimensions of existing openings on the same stream and by careful observation of the stream and the amount of water which it carries at flood times. This amount of water can be determined by measuring a cross-section of the stream at some narrow place near the culvert site.

30*c*. Length of Culverts.—The length of a culvert should depend upon the width of roadway and the depth of fill on top of the culvert. The slope of an earth fill can generally be taken as $1\frac{1}{2}:1$—that is, for every 1 ft. in height, the horizontal distance is $1\frac{1}{2}$ ft.

In highway construction the roadway should never be narrowed at a culvert since such a practice is dangerous and the construction unsightly.

[1] Selected papers of the Civil Engineers' Club of the University of Illinois, No. 2, p. 14.

30d. Design of Ends.—The arrangement of the headwalls or wings may be substantially the same for all arch and box culverts. The arrangement should be such that the embankment is protected and the flow of water facilitated. Wing walls may be built parallel with or at right angles to the axis of the culvert, or they may be so placed as to make an angle (usually 30 or 45 deg.) with this axis.

For the shorter spans (including spans for pipe culverts), wings parallel with the roadway are generally used for low fills and, in highway construction at least, these headwalls are carried up above the grade line to provide a low guard rail. This type of end, however, is not economical for the larger spans since the straight wings under such conditions need to be made of considerable length and height to retain the fill efficiently. A low guard rail may be formed with flared wings by raising and coping both head and wingwalls. The top of the wings, of course, should have a slope consistent with the slope of the earth fill.

Flared wings, especially at the upstream end, are the best for hydraulic reasons and, when used, the culvert is less likely to become choked than when either of the other two forms of wingwalls are employed. Straight wings—namely, wings parallel to the axis of the culvert—are of advantage in railroad construction when an extension of the culvert is likely to be made in the near future to accommodate another track.

It is common practice to design wing and headwalls of sufficient length to keep the culvert opening clear when the earth is assumed to fall around the ends on a $1\frac{1}{2}:1$ slope. In some cases a steeper slope could be assumed, but some soils take even flatter slopes than the standard.

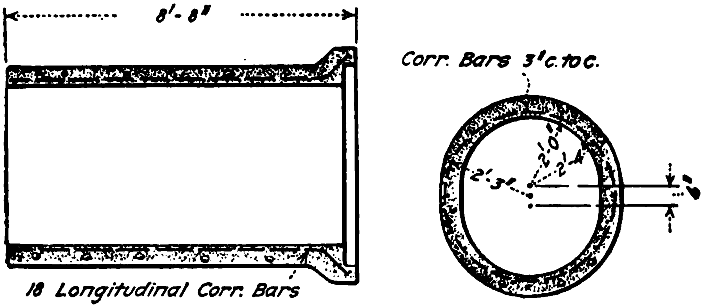


FIG. 69.—Reinforced-concrete pipe.

Box and arch culverts are built both with and without a floor, but in almost every case the smooth waterway that can be obtained by using a concrete floor will greatly increase the capacity of the culvert. A floor, if properly constructed, will also prevent any danger from erosion of the stream bed and undermining of the foundation. The floor at the ends of the culvert should be provided with an apron or baffle wall at its outer edge, and this wall should in all cases be carried as low as the bottom of the footings. If especially desirable, the floor should extend out to the end of the wingwalls.

31. Pipe Culverts.—One or more lines of pipe with suitable headwalls to protect the embankment is the simplest form of culvert. The pipe may be of burned clay, cast iron, plain concrete, or reinforced concrete; but, on account of frequent breakages, there seems to be a tendency at the present time to discontinue the use of vitrified and cast-iron pipe and also pipes of plain concrete. All kinds of pipe culverts have the same type of concrete headwalls, consequently the following articles treat only of pipe culverts of reinforced concrete.

Since it is desirable to make as few openings as possible through an embankment, water is usually conducted along the side of the roadway until at least a 15-in. diameter of pipe is required. In the following articles a pipe of at least this size is assumed.

Reinforced culvert pipes are usually made in from 4- to 8-ft. lengths, and with bell and spigot joints. The largest diameter of pipe yet made is 72 in. The pipes usually have a hoop reinforcement which is near the interior surface at the top and bottom of the pipe, and near the exterior surface at the sides (Fig. 69). Pipe with a double line of reinforcing is also used, as shown in Fig. 70. Longitudinal reinforcement is provided in pipes of the longer lengths due to the likelihood of beam action if settlement takes place.

31a. Pressure in Trenches.—The most elaborate and thorough investigation of the subject of pressure on pipes in ditches was made a few years ago at the Iowa State College of Agriculture and Mechanic Arts, the results of which were published in *Bulletin* No. 31[1]

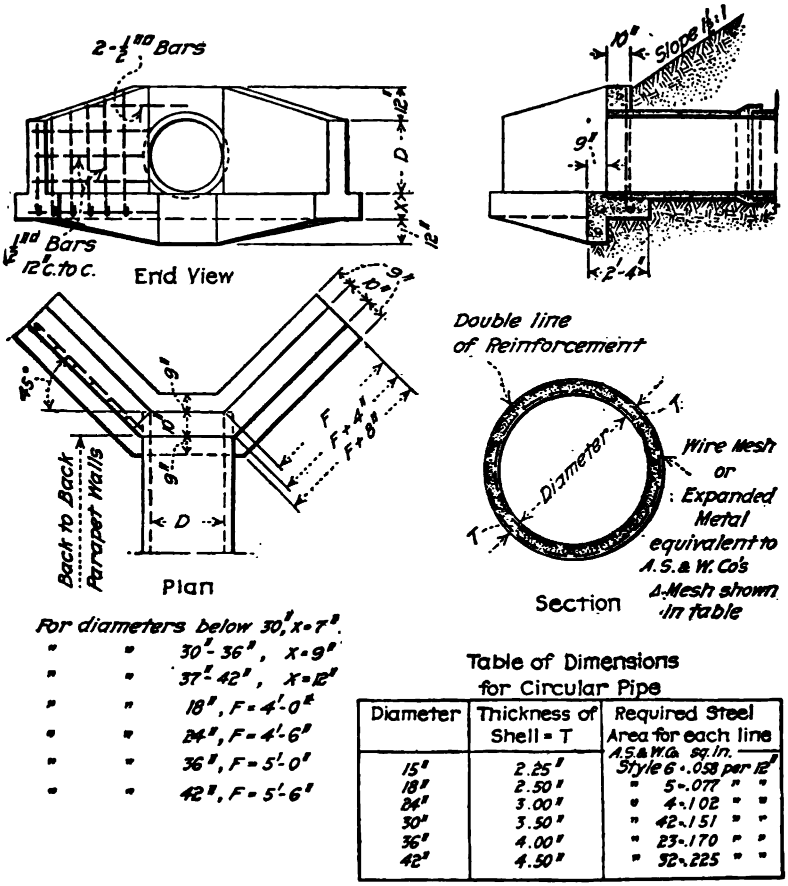


Table of Dimensions for Circular Pipe

Diameter	Thickness of Shell = T	Required Steel Area for each line A.S.&W.Co. sq. in.
15"	2.25"	Style 6 - .058 per 12"
18"	2.50"	" 5 - .077 " "
24"	3.00"	" 4 - .102 " "
30"	3.50"	" 42 - .151 " "
36"	4.00"	" 23 - .170 " "
42"	4.50"	" 32 - .225 " "

FIG. 70.—Standard dimensions for concrete pipe culverts with concrete head-walls, Iowa Highway Commission.

of the Engineering Experiment Station. The investigation was made with special reference to drain tile and sewer pipe, but the results apply equally well to culverts laid in trenches. The following tables and other data were taken from the bulletin above mentioned. The correctness and reliability of the theory which was developed by reason of this investigation were demonstrated with remarkable closeness by actual weighings of loads on pipes, the pipes ranging from 12 to 36 in. in internal diameter placed in ditches from 0 to 19 ft. in depth.

[1] Written by PROF. ANSON MARSTON and MR. A. O. ANDERSON.

TABLE 1.—APPROXIMATE MAXIMUM LOADS IN POUNDS PER LINEAR FOOT, ON PIPES IN DITCHES FROM COMMON DITCH-FILLING MATERIALS

H = height of fill above top of pipe	B = breadth of ditch a little below top of pipe									
	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.	1 ft.	2 ft.	3 ft.	4 ft.	5 ft.
	Partly compacted damp top soil 90 lb. per cu. ft.					Saturated top soil 110 lb. per cu. ft.				
2 ft.	130	310	490	670	830	170	380	600	820	1,020
4 ft.	200	530	880	1,230	1,580	260	670	1,090	1,510	1,950
6 ft.	230	690	1,190	1,700	2,230	310	870	1,500	2,140	2,780
8 ft.	250	800	1,430	2,120	2,790	340	1,030	1,830	2,660	3,510
10 ft.	260	880	1,640	2,450	3,290	350	1,150	2,100	3,120	4,150
	Dry sand, 100 lb. per cu. ft.					Saturated sand, 120 lb. per cu. ft.				
2 ft.	150	340	550	740	930	180	410	650	890	1,110
4 ft.	220	590	970	1,360	1,750	270	710	1,170	1,640	2,100
6 ft.	260	760	1,320	1,890	2,480	310	910	1,590	2,270	2,970
8 ft.	280	890	1,590	2,350	3,100	340	1,070	1,910	2,820	3,720
10 ft.	290	980	1,820	2,720	3,650	350	1,180	2,180	3,260	4,380
12 ft.	300	1,040	2,000	3,050	4,150	360	1,250	2,400	3,650	4,980
14 ft.	300	1,090	2,140	3,320	4,580	360	1,310	2,570	3,990	5,490
16 ft.	300	1,130	2,260	3,550	4,950	360	1,350	2,710	4,260	5,940
18 ft.	300	1,150	2,350	3,740	5,280	360	1,380	2,820	4,490	6,330
20 ft.	300	1,170	2,420	3,920	5,550	360	1,400	2,910	4,700	6,660
22 ft.	300	1,180	2,480	4,060	5,800	360	1,420	2,980	4,880	6,960
24 ft.	300	1,190	2,540	4,180	6,030	360	1,430	3,050	5,010	7,230
26 ft.	300	1,200	2,570	4,290	6,210	360	1,440	3,090	5,150	7,460
28 ft.	300	1,200	2,600	4,370	6,390	360	1,440	3,120	5,240	7,670
30 ft.	300	1,200	2,630	4,450	6,530	360	1,440	3,150	5,340	7,830
Infinity	300	1,210	2,730	4,850	7,580	360	1,450	3,270	5,820	9,090
	Partly compacted damp yellow clay, 100 lb. per cu. ft.					Saturated yellow clay, 130 lb. per cu. ft.				
2 ft.	160	350	550	750	930	210	470	730	1,000	1,240
4 ft.	250	620	1,010	1,400	1,800	340	840	1,330	1,870	2,370
6 ft.	300	830	1,400	1,990	2,580	430	1,140	1,900	2,630	3,410
8 ft.	330	990	1,720	2,500	3,250	490	1,380	2,360	3,360	4,400
10 ft.	350	1,110	2,000	2,920	3,880	520	1,570	2,760	3,980	5,270
12 ft.	360	1,200	2,220	3,320	4,450	540	1,730	3,100	4,560	6,050
14 ft.	370	1,280	2,410	3,650	4,950	560	1,850	3,410	5,050	6,760
16 ft.	370	1,330	2,570	3,950	5,400	570	1,940	3,660	5,510	7,440
18 ft.	380	1,390	2,710	4,210	5,810	570	2,020	3,880	5,930	8,060
20 ft.	380	1,410	2,830	4,450	6,180	580	2,090	4,070	6,280	8,610
22 ft.	380	1,430	2,920	4,640	6,500	580	2,140	4,240	6,610	9,130
24 ft.	380	1,450	3,000	4,820	6,800	580	2,180	4,380	6,910	9,590
26 ft.	380	1,470	3,060	4,980	7,080	580	2,210	4,500	7,160	10,010
28 ft.	380	1,480	3,120	5,100	7,310	580	2,240	4,610	7,380	10,430
30 ft.	380	1,490	3,170	5,230	7,530	580	2,260	4,700	7,590	10,780
Infinity	380	1,520	3,410	6,060	9,480	580	2,340	5,270	9,360	14,620

The width of the ditch above the pipe makes practically no difference in the load on the pipe, which is just as great for a vertical ditch as for one several times as wide at the top but of the same width a little below the top of the pipe.

In ditches of proportions customary in actual work, the diameter of the pipe used in any particular ditch of a fixed given width makes practically no difference in the load on the pipe. A 12-in. pipe will have to carry the same load as an 18-in. pipe, if both are placed in ditches 2 ft. wide under other similar conditions.

In case a wide ditch is necessary for constructive reasons, the load on the pipe can be diminished greatly, in firm soil, by stopping the wide ditch a few inches above the top of the pipe and digging in the bottom the narrowest ditch practicable to receive the pipe, making bell holes at the side for the sewer pipe, if necessary.

Grades or fills built over the surfaces of completed ditches, and piles of sand, gravel, and other materials having internal friction, operate to increase the loads on pipes in ditches to the same extent as an equal added height of ditch filling, for a breadth of ditch equal to that at a little below the top of the pipe.

A superload is any load applied to the upper surface of the ditch filling, except loads from fills or heaps of granular materials. A long superload is one extending a considerable length along a ditch, as compared with its depth and breadth, and may be caused by piles of paving brick, lumber, etc., over the ditch. Long superloads on completed ditches cause increases in the loads on pipes in ditches by percentages of the superload which decrease as depth increases, and safe values for which can be computed by Table 2. Table 2 has been closely checked by actual weighings of the increase in loads on pipes in ditches due to super loads.

Table 2.—Approximate Safe Values of C to Use in Formula $L = CL_1$

L = loads per unit of length, on pipes in ditches, due to L_1.
L_1 = long superloads on ditches, per unit of length.

$\frac{H}{B}$	Sand and damp top soil	Saturated top soil	Damp yellow clay	Saturated yellow clay
0.0	1.00	1.00	1.00	1.00
0.5	0.85	0.86	0.88	0.89
1.0	0.72	0.75	0.77	0.80
1.5	0.61	0.64	0.67	0.72
2.0	0.52	0.55	0.59	0.64
2.5	0.44	0.48	0.52	0.57
3.0	0.37	0.41	0.45	0.51
4.0	0.27	0.31	0.35	0.41
5.0	0.19	0.23	0.27	0.33
6.0	0.14	0.17	0.20	0.26
8.0	0.07	0.09	0.12	0.17
10.0	0.04	0.05	0.07	0.11

A short superload is one extending a short distance along a ditch as compared with the breadth and depth, and may come from the wheels of wagons, traction engines, steam road rollers, etc. Short superloads, on completed ditches, cause increases in the loads on pipes in ditches by percentages of the superload which decrease as the depth increases, and safe values which can be estimated, but not very reliably, by Table 3. Table 3 has not been checked by actual weighings of increase of loads on pipes in ditches.

Table 3.—Approximate Safe Values for C to Use in Formula $L = CL_1$

L = loads per unit of length, on pipes in ditches directly under L_1, due to L_1. L_1 = short superloads on ditches, per unit of length, of length A along ditch.

$\frac{H}{B}$	Sand and damp top soil				Saturated top soil				Damp yellow clay				Saturated yellow clay			
	$K_s = \frac{1}{2}K$		$K_s = K$		$K_s = \frac{1}{2}K$		$K_s = K$		$K_s = \frac{1}{2}K$		$K_s = K$		$K_s = \frac{1}{2}K$		$K_s = K$	
	$A =$		$A =$		$A =$		$A =$		$A =$		$A =$		$A =$		$A =$	
	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$	B	$\frac{B}{10}$
0.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.10
0.5	0.77	0.32	0.70	0.12	0.78	0.33	0.71	0.13	0.79	0.34	0.72	0.13	0.81	0.34	0.74	0.13
1.0	0.59	0.11	0.49	0.02	0.61	0.11	0.51	0.02	0.63	0.11	0.52	0.02	0.66	0.12	0.55	0.02
1.5	0.46	0.03	0.34	....	0.48	0.04	0.36	....	0.51	0.04	0.38	....	0.54	0.04	0.40	..
2.0	0.35	0.01	0.24	....	0.38	0.01	0.26	....	0.40	0.01	0.27	....	0.44	0.01	0.30	...
2.5	0.27	.....	0.17	....	0.29	.....	0.18	....	0.32	.....	0.20	....	0.35	.....	0.22	.
3.0	0.21	.....	0.12	....	0.23	.....	0.13	....	0.25	.....	0.14	....	0.29	.....	0.16	...
4.0	0.12	.....	0.06	....	0.14	.....	0.07	....	0.16	.....	0.08	....	0.19	.....	0.09	...
5.0	0.07	.....	0.03	....	0.09	.....	0.03	....	0.10	.....	0.04	....	0.13	.....	0.05	.
6.0	0.04	.....	0.01	....	0.05	.....	0.02	....	0.06	.....	0.02	....	0.08	.....	0.03	.
8.0	0.02	.....	....	....	0.02	.....	....	....	0.03	.....	0.01	....	0.04	.....	0.01	...
10.0	0.01	.....	....	...	C.01	.....	....	....	0.01	.....	....	....	0.02	.....	....	....

H = height of fill in ditch, above top of pipe.
B = breadth of ditch, a little below top of pipe.
K = ratio of lateral pressure to vertical in the ditch filling.
K_a = ratio of longitudinal pressure to vertical in the ditch filling.
Values of C for K_a = 0 are given in Table 2.

The formula $L = CL_1$ holds true only directly under L_1. Beyond L_1 in either direction, the intensity of load on the pipe diminishes rapidly.

Calculations made from Table 3 are not very reliable since we are usually uncertain as to the proper value to take for K_a, and there is great need of a series of tests of the actual loads on pipes caused by short superloads, but such tests would be very difficult to make and test results are not available.

In the meantime, Table 3 will be of some value to engineers of good judgment in assisting them to make reasonable safe allowances for the probable effect on the loads on pipes in ditches from heavy concentrated loads on wagon wheels, traction engines, and road rollers.

31b. Strength of Pipe.—The theoretical analysis of stresses in culvert pipe is that of thin elastic rings and is similar to the general method employed for arches. The difference in the intensity of the load at the crown and at the extremities of the horizontal diameter, due to the difference in the depths of the earth, is considered negligible, and the pressure and its distribution on the lower half of the ring is assumed to be the same as that on the upper half.

Theory gives the following values of the bending moment at the top and bottom sections of a pipe:

I. For single concentrated load (top and bottom), $M = 0.159Pd$.

II. For total uniformly distributed load over entire horizontal projection (top and bottom), $M = 0.0625Wd$.

III. For a uniformly distributed load over the top fourth of the circumference and with the pipe supported on its bottom quarter circumference, $M = 0.0845Wd$. Where

d = diameter of pipe.
P = concentrated load at top.
W = total uniformly distributed load above horizontal diameter.
M = bending moment in pipe in a unit length.

The bending moments at the ends of the horizontal diameters under the above conditions of loading are:

$$\text{I. } M = -0.091Pd$$
$$\text{II. } M = -0.0625Wd$$
$$\text{III. } M = -0.077Wd$$

The above moments will be reduced for any lateral restraint or lateral pressure. In fact for equal uniform horizontal and uniform vertical forces (which may be considered equivalent to a uniform radial pressure) the moments due to the lateral forces have equal but opposite signs to those given for Case II above, and it can be proved that the total moments at all points are zero. It is not good practice, however, to rely on any lateral restraint or pressure in the analysis of the strength of pipes. Mathematical analysis shows that the weight of pipes causes only five-eighths as much bending moment at the lowest point of the pipe as does an equal weight of earth.

Since the exact load and the nature of its distribution over pipe surface is usually uncertain, the probable range of bending moments under actual conditions of construction is all that laboratory tests can be expected to furnish. In a series of tests at the University of Illinois,[1] reinforced-concrete rings and circular pipe (48-in. internal diameter and 4 in. thick) were tested for concentrated loads at the top and bottom of the vertical diameter (Case I above), and for uniformly distributed loads above and below the entire horizontal diameter (Case II above). This latter loading was obtained by placing the pipe in a tight box so as to be entirely encircled with sand and then applying a load to the top surface of the sand. The reinforcement for most of the rings tested consisted of ¼-in. corrugated rods placed near the intrados at top and bottom and near the extrados at the sides. The rings were only 24 in. long, but the pipe

[1] See *Bulletin* 22 of the Engineering Experiment Station of the University of Illinois, written by Prof. A. N Talbot.

sections were from 102 to 104 in. in length with the usual bell and pigot points. To allow the circumferential reinforcement in the pipes to be circular in shape, the pipe cross-sections (Fig. 69. were made with the vertical diameter 4 in. longer than the horizontal diameter, thus bringing the reinforcement at the points of tension in the loaded pipe. Using the yield point of the steel in the common formula for the bending resistance of a reinforced section, there was found

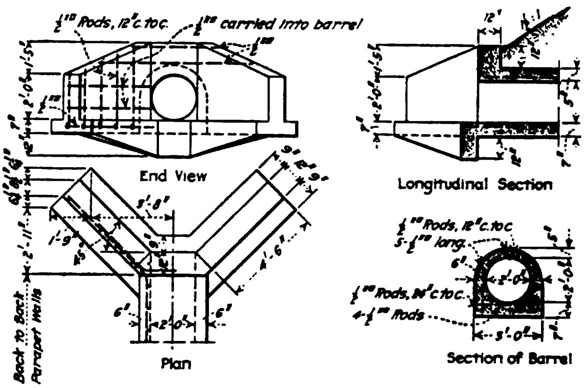


Fig. 71.—Standard design for 24-in. circular culvert, Iowa Highway Commission.

a close agreement between the theoretical and experimental values for the strength of pipe under these two methods of loading.

Marston and Anderson, in the investigation referred to in the preceding article, came to the conclusion that "the typical field bedding and loading of pipes in ditches are such that their effect on the pipe can be reproduced with practical exactness in laboratory tests by bedding

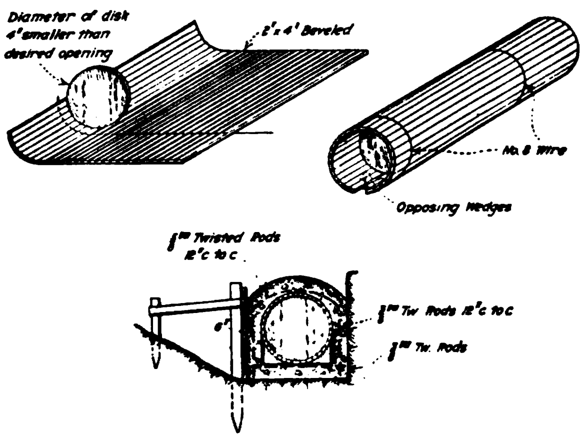


Fig. 72.—Collapsible, adjustable wood form for concrete culverts from 18 to 48 in. diameter.

the pipes in sand during the tests for 90 deg. of the circumference at the bottom and also for 90 deg. at the top." This method of loading is Case III above. In the Iowa investigation the weight of the pipe as well as that of the backfilling was taken into consideration.

31c. Circular Culverts Cast in Place.—A cast-in-place culvert with circular opening is shown in Fig. 71. Fig 72 shows an adjustable, collapsible wood form which can be very economically used for culverts of this type.

The method of constructing and using this form is described as follows in a booklet entitled "Small Concrete Bridges and Culverts," published by the Universal Portland Cement Co.:

This form is constructed of two by four's beveled and strung on wires, as shown in Fig. 72. The number of staves to be used, varying with the size of the culvert, are placed side by side with a wire drawn through each end of the stave as shown. The form is then rolled around a circular head size of the proposed culvert and wire bands are tied tightly around it on the outside. Wedges are then driven as shown in Fig. 72 to hold the staves firmly in position. After the culvert has been built the wedges are removed and the circular heads knocked in; the staves will

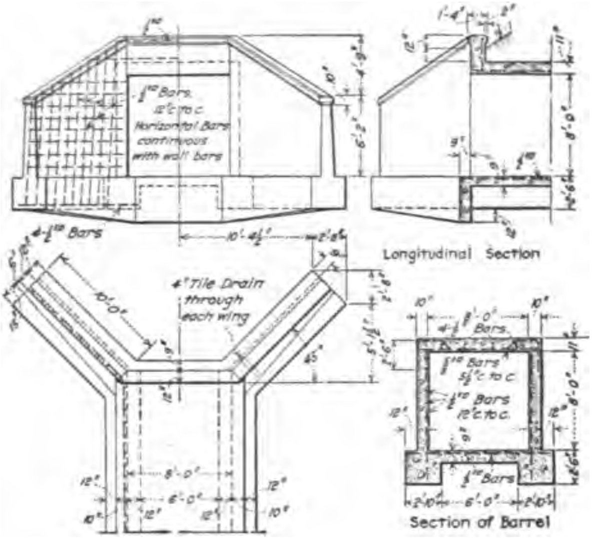


Fig. 73.—Standard design for 8 × 8-ft. box culvert, Iowa Highway Commission.

then collapse and are easily removed. This form can be used over and over again and Mr. Gearhart states that its cost should not exceed $15 or $20.

32. Box Culverts.—The box type of culvert is especially adapted to locations where the headroom is limited and, when planned for such locations, has a great advantage over the arch. A culvert of this type, for example, can be built with less excavation and less disturbance to the embankment and will give a greater distribution of load upon the foundation than the

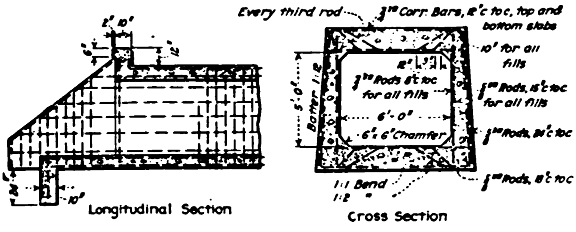


Fig. 74.—Standard culvert of 6-ft. span, Hampden R. R.

ordinary form of arch culvert. Also, the formwork for this type is much simpler and the cost of construction correspondingly lower, except perhaps in some cases where the number and size of culverts to be constructed warrants the use of commercial arch forms of collapsible steel. Box culverts are not always employed only under shallow fills, as is evident from Fig. 76.

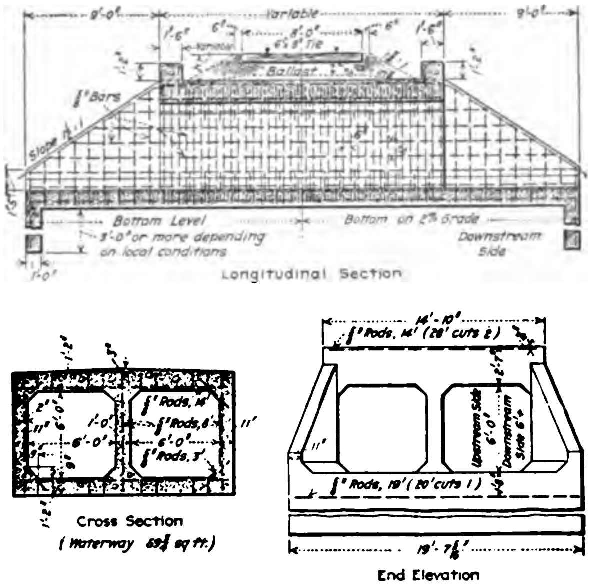


Fig. 75.—Double 6 × 6-ft. box culvert, A. T. & S. Fe. Ry. system.

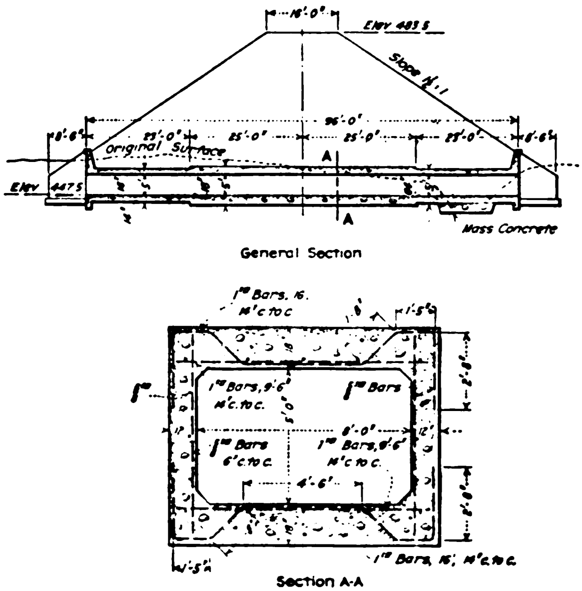


Fig. 76.—Details of box culvert on lines of Eastern-Texas Traction Co.

32a. Forms of Box Culverts.—There are two distinct forms of box culverts which may be distinguished by the terms *open-box* and *closed-box*. In the open-box (Fig. 73) the side walls have dependent footings, while in the closed-box (called simply box) the load is supported by the floor (Fig. 74). Double-box culverts (Fig. 75) are generally used where the span is equal to, or greater than, twice the height, as the use of a single box beyond these proportions greatly increases the cost.

32b. Loading.—For small box culverts built in trenches the load on the top slab may be approximately estimated by means of the tables in Art. 31a. For large box culverts and for all culverts not built by trench construction, no allowance should be made for the arching action of the material, which means that such culverts should be proportioned to carry the entire weight of the fill above the cover slab. The lateral pressure of the earth (including live-load surcharge) is usually assumed as that due to a fluid weighing about 30 lb. per cu. ft.—that is, a weight about one-fourth the weight of earth. It is obvious that the pressure due to live load does not spread out through the filling at the ordinary angle of repose of the material, but has a side slope, or line of zero stress, much more nearly vertical. It is frequently assumed that the live load is carried down at a slope of ½ horizontal: 1 vertical. In railroad embankments this slope is taken from the ends of the ties.

An allowance is usually made for impact of the live load in the design of railroad culverts. Some designers allow 50% for impact on all banks up to 40 ft. high. A more conservative plan often followed is to allow 100% for fills of less than 2 ft., 75% for fills between 2 ft. and 4 ft., and 50% for all fills over 4 ft.

32c. Design of Cross-section.—The top slab of a box culvert is partially fixed, but it is the general practice to design this slab as simply supported and to reinforce at the corners against negative bending. Negative reinforcement, however, is not always provided (see Fig. 75), in which case the sides and top act as simple beams and more or less cracking occurs on the outside near the corners. The walls or sides of a box culvert are usually designed somewhat empirically, but are always provided with sufficient strength to support the lateral earth pressure, neglecting any outward bending due to the bending of the cover slab. In open-box construction, cross struts are often used to assist in holding the footings against the pressures on the walls and also to provide bearing area in addition to that furnished by wall footings (Fig. 77). The struts are designed as beams with a span equal to the width of the culvert, and the struts are so spaced and proportioned as to obtain a uniform soil pressure throughout. When a bottom slab is employed and assumed to support the load, its thickness is made the same as that of the cover slab since the load for both slabs is substantially the same. Longitudinal reinforcement should be provided on account of the possibility of beam action due to unequal settlement. The amount of this reinforcement should depend upon the character of the foundation. Where the foundations are very bad, it is the practice of some engineers to figure the culvert as a whole to act as a beam. considering the length of beam as 12 times its depth. In long culverts under railroad tracks the load decreases beyond the ends of the ties and the cross-section should decrease whenever a material saving is found to result.

In Fig. 77, which illustrates a standard box culvert adopted by the Chicago, Milwaukee & St. Paul Railway, the cover is designed as a simply supported slab with span equal to clear span when fillets are used and to clear span plus one-half the maximum cover thickness (but not to exceed 1 ft.) when no fillets are provided. Stirrups and bent rods are employed to take care of two-thirds of the shear when the shear exceeds 40 lb., bent rods being also used to care for any negative moments which may develop due to connection with side walls. Longitudinal steel is employed with a sectional area of about $\frac{1}{150}$ of the area of the entire concrete section. The side walls, cross struts, and footings are proportioned in the same manner as above described. Keyways are formed in top of footings and side walls so as to offer shearing resistance to movement of side walls due to lateral earth pressure. For fills up to 40 ft., the load on the footings is assumed to include the live load and dead load of culvert, and the fill directly above the culvert for the width overall including footings. For fills over 40 ft. high the total weight resting on the footings is considered as 62½% of the culvert weight plus 62½% of the fill above the footings. The live load is disregarded and, for ease in computation, the weight of culvert is taken as equal to the fill it displaces. Wing walls over 8 ft. in height or making an angle of less than 60 deg. with the headwall are made self-supporting cantilevers with a joint at connection with the barrel. Wingwalls for all other conditions are made continuous with the main part of the culvert. The section of the culvert coming under the track is made approximately equal in length to the theoretical spread of the live load, which is equal to the distance out to out of ties (8 ft. for single track. 21 ft. for double track) plus the height

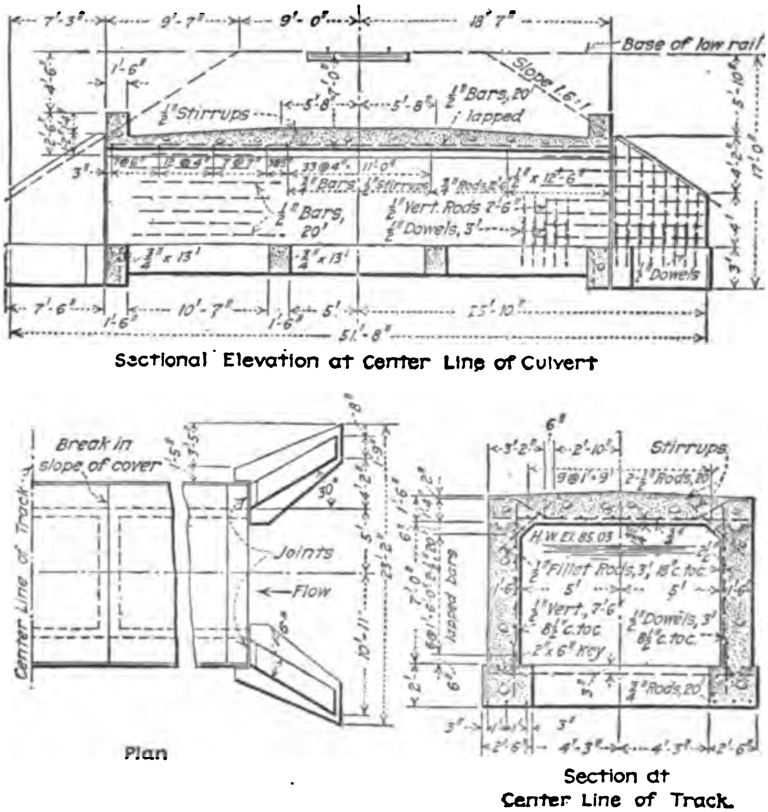


FIG. 77.—Single box culvert, C. M. & St. P. Ry.

of culvert plus one-half the height of fill above the cover slab. The invert is paved with a concrete slab in all spaces between struts so as to form a continuous concrete invert.

Accurate analysis may be made of the moments in box culverts when such culverts are reinforced so as to act monolithically. The following formulas and diagrams are given so that the moments from such analysis may be readily determined.

The open-box culvert may well be divided into two classes for analysis, each class depending on the material upon which the footings rest. If the structure rests upon rock, cemented gravel, or some other material having a high allowable bearing stress, the plane of the base of each side wall will be held at a practically constant slope; that is, the side may be considered fixed at the bottom. On the other hand, if the allowable bearing pressure of the supporting material is low, the plane of the base will become inclined until the moment at the footing has been reduced to a value that will be resisted by the moment of the upward pressure on the base. Since in such a case the resisting moment is small, an altogether different distribution of moment will occur throughout the structure. It will be slightly on the safe side to regard the sides as hinged at their base.

The following nomenclature will be used:

b = span of frame (c. to c. of side walls).
h = height of frame (c. to c. of top and bottom slabs).
$R = \frac{b}{h}$ = ratio of span to height.
I_C = moment of inertia of cross-section of top or bottom slab (considered alike) for one unit of length of the barrel.
I_D = moment of inertia of cross-section of either side wall (considered alike) for one unit of length of the barrel.
$S = \frac{I_C}{I_D}$ = ratio of the moment of inertia of the horizontal slabs to that of the sides.
w = uniform load per unit area on top or bottom slab.
p = uniform load per unit area on side walls.
P = concentrated load.
H_C = thrust at the center of the top slab.
V_C = zero for symmetrical loading, as is the case in the analysis of culverts.
M_C = moment at the center of the top slab.
M_D and
M_B = moment at center of side walls and center of bottom slab, respectively.
M_a and
M_b = moment at the upper and lower corners, respectively.

Type I. *The Closed Frame.*[1]—

[1] Assume the rectangular culvert of Fig. 78 to be rigidly fixed at the center of the bottom slab. With this condition assumed, a unit length of the culvert may be treated in the same manner as a symmetrical arch with fixed ends (see Art. 18, Sect. 16) and the following expressions result:

$$H_C = \frac{\int my\frac{ds}{I}\int\frac{ds}{I} - \int m\frac{ds}{I}\int y\frac{ds}{I}}{\int y^2\frac{ds}{I}\int\frac{ds}{I} - \left(\int y\frac{ds}{I}\right)^2}$$

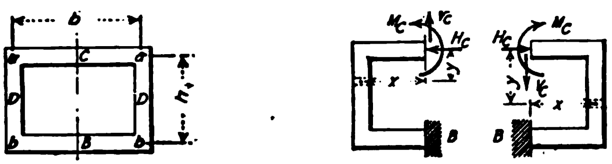


FIG. 78.

$$M_C = \frac{\int m\frac{ds}{I} - H_C\int y\frac{ds}{I}}{\int\frac{ds}{I}}$$

in which

m = moment at any point on either half of culvert of all external loads between point and crown section—all values of m to be substituted in equations as positive.

$\int$ = integration referred to one-half of culvert only.

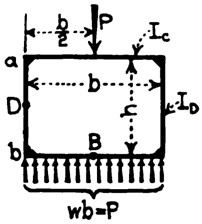
x, y = coördinates of any point.

From these equations were developed the formulas for Types I and II given above.

The bending moment at any point may be expressed as follows:

$$M_C = M_C + H_C y - m$$
$$M_R = M_C + H_C y - m$$

Case 1.—



$$H_C = \frac{1}{8} P \cdot \frac{R^2}{3R + S}$$

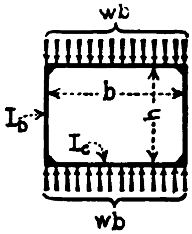
$$M_C = \frac{Pb}{24} \left[\frac{6S^2 + 20SR + 9R^2}{(S + 3R)(S + R)} \right]$$

$$M_B = M_C + H_C h - \frac{wb^2}{8} = M_b + \frac{Pb}{8}$$

$$M_a = M_C - \frac{Pb}{4}$$

$$M_b = M_C + H_C h - \frac{Pb}{4} = M_a + H_C h$$

Case 2.—

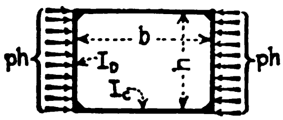


$$H_C = 0$$

$$M_C = M_B = \frac{wb^2}{24} \left(\frac{R + 3S}{R + S} \right)$$

$$M_D = M_a = M_b = M_C - \frac{wb^2}{8}$$

Case 3.—



$$H_C = \frac{ph}{2}$$

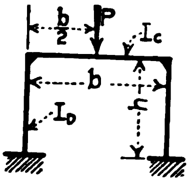
$$M_C = M_a = -\frac{ph^2}{12} \left(\frac{S}{S + R} \right)$$

$$M_D = M_C + \frac{ph^2}{4}$$

$$M_b = M_B = M_C$$

Type II.—Open Frame with Fixed Walls.[1]—

Case 1.—



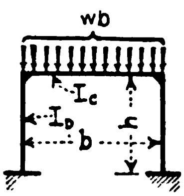
$$H_C = \frac{P}{8} \left(\frac{3R^2}{S + 2R} \right)$$

$$M_C = \frac{Pb}{4} \left(\frac{S + R}{S + 2R} \right)$$

$$M_a = M_C - \frac{Pb}{4}$$

$$M_b = M_a + H_C h$$

Case 2.—



$$H_C = \frac{wb}{4} \left(\frac{R^2}{2R + S} \right)$$

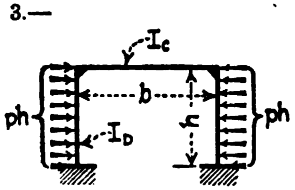
$$M_C = \frac{wb^2}{24} \left(\frac{2R + 3S}{2R + S} \right)$$

$$M_a = M_C - \frac{wb^2}{8}$$

$$M_b = M_C - \frac{wb^2}{8} + H_C h = M_a + H_C h.$$

[1] See footnote on p. 789.

Case 3.—



$$H_C = \frac{ph}{4}\left(\frac{3R + 2S}{2R + S}\right)$$

$$M_C = M_a = -\frac{ph^2}{12}\left(\frac{S}{2R + S}\right)$$

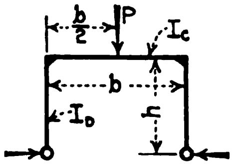
$$M_b = M_C + H_C h - \frac{ph^2}{2}$$

In sides M(max) occurs at $\frac{H_C}{p}$ from top.

$$M(\max) = M_C + \frac{H_C^2}{2p}$$

Type III.—Open Frame Hinged at the Base.[1]

Case 1.—

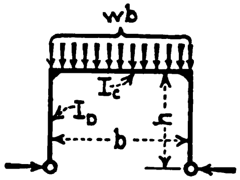


$$H = \frac{3P}{8}\left(\frac{R^2}{2S + 3R}\right)$$

$$M_C = \frac{Pb}{4} - Hh$$

$$M_a = -Hh$$

Case 2.—

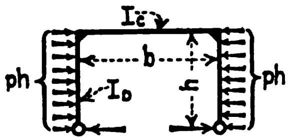


$$H = \frac{wb}{4}\left(\frac{R^2}{2S + 3R}\right)$$

$$M_C = \frac{wb^2}{8} - Hh$$

$$M_a = -Hh$$

Case 3.—



$$H = -\frac{3ph}{4}\left(\frac{S + 2R}{2S + 3R}\right)$$

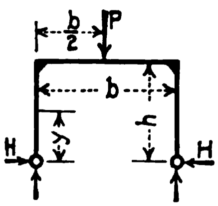
$$M_C = M_a = Hh - \frac{ph^2}{2} = -\frac{ph^2}{4}\left(\frac{S}{2S + 3R}\right)$$

$$M \text{ (max) for the sides } = \frac{H^2}{2p}$$

$$y \text{ at max. moment } = \frac{H}{p}$$

[1] The analysis of this type of frame follows that for a two-hinged arch. Let M' be the bending moment, at any section, of the vertical forces only, as for a simple horizontal beam. Then for any point there results the total moment $M = M' - Hy$ in which y is measured from the horizontal connecting the hinges, and H is the thrust acting along this line and upon the hinges. From "Curved Beams" (see Art. 15, Sect. 16) the increase in span is, with E constant,

$$\int My\frac{ds}{I}$$



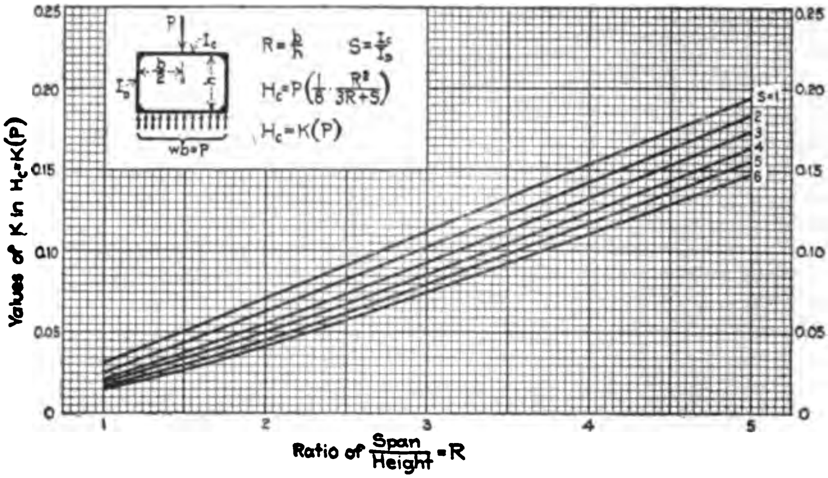
The span, however, is not permitted to increase, due to the resistance offered by H. Substituting for M its value given above,

$$\int (M' - Hy)y\frac{ds}{I} = 0$$

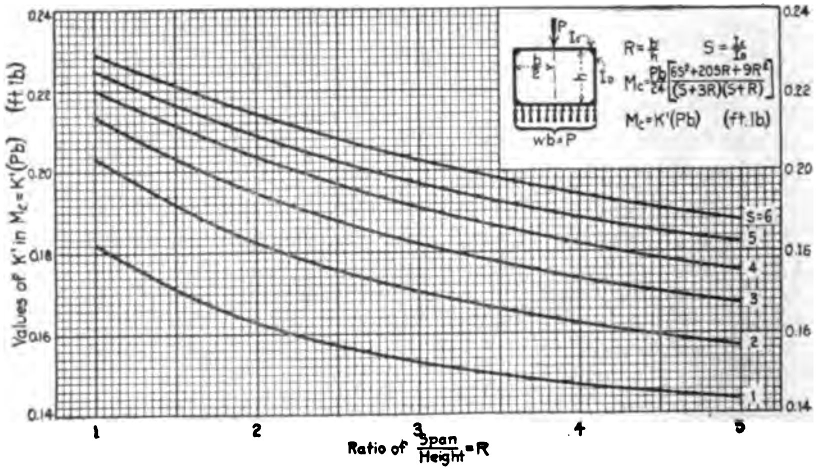
from which

$$H = \frac{\int M'y\frac{ds}{I}}{\int y^2\frac{ds}{I}}$$

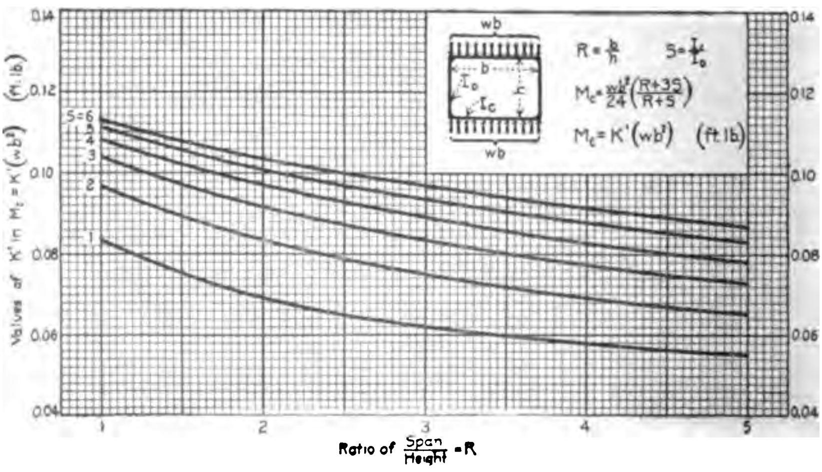
Since the thrust only is statically indeterminate, its solution for any case permits statical solution of moment by the equation for M given above.



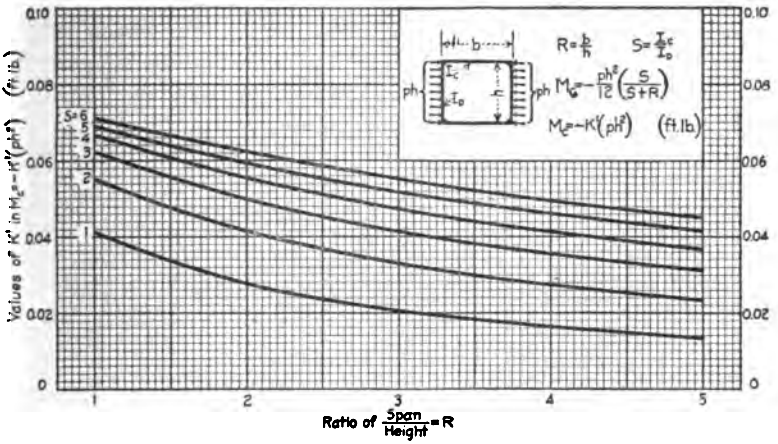
$R = \frac{b}{h}$ $S = \frac{I_c}{I_b}$
$H_c = P\left(\frac{1}{6} \cdot \frac{R^2}{3R+S}\right)$
$H_c = K(P)$
wb = P
Values of K in $H_c = K(P)$
Ratio of $\frac{\text{Span}}{\text{Height}} = R$

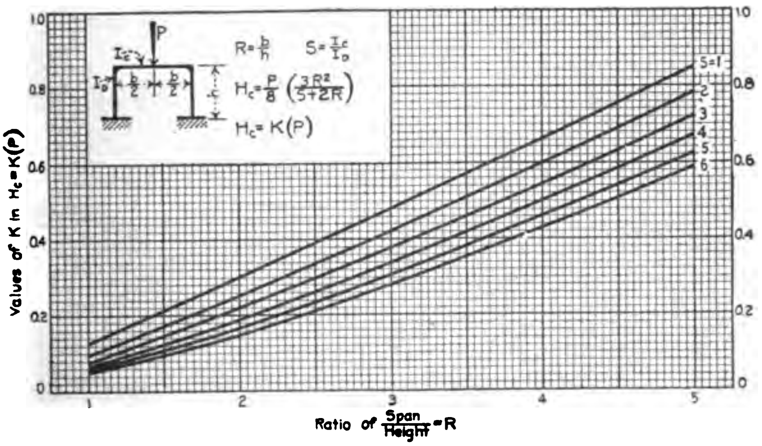


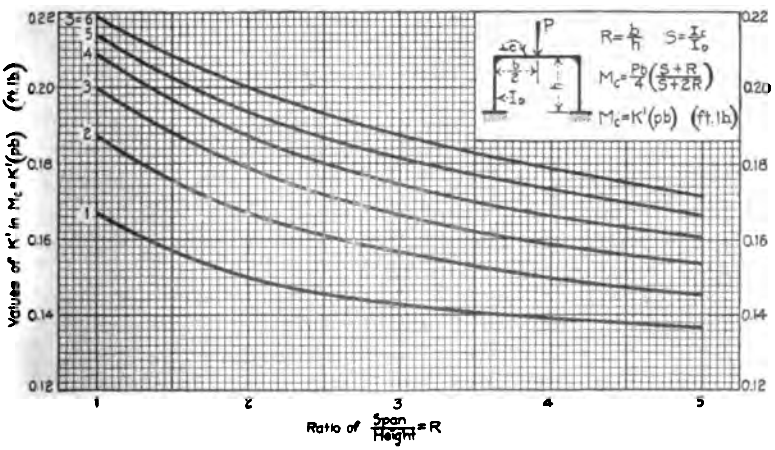
$R = \frac{b}{h}$ $S = \frac{I_c}{I_b}$
$M_c = \frac{Pb}{24}\left[\frac{6S^2+20SR+9R^2}{(S+3R)(S+R)}\right]$
$M_c = K'(Pb)$ (ft lb)
wb = P
Values of K' in $M_c = K'(Pb)$ (ft lb)
Ratio of $\frac{\text{Span}}{\text{Height}} = R$

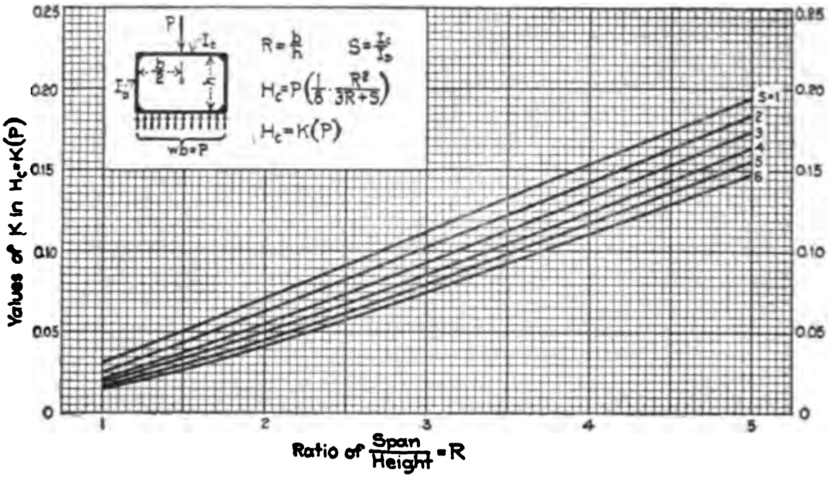


$R = \frac{b}{h}$ $S = \frac{I_c}{I_b}$
$M_c = \frac{wb^2}{24}\left(\frac{R+3S}{R+S}\right)$
$M_c = K'(wb^2)$ (ft lb)
wb
Values of K' in $M_c = K'(wb^2)$ (ft lb)
Ratio of $\frac{\text{Span}}{\text{Height}} = R$

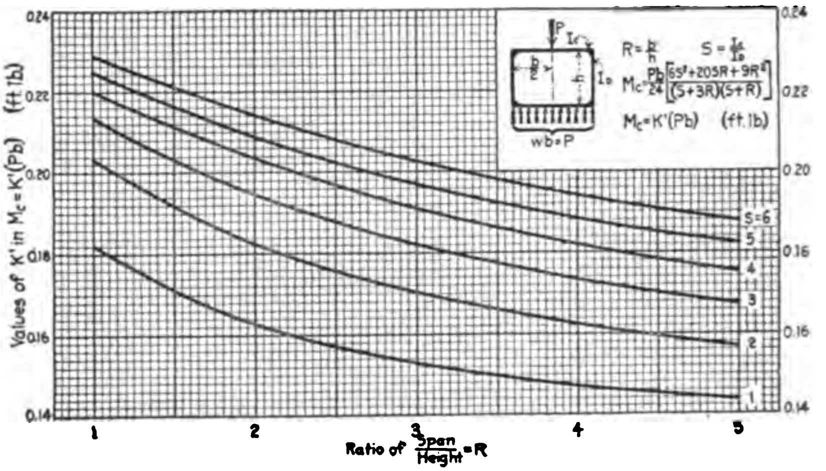

R = b/h
S = Ic/Io
Mc = −ph²/12 (S/(S+R))
Mc = −K'(ph²) (ft.lb)
Values of K' in Mc = −K'(ph²) (ft.lb)
Ratio of Span/Height = R


R = b/h
S = Ic/Io
Hc = P/8 (3R²/(S+2R))
Hc = K(P)
Values of K in Hc = K(P)
Ratio of Span/Height = R

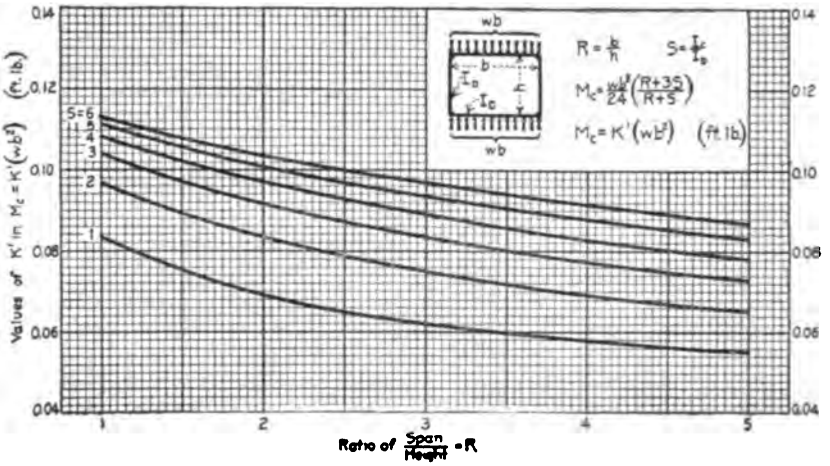

R = b/h
S = Ic/Io
Mc = Pb/4 ((S+R)/(S+2R))
Mc = K'(pb) (ft.lb)
Values of K' in Mc = K'(pb) (ft.lb)
Ratio of Span/Height = R



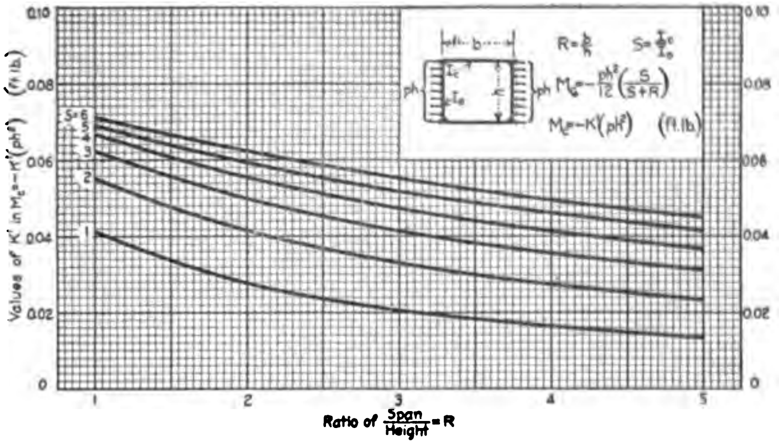
$R=\frac{b}{h}$ $S=\frac{I_c}{I_b}$
$H_c=P\left(\frac{1}{8}\cdot\frac{R^2}{3R+S}\right)$
$H_c=K(P)$
wb=P
Values of K in $H_c=K(P)$
Ratio of $\frac{\text{Span}}{\text{Height}}=R$

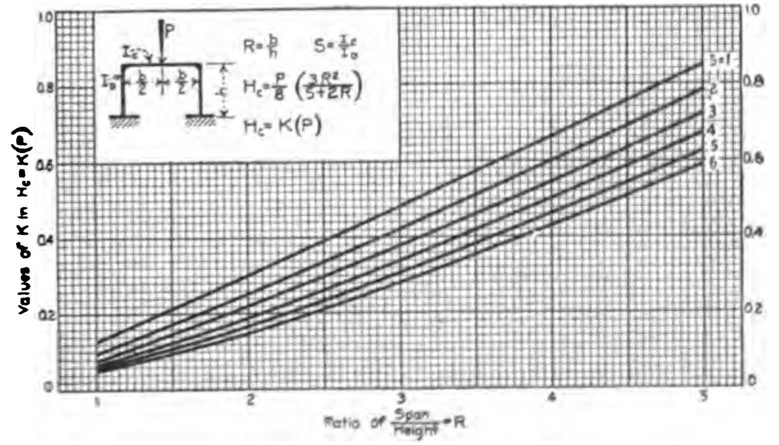


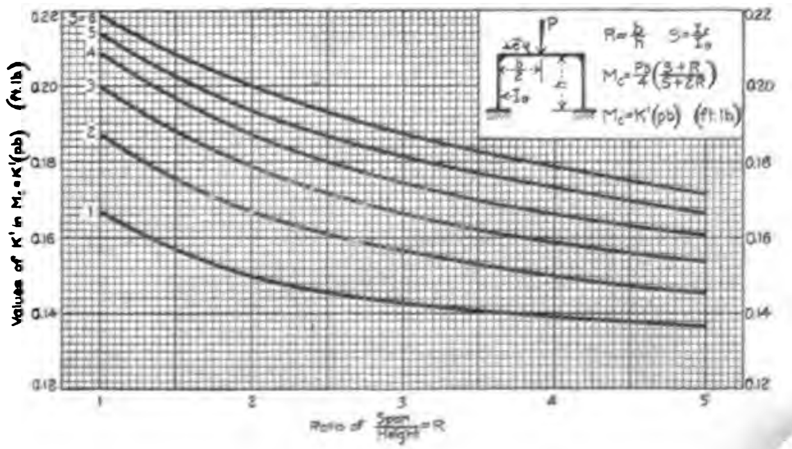
$R=\frac{b}{h}$ $S=\frac{I_c}{I_b}$
$M_c=\frac{Pb}{24}\left[\frac{6S^2+20SR+9R^2}{(S+3R)(S+R)}\right]$
$M_c=K'(Pb)$ (ft.lb)
wb=P
Values of K' in $M_c=K'(Pb)$ (ft.lb)
Ratio of $\frac{\text{Span}}{\text{Height}}=R$

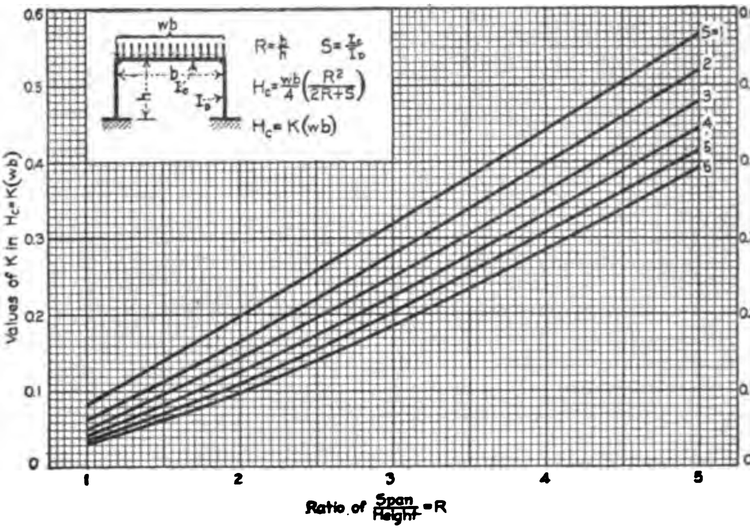


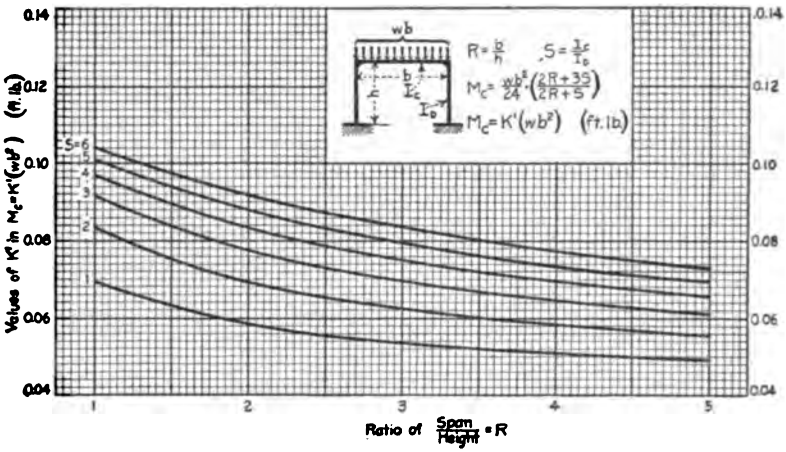
$R=\frac{b}{h}$ $S=\frac{I_c}{I_b}$
$M_c=\frac{wb^2}{24}\left(\frac{R+3S}{R+S}\right)$
$M_c=K'(wb^2)$ (ft.lb)
wb
Values of K' in $M_c=K'(wb^2)$ (ft.lb)
Ratio of $\frac{\text{Span}}{\text{Height}}=R$

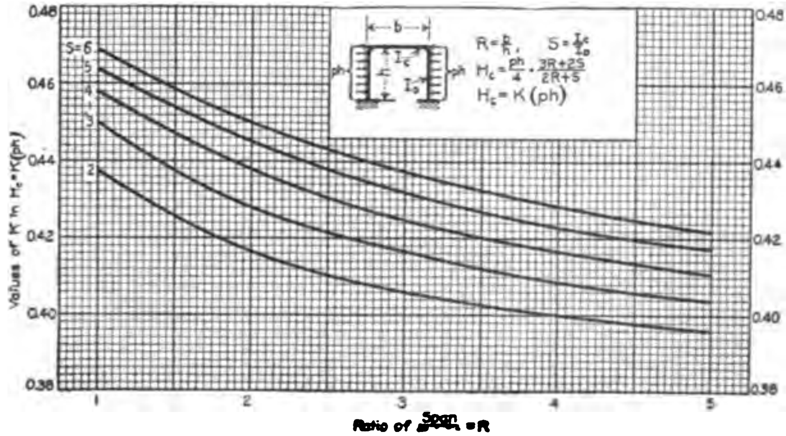

R = b/h S = Ic/Io
Mc = −(ph²/12)(S/(S+R))
Mc = −K′(ph²) (ft.lb)
Values of K′ in Mc = −K′(ph²) (ft.lb)
S=6
5
4
3
2
1
Ratio of Span/Height = R

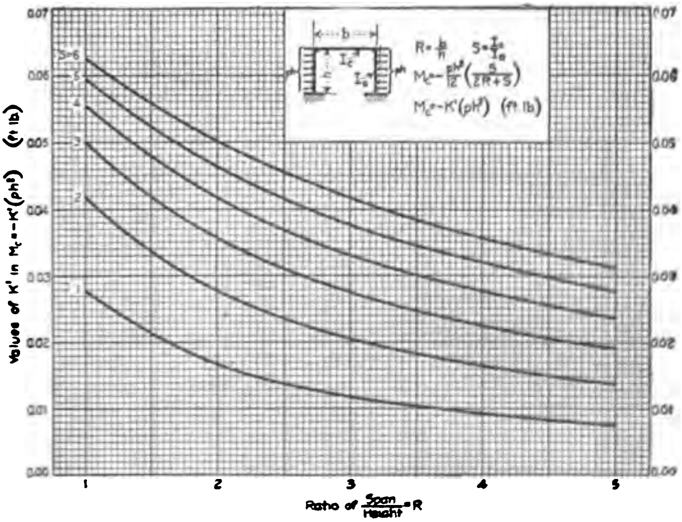

R = b/h S = Ic/Io
Hc = (P/8)(3R²/(S+2R))
Hc = K(P)
Values of K in Hc = K(P)
S=1
2
3
4
5
6
Ratio of Span/Height = R

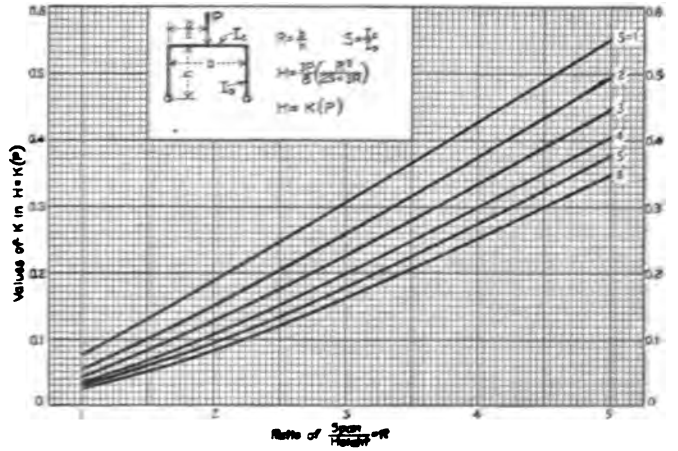

R = b/h S = Ic/Io
Mc = (Pb/4)((S+R)/(S+2R))
Mc = K′(pb) (ft.lb)
Values of K′ in Mc = K′(pb) (ft.lb)
S=6
5
4
3
2
1
Ratio of Span/Height = R

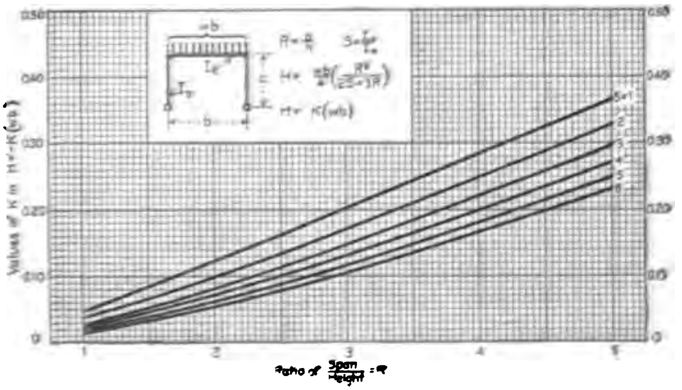

wb
b
$R=\frac{b}{h}$ $S=\frac{I_c}{I_b}$
$H_c=\frac{wb}{4}\left(\frac{R^2}{2R+S}\right)$
$H_c=K(wb)$
S=1
2
3
4
5
6
Values of K in $H_c=K(wb)$
0
0.1
0.2
0.3
0.4
0.5
0.6
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}}$ = R


wb
b
$R=\frac{b}{h}$ $S=\frac{I_c}{I_b}$
$M_c=\frac{wb^2}{24}\left(\frac{2R+3S}{2R+S}\right)$
$M_c=K'(wb^2)$ (ft.lb)
S=6
5
4
3
2
1
Values of K' in $M_c=K'(wb^2)$ (ft.lb)
0.04
0.06
0.08
0.10
0.12
0.14
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}}$ = R


b
ph
ph
$R=\frac{b}{h}$, $S=\frac{I_c}{I_b}$
$H_c=\frac{ph}{4}\cdot\frac{3R+2S}{2R+S}$
$H_c=K(ph)$
S=6
5
4
3
2
Values of K in $H_c=K(ph)$
0.38
0.40
0.42
0.44
0.46
0.48
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}}$ = R


Values of K' in $M_c = -K'(ph^2)$ (ft lb)
$R = \frac{b}{h}$ $S = \frac{I_c}{I_o}$
$M_c = -\frac{ph^2}{12}\left(\frac{S}{2R+S}\right)$
$M_c = -K'(ph^2)$ (ft lb)
S=6
5
4
3
2
1
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}} = R$


Values of K in H = K(P)
$R = \frac{b}{h}$ $S = \frac{I_c}{I_o}$
H = K(P)
S=1
2
3
4
5
6
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}} = R$


$R = \frac{b}{h}$ $S = \frac{I_c}{I_o}$
H = K(wb)
S=1
2
3
4
5
6
0.50
0.40
0.30
0.20
0.10
0
1
2
3
4
5
Ratio of $\frac{\text{Span}}{\text{Height}} = R$

Collapsible steel forms are now being used to a considerable extent in culvert construction, and in most cases with success. The Whalen steel form is shown in Fig. 83. It is built in but one size, yet by its unit method of construction, it readily lends itself to the building of various-sized culverts. Although built in arched top sections, a proper assemblage of these sections makes it possible to build most any size of flat top culvert.

If running water is encountered which cannot be temporarily diverted or dammed, the water in the case of small culverts should be carried in a new trench around one side of the back forms. In the case of the larger structures the trench excavated for the culvert should be arranged to flume the stream through between the abutments.

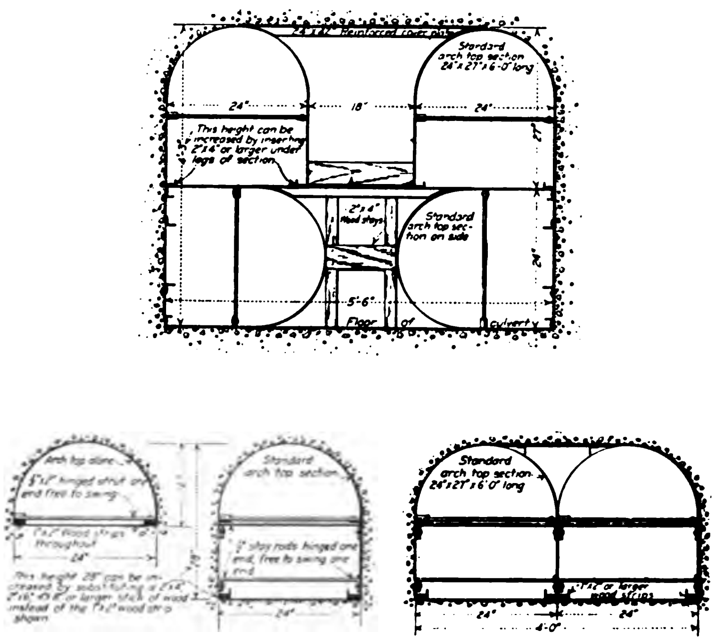


Fig. 83.—Whalen steel form.

33. Arch Culverts.—The arch type of culvert should be employed where an artistic design is especially desirable, and should also be used in all cases where the fill to be supported is excessively high and the foundations suitable. High fills over box culverts necessitate a slab of considerable thickness and the arch becomes the more economical under such conditions because of the fact that an increase in fill does not produce a correponding increase in ring stress. Arch culverts of reinforced concrete are not usually designed for spans less than about 8 ft., as plain concrete seems to answer the purpose for such small structures.

33a. Design of Cross-section.—In the ordinary type of arch culvert represented in Fig. 84, the arch ring may be analyzed in the same manner as described for arch bridges in Art. 16, Sect. 16. A uniform live load only is considered and this is placed over the whole span. Although steel is used, the line of pressure is usually kept everywhere within the middle third. In determining the dead load on the arch no allowance is made for the arch action of the fill, but the horizontal components of the earth pressure are taken into account. Longitudinal reinforcement is needed to prevent objectionable cracks caused by the shrinkage of concrete in hardening and the contraction due to a lowering of the temperature, and also to distribute the load.

Inverts are employed in the designs shown in Figs. 85 and 86 in order to provide additional bearing area and thus reduce the large abutments which would otherwise be needed in order to bring the pressure on the soil to a safe value. An invert also tends to prevent any possibility of water undermining the structure and the foundation need not always be carried to the same depth as when the abutments are independent of each other.

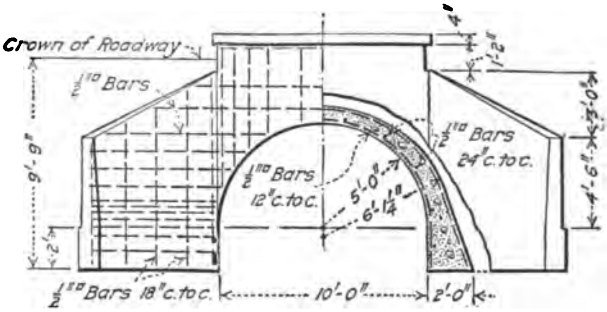


FIG. 84.—Standard design for 10-ft. arch culvert, State of Missouri Highway Department. (Actual dimensions and shape of foundations governed by conditions of soil at location of site.)

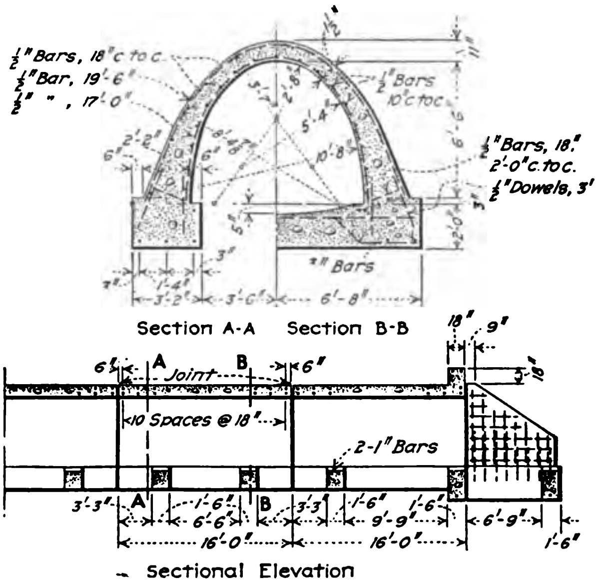
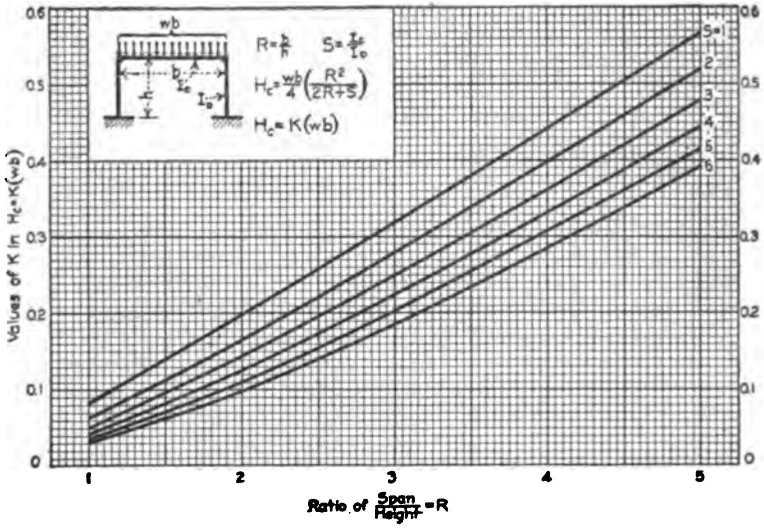
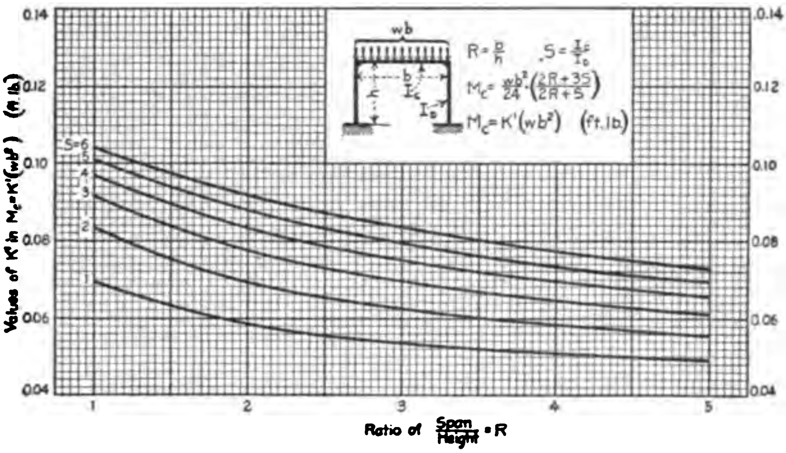


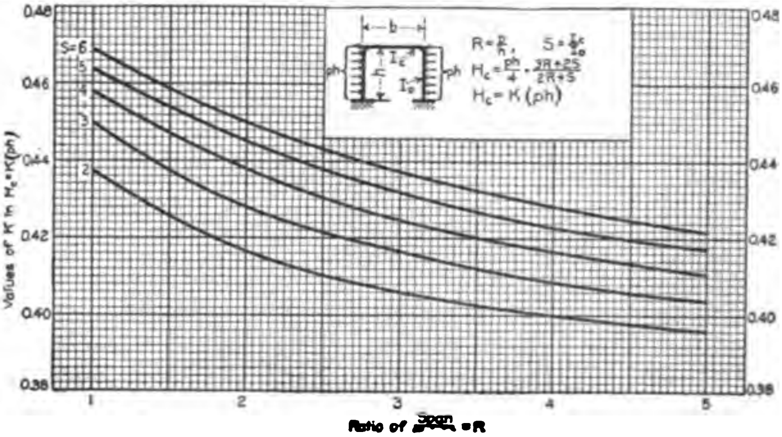
FIG. 85.—Standard arch culvert for fills up to 40 ft. high, C. M. & St. P. Ry.

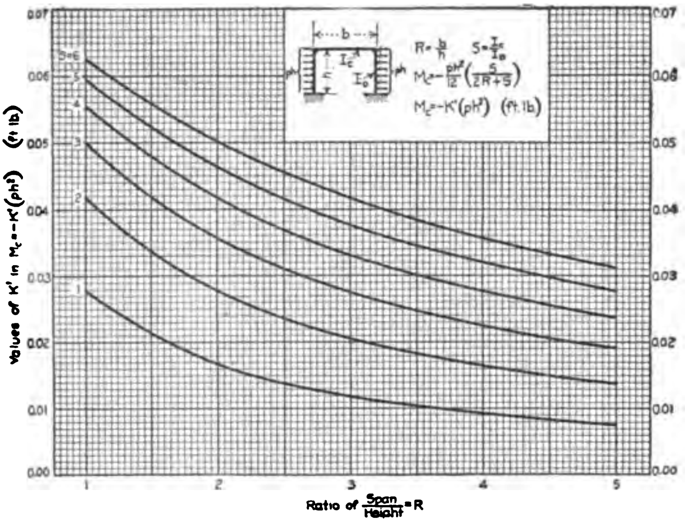
When the arch ring and invert are thoroughly tied together so as to act as a monolith (a rare case except in masonry sewer design), the entire culvert may be analyzed after the manner of the arch. For the analysis of this form of arch see Art. 33 on the design of conduits.

33b. Forms.—The centers and forms for the arch culvert are similar to those required for the arch bridge of small span. Where inverts are employed, the concrete for the

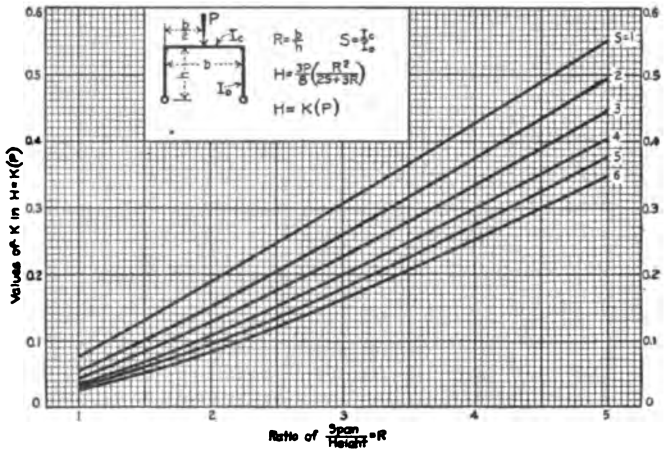

wb
R = b/h
S = Ic/Io
Hc = wb/4 (R²/(2R+S))
Hc = K(wb)
Values of K in Hc = K(wb)
S=1
2
3
4
5
6
Ratio of Span/Height = R


wb
R = b/h
S = Ic/Io
Mc = wb²/24 · (2R+3S)/(2R+S)
Mc = K'(wb²) (ft.lb)
Values of K' in Mc = K'(wb²) (ft.lb)
S=6
5
4
3
2
1
Ratio of Span/Height = R

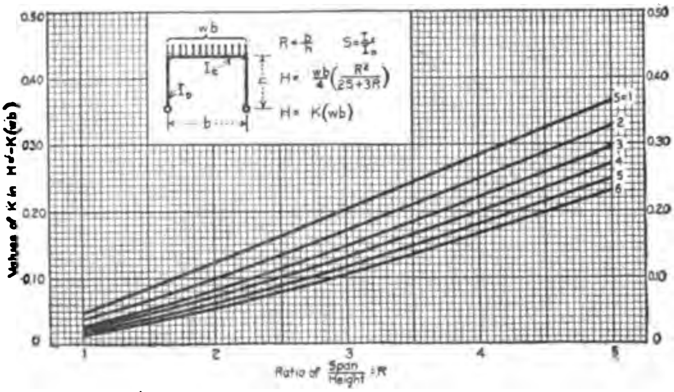

b
ph
R = b/h, S = Ic/Io
Hc = ph/4 · (3R+2S)/(2R+S)
Hc = K(ph)
Values of K in Hc = K(ph)
S=6
5
4
3
2
Ratio of Span/Height = R



$R=\frac{b}{h}$ $S=\frac{I_c}{I_o}$
$M_c=-\frac{ph^2}{12}\left(\frac{S}{2R+S}\right)$
$M_c=-K'(ph^2)$ (ft lb)
S=6
5
4
3
2
1
Values of K' in $M_c=-K'(ph^2)$ (ft lb)
Ratio of $\frac{\text{Span}}{\text{Height}}=R$



P
$R=\frac{b}{h}$ $S=\frac{I_c}{I_o}$
$H=\frac{3P}{8}\left(\frac{R^2}{2S+3R}\right)$
$H=K(P)$
S=1
2
3
4
5
6
Values of K in H=K(P)
Ratio of $\frac{\text{Span}}{\text{Height}}=R$



wb
$R=\frac{b}{h}$ $S=\frac{I_c}{I_o}$
$H=\frac{wb}{4}\left(\frac{R^2}{2S+3R}\right)$
$H=K(wb)$
S=1
2
3
4
5
6
Values of K in H=K(wb)
Ratio of $\frac{\text{Span}}{\text{Height}}=R$

from the crown around to BB, and treated precisely as an arch. It is assumed that there are vertical forces acting upward on the invert equal in amount to the total downward vertical forces, and uniformly distributed over the invert. The oblique resultant force on block 16 is

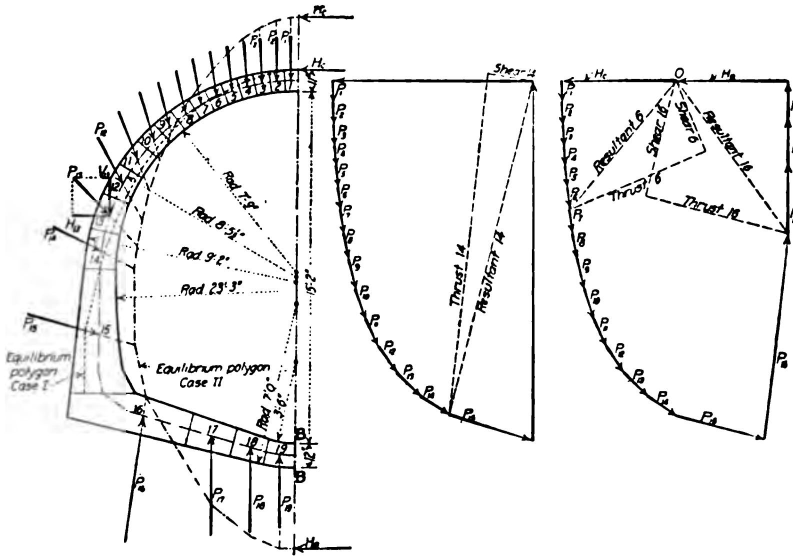


FIG. 91.

due to the combining of the upward vertical force with the earth pressure acting on the left side of the block. After having computed H_C, by the usual arch analysis, the force polygon may be drawn, and the closing line will be H_B. Rays may then be drawn, since the pole O

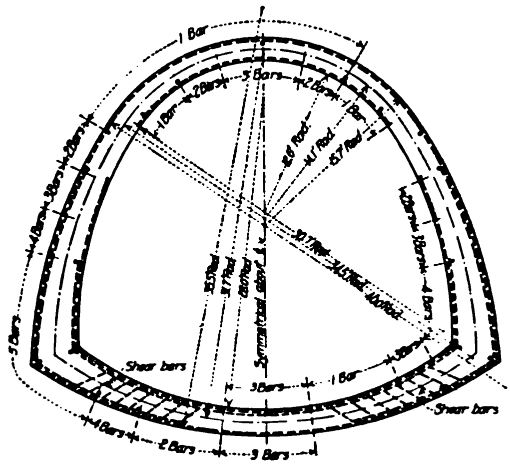


FIG. 92.

is now located and the pressure line drawn on the conduit section. The shears and moments may be computed as for arch analysis. Shear will usually be large near the outer ends of the invert, as is shown in Fig. 91. A typical method of placing reinforcement in horseshoe sections is shown in Fig. 92.

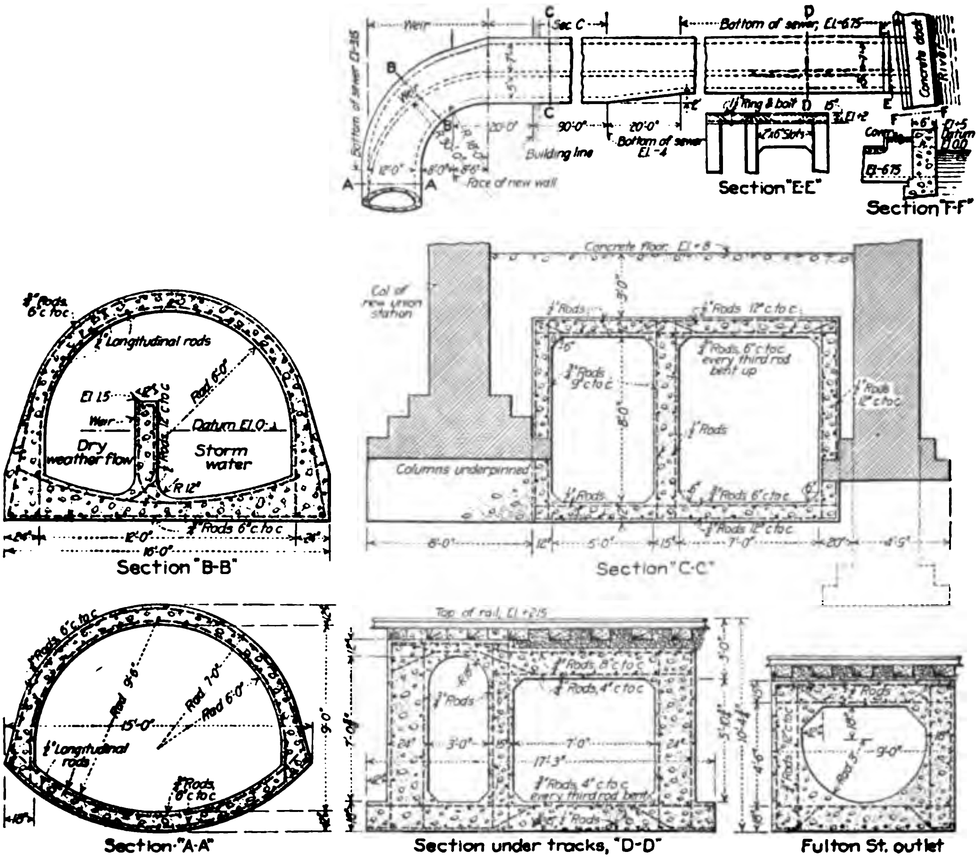
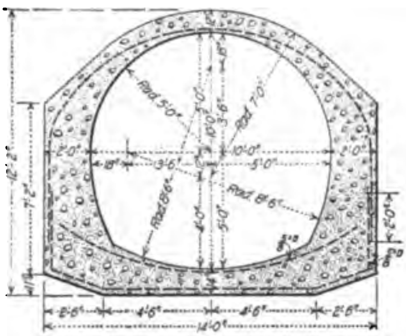


FIG. 93.—Monroe St. outlet sewer at Chicago.



Note:- On hard bottom no reinforcement required

FIG. 94.

37. Construction.—Some large sewer sections are shown in Figs. 93 and 94. Fig. 93 shows a typical section of the Monroe St. sewer in Chicago. Fig. 94 shows a horseshoe section in Evanston, Ill. A 1 : 1½ : 4½ mix was used in the latter case. A large number of typical sections are to be found in Metcalf and Eddy's "American Sewerage Practice," Vol. 1, Chap. XII, together with descriptions of each.

38. Longitudinal Reinforcement.—Temperature and shrinkage stresses require about 0.4% of longitudinal reinforcement in pipes and conduits.

Pipes resting at intervals on saddles are required to carry their own weight and should be reinforced longitudinally to resist the flexure set up. The bending moment may be assumed at support and center of span to be $\frac{wl^2}{12}$ where l is the span in feet, and w is the load per foot

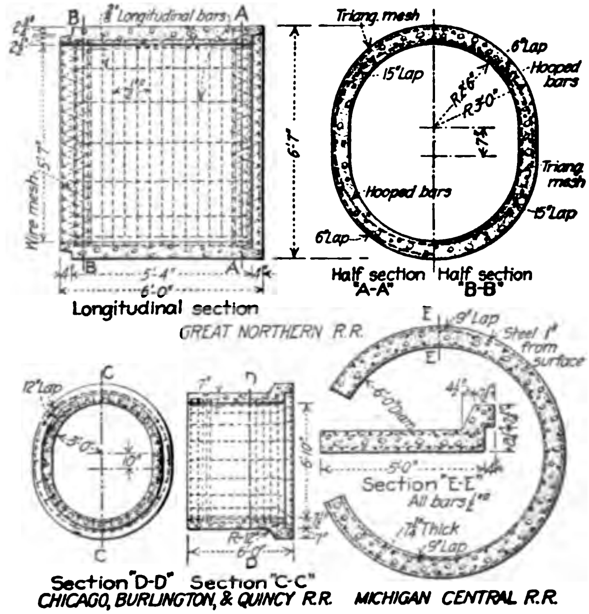


FIG. 95.—Three designs of reinforced-concrete pipe for railway culverts.

which the pipe is obliged to carry. Knowing the bending moment, and the dimensions of the pipe, the steel requirements and unit stresses in concrete and steel may be taken from Diagram 2, Sect. 18, page 816.

ILLUSTRATIVE PROBLEM.—Inside diameter of pipe 8 ft., thickness of shell 4 in. Find spacing of saddles for longitudinal shearing stress of 35 lb. per sq. in., and provide proper flexural reinforcement.

Weight of pipe $= 8.3 \times \pi \times \frac{1}{3} \times 150 =$ 650 lb. per ft.
Weight of water $= 16 \times \pi \times 62.5$ 3140 lb. per ft

Total, 3790 lb. per ft.

From Art. 15, Sect. 18, page 818, $\frac{V}{Ri} = 3.15v$.

Allowable $V = 50 \times 4 \times 3.15 \times 35 = 22{,}050$ lb. Clear spacing of saddles $= \frac{22{,}050}{3790} = 5.82$ ft., say 6.8 ft. on centers.

Bending moment $= \frac{1}{12}(3790)(6.8)^2(12) = 175{,}300$ in.-lb. $\frac{M}{R^3t} = \frac{175{,}350}{(2500)(4)} = 1.75$. Referring to Diagram 2, page 816, it is seen that at this extremely low value, the steel and concrete unit stresses are very low if a small amount of longitudinal steel is in place. Use 0.4% for temperature and flexure.

39. Examples of Reinforced-concrete Pipe.—Figs. 95 and 96 show several types of reinforced pre-cast pipes. Most of the pipe lines laid are of pre-cast units. For culverts and sewers the pipe may be oval or circular. For water under pressure the pipe should be circular, and the joints carefully made. Two types of joints are common, bell and spigot, and sleeve joints, both of which are shown in the figure.

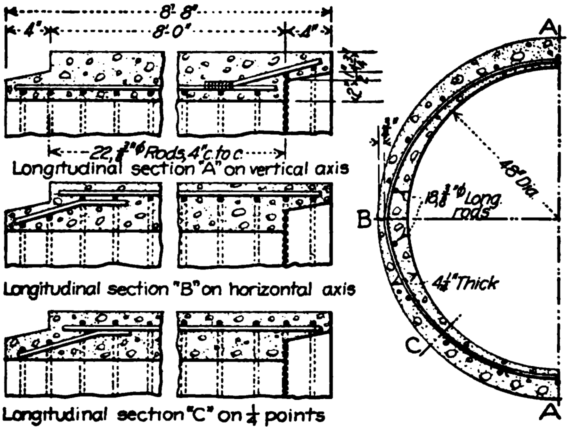


Fig. 96.—Reinforced-concrete culvert pipe, Chicago & Northwestern Ry.

40. Forms for Sewers.—Because of desirability to repeat the use of form material, an arrangement of collapsible form units is best. Steel forms are advantageous for this reason, especially for heavy work.

Usually the sewer or conduit is cast in stages, the invert being poured first, and if bricked, the pavement next laid. The arch centering and the steel for the arch are then placed. The wall sections are next set and the arch poured and finished. The joint with the floor should be well tied with dowel rods. The centering should be arranged so that it may be easily

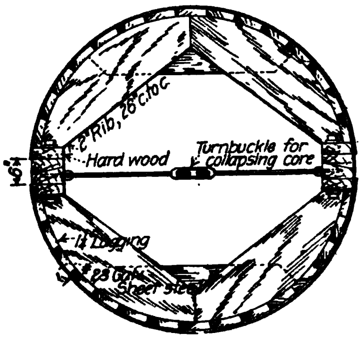


Fig. 97.

"struck," or loosened for removal. Fig. 97 shows a simple arrangement for circular sections. The same general principle is also used for arch forms both in wood and steel.

The backfill should cover the sewer before the centering, or core, is struck. General practice (*Eng. Rec.*, vol. 58, p. 664) indicates that under favorable circumstances forms may be struck in 36 to 48 hr. for sewers smaller than 10 ft., and 72 hr. for larger sewers. Temperature and humidity will greatly affect the setting of concrete, and should be considered as modifying the above rule.

SECTION 18

MISCELLANEOUS STRUCTURES

DEEP GRAIN BINS OR SILOS

Deep bins will be considered of such proportions that the plane of rupture drawn from the bottom of one side will not pass out at the free surface of the material retained. Analysis, therefore, cannot be made according to the theories of retaining walls.

1. Action of Grain in Deep Bins.—A study of the action of grain in a deep bin shows that the grain forms a dome from one side to the other, which is supported at its perimeter by the friction between the grain and the side of the bin. This dome supports a portion of the grain above it, the remainder being carried by further dome action to the sides. Compression thus exists upon the horizontal sections of the wall, which varies in some manner with the grain and the depth. The lateral radial pressure of the grain likewise varies in some manner with the depth and the grain.

2. Janssen's Formulas for Pressure in Deep Bins.[1]—

ϕ = angle of repose of the filling.
ϕ' = angle of friction of the filling on the bin walls.
$u = \tan \phi$ = coefficient of friction of filling on filling.
$u' = \tan \phi'$ = coefficient of friction of filling on the bin walls.
w = weight of filling in pounds per cubic foot.
V = vertical pressure of the filling in pounds per square foot.
L = lateral pressure of the filling in pounds per square foot.
$L = kV$ or
$k = \dfrac{L}{V}$.
A = area of bin in square feet.
U = circumference of bin in feet.
$R = \dfrac{A}{U}$ = hydraulic radius of bin.
h = depth of granular material at any point.

Then

$$V = \frac{wR}{ku'}\left[1 - \left(1 \div \text{number whose common log is } \frac{ku'h}{2.303R}\right)\right]$$

$$L = kV$$

Values of u' and k for different grains and bin materials are given in Tables 1 and 2.

Illustrative Problem.—Given wheat weighing 50 lb. per cu. ft.; $u' = 0.444$; $k = 0.5$; depth of material 50 ft.; diameter of bin 12 ft. Required vertical and horizontal pressures at base.

$$R = \frac{A}{U} = \frac{36}{12} = 3 \text{ ft.}$$

$$\frac{ku'h}{2.303R} = \frac{(0.5)(0.444)(50)}{(2.303)(3)} = 1.605$$

The number whose common logarithm is 1.605 is 40.3 and $1 \div 40.3 = 0.025$

$$V = \frac{wR}{ku'}(0.975) = \frac{(50)(3)(0.975)}{(0.5)(0.444)} = 660 \text{ lb. per sq. ft.}$$

$$L = kV = (0.5)(660) = 330 \text{ lb. per sq. ft.}$$

[1] For derivation of formulas see Ketchum's "Structural Engineers' Handbook," 1st Ed., p. 319.

TABLE 1.[1]—WEIGHTS AND COEFFICIENTS OF FRICTION OF VARIOUS KINDS OF GRAINS ON BIN WALLS (AIRY)

	Weight of a cubic foot loosely filled into measure (pounds)		Coefficients of friction			
		Grain on grain u (tan ϕ)	Grain on rough board u' (tan ϕ')	Grain on smooth board u' (tan ϕ')	Grain on iron u' (tan ϕ')	Grain on cement u' (tan ϕ')
Wheat...	49	0.466	0.412	0.361	0.414	0.444
Barley...	39	0.507	0.424	0.325	0.376	0.452
Oats.....	28	0.532	0.450	0.369	0.412	0.466
Corn.....	44	0.521	0.344	0.308	0.374	0.423
Beans....	46	0.616	0.435	0.322	0.366	0.442
Peas.....	50	0.472	0.287	0.268	0.263	0.296
Tares....	49	0.554	0.424	0.359	0.364	0.394
Flaxseed.	41	0.456	0.407	0.308	0.339	0.414

TABLE 2.[2]—VALUES OF $k = \frac{L}{V}$ FOR WHEAT AND OTHER GRAINS IN DIFFERENT BINS (PLEISNER)

Bins	$k = L/V$			
	Wheat	Rye	Rape	Flax-seed
Cribbed bin....................	0.4 to 0.5	0.23 to 0.32		
Ringed cribbed bin.............	0.4 to 0.5	0.3 to 0.34		
Small plank bin................	0.34 to 0.46	0.3 to 0.45	0.5 to 0.6	0.5 to 0.6
Large plank bin................	0.3	0.23 to 0.28		
Reinforced-concrete bin..........	0.3 to 0.35	0.3		

The formula for L has been used in plotting Diagram 1, for wheat retained in a reinforced-concrete bin. The above problem could have been solved directly from this diagram.

3. Conclusions from Tests.—Prof. Ketchum has drawn the following valuable conclusions[3] from tests[4] made by him and other experimenters upon model and full-sized grain bins:

1. The pressure of grain on bin walls and bottoms follows a law (which for convenience will be called the law of "semi-fluids"), which is entirely different from the law of the pressure of fluids.

2. The lateral pressure of grain on bin walls is less than the vertical pressure (0.3 to 0.6 of the vertical pressure, depending on the grain, etc.), and increases very little after a depth of 2½ to 3 times the width or diameter of the bin is reached.

[1] From KETCHUM'S "Walls, Bins and Grain Elevators," p. 327.
[2] From KETCHUM'S "Structural Engineers' Handbook," p. 321.
[3] KETCHUM'S "Structural Engineers' Handbook," p. 325.
[4] KETCHUM'S "Walls, Bins and Grain Elevators," Chap. XVII.

3. The ratio of lateral to vertical pressures k is not a constant, but varies with different grains and bins. The value of k can only be determined by experiment.

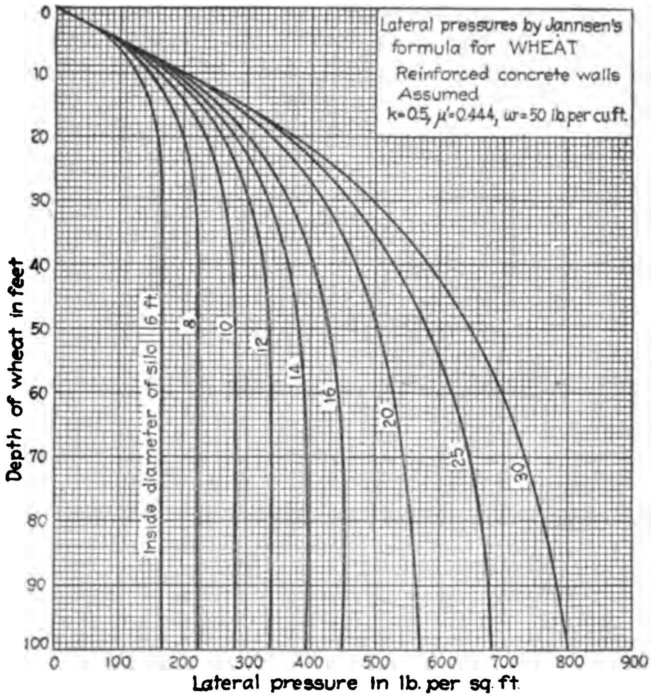
4. The pressure of moving grain is very slightly greater than the pressure of grain at rest (maximum variation for ordinary conditions is, probably, 10%).

5. Discharge gates in bins should be located at or near the center of the bin.

6. If the discharge gates are located in the sides of the bins, the lateral pressure due to moving grain is decreased near the discharge gate and is materially increased on the side opposite the gate (for common conditions this increased pressure may be 2 to 4 times the lateral pressure of grain at rest).

7. Tie rods decrease the flow but do not materially affect the pressure.

Diagram 1



8. The maximum lateral pressures occur immediately after filling, and are slightly greater in a bin filled rapidly than in a bin filled slowly. Maximum lateral pressures occur in deep bins during filling.

9. The calculated pressures by either Janssen's or Airy's formulas agree very closely with actual pressures.

10. The unit pressures determined on small surfaces agree very closely with unit pressures on large surfaces.

11. Grain bins designed by the fluid theory are in many cases unsafe as no provision is made for the side walls to carry the weight of the grain, and the walls are crippled.

12. Calculation of the strength of wooden bins that have been in successful operation shows that the fluid theory is untenable, while steel bins designed according to the fluid theory have failed by crippling the side plates.

Experiments by Willis Whited[1] and by Prof. Ketchum with wheat "have shown that the flow from an orifice is independent of the head and varies as the cube of the diameter of the orifice."

4. Design of Walls.

4a. Vertical Load Carried by Walls.—Prof. Ketchum[2] has shown that the vertical load of grain carried by 1 ft. of circumference of the wall at a depth y will be very approximately

$$Pu' = wR\left[y - \frac{R}{ku'}\right] \qquad \text{(approx.)}$$

where P is the total side pressure on a vertical strip y high and 1 ft. wide. The unit stress thus obtained must be added to that caused by the dead load of the structure.

4b. Wind Stresses on a Horizontal Section.—The wind stresses in a deep bin are very small as a rule, and it is common practice to permit the concrete to take tension on the windward side up to about 200 lb. per sq. in., assuming the tensile stress to be low, the unit stress in the concrete in tension and compression will be

$$f_c = \frac{MD}{2I}$$

Assuming a wind pressure of 30 lb. per sq. ft. of vertical projected area on a height of h ft. of the bin,

$$f_c = \frac{1080h^2D^2}{I_c + nI_s}$$

in which D = outside diameter or greatest dimension in feet, and I_c and I_s the moments of inertia of the concrete and steel sections, respectively, about a diameter, expressed in inches[4]. The steel unit stress is nf_c approximately. For a round bin of one cell, isolated, use two-thirds of the stress thus obtained.

If the tensile unit stress in the concrete exceeds 200 lb. per sq. in., the section should be analyzed as for a chimney of similar dimensions.

4c. Thickness of Walls.—The stresses obtained from vertical and wind loads are combined to obtain the maximum unit stress in the concrete. Common practice uses the same working stresses in this structure as in buildings. The thickness of wall is then proportioned to provide for these unit stresses. The same thickness of wall is usually maintained for the full height to permit the use of moving forms.

4d. Horizontal Reinforcement.—*Circular Sections.*—The lateral pressure tends to burst the circular bin along some vertical plane. Reinforcement against this is provided in the same manner as that for circular tanks. The area of steel A_s per foot of height, due to the lateral pressure of P lb. per sq. ft., is

$$A_s = \frac{Pr_s}{f_s}$$

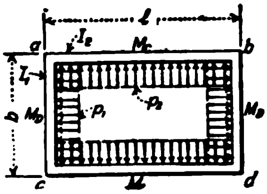


FIG. 1.

where r_s is the radius in feet of the steel hooping. The amount of steel would be proportional to the abscissas to the pressure curve (see Diagram 1, page 807).

4e. Rectangular Sections.—There is a tendency for the flat sides of the polygonal bin to bulge, and the angle made by two adjacent forces to open out (see Fig. 1). The moments, determined by the method of slope deflections (see Sect. 10) may be written readily in the following forms:

$$\text{Moment at any corner} = M_a = -\frac{1}{12}\,\frac{(p_1b^2I_2 + p_2l^2I_1)}{(I_1l + I_2b)}$$

[1] *Proc.* Eng. Soc. of West. Penn., April, 1901.

[2] Structural Engineers' Handbook," p. 324.

When $I_1 = I_2$

$$M_a = -\frac{1}{12}\left(\frac{p_1b^3 + p_2l^3}{l + b}\right)$$

When $p_1 = p_2$

$$M_a = -\frac{p}{12}(b^2 - bl + l^2)$$

For square cells, $b = l$ and $p_1 = p_2$

$$M_a = -\frac{pb^2}{12}$$

At the center of the sides,

$$M_c = \frac{p_2l^2}{8} - M_a$$

$$M_D = \frac{p_1b^2}{8} - M_a$$

in which M_a is chosen from the proper case above. For square cells,

$$M_c = M_D = \tfrac{1}{24}pb^2$$

The moments given above must be provided for by horizontal bars running across the inside of the corner and crossing to the outer face at the point of inflection. When the bins are grouped, the moments may be caused from pressure on either side of an intermediate wall.

Sufficient reinforcement should be provided in the walls *ac* and *bd* to take the pull of the wall *ab* or *cd* caused by the pressure p_2. The tension in *ac* or *bd* is $\tfrac{1}{2}p_2l$, and in *ab* or *cd* is $\tfrac{1}{2}p_1b$.

The pressure p_1 and p_2 may be found from the foregoing formulas and Diagram 1, page 807, since the pressure on the side of a rectangular bin may be computed by computing the pressure for a circular or square bin having the same hydraulic radius, $R = \frac{A}{U}$.

4*f*. Hexagonal Bins.—In groups of circular bins the interspaces are irregular in shape and do not hold as much as do the main cells. The hexagonal bin removes this difficulty. Moments at corners and sides:

$$M_{corner} = -\tfrac{1}{12}pb^2$$
$$M_{side} = \tfrac{1}{24}pb^2$$

in which b is the length of any side.

5. Construction.—The foundation for a group of deep bins is usually a mat, unless the foundation is unyielding. Pressures on the soil should be examined when half of the group is loaded, and wind is acting.

Round cells are commonly placed in such a manner that at the points of tangency the wall is of one thickness, so that the ring reinforcement from one cell laps over that of the adjacent one. This, with the vertical steel passing through the link thus made, provides bonding between cells.

Polygonal bins are arranged with walls in common, reinforced to take the pressure from either side. The steel may well be arranged as for a continuous slab, the two systems crossing each other at right angles at the corners.

Construction is usually continuous, the forms being jacked up on the vertical reinforcement, or on vertical rods for the purpose, imbedded in the walls. Metal forms are commonly used, since a smooth surface may be obtained. It is customary, when using moving forms, to make the walls the same thickness from top to bottom.

The concrete bins of the Great Northern Ry.'s grain elevator at West Superior, Wis.,[1]

[1] *Eng. News*, Aug. 4, 1910.

is shown in Fig. 2. The walls are of 1 : 2 : 4 concrete. The stresses used in the design were: steel in tension, 16,000; concrete in compression, 600; bond, 100 lb. per sq. in.; $n = 12$. The bins are 110 ft. high. The capacity of the 72 cells with interspaces is about 2,400,000 bu.

Fig. 3 is a typical section of rectangular bins in the elevator of the F. C. Ayres Mercantile Co., Denver, Colo. A detailed description will be found in Ketchum's "Walls, Bins and Grain Elevators," 2d Ed., page 454.

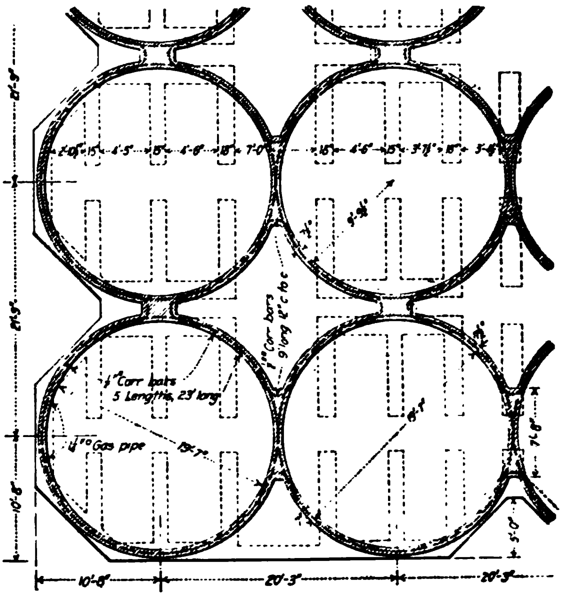


Fig. 2.—Concrete bins of Great Northern Ry's grain elevator, West Superior, Wis.

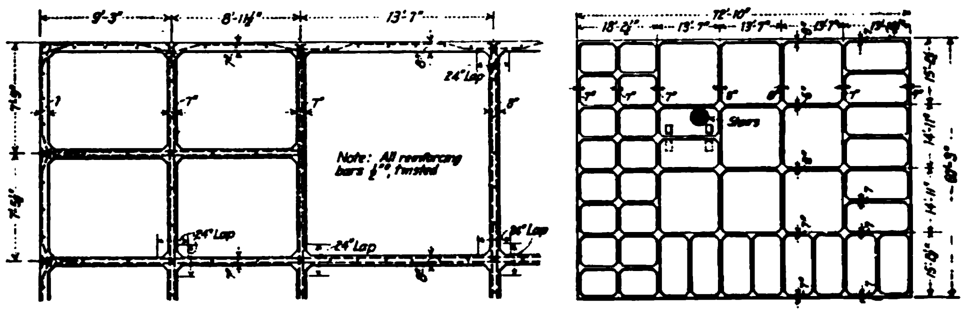


Fig. 3.—Rectangular bins in elevator of F. C. Ayres Mercantile Co., Denver, Colo.

SHALLOW BINS

If the plane of rupture, commencing from the bottom of one side of a bin, strikes the free surface of the retained material, the bin is said to be *shallow*. Such bins are commonly used for the storage of coal, coke, sand, ashes, etc. Pressures against the various sides are determined by the methods commonly employed to determine earth pressures against walls. Two conditions of loading will be given here: (1) level-full and (2) heaped bins. Bins with full sloped sides and with partially vertical sides will be analysed.

6. Sloped Sides—Level Full (Case I).—Let ABC, Fig. 4, represent a bin with sloped sides. On a vertical plane through B the pressure may at once be determined as for a vertical retaining wall holding the prism BDC (see Rebhann's construction, Art. 1b, Sect. 13). The triangle of pressure aBD indicates the total pressure acting on the plane BD, the resultant being P_1, acting $\frac{1}{3}BD$ above B. Let P_1 be produced to meet the weight W of the prism ABD applied at the center of gravity O of the triangle ABD. The resultant thus obtained is P_2. This may be resolved into the forces P_3 and P_4. P_3 is then the resultant normal pressure acting on AB. If the plane AB were smooth, P_4 would represent a component acting against DB; but since the plane AB is rough, P_4 becomes a thrust down the plane against B, if the angle between the forces P_3 and P_4 is equal to, or less than, the angle of friction between the plane and the material.

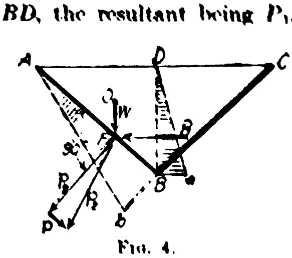


FIG. 4.

Since P_3 is the resultant pressure on AB, it passes through the centroid of the pressure triangle AbB, and its magnitude is equal to the area AbB. The unit pressure bB may thus readily be determined.

7. Partly Vertical Sides—Level Full (Case II).—So far as concerns the pressure on the side AB (Fig. 5), it will be seen to carry the same load as a similar portion of the sloping side AB, Fig. 4.

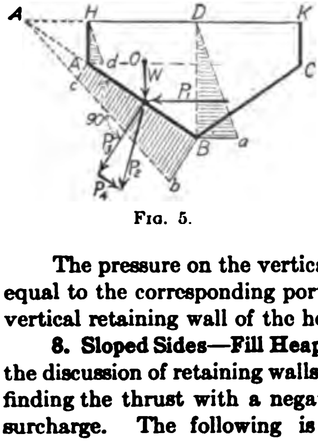


FIG. 5.

Locate the point A'. As before, determine P_1 and W (the latter applied at O, the center of gravity of $A'BD$), and find their resultant P_2. P_3 is the component of P_2 normal to $A'B$, and represents the resultant of normal pressure for the plane $A'B$. Its magnitude is equal to the area of $BA'b$. The unit pressures Bb and Ac may thus be found, defining the trapezoid $AcbB$, which represents the total pressure acting on, and normal to, AB.

The pressure on the vertical side HA is represented by the pressure triangle HAd, which is equal to the corresponding portion of the triangle BDa, and is therefore the pressure against a vertical retaining wall of the height HA, and supporting the same material.

8. Sloped Sides—Fill Heaped to Angle of Repose (Case III).—No method has been given in the discussion of retaining walls for finding the thrust with a negative surcharge. The following is an application of Rebhann's method.

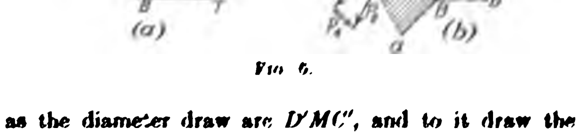


FIG. 6.

The pressures on a vertical plane through B, as BD (Fig. 6a), should balance either side. Assume that the thrust on BD acts horizontally. Lay off BC' making the angle ϕ (angle of internal friction) with the horizontal BT, and extending to DCC'. Draw DD' making the angle ϕ with BD (hence normal to BC'). With $D'C'$ as the diameter draw arc $D'MC'$, and to it draw the tangent BM from B. Make $BN = BM$, and draw NQ parallel to DD'. Make $RN = QN$. The area of the triangle RQN times the weight per cubic foot of the material in the fill equals the thrust P_1 (Fig. 6b) against BD.

Let P_1 be extended to meet W, the weight of the prism ABD. Their resultant is P_2, whose component normal to AB is P_3. The true position of P_3 should be $\frac{1}{3}AB$ from B. Its magnitude equals the area of the triangle ABa; and knowing the length AB, the unit · may be found.

9. Partly Vertical Sides—Fill Heaped (Case IV).—As in Case III, the pressure P_1 against BD is found, as in Fig. 6*a*. P_1 is then combined with W, the weight of a prism $A'DB$, to form P_2 (Fig. 7). The component normal to $A'B$ is P_3, and represents the area of the pressure triangle $A'Ba$. Knowing this area, and the length $A'B$, the unit pressures Ba and Ac may be determined. The total pressure on AB is given by the trapezoid $AcBa$.

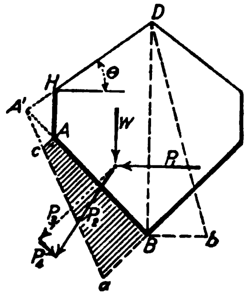


Fig. 7.

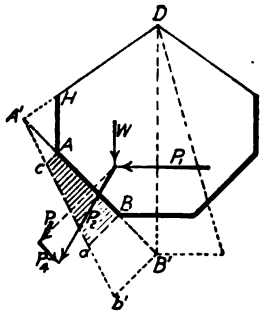


Fig. 8.

In some bins a flat portion is put into the bottom as in Fig. 8. In this case the vertical plane may be extended to meet the extended plane $A'ABB'$ at B'. The pressure P_1 on DB' is then found as in Case III, and combined with W, whence the normal pressure P_3 is found. This is the total normal pressure that would act on the plane $A'B'$, and hence represents in magnitude the area $A'B'b'$. The unit pressure $B'b'$ could be found, and likewise Ac and Ba. The actual normal pressure on AB is shown by the trapezoid $AcBa$.

10. Thrust due to P_4.—The thrust P_4 or the portion of P_4 actually acting on plane AB, should be provided for in the design of the slab AB and the supports at A and B. In Figs. 4 and 6*b* the thrust acting at B is equal to P_4. In Fig. 7 the thrust at B is equal to P_4 times the ratio of AB to $A'B$; and similarly in Fig. 8 the thrust at B is equal to P_4 times the ratio of AB to $A'B'$.

If the span of the slab AB is in the direction AB, the stresses in it should be determined by methods of combined flexure and direct stress. If the span is normal to the drawing, however, simple bending takes place.

11. Data for Bin Design.—Table 1 gives the weights and angle of repose of several materials commonly stored in shallow bins. For data on sand, earth, rock, etc., see table on page 575.

Table 1.[1]—Weight and Angle of Repose of Coal, Coke, Ashes and Ore

Material	Weight (lb. per cu. ft.)	Angle of repose ϕ (degree)	Authority
Bituminous coal	50	35	Link Belt Machinery Co.
Bituminous coal	47	35	Link Belt Engineering Co.
Bituminous coal	47 to 56	....	Cambria Steel
Anthracite coal	52	27	Link Belt Machinery Co.
Anthracite coal	52.1	27	Link Belt Engineering Co.
Anthracite coal fine	....	27	K. A. Muellenhoff
Anthracite coal	52 to 56	....	Cambria Steel
Slaked coal	....	45	Wellman-Seaver-Morgan Co.
Slaked coal	53	37½	Gilbert and Barth
Coke	23 to 32	....	Cambria Steel
Ashes	40	40	Link Belt Machinery Co.
Ashes, soft coal	40 to 45	.	Cambria Steel
Ore, soft iron	......	35	Wellman-Seaver-Morgan Co.

[1] From Ketchum's "Structural Engineers' Handbook," p. 311.

Table 2 gives some approximate friction angles for various materials against different bin linings.

TABLE 2.[1]—ANGLES OF FRICTION OF DIFFERENT MATERIALS ON BIN WALLS

Material	Steel plate ϕ' in degrees	Wood cribbed ϕ' in degrees	Concrete ϕ' in degrees
Bituminous coal	18	35	35
Anthracite coal	16	25	27
Ashes..........	31	40	40
Coke..........	25	40	40
Sand...........	18	30	30

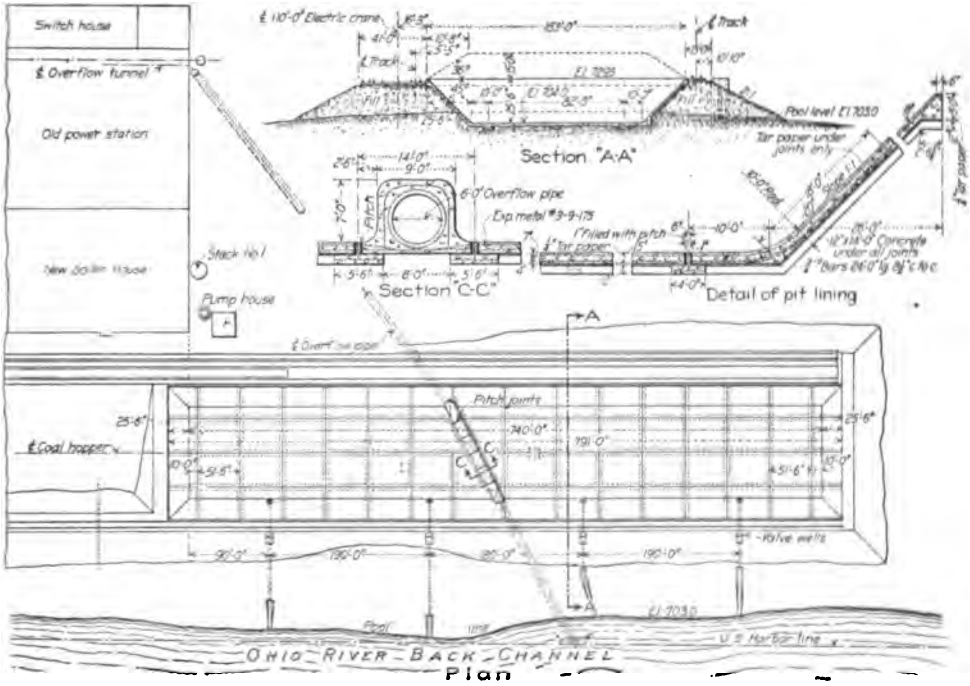


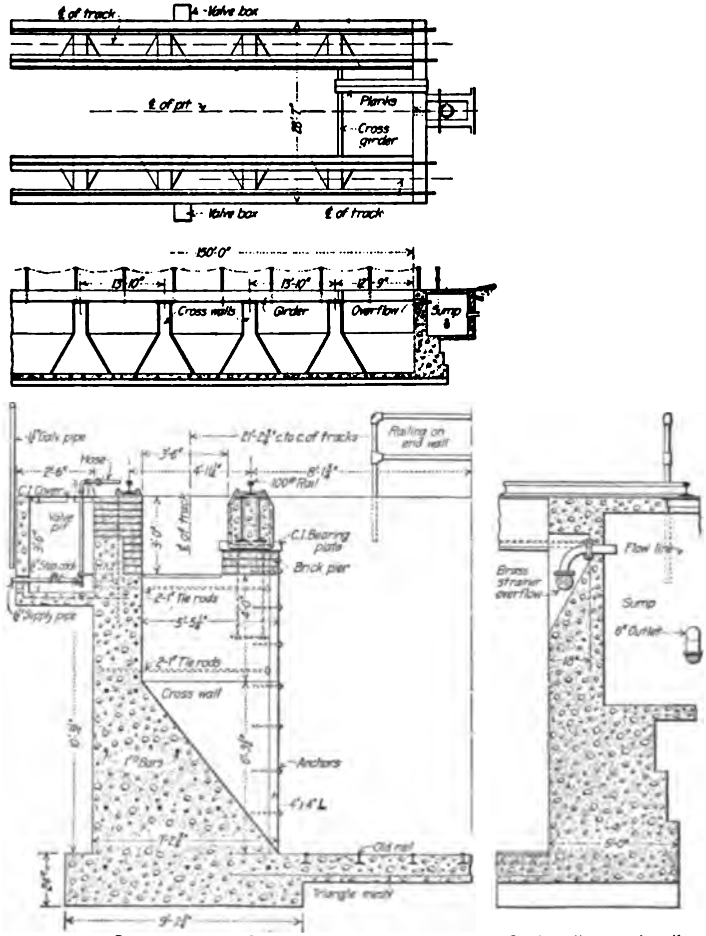
FIG. 9.—Plan and section of Duquesne Light Co.'s coal-storage pit at Pittsburgh.

12. Submerged Storage for Coal.—Coal stored in large quantities is commonly stored in water, to prevent combustion. Bins or pits filled with water and coal should be designed to resist the water pressure, as though full of water. Pits are then designed like open reservoirs, except that the pavement for the pit must be heavier than that for the reservoir, to withstand the dumping of the coal from the trestle overhead (see Fig. 9).

Ashes are frequently stored in pits of this nature, and very often water is run in (see Fig. 10).

When clam-shells or other forms of grab bucket are used to handle the material in the pits, rails are commonly embedded in the pavement about 3 ft. apart, with the heads protruding about ½ in. above the concrete, to protect it from the jaws of the bucket. This detail may be noted in Fig. 10.

[1] From KETCHUM'S "Structural Engineers' Handbook," p. 312.



Cross section of pit

Section thru end wall

FIG. 10.

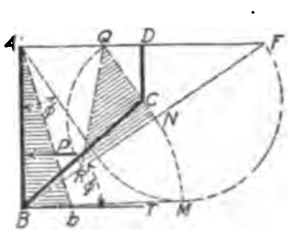


FIG. 11.

13. Dock Pockets.—Storage pockets in docks for ore, and coal, etc., are usually unsymmetrical. The commonest form is that in Fig. 11. The pressure upon the face AB may be found as for the vertical plane DB, Fig. 5. Thus P is the resultant of the pressures on AB, represented by triangle ABb, and equals the area of triangle QNR times the weight per cubic foot of the material retained.

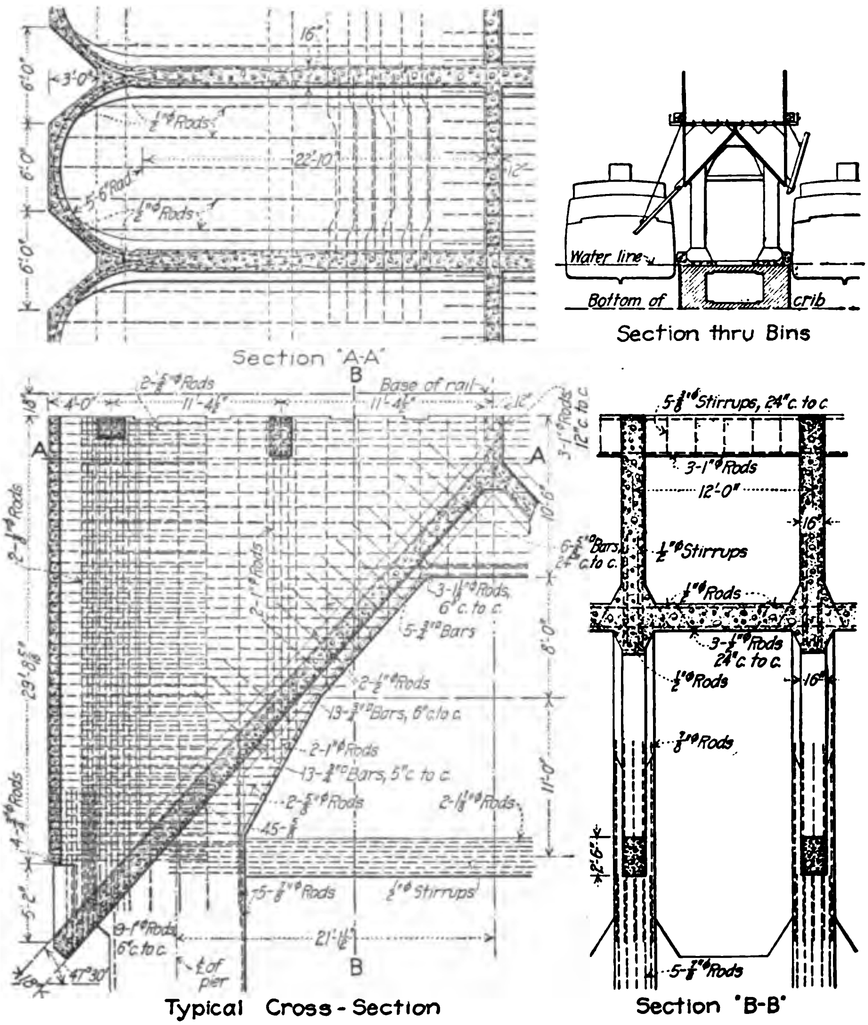


FIG. 12.—Details of superstructure of Minneapolis, St. Paul & Sault Ste. Marie R. R. dock.

The ore dock of the Minneapolis, St. Paul & Sault Ste. Marie R. R. at Ashland, Wis.,[1] is shown in Fig. 12. The front of the pockets are circular in plan, causing the wall stresses due to the thrust of the ore to be tensile. This feature is patented by Max Toltz, who was the designer of the dock. The concrete in the substructure was 1 : 3 : 5 while that in the superstructure was 1 : 2 : 4 mix. The horizontal thrust on the bin walls at a depth h was computed from $p = 30h$.

[1] *Eng. News*, vol. 76, Aug. 10, 1916, p. 243.

CHIMNEYS

The analysis of stresses in reinforced-concrete chimneys involves stresses due to (1) dead load, (2) wind, and (3) temperature.

14. Dead-load Stresses.—The dead-load stress in the concrete may be written

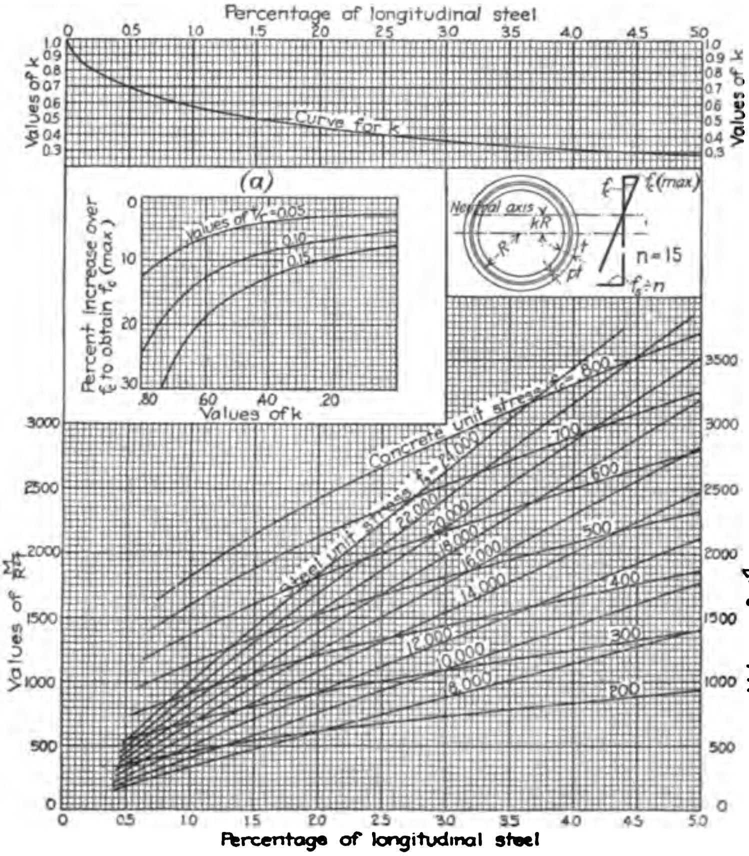
$$f_c = \frac{1.04h}{1 + 14p}$$

and

$$f_s' = nf_c$$

in which h is the height of chimney in feet above the section, and f_s' is the steel unit stress in compression. For preliminary estimate $f_c = 1.04h$ (approx.).

DIAGRAM 2



15. Stresses on Annular Sections in Flexure.—The following notation will be used (see sketch, Diagram 2):

R = radius of center of chimney wall (inches) = also radius of reinforcing steel (inches).
t = thickness of wall (inches).
p = % of longitudinal reinforcement, based on cross-sectional area (= $A_s + A_c$).
kR = perpendicular distance from center of chimney to neutral axis (inches).

The moment of the tension area of steel (arc NTA) about the neutral axis is

$$2nptR^2\int_{\pi-\sin^{-1}k}^{\frac{3\pi}{2}}(\sin\theta - k)d\theta = 2nptR^2\left[(1-k^2)^{1/2} + k\sin^{-1}k + \frac{k\pi}{2}\right] \tag{1}$$

The moment of the compression area, whose transformed stress width may be expressed by $[1+(n-1)p]t$, about the neutral axis is similarly

$$2R^2t[1+(n-1)p]\int_{\frac{\pi}{2}}^{\pi-\sin^{-1}k}(\sin\theta - k)d\theta = 2R^2t[1+(n-1)p]\left[(1-k^2)^{1/2} + k\sin^{-1}k - \frac{k\pi}{2}\right] \tag{2}$$

Since these two moments balance about the neutral axis, they may be equated, from whence is derived the expression

$$p = \frac{\left[(1-k^2)^{1/2} + k\sin^{-1}k - \frac{k\pi}{2}\right]}{\left[(1-k^2)^{1/2} + k\sin^{-1}k - \frac{k\pi}{2} - k\pi n\right]} \tag{3}$$

Equation (3) has been plotted at the top of Diagram 2, assuming $n = 15$.

The moment of inertia of the transformed section may be written

$$I = 2R^3t[1+(n-1)p]\int_{\frac{\pi}{2}}^{\pi-\sin^{-1}k}(\sin\theta - 2k\sin\theta + k^2)d\theta + 2R^3pnt\int_{\pi-\sin^{-1}k}^{\frac{3\pi}{2}}(\sin^2\theta - 2k\sin\theta + k^2)d\theta = R^3t\left\{(1-p)\left[(1+2k^2)\left(\frac{\pi}{2} - \sin^{-1}k\right) - 3k(1-k^2)^{1/2}\right] + \pi pn(1+2k^2)\right\} \tag{4}$$

Let c_s = distance (inches) from the neutral axis to the extreme fiber of steel in tension (on the transformed section).

c_s' = distance (inches) from the neutral axis to the extreme fiber of steel in compression (on the transformed section).

and c_c = distance (inches) from the neutral axis to the extreme fiber of concrete in compression.

Then $c_s = Rn(1+k)$; $c_s' = Rn(1-k)$; $c_c = R(1-k)$ (5)

If M is the bending moment in inch-pounds on any given cross-section due to external loads, then from mechanics,

$$f_s = \frac{Mc_s}{I} \qquad f_s' = \frac{Mc_s'}{I} \qquad f_c = \frac{Mc_c}{I} \tag{6}$$

in which f_s' = fiber stress of steel in compression, and f_s and f_c the same as in beam analysis. By substituting equations (4) and (5) into equation (6) the curves of Diagram 1 were plotted, assuming $n = 15$. This diagram does not include the stresses due to dead load, but gives only those stresses due to bending.

It may be noted that f_c relates to the compression in the concrete at the center of the chimney wall, and not at the outer face (see sketch, Diagram 2). The maximum compression in the concrete may be given by

$$f_c(\max) = f_c\left(1 + \frac{t}{2R(1-k)}\right) \tag{7}$$

The fractional term in the brackets is the increase over f_c to obtain f_c (max). The % increase in concrete compression stress over f_c to obtain f_c (max) is plotted on Diagram 2(a) for various values of t/R. The application of these curves relates to the stress due to flexure only.

Longitudinal Shear at Neutral Axis.—The intensity of longitudinal shearing stress at the neutral axis may be given by

$$v = \frac{VM_R}{4I} \tag{8}$$

for an annular section, in which V is the total transverse shear on the section and M_R equals the sum of the moments of the tension and compression areas as given in equations (1) and (2). Substituting into equation (8) there results

$$\frac{V}{Rt} = v\left[\frac{2\left\{(1-p)\left[(1+2k^2)\left(\frac{\pi}{2}-\sin^{-1}k\right) - 3k(1-k^2)^{1/2}\right] + \pi pn(1+2k^2)\right\}}{(1-p)\left[(1-k^2)^{1/2} + k\sin^{-1}k - \frac{k\pi}{2}\right] + 2pn[(1-k^2)^{1/2} + k\sin^{-1}k]}\right]$$

When $n = 15$ the term in the brackets becomes 3.15 for the usual ranges of k, whence

$$\frac{V}{Rt} = 3.15v \tag{9}$$

From this equation the longitudinal shearing stress v may be computed.

16. Wind Stresses in Chimneys of Reinforced Concrete.—The action of wind pressure alone upon a chimney is similar to the action of a distributed load on a cantilever beam of annular section. The force of the wind is usually assumed, for a cylindrical surface, to be two-thirds that on a plane surface. Thus, at 30 lb. per sq. ft., the pressure per foot of height of chimney would be $20D$, or $40R$.

The stresses caused by the wind may be determined by referring to Art. 15. A problem will best illustrate the procedure.

Illustrative Problem.—Required % of vertical steel, and thickness of shell at the base of a chimney 200 ft. high and 8 ft. mean diameter, such that the steel stress for wind shall not exceed 12,000 lb. per sq. in., and the concrete stress for wind shall not exceed 400 lb. per sq. in.

$$\text{Moment on section} = M = 20 \times 8 \times 200 \times 12 \times 100 = 38{,}400{,}000 \text{ in.-lb.}$$

$$\frac{M}{R^2t} = 1575 \text{ for } f_s = 12{,}000 \text{ and } f_c = 400 \text{ (from Diagram 2)}$$

$$t = \frac{38{,}400{,}000}{(4 \times 12)^2 \times 1575} = 10.6 \text{ in.}$$

The vertical steel required is 3.65%. Should a smaller percentage be desirable, the steel stress would govern. Thus with 3% vertical steel, $M/R^2t = 1320$ for $f_s = 12{,}000$. Then

$$t = \frac{38{,}400{,}000}{(48)^2 \times 1320} = 12.7 \text{ in.}$$

This serves to illustrate the effect of a governing stress when balanced stresses are not used. In the last instance $f_c = 370$ lb. per sq. in.

17. Chimney with No Vertical Reinforcement.—Short stacks, and the upper part of tall stacks, may not have sufficient moment due to wind to cause tension on the windward side. Assuming a wind pressure of 20 lb. per sq. ft. on a vertical projection, and noting that the boundary of the kern of a thin hollow circular section is $R/2$ from the center,

$$y = 236\,Rt \qquad \text{(ft.)}$$

in which y is the height of chimney in feet, above the section on which the resultant cuts the kernal boundary, and R and t are the mean radius and thickness of shell, respectively, also in feet. The compressive unit stress in the concrete, with no steel, is

$$f_c = 2.08\,\frac{W}{A}$$

where W is the weight above the section, and A is the sectional area in square inches.

The presence of vertical steel will affect the stresses, hence the formulas of Art. 15 will be found useful for determining the flexural stresses.

It should be noted that chimneys without vertical steel are subject to severe temperature stresses if used under conditions where inside temperatures are those of flue gases, or under conditions which give a temperature drop of 100°F. or more through the shell.

18. Longitudinal Shear in Chimneys.—Because the shell of concrete chimneys is relatively thin, the unit shear on a longitudinal section requires consideration. Equation (9), Art. 15, gives a means of solution of the longitudinal shearing stresses. The transverse shear $V = 2yR \times 20 = 40yR$, in which y is the height of chimney above the section considered and R is in feet. From equation (9),

$$\frac{40yR}{12Rt} = 3.15v$$

$$y = 0.079vt \times 12 = 0.948vt.$$

When $v = 40$, the following table will give the relation of t to h:

t	4	5	6	7	8	9	10	11	12
h	151.2	189.0	226.8	264.6	302.4	340.2	378.0	415.8	453.6

When $v = 30$:

t	4	5	6	7	8	9	10	11	12
h	113.4	141.8	170.1	198.5	226.8	255.2	283.5	311.9	340.2

The above heights will give the limits for the given shearing unit stresses and thicknesses, on the basis of a wind pressure of $\frac{2}{3}(30)$ lb. per sq. ft. of projected area.

19. Temperature Stresses in Chimneys.—Flue gases have a temperature sufficient to commonly give the chimney shell a temperature of 400° to 500°F. at the inner face near the flue, and seldom exceed 700°F.[1] At a point three-quarters of the height above the base it is found that the temperatures have not decreased more than 10 to 20% of the flue-level temperatures.

The fact that the inner face of the shell tends to expand laterally and vertically causes compression in the concrete and tension in the steel, in both directions. Assuming a constant modulus of elasticity for concrete in compression for the above temperature range, and assuming also a straight-line temperature gradient through the shell, Turneaure and Maurer have built up a theory for the estimation of these temperature stresses. Applications of this analysis yield a very high value of compression in the concrete vertically, and a moderate value (average about 400 to 500 lb. per sq. in.) in a lateral direction. Appearance of large cracks in a lateral, as well as vertical, direction, particularly near the top of the inner lining if one is present, bears out the fact that large stresses do exist.

Prevention of temperature cracks cannot alone be made by heavier reinforcement. The custom has been to extend a clay lining (in some instances though perhaps not in all cases warranted, a fire-clay lining) from the flue-line to one-third the chimney height, having an air space between the lining and the outer shell of 2 to 6 in. It is becoming evident, in the light of past experience, that the lining should extend at least two-thirds the height of the chimney. It is essential, also, that the air space between the lining and shell be provided with vents so that a good circulation may be obtained. This is as important as any other feature of design.

Because of the meagre information concerning the properties of concrete under high temperatures, it is not possible to build up a close theory of the stresses due to temperature. The

[1] Report on reinforced-concrete chimneys to Assoc. Am. Port. Cem. Mfrs. by SANFORD E. THOMPSON (1910).

discussion of Turneaure and Maurer[1] will offer a valuable guide to the estimation of the importance of these stresses, in the light of temperature data at present available.

20. Chimney Construction.—The shell is usually made with a smooth exterior in recently constructed chimneys, and changes in thickness to obtain lighter sections at the top are made by stepping the inner face. The forms used are usually steel either cylindrical or tapering, and are jacked up on the vertical reinforcement (see construction of deep grain bins or silos, Art. 5.), so that pouring is nearly continuous. Moderately dry 1 : 3 concrete is generally used, and is well tamped. The mix should not be so dry that upon tamping, moisture is not brought to the surface readily. Care should be taken to have a maximum silica (sand) surface on the inside. The steel used may well be deformed, to distribute the temperature cracks.

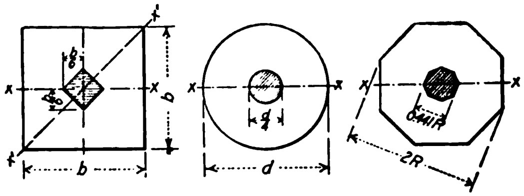


Fig. 13.

21. Bases for Chimneys.—Chimneys are often placed on yielding soil, using a large base slab. The common forms of bases are squares, circles, and octagons. Conditions of pressure under these slabs may be grouped in two cases: (1) resultant within the kern, (2) resultant outside the kern.

Kern sections for the forms of bases named are shown hatched in Fig. 13.

The section moduli, S, for these figures are given below:

Figure	Square	Circle	Octagon
S about axis $X-X$	$\frac{b^3}{6}$ (about $X'-X'$ $0.118b^3$)	$0.1d^3$ (approx.)	$0.6906R^3$

When the resultant pressure lies within the kern of the base figure, the soil pressure at the edge may be found by

$$p_1 = \frac{W}{A} + MS$$

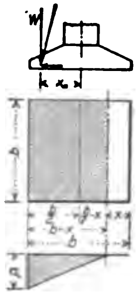


Fig. 14.

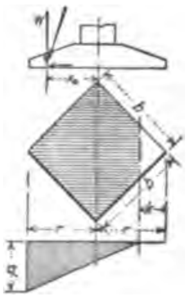


Fig. 15.

A table of allowable soil pressures is given in Art. 1, Sect. 12.

The design of the base slab for moment and shear may be found in the design of footings, Art. 7, Sect. 12.

[1] "Principles of Reinforced Concrete Construction," 2d Ed., p. 413.

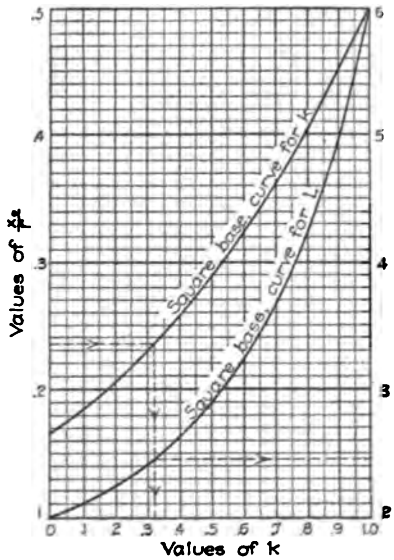
Square Bases.—When tension exists between the foundation and the windward toe, there is what might be considered a neutral axis and only part of the foundation is under pressure (see Figs. 14 and 15). The following formulas result:

For direction of wind parallel to a side of the square (Fig. 14)

$$x = 3x_0 - \frac{b}{2}$$

$$p_1 = \frac{4}{3\left(1 - 2\frac{x_0}{b}\right)} \cdot \frac{W}{b^2}$$

DIAGRAM 3



For direction of wind parallel to a long diameter of the square (Fig. 15)

$$\frac{x_0}{r} = \frac{1 - k^2 + \frac{k^4}{2}}{6 - 6k + k^3}$$

$$p_1 = \frac{2 - k}{1 - k + \frac{k^3}{6}} \cdot \frac{W}{b^2}$$

If the quantity $\frac{2 - k}{1 - k + \frac{k^3}{6}}$ be denoted by L, then $p_1 = L\frac{W}{A}$.

Diagram 3 gives values of k and L for various values of $\frac{x_0}{r}$. Dotted lines with arrows indicate how to obtain the values of k and L for a given value of $\frac{x_0}{r}$.

SECTION 19

ESTIMATING

By Leslie H. Allen[1]

ESTIMATING UNIT COSTS

1. Division of the Work.—Reinforced-concrete work may be considered under the following four main divisions:

1. The concrete itself.
2. The forms or falsework.
3. The steel reinforcement.
4. The finish of the exposed surfaces.

Each of these divisions should be considered separately when making an estimate.

In the prices mentioned in this chapter, the cost of labor is based upon the rates paid in the large cities at the present time (namely: carpenters 65 cts. per hr., laborers 35 cts. per hr., carpenter foremen $7 per day) and include the overhead expense of superintendent and timekeepers in charge of the work.

2. Estimating Unit Cost of Concrete.—With regard to the concrete itself, we shall take into account: (*a*) the cement, sand, stone, and water; (*b*) the labor of unloading, mixing, and placing of these materials; and (*c*) the plant necessary to accomplish this end.

2a. Materials.—The cost of cement in carload lots is, as a rule, about $1.50 per bbl. at the cement mill. If the cement is delivered in paper bags, there is an extra charge of 10 cts. per bbl. for bags. If delivered in cloth, the extra charge is 40 cts., but this 40 cts. is refunded by the mills if the cloth bags are returned in good condition. If furnished in wooden barrels, there is a charge of 40 cts., and the barrel is not returnable. It is usually considered the most economical to buy cement in cloth bags and return the bags when empty. The freight on a barrel of cement on a haul of say 500 miles would be about 40 cts., so that the total cost of a barrel of cement in cloth bags after returning and crediting the bags would be $1.90 per bbl. The cost of testing cement is from 3 to 5 cts. per bbl. It is customary among contractors and engineers to have the whole shipment of cement tested at a testing laboratory, and from $5 to $6 per carload is about the usual charge. The cost of unloading cement and placing it in a storehouse close to the track is about 5 cts. per bbl. If the railroad tracks do not run to the site of the construction work, there must also be added the cost of teaming, which would amount on a distance of 1 mile to about 5 cts. per bbl. In addition to this must be figured the cost of handling and returning empty sacks, the freight on same, and the loss of a few damaged or torn bags. This is usually estimated at about 3 cts. per bbl. Tabulating the above, the cost of cement per bbl. ready for use would appear as follows:

Cement	$1.50	
Freight	0.40	
Cloth sacks	0.40	
Total cost of cement f.o.b. cars at job	$2.30	per bbl.
Deduct credit for empty sacks	0.40	
	$1.90	
Add cost of testing	0.03	
Add cost of unloading	0.05	
Add cost of teaming, if any	0.05	
Add cost of bundling and returning empty bags, and loss on same	0.03	
Net price of cement ready for use in concrete	$2.06	per bbl.

[1] With Aberthaw Construction Co., Boston, Mass.

It is usual to obtain quotations from the cement companies for cement for jobs on which estimates are being made. These quotations always include the freight and the bags, and to arrive at the net cost it is necessary to deduct for the bags and add for the supplementary items, according to the above list.

Sand usually costs about 50 cts. per cu. yd. to dig and load on teams or cars. If it has to be screened or washed, it will cost from 60 to 70 cts. per cu. yd. Teaming or freight will vary according to the length of haul, but will usually bring the cost of sand, ready for use, up to $1.30 per cu. yd., f.o.b. the job. If it comes by rail, there should be added to this 15 cts. per cu. yd. for unloading from cars.

Crushed stone can be bought at from $1.00 to $1.25 per ton at the crusher, to which must be added the cost of teaming or freight, which will vary according to the length of haul. On a haul of moderate length it is usual to pay from 30 to 50 cts. per ton, so that the cost of crushed stone (f.o.b. the job ready for use) generally varies between $1.30 and $1.75 per ton. To this should be added about 25 cts. per ton if it has to be unloaded from railroad cars. If gravel of suitable size and quality is available for use, it can generally be obtained for $1.50 per cu. yd., a considerable saving on the price of crushed stone. In comparing the price of gravel and crushed stone, 1 cu. ft. of crushed trap rock or granite may be considered as weighing 100 lb.

Large bridges and other structures are sometimes built in places that are very difficult of access and in consequence the cost of teaming materials may be much greater than above mentioned. In some cases it may be found necessary to set up a crushing plant for the supply of stone. The contractor, however, always avoids this, if possible, as the cost of operating a small temporary plant is always greater than that of running a large permanent plant, and it pays to buy crushed stone from such a plant, even if rock from the excavations is available.

2b. Labor.—The labor of mixing and placing concrete varies considerably, according to the conditions of the job and the nature of the work. It is obvious that to mix and place concrete in heavy bridge abutments and concrete dams would cost a great deal less than to place concrete in floor slabs 3 or 4 in. thick, in arch ribs, or in beam and column forms, as the former would not require so much spading and spreading.

Assuming a well laid-out job and a machine mixer taking 4 bags to the batch, the cost of mixing concrete should be from 70 to 80 cts. per cu. yd. The operations will consist of loading wheelbarrows with sand and stone and wheeling them up to the mixer and charging same, bringing cement from the cement shed and putting it into the mixer, and the work of an engineer in running the mixer and discharging the concrete into wheelbarrows.

The cost of placing concrete will include the wheeling and dumping of the concrete in place, and the spreading and spading of the concrete in the forms. This should cost about 90 cts. per cu. yd. in average work. In columns and thin walls, where there is a lot of spading and where care has to be used to get a good surface on the concrete, this cost would be about doubled. On heavy masses of concrete, such as dams and thick retaining walls, these prices can be considerably reduced, especially if the plant is well laid out and the equipment is good. Concrete has been mixed and placed by mixer and derrick, or tracks and cars, for as low as 50 cts. per cu. yd.

The following is an approximate schedule of labor prices for mixing and placing concrete:

Mixing and placing in footings	$1.75 per cu. yd.
Mixing and placing in floor slabs not exceeding 4½ in. thick	2.00 per cu. yd.
Mixing and placing in floor slabs exceeding 5 in. thick	1.25 per cu. yd.
Mixing and placing in columns and thin walls	2.00 per cu. yd.
Mixing and placing in walls exceeding 18 in. in thickness	1.25 per cu. yd.
Mixing and placing in dams and thick retaining walls	1.00 per cu. yd.

On some jobs it is possible to unload sand and gravel direct from railroad cars to the wheelbarrows which charge the mixer. In such cases the materials would be handled once instead of twice before going into the mixer, and a saving of about 25 cts. per cu. yd. would result.

It is usual to specify that large stones may be embedded in massive concrete work to reduce the cost of same. These stones are generally placed not less than 6 in. apart and are kept at least 12 in. away from the face of the work. Some specifications will allow stones that one man can handle; others will allow any stone that the derricks can lift. It will be found that from 20 to 50% of the volume of a massive pier can be composed of large stone in this way. The cost of placing these stones, or "plums" as they are commonly called, should not exceed $1.25 per cu. yd. If the rock has first to be excavated for the purpose, the cost of the rock excavation must be added.

2c. Plant.—The cost of tools and plant and supplies on a job varies a good deal according to the nature and size of the work. But, assuming a building job containing 5000 cu. yd. of concrete work carried out by a contractor of ability, it will usually be found that the cost of plant, tools, and supplies, temporary buildings, office, cement shed, etc., will amount to about $6000. Of this amount, about $1000 would be spent on labor in setting up the plant and dismantling it; $300 for freight; $2000 for small tools and depreciation of mixer, hoisting engines and large tools; and $2700 for coal or power, small tools, supplies and sundries. The writer's practice is to estimate $1.25 for every yard of concrete on building jobs containing between 4000 and 10,000 yd. of concrete. On larger jobs than this, the proportion would be smaller, probably about 0.85 to $1.00 and, on smaller jobs than those containing 3000 yd., the proportion would be higher—from $1.40 to $2.00 per cu. yd. On jobs containing less than 600 cu. yd. of concrete, machine mixing is not usually economical, and in that case it will be necessary to estimate for mixing by hand, which will cost about $2 per cu. yd. more than the prices given above for labor instead of including a charge for plant.

As a general rule, the plant required in bridge construction is more costly than that for building construction. This is particularly the case where cableways are used. It is best to calculate separately the cost of labor and depreciation on each item of plant in each case and add the cost for supplies, fuel, and small tools. In general, however, it will be found that the total cost of plant for bridges, as in buildings, will vary pretty closely with the yardage of concrete. In the writer's practice he has found that figures of $7000 for plant on a 5000-yd. job, $8000 on a 6000-yd. job, $6000 on a 4000-yd. job, and so on, check up quite closely with actual costs. These figures, of course, do not include plant for excavating, drilling, etc.

2d. Summary.—The conditions on construction work do not approach those of laboratory work, and there is always a considerable waste of cement, sand, and stone. It has been found in practice, that, when estimating, it is not safe to allow less than the following amounts of cement for different proportions of mix:

1 : 1½ : 3 mix	2.00 bbl. per cu. yd.
1 : 2 : 4 mix	1.66 bbl. per cu. yd.
1 : 2½ : 5 mix	1.40 bbl. per cu. yd.
1 : 3 : 6 mix	1.20 bbl. per cu. yd.

The amount of sand and stone required varies considerably according to the percentage of voids. This variation cannot be taken into account in the usual methods of estimating—as it can only be ascertained by careful tests and varies from time to time, even when the source of supply is the same—and therefore it is usual to allow ½ cu. yd. of sand per cu. yd. of concrete, and 1 cu. yd. of crushed stone—figuring crushed stone to weigh 100 lb. per cu. ft.

The cost of 1 cu. yd. of concrete on a job containing 5000 cu. yd. of reinforced-concrete work in floors and columns, etc., may be estimated as follows:

Cement	1.66 bbl. at $2.06	$3.42
Sand	0.5 yd. at $1.30	0.65
Stone	1.35 tons at $1.60	2.16
Labor		1.65
Plant		1.25
Total		$9.13 per cu. yd.

The following illustrates the method of estimating the cost of concrete on a large typical bridge job:

Abutments and piers—1 : 2½ : 5 mix:	
Cement............1.4 bbl., @ $2.10 net	$2.94
Sand..............0.5 cu. yd., @ 1.30	0.65
Crushed stone.......1.35 tons, @ 1.60	2.16
Labor, mixing and placing	1.00
Plant	1.30
Total	$8.05 per cu. yd.
Abutments and piers—1 : 2½ : 5 mix—with 30% of large stones:	
7 cu. yd. concrete as above, @ $8.05	$56.35
3 cu. yd. placing large stones, @ 1.50	4.50
Cost of 10 cu. yd. of concrete and rock in place	$60.85
Average cost per cu. yd.	6.08
Arch ribs and deck slabs—1 : 2 : 4 mix:	
Cement............1.66 bbl., @ $2.10 net	$3.49
Sand..............0.5 cu. yd., @ 1.30	0.65
Crushed stone.......1.35 tons, @ 1.60	2.16
Labor, mixing and placing	1.30
Plant	1.30
Total	$8.90 per cu. yd.

The above costs do not include forms, steel, or finishing of surfaces.

3. Estimating Unit Cost of Forms.

3a. **Considerations Involved.**—Forms for building work should be measured by the square foot of surface (measuring all sides that touch the concrete) and priced according to the labor involved in erecting, studding, bracing and stripping. Strictly speaking, formwork is the labor of supporting wet concrete, and the material—that is, the lumber used—is only incidental to this labor; therefore it is not correct to take off the amount of lumber used and price it according to the number of board feet thus estimated. It is the practice of some firms to estimate forms by the latter method but such a practice is misleading, not only on the theoretical ground taken above, but on the practical ground that no two contractors would use the same amount of lumber in erecting a given piece of formwork. Besides, it is not possible to determine beforehand just how much lumber will be used for the same, since careful drawings of the forms are not usually available. In the writer's practice, he has always found that even though enough lumber to complete the work is ordered when a job is started it is always necessary, on account of loss and waste of lumber, to order a great deal more before the job is finished.

The forms for a building consist principally of:

- Forms to floors.
- Forms to beams and girders.
- Forms to columns.
- Forms to walls.
- Forms to footings.

Each of these should be measured separately and priced at separate and different rates.

The most uncertain and difficult item to estimate in bridge and similar construction is the formwork. It is best to estimate by the square foot of surface contact, as in building work, and to this add for the staging required for long arch spans.

3b. Materials.—The writer's experience, based on the cost accounts of many reinforced-concrete buildings, large and small, shows that a good rule for estimating the cost of lumber, nails, oil, and wire used in the construction of forms to a factory building, is to reckon from $3.50 to $5.50 per 100 sq. ft. for these items, a good average being $4.00 per 100 sq. ft. This figure is higher than that used by a good many estimators at the present day, but is based on actual experience of costs kept on a large number of jobs, and the writer is constantly proving that this figure is correct.

In estimating bridge formwork (the lumber, nails, oil, etc.) $5 per 100 sq. ft. should be a sufficient allowance.

3c. Labor.—The cost of forms to floor slabs in buildings will take into account:

1. Making up panels.
2. Setting up posts and bracing same.
3. Putting girts and ledgers on tops of the posts.
4. Laying the panels on the same.
5. Stripping the centering after the concrete has set.

The labor of making up form panels will average about 4½ cts. per sq. ft., and as these are generally used 3 or 4 times, 1½ ct. per sq. ft. on the whole area is a good figure to use in estimating. The labor of erecting studs and bracing will average about 3 cts. per sq. ft., and the cost of putting on joists and girts and laying down panels will also cost about 3 cts. per sq. ft., giving a total labor cost of erecting forms of 7½ cts. per sq. ft. Add to this 1¼ ct. per sq. ft. for stripping, and we get a labor cost of 8¾ cts. per sq. ft. as a total. On a small and irregular building, of course this cost will be much higher, but for a plain, large factory a low cost such as this can often be reached. If the height from floor to ceiling is over 16 ft., this price should be increased, as the studs will have to be spliced and extra bracing will have to be put in. Two or three cents a square foot should therefore be added in such a case.

The cost of beam and girder forms includes the cost of the following operations:

1. Making beam bottoms and beam sides.
2. Erecting same on the posts and joists which support the floor slab.
3. Stripping.

The cost of making up will be about 6 cts. per sq. ft., and as beam bottoms are left in longer than the slab forms, it is not safe to figure on using these more than twice, giving an average cost of 3 cts. per sq. ft. for making beam forms. Erecting the same will cost about 6 cts. per sq. ft., and stripping 1½ ct. per sq. ft., giving a total cost of 10½ cts. per sq. ft. for beam and girder forms. If beams and girders are haunched at the ends, 50 cts. more should be allowed each time for the haunching.

The labor of forming columns may be subdivided into:

1. Making up panels.
2. Erecting panels and placing yokes.
3. Plumbing.
4. Bolting.
5. Stripping.

The cost of making up panels, which are usually of 1½-in. stock, will be found to average between 5 and 6 cts. per sq. ft. Allowing that these will be used 3 times, the cost per square foot of formwork would be about 2 cts. The erecting, plumbing, and bolting will be about 9 cts. per sq. ft., and the cost of stripping about 1½ cts. per sq. ft., giving a total labor cost of 12½ cts. per sq. ft. for labor on column forms.

Columns less than 8 ft. high cost a good deal more per square foot than higher columns, owing to the fact that there is just as much time and labor spent in plumbing, erecting, and bolting up as if the columns were twice as high with twice the amount of surface area.

In a similar way, the cost of erecting wall forms and footing forms nay be found.

No general rules or instructions can be given for estimating bridge and other formwork, except to say that the labor should be estimated at not less than 13 cts. per sq. ft

3d. **Summary.**—The costs of forms per square foot to a reinforced-concrete building may be tabulated as follows:

Forms to floor slabs:

Lumber, nails, oil, etc.	$0.04
Labor making panels	0.015
Labor erecting studs and bracing	0.03
Labor laying panels	0.03
Labor stripping	0.0125
Total	$0.1275

Forms to beams and girders:

Lumber, nails and oil	$0.04
Labor making	0.03
Labor erecting	0.06
Labor stripping	0.015
Total	$0.145

Forms to columns:

Lumber, nails, oil, etc.	$0.04
Labor making panels	0.02
Labor erecting, plumbing, and bolting	0.09
Labor stripping	0.015
Total	$0.165

Forms to footings:

Lumber, nails, and oil	$0.04
Labor making and erecting	0.08
Labor stripping	0.015
Total	$0.135

Forms to walls:

Lumber, nails, oil, etc.	$0.04
Labor making	0.03
Labor erecting and plumbing	0.08
Labor stripping	0.015
Total	$0.165

4. Estimating Unit Cost of Steel Reinforcement.—The cost of steel reinforcement, if ordered in time to wait for delivery direct from the mill, will average about $3.00 per 100 lb. The freight rate on a haul of about 500 miles will be about 18 cts., giving a total of $3.18 per 100 lb. for steel bars, f.o.b. cars at the job. At the time of publication of this book (Feb. 1918), deliveries are so very slow that the universal practice is to buy steel only from stock at a delivered price of about $4.00 per 100 lb. The cost of steel bars varies according to the size of the bar. Bars of from 1½ to ¾ in. diameter are taken at the lowest rate, which is called the *base* price. Small bars take a higher rate as follows:

	From mill	From local warehouse
$\frac{5}{8}$ in. and $\frac{11}{16}$ in.	base plus 5 cts.	base plus 10 cts.
$\frac{1}{2}$ in. and $\frac{9}{16}$ in.	base plus 10 cts.	base plus 15 cts.
$\frac{7}{16}$ in.	base plus 20 cts.	base plus 30 cts.
$\frac{3}{8}$ in.	base plus 25 cts.	base plus 40 cts.
$\frac{5}{16}$ in.	base plus 35 cts.	base plus 55 cts.
$\frac{1}{4}$ in.	base plus 50 cts.	base plus 75 cts.

This differential is an important factor in design as well as in estimating. For example, assume a floor 200 by 100 ft. having a slab 6 in. thick and an area of steel per square foot of 0.462 sq. in. The total weight of steel required (allowing for laps) would be 33,301 lb.

Cost of $\frac{1}{2}$-in. square bars $6\frac{1}{2}$ in. on centers, 33,301 lb. @ \$4.15 per 100 lb. = \$1381.95.

Cost of $\frac{3}{8}$-in. round bars 3 in. on centers, 33,301 lb. @ \$4.40 per 100 lb. = \$1465.20.

Thus there is a difference of \$83.25 in favor of $\frac{1}{2}$-in. bars if taken from stock deliveries. It is usual in estimating on a building in normal times to figure on taking the steel for the footings and the lower part of the building, enough for the first 5 or 6 weeks work, out of local warehouse stock and buy the rest direct from the mill.

The cost of unloading steel and piling it on the job is about 50 cts. per ton, and, if it has to be teamed from the freight yards to the site of the work, this will cost 60 cts. per ton and upward.

The cost of bending and placing steel in a building will vary according to the amount of work that is done. Thus, placing steel bars $\frac{5}{8}$-in. diameter in a floor slab will cost about \$6 per ton. If the bars have to be bent up at the end, \$3 per ton should be added. Bending and placing steel bars and stirrups in beams will cost from \$8 to \$10 per ton. Wiring up and placing steel bars in columns and placing hoops around them will cost from \$10 to \$12 per ton. Placing steel of $\frac{1}{2}$-in. and $\frac{3}{8}$-in. diameter in walls will cost from \$15 to \$20 per ton.

These prices are sufficient to include the cost of wire, tools for bending, etc.; \$12 per ton is a good average price for labor on steel reinforcement all through the job.

The cost of steel reinforcement in a reinforced-concrete building may be estimated as follows:

Steel from warehouse stock @\$4.00 per 100 lb.	\$80.00 per ton
Unloading, teaming, and piling	1.10 per ton
Labor, bending and placing	12.00 per ton
Tools, wire, and sundries	0.75 per ton
Total	\$94.10 per ton

5. Estimating Unit Cost of Surface Finish.—One hundred square feet of granolithic finish laid 1 in. thick in the proportion of 1 of cement, 1 of sand, and 1 of fine crushed stone, will require 1 bbl. of cement, 4 cu. ft. of sand, and 4 cu. ft. of crushed stone. This may therefore be estimated as follows:

1 bbl. cement	@\$2.06 per bbl.	\$2.06
4 cu. ft. sand	@\$1.35 per cu. yd.	0.20
400 lb. fine crushed stone	@\$2.25 per ton	0.45
		\$2.71
Labor mixing and placing		1.00
Finishers' time trowelling surface		1.40
Total cost per 100 sq. ft.		\$5.11

If the surface of the concrete has to be cleaned off with acid or sand blasting, this will cost from 2 to 3 cts. per sq. ft. additional.

The exterior surfaces of concrete columns and beams are frequently rubbed smooth with carborundum stone, using a little water and cement.[1] The cost of this work, including hanging swing stages for the finishers, will be from 4 to 6 cts. per sq. ft.

For ornamental effect, external surfaces are sometimes picked with a pointed tool or crandall hammer. The cost of this runs between 6 and 10 cts. per sq. ft.

ESTIMATING QUANTITIES

6. Systematic Procedure Advisable.—The operation of estimating quantities is that of calculating (from plans supplied) quantities of labor and material which go to make up the completed building. This is usually called *taking off* or *scaling*. This should be done quite independently of the pricing or the arithmetical work of extending the quantities to obtain the totals of the quantities of work. The secret of accurate, speedy taking off is to be found in a systematic way of going about the work. No printed forms, tables, or special rules for taking off will insure against error, but the surest way of making an accurate estimate is to have a good system to work on and a clear and easily followed way of setting down the items. A good method is to use plain squared paper 8 by 10½ in., ruled in nine columns. In the first column is placed a description of the items measured; the next four columns are for the number, length, width, and height of the members of the building; the next two are for arithmetical calculations and totals; and the last two for unit and total price. It is very important to keep length, breadth, and height in the same order in every item. Each can then be readily identified. In taking off a reinforced-concrete building, start with the structural members in the order in which they are built; that is, first take concrete footings, then columns, then floor slabs, then beams and girders, then curtain walls and partitions, then cornice, and then stairs and landings. Take all the concrete first, one item at a time and complete it. When all the concrete is taken off, proceed to take off the forms, and after that take off the reinforcement, and then the finish to the surfaces. After that take off excavation, windows and doors, roofing and other incidental items necessary to complete the cost of the building.

In putting down the dimensions, it is well to put a note identifying each item, thus:

Concrete columns:

Basement,	(mark A)........	5 × 1½ × 1½ × 10
	(mark B)........	4 × 1½ × 1⅔ × 10
	(mark C)........	7 × 1½ × 1⅓ × 10
First floor,	(mark A)........	5 × 1⅓ × 1⅓ × 10
	(mark B)........	4 × 1⅓ × 1½ × 10
	(mark C)........	7 × 1⅓ × 1⅛ × 10

It takes a little more time to do this, but it is well worth the labor, and any item can be readily identified afterward. Also if an item is left out in error, it can be more easily detected. It is good practice to put all dimensions in feet and fractions. Some estimators work in feet and inches and some in feet and decimals. There seems to be the least chance for error in using fractions, but this is a matter of individual judgment.

7. Rules for Measurement of Concrete Work.—The following rules should govern the measurement of concrete work:

All concrete should be measured by the cubic foot or cubic yard, and in all cases forms should be measured separately. All concrete should be measured net as placed or poured in the structure or building, and an excess measurement of concrete should never be taken to pay for the cost of forms or extra labor in placing. All openings and voids in concrete should be deducted, but no deduction should be made for steel reinforcement, I-beams, bolts, etc.,

[1] See Art. 52*b*, Sect. 2.

embedded in the concrete, except where such have a sectional area of more than 1 sq. ft. No deduction should be made for chamfered, beveled, or splayed angles to columns, beams, and other work.

For beams and girders it is usual to show on the plan the depth of concrete from the top of the slab. Thus, if the quantity of concrete for the slab has been taken right through, it will be necessary to consider only the extra concrete below the slab in taking off beam and girder quantities. For example, in a floor 6 in. thick having 12 by 30-in. girders, the concrete to take off for the girders should be considered as 1 ft. wide by 2 ft. deep, since the other 6 in. is included in the slab.

Each class of concrete having a different proportion of cement, sand, or aggregate should be measured and described separately. Concrete in the different members of a building or structure should be measured and described separately according to the accessibility, location, or purpose of the work; concrete in floor slabs should be measured and priced separately from columns or walls, and so on. Concrete with large stones and rocks embedded in same (cyclopean masonry) should be measured as one item and described according to the richness of the mix and the percentage of rock in same. Concrete in stairs should be measured by the cubic foot, and it is usual to include surface finish with same in this case, as it forms such a small item in the cost.

8. Estimating Amount of Formwork.—Forms should be measured in square feet, taking the area of the surface of the concrete which is actually touched by the forms or falsework. Forms should in all cases be measured and described as a separate item and never included with the concrete. No deduction should be made in measurement of surface of concrete supported by forms because of forms being taken down and re-used 2 or 3 times in the course of construction.

It is not necessary to consider struts, posts, bracing, bolts, wire ties, oiling, cleaning, and repairing forms, as these should be covered by the price put on the square foot measurements.

Forms to the different parts of a building should be measured and described separately according to their nature; that is, forms to floor slabs, walls, columns, footings, etc., should be separated from each other. No allowance need be made for angle fillets or bevels to beams and columns, etc., but curved moldings should be measured and described separately.

No deduction in measurement of forms should be made for openings having an area of less than 25 sq. ft. as the labor in forming same is often greater than the cost of the omitted area. No deduction should be made in floor forms for heads of columns, or in column and girder forms for ends of girders, cross beams, etc. No allowance should be made for pockets in column forms for clearing out rubbish.

Fig. 1.

The correct measurement of column forms is the girth of the four sides, or circumference, multiplied by the height from the floor surface to the under side of floor slab above. Forms to octagonal, hexagonal, and circular columns should be measured and priced separately from forms to square columns. Caps and bases to columns and other ornamental work should be enumerated and fully described by sketches in the estimate with overall dimensions.

The correct measurement of beam forms is the net length between columns multiplied by the sum of the breadth (b) and twice the depth below the slab (d), except for beams at edge of floor or around openings, which shall have the thickness of the floor (t) added to the sum of the breadth and twice the depth (see Fig. 1).

Wall forms should be measured for both sides of concrete walls.

Forms to the upper side of sloping slabs such as saw-tooth roofs should be measured whenever the slope of such slab with the horizontal exceeds an angle of 25 deg.

Moldings in formwork should be measured by the linear foot. Forms to circular work should always be measured separately from forms to straight work.

No measurement or allowance should be made for construction joints in slabs, beams, etc., to stop the day's concreting, but construction joints or expansion joints in dams and other large masses of concrete should be measured by the square foot as they occur.

Forms to cornices should be measured by the linear foot and the girth stated. Plain forms to back of cornice should be measured separately. Forms to window sills, copings, and similar work should be measured by the linear foot. Forms to the underside of stairs should be measured by the superficial foot, and forms to the front edge by the linear foot. Forms to the ends of steps should be measured by number.

9. Estimating Amount of Steel.—Reinforcing bars should be measured by the linear foot and reduced to weight in pounds for pricing. The net weight placed in the building should be taken and no allowance made for waste and cutting, or wire ties and spacers, etc., but laps should be allowed for as called for by the plans or by the necessities of the design. Deformed bars should be measured separately from plain.

The cost of bending and placing in columns and beams is greater than in slabs, but as the difference is not great it is not usual to make any distinctions but to take off the whole of the steel together, except in special cases.

Pipe sleeves, turnbuckles, clamps, threaded ends, nuts, forgings, and other special items should be measured separately by number and size, and allowed for in addition to the weight. Wire cloth, expanded metal, and other steel fabrics sold in sheets are measured and described by the square foot. The size of mesh and weight per square foot of steel will govern the price, and should be stated. All laps should be measured and allowed for.

10. Estimating Amount of Surface Finish.—Finish of concrete surfaces should be measured by the square foot. Finish should always be measured and described separately. No measurement or allowance should be made for going over concrete work after removal of forms, and patching up voids and stone pockets, removing fins, etc., as this is part of the labor incidental to placing the concrete and the cost will depend upon the care used in spading the concrete into the forms.

Granolithic finish should be measured by the square foot and should include all labor and materials for the thickness specified. Finish laid integral with the slab should be measured separately from finish laid after the slab has set. No allowance should be made for protection of finish with sawdust, sand, or covering in to protect from weather. Grooved surfaces, gutters, curbing, etc., should be measured separately from plain granolithic and should be measured by the square foot or linear foot, as the case may require.

Putting on cement wash, rubbing with carborundum, scrubbing with wire brushes, tooling, and picking, are other surface labors that should each be separately measured and priced. The price should include the use of swing stages, tools, and materials required.

APPENDIX A

STANDARD SPECIFICATIONS AND TESTS FOR PORTLAND CEMENT[1]

These specifications are the result of several years' work of a special committee representing a United States Government Departmental Committee, the Board of Direction of the American Society of Civil Engineers, and Committee C-1 on Cement of the American Society for Testing Materials in coöperation with Committee C-1.

Specifications

1. Definition.—Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion, an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

I. Chemical Properties

2. Chemical Limits.—The following limits shall not be exceeded:

Loss on ignition, %	4.00
Insoluble residue, %	0.85
Sulphuric anhydride (SO_3), %	2.00
Magnesia (MgO), %	5.00

II. Physical Properties

3. Specific Gravity.—The specific gravity of cement shall be not less than 3.10 (3.07 for white Portland cement). Should the test of cement as received fall below this requirement a second test may be made upon an ignited sample. The specific gravity test will not be made unless specifically ordered.

4. Fineness.—The residue on a standard No. 200 sieve shall not exceed 22% by weight.

5. Soundness.—A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

6. Time of Setting.—The cement shall not develop initial set in less than 45 min. when the Vicat needle is used or 60 min. when the Gillmore needle is used. Final set shall be attained within 10 hr.

7. Tensile Strength.—The average tensile strength in pounds per square inch of not less than three standard mortar briquettes (see Sec. 51) composed of 1 part cement and 3 parts standard sand, by weight, shall be equal to or higher than the following:

Age at test, days	Storage of briquettes	Tensile strength, lb. per sq. in.
7	1 day in moist air, 6 days in water	200
28	1 day in moist air, 27 days in water	300

8. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

III. Packages, Marking and Storage

9. Packages and Marking.—The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net.

[1] These specifications and tests were adopted by letter ballot of the American Society for Testing Materials on Sept. 1, 1916, and became effective Jan. 1, 1917.

10. Storage.—The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness.

IV. Inspection

11. Inspection.—Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified by the purchaser. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test shall be waived only when specifically so ordered.

V. Rejection

12. Rejection.—The cement may be rejected if it fails to meet any of the requirements of these specifications.

13. Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100°C. for 1 hr. it meets this requirement.

14. Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter.

15. Packages varying more than 5% from the specified weight may be rejected: and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

Tests

VI. Sampling

16. Number of Samples.—Tests may be made on individual or composite samples as may be ordered. Each test sample should weigh at least 8 lb.

17. (a) *Individual Sample.*—If sampled in cars one test sample shall be taken from each 50 bbl. or fraction thereof. If sampled in bins one sample shall be taken from each 100 bbl.

(b) *Composite Sample.*—If sampled in cars one sample shall be taken from one sack in each 40 sacks (or 1 bbl. in each 10 bbl.) and combined to form one test sample. If sampled in bins or warehouses one test sample shall represent not more than 200 bbl.

18. Method of Sampling.—Cement may be sampled at the mill by any of the following methods that may be practicable, as ordered:

(a) *From the Conveyor Delivering to the Bin.*—At least 8 lb. of cement shall be taken from approximately each 100 bbl. passing over the conveyor.

(b) *From Filled Bins by Means of Proper Sampling Tubes.*—Tubes inserted vertically may be used for sampling cement to a maximum depth of 10 ft. Tubes inserted horizontally may be used where the construction of the bin permits. Samples shall be taken from points well distributed over the face of the bin.

(c) *From Filled Bins at Points of Discharge.*—Sufficient cement shall be drawn from the discharge openings to obtain samples representative of the cement contained in the bin, as determined by the appearance at the discharge openings of indicators placed on the surface of the cement directly above these openings before drawing of the cement is started.

19. Treatment of Sample.—Samples preferably shall be shipped and stored in air-tight containers. Samples shall be passed through a sieve having 20 meshes per linear inch in order to thoroughly mix the sample, break up lumps and remove foreign materials.

VII. Chemical Analysis

Loss on Ignition

20. Method.—One gram of cement shall be heated in a weighed covered platinum crucible, of 20 to 25-c.c. capacity, as follows, using either method (a) or (b) as ordered:

(a) The crucible shall be placed in a hole in an asbestos board, clamped horizontally so that about three-fifths of the crucible projects below, and blasted at a full red heat for 15 min. with an inclined flame; the loss in weight shall be checked by a second blasting for 5 min. Care shall be taken to wipe off particles of asbestos that may adhere to the crucible when withdrawn from the hole in the board. Greater neatness and shortening of the time of heating are secured by making a hole to fit the crucible in a circular disc of sheet platinum and placing this disc over a somewhat larger hole in an asbestos board.

(b) The crucible shall be placed in a muffle at any temperature between 900 and 1000°C. for 15 min. and the loss in weight shall be checked by a second heating for 5 min.

21. Permissible Variation.—A permissible variation of 0.25 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 4%.

Insoluble Residue

22. Method.—To a 1-gram sample of cement shall be added 10 c.c. of water and 5 c.c. of concentrated hydrochloric acid; the liquid shall be warmed until effervescence ceases. The solution shall be diluted to 50 c.c. and digested, on a steam bath or hot plate until it is evident that decomposition of the cement is complete. The residue shall be filtered, washed with cold water, and the filter paper and contents digested in about 30 c.c. of a 5% solution of sodium carbonate, the liquid being held at a temperature just short of boiling for 15 min. The remaining residue shall be filtered, washed with cold water, then with a few drops of hot hydrochloric acid, 1 : 9, and finally with hot water, and then ignited at a red heat and weighed as the insoluble residue.

23. Permissible Variation.—A permissible variation of 0.15 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 0.85%.

Sulphuric Anhydride

24. Method.—One gram of the cement shall be dissolved in 5 c.c. of concentrated hydrochloric acid diluted with 5 c.c. of water, with gentle warming; when solution is complete 40 c.c. of water shall be added, the solution filtered, and the residue washed thoroughly with water. The solution shall be diluted to 250 c.c., heated to boiling and 10 c.c. of a hot 10% solution of barium chloride shall be added slowly, drop by drop, from a pipette and the boiling continued until the precipitate is well formed. The solution shall be digested on the steam bath until the precipitate has settled. The precipitate shall be filtered, washed, and the paper and contents placed in a weighed platinum crucible and the paper slowly charred and consumed without flaming. The barium sulphate shall then be ignited and weighed. The weight obtained multiplied by 34.3 gives the percentage of sulphuric anhydride. The acid filtrate obtained in the determination of the insoluble residue may be used for the estimation of sulphuric anhydride instead of using a separate sample.

25. Permissible Variation.—A permissible variation of 0.10 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 2.00%.

Magnesia

26. Method.—To 0.5 gram of the cement in an evaporating dish shall be added 10 c.c. of water to prevent lumping and then 10 c.c. of concentrated hydrochloric acid. The liquid shall be gently heated and agitated until attack is complete. The solution shall then be evaporated to complete dryness on a steam or water bath. To hasten dehydration the residue may be heated to 150 or even 200°C. for ½ to 1 hr. The residue shall be treated with 10 c.c. of concentrated hydrochloric acid diluted with an equal amount of water. The dish shall be covered and the solution digested for 10 min. on a steam bath or water bath. The diluted solution shall be filtered and the separated silica washed thoroughly with water.[1] Five cubic centimeters of concentrated hydrochloric acid and sufficient bromine water to precipitate any manganese which may be present, shall be added to the filtrate (about 250 c.c.). This shall be made alkaline with ammonium hydroxide, boiled until there is but a faint odor of ammonia, and the precipitated iron and aluminum hydroxides, after settling, shall be washed with hot water, once by decantation and slightly on the filter. Setting aside the filtrate, the precipitate shall be transferred by a jet of hot water to the precipitating vessel and dissolved in 10 c.c. of hot hydrochloric acid. The paper shall be extracted with acid, the solution and washings being added to the main solution. The aluminum and iron shall then be reprecipitated at boiling heat by ammonium hydroxide and bromine water in a volume of about 100 c.c., and the second precipitate shall be collected and washed on the filter used in the first instance if this is still intact. To the combined filtrates from the hydroxides of iron and aluminum, reduced in volume if need be, 1 c.c. of ammonium hydroxide shall be added, the solution brought to boiling, 25 c.c. of a saturated solution of boiling ammonium oxalate added, and the boiling continued until the precipitated calcium oxalate has assumed a well-defined granular form. The precipitate after 1 hr. shall be filtered and washed, then with the filter shall be placed wet in a platinum crucible, and the paper burned off over a small flame of a Bunsen burner; after ignition it shall be redissolved in hydrochloric acid and the solution dilutod to 100 c.c. Ammonia shall be added in slight excess, and the liquid boiled. The lime shall then be reprecipitated by ammonium oxalate, allowed to stand until settled, filtered and washed. The combined filtrates from the calcium precipitates shall be acidified with hydrochloric acid, concentrated on the steam bath to about 150 c.c., and made slightly alkaline with ammonium hydroxide, boiled and filtered (to remove a little aluminum and iron and perhaps calcium). When cool, 10 c.c. of saturated solution of sodium-ammonium-hydrogen phosphate shall be added with constant stirring. When the crystallin ammonium-magnesium orthophosphate has formed, ammonia shall be added in moderate excess. The solution shall be set aside for several hours in a cool place, filtered and washed with water containing 2.5% of NH_3. The precipitate shall be dissolved in a small quantity of hot hydrochloric acid, the solution diluted to about 100 c.c., 1 c.c. of a saturated solution of sodium-ammonium-hydrogen phosphate added, and ammonia drop by drop, with constan stirring, until the precipitate is again formed as described and the ammonia is in moderate excess. The precipitate shall then be allowed to stand about 2 hr., filtered and washed as before. The paper and contents shall be placed in a weighed platinum crucible, the paper slowly charred, and the resulting carbon carefully burned off. The precipitate shall then be ignited to constant weight over a Meker burner, or a blast not strong enough to soften or melt the pyrophosphate. The weight of magnesium pyrophosphate obtained multiplied by 72.5 gives the percentage of magnesia. The precipitate so obtained always contains some calcium and usually small quantities of iron, aluminum, and manganese as phosphates.

27. Permissible Variation.—A permissible variation of 0.4 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 5.00%.

[1] Since this procedure does not involve the determination of silica, a second evaporation is unnecessary.

VIII. Determination of Specific Gravity

28. Apparatus.—The determination of specific gravity shall be made with a standardized Le Chatelier apparatus which conforms to the requirements illustrated in Fig. 1. This apparatus is standardized by the United States Bureau of Standard [illegible] erosene free from water, or benzine not lighter than 62°Bé., shall be used in making this determination.

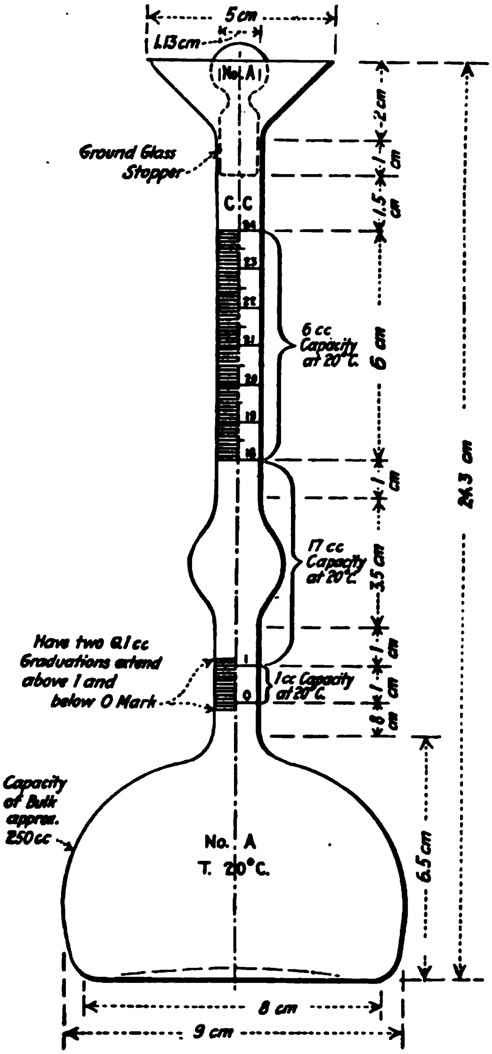


Fig. 1.—LeChatelier apparatus.

29. Method.—The flask shall be filled with either of these liquids to a point on the stem between zero and 1 c.c. and 64 grams of cement, of the same temperature as the liquid, shall be slowly introduced, taking care that the cement does not adhere to the inside of the flask above the liquid and to free the cement from air by rolling the flask in an inclined position. After all the cement is introduced, the level of the liquid will rise to some division of the graduated neck; the difference between readings is the volume displaced by 64 grams of the cement.

The specific gravity shall then be obtained from the formula

$$\text{Specific gravity} = \frac{\text{Weight of cement (grams)}}{\text{Displaced volume (c.c.)}}$$

30. The flask, during the operation, shall be kept immersed in water, in order to avoid variations in the temperature of the liquid in the flask, which shall not exceed 0.5°C. The results of repeated tests should agree within 0.01.

31. The determination of specific gravity shall be made on the cement as received; if it falls below 3.10, a second determination shall be made after igniting the sample as described in Sect. 20.

IX. Determination of Fineness

32. Apparatus.—Wire cloth for standard sieves for cement shall be woven (not twilled) from brass, bronze, or other suitable wire, and mounted without distortion on frames not less than 1½ in. below the top of the frame. The sieve frames shall be circular, approximately 8 in. in diameter, and may be provided with a pan and cover.

33. A standard No. 200 sieve is one having nominally an 0.0029-in. opening and 200 wires per inch standardized by the U. S. Bureau of Standards, and conforming to the following requirements:

The No. 200 sieve should have 200 wires per inch, and the number of wires in any whole inch shall not be outside the limits of 192 to 208. No opening between adjacent parallel wires shall be more than 0.0050 in. in width. The diameter of the wire should be 0.0021 in. and the average diameter shall not be outside the limits 0.0019 to 0.0023 in. The value of the sieve as determined by sieving tests made in conformity with the standard specification for these tests on a standardized cement which gives a residue of 25 to 20% on the No. 200 sieve, or on other similarly graded material, shall not show a variation of more than 1.5% above or below the standards maintained at the Bureau of Standards.

34. Method.—The test shall be made with 50 grams of cement. The sieve shall be thoroughly clean and dry. The cement shall be placed on the No. 200 sieve, with pan and cover attached, if desired, and shall be held in one hand in a slightly inclined position so that the sample will be well distributed over the sieve, at the same time gently striking the side about 150 times per minute against the palm of the other hand on the up stroke. The sieve shall be turned every 25 strokes about one-sixth of a revolution in the same direction. The operation shall continue until not more than 0.05 gram passes through in 1 min. of continuous sieving. The fineness shall be determined from the weight of the residue on the sieve expressed as a percentage of the weight of the original sample.

35. Mechanical sieving devices may be used, but the cement shall not be rejected if it meets the fineness requirement when tested by the hand method described in Sect. 34.

36. Permissible Variation.—A permissible variation of 1 will be allowed, and all results in excess of the specified limit but within this permissible variation shall be reported as 22 %.

X. Mixing Cement Pastes and Mortars

37. Method.—The quantity of dry material to be mixed at one time shall not exceed 1000 grams nor be less than 500 grams. The proportions of cement or cement and sand shall be stated by weight in grams of the dry materials; the quantity of water shall be expressed in cubic centimeters (1 c.c. of water = 1 gram). The dry materials shall be weighed, placed upon a non-absorbent surface, thoroughly mixed dry if sand is used, and a crater formed in the center, into which the proper percentage of clean water shall be poured; the material on the outer edge shall be turned into the crater by the aid of a trowel. After an interval of ½ min. for the absorption of the water the operation shall be completed by continuous, vigorous mixing, squeezing and kneading with the hands for at least 1 min.[1]

38. The temperature of the room and the mixing water shall be maintained as nearly as practicable at 21°C. (70°F.).

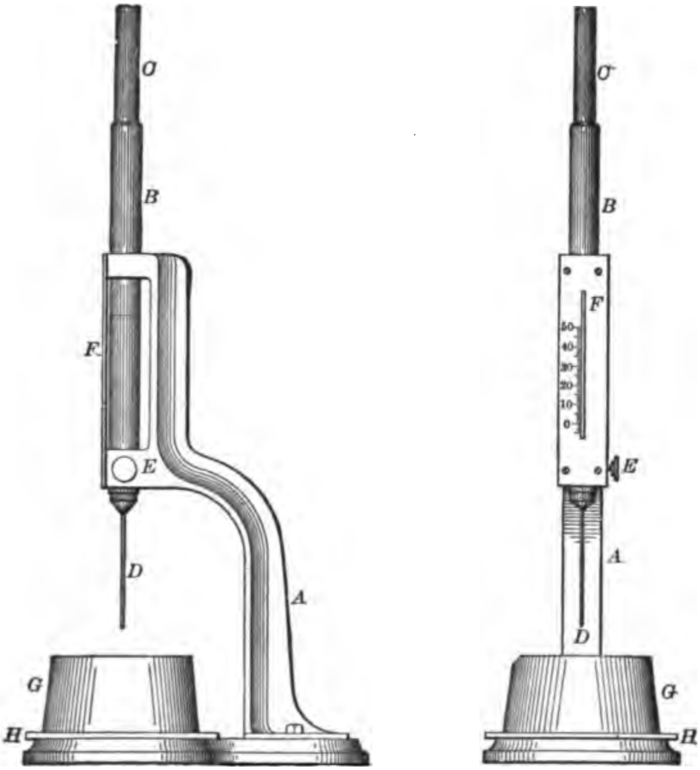


Fig. 2.—Vicat apparatus.

XI. Normal Consistency

39. Apparatus.—The Vicat apparatus consists of a frame *A* (Fig. 2) bearing a movable rod *B*, weighing 300 grams, one end *C* being 1 cm. in diameter for a distance of 6 cm., the other having a removable needle *D*, 1 mm. in diameter, 6 cm. long. The rod is reversible, and can be held in any desired position by a screw *E*, and has midway between the ends a mark *F* which moves under a scale (graduated to millimeters) attached to the frame *A*. The paste is held in a conical, hard-rubber ring *G*, 7 cm. in diameter at the base, 4 cm. high, resting on a glass plate *H* about 10 cm. square.

[1] In order to secure uniformity in the results of tests for the time of setting and tensile strength the manner of mixing above described should be carefully followed. At least one minute is necessary to obtain the desired plasticity which is not appreciably affected by continuing the mixing for several minutes. The exact time necessary is dependent upon the personal equation of the operator. The error in mixing should be on the side of c-
During the operation of mixing, the hands should be protected by rubber gloves.

40. **Method.**—In making the determination, 500 grams of cement, with a measured quantity of water, shall be kneaded into a paste, as described in Sect. 37, and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in. apart; the ball resting in the palm of one hand shall be pressed into the larger end of the rubber ring held in the other hand, completely filling the ring with paste; the excess at the larger end shall then be removed by a single movement of the palm of the hand; the ring shall then be placed on its larger end on a glass plate and the excess paste at the smaller end sliced off at the top of the ring by a single oblique stroke of a trowel held at a slight angle with the top of the ring. During these operations care shall be taken not to compress the paste. The paste confined in the ring, resting on the plate, shall be placed under the rod, the larger end of which shall be brought in contact with the surface of the paste; the scale shall be then read, and the rod quickly released. The paste shall be of normal consistency when the rod settles to a point 10 mm. below the original surface in ½ min. after being released. The apparatus shall be free from all vibrations during the test. Trial pastes shall be made with varying percentages of water until the normal consistency is obtained. The amount of water required shall be expressed in percentage by weight of the dry cement.

41. The consistency of standard mortar shall depend on the amount of water required to produce a paste of normal consistency from the same sample of cement. Having determined the normal consistency of the sample, the consistency of standard mortar made from the same sample shall be as indicated in Table I, the values being in percentage of the combined dry weights of the cement and standard sand.

TABLE I.—PERCENTAGE OF WATER FOR STANDARD MORTARS

Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement, three standard Ottawa sand	Percentage of water for neat cement paste of normal consistency	Percentage of water for one cement, three standard Ottawa sand
15	9.0	23	10.3
16	9.2	24	10.5
17	9.3	25	10.7
18	9.5	26	10.8
19	9.7	27	11.0
20	9.8	28	11.2
21	10.0	29	11.3
22	10.2	30	11.5

XII. DETERMINATION OF SOUNDNESS[1]

42. **Apparatus.**—A steam apparatus, which can be maintained at a temperature between 98 and 100°C., or one similar to that shown in Fig. 3, is recommended. The capacity of this apparatus may be increased by using a rack for holding the pats in a vertical or inclined position.

43. **Method.**—A pat from cement paste of normal consistency about 3 in. in diameter, ½ in. thick at the center, and tapering to a thin edge, shall be made on clean glass plates about 4 in. square, and stored in moist air for 24 hr. In molding the pat, the cement paste shall first be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center.

44. The pat shall then be placed in an atmosphere of steam at a temperature between 98 and 100°C. upon a suitable support 1 in. above boiling water for 5 hr.

45. Should the pat leave the plate, distortion may be detected best with a straight-edge applied to the surface which was in contact with the plate.

XIII. DETERMINATION OF TIME OF SETTING

46. The following are alternate methods, either of which may be used as ordered:

47. **Vicat Apparatus.**—The time of setting shall be determined with the Vicat apparatus described in Sect. 39 (see Fig. 2).

48. **Vicat Method.**—A paste of normal consistency shall be molded in the hard-rubber ring *G* as described in Sect. 40, and placed under the rod *B*, the smaller end of which shall then be carefully brought into contact with the surface of the paste, and the rod quickly released. The initial set shall be said to have occurred when the needle

[1] Unsoundness is usually manifested by change in volume which causes distortion, cracking, checking or disintegration.

Pats improperly made or exposed to drying may develop what are known as shrinkage cracks within the first 24 hours and are not an indication of unsoundness. These conditions are illustrated in Fig. 4.

The failure of the pats to remain on the glass or the cracking of the glass to which the pats are attached does not necessarily indicate unsoundness.

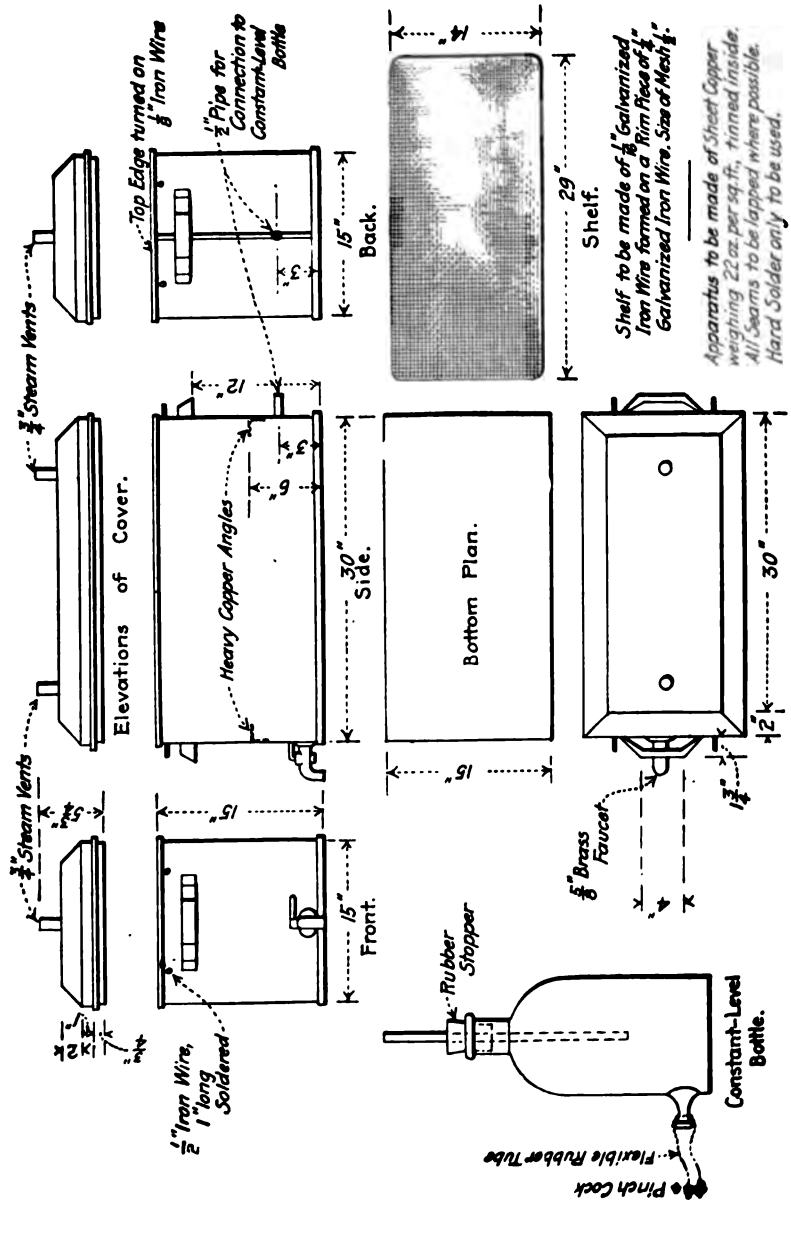


FIG. 3.—Apparatus for making soundness test of cement.

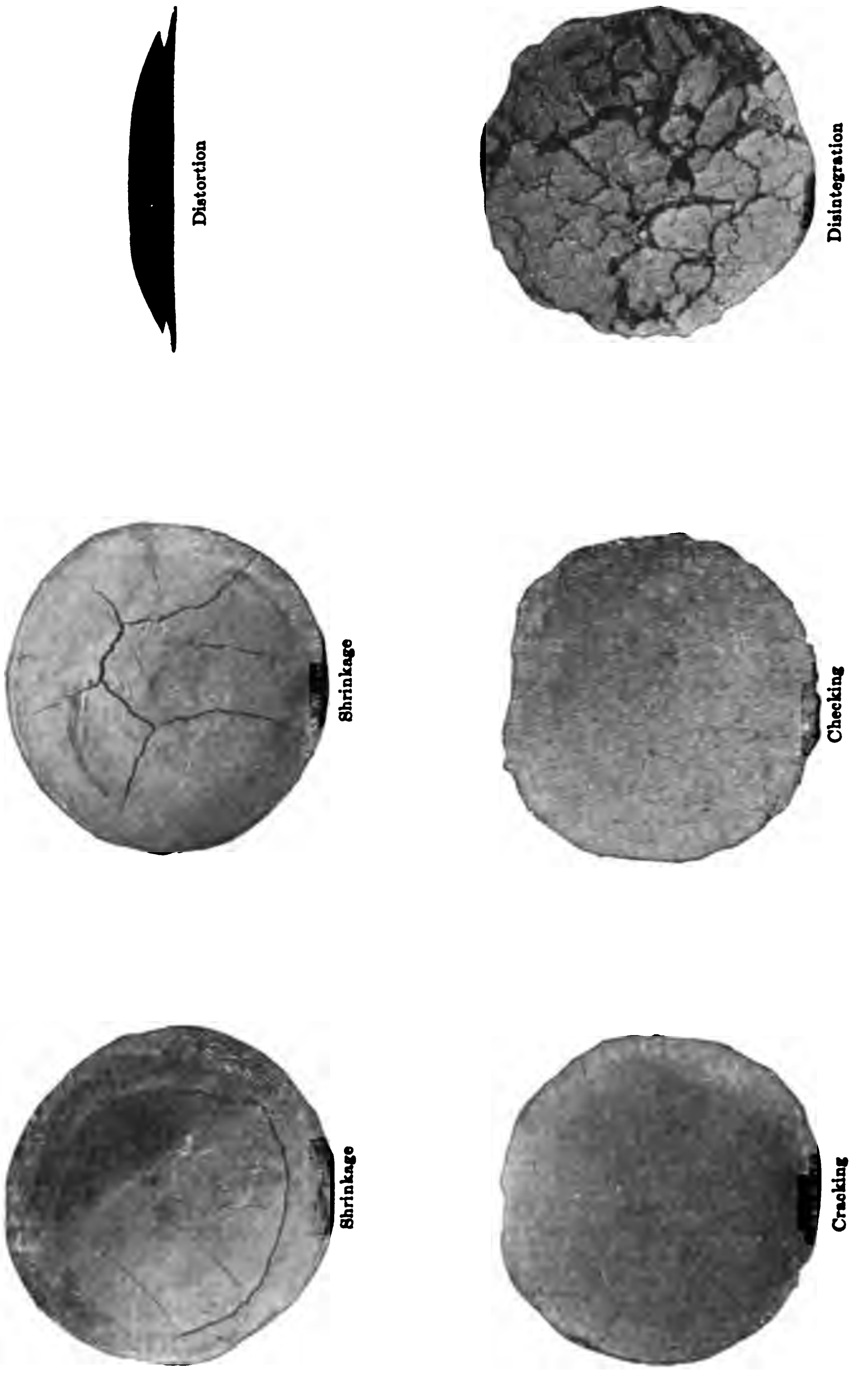
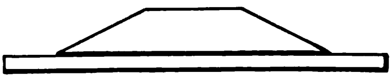


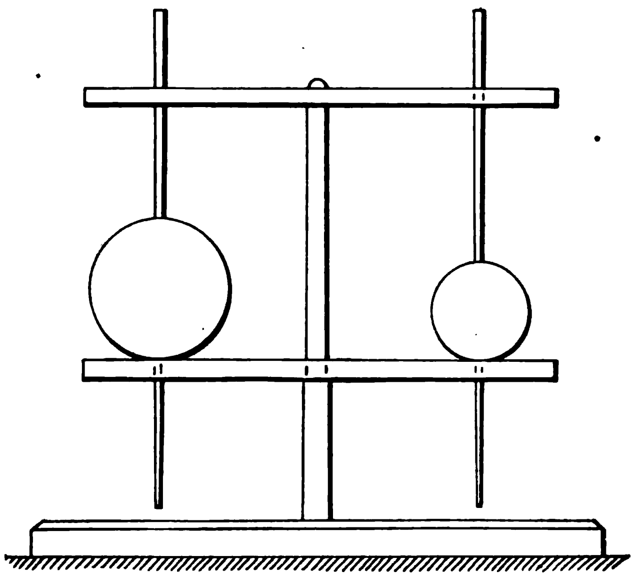
FIG. 4.—Typical failures in soundness test.

ceases to pass a point 5 mm. above the glass plate in ½ min. after being released; and the final set, when the needle does not sink visibly into the paste. The test pieces shall be kept in moist air during the test. This may be accomplished by placing them on a rack over water contained in a pan and covered by a damp cloth, kept from contact with them by means of a wire screen; or they may be stored in a moist closet. Care shall be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste receives, but by the temperature and humidity of the air, and its determination is therefore only approximate.

49. Gillmore Needles.—The time of setting shall be determined by the Gillmore needles. The Gillmore needles should preferably be mounted as shown in Fig. 5b.



(a) Pat with top surface flattened for determining time of setting by Gilmore method.



(b) Gillmore Needles.

FIG. 5.

50. Gillmore Method.—The time of setting shall be determined as follows: A pat of neat cement paste about 3 in. in diameter and ½ in. in thickness with a flat top Fig. 5a, mixed to a normal consistency, shall be kept in moist air at a temperature maintained as nearly as practicable at 21°C. (70°F.). The cement shall be considered to have acquired its initial set when the pat will bear, without appreciable indentation, the Gillmore needle 1/12 in. in diameter, loaded to weigh ¼ lb. The final set has been acquired when the pat will bear without appreciable indentation, the Gillmore needle 1/24 in. in diameter, loaded to weigh 1 lb. In making the test, the needles shall be held in a vertical position, and applied lightly to the surface of the pat.

XIV. TENSION TESTS

51. Form of Test Piece.—The form of test piece shown in Fig. 6 shall be used. The molds shall be made of non-corroding metal and have sufficient material in the sides to prevent spreading during molding. Gang molds when used shall be of the type shown in Fig. 7. Molds shall be wiped with an oily cloth before using.

52. Standard Sand.—The sand to be used shall be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve. This sand may be obtained from the Ottawa Silica Co., at a cost of 2 cts. per lb., f.o.b. cars, Ottawa, Ill.

53. This sand, having passed the No. 20 sieve, shall be considered standard when not more than 5 grains pass the No. 30 sieve after 1 min. continuous sieving of a 500-grain sample.

54. The sieves shall conform to the following specifications:

The No. 20 sieve shall have between 19.5 and 20.5 wires per whole inch of the warp wires and between 19 and 21 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0165 in. and the average diameter shall not be outside the limits of 0.0160 and 0.0170 in.

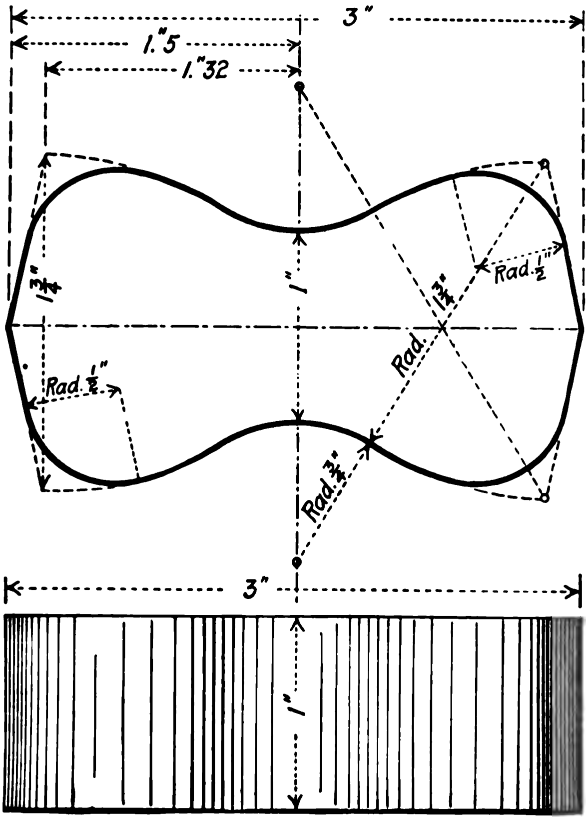


Fig. 6.—Details for briquette.

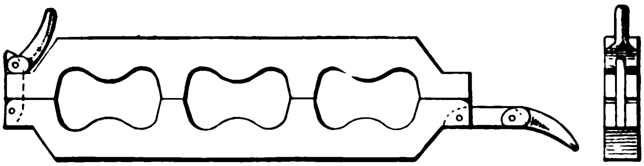


Fig. 7.—Gang mold.

The No. 30 sieve shall have bteween 29.5 and 30.5 wires per whole inch of the warp wires and between 28.5 and 31.5 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0110 in. and the average diameter shall not be outside the limits 0.0105 to 0.0115 in.

55. Molding.—Immediately after mixing, the standard mortar shall be placed in the molds, pressed in firmly with the thumbs and smoothed off with a trowel without ramming. Additional mortar shall be heaped above the mold and smoothed off with a trowel; the trowel shall be drawn over the mold in such a manner as to exert a moderate pressure on the material. The mold shall then be turned over and the operation of heaping, thumbing and smoothing off repeated.

56. Testing.—Tests shall be made with any standard machine. The briquettes shall be tested as soon as they are removed from the water. The bearing surfaces of the clips and briquettes shall be free from grains of sand or dirt. The briquettes shall be carefully centered and the load applied continuously at the rate of 600 lb. per min.

57. Testing machines should be frequently calibrated in order to determine their accuracy.

58. Faulty Briquettes.—Briquettes that are manifestly faulty, or which give strengths differing more than 15% from the average value of all test pieces made from the same sample and broken at the same period, shall not be considered in determining the tensile strength.

XV. Storage of Test Pieces

59. Apparatus.—The moist closet may consist of a soapstone, slate or concrete box, or a wooden box lined with metal. If a wooden box is used, the interior should be covered with felt or broad wicking kept wet. The bottom of the moist closet should be covered with water. The interior of the closet should be provided with non-absorbent shelves on which to place the test pieces, the shelves being so arranged that they may be withdrawn readily.

60. Methods.—Unless otherwise specified, all test pieces, immediately after molding, shall be placed in the moist closet for from 20 to 24 hr.

61. The briquettes shall be kept in molds on glass plates in the moist closet for at least 20 hr. After 24 hr. in moist air the briquettes shall be immersed in clean water in storage tanks of non-corroding material.

62. The air and water shall be maintained as nearly as practicable at a temperature of 21°C. (70°F.).

APPENDIX B

WORKING STRESSES[1]

1. General Assumptions.—The following working stresses are recommended for static loads. Proper allowances for vibration and impact are to be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts.

In selecting the permissible working stress on concrete, the designer should be guided by the working stresses usually allowed for other materials of construction, so that all structures of the same class composed of different materials may have approximately the same degree of safety.

The following recommendations as to allowable stresses are given in the form of percentages of the ultimate strength of the particular concrete which is to be used; this ultimate strength is that developed at an age of 28 days, in cylinders 8 in. in diameter and 16 in. long, of the consistency described,[2] made and stored under laboratory conditions. In the absence of definite knowledge in advance of construction as to just what strength may be expected, the Committee submits the following values as those which should be obtained with materials and workmanship in accordance with the recommendations of this report.

Although occasional tests may show higher results than those here given, the Committee recommends that these values should be the maximum used in design.

TABLE OF COMPRESSIVE STRENGTHS OF DIFFERENT MIXTURES OF CONCRETE
(In Pounds per Square Inch)

Aggregate	1 : 3*	1 : 4½*	1 : 6*	1 : 7½*	1 : 9*
Granite, trap rock	3,300	2,800	2,200	1,800	1,400
Gravel, hard limestone and hard sandstone	3,000	2,500	2,000	1,600	1,300
Soft limestone and sandstone	2,200	1,800	1,500	1,200	1,000
Cinders	800	700	600	500	400

NOTE.—For variations in the moduli of elasticity see Sect. 8.

* Combined volume fine and coarse aggregate measured separately.

2. Bearing.—When compression is applied to a surface of concrete of at least twice the loaded area, a stress of 35% of the compressive strength may be allowed in the area actually under load.

3. Axial Compression.—(*a*) For concentric compression on a plain concrete pier, the length of which does not exceed 4 diameters, or on a column reinforced with longitudinal bars only, the length of which does not exceed 12 diameters, 22.5% of the compressive strength may be allowed.

(*b*) Columns with longitudinal reinforcement to the extent of not less than 1% and not more than 4% and with lateral ties of not less than ¼ in. in diameter, 12 in. apart, nor more than 16 diameters of the longitudinal bar: the unit stress recommended for (*a*).

(*c*) Columns reinforced with not less than 1% and not more than 4% of longitudinal bars and with circular hoops or spirals not less than 1% of the volume of the concrete and as hereinafter specified:[3] a unit stress 55% higher than given for (*a*), provided the ratio of unsupported length of column to diameter of the hooped core is not more than 10.

4. Compression in Extreme Fiber.—The extreme fiber stress of a beam, calculated on the assumption of a constant modulus of elasticity for concrete under working stresses may be allowed to reach 32.5% of the compressive strength. Adjacent to the support of continuous beams, stresses 15% higher may be used.

5. Shear and Diagonal Tension.—In calculations on beams in which the maximum shearing stress in a section is used as the means of measuring the resistance to diagonal tension stress, the following allowable values for the maximum vertical shearing stress in concrete, calculated by the method given in formula (22)[4] are recommended:

[1] From Final Report of the Special Committee on Concrete and Reinforced Concrete of the American Society of Civil Engineers, presented before the Society, Jan. 17, 1917.

[2] The materials should be mixed wet enough to produce a concrete of such a consistency as will flow sluggishly into the forms and about the metal reinforcement when used, and which, at the same time, can be conveyed from the mixer to the forms without separation of the coarse aggregate from the mortar. The quantity of water is of the greatest importance in securing concrete of maximum strength and density; too much water is as objectionable as too little.

[3] See Art. 7, Sect. 8.

[4] $v = \frac{V}{bjd}$.

(a) For beams with horizontal bars only and without web reinforcement, 2% of the compressive strength.

(b) For beams with web reinforcement consisting of vertical stirrups looped about the longitudinal reinforcing bars in the tension side of the beam and spaced horizontally not more than one-half the depth of the beam; or for beams in which longitudinal bars are bent up at an angle of not more than 45 deg. or less than 20 deg. with the axis of the beam, and the points of bending are spaced horizontally not more than three-quarters of the depth of the beam apart, not to exceed 4½% of the compressive strength.

(c) For a combination of bent bars and vertical stirrups looped about the reinforcing bars in the tension side of the beam and spaced horizontally not more than one-half of the depth of the beam, 5% of the compressive strength.

(d) For beams with web reinforcement (either vertical or inclined) securely attached to the longitudinal bars in the tension side of the beam in such a way as to prevent slipping of bar past the stirrup, and spaced horizontally not more than one-half of the depth of the beam in case of vertical stirrups and not more than three-fourths of the depth of the beam in the case of inclined members, either with longitudinal bars bent up or not, 6% of the compressive strength.

The web reinforcement in case any is used should be proportioned by using two-thirds of the external vertical shear in formulas (24)[1] or (25).[2] The effect of longitudinal bars bent up at an angle of from 20 to 45 deg. with the axis of the beam, may be taken at sections of the beam in which the bent-up bars contribute to diagonal tension resistance, as defined under Chap. VII, Sect. 8, as reducing the shearing stresses to be otherwise provided for. The amount of reduction of the shearing stress by means of bent-up bars will depend upon their capacity, but in no case should be taken as greater than 4½% of the compressive strength of the concrete over the effective cross-section of the beam (formula 22).[3] The limit of tensile stress in the bent-up portion of the bar calculated by formula (25),[2] using in this formula an amount of total shear corresponding to the reduction in shearing stress assumed for the bent-up bars, may be taken as specified for the working stress of steel, but in the calculations the stress in the bar due to its part as longitudinal reinforcement of the beam should be considered. The stresses in stirrups and inclined members when combined with bent-up bars are to be determined by finding the amount of the total shear which may be allowed by reason of the bent-up bars, and subtracting this shear from the total external vertical shear. Two-thirds of the remainder will be the shear to be carried by the stirrups, using formulas (24)[1] or (25)[2].

Where punching shear occurs, provided the diagonal tension requirements are met, a shearing stress of 6% of the compressive strength may be allowed.

6. Bond.—The bond stress between concrete and plain reinforcing bars may be assumed at 4% of the compressive strength, or 2% in the case of drawn wire. In the best types of deformed bar, the bond stress may be increased, but not to exceed 5% of the compressive strength of the concrete.

7. Reinforcement.—The tensile or compressive stress in steel should not exceed 16,000 lb. per sq. in.

In structural steel members, the working stresses adopted by the American Railway Engineering Association are recommended.

8. Modulus of Elasticity.—The value of the modulus of elasticity of concrete has a wide range, depending on the materials used, the age, the range of stresses between which it is considered, as well as other conditions. It is recommended that, in computations for the position of the neutral axis, and for the resisting moment of beams, and for compression of concrete in columns, it be assumed as:

(a) One-fortieth that of steel, when the strength of the concrete is taken as not more than 800 lb. per sq. in

(b) One-fifteenth that of steel, when the strength of the concrete is taken as greater than 800 lb. per sq. in. and less than 2200 lb. per sq. in.

(c) One-twelfth that of steel, when the strength of the concrete is taken as greater than 2200 lb. per sq. in and less than 2900 lb. per sq. in., and

(d) One-tenth that of steel, when the strength of the concrete is taken as greater than 2900 lb. per sq. in.

Although not rigorously accurate, these assumptions will give safe results. For the deflection of beams which are free to move longitudinally at the supports, in using formulas for deflection which do not take into account the tensile strength developed in the concrete, a modulus of one-eighth of that of steel is recommended.

[1] Vertical web reinforcement,

$$T = \frac{V's}{jd}$$

[2] Bars bent up at angles between 20 and 45 deg. with the horizontal and web members inclined at 45 deg.,

$$T = \frac{3}{4}\frac{V's}{jd}$$

(See Standard Notation, *Appendix D.*)

[3] $v = \dfrac{V}{bjd}$.

APPENDIX C

RULINGS PERTAINING TO FLAT-SLAB DESIGN

Rulings have been adopted by the Cities of Pittsburgh and Chicago, by a Special Committee of the American Society of Civil Engineers, and by the American Concrete Institute, for the regulation of the design of flat-slab floors. These rulings are given below. Rulings have been adopted by other cities which are of merit, but those of Pittsburgh and Chicago have been most widely used and properly applied have given satisfactory results.

The ruling of the American Concrete Institute is in many respects similar to that of the City of Chicago, while the Report of the Special Committee of the American Society is similar as to method but more conservative as to reinforcement, and in the light of all the test data available would seem to be in some respects ultra conservative.

For the method of application and a comparison of designs, the reader is referred to Art. 20, Sect. 11, page 487.

RULING ON THE DESIGN OF CANTILEVER FLAT-SLAB CONSTRUCTION IN THE CITY OF PITTSBURGH

General.—The design and construction of reinforced-concrete flat slabs shall be carried out strictly in accordance with all the provisions of the ordinance "Authorizing and Regulating the Use of Concrete and Reinforced Concrete in the City of Pittsburgh," with special reference to Sect. 2, heading "Special Systems not Covered by this Ordinance."

Reason for Ruling.—Since there has been a number and are likely to be more applications for building permits, in which it is proposed to employ the flat-slab construction, and since there have been proposed several different proprietary systems, it has become advisable to set forth a ruling with which each one must conform, and which will state the interpretation of the ordinance and all its clauses which might relate to flat-slab construction as made by this Bureau.

The ruling is expected to provide uniformity of the requirements among the several systems and equity between them in preparing designs, and to so regulate their design and construction that they will be conducted under correct principles and in conformity with the spirit of the ordinance.

Statement.—Inasmuch as the above-mentioned clause of the ordinance specified that the tests shall be made to a breaking load in order to determine a factor of safety of 4, for loads for which the structure is planned, a statement should be made to explain the reason for placing the ruling in its following form.

On account of the practical impossibility of so conducting a breaking load test that it would provide a basis for a complete analysis which will prove the above required factor for safety and at the same time serve as a convincing guide in the proper analysis for safe design and construction, as well as omitting to require a severe, expensive and improper test which would not afford sufficient or consistent information, it has been formally decided that:

First.—The tests shall have been conducted in such a manner as to demonstrate what are the actual strains in the materials at the time working conditions are imposed upon the structure. This should also demonstrate clearly the effect of an uneven or partial distribution of design loading, and that such loads will not cause the strains in the materials to become substantially greater than those allowed for ordinary working conditions, and that an unusual load or uncertainties of manufacture and installation of building materials will not result in weakness or unsatisfactory result.

Second.—The test should be conducted in such a manner as to bring to light the relation which exists between the strength of construction designed according to the provisions of the ordinance, and according to the provisions of this ruling, and to show that all proper considerations have been provided for, therein.

Further, it is being understood that it is the intention of the ordinance to provide for safety of building construction and equity among the different systems, it is decided that the requirements for all special systems as to strains in the materials shall be the same as for ordinary construction as provided in the ordinance.

In view of the above reasoning, reports of strain-gage tests made by disinterested laboratories and testing engineers from which it is possible to derive formulas, have been accepted and studied as they were submitted by applicants. Inasmuch as the tests herein submitted have been conducted upon structures varying somewhat from those designed and built under this ruling, in the matter of strength, and as there are matters to be determined under this ruling, which have so far not been determined under this ruling, which have so far not been determined by test, it is sufficient that tests be conducted to determine:

First, the bending strains to which columns are subject within the interior of the structure as well as at the walls.

Second, the proper percentage of increase in strength which should be provided in outer floor panels or wall columns over that for interior construction, and for the effect upon the structure of uneven or partial distribution of floor loading, such as placing the load between two rows of columns stretching clear across the structure than on interior or exterior panels. If it should be disclosed by such test that any provision of this ruling is too severe or not severe enough, the ruling will be promptly revised only in so far as thus indicated.

Finally, it is herein determined that all systems admitted under this ruling shall have passed a strain-gage test to demonstrate its ability to conform to the requirements of this ruling before any further construction work can be passed to a permit.

If it should be decided by all parties interested to conduct jointly one complete loading test which will also be in all respects satisfactory to this Bureau, this test would be considered sufficient to allow all systems to be constructed without further test.

It being noted that the only additional information necessary will concern especially the points of uncertainty described immediately preceding. And that additional tests will not be requested by this Bureau of those whose systems are now approved. Tests by those whose systems are now approved will only be considered necessary whenever these requirements are objected to by applicants as being too severe in the above-mentioned points.

Flat slabs as understood by this ruling shall consist of reinforced-concrete columns with enlarged capitals on which is supported a flat reinforced slab floor with or without plates or depressed panels at the column cap. The construction may be such as to admit the use of hollow panels in the ceiling or smooth ceiling with depressed panel in the floor at the column cap.

The column capital shall be defined as the gradual flaring out of the column without any marked offset in the concrete.

The depressed panel shall be defined as a square, rectangular or approximately circular depression around the column capital extending below the adjacent slab.

The panel length shall be defined as the distance center to center of column of the side of a square panel, and the panel length and breadth of a rectangular panel as the distance center to center of column in the long and short directions respectively. The span length being defined as the distance in the clear from edge to edge of column cap where it intersects either the floor slab or the depressed panel, measured on the length or breadth of the panel as the case may be.

Requirements

Stresses.—All unit stresses shall be as specified in the ordinance governing the use of concrete and reinforced concrete. The resisting moment and coincident stresses shall be computed under the assumption set forth in the ordinance.

Moments.—The negative bending moment at the support shall be taken $\frac{W'L'}{11}$ in which W' equals the total load on one panel exclusive of any load within the area of the column capital, and L' is the clear span between column capitals measured along the side of the panel.

The positive moment at the center of the panel shall be taken as $\frac{WL}{16}$ in which W is the total load on a panel and L the distance center to center of columns measured along the side of the panel.

Resisting Sections.—The negative moment at the support shall be considered as acting on a vertical section passing through the slab along the periphery of the column capital. The compressive stress in the concrete on this section shall be calculated by the ordinary straight line assumptions of stress distribution, by the formulas given in the ordinance, taking the periphery of the column capital, as the width of the section and the depth from the lower face of the concrete adjoining the column capital to the center of gravity of the slab steel as the depth of the section.

The area of slab steel resisting the negative moment of $\frac{W'L'}{11}$ at the support shall be taken as the total section of all slab rods cutting a conical critical section starting at the periphery of the column capital and flaring outwards at a 45-deg. angle with the vertical. The spacing of rods thus determined for the width of the critical section shall be maintained for the full width of the bands.

The positive moment at the center shall be resisted by the steel and concrete cut by a vertical critical section through the slab having its center at the column center and its diameter equal to the main dimension of the sides of the panel.

Drop Construction.—The thickness of the slab adjacent to the column capital may be increased, if necessary, by means of a depending concrete drop panel centered on the column center. Where this drop panel is used the resisting moment of the slab at the periphery of the drop shall be not less than that calculated from the formula $W'\left(\frac{L'}{11} + \frac{X^2}{2L'} - \frac{X}{2}\right)$ in which W' and L' are as defined above and X is the distance between the edge of the column capital and circle of area equal to that of the drop used. This drop panel may be diminished in thickness at greater distance from the column capital if desired provided the resisting moment at any section shall not fall below the value determined by the above formula applied to that particular section.

Columns.—The columns shall be calculated for the unbalanced moment of the live floor load when the entire area in one side of a line through the column center is considered as loaded and the area on the other side unloaded. The live-load reaction producing this moment shall be considered as uniformly distributed along the periphery of the column capital.

Distribution of Slab Steel.—The computed area of steel per unit width of band determined from the moment at the support shall be maintained the same for the entire width of each band. Width of bands shall be sufficient to cause the whole area of the panel to be covered by reinforcing bars The total steel area as determined from the computations may be distributed equally between all the bands or somewhat more than one-half may be placed in the direct bands. In no case, however, shall the steel area of direct bands exceed that in diagonal bands by more than one-third for square panels.

Rectangular Panels.—The slab thicknesses and the steel area in the various bands shall be determined in rectangular panels by computations based on a square panel of the same dimensions. The steel area in the long direct band shall be that required in the same band if situated on the edge of a square panel of the long dimensions in size. The steel area in the short direct band shall be that required in the same band if situated on the edge of a square panel of the short dimension in size.

The steel area in the diagonal bands shall be that required in the same band if situated diagonally in a square panel whose size will be the average of the long and short dimensions.

Profile of Column Capital.—The profile of the column capital shall satisfy the moment of $\frac{W'L'}{11}$ at any concentric vertical section through it and the slab, the value of W' and L' being taken for the particular section considered.

Moments in Wall Panels.—The bending moment to be resisted by any band of reinforcing extending into an exterior panel shall be increased by 20% if concrete column supports are present at the wall, and by 40% if the slab rests on a brick wall at its exterior edge.

Concentrated Loads.—Girders and beams shall be provided where necessary to carry concentrated loads in excess of the safe capacity of the floor slab. Such girders and beams shall be calculated to carry the full concentrated load.

Openings Cut in Floors.—Girders and beams shall be provided on all sides of the opening wherever there are openings or holes in the slab. They shall be calculated to carry the reaction of the floor on all sides of the openings.

RULING COVERING DESIGN OF FLAT-SLAB CONSTRUCTION IN THE CITY OF CHICAGO

(For ruling as amended Jan. 1, 1918, see page 851.)

1. Definitions.—Flat slabs as understood by this ruling are reinforced-concrete slabs supported directly on reinforced columns with or without plates or capitals at the top, the whole construction being hingeless and monolithic without any visible beams or girders. The construction may be such as to admit the use of hollow panels in the ceiling or smooth ceiling with depressed panels in the floor.

2. The column capital shall be defined as the gradual flaring out of the top of the column without any marked offset.

3. The drop panel shall be defined as a square or rectangular depression around the column capital extending below the slab adjacent to it.

4. The panel length shall be defined as the distance center to center of columns of the side of a square panel, or the average distance center to center of columns of the long and short sides of a rectangular panel.

5. Columns.—The least dimensions of any concrete column shall be not less than one-twelfth the panel length, or one-twelfth the clear height of the column.

6. Slab Thickness.—The minimum total thickness of the slab in inches shall be determined by the formula:

$$t = 0.023L\sqrt{w}$$

where

t = total thickness of slab in inches.
L = panel length in feet.
w = total live and dead load in pounds per square foot.

7. In no case shall the slab thickness be less than one-thirty-second of the panel length for floors, and one-fortieth of the panel length for roofs, and also not less than 6 in.

8. Column Capital.—The diameter of the column capital shall be measured where its vertical thickness is at least 1½ in., and shall be at least 0.225 of the panel length.

9. The slope of the column capital shall nowhere make an angle with the vertical of more than 45 deg. Special attention shall be given to the design of the column capital in considering eccentric loads, and the effect of wind upon the structure.

10. Drop Panel.—The depth of the drop shall be determined by computing it as a beam, using the negative bending moment specified elsewhere in this ruling. The width and length shall be determined by the allowable unit shearing stresses on the perimeter, given below.

11. Shearing Stresses.—The allowable unit punching shear on the perimeter of the column capital shall be three-fiftieths of the ultimate compressive strength of the concrete as given in Sect. 546 of the building ordinance. The allowable unit shear on the perimeter of the drop panel shall be three one-hundredths of the ultimate compressive strength of the concrete. In computing shearing stress for the purpose of determining the resistance to diagonal tension the method specified by the ordinance shall be used.

12. Panel Strips.—For the purpose of establishing the bending moments and the resisting moments of a square panel, the panel shall be divided into strips known as strip A and strip B. Strip A shall include the reinforcement and slab in a width extending from the center line of the columns for a distance each side of this center line equal to one-fourth of the panel length. Strip B shall include the reinforcement and slab in the maining in the center of the panel. At right angles to these strips, the panel shall be divided into similar strips A and B, having the same widths and relations to the center line of the columns as the above strips. These strips shall be for designing purposes only, and are not intended as the boundary lines of any bands of steel used.

13. These strips shall apply to the system of reinforcement in which the reinforcing bars are placed parallel and at right angles to the center line of the columns, hereinafter known as the two-way system, and also to the system of reinforcement in which the reinforcing bars are placed parallel, at right angles to and diagonal to the center line of the columns hereinafter known as the four-way system.

Bending Moment Coefficients, Interior Panel, Two-way System

14. The negative bending moment taken at a cross-section of each strip A at the edge of a column capital or over it, shall be taken as $WL^2/15$. The positive bending moment taken at a cross-section of each strip A, midway between column centers shall be taken as $WL^2/30$. The positive bending moment taken at a cross-section of each strip B in the middle of the panel shall be taken at $WL^2/60$. The negative bending moment taken at a cross-section of each strip B on the center line of the columns shall be taken at $WL^2/60$. In the formulas hereinabove given

W = total live and dead load per lineal foot of each strip.
L = panel length in feet.

Bending Moment Coefficients, Interior Panel, Four-way System

15. The negative bending moment taken at a cross-section of each strip A at the edge of the column capital or over it, shall be taken as $WL^2/15$. The positive bending moment taken at a cross-section of each strip A, midway between column centers shall be taken as $WL^2/40$. The positive bending moment taken at a cross-section of each strip B in the middle of the panel shall be taken as $WL^2/60$. The negative bending moment taken at a cross-section of each strip B on the center line of the column shall be taken at $WL^2/60$.

Bending Moment Coefficients, Wall Panels

16. Wherever the coefficients $\frac{1}{15}$, $\frac{1}{30}$, $\frac{1}{40}$ or $\frac{1}{60}$ appear in the moments given for interior panels in either the two-way or the four-way systems, the coefficients $\frac{1}{12}$, $\frac{1}{25}$, $\frac{1}{33}$ and $\frac{1}{50}$ respectively shall be used in the moments for wall panels supported on concrete columns and girders.

17. When brick walls are used partly to support wall panels, these walls shall be stiffened by pilasters or piers as directed by the Commissioner of Buildings. Wherever the coefficients $\frac{1}{15}$, $\frac{1}{30}$, $\frac{1}{40}$ or $\frac{1}{60}$ appear in the moments given for interior panels in either the two-way or the four-way systems, the coefficients $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{27}$ and $\frac{1}{40}$ respectively shall be used in the moments for such panels resting on brick walls.

Point of Inflection

18. For the purpose of making the calculations of the bending moment at the sections away from the column capital, the point of inflection shall be considered as being one-quarter the distance center to center of columns, both crosswise and diagonally, from the center of the column.

Tensile Stress in Steel and Compressive Stress in Concrete

19. The tensile stress in steel and the compressive stress in the concrete to resist the bending moment shall be calculated on the basis of the reinforcement and slab in the width included in a given strip, and according to the assumptions and requirements given in Sects. 545 to 548 inclusive of the building ordinance.

20. The steel shall be considered as being concentrated at the center of gravity of all the bands of steel in a given strip.

21. For the four-way system of reinforcement the amount of steel to resist the negative bending moment over the support in each strip A shall be taken as the sum of the areas of steel in one cross band and one diagonal band The amount of steel to resist the positive bending moment of each strip B shall be considered as the area of the steel in a diagonal band. The amount of steel to resist the positive bending moment in each strip A shall be considered as the area of the steel in a cross-band, and the amount of steel to resist the negative moment in each strip B shall be the steel included in the width of strip B.

22. For the two-way system of reinforcement the amount of steel to resist the bending moment in any strip shall be considered as the area of steel included in the width of the strip.

23. In both systems of reinforcement the compressive stress in the concrete in any strip shall be calculated by taking the area of steel considered for each strip, and applying it in a beam formula based on the principles of Sect. 548 of the building ordinance.

24. When the length of a panel does not exceed the breadth by more than 5%, all computations shall be made on the basis of a square with sides equal to the mean of the length and breadth. In no rectangular panel shall the length exceed four-thirds the breadth.

25. For panels with length more than 5% in excess of the breadth, the slab shall first be designed for a bending moment based on an assumed square panel with sides equal to the mean of the length and breadth of the rectangular panel.

26. For the four-way system of reinforcement the amount of steel found for the positive moment of each strip B by designing in this manner shall be that used in the diagonal band. For the positive moment in each strip A, the required amount of steel in the cross-band shall be obtained by multiplying the steel used in the design of the assumed square panel by the cube of the ratio found by dividing the length or breadth of the rectangular panel by the side of the assumed square panel, for the long and short sides of the panel respectively. The compressive stresses shall be calculated on the basis of a width equal to one-half of the side of the assumed square panel, and on the assumptions used in the calculations of compressive stresses in square panels. In no case shall the amount of steel in the short side be less than two-thirds of that required for the long side.

27. For the two-way system of reinforcement, the amount of steel found for the positive and negative moment of each strip B by designing in this manner shall be obtained by multiplying the steel used in the design of the assumed square panel by the cube of the ratio found by dividing the length or breadth of the rectangular panel by the side of the assumed square panel, for the short and long side of the panel respectively. The method of obtaining the amount of steel required for each strip A, shall be the same as that given above for the four-way system.

28. **Walls and Openings.**—Girders or beams shall be constructed under walls, and around openings and to carry concentrated loads.

29. **Computations.**—Complete computations of interior and wall panels and such other portions of the building as may be required by the Commissioner of Buildings shall be left in the office of the Commissioner of Buildings when plans are presented for approval.

30. **Placing of Steel.**—In order that the slab bars shall be maintained in the position shown in the design during the work of pouring the slab, spacers and supports shall be provided satisfactory to the Commissioner of Buildings. All bars shall be secured in place at intersections by wire or other metal fastenings. In no case shall the spacing of the bars exceed 9 in. The steel to resist the negative moment in each strip B shall extend one-fourth of the panel length beyond the center line of the columns in both directions.

31. All splices in bars shall be made over the column head. The length of the splice beyond the center line of the column in both directions shall be at least 2 ft. nor less than that necessary for the full development of the strength of the bar as limited by the unit bond stresses given by the ordinance. The splicing of adjacent bars shall be avoided as far as possible.

32. Slab bars which are lapped over the column, the sectional area of both being included in the calculations for negative moment, shall extend not less than 0.25 of the panel length for cross-bands, and 0.35 of the panel length for diagonal bands, beyond the column center.

33. **Test of Workmanship.**—The Commissioner of Buildings or his representative may choose any two adjacent panels in the building for the purpose of ascertaining the character of workmanship. The test shall not be made sooner than the time required for the cement to set thoroughly, nor less than 6 weeks after the concrete had been poured.

34. All deflections under test load shall be taken at the center of the slab, and shall be measured from the normal unloaded position of the slab. The two panels selected shall be uniformly loaded over their entire area with a load equal to the dead load plus twice the live load, thus obtaining twice the total design load. The load shall remain in place not less than 24 hr. If the total deflection in the center of the panel under the test load does not exceed one eight-hundredth of the panel length, the slab may be placarded to carry the full design live load. If it exceeds this amount of deflection, and recovers not less than 80% of the total deflection within 7 days after the load is removed, the slab may be placarded to carry the full design live load. If the deflection exceeds the allowable amount above specified, and the recovery is less than 80% in 7 days after the removal of the test load, other tests shall be made on the same or other panels, the results of which will determine the amount of live load the slabs will be permitted to carry.

35. **General.**—The design and execution of the work shall conform to the provisions of the Chicago building ordinances, and to correct principles of construction.

CHICAGO REINFORCED-CONCRETE FLAT-SLAB RULING AMENDED

(New ordinance adopted Jan. 1, 1918, while this book was in press)

1. **Definitions.**—Flat slabs as understood by this ruling are reinforced-concrete slabs, supported directly on reinforced columns with or without plates or capitals at the top, the whole construction being hingeless and monolithic without any visible beams or girders. The construction may be such as to admit the use of hollow panels in the ceiling or smooth ceiling with depressed panels in the floor.

2. The column capital shall be defined as the gradual flaring out of the top of the column without any marked offset.

3. The drop panel shall be defined as a square or rectangular depression around the column capital extending below the slab adjacent to it.

4. The panel length shall be defined as the distance c. to c. of columns of the side of a square panel, or the average distance c. to c. of columns of the long and short sides of a rectangular panel.

5. **Columns.**—The least dimension of any concrete column shall be not less than one-twelfth the panel length, nor one-twelfth the clear height of the column.

6. **Slab Thickness.**—The minimum total thickness of the slab in inches shall be determined by the formula:

$t = W^{\frac{1}{2}}/44$ (= square root of W divided by 44), where t = total thickness of slab in inches, W = total live load and dead load in pounds on the panel, measured c. to c. of columns.

7. In no case shall the thickness be less than $1/32$ of the panel length ($L/32$) for floors, nor $1/40$ of the panel length ($L/40$) for roofs (L being the distance c. to c. of columns).

8. In no case shall the thickness of slab be less than 6 in. for floors or roofs.

9. Column Capital.—When used the diameter of the column capital shall be measured where its vertical thickness is at least 1½ in. and shall be at least 0.225 of the panel length.

The slope of the column capital shall nowhere make an angle with the vertical of more than 45 deg. Special attention shall be given to the design of the column capital in considering eccentric loads, and the effect of wind upon the structure.

10. Drop Panel.—When used, the drop panel shall be square or circular for square panels and rectangular or elliptical for oblong panels.

11. The length of the drop shall not be less than one-third of the panel length ($L/3$) if square, and not less than one-third of the long or short side of the panel respectively, if rectangular.

12. The depth of the drop panel shall be determined by computing it as a beam, using the negative moment over the column capital specified elsewhere in this ruling.

13. In no case, however, shall the dimensions of the drop panel be less than required for punching shear along its perimeter, using the allowable unit shearing stresses specified below.

14. Shearing Stresses.—The allowable unit punching shear on the perimeter of the column capital shall be $3/40$ of the ultimate compressive strength of the concrete as given in Sect. 533 of the building ordinance. The allowable unit shear on the perimeter of the drop panel shall be 0.03 of the ultimate compressive strength of the concrete. In computing shearing stress for the purpose of determining the resistance to diagonal tension the method specified by the ordinance shall be used.

15. Panel Strips.—For the purpose of establishing the bending moments and the resisting moments of a square panel, the panel shall be divided into strips known as strip A and strip B. Strip A shall include the reinforcement and slab in a width extending from the center line of the columns for a distance each side of this center line equal to one-quarter of the panel length. Strip B shall include the reinforcement and slab in the half width remaining in the center of the panel. At right angles to these strips, the panel shall be divided into similar strips A and B, having the same widths and relations to the center line of the columns as the above strips. These strips shall be for designing purposes only, and are not intended as the boundary lines of any bands of steel used.

16. These strips shall apply to the system of reinforcement in which the reinforcing bars are placed parallel and at right angles to the center line of the columns, hereinafter known as the two-way system, and also to the system of reinforcement in which the reinforcing bars are placed parallel, at right angles to and diagonal to the center line of the columns hereinafter known as the four-way system.

17. Any other system of reinforcement in which the reinforcing bars are placed in circular, concentric rings and radial bars, or systems with steel rods arranged in any manner whatsoever, shall comply with the requirements of either the two-way or the four-way system herein specified.

18. Bending Moment Coefficients, Interior Panel, Two-way System.—In panels where standard drops and column capitals are used as above specified, the negative bending moment, taken at a cross-section of each strip A at the edge of the column capital or over it, shall be taken as $WL/30$.

19. The positive bending moment taken at a cross-section of each strip A midway between column centers shall be taken as $WL/60$.

20. The positive bending moment taken at a cross-section of each strip B in the middle of the panel shall be taken as $WL/120$.

21. The negative bending moment taken at a cross-section of each strip B on the center line of the columns shall be taken as $WL/120$.

22. In the formulas hereinabove given W = total live and dead load on the whole panel in pounds, L = panel length, c. to c. of columns.

23. Bending Moment Coefficients, Interior Panel, Four-way System.—In panels where standard drops and column capitals are used as above specified, the negative bending moment, taken at a cross-section of each strip A at the edge of column capital or over it, shall be taken as $WL/30$.

24. The positive bending moment, taken at a cross-section of each strip A, midway between column centers, shall be taken as $WL/80$.

25. The positive bending moment, taken at a cross-section of each strip B, taken in the middle of the panel, shall be taken as $WL/120$.

26. The negative bending moment, taken at a cross-section of each strip B on the center line of the columns, shall be taken as $WL/120$.

27. Bending Moment Coefficients, Wall Panels.—Where wall panels with standard drops and capitals are carried by columns and girders built in walls, as in skeleton construction, the same coefficients shall be used as for an interior panel, except as follows: The positive bending moments on strips A and B midway between wall and first line of columns shall be increased 25%.

28. Where wall panels are carried on new brick walls, these shall be laid in Portland cement mortar and shall be stiffened with pilasters as follows: If a 16-in. wall is used, it shall have a 4-in. pilaster. If a 12-in. wall is used, it shall have an 8-in. pilaster. The length of pilasters shall be not less than the diameter of the column, nor less than one-eighth of the distance between pilasters. The pilasters shall be located opposite the columns as nearly as practicable, and shall be corbeled out 4 in. at the top, starting at the level of the base of the column capital. Not less than 8-in. bearing shall be provided for the slab, the full length of wall.

The coefficients of bending moments required for these panels shall be the same as those for the interior panels except as provided herewith: The positive bending moments on strips A and B midway between the wall and first line of columns shall be increased 50%.

29. Where wall panels are supported on old brick walls, there shall be columns with standard drops and capitals built against the wall, which shall be tied to the same in an approved manner, and at least an 8-in. bearing provided for the slab, the full length. Where this is impracticable, there shall be built a beam on the underside of slab adjacent to the wall between columns, strong enough to carry 25% of the panel load.

The coefficients of bending moments for the two cases of slab support herein described shall be the same as those specified in Sect. 27 and Sect. 28 for skeleton and wall bearing condition, respectively.

30. Nothing specified above shall be construed as applying to a case of slabs merely resting on walls or ledges, without any condition of restraint. These shall be figured as in ordinary beam-and-girder construction specified in the ordinances.

31. Bending Moment Coefficients, Wall and Interior Columns.—Wall columns in skeleton construction shall be designed to resist a bending moment of $WL/60$ at floors and $WL/30$ at roof. The amount of steel required for this moment shall be independent of that required to carry the direct load. It shall be placed as near the surfaces of the column as practicable on the tension sides, and the rods shall be continuous in crossing from one side to another. The length of rods below the base of the capital and above the floor line shall be sufficient to develop their strength through bond, but not less than 40 diameters, nor less than one-third the clear height between the floor line and the base of the column capital.

32. The interior columns must be analyzed for the worst condition of unbalanced loading. It is the intention of this ruling to cover ordinary cases of eccentric loads on the columns by the requirement of Sect. 5. Where the minimum size of column therein specified is found insufficient, however, the effect of the resulting bending moment shall be properly divided between the adjoining slab and the columns above and below according to best principles of engineering, and the columns enlarged sufficiently to carry the load safely.

33. Bending Moment Coefficients, Panels Without Drops, or Capitals, or Both.—In square panels where no column capital or no depressions are used, the sum total of positive and negative bending moments shall be equal to that computed by the following formula

$$B.M. = (WL/8)\ (1.53 - 4k + 4.18k^2)$$

where $B.M.$ = Numerical sum of positive and negative bending moments, regardless of algebraic signs;

W = Total live and dead load on the whole panel;

L = Length of side of a square panel, c. to c. of columns;

k = Ratio of the radius of the column or column capital to panel length, L.

This total bending moment shall be divided between the positive and the negative moments in the same proportion as in the typical square panels for two-way or four-way systems specified above for interior and wall panels respectively.

34. Point of Inflection.—For the purpose of making the calculations of the bending moment at the sections away from the column capital, the point of inflection shall be considered as being one-quarter the distance c. to c. of columns, both crosswise and diagonally, from the center of the column.

35. Tensile Stress in Steel and Compressive Stress in Concrete.—The tensile stress in steel and the compressive stress in the concrete to resist the bending moment shall be calculated on the basis of the reinforcement and slab in the width included in a given strip, and according to the assumptions and requirements given in Sects. 532 to 535 inclusive of the building ordinance. The steel shall be considered as being concentrated at the center of gravity of all the bands of steel in a given strip.

36. For the four-way system of reinforcement the amount of steel to resist the negative bending moment over the support in each strip A shall be taken as the sum of the areas of steel in one cross band and one diagonal band. The amount of steel to resist the positive bending moment of each strip B shall be considered as the area of the steel in a diagonal band. The amount of steel to resist the positive bending moment in each strip A shall be considered as the area of the steel in a cross band, and the amount of steel to resist the negative moment in each strip B shall be the steel included in the width of strip B.

37. For the two-way system of reinforcement the amount of steel to resist the bending moment in any strip shall be considered as the area of steel included in the width of the strip.

38. In both systems of reinforcement the compressive stress in the concrete in any strip shall be calculated by taking the area of steel considered for each strip and applying it in a beam formula based on the principles of Sect. 535 of the building ordinance.

39. Where drop panels are used, the width of beam assumed to resist the compressive stresses over the column capital shall be the width of the drop.

40. The width of beam, where no drop panels are used, shall be the width of steel bands. Where this is found insufficient, the area shall be increased by introducing compression steel in the bottom of slab.

41. Rectangular Panels.—When the length of panel in either two-way or four-way system does not exceed the breadth by more than 5%, all computations shall be based on a square panel whose side equals the mean of the length and breadth, and the steel equally distributed among the strips according to the coefficients above specified.

42. In no rectangular panel shall the length exceed the breadth by more than one-third of the latter.

43. Rectangular Panels, Four-way System.—In the four-way system of reinforcement, where length exceeds breadth by more than 5%, the amount of steel required in strip A, long direction, both positive and ne[illegible] the same as that required for the same strip in a square panel whose length is equal to the long side of the rectangular panel.

44. The amount of steel, strip *A*, short direction, positive and negative, shall be the same as that required for the same strip in a square panel, whose length is equal to the short side of the rectangular panel.

45. The amount of steel in strip *B*, positive and negative, shall be the same as that required for similar strip in a square panel whose length is equal to the mean of the long and the short side of the rectangular panel.

46. In no case shall the amount of steel in the short side be less than two-thirds of that required for the long side.

47. Rectangular Panels, Two-way System.—In the two-way system of reinforcement the amount of steel required for the positive and the negative moment of each strip *A* shall be determined in the same manner as indicated for the four-way system above.

48. The amount of steel in strip *B*, positive and negative, running in short direction, shall be equal to that required for the same strip in a square panel whose length equals the long side of the rectangular panel.

49. The amount of steel in strip *B*, long direction, positive and negative, shall be equal to that required for the same strip in a square panel, whose length equals the short side of the rectangular panel.

50. In no case shall the amount of steel in strip *B*, long direction, be less than two-thirds of that in the short direction.

51. Walls and Openings.—Girders and beams shall be constructed under walls, around openings and to carry concentrated loads.

52. Spandrel Beams.—The spandrel beams or girders shall, in addition to their own weight and the weight of the spandrel wall, be assumed to carry 20% of the wall panel load uniformly distributed upon them.

53. Placing of Steel.—In order that the slab bars shall be maintained in the position shown in the design during the work of pouring the slab, spacers and supports shall be provided satisfactory to the Commissioner of Buildings. All bars shall be secured in place at intersections by wire or other metal fastenings. In no case shall the spacing of the bars exceed 9 in. The steel to resist the negative moment in each strip *B* shall extend one-quarter of the panel length beyond the center line of the columns in both directions.

54. Splices in bars may be made wherever convenient, but preferably at points of minimum stress. The length of splice beyond the center point, in each direction, shall not be less than 40 diameters of the bars, nor less than 2 ft. The splicing of adjacent bars shall be avoided as far as possible.

55. Slab bars which are lapped over the column, the sectional area of both being included in the calculations for negative moment, shall extend not less than 0.25 of the panel length for cross bands and 0.35 of the panel length for diagonal bands, beyond the column center.

56. Computations.—Complete computations of interior and wall panels and such other portions of the building as may be required by the Commissioner of Buildings shall be left in the office of the Commissioner of Buildings when plans are presented for approval.

57. Test of Workmanship.—The Commissioner of Buildings or his representative may choose any two adjacent panels in the building for the purpose of ascertaining the character of workmanship. The test shall not be made sooner than the time required for the cement to set thoroughly, nor less than 6 weeks after the concrete had been poured.

58. All deflections under test load shall be taken at the center of the slab, and shall be measured from the normal unloaded position of the slab. The two panels selected shall be uniformly loaded over their entire area with a load equal to the dead load plus twice the live load, thus obtaining twice the total design load. The load shall remain in place not less than 24 hours. If the total deflection in the center of the panel under the test load does not exceed $\frac{1}{300}$ of the panel length, the slab may be placarded to carry the full design live load. If it exceeds this amount of deflection, and recovers not less than 80% of the total deflection within 7 days after the load is removed, the slab may be placarded to carry the full design live load. If the deflection exceeds the allowable amount above specified, and the recovery is less than 80% in 7 days after the removal of the test load, other tests shall be made on the same or other panels, the results of which will determine the amount of live load the slabs will be permitted to carry.

59. General.—The design and the execution of the work shall conform to the general provisions and the spirit of the Chicago Building Ordinances in points not covered by this Ruling and to the best engineering practice in general.

60. Enforcement.—This Ruling shall go into effect on and after Jan. 1, 1918. All previous rulings on flat slabs are hereby rescinded.

FINAL REPORT OF SPECIAL COMMITTEE OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

PART PERTAINING TO FLAT-SLAB DESIGN

The continuous flat slab reinforced in two or more directions and built monolithically with the supporting columns (without beams or girders) is a type of construction which is now extensively used and which has recognized advantages for certain types of structures as, for example, warehouses in which large, open floor space is desired. In its construction, there is excellent opportunity for inspecting the position of the reinforcement. The conditions attending depositing and placing of concrete are favorable to securing uniformity and soundness in the concrete. The recommendations in the following paragraphs relate to flat slabs extending over several rows of panels in each direction. Necessarily the treatment is more or less empirical.

The coefficients and moments given relate to uniformily distributed loads.

(a) *Column Capital.*—It is usual in flat-slab construction to enlarge the supporting columns at their top, thus forming column capitals. The size and shape of the column capital affect the strength of the structure in several ways. The moment of the external forces which the slab is called upon to resist is dependent upon the size of the capital; the section of the slab immediately above the upper periphery of the capital carries the highest amount of punching shear; and the bending moment developed in the column by an eccentric or unbalanced loading of the slab is greatest at the under surface of the slab. Generally, the horizontal section of the column capital should be round or square with rounded corners. In oblong panels the section may be oval or oblong, with dimensions proportional to the panel dimensions. For computation purposes, the diameter of the column capital will be considered to be measured where its vertical thickness is at least 1½ in., provided the slope of the capital below this point nowhere makes an angle with the vertical of more than 45 deg. In case a cap is placed above the column capital, the part of this cap within a cone made by extending the lines of the column capital upward at the slope of 45 deg. to the bottom of the slab or dropped panel may be considered as part of the column capital in determining the diameter for design purposes. Without attempting to limit the size of the column capital for special cases, it is recommended that the diameter of the column capital (or its dimension parallel to the edge of the panel) generally be made not less than one-fifth of the dimension of the panel from center to center of adjacent columns. A diameter equal to 0.225 of the panel length has been used quite widely and acceptably. For heavy loads or large panels, especial attention should be given to designing and reinforcing the column capital with respect to compressive stresses and bending moments. In the case of heavy loads or large panels, and where the conditions of the panel loading or variations in panel length or other conditions cause high bending stresses in the column, and also for column capitals smaller than the size herein recommended, especial attention should be given to designing and reinforcing the column capital with respect to compression and to rigidity of connection to floor slab.

(b) *Dropped Panel.*—In one type of construction the slab is thickened throughout an area surrounding the column capital. The square or oblong of thickened slab thus formed is called a dropped panel or a drop. The thickness and the width of the dropped panel may be governed by the amount of resisting moment to be provided (the compressive stress in the concrete being dependent upon both thickness and width), or its thickness may be governed by the resistance to shear required at the edge of the column capital and its width by the allowable compressive stresses and shearing stresses in the thinner portion of the slab adjacent to the dropped panel. Generally, however, it is recommended that the width of the dropped panel be at least four-tenths of the corresponding side of the panel as measured from center to center of columns, and that the offset in thickness be not more than five-tenths of the thickness of the slab outside the dropped panel.

(c) *Slab Thickness.*—In the design of a slab, the resistance to bending and to shearing forces will largely govern the thickness, and, in the case of large panels with light loads, resistance to deflection may be a controlling factor. The following formulas for minimum thicknesses are recommended as general rules of design when the diameter of the column capital is not less than one-fifth of the dimension of the panel from center to center of adjacent columns, the larger dimension being used in the case of oblong panels. For notation, let

t = total thickness of slab, in inches.
L = panel length, in feet.
w = sum of live load and dead load, in pounds per square foot.

Then, for a slab without dropped panels,

$$\text{minimum } t = 0.024\,L\sqrt{w} + 1\tfrac{1}{2}$$

for a slab with dropped panels,

$$\text{minimum } t = 0.02\,L\sqrt{w} + 1$$

for a dropped panel whose width is four-tenths of the panel length,

$$\text{minimum } t = 0.03\,L\sqrt{w} + 1\tfrac{1}{2}$$

In no case should the slab thickness be made less than 6 in., nor should the thickness of a floor slab be made less than one thirty-second of the panel length, nor the thickness of a roof slab less than one-fortieth of the panel length.

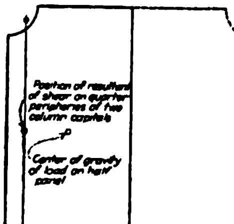


Fig. 1.

(d) *Bending and Resisting Moments in Slabs.*—If a vertical section of a slab be taken across a panel along a line midway between columns, and if another section be taken along an edge of the panel parallel to the first section, but skirting the part of the periphery of the column capitals at the two corners of the panels, the moment of the couple formed by the external load on the half panel, exclusive of that over the column capital (sum of dead and live loads) and the resultant of the external shear or reaction at the support at the two column capitals (see Fig. 1), may be found by ordinary static analysis. It will be noted that the edges of the area here considered are along lines of zero shear, except around the column capitals. This moment of the external forces acting on the half panel will be resisted by the numerical sum of (a) the moment of the internal stresses at the section of the panel midway between columns (positive resisting moment) and (b) the moment of the internal stresses at the section referred to at the end of the panel (negative resisting moment). In the curved portion of the end section (that skirting the column), the stresses considered are the components which act parallel to the normal stresses on the straight portion of the section. Analysis shows that, for a uniformly distributed load, and round columns, and square panels, the numerical sum of the positive moment and the negative moment at the two sections named is given quite closely by the equation

$$M_x = \tfrac{1}{8}wl\,(l - \tfrac{2}{3}c)^2$$

In this formula and in those which follow relating to oblong panels,

w = sum of the live and dead loads per unit of area.
l = side of a square panel measured from center to center of columns.
l_1 = one side of the oblong panel measured from center to center of columns.
l_2 = other side of oblong panel measured in the same way.
c = diameter of the column capital.
M_x = numerical sum of positive moment and negative moment in one direction.
M_y = numerical sum of positive moment and negative moment in the other direction.[1]

For oblong panels, the equation for the numerical sum of the positive moment and the negative moment at the two sections named becomes

$$M_x = \frac{1}{8}\, wl_2\left(l_1 - \frac{2}{3}c\right)^2$$

$$M_y = \frac{1}{8}\, wl_1\left(l_2 - \frac{2}{3}c\right)^2$$

where M_x is the numerical sum of the positive moment and the negative moment for the sections parallel to the dimension, l_2, and M_y is the numerical sum of the positive moment and the negative moment for the sections parallel to the dimension, l_1.

What proportion of the total resistance exists as positive moment and what as negative moment is not readily determined. The amount of the positive moment and that of the negative moment may be expected to vary somewhat with the design of the slab. It seems proper, however, to make the division of total resisting moment in the ratio of three-eighths for the positive moment to five-eighths for the negative moment.

With reference to variations in stress along the sections, it is evident from conditions of flexure that the resisting moment is not distributed uniformly along either the section of positive moment or that of negative moment. As the law of the distribution is not known definitely, it will be necessary to make an empirical apportionment along the sections; and it will be considered sufficiently accurate generally to divide the sections into two parts and to use an average value over each part of the panel section.

The relatively large breadth of structure in a flat slab makes the effect of local variations in the concrete less than would be the case for narrow members like beams. The tensile resistance of the concrete is less affected by cracks. Measurements of deformations in buildings under heavy load indicate the presence of considerable tensile resistance in the concrete, and the presence of this tensile resistance acts to decrease the intensity of the compressive stresses. It is believed that the use of moment coefficients somewhat less than those given in a preceding paragraph as derived by analysis is warranted, the calculations of resisting moment and stresses in concrete and reinforcement being made according to the assumptions specified in this report and no change being made in the values of the working stresses ordinarily used. Accordingly, the values of the moments which are recommended for use are somewhat less than those derived by analysis. The values given may be used when the column capitals are round, oval, square, or oblong.

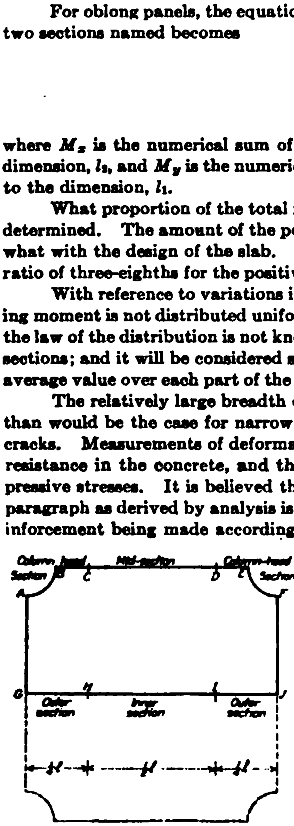


FIG. 2.

(e) *Names for Moment Sections.*—For convenience, that portion of the section across a panel along a line midway between columns which lies within the middle two quarters of the width of the panel (*HI*, Fig. 2) will be called the inner section, and that portion in the two outer quarters of the width of the panel (*GH* and *IJ*, Fig. 2) will be called the outer sections. Of the section which follows a panel edge from column capital to column capital and which includes the quarter peripheries of the edges of two column capitals, that portion within the middle two quarters of the panel width (*CD*, Fig. 2) will be called the mid-section, and the two remaining portions (*ABC* and *DEF*, Fig. 2), each having a projected width equal to one-fourth of the panel width, will be called the column-head sections.

(f) *Positive Moment.*—For a square interior panel, it is recommended that the positive moment for a section in the middle of a panel extending across its width be taken as $\frac{1}{25}\, wl\left(l - \frac{2}{3}c\right)^2$. Of this moment, at least 25% should be provided for in the inner section; in the two outer sections of the panel at least 55% of the specified moment should be provided for in slabs not having dropped panels, and at least 60% in slabs having dropped panels, except that in calculations to determine necessary thickness of slab away from the dropped panel at least 70% of the positive moment should be considered as acting in the two outer sections.

(g) *Negative Moment.*—For a square interior panel, it is recommended that the negative moment for a section which follows a panel edge from column capital to column capital and which includes the quarter peripheries of the edges of the two column capitals (the section altogether forming the projected width of the panel) be taken as $\frac{1}{15}wl\left(l - \frac{2}{3}c\right)^2$. Of this negative moment, at least 20% should be provided for in the mid-section and at least 65% in the two column-head sections of the panel, except that in slabs having dropped panels at least 80% of the specified negative moment should be provided for in the two column-head sections of the panel.

[1] See paper and closure, "Statical Limitations upon the Steel Requirement in Reinforced Concrete Flat Slab Floors," by JOHN R. NICHOLS, Jun. Am. Soc. C. E., *Trans.* Am. Soc. C. E., vol. 77.

(*h*) *Moments for Oblong Panels.*—When the length of a panel does not exceed the breadth by more than 5%, computation may be made on the basis of a square panel with sides equal to the mean of the length and the breadth.

When the long side of an interior oblong panel exceeds the short side by more than one-twentieth and by not more than one-third of the short side, it is recommended that the positive moment be taken as $\frac{1}{25}wl_2\left(l_1 - \frac{2}{3}c\right)^2$ on a section parallel to the dimension, l_2, and $\frac{1}{25}wl_1\left(l_2 - \frac{2}{3}c\right)^2$ on a section parallel to the dimenison, l_1; and that the negative moment be taken as $\frac{1}{15}wl_2\left(l_1 - \frac{2}{3}c\right)^2$ on a section at the edge of the panel corresponding to the dimension, l_2, and $\frac{1}{15}wl_1\left(l_2 - \frac{2}{3}c\right)^2$ at a section in the other direction. The limitations of the apportionment of moment between inner section and outer section and between mid-section and column-head sections may be the same as for square panels.

(*i*) *Wall Panels.*—The coefficient of negative moment at the first row of columns away from the wall should be increased 20% over that required for interior panels, and likewise the coefficient of positive moment at the section half way to the wall should be increased by 20%. If girders are not provided along the wall, or the slab does not project as a cantilever beyond the column line, the reinforcement parallel to the wall for the negative moment in the column-head section and for the positive moment in the outer section should be increased by 20%. If the wall is carried by the slab, this concentrated load should be provided for in the design of the slab. The coefficient of negative moments at the wall to take bending in the direction perpendicular to the wall line may be determined by the conditions of restraint and fixedness as found from the relative stiffness of columns and slab, but in no case should it be taken as less than one-half of that for interior panels.

(*j*) *Reinforcement.*—In the calculation of moments, all the reinforcing bars which cross the section under consideration and which fulfill the requirements given under paragraph (*l*) of this chapter may be used. For a column-head section, reinforcing bars parallel to the straight portion of the section do not contribute to the negative resisting moment for the column-head section in question. In the case of four-way reinforcement, the sectional area of the diagonal bars multiplied by the sine of the angle between the diagonal of the panel and the straight portion of the section under consideration may be taken to act as reinforcement in a rectangular direction.

(*k*) *Point of Inflection.*—For the purpose of making calculations of moments at sections away from the sections of negative moment to act integrally for a width equal to the width of the column-head section.

(*o*) *Provision for Diagonal Tension and Shear.*—In calculations for the shearing stress which is to be used as the means of measuring the resistance to diagonal tension stress, it is recommended that the total vertical shear on two column-head sections constituting a width equal to one-half the lateral dimension of the panel, for use in the formula for determining critical shearing stresses, be considered to be one-fourth of the total dead and live loads on a panel for a slab of uniform thickness, and to be three-tenths of the sum of the dead and live loads on a panel for a slab with dropped panels. The formula for shearing unit stress given in the Appendix to this report may then be written $v = \frac{0.25W}{bjd}$ for slabs of uniform thickness, and $v = \frac{0.30W}{bjd}$ for slabs with dropped panels, where W is the sum of the dead and live loads on a panel, b is half the lateral dimension of the panel measured from center to center of columns, and jd is the lever arm of the resisting couple at the section.

The calculation of what is commonly called punching shear may be made on the assumption of a uniform distribution over the section of the slab around the periphery of the column capital and also of a uniform distribution over the section of the slab around the periphery of the dropped panel, using in each case an amount of vertical shear greater by 25% than the total vertical shear on the section under consideration.

The values of working stresses should be those recommended for diagonal tension and shear in *Appendix B*.

(*p*) *Walls and Openings.*—Girders or beams should be constructed to carry walls and other concentrated loads which are in excess of the working capacity of the slab. Beams should also be provided in case openings in the floor reduce the working strength of the slab below the required carrying capacity.

(*q*) *Unusual Panels.*—The coefficients, apportionments, and thicknesses recommended are for slabs which have several rows of panels in each direction, and in which the size of the panels is approximately the same. For structures having a width of one, two, or three panels, and also for slabs having panels of markedly different sizes, an analysis should be made of the moments developed in both slab and columns, and the values given herein modified accordingly. Slabs with paneled ceiling or with depressed paneling in the floor are to be considered as coming under the recommendations herein given.

(*r*) *Bending Moments in Columns.*—Provision should be made in both wall columns and interior columns for the bending moment which will be developed by unequally loaded panels, eccentric loading, or uneven spacing of columns. The amount of moment to be taken by a column will depend upon the relative stiffness of columns and slab, and computations may be made by rational methods, such as the principle of least work, or of slope and deflection. Generally, the larger part of the unequalized negative moment will be transmitted to the columns, and the column should be designed to resist this bending moment. Especial attention should be given to wall columns and corner columns.

STANDARD BUILDING REGULATIONS FOR THE USE OF REINFORCED CONCRETE, AMERICAN CONCRETE INSTITUTE, 1917, PART PERTAINING TO FLAT-SLAB FLOORS

Continuous flat-slab floors, reinforced with steel rods or mesh and supported on spaced columns in orderly arrangement, shall conform to the following requirements:

(a) *Notation and Nomenclature.*—In the formulas let:

w = total dead and live load in pounds per square foot of floors.
l_1 = span in feet center to center of columns parallel to sections on which moments are considered.
l_2 = span in feet center to center of columns perpendicular to sections at which moments are considered.
c = average diameter of column capital in feet at point where its thickness is 1½ in.
q = distance from center line of the capital to the center of gravity of the periphery of the half capital divided by $\frac{1}{2}c$. For round capitals q may be considered as two-thirds and for square capitals as three-quarters.
t = total slab thickness in inches.
L = average span in feet center to center of columns, but not less than 0.9 of the greater span.

The column-head section, mid-section, outer section, and inner section, are located and dimensioned as shown in Fig. 3. Corresponding moments shall be figured on similar sections at right angles to those shown in Fig. 3.

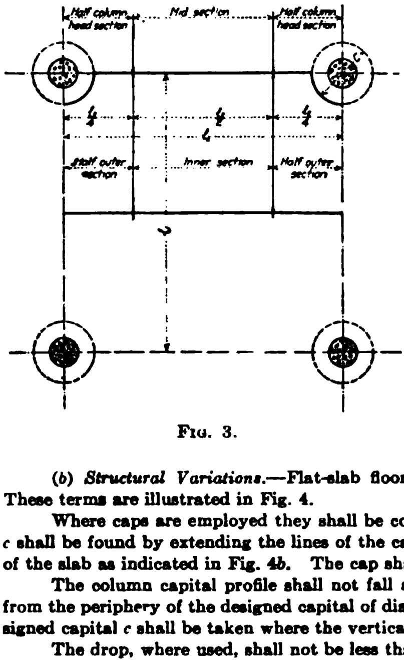


FIG. 3.

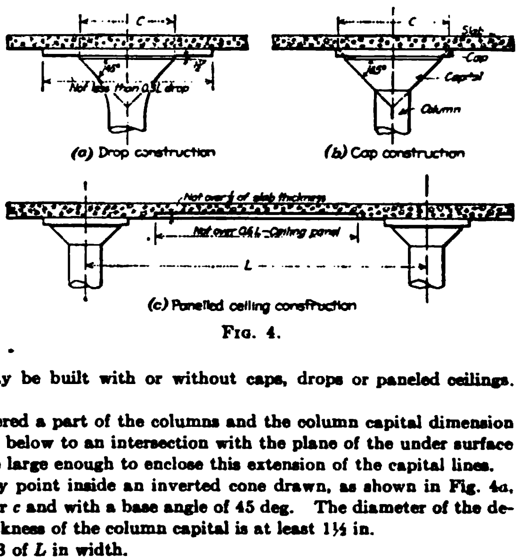


FIG. 4.

(b) *Structural Variations.*—Flat-slab floors may be built with or without caps, drops or paneled ceilings. These terms are illustrated in Fig. 4.

Where caps are employed they shall be considered a part of the columns and the column capital dimension c shall be found by extending the lines of the capital below to an intersection with the plane of the under surface of the slab as indicated in Fig. 4b. The cap shall be large enough to enclose this extension of the capital lines.

The column capital profile shall not fall at any point inside an inverted cone drawn, as shown in Fig. 4a, from the periphery of the designed capital of diameter c and with a base angle of 45 deg. The diameter of the designed capital c shall be taken where the vertical thickness of the column capital is at least 1½ in.

The drop, where used, shall not be less than 0.3 of L in width.

Where paneled ceilings are used the paneling shall not exceed one-third of the slab thickness in depth and the dimension of the paneling shall not exceed 0.6 of the paneled dimension (see Fig. 4c).

(c) *Slab Thickness.*—The slab thickness shall not be less than $t = 0.02L\sqrt{w} + 1$ in.

In no case shall the slab thickness be less that $\frac{1}{32}L$ for the floor slabs nor less than $\frac{1}{40}L$ for the roof slabs.

(d) *Design Moments.*—The numerical sum of the positive and negative moments in foot-pounds shall not be less than $0.09\, wl_2\, (l_1 - qc)^2$. Of this total amount not less than 40% shall be resisted in the column-head sections. Where a drop is used not less than 50% shall be resisted in the column-head sections.

Of the total amount not less than 10% shall be resisted in the mid-section.

Of the total amount not less than 18% shall be resisted in the outer sections.

Of the total amount not less than 12% shall be resisted on the inner sections.

(e) *Exterior Panels.*—The negative moments at the first interior row of columns and the positive moments at the center of the exterior panel on sections parallel to the wall, shall be increased 20% over those specified above for interior panels. If girders are not provided along the column line, the reinforcement parallel to the wall for negative moment in the column-head section and for positive moment in the outer section adjacent to the wall, shall be altered in accordance with the change in the value of c. The negative moment on sections at the wall and parallel thereto should be determined by the conditions of restraint, but must never be taken less than 50% of those for the interior panels.

(*f*) *Reinforcement.*—In the calculation of moments all the reinforcing bars which cross the section under consideration and which fulfil the requirements given under "Arrangement of Reinforcement" may be used. For a column-head section reinforcing bars parallel to the straight portion of the section do not contribute to the negative resisting moment for the column-head section in question. The sectional area of bars, crossing the section at an angle multiplied by the sine of the angle between those bars and the straight portion of the section under consideration may be taken to act as reinforcement in a rectangular direction.

(*g*) *Point of Inflection.*—For the purpose of making calculations of moment at sections away from the sections of negative moment and positive moment already specified, the point of inflection shall be taken at a distance from center line of columns equal to $\frac{1}{5}(l_1 - qc) + \frac{1}{2}qc$. This becomes $\frac{1}{5}(l_1 + c)$ where capital is circular. For slabs having drop panels the coefficient of $\frac{1}{4}$ should be used instead of $\frac{1}{5}$.

(*h*) *Arrangement of Reinforcement.*—The design should include adequate provision for securing the reinforcement in place so as to take not only the maximum moments but the moments of intermediate sections. If bars are extended beyond the column capital and are used to take the bending moment on the opposite side of the column, they must extend to the point of inflection. Bars in diagonal bands used as reinforcement for negative moment should extend on each side of the line drawn through the column center at right angles to the direction of the band a distance equal to 0.35 of the panel length, and bars in the diagonal bands used as reinforcement for positive moment, should extend on each side of the diagonal through the center of the panel a distance equal to 0.35 of the panel length. Bars spliced by lapping and counted as only one bar in tension shall be lapped not less than 80 diameters if splice is made at point of maximum stress and not more than 50% of the rods shall be so spliced at any point in any single band or in any single region of tensile stress. Continuous bars should not all be bent up at the same point of their length, but the zone in which this bending occurs should extend on each side of the assumed point of inflection.

(*i*) *Tensile and Compressive Stresses.*—The usual method of calculating the tensile and compressive stresses in the concrete and in the reinforcement, based on the assumptions for internal stresses, should be followed. In the case of the drop panel, the section of the slab and drop panel may be considered to act integrally for a width equal to the width of the column-head section. Within the column-head section the allowable compression may be increased by 10% over that prescribed in Sect. 41.[1]

[1] *Section* 41.—Reinforced-concrete structures shall be so designed that the stresses, figured in accordance with these regulations, in pounds per square inch, shall not exceed the following:

Extreme fiber stress in concrete in compression—37½% of the minimum compressive strength given in the table below. Adjacent to the support of continuous members, 41%, provided the member frames into a mass of concrete projecting at least 50% of the least dimension of the member on all sides of the compression area of the member.

Concrete in direct compression—25% of the minimum compressive strength given in the table.

Shearing stress in concrete when main steel is not bent and when steel is not provided to resist diagonal tension—2% of the minimum compressive strength given in the table.

Punching shear in concrete, 7½% of the minimum compressive strength given in the table.

Shearing stress in concrete when steel to assist in resisting diagonal tension is provided—7½% of the minimum compressive strength given in the table providing that sufficient web reinforcement is supplied to carry the stresses in excess of the value allowed for the unreinforced concrete; and providing further, that this web reinforcement extends from top to bottom of beam and is adequately anchored to the horizontal reinforcement. If main reinforcing bars are bent up and anchored, they may be considered as part of the web reinforcement.

Bond stress between concrete and plain reinforcing bars—4% of the compressive strength.

Bond stress between concrete and approved deformed bars—5% of the compressive strength.

Bearing upon a surface of concrete at least twice the loaded area—50% of the compressive strength of the concrete.

Tensile stress in steel—16,000 lb. per sq. in., except that for steel having an elastic limit of at least 50,000 lb., a working stress of 18,000 lb. per sq. in. will be allowed.

TABLE OF STRENGTHS OF DIFFERENT MIXTURES OF CONCRETE
(In Pounds per Square Inch)

Aggregate	1 : 3	1 : 4½	1 : 6	1 : 7½	1 : 9
Granite, trap rock	3,300	2,800	2,200	1,800	1,400
Gravel, hard limestone, hard sandstone and approved slag	3,000	2,500	2,000	1,600	1,300
Soft limestone and sandstone	2,200	1,800	1,500	1,200	1,000
Cinders	800	700	600	500	400

(*j*) *Provision for Diagonal Tension and Shear.*—In calculations for the shearing stress which is to be used as the means of measuring the resistance to diagonal tension stress, it shall be assumed that the total vertical shear on a column-head section constituting a width equal to one-half of the lateral dimension of the panel, for use in determining critical shearing stresses, shall be considered to be one-fourth of the total dead and live load on a panel for a slab of uniform thickness, and to be 0.3 of the sum of the dead and live loads on a panel for a slab with drop panels. The formula for shearing unit stress shall be $v = \frac{0.25W}{bjd}$ for slabs of uniform thickness and $v = \frac{0.30W}{bjd}$ for slabs with drop panels, where W is the sum of the dead and live load on a panel, b is half the lateral dimension of the panel measured from center to center of columns, and jd is the lever arm of the resisting couple at the section.

The calculation for punching shear shall be made on the assumption of a uniform distribution over the section of the slab around the periphery of the column capital and also of a uniform distribution over the section of the slab around the periphery of the drop panel, using in each case an amount of vertical shear greater by 50% than the total vertical shear on the section under consideration.

The values of working stresses should be those recommended for diagonal tension and shear in Sect. 41.

(*k*) *Walls and Openings.*—Girders or beams shall be constructed to carry walls and other concentrated loads which are in excess of the working capacity of the slab. Beams should also be provided in case openings in the floor reduce the working strength of the slab below the required carrying capacity.

(*l*) *Unusual Panels.*—The coefficients, steel distribution, and thicknesses recommended are for slabs which have three or more rows of panels in each direction and in which the sizes of the panels are approximately the same. For structures having a width of one or two panels and also for slabs having panels of markedly different size, an analysis should be made of the moments developed in both slab and columns and the values given herein modified accordingly.

(*m*) *Bending Moments in Columns.*—Provision shall be made in both wall columns and interior columns for the bending moment which will be developed by unequally loaded panels, eccentric loading, or uneven spacing of columns. The amount of moment to be taken by a column will depend on the relative stiffness of columns and slab, and computations may be made by rational methods such as the principle of least work or of slope and deflection. Generally the largest part of the unequalled negative moment will be transmitted to the columns and the columns should be designed, and reinforced where necessary, with these conditions in mind.

The resistance of any column to bending in a direction parallel to l_2 shall not be less than $0.022\, w_1 l_1 (l_2 - q_2)^2$, in which w_1 is the designed live load per square foot. In determining the resistance to be provided in exterior columns in a direction perpendicular to the wall the full dead and live load w shall be used in the above formula in place of w_1. The moment in such exterior column may be reduced by the balancing moment of the weight of the structure which projects beyond the supporting wall-column center line.

Where the column extends through the story above, the resisting moment shall be divided between the upper and the lower columns in proportion to their stiffness. The calculations of combined stresses due to bending and direct load shall not exceed by more than 50% the stresses allowed for direct load.

APPENDIX D

STANDARD NOTATION

(a) *Rectangular Beams.*

f_s = tensile unit stress in steel.
f_c = compressive unit stress in concrete.
E_s = modulus of elasticity of steel.
E_c = modulus of elasticity of concrete.
$n = \frac{E_s}{E_c}$
M = moment of resistance, or bending moment in general.
A_s = steel area.
b = breadth of beam.
d = depth of beam to center of steel.
k = ratio of depth of neutral axis to depth, d.
z = depth below top to resultant of the compressive stresses.
j = ratio of lever arm of resisting couple to depth, d.
$jd = d - z$ = arm of resisting couple.
p = steel ratio $= \frac{A_s}{bd}$

(b) **T**-*beams.*

b = width of flange.
b' = width of stem.
t = thickness of flange.

(c) *Beams Reinforced for Compression.*

A' = area of compressive steel.
p' = steel ratio for compressive steel.
f_s' = compressive unit stress in steel.
C = total compressive stress in concrete.
C' = total compressive stress in steel
d' = depth of center of compressive steel.
z = depth to resultant of C and C'.

(d) *Shear, Bond and Web Reinforcement.*

V = total shear.
V' = total shear producing stress in reinforcement.
v = shearing unit stress.
u = bond stress per unit area of bar.
o = circumference or perimeter of bar.
Σo = sum of the perimeters of all bars.
T = total stress in single reinforcing member.
s = horizontal spacing of reinforcing members.

(e) *Columns.*

A = total net area.
A_s = area of longitudinal steel.
A_c = area of concrete.
P = total safe load.

APPENDIX E

CONCRETE BARGES AND SHIPS

EXTRACT FROM REPORT OF THE JOINT COMMITTEE OF THE AMERICAN CONCRETE INSTITUTE AND PORTLAND CEMENT ASSOCIATION

The idea of building ships or other floating vessels of reinforced concrete is not new. For many years and in several different countries, attempts have been made from time to time to build small boats and barges of reinforced concrete. From the information at hand, apparently some of these attempts have been successful and the vessels thus built have been in use for considerable periods. However, no definite information regarding boats built prior to the war is at hand which would assist your Committee in solving the general problem of the concrete ship.

Since the beginning of the war, however, owing to the loss of the world's tonnage due to submarine sinkings, the attention of many naval architects, concrete engineers and others has been drawn to the question of concrete ships to replace those sunk. The scarcity of steel, wood, and labor, and the length of time necessary to construct ships of steel or wood directs attention generally to the substitution of reinforced concrete. Norway appears to have taken the lead in this work and two different companies are already building ships of concrete. The Porsgrund Cement Works, whose Vice President and General Manager, Jens Hauland, has recently been in this country, has already launched one or more ships of 100-tons cargo capacity and is reported to have under construction a ship of 1000-tons cargo capacity. The general design of these ships follows generally that of a framed steel ship, and your Committee has been informed by Mr. Hauland that the weight compares very favorably with that of a steel ship. No definite information is available, however, by which this statement can be verified.

The Fougner Steel Concrete Shipbuilding Company of Christiania, Norway, has been building vessels since June, 1916. About eighteen in all have been launched up to the present time. Several others are under construction. The "Namsenfjord" about 400 tons displacement launched some time ago has made a round trip between Norway and England. No detailed information is available at the present time regarding these vessels that would throw light on the general problem.

On this side of the Atlantic, at least two ships are under construction. At San Francisco, a large ship about 5000-tons capacity is being built and, from information at hand, will be shortly launched. Your Committee learns that this ship is 320 ft. long, 44 ft. 6 in. beam and 30 ft. 0 in. deep. At 24-ft. draft the displacement is said to be 8000 tons. The weight of the hull is said to be 2200 tons. At Montreal, Canada, a small concrete ship which will have about 300 tons carrying capacity has already been launched and is now being equipped. This vessel is being constructed by the Atlas Construction Co., Ltd., of which Chas. M. Morssen is President. Your Committee is indebted to Mr. Morssen for considerable data relating to this ship and it takes this opportunity of acknowledging the very courteous treatment shown its representatives by Mr. Morssen and the freedom with which he discussed the details of the design and construction. This ship is 126 ft. 0 in. long, between perpendiculars, 22 ft. 6 in. beam, with a depth of 12 ft. 6 in. The displacement is about 650 tons. The ship will be self-propelled and is now being equipped with boilers and engines. She will shortly make her first trip. No estimate of cost is at present available.

With the exception of the two ships noted above, little information has been gained from the ships now under construction which will assist in the solution of the concrete ship problem.

The present report will confine itself to a general statement of the several elements which make up the concrete ship problem and a discussion of the information obtained from the tentative design of a concrete ship.

In order to make any advance toward the solution of the concrete ship problem, information must be obtained concerning several points regarding which only meager information is now available. These points taken together constitute the concrete ship problem.

1. The Relation Between Carrying Capacity and Displacement.—The displacement of a ship is the weight of the water she displaces, and is therefore, the sum of the weight of the ship itself and its cargo capacity expressed in tons. The cargo capacity will hereafter be spoken of as the "dead weight"—following the usual practice with naval architects. Wherever displacement, dead weight or weight of ship is spoken of, it will be in terms of long tons (2240 lb.) which is the usual practice.

It is apparent that the efficiency of a ship as a cargo carrier depends upon the relationship between dead weight and displacement. Expressed in terms of %, in the average cargo ship built of steel, the dead weight is from 70 to 75% of the displacement—taking into account as weight of ship all spars, fittings, deck houses, anchors and chains, auxiliary engines and tanks, but not boilers, engines or coal. In a wooden ship, the dead weight is from 60 to 65%

of the displacement. It is quite evident that from the difference in weight of materials, it will be difficult to design a ship of concrete that will give a relationship between dead weight and displacement approaching that of steel. However, if ships are to be built of concrete for commercial use, the weight of the ship must be such as to provide a reasonable dead weight or cargo capacity for the displacement.

2. Transverse Strength.—The stresses in the transverse members of a ship are, in still water, functions of the draft and the stiffness, and may be computed by mathematical processes, although the computations are long and laborious. When the material is reinforced concrete, the problem becomes much more complicated. Experience has shown, however, that numerous elements other than draft effect the transverse strength of a ship, such as the effect of rolling in a sea way, impact with docks or other ships and stresses incident to going into dry-dock. The transverse members of cargo ships of today are, therefore, not designed to withstand computed stresses, but are designed in accordance with various rules which embody the result of long experience in the construction and use of ships. It should be noted in this connection that granting of insurance depends on compliance with these rules.

Steel ships are of two different types (*a*) framed ships in which transverse ribs or frames are spaced from 18 to 24 in. on centers, the plating being rivetted to these ribs without intermediate longitudinal members, excepting in the bottom, and (*b*) longitudinally framed ships (Isherwood) in which heavy frames are spaced from 10 to 15 ft. on centers with intermediate longitudinals to which the plating is rivetted.

From a comparison with the ordinary steel ship design, it would appear to be not difficult to design transverse members of reinforced concrete of equivalent strength to steel members—the question of strength only being considered.

3. Longitudinal Strength.—A ship must be able to meet conditions which are unlike any to which land structures are subject.

In determining the longitudinal strength of a ship, it is customary to assume two conditions. Under the first condition, the ship is assumed to be suspended between two wave crests, the length between crests being equal to the length of the ship between perpendiculars, the height of the wave being equal to one-twentieth of that length. In this case, the ship as a whole, is acting as a simple beam supported at the ends. This condition is termed "sagging." Under the second condition the ship is assumed to be supported amidships on one crest of the same wave. Under this condition, the ship as a whole acts as a cantilever. This condition is termed "hogging." It is apparent therefore, that when a ship is riding the waves both the deck and the bottom of the ship will be required to withstand tensile and compressive stresses alternately—the maximum tensile stress following the maximum compressive stress at very short intervals. In a steel ship the entire cross-sectional area of the midship section acts to resist these stresses, taking into account, in determining the moment of inertia, all of the continuous members such as continuous scantlings and deck, side and bottom plates. In the concrete ship, equivalent strength must be provided. In the case of the concrete ship, however, only the steel reinforcement can be relied upon to take tensile stresses. The concrete, assisted by the steel, will take the compressive stresses.

The effect of the rapid change of the character of the stress in either the deck or the bottom is much more serious in the case of concrete ship than in the steel ship for the reason that owing to the low tensile stress of concrete, cracks will occur at the extreme fiber under relatively low tensile stresses in the steel. These cracks, if any, alternately opening and closing, may tend to cause disintegration, with resulting leaks or impairment of the reinforcement.

At the present time, little information is available as to the effect of such reversal of stress, and but little can be hoped for until an actual trial has been made of a concrete ship in a sea.

4. Elasticity.—There is an almost unanimous opinion among naval architects and seafaring men generally that a concrete ship will be so inelastic that she will tear herself to pieces in a sea. While it is doubtless true that in a concrete ship there will not be the same readjustment of stresses as in a steel ship when subject to the action of a heavy sea, experience with reinforced concrete structures generally has shown that such structures have considerable elasticity and there is ample reason for the hope that reinforced concrete will prove a suitable material for ship building purposes.

5. Effects of Sea Water on Concrete and Reinforcing Steel.—Until very recently little information has been available as to the effect of sea water on concrete. The recent work of the Bureau of Standards, acting in cooperation with the Portland Cement Association, has thrown considerable light on what may be expected from the action of sea water. The result of their investigation tends to show that inferior concrete or concrete of which the surface skin has been impaired suffers serious disintegration when in contact with sea water. Inasmuch as in most instances the structures examined, which form the basis of the report of the Bureau of Standards, were built without thought as to the action of sea water (it being assumed that there would be no deleterious action) there is every reason to hope that where the effect of sea water is appreciated, and where steps are taken in the way of selected materials and adequate workmanship, assuring a good mix and a satisfactory surface skin, the concrete will not so deteriorate.

With regard to the effect of sea water on the reinforcing steel, there is some question as to whether a thin layer of concrete can be relied upon to protect the steel from corrosion. To provide a thick protective layer of concrete outside of the reinforcing steel is practically out of the question, owing to the large increase in weight. If the reinforcement, therefore, is to be maintained in perfect condition, the steel must be protected by galvanisation and by increasing the efficiency of the protective concrete by means of additional care in materials and workmanship and by a surface coating of a waterproofing character.

6. Relative Cost.—Just at the present time, the demand for tonnage is so great that any ship of reasonable capacity that can be used for carrying cargo will prove financially successful. The relative costs of ships of concrete and steel, or concrete and wood, is not therefore as important a consideration as it will be after the war when conditions again approach the normal. However, it is necessary to have an adequate idea of the probable cost of a concrete ship as well as a comparison with the cost of steel and wooden ships.

7. Speed of Construction.—Speed of construction is of vital importance in the ship building program today owing to the immediate requirements for tonnage. If it shall be found that the concrete ship can be constructed in much less time than a steel or wood ship, this fact will contribute largely to the success of the concrete ship.

Although there are some questions regarding the concrete ship which can only be answered by actual experiment, the studies which your Committee has made point to the commercial success of the concrete ship.

Your Committee suggests that specifications for a concrete vessel should embody the following principles:

(*a*) Both cement and aggregates should be selected with great care to insure a concrete of maximum efficiency.

(*b*) The concrete should be placed in one continuous operation to insure monolithic construction. The concrete mixture should be such as will develop a crushing strength in excess of 3000 lb. per sq. in. when tested in standard cylinders at 28 days. A concrete consisting of one part Portland cement, one part sand and two parts ½-in. aggregate may be expected to give such a concrete. The mixture and workmanship in placing must be such as will assure impermeability.

(*c*) The reinforcing steel should be in the form of deformed bars and should be galvanized.

(*d*) In parts of the vessel where cracks in the concrete would tend to cause leaks, the stress in the steel should be kept low (preferable less than 12,000 lb.).

e) Some form of elastic waterproofing coating should be applied to the hull below the deck.

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