THE SLIDE RULE

by

LEE H. JOHNSON
Dean of Engineering
University of Mississippi

D. VAN NOSTRAND COMPANY, INC.
TORONTO NEW YORK LONDON

NEW YORK
D. Van Nostrand Company, Inc., 250 Fourth Avenue, New York 3
TORONTO
D. Van Nostrand Company (Canada), Ltd., 228 Bloor Street, Toronto
LONDON
Macmillan & Company, Ltd., St. Martin's Street, London, W.C. 2

Published simultaneously in Canada by
D. VAN NOSTRAND COMPANY (Canada), LTD.

PRINTED IN THE UNITED STATES OF AMERICA

To
Eulalie

PREFACE

The engineer's slide rule is essential in the training of all students of engineering and in many phases of professional engineering and scientific work. As an efficient tool for calculation, it saves many tedious hours which would otherwise be spent in routine multiplications, divisions, and other computations. The modern Duplex slide rule with its characteristic arrangement of scales on both sides of the frame is the most versatile and efficient of the slide rules which have been devised. This book presents a technique of operation for the Duplex slide rule which makes full use of its versatility and efficiency.

The technique of operation presented is new. It is based on a new approach to the study of the slide rule and on new concepts of slide rule operation. The presentation has been carefully planned and developed. It is, therefore, highly important that the student study the subject matter in the order in which it is presented.

A chapter on the simpler type of slide rule used by engineers, the Mannheim type, is also included. Chapter I has been written to apply both to this type and to the Duplex slide rule.

The author is indebted to Keuffel and Esser Company for providing the photographic illustrations throughout the book. He also acknowledges the following trade names of Keuffel and Esser Company which are used in the text: *Log Log Duplex Trig*, *Log Log Duplex Decitrig*, *Polyphase Duplex Trig*, *Polyphase Duplex Decitrig*, and *Polyphase* slide rules. He wishes especially to thank his secretary, Mrs. Christine Reid, for valuable assistance in preparation of the manuscript; and Mr. R. F. Bowen, a student in the School of Engineering at the University of Mississippi, for a suggestion which enabled the author to improve the technique for mixed operations.

LEE H. JOHNSON

CONTENTS

I: Introduction

II: Multiplication and Division

III: Squares, Square Roots, Cubes, and Cube Roots

THE SLIDE RULE

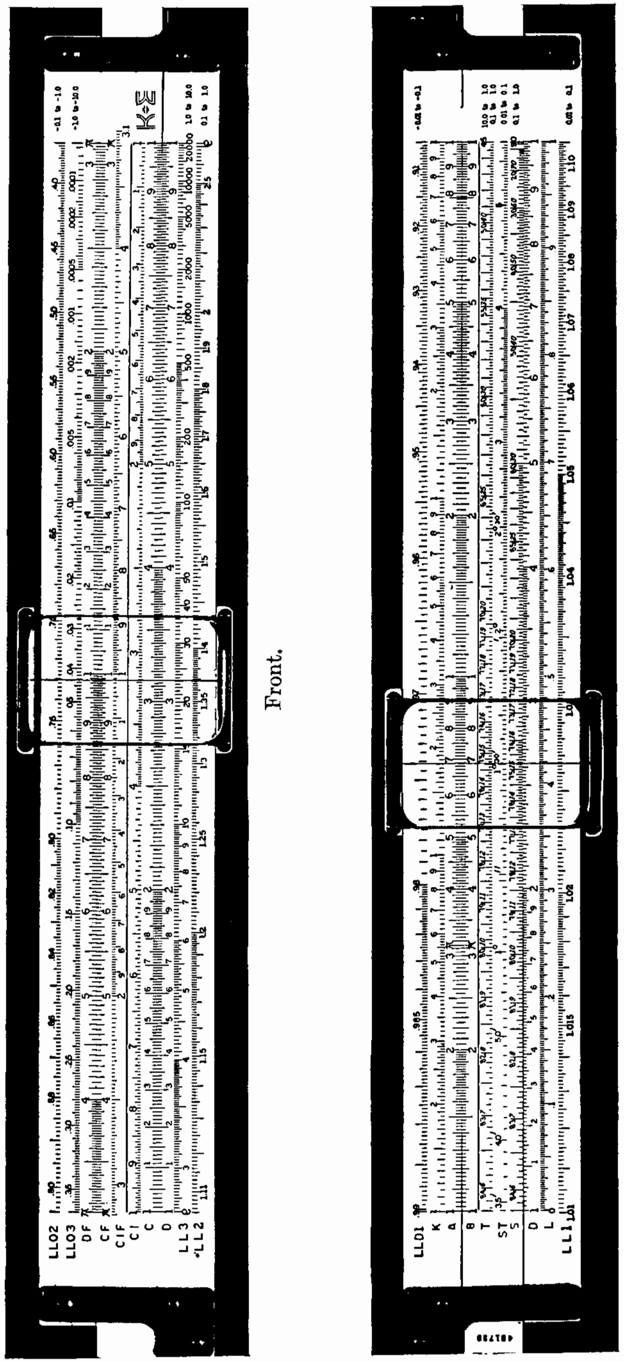
I

INTRODUCTION

1. Order of Study. This book presents a particular technique of slide rule operation in a particular manner. This technique applies to the duplex type of slide rule used by engineers and might be termed the "center-drift method." To learn this technique most effectively, Chapters I through VI must be studied in proper order without skipping. Techniques for other commonly used types are given in Chapter VII.

2. Function of the Slide Rule. The engineer's slide rule is used primarily to perform certain numerical calculations, such as multiplication, division, raising to powers, extracting roots, and various combinations of these operations, and to perform them rapidly. Addition and subtraction cannot be performed on the engineer's slide rule, nor can any operations involving them be done wholly upon it.

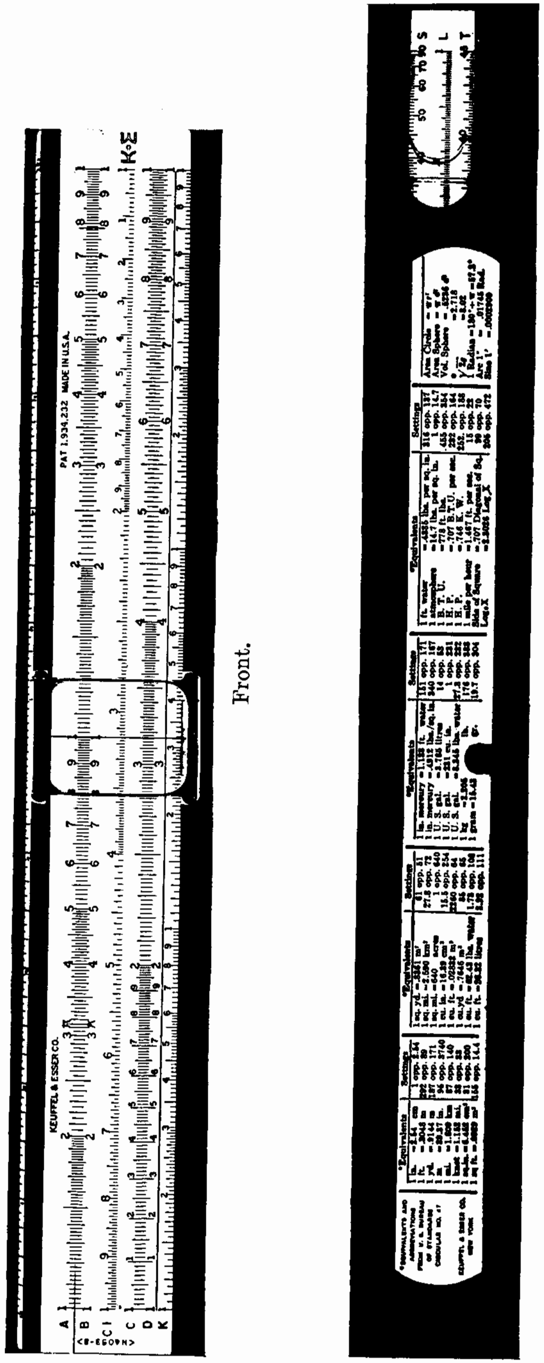
3. Description. The slide rule commonly used by engineering students consists of three parts: a fixed part called the "frame" or stock and two movable parts, the "slide" and the "runner" which is also called the indicator and the cursor. The words *frame*, *slide*, and *runner* will be used in this book. The frame and slide are marked in scales. The runner has a central line on it which is called the "hairline" and which is used to make settings on the scales. In the simpler rules made for beginners neither the frame nor the runner is adjustable, but in the better rules both can be adjusted to correct improper alignments. There are two types of slide rules commonly used by engineering students, the Mannheim type and the type exemplified by the Keuffel & Essex Co. Duplex series of rules. The Duplex rule has scales on both sides of the frame, and the Mannheim type has scales only on one side. There are many other types of slide rules, but this book will be confined to the study of the types just mentioned with particular emphasis on the Duplex rule, which



Front.

Back.

Fig. 1. Log Log Duplex Trig Slide Rule. (K. & E. Reg. U. S. Patent Off.)



Front.

Back.

Fig. 2. Polyphase Slide Rule. (K. & E. Reg. U. S. Patent Off.)

includes the Polyphase Duplex Trig, Polyphase Duplex Decitrig, Log Log Duplex Trig, and Log Log Duplex Decitrig slide rules. The Log Log Duplex Trig slide rule is illustrated in Fig. 1. The Polyphase slide rule, one of the rules of the Mannheim type, is illustrated in Fig. 2, which shows the chart of equivalents, usually placed on the back of the frame, and also the scales on the back of the slide. The Mannheim slide rule is similar to the Polyphase in construction but does not have the CI scale or the K scale. The Mannheim slide rule is the forerunner of the present engineer's slide rules and is named for its inventor, Amédée Mannheim, a French artillery officer who devised it in 1859.

Mannheim and Duplex slide rules are currently manufactured in three sizes which are designated according to the approximate lengths of their scales, as 5″, 10″, and 20″ sizes. Not all styles are manufactured in all three sizes. The most popular slide rule appears to be the 10″ rule.

The principal use of the slide rule by students of engineering is to multiply and divide numbers. The scales used in these operations on the Duplex slide rules are the D, DF, C, CF, CI, and CIF scales. The A and B scales are provided for squares and square roots, and the K scale for cubes and cube roots. The S, T, and ST scales are trigonometric scales providing values of the sine, cosine, tangent, and cotangent of an angle. The L scale is a scale of logarithms of numbers. On the Log Log Duplex Trig and Decitrig slide rules there are also the log log, or LL, scales which are used in calculations involving fractional powers or roots of numbers, such as, $(2.1)^{3.17}$, or $\sqrt[2.81]{51.6}$.

4. Reading the Scales. Before the student undertakes to perform multiplication, division, and other operations on the slide rule, it is necessary that he learn to read the scales correctly and rapidly. Since the 10″ slide rule is the most widely used, this section will discuss the reading of the scales only on that rule. The scales on the 5″ and 20″ rules are not subdivided in exactly the same manner as are those on the 10″ rule, but can be read without difficulty after one has learned to read the scales on the 10″ rule.

The scales which will be used most frequently are the D, DF, C, CF, CI, and CIF scales. All of them are subdivided in the same manner, the only difference between these scales being that the D, DF, C, and CF scales read from left to right, whereas the CI

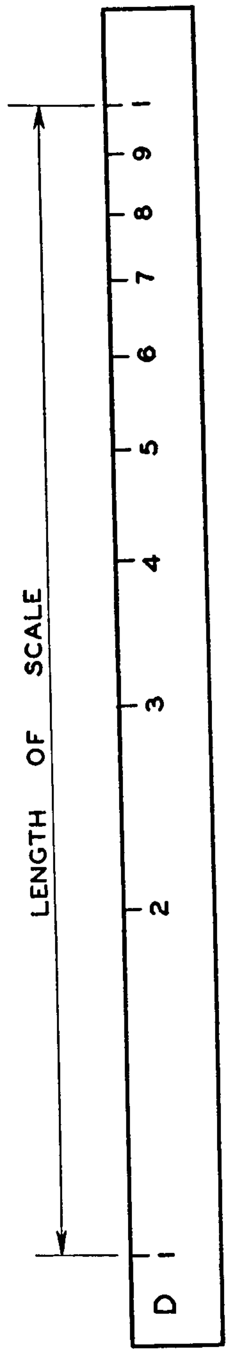


Fig. 3.

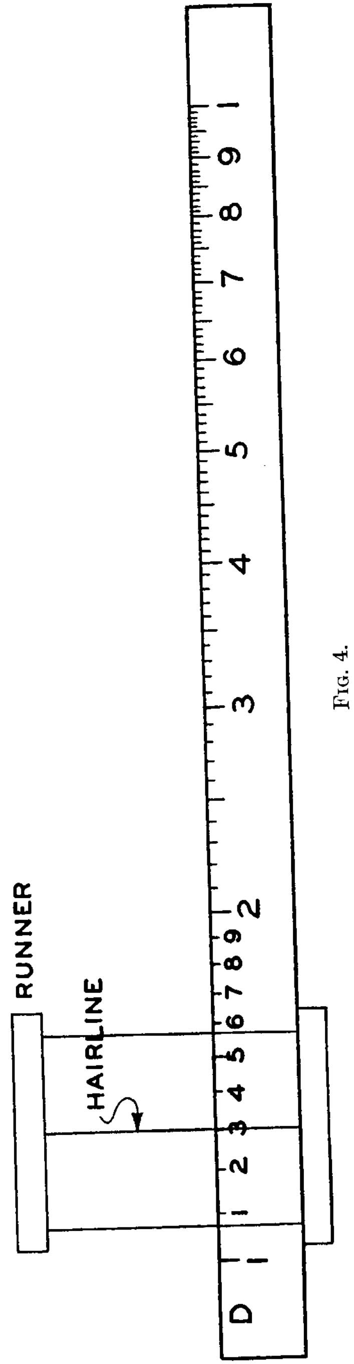


Fig. 4.

and CIF scales read in the reverse direction from right to left. Also, the D, DF, C, and CF scales are black, but the CI and CIF are red. These scales are divided into nine major divisions which are numbered as shown on the scale in Fig. 3 and which correspond to the nine integers, 1, 2, 3, 4, 5, 6, 7, 8, 9. Since all six scales are subdivided in exactly the same manner, only the divisions of the D scale will be used for purposes of illustration. The size of these major divisions diminishes from 1 to 9 because the scale is logarithmic, as will be explained in the next section on the fundamental principle of the slide rule. It is important here that the student learn that in multiplication or division *numbers are read or set on the scales without regard for decimal point.* The divisions as shown in Fig. 3 can represent numbers such as 2, 30, 500, 9000, or 0.007. The division, 2, can represent 20, 200, 2000, 0.02, or 0.0002.

Each major division of the D scale is subdivided into ten minor divisions which are not numbered themselves except between the major divisions 1 and 2, as shown in Fig. 4. Numbers such as 15, 33, and 92 are represented exactly by these minor divisions, and it is again emphasized that the hairline setting on 13 in Fig. 4 can represent 13, 1300, 1.30, or 0.0013. In other words, the only consideration in setting numbers on the scales is the *succession of integers* in the number. The decimal point has no significance here.

The scale is subdivided still further but in a different manner for different parts of the scale. Between 1 and 2 each of the ten minor divisions is again subdivided into ten parts, so that numbers such as 157, 183, 12.8, and 0.00167 are each represented by marks on the scale. The hairline in Fig. 5 is set on 143 for example. Between 2 and 4, however, each of the ten minor divisions is subdivided into only five parts, the reason being that the division lines would be crowded too closely together if it were attempted to subdivide into ten parts as between 1 and 2. In this portion of the scale, numbers such as 242, 37.6, and 288 are represented by subdivision lines, but numbers such as 237 and 345, ending in odd integers, have to be estimated by eye by placing the hairline midway between two division lines. For example, the hairline is set on 237 in Fig. 6. Only numbers ending in even integers are represented by marks on this portion of the scale. If subdivision into five parts were continued for the remainder of the scale from 4 on through 9, the division lines would again crowd too closely together, and so for this portion

of the scale the minor divisions are subdivided into only two parts. Here only numbers ending in five, such as 465, 735, and 0.00925, are represented by the subdivisions. Numbers such as 467 and 73.2

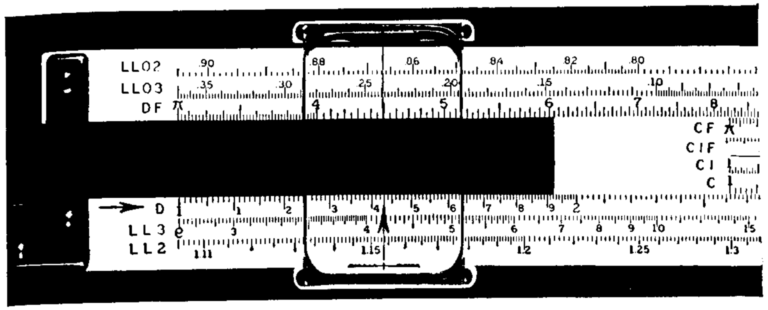


Fig. 5.

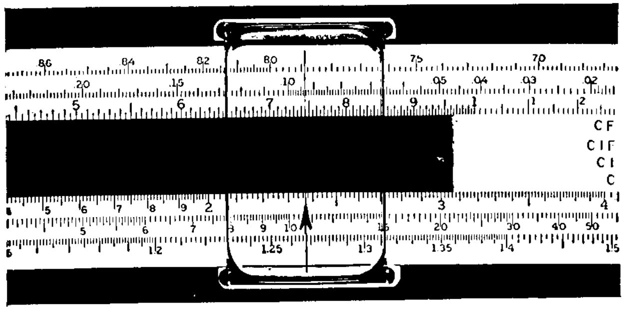


Fig. 6.

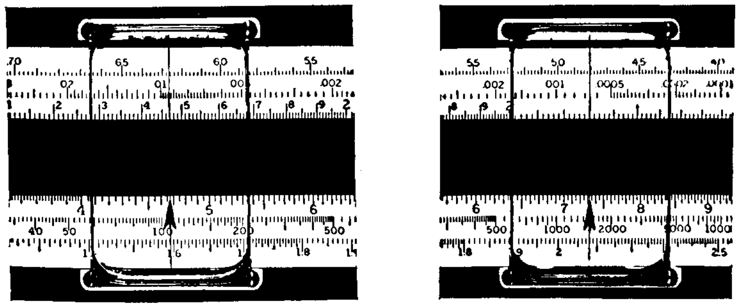
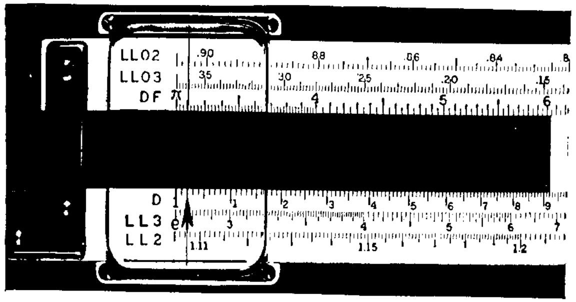


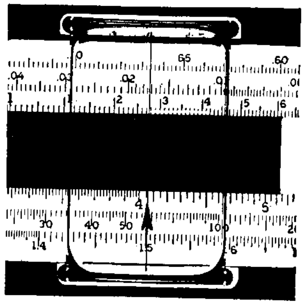
Fig. 7.

are set on the scale by estimating the correct position of the hairline as shown in Fig. 7.

It should be noted that the zeros which are used to indicate the position of the decimal point in numbers less than 1 are not significant in setting numbers on the scale, but that zeros which *follow* integers are significant. For instance, such numbers as 102 and 40.6 are shown by hairline settings in Fig. 8, and the significance of the zero in each number is apparent.



102.



40.6.

Fig. 8.

The A and B scales each consists of two complete scales extending from 1 through 9 and placed end to end. The space occupied by the nine major divisions on each scale is only half as long as on the D scale, consequently it is not subdivided as closely as on the D scale. The major divisions are subdivided into ten minor divisions each, but these in turn are not subdivided as closely as the minor divisions on the D scale. Between 1 and 2 the minor divisions are subdivided into five parts and between 2 and 5 into two parts. They are not subdivided at all from 5 through 9.

The K scale is composed of three scales end to end, each extending from 1 through 9, and there is correspondingly less space for sub-

division than on any of the scales previously mentioned. Between 1 and 4 the minor divisions are subdivided into two parts, and from 4 through 9 there is no further subdivision.

The trigonometric scales, S, T, and ST, and the log log scales, LL scales, will be discussed in later chapters. The L scale, a scale of logarithms, is subdivided uniformly.

5. Fundamental Principle of the Slide Rule. The fundamental principle of the slide rule is easily grasped if the reader will first forget the slide rule and then read carefully and without haste the following discussion. It is presumed that the reader is familiar with logarithms.

A simple scale such as a scale of inches and tenths on an engineer's scale is shown in Fig. 9*a*. Using this scale as a reference, plot the logarithms of numbers. The following logarithms are chosen as illustrations and are plotted in Fig. 9*b*.

log 1 = 0.000	log 10 = 1.000	log 100 = 2.000
log 2 = 0.301	log 20 = 1.301	log 200 = 2.301
log 3 = 0.477	log 30 = 1.477	log 300 = 2.477
log 4 = 0.602	log 40 = 1.602	log 400 = 2.602
		log 1000 = 3.000

Fig. 9*c* shows the numbers plotted in the position of their logarithms, and the scale thus formed is called a logarithmic scale. The scale shown is, of course, incomplete, not containing division marks for 5, 6, 7, 8, 9, and other numbers. On this scale the relative positions of 1, 2, 3, 4, and 10 are exactly the same as those for 10, 20, 30, 40, and 100, and similarly for 100, 200, 300, 400, 1000, i.e., the subdivision of one unit or cycle of the scale is the same as that of any other cycle. By "cycle" is meant the portion of the scale between any two successive integral powers of 10. The cycles continue indefinitely, to the right for increasing powers of 10 and to the left for decreasing powers of 10, as shown in Fig. 10.

If the decimal point is neglected, one cycle is sufficient to represent all numbers. The D, CI, and C scales of the engineer's slide rule are examples of this one cycle. These scales are shown in Figs. 1 and 2.

Turning now to the slide rule, how are these one-cycle scales used for multiplication and division? Any multiplication or division can

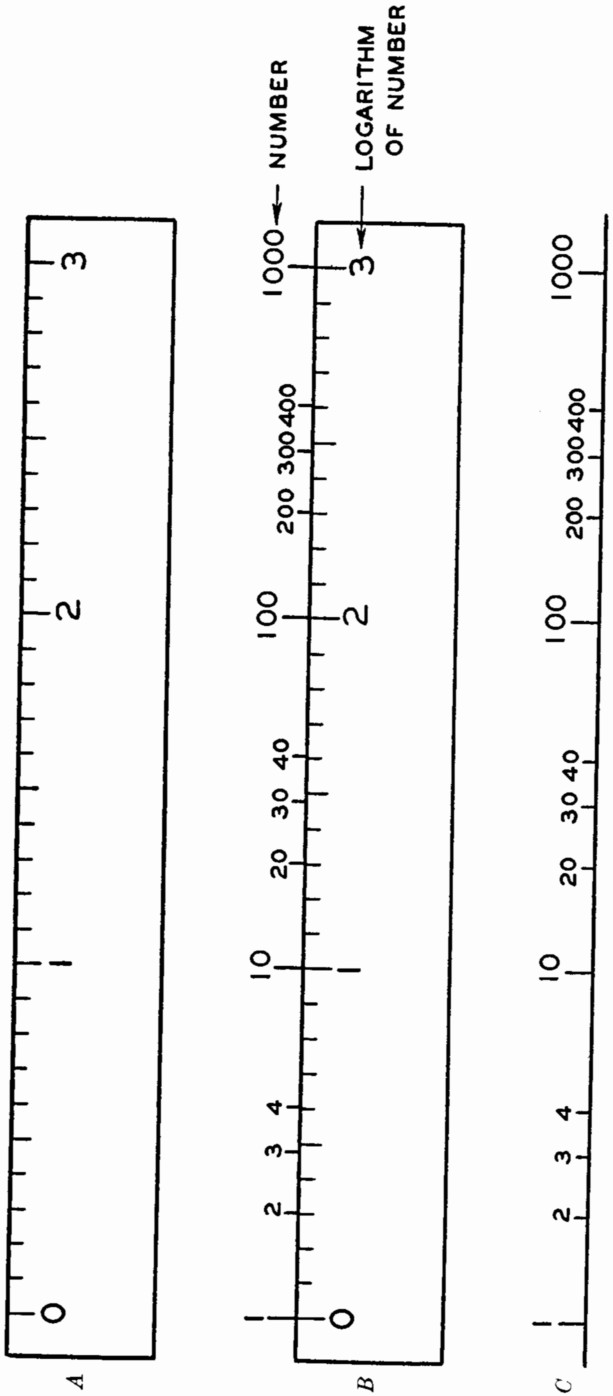


Fig. 9.

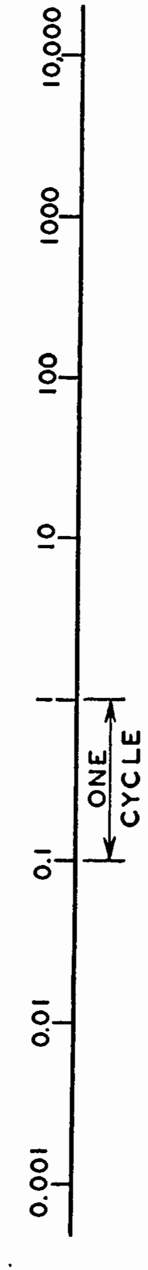


Fig. 10.

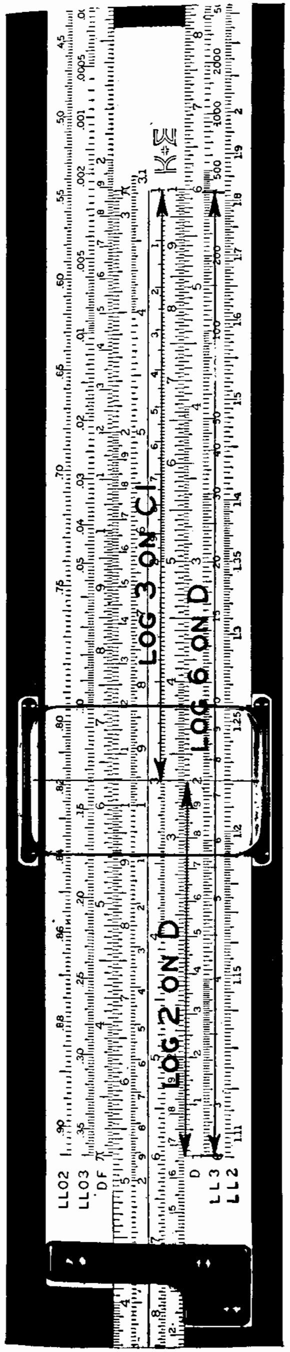


Fig. 11.

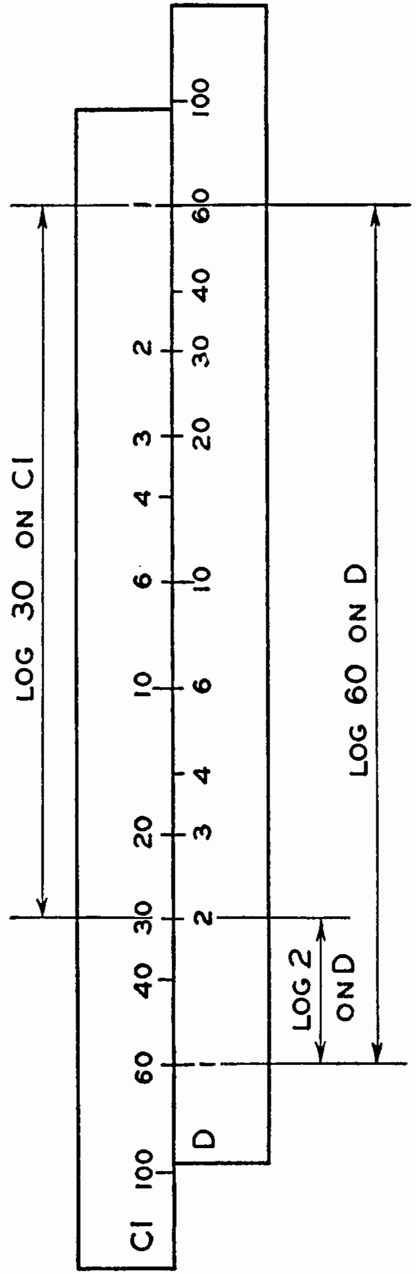


Fig. 12.

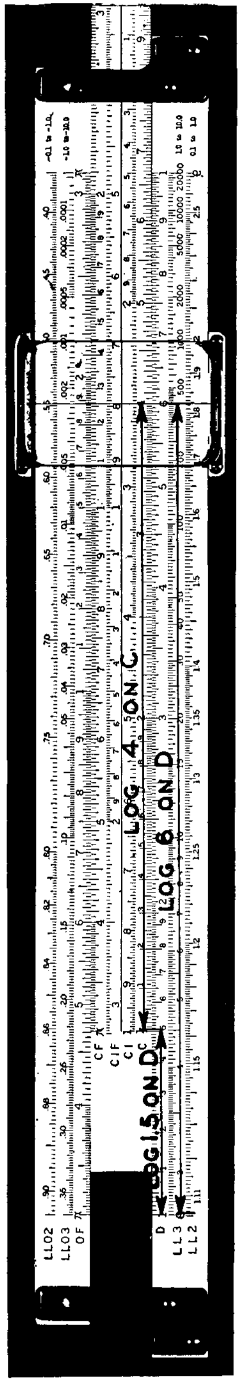


Fig. 13.

be done with logarithms. For example, the product of 2×3 is obtained with logarithms as follows:

$$\log (2 \times 3) = \log 2 + \log 3$$
$$= \log (\text{product}) = \log 6$$

Using the slide rule with its one-cycle scales, the sum of log 2 and log 3 is obtained in this instance by using the D and CI scales as shown in Fig. 11. These two logarithms could also be added using the D and C scales but with less efficiency, as will be shown in Chapter II.

The above operation could also represent $2 \times 30 = 60$, $20 \times 30 = 600$, or $0.2 \times 3 = 0.6$ since the scales are typical cycles representing all numbers. The decimal points are set by inspection. To illustrate this clearly, suppose the student did not wish to bother with setting the decimal point and had an imaginary two-cycle slide rule for numbers from 1 to 100. To multiply 2×30, he would add the respective logarithms as shown in Fig. 12. This would be fine for products less than 100 and greater than 1, but useless for all others. To use more cycles and extend the range would result either in a slide rule much too long to be practical or in cycles so small that the numbers could not be read with sufficient accuracy. Accordingly, such slide rules are seldom made and then only for special use in some routine procedure.

The division of one factor by another involves the difference of logarithms as seen in the solution of $\frac{6}{4}$:

$$\log \left(\tfrac{6}{4}\right) = \log 6 - \log 4$$
$$= \log (\text{quotient}) = \log 1.5$$

The difference of the logarithms in this instance is obtained by using the D and C scales as shown in Fig. 13. The D and C scales are more efficient for such division than the D and CI scales, whereas the reverse is true for multiplication.

6. Accuracy of the Slide Rule. The term "significant figures" must be defined and discussed before the student will understand the limitations in accuracy of the slide rule. Most problems in engineering involve measurements, and these measurements are not exact. "Significant figures" are the digits in these measurements which have been determined. A distance, say, is measured roughly to the nearest foot to be 24 feet. This number, or measurement, contains two significant figures which means that the distance is closer to 24 feet than 25 feet or 23 feet. A more accurate measurement to the nearest tenth of a foot may yield 24.3 feet, a measurement to three significant figures. A still more accurate measurement to four significant figures, or in this case to the nearest hundredth of a foot, may yield 24.28 feet. It is not possible to measure the distance exactly because there will always be an undetermined quantity too small to be measured. For example, if it should be possible to measure the above distance to the nearest millionth of a foot as 24.284379 feet, a number of eight significant figures, the decimals following the last integer are still undetermined. It is obviously impossible to measure any physical quantity exactly as this implies measurement to an infinite number of decimal places.

Zeros are significant figures in measurements unless they are used only to locate the decimal point.

The zeros are significant in the following numbers, each of which contains four significant figures: 204.6, 20.02, 21.30, 300.0. The zero in each place indicates that the measurement is closer to a zero division mark than it is to the adjoining marks for 9 or 1. For example, in 21.30, the zero indicates that the measurement is closer to 21.30 than to 21.29 or 21.31. In the case of 300.0, the measurement is closer to that than to 299.9 or 300.1.

The zeros are not significant in the following numbers, each of which contains three significant figures: 0.0124, 0.000467, 0.00217. The reason for this can be easily explained by contrasting two such numbers as 0.0034 which has two significant figures and 1.0034 which has five significant figures. These numbers are stated in units of ten-thousandths, and the first number represents 34 of these units while the second represents 10,034 of them. In fractional form the first number represents 34/10,000 or 34 ten-thousandths, and the second 10,034/10,000 or 10,034 ten-thousandths. Obviously the zeros in the first number are used only to place the decimal point in indicating the magnitude of the unit.

Zeros may or may not be significant in numbers such as 14,300 and 2750. If the numbers are given with no qualifying phrase or description, say as 14,300 feet or 2750 gallons, there is no way of determining whether or not the zeros are significant. If the quantities are specified as "about" or "roughly" 14,300 feet or 2750 gallons, then the zeros are not significant and are used only to point off the location of the decimal point. If the quantities are specified as 14,300 feet or 2750 gallons to the nearest foot or gallon, then the zeros are significant.

As previously stated, the term, "significant figures," usually refers to measured quantities, i.e., to inexact quantities. Integral numbers which are used in mathematics and in counting are exact and therefore are correct to an infinite number of decimal places. For instance, the number, 3, when used as a number in counting is exact and means 3.00000··· to an infinite number of places. Whenever these numbers occur, e.g., 3 columns, 37 cars, or 16 generators, they are exact numbers and the term "significant figures" has no meaning. Although it is possible to specify exact quantities to be measured such as 15 gallons, 25 pounds, or 100 yards, and to solve problems using these as exact quantities, it is not possible actually to make an exact measurement of them. A motorist buying 5 gallons of gasoline rarely if ever receives exactly 5 gallons.

The term "accuracy" as used in this discussion is a relative term. For instance, suppose that in measuring a distance of 100 feet an error of 1 foot is made. The measurement is said to be accurate to one part in a hundred, or 1 in 100. Now if in measuring a distance of 800 feet, an error of 8 feet is made, the accuracy of this measurement is the same as that of the preceding measurement, i.e., an error of 8 in 800 is relatively the same as one of 1 in 100. Accuracy, then, is a relative quantity and may be expressed as the ratio of the error in a measurement to the measurement itself, and also in per cent. In the measurements just discussed, the accuracy is 1 in 100 or 1 per cent.

Turning now to the slide rule, its accuracy depends upon how

accurately numbers can be set and read on the scales. This, in turn, depends upon the error made by the average person in setting with the naked eye. The D scale on a 10″ slide rule is commonly said to be accurate to one part in a thousand, 1 in 1000, or 0.1 per cent. How closely must the scale be read or the hairline be set to get this accuracy? Look at the left end of the D scale, Fig. 14, and let the first mark represent 1000. The next mark on the scale represents

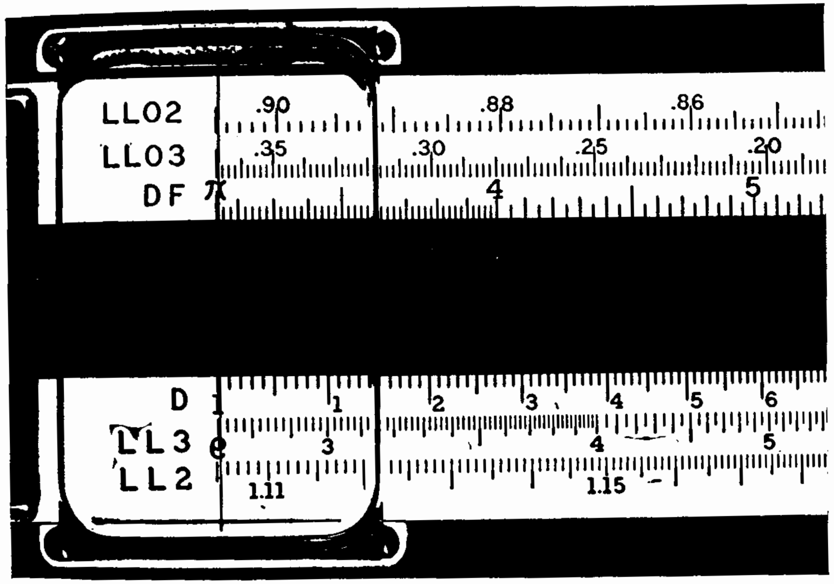


Fig. 14.

accordingly 1010. The interval between these marks corresponds to 10 in a 1000. To read or set to about 1 in 1000, the reading or setting must be made to about one-tenth of this interval. The hairline has been set approximately on 1001 in Fig. 14, or approximately to one-tenth of the interval. The distance from 1000 to the position for 1001 is slightly over four-thousandths of an inch. Accordingly, in stating that the 10″ slide rule is accurate to 1 in 1000, it is assumed that the average person will be able to read or to set the hairline to the nearest four-thousandths of an inch.

The accuracy of 1 in 1000 is constant at all points of the scale. This can be easily demonstrated. Suppose an engineer has lost his glasses and is attempting to use the slide rule. His eyesight is so poor that, in attempting to line up the C and D scales, he sets the first mark of the C scale opposite the second mark of the D scale as shown in Fig. 15. He has set 1000 on the C scale opposite 1010 on the D. His setting is off one entire division and is only accurate to 10 in 1000, or to 1 in 100. Looking at Fig. 15 or, better yet, at a slide rule which has been set as explained, the reader will notice that 2000 on the C scale is opposite 2020 on the D, an error of 2 in 200, or 1 in 100. Likewise 3000 on the C scale is opposite 3030 on the D and 5000 on the C is opposite 5050 on the D, both errors of 1 in 100. This is true at every point along the scale. Now if the engineer with better eyesight sets the C scale only one-tenth of a division in error instead of one division, so that the end of the C scale is opposite 1001, it will be found that 2000 on the C scale is opposite 2002 on the D, 3000 on the C scale opposite 3003 on the D, and so on as in Fig. 16. The error in all cases is 1 in 1000. That this error is constant can be proved mathematically with differential calculus.[1]

It is the author's conviction that attempts to secure greater accuracy with closer settings are not practical or efficient.

Reading or setting the slide rule to an accuracy of about 1 in 1000 means reading or setting to four significant figures in the 1's near the left end of the D scale. It also means reading or setting to only three significant figures in the 9's at the right end of the scale, since 1 in 997 or in 999 is very close to 1 in 1000. In between these extremes, however, reading to an accuracy of 1 in 1000 or to about four-thousandths of an inch does not give either the fourth signifi-

[1] The equation for the D scale on the 10″ slide rule is

$$x = 9.84 \log_{10} N$$

where x = distance from left end in inches
N = number on scale

Differentiating

$$dx = 9.84\, d(\log_{10} N)$$

$$dx = 9.84 \log_{10} e\, d(\log_e N)$$

$$dx = 4.27 \frac{dN}{N}$$

The accuracy, $\frac{dN}{N}$, is constant for an error in setting of dx.

On the D scale of the 10″ slide rule, the distance from 1000 to 1001 is 0.00427″. If this distance is taken as the error in setting, the error, $dx = 0.00427''$, corresponds to an accuracy $\frac{dN}{N} = \frac{dx}{4.27} = \frac{0.00427}{4.27} = 0.001 = \frac{1}{1000}$, which is constant throughout the scale for all numbers, N.

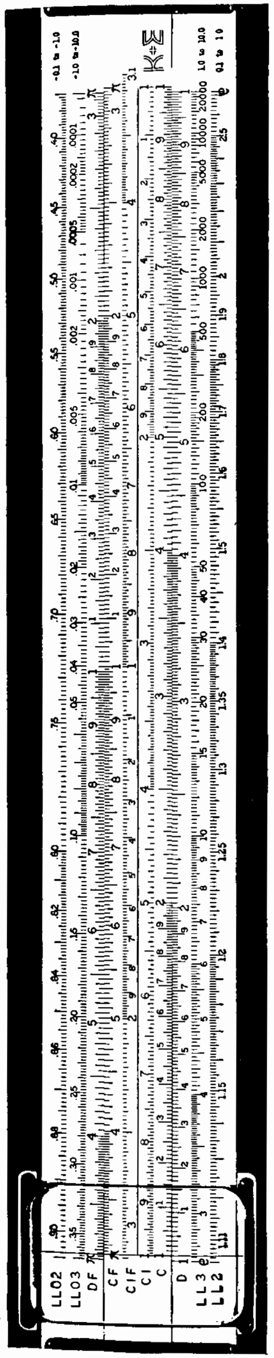


Fig. 15.

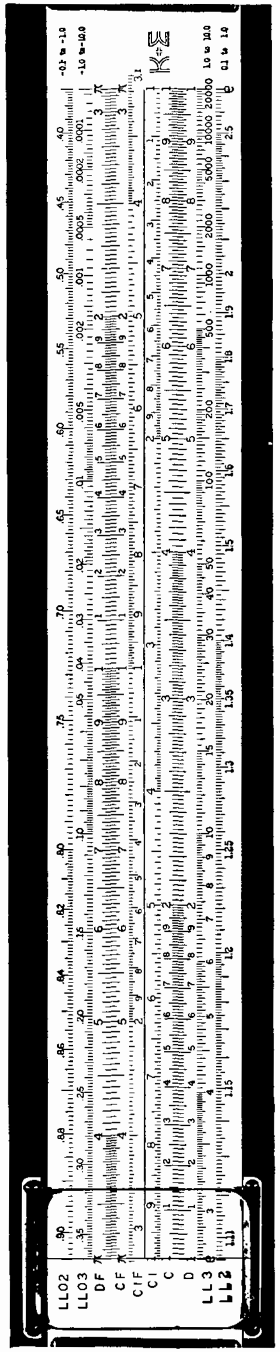


Fig. 16.

cant figure or the third. The interval of 0.00427″ represents 3 in the fourth place at the mark for 3, instead of 1 in the fourth place as at the left end or 1 in the third place (10 in the fourth place) as at the right end. (See Fig. 16 again.)

It is obviously not advisable to try to read to every third integer in the fourth place near 3, or to every seventh integer in the fourth place near 7. As a compromise, read or set only three significant figures on the scale from 2 through 9 and four significant figures on the scale between 1 and 2. Although the fourth place cannot be set accurately by most students near 2, such as in 1896 or 1927, the average student will probably not be in error more than one in the fourth place, and certainly not more than two, for careful setting and reading.

The slide rule is primarily useful for computations where no more than three significant figures are required. If the factors in a computation are given to three significant figures and none of them begins with 1, the answer on the slide rule may be read to four significant figures between 1 and 2 and to three significant figures from 2 through 9. If some of the factors do begin with 1, and if these are given to four significant figures, the answer may again be read to four significant figures between 1 and 2, and three significant figures otherwise. If the factors beginning with 1 are given only to three significant figures, then the answer should not be read to more than three significant figures anywhere on the scale. These statements are summarized in the following table:

Relation between the Significant Figures in the Factors of a Computation and in the Answer

Factors Beginning with	Significant Figures in Factors	Significant Figures to Be Read on D Scale from	
		1 to 2	2 through 9
2, 3, 4, ⋯ 9 only . .	3	4	3
2, 3, 4, ⋯ 9. . . . . . . 1.	3 4	4	3
1, 2, 3, 4, ⋯ 9. . . .	3	3	3

Actually if a number close to 1, such as 112, is given only to three significant figures in a computation and the answer to the computation falls in the 8's and 9's, the answer will not be reliable in the third significant figure. The number 112 represents any number of three significant figures between 111.5 and 112.5. This is a variation over a range of about 10 in 1000 which in the 8's and 9's would be respectively about 8 in 800 and 9 in 900. These are considerable variations in the third significant figure. However, reading to the third significant figure would be reading to a closer probable value than would be obtained by reading to two significant figures and is generally done by most engineers.

Similar rules would apply where factors are given only to two significant figures or to one significant figure.

If the factors in a computation have more than three significant figures, such as 3.148 × 41.671, they would be set on the slide rule to the nearest three significant figures, or as 3.15 × 41.7. The exception to this procedure, of course, is that factors beginning with 1 which have more than three significant figures could be set approximately to the nearest four significant figures. For numbers such as 3.125 × 41.55, the student has the choice of reducing these to 3.12 or to 3.13, and to 41.5 or to 41.6. In general, the error in the answer will be less if such numbers ending in 5 are reduced either to the nearest *even* number or to the nearest *odd* number. If the practice of reducing to the nearest even number is followed, the preceding numbers would be used as 3.12 × 41.6. In reducing to the nearest odd number, the factors would be 3.13 × 41.5. For numbers of four significant figures beginning with 4, 5, 6, 7, 8, or 9, and ending in 25 or 75, such as 51.75, no reduction is necessary. The hairline position for such a number as 51.75 is midway in a subdivision, a position which can be set more accurately than that for the reduced number.

Exercise 1

Determine the number of significant figures in each of the following numbers:

1. 42.3
2. 173.6
3. 9.02
4. 6.00
5. 10.042
6. 400.0
7. 8.635
8. 0.0917

9. 0.00042
10. 0.403
11. 0.05070
12. 16,421.2
13. 0.00008
14. 32.003
15. 0.900
16. 427.1
17. 6.2
18. 0.071
19. About 43,400
20. About 700,000

Reduce each of the following expressions for setting on the slide rule. *Example:* 13.458 × 6.927. *Answer:* Set as 13.46 × 6.93.

21. 2.163 × 41.71
22. 63.87 × 3.446
23. 107.32 × 8.428
24. $\frac{5722.4}{293.87}$
25. $\frac{64.89}{1.0072}$
26. 2.182 × 4.008 × 0.9263
27. $\frac{0.06848 \times 13.639}{5.0505}$
28. $\frac{4.875 \times 83.83}{36.15}$
29. $\frac{76{,}421}{11.555 \times 20.05}$
30. $\frac{1{,}374{,}200}{0.2927 \times 21.38 \times 986.7}$

7. The Decimal Point. As has been explained in the section on the fundamental principle of the slide rule, numbers are set on the scale without regard for decimal point. There are two general procedures which could be followed in setting the decimal point in the answer. One is to carry along the position of the decimal point from one factor to another during the computation. This is a relatively difficult procedure because it involves doing two things at the same time, following rules of operation and following rules for setting the decimal point. Consequently, its use is not advised.

The other general procedure is to forget the decimal point completely until the end of the operation and then to set it independently of the slide rule performance. This is done "by inspection" which is easily explained by the following examples. Consider the product 227.7 × 8.43. Performing this on the slide rule gives the answer in terms of integers without decimal point as 1 9 2. To set the decimal point by inspection, quickly approximate each factor mentally to get, say, 200 × 10 = 2000. The answer, 1920, is obviously much closer to 2000 than 192 or 19,200 and is the correct answer to three significant figures.

Of course, the rougher the approximations of the factors, the farther apart will be the estimated answer and the correct answer,

but, unless the approximations are unreasonably inaccurate, there will be no doubt as to the position of the decimal point.

In case of division, say 313/12.4, the integers in the answer are 2 5 2. Approximating the factors by a quick mental inspection,

TABLE 1

Computation	Integers in Answer without Decimal Point	Inspection	Answer
192.1 × 0.421	8 0 9	$200 \times \frac{1}{2} = 100$	80.9
3.16 × 21.7	6 8 6	$3 \times 20 = 60$	68.6
132.1 × 62.1	8 2 0	$100 \times 60 = 6000$	8200
57.2 × 0.0810	4 6 3	$50 \times \frac{1}{10} = 5$	4.63
$\frac{417}{6.72}$	6 2 0	$\frac{400}{10} = 40$	62.0
$\frac{81.7}{22.3}$	3 6 6	$\frac{80}{20} = 4$	3.66
$\frac{10.42}{0.316}$	3 3 0	$\frac{10}{\frac{1}{3}} = 30$	33.0
6.17 × 8.62 × 4.16	2 2 1	(a) $6 \times 8 \doteq 50$ (b) $50 \times 4 = 200$	221
212 × 3.16 × 8.12	5 4 5	(a) $200 \times 3 = 600$ (b) $600 \times 10 = 6000$	5450
$\frac{416 \times 1.72}{3.96}$	1 8 1	(a) $\frac{400}{4} = 100$ (b) $100 \times 1.72 = 172$	181
$\frac{8.21 \times 3.14}{42.6}$	6 0 5	(a) $3 \times 8 \doteq 20$ (b) $\frac{20}{40} = \frac{1}{2} = 0.5$	0.605
$\frac{4240}{12.1 \times 31.6}$	1 1 1	(a) $12 \times 30 \doteq 400$ (b) $\frac{4000}{400} = 10$	11.1
$\frac{17.24}{6.21 \times 8.43}$	3 2 9	(a) $\frac{17}{8} \doteq 2$ (b) $\frac{2}{6} = \frac{1}{3} \doteq 0.3$	0.329

300/10 = 30, and the answer is obviously 25.2, which is closer to 30 than 2.52 or 252.

The examples in Table 1 provide additional illustrations of the inspection method of setting the decimal point, especially for operations involving more than two factors.

In computations where division is involved, the setting of the decimal point can sometimes be simplified by "cancellation." For instance, consider $\frac{131 \times 4.42}{3.87}$. The ratio, $\frac{4.42}{3.87}$, is almost equal to 1. These two factors can therefore be "cancelled," and the answer at a glance is approximately 131 as is shown by $\frac{131 \times \cancel{4.42}}{\cancel{3.87}}$. The slide rule gives 150 for the correct answer. Again consider $\frac{17.2 \times 8.15}{7.41 \times 5.71}$.

Solving for decimal point by "cancellation," $\frac{\overset{3}{\cancel{17.2}} \times \cancel{8.15}}{\cancel{7.41} \times \cancel{5.71}} = 3$. The factors 8.15 and 7.41 cancel and 5 goes into 17 approximately 3 times. The slide rule gives 3.31 as the correct answer.

For very large or very small quantities, the procedure of setting the decimal point by inspection frequently involves rather unwieldy mental arithmetic. For example, 14,610 × 511 = ? This involves multiplying say 10,000 × 500, which is beyond everyday mental arithmetic. An easier procedure is to "reduce" each factor by moving the decimal point to the left, keeping in mind the number of places moved, e.g., reduce 14,610 to 1.461 by moving four places and reduce 500 to 5.11 by moving two places, or total of six places altogether. Now 1.461 × 5.11 ≐ 5, and restoring the six places as six zeros gives the approximation to the answer as 5,000,000. The slide rule gives the correct answer to be 7,470,000.

For a computation involving both very large and very small factors, such as 21,462 × 0.000416, an easy procedure to follow is to move the decimal point four places to the right in the small factor, giving 4.16, and four places to the left in the large factor, giving 2.15 (to three significant figures). The two shifts exactly neutralize each other, so that the answer is approximately 4 × 2 or 8. The slide rule gives 8.95 as the correct answer.

Taking another example, 114,200 × 13.1 × 0.0000612, by moving the decimal point an equal number of places, five to be specific, to both the right and left, the answer is approximately 1 × 10 × 6 = 60. The correct answer is 91.6.

Summarizing the section on setting the decimal point, the inspection procedure using mental arithmetic and approximations is easier than using a set of rules. Where division is involved, the "cancellation" device should be used as much as possible. For very large and very small factors, the "reduction" device of moving the decimal points in the factors should be used.

Exercise 2

Set the decimal points in the answers to the following computations by inspection:

1. 6.14 × 8.36 = 5 1 3
2. 12.2 × 9.71 = 1 1 8 4
3. 2.97 × 41.7 = 1 2 3 8
4. 5.63 × 1.820 = 1 0 2 5
5. 146 × 4.37 = 6 3 8
6. 29.7 × 11.8 = 3 5 0
7. 33.1 × 0.241 = 7 9 7
8. 4.03 × 472 = 1 9 0 3
9. 64.2 × 3.46 = 2 2 2
10. 7.32 × 0.816 = 5 9 7
11. $\frac{43.7}{6.21}$ = 7 0 4
12. $\frac{8.32}{14.64}$ = 5 6 8
13. $\frac{557}{2.17}$ = 2 5 7
14. $\frac{63.7}{19.2}$ = 3 3 2
15. $\frac{1463}{362}$ = 4 0 4
16. $\frac{7428}{2.93}$ = 2 5 3
17. 2.43 × 4.17 × 5.92 = 6 0 0
18. 6.25 × 21.7 × 1.83 = 2 4 8
19. 41.7 × 0.923 × 2.81 = 1 0 8 1
20. 11.61 × 3.37 × 9.24 = 3 6 2
21. 832 × 0.017 × 4.24 = 6 0 0
22. 2.61 × 1.62 × 61.3 = 2 5 9
23. 4.76 × 8.42 × 0.331 × 13.1 = 1 7 3 5
24. 6.42 × 51.3 × 1.37 × 4.75 = 2 1 4
25. 87.7 × 1.93 × 0.926 × 4.41 = 6 9 1
26. 3.77 × 9.23 × 109.2 × 0.505 = 1 9 1 8
27. 8.37 × 4.16 × 22.8 × 3.14 = 2 4 9
28. $\frac{849}{4.16 \times 12.42}$ = 1 6 4 2
29. $\frac{7092}{21.7 \times 33.6}$ = 9 7 2
30. $\frac{12{,}400}{61.7 \times 39.2}$ = 5 1 2

Set the decimal points in the answers to the following computations by cancellation:

31. $\dfrac{6.17 \times 3.14}{3.38} = 5\ 7\ 3$

32. $\dfrac{133 \times 82.4}{69.7} = 1\ 5\ 7\ 2$

33. $\dfrac{29.6}{32.4} = 9\ 1\ 3$

34. $\dfrac{0.0421 \times 17.37}{0.0374} = 1\ 9\ 5\ 5$

35. $\dfrac{50.6 \times 0.637}{0.819} = 3\ 9\ 4$

36. $\dfrac{8.77 \times 4.23}{10.06} = 3\ 6\ 9$

37. $\dfrac{607 \times 421 \times 0.592}{517 \times 0.643} = 4\ 5\ 6$

38. $\dfrac{92.6 \times 1.64 \times 37.2}{41.3 \times 105.1} = 1\ 3\ 0$

39. $\dfrac{327 \times 0.463 \times 17{,}212}{253 \times 0.361} = 2\ 8\ 5$

40. $\dfrac{6.31 \times 9.27 \times 18.43}{15.16 \times 11.72} = 6\ 0\ 7$

Set the decimal points in the answers to the following computations by reduction:

41. $1250 \times 437 = 5\ 4\ 6$
42. $16{,}720 \times 8320 = 1\ 3\ 9\ 3$
43. $425 \times 1622 \times 37.3 = 2\ 5\ 7$
44. $622 \times 47.1 \times 2460 = 7\ 2\ 1$
45. $0.0183 \times 0.00417 = 7\ 6\ 3$
46. $0.000631 \times 0.00283 = 1\ 7\ 9$
47. $0.0161 \times 0.00246 \times 0.371 = 1\ 4\ 7$
48. $5540 \times 0.00325 = 1\ 8\ 0$
49. $21{,}460 \times 0.00132 = 2\ 8\ 4$
50. $203 \times 0.0000297 = 5\ 9\ 5$
51. $130 \times 5270 \times 0.0000065 = 4\ 4\ 5$
52. $0.000084 \times 4160 \times 27.3 = 9\ 5\ 4$

Set the decimal points in the answers to the following computations by combinations of inspection, cancellation, and reduction:

53. $\dfrac{49.7 \times 8.62}{6.43 \times 18.4} = 3\ 6\ 2$

54. $\dfrac{232 \times 61.7}{11.41 \times 26.4} = 4\ 7\ 5$

55. $\dfrac{14.71 \times 7.24 \times 9.36}{2.17 \times 12.92} = 3\ 5\ 5$

56. $\dfrac{2 \times 162{,}400}{0.875 \times 0.375 \times 10 \times 225} = 4\ 4\ 0$

57. $\dfrac{6 \times 335{,}000}{3.13 \times 37.2} = 1\ 7\ 2\ 7$

58. $\dfrac{9910 \times 0.00427}{275} = 1\ 5\ 4$

59. $\dfrac{61.8 \times 26.7 \times 39.3}{572} = 1\ 1\ 3\ 3$

60. $\dfrac{18{,}300}{4 \times 47.2 \times 7.21} = 1\ 3\ 4\ 4$

8. Adjustments of the Slide Rule

The Duplex Slide Rule

It is sometimes necessary to adjust the slide rule so that the scales and hairline are in proper alignment and also so that the slide and runner move freely but not loosely. Before proceeding with adjustments of the Duplex slide rule, it should be noticed that the frame consists of two parts, a "lower" part to which two metal braces are permanently riveted at the ends, and an "upper" part which can be adjusted by loosening the screws which hold it at each end. In describing the two portions of the frame as "upper" and "lower," the slide rule is held in the position most frequently used, i.e., with the side having the D, DF, C, CF, CI, and CIF scales uppermost.

The adjustments of the slide rule are now given in the order in which they should be made.

(a) *Alignment of slide and lower frame.* Hold the lower frame at each end between the thumb and middle finger of each hand with the thumb on top. Place the forefinger of each hand on the ends of the slide and push it into position so that the divisions of the C scale and of the D scale are in line at one end. If they are not also in line at the other end, the alignment is faulty, no adjustment is possible, and the rule should be returned to the manufacturer.

It is important that the student master the technique of applying a push in opposite directions at each end of the slide, and pushing a little harder at one end than at the other in making a setting. It is very difficult to set the slide quickly and accurately by pushing or pulling from only one end.

(b) *Alignment of upper and lower frames.* Move the runner out of the way toward the center of the rule. Set the slide in alignment with the lower frame as in (a). Check the alignment of the CF scale divisions of the slide with the DF scale divisions on the upper frame. If they are not in alignment (Fig. 17), loosen the two screws which hold the upper frame at the ends about half a turn and push the upper frame or tap it lightly at one end to move it into alignment. Tighten the screws.

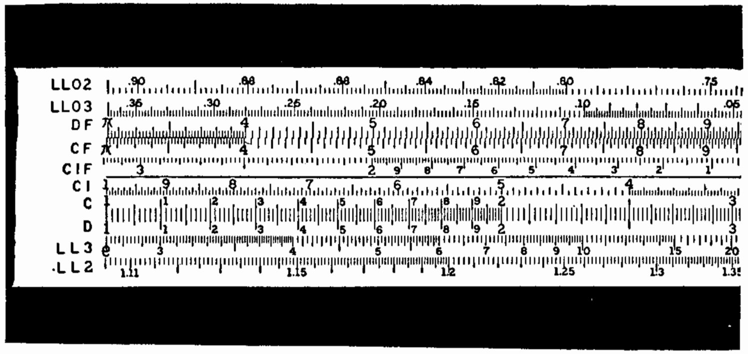


FIG. 17.

(c) *Adjustment of upper frame for tightness or looseness.* Test the slide for tightness or looseness by moving it back and forth. The slide should move smoothly without binding at any position and without effort on the part of the operator, but it should not be so loose that it will slip under its own weight when the slide rule is held in a vertical position.

If the slide is generally too tight or too loose, loosen the screw first at one end of the upper frame, moving that end a very slight amount either to decrease or to increase the pressure of the upper frame against the slide. Tighten the screw and repeat the performance at the other end. Several trials may be required.

Do not loosen both screws at the same time in any adjustments for tightness or looseness as this may result in shifting the upper frame out of alignment.

If the slide is too tight at one end of the rule, loosen the screw holding the upper frame at that end and move the upper frame slightly away from the slide until smooth action is obtained. This may require several trials.

If the slide is too tight at the center of the rule, it is necessary to pull the center of the upper frame away from the slide without changing the adjustment of the ends of the upper frame. This condition frequently occurs as good slide rules are usually made with a slight downward bow in the upper frame to insure a snug fit throughout the length of the rule when the ends are snugly fitted. Also it is easier to adjust for tightness at the center than for looseness there. To adjust for tightness, hold the slide rule in the left hand, palm upward, with slide rule parallel to the left forearm. Move the runner to the inner end, lay the outer end on a table top, and loosen the screw at the outer end with a small screwdriver. Grasping the rule firmly with left hand, pull the upper frame slightly away from the slide at the center, about the width of ordinary sewing thread, and clamp firmly in that position with heavy pressure from the thumb and middle finger of the left hand (Fig. 18). Press the loose end of the upper frame back against the slide by pushing with the forefinger of the left hand without relinquishing finger pressure at the center. This requires bracing the outer end of the rule against some heavy object on the table top (Fig. 19). Tighten the screw and release the finger pressure, permitting the upper frame to spring back against the slide. Repeat the same procedure with the other end of the upper frame.

If the upper frame is bowed away from the slide at the center, loosen the screw at one end of the upper frame, press the center of the frame firmly against the slide without moving the loose end, and tighten the screw. The loose end must not be pressed in against the slide along with the center of the upper frame as this will bind the slide. The loose end must be held slightly away from the slide as the center is pressed against it. This can be done by holding the loose end firmly between thumb and forefinger of the left hand and pressing the slide rule against the heel of the left hand with the third and little fingers. This is rather difficult and could be done more easily by having a second person to wield the screwdriver, so that one hand could be used on the loose end and the other on the center. Repeat the procedure at the other end. If this does not close the gap between the upper frame and the slide sufficiently, it will be necessary to repeat the procedure moving the slide almost entirely out of the rule at the end which is loosened so that the upper frame may be pressed further inward in making trial adjustments.

The preceding adjustments are all based on the assumption that tightness or looseness at the center is due to curvature of the upper frame. If the slide should be curved, no satisfactory adjustment can be made, and such a rule can be used only with the slide moving rather loosely.

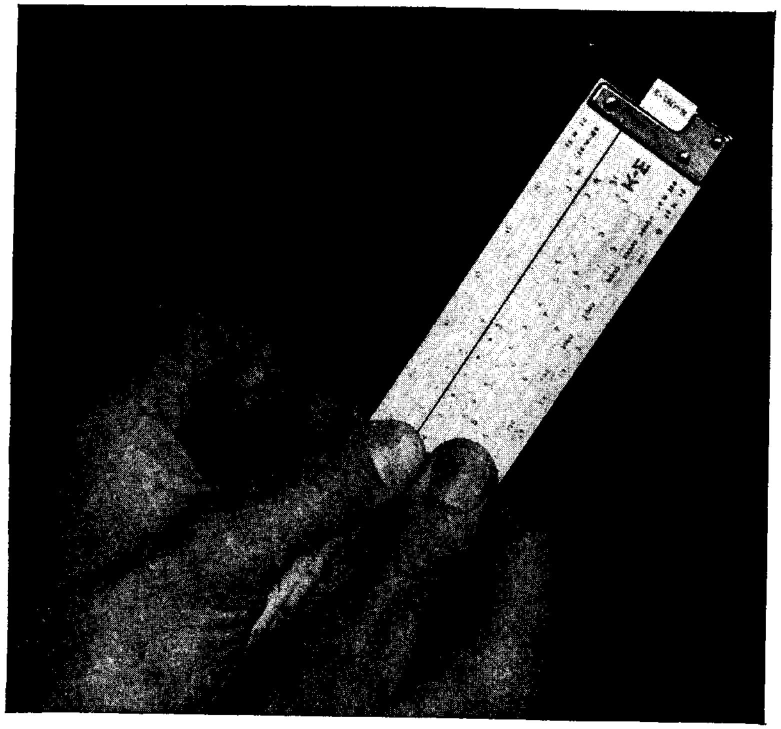


FIG. 18.

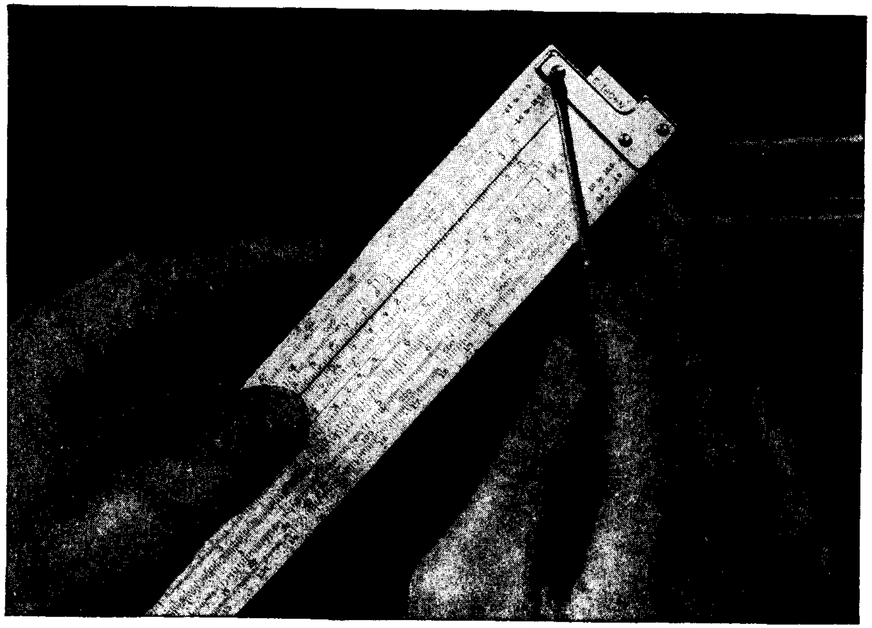


FIG. 19.

If either part of the frame or the slide should be warped at right angles to the face of the slide rule, no satisfactory adjustment can be made except straightening the member. In this event, the slide rule should be returned to the manufacturer.

(d) *Alignment of hairlines on runner.* The alignment of the hairlines is tested after the upper frame has been aligned with the slide and lower frame. Move the hairline to the left end of the D scale. If it also lies on the left end of the DF scale on the upper frame, the hairline is in adjustment. If it does not (Fig. 20),

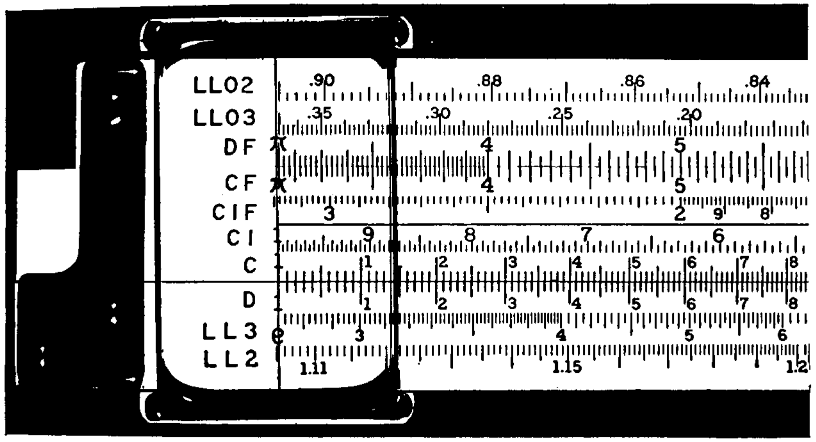


FIG. 20.

loosen slightly the four screws holding the metal frame of the runner and shift the glass plate until the hairline is in adjustment. With the "front" hairline on the ends of the D and DF scales, flip the rule over to the other side. If the "back" hairline on the other side is in alignment with the left ends of the D and A scales on that side, the back hairline is in adjustment. If it is not in alignment, adjust in the same manner as for the front hairline, taking care that the front hairline is not moved from its position at the ends of the D and DF scales on the front side.

The Mannheim Slide Rule

It is sometimes necessary to adjust the slide rule so that the scales and the hairline are in proper alignment and also so that the slide and runner move freely but not loosely. Before proceeding with the adjustments of this type of slide rule, it should be noticed that the frame consists of two parts, a "lower" frame which comprises the main body of the rule, and an "upper" frame which is attached to the lower frame by three screws and is adjustable (Fig. 2). The D scale is on the lower frame, and the A scale is on the upper frame.

The adjustments are now given in the order in which they should be made.

(a) *Alignment of slide and lower frame.* Hold the lower frame at each end between the thumb and middle finger of each hand with the thumb on top. Place the forefinger of each hand on the ends of the slide and push it into position so that the divisions of the C scale and of the D scale are in line at one end. If they are not also in line at the other end, the alignment is faulty, no adjustment is possible, and the rule should be returned to the manufacturer.

It is important that the student master the technique of applying a push in opposite directions at each end of the slide, and pushing a little harder at one end than at the other in making a setting. It is very difficult to set the slide quickly and accurately by pushing or pulling from only one end.

(b) *Alignment of upper and lower frames.* Move the runner out of the way toward the center of the rule. Set the slide in alignment with the lower frame as in (a). Check the alignment of the B scale divisions of the slide with the A scale divisions on the upper frame. If they are not in alignment, loosen the three screws which hold the upper frame about half a turn each and push the upper frame or tap it lightly at one end to move it into alignment. Tighten the screws.

(c) *Adjustment of upper frame for tightness or looseness.* Test the slide for tightness or looseness by moving it back and forth. The slide should move smoothly without binding at any position and without effort on the part of the operator, but it should not be so loose that it will slip under its own weight when the slide rule is held in a vertical position.

If the slide is generally too tight or too loose, loosen the screw first at one end of the upper frame, moving that end a very slight amount either to decrease or to increase the pressure of the upper frame against the slide. Tighten the screw and repeat the performance at the other end, and at the center. Several trials may be required. Another procedure is to loosen two screws at a time, keeping one screw tight.

Do not loosen all screws at the same time in any adjustments for tightness or looseness as this may result in shifting the upper frame out of alignment.

If the slide is too tight at one end of the rule, loosen the screw holding the upper frame at that end and move the upper frame slightly away from the slide until smooth action is obtained. This may require several trials. If the slide is too tight at the center of the rule, a similar procedure is followed.

(d) *Alignment of hairlines on runner.* The alignment of the hairline is tested after the upper frame has been aligned with the slide and lower frame. Move the hairline to the left end of the D scale. If it also lies on the left end of the A scale on the upper frame, the hairline is in adjustment. If it does not, loosen slightly the four screws holding the metal frame of the runner and shift the glass plate until the hairline is in adjustment. Tighten the screws carefully.

9. Care and Manipulation. It has been the author's experience that no lubrication of a well-made slide rule is necessary. The use of powders, waxes, and other substances results sooner or later in clogging the grooves with foreign material. Graphite such as comes from the lead in a soft pencil has been suggested by some writers. However, any foreign material seems undesirable in view of the fact that the slide rule is manipulated with the bare hands. Moisture from the hands sooner or later will combine with dust to form deposits in the grooves, and the presence of any dry lubricant would seem to accelerate this process.

Cleanliness is the key to good care of the slide rule. Keep the slide rule in its case when not in use. Protect it to the best of one's ability from dust and dirt. Always try to have clean hands when using it. A clean, well-adjusted slide rule needs no lubrication. If it should become dirty, clean the grooves of the frame and the tongues of the slide with a soft, dry cloth and clean the faces of the slide rule with a soft, dry, or slightly moist cloth. If particles of dust or dirt accumulate under the glasses of the runner, these may be removed without removing the runner. Tear a narrow strip from a sheet of paper, preferably a rougher paper such as mimeograph paper, insert this under the glass from one side, and pull it through to the other side, pressing the glass firmly against the strip of paper. Dust and dirt will come out with the paper.[2]

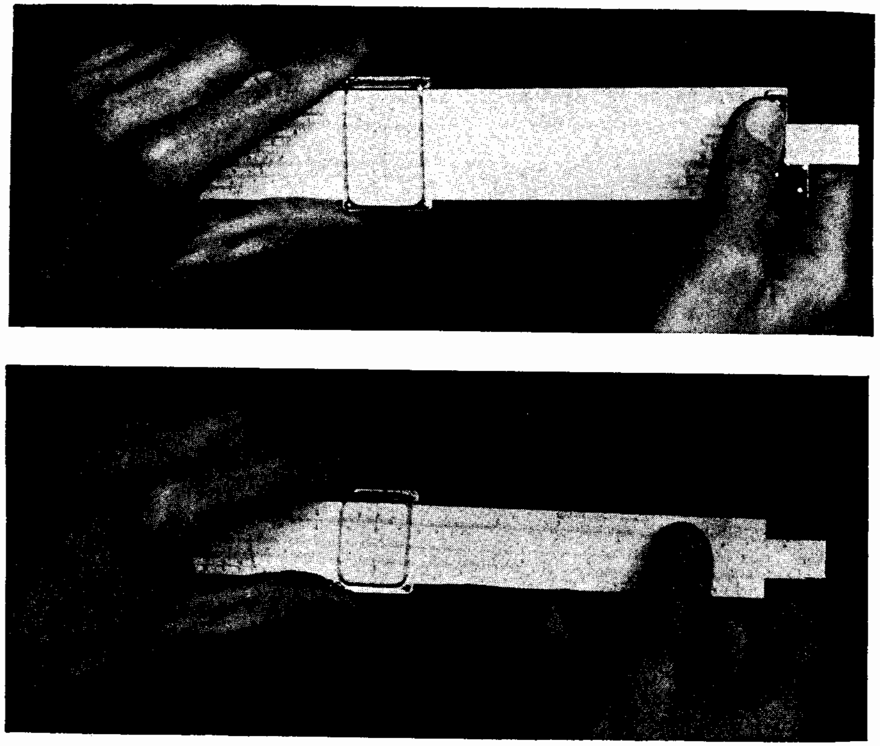


Fig. 21.

Manipulation of the slide rule involves setting the runner and the slide.

The runner may be set quickly either with one hand or with two. To set the runner with one hand, hold the rule toward the right end with the right hand and move the runner to the neighborhood of the setting with the thumb and forefinger of the left hand, the thumb at the bottom and the forefinger at the top. When close to the

[2] The runners of Mannheim slide rules may easily be slipped off of the rules for cleaning.

setting, clasp the rule lightly with the other fingers of the left hand to brace it, let the thumb rest lightly both against the runner and the lower frame, and ease the runner to the exact setting with the thumb and forefinger (Fig. 21). To set the runner with two hands, follow the same procedure through the approximate setting, but then move the right hand over so that the thumb and forefinger of each hand rest against the runner, the other fingers on both hands holding the rule. The final setting is made by pushing lightly on the runner from both sides, resting both thumbs against the lower frame and using slightly greater pressure from one side to move the runner in the desired direction.

Some students will prefer to reverse the hands in the preceding explanation, using the right hand to move the runner, and still others may develop variations of their own to suit themselves. There is nothing fixed about the procedure which has been explained, and it has been intended to serve primarily as a guide to the development of individual manipulation. The same is also true of the following suggestions for setting the slide.

The slide can be set efficiently with two hands, but not with one. The manipulation of the hands in setting the slide is subject to more variations than in setting the runner and can differ greatly from one individual to the other. In general, the slide is pulled or pushed with either hand into the neighborhood of the desired setting. The slide is then set on the desired factor using both hands to apply counterpressures from each end. In this process, the slide rule rests mainly upon the middle and third fingers of each hand. For setting the slide close to the central position in the frame, counterpressures are applied at each end with the forefingers of the hands (Fig. 22). If the slide projects to the right, the author finds it easiest to push with the thumb of the left hand against the left end of the slide and with the forefinger of the right hand against the right end, and vice versa if the slide projects to the left (Fig. 23). If the slide projects too far to the right for the right forefinger to reach the right end, he grasps the slide between the right thumb on top and right forefinger underneath. If the slide does not project far enough to the right to permit placing the left thumb against the left end because of interference from the metal bracket, he grasps the slide between the left thumb on top and left forefinger underneath close to the bracket. Some operators find it more natural to use both forefingers, one at each end of the slide, in making the final setting. No rigid rules for manipulation are suggested here, although it is possible that careful study may point to one particular manipulation as being more

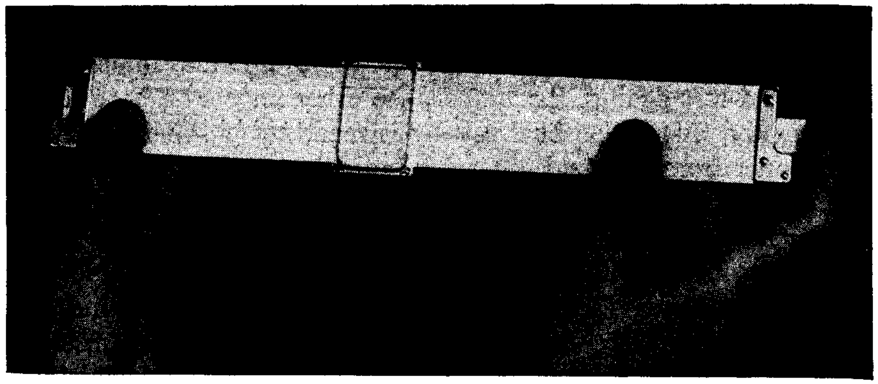


Fig. 22.

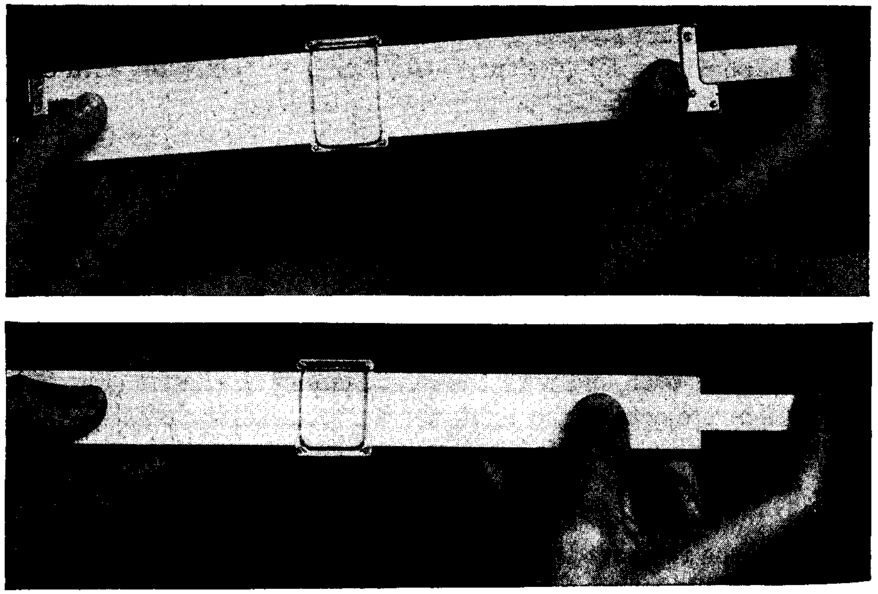


Fig. 23.

efficient than any other. The one thing which does seem to be certain is that a setting of the slide can be made more rapidly using counterpressures applied simultaneously at each end rather than by pushing or pulling only from one end.

10. Speed in Operation. The engineer's slide rule is useful only in certain types of computations mentioned on page 1 and is accurate primarily to three significant figures as has been explained. It has not been mentioned that the greatest usefulness of the slide rule is in saving time in these approximate computations. The reason it is so widely used by students of engineering is that it reduces tremendously the time required for routine calculations in solving problems and permits more time to be devoted to more important phases of an engineer's education. The method of slide rule operation presented in this book emphasizes rapid operation.

II

MULTIPLICATION AND DIVISION

11. General Principles. The secret of efficient operation of the slide rule lies in becoming familiar with all scales on the rule, particularly those used for multiplication and division, and with the relation between these scales. To be efficient an operator must feel equally at home on any group of scales on any part of the rule. This familiarity cannot be acquired overnight but comes from a gradual process of learning, beginning with the general principles of operation and continuing with step-by-step explanations of simple and then more complex multiplications and divisions. At the end of the chapter the entire method is reviewed and summarized.

In multiplication and division there are two groups of scales to be used concurrently, the lower group consisting of the D, C, and CI scales, and the upper group consisting of the DF, CF, and CIF scales. Factors may be set on *either group* of scales and neither group of scales should be regarded as being more important or more frequently used than the other group. It should be noted here that the arrangement of the scales in each group is similar in this way: in the lower group consisting of the D, C, and CI scales, the D scale is on the frame and the other two are on the slide with the C scale adjacent to the D scale; in the upper group consisting of the DF, CF, and CIF scales, the DF scale is on the frame and the other two are on the slide with the CF scale adjacent to the DF scale. The D and DF scales correspond, i.e., have the same function in an operation, the C and CF scales correspond, and the CI and CIF scales correspond. The upper scales are marked with the letter "F" to signify that they have been "folded" in the same manner that a flat 12-inch ruler might be cut at the center and the right half placed to the left of the other half. On most slide rules the folded scales

are formed in effect by cutting the normal logarithmic scale at the value for π, or 3.1416. The reason for this is explained later in the chapter on special operations.

Multiplication and division are performed on these two groups of scales by moving the runner and the slide to proper settings on them. Questions arise as to the order in which these two parts are moved and as to which scales are used. There is a standard technique of operation which must be learned first in answer to these questions.

Standard Technique of Operation

(a) *Always begin and end a computation with the runner. The first movement or setting is always made with the runner and the final movement or setting is likewise made with it.*

(b) *Always begin a computation with a setting and end a computation with a reading on one of the two D scales, either the lower D scale or the upper DF scale.*

This means that a computation is always begun by setting the runner on a factor either on the lower D scale or the upper DF scale, and a computation is always ended by setting the runner to read the answer either on the lower D scale or the upper DF scale.

The D and DF scales are not used in a computation except for setting the first factor and for reading the answer. Which of the two scales to choose for beginning or ending a computation will be explained later in this chapter.

(c) *Concerning movement of the runner, there is a choice, implicitly, of setting the runner either on a factor in the upper group of scales or on the same factor in the lower group of scales. This is generally true every time the hand is placed on the runner to set it.*

Keeping in mind rules (a) and (b), this means that for the initial setting the runner is set on one of the D scales. For subsequent settings it is set on one of the C scales or one of the CI scales, depending upon the computations involved. When the final setting is made with the runner on the upper or lower C or CI scales, the answer is read opposite the hairline on the *corresponding* upper or lower D scale.

(d) *Concerning movement of the slide, the runner and slide are always moved as a pair with the runner first and the slide next. The movements of runner and slide constitute a game of "Follow the Leader" with the runner as leader and the slide always following.* If the runner is set on the upper scales, the slide follows to the upper scales; if the runner drops to the lower scales the slide follows to the lower scales. Whichever group of scales the runner chooses, the slide follows it to that group. The runner is never moved twice in succession, nor is the slide, but they are moved as a pair to the same group of scales, except for the final movement or setting of the runner.

The student is reminded that the preceding standard technique applies primarily to computations involving multiplication, division, or a combination of the two.

12. Multiplication—Two Factors. It is presumed at the start of any computation that the runner and slide are approximately centered in the frame.

In multiplying two factors, they may be set either on the upper group of scales or on the lower group. The DF and CIF scales are used for multiplying two factors in the upper group. Correspondingly, the D and CI scales are used in the lower group. The procedure for the upper group is to set one factor on the DF scale with the runner, the other factor on the CIF scale with the slide, move the runner to 1 on the same scale, and read the answer opposite the hairline on the DF scale.

To explain the procedure in more detail, consider 11.1 × 8.00 as an example. The procedure is given step by step along with an abbreviated notation and is illustrated in Fig. 24.

Step 1: Move the runner to 11.1 on the DF scale, i.e., so that the hairline is opposite 11.1 on that scale. Abbreviation: Runner to 11.1 on DF.

Step 2: Move the slide so that 8.00 on the CIF (red) scale is opposite the hairline. Abbreviation: Slide to 8.00 on CIF.

Step 3: Move the runner to 1 on the CIF scale, which is also to 1 on the CF scale. These marks on the slide are called the "upper index." Abbreviation: Runner to upper index.

Step 4: Read the integers of the answer opposite the hairline on the DF scale as 8 8 8. Setting the decimal point gives the answer 88.8. Abbreviation: Read 88.8 on DF.

In abbreviated notation, the entire operation is as follows:

Runner to 11.1 on DF
Slide to 8.00 on CIF
Runner to upper index
Read 88.8 on DF

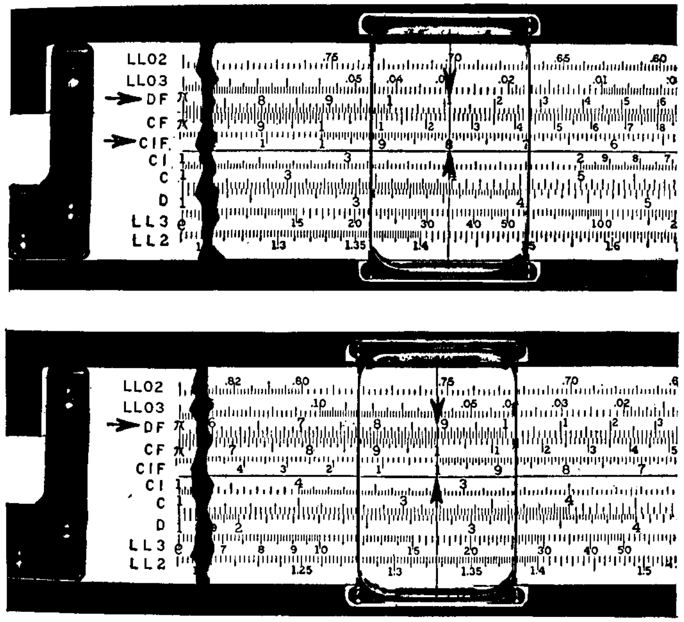


Fig. 24.

The question arises, "Can this same product of 11.1 × 8.00 be found on the lower scales?" It can be found as outlined in the following steps, and as shown in Fig. 25:

Step 1: Move the runner so that the hairline is opposite 11.1 on the D scale.

Step 2: Move the slide so that 8.00 on the CI (red) scale is opposite the hairline.

Steps 3 and 4: If the computation is completed on the lower scales, move the runner to the lower right index (the marks for 1 at the right end of the slide) and read 88.8 opposite the hairline on the D scale. However, notice that when the lower right index is opposite 88.8 on the D scale, the upper index is also opposite 88.8 on the DF scale and that less movement is required to move the runner to the upper index to read rather than to the lower right index. Consequently, the more efficient operation on the lower scales is:

Runner to 11.1 on D
Slide to 8.00 on CI
Runner to upper index
Read 88.8 on DF

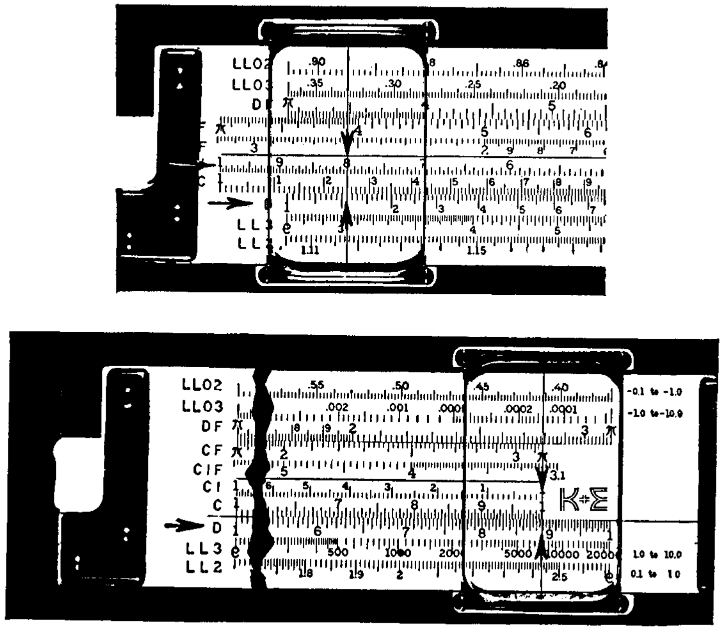


FIG. 25.

It should also be noticed at this point that when 8.00 on the CI scale is opposite 11.1 on the D scale, 8.00 on the CIF scale is also opposite 11.1 on the DF scale. This is why operations in general may be performed on either group of scales and why it is possible to shift with the runner from one group of scales to the other.

The difference between using the upper group and using the lower group of scales is in the distances required for the operations. To measure the distances, use the L scale on the slide rule which divides the length of the scales on the rule into tenths, hundredths, and five-hundredths. The distances required on the upper scales and on the lower scales are determined as follows for the product 11.1 × 8.00 in terms of the length of the scales:

Multiplication **11.1 × 8.00** *Runner and slide centered at start.*

Group of Scales	*Operation*	*Distance Moved in Terms of Scale Length*
Upper	Runner to 11.1 on DF	.05
	Slide to 8.00 on CIF	.05
	Runner to upper index	.10
	Read 88.8 on DF	. .
	Total	.20
Lower	Runner to 11.1 on D	.455
	Slide to 8.00 on CI	.05
	Runner to upper index	.405
	Read 88.8 on DF	. .
	Total	.91

It is apparent that the product of 11.1 × 8.00 can be found more efficiently on the upper scales. In considering why this is so, take another example:

Multiplication **2.52 × 3.01** *Runner and slide centered at start (Figs. 26 and 27).*

Group of Scales	*Operation*	*Distance Moved in Terms of Scale Length*
Upper	Runner to 2.52 on DF	.405
	Slide to 3.01 on CIF	.88
	Runner to lower left index	.025
	Read 7.59 on D	. .
	Total	1.31
Lower	Runner to 2.52 on D	.10
	Slide to 3.01 on CI	.12
	Runner to upper index	.02
	Read 7.59 on DF	. .
	Total	.24

In this instance the product is found more efficiently on the lower scales, the runner being moved to the nearest index to read in each case.

The reasons for the preceding results are probably apparent to the reader by this time. In 11.1 × 8.00, both factors are near the *center* of the rule on the upper scales but are near the ends on the lower scales. In 2.52 × 3.21, both factors are near the *center* of the rule on the lower scales but are near the ends on the upper scales. *The most efficient operation occurs when the factors are taken as near the center of the rule as possible,* which leads the author to the name *center-drift method.* This means that for factors beginning with 2, 3, 4, or 5, use the lower scales, and for factors beginning with 6, 7, 8, 9, or 1, use the upper scales. By most efficient operation is meant operation requiring a minimum of settings and movement. For the

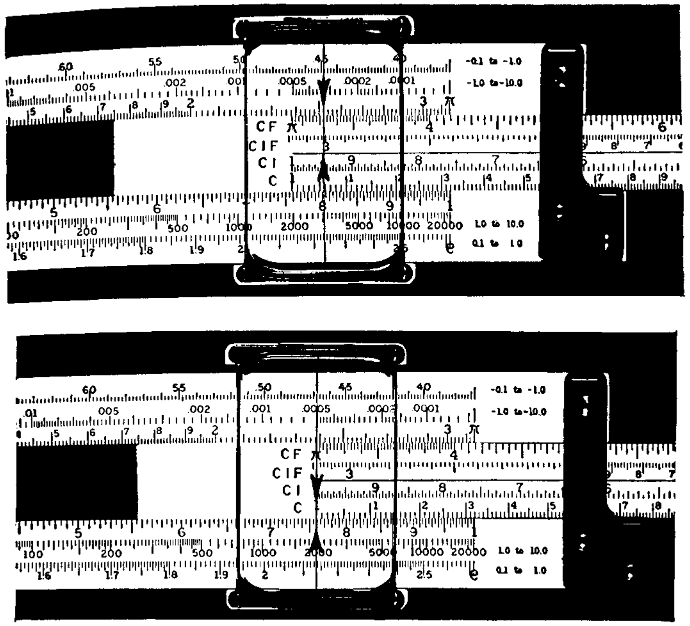


Fig. 26.

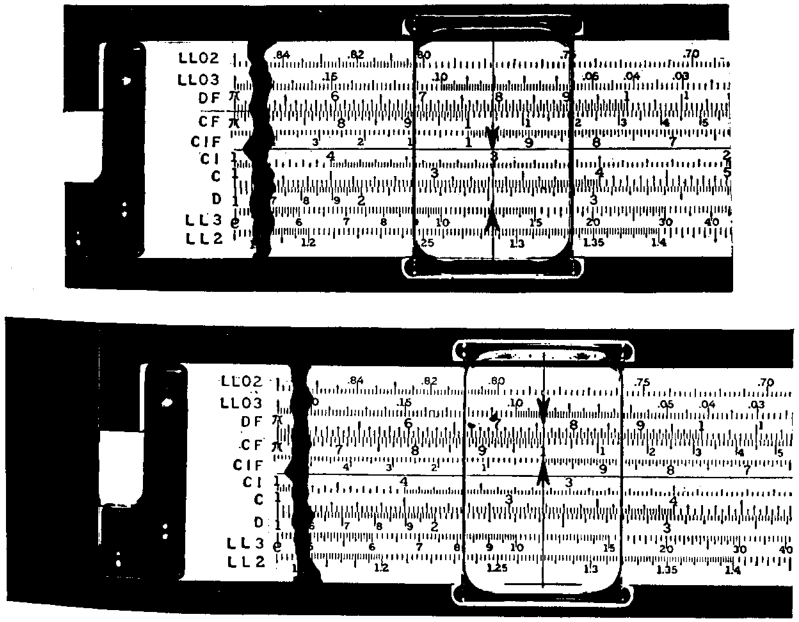


Fig. 27.

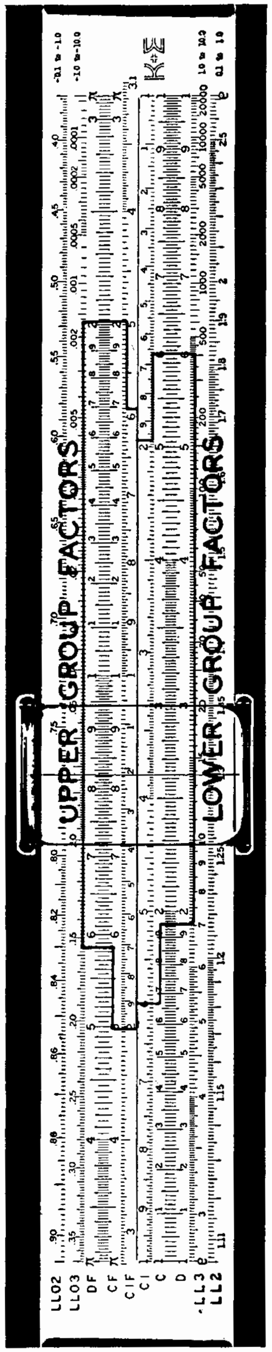


Fig. 28.

beginning student, it is sufficient that he remember the two groups of factors as the upper group composed of numbers beginning with 6, 7, 8, 9, or 1 and the lower group composed of numbers beginning with 2, 3, 4, or 5, as shown in Fig. 28.

The preceding grouping of factors into upper and lower groups does not fall precisely into the middle half of the rule. A more accurate grouping would give an upper group of numbers from 56 up through 9 and to 18, and a lower group of from 18 to 56. However, it is doubtful that attempts to use the more accurate grouping would lead to much greater efficiency as the slight decrease in movement required on one hand would probably be offset by an increase in time for choosing factors on the basis of the first two integers instead of just the first. The simple grouping as first presented will be used in this book and is illustrated by the following examples:

Typical Upper-scale Multiplications for Efficient Operation	*Typical Lower-scale Multiplications for Efficient Operation*
13 × 12	31 × 42
107 × 67	23 × 51
15 × 85	206 × 32
87 × 61	49 × 37
94 × 73	27 × 32
68 × 82	39 × 44

Before proceeding further, the student should practice the multiplication of two factors in the same group, either upper or lower, until he is thoroughly competent at picking the scales at a glance and in setting

the factors quickly on the proper scales. Patience and persistence are more important at this point than at any other place in the book.

For "mixed" multiplications in which one factor belongs to the upper group and the other to the lower group, a slight modification of the standard technique will be explained later in this chapter. It is not advisable to study this modification until the standard technique has been thoroughly mastered.

Other examples of the procedure are as follows:

Multiplication **7.28 × 16.9** (Fig. 29)

Runner to 7.28 on DF
Slide to 16.9 on CIF
Runner to upper index
Read 122.9 on DF

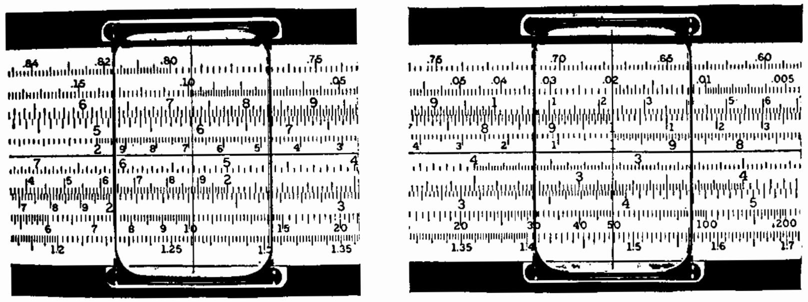


Fig. 29.

Multiplication **8.46 × 72.8** (Fig. 30)

Runner to 8.46 on DF
Slide to 72.8 on CIF
Runner to upper index
Read 616 on DF

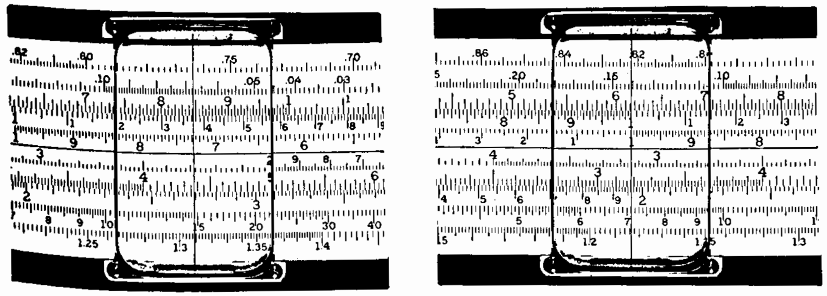


Fig. 30.

Multiplication **46.4 × 0.209** (Fig. 31)

Runner to 46.4 on D
Slide to 0.209 on CI
Runner to upper index
Read 9.70 on DF

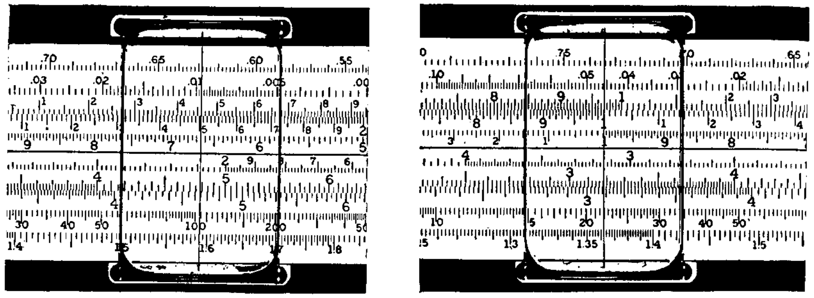


FIG. 31.

Multiplication **34.2 × 5.25** (Fig. 32)

Runner to 34.2 on D
Slide to 5.25 on CI
Runner to upper index
Read 179.5 on DF

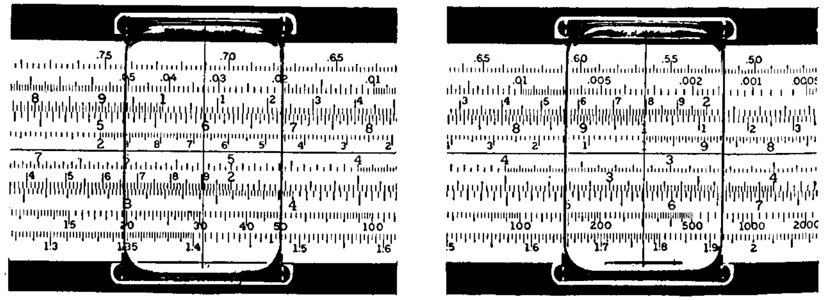


FIG. 32.

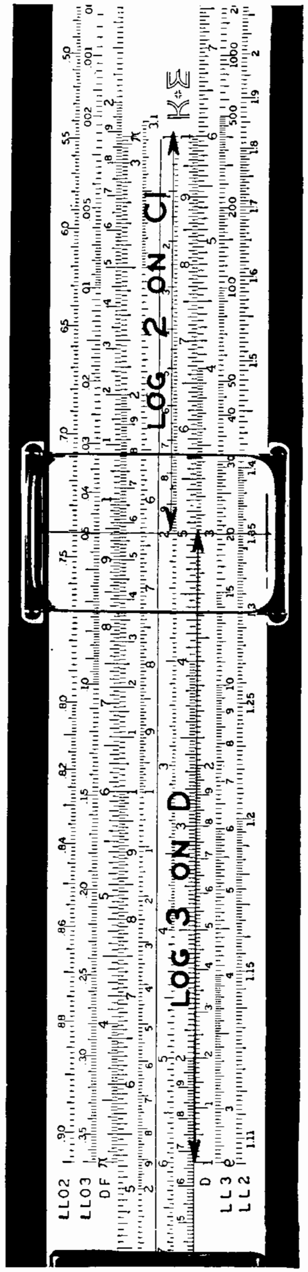
Substitution of the Eye for the Runner in Reading the Answer. In two-factor multiplication the last movement of the runner is to an index, either upper or lower, opposite which the answer is read. With practice the student can learn to read the answer opposite

the index by eye without setting the hairline on it, thus substituting a movement of the eye for the final movement of the runner. Although this may not seem important, it can become a great timesaver in performing many computations one after the other, particularly where only two factors are involved. Also it provides excellent training for the student's eyes in reading the scales quickly and easily.

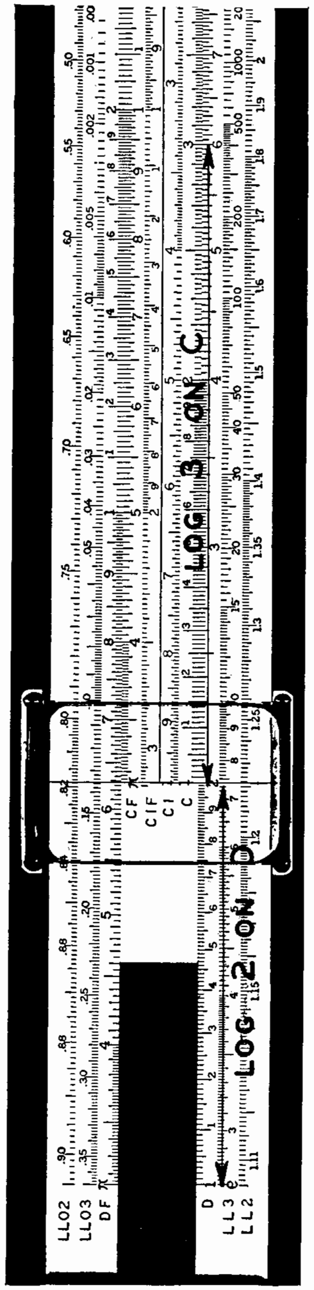
A word of caution should be added here. Not all of the answers to computations are read by moving the runner or the eye to an index. In fact, only those computations having an *even* number of factors, two, four, and so on, end in this manner. Computations with an *odd* number of factors end with the movement of the runner to the last factor, and the answer must be read opposite the hairline in that position and *not at the index*.

Principle of Multiplication of Two Factors. The procedure for using the so-called inverted scales or CI scales for multiplication rather than the C scale has been thoroughly explained. Why it is more advantageous to use these scales will not fully appear until the next section. The principle of operation is based, of course, on logarithms. As was explained on page 12 in the discussion of the fundamental principle of the slide rule, two numbers can be multiplied by adding their logarithms. This is clearly demonstrated in Fig. 33 which shows the product of 2 × 3 using the D and CI scales and using the D and C scales. In using the D and CI scales, the answer is read opposite an index, but in using the D and C scales the answer is read opposite the second factor. That this difference is significant will become apparent in the next section on the multiplication of three factors.

The CI and CIF scales are called inverted scales because they read in the opposite direction from the D and C scales and also because they represent reciprocals of the C and CF scales respectively. Looking at the CI and C scales on the slide in Fig. 33, it is observed that opposite 2 on the C scale is its reciprocal $\frac{1}{2} = 0.5$ on the CI scale. Opposite 5 on the C scale is its reciprocal $\frac{1}{5} = 0.2$ on the CI scale and opposite 4 on the C scale is its reciprocal $\frac{1}{4} = 0.25$ on the CI scale. The scales can be read in reverse if desired, e.g., opposite 2 on the CI scale is its reciprocal $\frac{1}{2} = 0.5$ on the C scale. The decimal point must be set by inspection in all cases, e.g., opposite 250 on the CI scale is its reciprocal 0.004 on the C scale.



2 × 3, using D and CI scales.



2 × 3, using D and C scales.

Fig. 33.

Exercise 3

Perform the following multiplications using the upper scales:

1. 91.7 × 0.826
2. 7.51 × 12.20
3. 6.41 × 7.85
4. 8.52 × 14.60
5. 9.61 × 69.7
6. 16.15 × 12.2
7. 1.08 × 13.9
8. 6.09 × 10.08
9. 16.2 × 91.1
10. 13.7 × 142
11. 8.41 × 8.29
12. 98.6 × 9.13
13. 8.81 × 9.67
14. 7.63 × 8.74
15. 88.7 × 7.28
16. 0.921 × 1.16
17. 8.43 × 6.72
18. 0.135 × 0.160
19. 90.6 × 7.72
20. 8.19 × 0.1870

Perform the following multiplications using the lower scales:

21. 32.3 × 51.4
22. 3.42 × 34.5
23. 2.27 × 3.88
24. 2.19 × 3.19
25. 293 × 5.20
26. 444 × 27.7
27. 55.1 × 3.33
28. 0.441 × 242
29. 415 × 2.48
30. 52.6 × 4.07
31. 43.2 × 32.4
32. 54.7 × 46.8
33. 2.96 × 21.2
34. 40.5 × 49.4
35. 0.311 × 0.423
36. 0.0482 × 3.49
37. 28.1 × 0.527
38. 0.319 × 0.0313
39. 4.15 × 0.245
40. 52.6 × 0.331

41. A rectangular field is 87.6 feet wide and 122.4 feet long. What is its area in square feet?

42. A city street is to be paved. The length of the street is 3270 feet and the width to be paved is 43.5 feet. How many square feet of pavement are required?

43. A manufacturing company can produce an average of 22.5 electric motors per day. What is its yearly production rate if it operates 294 days a year?

44. The unit cost of surfacing a stretch of road is $4.12 per linear yard. If this stretch is 5050 yards long, what is the approximate total cost?

45. It is estimated that 127 man-hours are required for the production of a piece of machinery. If the average labor cost is $1.41 per hour, what is the approximate total labor cost of producing the piece of machinery?

46. Compute:

(a) 92.4 per cent of 117.5
(b) 37.6 per cent of 293
(c) 14.1 per cent of 69.2
(d) 0.43 per cent of 3540

47. An automobile is moving at a speed of 78.6 miles per hour. How many miles will it go in 0.115 hours?

48. How far will an airplane go in 2.235 hours at a ground speed of 305 miles per hour.

49. Using the table of equivalents on page 226, compute the following equivalents:

(a) 42.6 inches in centimeters
(b) 21.7 meters in inches
(c) 13.6 square inches in square centimeters
(d) 9.42 square miles in acres
(e) 124.5 cubic feet of water in pounds
(f) 72.4 gallons of water in pounds
(g) 3.69 kilograms in pounds
(h) 49.1 gallons in cubic inches
(i) 85.8 miles per hour in feet per second
(j) 9.50 horsepower in kilowatts

13. Multiplication—Three Factors. The multiplication of three factors differs only in one movement from the multiplication of two factors. Instead of moving the runner finally to an index to read the answer, move the runner to the third factor on one of the C scales, either the C or the CF scale, and read the answer opposite the hairline. The procedure then is to set the first factor on a D scale with the runner, the second factor on the CI scale in the same group with the slide, the third factor on a C scale with the runner, and to read. The number of movements either for two-factor multiplication or for three-factor multiplication is the same, two movements of the runner and one of the slide.

For the most efficient operation, the same general principle which was used for the multiplication of two factors applies to three factors and, indeed, to any number of factors, namely, *choose the factors as near the center of the rule as possible.* A number of examples are given and the student, himself, should perform each of these, carefully noting the order of use of the scales, i.e., D, CI, and C, and also the choice of factors as near the center as possible when setting the runner and slide.

Multiplication **9.21 × 12.1 × 0.816** (Fig. 34)

Runner to 9.21 on DF
Slide to 12.1 on CIF
Runner to 0.816 on CF
Read 91.0 on DF

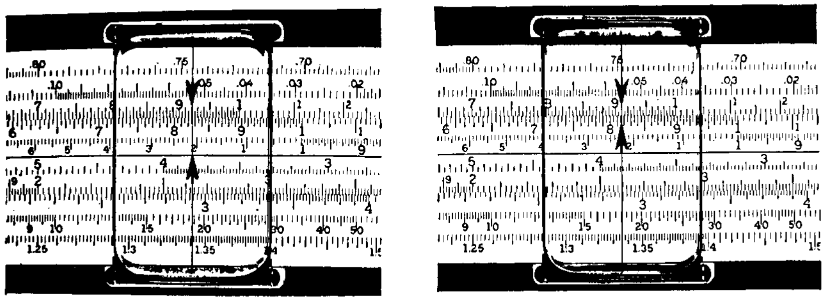


Fig. 34.

Multiplication **3.75 × 2.51 × 4.16** (Fig. 35)

Runner to 3.75 on D
Slide to 2.51 on CI
Runner to 4.16 on C
Read 39.2 on D

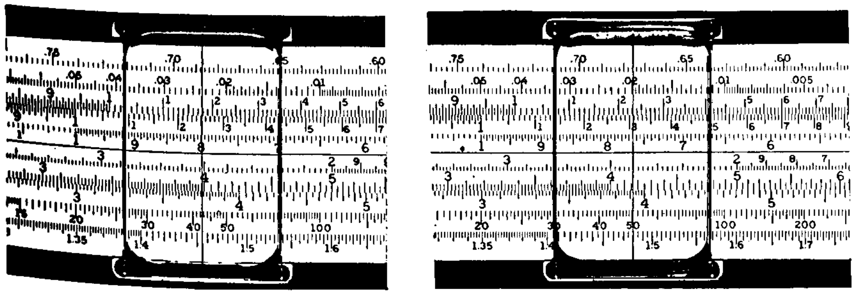


Fig. 35.

Multiplication **12.6 × 4.21 × 8.72** (Fig. 36)

Runner to 12.6 on DF
Slide to 8.72 on CIF
Runner to 4.21 on C
Read 462 on D

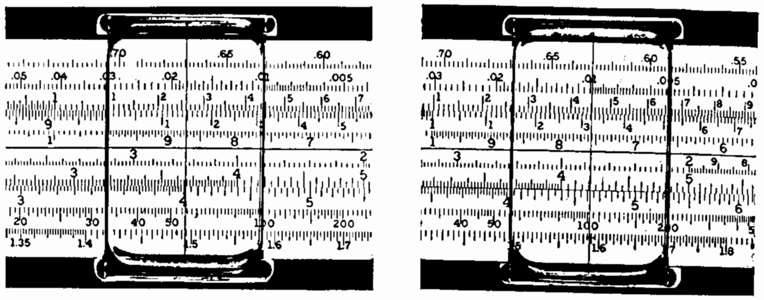


Fig. 36.

Note in this instance the two upper group factors are multiplied first on the upper scales to keep the runner and slide centered as much as possible, and the runner drops to the lower scales for the third factor.

Multiplication **4.07 × 2.19 × 1.03** (Fig. 37)

Runner to 4.07 on D
Slide to 2.19 on CI
Runner to 1.03 on CF
Read 9.20 on DF

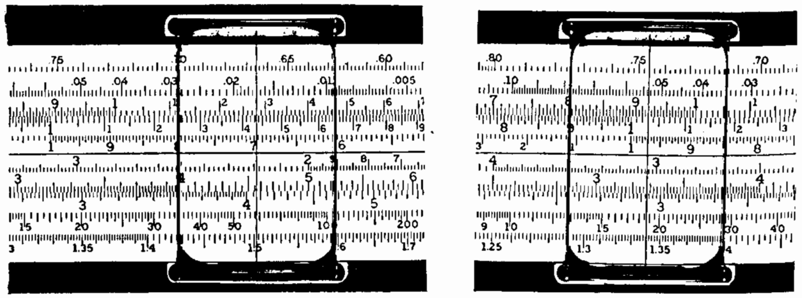


Fig. 37.

Multiplication **3.43 × 7.61 × 9.11** (Fig. 38)

Runner to 9.11 on DF
Slide to 7.61 on CIF
Runner to 3.43 on C
Read 238 on D

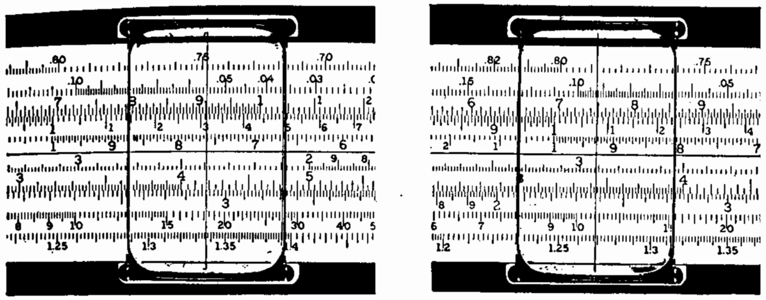


Fig. 38.

Reviewing the preceding examples, there are two things which the student should observe. First, if the runner is set finally on the C scale, the answer must be read opposite the hairline on the D scale, and, if set finally on the CF scale, the answer must be read on the DF scale. The runner cannot be set finally on the C scale and the answer read on the DF scale. The second observation is that runner and slide stay closer to the center when factors are used in pairs belonging to the same group. For instance, in the last example, using the two upper group factors, 7.61 and 9.11, for the first pair, the distance required is 0.28 scale lengths. If the multiplication were performed in the original order,

Runner to 3.43 on D
Slide to 7.61 on CI
Runner to 9.11 on CF
Read 238 on DF

the distance required would be 0.80 scale lengths, or about three times as much as for the first procedure.

In general then, it may be said that the runner and slide are kept closer to the center by choosing the factors in pairs belonging to the same group, upper or lower, rather than by choosing the factors at random. At this point, some students might conclude that if all three factors belong to the same group, such as in a previous example, 9.21 × 12.1 × 0.816, the operation should be performed entirely on the same group of scales, in this case, the upper scales. Although this is true for certain factors, it is not always true for factors which are far from the center of the rule. For instance, the following product of lower scale factors is performed most efficiently as shown:

Multiplication **2.53 × 22.1 × 2.07** (Fig. 39)

Runner to 2.53 on D
Slide to 22.1 on CI
Runner to 2.07 on CF
Read 116 on DF

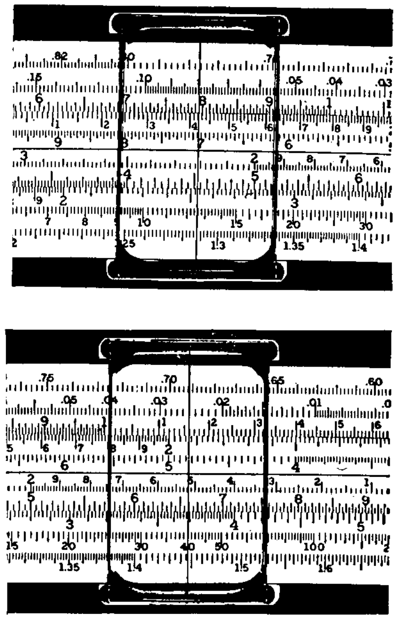


Fig. 39.

In performing this multiplication himself, the student will note after setting the first two factors that 2.07 on the C scale is almost

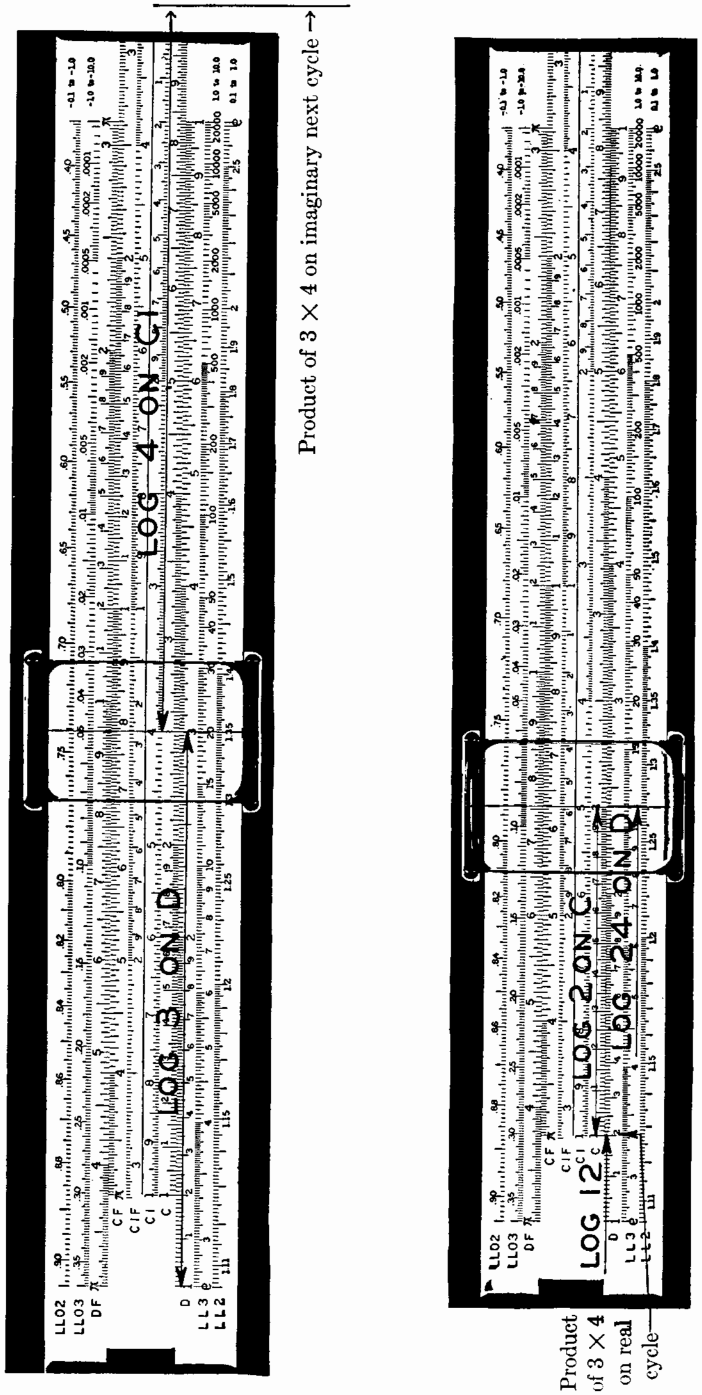
at the left end of the rule, but that on the CF scale it is very close to the center and also closer to the runner than on the other scale. Consequently, the better and simpler rule to follow is to choose the factors always as near to the center as possible regardless of every other consideration. Of course, at the beginning of an operation, a pair of upper group factors would naturally be placed on the upper scales, or similarly for a pair of lower group factors, but, once into an operation, this does not always result in moving toward the center.

Principle of Multiplication of Three Factors. As in the case of two factors, three factors are multiplied by adding their logarithms on the slide rule. Consider the simple example, 3 × 4 × 2. Fig. 40 shows the multiplication performed most efficiently using the D, CI, and C scales and beginning with the factor 3. Fig. 41 shows it performed inefficiently using only the D and C scales. These figures show how the logarithms are added in each case, but they do not show clearly the great advantage of one method over the other. This advantage is shown by the following procedures which contrast the two methods:

Multiplication **3 × 4 × 2**

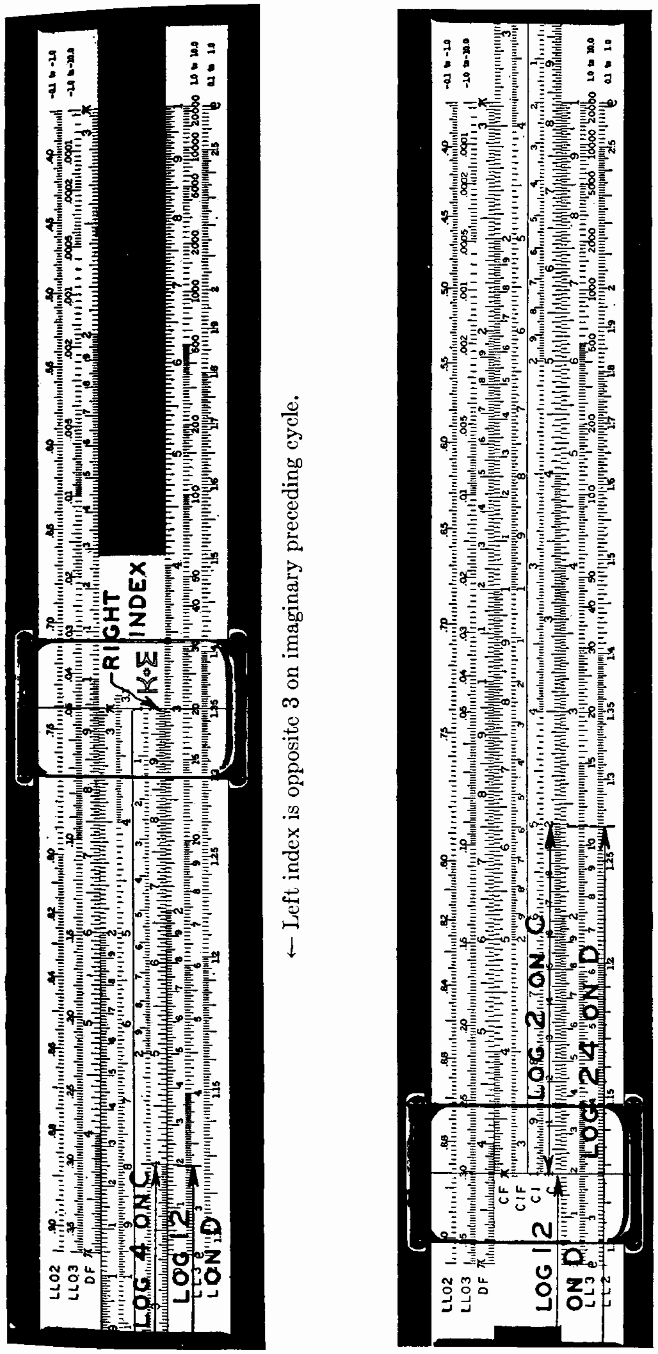
With D, CI, C Scales (Fig. 40)	*With D and C Scales* (Fig. 41)
Runner to 3 on D	Runner to 3 on D
Slide to 4 on CI	Slide to right index
Runner to 2 on C	Runner to 4 on C
Read 24 on D	Slide to left index
	Runner to 2 on C
	Read 24 on D
Distance: 0.2 scale length	Distance: 1.85 scale lengths

The inefficiency of the method using only the D and C scales is fully revealed as it requires two more settings and over nine times as much movement in this case as the method which uses three scales.



Product of 3 × 4 on imaginary next cycle →
LOG 4 ON CI
LOG 3 ON D
LOG 2 ON C
LOG 24 ON D
LOG 12
Product of 3 × 4 on real cycle

Fig. 40.



RIGHT INDEX
LOG 4 ON C
LOG 12 ON D
← Left index is opposite 3 on imaginary preceding cycle.
LOG 2 ON C
LOG 24 ON D
LOG 12 ON D

Fig. 41.

Exercise 4

Perform the following multiplications:

1. 9.92 × 8.73 × 7.96
2. 88.1 × 7.47 × 0.763
3. 8.86 × 16.1 × 0.680
4. 8.20 × 9.61 × 15.9
5. 106 × 0.0142 × 7.05
6. 1.01 × 1.17 × 0.108
7. 11.1 × 1.36 × 6.84
8. 9.55 × 0.932 × 8.18
9. 126.5 × 6.24 × 0.752
10. 1.202 × 6.91 × 0.801
11. 3.06 × 3.14 × 3.21
12. 291 × 0.336 × 2.06
13. 3.91 × 3.09 × 2.52
14. 0.329 × 5.10 × 1.870
15. 4.46 × 2.24 × 5.46
16. 50.1 × 4.91 × 2.01
17. 34.6 × 4.25 × 2.45
18. 5.24 × 32.4 × 0.489
19. 265 × 0.560 × 0.141
20. 460 × 0.351 × 0.137
21. 8.13 × 12.1 × 0.471
22. 9.31 × 6.48 × 0.333
23. 482 × 0.0359 × 9.99
24. 28.9 × 3.98 × 8.87
25. 11.25 × 22.3 × 3.34
26. 10.93 × 38.8 × 0.775
27. 9.09 × 8.08 × 3.03
28. 3.41 × 43.2 × 8.76
29. 2.39 × 2.15 × 0.0612
30. 7.71 × 8.00 × 0.426
31. 800 × 0.707 × 0.0667
32. 381 × 4.01 × 0.909
33. 4.39 × 57.1 × 0.0871
34. 0.083 × 0.93 × 34
35. 43.1 × 7.83 × 0.0522
36. 70.9 × 9.07 × 0.201
37. 0.222 × 1.11 × 0.0833
38. 8.93 × 3.37 × 99.4
39. 39.2 × 421 × 0.357
40. 43.1 × 777 × 0.0827

41. What is the volume of a rectangular box whose dimensions in feet are 2.53 × 3.82 × 8.66?

42. In making a highway fill, what volume of earth is used if the borrow-pit from which the earth was removed is 4.50 feet deep, 212 feet wide, and 1380 feet long?

43. If a man works 7.5 hours per day for 5 days per week, and works 47 weeks in a certain year, how many hours does he work in that year? If his wage rate is $1.50 per hour, approximately how much does he earn in the year?

44. The cost of manufacturing an electric refrigerator is $110.40. What is the approximate cost of a month's output of such refrigerators if the factory operates 22 days a month and produces 76 refrigerators per day.

45. The channel of the Mississippi River near Vicksburg is 3120 feet wide and has an average depth of 70.7 feet during a flood. If the average velocity of flow of the flood waters is 10.6 feet per second, what is the rate of flow in cubic feet per second?

14. Multiplication—Any Number of Factors. The multiplication of four factors adds two movements to the multiplication of three factors, the slide is moved to set the fourth factor and the runner (or the eye) is moved to an index to read. Again the controlling principle of operation is to *choose the factors as near the center of the rule as possible,* which results in most cases in taking the factors in pairs belonging to the same group. The following products serve as illustrations:

Multiplication **1.43 × 2.71 × 3.91 × 9.12** (Fig. 42)

Runner to 1.43 on DF
Slide to 9.12 on CIF
Runner to 2.71 on C
Slide to 3.91 on CI
Runner to upper index
Read 138 on DF

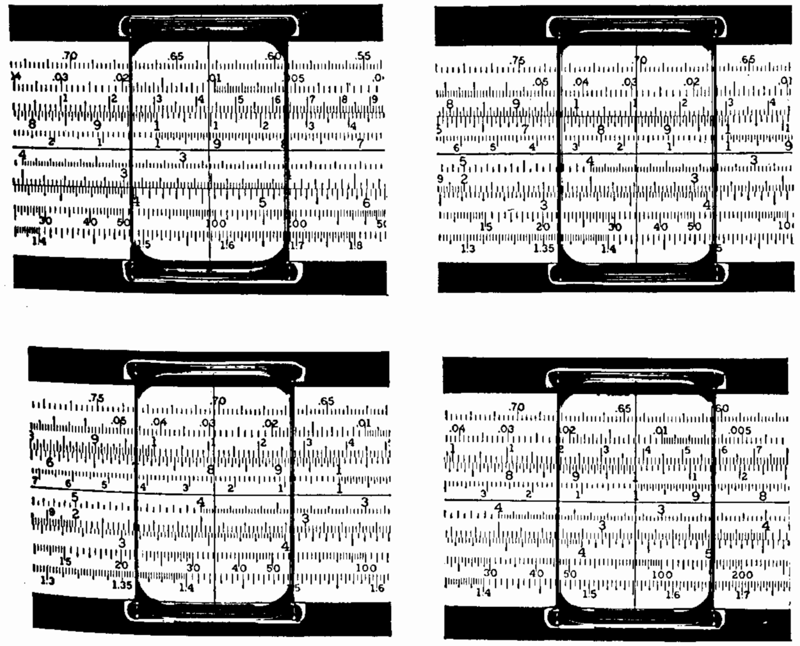


Fig. 42.

or beginning with the lower group factors,

Runner to 2.71 on D
Slide to 3.91 on CI
Runner to 9.12 on CF
Slide to 1.43 on CIF
Runner to upper index
Read 138 on DF

Multiplication **3.21 × 0.462 × 2.22 × 4.03**

Runner to 3.21 on D
Slide to 2.22 on CI
Runner to 0.462 on C
Slide to 4.03 on CI
Runner to upper index
Read 13.3 on DF

The significant fact which the student should recognize at this time is that after setting the first factor on one of the D scales, the remaining factors are set alternately on a CI scale and on a C scale. This is true regardless of the number of factors, the settings being first on a D scale, then a CI scale, a C scale, CI, C, CI, C, and so on until the last factor has been set either on a CI scale with the slide or on a C scale with the runner. Why this is so can be understood by the student from the preceding explanations of the theory of two- and three-factor multiplications; it is simply the successive addition of logarithms using the scales in such a way as to reduce the amount of movement required to a minimum or near-minimum. *There are two principles, then, which the student must keep in mind for most efficient multiplication: keeping close to the center, and alternating scales between the* CI *and the* C *scales in that order after setting the first factor.* The following multiplications illustrate these principles:

Multiplication **3.61 × 0.407 × 8.16 × 0.917 × 2.75**

Runner to 3.61 on D
Slide to 2.75 on CI
Runner to 0.917 on CF
Slide to 8.16 on CIF
Runner to 0.407 on C
Read 30.3 on D

Multiplication **1.09 × 3.55 × 2.71 × 8.16 × 4.39**

Runner to 1.09 on DF
Slide to 8.16 on CIF
Runner to 3.55 on C
Slide to 2.71 on CI
Runner to 4.39 on C
Read 376 on D

Multiplication **31.6 × 2.02 × 1.008 × 12.17 × 5.26**

Runner to 31.6 on D
Slide to 2.02 on CI
Runner to 12.17 on CF
Slide to 1.008 on CIF
Runner to 5.26 on C
Read 4120 on D

Setting the decimal point, the product of the first two factors is roughly 60, the product of the last two factors is roughly 60, and 60 × 60 = 3600.

Exercise 5

Perform the following multiplications:

1. 9.73 × 11.7 × 4.02 × 3.08
2. 20.5 × 0.439 × 8.05 × 9.03
3. 1.14 × 2.03 × 12.8 × 4.09

4. 30.4 × 9.06 × 2.48 × 8.01
5. 8.18 × 0.919 × 3.03 × 3.11
6. 0.376 × 16.57 × 0.851 × 4.07
7. 2.28 × 0.0371 × 98.1 × 1.50
8. 3380 × 0.0103 × 0.0490 × 8.01
9. 8.9 × 7.6 × 9.03 × 1.12
10. 0.443 × 0.0561 × 3.92 × 2.15
11. 1.70 × 1.80 × 1.90 × 2.00 × 0.961
12. 6.5 × 7.6 × 8.7 × 9.8 × 1.53
13. 12.1 × 2.02 × 4.12 × 0.0983
14. 7.27 × 0.828 × 9.39 × 1.50
15. 3.47 × 3.19 × 4.08 × 2.23
16. 7.52 × 8.54 × 9.57 × 3.51 × 0.465
17. 6.24 × 3.97 × 0.0716 × 4.27 × 9.45
18. 3.06 × 9.12 × 2.21 × 7.75 × 2.60
19. 6.65 × 1.93 × 0.717 × 3.62 × 8.22
20. 11.0 × 9.88 × 1.26 × 8.93 × 0.133

21. Water weighs 62.4 pounds per cubic foot. What is the total weight of water in a tank with inside dimensions in feet 2.44 × 3.63 × 8.02?

22. Mercury is approximately 13.6 times as heavy as water. What weight of mercury is required to fill a rectangular container 0.426 feet deep, 1.24 feet wide, and 1.463 feet long?

23. What is the approximate cost of paving an airport runway with a concrete slab 1.05 feet thick, 242 feet wide, and 5460 feet long if the concrete in place costs $0.612 per cubic foot?

24. In measuring the power of an electric motor using an electric dynamometer, horsepower is determined from the relation $HP = 0.000191\, LNW$. What is the horsepower of a motor for which L, the length of the brake arm, is 2.16 feet, N, the speed of the shaft, is 3060 revolutions per minute, and W, the load on the scale, is 72.2 pounds?

25. Suppose that the average use of electricity in a 7-room house is 0.550 kilowatts per room, and that the average time of use is 2.92 hours. What is the approximate monthly (31-day) cost of electricity in the house if the rate is $0.0225 per kilowatt-hour?

15. Division—Any Number of Factors. The same general principles as outlined at the beginning of this chapter apply to division as well as to multiplication. Also as in multiplication, the most efficient operation in division is obtained by choosing the factors as near the center of the rule as possible. In fact, the rules for multiplication as expressed on page 60 also apply to division with but two small modifications. One is that division *must begin* by placing the *numerator* on a *D scale*. The other is that factors after the first factor are set alternately on the C and CI scales in that order instead of alternately on the CI and C scales as for multiplication. In other words, the use of the CI and C scales is just reversed from multiplication to division.

It is desirable at this point to illustrate what is meant by two-factor division, three-factor division, and so on.

$$\text{Two-factor division: } \frac{17.2}{12.1}$$

$$\text{Three-factor division: } \frac{132.6}{8.71 \times 4.23}$$

$$\text{Four-factor division: } \frac{4743}{6.21 \times 3.43 \times 7.29}$$

$$\text{Five-factor division: } \frac{2170}{5.07 \times 7.12 \times 3.23 \times 1.09}$$

In multiplying four factors, the product of the first two is multiplied by the third, and that product multiplied in turn by the fourth factor. In a four-factor division, the quotient of the first two factors is divided by the third, and that quotient in turn divided by the fourth factor.

It should be emphasized that, although the multiplication of a series of factors may be begun with any one of the factors, the division of one factor by a series of others *must* begin by placing that factor first on a D scale. In the preceding examples of divisions with different numbers of factors, the operation in each instance *must* commence with the numerator: in the first example, place 17.2 on the DF scale; in the second, place 132.6 on the DF scale; in the third, place 4743 as 4740 on the D scale; in the last, place 2170 on the D scale. As in multiplication the factors should be taken in pairs belonging to the same group. This is illustrated in the following solutions for the preceding examples. Note the use of the scales on the slide in the order C, CI, C, CI, and so on.

Division (Fig. 43)

$$\frac{17.2}{12.1}$$

Runner to 17.2 on DF
Slide to 12.1 on CF
Runner to upper index
Read 1.42 on DF

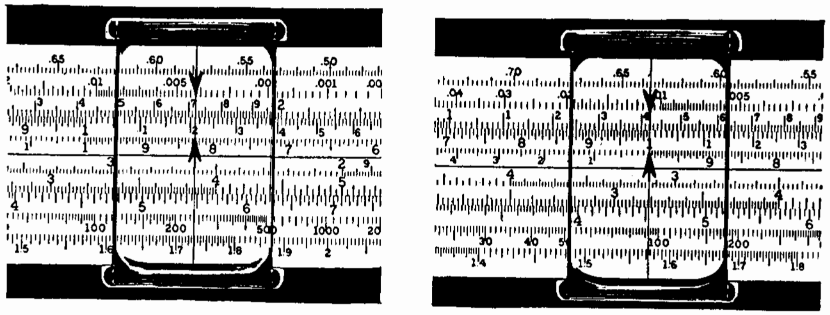


FIG. 43.

Division

$$\frac{385}{47.2}$$

Runner to 385 on D
Slide to 47.2 on C
Runner to upper index
Read 8.16 on DF

Division (Fig. 44)

$$\frac{132.6}{8.71 \times 4.23}$$

Runner to 132.6 on DF
Slide to 8.71 on CF
Runner to 4.23 on CI
Read 3.60 on D

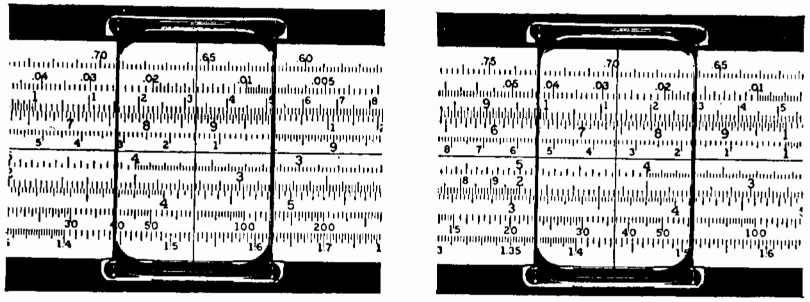


FIG. 44.

Division

$$\frac{4743}{6.21 \times 3.43 \times 7.29}$$

Runner to 4740 on D
Slide to 3.43 on C
Runner to 7.29 on CIF
Slide to 6.21 on CF
Runner to lower left index
Read 30.5 on D

Division

$$\frac{2170}{5.07 \times 7.12 \times 3.23 \times 1.09}$$

Runner to 2170 on D
Slide to 3.23 on C
Runner to 7.12 on CIF
Slide to 1.09 on CF
Runner to 5.07 on CI
Read 17.1 on D

Attention is called again to the almost identical methods applying to multiplication and division. Also, as in the multiplication of two factors in different groups, the division of two factors in different groups, "mixed divisions," is most efficient with the same slight modification of the standard technique. "Mixed divisions" include all cases where the numerator is of one group and *all* factors in the denominator are of another group. All of these cases will be explained later after the standard technique has been mastered.

Principle of Division with Two and Three Factors. When two quantities are divided by the use of logarithms, such as $\frac{a}{b} = c$, the solution is as follows:

$$\log \frac{a}{b} = \log a - \log b = \log c$$

Consequently in dividing one factor by another on the slide rule, a simple subtraction of logarithms is performed. On the slide rule the division of 6 by 3 to give 2 is done as log 6 − log 3 = log 2, as can be readily seen in Fig. 45.

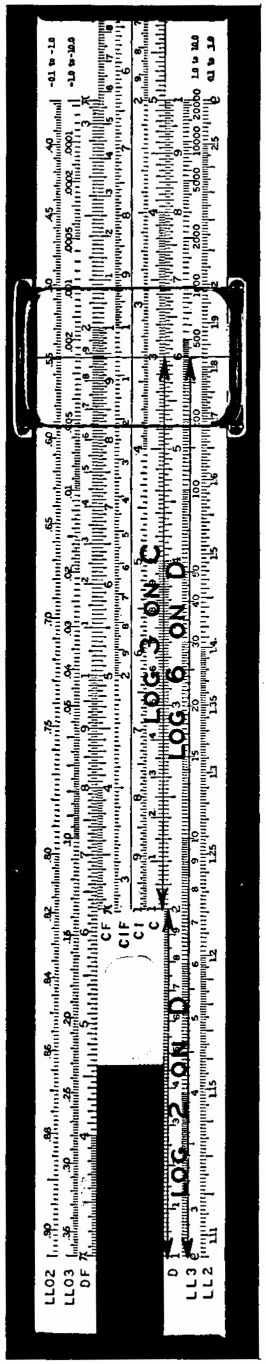
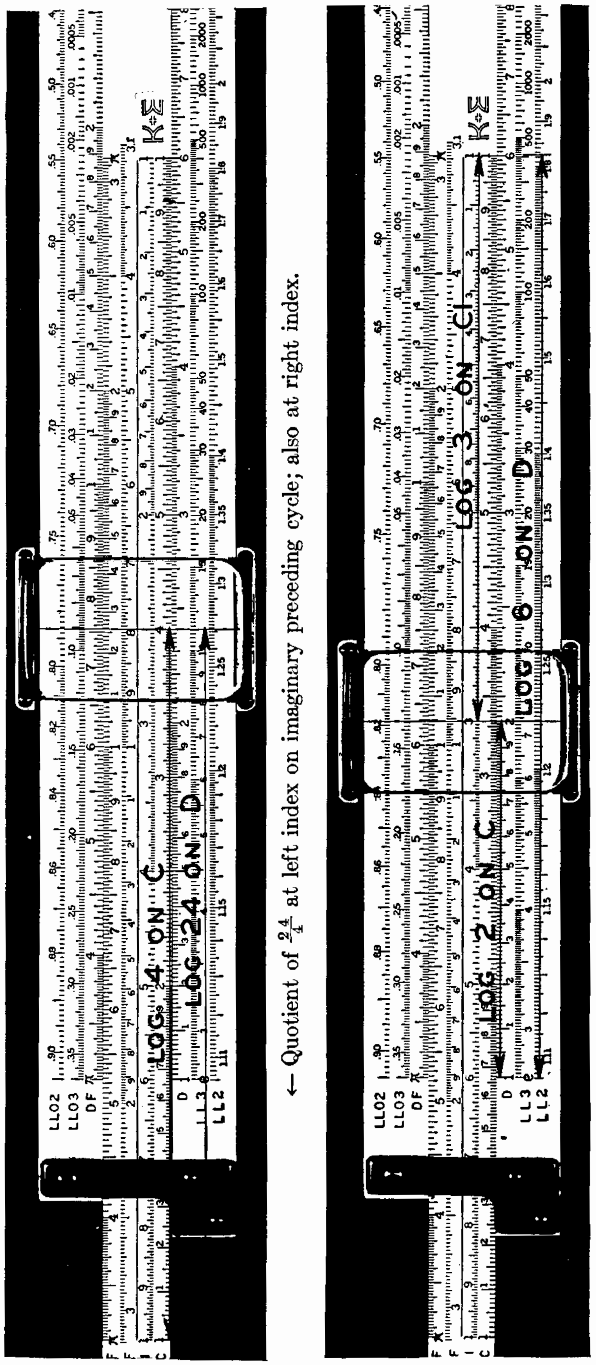


Fig. 45.



← Quotient of $\frac{24}{4}$ at left index on imaginary preceding cycle; also at right index.

Fig. 46.

In dividing one factor by two factors on the slide rule, such as $\frac{24}{4 \times 3} = 2$, the operation is done as log 24 − log 4 − log 3 = log 2, as is shown in Fig. 46.

Exercise 6

Perform the following divisions:

1. $\frac{75.1}{9.16}$
2. $\frac{6.71}{11.4}$
3. $\frac{131.2}{7.93}$
4. $\frac{47.8}{3.77}$
5. $\frac{212.4}{40.9}$
6. $\frac{37.1}{20.3}$
7. $\frac{7.92}{6.17}$
8. $\frac{38.2}{43.4}$
9. $\frac{760}{82.4}$
10. $\frac{49.6}{3250}$
11. $\frac{1463}{7.12 \times 11.5}$
12. $\frac{824.7}{12.22 \times 13.6}$
13. $\frac{97.6}{7.23 \times 1.09}$
14. $\frac{4970}{39.3 \times 2.64}$
15. $\frac{32.8}{4.03 \times 3.04}$
16. $\frac{37.2}{6.91 \times 4.32}$
17. $\frac{121.7}{3.75 \times 8.23}$
18. $\frac{8750}{91.1 \times 24.2}$
19. $\frac{52.6}{4.17 \times 128.5}$
20. $\frac{4375}{27.1 \times 3.18 \times 5.05}$
21. $\frac{11{,}210}{8.46 \times 16.2 \times 91.4}$
22. $\frac{291.2}{5.71 \times 2.34 \times 3.97}$
23. $\frac{879}{9.79 \times 6.21 \times 1.76}$
24. $\frac{3730}{1.24 \times 41.6 \times 7.74}$
25. $\frac{9250}{4.83 \times 3.71 \times 8.68}$
26. $\frac{2{,}750{,}000}{0.875 \times 0.375 \times 18.2 \times 576}$

27. $\dfrac{87{,}400}{1200 \times 2.43 \times 1.005 \times 0.812}$

28. $\dfrac{4440}{8.22 \times 2.36 \times 1.73 \times 31.1}$

29. $\dfrac{91.7}{1.43 \times 2.34 \times 0.30 \times 6.51}$

30. $\dfrac{3110}{4.27 \times 1.72 \times 2.04 \times 9.70}$

31. A rectangular plot has an area of 2620 square feet. What is the length of the long side if the short side is 38.6 feet?

32. How many miles of highway can be surfaced for $840,000 if the surfacing costs $0.523 per square foot and the roadway is 18 feet wide?

33. What is the density of the steel in a block which weighs 1925 pounds and has the following dimensions in feet: 1.37 × 0.785 × 3.69?

34. If mercury is 13.6 times as heavy as water and water weighs 62.4 pounds per cubic foot, what is the volume of 13.24 pounds of mercury?

35. A highway fill of 46,200 cubic yards is to be obtained from a rectangular borrow pit 121 yards wide and 202 yards long. What is the depth of the borrow pit in yards to yield this volume of fill?

36. The list price of a milling machine for metal working in a machine shop is $4750.00. How much discount is allowed the purchaser if it is sold for $3820.00?

37. A state institution asks the state legislature for an annual appropriation of $3,420,000. If the institution is granted $2,710,000, what per cent of its original request does it receive?

38. The revolution counter on a Diesel engine in a power plant shows that the engine has made 347,000 revolutions during a 24-hour run. What has been its average speed in revolutions per minute?

39. What is the average speed of a modern streamline train in traveling a distance of 281 miles in 3 hours and 57 minutes?

40. Find the time in minutes for one complete hauling cycle for a machine which hauls earth from a borrow pit to a fill under the following conditions:

	Haul Road Section	*Length in Feet*	*Average Speed in Miles per Hour*
LOADED HAUL	Borrow pit	400	6.0
	Haul road	2200	17.0
	Fill-packed	300	8.0
RETURN EMPTY	Fill-packed	400	7.0
	Haul road	2200	20.0
	Borrow pit	400	8.0

The total time for loading, turning, and dumping is 2.37 minutes.

16. Multiplication-division Combinations. By a multiplication-division combination is meant a fractional expression of at least three factors in which two are in the numerator. The following expressions are examples:

$$\frac{13.6 \times 8.17}{127}$$

$$\frac{3.37 \times 71.6}{2.68 \times 81.7}$$

$$\frac{14.1 \times 6.21 \times 0.82}{12.8}$$

The procedures used in solving such expressions follow the basic principles as set forth at the beginning of this chapter and also the rule of choosing the factors as close to the center as possible and in pairs belonging to the same group. The student has already learned how to perform a series of multiplications and also a series of divisions, but he has not learned how to proceed in changing from multiplication to division, or vice versa, during an operation. The rule is quite simple and is more easily grasped when stated immediately following the rule for a series of multiplications or divisions. In performing a series of multiplications or a series of divisions, the first factor is set on the D scales and the remaining factors are set *alternately* on the C and CI scales; in performing a multiplication-division combination in which the operations are *alternately* multiplication and division, the first factor is set on the D scales and the remaining factors are *all* set on the *same* scales, either the C scales if the first operation is a division, or the CI scales if the first operation is a multiplication. This rule is illustrated by the following examples:

$$\frac{13.6 \times 8.17}{127}$$ (Fig. 47)

Runner to 13.6 on DF

Division — Slide to 127 on CF

Multiplication — Runner to 8.17 on CF

Read 0.875 on DF

$$\frac{3.37 \times 71.6}{2.68 \times 81.7}$$ (Fig. 48)

Runner to 3.37 on D

Division — Slide to 2.68 on C

Multiplication — Runner to 71.6 on CF

Division — Slide to 81.7 on CF

Runner to upper index

Read 1.10 on DF

$$\frac{14.1 \times 6.21 \times 0.82}{12.8}$$ (Fig. 49)

Runner to 14.1 on DF

Multiplication — Slide to 6.21 on CIF

Division — Runner to 12.8 on CIF

Multiplication — Slide to 0.820 on CIF

Runner to upper index

Read 5.61 on DF

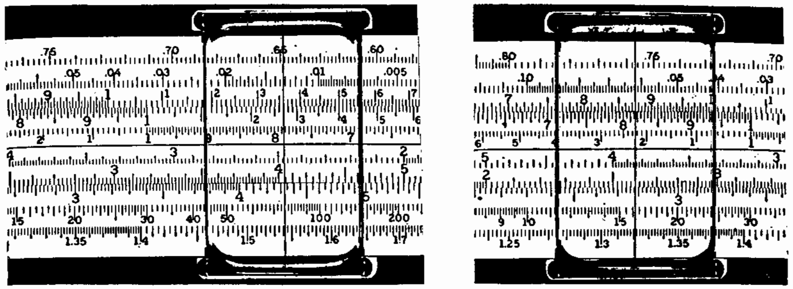


Fig. 47.

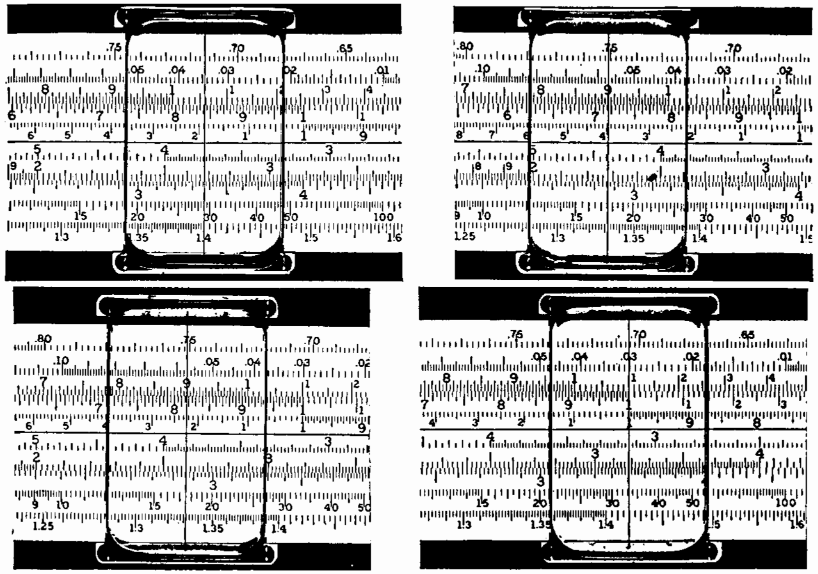


Fig. 48.

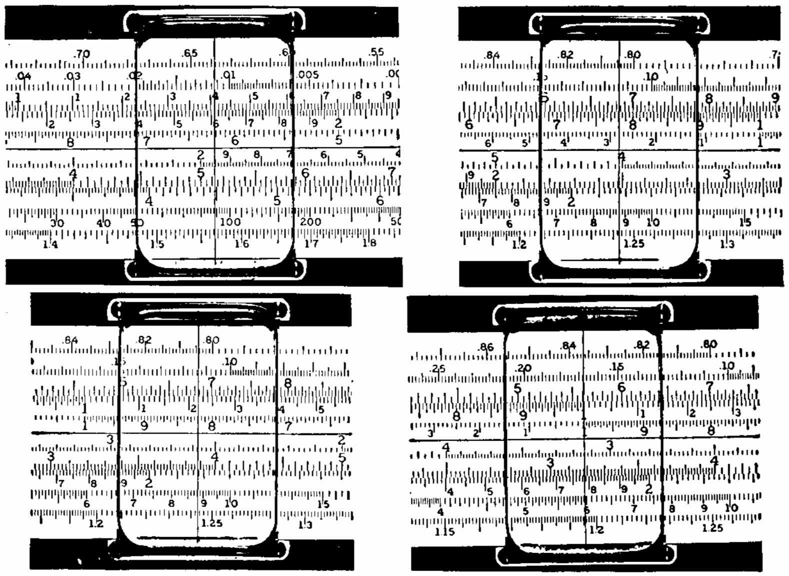


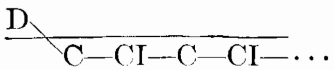
Fig. 49.

In the first two examples all factors after the first factor are set on the C scales because the operations alternate between multiplication and division. Similarly in the third example, all factors after the first factor are set on the CI scales.

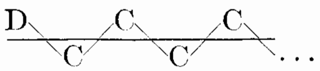
Diagrams of the procedures for multiplication, division, and combinations are useful in learning the procedures. Visualize the computations as proceeding horizontally or vertically. For example, the multiplication of a series of factors is a horizontal procedure, $a \times b \times c \times d$, whereas the alternating division and multiplication of a combination of factors is a vertical procedure, $\frac{a \times c}{b \times d}$, in which factors are taken alternately in numerator, denominator, numerator, and denominator. For horizontal or repeating operation, alternate slide scales, and for vertical or alternating operation, repeat slide scales. These rules are illustrated in the following diagrams:

Multiplication D—CI—C—CI—C—···

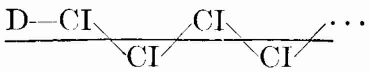
Division



Alternating Combination



Alternating Combination



It is understood that D, CI, and C in the preceding diagrams also represent the DF, CIF, and CF scales.

Keep in mind the general principle which states that an operation is always begun by setting the first factor on a D scale. Then referring to the remaining factors which are all placed on the C or the CI scales on the slide, *for repeating operations, alternate scales,* and *for alternating operations, repeat scales.* A series of multiplications is an example of repeating operations, therefore alternate between the CI and C scales. For a multiplication-division combination performed by alternating operations, repeat scales, either the C scales or the CI scales. A study of the following comparative illustrations will assist the student in grasping the ideas:

Multiplication $\mathbf{3.1 \times 4.2 \times 2.7 \times 4.9}$

Runner to 3.1 on D
Slide to 2.7 on CI
Runner to 4.2 on C
Slide to 4.9 on CI
Runner to upper index
Read 172 on DF

Combination $\mathbf{\frac{3.1 \times 4.2}{2.7 \times 4.9}}$

Runner to 3.1 on D
Slide to 2.7 on C
Runner to 4.2 on C
Slide to 4.9 on C
Runner to upper index
Read 0.98 on DF

Division $\mathbf{\frac{420}{3.1 \times 2.7 \times 4.9}}$

Runner to 420 on D
Slide to 4.9 on C
Runner to 2.7 on CI
Slide to 3.1 on C
Runner to upper index
Read 10.2 on DF

Combination $\dfrac{3.1 \times 4.9 \times 2.7}{4.2}$

Runner to 3.1 on D

Slide to 2.7 on CI

Runner to 4.2 on CI

Slide to 4.9 on CI

Runner to upper index

Read 9.8 on DF

Note in the multiplication and division that factors are placed alternately on the C and CI scales as the operation of multiplication or division is repeated. Note that the two combinations are performed with alternating multiplication and division and that factors are set on the same scale, either the C scale or the CI scale.

This completes the presentation of rules for multiplication and division. The principle of the use of the scales in multiplication-division combinations is again the simple addition and subtraction of logarithms as has been explained for a series of multiplications or divisions and it does not seem necessary or desirable to give additional explanations of this type. It is urged that the student, himself, work out the application of the principle to multiplication-division combinations as an exercise.

It is not always advisable to solve multiplication-division combinations with alternating operation. *The rule of choosing factors as near the center as possible for most efficient operation* should dictate the order of operation. In all of the illustrations of combinations which have been used so far in this section, the factors are paired in such a way that most efficient operation is obtained by alternating multiplication and division. However, the following examples illustrate the general application of the rules in keeping as close to the center as possible and in performing whatever order of operation is indicated in order to stay close to the center:

Combination $\dfrac{8.16 \times 11.4}{2.85 \times 3.29}$

Diagram DF—CIF ╲ CI—C (Fig. 50)

Solve by taking the two upper group factors first and then the two lower group factors.

Runner to 8.16 on DF

Multiplication Slide to 11.4 on CIF

Division Runner to 3.29 on CI

Division Slide to 2.85 on C

Runner to upper index

Read 9.94 on DF

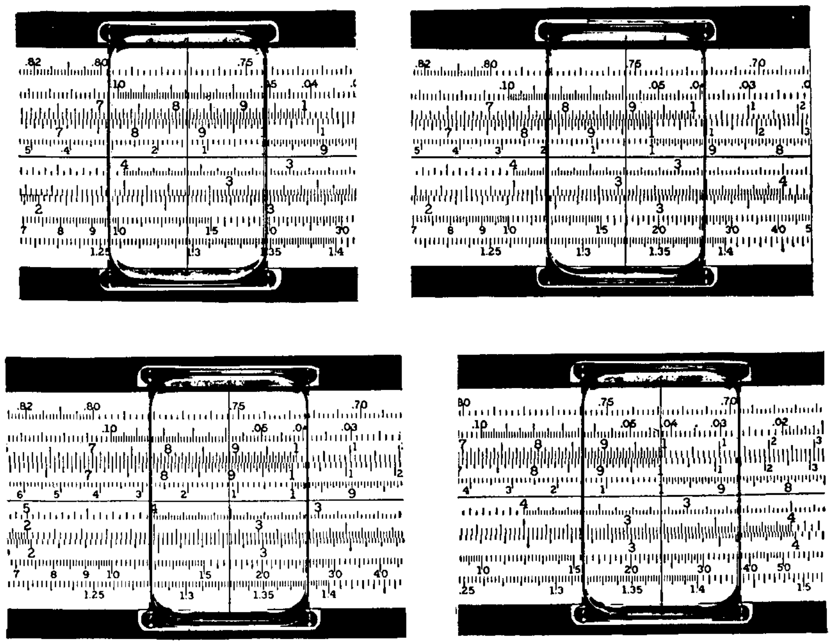


Fig. 50.

Combination $\dfrac{4.34 \times 47.2 \times 1.95}{13.77}$

Diagram

D—CI—C
C

	Runner to 4.34 on D
Multiplication	Slide to 47.2 on CI
Multiplication	Runner to 1.95 on C
Division	Slide to 13.77 on C
	Runner to lower left index
	Read 29.0 on D

Combination $\dfrac{7.67 \times 4.23}{0.931}$

Diagram

DF—CF
C

	Runner to 7.67 on DF
Division	Slide to 0.931 on CF
Multiplication	Runner to 4.23 on C
	Read 34.8 on D

Combination $\dfrac{32.8 \times 46.4}{6.12 \times 8.89 \times 0.122}$

Diagram

D—CI
CIF—CF—CI

	Runner to 32.8 on D
Multiplication	Slide to 46.4 on CI
Division	Runner to 0.122 on CIF
Division	Slide to 8.89 on CF
Division	Runner to 6.12 on CI
	Read 229 on D

Setting the decimal point in the last example, 6 × 8 in the denominator equals 48 which cancels 46 in the numerator, and 32 divided by 0.1 is approximately 300, hence the answer 229.

Exercise 7

Perform the following computations:

1. $\dfrac{12.3 \times 8.16}{1.42}$
2. $\dfrac{32.7 \times 4.21}{273}$
3. $\dfrac{89.7 \times 6.24}{7.37}$
4. $\dfrac{132.4 \times 9.09}{15.71}$
5. $\dfrac{50.6 \times 2.18}{4.72}$
6. $\dfrac{32.6 \times 3.81}{272}$
7. $\dfrac{8.97 \times 4.27}{11.6 \times 3.58}$
8. $\dfrac{36.3 \times 15.2}{10.9 \times 4.73}$
9. $\dfrac{7.17 \times 1.25}{83.6 \times 0.972}$
10. $\dfrac{47.2 \times 7.31}{3.66 \times 8.08}$
11. $\dfrac{2.47 \times 7.24}{3.94 \times 8.62}$
12. $\dfrac{13.6 \times 3.97 \times 9.03}{15.8 \times 2.62}$
13. $\dfrac{33.6 \times 6.18 \times 20.2}{4.17 \times 4.55}$
14. $\dfrac{1.06 \times 3.37 \times 8.16}{0.881 \times 2.51}$
15. $\dfrac{7.50 \times 8.62 \times 3.21}{9.73 \times 5.05}$
16. $\dfrac{2.47 \times 4.16}{7.76}$
17. $\dfrac{1320 \times 8.02}{324}$
18. $\dfrac{9.16 \times 7.23}{5.16}$
19. $\dfrac{3.42 \times 49.7}{1.07}$
20. $\dfrac{11.2 \times 8.63}{2.91 \times 3.17}$
21. $\dfrac{4.63 \times 3.27}{8.99 \times 12.62}$
22. $\dfrac{6.31 \times 7.29}{42.6 \times 0.373}$
23. $\dfrac{25.6 \times 54.7}{1130 \times 1.88}$
24. $\dfrac{2.27 \times 5.29}{2.05}$
25. $\dfrac{47.6 \times 6.21}{28.3 \times 0.731}$
26. $\dfrac{3.26 \times 1.05}{4.21 \times 8.93}$
27. $\dfrac{9.69 \times 22.43}{7.27 \times 3.25}$
28. $\dfrac{13.6 \times 8.21 \times 9.72}{32.7 \times 2.16}$
29. $\dfrac{43.6 \times 8.27}{3.07 \times 5.16 \times 7.71}$
30. $\dfrac{9.07 \times 7.92}{4.55 \times 2.64 \times 1.47}$

31. The area of a triangle is $\frac{1}{2} \times$ base $\times$ altitude. Compute the areas of triangles with the following dimensions:

	base	*altitude*
(a)	6.25 inches	2.69 inches
(b)	13.27 inches	36.3 inches
(c)	4.33 feet	11.68 feet
(d)	5.04 yards	7.37 yards

32. The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height. Compute the volumes of pyramids with rectangular bases with the following dimensions:

	base width	*base length*	*height*
(a)	2.63 feet	7.13 feet	12.27 feet
(b)	0.885 feet	1.462 feet	3.81 feet
(c)	4.27 inches	9.08 inches	1.46 inches

33. A pyramid with a triangular base is called a tetrahedron. Compute the volumes of the following tetrahedrons:

	base of tetrahedron		
	base of triangle	*altitude of triangle*	*height of tetrahedron*
(a)	1.64 feet	2.83 feet	4.08 feet
(b)	7.63 inches	32.5 inches	27.7 inches
(c)	3.31 feet	5.06 feet	8.28 feet

34. The indicated power of a steam engine is given by the relation:

$$HP = \frac{PLAN}{33{,}000}$$

where HP = horsepower
P = average cylinder pressure in pounds per square inch
L = length of stroke in feet
A = area of piston in square inches
N = strokes per minute (2 $\times$ r.p.m. per cylinder for a double-acting steam engine)

(a) What piston area is required in a double-acting steam engine to develop 1100 horsepower with an average steam pressure of 120 pounds per square inch? The speed is 220 revolutions per minute (440 strokes) and the length of stroke is 3 feet.

(b) What average pressure P is required in the steam engine of part (a) if it is to develop only 775 horsepower at 185 revolutions per minute?

35. Assuming that steel weighs 490 pounds per cubic foot, what is the height in feet of a pyramid of solid steel with a rectangular base 7.24 inches $\times$ 13.6 inches which weighs 418 pounds?

17. Mixed Operations. Mixed operations include the following:

(1) Multiplications of two factors of different groups.
(2) Divisions in which the factor in the numerator is of one group and *all* of the factors in the denominator are of the other group.

Mixed multiplications are solved by setting the lower scale factor first on the D scale and the upper scale factor next on the CIF scale. The runner is moved to the upper index and the answer is read back *down* on the lower D scale. For example:

Multiplication **2.50 $\times$ 1.40**

Runner to 2.50 on D
Slide to 1.40 on CIF
Runner to upper index
Read 3.50 on D

Fig. 51 shows how this modification of the standard technique still adds the logarithms properly to obtain the correct answer. In this modification the runner *must* follow the slide to the upper scales. There is no choice between scales for the runner as in the standard technique. Also, the answer *must* be read back down on the D scale although the runner is set finally on the upper scales on the slide.

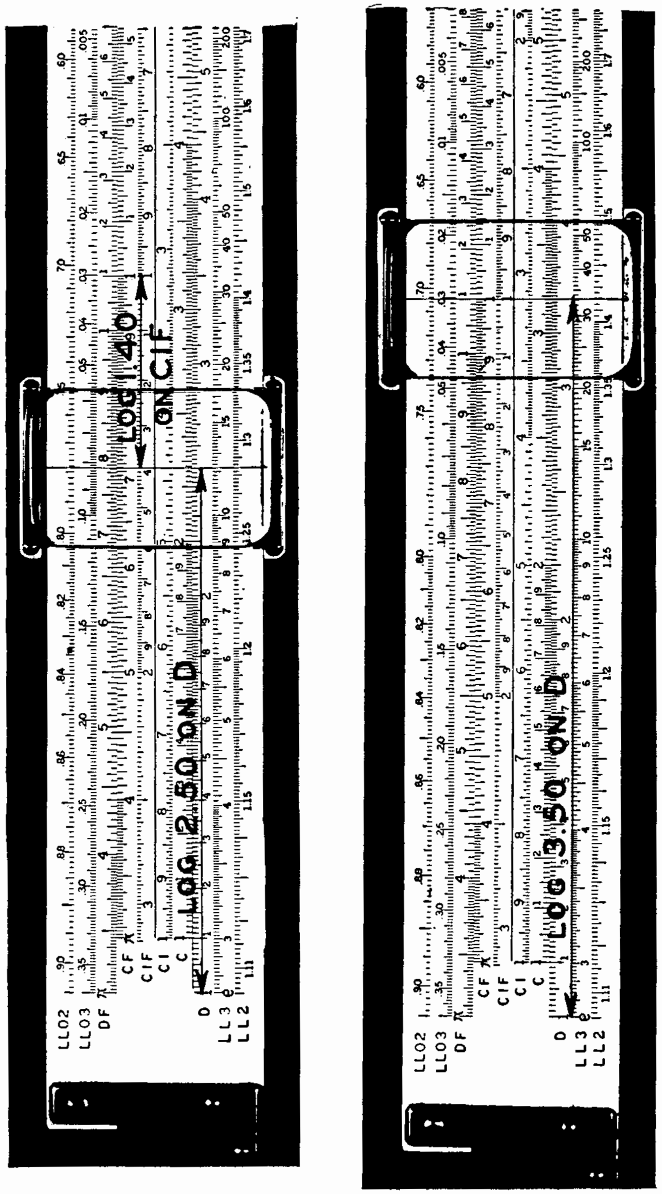


Fig. 51.

Another example:

Multiplication **4.31 × 7.22** (Fig. 52)

Runner to 4.31 on D
Slide to 7.22 on CIF
Runner to upper index
Read 31.1 on D

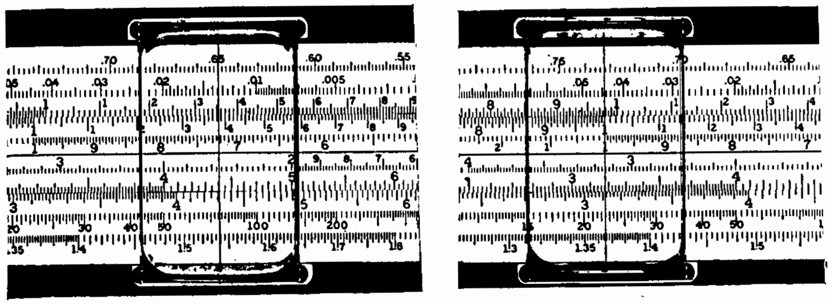


Fig. 52.

Mixed multiplications can occur in general in multiplications of an even number of factors where the last two factors are in different groups. For example, consider 1.24 × 7.27 × 5.29 × 8.12. Solve, beginning with the first pair of factors on the upper scales and with the last two as a mixed operation:

Runner to 1.24 on DF
Slide to 7.27 on CIF
Runner to 5.29 on C
Slide to 8.12 on CIF
Runner to upper index
Read 387 on D

It is not usually efficient to solve for such products beginning with the upper group factor on the DF scale because in such cases the runner would have to be moved to a lower index to read and the lower index would be near the end of the rule. Mixed multiplications should ordinarily be solved by setting the lower group factor first on the D scale and then the upper group factor on the CIF scale. Occasionally, however, the operation would be performed using the standard technique. For example: 4.80 × 1.70.

Runner to 4.80 on D

Slide to 1.70 on CI

Runner to upper index (or lower right index)

Read 8.16 on DF (or D)

In this example the slide is moved less than if the factor 1.70 had been set on the CIF scale and, correspondingly, is closer to the center position. Again it is emphasized that the "center-drift" idea should control the operation.

Mixed divisions are solved similarly. If the numerator is a lower group factor and the denominator consists of upper group factors, the numerator is set on the D scale and the other factors are set alternately on the CF and CIF scales. The answer is read back down on the D scale. For example:

Division $\dfrac{3.43}{9.17}$ (Fig. 53)

Runner to 3.43 on D

Slide to 9.17 on CF

Runner to upper index

Read 0.374 on D

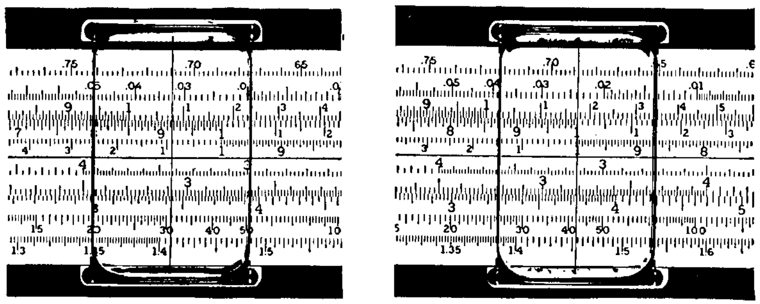


FIG. 53.

Division $\dfrac{286}{7.51 \times 9.42}$

Runner to 286 on D

Slide to 9.42 on CF

Runner to 7.51 on CIF

Read 4.04 on D

If the numerator is an upper group factor, begin on the upper scales. For example:

Division $\dfrac{142.1}{37.6}$ (Fig. 54)

Runner to 142.1 on DF

Slide to 37.6 on C

Runner to lower left index

Read 3.78 on DF

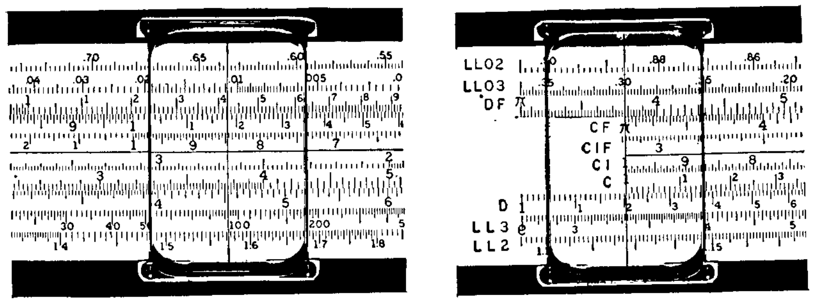


FIG. 54.

Division $\dfrac{85.7}{3.46 \times 2.49}$

Runner to 85.7 on DF

Slide to 2.49 on C

Runner to 3.46 on CI

Read 9.95 on DF

In the first of these two divisions where the numerator is an upper group factor and where the number of factors is even, the lower index must be read. This is not very efficient since the runner is moved close to the end of the rule. However, if the practice of reading the index by eye is followed instead of using the hairline, efficiency may be maintained. Note in such cases that the mark for π opposite the lower index is used. In the second division where the number of factors is odd and the runner is set on the last factor, all settings are confined to the central portion of the rule.

Mixed multiplications and mixed divisions may also occur in combination operations with an even number of factors where the last two factors are in different groups. They would be solved by the modified technique just discussed or by the standard procedure, whichever kept the runner and slide closer to the center. Usually the modified technique would result in less movement. An example of such a combination is the following:

$$\frac{\mathbf{27.6 \times 3.84}}{\mathbf{4.17 \times 9.25}}$$

Runner to 27.6 on D
Slide to 3.84 on CI
Runner to 4.17 on CI
Slide to 9.25 on CF
Runner to upper index
Read 2.75 on D

EXERCISE 8

Perform the following computations:

1. 4.36 × 7.82
2. 28.8 × 8.15
3. 13.62 × 31.4
4. 5.28 × 6.75
5. 2.07 × 124.8
6. 79.6 × 2.41
7. 18.25 × 21.3
8. 9.61 × 4.16
9. $\frac{27.3}{8.46}$
10. $\frac{429}{11.95}$
11. $\frac{36.3}{75.2}$
12. $\frac{5.88}{12.02}$
13. $\frac{84.6}{3.97}$

14. $\frac{113.5}{29.9}$
15. $\frac{445}{1.76 \times 9.29}$
16. $\frac{2460}{72.3 \times 8.48}$
17. $\frac{87.7}{22.5 \times 3.74}$
18. $\frac{136.4}{2.88 \times 20.3}$
19. $\frac{135.5 \times 2.86}{1.462 \times 0.848}$
20. $\frac{927}{4.63 \times 3.47 \times 2.82}$
21. $\frac{4060}{8.26 \times 13.09 \times 7.71}$
22. $\frac{3.57 \times 4.25}{2.66 \times 1.37}$
23. 3.73 × 3.14 × 4.45 × 6.07
24. 2.63 × 0.333 × 9.09 × 4.12
25. 8.16 × 1.44 × 3.64 × 1.04
26. $\frac{2470}{1.240 \times 8.63 \times 0.975}$
27. $\frac{9.63 \times 7.24}{11.69 \times 2.33}$
28. $\frac{2.63 \times 12.08}{3.45 \times 3.68}$
29. 10.08 × 36.3
30. $\frac{35.5}{8.59}$

18. Summary. The chapter on multiplication and division is summarized as follows:

1. On the duplex type of slide rule, two groups of scales are used, the upper group consisting of the DF, CF, and CIF scales and the lower group consisting of the D, C, and CI scales. The "upper group" numbers beginning with 6, 7, 8, 9, or 1 fall approximately within the middle one-half of the upper group scales. The "lower group" numbers beginning with 2, 3, 4, and 5 fall approximately within the middle one-half of the lower group scales.

2. In general, always choose the factors as near the center of the rule as possible. This usually requires taking the factors in pairs, either two upper group factors or two lower group factors.

3. Always begin and end a computation with the runner.
Always begin a computation with a setting and end with a reading on one of the two D scales.
There is always a choice, implicitly, of setting the runner either on the upper group or the lower group every time it is moved, except in mixed operations.
The runner and slide are always moved alternately beginning and ending with the runner. The slide follows the runner to

that group of scales to which the runner is moved except in mixed multiplications and mixed divisions.

4. After setting the first factor on a D scale, the remaining factors are all set on the slide scales as follows:

(a) To start with multiplication, use D and CI scales.
(b) To start with division, use D and C scales.
(c) For repeating operations, alternate scales (between C and CI).
(d) For alternating operations, repeat scales (either C or CI).

In the above rules, D, C, and CI are understood to mean D or DF, C or CF, and CI or CIF.

5. For mechanical ease of operation a color system for following multiplication or division in proper sequence may be preferred by some students. Visualize the computation in fractional form. If the operation proceeds horizontally, there is a change in the color of scales used; if the operation proceeds vertically, there is no change in color, speaking of the *black* C and CF scales and the *red* CI and CIF scales. The first factor is always set on one of the black D scales, hence an operation always begins with black. Examples of this rule are shown in general without regard to whether the factors are upper scale factors or lower scale factors. Beside each example is shown the corresponding operation in terms of the lower group of scales only so that the comparison may be seen at a glance.

Multiplication Black—Red—Black—Red—· · ·

D—CI—C—CI—· · ·

Division

Black\
———————
\Black—Red—Black—Red—· · ·

D\
———————
\C—CI—C—CI—· · ·

Combinations

Black\ /Black\
———————
\Black/ \Black

D\ /C\
———————
\C/ \C

Black—Red\ /Red\
———————
\Red/ \Red

D—CI\ /CI\
———————
\CI/ \CI

Black\ /Red
———————
\Black—Red/

D\ /CI
———————
\C—CI/

Black—Red\
———————
\Red—Black

D—CI\
———————
\CI—C

As the operation repeats, as is shown by horizontal operation, —— × —— × —— × ——, change from black to red, or vice versa.

As the operation changes, as is shown by vertical operation, \/\/\/\/, repeat the same color, either black or red.

EXERCISE 9

Perform the following review computations:

1. $\dfrac{9.63 \times 4.17}{1.234}$

2. 5.06×6.72

3. $\dfrac{82.8}{1320}$

4. $\dfrac{421 \times 303}{7940}$

5. $2.43 \times 4.17 \times 3.24 \times 6.92$

6. $\dfrac{41.6 \times 8.08}{113 \times 2.31}$

7. $\dfrac{1.62 \times 2.71}{3.43 \times 7.92}$

8. $\dfrac{4240}{7.55 \times 11.2 \times 27.4}$

9. 17.63×61.7

10. $\dfrac{38.7}{4.98}$

11. $\dfrac{3.72 \times 13.34 \times 4.07}{1.005}$

12. $23.6 \times 0.721 \times 5.03$

13. $\dfrac{14.1 \times 9.37}{4.11 \times 2.93}$

14. $\dfrac{13{,}200}{39.6}$

15. $1.766 \times 16.25 \times 1.880$

16. $\dfrac{333}{1.084 \times 12.35 \times 2.05}$

17. $7.07 \times 0.771 \times 76.7$

18. $\dfrac{2.17 \times 0.0983}{72.1 \times 0.870}$

19. $\dfrac{152.3}{8.53}$

20. $\dfrac{17.33 \times 31.6}{111.5}$

21. $0.535 \times 1.47 \times 3.44$

22. $\dfrac{24.7}{8.11 \times 2.02}$

23. $6.72 \times 2.76 \times 0.885 \times 3.47 \times 11.23$

24. $\dfrac{28{,}300}{42.1 \times 3.72 \times 5.05}$

25. 57.6×1.845

26. $\dfrac{17.6 \times 2.84}{27.3}$

27. $\dfrac{8.37 \times 6.04}{2.73}$

28. $\dfrac{7.28 \times 53.6 \times 1.377}{22.2 \times 0.882}$

29. $\dfrac{4.21 \times 1.170}{8.08 \times 0.312}$

30. $\dfrac{27.2}{4.73}$

31. $\dfrac{14.55 \times 9.04}{31.6 \times 2.33}$

32. 16.27×7.24

33. $3.72 \times 3.14 \times 28.2$

34. $\dfrac{57.6}{2.12 \times 4.37 \times 1.84}$

35. $\dfrac{4.95 \times 6.84}{9.73}$

36. $\dfrac{6.87}{1.92 \times 5.53}$

37. $\dfrac{15.65 \times 7.16}{4.29 \times 3.73}$

38. 19.92×0.383

39. $\dfrac{67.5}{892}$

40. $423 \times 6.18 \times 2.44$

III

SQUARES, SQUARE ROOTS, CUBES, AND CUBE ROOTS

19. Numbers Greater than One—Squares and Square Roots. Obtaining the squares and square roots of numbers is a much simpler process than that of multiplication and division and makes no use of the rules developed in the preceding chapter. The A and B scales are the square and square root scales and are usually on the reverse side of the duplex type slide rule from the side used for multiplication and division. Simple squares and square roots are obtained using only the A and D scales. The B scale is seldom used except in operations involving multiplication or division along with squares or square roots. The A and B scales are "two-cycle" scales, each having two portions with each portion extending from 1 through 9.[3] The left portion or cycle is designated as "cycle 1" and the right portion as "cycle 2," as is shown in Fig. 55.

The mechanics of operation in squaring a number or in extracting the square root are very simple. To square a number, set the number on the D scale with the runner and without further movement read its square on the A scale opposite the hairline. The reverse process is followed for square roots; set the number on the A scale and read its square root on the D scale. For example:

$(2)^2 = 4$ (Fig. 56)	$(6)^2 = 36$ (Fig. 57)
Runner to 2 on D	Runner to 6 on D
Read 4 on A	Read 36 on A
$\sqrt{4} = 2$	$\sqrt{36} = 6$
Runner to 4 on A (cycle 1)	Runner to 36 on A (cycle 2)
Read 2 on D	Read 6 on D

[3] The term "cycle" has been defined and explained in Section 5, p. 9.

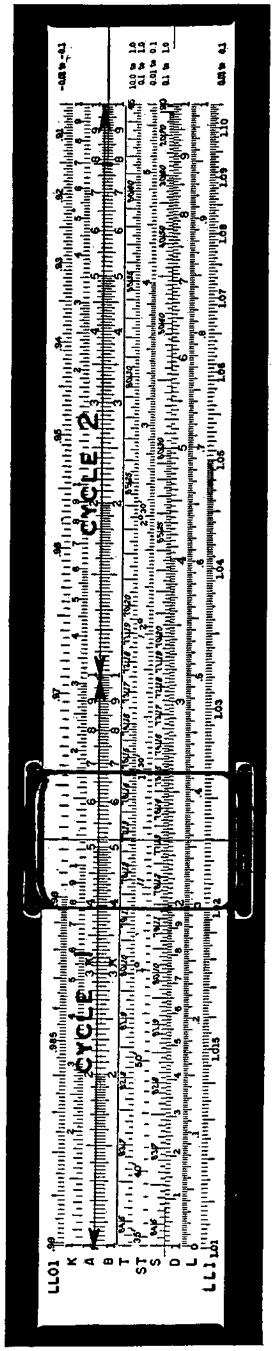


Fig. 55.

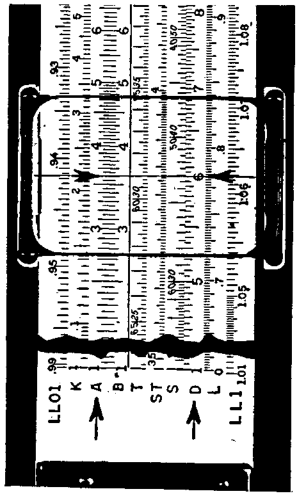


Fig. 57.

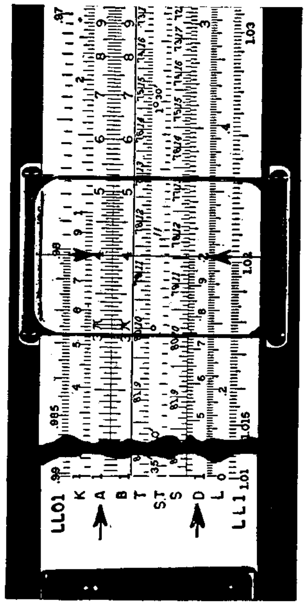


Fig. 56.

Briefly then, for squares, go from D to A; for square roots, go from A to D. Obviously the only matter requiring attention in so simple an operating procedure is the question of setting the decimal point, which also involves whether to use cycle 1 or cycle 2 of the A scale in extracting a square root.

The key to setting the decimal point in squares or square roots is in the idea of "two for one" or, vice versa, "one for two." In squaring a number there are always *two* places in the answer for *each* place in the number with but one exception. In extracting the square root of a number there is always *one* place in the answer for each *two* places in the number with but one exception.

Decimal Point Rules: Squares and Square Roots

These rules have to do with determining the number of places to the left of the decimal point in the square or square root of a number greater than one.

Squares: To locate the decimal point in the square of a number greater than one, count the places to the left of the decimal point in the number. There are two places in the answer for each place counted in the number except the last. If the answer falls on cycle 1, count one place for the last; if on cycle 2, count two places.

Square Roots: Count the places to the left of the decimal point in the number in groups of two. If the last group has only one place, set the number on cycle 1; if it has two places, set on cycle 2. There is one place to the left of the decimal point in the square root for every complete or partial group just counted.

These rules are illustrated in the following examples:

SQUARES

	Operation	*Decimal Point Count*	
(23.4)^2	Runner to 23.4 on D	(2 3. 4)^2	
	Read 5 4 8 on A, cycle 1	$1+2=3$	*Answer:* 548

Count two for the first place to the left of the decimal point and one for the second place since the answer falls on cycle 1 (Fig. 58).

$(473.6)^2$ Runner to 473.6 on D — $(4\ 7\ 3.\ 6)^2$

Read 2 2 4 on A, cycle 2 — 2+2+2 =6 — *Answer:* 224,000

Count two for each place to the left of the decimal point since the answer falls on cycle 2 (Fig. 59).

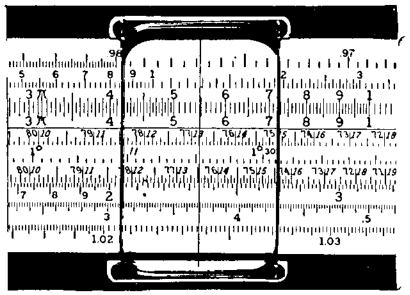


FIG. 58.

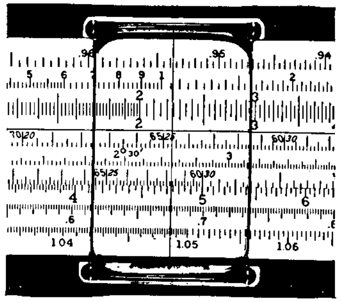


FIG. 59.

$(174)^2$ Runner to 174 on D — $(1\ 7\ 4)^2$

Read 3 0 3 on A, cycle 1 — 1+2+2 =5 — *Answer:* 30,300

Count two each for the first two places to the left of the decimal point and one for the third since the answer falls on cycle 1.

$(79.2)^2$ Runner to 79.2 on D — $(7\ 9.\ 2)^2$

Read 6 2 8 on A, cycle 2 — 2+2 =4 — *Answer:* 6280

Count two for each place to the left of the decimal point since the answer falls on cycle 2.

$(2.46)^2$ Runner to 2.46 on D — $(2.\ 4\ 6)^2$

Read 6 0 5 on A, cycle 1 — 1 — *Answer:* 6.05

Count one for the place to the left of the decimal point since the answer falls on cycle 1.

$(4.93)^2$ Runner to 4.93 on D — $(4.\ 9\ 3)^2$

Read 2 4 3 on A, cycle 2 — 2 — *Answer:* 24.3

Count two for the place to the left of the decimal point since the answer falls on cycle 2.

SQUARE ROOTS

	Cycle and Decimal Point	*Operation*
$\sqrt{1440}$	$\underline{14}\ \underline{40}$ 2 → cycle 2	Runner to 1440 on A, cycle 2 Read 3 8. 0 on D

There are two groups to the left of the decimal point, giving two places in the answer (Fig. 60).

$\sqrt{793.2}$	$\underline{7}\ \underline{93}.\ 2$ 1 → cycle 1	Runner to 793.2 on A, cycle 1 Read 2 8. 2 on D

There are two groups to the left of the decimal point, giving two places in the answer (Fig. 61).

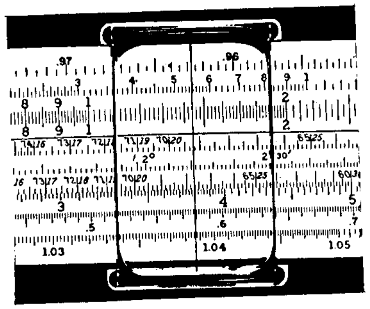


FIG. 60.

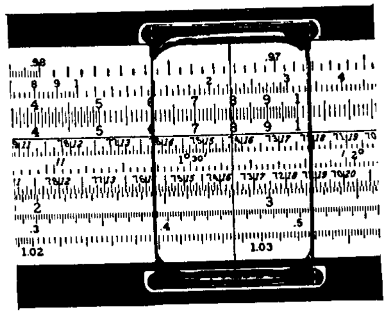


FIG. 61.

$\sqrt{375{,}000}$	$\underline{37}\ \underline{5{,}0}\ \underline{00}$ 2 → cycle 2	Runner to 375,000 on A, cycle 2 Read 6 1 2 on D

There are three groups to the left of the decimal point, giving three places in the answer.

$\sqrt{7.63}$	$\underline{7}.\ 63$ 1 → cycle 1	Runner to 7.63 on A, cycle 1 Read 2. 7 6 on D

There is one group to the left of the decimal point, giving one place in the answer.

$\sqrt{57.4}$	$\underline{57}.\ 4$ 2 → cycle 2	Runner to 57.4 on A, cycle 2 Read 7. 5 8 on D

There is one group to the left of the decimal point, giving one place in the answer.

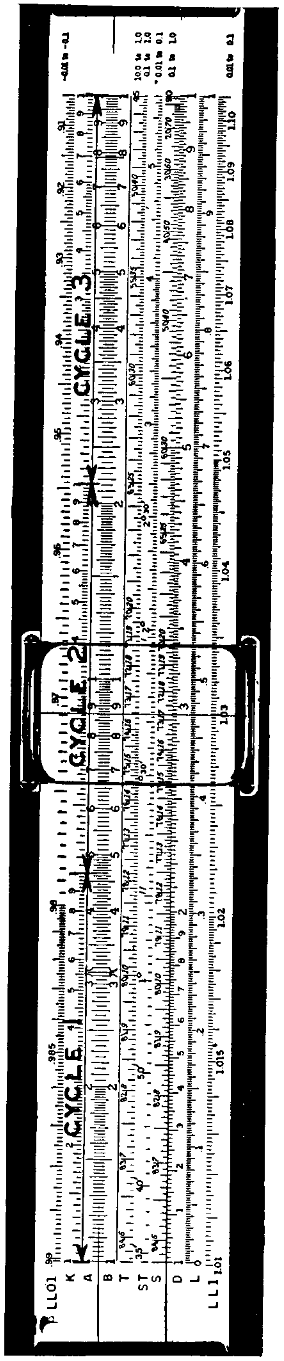


FIG. 62.

EXERCISE 10

Evaluate the following squares and square roots:

1. $(17.3)^2$
2. $(8.42)^2$
3. $(213)^2$
4. $(1.92)^2$
5. $(5.88)^2$
6. $(63.1)^2$
7. $(2.02)^2$
8. $(41.6)^2$
9. $(12.8)^2$
10. $(123)^2$
11. $(759)^2$
12. $(1710)^2$
13. $(4440)^2$
14. $(6.83)^2$
15. $(33.9)^2$
16. $\sqrt{8.13}$
17. $\sqrt{36.6}$
18. $\sqrt{449}$
19. $\sqrt{1680}$
20. $\sqrt{2.07}$
21. $\sqrt{13.48}$
22. $\sqrt{5.89}$
23. $\sqrt{83.7}$
24. $\sqrt{107.2}$
25. $\sqrt{23{,}600}$
26. $\sqrt{8480}$
27. $\sqrt{432{,}000}$
28. $\sqrt{6{,}250{,}000}$
29. $\sqrt{893}$
30. $\sqrt{57.6}$

20. Numbers Greater than One—Cubes and Cube Roots. The K scale is the cube scale of the engineer's slide rule and is usually on the same side of the duplex rule as the A and B scales. The mechanics of operation are the same as for the A scale. The K scale differs, however, from the A scale in having three cycles instead of two. These are designated from left to right as cycle 1, cycle 2, and cycle 3, as shown in Fig. 62. The decimal point rules are obtained simply by transposing the two-cycle rules for the A scale to rules for three cycles, using the ideas of *three for one* and *one for three.*

Decimal Point Rules: Cubes and Cube Roots

These rules are for obtaining the cubes and cube roots of numbers greater than one.

Cubes: To locate the decimal point in the cube of a number greater than one, count the places to the left of the decimal point in the number. There are three places in the answer for each place counted in the number except the last. If the answer falls on cycle 1, count one place for the last; if on cycle 2, count two places; if on cycle 3, count three places.

Cube Roots: Count the places to the left of the decimal point in the number in groups of three. If the last group has only one place, set the number on cycle 1; if two places, on cycle 2; if three places, on cycle 3. There is one place in the cube root for every complete or partial group just counted.

These rules are illustrated in the following examples:

CUBES

	Operation	*Decimal Point Count*	
$(14.62)^3$	Runner to 14.62 on D	$(1\ 4.\ 6\ 2)^3$	
	Read 3 1 2 on K, cycle 1	1+3 =4	*Answer:* 3120

Count three for the first place to the left of the decimal point and one for the second place since the answer falls on cycle 1 (Fig. 63).

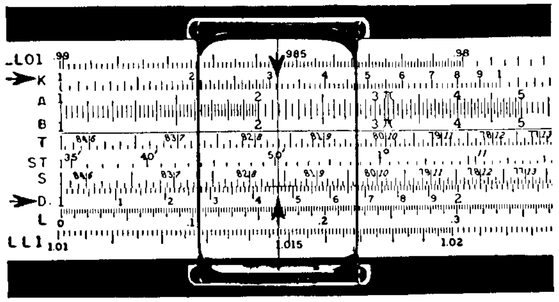


FIG. 63.

$(72.6)^3$ Runner to 72.6 on D $(7\ 2.\ 6)^3$

Read 3 8 3 on K, cycle 3 $3+3=6$ *Answer:* 383,000

Count three for each place to the left of the decimal point since the answer falls on cycle 3 (Fig. 64).

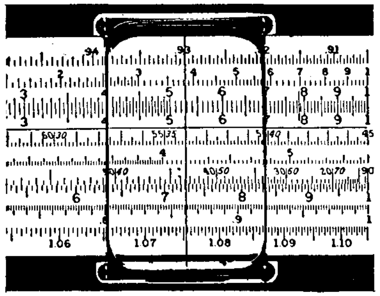


FIG. 64.

$(37.1)^3$ Runner to 37.1 on D $(3\ 7.\ 1)^3$

Read 5 1 0 on K, cycle 2 $2+3=5$ *Answer:* 51,000

Count three for the first place to the left of the decimal point and two for the second since the answer falls on cycle 2 (Fig. 65).

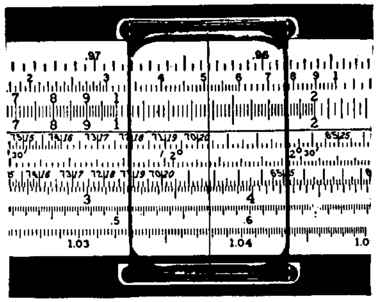


FIG. 65.

$(1.67)^3$ Runner to 1.67 on D $(1.\ 6\ 7)^3$

Read 4 6 5 on K, cycle 1 1 *Answer:* 4.65

Count one for the place to the left of the decimal point since the answer falls on cycle 1.

$(3.59)^3$ Runner to 3.59 on D $(3.\ 5\ 9)^3$

Read 4 6 3 on K, cycle 2 2 *Answer:* 46.3

Count two for the place to the left of the decimal point since the answer falls on cycle 2.

$(5.54)^3$ Runner to 5.54 on D $(5.\ 5\ 4)^3$

Read 1 7 0 on K, cycle 3 3 *Answer:* 170

Count three for the place to the left of the decimal point since the answer falls on cycle 3.

CUBE ROOTS

	Cycle and Decimal Point	*Operation*
$\sqrt[3]{16{,}400}$	$\underline{16}\ \underline{400}$ $2 \rightarrow$ cycle 2	Runner to 16,400 on K, cycle 2 Read 2 5. 4 on D

There are two groups to the left of the decimal point, giving two places in the answer (Fig. 66).

$\sqrt[3]{273.1}$	$\underline{273.}\ 1$ $3 \rightarrow$ cycle 3	Runner to 273.1 on K, cycle 3 Read 6. 4 9 on D

There is one group to the left of the decimal point, giving one place in the answer (Fig. 67).

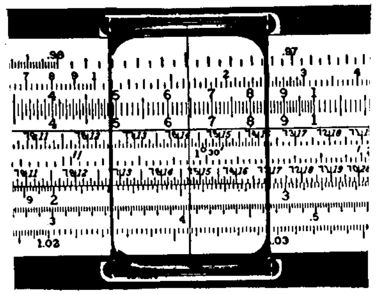


FIG. 66.

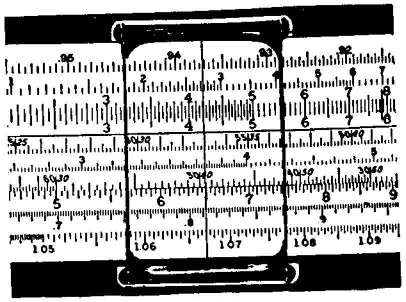


FIG. 67.

$\sqrt[3]{1752}$ 1 752   Runner to 1752 on K, cycle 1

1 → cycle 1   Read 1 2. 0 3 on D

There are two groups to the left of the decimal point, giving two places in the answer.

$\sqrt[3]{8.96}$ 8. 96   Runner to 8.96 on K, cycle 1

1 → cycle 1   Read 2. 0 8 on D

There is one group to the left of the decimal point, giving one place in the answer.

$\sqrt[3]{46.23}$ 46. 23   Runner to 46.23 on K, cycle 2

2 → cycle 2   Read 3. 5 9 on D

There is one group to the left of the decimal point, giving one place in the answer.

Exercise 11

Evaluate the following cubes and cube roots:

1. $(12.6)^3$
2. $(7.51)^3$
3. $(1.84)^3$
4. $(303)^3$
5. $(4.29)^3$
6. $(54.7)^3$
7. $(2.37)^3$
8. $(46.1)^3$
9. $(10.08)^3$
10. $(196)^3$
11. $(748)^3$
12. $(1330)^3$
13. $(50.7)^3$
14. $(9.59)^3$
15. $(62.4)^3$
16. $\sqrt[3]{46.9}$
17. $\sqrt[3]{327}$
18. $\sqrt[3]{9.29}$
19. $\sqrt[3]{2.71}$
20. $\sqrt[3]{3680}$
21. $\sqrt[3]{24.8}$
22. $\sqrt[3]{6.42}$
23. $\sqrt[3]{70.7}$
24. $\sqrt[3]{18{,}300}$
25. $\sqrt[3]{6440}$
26. $\sqrt[3]{278{,}000}$
27. $\sqrt[3]{1{,}728{,}000}$
28. $\sqrt[3]{49{,}900}$
29. $\sqrt[3]{29{,}800{,}000}$
30. $\sqrt[3]{1066}$

21. Numbers Less than One—Squares and Square Roots. The same rules which are used for numbers greater than one can also be used for numbers less than one by using a decimal point shift. Here, as before, the ideas of two for one and one for two are still useful.

Squares: Move the decimal point to the right to obtain a number between 1 and 10. Square this, using the rules for numbers greater than one. Move the decimal point in the square back to the left *twice* as many places as it was moved to the right.

Square Roots: Move the decimal point to the right *two* places at a time to obtain a number between 1 and 100. Obtain the square root of this number using the rules for numbers greater than one. Move the decimal point in the square root back to the left *one* place for every *group of two places* it was moved to the right.

These rules are illustrated in the following examples:

SQUARES

$(0.168)^2$ Move the decimal point *one* place to right
$(1.68)^2 = 2.82$
Move decimal point *two* places to left
Answer: 0.0282 [4]

$(0.503)^2$ Move decimal point *one* place to right
$(5.03)^2 = 25.3$
Move decimal point *two* places to left
Answer: 0.253

$(0.0172)^2$ Move decimal point *two* places to right
$(1.72)^2 = 2.96$
Move decimal point *four* places left
Answer: 0.000296

[4] The number of places moved to the left are underlined.

$(0.00797)^2$ Move decimal point *three* places to right
$(7.97)^2 = 63.5$
Move decimal point *six* places to left
Answer: 0.0000635

$(0.000056)^2$ Move decimal point *five* places to right
$(5.6)^2 = 31.4$
Move decimal point *ten* places to left
Answer: 0.00000000314

SQUARE ROOTS

$\sqrt{0.432}$ Move decimal point *two* places to right $\sqrt{0.\ 43\ 2}$
$\sqrt{43.2} = 6.57$
Move decimal point *one* place to left
Answer: 0.657

$\sqrt{0.0643}$ Move decimal point *two* places to right $\sqrt{0.\ 06\ 43}$
$\sqrt{6.43} = 2.54$
Move decimal point *one* place to left
Answer: 0.254

$\sqrt{0.00141}$ Move decimal point *four* places to right $\sqrt{0.\ 00\ 14\ 1}$
$\sqrt{14.1} = 3.75$
Move decimal point *two* places to left
Answer: 0.0375

$\sqrt{0.000383}$ Move decimal point *four* places to right $\sqrt{0.\ 00\ 03\ 83}$
$\sqrt{3.83} = 1.96$
Move decimal point *two* places to left
Answer: 0.0196

$\sqrt{0.00000942}$ Move decimal point *six* places to right $\sqrt{0.00\ 00\ 09\ 42}$
$\sqrt{9.42} = 3.07$
Move decimal point *three* places to left
Answer: 0.00307

$\sqrt{0.0000765}$ Move decimal point *six* places to right $\sqrt{0.\ 00\ 00\ 76\ 5}$
$\sqrt{76.5} = 8.75$
Move decimal point *three* places to left
Answer: 0.00875

EXERCISE 12

Evaluate the following squares and square roots:

1. $(0.828)^2$
2. $(0.309)^2$
3. $(0.165)^2$
4. $(0.0233)^2$
5. $(0.0716)^2$
6. $(0.528)^2$
7. $(0.00122)^2$
8. $(0.000923)^2$
9. $(0.00382)^2$
10. $(0.000222)^2$
11. $(0.0463)^2$
12. $(0.911)^2$
13. $(0.262)^2$
14. $(0.0107)^2$
15. $(0.669)^2$
16. $\sqrt{0.417}$
17. $\sqrt{0.863}$
18. $\sqrt{0.0870}$
19. $\sqrt{0.0205}$
20. $\sqrt{0.195}$
21. $\sqrt{0.000637}$
22. $\sqrt{0.0493}$
23. $\sqrt{0.00165}$
24. $\sqrt{0.0773}$

25. $\sqrt{0.000884}$
26. $\sqrt{0.000111}$
27. $\sqrt{0.00000607}$
28. $\sqrt{0.0000929}$
29. $\sqrt{0.000000235}$
30. $\sqrt{0.000807}$

22. Numbers Less than One—Cubes and Cube Roots. The decimal point rules are similar to those for squares and square roots.

Cubes: Move the decimal point to the right to obtain a number between 1 and 10. Cube this using the rules for numbers greater than one. Move the decimal point in the cube back to the left *three times* as many places as it was moved to the right.

Cube Roots: Move the decimal point to the right *three* places at a time to obtain a number between 1 and 1000. Obtain the cube root of this number using the rules for numbers greater than one. Move the decimal point in the cube root back to the left *one* place for every *group of three places* it was moved to the right.

These rules are illustrated in the following examples:

CUBES

$(0.426)^3$

Move decimal point *one* place to right
$(4.26)^3 = 77.5$
Move decimal point *three* places to left
Answer: 0.0775

$(0.187)^3$

Move decimal point *one* place to right
$(1.87)^3 = 6.55$
Move decimal point *three* places to left
Answer: 0.00655

$(0.658)^3$

Move decimal point *one* place to right
$(6.58)^3 = 285$
Move decimal point *three* places to left
Answer: 0.285

$(0.0132)^3$

Move decimal point *two* places to right
$(1.32)^3 = 2.30$
Move decimal point *six* places to left
Answer: 0.00000230

$(0.00519)^3$

Move decimal point *three* places to right
$(5.19)^3 = 140$
Move decimal point *nine* places to left
Answer: 0.000000140

$(0.0327)^3$

Move decimal point *two* places to right
$(3.27)^3 = 35.0$
Move decimal point *six* places to left
Answer: 0.0000350

CUBE ROOTS

$\sqrt[3]{0.0842}$

Move decimal point *three* places to right — $\sqrt[3]{0.\ \underline{084}\ 2}$
$\sqrt[3]{84.2} = 4.38$
Move decimal point *one* place to left
Answer: 0.438

$\sqrt[3]{0.76}$

Move decimal point *three* places to right — $\sqrt[3]{0.\ \underline{760}}$
$\sqrt[3]{760} = 9.13$
Move decimal point *one* place to left
Answer: 0.913

$\sqrt[3]{0.0043}$ Move decimal point *three* places to right $\sqrt[3]{0.\ 004\ 3}$

$\sqrt[3]{4.3} = 1.627$

Move decimal point *one* place to left

Answer: 0.1627

$\sqrt[3]{0.0000442}$ Move decimal point *six* places to right $\sqrt[3]{0.\ 000\ 044\ 2}$

$\sqrt[3]{44.2} = 3.53$

Move decimal point *two* places to left

Answer: 0.0353

$\sqrt[3]{0.0001232}$ Move decimal point *six* places to right $\sqrt[3]{0.\ 000\ 123\ 2}$

$\sqrt[3]{123.2} = 4.97$

Move decimal point *two* places to left

Answer: 0.0497

$\sqrt[3]{0.00000000665}$ Move decimal point *nine* places to right $\sqrt[3]{0.\ 000\ 000\ 006\ 65}$

$\sqrt[3]{6.65} = 1.88$

Move decimal point *three* places to left

Answer: 0.00188

EXERCISE 13

Evaluate the following cubes and cube roots:

1. $(0.183)^3$
2. $(0.426)^3$
3. $(0.817)^3$
4. $(0.0763)^3$
5. $(0.255)^3$
6. $(0.00684)^3$

7. $(0.01205)^3$
8. $(0.000831)^3$
9. $(0.00444)^3$
10. $(0.000241)^3$
11. $(0.0717)^3$
12. $(0.357)^3$
13. $(0.0909)^3$
14. $(0.116)^3$
15. $(0.00267)^3$
16. $\sqrt[3]{0.0456}$
17. $\sqrt[3]{0.693}$
18. $\sqrt[3]{0.00155}$
19. $\sqrt[3]{0.168}$
20. $\sqrt[3]{0.01375}$
21. $\sqrt[3]{0.00493}$
22. $\sqrt[3]{0.0000328}$
23. $\sqrt[3]{0.000187}$
24. $\sqrt[3]{0.00000561}$
25. $\sqrt[3]{0.00719}$
26. $\sqrt[3]{0.000000143}$
27. $\sqrt[3]{0.0000562}$
28. $\sqrt[3]{0.000456}$
29. $\sqrt[3]{0.01083}$
30. $\sqrt[3]{0.00697}$

23. Accuracy of Square and Cube Scales. The A and B scales which are used for squares are two-cycle scales, each cycle being one half as long as the cycle on the D scale. The accuracy of the D scale was found in Section 6 to be about 1 in 1000. The corresponding accuracy of the A and B scales is then about 2 in 1000 or 1 in 500. This means that the A and B scales can be read at the left end of a cycle to about 2 in the fourth significant figure and at the right end of a cycle to about 2 in the third significant figure. Numbers can be read to three significant figures on the A and B scales up to the 5's, where the accuracy is about 1 in the third significant figure. From the 5's through the 9's, it is very difficult to read closer than to 2 in the third significant figure.

The K scale, being a three-cycle scale, is correspondingly less accurate, having an accuracy of about 3 in 1000 or 1 in 333. Numbers on the K scale can be read to three significant figures up to the 3's where the accuracy is about 1 in the third significant figure. Readings to three significant figures in the 4's through the 9's on this scale will be inaccurate.

If the square or the cube of a number is desired more accurately than can be obtained on the A or K scales, the number should be multiplied two or three times in succession using the D, C, CI, DF, CF, and CIF scales.

24. Principle of the Square and Cube Scales. The square and cube scales are logarithmic scales, as are the scales which are used for multiplication and division. In solving for the square of a number by logarithms, we have

$$(D)^2 = A$$
$$\log (D)^2 = 2 \log D = \log A$$

Hence a distance on the A scale must represent twice as much as the corresponding distance on the D scale, which means that the A scale must have two cycles to the D scale's one.

In solving for the cube of a number by logarithms we have

$$(D)^3 = K$$
$$\log (D)^3 = 3 \log D = \log K$$

Here a distance on the K scale must represent three times as much as the corresponding distance on the D scale; hence the three cycles on the K scale.

IV

THE LOG LOG SCALES

25. Introduction. The log log scales are used for obtaining powers and roots of numbers. They serve a purpose similar to that of the square and cube scales but are not limited to just one power or root as are these scales. The log log scales are used to obtain powers and roots of numbers in general, such as $(4)^{7.2}$, $(1.07)^{23.4}$, $(75.3)^{\frac{1}{8.75}}$, $(0.092)^{4.37}$, and $(0.846)^{\frac{1}{0.425}}$. Of course, they could also be used to obtain the square or cube of a number or its square root or cube root. For numbers close to one, the log log scales can be read more accurately than the A and K scales and are sometimes used in their stead. Their principal use, however, is for finding fractional powers and exponents. The log log scales on the modern duplex slide rules are arranged in two groups, the LL1, LL2, and LL3 scales for obtaining powers and roots of numbers *greater than one* and the LL01, LL02, and LL03 scales for obtaining powers and roots of numbers *less than one.*

The LL1, LL2, and LL3 scales will be called the *LL scales.*

The LL01, LL02, and LL03 scales will be called the *LL0* scales.

The log log scales differ distinctly from the scales previously studied in that the decimal point is already fixed for numbers on the scale. The number 2 is in one place, 20 is in another, 200 is in another, and they cannot be interchanged. It also follows that the log log scales are not "cyclic" like the scales previously studied. Methods of operation will be presented for numbers greater than one with positive exponents, for numbers less than one with positive exponents, and for numbers in general with negative exponents. These methods apply respectively to the LL scales, the LL0 scales, and both groups of scales.

26. The LL1, LL2, and LL3 Scales. The LL1, LL2, and LL3 scales, or the LL scales, represent a single scale of numbers reading from left to right from 1.01 to approximately 22,000 which has been split into three parts, i.e., the LL2 scale begins at the left where the LL1 scale ends at the right, and similarly for the LL3 and LL2 scales. The LL2 and LL3 scales are on the front of the slide rule, but the LL1 scale is on the back. However, settings are made with the back hairline on the LL1 scale as if it were on the front of the rule, and operations are then continued on the front. These three scales are so arranged that the LL2 scale gives the tenth power of a number on the LL1 scale, and the LL3 scale gives the tenth power of a number on the LL2 scale. It follows that the LL3 scale gives the hundredth power of a number on the LL1 scale, the LL1 scale the tenth root of a number on the LL2 scale, and so on. For example:

Runner to 1.02 on LL1 (Fig. 68)

Read $(1.02)^{10}$ on LL2 as 1.219

Read $(1.02)^{100}$ on LL3 as 7.25

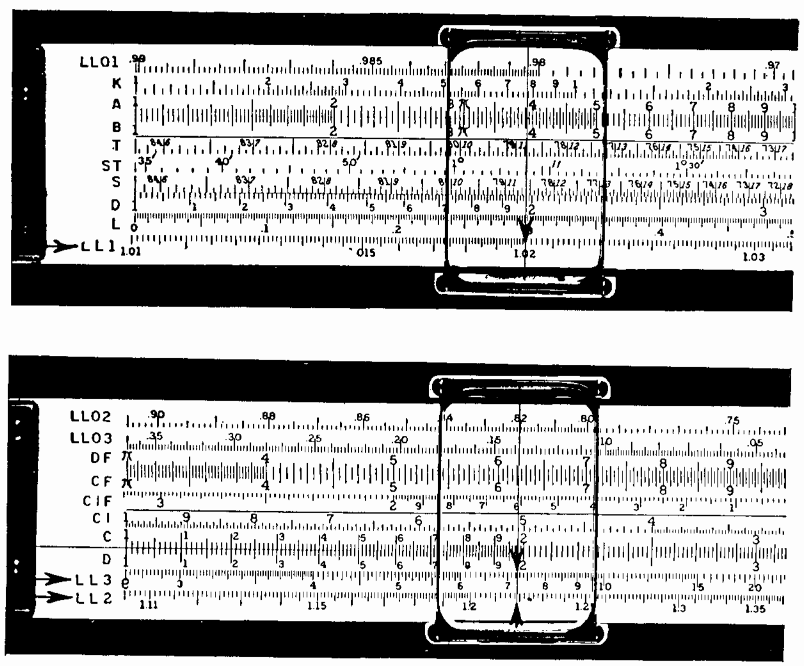


FIG. 68.

Runner to 1.36 on LL2 (Fig. 69)

Read $(1.36)^{10}$ on LL3 as 21.6

Read $\sqrt[10]{1.36}$ on LL1 as 1.0312

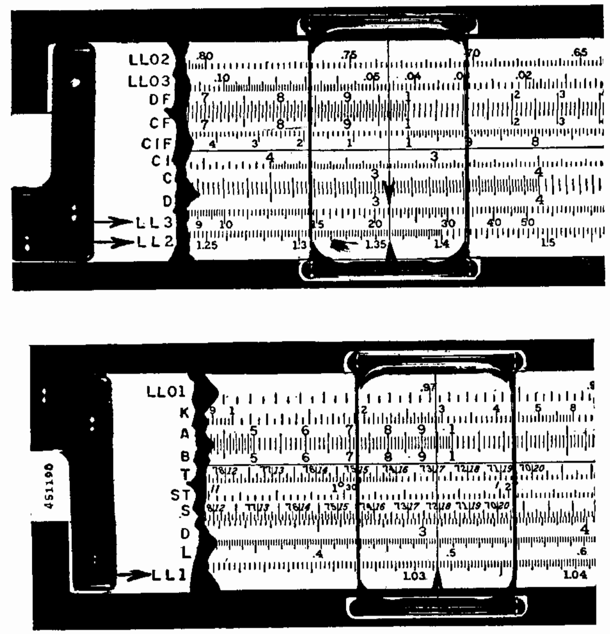


FIG. 69.

Runner to 5.50 on LL3

Read $\sqrt[10]{5.50}$ on LL2 as 1.186

Read $\sqrt[100]{5.50}$ on LL1 as 1.0172

The subdivision of the LL scales varies over an extremely wide range and the student must be very careful in reading these scales. In places the LL1 scale can be read to five significant figures and in other places the LL3 scale can be read only to two. The student must continually check the subdividing of that portion of the scale where he is reading by looking to both sides of the hairline and checking against the numbered divisions nearby on the scale.

In general, the LL scales are used for such computations as the following:

$$(1.27)^{2.57} \qquad (11.2)^{\frac{1}{0.42}} \qquad (1.018)^{\frac{5.78}{2.97}}$$

$$(2.14)^{0.643} \qquad (1.04)^{\frac{1}{3.72}} \qquad (53)^{\frac{3.09}{7.89}}$$

Note that in all cases the numbers are greater than one although the exponents may be either greater or less than one. The fractional notation for exponents will be used in this chapter because it is better adapted to the method of operation which is presented than the root notation. To refresh the reader's mind, if need be, the following equivalents are given:

$$(1.04)^{\frac{1}{3.72}} = \sqrt[3.72]{1.04}$$

$$(11.2)^{\frac{1}{0.42}} = \sqrt[0.42]{11.2}$$

$$(53)^{\frac{3.09}{7.89}} = \sqrt[7.89]{(53)^{3.09}}$$

General Rules of Operation. The rules developed for multiplication and division in Chapter II can be readily applied to the operation of the LL scales. One modification of the standard technique must be observed. Either the upper slide scales CIF and CF can be used or the lower slide scales CI and C can be used, but *not both groups*. In other words, it is not possible to change from one group of scales to the other with the runner or slide. Once the operation has been started on one group of scales, it must be continued on that group. Other than this modification, the standard technique for multiplication and division can be applied to exponential expressions *by placing the number on the LL (or LL0) scales instead of the D scales, and the exponents on the CI and C scales.* The operations are then performed as regular multiplications, divisions, or combinations. For example, $(1.27)^{2.57}$ is performed as 1.27×2.57 by placing 1.27 on the LL2 scale instead of the D scale. This and two other preceding computations are solved as follows:

$(1.27)^{2.57}$ Runner to 1.27 on LL2 (Fig. 70)

Slide to 2.57 on CI

Solve as 1.27×2.57 Runner to right index

Read 1.848 on LL2

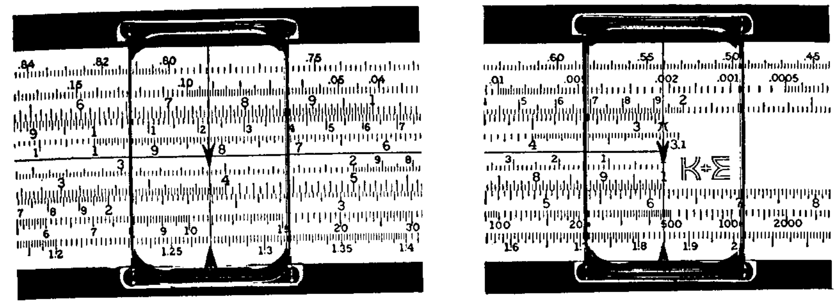


FIG. 70.

$(11.2)^{\frac{1}{0.42}}$ Runner to 11.2 on LL3 (Fig. 71)

Slide to **0.42** on C

Solve as $\frac{11.2}{0.42}$ Runner to right index

Read 315 on LL3

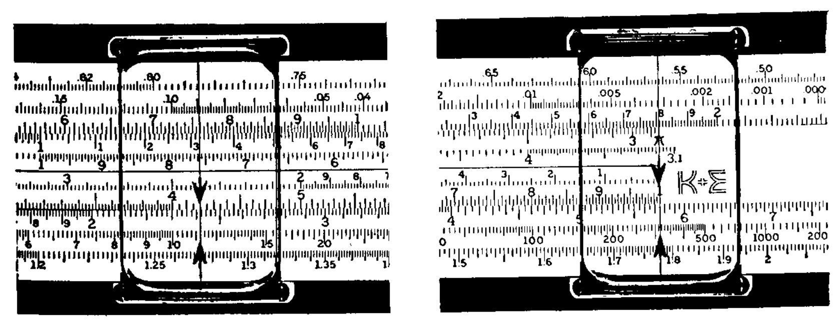


FIG. 71.

$(1.018)^{\frac{5.78}{2.97}}$ Runner to 1.018 on LL1 (Fig. 72)

Slide to 2.97 on C

Solve as $\frac{1.018 \times 5.78}{2.97}$ Runner to 5.78 on C

Read 1.0353 on LL1

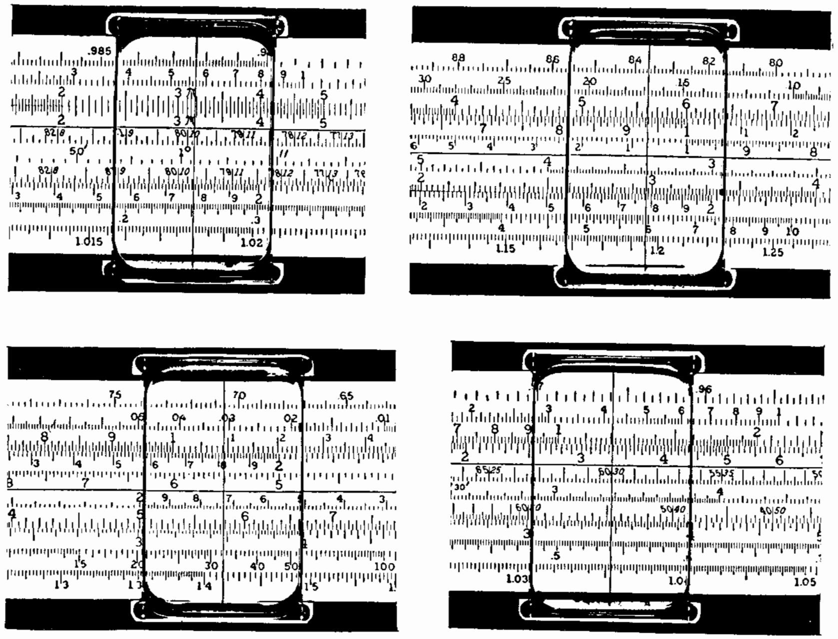


FIG. 72.

Note that the operation must begin and end on the LL scales, similar to beginning and ending on the D scales in multiplication or division. Also, the *operation must begin by setting the number*, and not the exponent, *on the LL scales*. The *exponents are always set on the C and CI scales or on the CF and CIF scales on the slide.*

Exponents between 1 and 10. Where the exponent of a number is greater than 1, the result of the computation is always greater than the number and is obtained by reading up the LL scales, or to the *right*. Such was the case for the three previous computations which were just solved where the *right* index was read further up the scale.

These computations should be reviewed at this time. Sometimes the right index will be found to be off of the rule when a scale is followed to the *right* or upward to read the answer. In such cases, begin at the left end of the next higher scale and follow it to the *right* until the left index is reached. Read the answer on this higher scale under the left index. The following examples illustrate these cases.

$(1.03)^{8.27}$ Runner to 1.03 on LL1 (Fig. 73)

Slide to 8.27 on CI

Solve as 1.03×8.27 Runner to left index

Read 1.277 on LL2

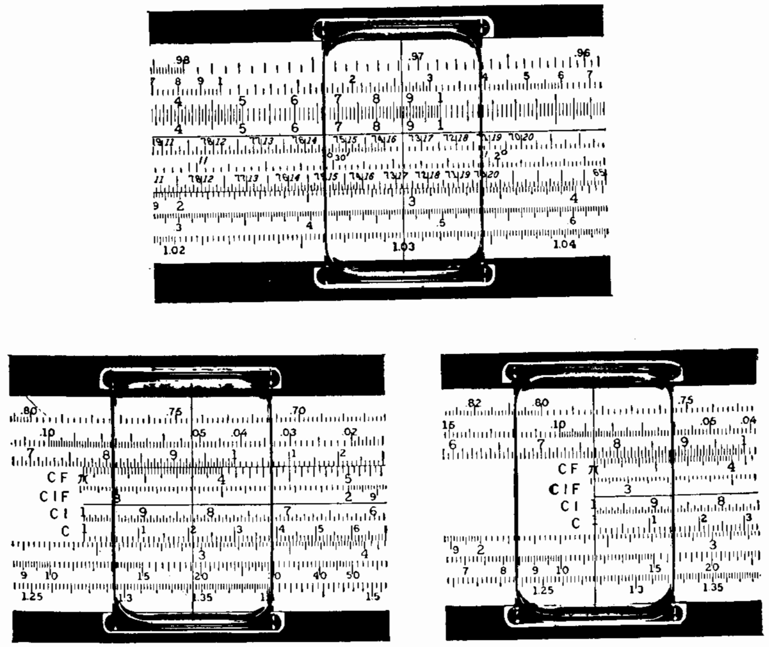
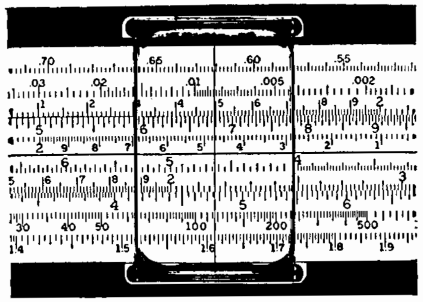


FIG. 73.

$(1.61)^{4.63}$	Runner to 1.61 on LL2	(Fig. 74)
	Slide to 4.63 on CI	
Solve as	Runner to left index	
1.61×4.63	Read 9.1 on LL3	



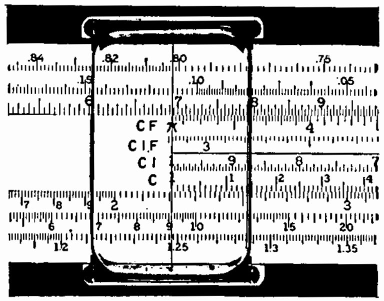


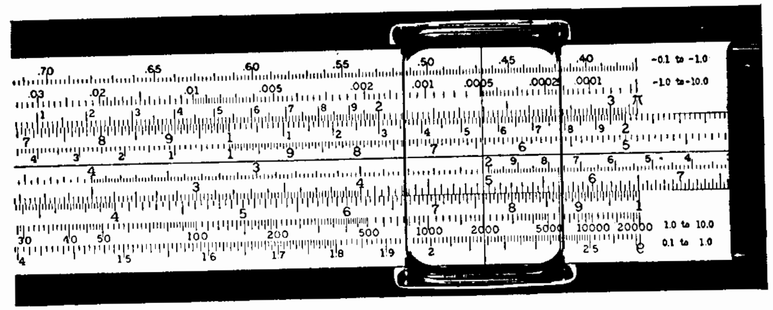
FIG. 74.

$(1.42)^{\frac{1}{0.21}}$	Runner to 1.42 on LL2
	Slide to 0.21 on C
Solve as	Runner to left index
$\frac{1.42}{0.21}$	Read 5.31 on LL3

The three preceding solutions were done on the lower scales to illustrate clearly the procedure in reading the answer on the next higher scale. However, the first two solutions can be done more efficiently using the upper scales after the procedure in transferring scales has been learned.

Exponents between 1 and 0.1. Where the exponent of a number is less than 1, the result of the computation is always less than the number and is obtained by reading down the LL scales, or to the *left.* The following examples are for exponents less than 1 but greater than 0.1:

$(2.14)^{0.643}$	Runner to 2.14 on LL2	(Fig. 75)
	Slide to 0.643 on CIF	
Solve as	Runner to upper index	
2.14×0.643	Read 1.631 on LL2	



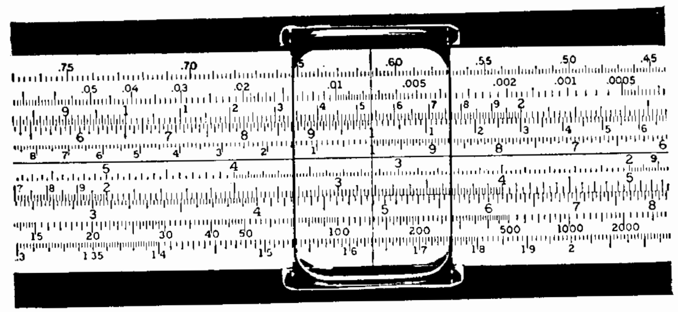


FIG. 75.

$(1.04)^{\frac{1}{3.72}}$	Runner to 1.04 on LL1
	Slide to 3.72 on C
Solve as	Runner to left index
$\frac{1.04}{3.72}$	Read 1.0106 on LL1

$(56)^{\frac{3.09}{7.89}}$ Runner to 56 on LL3 (Fig. 76)
Slide to 7.89 on C

Solve as $\frac{56 \times 3.09}{7.89}$ Runner to 3.09 on C
Read 4.84 on LL3

In those cases where the left index is off of the scale, begin at the right end of the next lower scale and go to the *left* until the right index is reached. Read the answer on this lower scale under the right index.

$(21.5)^{0.19}$ Runner to 21.5 on LL3 (Fig. 77)
Slide to 0.19 on CI

Solve as 21.5×0.19 Runner to right index
Read 1.792 on LL2

$(1.157)^{\frac{1}{3.41}}$ Runner to 1.157 on LL2
Slide to 3.41 on C

Solve as $\frac{1.157}{3.41}$ Runner to right index
Read 1.0437 on LL1

Exponents between 10 and 1000. Where the exponent of a number is between 10 and 1000, the answer can be readily obtained by the following procedure:

1. Move the decimal point mentally one or two places to the left to reduce the exponent to a number between 1 and 10.
2. Perform the operation for this exponent as previously explained.
3. Read the answer one or two scales higher, corresponding to the number of places the decimal point was moved.

These rules are illustrated in the following examples:

$(1.031)^{46}$ Perform as $(1.031)^{4.6}$ (Fig. 78)
Runner to 1.031 on LL1
Slide to 4.6 on CI
Runner to left index
Read LL3 instead of LL2 to get 4.07

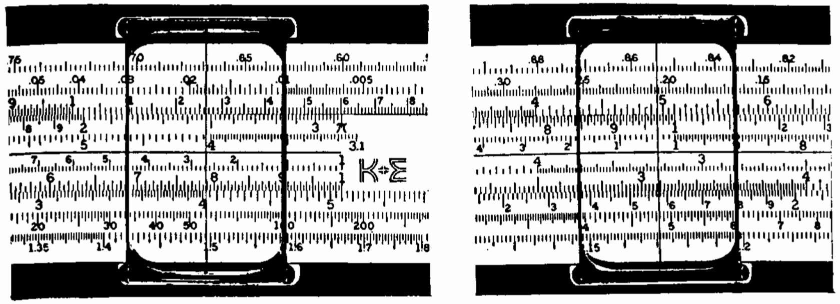


Fig. 76.

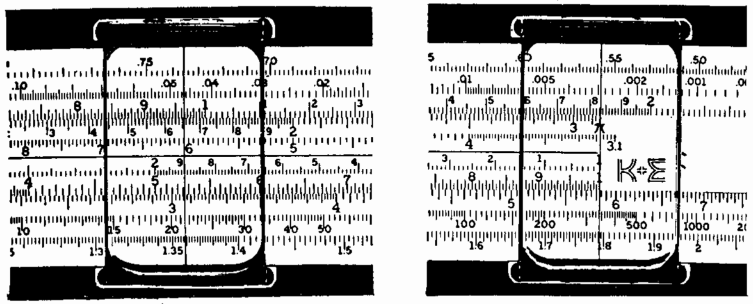


Fig. 77.

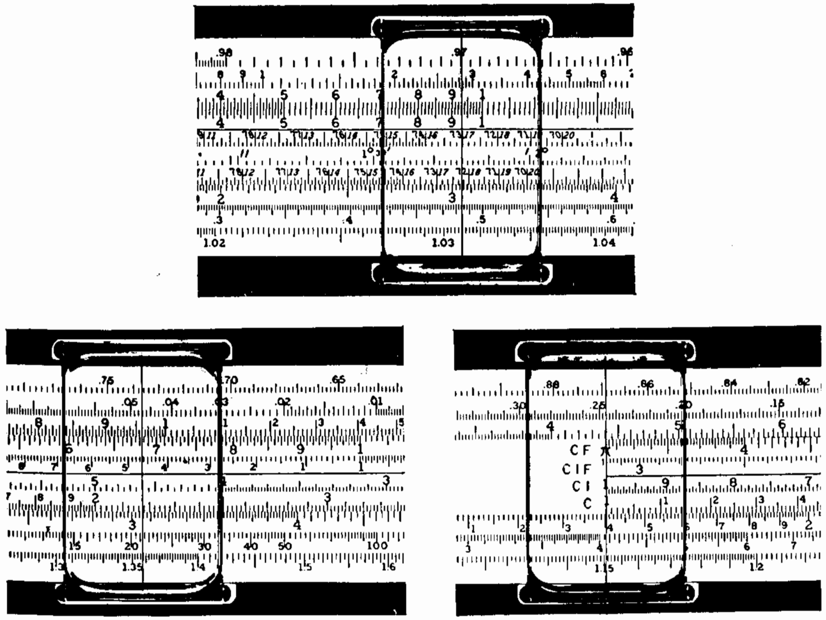


Fig. 78.

$(1.24)^{20}$ Perform as $(1.24)^{2.0}$

Runner to 1.24 on LL2

Slide to 2.0 on CIF

Runner to upper index

Read LL3 instead of LL2 to get 74

$(1.132)^{\frac{1}{0.037}}$ Perform as $(1.132)^{\frac{1}{0.37}}$ (Fig. 79)

Runner to 1.132 on LL2

Slide to 0.37 on C

Runner to right index

Read LL3 instead of LL2 to get 28.5

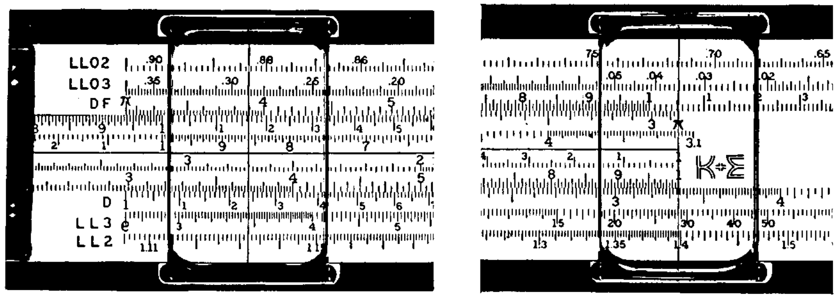


Fig. 79.

$(1.0164)^{335}$ Perform as $(1.0164)^{3.35}$

Runner to 1.0164 on LL1

Slide to 3.35 on CI

Runner to right index

Read LL3 instead of LL1 to get 232

Exponents between 0.1 and 0.001. For exponents between 0.1 and 0.001, the answer is obtained by procedures similar to those used for exponents between 10 and 1000.

1. Move the decimal point one or two places to the right to obtain an exponent between 0.1 and 1.

2. Perform the operation for this exponent as previously explained.

3. Read the answer one or two scales lower depending upon the number of places the decimal point was moved.

These rules are illustrated by the following examples:

$(47.5)^{0.041}$ Perform as $(47.5)^{0.41}$ (Fig. 80)

Runner to 47.5 on LL3

Slide to 0.41 on CI

Runner to left index

Read LL2 instead of LL3 to get 1.1715

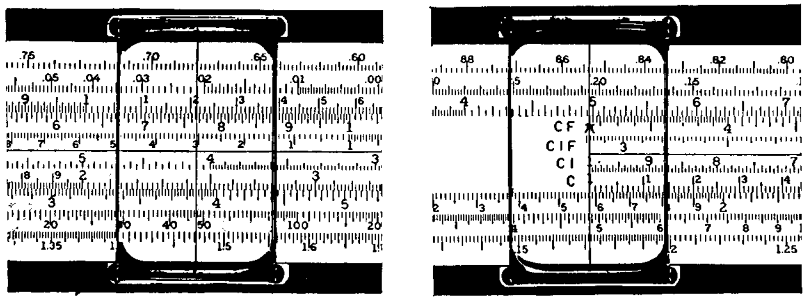


Fig. 80.

$(3200)^{\frac{1}{610}}$ Perform as $(3200)^{\frac{1}{6.10}}$

Runner to 3200 on LL3

Slide to 6.10 on C

Runner to left index

Read LL1 instead of LL3 to get 1.0133

Expressions of the Type, $3.12^x = 7.68$. This is an expression in which the exponent is unknown. Expressions of this type are solved by placing the slide rule in the same "set-up" as for computations where the number and exponent are known and the result is unknown, such as $4.02^{4.63} = x$. The solution is obtained by working backwards from the result, 7.68.

$\mathbf{3.12^x = 7.68}$ Runner to 7.68 on LL3 (Fig. 81)
$x = ?$ Upper index to hairline
Runner to 3.12 on LL3
Read x as 1.79 on CIF

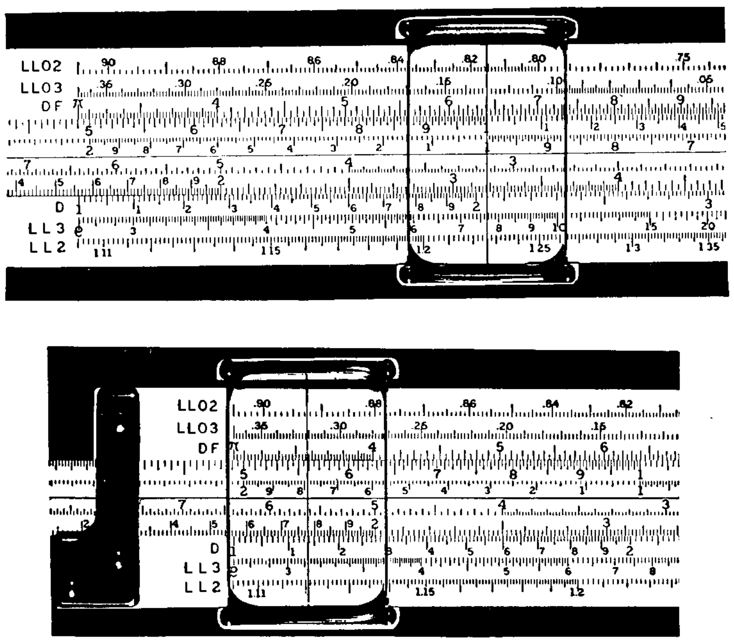


FIG. 81.

$\mathbf{1.87^x = 1.24}$ Runner to 1.24 on LL2
$x = ?$ Left index to hairline
Runner to 1.87 on LL2
Read x as 0.344 on CI

$\mathbf{220^{\frac{1}{x}} = 4.47}$ Runner to 4.47 on LL3
$x = ?$ Left index to hairline
Runner to 220 on LL3
Read x as 3.60 on C

$\mathbf{42.5^{\frac{1}{x}} = 1.072}$ Runner to 1.072 on LL1
$x = ?$ Right index to hairline
Runner to 42.5 on LL3
Read x as 53.9 on C

In this last example, the fact that the operation jumped a scale from LL3 to LL1 indicated in this instance that $1/x$ was less than 0.1, or that x was greater than 10.

Expressions of the Type, $x^{2.07} = 3800$. The solution of expressions of this type is similar to solutions in the preceding paragraph

$\mathbf{x^{2.07} = 3800}$ Runner to 3800 on LL3 (Fig. 82)
$x = ?$ Right index to hairline
Runner to 2.07 on CI
Read x as 54 on LL3

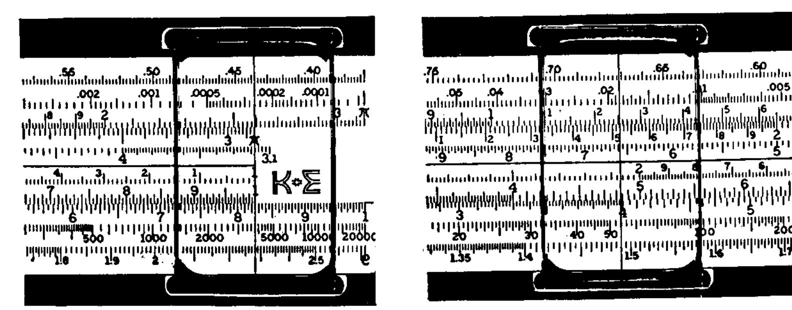


FIG. 82.

$\mathbf{x^{\frac{1}{4.23}} = 1.85}$ Runner to 1.85 on LL2
$x = ?$ Right index to hairline
Runner to 4.23 on C
Read x as 13.5 on LL3

$\mathbf{x^{72.5} = 4.2}$ Runner to 4.2 on LL3
$x = ?$ Left index to hairline
Runner to 72.5 on CI
Read 1.02 on LL1

Expressions for Which the Result Is Greater than 22,000. Such expressions can be solved in steps using such methods as are most advantageous. Where the exponent is integral, a useful method is to factor 10 from the number as in the following examples:

$(23.7)^7 = ?$ The answer is much greater than 22,000, but can be obtained by factoring 10 from the expression,

$$(23.7)^7 = (2.37 \times 10)^7 = (2.37)^7 \times (10)^7$$

Using the LL scales, $(2.37)^7 = 420$, hence

$$(23.7)^7 = 420 \times 10^7 = 4{,}200{,}000{,}000$$

Another example is

$$(12.1)^5 = (1.21)^5 \times (10)^5 = 2.59 \times (10)^5 = 259{,}000$$

Another useful method is to split the exponent into two or more smaller parts, compute each part separately using the LL scales, and make a final computation using the regular multiplication-division scales, or square scales.

$$(1.85)^{24.3} = (1.85)^{10} \times (1.85)^{14.3} = 470 \times 6600 = 3{,}100{,}000$$

Another solution would be to halve the exponent:

$$(1.85)^{24.3} = (1.85)^{12.15} \times (1.85)^{12.15} = 1760 \times 1760 = 3{,}100{,}000$$

There may be a slight difference in the answers because the operator is unable to read the LL scales throughout with the same degree of accuracy.

$$(17.3)^{4.7} = (17.3)^{2.35} \times (17.3)^{2.35} = (810)^2 = 655{,}000$$

$$(1.26)^{48} = (1.26)^{24} \times (1.26)^{24} = (256)^2 = 65{,}500$$

or

$$(1.26)^{48} = (1.26)^{16} \times (1.26)^{16} \times (1.26)^{16} = (40.3)^3 = 65{,}600$$

A third useful method is to factor the number itself.

$$(12)^{5.2} = (3 \times 4)^{5.2} = (3)^{5.2} \times (4)^{5.2} = 303 \times 1350 = 408{,}000$$

This method is generally less efficient than the method of splitting the exponent into parts.

EXERCISE 14

Evaluate the following expressions:

1. $(1.0342)^{10}$
2. $(1.063)^{100}$
3. $(1400)^{0.1}$
4. $(32.5)^{0.01}$
5. $(1.372)^{10}$
6. $(1.94)^{1.42}$
7. $(7.6)^{4.23}$
8. $(1.072)^{6.17}$
9. $(1.024)^{3.73}$
10. $(1.26)^{\frac{9.01}{2.66}}$
11. $(2.163)^{\frac{1}{0.47}}$
12. $(10.6)^{\frac{1}{0.363}}$
13. $(57.5)^{0.67}$
14. $(3.08)^{0.198}$
15. $(8400)^{\frac{1}{6.27}}$
16. $(1.046)^{\frac{1}{2.51}}$
17. $(1.11)^{\frac{1}{3.82}}$
18. $(99.9)^{\frac{2.42}{5.06}}$
19. $(3.03)^{\frac{0.86}{2.37}}$
20. $(1.28)^{23}$
21. $(1.027)^{45}$
22. $(1.151)^{\frac{1}{0.029}}$
23. $(1.033)^{\frac{237}{6.21}}$
24. $(1.092)^{\frac{126}{6.41}}$
25. $(43.7)^{0.052}$
26. $(4.63)^{\frac{1}{34}}$
27. $(1.67)^{\frac{1}{18.3}}$
28. $(2750)^{\frac{8.16}{433}}$
29. $(1.0216)^{337}$
30. $(11{,}200)^{\frac{1}{722}}$

Determine the value of x in the following expressions:

31. $(5.47)^x = 213$
32. $(1.073)^x = 3.24$
33. $(37.3)^x = 1.95$
34. $(2.47)^{\frac{1}{x}} = 1.243$
35. $(1.346)^{\frac{1}{x}} = 5.04$
36. $(x)^{17.4} = 20.9$
37. $(x)^{\frac{1}{2.43}} = 1.64$
38. $(x)^{0.22} = 1.039$

Evaluate the following expressions:

39. $(13.4)^5$
40. $(6.8)^{9.3}$
41. $(1.76)^{28.2}$
42. $(157)^{3.4}$
43. $(34.5)^{\frac{1}{0.17}}$
44. $(2.064)^{23}$
45. $(1.455)^{\frac{1}{0.02}}$
46. $(62.0)^4$

27. The LL01, LL02, and LL03 Scales. The LL01, LL02, and LL03 scales, or the LL0 scales, represent a single scale of numbers reading from right to left from approximately 0.00005 to 0.99 which has been split into three parts. The LL0 scales have also been arranged like the LL scales so that the LL02 scale gives the tenth power of a number on the LL01 scale, and the LL03 scale gives the tenth power of a number on the LL02 scale. Furthermore, the arrangement for operation of the LL0 scales is such that the same directions of movement for positive exponents apply both to the LL scales and to the LL0 scales. This is obtained by reversing the direction of the LL0 scales to show increasing magnitude from right to left instead of from left to right. Thus, raising a number on each group of scales to the same power, say to the 4.2 power, is done by reading in the *same direction*. The procedure is outlined for each group as follows:

$(1.19)^{4.2}$ *LL scales*	**$(0.861)^{4.2}$** *LL0 scales*
Runner to 1.19 on LL2	Runner to 0.861 on LL02
Slide to 4.2 on CI	Slide to 4.2 on CI
Runner to *right* index	Runner to *right* index
Read 2.08 on LL2	Read 0.533 on LL02

In each case, the answer is read to the *right* along each scale (Fig. 83).

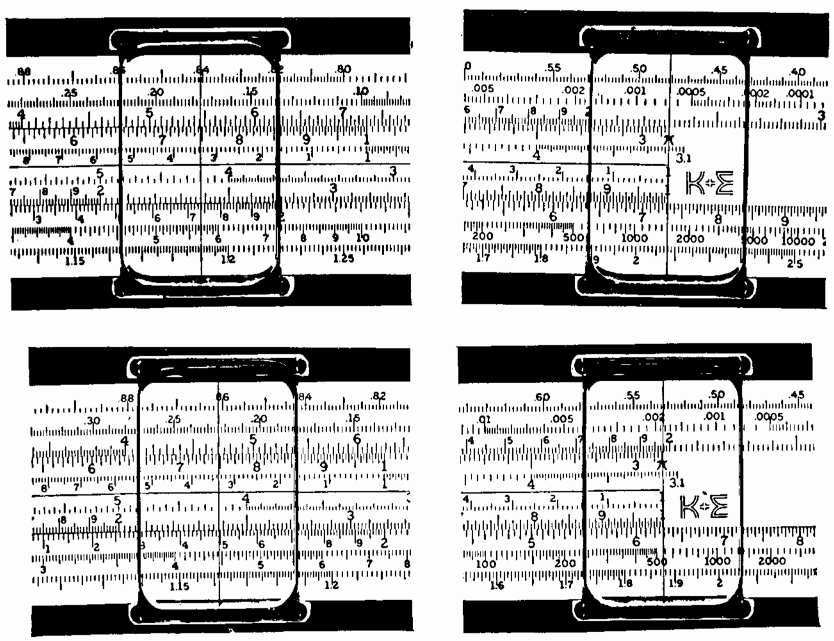


FIG. 83.

For both groups of scales, then, for powers greater than one, the answer is read to the *right*, and, for powers less than one, the answer is read to the *left*. The reason for this is obvious. Raising a number greater than one to a power greater than one increases the number, e.g., $(2)^3 = 8$. On the other hand, raising a number less than one to a power greater than one decreases the number, e.g. $(0.2)^3 = 0.008$. By reversing the direction of the LL0 scales with respect to the LL scales, a movement to the right corresponds to an increase on the LL scales but a decrease on the LL0 scales, or corresponds in each case to raising a number to a power greater than one. A movement to the left similarly corresponds in each case to raising a number to a power less than one.

The following examples are grouped according to exponents as was done in the preceding section and illustrate the application of the rules of procedure to the LL0 scales.

Exponents between 1 and 10

$(0.224)^{3.1}$	Runner to 0.224 on LL03	(Fig. 84)
	Slide to 3.1 on CI	
Solve as	Runner to right index	
0.224×3.1	Read 0.0096 on LL03	

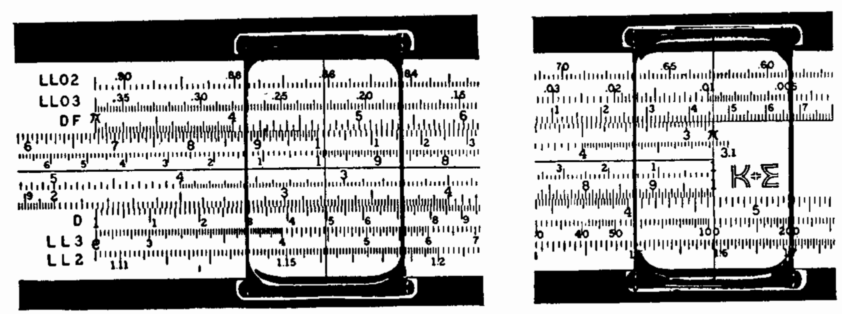


FIG. 84.

$(0.953)^{5.62}$	Runner to 0.953 on LL01
	Slide to 5.62 on CIF
Solve as	Runner to upper index
0.953×5.62	Read 0.763 on LL02

$(0.825)^{\frac{1}{0.627}}$ — Runner to 0.825 on LL02

Slide to 0.627 on CF

Solve as $\frac{0.825}{0.627}$ — Runner to upper index

Read 0.736 on LL02

$(0.209)^{\frac{6.23}{2.17}}$ — Runner to 0.209 on LL02

Slide to 2.17 on C

Solve as $\frac{0.209 \times 6.23}{2.17}$ — Runner to 6.23 on C

Read 0.0112 on LL03

Exponents between 1 and 0.1

$(0.944)^{0.462}$ — Runner to 0.944 on LL01

Slide to 0.462 on CIF

Solve as 0.944×0.462 — Runner to upper index

Read 0.9737 on LL01

$(0.00035)^{\frac{1}{4.03}}$ — Runner to 0.00035 on LL03

Slide to 4.03 on C

Solve as $\frac{0.00035}{4.03}$ — Runner to left index

Read 0.139 on LL03

$(0.883)^{0.254}$ — Runner to 0.883 on LL02

Slide to 0.254 on CIF

Solve as 0.883×0.254 — Runner to upper index

Read 0.9689 on LL01

Exponents between 10 and 1000

$(0.815)^{23.5}$ — Perform as $(0.815)^{2.35}$ (Fig. 85)

Runner to 0.815 on LL02

Slide to 2.35 on CI

Runner to right index

Read LL03 instead of LL02 to get 0.0082

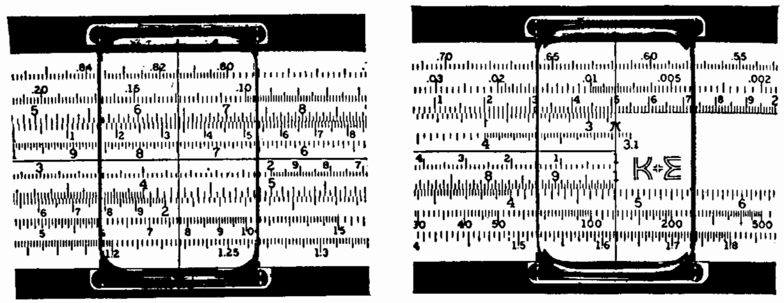


FIG. 85.

$(0.9795)^{346}$ — Perform as $(0.9795)^{3.46}$

Runner to 0.9795 on LL01

Slide to 3.46 on CI

Runner to right index

Read LL03 instead of LL01 to get 0.00078

$(0.942)^{\frac{1}{0.035}}$ — Perform as $(0.942)^{\frac{1}{0.35}}$

Runner to 0.942 on LL01

Slide to 0.35 on C

Runner to left index

Read LL03 instead of LL02 to get 0.182

Exponents between 0.1 and 0.001

(0.0028)$^{0.046}$ Perform as $(0.0028)^{0.46}$
Runner to 0.0028 on LL03
Slide to 0.46 on CIF
Runner to upper index
Read LL02 instead of LL03 to get 0.763.

(0.062)$^{0.019}$ Perform as $(0.062)^{0.19}$
Runner to 0.062 on LL03
Slide to 0.19 on CIF
Runner to upper index
Read LL01 instead of LL02 to get 0.9485

(0.707)$^{\frac{1}{21.3}}$ Perform as $(0.707)^{\frac{1}{2.13}}$ (Fig. 86)
Runner to 0.707 on LL02
Slide to 2.13 on C
Runner to left index
Read LL01 instead of LL02 to get 0.98385

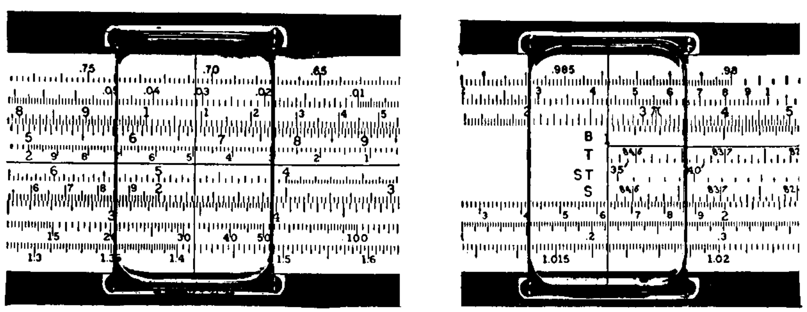


FIG. 86.

Expressions of the Type, $(0.846)^x = 0.0632$

(0.846)^x = **0.0632** Runner to 0.0632 on LL03
$x = ?$ Upper index to hairline
Runner to 0.846 on LL02
Read x as 16.5 on CIF

(0.037)$^{\frac{1}{x}}$ = **0.241** Runner to 0.241 on LL03
$x = ?$ Left index to hairline
Runner to 0.037 on LL03
Read x as 2.32 on C

Expressions of the Type, $x^{6.12} = 0.431$

$x^{6.12}$ = **0.431** Runner to 0.431 on LL02
$x = ?$ Right index to hairline
Runner to 6.12 on CI
Read x as 0.8715 on LL02

$x^{0.022}$ = **0.964** Runner to 0.964 on LL01
$x = ?$ Upper index to hairline
Runner to 0.022 on CIF
Read x as 0.189 on LL03

Expressions for Which the Result Is Less than 0.000045

$$(0.172)^{8.2} = (0.172)^{4.1} \times (0.172)^{4.1} = (0.00074)^2 = 0.000000548$$

$$(0.047)^5 = \frac{(0.47)^5}{10^5} = \frac{0.023}{10^5} = 0.00000023$$

Exercise 15

Evaluate the following expressions:

1. $(0.9811)^{10}$
2. $(0.984)^{100}$
3. $(0.515)^{0.1}$
4. $(0.027)^{0.01}$
5. $(0.9072)^{7.7}$
6. $(0.816)^{4.41}$
7. $(0.958)^{7.25}$
8. $(0.935)^{\frac{1}{0.39}}$
9. $(0.582)^{\frac{1}{0.53}}$
10. $(0.752)^{9.5}$
11. $(0.933)^{\frac{23.6}{3.2}}$
12. $(0.9884)^{73}$
13. $(0.847)^{27}$
14. $(0.028)^{\frac{1}{113}}$
15. $(0.927)^{\frac{1}{6.8}}$
16. $(0.024)^{0.21}$
17. $(0.707)^{0.096}$
18. $(0.101)^{\frac{1}{5.5}}$
19. $(0.00064)^{0.018}$
20. $(0.937)^{0.72}$

Determine the value of x in the following expressions:

21. $(0.842)^{x} = 0.507$
22. $(0.9735)^{x} = 0.025$
23. $(0.157)^{\frac{1}{x}} = 0.667$
24. $x^{6.41} = 0.00026$
25. $x^{\frac{1}{4.3}} = 0.868$
26. $x^{\frac{4.91}{1.70}} = 0.9295$

Evaluate the following expressions:

27. $(0.054)^{4.2}$
28. $(0.0082)^{3.9}$
29. $(0.825)^{70}$
30. $(0.306)^{15}$

28. Negative Exponents. In solving for numbers with negative exponents, such as $(10.43)^{-2.16}$, $(0.875)^{-7.2}$, and $(7.43)^{-\frac{1}{4.62}}$, both the LL scales and the LL0 scales are used. These two groups of scales are arranged so that the numbers on any scale in one group are the reciprocals of numbers on the corresponding scale in the other group. When the runner is set to 10 on the LL3 scale, it is also on 0.1 on the LL03 scale; when it is set to 2 on the LL2 scale, it is also on 0.5 on the LL02 scale. See Fig. 87. The significance of this relationship becomes apparent upon recalling the relationship between positive and negative exponents. A number raised to a negative power is the reciprocal of the number to the positive power, e.g. $(5)^{-4} = \frac{1}{5^4}$.

Consequently, in solving for 5^4 on the LL3 scale to obtain 625, the value of its reciprocal $\frac{1}{5^4}$, or of $(5)^{-4}$, is also obtained by reading the hairline on the corresponding LL03 scale to get 0.00160 (Fig. 88).

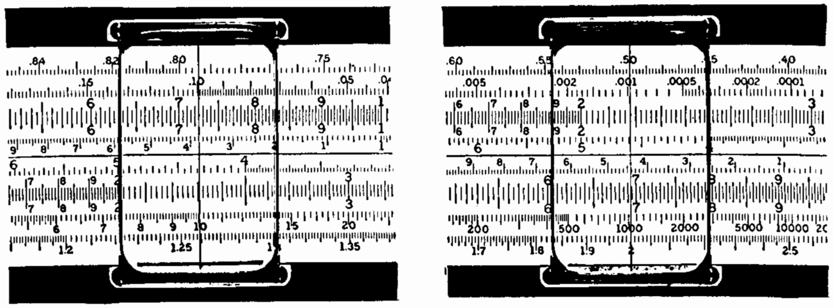


Fig. 87.

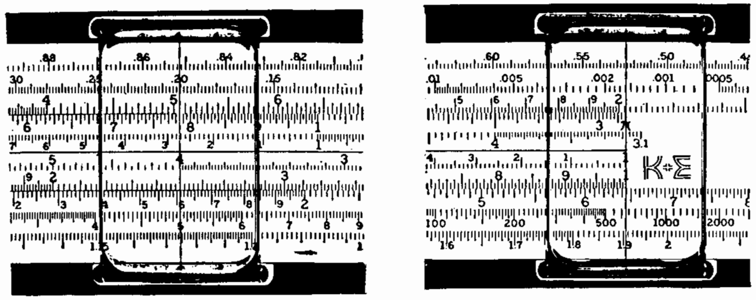


Fig. 88.

Therefore to evaluate a number raised to a negative power, simply evaluate the number raised to the positive power but read the answer on the corresponding scale in the other group of scales. The following examples illustrate the procedure in various cases:

$(10.16)^{-2.16}$

Solve as $(10.16)^{2.16}$ on LL scales, read LL0 scales

Runner to 10.16 on LL3

Slide to 2.16 on CI

Runner to right index

Read 0.0067 on LL03

$\mathbf{(0.875)^{-7.2}}$ Solve as $(0.875)^{7.2}$ on LL0 scales, read LL scales
Runner to 0.875 on LL02
Slide to 7.2 on CI
Runner to right index
Read 2.62 on LL2

$\mathbf{(7.43)^{-\frac{1}{4.62}}}$ Solve as $(7.43)^{\frac{1}{4.62}}$ on LL scales, read LL0 scales
Runner to 7.43 on LL3
Slide to 4.62 on C
Runner to right index
Read 0.648 on LL02

$\mathbf{(1.027)^{-0.49}}$ Solve as $(1.027)^{0.49}$ on LL scales, read LL0 scales
Runner to 1.027 on LL1
Slide to 0.49 on CI
Runner to left index
Read 0.9870 on LL01

$\mathbf{(0.933)^{-\frac{2.47}{4.92}}}$ Solve as $(0.933)^{\frac{2.47}{4.92}}$ on LL0 scales, read LL scales
Runner to 0.933 on LL01
Slide to 4.92 on C
Runner to 2.47 on C
Read 1.0354 on LL1

$\mathbf{(1.0182)^{-x} = 0.661}$ Runner to 0.661 on LL02
Upper index to hairline
Runner to 1.0182 on LL1
Read x as 22.9 on CIF

$\mathbf{(0.0037)^{-\frac{1}{x}} = 4.26}$ Runner to 4.26 on LL3
Left index to hairline
Runner to 0.0037 on LL03
Read x as 3.86 on C

$\mathbf{x^{-2.42} = 0.763}$ Runner to 0.763 on LL02
Upper index to hairline
Runner to 2.42 on CIF
Read x as 1.1183 on LL2

Exercise 16

Evaluate the following expressions:

1. $(1.25)^{-4.31}$
2. $(4.23)^{-3.79}$
3. $(1.0375)^{-46.4}$
4. $(293)^{-0.162}$
5. $(15.75)^{-\frac{1}{7.83}}$
6. $(1.567)^{-\frac{1}{12.2}}$
7. $(2.55)^{-0.287}$
8. $(2400)^{-0.0059}$
9. $(6.93)^{-\frac{3.62}{4.56}}$
10. $(1.273)^{-2.53}$
11. $(0.888)^{-2.19}$
12. $(0.0165)^{-0.333}$
13. $(0.9875)^{-11.7}$
14. $(0.556)^{-\frac{1}{2.47}}$
15. $(0.202)^{-\frac{6.27}{1.94}}$
16. $(0.939)^{-0.567}$
17. $(0.9476)^{-8.22}$
18. $(0.00017)^{-\frac{1}{6.99}}$
19. $(0.353)^{-0.441}$
20. $(0.9892)^{-57.6}$

29. Principle of the Log Log Scales. Regular multiplication is solved on the slide rule by the simple addition of logarithms using simple logarithmic scales such as the D, CI, and C scales. For example, the product $4.27 \times 3.21 = 13.7$ is solved as:

$$\log 4.27 + \log 3.21 = \log 13.7$$

where the log 4.27 and the log 3.21 are added by using the D and the CI scales.

For exponential expressions, this simple taking of logarithms is not sufficient to transform the computation into the sum of two quantities; for example:

$$(4.27)^{3.21} = 106$$

Taking logarithms once gives

$$\log (4.27)^{3.21} = 3.21 \times (\log 4.27) = \log 106$$

At this stage of the computation, there is no addition of quantities, but only the product of one number by the logarithm of another number. In order to transform the expression into an addition or subtraction of logarithms of one sort or another, it is necessary to take logarithms a second time:

$$\log [3.21 \times (\log 4.27)] = \log 3.21 + \log \log 4.27 = \log \log 106$$

Now the expression is in the general form in which two quantities are added to give a third quantity, and the computation can be solved by placing log log scales on the slide rule in addition to the regular log scales. Basically, the solution is obtained by adding the log log of 4.27 on the LL3 scale and the log of 3.21 on the CI scale, the result being read on the log log scale. And so exponential expressions in general can be solved in combination with the log scales. The student must remember that by log scales are meant such scales as the D, CI, and C scales which are logarithmically subdivided, and not the so-called "Log Scale" or L scale which is actually an arithmetic scale used in obtaining the numerical value of the common logarithm of a number in case such should be desired. For exponents in fractional form, the computation would be solved as a division:

Computation $(54)^{\frac{1}{2.72}} = 4.45$

Solution (1) $\log (54)^{\frac{1}{2.72}} = \frac{1}{2.72} \log 54 = \frac{\log 54}{2.72} = \log 4.45$

(2) $\log \log 54 - \log 2.72 = \log \log 4.45$

In the first step of the solution, the log 54 is divided by 2.72; hence the next step using the log log scales is the solution of this quotient which gives the difference rather than the sum of two quantities. This is performed by setting log log 54 on the LL3 scale, subtracting log 2.72 from it using the C scale, and reading the answer on the log log scales.

Actually the log log scales are not constructed by taking common logarithms, logarithms to the base 10, twice in succession. If this had been done, the mark for 10 on the LL3 scale would be opposite the left index of the D scale, since log log 10 = log 1 = 0. The first logarithm is a natural logarithm, i.e., to the base *e*, so that *e* is opposite the left index of the D scale. This is shown by $\log_{10} \log_e e = \log_{10} 1 = 0$. The log log scales, then, are actually $\log_{10} \log_e$ scales. The result of this construction is to bring both limits of the ranges of the LL and LL0 scales closer to 1 than they would be if common logarithms had been used entirely. The present ranges are for the LL scales from 1.01 to 22,000 and for the LL0 scales from 0.99 to 0.000045. If common logarithms had been used entirely, the ranges would have been for the LL scales from 1.023 to 10,000,000,000 and for the LL0 scales from 0.977 to 0.0000000001. In this event, the scales would have been much too inaccurate near the outer limits of the ranges to have been of any practical value.

V

TRIGONOMETRIC SCALES

30. Introduction. The trigonometric scales are the S, T, and ST scales, which are all graduated in degrees of an angle. On some rules the scales are divided into degrees, minutes, and seconds; on other rules they are divided into degrees, tenths, and hundredths. The S scale is for sines and related functions, the T scale is for tangents and cotangents, and the ST scale for both sines and tangents of small angles.

The numerical value of functions of an angle can be quickly determined with the trigonometric scales. The functions most frequently used are the sine, cosine, and tangent. The relations between these and the cotangent, secant, and cosecant are given for reference purposes.

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

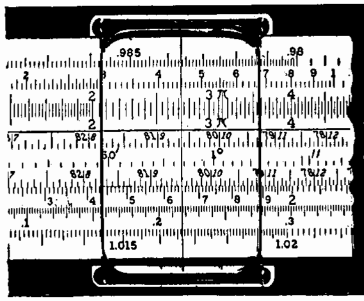
$$\tan x = \frac{1}{\cot x}$$

$$\cos x = \sin (90° - x)$$

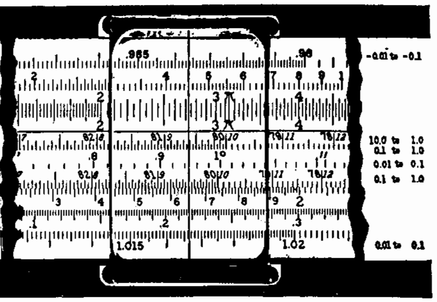
$$\cot x = \tan (90° - x)$$

31. The S and ST Scales—Sines, Cosines, Secants, and Cosecants. The S and ST scales are used to obtain the numerical values of the sines, cosines, secants, and cosecants of angles. The black numbers on the S and ST scales are for *sines* of angles which range from 0°34′(0.57°) to 5°44′(5.73°) on the ST scale and from 5°44′(5.73°) to 90° on the S scale. The values of the sines of these angles vary from 0.01 to 0.1 for the ST scale and from 0.1 to 1.0

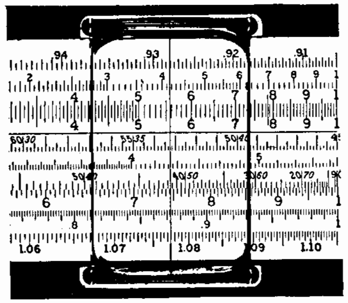
for the S scale, and are shown by the legends at the right ends of the scales. The ST scale is so designated because it is used for both sines and tangents of angles less than 5°44′(5.73°) for which the



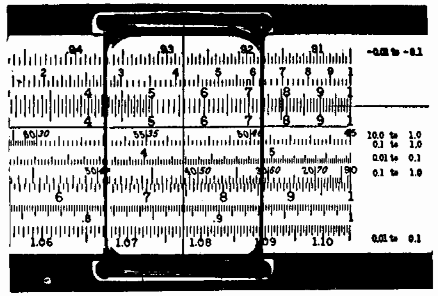
Sin 9°30′.



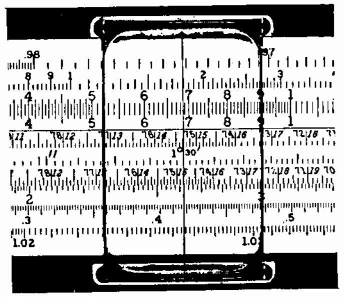
Sin 9.5°.



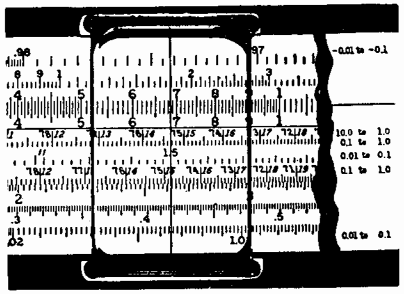
Sin 48°18′.



Sin 48.3°.



Sin 1°30′.



Sin 1.5°.

Fig. 89.

numerical values of the tangent do not differ by more than 1 in 200 from the numerical values of the sine. The value of the sine of an angle is read on the C scale, or the D scale if the slide is lined up with the D scale. The runner is set to the angle on the S or ST scale and the value of the sine is read on the D or C scale. The following examples are illustrated in Fig. 89.

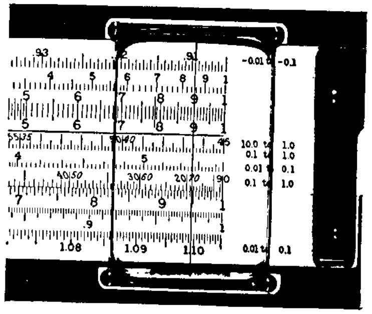
sin 9°30′(9.5°)	Runner to 9°30′(9.5°) on S black Read 0.165 on D or C
sin 48°18′(48.3°)	Runner to 48°18′(48.3°) on S black Read 0.746 on D or C
sin 1°30′(1.5°)	Runner to 1°30′(1.5°) on ST Read 0.0262 on D or C

The red numbers on the S scale are for cosines of angles, it being observed that, for example, the sin 25° = cos (90° − 25°) = cos 65°. There are no red numbers on the ST scale, but it can be used for cosines of angles between (90° − 5°44′) and (90° − 0°34′), i.e., between 84°16′(84.27°) and 89°25′(89.43°). The following examples are illustrated in Fig. 90:

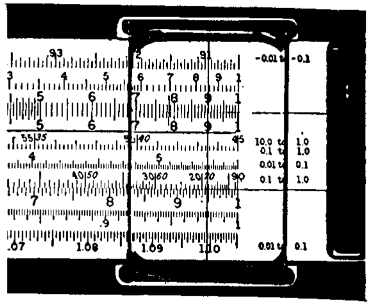
cos 18°30′(18.5°)	Runner to 18°30′(18.5°) on S red Read 0.948 on D or C
cos 87°36′(87.6°)	Runner to 2°24′(2.4°) on ST Read 0.0418 on D or C

Since the cosecant and secant are reciprocals of the sine and cosine respectively, the values of the cosecant or secant of an angle can be read directly on the CI scale by setting the runner on the respective sine or cosine values. The CI and C scales are reciprocal scales as explained on page 47. Accordingly, the numerical values of the cosecant and secant are from 10 to 100 for angles on the ST scale and from 1 to 10 for angles on the S scale.

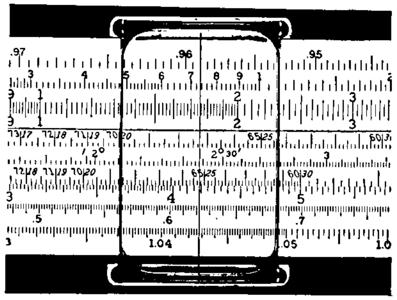
csc 25°	Runner to 25° on S black Read 2.36 on CI



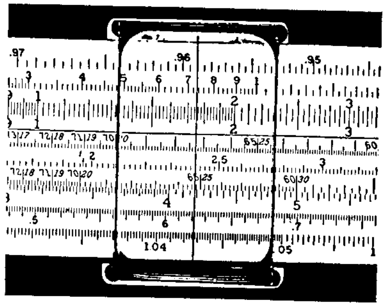
Cos 18°30′.



Cos 18.5°.



Cos 87°36′.



Cos 87.6°.

Fig. 90.

csc 76°	Runner to 76° on S black Read 1.03 on CI
sec 22°	Runner to 22° on S red Read 1.08 on CI
sec 83°	Runner to 83° on S red Read 8.20 on CI
csc 1°	Runner to 1° on ST Read 57.3 on CI
sec 87°	Runner to 3° on ST Read 19.1 on CI

The S and ST scales are also used in reverse to find the magnitudes of angles when the values of their functions are given.

sin x = 0.436 Runner to 0.436 on D or C (Fig. 91)
Read x as 25°51′(25.85°) on S black

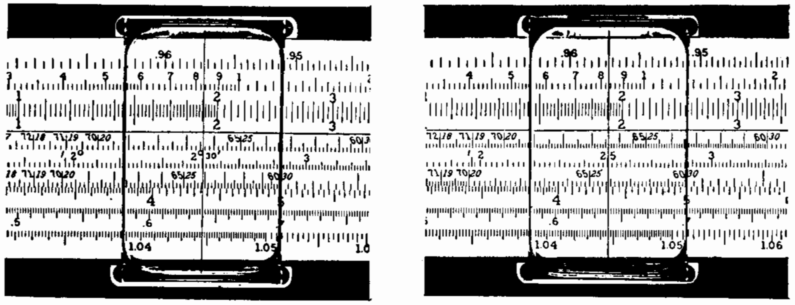


Fig. 91.

sin x = 0.0121 Runner to 0.0121 on D or C
Read x as 0°41′30″(0.692°) on ST

cos x = 0.812 Runner to 0.812 on D or C (Fig. 92)
Read x as 35°42′(35.7°) on S red

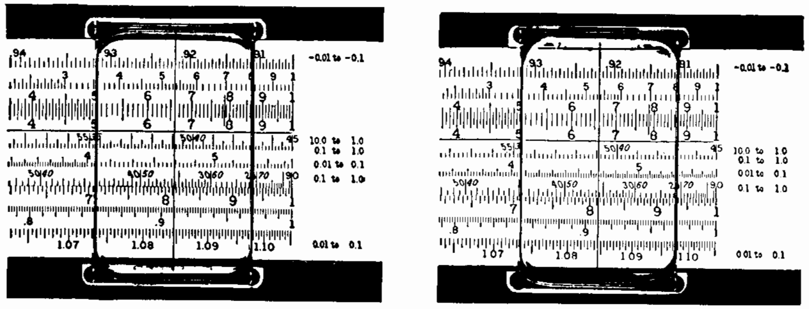


Fig. 92.

cos x = 0.055 Runner to 0.055 on D or C
Read x as (90° − 3°9′) on ST
or as 86°51′(86.85°)

sec x = 1.24 Runner to 1.24 on CI
Read x as 36°12′(36.2°) on S red

csc x = 28.2 Runner to 28.2 on CI
Read x as 2°1′48″(2.03°) on ST

Exercise 17

Evaluate the following sines and cosines:

1. sin 14°18′(14.30°)
2. sin 57°24′(57.40°)
3. sin 0°44′(0.733°)
4. cos 17°40′(17.67°)
5. cos 82°12′(82.20°)
6. sin 2°33′(2.55°)
7. cos 5°30′(5.50°)
8. sin 18°23′(18.38°)
9. sin 81°30′(81.50°)
10. sin 22°45′(22.75°)
11. sin 10°10′(10.17°)
12. cos 47°50′(47.83°)
13. cos 87°15′(87.25°)
14. cos 89°
15. sin 6°45′(6.75°)
16. sin 40°30′(40.50°)
17. cos 32°16′(32.27°)
18. sin 5°5′(5.083°)
19. cos 72°12′(72.20°)
20. sin 1°46′(1.767°)

Evaluate the following secants and cosecants:

21. sec 17°
22. csc 49°
23. csc 4°
24. sec 83°
25. csc 13°
26. csc 2°
27. sec 89°
28. csc 76°
29. csc 7°
30. sec 65°

Determine x in the following equations:

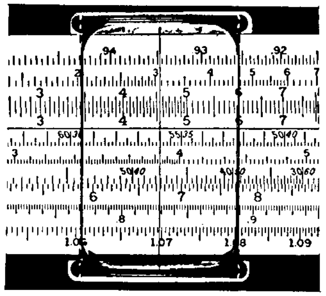
31. $\sin x = 0.629$
32. $\sin x = 0.0166$
33. $\cos x = 0.848$
34. $\sin x = 0.413$
35. $\cos x = 0.0772$
36. $\cos x = 0.185$
37. $\sin x = 0.970$
38. $\sin x = 0.0862$
39. $\cos x = 0.945$
40. $\cos x = 0.533$

32. The T and ST Scales—Tangents and Cotangents. The T and ST scales are used to obtain the numerical values of tangents and cotangents of angles. The black numbers on the ST and T scales are for tangents of angles from 0°34′(0.57°) to 5°44′(5.73°) on the ST scale and from 5°43′(5.71°) to 45° on the T scale. The numerical values for the tangents of these angles are read on the D or C scales. The black legends at the right ends give the ranges of values for the two scales. The red numbers on the T scale are for tangents of angles from 45° to 84°17′(84.29°). The numerical values are read on the CI scale and range from 1 to 10 as indicated by the red legend at the right end. The following examples are illustrated in Fig. 93:

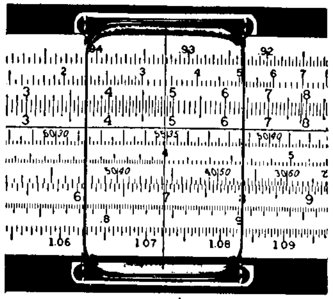
tan 34° Runner to 34° on T
Read 0.675 on D or C

tan 4° Runner to 4° on ST
Read 0.0698 on D or C

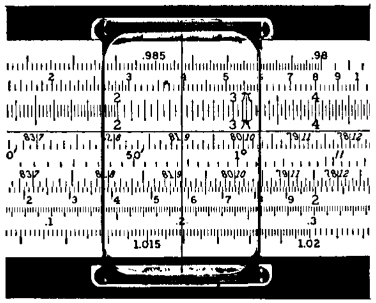
tan 81° Runner to 81° on T
Read 6.31 on CI

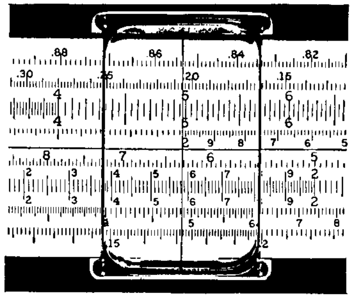


Tan 34°.



Tan 4°.





Tan 81°.

FIG. 93.

The cotangent of an angle is the reciprocal of the tangent and is also obtained on the T and ST scales. The cotangents of angles between 0°34′(0.57°) and 45° would be read on the CI scale and of angles between 45° and 84°17′(84.29°) on the D or C scale.

cot 5° Runner to 5° on ST
Read 11.48 on CI

cot 33° Runner to 33° on T
Read 1.54 on CI

cot 73° Runner to 73° on T
Read 0.306 on D or C

The tangent and cotangent of angles between 84°16′(84.27°) and 89°26′(89.43°) can also be found from the ST scale as follows:

cot 88° cot 88° = tan 2°
Runner to 2° on ST
Read 0.0349 on D or C

tan 87° tan 87° = cot 3°
Runner to 3° on ST
Read 19.1 on CI

The T scale is used in reverse as are the S and ST scales to find angles from given values of their functions.

tan x = 4.62 Runner to 4.62 on CI
Read x as 77°48′(77.8°) on T red

tan x = 0.752 Runner to 0.752 on D or C
Read x as 36°54′(36.9°) on T black

tan x = 0.0231 Runner to 0.0231 on D or C
Read x as 1°19′24″(1.32°) on ST

cot x = 0.307 Runner to 0.307 on D or C
Read x as 72°55′(72.92°) on T red

cot x = 6.42 Runner to 6.42 on CI
Read x as 8°51′(8.85°) on T black

cot x = 47.3 Runner to 47.3 on CI
Read x as 1°13′(1.21°) on ST

Exercise 18

Evaluate the following tangents and cotangents:

1. tan 16°15′(16.25°)
2. tan 36°30′(36.50°)
3. tan 2°40′(2.67°)
4. tan 49°45′(49.75°)
5. cot 6°30′(6.50°)
6. tan 81°20′(81.33°)
7. cot 22°30′(22.50°)
8. cot 83°12′(83.20°)
9. cot 38°24′(38.40°)
10. tan 67°48′(67.80°)
11. cot 0°54′(0.90°)
12. tan 87°30′(87.50°)
13. tan 89°18′(89.30°)
14. cot 86°42′(86.70°)
15. tan 1°28′(1.467°)
16. cot 19°15′(19.25°)
17. cot 69°20′(69.33°)
18. tan 6°6′(6.10°)
19. tan 66°12′(66.20°)
20. cot 47°24′(47.40°)

Determine x in the following equations:

21. $\tan x = 1.63$
22. $\cot x = 0.192$
23. $\tan x = 0.0427$
24. $\cot x = 4.79$
25. $\tan x = 0.667$
26. $\tan x = 7.79$
27. $\tan x = 23.4$
28. $\cot x = 0.0320$
29. $\tan x = 0.938$
30. $\tan x = 0.0122$

33. Functions of Angles of Less than 0°34′(0.57°). The functions of angles smaller than those on the ST scale can be obtained readily by using the relation that for small angles, $\sin x = \tan x = x$ (in radians), approximately. There are two gauge points on the ST scale which correspond to the numerical values in radians of an angle of one minute and an angle of one second on the CI scale. One gauge mark is called the minutes (′) gauge, and the other is called the seconds (″) gauge. Since $\sin 4' = 4'$ (in radians) approximately, or $4 \times$ radian measure of 1′, the sin 4′ is obtained by the corresponding procedure:

Runner to 4 on D (Fig. 94)

Slide to minutes gauge on ST

Runner to left index

Read 0.001164 on D

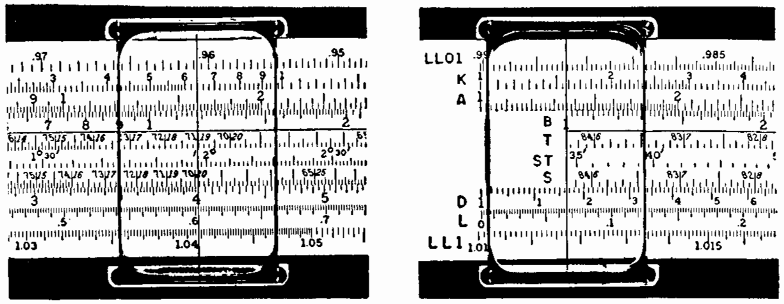


Fig. 94.

This is actually the multiplication of a number on the D scale by a number on the CI scale since the minutes gauge is the mark corresponding to the radian measure of 1′ on the CI scale. The decimal points for functions of small angles can be determined from the following data:

$$0.1° = 0.002 \text{ (2 zeros, 2) radians approximately}$$
$$1' = 0.0003 \text{ (3 zeros, 3) radians approximately}$$
$$1'' = 0.000005 \text{ (5 zeros, 5) radians approximately}$$

The following examples illustrate the procedure:

sin 2′ *Answer* $= 2 \times 0.0003 = 0.0006$ approx. (Fig. 95)

Runner to 2 on D

Slide to minutes gauge

Runner to right index

Read 0.000582 on D

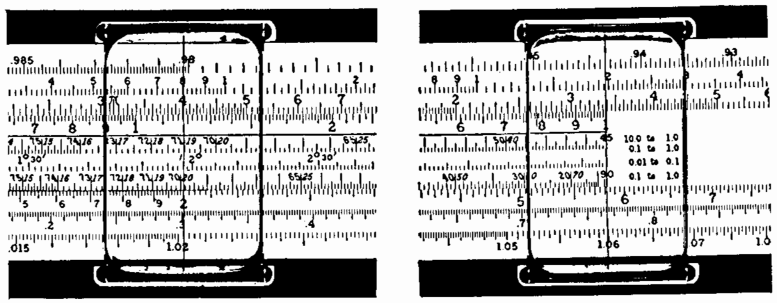


Fig. 95.

sin 33″ *Answer* = 33 × 0.000005 = 0.000165 approx. (Fig. 96)

or = 0.5 × 0.0003 = 0.00015 approx.

Runner to 33 on D

Slide to seconds gauge

Runner to left index

Read 0.0001602 on D

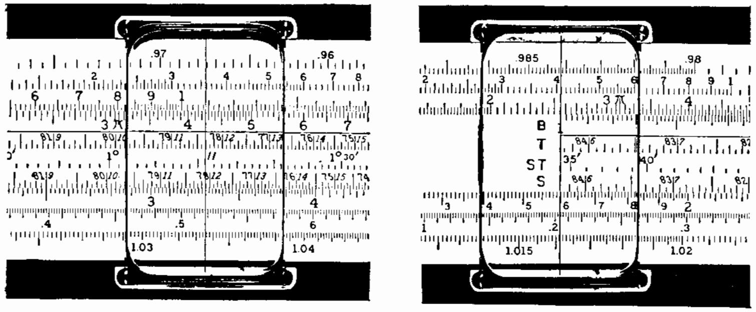


FIG. 96.

tan 7″ *Answer* = 7 × 0.000005 = 0.000035 approx.

Runner to 7 on D

Slide to seconds gauge

Runner to left index

Read 0.0000339 on D

sin 0.082° 0.082° = 60 × 0.082 = 4.92′

Answer ≑ 5 × 0.0003 ≑ 0.0015 approx.

Runner to 4.92 on D

Slide to minutes gauge

Runner to left index

Read 0.001433 on D

The reverse process of finding the magnitude of the angle from its function is also illustrated as follows:

tan x = 0.00082 $x \doteq \frac{8}{3}, \left(\frac{0.0008}{0.0003}\right) \doteq 2'$ (Fig. 97)

Runner to 0.00082 on D

Right index to hairline

Runner to minutes gauge

Read x as 2′49″(2.82′) on D

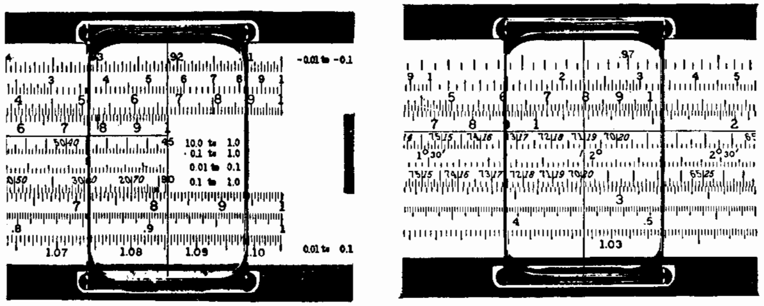


FIG. 97.

sin x = 0.0000149 $x \doteq \frac{15}{5}, \left(\frac{0.000015}{0.000005}\right) \doteq 3''$

Runner to 0.0000149 on D

Left index to hairline

Runner to seconds gauge

Read x as 3.07″ on D

Since the cos x = sin (90° − x) and cot x = tan (90° − x), the cosines and cotangents of angles greater than 89°26′(89.43°) can be found by computing the sines and tangents respectively of the complementary angles.

$$\cos 89°42' = \sin 18' = 0.00524$$

Also, since

$$\tan x = \frac{1}{\tan (90° - x)} \qquad \tan 89°42' = \frac{1}{\tan 18'} = \frac{1}{\sin 18'} = 191$$

Values of secants of angles close to 90° and of cosecants of angles close to 0° can be easily found from the corresponding values of the sine function.

$$\csc 0°18' = \frac{1}{\sin 0°18'} = 191$$

$$\sec 89°42' = \csc 0°18' = \frac{1}{\sin 0°18'} = 191$$

EXERCISE 19

Evaluate the following functions:

1. sin 17′
2. tan 4′30″
3. sin 43″
4. tan 22′
5. sin 6′15″
6. tan 8.5″
7. cos 89°46′
8. cot 89°59′10″
9. csc 24′45″
10. sec 89°53′
11. csc 6″
12. sin 31′

Determine x in the following equations:

13. $\sin x = 0.000872$
14. $\tan x = 0.0000436$
15. $\sin x = 0.00369$
16. $\sin x = 0.0000133$
17. $\cos x = 0.00207$
18. $\cot x = 0.000919$
19. $\tan x = 0.0000864$
20. $\cot x = 0.000371$

VI

SPECIAL OPERATIONS

34. Reciprocals. The CI scale and the C scale are reciprocals, i.e., if the runner is placed on a number on the C scale, such as 2, its reciprocal, $\frac{1}{2} = 0.5$, is read on the CI scale (Fig. 98). The scales can be used in two ways, either in setting a number on the C scale to read the reciprocal on the CI scale, or in setting a number on the CI scale to read the reciprocal on the C scale. The CF and CIF scales are also reciprocal scales. Thus, the reciprocal of any number can be found with these scales.

$\frac{1}{13.7}$ Runner to 13.7 on C
Read 0.073 on CI

$\frac{1}{6.8}$ Runner to 6.8 on CI
Read 0.147 on C

$\frac{1}{423}$ Runner to 423 on C
Read 0.00236 on CI

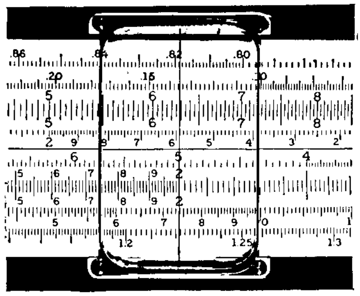


FIG. 98.

(Note that $\frac{1}{400} \doteq \frac{2}{1000} = 0.002$)

$\frac{1}{0.37}$ Runner to 0.37 on C
Read 2.70 on CI

The LL scales and the corresponding LL0 scales are also reciprocals as was mentioned in section 28. They cannot be used generally since the range of numbers over which they extend is limited. Also, the number of significant figures which can be read on each group of scales varies greatly. The LL scales cannot be read as accurately above e as can the C and CI scales. The number e is $2.718\cdots$, the base of natural logarithms. Likewise, the LL0 scales cannot be read as accurately below $\frac{1}{e}$, or 0.368, as can the C and CI scales. This means that the LL3 and LL03 scales are less accurate than the C and CI scales. However, the LL3 and LL03 scales can be read to three significant figures in many places. The LL1, LL2, LL01, and LL02 scales are more accurate than the C and CI scales, especially for values close to 1 where as many as six significant figures can be read. The LL and LL0 scales have the very obvious advantage of having the decimal point already placed. Although it may be desirable to use the C and CI scales instead of the LL3 and LL03 scales for the reciprocals of numbers between, say, 6 and 22,000 and between 0.10 and 0.000045, the log log scales do provide a ready means of setting the decimal point in such reciprocals. The following examples are for numbers on the LL1, LL2, LL01, and LL02 scales:

$\frac{1}{1.372}$ Runner to 1.372 on LL2 (Fig. 99)
Read 0.729 on LL02

$\frac{1}{0.563}$ Runner to 0.563 on LL02 (Fig. 100)
Read 1.776 on LL2

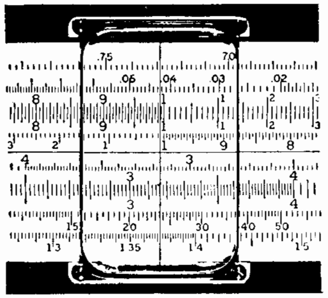


FIG. 99.

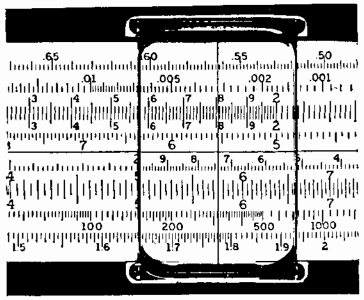


FIG. 100.

$\frac{1}{0.9864}$ Runner to 0.9864 on LL01
Read 1.01378 on LL1

$\frac{1}{1.0347}$ Runner to 1.0347 on LL1
Read 0.9664 on LL01

EXERCISE 20

Determine the reciprocals of the following quantities using the C and CI scales, or the CF and CIF scales:

1. 5.75	**6.** 4220
2. 186	**7.** 8.67
3. 23.6	**8.** 0.211
4. 0.0247	**9.** 0.0875
5. 0.00886	**10.** 152

Determine the reciprocals of the following quantities using the LL and LL0 scales:

11. 1.0165	**16.** 1.0859
12. 2.037	**17.** 0.853
13. 1.468	**18.** 0.9862
14. 0.97675	**19.** 1.898
15. 0.7168	**20.** 0.583

35. Proportions. The *ratio* of two numbers, say 2 and 5, is expressed as 2 divided by 5 or as $\frac{2}{5}$. A *proportion* is a statement of the equality of two ratios, for example $\frac{2}{5} = \frac{4}{10}$. If one of the quantities in a proportion is unknown, such as $\frac{3.7}{12} = \frac{x}{19}$, the proportion can be changed to give $x = \frac{3.7 \times 19}{12}$, and x can be determined by the standard procedure for multiplication and division. However, the solution can also be obtained as quickly by the device of setting the factors on the C and D scales in similar positions to those in the proportion. The procedure to determine x is as follows:

Runner to 12 on D (Fig. 101)
Slide to 3.7 on C
Runner to 19 on D
Read x as 5.86 on C

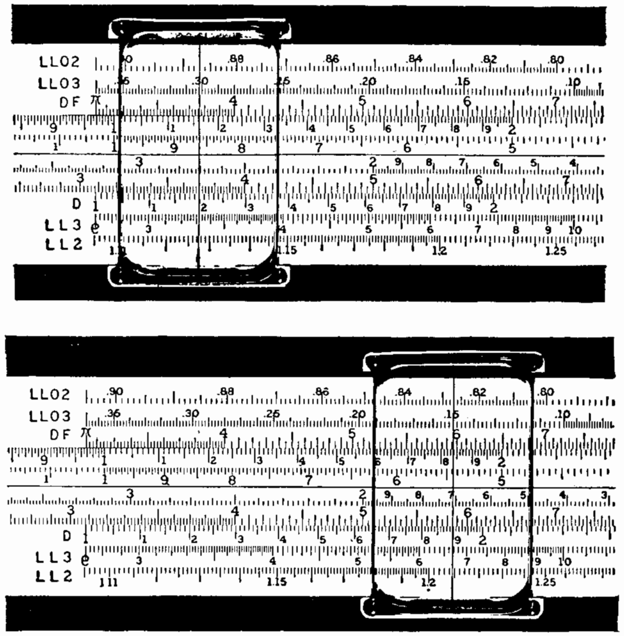


Fig. 101.

In this solution 3.7 is set directly over 12 and x is read directly over 19 on the C scale, $\frac{3.7 \text{ (on C)}}{12 \text{ (on D)}} = \frac{x \text{ (on C)}}{19 \text{ (on D)}}$.

In a proportion in which x is in the denominator, x would be read on the D scale. To determine x in the following proportion, the factors would be set on the scales as indicated:

$$\frac{6.12 \text{ (on C)}}{8.43 \text{ (on D)}} = \frac{21.7 \text{ (on C)}}{x \text{ (on D)}}$$

$$\text{whence } x = 29.9 \text{ on D}$$

The disadvantage of the proportion device is that frequently when the slide is set to the known ratio, the unknown ratio is off of the rule. For example in $\frac{1.26}{8.43} = \frac{x}{29.2}$, when 1.26 on the C scale is set opposite 8.43 on the D scale, the slide is shifted far to the right and nothing is opposite 29.2 on the D scale. To reset the slide is to lose the efficiency of the device of setting the factors like they are in the proportion. Using the folded scales is not always an alternative because in this particular case, the same situation occurs on the upper scales. Using the upper scales for one ratio and the lower scales for the other will solve this type of problem easily, but has the disadvantage that the ratio set on the upper scales must be inverted with respect to the ratio set on the lower scales.

Another method of solving such proportions as $\frac{1.26}{8.43} = \frac{x}{29.2}$ is to use a combination of upper and lower scales, such as the DF and C scales. The procedure is as follows:

Runner to 1.26 on DF

Slide to 8.43 on C

Runner to 29.2 on C

Read 4.36 on DF

The D and CF scales could also be used.

Exercise 21

Evaluate x in the following proportions:

1. $\frac{x}{32.7} = \frac{423}{67.7}$
2. $\frac{x}{6.51} = \frac{12.49}{27.7}$
3. $\frac{21.6}{x} = \frac{8.93}{47.7}$
4. $\frac{8.42}{x} = \frac{21.3}{3.72}$
5. $\frac{9.64}{72.8} = \frac{x}{243}$
6. $\frac{1.09}{4.93} = \frac{x}{18.18}$
7. $\frac{32.4}{48.63} = \frac{47.2}{x}$
8. $\frac{6.48}{0.927} = \frac{19.5}{x}$

9. A pole 12.5 feet tall casts a shadow 6.84 feet long. How tall is a monument which casts a shadow 47.2 feet long?

10. A man 6 feet 3 inches tall casts a shadow 15.40 feet long. How long a shadow will be cast at the same time by a man 5 feet 3 inches tall?

11. The ratio of height to width of a window to make the best appearance in a certain type of house is 17:11. If a window is to be 3 feet 2 inches wide, how high should it be?

12. A contractor has found that under certain conditions 5 bottom-dump earth hauling units can move 3350 cubic yards of earth per shift. How many units will he need if he is to haul 8650 cubic yards per shift under similar conditions?

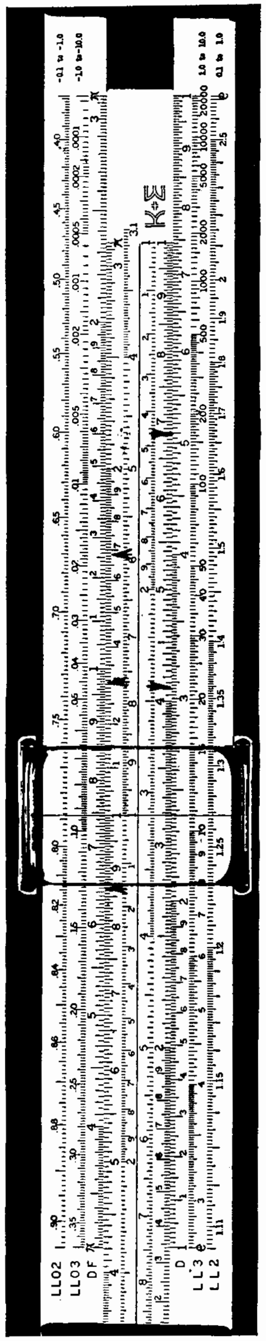


Fig. 102.

13. If 14,300 cubic yards of fill cost $4292 in place, what is the approximate cost of 9320 cubic yards of fill in place under similar conditions?

36. A Series of Numbers Multiplied or Divided by, or Divided into, One Factor.

1. *Multiplication of a Series of Numbers by One Factor.* Set the upper index on the single factor on the DF scale. Set the runner in turn on each number in the series either on the CF or the C scale, and read the answer either on the DF or the D scale respectively. For example: multiply the series of numbers 4.11, 6.83, 8.64, 13.07, 16.81 by 0.746.

Runner to 0.746 on DF (Fig. 102)
Upper index to hairline

Runner to 4.11 on C
Read 3.07 on D

Runner to 6.83 on C
Read 5.09 on D

Runner to 8.64 on CF
Read 6.44 on DF

Runner to 13.07 on CF
Read 9.75 on DF

Runner to 16.81 on CF
Read 12.54 on DF

The change from C to CF could also have been made with 6.83 or with 13.07.

2. *Division of a Series of Numbers by One Factor.* Set the single factor on the CF scale opposite the upper index. Set the runner in turn on each number on the CF or the C scale, and read the answer on the DF or the D scale respectively. For example: divide the series of 32.2, 24.15, 16.6, 11.9 by 8.05.

Runner to upper index (Fig. 103)
Slide to 8.05 on CF

Runner to 32.2 on C
Read 4.00 on D

Runner to 24.15 on C
Read 3.00 on D

Runner to 16.6 on C
Read 2.06 on D

Runner to 11.9 on C
Read 1.48 on D

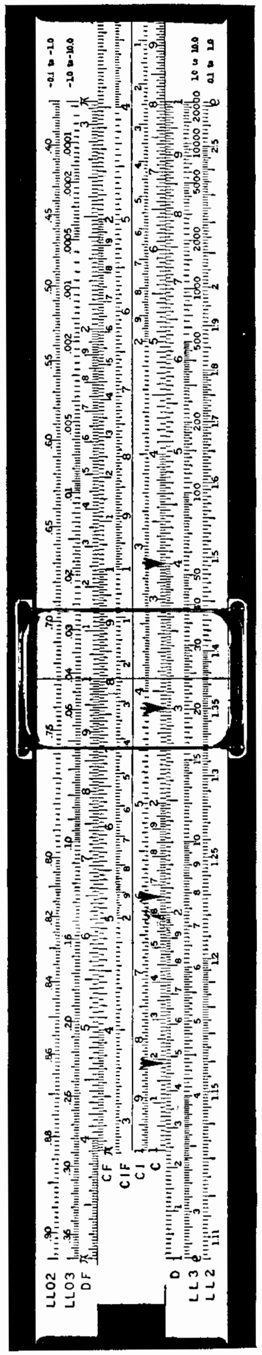


Fig. 103.

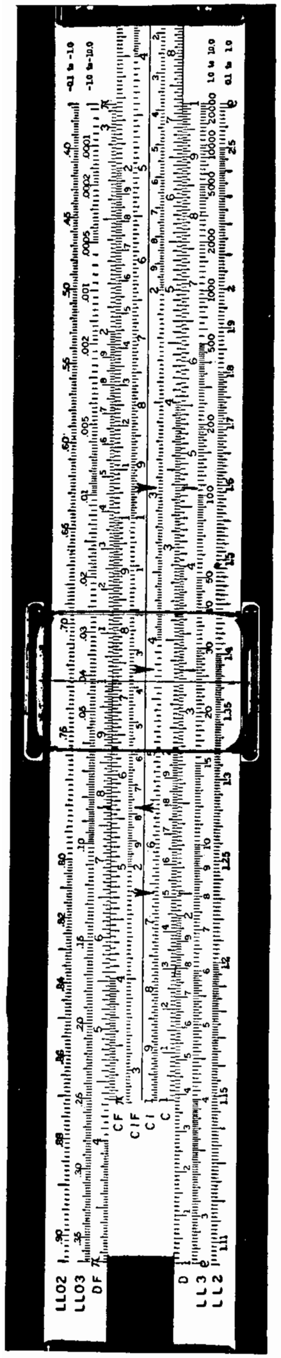


Fig. 104.

3. *Division of One Factor by a Series of Numbers.* Set the factor on the CIF to the upper index. Set the runner in turn on each number on the CIF or the CI scale, and read the answer on the DF or the D scale respectively. For example: divide 138.3 by the series 17.8, 29.6, 42.4, and 66.1.

Runner to upper index (Fig. 104)
Slide to 138.3 on CIF

Runner to 17.8 on CIF
Read 7.77 on DF

Runner to 29.6 on CI
Read 4.67 on D

Runner to 42.4 on CI
Read 3.26 on D

Runner to 66.1 on CI
Read 2.09 on D

Exercise 22

1. Multiply the series of numbers, 1.635, 2.08, 3.77, 5.15, 8.69, 12.06, by each of the following numbers:

(a) 3.72
(b) 7.63
(c) 11.15

2. Divide the series of numbers, 26.7, 49.9, 61.4, 97.3, 145.5, 316, by each of the following numbers:

(a) 6.98
(b) 13.25
(c) 27.8

3. Divide each of the following numbers by the series of numbers, 0.367, 0.724, 1.063, 2.88, 5.57, 8.81:

(a) 85.7
(b) 40.4
(c) 14.64

37. Multiplication and Division Involving π. The upper scales, DF, CIF, and CF, begin with π as previously explained in section 11. If the runner is set on a number on the D scale, it is also set on π times that number on the DF scale. The proof of this is simple. The product of π times a number, say πN, is solved by logarithms as $\log \pi N = \log \pi + \log N$. By placing π on the DF scale opposite the left index of the D scale, $\log \pi$ is correspondingly added to the logarithm of any number, N, on the D scale. The sum of the logarithms, i.e., the product of π and the number, is on the DF scale opposite the number on the D scale (Fig. 105). This arrangement permits the following rules to be stated:

To multiply a number, N, by π, set the number on D *and read πN on* DF (or on C and CF respectively).

To divide a number, N, by π, set the number on DF *and read $\frac{N}{\pi}$ on* D (or on CF and C respectively).

To divide π *by a number N, set N on* CF *and read* $\frac{\pi}{N}$ *on* CI.

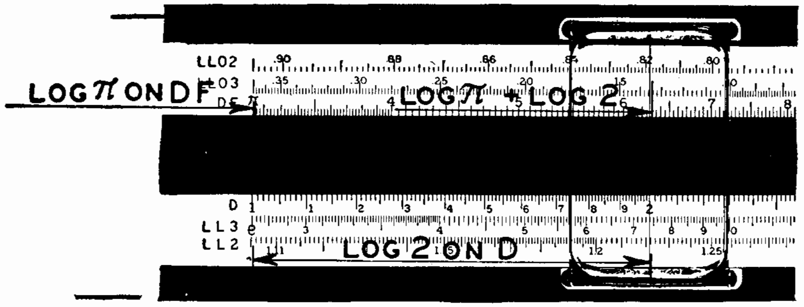


FIG. 105.

Examples:

15.4π	Runner to 15.4 on D Read 48.4 on DF or Runner to 15.4 on C Read 48.4 on CF
$\frac{24.7}{\pi}$	Runner to 24.7 on DF Read 7.86 on D
$\frac{\pi}{0.832}$	Runner to 0.832 on CF Read 3.78 on CI

For circles, the circumference is π times the diameter. Obviously, the DF and D scales, or the CF and C scales, are related to each other respectively as the circumference and diameter of a circle. To obtain the circumference, set the diameter on D and read on DF; to obtain the diameter, set the circumference on DF and read on D.

The area of a circle is πr^2 or $\frac{\pi d^2}{4}$, where r is the radius and d is the diameter. The D scale can be used with the A and B scales to obtain areas from diameters and vice versa with a single setting of the runner. Toward the right end of the A and B scales is placed a special mark representing $\frac{\pi}{4}$ or 0.7854. When this mark on the B scale is placed opposite the right index of the A scale, then the relation of B to D is as area to diameter. For example: *set* $\frac{\pi}{4}$ *on B to right index of A.*

Diameter 6.2	Runner to 6.2 on D Read area as 30.2 on B cycle 2	(Fig. 106)

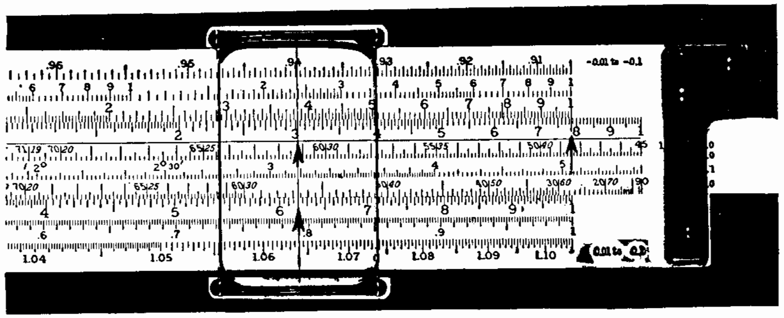


FIG. 106.

Diameter 2.5	Runner to 2.5 on D Read area as 4.91 on B cycle 1
Diameter 16.5	Runner to 16.5 on D Read area as 214 on B cycle 1
Area 57.6	Runner to 57.6 on B cycle 2 Read diameter as 8.56 on D
Area 143.1	Runner to 143.1 on B cycle 1 Read diameter as 13.5 on D

For diameters between the integers 1000 and 1130, and the corresponding areas between 7854 and 1000, the mark for $\frac{\pi}{4}$ on the B scale must be set opposite the center index of the A scale. As the B scale was originally set, it did not extend over the interval from 1000 to 1130 on the D scale, as can be seen in Fig. 106. In setting $\frac{\pi}{4}$ opposite the center index of A, the cycles of the B scale are thus shifted with regard to their normal designation. The left cycle now corresponds to cycle 2 and the right cycle to cycle 1 for setting the decimal point. This shift is illustrated in the following example:

Diameter 10.65 Runner to 10.65 on D

Read area as 89.1 on B cycle 2

There are 2π radians in a circle, i.e., in 360°. Radians may be converted into degrees by setting the ratio $\frac{2\pi}{360}$, or $\frac{\pi}{180}$, on the upper scales, and vice versa. For setting the decimal point the relationship that 1 radian is approximately 60° can be used. Examples of conversion are as follows: *set 180 on* CF *opposite* π *right on* DF.

1.5 radians Runner to 1.5 on DF

Read 85.9° on CF

0.243 radians Runner to 0.243 on DF

Read 13.92° on CF

0.468 radians Runner to 0.468 on D

Read 26.8° on C

173° Runner to 173 on CF

Read 3.02 radians on DF

Many multiplications, divisions, and combinations involving three factors of which one is π can be easily computed using the value of π on the folded scales, and also the relationship between the regular and the folded scales. For example:

Multiplication $4.68 \times 2\pi$

Runner to 4.68 on D

Slide to 2 on CI

Runner to π on CF

Read 29.4 on DF

Multiplication $2.27 \times 1.23\pi$

Runner to 2.27 on D

Slide to 1.23 on CIF

Runner to upper index

Read 8.78 on DF

When 2.27 is set on the D scale, 2.27π is also set on the DF scale.

Division $\frac{9.37}{3.77\pi}$

Runner to 9.37 on DF

Slide to 3.77 on C

Runner to lower right index

Read 0.791 on D

In this example, when 9.37 is set on the DF scale, $\frac{9.37}{\pi}$ is also set on the D scale.

Combination $\frac{14.4\pi}{6.27}$

Runner to 14.4 on DF

Slide to 6.27 on CF

Runner to π on CF

Read 7.21 on DF

Combination $\frac{8.16 \times 2.59}{\pi}$

Runner to 8.16 on DF

Slide to 2.59 on CI

Runner to upper index

Read 6.73 on DF

When 8.16 is set on the DF scale, $\frac{8.16}{\pi}$ is also set on the D scale.

Reciprocal $\frac{1}{5\pi}$

Runner to 5 on **C**

Read 0.0636 on **CIF**

Exercise 23

Evaluate the following expressions:

1. 4π

2. 0.735π

3. 15.72π

4. 3.2π

5. $\frac{8.75}{\pi}$

6. $\frac{19.27}{\pi}$

7. $\frac{0.523}{\pi}$

8. $\frac{6.08}{\pi}$

9. $3.42 \times 2\pi$

10. $6.27 \times 4.12\pi$

11. $1.65 \times 0.592\pi$

12. $\frac{2\pi}{14.75}$

13. $\frac{8.02\pi}{3.62}$

14. $\frac{17.6\pi}{4.95}$

15. $\frac{16.2 \times 7.16}{\pi}$

16. $\frac{22.8 \times 4.97}{\pi}$

17. $\frac{1}{2\pi}$

18. $\frac{1}{0.7\pi}$

19. $\frac{8.77}{2\pi}$

20. $\frac{77.6}{4.7\pi}$

Convert the following angles to radians:

21. 86°

22. 42°

23. 265°

24. 497°

25. 17°

26. 366°

Convert the following angles expressed in radians into degrees:

27. 1.49

28. 0.37

29. 2.06

30. 5.93

31. 12.62

32. 1.11

Compute the areas of circles which have the following diameters:

33. 2.49

34. 61.8

35. 0.374

36. 7.92

37. 0.0186

38. 10.4

Compute the diameters of circles which have the following areas:

39. 12.7

40. 149

41. 0.925

42. 37.3

43. 8.46

44. 529

45. What is the area of a circular garden plot inscribed in a plot of ground which is 27.5 feet square? What percentage of the area of the square is covered by the circle?

46. The lateral area of a cone of revolution is given by $A = \pi r l$, where A = lateral area, r = radius of circular base, l = slant height. Find the slant height of a cone whose base diameter is 17.6 inches and whose lateral area is 2450 square inches.

38. Multiplication and Division Involving the A, B, and K Scales. Many computations of the types $14.6(8.1)^2$, $(6.1)^3(2.3)^2$, $13\sqrt{12.4}$, $0.875\sqrt[3]{42.6}$, $\frac{\sqrt{18.7}\,\sqrt[3]{14.6}}{8.12}$, and so on can be solved directly in one continuous operation by using the A, B, and K scales along with the D, CI, and C scales. There are too many possible combinations like those above to try to include all of them in this book. A few selected examples will illustrate the procedures to be followed. The decimal point is set by inspection. Where the cycle on A, B, or K is not specified, it is not significant in the operation. In some operations of this type, it is necessary to re-set the slide since the operation "runs off the rule."

$14.6(8.1)^2$ Runner to 8.1 on D [$(8.1)^2$ is on A]

Right index of B to hairline

Runner to 14.6 on B

Read 958 on A

$\frac{(17.6)^2}{42.3}$ Runner to 17.6 on D [$(17.6)^2$ is on A] (Fig. 107)
Slide to 42.3 on B
Runner to center index of B
Read 7.32 on A

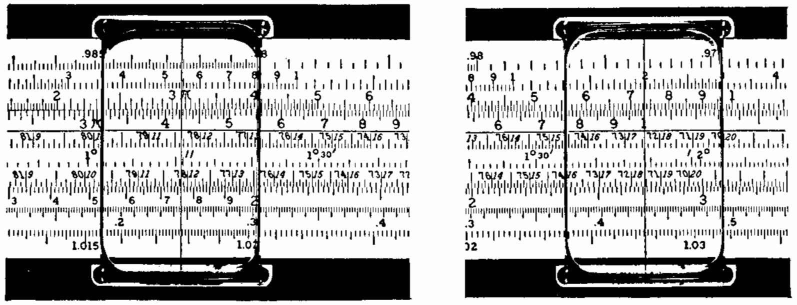


Fig. 107.

$\frac{1680}{(5.23)^2}$ Runner to 5.23 on D [$(5.23)^2$ is on A] (Fig. 108)
Slide to 1680 on B
Runner to center index of A
Read 615 on B

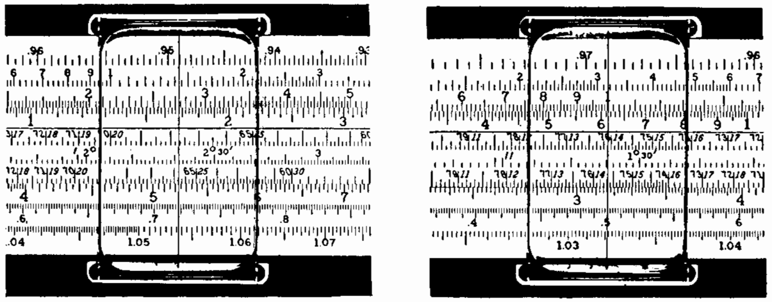


Fig. 108.

$32.1\sqrt{14.05}$ Runner to 14.05 on A cycle 2 [$\sqrt{14.05}$ is on D]
Slide to 32.1 on CI
Runner to upper index of CF
Read 120.3 on DF

$\frac{197.2}{\sqrt{576}}$ Runner to 197.2 on D
Slide to 576 on B cycle 1 [$\sqrt{576}$ is on C]
Runner to upper index of CF
Read 8.22 on DF

$\frac{\sqrt{6280}}{12.4}$ Runner to 6280 on A cycle 2
Slide to 12.4 on C
Runner to left index of C
Read 6.39 on D

$\frac{21.7\sqrt{8.23}}{3.24}$ Runner to 21.7 on D
Slide to 3.24 on C
Runner to 8.23 on B cycle 1
Read 19.2 on D

$\frac{\sqrt{529}}{4.21 \times 6.40}$ Runner to 529 on A cycle 1
Slide to 4.21 on C
Runner to 6.40 on CIF
Read 0.854 on DF

$\frac{8.02 \times 13.43}{\sqrt{1056}}$ Runner to 8.02 on D
Slide to 1056 on B cycle 2
Runner to 13.43 on C
Read 3.32 on D

$61.2 \times 0.238\sqrt{47.7}$ Runner to 47.7 on A cycle 2

Slide to 0.238 on CI

Runner to 61.2 on CF

Read 100.7 on DF

$1.82\sqrt{37.9}\sqrt{1.37}$ Runner to 37.9 on A cycle 2

Slide to 1.82 on CI

Runner to 1.37 on B cycle 1

Read 13.12 on D

$\dfrac{3.13(7.43)^2}{(14.6)^2}$ Runner to 7.43 on D

Slide to 14.6 on C

Runner to 3.13 on B

Read 0.810 on A

$\dfrac{776}{(4.12)^2(1.33)^2}$ Runner to 776 on A cycle 1

Slide to 4.12 on C

Runner to 1.33 on CI

Read 25.8 on A

$0.875\sqrt[3]{42.6}$ Runner to 42.6 on K cycle 2

[$\sqrt[3]{42.6}$ is on D]

Slide to 0.875 on CIF

Runner to upper index

Read 3.06 on D

$\dfrac{\sqrt[3]{6720}}{4.22}$ Runner to 6720 on K cycle 1

Slide to 4.22 on CF

Runner to upper index

Read 4.47 on D

$\dfrac{272}{\sqrt[3]{437}}$ Runner to 437 on K cycle 3 (Fig. 109)

Slide to 272 on C

Runner to right index *of D*

Read 35.8 on C

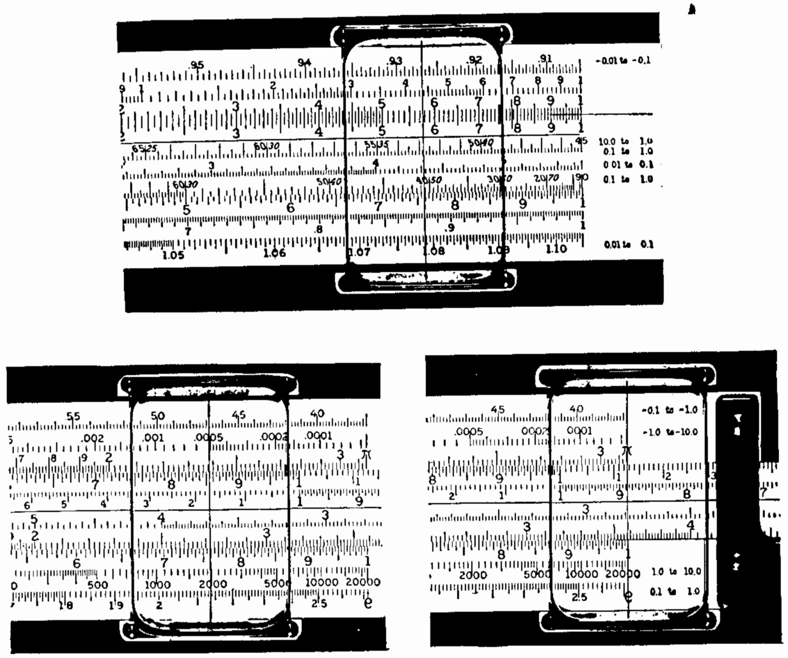


FIG. 109.

$1.76(4.23)^3$ Runner to 1.76 on K cycle 1

[$\sqrt[3]{1.76}$ on D]

Slide to 4.23 on CI

Runner to right index of C

Read 134 on K cycle 3

$\dfrac{48.4}{(0.816)^3}$ Runner to 48.4 on K cycle 2

Slide to 0.816 on C

Runner to right index of C

Read 89 on K cycle 2

$\frac{\sqrt{18.7}\sqrt[3]{14.6}}{8.12}$	Runner to 14.6 on K cycle 2 Slide to 8.12 on C Runner to 18.7 on B cycle 2 Read 1.30 on D
$\frac{5.09\sqrt[3]{100.7}}{\sqrt{41.2}}$	Runner to 100.7 on K cycle 3 Slide to 41.2 on B cycle 2 Runner to 5.09 on C Read 3.76 on D
$\frac{(14.6)^3}{(3.96)^2}$	Solve taking $(14.6)^3$ as $(14.6\sqrt{14.6})^2$ Runner to 14.6 on A cycle 2 Slide to 3.96 on C Runner to 14.6 on C Read 198 on A cycle 1
$(1.95)^3(6.27)^2$	Solve taking $(1.95)^3$ as $(1.95\sqrt{1.95})^2$ Runner to 1.95 on A cycle 1 Slide to 6.27 on CI Runner to 1.95 on C Read 292 on A cycle 1
$\frac{4.61(8.07)^2}{(2.44)^3}$	Solve using 4.61 as $(\sqrt{4.61})^2$ Runner to 4.61 on A cycle 1 Slide to 2.44 on B cycle 1 Runner to 8.07 on CF Slide to 2.44 on CF Runner to right index of C Read 20.7 on A cycle 2

$\frac{141}{6.23(3.74)^2}$	Runner to 141 on A cycle 2 Slide to 6.23 on B cycle 1 Runner to 3.74 on CI Read 1.62 on A

In this problem, 141 is divided by 6.23 on the square scales. The cycles in this quotient are not significant and are selected so as to keep the slide close to the center.

$\frac{643}{(4.62)^2(11.6)^2}$	Runner to 643 on A cycle 1 Slide to 4.62 on C Runner to 11.6 on CI Read 0.224 on A
$6.1\pi\sqrt{3.73}$	Runner to 3.73 on A cycle 1 Slide to 6.1 on CI Runner to left index of C Read 37.0 on DF
$\frac{297\pi}{\sqrt{1220}}$	Runner to 297 on D Slide to 1220 on B cycle 2 Runner to right index of C Read 26.7 on DF
$\sqrt{5\pi}$	Runner to π on A cycle 1 Left index of B to hairline Runner to 5 on B cycle 1 Read 3.96 on D
$\sqrt{81.3\pi}$	Runner to π on A cycle 1 Center index of B to hairline Runner to 81.3 on B cycle 1 Read 15.98 on D

Normally the right index of B should be set opposite π and 81.3 should be set on cycle 2. Actually, cycle 1 is used in place of cycle 2 and less movement is involved.

$\dfrac{\mathbf{19.3\sqrt{76.2}}}{\pi}$

Runner to 76.2 on A cycle 2
Slide to 19.3 on CI
Runner to upper index of CF
Read 53.6 on D

$\dfrac{\mathbf{741}}{\pi\sqrt{\mathbf{13.5}}}$

Runner to 741 on D
Slide to 13.5 on B cycle 2
Runner to upper index on CF
Read 64.2 on D

$\dfrac{\mathbf{5.26\sqrt{8.72}}}{\sqrt{\pi}}$

Runner to 8.72 on A cycle 1
Slide to π on B cycle 1
Runner to 5.26 on CF
Read 8.75 on DF

The C and D scales could also have been used in the last step, but with more movement.

$\dfrac{\mathbf{224}}{\sqrt{\mathbf{37.3}\pi}}$

Runner to 224 on D
Slide to 37.3 on B cycle 2
Runner to right index of C
Slide to π on B cycle 1
Runner to left index of C
Read 20.7 on D

Exercise 24

Evaluate the following expressions:

1. $16.1\sqrt{47.3}$

2. $47.9\sqrt{1.46}$

3. $8.42\sqrt{10.6}$

4. $2.17 \times 8.63\sqrt{0.562}$

5. $5.62 \times 41.7\sqrt{0.000975}$

6. $\dfrac{\sqrt{6.45}}{8.27}$

7. $\dfrac{\sqrt{22.25}}{1.86}$

8. $\dfrac{67.3}{\sqrt{123.4}}$

9. $\dfrac{8.47\sqrt{637}}{14.64}$

10. $\dfrac{2.21\sqrt{4.62}}{3.79}$

11. $\dfrac{15.7 \times 6.37}{\sqrt{127}}$

12. $\dfrac{3.19 \times 126.2}{\sqrt{1440}}$

13. $\dfrac{\sqrt{1836}}{6.23 \times 10.05}$

14. $\dfrac{\sqrt{436}}{2.08 \times 5.63}$

15. $\dfrac{6240}{5.15\sqrt{347}}$

16. $\dfrac{1}{42.7\sqrt{2.63}}$

17. $2.46\sqrt{6.23 \times 4.97}$

18. $\dfrac{8.98\sqrt{5.62}}{\sqrt{48.1}}$

19. $\dfrac{1722}{\sqrt{3.19 \times 20.6}}$

20. $\dfrac{\sqrt{673}}{4.22\sqrt{51.5}}$

21. $5.75\sqrt[3]{69.8}$

22. $17.52\sqrt[3]{8.63}$

23. $\dfrac{\sqrt[3]{499}}{4.22}$

24. $\dfrac{6720}{\sqrt[3]{1430}}$

25. $\sqrt{42.7}\sqrt[3]{12.3}$

26. $\dfrac{\sqrt[3]{1920}}{\sqrt{259}}$

27. $3.17\sqrt{13.6}\sqrt[3]{97.5}$

28. $\dfrac{22.4\sqrt[3]{649}}{\sqrt{1750}}$

29. $\dfrac{\sqrt{86.8}\sqrt[3]{9.70}}{21.1}$

30. $4.02\sqrt{12.25}\sqrt[3]{106}$

31. $4.1(6.1)^2$

32. $13.7(0.187)^2$

33. $\dfrac{(26.4)^2}{46.2}$

34. $\dfrac{(1.93)^2}{8.64}$

35. $\dfrac{529}{(7.76)^2}$

36. $\dfrac{11.75}{(0.165)^2}$

37. $\dfrac{4.27(3.66)^2}{1.69}$

38. $\dfrac{55.3}{8.17(2.59)^2}$

39. $\dfrac{6.25(21.63)^2}{(4.27)^2}$

40. $\dfrac{1340}{(5.92)^2(2.17)^2}$

41. $\dfrac{(2.29)^3}{(4.37)^2}$

42. $(6.77)^3(0.591)^2$

43. $\left(\frac{2.57}{1.69}\right)^2$

44. $\left(\frac{6.42}{9.69}\right)^3$

45. $\frac{(17.8)^2}{(2.04)^3}$

46. $4.92(7.09)^3$

47. $\frac{77.8}{(4.37)^3}$

48. $\frac{2.29(5.72)^2}{(3.73)^3}$

49. $\frac{0.842(5.69)^3}{(7.65)^2}$

50. $16.3\left(\frac{12.25}{7.37}\right)^3$

51. $2\pi\sqrt{37.7}$

52. $4.71\pi\sqrt{9.78}$

53. $\frac{\pi}{\sqrt{6.36}}$

54. $\frac{43.7\pi}{\sqrt{79.4}}$

55. $\sqrt{17.3\pi}$

56. $\frac{5.46\sqrt{183}}{\pi}$

57. $\frac{32.8}{\pi\sqrt{65.4}}$

58. $\frac{8.83\sqrt{4.97}}{\sqrt{\pi}}$

59. $4.63\sqrt{2\pi}$

60. $\frac{\sqrt{6\pi}}{5.22}$

The equations of motion for a freely falling body which starts from rest are

$$v = gt$$

$$s = \tfrac{1}{2}gt^2$$

$$v = \sqrt{2gs}$$

where g = 32.2 feet per second per second, the acceleration of gravity
v = velocity in feet per second
s = distance from starting point in feet
t = time in seconds

61. How far will a body fall from rest in 3.5 seconds?
62. What is the velocity of a body after falling for 1.37 seconds?
63. How long will it require for a body to fall 127 feet?
64. What is the velocity of a body after falling 384 feet?
65. How far has a body fallen when its velocity is 204 feet per second?
66. How long will it take a package thrown from an airplane at a height of 2720 feet to reach the ground? Neglect air resistance.
67. How hard must a baseball be thrown, i.e., with what velocity, from the ground to reach the top of a building 141 feet above the thrower? Convert this velocity to miles per hour.

68. The volume of a cubical packing box is 448 cubic inches. If 31 of such boxes are placed together in a single line, how long is this line in feet?
69. Find the weight of a cubical block of wood measuring 0.865 feet on a side. The density of the wood is 46.6 pounds per cubic foot.
70. The weight of a cubical block of wood is 108 pounds. If the density of the wood is 38.4 pounds per cubic foot, what is the dimension of the cube in feet? (Hint: work this problem by reversing the procedure of problem 69.)
71. The volume of a cone of revolution with a circular base is expressed by

$$V = \frac{\pi d^2 h}{12}$$

where V = volume
d = diameter of base
h = height

Find the volume of a cone of height 13.2 inches and base diameter 7.25 inches.
72. The surface area of a sphere of radius r is $4\pi r^2$, and the volume is $\frac{4}{3}\pi r^3$. Find the surface area of a sphere of diameter 15.8 inches.
73. A spherical water tank is 22 feet in diameter. What is its capacity in gallons? (Note: solve this problem in two steps, cubing 22 first.)
74. What is the diameter in feet of a spherical water tank which will hold 63,000 gallons? (Note: solve this problem in two steps, finding r^3 first.)
75. If the cost of aluminum paint is $0.075 per square foot, what is the approximate cost of painting a spherical water tank 33.5 feet in diameter?

39. Multiplication and Division Involving Trigonometric Functions. The trigonometric scales, S, ST, and T, can be used with the D and C scales to obtain the answers to multiplications and divisions involving trigonometric functions. Most of the trigonometric scales are used in the same way as C scales or cycles in such computations, the S scale and the T black scale representing numbers from 0.1 to 1 and the ST scale representing numbers from 0.01 to 0.1. However, the T red scale, which is used with the CI scale, is used in the same way as the CI scale in multiplication and division. In multiplication, the method using the D and C scales as shown in Fig. 33 must be used most of the time. The decimal point in such computations is set with the aid of the legends at the end of the trigonometric scales. The following computations are given as examples:

	Operation	Decimal Point
12 sin 25°	Runner to 12 on D	(Fig. 110)
	Left index to hairline	sin 25° ≐ 0.4 on C
	Runner to 25° on S black	12 × 0.4 ≐ 5
	Read 5 0 7 on D	*Answer:* 5.07

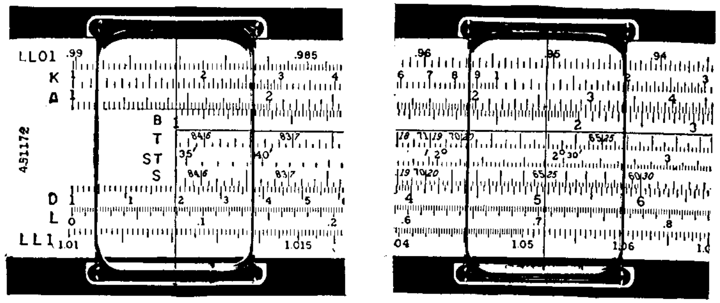


Fig. 110.

81.7 cos 76°	Runner to 81.7 on D	
	Right index to hairline	cos 76° ≐ 0.2 on C
	Runner to 76° on S red	80 × 0.2 = 16
	Read 1 9 7 6 on D	*Answer:* 19.76
$\dfrac{\mathbf{6.93}}{\mathbf{\cos 23°}}$	Runner to 6.93 on D	(Fig. 111)
	Slide to 23° on S red	cos 23° ≐ 0.9 on C
	Runner to right index	$\dfrac{7}{0.9} \doteq 7$
	Read 7 5 3 on D	*Answer:* 7.53

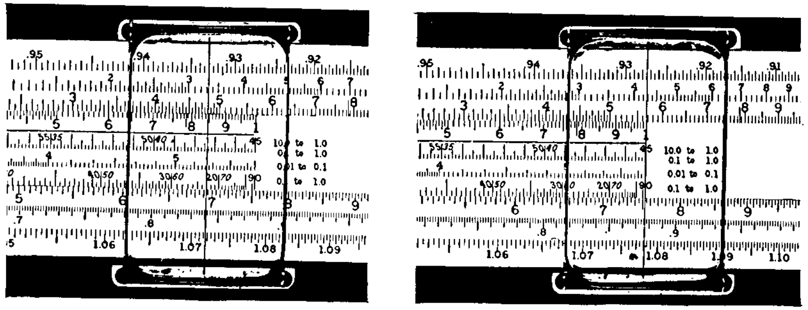


Fig. 111.

$\dfrac{\mathbf{18.7}}{\mathbf{\tan 3°}}$	Runner to 18.7 on D	
	Slide to 3° on ST	tan 3° ≐ 0.05 on C
	Runner to right index	$\dfrac{20}{.05} = 400$
	Read 3 5 8 on D	*Answer:* 358
4.62 tan 78°	Runner to 4.62 on D	
	Slide to 78° on T red	tan 78° ≐ 5
	Runner to left index	4 × 5 = 20
	Read 2 1 8 on D	*Answer:* 21.8
17 sec 27°	Solve as 17/cos 27°	
	Runner to 17 on D	
	Slide to 27° on S red	cos 27° ≐ 0.9
	Runner to right index	$\dfrac{17}{0.9} \doteq 17$
	Read 1 9 1 on D	*Answer:* 19.1
$\dfrac{\mathbf{18.64 \sin 32°}}{\mathbf{\tan 16°}}$	Runner to 18.64 on D	
	Slide to 16° on T	$\dfrac{20 \times 0.5}{0.3} \doteq 20$
	Runner to 32° on S black	
	Read 3 4 5 on D	*Answer:* 34.5

$$\sin x = \frac{0.621 \sin 21°}{\cos 37°}$$

Runner to 0.621 on D (Fig. 112)

Slide to 37° on S red

Runner to 21° on S black

Center the slide [5]

Read x as 16°12′(16.2°) on S black

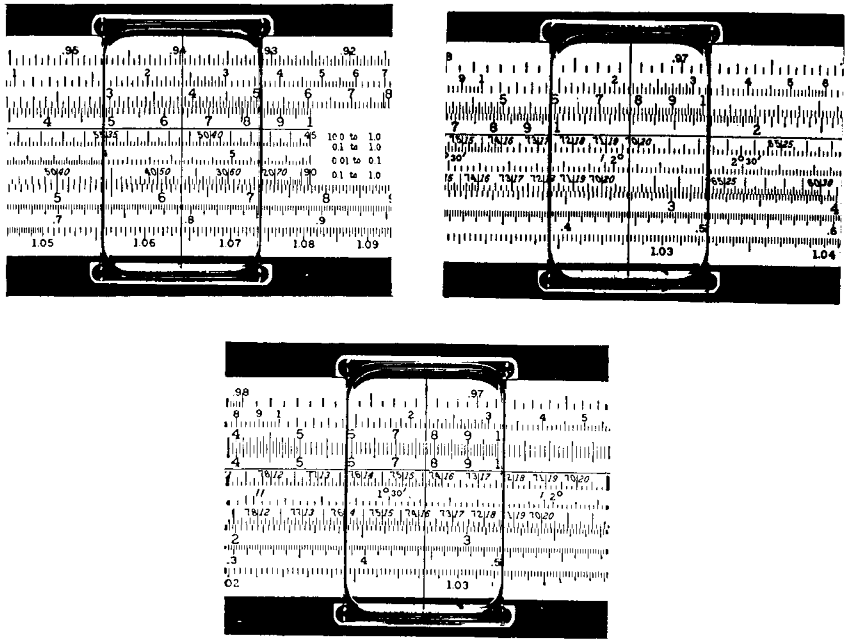


Fig. 112.

$$\cos x = \frac{437 \tan 2°}{62.4}$$

Runner to 437 on D

Slide to 62.4 on C

Runner to 2° on ST

Center the slide

Read x as 75°51′(75.85°) on S red

[5] Set 1 on the C scale opposite 1 on the D scale.

Certain problems lead to the setting up of an equation in the form of a proportion. Problems in trigonometry involving the law of sines are of this type. The law of sines stated in terms of the angles of a triangle and the corresponding opposite sides is

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Computations in this form can be solved as a proportion by setting the angles on the S scale and the corresponding sides on the D scale as shown in the following examples:

$$\frac{\sin 37°}{a} = \frac{\sin 62°}{71.7}$$

Runner to 71.7 on D (Fig. 113)

Slide to 62° on S black

Runner to 37° on S black

Read a as 48.9 on D

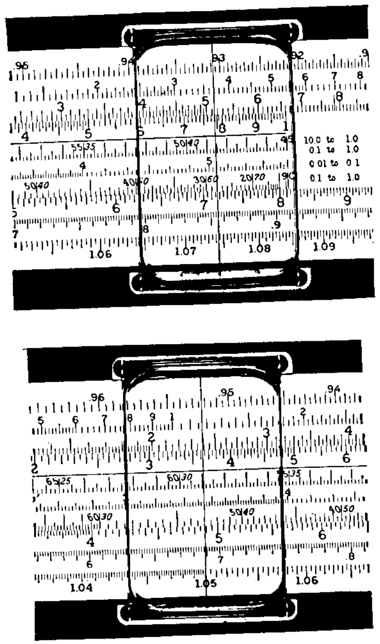


Fig. 113.

$$\frac{\sin A}{33.2} = \frac{\sin 42°}{57.9}$$

Runner to 57.9 on D (Fig. 114)

Slide to 42° on S black

Runner to 33.2 on D

Read A as 22°33′(22.55°) on S black

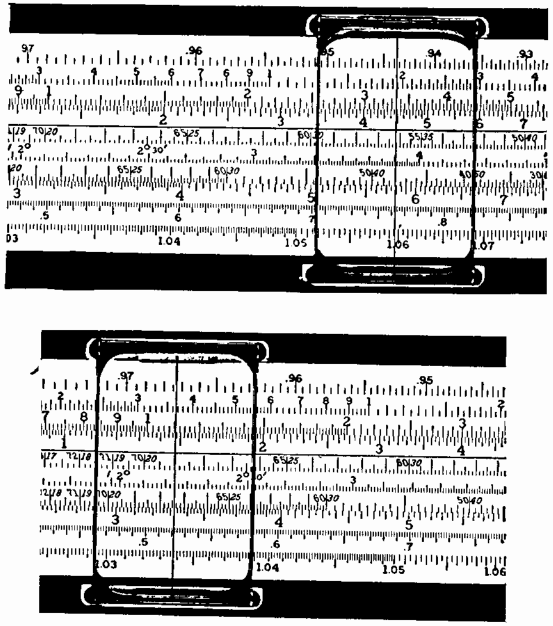


FIG. 114.

Note that the ratios are set on the slide rule just as they occur in the equation, for example, with 42° above 57.9 and A above 33.2.

In triangles which can be solved with the law of sines, the above procedure affords a very quick solution for unknown sides and angles. If two sides and the angle opposite one side are given, such as sides a and b and angle A, angle B is found from $\frac{\sin A}{a} = \frac{\sin B}{b}$. The third angle C is found by subtracting $A + B$ from 180°, and side c is found from the third term of the proportion $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. If two angles and any side are given, such as A, B, and c, the third angle C is found by subtracting $A + B$ from 180°, and

the other two sides are found from the proportion $\frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$. For example, solve for the unknown sides and angles of a triangle for which $a = 44.6$, $b = 34.3$, $A = 29°30'$.

$$\frac{\sin 29°30'}{44.6} = \frac{\sin B}{34.3} = \frac{\sin C}{c}$$

Runner to 44.6 on D

Slide to 29°30′ on S black

Runner to 34.3 on D

Read B as 22°15′ on S black

$C = 180° - 29°30' - 22°15'$

$= 180° - 51°45' = 128°15'$

$\sin 128°15' = \sin 51°45'$

Runner to 51°45′ on S black

Read c as 71.1 on D

As another example, solve for the unknown sides and angles of a triangle for which $a = 10.8$, $A = 53°$, $B = 42°$. Since 10.8 is close to the end of the D scale, use the DF scale in place of the D scale. $C = 180° - 53° - 42° = 85°$

$$\frac{10.8}{\sin 53°} = \frac{b}{\sin 42°} = \frac{c}{\sin 85°}$$

Runner to 10.8 on DF

Slide to 53° on S black

Runner to 42° on S black

Read b as 9.05 on DF

Runner to 85° on S black

Read c as 13.48 on DF

The following examples are computations involving trigonometric functions with π and the A, B, and K scales:

π tan 79° Runner to 79° on T
Read 16.4 on CIF

π tan 31° Runner to 31° on T
Read 1.89 on CF

π sin 1° Runner to 1° on ST
Read 0.0549 on CF

$\frac{\sin 17°}{\pi}$ Runner to upper index on DF
Left index of C to hairline
Runner to 17° on S black
Read 0.0931 on D

$\sin^2 27°$ Runner to 27° on S black
Read 0.206 on B cycle 2

$\pi \cos^2 32°$ Runner to π on A cycle 1
Right index (end) of S to hairline
Runner to 32° on S red
Read 2.26 on A cycle 1

$\frac{\tan^2 18°}{\pi}$ Center the slide
Runner to 18° on T
Slide to π on B cycle 1
Runner to left index of B
Read 0.0335 on A cycle 1

$\pi \tan^3 25°$ Runner to π on K cycle 1
Left index (end) of T to hairline
Runner to 25° on T
Read 0.319 on K cycle 3

$\sqrt{\pi} \cos 63°$ Runner to π on A cycle 1
Left index to hairline
Runner to 63° on S red
Read 0.804 on D

$\frac{\tan 4°}{\sqrt{\pi}}$ Center the slide
Runner to 4° on ST
Slide to π on B cycle 1
Runner to left index
Read 0.0396 on D

Exercise 25

Evaluate the following expressions:

1. 36.3 sin 47°
2. 4.12 × 2.70 sin 16°
3. 1.29 cos 62°15′(62.25°)
4. 6.09 sin 2°42′(2.70°)
5. 387 tan 4°45′(4.75°)
6. 8.69 cot 25°
7. 0.652 sec 87°
8. 1.37 csc 12°24′(12.40°)
9. $\frac{8.63}{\sin 19°}$
10. $\frac{10.63 \sin 42°}{3.64}$
11. $\frac{4.38 \times 2.77}{\sin 3°}$
12. $\frac{18.64}{\tan 79°}$
13. $\frac{72.6 \tan 36°}{\cos 57°}$
14. $\frac{5.46 \sin 69°}{\sin 12°}$
15. 0.059 tan 87°
16. $\frac{6.87}{\cos 27°}$
17. 3.43 tan 63° sin 15°
18. $\frac{4.46 \sin 26°}{\cos 12°}$
19. 287 sin 6′
20. 1130 tan 21″
21. $\cos^2 42°$
22. $\sin^3 4°30′(4.50°)$
23. $\tan^2 29°$
24. $\pi \cos^2 21°$
25. π tan 2°15′(2.25°)
26. $\pi \sin^3 37°$
27. $\sqrt{\pi} \tan 49°$
28. $\frac{\sin^2 67°}{\pi}$
29. $6.83 \tan^2 4°$
30. $12.2 \cos^3 25°$

Solve for the unknown sides and angles of the following triangles using the law of sines:

31. Given: $a = 37.5$ Find: c, A, C
$b = 49.5$
$B = 46°$

32. Given: $b = 76.2$ Find: a, c, B
$A = 32°$
$C = 79°$

33. Given: $c = 112$ Find: a, b, C
$A = 107°$
$B = 33°$

34. Given: $b = 5.63$ Find: a, A, B
$c = 6.49$
$C = 76°$

35. The length of a kite string is 940 feet and the angle of elevation of the kite (angle between the line of sight up to the kite and the horizontal) is 36°20′. Assume the string to be straight. How high is the kite?

36. What is the angle of elevation of the sun when it casts a shadow 493 feet long for a tower 379 feet high?

37. If the angle of elevation of the sun is 25°42′, how long a shadow will be cast by a man 5 feet $7\frac{1}{2}$ inches tall?

The grade of a slope is the tangent of the angle of elevation, or the ratio of the vertical *rise* in the slope for a corresponding horizontal *run*. The grade is usually expressed in per cent.

38. What is the length of a city street which has a 9.2 per cent grade if the upper end is 57.3 feet higher than the lower end?

39. The sloping face of a dam 84.6 feet high is 192 feet long on the slope. What is the grade of the slope?

40. If the maximum grade on a railroad is 2.25 per cent, what is the maximum angle of elevation of the track? (Note: Do not confuse this angle of elevation with the elevation of the outer rail on a curve.)

40. Powers of *e*; Hyperbolic Functions. *Powers of e.* In constructing the log log scales, the first logarithm was taken to the base e, as explained in section 29. As a result, e is in line with the index of the D scale and the numbers on the D scale represent powers of e. To obtain e^x, therefore, set the runner to x on the D scale and read e^x on an LL or an LL0 scale. The legend at the right end of each log log scale gives the range of values for the powers of e corresponding to the numbers on that scale. For instance, the legend for the LL3 scale is 1.0 to 10.0. The first mark on the scale

is $e^{1.0}$, or e, and the last mark on the scale is $e^{10.0}$, or about 22,000. Thus it is possible to read the powers of e from 0.01 to 10.0 and from -0.01 to -10.0 directly by setting the runner to the power on the D scale. Actually, six powers of e can be obtained with one setting. If the runner is set to 5 on the D scale, the following values of e are obtained:

$e^5 = 149$ on LL3 (Fig. 115)
$e^{0.5} = 1.649$ on LL2
$e^{0.05} = 1.0512$ on LL1
$e^{-0.05} = 0.9512$ on LL01
$e^{-0.5} = 0.6065$ on LL02
$e^{-5} = 0.00675$ on LL03

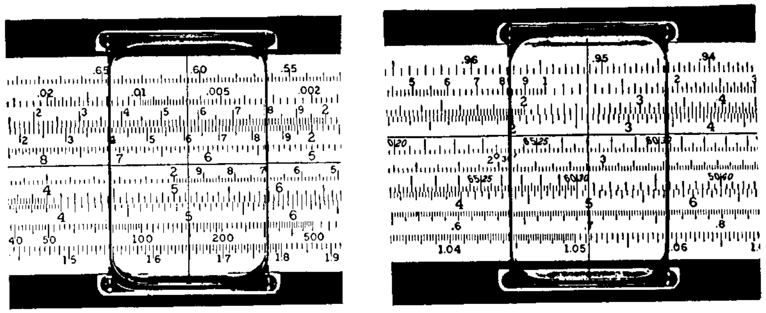


Fig. 115.

This is made possible by constructing the LL01, LL02, and LL03 scales as the reciprocals of the LL1, LL2, and LL3 scales respectively. Other examples are:

$e^{2.5} = 12.2$ on LL3
$e^{-2.5} = 0.084$ on LL03
$e^{0.81} = 2.25$ on LL2
$e^{-0.81} = 0.445$ on LL02

Fractional powers of e can be read directly by setting on the CI scale with the slide centered. For example:

$e^{\frac{1}{3.69}}$ Center the slide
Runner to 3.69 on CI
Read 1.311 on LL2

Powers of e involving roots, trigonometric functions, or π will be explained in the next section.

Hyperbolic Functions. The arrangement of the LL and LL0 scales as reciprocals on the modern slide rule makes possible rapid evaluation of hyperbolic functions. The hyperbolic sine and cosine are designated respectively as sinh and cosh and are defined by $\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\cosh x = \dfrac{e^x + e^{-x}}{2}$. To determine sinh 1.23 and cosh 1.23, for example, use the following procedure:

Runner to 1.23 on D

Read $e^x = 3.42$ on LL3

Read $e^{-x} = 0.293$ on LL03

$$\sinh 1.23 = \frac{3.42 - 0.293}{2} = \frac{3.127}{2} = 1.563$$

$$\cosh 1.23 = \frac{3.42 + 0.293}{2} = \frac{3.713}{2} = 1.856$$

A variation in the computing of the values of sinh and cosh after the values of e^x and e^{-x} have been found is to halve the values of e^x and e^{-x} first, and then subtract and add them. For example, compute sinh 2.16 and cosh 2.16:

Runner to 2.16 on D

Read $e^x = 8.68$ on LL3

Read $e^{-x} = 0.116$ on LL03

$$\sinh 2.16 = 4.34 - 0.058 = 4.286$$

$$\cosh 2.16 = 4.34 + 0.058 = 4.398$$

Since $\tanh x = \dfrac{\sinh x}{\cosh x}$, it can be computed from these two functions, or it can be computed independently by

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

EXERCISE 26

Evaluate the following expressions:

1. $e^{6.5}$
2. $e^{0.42}$
3. $e^{-0.14}$
4. $e^{1.18}$
5. $e^{0.036}$
6. $e^{-2.22}$
7. $e^{-0.015}$
8. $e^{0.95}$
9. $e^{\frac{1}{6.24}}$
10. $e^{-\frac{1}{4.71}}$
11. sinh 2.02
12. cosh 0.88
13. sinh 1.37
14. cosh 2.68
15. tanh 0.46
16. tanh 1.13

41. Exponents Involving Roots, π, or Trigonometric Functions. The log log scales are constructed with reference to the D scale and also with reference to its companion scales, the C and CI scales. They can be used with the other scales of the rule only if their use implicitly involves the transfer of a number from them to one of the three scales just mentioned. When the runner is set to 41 on the A scale, for example, it is also set to $\sqrt{41}$ on the D scale. Thus $e^{\sqrt{41}}$ can be found by this setting on the A scale without actually evaluating $\sqrt{41}$ on the D scale. The same reasoning applies for cube roots, trigonometric functions, and π. Such expressions as $e^{(1.5)^2}$ and $e^{(0.23)^3}$ cannot be evaluated in one operation since squaring or cubing the exponent transfers them *from* the D scale to the A or K scale, neither of which can be used directly with the log log scales. A number of selected examples of powers of e and powers of numbers in general are given.

$e^{\sqrt{41}}$ Runner to 41 on A cycle 2 (Fig. 116)

Read 600 on LL3

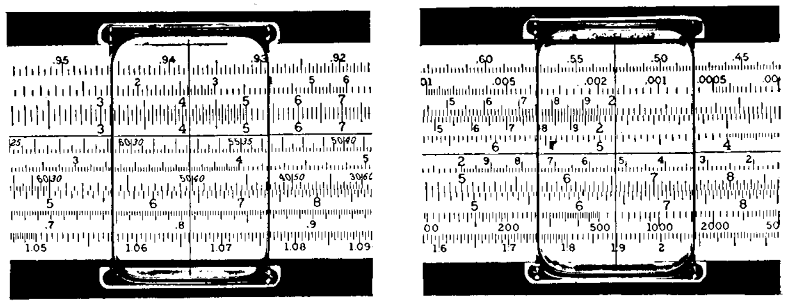


FIG. 116.

$e^{-\sqrt{0.192}}$	Runner to 0.192 on A cycle 2 Read 0.645 on LL02
$e^{-\sqrt[3]{426}}$	Runner to 426 on K cycle 3 Read 0.00054 on LL03
$e^{\frac{21.7}{\pi}}$	Runner to 21.7 on DF (Fig. 117) Read 1000 on LL3

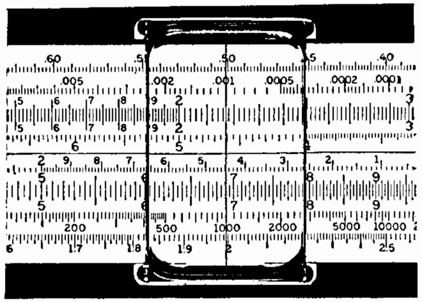


FIG. 117.

$e^{\frac{\pi}{13.3}}$	Runner to left index of D [6] Slide to upper index of CF Runner to 13.3 on CI Read 1.266 on LL2
$e^{\sqrt{\frac{39}{\pi}}}$	Runner to 39 on A cycle 2 Slide to π on B cycle 1 Runner to left index of B Read 34.0 on LL3
$e^{\sqrt{2\pi}}$	Runner to π on A cycle 1 Left index of B to hairline Runner to 2 on B cycle 1 Read 12.3 on LL3

[6] Although this procedure utilizes the value of π as marked on the folded scales, it is doubtful that it is superior to the straightforward procedure using only the C scale and setting π as 3.14+.

$e^{\sin 21°}$	Center the slide Runner to 21° on S black Read 1.432 on LL2
$e^{\tan 4°}$	Center the slide Runner to 4° on ST Read 1.0722 on LL1
$e^{\frac{1}{\cos 47°}}$	Runner to right index Slide to 47 on S red Runner to left index Read 4.34 on LL3
$(1.34)^{\sqrt{7.59}}$	Runner to 1.34 on LL2 Left index to hairline Runner to 7.59 on B cycle 1 Read 1.0838 on LL2
$(1.092)^{-\sqrt{0.043}}$	Runner to 1.092 on LL1 Right index to hairline Runner to 0.043 on B cycle 1 Read 0.9819 on LL01
$(654)^{\sqrt{\frac{2.17}{56.3}}}$	Runner to 654 on LL3 Slide to 56.3 on B cycle 2 Runner to 2.17 on B cycle 1 Read 3.57 on LL3
$(0.9871)^{\sqrt{\frac{6.42}{\pi}}}$	Runner to 0.9871 on LL01 Slide to π on B cycle 1 Runner to 6.42 on B cycle 1 Read 0.9816 on LL01

$(1.14)^{-\sqrt{13\pi}}$ Runner to 1.14 on LL2
Slide to left index of B
Runner to π on B cycle 1
Slide to left index of B
Runner to 13 on B cycle 2
Read 0.433 on LL02

$(0.473)^{\sin 12^\circ}$ Runner to 0.473 on LL02
Slide to right index
Runner to 12° on S black
Read 0.8558 on LL02

$(2.98)^{\frac{1}{\tan 32^\circ}}$ Runner to 2.98 on LL3
Slide to 32° on T
Runner to right index
Read 5.75 on LL3

$(0.632)^{\tan 71^\circ}$ Runner to 0.632 on LL02 [7]
Slide to 71° on T
Runner to left index
Read 0.264 on LL03

$(91.2)^{\frac{1}{\tan 82^\circ}}$ Runner to 91.2 on LL3 [7]
Slide to left index
Runner to 82° on T
Read 1.885 on LL2

$(1.029)^{\frac{\cos 87^\circ}{\sqrt{0.063}}}$ Runner to 1.029 on LL1
Slide to 0.063 on B cycle 1
Runner to 3° on ST
Read 1.0615 on LL1

[7] Note that tangents of angles greater than 45° are read on the CI scale instead of the C scale.

EXERCISE 27

Evaluate the following expressions:

1. $e^{\sqrt{6.42}}$
2. $e^{\sqrt{20.8}}$
3. $e^{-\sqrt{0.094}}$
4. $e^{-\sqrt{1.47}}$
5. $e^{\sqrt[3]{98.2}}$
6. $e^{-\sqrt[3]{0.0044}}$
7. $e^{\frac{\sqrt[3]{597}}{3.12}}$
8. $e^{\frac{8.64}{\sqrt{298}}}$
9. $e^{\frac{5.97}{\pi}}$
10. $e^{\frac{1}{\pi}}$
11. $e^{0.53\pi}$
12. $e^{\frac{4.22}{\sqrt{\pi}}}$
13. $e^{\sin 43^\circ}$
14. $e^{\frac{1}{\tan 69^\circ}}$
15. $e^{\frac{\sqrt{\pi}}{\tan 13^\circ}}$
16. $e^{\frac{\sqrt{3.68}}{\sin 14^\circ}}$
17. $(2.81)^{\sqrt{15.87}}$
18. $(47.7)^{\frac{1}{\sqrt{63.8}}}$
19. $(1.029)^{-\sqrt{475}}$
20. $(0.757)^{\sqrt{\frac{61.4}{9.83}}}$
21. $(245)^{\sqrt{\frac{2.47}{6.61}}}$
22. $(0.968)^{\frac{7.27}{\pi}}$
23. $(1.415)^{\frac{1}{2\pi}}$
24. $(20.2)^{\frac{\pi}{8.33}}$
25. $(0.9844)^{-\sqrt{\frac{10.4}{\pi}}}$
26. $(3.85)^{\sin 17^\circ}$
27. $(1.69)^{-\frac{1}{\cos 41^\circ}}$
28. $(1.595)^{\tan 86^\circ}$
29. $1.023^{\frac{\sqrt{11.84}}{\sin 3^\circ}}$
30. $3.06^{\frac{\cos 49^\circ}{\sqrt{0.864}}}$

42. Logarithms of Numbers; the L Scale. *Common Logarithms; the L Scale.* Common logarithms or logarithms to the base 10 can be determined directly with the aid of the L scale, which is a scale of the mantissas of common logarithms. To obtain the mantissa of the logarithm of a number, set the runner to the number on the D scale on the back of the frame and read the mantissa on the L scale. The characteristic is determined as usual. For numbers greater than one, it is positive and is one less than the number of places to the left of the decimal point. For numbers less than one, it is negative and is one more than the number of zeros to the right of the decimal point. The following table gives a number of evaluations:

Number on D	*Mantissa on L*	*Characteristic*	*Logarithm*
2	0.301	0	0.301
30	0.477	+1	1.477
75	0.875	+1	1.875
423	0.626	+2	2.626
0.164	0.215	−1	9.215—10
0.026	0.415	−2	8.415—10

Natural Logarithms. Natural logarithms or logarithms to the base e can be read directly on the D scale for numbers which are set on the LL and LL0 scales. The numbers on the D scale represent powers of e as was explained in section 40. Accordingly, they also represent the natural logarithms for numbers on the LL and LL0 scales. For example, e^4 is obtained by setting the runner to 4 on D and reading 54.5 on LL3 (Fig. 118). Since $e^4 = 54.5$, then $\log_e 54.5 = \ln 54.5 = 4$. The legends at the right end of the LL and LL0 scales give the ranges of values for the natural logarithms of the numbers on the respective scales.

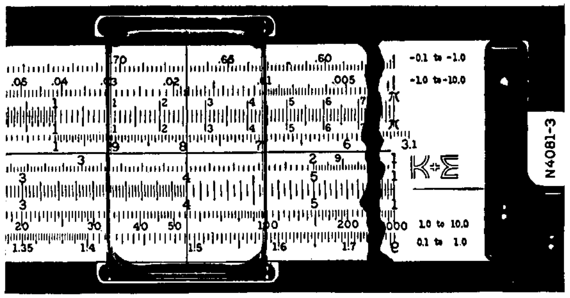


FIG. 118.

Ln 1000 Runner to 1000 on LL3
Read 6.91 on D

Ln 1.52 Runner to 1.52 on LL2
Read 0.419 on D

Ln 0.031 Runner to 0.031 on LL03
Read −3.48 on D

Ln 1.0292 Runner to 1.0292 on LL1
Read 0.0288 on D

Logarithms to Any Base. The log log scales not only are arranged to read directly the natural logarithms of numbers within the limits of the scales, but also can be used to read the logarithm of a number to any base. For instance, logarithms to the base 10 can be obtained completely, both characteristic and mantissa, by setting either

index of the C scale to 10 on the LL3 scale and reading the logarithms on the C scale. As an example:

Log 100 Runner to 10 on LL3
Left index to hairline
Runner to 100 on LL3
Read 2 on C

This setting leaves almost half of the slide out of the rule and requires re-setting with the right index for numbers from 4.3 through 9. By setting the upper index on the CF scale opposite 10 on the LL3 and reading the common logarithm on the CF scale, a much more efficient arrangement is obtained.

The initial setting for the following logs is—runner to 10 on LL3; upper index to hairline:

Log 6.1 Runner to 6.1 on LL3 (Fig. 119)
Read 0.785 on CF

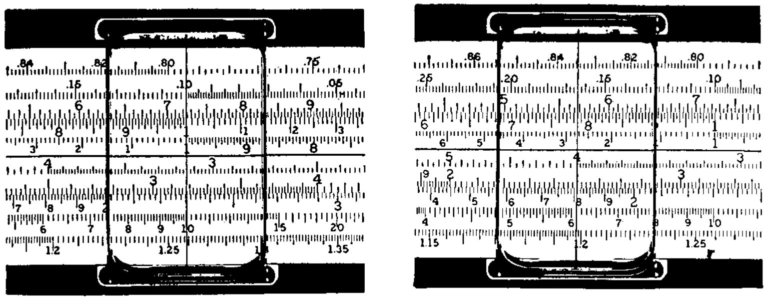


FIG. 119.

Log 20 Runner to 20 on LL3
Read 1.301 on CF

Log 0.026 Runner to 0.026 on LL03
Read −1.585 on CF
(or 8.415—10)

Log 423 Runner to 423 on LL3
Read 2.63 on CF

In setting the decimal point in logarithms to the base 10, logarithms for numbers to the right of 10 on the same scale are in the range from 1.0 to 10.0 and to the left of 10 on the same scale are in the range from 0.1 to 1.0. A similar relation is true for each of the other scales.

A similar procedure can be used for any base of logarithms and is illustrated in the following examples:

Log_8 4200 Runner to 8 on LL3 (Fig. 120)
Left index to hairline
Runner to 4200 on LL3
Read 4.01 on C
(i.e., $(8)^{4.01} = 4200$)

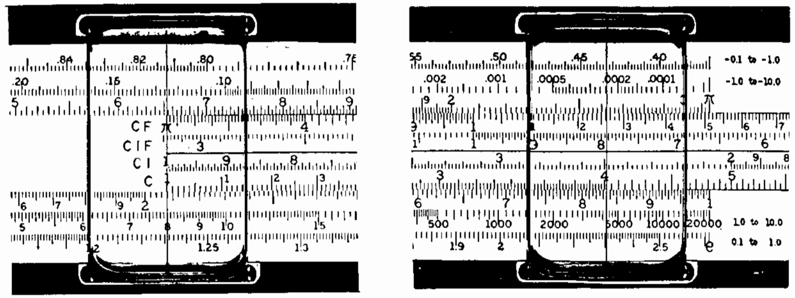


FIG. 120.

Log_4 1.037 Runner to 4 on LL3
Left index to hairline
Runner to 1.037 on LL1
Read 0.0262 on C

$\text{Log}_{1.02}$ 4.6 Runner to 1.02 on LL1
Upper index to hairline
Runner to 4.6 on LL3
Read 77.0 on CF

$\text{Log}_{0.63}$4.97 Runner to 0.63 on LL02
Upper index to hairline
Runner to 4.97 on LL3
Read −3.47 on CF

EXERCISE 28

Evaluate the common logarithms of the following numbers, using the L scale to determine the mantissas:

1. 4.93
2. 637
3. 15,900
4. 29.7
5. 0.551
6. 0.00693
7. 7320
8. 3.79
9. 56.3
10. 825

Evaluate the natural logarithms of the following numbers:

11. 8.65
12. 530
13. 72.2
14. 1.49
15. 12,500
16. 1.258
17. 1.0156
18. 0.9885
19. 0.624
20. 0.0013

Evaluate the following expressions:

21. Log_3 158
22. Log_2 3700
23. Log_{20} 9500
24. $\text{Log}_{1.62}$ 4.98
25. Log_6 1.037
26. Log_4 1.775
27. $\text{Log}_{1.03}$ 6.4
28. $\text{Log}_{0.83}$ 0.033
29. $\text{Log}_{0.42}$ 0.937
30. $\text{Log}_{0.75}$ 5.24

VII

THE POLYPHASE AND MANNHEIM SLIDE RULES

43. Introduction. This chapter on the Polyphase and Mannheim slide rules along with Chapters I, III, and VI provides complete instruction in their use for those who prefer these slide rules rather than the Duplex rule. The following order of study is recommended as a guide: Chapter I, Chapter VII, Chapter III, and Chapter VI, Sections 34–39, insofar as they apply to these slide rules.

44. Multiplication and Division. Multiplication and division are performed on the D and C scales by moving the runner and slide to proper settings on them. Questions arise as to the order in which these parts are moved and as to which scales are used. The following general principles of operation answer some of these questions, and the explanations of multiplication and division answer the others.

(a) *Always begin and end a computation with the runner. The first movement or setting is always made with the runner and the final movement or setting is likewise made with it.*

(b) *Always begin and end a computation on the* D *scale.*

This means that a computation is always begun by setting the runner on a factor on the D scale and is always ended by setting the runner to read the answer on the D scale.

(c) *The runner and slide are always moved alternately with the runner first and the slide following. The movements of runner and slide constitute a game of "Follow the Leader" with the runner as leader and the slide always following.* The runner is never moved twice in succession, nor is the slide.

Multiplication. The procedure of multiplying two factors, say 2 × 4, is explained in the following steps, along with an abbreviated notation.

Step 1: Move the runner to 2 on the D scale, i.e., so that the hairline is on 2 on that scale. Abbreviation: Runner to 2 on D.
Step 2: Move the left end of the C scale, called the *left index,* opposite the hairline. Abbreviation: left index to hairline.
Step 3: Move the runner to 4 on the C scale. Abbreviation: Runner to 4 on C.
Step 4: Read the integers of the answer opposite the hairline on the D scale as 8 0 0. Setting the decimal point gives 8.00. Abbreviation: Read 8.00 on D.

In abbreviated notation, the entire operation is as follows:

2 × 4	Runner to 2 on D	(Fig. 121)
	Left index to hairline	
	Runner to 4 on C	
	Read 8.00 on D	

Other examples are given as follows:

17.1 × 5.03	Runner to 17.1 on D	
	Left index to hairline	
	Runner to 5.03 on C	
	Read 86.0 on D	
4.21 × 6.37	Runner to 4.21 on D	(Fig. 122)
	Right index to hairline	
	Runner to 6.37 on C	
	Read 26.8 on D	

The right index, or the right end of the C scale, is used in the last example since 6.37 on the C scale would be "off the rule" if the left index were used. Use whichever index places the factor on the C scale on the rule. The principle of the multiplication of two factors has been explained in Section 5, Chapter I.

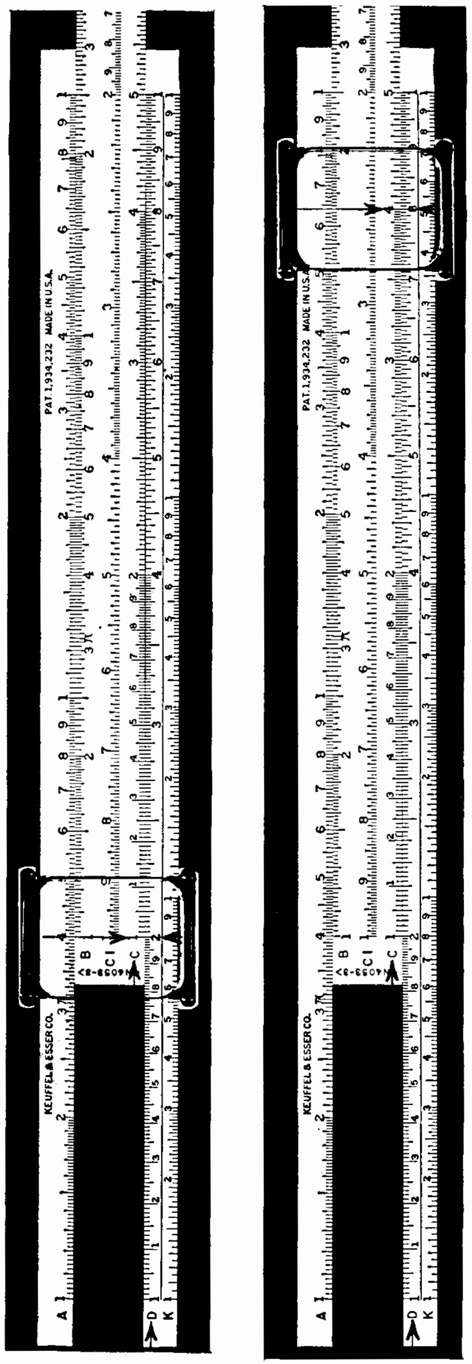


FIG. 121.

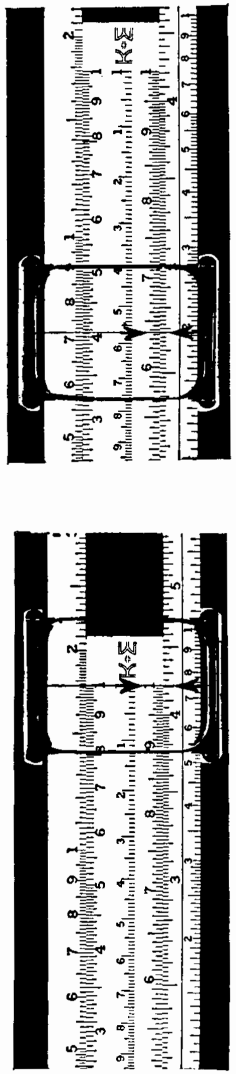


FIG. 122.

For three or more factors in multiplication, the same procedure is repeated until the last factor is reached. For example:

1.23 × 6.42 × 3.97	Runner to 1.23 on D	(Fig. 123)
	Left index to hairline	
	Runner to 6.42 on C	
	Right index to hairline	
	Runner to 3.97 on C	
	Read 31.4 on D	

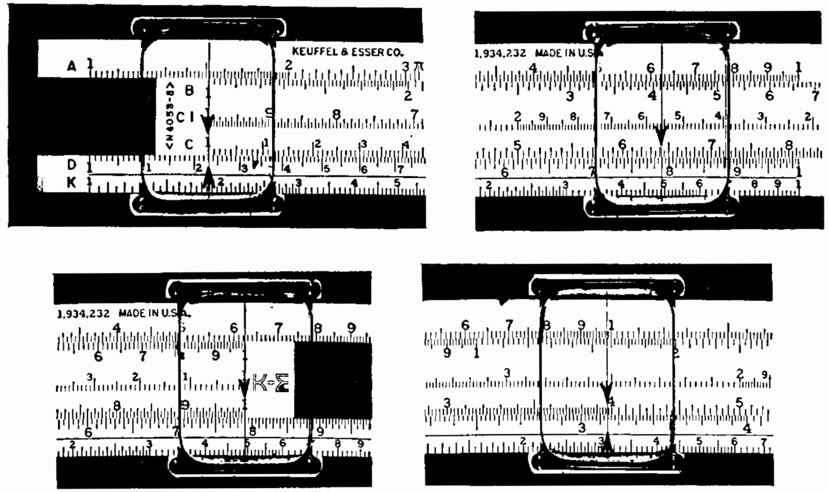


FIG. 123.

4.17×2.72×3.19 × 0.155	Runner to 4.17 on D
	Right index to hairline
	Runner to 2.72 on C
	Left index to hairline
	Runner to 3.19 on C
	Left index to hairline
	Runner to 0.155 on C
	Read 5.61 on D

Although the factors in the examples are taken in the order in which they are stated in the problems, this is not required. The factors may be taken in any order since the answer will be the same, i.e., $2 \times 3 \times 4 = 3 \times 2 \times 4 = 4 \times 2 \times 3 = 24$.

EXERCISE 29

Perform the following multiplications:

1. 4.36×7.82
2. 7.51×12.20
3. 2.19×3.19
4. 13.62×31.4
5. 242×0.441
6. 8.15×28.8
7. 9.61×4.16
8. 6.75×5.28
9. 13.9×1.08
10. 34.5×3.42
11. 1.16×0.921
12. 2.41×79.6
13. 2.96×21.2
14. 5.14×32.3
15. 415×2.48
16. 3.49×0.0482
17. 0.319×0.0313
18. $3.09 \times 3.91 \times 2.52$
19. $16.1 \times 0.680 \times 8.86$
20. $8.08 \times 3.03 \times 9.09$
21. $4.71 \times 8.13 \times 1.21$
22. $0.141 \times 265 \times 0.560$
23. $5.01 \times 4.91 \times 0.201$
24. $3.37 \times 8.93 \times 99.4$
25. $22.3 \times 11.25 \times 3.34$
26. $2.03 \times 12.8 \times 1.14 \times 4.09$
27. $2.28 \times 98.1 \times 0.0371 \times 1.50$
28. $8.05 \times 0.439 \times 9.03 \times 20.5$
29. $8.7 \times 7.6 \times 6.5 \times 1.53 \times 9.8$
30. $9.12 \times 7.75 \times 2.21 \times 3.06 \times 2.60$

31. A rectangular field is 87.6 feet wide and 122.4 feet long. What is its area in square feet?

32. A city street is to be paved. The length of the street is 3270 feet and the width to be paved is 43.5 feet. How many square feet of pavement are required?

33. The unit cost of surfacing a stretch of road is $4.12 per linear yard. If this stretch is 5050 yards long, what is the approximate total cost?

34. Compute:

(a) 92.4 per cent of 117.5
(b) 37.6 per cent of 293
(c) 14.1 per cent of 69.2
(d) 0.43 per cent of 3540

35. How far will an airplane go in 2.235 hours at a ground speed of 305 miles per hour?

36. Using the table of equivalents on page 226, compute the following equivalents:

(a) 42.6 inches in centimeters
(b) 21.7 meters in inches
(c) 13.6 square inches in square centimeters
(d) 9.42 square miles in acres
(e) 124.5 cubic feet of water in pounds
(f) 72.4 gallons of water in pounds
(g) 3.69 kilograms in pounds
(h) 49.1 gallons in cubic inches
(i) 85.8 miles per hour in feet per second
(j) 9.50 horsepower in kilowatts

37. What is the volume of a rectangular box whose dimensions in feet are $2.53 \times 3.82 \times 8.66$?

38. In making a highway fill, what volume of earth is used if the borrow-pit from which the earth was removed is 4.50 feet deep, 212 feet wide, and 1380 feet long?

39. If a man works 7.5 hours per day for 5 days per week, and works 47 weeks in a certain year, how many hours does he work in that year? If his wage rate is $1.50 per hour, approximately how much does he earn in the year?

40. Water weighs 62.4 pounds per cubic foot. What is the total weight of water in a tank with inside dimensions in feet $2.44 \times 3.63 \times 8.02$?

41. Mercury is approximately 13.6 times as heavy as water. What weight of mercury is required to fill a rectangular container 0.426 feet deep, 1.24 feet wide, and 1.463 feet long?

42. What is the approximate cost of paving an airport runway with a concrete slab 1.05 feet thick, 242 feet wide, and 5460 feet long if the concrete in place costs $0.612 per cubic foot?

Division. Division differs from multiplication in that the numerator or dividend *must* always be set first on the D scale. This is obvious when it is observed that, although in multiplication $3 \times 2 = 2 \times 3$, in division $\frac{3}{2} \neq \frac{2}{3}$. Also, the denominator or divisor is set on the C scale opposite the numerator on the D scale, instead of an index as was the case in multiplication. In this respect, division is the reverse procedure from multiplication. The procedure for obtaining the quotient of $\frac{42.5}{8.36}$ is shown as follows:

$\dfrac{42.5}{8.36}$ (Fig. 124)

Runner to 42.5 on D
Slide to 8.36 on C
Runner to right index
Read 5.08 on D

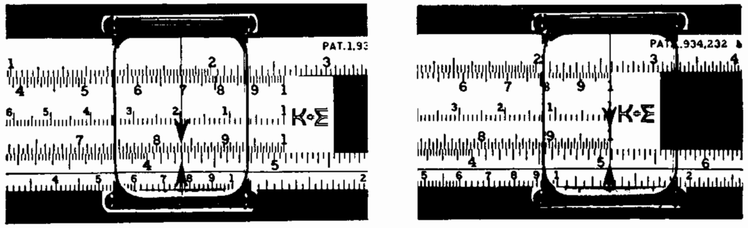


FIG. 124.

The abbreviation, slide to 8.36 on C, means to move the slide until 8.36 on the C scale is beneath the hairline. Other examples are given as follows:

$\dfrac{6.97}{1.83}$

Runner to 6.97 on D
Slide to 1.83 on C
Runner to left index
Read 2.81 on D

$\dfrac{247}{4.21}$

Runner to 247 on D
Slide to 4.21 on C
Runner to right index
Read 58.7 on D

The answer is read opposite whichever index is on the rule. The principle of division of two factors has been explained in Section 5, Chapter I.

The procedure for the division of one factor by another is repeated for the division of one factor by two or more factors as is shown by the following examples:

$\dfrac{293}{4.16 \times 6.07}$ (Fig. 125)

Runner to 293 on D
Slide to 4.16 on C
Runner to right index
Slide to 6.07 on C
Runner to left index
Read 11.6 on D

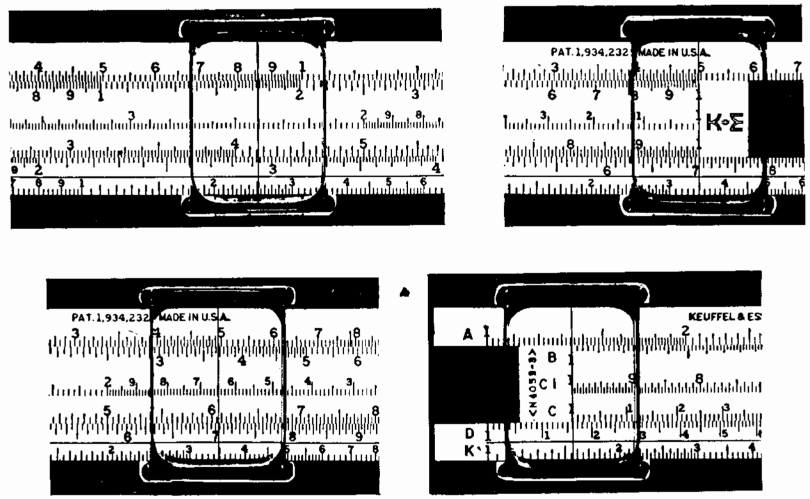


FIG. 125.

$\dfrac{416}{2.63 \times 11.4 \times 8.95}$

Runner to 416 on D
Slide to 2.63 on C
Runner to left index
Slide to 11.4 on C
Runner to left index
Slide to 8.95 on C
Runner to right index
Read 1.55 on D

The decimal point in the last example was estimated as follows: in the denominator, $10 \times 10 = 100$, $100 \times 2 = 200$, and 400 divided by 200 gives the answer as 2, or 1.55.

EXERCISE 30

1. $\frac{7.92}{4.17}$
2. $\frac{27.3}{8.46}$
3. $\frac{38.2}{13.4}$
4. $\frac{429}{11.95}$
5. $\frac{84.6}{3.97}$
6. $\frac{6.71}{11.4}$
7. $\frac{75.1}{9.16}$
8. $\frac{113.5}{29.9}$
9. $\frac{49.6}{1250}$
10. $\frac{212.4}{40.9}$
11. $\frac{5.88}{12.02}$
12. $\frac{37.1}{20.3}$
13. $\frac{131.2}{7.93}$
14. $\frac{32.8}{4.03 \times 3.04}$
15. $\frac{2460}{72.3 \times 8.48}$
16. $\frac{37.2}{6.91 \times 4.32}$
17. $\frac{87.7}{3.74 \times 22.5}$
18. $\frac{879}{9.79 \times 6.21 \times 1.76}$
19. $\frac{4375}{27.1 \times 3.18 \times 5.05}$
20. $\frac{3110}{2.04 \times 4.27 \times 1.72 \times 9.70}$

21. A rectangular plot has an area of 2620 square feet. What is the length of the long side if the short side is 38.6 feet?

22. How many miles of highway can be surfaced for $840,000 if the surfacing costs $0.523 per square foot and the roadway is 18 feet wide?

23. What is the density of the steel in a block which weighs 1925 pounds and has the following dimensions in feet: 1.37 × 0.785 × 3.69?

24. If mercury is 13.6 times as heavy as water and water weighs 62.4 pounds per cubic foot, what is the volume of 13.24 pounds of mercury?

25. A highway fill of 46,200 cubic yards is to be obtained from a rectangular borrow-pit 121 yards wide and 202 yards long. What is the depth of the borrow-pit in yards to yield this volume of fill?

26. What is the average speed of a modern streamline train in traveling a distance of 281 miles in 3 hours and 57 minutes?

Combinations of Multiplication and Division. By combinations of multiplication and division are meant such expressions as $\frac{6.31 \times 1.52}{4.63}$

and $\frac{8.49 \times 3.27}{2.05 \times 1.72}$. Such expressions can be solved most efficiently in the order of division, multiplication, division, and so on, rather than to multiply first all factors in the numerator, or to divide a factor by all factors in the denominator. When one factor is divided by a second, the answer is indicated at the index on the rule. To multiply by a third factor is now a simple matter since the index is already on the quotient of the first two. This quotient, of course, is not read because only the final answer is desired. Consider the example $\frac{6.31 \times 1.52}{4.63}$. Begin the solution as $\frac{6.31}{4.63}$.

Runner to 6.31 on D (Fig. 126)

Slide to 4.63 on C

The left index is now opposite the quotient (1.363) of these factors and the slide is set for the multiplication of this quotient by 1.52. The next movement, then, is not to the left index to read this quotient but to 1.52 on the C scale.

Runner to 1.52 on C

Read 2.07 on D

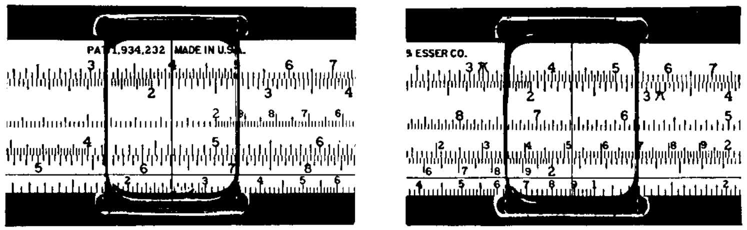


FIG. 126.

The following examples further illustrate the procedure:

$\frac{29.7 \times 6.63}{5.22}$

Runner to 29.7 on D

Slide to 5.22 on C

Runner to 6.63 on C

Read 37.7 on D

$\dfrac{76.2 \times 3.45}{8.99}$

Runner to 76.2 on D
Slide to 8.99 on C
Runner to 3.45 on C
Read 29.3 on D

In some instances the third factor is off of the rule and the slide must be *reset*. To reset the slide, move the runner to the index on the rule and then change indices, moving the other index to the hairline. This is illustrated in the computation of $\dfrac{8.27 \times 9.43}{5.62}$ (Fig. 127).

Runner to 8.27 on D
Slide to 5.62 on C

The factor 9.43 on the C scale is off of the rule. Therefore reset the slide and continue the operation.

Runner to left index
Right index to hairline
Runner to 9.43 on C
Read 13.9 on D

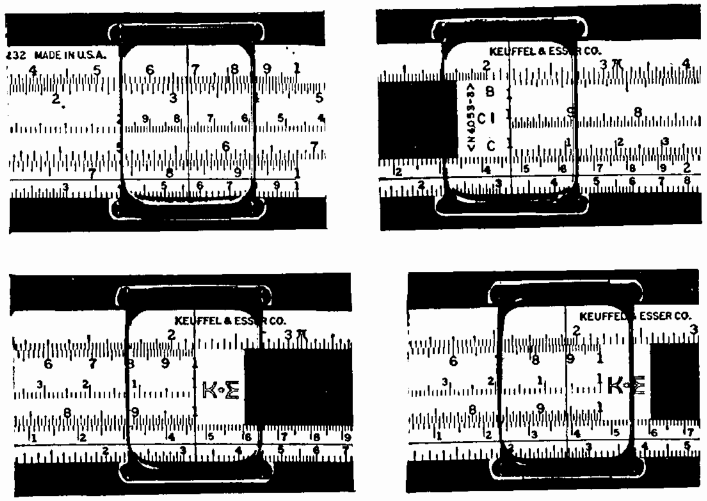


Fig. 127.

Another example of this type is given:

$\dfrac{1.57 \times 2.11}{6.46}$

Runner to 1.57 on D
Slide to 6.46 on C
Runner to right index
Left index to hairline
Runner to 2.11 on C
Read 0.513 on D

Other examples in general:

$\dfrac{6.49 \times 3.27}{2.85 \times 1.72}$

Runner to 3.27 on D
Slide to 2.85 on C
Runner to 6.49 on C
Slide to 1.72 on C
Runner to left index
Read 4.33 on D

$\dfrac{2.18 \times 49.7}{3.72 \times 0.643}$

Runner to 2.18 on D
Slide to 3.72 on C
Runner to 49.7 on C
Slide to 0.643 on C
Runner to right index
Read 45.4 on D

$\dfrac{7.06 \times 16.3 \times 2.85}{5.88 \times 3.12}$

Runner to 7.06 on D
Slide to 5.88 on C
Runner to 16.3 on C
Slide to 3.12 on C
Runner to 2.85 on C
Read 17.9 on D

EXERCISE 31

Perform the following computations:

1. $\dfrac{12.3 \times 8.16}{1.42}$

2. $\dfrac{32.7 \times 4.21}{273}$

3. $\dfrac{89.7 \times 6.24}{7.37}$

4. $\dfrac{132.4 \times 9.09}{15.71}$

5. $\dfrac{50.6 \times 2.18}{4.72}$

6. $\dfrac{32.6 \times 3.81}{272}$

7. $\dfrac{8.97 \times 4.27}{11.6 \times 3.58}$

8. $\dfrac{36.3 \times 15.2}{10.9 \times 4.73}$

9. $\dfrac{7.17 \times 1.25}{83.6 \times 0.972}$

10. $\dfrac{47.2 \times 7.31}{3.66 \times 8.08}$

11. $\dfrac{2.47 \times 7.24}{3.94 \times 8.62}$

12. $\dfrac{2.86 \times 135.5}{1.462 \times 0.848}$

13. $\dfrac{9.63 \times 7.24}{11.69 \times 2.33}$

14. $\dfrac{2.63 \times 12.08}{3.45 \times 3.68}$

15. $\dfrac{2.47 \times 4.16}{7.76}$

16. $\dfrac{3.42 \times 49.7}{1.07}$

17. $\dfrac{13.6 \times 3.97 \times 9.03}{15.8 \times 2.62}$

18. $\dfrac{33.6 \times 6.18 \times 20.2}{4.17 \times 4.55}$

19. $\dfrac{1.06 \times 3.37 \times 8.16}{0.881 \times 2.51}$

20. $\dfrac{7.50 \times 8.62 \times 3.21}{9.73 \times 5.05}$

21. The area of a triangle is $\frac{1}{2}$ × base × altitude. Compute tne areas of triangles with the following dimensions:

	base	*altitude*
(a)	6.25 inches	2.69 inches
(b)	13.27 inches	36.3 inches
(c)	4.33 feet	11.68 feet
(d)	5.04 yards	7.37 yards

22. The volume of a pyramid is $\frac{1}{3}$ × area of base × height. Compute the volumes of pyramids with rectangular bases with the following dimensions:

	base width	*base length*	*height*
(a)	2.63 feet	7.13 feet	12.27 feet
(b)	0.885 feet	1.462 feet	3.81 feet
(c)	4.27 inches	9.08 inches	1.46 inches

23. A pyramid with a triangular base is called a tetrahedron. Compute the volumes of the following tetrahedrons:

	base of tetrahedron		
	base of triangle	*altitude of triangle*	*height of tetrahedron*
(a)	1.64 feet	2.83 feet	4.08 feet
(b)	7.63 inches	32.5 inches	27.7 inches
(c)	3.31 feet	5.06 feet	8.28 feet

24. Assuming that steel weighs 490 pounds per cubic foot, what is the height in feet of a pyramid of solid steel with a rectangular base 7.24 inches × 13.6 inches which weighs 418 pounds?

45. The CI Scale. The CI scale on the Polyphase slide rule can be used to advantage frequently in multiplication and division. The principle of its use has been explained in Section 5, Chapter I. It is also a scale of reciprocals as is explained in Section 34 of Chapter VI, Special Operations. Two factors can be multiplied using the CI scale in the same way that two factors can be divided using the C scale. For example, 2 × 4 using the CI scale is performed as follows:

2 × 4 Runner to 2 on D (Fig. 128)

Slide to 4 on CI

Runner to right index

Read 8.00 on D

In this particular case, this procedure involves less movement of the slide than the standard procedure using the D and C scales. The

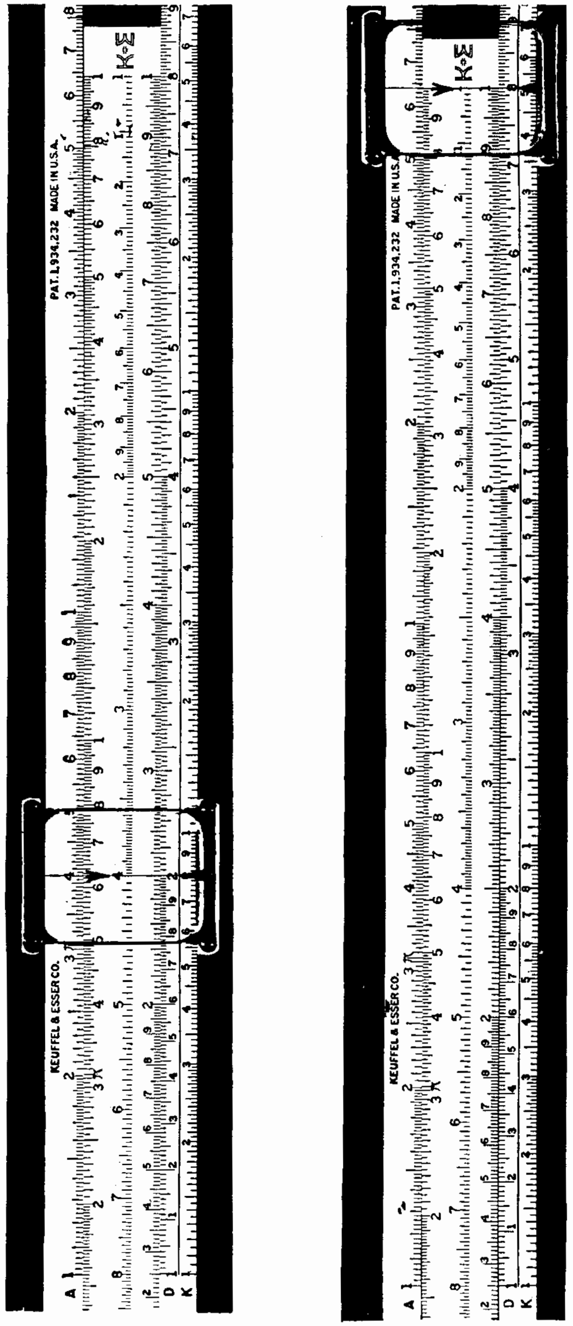


Fig. 128.

advantage of using the CI scale in certain cases is even more apparent in the following example:

14.1 × 8.16 Runner to 14.1 on D (Fig. 129)
Slide to 8.16 on CI
Runner to left index
Read 115 on D

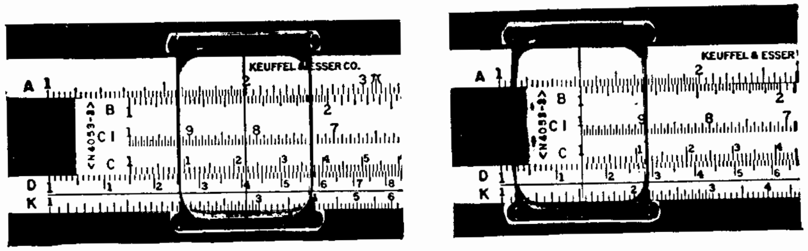


Fig. 129.

If this had been done using the D and C scales, the slide would have been moved to the left almost out of the frame. Note that 14.1 on the D scale and 8.16 on the C scale are at opposite ends of the rule, but that 14.1 on the D scale and 8.16 on the CI scale are at the same end of the rule. However, the CI scale would not be particularly useful for a product such as 11.7 × 12.4, since 11.7 on the D scale and 12.4 on the CI scale are at opposite ends of the rule. The CI scale can also be used to advantage in the division of two factors which are at opposite ends of the rule on the D and C scales. For example:

$$\frac{91.0}{11.6}$$

Runner to 91.0 on D (Fig. 130)
Right index to hairline
Runner to 11.6 on CI
Read 7.85 on D

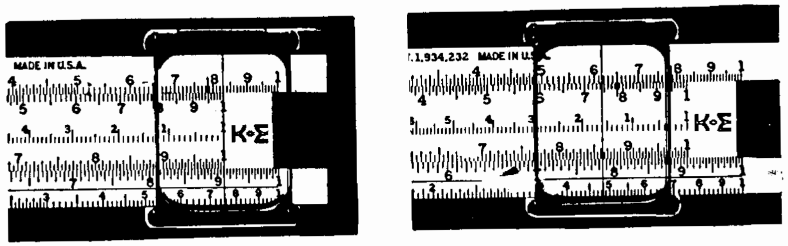


Fig. 130.

The CI scale does make possible a more efficient technique of operation in multiplications and also divisions which involve more than two factors. It eliminates the resetting of the slide for each factor and permits the setting of two factors for each setting of the slide. Consider the multiplications and divisions involving more than two factors in Section 44. The multiplication 1.23 × 6.42 × 3.97 required five movements, three of the runner and two of the slide. Using the CI scale as previously explained, the procedure will be

1.23 × 6.42 × 3.97 Runner to 1.23 on D (Fig. 131)

Slide to 6.42 on CI

Runner to 3.97 on C

Read 31.4 on D

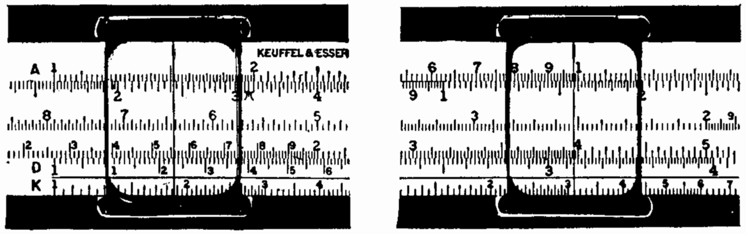


FIG. 131.

In contrast to the previous method, this requires only three movements, two of the runner and one of the slide. The following multiplication which is performed using the CI scale as well as the C scale required seven movements in the previous solution, four of the runner and three of the slide:

4.17 × 2.72 × 3.19 × 0.155 Runner to 4.17 on D

Slide to 2.72 on CI

Runner to 3.19 on C

Slide to 0.155 on CI

Runner to right index

Read 5.61 on D

This requires five movements, three of the runner and two of the slide. The procedures for the divisions of more than two factors in Section 44 are as follows:

$\frac{293}{4.16 \times 6.07}$ Runner to 293 on D (Fig. 132)

Slide to 4.16 on C

Runner to 6.07 on CI

Read 11.6 on D

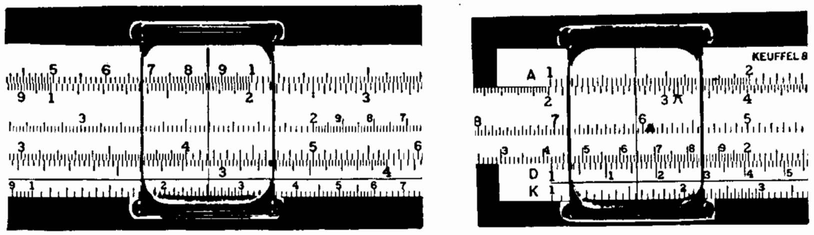


FIG. 132.

$\frac{416}{2.63 \times 11.4 \times 8.95}$ Runner to 416 on D

Slide to 2.63 on C

Runner to 8.95 on CI

Slide to 11.4 on C

Runner to left index

Read 1.55 on D

In all cases like the preceding where the CI scale is used, if the operation runs off the rule, the advantage of using the CI scale is lost. Frequently, it is possible to avoid running off the rule by choosing factors in the proper order. The last example given was one in which the factor 8.95 was taken out of order to avoid running off the rule.

Simple rules of procedure for the use of the CI scale with the C and D scales can be stated.

To start with multiplication, use D and CI scales.
To start with division, use D and C scales.
For repeating operations, alternate slide scales between C and CI.
For alternating operations, repeat slide scales, either C or CI.

Repeating operations are a multiplication of three or more factors and a division in which one factor is divided by two or more factors. Alternating operations occur in combinations of multiplication and division, such as $\frac{2 \times 4}{3 \times 7}$, which may be solved by alternately dividing and multiplying. This is shown schematically in the form 2 \ 3 / 4 \ 7 signifying a division by 3, a multiplication by 4, and a division by 7, or an alternating operation. In previous solutions of combinations of multiplication and division, the C scale was repeated. An example of an operation in which the CI scale is repeated is as follows:

$\frac{\mathbf{7.64 \times 13.9}}{\mathbf{2.52}}$	Runner to 7.64 on D
	Slide to 13.9 on CI
	Runner to 2.52 on CI
	Read 42.1 on D

This requires but slight movement of the slide. To do this operation using the C scale would require considerably more movement.

Exercise 32

1. 6.09×10.08
2. 1.16×0.921
3. 16.2×91.1
4. 7.51×12.20
5. 8.19×0.1870
6. 14.60×8.52
7. $\frac{131.2}{7.93}$
8. $\frac{6.71}{11.4}$
9. $\frac{82.8}{1320}$
10. $\frac{152.3}{8.53}$
11. $8.86 \times 16.1 \times 0.680$
12. $1.202 \times 0.801 \times 6.91$
13. $291 \times 0.336 \times 2.06$
14. $28.9 \times 3.98 \times 8.87$
15. $34.6 \times 2.45 \times 4.25$
16. $3.47 \times 2.23 \times 3.19 \times 4.08$
17. $12.1 \times 2.02 \times 4.12 \times 0.0983$
18. $\frac{4970}{39.3 \times 2.64}$
19. $\frac{1463}{11.5 \times 7.12}$
20. $\frac{52.6}{4.17 \times 128.5}$
21. $\frac{879}{9.79 \times 6.21 \times 1.76}$

22. $\frac{4375}{27.1 \times 3.18 \times 5.05}$
23. $\frac{12.3 \times 8.24}{3.42}$
24. $\frac{9.05 \times 1.325}{2.62}$
25. $\frac{11.6 \times 8.15}{2.84 \times 3.26}$

46. The S and T Scales. The S and T scales which are on the back of the slide of the Polyphase and Mannheim slide rules are trigonometric scales for obtaining respectively the sines and tangents of angles. Before explaining the use of these scales, the following trigonometric relations are given for reference purposes:

$$\sin x = \frac{1}{\csc x}$$
$$\cos x = \frac{1}{\sec x}$$
$$\tan x = \frac{1}{\cot x}$$
$$\cos x = \sin (90° - x)$$
$$\cot x = \tan (90° - x)$$

The S Scale. The S scale is a scale for obtaining the sines of angles from about 0°34′ to 90°. The angles are marked on the S scale and the corresponding sines are read on the A scale. Reversing the slide so that the S scale is on top, sines of angles can be read on the A scale directly opposite the angle on the S scale when the slide is centered in the frame. The sine of 16°, for example, is obtained using the following procedure:

sin 16°	Center the slide	(Fig. 133)
	Runner to 16° on S	
	Read 0.276 on A	

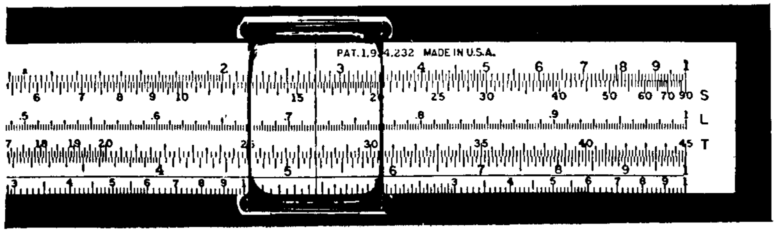


Fig. 133.

The decimal point is set from the knowledge that the numerical values of the sine of an angle are always less than one or equal to one. The sine of 5°44′ is 0.1, or the mid-point of the two-cycle A scale. The sines of angles adjacent to the left cycle of the A scale vary from 0.01 to 0.1 and the sines of angles adjacent to the right cycle of the A scale vary from 0.1 to 1.0. Consequently, the decimal point is definitely known for values of the sines of angles on the S scale. Other examples are given, assuming that the slide has been centered:

sin 30° Runner to 30° on S
Read 0.500 on A right

sin 62° Runner to 62° on S
Read 0.883 on A right

sin 0°50′ Runner to 0°50′ on S
Read 0.0147 on A left

sin 2°36′ Runner to 2°36′ on S
Read 0.0453 on A left

The cosines of angles can be obtained by using the relation, $\cos x = \sin (90° - x)$.

cos 17° $\cos 17° = \sin 73°$
Runner to 73° on S
Read 0.955 on A right

cos 79° $\cos 79° = \sin 11°$
Runner to 11° on S
Read 0.191 on A right

cos 87°20′ $\cos 87°20' = \sin 2°40'$
Runner to 2°40′ on S
Read 0.0465 on A left

The Polyphase and Mannheim slide rules are arranged with a hairline on the back of the frame so that sines and cosines can be evaluated without reversing the slide. The hairline is placed on a plastic insert so that it is in line with the right ends of the scales on the front of the frame. It will be called the *back index* in this book. When an angle on the S scale is set opposite the back index, the sine of the angle is read on the B scale opposite the right index of the A scale. The general principles of operation do not apply to this procedure because the slide is moved first. For example:

sin 23° Slide to 23° on S opposite back index (Fig. 134)
Runner to right index of A
Read 0.390 on B right

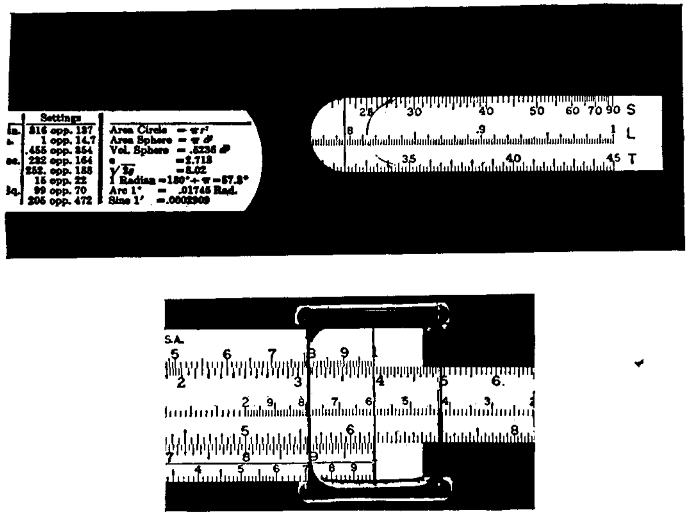


FIG. 134.

sin 3°35′ Slide to 3°35′ on S opposite back index
Runner to right index of A
Read 0.0624 on B left

One difficulty of this procedure lies in setting the runner so close to the end of the rule. A variation in the procedure would be to read the sine on the B scale opposite the right index of the A scale *by eye*, i.e., without setting the runner. This can be done quite well with a little practice.

Secants and cosecants can be evaluated directly using the back index and the trigonometric relations, $\sec x = \dfrac{1}{\cos x}$; $\csc x = \dfrac{1}{\sin x}$. For example, find csc 27°. Set 27° on the S scale opposite the back index. The sine of 27° is now on the B scale opposite the right index of the A scale. In this position, it is automatically divided into one on the A scale, and the result of this division is read on the A scale opposite the left index of the B scale. Since $\dfrac{1}{\sin 27°} = \csc 27°$, the cosecant is read directly on the A scale. The procedure is outlined as follows:

csc 27° Slide to 27° on S opposite back index (Fig. 135)
Read 2.20 on A opposite left index of B

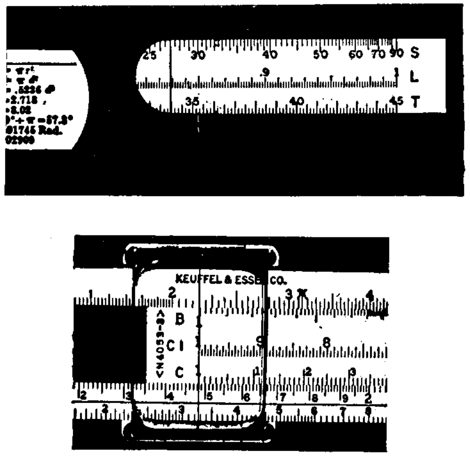


FIG. 135.

csc 4° Slide to 4° on S opposite back index
Read 14.3 on A opposite left index of B

In these two examples, the decimal points are set by inspection from the approximate values of the functions, sin 27° and sin 4°. Since the sine of 27° is between 0.1 and 1.0, the cosecant of 27° will be between 10 and 1. Since the sine of 4° is between 0.01 and 0.1, the cosecant of 4° will be between 100 and 10.

Secants are determined from the sines by converting the reciprocal cosines to sines. $\sec x = \dfrac{1}{\cos x} = \dfrac{1}{\sin (90° - x)}$.

sec 48° Slide to 42° on S opposite back index
Read 1.49 on A opposite left index of B

sec 17° Slide to 73° on S opposite back index
Read 1.042 on A opposite left index of B

In the preceding examples where the left index of the B scale is read each time, the values of the secant and cosecant will vary from 1 to 10 on the left cycle and from 10 to 100 on the right cycle of the A scale. The center index of the B scale could be read in many cases but this simple decimal point rule would not then apply.

The T Scale. The T scale is a scale for obtaining the tangents of angles from about 5°43′ to 45°. The angles are marked on the T scale and the corresponding tangents are read on the D scale. Reversing the slide so that the T scale is on top, tangents of angles can be read on the D scale directly opposite the angle on the T scale when the slide is centered in the frame. The numerical values of the tangents of angles between 5°43′ and 45° vary from 0.1 to 1.0. The following examples are given with the slide being centered in each case:

tan 43° Runner to 43° on T
Read 0.933 on D

tan 19°45′ Runner to 19°45′ on T
Read 0.359 on D

tan 8°25′ Runner to 8°25′ on T
Read 0.148 on D

The sines and tangents of small angles are very nearly equal. The tangent of 5°45′ differs from the sine of 5°45′ by about 1 in 200.

The smaller the angle, the less is the difference. Consequently, the tangents of angles between 0°34′ and about 5°45′ can be read to sufficient accuracy using the S scale and the left cycle of the A scale. This makes it possible to read directly the tangents of angles from 0°34′ to 45°. The cotangents of angles can be determined from the tangent scale using the relationship $\cot x = \tan (90° - x)$ and a procedure similar to that for evaluating cosines.

Tangents of angles less than 45° may also be evaluated directly using the back index and the right index as was done for sines. For example:

tan 37° Slide to 37° on T opposite back index

Read 0.754 opposite right index of D

tan 12°40′ Slide to 12°40′ on T opposite back index

Read 0.225 opposite right index of D

Tangents of angles greater than 45° may be evaluated directly by using the back index and a procedure similar to that for secants and cosecants, as is illustrated by the following examples.

tan 65° Slide to 25° on T opposite back index

Read 2.14 on D opposite left index of C

tan 79° Runner to 11° on T opposite back index

Read 5.15 on D opposite left index of C

tan 87° Runner to 3° on S opposite back index

Read 19.1 on A opposite left index of B

Note that the procedure uses the left portion of the S scale for angles less than 5°45′ where the values of the tangent and sine are approximately equal.

The CI scale on the Polyphase slide rule also may be used for evaluating tangents of angles greater than 45°. The procedure is as follows:

tan 65° Slide to 25° on T opposite back index

Runner to right index of D

Read 2.14 on CI

The disadvantage of this procedure lies in the difficulty of manipulation in setting the runner on the right index.

Functions of Angles Smaller than 0°34′. The functions of angles smaller than those on the S scale can be obtained readily by using the relation that for small angles, $\sin x = \tan x = x$ (in radians), approximately. There are two gauge points on the S scale which correspond to the numerical values in radians of an angle of one minute and an angle of one second. One gauge mark is called the minutes (′) gauge, and the other is called the seconds (″) gauge. Since $\sin 4' = 4'$ (in radians) approximately, or 4 × radian measure of 1′, the sin 4′ is obtained by the corresponding procedure:

Runner to 4 on A left (Fig. 136)

Slide to minutes gauge on S

Runner to left index

Read 0.001164 on A

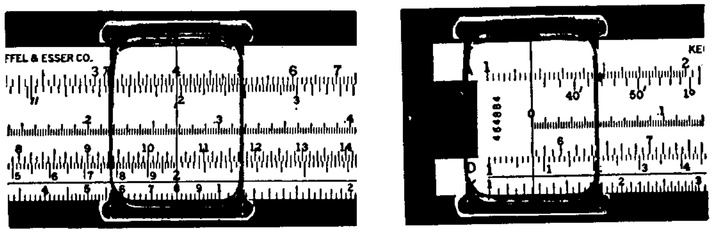


Fig. 136.

The cycles of A are of no significance in obtaining the functions of small angles. The left cycle is used because it is closer to the gauge marks. The decimal points for functions of small angles can be determined from the following data:

$$0.1° = 0.002 \text{ (2 zeros, 2) radians approximately}$$
$$1' = 0.0003 \text{ (3 zeros, 3) radians approximately}$$
$$1'' = 0.000005 \text{ (5 zeros, 5) radians approximately}$$

The following examples illustrate the procedure:

sin 2′ *Answer* $= 2 \times 0.0003 = 0.0006$ approx.
Runner to 2 on A left
Slide to minutes gauge
Runner to right index
Read 0.000582 on A

sin 33″ *Answer* $= 33 \times 0.000005 = 0.000165$ approx. (Fig. 137)
or $\doteq 0.5 \times 0.0003 = 0.00015$ approx.
Runner to 33 on A left
Slide to seconds gauge
Runner to left index
Read 0.000160 on A

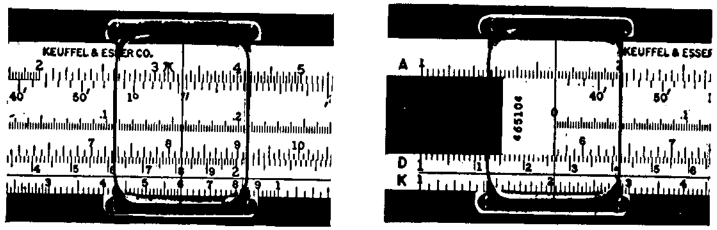


Fig. 137.

tan 7″ *Answer* $= 7 \times 0.000005 = 0.000035$ approx.
Runner to 7 on A left
Slide to seconds gauge
Runner to left index
Read 0.0000340 on A

sin 0.082° $0.082° = 60 \times 0.082 = 4.92'$
Answer $\doteq 5 \times 0.0003 \doteq 0.0015$ approx.
Runner to 4.92 on A left
Slide to minutes gauge
Runner to left index
Read 0.001433 on A

The reverse process of finding the magnitude of the angle from its function is also illustrated as follows:

tan x = 0.00082 $x \doteq \frac{8}{3}, \left(\frac{0.0008}{0.0003}\right) \doteq 2'$
Runner to 0.00082 on A right
Right index to hairline
Runner to minutes gauge
Read x as 2′49″(2.82′) on A

sin x = 0.0000149 $x \doteq \frac{15}{5}, \left(\frac{0.000015}{0.000005}\right) \doteq 3''$
Runner to 0.0000149 on A left
Left index to hairline
Runner to seconds gauge
Read x as 3.07″ on A

Since the $\cos x = \sin (90° - x)$ and $\cot x = \tan (90° - x)$, the cosines and cotangents of angles greater than 89°26′ can be found by computing the sines and tangents respectively of the complementary angles.

$$\cos 89°42' = \sin 18' = 0.00524$$

Also, since $\tan x = \dfrac{1}{\tan (90° - x)}$,

$$\tan 89°42' = \frac{1}{\tan 18'} = \frac{1}{\sin 18'} = 191.$$

Values of secants of angles close to 90° and of cosecants of angles close to 0° can be easily found from the corresponding values of the sine function.

$$\csc 0°18' = \frac{1}{\sin 0°18'} = 191$$

$$\sec 89°42' = \csc 0°18' = \frac{1}{\sin 0°18'} = 191$$

Exercise 33

Evaluate the following sines and cosines:

1. sin 14°18′
2. sin 0°44′
3. cos 82°12′
4. cos 5°30′
5. sin 22°45′
6. cos 47°50′
7. cos 89°
8. sin 40°30′
9. cos 72°12′
10. sin 1°46′

Evaluate the following secants and cosecants:

11. sec 17°
12. csc 49°
13. csc 4°
14. sec 83°
15. csc 2°
16. sec 89°

Determine x in the following equations:

17. $\sin x = 0.629$
18. $\cos x = 0.848$
19. $\sin x = 0.0862$
20. $\cos x = 0.0772$

Evaluate the following tangents and cotangents:

21. tan 16°15′
22. tan 36°30′
23. tan 2°40′
24. cot 6°30′
25. cot 83°12′
26. tan 89°18′
27. cot 0°54′
28. tan 66°12′

Determine x in the following equations:

29. $\tan x = 1.63$
30. $\tan x = 0.938$
31. $\cot x = 4.79$
32. $\tan x = 0.0427$

Evaluate the following functions:

33. sin 17′
34. tan 4′30″
35. sin 43″
36. cos 89°46′
37. cot 89°59′10″
38. sin 31′

Determine x in the following equations:

39. $\sin x = 0.000872$
40. $\tan x = 0.0000436$
41. $\sin x = 0.00369$
42. $\cos x = 0.00207$

47. The L Scale. The L scale, called the *log scale,* is a scale of the mantissas of the common logarithms of numbers, i.e., logarithms to the base 10. The mantissas are read directly on the L scale and the characteristics of the logarithms of numbers are evaluated as usual, by counting places to the left or the right of the decimal point. For a number greater than one, the characteristic is positive and is one less than the number of places to the left of the decimal point. For numbers less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.

The mantissa can be determined either by reversing the slide so that the L scale is on top, or by using the back index. Reversing the slide and centering it, the mantissa of the logarithm of a number is read directly on the L scale by setting the runner on the number on the D scale.

log 20 Mantissa: Runner to 20 on D (Fig. 138)
Read 0.301 on L

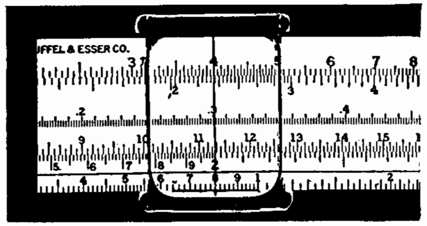


Fig. 138.

log 75 Mantissa: Runner to 75 on D
Read 0.875 on L

Using the back index, the procedure is as follows:

log 20 Mantissa: Slide to 20 opposite right index of D
Read 0.301 on L opposite back index

log 75 Mantissa: Slide to 75 opposite right index of D
Read 0.875 on L opposite back index

The following table gives a number of complete evaluations of logarithms:

Number on D	*Mantissa on L*	*Characteristic*	*Logarithm*
2	0.301	0	0.301
30	0.477	+1	1.477
75	0.875	+1	1.875
423	0.626	+2	2.626
0.164.........	0.215	−1	9.215—10
0.026.........	0.415	−2	8.415—10

The L scale can be used in the evaluation of such expressions as $(1.92)^{4.32}$ and $(18.47)^{\frac{1}{6.23}}$, as shown by the following procedures.

$x = (1.92)^{4.32}$ $\log x = 4.32 \log 1.92$

$\log 1.92 = 0.283$ using L scale

$4.32 \times 0.283 = 1.222$ using D and C scales

Runner to 0.222 on L

Read 1 6 7 on D

Characteristic 1 means 2 places to left of decimal point

$x = 16.7$

$x = (18.47)^{\frac{1}{6.23}}$ $\log x = \dfrac{\log 18.47}{6.23}$

$\log 18.47 = 1.266$ using L scale

$\dfrac{1.266}{6.23} = 0.203$ using D and C scales

Runner to 0.203 on L

Read 1 5 9 5 on D

Characteristic 0 means 1 place to left of decimal point

$x = 1.595$

For numbers less than one raised to powers, negative logarithms must be used as shown in the following example.

$x = (0.463)^{3.47}$ $\log x = 3.47 \log 0.463$

$\log 0.463 = 9.666—10$ using L scale

$= -0.334$

$-0.334 \times 3.47 = -1.158$ using D and C scales

$= 8.842—10$

Runner to 0.842 on L

Read 6 9 5 on D

Characteristic −2 means 1 zero to the right of the decimal point

$x = 0.0695$

Exercise 34

Evaluate the common logarithms of the following numbers using the L scale to determine the mantissas:

1. 4.93
2. 637
3. 15,900
4. 29.7
5. 0.551
6. 0.00693
7. 7320
8. 3.79
9. 56.3
10. 825

Evaluate the following expressions:

11. $(1.083)^{17.3}$
12. $(4.63)^{3.36}$
13. $(46.5)^{\frac{1}{4.37}}$
14. $(0.868)^{5.12}$

CONVERSION FACTORS

Multiply	*by*	*to obtain*
board feet	144	cubic inches
board feet	.0833	cubic feet
centimeters	.3937	inches
cubic feet	7.481	gallons, U. S.
cubic feet	28.32	liters
cubic feet	62.4	pounds, water
cubic inches	16.39	cubic centimeters
degrees, angular	.01745	radians
feet	30.48	centimeters
gallons, U. S.	8.337 (at 15°C)	pounds, water
gallons, U. S.	.1337	cubic feet
gallons, U. S.	231	cubic inches
horse-power	.746	kilowatts
inches	2.540	centimeters
kilograms	2.205	pounds
kilometers	.6214	miles
liters	.2642	gallons, U. S.
meters	39.37	inches
miles	5280	feet
miles	1.609	kilometers
miles per hour	1.467	feet per second
pounds	453.6	grams
radians	57.30	degrees, angular
square feet	.09290	square meters
square inches	6.452	square centimeters
square miles	640	acres
yards	.9144	meters

ANSWERS

Exercise 1

1. Three
2. Four
3. Three
4. Three
5. Five
6. Four
7. Four
8. Three
9. Two
10. Three
11. Four
12. Six
13. One
14. Five
15. Three
16. Four
17. Two
18. Two
19. Three
20. One
21. 2.16×41.7
22. 63.9×3.45
23. 107.3×8.43
24. $\frac{5720}{294}$
25. $\frac{64.9}{1.007}$
26. $2.18 \times 4.01 \times 0.926$
27. $\frac{0.0685 \times 13.64}{5.05}$
28. $\frac{4.875 \times 83.8}{36.1 \text{ or } 36.2}$
29. $\frac{76{,}400}{11.56 \times 20.0 \text{ or } 20.1}$
30. $\frac{1{,}374{,}000}{0.293 \times 21.4 \times 987}$

Exercise 2

1. 51.3
2. 118.4
3. 123.8
4. 10.25
5. 638
6. 350
7. 7.97
8. 1903
9. 222
10. 5.97
11. 7.04
12. 0.568
13. 257
14. 3.32
15. 4.04
16. 2530
17. 60.0
18. 248
19. 108.1
20. 362
21. 60.0
22. 259
23. 173.5
24. 2140
25. 691
26. 1918
27. 2490
28. 16.42
29. 9.72
30. 5.12
31. 5.73
32. 157.2
33. 0.913
34. 19.55
35. 39.4
36. 3.69
37. 456
38. 1.30
39. 28,500
40. 6.07
41. 546,000
42. 139,300,000
43. 25,700,000
44. 72,100,000
45. 0.0000763
46. 0.00000179
47. 0.0000147
48. 18.0
49. 28.4
50. 0.00595
51. 4.45
52. 9.54
53. 3.62
54. 47.5
55. 35.5
56. 440
57. 17,270
58. 0.154
59. 113.3
60. 13.44

Exercise 3

1. 75.7
2. 91.6
3. 50.3
4. 124.4
5. 670
6. 197
7. 15.0
8. 61.4
9. 1476
10. 1945
11. 69.7
12. 900
13. 85.2
14. 66.7
15. 646
16. 1.068
17. 56.6
18. 0.0216
19. 699
20. 1.532
21. 1660
22. 118.0
23. 8.81
24. 6.99
25. 1524
26. 12,300
27. 183.5
28. 106.7
29. 1029
30. 214
31. 1400
32. 2560
33. 62.8
34. 2000
35. 0.1316
36. 0.1682
37. 14.81
38. 0.00998
39. 1.017
40. 17.41
41. 10,720 ft.^{2}
42. 142,200 ft.^{2}
43. 6620 per yr.
44. $20,800
45. $179
46. (a) 108.5
(b) 110.2
(c) 9.76
(d) 15.22
47. 9.04 mi.
48. 682 mi.
49. (a) 108.2 cm.
(b) 854 in.
(c) 87.7 cm.^{2}
(d) 6030 acres
(e) 7770 lbs.
(f) 604 lbs.
(g) 8.14 lbs.
(h) 11,340 in.^{3}
(i) 125.9 ft. per sec.
(j) 7.09 kw.

Exercise 4

1. 689
2. 502
3. 97.0
4. 1253
5. 10.61
6. 0.1277
7. 103.2
8. 72.8
9. 594
10. 6.65
11. 30.8
12. 201
13. 30.4
14. 3.14
15. 54.5
16. 494
17. 360
18. 83.0
19. 20.9
20. 22.1
21. 46.3
22. 20.1
23. 172.8
24. 1020
25. 838
26. 329
27. 223
28. 1290
29. 0.314
30. 26.3
31. 37.7
32. 1389
33. 21.8
34. 2.6
35. 17.62
36. 129.3
37. 0.0205
38. 2990
39. 5890
40. 2770
41. 83.7 ft.^{3}
42. 1,317,000 ft.^{3}
43. (a) 1762 hrs.
(b) $2640
44. $184,600
45. 2,340,000 cfs.

Exercise 5

1. 1410
2. 654
3. 121
4. 5470
5. 70.8
6. 21.6
7. 12.45
8. 13.7
9. 684
10. 0.210
11. 11.2
12. 6400
13. 9.90
14. 84.8
15. 100.7
16. 1003
17. 71.6
18. 1243
19. 274
20. 163
21. 4430 lbs.
22. 656 lbs.
23. $849,000
24. 91.1 hp
25. $7.84

Exercise 6

1. 8.20
2. 0.588
3. 16.54
4. 12.68
5. 5.19
6. 1.828
7. 1.284
8. 0.880
9. 9.22
10. 0.01526
11. 17.9
12. 4.96
13. 12.38
14. 47.9
15. 2.68
16. 1.246
17. 3.94
18. 3.97
19. 0.0982
20. 10.05
21. 0.895
22. 5.49
23. 8.21
24. 9.35
25. 59.5
26. 800
27. 36.7
28. 4.25
29. 14.0
30. 21.4
31. 67.9 ft.
32. 16.9 mi.
33. 485 lbs. per ft.^{3}
34. 0.0156 ft.^{3}
35. 1.89 yd.
36. 19.6%
37. 79.2%
38. 241 rpm.
39. 71.1 mph.
40. 7.50 min.

Exercise 7

1. 70.7
2. 0.504
3. 75.9
4. 76.6
5. 23.4
6. 0.457
7. 0.922
8. 10.7
9. 0.110
10. 11.67
11. 0.526
12. 11.8
13. 221
14. 13.2
15. 4.23
16. 1.324
17. 32.7
18. 12.83
19. 159
20. 10.5
21. 0.1334
22. 2.89
23. 0.659
24. 5.86
25. 14.29
26. 0.0910
27. 9.20
28. 15.4
29. 2.95
30. 4.07
31. (a) 8.41 in.^{2}
(b) 241 in.^{2}
(c) 25.3 ft.^{2}
(d) 18.6 yd.^{2}
32. (a) 76.7 ft.^{3}
(b) 1.643 ft.^{3}
(c) 18.9 in.^{3}
33. (a) 3.16 ft.^{3}
(b) 1144 in.^{3}
(c) 23.1 ft.^{3}
34. (a) 229 in.^{2}
(b) 100.6 psi.
35. 3.74 ft.

Exercise 8

1. 34.1	**11.** 0.483	**21.** 4.87
2. 235	**12.** 0.489	**22.** 4.16
3. 428	**13.** 21.3	**23.** 316
4. 35.6	**14.** 3.80	**24.** 32.8
5. 258	**15.** 27.2	**25.** 44.5
6. 191.8	**16.** 4.01	**26.** 237
7. 389	**17.** 1.042	**27.** 2.56
8. 40.0	**18.** 2.33	**28.** 2.50
9. 3.23	**19.** 313	**29.** 366
10. 35.9	**20.** 20.5	**30.** 4.13

Exercise 9

1. 32.5	**15.** 54.0	**29.** 1.955
2. 34.0	**16.** 12.14	**30.** 5.75
3. 0.0627	**17.** 418	**31.** 1.785
4. 16.06	**18.** 0.00340	**32.** 117.8
5. 227	**19.** 17.85	**33.** 329
6. 1.29	**20.** 4.91	**34.** 3.38
7. 0.162	**21.** 2.71	**35.** 3.48
8. 1.83	**22.** 1.508	**36.** 0.647
9. 1088	**23.** 640	**37.** 7.00
10. 7.77	**24.** 35.8	**38.** 7.63
11. 201	**25.** 106.3	**39.** 0.0757
12. 85.6	**26.** 1.83	**40.** 6380
13. 11.0	**27.** 18.52	
14. 333	**28.** 27.4	

Exercise 10

1. 300	**11.** 576,000	**21.** 3.68
2. 71.0	**12.** 2,930,000	**22.** 2.42
3. 45,400	**13.** 19,700,000	**23.** 9.15
4. 3.69	**14.** 46.6	**24.** 10.35
5. 34.6	**15.** 1150	**25.** 153.8
6. 3980	**16.** 2.85	**26.** 92.0
7. 4.08	**17.** 6.05	**27.** 657
8. 1730	**18.** 21.2	**28.** 2500
9. 164	**19.** 41.0	**29.** 29.9
10. 15,150	**20.** 1.44	**30.** 7.59

Exercise 11

1. 2000	**11.** 419,000,000	**21.** 2.92
2. 423	**12.** 2,360,000,000	**22.** 1.86
3. 6.23	**13.** 130,000	**23.** 4.14
4. 27,900,000	**14.** 880	**24.** 26.4
5. 79	**15.** 243,000	**25.** 18.6
6. 164,000	**16.** 3.60	**26.** 65.2
7. 13.3	**17.** 6.88	**27.** 120
8. 98,000	**18.** 2.10	**28.** 36.8
9. 1025	**19.** 1.393	**29.** 310
10. 7,550,000	**20.** 15.42	**30.** 10.22

Exercise 12

1. 0.685	**11.** 0.00215	**21.** 0.0252
2. 0.0955	**12.** 0.830	**22.** 0.222
3. 0.0272	**13.** 0.0688	**23.** 0.0406
4. 0.000542	**14.** 0.0001145	**24.** 0.278
5. 0.00512	**15.** 0.448	**25.** 0.0297
6. 0.279	**16.** 0.646	**26.** 0.01053
7. 0.00000149	**17.** 0.929	**27.** 0.00246
8. 0.000000851	**18.** 0.295	**28.** 0.00963
9. 0.0000146	**19.** 0.143	**29.** 0.000485
10. 0.0000000493	**20.** 0.442	**30.** 0.0284

Exercise 13

1. 0.00615	**11.** 0.000369	**21.** 0.1702
2. 0.077	**12.** 0.0455	**22.** 0.0320
3. 0.545	**13.** 0.000750	**23.** 0.0572
4. 0.000445	**14.** 0.00156	**24.** 0.01777
5. 0.0166	**15.** 0.000000019	**25.** 0.193
6. 0.000000320	**16.** 0.357	**26.** 0.00523
7. 0.00000175	**17.** 0.885	**27.** 0.0383
8. 0.000000000575	**18.** 0.1157	**28.** 0.0769
9. 0.000000088	**19.** 0.551	**29.** 0.221
10. 0.000000000014	**20.** 0.240	**30.** 0.191

Exercise 14

1. 1.400	**4.** 1.0354	**7.** 5300
2. 450	**5.** 23.6	**8.** 1.536
3. 2.063	**6.** 2.56	**9.** 1.0925

10. 2.19
11. 5.16
12. 665
13. 15.1
14. 1.25
15. 4.22
16. 1.0181
17. 1.0277
18. 9.05
19. 1.495
20. 293
21. 3.32
22. 128
23. 3.46
24. 5.64
25. 1.217
26. 1.0460
27. 1.0284
28. 1.161
29. 1340
30. 1.0130
31. 3.16
32. 16.7
33. 0.1845
34. 4.16
35. 0.1840
36. 1.191
37. 3.33
38. 1.190
39. 432,000
40. 54,700,000
41. 8,400,000
42. 29,500,000
43. 1,100,000,000
44. 17,600,000
45. 140,000,000
46. 14,800,000

Exercise 15

1. 0.8263
2. 0.199
3. 0.9358
4. 0.9645
5. 0.472
6. 0.408
7. 0.733
8. 0.842
9. 0.360
10. 0.067
11. 0.600
12. 0.427
13. 0.0113
14. 0.9688
15. 0.9889
16. 0.457
17. 0.9673
18. 0.659
19. 0.876
20. 0.9542
21. 3.95
22. 137.2
23. 4.57
24. 0.276
25. 0.544
26. 0.9750
27. 0.00000475
28. 0.0000000072
29. 0.00000142
30. 0.000000019

Exercise 16

1. 0.382
2. 0.0042
3. 0.182
4. 0.398
5. 0.703
6. 0.9638
7. 0.764
8. 0.9551
9. 0.215
10. 0.542
11. 1.297
12. 3.92
13. 1.1585
14. 1.268
15. 176
16. 1.0364
17. 1.556
18. 3.46
19. 1.582
20. 1.868

Exercise 17

1. 0.247
2. 0.842
3. 0.0128
4. 0.953
5. 0.1357
6. 0.0445
7. 0.995
8. 0.315
9. 0.989
10. 0.387
11. 0.1766
12. 0.671

13. 0.0480
14. 0.01745
15. 0.1176
16. 0.649
17. 0.846
18. 0.0886
19. 0.306
20. 0.0308
21. 1.046
22. 1.325
23. 14.33
24. 8.20
25. 4.44
26. 28.6
27. 57.3
28. 1.030
29. 8.21
30. 2.37
31. 39°
32. 57′(0.95°)
33. 32°
34. 24°24′(24.4°)
35. 85°34′(85.57°)
36. 79°20′(79.33°)
37. 76°
38. 4°57′(4.95°)
39. 19°
40. 57°48′(57.8°)

Exercise 18

1. 0.291
2. 0.740
3. 0.0466
4. 1.181
5. 8.78
6. 6.56
7. 2.41
8. 0.1192
9. 1.262
10. 2.45
11. 63.7
12. 22.9
13. 81.8
14. 0.0577
15. 0.0256
16. 2.86
17. 0.377
18. 0.1069
19. 2.27
20. 0.920
21. 58°30′(58.5°)
22. 79°08′(79.13°)
23. 2°27′(2.45°)
24. 11°48′(11.8°)
25. 33°42′(33.7°)
26. 82°41′(82.68°)
27. 87°33′(87.55°)
28. 88°10′(88.17°)
29. 43°10′(43.17°)
30. 0°42′(0.70°)

Exercise 19

1. 0.00495
2. 0.00131
3. 0.000209
4. 0.00640
5. 0.00182
6. 0.0000412
7. 0.00407
8. 0.000242
9. 139
10. 491
11. 34,400
12. 0.00902
13. 3′
14. 9″
15. 12′42″(12.7′)
16. 2.74″
17. 89°52′53″(89.882°)
18. 89°57′20″(89.955°)
19. 17.8″
20. 89°58′43.5″ (89.979°)

Exercise 20

1. 0.174
2. 0.00538
3. 0.0424
4. 40.5
5. 112.9
6. 0.000237
7. 0.1153
8. 4.74
9. 11.43
10. 0.00659
11. 0.98380
12. 0.491
13. 0.681
14. 1.0238
15. 1.394
16. 0.9210
17. 1.172
18. 1.0140
19. 0.527
20. 1.714

Exercise 21

1. 204
2. 2.93
3. 115.4
4. 1.47
5. 32.2
6. 4.02
7. 70.9
8. 2.79
9. 86.2 ft.
10. 12.93 ft.
11. 4 ft. $10\frac{3}{4}$ in.
12. 13 units
13. $2,800

Exercise 22

1. (a) 6.08, 7.74, 14.02, 19.15, 32.3, 44.8
(b) 12.47, 15.87, 28.8, 39.3, 66.3, 92.0
(c) 18.22, 23.2, 42.0, 57.4, 96.8, 134.3

2. (a) 3.82, 7.15, 8.80, 13.93, 20.8, 45.3
(b) 2.02, 3.76, 4.64, 7.34, 10.99, 23.9
(c) 0.960, 1.795, 2.21, 3.50, 5.23, 11.38

3. (a) 234, 118.3, 80.6, 29.8, 15.4, 9.73
(b) 110, 55.8, 38.0, 14.02, 7.25, 4.58
(c) 39.9, 20.2, 13.77, 5.09, 2.63, 1.662

Exercise 23

1. 12.57
2. 2.31
3. 49.4
4. 10.06
5. 2.78
6. 6.13
7. 0.1665
8. 1.935
9. 21.5
10. 81.2
11. 3.07
12. 0.426
13. 6.96
14. 11.17
15. 36.9
16. 36.1
17. 0.159
18. 0.455
19. 1.395
20. 5.25
21. 1.502 radians
22. 0.734 rad.
23. 4.63 rad.
24. 8.68 rad.
25. 0.297 rad.
26. 6.39 rad.
27. 85.4°
28. 21.2°
29. 118.1°
30. 340°
31. 724°
32. 63.6°
33. 4.86
34. 3000
35. 0.110
36. 49.3
37. 0.000272
38. 85.0
39. 4.02
40. 13.78
41. 1.085
42. 6.89
43. 3.28
44. 26.0
45. 592 ft.^{2}
78.54% $\left(\frac{\pi}{4}\right)$
46. 88.6 in.

Exercise 24

1. 111
2. 58.0
3. 27.4
4. 14.05
5. 7.32
6. 0.307

7. 2.54
8. 6.05
9. 14.6
10. 1.255
11. 8.87
12. 10.6
13. 0.685
14. 1.782
15. 65.0
16. 0.01445
17. 13.7
18. 3.07
19. 212
20. 0.858
21. 23.7
22. 36.0
23. 1.88
24. 597
25. 15.1
26. 0.773
27. 53.9
28. 4.64
29. 0.941
30. 66.6
31. 153
32. 0.480
33. 15.1
34. 0.431
35. 8.80
36. 432
37. 33.8
38. 1.01
39. 161
40. 8.12
41. 0.630
42. 108.5
43. 2.32
44. 0.291
45. 37.3
46. 1750
47. 0.93
48. 1.45
49. 2.65
50. 75
51. 38.6
52. 46.3
53. 1.247
54. 15.4
55. 7.36
56. 23.5
57. 1.29
58. 11.1
59. 11.6
60. 0.832
61. 197 ft.
62. 44.1 ft. per sec.
63. 2.81 sec.
64. 157.2 ft. per sec.
65. 646 ft.
66. 13.0 sec.
67. 95.4 ft. per sec.
65.1 miles per hr.
68. 19.8 ft.
69. 30.0 lbs.
70. 1.41 ft.
71. 182 cu. in.
72. 785 sq. in.
73. 41,600 gal.
74. 25.3 ft.
75. $264

Exercise 25

1. 26.5
2. 3.07
3. 0.601
4. 0.287
5. 32.2
6. 18.6
7. 12.46
8. 6.38
9. 26.5
10. 1.95
11. 232
12. 3.62
13. 96.8
14. 24.5
15. 1.126
16. 7.71
17. 1.742
18. 2.00
19. 0.502
20. 0.115
21. 0.552
22. 0.000482
23. 0.307
24. 2.74
25. 0.1233
26. 0.68
27. 2.04
28. 0.270
29. 0.0333
30. 9.1
31. $c = 67.6$
$A = 33°$
$C = 101°$
32. $a = 43.3$
$c = 80.2$
$B = 69°$
33. $a = 166.6$
$b = 94.9$
$C = 40°$
34. $a = 4.86$
$A = 46°36'(46.6°)$
$B = 57°24'(57.4°)$
35. 557 ft.
36. 37°33′(37.55°)
37. 11.69 ft.
38. 623 ft.
39. 49%
40. 1°17′24″(1.29°)

Exercise 26

1. 660
2. 1.522
3. 0.8692
4. 3.26
5. 1.0366
6. 0.1085
7. 0.9851
8. 2.585
9. 1.174
10. 0.8085
11. 3.70
12. 1.412
13. 1.84
14. 7.33
15. 0.430
16. 0.812

Exercise 27

1. 12.6
2. 96
3. 0.736
4. 0.298
5. 101
6. 0.849
7. 14.9
8. 1.650
9. 6.69
10. 1.375
11. 5.30
12. 10.8
13. 1.977
14. 1.468
15. 2150
16. 2080
17. 61.5
18. 1.622
19. 0.536
20. 0.499
21. 28.8
22. 0.9284
23. 1.0568
24. 3.11
25. 1.0290
26. 1.483
27. 0.499
28. 810
29. 4.46
30. 2.20

Exercise 28

1. 0.693
2. 2.804
3. 4.201
4. 1.473
5. 9.741—10
6. 7.841—10
7. 3.865
8. 0.579
9. 1.750
10. 2.916
11. 2.16
12. 6.27
13. 4.28
14. 0.399
15. 9.44
16. 0.230
17. 0.0155
18. −0.01158
19. −0.471
20. −6.65
21. 4.61
22. 11.85
23. 3.06
24. 3.32
25. 0.0203
26. 0.414
27. 62.7
28. 18.32
29. 0.0750
30. −5.75

Exercise 29

1. 34.1
2. 91.6
3. 6.99
4. 428
5. 106.7
6. 235
7. 40.0
8. 35.6
9. 15.0
10. 118.0
11. 1.068
12. 191.8
13. 62.8
14. 166.0
15. 1029

16. 0.1682
17. 0.00998
18. 30.4
19. 97.0
20. 223
21. 46.3
22. 20.9
23. 4.94
24. 2990
25. 838
26. 121.1
27. 12.45
28. 654
29. 6400
30. 1243
31. 10,720 ft.^{2}
32. 142,200 ft.^{2}
33. $20,800
34. (a) 108.6
(b) 110.2
(c) 9.76
(d) 15.22
35. 682 mi.
36. (a) 108.2 cm.
(b) 854 in.
(c) 87.7 cm.^{2}
(d) 6030 acres
(e) 7770 lbs.
(f) 604 lbs.
(g) 8.14 lbs.
(h) 11,340 in.^{3}
(i) 125.9 ft. per sec.
(j) 7.09 kw.
37. 83.7 ft.^{3}
38. 1,317,000 ft.^{3}
39. (a) 1762 hrs.
(b) $2640
40. 4430 lbs.
41. 656 lbs.
42. $849,000

Exercise 30

1. 1.899
2. 3.23
3. 2.85
4. 35.9
5. 21.3
6. 0.589
7. 8.20
8. 3.80
9. 0.0397
10. 5.19
11. 0.489
12. 1.828
13. 16.54
14. 2.68
15. 4.01
16. 1.246
17. 1.042
18. 8.21
19. 10.05
20. 21.4
21. 67.9 ft.
22. 16.9 mi.
23. 485 lbs. per ft.^{3}
24. 0.0156 ft.^{3}
25. 1.89 yd.
26. 71.1 mph.

Exercise 31

1. 70.7
2. 0.504
3. 75.9
4. 76.6
5. 23.4
6. 0.457
7. 0.922
8. 10.7
9. 0.110
10. 11.67
11. 0.526
12. 313
13. 2.56
14. 2.50
15. 1.324
16. 159
17. 11.8
18. 221
19. 13.2
20. 4.23
21. (a) 8.41 in.^{2}
(b) 241 in.^{2}
(c) 25.3 ft.^{2}
(d) 18.6 yd.^{2}
22. (a) 76.7 ft.^{3}
(b) 1.643 ft.^{3}
(c) 18.9 in.^{3}
23. (a) 3.16 ft.^{3}
(b) 1144 in.^{3}
(c) 23.1 ft.^{3}
24. 3.74 ft.

Exercise 32

1. 61.4
2. 1.068
3. 1476
4. 91.6
5. 1.532
6. 124.4
7. 16.54
8. 0.589
9. 0.0627
10. 17.85
11. 97.0
12. 6.65
13. 201
14. 1020
15. 360
16. 100.7
17. 9.90
18. 47.9
19. 17.9
20. 0.0982
21. 8.21
22. 10.05
23. 29.6
24. 4.58
25. 10.21

Exercise 33

1. 0.247
2. 0.0128
3. 0.1358
4. 0.995
5. 0.387
6. 0.671
7. 0.01745
8. 0.650
9. 0.306
10. 0.0308
11. 1.046
12. 1.325
13. 14.33
14. 8.20
15. 28.6
16. 57.3
17. 39°
18. 32°
19. 4°57′
20. 85°34′
21. 0.291
22. 0.740
23. 0.0466
24. 8.78
25. 0.1192
26. 81.8
27. 63.7
28. 2.27
29. 58°30′
30. 43°10′
31. 11°48′
32. 2°27′
33. 0.00495
34. 0.00131
35. 0.000209
36. 0.00407
37. 0.000242
38. 0.00902
39. 3′
40. 9″
41. 12′42″
42. 89°52′53″

Exercise 34

1. 0.693
2. 2.804
3. 4.201
4. 1.473
5. 9.741—10
6. 7.841—10
7. 3.865
8. 0.579
9. 1.750
10. 2.916
11. 3.98
12. 173
13. 2.41
14. 0.484

INDEX

A

B

C

D

E

F

G

H

I

K

L

M

N

O

P

R

S

About the Author

LEE H. JOHNSON, Sc.D., is Dean of Engineering and Professor of Civil Engineering at the University of Mississippi.

Dean Johnson has had wide experience in engineering computations applied both to practical design problems and to research investigations. This experience includes the analysis and design of bridges, buildings, retaining walls and other structures, a mathematical simplification of the design of statically indeterminate structures, the design and construction of models for special soil studies, and flood routing and backwater computations for flood control projects.

Dean Johnson is a member of the American Society of Civil Engineers, American Society for Engineering Education, Mississippi Society of Professional Engineers, Phi Beta Kappa, and Omicron Delta Kappa. He is the author of a number of articles in professional journals.

HERE IS a new method of operation for the Duplex type of slide rule which makes full use of its versatility and efficiency.

The author has applied to the operation of the modern Duplex rule the kind of technique now familiar in time-and-motion studies for efficiency in production. The result has been to make available—for the first time since the invention of the Duplex slide rule – an integrated technique of operation for *all* of the scales on the rule. It is a time- and motion-saving technique in which movement is reduced by one-third to one-twentieth, and time reduced by one-half.

In addition to a full treatment of the Duplex series, including slide rules with the log log scales, a chapter has been devoted to the Mannheim rule.

No effort has been spared to make this text thoroughly self-teaching. The presentation is progressive, to ensure step-by-step comprehension of the principles and to promote facility in the skills of operation. The text is written in clear and direct terms, easy to follow. Many illustrative examples are included for each progressive step in manipulation and for each type of problem considered. Complete illustrations and specially designed notations are given for each of the operations and adjustments. Answers to all problems and exercises are included as a self-check of comprehension of the principles and adjustments essential to the *expert* use of this indispensable technical tool.