MECHANICAL DRAWING.

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W. Welch

THE PRACTICE

OF

MECHANICAL DRAWING

FOR

SELF-INSTRUCTION.

BY

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PREFACE.

This book is intended to enable mechanics and others to learn to make mechanical drawings when they cannot have the assistance of a teacher; and it is also intended as a text-book for beginners in mechanical drawing.

Geometrical terms and problems, which are not needed in ordinary work, have been avoided.

WMS. WELCH.

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CHAPTER II.

CONSTRUCTION OF GEOMETRICAL PROBLEMS.

A draftsman should be familiar with the problems in this chapter, and should be able to construct them very accurately when necessary. In practice, they are seldom used; but unless he *knows how* to draw them with precision, he will not be apt to get them so nearly correct when drawing them approximately.

Problems of this kind are about the best exercises for beginners in mechanical drawing.

The desired results will not be obtained unless extremely fine, hair-like lines are drawn, and all centres and other points taken *exactly on* these lines. Therefore it is important to use very hard pencils sharpened to a needle or chisel point, and to avoid making holes in the paper with the compasses.

Given and required lines should be drawn fine in ink, and construction lines drawn in pencil only; but, to distinguish them in these problems, *given* lines are *heavy*; *required* lines *fine;* and *construction* lines *broken.*

An ***ARC*** of a circle is drawn by sticking one point of the compasses, Fig. 15, in the paper and moving the other point around on the paper a short distance. The distance between the points of the compasses is the *radius* of the arc. An arc can be drawn when its centre and radius are given.

PROB. 1. Draw a line which will divide a given line into two equal parts and be perpendicular to it. (Take any line.)

With the ends of the given line as centres, and with equal radii, draw arcs which will intersect on both sides of the line. The line joining the points of intersection will be the one required.

PROB. 2. Draw a line perpendicular to a given line through a given point in the line. (Take any point in a line.)

With the given point as a centre, and the same radius, draw arcs intersecting the line. With the two points of intersection as centres and equal radii, draw arcs intersecting on both sides of the line. A line joining these two last points of intersection will be the perpendicular required.

PROB. 3. Draw a line perpendicular to a given line from any given point. (Take point 2 ins. from line.)

With the given point as a centre, draw an arc which intersects the line at two points. With these two points as centres, and equal radii, draw intersecting arcs. The line joining the given point and the last point of intersection will be the perpendicular required.

An ***ANGLE*** is formed by two straight lines meeting in a point. Shortening or lengthening the lines does not change the angle. For the purpose of measuring angles and arcs, the whole circumference of any circle is arbitrarily divided into

360 equal arcs, called *degrees.* If the *vertex* of an angle (point where the sides meet) is placed at the centre of the circle, the arc between the sides will contain the same number of degrees as the angle. A *right angle* contains 90 degrees. The radius of a circle will step around on the circumference exactly *six* times, dividing it into arcs of 60 degrees.

PROB. 4. Draw a line which will divide a given angle, or arc, into two equal parts. (Take any angle.)

With the vertex of the angle as a centre, draw an arc intersecting the sides of the angle. With the points of intersection as centres, and equal radii, draw intersecting arcs. The line joining the last point of intersection with the vertex, will bisect the angle and the arc contained between its sides.

Any point in this line will be equally distant from the sides of the angle.

PROB. 5. Divide a right angle into three equal parts, thereby constructing angles of 30 degrees.

With the vertex of the right angle as a centre, draw an arc intersecting the sides. With the points of intersection as centres, and the same radius used in drawing the arc, draw arcs intersecting the first arc. Lines drawn from the last points of intersection to the vertex will trisect the right angle, forming angles of 30 degrees each.

PROB. 6. Construct an angle at a given point, equal to a given angle. (Take any convenient angle and point.)

With the vertex of the given angle as a centre, draw an arc intersecting the sides; and, with the vertex of the required angle as a centre, draw an arc with the same radius. Make the two arcs between the sides equal and the angles will be equal.

Lines are **PARALLEL** when they lie in the same plane and never meet if produced indefinitely in both directions. The surface of the paper is the *plane* in which lines are drawn.

PROB. 7. Draw a line parallel to a given line, at a given distance from it. (Take 2 ins.)

With a radius equal to the given distance, and two points in the line—one near each end—as centres, draw arcs. A line just touching these arcs will be the parallel line required.

PROB. 8. Draw a line parallel to a given line, through a given point. (Take point about $3\frac{1}{2}$ ins. from line.)

Draw a line through the point and crossing the given line. Construct angles around the point equal to the corresponding angles formed by the intersection of the two lines. A side of these angles will be the parallel line required

PROB. 9. Divide a given line into any number of equal parts. (Take 7 parts.)

Draw a line through one end of the given line, and from that end measure off the required number of equal divisions on this second line. Draw a line through the last point of division and the other end of the given line, and draw lines through all the other points of division parallel with this last line. These parallel lines will divide the given line into the required number of equal parts.

A **TRIANGLE** is a plain figure bounded by three straight sides. When one angle is a right angle the figure is a *right triangle*; the longest side of which is the *hypotenuse*, and *the hypotenuse squared is equal to the sum of the squares of the other two sides.* When two of the sides are equal, the two angles opposite them are equal and the triangle is *isosceles.* The angles of any triangle added make *two right angles* The area of a triangle is equal to half its base multiplied by its height.

PROB. 10. Form a right angle at the end of a given line by constructing a right triangle on the line.

Measure off four equal divisions from the end of the line. With a radius equal to 3 of the divisions, and the end of the line as a centre, draw an arc. With a radius equal to 5 of the divisions, and the fourth point on the line as a centre, draw an arc intersecting the first arc. Draw a line from the point of intersection to the end of the given line and it will form a right angle with the line.

Proof.—The hypotenuse squared is 25; 3 squared is 9; 4 squared is 16; 9 and 16 make 25.

PROB. 11. Construct a right triangle containing two angles of 45 degrees each.

Construct a right angle, (Prob. 1, 2 or 10.) With the vertex as a centre, draw an arc intersecting the sides. Draw a line joining the points of intersection and it will form angles of 45 degrees with the sides.

PROB. 12. Construct a right triangle containing one angle of 30 degrees and another of 60.

Construct a right angle. With the vertex as a centre and any radius, draw an arc intersecting one side. With the point of intersection as a centre, and twice the first radius as a radius, draw an arc intersecting the other side. Draw a line through the two points of intersection, and it will form the required angles with the sides.

A **POLYGON** is a figure bounded by three or more straight lines in the same plane. When all the sides are equal, and all the angles are equal, it is a *regular* polygon; and a circle can be drawn around it, touching all the angles, and another can be drawn within it, touching all the sides at their middle points.

PROB. 13. Draw a regular polygon of 5 sides in a circle of a given diameter. (Take diameter of circle 2 ins.)

Draw a diameter of the circle, and a radius perpendicular to it. Take a point on the diameter, half way between the centre and the circumference, as a centre; and, with a radius equal to the distance from this point to the extremity of the radius, draw an arc intersecting the diameter. The distance between this point of intersection and the extremity of the radius will be a side of the required *pentagon*, and it can be completed by stepping the side around on the circumference and drawing lines joining these points.

PROB. 14. Draw a regular polygon of 6 sides in a circle. (Take diameter of circle 2 ins.)

The radius of the circle will be the sides of the inscribed *hexagon*, and it can be completed by stepping the radius around on the circumference and drawing lines joining these points.

PROB. 15. Divide the circumference of a circle into 24 equal parts; thereby making arcs and angles of 15 degrees each. (Take diameter of circle 2 ins.)

Draw two diameters perpendicular to each other. (Prob. 1.) With the radius of the circle as a radius and the extremities of the diameters as centres, draw arcs intersecting the circumference at 8 points. Bisect the arcs between all these points (Prob. 4) and the required number of divisions will be made.

PROB. 16. Draw a regular pentagon with sides of a given length. (Take sides 1¼ ins.)

Draw a side and erect a perpendicular to this side at one end (Prob. 2 or 10) equal in length to half the side. (Prob. 1.) From the other end of the side draw a line passing through the extremity of the perpendicular, and extend this line a distance beyond the perpendicular equal to half the side. The distance between the end of this line and the nearer end of the side will be the radius of the circumscribed circle. It may be drawn and the polygon completed by stepping the given side around on it 5 times.

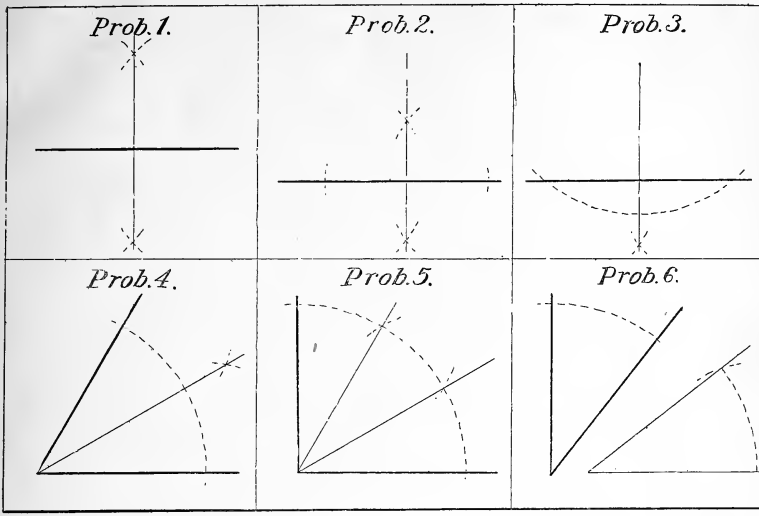
PROB. 17. Draw a regular 6 sided polygon, the distance between the opposite sides being given. (Take distance 1¾ ins.)

Draw parallel lines the given distance apart (Prob. 7), and draw a line making an angle of 60 degrees with them. (Prob. 5.) This line between the parallel lines will be the diameter of the circumscribed circle, and half of it will be equal to a side of the required hexagon. It may be quickly completed with a 60° triangle, Fig. 5.

PROB. 18. Draw a regular 8 sided polygon in a given square. (Take a square 2 x 2 ins.)

Find the centre of the square by drawing diagonal lines through the vertices (corners). With the vertices as centres, draw arcs passing through the centre of the square and intersecting the sides. These points of intersection will be the corners of the required *octagon.*

The *circumference* of a **CIRCLE** is drawn by a point moving around another point (the centre) and remaining the same distance from it in the same plane. A line drawn from the centre to the curve is the *radius*, and a line drawn across the circle through the centre is the *diameter.* A line drawn across elsewhere is a *chord*, and any part of the curve is an *arc.* The whole surface within the circumference is the *area*, one-half is a *semicircle*, and one-fourth is a *quadrant.* The surface enclosed by two radii and an arc is a *sector*, and that enclosed by a chord and an arc is a *segment.* The *circumference* is equal to about 3 1-7 times the diameter, or more exactly, 3.141592653590 times. The area of a sector is equal


Prob. 1.
Prob. 2.
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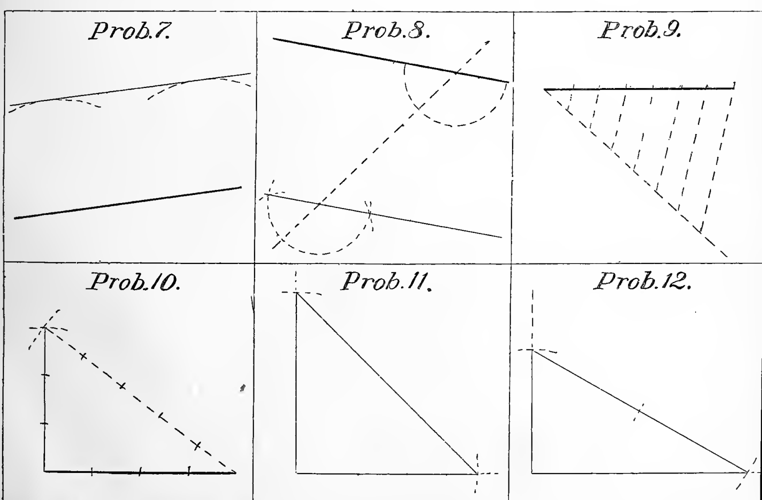
Prob. 7.
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Prob. 9.
Prob. 10.
Prob. 11.
Prob. 12.

to half its arc multiplied by the radius, and the area of the whole circle is equal to half the circumference multiplied by the radius.

PROB. 19. Draw a straight line, equal in length to the circumference of a given circle. (Take a circle with diam. 2 ins.)

Draw two diameters perpendicular to each other, and draw a chord from the extremity of one to the extremity of the other. Draw a perpendicular to this chord through its centre. (Prob. 4.) The part of this perpendicular between the chord and the arc, added to three times the diameter, will be very nearly equal to the circumference. It will be about 1-208 of the diameter too great.

PROOF.—Take the chord$=2$; the radius will be $\sqrt{2}=1.414$, and the part between the chord and arc will be $.414\div2.828=.1464$ which is slightly greater than .1416.

The chord is equal to the radius of a circle which has twice the area of the given circle.

PROB. 20. Draw a square with an area equal to the area of a given circle. (Take circle with diam. 2 ins.)

Draw a line equal in length to half the circumference of the given circles (Prob. 19) and extend the line a distance equal to the radius. With the middle point of the whole line as a centre, draw a semicircle passing through its ends. At the point where the two lines meet, erect a perpendicular. (Prob. 2.) The part of it between the line and semicircle will be a side of the required square.

PROOF.—The two parts of the diameter multiplied together equal the perpendicular squared; as it is a *mean proportional* between them. (See Wentworth's Geometry, page 157.)

A circle with an area two, three, four, five or more times as great as the area of a given circle may be drawn by making the shorter part of a diameter equal to the radius of the given circle and the longer part equal respectively to two, three, four, five or more times that radius. The perpendicular to the diameter, from the point where the two lines meet to the circumference, will be equal to the radius of the required circle.

PROOF.—Let r=given radius; p=perpendicular, and ar the longer part of the diameter. Then $ar^2=p^2$; $r^2 3.1416$=given area; $p^2 3.1416$=required area$=$ $ar^2 3.1416=a$ times the given area.

PROB. 21. Draw a circle equal in area to a given square. (Take side of square $1\frac{3}{4}$ inches.)

Draw a line from a vertex of the square to the centre of one side. This line will be very nearly equal to the diameter of the required circle. It will be about 1-100 of the side too small.

PROOF.—Take side of square$=1$; the diameter of circle will be 1.118, but it should be 1.128 to make the areas exactly the same.

PROB. 22. Draw a circle passing through 3 given points. (Take points 1, $1\frac{1}{2}$, and 2 ins. apart.)

With two of the given points as centres, and with equal radii, draw arcs which will intersect at two points. Draw a line through these two points of intersection. With the third given point and either one of the others as centres, draw arcs and another line as before. The point where these two lines cross will be the centre of the required circle.

By taking any three points in the circumference of a given circle or arc, its centre may be found in the same way.

PROB. 23. Draw, mechanically, an arc passing through 3 given points, without using a centre. (Take points $1\frac{1}{4}$, $1\frac{1}{2}$ and $2\frac{2}{3}$ ins. apart.)

Locate the three points on a piece of firm card-board, and cut straight lines passing from the intermediate point through the two extreme points. Stick fine pins in the two extreme points on the drawing paper, and hold a pencil in the vertex of the angle formed in the card-board while it is moved back and forth against the pins. The pencil will draw the required arc, and it can be inked with a curved ruler, Fig. 16.

PROB. 24. Draw an arc with a given radius without using a centre. (Take radius 6 inches.)

Part of a regular polygon of twelve sides can first be drawn, and the arc drawn through three or more of the corners. (Prob. 23.) The angles at the corners will be 150 degrees (Prob. 5); and the sides can be found by constructing a triangle with the base equal to half the given radius, and the angles at the base 90 and 15 degrees. (Prob. 12 and 4.) The hypotenuse will be the required side. If the radius is quite large, any regular polygon may be taken. The angles can be easily computed and the sides found by a table of chords.

A line is ***TANGENT*** to a circle when, however far produced, it passes through *but one point* in the circumference of the circle. It will be perpendicular to a radius of the circle drawn to the point of tangency.

PROB. 25. Draw a line tangent to an arc at a given point in the arc. (Take radius of arc 5 ins.)

With the given point as a centre, draw arcs intersecting the given arc at two points equally distant from the given point. With these two points as centres, draw intersecting arcs, and draw lines through the points of intersection. (Prob. 22.) A line drawn perpendicular to this line through the given point (Prob. 2) will be the tangent required.

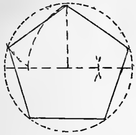
PROB. 26. Draw a line tangent to a circle from a given point outside of the circle. (Take circle 2 ins. in diam., point 3 ins. from centre.)

With the given point as a centre, draw an arc which will pass through the centre of the circle. With the centre of the circle as a centre, and with a radius equal to the diameter of the circle, draw an arc intersecting the first arc. Draw a line from the last point of intersection to the centre of the circle. This line will cross the circumference at the point of tangency; and a line drawn through it from the given point will be the tangent required. Two tangents can be drawn.

PROB. 27. Draw a line tangent to two circles of different diameters. (Take circles 1 and 2 ins. in diam., centres 2 ins. apart.)

Draw a line through the centres, and extend it beyond the smaller circle. Draw parallel lines through the centres of the circles.

Prob.13



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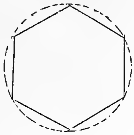

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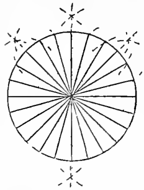

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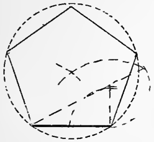
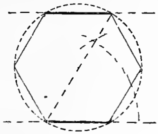
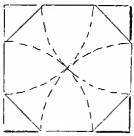


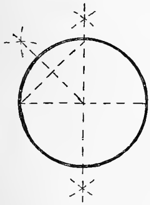
Prob.17



Prob.18



Prob.19



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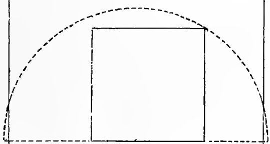

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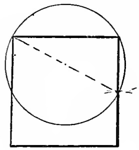

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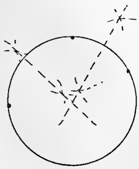
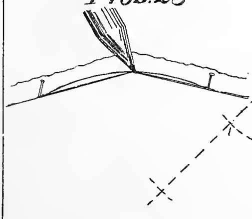
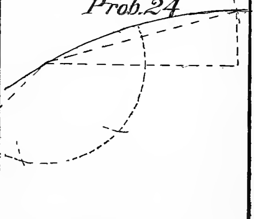


Prob.23



Prob.24



Through the points where these parallel lines intersect the circumferences draw a line, and extend it until it crosses the line passing through the centres. From this last point of intersection draw a tangent to either circle. (Prob. 26.) It will be the tangent required. Four such tangents can be drawn; two will be *interior*, and two *exterior* tangents.

PROB. 28. Draw a circle tangent to a line at a given point, and passing through another given point. (Take one point 1 in. from the line and 1½ ins. from the point of tangency.)

Draw a perpendicular to the line at the given point in it. (Prob. 2.) With the points as centres and with equal radius, draw arcs intersecting at two places, and draw a line through the points of intersection. (Prob. 22.) It will cross the perpendicular at the centre of the required circle.

PROB. 29. Draw a circle tangent to two lines and passing through a given point equally distant from them. (Take lines making an angle of 60 degrees, and a point 1 in. from vertex.)

Extend the lines until they meet, and draw a line bisecting the angle between them (Prob. 4.) Draw a perpendicular to this line through the given point (Prob. 2), and bisect the angle which the perpendicular makes with one of the sides. (Prob. 4.) The two bisecting lines will cross at the centre of the required circle.

NOTE.—If the lines cannot be made to meet on the paper, draw lines parallel to and equally distant from them (Prob. 7) and bisect the angle formed by these lines.

PROB. 30. Draw a circle tangent to three given lines. (Take one line 3 ins. long, and the others making angles of 45 and 60 degrees at its ends.)

Extend the lines until they meet, and draw lines bisecting any two of the angles formed. (Prob. 4.) These bisecting lines will cross at the centre of the required circle. Its radius can be found by drawing a perpendicular to one of the given lines. (Prob. 3.)

Two **CIRCLES ARE TANGENT** when the circumference of one passes through *but one point* in the circumference of the other. A line drawn through the centre of two tangent circles will pass through the point of tangency.

PROB. 31. Draw a circle with a given radius, tangent to a line and a given circle. (Take radius of given circle 1 in., with centre 1½ ins. from line; radius of required circle, ¾ ins.)

Draw a line parallel with the given line at a distance from it equal to the given radius (Prob. 7); and, with the centre of the given circle as a centre, and a radius equal to the radius of the given circle added to the radius of the required circle, strike an arc. It will cross the parallel line at the centre of the required circle.

PROB. 32. Draw a circle tangent to a circle and a line at a given point in the line. (Take line and circle, as in Prob 31, and point in line 2 ins. from centre of given circle.)

Draw a perpendicular to the line through the given point

(Prob. 2) and extend it on either side of the line a distance equal to the radius of the given circle. Draw a line from the extremity of the perpendicular to the centre of the circle, and draw a perpendicular to this line through its middle point. (Prob. 1.) It will cross the first perpendicular at the centre of the required circle. Two such circles can be drawn; the given circle will be tangent to the exterior of one and to the interior of the other.

PROB. 33. Draw a circle tangent to a line and to a circle at a given point in the circle. (Take line and circle, as in Prob. 31, and point in circle 1 in. from the line.)

Draw a line from the centre of the circle through the given point, and draw another through the point tangent to the circle. (Prob. 25.) Extend the tangent till it meets the given line, and draw a line bisecting the angle between them. (Prob. 4.) The bisecting line will cross the first line at the centre of the required circle. Two such circles can be drawn; the given circle will be tangent to the exterior of one and to the interior of the other.

PROB. 34. Draw a circle with a given radius, tangent to two given circles. (Take given circles 1 and 2 ins. in diameter, and centres 2 ins. apart; radius of required circle $\frac{3}{4}$ ins.)

With the centre of one circle as a centre, and with a radius equal to its radius added to the radius of the required circle, draw an arc. With the centre of the other circle as a centre, and with a radius equal to its radius added to the radius of the required circle, draw another arc. The two arcs will cross at the centre of the required circle.

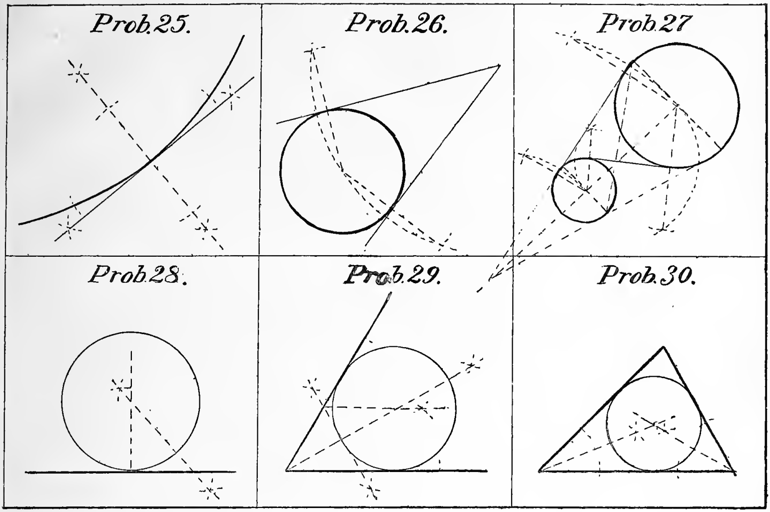
PROB. 35. Draw a circle tangent to two given circles at a given point in one circle. (Take given circles as in Prob. 34, and point in larger circle 30 degrees from a line between their centres.)

Draw a line through the given point and the centre of that circle. With the centre of that circle as a centre, draw three or four arcs crossing this line near the centre of the required circle. With the centre of the other circle as a centre, and a radius equal to its radius added to the distance from the given point to the first arc, draw an arc intersecting the first arc; then with the same centre and the same radius increased by the distance from the first arc to the second, draw an arc intersecting the second arc; then in the same way draw one intersecting the third arc and so on. With a curved ruler, Fig 16, draw a curved line through these points of intersection. It will cross the first line at the centre of the required circle. The curve is part of a hyperbola (Prob. 44) and curves towards the smaller circle.

PROB. 36. Draw a circle tangent to three given circles of different diameters. (Take centres of circles all 2 ins. apart; with radii of $\frac{1}{2}$, $\frac{3}{4}$ and 1 inch.)

Draw two curves as in Prob. 35, and they will cross each other at the centre of the required circle.

An ***ELLIPSE*** is drawn by a point moving around two other points in the same


Prob. 25.
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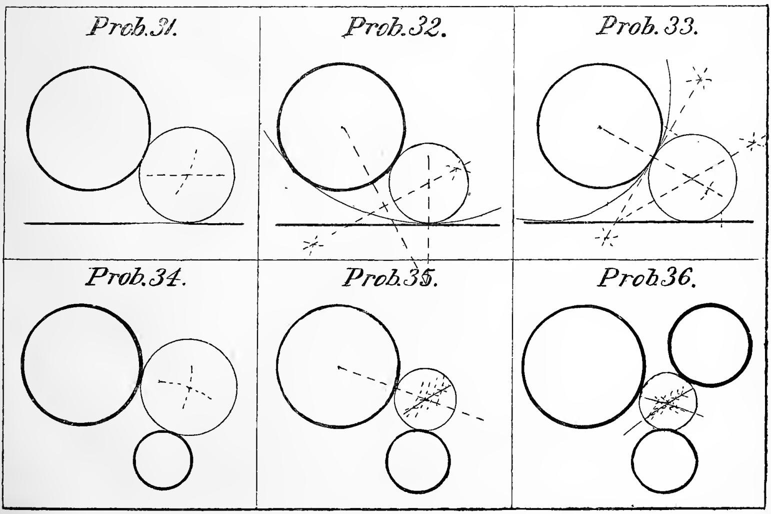

Prob. 31.
Prob. 32.
Prob. 33.
Prob. 34.
Prob. 35.
Prob. 36.

plane, so that the distance between it and one point, added to the distance between it and the other point, remains the same. The two fixed points are the *focii*. A line drawn through the focii is the *longer axis* of the ellipse, and a perpendicular to it through its middle point is the *shorter axis*.

PROB. 37. Draw an ellipse with pins and a string; the axes being given. (Take axes $1\frac{1}{2}$ and $2\frac{1}{2}$ ins.)

Draw the shorter axis. With its ends as a centres, and a radius equal to half the longer axis, draw arcs intersecting at two points. These points will be the focii. Stick pins in the focii and in one end of the shorter axis, and tie a linen thread around the three pins. Remove the pin from the shorter axis. The point of a pencil held tight against the string will draw the required ellipse.

The position of the focii and the length of the string can be computed: The distance from the point where the axes cross to the focii will be equal to the base of a right triangle; the altitude of which will be half the shorter axis, and the hypotenuse half the longer axis. The string will be equal in length to the longer axis added to the distance between the focii.

PROB. 38. Draw an ellipse by taking 3 points on a straight edge; the axes being given. (Take axis $1\frac{1}{2}$ and $2\frac{1}{2}$ ins.)

Draw the axes perpendicular to each other through their middle points. Take 3 points on the straight edge of a piece of paper or a scale, Fig. 17, and make the distance between the first and second points equal to half the shorter axis; and, between the first and third, equal to half the longer axis. Keep the second point on the longer axis, and the third point on the shorter axis. The first point will draw the required ellipse.

The straight edges of a square may be held on part of the axes, while a thin piece of wood, with pins in it for guides, is used for drawing the curve; or an instrument called a *trammel* may be used for drawing large ellipses, and an *ellipsograph* used for drawing small ones.

PROB. 39. Draw an ellipse approximately with 4 arcs; the axes being given. ($1\frac{1}{2}$ and $2\frac{1}{2}$ ins.)

Draw the axes as in Prob. 38 and from one end of the longer axis measure a distance equal to halt the shorter axis. Construct an angle of 45 degrees at this point meeting the shorter axes (Prob. 11). With this point as a centre, and half the hypotenuse of the triangle formed, as a radius, strike an arc intersecting the longer axis. With the point where the axes cross as a centre, and a radius equal to the distance from it to the further point of intersection, draw arcs crossing the axes at 4 points. Draw lines from the two points in the shorter axis through the two in the longer axis and extend them. These four points will be the centres, and the arcs will be drawn through the extremities of the axis, between the extended lines.

PROB. 40. Draw an ellipse approximately with 8 arcs; the axes being given. ($1\frac{1}{2}$ and $2\frac{1}{2}$ ins.)

Eight centres are used; two on each axis, and one in each angle between them. Draw the two axes as in Prob. 38. From an extremity of each axis draw a line, 1 and 2, parallel with the other axis and forming a rectangle on half the axes. Draw line 3 from the extremity of one axis to that of the other, diagonally across the rectangle. From the outside corner of the rectangle draw line 4 perpendicular to the diagonal and it will cross the axes at two of the required centers, *a* and *b*. With the intersection of the axes as a centre, and half the shorter axis as a radius, strike an arc intersecting the longer axis at 5. With a point half way between this point and the farther extremity of the longer axis, as a centre, draw a semicircle 6 passing through these two points. With the radius of this semicircle as a radius and the extremity of the shorter axis as a centre, strike an arc intersecting the shorter axis. With the centre *b* on the shorter axis extended, as a centre, draw arc 7 passing through the point of intersection on the shorter axis. With the extremity of the longer axis as a centre, and a radius equal to the length of the shorter axis between the centre and semicircle, as a radius, draw arc 8. It will intersect the last arc drawn at another one of the required centres *c*. Draw line 9 through the centres *b* and *c*, and line 10 through *c* and *a*. Three of the arcs will be drawn between these lines extended for one-fourth of the ellipse. The other centres and lines can be easily found from these.

PROB. 41. Draw a perpendicular and tangent to an ellipse at a given point on the curve. (Take axes $1\frac{1}{2}$ and $2\frac{1}{2}$ ins. and a point about equally distant from the ends of the axes.)

Draw a line from both focii through the given point, and bisect the angles they form (Prob. 4). One of the bisecting lines will be the required perpendicular, and the other the required tangent.

PROB. 42. Find the two axes of an ellipse; the curve only being given. (Take same sized ellipse.)

Draw any two parallel lines across the ellipse, and draw a line through the middle points of these lines. With the middle point of this line as a centre, draw a circle intersecting the curve at four points. With these four points as centres and equal radii, draw intersecting arcs and draw lines through the points of intersection. These lines will be the required axes.

A **PARABOLA** is drawn by a point moving in a plane, so that its distance from a given point remains equal to its distance from a given line. The fixed line is the *focus* and the line is the *directrix*.

PROB. 43. Draw a parabola by finding points in the curve. (Take focus $\frac{1}{2}$ in. from directrix.)

Draw lines parallel with the directrix (use T-square, Fig. 4.) With the distance from the directrix to the *first* parallel line as a radius, and the focus as a centre, draw an arc intersecting the *first* parallel line at two points. With the distance from the directrix to the *second* parallel line as a radius, and the focus as a centre, draw an arc intersecting the *second* parallel line. In the same way draw arcs intersecting the other parallel lines. These points of intersection will be points in the required curve. It may be drawn in ink, free-hand or with a curved ruler, Fig. 16, or with the compasses by finding centres and radii by trial, which will draw arcs through three of the points at a time.

A ***HYPERBOLA*** is drawn by a point moving in a plane, so that its distance from a given point remains equal to its distance from a given circle. The fixed point and the centre of the circle are the *focii.*

PROB. 44. Draw a hyperbola by finding points in the curve. (Take radius of circle 1 in.; focii $1\frac{1}{4}$ ins. apart.)

With the centre of the circle as a centre, draw a number of arcs where the required curve is to be drawn. With the distance from the circle to the *first* arc as a radius, and the other focus as a centre, draw an arc intersecting the *first* arc at two points. With the distance from the circle to the *second* arc as a radius, and the same focus as a centre, draw an arc intersecting the *second* arc. In the same way draw arcs intersecting the other arcs. These points of intersection will be points in the required curve, and it may be drawn in ink in the same way as the parabola.

A ***HELIX*** is generated by a point moving uniformly around a given line, and also moving in the direction of the line at a fixed distance from it. The given line is the *axis.* A corkscrew, a wire spring, and screw-threads are illustrations of a helix. Two views are necessary in a drawing to show a helix. The bottom view will be a circle and the side view will be reversed curves.

PROB. 45. Draw a helix with a given diameter and a given rise per revolution. (Take diameter 2 ins. and rise 1 in. per revolution.)

Draw a circle for the bottom view. From the centre of this circle draw the axis for the side view. Measure the rise per revolution on this axis. Divide the rise into 24 equal parts (Prob. 9 or with Scale, Fig. 17), and draw lines through these points of division perpendicular to the axis (use T-square). Divide half the circle unto 12 equal parts (Prob. 15). From the *first* point of division on the circle, draw a line parallel with the axis (use a triangle, Fig. 5) and intersecting the *first* line which is perpendicular to the axis; from the *second* point of division on the circle draw another parallel line intersecting the *second* perpendicular line; from the *third* draw one intersecting the *third*, and so on. These points of intersection will be points in the required curve. The curve may be traced on a piece of firm card-board or thin wood, and trimmed out smoothly with a keen pen-knife. With this curve a great many revolutions of the helix may be made neatly with ink in the drawing.

A ***SPIRAL*** is drawn by a point moving uniformly around a given point in the same plane, and moving away from it at the same time. A watch spring is an illustration of a spiral. This curve is used for drawing cams.

PROB. 46. Draw a spiral moving uniformly from the centre at given rate. (Take 1 in. per revolution.)

Draw lines through the centre, making angles of 15 degrees (Prob. 15). Measure the distance per revolution on one of these lines, and divide the distance into 24 equal parts. With the point as a centre and a radius equal to the distance to the *first* point of division, draw an arc intersecting one of the lines; with a radius equal to the distance to the *second* point of division, draw an arc intersecting the next line; from the *third* point of division draw an arc intersecting the third line, and so on. These points of division will be points in the required spiral. It may be drawn in ink with the compasses by finding centres and radii by trial, which will draw arcs through three of the points at a time.

The ***INVOLUTE*** of a circle is drawn by a point in a straight line which rolls on the circle. A pencil fastened to a string, which is kept stretched as it is unwound from a spool, will draw an involute of a circle. This curve is used for drawing the teeth of gear-wheels.

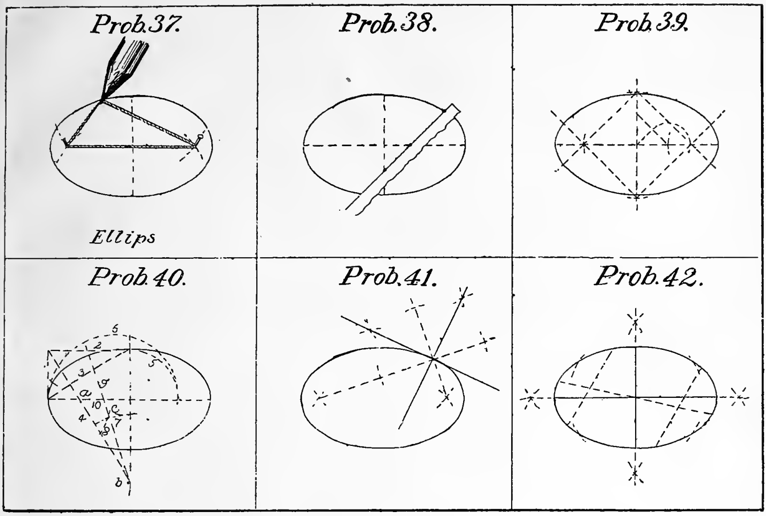
PROB. 47. Draw the involute of a given circle. (Take circle 1 in. in diameter.)

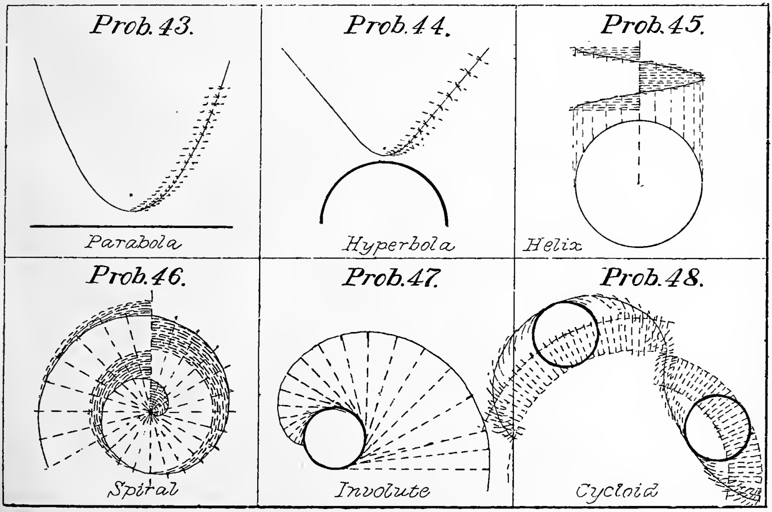
Divide the circle into 24 equal parts (Prob. 15), and draw lines tangent to the circle at these points of division (with triangles, Fig. 5). With one point of division as a centre, and with a radius equal to the length of the arc between the points of division, draw an arc from the circle to the first tangent line; with the next point of division as a centre, draw an arc from the end of this arc to the next tangent line; with the third point as a centre, continue the curve to the third tangent line, and so on. When 24 divisions are taken the curve will be more accurate, if these centres are taken on a circle whose diameter is about 1-94 greater than the diameter of the given circle.

A ***CYCLOID*** is drawn by a point in the circumference of a circle which rolls on a line. If the generating circle rolls on the outside of a circle, the curve is an *epicycloid;* and, if it rolls on the inside, it is a *hypocycloid.* The circle on which it rolls is the *pitch* circle. These curves are used for drawing the teeth of gear-wheels.

PROB. 48. Draw epi- and hypocycloids; the diameters of the circles being given. (Take diameter of pitch circle 4 ins., and diameter of generating circles 1 in., and let both curves start from the same point.)

Draw part of the pitch circle, and draw a generating circle tangent to it on the outside, and draw another tangent on the inside. With the centre of the pitch circle as a centre, draw arcs passing through the centres of the generating circles. Make a number of equal divisions on the pitch circle, and draw lines through them from the centre and intersecting the two arcs. With these points of intersection on the arcs as centres, draw *parts* of the generating circle where the curve is to be drawn. With a point of division on the pitch circle as a centre, and the distance between the points of division as a radius, draw an arc from the pitch circle to the nearest *part* of the generating circle; with the next point of division as a centre, draw an arc which will continue the curve to the next part of the gen-


Prob.37.
Ellips
Prob.38.
Prob.39.
Prob.40.
Prob.41.
Prob.42.


Prob.43.
Parabola
Prob.44.
Hyperbola
Prob.45.
Helix
Prob.46.
Spiral
Prob.47.
Involute
Prob.48.
Cycloid

erating circle; with the third point as a centre, continue the curve to the third part of the generating circle, and so on. Unless the points of division are quite small, the radii of the arcs will be slightly too great.

CHAPTER III.

PROJECTION AND DEVELOPMENT OF SOLIDS.

The problems in this chapter are given to show how the different views of an object are arranged and how patterns for curved surfaces are cut. Two views are usually sufficient to represent the dimensions of a solid. The top views, when drawn, are placed exactly above the principal views; the bottom views, exactly below them; and the side and end views, near the sides and ends which they represent.

In the figures, the outlines are drawn heavy on the bottom and right to shade them; and surfaces, which are cut by a plane, are distinguished by having oblique parallel lines drawn across them, called section-lines or hatching.

A good drawing board, T-square, triangle, and dividers or measuring scale, are essential for making these and similar drawings. The student should copy the developments on heavy paper and cut them out and bend them into shape to test the accuracy of his drawings.

A point is **PROJECTED** on a line or a plane when a straight line is drawn from it to the line or plane. The line drawn is the *projecting line*, and the point where the projecting line intersects the other line or pierces the plane is the *projection* of the given point on that line or plane.

A surface is **DEVELOPED** when it is removed from a solid and spead out on a plane.

A **PRISM** is a solid with two equal parallel *bases* which are polygons, and with *faces* (sides) which are all parallel to the same line. When the bases are regular polygons, the prism is *regular;* and when the faces are perpendicular to the bases, it is a *right* prism.

PROB. 49. Draw three views and the development of the faces of a regular 6-sided prism with its top cut away by a plane which makes an angle of 45 degrees with its base. (Take prism 1 inch across corners and 2 ins. high.)

Draw a regular hexagon for the bottom view. Project vertical lines upwards from the corners of the bottom view, as indicated in the figure by dotted lines with arrow points. These projecting lines determine the edges in the view which is above. Horizontal lines projected to the right from the corners determine the edges in the view which is at the right of the bottom view. At any convenient distance from the bottom view, draw straight lines across the projecting lines perpendicular to them, for the base in the other two views. Draw a line across the upper view at the required distance from the base and making the required angle with it. This last line represents the cutting plane. It crosses all the edges of the prism, and the length of each edge may be measured, and the corresponding edges in the view at the right made the same length.

The development of the faces may be drawn by drawing parallel lines, as far apart as the edges really are on the prism itself, and making each line equal in length to the corresponding edge of the prism. The distance between the parallel lines is equal to the distance between the corners in the bottom view. The length of each one may be projected from the upper view, as indicated in the figure by dotted lines.

PROB. 50. Draw three views and the development of the faces of a regular 8-sided prism with its bottom cut away by a plane which makes an angle of 30 degrees with its base. (Take prism 1 inch across corners and 2 ins. high.)

Draw a regular octagon for the top view. Project lines downwards from the corners and they will determine the edges in the view which is below it. Draw a line across this view making the required angle with the base. It will cut all the edges, and the length of each edge may be measured and the view at the right and the development of the faces drawn as in Prob. 49.

PROB. 51. Draw three views and the development of the faces of a regular 24-sided prism with its top cut away by a plane which makes an angle of 60 degrees with its base. (Take prism 1 inch across corners and 2 ins. high.)

Draw a regular polygon of 24 sides for the bottom view. Project lines from its corners for the edges in the other two views. Draw a line across the upper view, making the required angle with its base, and complete the other view and the development as in Prob. 49.

A **CYLINDER** is a solid with two equal, parallel *bases* which are circles or other plane curves, and with a curved lateral surface which is generated by a straight line moving parallel with itself and constantly touching the curves. Any position of the generating line is an *element* of the cylinder. The line joining the centres of the bases of a regular prism or a cylinder is its *axis*.

The *volume* of a prism or cylinder is equal to its base multiplied by its altitude (height).

PROB. 52. Draw three views and the development of the whole surface of a circular cylinder with its bottom cut away by a plane which makes an angle of 45 degrees with its axis. (Take cylinder 1 inch in diam. and 2 ins. high.)

Draw a circle for the top view and divide it into 24, 48, or any number of equal parts. Project lines drawn from these points of division and they will be elements of the cylinder in the lower view, which are equally distant from each other. Draw the base and draw a line across the lower view making the required angle with its axis. It will cut all the elements, and the length of each one may be measured, and the other view and the development of the surface drawn as in Prob. 51, with the exception that the width across the development is equal to the circumference of the base, and the outline is a curve traced through the extremities of the elements. The outline of the surface which is cut by the plane is known to be an ellipse. Its shorter axis is equal to the diameter of the cylinder and its longer axis is equal to the line drawn across the lower view.

A **PYRAMID** is a solid with one *base*, and triangular *faces* which meet at a point called the *apex* or *vertex*. When the base is a regular polygon and the faces are equal, the pyramid is *regular*.

PROB. 53. Draw three views and the development of the faces of a regular 6-sided pyramid with its top cut away by a plane which makes an angle of 45 degrees with its base. (Take pyramid 1½ ins. across corners of base and 2 ins. high with ½ inch of top cut off.)

Draw a regular hexagon for the outline of the base in the top view, and draw a horizontal line at a convenient distance below it for the base in the view below it. Project the corners of the base from the top view down to the base in the lower view. Locate the apex in the lower view at the required distance above the base and exactly below the centre of the top view. Draw lines from the corners of the base to the apex in both views. These lines are the edges of the pyramid. Draw a line across the lower view making the required angle with the base. It will cut all the edges. Project the points where they are cut up to the corresponding edges in the top view, as indicated in the figure by dotted lines. Draw lines joining these points on the edges in the top view, and its outline will be completed. The other view may be projected to the right of the lower view. The distance across its base is equal to the vertical distance across the top view; and the base, apex, and points where the edges are cut, can be projected from the lower view, as indicated in the figure by dotted lines.

The development of the faces of the complete pyramid is 6 equal isosceles triangles. Their bases are equal to the distance between the corners of the base of the pyramid in the top view, and their sides are equal to the true length of the edges of the pyramid. The development of the faces which are partly cut away may be drawn by finding the length of the remaining part of each edge. The edges on the right and left, in the lower view to the left in the figure, are parallel with the surface of the paper and are therefore drawn in their true lengths; but the other edges come towards the front at one end and are therefore shorter on the drawing than they really are on the pyramid. Their true lengths may be found by projecting their extremities to the line at the right or left, as shown in the figure by the dotted lines with arrow-points. The true length of the remaining part of each edge may be measured off on the corresponding edge of the development of the complete pyramid, and the required development completed as in Prob. 49.

PROB. 54. Draw three views and the development of the faces of a regular 8-sided pyramid with its bottom cut away by a plane which makes an angle of 30 degrees with its base. (Take pyramid 1½ ins. across corners of base and 2 ins. high.)

Draw a regular octagon for the outline of the base in the top view of the complete pyramid, and draw a horizontal line below it for the base in the side view below it. Draw the corners, as in Prob. 53, and draw a line across the side view making the required angle with the base. This line will cut all the edges, and the points where they are cut may be projected up to the corresponding edges in the top view. The view at the right and the development of the faces may be drawn as in Prob. 53.

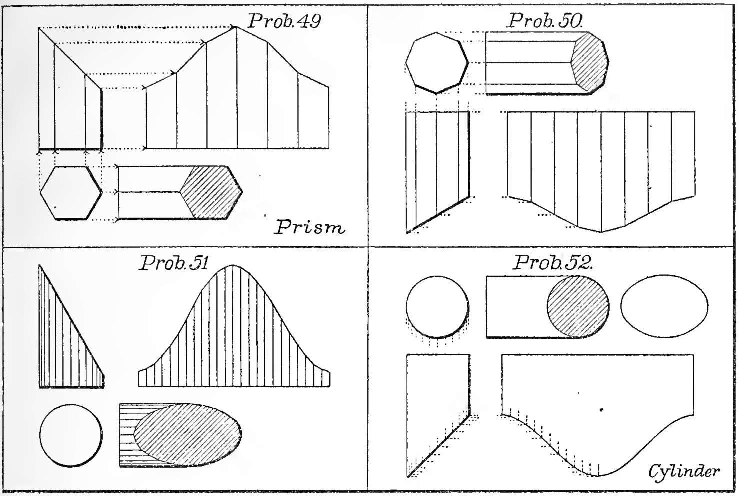
A **CONE** is a solid with one base which is a circle or other plane curve, and with a curved lateral surface generated by a straight line passing through a fixed point and constantly touching the curve. Any position of the generating line is an *element* of the cone. The line joining the centre of the base with the apex of a regular prism or a cone is its *axis*.

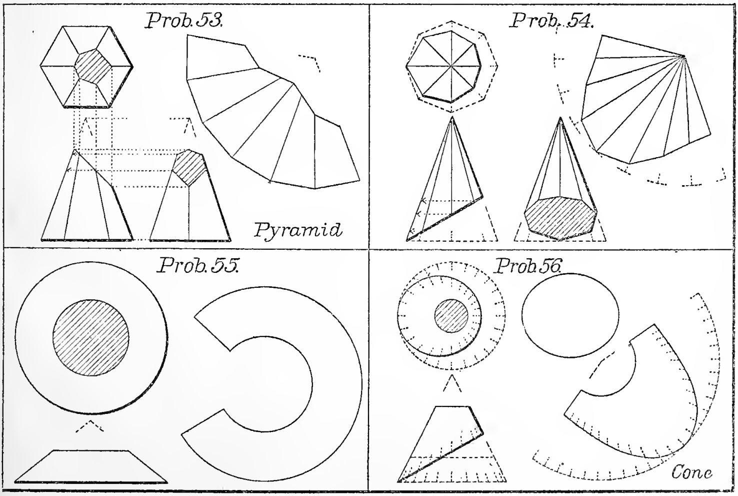
The volume of a pyramid or cone is equal to its base multiplied by one-third of its altitude.

PROB. 55. Draw two views and the development of the surface of a right circular cone, which has its top cut away by a plane parallel with its base. (Take cone 2½ ins. across base and 1 in. high with ½ in. of top cut off.)

Draw a circle for the outline of the base in the top view, and draw a horizontal line below it equal in length to the diameter of the circle, for the base in the lower view. Locate the apex at the required distance above the base in the lower view, and draw elements to it from the extremities of the base. Draw a line across the lower view parallel with the base and cutting away the required amount of the top. The outline of the cut surface in the top view is a circle with its diameter equal to the line drawn across the lower view.

The development of the surface of a cone is a *sector* of a circle whose radius is equal to an element of the cone, and the arc is equal in length to the circumference of the base of the cone. The part cut away from


Prob. 49
Prism
Prob. 50.
Prob. 51
Prob. 52.
Cylinder


Prob. 53.
Pyramid
Prob. 54.
Prob. 55.
Prob. 56.
Cone

the development is drawn with the same centre and with a radius equal to an element of the part which is cut away from the cone.

When the elements of a cone form angles of 60 degrees at its base and apex, the development is a semicircle.

NOTE.—This problem is used in cutting out flanges and such tin utensils as funnels, dippers, strainers, pails, coffee-pots, etc.

PROB. 56. Draw two views and the development of the whole surface of a cone which has its top cut away by a plane parallel with its base, and its bottom cut away by a plane which makes an angle of 30 degrees with its base. (Take cone 2 ins. across base and 2 ins. high with ½ in. of top cut off.)

Draw two views and the development of the *whole* cone as in Prob. 55. Divide the circle in the top view into 24, 48, or any number of equal parts and project these points of division down to the base in the lower view. From these points draw lines to the apex in both views. These lines are equally distant elements of the cone. Draw a horizontal line cutting off the top and an oblique line cutting off the bottom in the lower view. The two lines cut all the elements and the length of each one may be found and the development completed, as in Probs. 54 and 55, by tracing a curve through the extremities of the elements.

The surface of the cone which is cut by the oblique plane is known to be an ellipse. Its longer axis is equal to the oblique line which is drawn across this cone, and its shorter axis is equal to the diameter of the cone at the middle of the oblique line, as indicated in the figure by the broken horizontal line.

Solids **INTERSECT** when one pierces the other, or when they are cut so as to fit each other or become united. If they only touch they are *tangent* but not intersecting.

PROB. 57. Draw three views and the development of the whole surface of a cylinder intersected by a larger cylinder which is perpendicular to it. (Take diam. of one cylinder 1 in. and the other 1¼ ins.)

Draw three views of the smaller cylinder as in Prob. 52. In the lower view, draw a circle for the end view of the larger cylinder. Draw equally distant elements on the smaller cylinder, as in Prob. 52. These elements are all intersected by the larger circle, and the length of each one may be measured and the other two views and the development of the lateral surface completed as shown in the figure.

The development of the surface, which fits against the larger cylinder, is eliptical, and points in its outline may be found thus: Draw a straight line through the centre of the top view of the smaller cylinder and extend it. Take any convenient part of this extended line and make it equal in length to the part of the larger circle, in the lower view, against which the smaller cylinder fits. Make divisions on it equal to the corresponding divisions which are made on that part of the larger circle by the elements of the smaller cylinder. Draw perpendicular lines through these points of division. Project the nearest points of division, which are on the top view of the smaller cylinder, to the nearest perpendicular line; the next-nearest to the next line, and so on. The outline may be traced through these points of intersection.

PROB. 58. Draw three views and the development of the lateral surface of a cylinder intersected obliquely by a larger cylinder. (Take diam. of one cylinder 1 in. and the other 1⅛ ins. with their axes making an angle of 60 degrees with each other.)

Draw three views of the smaller cylinder and draw equally distant elements on it as in Prob. 52. Draw the larger cylinder making the required angle with the smaller one in the lower view. The elements drawn on the smaller cylinder are intersected by elements of the larger cylinder, and these elements and the points of intersection may be found thus: Draw arcs, in the upper and lower views, with radii equal to the radius of the larger cylinder, and their centres on the axis of the larger cylinder. In the top view, draw elements of the larger cylinder through the points of division on the top view of the smaller cylinder, and extend them to the arc. Measure off these points of division, which are made on the arc in the upper view, on the corresponding part of the arc in the lower view; and draw elements of the larger cylinder through them. These elements will intersect the elements drawn on the smaller cylinder. The same elements, which intersect in the top view, intersect in the lower one, and these points of intersection determine the length of each element of the smaller cylinder. The third view and the development may be drawn as in Prob. 57.

PROB. 59. Draw three views and the development of the surface of a regular 6-sided prism intersected by a cone whose axis coincides with the axis of the prism. (Take prism 1 in. across corners, and elements of cone forming an angle of 60 degrees at its apex.)

Draw three views of a regular 6-sided prism, as in Prob. 49. Locate the apex of the cone on the axis of the prism at the same distance from the base in both the side views. From these points draw elements of the cone forming equal angles with the axis of the prism. They intersect the *edges* of the prism in one view, and the *middle of the faces* in the other view. As all the edges are the same length, and all the faces go up the same distance in the middle, their extremities may be located and curves drawn through them, as shown in the figure. These curves are known to be hyperbolas (Prob. 44) but they are usually drawn as arcs of circles.

NOTE.—This problem is given to show how the chamfer is drawn on nuts and bolt-heads.

A **POLYHEDRON** is a solid with four or more faces. When the faces are equal, regular, polygons and the vertices are equal, the polyhedron is *regular*. There are but five regular polyhedrons. Three of them have respectively four, eight, and twenty faces, which are triangles; one (the cube) has six faces, which are squares; and one has twelve faces which are pentagons.

PROB. 60. Draw three views and the development of the faces of a regular polyhedron of 20 sides intersected by a square prism. (Take edges of polyhedron ⅞ ins. and prism ¾ ins. square.)

This problem is intended as an exercise which will give the student a clearer understanding of how the different views of an object are projected and arranged. He should first make the polyhedron by drawing the development, shown in the figure, on card-board, and cutting through the full lines and half through the dotted lines. The edges may then be brought together and held in place by pasting strips of paper over them. He can then study the problem out for himself.

Any point in the *top* view must be exactly *above* the same point in the view *below* it; and any point in one side view must be exactly to the right or left of the same point in the other side view. A side view is placed opposite and near the side it represents.

A **SPHERE** is a solid with a curved surface, every point of which is equally distant from the *centre*. The distance between the surface and the centre is the radius of the sphere; and the distance through the centre is the diameter.

A *zone* is part of the surface of a sphere between two circles which lie in parallel planes. A *lune* is part of the surface between two semicircles which meet at the extremities of a diameter.

The surface of a sphere is equal to its diameter multiplied by its circumference; and the volume of a sphere is equal to its surface multiplied by one-third its radius.

PROB. 61. Draw three views and the development of the surface of a regular 6-sided prism intersected by a sphere. (Take sphere 1½ ins. in diam. and prism 1 inch across corners with its axis passing through the centre of the sphere.)

The outline of the sphere in each view is a circle. At the centre of the top view draw a regular hexagon for the top view of the prism. Project lines from the corners of the hexagon to the other two views of the sphere. Draw the top base of the prism on each of the side views at an equal distance from the centre of the sphere. The outline of the sphere intersects the edges of the prism in one view, and the middle of the faces in the other. These points of intersection may be measured from the base and curves drawn through them as in Prob. 59.

PROB. 62. Draw three views and the development of the surface of a cylinder intersected by a sphere. (Take sphere 1½ ins. in diam. and cylinder 1 in. with its axis passing 3-16 ins. from centre of sphere.)

Draw three views of the sphere as in Prob. 61. Draw a circle in the top view, for the top view of the cylinder. Place the centre of this circle at the required distance to the right or left of the centre of the sphere. Draw the other two views of the cylinder and divide it into equally distant elements, as in the previous problems. Draw horizontal lines on the top view of the sphere through the points of division on the top view of the cylinder. These lines will be semicircles in the view below the top view, and will intersect the same elements which they intersect in the top view. These last points of intersection determine the length of each element, and the other view and the development may be completed as in the previous problems.

PROB. 63. Draw the development of the surface of a sphere, approximately, by dividing it into lunes. (Take sphere 1½ ins. in diam. and divide surface into 12 lunes.)

The length of a lune is equal to one-half the circumference of the sphere, and the width of it at its centre is equal to one of the parts into which the circumference is divided.

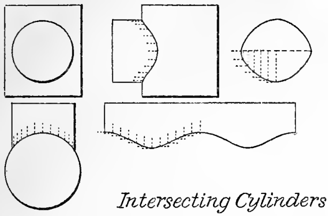
Draw a line equal in length to the circumference of the sphere and divide it into the required number of equal parts. Draw a perpendicular line half way between two of these points of division and extend it on both sides a distance equal to one-fourth the circumference of the sphere. Draw arcs passing through the extremities of the perpendicular line and the two points of division which are on each side of it. These arcs will be very nearly the outlines of the lune.

The lunes may all be drawn and cut out and fitted over a solid sphere and pressed or beaten into shape. In practice it is difficult to fasten all the points of the lunes together and it is therefore better to cut them off and fill out the place with a circular piece.

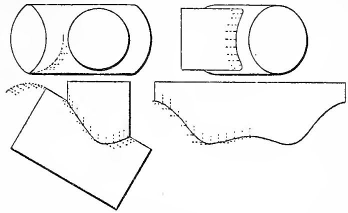
PROB. 64. Draw the development of the surface of a sphere approximately by dividing it into zones. (Take sphere 1½ ins. in diam. and divide surface into zones 15 degrees wide.)

Each zone may be treated as part of a cone, and its development drawn as in Prob. 55. The radius, with which the development of a zone is drawn, is an element of the complete cone, which would be tangent to the sphere at the centre of that zone.

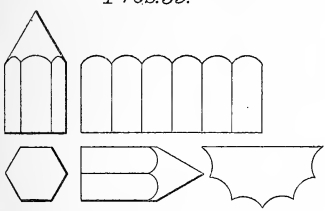
Prob. 57.



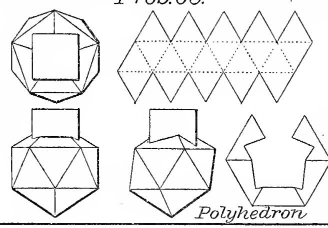
Prob. 58.



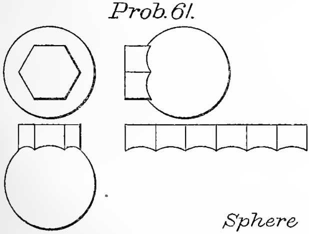
Prob. 59.



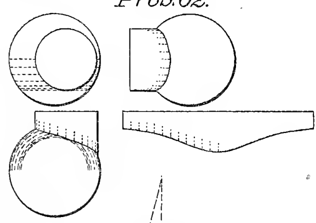
Prob. 60.



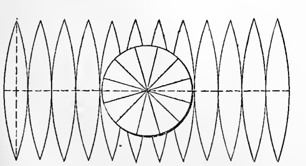
Prob. 61.



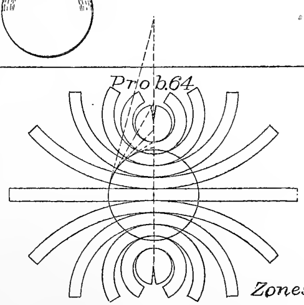
Prob. 62.



Prob. 63.



Prob. 64.



Draw a circle equal to the circumference of the sphere, and draw a vertical line through its centre. From one of the points where this line intersects the circle, divide one-fourth of the circle into 6 equal parts. Draw tangents to the circle from these points of division to the vertical line. These tangents form angles with the vertical line of 15, 30, 45, 60 and 75 degrees respectively. The width of each zone is equal to the distance between the points of division. The development of the zone at the centre is straight, and is equal in length to the circumference of the sphere. The *outside* arcs of the developments of the two zones next to it are drawn with a radius equal to the longest tangent added to half the width of the zone. Their length is made equal to the circumference of the sphere. The *inside* arcs and ends of the developments are determined as in Prob. 55. The developments of the next two zones are semicircles and their outside arcs are drawn with a radius equal to the next longest tangent added to half the width of the zone. The *outside* arcs of the next two are equal in length to the *inside* arcs of the semicircular ones. The outside arcs of the next smaller are equal in length to the inside arcs of the greater ones next to them. The openings in the smallest zones are drawn to their centres.

If the zones are all drawn touching each other, as in the figure, the distance between the centres of the smallest zones is equal to one-half the circumference of the sphere.

If the draftsman has a measuring scale by which he can measure decimally, and a protractor by which he can lay off angles; he can make the width of each zone equal to .262 times the radius of the sphere and the tangent lines equal respectively to 3.732, 1.732, 1.000, .577 and .268 times the radius. The middle development is straight and equal in length to 6.283 times the radius. The two developments next to it contain 93 1/6 degrees; the next two 180 degrees (semicircles). The next two have 105 degrees cut out; the next two 48 1/4, and the two smallest 12 1/4 cut out.