THE

INDUCTION MOTOR

ITS THEORY AND DESIGN

SET FORTH BY A PRACTICAL METHOD
OF CALCULATION

BY

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AUTHOR'S PREFACE.

WHEN we decided to publish this work, we were aware that several authors had already given attention to the subject of making calculations for polyphase non-synchronous (induction) motors, and that some of them had even given complete theories on the subject. We also knew that for a few years past many distinguished professors have been discoursing at length to their pupils on these machines.

It is not, therefore, for those engineers who have learned to design induction motors with these authors or with these professors, that this book has been written.

We have wished to come to the assistance of those who have not had occasion to take special courses, and who, owing to the lack of sufficiently extended knowledge, have not derived very much profit from all that has appeared in the scientific papers.

We have, therefore, endeavored to give a complete study of the polyphase non-synchronous (induction) motor, and to explain at length all the peculiarities of its operation, while remaining within the bounds of elementary mathematics.

The reader will find in this treatise a few new deductions and formulæ, such as, for instance, that which enables one to estimate the magnetizing action of wave windings, or, again, those which are necessary for defining judiciously the dimensions of the short-circuiting rings for rotors of the squirrel-cage type.

We also believe that we have been one of the first to show, by a practical example, the material influence of the thickness of the air-gap on the value of the magnetic leakage factor.

THE AUTHOR.

LYONS, Sept. 11, 1901.

TRANSLATOR'S PREFACE.

THE preparation of this translation, which was undertaken through friendship, has been an agreeable task; for the translator experienced an increasing pleasure, as the translation progressed, in becoming more familiar with the very original, highly ingenious, yet simple and comprehensive manner, in which the induction motor is discussed, and the features and peculiarities of its design are analyzed in this work by the author.

M. Boy de la Tour showed that he believed strongly in the induction motor, in the interesting remarks which he contributed to the animated discussion on electric motors, in August, 1901, at the Buffalo convention of the American Institute of Electrical Engineers, at which he was present, as a guest of the Institute. The present work furnishes convincing evidence of the fact that his faith and confidence in the induction motor are based upon a comprehensive and authoritative knowledge of this motor, both theoretically and practically. His contributions to the French electrical periodicals, during the last three years, on various topics, especially those related to alternating currents and alternating current motors, have been highly appreciated and have won him an enviable reputation in Europe. The translator is glad to find so good an opportunity as is offered by this work, to present M. de la Tour to a large circle of readers whose insufficient knowledge of the French language has prevented them hitherto from fully appreciating him and his work.

The French edition contains only eight chapters, which are the same as Chapters I. to VIII. of the translation. The additional chapter, (IX.), on the Heyland Motor has, at the suggestion of the translator, been especially written by the author, for the American edition. This chapter is a valuable addition to the book, dealing, as it does, opportunely, as well as intelligibly and intelligently, with a new development of the induction motor, which has excited a great deal of interest, and which is attracting considerable attention at present.

The translator believes in respecting the individuality and originality of an author; and, for this reason, he has endeavored to retain as perfectly as possible, those features of the original text which characterize the methods of thought, analysis, and expression, of the author.

One of the characteristic features of the original work, which is retained in the translation, is that none of the equations are numbered. The author does not hesitate to repeat the same equation as many times as necessary to bring it into the text at each point where it is referred to, thereby making each portion of the analysis, and each detail of the reasoning, self-contained and complete. These repetitions undoubtedly increase the volume of the book; but it must be admitted that they are of great convenience to the reader, as they obviate entirely the necessity of referring to other parts of the book for equations and formulæ, which is necessary when the same équations appear only once and are designated by a reference letter or number.

In constructing the equations themselves, the author prefers to let each separate factor or quantity entering into the equation, stand by itself, and retain its individuality, so to speak, instead of arranging the terms or factors in the simplest form. This peculiarity has the effect of making the equations longer and apparently more complicated; but it has the great advantage, which should not be underestimated, of enabling the reader to keep in mind, with much less effort, the physical significance or meaning of each factor. In this way, both the analytical and physical meanings of the equation are apt to be more easily grasped and retained.

The arrangement of the text in the form of short paragraphs, adopted by the author, is another characteristic feature which is retained in the translation. In translating the French text itself, the translator has endeavored, when a choice of words and phrases was possible, to adopt a wording giving the best idiomatic English phraseology compatible with the retention and the rendering of the literal meaning of the original text. A paraphrase translation was found desirable, however, and was adopted, in a very few cases, as the best way of conveying the author's meaning. In a few cases an additional word, sometimes a supplementing phrase, has been found desirable, and has been added to the original text, so as to make the meaning more clear, or to obviate ambiguity. Thus the words "axial" and "radial" have been interpolated into the text (usually in parentheses), at certain points, to give exactness to certain terms, such as, "length," "width," or "depth," which admit of different uses. Changes have also been made, where necessary, in the equations, so as to express magnetic fluxes in maxwells, and magnetic densities in gausses.

The translator has corrected various clerical or typographical errors found in the French edition, both in the text and in the equations.

The errors found in some of the diagrams, notably, Figs. 54, 55, 56, have also been corrected. The demonstration given in the foot-note on pp. 114 and 115 has been partly re-written. Additional short paragraphs have been added to the original text in Chapter III., and also in Chapter VII., to call attention to the "torque" system of rotative force units, and also to give modified formulæ partly based on non-metric units, such as, for instance, formulæ for expressing tangential effort, when dimensions are to be taken in inches instead of centimetres, and weights are to be taken in pounds instead of kilograms. It is thought that these formulæ may, in some cases, be more convenient to American readers than the original formulæ, based upon metric units throughout.

The translator soon discovered that the causes of ambiguity sometimes lay, to a great extent, in the peculiar meaning ascribed, either by convention or careless use, to certain English words used in a technical sense. He has thought that it would be opportune, and might serve to elucidate the English text, and, at the same time, give a key to the more exact meaning of the original French text, to say a word of explanation, and, in some cases, to utter a word of protest, regarding some of the terms and definitions, which are in current use in this country. These terms and definitions which are collated together in the Introduction, constitute a sort of glossary of the meanings and definitions of certain words used both in the French text and in the English translation.

Some of these notes were prepared especially for the convenience and benefit of students; and the notes on "Nomenclature," as a whole, are published as a contribution, by the translator, of data and opinions relative to terminology. There is urgent need of reform and of a movement in favor of correct and consistent terminology.

An explanatory note on "Notation" and a list of the principal symbols used in the book have also been included in the Introduction.

C. O. M.

NEW YORK, Feb. 2, 1903.

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INTRODUCTION.

BY THE TRANSLATOR.

A. NOMENCLATURE.

Air-Gap. There is, unfortunately, some ambiguity in the terms used to designate the thickness, measured radially, of the cylindrical layer of air interposed between the magnetic field poles and the armature of a Dynamo or Motor, and designated in English by the term "air-gap" (in French, "entrefer"). The author usually uses the word "depth," sometimes the word "thickness," to designate this *radial distance across* the "air-gap," whereas this distance is generally termed the "length," of the "air-gap" in this country. This use of the term "length," while it may be appropriate in the sense that it implies the length of magnetic circuit or magnetic path for the lines of magnetic force passing through the "air-gap," is unfortunate, inasmuch as the term "length" could also be used, and more logically (as it is, in fact, actually used by the author), to designate the mean circumferential distance measured over the periphery of the armature. It would seem almost as inappropriate to use the term "length" in referring to the distance ACROSS the air-gap as it would in referring to the distance across a river, a canyon, a chasm, or a valley, which latter are, so to speak, "air-gaps" of a certain kind. The word "length," used in connection with "air-gaps" of this kind, would certainly never be understood to mean the distance across. One would speak of the "distance across," or the "width" or "breadth" of the channel or gap between the two sides; and the "length" would invariably refer to the distance measured along a line midway between the two sides. In justice to the author, who has used what is at least a more logical term for the distance across the air-gap, the translator has not accepted the term "length" as an unqualified and satisfactory term for "distance across," but has tolerated it as a doubtful "substitute." The terms used by the author, meaning "depth" or "thickness," as applied to the air-gap, have been rendered in English, in the translation, by the dual expression "*radial distance across*" or "*length*" of the air-gap, the first definition being regarded as the more logical and preferred one, and the second being occasionally introduced, or rather retained, out of deference to the conventional use of the term "length" in this country. It is hoped that the use of the word "length," in this sense, will be discouraged and will in time disappear.

The (radial) "distance across the air-gap" is, in every case, designated, in the book, by the letter "d."

The translator is of the opinion that if it is desired to use a single word to express this dimension of the air-gap, the word "thickness," much used by the author, is, all things considered, preferable to any other. This word has been retained in the translation, in some cases.

The very fact that the term "length" has been used in this country, as just noted, in a conventional, though improper sense, for the "distance across the air-gap," has precluded the translator from using this term in its legitimate, proper sense, as it is used by the author, for the distance measured circumferentially around the air-gap. For this distance the translator has used the term "circumferential distance."

The translator would suggest, as a suitable term for this dimension of the air-gap, the word "development," whose geometrical significance is so definite and well understood as to leave no room for misunderstanding or ambiguity.

The third dimension of the air-gap, namely, the distance measured between one side and the other side of the magnetic core, on a line parallel with the centre of the motor shaft, has been variously termed, both in the original and in the translation, the "axial length," the "axial width," or the "axial dimension" of the air-gap, all of which terms have their partisans. This dimension is, in every case, designated by the letter "L," when introduced in the equations.

If the terms "thickness" and "development" were to be adopted for the other two dimensions of the air-gap, the word "depth," or the word "breadth," would seem to be a logical and satisfactory term for the axial dimension.

Armature and Field Magnet. These two terms, which have a perfectly definite significance and application in the case of direct-current motors, become, unfortunately, quite ambiguous and objectionable in the case of induction motors.

The portion of an induction motor which is directly acted upon by the polyphase currents sent into it, to produce the rotating magnetic field, and to supply the electric power applied to the motor, is not necessarily the "field magnet;" and the portion on which the rotating magnetic field operates, inductively, to produce mechanical motion, is not necessarily the "armature." Indeed, the reverse is more likely the case, if the terms "field magnet" and "armature" designate the same electromagnetic organizations and functions as in the case of direct current Dynamos and Motors. In French, the terms "inducteur" and "induit," used by the author, though having, conventionally, the meanings, respectively, of "field magnet" and "armature," may be still applicable to induction motors, because they also have literal meanings (i.e., "*inducing*" and "*induced*," respectively), which define appropriately enough the functions and relations of the two kinds of windings of an induction motor. In English, however, the terms "field magnet" and "armature" can have no such extension of meaning; and their use in connection with induction motors is, consequently, a source of confusion and misunderstanding. For this reason, these terms have been avoided as far as possible in the translation. It being the general practice among manufacturers, at the present time, to make the "inducing" portion stationary, and to make the "induced" portion movable, in induction motors, the terms "STATOR" and "ROTOR" which are already in general use in this country, have, though scarcely used by the author, been frequently used in the translation for designating the "inducing" and "induced" portions, respectively. The author does not mention any exceptions to this general rule, and, consequently, the terms "stator" and "rotor" would be perfectly correct, throughout the book, when used in the sense hereinabove indicated.

The term "primary" (core, circuit, or winding), applied by the author to the *inducing* portion (which is usually the stator) of the motor, and the term "secondary" applied to the *induced* portion (usually the rotor) are both consistent and conventional, by analogy with the corresponding parts of a transformer, and are in general accepted use, as such, in this country; consequently, these terms have been retained in the translation, and they have even been used in many cases in preference to "stator" and "rotor."

Armature Reaction. This term, although originally intended to designate the magnetic field-distortion phenomena occurring in direct-current dynamos and motors only, has been found sufficiently definite to describe the analogous phenomena occurring in alternators. Consequently, the term, which is that used by the author, has been retained in the translation to designate also the electromagnetic reactions taking place, in induction motors, between the primary and secondary cores, in consequence of the magnetizing effects of the primary and secondary currents. The idea conveyed by the term is equally definite whether we look upon the stator or the rotor as being the true analogue of the "armature" of a direct-current machine. The term "rotor reaction," used in one or two cases by the author, was retained in the translation as being also allowable, although the term "armature reaction" was deemed preferable.

Current. There is still a want of uniformity and agreement among electrical authorities in regard to some of the adjectives used to "qualify" the word "current" and to extend or specialize its meaning for different purposes. The terms "starting" current, "running" current, "full-load" current, "no-load" current, as applied to motors, are sufficiently definite, and are generally understood and recognized. Those terms have been used in the translation. The term "feeding" current, applied to the current passing through the primary of a transformer or induction motor, is not yet generally recognized, and has been avoided in the translation. The term "wattless" current is now generally accepted as the name of the current which is in phase with the *reactance* component of the impedance, and which produces the "reactance drop." (The term "idle" current, used by some authors, has not been generally accepted). There is, however, no generally recognized term for the current which is in phase with the *resistance* component of the impedance and which produces the "ohmic drop." This component has been variously termed the "power" current, the "energy" current, the "active" current, the "working" current, the "load" current, the "useful" current, the "watt" current, and even the "watty" current. All of these terms have their adherents, but with the exception of the last two, they lack the essential quality of symmetry with respect to the antithetical term "wattless." The French have adopted the terms "watté" and "déwatté," literally meaning, respectively, "watted," and "unwatted." The translator has thought that he could do nothing better than to take a suggestion from this fact, and translate literally the word "watté," making the word "watted," which is the adjective that has been used throughout the book, to designate the current which is in phase with the resistance component of the impedance.

The terms "parasite" currents and "Foucault" currents, used by the author, are perfectly definite, and have been retained, in some cases, in the translation, although the term "eddy" currents, which is becoming the more generally accepted term, has had the preference.

The letters I and i have, respectively, been used as generic symbols for "maximum" and "effective" values of current strength (in amperes) in the equations. The distinction between different kinds of current has been made by means of "subscripts." See "Notation."

Energy. The author is careful to distinguish between "energy" and "power." Hence, as might be expected from the fact that motors are machines for developing "power," the word "power" occurs quite frequently, while the word "energy" (taken in the sense of time-integral of power) occurs but rarely in the book. See "Power."

Joulean Effect. This term is used, both in the original text and in the translation, to denote the loss of power (watts), and the heating, due to ohmic resistance, usually designated in this country as the I^2R (formerly C^2R) loss.

Joulean Heat. This term is used to designate more specifically the heating effect of the I^2R loss.

Effective. This adjective is now generally accepted, and has been used both in the original and in the translation, as the proper term to denote that "mean" or "average" value of a quantity varying sinusoidally, which is equivalent to a certain "constant" value. Technically, the "effective" value is equal to the square root of the mean value of the squares, sometimes called the "root mean square" (R. M. S.) value. The effective value (x), in the case of a pure sine function, is related to the maximum or "amplitude" value (X) as follows:

$$x = X \times \frac{1}{\sqrt{2}} = \frac{X}{\sqrt{2}} = 0.707\, X.$$

$$X = x\sqrt{2} = 1.41\, x.$$

The term "effective" can be applied to electromotive force, magnetomotive force, magnetic density, magnetic flux, magnetic leakage (flux), hysteresis, electric current, ohmic losses, and, in general, to all quantities which vary sinusoidally in an induction motor.

In the text, and in the equations, "effective" values are usually designated by "lower case" letters, (the "maximum" values being designated by "capital" letters). See "Notation."

Induction. The term "induction magnétique" (often condensed into the single word "induction") has been used, in every case, by the author, to denote "strength of magnetic field," or the magnetic quantity measurable in units called "gausses." Although the corresponding terms "magnetic induction" and "induction," both literal translations from the French, have, to some extent, become current in English, and are still used by some authors, yet these terms have been avoided altogether in the translation, and the same idea has been rendered, in every case, in the translation, by the term "magnetic density," or, sometimes, briefly, by the word "density." This term, which is rapidly coming into general use in all countries, is more definite and intelligible than the term "magnetic induction."

The tendency has been to restrict the use of the word "induction" entirely to the electromagnetic phenomenon whereby electromotive forces are produced in electrical conductors cutting lines of magnetic force.

The magnetic density values, in gausses, have been designated symbolically, in the text, by the letters "B" and "b," used both with, and without subscripts. See "Notation."

Magnetic Density. (See Induction.)

Magnetic Flux. The expression "magnetic flux," literally translated from the French, has now become the accepted conventional term, and has been used as such in the translation, to designate the total number of lines of magnetic force present in a given magnetic path or circuit. This term represents the quantity measurable in units called "maxwells."

The Greek letter Φ has been used throughout the book, with various subscripts, as a characteristic symbol for magnetic flux. (The letter F has also been used for certain flux values, in chapters VII. and VIII. only. See F in list of symbols.)

Pitch. This word, which has long been used to denote the spacing of the teeth in gearing, is also used to denote the spacing of winding spaces and conductors.

The term "slot pitch" designates the distance between the centre lines of two consecutive "slots."

The term "pole pitch" designates the distance between the centre lines of two contiguous magnetic poles.

The term "winding pitch" designates the distance between the centre lines of two conductors or portions of winding connecting together to constitute a "coil," a "loop," or a "section" of the whole winding.

Power. The author uses this word always in its strictly correct general sense, meaning a time-rate of variation of energy $\left(\frac{dE}{dt}\right)$. When used specifically by the author in an electrical sense, it always refers to a quantity measurable and expressible in *watts;* when used in a mechanical sense, it always refers to the product of a (tangential) *force* by a *linear* velocity, or the product of a *torque* by an *angular* velocity.

The expression "useful power" is used to denote the quantity sometimes designated by the term "brake horse power." In some cases the "useful power" is expressed in "equivalent" watts.

The terms "watted power" and "wattless power" are self-explanatory.

The term "power input" has been used, in a few instances, to designate the total watts sent into and absorbed by the motor. By analogy the term "power output" might be used to designate the total power developed by the motor.

Slit. This term has been used, in the translation, to designate the (peripheral) opening into a "partially closed" winding slot (such as shown at X, in Fig. 50).

Slot. This word, which was originally applied only to the grooves in which the winding conductors of a direct current "toothed" armature are placed, has passed into conventional use as a general term for designating all forms whatsoever of "hollow winding spaces" between the "teeth" in magnetic cores.

These hollow spaces are called "open slots" when they are true slots of the "groove" form with a peripheral opening equal to the full width of the slot; they are called "partially closed slots" when this peripheral opening is less than the full width of the slot, the opening being then called the "slit"; and they are called "closed slots" when there is no "peripheral" opening (slit), the thin portions of magnetic core which extend between the "teeth" and close the openings being called "bridges."

Tension. The French word "tension," meaning either potential, potential difference, or electromotive force, and which is also occasionally used in English in the same sense, has been sedulously avoided, and has been rendered in the translation, sometimes by the term electromotive force (E.M.F.), usually by the expression "potential difference," or difference of potentials, and, in a few cases (when unavoidable), by the term "voltage."

The term "voltage," although convenient, is etymologically faulty, and a "good" word for the same idea is badly needed.

Torque. The author follows the general custom of Continental writers in discarding the use of the term "torque" altogether. The mechanical effort causing the motion of the rotating portion (or rotor) of the motor, is, in the original text, generally referred to as an electromagnetic "effort" or "couple" (sometimes as a "motive effort" or "motor couple") assumed to be exerted tangentially at the periphery of the said rotating portion. This effort is the mechanical "force" or "pull" known in mechanics as the "*tangential effort*," — a term which has been frequently used in the translation. The product of this tangential effort by the radius of the rotating portion is the "*torque*." The term "torque," therefore, *symbolizes* the important and useful concept of "leverage," and it expresses, in a single word, the idea of a rotating force equal to the "moment of the tangential effort."

While the term "torque" and the idea implied thereby are not indispensable, as the author has demonstrated by doing without them, yet they are so convenient, so well understood, and, especially, so generally used in this country, that the translator has deemed it expedient to introduce and to use the term in the translation. A paragraph giving the definition of "torque," and its derivation and distinction from "tangential effort," has been interpolated into the text (at the top of page 44); and the term "torque" has been used in the translation in all general or abstract statements wherein its substitution for the term "tangential effort" could occasion no ambiguity. In referring to the numerical calculations and values of the mechanical effort, and also in all references to these values as used, symbolically or numerically, in the equations, the terms "tangential effort" or "electromagnetic pull" have been retained in the translation.

It must be admitted that the author's preference for "tangential effort" instead of "torque" as the unit of rotative force has a logical basis. In the first place, the fundamental equation for the electromagnetic reaction taking place in the air-gap and causing the rotor to move, gives, directly, a numerical result which is a rectilinear force, and which is, in effect, a "tangential effort." In the second place, the "tangential effort," being a rectilinear force, is a quantity of exactly the same mathematical character as the force term or "factor" in the general equations for energy and power; and its use therefore simplifies the transition from the general to the specific case. This point will be made clear by a summary analysis of these fundamental equations.

The energy contained in a body is usually expressed as the product (fds) of two separate factors; namely, the distance (ds), and the force (f), acting on the body, — the total amount (E) of energy involved being equal to the summation of the whole series of such products comprised between the "initial" and the "final" states of position, or of motion (s' and s'') of the body. We therefore have

$$E = \int_{s'}^{s''} f ds, \qquad (a)$$

as the general equation for energy. Power may likewise be expressed as the product $\left(f\frac{ds}{dt}\right)$, of a force ($f$), by a distance "factor" $\left(\frac{ds}{dt}\right)$, which latter is somewhat different, however, in this case, being, in reality, a time-rate of distance variation; that is to say, a velocity $\left(\frac{ds}{dt} = v\right)$; so that power might be also defined as the product (fv) of a force by a velocity. We can therefore write

$$P = f\frac{ds}{dt}, \qquad (b)$$

or

$$P = fv, \qquad (c)$$

as the general equation for power. The first form (b) of the general equation for power shows at once that power is, in reality, the time-rate of variation of energy $\left(\frac{dE}{dt}\right)$. If time values ($dt$) were measured in seconds, power would be, in effect, the amount of energy transferred or transformed per second.

It is clearly seen that the factor (f) expressing the amount of force present is of precisely the same physical and mathematical character in both equations. This will remain true independently of any changes in the nature or character of the motion by which the distance involved is traversed, because the force factor (f) does not, itself, contain or suggest the notion of distance; that is to say, it is not a function of distance or motion. Hence, so far as the force applied is concerned, its effect and its measurement in a given case can be considered as being the same whether the motion produced be translatory (rectilinear) or rotatory (curvilinear). The distance "factor" *alone* would be affected by any change in the character of the motion. With rectilinear motion, the distance factor would enter into the equation for energy (a), as the measure of a *rectilinear distance* (or displacement), and it would enter into the equation for power (c), as the measure of a *rectilinear velocity.* With rotational motion, this factor would, similarly, represent peripheral or *circumferential distance*, in the case of energy, and peripheral or *circumferential velocity*, in the case of power. The distinction between the two kinds of distance and of motion is, however, more apparent than real. A circumferential distance or a circumferential velocity *is*, in reality, *measured as if it were rectilinear.* The units of measurement are, it is well known, exactly the same for both circumferential and rectilinear distances or velocities. The "circumferential" value can, in each case, be indicated precisely by a straight line, which is the exact geometrical "development" of the "circumference" representing the distance or velocity.

The final result is, consequently, as if the rotational motion were translated or converted into an equivalent rectilinear motion. Therefore, while the expedient of expressing the "force factor" in terms of "tangential effort" preserves the resemblance between the general and the specific equations, and makes the transition between them, in some respects, more simple, yet it is open to the objection that the force factor is not what it really should be, — a quantity implying and designating a *rotative* or *angular* force, — when used in the analysis or measurement of power or energy involving rotatory motion. It is precisely a quantity of this character that is designated by the term "torque." That is why the term and the idea of torque have proved so useful and convenient in connection with the analysis of power involving rotatory motion.

Torque is, in reality, a "rotational" or "polar" propulsive effort. Therefore, while tangential effort is a rectilinear or "translatory" force factor, *torque is a rotational or "angular" force factor.* Though "torque" includes the element of distance, and is of more complex nature than the rectilinear force factor, yet it actually simplifies the analysis of power involving rotational motion. Its great advantage lies in the fact that when the force factor of power or of energy (involving rotary motion) is expressed in "angular" or "torque" measure, the "distance factor" no longer needs to be expressed in "equivalent" *rectilinear* measure, but may be expressed directly in *rotational* measure, as *angular velocity* in the case of power, and as *angular distance* (or displacement) in the case of energy. In rectilinear measure, power is equal, as already seen, to the product of the (rectilinear) force factor (i.e., the tangential effort) by the distance factor (i.e., the peripheral velocity). In angular measure, power is, similarly, equal to the product of the (angular) force factor (the torque) by the distance factor (the angular velocity).

Let us take,

$F =$ tangential effort (or "rectilinear force factor").
$R =$ Radius of Rotor.
$T = FR =$ torque (or "angular" force factor).
$N =$ speed of motor (R.P.M.).
$\omega = 2\pi N =$ angular velocity.
$V = \omega R =$ peripheral velocity.
$P =$ Power.

Substituting the proper "factor" values in equation (c), we would have, as the measure of power,

$P = FV$ in "rectilinear" measure,

and $P = T\omega$ in "angular" measure.

If the value of P is assumed to be the same in both cases, we must necessarily have,

$$FV = T\omega.$$

Substituting for V and T, the preceding equation becomes,

$F \times \omega R = FR\omega$, which leads to the *identity:*

$$FR\omega = FR\omega.$$

showing that the two methods lead to the same result.

The preceding discussion shows that each system of "force" notation has its advantages. The conclusion to be drawn from the comparison of the two systems is that the first system of "force" notation, based, as it is, upon the assumption that the motion involved either takes place, or can be considered and treated as if taking place, along a straight line, is, primarily, best suited for dealing with translatory (rectilinear) motion; while the second, which is directly based upon the assumption that the motion involved is rotational, is specially suited for dealing with rotatory motion.

The reader will note that the velocity factor (ω) used with the torque notation system, in the equation for power, is exactly the same as used in the analysis of alternating current phenomena. This is an incidental advantage of that system.

The introduction of the term "torque" in the translation is, therefore, justified.

The terms "starting torque" and "running torque," which have been used in the translation, designate, respectively, the propulsive effort exerted while starting, and that exerted when running under full load.

The foregoing analysis of the relation between "tangential effort" and "torque" will enable the reader to readily understand the meaning and values of either quantity in terms of the other.

Windings. There is a considerable divergence of opinion and of practice in the nomenclature and classification of induction motor windings. The matter is one deserving consideration.

The author has found it convenient, for his purpose, in this work, to distinguish five different types of induction motor windings, as follows: —

1. "Long-coil" Windings, with overlapping conductors placed in slots.
2. "Ring" Windings.
3. "Wave" or "Progressive" Windings.
4. "Squirrel-cage" Windings.
5. Windings with short-circuited loops.

The first of these types of winding has been referred to, in the translation, either as a "definite" winding, or as a "polar" winding, both terms being accepted as conventional and being in current use, in this country, for designating such a winding. The term "definite" is apparently more popular and has, therefore, been given the preference, although the translator, himself, prefers the term "polar," which is "definite" enough when we remember the fact that all induction motor windings are "distributed" windings, and that, consequently, the expression "polar" really implies a "*distributed* polar winding."

The names given by the author to the other four types of windings have been retained in the translation.

The first three types can be used for both the "primary" and "secondary" windings of induction motors. The last two types are suitable for secondary windings only. In practice, the first type has been generally adopted, to the virtual exclusion of the others, for primary windings, and also for secondary windings which are intended for use with starting resistance; while the fourth type is the most frequently used for rotors which are to be started without extraneous resistances.

If considered with respect to their electrical or electromagnetic features, these five types could, in reality, be brought into three electrical classes, viz.: —

I. Definite or Polar Windings.
II. Wave or Progressive Windings.
III. Short-circuited Windings.

Class I. will then include the first two types. The characteristic features of a winding of this class are: first, it has, for each phase, a "definite," fixed, "polar relation," with respect to certain portions of the magnetic core; second, the centre lines of the magnetic poles produced by it in the air-gap circle are *exactly equidistant.*

Class II. has some analogy with Class I. It includes all windings of a certain type, such as used for direct-current multipolar dynamo armatures, in which the number of slots is not an even multiple of the number of poles. Windings of this class may have one, but never can have both, of the characteristic features of Class I. They have the first characteristic feature alone when used as primary windings, and the second alone when used, in a revolving magnetic field, as secondary windings. (There are special modifications of wave or progressive windings, made with an even number of slots, which also have the second characteristic, in which case they become substantially "definite" or "polar" windings, and may, therefore, be classed as such.)

Class III., which includes the fourth and fifth types of windings, has little or no analogy with the other two classes. In this class of windings, each individual conductor, or else, each pair of conductors (according to the particular type), constitutes, in reality, an independent winding. These individual windings may be considered as acting quite independently of each other; and the apparent co-ordination in their effects is merely a consequence of their being acted upon by a revolving magnetic field.

If, now, we also consider these windings with respect to their mechanical features, or peculiarities, we find still another classification and nomenclature.

Mr. August H. Kruesi has kindly prepared and placed at the translator's disposal the following statement, which may be considered authoritative: —

"In the first place, the inducing member of an induction motor, which is generally the stator, is termed the 'primary,' and the other member, which is generally the rotor, is termed the 'secondary.' There can be no possible confusion between these two terms; and, if one states that the secondary is the 'rotor' or the 'stator,' as the case may be, the whole arrangement of the motor becomes at once fixed.

"In regard to windings, it may be said that the general term 'distributed winding' would apply to and would include all forms of induction motor windings. (A winding for prominent poles, like the poles of direct-current dynamos, might be called a 'concentrated' winding). There are two general classes of distributed windings, namely, 'definite" windings and "squirrel-cage" windings. Definite windings may be of several types, designated, respectively, as "wave winding," "lap winding" "chain winding" and "basket winding." The terms "wave winding," and "lap winding" are used, as in the case of direct current armatures, to distinguish between two methods of progression in connecting together with conductors placed in the slots. Of these two, the "lap winding" is the more common. It is generally wound with a pitch equal to the full pole pitch, but it may also, in some cases, be carried out with a pitch which is less than the full pole pitch, in which case it is said to be a "fractional pitch winding." In consequence of the pitch being smaller, the end connections will be somewhat shorter, and, consequently, the ohmic loss (I^2R) will be reduced, and the efficiency will be improved. That part of the self induction which is due to the end connections is likewise reduced; but on the other hand, the magnetizing current will be increased, owing to the higher magnetic densities, so that, generally, the power factor of the motor will be slightly decreased.

"The variation in the winding pitch is often influenced and dictated by convenience of design or manufacture, as it enables the same winding coils to be used for different motors. As an example, it would be possible to use coils wound on the same frame, either for an 8-pole, 60-cycle, 900-R.P.M. motor, or for a 6-pole, 40-cycle, 600-R.P.M. motor. If the coils are 'full pitch' for the 8-pole motor, they would be only 'three-quarter pitch' in the case of the 6-pole motor.

"With regard to 'shop' names of induction motor windings, it may be said that the names generally describe a characteristic feature of the end-connections of the winding. Thus a winding having end-connections like those shown in Fig. 20 of this work, is known as a 'chain' winding, from the fact that the coils of the several phases are linked together. A winding like that shown in Fig. 25 is a 'squirrel-cage winding.' A winding such as shown in Fig. 26 is known as a 'barrel' winding. A winding such as shown in Fig. 28 is an 'evolute end-connection winding.' This winding, it may be noted in passing, is seldom used here. Most American machines are built with end-connections of the 'barrel' type because of its large radiating surfaces, and because the coils for this winding may be readily wound in 'formers.' In Continental practice, the chain type of winding is mostly used because, as a general rule, the slots are of the partially closed type, and the winding is, therefore, done by hand.

"There is still another kind of winding known as 'basket' winding, which is not illustrated in the book.

"The 'basket' winding is, in reality, a 'chain' winding, in which the 'links' are, so to speak, 'twisted' as they pass out of the slots. In the 'chain' winding, the conductors which are placed 'outermost' on one side of the frame become placed 'innermost' on the other side. In the 'basket' winding, each set of conductors as it passes out of a slot is slightly twisted so as to retain its relative place in the composite 'coil,' of which it is a part. For a 'full-pitch' winding, the 'loops' or 'turns' of the winding will all have exactly the full pitch, if of the 'basket' type, and they will all have a pitch alternately greater and smaller than the full pitch, if of the 'chain' type. The magnetizing effects and the distribution of magnetic flux are exactly the same for both of these two kinds of windings."

B. NOTATION.

General Description. The author has followed certain rules in the selection and use of symbols, whereby the notation is made systematic and consistent. By remembering these simple rules, the reader will find no difficulty in understanding and retaining the notation.

Each electrical or magnetic quantity is designated by a characteristic letter, in the equations. Thus the letter B always designates magnetic density (in gausses); the letter R always designates resistance (in ohms); the letter E always designates an E.M.F. or a P.D. value (in volts); etc. When this letter is a capital, it denotes, usually, a "maximum" value; when it is a "lower-case" letter, it denotes an "effective" value (or the value equal to the square root of the sum of the mean squares).

In a few cases the same letter has been used to denote two different quantities, but not in the same portion of the book, or in the same connection. Thus, in the early chapters, the letter a denotes the distance between two contiguous slots, while in Chapter VIII. it denotes the specific resistance, or the coefficient of resistivity, of copper.

The distinction between different kinds or sets of values of the same unit, both designated by the same letter, is made entirely by means of "subscript" figures and letters. Thus, the Roman figures I., II., III., are used as subscript symbols to distinguish "phases:" i_{I}, i_{II}, i_{III}, respectively, indicate the effective current strength for phases I., II., and III. The Arabic figures 1, 2, are used as subscript symbols to distinguish "primary" and "secondary" values, respectively; R_1 R_2, respectively, indicate the primary resistance and the secondary resistance. The Greek letter μ, used as subscript symbol for i, designates "magnetizing" current. The letter s is similarly used as a subscript symbol denoting "leakage," with ϕ to indicate a magnetic leakage flux, or with e, to indicate an equivalent E.M.F., due to magnetic leakage. Other subscript letters are also used, such as "u" with P, to denote "useful power" (P_u); w, with i, to denote "watted current" (i_w); d, with i, to denote "starting current" (i_d), and with B to denote maximum magnetic density in the core teeth (B_d); n with W to denote weight of cores (W_n); etc.

These symbols will be understood without difficulty, as the text in every case gives a clue to the meaning of the symbol.

The prime mark (′) and deuce mark (″) are used to denote "component" values, forms, or terms, which combine, algebraically or vectorially, to give the "resultant" values. The symbols having this mark therefore denote intermediary forms or values as distinguished from the definite values and terms appearing in the final forms of the equations.

List of Symbols. The meanings of the principal symbols used in the equations are as follows: —

A_1 = number of phases in the primary winding (p. 154).
A_2 = number of phases in the secondary winding.
a = distance between two contiguous slots.
a = the coefficient of resistivity for copper (value taken, $a = 1.8 \times 10^{-6}$ ohms-centimetres).
av. = subscript symbol denoting "average."
B = generic symbol for "Magnetic Density" (values always understood to be in gausses, in all cases).
B = magnetic density, usually maximum values.
B_{max} = magnetic density, always maximum values.
b = magnetic density, always "effective values." (Different sets of values of both B and b are distinguished by "subscripts," as indicated hereinafter.)
B_1 = maximum magnetic density, in primary core.
B_2 = maximum magnetic density, in secondary core.
B_d = maximum magnetic density, in "teeth" of core.
B_{d1} = maximum magnetic density, in primary teeth.
B_{d2} = maximum magnetic density, in secondary teeth.
b_1 = effective magnetic density, in primary core.

b_2 = effective magnetic density, in secondary core.
b_0 = effective magnetic density, in air-gap.
C = a mathematical constant.
C = electromagnetic couple (tangential effort).
D = (mean) diameter of air-gap circle, in centimetres. (Different values are distinguished by "subscripts.")
d = the (radial) distance across (sometimes called the "length") of the air-gap, in centimetres.
d = subscript symbol for "teeth" (from the French word "dent").
d = subscript symbol for "starting" (from the French word "démarrage").
E = Generic symbol for E.M.F or P.D. (Values always understood to be in volts.)
E = E.M.F. or P.D. (usually maximum values).
e = "effective" values of E.M.F. or P.D.
e_0 = "effective" values of P.D. applied at motor terminals.
e_1 = "effective" values of total E.M.F. induced in primary.
e_2 = "effective" values of resultant E.M.F. induced in secondary.
e_2 = $R_2 I_2$.
e'_1 = "effective" value of E.M.E. induced in primary, by main flux.
e'_2 = "effective" value of E.M.F. induced in secondary, by main flux.
e_{1s} = "effective" value of E.M.F. corresponding to magnetic leakage (stator).
e_{2s} = "effective" value of E.M.F. corresponding to magnetic leakage (rotor).
e''_s = "effective" value of E.M.F. corresponding to magnetic leakage flux Φ''.
e'_s = "effective" value of E.M.F. corresponding to magnetic leakage flux Φ'.
e_s = "effective" value of E.M.F. corresponding to magnetic leakage total flux Φ_{1s}.
F = symbol used only with subscripts 1 or 2 instead of Φ, in chapters VII. and VIII., to denote a "resultant" magnetic flux.
F_1 = resultant magnetic flux in stator.
F_2 = resultant magnetic flux in rotor.
F = total tangential effort, in kilogrammes. (Different kinds are indicated by subscript.)
f = tangential effort for one conductor.
I = generic symbol for electric current. (Values always understood to be in amperes. Different sets of values are designated by subscripts.)
I = strength of current, usually "maximum" values.

I_{max} = strength of current, always "maximum" values.

$I_{max} = i\sqrt{2}$.

i = strength of current, always "effective" values.

$i = \frac{I}{\sqrt{2}}$.

i_1 = effective primary current.

i_2 = effective secondary current.

$i\mu$ = effective magnetizing current.

$i_1\mu$ = effective primary magnetizing current.

$i\omega$ = effective watted current.

i_{1d} = effective starting (primary) current.

K = A constant depending upon the number of slots per coil in the primary winding, and whose value for different windings is given in special tables, in Chapters II. and VI.

The letter K, when used without any subscript, denotes general values, but in some cases it also denotes the constant used in the formulæ for the magnetizing action of windings; when used with the subscript "1," it usually denotes the constants used in the formulæ for the E.M.F. induced in the given winding.

L = axial dimension of magnetic core.

l = angular width of a loop of the winding.

l_m = mean length of a loop of winding.

M = magnetomotive force, maximum value, in C.G.S. units.

m = effective value of resultant magnetomotive force (obtained by vectorial addition of m_1 and m_2).

m_1 = effective value of M.M.F. (primary).

m_2 = effective value of M.M.F. (secondary).

m = subscript symbol for "mean" or "average."

max = subscript symbol for maximum value.

N = number of revolutions per minute (R.P.M.).

n = number of conductors per slot.

n = subscript symbol denoting "core" (French, "noyau").

P = power. (Values always expressed in watts.)

P_1 = power (watts) absorbed in primary.

P_2 = power (watts) absorbed in secondary.

P_2'' = power (watts) absorbed in secondary short-circuit rings.

P_{d1} = power (watts) absorbed by hysteresis in primary teeth.

P_{d2} = power (watts) absorbed by hysteresis in secondary teeth.

P_{n1} = power (watts) absorbed by hysteresis in primary core.

P_{n2} = power (watts) absorbed by hysteresis in secondary core.

p = number of poles of revolving magnetic field.

q = nt = number of turns per active side of coil.
$2q$ = number of slots per pole (two-phase winding).
$3q$ = number of slots per pole (three-phase winding).
R = Resistance, in ohms.
R_1 = Primary resistance.
R_2 = Secondary resistance.
$R_1 i_1$ = Ohmic drop in primary.
$R_2 i_2$ = Ohmic drop in secondary.
S = total number of active conductors on periphery of magnetic core (" primary " indicated by subscript " 1 " and " secondary " indicated by subscript " 2 ").
$\frac{S}{p}$ = number of slots per pole.
S = superficial area, when used with subscript (I. II. and 1,2,3,4).
$S,$ or $s,$ = sectional area (different values distinguished by subscript).
s = " slip," in per cents.
s = subscript symbol indicating " leakage."
T = time-duration of one current period, in seconds.
t = time values in seconds.
t = number of slots in which are placed the conductors forming the active side of each coil.
u = subscript symbol denoting " useful."
V = velocity, in centimetres per second.
V_1 = peripheral velocity factor of revolving field for primary winding.
V_2 = peripheral velocity factor of revolving field for secondary winding.
W = weight, in Kilogrammes.
W_1' = weight of primary core.
W_2' = weight of secondary core.
X = dimension of slot, in centimetres.
X = width of " slit " in centimetres.
X_0, or X_0, = width of slot.
x = depth of slot.
y_0 = depth of slot.
y = thickness of " bridges " between core teeth.
y = " pitch " of winding.
Z = total number of active conductors in series, per phase.
$\frac{Z}{2}$ = total number of conductors in series, per phase (two-phase motors).

$\frac{Z}{3}$ = total number of conductors in series, per phase (three-phase motors).

α = angles, expressed in degrees.

β = angles, expressed in degrees.

a = correcting factor. (Different values indicated by subscripts.)

a^2 = correcting factor. (Different values indicated by subscripts.)

η = efficiency (per cent).

μ = subscript symbol denoting "magnetizing."

σ = Magnetic leakage factor.

$\sigma = 1 - \frac{1}{V_1 V_2}$.

π = ratio of circumference to radius.

π = angular distance between centre lines of two contiguous magnetic poles, measured as a portion (one-half) of a complete cycle (2π). (It represents the angular displacement of the revolving magnetic field during one-half period, or 180°, of cyclical variation of the E.M.F.; which is exactly the distance between the centre lines of two contiguous magnetic poles.)

$\frac{\pi}{3q}$ = angular distance between two consecutive slots, for three-phase motor.

$\frac{\pi}{2q}$ = angular distance between two consecutive slots, for two-phase motor.

ϕ = angle whose cosine = power factor.

$\cos\phi$ = power factor.

Φ = generic symbol for "magnetic flux." (Values always expressed in maxwells.)

Φ_{av} = average or mean value of magnetic flux.

Φ_1 = primary magnetic flux.

Φ' = leakage magnetic flux across slots.

Φ'' = leakage magnetic flux across slot openings.

$\Phi_{1s} = \Phi' + \Phi''$ = leakage flux across (primary) slots and teeth.

Φ_{2s} = leakage flux across (secondary) slots and teeth.

$\omega = \frac{2\pi N}{60}$ = angular velocity, in radians per second.

ωt = angular distance or displacement in time t.

THE INDUCTION MOTOR.

CHAPTER I.

THE REVOLVING FIELD.

Polyphase Motors are also called revolving field motors, because the magnetic flux which is produced in the air-gap is not stationary, but is displaced with an angular velocity somewhat higher than that of the motor shaft when in normal operation. In order to show clearly how

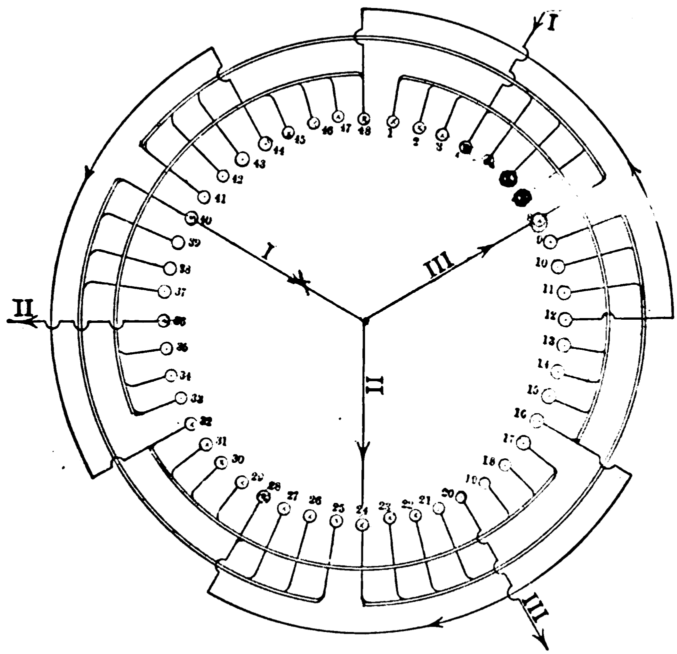


Fig. 1.

the rotation of a magnetic field is obtained, we shall examine the distribution of magnetic density, at a given moment, along the periphery of the "inducing" field (stator) of a three-phase motor having an ordinary winding with long coils, or with coils placed in slots, forming a "definite" or "polar" winding, arranged according to the diagram shown in Fig. 1.

It will be noticed that this winding consists of six coils, or two for each phase, the windings of each one of the six coils being distributed in eight holes in the iron core. The three circuits are identical, and any one of the three can be exactly superposed on the other two, by shifting it around, either 120 degrees, or 240 degrees.

Fig. 2 reproduces, on a larger scale, the development of a portion only of the winding.

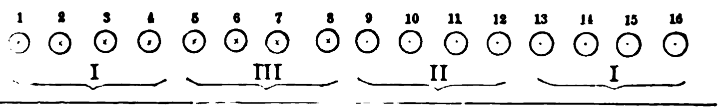


Fig. 2.

Let us now send, through this field winding, three-phase currents which we will suppose to be sinusoidal. Let us assume, moreover, that, at a given instant, the current in the first branch is going, as indicated by the arrow, from the stator winding terminal, toward the neutral point, or the point at which the three circuits come together, and that its highest value is indicated by the symbol I_{max}.

By hypothesis, since we are using three-phase currents, (Fig. 3), the value of the current in the second phase will be

$$i_{II} = -\tfrac{1}{2} I_{max},$$

and its direction (as shown by the arrow) will be from the neutral point toward the corresponding stator winding terminal (II).

In the third circuit there will be a current of the same value and of the same direction as in the second, directed toward the corresponding terminal (III).

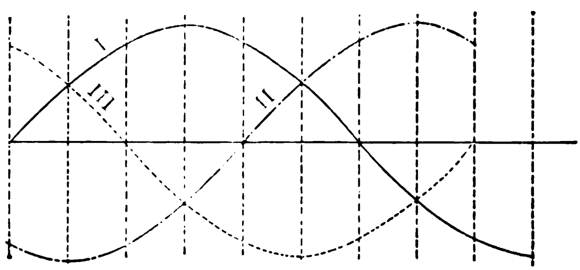


Fig. 3.

These conditions being assumed, it will now be easy to find the direction of the current in each of the holes in the stator core and to demonstrate the correctness of Fig. 1 in this respect.

If we examine more closely the distribution of the current in the various sets of conductors, we will see that the direction of the current remains the same in twelve consecutive holes, and that it is then reversed in the next twelve holes. The first four holes, as well as the last four holes, of each of these groups, contain conductors which belong to phases *II* and *III*. The four central holes contain the wires of the first set, which, at the particular instant under consideration, carry the current I_{max}, whose value is double that which is passing through the conductors in the neighboring holes.

If we place, concentrically, inside of this winding, an iron core of the same axial length as the stator core, but having an external diameter slightly smaller than the internal bore of the stator core, the windings of the latter will produce, in the air-gap, a magnetic field which we will now proceed to study.

For this purpose, let us consider, first, the wires placed in holes Nos. 3 and 14. Although the direction of the current in hole No. 3 is opposite to that in hole No. 14, nevertheless, the current value is

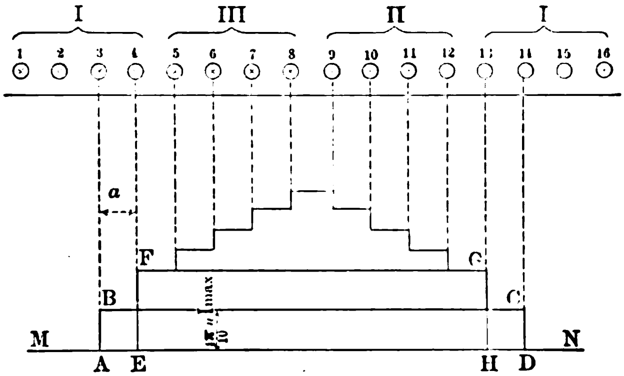


Fig. 4.

actually the same in the conductors placed in these two holes. We can, therefore, say that the windings in holes Nos. 3 and 14 constitute, together, a coil which gives rise to a magnetomotive force whose value is

$$\frac{4\pi}{10} nI_{max},$$

where n designates the number of wires in each hole, and I_{max} designates the current, in amperes, which is supposed, by hypothesis, to pass through each conductor. As we know, this magnetomotive force remains the same at all points in the plane of the coil. If we take as abscissæ the development of the air-gap, and as ordinates (Fig. 4), the values of the magnetomotive force, the magnetomotive force due to the windings 3–14 will be represented by the rectangle *ABCD* which rises to the height *AB* above the axis *MN*, and which extends a distance equal to that from *A* to *D* along that axis.

In this case *AD* represents the distance between the holes Nos. 3

and 14, measured along the air-gap, and AB represents, by its height, the value of the magnetomotive force, which we have just estimated to be

$$\frac{4\pi}{10} nI_{max}.$$

The wires in holes Nos. 4 and 13 likewise form a coil which produces a magnetomotive force equal to the preceding one, since the number of wires per opening and the current passing through them have the same values as before. This new magnetomotive force, which is constant between holes Nos. 4 and 13, adds itself to the preceding one in such manner that the total magnetomotive force between these holes, Nos. 4 and 13, will be represented by the rectangle $EFGH$. We must evidently have:

$$EF = 2\,AB.$$

We can apply the same reasoning to the coils resulting from the combination of the holes 5–12, 6–11, 7–10, and 8–9, all four of which give magnetomotive forces which are equal to each other and in the same direction, but which, however, are each only half of those produced by the coils 3–14 and 4–13, since the current is no longer I_{max} as in the case of these two coils, but only $\frac{1}{2}I_{max}$. We have already, in this manner, represented graphically the exact distribution of the magnetomotive force for one-fourth of the total development of the air-gap. It is now very easy to make the curve for the neighboring coils, 15–26, 16–25, 17–24, 18–23, 19–22, and 20–21; but it is important to notice that the magnetic effects of these new windings are opposed to those of the preceding windings, and that, consequently, the magnetomotive forces should be drawn below the axis MN. In order to become convinced of this, it is sufficient to note that the magnetic flux of the windings placed in the holes Nos. 3 to 14, inclusive, passes from the outer magnetic core (stator) into the central core (rotor), since, for an observer placed in the air-gap and facing these coils, the current circulates in counter-clock-wise direction, which is contrary to what takes place for the windings placed in holes Nos. 15 to 26. To state it differently, we will have a north pole from hole No. 3 to 14, and a south pole between holes Nos. 15 and 26.

We have just examined the distribution of magnetomotive force over one-half of the development of the air-gap, and, as the same process will be exactly repeated for the other half, we can say that the particular winding considered will produce four magnetic poles, that is to say, a number of poles double that of the number of coils per phase.

In polyphase motors, the magnetic density in the air-gap is always much lower than that which is found in direct current machines. This is also true for the cores and for the teeth of both the stationary portion of the magnetic circuit (stator) and the movable portion (rotor). The permeability of the teeth and of the cores always retains a very high value, so that one can neglect their reluctance in comparison with that of the air-gap. We will show, hereafter, that the error commited in so doing is very small, and that it can, in any event, be corrected by adding sufficiently to the distance across the air-gap to make up for this reluctance.

This consideration, which is perfectly justified, facilitates enormously the elucidation of what is to follow. Fig. 4 shows that the maximum magnetomotive force which our winding produces can be expressed as follows:—

$$\frac{4\pi}{10} n I_{max} (1 + 1 + 0.5 + 0.5 + 0.5 + 0.5) = \frac{4\pi}{10} n I_{max}\ 4.$$

We immediately deduce from this that the maximum magnetic density produced in the air-gap is expressed by the following relation:—

$$B_{max} = \frac{4\pi}{10} n I_{max} \frac{4}{d},$$

where d represents the (radial) distance across (or "length" of) the air-gap in centimetres.

The broken line, which represents the distribution of magnetomotive force along the air-gap, also gives that of the magnetic density, when the scale of ordinates is suitably changed. It is now easy to estimate the magnetic flux leaving one pole, by multiplying the area included between the broken line and the straight line AD, by the (axial) width, L, of the cores. We then have:

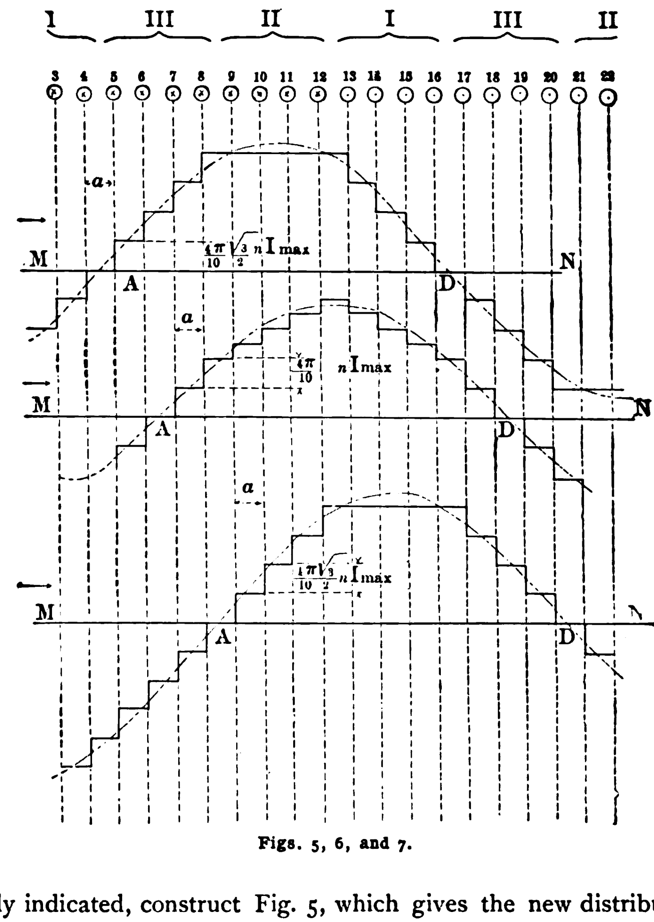
$$\Phi = 2 \times \frac{4\pi}{10} n I_{max} \frac{1}{d}\, a\,(1 + 2 + 2.5 + 3.0 + 3.5 + 2)\,L.$$

$$\Phi = \frac{4\pi}{10} n I_{max} \frac{1}{d}\, a \times 28 \times L,$$

in which a represents the distance between two consecutive holes.

All that has just been said applies only at the particular moment when the current in the first phase is at its point of highest value, I_{max}.

Let us now see what becomes of the magnetic flux, a moment later, when the current of the first phase will have decreased from I_{max} to $\frac{\sqrt{3}}{2} I_{max}$. If we draw (Fig. 3), three sine curves, of which the second one lags 120° and the third one 240° behind the first curve, all three having the same amplitude and the same length of wave, we will find that, at this particular moment, the current in the second phase is zero, while it is equal to $-\frac{\sqrt{3}}{2} I_{max}$ in the third phase. The direction of the current in the coils of the windings of phases I. and III. remains the same as indicated in Figs. 1 and 2. We can therefore, in the manner



Figs. 5, 6, and 7.

already indicated, construct Fig. 5, which gives the new distribution of the magnetomotive force in the air-gap, beginning, for example, with the windings placed in the holes 5–16, 6–15, 7–14, and 8–13. The openings 9, 10, 11, 12, produce no effect, since the current is zero, at that instant, in the conductors contained in them. The magnetomotive force, therefore, remains constant between holes Nos. 8 and 13. The difference of magnetic potential produced by these four coils has, for its maximum value,

$$\frac{4\pi}{10} n \frac{\sqrt{3}}{2} I_{max} 4,$$

and the corresponding maximum magnetic density becomes

$$B_{max} = \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I_{max} \frac{4}{d};$$

$$B_{max} = \frac{4\pi}{10} n I_{max} \frac{3.46}{d};$$

and the magnetic flux issuing from one of the poles is,

$$\Phi = \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I_{max} \frac{a}{d} (1 + 2 + 3 + 4 + 4 + 2) 2 L.$$

$$\Phi = \frac{4\pi}{10} n I_{max} \frac{a}{d} \sqrt{3} \times 16 \times L.$$

$$\Phi = \frac{4\pi}{10} n I_{max} \frac{a}{d} \times 27.68 \times L.$$

This relation, which is applicable at the moment when the current in the first phase becomes equal to $\frac{\sqrt{3}}{2} I_{max}$, differs slightly from the one which we had obtained when the current in this same phase had reached its maximum, I_{max}.

The magnetic flux produced in a pole is not, therefore, absolutely the same in both cases. The discrepancy is, nevertheless, very small, since it does not exceed

$$100 \frac{0.16}{28} = 0.5\%$$

of the mean value, which is

$$\Phi_{max} = \frac{4\pi}{10} n I^{max} \frac{a}{d} \times 27.84 \times L.$$

The magnetic flux is now distributed in a manner entirely different from what it was in the first case. The value found for B_{max} is here about 16% lower than before.

Let us again plot (Fig. 6) the distribution of the magnetic density in the air-gap at the instant when the current in the first phase will have decreased from $\frac{\sqrt{3}}{2} I_{max}$ to $\frac{1}{2} I_{max}$. The current in the second branch will have the following value:

$$i_{II} = + \tfrac{1}{2} I_{max},$$

and the current in the third branch will be

$$i_{III} = - I_{max}.$$

The new broken line is absolutely identical with the first one, represented in Fig. 4. The magnetic flux issuing from each pole is, therefore, exactly the same as in the first case; but the maximum magnetic density no longer occurs at the same point. The second broken line already seemed to indicate that the magnetic field was being displaced, since the maximum extended more toward the right than before, and the points A and D, which were originally opposite the slots 3 and 14, now come opposite the slots 5 and 16. In the third diagram, these points A and D have arrived opposite the conductors Nos. 7 and 18.

If now (see Fig. 7) we take

$$i_{I} = 0,$$

$$i_{II} = +\frac{\sqrt{3}}{2},$$

and

$$i_{III} = -\frac{\sqrt{3}}{2},$$

we obtain a fourth broken line absolutely identical with the second one already obtained. The points A and D will have again travelled towards the right, and, at the particular moment considered, will be opposite the slots 9 and 20. Thus, while the current in the first phase was passing from its maximum value to zero, that is to say, during the time of one-quarter of a period, the magnetic field became displaced in the air-gap by a distance equal to one-eighth of the circumference. By continuing these graphical representations, we would find that when the current in the first phase returns to the maximum value, after having passed through a complete period, the magnetic field has undergone a displacement equal to one-half of the circumference of the stator, and that, consequently, the number of turns per minute of the magnetic field in the case of this particular winding satisfies the following relation:

$$N = \frac{60}{2T} = \frac{30}{T},$$

in which N equals the number of complete revolutions of the magnetic field in the air-gap per minute, and T equals the duration of each period, in seconds.

If, for example, the three-phase currents used had a frequency of 50 periods per second, the duration of a period would be,

$$T = \tfrac{1}{50} \text{ of a second},$$

and the number of revolutions of the revolving field would be,

$$N = \frac{30}{\frac{1}{50}} = 30 \times 50 = 1500 \text{ revolutions per minute.}$$

The winding that we have just studied produces four magnetic poles, alternating in polarity. We have seen that when the current makes a complete period and returns to its original value, each magnetic flux is displaced in the air-gap by the space corresponding to two consecutive magnetic poles, and that it reaches the point at which the magnetic flux of same polarity which precedes it in the rotative movement was previously to be found. This circumstance indicates that the number of revolutions made by the magnetic field in each unit of time depends essentially on the number of poles of that field. They will be one-half less than before with a winding producing eight magnetic poles instead of four. In general, if we designate the number of poles by p, the angular velocity, N, of the magnetic field in turns per minute will be

$$N = \frac{60 \times 2}{pT} = \frac{120}{pT}. \qquad (1)$$

Thus, for example, a 10-pole winding would impart to the magnetic field an angular velocity of

$$N = \frac{120}{10 \times \frac{1}{50}} = 12 \times 50 = 600 \text{ revolutions per minute,}$$

when the frequency is equal to 50 periods per second.

We will return later to this equation, to show that it is not only true for three-phase motors, but that it applies, in general, to all polyphase motors.

We have seen that the mean magnetic flux issuing from one of the poles was, for the particular case illustrated in Figs. 4 and 5,

$$\Phi_{av} = 27.84 \frac{4}{10} n I_{max} \frac{a}{d} \times L.$$

If we take I_{max} in amperes, and a, d, and L in centimetres, this flux will be expressed in C.G.S. units, that is to say, in maxwells.

To facilitate future investigations, which would be long and difficult, if we wish to base them on the real distribution of the magnetic density in the air-gap, as given by the broken lines, we will suppose that the magnetic density varies, not in accordance with these broken lines, but in accordance with a sine curve, such that the resulting sinusoidal distribution of magnetic potential would produce a magnetic flux exactly equal to that which is actually produced. This is equivalent to saying that the area of a half-wave of the sine curve in question, multiplied by the width, L, of the magnetic cores should be equal to Φ_{av}; but we know that the area S comprised between this half-wave and the axis of abscissæ, is given by the equation,

$$S = \frac{2}{\pi} B_{max}\, l,$$

since $\frac{2}{\pi} B_{\max}$ is nothing else than the mean ordinate of any sine curve, and l represents half the wave length.

We can, therefore, write,

$$L\frac{2}{\pi} B_{\max} l = 27.84 \times \frac{4\pi}{10} n I_{\max} \frac{a}{d} L,$$

but inasmuch as, in the particular case,

$$l = 12\, a,$$

we therefore have

$$B_{\max} = \frac{27.84}{24} \pi \frac{4\pi}{10} n I_{\max} \frac{1}{d},$$

or,

$$B_{\max} = 3.642 \times \frac{4\pi}{10} n I_{\max} \frac{1}{d},$$

or again,

$$B_{\max} = 4.574\, n I_{\max} \frac{1}{d},$$

This equation gives us the maximum ordinate of the sine curve which replaces and is equal to the broken lines, and gives a theoretically ideal distribution of the magnetic density in the air-gap, while leading to the same actual mean value of the magnetic flux issuing from each of the magnetic field poles.

This equivalent sine curve has been drawn on the three broken lines in Figs. 5, 6, and 7, to show that this curve deviates but little from the broken lines, and that it, so to speak, covers them substantially.

All that has been said, thus far, relates exclusively to the winding illustrated in Fig. 1, in which each of the six coils is placed in eight slots.

This arrangement is in very general use; but it frequently happens that the designer may place the conductors of a coil in 4, 6, 8, 10, or even 12 slots, the active side of each coil then occupying one-half of these slots. The number of slots per coil naturally has no effect on the speed of rotation of the magnetic field, which depends only on the number of poles, since, during an entire period, a north pole has advanced only enough to take exactly the place previously occupied at the beginning of the period by the north pole which precedes it in rotation. The magnetic field will turn more slowly in proportion as the number of poles is increased, and as the number of pulsations of the currents is diminished. In general, the time required for a complete revolution of the magnetic field is equal to the duration of a complete period, multiplied by one-half the number of magnetic poles.

Thus, with a 12-pole winding, and a frequency of 50 periods per second, the time (t) required for a complete revolution of the magnetic field would be:

$$t = \tfrac{6}{50} \text{ seconds.}$$

From which it follows at once that the number of turns made by the magnetic field is:

$$N = \frac{1}{\frac{6}{50}} = \tfrac{50}{6} \text{ revolutions per second,}$$

or 500 revolutions per minute.

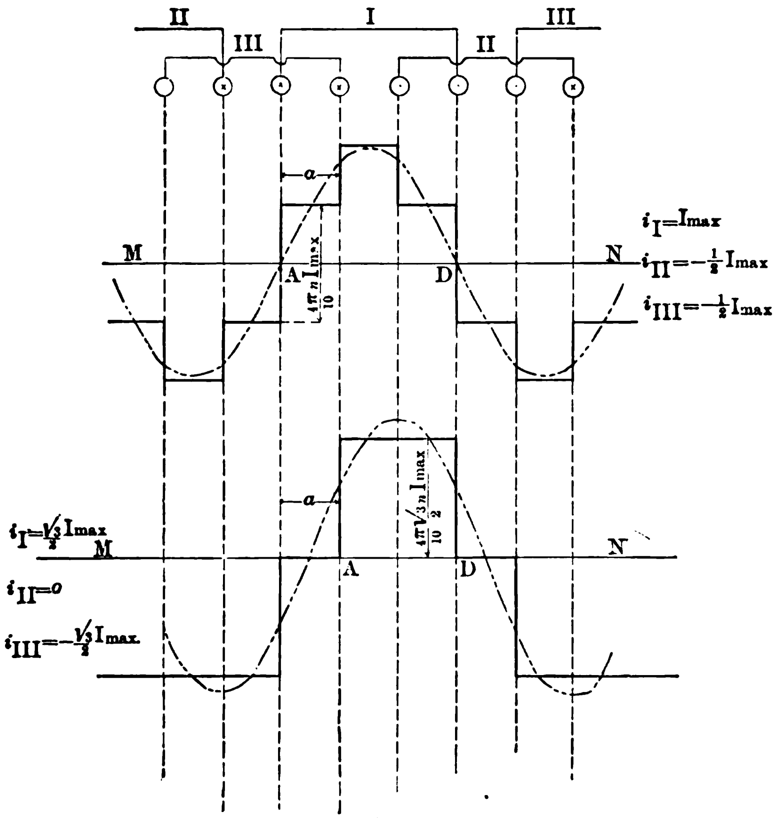


Fig. 8.

Fig. 8 represents a portion of a three-phase winding, in which the coils are placed each in two slots only.

The zig-zag curves, constructed in the manner previously explained, indicate the actual distribution of the magnetic density in the air-gap at the two different instants of time at which the maximum, and the minimum, magnetic flux occurs.

For the first broken line, we have,

$$\Phi = L \times \frac{4\pi}{10} nI_{\max} \frac{a}{d} (0.5 + 1 + 0.5)$$

$$\Phi = L \times \frac{4\pi}{10} nI_{\max} \frac{a}{d} \times 2,$$

and, for the second broken line, we have,

$$\Phi = L \frac{4\pi}{10} nI_{\max} \frac{a}{d} \left[2 \times \frac{\sqrt{3}}{2}\right],$$

$$\Phi = L \frac{4\pi}{10} nI_{\max} \frac{a}{d} \times 1.731,$$

so that the mean or average value is,

$$\Phi_{av} = 1.865 \frac{4\pi}{10} nI_{\max} \frac{a}{d} L.$$

We find at once, for the maximum ordinate of the sine curve giving the ideal distribution of the magnetic density in the air-gap, the following value :

$$l \frac{2}{\pi} B_{\max} L = 1.865 \frac{4\pi}{10} nI_{\max} \frac{a}{d} L;$$

but, since here,

$$l = 3a,$$

it follows that

$$B_{\max} = 1.227\, nI_{\max} \frac{1}{d}.$$

If we draw the equivalent sinusoidal curve over each of the two broken lines, we see that the latter depart materially from the ideal curve, and that in replacing them by these sinusoidal curves, we are making a somewhat rough approximation.

This last arrangement of slots, which is only used in motors of less than one-half horse-power, is not of great practical importance. It is, however, preferable to allow, even for small motors, two, or better still, three slots, for the active side of each coil.

Fig. 9 shows that with four slots per coil, the broken lines still depart considerably from the equivalent sinusoidal curve. With six slots per coil (Fig. 10), the approximation is already quite satisfactory.

We have already discussed at length the case where each coil is placed in eight slots. Fig. 11 shows the case where each active side is distributed in five slots, and Fig. 12 shows the distribution of the magnetic field in the air-gap when each coil occupies 12 slots.

In these two last cases, the equivalent sinusoidal curves absolutely cover the broken line.

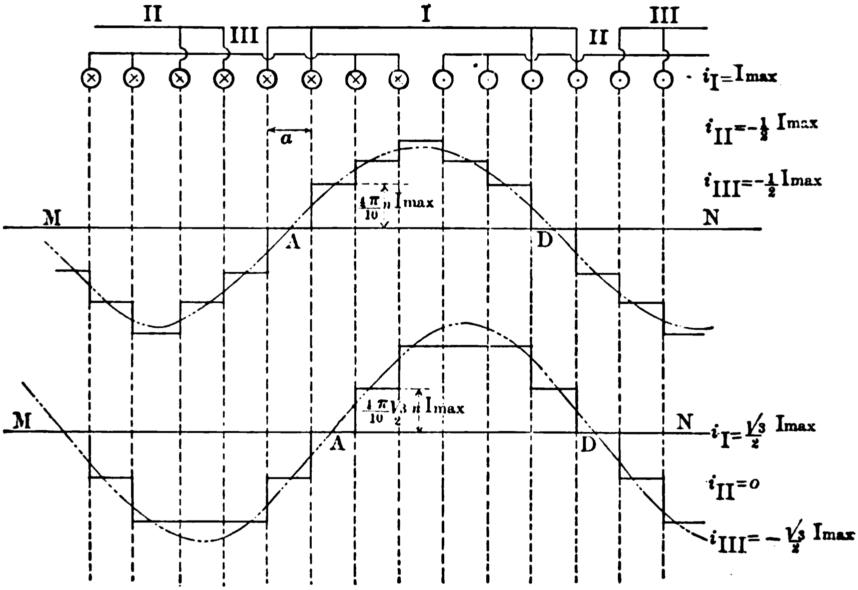

II
I
III
III
II
$i_I = I_{max}$
$i_{II} = -\frac{1}{2} I_{max}$
$i_{III} = -\frac{1}{2} I_{max}$
a
M
N
A
D
$\frac{4\pi}{10} n I_{max}$
$\frac{4\pi}{10} \frac{\sqrt{3}}{2} n I_{max}$
$i_I = \frac{\sqrt{3}}{2} I_{max}$
$i_{II} = o$
$i_{III} = -\frac{\sqrt{3}}{2} I_{max}$

Fig. 9.

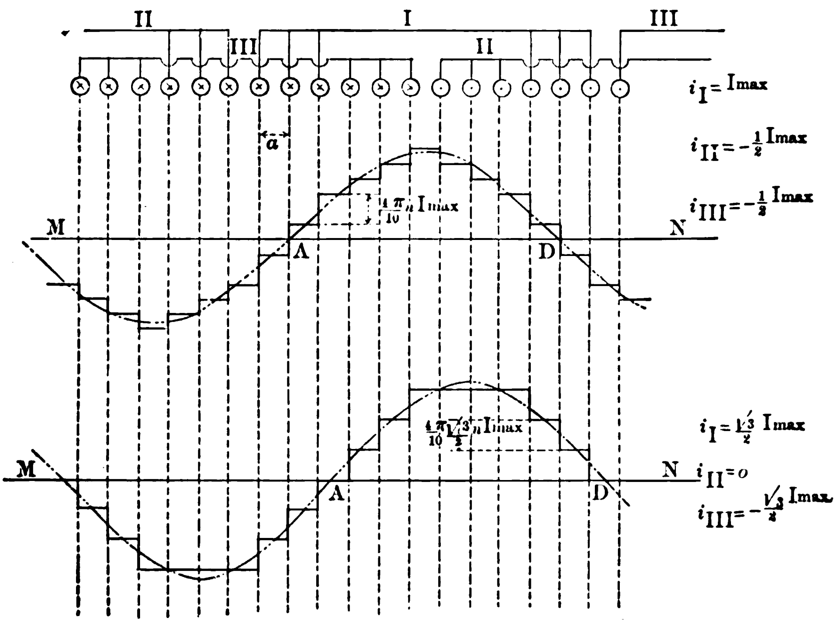

II
I
III
III
II
$i_I = I_{max}$
$i_{II} = -\frac{1}{2} I_{max}$
$i_{III} = -\frac{1}{2} I_{max}$
a
M
N
A
D
$\frac{4\pi}{10} n I_{max}$
$\frac{4\pi}{10} \frac{\sqrt{3}}{2} n I_{max}$
$i_I = \frac{\sqrt{3}}{2} I_{max}$
$i_{II} = o$
$i_{III} = -\frac{\sqrt{3}}{2} I_{max}$

Fig. 10.

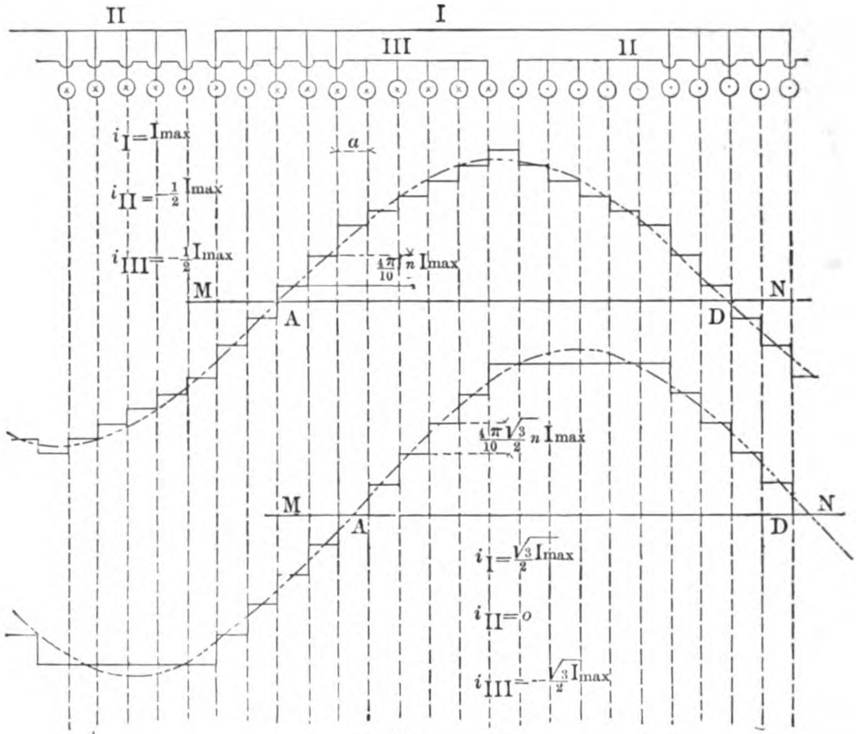

II
I
III
II
$i_I = I_{max}$
$i_{II} = -\frac{1}{2} I_{max}$
$i_{III} = -\frac{1}{2} I_{max}$
a
$\frac{4\pi}{10} n I_{max}$
M
N
A
D
$\frac{4\pi}{10} \frac{\sqrt{3}}{2} n I_{max}$
M
A
D
N
$i_I = \frac{\sqrt{3}}{2} I_{max}$
$i_{II} = o$
$i_{III} = -\frac{\sqrt{3}}{2} I_{max}$

Fig. 11.

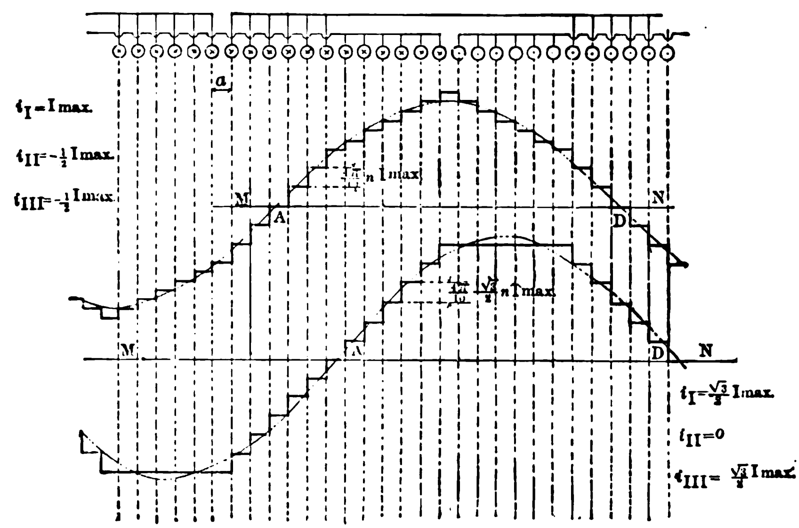

a
$i_I = I_{max}$
$i_{II} = -\frac{1}{2} I_{max}$
$i_{III} = -\frac{1}{2} I_{max}$
M
A
D
N
M
A
D
N
$i_I = \frac{\sqrt{3}}{2} I_{max}$
$i_{II} = o$
$i_{III} = \frac{\sqrt{3}}{2} I_{max}$

Fig. 12.

It is unnecessary to proceed further and to examine still other arrangements, for the reason that three-phase magnetic fields having more than six slots per active side of coil are but rarely met with. If, however, such a case should arise, the reader could, knowing what has already been stated, easily trace the broken lines corresponding to the maximum and minimum magnetic fluxes, and then estimate the maximum ordinate of the equivalent sinusoidal curve giving the ideal distribution of the magnetic flux.

From what precedes, we see that the maximum density in the air-gap can, in practice, be represented by the following equation: —

$$B_{max} = KnI_{max}\frac{1}{d},$$

in which

K = a coefficient.

n = the number of conductors per slot.

I_{max} = the maximum current, in amperes, passing through each conductor.

d = the (radial) distance across (or "length" of) the air-gap, in centimetres.

We know that the coefficient K depends only on the number of slots per coil. Its value is given in the following table: —

THREE-PHASE MOTORS.

Long Coil Windings.

SLOTS PER COIL.	VALUES OF K.
2	1.227
4	2.288
6	3.432
8	4.574
10	5.720
12	6.864

If we take effective values instead of maximum values, the preceding equation becomes

$$b = Kni\frac{1}{d},$$

in which b equals the effective magnetic density in the air-gap in gausses, and i, the effective current value, in amperes.

For Two-Phase Currents. (Long Coil Windings). — When we come to consider two-phase currents instead of three-phase currents, the values of the coefficient K are no longer the same as given in the preceding table.

It suffices, in order to demonstrate this and to determine the new values of the coefficient K, to examine the winding diagram of the stator of a two-phase motor having an ordinary winding with long coils or coils passing through sets of "polar" slots, and to note for any given instant, the value and the direction of the current in each slot. It will be remembered, for instance, that at the moment when the current value in the first-phase reaches its maximum, the current in the second phase is zero, and that an instant later these two currents have the same direction and are both equal to $\frac{I_{max}}{\sqrt{2}}$.

Bearing these facts in mind, it will be easy to trace the lines representing the actual distribution of the magnetomotive force, or of the magnetic density, in the air-gap, at different moments.

By means of these broken lines, we will be able to estimate the mean or average magnetic flux issuing from each pole, and, then, to deduce therefrom the maximum ordinate of the equivalent sinusoidal curve representing the ideal distribution of the magnetic density in the air-gap.

Making the necessary calculations, relative to Figs. 13 to 18 inclusive, which show two-phase windings in which each coil is placed in, respectively, 2, 4, 6, 8, 10, and 12 slots, we will obtain the following final relation:—

$$B_{max} = KnI_{max}\frac{1}{d},$$

in which the coefficient K takes the values indicated in the following table:—

TWO-PHASE MOTORS.

Long Coil Windings.

SLOTS PER COIL.	VALUES OF K.
2	0.842
4	1.400
6	2.160
8	2.800
10	3.625
12	4.200

The preceding relation is identical with the one already found for three-phase motors. It is evident that it also applies, without change, to four-phase or six-phase windings, which windings, however, are but rarely used.

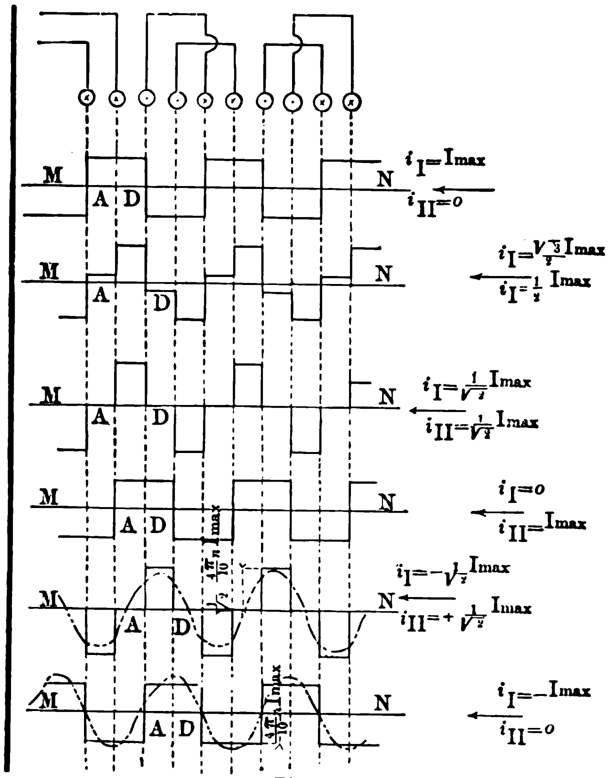

M
A
D
N
$i_I = I_{max}$
$i_{II} = o$
$i_I = \frac{\sqrt{3}}{2} I_{max}$
$i_I = \frac{1}{2} I_{max}$
$i_I = \frac{1}{\sqrt{2}} I_{max}$
$i_{II} = \frac{1}{\sqrt{2}} I_{max}$
$i_I = o$
$i_{II} = I_{max}$
$i_I = -\frac{1}{\sqrt{2}} I_{max}$
$i_{II} = +\frac{1}{\sqrt{2}} I_{max}$
$i_I = -I_{max}$
$i_{II} = o$

Fig. 13.

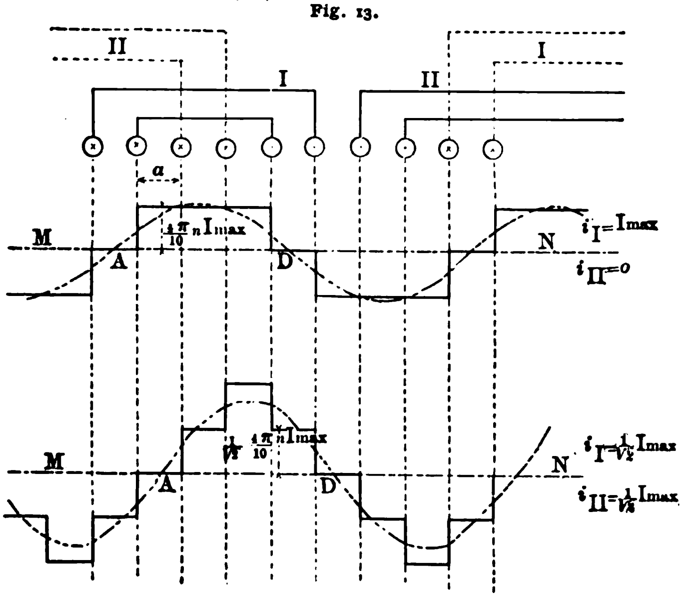

II
I
a
M
A
D
N
$\frac{4\pi}{10} n I_{max}$
$i_I = I_{max}$
$i_{II} = o$
$i_I = \frac{1}{\sqrt{2}} I_{max}$
$i_{II} = \frac{1}{\sqrt{2}} I_{max}$

Fig. 14.

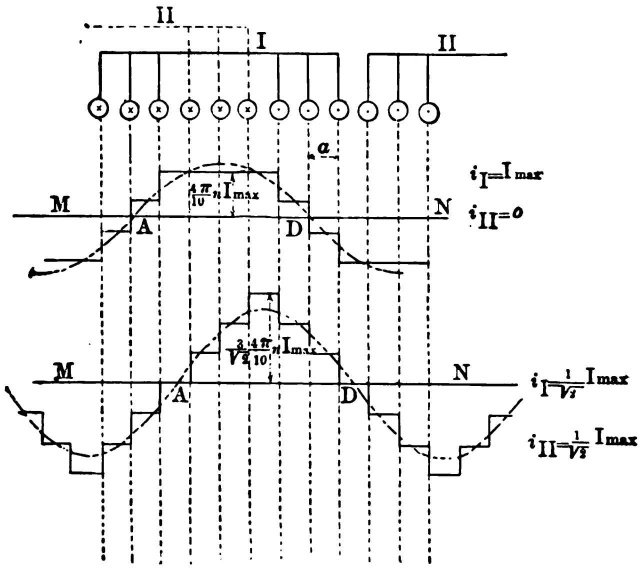

II
I
II
a
$i_I = I_{max}$
$i_{II} = 0$
M
N
A
D
$\frac{4\pi}{10} n I_{max}$
M
N
A
D
$\frac{3}{\sqrt{2}} \frac{4\pi}{10} n I_{max}$
$i_I = \frac{1}{\sqrt{2}} I_{max}$
$i_{II} = \frac{1}{\sqrt{2}} I_{max}$

Fig. 15.

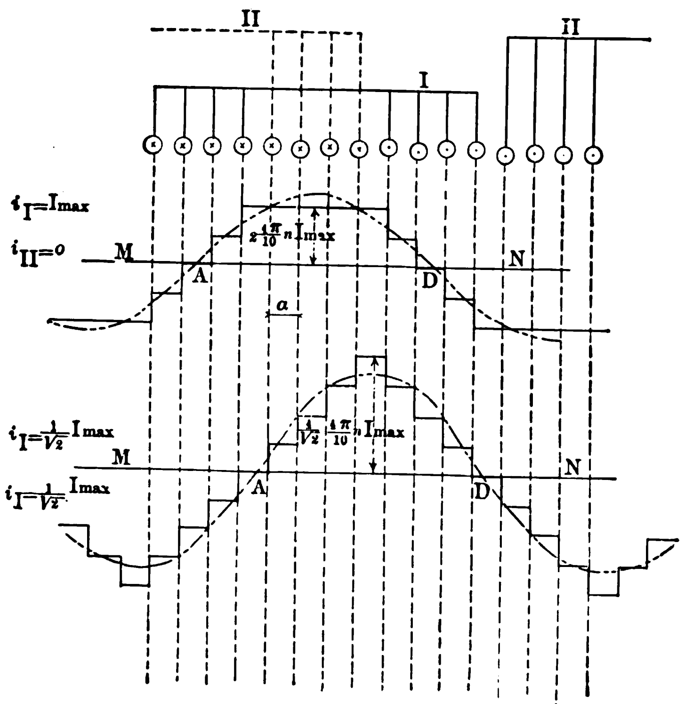

II
I
II
$i_I = I_{max}$
$i_{II} = 0$
M
N
A
D
$2 \frac{4\pi}{10} n I_{max}$
a
$\frac{4}{\sqrt{2}} \frac{4\pi}{10} n I_{max}$
$i_I = \frac{1}{\sqrt{2}} I_{max}$
$i_I = \frac{1}{\sqrt{2}} I_{max}$
M
N
A
D

Fig. 16.

This formula is, therefore, general.

Fig. 13 shows that, during each complete period, the magnetic field turns a distance equal to the distance between the centre lines of two consecutive poles of like polarity.

It follows from this, first, that the angular velocity of the field depends, here also, on the number of poles in the magnetic field, and on

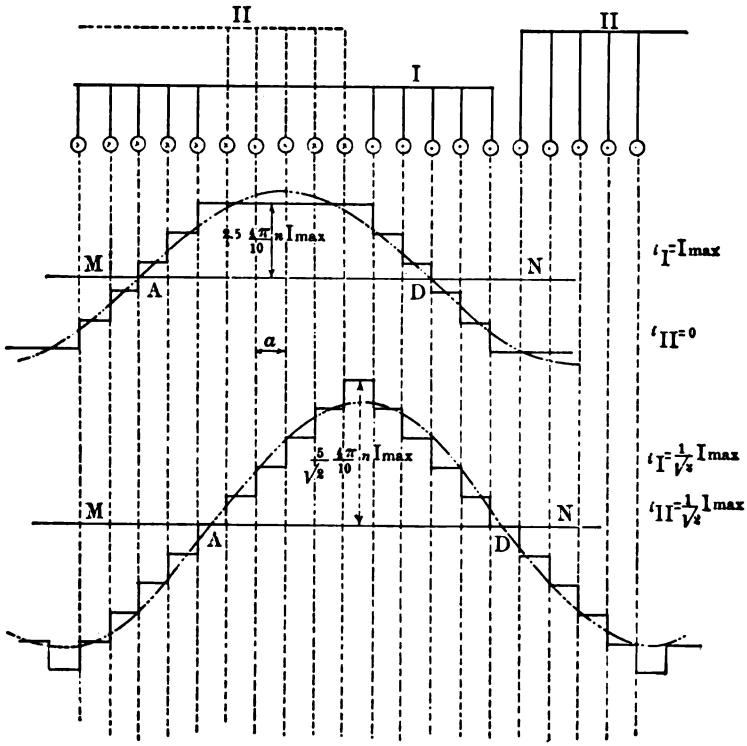


Fig. 17.

the number of pulsations of the current, and, second, that this angular velocity remains constant, whatever may be the number of phases of the motor.

The duration of a complete revolution of the magnetic field will always be:

$$= T \times \frac{p}{2},$$

in which T designates the duration of the period of the current, and p

the number of poles of the magnetic field. The number of revolutions of the magnetic field, per minute, will therefore be:

$$N=\frac{120}{Tp},$$

which is a general equation.

The primary cores of polyphase motors are not always provided with windings of the drum type with coils embedded in slots. They sometimes comprise windings with ring coils, or else progressive drum or wave windings.

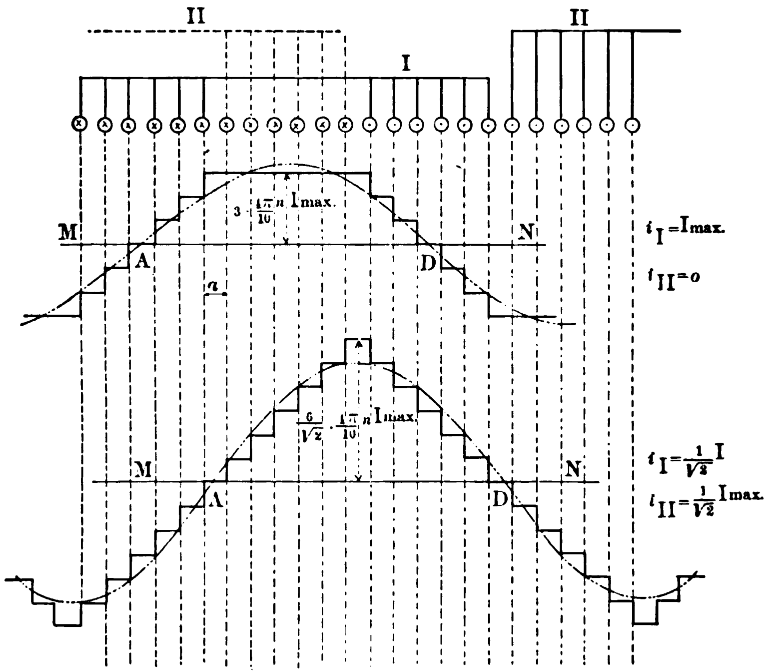


Fig. 18.

Polyphase Fields Having Ring Windings. By examining Fig. 19, which represents a three-phase, four-pole, winding of the ring type, it will be noticed that the magnetic flux is produced in the air-gap by connecting the coils in opposition.

If we compare this diagram with that in Fig. 1, we will see that these two windings are necessarily identical in their effects, since they both lead to an absolutely similar distribution of the conductors and currents in the slots.

A drum winding with long coils requires the use of two coils per phase to produce four poles in the air-gap. Each of these coils has two

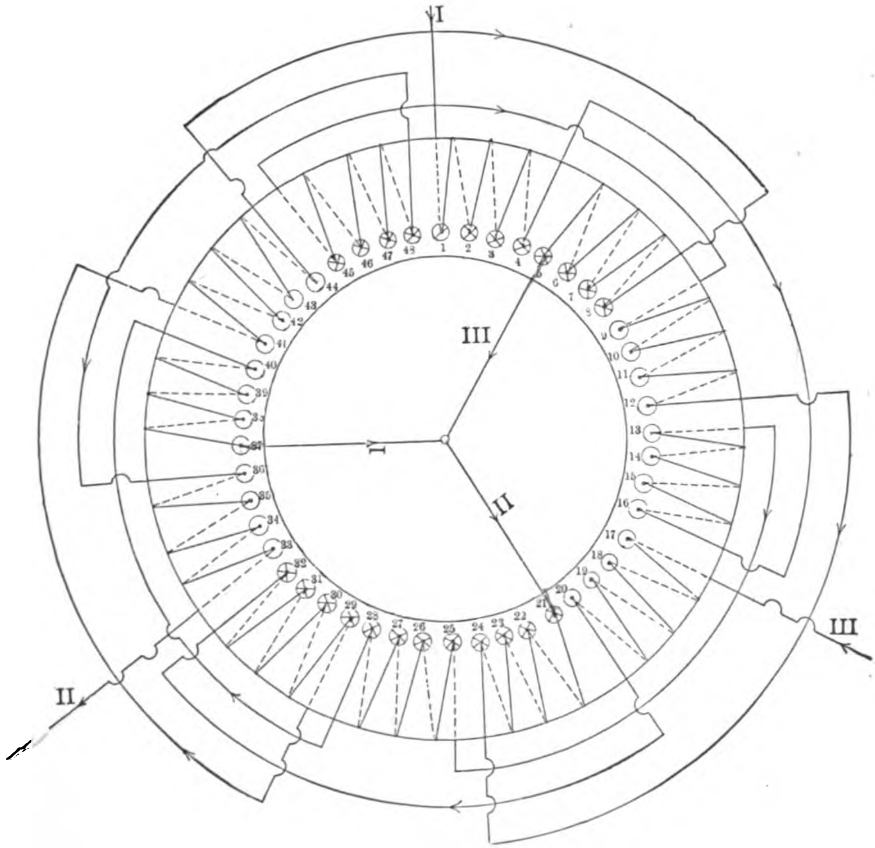


Fig. 19.

active sides. On the contrary, the ring winding requires four coils per phase, each having but one single active side.

The values of the coefficient *K*, therefore, remain the same as those already found, so that we at once obtain the two following tables:

THREE–PHASE MOTORS.

Ring Type Windings.

Slots per Coil.	Values of *K*.
1	1.227
2	2.228
3	3.432
4	4.574
5	5.720
6	6.864

TWO—PHASE MOTORS.

Ring Type Windings.

SLOTS PER COIL.	VALUES OF K.
1	0.842
2	1.400
3	2.160
4	2.800
5	3.625
6	4.200

Windings of the ring type are now used less and less frequently in the construction of polyphase motors, the reason being that the machines are heavier and more expensive than when the winding is of the drum type.

Polyphase Fields Comprising Progressive Windings. While this arrangement is rarely used for the stators, it is, on the contrary, very common for the rotors, where it is employed largely because it enables a simple and substantial winding to be obtained.

As we will see later, this winding is inferior to the preceding windings from the standpoint of the best utilization of materials.

A winding of this character is the same as an ordinary direct-current wave winding, designed for as many poles as the magnetic field has to produce. It suffices to open such a direct-current wave winding, which is normally closed upon itself, at three points, 120° apart, in order to transform it into a three-phase winding, or to open it at four points, 90° apart, to obtain a two-phase diagram.

If we take:

S = the total number of conductors in the winding;
y = the "pitch" of the winding;
p = the number of poles;

we know that in order to obtain a diagram closed upon itself and utilizing all the conductors once only, we must have the following relation:

$$y = \frac{S \pm 2}{p} = \text{an odd nnmber.}$$

This formula shows that it will not always be possible to obtain, with this arrangement, a perfectly regular three-phase winding. Indeed, since y is always an odd number, and since S must be divisible by 3, the numerator, $S \pm 2$, will not be a multiple of 3; but if the number of poles, p, is a multiple of 3, the quotient $\frac{S \pm 2}{p}$ cannot be a whole number.

This proves that a wave winding for direct current will not produce a perfectly regular and symmetrical three-phase winding, so long as the number of poles is a multiple of 3. The three branches will then necessarily have different numbers of conductors.

With a two-phase motor it will always be possible to obtain a regular construction, because S, $S + 2$, and p, are all even numbers; and it is therefore possible that the quotient $\frac{S \pm 2}{p}$ may give a whole number for the value of y. By thus cutting a direct-current wave winding at three or four points distant 120° or 90° from each other, we obtain 6 or 8 ends, which it is much more easy to arrange in the rotor than in the stator, where a winding of this kind is not at all advantageous. We will not here undertake to estimate the maximum ordinate of the sinusoidal curve giving the ideal distribution of the magnetic density in the air-gap for the different cases which can arise with progressive windings. We will return to this subject when we come to discuss armature reaction, for we will then have to examine various other kinds of winding which cannot be practically used in the construction of the stators for producing the revolving magnetic field.

CHAPTER II.

THE ELECTROMOTIVE FORCES INDUCED BY THE REVOLVING FIELD.

It is well known that a conductor moving in a magnetic field becomes the seat of an electromotive force. It is also well known that if the conductor be straight, of unit length (1 cm.), and if it be moving in a magnetic field whose magnetic density is equal to unity (1 gauss), in such manner that its displacement is in a line perpendicular to itself and to the magnetic field, and taking place at the rate of unit distance (1 cm.) in unit time (1 sec.), this electromotive force is, by definition, equal to the (C.G.S.) unit of E.M.F., or the one-hundred millionth of a volt ($=10^{-8}$ volt).

If this conductor had a length of L centimetres, if the magnetic field had a magnetic density of b units, and if the displacement took place at the rate (velocity) of V centimetres per second, the induced E.M.F., *in volts*, would be such as expressed by the following equation:—

$$e = LVb \times 10^{-8} \text{ volts.}$$

When L represents the (axial) width of (magnetic) core of the motor, and V the speed of the rotating field, and b the magnetic density in the air-gap at a given point and at a given moment, this equation gives the E.M.F. induced in *one* of the conductors at that particular time and place.

Let us note that no assumption has been made as to the distribution of the magnetic flux in the air-gap, and that, consequently, the preceding formula is applicable, no matter what may be the distribution of this flux.

The induced E.M.F. is always equal to the product of the active length of conductor by the magnetic flux and by the velocity at the instant under consideration.

If the distribution of the flux is known, it will be easy to determine the E.M.F. induced at each point, and to trace the form of the wave representing the variation of this E.M.F. as a function of the time. It is by this method that we determine the effective value of the potential difference at the terminals of polyphase generators, in which case the distribution of the flux in the air-gap could scarcely be expressed by an algebraical equation.

Definite Windings placed in Slots. This arrangement, which was mentioned in the preceding chapter, and which is shown in Figs. 1 to 18, comprises, for each phase, a number of coils equal to half the number of poles. These coils are all identical with each other, and they can be placed in one or in several slots.

Motors having coils each placed in more than six and less than two pairs of openings are rarely seen. The more the coil is subdivided, the more nearly does the field produced by it approach the sinusoidal curve. However, when the number of openings is made too large, the price of the motor is materially increased, because there is more punching to be done in the iron-core sheets, and also because the insulation of the numerous sets of wires involves more hand-work, while at the same time taking up valuable space, the loss of which has to be made up by increasing the diameter of the field-bore or the thickness of the cores.

The author recommends allowing not more than from 15 to 18 slots per pole for the largest motors, coming down to 9 or 10 for motors of less than 5 horse-power.

The coils of the same phase are generally connected in series. It may happen, however, that difficulties will arise in arranging the necessary windings, to avoid which difficulties, several sets of coils, or even all the coils, may have to be connected in multiple. The first arrangement is preferable, since it avoids altogether the occurrence of internal currents resulting from the possible inequality of the electromotive forces induced in the different coils of the same phase.

The windings for each pair of slots can, without inconvenience, be made up of several wires connected in multiple, but care should be taken to avoid connecting in multiple, in the same coil, wires not placed in the same slots, since these would produce electromotive forces of the same phase, but of unequal amplitudes, which would give rise to internal currents capable of producing excessive heating.

All the sets of conductors constituting one coil have a common symmetrical plane, and their widths are different with respect to each other; thus, in Fig. 1, the set of conductors occupying the openings 1 and 16, is the widest; that which is in the openings 4 and 13 is, on the contrary, the narrowest. The distance between the centres of the two active sides or the mean width of each coil, is always equal to that of the poles of the magnetic field.

Three-Phase Motors. It is sufficient to examine the diagram of the "polar" winding shown in Fig. 20 to ascertain that the active side of

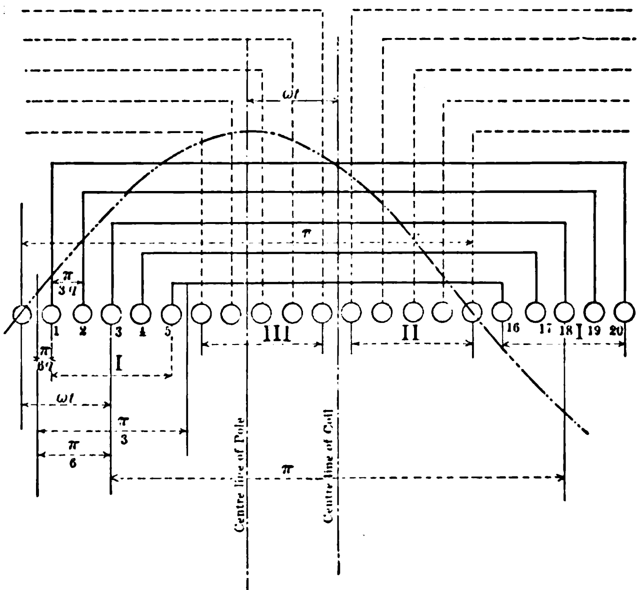


Fig. 20.

each coil occupies one-third of the width of the pole. If we suppose that, at a given moment, t, the axis of the coil, is distant from that of the pole by the angle ωt, and that the number of holes per coil be $2q$, the angular distance between the outermost hole, for any coil whatsoever, and the middle of the active side to which it belongs, will be, for one side

$$\omega t - \left(\frac{\pi}{6} - \frac{\pi}{6q}\right) = \omega t - \frac{\pi}{6q}(q - 1),$$

and for the other side,

$$\omega t + \left(\frac{\pi}{6} - \frac{\pi}{6q}\right) = \omega t + \frac{\pi}{6q}(q - 1),$$

since the angular distance between two consecutive openings is $\frac{\pi}{3q}$.

The electromotive force induced in each conductor placed in the external openings 1 and 20 of the coil will, therefore, be

$$e_1 = LVB \sin\left[\omega t + \frac{\pi}{6q}(q - 1)\right] \times 10^{-8}$$
$$+ LVB \sin\left[\omega t - \frac{\pi}{6q}(q - 1)\right] \times 10^{-8} \text{ volts,}$$

or

$$e_1 = LVB \left\{ \sin\left[\omega t + \frac{\pi}{6q}(q - 1)\right] + \sin\left[\omega t - \frac{\pi}{6q}(q - 1)\right]\right\} 10^{-8} \text{ volts,}$$

but it is well known that

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cos \beta,$$

from which it follows that

$$e_1 = 2\,LVB \sin(\omega t) \cos\left[\frac{\pi}{6q}(q - 1)\right] 10^{-8} \text{ volts.}$$

In like manner, we will have, for the E.M.F. induced in each of the wires placed in the second and in the next to the last opening of the coil,

$$e_2 = 2\,LVB \sin(\omega t) \cos\left[\frac{\pi}{6} - \left\{\frac{\pi}{6q} + \frac{\pi}{3q}\right\}\right] 10^{-8} \text{ volts,}$$

or

$$e_2 = 2\,LVB(\omega t) \cos\left[\frac{\pi}{6q}(q - 3)\right] 10^{-8} \text{ volts.}$$

By taking the sum of the simultaneous electromotive forces e_1, e_2, e_3, e_t, and then dividing by the number of holes of the coil, $2q$, we will have the mean E.M.F. induced, not in each turn, but in each conductor. This E.M.F. may be expressed as follows: —

$$e_{av} = LVB \sin(\omega t) \times$$

$$\left\{\frac{\cos\left[\frac{\pi}{6q}(q-1)\right] + \cos\left[\frac{\pi}{6q}(q-3)\right] + \cdots + \cos\left[\frac{\pi}{6q}\{q-(2q-1)\}\right]}{q}\right\} 10^{-8} \text{ volts.}$$

If we take

$$K_1 = \frac{\cos\left[\frac{\pi}{6q}(q-1)\right] + \cos\left[\frac{\pi}{6q}(q-3)\right] + \cdots + \cos\left[\frac{\pi}{6q}(1-q)\right]}{q}$$

and if we substitute K_1 in the previous equation, we will have

$$e_{av} = K_1 LVB \sin(\omega t)\ 10^{-8} \text{ volts.}$$

If we multiply this average E.M.F. by the number, Z, of all the conductors connected in series in the same phase, we will obtain the total E.M.F. induced in that phase. Its value is

$$E = K_1 ZLVB \sin(\omega t)\ 10^{-8} \text{ volts.}$$

This E.M.F. varies according to the sinusoidal law, and is a maximum when

$$\sin(\omega t) = 1,$$

or when

$$(\omega t) = 90°;$$

that is to say, when the centre lines of the active sides of the coils coincide with the centre lines of the magnetic poles.

Let us note, in passing, that the component electromotive forces induced in the windings placed in the different pairs of holes have different amplitudes but are of the same phase.

The values of the coefficient K_1, which depend on the number of slots per coil, are indicated in the following table:—

THREE-PHASE MOTORS.

Calculation of Induced E.M.F.

Number of Slots per Active Side, $q =$	Coefficient Values $K_1 =$
1	1.000
2	0.966
3	0.960
4	0.958
5	0.957
6	0.956
10	0.955

Two-Phase Motors. The wires forming one of the active sides of a coil here occupy one-half the width of the pole. If we assume, as before (Fig. 21), that the centre line of the coil is, at any given moment,

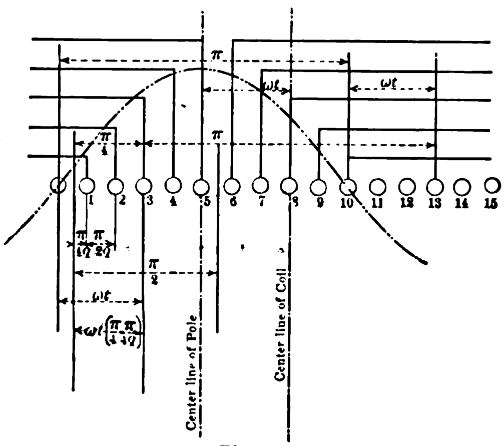


Fig. 21.

distant from that of the pole by the angle ωt, and if we again designate by $2q$ the number of slots per coil, and by π the angular width of the pole, then the angular distance between any two consecutive slots will be

$$\frac{\pi}{2q},$$

and that which separates the outermost slot from the middle of the active side to which it belongs will be as follows: —

On the right,

$$\omega t - \left\{\frac{\pi}{4} - \frac{\pi}{4q}\right\} = \omega t - \frac{\pi}{4q}(q-1);$$

and on the left,

$$\omega t + \left\{\frac{\pi}{4} - \frac{\pi}{4q}\right\} = \omega t + \frac{\pi}{4q}(q-1).$$

The induced E.M.F. in each of the loops placed in the outermost slots, 1 and 15, of the coil, will be

$$e_1 = LVB\left\{\sin\left[\omega t + \frac{\pi}{4q}(q-1)\right]\right\}10^{-8}$$
$$+ LVB\left\{\sin\left[\omega t - \frac{\pi}{4q}(q-1)\right]\right\}10^{-8}\text{ volts.}$$

We have seen that this equation can be reduced to

$$e_1 = 2\,LVB\sin(\omega t)\cos\left[\frac{\pi}{4q}(q-1)\right]10^{-8}\text{ volts.}$$

For the windings placed in the second and in the next to the last slot, we will likewise have

$$e_2 = 2LVB\sin(\omega t)\ \cos\left[\frac{\pi}{4q}(q-3)\right]10^{-8}\text{ volts.}$$

By summing up all the electromotive forces simultaneously induced in each pair of holes for the whole coil, and then dividing by the number, $2q$, of slots of which this coil is made up, we obtain the mean induced electromotive force per conductor, which is as follows: —

$$e_{av} = 2LVB\sin(\omega t)\times$$
$$\left\{\frac{\cos\left[\frac{\pi}{4q}(q-1)\right]+\cos\left[\frac{\pi}{4q}(q-3)\right]+\cdots+\cos\left[\frac{\pi}{4q}(1-q)\right]}{q}\right\}10^{-8}\text{ volts,}$$

and taking

$$K = \frac{\cos\left[\frac{\pi}{4q}(q-1)\right]+\cos\left[\frac{\pi}{4q}(q-3)\right]+\cdots\cdot+\cos\left[\frac{\pi}{4q}(1-q)\right],}{q}$$

we have, on substituting K in the previous equation,

$$e_{av} = K_1LVB\sin(\omega t)10^{-8}\text{ volts.}$$

If Z designates the number of active conductors connected in series in one of the branches of the winding, the total induced E.M.F. in this branch will be expressed by the equation,

$$E = K_1 L V B \sin (\omega t) \; 10^{-8} \text{ volts.}$$

This E.M.F. varies according to the sinusoidal law, and is maximum when

$$\omega t = 90^\circ,$$

that is to say, when the middle lines of the active sides of the coils coincide with the centre lines of the magnetic poles.

The values of the coefficient K_1, which depend on the number of slots allotted to each coil, are indicated in the following table:

TWO—PHASE MOTORS.

Calculation of Induced E.M.F.

NUMBER OF SLOTS PER ACTIVE SIDE OF EACH COIL. $q=$	VALUES OF THE COEFFICIENT. K_1
1	1,000
2	0,924
3	0,911
4	0,906
5	0,904
6	0,903
10	0,901

We have seen, both in the case of three-phase and two-phase motors, that the total induced E.M.F in one of the windings at any given moment is expressed by the equation:

$$E = K_1 L V Z B \sin (\omega t) \; 10^{-8} \text{ volts.}$$

It is easy to reduce this formula to another form, often used. Let us take

D = the diameter of the air-gap circle, in centimetres;
p = the number of poles;
N = the number of revolutions per minute of the magnetic field;
Φ = the magnetic flux issuing from each pole (in maxwells);
ω = the angular velocity of the revolving field.

We can write at once:

$$V = \frac{\pi D N}{60},$$

and

$$\Phi = \frac{2}{\pi} B L \frac{\pi D}{p} = \frac{2 B L D}{p},$$

$$\omega = \frac{p \pi N}{60};$$

and if we replace V by its value, in the equation for electromotive force, we have

$$E = K_1 LZB \sin(\omega t) \frac{\pi DN}{60} 10^{-8} \text{ volts},$$

which can also be written :

$$E = K_1 \frac{2\,LBD}{p} \times \frac{\pi Np}{60} \times \frac{Z}{2} \sin(\omega t)\, 10^{-8} \text{ volts},$$

or

$$E = K_1 \frac{Z}{2} \omega \Phi \sin(\omega t)\; 10^{-8} \text{ volts},$$

but $\frac{Z}{2}$ is nothing else than the number of winding turns in series per phase. The maximum induced electromotive force can, therefore, be likewise expressed as follows :

$$E_{max} = K_1 \frac{Z}{2} \omega \Phi \; 10^{-8} \text{ volts},$$

The value of the coefficient K_1 is naturally the same as before.

Ring Windings. In drum windings with coils placed in slots, each loop has two of its sides placed on the periphery of the core. In ring windings, on the contrary, each loop has but one active side, and the loop is in a plane passing through the axis of the motor. Fig. 19, which gives the diagram of a three-phase ring winding, shows that there are, in each of the phases, as many coils as poles. The coils are generally all connected in series, but they are also, sometimes, connected either wholly or partly in parallel. For reasons already given, it is preferable to connect them in series.

The conductors placed in the same opening may be composed of several wires connected in parallel, but it is not allowable to couple in parallel the turns corresponding to different holes. As the successive coils of the same branch are subjected alternately to magnetic fluxes of opposite polarity, it is important to connect them properly, in order that the electromotive forces induced may be cumulative. We may connect, for example, the end of the first coil with the end of the second, and the beginning of the latter with the beginning of the third, and so on.

This mode of winding, which was considerably used when the first revolving field motors appeared, now occurs more and more rarely. It has no advantage over the drum winding with long coils, but requires more copper, more insulating material, and more handwork in the winding. It also necessitates mounting the magnetic cores in bronze or brass frames, in order to avoid the parasite magnetic fluxes and the Foucault currents which the inactive sides of the coils might produce.

These windings are sometimes used for the secondary (rotor), in cases where the motor is to be started by means of resistances placed at some distance from the motor. To meet this requirement, the designer is obliged to increase considerably the working potential difference in the branches of the armature, so as to reduce the loss of potential in the circuits connecting the motor with the starting appliances. In cases of this character, the potential difference between two neighboring coils reaches a high value, especially at the time of starting; and the ring type has the advantage that it enables all the coils to be properly separated, while, at the same time, enabling the rotor to be made very solid.

Three-Phase Motors. If we designate by q the number of slots per coil, the number of slots per pole will be $3q$, so that the space occupied by any complete coil in the air-gap will be equal to one-third of the pole width. The angular distance between any two consecutive slots will be equal to $\frac{\pi}{3q}$ and the angular distance separating the outer slots from the middle of the active side to which they belong will be:

$$\frac{\pi}{6} - \frac{\pi}{6q}.$$

At the particular instant t, the centre lines of the coils and of the poles will be separated by an angle ωt, and the electromotive force induced in each of the conductors lodged in the extreme left-hand slot (Fig. 22) will be:

$$e_1 = LVB \cos\left[\omega t - \left(\frac{\pi}{6} - \frac{\pi}{6q}\right)\right] \times 10^{-8} \text{ volts},$$

or

$$e_1 = LVB \cos\left[\omega t - \frac{\pi}{6q}(q-1)\right] 10^{-8} \text{ volts},$$

or again

$$e_1 = LVB \times$$
$$\left\{\cos(\omega t)\left[\cos\frac{\pi}{6q}(q-1)\right] + \sin(\omega t)\cos\left[\frac{\pi}{6q}(q-1)\right]\right\} 10^{-8} \text{ volts}.$$

The conductors in the next slots will give, successively:

$$e_2 = LVB \times$$
$$\left\{\cos(\omega)\cos\left[\frac{\pi}{6q}(q-3)\right] + \sin(\omega t)\cos\left[\frac{\pi}{6q}(q-3)\right]\right\} 10^{-8} \text{ volts}.$$

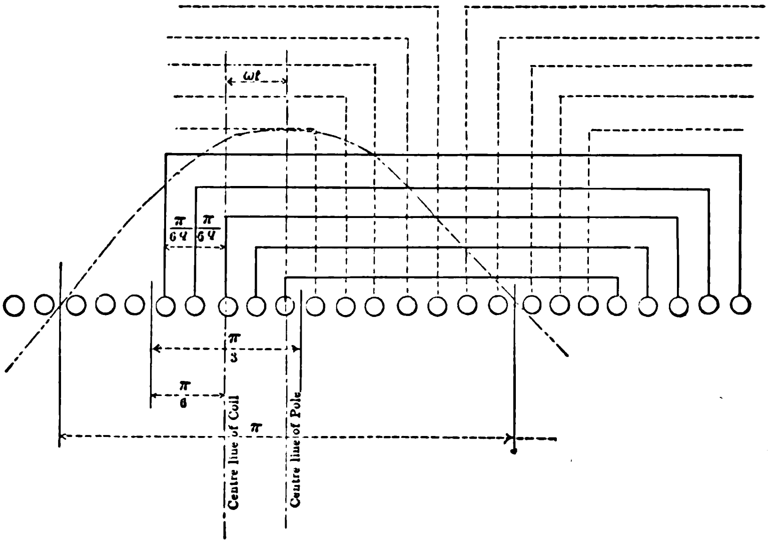


Fig. 22.

$$e_q' = LVB \times$$
$$\left\{ \cos(\omega t) \cos\left[\frac{\pi}{6q}(1 - q)\right] + \sin(\omega t) \cos\left[\frac{\pi}{6q}(1 - q)\right]\right\} \cdot 10^{-8} \text{ volts.}$$

By summing up these equations, and dividing the result by the number of holes (q), we obtain the mean electromotive force induced per conductor, the value of which will be

$$e_{av} = \frac{LVB}{l} \cos(\omega t) \times$$
$$\left\{ \cos\left[\frac{\pi}{6q}(q - 1)\right] + \cos\left[\frac{\pi}{6q}(q - 3)\right] + \cdots + \cos\left[\frac{\pi}{6q}(1 - q)\right]\right\}$$
$$+ \frac{LVB}{q} \sin(\omega t) \times$$
$$\left\{ \sin\left[\frac{\pi}{6q}(q - 1)\right] + \sin\left[\frac{\pi}{6q}(q - 3)\right] + \cdots + \sin\left[\frac{\pi}{6q}(1 - q)\right]\right\} 10^{-8} \text{ volts.}$$

But it is easy to ascertain that

$$\sin\left[\frac{\pi}{6q}(q - 1)\right] + \sin\left[\frac{\pi}{6q}(q - 3)\right] + \cdots + \sin\left[\frac{\pi}{6q}(1 - q)\right] = 0;$$

from which it therefore follows that

$$e_{av} = LVB \cos(\omega t) \times$$
$$\left\{ \frac{\cos\left[\frac{\pi}{6q}(q - 1)\right] + \cos\left[\frac{\pi}{6q}(q - 3)\right] + \cdots + \cos\left[\frac{\pi}{6q}(1 - q)\right]}{q} \right\} 10^{-8} \text{ volts.}$$

If we now take

$$K_1 = \frac{\cos\left[\frac{\pi}{6q}(q-1)\right] + \cos\left[\frac{\pi}{6q}(q-3)\right] + \cdots + \cos\left[\frac{\pi}{6q}(1-q)\right]}{q}$$

and if we substitute as before, in the preceding equation, we will finally have

$$e_{av} = K_1 L V B \cos(\omega t) \ 10^{-8} \text{ volts.}$$

If we designate by Z the number of active conductors connected in series per phase, the induced electromotive force in the whole branch will be given by the equation

$$E = K_1 L V Z B \cos \omega t. \ 10^{-8} \text{ volts.}$$

The maximum value of this E.M.F., namely,

$$E_{max} = K_1 L V Z B \ 10^{-8} \text{ volts,}$$

occurs when

$$\omega t = 0,$$

that is to say, when the centre lines of the active sides coincide with the centre lines of the poles of the magnetic field.

It will be observed that the value of the coefficient K_1 is absolutely identical with the value found in the case of a drum "polar" winding with coils placed in slots. This is explained by the fact that these two windings lead to an identical arrangement of the conductors on the periphery of the core, and that, other things being equal, the electromotive forces induced must be identically the same in the two cases.

Since these considerations also apply in the case of two-phase motors, it is, therefore, unnecessary to again determine the value of the coefficient K_1, which can be taken from the table giving its values for two-phase polar windings, with coils placed in slots.

Wave Windings. The rotors of polyphase motors of relatively large size are often provided with wave windings, made up by the subdivision of an ordinary winding, closed upon itself, similar to those which are used in ordinary direct-current machines.

It is known that if we denote by p the number of poles of the magnetic field, and by S the total number of conductors on the periphery of the armature, the "pitch" y, of the winding, must satisfy the relation

$$y = \frac{S \pm 2}{p},$$

in order that the winding may be closed upon itself after the entire number, S, of the conductors, has been utilized in the winding diagram. This pitch y must be, moreover, always an odd number.

Starting from any bar placed exactly in the axial line of one of the magnetic poles, and going to the next one connecting therewith according to the winding diagram, there will intervene the pitch distance y, which differs slightly from the distance separating the axial lines of two consecutive poles. This second bar will, therefore, not come, like the first, in the axial line of the next pole, but will be distant therefrom, to the right or to the left, by an amount equal to $\frac{2}{p}$ times the distance between two neighboring conductors. In fact, after having made, in the winding diagram, p times the pitch distance y, a complete turn plus or minus two winding spaces will have been made in the winding, since

$$py = S \pm 2.$$

After having reached the pth bar, the winding has advanced or receded in the field a distance equal to twice the space comprised between two consecutive conductors. By making the step y, only once, we advance or recede with respect to the axis of the next pole by a distance equal to $\frac{2}{p}$ of the "slot pitch," or the distance between two consecutive openings. Since all the magnetic poles are alike, we will not at all change the absolute value of the electromotive force induced in the second bar if we suppose it placed, not under the pole where it actually is, but under the preceding pole in the axis of which the first bar is already placed. These two conductors will then be separated by a distance equal to $\frac{2}{p}$ of that between any two consecutive holes.

It is equally clear that the E.M.F. induced in the entire winding of one of the phases would be the same as that which would be produced in all the bars supposed to be placed under the same pole, and separated from each other by a distance equal to $\frac{2}{p}$ times the distance between the slots.

These considerations simplify greatly the calculation of the induced E.M.F. In fact, it suffices to remember what has been said concerning ring windings, to arrive at the conclusion that this induced E.M.F. varies according to the sinusoidal law, and reaches its maximum value when the middle of the set composed of conductors of the same branch thus grouped under the same pole, coincides with the axial line of that pole.

Three-phase Motors. The bars of one of the phases of the winding when brought together under the same pole, are distant from each other $\frac{2}{p}$ times the slot pitch or the distance between slots, so that $\frac{p}{2}$ bars may be placed in a space equal to that which separates two consecutive slots.

If each winding were placed entirely under one pole, it would occupy

$$\frac{\frac{S}{3}}{\frac{p}{2}} = \frac{2}{3} \times \frac{S}{p} \text{ slots,}$$

since it comprises $\frac{S}{3}$ conductors.

$\frac{S}{p}$ being nothing more than the number of slots per pole, it follows that the conductors of each branch thus grouped extend over two-thirds of the space covered by the pole.

When the middle of the set of conductors coincides with the axis of one of the field magnet poles, the distance between the latter and the external bars of the set is equal to one-third of the width of the pole, and the induced E.M.F. then has its highest value. The mean magnetic field density which then acts on the $\frac{S}{3}$ bars of one of the circuits is equal to:

$$b_{av} = 0.826\ B_{max}.$$

This is because the partial area of the sinusoidal curve (Fig. 23) representing the variation of magnetic density in the air-gap, and

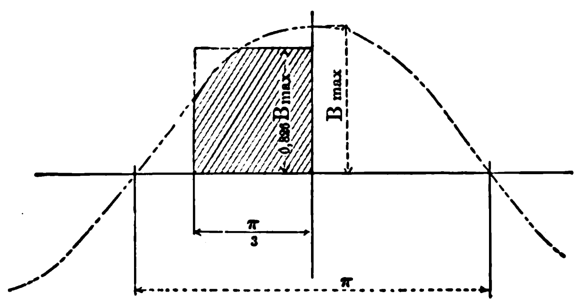


Fig. 23.

comprised between the maximum ordinate B_{max}, the ordinate situated at a distance $\frac{\pi}{3}$ therefrom, the axis of abscissæ, and the curve, is equal to:

$$B_{max} \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} B_{max}.$$

If we divide this area by the length of the base over which it extends, whose length is $\frac{\pi}{3}$, we obtain for the mean value of the magnetic density, the following:

$$\frac{\frac{\sqrt{3}}{2} B_{max}}{\frac{\pi}{3}} = \frac{3\sqrt{3}}{2\pi} B_{max} = 0.826\ B_{max}.$$

From this we conclude, at once, that the maximum induced E.M.F. in each phase will be:

$$E_{max} = 0.826 \frac{S}{3} BLV\, 10^{-8} \text{ volts.}$$

If we designate by Z the active conductors connected in series per phase, we have:

$$E = 0.826\ ZBLV\, 10^{-8} \text{ volts.}$$

The co-efficient K_1 therefore has, in this case, the value:

$$K_1 = 0.826.$$

Under like conditions the E.M.F. induced in the winding is weaker with progressive windings than with ring windings or with long coil windings. This is owing to the fact that, in the first arrangement, the conductors of the same phase are distributed over a width equal to two-thirds of that of the pole, while in the other arrangements the coils only occupy one-third of that width. The mean value of the magnetic density acting on the wires is, therefore, necessarily lower in the first case than in the second case.

We will see later that, other things being equal, the torque obtained with a progressive (wave) winding is materially smaller than that which can be obtained with other windings.

From the theoretical point of view it would be preferable, therefore, to abandon entirely the wave winding. This arrangement has, however, the advantage of leading to a simple and rational construction, which explains its very frequent use for the rotors, in motors of a certain size.

Two-phase Motors. If we bring together, as before, all the conductors of the same phase under one pole, placing them, with respect to the central line of that pole, at the same distance at which they are in reality placed with respect to the central line of the other poles, we will see that the bars or active sides of the same phase occupy half the width of one of the magnetic fluxes.

The induced E.M.F. will have its highest value, as we have already shown, when the middle of the set thus formed coincides with the axial line of the poles, at which time the extreme conductors will have an angular distance of $\frac{\pi}{4}$, or 45°, to the right or the left, from the axial line, when π represents the width of one of the magnetic fluxes. For this position of the winding, the mean value of the magnetic density acting on the bars is the mean ordinate of the area bounded, first, by the pole axis, second, by an ordinate distant $\frac{\pi}{4}$ from this axis, third, by the line of abscissæ, and last, by the sinusoidal curve representing the distribution of the magnetic flux in the air-gap.

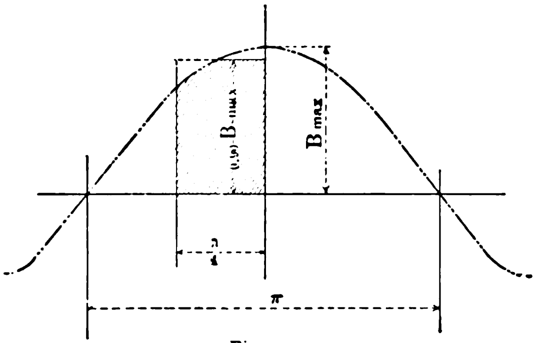


Fig. 2.

Since this surface is equal to:

$$\frac{B_{max}}{\sqrt{2}}$$

the mean ordinate will, therefore, be:

$$\frac{4}{\pi}\frac{1}{\sqrt{2}} B_{max} = 0.90\, B_{max}.$$

We will, therefore, have, for the maximum electromotive force induced in the Z conductors connected in series of each of the two phases:

$$E = 0.90 \frac{S}{2} BLV\, 10^{-8} \text{ volts}$$

or

$$E = 0.900\, ZBLV \times 10^{-8} \text{ volts.}$$

The coefficient K_1 in this case, will, therefore, have the following value:

$$K_1 = 0.900.$$

The use of the wave winding is more advantageous for two-phase than for three-phase motors. This is due to the fact that in order to obtain a two-phase diagram, one must open the wave winding at four points, and connect the four sections in series, in two sets, in order to avoid a four-phase arrangement. In the three-phase winding, the conductors of each set correspond to two-thirds of the width of the pole, while they occupy only half this width in the two-phase winding.

Squirrel Cage Windings. This kind of winding is used only for polyphase motors of small power which are to be started without any rheostat or special starting appliance.

The characteristic feature of the arrangement is that all the bars of the armature (Fig. 25) are connected together at the point where they leave the core, by means of two metallic rings. Each bar, at the moment when it passes across the axis of one of the poles of the magnetic field, becomes the seat of an induced E.M.F. whose maximum value at that instant is

$$E = BLV\,10^{-8} \text{ volts,}$$

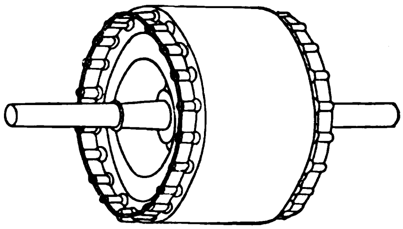


Fig. 25.

in which B designates, as before, the maximum density of the magnetic field, in gausses, and L, the (axial) length of the core in centimetres, and V, the speed of the bar with respect to the field, in centimetres per second.

As the conductors distributed within the width of a pole arrive only one after the other at the point where the magnetic density is a maximum, their E.M.F.'s are not in phase, but lag with respect to each other by a distance equal to

$$\frac{\frac{\pi}{S}}{p} = \frac{p\pi}{S} \text{ degrees,}$$

where S represents the number of bars in the rotor core, and p represents the number of poles of the magnetic field.

Windings With Short-Circuited Coils. This winding, which is only used in the construction of rotors of small power, differs from the pre-

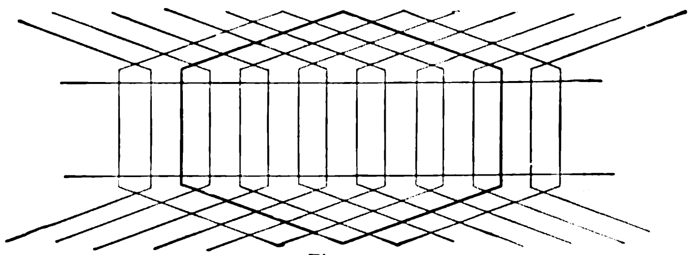


Fig. 26.

ceding in that the bars, instead of being all connected together at the two sides of the core, are connected in pairs forming independent closed circuits or "loops" (Fig. 26).

The connections between the two active sides of a rotor loop thus formed are generally made by means of connecting Y pieces or forks, usually either of copper, brass, or iron, sometimes of metal having still lower conductivity. This arrangement, which affords a large cooling surface, and which gives thorough ventilation, is especially useful

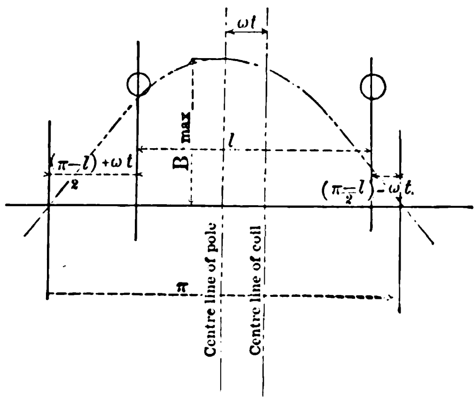


Fig. 27.

when the motors have to develop large starting torques without the use of rheostats or special starting arrangements.

The maximum E.M.F. induced in each of these loops depends naturally on their width compared with that of the pole.

If we designate by π the width of the pole, and by l the angular width of the loop, we have, retaining the preceding nomenclature, at the instant t, when the centre line of the loop is at the angular distance ωt away from the axial line of the pole (Fig. 27):

$$e = BLV\left[\sin\left\{\omega t + \frac{\pi - l}{2}\right\} + \sin\left\{\omega t - \frac{\pi - l}{2}\right\}\right] 10^{-8} \text{ volts.}$$

$$e = 2\, BLV \sin(\omega t) \cos\left\{\frac{\pi - l}{2}\right\} 10^{-8} \text{ volts.}$$

The E.M.F. induced in each loop follows the sinusoidal law, and its maximum value is:

$$e = 2BLV \cos\left\{\frac{\pi - l}{2}\right\} 10^{-8} \text{ volts.}$$

The factor $\cos\left\{\frac{\pi - l}{2}\right\}$ which depends only on the width l, tends to its maximum, which is unity, in proportion as the angular opening of the loop approaches π. It is, therefore, preferable to combine in pairs those bars whose respective angular distances are equal, or nearly equal, to the width of the magnetic field poles. The joints between the bars

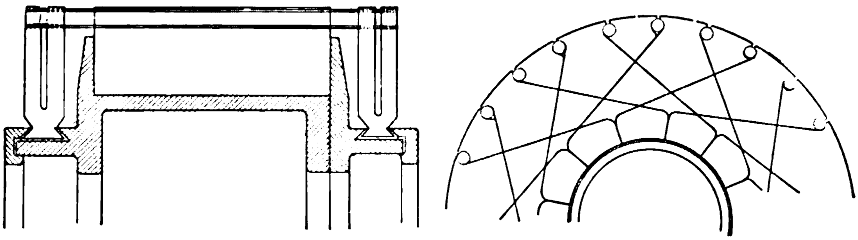


Fig. 28.

are made, as already stated, by means of metallic forks, whose blades are placed on each end of the armature in two parallel planes, as indicated in Fig. 28.

CHAPTER III.

THE ELECTROMAGNETIC COUPLE.

We have thus far studied windings which are capable of producing revolving magnetic fields, and we have developed methods whereby we can either estimate the magnetic flux produced in the air-gap, when the winding and the current passing through it are both given, or else determine the winding required for a given magnetic flux. We have also shown, and for widely varying arrangements, how the electromotive forces induced in the primary and secondary windings may be calculated when once the maximum density of the revolving magnetic field is known.

When the rotor circuits of a polyphase motor are open, and no current can be produced in them, we can make immediate application of the formulæ hitherto indicated, and we can calculate, for example, the electromotive force in each of the windings of the rotor when the latter is at rest. It will also be possible, since the counter electromotive force in the primary winding is substantially equal to the difference of potentials at the motor terminals, to predetermine the magnetic flux necessary for producing this counter electromotive force.

But in view of well-known electromagnetic principles, the revolving field of the stator could not, by itself alone, produce the mechanical couple, or the torque, which every motor must furnish. It is necessary that the conductors of the rotor should become the seat of electric currents in order that electromagnetic attractions may occur between the revolving field and these currents.

Since the revolving magnetic field induces, in the windings of the rotor, electromotive forces which are perfectly determinate, it will suffice to close the circuit of these windings in order that currents may at once be produced in these circuits. The measure of the total action of the magnetic field on these currents will represent the mechanical effort developed at the periphery of the rotor.

It is proper, however, to note that the secondary currents will, in their turn, give rise to magnetomotive forces, and that these will modify the magnetic field which the primary windings might produce, if they were acting alone.

To see in what way this modification of the primary magnetic field takes place, we could trace, along the air-gap, for both the rotor and the stator the theoretical distribution of the magnetomotive force.

We will obtain two sine curves, one of which lags behind the other. By taking the algebraical sum of the magnetomotive forces at each point of the air-gap, we will arrive at a new sine curve which will represent the true distribution of the magnetomotive force along the air-gap, and which, by reference to a different scale, will likewise represent the distribution of the resultant magnetic flux.

We will return to this subject when we come to discuss armature reaction, and, later, when we return to the elaboration of the "performance diagrams." We wished simply, in the preceding observations, to indicate to the reader that the resultant revolving field, that is to say, the only field which really exists during the operation of the motor, is substantially of sinusoidal character, so that whatever has been said heretofore concerning induced electromotive forces still applies perfectly, whether the motor be at rest or in motion. It will be sufficient to introduce into the formulæ for the magnetic flux, or the magnetic density, the values which really obtain in the air-gap.

It is known that, in 1820, Oersted discovered the action of an electric current on the magnetic needle, and that, later, La Place studied this phenomenon, and, basing himself on the experiments of Biot, gave a formula for expressing the mechanical effort of a magnetic pole on an element of electric current, and also for expressing the attraction or repulsion which takes place between a magnetic field and an element of electric current.

In the C.G.S. system the unit of current is, by definition, that which develops the unit of force between the magnetic field and the conductor, in a straight conductor having unit length placed in a uniform magnetic field of unit density.

From this it at once follows that a current of strength I, in a straight conductor of length L, will produce, in a uniform magnetic field whose density is B, a force,

$$F = LBI.$$

If the terms on the right side of the equation are expressed in C.G.S. units, the value of F will be in dynes. If we desire to express F in kilogrammes, and to take I in amperes, we should remember that the gramme is equivalent to 981 dynes, and that each C.G.S. unit of current is equal to 10 amperes ($\therefore$ 1 ampere = 0.1 C.G.S. current unit).

In this manner it will now be easy to calculate, for each type of winding, the electromagnetic effort exerted at the periphery of the rotor of polyphase motors.

[This electromagnetic effort, or electromagnetic pull, sometimes also called "tangential" effort or pull, or the "mechanical couple," should not be confounded with the "torque" of a motor, of which it is in reality only one of the factors.

The torque of a motor is, it is well known, its "moment of pull," and it is equal to the product of two factors, namely, the effort or pull exerted at the armature periphery, and the armature radius. When the pull is measured in pounds, and the armature radius in feet, the torque is expressed in *pound-feet*. In metric units the pull is usually measured either in kilogrammes, and the radius in metres or centimetres, the torque being then expressed in metre-kilogrammes or centimetre-kilogrammes, according to the case.]

Wave Windings, Polar Windings, or Ring Windings (Three-phase Rotors). We have seen, in the preceding chapter, that the coils or conductors of each of the branches, not only in the case of wave windings, but also in the case of polar windings and of ring windings, occupy a certain definite peripheral width under each pole, and that, for this reason, the maximum of the mean value of the magnetic field density acting upon the different wires of the winding, cannot be equal to the maximum density B of the magnetic field, but that it is given, in a general way, by the expression,

$$K_1 B,$$

in which K_1 represents a coefficient which varies according to the number of slots per coil in the case of ring or polar windings, and which remains constant in the case of wave windings. The different values of this coefficient have been given in the preceding chapter.

This mean density, whose amplitude is $K_1 B$, is a sine function of the time, and at any instant, t, when the centre lines of the active sides of the coils are separated by the angle ωt from the centre lines of the poles, its value will be given by the following equation (Fig. 29):

$$b_{av} = K_1 B \cos(\omega t).$$

We have just seen that, by definition, the electromagnetic action, f, produced between a conductor and the magnetic field, is

$$f = 0.1 \times Lib \text{ dynes},$$

in which

L = the active length of conductor, in centimetres.

i = the current, in amperes, passing through the said conductor.

b = the magnetic density of the magnetic field in C.G.S. units (or in gausses), at the point where the conductor happens to be at the instant of time considered (t).

But since the mean value of the magnetic density taken with respect to the total number, Z, of the active conductors of one of the windings at that instant, is

$$K_1 B \cos(\omega t),$$

we conclude that the total effort of the magnetic field on this winding will be:

$$F_{I} = 0.1\ K_1 Z L i_{I}\ B \cos(\omega t) \text{ dynes.}$$

The winding of the second phase having a lag in the magnetic field with respect to the first winding of $\frac{2}{3}\pi$, or 120°, we will have, for the winding of this second phase,

$$F_{II} = 0.1\ K_1 Z L i_{II}\ B \cos(\omega t + 120^\circ) \text{ dynes,}$$

and for the third branch,

$$F_{III} = 0.1\ K_1 Z L i_{III}\ B \cos(\omega t + 240^\circ) \text{ dynes.}$$

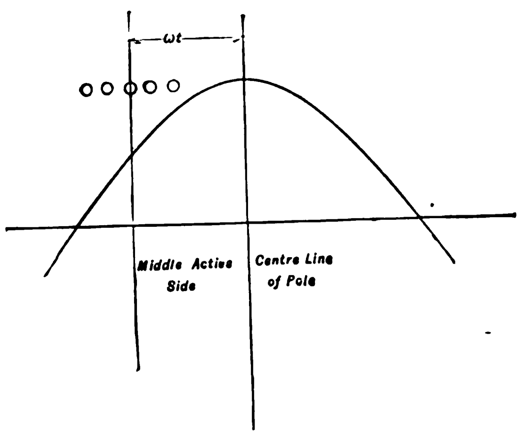


Fig. 29.

We have learned to calculate, for all cases, the electromotive force (E) which the magnetic field of the air-gap produces in a given winding.

If the secondary windings of polyphase motors were entirely free from self-induction, the currents produced therein would be governed by Ohm's law, and would always be in phase with the induced E.M.F. The amount of current, (i), in each case, would then be expressed by the relation:

$$i = \frac{E}{R},$$

in which R represents the ohmic resistance of each of the branches of the winding.

We know that the E.M.F. is a sine function of the time, and we deduce from this that such is also the case with the current, i.

If we call I the maximum value of this current, we can write

$$\begin{aligned} i_{\text{I}} &= I \cos \omega t, \\ i_{\text{II}} &= I \cos (\omega t + 120^\circ), \\ i_{\text{III}} &= I \cos (\omega t + 240^\circ). \end{aligned}$$

By introducing these values in the preceding equations, we have

$$\begin{aligned} F_{\text{I}} &= 0.1\ K_1 ZLIB \cos^2 (\omega t) \text{ dynes}, \\ F_{\text{II}} &= 0.1\ K_1 ZLIB \cos^2 (\omega t + 120^\circ) \text{ dynes}, \\ F_{\text{III}} &= 0.1\ K_1 ZLIB \cos^2 (\omega t + 240^\circ) \text{ dynes}. \end{aligned}$$

If we add these three simultaneous efforts, we will have the total action, F, of the magnetic field on the entire winding at the instant t;

$$F = 0.1\ K_1 ZLIB \times [\cos^2 (\omega t) + \cos^2 (\omega t + 120^\circ) + \cos^2 (\omega t + 240^\circ)] \text{ dynes};$$

but

$$\begin{aligned} \cos^2 (\omega t) &= \tfrac{1}{2} [1 + \cos (2\, \omega t)], \\ \cos^2 (\omega t + 120^\circ) &= \tfrac{1}{2} [1 + \cos (2\, \omega t + 240^\circ)], \\ \cos^2 (\omega t + 240^\circ) &= \tfrac{1}{2} [1 + \cos (2\, \omega t + 280^\circ)], \\ &= \tfrac{1}{2} [1 + \cos (2\, \omega t + 120^\circ)]. \end{aligned}$$

We can, therefore, write

$$F = 0.1\ KZLIB\ \tfrac{1}{2} \times [3 + \cos (2\, \omega t) + \cos (2\, \omega t + 120^\circ) + \cos (2\, \omega t + 240^\circ)] \text{ dynes};$$

but we know that the expression

$$\cos (2\, \omega t) + \cos (2\, \omega t + 120^\circ) + \cos (2\, \omega t + 240^\circ)$$

is likewise equal to zero.

We, therefore, have finally:

$$F = \frac{0.3}{2}\ K_1 ZLIB \text{ dynes}.$$

If now we designate by S the total number of active conductors disposed on the periphery of the core, and if we denote by b and i the effective values of the magnetic field and of the current, respectively, we can write

$$Z = \frac{S}{3}; \qquad B = b \sqrt{2}; \qquad I = i \sqrt{2},$$

and, finally,

$$F = 0.1\ K_1 SLib \text{ dynes},$$

or,

$$F = \frac{K_1 SLib}{9 \cdot 81 \times 10^6} \text{ kilogrammes}.$$

[When the effort is to be expressed in pounds, we have (taking L in inches instead of centimetres, and b being taken in gausses as before),

$$F = 8 \cdot 85\, K_1 SLib\ 10^{-8} \text{ pounds.]}$$

This last formula enables us to calculate (either in kilogrammes or in pounds, according to the case) the effort exerted, by the revolving magnetic field, on the secondary core, whether the latter be provided with a polar winding, a ring winding, or a wave winding. The coefficient, K_1, may be taken from the tables given in the preceding chapter for the values corresponding to these various kinds of windings.

The final equation to which we have arrived shows us that the couple acting between the rotor and the magnetic field is constant since time is entirely eliminated from the equation.

It will be remembered that the maximum value of the induced E.M.F., E, is

$$E_1 = KLVBZ\ 10^{-8} \text{ volts,}$$

from which we may conclude that the highest value which the current can reach will be

$$I = \frac{K_1 LVBZ\ 10^{-8}}{R} \text{ amperes,}$$

but since

$$I = i\sqrt{2}, \qquad B = b\sqrt{2}, \qquad \text{and} \qquad Z = \frac{S}{3},$$

$$\therefore i = \frac{K_1 LVSb}{3R}\ 10^{-8} \text{ amperes.}$$

If now we substitute in the final equation just found for the value of F, the value of the effective current strength, i, as given by the preceding equation, we have

$$F = \frac{K_1^2 S^2 L^2 V b^2}{30\, R}\ 10^{-8} \text{ dynes,}$$

or,

$$F = \frac{K_1^2 S^2 L^2 V b^2}{29.43 \times 10^{15} R} \text{ kilogrammes.}$$

[When this effort is to be expressed in pounds, we have (taking L in inches instead of centimetres),

$$F = \frac{1.16\, K_1^2 S^2 L^2 V b^2}{10^{17} R} \text{ pounds.]}$$

It is well to remember that R represents the resistance of each of the windings (phases) of the secondary, in ohms. All other values are to be taken in C.G.S. units. This last equation is used less frequently than that which was first given; but it is of some utility when we undertake the calculation of an existing motor, since it enables the effective value, b, of the magnetic density, to be immediately evaluated when the power and the speed of the motor are given.

Two-phase Rotor Windings. If we retain the preceding notation, we will have, for the mechanical effort exerted at a given moment, t, between the field and the Z conductors of one of the phases,

$$F_{\text{I}} = 0.1\ K_1 Z L i_{\text{I}} B \cos(\omega t) \text{ dynes.}$$

We will, likewise, have for the winding of the second phase,

$$F_{\text{II}} = 0.1\ K_1 S L i_{\text{II}} B \cos(\omega t + 90^\circ) \text{ dynes;}$$

we know that

$$i_{\text{I}} = I \cos(\omega t),$$

and that

$$i_{\text{II}} = I \cos(\omega t) + 90^\circ,$$

also,

$$\cos(\omega t + 90^\circ) = -\sin(\omega t).$$

Replacing i_{I}, i_{II}, and *cos* $(\omega t + 90^\circ)$ by their values, we have, for the total action of the magnetic field on the entire winding,

$$F = 0.1\ K_1 Z L I B\ [\cos^2(\omega t) + \sin^2(\omega t)] \text{ dynes,}$$

or,

$$F = 0.1\ K_1 Z L I B \text{ dynes.}$$

And taking

$$S = \frac{Z}{2}, \qquad I = i\sqrt{2}, \qquad \text{and} \qquad B = b\sqrt{2},$$

in which i and b are, respectively, the effective values of the current and of the magnetic field density, while S designates the total number of active conductors disposed on the periphery of the secondary core, we will have

$$F = 0.1\ K_1 S L i b \text{ dynes,}$$

or

$$F = \frac{K_1 S L i b}{9.81 \times 10^6} \text{ kilogrammes.}$$

[Taking L in inches, we also have

$$F = 8.85\ K_1 S L i b\ 10^{-8} \text{ pounds.]}$$

This equation being independent of the time, we may conclude that the couple, or the torque, exerted by the magnetic field on the rotor is constant.

The maximum induced E.M.F. being

$$E = K_1 L V B \frac{S}{2} \; 10^{-8} \text{ volts,}$$

the maximum current value will be

$$I = \frac{K_1 L V B S}{2\,R} \; 10^{-8} \text{ amperes.}$$

R being the resistance, in ohms, of each of the two branches of the secondary winding, we will likewise have for the effective current,

$$i = \frac{I}{\sqrt{2}} = \frac{K_1 L V B S}{2\sqrt{2}\,R} \; 10^{-8} \text{ amperes.}$$

If, in the previous equations, we replace i by its value as given in the preceding equation, the total electromagnetic effort or pull, F, will now appear under the following new form :

$$F = \frac{K_1^2 L^2 S^2 V b^2}{20\,R} \; 10^{-8} \text{ dynes,}$$

or,

$$F = \frac{K_1^2 L^2 S^2 V b^2}{19.62 \; 10^{15} R} \text{ kilogrammes.}$$

[We also have, taking L in inches,

$$F = \frac{1.74 \; K_1^2 \, S^2 L^2 V b^2}{10^{17} R} \text{ pounds.]}$$

The values of the coefficient, K_1, will be found in the tables given in Chapter II.

The various formulæ which have been derived for the electromagnetic pull, F, apply to wave windings as well as to all forms of ring windings or of polar windings.

Squirrel-Cage Windings. If we call b_0 the magnetic density of the magnetic field at a given point in the air-gap, the E.M.F. induced in a conductor (Fig. 30) having L centimetres of active length, which happens to be at that point at any particular instant, will be

$$E = L V b_0 \times 10^{-8} \text{ volts,}$$

where V designates the relative speed of the magnetic field in centimetres per second.

If we represent by R the ohmic resistance of the circuit formed by this conductor and by the corresponding portion of the two short-circuiting rings at the ends of the rotor, the current produced will be

$$i = \frac{1}{R} L V b_0 \times 10^{-8} \text{ amperes.}$$

The electromagnetic action of the magnetic field on this current will be equal to

$$f_1 = 0.1\ L i b_0 \text{ dynes,}$$

or

$$f_1 = \frac{0.1}{R} L^2 V b_0^2 \times 10^{-8} \text{ dynes.}$$

This effort is, therefore, proportional to the square of the magnetic density of the field which is acting on the conductor. But we know that the distribution of the magnetic flux along the air-gap follows the

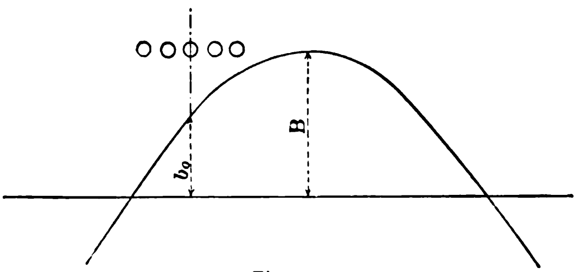


Fig. 30.

sinusoidal law, and that, consequently, the mean of the squares of the magnetic densities acting at different points of the air-gap, is nothing more than the effective value of this magnetic density raised to the second power; that is to say,

$$\left[\frac{B}{\sqrt{2}}\right]^2 = \frac{B^2}{2}.$$

We at once deduce the following expression for the mean effort exerted between the magnetic field and the conducting bars on the rotor,

$$f_{av} = \frac{0.1}{R} L^2 V \frac{B^2}{2} 10^{-8} \text{ dynes,}$$

so that the total action of the magnetic field upon the S active conductors distributed on the periphery of the rotor will be

$$F = \frac{0.1}{R} L^2 V \frac{B^2}{2} S\ 10^{-8} \text{ dynes.}$$

If we designate by b the effective density of the magnetic field at the point corresponding to the maximum ordinate, B, we have, from one of the equations just given,

$$b^2 = \frac{B^2}{2}, \text{ whence } b = \frac{B}{\sqrt{2}},$$

and, substituting this value for B in the preceding equation for F, we have

$$F = \frac{0.1}{R} L^2 V S b^2 \times 10^{-8} \text{ dynes.}$$

This last equation can be written under another form, if we remember that

$$I = \frac{LVB}{R} 10^{-8} \text{ amperes,}$$

and that

$$i = \frac{I}{\sqrt{2}} = \frac{LVb\sqrt{2}}{R\sqrt{2}} 10^{-8} = \frac{LVb}{R} 10^{-8} \text{ amperes.}$$

If, therefore, in the preceding formula for F, we replace by the symbols I and i the factors equivalent thereto, we have

$$F = 0.1 \, LSbi \text{ dynes,}$$

or

$$F = \frac{LSbi}{9.81 \times 10^6} \text{ kilogrammes.}$$

[We also have, taking L in inches,

$$F = 8.85 \, LSbi \times 10^{-8} \text{ pounds.}]$$

It is seen here again that the pulling effort is rigorously constant, and that the coefficient, K_1, is equal to unity.

We will investigate, later, the extent to which it is necessary to increase the ohmic resistance of each of the bars in order to take into account the resistance of the two short-circuiting rings.

Rotors with Separate Bars Short-Circuited Upon Themselves. Preserving the nomenclature employed in the preceding chapter and designating (Fig. 31) by ωt the angular distance existing at any given moment, t, between the centre line of one of the turns of the winding and the centre line of one of the magnetic poles, the E.M.F. induced in this turn of the winding will have, as we have already seen, the following expression:

$$e = \left[LVB \sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + LVB \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} \right] 10^{-8} \text{ volts.}$$

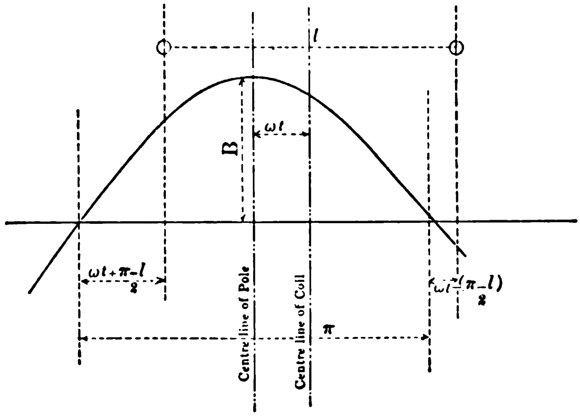


Fig. 31.

If R designates the ohmic resistance of the winding turn which is closed upon itself, the current produced therein at that instant, t, will be

$$c = \frac{1}{R} L V \left[B \sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + B \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} \right] 10^{-8} \text{ amperes.}$$

It is thus apparent that the effort, f, exerted by the magnetic field on this turn of the winding, at the instant considered, may be written

$$f = 0.1\, L i \left[B \sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + B \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} \right] \text{ dynes;}$$

and replacing i by its value, we have

$$f = \frac{0.1 L^2 V}{R} \left[B \sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + B \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} \right]$$
$$\times \left[B \sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + B \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} \right] 10^{-8} \text{ dynes;}$$

but

$$\sin \left\{ \omega t + \frac{\pi - l}{2} \right\} + \sin \left\{ \omega t - \frac{\pi - l}{2} \right\} = 2 \sin (\omega t) \cos \left\{ \frac{\pi - l}{2} \right\}.$$

We can therefore write

$$f = \frac{0.4 L^2 V}{R} \left[B^2 \sin^2 (\omega t) \cdot \cos^2 \left\{ \frac{\pi - l}{2} \right\} \right] 10^{-8} \text{ dynes,}$$

or, if we take

$$b_0 = B \sin (\omega t),$$

and substitute in the preceding equation, we will have

$$f = \frac{0.4\,L^2 V}{R} b_0^2 \cos^2 \left\{ \frac{\pi - l}{2} \right\} 10^{-8} \text{ dynes.}$$

The effort exerted between the magnetic field and the turn of the winding is proportional to the square of the magnetic density, b_0. But since the mean value of the squares of the magnetic field densities in the different portions of the air-gap is nothing more than the effective density raised to the second power, that is to say,

$$\left[\frac{B}{\sqrt{2}} \right]^2 = \frac{B^2}{2},$$

and since the number of winding turns closed upon themselves is $\frac{S}{2}$, in which S represents, as before, the total number of active sides, we therefore have, for the effort of the magnetic field on the entire rotor,

$$F = \frac{0.4 L^2 V}{R} \times \frac{S}{2} \times \frac{B^2}{2} \times \cos^2 \left\{ \frac{\pi - l}{2} \right\} 10^{-8} \text{ dynes.}$$

This equation can be written under another form. In fact, we know that the largest current which can be produced in the turn of the winding is

$$I = 2 \frac{LVB}{R} \cos \left\{ \frac{\pi - l}{2} \right\} 10^{-8} \text{ amperes.}$$

If we introduce this current, I, in the preceding formula, we have

$$F = \frac{0.1}{2} LSBI \cos \left\{ \frac{\pi - l}{2} \right\} \text{ dynes,}$$

and if we take

$$i = \frac{I}{\sqrt{2}} \text{ and } b = \frac{B}{\sqrt{2}}$$

as the effective values for current and magnetic density, respectively, we will have

$$F = 0.1\ LSbi \cos \left\{ \frac{\pi - l}{2} \right\} \text{ dynes,}$$

or

$$F = \frac{LSbi}{9.81 \times 10^6} \cos \left\{ \frac{\pi - l}{2} \right\} \text{ kilogrammes.}$$

[We also have, taking L in inches,

$$F = 8.85\ LSbi \times 10^{-8} \times \cos \left\{ \frac{\pi - l}{2} \right\} \text{pounds.]}$$

This effort is constant, since time no longer enters in the equation. When the width of the winding (l) becomes equal to π, which is the width of the poles of the magnetic field in the air-gap, the motor couple is the same as that which a squirrel-cage winding would produce under the same conditions.

In the preceding discussion we have assumed that the armature was free from self-induction, and that, consequently, the secondary currents reached their highest value at the moment when the centre lines of the active sides of the coils coincide with the centre lines of the magnetic poles in the air-gap.

This hypothesis is never practically realized. The magnetic leakage in the secondary winding always produces an electromotive force of self-induction which combines with the E.M.F. induced by the magnetic field to give a resultant, which, itself, produces the currents.

If we decompose the magnetic field in the air-gap into two magnetic fluxes, of which one would have its centre lines opposite the middle of the active sides of the coils at the precise moment when the current of these active sides is at its maximum, while the other would then lag one-quarter phase behind the first, it will be observed that only one of these magnetic fields is capable of producing a motor couple with the secondary currents. The effort due to the second component of the magnetic field is always equal to zero.

The value, b, which enters in the formulæ for the value of the motor couple, does not, therefore, represent the effective density of the magnetic field produced by the simultaneous action of the primary and secondary windings, but, rather, that of a component of this field. The axes of the poles of this component coincide with the centre lines of the active sides of the coils, at the moment when the current in these active sides reaches its maximum value.

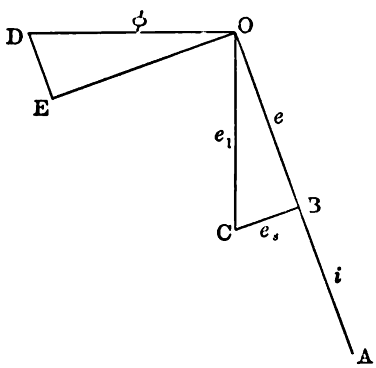


Fig. 32.

If, in Fig. 32, we represent by OA the secondary current, and by OB the induced E.M.F. which generates this current, BC will represent the electromotive force of self-induction, and OC the difference of potential produced by the magnetic field, OD, of the air-gap. This magnetic field, Φ, may be decomposed into two magnetic fluxes, including one, DE, parallel to OA, which produces no motive effort, and another, *OE*, perpendicular to *OA*, which produces the whole of the motive effort.

The value, b, which enters in the preceding formula, represents the effective magnetic density of that field, *OE*.

We see that magnetic leakage produces, first, a diminution of the secondary currents, and then causes a lag between these currents and the field. These two effects combine to reduce the motor couple.

CHAPTER IV.

THE MAGNETIZING ACTION OF WINDINGS.

As was stated before beginning the study of the electromagnetic couple exerted between the magnetic field and the rotor, the currents produced in the secondary winding by the influence of the revolving magnetic poles produce magnetomotive forces, of which we now proceed to study the distribution along the air-gap.

The currents which are sent through the windings of the primary core or stator give rise, likewise, and by virtue of the same principles, to differences of magnetic potential whose distribution in the air-gap has been studied at the beginning of this work, for another purpose, and only in the case of stators having windings of the ring type, or of the definite (polar) type.

When the motor is operating, we have to consider, therefore, a double distribution of magnetomotive forces, one due to the wires of the stator, and the other to those of the rotor. The action of these two windings will manifest itself at all points of the air-gap, so that it will suffice to take, at each point, the algebraical sum of the magnetomotive forces, to obtain the difference of magnetic potential which really exists at each of these points.

By proceeding in this manner, we will determine the true or resultant distribution of the magnetomotive force; and it will be easy to at once deduce therefrom the distribution of the magnetic flux, which will be the true or "resultant" magnetic field, due to the simultaneous action of the primary and secondary windings.

It seems evident at first glance, that a given winding placed on the core of the stator, and having definite currents passing through it, must cause a distribution of magnetic flux in the air-gap exactly like that which it would produce under the same conditions if placed on the core of the rotor.

From this consideration it follows that it will suffice to study the effects of a secondary winding, and that the conclusions reached will be equally valid in case the same winding were placed on the stator. The inverse reasoning being true, the partial study already made of the magnetic field produced in the air-gap of two-phase and three-phase motors by ring windings or definite windings placed on the stator, would, therefore, apply also, without change, to the reactions produced by two-phase and three-phase rotors having the same grouping of conductors.

We have seen that the ring type and the definite type of windings were absolutely identical in respect to the amount and distribution of magnetic flux which they can produce.

The preceding investigations demonstrated that it was possible, with the number of holes (slots) per coil employed in practice, to replace, without very appreciable error, by a sinusoidal distribution, the true distribution either of the magnetomotive force or of the magnetic density.

We will now return to, and we will proceed to complete, this first study, which was made in order to familiarize the reader, at the very beginning, with the idea of the revolving magnetic field, but which was extended (in Chapter I.) only to ring windings, and to definite windings for two-phase and three-phase motors.

Windings of Definite Type and of Ring Type. (*Three-Phase Motors.*)

As before, let

t = the number of openings in which are placed the wires forming one of the active sides of each coil,

n = the number of conductors per opening,

q = the number of turns per coil (such that $q \doteq n\,t$),

I = the maximum current in each conductor, in amperes,

Φ = the mean magnetic flux issuing from one of the poles of the rotating field, in maxwells,

M = the maximum magnetomotive force in the air-gap,

B = the maximum density of the magnetic field in the air-gap, in gausses,

L = the (axial) width of the air-gap, in centimetres,

d = the (radial) distance across (or "length" of) the air-gap, in centimetres,

D = the mean diameter of the cylindrical space constituting the air-gap, in centimetres,

p = the number of poles of the revolving magnetic field.

Let us first suppose that t is any even number whatever, and let us draw (Fig. 33), in the manner explained in Chapter I., the distribution, along the air-gap, of the magnetic density produced by the winding, at the particular instant when the current is zero in the first phase, and becomes equal to *minus* $\frac{\sqrt{3}}{2} I$ and to *plus* $\frac{\sqrt{3}}{2} I$, in the second and third phases, respectively.

If we multiply the core width L, by the area which is comprised between the axis AB, the broken line AM, and the vertical line MN (which is at the centre of the pole piece S_2), we will obtain the value of half the magnetic flux Φ issuing from each of the poles of the field.

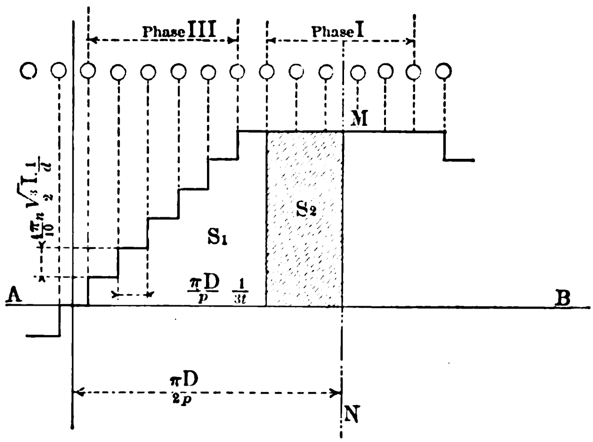


Fig. 33.

The area aforesaid may be decomposed into two portions, S_1 and S_2, and since the distance measured in the air-gap between the centre lines of two consecutive slots in the core has the following expression,

$$\frac{\pi D}{p} \times \frac{1}{3t}$$

the two component areas S_1 and S_2 may be expressed as follows:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} \times I \left\{1 + 2 + \cdots + t\right\} \frac{1}{d}.$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{t-1}{2} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} It \frac{1}{d}.$$

But we know that the arithmetical progression

$$1 + 2 + 3 + \cdots + t$$

may be summed up as follows:

$$\frac{t+1}{2} \times t = \frac{t^2}{2} + \frac{t}{2}.$$

We therefore have:

$$S_I = S_1 + S_2 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} \times n \frac{\sqrt{3}}{2} I \left[\frac{t^2}{2} + \frac{t}{2} + \frac{t^2}{2} - \frac{t}{2}\right] \frac{1}{d}$$

or:

$$S_I = \frac{\pi D}{3p} \times \frac{4\pi}{10} \times n \frac{\sqrt{3}}{2} It \frac{1}{d}.$$

This discloses the remarkable fact that the area in question is proportional to the number (t) of openings corresponding to the active side of each coil.

Let us now draw (Fig. 34), the distribution of the magnetic density in the air-gap at the moment when the current becomes a minimum, that is to say, when it becomes equal to $-I$ in the second phase.

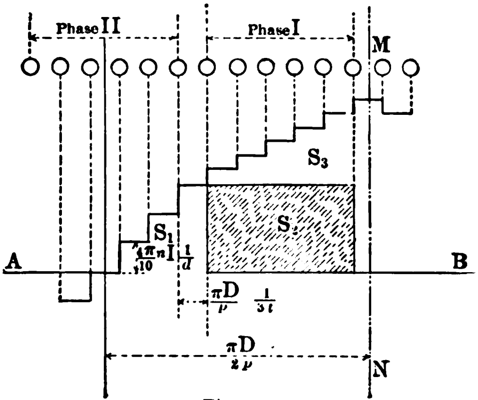


Fig. 34.

The current value in the first and third phases will then be $+\frac{I}{2}$. The area S_{II} can be decomposed into three portions, S_1, S_2, and S_3, for which we will have the following equations:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} \times nI \left\{ 1 + 2 + 3 + \cdots + \frac{t}{2} \right\} \frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{3t} \times (t-1) \frac{4\pi}{10} nI \frac{t}{2} \times \frac{1}{d},$$

$$S_3 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} \times n \frac{1}{2}(1 + 2 + 3 + \cdots + t) \frac{1}{d};$$

but

$$1 + 2 + 3 + \cdots + \frac{t}{2} = \frac{1 + \frac{t}{2}}{2} \times \frac{t}{2} = \frac{t_2}{8} + \frac{t}{4},$$

and

$$\frac{1}{2}(1 + 2 + 3 + \cdots t) = \frac{t+1}{2} \times \frac{t}{2} = \frac{t^2}{4} + \frac{t}{4};$$

we therefore have:

$$S_{II} = S_1 + S_2 + S_3 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} \times nI \times \left\{ \frac{t^2}{8} + \frac{t}{4} + \frac{t^2}{4} + \frac{t}{4} + \frac{t^2}{2} - \frac{t}{2} \right\} \frac{1}{d},$$

or

$$S_{II} = \frac{\pi D}{3p} \times \frac{4\pi}{10} \times nI \frac{7}{8} t \frac{1}{d}.$$

This area, although slightly different from that first obtained when the current was zero in the first phase, is, nevertheless, likewise proportional to the number (t) of openings per active side.

The expression "slightly different" is appropriate, since the factor $\frac{\sqrt{3}}{2}$, entering in the equation for S_{I} (p. 58), is only 1.14 % greater than the corresponding factor, $\frac{7}{8}$, which enters in the preceding equation for S_{II}. Since the product of each of these areas, by the core width L, gives the value of half of the magnetic flux issuing from one of the poles of the magnetic field, we may conclude that this magnetic flux (which, as we have seen at the beginning of this work, oscillates between the two limits, $S_{\text{I}}L$ and $S_{\text{II}}L$) is practically constant, for it varies only 0.6% from its mean value, which is:

$$\Phi = 2\,L\frac{\pi D}{3p} \times \frac{4\pi}{10} \times nI \left[\frac{\frac{\sqrt{3}}{2} + \frac{7}{8}}{2}\right] t \times \frac{1}{d}.$$

If the distribution of this magnetic flux in the air-gap were perfectly sinusoidal, we would have:

$$\Phi = \frac{2}{\pi}\,B\,\frac{\pi D}{p}\,L = 2\,\frac{BDL}{p},$$

from which we would have, for the maximum magnetic density of the equivalent sinusoidal curve:

$$B = \frac{p\Phi}{2\,DL}.$$

Replacing Φ by its value, we have:

$$B = \frac{p}{2\,DL}\;2\,L\frac{\pi D}{3p} \times \frac{4\pi}{10}\;nI\,(0.87)\;t\frac{1}{d},$$

or

$$B = \frac{4\pi^2}{30} \times 0.87\;nIt\frac{1}{d};$$

and, finally, since $q = nt$, we have:

$$B = 1.144\,qI\frac{1}{d}.$$

We would also have, for the maximum value of the magnetomotive force:

$$M = 1.144\,qI.$$

The maximum magnetic density in the air-gap is, therefore, proportional to the number (q) of wires comprised in the active side of each coil.

Let us bear in mind that the deductions just made only apply in cases where t is an EVEN number.

Let us now consider what happens when the conductors of the active sides of one of the coils are placed in an ODD number of slots.

At the precise moment when the current becomes zero in the first phase, and when it is passing through the values $-\frac{\sqrt{3}}{2}I$ and $+\frac{\sqrt{3}}{2}I$ in the second and third phases, the distribution of magnetic density along the air-gap is as indicated in Fig. 35.

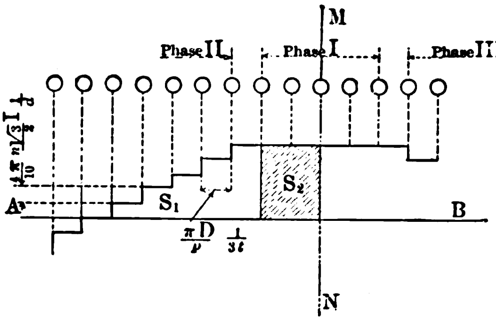


Fig. 35.

The area S_1, comprised between the axis AB, the "zig-zag" line, and the centre line MN of the pole, may be decomposed into two parts, S_1 and S_2, having the following values:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I(1+2+3+\cdots+t)\frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{t-1}{2} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I t \frac{1}{d};$$

but we know that

$$1+2+3+\cdots+t = \frac{t+1}{2}t = \frac{t^2}{2} + \frac{t}{2}.$$

We can therefore write:

$$S_1 = S_1 + S_2 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I \left\{ \frac{t^2}{2} + \frac{t}{2} + \frac{t^2}{2} - \frac{t}{2} \right\} \frac{1}{d}.$$

$$S_1 = \frac{\pi D}{3p} \times \frac{4\pi}{10} n \frac{\sqrt{3}}{2} I t \frac{1}{d}.$$

This area is also proportional to t.

At the moment when the current in the second phase attains its maximum negative value, $-I$, the current values in the other two phases are each equal to $+\frac{I}{2}$.

The distribution of magnetic density in the air-gap is then as indicated in Fig. 36. The total area S_{11} may be decomposed into four parts, S_1, S_2, S_3, S_4, having the following values:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI \left[1 + 2 + \cdots\cdots + \left\{ \frac{t-1}{2} \right\} \right] \frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{3t}(t-1)\frac{4\pi}{10} \cdot nI \left\{ \frac{t-1}{2} \right\} \frac{1}{d},$$

$$S_3 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI [1 + 2 + \cdots\cdots + t] \frac{1}{d},$$

$$S_4 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{3t-1}{2} \times \frac{4\pi}{10} n \frac{I}{2} \times \frac{1}{d}.$$

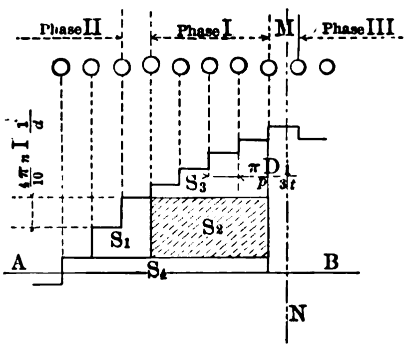


Fig. 36.

But the sum of the progression $t + 2 + 3 + \cdots\cdots \left\{ \frac{t-1}{2} \right\}$, being equal to the half-sum of the first and last figure, multiplied by the number of terms, we have:

$$1 + 2 + 3 + \cdots\cdots + \left\{ \frac{t-1}{2} \right\} = \frac{1 + \frac{t-1}{2}}{2} \times \frac{t-2}{2} = \frac{t^2}{8} - \frac{1}{8}.$$

We also have:

$$1 + 2 + 3 + \cdots\cdots + t = \frac{t+1}{2} \times t = \frac{t^2}{2} + \frac{t}{2}.$$

From this we conclude that:

$$S_{11} = S_1 + S_2 + S_3 + S_4 = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} \times nI \times$$

$$\left[\frac{t^2}{8} - \frac{1}{8} + \frac{t^2}{2} - t + \frac{1}{2} + \frac{t^2}{4} + \frac{t}{4} + \frac{3t}{4} - \frac{1}{4} \right] \frac{1}{d},$$

or:

$$S_{II} = \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI \left[\frac{7}{8} t^2 + \frac{1}{8}\right] \frac{1}{d}.$$

This area is no longer proportional to t. If we multiply each of the two equations just derived for the areas S_I and S_{II}, by the core width L, we will have the values for half the magnetic flux issuing from each pole at the two instants under consideration. We know that the half flux always oscillates between these two limits, which, however, differ but slightly from each other. The mean value, Φ, of the flux from each pole is:

$$\Phi = 2L \left\{ \frac{S_I + S_{II}}{2} \right\},$$

or

$$\Phi = 2L \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI \left[\frac{\frac{\sqrt{3}}{2} t^2 + \frac{7}{8} t^2 + \frac{1}{8}}{2} \right] \frac{1}{d}.$$

$$\Phi = 2L \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI \left[\frac{13.928\, t^2 + 1}{16} \right] \frac{1}{d}.$$

The smallest three-phase motors, of capacity less than 1 H.P., have windings the active side of which occupies at least two openings. It follows, therefore, that 3 is the smallest odd value which t can have in practice.

From 5 H.P. upward, in the case of three-phase motors, with ring windings or definite windings, there is almost always more than three openings for the active side of each coil.

From what precedes we may conclude that, in taking

$$\Phi = 2L \frac{\pi D}{p} \times \frac{1}{3t} \times \frac{4\pi}{10} nI \left[\frac{13.928 t^2}{16} \right] \frac{1}{d},$$

we introduce an error whose maximum value, when $t = 3$, will amount to

$$100 \frac{1}{13.928 t^2} = 100 \frac{1}{125.37} = 0.80\%$$

and which, when $t = 5$, will be reduced to

$$100 \frac{1}{348.25} = 0.29\%.$$

This error is not the only one to be considered. We must also consider the errors due to the fact that the wave form of the currents sent through the motors is never absolutely sinusoidal, and also that the distribution of magnetomotive force along the air-gap due to the stator or the rotor likewise no longer follows exactly the sinusoidal law; or, again, we must consider the errors due to the difficulty of making precise measurements of the losses due to hysteresis, eddy currents, and friction, or the errors incidental to measurements in testing the motors to determine their operation. These errors, and many others, are certainly more important than that which is introduced in our calculations by the approximation which we have just made, for the purpose of arriving at a simple general formula, applicable to all values of t.

Let us hasten to add, however, that the designer may always make use of the two equations given for the areas S_I and S_{II}, which are absolutely exact, and which will enable him to obtain, first, the expression for the mean flux Φ, and afterward, that for the maximum ordinate, B, of the sinusoidal curve giving the theoretical (ideal) distribution of the magnetic density in the air-gap.

Such exact calculation is wholly unnecessary, however; and we advise the reader always to employ the last formula, which, as already pointed out, is absolutely correct for all even values of t.

Let us now return to our simplified equation:

$$\Phi = 2 L \frac{\pi D}{p} \times \frac{1}{3 t} \times \frac{4 \pi}{10} n I \left[\frac{13{,}928}{16} t^2 \right] \frac{1}{d},$$

and let us find the value of the maximum ordinate, B, of the sinusoidal curve giving the theoretical distribution of magnetic density in the air-gap and producing the mean flux Φ. We already know that:

$$\Phi = \frac{2}{\pi} B \frac{\pi D}{p} L = \frac{2 BDL}{p}.$$

From this we derive:

$$B = \frac{p}{2 DL} \Phi,$$

or

$$B = \frac{p}{2 DL} 2 L \frac{\pi D}{3 p} \times \frac{4 \pi}{10} n I \frac{13.928}{16} t \frac{1}{d};$$

but since $nt = q$, we have finally:

$$B = 1.144\, q I \frac{1}{d} \text{ gausses.}$$

We would likewise have:

$$M = 1.144\, q I.$$

We see that the maximum values of magnetic density and of magnetomotive force in the air-gap are proportional to the number of turns comprised in each coil.

This last equation, which is only applicable in the case where t is an odd number, is exactly the same as the equation which was obtained when t was an even number. Consequently, whether t be an even or an odd number, we may always write:

$$B = 1.144\, qI \frac{1}{d},$$

and

$$M = 1.144\, qI,$$

which are the general formulæ giving the maximum values of the magnetic density and of the magnetomotive force in the air-gap as a function of the number (q) of turns in each coil, of the maximum current (I) passing through each conductor, and of the distance (d) across the air-gap.

The first general formula gives the magnetic density (B), in gausses, for three-phase windings of the ring type, or of the definite type, when the current (I) is taken in amperes, and when d is taken in centimetres.

If, instead of the maximum values, we take the effective values, b and i, for magnetic density and current, these general formulae become:

$$b = 1.144\, qi \frac{1}{d};$$

and

$$m = 1.144\, qi.$$

Two-phase Motors. We shall demonstrate, by proceeding as just indicated, that we can also establish, for two-phase motors having windings of the definite type and of the ring type, a general formula whereby it is possible to evaluate, for all practical cases, the maximum mag-

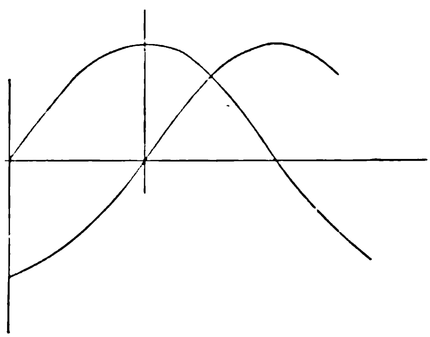


Fig. 37.

netic density in the air-gap as a function of the number (q) of conductors comprised in each coil, of the maximum value of the current (I) passing through these conductors, and of the distance (d) across the air-gap.

Let us suppose, in the beginning, that t is an even number, and let us first find the distribution of magnetic density in the air-gap at the precise moment when the current becomes zero in one of the phases.

As we are dealing with two currents differing in phase by a quarter wave (Fig. 37), the current in the other branch will, at that moment, attain its highest value (I). We can, therefore, immediately draw Fig. 38.

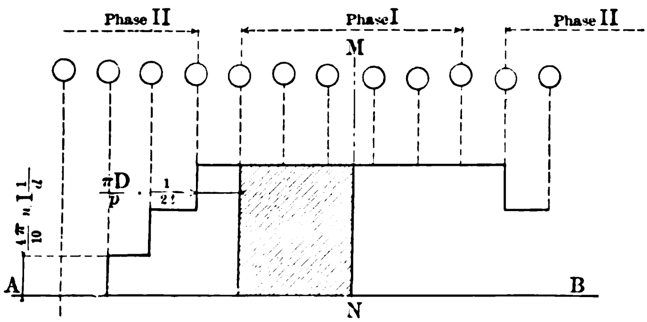


Fig. 38.

The area S_{I} comprised between the right line AB, the broken or "stepped" line, and the centre line MN of the magnetic pole, may be decomposed into two portions, S_1 and S_2, for which we will have the following equations:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} nI \left\{ 1 + 2 + \cdots + \frac{t}{2} \right\} \frac{1}{d}.$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{t-1}{2} \times \frac{4\pi}{10} nI \frac{t}{2} \times \frac{1}{d}.$$

But we know that

$$1 + 2 + \cdots + \frac{t}{2} = \frac{1 + \frac{t}{2}}{2} \cdot \frac{t}{2} = \frac{t^2}{8} + \frac{t}{4}.$$

We therefore have:

$$S_{I} = S_1 + S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} nI \left[\frac{t^2}{8} + \frac{t}{4} + \frac{t^2}{4} - \frac{t}{4} \right].$$

$$S_{I} = \frac{D}{2p} \times \frac{4\pi}{10} nI \frac{3}{8} t \frac{1}{d}.$$

A moment later the current will have the same value, $\left(+ \frac{1}{\sqrt{2}} I \right)$, in both phases, so that the distribution of magnetic density in the air-gap will be as represented in Fig. 39.

The area S_{II} may be decomposed into two parts, S_1 and S_2, and we will have:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} \times \frac{1}{\sqrt{2}} nI \left[1 + 2 + \cdots + (t-1)\right] \frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{1}{2} \times \frac{4\pi}{10} \times \frac{1}{\sqrt{2}} nIt \frac{1}{d};$$

but

$$1 - 2 + \cdots + (t-1) = \left(\frac{t-1+1}{2}\right)(t-1) = \frac{t^2}{2} - \frac{t}{2},$$

so that:

$$S_{II} = S_1 + S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} n \frac{1}{\sqrt{2}} I \left[\frac{t^2}{2} - \frac{t}{2} + \frac{t}{2}\right];$$

whence

$$S_{II} = \frac{\pi D}{2p} \times \frac{4\pi}{10} \times \frac{1}{\sqrt{2}} nI \frac{t}{2} \times \frac{1}{d}.$$

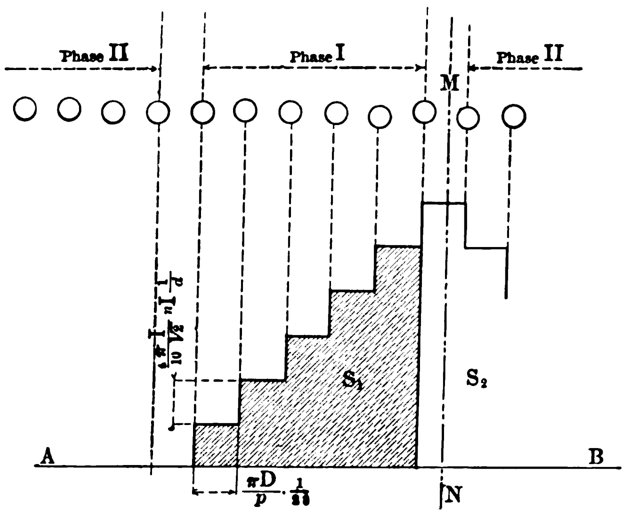


Fig. 39.

We know that the mean magnetic flux issuing from one of the poles of the magnetic field is:

$$\Phi = 2L \frac{S_I + S_{II}}{2},$$

or:

$$\Phi = 2L \frac{\pi D}{2p} \times \frac{4\pi}{10} nIt \left[\frac{\frac{3}{8} + \frac{1}{2\sqrt{2}}}{2}\right] \frac{1}{d},$$

$$\Phi = 2\,L\frac{\pi D}{2p} \times \frac{4\pi}{10}\,nIt\,[0.3645]\,\frac{1}{d}\,;$$

but the maximum ordinate, B, of the equivalent sinusoidal curve, which gives the same flux, Φ, is given by the equation,

$$B = \frac{p}{2\,DL}\,\Phi.$$

Since

$$\Phi = \frac{2}{\pi}\,B\,\frac{\pi D}{p}\,L,$$

the mean ordinate of any sinusoidal curve being $\frac{2}{\pi}\,B$, we can therefore write:

$$B = \frac{p}{2\,DL}\,2L\,\frac{\pi D}{2p} \times \frac{4\pi}{10}\,nIt\,[0.3645]\,\frac{1}{d},$$

or, finally, since $nt = q$,

$$B = 0.720\,qI\,\frac{1}{d}.$$

We also have:

$$M = 0.720\,qI.$$

When t is an odd number, the distribution of magnetic density along the air-gap, for the two instants already considered, will be as shown in Figs. 40 and 41.

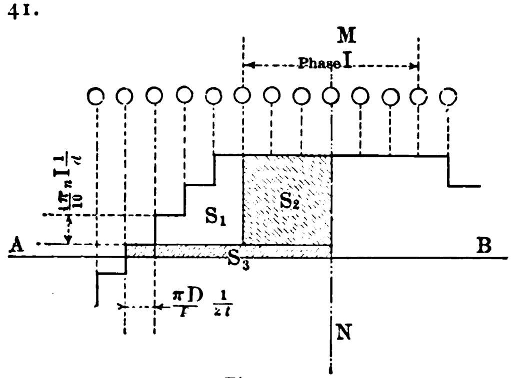


Fig. 40.

For the case shown in Fig. 40, we will have:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10}\,nI\left[1 + 2 + \cdots \left\{\frac{t-1}{2}\right\}\right]\frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10}\,nI\,\left\{\frac{t-1}{2}\right\} \times \left\{\frac{t-1}{2}\right\}\,\frac{1}{d},$$

$$S_3 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10}\,nt\,\frac{t}{2} \times \frac{1}{d},$$

but,

$$1 + 2 + \dots \left\{\frac{t-1}{2}\right\} = \frac{\frac{t-1}{2}+1}{2} \times \frac{t-1}{2} = \frac{t^2}{8} - \frac{1}{8};$$

therefore,

$$S_I = S_1 + S_2 + S_3 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} nI \left[\frac{3+2}{8} + \frac{1}{8}\right] \frac{1}{d}.$$

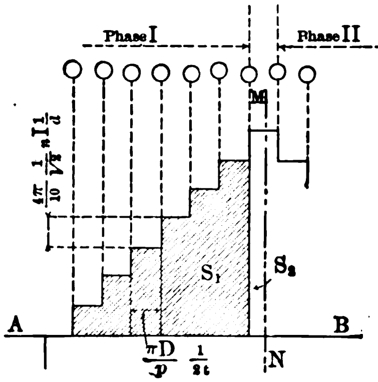


Fig. 41.

In similar manner, we will have, for Fig. 41:

$$S_1 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} n \frac{1}{\sqrt{2}} I \left\{1 + 2 + \dots + (t-1)\right\} \frac{1}{d},$$

$$S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} n \frac{1}{\sqrt{2}} It \frac{1}{2};$$

but,

$$1 + 2 + \dots + (t-1) = \frac{t-1+1}{2}(t-1) = \frac{t^2}{2} - \frac{t}{2},$$

so that,

$$S_{II} = S_1 + S_2 = \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4D}{10} n \frac{1}{\sqrt{2}} I \frac{t^2}{2}.$$

We know that the mean magnetic flux is:

$$\Phi = 2\frac{S_I + S_{II}}{2} L = \frac{2}{\pi} B \frac{\pi D}{p} L;$$

from which we have:

$$B = \frac{p}{2LD} 2L \frac{\pi D}{p} \times \frac{1}{2t} \times \frac{4\pi}{10} nI \left[\frac{\frac{3}{8}t^2 + \frac{1}{8} + \frac{1}{2\sqrt{2}}t^2}{2}\right] \frac{1}{d}.$$

Since, in practice, two-phase motors having less than four slots for the active side of each coil are never met with, the smallest odd number representing t will be 5. We can, therefore, replace the expression,

$$\frac{3}{8}t^2 + \frac{1}{8} + \frac{1}{2\sqrt{2}}t^2 \qquad \text{by} \qquad \frac{3}{8}t^2 + \frac{1}{2\sqrt{2}}t^2.$$

The greatest error made, when $t = 5$, is less than 0.6%. Making this approximation, and simplifying the preceding equation, we arrive at the final formulæ :

$$B = 0.720\, qI \frac{1}{d},$$

and

$$M = 0.720\, qI,$$

which are identically the same as the equations already obtained when t is made an even number.

It is seen, therefore, that the two preceding equations are applicable for any values which may be assigned to the number of slots (t) for the active side of each coil. This formula is, therefore, general for all two-phase windings, either of the ring type or of the definite type.

Wave or Progressive Windings. (*Three-phase Motors.*) In the various arrangements which have just been investigated, the distribution of magnetomotive force along the air-gap is perfectly symmetrical. Each pole can, at each instant, be superposed upon any other pole, and it will cover the same perfectly. This symmetry has enabled us, by the simplest analytical process, to discover a final formula which is applicable to all windings of the ring type or of the definite (polar) type.

In the case of wave windings, we could scarcely expect a uniform distribution of the magnetomotive force along the air-gap, for the reason that the number of active conductors in each phase is not always exactly equal, and also because these windings of necessity have a certain want of symmetry in the distribution of the wires of each phase under the different poles of the resultant field.

This peculiarity must render extremely difficult, not to say impossible, the search for a final formula which will give, for all cases, the maximum ordinate of the theoretical sinusoidal curve replacing the true distribution of the magnetomotive force or of the magnetic density in the air-gap.

The discovery of a general equation for that kind of winding is all the more desirable, since, owing to the want of symmetry of the revolving field, the designer must, in order to calculate the armature reaction, draw, not only for one but for all the poles, the true distribution of the lines of force. This work, which has to be done twice, for two different periods of time, so as to determine the mean magnetic field and replace it by the equivalent sinusoidal flux, is both long and useless.

Let us suppose that the rotor of a 40 H.P. three-phase motor has a wave winding for eight poles, consisting of 186 bars placed in 186 slots. In order that this winding may form a closed circuit after having utilized each of the bars only once, it is necessary that the pitch of the winding should be as given by the well-known equation:

$$y = \frac{S \pm 2}{p};$$

in which

y designates the pitch of the winding;
S designates the number of bars;
p designates the number of poles of the revolving field.

In the particular case assumed, we will have,

$$y = 23.$$

Let us now make a table of the winding, giving the numbers of the bars in the order in which they come in the diagram:

FIRST PHASE.

186 — 23 — 46 — 69 — 92 — 115 — 138 — 161
184 — 21 — 44 — 67 — 90 — 113 — 136 — 159
182 — 19 — 42 — 65 — 88 — 111 — 134 — 157
180 — 17 — 40 — 63 — 86 — 109 — 132 — 155
178 — 15 — 38 — 61 — 84 — 107 — 130 — 153
176 — 13 — 36 — 59 — 82 — 105 — 128 — 151
174 — 11 — 34 — 57 — 80 — 103 — 126 — 149
172 — 9 — 32 — 55 — 78 — 101

SECOND PHASE.

124 — 147
170 — 7 — 30 — 53 — 76 — 99 — 122 — 145
168 — 5 — 28 — 51 — 74 — 97 — 120 — 143
166 — 3 — 26 — 49 — 72 — 95 — 118 — 141
164 — 1 — 24 — 47 — 70 — 93 — 116 — 139
162 — 185 — 22 — 45 — 68 — 91 — 114 — 137
160 — 183 — 20 — 43 — 66 — 89 — 112 — 135
158 — 181 — 18 — 41 — 64 — 87 — 110 — 133
156 — 179 — 16 — 39

THIRD PHASE.

			— 62	— 85	— 108	— 131	
154	— 177	— 14	— 37	— 60	— 83	— 106	— 129
152	— 175	— 12	— 35	— 58	— 81	— 104	— 127
150	— 173	— 10	— 33	— 56	— 79	— 102	— 125
148	— 171	— 8	— 31	— 54	— 77	— 100	— 123
146	— 169	— 6	— 29	— 52	— 75	— 98	— 121
144	— 167	— 4	— 27	— 50	— 73	— 96	— 119
142	— 165	— 2	— 25	— 48	— 71	— 94	— 117
140	— 163						

The number of active bars being 186, each phase will have 62 bars. If we cut the winding at any point whatsoever, and suppress, for example, the connection which joins bars Nos. 163 and 186, we will also have to take away the connection between bars Nos. 101 and 124 on one hand, and 39 and 62 on the other hand. In this way the winding of the first phase will be made up of the bars 186–23–46–69 to 101,—the total number of bars being 62.

The 62 bars, 124–147–170 to 39, inclusive, will form the winding of the second phase, so that the third circuit will consist of the remaining 62 bars, 62–85–108–131 to 163.

It is easy to see that an arrangement of this character evidently constitutes a three-phase winding. In fact, if we suppose conductor No. 186 to be placed exactly in the centre line of a north pole, the beginning of the winding of the second phase will have to be 120° toward the right of any one of the north poles of the magnetic field, if we assume the rotation of the magnetic flux to take place in the direction of the hands of a watch. The bar No. 62 evidently complies with that essential condition. In fact, the peripheral distance between the centre lines of two consecutive north poles is:

$$= \frac{186}{4} a = 46.5\, a,$$

in which (a) designates the distance between two contiguous slots. Since this space also gives the length of a whole wave of magnetic flux, the third of a wave will be equal to:

$$\frac{46.5}{3} a = 15.5\, a;$$

but it is apparent that we must have

$$(46{,}5 + 15{,}5)\, a = 62.0a,$$

and that, consequently, the conductor 62, which is at the beginning of the winding of the second phase, really has a lag of one-third of a wave with respect to the conductor 186, which is at the starting point of the first phase. These two parts of the winding being similar to each other, we can bring them into similar position with respect to the field, by simply turning one of them around one-third of a wave.

We likewise see that, since

$$2 \times 62 = 124,$$

the conductor 124 will be shifted exactly two-thirds of a wave toward the right of a north pole. We therefore have before us a three-phase winding which is perfectly regular. This condition is not always exactly fulfilled. The equation:

$$y = \frac{S \pm 2}{p},$$

which must be satisfied by all wave windings, shows that the number *S*, of conductors on the rotor, can scarcely be divisible by 3, when the number of poles of the magnetic field is a multiple of 3.

If we designate this multiple by *m*, we may write:

$$p = m\,3$$

and

$$y = \frac{S \pm 2}{3\,m},$$

whence:

$$my \mp \frac{2}{3} = \frac{S}{3}$$

Since the term my is a whole number, it follows that $\frac{S}{3}$ cannot be a whole number. The three branches of the secondary winding do not comprise the same number of conductors, and the different windings cannot absolutely have a lag of one-third of a wave between them. The designer should seek to make the differences between the numbers of bars as small as possible. These differences should be of one, or, at most, of two bars. Thus, a wave winding for six poles with 208 conductors corresponding to the following equation:

$$y = \frac{208 \pm 2}{6} = 35,$$

will comprise closed circuits having respectively 69, 69, and 70, or 68, 70, and 70, active sides, if it is required that the connections of the winding with the collector rings shall be all placed at the same end of the rotor.

Let us return to our first arrangement; and, by means of the table of windings let us trace the diagram of the exact distribution of magnetomotive force along the air-gap at the moment when the current in the first phase reaches its highest positive value, $+I$, at which time the current in each of the other two phases will be equal to $\frac{I}{2}$.

Fig. 42 shows that the distribution of magnetomotive force and, consequently, of magnetic density, are not at all regular, and that the poles of the magnetic field have neither the same width nor the same maximum density. To find the points where the magnetomotive force is zero, it is necessary to remember that the magnetic flux which leaves the core to form a half north pole in the air-gap comes back wholly by the next half south pole, and that, consequently, the line corresponding to AB in Fig. 41, must be located in such a way that the surfaces AOC and BOD shall become equal. This condition determines the point O. In reality, all these points (O) fall on the same straight line, and the broken line should be disconnected at the crests C and D. It is not possible to proceed otherwise than was done, in constructing Fig. 42, since the position of the "zero" points, O, or the line of reference for each wave, can only be determined after the entire broken line has been drawn.

If we calculate separately each of the areas AOC, BOD, we find for the mean area:

$$S_{\text{I}} = \frac{4\pi}{10} Ia \frac{1}{d} [45.53].$$

If we repeat the process a second time, for the moment when the current becomes zero in the third phase and takes the values $\frac{\sqrt{3}}{2} I$ and $-\frac{\sqrt{3}}{2} I$, respectively, in the first and second phases (Fig. 42), we obtain the mean of the areas AOC and BOD:

$$S_{\text{II}} = \frac{4\pi}{10} Ia \frac{1}{d} \left[52.6 \frac{\sqrt{3}}{2}\right].$$

The mean value of the two areas S_{I} and S_{II}, which is a mean value with respect to time, has the following expression:

$$S = \frac{4\pi}{10} Ia \frac{1}{d} [45.56].$$

The magnetic field, which will oscillate between the values S_{I} and S_{II}, departs but little from the mean value S, and may be considered practically constant.

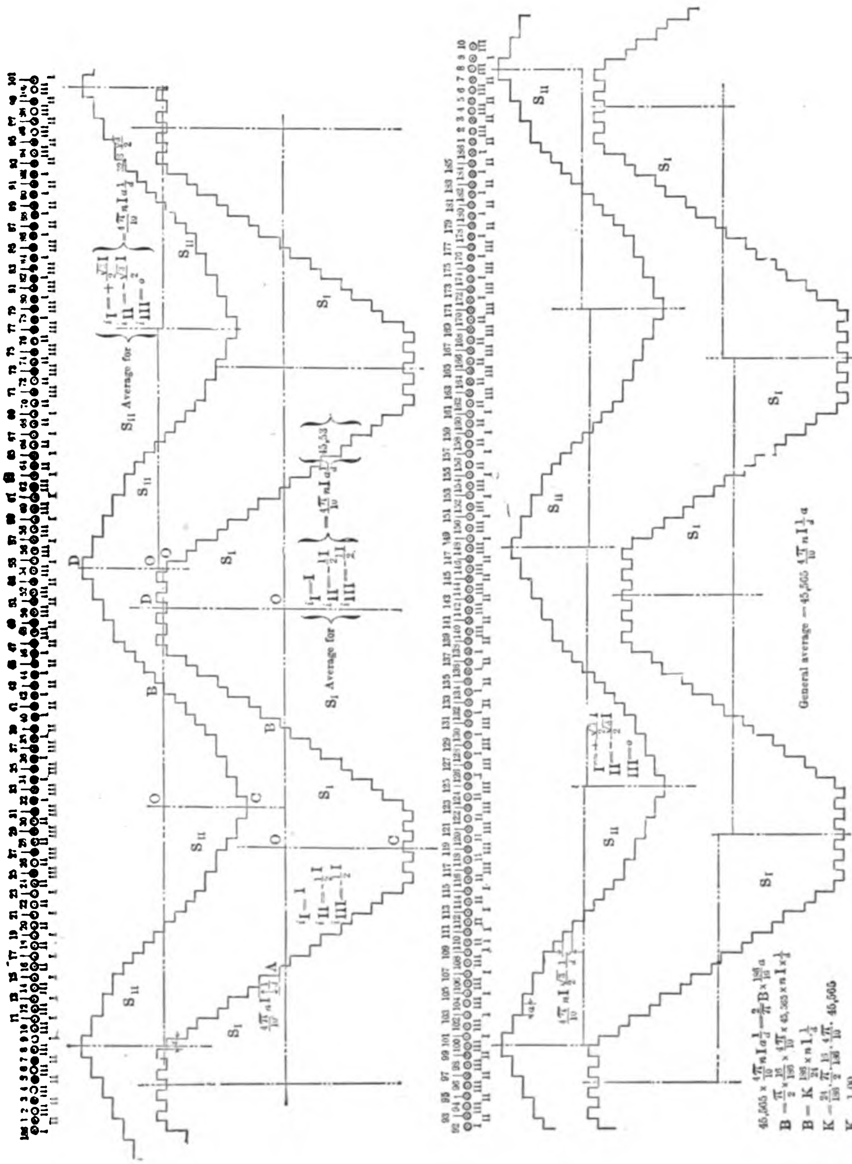


Fig. 42.

If we designate by M the amplitude of the sine curve enclosing an area equal to S, we have:

$$\frac{2}{\pi} M \frac{186}{16} a = \frac{4\pi}{10} Ia \; 45.56,$$

$$M = \frac{\pi}{2} \times \frac{16}{186} \times \frac{4\pi}{10} I \; 45.56.$$

We can write:

$$M = K \frac{186}{3 \times 8} \times I,$$

whence,

$$K = \frac{24}{186} \times \frac{\pi}{2} \times \frac{16}{186} \times \frac{4\pi}{10} \; 45.56,$$

or

$$K = 1.00+:$$

We would likewise have, for the magnetic density,

$$B = K \frac{186}{3 \times 8} \times I \frac{1}{d} \text{ gausses.}$$

It is worthy of note that the coefficient K is (substantially) equal to unity.

By repeating the preceding process for ten wave windings essentially different from each other in regard to the number of poles, as well as the number of bars per pole, the author has found values for K which are so near unity that the greatest discrepancy observed was under 0.5%.

This latter difference was obtained with a winding for six poles having three unequal windings comprising, respectively, 68, 68, and 70 conductors.

We can, therefore, always write, with great accuracy:

$$M = \frac{S}{3p} I,$$

in which

- M represents the maximum magnetomotive force in the air-gap.
- I represents the maximum current value, in amperes.
- S represents the total number of active conductors of the rotor.
- p represents the number of poles of the magnetic field.

If B is to designate the maximum density, in an air-gap whose distance across ("length") is d, we should write:

$$B = \frac{S}{3p} \times \frac{I}{d} \text{ gausses.}$$

The last two equations are general, and apply to all three-phase motors provided with wave windings which are opened at three points.

Two-phase Motors. By cutting a wave winding which is closed upon itself, at two different points which are the same distance apart as the space between two consecutive magnetic poles, we obtain two identical circuits which have a lag of a half-wave, or 180°, between them. The induced E.M.F.'s and currents which would be produced in an arrangement of this character, would always be equal and of contrary direction, and could not produce a revolving field.

To obtain currents having a phase difference of only one-quarter wave, it is necessary to subdivide the winding into four different circuits, in which the resultant field of the air-gap will induce four electromotive forces which lag 90° with respect to each other. Such an arrangement constitutes a four-phase winding. It is evident that the induced E.M.F. or the current of the first circuit will always be equal and of direction contrary to that in the third branch; and the same will be true for the second and fourth phases.

In order to convert this arrangement into a two-phase winding, it will be sufficient, therefore, to transpose the connections of the third and fourth branches, and to connect them, respectively, in series or in parallel with the first and second circuits. We will, in this manner, obtain a two-phase winding whereby E.M.F.'s and currents having a phase difference of 90° may be obtained.

The preceding observations being noted, we will select a number of bars, S, and a pitch, y, corresponding with the equation:

$$y = \frac{S \pm 2}{p},$$

in which y is always an odd number. We will then draw the distribution of magnetic density along the air-gap for the moment when the current in the first phase attains its highest value $+ I$. The current in the second branch is then zero.

This being done, we will determine, for the "zig-zag" line showing the variations of magnetic density, the "zero" points, O, where the magnetic density or the magnetomotive force is zero, and we will estimate, in the manner already indicated (Fig. 42), the mean area S_{I}. We will repeat the same operation for the instant at which the currents in the two windings are both equal to $\frac{I}{\sqrt{2}}$, and we will determine the mean area S_{II}. The quantity S_{I} being a maximum and the area S_{II} being a minimum, we will take the mean, S, of these two values, and we will have the value of the amplitude, M, of the equivalent sinusoidal curve.

If we then assume the relation

$$M = K\frac{S}{2p}I,$$

we can deduce the value of the coefficient K.

For a two-phase winding suitable for six poles and consisting of 208 bars, we have:

$$K = 0.719;$$

with eight poles and 186 bars, we have:

$$K = 0.718,$$

the latter value differing about 0.12% from the preceding value.

By repeating this operation for ten windings differing in the number of poles and in the number of bars per pole, the author has found that the coefficient K is but slightly different from the mean value:

$$K = 0.719.$$

The greatest discrepancy noted is less than 0.2%.

We can, therefore, write, with much accuracy,

$$B = 0.719\frac{S}{2p}I\frac{1}{d} \text{ gauss},$$

and

$$M = 0.719\frac{S}{2p}I.$$

This equation is general for all two-phase motors having wave windings.

Three-phase Motors Wound For Six Phases. It is sufficient to compare the two final formulæ which have been obtained for three-phase and two-phase motors, to note that, with an equal number of active conductors per pole, the armature reaction is higher in the two-phase motors. This is evidently due to the fact that in those wave windings which are opened at three points only, the conductors of each circuit are not entirely separate from each other, but are interspersed between those of the other branches. We have already indicated the disadvantages of such an arrangement with respect to the induced E.M.F. and the torque, and we noted that it was evidently preferable to open the wave winding, not at three points but at six points, so as to form six similar circuits having a phase difference of one-sixth of a wave with respect to each other.

The reader already knows, that instead of making a neutral point with one of the ends of each of the circuits and bringing the six others to six different rings, it is better to reverse the connections of the fourth, fifth, and sixth branches, and connect them in series or in parallel with those of the phases which are 180° apart from them, so as to form only three distinct windings, instead of six.

By proceeding as already indicated, for ten different windings thus arranged, the author has found that in the formula,

$$B = K \frac{S}{3p} I \frac{1}{d} \text{ gausses,}$$

the values of the coefficient K vary but slightly, not differing more than .01% from the following mean value,

$$K = 1.162.$$

We can, therefore, write for all three-phase motors provided with six-phase windings,

$$B = 1.162 \frac{S}{3p} I \frac{1}{d} \text{ gausses.}$$

Squirrel-Cage Windings. The distribution of currents being sinusoidal in the conductors on the periphery of the rotor, we can readily deduce the value of the maximum ordinate of the reaction which these currents would produce, since the effect of all the conductors situated under the same pole is cumulative. Since the mean current in the bars is

$$i_{av} = \frac{2}{\pi} I,$$

we will therefore have

$$B = \frac{4\pi}{10} \times \frac{2}{\pi} \times \frac{S}{p} I \frac{1}{2d} \text{ gausses,}$$

and

$$M = 0.4 \frac{S}{p} I,$$

in which

S designates the total number of bars on the rotor;
p designates the number of magnetic poles;
I designates the maximum current through each bar;
d designates the thickness of the layer of air separating the cores of rotor and stator (length of magnetic path, radially, through the air-gap).

Let us now take

$$B = K\frac{S}{p}I\frac{1}{d} \text{ gausses,}$$

we will have, as already seen,

$$K = 0.4.$$

We will therefore have, for all squirrel-cage windings:

$$M = 0.4\frac{S}{p}I$$

$$\text{and } B = 0.4\frac{S}{p}I\frac{1}{d} \text{ gausses.}$$

Windings Having Turns Closed Upon Themselves. If we designate, as before, by π, the width of pole of the field, and by l the width of the short-circuited loop, we know, as already demonstrated in Chapter III., that the distribution of current in the various windings follows the sine law, and that the current in each of them depends only on the width of the winding (l) and its instantaneous position with respect to the pole. It has also been seen that this width (l) has some influence on the maximum current value, which value attains its maximum when $l = \pi$, and diminishes as l becomes different from π.

If we designate by I the maximum current attainable in each coil, we can write, as in the case of squirrel-cage windings,

$$B = \frac{4\pi}{10} \times \frac{2}{\pi} \times \frac{S}{p} I \frac{1}{2d},$$

or

$$B = K\frac{S}{p} I \frac{1}{d} \text{ gausses,}$$

from which we have

$$K = 0.4.$$

We therefore have, in general, for all windings consisting of short-circuited turns,

$$M = 0.4\frac{S}{p} I,$$

and

$$B = 0.4\frac{S}{p} I \frac{1}{d} \text{ gausses.}$$

CHAPTER V.

THE RESISTANCE OF THE SHORT-CIRCUITING RINGS OF SQUIRREL-CAGE ROTORS.

THE determination of the resistance of windings, of the drum, ring, or polar types, or of windings comprising short-circuited coils, does not present the slightest difficulty. It is different in the case of squirrel-cage windings, in which the bars are all joined together at each of the two ends of the rotor.

Some manufacturers solve the problem by increasing greatly the cross-section of the short-circuiting rings, so as to make their resistance negligible, compared with that of the bars.

This method of proceeding cannot be recommended because the entire ohmic loss (Joulean effect) then occurs inside the secondary core, which it is very difficult to ventilate properly, owing to the smallness of the air-gap. The heating at the periphery of the iron core of the secondary (rotor) then tends to increase the heating of the primary (stator) core.

It does not follow that the decrease in cross-section of the squirrel-cage winding bars, obtained when the cross-section of the short-circuiting rings is increased, will enable the size of the motor to be reduced, because the dimensions of the latter must always be sufficient to allow proper space for the stator windings.

The author deems it more logical to give to the rotor bars the greatest cross-section which is compatible with the diameter of the core, and to leave to the *short-circuiting rings* the greater portion of the secondary resistance, R_2.

Considering the energetic ventilation to which these rings are always subjected, it will be easy to give them sufficient cooling surface to prevent excessive heating.

It sometimes happens that, for special purposes, motors of more than 20 H.P. are required, which can develop a heavy starting torque by simply closing the main switch, and without using any special starting outfit.

If very heavy starting currents are to be avoided, it is indispensable to make allowance for relatively large slips (up to 10%), and, conse-

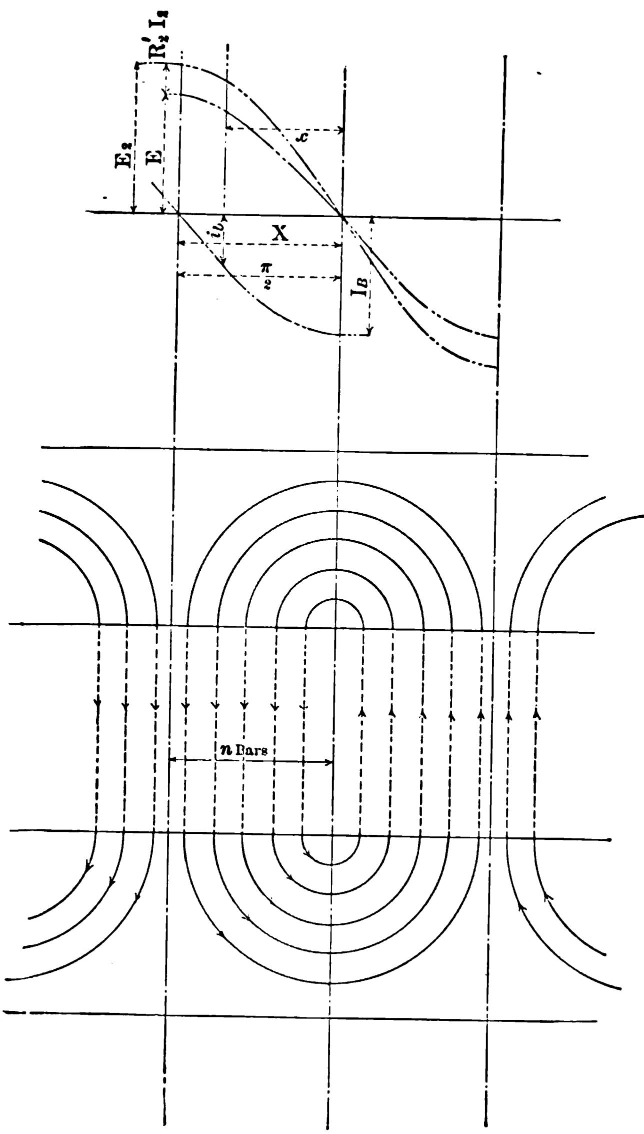


Fig. 43.

quently, it is necessary to have the rotor resistances relatively high. The loss by ohmic resistance (Joulean effect), and the consequent heating in the secondary, being considerable, it is absolutely necessary that this heat should be dissipated outside the core of the rotor, in order to avoid abnormal rises of temperature.

It is obvious that, in such cases, the proportions of the short-circuiting rings should not be fixed arbitrarily.

It may happen that, owing to a large amount of slip when the motor has to start rather slowly under a heavy load requiring a high starting torque, it is not possible to give sufficient cooling surface to the short-circuiting rings. In such cases, a winding consisting of insulated loops closed upon themselves should be adopted. The connecting forks serving to join the winding bars in pairs can always be designed in manner such as to provide sufficient ventilating surface.

The same result may be obtained, in a squirrel-cage winding, by making the short-circuiting rings of smaller diameter, and using thin wide strips for the connections between the short-circuiting rings and the winding bars.

The cost of these two arrangements being rather high, it is desirable to use the simplest arrangement possible in each case.

These considerations show that the short-circuiting rings for squirrel-cage windings deserve careful attention, and that it is, in any event, desirable to have a simple formula whereby they may be judiciously proportioned.

Fig. 43, which shows, diagrammatically, the development of a squirrel-cage winding, and which indicates the direction of the currents in the winding bars and in the short-circuiting rings, shows that it is precisely at the points where the current is zero in the active conductors on the periphery of the rotor, that the current density is greatest in the short-circuiting rings.

The current in the short-circuiting rings at these points is equal to the sum of the currents in the n bars which correspond to the half width of a field pole on one side of these points. The current in the short-circuiting rings is zero in the immediate vicinity of those bars wherein the current is attaining its highest value.

Let us take

E = the maximum ohmic drop in each of the short-circuiting rings, in volts.

I_2 = the maximum current in the bars, in amperes.

I_b = the maximum current in the short-circuiting ring, in amperes.

i_b = the current in this ring at any point whatever of the rotor periphery, in amperes.

$2n$ = the number of rotor winding bars per pole.

$2X$ = the width of each pole, in centimetres, measured along the periphery of the short-circuiting ring.

By examining Fig. 43 we see that

$$i_b = I_b \cos \left\{ \frac{\pi}{2X} x \right\}.$$

but

$$I_b = \frac{2}{\pi} I_2 n,$$

whence

$$i_b = \frac{2}{\pi} I_2 n \cos \left\{ \frac{\pi}{2X} x \right\}.$$

If we designate by E the total ohmic drop, in volts, in one of the short-circuiting rings, and by a the resistivity of the metal of which it is made, s being its cross-section, in square centimetres, we have

$$dE = i_b dr = \frac{2}{\pi} n I_2 \cos \left\{ \frac{\pi}{2X} x \right\} \frac{a}{s} dx;$$

from which, by integration, we obtain

$$E = \int_0^X i_b \, dr = \frac{2}{\pi} n I_2 \frac{a}{s} \int_0^X \cos \left\{ \frac{\pi}{2X} x \right\} dx,$$

or

$$E = \frac{2}{\pi} n I_2 \frac{a}{s} \times \frac{2X}{\pi} \left(\sin \left\{ \frac{\pi}{2X} x \right\} \right)_0^X,$$

and, finally

$$E = \frac{2}{\pi} n I_2 \frac{a}{s} \times \frac{2X}{\pi}.$$

By taking the effective values instead of the maximum values, we could write

$$e = \frac{2}{\pi} n i_2 \frac{a}{s} \times \frac{2X}{\pi}.$$

But, by dividing the effective loss of potential e, in the ring, by the effective current, i_2, passing through it, we have the ring resistance R, according to the following equation:

$$R = \frac{i_2}{e} = \frac{E}{I_2} = \frac{2}{\pi} n \frac{a}{s} \times \frac{2X}{\pi},$$

but

$$2X = \frac{\pi D}{p},$$

when D represents the diameter of the ring, in centimetres, and p the number of poles of the revolving field.

From this we deduce,

$$R = \frac{2n \times a \times D}{\pi \times p \times s}.$$

Designating by S the total number of winding bars on the secondary core, we may write

$$2np = S,$$

and finally, substituting, we have

$$R = \frac{SD}{\pi p^2} \times \frac{a}{s} \text{ ohms.}$$

This equation, which gives the resistance of only one of the two short-circuiting rings, can be written as follows:

$$s = \frac{SD}{\pi p^2} \times \frac{a}{R} \text{cm}^2 \cdot$$

The maximum current value in the ring is, evidently,

$$I_b = \frac{2}{\pi} n \ I_2 \text{ amperes,}$$

so that the mean square of the current in the ring will be

$$\tfrac{1}{2} I_b^2 \text{ amperes.}$$

If we multiply this value by the total resistance of the ring as given by the formula,

$$R_b = \frac{\pi Da}{s} \text{ ohm,}$$

we have, for the power (watts) lost in each of the rings,

$$P_b = \frac{\pi Da}{s} \times \frac{1}{2} \times \frac{4}{\pi} n^2 I_2^2,$$

or else, replacing I_2 by $i_2 \sqrt{2}$, and n by $\frac{S}{2p}$, since $2pn = S_1$,

$$P_b = \frac{S^2 Da}{sp^2} \ i_2^2 \text{ watts.}$$

This equation, which gives the total loss by ohmic resistance in each ring, will, at the same time, indicate the cooling surface which ought to be given to each ring, while retaining the cross-section s, required by the preceding formula.

In most cases a cooling surface of 3 square centimetres per watt of loss will be sufficient.

The magnetic field in the air-gap being of sinusoidal character, the induced E.M.F., E_2, will also be sinusoidal. In order that the distribution of the currents, i_2, in the winding bars, may also follow the sine law, it is indispensable that the ohmic drop along the short-circuiting ring should also be a sine function. This condition is fortunately fulfilled, for, as we have just seen,

$$E = dx = \frac{2}{\pi} n\, I_2 \cos \left\{ \frac{\pi}{2\, x}\, x \right\} \frac{a}{s}\, dx\,;$$

by integrating which, between limits corresponding to any point whose abscissa is x, we have, for the ohmic drop at said point:

$$E = \frac{2}{\pi} n I_2 \frac{a}{s} \times \frac{2\, x}{\pi} \left\{ \sin \frac{\pi}{2\, x}\, x \right\} + C;$$

but when $x = 0$, we also have

$$E = 0\,;$$

therefore, the constant C will be o. Consequently E is a sine function; and the distribution of currents, i_2, in the bars on the periphery of the rotor is really sinusoidal.

The formulæ which have been derived hereinabove are, therefore, perfectly exact.

CHAPTER VI.

RECAPITULATION OF PRECEDING CHAPTERS.

THE designer will find, in the preceding chapters, the derivation of all the formulæ necessary for calculating any polyphase motor whatever.

For the convenience of the reader and to facilitate calculations of polyphase non-synchronous (" induction ") motors as much as possible, we will briefly summarize the results which we have already obtained.

Induced E.M.F. Let

E = the maximum E.M.F., in volts, induced in the winding of each of the branches (phases);
Z = the number of conductors connected in series in the same phase;
L = the axial length of the iron core, in centimetres;
V = the speed of the magnetic field, in centimetres per second, with respect to the conductors, S;
B = the maximum magnetic density in the air-gap, in gausses;
K_1 = a co-efficient depending upon the kind of winding.

We can write for all polyphase motors:

$$E = K_1 ZLVB \, 10^{-8} \text{ volts.}$$

The value of the coefficient K_1 is given in the following tables:

THREE-PHASE MOTORS.

I. *Windings of Definite or of Ring Type.*

NUMBER OF SLOTS PER ACTIVE SIDE OF COIL, t.	VALUES OF THE COEFFICIENT, K_1.
1	1.000
2	0.966
3	0.960
4	0.958
5	0.957
6	0.956
∞	0.955

II. *Three-Phase Wave Windings.*

$$K_1 = 0.826,$$

$$E = 0.826 \times ZLVB\ 10^{-8} \text{ volts.}$$

III. *Six-Phase Wave Windings.*

$$K_1 = 0.955,$$

$$E = 0.955\ ZLVB\ 10^{-8} \text{ volts.}$$

TWO-PHASE MOTORS.

I. *Definite or Ring Windings.*

Number of Slots per Active Side of Coil, t.	Values of the Coefficient, K_1.
1	1.000
2	0.924
3	0.911
4	0.906
5	0 904
6	0 903
10	0.901
∞	0.900

II. *Four-Phase Wave Windings.*

$$K_1 = 0.900,$$

$$E = 0.900\ ZLVB\ 10^{-8} \text{ volts.}$$

FOR THE ROTORS OF POLYPHASE MOTORS.

I. *Squirrel-Cage Windings.*

We have in this case:

$$Z = 1$$

and

$$K_1 = 1,$$

$$\therefore E = LVB\ 10^{-8} \text{ volts.}$$

II. *Windings Composed of Short-circuited Turns.*

In this case we have: $Z = 2,$

and

$$K_1 = \cos\left\{\frac{\pi - l}{2}\right\},$$

$$E = 2LVB \cos\left\{\frac{\pi - l}{2}\right\} 10^{-8} \text{ volts,}$$

l being the angular width of each turn.

Electromagnetic Couple. Let

F = the electromagnetic (tangential) effort or pull, in kilogrammes, measured on the periphery of the core of the rotor;
S = the total number of active conductors on the periphery of the rotor;
i = the effective value of the current in these conductors, in amperes;
b = the effective magnetic density in the air-gap, in gausses;
K_1 = a coefficient depending upon the kind of winding.

We can write for all polyphase motors:

$$F = K_1 \frac{Sib}{9.81 \times 10^6} \text{ kilogrammes,}$$

$$= 2.35\, K_1 Sib \times 10^{-7} \text{ pounds,}$$

in which expression the coefficient K_1 takes exactly the same values as those given in the preceding tables and in the formulæ for calculating the induced E.M.F.

Magnetizing Actions of Primary and Secondary Windings. Let

S = the total number of active conductors of the winding;
p = the total number of poles of the revolving magnetic field;
I = the maximum current, in amperes, in the active conductors;
B = the maximum magnetic density, in gausses, which can be produced in the air-gap by the winding;
M = the maximum magnetomotive force, in C.G.S. units;
d = the (radial) distance across the air-gap, in centimetres;
K = a coefficient depending upon the kind of winding.

We can write for all polyphase motors:

$$B = K \frac{S}{p} \times I \times \frac{1}{d} \text{ gausses,}$$

and

$$M = K \times \frac{S}{p} \times I.$$

The values of the coefficient K are as designated in the following table:

VALUES OF THE COEFFICIENT K.

Three-phase winding of definite or of ring type	0.381
Two-phase winding of definite or of ring type	0.360
Three-phase wave winding	0.333
Four-phase wave winding	0.359
Six-phase wave winding	0.387
Squirrel-cage winding	0.400
Winding consisting of short-circuited coils	0.400

CHAPTER VII.

PERFORMANCE DIAGRAMS OF INDUCTION MOTORS.

LET us take any induction motor whatever, and let us supply it with a polyphase current of normal voltage, after having opened all the circuits of the rotor; the latter, since no current can circulate through it, will remain stationary.

To give more clearness to the deductions which follow, we will first suppose that the cores of the motor occasion no loss by parasite (Foucault) currents, or by hysteresis, and we will consider only the ohmic resistance, R_1, of the windings of each of the branches of the stator.

After we shall have constructed a performance diagram based upon this hypothesis, we will then proceed to construct a new diagram in which the losses in the iron of the cores will be taken into account.

As soon as the terminals of the stator winding are connected with the source of electric current supply, currents will circulate in the windings. These currents, by virtue of well-known principles, will produce a magnetic field which revolves in the air-gap.

We know that this revolving field should produce in each of the branches (phases) an electromotive force of which the value can be easily expressed. It seems evident, at first glance, that the two electromotive forces which act simultaneously in the winding, namely, the potential difference at the terminals, on the one hand, and the induced E.M.F. on the other hand, will combine, or what is equivalent to the same thing, will add themselves algebraically at each moment.

The sum, e, of this algebraical addition, gives the instantaneous value of the E.M.F. really acting in the winding and serving wholly, by virtue of Ohm's law, to produce the current, i_0 in the resistance, R_1. This current, i_0, is therefore, given by the formula:

$$i_0 = \frac{e}{R_1}.$$

Since the potential difference at the motor terminals, and the induced E.M.F., are both sine functions of time, their resultant, (e), will, likewise, follow the sine law.

If we designate by E the maximum value of the resultant E.M.F. the maximum current will be :

$$I_0 = \frac{E}{R_1}.$$

In order that it may be possible to find this resultant, E, it is necessary that we should know the phase difference which exists either between the current and the induced E.M.F., or between the latter and the E.M.F. producing the potential difference at the motor terminals, or, again, between the current and the applied E.M.F.

We have only to draw, in the manner which has already been described, a diagram of the distribution of magnetic density in the air-gap, in order to ascertain that the current becomes zero in one of the branches (phases), at the very moment when the active sides of the coils of said branch are the most favorably placed with reference to the production of the induced E.M.F.

We will also be easily convinced that this E.M.F. induced by the magnetic field is zero at the precise moments when the current attains its maximum and minimum values.

We must conclude, from what precedes, that the induced E.M.F. has a phase difference of 90° or of one-quarter of a wave, with respect to the current, and that it lags behind the latter.

Another mode of reasoning might also be adopted. After having observed that the magnetic flux threading through the turns of one of the windings is in phase with the current, and remembering that this flux is a sine function of time, it was easy to conclude that the induced E.M.F. must necessarily be a cosine function, and must consequently lag a quarter-phase with respect to the magnetic flux or the current.

If we represent (Fig. 44) the current at each instant of time by the projection, on the axis MN, of the vector OA, turning around the point O with the angular of velocity ω, and of length proportional to the maximum value of the current, the projection, on the line MN, of the vector OB, which lags 90° with respect to OA, will represent, at each instant, the value of the induced E.M.F., since OB is, for any given scale, the measure of the maximum value of this E.M.F.

As data of the problem, we have, in addition to the dimensions of the motor under consideration, the ohmic resistance of each winding and the maximum value of the difference of potential at the motor terminals.

It does not seem possible, *a priori*, to determine exactly, by means of these factors alone, the values of the no-load current and of the induced E.M.F.; for the latter, as well as the ohmic drop $R_1 I_0$, depend precisely on this current, I_0, whose value is not known.

This difficulty is, however, only apparent. It suffices, in fact, to assign to I_0 any value whatever, such as *OC*, in order to enable us to determine at once, by the scale, the ohmic drop *OD* and the corresponding induced E.M.F., *OE*.

Since *OD* must be the resultant of the induced E.M.F., *OE*, and of the initial difference of potentials, of the source of current supply, we find, for the latter, the vector *OF*, by completing the parallelogram *OEDF*.

This value, *OF*, is that which is necessary to produce, in the motor, the maximum current strength, I_0, which was previously assumed arbitrarily.

In order to obtain, now, the true value of the current I_0, that is to say, that which corresponds to the given difference of potential of the

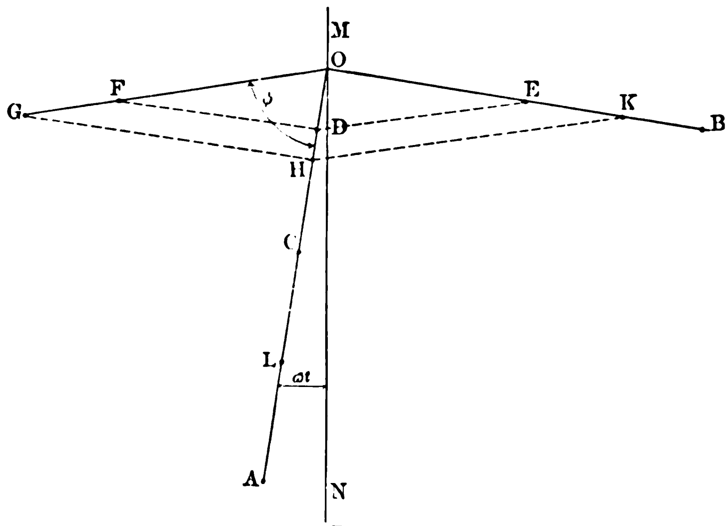


Fig. 44.

source of current supply, we must draw, on *OF* prolonged, the line *OG*, which will represent, to the same scale, the potential difference at the terminals.

We then draw *GH* parallel to *FD*, and *HK* parallel to *DE*.

The right lines *OH* and *OK* will then indicate the value and represent the phase relation of the maximum ohmic drop and of the E.M.F. of self-induction, corresponding to the voltage at the motor terminals.

Since *OH* is nothing more than I_0R_1, it will be easy to deduce from it the value of the current, *OL*, passing through the windings.

This construction of the diagram is absolutely rigorous; for, if the current increases from *OC* to *OL*, the ohmic drop, the magnetic flux, and, consequently, the induced E.M.F., will increase proportionally from OD to OH, and from OE to OK.

But, in order that OH may be the resultant of OK and of the potential difference at the motor terminals, it is necessary that the latter be represented, both in phase and amount, by OG. Now, this vector OG has been drawn exactly equal to the given potential difference at the motor terminals. This proves that when supplying an induction motor whose cores produce no loss, with a voltage OG, the current would have a value indicated by OL, its phase having a lag equal to the angle ϕ behind OG.

We have presumed, hitherto, that the vectors of the diagram represented the maximum values of currents and E.M.F.'s. Practically, since the maximum values are always equal to the effective values multiplied by $\sqrt{2}$, these same vectors may be considered as representing the effective values themselves.

The power absorbed in each branch of the winding is just equivalent to the Joulean effect (ohmic loss). We may, therefore, write:

$$I_0^2 R_1 = I_0 E_0 \cos \phi.$$

If we examine the diagram we see that the induced (counter) E.M.F., in combining with the potential difference at the motor terminals, gives a resultant potential difference which will be smaller in proportion as the winding resistance is reduced, and as the induced E.M.F. for a given current value is increased.

This explains why the currents which circulate in induction coils are very small in comparison with the terminal voltage and the resistance of their windings.

In all that precedes, we have assumed that there were no losses in the iron of the cores. As this hypothesis is never realized, it is necessary to take these losses into consideration in the diagram for induction motors running without load.

When a magnetic flux of variable magnetic density traverses a piece of iron, or of any other metal whatever, it must, by virtue of the laws of induction, create electromotive forces which, in turn, give rise to currents.

We know that these induced E.M.F.'s, and, consequently, the parasite (eddy) currents produced by them, lag a quarter phase behind the magnetic flux which generates them.

These currents, when they occur in the magnetic circuit, have, unavoidably, a magnetizing effect; and this effect combines with that of the main ampere-turns, which primarily cause the magnetic flux.

The resultant of these magnetic actions is, evidently, that which over-comes the reluctance of the magnetic circuit and produces the magnetic flux through it. This resultant is naturally in phase with the flux produced by it.

If, therefore, we assume that this flux is represented, in amount and in phase, by the vector *OA* (Fig. 45), the resultant of the magnetizing actions will fall in the direction of this vector. But we know that the eddy currents in the cores retard the flux by 90°. We therefore know their direction, which is that of the vector *OB*.

If, now, the magnetic flux has to retain the phase relation and the amount indicated by the vector, *OA*, it is necessary that the ampere-turns of the winding should have a first component in the direction *OA*, and a second component equal to *OB*, but of contrary direction thereto, in order to counterbalance the magnetizing action of the eddy currents.

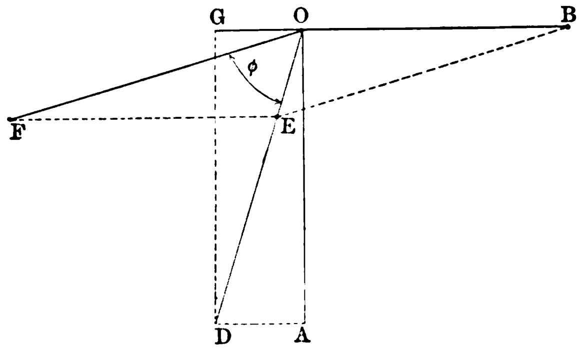


Fig. 45.

This method of looking at things is equivalent to considering the eddy currents as due to a secondary winding closed upon itself, whose magnetizing action must be, at each moment, destroyed by a component of the primary current having a phase difference of 180°, that is to say, opposed to *OB* and directed toward *OG*.

The value of the flux *OA* may be easily found, since we know that it must induce in the windings an E.M.F. very slightly less than the potential difference at the motor terminals.

The weights of the cores being easily determined, and assuming that the quality of the iron used is known, the losses by eddy currents may be estimated without difficulty.

If we divide these losses, expressed in watts, first, by the number of phases, then by the effective induced E.M.F., *OB*, we obtain the effective value, *OG*, of the component of the no-load current which destroys the magnetizing action of the eddy currents.

We can also calculate the second component of the inducing current which produces the flux *OA* with which it is in phase. Let *OA* be this component, when measured according to a scale of amperes.

The two vectors, *OG* and *OA*, being known, we need only to complete the parallelogram, *OGDA*, to obtain, in amount and in phase, the effective value, *OD*, of the actual current.

To complete the performance diagram for the no-load condition, we will plot, along *OD*, the distance, *OE*, equal to the ohmic drop, R_1I_0, then along *OB*, the electromotive force of self-induction, E_1; and, by completing the parallelogram, *OBEF*, we obtain the E.M.F., E_0, at the terminals. The angle, ϕ, between *OF* and *OD*, will represent the lag in phase between the current and the E.M.F. of the source of current supply.

Hitherto we have considered only a part of the losses caused by the iron cores, — that due to the presence of eddy currents. We have paid no attention to the hysteresis losses, in order to simplify and make as clear as possible the explanation of phenomena which, at first glance, might seem rather complicated. This decomposition of the total current into two components, one of which is called *wattless*, because it produces the magnetic field without giving energy, and the other *watted*, because it furnishes the power absorbed by the losses in the iron, seems irrefutable so long as we consider only the eddy currents.

The manner in which hysteresis reacts upon the primary windings is more difficult to conceive, and the authors who have hitherto dealt with this question do not appear to have made it absolutely clear.

In any event, the power lost by hysteresis is necessarily furnished by the external source of current, so that the no-load current, *I*, must contain a third "watted" component opposed to the induced E.M.F. and directed toward *OG*.

If, in Fig. 45, we admit that *OG* represents not only the watted component of no-load current due to the losses by parasite currents alone, but also that which corresponds to the total power dissipated in the cores, the diagram which has just been obtained will be that of the no-load operation of the motor under consideration.

In order to make the matter still more clear, we will take a numerical example, and calculate the no-load current, when the rotor circuits are opened, in a three-phase motor designed for 30 H.P., with 190 volts between phases, when running at 575 R.P.M. This motor was constructed according to the following data:

Axial width of cores	150 mm.
Interior bore of stator disks	552 "
Exterior diam. " " "	716 "

Exterior diam. of rotor disks	550 mm.
Interior " " " "	410 "
Number of periods per second	50
" " field poles	10
" " stator coils	75
" " rotor "	15
" " stator slots	90
" " rotor "	120
Turns per phase in stator	15
" " coil " "	15
Wires per slot in stator	5
Sectional area of wire	24 mm^2
Resistance of each branch of stator winding .	R_1 = 0.08 ohms.

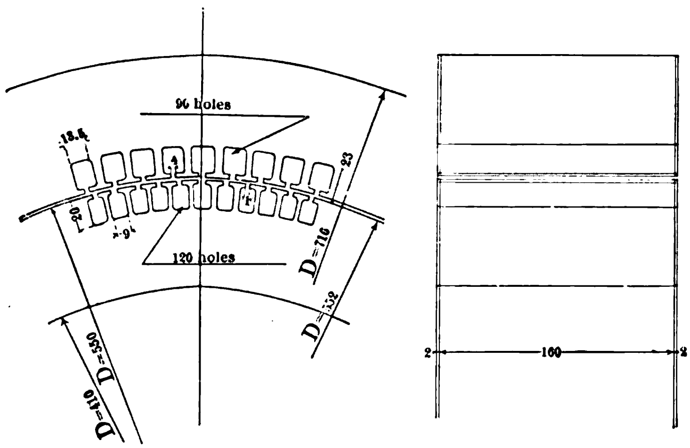


Fig. 46.

If, by means of these dimensions and those given in Fig. 46, we calculate the weight of the cores, making an allowance of 7 per cent loss in volume, owing to the buckling of the iron sheets and to the paper disks interposed between them, we find the following:

Weight of stator core	141	kilogrammes.
" " " teeth	15	"
" " rotor core	84	"
" " " teeth	13	"

The ohmic drop, in a three-phase motor working under full load, never exceeds 4 per cent of the voltage at its terminals. With no load, this ohmic drop is always below 1 per cent. Since the losses in the cores require a watted current which is very small in proportion to the total current, when the motor is running empty, we must conclude that the induced E.M.F. will be very nearly equal to the difference of potential at the motor terminals, and that the angle of lag, ϕ, between the latter and the current will be very nearly 90°.

Let us take, therefore, for the maximum value of the E.M.F. induced by the magnetic field,

$$E = 110\sqrt{2} \text{ volts};$$

but we know that

$$110\sqrt{2} = K_1 L V B \, 10^{-8}.$$

Now, when the active side of each coil occupies three slots, we have

$$K_1 = 0.96.$$

In the present case, we may take, for the length of these active sides,

$$L = 16.0^{cm},$$

and, for the total number of conductors per phase,

$$Z = 2 \times 75 = 150.$$

The velocity of the revolving magnetic field, at the middle of the air-gap, will be

$$V = \tfrac{600}{60} \times \pi \times 55.3 = 1{,}730 \text{ centimetres per second.}$$

If, in the last equation, we replace K_1, L, V, and Z by their numerical values, we have,

$$B = 3900 \text{ gausses},$$

for the maximum ordinate of the sine curve giving the theoretical distribution of magnetic density in the air-gap.

To put it differently, it is necessary, in order to produce a maximum E.M.F. of 110 $\sqrt{2}$ volts, that the highest magnetic density in the air-gap should attain the value of

$$B = 3900 \text{ gausses},$$

just found.

We have already seen that this maximum magnetic density satisfies the relation

$$B = K \times \frac{S}{p} \times I_0 \frac{1}{d} \text{ gauss};$$

whence we have

$$I_0 = \frac{Bdp}{KS}.$$

But, in the case of three-phase definite or polar windings, we have

$$K = 0.381;$$

and for the particular motor under consideration, we would have

$$p = 10, \qquad S = 2 \times 75 \times 3 = 450,$$

and

$$d = 0.1^{\text{cm}}.$$

A glance at Fig. 46 shows that the slots in the stator and rotor have the effect of reducing the surface of the air-gap through which the magnetic flux passes.

This decrease in surface involves a proportional increase in the magnetic density in the air-gap, because the magnetic flux necessary for producing the induced E.M.F. remains constant.

If there were no slits between the tips of the teeth, that is to say, if the slots were wholly closed instead of being only partially closed, the development of the periphery of the disks of the stator would be

$$\pi D_1 = \pi \times 55.2 = 173.4^{\text{cm}}.$$

In consequence of the slits (each 4 millimetres wide) between the teeth (see Fig. 46), it becomes reduced to

$$173.4 - 90 \times 0.4 = 137.4^{\text{cm}}.$$

In the case of the rotor the total development with closed slots would be

$$\pi \times 55.0 = 172.8^{\text{cm}};$$

but with 120 slits of one millimetre (Fig. 46), it becomes

$$172.8 - 120 \times 0.1 = 160.8^{\text{cm}}.$$

The mean circumferential or peripheral length of iron surface along the air-gap circle is therefore,

$$\frac{137.4 + 160.8}{2} = 149.1^{\text{cm}},$$

whereas it would be equal to

$$\pi \times 55.1 = 173.1^{\text{cm}},$$

if there were no slits.

The magnetic density, B, previously found, should therefore be increased in the ratio of $\frac{173.1}{149.1}$, and it will become

$$B = 3{,}900 \, \frac{173.1}{149.1} = 4525 \text{ gausses}.$$

Let us note, in passing, that we could also have made allowance for the decrease in the surface of the air-gap due to the slot openings, by increasing the distance across this air-gap (or its radial "length") in the same proportion, and that the result would not be changed.

If, in the preceding formula, we replace B, K, p, S, and d, by their numerical values, the maximum value of the no-load magnetizing current will be

$$I_\mu = \frac{4,525 \times 0.1 \times 10}{0.381 \times 450 \times 1} = 26.3 \text{ amperes.}$$

The effective value of this component would naturally be

$$i_\mu = \frac{26.3}{\sqrt{2}} = 18.6 \text{ amperes.}$$

Since, for each tooth of the stator, the ratio between the width at the base and at the narrowest point is $\frac{15.3}{5.9}$, the maximum magnetic density in the tooth will be

$$B_{d1} = 4,525 \times \frac{15.3}{5.9} = 11,720 \text{ gausses.}$$

We will have, likewise, for the teeth in the disks of the rotor,

$$B_{d2} = 4,525 \times \frac{13.4}{4.37} = 13,880 \text{ gausses.}$$

On the other hand, we know that the total magnetic flux issuing from one of the poles of the magnetic field is nothing more than the mean value of the magnetic density in the air-gap multiplied by the surface which this pole occupies in the air-gap. We therefore have

$$\Phi = \frac{2}{\pi} 3,900 \, \pi \, 55.1 \times 16,$$

or

$$\Phi = 688,000 \text{ maxwells.}$$

The cross section of the cores being

$$S_1 = 16 \times 0.93 \times 5.0 = 87.8^{\text{cm}^2}$$

for the stator, and

$$S_2 = 16 \times 0.93 \times 5.0 = 74.4^{\text{cm}^2}$$

for the rotor, we will have, for the maximum magnetic density,

$$B_1 = \frac{688,000}{87.8 \times 2} = 3,920 \text{ gausses,}$$

in the core of the stator, and

$$B_2 = \frac{688,000}{2 \times 74.4} = 4,630 \text{ gausses,}$$

in the core of the rotor.

If, now, the sheets constituting the cores are of such quality that the losses, in watts per 100 kilograms, measured in terms of the maximum magnetic density, are like those repsesented by the curve given in Fig. 47, we will have the following results for the core losses:

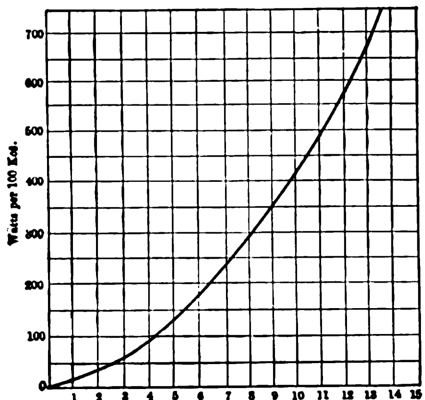


Fig. 57.

In the teeth of the stator,

$$P_{d1} = 0.15 \times 600 = 90 \text{ watts.}$$

In the teeth of the rotor,

$$P_{d2} = 0.13 \times 850 = 110 \text{ watts.}$$

In the core of the stator,

$$P_{n1} = 1.41 \times 75 = 105 \text{ watts.}$$

In the core of the rotor,

$$P_{n2} = 0.84 \times 100 = 84 \text{ watts.}$$

The total loss in the iron of the motor under consideration will be, consequently, equal to

$$P = 389 \text{ watts.}$$

Dividing this amount by the number of phases and by the effective induced E.M.F., the effective watted component of the no-load current will be:

$$i_w = \frac{389}{3 \times 110} = 1.18 \text{ amperes.}$$

Let us plot, along *OA* (Fig. 48), the value found for the effective magnetizing current, $i\mu = 18.6$ amperes, and along *OC*, the value of the watted component, $i_w = 1.18$ amperes.

By completing the rectangle $OADC$, we obtain the vector OD, which represents, in amount and phase relation, the effective value of the no-load current,

$$i_0 = 18.63 \text{ amperes.}$$

Let us plot, along OD, the ohmic drop, according to a suitable scale of volts, such that

$$OE = i_0 R_1,$$

and let us draw the induced E.M.F., perpendicularly to OA, such that

$$OB = 110 \text{ volts.}$$

By completing the parallelogram $OBEF$, we will obtain the effective difference of potential at the motor terminals:

$$OF = 110 \text{ volts.}$$

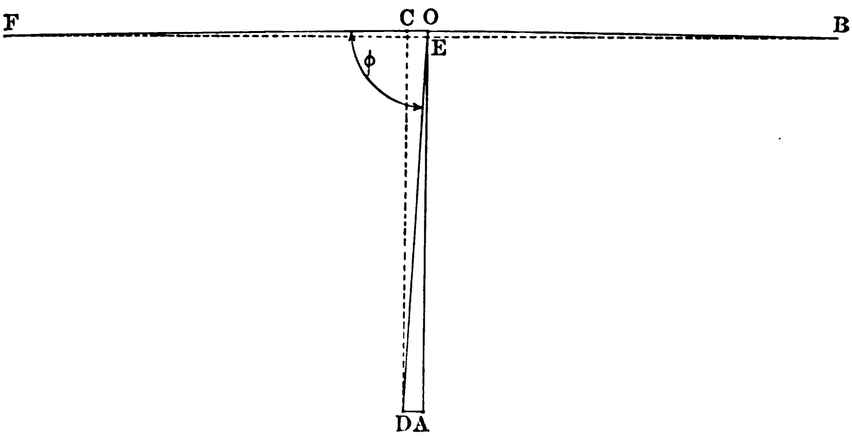


Fig. 48.

By subjecting the motor under consideration to an effective difference of potential of 110 volts at its terminals, the effective current through the stator winding, when the rotor is at rest and has no current passing through it, will be equal to 18.63 amperes.

The diagram shows that the phase difference between the no-load current and the difference of potential at the terminals, is almost equal to a quarter phase, and that the ohmic drop, when running without load, is sufficiently low to enable us to make the *a priori* assumption that the induced E.M.F. is equal to the potential difference at the motor terminals.

On testing the motor under consideration, it would be found that the current, when running without load, has a value nearly equal the calculated value already mentioned. We have, however, assumed, at the outset, that the reluctance of the magnetic circuit was the same as that of the air-gap. Now, this hypothesis is never realized, since the iron of the teeth and of the cores opposes a certain resistance to the magnetic flux. The magnetic permeability of the iron varies inversely with the magnetic density, and its variation causes, besides, a slight disturbance in the theoretical distribution of the magnetic flux along the air-gap. This disturbance is characterized as slight, because, while it is true that, at the points where the magnetic density is the greatest, the teeth have the highest reluctance, it cannot be denied that the magnetic flux, after passing through these teeth, enters into the least saturated portions of the core, whose great magnetic conductivity compensates more or less perfectly for the reduced permeability of the teeth. We have also neglected magnetic leakage, which tends to reduce the current i_μ, and which compensates, to some extent, for the effect of the reluctance of the iron.

It is not necessary to consider the modifications in the distribution of the magnetic flux in the air-gap, due to the unequal saturation of the iron; but it is proper, on the other hand, to take into account, in all calculations, the magnetic leakage, and also the mean reluctance of the magnetic cores and teeth, which reluctance is always additional to that of the air-gap. The simplest way of making this last correction consists in increasing slightly the distance across or the radial "length" of the air-gap; that is to say, it consists in replacing the magnetic resistance (reluctance) of the iron by that of an equivalent thickness or layer of air.

If the circuits of the rotor were closed, and if the mechanical effort necessary to overcome the friction of the air and of the bearings were excessively small, the rotor would turn with a velocity equal to that of the magnetic field. Its reaction would be inappreciable, and the diagram would remain the same. Nevertheless, the losses in the iron would amount to those produced by the core of the stator alone, since the core of the rotor, being motionless with respect to the revolving magnetic field, could not be the seat of eddy currents, or could not absorb any power through hysteresis.

Let us now examine how the performance diagram of induction motors becomes modified when these motors develop useful power.

In order that an induction motor may develop torque, it is necessary that the windings of the rotor may conduct currents whose presence in the magnetic field shall produce an electromagnetic effect.

We know that these currents are due to induced E.M.F.'s produced by the revolving magnetic field, and that, in order to generate them, it is necessary that the windings of the rotor have a certain speed with respect to the magnetic flux in the air-gap.

This displacement of the rotor with respect to the magnetic field has received the name of *slip*.

We have already explained, as clearly as possible, that the stator currents or primary currents, and the rotor currents or secondary currents, produce magnetomotive forces; and their distribution along the air-gap has been studied at length.

It has also been stated that the M.M.F.'s due to the two simultaneous actions of stator and rotor are added algebraically in order to give at each point a single resultant M.M.F. which produces a single magnetic flux through the air-gap and through the stator and rotor cores.

Since the actual distribution of the M.M.F., both in the rotcr and in the stator, can be reduced to a theoretical distribution following the sine law, it is evident that the resultant distribution will likewise follow the sine law.

If we assume that the magnetic reluctance of the cores is very small compared with that of the air-gap, which is generally the case, or if we make up for this reluctance by an equivalent increase in the distance across the air-gap, we only need to divide the resultant M.M.F., at each point, by the distance (d) across the air-gap, in order to obtain the magnetic density at that point.

The distribution of the magnetic flux along the air-gap is, therefore, likewise, of sinusoidal character.

In order to simplify the reasoning and make it as clear as possible, we will begin by disregarding the magnetic leakage flux, and we will first establish the performance diagram for full load as if there were no magnetic leakage.

We will naturally return to the case, later, to complete the diagram in this respect and make it harmonize with the actual facts.

We say, then, that the resultant magnetic effect of the primary and secondary windings produces a single resultant magnetic field which passes through the air-gap and the cores of both rotor and stator. It is that single flux which produces the currents in the rotor and which produces a counter E.M.F. in the stator winding.

Given the details of design of a polyphase motor, and also the normal frequency and voltage of the current with which it is to be operated, it is possible to determine, with very close approximation, the induced E.M.F. in the primary windings. We know that this E.M.F. combines with the difference of potential at the motor terminals to give the ohmic drop $R_1 i_1$, as a resultant. In all well-designed motors, the ohmic drop $R_1 i_1$, when the motor is operating at normal load, is under 4%. It therefore seems evident, at first glance, that the induced E.M.F. must differ but slightly from the voltage applied at the motor terminals, e_0.

We may assume, therefore, without fear of committing an appreciable error, that this induced E.M.F. is equal to 98% of the difference of potentials at the motor terminals.

Knowing the constants of the primary winding, we can estimate the velocity of the revolving field, V_1; then, by the formula with which we are already familiar, we have:

$$e_1 = K_1 b L V_1 Z_1 \times 10^{-8} \text{ volts};$$

and we also have, for the effective magnetic density (b) in the air-gap,

$$b = \frac{e \times 10^8}{K_1 L V_1 Z_1} \text{ gausses,}$$

the latter being plotted along OA in the diagram (Fig. 49).

The magnetic flux revolving in the air-gap, of which we have just determined the effective density, b, causes, in each of the windings of the rotor, an E.M.F., e_2, which depends upon the construction of the motor and the velocity of the magnetic field with respect to the secondary conductors. This slip is nothing more than the difference between the velocity of the revolving magnetic field and that of the rotor, whose speed (R.P.M.) is one of the given data of the problem.

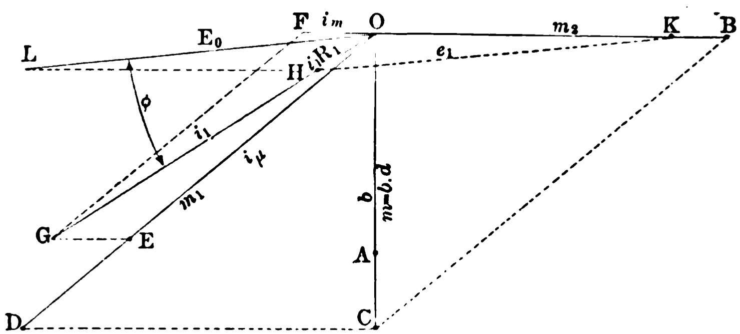


Fig. 49.

As soon as the effective magnetic density, b, of the magnetic field in the air-gap, has been found, we can determine without difficulty, first, the induced E.M.F., e_2, and then we can obtain the effective current, i_2, which passes through the secondary conductors, by merely dividing this induced E.M.F. by the resistance, R_2, of each of the windings. We will have

$$e_2 = K_1 b L V_2 Z_2 \times 10^{-8} \text{ volts}$$

and

$$i_2 = \frac{e_2}{R_2} \text{ amperes.}$$

By means of the data of the winding and of the current value, i_2, aforesaid, we will be able to determine the distribution along the air-gap of the magnetomotive force due to the rotor, or else, by applying the formulæ already known, we will be able to determine at once the effective value of this M.M.F., which is,

$$m_2 = K\frac{S}{p} \times i_2, \text{ in C.G.S. Units.}$$

The M.M.F. m_2 is in phase with the current i_2, producing it, and, consequently, it is also in phase with the induced E.M.F., e_2, since there is no magnetic leakage; but the induced E.M.F., e_2, lags 90° behind the magnetic induction, b, which produces it. We conclude from this that the effective value of the excitation due to the secondary or rotor winding may be plotted along OB, perpendicularly to the vector, OA. The magnetic density, b, of the single magnetic field passing through the air-gap and through the cores of the rotor and stator is due to the simultaneous and resultant effect of the primary and secondary windings; but the moment that we know this effective density, b, produced by the simultaneous action of the two windings, we know that the resultant M.M.F. of the two windings must have the value $b \times d$; that is to say, it must be equal to the product of the magnetic density, b, by the distance (d) across the air-gap.

The field excitation, $M = bd$, therefore, which is in phase with the magnetic flux produced thereby, and which, consequently, should be plotted along OC, on OA prolonged, is the resultant of the magnetizing effects of the stator and rotor. Having determined the component of the latter, OB, we only need to complete the parallelogram, $OBCD$, to obtain, in magnitude and in phase, the effective value of the M.M.F. produced along the air-gap by the primary or stator winding. This new vector, OD, enables us to obtain the effective value of the magnetizing current, $i_{1\mu}$, with which it is in phase. We have, in fact,

$$\overline{OD} = K\frac{S}{p} i_1\mu,$$

whence

$$i_1\mu = \frac{OD \times p}{KS} \text{ amperes.}$$

Let us suppose that OE represents the magnetizing current, $i_1\mu$, according to the scale of amperes, and let us plot, in a direction contrary to OB (which is the primary induced E.M.F.), the watted component, $OF = i_w$, which furnishes the energy consumed by the losses in the iron and required for neutralizing the magnetizing effect of eddy currents;

by completing the parallelogram, $OEGH$, we obtain, both in magnitude and in phase, the effective value of the current, i_1, in the stator. If, now, we take

$$OH = R_1 i_1,$$

and if OK represents the induced E.M.F., e_1, we will have the vector OL, representing the difference of potentials at the motor terminals. The angle, ϕ, between OL and OG, indicates the amount of lag between the current, i_1, and the E.M.F. of the source of current supply. The performance diagram of the motor, when under full load, is now complete.

To calculate the power developed by the motor, we will begin by calculating the electromagnetic effort produced between the rotating field and the secondary currents by means of the well-known formula:

$$C = \frac{K_1 b i_2 S_2 L}{9.81 \times 10^6} \text{ kilogrammes};$$

then, multiplying this value of C by the peripheral speed of the rotor, and deducting the losses caused by friction of bearings and windage, we obtain the power available at the motor-shaft.

We will not now go further in the study of the operation of induction motors; for, as we have already stated, the graphical method which has just been outlined is based on the assumption that there is no magnetic leakage. As magnetic leakage actually exists and plays a very important rôle, it is necessary to study its effects and its causes in order to be able to make allowance for it in the diagram.

We will therefore first consider magnetic leakage; and we will then return to our graphical diagram in order to examine, with all desired precision, not only the conditions of operation for any given power, but also the details of the performance of the motor from the time of starting it until it attains full synchronism.

Magnetic Leakage. While the estimation of a portion of the magnetic leakage is a simple matter, the calculation of the total magnetic leakage, even when it is only roughly approximated, is, in our opinion, an extremely difficult matter. However, in order to make what follows as clear as possible, we will not at once enter into the heart of the problem, but will begin by considering the magnetic leakage across a single one of the slots usually made in the cores of polyphase motors.

This slot, which is represented in Fig. 50, is placed near the periphery of the core disks, and the conductors contained therein are supposed to belong to the same single coil, through which passes an alternating current of given amplitude and frequency. These conductors and this current will naturally produce a magnetic flux which will endeavor to close itself by all the different paths possible. The value of this flux will depend on the number of wires in the slot, on the instantaneous value of the current passing through these wires, and on the magnetic conductivity of the different magnetic circuits available. A portion of the lines of magnetic force will close their circuit by passing through the air near the periphery of the core, between points such as *a* and *b*; others will pass across the narrow opening at the top of the slot, and others still will pass between the side walls of the slot across the space in the slot itself.

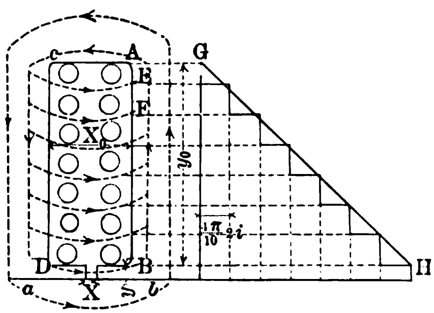


Fig. 50.

We will pay no attention to the lines of magnetic force which complete their magnetic circuit through the periphery of the core; for it is these lines which constitute, in induction motors, the revolving magnetic field which has already been studied. We will, therefore, confine ourselves to the estimation of the other two kinds of magnetic leakage.

If we suppose, for a moment, that only two wires at the bottom, *cA*, of the slot have a current, *i*, passing through them, while the others are inactive, we can conclude that these two wires together produce a M.M.F. equal to:

$$\frac{4\pi}{10} 2\, i.$$

But since the reluctance of the magnetic circuit in the iron is absolutely negligible compared with that presented by the width X_0 of the slot, it is evident that this M.M.F. is expended exclusively in forcing the passage of the lines of magnetic force through this slot; in other words, the difference of magnetic potentials will be almost wholly developed between the two sides *AB* and *cD*.

It is also clear that, since the magnetic flux must surround the two bottom wires just considered, the difference of magnetic potentials will not manifest itself at *A*, but it will only begin to be apparent from the point *E*, and it will remain constant as far as the point *B*.

If, now, we add, to the effect of these two wires, that of the two constituting the second layer, the M.M.F. will be double as far as the point *B*, but starting from the point *F* only. If we follow this reasoning, and successively introduce current into all the conductors placed in the slot, we will find that the distribution of the M.M.F. along the inner face *AB* of the slot has the general character of the broken line GH, which shows a progressive increase from A toward B. It is now apparent that the magnetic leakage flux inside of the slot, which is zero between A and c, increases as far as BD, where it attains its highest value,

$$\frac{4\pi}{10} ni \frac{1}{x_0},$$

in which n represents the number of wires in the slot, and x_0 represents the width of the slot. The total magnetic flux passing between the sides AB and cD will be equal to the product of the mean M.M.F. by the magnetic conductance of the slot, or,

$$\frac{4\pi}{10} \times \frac{n}{2} i.$$

Since the magnetic conductance of the slot is equal to

$$\frac{L \times y_0}{x_0},$$

where L equals the (axial) width of the iron core, the total leakage flux across the slot will be :

$$\Phi_1 = \frac{4\pi}{10} \times \frac{n}{2} \times i \times \frac{Ly_0}{x_0} \text{ maxwells.}$$

It is seen that this leakage flux varies with the current, i, and that consequently its highest value will be obtained by replacing, in this equation, the particular instantaneous value, i, by the maximum current value, I.

Having determined this portion of the magnetic leakage, we can now determine the second portion of the magnetic leakage flux, which passes across the slit or narrowed portion, X. Since the total M.M.F., $\frac{4\pi}{10} nI$, enters into play here, the value of this leakage flux will be :

$$\phi_2 = \frac{4\pi}{10} nI \frac{L \times y}{x} \text{ maxwells.}$$

Before applying the latter formula, it will be well to remember that it is rigorous only in cases where the reluctance in the iron portion of the circuit is negligible in comparison with that of the air portion of the circuit.

In the case of slots this condition is fulfilled; but that is not always the case for the peripheral opening of the slot, which is generally quite narrow.

The necessary correction is easily applied. All that is needed in the calculation is to increase this opening by an additional thickness of air whose reluctance is equal to that offered by the surfaces of the little protuberances of the teeth. This correction is so simple that it requires no further explanation.

Let us now suppose that a coil is arranged in such manner that the conductors of one of its active sides are placed in three consecutive slots, each slot having an equal number of wires through which a current of value, i, is passing.

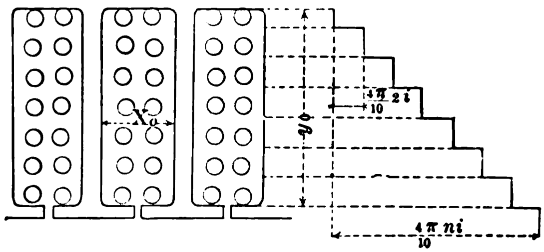


Fig. 51.

Fig. 51 shows clearly that the M.M.F. will increase from the bottom of the slot as far as the inner edge of the narrowed portion or slit, where it attains for all three slots, the value:

$$\frac{4\pi}{10} ni.$$

The wires in each slot will produce, between the two faces of the slot in which they are placed, a mean difference of magnetic potential equal to

$$\frac{4\pi}{10} \times \frac{n}{2} i.$$

It follows that the magnetic flux through each slot remains as before,

$$\Phi = \frac{4\pi}{10} \times \frac{n}{2} i \frac{Ly_0}{x_0}.$$

If the total M.M.F. of the wires in the three slots has a value three times as great as before, the air spaces which oppose the passage of the lines of force have also become three times greater, so that the magnetic leakage flux has not been increased and it remains equal to that which might be produced, by the conductors in each single slot, between the two sides of the slot.

This will naturally be the case likewise for the magnetic leakage at the openings or narrowed portions of the slot.

We see, by the preceding, that it is advantageous to distribute a coil having a given number of turns into as many slots as possible. In fact, the more numerous the sets of conductors, the smaller will be the maximum M.M.F. per slot, and consequently the smaller also will be the magnetic flux across the slot openings. The mean magnetic density in the slots may then also be decreased, but not in such a great proportion, because a smaller number of conductors per slot involves a smaller slot, having a lower reluctance. If, by reducing the number of slots per coil, the width of the slit at the opening were increased proportionally, the magnetic leakage across these openings would not increase.

It is proper to remember, before definitely designing the iron cores, that, in endeavoring to reduce the magnetic leakage across the slot openings by an increase in the width or in the number of these slots, we at the same time reduce the useful surface of the air-gap, and we soon lose all the advantage which might result from the reduction in magnetic leakage.

In many cases, and with the object of simplifying the winding, it is usual to give to the slot openings a width equal to, or slightly larger than, the outward diameter of the insulated wire. We do not believe that this method is advisable, in most cases. In a general way, the width of these openings should be in proportion with the ampere-turns per slot; and we should endeavor to reduce the magnetic leakage, not by increasing this width, but by reducing as much as possible the thickness, y, of the lips which partially close the slot.

It is also desirable to endeavor to reduce the depth, y_0, of the slot, and to increase its width, x_0.

Nevertheless, it is necessary to exercise discretion in doing this; for the cross-section of the teeth might become reduced unduly, thereby causing excessive magnetic density and a consequently low power factor, and perhaps excessive heating of the core iron also.

Let us now consider the case of an induction motor, and let us endeavor to determine the magnetic leakage flux between the teeth of the stator. Let us suppose, for the sake of greater clearness, that the rotor is stationary and without current.

The different magnetic leakages are all in phase with the ampere-turns which produce them, so that the total magnetic leakage, being the sum of these leakages, must also be in phase with the current, and must attain its highest value at the precise moment when the current value becomes a maximum.

Fig. 52, which represents a partial section through the cores of a three-phase motor, shows, by means of full lines, the paths followed by the lines of force of the principal magnetic flux, and, by means of dotted lines, the different paths of the magnetic leakage, in a case where the coils of the winding are each placed in three pairs of slots.

When the current passing through the $3n$ wires of the three consecutive slots forming the active side of each coil becomes a maximum, the leakage across these slots also becomes a maximum, its value then being:

$$\Phi' = \frac{4\pi}{10} \times \frac{3}{2} n I_{max} \frac{L y_0}{3 x_0},$$

which reduces to

$$\Phi' = \frac{4\pi}{10} n I_{max} \frac{L y_0}{2 x_0} \text{ maxwells.}$$

The highest value which the leakage flux can have, in the gap between the edges of the teeth of the stator core, is:

$$\Phi'' = \frac{4\pi}{10} 3 n I_{max} \frac{L y}{3 x},$$

or,

$$\Phi'' = \frac{4\pi}{10} n I_{max} \frac{L y}{x} \text{ maxwells.}$$

Other magnetic fluxes, besides the two leakage fluxes just considered and the main magnetic flux circulating in the cores of the stator and rotor, will be generated by the wires in the three slots above mentioned. These other magnetic leakage fluxes complete their circuits through the air-gap, through the slots of the rotor core, and especially in the air, at the two ends of the motor, and all along and around the wires of the winding.

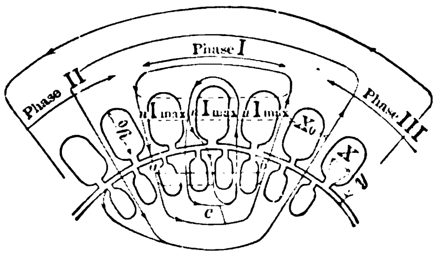


Fig. 52.

The magnetic leakage in the air-gap is evidently very small, owing to the thinness of the layer of air and the relatively long path which any lines of magnetic force closing their circuit through the air-gap would have to follow.

The leakage across the openings at the top of the slots in the rotor is negligible, since the magnetic reluctance at these openings is very high, and the M.M.F. available between the points *a* and *b* is very low, being due to the drop of magnetic potential resulting from the passage of the principal magnetic flux through the magnetic circuit, *a*, *b*, *c*. This "drop" or difference of magnetic potential is very small, since the teeth and the core of the rotor always have a high permeability owing to the low degree of magnetic saturation.

The same reasoning applies, with all the more force, to the leakage across the rotor slots themselves, since the difference of magnetic potential between points such as *a* and *b* diminishes more and more as the bottom of the slot is approached.

We may conclude, therefore, from what precedes, that the magnetic leakage flux circulating around the primary wires is limited to those lines of force only which pass between the inner faces of the slots of the stator, or through the narrow openings between the teeth of the stator, or which pass through the air at the two ends of the core. A portion of this magnetic flux may be calculated by the following formula:

$$\Phi_{1s} = \Phi' + \Phi'' = \frac{4\pi}{10} n I_{\max} L \left\{ \frac{y_0}{2x_0} + \frac{y}{x} \right\} \text{ maxwells.}$$

All the preceding deductions would still apply, if, instead of sending the current through the primary windings, it were sent through the secondary or rotor winding; hence, the maximum value of that portion of the secondary leakage flux which remains in the iron may be expressed by the same formula, thus:

$$\Phi_{2s} = \frac{4\pi}{10} n I_{\max} L \left(\frac{y_0}{x_0} + \frac{y}{x} \right) \text{maxwells,}$$

the quantities n, I, y_0, y, x_0, and x being, in this case, those which correspond to the slots in the rotor.

If the slots were closed at the periphery, that is to say, if there were no slits between the teeth, it would be necessary, in order to determine the magnetic leakage through the thin layer of iron extending between two consecutive teeth, to take into account the degree of magnetic saturation of this magnetic circuit.

We will show how we could estimate the E.M.F. induced by that portion of the magnetic leakage flux which passes between the teeth of the core disks.

The magnetic leakage across the slits between the teeth affects all of the turns of the winding to the same extent.

The effective E.M.F. induced by that portion of the magnetic leakage flux will naturally have the following expression:

$$e''_s = \frac{1}{\sqrt{2}} \times \frac{Z}{2} \omega\, 2\, \Phi''\, 10^{-8} \text{ volts,}$$

$$e''_s = \frac{Z\omega\Phi''}{\sqrt{2}} \times 10^{-8} \text{ volts,}$$

since $\frac{Z}{2}$ represents the number of turns connected in series in each phase.

The lines of magnetic force which have their path across the slots themselves do not all affect the same number of turns of the winding. Those which cross the slot near the periphery of the core act upon all the conductors, while those which cross the slots nearer to the bottom affect only a few of the conductors. This can be easily demonstrated.*

In the case under consideration, the effective E.M.F. induced by the magnetic leakage flux crossing the slots of the core will be:

$$e_s' = \frac{1}{\sqrt{2}} \times \frac{2}{3} \times \frac{Z}{2} 2\,\Phi' \omega \times 10^{-8} \text{ volts},$$

$$e_s' = \frac{1}{\sqrt{2}} \times \frac{2}{3} Z \Phi' \omega \times 10^{-8} \text{ volts}.$$

Adding e'_s and e''_s, the effective induced E.M.F. due to the magnetic leakage flux across the teeth will be:

$$e_s = \frac{1}{\sqrt{2}} Z \omega \left\{ \frac{2}{3} \Phi' + \Phi'' \right\} \times 10^{-8} \text{ volts},$$

in which the values of Φ' and Φ'' are such as may be determined by the formulæ already given for these quantities.

If we proceed to determine, by actual laboratory measurements, the

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* We have, in fact, (Fig. 53), $b = B \frac{x}{X}$,

and for the magnetic flux which affects a conductor of the winding situated at the distance x from the bottom of the slot, we have:

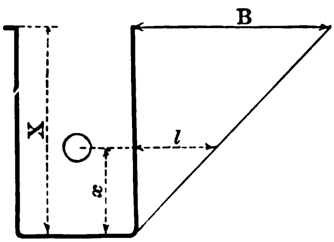

Fig. 53.

$$\Phi = 2\left\{B + B\frac{x}{X}\right\}\frac{1}{2}(X - x)\,L,$$

in which L equals the (axial) length of the core. Reducing, we have,

$$\Phi = B\left\{1 + \frac{x}{X}\right\}(X - x)\,L;$$

$$\Phi = \frac{BL}{X}(X^2 - x^2).$$

but the E.M.F. induced in this turn of the winding is

$$e = \omega\Phi \times 10^{-8} \text{ volts},$$

or, substituting for Φ,

$$e = \omega \frac{BL}{X}(X^2 - x^2)\,10^{-8} \text{ volts}.$$

This value of the induced E.M.F. would, obviously, vary with the distance (x). What we want is the mean value of e for the whole depth (X) of the slot. This total (primary and secondary) magnetic leakage for several existing machines, after having calculated the leakage through the teeth of the cores in the manner already indicated, it will be found that, unfortunately, this leakage across the slots in the cores constitutes but a small portion of the total leakage, which is due largely to the lines of magnetic force completing their magnetic circuit at the ends of the motors, and, in general, all over and around the wires of the winding.

Inasmuch as the magnetic circuits of these leakage fluxes cannot be exactly defined, in consequence of the disturbing effects of surrounding masses of metal, whether magnetic or non-magnetic, it would seem scarcely possible to make even an approximate calculation of this important portion of the magnetic leakage.

Owing to the very important rôle which magnetic leakage plays in the operation of induction motors, the designer may find himself at a loss to make proper allowance for it when designing a new machine. If he has, by numerous tests, determined the coefficients V_1 and V_2 for motors already built, he will have a sure basis by which he may be guided in selecting, with sufficient approximation, a suitable value for the coefficient σ (or leakage factor), which will be defined later.

In the absence of such data, in cases when the distance (d) across

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value (e_{av}) would, evidently, be equal to the mean ordinate of a curve showing the values of e as a function of x, or in symbols,

$$e_{av} = \frac{\int_0^X e\,dx}{X}.$$

Therefore, multiplying both terms of the last equation for e by dx, and integrating between the limits of 0 and X, we have

$$\int_0^X e\,dx = \int_0^X \frac{10^{-8}\omega BL}{X}(X^2 - x^2)\,dx$$

$$= \frac{10^{-8}\omega BL}{X}\left(X^2\int_0^X dx - \int_0^X x^2 dx\right)$$

$$= \frac{10^{-8}\omega BL}{X}\left(X^3 - \frac{X^3}{3}\right),$$

which finally reduces to

$$\int_0^X e\,dx = \tfrac{2}{3}\,\omega BLX^2 \times 10^{-8}.$$

Dividing by X we have the mean value of the E.M.F. induced per turn, which is:

$$e_{av} = \frac{1}{X}\int_0^X e\,dx = \tfrac{2}{3}\,\omega BLX\, 10^{-8} = \tfrac{2}{3}\,\omega\Phi \times 10^{-8} \text{ volts.}$$

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the air-gap differs but little from one millimetre, the machine may be designed on the assumption that:

$$V_1 = V_2 = 1.04 \text{ to } 1.05;$$

and, as a rule, the results of practical tests will not differ greatly from the calculated results, except in the case of the starting-torque.

Definite Diagram of Operation. Having discussed magnetic leakage and the E.M.F.'s produced thereby in the windings, we will proceed to construct a definite diagram showing the operation of polyphase motors.

Let us suppose, for the purpose of discussion, that we have the plans, and also the complete results of practical tests, of a given motor. It will be easy, by means of these data, to determine, for a given load, the precision of our diagram; and, after having made this verification, we will be able to determine the conditions of operation of the motor from the time it starts until it attains synchronous speed.

We will therefore begin by drawing (Fig. 54), in any position whatever, the vector OA, which will represent, by reference to an arbitrary

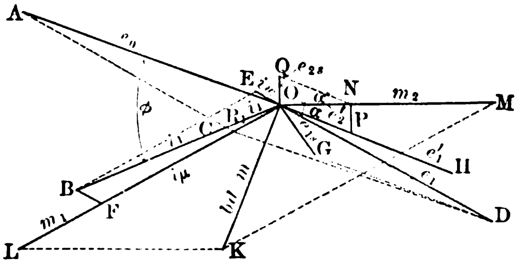

Fig. 54.

scale, the effective voltage, e_0, of the source of current supply. The straight line OB, making the angle ϕ with OA, will represent both the amount and the phase relation of the effective current i_1.

On the line OB, point off the distance $OC = R_1 i_1$; then, completing the parallelogram $OACD$, we will have the value of the induced E.M.F., e_1, in the prolongation, OE, of which, is found the watted component, i_w, of the current i, furnishing the power which supplies the core losses. By drawing BF parallel to OE, we find the magnetizing current $OF = i\mu$. Knowing $i\mu$, we draw, perpendicularly thereto, the E.M.F., $e_{1s} = OG$, which is due to the magnetic leakage in the stator; then, since the latter, together with the E.M.F., e_1', induced by the main magnetic field, gives the difference of potential, $e_1 = OD$, already known, we have, for the main induced E.M.F., $e_1' = OH$ obtained by completing the parallelogram $OGDH$. The magnetic flux which is common to the two cores produces, by itself, the E.M.F. e_1', and consequently leads OH by a phase difference of 90°. We know that this main magnetic flux is produced by the resultant of the primary and secondary magnetomotive forces m_1 and m_2. This resultant can, therefore, be determined by means of OH. The primary excitation, OL, is deduced from the constants of the winding and from the current $i\mu$, with which it is in phase. By completing the parallelogram $OLKM$, we determine the secondary excitation, OM, by means of which we can arrive at the value of the secondary effective current, i_2.

Since the E.M.F., e_2, serves wholly to create the current i_2, passing through the resistance R_2, we have:

$$e_2 = R_2 i_2 = ON; \text{ whence, } i_2 = \frac{e_2}{R_2}.$$

Now, since ON is the resultant of the E.M.F. produced by the main magnetic field which is common to both cores, and also of the E.M.F. produced by the magnetic leakage flux in the rotor, it follows that we only need to draw NP at right angles to OM, and to complete the parallelogram $OPNQ$, in order to finally determine the E.M.F., $e_2' = OP$, induced by the main magnetic flux, and the E.M.F., $e_{2s} = OQ$, due to the magnetic leakage in the secondary winding. It is evident that OQ must lag 90° with respect to OM, that is to say, with respect to the current i_2.

The diagram is now complete. All the electrical phenomena which occur in the operation of polyphase motors have been estimated and introduced in the diagram. We have, in particular, taken into account the ohmic drop, $R_1 i_1$, in the stator, the losses due to hysteresis and eddy currents in the iron cores, as well as the disturbances caused by magnetic leakage. The diagram is therefore complete and definite. It enables us to follow with very close approximation all the details of the operation of the motor, and to note the importance of all the factors which enter into play when the motor is in operation.

In order to verify the exactness of the diagram it will be sufficient to determine, by calculation, the slip of the rotor in the magnetic field, by means of the main magnetic field or the magnetic field density in the air-gap, and the induced E.M.F., $e_2' = OP$, which this magnetic field should produce.

This calculation for slip should lead to the value of $OQ = e_{2s}$, which is the value of the E.M.F. produced in the secondary windings by the magnetic leakage flux.

Before going further, let us note that the vector OQ should be always proportional to the product of the slip by the current i_2. But since the straight line $ON = R_2 i_2$, the ratio $\frac{OQ}{ON}$ is, of necessity, equal to the product of the slip by a constant. Now we have, from the diagram,

$$\frac{OQ}{ON} = \tan \alpha ;$$

consequently the trigonometrical tangent of the angle ONQ is always proportional to the slip.

Figure 54 may be simplified to the extent that it is not necessary to fully complete the different parallelograms. We have done it in this case, however, in order that the process might be better understood, and to show how really simple is the process of combining and composing the E.M.F.'s and M.M.F.'s which occur in the induction motor.

It is sufficient to remember the exact manner of drawing the polygon of forces, consisting in drawing the various components, one after the other, each always in its proper direction, and of proper length, and then completing the polygon by a straight line.

After the polygon is completed, or "closed," the system of forces thereby represented graphically is, as a whole, in a state of equilibrium and the resultant is zero. We conclude, from this, that the last straight line completing the polygon is, by itself, equal to the combined action of all the components. This straight line is, therefore, equal and opposed to the resultant of the other elements of the polygon. This resultant will, therefore, be obtained by simply reversing the direction of the last side of the polygon.

By following these elementary rules, the diagram shown in Fig. 55 may be constructed in a manner that will now be briefly described.

We begin by drawing, to a suitable scale, on a given vector OA, the difference of potential e_0 at the motor terminals. We then draw $OB = i$, the angle AOB being made equal to ϕ.

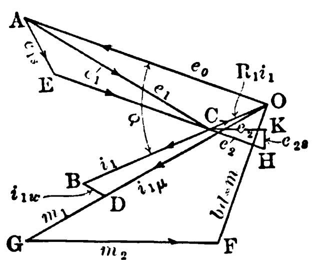

Fig. 55.

Knowing $OC = R_1 i_1$, which is the resultant of e_0 and e_1, we join AC, from which we get the value of the total induced E.M.F., e_1. Let us note that the E.M.F. e_0 must have a direction opposite to that of the two components $R_1 i_1$ and e_1 which both have the same direction in the triangle OAC.

The watted current i_{1w} being opposed to e_1, it will be sufficient to draw DB parallel to AC to determine, first, the point D, and, second, the vector OD representing the magnetizing current $i_{1\mu}$. The latter current produces the magnetic leakage flux which gives rise to the E.M.F., e_{1s}, whose lag behind $i_{1\mu}$ is 90°. We will therefore have $e_{1s} = AE$, and drawing AE perpendicular to OD, CE will represent the value of the E.M.F., e_1', induced by the magnetic flux which is common to the stator and rotor cores.

The resultant, m, of the primary and secondary magnetomotive forces will, therefore, be represented along OF, which is drawn in such manner as to be perpendicular to CE.

If we now take $OG = m_1$, that is to say, equal to the primary M.M.F., GF will then represent the value and the phase relation of the M.M.F. m_2 of the rotor.

The magnetic flux common to both cores, whose direction coincides with OF, will, as already seen, produce the E.M.F. e_1' in the stator and will likewise generate, in the secondary windings, the E.M.F. e_2' which is plotted along CH on the prolongation of EC, that is to say, perpendicular to OF.

The resultant E.M.F., e_2, in the rotor, should be equal to $R_2 i_2$, and should be represented by a vector CK, parallel to GF. The right line KH closing the triangle CKH is nothing more than the E.M.F. e_{2s} due to the magnetic leakage of the rotor. It should be perpendicular to the secondary current GF.

The angle α comprised between CK and CH possesses, as we have already seen, an interesting property in that sense that its trigonometrical tangent is always proportional to the slip of the rotor in the magnetic field.

We can now construct the diagram for any condition of operation between starting with full torque and running at full speed without load (Fig. 56). If we wish to know the operating conditions for a given speed, we begin by calculating the trigonometrical tangent for the angle HOK (which is equal, as already stated, to the "slip"), and by assuming a definite value for the vector OF representing the resultant M.M.F. in the air-gap. By means of OF and of the new slip, we will obtain the secondary induced E.M.F., $e_2' = OH$, whose direction is perpendicular to OF.

The right-angled triangle, OHK, can now be drawn, and the sides OH or HK will enable us to determine the current i_2 and, therefrom, the secondary M.M.F., m_2, which is represented along FG, parallel to OK.

The triangle OFG will give us the value, m_1, of the primary M.M.F., as well as of the magnetizing current, $i_1\mu = OD$.

The induced E.M.F., e_1', whose value depends on OF, may be drawn along OE, which is the prolongation of OK, and the E.M.F., e_{1s}, may be drawn along CE, perpendicular to $i_1\mu = OD$. This E.M.F., e_{1s}, which is generated by the magnetic leakage flux, is proportional to the magnetizing current $i_1\mu$, The triangle CEO determines the value $CO = e_1$, representing the resultant induced E.M.F. in the stator, which, in turn, leads to the watted component, $i_{1w} = BD$, having the opposite direction.

The right line OB will represent the primary current i_1, and AC, drawn parallel to OB, will represent the ohmic drop, or loss of potential, $R_1 i_1$.

On finally joining the points O and A we have the difference of potential at the motor terminals, $e_0 = OA$, and the angle of phase displacement, $AOB = \phi$.

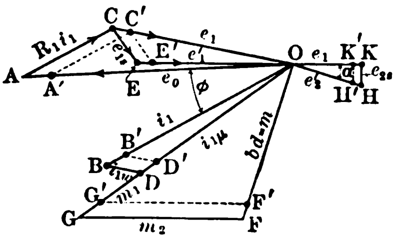

Fig. 56.

The potential difference, OA, at the motor terminals, is naturally that which corresponds to the vector OF, which has been arbitrarily taken for the resultant M.M.F. in the air-gap. It would be a matter of chance if it were really found exactly equal to that for which the motor under consideration was really designed.

In order to find the operating conditions corresponding to the actual potential difference at the motor terminals, as well as to the particular speed previously assumed, it would suffice to represent this potential difference by the distance OA' plotted on the right line OA, and to diminish all the vectors in the proportion: $\frac{OA}{OA'}$.

This diagram is rigorously correct; for it is evident, at a glance, that if we had begun the diagram by giving to the resultant M.M.F. in the air-gap, the dimension represented by the space OF' instead of OF, the potential differences e_1', e_2', e_2, e_{2s}, as well as the quantities m_2, m_1, $i_1\mu$, e_{1s}, e_1, $R_1 i_1$, e_0, would have diminished in the same ratio; and the value OA', of the potential difference at the motor terminals, which had already been obtained, would have actually been found to have the value previously determined.

It is true that the watted component, i_{1w}, is not strictly proportional to the induced E.M.F., and that the ratio $\frac{D'B'}{DB}$ diminishes more rapidly than the ratio $\frac{OC'}{OC}$.

But this slight error is insignificant, if the right line OF, with which we began the diagram, differs only slightly from the true value OF'; for, in all well designed motors, the quantity i_{1w} is very small. The proper correction could in any case easily be made, should it be found necessary.

By constructing the diagram anew for speeds varying between starting speed and synchronous speed (or full speed with no-load), and by plotting as ordinates the results thus obtained, and as abscissæ all the corresponding slips, it would be possible to draw all the curves of operation for any given motor.

We have shown, in the course of this work, how it is possible to calculate the electromagnetic effort exerted at the periphery of the rotor between the revolving magnetic field and the secondary or rotor currents. We use for this purpose the formula:

$$C = \frac{K_1 S_2 i_2 b \cos \alpha}{9.81 \times 10^6} \text{ kilogrammes,}$$

in which

C = the electromagnetic effort (pull) at the periphery of the rotor, in kilogrammes;

K_1 = a coefficient depending upon the kind of winding;

S_2 = the total number of conductors on the periphery of the rotor;

i_2 = the effective current in these conductors, in amperes;

b = the effective magnetic density in the air-gap;

α = the angle of lag existing between the centre lines of the poles and the active sides of the secondary coils at the time when the current in the rotor attains its maximum value I_2. This angle is precisely that existing between the induced E.M.F., e_2', and the resultant E.M.F., e_2.

The product $b \cos \alpha = b_2$ may be represented in the diagram by a line indicating its value according to a suitable scale. From the electromagnetic pull, C, representing the total action of the magnetic field on the rotor, there should be deducted an amount sufficient to allow for friction losses, or the frictional resistance of the rotor against the air (windage), and that of the shaft in the motor bearings.

It seems evident, at first glance, that this electromagnetic effort or pull, C, which exists between the revolving magnetic field and the secondary currents, must operate entirely on the stator, which constitutes the only portion of the motor against which the pull can react.

The reaction of the revolving magnetic field on the primary currents is therefore, itself, also equal to C. But the power supplied by the stator is evidently,

$$9.81 \times V_1 C \times 10^{-2} \text{ watts,}$$

in which V_1 is the speed of the revolving field, measured on the periphery of the rotor disks, in centimetres per second. The power which is mechanically transmitted by the rotor is, likewise, equal to:

$$9.81 \times 10^{-2} \times V_2 C \text{ watts,}$$

in which V_2, represents the peripheral velocity of the rotor, in centimetres per second. We therefore note a disappearance of power taking place in the secondary winding, having the value:

$$(V_1 - V_2)\, C \times 9.81 \times 10^{-2} \text{ watts.}$$

Now $V_1 - V_2$ is nothing more than the slip of the rotor in the magnetic field, so that, if this slip is expressed, according to the usual practice, in *percentages* of V_1, the same percentage will, likewise, represent the importance of the loss taking place in the secondary winding.

Thus, for example, a slip of 4% would cause a loss of:

$$0.04\, V_1 C \times 9.81 \times 10^{-2} \text{ watts,}$$

which would be transformed into heat by the action of the currents i_2 on the resistances R_2.

Before going further, let us note that, in proportion as the load is increased on the motor, the secondary currents and the slip will increase. The same thing occurs with the primary current and the ohmic drop $R_1\, i_1$. Since the difference of potential, e_0, at the motor terminals, remains constant, the (counter) E.M.F., e_1, must decrease. This increase becomes all the more apparent in the case of the induced potential difference e_1, owing to the increase of the magnetic leakage flux and of the E.M.F., e_{1s}, generated thereby. The principal magnetic field, which is common to the two cores, must, therefore, be weakened as the load is increased, since it is this magnetic field which causes the induced potential difference e_1.

In the secondary, the magnetic leakage, to which e_2 is proportional, contributes, together with the reduction in the density of the magnetic field, to lessen the currents i_2. By pushing things still further, a time will come when the ohmic drop $R_1 i_1$ and the magnetic leakage become so great that the torque of the motor will be notably diminished, notwithstanding the fact that the slip is attaining higher and higher values.

While the motor is starting, the currents i_2 will be very large, unless additional resistances are introduced in the secondary circuits.

The starting-torque itself, on the contrary, may be sometimes so small that it is not sufficient to move the rotor without load, that is to say, to overcome the friction of the motor bearings.

This is due partly to the decrease in density of the revolving magnetic field in the air-gap, and especially to the fact that since the angle α is very large, the product $i_2 b \cos \alpha$, to which the electromagnetic pull is proportional, becomes very small.

It suffices to construct the diagram with an angle α such as corresponds to the starting of the motor, in order to make this point very clear.

The magnetic saturation of the tips of the teeth of the magnetic cores, caused by the increase in the magnetic leakage flux, tends to reduce slightly this magnetic leakage; hence, so far as the starting-torque is concerned, the results will be in reality slightly more favorable than the diagram might indicate. We say "slightly," because this decrease in permeability will affect only the least important portion of the leakage, — that which takes place across the slits at the outer part of the slots. The greater portion of the lines of magnetic force of the leakage flux will always remain proportional to the product of the currents by the conductors in each coil.

We have seen that the principal or resultant magnetic field diminishes slightly when the load, and consequently the slip of the rotor, are increased.

The loss in the stator core is reduced in consequence of this, but the loss in the secondary core is increased in consequence of the increase in frequency in that portion of the motor. It often happens, for this reason, that the total power dissipated by hysteresis and by eddy currents in the magnetic cores remains substantially constant at all loads.

Circle Diagram. (*Approximate Construction.*) The graphical method which has just been described enables us to solve with accuracy all problems relating to the operation of polyphase motors, and to determine with ease the variations in current and voltage corresponding to all values of slip comprised between starting speed under full load and full speed with no load.

This diagram, however, is open to the objection that the effect of magnetic leakage is not very clearly shown, and that it is not easy to derive from it, by geometrical or analytical methods of reasoning, simple relations between the various factors which react on the operation of the motor. Another objection, of less importance it is true, since it causes only a slight loss of time, is the fact that a new diagram must be made for each value of the slip. There exists fortunately a graphical method whereby, by means of a single diagram, we not only can predetermine, with as much precision as rapidity, all the conditions of operation between starting speed and synchronous speed, but whereby we may, through simple reasoning, arrive at analytical relations which show, with all desirable clearness, the important rôle played by the magnetic leakage.

This method, which is certainly one of the most beautiful applications of graphical methods to the solution of electrical problems, is due to the work of Messrs. A. Blondel, B. Behrend, and A. Heyland.

Although M. Blondel may not have observed that a certain point of the diagram should move on a circumference, he has, nevertheless, in our opinion, contributed much to the discovery which was made independently and almost at the same time by Messrs. B. Behrend and A. Heyland, by his having given, ahead of all other authors, an exact analytical study of the operation of three-phase motors, based on a very simple diagram, which constitutes the starting-point of the diagram of these two engineers.

This diagram is based on the hypothesis or assumption that there are no losses in the copper and stator windings and in the iron of both

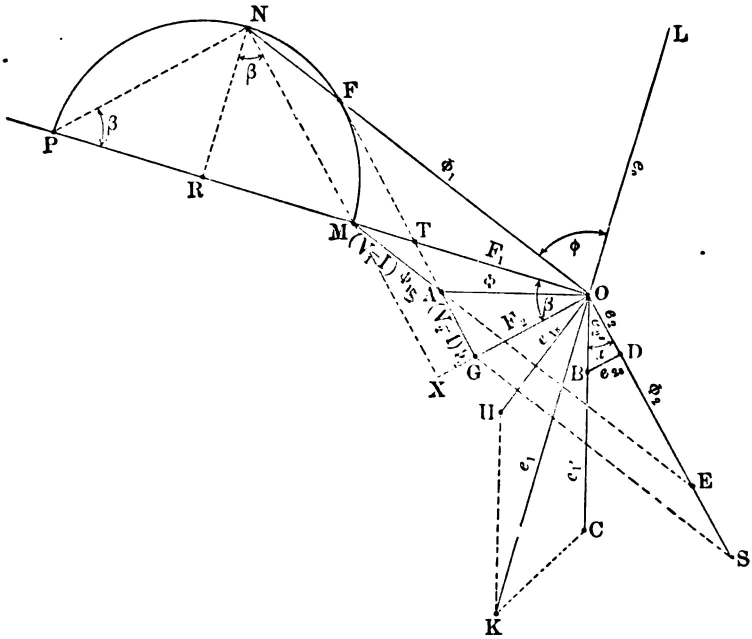

Fig. 57.

cores. Although these losses may always have but slight importance, a diagram which neglects them completely would lose much of its value. The graphical method which we are now going to discuss enables these losses to be subsequently taken into account with the greatest facility.

If we retain the hypothesis aforesaid, the diagram of Fig. 54 may be transformed into that of Fig. 57, in which OA represents the resultant excitation (M.M.F.) of the primary and secondary windings, giving rise to the magnetic flux which is common to both cores.

Along OC, which is perpendicular to OA, may be represented the two E.M.F.'s produced by the magnetic flux, which are:

$$e_2' = OB, \text{ and } e_1' = OC.$$

If OD is the value of the resultant potential difference in the rotor, BD, which is perpendicular to OD, will be that of the E.M.F., e_{2s}, produced in the rotor winding by the magnetic leakage flux.

The angle $BOD = \alpha$ will be that whose trigonometrical tangent is proportional to the slip.

OE and OF here represent, respectively, the secondary and primary M.M.F.'s.

OH represents the E.M.F., e_{1s}, due to magnetic leakage in the stator, while OK represents that which is due to the vector or geometrical addition of:

$$e_1' = OC \quad \text{and} \quad e_{1s} = OH.$$

Since we assume that the ohmic drop in the stator, as well as the losses in the iron cores, are equal to zero, the counter E.M.F., $e_1 = OK$ will be equal and opposed to the potential difference of the source of supply, whose value is:

$$e_0 = OL.$$

The angle $FOL = \phi$ will represent the phase difference between this potential difference OL and the current i_1 in the stator.

Thus far the construction is absolutely the same as that which has already been explained.

We will now follow the reasoning by which Mr. A. Blondel has been able to develop, and on which he has based, an analytical theory of the operation of three-phase motors.

The core of the rotor is subjected to two very distinct magnetic fluxes, of which one, $OA = \Phi$, is due to the resultant excitation (M.M.F.) of the two windings, while the other, or the magnetic leakage flux, $\Phi_{2s} = AG$, is produced only by the secondary currents. These two fluxes, when combined, give, as a resultant, a single flux, $OG = F_2$, which alone induces the resultant E.M.F.:

$$OD = e_2 = R_2 i_2.$$

In the stator we have also to deal with two magnetic fluxes, one of which is:

$$\Phi = OA,$$

which is also common to the rotor, while the other is:

$$AM = \Phi_{1s},$$

produced by the primary currents alone. This flux, $AM = \Phi_{1s}$, is naturally in phase with these currents; but it has a lead of 90° in advance of the E.M.F., ($e_{1s} = OH$), produced by it. It follows, therefore, that AM is parallel to OF and perpendicular to OH.

The resultant flux in the stator, due to the geometrical addition of the two fluxes, Φ_{1s} and Φ, just mentioned, will be:

$$OM = F_1.$$

This resultant flux induces the E.M.F., $e_1 = OK$, which lags 90° behind OM, and which is equal but opposed to the potential difference at the motor terminals, $e_0 = OL$.

This potential difference having a perfectly definite and constant value, the resultant flux F_1 in the stator will therefore be constant, independently of the motor load.

Instead of saying, as has been done hitherto, that the magnetic flux, $\Phi = OA$, circulating through the two cores, is created by the resultant action of the primary and secondary windings at each point of the air-gap (which, certainly, is in conformity with the facts of the case), we may assume that the primary excitation, OF, produces in the two cores a magnetic flux which is proportional to, and in phase with, OF, and that the secondary excitation, OE, likewise produces through the two cores a magnetic flux which is proportional to, and in phase with, this M.M.F., OE.

The superposition of these two hypothetical or "fictitious" magnetic fluxes, OE and OF, could then be regarded as combining and giving a single resultant magnetic field, $\Phi = OA$, which is common to the two windings. But in the stator there is also, in addition to the fictitious magnetic flux, whose value is $OF = \Phi_1$, a magnetic leakage flux, whose value is $AM = \Phi_{1s}$. The latter will combine with Φ to give, in the stator, a resultant flux F_1, which is the only one that really exists. Likewise, in the rotor, the flux Φ will give, together with $AG = \Phi_{2s}$, a resultant real magnetic flux, $OG = F_2$.

In the stator, the magnetic leakage, being in phase with the fictitious flux OF, combines therewith in such a manner that the stator currents would, if alone, produce a total flux, equal to:

$$\Phi_1 + \Phi_{1s} = OF + AM = OF + FN = ON.$$

Likewise, the currents of the rotor would produce a total magnetic field (flux) equal to:

$$\Phi_2 + \Phi_{2s} = OE + AG = OE + ES = OS.$$

If we designate by V_1 the quotient of the sum of the fictitious magnetic flux $OF = \Phi_1$ and the magnetic leakage flux $FN = \Phi_{1s}$, divided by this same fictitious flux (Φ_1), we will have:

$$\frac{\Phi_1 + \Phi_{1s}}{\Phi_1} = V_1;$$

It is evident that V_1 will have a constant value for each machine, for the magnetic fields Φ_1 and Φ_{1s} only depend on the details and data of construction and on the current i_1. But since i_1 enters and can be found in each of these terms its value will disappear from the ratio:

$$\frac{\Phi_1 + \Phi_{1s}}{\Phi_1}.$$

From the preceding equations we deduce:

$$\Phi_1 + \Phi_{1s} = V_1\Phi_1.$$

We will likewise have, for the rotor:

$$\frac{\Phi_2 + \Phi_{2s}}{\Phi_2} = V_2, \text{ and } \Phi_2 + \Phi_{2s} = V_2\Phi_2.$$

And we will also have:

$$\Phi_{1s} = \Phi_1 (V_1 - 1), \text{ and } \Phi_{1s} = \Phi_2(V_2 - 1).$$

The coefficients V_1 and V_2 are, by definition, larger than unity.

Always admitting that the losses in the copper of the stator and in the iron of the cores are both negligible, the vector $OM = F_1$ must retain a constant value, since it must be equal to the difference of potential at the motor terminals.

The points O and M are therefore fixed. If we draw, from the point N, a right line, NP, parallel to OG and extending as far as its intersection with OM prolonged, the angle PNM will be a right angle, for NM is parallel to AG and AG is perpendicular to OG.

The point X being the intersection of the right lines MN and OG, it is evident that:

$$\angle NPM = \angle TOG = \angle MOX = \angle \beta.$$

The triangles NPM, MXO, and TGO, being similar, we may write

$$PM = OM\,\frac{NM}{MX} \qquad \text{and} \qquad MX = TG\,\frac{OM}{OT},$$

from which we deduce:

$$PM = OM \times NM\,\frac{OT}{OM \times TG} = NM\frac{OT}{TG};$$

but:

$$NM = \Phi_2 \qquad \text{and} \qquad OT = \frac{OF}{ON} \times OM = \frac{\Phi_1}{V_1\Phi_1} \times F_1 = \frac{F_1}{V_1};$$

on the other hand:

$$TG = FG - FT = FG - \frac{OF}{ON} \times NM,$$

$$TG = V_2\Phi_2 - \frac{\Phi_1}{V_1\Phi_1} \times \Phi_2,$$

$$TG = \Phi_2\left(V_2 - \frac{1}{V_1}\right) = \Phi_2\left(\frac{V_1V_2 - 1}{V_1}\right)$$

If we replace *PM*, *NM*, *OT* and *TG* by their values, we have:

$$PM = \frac{\dfrac{\Phi_2 \times F_1}{V_1}}{\Phi_2\left(\dfrac{V_1 V_2 - 1}{V_1}\right)} = \frac{F_1}{V_1 V_2 - 1}.$$

F_1 being a constant, same as the coefficients V_1 and V_2, the length of the right line *MP* is constant, and the point *P* is a fixed point.

Now, we know that the angle *PNM* is a right angle, and we must conclude from this fact that the apex of the triangle (*N*) must move along a circumference whose diameter is *PM*. If we designate by the term "leakage factor" a factor or coefficient σ which satisfies the following relation:

$$\sigma = 1 = \frac{1}{V_1 V_2},$$

we will have:

$$1 - \quad = \frac{1}{V_1 V_2};$$

and, dividing the latter by the former, we have:

$$\frac{1-\sigma}{\sigma} = \frac{\dfrac{1}{V_1 V_2}}{1 - \dfrac{1}{V_1 V_2}} = \frac{1}{V_1 V_2 - 1}.$$

The diameter *PM* is then given by the following equation:

$$PM = F_1 \times \frac{1-\sigma}{\sigma}.$$

Let us note, in passing, that the ratio

$$\frac{MP}{OM} = \frac{F_1 \dfrac{1-\sigma}{\sigma}}{F_1} = \frac{1-\sigma}{\sigma}$$

only depends on the value of this leakage factor, σ. What is remarkable is that the vector *ON*, which represents the magnetic flux $V_1 \Phi_1$, or which represents the current i_1 in the stator, when drawn to some other scale, moves in a very characteristic manner, inasmuch as one of its extremities, *O*, remains fixed, while the other, *N*, is always on the periphery of a circle.

When the motor is running without load, the angular velocity of the rotor being then the same as that of the revolving magnetic field, the reaction of the secondary winding is zero. The magnetic flux, $V_1\Phi_1$, in the stator, becomes reduced to F_1. The current i_1 is then represented by the vector *OM*.

In proportion as the load increases, the current i_1 also increases. The greatest value which it can attain is OP.

The figure shows that:

$$\sin \beta = \frac{NM}{MP} = \frac{\Phi_2}{F_1 \frac{1-\sigma}{\sigma}} = \frac{\Phi_2}{F_1} \times \frac{\sigma}{1-\sigma}.$$

1. **Tangential Effort.** We know that the electromagnetic effort (pull) produced at the periphery of the rotor is proportional to the product of the resultant magnetic field F_2 by the currents i_2. This tangential effort or pull, C, will therefore have the following value:

$$C = K_1 F_2 i_2;$$

but i_2 is proportional to $V_2 \Phi_2$, and

$$F_2 = OG = OT \cos \beta = \frac{F_1}{V_1} \cos \beta;$$

therefore, we have:

$$C = K_1 \frac{F_1}{V_1} V_2 \Phi_2 \cos \beta.$$

The values K_1, E, V_1 and V_2 being constant, the tangential effort or pull is therefore proportional to $\Phi_2 \cos \beta$.

But:

$$\Phi_2 \cos \beta = NM \cos \beta = NR.$$

The length of the right line NR is therefore always proportional to the pull. It therefore indicates its amount.

[The same line, NR, will of course indicate *torques*, according to a scale of torque values, such that

$$T = Cr,$$

where T = torque, in kilogramme-centimetres.

r = radius of rotor, in centimetres.

when C = tangential effort or pull, in kilogrammes.]

2. **Slip.** The resultant difference of potential $e_2 = OD$, the current i_2, as well as the flux $V_2 \Phi_2$, all depend on the product of the resultant magnetic field F_2 by the slip. We will therefore have:

$$V_2 \Phi_2 = K'' F_2 s,$$

in which s represents the percentage of slip, and K'' represents a constant depending upon the dimensions of the motor.

We at once deduce:

$$s = \frac{V_2\Phi_2}{K''F_2} = K_2 \times \frac{V_2\Phi_2}{F_2}$$

$$s = K_2 \times \frac{V_1 V_2}{F_1} \times \frac{\Phi_2}{\cos\beta}\,.$$

If we prolong PN as far as Z, in the diagram (Fig. 58), and if we draw MY parallel to NR or to OL, the length of the right line MY will be precisely equal to $\frac{\Phi_2}{\cos\beta}$; but it is also clear that the right lines OZ and MY are always proportional to each other, so that by prolonging the right line PN until it intersects the vector OL, representing the

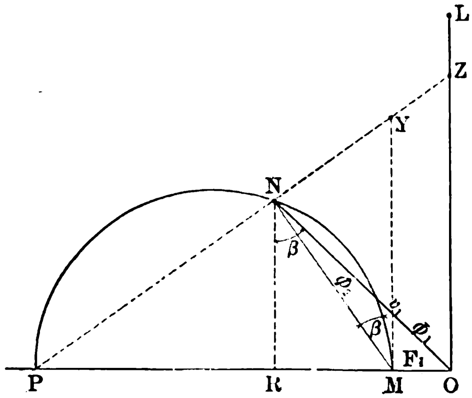

Fig. 58.

potential difference at the motor terminals, we obtain, on the latter, a distance OZ which will be a measure of the amount of slip, measured in percents.

3. Power Factor. The power factor is equal to the cosine of the angle $FOL = \phi$ which is comprised between the vector OL, representing the potential difference at motor terminals, and the vector ON, representing the current i_1. It becomes zero for the first time when the point N falls down to the point M, that is to say, when the motor is running without load, and a second time at the moment of theoretical short-circuiting, when the point N arrives at the point P.

Cos ϕ will have its maximum value in the case when the vector ON becomes tangential to the circle.

From Fig. 59 we see that:

$$\cos\phi = \frac{NR}{ON} = \frac{NR}{V_1\phi_1}\,.$$

But,
$$NR = \Phi_2 \cos \beta;$$

therefore:
$$\cos \phi = \frac{\Phi_2}{V_1 \Phi_1} \cos \beta.$$

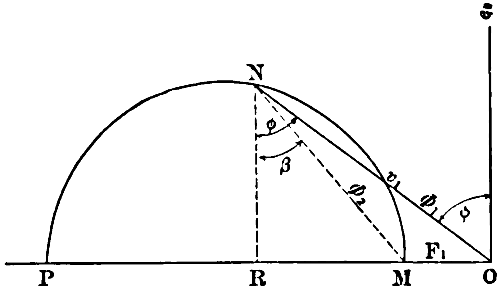

Fig. 59.

When the vector ON is tangent to the circle, we have (Fig. 60),

$$\cos \phi_{\max} = \frac{QN}{OQ}.$$

But,
$$OQ = QM + OM = QN + OM;$$

$$OQ = QN + F_1;$$

therefore:
$$\cos \phi_{\max} = \frac{QN}{QN + F_1}.$$

But,
$$QN = \frac{PM}{2},$$

and
$$PM = F_1 \frac{1 - \sigma}{\sigma}.$$

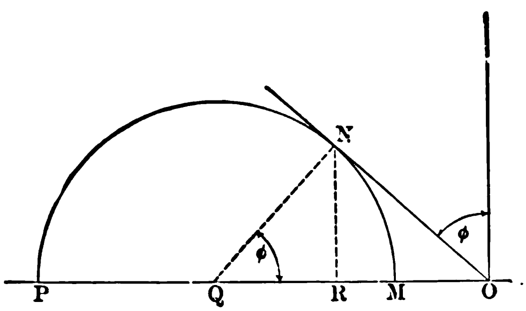

Fig. 60.

From which we deduce:

$$\cos\phi_{\max} = \frac{\frac{PM}{2}}{\frac{PM}{2} + F_1} = \frac{PM}{PM + 2F_1} = \frac{F_1\frac{1-\sigma}{\sigma}}{F_1\frac{1-\sigma}{\sigma} + 2F_1},$$

and, finally:

$$\cos\phi_{\max} = \frac{1-\sigma}{1-\sigma+2\sigma} = \frac{1-\sigma}{1+\sigma}.$$

Thus we find, curiously enough, that the highest value which the power factor $\cos\phi$ may have depends only on the leakage factor, σ.

4. **Power Absorbed.** The power which must be furnished to the machine is proportional to $e_0 i_1 \cos\phi$, that is to say, to $i_1 \cos\phi$, since e_0 is constant. But we have:

$$NR = i_1 \cos\phi.$$

The right line NR is always proportional to the power furnished to the motor. It may, therefore, be used as a measure of this power.

Effect of Magnetic Leakage. In order to obtain a clear idea of the important rôle played by magnetic leakage, it is necessary to compare

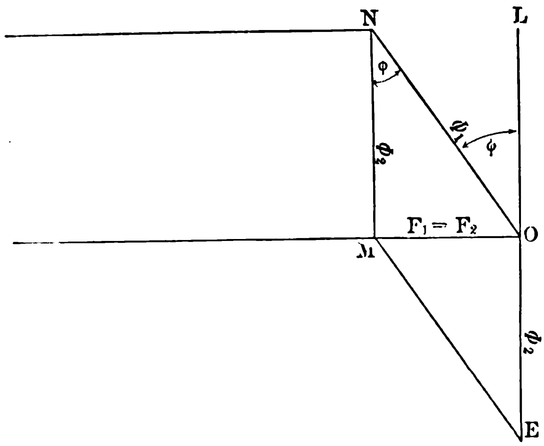

Fig. 61.

the expressions previously given for tangential effort, slip, angle of lag, and power absorbed, with those that would be obtained from a diagram made on the assumption that this leakage did not exist.

Such a diagram would be like that shown in Fig. 61.

The simultaneous action of the primary and secondary windings produces a single flux F_1, which is common to the stator and rotor cores, and whose value is the same as before, since it must induce an E.M.F. balancing and counteracting the difference of potential at the motor terminals. The distance OM, therefore, remains the same as in Fig. 57.

The angle MNP being 90°, and the right line MN being parallel to OL, the point P will be at an infinite distance.

The circle will be transformed into an infinitely long straight line, on which are placed the point M and all the points such as N.

The tangential effort which, in the case of Fig. 57, as already seen, was

$$C = K_1 F_1 \frac{V_2}{V_1} \Phi_2 \cos \beta,$$

now becomes:

$$C = K_1 F_1 \Phi_2.$$

It is therefore proportional to the area of the triangle OMN, which becomes very large at the time of starting, since the point N is then displaced in the direction of infinity.

It is therefore evident, from the comparison of the last diagram with the preceding ones, that magnetic leakage has the effect of diminishing the torque.

The slip, which we found previously to be

$$s = K_2 \frac{\Phi_2}{F_1} \times \frac{V_1 V_2}{\cos \beta},$$

now becomes

$$s = K_2 \times \frac{\Phi_2}{F_2}.$$

The presence of magnetic leakage therefore has the effect of materially increasing the slip.

We know that the value of the sine of the angle β is

$$\sin \beta = \frac{\Phi_2}{F_1} \times \frac{\sigma}{1 - \sigma}.$$

If the leakage were zero, V_1 and V_2 would be equal to unity. The coefficient σ would disappear, and we would evidently have

$$\sin \beta = 0, \text{ and } \beta = 0.$$

In the absence of any magnetic leakage, cos ϕ becomes equal to zero, when the motor is running without load at synchronous speed, and it would then increase in proportion as the load increases, and would attain the value of unity when the short-circuiting point is reached.

The motor would then operate like a real transformer, the power supplied to the primary being then entirely expended in the secondary. We would still have

$$\cos\phi = \frac{\Phi_2}{\Phi_1},$$

while, if there were magnetic leakage, we ought to have

$$\cos\phi = \frac{\Phi_2}{\Phi_1}\,\frac{\cos\beta}{V_1}.$$

Since V_1 will always be greater and $\cos\beta$ will always be smaller than unity, it follows that $\cos\phi$ will diminish rapidly in proportion as the magnetic leakage increases in importance.

The preceding formulæ are only approximative, owing to their being based on the hypothesis mentioned in the beginning, which assumes that there are no losses in the cores and in the copper of the stator winding.

These relations have been deduced solely to give as clear an idea as possible of the manner in which the operation of polyphase motors may become influenced by magnetic leakage.

These formulæ are not to be used in designing motors.

Before introducing into the diagram the corrections required to take into account the losses in the iron of the cores and in the copper of the stator winding, let us now indicate how we may represent in this same diagram, first, the available (useful) power of the motor, after deducting the losses in the secondary windings, the losses by bearing friction and windage, and, second, the available mechanical effort due to the difference between the total or gross mechanical effort and the mechanical efforts corresponding to the losses just mentioned.

To calculate the mechanical output which can be obtained from the rotor, we must deduct from the power furnished to the stator the (ohmic) loss $A_2\,R_2\,i^2_2$ due to Joulean effect in the secondary winding.

Of the total power applied to the stator, the amount corresponding to this loss is

$$\eta A e_0 i_1 \cos\phi.$$

The number of phases in the stator and rotor are represented by A_1 and A_2, respectively.

The coefficient η being the numerical ratio between the loss in the rotor and the total power applied to the motor, we have

$$\eta = \frac{A_2 R_2 e_2^2}{A_1 e_0 i_1 \cos\phi} = K \times \frac{i_2^2}{i_1 \cos\phi},$$

in which K is a constant, whose value is

$$K = \frac{A_2 R_2}{A_1 e_0}$$

But $i_1 \cos \phi$ being nothing more than NR (Fig. 62), and since $i_2 = MN$, we therefore have

$$\eta = K \frac{MN \times MN}{NR} = K \times \frac{MN}{\cos \beta} = K \times MX,$$

or again,

$$\eta = K \times PM \times \tan \beta.$$

The right line PM being of constant length, we may write

$$\eta = K_1 \tan \beta.$$

The ratio between the ohmic losses in the rotor and the total power absorbed by the motor is proportional to the trigonometrical tangent of the angle β.

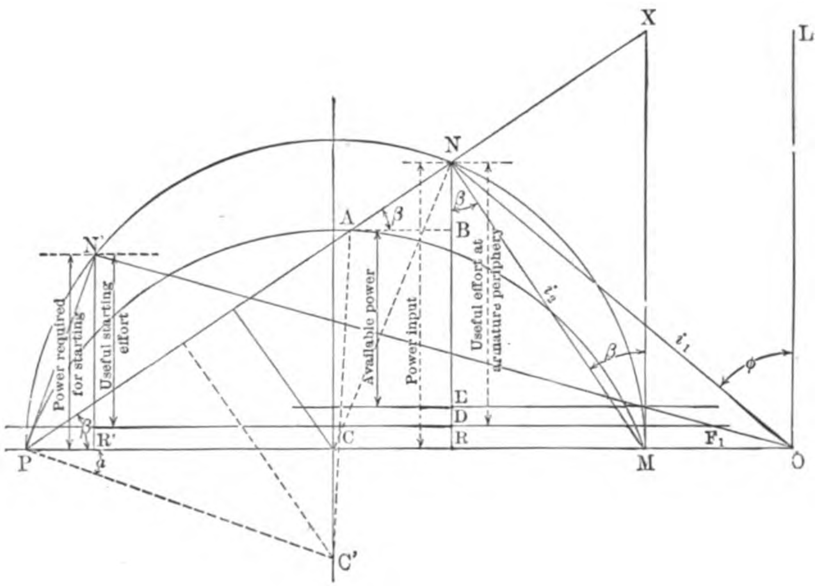

Fig. 62.

NR is, as we already know, a measure of the power furnished to the stator:

$$NR = A_1 e_0 i \cos \phi.$$

If NB represents, according to the same scale, the ohmic loss in the copper of the secondary winding, the difference,

$$NR - NB = BR,$$

will represent the power transformed and utilized through the winding of the rotor.

By hypothesis, we have

$$\eta = \frac{NB}{NR};$$

and we therefore have

$$\frac{NB}{NR} = K_1 \times \tan \beta.$$

If we draw, from the point B, a straight line parallel to OP, as far as its intersection with the line PX, at the point A, we will notice that the angle β is

$$NAB = \beta.$$

so that we may write:

$$\frac{NB}{NR} = K_1 \times \tan \beta = K_1 \frac{NB}{AB},$$

whence we have:

$$K_1 = \frac{AB}{NR}.$$

In other words, if NB is to represent the ohmic loss in the secondary, it is necessary that the ratio:

$$\frac{AB}{NR} = \text{a constant.}$$

We will now show that if the point A moves on a circumference passing through the points P and M, and having the radius R_1 and the point C' for its centre, the required condition, $\frac{AB}{NR} = \text{a constant}$, is always satisfied.

In fact,

$$PN = PM \cos \beta$$

and

$$PA = 2R_1 \cos (\beta + a);$$

but:

$$2 R_1 = \frac{PM}{\cos a}.$$

We therefore have:

$$PA = PM \frac{\cos (a + \beta)}{\cos a}.$$

Now, we find:

$$AB = AN \cos \beta = (PN - PA) \cos \beta,$$

$$AB = PM \left\{ \cos \beta - \frac{\cos (a + \beta)}{\cos a} \right\} \cos \beta.$$

We likewise have:

$$NR = PN \sin \beta = PM \cos \beta \sin \beta.$$

We can now write:

$$\frac{AB}{NR}=\frac{\cos\alpha\cos\beta-\cos(\alpha+\beta)\cos\beta}{\cos\beta\sin\beta\cos\alpha},$$

$$\frac{AB}{NR}=\frac{(\cos\alpha\cos\beta-\cos\alpha\cos\beta+\sin\alpha\sin\beta)\cos\beta}{\cos\beta\sin\beta\cos\alpha},$$

and, finally:

$$\frac{AB}{NR}=\tan\alpha=\text{constant}.$$

This shows that the required condition is attained when the point *A* moves on a circle passing through *A*, *P*, and *M*.

Thus, after having determined the loss in the rotor for any given value, *NR*, of the power applied to the stator, we can plot this loss, *NB*, along the straight line *NR*. We will then determine the point *A*, and then trace the circle *PAM*, by means of which we can at once determine all the current values i_1, for the entire range of speeds between synchronous speed and zero speed, and also the loss *NB* in the secondary windings, as well as the mechanical power *BR* developed by the rotor. It is apparent that this power is likewise measured by the distance of the point *A* from the line *PM*.

At the instant of starting, the secondary, being motionless, develops no mechanical power. The whole of the power absorbed by the stator is, therefore, transformed into heat in the rotor windings, since, by hypothesis, the losses in the iron and in the copper of the primary winding are assumed to be zero.

For this particular case the right line *PN′* becomes tangent to the circle at the point *P*, which then coincides with the point *A*.

This consideration enables us to determine immediately the currents i_1 and i_2, as well as the angle of phase difference produced at the time the rotor begins to move.

We know that the right line *NR* is a measure, not only of the power absorbed by the stator, but also of the total tangential effort acting on the rotor.

The right line *N′R′*, therefore, also gives the amount of tangential effort at the instant of starting.

If we now desire to know the useful mechanical effort at the periphery of the rotor, we must subtract from the straight lines *NR*, *N′R′*, ..., etc., the efforts expended to overcome the bearing friction and the friction due to air resistance (windage).

In the case of a direct-coupled motor, the friction of the bearings remains constant, since it depends only on the weight of the rotor, which does not vary.

With a belted motor, this friction is still substantially independent of the load. The load itself has merely the effect of increasing the difference between the working tensions on the tight and slack sides of the belt. This has more the effect of shifting the line of contact between the motor journals and bearings than of increasing the total bearing pressure.

As for the mechanical resistance due to air friction (windage), it increases with the speed. Nevertheless, as this resistance is always very small, and as the speed of motors varies but little with the load, it may be assumed, without appreciable error, that the opposing tangential effort represented by, and due to, bearing friction and windage, is constant for all loads.

It will, therefore, suffice to estimate the loss of torque from these causes for any given load whatever, and to represent the amount by a distance, such as *RD*, for example, drawn according to the scale of tangential efforts in the diagram (Fig. 62).

The useful tangential efforts will then be equal to the distance comprised between the circle *MNP*, and the right line parallel to *AM* and passing through the point *D*.

The opposing efforts just considered will cause a loss of power which will vary with the speed of the motor, this loss being zero at the time of starting, and being a maximum when the motor is running without load at synchronous speed.

Within the normal working limits of induction motors, that is to say, between running light and running fully loaded, the speed will increase from 3% to 5% at most. We can, therefore, assume with sufficient accuracy that between these two limits the losses due to bearing friction and windage will remain constant.

It will be sufficient, consequently, to estimate this loss, either for a motor running at full speed without load, or when running under full load, and to plot it according to the proper scale, such as equal to the distance *RE*, for example, in the diagram. The available power, that is to say, the power actually obtainable, will then be measured by the distance comprised between the circle *PAM* and the right line, parallel to *PM*, passing through the point *E*.

Fig. 62 shows that the current in the stator is much higher at the time of starting than while the motor is in normal operation.

If the starting of a motor of large size were to be accomplished without special precautions, by simply closing a switch, there would result a heavy flow of current which might cause a perceptible drop in the difference of potential at the motor terminals. This difficulty, which is especially apparent and objectionable when electric lamps are to be supplied from the same feeders, has led the Lighting and Power Companies to prohibit the use of motors not provided with special starting apparatus, in all cases when the power exceeds 5 horse-powers, and sometimes even when it exceeds 3 horse-powers.

The Motor Manufacturer is often obliged to turn out motors which can, at starting, develop a torque equal to the running torque without absorbing more current than that which corresponds to full normal load.

This condition can always be realized. It suffices, in fact, to prevent, by any means whatever, the secondary currents from reaching, at the time of starting, a value higher than that which they absorb while the motor is running under full load. If, on the other hand, arrangements are made so as to preserve the phase difference between the revolving magnetic field in the air-gap and the currents in the rotor, everything will take place as if the motor were running at full speed. The torque, the primary and secondary currents, and the power factor will retain the values which they attain when running at full load.

The height of the upper end, V, of the vector ON, above the right line PM representing the current i_1 for this full load, will indicate the power lost in the rotor at the time of starting. Since the current i_2 is given by the vector MN, it will be easy to determine the resistance which should be interposed in the circuits of the rotor in order to produce, when starting, the full running torque, while absorbing only a current i_1 equal to that required when running at full normal load and speed.

The starting of polyphase motors can be accomplished by other means than those which have just been mentioned. The means employed have for their object to produce, while starting, a sufficient torque with moderate stator currents and without making the normal operating conditions less favorable than they might be without such provision. It is possible to reduce the secondary currents and consequently those of the stator which counterbalance them, either by the temporary introduction of variable non-inductive resistances in the circuits of the rotor, as already stated, or by connecting the windings into opposing groups in which the induced E.M.F.'s may partially neutralize each other, or else by putting a portion of the winding out of use, or also by other more complicated arrangements.

The object of this treatise being to present a purely theoretical study of the design of induction motors, we will not dilate farther on the numerous ways which have been devised for the purpose of faciltating the starting of such motors. The reader will find numerous detailed descriptions of such methods which have been published within the last few years, in various scientific periodicals.

We may note, however, that the starting arrangement which is in most extensive use is that which consists in, and involves, the temporary introduction of variable ohmic resistances into the circuit of the rotor, these resistances being gradually cut out as the speed increases, and being completely excluded from the rotor circuit after the motor has attained full speed.

Definite and Exact Construction of Circle Diagram. In the preceding discussion of the circle diagram, we have supposed, for the purpose of making the matter simpler and more intelligible, that there were no losses due to hysteresis or Foucault currents in the iron cores, and that, likewise, there were no ohmic losses in the copper of the stator windings. Although these losses are generally very small, it is proper, since it can be easily done, to introduce into the diagram the necessary corrections for them. These corrections seem to be desirable, since a loss of potential in the primary has the effect of reducing the amount and of increasing the phase difference of the induced potential difference e_1, and also of causing variations in the resultant magnetic field F_1, represented by the vector OM, on whose fixed value the construction of the circle diagram depends altogether.

We will begin with the discussion of the core losses.

If a motor running without load at synchronous speed have a load applied to it in increasing amount until the rotor is brought to a full stop, the resultant flux F_1 in the stator will diminish in consequence of the increase in ohmic loss of potential, $R_1 i_1$. For this same reason, and in consequence of the increase in the magnetic leakage flux in the rotor, the resultant magnetic field F_2 will be likewise diminished.

But as the load becomes heavy, the slip of thê rotor becomes more marked, and the variations of flux in the rotor core become more rapid. The increase of iron losses resulting from this makes up quite nearly for the saving in power due to the reduction of the magnetic field fluxes F_1 and F_2.

As these losses have a very small value, compared to the power absorbed, we may assume, without fear of appreciable error, that they remain constant within the working limits of the motor.

There corresponds to this iron loss, as already explained at length (page 94), a watted component, i_{1w}, of the stator current, whose direction is naturally that of the induced potential difference e_1. We need only examine Fig. 54 to ascertain that this component has the effect of giving to the current i_1 a value very slightly higher than that of the magnetizing current $i_1\mu$, as well as of reducing slightly the angle of lag ϕ which would otherwise be obtained if this watted current i_{1w} did not exist.

Since the iron losses do not vary with the load, the watted component i_{1w} will always have the same value. The vector *OL*, which represents the E.M.F. e_1 in the circle diagram, having a fixed value, it follows that this watted component will always retain the same position.

Owing to these considerations, it will be easy to make allowance for these iron losses in the diagram. It will be sufficient, indeed, to plot along the prolongation of *OL* (Fig. 63), according to the scale of amperes, the watted component:

$$i_{1w} = OO';$$

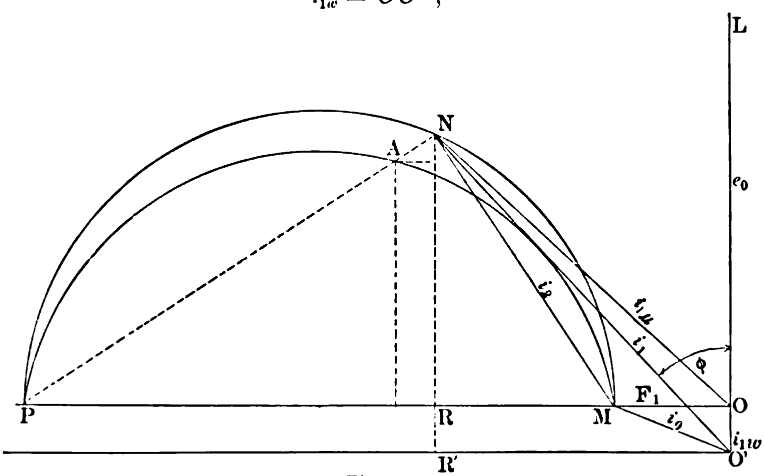

Fig. 63.

The vector *ON* will represent, as before, the magnetizing current $i_1\mu$. *O'N*, which results from the vectorial addition of *ON* and *OO'*, that is to say, the vectorial addition of $i_1\mu$ and i_{1w} will be the measure of the total stator current, i_1.

The straight line *MN* will represent, as before, the secondary current, i_2. The angle of phase difference, ϕ, which will be smaller than if i_{1w} were zero, will be measured, no longer between *NO* and *OL*, but between *NO'* and *O'L*.

When the motor is running without load, the primary current i_0 is no longer equal to *OM*, but to the resultant *O'M*, obtained by the vectorial addition of *OM* and *OO'*.

The power absorbed by the motor is no longer proportional to *NR* but to *NR'*.

The distance *NR*, however, will always represent the gross tangential effort obtained at the periphery of the rotor, from which, in order to obtain the net effort, there should be deducted the opposing effort due to friction of bearings and air resistance.

The height of the point A above the right line OP, as we already know, will indicate the mechanical force of the rotor, of which a portion is expended in overcoming the air friction and bearing friction of the rotor.

We thus see how easy it is to introduce into the diagram the hysteresis and eddy current losses occurring in the iron cores.

Let us now consider the matter of making a further correction which is necessary when taking into account the ohmic loss of potential, $R_1 i_1$, in the stator winding.

When we assume the resistance R_1 of the stator windings to be zero, the total induced E.M.F., e_1, should be exactly equal and opposed to, that is to say, it should exactly counterbalance, the potential difference at the motor terminals. In reality, since the primary windings always present a certain resistance, these two E.M.F.'s will never be along the same line, but they will produce a resultant, $R_1 i_1$, which is expended entirely in producing the current i_1 in the resistance R_1.

If the vectors ON and OL represent the current i_1 and the induced E.M.F., e_1 (Fig. 64), we will find the potential difference required at the

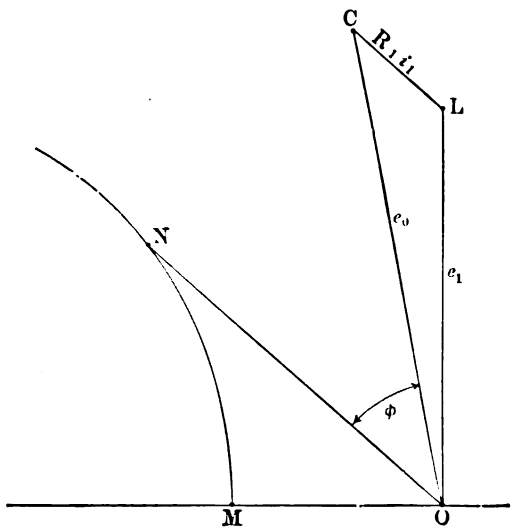

Fig. 64.

motor terminals to produce the current ON in the windings, when the induced E.M.F., e_1, already exists in said windings, by plotting the value of the ohmic drop, $R_1 i_1$, on a line LC, parallel to ON. The vector OC

will represent this potential difference at the motor terminals, both in amount and in phase. It should be noted that OC is always greater than OL.

However, if the potential difference of the source of current supply remains constant, the counter E.M.F., e_1, should diminish in proportion as $R_1 i_1$ increases in amount. This weakening of e_1 does not depend exclusively on the length of the vector LC, but also on its inclination with respect to OA.

It is evident that the angle of lag between OL and ON is greater than the angle NOC comprised between the two right lines OC and ON.

The presence, in the stators, of an appreciable resistance R_1 therefore causes, first, a diminution of the induced E.M.F., e_1, and second, an increase in the power factor, cos ϕ.

The circle diagram which has previously been worked out on the hypothesis that there are no ohmic losses, that is to say, on the assumption that both the E.M.F., e_1, and the distance OM, which is proportional thereto, are constant, could not now be utilized, since this E.M.F., e_1, is no longer, as before, always equal to the potential difference at the motor terminals, but takes a new value for each value of the primary current i_1.

For the same reason, the resultant field $F_1 = OM$, and the diameter MP of the circle MNP will no longer be constant.

Nevertheless, as all the vectors of the diagram vary proportionally with the total induced E.M.F., $e_1 = OL$, it will be sufficient to ascertain what ratio the terminal voltage OC bears to the new value of e_1. This ratio, which is:

$$\frac{OC}{OL} = \frac{e_0}{e_1} = a,$$

is not at all constant; on the contrary, it has a particular value for each value of the current, i_1.

In order to take into account the effect of the ohmic drop $R_1 i_1$, and to give exact expression to the results previously obtained by means of the circle diagram based upon the hypothesis $R_1 = o$, we would need to multiply all these results by the values of a or a^2 corresponding thereto.

We will make use of the coefficient a^2 for all values which are proportional to the product of two vectors, as, for example, the power applied to the primary (deducting the ohmic losses), or that of the secondary, or again, the tangential effort of the rotor, or the losses $R_2 i_2$ in the copper of the rotor.

The primary and secondary currents i_1 and i_2 will be multiplied only by the values of a which relate to them.

As the slip is proportional, not to a product, but to the quotient of the resultant magnetic field F_2 by the current i_1, it will not be changed in value, the two terms of the ratio being diminished in the same proportion.

We will now indicate the very simple manner in which the correcting factors a are to be determined.

In the circle diagram obtained by taking into account the watted component OO' which supplies the iron losses (but on the assumption of $R_1 = o$), we may draw (Fig. 65), a few vectors $O'M$, $O'N_1$, $O'N_2$ . . . ., etc., representing various values of the primary current.

We will then draw from the point L, along $O'L$, perpendicular to OP, the E.M.F., e_0, at motor terminals. From the point L we will draw the

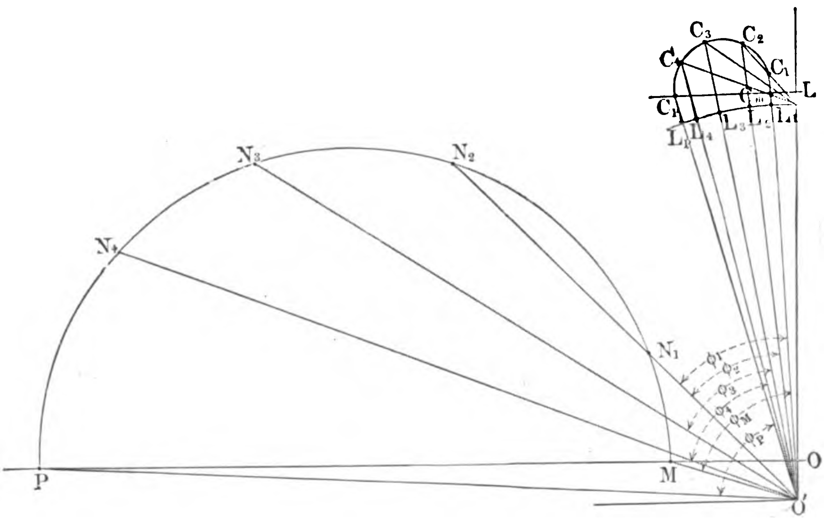

Fig. 65.

right lines, LC_1, LC_2, LC_3, LC_4, ... etc., parallel to the corresponding vectors $O'N_1$, $O'N_2$, $O'N_3$, $O'N_4$, ... etc. The distances LC_1, LC_2, LC_3, ... etc., are to be made equal to the corresponding ohmic drop, $R_1 i_1$.

The points L_1, L_2, L_3, are determined by the intersection of the circular arc drawn from L with O' as a centre, with the right lines $O'C_1$, $O'C_2$, ... etc., representing the values which the difference of potential at the terminals ought to have in order to maintain the induced E.M.F., e_1, constant, notwithstanding the resistance R_1.

But as the potential difference at the motor terminals always remains equal to $O'L$, we should multiply all the vectors of the diagram corresponding to a load defined by a given current strength, such as ON_1, by the ratio

$$\frac{O'C_1}{O'L_1} = a_1,$$

while all the vectors which relate to a load defined by the current strength ON_2, are to be multiplied by the ratio,

$$\frac{O'C_2}{O'L_2} = a_2,$$

and so on.

We thus see how easily the values of the correcting factors a_1, a_2, a_3, may be determined.

Since the lengths LC_1, LC_2, LC_3, ... etc., are nothing but the products of the current values by the same factor R_1, it is plain that the points C_1, C_2, ... etc., will all lie on a circle, same as the points N_1, N_2 . . . . etc.

It is sufficient, therefore, in order to obtain the correcting factors a, and the exact values of the angles of lag ϕ, to determine the straight lines,

$$LC_M = O'M \times R_1,$$

$$LC_P = O'P \times R_1,$$

and to draw a semicircle having the line $C_M C_P$ as a diameter.

The intersections of the right lines LC_1, LC_2, . . . , with this circle, will determine the values of the ohmic drop, and will give, for each case, the potential difference which it would be necessary to apply at the motor terminals if the E.M.F., $e_1 = O'L$, were to remain constant. The correcting factors a_1, a_2, ... , are to be determined by means of the ratios,

$$\frac{LC_1}{O'C_1}, \ \frac{LC_2}{O'C_2}, \ \ldots \text{ etc.}$$

We will have occasion again to return to all that has just been said concerning the circle diagram in the next chapter, where we make use of the theoretical study which forms the object of the present treatise in the complete calculation of some induction motors.

Before closing this chapter, it may be proper to add another observation.

In passing from the first diagram which was considered in this chapter to the circle diagram, the electromotive forces e_1' and e_2' induced by the resultant single magnetic field Φ, also the electromotive forces e_{1s} and e_{2s} produced by the magnetic leakage, as well as their resultants e_1 and e_2, have been replaced by magnetic fluxes which bear a constant proportion to them and which have a lead of a quarter-phase with respect to them.

Now, we know that all the lines of force of the leakage flux do not affect the entire number of conductors of the winding. The fluxes Φ_{1s} and Φ_{2s}, that is to say, $(V_1 - 1)\ \Phi_1$ and $(V_2 - 1)\ \Phi_2$ do not therefore represent the primary and secondary magnetic leakages which actually occur, but they represent leakage fluxes which are smaller, and whose lines of force are supposed to act on the entire number of conductors to produce E.M.F.'s which are precisely equal to e_{1s} and e_{2s}.

The consequence of this is that the resultant magnetic fields F_1 and F_2 are very slightly weaker than those which actually exist in the cores.

It is evident that this observation does not in the least diminish the value of the circle diagram, whose construction remains perfectly correct.

CHAPTER VIII.

PRACTICAL APPLICATIONS.

First Example. Before showing how the calculations relating to polyphase motor design should be made in order to meet certain given operating conditions, we will begin by verifying our theory by reference to some existing motor. We can do this by comparing the results obtained through the application of the deductions which form the basis of the preceding chapters with the results obtained from actual tests of the motor.

Let us take, for example, a three-phase motor of 10 H.P. capacity, running at 960 R.P.M. when supplied with current having a P.D. of 190 volts between phases, and having a frequency of 50 periods per second, the constructional data of the motor being as given in the table following:

STATOR.

Internal diameter of core	$D_1 = 331.5$ mm.	(13.05 in.)
External " " "	$d_1 = 484$ "	(19.06 in.)
Length (axial dimension) of core	$L = 125$ "	(4.92 in.)
Kind of winding: drum type, with coils in slots.		
Number of poles	6	
" " coils	9	
" " winding turns per phase	81	
" " series " " coil	27	
" " slots per active side of coil . . .	3	
" " wires per slot	9	
Winding conductor; $\frac{38}{10}$ mm. diam. bare, and $\frac{43}{10}$ mm. with double cotton covering (nearly equivalent to No. 7 B. & S. Gauge).		
Number of slots in core	54	
Ohmic resistance per phase	$R_1 =$ 0.125 ohm.	
The three phases are connected "star" fashion.		

ROTOR.

External diameter of core	$D_2 = 330$ mm.	(13.00 in.)
Internal " " "	$d_2 = 210$ "	(8.27 in.)
Length (axial dimension) of core	$L = 125$ "	(4.92 in.)
Kind of winding; drum type, with coils in slots.		

Number of coils		9
" " series turns per coil		12
" " slots per active side of coil . . .		4
" " wires per slot		3
Winding conductor consists of 37 wires each $\frac{8}{10}$ mm. diam. Diameter of bare cable, $\frac{53}{10}$ mm.		
Insulated diameter, with three cotton coverings, $\frac{60}{10}$ mm. (nearly equivalent to No. 4 B. & S. Gauge).		
Ohmic resistance per phase	$R_2 =$	0.0255 ohm.

The three branches of the winding are connected "star-fashion" at one end, and connect, respectively, with three collector rings at the other end. The motor is to be started by means of a liquid resistance interposed in the rotor circuit.

On opening the circuits of the rotor, and passing a current having a P.D. of 190 volts between phases in the stator winding, the P.D. between any two collector rings of the rotor was found by actual measurement to be 81 volts.

By applying to the rotor a P.D. of 80 volts, there was found, at the stator terminals, an E.M.F. of 172 volts.

If the magnetic flux F_1, in the primary, passed entirely through the secondary core, and if the rotor and stator windings were placed in the same number of openings, the mean E.M.F. induced in each of the turns of both windings would have the same value.

Since the turns per phase are 81 in number in the stator, while they are only 36 in the rotor, the total E.M.F.'s at the stator terminals and between the collector rings, respectively, ought to be in the ratio of 81 to 36.

As the two windings are not distributed in the same number of slots, the coefficients K_1 of the formula for calculating the induced E.M.F. are not equal, although they differ but slightly from each other.

We have, in fact, for the stator (with 3 slots per active side of coil),

$$K_1 = 0.960,$$

and for the rotor:

$$K_1 = 0.958.$$

The ratio of E.M.F.'s should, therefore, be, in reality, not $\frac{81}{36}$, but,

$$\frac{81}{36} \times \frac{0.96}{0.958}.$$

By subjecting the stator to a P.D. of 190 volts, if the magnetic flux F_1 passed entirely through the rotor core, the E.M.F. between the collector rings of the rotor ought to be:

$$e_2 = 190 \frac{36 \times 0.958}{81 \times 0.96} = 84.3 \text{ volts.}$$

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Now, by actual measurement, this E.M.F. is found to be only

$$e_2 = 81 \text{ volts.}$$

We therefore conclude that the flux F_1 in the stator core bears a relation to the flux which penetrates the rotor core, which is as shown by the following equation:

$$\frac{V_1\Phi_1}{\Phi} = \frac{84.3}{81.0} = 1.04, \text{ whence } V_1 = 1.04.$$

By repeating the same reasoning for the rotor, we would have:

$$\frac{V_2\Phi_2}{\Phi_2} = \frac{80\dfrac{81 \times 0.96}{36 \times 0.958}}{173} = 1.044,$$

whence:

$$V_2 = 1.044.$$

We found, in connection with the circle diagram:

$$\frac{MP}{OM} = \frac{1 - \sigma}{\sigma}.$$

If we write:

$$\sigma = 1 - \frac{1}{V_1V_2},$$

we at once deduce, for the case under consideration,

$$\sigma = 1 - \frac{1}{1.044 \times 1.04} = 0.00770,$$

so that,

$$\frac{PM}{OM} = \frac{0.923}{0.077} = 11.98.$$

If, in Fig. 66, we select arbitrarily the length OM, the point P will be determined, since

$$PM = 11.98\ \overline{OM}.$$

Let us trace on PM the well-known half-circle, and let us join the point O with the point 3, which occurs at some given point in the circle. The vector $O3$ will represent at will, and according to various scales, the magnetizing current, $i_1\mu$, or the fictitious magnetic flux, $V_1\Phi_1$. The vector $M3$ will represent either the secondary current, i_2, or the fictitious magnetic flux, Φ_2, or, again, the M.M.F. produced by the secondary winding.

The right line OM will represent, in amount and in phase, the magnetic flux, F_1, passing through the coils of the stator.

If from the point M we draw, parallel to $O3$, a right line MA, whose length represents the magnetic leakage flux,

$$\phi_{1s} = V_1 - 1,$$

the vector OA, as already explained, will then represent either the real magnetic flux, Φ, in the air-gap, or the magnetic density, or else the M.M.F. resulting from the simultaneous action of the primary and secondary windings.

If we assign to OM, or to the induced E.M.F., e_1, a constant value of 110 volts, which corresponds to a P.D. of 190 volts between phases, and if we for a moment neglect the ohmic resistance, R_1, and the iron losses, it is easy to determine the magnetic flux F_1, since it is that flue which produces, by itself alone, the effective induced E.M.F., $e_1 = 110$ volts.

We may therefore write:

$$E_1 \times 10^8 = K_1 Z_1 \omega F_1,$$

whence,

$$F_1 = \frac{E_1 \times 10^8}{K_1 Z_1 \omega}.$$

As each active side of coil occupies three slots, and as

$$K_1 = 0.96,$$

we will, therefore, have, for 50 periods per second,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{50}} = 314,$$

and (substituting actual values in the formula just found), the magnetic flux F_1 will be:

$$F_1 = \frac{110\sqrt{2} \times 10^8}{314 \times 81 \times 0.96} = 637{,}000 \text{ maxwells}.$$

Since the length of the right line OM represents a magnetic flux of 637,000 maxwells, we will have, for the scale of magnetic flux:

$$1_{\text{mm}} = \frac{637{,}000}{OM_{\text{mm}}} \text{ maxwells}.$$

The magnetic flux, Φ, in the air-gap will be:

$$\Phi = \frac{OA}{OM} \times 637{,}000 = 587{,}000 \text{ maxwells}.$$

We at once deduce therefrom the effective value of the magnetic density, b, in the middle of the air-gap, for:

$$\Phi = 0.9\, b \times \frac{\pi \times \frac{D_1 + D_2}{2}}{p} L,$$

whence: $$b = \frac{6 \times 587{,}000}{(0.9\pi \times 33.075) + 12.5} = 3{,}015 \text{ gausses.}$$

As the slot openings at the periphery of the core have a width of 0.1 cm. in the case of both the stator and rotor, the development, or the peripheral length, of the air-gap in front of the teeth will be, in the case of the stator disks:

$$S_1 = (\pi \times 33.15) - 5.4 = 98.6 \text{ cm. } (38.82 \text{ in.}),$$

and, in the case of the rotor disks,

$$S_2 = (\pi \times 33.0) - 7.2 = 96.5 \text{ cm. } (38.0 \text{ in.}),$$

while the mean peripheral length will be:

$$S = \frac{98.6 + 96.5}{2} = 97.6 \text{ cm. } (38.41 \text{ in.}).$$

If the slots were entirely closed, this peripheral length would be

$$S' = \pi \times 33.075 = 103.9 \text{ cm. } (40.91 \text{ in.});$$

so that the slot openings reduce the iron surface of the air-gap in the ratio of:

$$\frac{103.9}{97.6} = 1.065.$$

The radial distance across, or "length" of, the air-gap being

$$d = 0.075 \text{ cm. } (0.03 \text{ in.}),$$

the effective M.M.F., m, which is necessary to produce the magnetic flux Φ, must be:

$$m = 3.015 \times 0.075 \times 1.065;$$

$$m = 240.6 \text{ C.G.S. units.}$$

Knowing the length *OA* which represents 240.6 C.G.S. units of M.M.F., we can deduce the scale of the diagram, and at once estimate the magnetizing actions of the primary and secondary windings. The

C.G.S. units represented, according to that scale, by the right line $M3$, are:

$$m_2 = 1.071 \text{ C.G.S. units.}$$

The primary M.M.F. is not represented by the right line, $O3$, but only by the length:

$$\frac{O3}{V_1} = \frac{O3}{1.04}.$$

We are therefore enabled to take, according to the scale,

$$m_1 = 1.143 \text{ C.G.S. units.}$$

Now, we know that:

$$m_2 = K\frac{S_2}{p}i_2,$$

therefore:

$$i_2 = \frac{6 \times 1.054}{0.381 \times 71 \times 3} = 78.2 \text{ amperes.}$$

We would likewise have:

$$m_1 = 0.381 \times \frac{S_1}{p} \times i_1,$$

therefore:

$$i_1 = \frac{1.125 \times 6}{0.381 \times 9 \times 54} = 37.1 \text{ amperes.}$$

Since the lengths $O3$ and $M3$ represent the effective current values i_1 and i_2, we can determine the scale according to which these values are drawn in the diagram.

If we draw from the point A, parallel to $M3$, a right line AG, representing the magnetic leakage flux:

$$\Phi_{2'} = (V_2 - 1)\,\Phi_2,$$

we can determine the vector OG representing the magnetic flux, F_2, in the secondary winding. This vector should be perpendicular to $M3$.

The magnetic flux Φ in the air-gap may be decomposed into two magnetic fluxes, — one of which, OG, is perpendicular, while the other, AG, is parallel, to the current, $i_2 = M3$. The first one alone produces tangential effort (and torque), the electro-dynamic action of the second one being exactly zero. By means of the diagram we may write:

$$\frac{OA}{OG} = \frac{\Phi}{F_2} = \frac{587{,}000}{580{,}000} = 1.013;$$

The effective magnetic density of the magnetic field, *OG*, will therefore be:

$$b_2 = \frac{b}{1.013} = \frac{3.015}{1,013} = 2,970 \text{ gausses.}$$

The tangential effort exerted between the magnetic field and the rotor will be:

$$C = \frac{K_1 L S i_2 b_2}{9.81 \times 10^6} \text{ kilogrammes,}$$

or:

$$C = \frac{0.958 \times 2,970 \times 72 \times 3 \times 12.5 \times 78.2}{9.81 \times 10^6};$$

$$C = 61.25 \text{ kilogrammes.}$$

The tangential effort being represented in the diagram by the height of the point 3 above *PM*, we may thereby determine the scale of tangential efforts.

The power absorbed by the motor is, evidently,

$$P_1 = 3 e_1 i_1 \cos \phi.$$

The diagram gives:

$$\cos \phi = 0.848;$$

therefore

$$P_1 = 3 \times 110 \times 37.1 \times 0.848,$$

$$P_1 = 10,380 \text{ watts.}$$

This power being represented by the height of the point 3 above the right line *OP*, we can easily determine the scale of watts.

The ohmic loss in the rotor is:

$$P_2 = 3 e_2 i_2 = 3 i_2^2 R_2,$$

$$P_2 = 3 \times (78.2)^2 \times 0.0255 = 467 \text{ watts.}$$

This loss may be plotted by reference to the scale of watts, starting from the point 3 on the line dropped perpendicular to the right line *OP*. By drawing, from this new point, a line parallel to *OP*, as far as its intersection with the vector *P*3, we will determine the point *A*, whose height above *OP* measures the available mechanical power of the rotor.

We know that the geometrical locus of the point *A* is a half-circle passing through *M* and *P*. We can therefore immediately draw this semi-circle.

Let us draw, from the point P, a tangent to the semi-circle PAM, meeting, at the point 11, with the circle P, 11, $3M$. For that point, 11, the power applied to the primary is lost entirely by ohmic loss (Joulean effect) in the secondary winding. The rotor cannot then produce any mechanical power. It therefore remains motionless, and the slip attains the value of 100%. The point 11 corresponds to the point of maximum starting effort, and the tangential effort at that point is measured by the height of the point 11 above the line OP. The distance O–11 represents the value of the primary current, and the distance M–11 represents the value of the current i_2. The diagram therefore furnishes the following data regarding the starting of the motor:

Primary current	$i_1 = 106.5$ amperes.
Secondary current	$i_2 = 230$ "
Tangential effort measured at the middle of the air-gap	$C = 24.2$ kilogrammes (53.34 lbs.).
Angle of lag	$\phi = 83°, 20'$.
Power factor	$\text{Cos}\,\phi = 0.116$.
Electric power absorbed by motor .	$P_1 = 4{,}035$ watts.

The actual results give:

$$P_1 = 3e_1i_1 \cos \phi = 330 \times 106.5 \times 0.116$$

$$P_1 = 4{,}075 \text{ watts,}$$

and

$$P_2 = 3\, e_2i_2R_2 = 3i_2^2R_2$$

$$P_2 = 3 \times 230^2 \times 0.0255 = 4{,}060 \text{ watts.}$$

The concordance between the theoretical and the practical results is therefore quite satisfactory.

The slip being proportional to the sections cut off, on a vertical line perpendicular to the line PO, by means of lines such as P–11 and P–3, the scale of slips will be obtained by remembering that the right line P–11 prolonged will cut off from the vertical line prolonged a segment whose length represents the total number of revolutions per minute (R.P.M.) of the magnetic field.

If we divide this total distance into 100 equal parts, it will be possible to read directly on the diagram the slip, in per cents. We will thus find, for the point 3, a slip

$$s = 43.6\%,$$

or 43.6 R.P.M.

It is easy to verify this result.

We know, in fact, that the secondary induced E.M.F. is:

$$e_2 = i_2 R_2 = 78.2 \times 0.0255,$$

$$e_2 = 1.99 \text{ volt};$$

but:

$$e_2 = K_1 b_2 V_2 Z_2 L \times 10^{-} \text{ volts.}$$

$$V_2 = \pi \times 33.075 \frac{s}{60} \text{ centimetres.}$$

in which s represents the slip, in turns per minute; we have,

$$e_2 = 0.958 \times 2.970 \times \pi \times 33.075 \frac{s}{60} 72 \times 12.5 \times 10^{-8},$$

and since:

$$e_2 = 1.99 \text{ volt},$$

$$s = \frac{60 \times 1.99}{0.958 \times 2{,}970 \pi \times 33.075 \times 72 \times 12.5},$$

$$s = 44.8 \text{ turns per minute.}$$

The concordance between results is, therefore, very satisfactory.

The loss in the iron cores was found, during the tests, to amount to 412 watts, so that,

$$i_w = \frac{412}{3 \times 110} = 1.25 \text{ ampere.}$$

If we plot the current i_w along OO' according to the scale of primary current in amperes, and if we draw $O'M$, $O'3$, $O'11$, we obtain the values of the stator current i_1 corresponding, respectively, to these various loads.

By measurements taken from the diagram, we have,

For the motor running without load	$i_1 =$ 8.55 amperes.
Starting current	$i_1 =$ 106.9 "
For load corresponding to point 3 in Fig. 66 . . .	$i_1 =$ 38.0 "

The secondary currents i_2, the tangential effort, the amount of mechanical power available at the rotor, and the slip, retain, for all load conditions, the values previously determined.

Since it is evidently the external source of current which supplies the energy dissipated in the iron cores, the power absorbed should be reckoned no longer from the right line OP, but from a right line parallel thereto, passing through the point O'.

If we select arbitrarily certain points on the circle, such as 1, 2, 3, 4, ... 10, and if we draw the vectors $O'1$, $O'2$, ... $O'10$, the diagram may be used, by means of the various scales which have been determined, for measuring directly the results contained in the following table:

Load Condition.	Current: Amperes.		Tangential Effort (Kilograms), C.	Power Absorbed (Watts), P_1.	Mechanical Power Obtained (Watts).	Slip, s, R.P.M.
	Primary, i_1.	Secondary, i_2.				
0	8.5	0	0	412	0	0
1	19.5	35.6	29.0	5,320	4,830	19
2	28.9	57.3	46.2	8,220	7,575	31
3	38.1	78.0	61.0	10,780	9,900	43
4	47.3	98.3	74.0	12,970	11,880	58
5	61.2	129	89.1	15,520	13,900	82
6	72.4	153	95.6	16,600	14,490	108
7	77.2	164	96.4	16,720	14,400	122
8	89.8	200	83.7	14,530	11,620	213
9	102.5	220	64.9	10,160	6,140	354
10	106.3	229	29.0	5,290	1,100	798
Starting.	106.8	230	24.2	4,470	0	1,000

We only need now to introduce the corrections necessary to take into account the ohmic drop $R_1 i_1$ in the stator, which loss has thus far been neglected.

To this end, let us indicate the induced E.M.F., according to a suitable scale, on a line starting from the point O' and drawn perpendicular to OP, such that:

$$e_1 = OC = 110 \text{ volts},$$

which E.M.F. has hitherto been supposed constant.

From the point C let us draw two separate lines, CM', and CP', the former parallel to $O'M$, the latter parallel to $O'P$, and of lengths such as will be equal to the ohmic drop $R_1 i_{1M}$ and $R_1 i_{1P}$ which would be produced in the resistance R_1 of the winding, by the currents $O'M$ and $O'P$.

If we draw a semi-circle on the straight line $M'P'$, also a circular arc through the point C with O' as a radius, and also draw the right lines $C1'$, $C2'$, $C3'$, ... $C11'$, which are parallel and proportional to the vectors $O'1$, $O'2$, $O'3$, ..., $O'11$, and if we then join O' with the points $1'$, $2'$, $3'$, . . ., $11'$, we determine the phase angles ϕ and the correcting factors a_1, a_2, a_3, . . . , a_{11}. These factors, which are nothing more than the ratios between the potential difference which must be applied to the motor and the counter E.M.F. of the motor at the motor terminals (which latter is assumed to remain constant at 110 volts), will have the following values:

$a_M = 1{,}002$ and $a_M^2 = 1{,}004$	$a_6 = 1{,}060$ and $a_6^2 = 1{,}122$
$a_1 = 1{,}014$ " $a_1^2 = 1{,}026$	$a_7 = 1{,}061$ " $a_7^2 = 1{,}126$
$a_2 = 1{,}021$ " $a_2^2 = 1{,}045$	$a_8 = 1{,}058$ " $a_8^2 = 1{,}119$
$a_3 = 1{,}032$ " $a_3^2 = 1{,}065$	$a_9 = 1{,}044$ " $a_9^2 = 1{,}090$
$a_4 = 1{,}048$ " $a_4^2 = 1{,}098$	$a_{10} = 1{,}027$ " $a_{10}^2 = 1{,}053$
$a_5 = 1{,}054$ " $a_5^2 = 1{,}115$	$a_{11} = 1{,}025$ " $a_{11}^2 = 1{,}050$

If we divide by a the results obtained for the primary and secondary currents, and if we divide by a^2 the results obtained for the tangential effort, as well as those relating to the power absorbed and the mechanical power developed, we will obtain new exact values which are consigned in the following table of definite results:

Load Condition Steps.	Current: Amperes.		Useful Pull, C (Kilograms).	Total Power Absorbed, (Watts).	Power Available at Motor Shaft (Watts).	Efficiency, η %	Power Factor (cos ϕ).	Slip, s (r.p.m.)
	Primary, i_1.	Secondary, i_2.						
0	8.47	0	0	.437	0	0	0.157	0
1	19.2	35.1	26.8	5,328	4,470	84.0	0.84	19
2	28.3	56.2	42.8	8,170	7,060	86.5	0.875	31
3	36.9	75.6	55.9	10,630	9,100	85.5	0.874	44
4	45.2	94.0	66.0	12,585	10,580	84.2	0.844	58
5	58.1	122.3	78.5	15,185	12,200	80.5	0.792	82
6	68.3	144.4	83.7	16,535	12,690	76.8	0.734	108
7	73.8	154.6	84.3	16,920	12,810	72.1	0.695	122
8	84.2	189.2	73.4	15,660	9,820	62.7	0.563	213
9	98.2	211.0	52.8	12,940	5,800	44.8	0.409	354
10	103.8	223.0	26.0	9,050	855	9.8	0.264	798
Starting	104.0	224.0	21.6	8,310	0	0	0.242	1,000

The values given for tangential effort (useful pull) have all been reduced by 1.5 kilogramme to allow for friction, so that they indicate the net results, or the useful tangential effort available, when measured at the middle of the air-gap.

After multiplying by a^2 the values given in the first table for the power absorbed, there has been added to each value the corresponding ohmic loss,

$$3\, i_1^2 R_1.$$

The figures given therefore represent the total power supplied to the motor.

The power available at the motor-shaft was calculated by reference to the tangential effort and the speed of the rotor, using the following formula:

$$P_u = C \times \pi \times 0.33075 \frac{1.000 - s}{60} 9.81 \text{ watts.}$$

The angles of lag and the power factors were determined by means of the diagram.

The curves in Fig. 67 give a graphical representation of the results contained in the last table.

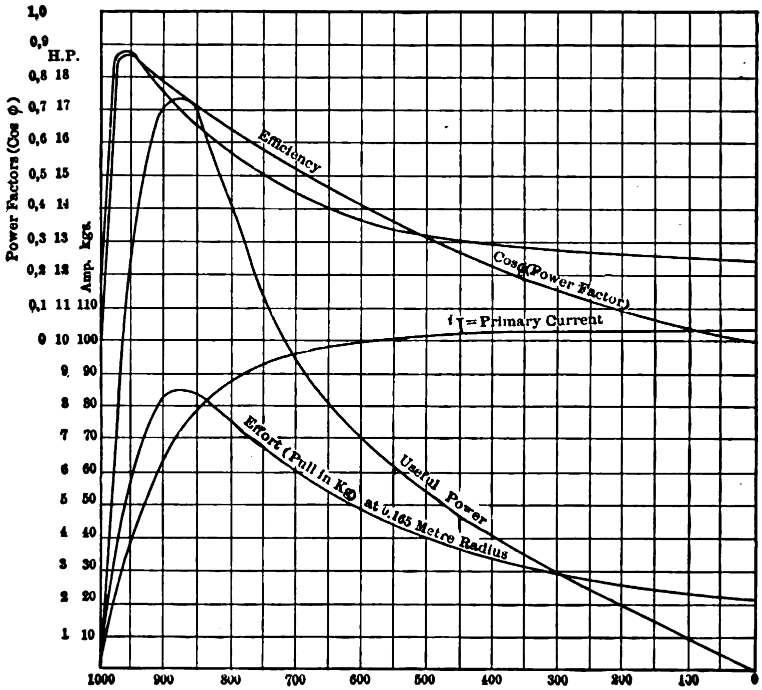

Fig. 67.

If we now compare these curves with those resulting from laboratory tests made with the machine under consideration, it will be noticed that the curves coincide so well as to cause astonishment at the close concordance obtained between theoretical and practical results.

The calculations which have just been made are somewhat lengthy. The designer can content himself with deriving from the diagram the various values of the primary and secondary currents, of the tangential effort, of the slip, the correcting factor, a, and also of the angles of lag, ϕ.

If we multiply the tangential effort by the speed of the rotor (both calculated with reference to the middle of the air-gap), we will obtain the mechanical power; and if we take the product of the current i_1 by the potential difference, in volts, applied at the motor terminals, and also by the number of phases and the power factor cos ϕ, we can determine the total power absorbed by the motor.

Second Example. It is required to design a three-phase motor capable of developing 3 H.P. when supplied with a polyphase current having a frequency of 50 periods per second and a potential difference of 190 volts between phases, the motor speed being 1440 R.P.M. This motor is to have an efficiency of 85%, and a power factor cos $\phi = 0.83$.

It is required that the motor shall start without special appliances, by merely closing the current supply circuit, and that it shall have a starting torque equal to twice the normal running torque.

The details of construction being optional with the designer, we will adopt for the stator a "definite" drum winding with the coils placed in slots. The active side of each coil will correspond to, and will be distributed in four slots.

The primary circuit connections are to be of the star or Y type.

The rotor winding is to be of the squirrel-cage type.

The useful power developed by the motor, when expressed in equivalent watts, will be

$$P_u = 3 \times 736 = 2{,}208 \text{ watts},$$

and the power absorbed by the motor will be

$$P_1 = \frac{2{,}208}{0.85} = 2{,}600 \text{ watts}.$$

Let us assume a loss of 4% through the friction of bearings and air resistance, equivalent to

$$0.04 \times 2{,}600 = 104 \text{ watts}.$$

The mechanical power developed by the rotor must, therefore, be equal to

$$2{,}208 + 104 = 2{,}312 \text{ watts}.$$

But the total power transmitted by the magnetic field to the secondary will be

$$2{,}312 \times \frac{1{,}500}{1{,}440} = 2{,}410 \text{ watts};$$

in which the numbers 1,500 and 1,440 represent, respectively, the speed

(R.P.M.) of the magnetic field and of the rotor; from which it follows that the losses by Joulean effect in the secondary winding will be

$$P_2 = 2{,}410 - 2{,}312 = 98 \text{ watts},$$

or (in percentage),

$$\frac{98}{2{,}600} \times 100 = 3.76\%.$$

Let us make an allowance of 4% for the ohmic drop in the copper of the primary winding, i.e.,

$$3R_1 i_1 = 104 \text{ watts.}$$

But

$$i_1 = \frac{P_1}{3e_0 \times \cos\phi} = \frac{2{,}600}{3 \times 110 \times 0.83} = 9.51 \text{ amperes.}$$

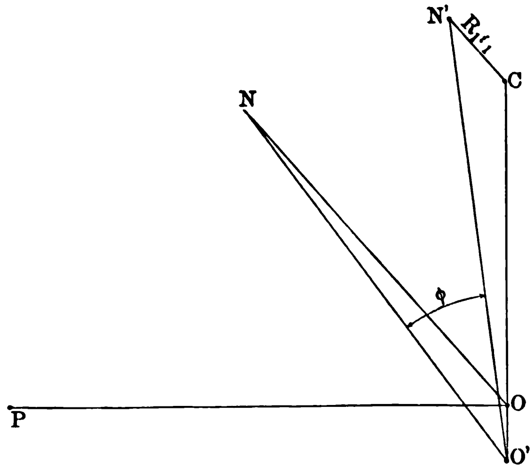

Fig. 68.

The primary resistance will, therefore, be,

$$R_1 = \frac{104}{3 \times (9.51)^2} = 0.384 \text{ ohms.}$$

Let us now draw (Fig. 68) two right lines, $O'N$ and $O'N'$, having between them an angle, ϕ, such that

$$\cos\phi = 0.83.$$

Let us suppose that $O'N$ represents the potential difference of the source of current supply, according to some given scale,

$$e_0 = 110 \text{ volts.}$$

Let us draw $N'C$ parallel to $O'N$ and equal to

$$R_1 i_1 = 9.51 \times 0.384 = 3.65 \text{ volts.}$$

The vector

$$O'C = 106.8 \text{ volts,}$$

will then represent, in magnitude and in phase, the induced E.M.F., e_1.

The losses thus far taken into account amount to 11.7 %, which therefore leaves 3.24% for the loss due to hysteresis and eddy currents, making the watted component of the current equal to:

$$i_w = \frac{0.0324 \times 2,600}{3 \times 106.8} = 0.263 \text{ ampere.}$$

The vector $O'N$, which corresponds to a current of 9.51 amperes, determines the scale of primary current values. If the watted component i_w, just calculated, be plotted according to that scale and in the direction of the E.M.F. e_1, we will determine the point O, in the diagram, through which is to be drawn the right line OP, perpendicular to $O'C$.

The vector $ON = 9.33$ amperes will represent the magnetizing current $i_1\mu$, both in magnitude and in phase.

As it is not possible to determine exactly by calculation the leakage coefficient:

$$\sigma = 1 - \frac{1}{V_1 V_2},$$

some assumptions will have to be made in regard to the values of V_1 and V_2.

When the designer has already constructed motors similar to the one under consideration, it will be easy to obtain, as the result of tests, the definite values of their magnetic leakage; and it will be possible, by comparison, to select the values V_1 and V_2 which are likely to obtain in the present case.

Although an error would modify but slightly the results corresponding to normal operation, even though this error were important, yet it would have a material influence on the value of the starting torque obtainable, according to the diagram.

When the designer is without precise data based upon laboratory-tests of a machine of the same type, it is very difficult, if not impossible, to determine with certainty the starting torque that will be obtained.

Nevertheless, inasmuch as motors which are to be started without special appliances are always of relatively small power, and as they are usually manufactured in various sizes, it will be well, before proceeding to construct a certain number of sizes of the new type, to begin by first completing and testing one of the machines.

If the starting torque should be greater than is necessary, we may endeavor to reduce it by diminishing the slip, or by reducing the resistance R_2. This may be done, if the speed permit it, by increasing the number of turns of the primary winding, thereby increasing the value of the power factor, cos ϕ.

On the other hand, should the starting torque be found too low, the designer may either increase the resistance R_2, or else reduce the number of turns in the stator winding. The latter modification will lead to a lower power factor and a lower efficiency, owing to the increase of the magnetic density in the air-gap, and the consequent increase in the core losses, which losses are not usually counterbalanced by the slight reduction in primary and secondary ohmic losses resulting from the reduction in the number of turns.

If the power factor is already low, and cannot stand any further reduction, it will be necessary either to increase the dimensions of the machine, or to diminish the thickness of the air-gap, while also reducing the number of turns of the primary winding.

When polyphase motors are to be started by special appliances, as in the majority of cases, and as is necessary, so to speak, whenever the useful power of the motor attains or exceeds 5 H.P., an error even of some importance in the value assigned to the coefficients V_1 and V_2, is not of great importance to, and has but little influence on, the results obtained, as shown by the diagram, between the no-load and full-load conditions.

It will therefore be possible to predetermine by calculation the details and data of motors capable of fulfilling certain given require ments with certainty and precision.

Let us suppose that, from the data of tests made with motors of similar type, we take the values of the two coefficients to be, respectively:

$$V_1 = 1.045,$$

and

$$V_2 = 1.033;$$

we immediately obtain:

$$V_1 V_2 = 1.08,$$

$$\frac{V_1 V_2}{1} = 0.9265,$$

and

$$\sigma = 1 - \frac{1}{V_1 V_2} = 0.0735,$$

$$\frac{1 - \sigma}{\sigma} = 12.6;$$

we therefore will have, for the diagram:

$$\frac{MP}{OM} = 12.6.$$

It is easy to construct, by trial, the well-known semi-circle, which must pass through the point N, and whose centre lies on the right line OP, while satisfying the condition:

$$\frac{MP}{OM} = 12.6.$$

This semi-circle having been drawn, it is necessary to ascertain whether, with the actual data, the starting torque will really be equal to twice the normal or full-load torque.

To this end, let us draw the right line PN, cutting off, on some line perpendicular to OP, a segment proportional to the value assumed for the slip, namely, 4%. If, after having determined, on that perpendicular line, the point corresponding to a slip of 100%, we draw a right line joining that point with the point P, the intersection of this line with the semi-circle will indicate, as we have already learned, the value of the tangential effort produced at starting, which, in the present case, should slightly exceed twice the tangential effort corresponding to normal load.

Since this value still has to be divided by the correcting factor a, in order to make allowance for the ohmic drop, $R_1 i_{1d}$, in the primary winding, the starting torque will become substantially equal to twice the normal running torque. The primary current i_{1d}, required at starting, when measured on the diagram, will be found to be:

$$i_{1d} = 58.6 \text{ amperes.}$$

In order to determine the exact value of the correcting factor a, besides, we will draw from the point C, parallel to the line representing the primary current i_{1d}, a right line whose length is equal to the ohmic drop, $R_1 i_{1d}$ volts. By completing the triangle having $O'C$ for its base, we obtain a vector, which, when divided by 110 volts, gives the correcting factor a. By proceeding in this manner, it will be found that the tangential effort at starting is, after all corrections have been made, equal to 2.1 times that due to normal load and speed.

It should be noted that we do not yet know the diameter, D, of the air-gap, or the axial distance (L) of the cores, or the radial distance, d, across the air-gap, and that, nevertheless, we can say that the primary current at starting will be:

$$i_{1d} = 58.6 \text{ amperes,}$$

also that the tangential effort developed at starting will be twice that corresponding to an output of 3 H.P. at a speed of 1440 R.P.M.; and, finally, that it will be possible to attain, at normal load, a power-factor value which has been set at 0.83.

Let us now proceed to set a value for the radial distance, d, across the air-gap. It will be seen later that, for a given motor, the power-factor $\cos \phi$ improves as the distance across the air-gap, d, is diminished, and that consequently it is desirable to make d as small as possible.

Assuming that a rotor-core disk and a stator-core disk be both punched concentrically out of the same sheet of iron, the two disks being detached from each other by punching or by a circular cutting-tool, and assuming that the active surfaces of both disks receive only a finishing cut in the turning-lathe, it would scarcely be possible to make the distance across the air-gap (or its radial "length") less than 0.5 mm.

This small clearance, between the fixed and movable parts of the motor, limits to a very small amount the wear which can be allowed at the bearings, and requires great precision and care in mechanical construction; for it must be remembered that the journals must turn freely in the bearings, that the bearings are centred in pedestals or bearing supports, and that the latter are in turn fastened to the stator frame.

The slightest defect in the fitting of these numerous parts involves or leads to an objectionable inequality in the distance across the air-gap if not an actual contact between the two cores.

Such a small amount of clearance involves very stiff journals and very long bearings, made of first-class material.

By making slit openings into the slots of the stator, of sufficient width to afford passage for the wire, the process of winding is thereby made simpler, quicker, and consequently cheaper.

But these openings diminish the section of iron available for the passage of the magnetic flux from the stator periphery into the air-gap, and they therefore have the same effect as an increase in the thickness d of the cylindrical layer of air in the air-gap.

In order to show clearly the effect of the radial distance across the air-gap (d), and of the width of the slits between the stator teeth, we will first make a calculation with

$$d = 0.1 \text{ cm.},$$

and then make the same calculation with

$$d = 0.05 \text{ cm.}$$

In the first instance, the width of the openings into the slots of the stator will be sufficient for the wire to pass through. In the second instance, this opening will be reduced to 0.1 cm. as in the case of the rotor.

Let us take, therefore, to begin with,

$$d = 0.1 \text{ cm.},$$

and let Z represent the number of conductors connected in series in one of the primary windings.

We may write at once for the primary M.M.F.:

$$m_1 = 0.381 \frac{S_1}{p} \times i_1;$$

but:

$$S_1 = 3Z_1,$$

whence:

$$m_1 = 0.381 \frac{3}{4} Z_1 \times 91.33.$$

This value of m_1 is represented in the diagram by the distance ON, so that we may measure:

$$m_2 = m_1 \frac{75.9}{89.0},$$

and

$$m = m_1 \frac{42.5}{89.0};$$

from which we deduce:

$$b = \frac{m}{d} = \frac{m}{0.12},$$

or

$$b = \frac{42.5}{89.0} \times 0.381 \frac{3}{4} 9.33 \frac{Z_1}{0.12} \text{ gausses},$$

$$b = 10.6\, Z_1 \text{ gausses}.$$

The clearance (air-space) between the two cores has been taken at:

$$d = 0.12 \text{ cm.},$$

or 0.02 cm. more than 0.10 cm., the actual air-gap distance assumed, in order to make allowance for the reduction of the iron surface on both sides of the air-gap caused by the slit openings into the slots of both the rotor and stator disks, and also for the reluctance of the teeth of both cores.

The magnetic flux issuing from one magnetic pole, and passing through the air-gap, will, therefore, be:

$$\Phi = 0.9\, b \frac{\pi D}{p} L = 0.9 \times 10.6\, Z_1 \frac{\pi D}{p} L.$$

But the diagram shows that between this magnetic flux Φ and the magnetic flux F_1 in the stator core, there exists the ratio:

$$\frac{F_1}{\Phi} = \frac{44.9}{42.5},$$

from which we have:

$$F_1 = \phi \frac{44.9}{42.5},$$

and, also,

$$F_1 = \frac{44.9}{42.5} 0.9 \times 10.6 \, Z_1 \frac{\pi}{4} DL \text{ maxwells.}$$

We know, moreover, that the induced primary E.M.F. is:

$$e_1 = 106.8 \text{ volts,}$$

and that, consequently,

$$e_1 \sqrt{2} \times 10^8 = K_1 \tfrac{1}{2} Z_1 \omega F_1 \text{ volts;}$$

but, in a case where there are four slots per active side of coil, we have:

$$K_1 = 0.958,$$

and for a frequency of 50 periods per second, we have:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{1}{50}} = 314;$$

therefore:

$$106.8 \sqrt{2} \times 10^8 = 0.958 \frac{Z_1}{2} \times 314 \times \frac{44.9}{42.5} \times 0.9 \times 10.6 \, Z_1 \frac{\pi}{4} DL;$$

from which we have:

$$Z_1^2 DL = \frac{42.5}{44.9} \times \frac{106.8 \times \sqrt{2} \times 10^8 \times 4 \times 2}{0.958 \times 314 \times 10.6 \times \pi \times 0.9},$$

or:

$$Z_1^2 DL = 1.268 \times 10^4.$$

If we select arbitrarily the ratio between D and L, and if we take, for example,

$$L = 0.6D,$$

the preceding equation becomes:

$$z_1 D_1 = \sqrt{2110 \times 10^4} = 4600.$$

This relation shows that the greater the number of conductors, Z, is made, the more the mean diameter D of the air-gap will be diminished.

The inferior limit of D is fixed by the necessity of making provision in the primary core for 48 slots of sufficient size to hold $3Z_1$ conductors, in addition to the insulation.

The number, Z_1, of conductors per branch, should not be taken too large, if we do not wish to exceed the value allowed for the leakage coefficient V_1, and if we do not wish to be disagreeably disappointed in regard to the starting torque.

The primary current i_1 being equal to 9.51 amperes, it can be predetermined at once that the wires of the stator winding will have a sectional area of about 3.5 sq. mm. (5425 circular mils), and a diameter not far from $\frac{21}{10}$ mm. (about No. 11 B. & S. G.). Allowing for a double cotton insulating covering, the external diameter of the wire will be about $\frac{26}{10}$ mm. (0.102 in.).

Let us assume that:

$$D = 20 \text{ cm. } (7.87 \text{ in.}).$$

We will have:

$$Z_1 = 227.5.$$

As these conductors must be placed in 16 slots, it is desirable that Z_1 should be a multiple of that number.

Let us, therefore, modify the preceding value, and take instead:

$$Z_1 = 224,$$

and

$$DL = \frac{1.268 \times 10^4}{224^2} = 253.$$

This will give:

$$D = 20 \text{ centimetres } (7.87 \text{ in.}),$$

and

$$L = \frac{2{,}520}{20} = 12.65 \text{ centimetres } (4.98 \text{ in.}).$$

There will therefore be 14 wires per slot, so that the core disks of the stator would have to be provided with an arrangement of slots such as represented in Fig. 69.

The space between the slots or the "pitch" of the teeth, when measured on a pitch circle of 219 mm. diameter, would be:

$$\text{pitch} = \frac{\pi \times 219}{48} = 14.33 \text{ mm. } (5.64 \text{ in.}).$$

Three wires per layer require a space of .	$3 \times 2.6 =$	7.8 mm.	(0.31 in.)
Two insulations require a space of . . .	$2 \times 0.75 =$	1.5 mm.	(0.06 in.)
Allowing a play of		0.3 mm.	(0.01 in.)
The width of the slot should therefore be .		9.6 mm.	(0.38 in.)
There will remain for the width of the teeth		4.72 mm.	(0.19 in.)

By arranging the 14 conductors in five layers, the total depth of the winding will be:

The five layers will require	$5 \times 2.6 =$ 13.0	mm.	(0.51 in.)
Allowance for two layers of insulation . .	$2 \times 0.75 =$ 1.5	mm.	(0.06 in.)
Allowance for additional space	1.0	mm.	(0.04 in.)
The total depth of the slot will be . . .	15.5	mm.	(0.61 in.)

The slit openings between the teeth will have a width of 3 mm. to afford passage for the wires, and to facilitate the winding.

A glance at Fig. 69 shows how difficult it would be to further reduce the diameter D, while increasing Z_1. The width of the teeth is already very small. By reducing it still further, the magnetic saturation

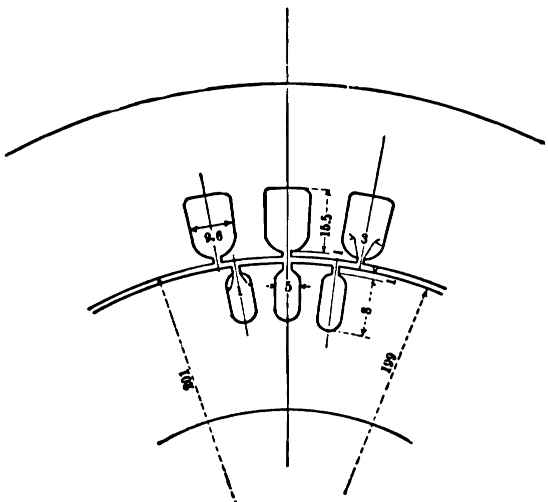

Fig. 69.

would increase all the more rapidly, since the magnetic density, b, increases in proportion with Z_1. This increase in magnetic saturation would cause excessive heating by hysteresis, and would, in any case, have the same effect as an increase in the distance across the air-gap, which latter also would influence the value of the power factor, cos ϕ. We therefore fix upon the following values:

$Z_1 = 224$
$D = 20$ cm. (7.87 in.)
$D_1 = 20.1$ cm. (7.91 in.)
$D_2 = 19.9$ cm. (7.83 in.)
and $L_2 = 12.7$ cm. (0.5 in.)

We then have:

$$b = 10.6 Z_1 = 2375 \text{ gausses},$$

and the resultant effective M.M.F. in the air-gap will be:

$$m = 2375 \times 0.12 = 285 \text{ C.G.S. units},$$

while the primary M.M.F will be:

$$m_1 = \frac{89.0}{42.5} \times 285 = 597.$$

Verifying by reference to the actual data of the motor, we have:

$$m_1 = 0.381 \frac{3 Z_1}{p} i_1 = 0.381 \tfrac{3}{4}\, 224 \times 9.33.$$

$$m_1 = 595.$$

The concordance is therefore very satisfactory.

If we adopt, for the rotor, a squirrel-cage winding consisting of bars of oval cross-section, having a radial depth of 0.7 cm., and a thickness of 0.4 cm., insulated by a layer of paper of 0.05 cm. thickness, the openings in the rotor disks should have dimensions of 0.8 cm. (depth) by 0.5 cm. (width) as shown in Fig. 69.

Since the diameter of the circumference passing through the middle of the slots is:

$$D_{2d} = 18.9 \text{ cm. } (7.44 \text{ in.}),$$

it will be possible to provide for 75 bars in the squirrel-cage winding. The "pitch" of the teeth being

$$\frac{\pi \times 189}{75} = 0.792 \text{ cm. } (0.31 \text{ in.}),$$

the width of the teeth will be 0.292 cm. (0.12 in.).

We have seen that the effective magnetic density, b, in the air-gap was

$$b = 2375 \text{ gausses};$$

it follows, therefore, that the maximum density in the teeth of the rotor will be

$$B_d = 2375 \sqrt{2} \frac{0.792}{0.292} = 9{,}220 \text{ gausses.}$$

This degree of saturation is not excessive; and we can therefore adopt, without fear, the dimensions just indicated for the slots and the teeth in the rotor.

Since the rotor must produce a total mechanical power equivalent to 2312 watts, the magnetic pull exerted between the magnetic field and the bars of the squirrel-cage winding, measured in the middle of the air-gap, will be given by the following formula:

$$F = \frac{2312}{9.81} \times \frac{60 \times 100}{1440 \times \pi \times 20.0} = 15.65 \text{ kilogrammes};$$

but we know that this pull, F, can also be expressed as follows:

$$F = K_1 \frac{LC_2 i_2 S_2}{9.81 \times 10^6} \text{ kilogrammes.}$$

In the particular case under consideration, the constants have the following values:

$$S_2 = 75; \; K_1 = 1.0; \; L = 12.7 \text{ cm. (5.0 in.)}.$$

The diagram gives

$$b_2 = b \frac{F_2}{\phi} = 2375 \frac{42.5}{42.6},$$

$$b_2 = 2365 \text{ gausses.}$$

From this we may calculate the rotor current:

$$i_2 = \frac{15.65 \times 9.81 \times 10^6}{2365 \times 75 \times 12.7} = 68.1 \text{ amperes};$$

and the E.M.F., e_2, will be:

$$e_2 = b_2 L V_2 \times 10^{-8} \text{ volts,}$$

$$e_2 = 2365 \times 12.7 \times \frac{\pi \times 20.0}{60} \times 60 \times 10^{-8},$$

$$e_2 = 0.01888 \text{ volt.}$$

We should, therefore, have, for the resistance of each bar and of the corresponding portion of the two short-circuiting rings, the following value:

$$R_2 = \frac{e_2}{i_2} = \frac{0.01888}{68.1} = 0.000277 \text{ ohm.}$$

We can also determine at once by means of the diagram,

$$m_2 = m_1 \frac{75.9}{89.0} = 595 \times \frac{75.9}{89.0} = 507,$$

and, verifying by calculation, we have

$$m_2 = K \frac{S_2}{p} i_2 = 0.4 \frac{75}{4} 68.1,$$

$$m_2 = 510,$$

which shows a very close agreement.

The cross-section of each bar is

$$s_2 = 24.56 \text{ sq. mm.,}$$

and its mean length is

$$l_m = 16.5 \text{ cm.}$$

The resistance of each bar will be,

$$R_2' = \frac{16.5 \times 1.8 \times 10^{-6}}{0.2456} \, 0.000121 \text{ ohm.}$$

Deducting this from the value found for the total resistance, we find that the resistance of the two short-circuiting rings must be 0.000156 ohm; and for one ring it must be

$$R_2'' = 0.000078 \text{ ohm.}$$

The mean diameter of this ring may be taken equal to 18.9 cm.

The relation between the resistance and the material, form, and dimensions, of each short-circuiting ring is given by the following equation:

$$0.0000780 = \frac{S_2 Da}{p^2 \pi \, s_2''},$$

from which we have, for the cross-section of the ring,

$$s_2'' = \frac{S_2 Da}{0.0000780 \, p^2 \, \pi}.$$

Assuming the ring to be of copper whose specific resistance, a, is

$$a = 1.8 \times 10^{-6} \text{ ohms-centimetres,}$$

and substituting, in the preceding equation, we have

$$s_2'' = \frac{75 \times 18.9 \times 1.8 \times 10^{-6}}{16 \times \pi \times 0.0000780} = 0.650 \text{ sq. cm.}$$

Consequently, the ring should have a sectional area of 65 mm.^{2}, which would correspond to a width of 26 mm. and a thickness of 2.5 mm. The ohmic loss in each of the short-circuiting rings being

$$P_2'' = \frac{S_2 Da}{\pi p^2 s_2''} \, i_2^2 = \frac{75^2 \times 18.9 \times 1.8 \times 68.1^2}{\pi \times 16 \times 0.65} \times 10^{-6},$$

$$P_2'' = 27.1 \text{ watts,}$$

and the surface of each ring being

$$\pi \times 18.9 \times 5.9 = 350 \text{ sq. centimetres,}$$

we will have a cooling surface equivalent to

$$= \frac{350}{27.1} = 12.9 \text{ cm.}^2 \text{ per watt,}$$

which is at least four times what would suffice to obviate excessive heating.

Since the resistance of the short-circuiting rings constitutes the greater portion of the total resistance, R_2, it would seem possible, at first glance, to further increase the number of conductors Z_1, in each winding, in order to still further reduce the size of the motor. It should be remembered, however, that a point would soon be reached where it would be no longer possible to place the winding copper properly in the core slots. Moreover, we have seen that the magnetic density, b, in the air-gap, increases with Z_1, so that in proportion as we would diminish the axial width, L, of the cores, their radial depth beyond the slots, and also the width of the teeth, would have to be increased, in order to avoid undue magnetic saturation. It follows that a decrease in the axial length, L, of the core, would not bring about a material reduction in the weight of the machine.

We also know that the magnetic leakage increases with the number of primary conductors Z_1, and with the amount of the secondary current i_2. Now, this magnetic leakage plays a rôle of the greatest importance in determining the value of the starting torque. It is therefore evident that in seeking to reduce the dimensions D and L of the air-gap by an increase in the number of primary winding turns, we will not materially reduce the price of the machine, and, at the same time, we run the risk of no longer being able to meet the requirement of a starting torque equal to twice the running torque, and a power factor at full load equal to

$$\cos \phi = 0.83.$$

These considerations show that the 3 H.P. motor which is the subject of this study will, in order to meet the prescribed requirements, have to be constructed according to the following specification:

Internal diameter of primary disks	201	mm. (7.91 in.)
External diameter of secondary disks	199	" (7.83 ")
(Axial) length of cores	127	" (5.00 ")
Number of slots in primary disks	48	
Number of slots in secondary disks	75	
Number of primary wires in active side of each coil	56	
Number of primary wires per slot	14	
Number of bars in rotor winding	75	
Sectional area of bars in rotor winding	24.56	mm.^{2} (0.077 in.^{2})
Sectional area of copper short-circuiting rings . .	67	mm.^{2} (0.210 ")
Diameter of copper short-circuiting rings	189	mm. (7.44 in.)

By means of these dimensions, we may ascertain that the mean length of each turn of the stator winding is:

$$l_m = 70 \text{ centimetres.}$$

As the resistance per phase is fixed at

$$R_1 = 0.384 \text{ ohm,}$$

it follows that the sectional area of the wire must be:

$$s_1 = \frac{70 \times 112 \times 1.8 \times 10^{-6}}{0.384} = 0.0368 \text{ cm.}^2$$

We will take a wire having a diameter of $\frac{22}{10}$ of a millimetre, bare, and $\frac{26}{10}$ of a millimetre with double cotton covering.

The slots are to be partially closed, with an opening of 3 millimetres in width between the teeth to leave passage for the wires in the winding.

We have seen that the pitch of the slots in the primary core was 14.32 mm., and that the teeth had a width of 4.72 mm. The maximum magnetic density in the teeth will, therefore, be:

$$B_d = b\sqrt{2}\,\frac{14.32}{4.72} = 2375\sqrt{2} \times \frac{14.32}{4.72},$$

$$B_d < 10400 \text{ gausses.}$$

If we limit the magnetic density to a maximum of 5000 gausses in the solid portion of the primary and secondary cores, their sectional area must be:

$$S_a = \frac{b \times 0.9\,\pi DL}{2 \times 5000 \times p} = \frac{2374 \times 0.9 \times \pi \times 20 \times 12.9}{10000 \times 4},$$

$$S_a = 41.2 \text{ cm.}^2$$

The radial depth of the cores, beyond the slots, will be:

$$S_c = \frac{41.6}{12.7 \times 0.93} = 3.52 \text{ cm.}$$

Allowing a loss of space equivalent to 7% due to the buckling of the core disks, the weight of the primary core will be:

$$P_1' = \pi \times 27.08 \times 0.93 \times 12.4 \times 3.52 \times 7.8 = 28 \text{ kilogrammes,}$$

and the weight of the teeth will be:

$$P_1'' = 48 \times 1.66 \times 0.472 \times 12.4 \times 0.93 \times 7.8 = 3.5 \text{ kilogrammes.}$$

The loss by hysteresis and Foucault currents in the primary disks will be:

Core	$0.28 \times 130 =$	37 watts
Teeth	$0.035 \times 430 =$	15 "
Total		52 watts.

It is to be expected that this figure will be increased at least 50% by the Foucault currents produced in consequence of the burrs due to punching or filing of the disks, and also the Foucault currents produced in the metal portions which happen to be near the primary coils.

If we assume, at the start, an amount equal to 3.24%, that is to say, 84 watts, for these losses, we will doubtless approach very closely to the results that are likely to be attained in normal operation.

We have not taken into consideration the losses in the core of the rotor because they are negligible owing to the low frequency of the magnetic flux in that portion of the motor.

The additional magnetic field density produced in the teeth and in the primary core by magnetic leakage has also been neglected, because it is only the least important portion of the magnetic leakage flux which closes its magnetic circuit through the disks of the stator core.

It remains to be shown that the ratio between the total surface of the air-gap and the surface actually utilized for the passage of lines of force is, in reality, equal to 1.20, as was assumed.

The whole circumference in the air-gap is:

$$\pi \times 20.0 = 62.8 \text{ cm.}$$

Deducting the surface corresponding to the slits between the teeth, the useful portion of the air-gap measured on the stator disks is:

$$\pi \times 20.1 - 48 \times 0.3 = 48.8 \text{ cm.},$$

while the useful portion measured on the periphery of the rotor core is:

$$\pi \times 19.9 - 75 \times 0.1 = 55.0 \text{ cm.},$$

on the assumption that the openings (slits) between the teeth in the rotor core have a width of 1 millimetre.

The useful portion of the air-gap, therefore, has a mean circumferential length of:

$$\frac{48.8 + 55.0}{2} = 51.9 \text{ cm.},$$

so that the ratio in question is:

$$\frac{62.8}{51.9} = 1.21.$$

This value is so close to 1.20 that it is unnecessary to introduce corrections in the preceding calculations.

The characteristic features of the motor having been all determined, we now proceed to show the great influence of the distance across the air-gap on the value of the power factor which may be attained with a motor of given size.

Let us suppose that the air-gap in the motor under consideration were reduced from 0.1 cm. to 0.05 cm., and that the openings between the teeth of the stator and rotor were reduced to 0.1 cm. The ratio between the total surface of the air-gap and that which is utilized by the lines of force would then be:

$$\frac{62.8}{\left(\frac{58.4 + 55.0}{2}\right)} = 1.108;$$

so that we would have to introduce in the formulæ for the distance across the air-gap:

$$d = 1.108 \times 0.05 = 0.0554 \text{ cm.}$$

Assuming the output, efficiency, power factor, leakage and losses, to be the same as previously assumed, the primary M.M.F. will be:

$$m_1 = 0.381 \times \tfrac{3}{4}\, Z_1 \times 9.33.$$

The diagram gives, as before,

$$m_2 = m_1 \frac{75.9}{89.0},$$

and

$$m = m_1 \frac{42.5}{89.0};$$

from which we deduce:

$$b = \frac{m}{d} = \frac{42.5}{89.0} \times 0.381 \times \tfrac{3}{4} \times \frac{Z_1}{0.554} \times 9.33.$$

$$b = 23.0 \times Z_1 \text{ gausses.}$$

Again making the same assumptions, we will also have:

$$Z_1^2\, DL = 585 \times 10^4.$$

If we were to take as before:

$$D = 20.0 \text{ cm.,}$$

and

$$L = 12.7 \text{ cm.,}$$

we would have:

$$Z_1 = \sqrt{\frac{585 \times 10^4}{20 \times 12.7}} = 151.8 \text{ conductors per phase.}$$

But since we have 16 slots in which to place these Z_1 conductors, we will take

$$Z_1 = 160,$$

and we will have 10 wires per slot. It will be necessary to change the width, L, of the cores, for which we will need:

$$L = \frac{585 \times 10^4}{20 \times 160^2} = 11.42 \text{ cm.}$$

The magnetic density in the air-gap would then be:

$$b = 23 \times Z_1 = 3680 \text{ gausses.}$$

This magnetic density is rather high, but it will be possible to increase the width of the teeth sufficiently to prevent the degree of magnetic saturation from materially exceeding the limits which were assumed in the first calculation that we made. This may be done by diminishing the thickness, or even the number, of the bars of the squirrel-cage winding on the rotor.

Since the number of turns has been decreased from 224 to 160, the induced E.M.F. remaining constant at 106.8 volts and the frequency remaining 50 periods per second, as before, it follows that the magnetic flux issuing from each pole and penetrating the stator core will increase in inverse proportion with the decrease in the number of conductors Z_1.

We may, therefore, conclude that if it is not desired to exceed a maximum magnetic density of 5000 gausses in the iron, as before, the depth of the cores will be much greater than before, and the weight of the disks will consequently be increased.

It therefore seems, at first glance, that the decrease in the radial distance across the air-gap is rather objectionable, since it increases the weight of the motor, as well as the difficulties of its mechanical construction.

It must not be forgotten, however, that owing to the decrease in the number Z_1 of conductors, the magnetic leakage will be materially diminished, and that the starting torque will certainly exceed greatly the value assumed at the outset.

It is useless to give to the starting torque a value higher than is required. We could, therefore, increase the number Z_1 of the conductors, even at the expense of increased magnetic leakage; but we should not expect to reduce the size of the motor, since for:

$$D = 20.0 \quad \text{and} \quad L = 12.7,$$

the magnetic density in the air-gap of

$$b = 3680 \text{ gausses,}$$

will already cause a degree of saturation which should not be exceeded in the teeth of the cores. A further diminution of the axial width of the core, L, would not only involve an increase in Z, but also an increase in the magnetic density, b.

Under the circumstances, it is best to retain the dimensions previously adopted for D and L, and to improve the power factor cos ϕ by an increase in the number of conductors Z_1. The increase in Z_1 will be limited by the requirement previously mentioned, of a starting torque equal to twice the running torque.

If we retain the values of D and L, and increase Z_1, the magnetic flux through the stator winding turns, and, consequently, the magnetic density in the air-gap, will be diminished.

Taking as before:

$$Z_1 = 224,$$

the result would be a motor of the same weight as that of the motor already worked out on the assumption of an air-gap of 0.1 cm. The efficiency and the starting torque would remain substantially the same, but the power factor would be materially improved.

It should not be forgotten that if, by reducing the distance across the air-gap, we diminish, for example, by one-half, the reluctance of the magnetic circuit, the fictitious magnetic fluxes, Φ_1 and Φ_2, will be nearly doubled. They will not be quite doubled, since, owing to the improvement in the power factor, cos ϕ, the primary current will be a little lower than it was before.

The resultant flux Φ, which is common to the two cores, remains nearly constant, since it alone, so to speak, induces the counter E.M.F., e_1, whose value always differs but very little from the potential difference at the motor terminals.

The primary and secondary magnetomotive forces m_1 and m_2 will vary but slightly. Their resultant, m, must be reduced about one-half, since the magnetic flux Φ remains nearly constant, and since the reluctance of the magnetic circuits is, by hypothesis, twice as small as it was before.

The magnetic leakage fluxes,

$$\Phi_1 (V_1 - 1) \quad \text{and} \quad \phi_2 (V_2 - 1),$$

will depart but slightly from their preceding values; first, because the number of ampere-turns of the rotor and stator coils has changed but little; and second, because the permeability of the different paths through which the magnetic leakage fluxes may pass is not affected by the diminution of the distance across the air-gap.

We may conclude from what precedes, that this decrease in the reluctance of the air-gap has an important influence on the primitive values of the factors V_1 and V_2, and, consequently, on the value of the magnetic leakage factor,

$$\sigma = 1 - \frac{1}{V_1 V_2}.$$

In order to make the matter more clear, we will again take up the calculation of the motor under consideration. We had brought back the distance across the air-gap to:

$$d = 0.0554 \text{ cm.},$$

and we had decided to retain the dimensions:

$$D = 20.0 \text{ cm.} \qquad \text{and} \qquad L = 12.7 \text{ cm.}$$

Let us now see if it will be possible to attain for the power factor, the value:

$$\cos \phi = 0.9,$$

while retaining, for the efficiency and the starting torque, the values previously fixed.

Let us assume a stator winding having

$$Z_1 = 256,$$

which corresponds to 16 conductors per slot.

We have, first:

$$i_1 = \frac{736 \times 3}{3 \times 110 \times 0.85 \times 0.9} = 8.75 \text{ amperes.}$$

$$\therefore \ R_1 i_1 = 8.75 \times 0.384 = 3.36 \text{ volts};$$

and we also have:

$$i_{1w} = \frac{84}{3 \times 107} = 0.262 \text{ amperes.}$$

The diagram gives:

$$e_1 = 107 \text{ volts,}$$

and

$$i_{1w} = 8.50 \text{ amperes.}$$

The primary leakage flux on which our first calculation was based was found equal to:

$$(V_2 - 1)\,\Phi_1 = 0.045\,\Phi_1.$$

The numbers of conductors per slot having been raised from 14 to 16, and the magnetizing current passing through these conductors having been reduced from 9.33 to 8.50 amperes, the magnetic leakage flux now becomes:

$$0.045 \times \frac{16}{14} \times \frac{8.5}{9.33}\,\Phi_1 = 0.047\,\Phi_1.$$

Between this fictitious flux Φ_1 and the flux Φ_1' which would result from the decrease in the distance across the air-gap, the following ratio exists:

$$\frac{\Phi_1}{\Phi_1'} = \frac{m_1}{m_1'} \times \frac{d'}{d},$$

$$\frac{\Phi_1}{\Phi_1'} = \frac{9.33 \times 224}{8.5 \times 256} \times \frac{0.0554}{0.12},$$

$$\Phi_1 = 0.443\ \Phi_1'.$$

The new leakage factor for the primary or stator winding will, therefore, be:

$$(V_1' - 1)\ \Phi_1' = 0.047\ \Phi_1 = 0.045 \times 0.443\ \Phi_1';$$

whence,

$$V_1' = 1.021.$$

The diameter, D, and the (axial) width, L, of the air-gap having retained the same dimensions as before, the magnetic density in the air-gap will be decreased in the ratio of $\frac{256}{224}$ in consequence of the increase in the number of turns in the stator winding.

Since the number of bars in the rotor winding and the speed of the rotor remain the same, and since, consequently, the magnetic pull exerted between the rotor and the revolving field remains the same, the secondary currents, i_2, should increase in the proportion of 256 to 224.

The ratio between the fictitious secondary magnetic flux Φ_2, obtained from the first calculation, and the new ratio Φ_2, obtained after the reluctance of the air-gap has been decreased, will, therefore, be:

$$\frac{\Phi_2}{\Phi_2'} = \frac{m_2}{m_2'} \times \frac{d'}{d},$$

$$\frac{\Phi_2}{\Phi_2'} = \frac{224}{256} \times \frac{0.0554}{1.2}.$$

But the magnetic leakage flux in the secondary has increased in the same proportion as the current i_2, and it will therefore be:

$$0.033 \times \tfrac{256}{224}\ \Phi_2 = 0.0377\ \Phi_2.$$

The leakage coefficient for the secondary will be:

$$V_2' = 1 + \frac{224}{256} \times \frac{0.0554}{1.2} \times 0.0377,$$

or

$$V_2' = 1.0152.$$

From the preceding, we deduce:

$$\sigma = 1 - \frac{1}{V_1 V_2} = 1 - \frac{1}{1.0208 \times 1.0152},$$

$$\sigma = 0.037.$$

So that, in the diagram, we will have the ratio

$$\frac{MP}{OM} = \frac{1 - \sigma}{\sigma} = \frac{0.963}{0.037} = 26.0.$$

If we draw the well known semi-circle on *MP*, it is easy to ascertain that, after having introduced the necessary correction to allow for the ohmic drop $R_1 i_1$, the starting torque obtained will still be more than twice as high as that corresponding to the running torque. We could, therefore, still further increase the number of conductors Z_1 and reduce slightly the axial width, *L*, of the cores.

It is well to observe, once more, that the decrease in the reluctance of the air-gap, while improving the power factor, substantially reduces the current absorbed by the motor, when running at full normal load, but that it increases, on the other hand, to a considerable extent, the primary and secondary starting currents.

Third Example. We will now show how a designer having relatively little familiarity with the subject of induction-motor design, could quickly determine the principal dimensions of a two-phase motor capable of developing, for example, 100 horse-power, when supplied with two-phase current having a frequency of 50 periods per second, and a potential difference of 3000 volts between phases, at the motor terminals, and when running at a speed of 485 R.P.M.

In induction motors, the effective magnetic density, b_2, of the resultant magnetic field in the rotor, ranges between 2000 and 3500 gausses, — the lower value being that which corresponds to smaller motors, and the higher value that which is usually found in the larger motors.

When the fictitious magnetic density of the secondary winding is about five times greater than b_2, the power factor generally approaches very closely to 0.9; and it is difficult to exceed this value, even with very large motors.

It is assumed that the rotor is to be designed so as to permit the use of starting resistances. In order to restrict the dimensions of the collector-rings, while yet making sufficient allowance for collecting secondary or rotor currents and conducting them to the starting resistance, it is advisable that these secondary or rotor currents should not exceed, but should, if possible, come under, 200 amperes.

If, now, we set the peripheral speed of the rotor at about 26 metres per second, we can very quickly determine all the dimensions of the motor required.

Let us take, then, for the peripheral speed of the rotor,

$$V = 2600 \text{ centimetres per second.}$$

The outer diameter of the rotor core (for a speed of 485 R.P.M.) will therefore be:

$$D_2 = \frac{2600 \times 60}{\pi \times 485} = 102.2 \text{ centimetres.}$$

Let us, therefore, assume,

$$D_2 = 102.7 \text{ centimetres,}$$

so that the bore diameter of the stator core disks will be,

$$D_1 = 103.0 \text{ centimetres,}$$

if we count upon an air-gap of 1.5 mm., which it does not seem practicable to further reduce. As the output is set at 100 H.P., we have, for the electromagnetic effort or pull required at the middle of the air-gap,

$$C = \frac{100 \times 75 \times 60}{1 \times 1.0285 \times 485} = 287.5 \text{ kilogrammes.}$$

If we add to this figure	5.5 "
That is to say, about 2%, for bearing friction and windage, the total mechanical effort which must be exerted between the magnetic field and the rotor will be	293.0 kilogrammes.

Let us now take, for the resultant effective magnetic density in the rotor,

$$b_2 = 3000 \text{ gausses,}$$

and for the rotor reaction,

$$B_2 = 15000 \text{ gausses.}$$

Now, we know that the fictitious magnetic density, B_2, which the secondary windings would produce, is given by the formula:

$$B_2 = K \times \frac{S_2}{p} \times i_2 \times \frac{1}{d} \text{ gausses.}$$

If we assign to the rotor current, i_2, the value,

$$i_2 = 208 \text{ amperes,}$$

and if we adopt a wave winding, the coefficient will be (p. 90):

$$K = 0.359.$$

From this, we have, for the total number of secondary conductors:

$$S_2 = \frac{15000 \times 0.175 \times 12}{0.359 \times 208} = 422.$$

We have introduced into this formula,

$$d = 0.175 \text{ centimetres,}$$

instead of

$$d = 0.15 \text{ centimetres,}$$

in order to allow for the decrease in the working surface of the air-gap, owing to the slits between the teeth of the core, and also to allow for the reluctance of the iron of the cores.

We know that in a given wave winding, we must always have,

$$y = \frac{S_2 \pm 2}{p},$$

in which the "pitch," y, should be an odd number.

The number of poles in this case being,

$$p = 12,$$

we will have,

$$S_2 = 422,$$

which happens to be a convenient number. We have, in fact,

$$y = \frac{422 - 2}{12} = \frac{420}{12} = 35.$$

We now proceed to predetermine the axial width, L, of the cores, by reference to the formula for the electromagnetic effort developed:

$$293 \times 9.81 \times 10^6 = K_1 \times b_2 \times L \times i_2 \times S_2.$$

In this particular instance we have,

$$K_1 = 0.9; \qquad b_2 = 3000 \text{ gausses;}$$

$$S_2 = 422; \qquad i_2 = 208 \text{ amperes.}$$

The axial width, L, will therefore be,

$$L = \frac{293 \times 9.81 \times 10^6}{0.9 \times 3000 \times 422 \times 208}.$$

$$L = 12.1 \text{ centimetres.}$$

The E.M.F., e_2, induced by the magnetic field will be:

$$e_2 = K_1 \times b_2 \times L \times \frac{422}{2} \times V_2 \times 10^{-8} \text{ volts,}$$

but

$$K_1 = 0.9,$$

$$b_2 = 3000,$$

$$L = 12.1 \text{ centimetres};$$

$$V_2 = \pi \times 102.85 \times \frac{15}{60} = 80.7 \text{ centimetres};$$

therefore,

$$e_2 = 0.9 \times 3000 \times 12.1 \times \frac{422}{2} \times 80.7 \times 10^{-8}.$$

$$e_2 = 5.56 \text{ volts.}$$

The resistance, R_2, of each of the secondary windings will be

$$R_2 = \frac{e_2}{i_2} = \frac{5.56}{208} = 0.0267 \text{ ohm.}$$

The "pitch" of the slots on the periphery of the rotor core being

$$\frac{\pi \times 102.7}{422} = 0.766 \text{ centimetres,}$$

we can give to the slots a width of 4.5 mm., and to the winding bars a rectangular cross-section of

$$3.5 \times 18 \text{ mm.} = 63 \text{ mm}^2.$$

These bars are to be insulated with paper one-half millimetre thick.

It is only necessary to make a rough sketch of the rotor to observe that the length of each winding bar will have to be about 19 cm., assuming that the connecting forks which are to be utilized for making the cross-connections are to have a width of 2 cm.

The resistance of 211 bars forming one of the branches of the winding will be

$$R_2' = \frac{211 \times 19 \times 2 \times 10^{-6}}{0.63} = 0.0127 \text{ ohm.}$$

There remains, therefore, for the end connectors,

$$R''_2 = 0.014 \text{ ohm.}$$

The mean length of the circuit in these connecting pieces will be 30 cm.; their cross-section will therefore be:

$$S_2'' = \frac{211 \times 30 \times 1.8 \times 10^{-6}}{0.014},$$

$$S''_2 = 0.818 \text{ cm.}^2$$

This corresponds to a strip 22 mm. wide and 4 mm. thick.

It being only necessary to multiply the values of the magnetic density by a constant in order to obtain the corresponding values of magnetic fluxes, we can draw along OA, according to any convenient scale (Fig. 70),

$$b_2 = 3{,}000 \text{ gausses};$$

and we may represent the armature reaction,

$$B_2 = 15{,}000 \text{ gausses},$$

by a line drawn along OB, perpendicular to OA, according to the same scale.

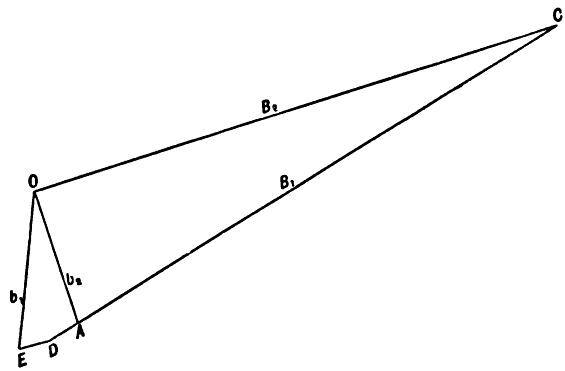

Fig. 70.

If, now, we assume that the magnetic leakage in the rotor represents 4% of the total magnetic flux OC in the rotor, we may draw

$$BC = 0.04 \times OC;$$

and likewise, taking for the magnetic leakage in the stator,

$$V_1 = 0.04,$$

the fictitious magnetic density generated in the air-gap by the primary (stator) core will be represented by the right line AC, prolonged to the point D, the distance AD being determined by the following equation:

$$AD = 0.04 \times CD.$$

By drawing ED equal and parallel to BC, the vector OE will represent the resultant effective magnetic density in the primary core.

Under these conditions Fig. 70 gives:

$$b_1 = OE = 3{,}350 \text{ gausses},$$

$$B_1 = AC = 15{,}900 \text{ gausses}.$$

But this effective magnetic density b_1 must produce, in the primary windings, an E.M.F., e_1, whose value differs but slightly from the potential difference at the motor terminals, with which it combines to produce a resultant which is utilized wholly in producing the current i_1 in the resistance R_1.

Since the difference of potentials at the terminals is 3,000 volts, we may take, without fear of making appreciable error,

$$e_1 = 0.98 \times 3{,}000 = 2{,}940 \text{ volts.}$$

We know that the E.M.F., e_1, corresponds to the formula:

$$e_1 = K_1 \times b_1 \times L \times V_1 \times Z_1 \times 10^{-8} \text{ volts,}$$

or, what amounts to the same thing, to the equation:

$$e_1 = K_1 \times \frac{Z_1}{2} \times \omega_1 \times \Phi\, 10^{-8} \text{ volts;}$$

but with a frequency of 50 periods per second, we have:

$$\omega = \frac{2\pi}{T} = 314;$$

on the other hand, the effective value of the magnetic flux issuing from each pole is:

$$\Phi_1 = 0.9 \times b_1 \times \frac{\pi \times D}{p} \times L \times \frac{1}{\sqrt{2}},$$

$$\Phi_1 = 0.9 \times 3350 \times \frac{\pi \times 102.85}{12} \times 12.1 \times \frac{1}{\sqrt{2}},$$

$$\Phi_1 = 692{,}000 \text{ maxwells.}$$

Let us adopt a "definite" or polar winding consisting of twelve coils which are distributed in twelve slots. The active side of each coil will comprise six slots; and we will consequently have, from the tables given in Chapter VI. (p. 88):

$$K_1 = 0.903.$$

The number of primary winding turns per phase will, therefore, be:

$$\frac{Z}{2} = \frac{2940 \times 10^8}{0.903 \times 314 \times 692000} = 1500.$$

We could arrange this winding thus:

42 turns per slot,

making 252 turns per coil,

and 1,512 turns per phase.

The diagram (Fig. 70) has given us, for the fictitious primary magnetic density, the value:

$$B_1 = 15{,}950 \text{ gausses.}$$

Now, we know that

$$B_1 = K \times \frac{S_1}{p} \times i_{1\mu} \times \frac{1}{d};$$

but for a "definite" winding we have

$$K = 0.36,$$

and, in the particular case, we have

$$S_1 = 6{,}048\ ;$$

so that

$$i_{1\mu} = \frac{15900 \times 12 \times 0.175}{0.36 \times 6048} = 15.4 \text{ amperes.}$$

In a motor of this kind the watted current, which supplies energy for the losses in the iron, is so small that we may assume, without material error, that the magnetizing current, $i_{1\mu}$, is equal to the total current, i_1.

We will, therefore, take:

$$i_1 = 15.4 \text{ amperes.}$$

By making a preliminary sketch, we will ascertain that the mean length of the winding turns of the stator winding is:

$$l_m = 102 \text{ centimetres.}$$

We will take wire having a diameter of $\frac{26}{10}$ of a millimetre, bare (about No. 10 B. & S. G.), or a cross-section:

$$S_1 = 5.31 \text{ square millimetres.}$$

The resistance of each winding will consequently be:

$$R_1 = \frac{102 \times 1512 \times 2 \times 10^{-8}}{0.053} = 5.82 \text{ ohms};$$

and the loss of potential ("ohmic drop") corresponding thereto will be:

$$R_1 i_1 = 5.82 \times 15.4 = 89.5 \text{ volts.}$$

The maximum magnetic density in the stator core may be taken equal to:

$$5{,}000 \text{ gausses.}$$

The sectional area of the stator core will be equal to:

$$S_a = \frac{692{,}000 \times \sqrt{2}}{2 \times 5000} = 98 \text{ cm.}^2$$

We conclude from this that the thickness of the iron, outside of the slots, ought to be:

$$\frac{98}{0.93 \times 12.1} = 8.7 \text{ cm.}$$

Since the stator winding is to be placed in 144 slots, the "pitch" of these slots, measured on a mean circle having a diameter of 107 cm., will be:

$$\frac{\pi \times 107.0}{144} = 2.34 \text{ cm.}$$

For a potential difference of 3,000 volts, it will be sufficient to give a thickness of 2 mm. to the micanite tubes in which the wires are to be placed. By placing in each slot 14 layers of 3 conductors, each tube will have, internally, a width of 9.5 mm., and a height of 44 mm.

The width of the slots will therefore be 14 mm., and their height will be 46.5 mm.

The maximum magnetic density will be:

$$B_{max} = 3000 \times \sqrt{2} \times \frac{0.766}{0.316} = 10300 \text{ gausses,}$$

in the teeth of the rotor; and,

$$B_{max} = 3000 \times \sqrt{2} \times \frac{23.4}{9.4} = 10600 \text{ gausses,}$$

in the teeth of the stator.

Let us now calculate the losses. These losses will be as follows:

In the copper of the rotor winding:

$$2 \times 0.0267 \times 208^2 = 2{,}310 \text{ watts.}$$

In the copper of the primary or stator winding:

$$2 \times 5.82 \times (15.4)^2 = 2{,}760 \text{ watts.}$$

For the stator core we will have:

Weight of core,

$$\pi \times 124.2 \times 12.1 \times 0.93 \times 7.8 \times (10)^3 = 342 \text{ kilogrammes.}$$

Weight of the teeth,

$$144 \times 4.65 \times 0.94 \times 12.1 \times 0.93 \times 7.8 \times 10^3 = 55 \text{ kilogrammes.}$$

Loss by hysteresis and eddy currents, per 100 kilogrammes:

For a magnetic density of

$B_{max} = 5000$, the loss will be . . . . 125 watts

For $B_{max} = 10600$, the loss will be . . . . . 500 "

Loss in the core,

$3.42 \times 125 =$ . . . . 428 watts

Loss in the teeth,

$0.55 \times 500 =$ . . . . 275 "

Loss in bearings and by air friction (windage),

$\pi \times 1.0285 \times \frac{485}{60} \times 5.5 \times 9.81 =$	1,418	"
Total Losses	7,191	"
Useful Power	73,600	"
Power Input	80,791	"

Efficiency, $\eta = \frac{73600}{80791} = 91\%$.

Total watted current,

$$i_{1w} = \frac{80791}{2 \times 3000} = 13.5 \text{ amperes.}$$

Power factor,

$$\cos \phi = \frac{13.5}{15.4} = 0.88.$$

CHAPTER IX.

THE HEYLAND INDUCTION MOTOR.

MUCH has been said, of late, concerning a new arrangement devised by Mr. A. Heyland, which makes it possible to design induction motors having a power factor equal to unity.

The articles published in various scientific periodicals by the inventor himself confine themselves to a summary description of the construction of the new machine, without attempting to discuss the theoretical considerations involved, and without seeking to explain the operation of the machine by means of diagrams.

The author has thought it expedient and opportune to complete the foregoing study of the induction motor by discussing the nature of this invention, and by showing what modifications it introduces in the operation of the ordinary induction motor, and also by giving a method of calculation for a machine of that type designed to meet certain given requirements.

In order to avoid complicating matters too much at the outset, we will suppose that we are dealing with an ordinary induction motor having a stator winding of negligible ohmic resistance.

We will also assume, and for the same reason, that there is no magnetic leakage, and that the diagram of operation is represented by Fig. 71, wherein, for a motor in normal operation, *OA* represents the

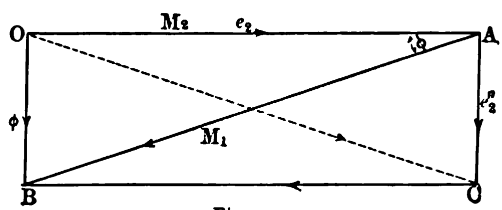

Fig. 71.

magnetizing effect M_2 of the rotor winding, while *AB* represents the magnetizing effect M_1 of the stator winding (the relative directions being indicated by the arrow heads), and, finally, *OB* represents the resultant of these two effects. *OB* will consequently indicate, but according to a different scale, the amount and direction of the resultant single magnetic flux ϕ, passing through the cores of both the rotor and the stator.

Since the primary current i_1 is in phase with AB (on the assumption that there are no losses in the iron), and since the E.M.F., e_1, induced in the primary winding by the resultant magnetic field, may be represented by a distance on the line OA, then (the resistance R_1 being assumed to be negligible) the angle of lag will be represented by the angle OAB.

If we now wish to neutralize this angle of lag, it is indispensable that the straight line AB should become parallel to OA, or, in other words, that the primary current i_1 should be in phase with the difference of potentials applied at the motor terminals, E_1, whose direction (phase relation) is exactly opposed to that of e_1, the E.M.F. induced by the resultant magnetic field.

But the difference of potentials at the motor terminals being constant, such is also, of necessity, the case with e_1, the induced E.M.F., and, consequently, it is also the case with the magnetic field, Φ, which produces this E.M.F.

We know that this magnetic field (Φ) is produced only by the resultant magnetic effects of the primary and secondary windings. It follows that if the current, i_1, or the primary M.M.F., M_1, undergoes a change in phase relation until it is directed along the line BC, the secondary M.M.F., M_2, will have its inclination changed from OA to OC.

The same result might be obtained if there were sent in the secondary winding, by any means whatsoever, a current, i''_2, whose phase relation is the same as that of the vector AC.

This auxiliary current, i''_2, will combine with the current, i'_2, produced by the resultant magnetic field, Φ, to give, in the rotor, a resultant magnetizing effect, represented in magnitude and in direction by OC.

It would seem, at first glance, as if we should arrive at the same result by producing the magnetizing effect AC in the stator.

Such is not the case, however; for the diagram shows, in fact, that the lag of the current BC would be 0°, while that of the current AC would, on the contrary, be 90°.

The resultant of these two currents would be BA, and the final power factor would be

$$\cos \phi = \cos OAB,$$

as before. Nothing would, therefore, have been gained thereby.

The current BC would, by itself, produce the whole output, while the current AC would produce the resultant magnetic field, the magnetizing effect of the secondary being, all the time, neutralized by that of the primary component BC.

We will see, later on, that by producing, in the rotor, the fictitious magnetic flux AC, the result is no longer the same.

Let us first note that if we want the torque to be the same as it was before changing anything in the machine, the point C must be placed at the intersection of the straight line BC (which is parallel to OA), and of the straight line AC (which must retain the same direction as OB).

The pull producing the torque is, in fact, proportional to the product of the magnetic field by that component of the resultant current of the rotor which is perpendicular thereto. This pull is therefore proportional to the area of the triangles such as OBA and OBC.

We might also reason in the following manner:

Since the lag must pass from ϕ° to 0°, we must have for the same output $BC = AB \cos \phi$. The diagram $OACB$ is therefore a rectangle.

It is evident that the application of the Heyland principle will not modify, in any respect, the importance of the slip.

In order to introduce the current i_2'' into the secondary, the inventor has made use of a rotor arranged like the armature of a direct current dynamo, that is to say, provided with an ordinary wave winding and with a commutator.

By applying to this commutator three sets of brushes placed 120° apart, it is possible to produce, in the armature, the effect of a three-phase winding with delta connection, which will remain fixed in space, notwithstanding the rotation of the conductors. On the end of the rotor opposite to the commutator, the winding is connected, at three points, to three collector rings; hence, if we remove the three sets of brushes, above mentioned, there remains only the armature of an ordinary induction motor.

The same rotor winding is thus made to combine the characteristic features of two distinct windings.

It is evident that in analyzing what takes place, we can study separately the effects of the two windings thus arranged, and then superpose the results.

The "first winding," being the ordinary winding which connects with the three collector rings and which is already well known, will not require extended study.

It is not at all fixed in space, but takes part in the rotation of the rotor. It therefore slips slowly in the magnetic field of the air-gap and produces a reaction, or, to put it differently, a fictitious flux, OA, which moves with the same speed as that of the stator field, and which, naturally, has an equal number of poles.

In thus slipping in the field OB, it gives rise to an E.M.F., e_2', producing, in each of the three branches, a current,

$$i_2' = \frac{e_2}{R_2}.$$

In this equation, R_2 = the ohmic resistance of each winding, and e'_2 is given by the well-known relation,

$$e_2' = 0.826\, ZBLV\, 10^{-8} \text{ volts,}$$

wherein

Z = the number of actual conductors connected in series, in each phase,

B = the effective density of the magnetic field of the air-gap, in gausses,

L = the axial length of the core of the rotor in cm.,

V = the peripheral velocity of the rotor in cm. per second, and

0.826 = a coefficient which applies to three-phase armatures having wave windings (see p. 36).

The fictitious flux produced by the reaction of this winding would cause, in the air-gap, an effective magnetic density,

$$B = \frac{S \times i_2'}{3p \times d} \text{ gausses,}$$

where

S = the total number of active conductors of the rotor,

p = the number of poles,

i_2' = the effective current, in amperes, in each of these conductors,

d = the (radial) distance across the air-gap, in cm., slightly increased to make up for the reluctance of the iron, and for the openings (slits) between the outer ends of the teeth of the cores.

The "second winding" is the same as a winding having delta connection, owing to the three sets of fixed brushes.

As already stated, this winding is the same as if fixed in space; and its position is, therefore, independent of the rotation of the armature (rotor).

Since the brushes are supplied by the same source of current as the winding of the stator, this rotor winding tends to produce a magnetic flux, revolving with the same velocity as that of the stator, or that in the air-gap, which, as pointed out in the previous chapter, is the only magnetic field resulting from the combination of all the magnetizing effects brought into play.

Since each of the branches of the second winding has a number of active sides equal to that of the phases of the first winding, the E.M.F. induced by the magnetic field of the air-gap will be the same in both cases, for the velocity of the conductors in this magnetic field always remains equal to that of the slip.

The currents in this second winding are naturally in phase with the reaction *AC*, whose direction is that of the magnetic flux, *OB*, in the air-gap.

It follows that the E.M.F. induced by the magnetic field in this second winding will have a lag of a quarter of a phase behind *AC*, and its direction will be along the line *AD* (Fig. 72).

If we plot the values of the product $i''R_2$, along *AC*, according to the scale of volts, we will find, by completing the parallelogram of

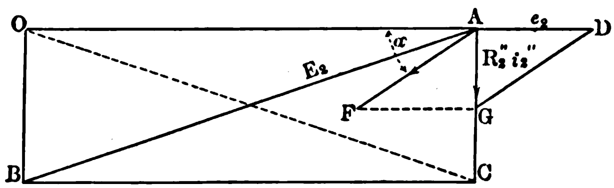

Fig. 72.

vectors, the magnitude and direction of the E.M.F. which should be applied to the brushes in order to produce the required current, i_2'', in the second winding aforesaid.

The potential difference, E_2, applied at the brushes, however, is obtained from the primary source of current, and, consequently, cannot have any phase difference with respect to the right line *OA*, which indicates the phase of the primary current at the instant of time under consideration. The E.M.F., E_2, as indicated in Fig. 72, lags by an amount equal to the angle *OAF* behind the E.M.F. in the primary circuit.

We must now ascertain by what means it will be possible to preserve both the magnitude and direction of the magnetic flux *AC*, while, at the same time, utilizing an E.M.F., E_2, which will always be in phase with that of the source supplying current to the motor.

We will show that the desired result may be obtained by simply displacing all three sets of brushes on the commutator in a direction opposite to that of the revolving magnetic field.

This displacement of the brushes (which involves that of the different branches of the winding) having taken place in a direction opposed to that of the rotation of the magnetic field, the E.M.F., e_2, induced by this revolving magnetic field in each of the branches will, therefore, advance by an angle corresponding to that which might have been produced before the brushes were moved (Fig. 73).

This E.M.F., e_2, retains the same magnitude as in Fig. 72.

Let us suppose the displacement of the brushes to have been such that the E.M.F., AF, which should be applied to the brushes in order

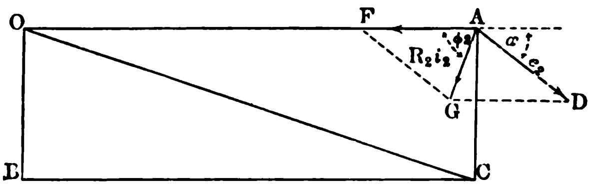

Fig. 73.

to preserve the current i_2'' at its preceding value, falls along the line OA, that is to say, in phase with the E.M.F. of the source of current supply.

It is easy to understand that this displacement of the brushes has in no way modified the magnitude and phase of the fictitious magnetic flux of the second winding.

In fact, when the brushes were moved backward in the revolving magnetic field, thereby making the phase of the E.M.F. applied to these brushes lead forward by an angle equal to the displacement of the brushes, a forward lead of the same amount with respect to the brushes was also, at the same time, given to the induced E.M.F., e_2, to the current i_2'', and, consequently, to the fictitious magnetic flux produced by that current.

It follows that the displacement of the brushes has not changed the instantaneous position of this fictitious magnetic flux with respect to the others.

This fictitious magnetic flux therefore remains in the direction AC, although the current is in the direction AG (Fig. 73).

The diagram shows that the potential difference AF at the brushes has a lead in phase with respect to the current i_2'' produced by the external source of rotor current.

We must not conclude, however, that this displacement of the brushes is capable of materially lowering the power factor of the magnetic field which, in all the diagrams given, has been assumed equal to unity.

It is apparent that the current i_2'' must always be smaller than the current i_2' when the motor is carrying a load.

On the other hand, the power $e_2 i_2'$ wasted by ohmic resistance (Joulean effect) in the ordinary winding itself amounts to only two or three per cent of the power which is applied to the stator.

We may conclude, therefore, that the power $e_2 i_2''$ taken from the external source of current, and sent through the brushes into each of the branches of the rotor, is very small, since it itself remains much below $e_2 i_2$, and, as a general rule, does not attain one per cent of the watted power supplied to the primary winding.

It follows that the definite value of cos ϕ for any machine will differ cnly by a negligible quantity from the value of the power factor for the stator circuits alone. This is all the more exact inasmuch as a considerable amount of the current sent into the rotor through the brushes is not wattless.

Let us note again that the resultant effect of the currents i_2' and i_2'' is given, both in magnitude and in phase, by the vector *OC*.

The potential difference, *AE*, which must be applied at the brushes, is generally very low compared to that applied at the motor terminals. This potential difference may be obtained either by means of a small auxiliary transformer connected in parallel across the motor terminals, or by means of a winding of a few turns placed on the stator core.

When the torque diminishes, the slip decreases. The area of the triangle *OAC* diminishes in the same proportion as the torque. The resultant magnetic flux *OB*, on the contrary, does not vary, since it must induce a constant E.M.F. in the stator.

If, for this new load, we wish to preserve a power factor equal to unity, the right line *AC* will have to retain its magnitude and direction. Since i_2'' cannot change, and since e_2 depends on the slip, the potential difference which must be now applied at the brushes will not only be smaller than in Fig. 72, but will also have a greater inclination with respect to the right line *OA*.

It will be necessary, therefore, in order to neutralize the phase difference in the primary, to again displace the brushes and to diminish the amount of the potential difference applied to the brushes. The latter adjustment may be obtained by means of variable resistances interposed in the leads conveying current to the brush connections.

If, on the other hand, the load were increased, these operations would naturally be reversed.

When the power factor is equal to unity, the stator current has, very nearly, a ratio which is substantially proportional to the torque.

It might perhaps be possible, therefore, in certain cases to avoid the necessity of adjusting the potential difference applied at the brushes by making use of a transformer whose primary is connected in series with the stator winding.

It is interesting to ascertain what will happen, if, without changing the potential difference applied to them, the brushes are displaced more than would be necessary to give $\cos \phi = 1$, in the primary winding.

The right line *OB* (Fig. 74) remains the same as it was in Fig. 73.

The distance *OA* will not vary; and, since the torque remains constant, the point *C* will be on a straight line passing through *A* and parallel to *OB*.

Since, by hypothesis, the brushes have been displaced more than was necessary, the vectors *A'D*, *A'G*, and *A'C* will have a greater inclination with respect to the right line *OA* than they have for the case illustrated in Fig. 73.

In order to change *OA* to *OA'*, the slip must increase. This circumstance will increase the E.M.F., e_2, as well as the current i_2''.

But since the fictitious magnetic flux *A'C* is produced by this current i_2'', its value will, therefore, be greater than was the case in Fig. 73.

The inclination of the vector *BC* resulting from the length of *A'C* shows that the power factor will become negative in the stator winding; that is to say, that the primary current through the stator will now lead the E.M.F. applied at the motor terminals.

We might expect this result, which could also be attained by retaining the same position of the brushes as in Fig. 73, and simply increasing the value *AF* of the potential difference applied to these brushes.

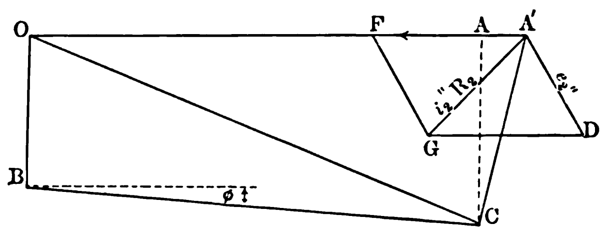

Fig. 74.

These points having been established, we will now show how easily the diagram may be corrected for magnetic leakage and for the resistance of the stator winding.

In designing an induction motor, we generally assume, at the beginning, the resultant magnetic density in the air-gap and the peripheral velocity of the rotor.

The latter seldom exceeds 25 metres (82 ft.) per second; the resultant magnetic density varies according to the size and character of motor, between 2,500 and 3,500 gausses.

Knowing the power and speed, the electromagnetic (tangential) effort (magnetic pull) at the periphery of the stator may be determined. We then calculate the reaction of the secondary winding, or, in other

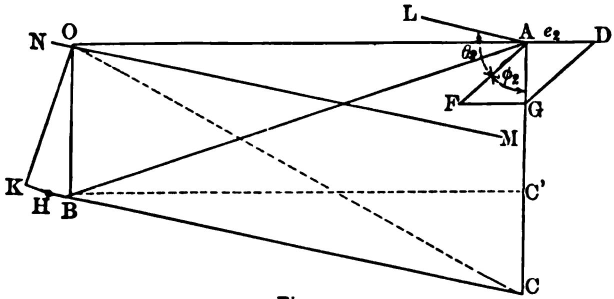

Fig. 75.

words, the value of the fictitious magnetic flux above mentioned, without paying attention, for the moment, to the Heyland principle.

It will then be possible to indicate, in Fig. 75, the value *OB* of the resultant flux, as well as the value *OA* of the fictitious magnetic flux, $V_2\ \Phi_2$, of the rotor, and to also determine the vector *AB*, which represents that portion, Φ_1, of the fictitious magnetic flux of the stator which penetrates the secondary core.

If the torque is not to be varied when the brushes are applied to the commutator, the point *C* must fall on a right line *AC* parallel to *OB*.

Owing to magnetic leakage, this point *C* will fall beyond the point *C′* obtained by the intersection of the right line *AC* with the right line *BC′*, parallel to *OA*.

Let us select the position of this point *C*. The vector *AC* will represent the fictitious magnetic flux produced by the current sent from an external source into the secondary or stator winding by means of the brushes.

The resultant fictitious magnetic flux of the rotor will be represented in magnitude and direction by *OC*.

In order to obtain equilibrium, *BC* must be equal to that portion of the primary fictitious magnetic flux which passes through the air-gap.

If now we plot, along *BC* prolonged, the distance *BH*, equal to the primary leakage, — that is to say, equal to $\Phi_1\ (V_1 - 1)$, — and if we plot, along *OC*, the distance *HK*, representing the secondary leakage, $\Phi_2\ (V_2 - 1)$, then the vector *OK* will indicate, both in magnitude and in direction, the resultant magnetic flux in the stator.

This resultant flux induces an E.M.F., *OM*, which lags 90° behind *OK*, and which differs but slightly from the potential difference, E_1, at the motor terminals. If it be required that the power factor shall be equal to unity, this induced E.M.F., *OM*, must be parallel to *BC*, which represents the primary current.

If this were not the case, it would be necessary to reconstruct the diagram, starting from a point *C*, situated at some other point along the line *AC'*.

When once this condition is fulfilled, the ohmic drop, R_1i_1, is to be plotted on *OM* prolonged, being indicated by the distance *ON*.

The difference of potentials at the terminals will then be represented by the length *NM*.

If we also draw *AD* equal to e_2, and $HG = R_2i_2''$, the difference of potentials which must be applied to the motor brushes will be represented by the distance *AF*. The right line *AL* being drawn parallel to *NM* the angle *LAF* will indicate the extent to which the brushes must be displaced.

Before closing this study, we may observe that, if the rotors were constructed as indicated hereinabove, serious sparking would inevitably result at the brushes.

These sparks are due to the high self-induction of the portions of the rotor-winding which are short-circuited at the brushes.

As the number of conductors per pole is always somewhat limited, a certain variation of the magnetic flux will occur when one of these conductors is cut out of circuit by the short-circuiting action of the brushes.

The effect of this variation is to retard the reversal of current in the portions of the winding which are at the commutation point, and to occasion an excessive potential difference between the collector and the trailing end of the brush.

It is this excessive difference of potential which is the direct cause of the sparking. To avoid this undesirable sparking, the commutator segments are shunted, i.e., partly circuited, by means of resistances.

It is evident that the shunting resistances not only complicate the construction of the motor, but also absorb some energy.

It would probably be difficult to predetermine by calculation the proper amount of resistance to be interposed between any two consecutive commutator segments. The resistance should be determined by experiment after the machine has been finished. It would seem that by increasing the number of active conductors per pole, as well as the number of commutator segments, the importance of these resistances, and of the currents which pass through them, might be materially reduced. These resistances, thus arranged, form a circuit which has much resemblance to the short-circuiting ring of the squirrel-cage winding, and practically the commutator segments and shunting resistances would doubtless take the form of such a ring, having a definite resistance.

Looking at the matter closely, we observe that the currents in these resistances modify the currents which would otherwise pass through the rotor winding.

We can, in the diagram, retain the action of these first currents, and superpose the magnetic effect of those which circulate through the winding and the commutator shunting resistances.

These currents must be in phase with the E.M.F. which produces them, and, consequently, they must lag 90° with the resultant field OB in the air-gap. They will therefore produce a fictitious magnetic flux, having the direction OA, and they will tend at the same time to increase the torque.

It will therefore be possible, thanks to their presence, to diminish slightly the secondary current i_2', in order that, finally, the length of the vector OA may retain its original value. It is seen, therefore, that the currents passing through the shunting resistance of the commutator are not useless. The ohmic drop which they produce in the conductors is compensated by a reduction in the amount of the current i_2'.

Let us note further, that since the slip at the instant of starting is easily thirty times greater than when running at full load, the currents in the commutator shunting resistances will increase in the same ratio.

It will, therefore, be necessary to make these resistances of large current carrying capacity, in order to prevent their burning out when starting the motor.

To sum up, the arrangement devised by Mr. Heyland enables a power factor equal to unity to be obtained at all loads. This property of the Heyland motor type, which is already an advantage in itself, would seem to be further enhanced by the possibility of its enabling motors to be designed with larger air-gaps than those of present motors.

But the constancy of the power factor under changes of load can only be obtained by regulating, in some way, the potential difference applied to the brushes, or by the simultaneous use of both means of regulation.

The ordinary induction motors, as a substitute for which the Heyland system is proposed, give at full load, when well designed, a power factor equal to 0.9, which decreases slightly with the load, and is still as high as about 0.87 when the torque falls down to about ¾ of its normal value.

In order to raise the power factor from 0.9 to 1.0, it is necessary, in the Heyland motor, to provide the rotor with the commutator and an outfit of brushes; it is also necessary that all the segments of this commutator be shunted by resistances mounted on the rotor. It is also necessary, except in a few particular cases, to transform the primary E.M.F., in order to obtain the necessary potential difference to be applied to the brushes; and this potential difference must be regulated by hand within rather wide limits.

The principal merit of induction motors consists, without doubt, in their great simplicity of construction, which enables them to operate without requiring, so to speak, any attention or repairs.

In order to slightly improve the power factor, and to raise it from 0.9 to 1.0, a motor even more complicated than a direct current motor has been devised, which, owing to the losses in the commutator shunting resistances, has an efficiency materially lower than that of the present induction motors; and, besides, it is by no means certain that in practice, when once sold and put in operation, the new machines will remain constantly regulated for a power factor equal to unity.

Under such conditions, what are the prospects of success for this invention? The future will tell.

The author prefers to abstain from answering this question, first, in order to avoid the appearance of being too pessimistic, and, second, because he has a firm belief that others will not fail to discover new improvements, perhaps new inventions even, which will eventually make the induction motor the ideal machine.