THE CARPENTER'S NEW GUIDE;
A COMPLETE BOOK OF LINES FOR CARPENTRY AND JOINERY:
TREATING FULLY ON
Practical Geometry,
Soffits, Groins, Niches, Roofs, and Domes,
AND CONTAINING
A GREAT VARIETY OF ORIGINAL DESIGNS.
ALSO A FULL EXEMPLIFICATION OF
THE THEORY AND PRACTICE OF STAIR BUILDING,
Cornices, Mouldings, and Dressings of Every Description.
INCLUDING ALSO
SOME OBSERVATIONS AND CALCULATIONS ON THE STRENGTH OF TIMBER,
BY PETER NICHOLSON,
AUTHOR OF "THE CARPENTER'S AND JOINER'S ASSISTANT," "THE STUDENT'S INSTRUCTOR TO THE FIVE ORDERS," ETC.
THE WHOLE BEING CAREFULLY AND THOROUGHLY REVISED
BY N. K. DAVIS.
AND CONTAINING NUMEROUS NEW, IMPROVED, AND ORIGINAL DESIGNS FOR ROOFS, DOMES, ETC.,
BY SAMUEL SLOAN, ARCHITECT,
AUTHOR OF "THE MODEL ARCHITECT."
SIXTEENTH EDITION.

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THE AUTHOR'S PREFACE.

To a book intended merely for the use of Practical Mechanics, much Preface is not necessary. It is proper, however, to say, that whatever rules by previous authors have on examination proved to be true and well explained, these have been selected and adopted, with such alterations as a very close attention has warranted for the more easily comprehending them, for their greater accuracy or facility of application; added to these, are many examples which are entirely of my own invention, and such as will, I am persuaded, conduce very much to the accuracy of the work, and to the ease of the workman.

The arrangement of the subjects in this work is gradual and regular, and such as a student should pursue who wishes to attain a thorough knowledge of his profession: and as it is Geometry that lays down all the first principles of building, measures of lines, angles, and solids, and gives rules for describing the various kinds of figures used in buildings; therefore, as a necessary introduction to the art treated of, I have first laid down, and explained in the terms of workmen, such problems of Geometry as are absolutely requisite to the well understanding and putting in practice the necessary lines for Carpentry. These problems duly considered, and their results well understood, the learner may proceed to the theoretical part of the subject, in which Soffits claim particular attention; for, by a thorough knowledge of these, the student will be enabled to lay down arches which shall stand exactly perpendicular over their plan, whatever form the plan may be: on this depends the well executing all groins, arches, niches, &c., constructed in circular walls, or which stand upon irregular bases; wherefore the importance of rightly understanding these I cannot too much insist upon, their construction being so various and intricate, and their uses so frequently required. The two plates of cuneoidal or winding soffits are new, and are constructed in a more simple and more accurate manner: yet this method is only a nearer approximation to truth than the former one; the surface of a conchoid cannot be developed; that is, it cannot be extended on a plane: it is therefore absurd to look for perfection on this subject.

The next subject which regularly presents itself is Groins; for the construction of which there will be found many methods entirely new; and besides the common figures, I have shown many which are difficult of execution, and not to be found in any other author. I have displayed a variety of methods for constructing spherical niches, a form more frequently wanted than the elliptic, which only has yet been explained.
THE AUTHOR'S PREFACE.

Among the various methods for finding the Lines for Roofs, I have given an entire new one for finding the down and side bevels of purlines, so that they shall exactly fit against the hip rafter; and by the same method the jack rafter will be made to fit.

Of Domes and Polygons, I have shown an entirely new method for finding their covering, within the space of the board, thereby avoiding the tedious and incommodious method of finding the lines on the dome itself, as has been always practised heretofore: also a method for finding the form of the boards near the bottom, when a dome is to be covered horizontally. Of dome-lights over stair-cases, or in the centre of groins, a rule upon true principles is given, for finding their proper curve against the wall, and the curve of the ribs; this has never before been made public.

Having gone thus far in the Art of Carpentry, it is necessary for me to say, by way of caution and guard to the ardent theorist, that there are some surfaces which cannot be developed; such as spherical or superoidal domes, where their coverings cannot be found by any other means than by supposing the curved surface to become polygonal; in which case such domes may be covered upon true principles, as may be demonstrated. Let us suppose a polygonal dome inscribed in a spherical one; then, the greater the number of sides of the polygonal dome, the nearer it will coincide with its circumscribing spherical one. Again, let us suppose that this polygonal dome has an infinite number of sides; then, its surface will exactly coincide with the spherical dome, and therefore in anything which we shall have occasion to practise, this method will be sufficiently near; as, for example, in a dome of one hundred sides, of a foot each, the rule for finding such a covering will give the practice so very near, that the variation from absolute truth could not be perceived.

Having gone through the constructive part of Carpentry, I next proceed to examples showing the best forms of floors, partitions, trusses for roofs, truss girders, domes, &c., which shall resist their own weight, or the addition of any adventitious load.

To conclude: as I pretend not to infallibility, I hope to be judged with candor, being always open to conviction, from a knowledge of the difficulty and intricacy of science; yet I hope that my labors may be of some use to others in shortening the road, and smoothing the path through which, for many years, I have been a persevering traveller for knowledge: I shall then be satisfied, and not deem my time misspent if my labors tend to the public good.

P. NICHOLSON.
Few words are necessary to explain the present revision. When the work originally appeared, it excited great interest among artizans, and at once took the foremost rank among works of the kind. By its intrinsic merit, it has ever since maintained this position, although, latterly, many professedly new and original works have been issued, with the avowed object of taking its place. They have failed to obtain foothold; and the limited sale which some few have been so fortunate as to secure, must be attributed to the extensive and valuable contributions levied on this work of Mr. Nicholson's. Indeed, it has been for many years the great source of supply to pseudo authors, who, having mutilated what they took to avoid recognition, represented their alterations as new and superior designs. Some have, to a limited extent, succeeded; but not among those who were acquainted with this, the original and standard work.

Recently, however, such great advances have been made in the arts of Carpentry and Joinery, that many things in the work are now almost obsolete. Notwithstanding this, the sales of the work were undiminished, showing the esteem in which it was held; but the publishers, unwilling that so valuable a treatise should be in any respect defective, determined upon a revision. The present editors have undertaken it—with what success, the public must judge.

All the plates have been re-engraved, and the matter they contained has been condensed, so that many valuable additions might be introduced, without increasing the bulk of the work. A few plates, those on Practical Carpentry in particular, have been thrown out, because they indicated methods entirely at variance with those in present use. A much larger number has been substituted, comprising every principle of construction which the carpenter will need.
PREFACE TO THE REVISED EDITION.

Some years ago, a few plates on Stair-Casing were withdrawn, and improved methods substituted by William Johnston, Architect, whose name appears on the title-page of the last edition. All these methods of his have been retained entire, together with his remarks upon them. Some plates, which were then rejected, have been replaced as too valuable to be lost. To these have been added some original plates of Stair-Lines, in which the principles are greatly simplified and may be applied to every kind of hand-railing. Many other additions may be observed by inspecting the work.

The text also has been entirely re-written. A great part of it was so very obscure, as to be almost unintelligible. This has been altered so as to be perspicuous; and in some parts the original explanations have been rejected entirely, and others introduced. It is not claimed that the work is free from errors in this respect, but that it is free from all practical error; and that the additions, substitutions, and alterations, will greatly improve its value, and render it a complete compendium of Carpentry and Joinery.

THE EDITORS.

Philadelphia, July, 1853.
CONTENTS.

PRACTICAL GEOMETRY.

<table>
<thead>
<tr>
<th>Definitions</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Problems</td>
<td>12</td>
</tr>
<tr>
<td>Drawing Instruments</td>
<td>20</td>
</tr>
<tr>
<td>Mensuration</td>
<td>22</td>
</tr>
</tbody>
</table>

CARPENTRY.

<table>
<thead>
<tr>
<th>Linings for Soffits</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
</tr>
<tr>
<td>Tracery</td>
<td>28</td>
</tr>
<tr>
<td>Arches</td>
<td>29</td>
</tr>
<tr>
<td>Groins</td>
<td>30</td>
</tr>
<tr>
<td>Niches</td>
<td>39</td>
</tr>
<tr>
<td>Roofs</td>
<td>43</td>
</tr>
<tr>
<td>Skylights</td>
<td>45</td>
</tr>
<tr>
<td>Domes</td>
<td>47</td>
</tr>
<tr>
<td>Practical Carpentry.—Designs</td>
<td>49</td>
</tr>
</tbody>
</table>

JOINERY.

<table>
<thead>
<tr>
<th>Stair-lines (Johnston)</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>68</td>
</tr>
<tr>
<td>Stair-lines (new)</td>
<td>76</td>
</tr>
<tr>
<td>Stair-lines (Nicholson)</td>
<td>78</td>
</tr>
<tr>
<td>Diminishing Columns</td>
<td>90</td>
</tr>
<tr>
<td>Sash-work</td>
<td>91</td>
</tr>
<tr>
<td>Architrave</td>
<td>92</td>
</tr>
<tr>
<td>Raking Mouldings, Cornices, &amp;c.</td>
<td>93</td>
</tr>
</tbody>
</table>

APPENDIX.—Strength of Timber........................................ 97
Extension has three dimensions, length, breadth, and thickness.

Geometry is the science which has for its object, first, the measurement of extension; and secondly, to discover, by means of such measurements, the properties and relations of geometrical figures.

Practical Geometry is that branch of the science which describes, without demonstration, the various methods of constructing angles, figures, curves, and geometrical solids, and comprises also the rules for mensuration.
PLATE 1.

DEFINITIONS.

A point is that which has neither length, breadth, nor thickness, but position only.
A line is that which has length, without breadth or thickness.
A right or straight line preserves the same direction between any two of its points. See Fig. A.
A curve or curved line changes its direction at every point. See Fig. B.
A surface is that which has length and breadth, without height or thickness.
A plane is a surface, such, that if any two of its points be joined by a straight line, that line will lie wholly in the surface.
Every surface which is not plane, or composed of plane surfaces, is a curved surface.
Two lines are said to be parallel, when, being situated in the same plane, they will not meet, how far soever, either way, both of them be produced. See Fig. C.
When two straight lines meet each other, their inclination is called an angle, which is greater or less according as the inclination is greater or less. The two straight lines are called the sides of the angle, and their common point of intersection the vertex.
When one straight line meets another straight line, without being inclined to it on the one side any more than on the other, the angle formed is called a right angle, and the two lines are said to be perpendicular to each other. See Fig. D.
An angle less than a right angle is an acute angle. See Fig. E.
An angle greater than a right angle is an obtuse angle. See Fig. F.
An angle is designated by a single letter placed at the vertex, or by three letters, two of them upon the sides, and the other at the vertex, the letter at the vertex being always placed in the middle, as the angle \( c \) or \( a c b \), in Fig. E.
A polygon is a portion of a plane terminated on all sides by lines.
A polygon of three sides is a triangle; one of four sides, a quadrilateral; one of five, a pentagon; one of six, a hexagon; one of seven, a heptagon; one of eight, an octagon; one of nine, a nonagon; one of ten, a decagon.
An equilateral triangle has all its sides equal. See Fig. G.
An isosceles triangle has two of its sides equal. See Fig. H.
A scalene triangle has all its sides unequal. See Fig. I.
A right-angled triangle has one of its angles a right angle. See Fig. J.
A trapezium is a quadrilateral which has no two of its sides parallel. See Fig. K.
A trapezoid is a quadrilateral which has two of its sides parallel. See Fig. L.
A parallelogram has its opposite sides parallel. See Fig. M.
A rhombus has its opposite sides equal and parallel—its angles not right angles. See Fig. N.
A rectangle has its opposite sides parallel, and its angles right angles. See Fig. 0.
A square has all its sides equal, and its angles right angles. See Fig. P.
A regular polygon is one whose sides and angles are equal each to each. Thus, Fig. R is a regular pentagon, S a regular hexagon, T a regular octagon.
An irregular polygon is one whose sides and angles are not equal. See Fig. Q.
A polygon is said to be inscribed in a circle when the vertices of its angles lie in the circumference. Thus, $a c d$, Fig. X, is an inscribed triangle. The circle is also said to be circumscribed about the triangle.
A circle is a portion of a plane bounded on all sides by a curved line, every point of which is equally distant from a point within, called the centre. See Fig. U.
The radius of a circle is a right line drawn from the centre to the circumference, as $c d$, $c a$, or $c b$, Fig. U.
The diameter of a circle is a line passing through the centre, and terminated on both sides by the circumference, as $a b$, Fig. U.
An arc is any part of the circumference, as $a b$, Fig. V.
A chord is a right line which joins the extremities of an arc, as $a b$, Fig. V.
A segment is the part of a circle included between an arc and its chord. See Fig. V.
A sector is the part of a circle included between an arc and two radii drawn to its extremities. See Fig. W.
A line is tangent to a circle when it has but one point in common with the circumference, and does not intersect it. Thus, $c a b$, in Fig. X, is a tangent.
The circumference of a circle is divided into 360 equal parts, called degrees. Arcs are estimated according to the number of these equal parts which they contain. Thus, in Fig. Y, if the arc $d e$ contains 30 of these parts, it is an arc of 30 degrees, written $30^\circ$. If $a b$ contains 90 parts, it is an arc of $90^\circ$, or a quadrant, that is, one-fourth of a circumference.
Arcs are also measures of angles, the vertices of the angles being supposed to be at the centre of the circle. Thus, if the arc $d e$, in Fig. Y, contains $30^\circ$, the angle $d e e$ is an angle of $30^\circ$, and $d e b$ is an angle of $90^\circ$, or a right angle.
It will be observed that an angle of $45^\circ$ is the half of a right angle. Thus, in Fig. Z, the angle $f e e$, or $f c a$, is an angle of $45^\circ$. Also, an angle of $60^\circ$ is two-thirds of a right angle. Thus, the angle $g e b$ is an angle of $60^\circ$. The chord of $60^\circ$ is equal to the radius of the circle. Hence, the triangle $g e b$ is equilateral.
PLATE 2.

PROBLEMS.

Fig. 1. To draw a Perpendicular at a given Point in a Line.

Having taken two points, \(a\) and \(b\), equally distant from \(c\), the given point, as centres, describe arcs intersecting each other at \(d\), and then draw \(d e\); it will be the perpendicular required.

Fig. 2. From a given Point without a Line to let fall a Perpendicular upon the Line.

From the given point \(d\), as a centre, describe an arc cutting the given line in two points, \(a\) and \(b\); with those points as centres, describe two arcs intersecting each other at \(e\); then draw \(d e\), and it will be the perpendicular required.

Fig. 3. To bisect a given Line by a Perpendicular.

From \(a\) and \(b\), the extremities of the line, as centres, describe two arcs, intersecting each other at \(d\) and \(e\); draw \(d e\), and it will bisect the given line at \(e\), and be perpendicular to it.

Second method. From \(a\) and \(b\), as centres, describe two arcs, intersecting each other at \(g\), and from \(g\) let fall a perpendicular upon \(a b\).

Fig. 4. To draw a Perpendicular to a Line at its Extremity.

Let \(b\) be the extremity of the given line; from any point \(c\) above the line as a centre, describe the arc \(a b d\); draw \(a c d\) and \(d b\); it will be the perpendicular required.

Second method. Upon the given line take \(a b\), equal to six units of measure, as inches, feet, or rods, then, with a radius of two units and centre \(b\), draw a dotted arc above \(a b\), as at \(d\), and with a radius of two units and the centre \(a\), draw an intersecting arc at \(d\); from \(d\), the point of intersection, draw \(d b\), and it will be the perpendicular required.

Fig. 5. To draw a Perpendicular to a Line at or near its Extremity.

From the points \(a\) and \(g\), as centres, describe two arcs intersecting each other at \(d\) and \(e\); draw \(d e\), and it will be perpendicular to the given line at \(e\).

Fig. 6. Through a given Point to draw a Line parallel to a given Line.

Let \(a\) be the given point and \(a b\) the given line; from \(a\) and \(c\) as centres describe the arcs \(a c\) and \(c b\); make \(a c\) equal to \(b c\), and draw \(c d\); it will be parallel to \(a b\).

Fig. 7. To bisect a given Angle.

From \(c\), the vertex of the angle, as a centre, describe an arc \(a b\); from \(a\) and \(b\), as centres, describe two arcs intersecting each other at \(e\); draw \(e c\), and it will bisect the angle.

Fig. 8. To make an Angle at a given Point on a Line equal to a given Angle.

Let \(y e\) be the line, \(y\) the given point, and \(f e g\) the given angle; from \(e\) as a centre with any radius, as \(e a\), describe the arc \(a b\); from the given point \(y\) as a centre, and with the same radius, describe the arc \(x z\) and make it equal to \(a b\); then draw \(y z\), and \(x y z\) will be the required angle.

Fig. 8. To divide a Line into Parts proportional to the Parts of a given Line.

Let \(y e\) be the given line, divided into parts at the points \(a\), \(b\), \(c\), \(d\); let \(y n\) be the line to be divided; join the extremities \(e n\), and draw the lines \(d k, c i\), \&c., parallel to \(e n\).

Second method. Let \(f g\) be the given line, divided at the points \(i\) and \(k\), and \(d e\) the one to be divided. From \(e\), the vertex of the triangle \(c f g\), draw the lines \(e i\) and \(e k\).

Fig. 9. Two Angles of a Triangle being given, to find the Third.

On the straight line \(a b\) lay off \(b c e\) equal to one of the given angles, and \(d c e\) equal to the other; and \(a c d\) will be the third angle required.
PLATE 3.

PROBLEMS—(continued).

Fig. 1. To construct an equilateral Triangle on a given Line.
From c and b, the extremities of the given line, as centres, and with the radius c b, describe two arcs intersecting at a, and draw a b, a c.

Fig. 2. The three Sides of a Triangle being given, to construct the Triangle.
Make the side a b equal to C, one of the given lines. From the extremities a and b, with radii equal to A and B, the other given lines, describe two arcs intersecting at c, and draw a c and b c.

Fig. 3. To construct a Rectangle, equivalent to a given Parallelogram.
Let a b d c be the given parallelogram. Construct upon its base, c d, a rectangle having the same altitude as the parallelogram.

Fig. 4. To construct a Rhombus with a given Angle.
Make the angle b a d equal to the given angle. Make a b and a d equal, and draw b c parallel to a d, and d c parallel to a b.

Fig. 5. To construct a Square on a given Line.
From c and d, the extremities of the line, as centres, describe the arcs c c b, and d c a. From e set off e a, and e b equal to e f; the half of c e; then draw d b, a b, and a c.

Second Method. At c and d, the extremities of the given line, erect perpendiculars. From c and d, as centres, with the radius c d, describe arcs intersecting the perpendiculars at b and e, and then draw b c.

Fig. 6. To construct a Rectangle equivalent to a given Triangle.
At b and c, the extremities of the base of the given triangle, erect the perpendiculars b d and c e. Intersect those perpendiculars by the line d e, parallel to the base b c, and drawn through f; the middle point of the altitude of the triangle.

Fig. 7. To construct a Square equivalent to a given Rectangle.
Produce d c, one side of the rectangle, until c f, the part produced, is equal to c b, the other side of the rectangle. Bisect d f at n; from n, as a centre, with the radius n d, describe a semicircumference. Produce c b to e, and on c e describe a square.

Fig. 8. To inscribe in a Circle a regular Hexagon and an equilateral Triangle.
Apply the radius c e six times to the circumference, and then will be inscribed a regular hexagon. Join the alternate angles of the hexagon, and there will be inscribed an equilateral triangle.

Fig. 9. To inscribe in a Circle a regular Pentagon.
Draw two diameters, a h and b i, perpendicular to each other. Bisect the radius b c at e; take e d, equal to a e; then from a, as a centre, and with the radius a d, describe the arc d f, and the chord a f will be one side of the required pentagon.

Fig. 10. To inscribe in a Circle a Square and a regular Octagon.
To inscribe a square, draw two diameters at right angles to each other, and join their extremities. Bisect the arc subtended by one of the sides of the square, and the chord a b, of half the arc, will be the side of the octagon required.

Fig. 11. To make an Octagon out of a Square.
Draw the diagonals f e and d g; from f and e, as centres, and with a radius equal to one-half of the diagonal, describe arcs cutting the sides of the square in a and b; remove from each corner of the square a triangle equal to a b g.

Fig. 12. To make a Square equal to two given Squares.
Let b d and b g be the two given squares. Having placed them so that the angle at b shall be vertical, as represented in the figure, join a and c, and construct a square on a c.

Note.—The proposition upon which the solution of the above problem depends, viz., that in every right-angled triangle the square upon the hypothenuse is equal to the sum of the squares upon the other two sides, is of great practical value. A new method of demonstrating the proposition is shown in Fig. 13. Let a b c be a right-angled triangle. Produce b a until a y is equal to b e; construct squares on b g, b a, and c e; draw k h parallel to f e, and e f k h will be a square on b e; then the whole figure, minus the four triangles, A, B, C, and D, is the square upon the hypothenuse. The same figure, minus the four equal triangles, C, D, E, and F, is the sum of the squares upon the other two sides; hence, the proposition is true.

The solution of the problem in Plate 2, Fig. 4, depends also, upon this proposition. The distances a b, and b d, are the base and perpendicular of a right-angled triangle, of which the distance a d is the hypothenuse. It will be observed, the sum of the squares of 6 and 8, equal to 36 and 64, is equal to the square of 10, which is 100.

Various other valuable practical results may be obtained from the demonstration. As, for instance, the square on the hypothenuse is equal to twice the rectangle contained by the sides, plus the square, upon the difference of the sides; all of which may be readily seen by a reference to the figure.

The above demonstration of this celebrated proposition has never before been published.—Ep.
PLATE 4.

PROBLEMS (CONTINUED).

Fig. 1. To find the Centre of a Circle.
Draw any chord, as e f. At i, the middle point of e f, erect the perpendicular a i, and produce it to b. Then c, the middle point of a b, is the centre of the circle.

Fig. 2. To find the Centre of an Arc.
Draw two chords, a b and a d; bisect them by perpendiculars; and e, their point of intersection, is the centre of the arc.

Fig. 3. To find the Length of an Arc.
Let a b c be any arc; bisect it at b, and draw the chord a b; produce the chord a c, until it is equal to twice a b; again, produce c n, until n i is equal to one-fourth of e n; and a i is the length of the arc.

Fig. 4. To find the Length or Stretch-out of a Semicircumference.
Construct an equilateral triangle, m n i, on the diameter m n; draw the tangent-line b a d parallel to m n, until it meets i m and i n produced. Then b a d is the stretch-out of the semicircumference m a n.

Fig. 5. To draw a Tangent to a Circle at a given Point.
Let a be the given point; draw b a d perpendicular to the radius c a, at its extremity, and it will be the tangent required.

Fig. 6. A Tangent-line being given, to find the Point where it touches the Circumference.
Take d, any point of the tangent-line, and draw d c to the centre of the circle; on c d, as a diameter, describe a semicircumference, and the point a, where it intersects the circumference of the circle, is the point of tangency.

Fig. 7. To describe a Segment of a Circle having a given Base and Height.
Let b d be the base, and a f the height; bisect b d by the perpendicular a f c, and draw b a; bisect b a by the perpendicular e c, and c, the intersection of the two perpendiculars, is the centre of the circle, from which, with the radius c a, the segment b a d may be described.

Fig. 8. To draw a Segment by Rods to any Length and Height.
Make two rods, a b and a d, each being equal to the base b d of the segment, to form the angle b a d; then, having them secured, and placed as in the figure, put a nail at b and one at d. Now, place a pencil-point at a, and move the frame either way, sliding against the nails at b and d, and the point a will mark the arc of the required segment.

Second Method. — If the segment required is too large to be conveniently drawn in this way, we may cut a triangular piece of board, as shown at Fig. 9, the height i e of the triangle being half the height of the segment. Now, by putting a nail also at a, we may, with this triangle, draw half the arc of the required segment at a time, in a manner similar to the above, placing it, as shown by the rods, at e a and e b. The angle a e b of the rods is equal to b i d.

Fig. 10. To draw the Segment of a Circle by the Method of intersecting Lines.
Let b d be the base of the segment, and a 6 its height; draw the chord a b, and erect b m perpendicular to it, and e b perpendicular to b d; divide b 6 and 6 d, each into six equal parts, at the points 1, 2, &c.; divide, also, a m into six equal parts, at the points 1', 2', &c., and draw the lines 1 1', 2 2', &c.; and their points of intersection with the lines a 1, a 2, &c., are points of the curve; trace the curve through them, and you will have the half-segment a b. The other half may be drawn in the same way.
PLATE 5.

OF SOLIDS.

DEFINITIONS.

A cube is a solid bounded by six equal squares. See Fig. 1.

A prism is a solid whose lateral faces are parallelograms, and whose upper and lower bases are equal polygons. See Figs. 2 and 3, Plate 4.

A prism is triangular, quadrangular, or pentagonal, according as its base is a triangle, quadrilateral, or pentagon.

The altitude of a prism is the perpendicular distance between its upper and lower bases. Thus, $a \, b$, in Fig. 3, represents the altitude of the prism.

A prism is right when its altitude is equal to its side, or when the side is perpendicular to the plane of the base; otherwise it is oblique.

A pyramid is a solid formed by several plane angles proceeding from a common point, called its vertex, and terminating in a regular polygon, which forms its base. See Figs. 5 and 6.

A truncated pyramid, or the frustum of a pyramid, is that part of a pyramid which is left after the upper part has been cut off by a plane parallel to its base. See Fig. 5.

The altitude of a pyramid is the perpendicular distance from its vertex to its base. Thus, $b \, c$, in Fig. 6, is the altitude of the pyramid.

A cylinder is a solid generated by the revolution of a rectangle about one of its sides. See Fig. 7. The side of the rectangle about which it is supposed to revolve, is the axis or altitude of the cylinder. See $a \, b$, in Fig. 8.

A sphere is a solid generated by the revolution of a semicircle around its diameter. See Fig. 9. The diameter of a sphere, is a line passing through the centre and terminating on both sides in the surface. See $a \, d$, Fig. 10. The radius of a sphere is half of the diameter, or a line drawn from the centre to any point of the surface. See $b \, c$, Fig. 10.

A cone is a solid generated by the revolution of a right-angled triangle around one of its sides. Thus, in Fig. 12, if the right-angled triangle be revolved around the side $a \, b$, it will generate a cone, as represented in Fig. 11. In Fig. 12, $a \, b$ is the altitude of the cone, and $b \, d$ the radius of the base.

If a cone be cut by a plane, making, with the plane of the base, an angle less than the angle included between the side of the cone and its base, the section will be an ellipse. Thus, the section formed by a plane passing through the line $a \, b$, in Fig. 11, is an ellipse. It is also represented in Fig. 13.

If a cone be cut by a plane, making, with the plane of the base, an angle equal to the angle included between the side and the base, the section will be a parabola. Thus, the section formed by a plane passing through the line $a \, c$, Fig. 11, is a parabola. It is represented in Fig. 14.

If a cone be cut by a plane, making, with the plane of the base, an angle greater than the angle included between the side and the base, the section will be an hyperbola. Thus, the section formed by a plane passing through the line $a \, d$, in Fig. 11, is an hyperbola. It is represented in Fig. 15.
PRACTICAL GEOMETRY.

PLATE 6.
THE ELLIPSE.

DEFINITIONS.

An ellipse is a curve, such that if from any point two lines be drawn to two fixed points, their sum will be always equal to a given line. Thus, let a, Fig. 1, be any point of the curve, and o and o' the two fixed points; then, if o a + o' a is always equal to a given line, the curve is an ellipse.

The two fixed points, o and o', are called foci.

A diameter is any line passing through the centre, and terminating in the curve.

The diameter which passes through the foci is called the transverse axis; and the one perpendicular to it is the conjugate axis. Thus, d e, Fig. 1, is the transverse axis, and A B the conjugate axis.

PROBLEMS.

Fig. 1. To describe an Ellipse with a String, the Foci and transverse Axes being given.

Take a string equal to the transverse axis, and fasten its extremities at the foci; then place a pencil against the string, and move it around, keeping the string constantly stretched.

Fig. 1. To do the same with the Trammel, the Centre and Axes being given.

Place the trammel at the centre, as seen in the figure, and so arrange the rod e f g upon the arms, and the pencil g upon the rod, that e g will be equal to the transverse, and f g equal to the conjugate axis. Move the pencil around, and it will describe an ellipse.

Fig. 2. To describe an Ellipse by means of intersecting Lines, the Axes being given.

Describe a rectangle upon the axes, and divide the conjugate axis into a number of equal parts, at the points 1, 2, 3, &c.; divide the transverse axis into the same number of equal parts, at the points 1', 2', 3', &c.; then draw the lines a 1, a 2, &c., b 1', b 2', &c., and their intersections will be points of the curve. Trace the curve through these points.

Fig. 3. To describe a rampant Ellipse.

This problem is performed in the same way as the preceding, except that the parallelogram a b e d is used instead of the rectangle a b d c, in Fig. 2.

Fig. 4. The transverse and conjugate Axis of an Ellipse being given, to draw its representation.

Draw b e parallel and equal to a c; bisect it at f, and draw a f and b e, intersecting each other at k; bisect a k by a perpendicular, meeting a b produced in c, and draw b c, meeting c e in e; then from e, as a centre, describe the arc e k, and from c, as a centre, describe the arc a k, and you will have one-fourth of the curve. Draw the other parts in the same way.

Fig. 5. An Ellipse being given, to describe within it another, having the same Eccentricity, or the same Proportions in respect to Length and Width.

Describe the rectangle a b d c on the transverse and conjugate axis, and draw the diagonals a d and b c; let a' b' be the conjugate axis of the required ellipse, and through a' b' draw a' b' and c' d' parallel to d e; join a' c' and b' d', and d' e' will be the transverse axis of the required ellipse.

Fig. 6. An Ellipse being given, to find the Centre, Axes, and Foci.

Draw any two lines, a c and d e, parallel to each other, and draw i k through their middle points; bisect i k at c, and c will be the centre of the ellipse.

From c, as a centre, describe two arcs, intersecting the curve at m and n; draw m n, and d c e, perpendicular to it, will be the transverse axis, and a c b, perpendicular to it, will be the conjugate axis.

From b, as a centre, with a radius equal to c e, describe two arcs, intersecting the transverse axis, and the points of intersection, o and o', will be the foci.

Fig. 6. To draw a Tangent to an Ellipse, at a given Point.

Let b be the given point; draw b o and b o' to the foci, and produce o b to o; bisect the angle o b o', and the bisecting line will be the required tangent.
PLATE 7.
THE PARABOLA, HYPERBOLA, AND CYCLOID.

DEFINITIONS.

A parabola is a curve, any point of which is equally distant from a fixed point and a given line. Let \(A B\), Fig. 1, \(b\) the given line, and \(F\) the fixed point; then, for any point of the curve, as \(G\), the distances \(G F\) and \(G C\) are equal.
The given line, \(A B\), is called the directrix.
The fixed point, \(F\), is called the focus.
The line, \(FD\), drawn through the focus and perpendicular to \(A B\), is called the axis.
The line, \(m n\), drawn through the focus, perpendicular to the axis, is called the parameter.
An hyperbola is a curve, in which the difference of two lines, drawn from any of its points to two fixed points, is constantly equal to a given line. Let \(n\), Fig. 4, be any point of the curve, and \(A\) and \(F\) the two fixed points; then the difference between \(A B\) and \(B F\) is always equal to a given line.
The points \(A\) and \(F\) are called the foci, and \(M\) the centre, of the hyperbola.
The two curves described around the foci \(A\) and \(F\), are called branches of the hyperbola.
In common language, the term hyperbola is applied to a single branch of the curve.
A diameter is any line passing through the centre and terminating on both sides of the curve. Thus, \(IL\) is a diameter.
The diameter \(KH\), which, being produced, passes through the foci, is called the transverse axis.
The line \(NO\), perpendicular to \(i\), is the conjugate axis.
If a circle, \(EFD\), Fig. 6, be rolled along a right line, \(AB\), any point of the circumference, as \(D\), will describe an arc, as \(ab\), which is called a cycloid.
The circle, \(EFD\), is called the generating circle, and the point \(D\), the generating point.
The right line, \(a b\), equal to the circumference of the generating circle, is called the base of the cycloid.
The line, \(DF\), is called the axis of the cycloid.

PROBLEMS.

Fig. 1. To describe a Parabola.

Take a straight edge, \(AB\), and T-square, \(CG\); fasten at one end of a string, equal to \(GC\), and the other end at \(F\); place a pencil against the string, keeping it always stretched, and move the square along the straight edge. The pencil will describe a parabola.

Fig. 2. To describe a Parabola by intersecting Lines.

Take the rectangle \(AHCA\), and divide the sides \(a c\) and \(c H\) into the same number of equal parts at the points \(1, 2, 3, 1', 2', 3', \&c.\); draw perpendiculars to \(c H\), at the points \(1', 2', 3', \&c.\), and also, the lines \(A 1, A 2, \&c.\), intersecting them; trace the curve through the points of intersection.

Fig. 3. To do the same by another Method.

Take the triangle \(CCD\), and divide the sides \(c c\) and \(C D\) into the same number of equal parts at the points \(1, 2, 3, \&c.\); draw the lines \(1 1, 2 2, 3 3, \&c.\), and trace the curve so that those lines shall be tangent to it, as represented in the figure.

Fig. 4. To describe an Hyperbola.

Fasten one end of a rod at \(A\), and attach a string to the other end, at \(C\); fasten the other end of the string at \(F\); place a pencil against it, and move it round the point \(F\), keeping the string always stretched.

Fig. 5. To do the same by intersecting Lines.

Divide the sides \(a c\) and \(c b\) of the rectangle \(ABCA\) into the same number of equal parts at the points \(1, 2, 3, 1', 2', 3', \&c.\); produce \(BA\) to \(C\), and trace the curve through the intersection of the lines \(c 1, A 1, c 2, A 2, \&c.\).

Fig. 6. To describe a Cycloid.

Upon \(a e\), half the base of the cycloid, and \(e c\), the radius of the generating circle, construct the rectangle \(a e c c\); divide \(a e\), and the semicircumference \(EFD\), into the same number of equal parts at the points \(1, 2, 3, 1', 2', 3', \&c.\), and erect the perpendiculars \(1' F, 2' F, \&c.\); from \(6'\), as a centre, with a radius equal to \(c e\), describe the arc \(6' a\); from \(5'\), as a centre, with the same radius, describe the arc \(5' i\), and so on; from the points \(6', 5', \&c.\), lay off, on these arcs, the chords \(e 1, e 2, \&c.\), and through their extremities, \(a, i, h, \&c.\), trace the curve.
PLATE 8.

THE SECTIONS OF A SEMI-CYLINDER AND HEMISPHERE.

The section of a cylinder, made by a plane oblique to the axis, is an ellipse.

Every section of a globe is a circle.

Every section of a globe, made by a plane passing through its centre, is called a great circle, being the largest that can be obtained by cutting the globe. Its radius is equal to the radius of the globe.

PROBLEMS.

Fig. 1. To find the Section of a Semi-cylinder, made by a Plane at right angles to the Plane passing through its Axis.

The section is a semi-ellipse, whose conjugate axis is the diameter of the cylinder, and whose transverse axis is the intersection of the two perpendicular planes; thus, taking $\epsilon \alpha'$, equal to $eo$, the radius of the cylinder, as the semi-conjugate axis, and $ab$, the intersection of the planes, as the transverse axis, the semi-ellipse may be described with the trammel, as represented in the figure.

Second Method. Divide the semicircumference $deo$ into any number of parts, and draw, from the points of division, perpendiculars to $dc$, meeting $ab$ at the points $1' 2' 3'$, &c.; erect the perpendiculars $1' 1$, $2' 2'$, &c., respectively equal to the lines $11, 22, 33, &c.$, and trace the curve through $1' 1$, $2' 2'$, $3' 3'$, &c.

For, conceive the semicircle and the semi-ellipse to be turned around $dc$ and $ab$, so as to be perpendicular to the plane $abc$, then will they occupy the same position as in the solid, and the lines $1' 1$, $2' 2'$, &c., will be respectively parallel to $11, 22, &c.$; hence, the semi-ellipse thus found is the true section of the cylinder.

Fig. 2. To find the Section of a Semi-cylinder, made by a Plane, forming an acute Angle with the Plane passing through its Axis.

In the right-angled triangle $i' i' \ell'$, at $A$, make the angle $i' \ell' i'$ equal to the angle of the planes, and $i'x'$ equal to the radius of the base of the cylinder; from $s$ draw $sx$, parallel to $cd$; draw $tx$, equal to $t x'$ at $A$, and perpendicular to $ab$; produce it until $t i'$ is equal to $t i'$ at $A$, and draw $i's$; draw $xg$ perpendicular to the tangent $fg$, which is parallel to $cd$; join $c$ and $g$; draw the tangent $Vh$ parallel to $eg$, and from $V$ draw $Vy$ perpendicular to $cd$, until it meets $ba$ produced; draw $ym$ parallel to $si'$, and then draw $sm$ perpendicular to $si'$ and $ym$; then the section will be an ellipse of which $sm$ is the semi-transverse axis, and $s4'$ equal to $ef$, the semi-conjugate axis. The ellipse may be described with the trammel.

Second Method. Draw $11, 22, 33, &c.$, parallel to $eg$, and from the points $1, 2, 3, &c.$, in $cd$, erect the perpendiculars $11', 22', 33'$, &c.; then, draw $1'1', 2'2', 3'3'$, &c., parallel to $si'$, making them respectively equal to $11, 22, 33, &c.$; trace the curve through the points $1'1', 2'2', 3'3'$, &c.

Fig. 3. To find the Section of a Segment of a Cylinder, made by a Plane forming an obtuse Angle with the Plane of the Segment.

Make the angle $b' a' x$, at $B$, equal to the angle of the planes, and draw $a' \ell'$ perpendicular to $a' x$, and make it equal to $gh$, the height of the segment; draw $b' \ell'$ perpendicular to $a' \ell'$; draw $ab$ parallel to $mn$, and $bc$ perpendicular to $ef$, and equal to $b' \ell'$ at $B$; produce $bc$ until $c4'$ is equal to $c' a'$ at $B$; join $4'$ and $a$. This line corresponds to $is$, Fig. 2, and the curve may be traced by the second method of the preceding problem.
At Fig. 4 is exhibited the method of finding an oblique section of a solid of irregular form; which is the same as that employed in the second method of the problem preceding the last.

Fig. 5.  Given, the Position of three Points in the Circumference of a Cylinder, and their respective Heights from the Base, to find the Section of the Segment of the Cylinder, through these three Points.

Let \( c d e \) be three points in the circumference of the base of the cylinder, immediately under the three given points, and \( Z Y X \) the height of the given points respectively, above the base; join the points \( c \) and \( d \), and draw \( c a \), \( e A \), and \( d b \), perpendicular to \( c d \), equal to \( Z \), \( Y \), and \( X \), respectively; produce \( c d \) and \( a b \) to meet each other in \( O \); draw \( e D \) parallel to \( d O \), and \( A D \) parallel to \( O b \); join \( D O \). In \( D O \), take any point, as \( D \), and draw \( D H \) perpendicular to \( D c \), cutting \( O d \) in \( H \); from the point \( H \), draw \( H I \) perpendicular to \( O b \), cutting it at \( K \); from \( O \), with the radius \( O D \), describe the arc \( I D \), cutting \( H I \) in \( I \), and join \( O I \); divide the circumference \( c e d \) into any number of equal parts, and from the points of division draw lines to \( c d \) parallel to \( O D \), cutting \( c d \) in \( 1, 2, 3, \&c. \); from the points \( 1, 2, 3, \&c. \), in \( c d \), draw lines parallel to \( d b \), cutting the line \( a b \) in \( 1', 2', 3', \&c. \); from the points \( 1', 2', 3', \&c. \), in \( a b \), draw lines parallel to \( O I \), and make \( 1' 1' \) equal to \( 1 1 \) on the base of the cylinder; make \( 2' 2' \) equal to \( 2 2, 3' 3' \), equal to \( 3 3, \&c. \) Through the points \( 1' 2' 3', \&c. \), trace the curve, which will be the contour of the section required.

Fig. 6.  To find the Section of a Hemisphere made by a Plane perpendicular to its Base.

Let \( f d \) be the radius of the sphere, and \( a b \) the diameter of the section; describe a semicircumference upon \( a b \), as a diameter, and it will be the section required.

Fig. 7.  To find the Section of a Semi-ellipsoid, which is a Solid generated by the Revolution of a Semi-ellipse about its Axis, made by a Plane perpendicular to its Base.

Let \( c d \) be the diameter of the circular base, and the curve \( c o d \) a section perpendicular to the base, and passing through the centre; then, let \( a b \), parallel to \( c d \), be the diameter of the required section. The curve \( a o' b \) may be traced by means of the lines in the figure, according to principles heretofore developed.

At Fig. 8, is exhibited the method of finding any section of an irregular solid, generated by revolution. Let \( s f \) be the axis of revolution, \( c f \) the base of the generating figure, and \( a b \) the base of the section. The method will be readily understood by inspection of the figure.
PLATE 9.

DRAWING INSTRUMENTS.

The Geometrical Drawing of an object is usually made on a much smaller scale than the real size of the object. Thus, the drawing of a house, for instance, is much smaller than the house. In order that the drawing may be a correct representation of the object, the relative size of the various parts of the drawing must be the same as the corresponding parts of the object itself. In other words, the parts of the drawing must all be made on the same scale. There are a variety of instruments used in drawing, some of the most important of which are represented in the plate.

Fig. 1. This figure represents a drawing-board, a T-square, and a triangle. Drawing-boards are sometimes made with a frame surrounding the board, by which to confine the paper; but experience proves that the kind here exhibited are cheapest and best. Any ordinary workman can make one. Take a board about two feet wide and three feet long, and screw cleats beneath, to keep it from warping. Dress the top perfectly smooth and level, and see that the ends are perfectly parallel and at right angles to the sides. To prepare it for use, cut your paper somewhat smaller than the board. Sponge it all over, and having pasted or glued the edges of the paper to the upper surface of the board, put it aside. In drying, the paper will stretch tight, and present a smooth firm surface to draw upon, which being done, the drawing may be cut out, and the board cleaned off with hot water.

The T-square is used to draw parallel lines. By moving the shoulder of the cross-piece along the end of the board, any number of parallels may be produced.

In using the triangle, the T-square is held firmly on the board, and by placing the triangle, with either of its edges, against the rule of the square, oblique or perpendicular lines may be drawn to it, and by moving the triangle along the rule, lines parallel to these may be produced.

Fig. 2 is a flat instrument, usually made of ivory, upon one side of which is a variety of scales; those following the numbers 20, 25, &c., are scales of equal parts, differing only in respect to the length of the unit of the scale. The first unit, which is usually not numbered, is divided on the upper side into twelve, and on the lower side into ten, equal parts. In drawing an object, having determined which of the scales shall be used, and how many units of measure of any given denomination on the object, as inches or feet, shall be represented by a unit of the scale, take from the scale, with the divisions, the number of units corresponding to the length of any line on the object, and lay them down upon the drawing. The fractional parts of the unit of the scale may be taken nearly by means of the divisions of the first unit in the scale. When greater accuracy is required in the fractional parts, the diagonal scale of equal parts, which is represented on the lower side of the figure, is used. The first unit in this scale is divided, both on the upper and lower side, into ten equal parts, and
oblique lines drawn from the first point of division on the one side to the second on the other, from the second on the one to the third on the other, &c. It is, also, divided into ten equal parts by horizontal lines. By this means, the length of a line can be taken off accurately to one one-hundredth of the unit of the scale. If the fractional part of the unit consists of any number of tenths and hundredths, open the dividers till one foot reaches the oblique line, denoted by the tenths, counting from the right, on the horizontal line, denoted by the hundredths, counting from the bottom.

The scale marked \( O H O \) is a scale of chords. It is formed, as the figure indicates, by laying down, upon a right line, the chords of one, two, three degrees, &c., to ninety degrees. This scale is useful in laying down an angle containing any given number of degrees, and in measuring the number of degrees in a given angle. To lay down an angle containing any number of degrees, draw one side of the angle; from one extremity of the side, as a centre, with a radius equal to the chord of sixty degrees, describe an arc; lay off, from the side on this arc, the chord of the given number of degrees, and, through the extremity of the chord, draw from the centre the other side of the angle. To measure the number of degrees in a given angle, from the vertex of the angle as a centre, with a radius equal to the chord of sixty degrees, describe an arc intersecting the sides, and the chord of the intercepted arc will indicate the number of degrees in the angle.

The other side of this instrument is represented in Fig. 4. By means of the graduation of its perimeter, it is made to answer the same purpose as the semicircular Protractor, which will soon be explained.

Fig. 3 is a Sectoral scale of equal parts. It consists of two arms, which turn upon a joint, and upon each of which is a scale of equal parts. Its use is the same as that of the above scales of equal parts. Having determined upon a scale for drawing, as, for instance, five feet to the inch, open the dividers to the distance of an inch, and then open the arms of the instrument, until the dividers will extend from 5 on one arm to 5 on the other. Then, to take off any number of feet with the dividers, as, for instance, nine, the angles of the arms remaining unchanged, open the dividers until its feet extend from 9 on the one side to 9 on the other. Other scales are usually laid down on this instrument, but they are of no importance to the student of practical carpentry.

Fig. 4 is a Semicircular Protractor. It is used, like the scale of chords, in laying down and measuring angles. To lay down an angle of any number of degrees, place the centre of a circle at the vertex of the angle; make the diameter coincide with one side of the angle, and then count off on the circumference the required number of degrees, and draw from the vertex the other side of the angle. To measure an angle, place the centre at the vertex of the angle, and count on the circumference the number of degrees included between the sides of the angle.
MENSURATION.

OF SURFACES.

We determine the area or contents of a surface, by finding how many times the given surface contains some other surface which is assumed as the unit of measure. Thus, when we say that a square yard contains 9 square feet, we should understand that one square foot is taken as the unit of measure, and that this unit is contained 9 times in the square yard.

The most convenient unit of measure for a surface is a square whose side is a linear unit in which the linear dimensions of the figure are estimated. Thus, if the linear dimensions are feet, it will be most convenient to express the area in square feet; if the linear dimensions are yards, it will be most convenient to express the area in square yards, &c.

I. To find the Area of a Square, a Rectangle, or a Parallelogram.

Multiply the base by the altitude, and the product will be the area.

Ex. 1. What is the area of a square whose side is 204-3 feet? Ans. 41738-49 sq. ft.
Ex. 2. To find the area of a rectangular board, whose length is 12$\frac{1}{2}$ feet, and breadth 9 inches. Ans. 9$\frac{3}{8}$ sq. ft.
Ex. 3. To find the number of square yards of plastering in a parallelogram, whose base is 37 feet, and altitude 5 feet 3 inches. Ans. 21$\frac{1}{4}$ sq. yds.

II. To find the Area of a Triangle.

Multiply the base by the altitude, and take half the product. Or, multiply one of these dimensions by half the other.

Ex. 1. To find the number of square yards in a triangle, whose base is 40 feet, and whose altitude is 30 feet. Ans. 66$\frac{3}{4}$ sq. ft.

III. To find the Area of a Trapezoid.

Add together the two parallel sides; then multiply their sum by the altitude of the trapezoid, and half the product will be the required area.

Ex. 1. How many square feet are contained in a plank, whose length is 12 feet 6 inches, the breadth at the greater end 15 inches, and at the less end 11 inches? Ans. 13$\frac{3}{4}$ sq. ft.

IV. To find the Area of a Quadrilateral.

Join two of the angles by a diagonal, dividing the quadrilateral into two triangles; then, from each of the other angles, let fall a perpendicular on the diagonal; then multiply the diagonal by half the sum of the two perpendiculars, and the product will be the area.

Ex. 1. How many square yards of flooring in the quadrilateral, whose diagonal is 65 feet, and the two perpendiculars let fall upon it, 28 and 33$\frac{1}{2}$ feet? Ans. 222$\frac{1}{2}$ sq. yds.

V. To find the Area of a regular Polygon.

Multiply half the perimeter of the polygon by the apothegm or perpendicular, let fall from the centre on one of its sides, and the product will be the area required.

Ex. 1. To find the area of a regular hexagon whose sides are 20 feet each, and whose apothegm is 17-3205 feet. Ans. 1039.23 sq. ft.

* Davies' Legendre.
MENSURATION.

VI. To find the Circumference of a Circle when the Diameter is given, or the Diameter when the Circumference is given. Multiply the diameter by 3·1416, and the product will be the circumference; or, divide the circumference by 3·1416, and the quotient will be the diameter.
Ex. 1. What is the circumference of a circle whose diameter is 25? Ans. 78·54.
Ex. 2. What is the diameter of a circle whose circumference is 6850? Ans. 2180·41.

VII. To find the Length of an Arc of a Circle containing any Number of Degrees. Multiply the number of degrees in the given arc by 0·0087266, and this product by the diameter of the circle.
Remark. — When the arc contains degrees and minutes, reduce the minutes to the decimal of a degree, which is done by dividing them by 60.
Ex. 1. To find the length of an arc of thirty degrees (30°), the diameter being 18 feet. Ans. 4·712364 ft.

VIII. To find the Area of a Circle.
Multiply the circumference by half the radius. Or, multiply the square of the radius by 3·1416.
Ex. 1. How many square yards in a circle whose diameter is 3½ feet? Ans. 1·069016.

IX. To find the Area of a Sector of a Circle.
Multiply the arc of the sector by half the radius.
Ex. 1. To find the area of a sector whose arc is 20 feet, the radius being 10 feet. Ans. 100 sq. ft.

X. To find the Area of a Segment of a Circle.
Find the area of a sector having the same arc; find the area of the triangle formed by the chord of the segment and two radii of the circle; then add these two areas together when the segment is greater than a semicircle, and subtract the triangle from the sector when it is less.
Ex. 1. Required the area of the segment whose cord is 16, the diameter being 20. Ans. 44·764.

OF SOLIDS.

The Mensuration of Solids is divided into two parts: first, the mensuration of their surfaces; and, secondly, the mensuration of their solidities.

We have already seen, that the unit of measure for plane surfaces is a square whose side is the unit of length. A curved line, which is expressed by numbers, is also referred to a unit of length, and its numerical value is the number of times which the line contains its unit. If, then, we suppose the linear unit to be reduced to a right line, and a square constructed on this line, this square will be the unit of measure for curved surfaces.

The unit of solidity is a cube, the face of which is equal to the superficial unit in which the surface of the solid is estimated, and the edge is equal to the linear unit in which the linear dimensions of the solid are expressed.

The following is a table of solid measures:

<table>
<thead>
<tr>
<th>Cubic Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1728 cubic inches</td>
</tr>
<tr>
<td>27 cubic feet</td>
</tr>
<tr>
<td>44921 cubic feet</td>
</tr>
</tbody>
</table>

I. To find the Surface of a right Prism.

Multiply the perimeter of the base by the altitude, and the product will be the convex surface; to this add the area of the two bases, when the whole surface is required.

Ex. 1. What must be paid for lining a rectangular cistern with lead, at 2d. a pound, the thickness of the lead being such as to weigh 7 lbs. to each square foot of surface; the inner dimensions of the cistern being as follows, viz.: the length 3 feet 2 inches, the breadth 2 feet 8 inches, and the depth 2 feet 6 inches? Ans. 2l. 3s. 10½d.
II. To find the Surface of a right Pyramid.

Multiply the perimeter of the base by half the slant height, and the product will be the convex surface; to this add the area of the base.

Ex. 1. What is the entire surface of a right pyramid whose slant height is 15 feet, and the base a pentagon whose sides are each 25 feet? Ans. 2012.798 sq. ft.

III. To find the Solidity of a Prism.

Find the area of the base; multiply this area by the altitude, and the product will be the solidity of the prism.

Ex. 1. What are the solid contents of a cube whose side is 24 inches? Ans. 13824 cu. in.

Ex. 2. How many cubic feet in a block of marble of which the length is 3 feet 2 inches, breadth 2 feet 8 inches, and height or thickness 2 feet 6 inches? Ans. 21\(\frac{1}{2}\) cu. ft.

Ex. 3. Required the solidity of a triangular prism whose height is 10 feet, and the sides of its triangular base 3, 4, and 5 feet. Ans. 60 cu. ft.

IV. To find the Solidity of a Pyramid.

Multiply the area of the base by one-third of the altitude, and the product will be the solidity.

Ex. 1. What is the solidity of a pyramid, each side of its square base being 30 feet, and the altitude 25 feet? Ans. 7500 cu. ft.

V. To find the Surface of a Cylinder.

Multiply the circumference of the base by the altitude, and the product will be the convex surface; to this add the areas of the two bases.

Ex. 1. Required the entire surface of a cylinder, the diameter of whose base is 2, and whose altitude is 20. Ans. 1319.9472.

VI. To find the Surface of a Cone.

Multiply the circumference of the base by half the slant height; to this add the area of the base.

Ex. 1. Required the entire surface of a cone whose slant height is 36 feet, and the diameter of its base 18 feet. Ans. 1272.848 sq. ft.

VII. To find the Solidity of a Cylinder.

Multiply the area of the base by the altitude.

Ex. 1. Required the solidity of a cylinder whose altitude is 12 feet, and the diameter of its base 15 feet. Ans. 2120.58 cu. ft.

VIII. To find the Solidity of a Cone.

Multiply the area of the base by the altitude, and take one-third of the product.

Ex. 1. Required the solidity of a cone whose altitude is 27 feet, and the diameter of the base 10 feet. Ans. 706.86 cu. ft.

IX. To find the convex Surface of a spherical Zone.

Multiply the altitude of the zone by the circumference of a great circle of the sphere, and the product will be the convex surface.

Ex. 1. The diameter of a sphere being 42 inches, what is the convex surface of a zone whose altitude is 9 inches? Ans. 1187.5248 sq. in.

X. To find the Solidity of a Sphere.

Multiply the surface, found by the preceding rule, by one-third the radius. Or, cube the diameter, and multiply the number thus found by 0.5236.

Ex. 1. What is the solidity of a sphere whose diameter is 12? Ans. 904.7808.

XI. To find the Solidity of a spherical Segment.

Find the areas of the two bases, and multiply their sum by half the height of the segment; to this product add the solidity of a sphere whose diameter is equal to the height of the segment.

Remark.—When the segment has but one base, the other is to be considered equal to 0.

Ex. 1. What is the solidity of a spherical segment, the diameter of the sphere being 40, and the distances from the centre to the bases being 16 and 10? Ans. 4297.7088.
OF CARPENTRY.

LININGS FOR SOFFITS.

DEFINITIONS.

The Lining of a Soffit, in Carpentry, signifies the covering of any concave surface. The lining is drawn as if spread out on a plane, so that when bent to the curve it will exactly fit the Soffit.

A Soffit, in Architecture, is the under side of the head of a door, window, or the intrados of an arch. It may be either plane or curved.

PLATE 10.

Fig. 1. To draw the Lining for the Soffit of a Window or Door, having parallel Jambs and a semicircular Head, which cuts obliquely through a straight Wall.

Let C be the plan or opening of the window, and let the base of the semicircle B be drawn at right angles to the jambs or sides of the plan C. Divide the circumference into any number of equal parts, as ten, and from these points draw perpendiculars to its base across the plan; extend the parts around B on the stretch-out line, and from these points erect perpendiculars; now make 1'a' = 1 a, 2'b' = 2 b, and so on. Trace a curve through the points a' b' c' d', &c., and it will be one edge of the lining; the other is obtained in a similar manner.

If it is desired to make the cylinder, whose length shall be only the thickness of the wall, the form of the end of it is seen at D, which is of course a semi-ellipse, traced by ordinates from the semicircle B.

Fig. 2 shows the method of getting the lining for the soffit of a semicircular window or door-head, cutting right into a circular wall. The principle of this, and the following, is similar to the preceding, and requires no further explanation.

Fig. 3 shows the lining for a soffit of an opening, which cuts obliquely into a circular wall.
PLATE 11.

Fig. 1.  To draw the Lining for the Soffit of an Opening, with a circular Head splaying equally all around, in a straight Wall.

Let A be the plan of the opening; continue the sides or jambs a c and d b until they meet in e; then, with the centre e, describe the edges of the lining C; make the curve a b' equal in length to the semicircumference at B, and the lining C, of the soffit, will be determined.

For, conceive the semicircle B to be turned at right angles to the plan A, then every point in the semicircumference at B will be equidistant from the point e. But C is described with the same radius = a e; therefore, the edge a b' of the lining C, may be brought to coincide with the arch of the semicircle B.

Fig. 2.  To draw the Lining for the Soffit of an Opening, with a circular Head splaying equally all around, in a circular Wall.

Divide the semicircumference at B into any number of equal parts, as ten, and draw perpendiculars from these points to the base o o; thence draw lines to meet in f; from the points 1' 2' 3' 4', &c., in the stretch-out line, which correspond to 1 2 3 4, &c., at B, draw lines also converging to f; now, from the points of intersection a b c d e, on the plan of the opening A, draw lines to o f, parallel to the base o o, and with the centre f, and radius f i, draw the arc i e'; also, with f h, draw the arc h d', and so on; then, by tracing a curve through the points a' b' c', &c., we have half the outer edge of the lining, from which the other half is easily obtained. The lower edge is found on the same principle, and thus the lining C of the soffit is determined.

For it is easy to conceive that, if the semicircle B be turned at right angles to the plan A, the points 1' 2' 3', &c., may be brought to coincide with the points 1 2 3, &c., on the arch B; then, the points a' b' c', &c., will stand perpendicularly over the points a b c, &c., on the plan; because the arcs e a', f b', g c', &c., will lie directly over the lines e a, f b, g c, &c.

Note.  The learner is advised to cut this, and the following soffits, out of pasteboard, which will familiarise the principles.
PLATE 12.

Fig. 1. To draw the Lining of a cylindrical Soffit cutting right in a Wall which does not stand perpendicular to the Ground, so that the Edge of the Lining will stand directly over the base Line of the Aperture.

Let a g, at A, be the level of the ground, and a m the line of the wall, equal to the radius of the cylinder; with this radius describe the semicircle at B; take the distance a b to the foot of the perpendicular m b, at A, and place it from a to 4, at B. Now, upon the lines o a and a 4, as axes, draw, by means of the intersecting lines exhibited, the semi-ellipse 0, 1, 2, 3, 4, &c.; divide the semicircumference into any number of equal parts, as four, and let fall the perpendiculares to the base line o c; also, at C, make the line o o equal to the stretch-out of the semicircumference, at B, and lay down the corresponding division. Now, at A, draw m g perpendicular to m a, and take from B the distances d 1, c 2, b 3, a 4, and set them, at A, from g, thus, g f, f e, d d, d c; erect perpendiculars from each of the points f, e, d, c, towards the line g m; on the lining, C, make the distances 1 d, 2 e, 3 b, 4 a, equal to the distances g h, h i, i k, k l, at A, respectively; then, trace a curve, at C, through the points o, d, c, b, a, and these points of the lining, when it is bent around the curve on the plan B, will stand directly over o, d, c, b, a, at B, and the points 1, 2, 3, 4, of the lining C, will coincide with the points 1, 2, 3, 4, at B.

Fig. 2. To find the Lining for the Soffit of an Aperture in a straight Wall, whose Plan is a Trapezoid, and whose Elevation on the inside is a Semi-ellipse, and on the outside a Semicircle, so related to each other, that a Straight-edge coinciding with the lined Surface may everywhere be horizontal.

Let the trapezoid A B C D be the plan of the aperture, A B being the inner side, and C D being the outer side, while A C and B D represent the jambs; with the centre G and radius G D describe the semicircle D E' C, which will be the outside elevation; also, taking E F = G E', describe the semi-ellipse A E B, which will be the inside elevation; continue the jambs A C and B D until they meet in H, and through H and G draw the line H E. Now, divide the quarter circumference E E' into any number of equal parts, as five, and from these points 1, 2, 3, 4, let fall perpendiculars to C G. From H, through the points e, f, g, h, the foot of each perpendicular, draw lines intersecting the line A B.

Draw the line H K perpendicular to A H, and equal to C G, and divide it like to C G, making H l = a h, l m, h g, &c. Now, with the radius H e and the centre l, describe an arc towards e'; also, with the radius H f and the centre m, describe an arc towards f', and so on. Having taken the fifth part of the arc C E', such as C 1, fix one foot of the dividers in C, and with the other intersect the first arc at e'; then, place the foot in e', and intersect the next arc at f', and so on, to o'. Trace a line through the points c, e', f', g', h', o', and this will be half the outer edge of the lining, which will lie over the line C, e, f, g, h, o, of the plan, and will coincide with the arc c, 1, 2, 3, 4, e', which is turned around C G, so as to stand at right angles to the plan. The other half of the outer edge of the lining c' d' is similar, and found by inversion, making the angle I K L = I K H, and the divisions on K L equal to those on K H.

Extend the lines l e', m f', n g', and o h', towards a', b', c', d', and take e' a' = e a on the plan; also, f' b' = f b, and so on. Then, through the points A, a', b', c', d', f', trace the line A F', and, having obtained in like manner its prolongation f' b', the line A b' will be the outer edge of the lining. Now, the figure A b' d' c will be the entire development of the lining, which, when bent over the plan, will exactly fit the soffit.
LININGS FOR SOFFITS.—TRACERY.

PLATE 13.

Fig. 1. To find the Lining for the Soffit of an Aperture in a circular Wall, the Jambs being inclined to each other, and the covered Surface splaying, so that a Straight-edge, coinciding with it, may everywhere be horizontal.

The method for obtaining these lines is similar to that exhibited on the last plate. The nature of the aperture is the same, and the principle explained there, as applying to a straight wall, is here applied to a circular wall. The curve of the wall is drawn upon the plate with a very short radius, in order to exhibit the method more plainly. Find the line $s c' o'$, as in the last plate; take the distances from the line $s c o$ to the arcs representing the faces of the wall, and transfer these distances to the lining, and the edges will be found as in the last.

TRACERY.

Fig. 2. Having given any Gothic Arch, to draw another, either right or rampant, so that the two shall intersect or mitre truly together.

Let $A$ be the given arch. Draw the chord $a o$, and divide it into any number of equal parts, as four; then, from the point $e$, draw lines through 1, 2, 3, intersecting the arch at $h g f$. Erect $a d$ perpendicular to $a e$, and from $o$, through $f g h$, draw lines intersecting the perpendicular in $d, c, b$. Now, let the arch $B$, which we wish to draw in correspondence with $A$, have the same height, $e o$, and a greater base, $a e$; draw the line $a o$, and divide it also into four equal parts; make the divisions on $a d$ equal to those on $a d$ at $A$, and draw lines from $o$ to the points $b, c, d$. Having drawn lines from $e$, through 1, 2, 3, trace the curve $o f g h a$, through the points of intersection. This will give the desired arch.

By similar construction, the rampant arch $C$ may be made to correspond with either $A$ or $B$. 
PLATE 14.

To draw Arches of various Forms, and to find the Lines of the Joints between the Arch-stones.

As carpenters are frequently called upon to prepare centring for arches, and also to cut out patterns of the arch-stones, to be used by the stone-cutter, we have thought best to introduce a plate which will familiarise the student with the best methods of drawing and dividing arches.

Fig. 1. This is the semicircular, or perfect arch. It is drawn from the centre c, and the joints between the voussoirs are parts of the radii. If it is not convenient to draw the radii, make each line perpendicular to a tangent, as at t.

Fig. 2. This is a diminished, surbased, segmental, or imperfect arch; being composed of an arc less than a semicircle. An easier method of drawing the joint is here exhibited. From the points 1 and 3, as centres, draw small intersecting arcs above the arch; and from the point of intersection, draw a line through the point 2, bisecting 1, 3; this will give the line of the joint correctly.

Fig. 3. This is an Arabesque, Saracenic, or horse-shoe arch, sometimes also called the Oriental arch. It consists of an arc greater than a semicircle. The joints between the springers lower than the centre, must not be drawn from the centre, as c c, but must be made parallel to the impost, or base line d e, as a b.

Fig. 4. The elliptical arch. Various methods for drawing the ellipse are laid down in the first part of the work, and it is unnecessary to repeat them. To draw the joints, let f f' be the foci of the ellipse; then, a line bisecting the angle f 1 f' will give the first joint, and so on. If the curve is to be composed of a series of arcs of a circle, the points o, a, b, c, may be used as centres.

Fig. 5. The Gothic lancet arch consists of two arcs, the radii of which are longer than the span a b. The joints are drawn from the points o o'.

Fig. 6. The equilateral arch is described by radii equal to the span.

Fig. 7. The obtuse pointed arch is described by radii shorter than the span.

Fig. 8. The ogee, contrasted, or reflected arch, is described from four centres, two within, and two without the arch, a, b, o, o'. The proportions may be varied at pleasure.

Fig. 9. The Tudor arch is described from four centres within the arch, o, o', o, o'. For an arch whose height is half its span, they may be found thus: divide the base line a b into three equal parts at o and o'; then will o' be the centre of the arc b n; from d, through o, draw a line, and make the distance from c to o' equal to c d; then is o' the centre with which to describe the arc d n. Those arches which have their height greater or less than half the span, are found by other rules; but the proportions here given are perhaps the best. The joints between the voussoirs, in this case also, lie in the direction of the radii.
PLATE 15.

GROINS.

DEFINITION.

GROINS are formed by the intersection of arches or vaults, and the surfaces where they meet may be considered as the sections of cylinders, &c.

DESCRIPTION OF THE CENTRING FOR BRICK-GROINS.

Fig. 1. $P, P, P, P$, is the plan of the piers which the vault is to stand upon; $a b$, Fig. 2, is the end opening, which is a given semicircle; and $b c$ is the opening of the side-arch, which is to come to the same height as the end-arch $a b$: fix your centres over the body-vault, Fig. 1, as shown in the section at $C$, then board them over. In Fig. 1 is the manner of fixing the jack-ribs upon the boards, which is likewise shown at $C$.

*To find the Mould for the Jack-ribs.*

Take the opening of your arches in Fig. 1, that is, $a b$ and $a d$, and lay them down in Fig. 2, at $a b$ and $b c$, to make a right-angle. Divide one half of the given semicircle, $E$, into five parts, and draw perpendiculars across, at 1, 1, 1, &c., to cut $b d$ and $d c$, the diagonals in 2, 2, 2, &c. Now, through the points 2, 2, 2, &c., draw lines parallel to 1, 1, 1, &c., at the base of $E$, both ways towards $F$ and $G$; stick in nails at 1, 2, 3, 4, 5, in the arc of $E$, and bend a thin slip of wood round them, which mark with a pencil at every nail; this slip of wood being stretched out from $d$, 1', 2', 3', 4', 5', and perpendiculars drawn towards $G$, will intersect the other lines, making small rectangles: a curve being traced through the intersections, will give a mould to set the jack-ribs.

*To place the Jack-ribs.*

Bend your mould, $G$, from $a$ to the crown at $e$, in Fig. 1, which will give the edges of your boards; then fix a temporary piece of wood, level upon the crown, in the direction of $f f$, and let it come the thickness of the boards lower than the crown, and it will give the height of the jack-ribs, which is a very sure method of placing them.

*To find a Mould to cut the Ends of the Boards.*

The semi-ellipse $F$ is traced to the height of $E$, or drawn by a trammel. Take the parts round $F$, and lay them out to 1', 2', 3', 4', 5'; then $H$ will be found in the same manner as $G$, which will be a mould to cut the ends of the board that goes upon the jack-ribs against the body-vault.

*To find the Moulds when both Arches have the same Opening.*

Fig. 3. Take half the opening of the arches, whatever they are, and draw a quarter-circle, and divide it into six parts; bend a slip round it to take its parts, then stretch it out upon the base from 0 to 6', and draw perpendiculars from the points 1', 2', 3', &c. Through the points in the arc, draw the lines on both sides, parallel to 0, 6'; the curve being traced as before, gives both moulds of an equal and similar form.
PLATE 16.
The Plan and Inclination of an ascending Groin, one of the Body-ribs, and the Place of the
Intersection on the Plan, being given, to find the Form of the Side-ribs, so that the Intersection
of the Arches shall lie in a perpendicular Plane.

Divide half the circumference of the body-rib, at B, into any number of equal parts; draw lines from these points perpendicular to its base, and continue them to the line of
intersection on the plan; from thence, let them be drawn at right-angles towards C, and
make the distances 1 b, 1 c, 1 d, 1 e, &c., at C, equal to the corresponding distances at B; then
will the curve a b c d, &c., be the true curve of the side-rib. This curve is a semi-ellipse, and
may be found by intersecting lines, as at F, according to the rule for describing a rampant
ellipse.

To find the Moulds for placing the Jack-ribs.

At c, draw lines from the points a, b, c, d, &c., perpendicular to the line of ascent h g,
towards D and E; draw the semi-ellipse A as wide as the body-range, and as high as a h at
c; continue the ordinates 1 b, 1 c, 1 d, &c., up to A; bend a slip around A, and mark upon
it the points 0, 1, 2, 3, &c.; extend the slip upon the line k 6, at D and E, and divide it
correspondingly; now, through each of these points, draw perpendiculars across k 6 to inter-
sect the lines drawn perpendicular to the rake; then, curves traced through the points of
intersection will give the moulds for placing the jack-ribs. The edges of these moulds, bent
over the body-vault when boarded in, will exactly coincide with the intersection of the side
and body vaults.

To find the Jack-ribs of the Side-groins.

Draw the number of the jack-ribs upon the arch B, at their proper distances, and take
their several heights, h i, k l, m n, &c., and set them upon the arch G from a to b, from b to
c, from c to d; draw lines through b c d, parallel to the rake, and they will show on the curve
the proper length and form of the jack-ribs.

To bevel the Body-ribs.

Since all the body-ribs stand perpendicular to the plan, the upper edge must be bevelled
to correspond to the rake of the groin. To do this, let the under edge 1 1 1 1, at B, of the
body-ribs, be bevelled according to the rake, so that they may stand perpendicular; then,
take a mould from B, or one of the body-ribs will answer, and place it on each side of the
rib to be cut, making the lower bevelled edges correspond. The upper edges may now be
marked and bevelled.
PLATE 17.

The two Arches and Inclination of an ascending Groin being given, to find the Place of the Intersection on the Plan.

Divide half of the body-rib $B$ into equal parts, and thence draw lines, parallel to its base, to the line $a f$. With $a$ as a centre, draw the concentric arcs from the points $b, c, d, e, f$, around to $h, g, i, k, l$, and thence draw lines, parallel to the rake, to cut the arch $C$ at 1, 2, 3, 4, and $a, b, c, d$. From these points, draw lines through the plan, perpendicular to the direction of the body-vault. From the centre $g$, erect a perpendicular to the rake, cutting the arch at 5. From 5, draw another line through the plan. Also, from 1, 2, 3, 4, 5, at $B$, draw lines through the plan, parallel to the direction of the body-vault, and trace curves from the piers to $h$, through the points of intersection. These curves will be the place on the plan of the intersection of the vaults.

The moulds $D$ and $E$, and the jack-ribs, are found in the same way as those on the last plate.

Remarks.—Groins similar to these last may be seen under the Adelphi Buildings in the Strand, London, where the declivity in going to the river is very steep.

In practice, there is no occasion for tracing these intersections on the plan, since the moulds $D$ and $E$ are sufficient, without reference to the plan. To use these moulds, for this and all other kinds of brick-groins, the centres or body-ribs must be placed first, as if there were no side-arches cutting across them, and then closely boarded over. The moulds $D$ and $E$ must both be bent over the body-vault together. Place the points $l$ and $e$ upon the piers at $o$ and $e$. Then, keeping the top-points at $5'$ together, bend the moulds over the body-vault, and the point $5'$ will stand perpendicularly over $h$ on the plan. Now, along the inner edges of the moulds, draw a curve on the boards of the body-vault, and this will be the line of intersection. The jack-ribs of the side-vault are set to this line at the proper distances from each other.

It must further be observed, that the arch $B$ must not be used instead of the arch $A$, since this would produce a very great error in the moulds $D$ and $E$; for it is evident that a section of the body-vault, perpendicular to its axis, must be less in height than the vertical section $B$, since this is oblique to the cylinder. Hence, the moulds being drawn by means of perpendiculars to the rake, the arch $A$ must necessarily be used in finding them. If all these things are properly understood, no difficulty can occur in brick-groins which may not easily be surmounted.
GROINS.

PLATE 18.

Fig. 1. The Diagonals of a plaster Groin, which are straight upon the Plan, and one of the Side-arches being given, to find the Jack-ribs.

Lay down the plan of the ribs, as at B, and draw a rib upon each opening; draw perpendicular lines from the plan of each opening, at the extremities a e, to cut its corresponding ribs at b d f; then, the distance from b to b shows the length of the first jack-rib, from d to d the length of the second, and from f to f, the third.

How to bevel the Angle-ribs, so that they shall range with the Opening of the Groin.

First, get out the ribs in two halves, or thicknesses, as at E and F; then, draw the plan of the angle-rib, which, placed between E and F, will show the true ranging upon the bottom of the rib; now shift your rib-mould parallel to the base of E and F, and it will show how much wood there is to be bevelled off; then nail the two halves together, and the rib will be complete.

To find the Jack-ribs, when the given Arch is the Segment of a Circle.

The ribs in this case may be found by the method explained on Plate 13, Fig. 2, as shown upon this plate at B, E, and F, Fig. 2; also, we may take the height of the segment A, Fig. 2, and place it from b to c, at C and D; now take twice the radius a c, at A, and place it from c and c, the crowns of C and D, to a and d; the arches C and D, which are parts of ellipses, may then be drawn by intersecting lines, as explained on Plate 6, Fig. 2. Either of these methods is much easier, in practice, than to trace the ribs through ordinates.
PLATE 19.

Given one of the Body-ribs, the Angles straight upon the Plan, and the Ascent of a Groin not standing upon level Ground, to find the Form of the ascending Arches, and the Angle-ribs.

Let \( b a c \) at \( B \) be the angle of the ascent; from the point \( b \) make \( b c \) perpendicular to \( a b \), and describe the rampant curve \( B \); then draw the diagonal \( a b \) at \( E \), and make \( b c \) perpendicular to it, and equal to \( b c \) at \( B \); then draw the hypothenuse \( a c \), and describe the angle-rib \( E \), in the same manner as that of \( B \).

To find the Length of the Jack-ribs, so that they shall fit to the Rake of the Groin.

Draw lines up from the plan to the arch, as at \( D \), in the same manner as explained heretofore; then the arch from \( a \) to \( a \) is the first jack-rib, from \( b \) to \( b \) the second, and from \( c \) to \( c \) the third, &c.

To range the Angle-ribs for these Groins.

Get the ribs out in two halves, as in the last plate; then the bottom of the ribs must be bevelled agreeably to the ascent of the groin, and the plan of it must be drawn upon the level, and from thence they may be drawn perpendicular from the plan to the rake of the rib; then take a mould to the form of the rib, or the rib itself, and slide this agreeably to the rake to the distance that is marked upon the bottom to be backed off; this will show how much the rib is to bevel all around.
Given, one of the Body-ribs, and the Width and Height of the Side-arch, to find the Side and Angle-ribs, so that the Place of the Intersection, on the Plan, shall be a straight Line.

Draw $ce$, at $F$, perpendicular to the base of the body-rib $B$, and equal to the height of the side-arch $D$; from $e$, draw $ea$ parallel to $cb$, and from $a$ let fall the perpendicular $abk$; draw $km$, and $kl$, which are the places on the plan of the intersection; then, the ribs $D$ and $E$ may be described from $F$, as directed in the problem at Fig. 2, Plate 13; or they may be obtained by intersecting lines, as at $A$ and $C$. The first method is, however, much the easiest in practice; for by using the points $k$, $g$, and $f$, at $E$ and $D$, as centres, the lines may be struck by a chalk-line much sooner than the parallels at $A$ can be drawn, and with greater accuracy. But it must be recollected that four or five points will not be sufficient, in practice, for tracing the curve with accuracy; and, therefore, a greater number must be found.

The way in which the groin is put together is shown below, in a manner sufficiently intelligible for any workman. These groins are applicable when a window or door is to be made in the side of a barrel-vault.
PLATE 21.

DEFINITION.

A Welsh groin is a groin whose side-arches are less in height than the body-arch, and both the side and body-arches are given semicircles, or similar segments.

*Given, the Body and Side-ribs of a Welsh Groin, to find a Mould for the intersecting Ribs.*

Divide half the side-rib B into any number of parts, and from these points 1, 2, 3, d, let fall perpendiculars to a b, its base, and produce them beyond; also, from the same points, draw lines parallel to a b, to intersect e f; transfer the divisions from e f to e g, and from the divisions of e g draw lines parallel to p q, the base of A, to intersect the body-rib A at h, u, w, y; from these points, draw perpendiculars to p q, and continue them to intersect the perpendiculars from B, at the points k, v, m, n; now trace a curve through these points, and this will be the place of the intersecting rib on the plan; then draw two other curve lines, on either side of, and parallel to this, to show the thickness of the rib on the plan; on the inside curve draw chords from l, and on the outside curve draw tangents parallel to these chords. The distance between the chord and tangent shows the thickness of the stuff for the intersecting rib. Draw perpendicular lines to the chords, and make the heights 1 1, 2 2, 3 3, &c., at D, equal the corresponding heights at B. In a similar manner draw C; then will C and D be the moulds by which to cut the intersecting rib.

*To range the Ribs so that they will stand perpendicular over the Plan.*

At the points x, v, t, i, in the base of C, erect perpendiculars parallel to the ordinates of C and D, and make their corresponding heights equal to the ordinates of B or A; draw the dotted curve h, u, v, y, at C, and it will show how much is to be bevelled off that side of the rib. In like manner, the other side D is to be bevelled.

*To find a Mould to bend under the intersecting Rib, so that the Line of the intersecting Angle may be marked on the Rib according to the Plan.*

Take the stretch-out of the under side of the rib D, by bending a thin slip of wood around it; mark the different points, and transfer them to the straight line b c, at E; then make 1 n, 2 m, &c., at E, equal to 1 n, 2 m, &c., on the plan. A curve drawn through n, m, l, k, at E, will give the mould. The straight edge of the mould, when bent under, must be made to correspond to the chord-line 1 2 3 c, at D; then, by drawing a line by the curved edge of the mould, on the under side of the rib, we have the true place of the intersecting angle.

The method of framing the ribs is shown below, on the Plate.
The intersecting or Angle-ribs of a Groin standing upon an octagon Plan, the Side and Body-ribs being given both to the same height.

Fig. 1. $E$ is a given body-rib, which may be either a semicircle or a semi-ellipse, and $A$ is a side-rib given of the same height; $D$ is a rib across the angles. Trace from $E$, the base of both $E$ and $D$ being divided into a like number of equal parts, and divide the base of the given rib $A$ into the same number of parts. From these points, draw lines across the groin to its centre at $m$, and from the divisions of the base of the rib $D$, draw lines parallel to the side of the groin. Then trace the angle-curves through the quadrilaterals, and the result will give the place of the intersecting ribs. Draw the chords $a \ b$ and $b \ c$, then mark the moulds $B$ and $C$ from $E$ or $D$, taking care not to mark them from the crooked line at the base, but from the straight chords $a \ b$ and $b \ c$.

To describe and range the Angle-ribs of a Groin upon a circular Plan, the Side and Body-arches being given, as in the last Groin.

Fig. 2. The ribs are described in the same manner as in the last example for the octagon groin, or in the same manner as in Plate 21; and the ranging is found in the same manner as is described in that Plate. $E$ and $F$ are the same moulds as are shown at $B$ and $D$. 
PLATE 23.

The Side-rib of a Groin and Angles of Intersection being given straight upon a circular Plan, to find the Angle-rib, and the Body-rib.

Fig. 1. Let the rib \( A \) be supposed to be placed over the straight line \( a \ b \), as its base, which divide into any number of equal parts, as eight; from the points of division draw lines to the centre of the plan, to intersect the angles at \( a, b, c, d, e, f, g \). From these points, erect perpendiculars to the line \( b \ d \), and these being made respectively equal to those at \( A \), will give the curve of the rib \( G \). If from the points \( a, b, c, &c. \), arcs be drawn with the centre of the groin to intersect the base of \( C \), at 1, 2, 3, 4, 3, 2, 1, and perpendiculars be drawn and made correspondingly equal to those of \( A \), and \( C \) be traced to these points, then \( C \) will be the body-rib.

To describe the Ribs of a Groin over Stairs upon a circular Plan, the Body-rib being given.

Fig. 2. Take the tread of as many steps as you please, suppose nine, from \( E \), and the heights corresponding to them, which lay down at \( F \); draw the plan of the angles as in the other groins, and take the stretch round the middle of the steps at \( E \), and lay it from \( a \) to \( b \) at \( F \). Make \( d \ e \) perpendicular to \( d \ c \) at \( B \), equal to \( d \ e \) at \( F \); draw the hypothenuse \( e \ c \); draw perpendiculars from \( d \ c \) up to \( B \), and mark \( B \) from \( A \), as the figures direct; then \( B \) is the mould to stand over \( a \ b \) at \( F \). Draw the chords \( a \ 4 \) and \( 4 \ m \) at the angles; make \( a \ g, 4 \ h \), perpendicular to them, each equal to half the height \( d \ e \), at \( B \) or \( F \); draw the hypothenuse \( g \ 4 \), and \( h \ m \); draw the perpendicular ordinates from the cords through the intersection of the other lines that meet at the angles; then trace the moulds \( D \) and \( C \), from the given rib \( A \), and they will form the moulds for the angle or intersecting ribs.
PLATE 24.

As all the sections of a sphere are circles, and those passing through its centre are equal, and the greatest which can be formed by cutting the sphere, it is evident that if the head of a niche is intended to form a spherical surface, the best method is to make the plane of the back-ribs pass through the centre. This may be done in an infinite variety of positions; but perhaps the best, and that which would be easiest understood, is to dispose them in vertical planes. If the head is a quarter of a sphere, the front-rib, and the still-plate or springing, on which the back-ribs stand, will curve equally with the vertical ones; but if otherwise, they will be portions of less circles. But it is evident, if the front and springing ribs are intended to be arcs less than those of semicircles, either equal to each other or unequal, that as they are placed at right-angles to each other, there is only one sphere which can pass through them; consequently, if the places of the vertical ribs are marked on the plan, these ribs can have only one curve. In the former case no diagram is necessary; but in the latter it may be proper to show how the vertical ribs and their situation on the front-rib are found.

To find the Ribs for the Head of a Spherical Niche, the Elevation being a Semicircle, and the Plan a Segment of a Circle.

From the centre $C$ draw the ground-plan of the ribs as at $A$, and set out as many ribs upon the plan as you intend to have in the head of the niche, and draw them all out towards the centre at $C$. Place the foot of your compass in the centre $C$, and from the ends of each rib, at $e$ and $c$, draw the small concentric dotted arcs round to the centre rib at $m$ and $n$; and draw $mg$ and $ni$ parallel to $rs$, the face of the wall; then from $q$ round to $o$ upon the plan, is the length and sweep of the centre-rib, to stand over $ab$; and from $i$ round to $o$, the length and sweep of the rib that stands from $e$ to $d$ upon the plan; and from $g$ round to $o$ is the sweep of the shortest rib, that stands from $e$ to $f$ upon the plan.

To bevel the Ends of the Back-ribs against the Front-rib.

The back-ribs are laid down distinct by themselves at $C, D$, and $E$, from the plan. Take 1 to 1', in $A$, and set it from 1 to 1' in $D$; it will give the bevel of the top of the rib $D$. And from $A$, take from 2 to 2' upon the plan, and set from 2 to 2' in the rib $E$; it will give the bevel of the top.

To find the Places of the Back-ribs where they are fixed upon the Front.

From the points $a, c$, and $e$, at the ends of the ribs, in the plan, at $A$, draw the dotted lines up to the front-rib, to $a', c'$, and $e'$, which will show where they are to be fixed upon the front-rib. The double circle upon the front-rib shows the ranging.
To find the Ribs of a Spherical Niche, the Plan and Elevation being given Segments of Circles.

The elevation of the niche is shown at $A$, being the segment of a circle whose centre is $t$. At $B$ is the plan of the same width, which may be made to any depth, according to the place it is intended for; its centre is $c$. On the plan $B$, lay out as many ribs as it will require, and draw them all tending to the centre at $c$; they will cut the plan of the front-rib in $g, f, e, d$. Through the centre $c$, draw the line $m n$, parallel to $a b$, the plan of the front-rib; put the foot of your compass in the centre at $c$, draw the circular lines from $a, g, f, e, d$, to the line $m n$, and make $c s$ equal to $u t$; that is, make the distance from the middle of the chord-line $m n$ to $s$, the centre of the arch at $C$, equal to the distance from the middle of the chord $A B$ at $A$, to the centre at $t$; then place the foot of your compass in $s$, as a centre, and from the extremities $m$ or $n$, describe the arch at $C$. With the same centre, draw another line parallel to it, to any breadth that you intend your ribs shall be; then $C$ is the true sweep of all the back-ribs in the niche.

The points $l, k, i, h$, show what length of each rib will be sufficient from the point $m$; from $h$ to $m$ is the rib that will stand over $d x$; from $i$ to $m$ is the rib that will stand over $e y$; from $k$ to $m$ over $f v$; and from $l$ to $m$ over $g w$; the other half is the same.
The Plan of a Niche in a circular Wall being given, to find the Front-rib.

$B$ is the plan given, which is a semicircle whose diameter is $a\ b$, and $a, i, k, l, m, h$, the front of the circular wall. Suppose the semicircle $B$, to be turned round its diameter $a\ b$, so that the point $v$ may stand perpendicular over $h$ in the front of the wall, the seat of the semicircle standing in this position upon the plan will be an ellipse. Then divide half the arch of $B$ upon the plan into any number of equal parts, as five; draw the perpendiculars $1\ d, 2\ e, 3\ f, 4\ g$. Upon the centre $c$ with the radius $c\ h$, describe the quadrant of a smaller circle, which divide into the same number of equal parts as are round $B$. Through the points $1, 2, 3, 4, 5$, draw parallel lines to $a\ b$, to intersect the others at the points $d, e, f, g, h$. Through these points draw a curve: it will be an ellipse. Then take the stretch-out of the rib $B$, round $1, 2, 3, 4, 5$, and lay the divisions from the centre both ways at $F$, stretched out; take the same distances $d\ i, e\ k, f\ l, g\ m$, from the plan, and at $F$ make $d\ i, e\ k, f\ l$, equal to them, which will give a mould to bend under and mark the front-rib, so that the edge of the front-rib will be perpendicular to $a, i, k, l, m$.

The curve of the front-rib at $A$ is a semicircle, the same as the ground-plan; and the back-ribs at $C\ D$ and $E$ are likewise of the same curvature.

The curve of the mould $F$ will not be exactly true, as the distances $d\ i, e\ k, f\ l, \&c.$, at $B$, are rather too short for the same corresponding distances upon the mould at $F$; but in practice it will be sufficiently near for plaster-work; but those who would wish to see a method more exact, may examine Plate 12, fig. 1, where $C$ is the exact soffit that will bend over its plan at $B$.

In applying the mould $F$ when bent round the under edge of the front-rib, the straight side of the mould $F$ must keep close to the back-edge of the front-rib; and the rib being drawn by the other edge of the mould, will give its place over the plan.
Plate 27.

The Plan and Elevation of an Ellipsoidal Niche being given, to find the Curve of the Ribs.

Fig. A. Describe every rib with a trammel, by taking the extent of each base from the plan whereon the ribs stand to its centre, and the height of each rib to the height of the top of the niche; it will give the true sweep of each rib.

To back the Ribs of the Niche.

There will be no occasion for making any moulds for these ribs, but make the ribs themselves; there will be two ribs of each kind. Take the small distances 1 e, 2 d, from the plan at B, and put it at the bottom of the ribs D and E, from d to 2, and e to 1; then the ranging may be drawn off by the other corresponding rib; or with the trammel, as for example, at the rib E, by moving the centre of the trammel towards e, upon the line e c, from the centre c, equal to the distance 1 e, the trammel-rod remaining the same as when the inside of the curve was struck.

Given one of the common Ribs of the Bracketing of a Cove, to find the Angle-bracket for a rectangular Room.

Let H be the common bracket, b c its base; draw b a perpendicular to b c, and equal to it. Draw the hypothenuse a c, which will be the place of the mitre; take any number of ordinates in H, perpendicular to b c, its base, and continue them to meet the mitre-line a c, that is, the base of the bracket at I; draw the ordinates of I at right-angles to its base; then the bracket at I, being pricked from H, as may be seen by the figures, will be the form of the angle-rib required.

The way to obtain the angle-bracket of a common plaster cornice is similar, and may be seen at K L.
ROOFS.

PLATE 28.

ROOFS.

To find the principal Lines of a Roof.

Let $ABCD$ be the plan of a roof; take $DL$ half the width of the roof, and make $DF$ and $CE$ equal to it. Join $EF$; and having bisected $EF$ at $H$, take $GH$ equal to the height of the roof, and perpendicular to $EF$, and join $GE$ and $GF$, and these lines will be the length of the principal rafters. Join $HC$ and $HD$; produce $HD$, and take $HI$, equal to $HG$; join $IC$, and it will be the length of each hip-rafter. Draw a line, as $a'b'$, perpendicular to $HC$; and with the centre $c'$, draw an arc tangent to $CI$, cutting $GH$ at $d'$. Join $d'a'$ and $d'b'$, and the angle $a'd'b'$ is what is generally termed the backing of the hip-rafter. The bevel is shown at $R$, $st$ being parallel to $CH$.

To find the Bevels of a Purline against a Hip-rafter.

Two cases are here exhibited; the first is when the purline lies level, having two sides horizontal. This is an easy case. The down bevel is shown at $M$, and the other at $N$. The second case is that in which the purline stands at right-angles to the rafter. From the point $c$, the uppermost edge of the purline, as a centre, describe a circle. Draw the two lines $bh$ and $en$ tangent to the circle, and parallel to $FD$. Then from the points $g$ and $k$, where the sides of the purline produced meet the circle, draw $gi$ and $km$ also parallel to $FD$. Draw $ki$ and $mn$ perpendicular to these lines, and join $kf$ and $fn$. The down bevel is shown at $O$, and the side bevel at $P$.

To find the Bevels of a Jack-rafter against the Hip-rafter.

The angle $xfn$ is the side bevel, $fx$ being perpendicular to $FD$. The down bevel is the angle $FGH$. 

PLATE 29.

The lines for a roof having a square or rectangular plan are found by the methods developed on Plate 28.

Fig. 1 is the plan of a roof. Since the plan is a trapezoid, the inclinations of the hips will differ. Upon the lines AC and BD, as diameters, describe semicircles, cutting the ridge-line at E and F. From these points, draw lines to the extremities of the respective diameters, and these lines will be the bases of right-angled triangles, whose height must equal that of the roof, and whose hypothenuse will equal in length the hip-rafters. The bevels for backing the hips are found as in Plate 28.

Fig. 2 is an octagonal plan for a roof, whose height is c b. The lines are obtained as in the preceding cases.

The Plan of a polygonal Roof being given, and a Side-rib of any Form, to find the Angle-rib and the Form of the Covering.

Figs. 3 and 4. Let A B D be the given plan, and a 1 2 3 F be the given side-rib. Divide a F into any number of parts, and, by means of lines drawn and measured as in the figure, describe the angle-rib B 1' 2' 3' C. Draw a E perpendicular to A B, and mark upon it the divisions of a F. Make f f', g g', h h', respectively, equal to d d', c c', b b', and through f', g', h', trace a curve; then the figure E a B will be that part of the covering which is to be bent over a B C on the plan.
PLATE 30.

SKYLIGHTS.

One of the Ribs being given, the square Plan of the Opening or Well, and the octagon Curb above, to find the Ribs and the springing Curve, where the Foot of the Ribs come, so that the interior Finish may correspond with the Curb.

Fig. 1. Let C be the given rib. From various points in it, draw lines perpendicular to eb, and continue them around the curb, parallel to its sides. Erect the perpendiculars to gf at D, and make them respectively equal to those at C; then is D the angle-rib. From the points where the perpendiculars to eb intersect a b, erect others to a b, and make them respectively equal to those at C; then will the curve at S be the springing curve.

The Plan of the Opening being square, to find the springing Curve and the Ribs, so that the interior Finish may be spherical.

Fig. 2. Upon the side mn of the plan, describe the semicircle B; this will be the springing curve (for all sections of a hemisphere, perpendicular to its base, are semicircles). From the centre of the circular curb at A, draw the rib at C, with a radius equal to half the diagonal of the plan. All the ribs will have the same curve. Their several lengths may be determined by drawing, with the centre of A, small concentric arcs from the points a, b, c, around to 1, 2, 3. Thence, draw perpendiculars to p s, meeting C at 1', 2', 3'. The distances from these points towards i will be the lengths of the ribs.

The vertical section of a segmental Dome passing through its Centre, the Plan of the square Opening, and the circular Curb, being given, to find the springing Curve and the Ribs.

Fig. 3. Let the section over the diagonal q m, as shown in the figure, be given, and find its centre s. Bisect m n by a perpendicular, and take a o, equal to s i. With the radius o m, draw the curve m b n, and it will be the springing curve. The angle-rib is shown at C, and the lesser ribs, all having the same curve, are obtained as explained in the last problem.

Fig. 4. These lines are obtained on the same principle as the others. The springing curve is traced from C.
PLATE 31.

Having given the rectangular Plan of the Well and the elliptical Curb of a Skylight, to find the Ribs and springing Curves, so that the Finish may be ellipsoidal.

Since all sections of a prolate-ellipsoid perpendicular to its axis are circles, the rib that stands over $eb$ at $A$ is part of a circumference whose radius is the semi-conjugate diameter. This rib is shown at $D$.

Since all sections of the ellipsoid not perpendicular to the axis are ellipses, all other ribs will be parts of ellipses. The conjugate axis of these is in every case the same, and equals the conjugate diameter of the ellipsoid. The semi-transverse axis in each case equals the distance from the centre $s$, through the desired rib, to the circumscribing ellipse, which is described proportionate to the curb. The rib which stands over $sp$ is shown at $E$. The distance $cd$ at $E$ equals that from $p$ to the curb on the plan. The rib which stands over $os$ is shown at $F$.

Since all parallel sections of the ellipsoid are similar, the springing curves are similar to the parallel section passing through the centre. Upon $ng$, as a diameter, describe the semi-circle at $G$; this is one springing curve. Taking the height of this as a semi-conjugate axis, and $mn$ as a transverse axis, describe the semi-ellipse at $B$; this gives the other springing curve.
PLATE 32.
OF DOMES.

To find the Covering of a hemispherical Dome.

Fig. 1. Describe a circle for the plan of the dome. Take the stretchout, a b, of a quartercircumference, b d, and place it from c to 4' at B. Take a b at B, of any length which the board will admit, and describe upon it a semicircle. Divide half of this into any number of equal parts, as four, and divide also 4' into four equal parts, and through all the points of division draw lines parallel to the base a b. Make the lines 1' d, 2' e, 3' f, each equal to the corresponding lines within the semicircle, and trace the curve a d e f 4'. By this means, we obtain the form of the boards, which, when bent over, will exactly cover the hemisphere.

To find the Covering of a segmental Dome.

Fig. 1. Let c h be the height of the segment. Draw the cord g h, and the line 1 2 perpendicular to c g. Take the stretchout of the arc g h, and place it from c to 4' at C. Take the line a b at C, equal to twice 1 g at A, and upon it describe a small segment of a circle whose height shall equal 1 2 at A. Proceed as in the last case.

In a similar manner, the covering for various domical figures may be developed. The method for obtaining the covering for an ogee figure is exhibited.

To find the Form of the Boards to cover a hemispherical Dome horizontally.

Fig. 2. Let the semicircle at A, whose centre is c, be a vertical section through the centre of the dome. Divide the quarter-circumference d b into a number of equal parts, each being equal to the width of a board. Extend the radius c d to m, and through the points 1 and 2 draw a line to t. Then, with t as a centre, describe arcs from the points 1 and 2, which gives the form of the board. The others may be found in like manner.

Now, suppose that the centre of the board which lies between 5 and 6 is the last that can conveniently be found — to find the others: with the centre y, continue the arc from 5 around to o, and through the points 5 and 0 draw a line to meet the circumference at e. From e draw radial lines to 5, 6, 7, 8, &c. Also, draw lines from 6, 7, 8, &c., parallel to a b, meeting c d at 6', 7', &c.

Transfer 6' 6 to a b at B, and take b d, equal to 6' f at A. Draw a d; make b c equal to a b; draw d e, and bisect the angle d c b. At the middle point of the line a d erect f e perpendicular, meeting the bisecting line at e. Through the points a, e, and d, draw an arc.

If the line a d becomes too long to use conveniently, bisect also the angle e c a, as at C, and describe the arc a e upon the same principle.
**PLATE 33.**

**ELLIPSOIDAL DOMES.**

Fig. A is the plan of a ellipsoidal dome; B is the longest section, C the shortest section: a a in B, and b b in C show how to square the purlines, so that one side may be fair with the surface of the dome; the dotted lines from a a in B, and b b in C, show how to get the length and width of the purline in A; but if the sides of the purline were made to stand perpendicular over the plan, the curve of it would be found in the same manner; then it would require no more than half the stuff that the other would, and take only half the time.

*How to proportion the inside Curb for the Skylight, so that it will answer to the Surface of the Dome.*

Draw the diagonals i l and k h at A, and let d e, or g f, be the length; then e g, or d f, will be the breadth of the curb; because every section parallel to the base will be proportional to the base.

The ribs (Figs. 4 and 5) in this case are obtained in the same manner as the ribs for an ellipsoidal niche.

*To find the Form of a Board to cover any Part of the Dome when bent up to the Crown.*

Divide one-quarter of the base of the dome at D into three equal parts, r o, o p, p k'. In finding a board over s r o in the plan, take the triangle s r o in D, and lay it down at Fig. 2; then draw the line s b a at right-angles to r o, and describe a rib, s b c, to the height of the dome, and to the length of the perpendicular of the triangle s r o. Divide it into five equal parts, lay them along the line b a, and find the mould from the triangle s r o, as the letters are marked. The board (Fig. 2) will be found in the same manner.

*Remark.* — In practice, you are to divide one-quarter of this dome into as many parts as you think the breadth of the boards will require; and the boards, when obtained by this method, will fit with very great exactness. This is divided only into three, that the parts may be clearly seen by learners.

If the boards are obtained for one-quarter of the circuit, the corresponding boards in the other three-quarters will not require other lines; for every board in the first quarter will be a mould for three more boards.
PRACTICAL CARPENTRY.

[The designs which heretofore occupied this portion of the work have been laid aside. They were, in their time, excellent; but such advances have since been made in the art as to render them useless to the carpenter of the present day. The designs substituted are not offered as a complete series, to illustrate every case likely to arise, but are intended to comprehend all the most approved principles of framing in use at this time. These principles are difficult and important, and the designs which exhibit them will afford great practical assistance to both architects and carpenters.—Editor.]

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PLATE 34.

SCARFING.

On this plate are exhibited ten different methods for scarfing timber. Some are secured by iron pins, and in a few cases iron plates are also added. In cases where vertical pressure, and not traction, is to be sustained, the best, but more expensive plan, is to bind the scarf with iron straps and dispense with the bolts. Figs. 5 and 6 have no such aid, but are brought tightly together by wedges. Many other designs for scarfing have been offered, but being more complicated, they are inferior to these simple forms.
This plate exhibits additional methods of scarfing. Perspective views of the pieces are given, that there may be no difficulty in understanding the work. We subjoin some selected rules, which have reference to the kind of timber used.

In oak, ash, or elm, the whole length of the scarf should be six times the depth or thickness of the beam, when there are no bolts or straps.

In fir (or pine), the whole length of the scarf should be about twelve times the thickness of the beam, when there are no bolts or straps.

In oak, ash, or elm, the whole length of a scarf depending on bolts only should be about three times the breadth of the beam; and for fir beams, it should be about six times the breadth.

When both bolts and indents are combined, the whole length of the scarf for oak and hard woods may be twice the depth; and for fir and soft woods, four times the depth.
Figs. 1 and 2 show the methods of giving the requisite stiffness to joists. Stout oak laths, nailed on in the manner represented, improve their qualities in this respect very much.

Figs. 3 and 4 exhibit a method for trussing girders. A is the king-bolt, and B is the butt-bolt.

Fig. 5 shows a simpler and more effective method. The girder is hung on each side by iron bars, secured by nuts at the ends. By turning these, any degree of camber may be given to the girder.

Fig. 6 presents a still more effective method, though its use is limited to certain parts of a building. The use of iron, in the manner here represented, is the great characteristic feature in modern Carpentry. A girder, well constructed on this plan, will support a brick wall built upon it.
PLATE 37.

This Plate exhibits two designs for trussed partitions. It is a matter of great difficulty to construct partitions so as not to spring and crack the plastering; especially when there is no support beneath, and when they are pierced by doors. In all such cases, they must be trussed in a manner at least somewhat similar to those shown in the Plate. These will span from twenty to forty feet.
Fig. 1 is calculated for a small building. The ends of the ridge-pole, purlins, and plates, are shown in this and the following cases. The span may be twenty or twenty-five feet.

Fig. 2 is a roof for the purlins to be framed in. The dotted lines show the lower edge of the common rafters. The span may be from twenty to thirty-five feet.

Fig. 3 is a much heavier roof, the tie-beam being supported by two queen-posts, so as to give room for a passage or apartment in the roof. It is calculated for a span of forty or fifty feet.
PLATE 39.

Fig. 1. This roof is somewhat similar to the last, but essentially different in the construction of the queen-posts. The span is forty or sixty feet.

Fig. 2 is a very light roof, having forty or sixty feet span. It may be used when the ceiling below is to be arched. The sheathing may be laid directly on the purlins, for a metal covering.
Plate 39

Fig. 1.

Fig. 2.
PLATE 40.

Fig. 1 is a very simple roof, designed upon the more recent principles which have revolutionized Carpentry. An oaken block is placed between the upper ends of the principal rafters, after the manner of a key-stone; the grain of the block running in the same direction as that of the rafters. By this arrangement, the effect of shrinkage is rendered imperceptible. An iron bar is passed through the block, and gives support and camber to the tie-beam. The foot of each principal rafter is bolted with iron. The span of this roof may be from twenty to thirty feet.

Fig. 2 is another roof, designed to contain an apartment. A horizontal piece is placed between the bases of the queen-posts, that they may not be displaced by the lateral thrust of the trusses. The span may be from forty to fifty feet.
PLATE 41.

Fig. 1 is a design for a roof having very considerable span. It may extend from fifty to eighty feet. We may here see the method of suspending the tie-beam by iron bars, instead of by king and queen-posts. Every purlin has its corresponding truss. Between all pieces which do not come fairly end to end, oaken blocks are placed, as before described. The blocks are shaded, in this Plate. But what is here more particularly worthy of remark, is the character given the design by the position of the trusses. It will be observed that they have somewhat the appearance of lattice-work. This principle, we believe, is of American origin; and is one of the most important improvements which have been introduced into Carpentry of late years. The arches for domes and arched roofs, when trussed by this lattice-work, are lighter, and more durable, than when constructed on any other plan. The example here given exhibits the principle satisfactorily; but it is capable of many important modifications and applications, which should be thoroughly studied by our carpenters.

Fig. 2 is another roof, similar in principle, but heavier in proportion, and of shorter span. It may extend from fifty to sixty-five feet.

Where architectural effect is not an object, preference is now almost universally given to flat roofs. The way in which this is managed in large roofs, is exhibited in both these designs. The purlins, when necessary, are elevated on short, upright supports, standing on the principal rafters; and the walls are built considerably above the ends of the tie-beam. By such an arrangement, any degree of weight may be given to the cornice.
PLATE 42.

Fig. 1 is another design for a large, heavy roof, having a span of about thirty or sixty feet. The oaken blocks are, in this case, dispensed with. The ceiling-joists are shown against the tie-beam.

Fig. 2 is a floor, designed to have a span of twenty or forty feet. The ends of both floor and ceiling-joists are exhibited. The trussed girder which supports the floor, combines, as may be seen in the figure, both the methods described in Plate 36, Figs. 5 and 6, with a slight modification.

Fig. 3 exhibits a section of the same floor, at right-angles to the above, through the line a b, Fig. 2. The shaded parts are the sections of the girders, and show their proper distance apart.
PLATE 43.

Fig. 1 represents the framing of a church spire. It is quite light, but very strong. The body is square, but the spire proper has a conical finish. The whole rests upon frame-work supported by the outer walls; and in this case there need be no support immediately beneath the spire.

Fig. 2 shows a horizontal section below the first window. Fig. 3 is a section through the second window. Figs. 4 and 5 are the naked flooring.
ROOFS.

Fig. 1 exhibits the framing for the roof of a church having a vaulted nave. It will be observed that the opposite sides of this figure furnish two designs, either of which are sufficiently substantial.

Fig. 2 is a similar design, on a smaller scale. The roof, in this case, has a higher pitch, but about the same span as the other.
PLATE 45.

On this Plate is shown a design for the roof of a small Gothic church, or hall. The vault is ribbed, and the finish between the ribs is also exhibited. The ribs terminate in some tracery on the upper portion of the walls, which is represented on the lower portion of the Plate.
PLATE 46.

This exhibits a design for an open-work Gothic roof for a church, hall, or open shed, for large assemblies. The design is simple, and needs no remark. A different view of the same design is shown on the bottom of the Plate.
PLATE 47.

We have here exhibited a design for an open-work roof for a small Gothic church or hall, in the perpendicular style. A considerable degree of ornament always accompanies this style. Below is shown the inner-wall decoration.
PLATE 48.

This Plate presents a design for an arched roof of very great span. It may extend as much as one hundred and twenty feet, or even more, with safety, if the buttresses are very strong. The arch is made by bending stout planks to the desired curve, and bolting them together, which forms an arch of great strength and stability. Oftentimes, the arch is with advantage made of the lattice-work before referred to. The stays which connect the arch and principal rafter, consist of two pieces of plank, one on each side of the rafter and arch, secured by long bolts.
PLATE 49.

DOMES.

This Plate exhibits a design for a dome. The framing is simple, although, from the nature of the sectional drawing, it appears complicated. A lantern is arranged for external effect, but does not extend to the interior.
PLATE 50.

Fig. 1 shows the plan of the preceding design, taken from just above the ceiling-line. Fig. 2 is a horizontal section of the lantern, and Fig. 3 the roof of the same.
On this Plate is shown the centring for an arch, as applied to a bridge. It is so constructed as to have no other support than at the abutments.

Fig. 1 is a vertical section, passing between the key-stone and the adjacent voissoir.

Fig. 2 is a side view of the cradling.
PLATE 52.

This Plate exhibits a design for the construction of a window-case, arranged for inside shutters.

Fig. 1 is a horizontal section of the jamb. The shutter is folded into the jamb. The circles above represent the weights with which the sash is hung.

Fig. 2 is a vertical section of the window-sill, having pannelling below.
JOINERY.

HAND-RAILING.

PREFACE.

In that elegant branch of the building art called Joinery, Stairs and Hand-railing take precedence. For the manner of finding the face and falling moulds, I have laid down correct methods founded on the most obvious principles, and which have been put in practice by myself, and by those who attended the instructions given by me, in this art, some years since, and found to answer well in every case.

The superior advantages, in every respect, of the new plates on hand-railing, over those published in the former editions of this work, will, I trust, be deemed a sufficient reason for the change made by me in this department of the Carpenter's Guide. I have retained those plates on hand-railing, by P. Nicholson, which are considered useful; and hope that the alterations made in this department of his work, will meet the approbation of Carpenters generally. In conclusion, I think it proper to say that, for the method of finding the butt joints, Plate 55, I am indebted to an eminent stair-builder of this city, whose mechanical skill in joinery, I, with others, hold in high estimation.

WILLIAM JOHNSTON,
ARCHITECT, Philadelpbia.

PLATE 53.

Plan and Elevation of a Newel Post Stair-case. Scale 1/2 an inch to a Foot.

To draw the Ramp.

Make $AB$ equal to $AC$, draw $CD$ at right-angles with the rail, also produce the horizontal line $E$ until it intersects at $D$, which is the centre of the ramp.
PLATE 54.

Plan of a stair-case, showing how to arrange the steps at the circle, so as to allow the string-board to be formed without the easing (usually made) on its lower edge, caused by making the tread of the steps greater or less at the circle than at the flyers; this plan also admits of the balusters being the same length, and nearly the same distance apart, as at the flyers.

To draw the Plan.

Describe the equilateral triangle $ABC$; draw the right line $ED$, touching the face of the string-board; produce $CA$ and $CB$, to intersect it at $D$ and $E$; then is the right line $ED$ equal to the semicircumference of the circle. Upon this line lay off the tread of a step, $OP$, and draw lines from these points to the centre $C$; which gives the position of the risers $KK$ on the circular part of the string-board, at the points $SS$. The position of the same risers on the carriage, is found by adding to the distance $DP$ or $OE$, whatever may be requisite to make a full tread, and place it from $A$ to $W$, which gives the position of the risers $NN$ and $KK$ on the carriages.
Fig. 1 is a plan of the rail for Plate 54; the stretch-out is found by the same method as in the foregoing Plate.

To draw the falling Moulds for the Rail.

Fig. 2. Let \( P \) be the pitch-board, and \( A \) the beginning of the circular part of the rail; from the line \( A H \), place the stretch-out of the outside and inside of the rail, to \( C \) and \( D \); and from the line \( D B \), set up the height \( DF \) or \( CE \), which is equal to a rise and a half. Having found the stretch-out and height, place half the thickness of the rail on each side of the central points \( F; E \) and \( H \), as denoted by the small circles; then connect the upper heights, \( EF \), with the lower one, \( H \), which completes the falling moulds.

To cut the centre or butt joints, divide the distance \( FE \) into two equal parts; draw the line \( II L \), and at right-angles with this line, draw the lines \( FJ \) and \( EK \), which gives the joints required.

To find the Spring of the Plank.

Fig. 2. Take \( BG \), equal to \( CJ \), Fig. 1, and erect at \( G \) a perpendicular to meet the hypothenuse of the pitch-board produced at \( I \); draw \( IO \) parallel to \( BG \); then is the distance \( OE \) the spring of the plank.
PLATE 56.

To draw the Face-mould for the Rail.

Fig. 1. Draw the chord-line $AD$ to the plan of the rail, also the base-line $OP$, at any convenient distance above it; make the length of this line equal to the dotted line on the plan, and set the height $NE$ of the falling moulds, Plate 55, from $O$ to $R$; draw $RP$; then draw ordinates through the plan, perpendicular to the base-line, touching the chord $RP$; also draw the ordinates $BB$, $BB$, &c., at right-angles with this chord, and make the distances $12$, $34$, &c., on the face-mould, equal to $12$, $34$, &c., on the plan.

The circular part being found, draw the straight part by keeping it parallel to the line of the joint $S$. In applying this mould to the plank, care is to be taken that the chord $RP$ be kept parallel to its edge.

As this method of finding the face-mould does not admit (if three inch plank be used) of more than four or five inches of straight rail being attached to the circular part, another method is shown below, by which the straight rail can be extended at pleasure.

Fig. 2. Let $Q$ be the plan of the rail, and $ABC$ the pitch-board; draw the ordinates $12$, $34$, &c., at right-angles to $CB$; place the spring $OE$, Plate 55, from $A$ to $D$; draw $DE$ at right-angles to $AC$. Make $DE$, at $N$, equal to $ED$; draw $EF$, also $CG$, parallel to it; draw the lines $0$, $0$, $0$, &c., from the points made by the intersection of the ordinates with the convex and concave sides of the rail, and produce them to the line $CG$; draw $CI$ at right-angles with $CA$, make $IJ$ equal to $EA$, join $JK$, parallel to $JK$, draw the ordinates $LL$, &c., and with $C$ as a centre, draw all the concentric lines to intersect with the line $CH$, and continue those lines parallel to $AC$, and at the points of intersection with the ordinates, trace the face-mould $P$. The dotted line $S$ shows the over wood for cutting the square or butt joint.
Fig. 1 is the plan of a staircase having eight winders.

To draw the falling Moulds.

Fig. 2. Let $AB$ be the stretch-out of the convex side of the rail, and $AI$ the straight portion attached to the circular part; draw the flyers $H$ and $G$, and the four winders $CD$ and $E$; also draw the base-line $BL$, at any convenient distance below the point $O$; raise the rail five inches above the line of the nosings, from the point $K$ to $J$; also make the distance $AL$ equal to the distance $AL$, Plate 58, and proceed to draw the moulds as before described in Plate 55.
PLATE 58.

To draw the Face-mould.

Let $A m B$ be the plan of the rail; draw the chord $A B$, also the line $C D$, parallel to it; make the lines $E F G$ equal in height to the lines $B L I$, Plate 57. Join $G H$; also draw $I J$ parallel to $C D$, and at the point $J$ let fall the perpendicular line $J K$. Join $K M$, which gives the directing ordinate on the plan; draw a sufficient number of ordinates to meet the chord $N O$; also draw $I P$ at right-angles with the line $H G$, and with the distance $K M$, from the point $J$ as a centre, bisect the line $I P$ at $P$. Join $J P$, which gives the directing ordinate for the face-mould; transfer the ordinates from the plan, and set them from the chord $O N$, and through the points trace the face-mould. Take the distance $I S$, set it from $Y$ to $Z$ on the chord, and join $Z m$; with this bevel, the edge of the plank must correspond, before you apply the face-mould to its upper and lower surface.
To draw the Scroll of a Hand-rail.

Fig. 1. Make a circle 3\ 1\ inches diameter; divide the diameter into three equal parts; make the square in the centre equal one of these parts, and divide each of its sides into six equal divisions, with the centres 1, 2, 3, 4, &c.; complete the outside revolution, set the width of the rail from $E$ to $B$, and go the reverse way to complete the inside revolution.

If a scroll of less diameter be required, draw the dotted lines $CA$ for the straight part of the rail, and a scroll of one-quarter revolution less is given.

To draw the falling Mould for the Scroll.

Fig. 2. Take the pitch-board, and let $AB$ be the stretch-out of the convex side of the rail. Lay half the thickness of the rail on each side of the upper line of the pitch-board, shown by the small circle; draw the line $EC$, and the horizontal line $FE$, and complete the under edge of the mould by intersecting lines.

In forming this easing, the mould should not come to a level at the joint but continue to descend for a few inches past it.
PLATE 60.

To draw the Face-mould for the Twist of the Scroll.

Fig. 1. Draw a sufficient number of ordinates through the plan, and from the points 1', 2', 3', &c., draw ordinates at right-angles to the line 1'E, and transfer the distances 11, 22, 33, &c., upon these lines, and trace the face-mould through the points.

The plumb-bevel is shown at D. BE1' is the pitch-board.

To draw the Plan of the curtail Riser.

Fig. 2. When the scroll is drawn, set the projection of the noseing without, and draw it equally distant from it, which will give the form of the curtail step; then set the distance which the riser is back from the line of noseings, and you will get the form of the curtail riser. This figure is drawn to a smaller scale.
PLATE 61.

The methods for obtaining the moulds of hand-railing, exhibited in the two following Plates, have never heretofore been published. They are founded on mathematical principles, and will be found to produce extremely accurate results. The lines are more simple, and much easier to draw, in practice, than those now in use; and the few principles here developed may be applied to every ordinary case.—Ed.

Having given the Plan of the Railing, and the Tread and Position of Steps leading to a Landing, to find the Face-mould.

Fig. 1. Let OAD be the plan of the rail, either turning up towards I, or continuing towards II; draw the noseings in their proper positions; lay down the distance DC at D'C, Fig. 2, and take the angle DCD', equal to the inclination of the railing, and draw DD' perpendicular to DC; take CA perpendicular to DC, and equal to DC, Fig. 1. Then, with CA as a semi-conjugate, and DC as a semi-transverse axis, draw, with a trammel, the quarter ellipse DA; now, with BA half the breadth of the rail, draw a small circle, tangent to DD' at D'; draw GG', &c., parallel to DD'; and then, with CF as a semi-transverse axis, and the same semi-conjugate, draw the inner edge of the mould, and it will be complete. The plumb bevel is shown at K.

To find the falling Mould.

Fig. 3. Lay down a step at abcd, and take cD', equal to the stretch-out of the quadrant AD, Fig. 1; take bh, equal to the distance on the plan, from A (where the rail begins to bend), to the riser of the top step, and erect hA at right-angles to ab; take D'i, equal to six inches, or whatever distance you wish to elevate the rail on the landing, and draw a line through i, parallel to D'c; also join ac, and continue it to meet the other line at d; take se and id, each equal to the thickness of the rail, and draw Ef parallel to sd; now from A, trace any curve around to D (the arc of a circle is perhaps the best and easiest), and from A' draw one parallel to this, and the falling mould is complete. The point A is applied to A, Fig. 2, and the falling mould is bent around the piece, until the point D falls immediately beneath D, Fig. 2.

To find the Thickness of the Stuff.

Fig. 3. Draw A'l parallel to cd', and take A'n equal to AC, Fig. 1; erect nf perpendicular to A'l, and from e let fall eg, perpendicular to fe; then will eg be the thickness of the stuff required to produce the rail.

Remark.—It will be observed that any length of straight rail may be formed in connection with this piece; and by using sufficiently thick plank, the rail may be formed without any joint whatever, except at D, Fig. 1. If a butt-joint is made at E, Fig. 3, the length of the stuff required is equal to Ef.
PLATE 62.

To find the Face-mould for the Railing of a Stairway having a semicircular Well, and Steps of equal Tread throughout.

Fig. 1. The plan of the rail and stairs is here exhibited. The steps do not wind, but go up to a landing; and the rail continues an even ascent without any horizontal part, as in the last case.

Fig. 2 exhibits the face-mould. Take $Cb$, equal to $EF$, Fig. 1, the difference between the radius $CB$, of the well, and the stretch-out of $BG$; make the angle $Cb\alpha$ equal to the pitch of the stairs, and draw $\alpha C$ at right-angles to $Cb$; extend $\alpha C$, making $Cc$ equal to the radius $BC$ of the well; join $c$ and $b$; from $C$ let fall a perpendicular upon $bc$, and extend it beyond $T$; perpendicular to this, draw $CE$, and take $Ca'$ equal to $Ca$; through $a'$, from $O$, draw $OS$, equal to $BC$, Fig. 1, and at $S$ erect the perpendicular $ST$; now, with $O$ as a centre, and half the breadth of the rail as a radius, describe a small circle, and draw the tangents $m$ and $n$ to the points where $SQ$ intersects the circumference; then take the distance $OT$, and set it from $C$ to $F$; extend the dividers from $O$ to $F$; then place one leg at $m$, and mark $m'$; and with one leg at $n$, mark $n'$; around $E$ describe a circle to the breadth of the rail; now $FC$ and $EC$, which equals $BC$, Fig. 1, are trammel lines. Describe the quarter ellipse $FGE$, and it will be the central line of the face-mould. Describe from $m'$ and $n'$ the edges of the mould, tangent to the circle at $E$.

It now remains to find the butt-joints. Make $Co$ equal to $Oa'$, and through $o$ draw $BV$ parallel to $bc$; draw $Be$ and $b'b$ at right-angles to $bc$, and through $c$ and $C$ draw a line cutting the mould in $LD$; this will be the upper butt. Also draw a line through $C$ and $B$, and from $A$ draw downwards the outer edge of the mould parallel to $CL$, and make the inner edge parallel to this; now, through $L$, draw a line (not represented on the Plate) parallel to $OF$, cutting the mould at $K$, and the lower butt joint may be made at $K$, or any point above. The mould is now complete. To use it, the points $L$ and $K$ must be fixed to the edge of the plank, and the mould marked; then place the plumb bevel, shown at $T$, on the edge of the plank at $L$, and it will give the joint on the other side to which $L$ must be affixed.

No falling mould is requisite. A straight-edge bent around the piece, when cut from the board by the face-mould, gives the correct lines for dressing off the upper and lower sides of the rail.

Remark.—This case is also applicable to a winding stairway. Instead of taking $EF$, Fig. 1, equal to $Cb$, Fig. 2, take the sum of the heights of the risers between $B$ and $G$, Fig. 1, and having added five inches, lay down this length from $A$ to $B$, Fig. 3; then make the angle $ACB$ equal to the inclination of the rail ($AB$ being a right-angle), and take $BE$, equal to $BC$, Fig. 1, and erect $EF$ perpendicular to $BC$; now use the right-angle triangle $EFC$, Fig. 3, in the place of $Ca'b$, Fig. 2, and it will give the face-mould for the rail of the winding stairs. In this case, however, falling moulds will be necessary. They may be found in the way indicated upon Plate 57.
**PLATE 63.**

*To draw the Form of a Hand-rail.*

Fig. 1. Make an equilateral triangle, \( vw t \), upon the width of the rail, and divide it into five equal parts, and from one part on each side draw \( z s \) and \( y u \); then \( t, g, \) and \( m \), are the centres, \( l m \) being made equal to \( lg \). The centres are found the same for the other side.

*The Form of a Rail being given, to draw the Mitre-cap.*

Fig. 1. Let the projection of the cap be three inches and a half, and make the distance of the inside circle from the outside circle, equal to the projection of the nose on each side of the rail, and draw the mitre \( n o \) and \( n' o \); then continue parallel lines down to the mitre \( n' o \), put the foot of your compass in the centre of the cap, and circle the parallel lines round to \( a', c', e', g' \), and \( i' \), and draw the ordinates \( a' b', c' d', e' f' \), &c., and then mark out the cap for the rail according to the letters.

*To draw the Form of the Cap; the Mitre to come to the Centre.*

Fig. 2. In every case draw the lines which are perpendicular to the diameter of the rail, to the mitre; and then, with the apex of the mitre, as \( a' \), for a centre, describe the arcs around to the diameter of the cap; then make \( a' b', c' d', e' f' \), &c., respectively, equal to \( a b, c d, e f \), &c., and it will give the form of the cap.
HAND-RAILING.

PLATE 64.

To draw the Scroll of a Hand-rail.

Fig. 1. Make a circle with the centre c, three inches and a half in diameter, and divide the diameter into three equal parts; make a square about the centre of the circle, whose sides are equal to one of these parts, and divide each of its sides into six equal parts. The square must then be divided by lines, as shown, full size, at Fig. 2. Now, with the points 1, 2, 3, 4, 5, 6, as successive centres, describe first the arc from i to k, Fig. 1, then from k to b', from b' to m, and so on around to a. This will give the outside spiral. Set the width of the rail from a to f, and go the reverse way for the inside spiral. This will complete the scroll.

To draw the curtail Step.

Fig. 1. Set the ballusters in their proper places on each quarter of the scroll; the first balluster shows the return of the noseing round the step, the second balluster is placed at the beginning of the twist, and the third balluster a quarter distant, and straight with the front of the last riser; then set the projection of your noseing without, and draw it all round equally distant from the scroll, which will give the form of the curtail step.

To find the falling Mould.

Fig. 3. At a b c is the pitch-board; the height is divided into six parts, to give the level of the scroll; the distance a d is from the face of the riser to the beginning of the twist; and the distance from d to k is the stretch-out from a, the beginning of the twist round to b, Fig. 1; divide the level of the scroll, and the rake of the pitch-board, into a like number of parts, and complete the top edge of the mould by intersecting lines, and the under edge parallel to it to the depth of the rail.

The outside falling mould, Fig. 4, is found in like manner.

The method for obtaining the thickness of the stuff requisite for cutting the scroll is also shown at Fig. 3.
PLATE 65.

As the method of getting a scroll out of a solid piece of wood, having the grain of the wood to run in the same direction with the rail, is far preferable to any of the other methods with joints in them, being much stronger than any other scroll with one or two joints, and much more beautiful when executed, as no joint can be seen, and consequently no difference in the grain of the wood at the same place; I shall here give a specimen, the method for describing a scroll being already given in the last Plate; and likewise the falling mould.

How to find the Face-mould.

Fig. 1. Place your pitch-board, \(a'b'c'\), as in the last Plate; then draw ordinates across the scroll at discretion, and take the length of the line \(d'b'\), with its divisions on the longest side of the pitch-board, and lay it on \(d'b'\) in Fig. 2; then, the perpendiculars being drawn, it may be traced from Fig. 1, as the letters direct.

How to find the Thickness of the Stuff.

Fig. 3. Let \(a'b'c'\) be the pitch-board, and let the level of the scroll rise one-sixth, as in the last Plate; and from the end of the pitch-board, at \(b\), set from \(b\) to \(d\) half the thickness of the balluster, to the inside; then set from \(d\) to \(e\) half the width of the rail, and draw the form of the rail on the end at \(e\), the point \(b\) being where the front of the riser comes, then the point \(e\) will be the projection of the rail before it; then draw a dotted line to touch the nose of the scroll, parallel with \(c'b\), the longest side of the pitch-board; then will the distance between \(m\) and \(n\) be the thickness required. Much lighter stuff may be used, by gluing a piece on the under side, as shown at \(m\).

Figs. 4, 5, and 6, show the plan, face-mould, and thickness, of another scroll, of a different pattern. The principles of the drawing are the same as in the other case.
HAND-RAILING.

PLATE 66.

Fig. 1. Part of the plan of the rail for a stair, consisting of eight winders round the semicircular part, and flyers below and above.

Fig. 2. The elevation of the convex side of the semicylinder, showing the winders.

Fig. 3. The elevation of the concave side, with the delineation of the winders, drawn from the equal parts on the concave side of the plan.

Fig. 4. The steps stretched out for the semicircumference of the convex side of the rail; ABC being the triangle of the winders, and ADE and CFG flyers, one below and the other above the winders in position.

To draw the falling Mould.

Produce DA and BC upwards to H and O; make AH and CO of any equal heights; join HO. Draw HK parallel to AE, and ON parallel to GC; therefore NO and HK will be parallel to each other. Make ON and OM, also HL and HK, equal to AE or GC, and describe two tanged curves, NM and KUL, and KULMIN will be the under side of the falling mould; then, the upper curve PQRST being drawn parallel to a given distance, will complete the falling mould. Ordinarily, the distance between the under and upper edges of the falling mould, is two inches; the breadth of the rail, two and a quarter, or three inches. Much in the same manner most falling moulds are to be formed; viz., by placing a flyer before and after the winders. In semicircular rails upon level landings and half spaces, the winders are reduced to a single step with a broad tread equal to the circumference of the rail, and the height the common height of the flyers: also in quarter spaces, the quadrantal periphery of the rail is considered as the tread of a step, in addition to the other winders, which are necessary in the other quarter.

Fig. 5. AB is the pitch of the steps or falling mould for the concave side of the rail, CD the pitch of the steps or of the falling mould for the convex side of the rail; let these two pitches intersect one another in E, in the middle of AB: it must be obvious that neither of these positions gives the true pitch of the rail piece. If the rail were considered in depth only, without thickness, then the middle part of the cylindric projection, as shown in the elevation, Fig. 2, would give the pitch, provided the radius of the rail were the same as the outside of the plan. Therefore, neither the outside nor inside falling moulds, nor the elevations of the concave or convex projections of the rail, will give its pitch; but from what has now been shown of the projections of the cylinder, it will be no difficult matter to form an idea of the complete projection of a rail piece, showing the entire solid in order to form the true pitch. This will be described in the next Plate.

Fig. 6, the pitch-board of the flyers. Fig. 7, the pitch-board of the winders.
PLATE 67.

To show the proper Twist of a Rail and Thickness of Stuff, and the Face-moulds.

Fig. 1. Let $A$ be the plan of a rail, and Fig. 2 the falling mould, as completed. Let the plan of the rail be so placed that the chord $AO$ be parallel to the base $MO$ of the falling mould, Fig. 2, and let $BB'$ be the separation of the straight and circular parts of the rail, $O B$, $B'B'$ being the quadrant part of the rail, and $B'A$, $AB$ the straight part; divide the concave side $BCDO$, of the quadrant part, into equal parts at the points $B$, $C$, $D$, and $O$; through all the points $B$, $C$, $D$, &c., draw lines at right-angles to the chord $AO$, cutting it in $1$, $2$, $3$, &c., and produce them upwards to the points $f g h$, &c. Let $MBN$, Fig. 2, be the section of a step, and upon $MO$, make $BA$ equal to $BA$, Fig. 1. Extend the parts $AB$, $BC$, $CD$, &c., upon the base $MO$, Fig. 2, from $A$ to $B$, from $B$ to $C$, from $C$ to $D$, &c.; from the points $A$, $B$, $C$, $D$, &c., draw lines perpendicular to $MO$, cutting the lower edge of the falling mould at $E$, $F$, $G$, $H$, &c., and the upper edge at $I$, $J$, $K$, $L$, &c. In Fig. 1, draw any line, $abcld$, &c., parallel to the chord $AO$; make $ae$, $ef$, $fg$, $dh$, &c., respectively equal to $AE$, $EF$, $FG$, $DH$, &c., Fig. 2; also in Fig. 1, make $ai$, $bj$, $ck$, $dl$, &c., equal to $AI$, $BJ$, $CK$, $DL$, &c., Fig. 2. Through the points $e$, $f$, $g$, $h$, &c., Fig. 1, draw a curve; also through the points $i$, $j$, $k$, $l$, &c., draw another curve, and these two curves will complete the projection of the falling mould. From the point $s$, Fig. 1, radiate the lines $c'c$, $d'd'$, &c., cutting the convex side at $c'd'$, &c.; from the points $A'$, $B'$, $C'$, $D'$, &c., draw the lines $A'j'$, $B'k'$, $C'l'$, $D'm'$, &c.; draw $ee'$, $ii'$, $jj'$, $kk'$, $ll'$, &c., parallel to the chord $AO$. Through the points $i'$, $j'$, $k'$, &c., draw a curve until it intersects with the curve $i$, $j$, $k$, &c.; this will form the projection of the top of the rail piece. In like manner the under parts which appear at $qq'$, $hh'$, will be completed in the same manner, so that $b$ is the whole appearance of the solid, $q r' r q'$ and $i' e' e'$ being sections, or the ends which join the contiguous parts of the rail.

To trace the face-mould at $C'$; join $i' r'$, and let the perpendiculars cut $i r$ at $1$, $2$, $3$, &c. Draw the ordinates $1 b u$, $2 c v$, $3 d w$, &c., perpendicular to $i' r'$; make $1 b$, $2 c$, $3 d$, &c., equal to $1 B$, $2 C$, $3 D$, &c., at $A$; also, at $C$, make $1 u$, $2 v$, $3 w$, &c., equal to $1 U$, &c.; also make $x o$ equal to $x o'$, $i a'$ equal to $1 A'$; draw $by$ parallel to $i a'$, and $ay$ parallel to $i b$, and $ta'y b$ will complete the straight part of the face-mould: join $r'o'$; draw the concave curve $bedr$, and the convex curve $yu v w o$, which completes the curve part, and the whole of the face-mould.

Fig. 3 shows the projection and face-mould for the quadrant part of a rail. The principles of projection, and manner of drawing the face-mould, is the same as what has now been shown. The figure is introduced to show how much less wood the part of the rail requires from a quadrant plan, than when a straight part of the rail is taken in; and the more of the straight rail that is taken in, the greater will be the deflection from the chord; thus, in Fig. 3, the distance between the chord $f' r$, at any point to the nearest point of the projection, is much less than the distance, Fig. 1, from a corresponding point in $i' r$ to the nearest point of the projection.
PLATE 68.

To find the Moulds for executing a Rail with a Semicircle of Winders.

Fig. 1. The falling mould as here drawn does not follow the line of noseings, but is raised six inches, as is the practice with several hand-railers, in order that the rail should not approach nearer to the noseings of the winders than to the flyers. This example is adapted to a stair with ten winders in the semicircumference.

Fig. 2. The plan and face-mould of the lower quarter winders.

Fig. 3. The plan and face-mould of the upper quarter winders. A B C the convex side of the quadrantal part of the plan, C D a straight part intended to be wrought on the same piece with the twisted part. In this method, the plane of the top of the face-mould is supposed to rest upon the upper extremities of three straight lines or slender rods perpendicular to the plane of the seat of the said face, and these three perpendicular lines to rise from three points, in a line dividing the breadth of the plan everywhere into two equal parts of the rail piece; and these three points to be so situated that one may be at each extremity, one at the end of the quadrant, and one at the end of the straight piece, and the intermediate point at the intersection of a perpendicular, drawn from the centre of the plan of the rail piece to a straight line joining the two extreme points. Let each of the upper extremities of the three perpendicular lines be called resting-points, and let the feet of the perpendiculars be called the foot of the heights of the rail piece, which will therefore be the same as the seats of the resting-points; and let the three perpendicular lines themselves be called the heights of the rail piece, and their places distinguished by the lower height, the middle height, and the upper height.

Let a, Fig. 2, be the foot of the upper height, b the foot of the middle height, and d the foot of the lower height; join a d; draw a f, b' g, and d h, perpendicular to a d; the middle one, b g, being drawn from the centre E. Let A, B, C, D, Fig. 1, be the points corresponding to A, B, C, D, in the convex side of the plan, Fig. 2, and let A F, B G, and D H, be the heights of the rail piece, Fig. 1; make a f, b' g, d h, Fig. 2, respectively equal to A F, B G, D H, Fig. 1: join f h, Fig. 2: draw g i parallel to b d, cutting f h at i; draw i k parallel to g b, cutting b d at k: draw g l m perpendicular to f h, cutting f h at l; join b k; from i, with the distance b k, describe an arc at m; join i m: draw d' P parallel to d a from the extremity D of the concave side; produce the convex quadrant C B A to Q, and the concave quadrant C' B' A to P, and radiate the line P Q from the centre E, which will complete the whole plan of the rail piece; the part P A' A Q will make a sufficient allowance for the cutting of the joint. Draw ordinates parallel to b k, cutting the chord d' P, the concave side of the plan, and the convex side of the same: produce b k to meet d' P in t, and produce m i to u, making i u equal to k t; through
u draw \( vw \) parallel to \( fh \); from the points where the ordinates intersect \( v'p \), draw lines parallel to \( af, bg, \) or \( dh \), cutting \( vw \): from the cutting points in \( vw \), draw lines parallel to \( um \) as ordinates: transfer the interior ordinates from the plan to the face-mould, also transfer the exterior ordinates of the plan to the face-mould, applying them from the intersected points in the chord \( vw \), and through the points thus set off, trace the concave and convex curves, as also the straight part of the mould; observe, however, that as the straight part of the mould is a parallelogram, if three points on two contiguous sides are found, joining the middle point to each of the other two, gives two sides; each of the other two remaining sides is found by drawing a line parallel to its opposite side.

In the same manner, the mould, Fig. 3, is to be found; but the base line of the heights is taken upon any convenient line, \( sr \), parallel to \( ab \), Fig. 1, so as to shorten the height lines, as otherwise Fig. 3 would occupy more space than might be found at all times convenient; and at any rate, the shortening of the heights will shorten the time of drawing Fig. 3, as shorter lines can be drawn sooner than longer ones. The distance between the height lines of the upper rail piece is the same as those for the under rail piece.

To find the spring of the plank, Fig. 4. Draw any straight line, \( ab \); from which cut off \( bd \), equal to \( gl \), Fig. 2; draw \( dc \), Fig. 4, perpendicular to \( ab \); make \( dc \) equal to \( bv \), Fig. 2, and join \( bc \), Fig. 4; then the angle \( abc \) is denominated the spring of the plank, and the angle is said to be acute, when the planes of the top and edge of the plank, form an acute angle with each other; but when these two form an obtuse angle with each other, the spring is said to be obtuse.

To find the Spring of the Plane at an obtuse Angle.

In Fig. 5, let \( ab \) be any straight line, which produce to \( d \): make \( bd \) equal to \( bd \), Fig. 3; in Fig. 5, draw \( dc \) perpendicular to \( ab \); make \( dc \) equal to \( ace \), Fig. 3; then \( abc \) will be the spring of the plank at the obtuse angle.

Both these bevels are supposed to be applied from the top of the plank; but if the complementary angle of the obtuse spring bevel, which answers to the upper wreath piece, be taken, then the lower spring is applied from the top in order to give the spring of the lower wreath piece, and the complementary spring of the upper wreathed piece to the lower edge of the said piece.

The reader will perceive that in Figs. 2 and 3, though the plan is the same in both, with its position inverted, the face-mould of the lower wreath piece is much longer than that of the upper one. This circumstance is owing to the middle parts of the falling mould being raised over the nosings of the winders; and the more it is raised above the winders, the greater will the face-mould of the lower wreathed piece exceed the length of the mould of the upper wreathed piece; and unless that (if supposing a line passing through the middle of the falling mould bisecting the breadth of the same), the distance of the line thus passing be the same over the winders as over the flyers, the two face-moulds can never be equal.
PLATE 69.

To find the Face-mould for the Rail of a Stair-case having a Landing, so that, when set to its proper Rake, it will stand directly over the given Plan.

Fig. 1. Draw the central line $bq$ parallel to the sides of the rail; on the right line $bq$ apply the pitch-board $qca$; from $q$ to $a$, draw ordinates $nd', me, lf, kg$, and $ih$, at discretion, taking care that one of the lines, as $kg$, touch the inside of the rail at the point $g$, so that you may obtain the same point exactly in the face-mould; then take the parts $qu, uv, vw, wx, xy$, and apply them at Fig. 2, from $qu, uv, vw, wx, xy$; from these points draw perpendiculars to $qy$, and mark their lengths from the plan, Fig. 1; then Fig. 2 will be the mould required.

To find the falling Mould.

Divide the radius of the circle, Fig. 1, into four equal parts, and set three of these parts from 4 to $b$; through $n$ and $v$, the extremities of the diameter of the rail, draw $bn$ and $bv$, to cut the tangent line at the points $o$ and $d$; then will $od$ be the circumference of the rail, which is applied from $c$ to $d$, at Fig. 3, as a base-line: make $ce$ the height of a step; draw the hypothenuse $ed$; at the points $e$ and $d$, apply the pitch-board of a common step at each end of their bases, parallel to $cd$; make $df$ equal to $dg$, if it will admit of it, and by these lengths curve off the angles by the common method of intersecting lines; then draw a line parallel to it, for the upper edge of the mould.
PLATE 70.

To draw a falling Mould for a Rail having Winders all round the circular Part; thence to find the Face-mould.

To describe every particular in this, would almost be repeating what has been already described. The heights are marked the same upon the falling mould, Fig. 2, as they are at the face-mould, which will give the heights of the sections of the rail; and the face-mould, Fig. 1, is traced from the plan according to the lefters. Fig. 3 shows the application of the mould to the plank; take the bevel at h, Fig. 1, and apply it to the edge of the plank at Fig. 3, and draw the line b c; then apply your mould to the top of the plank, keeping one corner of it to the point b, and the other corner close to the same edge of the plank; then draw the top face of the plank by your mould; then take your mould, and apply it to the under side at c, in the same manner.
PLATE 71.

To form the Line of Noseings of the Steps of a semicircular Winding Stair-case, into a regular Curve of contrary Flexure; thence to find the Plan of the Steps, the Form of the String, and the Rail stretched out.

Let $A B C D E$, Fig. 1, be a part of the rail, $B C D$ the semicircular part, $A B$ and $D E$ straight parts, each equal to the breadth of a flyer; let $A I$ be the seat of the last riser. Draw $B F$ parallel to $A I$, and make $B F$ equal to 18 inches; describe a semicircumference $F G H$: divide the breadth of the semicircumference into equal chords, each equal to the breadth of a step, or as nearly equal to the breadth of a step as a semicircular arc will divide. The number thus contained in this example is nine: produce $A B$ to $N$; make $B N$ equal to half the stretch-out of the semicircular convexity $B C D$ of the rail. Draw $N O$ perpendicular to $B N$; make $N O$ equal to half the number of steps in the semicircumference, which are here 4½: draw $A L$ parallel to $A N$, make $B K$ equal to the height of a step; join $A K$: produce $A K$, cutting $N O$ at $M$; bisect $A M$ at $L$: join $L O$: make $L t$ equal to $L A$; describe an arc to tange the straight lines $L A$ and $L t$ at $A$ and $t$: parallel to $N A$, draw $1 a, 2 b, 3 c, 4 d, 5 e$, cutting the curve at $a b c$; draw $a f, b g, c h, d i, e k$; cutting $A N$ at $f, g, h, i, k$; transfer the distances $f g, g h, h i, i k, k N$, to the arc $B C D$, from $B$ to $l$, from $l$ to $m$, from $m$ to $n$, from $n$ to $o$, from $o$ to $c$; make $c p, c q, c r$, &c., equal to $c o, c n, c m$, &c., and through the points $r, 1, 2, 3, &c.$ draw $f v, g p, m q, n r$, &c., which will represent the winders; then $A a b c d e o$ is half the line of noseings, which may be transferred to Fig. 2, at $A a b c d e f$, &c.; thence the form of the steps and string are drawn. The under edge of the falling mould, Fig. 3, is also of the same form, and the upper edge is drawn parallel to the under edge.

This formation of the string-board is certainly a very great improvement in stair-casing, as it gives one universal curvature to the soffit; that is, without any breaking of surface so as to form an angle, as is the case with all stair-cases with winders when the ends of the steps at the rail are equal, and where the breadths of the threads next to the rail differ from the flyers. The angle at the junction of the flyers and winders increases as this difference is greater.
In every kind of stair-case whatever, the breadths of the heads of the steps are always reckoned on a line, bisecting their length, or at 18 inches distant from the rail. In this example, the steps are divided into equal parts, both at the rail and at the wall. This division will make the falling mould straight on the edges, and consequently will form an easy skirting as well as an easy rail.
PLATE 73.

To draw the Face-moulds for the Railing of an elliptic Stair-case.

The plan and section being laid down as in the preceding plates, it will be observed that the ends of the steps are equally divided at each end; that is, they are equally divided round the elliptic wall, and also at the rail. In this Plate, the rail is laid down to a larger size than that in the last Plate; the plan of this rail must be divided round, into as many equal parts as there are steps; then take the treads of as many steps as you please, suppose 8, and let \( hh \), at Fig. 1, be the tread of eight steps; on the perpendicular \( hm \) set up the height of as many steps, that is, 8; and draw the hypothenuse \( mh \), which will give the under edge of the falling mould. The reader will observe that this falling mould will be a straight line, excepting a little turn at the landing and at the scroll, where the rail must have a little bend at these places, in order to bring it level to the landing, and to the scroll; then mark the plan of your rail in as many places as you would have pieces in your rail (in this plan are three); then draw a chord line for each piece to the joints; also draw lines parallel to the chords, to touch the convex side of the plan of the rail; from every joint draw perpendiculars to their respective chords. Now the tread of the middle piece at \( c \) being just 8 steps, the height of the section from \( h \) to \( m \) is 8 steps; and the section \( mn \) is the same as \( mn \) on the falling mould, and the section \( hi \) is the same height as \( hi \) upon the falling mould; draw a line to touch the sections, and complete your face-mould as in the foregoing plates: in each of the other pieces at \( e \) and \( g \), the number of treads being 6; therefore, from your falling mould set the stretch of six steps, from \( h \) to \( h' \). Draw \( h' l \) parallel to \( hn \), then \( h' h l \) will give the height of the sections at \( d \) and \( e \): everything else agreeably to the letters.

The stretch-out of 8 steps, or any other number, is not reckoned on the chord; but it is the stretch-out round the convex side of the rail, or what most people call the inside.
PLATE 74.

How to diminish the Shaft of a Column by the ancient Method.

Fig. 1. Describe a semicircle at the bottom; let a line be drawn through the diameter at the top, parallel with the axis of the column, till it intersects the semicircle at 1, at the bottom; then 1, 1 at the bottom will be equal to 1, 1 at the top; divide the arch into four equal parts, and through these points draw lines parallel to the base, the height of the column being also divided into the same number of parts, and lines drawn parallel to the base, then the column is to be traced from the semicircle, according to the figures.

How to describe the Column by another Method.

Fig. 2. Take the semi-diameter $ab$ at bottom, and set the foot of your compass in the top at $c$, and cross the axis at $g$, and draw the line $a$ 8 on the outside, parallel to $b$ 8 on the axis, and divide each of these lines into eight equal parts, and set the diameter $ab$ at the bottom, along the slant lines 1 1, 2 2, 3 3, &c., from the axis; this will give the diminishing of the column.

How to diminish the Column by Lines drawn from a Centre at a Distance.

Fig. 3. Take the semi-diameter $ab$ at the bottom, set the foot of your compass in $c$ at the top, and cross the axis in the point $d$; continue $cd$ at the top, and $ab$ at the bottom, to meet at $e$; then draw from $e$ as many lines across the column as you please, and take the diameter $ab$ at the bottom, and prick each line upon the axis equal to $ba$, which will give the swell of the column.

To diminish a Column by Laths, upon the same Principle.

Fig. 4. The point $e$ being found, as in Fig. 3, groove a rod $db$, and lay the groove upon the axis of the column, and groove the describing rod upon the under side, and lay the groove upon a pin fixed at $e$; then fix a pin at $g$, to run in the groove upon the axis of the column, and the distance of the pencil at $f$, equal to $ba$, then move the pencil at $f$; it will describe the curve.

How to make a diminishing Rule.

Fig. 5. Divide the height of your rule into any number of equal parts, as 6; draw lines at right-angles from these points across the rule, and divide the projection of the rule at the top, that is, half of what the column diminishes, into the same number of equal parts. Put a pin or brad-awl at $a$; lay a ruler from $a$ to 5, mark the cross line at $f$; then lay a ruler from 4 to $a$, and mark the next cross line at $c$; then lay the ruler from 3 to $a$, mark the next at $d$, and so on to the bottom; bend a slip round these points, and draw the curve by it, and it will give a proper curve for the side of the column.

This is the readiest method, and gives the best curve of any that I have tried.
PLATE 75.

The Plan and Elevation of a circular Sash, in a circular Wall, being given, to find the Mould for the radial Bars, so that they shall be perpendicular to the Plan.

Fig. 1. Draw perpendiculars from the points 1, 1, 1, 1, &c., at A and B, in the radial bars, either equally divided, or taken at discretion, down upon the plan to 1, 2, 3, 4, 5, 6, 7, at C and D; and draw a line from the first division upon the convex side parallel to the base; then draw ordinates from 1, 1, 1, 1, &c., at right-angles to the radial bars, at A and B, which being pricked from the plans at D and C, will give a mould for each bar; and the bevels upon the end will show the application of the moulds.

To find the Veneer of the Arch-bar.

Fig. 2. To avoid confusion, I have laid down the plan and elevation for the head of the sash below. The stretch-out of the veneer is got round, 1, 2, 3, 4, 5, 6, on the arch-bar, which, being pricked from the small distance on the plan at M, will give the veneer above, at E.

To find the Face-mould for the Sash-head.

Fig. 2. Divide the sash-head round into any number of equal parts, at G, and draw them perpendicular to the base at H; draw the chord of one-half of the plan at H, and draw a line parallel to it to touch the plan upon the back side; then the distance between these lines at H, will show what thickness of stuff the head is to be made out of; and from the intersecting points on the back side, draw perpendiculars from the base of the face-mould, which, being pricked from the elevation, as the figures direct, will give the face-mould.

To find the Moulds for giving the Form of the Head, perpendicular to the Plan.

Fig. 3. The base of L is got round the arch 1 2 3 4 5 6, at F, Fig. 2, and the base of K is got round a b c d e f g, also at F, and the heights of the ordinates of each are pricked either from H or I, which will give both moulds.

Note.—The face-mould at G, Fig. 2, must be applied in the same manner as in groins; so that the sash-head must be bevelled by shifting the mould G, on each side, before you can apply the moulds K and L, Fig. 3; the black lines at K and L are pricked from the plan, at H; these black lines will exactly coincide with the front of the rib when bent round; a line being drawn by the other edge of the moulds, will be perpendicular over its plan, and the thickness of the sash-frame towards the inside will be found near enough by guaging from the outside.
Fig. 1. The plan and elevation of an architrave or archivolt for a circular window, in a circular wall.

To find the Form of the Veneer.

Fig. 2. Lay down the line 6' 6', equal to the double stretch-out of the line a b c d, &c., Fig. 1; erect perpendiculairs from the corresponding points, and by measurement we may obtain the arch 5 4 3 2, &c., which is the form of the veneer.

The first veneer is to be partly cut out of the solid architrave as far as h 3 and k 3, on each side, to join to the middle piece that lays between these joints; break the joint with the next veneer that is between p 6' and m n, or 6' s, &c.
To describe the Angle Bars for Shop Fronts.

Fig. 1. B is a common bar, and c is the angle bar of the same thickness; take the raking projection 1, 1, in c, and set the foot of your compass in 1 at b, and cross the middle of the base at the other 1; then draw the lines 2, 2; 3, 3, &c., parallel to 1, 1; then prick your bar at c from the ordinates so drawn at b, which being traced will give the angle bar.

To draw the Mitre Angle of a Commode Front for a Shop.

Fig. 2. Divide the projection each way in a like number of equal parts; then the parallel lines continued each way will give the mitre.

To find the Raking Mouldings of a Pediment.

Fig. 4. Let the simarecta on the under side be the given moulding, and let lines be drawn upon the rake at discretion; but if you please, let them be equally divided upon the simarecta, and drawn parallel to the rake; then the mould at the middle being pricked off from these level lines at the bottom, will give the form of the face. The return moulding at the top must be pricked upon the rake, according to the letters.

The cavetto, Fig. 3, is drawn in the same manner.

N. B. If the middle moulding, Fig. 4, be given, perpendiculars must be drawn to the top of it; then horizontal lines must be drawn over the mouldings at each end, with the same divisions as are over the mouldings; and lines being drawn perpendicularly down, as above, will show how to trace the end mouldings.
Figures 1 and 2 show how to trace base mouldings for skirting to stairs, upon the same principles as shown in the last Plate; at the bottom are given two methods of mitring mouldings of different projections together.
PLATE 79.

Given the Form of a Cornice, to draw it to a greater Proportion.

Fig. 1. Let the given height of the cornice be \( a \, a \, \), set one foot of your compass in \( a \), and cross the under side at \( b \) with that height, and from the point \( c \) draw the line \( c \, a \) at right-angles to \( a \, b \); then the height of all your mouldings will be the parts of \( a \, b \), and the projections the parts of \( c \, a \) in proportion.

Note.—\( a \, f \) shows another height; \( c \, e \) its projection in proportion to that height.

To diminish a Cornice in the Proportion of a greater.

Fig. 2. Describe equilateral triangles on the base and projection, as at \( A \), and make \( i \, f \) and \( i \, g \) equal to the intended height, and draw the line \( f \, g \) across the triangle, which will give the heights in proportion to \( a \, b \); put the foot of your compass in \( b \) as a centre, and circle \( b \, c \) round \( b \, h \), and draw the dotted line \( h \, i \), cutting \( f \, g \) in \( k \); then set off \( i \, e \) and \( i \, d \), each equal to \( g \, k \); draw \( d \, e \); then take the divisions of \( e \, d \), and set them from \( f \) to \( m \); in the same order draw perpendiculars: it will give the diminished cornice at \( A \).

To do this by another method, as shown at \( B \), let the given height be \( a \, b \), and draw the hypothenuse \( a \, g \), and lines being squared up to \( a \, b \), from the divisions of \( a \, g \), will give the heights; and if you draw the line \( g \, d \) at a right-angle with \( a \, g \), then \( d \, c \) will give the projection in proportion, when returning upon \( d \, e \).
MOULDINGS.

PLATE 80.

MOULDINGS UPON THE SPRING.

To find the Sweep of a Moulding to be bent upon the Spring round a circular Cylinder.

In Fig. 1, which stands upon a semicircular plan, make $ac$ equal to the height of your moulding, and make $ab$ equal to the projection; describe the form of the moulding, and draw a dotted line to touch the face of it; then draw the line $ed$ to meet in the centre of the body at $d$, so as to keep your moulding to a sufficient parallel thickness; from the centre $d$ describe the several concentric circles, which are the arrises of the moulding required.

To find the Sweep of the Moulding when the Plan is a Segment.

Fig. 2. Complete the semicircle; then proceed as described under Fig. 1.

Figs. 3 and 4 show the method for bending a moulding round the inside, which is performed the same as above.

The demonstration may easily be conceived from the covering of a cone.
APPENDIX.

STRENGTH OF TIMBER.

PROPOSITION I.

The strengths of the different pieces of timber, each of the same length and thickness, are in proportion to the square of the depth; but if the thickness and depth are both to be considered, then the strength will be in proportion to the square of the depth, multiplied into the thickness; and if all the three dimensions are taken jointly, then the weights that will break each will be in proportion to the square of the depth multiplied into the thickness, and divided by the length; this is proved by the doctrine of mechanics. Hence a true rule will appear for proportioning the strength of timbers to one another.

RULE.

Multiply the square of the depth of each piece of timber into the thickness; and each product being divided by the respective lengths, will give the proportional strength of each.

EXAMPLE.

Suppose three pieces of timber, of the following dimensions:
The first, 6 inches deep, 3 inches thick, and 12 feet long.
The second, 5 inches deep, 4 inches thick, and 8 feet long.
The third, 9 inches deep, 8 inches thick, and 15 feet long. The comparative weight that will break each piece is required.

Ans. 9, 12½, and 43½.
Therefore the weights that will break each are nearly in proportion to the numbers 9, 12, and 43, leaving out the fractions, in which you will observe, that the number 43 is almost 5 times the number 9; therefore the third piece of timber will almost bear 5 times as much weight as the first; and the second piece nearly once and a third the weight of the first piece; because the number 12 is once and a third greater than the number 9.

The timber is supposed to be everywhere of the same texture, otherwise these calculations cannot hold true.

**PROPOSITION II.**

Given the length, breadth, and depth of a piece of timber; to find the depth of another piece whose length and breadth are given, so that it shall bear the same weight as the first piece, or any number of times more.

**RULE.**

*Multiply the square of the depth of the first piece into its breadth, and divide that product by its length; multiply the quotient by the number of times as you would have the other piece to carry more weight than the first, and multiply that by the length of the last piece, and divide it by its width; out of this last quotient extract the square root, which is the depth required.*

**EXAMPLE I.**

Suppose a piece of timber 12 feet long, 6 inches deep, 4 inches thick; another piece 20 feet long, 5 inches thick; requireth its depth, so that it shall bear twice the weight of the first piece.  

*Ans. 9.8 inches, nearly.*

**EXAMPLE II.**

Suppose a piece of timber 14 feet long, 8 inches deep, 3 inches thick; requireth the depth of another piece 18 feet long, 4 inches thick, so that the last piece shall bear five times as much weight as the first.  

*Ans. 17.5 inches, nearly.*

**Note.**—As the length of both pieces of timber is divisible by the number 2, therefore half the length of each is used instead of the whole; the answer will be the same.

**PROPOSITION III.**

Given the length, breadth, and depth, of a piece of timber; to find the breadth of another piece whose length and depth is given, so that the last piece shall bear the same weight as the first piece, or any number of times more.
APPENDIX.

RULE.

Multiply the square of the depth of the first piece into its thickness; that divided by its length, multiply the quotient by the number of times as you would have the last piece bear more than the first; that being multiplied by the length of the last piece, and divided by the square of its depth, this quotient will be the breadth required.

EXAMPLE I.

Given a piece of timber 12 feet long, 6 inches deep, 4 inches thick; and another piece 16 feet long, 8 inches deep; requireth the thickness, so that it shall bear twice as much weight as the first piece. \( \text{Ans. 6 inches.} \)

EXAMPLE II.

Given a piece of timber 12 feet long, 5 inches deep, 3 inches thick; and another piece 14 feet long, 6 inches deep; requireth the thickness, so that the last piece may bear four times as much weight as the first piece. \( \text{Ans. 9.722 inches.} \)

PROPOSITION IV.

If the stress does not lie in the middle of the timber, but nearer to one end than the other, the strength in the middle will be to the strength in any other part of the timber, as 1 divided by the square of half the length is to 1 divided by the rectangle of the two segments, which are parted by the weight.

EXAMPLE I.

Suppose a piece of timber 20 feet long, the depth and width is immaterial; suppose the stress or weight to lie five feet distant from one of its ends, consequently from the other end 15 feet, then the above portion will be \( \frac{1}{10 \times 10} : \frac{1}{100} : \frac{1}{5 \times 15} = \frac{1}{75} \) as the strength at five feet from the end is to the strength at the middle, or ten feet, or as \( \frac{100}{100} = 1 : \frac{100}{75} = 1 \frac{1}{3} \).

Hence it appears that a piece of timber 20 feet long is one-third stronger at 5 feet distance for the bearing, than it is in the middle, which is 10 feet, when cut in the above proportion.

EXAMPLE II.

Suppose a piece of timber 30 feet long; let the weight be applied 4 feet distant from one end, or more properly from the place where it takes its bearing, then from the other end it
APPENDIX.

will be 26 feet, and the middle is 15 feet; then, \( \frac{1}{15 \times 15} = \frac{1}{225} : \frac{1}{4 \times 26} = \frac{1}{104} \) or as 
\[
\frac{225}{225} = \frac{1}{104} = \frac{2}{17} or nearly 2 \frac{1}{6}.
\]

Hence it appears that a piece of timber 30 feet long will bear double the weight, and one-sixth more, at four feet distance from one end, than it will do in the middle, which is 15 feet distant.

EXAMPLE III.

Allowing that 266 pounds will break a beam 26 inches long, requireth the weight that will break the same beam when it lies at 5 inches from either end; then the distance to the other end is 21 inches; 21 x 5 = 105, the half of 26 inches is 13: : 13 x 13 = 169; therefore the strength at the middle of the piece is to the strength at 5 inches from the end, as 
\[
\frac{169}{105} : \frac{169}{105} or as 1 : \frac{169}{105} the proportion is stated thus: 1: \frac{169}{105} : 266: 428 +, Ans.
\]

From this calculation it appears, that rather more than 428 pounds will break the beam at 5 inches distance from one of its ends, if 266 pounds will break the same beam in the middle.

By similar propositions the scantlings of any timber may be computed, so that they shall sustain any given weight; for if the weight one piece will sustain be known, with its dimensions, the weight that another piece will sustain, of any given dimensions, may also be computed. The reader must observe, that although the foregoing rules are mathematically true, yet it is impossible to account for knots, cross-grained wood, &c., such pieces being not so strong as those which are straight in the grain; and if care is not taken in choosing the timber for a building, so that the grain of the timbers run nearly equal to one another, all rules which can be laid down will be baffled, and consequently all rules for just proportion will be useless in respect to its strength. It will be impossible, however, to estimate the strength of timber fit for any building, or to have any true knowledge of its proportions, without some rule; as without a rule everything must be done by mere conjecture.

Timber is much weakened by its own weight, except it stands perpendicular, which will be shown in the following problems; if a mortice is to be cut in the side of a piece of timber, it will be much less weakened when cut near the top, than it will be if cut at the bottom, provided the tenon is drove hard in to fill up the mortice.

The bending of timber will be nearly in proportion to the weight that is laid on it; no beam ought to be trusted for any long time with above one-third or one-fourth part of the weight it will absolutely carry: for experiment proves, that a far less weight will break a piece of timber when hung to it for any considerable time, than what is sufficient to break it when first applied.
PROBLEM I.

Having the length and weight of a beam that can just support a given weight, to find the length of another beam of the same scantling that shall just break with its own weight.

Let \( l = \) the length of the first beam,
\[ L = \] the length of the second;
\[ a = \] the weight of the first beam,
\[ w = \] the additional weight that will break it.

And because the weights that will break beams of the same scantling are reciprocally as their lengths,
\[
\frac{1}{L} : \frac{1}{L} = \frac{a}{2} : \frac{w + \frac{a}{2}}{2} \]

therefore \( \frac{1}{L} : \frac{1}{L} = \frac{w + \frac{a}{2}}{2} = W = \) the weight that will break the greater beam; because \( w + \frac{a}{2} \) is the whole weight that will break the lesser beam.

But the weights of beams of the same scantling are to one another as their lengths:
Whence, \( l : L :: \frac{a}{2} : L \frac{a}{2} = W \) half the weight of the greater beam.

Now the beam cannot break by its own weight, unless the weight of the beam be equal to the weight that will break it:
\[
L \frac{a}{2} = \frac{w + \frac{a}{2}}{2} \]

Wherefore, \( \frac{L a}{2} = \frac{w + \frac{a}{2}}{L} = \frac{2 w + a}{2 L} : L = L :: L^2 a = 2 w l^2 + a l^2; \)
\[
\therefore a : 2 w + a : l^2 : L^2, \text{ consequently } \sqrt{L^2} = L = \text{the length of the beam that can just sustain its own weight.} \]

PROBLEM II.

Having the weight of a beam that can just support a given weight in the middle, to find the depth of another beam similar to the former, so that it shall just support its own weight.

Let \( d = \) the depth of the first beam;
\[ x = \] the depth of the second;
\[ a = \] the weight of the first beam;
\[ w = \] the additional weight that will break the first beam;
then will \( w + \frac{a}{2} \) or \( \frac{2 w + a}{2} \) = the whole weight that will break the lesser beam.

And because the weights that will break similar beams are as the squares of their lengths,
\[
\therefore d^2 : x^2 :: \frac{2 w + a}{2} : \frac{2 x^2 w + a x^2}{2 a l^2} = W
\]
the weights of similar beams are as the cubes of their corresponding sides:

\[
\text{Hence } d^3 : x^3 : \frac{a}{2} : \frac{ax^3}{2d^3} = W
\]

\[
\text{:. } \frac{ax^3}{2d^3} = \frac{2x^2w + x^2 a}{2d^2} : a x = 2w d + a d
\]

\[
\therefore a : a + 2w : d : x = \text{the depth required.}
\]

As the weight of the lesser beam is to the weight of the lesser beam together with the additional weight, so is the depth of the lesser beam to the depth of the greater beam.

Note.—Any other corresponding sides will answer the same purpose, for they are all proportioned to one another.

**EXAMPLE.**

Suppose a beam whose weight is one pound, and its length 10 feet, to carry a weight of 399·5 pounds, requireth the length of a beam similar to the former, of the same matter, so that it shall break with its own weight.

here \(a = 1\)
and \(w = 399·5\)
then \(a + 2w = 800 = 1 + 2 \times 399·5\)
\(d = 10\)

Then by the last problem it will be \(1 : 800 : 10 : 8000 = x\) for the length of a beam that will break by its own weight.

**PROBLEM III.**

The weight and length of a piece of timber being given, and the additional weight that will break it, to find the length of a piece of timber similar to the former, so that this last piece of timber shall be the strongest possible:

Put \(l\) = the length of the piece given

\(w = \text{half its weight,}\)

\(W = \text{the weight that will break it;}\)

\(x = \text{the length required.}\)

Then, because the weights that will break similar pieces of timber are in proportion to the squares of their lengths,

\[
\therefore l^2 : x^2 : W + w : \frac{Wx^2 + wx^2}{l^2} = \text{the whole weight that breaks the beam;}
\]

and because the weights of similar beams are as the cubes of their lengths, or any other corresponding sides,
APPENDIX.

then \( l^3 : x^3 : : w : \frac{w x^3}{l^2} \) the weight of the beam;
consequently \( \frac{W x^2 + w x^2}{l^2} \) less \( \frac{w x^3}{l^3} \) is the weight that breaks the beam = a maximum;
therefore its fluxion is nothing.

\[ \therefore \text{that is, } 2 W x x + 2 w x x - \frac{3 w x^2}{l} = \text{nothing.} \]

\[ 2 W + 2 w = \frac{3 w x}{l} \text{ therefore, } x = l \times \frac{2 W + 2 w}{3 w} \]

Hence it appears from the foregoing problems, that large timber is weakened in a much greater proportion than small timber, even in similar pieces, therefore a proper allowance must be made for the weight of the pieces, as I shall here show by an

EXAMPLE.

Suppose a beam 12 feet long, and a foot square, whose weight is 3 hundred weight, to be capable of supporting 20 hundred weight, what weight will a beam 20 feet long, 15 inches deep, and 12 thick, be able to support?

<table>
<thead>
<tr>
<th>12 inches square</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>144</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>75</td>
</tr>
<tr>
<td>12)1728</td>
<td>225</td>
</tr>
<tr>
<td>144</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>2070</td>
</tr>
<tr>
<td></td>
<td>2700</td>
</tr>
<tr>
<td></td>
<td>135</td>
</tr>
</tbody>
</table>

But the weights of both beams are as their solid contents:

<table>
<thead>
<tr>
<th>12 inches square</th>
<th>15 deep</th>
<th>12 wide</th>
</tr>
</thead>
<tbody>
<tr>
<td>144</td>
<td>180</td>
<td>240</td>
</tr>
<tr>
<td>144 inches = 12 feet long</td>
<td>576</td>
<td>7200</td>
</tr>
<tr>
<td>576</td>
<td>360</td>
<td></td>
</tr>
<tr>
<td>144</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20736 solid contents of the 1st beam</td>
<td>43200 solid contents of the 2d beam</td>
<td></td>
</tr>
</tbody>
</table>
20736:43200::3 cwt. lb.

\[ \frac{20736}{124416} \times 28 = \text{weight of the 2d beam} \]

\[ \frac{67.5}{135} = 0.5 \\
\frac{270}{20736} = 0.012 \]

\[ \frac{5184}{112} = 46.0 \\
\frac{10368}{5184} = 2.0 \\
\frac{20736}{3125} = 6.628 \]

\[ \frac{144}{17} \times 3 = 21.0 \\
\frac{17\frac{3}{8}}{144} = 0.125 \]

If several pieces of timber of the same scantling and length are applied one above another, and supported by props at each end, they will be no stronger than if they were laid side by side; or this, which is the same thing, the pieces that are applied one above another are no stronger than one single piece whose width is the width of the several pieces collected into one, and its depth the depth of one of the pieces; it is therefore useless to cut a piece of timber lengthways, and apply the pieces so cut one above another, for these pieces are not so strong as before, even if bolted.

EXAMPLE.

Suppose a girder 16 inches deep, 12 inches thick, the length is immaterial, and let the depth be cut lengthways in two equal pieces; then will each piece be 8 inches deep, and 12...
inches thick. Now, according to the rule of proportioning timber, the square of 16 inches, that is, the depth before it was cut, is 256, and the square of 8 inches is 64; but twice 64 is only 128, therefore it appears that the two pieces applied one above another, are but half the strength of the solid piece, because 256 is double 128.

If a girder be cut lengthways in a perpendicular direction, the ends turned contrary, and then bolted together, it will be but very little stronger than before it was cut; for although the ends being turned give to the girder an equal strength throughout, yet wherever a bolt is, there it will be weaker, and it is very doubtful whether the girder will be any stronger for this process of sawing and bolting; and I say this from experience.

If there are two pieces of timber of an equal scantling (Pl. 51, Fig. B), the one lying horizontal, and the other inclined, the horizontal piece being supported at the points e and f, and the inclined piece at c and d, perpendicularly over e and f, according to the principles of mechanics, these pieces will be equally strong. But, to reason a little on this matter, let it be considered, that although the inclined piece D is longer, yet the weight has less effect upon it when placed in the middle, than the weight at h has upon the horizontal piece C, the weights being the same; it is therefore reasonable to conclude, that in these positions the one will bear equal to the other.

The foregoing rules will be found of excellent use when timber is wanted to support a great weight; for, by knowing the superincumbent weight, the strength may be computed to a great degree of exactness, so that it shall be able to support the weight required. The consequence is as bad when there is too much timber, as when there is too little, for nothing is more requisite than a just proportion throughout the whole building, so that the strength of every part shall always be in proportion to the stress; for when there is more strength given to some pieces than others, it encumbers the building, and consequently the foundations are less capable of supporting the superstructure.

No judicious person, who has the care of constructing buildings, should rely on tables of scantlings, such as are commonly in books; for example, in story posts the scantlings, according to several authors, are as follows:

For 9 feet high 6 inches square.
12 ———— 8
15 ———— 10
18 ———— 12
Now, according to this table, the scantlings are increased in proportion to the height; but there is no propriety in this, for each of these will bear weight in proportion to the number 9, 16, 25, and 36, that is, in proportion to the square of their heights, 36 being 4 times 9; therefore the piece that is 18 feet long, will bear four times as much weight as that piece which is 9 feet long; but the 9 feet piece may have a much greater weight to carry than an 18 feet piece, suppose double: in this case it must be near 12 inches square instead of 6. The same is also to be observed in breast-summers, and in floors where they are wanted to support a great weight; but in common buildings, where there are only customary weights to support, the common tables for floors will be near enough for practice.

To conclude the subject, it may be proper to notice the following observations which several authors have judiciously made, viz.; that in all timber there is moisture, wherefore all bearing timber ought to have a moderate camber, or roundness on the upper side, for till that moisture is dried out, the timber will swag with its own weight.

But then observe, that it is best to truss girders when they are fresh sawn out, for by their drying and shrinking, the trusses become more and more tight.

That all beams or ties be cut, or in framing forced to a roundness, such as an inch in twenty feet in length, and that principal rafters also be cut or forced in framing, as before; because all joists, though ever so well framed, by the shrinking of the timber and weight of the covering will swag, sometimes so much as not only to be visible, but to offend the eye: by this precaution the truss will always appear well.

Likewise observe, that all case-bays, either in floors or roofs, do not exceed twelve feet if possible; that is, do not let your joints in floors exceed twelve feet, nor your purlines in roofs, &c., but rather let their bearing be eight, nine, or ten feet. This should be regarded in forming the plan.

Also, in bridging floors, do not place your binding or strong joists above three, four, or five feet apart, and take care that your bridging or common joists are not above ten or twelve inches apart, that is, between one joist and another.

Also, in fitting down tie-beams upon the wall plates, never make your cocking too large, nor yet too near the outside of the wall plate, for the grain of the wood being cut across in the tie-beam, the piece that remains upon its end will be apt to split off, but keeping it near the inside will tend to secure it.

Likewise observe, never to make double tenons for bearing uses, such as binding joists,
common joists, or purlines; for, in the first place, it very much weakens whatever you frame it into, and in the second place, it is a rarity to have a draught to both tenons, that is, to draw both joints close; for the pin in passing through both tenons, if there is a draught in each, will bend so much, that unless it be as tough as wire, it must needs break in driving, and consequently do more hurt than good.

Roofs will be much stronger if the purlines are notched above the principal rafters, than if they are framed into the side of the principals; for by this means, when any weight is applied in the middle of the purline, it cannot bend, being confined by the other rafters; and if it do, the sides of the other rafters must needs bend along with it; consequently it has the strength of all the other rafters sideways added to it.

THE END.
Neutralizing agent: Magnesium Oxide
Deacidified using the Bookkeeper process.
Neutralizing agent: Magnesium Oxide
Treatment Date: May 2004