THE ARITHMETIC

OF

THE STEAM ENGINE

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PREFACE.

THE object of this small volume is to furnish a clear and concise digest of the fundamental principles of the steam-engine, and the practical calculations based upon them. While not entering into the more abstruse mathematics of the subject, it is believed that these few pages contain all that is necessary to solve the ordinary problems relating to steam in its applications to the steam-engine.

The preparation of this work has grown out of the author's own need for a collection of simple and accurate facts and rules, such as he has been unable to find in any single treatise of the many he has consulted. Its merit, if it has any, is not in presenting anything new, but merely in putting the accepted facts of the subject in readily accessible shape for practical use.

E. S. G.

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ARITHMETIC OF THE STEAM-ENGINE.

HEAT.

HEAT is a form of motion, and may be defined as a state of intermolecular activity.

Heat is imponderable: the *weight* of a cubic foot of water is not increased by having additional heat imparted to it, although its *temperature* is increased.

The unit of *temperature* in this country and England is the Fahrenheit degree.

The unit of *heat* in this country and England is the British Thermal Unit (B. T. U.). It is the quantity of heat necessary to raise the temperature of 1 lb. water from 39° (degree of maximum density) to 40°. In general it is considered as that quantity of heat necessary to raise 1 lb. of water 1° from or to any temperature between the limits of 32° and 212°.

If equal quantities of combustible be burned in heating two different volumes of water of equal initial temperature, their respective augmentations of temperature will be inversely as the volumes or weights of water heated.

Thus, if one vessel contains 1 lb. and another similar one contains 10 lbs. of water at 40°, and equal weights of alcohol are burned under both, then if the temperature of the 1 lb. water rises to 50°, that of the 10 lbs. would rise to 41°.

If the quantities of combustible consumed were proportional to the weights of water heated, then the temperatures would receive equal increments, and remain the same. The water in each vessel would then contain and could communicate quantities of heat, or numbers of B. T. U., directly as their weights.

Thus, in the above case, if 10 times as much alcohol were burned under the larger vessel as under the smaller, the temperature of the water in both would rise to 50°; and the larger volume would contain 10 times as much heat, and could communicate or extract 10 times as much to or from another body of lower or higher temperature.

Naturally, such exactness of proportion could not be realized in practice; but the actual results would always be in the direction indicated, between 32° and 212°.

The temperature of a body is the degree of heat which it communicates to, and which is recorded by, a thermometer. The quantity of heat which it contains is its temperature into its weight into its specific heat.

The specific heat of any substance is the quantity of heat necessary to raise the temperature of a given weight of the substance through a given number of degrees, compared with that necessary to raise the same weight of water through the same number of degrees. It is therefore that fraction of a B. T. U. necessary to raise the temperature of 1 lb. of the substance through 1°. Water having the greatest capacity for heat of any known substance, its specific heat is taken as unity, and that of all other substances is a fraction less than unity.

Thus, it takes $\frac{1}{10}$th B. T. U. to raise the temperature of 1 lb. copper 1°. The specific heat of copper is therefore 0.10.

In mixtures of substances, liquid or solid, of different (or the same) temperatures, weights, and specific heats, the total quantity of heat before mixing is equal to the total quantity of heat after mixing; the bodies of higher temperature losing heat which those of lower temperature gain, until a uniform temperature is attained throughout the mass, the total loss being equal to the total gain. The multiplication of the weight of a body of given temperature by its specific heat reduces it to an equivalent weight of water of equal temperature, so that, practically, mixtures of all bodies, solid or liquid, are supposed to take place between given weights of water at given temperatures.

For the mixture of two substances, therefore, we have the equations:—

$$TWS + T'W'S' = t(WS + W'S'). \qquad (1)$$

$$t = \frac{TWS + T'W'S'}{WS + W'S'}. \qquad (2)$$

In which:

W = weight in lbs. of one substance of temperature T and sp. ht. S; W', T', S', = same for the other substance, and t = uniform temperature of mixture.

The above formulæ can obviously be extended to any number of substances.

Example 1. Twenty lbs. of water at 73°; 341 lbs. at 87°.5, and 50 at 200° are mixed together. What is the uniform temperature of mixture? (Sp. ht. water = 1).

$$t = \frac{20 \times 73 + 341 \times 87.5 + 50 \times 200}{20 + 341 + 50} = 100°.48.$$

Example 2. From a vessel containing 10 lbs. water at 180° there is taken 1 lb., which is mixed with 9 lbs. water at 40°. The remaining 9 lbs. at 180° are then mixed with 1 lb. water at 40°. The two separate mixtures are then mixed together. What are the respective temperatures of the three mixtures? 54°; 166°; 110°.

Example 3. 2 lbs. copper (sp. ht. = 0.10) at 500° and 3 lbs. cast iron (sp. ht. 0.13) at 650° are immersed in 100 lbs. water at 60°. What is the resultant temperature of the three bodies when it has become uniform?

$$t = \frac{500 \times 2 \times 0.10 + 650 \times 3 \times 0.13 + 60 \times 100}{2 \times 0.10 + 3 \times 0.13 + 100} = 63^\circ.16.$$

Example 4. — How many pounds of copper at 500° must be added to 100 lbs. of water at 75°, to heat it to a temperature of 100°?

$$500 \times 0.10\,x + 75 \times 100 = 100\,(0.10\,x + 100);$$
$$x = 62.5 \text{ lbs.}$$

Example 5. — 20 lbs. of alcohol at 90° are mixed with 10 lbs. of water at 32°. The resultant temperature is 65°. What is the specific heat of the alcohol?

$$90 \times 20\,x + 32 \times 10 = 65\,(20\,x + 10);$$
$$x = 0.66 = \text{sp. ht. alcohol.}$$

Example 6. — What must be the temperature of 10 lbs. water which, added to 70 lbs. at 100°, raises the temperature to 101°?

$$10\,x + 7000 = 8080; \; x = 108^\circ.$$

Practically all such calculations relating to mixtures give only approximate results, because they are based on the unrealizable assumption that the interchange of heat takes place wholly and only between the bodies mixed, taking no account of the temperature or conductibility of the containing vessel, or of the temperature of the surrounding air.

THE ABSOLUTE ZERO OF TEMPERATURE.

This is the imaginary temperature at which the volume of a perfect gas, contracting 0.268 of its bulk in passing from 212° to 32°, would shrink to zero. It is determined from the above data, by the following proportion : —

$$\frac{x}{180} = \frac{1}{0.268}; \quad x = 671.7.$$

Then absolute zero, Z, below 0 Fahr., is given by the equation : —

$$Z = 671.7 - 212 = 459^\circ.7.$$

It may be taken practically as 460°. The absolute temperature corresponding to any given reading of the Fahrenheit thermometer is obtained by adding, algebraically, 460° to such reading. Thus, if steam has a temperature of 300°, its absolute temperature is 760°. The absolute temperature corresponding to $-50°$ is 410°.

Obviously we can conceive of the possibility of an absolute minimum temperature, but not of a maximum, because converging lines may ultimately meet, while there is no limit to diverging ones.

ICE.

THE temperature of ice can be indefinitely lowered, but cannot be raised at atmospheric pressure above 32°, at which pressure and temperature ice tends to melt and water to freeze.

A pound of ice at 32° requires the addition of 143 B. T. U. to convert it to water at 32°, and 143 B. T. U. must be extracted from a pound of water at 32° in order to convert it into ice at 32°. In mixtures containing ice at 32° therefore, 143 times the weight of ice must first be subtracted, and then 32 times its weight added to the mixture. This is equivalent to subtracting 111 times its weight from the total heat of the mixture.

Example 7. — 5 lbs. of ice at 32° are immersed in 7 lbs. of water at 160°. What is the temperature of the mixture when all the ice has just melted?

$$t = \frac{160 - 111 \times 5}{7 + 5} = 47°.$$

Ice presents the peculiarity of having a specific heat of about 0.50 between 0° and 32°.

Example 8. — 10 lbs. of ice at 0° are immersed in 100 lbs. of water at 212°; what is the resultant temperature of the mixture when all the ice is just melted?

The consequence of this mixture would be first to raise the temperature of the ice at 0° to ice at 32°, and it then becomes the same as in previous example. Thus:

$$t = \frac{212 \times 100 - 32 \times 10 \times 0.50 - 111 \times 10}{110} = 181^{\circ}.18.$$

If the ice had been at 32° instead of at 0°:

$$t = 182.64.$$

Practically, ice is assumed to be at 32° in calculations of mixtures.

STEAM.

JUST as 1 lb. of ice at 32° requires the addition of 143 B. T. U. to convert it into water at 32°, so 1 lb. of water at 212° requires, at atmospheric pressure, the addition of 966 B. T. U. to convert it into steam at 212°.

The total quantity of heat contained in 1 lb. of steam at 212° is therefore 212° + 966° = 1178°. A thermometer placed in the steam or the water from which it is produced would register only 212°. The 966°, which have no effect upon the thermometer, constitute the latent heat of the steam, and 1178° is the total heat. The latent and total heats pertain only to the steam; the water contains only the heat indicated by its temperature.

At the sea-level, or under a normal atmospheric pressure of about 15 lbs. per sq. in., water cannot be reduced below 32°, or raised above 212° without becoming, or tending to become, ice in the one case and steam in the other.

Temperature and sensible heat are generally considered as synonymous terms. Sensible heat is, however, always a little higher than the temperature, as is shown by Porter's Tables. The difference is so slight that in practical calculations no appreciable error ensues from considering them the same. Throughout the following pages the term *Temperature* will be uniformly used.

If steam be generated in a closed or partially closed vessel, so that it cannot freely escape as fast as it is formed, the pressure increases beyond that of the atmosphere, and the temperature (and sensible heat) increase also, while the latent heat diminishes, but not in so rapid a proportion. The total heat of steam does not therefore greatly vary, even when the pressures are considerably increased. This fact has a most important bearing upon the economics of the steam-engine; for the result is, that it costs very little more fuel to maintain steam at a high pressure than at a low one.

The three characteristics, Temperature, Latent Heat, and Total Heat, corresponding to various pressures, are given in the published tables of the properties of dry saturated steam, by which rather contradictory title is meant steam without admixture of entrained water, generated in contact with unevaporated water of its own temperature, as is the case of steam in a boiler. If heat be added to steam after all the water from which it is formed has been evaporated, it becomes what is called *Superheated Steam.*

Such tables should always be consulted when making the various calculations relating to steam; but there are also formulæ which give, some very closely and others much less so, the same properties. Thus, very closely: —

$$H = 0.30\ T + 1115. \tag{3}$$

$$L = 1114 - 0.70\ T. \tag{4}$$

In which H = total heat; T = temperature, and L = latent heat.

The following give approximately the total heat and temperature of steam at different gauge pressures. For those below 15 lbs. per sq. in.: —

$$H = 4\sqrt{G} + 1178. \tag{5}$$

$$T = 12.5\sqrt{G} + 212. \tag{6}$$

For those above 15 lbs.: —

$$H = 4.2\sqrt{G} + 1175. \tag{7}$$

$$T = 14\sqrt{G} + 199. \tag{8}$$

In which G = gauge or boiler pressure, or pressure above the atmosphere. The absolute pressure, which is most commonly used, is obtained from the gauge pressure by adding that of the atmosphere, or say 15 lbs. To use the above formulæ when the absolute pressure is given, subtract 15 to reduce it to gauge pressure.

Should the absolute pressure be less than 15 lbs., subtract it from 15, extract the square root of the remainder, and give it the negative sign.

Example 9. — Temperature of steam is 300°. What are total and latent heats?

$$H = 0.30 \times 300 + 1115 = 1205°.$$

$$L = 1114 - 0.70 \times 300 = 904°.$$

Example 10. — Gauge pressure = 121. What is total heat?

$$H = 4.2 \times 11 + 1175 = 1221°.20.$$

The correct answer, per Porter's Tables, is 1220°.85.

Example 11. — Absolute pressure 6 lbs. What is temperature?

$$T = 212 - 12.5 \times 3 = 174°.5.$$

The correct answer, per Porter's Tables, is 170°.57.

When mixtures of steam and water are made, the amount of heat contributed by the steam is its total heat into its weight.

Example 12. — The temperature of the exhaust steam of a condensing engine is 212°. How much injection water at 60° must the jet condenser furnish per pound of steam, to maintain the temperature of the hot well at 105°?

The general formula for all such problems, derived from (2) is —

$$W = \frac{H - C}{C - I}. \tag{9}$$

In which W = weight of injection water in pounds, per pound of steam; H = total heat of steam above 0° Fahr.; C = temperature of condenser or hot well, and I = temperature of injection water.

Substituting the data of the example in (9):

$$W = \frac{1178 - 105}{105 - 60} = 23.90 \text{ lbs.}$$

Formula (9) relates to the jet condenser. When a surface condenser is used, no mixture of water and steam takes place. The data required are: Total heat of steam; temperature of hot well; weight of circulating water per pound of steam (instead of weight of injection water); its initial temperature (corresponding to temperature of injection), and its final temperature. This is the only additional factor.

In the surface condenser, the heat necessary to raise the temperature of the circulating water from its initial to its final temperature is furnished by the steam, and the quantity of circulating water must be sufficient to extract enough heat from the steam on passing from its initial to its final temperature to reduce the steam to water of the temperature of the condenser or hot well. The formula is:

$$W = \frac{H - C}{F - I}. \qquad (10)$$

In which I = initial, and F = final temperature of circulating water.

Example 13. — Temperature of steam and hot well being as in previous example, what weight of circulating water per pound of steam is required, when $I = 60°$ and $F = 105°$?

Inserting data in (1):

$$W = \frac{1178 - 105}{105 - 60} = 23.90 \text{ lbs.}$$

This is the same as in previous example of the action of the jet condenser, as will always happen when the circulating water enters at the temperature of the injection water, and leaves it at that of the hot well; but this rarely occurs in practice, because the final temperature will almost always be lower than that of the hot well, consequently rather more water is required in surface condensation than by jet.

The surface condenser is used when it is desirable to prevent the condensing water from entering the hot well and being pumped back into the boiler. Its use is essential in the modern marine engine working with high-pressure steam. It is also frequently used for pumping-engines, when there is not sufficient water to waste in a jet condenser. In such cases part of the water supply taken either from the suction or force main is passed as circulating water through the surface condenser.

PRESSURE AND VOLUME.

A perfect gas, maintained at a constant temperature, contracts and expands in volume in inverse ratio to the pressure. Thus, if it occupies a volume 2 under a pressure 1, it will occupy a volume 1 under a pressure 2. Hence the invariable relation, —

$$P' V' = P V. \tag{11}$$

In which P and P' are two different pressures, and V and V', the corresponding volumes.

Although saturated steam as used in the steam cylinder by no means complies with the conditions of this law, being neither a perfect gas nor maintaining a constant temperature during changes of pressure and volume, the above formula is used in steam calculations as a convenient approximation with practical results of the greatest utility. In fact, it may be said that much of the progress made in steam engineering has been due to the bold use of simple approximate calculations, by which the leading principles of the art have been kept clearly in view, not only of the educated engineer, but also of the simple artisan.

The volumes correctly corresponding to different pressures are given in the published tables of the properties of saturated steam, but for ordinary pressures, ranging from 50 to 150 lbs. absolute, it is never far wide of the mark to assume a constant value of 420 for PV. Hence, within these limits:

$$P'V' = 420. \qquad (12)$$

Example 14. — What volume does 1 lb. of steam at 100 lbs. absolute pressure occupy?

$$V' = \frac{420}{100} = 4.20 \text{ cu. ft.}$$

The correct volume is 4.33 cu. ft.

For lower pressures, say around 30 lbs. absolute, the constant is about 405, and for those between 150 and 200, about 450.

From (11) we have, for saturated steam:

$$\frac{P' V'}{P V} = 1.$$

For superheated steam the relation is:

$$\frac{P' V'}{P V} = \frac{r'}{r}. \qquad (13)$$

Also:

$$P V = 85.5\, r \qquad (14)$$

In which r and r' are the absolute temperatures, corresponding respectively to PV and $P'V'$.

By consulting a table of the properties of saturated steam it will appear that the pressure, temperature, and volume of a given weight of saturated steam are invariably related, so that one element being given, the other two follow of necessity. For a given weight of saturated steam, the temperature and density increase, and the volume diminishes with an increase of pressure. Professor Yeo says:—

1. "The steam possesses only as much heat as is absolutely necessary for its maintenance as steam at the particular pressure.

2. "The temperature and the volume per pound correspond with the pressure, as in the process of evaporation, being respectively the *lowest temperature* and *the smallest volume per pound* at which steam of the particular pressure can exist; in other words, saturated steam has the lowest temperature and the greatest density consistent with its pressure."

A given weight of superheated steam increases in volume with an increase of temperature, the pressure remaining constant and the density diminishing.

Superheated steam acquires "an amount of heat in excess of that possessed by ordinary steam of the same pressure; and it could then part with heat, until reduced to the ordinary or saturated condition, without any liquefaction occurring." — YEO.

Superheated steam is dry steam not in contact with water, to which additional heat is applied.

With the low pressures of former days, steam was advantageously superheated. As the pressures were increased, it fell into disuse except to the very limited extent necessary to secure dry steam. Of late, however, its use is being revived. Professor Unwin cites instances of its being used with good results in Alsatia.

COMBUSTION AND COMBUSTIBLES.

A combustible is a substance capable of combining with the oxygen of the atmosphere in the presence of a sufficient degree of heat. Such combination is called *combustion.*

"When coal is brought to a high temperature it acquires a strong affinity for oxygen; and combination with oxygen will produce more than sufficient heat to maintain the original temperature, so that part of the heat is rendered applicable to other purposes." — BOURNE.

The complete combustion of 1 lb. of pure carbon evolves 14,500 B. T. U. Since 1 lb. of water at 212° requires the addition of 966 B. T. U. to convert it into steam at same temperature, 1 lb. of pure carbon should be capable of evaporating $\frac{14500}{966} = 15$ lbs. of water "from and at" 212°. Although a pound of good steam coal will, in the laboratory, furnish nearly 14,000 B. T. U., and should therefore evaporate about 14 lbs. of water, it is a very good practical result when a pound of coal burned under or within a boiler develops about 10,000 B. T. U., and evaporates about 10 lbs. of water at ordinary temperatures of feed and steam.

In comparing the efficiency of boilers, the weight of water actually evaporated per pound of coal, from feed water at the given temperature to steam at the given temperature, is reduced to the equivalent weight with both at 212°, or as it is called, "from and at" 212°, as a standard of comparison.

The formula of reduction is:

$$W' = W\frac{(H-F)}{966}. \tag{15}$$

In which W = lbs. of water evaporated per lb. of coal, from feed at $F°$ to steam of total heat $H°$, and W' = the corresponding weight of water at 212°, which would be evaporated into steam at 212° by the same number of heat units.

Example 15. — Feed water, 90°; steam, 95 lbs. absolute. Amount of water evaporated per lb. of coal, 9 lbs. What is the equivalent evaporation per lb. from and at 212°?

$$W' = 9\frac{(1213-90)}{966} = 10.46 \text{ lbs.}$$

EFFICIENCY OF THE STEAM-ENGINE.

One horse-power = 33,000 ft. lbs. per minute, or 1,980,000 per hour. One B. T. U. = 772 ft. lbs. Consequently the heat-equivalent of a horse-power is $\frac{1980000}{772}$ = 2,565 B. T. U. per hour. If 10,000 B. T. U. be developed per hour, they should furnish $\frac{10000}{2565}$ = 3.90 H. P.

10,000 B. T. U. should evaporate, from and at 212°, $\frac{10000}{966}$ = 10.35 lbs. of water. If evaporated in an hour, developing, as above 3.90 horse-power, this would be at the rate of 2.65 lbs. of water or steam from and at 212° per hour per horse-power.

We have seen that 1 lb. good coal will develop in the furnace about 10,000 B. T. U. Hence, 1 lb. coal should develop 3.90 H. P. per hour. In round numbers we can say that theoretically $\frac{1}{4}$ lb. coal and $2\frac{2}{3}$ lbs. water per hour should furnish 1 H. P.

Now, the best type of single-cylinder condensing engine requires at least 25 lbs. water evaporated from and at 212° per hour per horse-power, or say 26.60 lbs., as against the 2.66 which should suffice. Its efficiency is therefore —

$$\frac{2.66}{26.6} = 0.10, \text{ or } 10\%.$$

The most refined type of quadruple-expansion engine will require 14 lbs., or say 13.30 lbs., water evaporated into steam from and at 212° per hour per horse-power, showing an efficiency of $\frac{2.66}{13.30} = 20\%$.

These figures show that the efficiency of the best quadruple-cylinder engine is to that of the best single-cylinder engine about as 2 to 1.

If we admit 10,000 B. T. U. per lb. of coal as a working average, the most perfect single-cylinder condensing engine will require about 2½ lbs. coal per hour per horse-power, and the quadruple about 1⅓ lbs.

The above efficiencies have been figured between the boiler and the cylinder; that is, between the B. T. U. furnished and the indicated horse-power returned. If, however, the work "done on the shaft," or the actual useful work performed, should be taken instead of the indicated horse-power given off by the cylinder, the efficiency would be still less. The efficiency as between the work delivered by the piston, or the indicated horse-power, and that taken off the shaft, or the brake horse-power, varies greatly, according to the power necessary to run the unloaded engine, which varies with the type of engine used; 85% is a good result.

Of all the losses between the coal heap and the shaft, that is to say, between the 14,000 B. T. U. actually contained in a pound of very superior coal which, if consumed in an hour, is theoretically equivalent to nearly 5½ horse-power, and the ½ horse-power, which may perhaps be taken off the shaft of a very perfect engine, or say a total loss of about 90%, it will be seen from what precedes that the loss between the furnace and boiler is small, the loss between the boiler and the cylinder very large, and the loss between the cylinder and shaft small. The main loss is between boiler and cylinder.

It is evident, however, that the computation has not been fair to the cylinder, because it is charged with *all* the heat furnished by the boiler. But some of this heat is not and cannot be used; it is *rejected* by the cylinder to the condenser, and the temperature of the condenser or hot well, together with the weight and initial temperature of the injection water, enable us to calculate the heat thus rejected. This loss of rejected heat can never be wholly got rid of; in other words, the temperature of the hot well can never be reduced to absolute zero.

Carnot's theory of the theoretically possible efficiency of a perfect heat engine, while based upon an ideal state of things not only unrealizable in practice, but to a certain extent self-contradictory in theory, is still our surest guide to the increase of efficiency of the steam-engine; another instance of the indebtedness of steam-engineering to intelligent approximations.

Carnot's theory is thus expressed:

$$E = \frac{r - r'}{r}. \qquad (16)$$

In which E = theoretical maximum of efficiency; r = absolute temperature of heat received, and r' = absolute temperature of heat rejected. This equation brings out in bold relief the central fact that the higher the temperature of the heat received and the lower that of the heat rejected the greater the efficiency. If r' were reduced to absolute zero, the whole of the heat received would be utilized, and the efficiency would = 1.

To make use of (16) practically, it must be borne in mind that the right-hand member is a coefficient to be applied to the total heat (not temperature) furnished. The complete practical formula is therefore:

$$H' = H\frac{(t - t')}{t + 460}. \qquad (17)$$

In which H' = maximum utilizable heat; H = total heat supplied; t = temperature above 0° Fahr. of heat received; t' = temperature above 0° Fahr. of heat rejected, and $t + 460$ = absolute temperature of heat received. [Note that it is unnecessary in (16) to reduce the numerator to absolute temperature.]

As applied to the steam-engine—the only application that we are at present concerned with—the practical formula is thus written:

$$\text{B. T. U.} = L\,\frac{(T - T')}{T + 460}. \qquad (18)$$

In which B. T. U. = heat units theoretically utilizable per pound of steam; L = latent heat of steam; T = temperature of steam; T' = temperature of hot well.

Example 16. — Steam 60 lbs. absolute pressure; temperature of hot well 100°. What is the theoretically possible work of engine in B. T. U. per lb. of steam?

$$\text{B. T. U.} = 908\,\frac{(295 - 100)}{755} = 234.26.$$

It has been already shown that 2,565 B. T. U. per hour = 1 H. P. The above engine should therefore run on $\frac{2565}{234} = 11$ lbs. steam (or water) per hour per horse-power. Practically, it would require at least 25 lbs. of water. Its efficiency therefore is 44%.

Example 17. — Steam 200 lbs. absolute. Hot well, 100°.

$$\text{B. T. U.} = 843.40\,\frac{(387 - 100)}{847} = 286.$$

This would call for 8.95 lbs. water as against at least 13.30 lbs. which such an engine would practically require per hour. Its efficiency therefore is about 67%.

At high temperatures, the latent heat of steam is nearly equal to its absolute temperature, so the efficiency of an engine working with high-pressure steam can be rapidly approximated thus:

$$E = \frac{2565}{W(T - T')}. \qquad (19)$$

In which $W =$ weight of water consumed per indicated horse-power per hour.

This formula is exactly correct for steam at 195 lbs. absolute.

STEAM USED EXPANSIVELY.

Steam is generally used expansively; that is to say, it is cut off in the cylinder before the piston has completed its stroke, and is allowed to expand during the rest of the stroke.

There are three essential elements connected with the expansive working of steam, any two of which being given the third may be deduced. They are: Initial Pressure, Terminal Pressure, and Point of Cut-off. The calculations are made by the use of Boyle's equation, (11), already given:

$$P'V' = PV.$$

Referring to Fig. 1 we have, since cylinder volumes are proportional to length of stroke:

$$Pc = P'l. \qquad (20)$$

In which $P =$ initial absolute pressure of steam, represented to scale by ab; $P' =$ terminal absolute pressure represented on same scale by de; $c =$ length of stroke to point of cut-off $= bc$ in figure; and $l =$ full stroke $= ae$ in figure.

Example 18. — Length of stroke 48 inches. Length to point of cut-off 9 inches. Initial pressure 75 lbs. absolute. What is terminal pressure?

$$P' = \frac{75 \times 9}{48} = 14 \text{ lbs.}$$

To find the pressure, p, at any intermediate length of stroke, s, either Pc or $P'l$ is taken as the constant, according as the initial pressure and length of cut-off, or terminal pressure and length of full stroke, are given. Thus:

$$p = \frac{Pc}{s} = \frac{P'l}{s}. \qquad \text{(20 bis.)}$$

Example 19. — In the above example, given initial pressure and cut-off, what is the pressure, p, at 12-inch stroke?

$$12\,p = 75 \times 9;$$

$$p = \frac{675}{12} = 56.25 \text{ lbs.}$$

Example 20. — In the above, given terminal pressure = 14 lbs. and length of stroke 48 inches, what is the pressure, p, at 25-inch stroke?

$$25\,p = 14 \times 48;$$

$$p = \frac{672}{25} = 27 \text{ lbs.}$$

The point of cut-off is most conveniently given, when it can be done in whole numbers, as a fraction of the full stroke, in which the length of cut-off is taken as unity, and made the numerator of the fraction, the denominator being the full stroke expressed in "times the cut-off." This fraction gives also the rate of expansion, or number of times the steam is expanded in the cylinder. Thus, if the full stroke were 30 inches, and the cut-off took place at 6 inches, we should say that the steam was cut off at $\frac{1}{5}$ stroke, and the rate of expansion was 5. Hence, point of cut-off is the reciprocal of the rate of expansion.

There is, then, the simple relation between initial and terminal pressure and point of cut-off, or rate of expansion:

$$P' = \frac{P}{R}. \qquad (21)$$

In which R = rate of expansion.

Example 21. — Initial pressure, 75 lbs. Rate of expansion 5. What is the terminal pressure?

$$P' = \frac{75}{5} = 15 \text{ lbs.}$$

BACK PRESSURE.

While the piston is being urged forward by the pressure of steam behind it, its progress is opposed by the *back pressure* of the steam in front of it, due to obstructions to its free escape into the atmosphere in the case of non-condensing engines, and in the case of condensing engines due to the temperature of the hot well, or the rejected heat which was not only not utilized on the previous stroke, but which also exercises a retarding influence on the return. About 100° is the very lowest temperature to which the hot well can be reduced, corresponding to about 1 lb. per sq. in. of back pressure; and it is doubtful if even this has ever been realized in practice. The back pressure is ordinarily measured directly by the vacuum gauge. If the gauge reads 26 in. of mercury, the back pressure is $\frac{30-26}{2} =$ 2 lbs., supposing 30 inches of mercury to indicate a perfect vacuum.

It is clear that steam must never be expanded in the cylinder to below the back pressure. Should this be done, then, when the piston reached the point (Fig. 1) where the pressure, p, just equalled the back pressure, the engine would stop, unless carried over by a fly-wheel or the momentum of the moving parts, or the direct action of a second engine working at right angles to the first. The theoretical number of advantageous expansions would therefore be $\frac{\textit{initial pressure}}{\textit{back pressure}}$, but in practice only 75% of this can be taken. Practically steam cannot be expanded to below 8 lbs. absolute terminal pressure, and

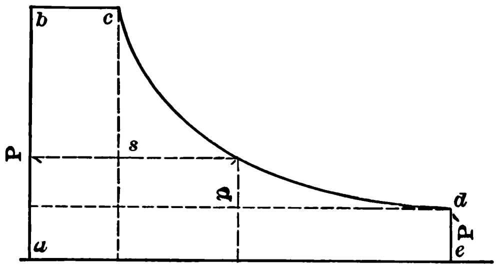

Fig. 1.

10 lbs. is nearer general practice. On the other hand, the vacuum can never be perfect; nor should it be even if that were possible, for there must be always sufficient pressure in the hot well to actuate the valves of the air-pump.

MEAN EFFECTIVE PRESSURE.

When an engine works expansively, the pressure varies at different parts of the stroke. In calculations relating to horse-power, it is necessary to know the average pressure, or that pressure in lbs. per sq. in. which, multiplied by the area of the piston in sq. ins. and length of stroke in feet, shall give the foot pounds developed by a piston stroke. Knowing the initial and terminal pressures, a diagram may be constructed by means of the formulæ already given which shall represent by an area the total pressure exerted, which, being divided by the length of stroke, will give the mean or average pressure. This method is not, however, generally resorted to in practice, because the mean effective pressure can be more readily and accurately determined by the formula:

$$M.\,E.\,P. = \frac{P}{R}(1 + \text{hyp. log. } R) - B. \qquad (22)$$

In which $M.\,E.\,P.$ = mean effective absolute pressure; R = rate of expansion; hyp. log. R = hyperbolic logarithm of R, and B = back pressure. It is well to remember that the hyp. log. of a number equals very closely its common log. multiplied by 2.30.

Example 22. — Initial pressure 75 lbs., rate of expansion 5, back pressure 3 lbs.; what is *M. E. P.?*

$M.\,E.\,P. = \frac{75}{5}(1 + 1.6094) - 3 = 36.14$ lbs.

CLEARANCE.

Clearance comprises all the spaces remaining between the end of the piston and the face of the steam valves, at the end of each stroke. These spaces practically increase the length of the cylinder and lessen the rate of expansion. Their measurement is sometimes difficult, and the result is generally expressed as a percentage of the length of cylinder, which is added as a constant both to the actual length of cylinder and to the distance covered by the piston to point of cut-off as shown in Fig. 1, and expressed by (20).

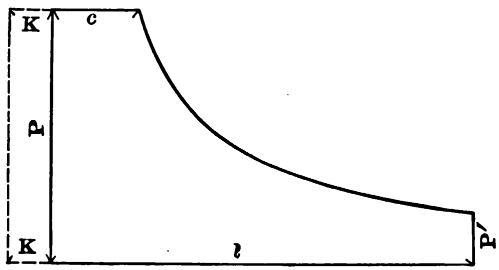

Fig. 2.

Referring to Fig. 2, we have (20) modified by the effects of clearance, thus:

$$P(c + K) = P'(l + K). \qquad (23)$$

In which P = initial pressure; P' = terminal pressure; c = length of stroke to point of cut-off; l = length of full stroke, and K = equivalent of clearance spaces. The true rate of expansion is then, —

$$R = \frac{l + K}{c + K}. \tag{24}$$

and :

$$P' = P\left(\frac{c + K}{l + K}\right). \tag{25}$$

Example 23. — Length of cylinder, 40 in.; cut-off, 8 in.; clearance 10% = 4 in.; $P = 75$ lbs.; $B = 3$ lbs. What is the rate of expansion, and the terminal and mean effective pressures with and without clearance?

With clearance:

$$R = \frac{40 + 4}{8 + 4} = 3.67.$$

$$P' = 75\left(\frac{8 + 4}{40 + 4}\right) = 20.45 \text{ lbs.}$$

$$M.\,E.\,P. = \frac{75}{3.67}(1 + 1.3002) - 3 = 44 \text{ lbs.}$$

Without clearance:

$$R = \frac{40}{8} = 5.$$

$$P' = \frac{75}{5} = 15 \text{ lbs.}$$

$$M.\,E.\,P. = 36.14 \text{ lbs.}$$

Clearance, then, increases mean and terminal pressures, and therefore power. It also increases consumption of steam, and therefore lessens economy. The consumption of steam increases at a more rapid rate than the increase of power; for in the above example, the consumption of steam in the two cases is as $\frac{12}{8} = 1.5$, while the increase of power is as $\frac{44}{36} = 1.22$.

In a word, clearance has the effect of making the engine work less expansively and therefore more expensively.

The pressure at any point of stroke, with clearance, is found as in previous examples.

Example 24. — In the previous example, with clearance, what is the pressure at half stroke?

$$p = \frac{75\,(8 + 4)}{20 + 4} = 37.50 \text{ lbs.}$$

The preceding rules afford the means of calculating approximately the consumption of steam in the cylinders per stroke.

Example 25. — Cylinder, 20 in. diameter; stroke, 40 in.; cut-off, 8 in.; clearance, 4 in.; initial pressure, 75 lbs. How many pounds of steam are admitted and exhausted each stroke?

Capacity of cylinder to point of cut-off,

$$\frac{314.16 \times 12}{1728} = 2.18 \text{ cu. ft.}$$

Weight of cubic foot of steam of 75 lbs. pressure = 0.1792 lbs. Weight of steam admitted at each stroke, $2.18 \times 0.1792 = 0.391$ lbs.

Also, terminal pressure being 20.45 lbs., and capacity of cylinder, including clearance, being 8 cu. ft., and weight of one cubic foot of steam of 20.45 being about 0.052, the steam exhausted as reckoned from terminal pressure is 0.416 lbs., a close agreement with the amount as reckoned from initial pressure.

The above calculation is of course only an approximation, because, for one reason, the clearance spaces are not entirely filled and emptied at each stroke. A closer approximation may be obtained from an indicator diagram.

HORSE-POWER.

Knowing the mean effective pressure, stroke, and area of piston, and number of revolutions per minute, the horse-power of an engine is readily calculated thus:

$$\text{H. P.} = \frac{2\,PLAN}{33000} \qquad (26)$$

Also,

$$\text{H. P.} = PLD^2N \times 0.000048. \quad (27)$$

$$\text{H. P.} = PlD^2N \times 0.000004. \quad (28)$$

In which P = mean effective pressure per square inch; L = length of stroke in feet; A = area of piston in square inches, N = number of revolutions per minute; D = diameter of piston in inches; l = length of stroke in inches.

INDICATOR DIAGRAMS.

It is by means of indicator diagrams that the best knowledge is obtained of the working of the valves and the distribution of steam, as well as the most accurate determination of the mean effective pressure, and therefore of the horse-power of a steam-engine.

If there were no cut-off, and the engine were worked entirely without expansion, the diagram would be a rectangle, $ABEF$ (Fig. 3), and the work per square inch of piston would be represented by the area of the rectangle.

If, however, steam were cut off at some point C, and allowed to expand down to pressure DE, then the work done by the piston per square inch of its area would be equal to the area $ABCDE$, which would be equal to $M.E.P.$ into AE.

An indicator card is satisfactory in proportion as it approaches the theoretical figure shown in Fig. 3, which is obtained by first laying off the work of the steam up to point of cut-off, and then plotting the expansion

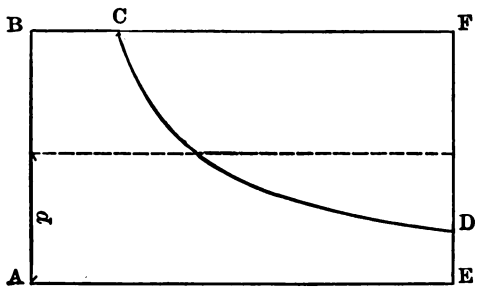

Fig. 3.

curve by means of (20 bis.). It is usual in using this formula for this purpose to divide the stroke into ten equal parts, when:

$$p = \frac{10\,P}{s}.$$

In which s has successive values from the division nearest to the point of cut-off to 9.

The "saturation curve" is now often used as a standard of comparison; but the hyperbolic curve, as above, so identified with steam-engineering practice, is still an excellent criterion of the performance of the engine.

The spring of the indicator is so adjusted that the lines *AB* and *DE* (Fig. 3) represent pounds per square inch to a certain scale.

The mean pressure is obtained from a diagram by ascertaining its area in any way, or to any scale, and dividing the same by the stroke *AE* expressed in the same scale. The quotient is the mean effective pressure taken upon the same scale as *AB*.

In an indicator diagram there is always a base line, *AB* (Fig. 4), traced with the pencil before the steam

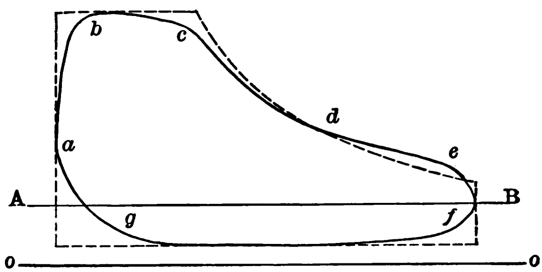

Fig. 4.

is turned on. This is called the atmospheric line, and in a non-condensing-engine the diagram lies entirely above it; but the more perfect the engine, the nearer will the return line approach to the atmospheric line.

In a condensing engine the line of the return stroke, *fg* (Fig. 4), is always below the atmospheric line *AB*, but can never get down to the zero line, 00, which represents a perfect vacuum, and which is laid off = 14.70 lbs. to scale below *AB.*

In obtaining mean pressures from a diagram, the net area, *a b c d e f g* only, is generally taken; whereas in obtaining it by calculation from (22) the total area above the zero line, 00, is first taken, and then the back pressure, equivalent to the space between *agf* and 00, deducted from it. This is the only way in which the logarithmic formula can be applied.

The actual diagram shown in full lines (Fig. 4) will differ more or less from the theoretical one shown in dotted lines. The rounded corners at *b* and *c* show imperfect action of the valves, as do the sloping lines *ab* and *bc.* The sloping line, *bc*, shows wire drawing. The portion *de*, lying above the theoretical line shows re-evaporation or leakage, or both. The rounded corner at *e* shows exhaust opening behind advancing piston; that at *f* shows incomplete exhaust opening; that at *g* shows cushioning and compression, which have practical advantages, although reducing amount of work.

In a well-constructed engine the back pressure on the forward stroke is nearly identical with that of the return stroke; and so the diagram, as produced by an indicator placed at one end of the cylinder, may be used to determine, with sufficient accuracy in most cases, the mean pressure. But in point of fact, a single diagram does not show exactly what is going on in the cylinder. To ascertain this, simultaneous diagrams must be taken from both ends of the cylinder, and the back pressure of one applied to the forward pressure of the other, as shown in Fig. 5. The true area representing total pressure is obtained by taking the whole area, *a b c d e*, over the zero line 00 (Fig. 5), and then deducting the back pressure, *a f d g e*, against which it

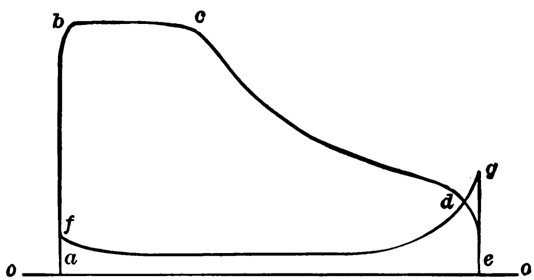

Fig. 5.

was working. The diagram thus prepared also shows the relation between the forward and back pressures at any point of the stroke. Thus, at the point *d* the forward and back pressures are equal. From *d* on to the end of the stroke, the back exceeds the forward pressure, and the piston is carried on by the momentum of the moving parts, or by some other means.

An interesting diagram can be constructed, as shown in Fig. 6, by setting up ordinates from a line, *AB*, making them equal to the corresponding ordinates on Fig. 5, measured above the line of back pressure; that is, representing the projection of the line of back pressure by the straight line *AB*. (See "The Steam Engine," by G. C. V. Holmes.)

The points to be aimed at in an engine, and the approaches to which are exhibited by the indicator card, are: a quick and wide action of the valves, getting full

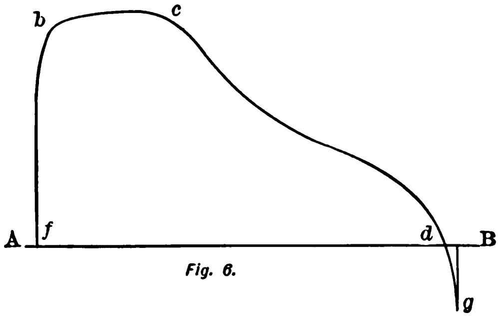

Fig. 6.

pressure behind the piston as rapidly as possible, and maintaining the same intact up to the point of cut-off, when the admission should be instantly and completely arrested. The exhaust should also be free and rapid in front of the advancing piston, with a proper degree of cushioning, however, to bring the piston gradually to rest, and fill the clearances with compressed steam. This perfect action can never be realized with the slide-valve, but is very closely approached by valves of the Corliss type. The great practical advantages of the slide-valve, however, give it the preference in many cases.

THE COMPOUND ENGINE.

Carnot's law already referred to shows that the efficiency of a heat-engine depends exclusively upon the difference of temperatures of the heat received and rejected; that is to say, as regards the steam-engine, the difference between the temperature of the steam in the boiler and that of the hot well. The lowest practical temperature of the latter is nearly a fixed quantity, and may be stated as 100°, corresponding to a pressure of about 1 lb. But to fulfill Carnot's law, the steam should be expanded in the cylinder down to the temperature of the hot well, which is impossible. Practically, steam cannot be expanded in a cylinder below 8 lbs., corresponding to a temperature of about 183°; and this is the temperature at which the steam-engine actually rejects its heat. The temperature at which it receives the steam, or the boiler pressure, is also confined to a practical limit, but less rigorously; and improvement in the efficiency of the steam-engine lies in the direction of facilitating the production and utilization of steam at higher and higher pressures. This leads to the use of the compound or multi-cylinder engine.

In order to approach as nearly as possible to the perfect heat engine, the maximum of expansion is necessary. But there are very narrow practical limits to the degree of expansion possible in a single cylinder, owing to the phenomena of cylinder condensation and re-evaporation, as well as the bad results of great variation of initial and terminal pressures in a single cylinder. Five expansions are as many as can be advantageously realized in a single cylinder, corresponding to a cut-off of one-fifth. If, therefore, the steam be expanded down to say 10 lbs., the maximum economical initial pressure will be 50 lbs. absolute, or 35 "by the gauge."

If it be desired to develop 10 expansions, corresponding to 100 lbs. initial pressure, a two-cylinder engine would be necessary; if 15 expansions, with 150 lbs. initial pressure, a triple expansion or three-cylinder engine would be needed; while 20 expansions, from 200 lbs. initial pressure down to 10 lbs. terminal pressure, requires a quadruple or four-cylinder engine, which represents the highest type of perfect steam-engine as yet realized. The above are round numbers, but they closely agree with actual practice.

The cardinal points in economical steam engineering are, to increase initial pressure, and, by steam-jacketing, to endeavor to "keep the cylinder as hot as the steam that enters it." This is for the engine; good boiler practice involves the greatest evaporative power per pound of coal in the production of dry high pressure steam. All these refinements increase the cost of engine and boiler, as well as that of running and maintenance, so that the question of relative economy turns on the price of a ton of coal, as against cost and care of engine and boiler. The highest type of quadruple-expansion engine consumes about half the steam — or in other words, fuel — required by the best single-cylinder type of the same power. The first cost of the former would probably be more than double that of the latter.

The point of cut-off and rate of expansion in a compound engine are determined by the following considerations: If steam were admitted full stroke in the small or high-pressure cylinder, and allowed to expand in the large or low-pressure one, the rate of expansion, or number of expansions, would be the quotient of the volume of the large, divided by that of the small, cylinder. If, however, as is usually the case, the steam is cut off in the high-pressure cylinder at some fraction of its stroke, then the rate of expansion is the quotient obtained by dividing the total cylinder volume in which the steam is expanded by the volume of high-pressure cylinder to the point of cut-off. In a close calculation, all the clearance spaces, receiver capacity, etc., must be considered, and the volumes and pressures of the steam followed step by step from admission to exhaust. For ordinary approximate calculations, however, the rate of expansion is taken as the quotient of the volume of large cylinder divided by that of small one, up to point of cut-off, neglecting clearance. We have, therefore:

$$R = \frac{V}{V'} R'. \tag{29}$$

In which R = rate of expansion or number of expansions of compound engine; V = volume (or square of diameter) of low-pressure cylinder; V' = volume (or square of diameter) of high-pressure cylinder, and R' = rate of expansion, or number of expansions in high-pressure cylinder.

The total expansion can be determined by dividing the initial absolute pressure by the final (21), when these two pressures are known or assumed; and the proper point of cut-off, if any, in the high-pressure cylinder can then be determined by (29).

Example 26. — Ratio of diameters of two-cylinder engine 1.75 to 1. Initial and terminal pressures, absolute, 90 and 10. Where is steam cut-off in high-pressure cylinder?

Here,
$$R = \frac{90}{10} = 9.$$

$$9 = \left(\frac{1.75}{1}\right)^2 R'.$$

$$R' = \frac{9}{3} = 3.$$

Hence, cut-off occurs at one-third stroke.

In designing a compound engine, the initial and terminal pressures being assumed, the low-pressure cylinder is first considered, and the whole work and expansion is supposed to be carried on in that cylinder alone, and its dimensions proportioned accordingly. The high-pressure cylinder, with point of cut-off, is then dimensioned. That is to say, assuming initial and terminal pressures, back pressure, and number of strokes per minute, the dimensions of a single cylinder (the low-pressure) are first determined, which would furnish the desired power; and then the second, or high-pressure cylinder, is so proportioned as to effect a proper steam distribution.

Example 27. — I have a single-cylinder condensing-engine, piston 30 in. diameter, and 40 in. stroke. Initial and terminal pressures, 50 and 10 lbs. absolute. Back pressure, 2 lbs. Number of revolutions, 80 per minute. Neglecting clearance, the *M. E. F.* is, by (22), = 24 lbs., and the horse-power, therefore, 276.

Now, I wish to reduce the amount of fuel consumed, keeping the horse-power the same. This I propose to do by converting my single-cylinder into a tandem compound engine, adding for this purpose another high-pressure cylinder, and using steam at 100 lbs. initial pressure, the terminal and back pressures remaining the same.

Neglecting clearance, the number of expansions will be 10, and the mean effective pressure 31 lbs.

The previous mean pressure was 24 lbs.; therefore, to maintain the same power as before, and averaging the whole work as if done in the low-pressure cylinder alone, the number of revolutions must be reduced in the proportion of $\frac{24}{31}$, or from 80 to 62 per minute. Since, as we have already seen, it requires but little more fuel to maintain steam at a high pressure than at a low one, we see that the saving of fuel will be, roughly, in the same proportion. But the question of economy will be reverted to later on.

As regards dimensions and point of cut-off of the added high-pressure cylinder, the stroke must of course remain the same. A common ratio of diameters is one-half. Then, by (29), the number of expansions being $\frac{100}{10} = 10$, the rate of expansion in high-pressure cylinder is:

$$R = \frac{10}{2^2} = \frac{10}{4};$$

and the point of cut-off, being the reciprocal of the rate of expansion, occurs at 0.40 stroke, or 16 inches.

The relative volumes of steam consumed by the two engines, and their consequent relative economy of fuel, may be computed as per EXAMPLE **25**; but this is not necessary, nor would it be practically the most correct way. We know by precedent that the average consumption of water per hour per horse-power would be in the one case about 25 lbs. and in the other about 18 lbs. In 24 hours, therefore, the single-cylinder engine would consume 165,600 lbs. of water in the form of steam, and the compound 119,232 lbs. If the boilers evaporated 10 lbs. water per pound of coal, the respective coal consumptions would be 8.25 tons and 6 tons per 24 hours, the saving being $2\frac{1}{4}$ tons per day.

Incidentally there would be a reduced amount of condensing water required; and if the same boilers were found able to produce steam at the higher temperatures, the boiler capacity would be increased, since a smaller volume of steam at higher pressure would be required.

Compound engines are of two types, the tandem, with one cylinder directly in line with the other, the two pistons being upon the same rod; and the cross-compound, with the cylinders placed side by side.

In the tandem type, the pistons commence and end their strokes simultaneously. This type is adapted to cases where there is only one crank; if there are two cranks, there must be a separate engine for each, with high- and low-pressure cylinders, making four in all. The tandem type is well adapted to direct-acting engines, even when triple expansion is practiced. In this case the low-pressure cylinder is sometimes placed between the high and intermediate one. The construction of this type is very simple, and it admits of easily transforming a single-cylinder into a compound engine. It presents the disadvantage of long steam passages on the high-pressure cylinder, owing to the necessity of placing the valve-rod of both cylinders in the same straight line. These long passages increase the clearance spaces.

The cross-compound type is suited to cases where two cranks are used, set at right angles to each other. Each piston drives its own crank, the two pistons receiving steam on opposite faces. Therefore, when the high-pressure piston is at the commencement or end of its stroke, the low-pressure one is at half-stroke, and consequently not ready to receive the exhaust steam from the high-pressure cylinder. The latter exhausts therefore into a receiver, where it is held in reserve until the low-pressure piston reaches the end of its stroke. It is then allowed to escape from the receiver, and act upon the low-pressure piston. Generally, however, it is found that sufficient reserve space is afforded by the exhaust pipe of the high- and the valve chest of the low-pressure cylinder, so that no special receptacle is needed.

In the triple expansion, if of the cross-compound type, the cranks are placed at 120° to each other, which gives increased regularity of stress.

It may be here remarked that even a tandem compound, acting on a single crank, gives a smaller range of stress on the crank-pin than a single cylinder, the initial stress being less and the terminal greater in the former than in the latter, owing to the difference of diameter of the two cylinders.

The range of temperature is less in the high-pressure cylinder of a cross-compound engine than in that of a tandem.

Upon the whole, the cross-compound seems to be the more perfect, and the tandem the simpler form of the compound engine.

The extraordinary economy of some highly refined types of modern engines, when running at their maximum speed in a duty trial, cannot be realized in the every-day working of the same engines, when running at varying speeds under average conditions. It may be broadly asserted that the high-water mark of steam-engineering is reached when a steam plant, running as above, under daily actual use, realizes as to the engine, one horse-power per 20 lbs. of water evaporated per hour, and as to the boiler, an evaporation of 10 lbs. water per lb. of good coal. The combination of such a boiler and engine results in a consumption of 2 lbs. coal per hour per horse-power; and this represents the best economy that can be expected, year in and out, from a steam-plant under ordinary circumstances.

WORK.

In what precedes we have studied the action of the steam-engine within itself, from the boiler to the shaft. Broadly stated, the object of the steam-engine is to utilize the heat which it receives, for the purpose of moving objects which would otherwise remain stationary, such as raising coal from a mine or water from a well, or transporting freight from one place to another, or doing any other useful service involving the performance of what is called "*Work.*"

It is now in order to ascertain how the amount of such useful work can be calculated, and the first thing to do will be to get a clear idea of what "Work" is.

By definition, Work is weight into distance, irrespective of time. It is expressed in foot pounds.

It is found, both by theory and experiment, that exactly as much work is required to move a resistance of 33,000 lbs. a distance of 1 foot as to move a resistance of 1 lb. a distance of 33,000 feet, the amount of foot-pounds being the same in either case.

If, therefore, one force, weight, or resistance, P, moves or is moved a distance, d, the work performed by or on it is exactly equal to that performed by or on another force, weight, or resistance, P', moving or moved a distance, d', provided the product, Pd, is equal to the product $P'd'$.

But in order to apply practically this property of work, it is necessary to stipulate also that the work in both cases must be performed in *equal*, although not necessarily *stated*, times. With this proviso we can write the formula:

$$Pd = P'd'. \tag{30}$$

This is the expression of the equality of work simultaneously performed, the most important, indeed, the fundamental, principle of the mechanics of the steam-engine.

Although the force of an engine is always expressed in horse-power, there are many problems which can be more readily solved by recourse to the above principle of work. The force which an engine can exercise is expressed by the mean effective pressure in pounds upon the piston into the distance travelled by the piston during one revolution. This can be equated with the weight raised, or resistance moved, in foot-pounds, also during one revolution.

Example 27.—The following data belong to a locomotive on an elevated railroad.

Circumference of driving-wheels . .	11	ft.
Joint area of both pistons	190	sq. in.
Length of double stroke	$2\frac{2}{3}$	ft.
Weight of train, including engine and tender	140,000	lbs.
Total train-resistance to traction at given speed	1,000	lbs.

What is the mean effective pressure, *P*, per square inch of pistons theoretically necessary to maintain a steady speed at given rate against train resistance on the level?

Here the work to be done during one revolution is the overcoming of the resistance of 1,000 lbs. applied to the circumference of the drivers, and which may be considered as concentrated upon a single driver. The pressure theoretically necessary to overcome this, and maintain the exact speed which produces the given resistance, is the total pressure upon both pistons into the length of a double stroke.

$$P \times \frac{8}{3} \times 190 = 1000 \times 11.$$

$$P = 21.71 \text{ lbs. per sq. in.}$$

If, instead of being on the level, the track had a grade of $\frac{1}{100}$, the pressure must be sufficient not only to overcome train resistance, but also to lift the whole train bodily 0.11 ft. at each revolution. Then the total pressure, P', would be:

$$P' \times \tfrac{8}{3} \times 190 = 1000 \times 11 + 140000 \times 0.11.$$

$$P' = 52.10 \text{ lbs. per sq. in.}$$

Example 28. — A pumping-engine has a piston of 200 sq. ins. area, the mean effective pressure being 30 lbs., and the length of double stroke 4 feet. Its elements are such that at every revolution it raises the water lifted 60 feet high. Neglecting all other resistances, what weight of water will be raised each revolution?

$$60\ W = 30 \times 200 \times 4;$$

$$W = 400 \text{ lbs.}$$

In applying all such calculations to practical problems, three points must be carefully borne in mind: First, that our equations express an exact equilibrium or balance of force and resistance, whereas to produce motion a slight excess of force over resistance is necessary. Secondly, the resistance of useful work only is considered, whereas a considerable amount of force is necessary to move the machine itself, all of which is loss as far as useful work is concerned. Thirdly, only the force necessary to *maintain* motion is shown by the equation, whereas a considerable excess would be necessary to overcome the inertia of the machine and other resistances on starting. This is a most important point, and shows that, just as a certain amount of heat is absorbed or rendered latent in effecting the change of state from water to steam, so a certain amount of force is absorbed or rendered latent in effecting a change of state from rest to motion.

In (30) equality of work has been expressed in terms of weight and distance in the unit of time. It is evident that it might be also expressed in mean velocity, which is merely distance divided by time. When thus expressed, the formula of work becomes that of VIRTUAL VELOCITY.

If a body of weight, P, moves with a mean velocity, V^m, and another body, P', moves with a mean velocity, V'^m, then the equation of equivalent virtual velocities obtains when:

$$P V^m = P' V'^m. \tag{31}$$

Example 29. — A force, weight, or resistance, $P =$ 1,000 lbs., is moving with a steady velocity of 20 ft. per second. What force, weight, or resistance, P', moving with a steady velocity of 30 ft. per second, is its equivalent?

$$P' = \frac{1000 \times 20}{30} = 666\tfrac{2}{3} \text{ lbs.}$$

The engineering unit of virtual velocity is the horse-power, which is 33,000 ft. pds. per minute, or 550 ft. pds. per second.

The formula for the reduction of virtual velocity to horse-power is, therefore,

$$\text{H. P.} = \frac{Pd}{33000\,m};$$

or:

$$\text{H. P.} = \frac{Pd}{550\,s}.$$

In which m = number of minutes and s = number of seconds in which the given foot pounds, Pd, are developed.

Example 30. — If in Example 27 the driving-wheels make 2 revolutions per second, what is the theoretical horse-power developed on the level, and what on the grade?

On the level:

$$\text{H. P.} = \frac{1000 \times 11 \times 2}{550} = 40.$$

On the grade:

$$\text{H. P.} = \frac{22000 + 140000 \times 0.22}{550} = 96.$$

Example 31. — 1,000,000 lbs. are raised 100 ft. high in 3 minutes. What horse-power does this represent?

$$\text{H. P.} = \frac{1000000 \times 100}{33000 \times 3} = 1010.10.$$

When the work performed upon a heavy body is just sufficient to produce motion, as in the case of a weight slowly raised by means of a winch, we may consider that the work is expended as fast as it is produced; and when the force is withdrawn, motion will immediately cease. When, however, as in the case of a rapidly revolving fly-wheel, much more energy is put in it than that just sufficient to produce motion, — in this case, that just sufficient to turn the wheel over — the excess takes the form of velocity, and is stored in the moving mass. If the force were withdrawn, the body would continue its motion, expending its stored energy in performing work against any force which tended to retard it, until it finally came to rest. Such cases give rise to problems in MOMENTUM and VIS VIVA.

The principles upon which these expressions of energy depend are based upon the laws of FALLING BODIES, which we will now proceed to study.

FALLING BODIES.

The properties of heavy bodies falling freely under the action of terrestrial gravitation, as determined by actual experiment, are expressed as follows:

$$v = gt = 32\,t \quad \text{(approximately).} \qquad (32)$$

$$v = \sqrt{2gh} = 8\sqrt{h} \quad \text{(approximately).} \qquad (33)$$

$$h = \frac{v^2}{2g} = \frac{v^2}{64} \quad \text{(approximately).} \qquad (34)$$

$$h = \frac{gt^2}{2} = 16\,t^2 \quad \text{(approximately).} \qquad (35)$$

$$t = \frac{v}{g} = 0.03\,v \quad \text{(approximately).} \qquad (36)$$

$$t = \sqrt{\frac{2h}{g}} = \frac{\sqrt{h}}{4} \quad \text{(approximately).} \qquad (37)$$

In which v = velocity in feet per second, at a given instant; t = corresponding time of fall, in seconds; h = corresponding height of fall, or distance fallen through;

g = acceleration due to gravity in feet per second, per second, = 32.2. In most cases the approximate values above given suffice.

If the falling body has an initial velocity, v', then:

$$v = v' + gt. \tag{38}$$

$$h = v't + \frac{gt^2}{2}. \tag{39}$$

From (38) and (39):

$$v = \sqrt{v'^2 + 2gh}. \tag{40}$$

Example 32. — A stone dropped into a well is heard to strike the water after an interval of 3 seconds. What is distance to surface of water? From (35):

$$h = 16 \times 9 = 144 \text{ feet.}$$

Example 33. — A bullet is fired vertically upward, and reaches a height of 961 feet. With what velocity did it leave the muzzle of the gun? and how long will it take to reach the ground, after it commences to fall? From (33) and (37):

$$v = 8\sqrt{961} = 248 \text{ ft. per second;}$$

$$t = \frac{\sqrt{961}}{4} = 7.75 \text{ seconds.}$$

Example 34. — A ball is thrown vertically upward, and caught again by the thrower. It occupies 5 seconds from the time of leaving his hand to the time of reaching it again. How high did it rise? From (35):

$$h = 16 \times \left(\frac{5}{2}\right)^2 = 100 \text{ feet.}$$

In all practical applications of the above formulæ, the resistance of the air greatly modifies the results, particularly in high falls.

Example 35. — A bullet is shot vertically downward from a height of 961 ft., leaving the muzzle of the gun with an initial velocity of 1000 ft. per second. With what velocity will it strike the ground? From (40):

$$v = \sqrt{1000000 + 61504} = 1030 \text{ ft. per second, per second.}$$

The above formulæ apply especially to bodies falling vertically downward under the action of terrestrial gravitation, the acceleration due to which is 32.2 ft. per second, per second. They apply also to any heavy body urged freely in any direction by a constant force, acting in such direction, of which the coefficient of acceleration is a. All of the formulæ from (32) to (40) can therefore be generalized by substituting a for g.

One of the most important laws of dynamics is that constant or uniform forces, acting freely upon heavy bodies, impart to such bodies accelerations proportional to the intensities of the forces themselves. This law enables us to use terrestrial gravitation as a standard by which to measure the intensity of other forces, moving in any direction, under the influence of any other acceleration, a.

The law is thus expressed:

$$\frac{F}{a} = \frac{W}{g}. \tag{41}$$

In which $F =$ any uniform force in lbs.; $a =$ its acceleration in feet per second, per second, or the velocity imparted at the end of the first second; $W =$ weight in lbs. of any heavy body freely acted upon by terrestrial gravitation, the coefficient of acceleration of which is $g = 32.2$.

We have already seen that (32) may be generalized, thus: $v = at$. From this we deduce, $a = v / t$. Substituting this value of a, in (41), gives:

$$Ft = \frac{WV}{g}. \tag{42}$$

This is the formula of MOMENTUM. Momentum, or "Quantity of Motion," is weight into time, and is expressed in second-pounds. The right-hand member of (42) shows that the momentum of a body moving with the velocity, V, is equal to its weight, W, into the time during which it would have to fall under the influence of gravity in order to acquire that velocity. Momentum represents the stored or acquired energy of weight acting through time.

Example 36. — A detached railway car of $W =$ 10,000 lbs. is running away on a level track, with an initial velocity of 20 ft. per second. The train resistance is 30 lbs. per ton, or a total of 150 lbs. In how many seconds will it come to rest?

$$150\,t = \frac{10000 \times 20}{32}.$$

$$t = 41\tfrac{2}{3} \text{ seconds.}$$

Example 37. — Two weights, $W = 100$ lbs. and $W' = 90$ lbs., are suspended from the ends of a flexible cord, passing over a smoothly running pulley. The heavier weight will naturally descend, raising the lighter one, but not with as great velocity as if falling freely. What velocity will it have acquired at the end of 2 seconds, from rest?

To solve this problem, deduce the value of V from (42), thus:

$$V = \frac{Ftg}{W}.$$

In the present case, $F = 10$ lbs., the difference of the two weights producing motion; and $W = 190$ lbs., the sum of the two weights, which is the total weight of the mass set in motion. Hence:

$$V = \frac{10 \times 2 \times 32}{190} = 3.36 \text{ ft. per second.}$$

The mean velocity of a falling body, or of a body moving in any direction, under the influence of a uniformly accelerating force, is equal to half its terminal velocity; and reciprocally, the final velocity is equal to twice the mean velocity.

Example 38. — A weight or resistance, $W = 200$ lbs., is moved from rest, and at the end of 3 seconds has passed over 60 ft. What uniform force was expended in producing this velocity?

Here, the velocity acquired at the end of 3 seconds is $60 \times 2 = 120$ ft. Inserting the data in (42):

$$F = \frac{200 \times 120}{3 \times 32} = 250 \text{ lbs.}$$

Again (34) may be generalized thus:

$$a = \frac{V^2}{2h}.$$

Substituting this value of a in (41), we deduce:

$$Fh = \frac{WV^2}{2g}. \tag{43}$$

This is the formula for VIS VIVA, or weight into distance, expressed in foot pounds. The right-hand side of the equation shows that the vis viva of a body moving with the velocity, V, is the product of its weight into the distance through which it must fall, under the influence of gravity, to acquire such velocity. Vis viva represents the stored or acquired energy of weight acting through distance.

Example 39. — In Example 36, in what distance would the car stop? From (43):

$$150\,h = \frac{10000 \times 400}{64};$$

$$h = 416\tfrac{2}{3} \text{ ft.}$$

Example 40. — Mean diameter of cast-iron fly-wheel = 8 ft. Weight of rim = 5,000 lbs. When making 1 revolution per second, what is its vis viva, or stored energy?

$$Fh = \frac{5000 \times \overline{25.13}^{\,2}}{64} = 9337 \text{ ft. pds.}$$

If steam were shut off, and a retarding force in the shape of a steady pressure, $F = 500$ lbs., were applied to the circumference of a break-drum 2 feet in diameter, in how many revolutions would the engine stop?

The circumference of the drum being 6.280 ft., there is set up at every revolution a retarding force of $500 \times 6.28 = 3140$ ft. pds. The total foot pounds stored up in the rim being 49,337, it is clear that we require $\frac{49337}{3140} = 15.71$ revolutions in order to extinguish them.

If a weight, W, falls freely from a known height, h, its stored energy at the moment of striking the ground is Wh, and it is not necessary to know its velocity in order to calculate that energy. If, however — by means of a parachute, for instance — its velocity is rendered uniform, we cannot tell what its stored energy is, unless we know its velocity of descent. The height of fall in this case does not enter into the problem. But should its velocity be rendered uniform, or nearly so, by the resistance of another weight, W', — a heavier one, for instance, which it raises by means of gearing to a lesser height, h', — its work is independent of its velocity and time of falling, and is, as before, Wh. When motion, however slow, is thus produced, there will still be some energy stored in the moving parts. If the motion of the descending weight were suddenly arrested, its stored energy would be absorbed by the shock; and the stored energy of the ascending weight would be expended in raising the weight a greater or less distance after the descending weight had come to a stop.

Since the average velocity of a falling body is half its terminal velocity, we have:

$$\text{H. P.} = \frac{W}{550} \times \frac{V}{2} = \frac{WV}{1100};$$

which represents the horse-power developed by a falling weight, W, which strikes the ground with the terminal velocity, V, which is also the horse-power necessary to raise the weight to the height from which it fell in the time occupied in falling.

To complete our study of the mechanics of the steam-engine, we must now consider CENTRIFUGAL FORCE, or that with which a body revolving in a circle tends to leave its orbit, and escape in the direction of the radius. It is expressed in radius-pounds.

The formula for centrifugal force, of somewhat intricate demonstration is:

$$FR = \frac{WV^2}{g}; \tag{44}$$

in which R = radius of circle in which rotation takes place. This formula much resembles that of vis viva.

There is also the closely approximate formula:

$$F = \frac{WN^2D}{6000}; \qquad (45)$$

in which F = centrifugal force in pounds, N = number of revolutions per minute, and D = mean diameter of circle of revolution.

Example 41. — Mean diameter of fly-wheel 30 ft.; weight of rim = 15,718 lbs.; number of revolutions per minute = 17.5. What is bursting or centrifugal force?

$$F = \frac{15718 \times \overline{17.5}^{2} \times 30}{6000} = 24600 \text{ lbs.}$$

Example 42. — A locomotive passes around a curve of radius 500 ft., at a speed of 30 miles an hour = 44 ft. per second. Weight upon one pair of driving-wheels = 32,000 lbs. With what force does that pair of wheels press radially against the rails? From (44):

$$F = \frac{32000 \times 1936}{32 \times 500} = 3872 \text{ lbs.}$$

TABLES.

TABLE NO. 1. — PROPERTIES OF SATURATED STEAM.

TABLE NO. 2. — HYPERBOLIC LOGARITHMS.

TABLE No. 1.

PROPERTIES OF SATURATED STEAM.

[From Charles T. Porter's treatise on *The Richards Steam-Engine Indicator.*]

Pressure above zero.	Temperature.	Sensible Heat above zero Fahr.	Latent Heat.	Total Heat above zero Fahr.	Weight of One Cubic Foot.
Lbs. per sq. in.	Fahr. Deg.	B.T.U.	B.T.U.	B.T.U.	Lbs.
1	102.00	102.08	1042.96	1145.05	.0030
2	126.26	126.44	1026.01	1152.45	.0058
3	141.62	141.87	1015.25	1157.13	.0085
4	153.07	153.39	1007.22	1160.62	.0112
5	162.33	162.72	1000.72	1163.44	.0137
6	170.12	170.57	995.24	1165.82	.0163
7	176.91	177.42	990.47	1167.89	.0189
8	182.91	183.48	986.24	1169.72	.0214
9	188.31	188.94	982.43	1171.37	.0239
10	193.24	193.91	978.95	1172.87	.0264
11	197.76	198.49	975.76	1174.25	.0289
12	201.96	202.73	972.80	1175.53	.0313
13	205.88	206.70	970.02	1176.73	.0337
14	209.56	210.42	967.42	1177.85	.0362
15	213.02	213.93	964.97	1178.91	.0387
16	216.29	217.25	962.65	1179.90	.0413
17	219.41	220.40	960.45	1180.85	.0437
18	222.37	223.41	958.34	1181.76	.0462
19	225.20	226.28	956.34	1182.62	.0487
20	227.91	229.03	954.41	1183.45	.0511
21	230.51	231.67	952.57	1184.24	.0536
22	233.01	234.21	950.79	1185.00	.0561
23	235.43	236.67	949.07	1185.74	.0585
24	237.75	239.02	947.42	1186.45	.0610
25	240.00	241.31	945.82	1187.13	.0634
26	242.17	243.52	944.27	1187.80	.0658
27	244.28	245.67	942.77	1188.44	.0683
28	246.32	247.74	941.32	1189.06	.0707
29	248.31	249.76	939.90	1189.67	.0731
30	250.24	251.73	938.92	1190.26	.0755
31	252.12	253.64	937.18	1190.83	.0779
32	253.95	255.51	935.88	1191.39	.0803
33	255.73	257.32	934.60	1191.93	.0827
34	257.47	259.10	933.36	1192.46	.0851
35	259.17	260.83	932.15	1192.98	.0875
36	260.83	262.52	930.96	1193.49	.0899
37	262.45	264.18	929.80	1193.98	.0922
38	264.04	265.80	928.67	1194.47	.0946
39	265.59	267.38	927.56	1194.94	.0970
40	267.12	268.93	926.47	1195.41	.0994

TABLE No. 1. — *Continued.*

Pressure above zero.	Temperature.	Sensible Heat above zero Fahr.	Latent Heat.	Total Heat above zero Fahr.	Weight of One Cubic Foot.
Lbs. per sq. in.	Fahr. Deg.	B.T.U.	B.T.U.	B.T.U.	Lbs.
41	268.61	270.46	925.40	1195.86	.1017
42	270.07	271.95	924.35	1196.31	.1041
43	271.50	273.41	923.33	1196.74	.1064
44	272.91	274.85	922.32	1197.17	.1088
45	274.29	276.26	921.33	1197.60	.1111
46	275.65	277.65	920.36	1198.01	.1134
47	276.98	279.01	919.40	1198.42	.1158
48	278.29	280.35	918.46	1198.82	.1181
49	279.58	281.67	917.54	1199.21	.1204
50	280.85	282.96	916.63	1199.60	.1227
51	282.09	284.24	915.73	1199.98	.1251
52	283.32	285.49	914.85	1200.35	.1274
53	284.53	286.73	913.98	1200.72	.1297
54	285.72	287.95	913.13	1201.08	.1320
55	286.89	289.15	912.29	1201.44	.1343
56	288.05	290.33	911.46	1201.79	.1366
57	289.11	291.50	910.64	1202.14	.1388
58	290.31	292.65	909.83	1202.48	.1411
59	291.42	293.79	909.03	1202.82	.1434
60	292.52	294.91	908.24	1203.15	.1457
61	293.59	296.01	907.47	1203.48	.1479
62	294.66	297.10	906.70	1203.81	.1502
63	295.71	298.18	905.94	1204.13	.1524
64	296.75	298.24	905.20	1204.44	.1547
65	297.77	300.30	904.46	1204.76	.1569
66	298.78	301.33	903.73	1205.07	.1592
67	299.78	302.36	903.01	1205.37	.1614
68	300.77	303.37	902.29	1205.67	.1637
69	301.75	304.38	901.59	1205.97	.1659
70	302.71	305.37	900.89	1206.26	.1681
71	303.67	306.35	900.21	1206.56	.1703
72	304.61	307.32	899.52	1206.84	.1725
73	305.55	308.27	898.85	1207.13	.1748
74	306.47	309.22	898.18	1207.41	.1770
75	307.38	310.16	897.52	1207.69	.1792
76	308.29	311.09	896.87	1207.96	.1814
77	309.18	312.01	896.23	1208.24	.1836
78	310.06	312.92	895.59	1208.51	.1857
79	310.94	313.82	894.95	1208.77	.1879
80	311.81	314.71	894.33	1209.04	.1901
81	312.67	315.59	893.70	1209.30	.1923
82	313.52	316.46	893.09	1209.56	.1945

TABLE No. 1. — *Continued.*

Pressure above zero.	Temperature.	Sensible Heat above zero Fahr.	Latent Heat.	Total Heat above zero Fahr.	Weight of One Cubic Foot.
Lbs. per sq. in.	Fahr. Deg.	B.T.U.	B.T.U.	B.T.U.	Lbs.
83	314.36	317.33	892.48	1209.82	.1967
84	315.19	318.19	891.88	1210.07	.1988
85	316.02	319.04	891.28	1210.32	.2010
86	316.83	319.88	890.69	1210.57	.2032
87	317.65	320.71	890.10	1210.82	.2053
88	318.45	321.54	889.52	1211.06	.2075
89	319.24	322.36	888.94	1211.31	.2097
90	320.03	323.17	888.37	1211.55	.2118
91	320.82	323.98	887.80	1211.79	.2139
92	321.59	324.78	887.24	1212.02	.2160
93	322.36	325.57	886.68	1212.26	.2182
94	323.12	326.35	886.13	1212.49	.2204
95	323.88	327.13	885.58	1212.72	.2224
96	324.63	327.90	885.04	1212.95	.2245
97	325.37	328.67	884.50	1213.18	.2266
98	326.11	329.43	883.97	1213.40	.2288
99	326.84	330.18	883.44	1213.62	.2309
100	327.57	330.93	882.91	1213.84	.2330
101	328.29	331.67	882.39	1214.06	.2351
102	329.00	332.41	881.87	1214.28	.2371
103	329.71	333.14	881.35	1214.50	.2392
104	330.41	333.86	880.84	1214.71	.2413
105	331.11	334.58	880.34	1214.92	.2434
106	331.80	335.30	879.84	1215.14	.2454
107	332.49	336.00	879.34	1215.35	.2475
108	333.17	336.71	878.84	1215.55	.2496
109	333.85	337.41	878.35	1215.76	.2516
110	334.52	338.10	877.86	1215.97	.2537
111	335.19	338.79	877.37	1216.17	.2558
112	335.85	339.47	876.89	1216.37	.2578
113	336.51	340.15	876.41	1216.57	.2599
114	337.16	340.83	875.94	1216.77	.2619
115	337.81	341.50	875.47	1216.97	.2640
116	338.45	342.16	875.00	1217.17	.2661
117	339.10	342.83	874.53	1217.36	.2681
118	339.73	343.48	874.07	1217.56	.2702
119	340.36	344.14	873.61	1217.75	.2722
120	340.99	344.78	873.15	1217.94	.2742
121	341.61	345.43	872.70	1218.13	.2762
122	342.23	346.07	872.25	1218.32	.2782
123	342.85	346.70	871.80	1218.51	.2802
124	343.46	347.34	871.35	1218.69	.2822

TABLE No. 1. — *Continued.*

Pressure above zero.	Temperature.	Sensible Heat above zero Fahr.	Latent Heat.	Total Heat above zero Fahr.	Weight of One Cubic Foot.
Lbs. per sq. in.	Fahr. Deg.	B.T.U.	B.T.U.	B.T.U.	Lbs.
125	344.07	347.97	870.91	1218.88	.2842
126	344.67	348.59	870.47	1219.06	.2862
127	345.27	349.21	870.03	1219.25	.2882
128	345.87	349.83	869.59	1219.43	.2902
129	346.45	350.44	869.16	1219.61	.2922
130	347.05	351.05	868.73	1219.79	.2942
131	347.64	351.66	868.30	1219.97	.2961
132	348.22	352.26	867.88	1220.15	.2981
133	348.80	352.86	867.46	1220.32	.3001
134	349.38	353.46	867.03	1220.50	.3020
135	349.95	354.05	866.62	1220.67	.3040
136	350.52	354.64	866.20	1220.85	.3060
137	351.08	355.23	865.79	1221.02	.3079
138	351.75	355.81	865.38	1221.19	.3099
139	352.21	356.39	864.97	1221.36	.3118
140	352.76	356.96	864.56	1221.53	.3138
141	353.31	357.54	864.16	1221.70	.3158
142	353.86	358.11	863.76	1221.87	.3178
143	354.41	358.67	863.36	1222.03	.3199
144	354.96	359.24	862.96	1222.20	.3219
145	355.50	359.80	862.56	1222.36	.3239
146	356.03	360.85	862.17	1222.53	.3259
147	356.57	360.91	861.78	1222.69	.3279
148	357.10	361.46	861.39	1222.85	.3299
149	357.63	362.01	861.00	1223.01	.3319
150	358.16	362.55	860.62	1223.18	.3340
151	358.68	363.10	860.23	1223.33	.3358
152	359.20	363.64	859.85	1223.49	.3376
153	359.72	364.17	859.47	1223.65	.3394
154	360.23	364.71	859.10	1223.81	.3412
155	360.74	365.24	858.72	1223.97	.3430
156	361.26	365.77	858.35	1224.12	.3448
157	361.76	366.30	857.98	1224.28	.3466
158	362.27	366.82	857.61	1224.43	.3484
159	362.77	367.34	857.24	1224.58	.3502
160	363.27	367.86	856.87	1224.74	.3520
161	363.77	368.38	856.50	1224.89	.3539
162	364.27	368.89	856.14	1225.04	.3558
163	364.76	369.41	855.78	1225.19	.3577
164	365.25	369.92	855.42	1225.34	.3596
165	365.74	370.42	855.06	1225.49	.3614
166	366.23	370.93	854.70	1225.64	.3633
167	366.71	371.43	854.35	1225.78	.3652

TABLE No. 1. — *Concluded.*

Pressure above zero.	Temperature.	Sensible Heat above zero Fahr.	Latent Heat.	Total Heat above zero Fahr.	Weight of One Cubic Foot.
Lbs. per sq. in.	Fahr. Deg.	B.T.U.	B.T.U.	B.T.U.	Lbs.
168	367.19	371.93	853.99	1225.93	.3671
169	367.68	372.43	853.64	1226.08	.3690
170	368.15	372.93	853.29	1226.22	.3709
171	368.63	373.42	852.94	1226.37	.3727
172	369.10	373.91	852.59	1226.51	.3745
173	369.57	374.30	852.25	1226.66	.3763
174	370.04	374.89	851.90	1226.80	.3781
175	370.51	375.38	851.56	1226.94	.3799
176	370.97	375.86	851.22	1227.08	.3817
177	371.44	376.34	850.88	1227.23	.3835
178	371.90	376.82	850.54	1227.37	.3853
179	372.36	377.30	850.20	1227.51	.3871
180	372.82	377.78	849.86	1227.65	.3889
181	373.27	378.25	849.53	1227.78	.3907
182	373.73	378.72	849.20	1227.92	.3925
183	374.18	379.19	848.86	1228.06	.3944
184	374.63	379.66	848.53	1228.20	.3962
185	375.08	380.13	848.20	1228.33	.3980
186	375.52	380.59	847.88	1228.47	.3999
187	375.97	381.05	847.55	1228.61	.4017
188	376.41	381.51	847.22	1228.74	.4035
189	376.85	381.97	846.90	1228.87	.4053
190	377.29	382.42	846.58	1229.01	.4072
191	377.72	382.88	846.26	1229.14	.4089
192	378.16	383.33	845.94	1229.27	.4107
193	378.59	383.78	845.62	1229.41	.4125
194	379.02	384.23	845.30	1229.54	.4143
195	379.45	384.67	844.99	1229.67	.4160
196	379.97	385.12	844.68	1229.80	.4178
197	380.30	385.56	844.36	1229.93	.4196
198	380.72	386.00	844.05	1230.06	.4214
199	381.15	386.44	843.74	1230.19	.4231
200	381.57	386.88	843.43	1230.31	.4249
201	381.99	387.32	843.12	1230.44	.4266
202	382.41	387.76	842.81	1230.57	.4283
203	382.82	388.19	842.50	1230.70	.4300
204	383.24	388.62	842.20	1230.82	.4318
205	383.65	389.05	841.89	1230.95	.4335
206	384.06	389.48	841.59	1231.07	.4352
207	384.47	389.91	841.29	1231.20	.4369
208	384.88	390.33	840.99	1231.32	.4386
209	385.28	390.75	840.69	1231.45	.4403
210	385.67	391.17	840.39	1231.57	.4421

TABLE No. 2. — HYPERBOLIC LOGARITHMS.

Numb.	Log.	Numb.	Log.	Numb.	Log.	Numb.	Log.
1·05	·049	2·55	·936	4·05	1·399	5·55	1·714
1·1	·095	2·6	·956	4·1	1·411	5·6	1·723
1·15	·140	2·65	·975	4·15	1·423	5·65	1·732
1·2	·182	2·7	·993	4·2	1·435	5·7	1·740
1·25	·223	2·75	1·012	4·25	1·447	5·75	1·749
1·3	·262	2.8	1·030	4·3	1·459	5·8	1·758
1·35	·300	2·85	1·047	4·35	1·470	5·85	1·766
1·4	·336	2·9	1·065	4·4	1·482	5·9	1·775
1·45	·372	2·95	1·082	4·45	1·493	5·95	1·783
1·5	·405	3·0	1·099	4·5	1·504	6·0	1·792
1·55	·438	3·05	1·115	4·55	1·515	6·05	1·800
1·6	·470	3·1	1·131	4·6	1·526	6·1	1·808
1·65	·501	3·15	1·147	4·65	1·537	6·15	1·816
1·7	·531	3·2	1·163	4·7	1·548	6·2	1·824
1·75	·560	3·25	1·179	4·75	1·558	6·25	1·833
1·8	·588	3·3	1·194	4·8	1·569	6·3	1·841
1·85	·615	3·35	1·209	4·85	1·579	6·35	1·848
1·9	·642	3·4	1·224	4·9	1·589	6·4	1·856
1·95	·668	3·45	1·238	4·95	1·599	6·45	1·864
2·0	·693	3·5	1·253	5·0	1·609	6·5	1·872
2·05	·718	3·55	1·267	5·05	1·619	6·55	1·879
2·1	·742	3·6	1·281	5·1	1·629	6·6	1·887
2·15	·765	3·65	1·295	5·15	1·639	6·65	1·895
2·2	·788	3·7	1·308	5·2	1·649	6·7	1·902
2·25	·811	3·75	1·322	5·25	1·658	6·75	1·910
2·3	·833	3·8	1·335	5·3	1·668	6·8	1·917
2·35	·854	3·85	1·348	5·35	1·677	6·85	1·924
2·4	·875	3·9	1·361	5·4	1·686	6·9	1·931
2·45	·896	3·95	1·374	5·45	1·696	6·95	1·939
2·5	·916	4·0	1·386	5·5	1·705	7·0	1·946

TABLE No. 2. — **HYPERBOLIC LOGARITHMS.** — *Continued.*

Numb.	Log.	Numb.	Log.	Numb.	Log.	Numb.	Log.
7·05	1·953	8·05	2·086	9·05	2·203	15·	2·708
7·1	1·960	8·1	2·092	9·1	2·208	20·	2·996
7·15	1·967	8·15	2·098	9·15	2·214	25·	3·219
7·2	1·974	8·2	2·104	9·2	2·219	30·	3·401
7·25	1·981	8·25	2·110	9·25	2·225	35·	3·555
7·3	1·988	8·3	2·116	9·3	2·230	40·	3·689
7·35	1·995	8·35	2·122	9·35	2·235	45·	3·807
7·4	2·001	8·4	2·128	9·4	2·241	50·	3·912
7·45	2·008	8·45	2·134	9·45	2·246	55·	4·007
7·5	2·015	8·5	2·140	9·5	2·251	60·	4·094
7·55	2·022	8·55	2·146	9·55	2·257	65·	4·174
7·6	2·028	8·6	2·152	9·6	2·262	70·	4·248
7·65	2·035	8·65	2·158	9·65	2·267	75·	4·317
7·7	2·041	8·7	2·163	9·7	2·272	80·	4·382
7·75	2·048	8·75	2·169	9·75	2·277	85·	4·443
7.8	2·054	8·8	2·175	9·8	2·282	90·	4·500
7·85	2·061	8·85	2·180	9·85	2·287	95·	4·554
7·9	2·067	8·9	2·186	9·9	2 293	100·	4·605
7·95	2·073	8·95	2·192	9·95	2·298	1000	6·908
8·0	2·079	9·0	2·197	10·0	2·303	10,000	9·210