TECHNICAL MATHEMATICS

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Contents

UNIT ONE **SLIDE RULE AND REVIEW OF ARITHMETIC AND GEOMETRY**

Efficient computing methods applied to arithmetic, mensuration, and geometry—how to use the slide rule

Chapter 1 TREATMENT OF MEASURED DATA

In the first few chapters of this book we shall review some of the methods whereby much of the drudgery of computation may be eliminated and the effectiveness of the work increased. We shall discuss the efficient use of the slide rule, of tables, and (in the Appendix) of short cuts, approximations, formulas, and graph paper.

1·1 Measured Data. Many of the data with which the average technical man works are obtained experimentally. There is a definite limit to their reliability. The *reliability* of a number may be expressed in terms of either precision or accuracy. *Precision* is gauged by the position of the last reliable digit relative to the decimal point, whereas *accuracy* is measured by the number of significant figures. *Significant figures* are those known to be reliable and include any zeros not merely used to locate the decimal point.

For instance, if the diameters of several wires had been measured with a micrometer and found to be 0.118, 0.056, 0.008, and 0.207 in., one might say that these diameters had been measured to a precision of 0.001 in., and to an accuracy of three, two, one, and three figures, respectively.

Should the definition of significant figures seem somewhat arbitrary, let us consider the computation of the volume of a rectangular sheet of metal. Suppose that the measured length, width, and thickness are 165.2, 5.07, and 0.0021 in., respectively, and that these measurements are correct to the last digit given. Let us now compare the effect on the volume of changing the last digit of each measurement by one. It will be seen that such a change introduces respective errors of about one-sixteenth of 1 per cent, one-fifth of 1 per cent, and 5 per cent. The length, then, is the most accurate and the thickness the least accurate.

1·2 Rounding Off Numbers. Frequently a result will be rounded off because the last several digits either are in doubt or are not required in that particular computation. The operation of rounding off is governed by the following rule:

If the figures to be rejected represent less than half a unit in the last place to be retained, they are dropped. If they represent more than half a unit in the last place to be retained, the last retained digit is increased by one. If the rejected part represents just half a unit in the last place to be retained, the last retained significant digit is left even or raised to the nearest even number.

EXAMPLE 1

Number	Rounded off to: Four figures	Three figures	Two figures
3.1416	3.142	3.14	3.1
14.815	14.82	14.8	15.
321.35	321.4	321	320
6,274.5	6,274	6,270	6,300

In addition and subtraction the precision of the answer corresponds to the least precise of the quantities involved. *Perform the addition or subtraction, and round off by eliminating any digits resulting from operations on broken columns on the right.*

EXAMPLE 2. Add:

175.6
2.126
13.04
0.0028
190.7688 (Round off to 190.8.)

In multiplication and division the accuracy of the answer corresponds to the least accurate of the quantities involved. *Perform the multiplication or division and round off the answer to a number of significant figures equal to that in the least accurate quantity in the computation.*

EXAMPLE 3. Multiply:

$$3.14159 \times 47.82 = 150.2308338$$

Although the multiplicand has six significant figures, the multiplier has only four; therefore, we round off the product to four significant figures and get 150.2.

1·3 Scientific Notation. In scientific work very large or very small numbers are expressed as a number between 1 and 10 times an integral power of 10. Thus 2,580,000 would be written 2.58×10^6, and 0.0000258 would be written 2.58×10^{-5}. The magnitude of the

number is revealed by a glance at the exponent (see Table 14·4, page 271).

Several other advantages in this notation will become apparent. Space is saved, a particularly important point in tabulating data. The labor of counting figures to the right or left of the decimal point —a labor attended by risk of error—is eliminated. The accuracy with which a quantity is known is indicated by the number of figures to the right of the decimal point. For example, when we consider the number 72,000, we cannot tell whether there are two, three, four, or five significant figures. No uncertainty exists when we write 7.2×10^4, 7.20×10^4, 7.200×10^4, or 7.2000×10^4.

The ease of dealing with large and small quantities in this manner is illustrated by the following problem.

Simplify the expression

$$\frac{400{,}000 \times 8{,}000{,}000 \times 0.0045}{60{,}000 \times 0.025 \times 100} = \frac{4 \times 10^5 \times 8 \times 10^6 \times 4.5 \times 10^{-3}}{6 \times 10^4 \times 2.5 \times 10^{-2} \times 10^2}$$

$$= \frac{4 \times 8 \times 4.5}{6 \times 2.5} \times 10^{(5+6-3)-(4-2+2)} = 9.6 \times 10^4 = 96{,}000$$

There are two instances in which we depart from the rule of expressing a quantity as a number between 1 and 10 times a suitable power of 10. If we were to extract the square root of 2.5×10^{-7}, we should write this as 25×10^{-8} in order to make the exponent of 10 divisible by the index of the root. The square root is readily seen to be 5×10^{-4}. Also, when quantities are to be added and subtracted, they must have the same exponents. Thus $4 \times 10^{-7} + 7 \times 10^{-5} = 4 \times 10^{-7} + 700 \times 10^{-7} = 704 \times 10^{-7} = 7.04 \times 10^{-5}$.

EXERCISE

1. Translate into ordinary notation: 5.18×10^6; 3.76×10^{-4}; 7.5×10^{-8}; 4.375×10^2.
2. Translate into scientific notation:

   1 year = 31,500,000 sec (approx)
   1 light-second = 186,000 miles
   Wavelength of blue light = 0.000047 cm

3. $\dfrac{5 \times 10^7 \times 9 \times 10^3 \times 400}{4.8 \times 10^4} = ?$
4. $\dfrac{6 \times 10^2 \times 15 \times 10^4 \times 4 \times 10^{-1}}{8 \times 10^3 \times 25 \times 10^5} = ?$

5. $\dfrac{280{,}000 \times 16{,}000 \times 0.009}{2{,}100 \times 2{,}400{,}000 \times 0.04} = ?$

6. $\dfrac{0.12 \times 5{,}000 \times 33{,}000}{550 \times 18 \times 0.0002} = ?$

7. $\dfrac{1.2 \times 10^5 \times 9 \times 10^{-2}}{16 \times 10^3 \times 1.5 \times 10^{-4}} - 20{,}000 + \dfrac{7 \times 10^6}{2 \times 10^2} = ?$

Chapter 2 THE SLIDE RULE

In the last chapter we mentioned four of the most important aids to computation—slide rule, formulas, tables, and graph paper. In this chapter we shall obtain practice in the use of the first of these.

2·1 Scope of the Slide Rule. The student should have a clear idea of what can and what cannot be done on a slide rule. The slide rule is an instrument used for processes of multiplication, division, proportion, and calculation of simple powers and roots. Further discussion of the slide rule will be found in Secs. 14·31, 14·32, and 18·21 to 18·23, but this chapter will deal only with the processes named above. Addition and subtraction cannot be performed on the slide rule. One of the most frequently asked questions is, "Is slide-rule computation good enough?" The answer depends upon the nature of the problem. Many experimental data are accurate to no more than three significant figures. In such cases slide-rule computation is sufficiently accurate. Even in problems requiring more precise methods, the slide rule is accurate enough to detect gross errors and to estimate the order of magnitude of the answer.

The slide rule will not think for you. Like an automobile, it will take you to your destination if you provide the proper direction.

There are many types of slide rule, but our discussion will be limited to the general technique of operation common to most 10-in. slide rules.

2·2 Description of the Slide Rule. The slide rule consists of three parts (see Fig. 2·1): the *slide*, or central sliding part; the *body*, or the upper and lower bars between which the slide operates; and the *indicator*, which is the movable glass plate marked with a hairline.

The mark associated with the numeral 1 at an end of a scale is called the *index* of the scale. Two positions on two different scales are said to be *opposite* if the hairline can be made to cover both simultaneously without moving the slide.

In order to perform the operations mentioned in Sec. 2·1, we need use only the C, D, A, B, and K scales. The C and D scales are identical. The A and B scales are identical with each other, and the distance

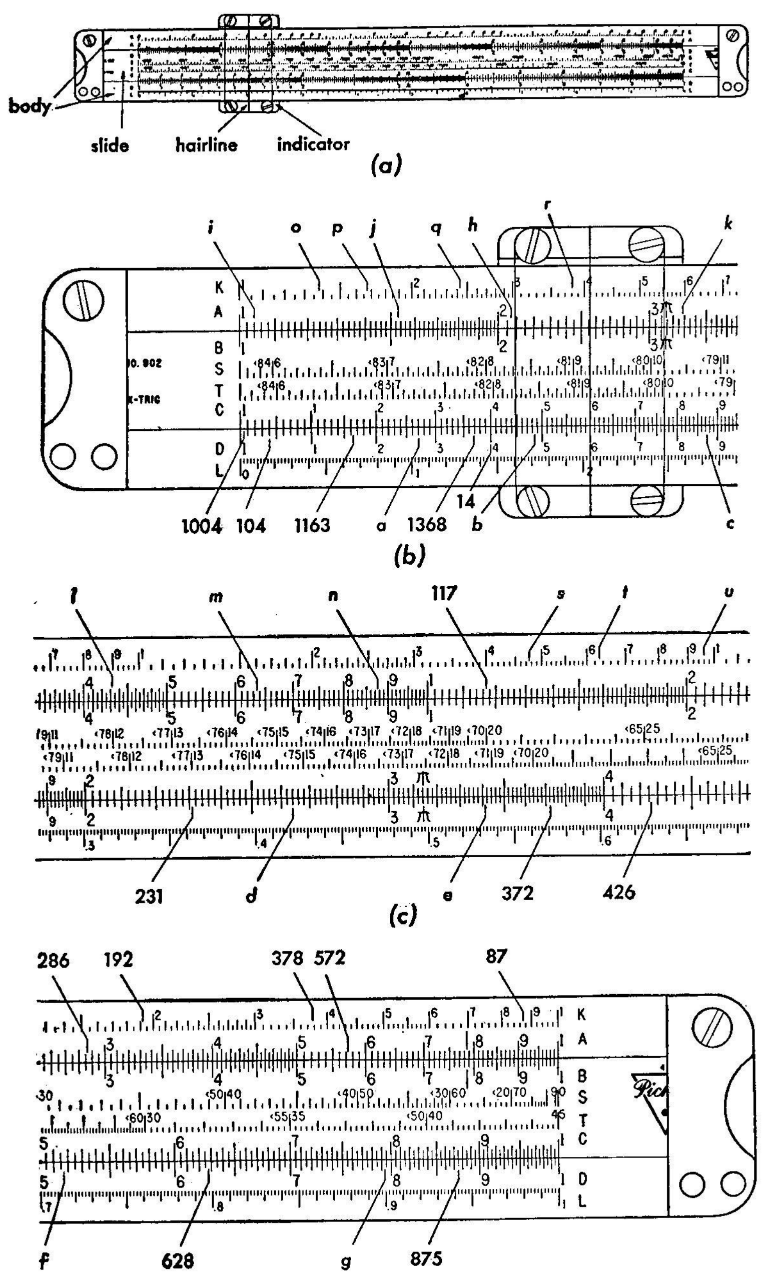


Fig. 2·1

between two successive integers is one-half the distance between the same two integers on the C and D scales. The K scale is compressed even further, so that the distance between any two integers is one-third the distance between the same two integers on scales C and D.

The C and D scales are divided into nine principal divisions by *primary marks* bearing the large numbers 1, 2, 3, . . . , 8, 9, 1. The space between any two primary marks is divided into ten parts by *secondary marks.* These are not numbered, except between the primary marks 1 and 2, where they bear the small numerals 1 through 9. The space between two successive secondary marks is divided into two, five, or ten spaces by unnumbered *tertiary marks.*

The A and B scales each have two identical portions. Each portion is divided into nine principal divisions by numbered primary marks. Unnumbered secondary marks further divide each principal division, and still finer division is supplied by tertiary marks between the primary marks numbered 1 and 5.

The K scale is divided into three identical portions. They are divided much as the A and B scales are, but more coarsely.

2·3 Location of Numbers on the Scales. Accuracy of the Slide Rule. It must be remembered that the decimal point plays no part in locating a number on the C and D scales. The first significant digit is located by reference to the primary marks, the second by reference to the secondary marks, and the third by reference to the tertiary marks, or some point between tertiary marks, according to the portion of the scale being used. By estimating fractions of a graduation, a fourth digit may be located for numbers occurring between primary marks numbered 1 and 2 on the C and D scales. This means that the maximum accuracy ordinarily available with a 10-in. slide rule is one part in 1,000. Location of actual numbers will be best illustrated by referring to the examples in Fig. 2·1(*b–d*).

EXERCISE 1

Read as closely as possible the points indicated in Fig. 2·1(*b–d*).

D scale	*a b c d e f g*
A scale	*h i j k l m n*
K scale	*o p q r s t u*

2·4 Multiplication. Either the C and D scales or the A and B scales may be used. Ordinarily the C and D scales are preferable, since their larger-scale divisions make for greater accuracy. Their use is illustrated in Fig. 2·2 and in the following example.

EXAMPLE 1. Multiply 8 × 5.

Set the right index of C opposite 8 on the D scale. Move the indicator so the hairline covers 5 on the C scale. Directly below this 5 will be found the "slide-rule product" 4 on the D scale. The student must locate the decimal place himself, and common sense indicates that the answer is 40, not 4.

It will be appreciated that the same setting would be used for multiplying 80 by 50, 8,000 by 0.5, 0.08 by 500, etc.

If we were to try to use the above setting of the right index to multiply 8 by 12, a reading would be impossible, as the 12 on the C scale is located beyond the end of the D scale. In this case we would

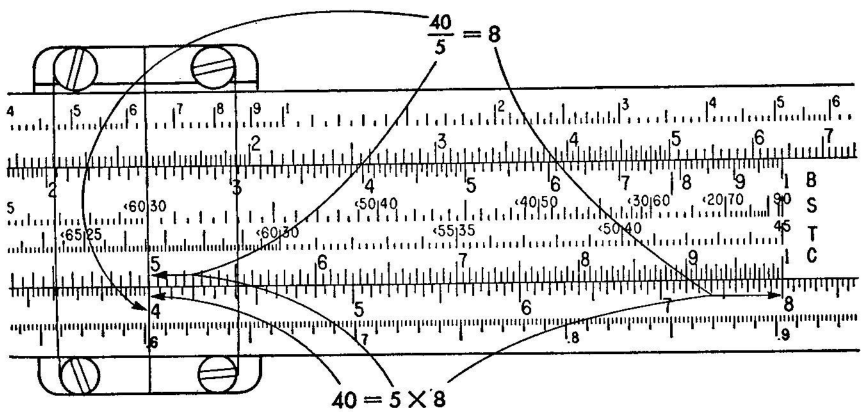


Fig. 2·2

set the *left* index of the C scale over 8 on the D scale and locate on the D scale our answer 96, immediately under 12 on the C scale. It will be useful to remember that when the product of the first significant figures in multiplicand and multiplier is less than 10, the student should use the left index; when it is greater than 10, he should use the right index.

When multiplying more than two numbers, use the hairline to mark the position of the product of the first two. Without reading this product, use it as the multiplicand for the next multiplication.

If the magnitude of the product is not immediately obvious, a rough mental check will suffice.

EXAMPLE 2. The slide-rule product of 1440 × 37.5 × 0.08125 is 439. It is evident that our product is approximately 1,500 × 40 × 0.08, or 4,800. Hence the answer must be 4,390.

To multiply two numbers:

1. *Set the proper index of C opposite the multiplicand on the D scale.*

2. *Place the hairline over the multiplier on the C scale, and read the significant digits of the product under the hairline on the D scale.*
3. *Determine the position of the decimal point by a rough mental estimate.*
4. *In the above steps, C and D may be replaced by B and A, respectively.*

EXERCISE 2

Perform the following multiplications, retaining in the product as many significant figures as you think are justified:

1. 4.00 × 17.00
2. 7.50 × 1.20
3. 3.30 × 9.0
4. 64.0 × 0.375
5. 288 × 382
6. 321 × 1,069
7. 728 × 1,218
8. 617 × 1,645
9. 1,006 × 902
10. 1,258 × 1,562
11. 862 × 482
12. 66.2 × 10.3
13. 1.475 × 1,520
14. 0.981 × 0.693
15. 329 × 0.00352
16. 0.342 × 1.306
17. 8.14 × 0.0309
18. 2.46 × 330,000 × 3.14
19. 3.1 × 920 × 0.486 × 1,520
20. 0.1038 × 0.0063 × 28 × 9.82
21. 512 × 62.5 × 0.0027 × 87
22. 0.1047 × 0.00774 × 0.349 × 0.0562

2·5 Division. Since division is the inverse of multiplication, Fig. 2·2 may be used to illustrate division as well as multiplication. In this example we have the setting for 40 ÷ 5 = 8.

To divide one number by another:

1. *Bring the dividend on the D scale opposite the divisor on the C scale by means of the hairline.*
2. *Opposite an index of the C scale, read the significant figures of the quotient on the D scale. If desired, the indicator may be used to aid in this reading.*
3. *Determine the position of the decimal point by a rough mental estimate.*
4. *C and D may be replaced by B and A, respectively, as in multiplication.*

EXERCISE 3

Perform the following divisions, retaining in the quotient as many significant figures as you believe justified:

1. 18.00 ÷ 50.0 **2.** 25.0 ÷ 3.00 **3.** 750 ÷ 5.50
4. 12.8 ÷ 72 **5.** 69.8 ÷ 4.78 **6.** 197.2 ÷ 858
7. 0.924 ÷ 21.0 **8.** 17.5 ÷ 1,646 **9.** 1 ÷ 37.5
10. 0.0752 ÷ 0.000718 **11.** 0.1804 ÷ 363 **12.** 1 ÷ 2.73
13. 0.1875 ÷ 0.078125 **14.** 0.005632 ÷ 18.432

2·6 Location of Decimal Point in Scientific Notation. Use of scientific notation makes work with very large or very small numbers easier and reduces the likelihood of error.

EXAMPLE 3. Multiply 538,000 × 0.00377.

In scientific notation, we have $5.38 \times 10^5 \times 3.77 \times 10^{-3}$. This product may be written $5.38 \times 3.77 \times 10^2$. Since the product of 5.38 and 3.77 on the slide rule is 20.3, our answer is 20.3×10^2, or 2,030.

EXAMPLE 4. Divide 538,000 by 0.00377.

In scientific notation, we have

$$\frac{5.38 \times 10^5}{3.77 \times 10^{-3}} = 1.427 \times 10^{5-(-3)} = 1.427 \times 10^8 = 142,700,000$$

2·7 Combined Multiplication and Division. The easiest method of computing the quotient of two products is to alternate between multiplication and division. If we do all the multiplying first, then all the dividing, more moves are required, with greater chance of error.

EXAMPLE 5. Evaluate $\frac{825 \times 184}{227 \times 316}$.

Divide 825 by 227 in the usual way. The quotient will be found on the D scale under the C index. However, we do not read the value, since it serves merely as the multiplicand for 184. Without moving the slide, we move the indicator so that the hairline covers 184 on the C scale. The product will be found on the D scale under the hairline. Since this product is to be the dividend for the division by 316, we do not read the value, nor do we move the hairline, but we move the slide so that 316 is under the hairline. Directly beneath the C index read approximately 2115. The answer is 2.115, with the last figure in doubt.

Sometimes the slide must be moved so that one C index is moved to the spot formerly occupied by the other C index (as marked with the hairline). This situation might have been avoided, with some sacrifice of accuracy, by using the A and B scales.

EXERCISE 4

Perform the following computations, expressing the answer in scientific notation. Retain in the answer as many significant figures as conditions justify.

1. 2.4 × 6.5 × 10.37 **2.** 1476 × 37.8 × 54.0
3. 0.00842 × 0.295 × 6.1875 **4.** 67.1 × 0.000418 × 3.0
5. 32.00 × 5.000 × 1.900 × 0.4000 **6.** $\frac{1}{0.00532 \times 0.0612}$
7. $\frac{1.28 \times 3.56}{74.4}$ **8.** $\frac{15.8 \times 1.35}{0.031}$
9. $\frac{21.3 \times 0.054}{97.4 \times 3.80}$ **10.** $\frac{1,927}{412 \times 0.00592 \times 483}$
11. $\frac{24.6 \times 0.359}{296 \times 4.61 \times 98.7}$ **12.** $\frac{560,000 \times 0.0045 \times 12,500}{1,050,000 \times 0.072}$

2·8 Proportion. Proportions may usually be solved with only one setting of the slide. Observe in Fig. 2·3 that when 8 on the C scale

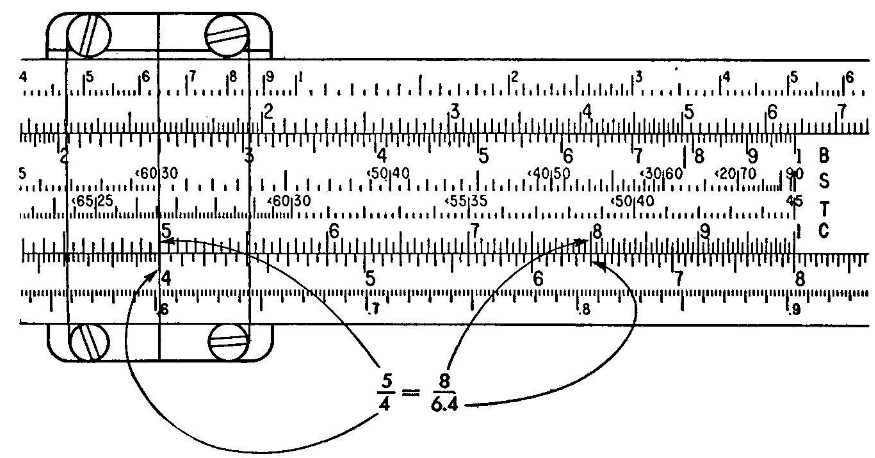


Fig. 2·3

is opposite 64 on the D scale, we find 5 opposite 4. This setting could therefore illustrate the solution of the proportion $5/x = 8/6.4$, in which we set 8 opposite 64 and opposite 5 read $x = 4$. Note that all other pairs of numbers that are opposite each other have the same ratio (*e.g.*, $^{60}/_{48}$, $^{70}/_{56}$, $^{75}/_{60}$, $^{90}/_{72}$).

To solve a proportion, locate the numbers on the C and D scales in the same relative position as in the proportion $a/b = c/d$ (or $c/d =$

a/b if the setting for a falls to the right of the setting for c). When the C and D scales cannot accommodate the simultaneous settings of a over b and c over d, the A and B scales may be used.

EXERCISE 5

In the following proportions, calculate x to three significant figures:

1. $\frac{x}{8.5} = \frac{32}{28.9}$ $x = ?$
2. $\frac{x}{21.5} = \frac{89}{79}$ $x = ?$
3. $\frac{372}{x} = \frac{637}{9.31}$ $x = ?$
4. $\frac{8.2}{377} = \frac{0.323}{x}$ $x = ?$
5. $\frac{18.3}{63.6} = \frac{x}{29}$ $x = ?$
6. $\frac{267}{8.75} = \frac{x}{192}$ $x = ?$
7. $\frac{0.716}{x} = \frac{10.1}{168}$ $x = ?$
8. $\frac{795}{0.109} = \frac{42.3}{x}$ $x = ?$

2·9 Squares and Square Roots. To find the square of a number, set the hairline of the indicator over that number on the D Scale, and

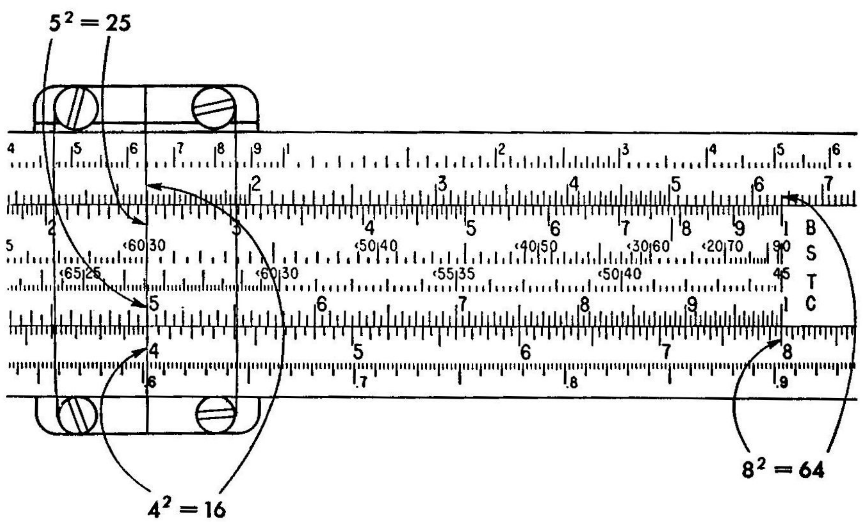


Fig. 2·4

under the hairline read the square of that number on the A scale. We can also read from the C to the B scale in the same way. Figure 2·4 shows that $4^2 = 16$ (D to A scale), that $5^2 = 25$ (C to B scale), and that $8^2 = 64$ (right C index reading to right B index setting).

Figure 2·5 illustrates that $(1.428)^2 = 2.04$, also that $(2.53)^2 = 6.4$.

EXAMPLE 6. Find $(6)^2$. Set the hairline over 6 on the D scale. Read 36 under the hairline on the A scale.

EXAMPLE 7. Find $(8.62)^2$. Set the hairline over 8.62 on the D scale. Read 74.3 under the hairline on the A scale.

EXAMPLE 8. Find $(71{,}700)^2$. On the A scale read 514 directly above the 717 on the D scale. Since $(71{,}700)^2 = (7.17 \times 10^4)^2$, the answer must be $51.4 \times 10^{4\times2} = 51.4 \times 10^8 = 5{,}140{,}000{,}000$.

EXAMPLE 9. Find $(0.00386)^2$. On the A scale read 149 directly above the 386 on the D scale. Since $(0.00386)^2 = (3.86 \times 10^{-3})^2$, the answer must be $14.9 \times 10^{-3\times2} = 14.9 \times 10^{-6} = 0.0000149$.

The process of obtaining square roots is the reverse of that used in calculating squares. Therefore Figs. 2·4 and 2·5 may be used to show

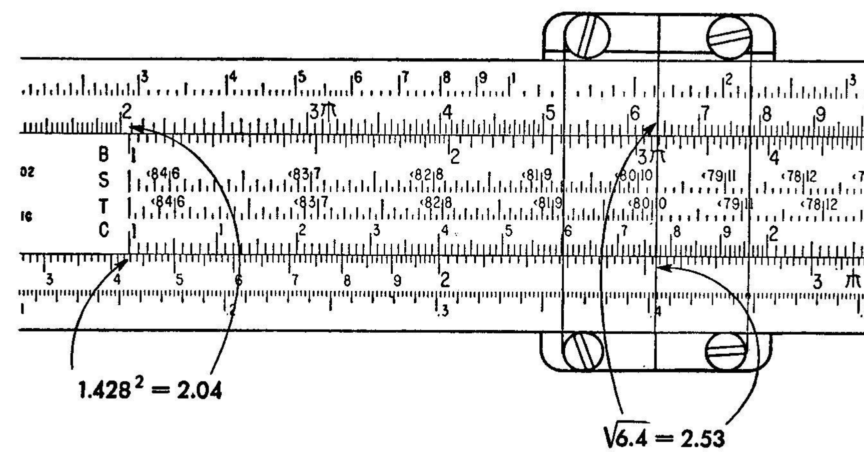


Fig. 2·5

that $\sqrt{16} = 4$, $\sqrt{25} = 5$, $\sqrt{64} = 8$, $\sqrt{2.04} = 1.428$, and $\sqrt{6.4} = 2.53$. We may indeed use Fig. 2·4 to read $\sqrt{64} = 8$; but if we had not had the operation of squaring 8 to guide us, we might have made an unfortunate choice in working from the 64 on the left half of the A scale; *i.e.*, we might have read 253 directly below on the D scale. A quick check will show that 2.53 is the square root of 6.4. (This procedure is opposite to that of squaring 2.53, an operation originally illustrated in Fig. 2·5.)

It is evident that in reversing the procedure for squaring we must first determine which half of the A scale to choose as a starting point. There are a number of ways of making this choice. Perhaps the simplest is that used in the tables on page 25. The procedure is as follows: Write down the number whose square root is to be found. Indicate the grouping of the digits as for longhand extraction of square root. Write in their proper positions the decimal point and the first significant digit in the square root. Select that half of the A scale which lies above the first significant figure (just determined) on the D scale.

EXAMPLE 10. Find $\sqrt{6{,}870{,}000}$. Indicating the grouping, the decimal point, and the first significant figure, we write $\overset{2\quad\quad\;\;\cdot}{\sqrt{6\ 87\ 00\ 00.}}$ It is apparent that we read from 687 on the *left* half of the A scale, since we find 2 on the D scale under that half. Accordingly we read from 687 on the left half of the A scale directly below to 262 on the D scale. This indicates that the square root of 6,870,000 will be written $\overset{2\ \ 6\ \ 2\ \ 0.}{\sqrt{6\ 87\ 00\ 00.}}$

EXAMPLE 11. Find $\sqrt{0.0000687}$. A rough indication of the answer is found by proceeding as before and writing $\overset{0.\ \ 0\ \ 0\ \ 8}{\sqrt{0.\ 00\ 00\ 68\ 7}}$ In this case we evidently read down from the 687 on the *right* half of the A scale. We read 829 on the D scale, and the answer is clearly 0.00829.

2.10 Cubes and Cube Roots. To find the cube of a number, set the hairline over that number on the D scale and read its cube on the K scale under the hairline.

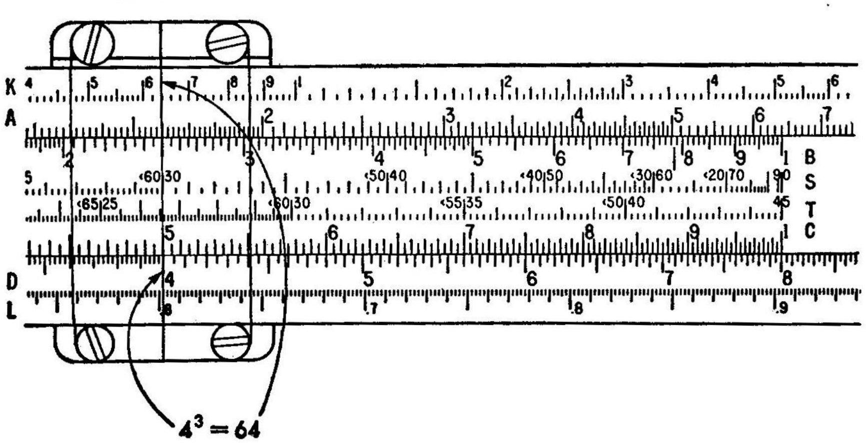


Fig. 2·6

EXAMPLE 12. Find $(6.35)^3$. Set the hairline over 6.35 on the D scale. Read 256 under the hairline on the K scale.

EXAMPLE 13. Find $(0.0439)^3$. Read 846 on the K scale directly opposite 439 on the D scale. Since $(0.0439)^3 = (4.39 \times 10^{-2})^3$, the answer must be $84.6 \times 10^{-2\times 3} = 84.6 \times 10^{-6} = 0.0000846$.

To find cube root we reverse the procedure for cubing and work from the K scale to the D scale. When working from the A scale to the D scale, we had to choose between two sections. On the K scale we must choose the proper section out of three. The process is exactly comparable to that used in finding square root. Figures 2·6, 2·7, and

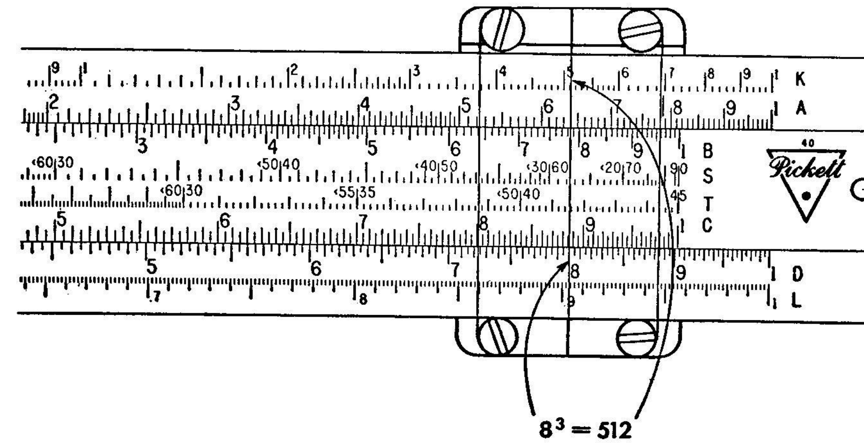


Fig. 2·7

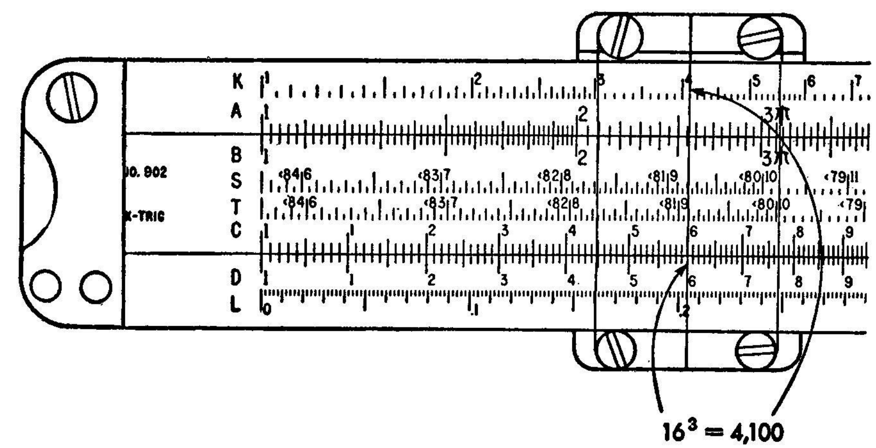


Fig. 2·8

2·8 show the settings for 4^3, 8^3, and 16^3 and therefore also the settings for $\sqrt[3]{64}$, $\sqrt[3]{512}$, and $\sqrt[3]{4{,}100}$.

EXAMPLE 14. Find $\sqrt[3]{0.0000048}$. Indicating the grouping, the decimal point, and the first significant figure, we write $\overset{0.\ \ 0\ \ \ \ 1\quad\cdot}{\sqrt[3]{0.000\ 004\ 8}}$ Evidently we work from the *left* third of the K scale, since it lies opposite the 1+ on the D scale. Therefore, we read from 48 on the left third of the K scale directly to

1,687 on the D scale. This indicates that the cube root of 0.0000048 will be written

$$\begin{array}{l} \text{0. 0 \quad 1 6 8 7} \\ \sqrt[3]{0.000\ 004\ 8} \end{array}$$

EXAMPLE 15. Find $\sqrt[3]{58{,}500{,}000}$. Proceeding as before, we write

$$\begin{array}{l} \quad 3 \qquad\quad . \\ \sqrt[3]{58\ 500\ 000.} \end{array}$$

and find that the *middle* third of the K scale lies opposite the 3 on the D scale. Reading, therefore, from 585 in the middle section of the K scale, we find 388 on the D scale directly opposite. Hence the answer must be 388.

EXERCISE 6

Evaluate the following as accurately as the slide rule will allow:

1. $(18.00)^2$ **2.** $(2{,}730)^2$
3. $(6.05)^2$ **4.** $(34.5)^2$
5. $(167.8)^2$ **6.** $(0.854)^2$
7. $(0.1054)^2$ **8.** $(0.00782)^2$
9. $(517.23)^2$ **10.** $(0.030448)^2$
11. $(63 \times 426)^2$ **12.** $(9.1 \times 0.0119)^2$
13. $(0.027 \times 1.72 \times 7.95)^2$ **14.** $(5.1 \times 0.438 \times 14.12)^2$
15. $\left(\frac{1{,}647}{658}\right)^2$ **16.** $\left(\frac{69.8}{858}\right)^2$
17. $\left(\frac{852}{658}\right)^2$ **18.** $\left(\frac{61.4 \times 0.673}{2.16}\right)^2$
19. $\left(\frac{582 \times 1.104}{37.2 \times 8}\right)^2$ **20.** $\sqrt{49.2}$
21. $\sqrt{0.00702}$ **22.** $\sqrt{2{,}980}$
23. $\sqrt{0.837}$ **24.** $\sqrt{17{,}840}$
25. $\sqrt{0.0542}$ **26.** $\sqrt{0.000347}$
27. $\sqrt{45{,}897}$ **28.** $\sqrt{103.6}$
29. $\sqrt{7{,}080{,}000}$ **30.** $\sqrt{\frac{1.316}{0.016}}$
31. $\sqrt{\frac{1{,}127 \times 5.47}{21 \times 0.0025}}$ **32.** $\sqrt{\frac{11.26}{0.787 \times 24.8}}$
33. $(11.9)^3$ **34.** $(1.33)^3$
35. $(0.157)^3$ **36.** $(118.5)^3$
37. $(23.19)^3$ **38.** $(0.0342)^3$
39. $(478)^3$ **40.** $(6.375)^3$

41. $(783)^3$ **42.** $(0.0876)^3$
43. $(15.45 \times 0.132)^3$ **44.** $(8.75 \times 0.037)^3$
45. $\left(\frac{10.12}{17.58}\right)^3$ **46.** $\left(\frac{48.8}{4.21}\right)^3$
47. $\sqrt[3]{13.5}$ **48.** $\sqrt[3]{0.0474}$
49. $\sqrt[3]{76{,}215}$ **50.** $\sqrt[3]{2.85}$
51. $\sqrt[3]{9{,}384{,}260}$ **52.** $\sqrt[3]{0.00625}$
53. $\sqrt[3]{652 \times 0.725}$ **54.** $\sqrt[3]{1.447 \times 0.0298}$
55. $\sqrt[3]{\frac{8{,}747}{0.212}}$ **56.** $\sqrt[3]{\frac{158.5}{255}}$

2·11 Circumference and Area of a Circle. Since the problem of finding the circumference of a circle is simply one of finding the

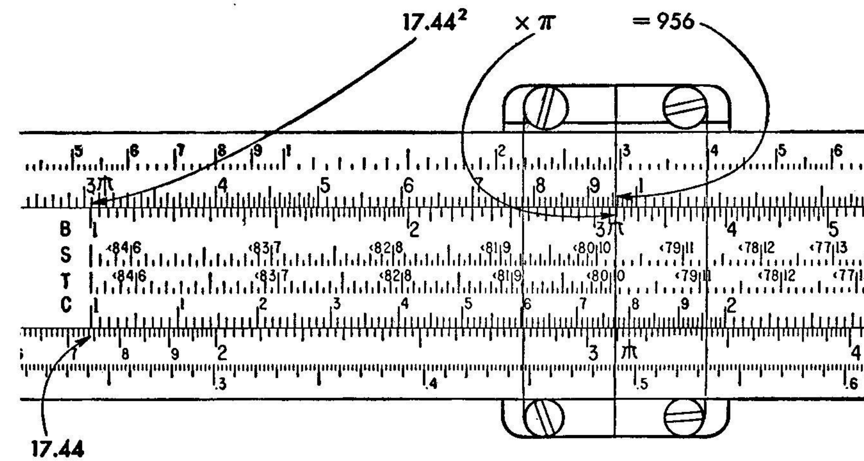


Fig. 2·9

product of a given diameter and 3.14, we follow the usual procedure for multiplication.

To find the area of a circle when the radius is given, set the index of C opposite the radius on the D scale. Place the hairline to cover π on the B scale. The answer is found under the hairline on the A scale. Fig. 2·9 shows that the area of a circle of radius 17.44 in. is about 956 sq in.

To find the area of a circle when the diameter is given, set the index of C opposite the diameter on the D scale. Place the hairline to cover $\pi/4$ or 0.785 on the B scale. The area is found under the hairline on the A scale. Figure 2·10 shows that the area of a 9.03-in.-diameter circle is 64.0 sq in.

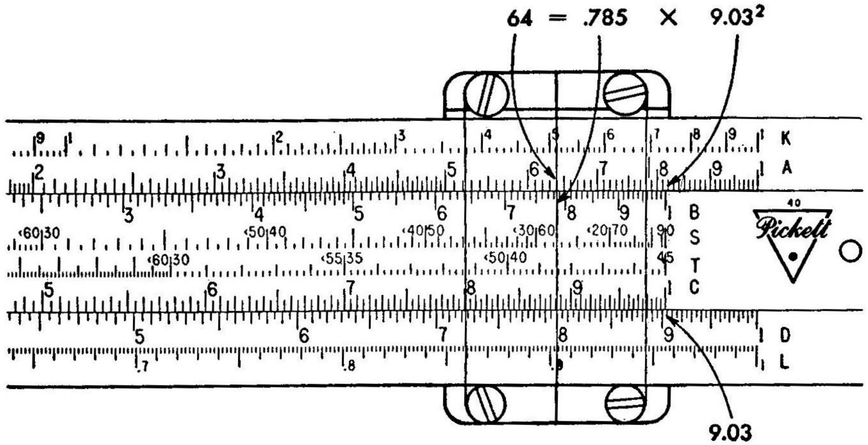


Fig. 2·10

EXERCISE 7

Make a copy of the following table of circles and fill in the blanks.

	Radius	*Diameter*	*Circumference*	*Area*
1.	6.3 miles			
2.	5¼ in.			
3.	78 ft			
4.	9/16 in.			
5.		11.2 yd		
6.		31.6 ft		
7.		7/8 in.		
8.		14½ in.		
9.			8.43 in.	
10.			26.7 ft	
11.				10.00 sq in.
12.				63.8 sq ft

2·12 General Suggestions for Slide-rule Operation. As the student becomes skillful in the use of the slide rule, he will acquire various tricks of the trade which will make this tool even more effective. A few suggestions are outlined in the following paragraphs.

Before setting numbers on the slide rule, cancel or combine simple numbers to reduce the number of moves required.

EXAMPLE 16. $\dfrac{2 \times 43 \times 3}{17 \times 61} = \dfrac{6 \times 43}{17 \times 61}$. A move is saved.

EXAMPLE 17. $\dfrac{60 \times 105}{5 \times 2 \times 97} = \dfrac{6 \times 105}{97}$. Two moves are saved.

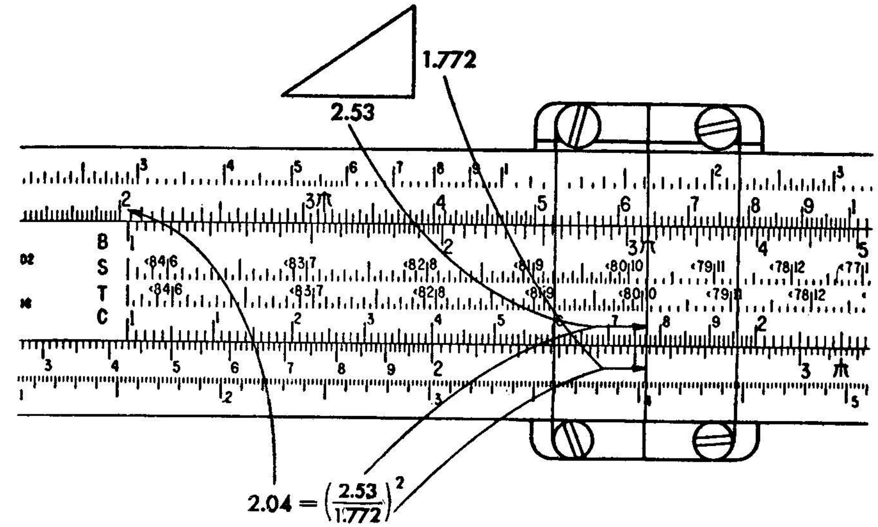


Fig. 2·11

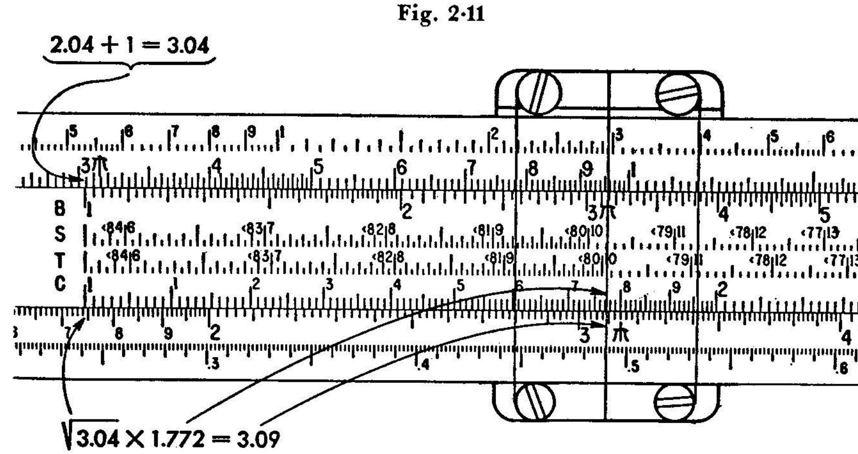


Fig. 2·12

Preliminary rearrangement of formulas is often helpful, as in logarithmic computation (see page 289).

As an alternative to the factoring method, it is suggested that the familiar formulas of the Pythagorean theorem be modified before using on the slide rule, as indicated below:

$$a = \sqrt{c^2 - b^2} \quad \text{becomes} \quad a = b\sqrt{(c/b)^2 - 1}$$

$$b = \sqrt{c^2 - a^2} \quad \text{becomes} \quad b = a\sqrt{(c/a)^2 - 1}$$

$$c = \sqrt{a^2 + b^2} \quad \text{becomes} \quad c = b\sqrt{(a/b)^2 + 1} \quad (a > b)$$

$$c = a\sqrt{(b/a)^2 + 1} \quad (b > a)$$

With the subtraction or addition of 1 done mentally, a minimum of moves is required.

EXAMPLE 18. Find the hypotenuse of a right triangle whose sides are 2.53 and 1.772. Letting 2.53, the larger side, be represented by a and 1.772 by b and referring to Fig. 2·11, we have the setting for obtaining $(2.53/1.772)^2$, which is seen to be 2.04. Adding 1 mentally to obtain 3.04, or $(a/b)^2 + 1$, we then refer to Fig. 2·12, which shows the setting for $1.772\sqrt{3.04}$, the value of which we read on the D scale as approximately 3.09.

EXERCISE 8

Compute the missing side in the following right triangles where c is the hypotenuse:

	a	b	c
1.	38	47	
2.	7.52	10.36	
3.	523	408	
4.	1⅞ in.	2¾ in.	
5.	9.85	25.6	
6.		0.0645	0.0892
7.		12.33	20.7
8.		3⅛ in.	4¼ in.
9.	5,670		7,030
10.	0.1525		0.291
11.	4⅞ in.		5½ in.

In finding the hypotenuse of right triangles containing a small acute angle, the approximation $c \approx a + (b^2/2a)$ is useful where b is the smallest side (see page 589). The smaller b becomes, in relation to a, the closer the approximation. Where b is as large as $0.2a$, c, as calculated by the approximation, will be about 0.02 per cent too large.

EXERCISE 9

Find c, evaluating $b^2/2a$ to three significant figures by slide rule.

	a	b
1.	52	3
2.	109	4
3.	15 in.	⅜ in.
4.	213	7
5.	1.75	0.08

An interesting application of proportion to gear ratios is as follows: Suppose we must have a gear ratio of approximately 1/0.7933, the number of teeth on each gear not to exceed 100. Referring to Fig. 2·13, in which 1 is set opposite 0.7933, we read (in addition to the pair 92 over 73 indicated by the hairline) the following pairs of integers which are approximately matched: 97 over 77; 87 over 69; 63 over 50; 58 over 46 (58 over 46 is a variation of 87 over 69 and would not be considered an independent pair). These results indicate in a rough

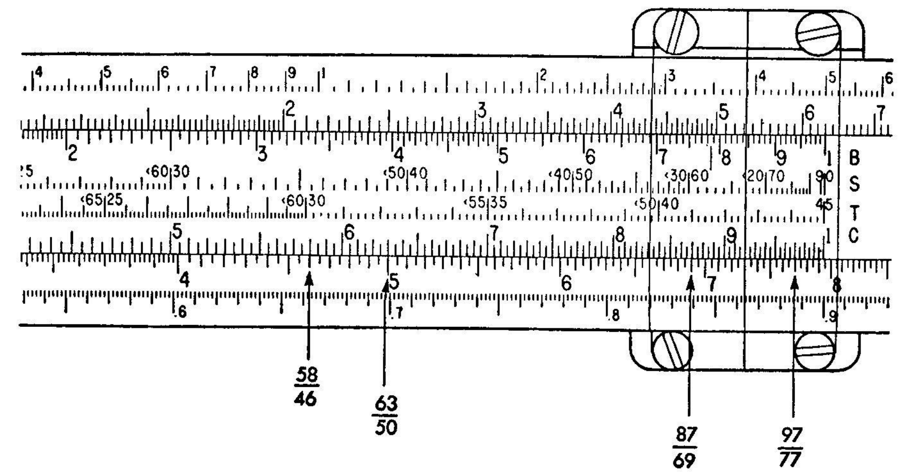


Fig. 2·13

way the number of teeth in gears, giving us the ratio 1:0.7933. The decimal values and accuracy of these ratios are tabulated below.

$$\frac{77}{97} = 0.7938, \text{ or } 0.06 \text{ per cent too large}$$

$$\frac{73}{92} = 0.7935, \text{ or } 0.02 \text{ per cent too large}$$

$$\frac{69}{87} = 0.7931, \text{ or } 0.02 \text{ per cent too small}$$

$$\frac{50}{63} = 0.7936, \text{ or } 0.04 \text{ per cent too large}$$

More precise results can be obtained by using logarithms or continued fractions.

EXERCISE 10

Determine for each of the following ratios all independent pairs of integers, less than 100, whose ratio does not differ from the given value by more than 0.05 per cent:

1. 1.369:1	**2.** 1.841:1	**3.** 2.073:1	**4.** 1.5847:1
5. 0.881:1	**6.** 0.82367:1	**7.** 0.763:1	**8.** 0.6833:1

In the following exercise a set of miscellaneous practice problems involving slide-rule operation is given. If further drill seems advisable, mensuration problems may be selected from Chap. 4 for computation by slide rule.

EXERCISE 11

1. Find the surface area of 1 oz of glass wool whose threads are 0.0008 in. in diameter. Neglect the area of the ends. (One cubic foot of glass weighs 160 lb.)
2. If the national budget is $71.5 billion, for how many miles would a line of dollar bills represented in this sum extend if laid end to end? Each bill is 6.1 in. long.
3. The product of the wavelength of light and its frequency gives its velocity. Find the wavelength (in centimeters) of red light if its frequency = 4.00×10^{14} vibrations per second. The velocity of light is 186,000 miles per sec.
4. How many miles away is the nearest star whose light takes 4 years to reach the earth? (Refer to velocity of light given in Prob. 3.)
5. How far away from the earth would such a star be in a small-scale model of the solar system? In this model the earth, which has an approximate diameter of 8,000 miles, is represented by a spherical pellet ¼ in. in diameter.
6. During a storm 1.5 in. of rain fell. How many gallons of water fell on a square mile of land? 1 gal = 231 cu in.
7. A reservoir having 5 sq miles of surface dropped 2 in. in a week. If it supplies a city of 800,000 inhabitants, how many gallons per week per inhabitant does the consumption amount to?
8. The Mississippi River carries about 3×10^7 cu ft of sediment to the Gulf of Mexico daily. What average yearly erosion (in inches) does this represent if the area of the Mississippi watershed is taken as 1,300,000 sq mile?
9. The earth receives energy from a beam of sunlight 1 sq mile in cross section at the rate of 4.9×10^6 hp approximately. At what rate does a solar-heated house receive energy if the heat-collecting roof surface is 18 by 25 ft? The sun's rays are assumed to be perpendicular to the roof surface.

10. The polar regions have been estimated to be covered with a sheet of ice averaging 1,500 ft thick over an area of 4,500,000 sq miles. The return of a "tropical age" would melt the ice and raise the level of the oceans. How many feet of rise would this amount to if the water surface is taken as 150,000,000 sq miles? The specific gravity of ice = 0.9.
11. Assuming an average rate of 72 beats per minute, about how many times will the heart beat during a lifetime of 70 years?
12. A manufacturer's advertisement states that a cubic mile of sea water contains 5,000,000 tons of magnesium. If the density of sea water is 64.0 lb per cu ft, find the percentage by weight of magnesium in sea water.
13. It has been stated that a teaspoon of fuel oil will furnish enough energy to draw a 1-ton load for 1 mile along a level track. What over-all coefficient of friction is assumed if a teaspoon contains 4 g of oil? Heat value of oil = 18,000 Btu per lb (1 Btu = 778 ft-lb). (Divide foot-pounds of energy equivalent in the oil by the foot-pounds in a ton-mile.)

used for drying, *i.e.*, E represents the limit to the drying possible under the given conditions

K = a constant

If the moisture content drops from 1.2 to 0.8 in 3 hr, with a further drop to 0.64 in another 6 hr, find the amount of moisture remaining in the stock after a drying period of indefinitely great length.

56. The resistance of a tungsten lamp filament is given by the relationship

$$\frac{R_1}{R_2} = \left(\frac{T_1}{T_2}\right)^{1.2}$$

where R_1 = resistance at room temperature (T_1)

R_2 = resistance at operating temperature (T_2)

Temperatures are both Kelvin (Centigrade absolute) or both Rankine (Fahrenheit absolute). If the resistance at 20°C is 16 ohms and the operating resistance is 232 ohms, find the operating temperature in degrees Centigrade.

57. A tungsten lamp is rated at 60 watts at 115 volts. If the resistance measured at 75°F is 15.2 ohms, find the operating temperature in degrees Fahrenheit. Ohms $= \dfrac{(\text{volts})^2}{\text{watts}}$ (see Prob. 56).

58. The horsepower necessary to compress a gas in a single-stage compressor is given by the formula

$$\text{hp} = \frac{144nP_1V_1}{33{,}000(n-1)}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

Find the horsepower necessary to compress 240 cu ft of helium from atmospheric pressure (14.7 psia) to 250 psig. For helium, $n = 1.66$. Other symbols have same meaning as in Probs. 28 to 31, page 293?

59. The weight P in pounds that will crush a solid cylindrical cast-iron column is given by the formula

$$P = 6.759 \times 10^6 \frac{d^{3.55}}{l^{1.7}}$$

where d and l are the diameter and length of the cylinder in inches.

What weight will crush a cast-iron column 72 in. high and 4.3 in. in diameter?

60. The weight in pounds W of a cubic foot of saturated steam depends upon the boiler pressure P (pounds per square inch abso-

lute) according to the formula

$$W = \frac{P^{0.941}}{330.36}$$

Find W when P is 265 psig. Assume atmospheric pressure is 15 psi.

14·31 Relation of Logarithms to the Slide Rule. An arrangement of uniform scales similar to those illustrated in Fig. 14·4 could be used for mechanically adding or subtracting numbers.

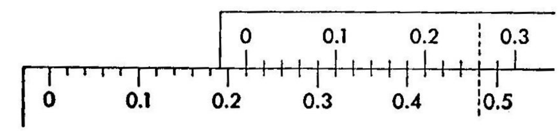


Fig. 14·4

This figure illustrates the addition $0.22 + 0.26 = 0.48$. It also illustrates the subtraction $0.48 - 0.26 = 0.22$.

As an example of multiplication, multiply 1.66 by 1.82. Looking up three-place logarithms, we find 0.220 and 0.260 to be the respective mantissas. We may use the scales to add 0.220 and 0.260, obtaining

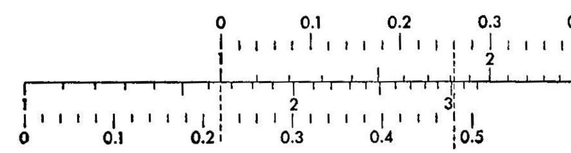


Fig. 14·5

0.480, as in Fig. 14·4. Looking up the antilogarithm of 0.480, we obtain 3.02.

Now, such a slide rule would not be of much help to us. However, if we replace the uniform scales which we have been using to represent mantissas by scales bearing the numbers to which these mantissas

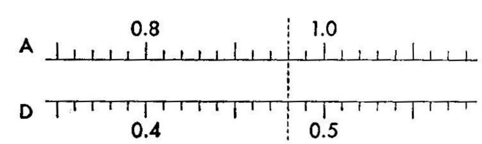


Fig. 14·6

correspond, we will have eliminated the necessity of looking up logarithms and antilogarithms (Fig. 14·5). Thus the scheme for mechanically adding 0.22 and 0.26 becomes a means of multiplying the antilogarithms 1.66 and 1.82 to get 3.02. Evidently, by reversing the procedure, $0.48 - 0.26 = 0.22$ corresponds to $3.02 \div 1.82 = 1.66$.

In Fig. 14·6 it will be observed that opposite any given number n on the D scale we read $2n$ on the A scale.

If we wished to square 3.02, we could look up the (three-place) mantissa = 0.480, double by the arrangement in Fig. 14·6, and look up the antilogarithm of 0.96, obtaining 9.12. Obviously we stand to gain here also by placing the antilogarithms in the place of the corresponding mantissas. Thus the process of mechanically multiplying 0.48

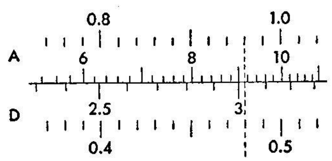


Fig. 14·7

by 2 to get 0.96 becomes the operation of squaring 3.02 to obtain 9.12. Likewise the operation $0.96/2 = 0.48$ is replaced by $\sqrt{9.12} = 3.02$ (Fig. 14·7).

These illustrations will serve to indicate that the slide rule is basically a device to add or subtract quantities (mantissas) mechanically. A knowledge of the laws of logarithms (Sec. 14·5) will be naturally helpful in developing facility in the use of the slide rule.

14·32 Logarithmic Computations on the Slide Rule. It will be noticed that the uniform scale we have been discussing in Sec. 14·31

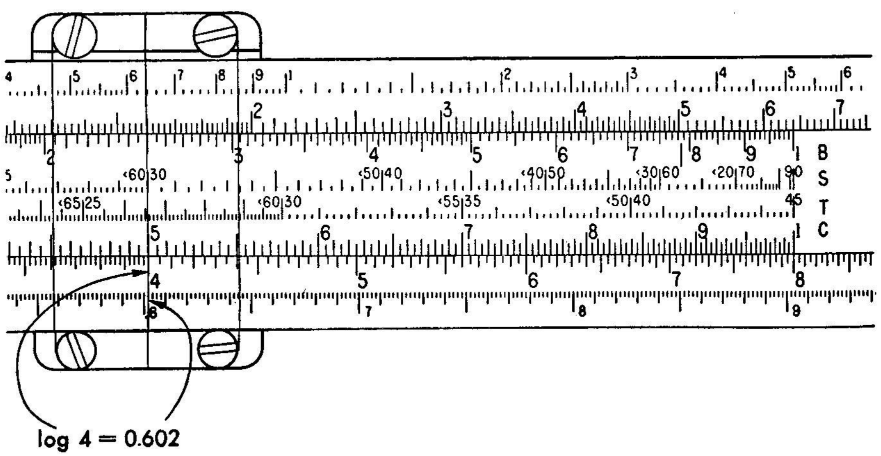


Fig. 14·8

is engraved on the slide rule as the L scale. If we set the hairline over a given number on the D scale, we shall at the same time find under the hairline on the L scale the mantissa of the logarithm of that

number. Thus in Fig. 14·8 we find illustrated log 4 = 0.602; in Fig. 14·9, log 8 = 0.903; in Fig. 14·10, the mantissa of log 16 = 0.204, or log 16 = 1.204.

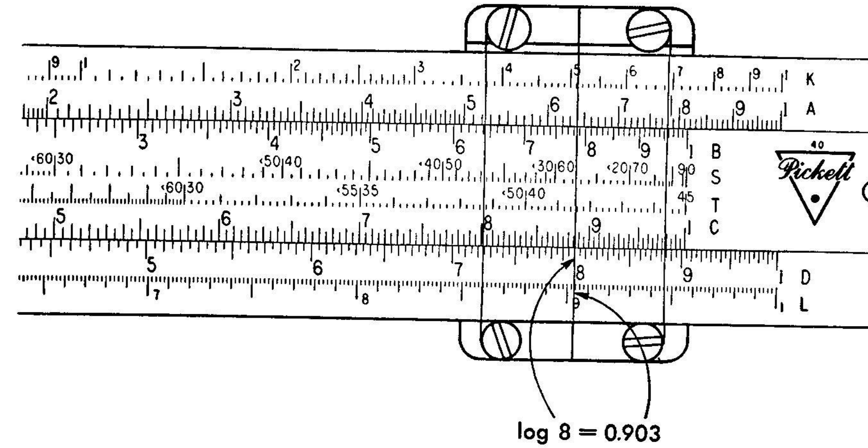


Fig. 14·9

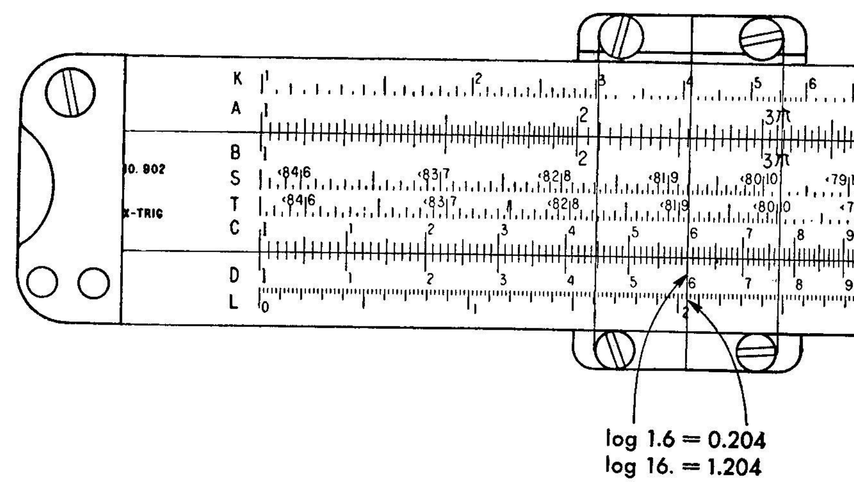


Fig. 14·10

Let us illustrate the use of the L scale in certain logarithmic computations.

EXAMPLE 27. Solve the equation $4^{3/2} = x$. Taking logarithms, we have 1.5 log 4 = log x (Sec. 14·25). In Fig. 14·8 we have shown that log 4 = 0.602. Figure 14·11 shows that $1.5 \times 0.602 = 0.903$. If log x = 0.903, then antilog $0.903 = x = 8$. This step is illustrated in Fig. 14·9.

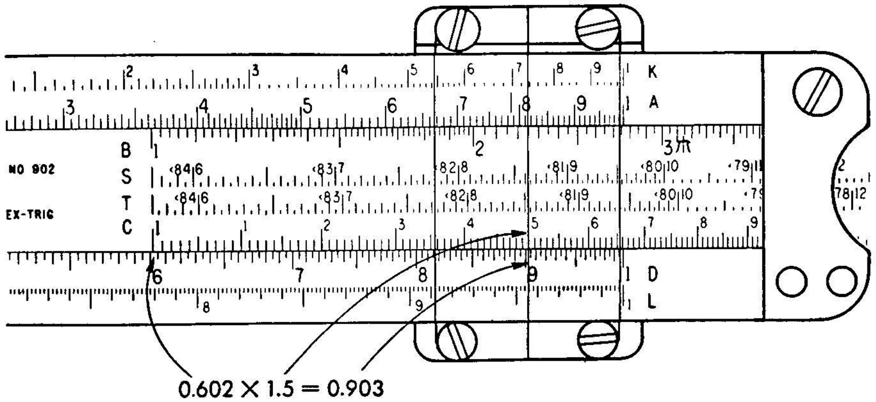


Fig. 14·11

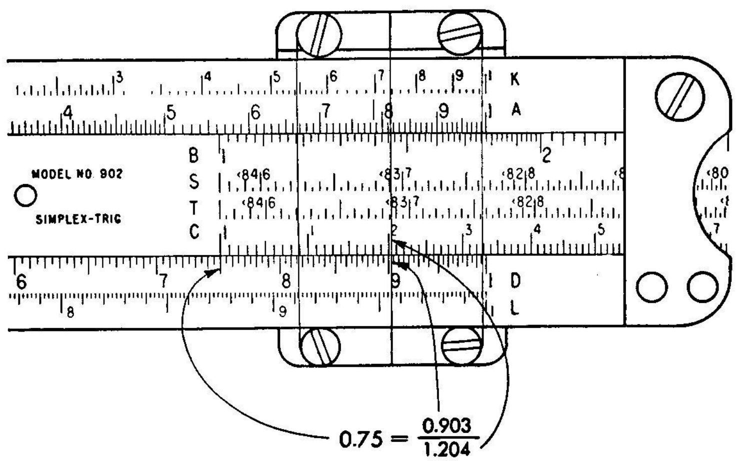


Fig. 14·12

EXAMPLE 28. Solve the equation $16^x = 8$. Taking logs of both sides, we have $x \log 16 = \log 8$. Since the settings in Fig. 14·10 and 14·9 show that $\log 16 = 1.204$ and $\log 8 = 0.903$, we may write $1.204x = 0.903$. Finally Fig. 14·12 illustrates the division $0.903 \div 1.204 = 0.75 = x$.

18·21 The Slide-rule Solution of Right Triangles. Within its inherent limit of accuracy, the slide rule may be used to solve trigonometry problems.

The following table gives a good working idea of the relation between the number of significant digits in the linear dimensions of a triangle and the corresponding precision in angular measure.

Significant digits in linear dimensions	*Precision in angular measure*
2	Nearest degree
3	Nearest 10 minutes
4	Nearest minute
5	Nearest tenth of a minute

On the slide rule, linear dimensions can be used and computed to about one part in a thousand. Angles can be used and computed to about the nearest 0.05°. This is not strictly true over the entire scale, but it is a good working average.

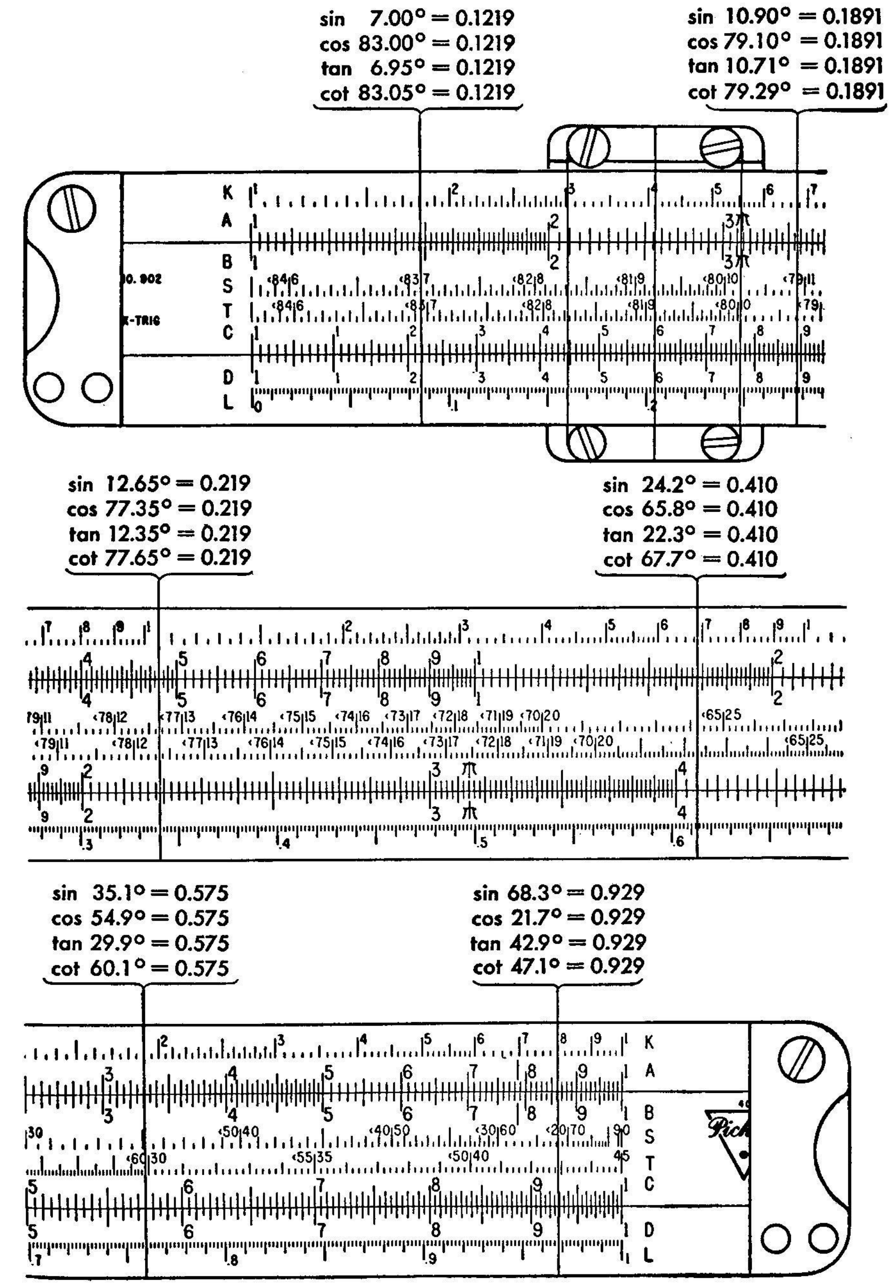


Fig. 18·18

On the particular rule used here for illustration, the S scale (see Fig. 18·18) is calibrated in degrees and decimal parts of a degree. The long markers are double-numbered in pairs of complementary angles, *e.g.*, 82/8, 70/20, 60/30.

When the *right-hand* numerals are used, the hairline simultaneously indicates an angle on the S scale and the *sine* of that angle on the C scale.

When the *left-hand* numerals are used, the hairline simultaneously indicates an angle on the S scale and the cosine of that angle on the C scale.

EXERCISE 16

1. Using a slide rule, find the sine and cosine of the following angles:

a. 28°	**b.** 8°	**c.** 17°	**d.** 13°	**e.** 20°	**f.** 30°
g. 7.2°	**h.** 12.2°	**i.** 26.2°	**j.** 7.63°	**k.** 16.75°	**l.** 41.4°

The slide-rule setting illustrated in Fig. 18·19 is adequate for solving Examples 39 to 42.

EXAMPLE 39. The hypotenuse of a right triangle is 8 in., and one angle is 30°. Find the side opposite the 30° angle. Here the slide rule is set to multiply 8 in. by the sine of 30°, giving 4 in. as the length of the opposite side.

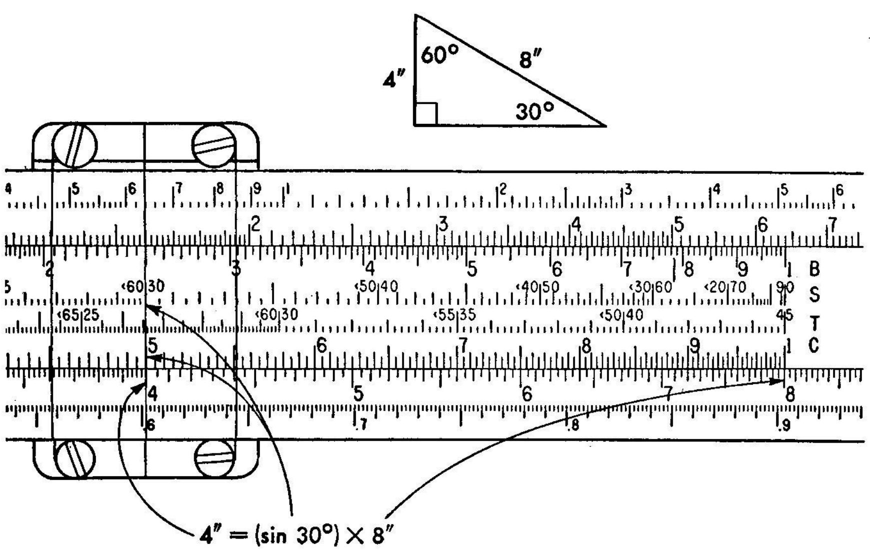


Fig. 18·19

EXAMPLE 40. The hypotenuse of a right triangle is 8 in., and one angle is 60°. Find the side adjacent to the 60° angle. In Fig. 18·19 the slide rule is set to multiply 8 in. by the cosine of 60°.

EXAMPLE 41. One side of a right triangle is 4 in., and the hypotenuse is 8 in. Find the angle opposite the 4-in. side. Instead of dividing 4 by 8 to obtain the sine of the required angle, it is more convenient to answer the question, "Eight times the sine of what angle equals 4?" Therefore the slide rule is set to multiply by 8, and the hairline is set over the product 4. The angle whose sine gives this product is read under the hairline on the S scale. A division of 4 by 8 would give a quotient of 0.5 on the D scale, and a new setting would have to be made to find the angle whose sine is this quotient.

EXAMPLE 42. One side of a right triangle is 4 in., and the hypotenuse is 8. Find the angle adjacent to the 4-in. side. In this problem we answer the question, "Eight times the cosine of what angle yields 4 as the product?" (see Fig. 18·19).

EXERCISE 17

1. Verify the statement that the slide-rule setting shown in Fig. 2·5 is consistent with the following relations:

   a. $\dfrac{2.53}{14.28} = \sin 10.21°$

   b. $\dfrac{2.53}{14.28} = \cos 79.79°$

   c. $14.28 \cos 79.79° = 2.53$

   d. $14.28 \sin 10.21° = 2.53$

2. Verify the statement that the slide-rule setting in Fig. 2·7 is consistent with the following relations:

   a. $\dfrac{8}{9.03} = \sin 62.4°$

   b. $\dfrac{8}{9.03} = \cos 27.6°$

   c. $9.03 \sin 62.4° = 8$

   d. $9.03 \cos 27.6° = 8$

3. Write a series of four equations similar to those given in Probs. 1 and 2 above, but consistent with Figs. 2·9, 2·11, and 2·13.
4. Using a slide rule, solve Probs. 1 to 10 and 31 to 40, page 390.

The T scale is calibrated in degrees and decimal parts of a degree. The long markers are double-numbered in pairs of complementary angles in a way similar to the S scale.

When the *right-hand* numerals are used, the hairline simultaneously indicates an angle on the T scale and the *tangent* of that angle on the C scale. When the *left-hand* numerals are used, the hairline simultaneously indicates an angle on the T scale and the *cotangent* of that angle on the C scale.

The C scale is used for sines, cosines, tangents, and cotangents. The left-hand index of the C scale is used as 0.1, and the right-hand index is used as 1.0. Therefore on this particular rule we are limited to angles whose sine is between 0.1 and 1.0, angles whose cosine is between 0.1 and 1.0, and angles whose tangent or cotangent is between 0.1 and 1.0. The smallest angle we can use directly is about 5.7°. When using sines, we can process angles up to 90°, although the scale is crowded near 90°. When using cosines, we can use angles up to about 84.3°. The upper limit of the tangent scale and the lower limit of the cotangent scale is 45°. However, as will be illustrated in subsequent examples, this does not put any additional limitations on the usefulness of the slide rule.

If it should be necessary to find the tangent of an angle greater than 45°, the reciprocal of its cotangent may be found.

EXERCISE 18

1. Using a slide rule, find

a. tan 20°	**b.** tan 35.76°	**c.** tan 44.2°
d. tan 7.9°	**e.** tan 15.82°	**f.** tan 29.62°
g. cot 48.2°	**h.** cot 82.1°	**i.** cot 52.7°

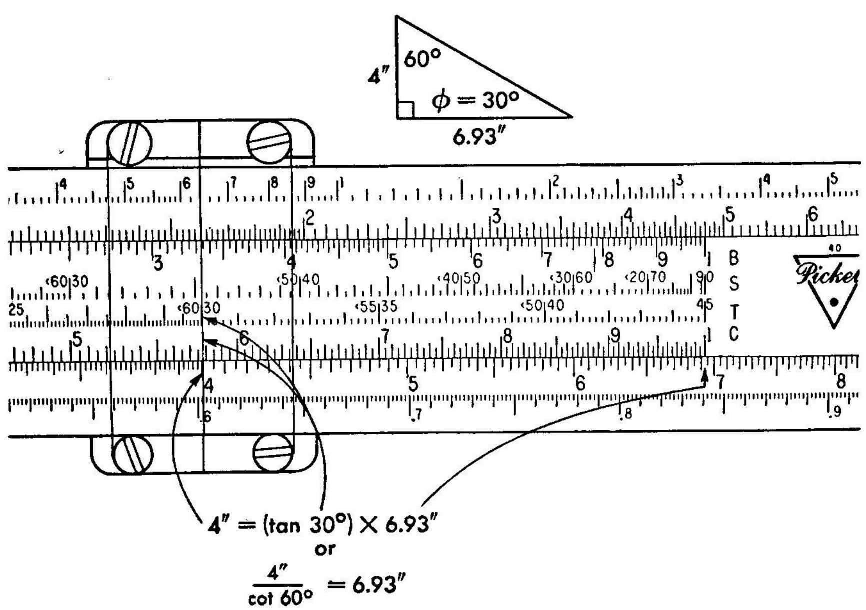


Fig. 18·20

The slide-rule setting illustrated in Fig. 18·20 is adequate for solving the following examples:

EXAMPLE 43. One side of a right triangle is 6.93 in., and the adjacent angle is 30°. Find the side opposite the 30° angle.

Use the equation

$$\text{Opposite side} = \text{adjacent side} \times \text{tangent } \phi$$

The slide-rule setting in Fig. 18·20 accomplishes this multiplication.

EXAMPLE 44. One side of a right triangle is 4 in., and the adjacent angle is 60°. Find the side opposite the 60° angle.

Use the equation

$$\text{Opposite side} = \frac{\text{adjacent side}}{\cot \phi}$$

The slide-rule setting shown in Fig. 18·20 accomplishes this division. Thus we avoid using tangents of angles greater than 45°.

EXAMPLE 45. One side of a right triangle is 6.93 in., and the other side is 4 in. Find the angle opposite the 4-in. side. Again refer to Fig. 18·20. If we divided 4 in. by 6.93 in. directly, the quotient would appear on the C scale and the angle could not be read directly. Instead we find the angle whose tangent multiplied by 6.93 in. equals 4 in. This angle is read directly on the T scale.

EXAMPLE 46. One side of a right triangle is 6.93 in., and the other side is 4 in. Find the angle opposite the 6.93-in. side. Here we find the angle whose cotangent divided into 4 in. yields 6.93 in. as a quotient (see Fig. 18·20).

Observe that in the preceding examples, two linear dimensions appear on the D scale. The longest linear dimension appears under a C index and the shorter appears under the hairline. The angle opposite the shorter side is under the hairline on the S scale if the hypotenuse is involved and on the T scale if the hypotenuse is *not* involved.

EXERCISE 19

1. Verify the statement that the slide-rule setting shown in Fig. 2·5 is consistent with the following relations:

a. $\dfrac{2.53}{14.28} = \tan 10.05°$

b. $\dfrac{2.53}{14.28} = \cot 79.95°$

c. $2.53 = 14.28 \tan 10.05°$

d. $2.53 = 14.28 \cot 79.95°$

2. Verify the statement that the slide-rule setting shown in Fig. 2·7 is consistent with the following relations:

a. $\dfrac{8}{9.03} = \tan 41.55°$

b. $\dfrac{8}{9.03} = \cot 48.45°$

c. $8 = 9.03 \tan 41.55°$

d. $8 = 9.03 \cot 48.45°$

3. Write a series of four equations similar to those given in Probs. 1 and 2 above but consistent with Figs. 2·9, 2·11, and 2·13.

4. Using a slide rule, solve Probs. 11 to 30, page 390.

Section 2·12 describes a method of solving Pythagorean theorem problems on the slide rule. The method illustrated in the following example is somewhat more convenient for those with a knowledge of trigonometry.

EXAMPLE 47. The hypotenuse of a right triangle is 8 in., and one side is 4 in. Find the other side. First find the smaller angle, as in Fig. 18·19 and Example 41. Then, having found the smaller angle (in this case 30°), divide the smaller side by the tangent of the smaller angle, as in Fig. 18·20.

EXAMPLE 48. The two sides of a right triangle are 4 and 6.93 in., respectively. Find the hypotenuse. First find the smaller angle, as in Example 45 and Fig. 18·20. Then, having found the smaller angle, divide the sine of this angle into the shorter side to find the hypotenuse, as in Fig. 18·19.

EXERCISE 20

1. Using a slide rule, solve Probs. 21 to 30 in Exercise 11, page 390.

2. Using a slide rule, solve Probs. 31 to 40 in Exercise 11, page 391.

18·22 The Slide-rule Solution of Oblique Triangles. The slide rule can be used to solve oblique triangles, using methods similar to those described in Secs. 18·18 to 18·20.

EXAMPLE 49. Given the triangle shown in Fig. 18·21, where $b = 8$ in., $A = 30°$, and $B = 34°$, find a, c, and C.

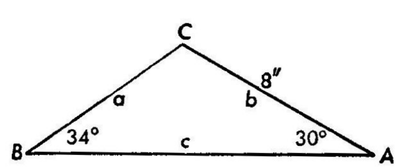


Fig. 18·21

1. Set the rule to indicate h, as in Fig. 18·22.

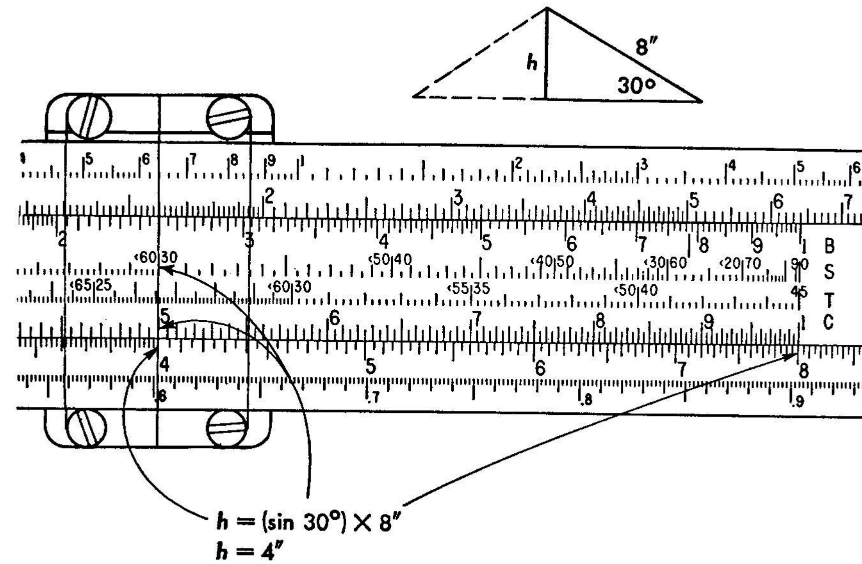
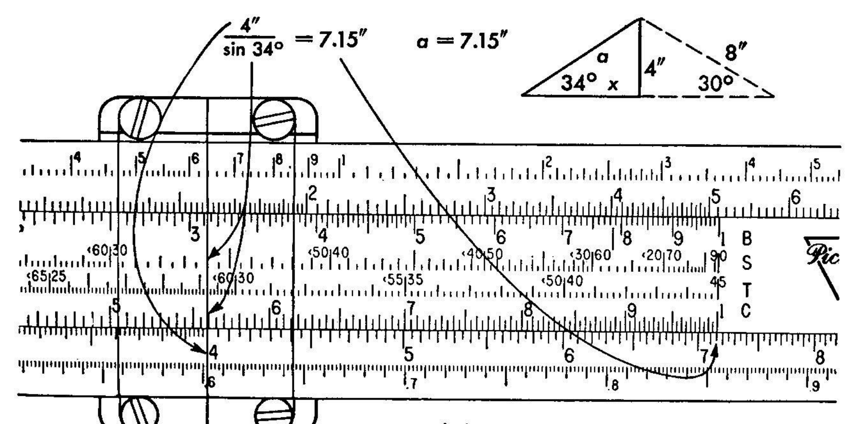


Fig. 18·22

2. Set the rule to indicate a, as in Fig. 18·23a.



(a)

Fig. 18·23a

3. Set the rule to indicate x, as in Fig. 18·23*b*.

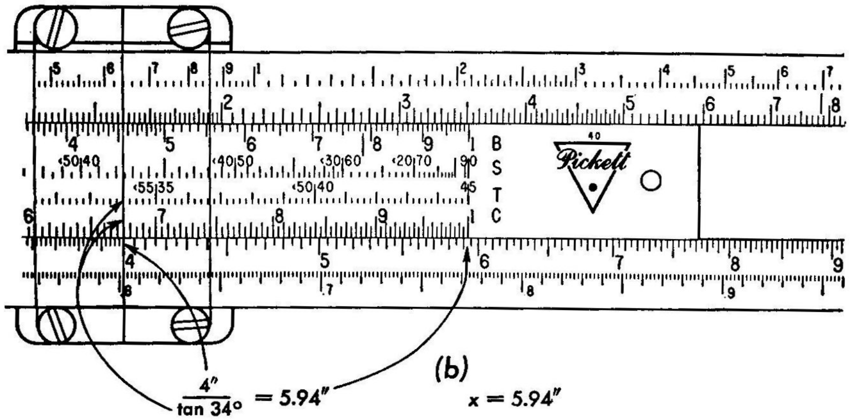


Fig. 18·23*b*

4. Set the rule to indicate y, as in Fig. 18·24.

$$\text{Side } c = x + y, \text{ angle } C = 180° - (A + B)$$

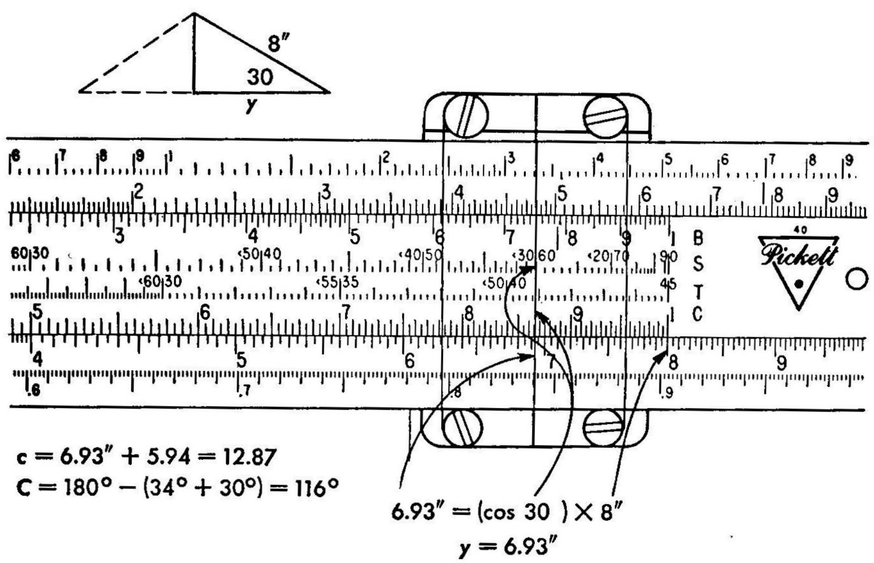


Fig. 18·24

EXAMPLE 50. Given the triangle shown in Fig. 18·25, if b = 8 in., c = 12.87 in., and A = 30°, find a, B, and C.

1. Find h and y, as in Figs. 18·22 and 18·24, respectively.
2. Since x = 12.87 − y, the length x may be found by subtraction.

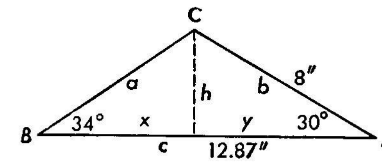


Fig. 18·25

3. Knowing h and x, find B, as in Fig. 18·23*b*.
4. Knowing h and B, find a, as in Fig. 18·23*a*.

EXERCISE 21

Solve Probs. 1 to 11 in Exercise 15 on a slide rule.

18·23 The Solution of Right Triangles Involving Small Angles. The slide rule illustrated in this book does not indicate angles less than about 5.7°. However, with one exception the slide rule can be used to solve triangles involving smaller angles by using special methods described below.

The exception occurs when the hypotenuse and long side are known and are nearly equal. In this case, the method required for solution is inherently quite inaccurate. It is inherently inaccurate regardless of whether a table of trigonometric functions or the slide rule is used. However, since tables are in general more precise than slide rules, the inherent inaccuracies in the method of solution are not so noticeable when using tables as when using a slide rule.

EXAMPLE 51. Find the short side of a right triangle in which the hypotenuse is 134.4 in. and the long side is 133.6 in.

Let the hypotenuse be c, the long side be b, and the short side be a.

$$a = \sqrt{c^2 - b^2} = \sqrt{(c - b)(c + b)} \qquad (41)$$

$$c - b = 134.4 - 133.6 = 0.8 \text{ in.}$$

Notice that the difference (0.8 in.) is expressed to only one significant figure, whereas the c and b are given to four significant figures.

$$c + b = 134.4 + 133.6 = 268.0 \text{ in.}$$

$$a = \sqrt{0.8 \times 268} = \sqrt{214.4} = 14.64 \text{ in.}$$

If the length of the side b is changed by 0.1 in. to make b = 133.7, then

$$c = \sqrt{(134.4 - 133.7)(134.4 + 133.7)} = 13.70 \text{ in.}$$

Thus a change of 0.1 in. in b makes a change of 0.94 in. in a. Consequently the slide rule would hardly be an appropriate instrument for this type of problem.

The examples given below illustrate problems in which the use of the slide rule is appropriate.

EXAMPLE 52. Solve the right triangle shown in Fig. 18·26.

The side a in Fig. 18·26 is nearly equal to the arc length BX in Fig. 18·27. Of course this approximation will be close only when A is small. The arc

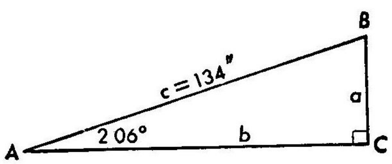


Fig. 18·26

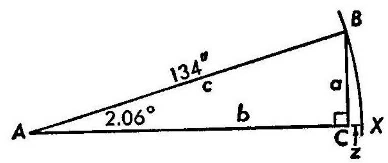


Fig. 18·27

length BX is a fraction of the circumference of a circle of radius 134 in. whose center is at A.

This fraction is 2.06°/360°.

In other words,

$$\frac{2.06°}{360°} \times 2\pi \times 134 = \text{arc length } BX = \frac{\pi}{180°} \times 2.06° \times 134 \qquad (42)$$

Or the side a is given by the approximate formula

$$a \approx \frac{2\pi}{360°} \times 2.06° \times 134 = \frac{\pi}{180°} \times 2.06° \times 134 \qquad (43)$$

$$a \approx 0.0359 \times 134 = 4.82 \text{ in.}$$

See Fig. 18·28 for the appropriate settings to solve Eq. (43).

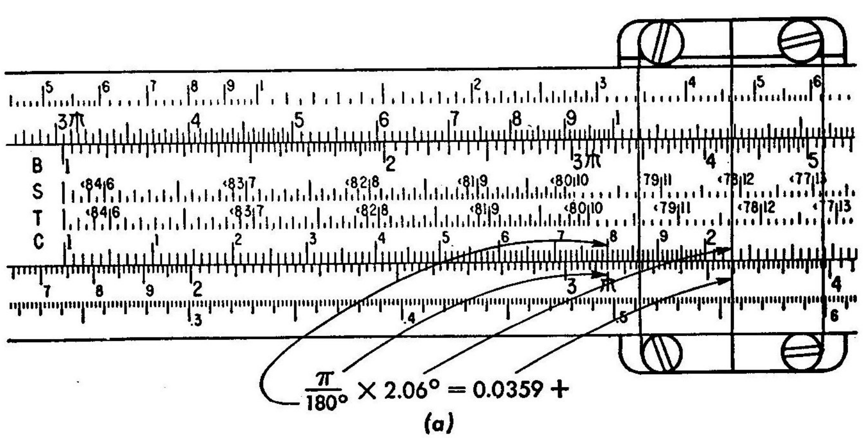


Fig. 18·28a

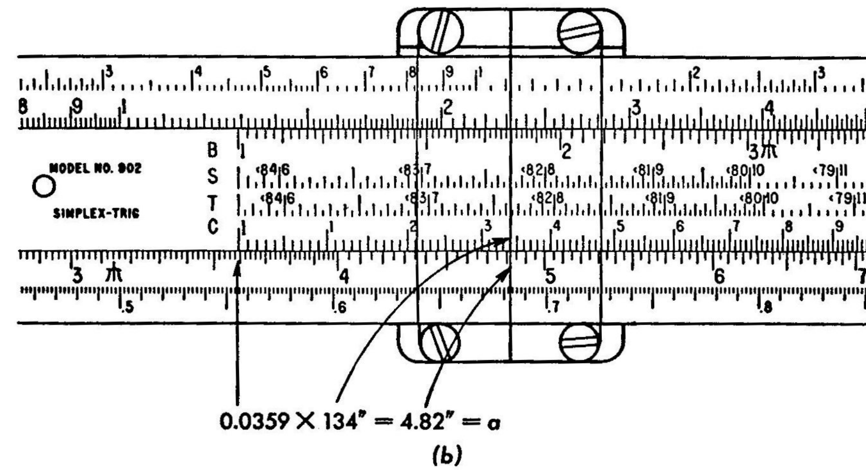


Fig. 18·28b

The general formula for a is

$$a \approx \frac{\pi}{180} \times A \times c \qquad (44)$$

To find b we first find the length z (Fig. 18·27). In Fig. 18·27,

$$c = b + z \qquad (45)$$

and

$$c - b = z \qquad (46)$$

since both c and $b + z$ are radii of the same circle. But, by the Pythagorean theorem,

$$a^2 = c^2 - b^2 = (c - b)(c + b) \qquad (47)$$

$$a^2 = z(c + b) \qquad (48)$$

or

$$z = \frac{a^2}{c + b} \qquad (49)$$

If c and b are nearly equal, we may unite

$$z \approx \frac{a^2}{2c} \quad \text{(See Table A·2, page 589)} \qquad (50)$$

In the present example

$$a = 4.82 \text{ in.}$$
$$c = 134 \text{ in.}$$

Therefore

$$z \approx \frac{4.82^2}{2 \times 134} = 0.0867 \text{ in.} \qquad (51)$$

and

$$b \approx 134.0 \text{ in.} - 0.0867 \text{ in.} \tag{52}$$

or, within slide-rule precision,

$$b \approx 134.0 \text{ in.} - 0.1 \text{ in.} = 133.9 \text{ in.}$$

The slide-rule settings used to solve Eq. (51) are illustrated in Fig. 18·29.

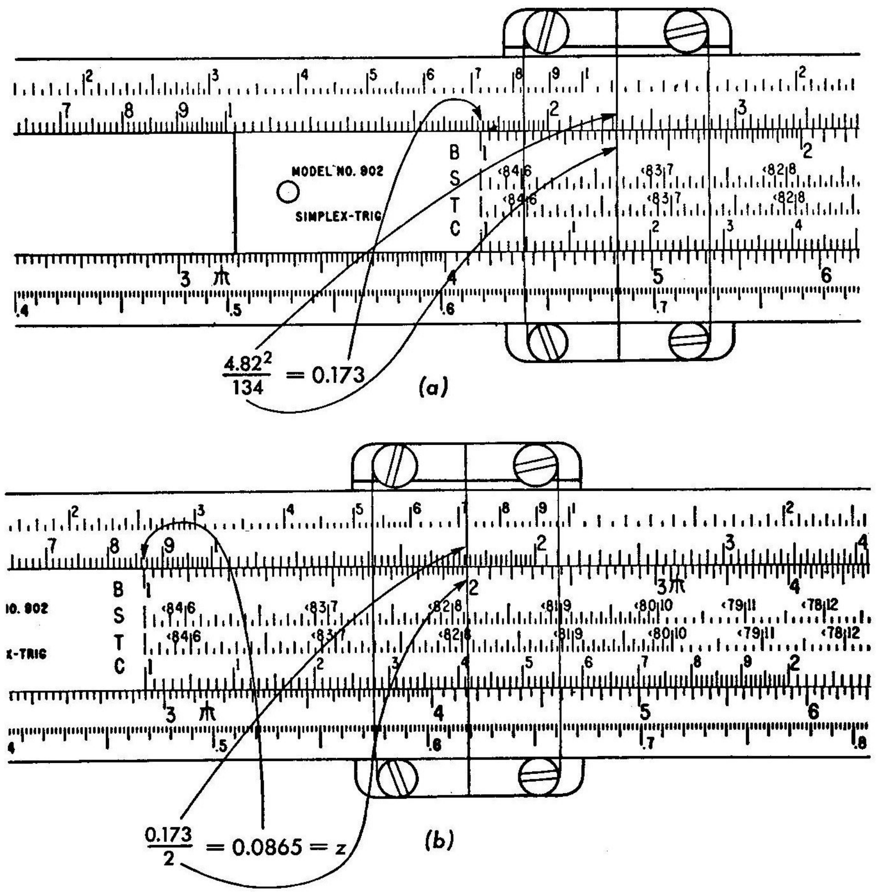


Fig. 18·29

EXAMPLE 53. Solve the right triangle shown in Fig. 18·30.

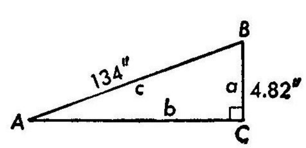


Fig. 18·30

Following Eq. (44),

$$\frac{a}{c} \approx \frac{\pi}{180} \times A \tag{53}$$

The ratio a to c is set on the slide rule shown in Fig. 18·28b. To this ratio we match the ratio $\frac{\pi}{180}$ multiplied by the value of A (as yet unknown) (see Fig. 18·28a). This requires that A be 2.06°.

The side b is calculated as in Example 52.

EXAMPLE 54. Solve the triangle shown in Fig. 18·31.

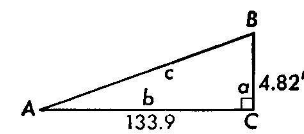


Fig. 18·31

Since b and c are nearly equal, Eq. (44) may be altered to read

$$\frac{a}{b} \approx \frac{\pi}{180°} \times A \tag{54}$$

To solve for c, we alter Eq. (50) as follows:

$$z \approx \frac{a^2}{2b} \tag{55}$$

Therefore

$$c \approx b + \frac{a^2}{2b} \quad \text{(See Table A·2, page 589)} \tag{56}$$