REINFORCED CONCRETE

THEORY AND PRACTICE

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PREFACE

Much has been written upon the subject of reinforced concrete, and the design of structures in this material no doubt still affords opportunity for invention and improvement. New systems, new bars, new details of various kinds are constantly being patented in many countries, but the leading features and ideas remain the same. Generally speaking, one may say that there are as many systems as there are specialists, each naturally insisting upon the superiority of his own favourite ideas.

The Author had occasion to see reinforced concrete constructions designed and executed for many years, and has closely followed its development. His principal object in writing this book was not to put forward any particular method of construction, but to collect in a concise form what seemed to him best of the many formulæ and systems used in various countries, and to deal with the subject in such a manner as to be intelligible to average students of architecture who have not been required to devote that amount of study to the theory of construction which is demanded of the young engineer. At the same time, it is hoped that the present volume may be useful also to the latter.

As no mere series of unexplained formulæ can give any useful idea of the subject to a beginner, and as, as has been indicated, the intention is to treat the subject in an elementary manner, an effort has been made to afford brief explanations of the calculations given and to further elucidate them by numerical examples. Thus it is hoped the reader will be enabled to acquire a methodical knowledge of the principles upon the application of which all the varied systems alike depend.

No doubt the design and execution of reinforced concrete work will always remain to a great extent in the hands of specialists, but the average architect or engineer should have sufficient knowledge of the subject to himself decide where this form of construction can be most usefully employed and what kind of reinforcement is most suitable to the particular case in hand. Each patent bar and system has its advantages, and after a careful study of the principles set forth in the following pages it should be possible for the designer to himself decide which is the most suitable for use in any special case, and to hand over to the specialist only the task of properly working out the details upon general lines already laid down. Thus will be avoided the risks inherent in having to leave the whole design in the hands of one whose financial interests may incline him to use methods not quite the best for the special work under consideration.

The formulæ are based on the assumption that ordinary round bars, such as are obtainable everywhere from stock, are used. Some tables and extracts are reproduced from the R. I. B. A. Report on Reinforced Concrete, by kind permission of the Institute. The history of reinforced concrete is partly compiled from the data given in Tozer's *Handbook on the Lock Woven Mesh System*, and facts relating to the manufacture and qualities of Portland cement and its use are chiefly from *Everyday Uses of Portland Cement*, published by the Associated Portland Cement Manufacturers (1900) Ltd. The author is indebted to the various specialists mentioned for the loan of interesting photographs, etc., dealing with work executed in reinforced concrete.

The Figs. marked [1] are reproduced from Kersten's *Der Eisenbetonbau*, except where otherwise stated.

It is hoped that the tables at the end of the book, together with the Ready Reckoner, will be a help to designers and others for reference, calculation, and the checking of designs.

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LIST OF SYMBOLS

BASED ON THE STANDARD NOTATION SUGGESTED BY THE SCIENCE STANDING COMMITTEE OF THE CONCRETE INSTITUTE.

a	Area of the couple formed by compressive and tensile forces in a beam.
a_c	Area of compressive force measured from neutral axis in ribbed slabs.
a_t	Area of tensile reinforcement measured from neutral axis.
b	Breadth generally in inches.
b_r	Breadth of rib in a tee-beam in inches.
b_s	Effective breadth of slab in tee-beam in inches.
c	Compressive stress intensity on concrete.
c_s	Compressive stress intensity on steel.
c_x, c_y	Stresses in concrete of columns eccentrically loaded.
d	Depth generally in rectangular sections.
d	Effective depth of beam or slab from top to axis of tensile reinforcement in inches.
d	Diameter in circular sections in inches.
d_c	Depth or distance of centre of compressive reinforcement from compressed edge of beams in inches.
d_c	Diameter of core of pillars in inches.
d_c	Depth of arch ring at crown of arch in inches.
d_d	Distance of bottom of reinforcement of rib from centre of gravity of reinforcement in inches.
d_h	Diameter of a helical reinforcing rod in any compression piece in inches.

d_l Diameter of a longitudinal reinforcing rod of a pillar in inches.

d_n Deflection of a beam in inches.

d_r Distance of rods centre to centre in inches.

d_s Total depth of slab in tee-beam in inches.

d_t Total depth in inches.

e Eccentricity of load in inches.

e Distance of centre of rod from axis of column in inches.

f Friction or adhesion of concrete and steel.

h Height generally in inches.

i Inset of centre of reinforcement from bottom of slab or rib in inches.

i Inset of rod centres from outer edge of column section in inches.

i Inset of centre of gravity of column section from outer edge in inches.

i Distance of eccentric load from outer edge of column section in inches. $i = d - e$ (diameter − eccentricity).

l Length generally in inches.

l Effective length or span of beam or arch.

m Modular ratio, *i.e.* the ratio between the elastic moduli of steel and concrete $= \frac{E_s}{E_c}$.

n Distance of neutral axis from compressed edge in inches.

p Intensity of pressure per unit of length or area.

r Radius in inches.

s Shearing stress intensity.

s_h Spacing of hoops round columns in inches.

$s_r = \frac{t}{c}$ Stress ratio in ribbed slabs.

t Tensile stress intensity on steel.

t_c Tensile stress intensity on concrete.

$\left.\begin{array}{l} t_x \\ t_y \end{array}\right\}$ Stresses in steel in columns eccentrically loaded.

v Versine or camber of a curve or rise of an arch in inches.

w	Weight or load generally, per unit of length or area.
w	Superimposed load uniformly distributed on arch.
w_d	Dead load above arch ring at crown.
$\left.\begin{matrix} x \\ y \end{matrix}\right\}$	Co-ordinates in arch calculations in inches.
x	Distance of hangers or bending up of rods from support in inches.
y	Height of shear triangle.
β	Distance of compressive force from neutral axis in ribbed slabs in inches.
$\gamma = \frac{t}{c}$	In ribbed slabs.
π	Ratio of circumference of a circle to its diameter.
O	Perimeter of steel rods in inches.

A	Total cross-sectional area of beam or pillar in inches.
A_C	Area of compressive reinforcements of beams in inches.
A_L	Cross-sectional area of longitudinal steel rods of pillar in inches.
A_r	Sectional area of one rod in ins.^{2}
A_S	Area of shear reinforcement in ins.^{2}
A_T	Area of tensile reinforcement in beams in ins.^{2}
B	Bending moment generally.
B	Maximum bending moment of the external forces or loads on a beam.
B	Bending moment at crown of arch.
B_C	Bending moment at centre of beam.
B_E	Bending moment at end of beam.
B_L	Bending moment left half of arch.
B_R	Bending moment right half of arch.
C	Total compressive force or stress.
C_C	Total compression on concrete.
C_S	Total compression on steel.
E_C	Elastic modulus of concrete in compression in lbs./in.^{2}

E_S Elastic modulus of steel in lbs./in.^{2}

G Centre of gravity of column section.

I_C Moment of inertia for concrete.

I_S Moment of inertia for steel.

N_d Number of divisions in one half of arch.

N_r Number of rods.

P_H Horizontal pressure.

P_V Vertical pressure.

R Moment of resistance of internal stresses in a beam at a given cross-section.

R_L Left reaction.

R_R Right reaction.

S Total shearing force across a section.

S_C Shear at crown of arch.

S_C Total shear taken up by concrete.

S_S Total shear taken up by steel.

S_F Safety factor.

T Total tensile force.

T_C Thrust at crown of arch.

W Weight or load.

CONTENTS

REINFORCED CONCRETE

THEORY AND PRACTICE

CHAPTER I

INTRODUCTORY

Reinforced concrete, although considered a modern building construction, is really very old in principle, and it has been proved that the Romans, many years before Christ, used it,—naturally only in a very crude form, but evidently fully understanding the principle of the combination of metal and concrete. There are examples of Roman reinforced concrete in many parts, the reinforcement consisting as a rule of bronze rods placed crossing each other in the centre of the slab. The concrete consisted of lime with occasionally other additions of hydraulic materials and aggregate, which latter was, as a rule, rather coarse. The Roman system of strengthening concrete with tiles is well known, and there are still many samples of their work in existence. The reinforced concrete of old times cannot, of course, be compared at all with our modern concrete as regards properties of strength and resistance, as the manufacture of Portland cement was not then known. In the Middle Ages concrete of lime mortar and stones was also used to a certain extent, but it was not before about the middle of the nineteenth century that the idea was more fully explored. About this time we trace various patents relating to the construction, like Louis Leconte's patent protecting the use of iron plate trusses for floors. He suspended iron rods from these plates, the rods carrying a meshwork of wire, which in its turn supported the ceiling plaster. Other patents of this period are the Vaux and Thuasné systems. Vaux used round rods, hooked on flat iron bars placed edgewise in the concrete slabs. Thuasné's system consisted of small iron joists having hangers placed over them, with round iron bars suspended through a hole in the hanger. In these systems plaster of Paris was used. This material does, however, not protect the iron from rusting, and consequently the constructions were not lasting.

In these specimens of reinforcement no attention was paid to what is now the leading principle of reinforced concrete constructions, namely, to use the iron reinforcement to resist the tensile stresses while the concrete resists the compressive stresses.

No substantial improvement can be recorded before the invention of Portland cement. This was discovered in 1824 by Joseph Aspdin of Leeds, and improved by William Aspdin, who took out a patent relating to the manufacture of Portland cement in 1852. Wilkinson in 1854 used a layer of wet sand on the surface of fresh concrete, keeping the sand wet in order to get the concrete as hard as possible. The same inventor also took out a patent for hollow partition blocks and for fireproof floors. These latter he reinforced with flat iron bars placed on edge, and he described these bars as taking the tensile stresses, thus coming nearer to our modern ideas of reinforced concrete.

François Coignet of Paris invented about the same period his "Béton-Coignet," a concrete composed of hydraulic lime and aggregates mixed mechanically in certain proportions. In constructing slabs he put rods crosswise, similar to the Monier system. A good specimen of his work is the aqueduct of the River Vanne, which still exists at the present day.

In 1857 Dennett, a Nottingham contractor, introduced concrete arch floors between ⊥ iron joists.

In 1867 Scott took out a patent for a fireproof floor consisting of a lacework of rods, hoop irons or wire embedded in the concrete, and he states in his specification that the concrete takes the compression while the ironwork resists the tension in the slab.

FIG. 1.—AQUEDUCT

This remarkable Aqueduct for the Paris Water Supply was executed by the late Frar
has a span of

(*Reproduced from Coignet's Handbook.*)

HE RIVER VANNE.

Coignet in moulded concrete. The principal arch shown in the above photograph
it 132 feet.

[*To come between pages 2 and 3.*]

The introduction of reinforced concrete is usually attributed to Monier, who patented in France in 1867 a method for making large tubs for shrubs, using a meshwork of wires and rods embedded in concrete. Later on he took out further protection for other applications of his idea, and, on exhibiting his inventions at the Antwerp Exhibition, 1879, he came in touch with Wayss of Berlin, a civil engineer, who took Monier's patents up and worked them extensively. Wayss and his partner Koenen are responsible for the first method of calculating the strength of reinforced concrete floors. In these calculations they assumed the neutral axis to lie half-way up the beam and that the steel rods are equivalent to the bottom flange of an ordinary steel girder, while the concrete was considered to take the place of the top flange.

Lascelles in 1877 erected a number of cottages, the walls of which consisted of concrete slabs reinforced with iron rods placed diagonally.

The first reinforced concrete building in America was built by Ward of New York in 1875, the whole of the walls, floors and roof being composed of concrete reinforced with metal rods.

Further important inventions are the patents of Golding (1884) for expanded metal, Ransome (1884) for a twisted bar, and Lindsay's patent (1885) for reinforced concrete floors consisting of passing rods over and under the iron joists to form a continuous truss.

In 1894 Edmond Coignet published a booklet setting forth a theory of the distribution of stresses based on the different moduli of elasticity of iron and concrete, thus establishing the modern theory of calculating the stresses of reinforced concrete.

A further important advance was made by Wayss and Koenen of Berlin in 1892, who patented a reinforced concrete floor having the rods cranked up at the point of contraflexure.

About the same time Hennebique patented a construction of reinforced beams having stirrups to resist shear, and later, in 1897, the same inventor introduced the system of rods cranked up placed one above the other to reduce the width of the beam.

Further important patents were taken out in quick succession in various countries—like the Ast patent largely in use on the Continent and many others; and the introduction of various patent bars, mention of which will be made later, rapidly put the important subject of reinforced concrete on strong bases, and the engineering and architectural professions of almost every civilized country were induced to look upon reinforced concrete as what it really means, *viz.*, an ideal building construction tending to sound stability and, if properly designed, considerable economy as compared with solid brick and iron buildings, the most important feature being its fireproof properties.

It naturally became necessary for the building authorities in the various countries to safeguard the public against improper usage of the new method of building, and the German Government passed some very stringent building laws dealing with the calculating of stresses and the execution of the work, mention of which will be made in due course.

The Royal Institute of British Architects, recognising the great importance of the subject, appointed a committee who in 1907 issued a report laying down various recommendations and suggestions for the calculation of stresses, to which reference is made hereafter.

The leading idea of the construction is to use the concrete, the tensile resistance of which is considerably less than its compressive resistance, to take the compressive stresses of the combined material while the steel work resists the tensile and shearing stresses. Consequently round or square rods are placed in the concrete in such positions and in such dimensions as is necessary to resist the tensile and shearing stresses at the various points of stress, while the concrete is left to take the compression.

The three principal qualities of the two materials making it possible to gain the particular result are:—

1. The adhesion of the concrete to the steel is considerable (100 lbs. per square inch: see later).

2. The coefficient of expansion of concrete has been shown to be practically the same as that of steel.

3. The protection of the steel is such, that the formation of rust is quite impossible.

ADVANTAGES OF REINFORCED CONCRETE.

Reinforced concrete has been used so frequently and for so many purposes that practical conclusions can be arrived at, and it is now universally granted that the construction possesses many advantages over the method of building as used heretofore. There is hardly a branch of construction where reinforced concrete has not been used to decided advantage.

The principal recommendation is the fact that it is highly fire-resisting.

The vast expansion of our big cities, the huge factories, where hundreds of people work in close proximity, the massing of people in theatres, schools, churches, and public buildings, make it imperative to study the prevention and spreading of fire and to use every possible means to this end in designing a building. Steel in itself, as used for stanchions, columns and girders, does not guarantee a protection at all; in fact, the contrary effect is more likely to happen, as the destruction by fire of a beam does not only involve the collapse of a floor or other superincumbent load, but very often the demolition of the walls as well. The heated steel loses its power of resistance and bends and fails altogether, bringing down everything with it. Various big fires have repeatedly shown this, where heavy girders were bent to all sorts of fantastic shapes. The failing naturally makes the extinction of the fire and the salvage almost impossible. It is absolutely necessary to consider the fire danger, even if everything in a room or building is carried by steel constructions. The only remedy is reinforced concrete, as the protection afforded by the concrete does away with the danger of the steel failing, and even if the whole building is burnt out, the carrying frame remains unhurt and rebuilding can start at once, be carried on at a greater speed, and the cost of rebuilding is reduced to the reconstruction of the fittings and decorations. The danger of collapse during a fire is almost entirely removed and thus salvage operations made possible.

It is consequently necessary to protect all steel stanchions and girders with a fire-resisting material, and cement concrete has for some considerable time past been used for this purpose. In ordinary steel constructions, however, this is rather costly, as the concrete mantel does not take any stresses, and, therefore, does not make it possible to reduce the thicknesses and weights of the protected stanchions or girders. In fact, the material used is simply superfluous and only of use in case of a fire which may never occur. Reinforced concrete, on the other hand, does away with all heavy steel work and the concrete is made to do part of the duty of the member protected, thus effecting a considerable saving in cost, while at the same time affording full protection against fire.

The concrete does not crack nor split under the influence of fire, nor when water is thrown on while heated, thus effectively protecting the embedded steel from all dangerous influences.

It must be admitted that when exposed to great heat the concrete loses somewhat of its strength. The hardening of the material took place under the influence of water, and it is obvious that, if this is lost under fire, the concrete must become a little less compact and perfect, but this shortcoming is easily outbalanced by the advantage of keeping the whole structure intact, and as the influence of the heat can only be destructive to a very little depth, the various parts are easily repaired at small cost.

Furthermore, it has been repeatedly proved that the fire does not affect the complete adhesion of the concrete to the steel, so that, as far as the strength of the structure is concerned, little need be feared in consequence of a fire.

Objections have been raised repeatedly that the moisture contained in the concrete during construction would cause the steel work to rust. But this supposition has been proved wrong over and over again. The famous French architect, Viollet le Duc, removed some iron clamps that had been built into the stonework

(*Reproduced from Expanded Metal Co. Handbook.*)

FIG. 2.—THE "KOHL" BUILDING, SAN FRANCISCO.

(View taken immediately after the Great Earthquake and Fire in San Francisco, California, April, 1906.)

[*To face page* 6.]

of the church of Notre Dame at Paris, and they were found to be as bright as when they were put in some 500 years ago. Some reinforced concrete mortar pipes ($1\frac{3}{8}$ in. thick) were constructed in Grenoble twenty-two years ago. After fifteen years two lengths of pipe were raised for inspection, and it was found that, although the water had been flowing through them and they had been embedded in soil for these fifteen years with only $\frac{3}{8}$ in. of Portland cement concrete protecting the steel, the metal was as bright as on the day it had been put in. Many other instances could be mentioned, and we might take it for granted that experience has shown how perfect is the protection afforded by the concrete.

The mixing of the concrete should be as perfect as possible with a sufficiency but not superabundance of water, as the latter has a weakening effect on the strength of the concrete. The proportion should be 1 part of water to 3 or 4 parts of solids; in no case less.

It is very important that the reinforcements should be fully protected against rust. Painting with oil would seriously interfere with the adhesion and must, therefore, not be employed. Many experts recommend painting the steel rods first with a thin mixture of cement and water, and this course is doubtless highly satisfactory. There is no necessity to free the rods from any rust as this is not detrimental at the initial stage, on the contrary, it may improve the adhesion. The point is to prevent the formation of rust after the rods are built in.

A further great advantage in using reinforced concrete is the *rapidity of erection*. The raw material is deposited on the site in a simple fashion and worked up by machinery in a very short time. In cases of large buildings, and particularly where it is of importance that they should be erected as quickly as possible, reinforced concrete will decidedly be preferable to brickwork.

The *saving of space* is another important item. The thickness of the external walls is much reduced, especially in cases of tall buildings, thus giving an increased floor area. Columns and stanchions can be spaced considerable distances apart, particularly where ribbed ceilings are used, while owing to the reduced weight of the structure, supports need be less frequent than is the case with ordinary iron stanchions and girders. Furthermore, a reinforced concrete column, as a rule, takes up less room than an iron column or stanchion with its casing of concrete, besides affording greater protection against fire. A heavy iron stanchion is naturally liable to considerable expansion when exposed to fire, and the casing must be of appreciable thickness to resist this and protect the stanchion sufficiently to avoid expansion, quite apart from the consideration that a casing is very likely to crack under the influence of fire and the sudden exposure to water, when heated. In reinforced concrete stanchions the casing forms part and parcel of the stanchion itself, while the reinforcement is of such a small comparative sectional area that expansion is hardly possible.

The concrete lends itself to all irregular shapes and outlines, and there is no cutting as with brickwork, and perfect level and smooth surfaces are obtainable.

The *carrying capacity* of reinforced concrete beams and stanchions makes it possible to effect a great saving in the number of columns or stanchions required, and thus better light, more air, and better superintendence in case of factories are gained. This is particularly important where the heights of floors or the extent of buildings are limited.

The *resistance against vibration or oscillation* owing to the monolithic or homogeneous nature of the construction is also a very important feature. In case of factories this is particularly noticeable.

Experience has shown that sudden shocks such as, for instance, railway bridges or the like structures are subject to, cause no bad effects. While in solid masonry a crack is often caused which acts detrimentally on the structure, reinforced concrete constructions cause, through the elasticity and continuity of the steel work, the shock to be distributed evenly over a large surface instead of being taken up by a very confined portion only. This advantage is very important also in cases of fire, as the floors are able to resist any vibrations caused by falling machinery or débris much better than any other floors.

In comparing the *cost* of reinforced concrete buildings with that of brick or stone buildings the advantage is usually with the former. They naturally require less material and labour. The thicknesses of walls are considerably less, as brick walls must be increased in thickness according to their height to prevent bending or failure.

The only weak point in this respect is the considerable expense of centering and boarding and extra supervision. As the soundness and quality of the work very largely depends on good workmanship, it is essential that the supervision should be strict and general. It is also necessary that the superintending clerk of the works or foreman should be fully acquainted with the construction and alive to the great responsibility he incurs. The cost of centering and boarding is naturally appreciable, and although these materials may be reused three times or more according to quality of timber, full allowance must be made for cutting and waste. A judicious superintending of the work and looking after the workmen goes a long way towards reducing this item. Furthermore, the centering and boarding used should be of ample thickness and scantling. Although the initial expense in establishing the plant is thus increased, it will pay in the long run, as the plant can be reused oftener and splitting and consequent loss is avoided.

The expensive cartage of heavy ironwork and the haulage of heavy members into place is done away with, and it must not be overlooked that the encasing of ironwork becomes unnecessary.

The *cost of maintenance* is decidedly less than is the case with brick buildings. There is no pointing as with brickwork, nor repairs to stonework or any of the many costly items of repairs of an ordinary building, while the life of a reinforced concrete building is almost permanent, the structures being indestructible. Age has no bad influence, there is no decay; in fact, the work becomes stronger in course of time.

In order to avoid the necessity of repairing cracks great care should be taken to arrange for sufficient thickness of the concrete covering the reinforcements, particularly in external work exposed to the influence of wet and frost. If the layers are made too thin, cracks are caused, and it will be a costly item to remedy this shortcoming by future repairs. It is decidedly more economical to avoid this by allowing ample thickness.

From a *hygienic point of view* reinforced concrete buildings are also preferable, particularly for hospitals and schools. Formation of fungus is impossible, and there are no hiding-places for insects or microbes and bacilli as is the case with wooden floors. The absence of projecting girder flanches prevents the accumulation of dust and the buildings are easily kept clean and sanitary.

As regards the *architectural treatment* of reinforced concrete, there are already many examples, as buildings, bridges, towers, etc., proving its adaptability for ornamental work. Artificial stone has been used for many years to the greatest advantage. There is no fear of sandholes, shales or other defects spoiling the appearance of many of our best designed buildings. Owing to the compactness and hardness of the material decay of delicate architectural features is almost impossible, quite apart from the saving in cost of material and workmanship. A great variety of designs is obtainable by removing the surface film of cement and showing the grain of the aggregate, thus overcoming the monotonous colour of the cement concrete. For the outer layers aggregate composed of small chips of any natural stone may be employed, giving plenty of opportunity for varied design. Mouldings and ornaments can be either cast in the moulds as the work proceeds or fixed in afterwards, and owing to the nature of the material it is possible to execute the most delicate designs.

For *waterproofing* concrete many methods have been advocated. It stands to reason that a greater proportion of cement tends to a more waterproof mixture. This is, of course, expensive, and a small addition of lime has been used with good results. A mixture of 1 part of Portland cement, $\frac{1}{2}$ part of lime and 3 parts of sand was found to be perfectly waterproof after six days. The usual method is to apply a wash of soft soap to the surface after the concrete has become set. This serves, at least, as a temporary measure until the surface becomes hard enough in itself. It is not advisable to mix the soap with the concrete. The concrete very often shows fine surface or hair cracks, and in such cases mastic asphalte has often been used for waterproofing.

TEMPERATURE AND HAIR CRACKS.

Temperature cracks usually occur in large and bulky work, such as reservoir and dam construction, and arise from the effect of thermal variations. Although these cracks often appear to be of a serious nature, this is, as a rule, not so, and simple filling-in with mortar, lead or neat cement remedies the defect. As previously pointed out the reinforcements should be well distributed, and long walls or conduits require reinforcements in both directions to prevent cracks.

Fine surface or hair cracks are usually due to the circumstance that the surface of the work dries more rapidly than the bulk of the concrete. They are not, as is often supposed, due to faulty cement, but rather to a too rich mortar. All cement used in dressing concrete should be well mixed with sand or other very fine aggregate, and the surface work or veneer must be well rubbed down and washed.

CHAPTER II

THE MATERIALS

A. PORTLAND CEMENT.

PORTLAND cement derives its name from its resemblance, when hard set, to Portland stone, and was invented, as before mentioned, by Joseph Aspdin in 1824. It was first commercially manufactured at Swanscombe, Northfleet, Faversham and Cliffe, at the works of J. B. White & Bros., Robins & Co., Knight, Bevan & Sturge, Hilton, Anderson & Co., Francis & Co., and others.

While formerly the manufacturing process was somewhat crude, the superintendence is now usually in the hands of experienced chemists and the process of manufacture is carefully watched. Generally speaking, it may be taken that any modern Portland cement hailing from one of the recognised works is reliable, if properly treated and used. There is a great deal of so-called "natural" cement on the market, made principally in Belgium and sold as "Portland cement," and care should be taken that only best British Portland cement is used for reinforced concrete work to secure perfect results. The standard specification of Portland cement drawn up by the Engineering Standards Committee defines Portland cement as follows: "The cement shall be prepared by intimately mixing together calcareous and argillaceous materials, burning them at a clinkering temperature and grinding the resulting clinker". This definition shows that genuine Portland cement must be prepared by the mixture of separate raw materials. To ensure accurate results, great care must be exercised in the mixing and a complete chemical combination during the process of calcination attained.

Another cement to be avoided is that made from blast furnace slag. This is of different composition and cannot be relied on. It is only satisfactory if used quite fresh, and quickly deteriorates.

Genuine Portland cement is made from chalk and clay or suitable limestone and shales. After being accurately proportioned and mixed the mixture is burnt to a hard clinker. This clinker is then finely ground and the result is the Portland cement. Very finely ground Portland cement will go further than a coarsely-ground Portland cement, as a more intimate and perfect mixture is obtained. Except for special work it is advisable to use either a "medium" or "slow" setting Portland cement. The Engineering Standards Committee defines the former as a cement which sets, when gauged neat, in not less than half an hour nor in more than two hours at normal atmospheric temperature; the latter is one which takes not less than two nor more than seven hours to set.

To ascertain whether the cement is of good quality and condition in a rough and ready manner, a pat of cement $\frac{1}{2}$ in. in thickness should be gauged with about 25 per cent. by weight of clean water and placed on a piece of glass, iron or slate. At the end of twenty-four hours the pat on the glass should be placed in still water and left there for inspection during the progress of the work. If the cement continues to increase in hardness, and its appearance is satisfactory, the user may look to other causes if the work is not good.

Another rough test is to mix cement to the consistency of stiff treacle and fill a bottle with the mixture. If the bottle cracks the cement is over-limed or contains too much free lime. If the mixture shrinks or becomes loose it is over-clayed.

Portland cement should not expand to any great extent.

The initial setting of the cement is the commencement of the chemical action which is set up when the water combines with the cement; the hardening process is a much slower one. Care should be taken that the work is not disturbed during setting.

The atmospheric temperature greatly influences the setting. The warmer the weather and water, the more quickly will the cement set. A temperature below freezing point practically stops the chemical action, and many other causes may retard the setting. If, however, treated properly, the cement will set ultimately.

When Portland cement concrete is subjected to sea-water, particular care should be taken to get a close and compact mixture.

B. CONCRETE.

All aggregates used for mixing with Portland cement to form concrete should be perfectly clean and only clear water must be used. A good many materials are suitable for concrete, as ballast, broken stone, crushed granite, broken brick, burnt ballast and pumice stone.

Coke breeze is cheap and largely used, but must be carefully selected. Pan breeze or ashes are unsuitable. The coke breeze must be free from particles of coal dust, ammonia or sulphur and organic impurities. Pure vitrified furnace clinker is a good aggregate but makes a porous concrete. The concrete thus gained is light and cannot resist the same compression as that made with more substantial aggregate, as ballast, stone or brick.

Ballast concrete is likely to splinter, particularly when water is poured on it while heated, as in the case of a fire, and should, therefore, not be used for fire-resisting floors.

Pumice stone is also objectionable on account of its making very cellular concrete. It absorbs moisture and may induce rusting of the reinforcing steel work.

Ballast concrete resists a great crushing strain. For floor construction it should be crushed so as to be not larger than to pass through a mesh $\frac{3}{4}$ in. square; if reduced to $\frac{1}{2}$ in. the concrete will be more fire-resisting. For heavier work and foundations the size may be from 1 to 2 ins. mixed with smaller particles. It should be well washed before use to ensure best results. Angular ballast will naturally give better concrete than that composed of round particles.

Broken, hard limestone makes a good concrete, if clean, but is not very fire-resisting, as limestone is subject to calcination at a high temperature.

Sandstone concrete is somewhat inferior in strength to limestone.

Diorites give a very good concrete.

Granite chips are to be recommended, particularly for floor constructions, giving a good wearing surface.

Broken brick is highly fire-resisting and an excellent aggregate for concrete. It affords plenty of adhesion and does not splinter at high temperatures.

Burnt hard clay ballast is also suitable for concrete but inferior to broken brick.

Pumice is a cellular volcanic product, and concrete made of this material is somewhat stronger than coke breeze or clinker.

The breaking of the aggregate is done either by hand or machinery. If broken by hand the results are somewhat better, but it is, of course, more expensive. If a stone-breaking machine is used, care should be taken that the fine dust produced in the breaking is eliminated, as the presence of this dust will naturally weaken the concrete.

The washing of the aggregate is done advantageously with a washing machine, which should be so constructed as to avoid any sediment.

In mixing the aggregate with cement there will naturally be a large number of voids varying according to the nature and size of the aggregate used. It has been proved that if sand be added sufficiently to fill up these voids, and only just sufficient cement is added to fill the interstices between the sand, a much smaller quantity of cement is needed than if the sand is omitted, while at the same time a strong, heavier and more impervious concrete is obtained. Very fine sand will make the concrete weak, but too coarse a sand is also a mistake, as more cement is required to fill in the interstices or these remain and weaken the concrete. Medium sized sand is, therefore, the material to be used.

Particular attention must be paid to the selection of the sand. It must be perfectly clean, as any organic or loamy matter is detrimental to the strength of the concrete. If a loamy pit sand be used for economic or other reasons, it should be well washed. River sand is preferable to pit sand, and it is bound to be cleaner. Sea sand may be used without any bad effects. The presence of the salt will retard the setting of the cement to a certain extent and may cause discolorations, which can, however, be easily removed by a wash with a solution of sulphuric acid, much diluted with water.

The British Fire Prevention Committee carried out a number of tests with concrete floor slabs composed of slag, broken brick, granite, burnt ballast, coke breeze, clinker and Thames ballast, in order to find the most suitable aggregate to resist fire. The cement used was the "Ferrocrete" Brand, manufactured by the Associated Portland Cement Manufacturers (1900) Ltd. The results of these tests are set forth in Report No. 101 of the British Fire Prevention Committee, the following "Object of Test" and "Summary of Effect" with table giving a concise view of the relative efficiency of the aggregates.

OBJECT OF TEST.

To record the effect of a fire of three hours' duration, the temperature to reach 1800° Fahr. (982·2° C.) but not to exceed 2200° Fahr. (1204·4° C.) followed by the application of water for two minutes.

The area of the floor under investigation was to be divided into seven equal bays of different aggregates, the quantity and quality of Portland cement used being identical for each bay, and the nature of the concrete used being as follows :—

No.	Parts by Volume.	
1. Slag concrete	Blast furnace slag	3
	Clean sand	2
	Cement	1
2. Broken brick concrete	Broken brick	3
	Clean sand	2
	Cement	1
3. Granite concrete	Broken granite	3
	Clean sand	2
	Cement	1
4. Burnt ballast concrete	Burnt ballast	5
	Cement	1
5. Coke breeze concrete	Coke breeze	5
	Cement	1
6. Clinker concrete	Furnace clinker	3
	Clean sand	2
	Cement	1
7. Thames ballast concrete	Thames ballast	3
	Clean sand	2
	Cement	1

The total area of the floor under investigation was to be at least 200 ft. sup. (18·58 sq. m.).

The soffit of each bay exposed was to be about 10′·0″ by 2′·7″ (3·04 m. by ·787 m.), the thickness being 5½ ins. (·139 m.).

The floor was to be loaded with 224 lbs. per ft. sup. (1093·76 kg. per sq. m.).

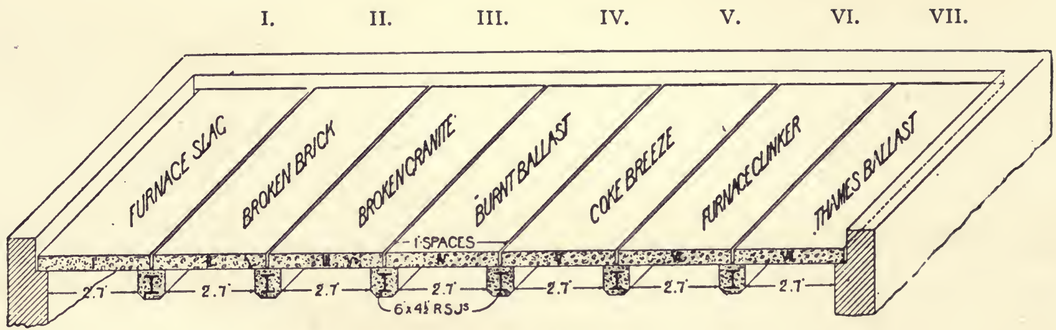
The centering was to be struck fourteen days after completion of the floor. The time allowed for drying was forty days (autumn).

Summary of Effect.

In ten minutes after the gas was lighted the plaster began to fall off the beams and continued to do so until the end of the test.

Towards the end of the test it was observed, from the top of the hut, that the edges of Bays 1, 6 and 7 were red-hot, No. 7 being the worst.

On the application of water, more plaster was washed off the beams than had fallen during the fire test, and some of the concrete from the underside of Bays Nos. 3, 4, 5, 6 and 7 was washed off. All the slabs remained in position.



No. I.	No. II.	No. III.	No. IV.	No. V.	No VI.	No. VII.
Top: Slab cracked across in two places.	Top: Slab cracked across in three places; slight curve downwards.	Top: Slab cracked across in three places; curved downwards about ½ in.	Top: No cracks; Not curved downwards.	Top: No cracks; not curved downwards.	Top: Slab cracked across in two places; curved downwards about ⅜ in.	Top: Slab cracked across in very many places; curved downwards about 2 in.
Underside: Curved downwards ¼ in.; slight cracks visible.	Underside: Curved downwards ½ in.; slight cracks visible.	Underside: Curved downwards ½ in.; no cracks visible; about 1 in. washed off by water.	Underside: Not curved downwards; no cracks visible; about 3 in. washed off underside (in parts) by water.	Underside: Not curved: no cracks visible; about 1 in. washed off underside (in parts) by water.	Underside: Not curved; one slight crack visible; pitted in places about 1 in. deep by water.	Underside: Curved downward 1¼ in., and bad cracks all over in all directions, mainly longitudinally; much washed off by water.

FIG. 3.—The Seven Concrete Bays. Diagram and Table illustrating Summary of an official Fire Test of the British Fire Prevention Committee.

Bays Nos. 4, 5 and 6 were flat on the soffit, the others were convex on the underside, No. 7 (the worst) to the extent of 1½ in. On the removal of the load it was found that Bays Nos. 1, 2, 3, 6 and 7 were cracked across, No. 7 being worst.

THE MANUFACTURE OF CONCRETE.

For the making of good concrete it is essential that the aggregate should be perfectly clean, and should vary in size from a Spanish nut to a hen's egg, at any rate it should not exceed this latter size. The sand should be clean, sharp and of medium coarseness to fill the voids between the aggregates, and, lastly, the Portland cement should be as finely ground as possible to fill the interstices between the sand and to be plentiful enough, in addition, to adhere properly to the aggregates.

The ideal concrete should be so composed as to give the best results as regards strength at the least expenditure.

Experience has shown that it is not feasible to lay down a hard and fast rule as to the proportioning of the components of concrete ; this largely depends on the aggregates and nature of the sand used.

The greatest possible density is more likely to secure perfect concrete than an increased portion of Portland cement. The various components must fit into each other to perfection, and it has been proved that a concrete well mixed with a moderate proportion of Portland cement is stronger than a concrete having cavities due to improper mixing, but containing a larger proportion of cement. Only by perfect density it becomes possible to distribute the pressure evenly throughout. Where great strength is desired, the proportion of cement may be increased, but it must not be overlooked that a mixture of perfect density having a small proportion of cement gives a stronger concrete than a mixture of less density having a greater proportion of cement. In the mixture there should be a certain amount of smaller stones to fill the voids between the larger stones, an amount of still smaller stones to fill the voids between the small stones, the sand filling the voids between the latter and the Portland cement being added to bind the whole together and fill the voids in the sand. Attention should consequently be paid not only to the hardness of the aggregate, but to secure aggregate of such a nature that the particles are of various sizes proportioned so as to form themselves into a solid mass with the smallest voids possible. Naturally, the more angular and rough the particles of aggregate are, the better will be the adhesion and consequently the stronger the concrete.

The size of the aggregate depends largely on the work the concrete is destined for. For foundations, thick walls, etc., the size may be up to say $2\frac{1}{2}$ ins. in diameter, while for floors, partitions, and walls less than 12 ins., the aggregate should not be more than $\frac{3}{4}$ in. in diameter. We may assume that each particle of aggregate is able to resist the same crushing strain proportionately as a bigger cube of the same material. The material should be well sifted so as to remove the loose dust. The dust resulting very largely from the crushing of the aggregate forms a coating round the small stones and thus prevents these coming in direct contact with the cement, thus preventing thorough adhesion. Particles of loam or mould will necessarily weaken the concrete, and must be removed in any case, but the coarser dust resulting from breaking up must be considered as forming portion of the sand to be incorporated and duly allowed for in deciding the rate of proportioning.

The best aggregates to be used are, no doubt, crushed ballast or stone. Concrete of small aggregate is more fire-resisting than that composed of larger aggregate, and the smaller aggregate is also more suitable from a practical point of view, as it is easier to get it into all crevices round the reinforcements, and, furthermore, voids cannot so easily occur with the fine material.

As regards the proportion of Portland cement required, this depends largely on the nature of the sand. The sand should be clean and sharp and siliceous. Fine sand naturally means more voids and consequently more cement to fill same, while it is more difficult to fill the voids than with a coarser sand.

In deciding what aggregate should be used for a particular contract, it must from an economical point of view first be ascertained what in the nature of aggregate can be procured on the site or in the immediate neighbourhood, in order to cheapen the cost of the work. If the concrete is for walls or other exposed parts of the structure, care must be taken to select an aggregate which is able to resist frost, and for this reason no porous material should be used, quite apart from the fact that porous aggregate makes poor concrete. If, however, for purposes of economy it is necessary to use porous aggregates, these should be well soaked before using, so as to avoid the absorption of moisture from the cement mortar. A good and cheap aggregate very often met with on the site is gravel, and as it is found in various sizes mixed together, the proportion of cement required is not excessive. But the concrete composed of gravel can naturally not be expected to afford the same strength of resistance as such made of broken stone or granite. Gravel contains always a proportion of sand or material which must be considered as sand, and, if gravel is to be used, the proportion of this sand must be carefully ascertained and the decision of how the concrete should be composed made accordingly. This is done by passing and repassing the material through sieves of various mesh.

The sand is also often found on the site, and it should be decided if it is suitable and particularly if it is clean. This is easily ascertained by placing a quantity of sand into a glass tumbler and filling this with clean water. If the water remains clear after shaking, the sand is fit for use, but if the water becomes cloudy or dirty, the material must be washed until, on further testing it, the water remains clear.

The water used for the concrete must be clean, and free from impurities and of a medium temperature. If the water is too warm, the concrete sets too quickly, while very cold water delays setting. The quantity of water required depends partly on the nature of the aggregate—porous material requiring more water than compact and solid aggregate—and partly on the weather conditions. If the atmosphere is damp, less water is required than on a hot, dry day. Too little water causes imperfect setting of the cement, while too much water forms small voids in the concrete, which later on will come up to the surface. A practical test is to take up a handful of the concrete, when mixed, and press it together. The water should then drip out and, on opening the hand, the sample should retain the shape thus given to it. Broadly speaking, the concrete should be of such consistency as to be easily workable for whatever purpose used.

As regards finding the proper proportioning of the various materials in order to get a dense concrete many methods are advocated.

The simplest form is to fill a tumbler with the aggregate decided upon, level it at top and then add as much water as possible, *viz.*, until it runs over the brim; the water to be taken out of a graded glass. The proportion of water thus used would be the amount of sand required, on the assumption that the water fills up the voids between the aggregate which, in the concrete, are to be filled with sand. The same process is then repeated with the sand by filling the tumbler again with the sand to be used and adding as much water as the tumbler will hold. The proportion of water used will represent the amount of Portland cement necessary. The difficulty here is that some aggregates, particularly those of a porous nature, will absorb a great deal of water, and in order to get as true a result as possible, the aggregate should be well wetted before being placed into the tumbler or measure used.

This method does not, however, accurately determine the true proportions required, owing to the fact that the various materials differ in compactness under various methods of handling. As the grains of sand tend to thrust the particles of the larger aggregate apart, and a portion of the sand is often too coarse to enter the voids of the coarser material, the test has its drawbacks. Again, with some of the aggregates, the voids are smaller than the particles of sand, which, therefore, get between the larger aggregate and thus increase the bulk of the mass. To obviate this, the following method is recommended: Determine the proportion of voids in the larger aggregate by filling a measure therewith and pouring in water as described above. Also determine the percentage of voids in sand by weighing a cubic foot of packed sand and subtracting from 165 lbs. (the weight of a cubic foot of quartz), multiplying by 100, and dividing the product by 165. Then proportion the cement and sand so that the cement paste will be 10 per cent. in excess of the voids in the sand, and allow sufficient of this mortar to fill the voids in the large aggregate with an excess of 10 per cent. Thus: Supposing a sand contains 38 per cent. voids and the large aggregate 48 per cent. voids, then cement paste required per c. ft. of sand = $0{\cdot}38 + (\frac{1}{10} \times 0{\cdot}38) = 0{\cdot}42$ c. ft. (approximately). By trial 1 c. ft. of loose cement, lightly shaken, makes 0·85 c. ft. of cement paste, and requires $\frac{0{\cdot}85}{0{\cdot}42}$, or approximately, 2 c. ft. of sand, producing an amount of mortar equal to $0{\cdot}85 + 2\,(1 - 0{\cdot}38) = 2{\cdot}09$ c. ft. Mortar required per c. ft. of large aggregate = $0{\cdot}48 + \frac{1}{10} \times 0{\cdot}48 = 0{\cdot}528$ c. ft. Therefore 2·09 c. ft. mortar will require $\frac{2{\cdot}09}{0{\cdot}528}$ = approximately 4 c. ft. of aggregate. The proportions are, therefore, 1 part of cement, 2 parts of sand, 4 parts large aggregate.

The foregoing method is recommended by the Associated Portland Cement Manufacturers (1900) Ltd., and the following tables, etc., are taken from their book on *Everyday Uses of Portland Cement.*

As the principal object in proportioning the various materials is to get a concrete of maximum density, the proportioning should be found by trial mixtures.

The following table is fairly reliable as regards the percentage of voids in various materials, and may be used where it is not convenient to determine the exact percentage of voids. A box, whose weight has been ascertained, say 1′·0″ × 1′·6″ × 2′·0 (containing 3 c. ft.), should be filled with the materials after they have been heated to 212° F. to drive off any moisture. The materials should be put in the box loosely and the top levelled off with a straight-edge. The box should be weighed when full. Deduct the weight of box to ascertain net weight, and divide this by the number of cubic feet in the contents (*viz.*, 3 in this case). The result is the actual weight of 1 c. ft. of the concrete.

By reference to the table below, the percentage of voids may be ascertained. The table does not apply to fine materials, such as sand, or particles fine enough to pass a $\frac{1}{4}$ in. mesh sieve, and, therefore, an aggregate that contains fine particles must be sifted before its percentage of voids can be determined by the table. The finer particles must be figured as a portion of the mortar.

PERCENTAGE OF VOIDS.

Weight per c. ft. %	Ballast. %	Sand-stone. %	Lime-stone, medium soft. %	Lime-stone, medium hard. Sand-stone, hard. %	Granite. Blue stone. Lime-stone, hard. %	Granite, hard. Trap rock, medium. %	Trap rock, hard. %
70	57	53	55	57	58	60	61
80	51	47	49	51	52	54	56
90	45	40	42	45	47	48	50
100	39	33	36	38	41	43	45
110	33	26	29	32	35	37	39
120	27	20	23	26	29	31	34
130	20	13	17	20	23	26	28
140	14	6	10	14	17	20	23

The stones having been measured loose, the percentage of voids is slightly more than would be the case in actually rammed or tamped concrete.

A convenient way of ascertaining the percentage of sand required is as follows :—

Moisten the sand intended for use, so that, when squeezed in the hand, it will retain its form without pressing out any excess water. Measure 50 c.c. by tamping it into a graduated glass tube marked with cubic centimetres. From the character of the sand, estimate approximately the quantity of Portland cement required to make a concrete of the desired plasticity, density, or strength. If this estimate is, say, 1 part of cement to 2 parts sand, 25 c.c. of Portland cement will be required for admixture with 50 c.c. of sand. This quantity of Portland cement may be obtained by weighing, with reference to the weight of a specific volume of Portland cement. With another sample try another proportion, say, 2½ parts of sand to 1 part of Portland cement and so on. After each sample has been measured out and the cement thoroughly mixed with the sand, sufficient water should be added to each to make a mortar of about the same consistency as will be required for the concrete.

Each sample should then be experimented upon by placing a little at a time in a graduated glass and tamping as before, the space occupied by each sample being noted. If the total quantity in any case should be greater than the volume of sand, probably too much cement has been added.

If the concrete requires a dense, strong mortar, samples should be used which contain the most Portland cement. Should, however, a very dense or strong mortar not be required for the concrete, the proportions are determined by one of the samples containing the least Portland cement and sufficiently plastic to give a good bond in the concrete.

Dense mortar must be used to produce a concrete that shall be almost impervious to water.

The following table may be used to show the proportion of aggregates which will give the maximum density with the minimum of Portland cement, the unit of measurement being that 1 ft. of Portland cement weighs 95 lbs. The figures given for the proportions of mortar, such as 1 : 3, signify 1 Portland cement, 3 sand.

Voids in Aggregate.	Proportions of Aggregate. (Expressed in c. ft.)									
	Proportions of Mortar.									
%	1 : 1	1 : 2	1 : 2½	1 : 3	1 : 3½	1 : 4	1 : 4½	1 : 5	1 : 5½	1 : 6
20	5	10	12½	15	17½	20	22½	25	27½	30
22	4½	9	11⅓	13⅔	16	18¼	20½	22¾	25	27¼
24	4	8⅓	10½	12½	14⅔	16⅔	18¾	20¾	23	25
26	3¾	7⅔	9⅔	11½	13½	15⅓	17⅓	19¼	21¼	23
28	3½	7¼	9	10¾	12½	14¼	16	17¾	19⅔	21½
30	3⅓	6⅔	8⅓	10	11⅔	13⅓	15	16⅔	18⅓	20
32	3	6¼	7¾	9⅓	11	12½	14	15½	17¼	18¾
34	3	6	7⅓	8¾	10¼	11¾	13¼	14¾	16¼	17⅔
36	2¾	5½	7	8⅓	9¾	11	12½	14	15¼	16⅔
38	2⅔	5¼	6½	8	9¼	10½	11¾	13¼	14½	15¾
40	2½	5	6¼	7½	8¾	10	11¼	12½	13¾	15
42	2⅓	4¾	6	7¼	8⅓	9½	10¾	12	13	14¼
44	2¼	4½	5⅔	6¾	8	9	10¼	11⅓	12½	13⅔
46	2¼	4⅓	5½	6½	7⅔	8⅔	9¾	10¾	12	13
48	2	4	5¼	6¼	7⅓	8⅓	9⅓	10⅓	11½	12½
50	2	4	5	6	7	8	9	10	11	12
52	2	2¾	4¾	5¾	6¾	7⅔	8⅔	9⅔	10½	11½
54	1¾	3½	4⅔	5½	6½	7⅓	8⅓	9¼	10¼	11
56	1¾	3½	4½	5⅓	6¼	7¼	8	9	9¾	10¾
58	1¾	3½	4⅓	5¼	6	7	7¾	8⅔	9½	10⅓
60	1⅔	3⅓	4	5	5¾	6⅔	7½	8⅓	9¼	10

Good mixing is absolutely essential, and it is best to use a machine mixer wherever the work is large enough to warrant it.

Practical experience has shown that the normal proportions for reinforced concrete work should be 1 part of Portland cement to 2 of sand to 4 of aggregate. In concrete used in foundations, walls, arches, stairs, floors, etc., 1 : 2½ : 5. For heavy and bulky work like retaining walls, piers, abutments, etc., 1 : 3 : 6. Where very bulky masses are used and the concrete is subjected to compression only, 1 : 4 : 8 would be enough, and it is economy to mix up large stones well spaced out and thoroughly embedded in the concrete.

As regards the quantity of concrete obtained from various proportions it must not be overlooked that the sand goes to fill the voids in the aggregate and the cement those in the sand; consequently it does not follow that concrete mixed of 1 part of cement to 2 of sand and 4 of aggregate gives 7 parts of concrete.

The following are results obtained with various mixtures at the construction of the Connecticut Avenue Bridge in Washington, U.S.A.:—

1 : 2 : 4½ concrete—378·25 lbs. cement measuring 4·5 c. ft. loose, 9 c. ft. sand, and 20·25 c. ft. broken stone, yielded 21·4 c. ft. of concrete when rammed in place.

1 : 2½ : 6 concrete—378·25 lbs. cement measuring 4·5 c. ft. loose, 11·25 c. ft. sand, and 27 c. ft. broken stone (or in another case 13·5 c. ft. ballast and 13·5 c. ft. stone), yielded 27·66 c. ft. of concrete when rammed in place.

1 : 3 : 10 concrete—378·25 lbs. cement measuring 4·5 c. ft. loose, 13·5 c. ft. sand, and 45 c. ft. ballast, yielded 45 c. ft. of concrete when rammed in place.

The Mixing of Concrete.

Portland cement must, until it is used, be kept in a dry place and not left in the open, and no concrete that is not absolutely mixed fresh should be used. Concrete that has begun to set may, however, be used as an aggregate.

To secure a good result the mixing must be thorough; all parts being carefully measured and weighed out. A box without top or bottom and of proportionate dimensions is the most convenient measure for aggregate and sand. The cement should be weighed and the water measured by a pail.

The mixing should take place on a clean wooden platform, the sand being measured first and spread over the platform in a layer of uniform thickness, and it should be dry, as wet sand does not mix properly, except where the mixing is done by machinery. If a great quantity of concrete is to be mixed by hand, the platform is best covered with a sheet of zinc or iron.

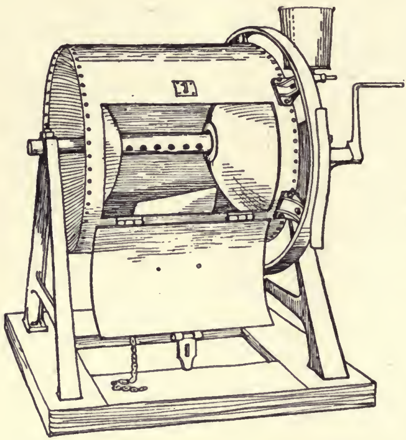


FIG. 4.[1]

No more concrete should be mixed at a time than can be immediately disposed of, and the mixing should be done as near to the place of destination as possible.

When the sand is levelled down, the Portland cement should be evenly distributed over the surface and the whole turned over at least three times with the shovel and until the uniformity of colour indicates a thorough mixing. Then the aggregate should be added to the mixture, the whole turned over again three times, and water gradually added under constant turning over of the materials. The concrete should only be sufficiently wet

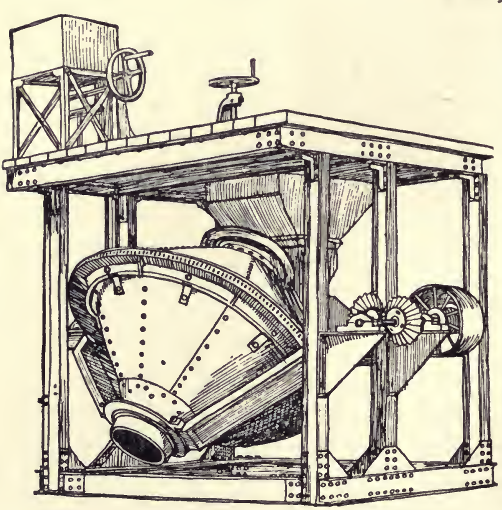


FIG. 5.[1]

[1] From *Everyday Uses of Portland Cement.*

to show water on the surface when it is well rammed in position with a wooden or iron hammer. The best way of adding the water is by sprinkling it over the mixture with a watercan having a proper rosehead.

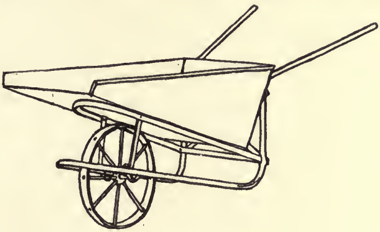


FIG. 6.[1]

Wherever the size of the job allows it, the use of a mixing machine is preferable. Figs. 4 and 5[1] illustrate such machines, many patterns of which are on the market.

In placing the concrete in position, it should not be thrown from a height, but carefully tipped out of a barrow or truck as the case may be. It should then be well and evenly rammed. Not sufficient importance can be attached to this proceeding, as it is of the greatest moment in order to get good results. Figs. 6 and 7[1] illustrate a handtip cart and a tipping truck used for the work.

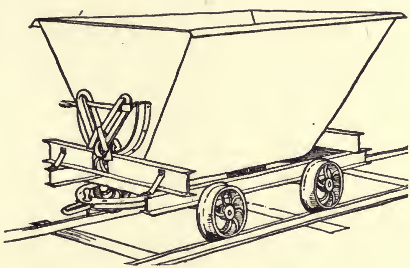


FIG. 7.[1]

CONCRETING DURING FROSTY WEATHER AND HOT WEATHER.

It is not advisable to execute concrete work during frosty weather, as the frost prevents proper and uniform setting. If, however, urgency makes this necessary, it is well to add to the water 1 per cent. by weight of salt for every degree Fahr. below the freezing-point.

During the erection of a building at Rochester, N.J., the water

[1] From *Everyday Uses of Portland Cement.*

was heated to about 90° Fahr. and salt added in about the proportion of 1·6 lb. per c. ft. of Portland cement. The water was heated by passing live steam through perforated pipes in storage tanks, and the sand and gravel were heated in the storage bins by means of steam pipes and hot air pipes.

Certain experts on the Continent advise the addition of a small percentage of soda or chlor. calcium.

On the other hand, exposure to intense heat is also detrimental. The heat causes the upper layers of the concrete to set quicker than the lower and, naturally, withdraws the moisture too quickly. In hot weather it is therefore advisable to keep the surface of the concrete damp by sprinkling water or by covering it with a layer of wet sand, which will counteract the heat of the sun rays and cause the concrete to set in due time.

In the United Kingdom cases of extreme heat or cold rarely happen and, as a rule, only last a very short time, so that the work can be suspended.

C. STEEL REINFORCEMENTS.

The committee appointed by the R. I. B. A. in their report on reinforced concrete recommended as follows :—

The metal used should be steel, having the following qualities :—

(*a*) An ultimate strength of not less than 60,000 lbs./in.^{2}

(*b*) An elastic limit of not less than 50 per cent. or more than 60 per cent. of the ultimate.

(*c*) An elongation of not less than 22 per cent. in the lengths stated below.

(*d*) It must stand bending cold 180° to a diameter of the thickness of pieces tested without fracture on outside of bent portion.

In the case of round bars the elongation should not be less than 22 per cent. measured on a gauge-length of eight diameters. In the case of bars over one inch in diameter, the elongation may be measured on a gauge of four diameters, and should then be not less than 27 per cent. For other sectional material the tensile and elongation tests should be those prescribed in the British Standard Specification for structural steel.

Before use in the work the metal must be clean and free from scale or loose rust. It should not be oiled or painted, but a wash of thick Portland cement grout is desirable.

FIGS. 8[1] and 9[1].

Welding should in general be forbidden; if it is found necessary, it should be at points where the metal is least stressed, and it should never be allowed without the special sanction of the architect or engineer responsible for the design.

The reinforcements should be placed and kept exactly in the positions marked on the drawings, and apart from any consideration of fire-resistance, ought not to be nearer the surface of the concrete at any point than 1 inch in beams and ½ inch in floor slabs or other thin structures.

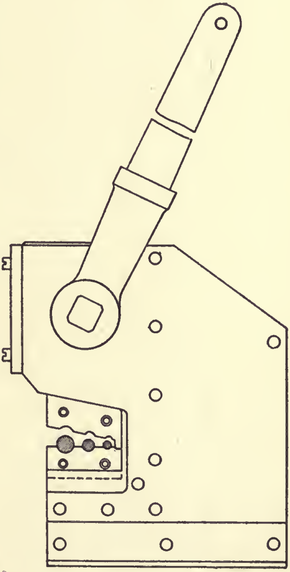


FIG. 10.

As regards rust, experience shows that, if not loose, it has the tendency to increase the adhesion to the steel of the mortar. Dirt or fat, on the other hand, acts detrimentally. Wherever the rods have to resist tensile stresses, it is advisable to bend the ends over to form hooks, so as to prevent any sliding tendency and give a better fixing in the concrete.

In columns or stanchions, where rods are continuous, and it is necessary to join them, it is a good practice to form a cup at the end of the lower rod, the upper rod finding its base in the cup. Rods are usually jointed by lapping them 3 or 4 ins. and winding wire round the joint (see Figs. 8 and 9). The ends should be well bent over and well incased with concrete.

The cutting of the rods is done by hand, with a chisel, stouter rods being heated first. A very handy little machine (Figs. 10 and 11) for round and square bars is now on the market (The Concave Floor Co., 1 Hawstead Road, Catford, S.E.) by means of which rods can be cut in a cold state with great rapidity. The machines are screwed down to a bench or other firm platform. The same firm supply also a machine for bending rods (Fig. 12) for various purposes by means of which it is easy to bend the rods in exactly the same places uniformly.

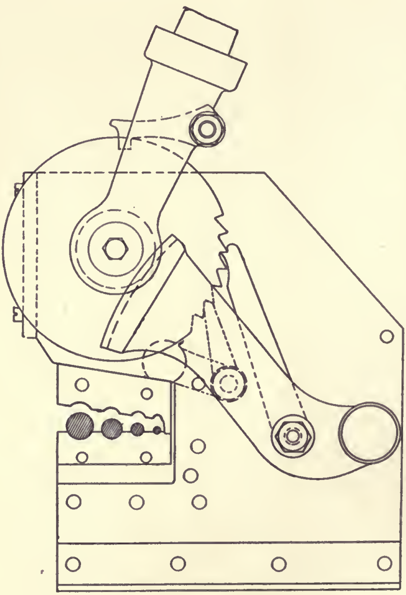


Fig. 11.

These machines can easily be taken from one job to another, and thus do away with the necessity of preparing the rods beforehand and facilitate transport and handling before use.

The small waste pieces, which amount to some 10 per cent., can be utilised for hangers, straps and other connexions (see Figs. 13, 14, 15, 16), and machines for these purposes are also supplied by this company.

The advantages are obvious. The rods can be delivered on the site in stock lengths and the cutting and bending be done on the spot from dimensions taken on the site and under the direct supervision of the clerk of works or foreman, and mistakes are avoided.

The rods designed to resist the tensile stresses may be termed tension rods. In case of a slab supported on all sides these rods are best placed in the direction of the shortest span. If the slab is approximately square, it is advisable to let them cross each other. The selection of the diameter depends on the load to be carried, the spacing, and the span of the slab. Round rods are commonly used, spaced certain distances apart. The distance can easily be ascertained according to formulæ mentioned hereafter. Care must be taken not to join rods where great bending moments occur.

Another series of rods, termed distributing rods, connect the tension rods in the opposite direction and are designed to give the tension rods a better hold, to distribute the stresses uniformly over the

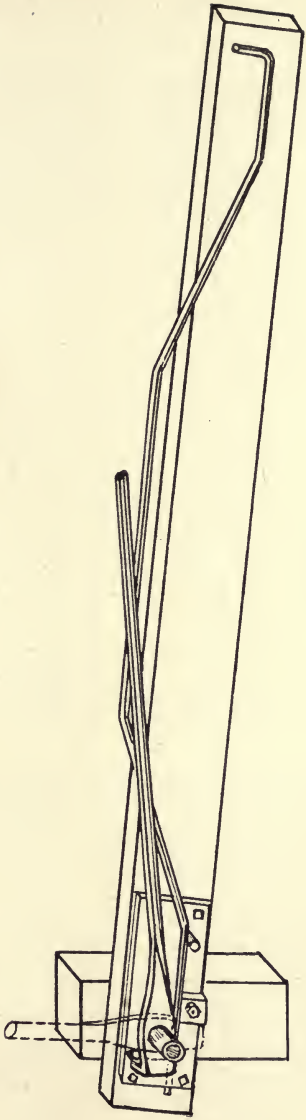
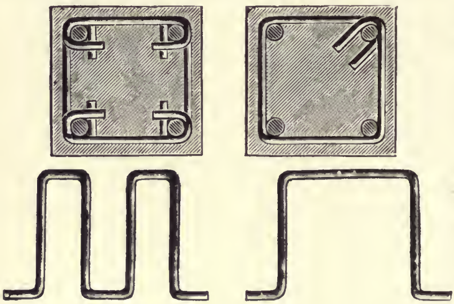


Fig. 12.

tension rods, and to increase the strength of the slab against shear. These rods are usually selected of a smaller diameter and placed over the tension rods so that the latter come as close as possible to the fibres in greatest tension. Fig. 17[1] shows the arrangement of rods in a single reinforced slab, Fig. 18[1] those in a double reinforced slab. At the points of crossing the two sets of rods are connected alternately with wire so that the whole reinforcement forms an iron netting. The width of the mesh varies according to circumstances. As a rule, in case of ordinary floor slabs, the rods are spaced from 4 to 12 ins. apart and of various diameters. Where the span is large and there are great loads to carry, the slab must either be thicker or the reinforcing rods stouter, as the case may be. From a practical point of view it is always more advisable to choose thin rods, closely spaced, rather than stout rods, spaced very much apart. The round rods are the more frequently used, they facilitate the escape of air-bubbles and the tamping of the concrete; furthermore, they have no sharp arrises cutting into the concrete. On the other hand, the circular section offers a smaller coefficient of adhesion than is the case with square rods. Square bars, flat or hoop irons are



FIGS. 13-16.

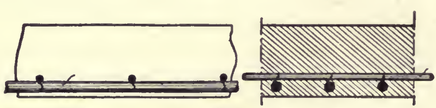


FIG. 17.[1]

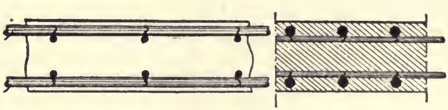


FIG. 18.[1]

also used, often twisted, in order to get better adhesion. Other sections used are of + ⊥ I L S △ shape, and many patent bars of peculiar sections, twists and bends, of which more will be said hereafter. Expanded metal, wire meshing, dove-tailed sheeting, etc., are also used for floors, foundations, roofs, etc., and will be dealt with in due course.

CHAPTER III

EXECUTION OF WORK

It has already been mentioned, that it is essential to store the Portland cement in a dry place and protect it from the action of moisture in the atmosphere, until it is to be used.

It is also advisable to keep the sand and gravel or other loose materials under cover, as they will get wet, and it is then difficult to accurately ascertain the proper amount of cement and water required.

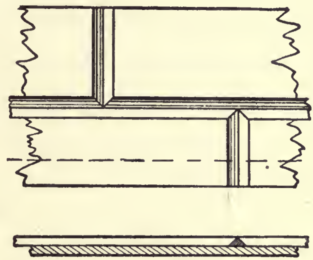
While the centering is being prepared and erected in place, there is opportunity and time for testing the cement, deciding the proportions of aggregates and sand to be used and make all the preliminary investigations and tests before mentioned.

The centering and moulds, usually termed "forms," are necessarily an expensive item, and special consideration should be given to their design, and all unnecessary cutting avoided.

Well-seasoned timber is not particularly suitable, as it is likely to swell and warp and absorb the moisture from the concrete. For this reason green, or almost green, timber is preferable. Any kind of timber may be used, fir, yellow pine or spruce, or indeed any timber most cheaply and conveniently obtained.

To secure a smooth surface the boarding next to the concrete should be planed. Where forms are required to be used several times over, the inside surface of the timber is coated or painted with a mixture of soft soap, linseed oil or crude petroleum oil. Others recommend limewhiting to prevent the sticking of the concrete to the forms and thus causing rough surfaces of the work. Where it is intended to plaster the concrete afterwards, no oily or fatty matter should be used, and, in fact, for that purpose the concrete is best left rough, so as to form a key for the plaster, and it is sufficient to wet the forms before concreting begins.

Forms are constructed of timber, boards and battens of small scantling. The boarding is usually from 1 in. to 2 ins. thick, and, according to the thickness used, the battens are spaced. Roughly speaking, the studding should not be more than 2 ft. apart for inch boarding nor more than 5 ft. for 2 in. boarding. The battens must be thoroughly braced to withstand the pressure of the soft concrete and the stress of ramming and tamping. Tongued and grooved boards are better than square-edged boards. For walls the boarding should be 1½ or 2 ins. thick, 1 in. boards being used for small panels only and for beams, girders and small floor panels, although, if there is a good deal of flooring to be done and the boarding



Figs. 19[1] and 20.[1]

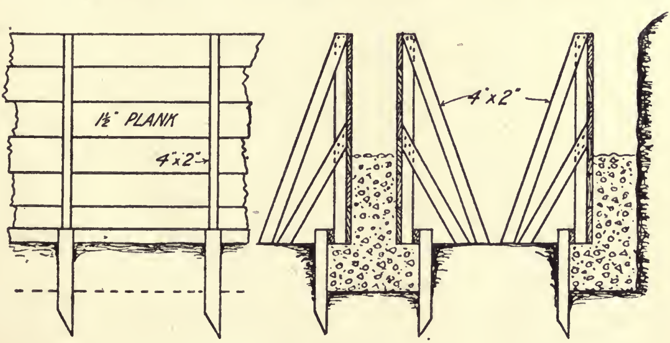


Fig. 21.[1]

used over and over again, it is naturally more economical to use thicker stuff. Wherever great weights are temporarily to be

[1] From *Everyday Uses of Portland Cement.*

carried—as in the case of the underside of beams and girders, and in forming centering for columns or posts—2 in. boards should be

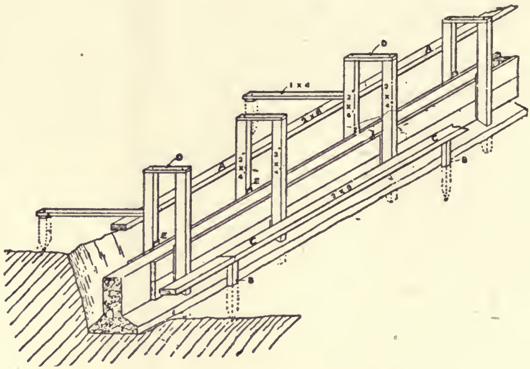


FIG. 22.

used. Timber ends may be run beyond the work they enclose so as to save waste caused by sawing.

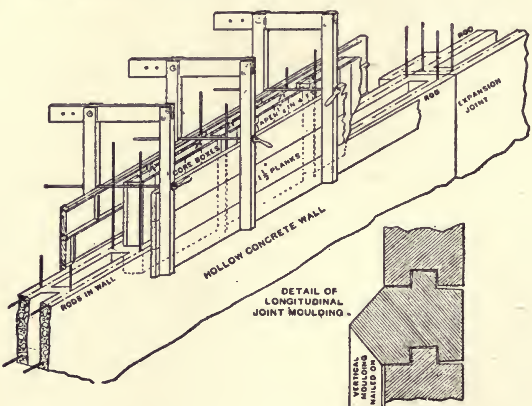


FIG. 23. FIG. 24.

By nailing arris rails to the boarding the external walls are given the appearance of a building built with heavy masonry. See Figs. 19, 20.

The Associated Portland Cement Manufacturers (1900) Ltd. in their book on *Everyday Uses of Portland Cement* illustrate some useful forms for reinforced concrete work. Fig. 21 shows forms for low wall and cellar wall, Fig. 22 form for a low wall, Fig. 23 form for a hollow wall with Fig. 24, a detail of longitudinal joint moulding; Fig. 25 is a form for a solid wall.

For girder forms hardwood wedges should be used at top and bottom of each strut, as these can be loosened for resting if there is any deflection. If possible, the wedges should be loosened 24

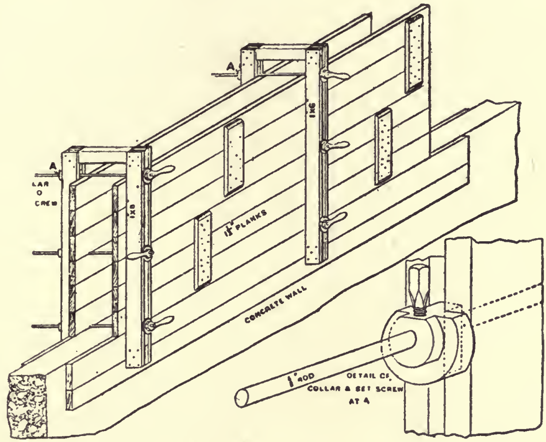


FIG. 25.[1]

hours in advance of the struts. As a rule, light joists, 2 by 8 ins. or 2 by 10 ins. are used in preference to heavier timbers. Experience has shown that the maximum unsupported distance for 1 in. boards is 2′·0″, for 1½ in. planks 4′·0″, and for 2 in. planks the studding usually varies from 3′·0″ to 4′·6″ apart, according to circumstances.

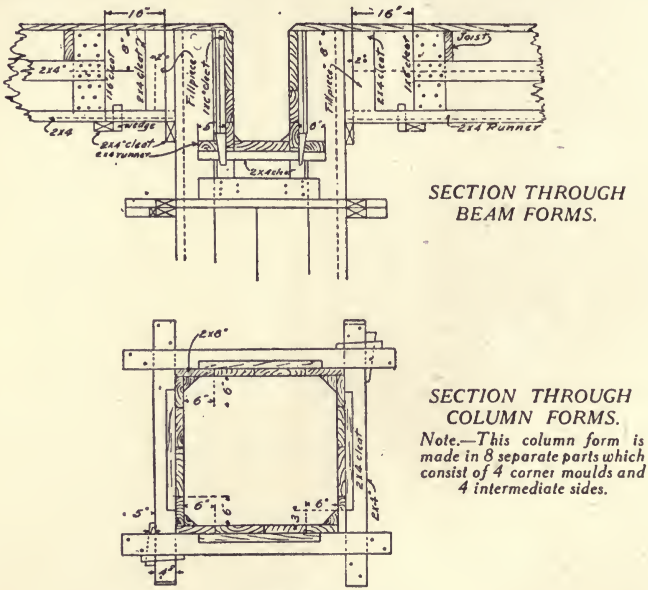
Fig. 26[1] shows the arrangement of a beam form and Fig. 27[1] that of a column form.

[1] From *Everyday Uses of Portland Cement.*

The same author gives the safe strength of struts for floor forms in lbs. per sq. in. of section for different sized timber.

Length of Strut	3″ × 4″	4″ × 4″	6″ × 6″	8″ × 8″	
14′ 0″	500	700	900	1,100	lbs. per square inch.
12′ 0″	600	800	1,000	1,200	
10′ 0″	700	900	1,100	1,200	
8′ 0″	850	1,050	1,200	1,200	
6′ 0″	1,000	1,200	1,200	1,200	

Special care must be taken that the forms are quite strong



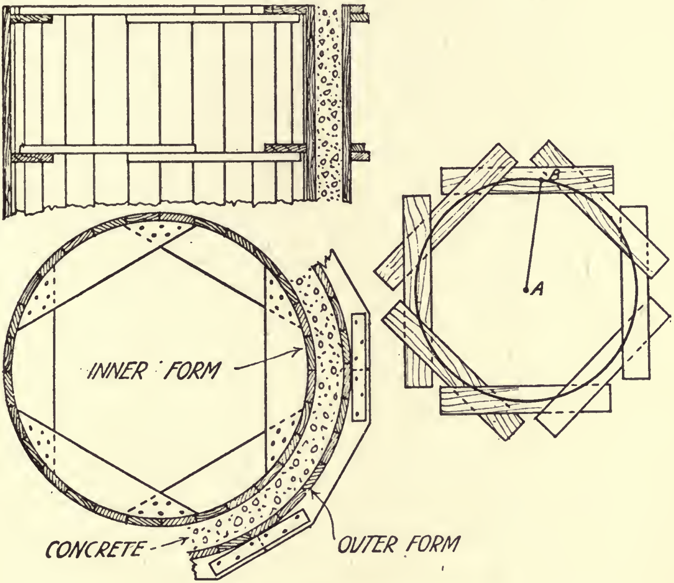
FIGS. 26[1] and 27.[1]

enough to do all the work they are called upon to do, and furthermore, that they are not removed too early, as accidents might very easily happen on account of this. The time for the centering to remain under a floor should be from one week to six weeks or longer according to the composition of the concrete, and the conditions, atmospheric and otherwise, under which it was prepared.

[1] From *Everyday Uses of Portland Cement.*

Broadly speaking, the centering should remain for twenty-eight days, by which time the concrete has gained about 60 per cent. of its ultimate strength. Fig. 28[1] shows form for circular work and Fig. 29[1] the setting out of the same.

The report of the R. I. B. A. Committee recommends as to striking of centres as follows :—



FIGS. 28[1] and 29.[1]

The time during which the centres should remain up depends on various circumstances, such as the dimensions or thickness of the parts of the work, the amount of water used in mixing, the state of the weather during laying and setting, etc., and must be left to the judgment of the person responsible for the work. The casing for columns, for the sides of beams and for the soffits of floor slabs not more than 4 ft. span must not be removed under

[1] From *Everyday Uses of Portland Cement.*

eight days, soffits of beams and of floors of greater span should remain up for at least fourteen days, and for large span arches for at least twenty-eight days. The centering of floors in buildings, which are not loaded for some time after the removal of same, may be removed in a short time; the centering for structures which are to be used as soon as completed must remain in place much longer. If frost occurs during the setting, the time should be increased by the duration of the frost.

As before mentioned, the concrete should not be too wet nor too dry when being brought in. It should be placed in layers of from 6 to 8 ins. in depth and of such consistency that, when it is tamped lightly with a wooden or iron rammer, the water shows on the top and the tamping should continue until every particle of the aggregate is entirely covered with mortar. In preparing the concrete no more material must be made than can be disposed of at once, and in no case should any concrete lie longer than one hour before being used.

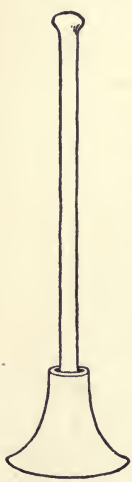


Fig. 30.[1]

In warehouses, factories or other bigger buildings the mixing machines, etc., are most conveniently placed in the basement or lowest story, as the materials are thus at once protected. The saving of labour should be studied as much as possible. If the ground varies in level, the mixing should be done at a high level, so that barrows run down the hill when full and up the hill when empty, thus saving labour and energy.

As soon as the concrete is placed in position, the tamping and ramming begins. Fig. 30[1] shows a rammer made of cast iron with wooden shaft. The tamping should be carried out with the object in view of consolidating the mass, bringing up any air, and getting the various particles to slip into their proper places and filling the voids. The outer portions require special attention, as they have to resist the tensile stresses. For

[1] From *Everyday Uses of Portland Cement.*

this reason it is also advisable to take care that no larger aggregates come to the outside, unless they are well covered with the cement mortar.

Arches or vaults should be tamped in the direction of the stress curves, working up from the springing. The weight and size of the rammers depend on the nature and size of the work. They should have preferably a square base from 4 to 7 ins. and varying weight, according to the purpose they are used for. For light work wooden rammers are often employed.

Where the concrete is brought in in layers, it may be necessary to roughen the surface of the first layer before placing the second, to form a key and attain better adhesion of the whole. It is also recommended to make the bottom layers somewhat wetter than the upper ones, to avoid the draining of moisture out of the concrete. Should the work have to be interrupted temporarily, as at meal times, the concrete should be covered over with wet sacks and cleaned down before work is resumed. It is also of advantage to step the concrete in case of foundations or walls, as the solidity will thus be improved. If the interruption is more than an hour or two a thin layer of Portland cement mortar is advisable on top of the layer last brought in.

When the tamping and ramming is finished, the concrete should be left to set undisturbed, and it is advisable to wet it at intervals, particularly in warm or dry weather. It should also be protected from strong winds.

Care should be taken not to leave openings and holes for piping in places where great bending moments occur.

The striking of the centering has already been dealt with. Should it be decided to plaster the concrete, it should be done immediately after striking the centering. In any case the concrete should receive its final treatment on the surface before it becomes too hard, although it is even then difficult to prevent hair cracks in the plaster. It is, therefore, better to give the concrete the desired appearance *en bloc* without plastering it over. In case plastering is decided on, the surface should be well wetted before this is done.

As has already been said, various finishes can be given to the concrete by treating the centering in a special way. If it is desired to give the concrete a rough appearance, the surface is washed and rinsed as soon as the forms are removed. The thin cement film on the surface comes off and the aggregate is thus laid bare. The roughness depends, of course, on the size of the aggregate used and the mixing of the whole concrete. In cases where a very rough aggregate must be used, yet a finely coarse surface is required, a special mixture of small aggregate, sand and cement may be put in first against the forms, before the main body of concrete is brought in, care being taken to get a perfect union of the two.

Mineral oxides may be added to give a colour effect.

To get the appearance of a washed surface, it is also possible to chip the surface with a sharp hammer and wash off with diluted spirits of salts, which must of course be well rinsed off afterwards.

Fig. 31 [1] shows a finished surface of concrete, composed of 1 part of Portland cement, 2 parts of yellow sand, and 3 parts of $\frac{3}{8}$ in. screeded stone, after being scrubbed.

Fig. 32 [1] shows yellow bar sand mortar, composed of 1 part of Portland cement to 3 parts of yellow sand.

The expansion and contraction of concrete, specially if the areas are large, is considerable and the occurrence of cracks should be avoided by expansion joints. These are made by inserting greased boards between the various sections of the work and withdrawing them just as the concrete is setting and filling the cavity with sand. Several thicknesses of tarred paper may also be inserted between the different sections and left in the concrete. The presence of the iron reinforcement largely prevents cracking, or at least causes the cracks to be so small as to be barely visible. For this reason the more meshwork there is in a slab, the more perfect the concrete surface is likely to be.

[1] From *Everyday Uses of Portland Cement.*

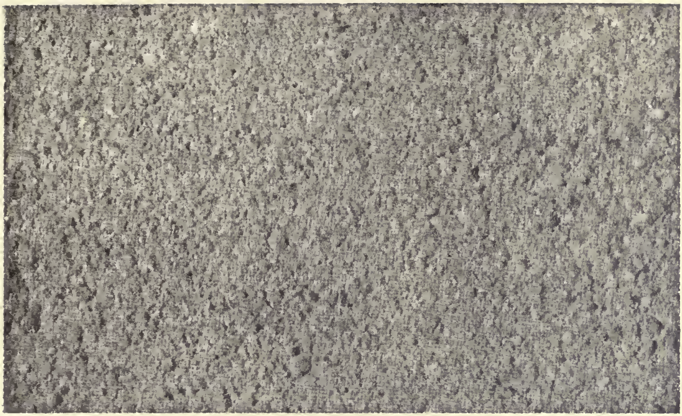


Fig. 31.—Surface Finish.

Fig. 32.—Surface Finish.

[*To face page* 44.]

Floors of reinforced concrete are finished by screeding in the usual way. Battens are embedded in the concrete, a few feet apart, and on top of these a board moved backwards and forwards. If a fine finish is required the surface is steel trowelled.

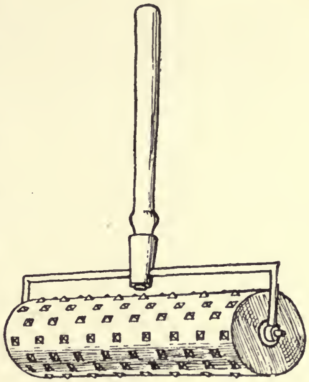


FIG. 33.[1]

The surface may be made rough to give a better foothold, and for this purpose an indenting roller (Fig. 33 [1]) is used. Or the surface may be cut up into squares by means of a joint cutter (Fig. 34 [1]).

As regards the *testing of the concrete*. The report of the R. I. B. A. Committee says as follows:—

Before the detailed designs for an important work are prepared and during the execution of such a work, test pieces of concrete should be made from the cement, sand and aggregate to be used in the work, mixed in the proportions specified. These pieces should be either cubes of not less than 4 ins. each way, or cylinders not less than 4 ins. diameter, and of a length not less than the diameter. They should be prepared in moulds, and punned as described for the work. Not less than 4 cubes or cylinders should be used for each test, which should be made twenty-eight days after moulding. The pieces should be tested by compression, the load being slowly and uniformly applied. The average of the results should be taken as the strength of the concrete for the purposes of calculation, and in the case of concrete made in proportions of 1 cement: 2 sand: 4 hard stone, the strength should not be less than 2,400 lbs./in.^{2}

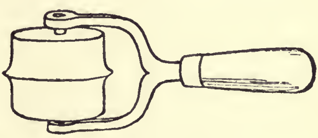
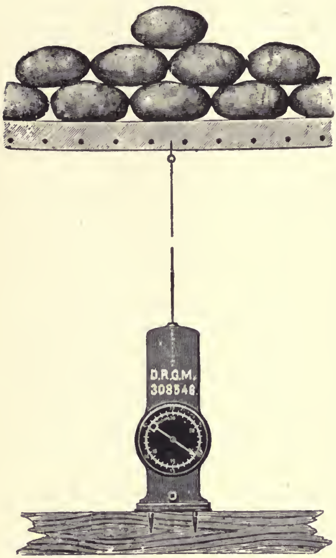


FIG. 34.[1]

[1] From *Everyday Uses of Portland Cement.*

Loading tests on the structure itself should not be made until at least two months have elapsed since the laying of the concrete. The test load should not exceed one and a half times the accidental load. Consideration must also be given to the action of the adjoining parts of the structure in cases of partial loading. In no case should any test load be allowed which would cause the stress in any part of the reinforcement to exceed 2/3 of that at which the steel reaches its elastic limit. There is a decided tendency in this country to impose tests greatly exceeding all practical contingencies.



Figs. 35 and 36.

Figs. 35 and 36 illustrate an apparatus for measuring the deflection of floors under test, which is in general use on the Continent (Agent, The Concave Floor Co., 1 Hawstead Rd., Catford, S.E.). The same apparatus can also be used for measuring horizontally, as for instance in loading tests of walls or other upright structures.

The same firm also supply a very handy patent bracket which supports centering and thus saves a great deal of cutting (Fig. 37).

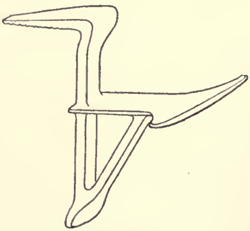


Fig. 37.

CHAPTER IV

LOADS, MOMENTS, STRESSES AND VARIOUS APPLICATIONS

A. FLOOR SLABS.

ASSUMING the crushing strength of the concrete to be 2,400 to 3,000 lb./in.^{2} after twenty-eight days, and the steel to have a tenacity of not less than 60,000 lb./in.^{2}, the following stresses may be allowed:—

	lbs./in.^{2}
Concrete in compression in beams subjected to bending	600
Concrete in columns under simple compression . .	500
Concrete in shear in beams	60
Adhesion of concrete to steel	100
Steel in tension	15,000 to 17,000

If the concrete is differently proportioned than stated above (1 : 2 : 4) the stress in compression allowed in beams may be taken at $\frac{1}{4}$ and that in columns at $\frac{1}{5}$ of the crushing stress of concrete cubes of sufficient size at twenty-eight days after gauging. If stronger steel is used, the allowable tensile stress may be taken at $\frac{1}{2}$ of the stress at the yielding point of the steel. The "yield-point" or yielding point is determined by careful observation of the drop of the beam or belt in the gauge of the testing machine. In mild steel the yielding point (the true elastic limit being several thousand pounds lower) is safely taken at 30,000 lbs./in.^{2}

High carbon steel has a yielding point of 50,000 to 55,000 lbs./in.[2] The cold-rolling or drawing of mild steel increases the yielding point, 65,000 lbs. often being obtained.

As has been previously stated, the fundamental principles of reinforced concrete are that the concrete resists the compression and the steel the tension, the tensional resistance of concrete being neglected.

An ordinary floor slab is the simplest form of a structure exposed to tension and compression, yet it depends very much whether the slab is freely supported or continuous or built in at both ends, and the reinforcement must be placed in such positionand be of such strength as to fully do its required work.

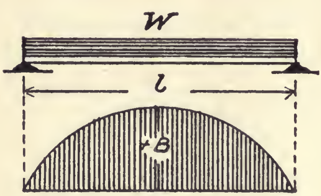


FIG. 38.

If the slab is *supported at both* ends and uniformly loaded the following facts must be considered: The bending moments at the supports are o, they increase towards the centre and are greatest at the centre, or the compressive stresses above the neutral axis and the tensile stresses below it increase towards the centre (Fig. 38).

FIG. 39.[1]

FIG. 40.[1]

FIG. 41.[1]

Fig. 39[1] shows the simplest form of a concrete slab; the reinforcement is placed in the line of tension and all the compressive stresses are taken by the concrete. The reinforcements are best placed as near to the most stressed fibre as possible.

Fig. 40[1] shows another form of simple reinforcement, the latter following the line of stress, which increases from the supports towards the centre. Wire mesh reinforcements in floors are placed in this fashion.

The bending up of rods towards the supports is most important, as will be explained later on, to resist the shearing stresses.

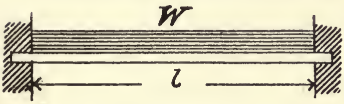


FIG. 42.

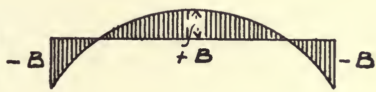


FIG. 43.

Fig. 41[1] shows an arrangement to be used where economy of concrete and reduced thickness is desired, the rods not only taking the tension but also supporting the concrete to resist compression, although the latter effect is not very great.

If the slabs are so arranged that *both ends are fixed* the effect is much more favourable. The tension is considerably less and the elastic line shows two turning-points, *viz.*, the bending moment is in two places = o. There are in this case positive and negative moments, and the former is greatest in the centre while the greatest negative moments are at the fixed ends. Consequently in the centre portion of the slab the

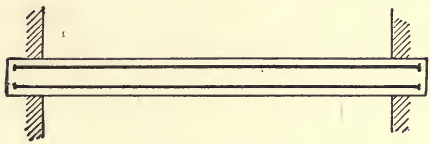


FIG. 44.[1]

FIG. 45.[1]

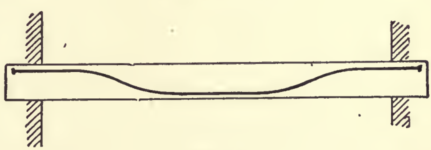


FIG. 46.[1]

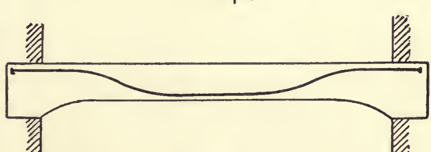


FIG. 47.[1]

lower fibres are in tension and the upper fibres in compression, while at the fixed ends the tension is in the upper and the compression in the lower fibres. See Figs. 42 and 43.

Fig. 44[1] shows a simple arrangement of reinforcements for such a slab. The reinforcement is placed at top as well as bottom. If the turning point can be ascertained, that is, if it can be shown at what point in the upper fibres the tension ceases and the compression begins, the reinforcement as shown in Fig. 45[1] can be adopted.

Figs. 46[1] and 47[1] show a very good arrangement of the reinforcements. Only one rod is used, which, however, resists the tensile stresses in the upper fibres as well as those in the lower fibres. The arrangement in Fig. 47 gives a better fixing and better results.

Figs. 48[1] and 49[1] show other forms of reinforcement, the arrangement in Fig. 48 being very useful for slabs supported at both ends as well as those securely fixed. In the latter case the moments towards the supports become theoretically = 0, and consequently only a part of the rods calculated for centre of slab is necessary in the lower fibres. The other parts are bent upwards and considerably strengthen the slab against shear.

FIG. 48.[1]

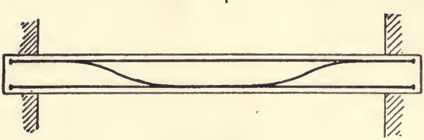


FIG. 49.[1]

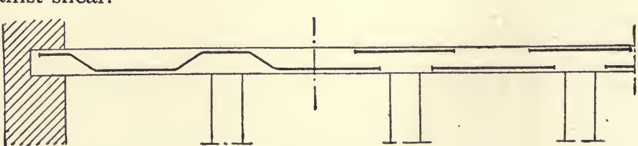


FIG. 50.[1]

If the slabs are designed as continuous over several supports negative moments are created over these supports and consequently reinforcements must be arranged at these points near the outer fibres to resist the tension (Fig. 50 [1]).

In the case of cantilevers the slabs are considered as securely fixed at one end (Fig. 51 [1]). The stresses are opposite to the stresses in slabs supported at both ends; the upper fibres are in tension and the lower in compression. Consequently the reinforcements must be arranged in the upper fibres. If the projection is considerable as compared with the section of the slab, it is advisable to place the reinforcement also in the compressed fibres (Fig. 52 [1]). Fig. 53 [1] shows another arrangement which at the same time effects saving of material.

Fig. 51.[1]

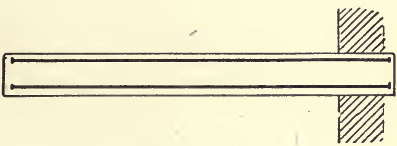


Fig. 52.[1]

In ordinary reinforced slabs the rods are simply arranged to run through the slab, but where any tension rods are used, it is advisable, particularly in cases of greater spans, to build in straps or hangers as shown in Fig. 54[1],

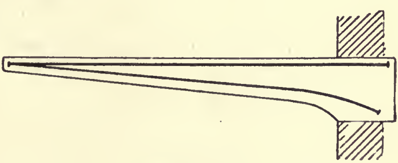


Fig. 53.[1]

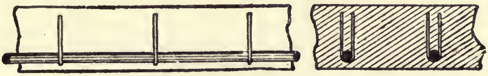


Fig. 54.[1]

and where compression as well as tension rods are used, they can

Fig. 55.[1]

be joined together by means of straps (Fig. 55 [1]), the straps in either case resisting the shearing stresses.

B. RIBBED OR BEAM CEILINGS.

These are used when larger rooms are to be covered in. The beams are arranged parallel to the shorter side of the room and connected with slabs. If the spans are too great, the beams are supported at intervals with columns or piers.

Fig. 56[1] shows the arrangement of an ordinary beam ceiling.

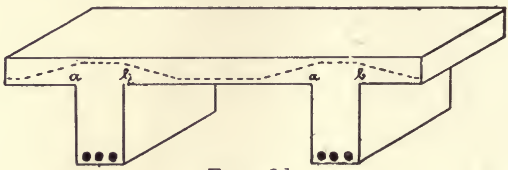


FIG. 56.[1]

The shearing stresses between slab and beam are considerable, and consequently the section *a* to *b* is usually strengthened with straps or hangers (see Fig. 57[1]).

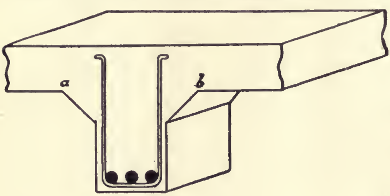


FIG. 57.[1]

As regards the reinforcement of beams, the same rules apply here as previously laid down for slabs, the reinforcement depending on the means and kind of support, *viz.*, whether the beam is freely supported at both ends, continuous, or fixed at ends. When positive moments occur, the rods are, therefore, placed in the lower part as close to the most stressed fibre as possible, and where negative moments are to be dealt with in the upper fibres.

The distance of the beams depends on the dimensions of the room, the spans, and the loads to be carried. If spaced short distances apart, the slabs can be made thinner, while with large distances stronger slabs are necessary. If large rooms have to be covered, main beams and subsidiary beams may be arranged, the slab being continuous over both. The slabs as well as the beams are continued and built into the brickwork, the same as is the practice with ordinary steel girders and fireproof floors. The

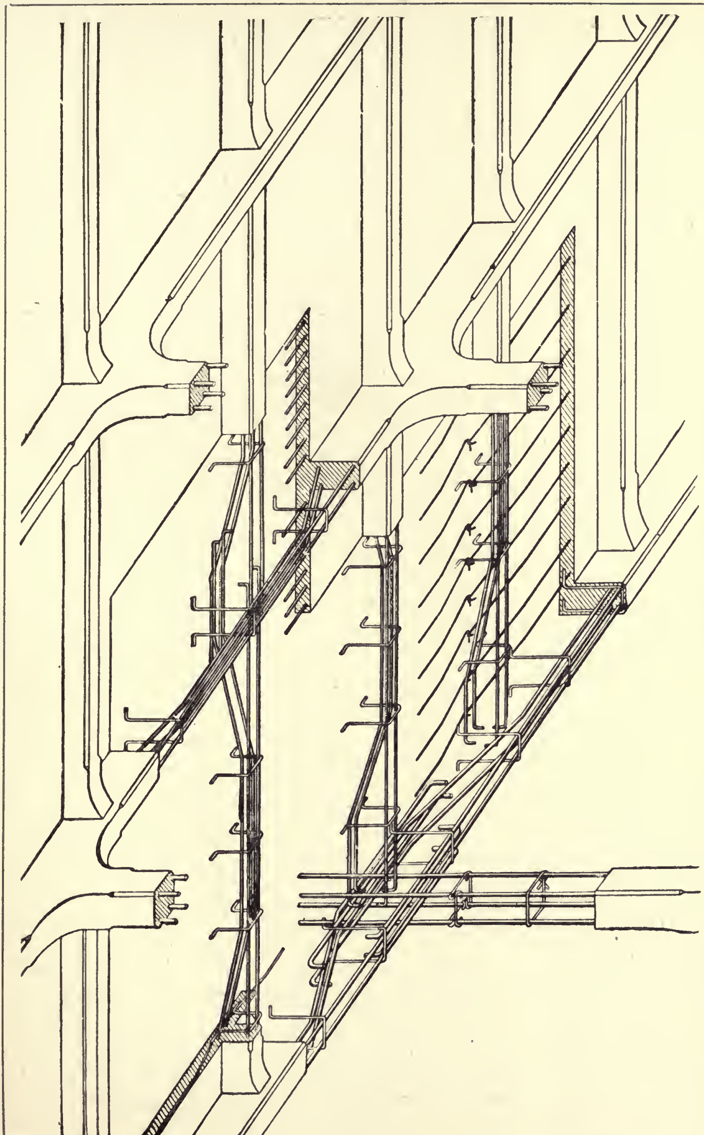


Fig. 58.

centering for beam ceilings is somewhat more expensive than that for simple slab ceilings, but the former will, as a rule, be more economical.

Fig. 58 shows a typical arrangement of a beam ceiling with main and secondary beams and continuous floor slabs, the main beams being supported by reinforced concrete columns.

C. STANCHIONS AND COLUMNS.

These are, as a rule, required to take up as little room as possible. They are reinforced with square or round rods, placed near the quoins and usually made with a square section and chamfered corner. The columns have to support, generally, simply a crushing load. The tendency to burst outward is resisted by placing steel horizontally in the columns in the shape of hoops. The upright rods are designed to resist partly the compression and thus reduce the thickness of concrete. Very often a spiral reinforcement is used. Fig. 59[1] shows the arrangement of a column. Sufficient concrete must be between the outside and the steel reinforcement to protect the latter from moisture and fire.

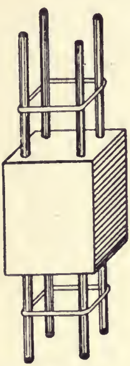
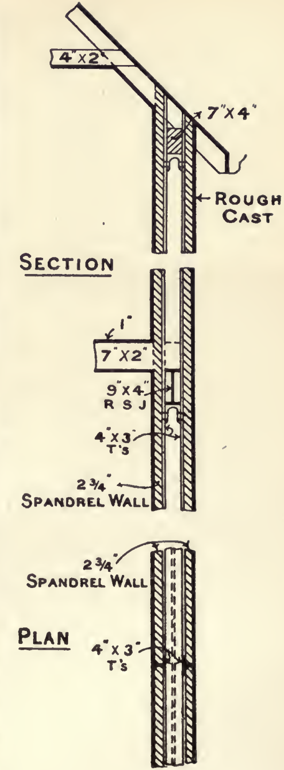


FIG. 59.[1]

D. WALLS.

Walls are constructed with an arrangement of rods placed lattice-wise and are otherwise constructed on the same principles as columns or slabs.

Mention must be made of the spandrel patent system of reinforced brickwork (The Fireproof Partition and Spandrel Wall Co., Bank Chambers, 92 Tooley Street, London Bridge, S.E.). These walls are particularly useful for enclosing buildings. The whole area of the wall is divided into squares (about 18 ins.) formed by hoop iron netting, without penetration or fixing at the points of crossing. The squares thus formed are filled with concrete in situ or with slabs.



FIGS. 60, 61.—BUILDING UNDER CONSTRUCTION.

[To face page 55.]

In case of dwelling-houses it has often been found that con-

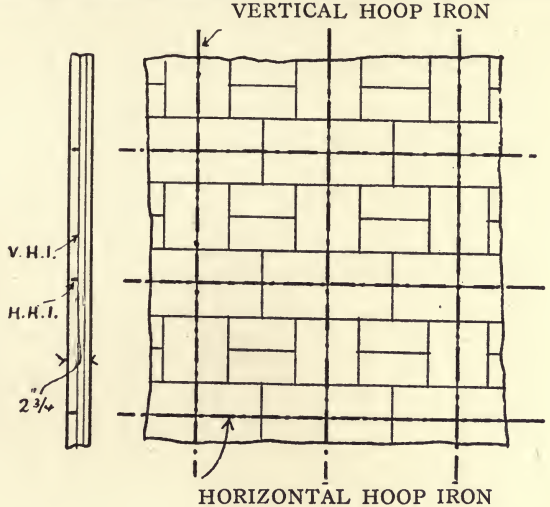


Fig. 62.

crete walls are cold and may cause condensation, and for this reason the hoop iron netting work is often filled in with brickwork instead of concrete. As the netting practically forms a lattice-girder, the walls support themselves between stanchions or piers and a great saving in excavating and foundations is effected. The peculiar arrangement of the hoops give maximum strength and resistance against side pressure (Fig. 62).

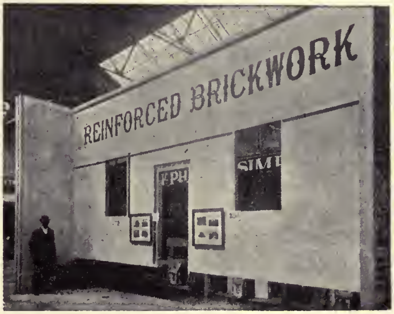


Fig. 63.

Fig. 63 illustrates a self-supporting wall 3 inches thick unsupported for 30 feet, and Figs. 60 and 61 a building on this system during erection.

FIG. 64.[1]

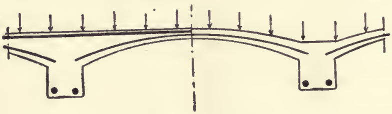


FIG. 65.[1]

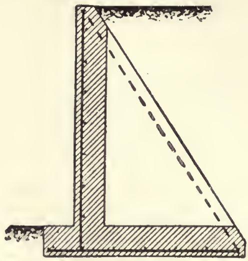


FIG. 66.[1]

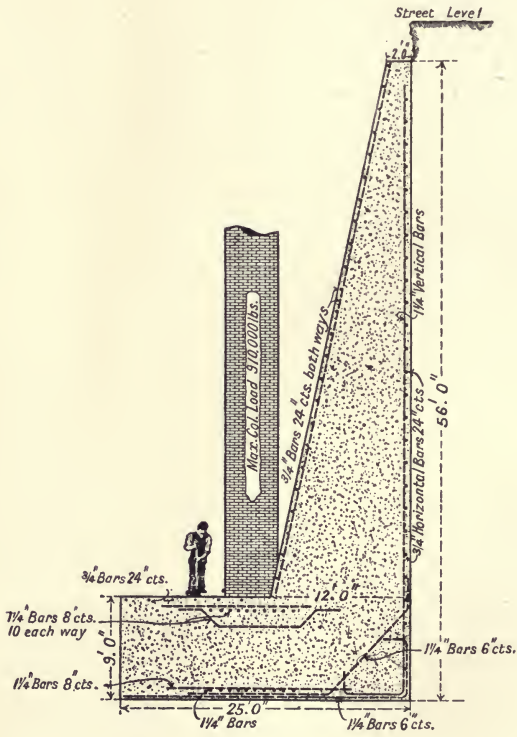


FIG. 67.

Retaining walls are usually designed as slabs between the buttresses (Figs. 64 and 65 [1]). For bigger walls a section as shown in Fig. 66 [1] is often adopted by means of which the soil is made to act on the groundplate and thus strengthen the construction. The groundplate is connected with the wall slab by means of reinforced struts, the reinforcement of the slab being calculated to resist the pressure of the earth.

The striking illustration of a retaining wall (Fig. 67 [1]) is taken from the Indented Steel Bar Co. handbook and forms part of Selfridge's Stores Building, Oxford Street, London.

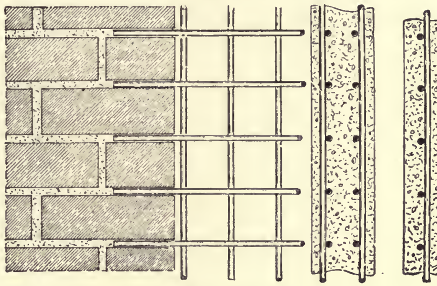


FIG. 68.[1]

Fig. 68 [1] shows a reinforced concrete wall in the Monier system. The reinforcement consists of strong wire and is, as a rule, placed in the centre of wall. Where exceptional stresses, such as wind pressure, must be resisted, a double system of wire-netting is used, placed near the outsides of the wall. In case of hollow walling the outer wall is made thicker than the inner wall.

E. ARCHES, VAULTS AND BRIDGES.

The axis of arches may occur in different planes, horizontal, vertical or at an inclination (Figs. 69, 70, 71 [1]).

FIG. 69.[1]

FIG. 70.[1]

FIG. 71.[1]

If the spans and loads to be carried are not appreciable, reinforcement of the lower fibres near the soffit is sufficient, special care being taken that the rods are well fixed in the abutment.

Fig. 72.[1]

For heavier work the upper fibres are also reinforced (Figs. 72, 73[1]), but it is often sufficient to reinforce the upper fibres only towards the supports (Figs. 74, 75[1]).

Fig. 73.[1]

The reinforcement can be arranged at equal distances throughout, but for heavier work it is advisable to increase the thickness towards the supports (Fig. 76[1]). A still stronger arrangement is shown in Fig. 77[1], where stirrups further strengthen the arch and take the shearing stresses. A similar arrangement as used in ribbed ceilings may also be adopted with main and subsidiary beams and a continuous slab.

Fig. 74.[1]

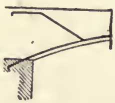
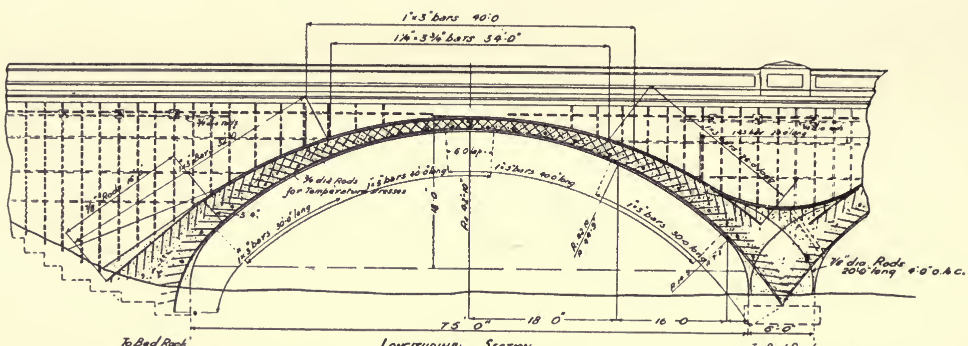


Fig. 75.[1]

Fig. 76.[1]

Fig. 77.[1]



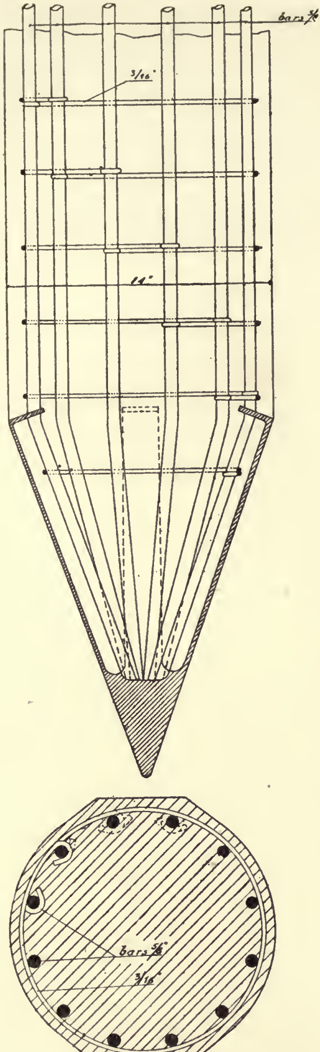
FIGS. 78, 79.—VIEW AND LONGITUDINAL SECTION OF CHARLEY CREEK BRIDGE.

[To face page 58.]

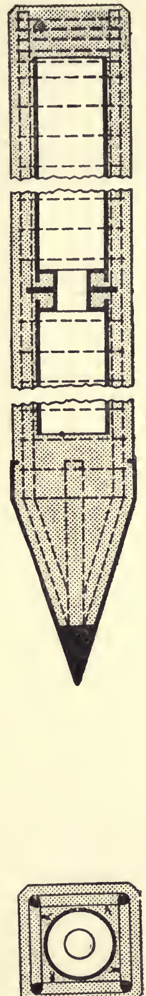
Fig. 80.—Railway Bridge over the River Sée

AVRANCHES, FRANCE. TOTAL LENGTH, 281 FEET.

[*To come between pages 58 and 59.*]



FIGS. 81 and 82.



FIGS. 83 and 84.

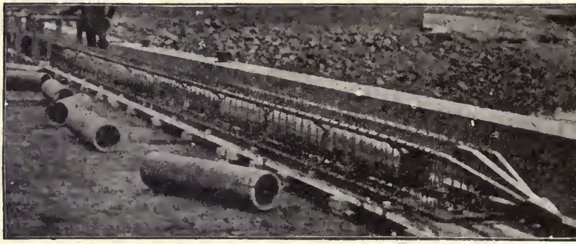


FIG. 85.

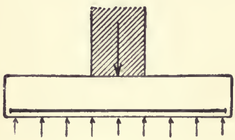


FIG. 86.

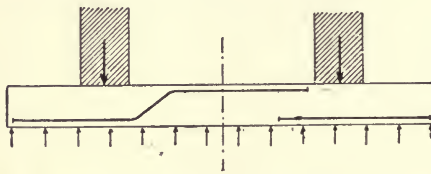


FIG. 87.

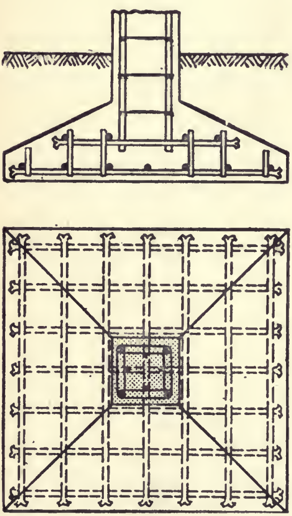


FIG. 88.

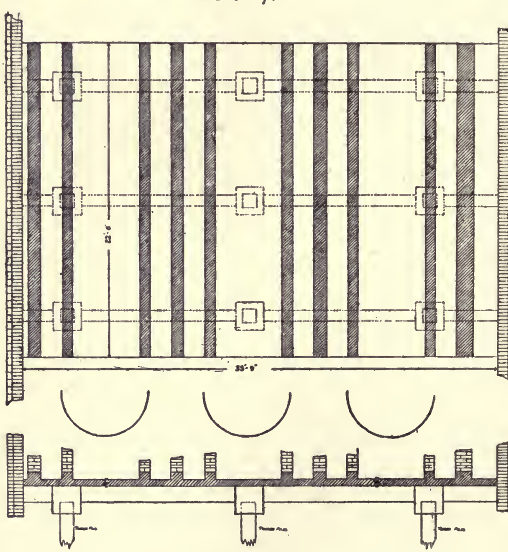
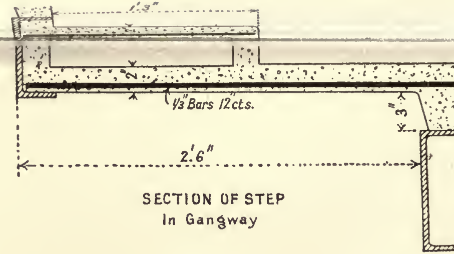


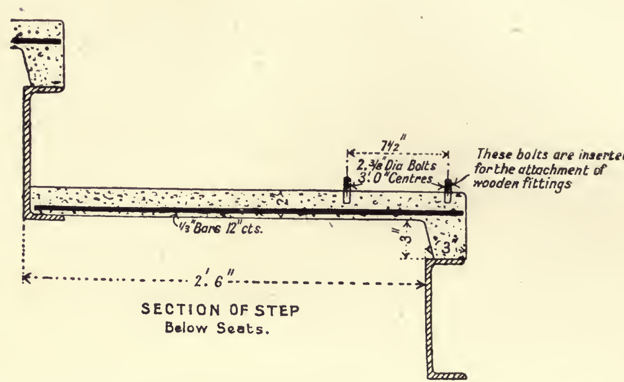
FIG. 89.


FERRO CONCRETE
SEATING TO STADIUM
CONSTRUCTED WITH
PATENT INDENTED STEEL BARS
BY EMPIRE STONE Co LTD
THANET HOUSE.
STRAND. W.C.


1'3"



SECTION OF STEP
In Gangway



SECTION OF STEP
Below Seats.

FIGS. 90-92.—PROSPECTIVE VIEW AND SECTIONS SHOWING SEATING ACCOMMODATION AT THE STADIUM, LONDON.

[To come between pages 60 and 61.]

For bridge building reinforced concrete is now being fairly generally adopted owing to the great stability obtained and the great saving in up-keep and repair. Figs. 78, 79 illustrate a bridge reinforced with Kahn bars.

F. FOUNDATIONS AND PILES.

Reinforced concrete is now largely used for foundation work.

Piles are made similarly to columns; they usually receive a wooden cap during driving operations to prevent splintering. There are a great variety of systems and constructions. Figs. 83 and 84 show a Coignet pile as used in the foundations of a tobacco warehouse at Bristol. It is interesting to note that these piles, weighing 5 tons each, and being some 45 feet long by 15 ins. in diameter, could be lifted at one end, the other resting on the ground, thus demonstrating the great strength and resistance of the construction. Figs. 81, 82, 85 illustrate a Hennebique pile.

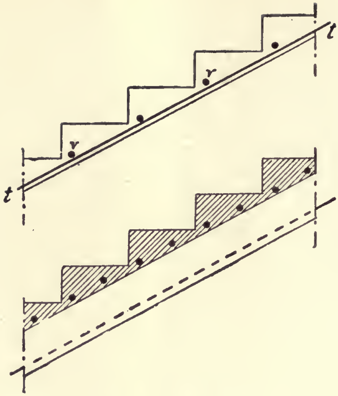
For ordinary level or raft foundations wire meshing or expanded metal are extremely useful. Wherever the columns, piers or concentrated loads occur, the rods must be so arranged as to resist the compressive or tensile stresses as the case may be (Figs. 86, 87).

A Hennebique column base is shown in Fig. 88, and a boiler foundation in the Coignet system in Fig. 89.

G. STAIRS, ETC.

Concrete stairs are reinforced as shown in Figs. 93-95.[1] The tension and distributing rods are placed in the lower fibres and the stairs are either cast in situ or made independently before fixing. In the latter case the steps are built into the walls and the rods placed near the surface, the tension being in the upper fibres. If resting on strings, the tension is again in the lower fibres and the reinforcements placed accordingly.

An interesting piece of work is the Stadium at the Franco-British Exhibition (Figs. 90-92), the reinforcement used being the indented steel bar.



Figs. 93[1] and 94.[1]

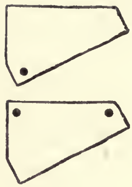


Fig. 95.[1]

H. PIPES, WATER MAINS, SEWERS, etc.

The reinforcement is similar to that of columns, expanded metal or wire reinforcement being also largely used. Fig. 96 shows reinforcement for a water main.

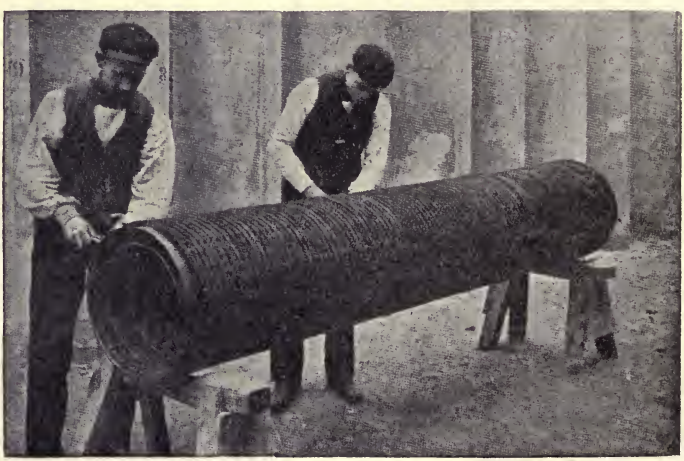


Fig. 96.

Telegraph poles, fence posts, etc., are also made of reinforced concrete and are constructed in a similar manner.

Fig. 97 [1] illustrates a simple reinforcement for water tanks. The rods are spaced closer towards the bottom where the stresses increase.

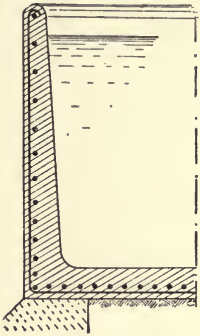


Fig. 97.[1]

I. ROOFS.

The construction of flat roofs is done on the same principles as that of floor slabs. The material opens up a new field for the design of curved and ornamental roofs of any shape desired, very fine examples of which are to be found in Indian architecture. Concrete being a non-conductor, an even temperature is maintained in buildings. Sheet and wire reinforcements are naturally most economical and practicable as the concave system, expanded metal or lock-woven mesh, Fig. 98 illustrating a roof constructed in the latter system.

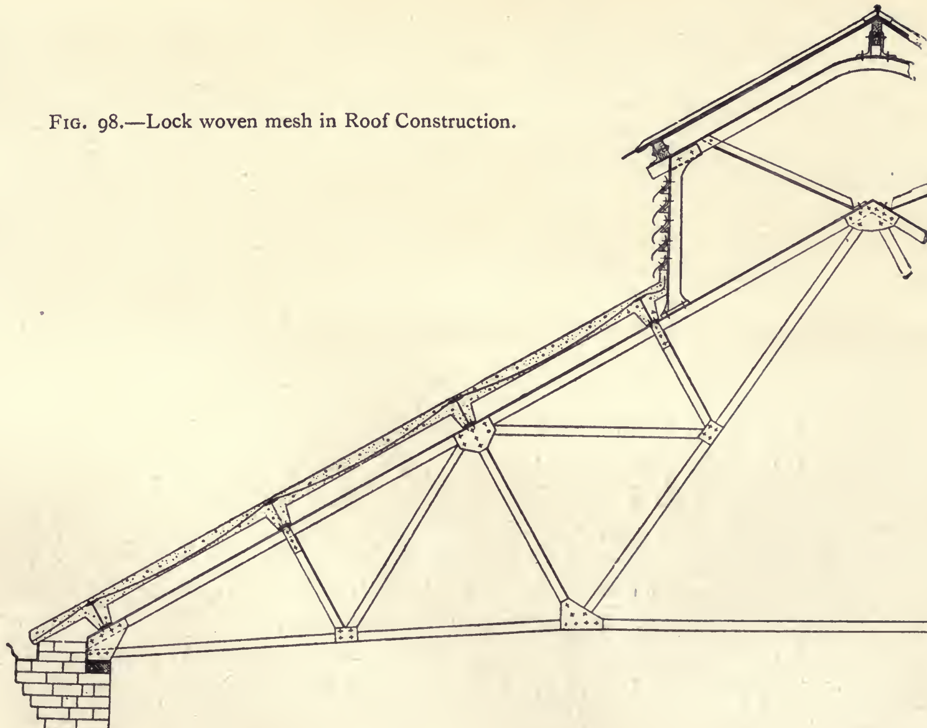
The Visintini system lends itself particularly well for great spans, and Fig. 99 is a photo of a roof constructed on this principle, during erection, the span being 11·80 metres or about 38 ft., and the distance of principals 4·68 metres or about 15 ft. 6 ins.

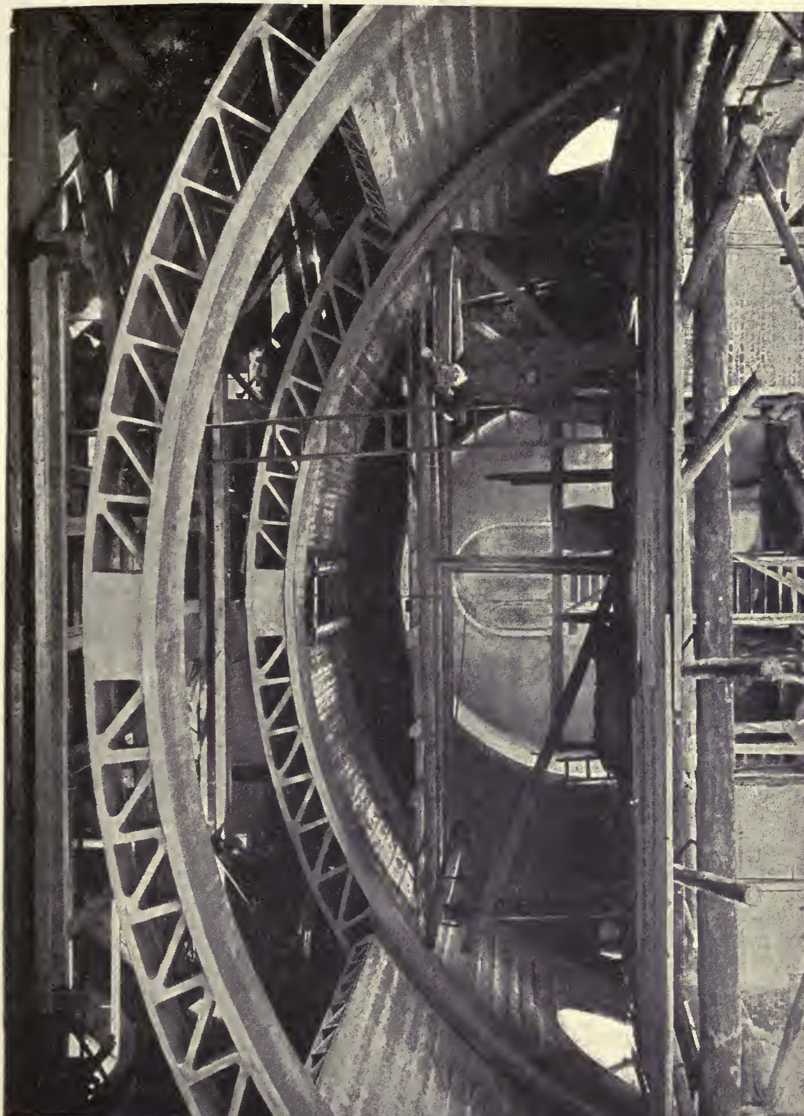
Figs. 100-107 [1] illustrate details and connexions of the various roof members to the reinforced concrete which is constructed in the Monier system, and Fig. 108 [1] is a flat roof self-supporting without principals or binders.

When deciding on the roof covering, the material used must secure protection from change of temperature and extreme heat and cold. It is advisable to arrange for some isolating layer of cork, roof felt or the like, and openings should be left at bottom of rafters to create a constant current of air and ventilation to prevent condensation.

For flat roofs a hollow construction like the concave system (p. 134) is to be highly recommended. The air space effectively counteracts the influence of extreme heat and cold and secures a

Fig. 98.—Lock woven mesh in Roof Construction.





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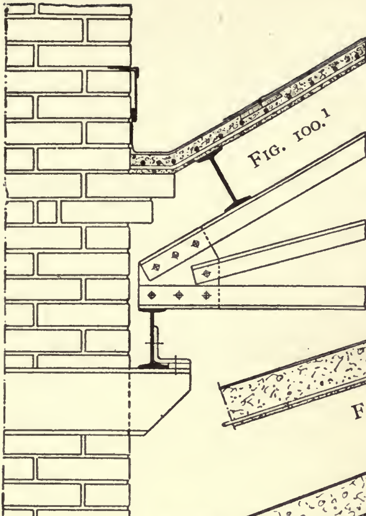


FIG. 100.[1]

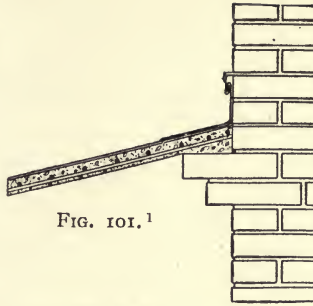


FIG. 101.[1]

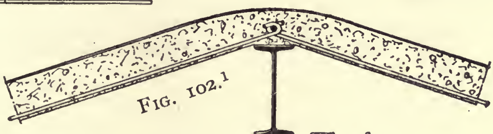


FIG. 102.[1]

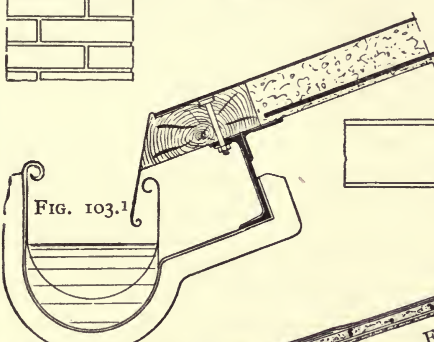


FIG. 103.[1]

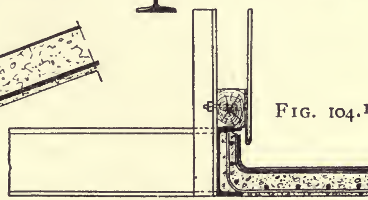


FIG. 104.[1]

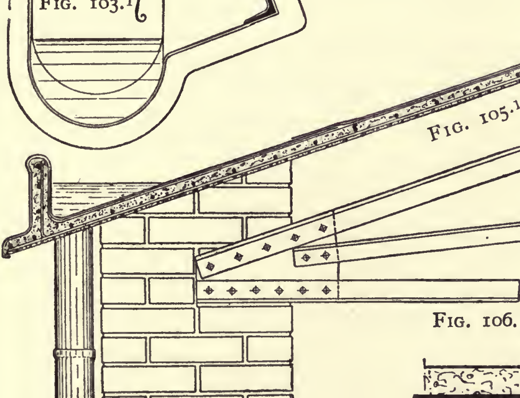


FIG. 105.[1]

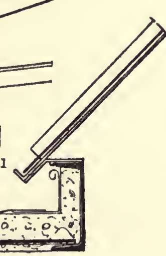


FIG. 106.[1]

perfect ventilation and constant circulation of air. The Vulcanite

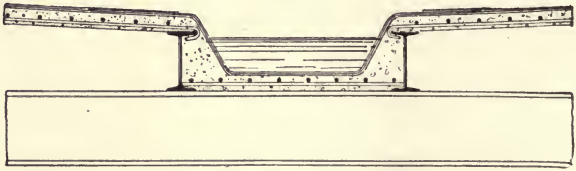


FIG. 107.[1]

system of roofing is also largely used, but can necessarily not give the same advantages as a hollow roof construction.

FIG. 108.[1]

CHAPTER V

RESISTANCE AND SAFE STRESSES, ETC.

THE various factors to be taken into account when designing reinforced concrete work are the following:—

As regards loads:—

1. The weight of the structure.

2. The permanent load to be carried.

3. The accidental load or the imposed load in addition to the weight of the structure.

4. The vibration, oscillation and shock.

In calculating the stresses, the member under consideration must be taken under the worst conditions, *viz.*, the calculation must be based on the greatest straining action the member may be subjected to.

The weight of reinforced concrete may be taken at **150 lbs./ft.^{3}** (many advocate to allow 156 lbs./ft.^{3}).

In structures subjected to very varying loads, together with a certain amount of vibration and shock, like factories, public halls, etc., the *factor for shock should be taken equal to half the accidental load.*

Where *machinery* has to be carried and the structure is, therefore, under *considerable vibration and shock*, the *factor for shock should be taken equal to the accidental load.*

For *columns and piers* of buildings having several stories, the *structures carrying the top floor* should be calculated to take *the full accidental load of floor and roof.* For the story below *10 per cent. less than* the figure allowed for the top floor, for the floor below this *20 per cent. less*, and so on to the floor at which the reduction amounts to *50 per cent.* of the assumed load on the floor. For all *lower floors* the *accidental loads on columns or piers* should be taken at *50 per cent.* of the loads assumed in calculating these floors.

As regards spans :—

Measure the spans as follows :—

For *beams*, the distance from *centre to centre* of bearings.

For *slabs supported at ends*, the *clear span* and *the thickness of slab.*

For *slabs continuous* of over more than one span *the distance from centre to centre of beams.*

As regards bending moments :—

The bending moments in case of a uniformly distributed load of w lb. per inch run of span are as follow :—

For *beams or slabs supported at the ends*, the greatest bending moment at centre of span of l inches is equal to $\frac{wl^2}{8}$.

For *beams continuous over several spans or fixed in direction at each end*, the bending moments are at the ends of span, and the beam should be reinforced at its upper side near the ends. If continuity can be relied on, the bending moment at the centre of span is $\frac{wl^2}{24}$ and that over the supports $= \frac{wl^2}{12}$. If the continuity is not quite perfect, the bending moment at the centre will be greater, and that at the supports less. Generally speaking, the *centre bending moment* should not be taken less than $\frac{wl^2}{12}$. These values are recommended by the R. I. B. A. Committee and now largely adopted in this country.

The Prussian Government regulations for continuous slabs or beams are as follows :—

"Slabs and beams, continuous over several spans, may, if the actual moment and the reactions at supports are not statically ascertained according to the rules for continuous beams freely supported in the centre and at the ends or proved by experiments, be calculated with a bending moment equal to **four-fifths** of the value, which would be applicable to a slab freely supported at both ends. The negative bending moment over the supports is to be taken equal to the moment of span for slab freely supported at both ends. Slabs and beams can only be considered as continuous if they rest on firm stanchions or reinforced concrete beams, level throughout. In arranging the reinforcing rods the possibility of negative moments occurring must be carefully considered. Beams may be considered fixed at the ends, only if special structural arrangements guarantee secure fixing.

In calculations *the continuity* must not be considered as extending to *more than over 3 spans.* Where the live load exceeds

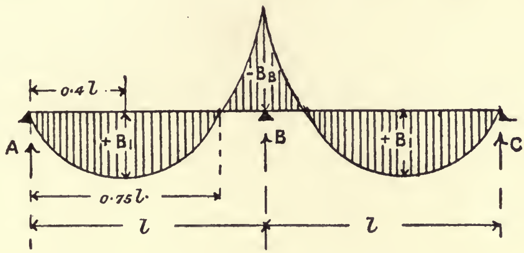


FIG. 109.[1]

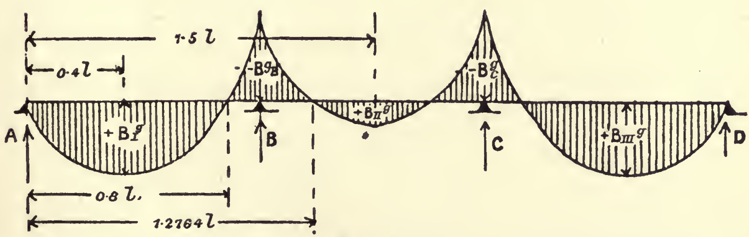


FIG. 110.[1]

1,000 kg. per sq. metre (1 ton per 10·76 ft.^{2} or 208·18 lbs./ft.^{2}), a calculation for the most unfavourable position of the load must also be made.

This would give the following values for the moment for a uniformly distributed load $W = g + p$ where g represents the self load and p the live load. (See Figs. 109 and 110.[1])

Moments.		Approximate calculation with $+\frac{Wl}{10}$ and $-\frac{Wl}{8}$ $p < 208$ lbs./ft.^{2}	Loads g and p uniformly distributed over the beam. $p < 208$ lbs./ft.^{2}	g uniformly distributed, p variable (exact calculation) $p > 208$ lbs./ft.^{2}
2 spans (3 supports)	Moment on span	$+0{\cdot}1\,(g+p)\,l^2$	$+0{\cdot}07\,(g+p)\,l^2$	$+0{\cdot}07\,g\,l^2 - 0{\cdot}095\,p\,l^2$
	Moment at support	$-0{\cdot}125\,(g+p)\,l^2$	$-0{\cdot}125\,(g+p)\,l^2$	$-0{\cdot}125\,g\,l^2 - 0{\cdot}125\,p\,l^2$
3 or more spans (4 or more supports).	Moment on end span	$+0{\cdot}1\,(g+p)\,l^2$	$+0{\cdot}08\,(g+p)\,l^2$	$+0{\cdot}08\,g\,l^2 + 0{\cdot}10\,p\,l^2$
	Moment on centre span	$+0{\cdot}1\,(g+p)\,l^2$	$+0{\cdot}025\,(g+p)\,l^2$	$+0{\cdot}025\,g\,l^2 + 0{\cdot}075\,p\,l^2$ and $+0{\cdot}025\,g\,l^2 - 0{\cdot}05\,p\,l^2$
	Moment at support	$-0{\cdot}125\,(g+p)\,l^2$	$-0{\cdot}1\,(g+p)\,l^2$	$-0{\cdot}10\,g\,l^2 - 0{\cdot}117\,p\,l^2$

As mentioned before, it is advisable to reinforce floor slabs over rectangular or nearly rectangular rooms diagonally, particularly strengthening the centre of the slab. This method is advantageous if the slab is quite square or one side slightly longer than the other, but does not give special advantages as soon as one side of the square becomes nearly double or more than double the other side.

Where the *slab is quite square*, experience has shown that the centre bending moment may be taken with safety at $\frac{wl^2}{16}$, the factor 16 being reduced gradually to 12 in cases where one side measures 2 of the other side; always provided that the slab is uniformly loaded and supported all round.

Foundation slabs are considered as beams supported at both ends and uniformly loaded. The walls or columns to be supported represent the supports and the soil pressure the load. Thus negative moments are created between the supports and positive moment near the supports. (See Figs. 111 and 112.[1])

FIG. 111.[1]

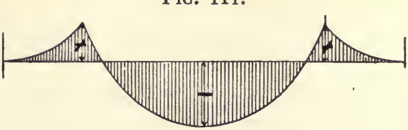


FIG. 112.[1]

As the concrete may be very differently proportioned according to the aggregate and sand used, it is impossible to adopt a uniform coefficient of elasticity. The strength of the material should be ascertained by tests in every case.

At any rate it is not advisable to operate with a *factor of safety less than 6*, that is, where reinforced concrete is exposed to compressive stresses it should not be loaded or stressed more than to the extent of **one-sixth** of its breaking moment, while in cases of columns or stanchions it should not be stressed more than **one-tenth** of its breaking moment.

The resistance of *concrete to tension* is very difficult to determine, and is so small that in reinforced concrete construction it is, as a rule, not taken into consideration at all.

The *resistance of concrete to shear* is also very difficult to ascertain. Tests have proved that it is at any rate greater than its resistance to tension and depends very much on the composition of the concrete. Broadly speaking, tests have shown it to be about **300 lbs./in.**^{2}, so that allowing for a factor of safety of 5 a stress of **60 lbs./in.**^{2} may be adopted in case of concrete mixed 1 : 2 : 4.

The adhesion of the concrete to the steel is best proved by tests with ordinary concrete slabs compared with such reinforced with steel. It has been found that the latter resist a much greater tension, which can only be attributed to the adhesion between the two materials. The cause is probably a purely mechanical effect, resulting from the circumstance that the concrete in setting contracts and thus gets a firmer grip on the iron or steel. Certain experts attribute it to a chemical action. Whatever the cause may be, the fact certainly remains that concrete is considerably strengthened on account of this adhesion. It increases proportionately with the percentage of reinforcing rods and the circumference of same. Consequently *it is better to use more rods of a small diameter than a reduced number of a greater diameter.* Small diameter rods are also more easily manipulated. Experiments have proved that the surface of the reinforcement has very little to do with the amount of the adhesion. Rods with smooth surfaces exhibited almost the same adhesion as those with a rough surface. As a rule, round rods showed a better adhesion than rods of another section. The amount of adhesion depends also largely on the quality and composition of the concrete and the proportion of water used, and it may be taken that the adhesion is the greater the stronger the composition, the slower the setting of the cement takes place, and the older the concrete is. It also is increased with coarser grain of sand and reduction of the quantity of water used. Practical experience has also shown that vibrations and similar shocks do not interfere with the adhesion.

The adhesion is greater than the resistance of concrete to shear, as in testing operations where rods were pulled out of the concrete, small particles of the concrete still adhered to the steel. The adhesion has been ascertained to be some 500 lbs./in.^{2}, so that allowing a safety factor of 5, 100 lbs./in.^{2} may with confidence be adopted. This is really more than ample, considering that in calculations the resistance of the concrete to tension is neglected, and, as a rule, only the straight rods are taken into account while the bent rods and stirrups or hangers are also neglected. Furthermore, the ends of the rods, if bent over, as a matter of course considerably increase the resistance to sliding of the rods through the concrete.

As before mentioned, particular care must be taken that all reinforcements are perfectly embedded in the concrete and no voids left. It is not always necessary to join the ends of the rods except where great bending moments occur. As a rule, it is advisable to effect the joins as shown in Fig. 8, page 31. Wherever necessary or desirable the free ends should be well bent over or so arranged as to make slipping impossible. Many of the patent bars (Kahn, Indented steel bar, etc.) are designed to prevent this slipping and to get better adhesion and hold on the concrete by means of wings or indentations in the rods.

As regards *expansion and contraction*, concrete, if the setting takes place in the open, will contract, while, if under water, it will expand. There are in consequence certain stresses in reinforced concrete during setting. In the first case tensile stresses are created in the concrete and compressive stresses in the steel, while in the second case (under water) the stresses are opposite, compressive in the concrete and tensile in the steel. This circumstance often causes fine cracks, but the stresses are so small that they are not considered in calculations except in special cases like water tanks, etc.

A great objection to the new method of building, namely, that in case of fire the expansion of concrete and steel would be very different and thus cause failure of the structure, has now been proved entirely erroneous. Many experiments and tests have shown that the coefficient of expansion of the two materials is practically the same. That of steel is about ·0000066 per degree Fahrenheit. Concrete mixed 1 : 2 : 4 expands between ·0000060 and ·0000065 per degree Fahrenheit, and it is this circumstance particularly that makes reinforced concrete so desirable for fireproof buildings.

As regards the elasticity of the reinforcement, wrought-iron rods have practically gone out of use, and been replaced by mild steel, high carbon steel and cold drawn steel. Mild steel is usually used now. The elastic limit of mild steel is about 30,000 lbs./in.^{2}, that of high carbon steel about 55,000 lbs., while that of cold rolled or drawn mild steel is about 65,000 lbs. It is largely a question of price against quantity of material. The *modulus of elasticity* of all three steels is about 30,000,000 lbs./in.^{2}, or *15 times* that of concrete.

Subjoined is an extract from the report of the R. I. B. A. Committee on reinforced concrete showing the various values. The subsequent calculations are based on these figures adopted by the Institute.

The internal stresses are determined, as in the case of a homogeneous beam, on these approximate assumptions :—

(*a*) The coefficient of elasticity in compression of stone or gravel concrete, not weaker than 1 : 2 : 4, is treated as constant and taken at one-fifteenth of the coefficient of elasticity of steel.

		lbs./in.²
Coefficient for concrete	$= E_C =$	2,000,000
,, ,, steel	$= E_S =$	30,000,000

$$\frac{E_S}{E_C} = 15.$$

It follows that at any given distance from the neutral axis, the stress per square inch on steel will be fifteen times as great as on concrete.

(*b*) The resistance of concrete to tension is neglected, and the steel reinforcement is assumed to resist all the tension.

(*c*) The stress on the steel reinforcement is taken as uniform on a cross-section, and that on the concrete as uniformly varying.

Working stresses.—If the concrete is of such a quality that its crushing strength is **2,400 to 3,000 lbs./in.²** after twenty-eight days, and the steel has a tenacity of not less than **60,000 lbs./in.²**, the following stresses may be allowed:—

	lbs./in.²
Concrete, in compression in beams subjected to bending	600
Concrete in columns under simple compression	500
Concrete in shear in beams	60
Adhesion of concrete to metal	100
Steel in tension	15,000 to 17,000

When the proportions of the concrete differ from those stated above the stresses in compression allowed in beams may be taken at one-fourth, and that in columns at one-fifth of the crushing stress of cubes of the concrete of sufficient size at twenty-eight days after gauging. If stronger steel is used than that stated above, the allowable tensile stress may be taken at one-half the stress at the yield point of the steel.

CHAPTER VI

FORMULAE FOR FLOOR SLABS AND BEAMS (SINGLE REINFORCEMENT)

If a concrete slab or beam, *supported at both ends*, is loaded, the various particles comprising the slab are shifted and the shape of the slab is consequently slightly altered. The upper fibres of the slab are compressed and the lower fibres stretched. These stresses are greatest in the external fibres (top and bottom of slab) and become less towards the centre of slab, until they become = 0 at the line of the "neutral axis" (see Fig. 113 [1]).

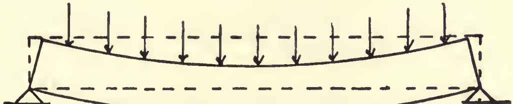


Fig. 113.[1]

All the fibres remain parallel to the neutral axis, which, owing to the stress, takes the form of a curve.

If we consider the slab first as a simple concrete slab without reinforcement and of a rectangular section, we find that, although all the various sections of the slab remain even, the sections are

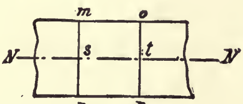


Fig. 114.[1]

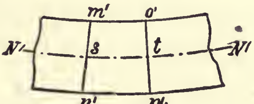


Fig. 115.[1]

not parallel to one another. There is a turning action with the neutral axis as the turning point. Figs. 114 and 115 [1] illustrate this;

Fig. 114 shows the slab before the stresses attack it, and Fig. 115 shows the same slab under stress. NN is the neutral axis, namely, the layer of fibres neither in compression nor tension. The originally parallel sections *mn* and *op* are moved into the places *m'n'* and *o'p'*. The distances *st* have remained the same, as the neutral axis has been neither lengthened nor shortened.

To counteract these stresses it is clear that steel should be inserted in the portion of the beam which is in tension, and it may also be desirable to reinforce the compressive layers.

The forces cause a variation of the fibres, the fibres of the compressive area becoming shorter and those of the tensile area longer.

The relative elasticity of the materials is quite different, the comparison being made by the ratio of the "coefficients of elasticity," which is the stress per sq. in. that would be necessary to stretch a material to double its original length, or compress it to half its original length if it retained its true elasticity up to that stress.

The elastic coefficient, E_S, for steel is constant until the elastic limit is reached, and in case of mild steel is taken at **30,000,000 lbs./in.²**

The elastic coefficient, E_C, for concrete, however, has a varying value, but for stresses up to 400 or 600 lbs./in.² — the maximum safety stresses allowed — may be taken as constant at **2,000,000 lbs./in.²**, or $\frac{1}{15}$th that of steel.

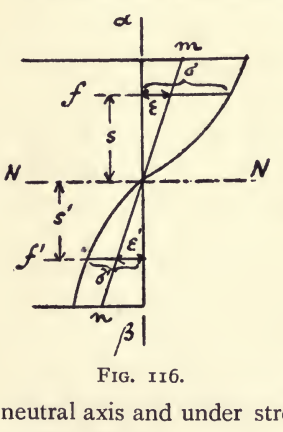


FIG. 116.

The ratio m of the two materials is, therefore,

$$m = \frac{E_S}{E_C} = 15.$$

In Fig. 116 the stresses are graphically illustrated. If the two fibres f and f' are at the distances s and s' from the neutral axis and under stress are altered in length to the extent ϵ and ϵ', we get

$$\epsilon : \epsilon' = s : s',$$

that is, the stresses are proportional to the distances from the neutral axis.

As before stated, the resistance of the concrete to tension is neglected for many reasons. Being of a very varying nature, true and reliable results are not available at present. Furthermore, the omission simplifies calculations very much, while practically giving an extra factor of safety. It stands to reason that only the fibres of concrete close to the neutral axis can be relied upon to resist the tension, and as this depends very largely on the workmanship in placing the concrete in position so that the cement perfectly embeds the reinforcements, it is better to allow for some errors of judgment and small voids which may occur.

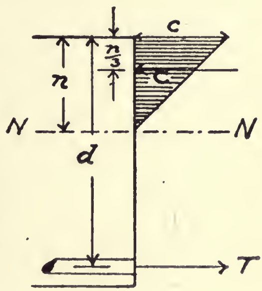


FIG. 117.

Consequently we omit the stress diagram below the neutral axis from consideration (Fig. 117).

As the calculation of the stresses depends largely on the position of the neutral axis, it becomes necessary to show how this position can be ascertained.

Supposing d to be the effective thickness of slab in inches,
n the distance of the neutral axis from the top of slab in inches,
b the width of strip of slab under discussion in inches,
c the compressive stress intensity on concrete,
t the tensile stress intensity on steel,
A_T the area of tensile reinforcement,

we get the compression,

$$C = \frac{c \cdot n}{2} \cdot b \tag{1}$$

and the tension,

$$T = t \cdot A_T \tag{2}$$

The internal resisting forces in compression and tension must balance each other, so that (Fig. 118)

$$\frac{c \cdot n}{2} \cdot b = t \cdot A_T \qquad (3)$$

The bending moment must equal the resistance of the concrete or reinforcement multiplied by the lever arm of the resisting forces, namely—

$$d - \frac{n}{3}$$

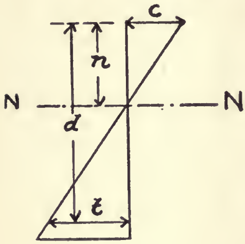


FIG. 118.

therefore (Figs. 119-120)

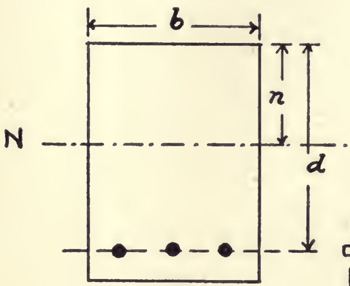


FIG. 119.

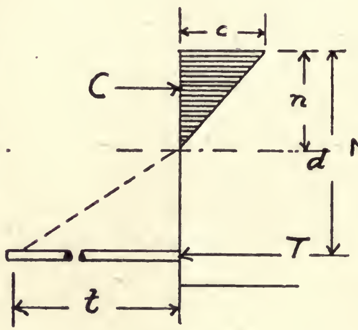


FIG. 120.

$$B = \frac{c \cdot n \cdot b}{6}(3d - n) \text{ or} \qquad (4)$$

$$B = \frac{t \cdot A_T}{3}(3d = n) \qquad (5)$$

As before illustrated, the stresses are proportional to the distances from the neutral axis multiplied by the coefficient of elasticity, or

$$c : t = n \cdot E_C : (d - n)E_S, \text{ or}$$

$$t = m \cdot c\frac{d - n}{n} \qquad (6)$$

Substituting this value in the former equation (formula 3), we get

$$\frac{n^2 . b}{2} = m . A_T(d - n), \text{ from which}$$

$$n = \frac{mA_T}{b}\left[\sqrt{1 + \frac{2bd}{mA_T}} - 1\right] \qquad (7)$$

If the values of c and t are to be checked in work already designed, the value for n may be inserted in the above formulæ 4 and 5.

The formula 7 thus fixes the position of the neutral axis, and it is clear that this position depends on the sectional area of the reinforcements and not on the load to be carried.

To ascertain the greatest stress of the concrete, c, we put the greatest bending moment B in lbs./in.² equal to the moment of resistance R, so that

$$B = \frac{c . n}{2} \cdot b\left(d - \frac{n}{3}\right) \text{ or}$$

$$c = \frac{2B}{b . n\left(d - \frac{n}{3}\right)} \text{ lbs./in.}^2 \qquad (8)$$

To find the stress of the steel we equate the moments of the outer and inner forces.

$$B = t . A_T\left(d - \frac{n}{3}\right) \text{ or}$$

$$t = \frac{B}{A_T\left(d - \frac{n}{3}\right)} \text{ lbs./in.}^2 \qquad (9)$$

In *designing a structure* the values of n, c and t are, of course, not known, as, to arrive at their values, the thickness of slab and sectional area of steel must be available. Consequently, it is necessary to find means of calculating the values from the bending moment or other values given.

The following formulæ enable us to design a slab without these data.

The greatest bending moment is ascertained as before described. We know that

$$\frac{c}{E_C} : n = \frac{t}{E_S} : (d - n)$$

As we have decided to adopt the various values recommended by the R. I. B. A. Committee's report, we have $\frac{E_S}{E_C} = m = 15$, $c = 500$ lbs./in.², $t = 15{,}000$ lbs./in.², so that

$$\frac{c(d - n)}{E_c} = \frac{t \cdot n}{E_s}$$

$$c(d - n)E_s = t \cdot n \cdot E_c$$

$$c(d - n)15 = t \cdot n$$

$$500\,(d - n)15 = 15{,}000\,n$$

$$\mathbf{d = 3\,n} \qquad \textbf{(10)}$$

or the effective depth of a slab is 3 times the distance from the top of slab to the neutral axis. To get the total depth d_t, sufficient thickness of concrete must be added to protect the steel from fire as before described (see page 31).

If the effective depth d is to be calculated immediately from the bending moment, we insert the values $c = 500$ lbs./in.², $b = 12$ inches, $n = \frac{1}{3}\,d$ in the formula 8, and get

$$c = \frac{2\,B}{b \cdot n\left(d - \frac{n}{3}\right)} \text{ or}$$

$$\mathbf{d = 0{\cdot}0335\,\sqrt{B}} \text{ and} \qquad \textbf{(11)}$$

$$\mathbf{A_T = 0{\cdot}066\,d} \text{ and} \qquad \textbf{(12)}$$

$$\text{from formula (9) } \mathbf{A_T = 0{\cdot}002261\,\sqrt{B}} \qquad \textbf{(13)}$$

The value A_T may also be ascertained from the distance n, as follows :—

$$C = T$$

$$\mathbf{c \cdot \frac{n}{2} \cdot b = A_T \cdot t} \qquad \textbf{(14)}$$

or if $b = 12$ inches, $c = 500$, and $t = 15{,}000$,

$$\frac{500\,n}{2} \cdot 12 = A_T\,15{,}000$$

$$\mathbf{A_T = \frac{1}{5}\,n = 0{\cdot}20\,n} \qquad \textbf{(15)}$$

If the total thickness of slab is calculated and found to be less than $3\frac{1}{2}$ ins., it should be made that thickness, as from a practical point of view anything less in substance is not reliable enough.

Note. The Prussian Government regulations fix the least allowable thickness of floor slab at 8 cm. (or 3·15 ins.)

The number of rods required and their distances d_r apart is derived from the formula

$$N_r = \frac{A_T}{A_r} \qquad (16)$$

where A_r is the sectional area of the rod selected.

If, therefore, by the previous formulæ the value of A_T has been found, the section is selected from the tables at end of book, and the above equation (16) gives at once the number of rods required for the width of slab = 12 ins., and from this and the total width of slab the distance from centre to centre is fixed.

Having thus provisionally fixed the area of steel required, the stresses c and t must be ascertained by means of formulæ 8 and 9. If on investigation these stresses are found to exceed the allowable figures, either the sectional area A_T can be increased, keeping the dimensions of slab as found, or the thickness of slab d may be increased and the value of A_T adhered to, or, lastly, both may be increased. In either case the tensile stresses are reduced, particularly if the slab is made thicker.

If it is found that neither of the two materials is stressed to its allowable figure, *viz.*, c = 500 lbs./in.^{2}, t = 1,500 lbs./in.^{2}, it is economical to increase the stress of one to its full limit, thus reducing the other in quantity.

Note. Sectional areas and weights, etc., of sundry reinforcements are given in the tables at end of book, where also other useful information relating to loads, etc., will be found.

A table is attached giving the comparative values for the various dimensions based on various allowable stresses of steel and concrete.

A ready reckoner for slabs and beams is added to the book, which will be found extremely useful for designing and checking of slabs and beams.

Example I.

A floor slab is to be designed over a room 12 ft. wide. The live load is to be taken at 60 lbs./ft.^{2}, the weight of flooring at 10 lbs./ft.^{2}, the weight of floor slab at 150 lbs./ft.^{3}

Assuming a depth for the slab of 6 ins., we get the span as $144 + 6 = 150$ ins.

The total load is then

Live load	60
Flooring .	10
Slab . .	75
	145 lbs./ft.^{2}, or $145 \cdot 12{\cdot}5 = 1813$ lbs.

for a strip 12 ins. wide.

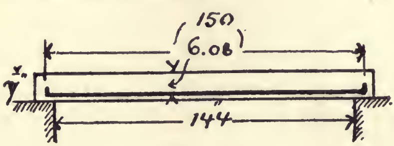


Fig. 121.

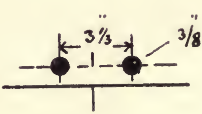


Fig. 122.

If we consider this as a slab freely supported at both ends, the greatest bending moment is

$$B = \frac{Wl}{8} = \frac{1813 \cdot 150}{8} = 33994 \text{ lbs./in.}^2$$

$$\sqrt{33994} = 184{\cdot}38$$

$$d = 0{\cdot}033\ \sqrt{B} = 0{\cdot}033 \cdot 184{\cdot}38 = 6{\cdot}08$$

$$A_T = 0{\cdot}066 \cdot 6{\cdot}08 = 0{\cdot}40 \text{ in.}^2$$

$$N_r = \frac{0{\cdot}40}{0{\cdot}1104} = 3{\cdot}62$$

if we select rods of $\frac{3}{8}$ in. diameter with an area of $0{\cdot}1104$ in.^{2}, that is, we space the rods $3\frac{1}{3}$ ins. apart centre to centre and the slab would be $6{\cdot}08 + 1 = 7$ ins. thick.

Figs. 121 and 122 illustrate the slab thus designed.

In order to see that this section is correct, *viz.*, that neither of the materials are stressed beyond their limit, we ascertain the stresses as follows :—

Live load	. . .	60
Flooring	. . .	10
Slab 150 . 0·58 =		87
		157 lbs./ft.^{2}

$$B = \frac{wl^2}{8} = \frac{157 \cdot 125^2}{8} \cdot 12 = 36797 \text{ lbs./in.}^2$$

$$d = 6{\cdot}08 \text{ ins.}$$

$$A_T = 0{\cdot}40 \text{ in.}^2$$

$$b = 12 \text{ ins.}$$

$$m = 15$$

To fix the position of neutral axis we use formula 7.

$$n = \frac{mA_T}{b}\left[\sqrt{1 + \frac{2bd}{mA}} - 1\right]$$

$$n = \frac{15 \cdot 0{\cdot}40}{12}\left[\sqrt{1 + \frac{2 \cdot 12 \cdot 6{\cdot}08}{15 \cdot 0{\cdot}40}} - 1\right]$$

$$n = 0{\cdot}5[\sqrt{1 + 24{\cdot}32} - 1] = 0{\cdot}5[\sqrt{25{\cdot}32} - 1]$$

$$n = 0{\cdot}5[5{\cdot}03 - 1] = 0{\cdot}5 \cdot 4{\cdot}03 = 2{\cdot}015$$

$$d - \frac{n}{3} = 6{\cdot}08 - 0{\cdot}67 = 5{\cdot}41$$

According to formulæ 8 and 9,

$$c = \frac{2B}{b \cdot n\left(d - \frac{n}{3}\right)} \text{ and } t = \frac{B}{A_T\left(d - \frac{n}{3}\right)}$$

$$c = \frac{73594}{12 \cdot 2 \cdot 0{\cdot}15 \cdot 5{\cdot}41} = \frac{73594}{130} = 566 \text{ lbs./in.}^2$$

$$t = \frac{36797}{0{\cdot}40 \cdot 5{\cdot}41} = \frac{36797}{2{\cdot}164} = 17000 \text{ lbs./in.}^2$$

The stresses allowable for concrete and steel in beams being 600 lbs./in.^{2} and 15000 to 17000 lbs./in.^{2} respectively, the slab may be carried out as designed.

As has been previously described, it is more economical to put the rods diagonally if the slab is nearly square. The present example being thus, we could make

$$B = \frac{Wl}{16} \text{ or}$$

$$B = \frac{1813 \cdot 150}{16} = 16997 \text{ lbs./in.}^2$$

$$\sqrt{B} = 130 \cdot 67$$

$$d = 0 \cdot 033 \sqrt{B} = 0 \cdot 033 \cdot 130 \cdot 67 = 4 \cdot 31 \text{ in.}$$

$$A_T = 0 \cdot 066 \cdot 4 \cdot 31 = 0 \cdot 284 \text{ in.}^2$$

$$N_r = \frac{0 \cdot 284}{0 \cdot 1104} = 2 \cdot 57$$

So that $\frac{3}{8}$ in. rods would have to be spaced $4\frac{3}{4}$ in. apart by a depth of slab $4 \cdot 31 + 1 = 5\frac{1}{2}$ in.

For practical reasons the rods should be spaced somewhat closer towards the centre of the slab, while the distances may be increased towards the end of the diagonals, *viz.*, near the supports.

EXAMPLE II.

To construct a reinforced concrete ceiling between iron girders 6 ft. apart. The live load to be 100 lbs./ft.²

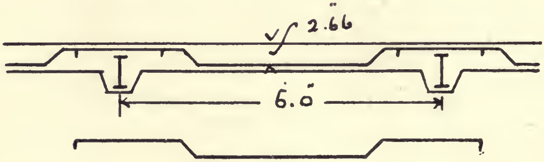


FIG. 123.

Assuming a thickness of slab of 4 ins.

$$W = (100 + 50) \cdot 6 \cdot 0 = 900 \text{ lbs.}$$

$$B = \frac{900 \cdot 6 \cdot 0}{10} \cdot 12 = 6480 \text{ lbs./in.}^2$$

$$\sqrt{B} = \sqrt{6480} = 80 \cdot 49$$

$$d = 0 \cdot 033 \cdot 80 \cdot 49 = 2 \cdot 66 \text{ ins.}$$

$$A_T = 0 \cdot 066 \cdot 2 \cdot 66 = 0 \cdot 18 \text{ in.}^2$$

$$N_r = \frac{0 \cdot 18}{0 \cdot 0491} = 3 \cdot 68$$

$\frac{1}{4}$ in. rods spaced $3\frac{1}{4}$ in. apart by the thickness of slab = $2{\cdot}66 + 1$ = say $3\frac{3}{4}$ ins.

The stresses are as follows :—

$$\left.\begin{array}{ll}\text{Live load} & 100 \\ \text{Slab} & 47\end{array}\right\} = 147 \text{ lbs.}$$

$$B = \frac{wl^2}{8} = \frac{147 \cdot 6{\cdot}0^2}{8} \cdot 12 = 7938 \text{ lbs./in.}^2$$

$$d = 2{\cdot}66 \text{ ins.}$$

$$A_T = 0{\cdot}18 \text{ in.}^2$$

$$b = 12 \text{ ins.}$$

$$m = 15$$

$$n = \frac{15 \cdot 0{\cdot}18}{12}\left[\sqrt{1 + \frac{2 \cdot 12 \cdot 2{\cdot}66}{15 \cdot 0{\cdot}18}} - 1\right] = 0{\cdot}225[\sqrt{24{\cdot}65} - 1] =$$

$$0{\cdot}225 \cdot 4{\cdot}961 = 1{\cdot}12$$

$$d - \frac{n}{3} = 2{\cdot}66 - 0{\cdot}37 = 2{\cdot}29$$

$$c = \frac{15876}{12 \cdot 1{\cdot}12 \cdot 2{\cdot}29} = 515 \text{ lbs./in.}^2$$

$$t = \frac{7938}{0{\cdot}18 \cdot 2{\cdot}29} = 19361 \text{ lbs./in.}^2$$

This latter being too high, we must increase the sectional area of the steel. We can easily effect this by spacing the rods somewhat closer. If we space them 3 ins. apart we get a A_T of 0·20, which would give us a greatest stress of 17235 lbs./in.² The spacing should, therefore, be slightly less than 3 ins., or a stronger section of rod with a wider spacing may be used.

Example III.

A window lintel to be designed over an opening 8 ft. clear, the thickness of wall to be 14 ins. and the load to be carried 12 tons.

Assuming the depth of the lintel for architectural effect to be restricted to 9 ins.

Span 8′0″ + 9″	= 8′9″
Load 12 tons	= 26880 lbs.
Lintel 9″ . 1′2″ . 8′9″ . 150	= 1050 lbs.
	27930 lbs.

Load to be carried by a strip 12″ wide $= \frac{1}{6}$ less or

$$= 23275 \text{ lbs.}$$

$$\frac{23275 \cdot 8{\cdot}9}{8} \cdot 12 = 305484 \text{ lbs./in.}^2$$

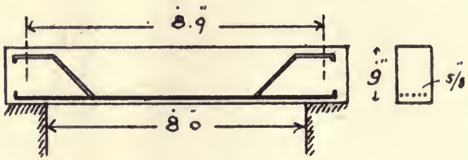


Fig. 124.

$$\sqrt{B} = 552{\cdot}71$$

$$A_T = 1{\cdot}20 \text{ in.}^2$$

$$\text{or for } b = 1'2'' = 1{\cdot}40 \text{ in.}^2$$

The reinforcement required is therefore,

No. 5 $\frac{5}{8}$ in. rods.

CHAPTER VII

FORMULAE FOR SLABS WITH DOUBLE REINFORCEMENT

WHERE positive as well as negative bending moments are bound to occur, it is advisable to have distributing rods as well as tensile rods. The distributing rods resist then the negative moments. Double reinforcement is also useful where it is desirable to restrict the height of construction, and, lastly, where on ascertaining the

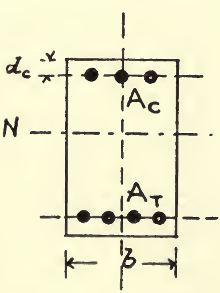


FIG. 125.

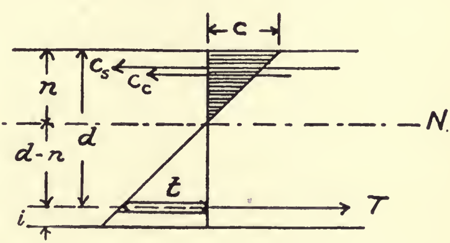


FIG. 126.

stresses after calculating it is found that the concrete is put under too great a compression.

If the sectional area of the tension rods is less than 0·5 to 0·6 per cent. of the total section, it is not economical to use distributing rods.

The calculation of a slab with double reinforcement is similar to that of a slab with single reinforcement.

d = effective depth of slab,
n = distance of neutral axis from top of slab,
A_T & A_C = the sectional area of reinforcements in in.^{2},
t & c_s = the stresses of reinforcements in lbs./in.^{2},
c = the stress of concrete in lbs./in.^{2}

The position of neutral axis will be found as before by the formula,

$$C_s = A_c \cdot c_s$$

acting at a distance d_c from top of slab.

As before, the compression of concrete,

$$C_c = \frac{c \cdot n}{2} \cdot b$$

acting at a distance $\frac{n}{3}$ from top of slab.

Both forces C_s and C_c together must be equal to the tensile force T, or

$$C_s + C_c = T$$

$$\frac{c \cdot n}{2} \cdot b + A_c \cdot c_s = A_T \cdot t$$

$$\frac{c}{E_c} : \frac{t}{E_s} = \frac{n}{d - n} \text{ and } \frac{c}{E_c} : \frac{c_s}{E_s} = \frac{n}{d - d_c}$$

$$\frac{E_s}{E_c} = m \text{ and } \frac{c \cdot m}{t} = \frac{n}{d - n}, \text{ therefore}$$

$$t = \frac{c \cdot m(d - n)}{n} \text{ and } c_s = \frac{c \cdot m(n - d_c)}{n}$$

$$\frac{c \cdot n}{2} \cdot b + A_C \frac{c \cdot m(n - d_c)}{n} = A_T \frac{c \cdot m(d - n)}{n}$$

$$n = \sqrt{\left(m\frac{(A_T + A_C)}{b}\right)^2 + \frac{2m}{b}[A_C \cdot d_c + A_T \cdot d]} - \frac{m(A_T + A_C)}{b} \quad (17)$$

To ascertain the greatest stresses of the concrete, we again put the greatest bending moment B equal to the moment of resistance R, and find

$$c = \frac{B}{\frac{b.n}{2}\left(d - \frac{n}{3}\right) + m.A_C\frac{n - d_c}{n}(d - d_c)} \qquad (18)$$

$$t = \frac{c.m(d - n)}{n} \qquad (19)$$

$$c_s = \frac{c.m(n - d_c)}{n} \qquad (20)$$

As regards formulæ for the design of slabs with double reinforcements, the moment B and thickness of slab d_t is usually known. If we assume certain maximum stresses c and t and from these find B° and A_T°, using the figures given in the table annexed for slabs with single reinforcement, we find

$$A_T = \frac{B}{B^\circ}.A_T^\circ \qquad (21)$$

$$A_C = 3\left(\frac{B}{B^\circ} - 1\right).A_T^\circ \qquad (22)$$

As a matter of fact, n is constant for fixed values of c and t such as 500 and 15,000, whatever the values of A_T and A_C may be. From this it follows that if we are restricted to a certain depth of beam or slab, we need only find the amount of reinforcement in tension which can be used for the required depth, and from this the bending moment it will resist. All that is needed then is to calculate the extra amount of steel required for the excess of bending moment, and this will be the section of steel required for the distributing rods.

Example.

A slab ceiling of 6 ins. effective depth has to support a moment of 40,000 lbs./in.^{2}; the materials are to be stressed to their full limit, *viz.*, the concrete to 500 lbs./in.^{2} and the steel to 15,000 lbs./in.^{2} What sectional areas are the two sets of rods to receive?

According to table, page 148,

$$d = 0{\cdot}0335\ \sqrt{B^\circ} \text{ or}$$
$$6 = 0{\cdot}0335\ \sqrt{B^\circ}$$
$$\sqrt{B^\circ} = 181.81 \text{ or } B^\circ = 32955 \text{ lbs./in.}^2$$

According to the same table,

$$A_T^\circ = 0{\cdot}002261\sqrt{B^\circ} = 0{\cdot}40 \text{ in.}^2$$

Formula 21 $$A_T = \frac{40000}{32955} \,.\, 0{\cdot}40 = 0{\cdot}488 \text{ in.}^2$$

$$A_C = 3\left(\frac{40000}{32955} - 1\right)0{\cdot}40 = 0{\cdot}25 \text{ in.}^2$$

for a width $b = 12$ inches.

CHAPTER VIII

FORMULAE FOR RIBBED CEILINGS OR T BEAMS

According to the position of the neutral axis, there are 3 cases possible (Fig. 127[1]).

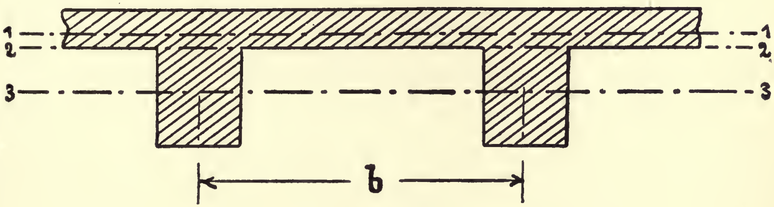


Fig. 127.[1]

1. The neutral axis falls within the slab.
2. ,, ,, ,, ,, at bottom of slab.
3. ,, ,, ,, ,, below the slab.

1. *If the neutral axis falls within the slab* the conditions are the same as in the case of slabs with single reinforcement.

The section to be considered is $b_s\ d_s$, but whereas we have dealt so far with a width of slab $b = 12$ ins., we have now various values for b_s, according to circumstances as explained hereafter.

If d_s is the depth of slab in inches,

d the effective depth in inches,

b_r the width of rib,

A_T the sectional area of steel in in.^{2},

$\left.\begin{array}{l}c\\t\end{array}\right\}$ the stresses of concrete and steel respectively, we get

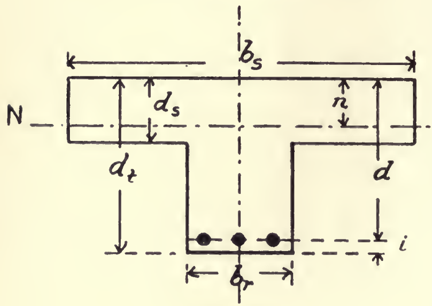


FIG. 128.

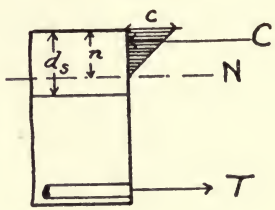


FIG. 129.

$$n = \frac{m \cdot A_T}{b}\left[\sqrt{1 + \frac{2b_s \cdot d}{m \cdot A_T}} - 1\right] \qquad \text{(formula 7)}$$

$$c = \frac{2B}{b_s \cdot n\left(d - \frac{n}{3}\right)} \qquad \text{(formula 8)}$$

$$t = \frac{M}{A_T\left(d - \frac{n}{3}\right)} \qquad \text{(formula 9)}$$

2. *If the neutral axis falls at bottom of slab* (Fig. 130), the distance n becomes $= d_s$, consequently,

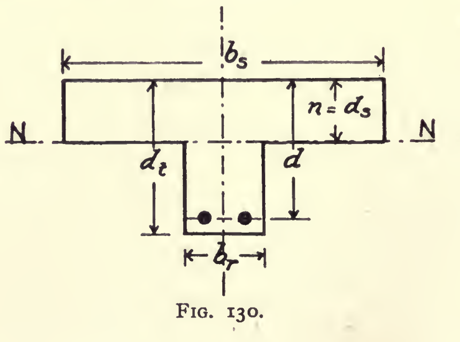


FIG. 130.

$$n = d_s = \frac{m \cdot A_T}{b_s}\left[\sqrt{1 + \frac{2b_s \cdot d}{m \cdot A_T}} - 1\right] \quad \text{(formula 7)}$$

$$c = \frac{2B}{b_s d\left(d - \frac{d_s}{3}\right)} \tag{23}$$

$$t = \frac{B}{A_T\left(d - \frac{d_s}{3}\right)} \tag{24}$$

3. *If the neutral axis falls below the slab*, the small compressive stresses in the rib may be neglected (Figs. 132, 133).

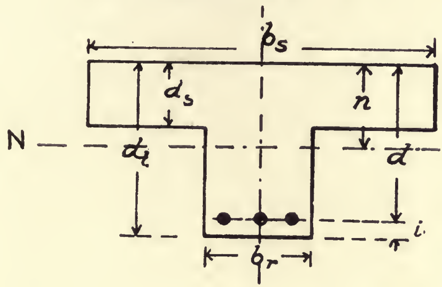


FIG. 131.

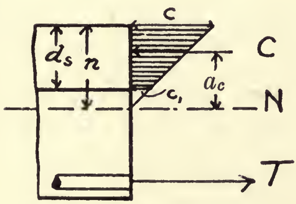


FIG. 132.

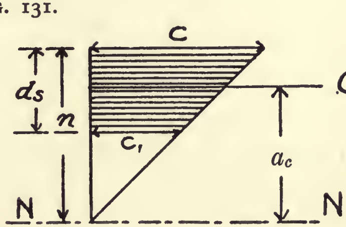


FIG. 133.

$$C = \frac{c + c_1}{2} \cdot d_s \cdot b_s$$

$$T = A_T \cdot t \text{ or } C = T \text{ and}$$

$$\frac{c}{c_1} = \frac{n}{n - d_s} \text{ or } c_1 = c\,\frac{n - d_s}{n}$$

$$C = \frac{c + c\,\frac{n - d_s}{n}}{2} \cdot d_s \cdot b_s \text{ and} \tag{25}$$

$$t = m \cdot c \frac{d - n}{n} \tag{26}$$

Inserting the values for c_1 and t in formula 25, we find

$$\frac{c + c \dfrac{n - d_s}{n}}{2} d_s \cdot b_s = A_T \cdot m \cdot c \frac{d - n}{n}$$

from which it follows that

$$n = \frac{m \cdot A_T \cdot d + \dfrac{d_s^2 \cdot b_s}{2}}{d_s \cdot b_s + m \cdot A_T} \tag{27}$$

If we call a_c = the distance of the compressive force from neutral axis (Fig. 133), we find that

$$n - a_c = \frac{d_s}{3} \cdot \frac{c + 2c_1}{c + c_1}, \text{ and as}$$

$$c_1 = c \cdot \frac{n - d_s}{n}, \text{ it follows that}$$

$$n - a_c = \frac{d_s}{3} \cdot \frac{3n - 2d_s}{2n - d_s}, \text{ or}$$

$$a_c = n - \frac{d_s}{2} + \frac{d_s^2}{6(2n - d_s)} \tag{28}$$

If $n = d_s$, that is, if the neutral axis falls at bottom of slab, $\beta = \frac{2}{3} d_s$.

The greatest stresses of steel and concrete are ascertained again by putting the greatest bending moment equal to the moment of resistance.

$$B = T(d - n + a_c) = t \cdot A_T(d - n + a_c), \text{ or}$$

$$t = \frac{B}{A_T(d - n + a_c)} \text{ and} \tag{29}$$

$$c = t \frac{n}{m(d - n)} \tag{30}$$

As regards formulæ for designing the slab.

The neutral axis usually falling within the depths of the slab (case 1), the thickness of slab and the area of steel required are easily calculated from the bending moment.

$$\frac{c}{E_c} : n = \frac{t}{E_s} : (d_s - n), \text{ or}$$

$$n \cdot \frac{t}{c} = (d_s - n \frac{E_s}{E_c} \text{ and as, } \frac{E_s}{E_c} = m = 15$$

and calling $\frac{t}{c} = s_r$, we get

$$n = \frac{m}{\gamma + m} d_s \qquad (31)$$

And as C = T and $\frac{c \cdot n}{2} \cdot b_s = A_T \cdot t$, it follows that

$$A_T = \frac{c \cdot n \cdot b_s}{2t} = \frac{b_s \cdot n}{2 s_r} \qquad (32)$$

Inserting these values in formula 9, we get

$$t = \frac{B}{\frac{b_s \cdot m \cdot d}{2 s_r(s_r + m)}\left[d - \frac{m - d}{3(s_r + m)}\right]} \text{ from which}$$

$$d = \sqrt{\frac{2 s_r(s_r + m)}{b_s \cdot m \cdot t\left[1 - \frac{m}{3(s_r + m)}\right]}} \cdot \sqrt{B} \qquad (33)$$

and from formulæ 31 and 32,

$$A_T = \frac{m \cdot b_s}{2 s_r(s_r + m)} d \qquad (34)$$

With the aid of the following formulæ, A_T and d may be calculated direct from B :—

$$t = \frac{B}{A_T\left(d - \frac{n}{3}\right)}$$

$$d = \frac{n(s_r + m)}{m} \text{ and}$$

$$n = \frac{2s_r \cdot A_T}{b_s}$$

$$t = \frac{B}{A_T\left[\frac{2s_r \cdot A_T(s_r + m)}{m \cdot b_s} - \frac{2s_r \cdot A_T}{3 \cdot b_s}\right]} \text{ or}$$

$$A_T = \sqrt{\frac{b_s}{2s_r \cdot t\left[\frac{s_r + m}{m} - \frac{1}{3}\right]}} \cdot \sqrt{B} \qquad (35)$$

In designing a ribbed slab construction it is first necessary to decide the span. This should be taken at about **one-twenty-fifth** more than the clear width of the room to be covered in.

The weight of the ribbed slab has also to be assumed in order to get the bending moment, and for this purpose the contents of a plain slab of B width and **1·5 to** $2d_s$ depth ($2d_s$ in case of deep ribs and small thickness of slab), d_s being the thickness of the slab. Usually d_s is a known value, through calculating the continuous slab over the ribs for B span. If the thickness d_s is not known, it must be assumed from **3 to 6 ins.**, according to the load and spans.

A ribbed slab is practically a T beam, the slab or part of the slab being the table of the T. Opinions vary as to what extent the slab may be assumed to form part of the T.

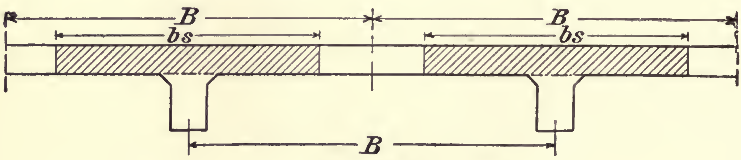


FIG. 134.[1]

The Prussian Government regulations stipulate that the width of the slab forming part of the T for calculating purposes measured from the centre of the rib on either side must not exceed **one-sixth** of the length of beam.

If, for instance, the span l = 30 ft. and the ribs are 12 ft. centre to centre, the whole of the 12 ft. must not be considered the width of the T but only $2 \cdot l/6$ = 10 ft. In ascertaining the bending moment the full width of 12 ft. is of course retained.

If half the distance between ribs is less than $\frac{1}{6}$ of the span, b_s must be taken equal to B. *If the width between the ribs is optional it is economical to make it* $\frac{1}{3}$ *of the span.*

Care should be taken that the *width of rib* is not taken too small. It depends largely on the strength of the reinforcement. In ordinary cases **7 to 12 ins.** and for heavy work **11 to 16 ins.**, suffice. The thickness of concrete from bottom of reinforcement to bottom of beam should in no case be less than 1 in.

From an economical point of view, it is, of course, desirable to make the ribs as deep as possible, as the deeper the rib the less reinforcement required. In many cases, however, the depth is governed by the height of construction and dimensions of building, and thus it merely remains to ascertain the section of steel required. To stress the concrete to its limit is not often possible, as it would mean low ribs and consequently heavy steel reinforcements. Where the neutral axis falls into the bottom line of slab the most economical use of the concrete is secured.

If B is the bending moment in inch pounds, the distance of the centre of reinforcement from top of slab in inches, and d_s the thickness of slab in inches, the following formula is very useful for the design in ordinary cases :—

$$A_T = \frac{B}{t\left(d - \frac{d_s}{2}\right)} \qquad (36)$$

To provisionally determine the bending moment, we again take the contents of a plain slab of B width and 1·6 to 2·0 d_s thickness.

If, however, it is desired to at once ascertain definite stresses, the following formulæ may be used for the thickness d and the sectional area A_T of steel required, assuming

$$s_r = \frac{t}{c}, \text{ the effective depth,}$$

$$d = d_d + \sqrt{d_d^2 - \beta} \qquad (37)$$

d_d being the distance of bottom edge of reinforcements from centre of gravity of reinforcements in rib.

$$d_d = \frac{B}{2c \, . \, d_s \, . \, b_s} + \frac{d_s}{4}\left(1 + \frac{1}{\alpha}\right) \text{ where}$$

$$\alpha = \frac{m}{m + s_r} \text{ and } \beta = \frac{d_s^2}{3\alpha}$$

$$A_T = \frac{6(2\alpha \, . \, d - d_s)}{3(2\alpha \, . \, d - d_s)\,(2d - d_s) + d_s^2} \cdot \frac{B}{t} \qquad (38)$$

In cases of fixed slabs negative moments occur near the supports, and in continuous slabs over the supports, so that tensile stresses are in the slab and compressive stresses in the lower fibres of ribs. Double reinforcements are useful here as they reduce the depth of rib.

As the span l, the widths B and b_s and the load are usually known, the design of ribbed slabs can easily be accomplished in various ways with the help of the different formulæ given, as for instance—

The weight of slab may be assumed and the greatest bending moment calculated accordingly, from which the values d, n and A_T are then ascertained; or

The slab dimensions may be calculated first, then the weight of same, and after that the greatest moment from which d and A_T are obtained; or

The slab is calculated, assuming a certain thickness, b_s and d are selected to suit the particular case under calculation, and then the greatest moment fixed, A_T being found by means of formula 36; or

Lastly, if it is apparent that the neutral axis must be below the slab, the formulæ 37 and 38 may be used.

Example.

To construct a ribbed ceiling over a room 20 ft. wide. The distance of ribs to be 5 ft., the live load including weight of flooring and ceiling plaster to be 75 lbs./ft.^{2}

As the distance of ribs is less than $\frac{1}{3}$ l, $b_s = 5{\cdot}0$.

Width l = 1·04, l = 20·80.

If we assume the thickness of slab to be 5 ins., the approximate weight of ribbed slab is

5′ 0″ . ((say) 1·75 . 5″) × 150 = 3·65 × 150 = 548 lbs. per foot run.

Live load 75 . 5 = 375 lbs.

Total load 20′ 8″ (548 + 375) = 19198 lbs.

$$B = \frac{19198 \cdot 20{\cdot}8}{8} \cdot 12 = 598978 \text{ lbs./in.}^2$$

$$\sqrt{B} = 773{\cdot}94$$

Taking $t = 15000$ and $c = 500$, we get from (33)

$$d = \sqrt{\frac{60 \cdot (30 + 15)}{60 \cdot 15 \cdot 15000\left[1 - \frac{15}{3(30 + 15)}\right]}} \cdot \sqrt{598978} = 11{\cdot}61 \text{ ins.}$$

$$n = \frac{15}{30 + 15} \cdot 11{\cdot}61 \text{ from (31)} = 3{\cdot}87 \text{ ins.}$$

$$A_T = \frac{60 \cdot 3{\cdot}87}{60} \text{ from (34)} = 3{\cdot}87 \text{ in.}^2$$

$$\tfrac{7}{8} \text{ rods with } 0{\cdot}60 = 3{\cdot}60 \text{ in.}^2$$

If we select No. 6, we get the following dimensions of ribbed slab:—

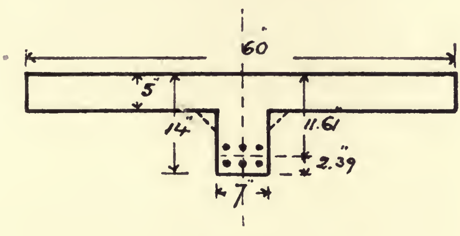


FIG. 135.

The greatest moments are then as follows:—

$$\text{Live load } 75 \cdot 5 = 375 \text{ lbs.}$$

$$\text{Weight of ribbed slab} = \frac{60 \cdot 5 + 9 \cdot 7}{144} \cdot 150 = 377 \text{ lbs.}$$

$$20{\cdot}8 \cdot (375 + 377) = 15642 \text{ lbs.}$$

$$B = \frac{15642 \cdot 20{\cdot}8}{8} \cdot 12 = 488028 \text{ lbs./in.}^2$$

$$c = \frac{\not{2} \cdot 488028}{\not{6}\not{0}\,_{30} \cdot 3{\cdot}87\left(11{\cdot}61 - \frac{3{\cdot}87}{3}\right)} = \frac{488028}{116{\cdot}1 \cdot 10{\cdot}32} = \frac{488028}{1198} = 407 \text{ lbs./in.}^2$$

$$t = \frac{488028}{37} = 13189 \text{ lbs./in.}^2$$

This result is somewhat too extravagant,—the cause being that in estimating the weight of slab we have taken the figure 1·75 while 1·5 would have been enough. We could, therefore, reduce the area of steel without detriment.

If we adopt 6 rods of $\frac{13}{16}$ in. diam., which would give us a A_T of 3·11, t would become 15250 lbs./in.^{2} This would be quite safe enough.

Example.

A floor to be constructed over a room 32 ft. wide; live load including flooring 250 lbs./ft.^{2} The ribs to be 8 ft. apart and the floor 7 ins. thick.

Span 1·04 . 32 = 33·28′

Width of T = 8·00′ = 96 ins.

Approximate weight of ribbed slab :—

$$\frac{96 \cdot (1{\cdot}9 \cdot 7)}{144} \cdot 150 = 1320 \text{ lbs. per foot run.}$$

Live load = 2000 lbs. per foot run.

Total load = 33·28(1320 + 2000) = 110490 lbs.

$$B = \frac{110490 \cdot 33{\cdot}28}{8} \cdot 12 = 5515656 \text{ lbs./in.}^2$$

$$\sqrt{B} = 2349 \cdot \sqrt{b_s} = 9{\cdot}8$$

According to the table on page 148, we get the following values :—

$$d = 0{\cdot}116 \sqrt{\frac{B}{b_s}};\ d = 0{\cdot}116 \cdot \frac{2349}{9{\cdot}8} = 27{\cdot}7 \text{ inches.}$$

$$A_T = 0{\cdot}00064 \sqrt{B \cdot b} = 0{\cdot}00064 \cdot 2349 \cdot 9{\cdot}8 = 14{\cdot}73 \text{ in.}^2$$

$$n = 0{\cdot}333\, d = 0{\cdot}333 \cdot 27{\cdot}7 = 9{\cdot}2 \text{ inches.}$$

If we select No. 4 rods $1\frac{3}{4}$ diam. = 9·620 and

No. 3 rods $1\frac{1}{2}$ diam. = 5·301 we get

Area of steel A_T = 14·921 in.^{2}

This would give the section of ribbed ceiling as below :—

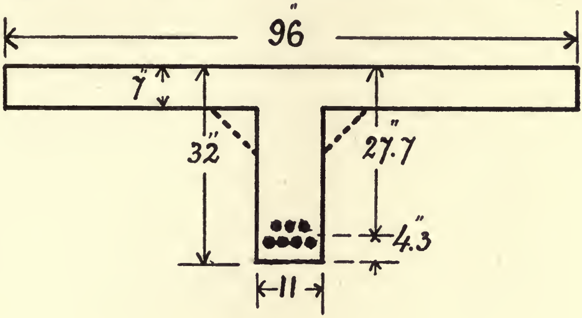


FIG. 136.

This gives a weight of ribbed slab :—

$$\frac{96 \cdot 7 + 25 \cdot 11}{144} \cdot 150 = 987$$

$$\begin{aligned} \text{Live load} &= 2000 \\ \text{Total load} &= \overline{2987} \cdot 33{\cdot}28 = 99407 \text{ lbs.} \end{aligned}$$

$$B = \frac{99407 \cdot 33{\cdot}28}{8} \cdot 12 = 4962396 \text{ lbs./in.}^2$$

$$a_c = 9{\cdot}2 - 3{\cdot}5 + \frac{49}{6(18{\cdot}4 - 7)} = 6{\cdot}41 \text{ (formula 28).}$$

$$t = \frac{4962396}{14{\cdot}9(27{\cdot}7 - 9{\cdot}2) + 6{\cdot}41} = 13376 \text{ lbs./in.}^2 \text{ (formula 29).}$$

$$c = 13376 \frac{9{\cdot}2}{15(27{\cdot}7 - 9{\cdot}2)} = 442 \text{ lbs./in.}^2 \text{ (formula 30).}$$

CHAPTER IX

FORMULAE FOR RIBBED SLABS WITH DOUBLE REINFORCEMENT

DOUBLE reinforcement of ribbed slabs is advantageous where the height of construction is very limited. Necessarily, the cost is greater as more steel is required.

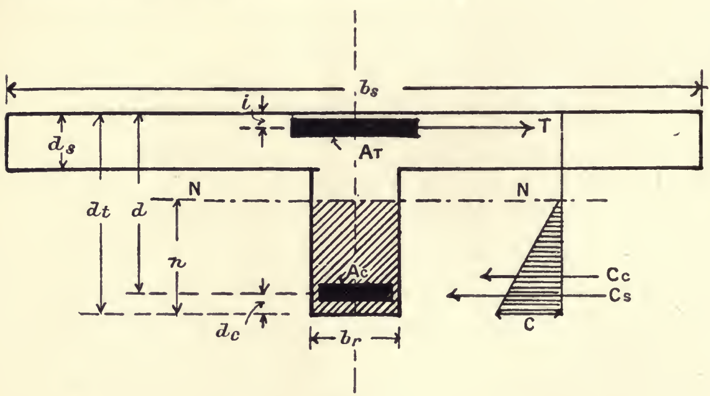


FIG. 137.

When ribbed slabs are built in all round and where they are continuous, negative moments occur over the supports. These are, as a rule, greater than the positive moments. Consequently the lower part of the beam or slab resists compression. The advantages of the compressive steel reinforcements can mostly be utilized and taken into account when calculating the steel required, and it is not always advisable to neglect this as we have done before.

The greatest stresses are again depending on the position of neutral axis, which may be within, at the bottom, or below the slab.

In the former two cases the conditions are the same as described for double reinforced slabs, and the formulæ 17, 18, 19 and 20 may be used.

If *the neutral axis falls below the slab* and we neglect the compressive stresses in the rib, we get

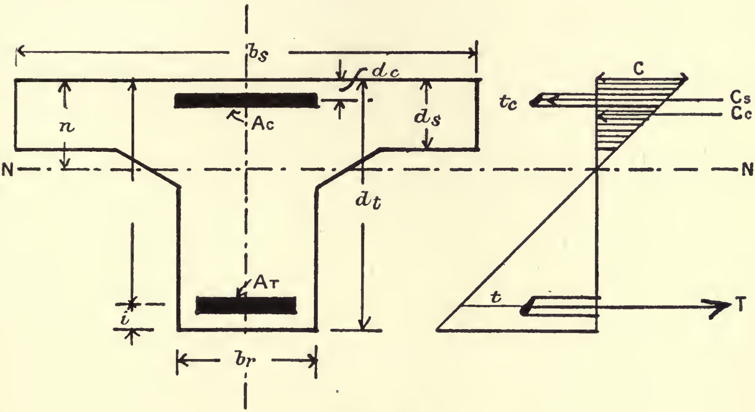


FIG. 138.

$$n = \frac{b_s \cdot d_s^2 + 2m(A_T \cdot d + A_C \cdot d_c)}{2[m(A_T + A_C) + b_s \cdot d_s]} \qquad (39)$$

a_c according to formula 28, and

$$c = \frac{B \cdot n}{(n - \frac{d_s}{2})d_s \cdot b_s \cdot a_c + m[A_T(d_s - n)^2 + A_C(n - d_c)^2]} \qquad (40)$$

t according to formula 19,

t_c according to formula 20.

The sectional area of steel is then found as follows:—

The values for b_s, d_s b d_s, are fixed from practical considerations, the weight of ribbed slab is ascertained and a value for B found.

The relation $s_r = \frac{t}{c}$ is decided on and values for d_c and i determined.

$$n = d_s \frac{15}{s_r + 15}$$

a_c according to formula 28,

c according to formula 25.

Compressive stress of steel C_s from the equation

$$B = C_c(d_s - n + a_c) + C_s(d - d_c)$$

Tensile stress $T = C_s + C_c$ and

$$A_T = \frac{T}{t} \qquad (41)$$

Stress in the compressive reinforcement from formula 20 and

$$A_C = \frac{C_s}{t_c} \qquad (42)$$

CHAPTER X

SHEARING STRESSES AND ADHESION

If a slab is loaded, two different kinds of shearing stresses occur, some of which are parallel to its length and some parallel to its width.

It is clear that the shearing stresses are smallest in the centre of the slab and increase towards the supports where they become greatest.

Consequently, it is necessary under certain conditions to reinforce slabs near the points of fixture or support so as to prevent the slab being destroyed by shear.

If the slab consisted of uniform material, like ordinary concrete, the shearing stress would be

$$s = \frac{S}{A}$$

where S is the greatest shearing moment in lbs./in.[2] and A the section of slab, S being also in lbs./in.[2]

In reinforced concrete the slab is composed of two materials having different moduli of elasticity, and as the shearing moment is attacking both in the same proportion the shearing stress of the concrete must be

$$\frac{s_c}{E_c} = \frac{S}{A.E_c + A_T E_s} \text{ or}$$

$$s_c = \frac{S}{A + mA_T} \qquad (43)$$

The shearing stress of the steel is then,

$$S_s = \frac{S}{A_T + \frac{A}{m}} = m \cdot s_c \qquad (44)$$

A and A_T are expressed in in.^{2}, and for m the value 15 is to be taken as before.

It is *not* necessary to calculate the shearing stresses in the direction of the *width of slab*, at least in cases of ceilings, as the stresses in these cases never reach the allowable greatest stresses for the two materials.

The shearing stresses in the direction of the *length of slab* must, however, be taken into account. They have the tendency to cut the slab into two as shown in Fig. 139.[1] The two fibres *mn* and

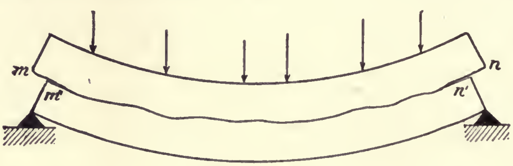


FIG. 139.[1]

m'n' show after destruction different lengths. The fibre *mn* is subjected to tension and the fibre *m'n'* to compression, while before they were of equal lengths and equally stressed. The form of the shear diagram is seen from Fig. 140. At the upper surface of slab the shear = 0 and also at the bottom surface. The shearing stresses increase from the outer surfaces towards the centre and reach their greatest moment at the line of the neutral axis. Consequently, the greatest shearing moment must be equal to the adhesion of the iron and the concrete, and the shear is greatest at the points of fixing.

If the slab Fig. 139 is cut vertically at a distance x from point A of fixing or support,

$$C \cdot a = S \cdot x$$

If we make $x = 1$ we get

$$C = \frac{S}{a}$$

as the greatest shear is in the neutral axis and must be equal to the moment of resistance.

$$C = s \cdot 1 \cdot b$$

$$s = \frac{S}{b \cdot a} \qquad (45)$$

As shown, the shearing stresses must be equal to the adhesion f of steel to concrete, *viz.*, these stresses affect the circumference of the reinforcement only. If we again make $x = 1$ in. and the circumference of all rods in C width of slab = O, we get

$$s \cdot 1 \cdot b = f \cdot 1 \cdot O$$

$$f = \frac{s \cdot b}{O} = \frac{S}{O \cdot a} \qquad (46)$$

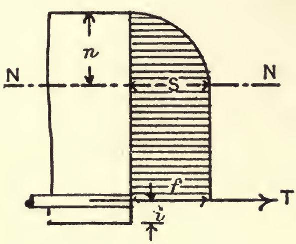


FIG. 140.

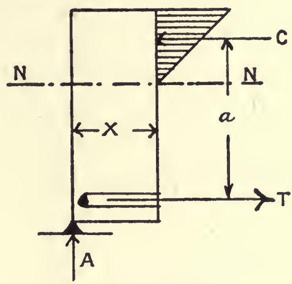


FIG. 141.

If O is expressed in inches, f will be in lbs./in.^{2} The required circumference must therefore be,

$$O = \frac{S}{f \cdot a}$$

For constructions of ordinary dimensions it is not necessary to go into this question at all, and the calculation of tensile stresses is decisive for the dimensions of steel required. Consequently, there is no necessity to arrange hangers or straps in slabs, particularly as in most cases some of the rods will be bent up towards the supports to resist the shear, and the resistance of the concrete to tension which is entirely neglected in the calculations acts as a useful agent. Furthermore, experience has shown that the adhesion of the steel to concrete is greater than the shear. This is proved by the fact that if a rod is pulled out of the concrete, particles of the concrete still adhere to the steel.

Where great shearing stresses are anticipated, rods or bars with uneven surface, like, for instance, the indented steel bar, may be adopted.

For *ribbed slabs* the shearing stresses must, however, be ascertained and counteracted, as in consequence of these stresses a failure may be more possible near the supports than in the centre of the slab. Particularly also is it likely that the slab might glide away over the rib.

Shearing and adhesive stresses in ribbed slabs are calculated as described for ordinary slabs. If the neutral axis occurs within the area of slab, the formulæ 45 and 46 are used with the modification that for b the width of the T must be inserted. If the neutral axis occurs at bottom of slab, b must be the distance of ribs. If the neutral axis falls below the slab, b must be again the width of T, and the distance of the compressive force C from the centre of reinforcement must be used, in which case

$$f = s\frac{b_r}{O}$$

It follows that the shearing stress does not depend on the amount of shear only, but also on the width and the height of the rib, as s will increase according to the increase of S or the decrease of b_r and $(d_t - d_s)$.

When simple ribbed ceilings are used, hangers and bending up of rods becomes necessary when

$$b_r = \frac{S}{s \cdot a}$$

Practically speaking, in case of *ribbed slabs* the *circumference of the reinforcing rods* should be about *equal to the width of the rib.*

CALCULATION OF HANGERS OR STRAPS.

Wherever the *shearing stresses exceed* **50 lbs./in.^{2}** it becomes advisable to arrange a series of hangers or straps connecting the rib with the slab and having a firm grip on the reinforcement

(Fig. 142[1]). As a rule, round rods or hoop irons are used. By thus connecting the essential parts of a ribbed floor the danger of cracks or failure of the concrete in the compressive area is considerably lessened, particularly under sudden shock or oscillation. For factory floors, bridges and other structures subjected to sudden shocks the arrangement of hangers is unavoidable.

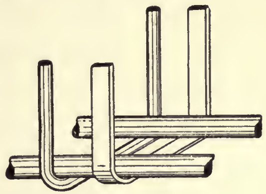


FIG. 142.[1]

In case of a uniformly distributed load the shear diagram is a triangle of the height y and a width $\frac{l}{2}$. The hatched portion of this triangle has a height $y - 50$ and a width x. To obtain the shearing stress the hangers have to resist, the area of this triangle is multiplied by the width b_s of the ribbed floor.

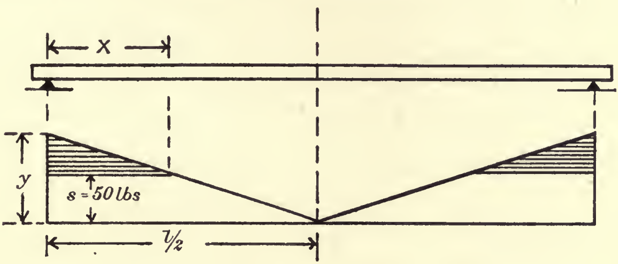


FIG. 143.

$$\frac{x}{y - 50} = \frac{\frac{l}{2}}{y}$$

$$x = \frac{(y - 50)}{y} \cdot \frac{l}{2} \qquad (47)$$

Assuming the allowable shearing stress of steel as 12000 lbs./in.^{2}, the sectional area of hangers required for half the width of ribbed slab is

$$As = \frac{(y - 50) \cdot x \cdot b_s}{2 \cdot 12000} \qquad (48)$$

Fig. 144 shows how the distances of hangers may be ascertained graphically.

The distance $\frac{1}{2}$ l AA_1 is divided in equal parts and perpendiculars erected in these points intersecting a semicircle over AA_1. Taking a pair of dividers these points of intersection are transferred to AA_1, and the distances thus obtained represent the position of the hangers.

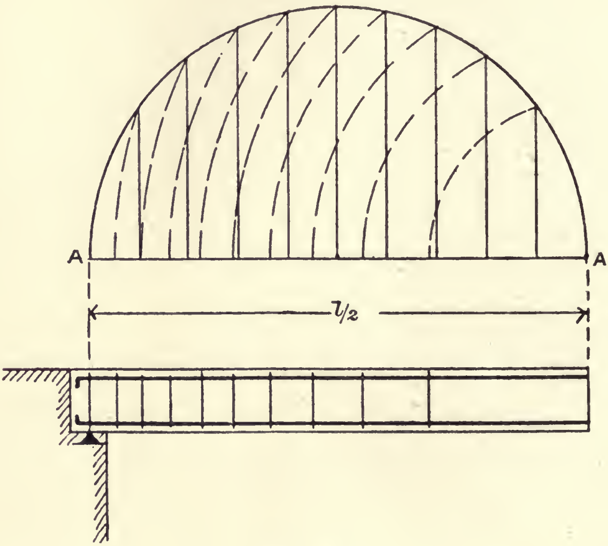


FIG. 144.

It is advisable to continue the hangers also through the centre portion of slab in equal distances as, if only half the slab is loaded, shearing stresses occur in the centre as well.

The hangers are usually arranged vertically, as for practical reasons it is difficult to arrange them obliquely, unless they form part of the bar, as for instance on the Kahn bar, skeleton bar and others.

Tests with beams have shown that cracks occur at angles of about 45°, thus proving that the shearing stresses take this inclination. For this reason, it is advisable to bend some of the rods up towards the supports under an angle of 45°.

The distance from the support is again found from formula 47,

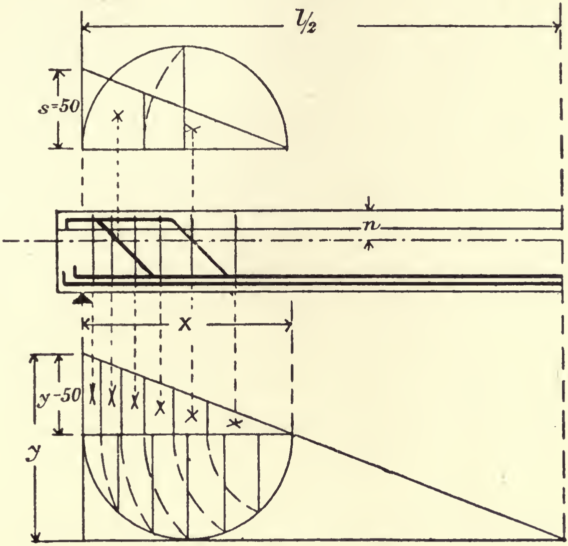


FIG. 145.

and the points where the rods are to be bent up may again be graphically ascertained (Fig. 145). The shear triangle is divided into equal areas and the centres of gravity of these connected by perpendiculars with the axis. The points of intersection are the points of bending the rods. Figs. 146, 147 show a typical arrangement of a beam with hangers, etc.

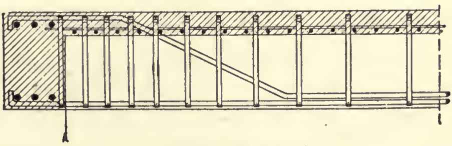


FIG. 146.[1]

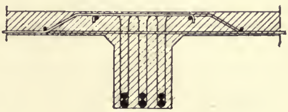


FIG. 147.[1]

CHAPTER XI

FORMULAE FOR COLUMNS

CALCULATION OF COLUMNS AXIALLY LOADED.

If we first consider a column without any reinforcement of the section A in.^{2} supporting a load of W lbs., this load is uniformly distributed over the whole sectional area and parallel to the length of the column.

The compressive stress is then,

$$C = \frac{W}{A} \text{ lbs./in.}^2$$

If the concrete column is reinforced with steel rods, parallel to the length of column, the two materials compress at the same rate, so that,

$$\frac{c}{E_c} = \frac{c_s}{E_s}$$

and as we call,

$$\frac{E_s}{E_c} = m$$

or as the steel can resist the compression m times more than the concrete, it is only then compressed at the same rate as the concrete, when the load W is m times bigger, so that,

$$C = \frac{c_s}{m} \text{ and } c_s = m \cdot c$$

Allowing as before $m = 15$ and the safe stress of concrete in columns at 500 lbs./in.^{2}, we get

$$c_s = 15 \cdot 500 = 7500 \text{ lbs./in.}^2$$

Consequently in designing a column or checking the design we have to deal with the stress of the concrete only, as the steel can never reach its highest safe stress of 15000 lbs./in.^{2}

If A is the sectional area of the concrete column under compression, without deducting the small area of steel, the total stress is,

$$C_c = c \cdot A$$

and the stress of the steel,

$$C_s = c_s \cdot A = m \cdot c \cdot A$$

and as the stresses must be equal to the load,

$$C_c + C_s = W$$

$$c \cdot A_c + m \cdot c \cdot A_L = W \text{ or}$$

$$c = \frac{W}{A_c + m \cdot A_L} \qquad (49)$$

$$c_s = m \cdot c = \frac{mW}{A_c + m \cdot A_L} \qquad (50)$$

When the column *exceeds* 18 *times its smallest diameter* there is danger of bending, and the column must, therefore, be calculated so as to resist the tendency to bend outwards.

For this Euler's formula is usually used.

$$W = \frac{\pi^2}{S_F \cdot l^2} \cdot E_c \cdot I$$

S_F is the factor of safety and may be taken as 6. (The Prussian Government regulations insist on a factor of safety of 10, which is, however, generally considered much too high.)

In calculating I, the moment of inertia, the sectional area of the steel rods is to be multiplied by $m = 15$ when used for calculating

$$W = \frac{\pi^2}{S_F \cdot l^2}\left[E_c\, I_c + E_s \cdot I_s\right] \text{ and as } \frac{E_s}{E_c} = m$$

$$W = \frac{\pi^2}{S_F \cdot l^2} E_c\left(I_c + m \cdot I_s\right)$$

If we take $E_c = \dfrac{30000000}{15} = 2000000$ lbs./in.^{2}

$m = 15$, $S_F = 6$ and $\pi^2 = 10$, we get

$$W = \frac{10 \cdot 2000000}{6 \cdot l^2}(I_c + 15\, I_s)$$

$$W = \frac{20000000}{6l^2}(I_c + 15\, I_s) \qquad (51)$$

or for W in tons and l in feet,

$$W = \frac{10 \times 2000000}{2240 \times 6 \times l^2 \times 144}(I_c + 15 . I_s) \text{ or}$$

$$\mathbf{W = \frac{10{\cdot}33\,(I_c + 15\,I_s)}{l^2}} \qquad \textbf{(52)}$$

This formula is based on the assumption that the column is fixed as shown in Fig. 148 and gives a very high factor of safety, as ordinary columns may be considered as fixed at both ends, which would mean that their carrying capacity is about four times more.

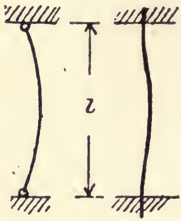


Fig. 148.

The iron rods being close to the outside and tending to destroy the concrete, it is necessary to investigate also, if the rods themselves are strong enough to resist bending outwards. The concrete in question is not thick enough to form any proper help, and it is therefore necessary to prevent the bending of rods by an arrangement of hoops or similar means.

The distance of these hoops should be equal to the smallest diameter of the column, but must not exceed thirty times the diameter of the rods.

The factor of safety should be 5.

$$W = A_L . t_s = \frac{\pi . d^2}{4} . c_s$$

$$\text{If } \pi^2 = 10 \,;\; S_F = 5 \,;\; E_c = 2000000 \text{ lbs./in.}^2$$

$$I = \frac{\pi . d^4}{64},\; c_s = m . c, \text{ we get}$$

$$\frac{\pi . d^2}{4} = \frac{10 . 2000000\, \pi . d^4}{5 . s_h^2 . 64}$$

Where s_h^2 is the distance of the hoops,

$$s_h^2 = \frac{10 . 2000000 . \pi . d^4 . 4}{5 . 64 . \pi . d^2 . c_s}$$

$$s_h^2 = 222222 \frac{d^2}{c_s} = 14814 \frac{d^2}{c} \text{ or}$$

$$\mathbf{s_h^2 = 471{\cdot}4 \frac{d}{\sqrt{c_s}} = 121{\cdot}7 \frac{d}{\sqrt{c}}} \qquad \textbf{(53)}$$

In designing a column the load to be carried and the length of the column is known.

Accordingly we take as diameter $\frac{1}{18}$ of the length. We get then

$$A_L = \frac{W - c' . A_c}{m . c} \tag{54}$$

$$A_c = \frac{W - m . c . A_s}{c} \tag{55}$$

If we make $c = 500$ lbs./in.^{2}, $m = 15$, and take for A_c a sectional area based on a length of square equal to $\frac{1}{18}$ the length of column, the sectional area A_s is found from 54.

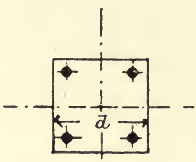


FIG. 149.

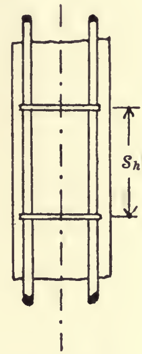


FIG. 150.

Practically speaking, the area of steel required is about **1·75 per cent.** of the total sectional area.

Note. At the end of book is attached the table recommended by the R. I. B. A. report on reinforced concrete, from which the required values of A and A_s can be readily found.

Another formula for a square section is as follows :—

$$m = 15, \; c = 500 \text{ lbs./in.}^2, \; A = \left(\frac{l}{18}\right)^2$$

$$A_L = \frac{W - 500\left(\frac{l}{18}\right)^2}{15 . 500}$$

$$A_L = \frac{W - 1{\cdot}55 l^2}{7500} \text{ in.}^2 \tag{56}$$

If W is taken in lbs. and l in inches, or if W is taken in tons and l in feet, the formula is

$$A_L = \frac{22{\cdot}4\,W - 2{\cdot}23l^2}{75} \qquad (57)$$

EXAMPLE.

A column of 12 ft. length supporting a load of 20 tons to be constructed.

$$12 \text{ feet} = 144 \text{ inches}, \; 20 \text{ tons} = 44800 \text{ lbs.}$$

$$d = \frac{l}{18} = 8 \text{ ins.}$$

$$A_L = \frac{44800 - 500 \cdot 64}{15 \cdot 500} = \frac{12800}{7500} = 1{\cdot}70 \text{ in.}^2$$

If we select No. 4 $\frac{3}{4}$ in. rods with an area of $0{\cdot}4418$ in.^{2}

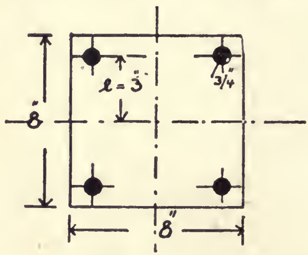


FIG. 151.

$$A_L = 4 \cdot 0{\cdot}4418 = 1{\cdot}76 \text{ in.}^2$$

$$c = \frac{44800}{64 + 15 \cdot 1{\cdot}76} = 495 \text{ lbs./in.}^2$$

The moment of inertia of the concrete section is

$$I_c = \frac{d \cdot d^3}{12} = \frac{8^4}{12} = \frac{4096}{12} = 341 \text{ in.}^4$$

and the moment of inertia of the steel

$$I_s = 4\left(\frac{\pi \cdot d^4}{64} + A_r \cdot e^2\right)$$

Where A_r is the area of one rod and e the distance between centre of rod and axis of column,

$$I_s = 4\left(\frac{3{\cdot}14 \cdot 0{\cdot}75^4}{64} + 0{\cdot}44 \cdot 3^2\right)$$

The value $\frac{3{\cdot}14 \cdot 0{\cdot}75^4}{64}$, namely, the moment of inertia of one rod section, is so small that it need not be considered, consequently

$$I_s = 4{\cdot}044 \cdot 9 = 15{\cdot}84 \text{ in.}^4$$

We find then the load which the column can support without bending from formula 52.

$$W = \frac{10{\cdot}33(341 + 15 \cdot 15{\cdot}84)}{12^2} = 41 \text{ tons,}$$

so that there is no danger of bending, as we have only half that load to carry.

As regards the danger of the rods bending out, we have $d =$ 0·75 in.

$$c_s = 15 \cdot 495 = 7425 \text{ lbs./in.}^2$$

consequently the distance of hoops is

$$s_h = 471{\cdot}4 \frac{d}{\sqrt{7425}} = 121{\cdot}7 \frac{d}{\sqrt{495}} = 4{\cdot}1 \text{ ins.}$$

the cross-bindings, or hoops, should therefore be 4·1 ins. apart.

COLUMNS ECCENTRICALLY LOADED.

Where the load does not act in the centre of gravity of the column section, there are three cases possible. The force can either act within the core, or at the extreme point of it, or, lastly, outside of it.

The core is the centre portion of column section, and its distance from either axis of the column is,

$$d_c = \frac{R}{A} \text{ ins.}$$

where R is the moment of resistance of the section and A the sectional area of the concrete section plus m times the area of steel,

$$m(A_{L1} + A_{L2})$$

(Figs. 152, 153).

Where the section is not symmetrical the centre of gravity has to be found by the following formula,

$$i = \frac{\frac{d \cdot d_1^2}{2} + m[A_{L1} \cdot i_2 + A_{L2}(d_1 - i_1)]}{d \cdot d_1 + m(A_{L1} + A_{L2})} \quad (58)$$

The moment of inertia of the total section relative to the axis XX and omitting the very small moment of inertia of the steel as being insignificant, is then

$$I_x = \frac{d}{3}[i^3 + (d_1 - i)^3] + m[A_{L1}(i - i_2)^2 + A_{L2}(d_1 - i - i_1)^2]$$

In a symmetrical section

$$A_{L1} = A_{L2} = \frac{A_L}{2} \text{ and } i_1 = i_2$$

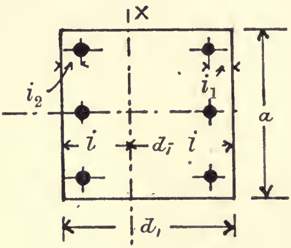


FIG. 152.

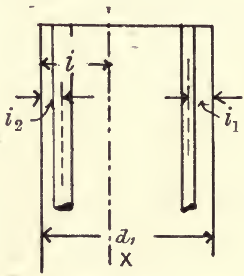


FIG. 153.

$$i = \frac{d_1}{2} \text{ and } I_x = \frac{d \cdot d_1^3}{12} + m \cdot A_L\left(\frac{d_1}{2} - i_2\right)^2$$

Consequently,

$$d_c = \frac{I_x \cdot 2}{A \cdot d_1} = \frac{d \cdot d_1^2}{6 \cdot A} + \frac{2\,m\,A_L\left(\frac{d}{2} - i_1\right)^2}{A \cdot d_1} \quad (59)$$

See Figs. 153, 154.

In the following formulæ,

$A = d \cdot d_1 + m(A_{L1} + A_{L2})$ = Total sectional area in in.^{2}

A_{L1} & A_{L2} = Sectional areas of the reinforcements under compression and tension respectively.

$A_L = A_{L1} + A_{L2}$ in in.^{2}

i_1 & i_2 = the distances of A_{L1} and A_{L2} from the outer edge under compression and tension respectively.

d_c = the width or diameter of core.

e = eccentricity of column in in.

$\frac{E_s}{E_c} = m = 15.$

n = distance of neutral axis from compressed edge.

I = moment of inertia relative to axis of gravity in in.[4]

c_x & c_y = the stresses in the concrete in lbs./in.[2]

t_x & t_y = the stresses in the steel in lbs./in.[2]

1. *The Force Acts within the Core.*

In this case the eccentricity e is smaller than the width of the core d_c and the neutral axis falls outside of the section.

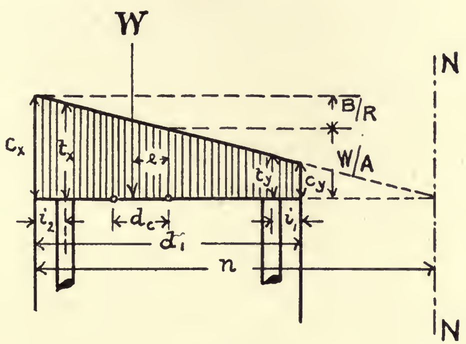


FIG. 154.

$$c_x = \frac{W}{A} + \frac{W.e.d_1}{2I_s} \qquad (60)$$

$$c_y = \frac{W}{A} - \frac{W.e.d_1}{2I_s} \qquad (61)$$

$$A_{L_1} = m\left[\frac{(c_x - c_y)(d_1 - i_2)}{d_1}\right] + c_y \qquad (62)$$

$$A_{L_2} = m\left[\frac{(c_x - c_y).i_1}{d_1}\right] + c_y \qquad (63)$$

2. *The Force Acts at the Extreme Edge of Core* (Fig. 155).

In this case d_c becomes $= e$,

$$d_c = e = \frac{I_s \cdot 2}{A \cdot d_1} \text{ and therefore}$$

$$c_x = \frac{W}{A} + \frac{W \cdot d_c \cdot d_1}{2 I_s} \text{ or}$$

$$c_x = \frac{2W}{A} \qquad (64)$$

$$c_y = 0 \qquad (65)$$

As n becomes $= d$, the stresses are

$$A_{L_1} = m \cdot c_x \frac{(d_1 - i_2)}{d_1} \qquad (66)$$

$$A_{L_2} = m \cdot c_x \frac{i_2}{d_1} \qquad (67)$$

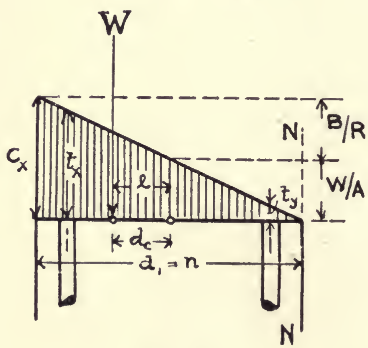


FIG. 155.

These formulæ may also be used if the force acts immediately near the border of the core.

3. *The Force Acts without the Core.*

In that case e is $>$ than d_c and the neutral axis occurs within the section so that $n > d$, Fig. 156, so that,

$$\frac{c_x}{E_c} : n = \frac{A_{L1}}{E_s} : (d_1 - n - i_2)$$

$$t_x = m \cdot c_x \frac{n - i_2}{n} \qquad (68)$$

$$t_y = m \cdot c_x \frac{d_1 - n - i_2}{n} \qquad (69)$$

$$W = c_x\left[\frac{d \cdot n}{2} + \frac{m \cdot t_x}{n}(2n - d_1)\right. \qquad (70)$$

$$\text{and } n = \frac{d}{3 \cdot m \cdot A_L} \cdot n^3 + \frac{d \cdot i_3}{m \cdot A_L} \cdot n^2 - (d_1 + 2i_3) \cdot n \text{ or}$$

$$n = 2i_2^2 + d_1^2 - d(2i_1 - di_3) \qquad (71)$$

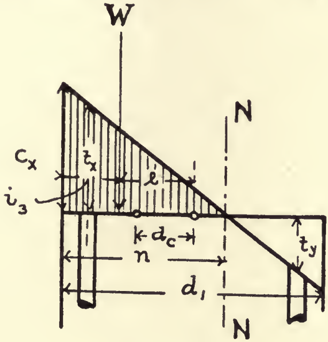


FIG. 156.

EXAMPLE.

A column 14 by 14 ins. carrying a load of 42 tons is eccentrically loaded, the load occurring at a point 1·5″ from the centre of

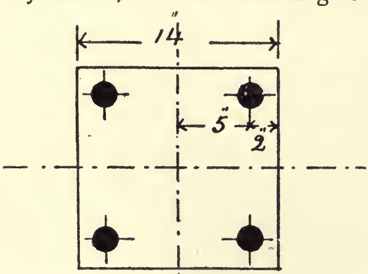


FIG. 157.

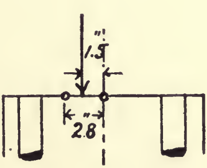


FIG. 158.

column. The column is reinforced with 4 rods of $1\frac{1}{2}$ in. diam. and 1·76 in.² sectional area each. What are the stresses?

$d = d_1 = 14$ ins.

$i_1 = i_2 = 2$ ins.

$A_L = 4 \cdot 1{\cdot}76 = 7{\cdot}04$ in.²

$A = 196 + 15 \cdot 7{\cdot}04 = 301{\cdot}6$ in.²

$$I_s = \frac{14^4}{12} + 15 \cdot 7{\cdot}04 \cdot 5^2 = 5841{\cdot}3 \text{ in.}^4$$

$$d_c = \frac{5841{\cdot}3 \cdot 2}{301{\cdot}6 \cdot 14} = 2{\cdot}8 \text{ ins.}$$

and as $e = 1{\cdot}5$ in. and $W = 94080$ lbs.

$$c_x = \frac{94080}{301{\cdot}6} + \frac{94080 \cdot 1{\cdot}5 \cdot 14}{2 \cdot 5841{\cdot}3} = 480 \text{ lbs./in.}^2$$

$$c_y = \frac{94080}{301{\cdot}6} - \frac{94080 \cdot 1{\cdot}5 \cdot 14}{2 \cdot 5841{\cdot}3} = 144 \text{ lbs./in.}^2$$

$$t_x = 15\left[\frac{(480 - 144)(14 - 2)}{14} + 144\right] = 6480 \text{ lbs./in.}^2$$

$$t_y = 15\left[\frac{(480 - 144) \cdot 2}{14} + 144\right] = 2880 \text{ lbs./in.}^2$$

What load could the same column support if the load was acting at a distance of 4 inches from the centre?

As the distance 4 is less than $\frac{1}{2} d_1 = 7$, the load acts still within the section

$$i_3 = d_1 - e = 7 - 4 = 3 \text{ ins.}$$

From formula 71 we find the position of neutral axis as follows:—

$$\frac{14}{3{\cdot}15 \cdot 7{\cdot}04} \cdot n^3 - \frac{14{\cdot}3}{15 \cdot 7{\cdot}04} \cdot n^2 + (14 - 2{\cdot}3) \cdot n = 2{\cdot}2^2 + 14^2 - 14(2 \cdot 2 + 3)$$

$$0{\cdot}044\, n^3 - 0{\cdot}39\, n^2 + 8\, n = 8 + 196 - 98 = 106$$

$$n = 8 \text{ ins.}$$

If we allow for c the value of 500 lbs./in.^{2}, we get from formula 70,

$$W = 500\left[\frac{14{\cdot}8}{2} + \frac{15 \cdot 3{\cdot}5^2}{8}(16 - 14)\right]$$

$$W = 34600 \text{ lbs.} = 15 \text{ tons.}$$

The stress in the steel we find from formula 68,

$$t_x = 15 \cdot 500 \frac{8 - 2}{8} = 5625 \text{ lbs./in.}^2$$

CHAPTER XII

FORMULAE FOR ARCHES, VAULTS, ETC.

The reinforcing rods are usually spaced symmetrically parallel to the longitudinal axis, and consequently the core of the arch and the moment of resistance at any point can be determined. To ascertain the stresses the formulæ already developed for axial and eccentrical loading are used.

The stresses are as a rule graphically ascertained by finding the line of resistance, which must on no account fall outside the arch section and should be within the inner third of section.

The depth of arch ring at crown may be assumed from experience or determined from the formula,

$$d_c = \sqrt{l} + 0{\cdot}1\,l = 0{\cdot}005\,w + 0{\cdot}0025\,w_d \qquad (72)$$

wherein d_c = depth at crown in in.

l = clear span in feet.

w = superimposed load uniformly distributed in lbs./ft.^{2}

w_d = dead load above arch ring at crown in lbs./ft.^{2}

The radial depth at quarter points is usually made = $1\frac{1}{3}$ that at the crown.

The rise of arch is preferably made = $\frac{1}{4}$ to $\frac{1}{6}$ of the span.

Fig. 159 illustrates a simple form of arch; the stresses in any particular joint are found as follows:—

If $\frac{W}{2}$ = weight of half the arch.

W_J = weight of arch up to the joint under observation,

W = live load on half arch for 12 ins.

l = effective span of arch.

V = rise of arch.

d_1 & d_2 = the distances of $\frac{W}{2}$ and W_J from the centre of abutment.

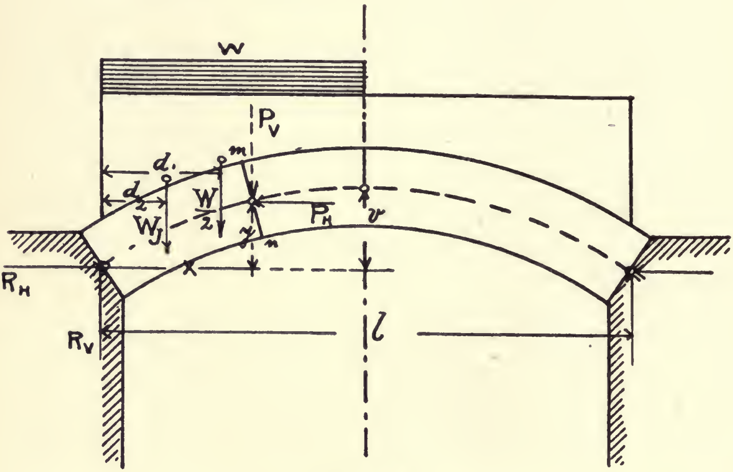


FIG. 159.

x and y = the co-ordinates of centre of section mn referred to centre of abutment.

Assuming that the line of resistance passes through the centres of the joints at abutments and crown and that the live load occurs on one-half of the arch only but allowing for the self load of the whole arch, we get the components of the left reaction as follows :—

$$R_V^1 = \frac{W}{2} \text{ (on account of self load)}$$

$$R_V^2 . l - w . \frac{l}{2} . \frac{3}{4} l = o$$

$R_V^2 = \frac{3}{8} w \,.\, l$ (on account of live load), therefore

$$\mathbf{R_V = \frac{W}{2} + \frac{3}{8} wl} \qquad (73)$$

$$R_V \,.\, \frac{l}{2} - R_H^1 \,.\, v - \frac{W}{2}\left(\frac{l}{2} - d_1\right) = o$$

$$R_H^1 = \frac{R_V \,.\, \frac{l}{2} - \frac{W}{2} \,.\, \frac{l}{2} + \frac{W}{2} \,.\, d_1}{v}$$

$$R_H^1 = \frac{\frac{W}{2} \,.\, d_1}{v} \text{ (on account of self load)}$$

$$R_H^2 \,.\, v = R_V \,.\, \frac{l}{2} - \frac{w \,.\, l}{2} \,.\, \frac{l}{4}$$

$$R_H^2 = \frac{1}{16} \frac{w \,.\, l^2}{v} \text{ (on account of live load)}$$

$$\mathbf{R_H = \frac{1}{v}\left(\frac{W}{2} \,.\, d_1 + \frac{w \,.\, l^2}{16}\right)} \qquad (74)$$

The vertical and horizontal components of the force acting in centre of section *mn* are as follows :—

$$P_V = R_V - W_J = \frac{W}{2} - W_J \text{ (on account of self load)}$$

$$P_V = R_V - w \,.\, x = w\left(\frac{3}{8} l - x\right) \text{ (on account of live load)}$$

therefore $\mathbf{P_V = \frac{W}{2} - W_J + w\left(\frac{3}{8} l - x\right)}$ (75)

$$P_H = R_H = \frac{\frac{W}{2} \,.\, d_1}{v} \text{ (on account of self load)}$$

$$P_H = R_H = \frac{w \,.\, l^2}{16 v} \text{ (on account of live load)}$$

therefore $\mathbf{P_H = \frac{1}{v}\left(W_J \,.\, d_1 + \frac{w \,.\, l^2}{16}\right)}$ (76)

The bending moment is then as follows :—

$$\mathbf{B = R_V x - R_H \,.\, y - W_J (x - d_2) - \frac{w \,.\, x^2}{2}} \qquad (77)$$

and from this the stresses in concrete and steel can be ascertained according to the formulæ for eccentrical loading.

The arch should be investigated for reverse positions of the load to obtain the maximum stresses. It should in any case be considered under full load, half load and centre third load.

Another way of calculating the reinforcement required is as follows. We ascertain the thrust and bending moments, and in order to determine the amount and position of reinforcement we find first the compressive stress of the concrete due to thrust, and deduct this from the safe stress of the concrete. The amount of reinforcement required to resist the bending moment is then arrived at by using the formulæ for beams. The compressive value for the concrete must in this case be reduced by the amount obtained to resist the thrust, and the safe tensile stress for steel increased m times the unit compression due to thrust. Similarly the formulæ for double reinforced beams may be used for arches with double reinforcements.

Temperature stresses must be carefully considered. Considering the abutments as rigid, these stresses create a thrust together with a negative bending moment at the crown.

If the abutments cannot be considered as perfectly rigid the horizontal thrust must be taken by tension rods, this form of construction being quite usual in arched roofs.

CHAPTER XIII

PATENT BARS AND SYSTEMS

A GREAT variety of systems of reinforced concrete construction, patent bars, etc., have been invented within the last few years, and the following is a condensed review of those principally used in this country at the present moment. They are arranged alphabetically. All of these bars and systems have certain advantages under certain conditions and circumstances, and if used in their right places may tend to improve the soundness of the construction and reduce cost.

The Armoured Tubular Flooring Co. Ltd., 153 Victoria Street, Westminster, S.W.

The armoured tubular floor known as the "Herbst" system consists of the concrete webs AA, concrete tube B and top layer of concrete C (Fig. 160), the concrete webs A having steel reinforcements made of mild steel of 28 to 32 tons tensile resistance.

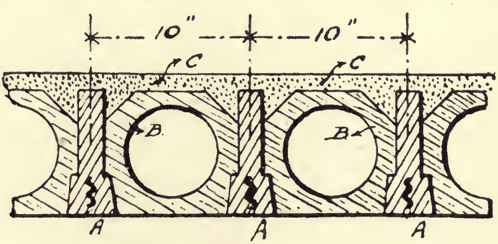


FIG. 160.

Fig. 161 shows the reinforcement; the floor has been constructed in spans up to 30 feet.

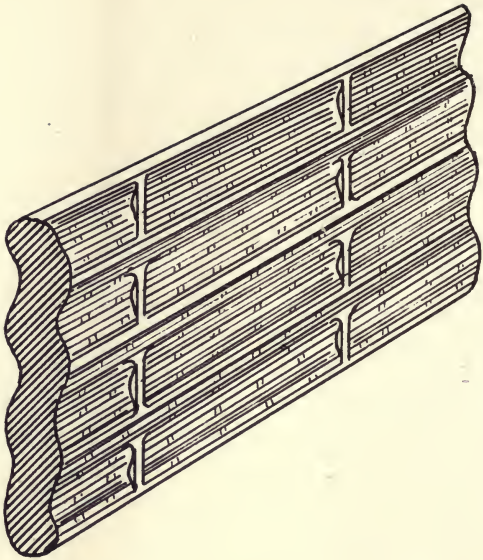


Fig. 161.

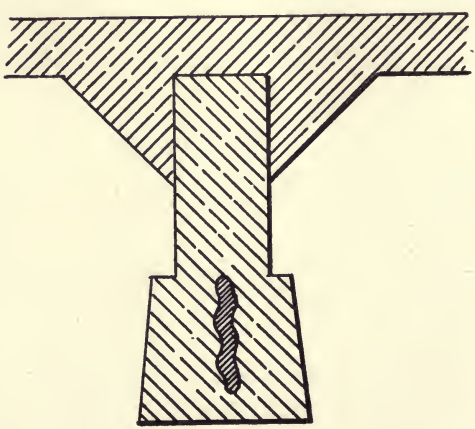


Fig. 162.

A special feature is the grip on the concrete webs obtained by shaping the top layer as shown in Fig. 162, which should be an

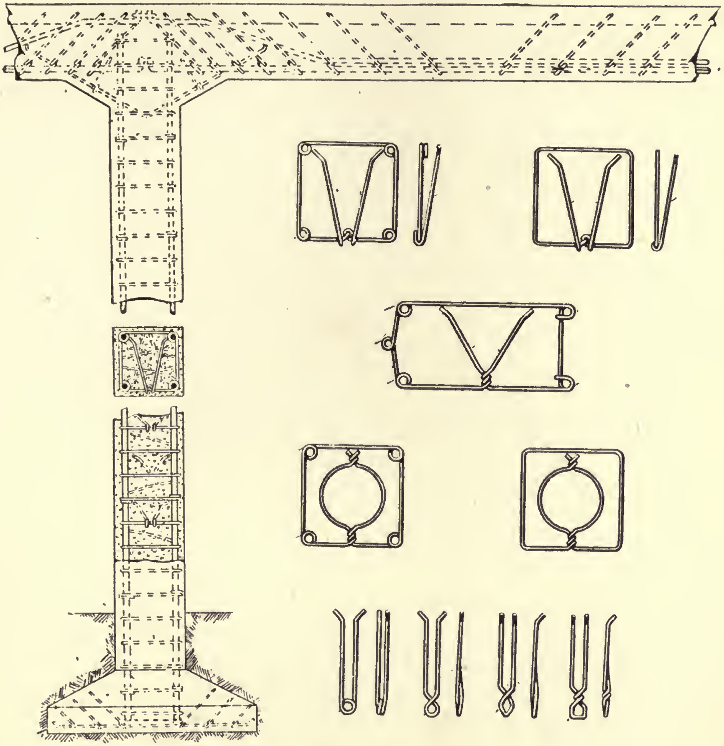


FIG. 163.

excellent protection against shear. The floor does not require centering during construction.

FIG. 164.

(For stock sections of reinforcement, see p. 151.)

The British Reinforced Concrete Engineering Co. Ltd., 196 Deansgate, Manchester, use clips and stirrups made of high carbon steel of various shapes. Fig. 163 shows general arrangement and details.

The stirrups are sprung on the tension bar by squeezing the

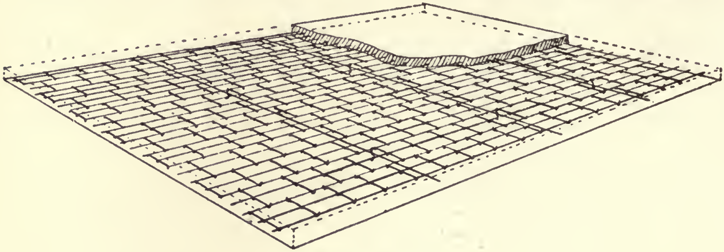


FIG. 165.

arms and, when released, retain a tight grip in the required

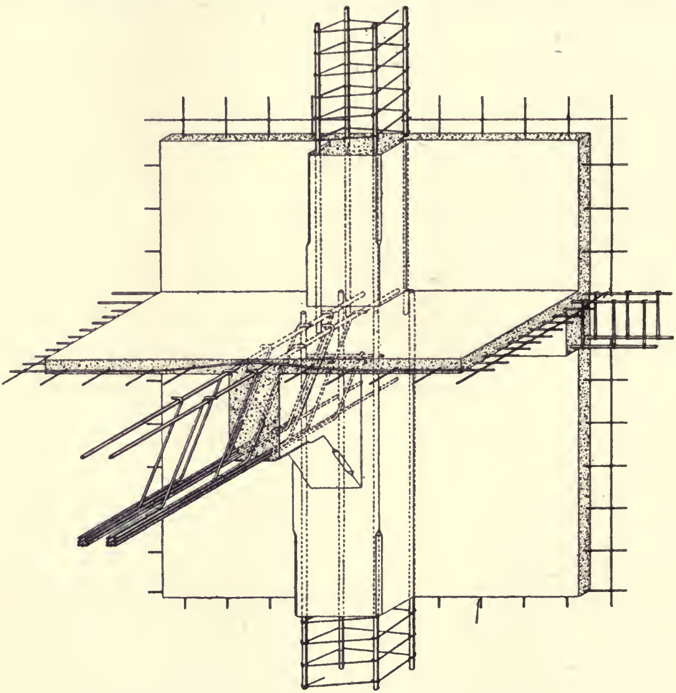


FIG. 166.

position, tending to come tighter when the concrete is rammed.

The ends of the hoop rods are arranged to lie through the core in such a manner as to be securely anchored in the concrete and to bond the core in every direction against bulging action set

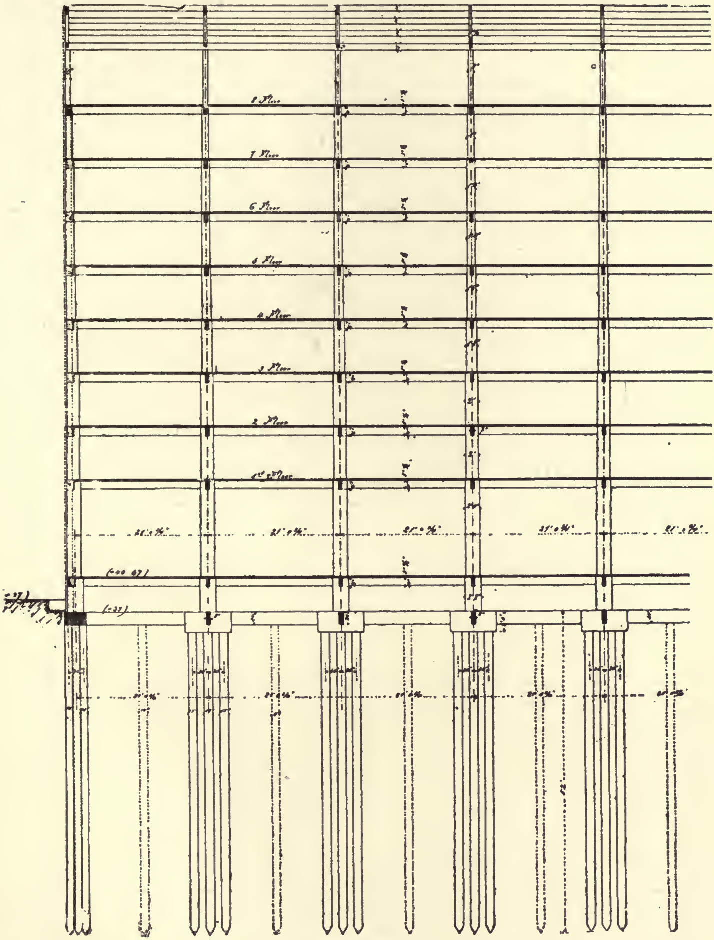


FIG. 167.

up under heavy loading. The illustration shows how and where the various fitments *b* and *c* are used.

The Chain Concrete Syndicate, 1 Basinghall Square, Leeds, use ordinary round mild steel bars of such dimensions as to produce sufficient tensile stress. The leading feature of the system is that all bars are connected by steel clips of patterns and weights to suit requirements. These clips (Fig. 164) are made from flat bar steel ¼ in. thick and from ½ to 1½ in. broad and cut and bent by machinery. The company claim that owing to the fact that the reinforcement is distributed uniformly in all directions larger floor panels can be constructed without the necessity of beams. Fig. 165 shows the arrangement of reinforcement in floor.

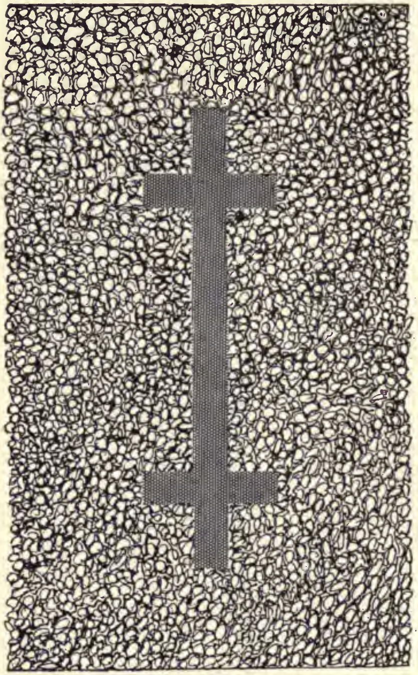


FIG. 168.

Edward Coignet Ltd., 20 Victoria Street, S.W.

The Coignet system of armoured concrete is one of the oldest forms of reinforced concrete. The principal feature of the construction is the connexion of tension and compression rods with stirrups (Fig. 166).

In upright structures like columns, piles, etc., the rods are bound by special ties to prevent bursting. Fig. 81, p. 59, illustrates a Coignet pile usually of a circular section, varying between 10 and 16 ins. in diameter. A Coignet pile during construction has been previously mentioned (see p. 61). Fig. 167 shows section through a tobacco warehouse at Bristol in the Coignet system.

A boiler foundation supported on piles has already been mentioned (see p. 60), also an early piece of work in moulded concrete, the aqueduct for the Paris water supply, executed by the late Mons. François Coignet (see p. 2). The principal arch has a span of about 132 feet, the total aqueduct being about 5 miles long and comprising twenty-eight arches.

The Columbian Fireproofing Co. Ltd., 37 King William Street, E.C., use special ribbed bars, suspended in steel stirrups over joists or resting on walls. The ribbed bars are embedded in the concrete (Fig. 168), the thickness of concrete and depth of bars being governed by the width of spans, etc. Fig. 169 illustrates the system.

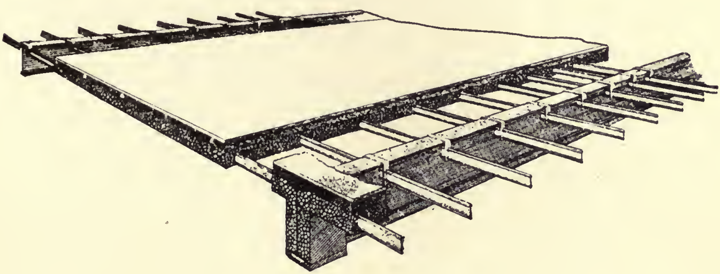


Fig. 169.

The same firm are also the makers of the "Bonna" reinforced concrete pipes. These have a thin steel tube to make them perfectly watertight. The spiral reinforcement consists of steel bars cruciform in section and round similar bars running longitudinally so that a complete circular network of steel bars is formed.

The Concave Floor Co., 1 Hawstead Road, Catford, S.E., use ordinary wire meshing of various thickness and gauge; according to spans, 1, 2 or 3 layers being used. Fig. 170 shows an arrangement of hollow flooring to facilitate drying out in case the floor is constructed at the ground level and thus prevent expansion and cracking of parquet, wood-block and other finish. The floor is a centering in itself and can be constructed either hollow or solid. The mode of construction is to first place ordinary large mesh

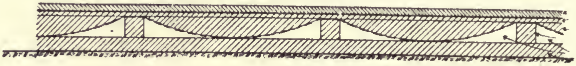


FIG. 170.

wire netting over the beams covered with brown paper to prevent the concrete squeezing through. A thin layer of concrete is then

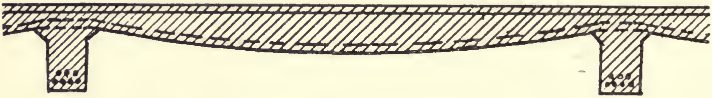


FIG. 171.

spread over this and the reinforcing wire mesh follows, after which the bulk of the concrete is brought in. The first layer of

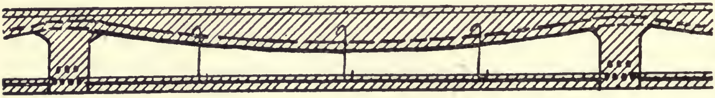


FIG. 172.

wire can afterwards be cut away, together with the paper, and the floor finished with level soffit or it may remain where a hollow

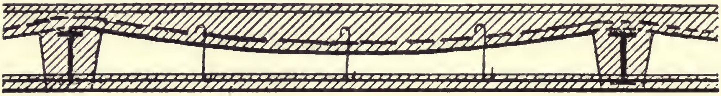


FIG. 173.

floor is desired. Figs. 171, 172, 173 show types of this floor which is also very suitable for flat roofs. The whole area being cut up into very small squares of minute reinforcement the formation of hair cracks is made almost impossible while possibility of failure is practically avoided particularly where the meshing continues over several spans.

The Considère Construction Co. Ltd., 5 Victoria Street, S.W. The principal feature of the Considère system is the spiral

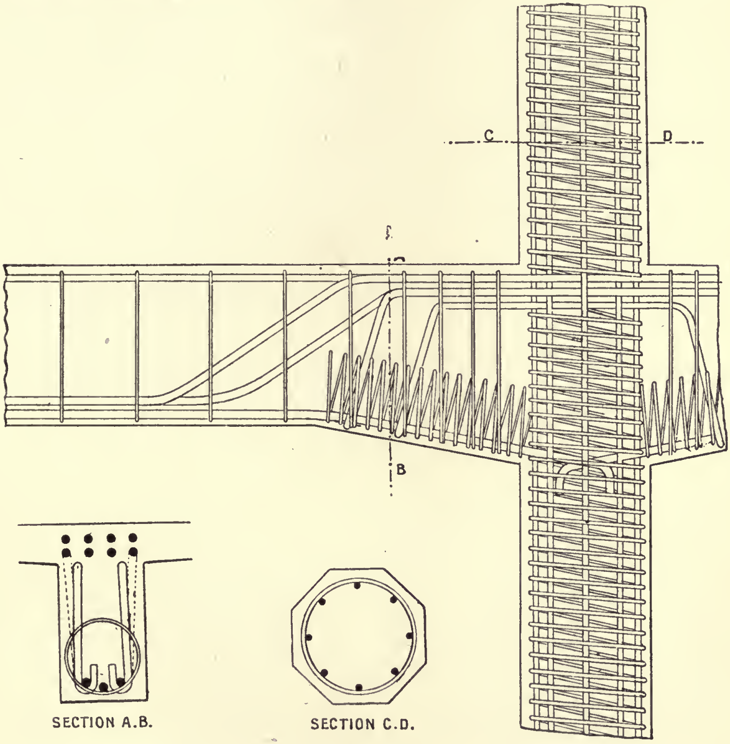


FIG. 174.

armouring of the concrete. It is claimed that a much greater resistance is obtained. Fig. 174 shows details of a continuous spirally armoured girder. The system lends itself particularly also to pile making and, furthermore, some excellent work has been done in bridge building (see p. 59).

H. Kempton Dyson's Patent Bar. This is a recent invention and not yet commercially worked. The bar provides for rigid attachment of shear members to top as well as bottom rods, and forms practically a lattice girder with the concrete in which it is embedded. Owing to the rigid attachment of the various parts there is no fear of displacement during concreting operations, while a mechanical bond is also created.

The bar can be rolled up to a length of 60 or 80 ft., and all cutting being done while the metal is hot, the expense of cutting cold and consequent danger of splitting is done away with. The only processes entailed in its manufacture are rolling, cutting to length and expansion. That done it can be put in place straight away. The bar has many other advantages, such as easy handling, etc. (Fig. 175.)

The cutting is done by means of spiral cutting edges on the rolls.

Fig. 175.

This patented process has been applied to the making of expanded metal for reinforcing floor or wall slabs, pipes, etc., and to the reinforcing of columns, piles, etc.

The Empire Stone Co. Ltd., 231 Strand, W.C., are the makers of the *Siegwart floor*. This consists of hollow beams made of granite concrete and reinforced with steel rods. The beams are placed side by side on the supports, walls, or girders, and then grouted in with cement mortar. (Figs. 176, 177.)

The Expanded Metal Co. Ltd., York Mansions, Westminster, S.W., manufacture an expanded steel lathing from sheets of rolled metal of various thicknesses, cut and expanded by machinery into meshes of various shapes.

This material is a very useful reinforcement for floor and foundation slabs, partitions and particularly also for encasing steel work as a protection against fire.

Fig. 178 shows a typical floor reinforced and generally treated

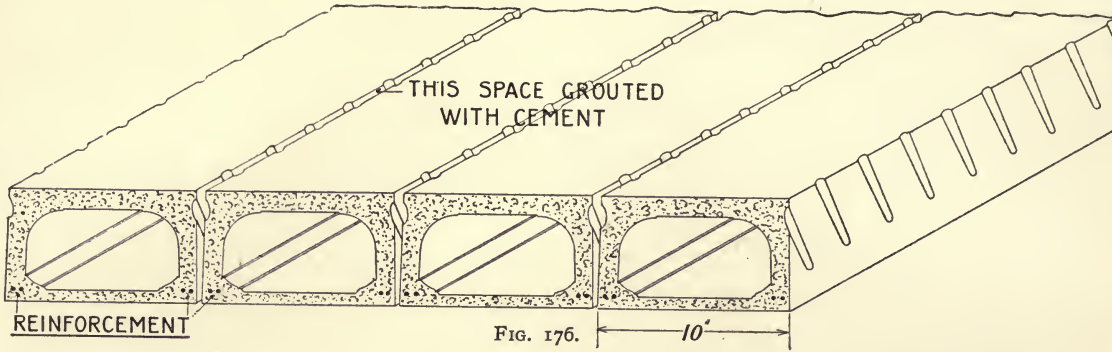


Fig. 176.

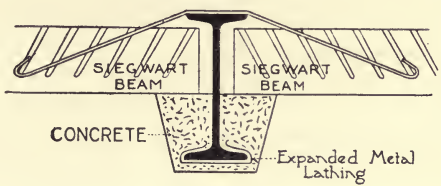


Fig. 177.

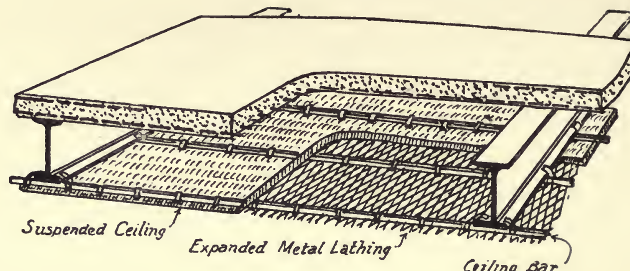
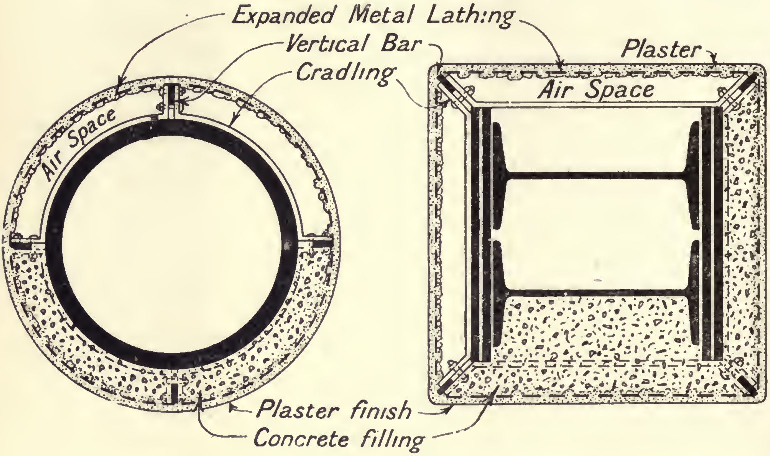


Fig. 178.

with expanded metal, while Figs. 179, 180 illustrate how by means of this material columns and stanchions may be protected from fire. (For stock sizes, see p. 151.)

The Hennebique system (L. P. Mouchel & Partners, 38



FIGS. 179 and 180.

Victoria Street, S.W.) is one of the oldest systems of reinforced concrete known and has been used for many important works in many countries. Ordinary round rods are used, together with a series of hangers or stirrups, Fig. 181 showing the usual arrange-

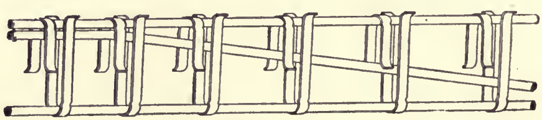


FIG. 181.

ment. Fig. 184 shows the reinforcement of columns, the longitudinal bars having closely spaced steel wire links of $\frac{3}{16}$ in. steel wire applied in sets of four. A Mouchel Hennebique pile has already been mentioned (see p. 60) which, in addition to the longitudinal bars and transverse links, has diaphragms, further connecting the bars. These diaphragms hold in place a consecutive

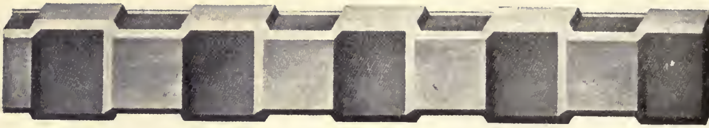


Fig. 182.—Indented Bar.

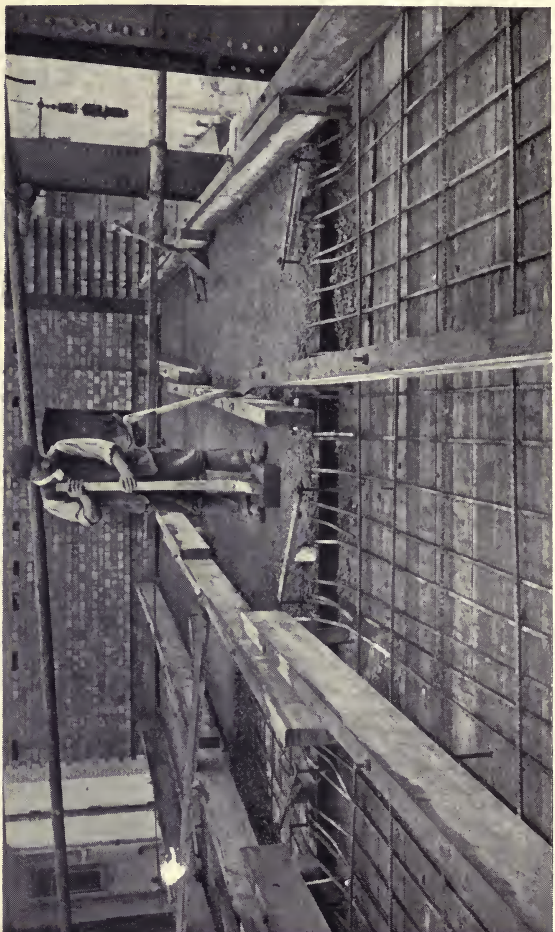


Fig. 183.—View of Floor showing use of Indented Bar.

[*To face page* 139.]

series of tubes, each about 4 ft. long, their object being to form the hollow core of the finished pile (p. 59). A column base is shown on p. 60. The lower portion of the concrete is reinforced by a double system of bars laid in two directions so as to provide for the tensile stresses caused by the bending moments developed by the central load and the vertical reaction of the ground.

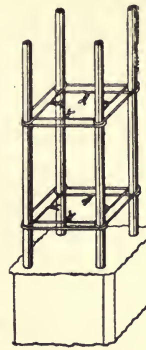


FIG. 184.

The *indented steel bar* is manufactured by the *Patent Indented Steel Bar Co. Ltd.*, Queen Anne's Chambers, Westminster, S.W. This bar gives a great bonding efficiency. It is of uniform cross section throughout, but in longitudinal section there are a series of projections, the edges of which are inclined at an angle exceeding the angle of friction between concrete and steel, so as to prevent splitting; a mechanical bond is thus given throughout, without any waste of material (Fig. 185). The bars are easy to handle and can be bent to any required shape. Where shearing stresses occur, the nature of the surface of bar greatly increases the adhesion of the concrete and prevents

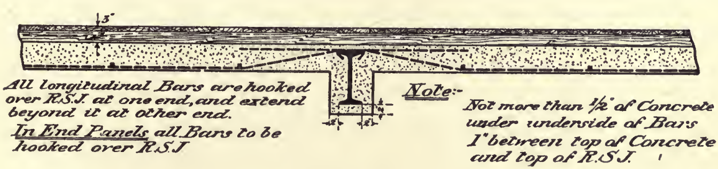


FIG. 185.

slipping. A retaining wall reinforced with these bars has already been mentioned (see p. 56).

Figs. 182 and 183 show a floor during construction and section of bar.

On p. 61 the stadium at the Franco-British Exhibition is reproduced, in the construction of which these bars were used. (For stock sizes, see p. 152.)

The Improved Construction Co. Ltd., of 47 Victoria Street, Westminster, S.W., manufacture a variety of articles by a special process, called after the inventor, the Jagger process. The principal feature of this is a vibrating oscillating table by means of which a perfect mixing of the concrete is obtained giving maximum density. Mention must be made of railway sleepers

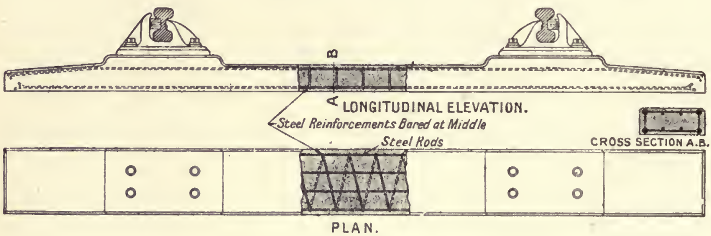


FIG. 186.

made in this system (Fig. 186) which should prove a great improvement on the present wooden and iron sleepers.

Johnson's wire lattice, manufactured by *R. Johnson*, Clapham, and *Morris, Ltd.*, Lever Street, Manchester, is made in sheets or

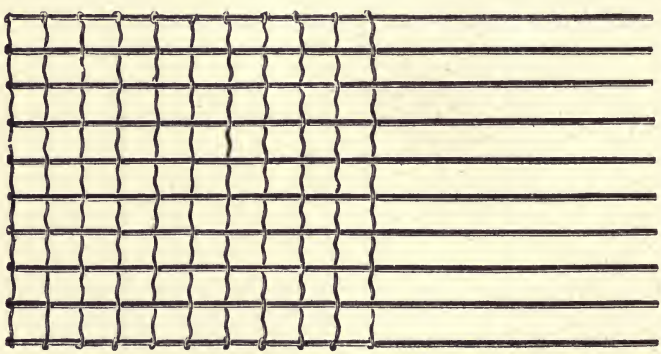


FIG. 187.

rolls of practically any length by any width up to 8 ft. 6 ins. It is made up of tension and binding wires, woven to form a rectangular mesh. The tension wires are straight and the binding wires crimped. This material is a useful reinforcement for floor slabs and similar structures (Fig. 187). (For stock sizes, see p. 153.)

The "*Kahn*" *bar* is manufactured by the *Trussed Concrete Steel Co. Ltd.*, Caxton House, Westminster, S.W. The bar is of diamond shape section (Fig. 188), having side

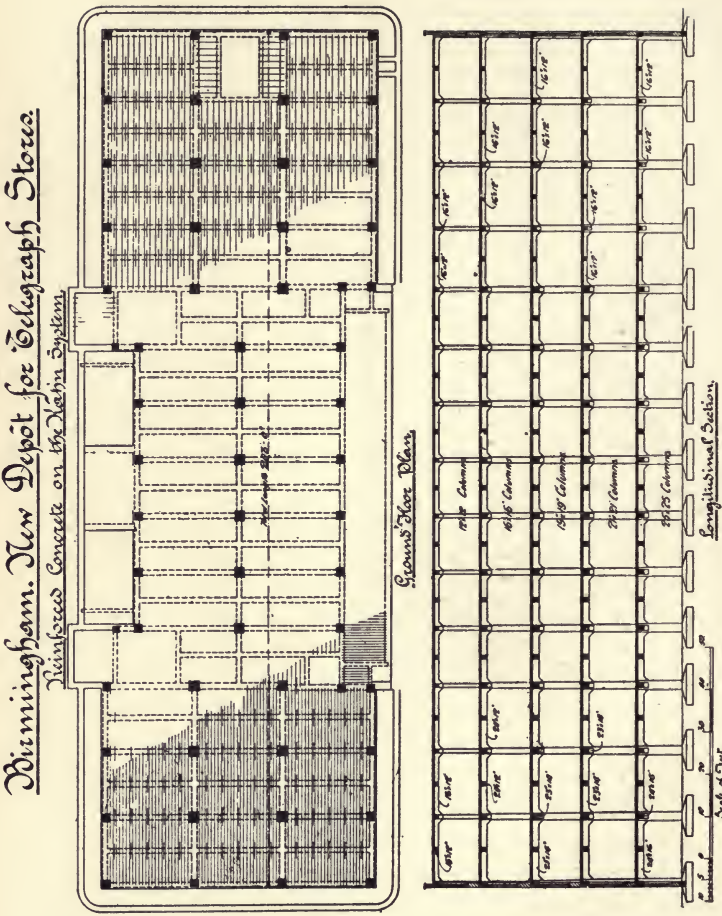
Fig. 188.

wings turned up as shown to form shear members and to give a mechanical bond. The bar is supplied in 4 different sizes and various patterns, some having the wings all one way, others in opposite directions, either the whole bar being sheared or the centre left unsheared. The advantages are obvious, and wherever shearing stresses occur the bar is used to great advantage.

As the shearing members are rigidly connected with the main bar, displacement during concreting is made impossible. Figs. 189, 190 illustrate plan and section and part elevation of new telegraph stores, Birmingham. Elevation and section of one span of Charles Creek Bridge has already been mentioned (see p. 55). (For stock sizes of bar, see p. 153.)

Leslie & Co. Ltd., Kensington Square, W. In this system the main members are connected with strips of flat metal or wire looped to engage the bars. Hooks are driven on the stirrups, which owing to the wedge-shaped, bent over ends tighten the strip or rod upon the bar. This system has the great advantage that the whole of the steel work is framed up completely as a unit and dropped into place, and owing to the rigid connexion of the various members displacement during concreting is prevented. Fig. 191 shows a typical arrangement of floor, beam and column construction.

The *lock woven mesh system*, by *James H. Tozer & Son, Ltd.*, York Mansions, Westminster, S.W., is suitable for floors, roofs, raft foundations, walls, sewers, etc.; in fact, wherever large areas have to be reinforced. As the name implies, the material consists of wire, woven together to form a square mesh and lock jointed at the points of intersection. The material, being made in continuous sheets, gives a uniform distribution of stresses and a mechanical bond.



FIGS. 189 and 190.

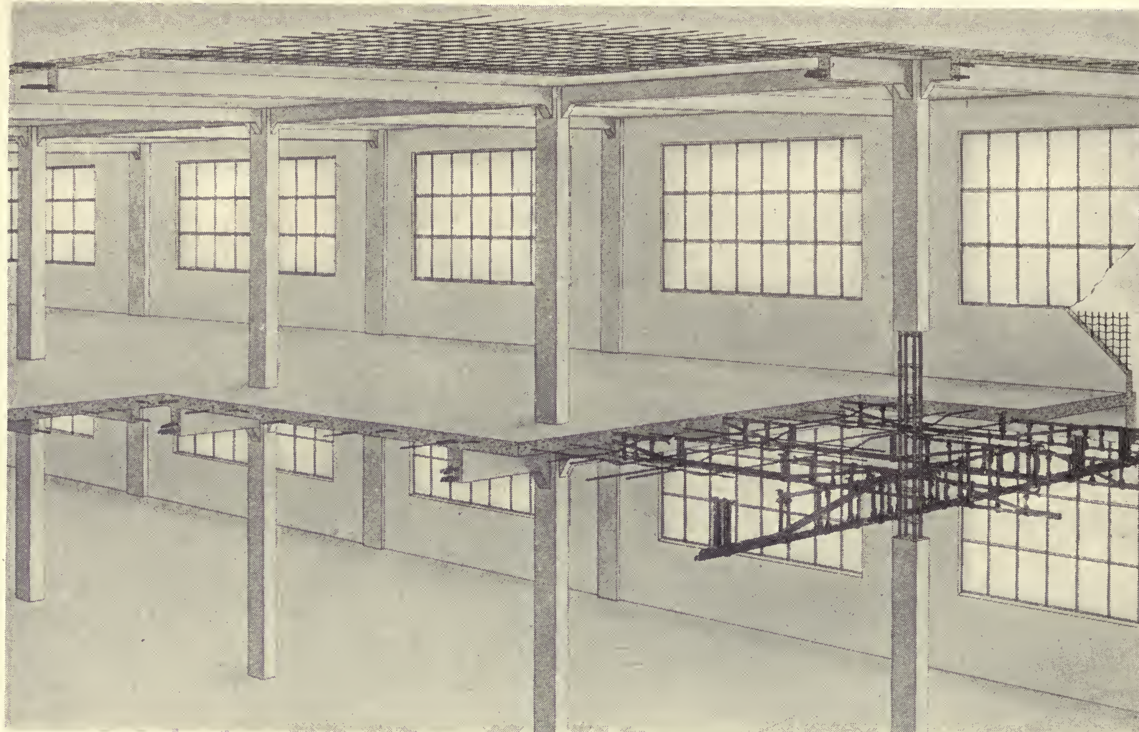


FIG. 191.—VIEW OF FLOOR CONSTRUCTION UNDER THE LESLIE SYSTEM.

[To face page 142.]

FIG. 192.—TOZERS FLOOR.

[*To face page* 143.]

Fig. 192 shows a floor being laid of 15 ft. span. An appli-

Fig. 193.

cation of the material for roof construction has already been dealt with (see p. 64). (For stock sizes, see p. 154.)

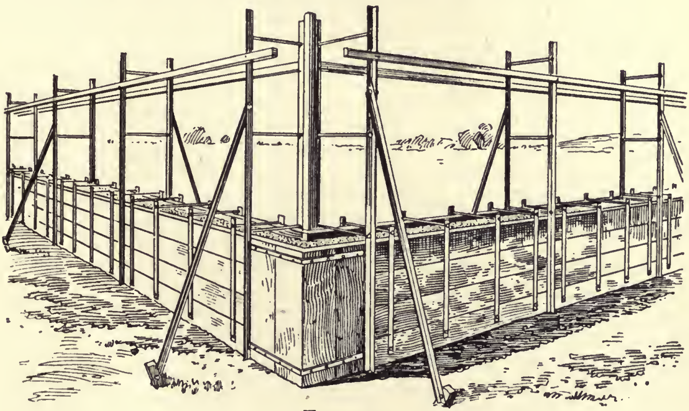


Fig. 194.

Potter & Co. Ltd., 66 Victoria Street. Fig. 194 illustrates a system of forms for concrete walls, designed to reduce the cost of forms and waste of timber used. The appliances consist of steel girders secured together by bars and pins to suit walls of any thickness and they are raised as the walls grow in height. The trough boards are attached to smaller girders. After the concrete has been deposited for some 24 hours the appliance is raised, and so on until the top of wall is reached, when it is finally taken down and ready for re-use. Thus a great saving in timbering is effected. Mr. Potter has also just brought out a new reinforcing arrangement for beams (Fig. 195). The system creates rigid and immovable attachment and practically forms a truss arrangement. The tensile member is not weakened by holes, and

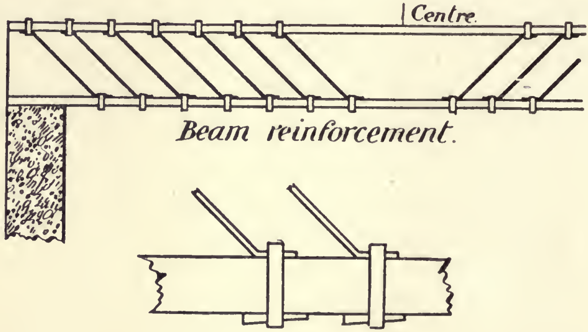


FIG. 195.

the shear members can be quickly attached on the job, while displacement during concreting and sliding of tensile member when under severe stress is made impossible.

"*Sideolith*," 19 Temple Street, Birmingham, are the makers of the "*skeleton*" reinforcement, a bar stamped out of steel, split and expanded into girder-like form. This bar would appear to be particularly useful for beams, lintels and the like, the perfect connexion of shear members to tension and compression rods preventing any possible displacement during concreting. Fig. 196 illustrates the skeleton bar which is made in sections from $3\frac{1}{4}$ to 6 ins. width and a proportionate depth of $4\frac{1}{2}$ to 14 ins.

The Visintini system, largely used on the Continent, is particularly suitable for large spans such as occur in roof and bridge constructions.

Figs. 197 - 199 show the arrangement of the reinforcements, the whole beam being a lattice girder, and the various rods are calculated in the same fashion as such a girder. A typical application of this type of beam has already been mentioned (see p. 64).

E. P. Wells, 94 Larkhall Rise, Clapham. Wells' twin rod has the shape of the figure 8, being composed of two round rods (Fig. 200).

In beams the twin rods are placed flat and one of them bent up towards the support and continued over same in the usual way. The web between the two rods is slit and stirrups inserted to form shear members. Figs. 201-203 illustrate a column and base and floor together with details of the Wells system.

Fig. 196.

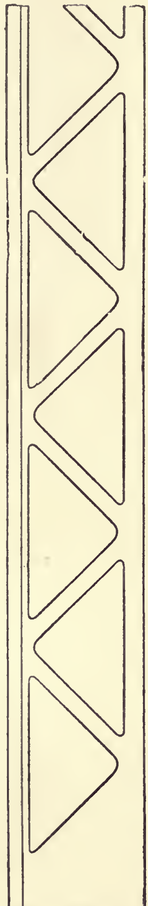


Fig. 197.

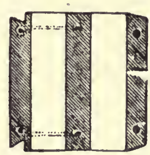


Fig. 199.

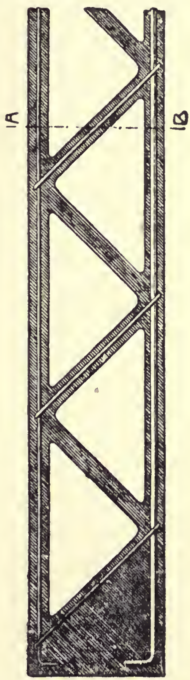


Fig. 198.

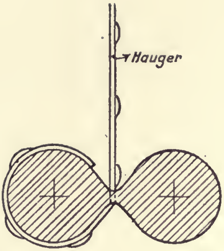


FIG. 200.

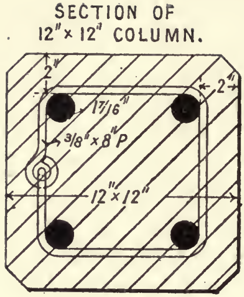


FIG. 201.

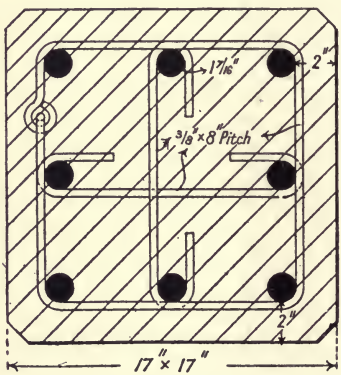
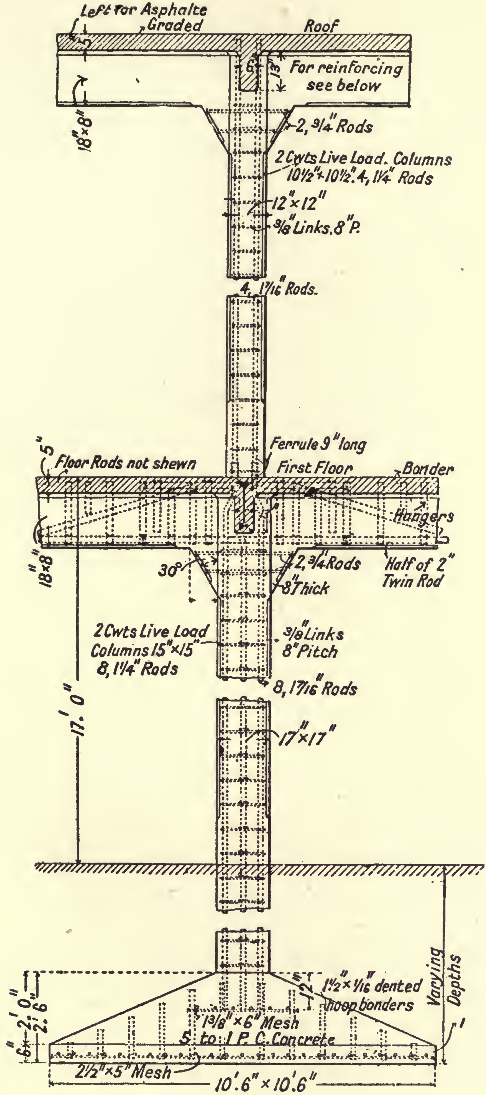


FIG. 202.



Left for Asphalte
Graded
Roof
5"
6"
13"
For reinforcing see below
18"×8"
2, 3/4" Rods
2 Cwts Live Load. Columns 10½"×10½". 4, 1¼" Rods
12"×12"
3/8" Links. 8" P.
4, 17/16" Rods.
Ferrule 9" long
5"
Floor Rods not shewn
First Floor
Bonder
Hangers
13"
18"×8"
30°
2, 3/4" Rods
8" Thick
Half of 2" Twin Rod
2 Cwts Live Load Columns 15"×15" 8, 1¼" Rods
3/8" Links 8" Pitch
8, 17/16" Rods
17'. 0"
17"×17"
Varying Depths
1½"×1/16" dented hoop bonders
12"
2'. 0"
2'. 6"
6"
1 3/8"×6" Mesh
5 to 1 P. C. Concrete
2½"×5" Mesh
10'. 6"×10'. 6"

FIG. 203.

Values d, n and A_T for a Width of Slab = 12 Inches for Various Proportionate Stresses t and c.

Allowable max. stresses in lbs./in.[2]		Effective depth in ins.	Distance of neutral axis in ins.	Sectional area of steel required in in.[2]		
Steel t	Concrete c	d	n	A_T	A_T	
15000	300	0·0511 a 0·1767 b	0·230 d	0·12 n	0·001413 a 0·000408 c	α β
15000	350	0·0449 a 0·1573 b	0·259 d	0·14 n	0·001625 a 0·000469 c	α β
15000	400	0·0402 a 0·1391 b	0·285 d	0·16 n	0·001832 a 0·000528 c	α β
15000	450	0·0365 a 0·1263 b	0·310 d	0·18 n	0·002037 a 0·000588 c	α β
15000	500	0·0335 a 0·1160 b	0·333 d	0·20 n	0·002261 a 0·000640 c	α β
15000	550	0·0304 a 0·1080 b	0·354 d	0·22 n	0·002486 a 0·000700 c	α β
15000	600	0·0290 a 0·1008 b	0·375 d	0·24 n	0·002624 a 0·000756 c	α β
14000	550	0·0305 a 0·1059 b	0·370 d	0·23 n	0·002678 a 0·000768 c	α β
14000	500	0·0328 a 0·1138 b	0·349 d	0·21 n	0·002453 a 0·000708 c	α β
14000	450	0·0357 a 0·1212 b	0·325 d	0·19 n	0·002244 a 0·000648 c	α β
13000	550	0·0299 a 0·1039 b	0·388 d	0·25 n	0·002955 a 0·000854 c	α β
13000	500	0·0323 a 0·1113 b	0·366 d	0·23 n	0·002712 a 0·000785 c	α β
13000	450	0·0350 a 0·1208 b	0·342 d	0·21 n	0·002483 a 0·000717 c	α β
12000	550	0·0293 a 0·1015 b	0·407 d	0·27 n	0·003291 a 0·000940 c	α β
12000	500	0·0315 a 0·1086 b	0·385 d	0·25 n	0·003034 a 0·000874 c	α β
12000	450	0·0342 a 0·1179 b	0·360 d	0·23 n	0·002768 a 0·000804 c	α β

Note.—The values α apply to slabs with single reinforcement.
The values β apply to ribbed slabs.

The symbols a, b, c, stand for $\sqrt{B}$, $\sqrt{\frac{B}{b_s}}$, $\sqrt{B \,.\, b_s}$ respectively.

The following are the Sizes and Properties of such sections as are generally used in reinforced concrete works.

ONE CUBIC FT. OF STEEL WEIGHS 48·6 LB.

Thickness or diam. in ins.	Weight of ● Bar 1 foot long	Weight of ■ Bar 1 foot long	Area of ● Bar in sq. ins.	Area of ■ Bar in ins.	Circumference of ● Bar in in.
0 $\frac{3}{16}$	·094	·119	·0276	·0352	·589
$\frac{1}{4}$	·167	·212	·0491	·0625	·7854
$\frac{5}{16}$	·261	·333	·0767	·0977	·9817
$\frac{3}{8}$	·375	·478	·1104	·1406	1·1781
$\frac{7}{16}$	·511	·651	·1503	·1914	1·3744
$\frac{1}{2}$	·667	·850	·1963	·2500	1·5708
$\frac{9}{16}$	·845	1·076	·2485	·3164	1·7671
$\frac{5}{8}$	1·043	1·328	·3068	·3906	1·9635
$\frac{11}{16}$	1·262	1·608	·3712	·4727	2·1598
$\frac{3}{4}$	1·502	1·913	·4418	·5625	2·3562
$\frac{13}{16}$	1·763	2·245	·5185	·6602	2·5525
$\frac{7}{8}$	2·044	2·603	·6013	·7656	2·7489
$\frac{15}{16}$	2·347	2·989	·6903	·8789	2·9452
1	2·67	3·4	·7854	1·000	3·1416
$\frac{1}{16}$	3·014	3·838	·8866	1·1289	3·3379
$\frac{1}{8}$	3·379	4·303	·9940	1·2656	3·5343
$\frac{3}{16}$	3·766	4·795	1·1075	1·4102	3·7306
$\frac{1}{4}$	4·173	5·312	1·2272	1·5625	3·927
$\frac{5}{16}$	4·6	5·857	1·353	1·7227	4·1233
$\frac{3}{8}$	5·049	6·428	1·4849	1·8906	4·3197
$\frac{7}{16}$	5·518	7·026	1·623	2·0664	4·5160
$\frac{1}{2}$	6·008	7·65	1·7671	2·25	4·7124
$\frac{9}{16}$	6·52	8·301	1·9175	2·4414	4·9087
$\frac{5}{8}$	7·051	8·978	2·0739	2·6406	5·1051
$\frac{11}{16}$	7·604	9·682	2·2365	2·8477	5·3014
$\frac{3}{4}$	8·178	10·41	2·4053	3·0625	5·4978
$\frac{13}{16}$	8·773	11·17	2·5802	3·2852	5·6941
$\frac{7}{8}$	9·338	11·95	2·7612	3·5156	5·8905
$\frac{15}{16}$	10·02	12·76	2·9483	3·7539	6·0868
2	10·68	13·6	3·1416	4·000	6·2832

Hoops, Bands and Flats of small section are also used. Such reinforcements are particularly suitable for placing in the joints between hollow terra-cotta or concrete blocks or bricks in floor slabs. Hoops and bands are obtainable from a minimum width of $\frac{3}{8}$ in. in the following thicknesses: Gauges 1 to 26, and $\frac{1}{32}$, $\frac{3}{64}$, $\frac{1}{16}$, $\frac{5}{64}$, $\frac{3}{32}$, $\frac{7}{64}$, $\frac{1}{8}$, $\frac{9}{64}$, $\frac{5}{32}$, $\frac{11}{64}$, $\frac{3}{16}$ in. Flats are obtainable as follows:—

Width	Thickness	Width	Thickness
In.	In.	In.	In.
$\frac{5}{8}$	$\frac{3}{16}$ to $\frac{5}{8}$	$1\frac{1}{2}$, $1\frac{5}{8}$, $1\frac{3}{4}$, $1\frac{7}{8}$	
$\frac{3}{4}$	$\frac{3}{16}$ to $\frac{3}{4}$	2, $2\frac{1}{4}$, $2\frac{3}{8}$, $2\frac{1}{2}$	$\frac{3}{16}$ to 1
$\frac{7}{8}$, 1, $1\frac{1}{8}$, $1\frac{1}{4}$, $1\frac{3}{8}$	$\frac{3}{16}$ to $\frac{7}{8}$	$2\frac{5}{8}$, $2\frac{3}{4}$, 3	

IRON WIRE.

Sizes, Weights, Lengths, and Breaking Strains, Imperial Standard Wire Gauge.

Sizes on wire gauge	Diameter in in.	Weight		Length per cwt.	Breaking strain	
		100 yds.	1 mile		Annealed	Bright
		lbs.	lbs.	yards	lbs.	lbs.
7/0	·500	193·4	3,404	58	10,470	15,700
6/0	·464	166·5	2,930	67	9,017	13,525
5/0	·432	144·4	2,541	78	7,814	11,725
4/0	·400	123·8	2,179	91	6,702	10,052
3/0	·372	107·1	1,885	105	5,796	8,694
2/0	·348	93·7	1,649	120	5,072	7,608
1/0	·324	81·2	1,429	138	4,397	6,595
1	·300	69·6	1,225	161	3,770	5,655
2	·276	58·9	1,037	190	3,190	4,785
3	·252	49·1	864	228	2,660	3,990
4	·232	41·6	732	269	2,254	3,381
5	·212	34·8	612	322	1,883	2,824
6	·192	28·5	502	393	1,544	2,316
7	·176	24	422	467	1,298	1,946
8	·160	19·8	348	566	1,072	1,608
9	·144	16	282	700	869	1,303
10	·128	12·7	223	882	687	1,030
11	·116	10·4	183	1,077	564	845
12	·104	8·4	148	1,333	454	680
13	·092	6·5	114	1,723	355	532
14	·080	5	88	2,240	268	402
15	·072	4	70	2,800	218	326
16	·064	3·2	56	3,500	172	257
17	·056	2·4	42	4,667	131	197
18	·048	1·8	32	6,222	97	145
19	·040	1·2	21	9,333	67	100
20	·036	1	18	11,200	55	82

Armoured Tubular Floor "Herbst" Bar.

No.	Section in ins.	Sectional area in sq. in.	Per foot weight in lbs.	Sectional area sq. centimetre	Per metre weight in kilograms
1	1 × 1/8	0·156	0·56	1·00	0·83
2	1 × 3/16	0·234	0·84	1·52	1·25
3	2 × 1/8	0·313	1·12	2·02	1·66
4	2 × 5/32	0·387	1·38	2·52	2·07
5	2 × 3/16	0·468	1·68	3·03	2·49
6	2 × 7/32	0·545	1·96	3·53	2·90
7	2 × 1/4	0·625	2·25	4·04	3·31
8	2 × 9/32	0·695	2·48	4·53	3·72
9	2½ × 1/4	0·781	2·79	5·05	4·14
10	2½ × 9/32	0·879	3·12	5·67	4·65
11	2½ × 5/16	0·976	3·48	6·30	5·17
12					

Expanded Metal Co. Ltd.

Expanded Metal Diamond Mesh Lathing.

Note.—Sheets, 8 ft. long × under 2 ft. 3 ins. wide, and sheets 6 ft. or 7 ft. long × under 2 ft. wide, are cut and charged as standard sizes. Other sizes can be cut for special requirements, and quotations for such sheets will be furnished on application.

No.	Size of mesh shortway	Gauge of metal	Sizes of sheets keep in stock	Approx. weight per super. yard
	Inches		Longway of mesh ft. ins. × Shortway of mesh ft. ins.	
1	3/8	24G	8 0 × 2 3	3½ lbs.
91	3/8	22G	7 0 × 2 0	4¼ "
92	3/8	20G	6 0 × 2 0	5 "
26	1/4	24G		3½ "
93	1/4	22G		4¼ "
94	1/4	20G		5 "

Expanded Metal Cup Mesh Lathing.

NOTE.—These Lathings are supplied in standard size sheets only.

81	$\frac{1}{4}$	27G	8 0 × 1 3	$3\frac{1}{2}$ lbs.
82	$\frac{3}{8}$	27G	7 8 × 1 8	$2\frac{3}{4}$ "
83	$\frac{1}{4}$	24G	8 0 × 1 3	$4\frac{2}{3}$ "
84	$\frac{3}{8}$	24G	7 8 × 1 8	$3\frac{2}{3}$ "

Expanded Metal Square Mesh Lathing.

NOTE.—These Lathings are supplied in standard size sheets only.

200	$\frac{7}{16}$ square	27G	8 0 × 2 0	$2\frac{1}{3}$ lbs.
201	$\frac{7}{16}$ "	24G	8 0 × 2 0	$3\frac{1}{4}$ "

THE PATENT INDENTED STEEL BAR.

Size of bar	Net section sq. ins.	Weight per foot run lbs.	No. of lineal feet in a ton feet	Normal lengths to which bars are ordinarily rolled feet	Abnormal lengths to which they can be rolled if required feet
$\frac{1}{4}$″ □ Bar	0·06	0·24	9,333	30	45
$\frac{1}{3}$″ □ Bar	0·11	0·38	5,894	40	45
$\frac{1}{2}$″ □ Bar	0·25	0·85	2,635	50	60
$\frac{5}{8}$″ □ Bar	0·39	1·33	1,684	50	70
$\frac{3}{4}$″ □ Bar	0·56	1·91	1,172	50	80
$\frac{7}{8}$″ □ Bar	0·77	2·60	861	40	80
1″ □ Bar	1·00	3·40	658	40	70
$1\frac{1}{4}$″ □ Bar	1·56	5·31	422	40	70

A variation of 3 per cent. either way is allowed in the weight of bars.

TABLE OF JOHNSON'S WIRE LATTICE. SPECIAL CONCRETE MESHES.

Number	Mesh	Gauge of wires	Sectional area sq. in. per ft. of cross section
7	$1\frac{1}{2}'' \times 3''$	13 × 13	·0528
8	$1\frac{1}{2}'' \times 3''$	11 × 11	·0848
9	$1\frac{1}{2}'' \times 3''$	10 × 10	·1032
17	$2'' \times 4''$	8 × 11	·1206
18	$1\frac{1}{2}'' \times 3''$	9 × 11	·1304
19	$2'' \times 4''$	7 × 11	·1458
16	$1\frac{1}{2}'' \times 3''$	8 × 11	·1608
10	$2'' \times 4''$	6 × 11	·1740
20	$1\frac{1}{2}'' \times 3''$	7 × 11	·1944
11	$1\frac{1}{2}'' \times 3''$	6 × 11	·2320
21	$1\frac{1}{4}'' \times 3''$	7 × 11	·2430
22	$1\frac{1}{2}'' \times 3''$	9T × 11	·2608
12	$2'' \times 4''$	3 × 11	·2994
23	$2'' \times 4''$	6T × 11	·3480
24	$1\frac{1}{2}'' \times 3''$	3 × 11	·3990
25	$2'' \times 4''$	13 × 13	·0396

KAHN TRUSSED BAR.

Size ins.	Weight per foot lbs.	Area in sq. inches	Standard length of diagonals ins.
$\frac{1}{2} \times 1\frac{1}{2}$	1·4	0·41	6
$\frac{3}{4} \times 2\frac{3}{16}$	2·7	0·79	12
1×3	4·8	1·41	18
$1\frac{3}{4} \times 2\frac{3}{4}$	6·8	2·00	24

Lock Woven Mesh.

Table of Weights, Gauges and Sectional Areas.

Pattern number	Size of mesh centres of wires in ins.	Gauge of wires		Sq. in. sectional areas per ft. of cross section	Equivalent breaking strains in pounds	Approximate weight lbs. per sq. yd.	Stock sizes		Style of knots
		Tension wires	Tran-verse wires				Width of roll	Length of roll	
3	4 × 6·6	11	11	·0318	2,849	2·39			
4	4 × 6·6	10	11	·0387	3,467	2·62		100′ 0″	"Lock"
5	4 × 6·6	9	11	·0489	4,381	2·96		and	knot
6	4 × 6·6	8	11	·0603	5,402	3·35		200′ 0″	style
14	4 × 13·2	6	11	·0870	7,795	3·47	4′ 0″		
17	4 × 13·2	5	11	·1059	9,488	4·10	5′ 0″	100′ 0″	
20	4 × 13·2	4	11	·1269	11,370	4·80	and	and	"Spiral"
23	4 × 13·2	3	11	·1497	13,413	5·55		150′ 0″	knot
25	3 × 13·2	4	11	·1692	15,160	6·17	6′ 0″		
28	3 × 13·2	3	11	·1996	17,884	7·20			style
30	3 × 13·2	T 6	11	·2320	20,787	8·26		50′ 0″	
32	3 × 13·2	T 5	11	·2824	25,303	9·92		and	
34	3 × 13·2	T 4	11	·3384	30,321	11·75		100′ 0″	

Superimposed Floor Loads in Various Buildings.

	lbs. per sq. foot
Dwellings	40
Schools	50
Offices	60 to 80
Stables	65
Banks, Churches and Theatres	80
Assembly Halls, Corridors and Hotels, etc.	120
Shops	120
Drill Halls	150
Warehouses and Factories	150 to 400

Bearing Power of Soils.

	lbs. per sq. foot
Rock	10,000 to 100,000
Gravel	16,000 to 20,000
Sand	4,000 to 12,000
Clay, dry and thick bed	8,000 to 12,000
„ moderately dry	4,000 to 8,000
„ soft	2,000 to 4,000
Quicksand	1,000 to 1,500

Weight of Various Substances.

Forage.

1 truss of hay weighs 60 lbs. and contains 11 ft. cube
1 „ „ straw „ 36 „ „ „ „
1 cwt. of oats = 3·64 ft. cube
1 „ „ barley = 2·38 „
1 „ „ wheat = 2·20 „

Earth, etc.

		ft. cube
1 ton of chalk	=	13½
1 „ clay	=	17½
1 „ gravel	=	19
1 „ river sand	=	19
1 „ pit sand	=	21¾
1 „ loam	=	21
1 „ Thames ballast	=	20
1 „ shingle	=	23½

Metals.

	lbs. per ft. cube
Zinc	= 450
Cast iron	= 450
Wrought iron	= 485
Steel	= 490
Copper	= 550
Lead (milled)	= 712

Timber.

		lbs. per ft. cube
Yellow pine	=	33
Fir	=	35 to 38
Baltic oak	=	47
English oak	=	50
Mahogany	=	50

Stones.

		ft. cube
1 ton of marble	=	13
1 ,, granite	=	13½
1 ,, Kentish rag	=	13½
1 ,, Yorkshire	=	14½
1 ,, blue lias limestone	=	14½
1 ,, Portland	=	15
1 ,, Bath	=	16

Men and Horses.

Men closely packed	=	84 lbs. per ft. super.
1 cart horse	=	18 cwts.

Sundries.

1 gallon of water weighs	=	10 lbs.
1 foot cube of water	=	6·232 gall.
1 cwt. of water	=	1·8 ft. cube.
1 sack of flour of 2 bolls	=	280 lbs.
1 tun of oil (vegetable)	=	236 galls.
1 ,, ,, (animal)	=	252 galls.
1 sack of wool	=	364 lbs.
1 pocket of hops	=	1¾ cwt. (abt.)
Brickwork in lime mortar	=	100 lbs. per ft. cube.
,, ,, cement	=	110 ,, ,,
Concrete	=	112 ,, ,,
Reinforced concrete 1 : 2 : 4	=	150 ,, ,,
Gypsum	=	140 ,, ,,
Chalk lime	=	45 ,, ,,
Masonry	=	140-160 ,,
River sand	=	118 ,, ,,
Thames ,,	=	103 ,, ,,
Pit ,,	=	100 ,, ,,
Portland cement	=	90 ,, ,,

1 ton of Portland cement = 10 sacks of 2 cwt. each.
54 cubic feet = 1 double load.
A wheelbarrow contains 2 ft. 9 ins. or $\frac{1}{10}$ yd. cube.
A small earth waggon holds 1½ yds. cube.
A large ,, ,, 3 ,,
A run is 22 yards.

A rod of reduced brickwork = 272 ft. supl. $1\frac{1}{2}$ bk. thick and is 306 ft. cube or $11\frac{1}{3}$ yards cube.

500 bricks = 1 cartload.

A rod of brickwork weighs about 13 tons.

Plain tiles laid to $3\frac{1}{2}$ gauge require 700 tiles and weigh $14\frac{1}{2}$ cwt.

Battens are 7″ × $2\frac{1}{2}$″ and 7″ × 3″

Deals are 9″ × $2\frac{1}{2}$″ and 9″ × 3″

Planks are 11″ × $2\frac{1}{2}$″ and 11″ × 3″

120 deals = 1 hundred.

50 feet cube squared timber = 1 load.

600 feet sup. of 1 board = 1 ,,

The waste in sawing timber = $\frac{1}{10}$th.

Roof covered with lead weighs	7	cwt.	per sq.	of 100 ft. sup.	Exclusive of framing
,, zinc ,,	$1\frac{1}{2}$	,,	,,	,,	
,, corrugated iron ,,	3	,,	,,	,,	
,, slates ,,	$7\frac{1}{2}$ to 9	,,	,,	,,	
,, tiles ,,	8 to 15	,,	,,	,,	
Boarding $\frac{3}{4}$ thick ,,	$2\frac{1}{2}$	,,	,,	,,	
Timber framing for slated roof	5 to 6	,,	,,	,,	
Additional load for pressure of wind	36	,,	,,	,,	
Additional load in hurricane, say	80	,,	,,	,,	

Diagrams for ascertaining the cost of stone, sand and cement per cube yard of concrete for various mixtures.

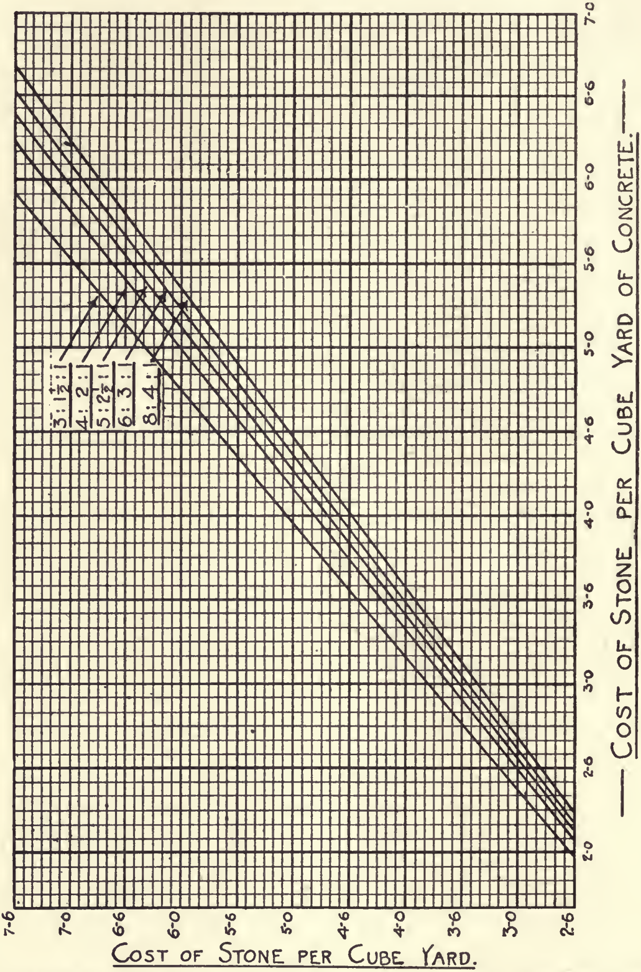
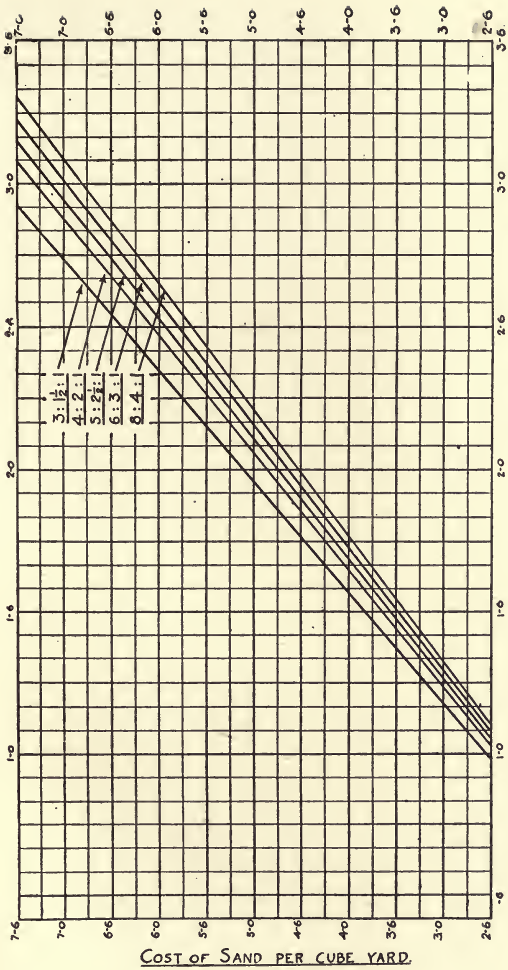


Fig. 204.

Directions : Follow the horizontal line corresponding to the cost of stone or sand per cube yard, until it intersects the heavy line corresponding to the proportions in which the materials are to be mixed. The figure at the end of the vertical line intersecting this point is the cost of stone or sand per cube yard of well-rammed concrete.



Cost of Sand per Cube Yard of Concrete.

Fig. 205.

Follow the horizontal line corresponding to the cost of cement per ton, until it intersects the heavy line corresponding to the proportions in which the materials are to be mixed. The figure at the end of the vertical line intersecting this point is the cost of cement per cube yard of well-rammed concrete.

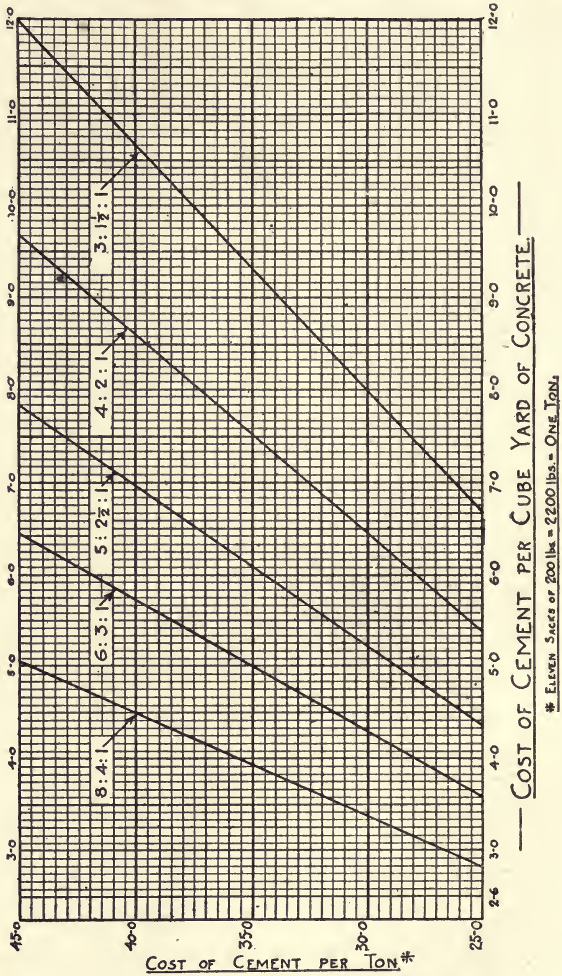
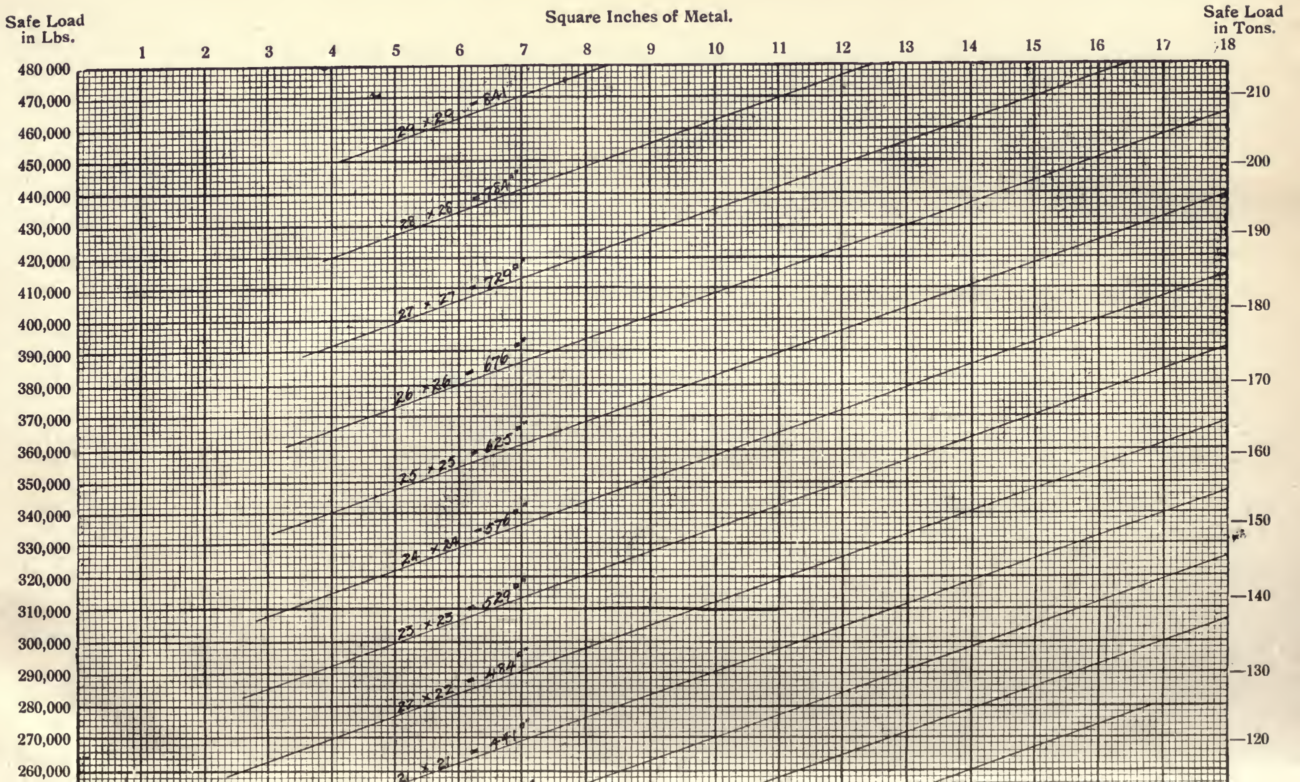


Fig. 203.

The foregoing diagrams have been based on 1·40 cube yards of dry materials being required to make 1 cube yard of well-rammed concrete. These figures have been arrived at after numerous experiments.


Safe Load in Lbs.
Square Inches of Metal.
Safe Load in Tons.
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
480,000 470,000 460,000 450,000 440,000 430,000 420,000 410,000 400,000 390,000 380,000 370,000 360,000 350,000 340,000 330,000 320,000 310,000 300,000 290,000 280,000 270,000 260,000
210 200 190 180 170 160 150 140 130 120
29 × 29 = 841
28 × 28 = 784
27 × 27 = 729
26 × 26 = 676
25 × 25 = 625
24 × 24 = 576
23 × 23 = 529
22 × 22 = 484
21 × 21 = 441

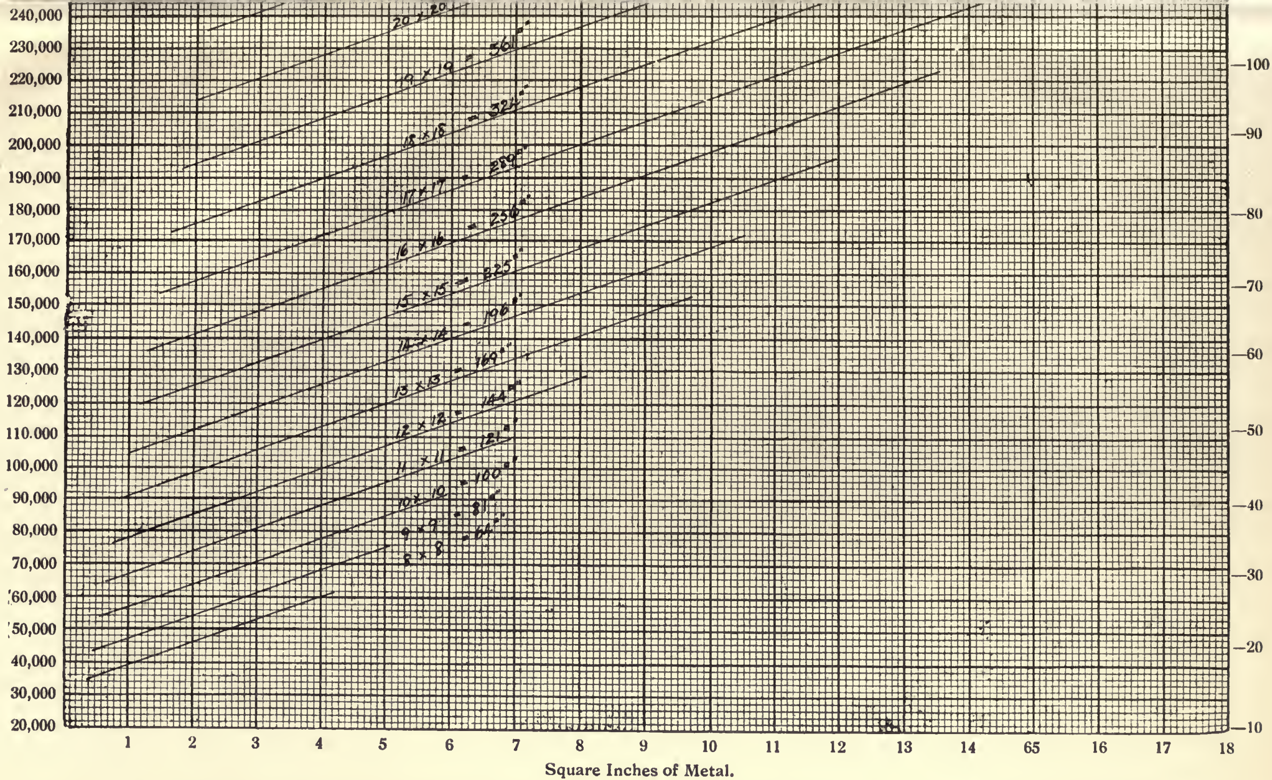


Diagram showing Sap Load on Concrete Columns Reinforced with Longitudinal Rods, with Cross Bindings.

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
1	1	1	1,0000	1,0000	1000,000	3,142	0,78 54	1
2	4	8	1,4142	1,2599	500,000	6,283	3,14 16	2
3	9	27	1,7321	1,4422	333,333	9,425	7,06 86	3
4	16	64	2,0000	1,5874	250,000	12,566	12,56 64	4
5	25	125	2,2361	1,7100	200,000	15,708	19,63 50	5
6	36	216	2,4495	1,8171	166,667	18,850	28,27 43	6
7	49	343	2,6458	1,9129	142,857	21,991	38,48 45	7
8	64	512	2,8284	2,0000	125,000	25,133	50,26 55	8
9	81	729	3,0000	2,0801	111,111	28,274	63,61 73	9
10	1 00	1 000	3,1623	2,1544	100,000	31,416	78,53 98	**10**
11	1 21	1 331	3,3166	2,2240	90,9091	34,558	95,03 32	11
12	1 44	1 728	3,4641	2,2894	83,3333	37,699	1 13,09 7	12
13	1 69	2 197	3,6056	2,3513	76,9231	40,841	1 32,73 2	13
14	1 96	2 744	3,7417	2,4101	71,4286	43,982	1 53,93 8	14
15	2 25	3 375	3,8730	2,4662	66,6667	47,124	1 76,71 5	15
16	2 56	4 096	4,0000	2,5198	62,5000	50,265	2 01,06 2	16
17	2 89	4 913	4,1231	2,5713	58,8235	53,407	2 26,98 0	17
18	3 24	5 832	4,2426	2,6207	55,5556	56,549	2 54,46 9	18
19	3 61	6 859	4,3589	2,6684	52,6316	56,690	2 83,52 9	19
20	4 00	8 000	4,4721	2,7144	50,0000	62,832	3 14,15 9	**20**
21	4 41	9 261	4,5826	2,7589	47,6190	65,973	3 46,36 1	21
22	4 84	10 648	4,6904	2,8020	45,4545	69,115	3 80,13 3	22
23	5 29	12 167	4,7958	2,8439	43,4783	72,257	4 15,47 6	23
24	5 76	13 824	4,8990	2,8845	41,6667	75,398	4 52,38 9	24
25	6 25	15 625	5,0000	2,9240	40,0000	78,540	4 90,87 4	25
26	6 76	17 576	5,0990	2,9625	38,4615	81,681	5 30,92 9	26
27	7 29	19 683	5,1962	3,0000	37,0370	84,823	5 72,55 5	27
28	7 84	21 952	5,2915	3,0366	35,7143	87,965	6 15,75 2	28
29	8 41	24 389	5,3852	3,0723	34,4828	91,106	6 60,52 0	29
30	9 00	27 000	5,4772	3,1072	33,3333	94,248	7 06,85 8	**30**
31	9 61	29 791	5,5678	3,1414	32,2581	97,389	7 54,76 8	31
32	10 24	32 768	5,6569	3,1748	31,2500	100,531	8 04,24 8	32
33	10 89	35 937	5,7446	3,2075	30,3030	103,673	8 55,29 9	33
34	11 56	39 304	5,8310	3,2396	29,4118	106,814	9 07,92 0	34
35	12 25	42 875	5,9161	3,2711	28,5714	109,956	9 62,11 3	35
36	12 96	46 656	6,0000	3,3019	27,7778	113,097	10 17,88	36
37	13 69	50 653	6,0828	3,3322	27,0270	116,239	10 75,21	37
38	14 44	54 872	6,1644	3,3620	26,3158	119,381	11 34,11	38
39	15 21	59 319	6,2450	3,3912	25,6410	122,522	11 94,59	39
40	16 00	64 000	6,3246	3,4200	25,0000	125,66	12 56,64	**40**
41	16 81	68 921	6,4031	3,4482	24,3902	128,81	13 20,25	41
42	17 64	74 088	6,4807	3,4760	23,8095	131,95	13 85,44	42
43	18 49	79 507	6,5574	3,5034	23,2558	135,09	14 52,20	43
44	19 36	85 184	6,6332	3,5303	22,7273	138,23	15 20,53	44
45	20 25	91 125	6,7082	3,5569	22,2222	141,37	15 90,43	45
46	21 16	97 336	6,7823	3,5830	21,7391	144,51	16 61,90	46
47	22 09	103 823	6,8557	3,6088	21,2766	147,65	17 34,94	47
48	23 04	110 592	6,9282	3,6342	20,8333	150,80	18 09,56	48
49	24 01	117 649	7,0000	3,6593	20,4082	153,94	18 85,74	49
50	25 00	125 000	7,0711	3,6840	20,0000	157,08	19 63,50	**50**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
50	25 00	125 000	7,0711	3,6840	20,0000	157,08	19 63,50	**50**
51	26 01	132 651	7,1414	3,7084	19,6078	160,22	20 42,82	51
52	27 04	140 608	7,2111	3,7325	19,2308	163,36	21 23,72	52
53	28 09	148 877	7,2801	3,7563	18,8679	166,50	22 06,18	53
54	29 16	157 464	7,3485	3,7798	18,5185	169,65	22 90,22	54
55	30 25	166 375	7,4162	3,8030	18,1818	172,79	23 75,83	55
56	31 36	175 616	7,4833	3,8259	17,8571	175,93	24 63,01	56
57	32 49	185 193	7,5498	3,8485	17,5439	179,07	25 51,76	57
58	33 64	195 112	7,6158	3,8709	17,2414	182,21	26 42,08	58
59	34 81	205 379	7,6811	3,8930	16,9492	185,35	27 33,97	59
60	36 00	216 000	7,7460	3,9149	16,6667	188,50	28 27,43	**60**
61	37 21	226 981	7,8102	3,9365	16,3934	191,64	29 22,47	61
62	38 44	238 328	7,8740	3,9579	16,1290	194,78	30 19,07	62
63	39 69	250 047	7,9373	3,9791	15,8730	197,92	31 17,25	63
64	40 96	262 144	8,0000	4,0000	15,6250	201,06	32 16,99	64
65	42 25	274 625	8,0623	4,0207	15,3846	204,20	33 18,31	65
66	43 56	287 496	8,1240	4,0412	15,1515	207,35	34 21,19	66
67	44 89	300 763	8,1854	4,0615	14,9254	210,49	35 25,65	67
68	46 24	314 432	8,2462	4,0817	14,7059	213,63	36 31,68	68
69	47 61	328 509	8,3066	4,1016	14,4928	216,77	37 39,28	69
70	49 00	343 000	8,3666	4,1213	14,2857	219,91	38 48,45	**70**
71	50 41	357 911	8,4261	4,1408	14,0845	223,05	39 59,19	71
72	51 84	373 248	8,4853	4,1602	13,8889	226,19	40 71,50	72
73	53 29	389 017	8,5440	4,1793	13,6986	229,34	41 85,39	73
74	54 76	405 224	8,6023	4,1983	13,5135	232,48	43 00,84	74
75	56 25	421 875	8,6603	4,2172	13,3333	235,62	44 17,86	75
76	57 76	438 976	8,7178	4,2358	13,1579	238,76	45 36,46	76
77	59 29	456 533	8,7750	4,2543	12,9870	241,90	46 56,63	77
78	60 84	474 552	8,8318	4,2727	12,8205	245,04	47 78,36	78
79	62 41	493 039	8,8882	4,2908	12,6582	248,19	49 01,67	79
80	64 00	512 000	8,9443	4,3089	12,5000	251,33	50 26,55	**80**
81	65 61	531 441	9,0000	4,3267	12,3457	254,47	51 53,00	81
82	67 24	551 368	9,0554	4,3445	12,1951	257,61	52 81,02	82
83	68 89	571 787	0,1104	4,3621	12,0482	260,75	54 10,61	83
84	70 56	592 704	9,1652	4,3795	11,9048	263,89	55 41,77	84
85	72 25	614 125	9,2195	4,3968	11,7647	267,04	56 74,50	85
86	73 96	636 056	9,2736	4,4140	11,6279	270,18	58 08,80	86
87	75 69	658 503	9,3274	4,4310	11,4943	273,32	59 44,68	87
88	77 44	681 472	9,3808	4,4480	11,3636	276,46	60 82,12	88
89	79 21	704 969	9,4340	4,4647	11,2360	279,60	62 21,14	89
90	81 00	729 000	9,4868	4,4814	11,1111	282,74	63 61,73	**90**
91	82 81	753 571	9,5394	4,4979	10,9890	285,88	65 03,88	91
92	84 64	778 688	9,5917	4,5144	10,8696	289,03	66 47,61	92
93	86 49	804 357	9,6437	4,5307	10,7527	292,17	67 92,91	93
94	88 36	830 584	9,6954	4,5468	10,6383	295,31	69 39,78	94
95	90 25	857 375	9,7468	4,5629	10,5263	298,45	70 88,22	95
96	92 16	884 736	9,7980	4,5789	10,4167	301,59	72 38,23	96
97	94 09	912 673	9,8489	4,5947	10,3093	304,73	73 89,81	97
98	96 04	941 192	9,8995	4,6104	10,2041	307,88	75 42,96	98
99	98 01	970 299	9,9499	4,6261	10,1010	311,02	76 97,69	99
100	1 00 00	1 000 000	10,0000	4,6416	10,0000	314,16	78 53 98	**100**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
100	1 00 00	1 000 000	10,0000	4,6416	10,0000	314,16	78 53,98	**100**
101	1 02 01	1 030 301	10,0499	4,6570	9,90099	317,30	80 11,85	101
102	1 04 04	1 061 208	10,0995	4,6723	9,80392	320,44	81 71,28	102
103	1 06 09	1 092 727	10,1489	4,6875	9,70874	323,58	83 32,29	103
104	1 08 16	1 124 864	10,1980	4,7027	9,61538	326,73	84 94,87	104
105	1 10 25	1 157 625	10,2470	4,7177	9,52381	329,87	86 59,01	105
106	1 12 36	1 191 016	10,2956	4,7326	9,43396	333,01	88 24,73	106
107	1 14 49	1 225 043	10,3441	4,7475	9,34579	336,15	89 92,02	107
108	1 16 64	1 259 712	10,3923	4,7622	9,25926	339,29	91 60,88	108
109	1 18 81	1 295 029	10,4403	4,7769	9,17431	342,43	93 31,32	109
110	1 21 00	1 331 000	10,4881	4,7914	9,09091	345,58	95 03,32	**110**
111	1 23 21	1 367 631	10,5357	4,8059	9,00901	348,72	96 76,89	111
112	1 25 44	1 404 928	10,5830	4,8203	8,92857	351,86	98 52,03	112
113	1 27 69	1 442 897	10,6301	4,8346	8,84956	355,00	1 00 28,7	113
114	1 29 96	1 481 544	10,6771	4,8488	8,77193	358,14	1 02 07,0	114
115	1 32 25	1 520 875	10,7238	4,8629	8,69565	361,28	1 03 86,9	115
116	1 34 56	1 560 896	10,7703	4,8770	8,62069	364,42	1 05 68,3	116
117	1 36 89	1 601 613	10,8167	4,8910	8,54701	367,57	1 07 51,3	117
118	1 39 24	1 643 032	10,8628	4,9049	8,47458	370,71	1 09 35,9	118
119	1 41 61	1 685 159	10,9087	4,9187	8,40336	373,85	1 11 22,0	119
120	1 44 00	1 728 000	10,9545	4,9324	8,33333	376,99	1 13 09,7	**120**
121	1 46 41	1 771 561	11,0000	4,9461	8,26446	380,13	1 14 99,0	121
122	1 48 84	1 815 848	11,0454	4,9597	8,19672	383,27	1 16 89,9	122
123	1 51 29	1 860 867	11,0905	4,9732	8,13008	386,42	1 18 82,3	123
124	1 53 76	1 906 624	11,1355	4,9866	8,06452	389,56	1 20 76,3	124
125	1 56 25	1 953 125	11,1803	5,0000	8,00000	392,70	1 22 71,8	125
126	1 58 76	2 000 376	11,2250	5,0133	7,93651	395,84	1 24 69,0	126
127	1 61 29	2 048 383	11,2694	5,0265	7,87402	398 98	1 26 67,7	127
128	1 63 84	2 097 152	11,3137	5,0397	7,81250	402,12	1 28 68,0	128
129	1 66 41	2 146 689	11,3578	5,0528	7,75194	405,27	1 30 69,8	129
130	1 69 00	2 197 000	11,4018	5,0658	7,69231	408,41	1 32 73,2	**130**
131	1 71 61	2 248 091	11,4455	5,0788	7,63359	411,55	1 34 78,2	131
132	1 74 24	2 299 968	11,4891	5,0916	7,57576	414,69	1 36 84,8	132
133	1 76 89	2 352 637	11,5326	5,1045	7,51880	417,83	1 38 92,9	133
134	1 79 56	2 406 104	11,5758	5,1172	7,46269	420,97	1 41 02,6	134
135	1 82 25	2 460 375	11,6190	5,1299	7,40741	424,12	1 43 13,9	135
136	1 84 96	2 515 456	11,6619	5,1426	7,35294	427,26	1 45 26,7	136
137	1 87 69	2 571 353	11,7047	5,1551	7,29927	430,40	1 47 41,1	137
138	1 90 44	2 628 072	11,7473	5,1676	7,24638	433,54	1 49 57,1	138
139	1 93 21	2 685 619	11,7898	5,1801	7,19424	436,68	1 51 74,7	139
140	1 96 00	2 744 000	11,8322	5,1925	7,14286	439,82	1 53 93,8	**140**
141	1 98 81	2 803 221	11,8743	5,2048	7,09220	442,96	1 56 14,5	141
142	2 01 64	2 863 288	11,9164	5,2171	7,04225	446,11	1 58 36,8	142
143	2 04 49	2 924 207	11.9583	5,2293	6,99301	449,25	1 60 60,6	143
144	2 07 36	2 985 984	12,0000	5,2415	6,94444	452,39	1 62 86,0	144
145	2 10 25	3 048 625	12,0416	5,2536	6,89655	455,53	1 65 13,0	145
146	2 13 16	3 112 136	12,0830	5,2656	6,84932	458,67	1 67 41,5	146
147	2 16 09	3 176 523	12,1244	5,2776	6,80272	461,81	1 69 71,7	147
148	2 19 04	3 241 792	12,1655	5,2896	6,75676	464,96	1 72 03,4	148
149	2 22 01	3 307 949	12,2066	5,3015	6,71141	468,10	1 74 36,6	149
150	2 25 00	3 375 000	12,2474	5,3133	6,66667	471,24	1 76 71,5	**150**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
150	2 25 00	3 375 000	12,2474	5,3133	6,66667	471,24	1 76 71,5	**150**
151	2 28 01	3 442 951	12,2882	5,3251	6,62252	474,38	1 79 07,9	151
152	2 31 04	3 511 808	12,3288	5,3368	6,57895	477,52	1 81 45,8	152
153	2 34 09	3 581 577	12,3693	5,3485	6,53595	480,66	1 83 85,4	153
154	2 37 16	3 652 264	12,4097	5,3601	6,49351	483,81	1 86 26,5	154
155	2 40 25	3 723 875	12,4499	5,3717	6,45161	486,95	1 88 69,2	155
156	2 43 36	3 796 416	12,4900	5,3832	6,41026	490,09	1 91 13,4	156
157	2 46 49	3 869 893	12,5300	5,3947	6,36943	493,23	1 93 59,3	157
158	2 49 64	3 944 312	12,5698	5,4061	6,32911	496,37	1 96 06,7	158
159	2 52 81	4 019 679	12,6095	5,4175	6,28931	499,51	1 98 55,7	159
160	2 56 00	4 096 000	12,6491	5,4288	6,25000	502,65	2 01 06,2	**160**
161	2 59 21	4 173 281	12,6886	5,4401	6,21118	505,80	2 03 58,3	161
162	2 62 44	4 251 528	12,7279	5,4514	6,17284	508,94	2 06 12,0	162
163	2 65 69	4 330 747	12,7671	5,4626	6,13497	512,08	2 08 67,2	163
164	2 68 96	4 410 944	12,8062	5,4737	6,09756	515,22	2 11 24,1	164
165	2 72 25	4 492 125	12,8452	5,4848	6,06061	518,36	2 13 82,5	165
166	2 75 56	4 574 296	12,8841	5,4959	6,02410	521,50	2 16 42,4	166
167	2 78 89	4 657 463	12,9228	5,5069	5,98802	524,65	2 19 04,0	167
168	2 82 24	4 741 632	12,9615	5,5178	5,95238	527,79	2 21 67,1	168
169	2 85 61	4 826 809	13,0000	5,5288	5,91716	530,93	2 24 31,8	169
170	2 89 00	4 913 000	13,0384	5,5397	5,88235	534,07	2,26 98,0	**170**
171	2 92 41	5 000 211	13,0767	5,5505	5,84795	537,21	2 29 65,8	171
172	2 95 84	5 088 448	13,1149	5,5613	5,81395	540,35	2 32 35,2	172
173	2 99 29	5 177 717	13,1529	5,5721	5,78035	543,50	2 35 06,2	173
174	3 02 76	5 268 024	13,1909	5,5828	5,74713	546,64	2 37 78,7	174
175	3 06 25	5 359 375	13,2288	5,5934	5,71429	549,78	2 40 52,8	175
176	3 09 76	5 451 776	13,2665	5,6041	5,68182	552,92	2 43 28,5	176
177	3 13 29	5 545 233	13,3041	5,6147	5,64972	556,06	2 46 05,7	177
178	3 16 84	5 639 752	13,3417	5,6252	5,61798	559,20	2 48 84,6	178
179	3 20 41	5 735 339	13,3791	5,6357	5,58659	562,35	2 51 64,9	179
180	3 24 00	5 832 000	13,4164	5,6462	5,55556	565,49	2 54 46,9	**180**
181	3 27 61	5 929 741	13,4536	5,6567	5,52486	568,63	2 57 30,4	181
182	3 31 24	6 028 568	13,4907	5,6671	5,49451	571,77	2 60 15,5	182
183	3 34 89	6 128 487	13,5277	5 6774	5,46448	574,91	2 63 02,2	183
184	3 38 56	6 229 504	13,5647	5,6877	5,43478	578,05	2 65 90,4	184
185	3 42 25	6 331 625	13,6015	5,6980	5,40541	581,19	2 68 80,3	185
186	3 45 96	6 434 856	13,6382	5,7083	5,37634	584,34	2 71 71,6	186
187	3 49 69	6 539 203	13,6748	5,7185	5,34759	587,48	2 74 64,6	187
188	3 53 44	6 644 672	13,7113	5,7287	5,31915	590,62	2 77 59,1	188
189	3 57 21	6 751 269	13,7477	5,7388	5,29101	593,76	2 80 55,2	189
190	3 61 00	6 859 000	13,7840	5,7489	5,26316	596,90	2 83 52,9	**190**
191	3 64 81	6 967 871	13,8203	5,7590	5,23560	600,04	2 86 52,1	191
192	3 68 64	7 077 888	13,8564	5,7690	5,20833	603,19	2 89 52,9	192
193	3 72 49	7 189 057	13,8924	5,7790	5,18135	606,33	2 92 55,3	193
194	3 76 36	7 301 384	13,9284	5,7890	5,15464	609,47	2 95 59,2	194
195	3 80 25	7 414 875	13,9642	5,7989	5,12821	612,61	2 98 64,8	195
196	3 84 16	7 529 536	14,0000	5,8088	5,10204	615,75	3 01 71,9	196
197	3 88 09	7 645 373	14,0357	5,8186	5,07614	618,89	3 04 80,5	197
198	3 92 04	7 762 392	14,0712	5,8285	5,05051	622,04	3 07 90,7	198
199	3 96 01	7 880 599	14,1067	5,8383	5,02513	625,18	3 11 02,6	199
200	4 00 00	8 000 000	14,1421	5,8480	5,00000	628,32	3 14 15,9	**200**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
200	4 00 00	8 000 000	14,1421	5,8480	5,00000	628,32	3 14 15,9	**200**
201	4 04 01	8 120 601	14,1774	5,8578	4,97512	631,46	3 17 30,9	201
202	4 08 04	8 242 408	14,2127	5,8675	4,95050	634,60	3 20 47,4	202
203	4 12 09	8 365 427	14,2478	5,8771	4,92611	637,74	3 23 65,5	203
204	4 16 16	8 489 664	14,2829	5,8868	4,90196	640,88	3 26 85,1	204
205	2 20 25	8 615 125	14,3178	5,8964	4,87805	644,03	3 30 06,4	205
206	4 24 36	8 741 816	14,3527	5,9059	4,85437	647,17	3 33 29,2	206
207	4 28 49	8 869 743	14,3875	5,9155	4,83092	650,31	3 36 53,5	207
208	4 32 46	8 998 912	14,4222	5,9250	4,80769	653,45	3 39 79,1	208
209	4 36 81	9 129 329	14,4568	5,9345	4,78469	656,59	3 43 07,0	209
210	4 41 00	9 261 000	14,4914	5,9439	4,76190	659,73	3 46 36,1	**210**
211	4 45 21	9 393 931	14,5258	5,9533	4,73934	662,88	3 49 66,7	211
212	4 49 44	9 528 128	14,5602	5,9627	4,71698	666,02	3 52 98,9	212
213	4 53 69	9 663 597	14,5945	5,9721	4,69484	669,16	3 56 32,7	213
214	4 57 96	9 800 344	14,6287	5,9814	4,67290	672,30	3 59 68,1	214
215	4 62 25	9 938 375	14,6629	5,9907	4,65116	675,44	3 63 05,0	215
216	4 66 56	10 077 696	14,6969	6,0000	4,62963	678,58	3 66 43,5	216
217	4 70 89	10 218 313	14,7309	6,0092	4,60829	681,73	3 69 83,6	217
218	4 75 24	10 360 232	14,7648	6,0185	4,58716	684,87	3 73 25 3	218
219	4 79 61	10 503 459	14,7986	6,0277	4,56621	688,01	3 76 68,5	219
220	4 84 00	10 648 000	14,8324	6,0368	4,54545	691,15	3 80 13,3	**220**
221	4 88 41	10 793 861	14,8661	6,0459	4,52489	694,29	3 83 59,6	221
222	4 92 84	10 941 048	14,8997	6,0550	4,50450	697,43	3 87 07,6	222
223	4 97 29	11 089 567	14,9332	6,0641	4,48430	700,58	3 90 57,1	223
224	5 01 76	11 239 424	14,9666	6,0732	4,46429	703,72	3 94 08,1	224
225	5 06 25	11 390 625	15,0000	6,0822	4,44444	706,86	3 97 60,8	225
226	5 10 76	11 543 176	15,0333	6,0912	4,42478	710,00	4 01 15,0	226
227	5 15 29	11 697 083	15,0665	6,1002	4,40529	713,14	4 04 70,8	227
228	5 19 84	11 852 352	15,0997	6,1091	4,38596	716,28	4 08 28,1	228
229	5 24 41	12 008 989	15,1327	6,1180	4,36681	719,42	4 11 87,1	229
230	5 29 00	12 167 000	15,1658	6,1269	4,34783	722,57	4 15 47,6	**230**
231	5 33 61	12 326 391	15,1987	6,1358	4,32900	725,71	4 19 09,6	231
232	5 38 24	12 487 168	15,2315	6,1446	4,31034	728,85	4 22 73,3	232
233	5 42 89	12 649 337	15,2643	6,1534	4,29185	731,99	4 26 38,5	233
234	5 47 56	12 812 904	15,2971	6,1622	4,27350	735,13	4 30 05,3	234
235	5 52 25	12 977 875	15,3297	6,1710	4,25532	738,27	4 33 73,6	235
236	5 56 96	13 144 256	15,3623	6,1797	4,23729	741,42	4 37 43,5	236
237	5 61 69	13 312 053	15,3948	6,1885	4,21941	744,56	4 41 15,0	237
238	5 66 44	13 481 272	15,4272	6,1972	4,20168	747,70	4 44 88 1	238
239	5 71 21	13 651 919	15,4596	6,2058	4,18410	750,84	4 48 62,7	239
240	5 76 00	13 824 000	15,4919	6,2145	4,16667	753,98	4 52 38,9	**240**
241	5 80 81	13 997 521	15,5242	6,2231	4,14938	757,12	4 56 16,7	241
242	5 85 64	14 172 488	15,5563	6,2317	4,13223	760,27	4 59 96,1	242
243	5 90 49	14 348 907	15,5885	6,2403	4,11523	763,41	4 63 77,0	243
244	5 95 36	14 526 784	15,6205	6,2488	4,09836	766,55	4 67 59,5	244
245	6 00 25	14 706 125	15,6525	6,2573	4,08163	769,69	4 71 43,5	245
246	6 05 16	14 886 936	15,6844	6,2658	4,06504	772,83	4 75 29,2	246
247	6 10 09	15 069 223	15,7162	6,2743	4,04858	775,97	4 79 16,4	247
248	6 15 04	15 252 992	15,7480	6,2828	4,03226	779,11	4 83 05,1	248
249	6 20 01	15 438 249	15,7797	6,2912	4,01606	782,26	4 86 95,5	249
250	6 25 00	15 625 000	15,8114	6,2996	4,00000	785,40	4 90 87,4	**250**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
250	6 25 00	15 625 000	15,8114	6,2996	4,00000	785,40	4 90 87,4	**250**
251	6 30 01	15 813 251	15,8430	6,3080	3,98406	788,54	4 94 80,9	251
252	6 35 04	16 003 008	15.8745	6,3164	3,96825	791,68	4 98 75,9	252
253	6 40 09	16 194 277	15,9060	6,3247	3,95257	794,82	5 02 72,6	253
254	6 45 16	16 387 064	15,9374	6,3330	3,93701	797,96	5 06 70,7	254
255	6 50 25	16 581 375	15,9687	6,3413	3,92157	801,11	5 10 70,5	255
256	6 55 36	16 777 216	16,0000	6,3496	3,90625	804,25	5 14 71,9	256
257	6 60 49	16 974 593	16,0312	6,3579	3,89105	807,39	5 18 74,8	257
258	6 65 64	17 173 512	16,0624	6,3661	3,87597	810,53	5 22 79,2	258
259	6 70 81	17 373 979	16,0935	6,3743	3,86100	813,67	5 26 85,3	259
260	6 76 00	17 576 000	16,1245	6,3825	3,84615	816,81	5,30 92,9	**260**
261	6 81 21	17 779 581	16,1555	6,3907	3,83142	819,96	5 35 02,1	261
262	6 86 44	17 984 728	16,1864	6,3988	3,81679	823,10	5 39 12,9	262
263	6 91 69	18 191 447	16,2173	6,4070	3,80228	826,24	5 43 25,2	263
264	6 96 96	18 399 744	16,2481	6,4151	3,78788	829,38	5 47 39,1	264
265	7 02 25	18 609 625	16,2788	6,4232	3,77358	832,52	5 51 54,6	265
266	7 07 56	18 821 096	16,3095	6,4312	3,75940	835,66	5 55 71,6	266
267	7 12 89	19 034 163	16,3401	6,4393	3,74532	838,81	5 59 90,2	267
268	7 18 24	19 248 832	16,3707	6,4473	3,73134	841,95	5 64 10,4	268
269	7 23 61	19 465 109	16,4012	6,4553	3,71747	845,09	5 68 32,2	269
270	7 29 00	19 683 000	16,4317	6,4633	3,70370	848,23	5 72 55,5	**270**
271	7 34 41	19 902 511	16,4621	6,4713	3,69004	851,37	5 76 80,4	271
272	7 39 84	20 123 648	16,4924	6,4792	3,67647	854,51	5 81 06,9	272
273	7 45 29	20 346 417	16,5227	6,4872	3,66300	857,65	5 85 34,9	273
274	7 50 76	20 570 824	16,5529	6,4951	3,64964	860,80	5 89 64,6	274
275	7 56 25	20 796 875	16,5831	6,5030	3,63636	863,94	5 93 95,7	275
276	7 61 76	21 024 576	16,6132	6,5108	3.62319	867,08	5 98 28,5	276
277	7 67 29	21 253 933	16,6433	6,5187	3,61011	870,22	6 02 62,8	277
278	7 72 84	21 484 952	13,6733	6,5265	3,59712	873,36	6 06 98,7	278
279	7 78 41	21 717 639	16,7033	6,5343	3,58423	876,50	6 11 36,2	279
280	7 84 00	21 952 000	16,7332	6,5421	3,57143	879,65	6 15 75,2	**280**
281	7 89 61	22 188 041	16,7631	6,5499	3,55872	882,79	6 20 15,8	281
282	7 95 24	22 425 768	16,7929	6,5577	3,54610	885,93	6 24 58,0	282
283	8 00 89	22 665 187	16,8226	6,5654	3.53357	889,07	6 29 01,8	283
284	8 06 56	22 906 304	16,8523	6,5731	3,52113	892,21	6 33 47,1	284
285	8 12 25	23 149 125	16,8819	6,5808	3,50877	895,35	6 37 94,0	285
286	8 17 96	23 393 656	16 9115	6,5885	3,49650	898,50	6 42 42,4	286
287	8 23 69	23 639 903	16,9411	6,5962	3,48432	901,64	6 46 92,5	287
288	8 29 44	23 887 872	16,9706	6,6039	3,47222	904,78	6 51 44,1	288
289	8 35 21	24 137 569	17,0000	6,6115	3,46021	907,92	6 55 97,2	289
290	8 41 00	24 389 000	17,0294	6,6191	3,44828	911,06	6 60 52,0	**290**
291	8 46 81	24 642 171	17,0587	6,6267	3,43643	914,20	6 65 08,3	291
292	8 52 64	24 897 088	17,0880	6,6343	3,42466	917,35	6 69 66,2	292
293	8 58 49	25 153 757	17,1172	6,6419	3,41297	920,49	6 74 25,6	293
294	8 64 36	25 412 184	17,1464	6,6494	3,40136	923,63	6 78 86,7	294
295	8 70 25	25 672 375	17,1756	6,6569	3,38983	926,77	6 83 49,3	295
296	8 76 16	25 934 336	17,2047	6.6644	3,37838	929,91	6 88 13,4	296
297	8 82 09	26 198 073	17,2337	6,6719	3,36700	933,05	6 92 79,2	297
298	8 88 04	26 463 592	17,2627	6,6794	3,35570	936,19	6 97 46,5	298
299	8 94 01	26 730 899	17,2916	6,6869	3,34448	939,34	7 02 15,4	299
300	9 00 00	27 000 000	17,3205	6,6943	3,33333	942,48	7 06 85,8	**300**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
300	9 00 00	27 000 000	17,3205	6,6943	3,33333	942,48	7 06 85,8	**300**
301	9 06 01	27 270 901	17,3494	6,7018	3,32226	945,62	7 11 57,9	301
302	9 12 04	27 543 608	17,3781	6,7092	3,31126	948,76	7 16 31,5	302
303	9 18 09	27 818 127	17,4069	6,7166	3,30033	951,90	7 21 06,6	303
304	9 24 16	28 094 464	17,4356	6,7240	3,28947	955,04	7 25 83,4	304
305	9 30 25	28 372 625	17,4642	6,7313	3,27869	958,19	7 30 61,7	305
306	9 36 36	28 652 616	17,4929	6,7387	3,26797	961,33	7 35 41,5	306
307	9 42 49	28 934 443	17,5214	6,7460	3,25733	964,47	7 40 23,0	307
308	9 48 64	29 218 112	17,5499	6,7533	3,24675	967,61	7 45 06,0	308
309	9 54 81	29 503 629	17,5784	6,7606	3,23625	970,75	7 49 90,6	309
310	9 61 00	29 791 000	17,6068	6,7679	3,22581	973,89	7 54 76,8	**310**
311	9 67 21	30 080 231	17,6352	6,7752	3,21543	977,04	7 59 64,5	311
312	9 73 44	30 371 328	17,6635	6,7824	3,20513	980,18	7 64 53,8	312
313	9 79 69	30 664 297	17,6918	6,7897	3,19489	983,32	7 69 44,7	313
314	9 85 96	30 959 144	17,7200	6,7969	3,18471	986,46	7 74 37,1	314
315	9 92 25	31 255 875	17,7482	6,8041	3,17460	989,60	7 79 31,1	315
316	9 98 56	31 554 496	17,7764	6,8113	3,16456	992,74	7 84 26,7	316
317	10 04 89	31 855 013	17,8045	6,8185	3,15457	995,88	7 89 23,9	317
318	10 11 24	32 157 432	17,8326	6,8256	3,14465	999,03	7 94 22,6	318
319	10 17 61	32 461 759	17,8606	6,8328	3,13480	1002,2	7 99 22,9	319
320	10 24 00	32 768 000	17,8885	6,8399	3,12500	1005,3	8 04 24,8	**320**
321	10 30 41	33 076 161	17,9165	6,8470	3,11526	1008,5	8 09 28,2	321
322	10 36 84	33 386 248	17,9444	6,8541	3,10559	1011,6	8 14 33,2	322
323	10 43 29	33 698 267	17,9722	6,8612	3,09598	1014,7	8 19 39,8	323
324	10 49 76	34 012 224	18,0000	6,8683	3,08642	1017,9	8 24 48,0	324
325	10 56 25	34 328 125	18,0278	6,8753	3,07692	1021,0	8 29 57,7	325
326	10 62 76	34 645 976	18,0555	6,8824	3,06748	1024,2	8 34 69,3	326
327	10 69 29	34 965 783	18,0831	6,8894	3,05810	1027,3	8 39 81,8	327
328	10 75 84	35 287 552	18,1108	6,8964	3,04878	1030,4	8 44 96,3	328
329	10 82 41	35 611 289	18,1384	6,9034	3,03951	1033,6	8 50 12,3	329
330	10 89 00	35 937 000	18,1659	6,9104	3,03030	1036,7	8 55 29,9	**330**
331	10 95 61	36 264 691	18,1934	6,9174	3,02115	1039,9	8 60 49,0	331
332	11 02 24	36 594 368	18,2209	6,9244	3,01205	1043,0	8 65 69,7	332
333	11 08 89	36 926 037	18,2483	6,9313	3,00300	1046,2	8 70 92,0	333
334	11 15 56	37 259 704	18,2757	6,9382	2,99401	1049,3	8 76 15,9	334
335	11 22 25	37 595 375	18,3030	6,9451	2,98507	1052,4	8 81 41,3	335
336	11 28 96	37 933 056	18,3303	6,9521	2,97619	1055,6	8 86 68,3	336
337	11 35 69	38 272 753	18,3576	6,9589	2,96736	1058,7	8 91 96,2	337
338	11 42 44	38 614 472	18,3848	6,9658	2,95858	1061,9	8 97 27,0	338
339	11 49 21	38 958 219	18,4120	6,9727	2,94985	1065,0	9 02 58,7	339
340	11 56 00	39 304 000	18,4391	6,9795	2,94118	1068,1	9 07 92,0	**340**
341	11 62 81	39 651 821	18,4662	6,9864	2,93255	1071,3	9 13 26,9	341
342	11 69 64	40 001 688	18,4932	6,9932	2,92398	1074,4	9 18 63,3	342
343	11 76 49	40 353 607	18,5203	7,0000	2,91545	1077,6	9 24 01,3	343
344	11 83 36	40 707 584	18,5472	7,0068	2,90698	1080,7	9 29 40,9	344
345	11 90 25	41 063 625	18,5742	7,0136	2,89855	1083,8	9 34 82,0	345
346	11 97 16	41 421 736	18,6011	7,0203	2,89017	1087,0	9 40 24,7	346
347	12 04 09	41 781 923	18,6279	7,0271	2,88184	1090,1	9 45 69,0	347
348	12 11 04	42 144 192	18,6548	7,0338	2,87356	1093,3	9 51 14,9	348
349	12 18 01	42 508 549	18,6815	7,0406	2,86533	1096,4	9 56 62,3	349
350	12 25 00	42 875 000	18,7083	7,0473	2,85714	1099,6	9 62 11,3	**350**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
350	12 25 00	42 875 000	18,7083	7,0473	2,85714	1099,6	9 62 11,3	**350**
351	12 32 01	43 243 551	18,7350	7,0540	2,84900	1102,7	9 67 61,8	351
352	12 39 04	43 614 208	18,7617	3,0607	2,84091	1105,8	9 73 14,0	352
353	12 46 09	43 986 977	18,7883	7,0674	2,83286	1109,0	9 78 67,7	353
354	12 53 16	44 361 864	18,8149	7,0740	2,82486	1112,1	9 84 23,0	354
355	12 60 25	44 738 875	18,8414	7,0807	2,81690	1115,3	9 89 79,8	355
356	12 67 36	45 118 016	18,8680	7,0873	2,80899	1118,4	9 95 38,2	356
357	12 74 49	45 499 293	18,8944	7,0940	2,80112	1121,5	10 00 98	357
358	12 81 64	45 882 712	18,9209	7,1006	2,79330	1124,7	10 06 60	358
359	12 88 81	46 268 279	18,9473	7,1072	2,78552	1127,8	10 12 23	359
360	12 96 00	46 656 000	18,9737	7,1138	2,77778	1131,0	10 17 88	**360**
361	13 03 21	47 045 881	19,0000	7,1204	2,77008	1134,1	10 23 54	361
362	13 10 44	47 437 928	19,0263	7,1269	2,76243	1137,3	10 29 22	362
363	13 17 69	47 832 147	19,0526	7,1335	2,75482	1140,4	10 34 91	363
364	13 24 96	48 228 544	19,0788	7,1400	2,74725	1143,5	10 40 62	364
365	13 32 25	48 627 125	19,1050	7,1466	2,73973	1146,7	10 46 35	365
366	13 39 56	49 027 896	19,1311	7,1531	2,73224	1149,8	10 52 09	366
367	13 46 89	49 430 863	19,1572	7,1596	2,72480	1153,0	10 57 85	367
368	13 54 24	49 836 032	19,1833	7,1661	2,71739	1156,1	10 63 62	368
369	13 61 61	50 243 409	19,2094	7,1726	2,71003	1159,2	10 69 41	369
370	13 69 00	50 653 000	19,2354	7,1791	2,70270	1162,4	10 75 21	**370**
371	13 76 41	51 064 811	19,2614	7,1855	2,69542	1165,5	10 81 03	371
372	13 83 84	51 478 848	19,2873	7,1920	2,68817	1168,7	10 86 87	372
373	13 91 29	51 895 117	19,3132	7,1984	2,68097	1171,8	10 92 72	373
374	13 98 76	52 313 624	19,3391	7,2048	2,67380	1175,0	10 98 58	374
375	14 06 25	52 734 375	19,3649	7,2112	2,66667	1178,1	11 04 47	375
376	14 13 76	53 157 376	19,3907	7,2177	2,65957	1181,2	11 10 36	376
377	14 21 29	53 582 633	19,4165	7,2240	2,65252	1184,4	11 16 28	377
378	14 28 84	54 010 152	19,4422	7,2304	2,64550	1187,5	11 22 21	378
379	14 36 41	54 439 939	19,4679	7,2368	2,63852	1190,7	11 28 15	379
380	14 44 00	54 872 000	19,4936	7,2432	2,63158	1193,8	11 34 11	**380**
381	14 51 61	55 306 341	19,5192	7,2495	2,62467	1196,9	11 40 09	381
382	14 59 24	55 742 968	19,5448	7,2558	2,61780	1200,1	11 46 08	382
383	14 66 89	56 181 887	19,5704	7,2622	2,61097	1203,2	11 52 09	383
384	14 74 56	56 623 104	19,5959	7,2685	2,60417	1206,4	11 58 12	384
385	14 82 25	57 066 625	19,6214	7,2748	2,59740	1209,5	11 64 16	385
386	14 89 96	57 512 456	19,6469	7,2811	2,59067	1212,7	11 70 21	386
387	14 97 69	57 960 603	19,6723	7,2874	2,58398	1215,8	11 76 28	387
388	15 05 44	58 411 072	19,6977	7,2936	2,57732	1218,9	11 82 37	388
389	15 13 21	58 863 869	19,7231	7,2999	2,57069	1222,1	11 88 47	389
390	15 21 00	59 319 000	19,7484	7,3061	2 56410	1225,2	11 94 59	**390**
391	15 28 81	59 776 471	19,7737	7,3124	2,55754	1228,4	12 00 72	391
392	15 36 64	60 236 288	19,7990	7,3186	2,55102	1231,5	12 06 87	392
393	15 44 49	60 698 457	19,8242	7,3248	2,54453	1234,6	12 13 04	393
394	15 52 36	61 162 984	19,8494	7,3310	2,53807	1237,8	12 19 22	394
395	15 60 25	61 629 875	19,8746	7,3372	2,53165	1240,9	12 25 42	395
396	15 68 16	62 099 136	19,8997	7,3434	2,52525	1244,1	12 31 63	396
397	15 76 09	62 570 773	19,9249	7,3496	2,51889	1247,2	12 37 86	397
398	15 84 04	63 044 792	19,9499	7,3558	2,51256	1250,4	12 44 10	398
399	15 92 01	63 521 199	19,9750	7,3619	2,50627	1253,5	12 50 36	399
400	16 00 00	64 000 000	20,0000	7,3681	2,50000	1256,6	12 56 64	**400**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
400	16 00 00	64 000 000	20,0000	7,3681	2,50000	1256,6	12 56 64	**400**
401	16 08 01	64 481 201	20,0250	7,3742	2,49377	1259.8	12 62 93	401
402	16 16 04	64 964 808	20.0499	7,3803	2,48756	1262,9	12 69 23	402
403	16 24 09	65 450 827	20,0749	7,3864	2,48139	1266,1	12 75 56	403
404	16 32 16	65 939 264	20,0998	7,3925	2,47525	1269,2	12 81 90	404
405	16 40 25	66 430 125	20,1246	7,3986	2,46914	1272,3	12 88 25	405
406	16 48 36	66 923 416	20,1494	7,4047	2,46305	1275,5	12 94 62	406
407	16 56 49	67 419 143	20,1742	7,4108	2,45700	1278,6	12 01 00	407
408	16 64 64	67 917 312	20,1990	7,4169	2,45098	1281,8	12 07 41	408
409	16 72 81	68 417 929	20,2237	7,4229	2,44499	1284,9	12 13 82	409
410	16 81 00	68 921 000	20 2485	7,4290	2,43902	1288,1	13 20 25	**410**
411	16 89 21	69 426 531	20,2731	7,4350	2,43309	1291,2	13 26 70	411
412	16 97 44	69 934 528	20,2978	7,4410	2,42718	1294,3	13 33 17	412
413	17 05 69	70 444 997	20,3224	7,4470	2,42131	1297,5	13 39 65	413
414	17 13 96	70 957 944	20,3470	7,4530	2,41546	1300,6	13 46 14	414
415	17 22 25	71 473 375	20,3715	7,4590	2,40964	1303,8	13 52 65	415
416	17 30 56	71 991 296	20,3961	7,4650	2,40385	1306,9	13 59 18	416
417	17 38 89	72 511 713	20,4206	7,4710	2,39808	1310,0	13 65 72	417
418	17 47 24	73 034 632	20,4450	7.4770	2,39234	1313,2	13 72 28	418
419	17 55 61	73 560 059	20,4695	7,4829	2,38663	1316,3	13 78 85	419
420	17 64 00	74 088 000	20,4939	7,4889	2,38095	1319,5	15 85 44	**420**
421	17 72 41	74 618 461	20,5183	7,4948	2,37530	1322,6	13 92 05	421
422	17 80 84	75 151 448	20,5426	7,5007	2,36967	1325,8	13 98 67	422
423	17 89 29	75 686 967	20,5670	7,5067	2,36407	1328,9	14 05 31	423
424	17 97 76	76 225 024	20,5913	7,5126	2,35849	1332,0	14 11 96	424
425	18 06 25	76 765 625	20,6155	7,5185	2,35294	1335,2	14 18 63	425
426	18 14 76	77 308 776	20,6398	7,5244	2,34742	1338,3	14 25 31	426
427	18 23 29	77 854 483	20,6640	7,5302	2,34192	1341,5	14 32 01	427
428	18 31 84	78 402 752	20,6882	7,5361	2,33645	1344,6	14 38 72	428
429	18 40 41	78 953 589	20,7123	7,5420	2,33100	1347,7	14 45 45	429
430	18 49 00	79 507 000	20,7364	7,5478	2,32558	1350,9	14 52 20	**430**
431	18 57 61	80 062 991	20,7605	7,5537	2,32019	1354,0	14 58 96	431
432	18 66 24	80 621 568	20,7846	7,5595	2,31481	1357,2	14 65 74	432
433	18 74 89	81 182 737	20,8087	7,5654	2,30947	1360,3	14 72 54	433
434	18 83 56	81 746 504	20,8327	7,5712	2,30415	1363,5	14 79 34	434
435	18 92 25	82 312 875	20,8567	7,5770	2,29885	1366,6	14 86 17	435
436	19 00 96	82 881 856	20,8806	7,5828	2,29358	1369,7	14 93 01	436
437	19 09 69	83 453 453	20,9045	7,5886	2,28833	1372,9	14 99 87	437
438	19 18 44	84 027 672	20,9284	7,5944	2,28311	1376,0	15 06 74	438
439	19 27 21	84 604 519	20,9523	7,6001	2,27790	1379,2	15 13 63	439
440	19 36 00	85 184 000	20,9762	7,6059	2,27273	1382,3	15 20 53	**440**
441	19 44 81	85 766 121	21,0000	7,6117	2 26757	1385,4	15 27 45	441
442	19 53 64	86 350 888	21,0238	7,6174	2,26244	1388,6	15 34 39	442
443	19 62 49	86 938 307	21,0476	7,6232	2,25734	1391,7	15 41 34	443
444	19 71 36	87 528 384	21,0713	7,6289	2,25225	1394,9	15 45 30	444
445	19 80 25	88 121 125	21,0950	7,6346	2,24719	1398,0	15 58 28	445
446	19 89 16	88 716 536	21,1187	7,6403	2,24215	1401,2	15 62 28	446
447	19 98 09	89 314 623	21,1424	7,6460	2,23714	1404,3	15 69 30	447
448	20 07 04	89 915 392	21,1660	7,6517	2,23214	1407,4	15 76 33	448
449	20 16 01	90 518 849	21,1896	7,6574	2.22717	1410,6	15 83 37	449
450	20 25 00	91 125 000	21,2132	7,6631	2,22222	1413,7	15 90 43	**450**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
450	20 25 00	91 125 000	21,2132	7,6631	2,22222	1413,7	15 90 43	**450**
451	20 34 01	91 733 851	21,2368	7,6688	2,21729	1416,9	15 97 51	451
452	20 43 04	92 345 408	21,2603	7,6744	2,21239	1420,0	16 04 60	452
453	20 52 09	92 959 677	21,2838	7,6801	2,20751	1423,1	16 11 71	453
454	20 61 16	93 576 664	21,3073	7,6857	2,20264	1426,3	16 18 83	454
455	20 70 25	94 196 375	21,3307	7,6914	2,19780	1429,4	16 25 97	455
456	20 79 36	94 818 816	21,3542	7,6970	2,19298	1432,6	16 33 13	456
457	20 88 49	95 443 993	21,3776	7,7026	2,18818	1435,7	16 40 30	457
458	20 97 64	96 071 912	21,4009	7,7082	2,18341	1438,8	16 47 48	458
459	21 06 81	96 702 579	21,4243	7,7138	2,17865	1442,0	16 54 68	459
460	21 16 00	97 336 000	21,4476	7,7194	2,17391	1445,1	16 61 90	**460**
461	21 25 21	97 972 181	21,4709	7,7250	2,16920	1448,3	16 69 14	461
462	21 34 44	98 611 128	21,4942	7,7306	2,16450	1451,4	16 76 39	462
463	21 43 69	99 252 847	21,5174	7,7362	2,15983	1454,6	16 83 65	463
464	21 52 96	99 897 344	21,5407	7,7418	2,15517	1457,7	16 90 93	464
465	21 62 25	100 544 625	21,5639	7,7473	2,15054	1460,8	16 98 23	465
466	21 71 56	101 194 696	21,5870	7,7529	2,14592	1464,0	17 05 54	466
467	21 80 89	101 847 563	21,6102	7,7584	2,14133	1467,1	17 12 87	467
468	21 90 24	102 503 232	21,6333	7,7639	2,13675	1470,3	17 20 21	468
469	21 99 61	103 161 709	21,6564	7,7695	2,13220	1473,4	17 27 57	469
470	22 09 00	103 823 000	21,6795	7,7750	2,12766	1476,5	17 34 94	**470**
471	22 18 41	104 487 111	21,7025	7,7805	2,12314	1479,7	17 42 34	471
472	22 27 84	105 154 048	21,7256	7,7860	2,11864	1482,8	17 49 74	472
473	22 37 29	105 823 817	21,7486	7,7915	2,11416	1486,0	17 57 16	473
474	22 46 76	106 496 424	21,7715	7,7970	2,10970	1489,1	17 64 60	474
475	22 56 25	107 171 875	21,7945	7,8025	2,10526	1492,3	17 72 05	475
476	22 65 76	107 850 176	21,8174	7,8079	2,10084	1495,4	17 79 52	476
477	22 75 29	108 531 333	21,8403	7,8134	2,09644	1498,5	17 87 01	477
478	22 84 84	109 215 352	21,8632	7,8188	2,09205	1501,7	17 94 51	478
479	22 94 41	109 902 239	21,8861	7,8243	2,08768	1504,8	18 02 03	479
480	23 04 00	110 592 000	21,9089	7,8297	2,08333	1508,0	18 09 56	**480**
481	23 13 61	111 284 641	21,9317	7,8352	2,07900	1511,1	18 17 11	481
482	23 23 24	111 980 168	21,9545	7,8406	2,07469	1514,2	18 24 67	482
483	23 32 89	112 678 587	21,9773	7,8460	2,07039	1517,4	18 32 25	483
484	23 42 56	113 379 904	22,0000	7,8514	2,06612	1520,5	18 39 84	484
485	23 52 25	114 084 125	22,0227	7,8568	2,06186	1523,7	18 47 45	485
486	23 61 96	114 791 256	22,0454	7,8622	2,05761	1526,8	18 55 08	486
487	23 71 69	115 501 303	22,0681	7,8676	2,05339	1530,0	18 62 72	487
488	23 81 44	116 214 272	22,0907	7,8730	2,04918	1533,1	18 70 38	488
489	23 91 21	116 930 169	22,1133	7,8784	2,04499	1536,2	18 78 05	489
490	24 01 00	117 649 000	22,1359	7,8837	2,04082	1539,4	18 85 74	**490**
491	24 10 81	118 370 771	22,1585	7,8891	2,03666	1542,5	18 93 45	491
492	24 20 64	119 095 488	22,1811	7,8944	2,03252	1545,7	19 01 17	492
493	24 30 49	119 823 157	22,2036	7,8998	2,02840	1548,8	19 08 90	493
494	24 40 36	120 553 784	22,2261	7,9051	2,02429	1551,9	19 16 65	494
495	24 50 25	121 287 375	22,2486	7,9105	2,02020	1555,1	19 24 42	495
496	24 60 16	122 023 936	22,2711	7,9158	2,01613	1558,2	19 32 21	496
497	24 70 09	122 763 473	22,2935	7,9211	2,01207	1561,4	19 40 00	497
498	24 80 04	123 505 992	22,3159	7,9264	2,00803	1564,5	19 47 82	498
499	24 90 01	124 251 499	22,3383	7,9317	2,00401	1567,7	19 55 65	499
500	25 00 00	125 000 000	22,3607	7,9370	2,00000	1570,8	19 63 50	**500**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\ln n$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
500	25 00 00	125 000 000	22,3607	7,9370	6,21461	2,00000	1570,8	19 63 50	**500**
501	25 10 01	125 751 501	22,3830	7,9423	6,21661	1,99601	1573,9	19 71 36	501
502	25 20 04	126 506 008	22,4054	7,9476	6,21860	1,99203	1577,1	19 79 23	502
503	25 30 09	127 263 527	22,4277	7,9528	6,22059	1,98807	1580,2	19 87 13	503
504	25 40 16	128 024 064	22,4499	7,9581	6,22258	1,98413	1583,4	19 95 04	504
505	25 50 25	128 787 625	22,4722	7,9634	6,22456	1,98020	1586,5	20 02 96	505
506	25 60 36	129 554 216	22,4944	7,9686	6,22654	1,97628	1589,6	20 10 90	506
507	25 70 49	130 323 843	22,5167	7,9739	6,22851	1,97239	1592,8	20 18 86	507
508	25 80 64	131 096 512	22,5389	7,9791	6,23048	1,96850	1595,9	20 26 83	508
509	25 90 81	131 872 229	22,5610	7,9843	6,23245	1,96464	1599,1	20 34 82	509
510	26 01 00	132 651 000	22,5832	7,9896	6,23441	1,96078	1602,2	20 42 82	**510**
511	26 11 21	133 432 831	22,6053	7,9948	6,23637	1,95695	1605,4	20 50 84	511
512	26 21 44	134 217 728	22,6274	8,0000	6,23832	1,95312	1608,5	20 58 87	512
513	26 31 69	135 005 697	22,6495	8,0052	6,24028	1,94932	1611,6	20 66 92	513
514	26 41 96	135 796 744	22,6716	8,0104	6,24222	1,94553	1614,8	20 74 99	514
515	26 52 25	136 590 875	22,6936	8,0156	6,24417	1,94175	1617,9	20 83 07	515
516	26 62 56	137 388 096	22,7156	8,0208	6,24611	1,93798	1621,1	20 91 17	516
517	26 72 89	138 188 413	22,7376	8,0260	6,24804	1,93424	1624,2	20 99 28	517
518	26 83 24	138 991 832	22,7596	8,0311	6,24998	1,93050	1627,3	21 07 41	518
519	26 93 61	139 798 359	22,7816	8,0363	6,25190	1,92678	1630,5	21 15 56	519
520	27 04 00	140 608 000	22,8035	8,0415	6,25383	1,92308	1633,6	21 23 72	**520**
521	27 14 41	141 420 761	22,8254	8,0466	6,25575	1,91939	1636,8	21 31 89	521
522	27 24 84	142 236 648	22,8473	8,0517	6,25767	1,91571	1639,9	21 40 08	522
523	27 35 29	143 055 667	22,8692	8,0569	6,25958	1,91205	1643,1	21 48 29	523
524	27 45 76	143 877 824	22,8910	8,0620	6,26149	1,90840	1646,2	21 56 51	524
525	27 56 25	144 703 125	22,9129	8,0671	6,26340	1,90476	1649,3	21 64 75	525
526	27 66 76	145 531 576	22,9347	8,0723	6,26530	1,90114	1652,5	21 73 01	526
527	27 77 29	146 363 183	22,9565	8,0774	6,26720	1,89753	1655,6	21 81 28	527
528	27 87 84	147 197 952	22,9783	8,0825	6,26910	1,89394	1658,8	21 89 56	528
529	27 98 41	148 035 889	23,0000	8,0876	6,27099	1,89036	1661,9	21 97 87	529
530	28 09 00	148 877 000	23,0217	8,0927	6,27288	1,88679	1665,0	22 06 18	**530**
531	28 19 61	149 721 291	23,0434	8,0978	6,27476	1,88324	1668,2	22 14 52	531
532	28 30 24	150 568 768	23,0651	8,1028	6,27664	1,87970	1671,3	22 22 87	532
533	28 40 89	151 419 437	23,0868	8,1079	6,27852	1,87617	1674,5	22 31 23	533
534	28 51 56	152 273 304	23,1084	8,1130	6,28040	1,87266	1677,6	22 39 61	534
535	28 62 25	153 130 375	23,1301	8,1180	6,28227	1,86916	1680,8	22 48 01	535
536	28 72 96	153 990 656	23,1517	8,1231	6,28413	1,86567	1683,9	22 56 42	536
537	28 83 69	154 854 153	23,1733	8,1281	6,28600	1,86220	1687,0	22 64 84	537
538	28 94 44	155 720 872	23,1948	8,1332	6,28786	1,85874	1690,2	22 73 29	538
539	29 05 21	156 590 819	23,2164	8,1382	6,28972	1,85529	1693.3	22 81 75	539
540	29 16 00	157 464 000	23,2379	8,1433	6,29157	1,85185	1696,5	22 90 22	**540**
541	29 26 81	158 340 411	23,2594	8,1483	6,29342	1,84843	1699,6	22 98 71	541
542	29 37 64	159 220 088	23,2809	8,1533	6,29527	1,84502	1702,7	23 07 22	542
543	29 48 49	160 103 007	23,3024	8,1583	6,29711	1,84162	1705,9	23 15 74	543
544	29 59 36	160 989 184	23,3238	8,1633	6,29895	1,83824	1709,0	23 24 28	544
545	29 70 25	161 878 625	23,3452	8,1683	6,30079	1,83486	1712,2	23 32 83	545
546	29 81 16	162 771 336	23,3666	8,1733	6,30262	1,83150	1715,3	23 41 40	546
547	29 92 09	163 667 323	23,3880	8,1783	6,30445	1,82815	1718,5	23 49 98	547
548	30 03 04	164 566 592	23,4094	8,1833	6,30628	1,82482	1721,6	23 58 58	548
549	30 14 01	165 469 149	23,4307	8,1882	6,30810	1,82149	1724,7	23 67 20	549
550	30 25 00	166 375 000	23,4521	8,1932	6,30992	1,81818	1727,9	23 75 83	**550**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
550	30 25 00	166 375 000	23,4521	8,1932	1,81818	1727,9	23 75 83	**550**
551	30 36 01	167 284 151	23,4734	8,1982	1,81488	1731,0	23 84 48	551
552	30 47 04	168 196 608	23,4947	8,2031	1,81159	1734,2	23 93 14	552
553	30 58 09	169 112 377	23,5160	8,2081	1,80832	1737,3	24 01 82	553
554	30 69 16	170 031 464	23,5372	8,2130	1,80505	1740,4	24 10 51	554
555	30 80 25	170 953 875	23,5584	8,2180	1,80180	1743,6	24 19 22	555
556	30 91 36	171 879 616	23,5797	8,2229	1,79856	1746,7	24 27 95	556
557	31 02 49	172 808 693	23,6008	8,2278	1,79533	1749,9	24 36 69	557
558	31 13 64	173 741 112	23,6220	8,2327	1,79211	1753,0	24 45 45	558
559	31 24 81	174 676 879	23,6432	8,2377	1,78891	1756,2	24 54 22	559
560	31 36 00	175 616 000	23,6643	8,2426	1,78571	1759,3	24 63 01	**560**
561	31 47 21	176 558 481	23,6854	8,2475	1,78253	1762,4	24 71 81	561
562	31 58 44	177 504 328	23,7065	8,2524	1,77936	1765,6	24 80 63	562
563	31 69 69	178 453 547	23,7276	8,2573	1,77620	1768,7	24 89 47	563
564	31 80 96	179 406 144	23,7487	8,2621	1,77305	1771,9	24 98 32	564
565	31 92 25	180 362 125	23,7697	8,2670	1,76991	1775,0	25 07 19	565
566	32 03 56	181 321 496	23,7908	8,2719	1,76678	1778,1	25 16 07	566
567	32 14 89	182 284 263	23,8118	8,2768	1,76367	1781,3	25 24 97	567
568	32 26 24	183 250 432	23,8328	8,2816	1,76056	1784,4	25 33 88	568
569	32 37 61	184 220 009	23,8537	8,2865	1,75747	1787,6	25 42 81	569
570	32 49 00	185 193 000	23,8747	8,2913	1,75439	1790,7	25 51 76	**570**
571	32 60 41	186 169 411	23,8956	8,2962	1,75131	1793,8	25 60 72	571
572	32 71 84	187 149 248	23,9165	8,3010	1,74825	1797,0	25 69 70	572
573	32 83 29	188 132 517	23,9374	8,3059	1,74520	1800,1	25 78 69	573
574	32 94 76	189 119 224	23,9583	8,3107	1,74216	1803,3	25 87 70	574
575	33 06 25	190 109 375	23,9792	8,3155	1,73913	1806,4	25 96 72	575
576	33 17 76	191 102 976	24,0000	8,3203	1,73611	1809,6	26 05 76	576
577	33 29 29	192 100 033	24,0208	8,3251	1,73310	1812,7	26 14 82	577
578	33 40 84	193 100 552	24,0416	8,3300	1,73010	1815,8	26 23 89	578
579	33 52 41	194 104 539	24,0624	8,3348	1,72712	1819,0	26 32 98	579
580	33 64 00	195 112 000	24,0832	8,3396	1,72414	1822,1	26 42 08	**580**
581	33 75 61	196 122 941	24,1039	8,3443	1,72117	1825,3	26 51 20	581
582	33 87 24	197 137 368	24,1247	8,3491	1,71821	1828,4	26 60 33	582
583	33 98 89	198 155 287	24,1454	8,3539	1,71527	1831,6	26 69 48	583
584	34 10 56	199 176 704	24,1661	8,3587	1,71233	1834,7	26 78 65	584
585	34 22 25	200 201 625	24,1868	8,3634	1,70940	1837,8	26 87 83	585
586	34 33 96	201 230 056	24,2074	8,3682	1,70648	1841,0	26 97 03	586
587	34 45 69	202 262 003	24,2281	8,3730	1,70358	1844,1	27 06 24	587
588	34 57 44	203 297 472	24,2487	8 3777	1,70068	1847,3	27 15 47	588
589	34 69 21	204 336 469	24,2693	8,3825	1,69779	1850,4	27 24 71	589
590	34 81 00	205 379 000	24 2899	8,3872	1,69492	1853,5	27 33 97	**590**
591	34 92 81	206 425 071	24,3105	8,3919	1,69205	1856,7	27 43 25	591
592	35 04 64	207 474 688	24,3311	8,3967	1,68919	1859,8	27 52 54	592
593	35 16 49	208 527 857	24,3516	8,4014	1,68634	1863,0	27 61 84	593
594	35 28 36	209 584 584	24,3721	8,4061	1,68350	1866,1	27 71 17	594
595	35 40 25	210 644 875	24,3926	8,4108	1,68067	1869,2	27 80 51	595
596	35 52 16	211 708 736	24,4131	8.4155	1,67785	1872,4	27 89 86	596
597	35 64 09	212 776 173	24,4336	8,4202	1,67504	1875,5	27 99 23	597
598	35 76 04	213 847 192	24,4540	8,4249	1,67224	1878,7	28 08 62	598
599	35 88 01	214 921 799	24,4745	8,4296	1,66945	1881,8	28 18 02	599
600	36 00 00	216 000 000	24,4949	8,4343	1,66667	1885,0	28 27 43	**600**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
600	36 00 00	216 000 000	24,4949	8,4343	1,66667	1885,0	28 27 43	**600**
601	36 12 01	217 081 801	24,5153	8,4390	1,66389	1888,1	28 36 87	601
602	36 24 04	218 167 208	24,5357	8,4437	1,66113	1891,2	28 46 31	602
603	36 36 09	219 256 227	24,5561	8,4484	1,65837	1894,4	28 55 78	603
604	36 48 16	220 348 864	24,5764	8,4530	1,65563	1897,5	28 65 26	604
605	36 60 25	221 445 125	24,5967	8,4577	1,65289	1900,7	28 74 75	605
606	36 72 36	222 545 016	24,6171	8,4623	1,65017	1903,8	28 84 26	606
607	36 84 49	223 648 543	24,6374	8,4670	1,64745	1906,9	28 93 79	607
608	36 96 64	224 755 712	24,6577	8,4716	1,64474	1910,1	29 03 33	608
609	37 08 81	225 866 529	24,6779	8,4763	1,64204	1913,2	29 12 89	609
610	37 21 00	226 981 000	24,6982	8,4809	1,63934	1916,4	29 22 47	**610**
611	37 33 21	228 099 131	24,7184	8,4856	1,63666	1919,5	29 32 06	611
612	37 45 44	229 220 928	24,7386	8,4902	1,63399	1912,7	29 41 66	612
613	37 57 69	230 346 397	24,7588	8,4948	1,63132	1925,8	29 51 28	613
614	37 69 96	231 475 544	24,7790	8,4994	1,62866	1928,9	29 60 92	614
615	37 82 25	232 608 375	24,7992	8,5040	1,62602	1932,1	29 70 57	615
616	37 94 56	233 744 896	24,8193	8,5086	1,62338	1935,2	29 80 24	616
617	38 06 89	234 885 113	24,8395	8,5132	1,62075	1938,4	29 89 92	617
618	38 19 24	236 029 032	24,8596	8,5178	1,61812	1941,5	29 99 62	618
619	38 31 61	237 176 659	24,8797	8,5224	1,61551	1944,6	30 09 34	619
620	38 44 00	238 328 000	24,8998	8,5270	1,61290	1947,8	30 19 07	**620**
621	38 56 41	239 483 061	24,9199	8,5316	1,61031	1950,9	30 28 82	621
622	38 68 84	240 641 848	24,9399	8,5362	1,60772	1954,1	30 38 58	622
623	38 81 29	241 804 367	24,9600	8,5408	1,60514	1957,2	30 48 36	623
624	38 93 76	242 970 624	24,9800	8,5453	1,60256	1960,4	30 58 15	624
625	39 06 25	244 140 625	25,0000	8,5499	1,60000	1963,5	30 67 96	625
626	39 18 76	245 314 376	25,0200	8,5544	1,59744	1966,6	30 77 79	626
627	39 31 29	246 491 883	25,0400	8,5590	1,59490	1969,8	30 87 63	627
628	39 43 84	247 673 152	25,0599	8,5635	1,59236	1972,9	30 97 48	628
629	39 56 41	248 858 189	25,0799	8,5681	1,58983	1976,1	31 07 36	629
630	39 69 00	250 047 000	25,0998	8,5726	1,58730	1979,2	31 17 25	**630**
631	39 81 61	251 239 591	25,1197	8,5772	1,58479	1982,3	31 27 15	631
632	39 94 24	252 435 968	25,1396	8,5817	1,58228	1985,5	31 37 07	632
633	40 06 89	253 636 137	25,1595	8,5862	1,57978	1988,6	31 47 00	633
634	40 19 56	254 840 104	25,1794	8,5907	1,57729	1991,8	31 56 96	634
635	40 32 25	256 047 875	25,1992	8,5952	1,57480	1994,9	31 66 92	635
636	40 44 96	257 259 456	25,2190	8,5997	1,57233	1998,1	31 76 90	636
637	40 57 69	258 474 853	25,2389	8,6043	1,56986	2001,2	31 86 90	637
638	40 70 44	259 694 072	25,2587	8,6088	1,56740	2004,3	31 96 92	638
639	40 83 21	260 917 119	25,2784	8,6132	1,56495	2007,5	32 06 95	639
640	40 96 00	262 144 000	25,2982	8,6177	1,56250	2010,6	32 16 99	**640**
641	41 08 81	263 374 721	25,3180	8,6222	1,56006	2013,8	32 27 05	641
642	41 21 64	264 609 288	25,3377	8,6267	1,55763	2016,9	32 37 13	642
643	41 34 49	265 847 707	25,3574	8,6312	1,55521	2020,0	32 47 22	643
644	41 47 36	267 089 984	25,3772	8,6357	1,55280	2023,2	32 57 33	644
645	41 60 25	268 336 125	25,3969	8,6401	1,55039	2026,3	32 67 45	645
646	41 73 16	269 586 136	25,4165	8,6446	1,54799	2029,5	32 77 59	646
647	41 86 09	270 840 023	25,4362	8,6490	1,54560	2032,6	32 87 75	647
648	41 99 04	272 097 792	25,4558	8,6535	1,54321	2035,8	32 97 92	648
649	42 12 01	273 359 449	25,4755	8,6579	1,54083	2038,9	33 08 10	649
650	42 25 00	274 625 000	25,4951	8,6624	1,53846	2042,0	33 18 31	**650**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
650	42 25 00	274 625 000	25,4951	8,6624	1,53846	2042,0	33 18 31	**650**
651	42 38 01	275 894 451	25,5147	8,6668	1,53610	2045,2	33 28 53	651
652	42 51 04	277 167 808	25,5343	8,6713	1,53374	2048,3	33 38 76	652
653	42 64 09	278 445 077	25,5539	8,6757	1,53139	2051,5	33 49 01	653
654	42 77 16	279 726 264	25,5734	8,6801	1,52905	2054,6	33 59 27	654
655	42 90 25	281 011 375	25,5930	8,6845	1,52672	2057,7	33 69 55	655
656	43 03 36	282 300 416	25,6125	8,6890	1,52439	2060,9	33 79 85	656
657	43 16 49	283 593 393	25,6320	8,6934	1,52207	2064,0	33 90 16	657
658	43 29 64	284 890 312	25,6515	8,6978	1,51976	2067,2	34 00 49	658
659	43 42 81	286 191 179	25,6710	8,7022	1,51745	2070,3	34 10 84	659
660	43 56 00	287 496 000	25,6905	8,7066	1,51515	2073,5	34 21 19	**660**
661	43 69 21	288 804 781	25,7099	8,7110	1,51286	2076,6	34 31 57	661
662	43 82 44	290 117 528	25,7294	8,7154	1,51057	2079,7	34 41 96	662
663	43 95 69	291 434 247	25,7488	8,7198	1,50830	2082,9	34 52 37	663
664	44 08 96	292 754 944	25,7682	8,7241	1,50602	2086,0	34 62 79	664
665	44 22 25	294 079 625	25,7876	8,7285	1,50376	2089,2	34 73 23	665
666	44 35 56	295 408 296	25,8070	8,7329	1,50150	2092,3	34 83 68	666
667	44 48 89	296 740 963	25,8263	8,7373	1,49925	2095,4	34 94 15	667
668	44 62 24	298 077 632	25,8457	8,7416	1,49701	2098,6	35 04 64	668
669	44 75 61	299 418 309	25,8650	8,7460	1,49477	2101,7	35 15 14	669
670	44 89 00	300 763 000	25,8844	8,7503	1,49254	2104,9	35 25 65	**670**
671	45 02 41	302 111 711	25,9037	8,7547	1,49031	2108,0	35 36 18	671
672	45 15 84	303 464 448	25,9230	8,7590	1,48810	2111,2	35 46 73	672
673	45 29 29	304 821 217	25,9422	8,7634	1,48588	2114,3	35 57 30	673
674	45 42 76	306 182 024	25,9615	8,7677	1,48368	2117,4	35 67 88	674
675	45 56 25	307 546 875	25,9808	8,7721	1,48148	2120,6	35 78 47	675
676	45 69 76	308 915 776	26,0000	8,7764	1,47929	2123,7	35 89 08	676
677	45 83 29	310 288 733	26,0192	8,7807	1,47710	2126,9	35 99 71	677
678	45 96 84	311 665 752	26,0384	8,7850	1,47493	2130,0	36 10 35	678
679	46 10 41	313 046 839	26,0576	8,7893	1,47275	2133,1	36 21 01	679
680	46 24 00	314 432 000	26,0768	8,7937	1,47059	2136,3	36 31 68	**680**
681	46 37 61	315 821 241	26,0960	8,7980	1,46843	2139,4	36 42 37	681
682	46 51 24	317 214 568	26,1151	8,8023	1,46628	2142,6	36 53 08	682
683	46 64 89	318 611 987	26,1343	8,8066	1,46413	2145,7	36 63 80	683
684	46 78 56	320 013 504	26,1534	8,8109	1,46199	2148,8	36 74 53	684
685	46 92 25	321 419 125	26,1725	8,8152	1,45985	2152,0	36 85 28	685
686	47 05 96	322 828 856	26,1916	8,8194	1,45773	2155,1	36 96 05	686
687	47 19 69	324 242 703	26,2107	8,8237	1,45560	2158,3	37 06 84	687
688	47 33 44	325 660 672	26,2298	8,8280	1,45349	2161,4	37 17 64	688
689	47 47 21	327 082 769	26,2488	8,8323	1,45138	2164,6	37 28 45	689
690	47 61 00	328 509 000	26,2679	8,8366	1,44928	2167,7	37 39 28	**690**
691	47 74 81	329 939 371	26,2869	8,8408	1,44718	2170,8	37 50 13	691
692	47 88 64	331 373 888	26,3059	8,8451	1,44509	2174,0	37 60 99	692
693	48 02 49	332 812 557	26,3249	8,8493	1,44300	2177 1	37 71 87	693
694	48 16 36	334 255 384	26,3439	8,8536	1,44092	2180,3	37 82 76	694
695	48 30 25	335 702 375	26,3629	8,8578	1,43885	2183,4	37 93 67	695
696	48 44 16	337 153 536	26,3818	8,8621	1,43678	2186,5	38 04 59	696
697	48 58 09	338 608 873	26,4008	8,8663	1,43472	2189,7	38 15 53	697
698	48 72 04	340 068 392	26,4197	8,8706	1,43266	2192,8	38 26 49	698
699	48 86 01	341 532 099	26,4386	8,8748	1,43062	2196,0	38 37 46	699
700	49 00 00	343 000 000	26,4575	8,8790	1,42857	2199,1	38 48 45	**700**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
700	49 00 00	343 000 000	26,4575	8,8790	1,42857	2199,1	38 48 45	**700**
701	49 14 01	344 472 101	26,4764	8,8833	1,42653	2202,3	38 59 45	701
702	49 28 04	345 948 408	26,4953	8,8875	1,42450	2205,4	38 70 47	702
703	49 42 09	347 428 927	26,5141	8,8917	1,42248	2208,5	38 81 51	703
704	49 56 16	348 913 664	26,5330	8,8959	1,42045	2211,7	38 92 56	704
705	49 70 25	350 402 625	26,5518	8,9001	1,41844	2214,8	39 03 63	705
706	49 84 36	351 895 816	26,5707	8,9043	1,41643	2218,0	39 14 71	706
707	49 98 49	353 393 243	26,5895	8,9085	1,41443	2221,1	39 25 80	707
708	50 12 64	354 894 912	26,6083	8,9127	1,41243	2224,2	39 36 92	708
709	50 26 81	356 400 829	26,6271	8,9169	1,41044	2227,4	39 48 05	709
710	50 41 00	357 911 000	26,6458	8,9211	1,40845	2230,5	39 59 19	**710**
711	50 55 21	359 425 431	26,6646	8,9253	1,40647	2233,7	39 70 35	711
712	50 69 44	360 944 128	26,6833	8,9295	1,40449	2236,8	39 81 53	712
713	50 83 69	362 467 097	26,7021	8,9337	1,40252	2240,0	39 92 72	713
714	50 97 96	363 994 344	26,7208	8,9378	1,40056	2243,1	40 03 93	714
715	51 12 25	365 525 875	26,7395	8,9420	1,39860	2246,2	40 15 15	715
716	51 26 56	367 061 696	26,7582	8,9462	1,39665	2249,4	40 26 39	716
717	51 40 89	368 601 813	26,7769	8,9503	1,39470	2252,5	40 37 65	717
718	51 55 24	370 146 232	26,7955	8,9545	1,39276	2255,7	40 48 92	718
719	51 69 61	371 694 959	26,8142	8,9587	1,39082	2258,8	40 60 20	719
720	51 84 00	373 248 000	26,8328	8,9628	1,38889	2261,9	40 71 50	**720**
721	51 98 41	374 805 361	26,8514	8,9670	1,38696	2265,1	40 82 82	721
722	52 12 84	376 367 048	26,8701	8,9711	1,38504	2268,2	40 94 15	722
723	52 27 29	377 933 067	26,8887	8,9752	1,38313	2271,4	41 05 50	723
724	52 41 76	379 503 424	26,9072	8,9794	1,38122	2274,5	41 16 87	724
725	52 56 25	381 078 125	26,9258	8,9835	1,37931	2277,7	41 28 25	725
726	52 70 76	382 657 176	26,9444	8,9876	1,37741	2280,8	41 39 65	726
727	52 85 29	384 240 583	26,9629	8,9918	1,37552	2283,9	41 51 06	727
728	52 99 84	385 828 352	26,9815	8,9959	1,37363	2287,1	41 62 48	728
729	53 14 41	387 420 489	27,0000	9,0000	1,37174	2290,2	41 73 93	729
730	53 29 00	389 017 000	27,0185	9,0041	1,36986	2293,4	41 85 39	**730**
731	53 43 61	390 617 891	27,0370	9,0082	1,36799	2296,5	41 96 86	731
732	53 58 24	392 223 168	27,0555	9,0123	1,36612	2299,6	42 08 35	732
733	53 72 89	393 832 837	27,0740	9,0164	1,36426	2302,8	42 19 86	733
734	53 87 56	395 446 904	27,0924	9,0205	1,36240	2305,9	42 31 38	734
735	54 02 25	397 065 375	27,1109	9,0246	1,36054	2309,1	42 42 93	735
736	54 16 96	398 688 256	27,1293	9,0287	1,35870	2312,2	42 54 47	736
737	54 31 69	400 315 553	27,1477	9,0328	1,35685	2315,4	42 66 04	737
738	54 46 44	401 947 272	27,1662	9,0369	1,35501	2318,5	42 77 62	738
739	54 61 21	403 583 419	27,1846	9,0410	1,35318	2321,6	42 89 22	739
740	54 76 00	405 224 000	27,2029	9,0450	1,35135	2324,8	43 00 84	**740**
741	54 90 81	406 869 021	27,2213	9,0491	1,34953	2327,9	43 12 47	741
742	55 05 64	408 518 488	27,2397	9,0532	1,34771	2331,1	43 24 12	742
743	55 20 49	410 172 407	27,2580	9,0572	1,34590	2334,2	43 35 78	743
744	55 35 36	411 830 784	27,2764	9,0613	1,34409	2337,3	43 47 46	744
745	55 50 25	413 493 625	27,2947	9,0654	1,34228	2340,5	43 59 16	745
746	55 65 16	415 160 936	27,3130	9,0694	1,34048	2343,6	43 70 87	746
747	55 80 09	416 832 723	27,3313	9,0735	1,33869	2346,8	43 82 59	747
748	55 95 04	418 508 992	27,3496	9,0775	1,33690	2349,9	43 94 33	748
749	56 10 01	420 189 749	27,3679	9,0816	1,33511	2353,1	44 06 09	749
750	56 25 00	421 875 000	27,3861	9,0856	1,33333	2356,2	44 17 86	**750**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
750	56 25 00	421 875 000	27,3861	9,0856	1,33333	2356,2	44 17 86	**750**
751	56 40 01	423 564 751	27,4044	9,0896	1,33156	2359,3	44 29 65	751
752	56 55 04	425 259 008	27,4226	9,0937	1,32979	2362,5	44 41 46	752
753	56 70 09	426 957 777	27,4408	9,0977	1,32802	2365,6	44 53 28	753
754	56 85 16	428 661 064	27,4591	9,1017	1,32626	2368,8	44 65 11	754
755	57 00 25	430 368 875	27,4773	9,1057	1,32450	2371,9	44 76 97	755
756	57 15 36	432 081 216	27,4955	9,1098	1,32275	2375,0	44 88 83	756
757	57 30 49	433 798 093	27,5136	9,1138	1,32100	2378,2	45 00 72	757
758	57 45 64	435 519 512	27,5318	9,1178	1,31926	2381,3	45 12 62	758
759	57 60 81	437 245 479	27,5500	9,1218	1,31752	2384,5	45 24 53	759
760	57 76 00	438 976 000	27,5681	9,1258	1,31579	2387,6	45 36 46	**760**
761	57 91 21	440 711 081	27,5862	9,1298	1,31406	2390,8	45 48 41	761
762	58 06 44	442 450 728	27,6043	9,1338	1,31234	2393,9	45 60 37	762
763	58 21 69	444 194 947	27,6225	9,1378	1,31062	2397,0	45 72 34	763
764	58 36 96	445 943 744	27,6405	9,1418	1,30890	2400,2	45 84 34	764
765	58 52 25	447 697 125	27,6586	9,1458	1,30719	2403,3	45 96 35	765
766	58 67 56	449 455 096	27,6767	9,1498	1,30548	2406,5	46 08 37	766
767	58 82 89	451 217 663	27,6948	9,1537	1,30378	2409,6	46 20 41	767
768	58 98 24	452 984 832	27,7128	9,1577	1,30208	2412,7	46 32 47	768
769	59 13 61	454 756 609	27,7308	9,1617	1,30039	2415,9	46 44 54	769
770	59 29 00	456 533 000	27,7489	9,1657	1,29870	2419,0	46 56 63	**770**
771	59 44 41	458 314 011	27,7669	9,1696	1,29702	2422,2	46 68 73	771
772	59 59 84	460 099 648	27,7849	9,1736	1,29534	2425,3	46 80 85	772
773	59 75 29	461 889 917	27,8029	9,1775	1,29366	2428,5	46 92 98	773
774	59 90 76	463 684 824	27,8209	9,1815	1,29199	2431,6	47 05 13	774
775	60 06 25	465 484 375	27,8388	9,1855	1,29032	2434,7	47 17 30	775
776	60 21 76	467 288 576	27,8568	9,1894	1,28866	2437,9	47 29 48	776
777	60 37 29	469 097 433	27,8747	9,1933	1,28700	2441,0	47 41 68	777
778	60 52 84	470 910 952	27,8927	9,1973	1,28535	2444,2	47 53 89	778
779	60 68 41	472 729 139	27,9106	9,2012	1,28370	2447,3	47 66 12	779
780	60 84 00	474 552 000	27,9285	9,2052	1,28205	2450,4	47 78 36	**780**
781	60 99 61	476 379 541	27,9464	9,2091	1,28041	2453,6	47 90 62	781
782	61 15 24	478 211 768	27,9643	9,2130	1,27877	2456,7	48 02 90	782
783	61 30 89	480 048 687	27,9821	9,2170	1,27714	2459,9	48 15 19	783
784	61 46 56	481 890 304	28,0000	9,2209	1,27551	2463,0	48 27 50	784
785	61 62 25	483 736 625	28,0179	9,2248	1,27389	2466,2	48 39 82	785
786	61 77 96	485 587 656	28,0357	9,2287	1,27226	2469,3	48 52 16	786
787	61 93 69	487 443 403	28,0535	9,2326	1,27065	2472,4	48 64 51	787
788	62 09 44	489 303 872	28,0713	9,2365	1,26904	2475,6	48 76 88	788
789	62 25 21	491 169 069	28,0891	9,2404	1,26743	2478,7	48 89 27	789
790	62 41 00	493 039 000	28,1069	9,2443	1,26582	2481,9	49 01 67	**790**
791	62 56 81	494 913 671	28,1247	9,2482	1,26422	2485,0	49 14 09	791
792	62 72 64	496 793 088	28,1425	9,2521	1,26263	2488,1	49 26 52	792
793	62 88 49	498 677 257	28,1603	9,2560	1,26103	2491,3	49 38 97	793
794	63 04 36	500 566 184	28,1780	9,2599	1,25945	2494,4	49 51 43	794
795	63 20 25	502 459 875	28,1957	9,2638	1,25786	2497,6	49 63 91	795
796	63 36 16	504 358 336	28,2135	9,2677	1,25628	2500,7	49 76 41	796
797	63 52 09	506 261 573	28,2312	9,2716	1,25471	2503,8	49 88 92	797
798	63 68 04	508 169 592	28,2489	9,2754	1,25313	2507,0	50 01 45	798
799	63 84 01	510 082 399	28,2666	9,2793	1,25156	2510,1	50 13 99	799
800	64 00 00	512 000 000	28,2843	9,2832	2,15000	2513,3	50 26 55	**800**

n	n^2	n^3	$\sqrt{n}$	$\sqrt{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
800	64 00 00	512 000 000	28,2843	9,2832	1,25000	2513.3	50 26 55	**800**
801	64 16 01	513 922 401	28,3019	9,2870	1,24844	2516,4	50 39 12	801
802	64 32 04	515 849 608	28,3196	9,2909	1,24688	2519,6	50 51 71	802
803	64 48 09	517 781 627	28,3373	9,2948	1,24533	2522,7	50 64 32	803
804	64 64 16	519 718 464	28,3549	9,2986	1,24378	2525,8	50 76 94	804
805	64 80 25	521 660 125	28,3725	9,3025	1,24224	2529,0	50 89 58	805
806	64 96 36	523 606 616	28,3901	9,3063	1,24069	2532,1	51 02 23	806
807	65 12 49	525 557 943	28,4077	9,3120	1,23916	2535.3	51 14 90	807
808	65 28 64	527 514 112	28,4253	9,3140	1,23762	2538,4	51 27 58	808
809	65 44 81	529 475 129	28,4429	9,3179	1,23609	2541,5	51 40 28	809
810	65 61 00	531 441 000	28,4605	9,3217	1,23457	2544,7	51 53 00	**810**
811	65 77 21	533 411 731	28,4781	9,3255	1,23305	2547,8	51 65 73	811
812	65 93 44	535 387 328	28,4956	9,3294	1,23153	2551,0	51 78 48	812
813	66 09 69	537 367 797	28,5132	9,3332	1,23001	2554,1	51 91 24	813
814	66 25 96	539 353 144	28,5307	9,3370	1,22850	2557,3	52 04 02	814
815	66 42 25	541 343 375	28,5482	9,3408	1,22699	2560,4	52 16 81	815
816	66 58 56	543 338 496	28,5657	9,3447	1,22549	2563,5	52 29 62	816
817	66 74 89	545 388 513	28,5832	9,3485	1,22399	2566,7	52 42 45	817
818	66 91 24	547 343 432	28,6007	9,3523	1,22249	2569,8	52 55 29	818
819	67 07 61	549 353 259	28,6182	9,3561	1,22100	2573,0	52 68 14	819
820	67 24 00	551 368 000	28,6356	9,3599	1,21951	2576,1	52 81 02	**820**
821	67 40 41	553 387 661	28.6531	9,3637	1,21803	2579,2	52 93 91	821
822	67 56 84	555 412 248	28,6705	9,3675	1,21655	2582,4	53 06 81	822
823	67 73 29	557 441 767	28,6880	9,3713	1,21507	2585,5	53 19 73	823
824	67 89 76	559 476 224	28,7054	9,3751	1,21359	2588,7	53 32 67	824
825	68 06 25	561 515 625	28,7228	9,3789	1,21212	2591,8	53 45 62	825
826	68 22 76	563 559 976	28,7402	9,3827	1,21065	2595.0	53 58 58	826
827	68 39 29	565 609 283	28,7576	9,3865	1,20919	2598,1	53 71 57	827
828	68 55 84	567 663 552	28,7750	9,3902	1,20773	2601,2	53 84 56	828
829	68 72 41	569 722 789	28,7924	9,3940	1,20627	2604,4	53 97 58	829
830	68 89 00	571 787 000	28,8097	9,3978	1,20482	2607,5	54 10 61	**830**
831	69 05 61	573 856 191	28,8271	9,4016	1,20337	2610,7	54 23 65	831
832	69 22 24	575 930 368	28,8444	9,4053	1,20192	2613,8	54 36 71	832
833	69 38 89	578 009 537	28,8617	9,4091	1,20048	2616,9	54 49 79	833
834	69 55 56	580 093 704	28,8791	9,4129	1,19904	2620,1	54 62 88	834
835	69 72 25	582 182 875	28,8964	9,4166	1,19760	2623,2	54 75 99	835
836	69 88 96	584 277 056	28,9137	9,4204	1,19617	2626,4	54 89 12	836
837	70 05 69	586 376 253	28,9310	2,4241	1,19474	2629,5	55 02 26	837
838	70 22 44	588 480 472	28.9482	9,4279	1,19332	2632,7	55 15 41	838
839	70 39 21	590 589 719	28,9655	9,4316	1,19190	2635,8	55 28 58	839
840	70 56 00	592 704 000	28,9828	9,4354	1,19048	2638,9	55 41 77	**840**
841	70 72 81	594 823 321	29,0000	9,4391	1,18906	2642,1	55 54 97	841
842	70 89 64	596 947 688	29,0172	9,4429	1,18765	2645,2	55 68 19	842
843	71 06 49	599 077 107	29,0345	9,4466	1,18624	2648,4	55 81 42	843
844	71 23 36	601 211 584	29,0517	9,4503	1,18483	2651,5	55 94 67	844
845	71 40 25	603 351 125	29,0689	9,4541	1,18343	2654,6	56 07 94	845
846	71 57 16	605 495 736	29,0861	9,4578	1,18203	2657,8	56 21 22	846
847	71 74 09	607 645 423	29,1033	9,4615	1,18064	2660,9	56 34 52	847
848	71 91 04	609 800 192	29,1204	9,4652	1,17925	2664,1	56 47 83	848
849	72 08 01	611 960 049	29.1376	9,4690	1,17786	2667,2	56 61 16	849
850	72 25 00	614 125 000	29,1548	9,4727	1,17647	2670,4	56 74 50	**850**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
850	72 25 00	614 125 000	29,1548	9,4727	1 17647	2670,4	56 74 50	**850**
851	72 42 01	616 295 051	29,1719	9,4764	1,17509	2673,5	56 87 86	851
852	72 59 04	618 470 208	29,1890	9,4801	1,17371	2676,6	57 01 24	852
853	72 76 09	620 650 477	29,2062	9,4838	1,17233	2679,8	57 14 63	853
854	72 93 16	622 835 864	29,2233	9,4875	1,17096	2682,9	57 28 03	854
855	73 10 25	625 026 375	29,2404	9,4912	1,16959	2686,1	57 41 46	855
856	73 27 36	627 222 016	29,2575	9,4949	1,16822	2689,2	57 54 90	856
857	73 44 49	629 422 793	29,2746	9,4986	1,16686	2692,3	57 68 35	857
858	73 61 64	631 628 712	29,2916	9,5023	1,16550	2695,5	57 81 82	858
859	73 78 81	633 839 779	29,3087	9,5060	1,16414	2698,6	57 95 30	859
860	73 96 00	636 056 000	29,3258	9,5097	1,16279	2701,8	58 08 80	**860**
861	74 13 21	638 277 381	29,3428	9 5134	1,16144	2704,9	58 22 32	861
862	74 30 44	640 503 928	29,3598	9,5171	1,16009	2708,1	58 35 85	862
863	74 47 69	642 735 647	29,3769	9,5207	1,15875	2711,2	58 49 40	863
864	74 64 96	644 972 544	29,3939	9,5244	1,15741	2714,3	58 62 97	864
865	74 82 25	647 214 625	29,4109	9,5281	1,15607	2717,5	58 76 55	865
866	74 99 56	649 461 896	29,4279	9,5317	1,15473	2720,6	58 90 14	866
867	75 16 89	651 714 363	29.4449	9,5354	1,15340	2723,8	59 03 75	867
868	75 34 24	653 972 032	29,4618	9,5391	1,15207	2726,9	59 17 38	868
869	75 51 61	656 234 909	29,4788	9,5427	1,15075	2730,0	59 31 02	869
870	75 69 00	658 503 000	29,4958	9,5464	1,14943	2733,2	59 44 68	**870**
871	75 86 41	660 776 311	29,5127	9,5501	1,14811	2736,3	59 58 35	871
872	76 03 84	663 054 848	29,5296	9,5537	1,14679	2739,5	59 72 04	872
873	76 21 29	665 338 617	29,5466	9,5574	1,14548	2742,6	59 85 75	873
874	76 38 76	667 627 624	29,5635	9,5610	1,14416	2745,8	59 99 47	874
875	76 56 25	669 921 875	29,5804	9,5647	1,14286	2748,9	60 13 20	875
876	76 73 76	272 221 376	29,5973	9,5683	1,14155	2752,0	60 26 96	876
877	76 91 29	674 526 133	29,6142	9,5719	1,14025	2755,2	60 40 73	877
878	77 08 84	676 836 152	29,6311	9,5756	1,13895	2758,3	60 54 51	878
879	77 26 41	679 151 439	29,6479	9,5792	1,13766	2761,5	60 68 31	879
880	77 44 00	681 472 000	29,6648	9,5828	1,13636	2764,6	60 82 12	**880**
881	77 61 61	683 797 841	29,6816	9,5865	1,13507	2767,7	60 95 95	881
882	77 79 24	686 128 968	29,6985	9,5901	1,13379	2770,9	61 09 80	882
883	77 96 89	688 465 387	29,7153	9,5937	1,13250	2774,0	61 23 66	883
884	78 14 56	690 807 104	29,7321	9,5973	1,13122	2777,2	61 37 54	884
885	78 32 25	693 154 125	29,7489	9,6010	1,12994	2780,3	61 51 43	885
886	78 49 96	695 506 456	29,7658	9,6046	1,12867	2783,5	61 65 34	886
887	78 67 69	697 864 103	29,7825	9,6082	1,12740	2786,6	61 79 27	887
888	78 85 44	700 227 072	29,7993	9,6118	1,12613	2789,7	61 93 21	888
889	79 03 21	702 595 369	29,8161	9,6154	1,12486	2792,9	62 07 17	889
890	79 21 00	704 969 000	29,8329	9,6190	1,12360	2796,0	62 21 14	**890**
891	79 38 81	707 347 971	29,8496	9,6226	1,12233	2799,2	62 35 13	891
892	79 56 64	709 732 288	29,8664	9,6262	1,12108	2802,3	62 49 13	892
893	79 74 49	712 121 957	29,8831	9,6298	1,11982	2805,4	62 63 15	893
894	79 92 36	714 516 984	29,8998	9,6334	1,11857	2808,6	62 77 18	894
895	80 10 25	716 917 375	29,9166	9,6370	1,11732	2811,7	62 91 24	895
896	80 28 16	719 323 136	29,9333	9,6406	1,11607	2814,9	63 05 30	896
897	80 46 09	721 734 273	29,9500	9,6442	1,11483	2818,0	63 19 38	897
898	80 64 04	724 150 792	29,9666	9,6477	1,11359	2821,2	63 33 48	898
899	80 82 01	726 572 699	29,9833	9,6513	1,11235	2824,3	63 47 60	899
900	81 00 00	729 000 000	30,0000	9,6549	1,11111	2827,4	63 61 73	**900**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
900	81 00 00	729 000 000	30,0000	9,6549	1,11111	2827,4	63 61 73	**900**
901	81 18 01	731 432 701	30,0167	9,6585	1,10988	2830,6	63 75 87	901
902	81 36 04	733 870 808	30,0333	9,6620	1,10865	2833,7	63 90 03	902
903	81 54 09	736 314 327	30,0500	9,6656	1,10742	2836,9	64 04 21	903
904	81 72 16	738 763 264	30,0666	9,6692	1,10619	2840,0	64 18 40	904
905	81 90 25	741 217 625	30,0832	9,6727	1,10497	2843,1	64 32 61	905
906	82 08 36	743 677 416	30,0998	9,6763	1,10375	2846,3	64 46 83	906
907	82 26 49	746 142 643	30,1164	9,6799	1,10254	2849,4	64 61 07	907
908	82 54 64	748 613 312	30,1330	9,6834	1,10132	2852,6	64 75 33	908
909	82 62 81	751 089 429	30,1496	9,6870	1,10011	2855,7	64 89 60	909
910	82 81 00	753 571 000	30,1662	9,6905	1,09890	2858,8	65 03 88	**910**
911	82 99 21	756 058 031	30,1828	9,6941	1,09769	2862,0	65 18 18	911
912	83 17 44	758 550 528	30,1993	9,6976	1 09649	2865,1	65 32 50	912
913	83 35 69	761 048 497	30,2159	9,7012	1,09529	2868,3	65 46 84	913
914	83 53 96	763 551 944	30,2324	9,7047	1,09409	2871,4	65 61 18	914
915	83 72 25	766 060 875	30,2490	9,7082	1,09290	2874,6	65 75 55	915
916	83 90 56	768 575 296	30,2655	9,7118	1,09170	2877,7	65 89 93	916
917	84 08 89	771 095 213	30,2820	9,7153	1,09051	2880,8	66 04 33	917
918	84 27 24	773 620 632	30,2985	9,7188	1,08932	2884,0	66 18 74	918
919	84 45 61	776 151 559	30,3150	9,7224	1,08814	2887,1	66 33 17	919
920	84 64 00	778 688 000	30,3315	9,7259	1,08696	2890,3	66 47 61	**920**
921	84 82 41	781 229 961	30,3480	9,7294	1,08578	2893,4	66 62 07	921
922	85 00 84	783 777 448	30,3645	9,7329	1,08460	2896,5	66 76 54	922
923	85 19 29	786 330 467	30,3809	9,7364	1,08342	2899,7	66 91 03	923
924	85 37 76	788 889 024	30,3974	9,7400	1,08225	2902,8	67 05 54	924
925	85 56 25	791 453 125	30,4138	9,7435	1,08108	2906,0	67 20 06	925
926	85 74 76	794 022 776	30,4302	2,7470	1,07991	2909,1	67 34 60	926
927	85 93 29	796 597 983	30,4467	9,7505	1,07875	2912,3	67 49 15	927
928	86 11 84	799 178 752	30,4631	9,7540	1,07759	2915,4	67 63 72	928
929	86 30 41	801 765 089	30,4795	9,7575	1,07643	2918,5	67 78 31	929
930	86 49 00	804 357 000	30,4959	9,7610	1,07527	2921,7	67 92 91	**930**
931	86 67 61	806 954 491	30,5123	9,7645	1,07411	2924,8	68 07 52	931
932	86 86 24	809 557 568	30,5287	9,7680	1,07296	2928,0	68 22 16	932
933	87 04 89	812 166 237	30,5450	9,7715	1,07181	2931,1	68 36 80	933
934	87 23 56	814 780 504	30,5614	9,7750	1,07066	2934,2	68 51 47	934
935	87 42 25	817 400 375	30,5778	9,7785	1,06952	2937,4	68 66 15	935
936	87 60 96	820 025 856	30,5941	9,7819	1,06838	2940,5	68 80 84	936
937	87 79 69	822 656 953	30,6105	9,7854	1,06724	2943,7	68 95 55	937
938	87 98 44	825 293 672	30,6268	9,7889	1,06610	2946,8	69 10 28	938
939	88 17 21	827 936 019	30,6431	9,7924	1,06496	2950,0	69 25 02	939
940	88 36 00	830 584 000	30,6594	9,7959	1,06383	2953,1	69 39 78	**940**
941	88 54 81	833 237 621	30,6757	9 7993	1,06270	2956,2	69 54 55	941
942	88 73 64	835 896 888	30,6920	9,8028	1,06157	2959,4	69 69 34	942
943	88 92 49	838 561 807	30,7083	9,8063	1,06045	2962,5	69 84 15	943
944	89 11 36	841 232 384	30,7246	9,8097	1,05932	2965,7	69 98 97	944
945	89 30 25	843 908 625	30,7409	9,8132	1,05820	2968,8	70 13 80	945
946	89 49 16	846 590 536	30,7571	9,8167	1,05708	2971,9	70 28 65	946
947	89 68 09	849 278 123	30,7734	9,8201	1,05597	2975,1	70 43 52	947
948	89 87 04	851 971 392	30,7896	9,8236	1,05485	2978,2	70 58 40	948
949	90 06 01	854 670 349	30,8058	9,8270	1,05374	2981,4	70 73 30	949
950	90 25 00	857 375 000	30,8221	9,8305	1,05263	2984,5	70 88 22	**950**

n	n^2	n^3	$\sqrt{n}$	$\sqrt[3]{n}$	$\frac{1000}{n}$	πn	$\frac{\pi n^2}{4}$	n
950	90 25 00	857 375 000	30,8221	9,8305	1,05263	2984,5	70 88 22	**950**
951	90 44 01	860 085 351	30,8383	9,8339	1,05152	2987,7	71 03 15	951
952	90 63 04	862 801 408	30,8545	9,8374	1,05042	2990,8	71 18 09	952
953	90 82 09	865 523 177	30,8707	9,8408	1,04932	2993,9	71 33 06	953
954	91 01 16	868 250 664	30,8869	9,8443	1,04822	2997,1	71 48 03	954
955	91 20 25	870 983 875	30,9031	9,8477	1,04712	3000,2	71 63 03	955
956	91 39 36	873 722 816	30,9192	9,8511	1,04603	3003,4	71 78 04	956
957	91 58 49	876 467 493	30,9354	9,8546	1,04493	3006,5	71 93 06	957
958	91 77 64	879 217 912	30,9516	9,8580	1,04384	3009,6	72 08 10	958
959	91 96 81	881 974 079	30,9677	9,8614	1,04275	3012,8	72 23 16	959
960	92 16 00	884 736 000	30,9839	9,8648	1,04167	3015,9	42 38 23	**960**
961	92 35 21	887 503 681	31,0000	9,8683	1,04058	3019,1	72 53 32	961
962	92 54 44	890 277 128	31,0161	9,8717	1,03950	3022,2	72 68 42	962
963	92 73 69	893 056 347	31,0322	9,8751	1,03842	3025,4	72 83 54	963
964	92 92 96	895 841 344	31,0483	9,8785	1,03734	3028,5	72 98 67	964
965	93 12 25	898 632 125	31,0644	9.8819	1,03627	3031,6	73 13 82	965
966	93 31 56	901 428 696	31,0805	9,8854	1,03520	3034,8	73 28 99	966
967	93 50 89	904 231 063	31,0966	9,8888	1,03413	3037,9	73 44 17	967
968	93 70 24	907 039 232	31,1127	9,8922	1,03306	3041,1	73 59 37	968
969	93 89 61	909 853 209	31,1288	9,8956	1,03199	3044,2	73 74 58	969
970	94 09 00	912 673 000	31,1448	9,8990	1,03093	3047,3	73 89 81	**970**
971	94 28 41	915 498 611	31,1609	9,9024	1,02987	3050.5	74 05 06	971
972	94 47 84	918 330 048	31,1769	9,9058	1,02881	3053,6	74 20 32	972
973	94 67 29	921 167 317	31,1929	9,9092	1,02775	3056,8	74 35 59	973
974	94 86 76	924 010 424	31,2090	9,9126	1,02669	3059,9	74 50 88	974
975	95 06 25	926 859 375	31,2250	9,9160	1,02564	3063,1	74 66 19	975
976	95 25 76	929 714 176	31,2410	9,9194	1,02459	3066,2	74 81 51	976
977	95 45 29	932 574 833	31,2570	9,9227	1,02354	3069,3	74 96 85	977
978	95 64 84	935 441 352	31,2730	9,9261	1,02249	3072,5	75 12 21	978
979	95 84 41	938 313 739	31,2890	9,9295	1,02145	3075,6	75 27 58	979
980	96 04 00	941 192 000	31,3050	9,9329	1,02041	3078,8	75 42 96	**980**
981	96 23 61	944 076 141	31,3209	9,9363	1,01937	3081,9	75 58 37	981
982	96 43 24	946 966 168	31,3369	9,9396	1,01833	3085,0	75 73 78	982
983	96 62 89	949 862 087	31,3528	9,9430	1,01729	3088,2	75 89 22	983
984	96 82 56	952 763 904	31,3688	9,9464	1,01626	3091,3	76 04 66	984
985	97 02 25	955 671 625	31,3847	9,9497	1.01523	3094,5	76 20 13	985
986	97 21 96	958 585 256	31,4006	9,9531	1,01420	3097,6	76 35 61	986
987	97 41 69	961 504 803	31,4166	9,9565	1,01317	3100,8	76 51 11	987
988	97 61 44	964 430 272	31,4325	9,9598	1,01215	3103,9	76 66 62	988
989	97 81 21	967 361 669	31,4484	9,9632	1,01112	3107,0	76 82 14	989
990	98 01 00	970 299 000	31,4643	9,9666	1,01010	3110,2	76 97 69	**990**
991	98 20 81	973 242 271	31,4802	9,9699	1,00908	3113,3	77 13 25	991
992	98 40 64	976 191 488	31,4960	9,9733	1,00806	3116,5	77 28 82	992
993	98 60 49	979 146 657	31,5119	9,9766	1,00705	3119,6	77 44 41	993
994	98 80 36	982 107 784	31,5278	9,9800	1,00604	3122,7	77 60 02	994
995	99 00 25	985 074 875	31,5436	9,9833	1,00503	3125,9	77 75 64	995
996	99 20 16	988 047 936	31,5595	9,9866	1,00402	3129,0	77 91 28	996
997	99 40 09	991 026 973	31 5753	9.9900	1.00301	3132,2	78 06 93	997
998	99 60 04	994 011 992	31,5911	9,9933	1,00200	3135,3	73 22 60	998
999	99 80 01	997 002 999	31,6070	9,9967	1,00100	3138,5	78 38 28	999

LIST OF SYMBOLS

BASED ON THE STANDARD NOTATION SUGGESTED BY THE SCIENCE STANDING COMMITTEE OF THE CONCRETE INSTITUTE.

a Area of the couple formed by compressive and tensile forces in a beam.

a_c Area of compressive force measured from neutral axis in ribbed slabs.

a_t Area of tensile reinforcement measured from neutral axis.

b Breadth generally in inches.

b_r Breadth of rib in a tee-beam in inches.

b_s Effective breadth of slab in tee-beam in inches.

c Compressive stress intensity on concrete.

c_s Compressive stress intensity on steel.

c_x, c_y Stresses in concrete of columns eccentrically loaded.

d Depth generally in rectangular sections.

d Effective depth of beam or slab from top to axis of tensile reinforcement in inches.

d Diameter in circular sections in inches.

d_c Depth or distance of centre of compressive reinforcement from compressed edge of beams in inches.

d_c Diameter of core of pillars in inches.

d_c Depth of arch ring at crown of arch in inches.

d_d Distance of bottom of reinforcement of rib from centre of gravity of reinforcement in inches.

d_h Diameter of a helical reinforcing rod in any compression piece in inches.

d_l Diameter of a longitudinal reinforcing rod of a pillar in inches.

d_n Deflection of a beam in inches.

d_r Distance of rods centre to centre in inches.

d_s Total depth of slab in tee-beam in inches.

d_t Total depth in inches.

e Eccentricity of load in inches.

e Distance of centre of rod from axis of column in inches.

f Friction or adhesion of concrete and steel.

h Height generally in inches.

i Inset of centre of reinforcement from bottom of slab or rib in inches.

i Inset of rod centres from outer edge of column section in inches.

i Inset of centre of gravity of column section from outer edge in inches.

i Distance of eccentric load from outer edge of column section in inches. $i = d - e$ (diameter – eccentricity).

l Length generally in inches.

l Effective length or span of beam or arch.

m Modular ratio, *i.e.* the ratio between the elastic moduli of steel and concrete $= \frac{E_s}{E_c}$.

n Distance of neutral axis from compressed edge in inches

p Intensity of pressure per unit of length or area.

r Radius in inches.

s Shearing stress intensity.

s_h Spacing of hoops round columns in inches.

$s_r = \frac{t}{c}$ Stress ratio in ribbed slabs.

t Tensile stress intensity on steel.

t_c Tensile stress intensity on concrete.

$\left.\begin{matrix} t_x \\ t_y \end{matrix}\right\}$ Stresses in steel in columns eccentrically loaded.

v Versine or camber of a curve or rise of an arch in inches.

w Weight or load generally, per unit of length or area.
w Superimposed load uniformly distributed on arch.
w_d Dead load above arch ring at crown.
$\left.\begin{matrix} x \\ y \end{matrix}\right\}$ Co-ordinates in arch calculations in inches.
x Distance of hangers or bending up of rods from support in inches.
y Height of shear triangle.
β Distance of compressive force from neutral axis in ribbed slabs in inches.
$\gamma = \frac{t}{c}$ In ribbed slabs.
π Ratio of circumference of a circle to its diameter.
O Perimeter of steel rods in inches.

A Total cross-sectional area of beam or pillar in inches.
A_C Area of compressive reinforcements of beams in inches.
A_L Cross-sectional area of longitudinal steel rods of pillar in inches.
A_r Sectional area of one rod in ins.^{2}
A_S Area of shear reinforcement in ins.^{2}
A_T Area of tensile reinforcement in beams in ins.^{2}
B Bending moment generally.
B Maximum bending moment of the external forces or loads on a beam.
B Bending moment at crown of arch.
B_C Bending moment at centre of beam.
B_E Bending moment at end of beam.
B_L Bending moment left half of arch.
B_R Bending moment right half of arch.
C Total compressive force or stress.
C_C Total compression on concrete.
C_S Total compression on steel.
E_C Elastic modulus of concrete in compression in lbs./in,^{2}

E_S Elastic modulus of steel in lbs./in.^{2}

G Centre of gravity of column section.

I_C Moment of inertia for concrete.

I_S Moment of inertia for steel.

N_d Number of divisions in one half of arch.

N_r Number of rods.

P_H Horizontal pressure.

P_V Vertical pressure.

R Moment of resistance of internal stresses in a beam at a given cross-section.

R_L Left reaction.

R_R Right reaction.

S Total shearing force across a section.

S_C Shear at crown of arch.

S_C Total shear taken up by concrete.

S_S Total shear taken up by steel.

S_F Safety factor.

T Total tensile force.

T_C Thrust at crown of arch.

W Weight or load.

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